

# Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/104-4.3.3.1-a+b-tan-<sup>m</sup>-c+d-tan-<sup>n</sup>-  
A+B-tan-

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# Chapter 1

## Introduction

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This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 855 ]. This is test number [ 104 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.



## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 855 )	0.00 ( 0 )
Mathematica	93.22 ( 797 )	6.78 ( 58 )
Maple	91.23 ( 780 )	8.77 ( 75 )
Fricas	72.63 ( 621 )	27.37 ( 234 )
Mupad	61.87 ( 529 )	38.13 ( 326 )
Maxima	50.06 ( 428 )	49.94 ( 427 )
Giac	31.58 ( 270 )	68.42 ( 585 )
Sympy	24.44 ( 209 )	75.56 ( 646 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

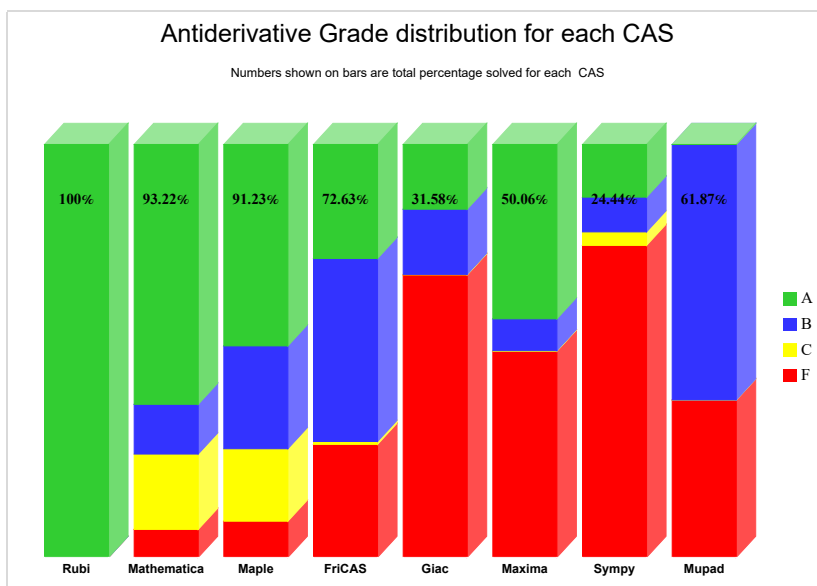
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

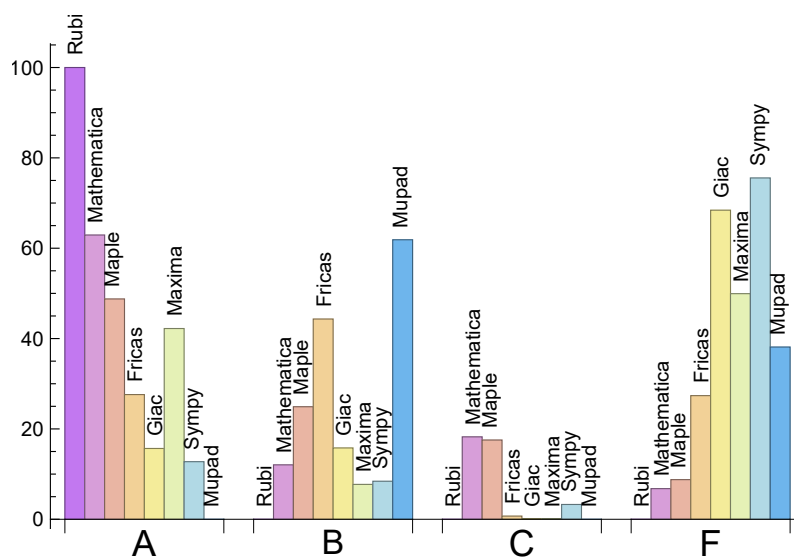
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	62.92	12.05	18.25	6.78
Maple	48.77	24.91	17.54	8.77
Maxima	42.22	7.72	0.12	49.94
Fricas	27.60	44.33	0.70	27.37
Giac	15.67	15.79	0.12	68.42
Sympy	12.75	8.42	3.27	75.56
Mupad	N/A	61.87	0.00	38.13

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	58	81.03 %	18.97 %	0.00 %
Maple	75	100.00 %	0.00 %	0.00 %
Fricas	234	30.77 %	66.67 %	2.56 %
Giac	585	55.38 %	35.04 %	9.57 %
Maxima	427	43.79 %	11.48 %	44.73 %
Sympy	646	70.74 %	16.10 %	13.16 %
Mupad	326	99.08 %	0.92 %	0.00 %

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

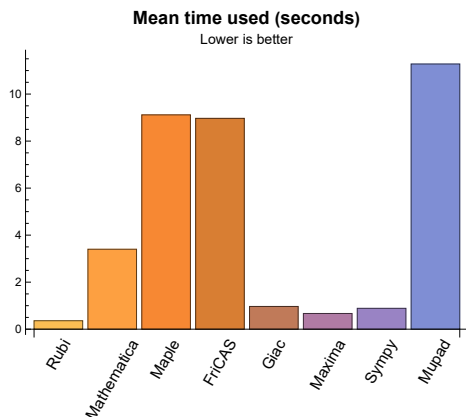
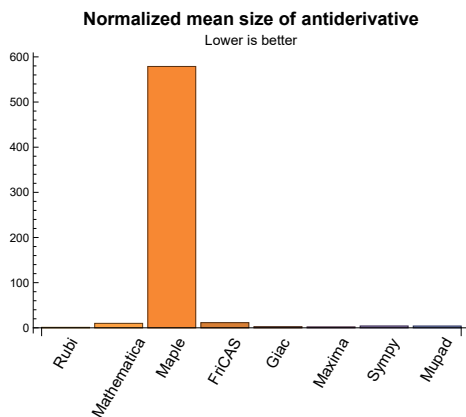
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.36	202.72	1.00	189.00	1.00
Mathematica	3.40	2499.25	9.85	215.00	1.10
Maple	9.12	136785.57	578.75	275.00	1.32
Maxima	0.67	331.22	1.95	185.50	1.05
Fricas	8.97	2624.25	11.43	421.00	2.23
Sympy	0.89	508.45	3.98	258.00	2.06
Giac	0.97	338.19	2.48	189.00	1.82
Mupad	11.28	748.05	3.85	221.00	1.39

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}



## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {58, 74, 80, 81, 85, 86, 87, 88, 89, 112, 119, 127, 128, 129, 132, 133, 149, 150, 169, 170, 171, 172, 173, 204, 205, 206, 209, 210, 211, 347, 437, 445, 446, 458, 463, 532, 546, 551, 552, 553, 623, 631, 632, 646, 652, 716, 805, 816, 818, 819, 822, 823}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

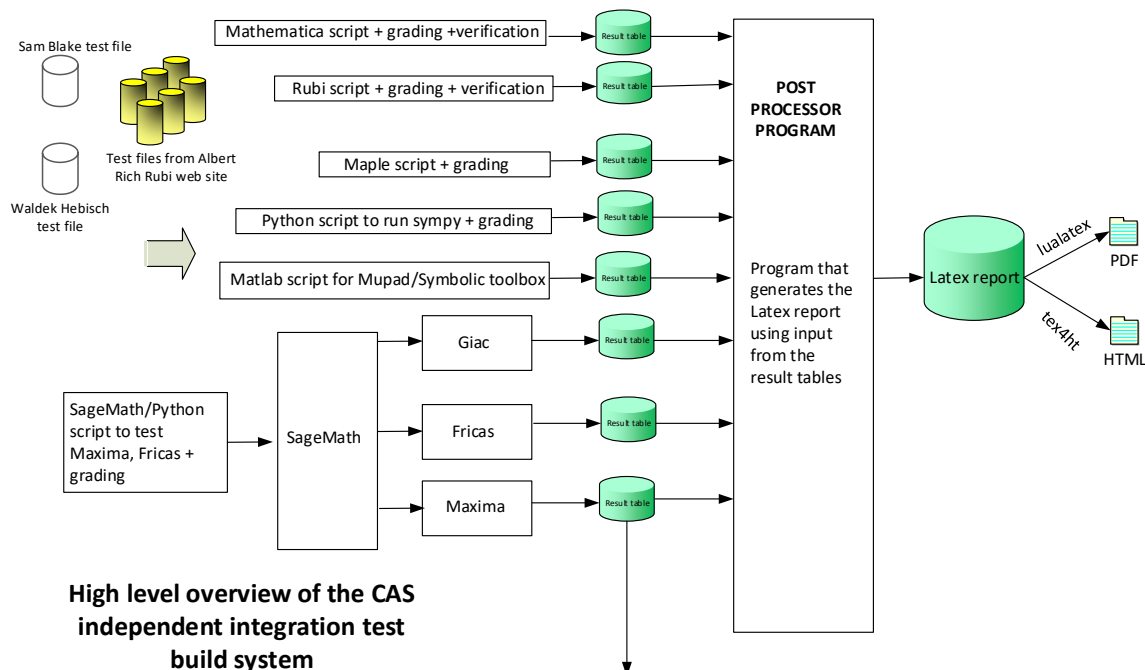
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



### High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax



# Chapter 2

## detailed summary tables of results

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## 2.1 List of integrals sorted by grade for each CAS

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### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

B grade: { }

C grade: { }

F grade: { }

## 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 33, 45, 46, 47, 48, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 119, 120, 122, 123, 124, 126, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 232, 233, 234, 235, 270, 298, 299, 300, 301, 302, 304, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 346, 348, 349, 352, 354, 355, 356, 360, 362, 363, 364, 367, 369, 370, 373, 374, 382, 383, 384, 400, 401, 417, 419, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 439, 440, 441, 442, 443, 444, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 494, 495, 496, 497, 498, 499, 500, 501, 504, 505, 506, 507, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 607, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 648, 649, 650, 651, 653, 654, 655, 665, 668, 669, 670, 672, 673, 674, 676, 678, 679, 680, 681, 685, 686, 687, 688, 690, 691, 692, 693, 694, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 711, 712, 713, 714, 719, 720, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 767, 768, 769, 770, 774, 775, 776, 777, 778, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 819, 820, 821, 822, 823, 824, 825, 826, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 57, 58, 65, 66, 71, 72, 80, 81, 85, 86, 87, 88, 89, 112, 113, 118, 121, 125, 127, 128, 132, 133, 155, 156, 173, 204, 205, 206, 209, 210, 211, 221, 222, 314, 324, 336, 337, 486, 502, 503, 508, 511, 666, 667, 671, 675, 677, 682, 683, 684, 689, 695, 696, 697, 698, 709, 710, 716, 717, 718, 817, 818, 827, 828, 829 }

C grade: { 5, 6, 7, 8, 197, 198, 199, 200, 202, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 316, 341, 342, 347, 350, 351, 353, 357, 358, 359, 361, 365, 366, 368, 371, 372, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 420, 421, 422, 423, 424, 425, 426, 437, 445, 446, 458, 463, 477, 606, 608, 610, 611, 623, 631, 632, 646, 652 }

F grade: { 154, 196, 201, 203, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225,

226, 227, 228, 229, 230, 231, 487, 488, 489, 490, 491, 492, 493, 540, 567, 568, 569, 570, 571, 572, 573, 656, 657, 658, 659, 660, 661, 662, 663, 664, 715, 727, 764, 765, 766, 771, 772, 773, 779, 780 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 82, 83, 84, 90, 91, 92, 93, 97, 98, 99, 100, 104, 105, 106, 107, 108, 119, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 196, 197, 198, 199, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 341, 365, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 801, 802, 803, 804, 806, 807, 808, 812, 813, 814, 815, 817, 819, 820, 825, 826, 827, 828, 829, 833, 834, 835, 839, 840, 841, 842, 846, 847, 848, 849, 850, 854, 855 }

B grade: { 71, 72, 73, 74, 78, 79, 80, 81, 85, 86, 87, 88, 89, 94, 95, 96, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 129, 130, 131, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 317, 318, 319, 320, 325, 326, 327, 332, 333, 334, 340, 342, 343, 344, 345, 346, 350, 351, 352, 353, 357, 358, 359, 360, 361, 366, 368, 370, 371, 372, 373, 374, 375, 376, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 791, 799, 800, 805, 809, 810, 811, 816, 818, 821, 822, 823, 824, 830, 831, 832, 836, 837, 838, 843, 844, 845 }

C grade: { 321, 322, 323, 324, 328, 329, 330, 331, 335, 336, 337, 338, 339, 347, 348, 349, 354, 355, 356, 362, 363, 364, 367, 369, 473, 474, 475, 476, 477, 478, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 689, 711, 723, 736 }

F grade: { 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219,

220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 567, 568, 569, 570, 571, 572, 573, 656, 657, 658, 659, 660, 661, 662, 663, 664, 705, 715, 727, 851, 852, 853 }

#### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 119, 120, 121, 125, 126, 127, 128, 129, 130, 131, 132, 133, 196, 197, 198, 199, 200, 201, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 508, 512, 513, 514, 515, 516, 517, 518, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 666, 667, 668, 669, 670, 677, 678, 679, 680, 681, 682, 690, 691, 692, 693, 694, 695, 696, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 800, 801, 802, 803, 804, 811, 813, 814, 815, 824, 826, 827, 828, 829, 832, 833, 838, 841, 845, 846, 855 }

B grade: { 112, 113, 114, 115, 116, 117, 118, 122, 123, 124, 292, 293, 294, 302, 373, 502, 503, 504, 505, 506, 507, 509, 510, 511, 536, 537, 538, 541, 542, 543, 547, 548, 549, 665, 676, 689, 787, 788, 789, 790, 795, 796, 797, 798, 799, 805, 806, 807, 808, 809, 810, 812, 816, 817, 818, 819, 820, 821, 822, 823, 825, 830, 831, 836, 844, 850 }

C grade: { 301 }

F grade: { 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 539, 540, 544, 545, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650,

651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 671, 672, 673, 674, 675, 683, 684, 685, 686, 687, 688, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 791, 792, 793, 794, 834, 835, 837, 839, 840, 842, 843, 847, 848, 849, 851, 852, 853, 854 }

### 2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 298, 299, 300, 301, 302, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 375, 665, 667, 668, 670, 671, 672, 673, 674, 677, 678, 679, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 708, 709, 710, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 737, 738, 739, 742, 743, 744, 745, 746, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 785, 786, 792, 793, 794, 801, 802, 803, 804, 812, 813, 814, 815, 825, 826, 827, 828, 829, 833, 834, 835, 839, 840, 841, 842, 846, 847, 848, 849, 850, 853, 854 }

B grade: { 1, 7, 8, 29, 30, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 303, 305, 317, 318, 319, 320, 321, 322, 323, 324, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 474, 477, 478, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 666, 675, 676, 689, 695, 706, 707, 740, 741, 747, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 795, 796, 797, 798, 799, 800, 805, 806, 807, 808, 809, 810, 811, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 832, 836, 837, 838, 843, 844, 845, 855 }

C grade: { 669, 680, 693, 711, 723, 736 }

F grade: { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 362, 363, 364, 392, 393, 394, 395, 396, 397, 409, 410, 411, 412, 413, 414, 415, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493,

494, 495, 496, 497, 498, 499, 500, 501, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 705, 715, 727, 851, 852 }

## 2.1.6 Sympy

A grade: { 2, 3, 5, 6, 9, 11, 12, 13, 14, 15, 16, 17, 19, 23, 24, 25, 26, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 232, 233, 240, 241, 242, 243, 250, 251, 252, 259, 260, 261, 262, 263, 265, 266, 301, 305, 670, 671, 682, 683, 684, 696, 697, 698, 699, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726, 728, 729, 730, 733, 734, 735, 736, 737, 738, 739, 854 }

B grade: { 1, 4, 7, 8, 10, 18, 20, 21, 22, 27, 28, 29, 30, 32, 234, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 249, 253, 254, 255, 256, 257, 258, 264, 298, 299, 300, 302, 303, 304, 314, 665, 666, 667, 668, 672, 673, 674, 675, 676, 677, 678, 679, 681, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 700, 701, 702, 703, 704, 720, 731, 732, 853 }

C grade: { 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 669, 680, 693 }

F grade: { 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644,

645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 705, 715, 727, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 855 }

### 2.1.7 Giac

A grade: { 32, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 235, 243, 244, 251, 252, 253, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 288, 297, 299, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 475, 668, 672, 679, 692, 711, 712, 713, 714, 720, 721, 722, 723, 724, 725, 726, 733, 734, 735, 736, 737, 738, 739 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 39, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 254, 255, 256, 257, 258, 259, 264, 265, 266, 277, 278, 279, 284, 285, 286, 287, 289, 290, 291, 292, 293, 294, 295, 296, 298, 300, 302, 303, 304, 305, 373, 374, 375, 376, 477, 666, 667, 669, 670, 671, 673, 674, 675, 677, 678, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 716, 717, 718, 719, 728, 729, 730, 731, 732, 798, 854 }

C grade: { 301 }

F grade: { 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603,

604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 676, 689, 705, 715, 727, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 855 }

## 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 180, 181, 196, 197, 198, 199, 200, 201, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 453, 454, 473, 474, 475, 476, 477, 478, 606, 607, 608, 609, 610, 611, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 790, 791, 792, 793, 794, 801, 802, 803, 804, 812, 813, 814, 815, 825, 826, 827, 828, 829, 832, 833, 834, 835, 839, 840, 841, 842, 846, 847, 848, 849, 850, 853, 854, 855 }

C grade: { }

F grade: { 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466,



467, 468, 469, 470, 471, 472, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 705, 715, 727, 787, 788, 789, 795, 796, 797, 798, 799, 800, 805, 806, 807, 808, 809, 810, 811, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 836, 837, 838, 843, 844, 845, 851, 852 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, **Mathematica** was abbreviated to **MMA**.

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA.	grade	A	A	A	A	A	B	B	B	B
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	91	91	86	97	82	170	167	284	82
	N.S.	1	1.00	0.95	1.07	0.90	1.87	1.84	3.12	0.90
	time (sec)	N/A	0.077	0.920	0.102	0.503	0.977	0.356	0.624	6.113

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	72	68	109	109	194	59
N.S.	1	1.00	1.01	1.04	0.99	1.58	1.58	2.81	0.86
time (sec)	N/A	0.038	0.336	0.034	0.685	0.946	0.284	0.451	6.075

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	50	50	64	53	103	38
N.S.	1	1.00	1.43	1.09	1.09	1.39	1.15	2.24	0.83
time (sec)	N/A	0.020	0.036	0.025	0.513	1.615	0.214	0.440	6.065

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	49	46	49	36	94	74	36
N.S.	1	1.00	1.22	1.15	1.22	0.90	2.35	1.85	0.90
time (sec)	N/A	0.050	0.069	0.096	0.506	1.603	1.080	0.548	6.231

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	84	57	64	62	53	104	39
N.S.	1	1.00	1.91	1.30	1.45	1.41	1.20	2.36	0.89
time (sec)	N/A	0.058	0.212	0.069	0.532	1.822	0.240	0.629	6.210

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	81	84	111	109	162	60
N.S.	1	1.00	1.12	1.19	1.24	1.63	1.60	2.38	0.88
time (sec)	N/A	0.086	0.393	0.089	0.515	1.092	0.345	0.733	6.213

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	99	103	166	168	221	80
N.S.	1	1.00	1.15	1.11	1.16	1.87	1.89	2.48	0.90
time (sec)	N/A	0.110	0.741	0.092	0.531	0.948	0.354	0.818	6.246

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	96	112	116	206	218	282	100
N.S.	1	1.00	0.86	1.01	1.05	1.86	1.96	2.54	0.90
time (sec)	N/A	0.139	0.882	0.103	0.518	0.853	0.914	0.981	6.480

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	305	121	112	236	236	408	153
N.S.	1	1.00	2.16	0.86	0.79	1.67	1.67	2.89	1.09
time (sec)	N/A	0.175	5.293	0.050	0.549	0.695	0.501	0.713	6.102

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	273	99	92	175	178	312	111
N.S.	1	1.00	2.55	0.93	0.86	1.64	1.66	2.92	1.04
time (sec)	N/A	0.082	3.344	0.040	0.951	0.664	0.354	0.603	6.132

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	263	76	71	121	122	214	76
N.S.	1	1.00	3.29	0.95	0.89	1.51	1.52	2.68	0.95
time (sec)	N/A	0.048	1.834	0.035	0.816	0.847	0.347	0.539	6.096

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	201	86	67	97	109	174	70
N.S.	1	1.00	2.68	1.15	0.89	1.29	1.45	2.32	0.93
time (sec)	N/A	0.111	2.185	0.092	0.500	0.913	1.348	0.765	6.195

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	202	88	74	102	109	155	87
N.S.	1	1.00	2.56	1.11	0.94	1.29	1.38	1.96	1.10
time (sec)	N/A	0.128	2.308	0.089	0.533	0.566	1.370	0.941	6.319

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	302	113	94	123	119	186	67
N.S.	1	1.00	3.21	1.20	1.00	1.31	1.27	1.98	0.71
time (sec)	N/A	0.146	1.763	0.095	0.648	0.580	0.364	1.139	6.199

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	435	142	111	181	182	255	93
N.S.	1	1.00	3.72	1.21	0.95	1.55	1.56	2.18	0.79
time (sec)	N/A	0.175	2.220	0.102	0.546	0.472	0.714	1.332	6.280

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	902	168	132	227	235	322	113
N.S.	1	1.00	6.49	1.21	0.95	1.63	1.69	2.32	0.81
time (sec)	N/A	0.226	7.135	0.110	0.500	0.608	0.579	1.257	6.441

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	847	146	132	291	292	504	230
N.S.	1	1.00	4.65	0.80	0.73	1.60	1.60	2.77	1.26
time (sec)	N/A	0.295	7.251	0.055	0.507	0.434	0.567	0.845	6.150

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	980	123	114	227	235	408	176
N.S.	1	1.00	7.10	0.89	0.83	1.64	1.70	2.96	1.28
time (sec)	N/A	0.096	7.013	0.047	0.528	0.462	0.510	0.690	6.044

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	331	100	96	175	184	312	125
N.S.	1	1.00	3.01	0.91	0.87	1.59	1.67	2.84	1.14
time (sec)	N/A	0.065	3.204	0.044	0.614	0.520	0.411	0.587	6.126

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	281	133	89	172	224	264	87
N.S.	1	1.00	2.63	1.24	0.83	1.61	2.09	2.47	0.81
time (sec)	N/A	0.203	6.808	0.104	0.517	0.565	1.702	0.935	6.155

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	291	124	84	141	219	257	76
N.S.	1	1.00	2.51	1.07	0.72	1.22	1.89	2.22	0.66
time (sec)	N/A	0.206	2.644	0.099	0.499	0.678	1.122	1.258	6.431

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	1010	139	96	179	226	223	88
N.S.	1	1.00	8.21	1.13	0.78	1.46	1.84	1.81	0.72
time (sec)	N/A	0.221	6.962	0.112	0.496	0.632	1.176	1.251	6.443

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	442	168	115	181	182	254	93
N.S.	1	1.00	3.30	1.25	0.86	1.35	1.36	1.90	0.69
time (sec)	N/A	0.257	3.604	0.104	0.507	0.627	0.548	0.875	6.317

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	1007	205	134	228	235	322	114
N.S.	1	1.00	6.41	1.31	0.85	1.45	1.50	2.05	0.73
time (sec)	N/A	0.286	6.958	0.113	0.500	0.501	1.532	0.952	6.571

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	943	246	151	287	296	391	140
N.S.	1	1.00	5.24	1.37	0.84	1.59	1.64	2.17	0.78
time (sec)	N/A	0.309	7.174	0.122	0.494	0.479	0.900	1.006	6.940

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	951	168	150	345	348	600	308
N.S.	1	1.00	4.23	0.75	0.67	1.53	1.55	2.67	1.37
time (sec)	N/A	0.435	7.401	0.111	0.664	0.429	0.969	0.909	6.212

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	589	145	132	279	291	504	240
N.S.	1	1.00	3.51	0.86	0.79	1.66	1.73	3.00	1.43
time (sec)	N/A	0.117	3.803	0.094	0.930	0.484	0.577	0.755	6.205

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	448	122	114	227	241	408	181
N.S.	1	1.00	3.20	0.87	0.81	1.62	1.72	2.91	1.29
time (sec)	N/A	0.083	2.704	0.083	0.481	0.455	0.514	0.687	6.117

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	429	184	107	246	289	332	133
N.S.	1	1.00	3.02	1.30	0.75	1.73	2.04	2.34	0.94
time (sec)	N/A	0.290	5.663	0.208	0.480	0.459	2.142	1.271	6.219

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	1122	166	102	254	260	336	110
N.S.	1	1.00	7.79	1.15	0.71	1.76	1.81	2.33	0.76
time (sec)	N/A	0.288	7.187	0.203	0.515	0.527	1.530	0.902	6.562

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	1116	171	108	255	252	317	102
N.S.	1	1.00	7.15	1.10	0.69	1.63	1.62	2.03	0.65
time (sec)	N/A	0.303	7.261	0.211	0.509	0.536	1.345	0.979	6.791

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	1138	191	117	249	292	291	113
N.S.	1	1.00	6.98	1.17	0.72	1.53	1.79	1.79	0.69
time (sec)	N/A	0.311	7.110	0.207	0.619	0.504	2.952	1.059	6.715

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	319	227	136	228	235	322	114
N.S.	1	1.00	1.80	1.28	0.77	1.29	1.33	1.82	0.64
time (sec)	N/A	0.358	4.463	0.216	0.642	0.513	0.759	1.204	6.568



Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	542	279	153	287	296	391	140
N.S.	1	1.00	2.71	1.40	0.76	1.44	1.48	1.96	0.70
time (sec)	N/A	0.396	6.490	0.233	0.556	0.467	3.439	1.280	6.850

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	1009	332	172	332	347	459	162
N.S.	1	1.00	4.52	1.49	0.77	1.49	1.56	2.06	0.73
time (sec)	N/A	0.433	7.296	0.253	0.658	0.455	1.416	1.350	7.586

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	898	98	0	186	196	125	141
N.S.	1	1.00	6.96	0.76	0.00	1.44	1.52	0.97	1.09
time (sec)	N/A	0.119	6.813	0.130	0.000	0.466	0.462	0.770	6.377

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	240	78	0	127	151	101	95
N.S.	1	1.00	2.38	0.77	0.00	1.26	1.50	1.00	0.94
time (sec)	N/A	0.087	4.635	0.103	0.000	0.478	0.315	0.627	6.297

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	148	68	0	66	119	82	81
N.S.	1	1.00	2.21	1.01	0.00	0.99	1.78	1.22	1.21
time (sec)	N/A	0.066	0.809	0.086	0.000	0.442	0.237	0.488	6.235

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	102	68	0	42	87	85	45
N.S.	1	1.00	2.17	1.45	0.00	0.89	1.85	1.81	0.96
time (sec)	N/A	0.031	0.414	0.079	0.000	0.449	0.121	0.484	6.167

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	150	77	0	68	116	99	98
N.S.	1	1.00	2.42	1.24	0.00	1.10	1.87	1.60	1.58
time (sec)	N/A	0.078	0.886	0.228	0.000	0.439	0.207	0.555	6.262

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	225	93	0	129	158	135	126
N.S.	1	1.00	2.21	0.91	0.00	1.26	1.55	1.32	1.24
time (sec)	N/A	0.122	2.701	0.210	0.000	0.509	0.328	0.678	6.464

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	902	111	0	188	199	165	153
N.S.	1	1.00	6.89	0.85	0.00	1.44	1.52	1.26	1.17
time (sec)	N/A	0.152	6.956	0.242	0.000	0.483	0.499	0.807	6.510

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	1062	129	0	249	253	186	174
N.S.	1	1.00	6.85	0.83	0.00	1.61	1.63	1.20	1.12
time (sec)	N/A	0.178	7.052	0.244	0.000	0.474	0.434	0.989	6.554

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	956	98	0	150	262	120	141
N.S.	1	1.00	6.73	0.69	0.00	1.06	1.85	0.85	0.99
time (sec)	N/A	0.196	6.751	0.158	0.000	0.490	0.462	0.832	6.414

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	185	89	0	84	223	107	114
N.S.	1	1.00	1.80	0.86	0.00	0.82	2.17	1.04	1.11
time (sec)	N/A	0.150	0.885	0.128	0.000	0.463	0.364	0.678	6.294

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	92	89	0	52	167	109	106
N.S.	1	1.00	1.21	1.17	0.00	0.68	2.20	1.43	1.39
time (sec)	N/A	0.092	0.561	0.099	0.000	0.474	0.202	0.550	6.170

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	89	0	54	162	110	70
N.S.	1	1.00	1.18	1.11	0.00	0.68	2.02	1.38	0.88
time (sec)	N/A	0.047	0.533	0.097	0.000	0.403	0.184	0.571	6.166

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	184	98	0	86	219	121	129
N.S.	1	1.00	1.94	1.03	0.00	0.91	2.31	1.27	1.36
time (sec)	N/A	0.163	1.015	0.248	0.000	0.470	0.290	0.737	6.307

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	960	114	0	152	267	161	164
N.S.	1	1.00	6.81	0.81	0.00	1.08	1.89	1.14	1.16
time (sec)	N/A	0.239	6.791	0.263	0.000	0.479	0.543	0.916	6.534

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	1112	132	0	215	323	176	188
N.S.	1	1.00	6.54	0.78	0.00	1.26	1.90	1.04	1.11
time (sec)	N/A	0.271	7.058	0.299	0.000	0.481	0.528	1.247	6.619

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	1251	120	0	174	337	144	184
N.S.	1	1.00	6.55	0.63	0.00	0.91	1.76	0.75	0.96
time (sec)	N/A	0.321	6.864	0.181	0.000	0.465	0.654	1.215	6.757

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	178	110	0	104	296	130	146
N.S.	1	1.00	1.20	0.74	0.00	0.70	2.00	0.88	0.99
time (sec)	N/A	0.245	1.255	0.148	0.000	0.514	0.714	0.961	6.638

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	147	110	0	78	258	131	111
N.S.	1	1.00	1.19	0.89	0.00	0.63	2.08	1.06	0.90
time (sec)	N/A	0.340	1.091	0.125	0.000	0.459	0.301	0.813	6.522

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	110	0	74	260	130	147
N.S.	1	1.00	1.35	1.00	0.00	0.67	2.36	1.18	1.34
time (sec)	N/A	0.113	1.408	0.119	0.000	0.459	0.316	0.663	6.500

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	150	110	0	76	258	131	111
N.S.	1	1.00	1.34	0.98	0.00	0.68	2.30	1.17	0.99
time (sec)	N/A	0.062	0.782	0.118	0.000	0.494	0.271	0.681	6.464

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	180	119	0	104	292	145	164
N.S.	1	1.00	1.37	0.91	0.00	0.79	2.23	1.11	1.25
time (sec)	N/A	0.248	1.135	0.279	0.000	0.869	0.444	1.087	6.579

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	1282	135	0	173	340	186	197
N.S.	1	1.00	7.01	0.74	0.00	0.95	1.86	1.02	1.08
time (sec)	N/A	0.363	6.954	0.305	0.000	0.608	0.557	1.021	6.882

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	1448	153	0	235	398	211	221
N.S.	1	1.00	6.70	0.71	0.00	1.09	1.84	0.98	1.02
time (sec)	N/A	0.406	7.158	0.345	0.000	0.513	1.207	1.361	7.140

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	195	131	0	120	359	154	178
N.S.	1	1.00	1.05	0.71	0.00	0.65	1.94	0.83	0.96
time (sec)	N/A	0.344	1.229	0.200	0.000	2.302	1.559	1.307	6.767

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	158	131	0	88	301	153	178
N.S.	1	1.00	0.99	0.82	0.00	0.55	1.89	0.96	1.12
time (sec)	N/A	0.312	1.262	0.163	0.000	2.586	0.448	1.054	6.619

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	144	131	0	78	241	151	135
N.S.	1	1.00	0.99	0.90	0.00	0.54	1.66	1.04	0.93
time (sec)	N/A	0.197	1.504	0.158	0.000	2.862	0.570	0.893	6.396

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	141	131	0	78	246	154	172
N.S.	1	1.00	0.99	0.92	0.00	0.55	1.72	1.08	1.20
time (sec)	N/A	0.137	1.279	0.152	0.000	3.506	0.332	0.788	6.509

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	160	131	0	88	299	154	143
N.S.	1	1.00	1.10	0.90	0.00	0.61	2.06	1.06	0.99
time (sec)	N/A	0.079	0.848	0.150	0.000	1.887	0.357	0.723	6.681

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	193	140	0	122	359	165	196
N.S.	1	1.00	1.19	0.86	0.00	0.75	2.22	1.02	1.21
time (sec)	N/A	0.344	1.227	0.334	0.000	1.866	0.465	1.028	6.674

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	1466	156	0	190	406	205	226
N.S.	1	1.00	6.66	0.71	0.00	0.86	1.85	0.93	1.03
time (sec)	N/A	0.487	7.037	0.362	0.000	2.083	1.161	1.327	7.050

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	1625	174	0	253	466	228	251
N.S.	1	1.00	6.37	0.68	0.00	0.99	1.83	0.89	0.98
time (sec)	N/A	0.537	7.183	0.412	0.000	1.984	0.830	1.178	7.636

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	201	162	153	440	0	0	216
N.S.	1	1.00	1.04	0.84	0.79	2.27	0.00	0.00	1.11
time (sec)	N/A	0.339	2.951	0.247	0.486	1.203	0.000	0.000	1.272

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	184	124	130	383	0	0	168
N.S.	1	1.00	1.29	0.87	0.91	2.68	0.00	0.00	1.17
time (sec)	N/A	0.203	2.308	0.111	0.533	1.692	0.000	0.000	7.119

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	132	82	107	332	0	0	120
N.S.	1	1.00	1.26	0.78	1.02	3.16	0.00	0.00	1.14
time (sec)	N/A	0.089	1.179	0.092	0.494	1.567	0.000	0.000	6.933

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	87	63	87	272	0	0	96
N.S.	1	1.00	1.16	0.84	1.16	3.63	0.00	0.00	1.28
time (sec)	N/A	0.049	1.117	0.089	0.561	1.495	0.000	0.000	0.529

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	192	312	113	447	0	0	493
N.S.	1	1.00	2.23	3.63	1.31	5.20	0.00	0.00	5.73
time (sec)	N/A	0.148	2.029	0.889	0.487	1.416	0.000	0.000	6.792

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	293	1179	145	649	0	0	168
N.S.	1	1.00	2.38	9.59	1.18	5.28	0.00	0.00	1.37
time (sec)	N/A	0.245	4.184	0.712	0.579	1.745	0.000	0.000	7.063

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	230	2240	202	730	0	0	702
N.S.	1	1.00	1.36	13.25	1.20	4.32	0.00	0.00	4.15
time (sec)	N/A	0.380	3.265	0.636	0.510	1.749	0.000	0.000	6.905



Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	414	1783	249	823	0	0	735
N.S.	1	1.00	1.97	8.49	1.19	3.92	0.00	0.00	3.50
time (sec)	N/A	0.496	4.412	0.513	0.578	1.907	0.000	0.000	7.089

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	239	164	153	467	0	0	211
N.S.	1	1.00	1.21	0.83	0.78	2.37	0.00	0.00	1.07
time (sec)	N/A	0.361	3.801	0.106	0.512	2.266	0.000	0.000	7.417

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	204	123	130	415	0	0	163
N.S.	1	1.00	1.49	0.90	0.95	3.03	0.00	0.00	1.19
time (sec)	N/A	0.117	3.688	0.092	0.498	3.120	0.000	0.000	6.931

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	190	99	111	359	0	0	139
N.S.	1	1.00	1.78	0.93	1.04	3.36	0.00	0.00	1.30
time (sec)	N/A	0.070	2.612	0.088	0.512	2.634	0.000	0.000	6.639

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	157	467	130	514	0	0	553
N.S.	1	1.00	1.39	4.13	1.15	4.55	0.00	0.00	4.89
time (sec)	N/A	0.249	2.046	0.468	0.501	3.059	0.000	0.000	6.598

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	201	1117	145	685	0	0	2338
N.S.	1	1.00	1.61	8.94	1.16	5.48	0.00	0.00	18.70
time (sec)	N/A	0.261	2.895	0.561	0.509	3.390	0.000	0.000	8.081

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	400	1290	203	762	0	0	2500
N.S.	1	1.00	2.34	7.54	1.19	4.46	0.00	0.00	14.62
time (sec)	N/A	0.391	5.847	0.586	0.492	2.751	0.000	0.000	7.819

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	439	1804	253	856	0	0	2500
N.S.	1	1.00	2.06	8.47	1.19	4.02	0.00	0.00	11.74
time (sec)	N/A	0.499	6.283	0.693	0.505	3.284	0.000	0.000	7.858

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	284	206	176	531	0	0	258
N.S.	1	1.00	1.15	0.84	0.72	2.16	0.00	0.00	1.05
time (sec)	N/A	0.485	5.249	0.108	0.508	3.534	0.000	0.000	7.571

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	268	165	153	477	0	0	212
N.S.	1	1.00	1.57	0.96	0.89	2.79	0.00	0.00	1.24
time (sec)	N/A	0.136	3.695	0.093	0.495	2.903	0.000	0.000	7.318

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	236	141	134	421	0	0	188
N.S.	1	1.00	1.67	1.00	0.95	2.99	0.00	0.00	1.33
time (sec)	N/A	0.087	3.058	0.092	0.490	2.819	0.000	0.000	0.962

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	429	965	154	610	0	0	597
N.S.	1	1.00	2.92	6.56	1.05	4.15	0.00	0.00	4.06
time (sec)	N/A	0.352	8.014	0.553	0.518	2.780	0.000	0.000	6.974

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	413	1141	163	705	0	0	2500
N.S.	1	1.00	2.61	7.22	1.03	4.46	0.00	0.00	15.82
time (sec)	N/A	0.365	7.221	0.559	0.493	2.794	0.000	0.000	8.377

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	427	1292	206	774	0	0	2500
N.S.	1	1.00	2.47	7.47	1.19	4.47	0.00	0.00	14.45
time (sec)	N/A	0.401	8.721	0.550	0.563	2.124	0.000	0.000	8.469

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	634	2506	249	868	0	0	2500
N.S.	1	1.00	2.92	11.55	1.15	4.00	0.00	0.00	11.52
time (sec)	N/A	0.530	8.670	0.628	0.500	1.998	0.000	0.000	8.467

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	698	3444	293	944	0	0	2500
N.S.	1	1.00	2.67	13.20	1.12	3.62	0.00	0.00	9.58
time (sec)	N/A	0.659	8.923	0.529	0.548	1.976	0.000	0.000	8.591

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	176	168	157	447	0	0	236
N.S.	1	1.00	0.86	0.82	0.77	2.18	0.00	0.00	1.15
time (sec)	N/A	0.340	3.175	0.114	0.501	2.274	0.000	0.000	7.953

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	147	127	134	391	0	0	188
N.S.	1	1.00	0.92	0.80	0.84	2.46	0.00	0.00	1.18
time (sec)	N/A	0.222	2.230	0.109	0.506	1.833	0.000	0.000	7.666

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	140	88	110	327	0	0	141
N.S.	1	1.00	1.28	0.81	1.01	3.00	0.00	0.00	1.29
time (sec)	N/A	0.096	1.434	0.099	0.540	2.719	0.000	0.000	7.383

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	129	71	91	328	0	0	117
N.S.	1	1.00	1.57	0.87	1.11	4.00	0.00	0.00	1.43
time (sec)	N/A	0.054	1.110	0.096	0.542	2.351	0.000	0.000	0.755

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	208	948	133	575	0	0	515
N.S.	1	1.00	1.82	8.32	1.17	5.04	0.00	0.00	4.52
time (sec)	N/A	0.240	2.494	0.942	0.558	2.087	0.000	0.000	7.137

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	224	2727	184	742	0	0	2500
N.S.	1	1.00	1.34	16.33	1.10	4.44	0.00	0.00	14.97
time (sec)	N/A	0.380	4.013	0.714	0.602	1.660	0.000	0.000	8.621

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	363	2751	232	835	0	0	2500
N.S.	1	1.00	1.66	12.56	1.06	3.81	0.00	0.00	11.42
time (sec)	N/A	0.496	4.388	0.533	0.514	1.896	0.000	0.000	8.654

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	176	153	160	440	0	0	233
N.S.	1	1.00	0.84	0.73	0.77	2.11	0.00	0.00	1.11
time (sec)	N/A	0.359	4.027	0.113	0.523	1.787	0.000	0.000	7.437

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	167	116	137	375	0	0	186
N.S.	1	1.00	1.00	0.69	0.82	2.25	0.00	0.00	1.11
time (sec)	N/A	0.239	2.828	0.108	0.572	1.909	0.000	0.000	0.698

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	145	96	116	369	0	0	163
N.S.	1	1.00	1.22	0.81	0.97	3.10	0.00	0.00	1.37
time (sec)	N/A	0.133	2.346	0.097	0.504	2.069	0.000	0.000	7.067

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	143	96	113	372	0	0	162
N.S.	1	1.00	1.18	0.79	0.93	3.07	0.00	0.00	1.34
time (sec)	N/A	0.074	2.308	0.095	0.510	1.723	0.000	0.000	6.988

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	192	1026	161	624	0	0	563
N.S.	1	1.00	1.23	6.58	1.03	4.00	0.00	0.00	3.61
time (sec)	N/A	0.351	4.121	0.477	0.496	1.727	0.000	0.000	6.700

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	259	2818	215	814	0	0	2500
N.S.	1	1.00	1.19	12.99	0.99	3.75	0.00	0.00	11.52
time (sec)	N/A	0.534	4.763	0.540	0.501	6.072	0.000	0.000	8.229

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	283	2818	259	903	0	0	2500
N.S.	1	1.00	1.06	10.51	0.97	3.37	0.00	0.00	9.33
time (sec)	N/A	0.654	5.341	0.566	0.524	6.576	0.000	0.000	8.410

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	191	181	185	457	0	0	279
N.S.	1	1.00	0.75	0.71	0.73	1.79	0.00	0.00	1.09
time (sec)	N/A	0.499	5.124	0.113	0.538	5.863	0.000	0.000	6.956

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	193	142	162	392	0	0	230
N.S.	1	1.00	0.91	0.67	0.77	1.86	0.00	0.00	1.09
time (sec)	N/A	0.369	4.218	0.110	0.495	2.171	0.000	0.000	6.893

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	176	124	141	393	0	0	187
N.S.	1	1.00	1.05	0.74	0.84	2.35	0.00	0.00	1.12
time (sec)	N/A	0.263	3.181	0.108	0.489	4.953	0.000	0.000	6.743

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	176	121	136	391	0	0	186
N.S.	1	1.00	1.15	0.79	0.89	2.56	0.00	0.00	1.22
time (sec)	N/A	0.153	2.639	0.098	0.517	2.553	0.000	0.000	6.727

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	176	123	139	392	0	0	205
N.S.	1	1.00	1.14	0.79	0.90	2.53	0.00	0.00	1.32
time (sec)	N/A	0.089	2.442	0.094	0.590	3.078	0.000	0.000	6.584

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	233	1084	186	644	0	0	528
N.S.	1	1.00	1.21	5.65	0.97	3.35	0.00	0.00	2.75
time (sec)	N/A	0.452	4.383	0.512	0.500	3.442	0.000	0.000	6.584

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	287	2858	240	834	0	0	2500
N.S.	1	1.00	1.11	11.03	0.93	3.22	0.00	0.00	9.65
time (sec)	N/A	0.689	7.704	0.591	0.502	2.770	0.000	0.000	8.254

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	317	2876	283	923	0	0	2500
N.S.	1	1.00	1.02	9.22	0.91	2.96	0.00	0.00	8.01
time (sec)	N/A	0.806	8.824	0.634	0.512	2.317	0.000	0.000	8.386

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	280	272	202	480	0	142	161
N.S.	1	1.00	2.15	2.09	1.55	3.69	0.00	1.09	1.24
time (sec)	N/A	0.142	4.060	0.054	0.607	2.188	0.000	0.789	10.116

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	266	251	186	432	0	112	130
N.S.	1	1.00	2.53	2.39	1.77	4.11	0.00	1.07	1.24
time (sec)	N/A	0.122	2.552	0.038	0.546	2.708	0.000	0.678	8.399



Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	112	226	170	372	0	82	99
N.S.	1	1.00	1.40	2.82	2.12	4.65	0.00	1.02	1.24
time (sec)	N/A	0.088	1.653	0.036	0.596	1.800	0.000	0.602	7.709

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	92	201	151	314	0	47	68
N.S.	1	1.00	1.67	3.65	2.75	5.71	0.00	0.85	1.24
time (sec)	N/A	0.065	1.377	0.038	0.508	2.039	0.000	0.608	7.081

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	76	202	151	367	0	47	67
N.S.	1	1.00	1.43	3.81	2.85	6.92	0.00	0.89	1.26
time (sec)	N/A	0.066	2.027	0.040	0.510	1.641	0.000	0.706	6.965

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	94	220	171	427	0	69	99
N.S.	1	1.00	1.21	2.82	2.19	5.47	0.00	0.88	1.27
time (sec)	N/A	0.092	1.428	0.043	0.501	1.473	0.000	0.744	7.533

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	265	236	187	486	0	93	123
N.S.	1	1.00	2.57	2.29	1.82	4.72	0.00	0.90	1.19
time (sec)	N/A	0.115	3.434	0.043	0.596	2.171	0.000	0.917	8.742

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	315	298	230	555	0	194	326
N.S.	1	1.00	1.72	1.63	1.26	3.03	0.00	1.06	1.78
time (sec)	N/A	0.252	4.989	0.041	0.504	2.453	0.000	1.186	11.031

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	307	275	212	501	0	160	291
N.S.	1	1.00	1.97	1.76	1.36	3.21	0.00	1.03	1.87
time (sec)	N/A	0.217	4.119	0.041	0.500	3.190	0.000	0.992	9.281

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	272	252	194	441	0	126	256
N.S.	1	1.00	2.11	1.95	1.50	3.42	0.00	0.98	1.98
time (sec)	N/A	0.182	4.175	0.041	0.580	3.717	0.000	0.874	7.731

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	110	229	174	389	0	92	221
N.S.	1	1.00	1.06	2.20	1.67	3.74	0.00	0.88	2.12
time (sec)	N/A	0.157	2.607	0.041	0.616	1.466	0.000	0.840	6.844

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	217	170	386	0	70	203
N.S.	1	1.00	0.87	2.21	1.73	3.94	0.00	0.71	2.07
time (sec)	N/A	0.149	2.518	0.044	0.513	1.650	0.000	0.892	6.719

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	222	177	441	0	79	222
N.S.	1	1.00	0.94	2.18	1.74	4.32	0.00	0.77	2.18
time (sec)	N/A	0.150	2.133	0.045	0.550	1.852	0.000	0.978	7.117

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	272	240	195	502	0	108	258
N.S.	1	1.00	2.14	1.89	1.54	3.95	0.00	0.85	2.03
time (sec)	N/A	0.184	3.976	0.046	0.514	2.691	0.000	1.211	7.991

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	296	258	213	561	0	136	293
N.S.	1	1.00	1.92	1.68	1.38	3.64	0.00	0.88	1.90
time (sec)	N/A	0.211	5.476	0.045	0.505	2.570	0.000	1.190	9.520

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	496	299	234	561	0	194	327
N.S.	1	1.00	2.51	1.51	1.18	2.83	0.00	0.98	1.65
time (sec)	N/A	0.323	8.044	0.042	0.501	2.981	0.000	1.181	10.946

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	452	276	216	498	0	160	292
N.S.	1	1.00	2.64	1.61	1.26	2.91	0.00	0.94	1.71
time (sec)	N/A	0.287	8.131	0.041	0.499	3.287	0.000	1.001	8.921

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	273	253	196	447	0	127	257
N.S.	1	1.00	1.87	1.73	1.34	3.06	0.00	0.87	1.76
time (sec)	N/A	0.253	5.836	0.040	0.517	2.348	0.000	0.939	7.161

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	151	241	190	405	0	110	239
N.S.	1	1.00	1.13	1.80	1.42	3.02	0.00	0.82	1.78
time (sec)	N/A	0.233	5.197	0.044	0.507	2.209	0.000	1.079	6.896

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	266	234	191	438	0	97	240
N.S.	1	1.00	1.96	1.72	1.40	3.22	0.00	0.71	1.76
time (sec)	N/A	0.244	5.083	0.046	0.526	1.814	0.000	1.100	7.057

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	449	240	197	504	0	107	258
N.S.	1	1.00	3.12	1.67	1.37	3.50	0.00	0.74	1.79
time (sec)	N/A	0.250	7.908	0.046	0.501	2.166	0.000	1.382	7.386

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	495	258	215	561	0	136	293
N.S.	1	1.00	2.93	1.53	1.27	3.32	0.00	0.80	1.73
time (sec)	N/A	0.287	8.885	0.047	0.514	2.666	0.000	1.460	9.520

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	248	166	0	707	0	165	305
N.S.	1	1.00	0.81	0.54	0.00	2.31	0.00	0.54	1.00
time (sec)	N/A	0.286	2.517	0.116	0.000	2.672	0.000	0.675	11.328

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	220	143	0	621	0	118	270
N.S.	1	1.00	0.80	0.52	0.00	2.26	0.00	0.43	0.98
time (sec)	N/A	0.244	1.026	0.094	0.000	1.061	0.000	0.613	10.609

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	198	124	0	570	0	97	184
N.S.	1	1.00	0.84	0.53	0.00	2.42	0.00	0.41	0.78
time (sec)	N/A	0.195	0.818	0.174	0.000	1.349	0.000	0.563	8.508

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	199	121	0	571	0	98	184
N.S.	1	1.00	0.85	0.52	0.00	2.44	0.00	0.42	0.79
time (sec)	N/A	0.194	0.942	0.115	0.000	2.175	0.000	0.713	7.204

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	217	140	0	703	0	111	266
N.S.	1	1.00	0.81	0.52	0.00	2.63	0.00	0.42	1.00
time (sec)	N/A	0.258	1.076	0.095	0.000	2.635	0.000	0.864	7.366

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	241	160	0	795	0	141	303
N.S.	1	1.00	0.81	0.54	0.00	2.69	0.00	0.48	1.02
time (sec)	N/A	0.283	1.410	0.092	0.000	2.255	0.000	1.011	9.816

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	255	159	0	664	0	145	334
N.S.	1	1.00	0.81	0.50	0.00	2.10	0.00	0.46	1.06
time (sec)	N/A	0.390	1.294	0.102	0.000	2.443	0.000	0.848	9.780

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	243	147	0	664	0	123	318
N.S.	1	1.00	0.88	0.53	0.00	2.40	0.00	0.44	1.15
time (sec)	N/A	0.337	1.238	0.129	0.000	1.983	0.000	0.730	9.814

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	241	148	0	658	0	127	318
N.S.	1	1.00	0.86	0.53	0.00	2.36	0.00	0.46	1.14
time (sec)	N/A	0.312	1.123	0.124	0.000	2.920	0.000	0.666	9.750

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	243	144	0	664	0	126	318
N.S.	1	1.00	0.85	0.51	0.00	2.33	0.00	0.44	1.12
time (sec)	N/A	0.340	1.258	0.121	0.000	2.013	0.000	0.963	9.680

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	250	157	0	763	0	143	338
N.S.	1	1.00	0.79	0.49	0.00	2.40	0.00	0.45	1.06
time (sec)	N/A	0.389	1.424	0.101	0.000	0.905	0.000	1.251	9.825

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	282	176	0	861	0	162	373
N.S.	1	1.00	0.81	0.51	0.00	2.48	0.00	0.47	1.07
time (sec)	N/A	0.430	1.909	0.102	0.000	0.858	0.000	1.410	10.479

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	300	204	0	779	0	210	431
N.S.	1	1.00	0.76	0.52	0.00	1.98	0.00	0.53	1.10
time (sec)	N/A	0.571	2.631	0.110	0.000	0.743	0.000	1.052	10.215

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	286	181	0	685	0	165	395
N.S.	1	1.00	0.79	0.50	0.00	1.88	0.00	0.45	1.09
time (sec)	N/A	0.510	1.929	0.107	0.000	0.728	0.000	1.008	9.971

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	254	163	0	681	0	135	308
N.S.	1	1.00	0.83	0.53	0.00	2.22	0.00	0.44	1.00
time (sec)	N/A	0.420	1.409	0.136	0.000	0.627	0.000	0.950	8.168

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	274	152	0	635	0	131	239
N.S.	1	1.00	0.89	0.49	0.00	2.06	0.00	0.42	0.77
time (sec)	N/A	0.418	2.150	0.128	0.000	0.638	0.000	0.862	6.688

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	272	152	0	631	0	131	239
N.S.	1	1.00	0.86	0.48	0.00	1.99	0.00	0.41	0.75
time (sec)	N/A	0.420	2.007	0.130	0.000	0.593	0.000	0.827	6.593

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	258	157	0	682	0	137	308
N.S.	1	1.00	0.82	0.50	0.00	2.17	0.00	0.43	0.98
time (sec)	N/A	0.433	1.702	0.132	0.000	0.569	0.000	1.363	6.672

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	278	177	0	784	0	167	389
N.S.	1	1.00	0.76	0.49	0.00	2.15	0.00	0.46	1.07
time (sec)	N/A	0.532	1.843	0.110	0.000	0.666	0.000	1.710	6.829

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	306	196	0	875	0	182	425
N.S.	1	1.00	0.78	0.50	0.00	2.23	0.00	0.46	1.08
time (sec)	N/A	0.571	2.300	0.112	0.000	0.555	0.000	0.798	8.532



Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	0	838	0	771	0	0	-1
N.S.	1	1.00	0.00	4.19	0.00	3.86	0.00	0.00	-0.00
time (sec)	N/A	0.452	4.663	0.303	0.000	0.479	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	560	713	0	673	0	0	2225
N.S.	1	1.00	3.68	4.69	0.00	4.43	0.00	0.00	14.64
time (sec)	N/A	0.316	2.701	0.123	0.000	0.522	0.000	0.000	24.348

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	238	502	0	539	0	0	372
N.S.	1	1.00	2.12	4.48	0.00	4.81	0.00	0.00	3.32
time (sec)	N/A	0.209	2.316	0.118	0.000	0.457	0.000	0.000	10.836

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	156	434	0	435	0	0	-1
N.S.	1	1.00	1.73	4.82	0.00	4.83	0.00	0.00	-0.01
time (sec)	N/A	0.119	3.369	0.116	0.000	0.439	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	174	553	0	483	0	0	-1
N.S.	1	1.00	1.29	4.10	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.224	3.603	0.116	0.000	0.462	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	211	630	0	543	0	0	-1
N.S.	1	1.00	1.19	3.54	0.00	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.363	3.850	0.118	0.000	1.566	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	239	707	0	593	0	0	-1
N.S.	1	1.00	1.08	3.20	0.00	2.68	0.00	0.00	-0.00
time (sec)	N/A	0.463	5.725	0.117	0.000	0.914	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	420	652	0	911	0	0	-1
N.S.	1	1.00	1.69	2.63	0.00	3.67	0.00	0.00	-0.00
time (sec)	N/A	0.600	4.775	0.148	0.000	1.391	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	389	565	0	829	0	0	-1
N.S.	1	1.00	1.91	2.77	0.00	4.06	0.00	0.00	-0.00
time (sec)	N/A	0.462	3.781	0.128	0.000	1.228	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	221	486	0	743	0	0	-1
N.S.	1	1.00	1.42	3.12	0.00	4.76	0.00	0.00	-0.01
time (sec)	N/A	0.334	2.086	0.121	0.000	1.019	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	234	521	0	748	0	0	-1
N.S.	1	1.00	1.60	3.57	0.00	5.12	0.00	0.00	-0.01
time (sec)	N/A	0.315	2.494	0.119	0.000	1.168	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	221	618	0	524	0	0	-1
N.S.	1	1.00	1.61	4.51	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.244	3.600	0.119	0.000	0.780	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	237	707	0	593	0	0	-1
N.S.	1	1.00	1.31	3.91	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.368	5.597	0.119	0.000	0.736	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	261	796	0	638	0	0	-1
N.S.	1	1.00	1.16	3.54	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	0.483	7.141	0.119	0.000	0.941	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	242	885	0	701	0	0	-1
N.S.	1	1.00	0.90	3.29	0.00	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.619	8.775	0.122	0.000	1.035	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	459	742	0	1017	0	0	-1
N.S.	1	1.00	1.54	2.49	0.00	3.41	0.00	0.00	-0.00
time (sec)	N/A	0.750	7.445	0.126	0.000	0.999	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	426	653	0	932	0	0	-1
N.S.	1	1.00	1.69	2.59	0.00	3.70	0.00	0.00	-0.00
time (sec)	N/A	0.604	6.508	0.124	0.000	0.794	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	394	566	0	849	0	0	-1
N.S.	1	1.00	1.91	2.75	0.00	4.12	0.00	0.00	-0.00
time (sec)	N/A	0.465	6.530	0.121	0.000	0.760	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	389	565	0	857	0	0	-1
N.S.	1	1.00	1.98	2.88	0.00	4.37	0.00	0.00	-0.01
time (sec)	N/A	0.453	6.891	0.122	0.000	0.704	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	456	620	0	846	0	0	-1
N.S.	1	1.00	2.40	3.26	0.00	4.45	0.00	0.00	-0.01
time (sec)	N/A	0.445	7.180	0.119	0.000	0.686	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	323	709	0	608	0	0	-1
N.S.	1	1.00	1.75	3.83	0.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.381	8.323	0.120	0.000	0.614	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	363	798	0	657	0	0	-1
N.S.	1	1.00	1.57	3.45	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	0.509	9.470	0.120	0.000	0.654	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	246	887	0	722	0	0	-1
N.S.	1	1.00	0.89	3.20	0.00	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.641	9.252	0.123	0.000	0.578	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	328	976	0	771	0	0	-1
N.S.	1	1.00	1.02	3.02	0.00	2.39	0.00	0.00	-0.00
time (sec)	N/A	0.776	11.346	0.122	0.000	0.543	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	326	620	0	900	0	0	-1
N.S.	1	1.00	1.72	3.26	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.485	6.682	0.179	0.000	0.604	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	277	1135	0	807	0	0	-1
N.S.	1	1.00	1.35	5.54	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.452	3.047	0.149	0.000	0.737	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	183	894	0	718	0	0	2500
N.S.	1	1.00	1.17	5.73	0.00	4.60	0.00	0.00	16.03
time (sec)	N/A	0.308	2.280	0.134	0.000	0.784	0.000	0.000	24.205

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	123	639	0	416	0	0	426
N.S.	1	1.00	1.24	6.45	0.00	4.20	0.00	0.00	4.30
time (sec)	N/A	0.126	1.901	0.129	0.000	0.845	0.000	0.000	11.615

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	181	701	0	473	0	0	-1
N.S.	1	1.00	1.27	4.90	0.00	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.242	2.037	0.136	0.000	0.720	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	221	746	0	534	0	0	-1
N.S.	1	1.00	1.16	3.91	0.00	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.366	2.398	0.136	0.000	1.464	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	241	821	0	596	0	0	-1
N.S.	1	1.00	1.02	3.46	0.00	2.51	0.00	0.00	-0.00
time (sec)	N/A	0.497	3.156	0.138	0.000	1.400	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	285	1223	0	758	0	0	-1
N.S.	1	1.00	1.40	6.02	0.00	3.73	0.00	0.00	-0.00
time (sec)	N/A	0.441	3.890	0.142	0.000	0.868	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	228	868	0	443	0	0	-1
N.S.	1	1.00	1.52	5.79	0.00	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.247	3.059	0.126	0.000	0.739	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	230	868	0	441	0	0	-1
N.S.	1	1.00	1.55	5.86	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.251	3.253	0.134	0.000	1.129	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	237	931	0	511	0	0	-1
N.S.	1	1.00	1.22	4.80	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.378	3.009	0.135	0.000	1.195	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	244	1012	0	569	0	0	-1
N.S.	1	1.00	1.02	4.22	0.00	2.37	0.00	0.00	-0.00
time (sec)	N/A	0.507	4.146	0.134	0.000	1.186	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	275	1542	0	777	0	0	-1
N.S.	1	1.00	1.10	6.19	0.00	3.12	0.00	0.00	-0.00
time (sec)	N/A	0.572	4.480	0.143	0.000	0.982	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	214	1096	0	459	0	0	-1
N.S.	1	1.00	1.10	5.65	0.00	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.400	4.430	0.132	0.000	0.685	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	215	1096	0	461	0	0	-1
N.S.	1	1.00	1.10	5.59	0.00	2.35	0.00	0.00	-0.01
time (sec)	N/A	0.400	3.516	0.131	0.000	0.747	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	216	1096	0	460	0	0	-1
N.S.	1	1.00	1.11	5.65	0.00	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.394	3.808	0.134	0.000	0.818	0.000	0.000	0.000



Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	288	1158	0	529	0	0	-1
N.S.	1	1.00	1.20	4.82	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.536	6.186	0.133	0.000	0.883	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	268	1239	0	588	0	0	-1
N.S.	1	1.00	0.94	4.33	0.00	2.06	0.00	0.00	-0.00
time (sec)	N/A	0.677	4.754	0.134	0.000	0.825	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	0	159	167	405	0	0	365
N.S.	1	1.00	0.00	0.79	0.83	2.01	0.00	0.00	1.82
time (sec)	N/A	0.123	180.004	0.059	0.567	0.675	0.000	0.000	1.044

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	104	222	210	660	0	0	436
N.S.	1	1.00	0.39	0.82	0.78	2.44	0.00	0.00	1.61
time (sec)	N/A	0.315	1.529	0.071	0.526	0.686	0.000	0.000	7.748

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	115	181	187	571	0	0	390
N.S.	1	1.00	0.50	0.78	0.81	2.46	0.00	0.00	1.68
time (sec)	N/A	0.163	0.921	0.054	0.540	0.645	0.000	0.000	7.449

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	91	159	168	489	0	0	367
N.S.	1	1.00	0.45	0.79	0.83	2.42	0.00	0.00	1.82
time (sec)	N/A	0.113	0.681	0.048	0.512	0.663	0.000	0.000	7.300

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	127	0	252	711	0	0	1761
N.S.	1	1.00	0.44	0.00	0.87	2.46	0.00	0.00	6.09
time (sec)	N/A	0.260	1.053	0.248	0.527	0.782	0.000	0.000	6.749

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	0	0	298	1096	0	0	2500
N.S.	1	1.00	0.00	0.00	0.87	3.20	0.00	0.00	7.31
time (sec)	N/A	0.416	4.186	0.227	0.548	0.638	0.000	0.000	8.072

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	137	166	172	547	0	0	383
N.S.	1	1.00	0.64	0.78	0.81	2.57	0.00	0.00	1.80
time (sec)	N/A	0.123	0.731	0.054	0.548	0.901	0.000	0.000	6.985

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	0	166	171	493	0	0	390
N.S.	1	1.00	0.00	0.78	0.80	2.31	0.00	0.00	1.83
time (sec)	N/A	0.117	0.370	0.050	0.521	1.811	0.000	0.000	0.669

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	1805	0	0	0	0	0	-1
N.S.	1	1.00	6.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.753	9.965	0.522	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	1305	0	0	0	0	0	-1
N.S.	1	1.00	6.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	8.729	0.327	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	1587	0	0	0	0	0	-1
N.S.	1	1.00	12.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	8.143	0.327	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	2.015	0.242	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	4.259	0.386	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	565	0	0	0	0	0	-1
N.S.	1	1.00	2.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	4.403	0.734	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	712	0	0	0	0	0	-1
N.S.	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	49.528	0.890	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	921	0	0	0	0	0	-1
N.S.	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.857	50.772	0.507	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	4.306	0.461	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	2.980	0.375	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	2.287	0.455	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	180.001	0.390	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	10.266	0.371	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	45.801	0.372	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	12.166	0.290	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	40.086	0.263	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	14.306	0.442	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	270	0	0	0	0	0	-1
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	21.322	0.434	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	171	0	0	0	0	0	-1
N.S.	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	4.171	0.250	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	15.325	0.272	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	27.578	0.368	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.404	39.618	0.580	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.788	9.222	0.323	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	10.544	0.326	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	11.898	0.338	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	13.072	0.310	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.326	5.993	0.293	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	7.667	0.289	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	99	86	85	136	1017	84
N.S.	1	1.00	0.99	1.14	0.99	0.98	1.56	11.69	0.97
time (sec)	N/A	0.082	0.399	0.056	0.573	0.707	0.111	0.972	6.204

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	74	66	66	104	616	63
N.S.	1	1.00	1.03	1.14	1.02	1.02	1.60	9.48	0.97
time (sec)	N/A	0.040	0.215	0.047	0.526	0.653	0.086	0.628	6.173



Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	51	50	50	73	329	55
N.S.	1	1.00	1.40	1.21	1.19	1.19	1.74	7.83	1.31
time (sec)	N/A	0.018	0.026	0.033	0.563	0.520	0.069	0.519	6.322

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	43	52	59	78	53	69
N.S.	1	1.00	1.19	1.16	1.41	1.59	2.11	1.43	1.86
time (sec)	N/A	0.050	0.056	0.128	0.547	0.611	0.208	0.532	6.466

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	78	53	68	73	122	119	87
N.S.	1	1.00	1.81	1.23	1.58	1.70	2.84	2.77	2.02
time (sec)	N/A	0.058	0.125	0.106	0.562	0.561	0.458	0.633	6.209

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	77	77	86	95	150	179	108
N.S.	1	1.00	1.17	1.17	1.30	1.44	2.27	2.71	1.64
time (sec)	N/A	0.088	0.331	0.136	0.558	0.587	0.668	0.737	6.252

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	101	95	104	121	180	237	127
N.S.	1	1.00	1.16	1.09	1.20	1.39	2.07	2.72	1.46
time (sec)	N/A	0.112	0.724	0.137	0.564	0.534	0.980	0.830	6.409

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	100	108	122	138	211	299	145
N.S.	1	1.00	0.93	1.00	1.13	1.28	1.95	2.77	1.34
time (sec)	N/A	0.141	0.816	0.155	0.512	0.536	1.492	0.991	6.395

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	221	176	147	146	246	2228	151
N.S.	1	1.00	1.49	1.19	0.99	0.99	1.66	15.05	1.02
time (sec)	N/A	0.188	6.135	0.074	0.508	0.529	0.151	1.942	6.197

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	172	135	120	119	192	1509	121
N.S.	1	1.00	1.54	1.21	1.07	1.06	1.71	13.47	1.08
time (sec)	N/A	0.090	1.221	0.062	0.573	0.524	0.120	1.118	6.210

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	96	97	91	91	143	901	91
N.S.	1	1.00	1.10	1.11	1.05	1.05	1.64	10.36	1.05
time (sec)	N/A	0.058	0.295	0.046	0.508	0.444	0.100	0.808	6.228

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	82	85	92	129	86	90
N.S.	1	1.00	1.33	1.17	1.21	1.31	1.84	1.23	1.29
time (sec)	N/A	0.081	0.198	0.148	0.527	0.459	0.356	0.805	6.366

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	100	84	93	112	167	118	100
N.S.	1	1.00	1.39	1.17	1.29	1.56	2.32	1.64	1.39
time (sec)	N/A	0.095	0.178	0.144	0.508	0.482	0.652	0.964	6.371

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	123	107	120	122	214	237	127
N.S.	1	1.00	1.40	1.22	1.36	1.39	2.43	2.69	1.44
time (sec)	N/A	0.135	0.238	0.167	0.513	0.499	0.958	1.126	6.399

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	152	136	149	157	260	334	156
N.S.	1	1.00	1.29	1.15	1.26	1.33	2.20	2.83	1.32
time (sec)	N/A	0.172	0.916	0.159	0.522	0.450	1.407	1.465	6.374

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	162	175	191	313	435	182
N.S.	1	1.00	1.19	1.07	1.16	1.26	2.07	2.88	1.21
time (sec)	N/A	0.215	1.892	0.184	0.513	0.484	2.251	1.269	6.344

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	241	271	214	213	384	3997	217
N.S.	1	1.00	1.20	1.35	1.06	1.06	1.91	19.89	1.08
time (sec)	N/A	0.258	1.451	0.097	0.543	0.531	0.196	3.814	6.389

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	209	213	179	178	311	2870	181
N.S.	1	1.00	1.27	1.29	1.08	1.08	1.88	17.39	1.10
time (sec)	N/A	0.139	0.988	0.080	0.566	1.207	0.170	2.279	6.331

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	130	159	143	142	240	1911	142
N.S.	1	1.00	0.93	1.14	1.02	1.01	1.71	13.65	1.01
time (sec)	N/A	0.107	0.652	0.061	0.528	0.795	0.131	1.504	6.241

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	115	131	124	133	204	129	118
N.S.	1	1.00	0.98	1.12	1.06	1.14	1.74	1.10	1.01
time (sec)	N/A	0.185	0.400	0.182	0.507	0.776	0.566	1.185	6.417

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	123	125	145	223	152	114
N.S.	1	1.00	0.95	1.03	1.05	1.22	1.87	1.28	0.96
time (sec)	N/A	0.184	0.360	0.160	0.512	0.684	0.961	1.443	6.407

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	126	138	142	162	262	193	135
N.S.	1	1.00	0.99	1.09	1.12	1.28	2.06	1.52	1.06
time (sec)	N/A	0.196	0.290	0.201	0.514	0.651	1.353	1.433	6.391

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	164	166	180	181	332	390	169
N.S.	1	1.00	1.06	1.08	1.17	1.18	2.16	2.53	1.10
time (sec)	N/A	0.254	0.797	0.173	0.503	0.761	2.216	1.057	6.464

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	199	203	215	225	400	528	204
N.S.	1	1.00	1.04	1.06	1.13	1.18	2.09	2.76	1.07
time (sec)	N/A	0.311	0.501	0.208	0.501	0.819	2.971	1.121	6.531

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	237	244	250	266	471	670	238
N.S.	1	1.00	1.02	1.05	1.07	1.14	2.02	2.88	1.02
time (sec)	N/A	0.341	0.755	0.222	0.506	0.773	5.080	1.188	6.572

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	290	386	290	289	536	6392	300
N.S.	1	1.00	1.10	1.47	1.10	1.10	2.04	24.30	1.14
time (sec)	N/A	0.299	3.763	0.121	0.510	0.766	0.270	7.374	6.368

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	257	309	246	245	437	4789	251
N.S.	1	1.00	1.14	1.37	1.09	1.08	1.93	21.19	1.11
time (sec)	N/A	0.189	2.385	0.104	0.521	0.822	0.218	4.623	6.319

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	240	237	202	201	347	3383	205
N.S.	1	1.00	1.19	1.17	1.00	1.00	1.72	16.75	1.01
time (sec)	N/A	0.159	2.207	0.079	0.510	0.722	0.164	2.867	6.269

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	149	190	175	185	291	191	151
N.S.	1	1.00	0.87	1.10	1.02	1.08	1.69	1.11	0.88
time (sec)	N/A	0.318	0.932	0.227	0.557	1.256	0.901	1.745	6.491

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	134	172	164	193	289	195	142
N.S.	1	1.00	0.77	0.98	0.94	1.10	1.65	1.11	0.81
time (sec)	N/A	0.324	0.675	0.211	0.531	1.062	1.389	1.332	6.441

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	140	177	173	199	309	224	149
N.S.	1	1.00	0.75	0.95	0.93	1.07	1.66	1.20	0.80
time (sec)	N/A	0.350	0.457	0.235	0.491	0.802	2.164	1.461	6.453

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	167	197	202	222	369	281	177
N.S.	1	1.00	0.89	1.05	1.08	1.19	1.97	1.50	0.95
time (sec)	N/A	0.356	0.722	0.227	0.518	0.680	2.972	1.587	6.547

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	211	233	246	249	459	584	218
N.S.	1	1.00	0.94	1.04	1.09	1.11	2.04	2.60	0.97
time (sec)	N/A	0.430	0.612	0.230	0.569	0.784	4.929	1.703	6.467

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	257	285	289	300	546	763	263
N.S.	1	1.00	0.94	1.04	1.06	1.10	2.00	2.79	0.96
time (sec)	N/A	0.488	0.992	0.260	0.510	0.676	6.526	1.699	6.683

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	299	338	333	350	643	943	307
N.S.	1	1.00	0.93	1.05	1.03	1.08	1.99	2.92	0.95
time (sec)	N/A	0.580	0.827	0.290	0.515	0.921	10.001	1.887	6.955

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	138	127	130	190	1297	135	144
N.S.	1	1.00	1.09	1.00	1.02	1.50	10.21	1.06	1.13
time (sec)	N/A	0.272	1.007	0.203	0.579	1.513	0.774	0.754	6.515

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	118	101	109	149	1015	110	117
N.S.	1	1.00	1.17	1.00	1.08	1.48	10.05	1.09	1.16
time (sec)	N/A	0.136	0.411	0.129	0.541	0.966	0.595	0.587	6.412

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	98	87	94	110	700	95	100
N.S.	1	1.00	1.22	1.09	1.18	1.38	8.75	1.19	1.25
time (sec)	N/A	0.090	0.126	0.132	0.504	1.024	0.525	0.497	6.655

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	66	82	88	76	524	94	93
N.S.	1	1.00	1.14	1.41	1.52	1.31	9.03	1.62	1.60
time (sec)	N/A	0.052	0.073	0.080	0.500	1.356	0.460	0.497	6.665

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	113	101	107	118	952	113	115
N.S.	1	1.00	1.41	1.26	1.34	1.48	11.90	1.41	1.44
time (sec)	N/A	0.077	0.260	0.244	0.501	1.150	1.126	0.551	7.025

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	138	123	131	177	2067	157	140
N.S.	1	1.00	1.34	1.19	1.27	1.72	20.07	1.52	1.36
time (sec)	N/A	0.173	0.596	0.232	0.587	1.970	2.141	0.664	7.698

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	163	152	158	234	2596	214	175
N.S.	1	1.00	1.19	1.11	1.15	1.71	18.95	1.56	1.28
time (sec)	N/A	0.366	0.939	0.285	0.541	1.575	3.256	0.807	8.153



Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	194	189	200	293	3012	285	208
N.S.	1	1.00	1.15	1.12	1.18	1.73	17.82	1.69	1.23
time (sec)	N/A	0.558	1.637	0.281	0.498	2.155	5.506	0.934	8.344

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	444	172	220	434	4534	290	210
N.S.	1	1.00	2.13	0.83	1.06	2.09	21.80	1.39	1.01
time (sec)	N/A	0.301	2.793	0.227	0.512	1.543	1.102	0.820	7.185

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	323	155	197	311	3485	244	165
N.S.	1	1.00	2.06	0.99	1.25	1.98	22.20	1.55	1.05
time (sec)	N/A	0.184	1.476	0.180	0.538	1.712	0.953	0.665	6.556

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	140	145	185	221	2987	241	163
N.S.	1	1.00	1.22	1.26	1.61	1.92	25.97	2.10	1.42
time (sec)	N/A	0.110	1.462	0.135	0.509	1.284	0.821	0.556	6.601

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	190	141	177	222	2878	234	153
N.S.	1	1.00	1.71	1.27	1.59	2.00	25.93	2.11	1.38
time (sec)	N/A	0.097	1.483	0.125	0.516	1.454	0.786	0.535	6.477

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	183	163	208	323	4447	279	180
N.S.	1	1.00	1.34	1.19	1.52	2.36	32.46	2.04	1.31
time (sec)	N/A	0.217	0.582	0.308	0.549	1.721	1.800	0.751	8.001

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	193	196	262	465	8102	362	230
N.S.	1	1.00	1.01	1.02	1.36	2.42	42.20	1.89	1.20
time (sec)	N/A	0.363	2.296	0.324	0.515	1.745	2.785	0.920	9.272

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	220	227	325	590	9840	402	284
N.S.	1	1.00	0.88	0.91	1.30	2.36	39.36	1.61	1.14
time (sec)	N/A	0.563	2.943	0.387	0.534	1.474	4.347	0.985	10.661

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	1146	263	389	890	0	505	335
N.S.	1	1.00	3.46	0.79	1.18	2.69	0.00	1.53	1.01
time (sec)	N/A	0.539	6.593	0.354	0.596	1.951	0.000	1.198	7.868

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	462	242	366	666	0	458	307
N.S.	1	1.00	1.85	0.97	1.46	2.66	0.00	1.83	1.23
time (sec)	N/A	0.329	3.146	0.317	0.513	1.917	0.000	0.954	6.864

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	288	223	333	478	0	410	280
N.S.	1	1.00	1.52	1.18	1.76	2.53	0.00	2.17	1.48
time (sec)	N/A	0.247	3.522	0.192	0.579	1.649	0.000	0.785	6.666

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	188	213	330	488	0	410	282
N.S.	1	1.00	1.05	1.19	1.84	2.73	0.00	2.29	1.58
time (sec)	N/A	0.181	2.609	0.201	0.519	1.469	0.000	0.696	6.628

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	243	208	321	482	0	409	279
N.S.	1	1.00	1.39	1.19	1.83	2.75	0.00	2.34	1.59
time (sec)	N/A	0.179	2.472	0.188	0.527	1.236	0.000	0.667	6.524

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	254	243	372	683	0	479	315
N.S.	1	1.00	1.18	1.13	1.73	3.18	0.00	2.23	1.47
time (sec)	N/A	0.418	2.347	0.472	0.527	1.537	0.000	1.030	8.380

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	288	289	454	917	0	560	380
N.S.	1	1.00	1.00	1.01	1.58	3.20	0.00	1.95	1.32
time (sec)	N/A	0.590	6.292	0.488	0.612	1.770	0.000	1.020	11.230

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	320	320	541	1065	0	812	434
N.S.	1	1.00	0.91	0.91	1.54	3.03	0.00	2.31	1.23
time (sec)	N/A	0.828	6.329	0.581	0.545	1.711	0.000	1.221	12.870

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	1812	343	583	1113	0	719	486
N.S.	1	1.00	5.16	0.98	1.66	3.17	0.00	2.05	1.38
time (sec)	N/A	0.551	6.623	0.485	0.551	1.893	0.000	1.327	7.354

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	465	318	550	813	0	670	470
N.S.	1	1.00	1.56	1.07	1.85	2.73	0.00	2.25	1.58
time (sec)	N/A	0.387	6.242	0.340	0.546	1.492	0.000	1.070	7.193

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	357	301	526	836	0	632	446
N.S.	1	1.00	1.37	1.15	2.02	3.20	0.00	2.42	1.71
time (sec)	N/A	0.325	6.022	0.315	0.512	2.282	0.000	0.892	7.032

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	248	287	523	838	0	638	447
N.S.	1	1.00	0.99	1.15	2.09	3.35	0.00	2.55	1.79
time (sec)	N/A	0.279	0.850	0.346	0.558	1.463	0.000	0.766	6.803

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	327	284	514	815	0	630	442
N.S.	1	1.00	1.32	1.15	2.08	3.30	0.00	2.55	1.79
time (sec)	N/A	0.275	6.180	0.318	0.566	1.865	0.000	0.751	6.816

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	308	335	580	1126	0	722	484
N.S.	1	1.00	1.02	1.11	1.92	3.73	0.00	2.39	1.60
time (sec)	N/A	0.613	2.097	0.632	0.571	2.402	0.000	1.032	9.756

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	357	400	698	1510	0	846	576
N.S.	1	1.00	0.89	1.00	1.75	3.78	0.00	2.12	1.44
time (sec)	N/A	0.858	3.890	0.779	0.614	2.573	0.000	1.332	11.019

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	417	429	815	1732	0	903	664
N.S.	1	1.00	0.87	0.90	1.71	3.63	0.00	1.89	1.39
time (sec)	N/A	1.121	6.459	0.880	0.551	2.287	0.000	1.196	12.378

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	30	30	31	53	187	28
N.S.	1	1.00	0.90	1.03	1.03	1.07	1.83	6.45	0.97
time (sec)	N/A	0.012	0.020	0.066	0.536	1.897	0.392	0.720	6.206

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	22	22	19	36	22	16
N.S.	1	1.00	1.56	1.38	1.38	1.19	2.25	1.38	1.00
time (sec)	N/A	0.007	0.008	0.050	0.560	1.309	0.312	0.571	6.195

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	17	19	37	99	17
N.S.	1	1.00	1.00	1.38	1.31	1.46	2.85	7.62	1.31
time (sec)	N/A	0.004	0.006	0.047	0.532	1.687	0.284	0.469	6.240

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	10	3	2	10	3
N.S.	1	1.00	1.00	1.33	3.33	1.00	0.67	3.33	1.00
time (sec)	N/A	0.001	0.000	0.033	0.520	1.248	0.057	0.427	6.264

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	29	20	49	59	27
N.S.	1	1.00	1.67	1.08	2.42	1.67	4.08	4.92	2.25
time (sec)	N/A	0.004	0.009	0.101	0.509	1.278	0.449	0.489	6.274

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	22	23	42	37	39	16
N.S.	1	1.00	1.76	1.29	1.35	2.47	2.18	2.29	0.94
time (sec)	N/A	0.007	0.014	0.088	0.671	2.621	0.409	0.487	6.253

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	35	26	40	53	80	124	37
N.S.	1	1.00	1.17	0.87	1.33	1.77	2.67	4.13	1.23
time (sec)	N/A	0.011	0.057	0.115	0.576	1.511	0.846	0.521	6.237

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	27	38	90	49	69	32
N.S.	1	1.00	1.10	0.87	1.23	2.90	1.58	2.23	1.03
time (sec)	N/A	0.013	0.014	0.105	0.532	2.433	0.963	0.529	6.286

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	93	104	144	782	105	114
N.S.	1	1.00	1.06	0.91	1.02	1.41	7.67	1.03	1.12
time (sec)	N/A	0.168	0.315	0.142	0.504	2.016	0.965	1.043	6.339

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	80	89	119	660	90	98
N.S.	1	1.00	1.11	0.96	1.07	1.43	7.95	1.08	1.18
time (sec)	N/A	0.102	0.281	0.141	0.553	1.701	0.794	0.777	6.370

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	69	75	95	442	76	81
N.S.	1	1.00	0.98	0.85	0.93	1.17	5.46	0.94	1.00
time (sec)	N/A	0.075	0.066	0.115	0.516	1.575	0.654	0.598	6.409

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	67	64	71	65	282	76	79
N.S.	1	1.00	1.40	1.33	1.48	1.35	5.88	1.58	1.65
time (sec)	N/A	0.052	0.086	0.097	0.503	1.333	0.570	0.520	6.379

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	77	63	72	64	272	77	76
N.S.	1	1.00	1.64	1.34	1.53	1.36	5.79	1.64	1.62
time (sec)	N/A	0.038	0.043	0.075	0.599	2.894	0.540	0.470	6.313

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	79	81	88	104	672	92	99
N.S.	1	1.00	1.14	1.17	1.28	1.51	9.74	1.33	1.43
time (sec)	N/A	0.060	0.085	0.203	0.587	1.736	1.454	0.566	6.359

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	97	95	105	147	1137	122	113
N.S.	1	1.00	1.14	1.12	1.24	1.73	13.38	1.44	1.33
time (sec)	N/A	0.126	0.286	0.197	0.537	1.543	2.631	0.619	6.380

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	115	130	192	1397	165	143
N.S.	1	1.00	0.96	1.03	1.16	1.71	12.47	1.47	1.28
time (sec)	N/A	0.224	0.439	0.236	0.552	1.794	4.184	0.708	6.430



Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	62	37	35	44	39	36	49
N.S.	1	1.00	2.48	1.48	1.40	1.76	1.56	1.44	1.96
time (sec)	N/A	0.034	0.028	0.062	0.598	2.007	0.169	0.482	6.227

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	65	85	95	78	235	99	112
N.S.	1	1.00	1.12	1.47	1.64	1.34	4.05	1.71	1.93
time (sec)	N/A	0.053	0.069	0.099	0.709	1.158	0.439	0.520	6.570

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	187	125	161	191	1346	199	152
N.S.	1	1.00	1.85	1.24	1.59	1.89	13.33	1.97	1.50
time (sec)	N/A	0.089	1.637	0.125	0.549	1.524	0.697	0.598	6.626

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	212	740	0	9053	0	0	1093
N.S.	1	1.00	0.91	3.18	0.00	38.85	0.00	0.00	4.69
time (sec)	N/A	0.458	1.856	0.323	0.000	20.359	0.000	0.000	56.406

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	169	692	0	8926	0	0	938
N.S.	1	1.00	0.91	3.72	0.00	47.99	0.00	0.00	5.04
time (sec)	N/A	0.301	1.292	0.140	0.000	19.039	0.000	0.000	21.566

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	140	651	0	8737	0	0	864
N.S.	1	1.00	0.96	4.46	0.00	59.84	0.00	0.00	5.92
time (sec)	N/A	0.186	0.341	0.116	0.000	17.606	0.000	0.000	12.052

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	120	632	0	8608	0	0	845
N.S.	1	1.00	0.98	5.18	0.00	70.56	0.00	0.00	6.93
time (sec)	N/A	0.139	0.090	0.108	0.000	14.519	0.000	0.000	8.636

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	219	29038	0	17257	0	0	2500
N.S.	1	1.00	1.67	221.66	0.00	131.73	0.00	0.00	19.08
time (sec)	N/A	0.237	0.384	1.989	0.000	26.073	0.000	0.000	8.474

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	235	50546	0	18318	0	0	2500
N.S.	1	1.00	1.41	302.67	0.00	109.69	0.00	0.00	14.97
time (sec)	N/A	0.338	1.607	1.362	0.000	24.476	0.000	0.000	7.575

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	271	81276	0	18627	0	0	2500
N.S.	1	1.00	1.24	371.12	0.00	85.05	0.00	0.00	11.42
time (sec)	N/A	0.561	3.165	1.819	0.000	30.655	0.000	0.000	7.897

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	564	118304	0	20228	0	0	2500
N.S.	1	1.00	2.02	424.03	0.00	72.50	0.00	0.00	8.96
time (sec)	N/A	0.740	6.292	2.388	0.000	34.913	0.000	0.000	8.465

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	252	984	0	0	0	0	2993
N.S.	1	1.00	1.18	4.60	0.00	0.00	0.00	0.00	13.99
time (sec)	N/A	0.398	1.681	0.161	0.000	0.000	0.000	0.000	69.452

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	192	950	0	0	0	0	2868
N.S.	1	1.00	1.10	5.43	0.00	0.00	0.00	0.00	16.39
time (sec)	N/A	0.248	0.923	0.148	0.000	0.000	0.000	0.000	29.598

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	140	920	0	0	0	0	2823
N.S.	1	1.00	0.93	6.13	0.00	0.00	0.00	0.00	18.82
time (sec)	N/A	0.206	0.357	0.117	0.000	0.000	0.000	0.000	17.717

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	144	41721	0	0	0	0	2500
N.S.	1	1.00	0.95	274.48	0.00	0.00	0.00	0.00	16.45
time (sec)	N/A	0.410	0.234	1.707	0.000	0.000	0.000	0.000	9.441

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	282	69532	0	0	0	0	2500
N.S.	1	1.00	1.67	411.43	0.00	0.00	0.00	0.00	14.79
time (sec)	N/A	0.405	0.386	1.681	0.000	0.000	0.000	0.000	8.507

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	195	102706	0	0	0	0	2500
N.S.	1	1.00	0.89	468.98	0.00	0.00	0.00	0.00	11.42
time (sec)	N/A	0.628	1.666	2.183	0.000	0.000	0.000	0.000	8.649

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	241	145176	0	0	0	0	2500
N.S.	1	1.00	0.87	522.22	0.00	0.00	0.00	0.00	8.99
time (sec)	N/A	0.847	3.708	2.793	0.000	0.000	0.000	0.000	9.051

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	296	1325	0	0	0	0	2500
N.S.	1	1.00	1.17	5.26	0.00	0.00	0.00	0.00	9.92
time (sec)	N/A	0.462	3.038	0.176	0.000	0.000	0.000	0.000	168.237

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	258	1282	0	0	0	0	2500
N.S.	1	1.00	1.21	6.02	0.00	0.00	0.00	0.00	11.74
time (sec)	N/A	0.330	1.080	0.159	0.000	0.000	0.000	0.000	79.739

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	233	1249	0	0	0	0	2500
N.S.	1	1.00	1.24	6.64	0.00	0.00	0.00	0.00	13.30
time (sec)	N/A	0.274	0.730	0.129	0.000	0.000	0.000	0.000	34.445

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	177	55566	0	0	0	0	2500
N.S.	1	1.00	0.97	305.31	0.00	0.00	0.00	0.00	13.74
time (sec)	N/A	0.558	0.776	3.065	0.000	0.000	0.000	0.000	12.288

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	400	88645	0	0	0	0	2500
N.S.	1	1.00	2.04	452.27	0.00	0.00	0.00	0.00	12.76
time (sec)	N/A	0.574	0.699	2.469	0.000	0.000	0.000	0.000	9.768

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-1)	F(-1)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	448	128221	0	0	0	0	2500
N.S.	1	1.00	2.04	582.82	0.00	0.00	0.00	0.00	11.36
time (sec)	N/A	0.603	1.594	2.634	0.000	0.000	0.000	0.000	9.798

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F(-1)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	240	171974	0	0	0	0	2500
N.S.	1	1.00	0.87	620.84	0.00	0.00	0.00	0.00	9.03
time (sec)	N/A	0.834	4.341	3.346	0.000	0.000	0.000	0.000	10.071

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F(-1)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	622	227162	0	0	0	0	2500
N.S.	1	1.00	1.82	664.22	0.00	0.00	0.00	0.00	7.31
time (sec)	N/A	1.091	6.321	4.308	0.000	0.000	0.000	0.000	10.670

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	193	682	0	7704	0	0	2500
N.S.	1	1.00	1.28	4.52	0.00	51.02	0.00	0.00	16.56
time (sec)	N/A	0.174	0.888	0.144	0.000	6.447	0.000	0.000	27.935

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	183	390	0	4304	0	0	2529
N.S.	1	1.00	0.45	0.96	0.00	10.55	0.00	0.00	6.20
time (sec)	N/A	0.359	0.314	0.135	0.000	3.043	0.000	0.000	17.764

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	157	802	0	3055	0	0	581
N.S.	1	1.00	0.37	1.90	0.00	7.24	0.00	0.00	1.38
time (sec)	N/A	0.343	0.164	0.135	0.000	2.364	0.000	0.000	9.021

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	170	1503	0	8574	0	0	3054
N.S.	1	1.00	0.80	7.06	0.00	40.25	0.00	0.00	14.34
time (sec)	N/A	0.349	2.867	0.153	0.000	8.309	0.000	0.000	13.973

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	139	1463	0	8468	0	0	2981
N.S.	1	1.00	0.84	8.81	0.00	51.01	0.00	0.00	17.96
time (sec)	N/A	0.240	1.110	0.137	0.000	8.123	0.000	0.000	10.546

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	118	1417	0	8365	0	0	2930
N.S.	1	1.00	0.95	11.43	0.00	67.46	0.00	0.00	23.63
time (sec)	N/A	0.143	0.365	0.116	0.000	6.537	0.000	0.000	9.351

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	1407	0	8282	0	0	2909
N.S.	1	1.00	0.99	13.79	0.00	81.20	0.00	0.00	28.52
time (sec)	N/A	0.099	0.073	0.359	0.000	7.613	0.000	0.000	8.773

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	11296	33052	0	16835	0	0	2500
N.S.	1	1.00	86.23	252.31	0.00	128.51	0.00	0.00	19.08
time (sec)	N/A	0.213	29.411	1.178	0.000	16.029	0.000	0.000	10.742

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	201	69579	0	17527	0	0	2500
N.S.	1	1.00	1.19	411.71	0.00	103.71	0.00	0.00	14.79
time (sec)	N/A	0.347	1.874	1.618	0.000	14.880	0.000	0.000	8.678

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	241	111103	0	17769	0	0	2500
N.S.	1	1.00	1.08	496.00	0.00	79.33	0.00	0.00	11.16
time (sec)	N/A	0.527	5.922	2.288	0.000	27.352	0.000	0.000	9.401

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	300	2324	0	21128	0	0	2500
N.S.	1	1.00	1.14	8.80	0.00	80.03	0.00	0.00	9.47
time (sec)	N/A	0.464	2.224	0.183	0.000	44.656	0.000	0.000	20.575

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	248	2298	0	20869	0	0	2500
N.S.	1	1.00	1.49	13.76	0.00	124.96	0.00	0.00	14.97
time (sec)	N/A	0.282	0.837	0.158	0.000	46.863	0.000	0.000	13.281

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	229	2275	0	20761	0	0	2500
N.S.	1	1.00	1.62	16.13	0.00	147.24	0.00	0.00	17.73
time (sec)	N/A	0.190	0.900	0.126	0.000	65.117	0.000	0.000	12.185

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	113	2274	0	20729	0	0	2500
N.S.	1	1.00	0.82	16.48	0.00	150.21	0.00	0.00	18.12
time (sec)	N/A	0.164	0.133	0.123	0.000	52.641	0.000	0.000	12.071



Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	186	63939	0	42091	0	0	2500
N.S.	1	1.00	1.09	373.91	0.00	246.15	0.00	0.00	14.62
time (sec)	N/A	0.411	0.789	1.671	0.000	103.705	0.000	0.000	12.913

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	208	119757	0	45424	0	0	2500
N.S.	1	1.00	0.95	546.84	0.00	207.42	0.00	0.00	11.42
time (sec)	N/A	0.591	2.513	2.974	0.000	119.326	0.000	0.000	10.327

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	409	174418	0	46017	0	0	2500
N.S.	1	1.00	1.44	611.99	0.00	161.46	0.00	0.00	8.77
time (sec)	N/A	0.806	6.145	3.490	0.000	173.176	0.000	0.000	10.279

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	450	3303	0	36379	0	0	2500
N.S.	1	1.00	1.21	8.90	0.00	98.06	0.00	0.00	6.74
time (sec)	N/A	0.708	6.224	0.204	0.000	169.428	0.000	0.000	40.794

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	309	3275	0	35969	0	0	2500
N.S.	1	1.00	1.18	12.55	0.00	137.81	0.00	0.00	9.58
time (sec)	N/A	0.484	2.312	0.197	0.000	192.049	0.000	0.000	31.392

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	260	3249	0	35709	0	0	2500
N.S.	1	1.00	1.31	16.41	0.00	180.35	0.00	0.00	12.63
time (sec)	N/A	0.346	0.692	0.143	0.000	199.075	0.000	0.000	23.117

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	325	3246	0	35726	0	0	2500
N.S.	1	1.00	1.73	17.27	0.00	190.03	0.00	0.00	13.30
time (sec)	N/A	0.271	2.309	0.131	0.000	140.897	0.000	0.000	22.344

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	115	3236	0	35653	0	0	2500
N.S.	1	1.00	0.62	17.49	0.00	192.72	0.00	0.00	13.51
time (sec)	N/A	0.250	0.137	0.128	0.000	151.700	0.000	0.000	22.950

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	242	185603	0	0	0	0	2500
N.S.	1	1.00	1.08	828.58	0.00	0.00	0.00	0.00	11.16
time (sec)	N/A	0.635	3.269	5.095	0.000	0.000	0.000	0.000	15.744

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	306	339366	0	0	0	0	2500
N.S.	1	1.00	1.06	1174.28	0.00	0.00	0.00	0.00	8.65
time (sec)	N/A	0.839	3.267	8.956	0.000	0.000	0.000	0.000	12.392

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	373	467697	0	0	0	0	2500
N.S.	1	1.00	1.02	1284.88	0.00	0.00	0.00	0.00	6.87
time (sec)	N/A	1.104	4.491	9.667	0.000	0.000	0.000	0.000	12.401

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	88	364	0	2049	0	0	3033
N.S.	1	1.00	0.24	1.01	0.00	5.66	0.00	0.00	8.38
time (sec)	N/A	0.298	0.076	0.172	0.000	2.591	0.000	0.000	8.273

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	88	778	0	2127	0	0	2500
N.S.	1	1.00	0.22	1.92	0.00	5.24	0.00	0.00	6.16
time (sec)	N/A	0.302	0.042	0.146	0.000	1.186	0.000	0.000	10.809

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	112	20195	0	5387	0	0	2142
N.S.	1	1.00	0.94	169.71	0.00	45.27	0.00	0.00	18.00
time (sec)	N/A	0.197	0.099	0.816	0.000	2.257	0.000	0.000	8.768

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	106	810	0	6756	0	0	2500
N.S.	1	1.00	0.86	6.59	0.00	54.93	0.00	0.00	20.33
time (sec)	N/A	0.137	0.104	0.139	0.000	1.675	0.000	0.000	18.415

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	166	39351	0	13991	0	0	2500
N.S.	1	1.00	1.08	255.53	0.00	90.85	0.00	0.00	16.23
time (sec)	N/A	0.345	0.782	1.162	0.000	2.892	0.000	0.000	11.764

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	783	0	3540	0	0	2731
N.S.	1	1.00	1.07	7.68	0.00	34.71	0.00	0.00	26.77
time (sec)	N/A	0.104	0.113	0.152	0.000	1.469	0.000	0.000	8.509

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	154	928	0	6318	0	0	2500
N.S.	1	1.00	1.17	7.03	0.00	47.86	0.00	0.00	18.94
time (sec)	N/A	0.158	0.251	0.146	0.000	1.657	0.000	0.000	11.922

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	156	1182	0	10036	0	0	2500
N.S.	1	1.00	0.90	6.79	0.00	57.68	0.00	0.00	14.37
time (sec)	N/A	0.241	0.214	0.158	0.000	3.156	0.000	0.000	21.257

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	729	6480	249	0	153	1410
N.S.	1	1.00	1.56	16.20	144.00	5.53	0.00	3.40	31.33
time (sec)	N/A	0.036	0.848	0.133	0.719	1.178	0.000	0.511	8.704

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	741	0	267	0	153	1410
N.S.	1	1.00	1.00	16.47	0.00	5.93	0.00	3.40	31.33
time (sec)	N/A	0.036	0.033	0.108	0.000	1.172	0.000	0.515	7.625

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	69	54	0	28	0	65	31
N.S.	1	1.00	2.30	1.80	0.00	0.93	0.00	2.17	1.03
time (sec)	N/A	0.022	0.116	0.191	0.000	1.321	0.000	0.447	0.677

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	65	52	0	47	0	57	35
N.S.	1	1.00	2.41	1.93	0.00	1.74	0.00	2.11	1.30
time (sec)	N/A	0.021	0.079	0.062	0.000	1.200	0.000	0.427	7.114

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	134	0	855	0	0	147
N.S.	1	1.00	0.88	1.58	0.00	10.06	0.00	0.00	1.73
time (sec)	N/A	0.073	0.062	0.225	0.000	1.114	0.000	0.000	7.231

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	151	275	227	13690	0	0	1522
N.S.	1	1.00	0.54	0.99	0.82	49.24	0.00	0.00	5.47
time (sec)	N/A	0.224	1.128	0.062	0.510	20.184	0.000	0.000	15.345

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	134	253	210	13541	0	0	1492
N.S.	1	1.00	0.53	1.00	0.83	53.31	0.00	0.00	5.87
time (sec)	N/A	0.182	0.677	0.047	0.523	26.639	0.000	0.000	11.620

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	114	229	192	13378	0	0	1456
N.S.	1	1.00	0.50	1.00	0.84	58.42	0.00	0.00	6.36
time (sec)	N/A	0.152	0.277	0.046	0.540	24.188	0.000	0.000	9.523

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	94	203	174	13180	0	0	1420
N.S.	1	1.00	0.46	0.99	0.85	64.29	0.00	0.00	6.93
time (sec)	N/A	0.128	0.107	0.046	0.535	24.920	0.000	0.000	8.889

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	158	203	174	13523	0	0	1420
N.S.	1	1.00	0.77	0.99	0.85	65.97	0.00	0.00	6.93
time (sec)	N/A	0.132	0.406	0.049	0.529	31.721	0.000	0.000	8.808

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	178	222	192	13671	0	0	1448
N.S.	1	1.00	0.78	0.97	0.84	59.70	0.00	0.00	6.32
time (sec)	N/A	0.154	0.487	0.052	0.502	26.064	0.000	0.000	10.267

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	198	240	211	14358	0	0	1473
N.S.	1	1.00	0.78	0.94	0.83	56.53	0.00	0.00	5.80
time (sec)	N/A	0.179	0.766	0.050	0.506	28.891	0.000	0.000	12.495

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	205	374	329	26023	0	0	2500
N.S.	1	1.00	0.52	0.95	0.84	66.05	0.00	0.00	6.35
time (sec)	N/A	0.430	3.901	0.048	0.513	197.877	0.000	0.000	25.931

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	178	333	302	25761	0	0	2500
N.S.	1	1.00	0.49	0.92	0.84	71.56	0.00	0.00	6.94
time (sec)	N/A	0.381	1.417	0.049	0.515	152.955	0.000	0.000	18.478

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	151	292	275	25469	0	0	2500
N.S.	1	1.00	0.46	0.90	0.84	78.13	0.00	0.00	7.67
time (sec)	N/A	0.343	0.810	0.047	0.514	196.120	0.000	0.000	12.964

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	119	251	248	25195	0	0	2500
N.S.	1	1.00	0.40	0.85	0.84	85.70	0.00	0.00	8.50
time (sec)	N/A	0.303	0.353	0.048	0.523	163.166	0.000	0.000	9.785

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	211	238	240	25667	0	0	2500
N.S.	1	1.00	0.76	0.86	0.87	93.00	0.00	0.00	9.06
time (sec)	N/A	0.219	0.600	0.054	0.517	193.330	0.000	0.000	8.889

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	119	243	248	25680	0	0	2500
N.S.	1	1.00	0.42	0.86	0.88	90.74	0.00	0.00	8.83
time (sec)	N/A	0.246	0.468	0.053	0.530	222.956	0.000	0.000	9.743

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	120	270	276	26849	0	0	2500
N.S.	1	1.00	0.38	0.85	0.87	84.70	0.00	0.00	7.89
time (sec)	N/A	0.262	0.406	0.053	0.523	151.030	0.000	0.000	12.726

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	221	432	398	0	0	0	2500
N.S.	1	1.00	0.48	0.93	0.86	0.00	0.00	0.00	5.40
time (sec)	N/A	0.559	2.292	0.049	0.507	0.000	0.000	0.000	32.652

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	197	372	363	0	0	0	2500
N.S.	1	1.00	0.47	0.88	0.86	0.00	0.00	0.00	5.94
time (sec)	N/A	0.484	1.306	0.049	0.531	0.000	0.000	0.000	20.833



Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	153	314	327	0	0	0	2500
N.S.	1	1.00	0.40	0.83	0.86	0.00	0.00	0.00	6.58
time (sec)	N/A	0.440	0.952	0.050	0.509	0.000	0.000	0.000	11.982

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	264	286	310	0	0	0	2500
N.S.	1	1.00	0.71	0.76	0.83	0.00	0.00	0.00	6.68
time (sec)	N/A	0.435	1.710	0.053	0.503	0.000	0.000	0.000	9.930

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	165	278	310	0	0	0	2500
N.S.	1	1.00	0.44	0.75	0.83	0.00	0.00	0.00	6.72
time (sec)	N/A	0.396	0.893	0.055	0.502	0.000	0.000	0.000	10.145

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	166	290	327	0	0	0	2500
N.S.	1	1.00	0.44	0.76	0.86	0.00	0.00	0.00	6.58
time (sec)	N/A	0.397	0.882	0.055	0.512	0.000	0.000	0.000	12.196

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	187	292	258	25248	0	0	2500
N.S.	1	1.00	0.58	0.90	0.79	77.69	0.00	0.00	7.69
time (sec)	N/A	0.639	0.753	0.105	0.500	62.244	0.000	0.000	12.990

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	165	265	234	24896	0	0	2500
N.S.	1	1.00	0.56	0.89	0.79	83.82	0.00	0.00	8.42
time (sec)	N/A	0.413	0.232	0.100	0.553	30.102	0.000	0.000	11.821

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	195	244	216	24744	0	0	2500
N.S.	1	1.00	0.70	0.88	0.78	89.01	0.00	0.00	8.99
time (sec)	N/A	0.235	0.282	0.104	0.529	22.656	0.000	0.000	11.116

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	194	244	217	24848	0	0	2500
N.S.	1	1.00	0.70	0.88	0.78	89.38	0.00	0.00	8.99
time (sec)	N/A	0.239	0.264	0.093	0.520	26.611	0.000	0.000	11.844

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	153	265	235	25852	0	0	2500
N.S.	1	1.00	0.52	0.89	0.79	87.04	0.00	0.00	8.42
time (sec)	N/A	0.416	0.389	0.095	0.526	72.601	0.000	0.000	11.600

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	174	286	256	26114	0	0	2500
N.S.	1	1.00	0.54	0.88	0.79	80.35	0.00	0.00	7.69
time (sec)	N/A	0.643	2.241	0.096	0.528	63.762	0.000	0.000	12.967

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	275	351	377	50230	0	0	2500
N.S.	1	1.00	0.63	0.81	0.86	115.21	0.00	0.00	5.73
time (sec)	N/A	0.767	1.552	0.113	0.517	123.050	0.000	0.000	36.193

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	230	336	354	49950	0	0	2500
N.S.	1	1.00	0.59	0.86	0.91	127.75	0.00	0.00	6.39
time (sec)	N/A	0.518	1.212	0.109	0.513	111.305	0.000	0.000	33.224

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	220	332	341	50058	0	0	2500
N.S.	1	1.00	0.56	0.85	0.87	128.03	0.00	0.00	6.39
time (sec)	N/A	0.515	0.916	0.113	0.519	123.472	0.000	0.000	32.551

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	204	336	356	50087	0	0	2500
N.S.	1	1.00	0.52	0.86	0.91	128.10	0.00	0.00	6.39
time (sec)	N/A	0.525	0.735	0.108	0.532	117.143	0.000	0.000	33.534

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	239	352	396	53933	0	0	2500
N.S.	1	1.00	0.54	0.80	0.90	122.85	0.00	0.00	5.69
time (sec)	N/A	0.743	1.533	0.106	0.521	134.736	0.000	0.000	25.690

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	287	373	449	0	0	0	2500
N.S.	1	1.00	0.58	0.76	0.91	0.00	0.00	0.00	5.07
time (sec)	N/A	0.982	2.579	0.107	0.534	0.000	0.000	0.000	22.901

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	1194	466	571	0	0	0	2500
N.S.	1	1.00	1.99	0.78	0.95	0.00	0.00	0.00	4.17
time (sec)	N/A	1.103	5.060	0.124	0.533	0.000	0.000	0.000	60.842

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	372	451	551	0	0	0	2500
N.S.	1	1.00	0.70	0.84	1.03	0.00	0.00	0.00	4.68
time (sec)	N/A	0.792	4.467	0.119	0.545	0.000	0.000	0.000	52.613

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	333	450	538	0	0	0	2500
N.S.	1	1.00	0.62	0.84	1.01	0.00	0.00	0.00	4.69
time (sec)	N/A	0.796	3.755	0.122	0.529	0.000	0.000	0.000	53.109

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	344	451	537	0	0	0	2500
N.S.	1	1.00	0.65	0.85	1.01	0.00	0.00	0.00	4.71
time (sec)	N/A	0.837	4.007	0.123	0.533	0.000	0.000	0.000	52.404

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	288	452	551	0	0	0	2500
N.S.	1	1.00	0.54	0.85	1.03	0.00	0.00	0.00	4.68
time (sec)	N/A	0.826	2.943	0.121	0.518	0.000	0.000	0.000	53.703

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	341	466	607	0	0	0	2500
N.S.	1	1.00	0.57	0.78	1.01	0.00	0.00	0.00	4.16
time (sec)	N/A	1.083	5.722	0.123	0.531	0.000	0.000	0.000	48.476

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	38	102	123	592	0	0	2500
N.S.	1	1.00	0.24	0.65	0.79	3.79	0.00	0.00	16.03
time (sec)	N/A	0.069	0.037	0.048	0.511	1.729	0.000	0.000	11.607

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	138	102	123	529	0	0	2500
N.S.	1	1.00	0.90	0.66	0.80	3.44	0.00	0.00	16.23
time (sec)	N/A	0.066	0.096	0.046	0.508	1.490	0.000	0.000	11.290

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	36	90	109	525	0	0	2500
N.S.	1	1.00	0.26	0.65	0.79	3.80	0.00	0.00	18.12
time (sec)	N/A	0.061	0.015	0.079	0.510	1.223	0.000	0.000	11.083

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	110	90	112	501	0	0	2500
N.S.	1	1.00	0.80	0.65	0.81	3.63	0.00	0.00	18.12
time (sec)	N/A	0.065	0.023	0.069	0.531	2.071	0.000	0.000	11.121

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	34	102	122	642	0	0	2500
N.S.	1	1.00	0.22	0.66	0.79	4.17	0.00	0.00	16.23
time (sec)	N/A	0.066	0.017	0.049	0.516	1.560	0.000	0.000	10.887

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	36	102	124	615	0	0	2500
N.S.	1	1.00	0.23	0.65	0.79	3.94	0.00	0.00	16.03
time (sec)	N/A	0.065	0.021	0.050	0.499	1.324	0.000	0.000	11.587

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	156	242	188	8974	0	0	2500
N.S.	1	1.00	0.61	0.95	0.73	35.05	0.00	0.00	9.77
time (sec)	N/A	0.304	0.136	0.088	0.501	11.976	0.000	0.000	35.550

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	228	226	173	8860	0	0	2500
N.S.	1	1.00	0.96	0.95	0.73	37.38	0.00	0.00	10.55
time (sec)	N/A	0.191	0.174	0.086	0.496	13.260	0.000	0.000	32.871

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	225	172	8971	0	0	2500
N.S.	1	1.00	0.86	0.95	0.73	37.85	0.00	0.00	10.55
time (sec)	N/A	0.181	0.109	0.089	0.511	12.522	0.000	0.000	31.520

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	226	226	172	8968	0	0	2500
N.S.	1	1.00	0.95	0.95	0.73	37.84	0.00	0.00	10.55
time (sec)	N/A	0.186	0.127	0.087	0.520	12.126	0.000	0.000	31.620

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	132	242	188	9596	0	0	2500
N.S.	1	1.00	0.52	0.95	0.73	37.48	0.00	0.00	9.77
time (sec)	N/A	0.296	0.349	0.086	0.500	12.921	0.000	0.000	24.713

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	304	2182154	0	0	0	0	-1
N.S.	1	1.00	1.15	8265.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.359	2.271	1.584	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	239	2178538	0	0	0	0	2500
N.S.	1	1.00	1.19	10838.50	0.00	0.00	0.00	0.00	12.44
time (sec)	N/A	1.018	1.845	296.351	0.000	0.000	0.000	0.000	118.045

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	204	2175963	0	0	0	0	1141
N.S.	1	1.00	1.21	12875.52	0.00	0.00	0.00	0.00	6.75
time (sec)	N/A	0.417	0.562	0.783	0.000	0.000	0.000	0.000	17.994

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	169	2178365	0	0	0	0	-1
N.S.	1	1.00	1.10	14145.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.552	0.922	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	194	2181119	0	0	0	0	-1
N.S.	1	1.00	0.97	10960.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.500	0.980	0.476	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	226	2183172	0	0	0	0	-1
N.S.	1	1.00	0.90	8732.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.687	1.650	1.016	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	265	2184224	0	0	0	0	-1
N.S.	1	1.00	0.84	6956.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	2.572	0.513	0.000	0.000	0.000	0.000	0.000



Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	347	2401996	0	0	0	0	-1
N.S.	1	1.00	1.07	7436.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.821	2.802	0.531	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	290	2400957	0	0	0	0	-1
N.S.	1	1.00	1.08	8958.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.685	1.634	0.508	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	243	2398415	0	0	0	0	-1
N.S.	1	1.00	1.19	11756.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.236	0.658	0.540	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	121803	2397265	0	0	0	0	-1
N.S.	1	1.00	582.79	11470.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.208	38.236	0.535	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	238	2398858	0	0	0	0	-1
N.S.	1	1.00	1.21	12239.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.581	0.664	0.537	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	286	2400946	0	0	0	0	-1
N.S.	1	1.00	1.10	9270.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	1.999	0.535	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	346	2403086	0	0	0	0	-1
N.S.	1	1.00	1.11	7726.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.995	3.528	0.550	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	411	2405433	0	0	0	0	-1
N.S.	1	1.00	1.08	6296.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.225	6.397	0.521	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	411	2659561	0	0	0	0	-1
N.S.	1	1.00	1.04	6699.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.210	3.001	0.601	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	345	2657119	0	0	0	0	-1
N.S.	1	1.00	1.09	8408.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.172	2.923	0.580	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	291	2654895	0	0	0	0	-1
N.S.	1	1.00	1.12	10211.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.648	1.771	0.572	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	209298	2653774	0	0	0	0	-1
N.S.	1	1.00	868.46	11011.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.667	39.622	0.586	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	139636	2654078	0	0	0	0	-1
N.S.	1	1.00	581.82	11058.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.453	38.642	0.599	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	321	2654930	0	0	0	0	-1
N.S.	1	1.00	1.30	10748.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	1.366	0.622	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	381	2657093	0	0	0	0	-1
N.S.	1	1.00	1.23	8599.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.033	4.126	0.663	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	458	2659448	0	0	0	0	-1
N.S.	1	1.00	1.21	7035.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.268	6.507	0.620	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	632	2660696	0	0	0	0	-1
N.S.	1	1.00	1.37	5784.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.547	6.708	0.668	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	356	1491744	0	0	0	0	-1
N.S.	1	1.00	1.41	5896.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.703	2.972	0.448	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	245	1890398	0	0	0	0	-1
N.S.	1	1.00	1.19	9176.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.965	1.243	1.009	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	205	1886894	0	0	0	0	2500
N.S.	1	1.00	1.22	11231.51	0.00	0.00	0.00	0.00	14.88
time (sec)	N/A	0.377	0.851	0.591	0.000	0.000	0.000	0.000	91.162

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	137	1879756	0	0	0	0	2500
N.S.	1	1.00	1.11	15282.57	0.00	0.00	0.00	0.00	20.33
time (sec)	N/A	0.237	0.151	0.620	0.000	0.000	0.000	0.000	54.396

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	172	1887172	0	0	0	0	-1
N.S.	1	1.00	1.08	11869.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.322	0.627	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	195	1890767	0	0	0	0	-1
N.S.	1	1.00	0.96	9314.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	1.243	0.943	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	227	1891860	0	0	0	0	-1
N.S.	1	1.00	0.89	7390.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.667	3.574	0.606	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	177751	1561438	0	0	0	0	-1
N.S.	1	1.00	811.65	7129.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.294	39.014	4.061	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	239	1561041	0	0	0	0	-1
N.S.	1	1.00	1.41	9182.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.406	1.039	1.113	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	202	1561075	0	0	0	0	-1
N.S.	1	1.00	1.15	8920.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	0.525	1.055	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	248	1561981	0	0	0	0	-1
N.S.	1	1.00	1.15	7231.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	1.389	1.015	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	299	1564046	0	0	0	0	-1
N.S.	1	1.00	1.08	5666.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	1.930	1.118	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	265550	2979638	0	0	0	0	-1
N.S.	1	1.00	941.67	10566.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.760	40.109	1.399	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	308	2976654	0	0	0	0	-1
N.S.	1	1.00	1.26	12199.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.647	2.044	1.476	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	320	2978176	0	0	0	0	-1
N.S.	1	1.00	1.31	12205.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.647	2.244	1.372	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	273	2978185	0	0	0	0	-1
N.S.	1	1.00	1.11	12057.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.613	1.659	4.347	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	326	2979708	0	0	0	0	-1
N.S.	1	1.00	1.08	9899.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.790	2.429	4.229	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	383	2982515	0	0	0	0	-1
N.S.	1	1.00	1.07	8307.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.058	2.289	1.434	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	190	944366	0	0	0	0	-1
N.S.	1	1.00	1.23	6092.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.603	0.389	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	124	940499	0	0	0	0	-1
N.S.	1	1.00	1.06	8038.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.066	0.428	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	125	940264	0	0	0	0	-1
N.S.	1	1.00	1.13	8470.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.070	0.366	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	158	944401	0	0	0	0	-1
N.S.	1	1.00	1.05	6296.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.282	0.422	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	263	99	0	0	0	0	2500
N.S.	1	1.00	0.69	0.26	0.00	0.00	0.00	0.00	6.60
time (sec)	N/A	0.302	0.637	2.108	0.000	0.000	0.000	0.000	21.867



Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	347	97	0	34380	0	0	2537
N.S.	1	1.00	0.92	0.26	0.00	91.19	0.00	0.00	6.73
time (sec)	N/A	0.267	0.593	0.152	0.000	152.028	0.000	0.000	17.683

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	227	72	0	0	0	147	2500
N.S.	1	1.00	0.64	0.20	0.00	0.00	0.00	0.41	7.00
time (sec)	N/A	0.192	0.297	0.260	0.000	0.000	0.000	3.167	18.252

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	305	69	0	0	0	0	2500
N.S.	1	1.00	0.85	0.19	0.00	0.00	0.00	0.00	7.00
time (sec)	N/A	0.197	0.160	0.300	0.000	0.000	0.000	0.000	21.697

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	42	0	284	0	910	2500
N.S.	1	1.00	0.74	0.28	0.00	1.92	0.00	6.15	16.89
time (sec)	N/A	0.090	0.978	0.340	0.000	1.934	0.000	0.616	20.672

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	330	72	0	2784	0	0	2500
N.S.	1	1.00	1.10	0.24	0.00	9.31	0.00	0.00	8.36
time (sec)	N/A	0.242	0.312	0.297	0.000	1.882	0.000	0.000	22.174

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	355	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	3.677	0.479	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	232	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	1.754	0.326	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	155	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.510	0.271	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	108	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.317	0.218	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	144	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.638	0.430	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	239	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	2.014	0.512	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	370	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.880	5.529	0.642	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	1901	0	0	0	0	0	-1
N.S.	1	1.00	2.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.646	6.191	0.686	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	30.831	0.369	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	9.760	0.314	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	3.500	0.313	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.259	5.688	0.328	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	13.936	0.327	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	60.479	0.355	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	1.687	0.313	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	385	384	0	0	0	0	0	-1
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.730	3.929	0.283	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	289	281	0	0	0	0	0	-1
N.S.	1	0.99	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	1.606	0.254	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	169	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.946	0.258	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	125	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.194	0.231	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	120	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.133	0.219	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	169	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.273	0.239	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	202	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.289	0.275	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	230	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.558	0.365	0.275	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	263	2945	190	434	0	0	-1
N.S.	1	1.00	2.55	28.59	1.84	4.21	0.00	0.00	-0.01
time (sec)	N/A	0.162	1.573	24.110	0.515	1.341	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	161	1538	174	382	0	0	-1
N.S.	1	1.00	2.06	19.72	2.23	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.912	22.710	0.509	1.234	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	92	1425	155	316	0	0	-1
N.S.	1	1.00	1.74	26.89	2.92	5.96	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.983	23.961	0.520	1.937	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	108	784	155	364	0	0	-1
N.S.	1	1.00	1.96	14.25	2.82	6.62	0.00	0.00	-0.02
time (sec)	N/A	0.100	1.937	63.471	0.513	1.199	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	96	889	177	429	0	0	-1
N.S.	1	1.00	1.20	11.11	2.21	5.36	0.00	0.00	-0.01
time (sec)	N/A	0.125	1.347	46.207	0.522	2.347	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	133	971	191	482	0	0	-1
N.S.	1	1.00	1.27	9.25	1.82	4.59	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.710	49.458	0.520	1.490	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	272	2947	198	448	0	0	-1
N.S.	1	1.00	2.12	23.02	1.55	3.50	0.00	0.00	-0.01
time (sec)	N/A	0.239	2.430	25.401	0.519	2.306	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	174	1541	180	394	0	0	-1
N.S.	1	1.00	1.69	14.96	1.75	3.83	0.00	0.00	-0.01
time (sec)	N/A	0.214	2.122	25.379	0.519	1.231	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	163	1416	174	385	0	0	-1
N.S.	1	1.00	1.65	14.30	1.76	3.89	0.00	0.00	-0.01
time (sec)	N/A	0.207	2.134	67.122	0.530	2.294	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	254	888	180	439	0	0	-1
N.S.	1	1.00	2.42	8.46	1.71	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.215	1.605	70.696	0.510	1.890	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	133	971	199	505	0	0	-1
N.S.	1	1.00	1.02	7.47	1.53	3.88	0.00	0.00	-0.01
time (sec)	N/A	0.245	2.627	47.801	0.534	1.351	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	161	3132	218	508	0	0	-1
N.S.	1	1.00	0.94	18.32	1.27	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.349	3.987	24.181	0.535	2.365	0.000	0.000	0.000



Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	132	2947	200	449	0	0	-1
N.S.	1	1.00	0.90	20.18	1.37	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.315	3.218	21.358	0.561	1.536	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	146	1562	194	409	0	0	-1
N.S.	1	1.00	1.06	11.32	1.41	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.301	2.692	68.579	0.549	1.405	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	1539	197	442	0	0	-1
N.S.	1	1.00	0.93	10.84	1.39	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.310	2.779	85.028	0.511	1.666	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	140	961	203	502	0	0	-1
N.S.	1	1.00	0.95	6.49	1.37	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.327	3.067	69.447	0.550	1.757	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	298	1043	221	563	0	0	-1
N.S.	1	1.00	1.72	6.03	1.28	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.361	5.250	62.241	0.518	1.850	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	247	2598	0	716	0	0	-1
N.S.	1	1.00	0.83	8.75	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.347	1.494	25.796	0.000	1.671	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	223	2431	0	623	0	0	-1
N.S.	1	1.00	0.83	9.07	0.00	2.32	0.00	0.00	-0.00
time (sec)	N/A	0.299	1.106	24.352	0.000	1.143	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	199	1139	0	570	0	0	-1
N.S.	1	1.00	0.85	4.85	0.00	2.43	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.776	21.006	0.000	2.367	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	198	6977	0	571	0	0	-1
N.S.	1	1.00	0.84	29.44	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.911	23.524	0.000	1.228	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	214	8548	0	700	0	0	-1
N.S.	1	1.00	0.78	30.97	0.00	2.54	0.00	0.00	-0.00
time (sec)	N/A	0.300	1.103	72.585	0.000	1.803	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	242	8761	0	794	0	0	-1
N.S.	1	1.00	0.79	28.54	0.00	2.59	0.00	0.00	-0.00
time (sec)	N/A	0.333	1.369	83.833	0.000	1.533	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	256	2507	0	669	0	0	-1
N.S.	1	1.00	0.81	7.91	0.00	2.11	0.00	0.00	-0.00
time (sec)	N/A	0.452	1.344	29.430	0.000	1.657	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	243	1517	0	666	0	0	-1
N.S.	1	1.00	0.86	5.34	0.00	2.35	0.00	0.00	-0.00
time (sec)	N/A	0.395	1.154	25.353	0.000	1.040	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	243	13774	0	663	0	0	-1
N.S.	1	1.00	0.89	50.27	0.00	2.42	0.00	0.00	-0.00
time (sec)	N/A	0.372	1.181	23.589	0.000	1.949	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	241	13781	0	666	0	0	-1
N.S.	1	1.00	0.85	48.52	0.00	2.35	0.00	0.00	-0.00
time (sec)	N/A	0.392	1.384	21.326	0.000	1.240	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	249	13802	0	761	0	0	-1
N.S.	1	1.00	0.78	43.27	0.00	2.39	0.00	0.00	-0.00
time (sec)	N/A	0.448	1.560	73.334	0.000	2.152	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	284	2577	0	688	0	0	-1
N.S.	1	1.00	0.77	7.02	0.00	1.87	0.00	0.00	-0.00
time (sec)	N/A	0.605	2.031	28.934	0.000	1.149	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	258	1581	0	683	0	0	-1
N.S.	1	1.00	0.81	4.97	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.506	1.414	25.103	0.000	1.656	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	272	15195	0	635	0	0	-1
N.S.	1	1.00	0.86	48.09	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.493	2.196	24.881	0.000	1.457	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	274	13448	0	637	0	0	-1
N.S.	1	1.00	0.89	43.66	0.00	2.07	0.00	0.00	-0.00
time (sec)	N/A	0.475	2.071	22.403	0.000	1.686	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	415	17109	0	685	0	0	-1
N.S.	1	1.00	1.34	55.19	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.496	2.524	25.589	0.000	1.573	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	280	18612	0	783	0	0	-1
N.S.	1	1.00	0.76	50.71	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.603	2.395	77.658	0.000	1.596	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	188	2243	1409	481	0	0	-1
N.S.	1	1.00	0.95	11.33	7.12	2.43	0.00	0.00	-0.01
time (sec)	N/A	0.423	1.612	60.807	1.048	1.462	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	162	1984	1157	430	0	0	-1
N.S.	1	1.00	1.05	12.80	7.46	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.305	1.171	60.724	0.743	1.838	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	112	1032	558	370	0	0	-1
N.S.	1	1.00	1.02	9.38	5.07	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.194	1.166	61.796	0.621	1.573	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	241	879	0	576	0	0	-1
N.S.	1	1.00	1.59	5.78	0.00	3.79	0.00	0.00	-0.01
time (sec)	N/A	0.282	1.929	63.570	0.000	1.240	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	0	3841	0	793	0	0	-1
N.S.	1	1.00	0.00	20.01	0.00	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.387	4.722	65.857	0.000	1.541	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	320	3124	3787	568	0	0	-1
N.S.	1	1.00	1.31	12.75	15.46	2.32	0.00	0.00	-0.00
time (sec)	N/A	0.579	4.164	69.306	3.028	1.407	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	289	2244	1457	510	0	0	-1
N.S.	1	1.00	1.44	11.16	7.25	2.54	0.00	0.00	-0.00
time (sec)	N/A	0.459	2.969	57.478	0.908	1.227	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	259	1985	1113	460	0	0	-1
N.S.	1	1.00	1.65	12.64	7.09	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.323	2.706	66.978	0.682	1.625	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	286	1366	0	674	0	0	-1
N.S.	1	1.00	1.54	7.34	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.395	2.530	68.708	0.000	1.495	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	360	1290	0	827	0	0	-1
N.S.	1	1.00	1.84	6.58	0.00	4.22	0.00	0.00	-0.01
time (sec)	N/A	0.406	4.575	60.917	0.000	1.370	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	441	4490	0	913	0	0	-1
N.S.	1	1.00	1.81	18.40	0.00	3.74	0.00	0.00	-0.00
time (sec)	N/A	0.557	4.017	63.257	0.000	1.525	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	354	3412	4543	629	0	0	-1
N.S.	1	1.00	1.19	11.49	15.30	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.705	6.171	61.642	5.349	1.540	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	332	3126	4087	577	0	0	-1
N.S.	1	1.00	1.32	12.45	16.28	2.30	0.00	0.00	-0.00
time (sec)	N/A	0.604	5.375	68.288	2.132	1.895	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	306	2214	1545	515	0	0	-1
N.S.	1	1.00	1.49	10.80	7.54	2.51	0.00	0.00	-0.00
time (sec)	N/A	0.471	4.282	62.414	0.959	1.215	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	396	2597	0	780	0	0	-1
N.S.	1	1.00	1.72	11.29	0.00	3.39	0.00	0.00	-0.00
time (sec)	N/A	0.515	7.568	71.145	0.000	1.473	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	387	1896	0	849	0	0	-1
N.S.	1	1.00	1.64	8.03	0.00	3.60	0.00	0.00	-0.00
time (sec)	N/A	0.544	5.718	65.303	0.000	1.347	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	447	1530	0	923	0	0	-1
N.S.	1	1.00	1.82	6.22	0.00	3.75	0.00	0.00	-0.00
time (sec)	N/A	0.552	5.097	63.796	0.000	0.837	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	484	4851	0	1013	0	0	-1
N.S.	1	1.00	1.66	16.61	0.00	3.47	0.00	0.00	-0.00
time (sec)	N/A	0.703	5.972	69.047	0.000	0.664	0.000	0.000	0.000



Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	166	683	0	487	0	0	-1
N.S.	1	1.00	0.79	3.24	0.00	2.31	0.00	0.00	-0.00
time (sec)	N/A	0.443	2.472	65.635	0.000	0.883	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	165	482	0	426	0	0	-1
N.S.	1	1.00	1.01	2.96	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.316	1.703	65.509	0.000	0.719	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	156	431	0	423	0	0	-1
N.S.	1	1.00	1.31	3.62	0.00	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.203	1.381	60.098	0.000	1.179	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	227	805	0	736	0	0	-1
N.S.	1	1.00	1.16	4.11	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.400	2.458	68.159	0.000	1.363	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	195	648	0	465	0	0	-1
N.S.	1	1.00	0.91	3.03	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.479	2.642	66.185	0.000	0.940	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	192	853	0	463	0	0	-1
N.S.	1	1.00	1.14	5.08	0.00	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.345	2.179	63.083	0.000	1.510	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	190	867	0	461	0	0	-1
N.S.	1	1.00	1.12	5.10	0.00	2.71	0.00	0.00	-0.01
time (sec)	N/A	0.344	2.194	64.296	0.000	1.093	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	388	1516	0	782	0	0	-1
N.S.	1	1.00	1.60	6.24	0.00	3.22	0.00	0.00	-0.00
time (sec)	N/A	0.532	4.511	66.830	0.000	1.415	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	200	764	0	482	0	0	-1
N.S.	1	1.00	0.77	2.94	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.637	5.144	68.516	0.000	1.425	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	167	1078	0	481	0	0	-1
N.S.	1	1.00	0.78	5.04	0.00	2.25	0.00	0.00	-0.00
time (sec)	N/A	0.493	3.786	65.528	0.000	1.380	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	168	1092	0	482	0	0	-1
N.S.	1	1.00	0.78	5.06	0.00	2.23	0.00	0.00	-0.00
time (sec)	N/A	0.486	4.148	64.151	0.000	1.184	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	169	1212	0	482	0	0	-1
N.S.	1	1.00	0.79	5.66	0.00	2.25	0.00	0.00	-0.00
time (sec)	N/A	0.488	4.220	73.488	0.000	1.221	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	426	2158	0	799	0	0	-1
N.S.	1	1.00	1.47	7.47	0.00	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.672	5.698	69.965	0.000	1.283	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.280	12.050	3.790	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	6.669	0.615	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.385	12.892	0.605	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	11.777	0.648	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.401	7.687	0.592	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	10.477	0.521	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.875	13.520	0.546	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	198	4490	196	0	0	0	-1
N.S.	1	1.00	0.86	19.61	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.612	26.669	0.544	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	179	4230	178	0	0	0	-1
N.S.	1	1.00	0.87	20.63	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.313	27.582	0.517	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	178	2223	178	0	0	0	-1
N.S.	1	1.00	0.87	10.84	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.139	80.470	0.592	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	198	2401	198	0	0	0	-1
N.S.	1	1.00	0.86	10.48	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.333	86.372	0.524	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	255	13170	279	0	0	0	-1
N.S.	1	1.00	0.78	40.40	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	1.183	28.245	0.598	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	226	6783	252	0	0	0	-1
N.S.	1	1.00	0.77	23.07	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.742	28.577	0.508	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	221	6315	244	0	0	0	-1
N.S.	1	1.00	0.80	22.88	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.532	76.867	0.518	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	226	3528	254	0	0	0	-1
N.S.	1	1.00	0.80	12.47	0.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.340	68.860	0.519	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	255	3748	282	0	0	0	-1
N.S.	1	1.00	0.80	11.82	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	0.712	70.127	0.579	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	326	18343	366	0	0	0	-1
N.S.	1	1.00	0.77	43.57	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	2.295	44.041	0.506	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	286	17340	330	0	0	0	-1
N.S.	1	1.00	0.75	45.63	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	1.504	41.646	0.509	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	270	9099	314	0	0	0	-1
N.S.	1	1.00	0.72	24.33	0.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	1.289	66.625	0.505	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	270	8811	316	0	0	0	-1
N.S.	1	1.00	0.73	23.69	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	1.308	69.729	0.510	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	287	4875	334	0	0	0	-1
N.S.	1	1.00	0.76	12.83	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	0.793	64.797	0.528	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	327	5039	370	0	0	0	-1
N.S.	1	1.00	0.78	11.97	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	1.594	75.603	0.555	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	272	22300	262	0	0	0	-1
N.S.	1	1.00	0.84	68.62	0.81	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.742	1.021	38.599	0.529	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	249	20614	239	0	0	0	-1
N.S.	1	1.00	0.84	69.41	0.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	0.581	42.860	0.526	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	215	4035	221	0	0	0	-1
N.S.	1	1.00	0.77	14.51	0.79	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.286	38.318	0.521	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	215	3684	220	0	0	0	-1
N.S.	1	1.00	0.77	13.25	0.79	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.295	25.599	0.513	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	251	9725	238	0	0	0	-1
N.S.	1	1.00	0.85	32.74	0.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.379	76.530	0.546	0.000	0.000	0.000	0.000



Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	272	11951	262	0	0	0	-1
N.S.	1	1.00	0.84	36.77	0.81	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.719	0.653	74.209	0.546	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	383	57122	383	0	0	0	-1
N.S.	1	1.00	0.87	130.42	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.868	3.393	39.523	0.573	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	341	35577	362	0	0	0	-1
N.S.	1	1.00	0.87	90.76	0.92	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	1.635	31.125	0.573	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	336	40165	347	0	0	0	-1
N.S.	1	1.00	0.86	102.99	0.89	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	1.796	42.057	0.534	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	342	40146	360	0	0	0	-1
N.S.	1	1.00	0.87	102.41	0.92	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.606	1.598	43.846	0.537	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	390	42740	402	0	0	0	-1
N.S.	1	1.00	0.89	97.80	0.92	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	2.097	72.480	0.559	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	578	156936	577	0	0	0	-1
N.S.	1	1.00	0.96	261.12	0.96	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.250	5.872	42.259	0.548	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	520	100811	558	0	0	0	-1
N.S.	1	1.00	0.97	188.78	1.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.900	4.394	27.680	0.544	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	558	102181	544	0	0	0	-1
N.S.	1	1.00	1.04	191.35	1.02	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.916	6.238	26.711	0.549	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	518	102237	545	0	0	0	-1
N.S.	1	1.00	0.98	192.90	1.03	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.922	4.308	26.020	0.541	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	547	102127	556	0	0	0	-1
N.S.	1	1.00	1.02	191.25	1.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.927	4.519	23.702	0.546	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	599	104911	612	0	0	0	-1
N.S.	1	1.00	1.00	174.85	1.02	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.221	5.967	51.420	0.547	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	38	1275	127	0	0	0	64
N.S.	1	1.00	0.24	8.17	0.81	0.00	0.00	0.00	0.41
time (sec)	N/A	0.072	0.049	28.288	0.535	0.000	0.000	0.000	9.988

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	138	969	127	0	0	0	65
N.S.	1	1.00	0.90	6.29	0.82	0.00	0.00	0.00	0.42
time (sec)	N/A	0.072	0.115	26.598	0.518	0.000	0.000	0.000	9.401

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	36	324	113	0	0	0	42
N.S.	1	1.00	0.26	2.35	0.82	0.00	0.00	0.00	0.30
time (sec)	N/A	0.080	0.016	22.888	0.536	0.000	0.000	0.000	8.893

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	110	284	116	0	0	0	47
N.S.	1	1.00	0.80	2.06	0.84	0.00	0.00	0.00	0.34
time (sec)	N/A	0.062	0.021	24.374	0.523	0.000	0.000	0.000	8.955

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	34	658	126	0	0	0	64
N.S.	1	1.00	0.22	4.27	0.82	0.00	0.00	0.00	0.42
time (sec)	N/A	0.072	0.026	49.607	0.554	0.000	0.000	0.000	8.983

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	36	546	128	0	0	0	65
N.S.	1	1.00	0.23	3.50	0.82	0.00	0.00	0.00	0.42
time (sec)	N/A	0.071	0.033	47.769	0.538	0.000	0.000	0.000	9.490

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	291	44507	0	0	0	0	-1
N.S.	1	1.00	0.82	125.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.969	2.657	30.512	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	252	43145	0	0	0	0	-1
N.S.	1	1.00	0.87	148.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.795	1.468	31.064	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	216	21822	0	0	0	0	-1
N.S.	1	1.00	0.90	91.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	1.251	29.122	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	189	21149	0	0	0	0	-1
N.S.	1	1.00	0.97	109.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.421	0.320	27.974	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	238	8336	0	0	0	0	-1
N.S.	1	1.00	1.04	36.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	0.513	29.658	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	293	23915	0	0	0	0	-1
N.S.	1	1.00	1.12	91.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.105	2.278	31.361	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	356	28578	0	0	0	0	-1
N.S.	1	1.00	1.10	88.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.414	3.126	30.200	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	411	74462	0	0	0	0	-1
N.S.	1	1.00	0.97	176.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.325	5.997	31.513	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	346	49793	0	0	0	0	-1
N.S.	1	1.00	0.99	141.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.103	3.206	31.900	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	286	48985	0	0	0	0	-1
N.S.	1	1.00	0.96	163.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.858	1.918	29.463	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	244	24872	0	0	0	0	-1
N.S.	1	1.00	1.03	105.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	0.653	29.737	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	114092	42482	0	0	0	0	-1
N.S.	1	1.00	424.13	157.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.322	31.295	27.778	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	263	27748	0	0	0	0	-1
N.S.	1	1.00	1.00	105.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.314	0.901	32.957	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	310	31112	0	0	0	0	-1
N.S.	1	1.00	0.95	94.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.716	2.121	29.260	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	367	34370	0	0	0	0	-1
N.S.	1	1.00	0.96	89.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.847	3.707	29.394	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	653	103896	0	0	0	0	-1
N.S.	1	1.00	1.31	207.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.657	6.620	31.439	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	564	101204	0	0	0	0	-1
N.S.	1	1.00	1.35	242.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.372	6.538	29.566	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	382	67595	0	0	0	0	-1
N.S.	1	1.00	1.09	193.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.093	3.405	31.671	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	321	66799	0	0	0	0	-1
N.S.	1	1.00	1.12	232.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.847	1.825	29.212	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	130606	47378	0	0	0	0	-1
N.S.	1	1.00	435.35	157.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.612	38.938	27.875	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	196709	57756	0	0	0	0	-1
N.S.	1	1.00	653.52	191.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.757	39.563	29.629	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	311	32917	0	0	0	0	-1
N.S.	1	1.00	0.97	102.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.619	2.335	30.765	0.000	0.000	0.000	0.000	0.000



Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	365	36039	0	0	0	0	-1
N.S.	1	1.00	0.97	95.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.213	3.897	30.419	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	431	39827	0	0	0	0	-1
N.S.	1	1.00	0.94	87.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.238	3.663	31.100	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	244	29195	0	0	0	0	-1
N.S.	1	1.00	0.82	98.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.758	4.044	29.687	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	213	14834	0	0	0	0	-1
N.S.	1	1.00	0.88	61.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.563	1.788	34.737	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	193	14215	0	0	0	0	-1
N.S.	1	1.00	0.97	71.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.402	1.049	34.405	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	157	3484	0	0	0	0	-1
N.S.	1	1.00	0.96	21.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	0.248	29.866	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	225	6698	0	0	0	0	-1
N.S.	1	1.00	0.99	29.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	0.991	32.444	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	354	21473	0	0	0	0	-1
N.S.	1	1.00	1.33	80.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.038	1.678	28.127	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	301	19809	0	0	0	0	-1
N.S.	1	1.00	0.95	62.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.878	2.739	29.591	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	255	18997	0	0	0	0	-1
N.S.	1	1.00	1.00	74.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	1.503	31.144	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	222	9577	0	0	0	0	-1
N.S.	1	1.00	1.03	44.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.474	0.812	28.154	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	259	9705	0	0	0	0	-1
N.S.	1	1.00	1.23	46.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.480	1.391	30.194	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	167374	21787	0	0	0	0	-1
N.S.	1	1.00	599.91	78.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.372	39.004	28.826	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	385	82067	0	0	0	0	-1
N.S.	1	1.00	0.96	205.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.163	2.665	29.982	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	334	54581	0	0	0	0	-1
N.S.	1	1.00	0.98	160.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.874	2.694	31.545	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	293	40999	0	0	0	0	-1
N.S.	1	1.00	1.02	142.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	2.389	32.163	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	340	40915	0	0	0	0	-1
N.S.	1	1.00	1.20	144.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.742	2.925	31.224	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	328	40927	0	0	0	0	-1
N.S.	1	1.00	1.15	144.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	2.458	30.667	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	250233	77867	0	0	0	0	-1
N.S.	1	1.00	731.68	227.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.780	31.854	31.627	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	145	2085	0	0	0	0	-1
N.S.	1	1.00	0.96	13.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.200	31.075	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	144	1647	0	0	0	0	-1
N.S.	1	1.00	0.92	10.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.128	31.192	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	210	4706	0	0	0	0	-1
N.S.	1	1.00	0.98	21.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.752	30.645	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.298	4.846	0.517	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.313	5.142	0.530	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	4.526	0.563	0.000	0.000	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	8.547	0.525	0.000	0.000	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	9.449	0.505	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	1.972	0.491	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	1.016	0.488	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.936	0.508	0.000	0.000	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	1.640	0.451	0.000	0.000	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	75	80	326	96	394	0	128
N.S.	1	1.00	1.19	1.27	5.17	1.52	6.25	0.00	2.03
time (sec)	N/A	0.063	1.775	1.346	0.579	1.638	0.664	0.000	1.109

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	226	85	100	106	155	119	100
N.S.	1	1.00	3.83	1.44	1.69	1.80	2.63	2.02	1.69
time (sec)	N/A	0.055	1.184	0.100	0.497	1.713	0.360	0.928	8.505

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	63	76	91	136	105	76
N.S.	1	1.00	2.73	1.07	1.29	1.54	2.31	1.78	1.29
time (sec)	N/A	0.062	1.300	0.099	0.522	1.568	0.311	0.749	8.472

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	109	49	58	78	117	92	50
N.S.	1	1.00	1.65	0.74	0.88	1.18	1.77	1.39	0.76
time (sec)	N/A	0.059	0.897	0.086	0.505	3.613	0.218	0.646	8.566

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	31	57	82	113	25
N.S.	1	1.00	1.00	0.84	0.97	1.78	2.56	3.53	0.78
time (sec)	N/A	0.026	0.031	0.046	0.502	2.824	0.155	0.537	8.419

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	50	53	67	53	110	38
N.S.	1	1.00	1.43	1.09	1.15	1.46	1.15	2.39	0.83
time (sec)	N/A	0.021	0.024	0.076	0.555	3.558	0.218	0.491	8.469

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	123	44	0	45	88	130	51
N.S.	1	1.00	2.28	0.81	0.00	0.83	1.63	2.41	0.94
time (sec)	N/A	0.062	0.841	0.183	0.000	4.651	0.189	0.561	8.570

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	62	46	0	48	153	84	51
N.S.	1	1.00	1.35	1.00	0.00	1.04	3.33	1.83	1.11
time (sec)	N/A	0.052	0.798	0.207	0.000	3.187	0.195	0.660	8.488

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	72	43	0	62	201	149	63
N.S.	1	1.00	1.31	0.78	0.00	1.13	3.65	2.71	1.15
time (sec)	N/A	0.062	0.656	0.212	0.000	5.664	0.255	0.730	8.634



Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	97	44	0	85	304	213	73
N.S.	1	1.00	1.70	0.77	0.00	1.49	5.33	3.74	1.28
time (sec)	N/A	0.064	0.805	0.234	0.000	13.342	0.325	0.949	8.665

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	124	45	0	99	348	277	82
N.S.	1	1.00	2.25	0.82	0.00	1.80	6.33	5.04	1.49
time (sec)	N/A	0.061	1.278	0.277	0.000	3.029	0.431	1.052	8.600

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	146	167	689	211	1482	0	193
N.S.	1	1.00	1.34	1.53	6.32	1.94	13.60	0.00	1.77
time (sec)	N/A	0.109	2.665	1.488	0.649	4.972	1.502	0.000	11.375

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	254	147	158	161	243	191	158
N.S.	1	1.00	2.57	1.48	1.60	1.63	2.45	1.93	1.60
time (sec)	N/A	0.126	1.904	0.135	0.510	3.442	0.774	1.191	8.678

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	177	116	122	144	224	177	120
N.S.	1	1.00	1.79	1.17	1.23	1.45	2.26	1.79	1.21
time (sec)	N/A	0.102	1.438	0.110	0.526	3.613	0.625	1.060	8.595

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	146	98	106	131	206	164	108
N.S.	1	1.00	1.47	0.99	1.07	1.32	2.08	1.66	1.09
time (sec)	N/A	0.101	1.660	0.126	0.515	4.190	0.483	0.849	9.019

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	53	76	111	160	411	82
N.S.	1	1.00	0.85	0.85	1.23	1.79	2.58	6.63	1.32
time (sec)	N/A	0.073	0.117	0.085	0.518	2.990	0.316	0.693	8.493

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	109	51	56	103	158	127	50
N.S.	1	1.00	1.70	0.80	0.88	1.61	2.47	1.98	0.78
time (sec)	N/A	0.055	0.903	0.102	0.541	1.985	0.222	0.593	8.535

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	263	76	75	127	122	228	76
N.S.	1	1.00	3.29	0.95	0.94	1.59	1.52	2.85	0.95
time (sec)	N/A	0.049	1.155	0.108	0.550	1.729	0.320	0.576	8.542

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	418	62	0	116	134	283	105
N.S.	1	1.00	4.49	0.67	0.00	1.25	1.44	3.04	1.13
time (sec)	N/A	0.105	2.631	0.268	0.000	3.251	0.384	0.631	8.660

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	184	64	0	65	160	201	104
N.S.	1	1.00	2.02	0.70	0.00	0.71	1.76	2.21	1.14
time (sec)	N/A	0.102	1.348	0.253	0.000	2.516	0.316	0.748	8.707

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	69	0	51	167	165	87
N.S.	1	1.00	0.87	0.74	0.00	0.55	1.80	1.77	0.94
time (sec)	N/A	0.106	1.051	0.303	0.000	3.075	0.304	0.928	8.689

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	68	0	67	218	201	78
N.S.	1	1.00	1.00	0.75	0.00	0.74	2.40	2.21	0.86
time (sec)	N/A	0.103	1.128	0.254	0.000	5.130	0.372	1.020	8.710

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	116	69	0	93	332	309	108
N.S.	1	1.00	1.22	0.73	0.00	0.98	3.49	3.25	1.14
time (sec)	N/A	0.100	1.409	0.439	0.000	3.941	0.456	1.251	8.772

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	143	66	0	109	379	381	118
N.S.	1	1.00	1.57	0.73	0.00	1.20	4.16	4.19	1.30
time (sec)	N/A	0.101	2.030	0.297	0.000	2.070	0.600	1.343	8.937

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	822	4339	1099	342	3665	0	323
N.S.	1	1.00	5.44	28.74	7.28	2.26	24.27	0.00	2.14
time (sec)	N/A	0.134	8.442	1.324	0.699	2.718	3.472	0.000	13.877

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	262	211	202	206	325	254	208
N.S.	1	1.00	1.94	1.56	1.50	1.53	2.41	1.88	1.54
time (sec)	N/A	0.138	3.067	0.141	0.518	1.324	1.228	1.444	8.697

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	215	177	174	193	306	241	174
N.S.	1	1.00	1.59	1.31	1.29	1.43	2.27	1.79	1.29
time (sec)	N/A	0.134	2.030	0.149	0.570	2.326	1.056	1.298	8.646

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	172	159	158	180	287	228	156
N.S.	1	1.00	1.30	1.20	1.20	1.36	2.17	1.73	1.18
time (sec)	N/A	0.118	1.528	0.132	0.604	4.034	0.818	1.097	8.594

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	75	112	156	224	793	120
N.S.	1	1.00	0.77	0.89	1.33	1.86	2.67	9.44	1.43
time (sec)	N/A	0.083	0.189	0.087	0.519	13.189	0.581	1.104	8.464

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	146	94	110	160	250	203	108
N.S.	1	1.00	1.45	0.93	1.09	1.58	2.48	2.01	1.07
time (sec)	N/A	0.099	1.433	0.117	0.564	8.003	0.482	0.813	8.419

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	161	63	76	139	218	174	72
N.S.	1	1.00	2.64	1.03	1.25	2.28	3.57	2.85	1.18
time (sec)	N/A	0.058	1.374	0.104	0.545	5.611	0.313	0.695	8.475

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	331	100	101	184	184	333	125
N.S.	1	1.00	3.01	0.91	0.92	1.67	1.67	3.03	1.14
time (sec)	N/A	0.066	2.033	0.111	0.499	6.677	0.383	0.647	8.913

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	944	81	0	172	206	322	139
N.S.	1	1.00	7.93	0.68	0.00	1.45	1.73	2.71	1.17
time (sec)	N/A	0.119	7.241	0.213	0.000	5.613	0.498	0.729	9.003

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	285	83	0	144	236	357	182
N.S.	1	1.00	2.32	0.67	0.00	1.17	1.92	2.90	1.48
time (sec)	N/A	0.122	5.343	0.297	0.000	4.184	0.539	0.817	9.164

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	167	88	0	81	212	255	141
N.S.	1	1.00	1.29	0.68	0.00	0.63	1.64	1.98	1.09
time (sec)	N/A	0.104	1.648	0.296	0.000	5.146	0.484	1.005	8.975

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	90	0	51	167	237	118
N.S.	1	1.00	0.82	0.91	0.00	0.52	1.69	2.39	1.19
time (sec)	N/A	0.101	1.215	0.337	0.000	3.958	0.427	1.147	8.933

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	91	87	0	67	218	309	128
N.S.	1	1.00	0.75	0.71	0.00	0.55	1.79	2.53	1.05
time (sec)	N/A	0.118	1.549	0.396	0.000	1.879	0.556	1.412	9.006

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	112	90	0	93	332	345	140
N.S.	1	1.00	0.88	0.71	0.00	0.73	2.61	2.72	1.10
time (sec)	N/A	0.121	2.248	0.292	0.000	1.011	0.626	0.960	9.089

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	143	89	0	109	379	453	151
N.S.	1	1.00	1.14	0.71	0.00	0.87	3.03	3.62	1.21
time (sec)	N/A	0.122	2.531	0.662	0.000	1.471	0.872	1.166	9.296

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	182	90	0	137	496	525	160
N.S.	1	1.00	1.43	0.71	0.00	1.08	3.91	4.13	1.26
time (sec)	N/A	0.122	3.299	0.347	0.000	1.123	0.895	1.215	9.473

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	9.529	1.076	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	260	106	0	311	328	445	205
N.S.	1	1.00	1.66	0.68	0.00	1.98	2.09	2.83	1.31
time (sec)	N/A	0.146	1.600	0.217	0.000	2.835	0.568	0.833	8.800

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	212	81	0	229	265	323	136
N.S.	1	1.00	1.75	0.67	0.00	1.89	2.19	2.67	1.12
time (sec)	N/A	0.120	3.279	0.221	0.000	3.239	0.494	0.694	8.705

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	184	62	0	160	194	283	110
N.S.	1	1.00	1.92	0.65	0.00	1.67	2.02	2.95	1.15
time (sec)	N/A	0.105	1.750	0.226	0.000	4.466	0.384	0.643	8.567

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	124	44	0	71	112	130	54
N.S.	1	1.00	2.18	0.77	0.00	1.25	1.96	2.28	0.95
time (sec)	N/A	0.062	0.671	0.164	0.000	3.349	0.194	0.556	8.457

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	102	68	0	44	87	90	45
N.S.	1	1.00	2.17	1.45	0.00	0.94	1.85	1.91	0.96
time (sec)	N/A	0.031	0.264	0.157	0.000	4.516	0.123	0.538	8.586

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	82	0	61	165	53	40
N.S.	1	1.00	0.96	1.82	0.00	1.36	3.67	1.18	0.89
time (sec)	N/A	0.088	0.067	0.168	0.000	3.802	0.191	0.554	8.451

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	166	106	0	84	284	169	129
N.S.	1	1.00	1.47	0.94	0.00	0.74	2.51	1.50	1.14
time (sec)	N/A	0.133	1.245	0.243	0.000	4.968	0.282	0.689	9.062

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	203	128	0	97	328	192	161
N.S.	1	1.00	1.36	0.86	0.00	0.65	2.20	1.29	1.08
time (sec)	N/A	0.145	1.115	0.308	0.000	8.488	0.341	0.793	9.112



Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	221	148	0	121	439	221	204
N.S.	1	1.00	1.22	0.82	0.00	0.67	2.43	1.22	1.13
time (sec)	N/A	0.166	1.144	0.339	0.000	6.130	0.395	0.951	9.258

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	67.803	1.209	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1357	127	0	343	445	517	282
N.S.	1	1.00	6.99	0.65	0.00	1.77	2.29	2.66	1.45
time (sec)	N/A	0.172	7.532	0.309	0.000	2.839	0.821	1.067	8.811

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	1079	103	0	260	377	444	207
N.S.	1	1.00	6.83	0.65	0.00	1.65	2.39	2.81	1.31
time (sec)	N/A	0.144	7.115	0.273	0.000	2.869	0.693	0.951	8.774

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	413	83	0	185	309	357	194
N.S.	1	1.00	3.23	0.65	0.00	1.45	2.41	2.79	1.52
time (sec)	N/A	0.127	3.645	0.242	0.000	3.947	0.555	0.813	9.008

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	140	64	0	93	206	201	104
N.S.	1	1.00	1.44	0.66	0.00	0.96	2.12	2.07	1.07
time (sec)	N/A	0.106	1.073	0.292	0.000	3.959	0.329	0.685	8.509

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	58	46	0	49	158	84	50
N.S.	1	1.00	1.21	0.96	0.00	1.02	3.29	1.75	1.04
time (sec)	N/A	0.054	0.665	0.240	0.000	2.668	0.199	0.623	8.536

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	89	0	57	162	117	70
N.S.	1	1.00	1.18	1.11	0.00	0.71	2.02	1.46	0.88
time (sec)	N/A	0.046	0.357	0.199	0.000	2.033	0.186	0.536	8.542

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	129	106	0	83	296	169	129
N.S.	1	1.00	1.10	0.91	0.00	0.71	2.53	1.44	1.10
time (sec)	N/A	0.137	1.078	0.230	0.000	1.798	0.277	0.656	8.772

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	124	0	95	360	67	53
N.S.	1	1.00	0.75	1.75	0.00	1.34	5.07	0.94	0.75
time (sec)	N/A	0.098	0.094	0.173	0.000	1.141	0.369	0.641	8.501

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	217	148	0	121	454	219	208
N.S.	1	1.00	1.19	0.81	0.00	0.66	2.48	1.20	1.14
time (sec)	N/A	0.164	1.081	0.298	0.000	1.766	0.481	0.925	9.644

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	232	170	0	134	498	241	247
N.S.	1	1.00	1.05	0.77	0.00	0.61	2.25	1.09	1.12
time (sec)	N/A	0.186	1.193	0.359	0.000	0.825	0.621	1.047	10.174

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	274	190	0	159	605	269	291
N.S.	1	1.00	1.09	0.76	0.00	0.63	2.41	1.07	1.16
time (sec)	N/A	0.212	1.564	0.446	0.000	2.749	0.706	0.966	10.238

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-2)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	180.001	1.388	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	347	123	0	279	473	516	233
N.S.	1	1.00	1.82	0.64	0.00	1.46	2.48	2.70	1.22
time (sec)	N/A	0.165	5.627	0.349	0.000	3.303	0.935	1.295	8.976

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	319	104	0	206	403	429	266
N.S.	1	1.00	1.95	0.63	0.00	1.26	2.46	2.62	1.62
time (sec)	N/A	0.151	7.073	0.316	0.000	5.028	0.754	1.091	11.011

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	145	88	0	109	258	255	149
N.S.	1	1.00	1.07	0.65	0.00	0.81	1.91	1.89	1.10
time (sec)	N/A	0.109	1.408	0.278	0.000	3.157	0.463	0.976	8.962

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	79	69	0	51	172	165	87
N.S.	1	1.00	0.80	0.70	0.00	0.52	1.74	1.67	0.88
time (sec)	N/A	0.110	1.180	0.320	0.000	4.610	0.296	0.874	8.928

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	43	0	63	206	149	62
N.S.	1	1.00	1.37	0.73	0.00	1.07	3.49	2.53	1.05
time (sec)	N/A	0.062	0.601	0.216	0.000	5.192	0.269	0.772	8.843

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	150	110	0	80	258	140	111
N.S.	1	1.00	1.34	0.98	0.00	0.71	2.30	1.25	0.99
time (sec)	N/A	0.061	0.466	0.215	0.000	6.145	0.271	0.699	9.132

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	164	128	0	96	340	192	161
N.S.	1	1.00	1.07	0.84	0.00	0.63	2.22	1.25	1.05
time (sec)	N/A	0.154	0.982	0.308	0.000	6.826	0.363	0.781	9.061

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	217	148	0	121	452	219	208
N.S.	1	1.00	1.17	0.80	0.00	0.65	2.44	1.18	1.12
time (sec)	N/A	0.184	1.132	0.304	0.000	3.014	0.460	0.942	9.473

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	63	166	0	133	508	79	64
N.S.	1	1.00	0.64	1.68	0.00	1.34	5.13	0.80	0.65
time (sec)	N/A	0.099	0.108	0.240	0.000	6.846	0.595	0.899	8.717

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	267	190	0	159	605	271	286
N.S.	1	1.00	1.06	0.76	0.00	0.63	2.41	1.08	1.14
time (sec)	N/A	0.213	1.533	0.334	0.000	4.462	0.691	0.913	10.396

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	280	212	0	168	646	289	319
N.S.	1	1.00	0.98	0.74	0.00	0.59	2.25	1.01	1.11
time (sec)	N/A	0.232	1.796	0.438	0.000	4.104	0.911	1.026	10.734

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	321	232	0	195	753	319	352
N.S.	1	1.00	1.01	0.73	0.00	0.61	2.36	1.00	1.10
time (sec)	N/A	0.260	2.298	0.587	0.000	4.263	0.951	1.042	11.296

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	90	55	50	114	0	0	101
N.S.	1	1.00	1.45	0.89	0.81	1.84	0.00	0.00	1.63
time (sec)	N/A	0.070	2.112	0.529	0.297	5.478	0.000	0.000	14.384

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	88	55	50	101	0	0	101
N.S.	1	1.00	1.42	0.89	0.81	1.63	0.00	0.00	1.63
time (sec)	N/A	0.068	1.430	0.456	0.280	2.708	0.000	0.000	15.529

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	97	55	50	82	0	0	99
N.S.	1	1.00	1.56	0.89	0.81	1.32	0.00	0.00	1.60
time (sec)	N/A	0.069	1.104	0.449	0.291	0.976	0.000	0.000	11.948

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	66	50	69	0	0	102
N.S.	1	1.00	0.75	1.10	0.83	1.15	0.00	0.00	1.70
time (sec)	N/A	0.062	0.810	0.266	0.284	1.182	0.000	0.000	0.685

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	82	53	50	57	0	0	164
N.S.	1	1.00	1.41	0.91	0.86	0.98	0.00	0.00	2.83
time (sec)	N/A	0.065	0.830	0.219	0.300	3.114	0.000	0.000	10.164

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	98	53	47	78	0	0	170
N.S.	1	1.00	1.63	0.88	0.78	1.30	0.00	0.00	2.83
time (sec)	N/A	0.069	1.235	0.234	0.292	2.367	0.000	0.000	10.189

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	100	53	48	96	0	0	232
N.S.	1	1.00	1.61	0.85	0.77	1.55	0.00	0.00	3.74
time (sec)	N/A	0.075	2.188	0.239	0.284	2.597	0.000	0.000	10.953

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	100	53	48	118	0	0	157
N.S.	1	1.00	1.61	0.85	0.77	1.90	0.00	0.00	2.53
time (sec)	N/A	0.074	3.345	0.263	0.294	4.650	0.000	0.000	12.227

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	119	83	81	157	0	0	132
N.S.	1	1.00	1.13	0.79	0.77	1.50	0.00	0.00	1.26
time (sec)	N/A	0.133	3.212	0.444	0.290	4.419	0.000	0.000	13.892

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	112	83	81	140	0	0	132
N.S.	1	1.00	1.07	0.79	0.77	1.33	0.00	0.00	1.26
time (sec)	N/A	0.122	2.063	0.453	0.303	5.440	0.000	0.000	15.338

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	116	83	81	125	0	0	130
N.S.	1	1.00	1.10	0.79	0.77	1.19	0.00	0.00	1.24
time (sec)	N/A	0.121	1.809	0.429	0.302	4.138	0.000	0.000	13.672

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	83	83	81	109	0	0	241
N.S.	1	1.00	0.81	0.81	0.79	1.06	0.00	0.00	2.34
time (sec)	N/A	0.115	1.309	0.444	0.297	7.066	0.000	0.000	11.989

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	138	93	84	97	0	0	176
N.S.	1	1.00	1.37	0.92	0.83	0.96	0.00	0.00	1.74
time (sec)	N/A	0.123	1.445	0.257	0.290	4.125	0.000	0.000	10.774

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	112	80	82	84	0	0	158
N.S.	1	1.00	1.11	0.79	0.81	0.83	0.00	0.00	1.56
time (sec)	N/A	0.128	2.080	0.216	0.287	4.422	0.000	0.000	10.147



Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	118	80	79	106	0	0	208
N.S.	1	1.00	1.15	0.78	0.77	1.03	0.00	0.00	2.02
time (sec)	N/A	0.125	3.458	0.253	0.286	3.608	0.000	0.000	11.362

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	80	79	128	0	0	167
N.S.	1	1.00	1.16	0.76	0.75	1.22	0.00	0.00	1.59
time (sec)	N/A	0.130	5.303	0.253	0.294	3.483	0.000	0.000	11.808

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	127	121	108	189	0	0	349
N.S.	1	1.00	0.88	0.84	0.75	1.31	0.00	0.00	2.42
time (sec)	N/A	0.148	4.037	0.498	0.287	4.953	0.000	0.000	15.067

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	139	121	108	182	0	0	349
N.S.	1	1.00	0.97	0.84	0.75	1.26	0.00	0.00	2.42
time (sec)	N/A	0.146	3.760	0.470	0.290	4.234	0.000	0.000	14.475

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	130	121	108	161	0	0	335
N.S.	1	1.00	0.90	0.84	0.75	1.12	0.00	0.00	2.33
time (sec)	N/A	0.138	3.095	0.463	0.317	4.960	0.000	0.000	14.078

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	124	121	108	144	0	0	313
N.S.	1	1.00	0.87	0.85	0.76	1.01	0.00	0.00	2.20
time (sec)	N/A	0.126	2.105	0.483	0.308	3.969	0.000	0.000	13.583

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	152	135	112	133	0	0	351
N.S.	1	1.00	1.09	0.96	0.80	0.95	0.00	0.00	2.51
time (sec)	N/A	0.128	2.469	0.254	0.297	2.991	0.000	0.000	12.430

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	118	109	122	0	0	221
N.S.	1	1.00	1.20	0.84	0.78	0.87	0.00	0.00	1.58
time (sec)	N/A	0.146	3.151	0.257	0.298	1.636	0.000	0.000	10.706

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	105	111	106	0	0	208
N.S.	1	1.00	0.96	0.75	0.79	0.76	0.00	0.00	1.49
time (sec)	N/A	0.147	4.495	0.240	0.302	1.638	0.000	0.000	10.278

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	141	105	106	128	0	0	161
N.S.	1	1.00	0.99	0.74	0.75	0.90	0.00	0.00	1.13
time (sec)	N/A	0.142	6.201	0.255	0.290	1.463	0.000	0.000	10.727

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	0	192	197	485	0	0	298
N.S.	1	1.00	0.00	0.87	0.90	2.20	0.00	0.00	1.35
time (sec)	N/A	0.189	180.004	0.387	0.507	2.777	0.000	0.000	1.483

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	0	150	173	421	0	0	245
N.S.	1	1.00	0.00	0.83	0.96	2.34	0.00	0.00	1.36
time (sec)	N/A	0.160	180.004	0.387	0.497	3.026	0.000	0.000	1.256

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	111	143	341	0	0	189
N.S.	1	1.00	0.00	0.77	0.99	2.37	0.00	0.00	1.31
time (sec)	N/A	0.152	180.003	0.367	0.508	3.132	0.000	0.000	9.773

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	168	90	120	337	0	0	159
N.S.	1	1.00	1.54	0.83	1.10	3.09	0.00	0.00	1.46
time (sec)	N/A	0.126	0.843	0.435	0.506	2.979	0.000	0.000	9.560

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	160	121	148	367	0	0	212
N.S.	1	1.00	1.13	0.86	1.05	2.60	0.00	0.00	1.50
time (sec)	N/A	0.148	1.334	0.333	0.508	4.294	0.000	0.000	9.853

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	239	141	173	401	0	0	261
N.S.	1	1.00	1.30	0.77	0.94	2.18	0.00	0.00	1.42
time (sec)	N/A	0.178	2.158	0.306	0.508	4.659	0.000	0.000	10.325

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	213	168	202	432	0	0	308
N.S.	1	1.00	0.96	0.75	0.91	1.94	0.00	0.00	1.38
time (sec)	N/A	0.199	2.495	0.328	0.522	4.191	0.000	0.000	10.835

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	0	220	253	533	0	0	402
N.S.	1	1.00	0.00	0.80	0.92	1.94	0.00	0.00	1.46
time (sec)	N/A	0.214	180.003	0.357	0.495	3.768	0.000	0.000	9.563

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	178	229	474	0	0	349
N.S.	1	1.00	0.00	0.75	0.96	1.99	0.00	0.00	1.47
time (sec)	N/A	0.185	180.004	0.356	0.519	8.181	0.000	0.000	9.394

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	0	140	200	393	0	0	294
N.S.	1	1.00	0.00	0.70	1.01	1.97	0.00	0.00	1.48
time (sec)	N/A	0.164	180.002	0.354	0.505	3.737	0.000	0.000	9.359

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	205	118	172	384	0	0	267
N.S.	1	1.00	1.28	0.74	1.08	2.40	0.00	0.00	1.67
time (sec)	N/A	0.161	1.404	0.388	0.521	5.053	0.000	0.000	9.393

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	206	120	178	374	0	0	264
N.S.	1	1.00	1.30	0.75	1.12	2.35	0.00	0.00	1.66
time (sec)	N/A	0.145	1.102	0.404	0.544	4.723	0.000	0.000	9.181

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	160	151	202	403	0	0	305
N.S.	1	1.00	0.82	0.77	1.04	2.07	0.00	0.00	1.56
time (sec)	N/A	0.166	2.137	0.353	0.538	3.916	0.000	0.000	9.441

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	204	178	225	434	0	0	353
N.S.	1	1.00	0.90	0.79	1.00	1.92	0.00	0.00	1.56
time (sec)	N/A	0.191	2.346	0.348	0.517	4.112	0.000	0.000	9.723

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	209	198	254	461	0	0	399
N.S.	1	1.00	0.77	0.73	0.93	1.69	0.00	0.00	1.46
time (sec)	N/A	0.221	3.327	0.342	0.513	2.931	0.000	0.000	10.143

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	0	207	276	488	0	0	441
N.S.	1	1.00	0.00	0.71	0.95	1.68	0.00	0.00	1.52
time (sec)	N/A	0.204	180.003	0.371	0.526	3.777	0.000	0.000	9.591

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	A	B	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	0	168	249	415	0	0	386
N.S.	1	1.00	0.00	0.67	0.99	1.65	0.00	0.00	1.53
time (sec)	N/A	0.183	180.002	0.349	0.509	4.535	0.000	0.000	9.427

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	227	147	220	416	0	0	360
N.S.	1	1.00	1.07	0.69	1.03	1.95	0.00	0.00	1.69
time (sec)	N/A	0.174	2.429	0.399	0.515	3.583	0.000	0.000	9.422

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	224	139	220	408	0	0	360
N.S.	1	1.00	1.06	0.66	1.04	1.93	0.00	0.00	1.71
time (sec)	N/A	0.167	2.067	0.403	0.527	3.327	0.000	0.000	9.407

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	225	147	228	399	0	0	355
N.S.	1	1.00	1.08	0.70	1.09	1.91	0.00	0.00	1.70
time (sec)	N/A	0.162	1.473	0.402	0.542	1.511	0.000	0.000	9.363

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	181	178	250	422	0	0	394
N.S.	1	1.00	0.74	0.73	1.02	1.72	0.00	0.00	1.61
time (sec)	N/A	0.193	2.470	0.396	0.533	1.229	0.000	0.000	9.666

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	206	205	273	453	0	0	443
N.S.	1	1.00	0.75	0.75	1.00	1.65	0.00	0.00	1.62
time (sec)	N/A	0.210	3.314	0.352	0.541	2.360	0.000	0.000	9.978

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	256	232	302	480	0	0	490
N.S.	1	1.00	0.82	0.75	0.97	1.54	0.00	0.00	1.58
time (sec)	N/A	0.242	4.064	0.367	0.527	1.580	0.000	0.000	10.804

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	257	349	1419	631	0	0	-1
N.S.	1	1.00	0.94	1.28	5.22	2.32	0.00	0.00	-0.00
time (sec)	N/A	0.209	3.377	0.520	1.530	2.940	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	226	285	1147	570	0	0	-1
N.S.	1	1.00	1.04	1.31	5.29	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.187	2.395	0.388	0.910	3.338	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	223	819	478	0	0	-1
N.S.	1	1.00	0.97	1.36	4.99	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.166	2.353	0.404	0.696	3.580	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	121	481	308	0	0	133
N.S.	1	1.00	0.98	1.16	4.62	2.96	0.00	0.00	1.28
time (sec)	N/A	0.136	1.329	0.394	0.628	5.103	0.000	0.000	11.089

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	127	321	0	348	0	0	266
N.S.	1	1.00	1.17	2.94	0.00	3.19	0.00	0.00	2.44
time (sec)	N/A	0.147	1.573	0.458	0.000	4.143	0.000	0.000	12.406

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	100	0	101	0	0	145
N.S.	1	1.00	0.99	0.98	0.00	0.99	0.00	0.00	1.42
time (sec)	N/A	0.138	1.646	0.420	0.000	7.746	0.000	0.000	1.238

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	114	125	0	119	0	0	171
N.S.	1	1.00	0.74	0.81	0.00	0.77	0.00	0.00	1.10
time (sec)	N/A	0.160	2.744	0.422	0.000	9.577	0.000	0.000	9.859



Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	136	147	0	138	0	0	246
N.S.	1	1.00	0.65	0.71	0.00	0.66	0.00	0.00	1.18
time (sec)	N/A	0.185	4.322	0.397	0.000	6.381	0.000	0.000	10.798

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	257	412	1745	713	0	0	-1
N.S.	1	1.00	0.92	1.48	6.25	2.56	0.00	0.00	-0.00
time (sec)	N/A	0.227	5.481	0.427	3.005	3.962	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	241	350	1451	641	0	0	-1
N.S.	1	1.00	1.07	1.55	6.42	2.84	0.00	0.00	-0.00
time (sec)	N/A	0.193	4.339	0.444	1.540	4.329	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	109	186	915	456	0	0	-1
N.S.	1	1.00	0.69	1.18	5.83	2.90	0.00	0.00	-0.01
time (sec)	N/A	0.165	3.128	0.412	0.674	4.411	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	220	223	819	478	0	566	-1
N.S.	1	1.00	1.38	1.39	5.12	2.99	0.00	3.54	-0.01
time (sec)	N/A	0.174	2.986	0.418	0.668	3.710	0.000	1.705	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	190	497	637	454	0	0	-1
N.S.	1	1.00	1.12	2.94	3.77	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.179	3.021	0.418	0.644	4.435	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	123	406	184	397	0	0	-1
N.S.	1	1.00	0.79	2.62	1.19	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.168	2.841	0.431	0.600	4.073	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	117	90	162	103	0	0	190
N.S.	1	1.00	1.15	0.88	1.59	1.01	0.00	0.00	1.86
time (sec)	N/A	0.149	3.538	0.405	0.669	3.870	0.000	0.000	10.258

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	131	113	198	123	0	0	215
N.S.	1	1.00	0.85	0.73	1.28	0.79	0.00	0.00	1.39
time (sec)	N/A	0.167	5.580	0.434	0.610	5.390	0.000	0.000	10.550

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	148	136	276	143	0	0	290
N.S.	1	1.00	0.71	0.65	1.33	0.69	0.00	0.00	1.39
time (sec)	N/A	0.186	3.570	0.422	0.649	3.001	0.000	0.000	11.648

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	179	158	332	163	0	0	315
N.S.	1	1.00	0.69	0.61	1.27	0.62	0.00	0.00	1.21
time (sec)	N/A	0.217	6.142	0.414	0.619	2.222	0.000	0.000	14.506

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	568	478	2137	791	0	0	-1
N.S.	1	1.00	1.97	1.66	7.42	2.75	0.00	0.00	-0.00
time (sec)	N/A	0.229	9.209	0.442	5.902	2.716	0.000	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	119	252	1535	616	0	0	-1
N.S.	1	1.00	0.56	1.18	7.21	2.89	0.00	0.00	-0.00
time (sec)	N/A	0.178	4.919	0.434	1.192	1.233	0.000	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	145	350	1451	641	0	0	-1
N.S.	1	1.00	0.65	1.58	6.54	2.89	0.00	0.00	-0.00
time (sec)	N/A	0.202	5.727	0.417	1.430	2.406	0.000	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	253	285	1149	570	0	0	-1
N.S.	1	1.00	1.17	1.31	5.29	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.186	3.949	0.411	0.866	2.334	0.000	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	239	565	1049	550	0	0	-1
N.S.	1	1.00	1.05	2.49	4.62	2.42	0.00	0.00	-0.00
time (sec)	N/A	0.198	4.717	0.423	0.733	4.057	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	227	667	841	510	0	0	-1
N.S.	1	1.00	1.00	2.95	3.72	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.201	4.910	0.420	0.646	4.756	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	203	555	230	436	0	0	-1
N.S.	1	1.00	1.00	2.73	1.13	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.189	6.522	0.413	0.605	3.079	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	121	115	174	109	0	0	192
N.S.	1	1.00	1.19	1.13	1.71	1.07	0.00	0.00	1.88
time (sec)	N/A	0.147	6.408	0.446	0.616	5.731	0.000	0.000	10.281

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	135	138	210	131	0	0	217
N.S.	1	1.00	0.87	0.89	1.35	0.85	0.00	0.00	1.40
time (sec)	N/A	0.173	3.628	0.418	0.635	3.957	0.000	0.000	11.001

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	156	161	292	153	0	0	292
N.S.	1	1.00	0.75	0.77	1.40	0.74	0.00	0.00	1.40
time (sec)	N/A	0.189	6.557	0.403	0.630	4.784	0.000	0.000	12.853

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	183	183	352	175	0	0	191
N.S.	1	1.00	0.70	0.70	1.35	0.67	0.00	0.00	0.73
time (sec)	N/A	0.217	11.318	0.474	0.644	3.103	0.000	0.000	13.729

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	666	604	2735	925	0	0	-1
N.S.	1	1.00	1.90	1.73	7.81	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.240	10.983	0.433	23.310	5.646	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	535	314	2019	732	0	0	-1
N.S.	1	1.00	2.00	1.18	7.56	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.198	9.669	0.446	3.304	2.642	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	572	478	2137	791	0	0	-1
N.S.	1	1.00	2.01	1.68	7.52	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.218	9.164	0.444	5.836	2.272	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	507	412	1747	713	0	0	-1
N.S.	1	1.00	1.82	1.48	6.26	2.56	0.00	0.00	-0.00
time (sec)	N/A	0.225	8.724	0.399	2.929	4.759	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	465	349	1421	631	0	0	-1
N.S.	1	1.00	1.71	1.28	5.22	2.32	0.00	0.00	-0.00
time (sec)	N/A	0.212	8.243	0.468	1.477	5.707	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	481	627	1403	614	0	0	-1
N.S.	1	1.00	1.70	2.22	4.96	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.225	8.914	0.447	0.957	3.435	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	517	731	1240	592	0	0	-1
N.S.	1	1.00	1.81	2.56	4.35	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.232	9.894	0.416	0.742	2.060	0.000	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	528	833	990	533	0	0	-1
N.S.	1	1.00	1.87	2.94	3.50	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.232	11.759	0.431	0.662	1.618	0.000	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	226	638	264	453	0	0	-1
N.S.	1	1.00	0.90	2.54	1.05	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.211	13.771	0.411	0.621	1.784	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	121	134	174	109	0	0	192
N.S.	1	1.00	1.19	1.31	1.71	1.07	0.00	0.00	1.88
time (sec)	N/A	0.150	4.514	0.421	0.626	2.392	0.000	0.000	11.478

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	135	161	210	131	0	0	217
N.S.	1	1.00	0.87	1.04	1.35	0.85	0.00	0.00	1.40
time (sec)	N/A	0.174	7.622	0.419	0.666	3.143	0.000	0.000	11.752

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	495	184	292	153	0	0	167
N.S.	1	1.00	2.38	0.88	1.40	0.74	0.00	0.00	0.80
time (sec)	N/A	0.186	14.523	0.459	0.669	3.786	0.000	0.000	13.485

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	577	206	352	175	0	0	191
N.S.	1	1.00	2.21	0.79	1.35	0.67	0.00	0.00	0.73
time (sec)	N/A	0.211	16.806	0.447	0.696	5.762	0.000	0.000	13.485

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	655	230	436	197	0	0	229
N.S.	1	1.00	2.09	0.73	1.39	0.63	0.00	0.00	0.73
time (sec)	N/A	0.240	17.066	0.462	0.736	3.424	0.000	0.000	14.485

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	185	566	1403	570	0	0	-1
N.S.	1	1.00	0.81	2.48	6.15	2.50	0.00	0.00	-0.00
time (sec)	N/A	0.196	2.941	0.466	1.014	4.417	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	161	499	960	474	0	0	-1
N.S.	1	1.00	0.95	2.95	5.68	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.173	2.567	0.460	0.656	5.027	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	152	323	150	366	0	0	250
N.S.	1	1.00	1.38	2.94	1.36	3.33	0.00	0.00	2.27
time (sec)	N/A	0.139	1.595	0.418	0.562	13.111	0.000	0.000	12.161

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	77	99	132	121	0	0	143
N.S.	1	1.00	0.84	1.08	1.43	1.32	0.00	0.00	1.55
time (sec)	N/A	0.131	1.099	0.451	0.604	6.153	0.000	0.000	0.799



Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	103	151	0	155	0	0	146
N.S.	1	1.00	0.66	0.96	0.00	0.99	0.00	0.00	0.93
time (sec)	N/A	0.166	1.848	0.419	0.000	4.675	0.000	0.000	0.732

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	128	184	0	170	0	0	186
N.S.	1	1.00	0.60	0.86	0.00	0.80	0.00	0.00	0.87
time (sec)	N/A	0.187	3.013	0.403	0.000	3.415	0.000	0.000	9.886

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	255	733	1459	612	0	0	-1
N.S.	1	1.00	0.89	2.55	5.08	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.231	5.224	0.428	1.214	3.364	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	174	669	0	530	0	0	-1
N.S.	1	1.00	0.76	2.92	0.00	2.31	0.00	0.00	-0.00
time (sec)	N/A	0.197	3.737	0.407	0.000	4.491	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	114	408	182	419	0	0	-1
N.S.	1	1.00	0.73	2.60	1.16	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.169	2.649	0.417	0.590	4.578	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	81	103	0	97	0	0	195
N.S.	1	1.00	0.78	0.99	0.00	0.93	0.00	0.00	1.88
time (sec)	N/A	0.140	1.186	0.391	0.000	1.979	0.000	0.000	10.131

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	85	152	0	153	0	0	170
N.S.	1	1.00	0.56	1.00	0.00	1.01	0.00	0.00	1.12
time (sec)	N/A	0.163	1.392	0.434	0.000	0.915	0.000	0.000	9.772

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	150	120	113	212	159	0	0	198
N.S.	1	0.99	0.79	0.74	1.39	1.05	0.00	0.00	1.30
time (sec)	N/A	0.161	2.227	0.389	0.599	1.606	0.000	0.000	9.941

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	170	199	0	191	0	0	196
N.S.	1	1.00	0.63	0.74	0.00	0.71	0.00	0.00	0.73
time (sec)	N/A	0.219	4.040	0.398	0.000	1.240	0.000	0.000	9.992

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	247	899	0	634	0	0	-1
N.S.	1	1.00	0.72	2.62	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.262	8.917	0.429	0.000	2.331	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	205	835	1091	553	0	0	-1
N.S.	1	1.00	0.72	2.94	3.84	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.235	6.260	0.418	0.788	2.778	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	129	557	232	456	0	0	-1
N.S.	1	1.00	0.63	2.72	1.13	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.187	4.046	0.416	0.594	3.535	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	92	161	103	0	0	240
N.S.	1	1.00	0.88	0.88	1.55	0.99	0.00	0.00	2.31
time (sec)	N/A	0.154	2.270	0.550	0.621	4.636	0.000	0.000	11.069

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	106	127	0	118	0	0	246
N.S.	1	1.00	0.68	0.81	0.00	0.75	0.00	0.00	1.57
time (sec)	N/A	0.156	1.403	0.417	0.000	3.065	0.000	0.000	10.956

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	132	186	0	170	0	0	246
N.S.	1	1.00	0.62	0.88	0.00	0.80	0.00	0.00	1.16
time (sec)	N/A	0.189	2.070	0.425	0.000	3.772	0.000	0.000	10.716

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	216	133	199	0	191	0	0	249
N.S.	1	0.99	0.61	0.91	0.00	0.88	0.00	0.00	1.14
time (sec)	N/A	0.198	3.377	0.402	0.000	2.145	0.000	0.000	10.637

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	204	151	124	354	196	0	0	249
N.S.	1	0.99	0.73	0.60	1.72	0.95	0.00	0.00	1.21
time (sec)	N/A	0.185	4.583	0.425	0.622	3.593	0.000	0.000	10.942

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	216	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	16.700	0.980	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	177	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	59.466	1.566	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	56	0	0	58	66	0	90
N.S.	1	1.00	1.70	0.00	0.00	1.76	2.00	0.00	2.73
time (sec)	N/A	0.073	1.225	0.733	0.000	6.659	0.690	0.000	9.380

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	201	134	0	87	296	200	159
N.S.	1	1.00	1.93	1.29	0.00	0.84	2.85	1.92	1.53
time (sec)	N/A	0.164	1.139	0.241	0.000	7.245	0.280	0.581	9.497

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	206	131	146	633	0	0	245
N.S.	1	1.00	1.40	0.89	0.99	4.31	0.00	0.00	1.67
time (sec)	N/A	0.221	3.247	0.260	0.521	6.657	0.000	0.000	10.267

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [852] had the largest ratio of [47]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	32	0.125
2	A	3	3	1.00	30	0.100
3	A	2	2	1.00	24	0.083
4	A	4	3	1.00	30	0.100
5	A	3	3	1.00	32	0.094
6	A	4	4	1.00	32	0.125
7	A	5	4	1.00	32	0.125
8	A	6	4	1.00	32	0.125
9	A	5	5	1.00	34	0.147
10	A	4	4	1.00	32	0.125
11	A	3	3	1.00	26	0.115
12	A	5	4	1.00	32	0.125
13	A	5	4	1.00	34	0.118
14	A	4	4	1.00	34	0.118
15	A	5	5	1.00	34	0.147
16	A	6	5	1.00	34	0.147
17	A	6	5	1.00	34	0.147
18	A	5	5	1.00	32	0.156
19	A	4	4	1.00	26	0.154
20	A	6	4	1.00	32	0.125
21	A	6	5	1.00	34	0.147
22	A	6	4	1.00	34	0.118
23	A	5	4	1.00	34	0.118
24	A	6	5	1.00	34	0.147
25	A	7	5	1.00	34	0.147

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	5	1.00	34	0.147
27	A	6	5	1.00	32	0.156
28	A	5	4	1.00	26	0.154
29	A	7	4	1.00	32	0.125
30	A	7	5	1.00	34	0.147
31	A	7	5	1.00	34	0.147
32	A	7	4	1.00	34	0.118
33	A	6	4	1.00	34	0.118
34	A	7	5	1.00	34	0.147
35	A	8	5	1.00	34	0.147
36	A	4	4	1.00	34	0.118
37	A	3	3	1.00	34	0.088
38	A	5	5	1.00	32	0.156
39	A	2	2	1.00	26	0.077
40	A	3	3	1.00	32	0.094
41	A	4	4	1.00	34	0.118
42	A	5	4	1.00	34	0.118
43	A	6	4	1.00	34	0.118
44	A	4	3	1.00	34	0.088
45	A	6	6	1.00	34	0.176
46	A	3	3	1.00	32	0.094
47	A	3	3	1.00	26	0.115
48	A	4	3	1.00	32	0.094
49	A	5	4	1.00	34	0.118
50	A	6	4	1.00	34	0.118
51	A	5	3	1.00	34	0.088
52	A	7	6	1.00	34	0.176
53	A	4	4	1.00	34	0.118
54	A	4	4	1.00	32	0.125
55	A	4	3	1.00	26	0.115
56	A	5	3	1.00	32	0.094
57	A	6	4	1.00	34	0.118
58	A	7	4	1.00	34	0.118
59	A	8	6	1.00	34	0.176
60	A	5	4	1.00	34	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	34	0.147
62	A	5	4	1.00	32	0.125
63	A	5	3	1.00	26	0.115
64	A	6	3	1.00	32	0.094
65	A	7	4	1.00	34	0.118
66	A	8	4	1.00	34	0.118
67	A	6	5	1.00	36	0.139
68	A	5	5	1.00	36	0.139
69	A	4	4	1.00	34	0.118
70	A	3	3	1.00	28	0.107
71	A	6	6	1.00	34	0.176
72	A	7	7	1.00	36	0.194
73	A	8	7	1.00	36	0.194
74	A	9	7	1.00	36	0.194
75	A	6	6	1.00	36	0.167
76	A	5	5	1.00	34	0.147
77	A	4	4	1.00	28	0.143
78	A	7	7	1.00	34	0.206
79	A	7	7	1.00	36	0.194
80	A	8	8	1.00	36	0.222
81	A	9	8	1.00	36	0.222
82	A	7	6	1.00	36	0.167
83	A	6	5	1.00	34	0.147
84	A	5	4	1.00	28	0.143
85	A	8	7	1.00	34	0.206
86	A	8	8	1.00	36	0.222
87	A	8	7	1.00	36	0.194
88	A	9	8	1.00	36	0.222
89	A	10	8	1.00	36	0.222
90	A	6	6	1.00	36	0.167
91	A	5	5	1.00	36	0.139
92	A	4	4	1.00	34	0.118
93	A	3	3	1.00	28	0.107
94	A	7	7	1.00	34	0.206
95	A	8	8	1.00	36	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	9	8	1.00	36	0.222
97	A	6	5	1.00	36	0.139
98	A	5	5	1.00	36	0.139
99	A	4	4	1.00	34	0.118
100	A	4	4	1.00	28	0.143
101	A	8	7	1.00	34	0.206
102	A	9	8	1.00	36	0.222
103	A	10	8	1.00	36	0.222
104	A	7	5	1.00	36	0.139
105	A	6	5	1.00	36	0.139
106	A	5	5	1.00	36	0.139
107	A	5	5	1.00	34	0.147
108	A	5	4	1.00	28	0.143
109	A	9	7	1.00	34	0.206
110	A	10	8	1.00	36	0.222
111	A	11	8	1.00	36	0.222
112	A	6	4	1.00	34	0.118
113	A	5	4	1.00	34	0.118
114	A	4	4	1.00	34	0.118
115	A	3	3	1.00	34	0.088
116	A	3	3	1.00	34	0.088
117	A	4	4	1.00	34	0.118
118	A	5	4	1.00	34	0.118
119	A	7	5	1.00	36	0.139
120	A	6	5	1.00	36	0.139
121	A	5	5	1.00	36	0.139
122	A	4	4	1.00	36	0.111
123	A	4	4	1.00	36	0.111
124	A	4	4	1.00	36	0.111
125	A	5	5	1.00	36	0.139
126	A	6	5	1.00	36	0.139
127	A	7	5	1.00	36	0.139
128	A	6	5	1.00	36	0.139
129	A	5	4	1.00	36	0.111
130	A	5	5	1.00	36	0.139

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	4	1.00	36	0.111
132	A	5	4	1.00	36	0.111
133	A	6	5	1.00	36	0.139
134	A	13	9	1.00	36	0.250
135	A	12	9	1.00	36	0.250
136	A	11	8	1.00	36	0.222
137	A	11	8	1.00	36	0.222
138	A	12	9	1.00	36	0.250
139	A	13	9	1.00	36	0.250
140	A	13	9	1.00	36	0.250
141	A	12	8	1.00	36	0.222
142	A	12	9	1.00	36	0.250
143	A	12	8	1.00	36	0.222
144	A	13	9	1.00	36	0.250
145	A	14	9	1.00	36	0.250
146	A	15	9	1.00	36	0.250
147	A	14	9	1.00	36	0.250
148	A	13	8	1.00	36	0.222
149	A	13	9	1.00	36	0.250
150	A	13	9	1.00	36	0.250
151	A	13	8	1.00	36	0.222
152	A	14	9	1.00	36	0.250
153	A	15	9	1.00	36	0.250
154	A	9	8	1.00	38	0.210
155	A	8	8	1.00	38	0.210
156	A	7	7	1.00	38	0.184
157	A	4	4	1.00	38	0.105
158	A	5	4	1.00	38	0.105
159	A	6	4	1.00	38	0.105
160	A	7	4	1.00	38	0.105
161	A	10	9	1.00	38	0.237
162	A	9	9	1.00	38	0.237
163	A	8	8	1.00	38	0.210
164	A	8	8	1.00	38	0.210
165	A	5	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	5	1.00	38	0.132
167	A	7	5	1.00	38	0.132
168	A	8	5	1.00	38	0.132
169	A	11	9	1.00	38	0.237
170	A	10	9	1.00	38	0.237
171	A	9	8	1.00	38	0.210
172	A	9	9	1.00	38	0.237
173	A	9	8	1.00	38	0.210
174	A	6	5	1.00	38	0.132
175	A	7	5	1.00	38	0.132
176	A	8	5	1.00	38	0.132
177	A	9	5	1.00	38	0.132
178	A	9	8	1.00	46	0.174
179	A	9	9	1.00	38	0.237
180	A	8	8	1.00	38	0.210
181	A	4	4	1.00	38	0.105
182	A	5	5	1.00	38	0.132
183	A	6	5	1.00	38	0.132
184	A	7	5	1.00	38	0.132
185	A	9	8	1.00	38	0.210
186	A	5	5	1.00	38	0.132
187	A	5	4	1.00	38	0.105
188	A	6	5	1.00	38	0.132
189	A	7	5	1.00	38	0.132
190	A	10	8	1.00	38	0.210
191	A	6	5	1.00	38	0.132
192	A	6	5	1.00	38	0.132
193	A	6	4	1.00	38	0.105
194	A	7	5	1.00	38	0.132
195	A	8	5	1.00	38	0.132
196	A	6	6	1.00	28	0.214
197	A	8	8	1.00	36	0.222
198	A	7	7	1.00	34	0.206
199	A	6	6	1.00	28	0.214
200	A	11	7	1.00	34	0.206

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	12	8	1.00	36	0.222
202	A	6	6	1.00	28	0.214
203	A	6	6	1.00	28	0.214
204	A	7	5	1.00	34	0.147
205	A	6	5	1.00	34	0.147
206	A	5	5	1.00	34	0.147
207	A	3	3	1.00	32	0.094
208	A	6	4	1.00	34	0.118
209	A	7	4	1.00	34	0.118
210	A	8	4	1.00	34	0.118
211	A	9	4	1.00	34	0.118
212	A	9	8	1.00	36	0.222
213	A	8	8	1.00	36	0.222
214	A	7	7	1.00	36	0.194
215	A	8	8	1.00	36	0.222
216	A	9	8	1.00	36	0.222
217	A	10	8	1.00	36	0.222
218	A	7	7	1.00	34	0.206
219	A	6	5	1.00	34	0.147
220	A	5	5	1.00	34	0.147
221	A	4	4	1.00	32	0.125
222	A	3	3	1.00	26	0.115
223	A	5	5	1.00	32	0.156
224	A	6	6	1.00	34	0.176
225	A	7	6	1.00	34	0.176
226	A	11	9	1.00	36	0.250
227	A	10	9	1.00	36	0.250
228	A	9	9	1.00	36	0.250
229	A	8	8	1.00	36	0.222
230	A	9	9	1.00	36	0.250
231	A	10	9	1.00	36	0.250
232	A	4	4	1.00	29	0.138
233	A	3	3	1.00	27	0.111
234	A	2	2	1.00	21	0.095
235	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	29	0.103
237	A	4	4	1.00	29	0.138
238	A	5	4	1.00	29	0.138
239	A	6	4	1.00	29	0.138
240	A	5	5	1.00	31	0.161
241	A	4	4	1.00	29	0.138
242	A	3	3	1.00	23	0.130
243	A	4	3	1.00	29	0.103
244	A	4	3	1.00	31	0.097
245	A	4	4	1.00	31	0.129
246	A	5	5	1.00	31	0.161
247	A	6	5	1.00	31	0.161
248	A	6	5	1.00	31	0.161
249	A	5	4	1.00	29	0.138
250	A	4	3	1.00	23	0.130
251	A	5	4	1.00	29	0.138
252	A	5	4	1.00	31	0.129
253	A	5	4	1.00	31	0.129
254	A	5	5	1.00	31	0.161
255	A	6	6	1.00	31	0.194
256	A	7	6	1.00	31	0.194
257	A	7	5	1.00	31	0.161
258	A	6	4	1.00	29	0.138
259	A	5	3	1.00	23	0.130
260	A	6	5	1.00	29	0.172
261	A	6	5	1.00	31	0.161
262	A	6	5	1.00	31	0.161
263	A	6	5	1.00	31	0.161
264	A	6	6	1.00	31	0.194
265	A	7	7	1.00	31	0.226
266	A	8	7	1.00	31	0.226
267	A	6	6	1.00	31	0.194
268	A	5	5	1.00	31	0.161
269	A	5	5	1.00	29	0.172
270	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	3	1.00	29	0.103
272	A	4	4	1.00	31	0.129
273	A	5	5	1.00	31	0.161
274	A	6	5	1.00	31	0.161
275	A	6	6	1.00	31	0.194
276	A	5	5	1.00	31	0.161
277	A	3	3	1.00	29	0.103
278	A	3	3	1.00	23	0.130
279	A	4	4	1.00	29	0.138
280	A	5	5	1.00	31	0.161
281	A	6	5	1.00	31	0.161
282	A	7	7	1.00	31	0.226
283	A	6	6	1.00	31	0.194
284	A	4	4	1.00	31	0.129
285	A	4	4	1.00	29	0.138
286	A	4	3	1.00	23	0.130
287	A	5	5	1.00	29	0.172
288	A	6	5	1.00	31	0.161
289	A	7	5	1.00	31	0.161
290	A	7	7	1.00	31	0.226
291	A	5	5	1.00	31	0.161
292	A	5	5	1.00	31	0.161
293	A	5	4	1.00	29	0.138
294	A	5	3	1.00	23	0.130
295	A	6	5	1.00	29	0.172
296	A	7	5	1.00	31	0.161
297	A	8	5	1.00	31	0.161
298	A	3	3	1.00	34	0.088
299	A	3	3	1.00	34	0.088
300	A	2	2	1.00	32	0.062
301	A	2	2	1.00	26	0.077
302	A	2	2	1.00	32	0.062
303	A	3	3	1.00	34	0.088
304	A	3	3	1.00	34	0.088
305	A	4	3	1.00	34	0.088

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	7	1.00	34	0.206
307	A	6	6	1.00	34	0.176
308	A	5	5	1.00	34	0.147
309	A	3	3	1.00	32	0.094
310	A	3	3	1.00	26	0.115
311	A	4	4	1.00	32	0.125
312	A	5	5	1.00	34	0.147
313	A	6	6	1.00	34	0.176
314	A	2	2	1.00	21	0.095
315	A	2	2	1.00	28	0.071
316	A	3	3	1.00	23	0.130
317	A	11	8	1.00	33	0.242
318	A	10	7	1.00	33	0.212
319	A	9	6	1.00	31	0.194
320	A	8	5	1.00	25	0.200
321	A	11	6	1.00	31	0.194
322	A	12	7	1.00	33	0.212
323	A	13	8	1.00	33	0.242
324	A	14	8	1.00	33	0.242
325	A	11	7	1.00	33	0.212
326	A	10	6	1.00	31	0.194
327	A	9	5	1.00	25	0.200
328	A	12	7	1.00	31	0.226
329	A	12	7	1.00	33	0.212
330	A	13	8	1.00	33	0.242
331	A	14	8	1.00	33	0.242
332	A	12	7	1.00	33	0.212
333	A	11	6	1.00	31	0.194
334	A	10	5	1.00	25	0.200
335	A	13	8	1.00	31	0.258
336	A	13	8	1.00	33	0.242
337	A	13	8	1.00	33	0.242
338	A	14	9	1.00	33	0.273
339	A	15	9	1.00	33	0.273
340	A	10	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	13	9	1.00	27	0.333
342	A	13	9	1.00	27	0.333
343	A	10	7	1.00	33	0.212
344	A	9	6	1.00	33	0.182
345	A	8	5	1.00	31	0.161
346	A	7	4	1.00	25	0.160
347	A	11	6	1.00	31	0.194
348	A	12	7	1.00	33	0.212
349	A	13	8	1.00	33	0.242
350	A	10	7	1.00	33	0.212
351	A	9	6	1.00	33	0.182
352	A	8	5	1.00	31	0.161
353	A	8	5	1.00	25	0.200
354	A	12	7	1.00	31	0.226
355	A	13	8	1.00	33	0.242
356	A	14	8	1.00	33	0.242
357	A	11	8	1.00	33	0.242
358	A	10	7	1.00	33	0.212
359	A	9	6	1.00	33	0.182
360	A	9	6	1.00	31	0.194
361	A	9	5	1.00	25	0.200
362	A	13	8	1.00	31	0.258
363	A	14	8	1.00	33	0.242
364	A	15	8	1.00	33	0.242
365	A	12	8	1.00	28	0.286
366	A	12	8	1.00	28	0.286
367	A	12	7	1.00	34	0.206
368	A	9	6	1.00	28	0.214
369	A	13	8	1.00	34	0.235
370	A	7	4	1.00	27	0.148
371	A	8	5	1.00	27	0.185
372	A	9	5	1.00	27	0.185
373	A	3	3	1.00	27	0.111
374	A	3	3	1.00	27	0.111
375	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	2	2	1.00	17	0.118
377	A	5	4	1.00	25	0.160
378	A	14	9	1.00	31	0.290
379	A	13	9	1.00	31	0.290
380	A	12	9	1.00	31	0.290
381	A	11	8	1.00	31	0.258
382	A	11	8	1.00	31	0.258
383	A	12	9	1.00	31	0.290
384	A	13	9	1.00	31	0.290
385	A	15	10	1.00	33	0.303
386	A	14	10	1.00	33	0.303
387	A	13	10	1.00	33	0.303
388	A	12	9	1.00	33	0.273
389	A	12	9	1.00	33	0.273
390	A	12	9	1.00	33	0.273
391	A	13	10	1.00	33	0.303
392	A	15	11	1.00	33	0.333
393	A	14	11	1.00	33	0.333
394	A	13	10	1.00	33	0.303
395	A	13	10	1.00	33	0.303
396	A	13	10	1.00	33	0.303
397	A	13	10	1.00	33	0.303
398	A	16	13	1.00	33	0.394
399	A	15	12	1.00	33	0.364
400	A	14	11	1.00	33	0.333
401	A	14	11	1.00	33	0.333
402	A	15	12	1.00	33	0.364
403	A	16	13	1.00	33	0.394
404	A	16	13	1.00	33	0.394
405	A	15	12	1.00	33	0.364
406	A	15	12	1.00	33	0.364
407	A	15	12	1.00	33	0.364
408	A	16	13	1.00	33	0.394
409	A	17	13	1.00	33	0.394
410	A	17	14	1.00	33	0.424

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	16	13	1.00	33	0.394
412	A	16	13	1.00	33	0.394
413	A	16	13	1.00	33	0.394
414	A	16	13	1.00	33	0.394
415	A	17	13	1.00	33	0.394
416	A	13	10	1.00	36	0.278
417	A	13	10	1.00	36	0.278
418	A	12	9	1.00	36	0.250
419	A	12	9	1.00	36	0.250
420	A	13	10	1.00	36	0.278
421	A	13	10	1.00	36	0.278
422	A	16	13	1.00	36	0.361
423	A	15	12	1.00	36	0.333
424	A	15	12	1.00	36	0.333
425	A	15	12	1.00	36	0.333
426	A	16	13	1.00	36	0.361
427	A	14	10	1.00	35	0.286
428	A	13	9	1.00	35	0.257
429	A	12	9	1.00	35	0.257
430	A	8	6	1.00	35	0.171
431	A	9	7	1.00	35	0.200
432	A	10	7	1.00	35	0.200
433	A	11	7	1.00	35	0.200
434	A	15	10	1.00	35	0.286
435	A	14	10	1.00	35	0.286
436	A	13	9	1.00	35	0.257
437	A	13	9	1.00	35	0.257
438	A	9	7	1.00	35	0.200
439	A	10	7	1.00	35	0.200
440	A	11	7	1.00	35	0.200
441	A	12	7	1.00	35	0.200
442	A	16	10	1.00	35	0.286
443	A	15	10	1.00	35	0.286
444	A	14	10	1.00	35	0.286
445	A	14	10	1.00	35	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	14	10	1.00	35	0.286
447	A	10	8	1.00	35	0.229
448	A	11	8	1.00	35	0.229
449	A	12	8	1.00	35	0.229
450	A	13	8	1.00	35	0.229
451	A	14	10	1.00	43	0.233
452	A	13	9	1.00	35	0.257
453	A	12	9	1.00	35	0.257
454	A	7	5	1.00	35	0.143
455	A	8	6	1.00	35	0.171
456	A	9	7	1.00	35	0.200
457	A	10	7	1.00	35	0.200
458	A	13	9	1.00	35	0.257
459	A	8	6	1.00	35	0.171
460	A	8	6	1.00	35	0.171
461	A	9	7	1.00	35	0.200
462	A	10	7	1.00	35	0.200
463	A	14	10	1.00	35	0.286
464	A	9	7	1.00	35	0.200
465	A	9	7	1.00	35	0.200
466	A	9	7	1.00	35	0.200
467	A	10	7	1.00	35	0.200
468	A	11	7	1.00	35	0.200
469	A	13	10	1.00	38	0.263
470	A	8	6	1.00	38	0.158
471	A	8	6	1.00	38	0.158
472	A	10	8	1.00	38	0.210
473	A	12	7	1.00	25	0.280
474	A	12	7	1.00	25	0.280
475	A	11	6	1.00	25	0.240
476	A	11	6	1.00	25	0.240
477	A	5	5	1.00	27	0.185
478	A	11	6	1.00	26	0.231
479	A	9	7	1.00	31	0.226
480	A	8	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	7	5	1.00	31	0.161
482	A	6	4	1.00	29	0.138
483	A	8	6	1.00	31	0.194
484	A	9	7	1.00	31	0.226
485	A	10	8	1.00	31	0.258
486	A	11	8	1.00	31	0.258
487	A	7	4	1.00	33	0.121
488	A	7	4	1.00	33	0.121
489	A	7	4	1.00	33	0.121
490	A	7	4	1.00	33	0.121
491	A	7	4	1.00	33	0.121
492	A	7	4	1.00	33	0.121
493	A	7	4	1.00	31	0.129
494	A	9	6	0.99	31	0.194
495	A	8	6	0.99	31	0.194
496	A	7	5	1.00	31	0.161
497	A	6	4	1.00	29	0.138
498	A	5	3	1.00	23	0.130
499	A	8	6	1.00	29	0.207
500	A	9	7	1.00	31	0.226
501	A	10	8	1.00	31	0.258
502	A	6	5	1.00	34	0.147
503	A	5	5	1.00	34	0.147
504	A	4	4	1.00	34	0.118
505	A	4	4	1.00	34	0.118
506	A	5	5	1.00	34	0.147
507	A	6	5	1.00	34	0.147
508	A	6	6	1.00	36	0.167
509	A	5	5	1.00	36	0.139
510	A	5	5	1.00	36	0.139
511	A	5	5	1.00	36	0.139
512	A	6	6	1.00	36	0.167
513	A	7	6	1.00	36	0.167
514	A	6	5	1.00	36	0.139
515	A	6	6	1.00	36	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	6	5	1.00	36	0.139
517	A	6	5	1.00	36	0.139
518	A	7	6	1.00	36	0.167
519	A	14	10	1.00	36	0.278
520	A	13	10	1.00	36	0.278
521	A	12	9	1.00	36	0.250
522	A	12	9	1.00	36	0.250
523	A	13	10	1.00	36	0.278
524	A	14	10	1.00	36	0.278
525	A	14	10	1.00	36	0.278
526	A	13	9	1.00	36	0.250
527	A	13	10	1.00	36	0.278
528	A	13	9	1.00	36	0.250
529	A	14	10	1.00	36	0.278
530	A	15	10	1.00	36	0.278
531	A	14	9	1.00	36	0.250
532	A	14	10	1.00	36	0.278
533	A	14	10	1.00	36	0.278
534	A	14	9	1.00	36	0.250
535	A	15	10	1.00	36	0.278
536	A	7	5	1.00	38	0.132
537	A	6	5	1.00	38	0.132
538	A	5	5	1.00	38	0.132
539	A	8	8	1.00	38	0.210
540	A	9	9	1.00	38	0.237
541	A	8	6	1.00	38	0.158
542	A	7	6	1.00	38	0.158
543	A	6	6	1.00	38	0.158
544	A	9	9	1.00	38	0.237
545	A	9	9	1.00	38	0.237
546	A	10	10	1.00	38	0.263
547	A	9	6	1.00	38	0.158
548	A	8	6	1.00	38	0.158
549	A	7	6	1.00	38	0.158
550	A	10	9	1.00	38	0.237

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	10	10	1.00	38	0.263
552	A	10	9	1.00	38	0.237
553	A	11	10	1.00	38	0.263
554	A	7	6	1.00	38	0.158
555	A	6	6	1.00	38	0.158
556	A	5	5	1.00	38	0.132
557	A	9	9	1.00	38	0.237
558	A	7	6	1.00	38	0.158
559	A	6	5	1.00	38	0.132
560	A	6	6	1.00	38	0.158
561	A	10	9	1.00	38	0.237
562	A	8	6	1.00	38	0.158
563	A	7	5	1.00	38	0.132
564	A	7	6	1.00	38	0.158
565	A	7	6	1.00	38	0.158
566	A	11	9	1.00	38	0.237
567	A	8	8	1.00	34	0.235
568	A	11	10	1.00	36	0.278
569	A	10	10	1.00	36	0.278
570	A	9	9	1.00	36	0.250
571	A	10	10	1.00	36	0.278
572	A	11	10	1.00	36	0.278
573	A	12	10	1.00	36	0.278
574	A	13	10	1.00	31	0.323
575	A	12	9	1.00	31	0.290
576	A	12	9	1.00	31	0.290
577	A	13	10	1.00	31	0.323
578	A	14	11	1.00	33	0.333
579	A	13	10	1.00	33	0.303
580	A	13	10	1.00	33	0.303
581	A	13	10	1.00	33	0.303
582	A	14	11	1.00	33	0.333
583	A	15	12	1.00	33	0.364
584	A	14	11	1.00	33	0.333
585	A	14	11	1.00	33	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	14	11	1.00	33	0.333
587	A	14	11	1.00	33	0.333
588	A	15	12	1.00	33	0.364
589	A	17	14	1.00	33	0.424
590	A	16	13	1.00	33	0.394
591	A	15	12	1.00	33	0.364
592	A	15	12	1.00	33	0.364
593	A	16	13	1.00	33	0.394
594	A	17	14	1.00	33	0.424
595	A	17	14	1.00	33	0.424
596	A	16	13	1.00	33	0.394
597	A	16	13	1.00	33	0.394
598	A	16	13	1.00	33	0.394
599	A	17	14	1.00	33	0.424
600	A	18	15	1.00	33	0.454
601	A	17	14	1.00	33	0.424
602	A	17	14	1.00	33	0.424
603	A	17	14	1.00	33	0.424
604	A	17	14	1.00	33	0.424
605	A	18	14	1.00	33	0.424
606	A	13	10	1.00	36	0.278
607	A	13	10	1.00	36	0.278
608	A	12	9	1.00	36	0.250
609	A	12	9	1.00	36	0.250
610	A	13	10	1.00	36	0.278
611	A	13	10	1.00	36	0.278
612	A	12	8	1.00	35	0.229
613	A	11	8	1.00	35	0.229
614	A	10	8	1.00	35	0.229
615	A	9	7	1.00	35	0.200
616	A	13	10	1.00	35	0.286
617	A	14	10	1.00	35	0.286
618	A	15	11	1.00	35	0.314
619	A	13	8	1.00	35	0.229
620	A	12	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	11	8	1.00	35	0.229
622	A	10	8	1.00	35	0.229
623	A	14	10	1.00	35	0.286
624	A	14	10	1.00	35	0.286
625	A	15	11	1.00	35	0.314
626	A	16	11	1.00	35	0.314
627	A	14	9	1.00	35	0.257
628	A	13	9	1.00	35	0.257
629	A	12	9	1.00	35	0.257
630	A	11	9	1.00	35	0.257
631	A	15	11	1.00	35	0.314
632	A	15	11	1.00	35	0.314
633	A	15	11	1.00	35	0.314
634	A	16	11	1.00	35	0.314
635	A	17	11	1.00	35	0.314
636	A	11	8	1.00	35	0.229
637	A	10	8	1.00	35	0.229
638	A	9	7	1.00	35	0.200
639	A	8	6	1.00	35	0.171
640	A	13	10	1.00	35	0.286
641	A	14	10	1.00	35	0.286
642	A	11	8	1.00	35	0.229
643	A	10	8	1.00	35	0.229
644	A	9	7	1.00	35	0.200
645	A	9	7	1.00	35	0.200
646	A	14	10	1.00	35	0.286
647	A	12	8	1.00	35	0.229
648	A	11	8	1.00	35	0.229
649	A	10	8	1.00	35	0.229
650	A	10	8	1.00	35	0.229
651	A	10	8	1.00	35	0.229
652	A	15	11	1.00	35	0.314
653	A	9	7	1.00	38	0.184
654	A	9	7	1.00	38	0.184
655	A	14	11	1.00	38	0.290

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	8	5	1.00	31	0.161
657	A	10	6	1.00	33	0.182
658	A	10	6	1.00	33	0.182
659	A	10	6	1.00	33	0.182
660	A	10	6	1.00	33	0.182
661	A	9	5	1.00	33	0.152
662	A	9	5	1.00	33	0.152
663	A	9	5	1.00	33	0.152
664	A	9	5	1.00	33	0.152
665	A	3	2	1.00	39	0.051
666	A	3	2	1.00	39	0.051
667	A	3	2	1.00	39	0.051
668	A	3	2	1.00	39	0.051
669	A	2	1	1.00	37	0.027
670	A	2	2	1.00	24	0.083
671	A	3	2	1.00	39	0.051
672	A	2	2	1.00	39	0.051
673	A	3	2	1.00	39	0.051
674	A	3	2	1.00	39	0.051
675	A	3	2	1.00	39	0.051
676	A	3	2	1.00	41	0.049
677	A	3	2	1.00	41	0.049
678	A	3	2	1.00	41	0.049
679	A	3	2	1.00	41	0.049
680	A	4	3	1.00	41	0.073
681	A	3	2	1.00	39	0.051
682	A	3	3	1.00	26	0.115
683	A	3	2	1.00	41	0.049
684	A	3	2	1.00	41	0.049
685	A	3	2	1.00	41	0.049
686	A	3	2	1.00	41	0.049
687	A	3	2	1.00	41	0.049
688	A	3	2	1.00	41	0.049
689	A	3	2	1.00	41	0.049
690	A	3	2	1.00	41	0.049

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	3	2	1.00	41	0.049
692	A	3	2	1.00	41	0.049
693	A	5	4	1.00	41	0.098
694	A	3	2	1.00	41	0.049
695	A	3	2	1.00	39	0.051
696	A	4	4	1.00	26	0.154
697	A	3	2	1.00	41	0.049
698	A	3	2	1.00	41	0.049
699	A	4	3	1.00	41	0.073
700	A	3	3	1.00	41	0.073
701	A	3	2	1.00	41	0.049
702	A	3	2	1.00	41	0.049
703	A	3	2	1.00	41	0.049
704	A	3	2	1.00	41	0.049
705	A	3	3	1.00	41	0.073
706	A	3	2	1.00	41	0.049
707	A	3	2	1.00	41	0.049
708	A	3	2	1.00	41	0.049
709	A	3	2	1.00	39	0.051
710	A	2	2	1.00	26	0.077
711	A	4	4	1.00	41	0.098
712	A	4	3	1.00	41	0.073
713	A	4	3	1.00	41	0.073
714	A	4	3	1.00	41	0.073
715	A	3	3	1.00	41	0.073
716	A	3	2	1.00	41	0.049
717	A	3	2	1.00	41	0.049
718	A	3	2	1.00	41	0.049
719	A	3	2	1.00	41	0.049
720	A	2	2	1.00	39	0.051
721	A	3	3	1.00	26	0.115
722	A	4	3	1.00	41	0.073
723	A	5	5	1.00	41	0.122
724	A	4	3	1.00	41	0.073
725	A	4	3	1.00	41	0.073

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	4	3	1.00	41	0.073
727	A	3	3	1.00	41	0.073
728	A	3	2	1.00	41	0.049
729	A	3	2	1.00	41	0.049
730	A	4	3	1.00	41	0.073
731	A	3	2	1.00	41	0.049
732	A	3	2	1.00	39	0.051
733	A	4	3	1.00	26	0.115
734	A	4	3	1.00	41	0.073
735	A	4	3	1.00	41	0.073
736	A	6	5	1.00	41	0.122
737	A	4	3	1.00	41	0.073
738	A	4	3	1.00	41	0.073
739	A	4	3	1.00	41	0.073
740	A	3	2	1.00	41	0.049
741	A	3	2	1.00	41	0.049
742	A	3	2	1.00	41	0.049
743	A	3	2	1.00	41	0.049
744	A	3	2	1.00	41	0.049
745	A	3	2	1.00	41	0.049
746	A	3	2	1.00	41	0.049
747	A	3	2	1.00	41	0.049
748	A	3	2	1.00	43	0.047
749	A	3	2	1.00	43	0.047
750	A	3	2	1.00	43	0.047
751	A	3	2	1.00	43	0.047
752	A	3	2	1.00	43	0.047
753	A	3	2	1.00	43	0.047
754	A	3	2	1.00	43	0.047
755	A	3	2	1.00	43	0.047
756	A	3	2	1.00	43	0.047
757	A	3	2	1.00	43	0.047
758	A	3	2	1.00	43	0.047
759	A	3	2	1.00	43	0.047
760	A	3	2	1.00	43	0.047

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	3	2	1.00	43	0.047
762	A	3	2	1.00	43	0.047
763	A	3	2	1.00	43	0.047
764	A	7	5	1.00	43	0.116
765	A	6	5	1.00	43	0.116
766	A	5	5	1.00	43	0.116
767	A	4	4	1.00	43	0.093
768	A	5	5	1.00	43	0.116
769	A	6	5	1.00	43	0.116
770	A	7	5	1.00	43	0.116
771	A	8	6	1.00	43	0.140
772	A	7	6	1.00	43	0.140
773	A	6	6	1.00	43	0.140
774	A	5	5	1.00	43	0.116
775	A	5	5	1.00	43	0.116
776	A	6	6	1.00	43	0.140
777	A	7	6	1.00	43	0.140
778	A	8	6	1.00	43	0.140
779	A	8	6	1.00	43	0.140
780	A	7	6	1.00	43	0.140
781	A	6	5	1.00	43	0.116
782	A	6	6	1.00	43	0.140
783	A	6	5	1.00	43	0.116
784	A	7	6	1.00	43	0.140
785	A	8	6	1.00	43	0.140
786	A	9	6	1.00	43	0.140
787	A	8	6	1.00	45	0.133
788	A	7	6	1.00	45	0.133
789	A	6	6	1.00	45	0.133
790	A	5	5	1.00	45	0.111
791	A	5	5	1.00	45	0.111
792	A	3	3	1.00	45	0.067
793	A	4	4	1.00	45	0.089
794	A	5	4	1.00	45	0.089
795	A	8	7	1.00	45	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	7	7	1.00	45	0.156
797	A	6	6	1.00	45	0.133
798	A	6	6	1.00	45	0.133
799	A	6	6	1.00	45	0.133
800	A	6	6	1.00	45	0.133
801	A	3	3	1.00	45	0.067
802	A	4	4	1.00	45	0.089
803	A	5	4	1.00	45	0.089
804	A	6	4	1.00	45	0.089
805	A	8	7	1.00	45	0.156
806	A	7	6	1.00	45	0.133
807	A	7	7	1.00	45	0.156
808	A	7	6	1.00	45	0.133
809	A	7	6	1.00	45	0.133
810	A	7	7	1.00	45	0.156
811	A	7	6	1.00	45	0.133
812	A	3	3	1.00	45	0.067
813	A	4	4	1.00	45	0.089
814	A	5	4	1.00	45	0.089
815	A	6	4	1.00	45	0.089
816	A	9	7	1.00	45	0.156
817	A	8	6	1.00	45	0.133
818	A	8	7	1.00	45	0.156
819	A	8	7	1.00	45	0.156
820	A	8	6	1.00	45	0.133
821	A	8	6	1.00	45	0.133
822	A	8	7	1.00	45	0.156
823	A	8	7	1.00	45	0.156
824	A	8	6	1.00	45	0.133
825	A	3	3	1.00	45	0.067
826	A	4	4	1.00	45	0.089
827	A	5	4	1.00	45	0.089
828	A	6	4	1.00	45	0.089
829	A	7	4	1.00	45	0.089
830	A	7	6	1.00	45	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	6	6	1.00	45	0.133
832	A	5	5	1.00	45	0.111
833	A	3	3	1.00	45	0.067
834	A	4	4	1.00	45	0.089
835	A	5	4	1.00	45	0.089
836	A	8	7	1.00	45	0.156
837	A	7	7	1.00	45	0.156
838	A	6	6	1.00	45	0.133
839	A	3	3	1.00	45	0.067
840	A	4	4	1.00	45	0.089
841	A	4	4	0.99	45	0.089
842	A	6	4	1.00	45	0.089
843	A	9	7	1.00	45	0.156
844	A	8	7	1.00	45	0.156
845	A	7	6	1.00	45	0.133
846	A	3	3	1.00	45	0.067
847	A	4	4	1.00	45	0.089
848	A	5	4	1.00	45	0.089
849	A	5	4	0.99	45	0.089
850	A	5	5	0.99	45	0.111
851	A	4	4	1.00	41	0.098
852	A	4	4	1.00	47	0.085
853	A	2	2	1.00	46	0.043
854	A	3	3	1.00	36	0.083
855	A	4	4	1.00	38	0.105

# Chapter 3

## Listing of integrals

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3.5	$\int \cot^2(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$	243
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3.8	$\int \cot^5(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$	255
3.9	$\int \tan^2(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx$	260
3.10	$\int \tan(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx$	265
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3.14	$\int \cot^3(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx$	281
3.15	$\int \cot^4(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx$	285
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3.17	$\int \tan^2(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	295
3.18	$\int \tan(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	300
3.19	$\int (a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	305
3.20	$\int \cot(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	309
3.21	$\int \cot^2(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	314
3.22	$\int \cot^3(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	319
3.23	$\int \cot^4(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	324
3.24	$\int \cot^5(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	329
3.25	$\int \cot^6(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx$	334
3.26	$\int \tan^2(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx$	340
3.27	$\int \tan(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx$	346
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3.29	$\int \cot(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	356
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3.31	$\int \cot^3(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	367
3.32	$\int \cot^4(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	372
3.33	$\int \cot^5(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	377
3.34	$\int \cot^6(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	382
3.35	$\int \cot^7(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	387
3.36	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	393
3.37	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	398
3.38	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	402
3.39	$\int \frac{A+B \tan(c+dx)}{a+ia \tan(c+dx)} dx$	406
3.40	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	409
3.41	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	413
3.42	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	417
3.43	$\int \frac{\cot^4(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	422
3.44	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	427
3.45	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	431
3.46	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	435
3.47	$\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^2} dx$	439
3.48	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	443
3.49	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	447
3.50	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	452
3.51	$\int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	457
3.52	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	462
3.53	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	467
3.54	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	471
3.55	$\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^3} dx$	475
3.56	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	479
3.57	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	483
3.58	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	488
3.59	$\int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	493
3.60	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	498
3.61	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	502
3.62	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	506
3.63	$\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^4} dx$	510



3.64	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	514
3.65	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	518
3.66	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	523
3.67	$\int \tan^3(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	529
3.68	$\int \tan^2(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	534
3.69	$\int \tan(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	539
3.70	$\int \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	543
3.71	$\int \cot(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	547
3.72	$\int \cot^2(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	552
3.73	$\int \cot^3(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	558
3.74	$\int \cot^4(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	565
3.75	$\int \tan^2(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	572
3.76	$\int \tan(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	577
3.77	$\int (a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	582
3.78	$\int \cot(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	586
3.79	$\int \cot^2(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	592
3.80	$\int \cot^3(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	599
3.81	$\int \cot^4(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	607
3.82	$\int \tan^2(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	615
3.83	$\int \tan(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	621
3.84	$\int (a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	626
3.85	$\int \cot(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	630
3.86	$\int \cot^2(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	636
3.87	$\int \cot^3(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	643
3.88	$\int \cot^4(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	651
3.89	$\int \cot^5(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	660
3.90	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	669
3.91	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	674
3.92	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	679
3.93	$\int \frac{A+B \tan(c+dx)}{\sqrt{a+ia \tan(c+dx)}} dx$	683
3.94	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	687
3.95	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	693
3.96	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	701
3.97	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	710
3.98	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	715
3.99	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	720

3.100	$\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^{3/2}} dx$	724
3.101	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	728
3.102	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	734
3.103	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	743
3.104	$\int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	752
3.105	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	758
3.106	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	763
3.107	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	768
3.108	$\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^{5/2}} dx$	773
3.109	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	777
3.110	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	783
3.111	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	792
3.112	$\int \tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	801
3.113	$\int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	806
3.114	$\int \sqrt{\tan(c+dx)}(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	811
3.115	$\int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	816
3.116	$\int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	820
3.117	$\int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	824
3.118	$\int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$	829
3.119	$\int \tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	834
3.120	$\int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	839
3.121	$\int \sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	844
3.122	$\int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	849
3.123	$\int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	854
3.124	$\int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	859
3.125	$\int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$	864
3.126	$\int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$	869
3.127	$\int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	874
3.128	$\int \sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	880
3.129	$\int \frac{(a+ia \tan(c+dx))^3(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	885
3.130	$\int \frac{(a+ia \tan(c+dx))^3(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	890
3.131	$\int \frac{(a+ia \tan(c+dx))^3(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	895

3.132	$\int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$	900
3.133	$\int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$	905
3.134	$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	910
3.135	$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	916
3.136	$\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	922
3.137	$\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))} dx$	928
3.138	$\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))} dx$	934
3.139	$\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))} dx$	940
3.140	$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	946
3.141	$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	952
3.142	$\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	958
3.143	$\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^2} dx$	964
3.144	$\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^2} dx$	970
3.145	$\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^2} dx$	976
3.146	$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	982
3.147	$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	989
3.148	$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	995
3.149	$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	1001
3.150	$\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	1007
3.151	$\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^3} dx$	1013
3.152	$\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx$	1019
3.153	$\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx$	1025
3.154	$\int \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	1032
3.155	$\int \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$	1038
3.156	$\int \frac{\sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	1045
3.157	$\int \frac{\sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	1051
3.158	$\int \frac{\sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	1056

- 3.159  $\int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx \dots \dots \dots 1061$
- 3.160  $\int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx \dots \dots \dots 1066$
- 3.161  $\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx \dots \dots \dots 1071$
- 3.162  $\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx \dots \dots \dots 1077$
- 3.163  $\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \dots \dots \dots 1083$
- 3.164  $\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx \dots \dots \dots 1089$
- 3.165  $\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx \dots \dots \dots 1095$
- 3.166  $\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx \dots \dots \dots 1100$
- 3.167  $\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx \dots \dots \dots 1106$
- 3.168  $\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)} dx \dots \dots \dots 1112$
- 3.169  $\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx \dots \dots \dots 1118$
- 3.170  $\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx \dots \dots \dots 1125$
- 3.171  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \dots \dots \dots 1131$
- 3.172  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx \dots \dots \dots 1137$
- 3.173  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx \dots \dots \dots 1143$
- 3.174  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx \dots \dots \dots 1149$
- 3.175  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx \dots \dots \dots 1155$
- 3.176  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)} dx \dots \dots \dots 1161$
- 3.177  $\int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\tan^{\frac{13}{2}}(c + dx)} dx \dots \dots \dots 1167$
- 3.178  $\int \frac{(a + ia \tan(c + dx))^{5/2} \left( \frac{3bE}{2a} + B \tan(c + dx) \right)}{\tan^{\frac{5}{2}}(c + dx)} dx \dots \dots \dots 1173$
- 3.179  $\int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} dx \dots \dots \dots 1179$
- 3.180  $\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} dx \dots \dots \dots 1185$
- 3.181  $\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} dx \dots \dots \dots 1192$
- 3.182  $\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} dx \dots \dots \dots 1197$
- 3.183  $\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} dx \dots \dots \dots 1202$
- 3.184  $\int \frac{A + B \tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} dx \dots \dots \dots 1208$
- 3.185  $\int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx \dots \dots \dots 1214$

3.186	$\int \frac{\sqrt{\tan(c+dx)}(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	1220
3.187	$\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{3/2}} dx$	1225
3.188	$\int \frac{A+B \tan(c+dx)}{\tan^{3/2}(c+dx)(a+ia \tan(c+dx))^{3/2}} dx$	1230
3.189	$\int \frac{A+B \tan(c+dx)}{\tan^{5/2}(c+dx)(a+ia \tan(c+dx))^{3/2}} dx$	1236
3.190	$\int \frac{\tan^{5/2}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	1242
3.191	$\int \frac{\tan^{3/2}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	1248
3.192	$\int \frac{\sqrt{\tan(c+dx)}(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	1254
3.193	$\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{5/2}} dx$	1260
3.194	$\int \frac{A+B \tan(c+dx)}{\tan^{3/2}(c+dx)(a+ia \tan(c+dx))^{5/2}} dx$	1265
3.195	$\int \frac{A+B \tan(c+dx)}{\tan^{5/2}(c+dx)(a+ia \tan(c+dx))^{5/2}} dx$	1271
3.196	$\int \sqrt[3]{a+ia \tan(c+dx)}(A+B \tan(c+dx)) dx$	1277
3.197	$\int \tan^2(c+dx)(a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$	1282
3.198	$\int \tan(c+dx)(a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$	1288
3.199	$\int (a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$	1294
3.200	$\int \cot(c+dx)(a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$	1299
3.201	$\int \cot^2(c+dx)(a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$	1305
3.202	$\int \frac{A+B \tan(c+dx)}{\sqrt[3]{a+ia \tan(c+dx)}} dx$	1312
3.203	$\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^{2/3}} dx$	1318
3.204	$\int \tan^m(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$	1323
3.205	$\int \tan^m(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	1329
3.206	$\int \tan^m(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	1334
3.207	$\int \tan^m(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	1339
3.208	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	1342
3.209	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	1346
3.210	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	1350
3.211	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$	1354
3.212	$\int \tan^m(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	1359
3.213	$\int \tan^m(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1364
3.214	$\int \tan^m(c+dx)\sqrt{a+ia \tan(c+dx)}(A+B \tan(c+dx)) dx$	1369
3.215	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$	1373
3.216	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$	1378
3.217	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$	1383
3.218	$\int \tan^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$	1388
3.219	$\int \tan^3(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$	1392

3.220	$\int \tan^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1396
3.221	$\int \tan(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1400
3.222	$\int (a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1404
3.223	$\int \cot(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1407
3.224	$\int \cot^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1411
3.225	$\int \cot^3(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1415
3.226	$\int \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1419
3.227	$\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1424
3.228	$\int \sqrt{\tan(c + dx)}(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$	1429
3.229	$\int \frac{(a + ia \tan(c + dx))^n(A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$	1434
3.230	$\int \frac{(a + ia \tan(c + dx))^n(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$	1439
3.231	$\int \frac{(a + ia \tan(c + dx))^n(A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$	1444
3.232	$\int \tan^2(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1449
3.233	$\int \tan(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1454
3.234	$\int (a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1458
3.235	$\int \cot(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1461
3.236	$\int \cot^2(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1465
3.237	$\int \cot^3(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1469
3.238	$\int \cot^4(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1473
3.239	$\int \cot^5(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx$	1477
3.240	$\int \tan^2(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1482
3.241	$\int \tan(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1488
3.242	$\int (a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1493
3.243	$\int \cot(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1497
3.244	$\int \cot^2(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1501
3.245	$\int \cot^3(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1505
3.246	$\int \cot^4(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1509
3.247	$\int \cot^5(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx$	1514
3.248	$\int \tan^2(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1519
3.249	$\int \tan(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1525
3.250	$\int (a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1531
3.251	$\int \cot(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1536
3.252	$\int \cot^2(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1540
3.253	$\int \cot^3(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1544
3.254	$\int \cot^4(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1548
3.255	$\int \cot^5(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1553
3.256	$\int \cot^6(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx$	1559
3.257	$\int \tan^2(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx$	1565
3.258	$\int \tan(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx$	1572
3.259	$\int (a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx$	1578
3.260	$\int \cot(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx$	1584
3.261	$\int \cot^2(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx$	1589

3.262	$\int \cot^3(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$	1594
3.263	$\int \cot^4(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$	1599
3.264	$\int \cot^5(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$	1604
3.265	$\int \cot^6(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$	1610
3.266	$\int \cot^7(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$	1616
3.267	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1623
3.268	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1629
3.269	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1634
3.270	$\int \frac{A+B \tan(c+dx)}{a+b \tan(c+dx)} dx$	1639
3.271	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1643
3.272	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1647
3.273	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1652
3.274	$\int \frac{\cot^4(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	1658
3.275	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1665
3.276	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1672
3.277	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1678
3.278	$\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^2} dx$	1684
3.279	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1690
3.280	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1696
3.281	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1703
3.282	$\int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1710
3.283	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1717
3.284	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1722
3.285	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1727
3.286	$\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^3} dx$	1732
3.287	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1737
3.288	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1742
3.289	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	1748
3.290	$\int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1754
3.291	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1761
3.292	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1767
3.293	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1773
3.294	$\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^4} dx$	1778
3.295	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1783
3.296	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1789

3.297	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	1796
3.298	$\int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1803
3.299	$\int \frac{\tan^2(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1807
3.300	$\int \frac{\tan(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1810
3.301	$\int \frac{aB+bB \tan(c+dx)}{a+b \tan(c+dx)} dx$	1813
3.302	$\int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1816
3.303	$\int \frac{\cot^2(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1819
3.304	$\int \frac{\cot^3(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1822
3.305	$\int \frac{\cot^4(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$	1826
3.306	$\int \frac{\tan^4(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1830
3.307	$\int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1836
3.308	$\int \frac{\tan^2(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1841
3.309	$\int \frac{\tan(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1846
3.310	$\int \frac{aB+bB \tan(c+dx)}{(a+b \tan(c+dx))^2} dx$	1850
3.311	$\int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1854
3.312	$\int \frac{\cot^2(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1858
3.313	$\int \frac{\cot^3(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	1863
3.314	$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)} dx$	1869
3.315	$\int \frac{\frac{bB}{a}+B \tan(c+dx)}{a+b \tan(c+dx)} dx$	1872
3.316	$\int \frac{a+b \tan(c+dx)}{(b+a \tan(c+dx))^2} dx$	1876
3.317	$\int \tan^3(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1881
3.318	$\int \tan^2(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1889
3.319	$\int \tan(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1896
3.320	$\int \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1903
3.321	$\int \cot(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1909
3.322	$\int \cot^2(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1916
3.323	$\int \cot^3(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1924
3.324	$\int \cot^4(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	1932
3.325	$\int \tan^2(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1940
3.326	$\int \tan(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1947
3.327	$\int (a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1954
3.328	$\int \cot(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1960
3.329	$\int \cot^2(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1966
3.330	$\int \cot^3(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1972
3.331	$\int \cot^4(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	1979
3.332	$\int \tan^2(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	1986
3.333	$\int \tan(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	1993



3.334	$\int (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$	2000
3.335	$\int \cot(c + dx) (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$	2006
3.336	$\int \cot^2(c + dx) (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$	2012
3.337	$\int \cot^3(c + dx) (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$	2019
3.338	$\int \cot^4(c + dx) (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$	2026
3.339	$\int \cot^5(c + dx) (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$	2033
3.340	$\int (-a + b \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$	2041
3.341	$\int (-a + b \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$	2050
3.342	$\int (-a + b \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$	2058
3.343	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a + b \tan(c + dx)}} dx$	2066
3.344	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a + b \tan(c + dx)}} dx$	2075
3.345	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{\sqrt{a + b \tan(c + dx)}} dx$	2083
3.346	$\int \frac{A+B \tan(c+dx)}{\sqrt{a + b \tan(c + dx)}} dx$	2090
3.347	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{\sqrt{a + b \tan(c + dx)}} dx$	2097
3.348	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a + b \tan(c + dx)}} dx$	2104
3.349	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a + b \tan(c + dx)}} dx$	2112
3.350	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2120
3.351	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2129
3.352	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2138
3.353	$\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx$	2146
3.354	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2154
3.355	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2162
3.356	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2170
3.357	$\int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2179
3.358	$\int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2190
3.359	$\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2199
3.360	$\int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2208
3.361	$\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^{5/2}} dx$	2217
3.362	$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2226
3.363	$\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2233
3.364	$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2240
3.365	$\int \frac{aB+bB \tan(c+dx)}{\sqrt{a + b \tan(c + dx)}} dx$	2248
3.366	$\int \frac{aB+bB \tan(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx$	2256

3.367	$\int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	2264
3.368	$\int \frac{aB+bB \tan(c+dx)}{(a+b \tan(c+dx))^{5/2}} dx$	2271
3.369	$\int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	2280
3.370	$\int \frac{-a+b \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$	2288
3.371	$\int \frac{-a+b \tan(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx$	2295
3.372	$\int \frac{-a+b \tan(c+dx)}{(a+b \tan(c+dx))^{5/2}} dx$	2303
3.373	$\int \frac{1+i \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$	2311
3.374	$\int \frac{1-i \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$	2318
3.375	$\int \frac{3+\tan(x)}{\sqrt{4+3 \tan(x)}} dx$	2323
3.376	$\int \frac{1-3 \tan(x)}{\sqrt{4+3 \tan(x)}} dx$	2326
3.377	$\int \frac{4-3 \tan(a+bx)}{\sqrt{4+3 \tan(a+bx)}} dx$	2329
3.378	$\int \tan^{5/2}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$	2334
3.379	$\int \tan^{3/2}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$	2342
3.380	$\int \sqrt{\tan(c+dx)}(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$	2349
3.381	$\int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	2356
3.382	$\int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\tan^{3/2}(c+dx)} dx$	2363
3.383	$\int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\tan^{5/2}(c+dx)} dx$	2370
3.384	$\int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\tan^{7/2}(c+dx)} dx$	2377
3.385	$\int \tan^{5/2}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	2385
3.386	$\int \tan^{3/2}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	2394
3.387	$\int \sqrt{\tan(c+dx)}(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	2403
3.388	$\int \frac{(a+b \tan(c+dx))^2(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	2412
3.389	$\int \frac{(a+b \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{3/2}(c+dx)} dx$	2420
3.390	$\int \frac{(a+b \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{5/2}(c+dx)} dx$	2428
3.391	$\int \frac{(a+b \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{7/2}(c+dx)} dx$	2436
3.392	$\int \tan^{3/2}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	2445
3.393	$\int \sqrt{\tan(c+dx)}(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	2453
3.394	$\int \frac{(a+b \tan(c+dx))^3(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	2461
3.395	$\int \frac{(a+b \tan(c+dx))^3(A+B \tan(c+dx))}{\tan^{3/2}(c+dx)} dx$	2469
3.396	$\int \frac{(a+b \tan(c+dx))^3(A+B \tan(c+dx))}{\tan^{5/2}(c+dx)} dx$	2477

- 3.397  $\int \frac{(a+b \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^2(c+dx)} dx \dots\dots\dots 2485$
- 3.398  $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 2493$
- 3.399  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 2502$
- 3.400  $\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 2511$
- 3.401  $\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))} dx \dots\dots\dots 2520$
- 3.402  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))} dx \dots\dots\dots 2529$
- 3.403  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx \dots\dots\dots 2538$
- 3.404  $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx \dots\dots\dots 2548$
- 3.405  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx \dots\dots\dots 2558$
- 3.406  $\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx \dots\dots\dots 2567$
- 3.407  $\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^2} dx \dots\dots\dots 2576$
- 3.408  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2} dx \dots\dots\dots 2585$
- 3.409  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2} dx \dots\dots\dots 2595$
- 3.410  $\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx \dots\dots\dots 2603$
- 3.411  $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx \dots\dots\dots 2613$
- 3.412  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx \dots\dots\dots 2622$
- 3.413  $\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx \dots\dots\dots 2631$
- 3.414  $\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^3} dx \dots\dots\dots 2640$
- 3.415  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3} dx \dots\dots\dots 2649$
- 3.416  $\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 2658$
- 3.417  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 2665$
- 3.418  $\int \frac{\sqrt{\tan(c+dx)} (aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 2672$
- 3.419  $\int \frac{aB+bB \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))} dx \dots\dots\dots 2679$
- 3.420  $\int \frac{aB+bB \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))} dx \dots\dots\dots 2686$
- 3.421  $\int \frac{aB+bB \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx \dots\dots\dots 2693$
- 3.422  $\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx \dots\dots\dots 2700$
- 3.423  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx \dots\dots\dots 2709$

3.424	$\int \frac{\sqrt{\tan(c+dx)} (aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	2718
3.425	$\int \frac{aB+bB \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^2} dx$	2727
3.426	$\int \frac{aB+bB \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2} dx$	2736
3.427	$\int \tan^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	2745
3.428	$\int \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	2751
3.429	$\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	2758
3.430	$\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	2763
3.431	$\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	2768
3.432	$\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$	2773
3.433	$\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$	2778
3.434	$\int \tan^{\frac{3}{2}}(c+dx) (a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx$	2783
3.435	$\int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx$	2789
3.436	$\int \frac{(a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	2795
3.437	$\int \frac{(a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	2800
3.438	$\int \frac{(a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	2805
3.439	$\int \frac{(a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$	2810
3.440	$\int \frac{(a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$	2815
3.441	$\int \frac{(a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)} dx$	2820
3.442	$\int \tan^{\frac{3}{2}}(c+dx) (a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx)) dx$	2826
3.443	$\int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx)) dx$	2832
3.444	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$	2838
3.445	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$	2844
3.446	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$	2850
3.447	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$	2856
3.448	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$	2861
3.449	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)} dx$	2867
3.450	$\int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)} dx$	2873

- 3.451  $\int \frac{(a+b \tan(c+dx))^{5/2} \left(\frac{3bE}{2a} + B \tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots 2880$
- 3.452  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 2886$
- 3.453  $\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 2891$
- 3.454  $\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 2897$
- 3.455  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 2902$
- 3.456  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 2907$
- 3.457  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 2912$
- 3.458  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2917$
- 3.459  $\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2922$
- 3.460  $\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2927$
- 3.461  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2932$
- 3.462  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2937$
- 3.463  $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 2942$
- 3.464  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 2948$
- 3.465  $\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 2953$
- 3.466  $\int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 2958$
- 3.467  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 2963$
- 3.468  $\int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 2968$
- 3.469  $\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2973$
- 3.470  $\int \frac{\sqrt{\tan(c+dx)} (aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2978$
- 3.471  $\int \frac{aB+bB \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2983$
- 3.472  $\int \frac{aB+bB \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 2987$
- 3.473  $\int (a+b \tan(c+dx))^{2/3} (A+B \tan(c+dx)) dx \dots\dots\dots 2992$
- 3.474  $\int \sqrt[3]{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx \dots\dots\dots 2998$
- 3.475  $\int \frac{A+B \tan(c+dx)}{\sqrt[3]{a+b \tan(c+dx)}} dx \dots\dots\dots 3006$
- 3.476  $\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^{2/3}} dx \dots\dots\dots 3012$
- 3.477  $\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d \tan(e+fx)}} dx \dots\dots\dots 3018$

3.478	$\int \frac{d-c \tan(e+fx)}{(c+d \tan(e+fx))^{2/3}} dx$	3025
3.479	$\int \tan^m(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$	3033
3.480	$\int \tan^m(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3038
3.481	$\int \tan^m(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3043
3.482	$\int \tan^m(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$	3047
3.483	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	3051
3.484	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	3055
3.485	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	3060
3.486	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$	3065
3.487	$\int \tan^m(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$	3071
3.488	$\int \tan^m(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	3075
3.489	$\int \tan^m(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$	3079
3.490	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$	3083
3.491	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$	3087
3.492	$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$	3091
3.493	$\int \tan^m(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3095
3.494	$\int \tan^4(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3099
3.495	$\int \tan^3(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3104
3.496	$\int \tan^2(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3109
3.497	$\int \tan(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3113
3.498	$\int (a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3117
3.499	$\int \cot(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3120
3.500	$\int \cot^2(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3124
3.501	$\int \cot^3(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$	3128
3.502	$\int \cot^{7/2}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	3133
3.503	$\int \cot^{5/2}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	3139
3.504	$\int \cot^{3/2}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	3144
3.505	$\int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$	3149
3.506	$\int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3154
3.507	$\int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\cot^{3/2}(c+dx)} dx$	3159
3.508	$\int \cot^{7/2}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3164
3.509	$\int \cot^{5/2}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3170
3.510	$\int \cot^{3/2}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3175
3.511	$\int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3180
3.512	$\int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3185
3.513	$\int \cot^{9/2}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3191
3.514	$\int \cot^{7/2}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3198
3.515	$\int \cot^{5/2}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3204

3.516	$\int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3210
3.517	$\int \sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3215
3.518	$\int \frac{(a+ia \tan(c+dx))^3(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3220
3.519	$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	3226
3.520	$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	3233
3.521	$\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$	3240
3.522	$\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))} dx$	3246
3.523	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))} dx$	3251
3.524	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))} dx$	3257
3.525	$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	3264
3.526	$\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$	3272
3.527	$\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^2} dx$	3278
3.528	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^2} dx$	3284
3.529	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^2} dx$	3290
3.530	$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	3296
3.531	$\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$	3304
3.532	$\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^3} dx$	3310
3.533	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx$	3316
3.534	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx$	3322
3.535	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx$	3328
3.536	$\int \cot^{\frac{7}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}(A+B \tan(c+dx)) dx$	3334
3.537	$\int \cot^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}(A+B \tan(c+dx)) dx$	3341
3.538	$\int \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}(A+B \tan(c+dx)) dx$	3347
3.539	$\int \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}(A+B \tan(c+dx)) dx$	3352
3.540	$\int \frac{\sqrt{a+ia \tan(c+dx)}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3358
3.541	$\int \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	3365
3.542	$\int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	3373
3.543	$\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	3380
3.544	$\int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	3387
3.545	$\int \sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$	3393

- 3.546  $\int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 3399$
- 3.547  $\int \cot^{\frac{11}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3407$
- 3.548  $\int \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3415$
- 3.549  $\int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3423$
- 3.550  $\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3430$
- 3.551  $\int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3437$
- 3.552  $\int \sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3444$
- 3.553  $\int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 3451$
- 3.554  $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx \dots\dots\dots 3459$
- 3.555  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx \dots\dots\dots 3465$
- 3.556  $\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx \dots\dots\dots 3470$
- 3.557  $\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia \tan(c+dx)}} dx \dots\dots\dots 3475$
- 3.558  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx \dots\dots\dots 3481$
- 3.559  $\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx \dots\dots\dots 3487$
- 3.560  $\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^{3/2}} dx \dots\dots\dots 3492$
- 3.561  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}} dx \dots\dots\dots 3498$
- 3.562  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx \dots\dots\dots 3505$
- 3.563  $\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx \dots\dots\dots 3511$
- 3.564  $\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^{5/2}} dx \dots\dots\dots 3517$
- 3.565  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}} dx \dots\dots\dots 3523$
- 3.566  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}} dx \dots\dots\dots 3529$
- 3.567  $\int \cot^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 3536$
- 3.568  $\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 3540$
- 3.569  $\int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 3546$
- 3.570  $\int \sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 3551$
- 3.571  $\int \frac{(a+ia \tan(c+dx))^n(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 3556$
- 3.572  $\int \frac{(a+ia \tan(c+dx))^n(A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 3561$
- 3.573  $\int \frac{(a+ia \tan(c+dx))^n(A+B \tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots 3567$
- 3.574  $\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx \dots\dots\dots 3573$
- 3.575  $\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx \dots\dots\dots 3579$



3.576	$\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))(A+B \tan(c+dx)) dx$	3585
3.577	$\int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3591
3.578	$\int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3597
3.579	$\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3603
3.580	$\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3608
3.581	$\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$	3613
3.582	$\int \frac{(a+b \tan(c+dx))^2(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3620
3.583	$\int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3628
3.584	$\int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3634
3.585	$\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3640
3.586	$\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3646
3.587	$\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$	3652
3.588	$\int \frac{(a+b \tan(c+dx))^3(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$	3660
3.589	$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	3666
3.590	$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	3672
3.591	$\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$	3678
3.592	$\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))} dx$	3685
3.593	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))} dx$	3692
3.594	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx$	3698
3.595	$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	3705
3.596	$\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$	3712
3.597	$\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))^2} dx$	3718
3.598	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2} dx$	3724
3.599	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2} dx$	3730
3.600	$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	3737
3.601	$\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$	3744
3.602	$\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))^3} dx$	3751
3.603	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3} dx$	3758
3.604	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^3} dx$	3765
3.605	$\int \frac{A+B \tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^3} dx$	3772

- 3.606  $\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 3779$
- 3.607  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 3785$
- 3.608  $\int \frac{\sqrt{\cot(c+dx)}(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx \dots\dots\dots 3790$
- 3.609  $\int \frac{aB+bB \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b \tan(c+dx))} dx \dots\dots\dots 3795$
- 3.610  $\int \frac{aB+bB \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))} dx \dots\dots\dots 3800$
- 3.611  $\int \frac{aB+bB \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx \dots\dots\dots 3805$
- 3.612  $\int \cot^{\frac{9}{2}}(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx \dots\dots\dots 3810$
- 3.613  $\int \cot^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx \dots\dots\dots 3816$
- 3.614  $\int \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx \dots\dots\dots 3821$
- 3.615  $\int \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx \dots\dots\dots 3826$
- 3.616  $\int \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx \dots\dots\dots 3831$
- 3.617  $\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 3836$
- 3.618  $\int \frac{\sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 3842$
- 3.619  $\int \cot^{\frac{11}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3848$
- 3.620  $\int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3854$
- 3.621  $\int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3860$
- 3.622  $\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3865$
- 3.623  $\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3870$
- 3.624  $\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3876$
- 3.625  $\int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 3882$
- 3.626  $\int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 3888$
- 3.627  $\int \cot^{\frac{13}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3894$
- 3.628  $\int \cot^{\frac{11}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3901$
- 3.629  $\int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3908$
- 3.630  $\int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3914$
- 3.631  $\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3920$
- 3.632  $\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3926$
- 3.633  $\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx \dots\dots\dots 3932$
- 3.634  $\int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 3938$
- 3.635  $\int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 3944$
- 3.636  $\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 3950$
- 3.637  $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 3955$

- 3.638  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 3960$
- 3.639  $\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 3965$
- 3.640  $\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 3971$
- 3.641  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \dots\dots\dots 3976$
- 3.642  $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 3982$
- 3.643  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 3988$
- 3.644  $\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 3993$
- 3.645  $\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 3998$
- 3.646  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 4003$
- 3.647  $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 4009$
- 3.648  $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 4015$
- 3.649  $\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 4021$
- 3.650  $\int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 4026$
- 3.651  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 4031$
- 3.652  $\int \frac{A+B \tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}} dx \dots\dots\dots 4036$
- 3.653  $\int \frac{\sqrt{\cot(c+dx)}(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 4042$
- 3.654  $\int \frac{aB+bB \tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 4048$
- 3.655  $\int \frac{aB+bB \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}} dx \dots\dots\dots 4054$
- 3.656  $\int \cot^m(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 4061$
- 3.657  $\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 4065$
- 3.658  $\int \sqrt{\cot(c+dx)}(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 4069$
- 3.659  $\int \frac{(a+b \tan(c+dx))^n(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx \dots\dots\dots 4073$
- 3.660  $\int \frac{(a+b \tan(c+dx))^n(A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 4078$
- 3.661  $\int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 4083$
- 3.662  $\int \sqrt{\tan(c+dx)}(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx \dots\dots\dots 4087$
- 3.663  $\int \frac{(a+b \tan(c+dx))^n(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx \dots\dots\dots 4091$
- 3.664  $\int \frac{(a+b \tan(c+dx))^n(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 4095$
- 3.665  $\int (a+ia \tan(e+fx))(A+B \tan(e+fx))(c-ict \tan(e+fx))^n dx \dots\dots\dots 4099$
- 3.666  $\int (a+ia \tan(e+fx))(A+B \tan(e+fx))(c-ict \tan(e+fx))^4 dx \dots\dots\dots 4103$

3.667	$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ict \tan(e + fx))^3 dx$	4107
3.668	$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ict \tan(e + fx))^2 dx$	4111
3.669	$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ict \tan(e + fx)) dx$	4115
3.670	$\int (a + ia \tan(e + fx))(A + B \tan(e + fx)) dx$	4118
3.671	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{c-ic \tan(e+fx)} dx$	4121
3.672	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx$	4125
3.673	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^3} dx$	4129
3.674	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^4} dx$	4133
3.675	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^5} dx$	4137
3.676	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ict \tan(e + fx))^n dx$	4141
3.677	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ict \tan(e + fx))^5 dx$	4146
3.678	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ict \tan(e + fx))^4 dx$	4150
3.679	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ict \tan(e + fx))^3 dx$	4154
3.680	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ict \tan(e + fx))^2 dx$	4158
3.681	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ict \tan(e + fx)) dx$	4162
3.682	$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) dx$	4166
3.683	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{c-ic \tan(e+fx)} dx$	4170
3.684	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx$	4174
3.685	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^3} dx$	4178
3.686	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^4} dx$	4182
3.687	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^5} dx$	4186
3.688	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^6} dx$	4190
3.689	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx))^n dx$	4194
3.690	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx))^6 dx$	4202
3.691	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx))^5 dx$	4206
3.692	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx))^4 dx$	4210
3.693	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx))^3 dx$	4214
3.694	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx))^2 dx$	4219
3.695	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ict \tan(e + fx)) dx$	4223
3.696	$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) dx$	4227
3.697	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{c-ic \tan(e+fx)} dx$	4231
3.698	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx$	4236
3.699	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^3} dx$	4240
3.700	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^4} dx$	4244
3.701	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^5} dx$	4248
3.702	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^6} dx$	4252
3.703	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^7} dx$	4256
3.704	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^8} dx$	4260

3.705	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^n}{a+ia \tan(e+fx)} dx$	4264
3.706	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^4}{a+ia \tan(e+fx)} dx$	4268
3.707	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^3}{a+ia \tan(e+fx)} dx$	4272
3.708	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^2}{a+ia \tan(e+fx)} dx$	4276
3.709	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))}{a+ia \tan(e+fx)} dx$	4280
3.710	$\int \frac{A+B \tan(e+fx)}{a+ia \tan(e+fx)} dx$	4284
3.711	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))} dx$	4287
3.712	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^2} dx$	4291
3.713	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^3} dx$	4295
3.714	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^4} dx$	4299
3.715	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^n}{(a+ia \tan(e+fx))^2} dx$	4303
3.716	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^5}{(a+ia \tan(e+fx))^2} dx$	4307
3.717	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^4}{(a+ia \tan(e+fx))^2} dx$	4312
3.718	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^3}{(a+ia \tan(e+fx))^2} dx$	4317
3.719	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^2}{(a+ia \tan(e+fx))^2} dx$	4321
3.720	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))}{(a+ia \tan(e+fx))^2} dx$	4325
3.721	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2} dx$	4329
3.722	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))} dx$	4333
3.723	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^2} dx$	4337
3.724	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^3} dx$	4341
3.725	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^4} dx$	4345
3.726	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^5} dx$	4350
3.727	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^n}{(a+ia \tan(e+fx))^3} dx$	4355
3.728	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^5}{(a+ia \tan(e+fx))^3} dx$	4359
3.729	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^4}{(a+ia \tan(e+fx))^3} dx$	4364
3.730	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^3}{(a+ia \tan(e+fx))^3} dx$	4368
3.731	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^2}{(a+ia \tan(e+fx))^3} dx$	4372
3.732	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))}{(a+ia \tan(e+fx))^3} dx$	4376
3.733	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3} dx$	4380
3.734	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))} dx$	4384
3.735	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^2} dx$	4388
3.736	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^3} dx$	4392
3.737	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^4} dx$	4397
3.738	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^5} dx$	4402

3.739	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^6} dx$	4407
3.740	$\int (a+ia \tan(e+fx))(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx$	4412
3.741	$\int (a+ia \tan(e+fx))(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$	4416
3.742	$\int (a+ia \tan(e+fx))(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx$	4420
3.743	$\int (a+ia \tan(e+fx))(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)} dx$	4424
3.744	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx$	4428
3.745	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$	4432
3.746	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$	4436
3.747	$\int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$	4440
3.748	$\int (a+ia \tan(e+fx))^2(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx$	4444
3.749	$\int (a+ia \tan(e+fx))^2(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$	4448
3.750	$\int (a+ia \tan(e+fx))^2(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx$	4452
3.751	$\int (a+ia \tan(e+fx))^2(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)} dx$	4456
3.752	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx$	4460
3.753	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$	4464
3.754	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$	4468
3.755	$\int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$	4472
3.756	$\int (a+ia \tan(e+fx))^3(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx$	4476
3.757	$\int (a+ia \tan(e+fx))^3(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$	4480
3.758	$\int (a+ia \tan(e+fx))^3(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx$	4484
3.759	$\int (a+ia \tan(e+fx))^3(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)} dx$	4488
3.760	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx$	4492
3.761	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$	4496
3.762	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$	4500
3.763	$\int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$	4504
3.764	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2}}{a+ia \tan(e+fx)} dx$	4508
3.765	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2}}{a+ia \tan(e+fx)} dx$	4514
3.766	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}{a+ia \tan(e+fx)} dx$	4519
3.767	$\int \frac{(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)}}{a+ia \tan(e+fx)} dx$	4524
3.768	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))\sqrt{c-ic \tan(e+fx)}} dx$	4529
3.769	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^{3/2}} dx$	4534
3.770	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^{5/2}} dx$	4540
3.771	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{9/2}}{(a+ia \tan(e+fx))^2} dx$	4546
3.772	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^2} dx$	4552

3.773	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^2} dx$	4558
3.774	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^2} dx$	4564
3.775	$\int \frac{(A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}}{(a+ia \tan(e+fx))^2} dx$	4569
3.776	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2 \sqrt{c-ic \tan(e+fx)}} dx$	4574
3.777	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2 (c-ic \tan(e+fx))^{3/2}} dx$	4580
3.778	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2 (c-ic \tan(e+fx))^{5/2}} dx$	4586
3.779	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{9/2}}{(a+ia \tan(e+fx))^3} dx$	4592
3.780	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^3} dx$	4598
3.781	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^3} dx$	4604
3.782	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^3} dx$	4609
3.783	$\int \frac{(A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}}{(a+ia \tan(e+fx))^3} dx$	4615
3.784	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3 \sqrt{c-ic \tan(e+fx)}} dx$	4621
3.785	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3 (c-ic \tan(e+fx))^{3/2}} dx$	4627
3.786	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3 (c-ic \tan(e+fx))^{5/2}} dx$	4633
3.787	$\int \sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx$	4639
3.788	$\int \sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$	4646
3.789	$\int \sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx$	4652
3.790	$\int \sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)} dx$	4658
3.791	$\int \frac{\sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx$	4663
3.792	$\int \frac{\sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$	4668
3.793	$\int \frac{\sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$	4672
3.794	$\int \frac{\sqrt{a+ia \tan(e+fx)} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$	4676
3.795	$\int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx$	4681
3.796	$\int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$	4688
3.797	$\int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx$	4694
3.798	$\int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)} dx$	4700
3.799	$\int \frac{(a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx$	4706
3.800	$\int \frac{(a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$	4712
3.801	$\int \frac{(a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$	4718
3.802	$\int \frac{(a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$	4722
3.803	$\int \frac{(a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{9/2}} dx$	4726

- 3.804  $\int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{11/2}} dx \dots\dots\dots 4731$
- 3.805  $\int (a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx \dots\dots 4736$
- 3.806  $\int (a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx \dots\dots 4743$
- 3.807  $\int (a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx \dots\dots 4749$
- 3.808  $\int (a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)} dx \dots\dots 4755$
- 3.809  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx \dots\dots\dots 4761$
- 3.810  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx \dots\dots\dots 4767$
- 3.811  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx \dots\dots\dots 4773$
- 3.812  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx \dots\dots\dots 4779$
- 3.813  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{9/2}} dx \dots\dots\dots 4783$
- 3.814  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{11/2}} dx \dots\dots\dots 4787$
- 3.815  $\int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{13/2}} dx \dots\dots\dots 4792$
- 3.816  $\int (a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{9/2} dx \dots\dots 4797$
- 3.817  $\int (a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2} dx \dots\dots 4805$
- 3.818  $\int (a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx \dots\dots 4812$
- 3.819  $\int (a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2} dx \dots\dots 4819$
- 3.820  $\int (a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)} dx \dots\dots 4826$
- 3.821  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{\sqrt{c-ic \tan(e+fx)}} dx \dots\dots\dots 4833$
- 3.822  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx \dots\dots\dots 4840$
- 3.823  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx \dots\dots\dots 4847$
- 3.824  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx \dots\dots\dots 4853$
- 3.825  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{9/2}} dx \dots\dots\dots 4859$
- 3.826  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{11/2}} dx \dots\dots\dots 4863$
- 3.827  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{13/2}} dx \dots\dots\dots 4867$
- 3.828  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{15/2}} dx \dots\dots\dots 4872$
- 3.829  $\int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{17/2}} dx \dots\dots\dots 4877$
- 3.830  $\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2}}{\sqrt{a+ia \tan(e+fx)}} dx \dots\dots\dots 4883$
- 3.831  $\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}{\sqrt{a+ia \tan(e+fx)}} dx \dots\dots\dots 4890$
- 3.832  $\int \frac{(A+B \tan(e+fx))\sqrt{c-ic \tan(e+fx)}}{\sqrt{a+ia \tan(e+fx)}} dx \dots\dots\dots 4896$
- 3.833  $\int \frac{A+B \tan(e+fx)}{\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}} dx \dots\dots\dots 4901$
- 3.834  $\int \frac{A+B \tan(e+fx)}{\sqrt{a+ia \tan(e+fx)}(c-ic \tan(e+fx))^{3/2}} dx \dots\dots\dots 4905$



3.835	$\int \frac{A+B \tan(e+fx)}{\sqrt{a+ia \tan(e+fx)} (c-ic \tan(e+fx))^{5/2}} dx$	4909
3.836	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^{3/2}} dx$	4913
3.837	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^{3/2}} dx$	4920
3.838	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^{3/2}} dx$	4926
3.839	$\int \frac{(A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}}{(a+ia \tan(e+fx))^{3/2}} dx$	4932
3.840	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{3/2} \sqrt{c-ic \tan(e+fx)}} dx$	4936
3.841	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{3/2} (c-ic \tan(e+fx))^{3/2}} dx$	4940
3.842	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{3/2} (c-ic \tan(e+fx))^{5/2}} dx$	4944
3.843	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{9/2}}{(a+ia \tan(e+fx))^{5/2}} dx$	4949
3.844	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^{5/2}} dx$	4955
3.845	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^{5/2}} dx$	4961
3.846	$\int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^{5/2}} dx$	4967
3.847	$\int \frac{(A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}}{(a+ia \tan(e+fx))^{5/2}} dx$	4971
3.848	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{5/2} \sqrt{c-ic \tan(e+fx)}} dx$	4975
3.849	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{5/2} (c-ic \tan(e+fx))^{3/2}} dx$	4979
3.850	$\int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{5/2} (c-ic \tan(e+fx))^{5/2}} dx$	4983
3.851	$\int (a+ia \tan(e+fx))^m (A+B \tan(e+fx))(c-ic \tan(e+fx))^n dx$	4988
3.852	$\int (a+ia \tan(e+fx))^{1+m} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{-1-m} dx$	4992
3.853	$\int \frac{(c-ic \tan(e+fx))^n (-i(2+n)+(-2+n) \tan(e+fx))}{(-i+\tan(e+fx))^2} dx$	4997
3.854	$\int \frac{(A+B \tan(e+fx))(c+d \tan(e+fx))}{(a+ia \tan(e+fx))^2} dx$	5000
3.855	$\int \frac{(A+B \tan(e+fx))(c+d \tan(e+fx))}{(a+ia \tan(e+fx))^{3/2}} dx$	5004

### 3.1 $\int \tan^2(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=91

$$-a(A-iB)x + \frac{a(iA+B)\log(\cos(c+dx))}{d} + \frac{a(A-iB)\tan(c+dx)}{d} + \frac{a(iA+B)\tan^2(c+dx)}{2d} + \frac{iaB\tan^3(c+dx)}{3d}$$

[Out]  $-a*(A-I*B)*x+a*(I*A+B)*\ln(\cos(d*x+c))/d+a*(A-I*B)*\tan(d*x+c)/d+1/2*a*(I*A+B)*\tan(d*x+c)^2/d+1/3*I*a*B*\tan(d*x+c)^3/d$

Rubi [A]

time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3673, 3609, 3606, 3556}

$$\frac{a(B+iA)\tan^2(c+dx)}{2d} + \frac{a(A-iB)\tan(c+dx)}{d} + \frac{a(B+iA)\log(\cos(c+dx))}{d} - ax(A-iB) + \frac{iaB\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(a*(A - I*B)*x) + (a*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d + (a*(A - I*B)*\text{Tan}[c + d*x])/d + (a*(I*A + B)*\text{Tan}[c + d*x]^2)/(2*d) + ((I/3)*a*B*\text{Tan}[c + d*x]^3)/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3606

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3609

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \tan^2(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{iaB \tan^3(c + dx)}{3d} + \int \tan^2(c + dx)(a(A - iB) \\ &= \frac{a(iA + B) \tan^2(c + dx)}{2d} + \frac{iaB \tan^3(c + dx)}{3d} \\ &= -a(A - iB)x + \frac{a(A - iB) \tan(c + dx)}{d} + \frac{a(iA + B) \log(\cos(c + dx))}{d} \end{aligned}$$

### Mathematica [A]

time = 0.92, size = 86, normalized size = 0.95

$$\frac{a(-6(A - iB)\text{ArcTan}(\tan(c + dx)) + 6(iA + B)\log(\cos(c + dx)) + 6(A - iB)\tan(c + dx) + 3(iA + B)\tan^2(c + dx) + 2iB\tan^3(c + dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] (a*(-6*(A - I*B)*ArcTan[Tan[c + d*x]] + 6*(I*A + B)*Log[Cos[c + d*x]] + 6*(
A - I*B)*Tan[c + d*x] + 3*(I*A + B)*Tan[c + d*x]^2 + (2*I)*B*Tan[c + d*x]^3
))/(6*d)
```

### Maple [A]

time = 0.10, size = 97, normalized size = 1.07

method	result
norman	$(iaB - aA)x + \frac{(-iaB + aA)\tan(dx+c)}{d} + \frac{(Aai + aB)(\tan^2(dx+c))}{2d} + \frac{iaB(\tan^3(dx+c))}{3d} - \frac{(Aai + aB)\ln(1+\tan^2(dx+c))}{2d}$
derivativedivides	$a\left(\frac{iB(\tan^3(dx+c))}{3} + \frac{iA(\tan^2(dx+c))}{2} - iB\tan(dx+c) + \frac{B(\tan^2(dx+c))}{2} + A\tan(dx+c) + \frac{(-iA-B)\ln(1+\tan^2(dx+c))}{2}\right) + (iB - aA)x$
default	$a\left(\frac{iB(\tan^3(dx+c))}{3} + \frac{iA(\tan^2(dx+c))}{2} - iB\tan(dx+c) + \frac{B(\tan^2(dx+c))}{2} + A\tan(dx+c) + \frac{(-iA-B)\ln(1+\tan^2(dx+c))}{2}\right) + (iB - aA)x$

risch	$-\frac{2iaBc}{d} + \frac{2aAc}{d} + \frac{2a(6iAe^{4i(dx+c)} + 9Be^{4i(dx+c)} + 9iAe^{2i(dx+c)} + 9Be^{2i(dx+c)} + 3iA + 4B)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{a \ln(e^{2i(dx+c)} + 1)B}{d}$
-------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS E)`

[Out] `1/d*a*(1/3*I*B*tan(d*x+c)^3+1/2*I*A*tan(d*x+c)^2-I*B*tan(d*x+c)+1/2*B*tan(d*x+c)^2+A*tan(d*x+c)+1/2*(-I*A-B)*ln(1+tan(d*x+c)^2)+(-A+I*B)*arctan(tan(d*x+c)))`

**Maxima** [A]

time = 0.50, size = 82, normalized size = 0.90

$$\frac{-2iBa \tan(dx+c)^3 + 3(-iA-B)a \tan(dx+c)^2 + 6(dx+c)(A-iB)a + 3(iA+B)a \log(\tan(dx+c)^2 + 1) - 6(A-iB)a \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `-1/6*(-2*I*B*a*tan(d*x+c)^3 + 3*(-I*A-B)*a*tan(d*x+c)^2 + 6*(d*x+c)*(A-I*B)*a + 3*(I*A+B)*a*log(tan(d*x+c)^2 + 1) - 6*(A-I*B)*a*tan(d*x+c))/d`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(77) = 154.

time = 0.98, size = 170, normalized size = 1.87

$$\frac{6(-2iA-3B)ae^{4i(dx+4i)} + 18(-iA-B)ae^{2i(dx+2i)} + 2(-3iA-4B)a + 3((-iA-B)ae^{6i(dx+6i)} + 3(-iA-B)ae^{4i(dx+4i)} + 3(-iA-B)ae^{2i(dx+2i)} + (-iA-B)a \log(e^{2i(dx+2i)} + 1))}{3(de^{6i(dx+6i)} + 3de^{4i(dx+4i)} + 3de^{2i(dx+2i)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `-1/3*(6*(-2*I*A-3*B)*a*e^(4*I*d*x+4*I*c) + 18*(-I*A-B)*a*e^(2*I*d*x+2*I*c) + 2*(-3*I*A-4*B)*a + 3*((-I*A-B)*a*e^(6*I*d*x+6*I*c) + 3*(-I*A-B)*a*e^(4*I*d*x+4*I*c) + 3*(-I*A-B)*a*e^(2*I*d*x+2*I*c) + (-I*A-B)*a)*log(e^(2*I*d*x+2*I*c)+1)/(d*e^(6*I*d*x+6*I*c) + 3*d*e^(4*I*d*x+4*I*c) + 3*d*e^(2*I*d*x+2*I*c) + d)`

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(75) = 150.

time = 0.36, size = 167, normalized size = 1.84

$$\frac{ia(A-iB) \log(e^{2idx} + e^{-2ic})}{d} + \frac{6iAa + 8Ba + (18iAae^{2ic} + 18Bae^{2ic})e^{2idx} + (12iAae^{4ic} + 18Bae^{4ic})e^{4idx}}{3de^{6ic}e^{6idx} + 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] I\*a\*(A - I\*B)\*log(exp(2\*I\*d\*x) + exp(-2\*I\*c))/d + (6\*I\*A\*a + 8\*B\*a + (18\*I\*A\*a\*exp(2\*I\*c) + 18\*B\*a\*exp(2\*I\*c))\*exp(2\*I\*d\*x) + (12\*I\*A\*a\*exp(4\*I\*c) + 18\*B\*a\*exp(4\*I\*c))\*exp(4\*I\*d\*x))/(3\*d\*exp(6\*I\*c)\*exp(6\*I\*d\*x) + 9\*d\*exp(4\*I\*c)\*exp(4\*I\*d\*x) + 9\*d\*exp(2\*I\*c)\*exp(2\*I\*d\*x) + 3\*d)

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs.  $2(77) = 154$ .  
time = 0.62, size = 284, normalized size = 3.12

$\frac{3i A a e^{2i d x} \log(e^{2i d x + 2i c} + 1) + 3 B a e^{2i d x} \log(e^{2i d x + 2i c} + 1) + 9i A a e^{4i d x} \log(e^{2i d x + 2i c} + 1) + 9 B a e^{4i d x} \log(e^{2i d x + 2i c} + 1) + 9i A a e^{6i d x} \log(e^{2i d x + 2i c} + 1) + 9 B a e^{6i d x} \log(e^{2i d x + 2i c} + 1) + 12i A a e^{8i d x} \log(e^{2i d x + 2i c} + 1) + 18 B a e^{8i d x} \log(e^{2i d x + 2i c} + 1) + 18i A a e^{10i d x} \log(e^{2i d x + 2i c} + 1) + 18 B a e^{10i d x} \log(e^{2i d x + 2i c} + 1) + 3i A a \log(e^{2i d x + 2i c} + 1) + 3 B a \log(e^{2i d x + 2i c} + 1) + 6i A a + 8 B a}{3(d e^{2i d x + 2i c} + 3 d e^{4i d x + 4i c} + 3 d e^{6i d x + 6i c} + d)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/3\*(3\*I\*A\*a\*e^(6\*I\*d\*x + 6\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 3\*B\*a\*e^(6\*I\*d\*x + 6\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 9\*I\*A\*a\*e^(4\*I\*d\*x + 4\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 9\*B\*a\*e^(4\*I\*d\*x + 4\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 9\*I\*A\*a\*e^(2\*I\*d\*x + 2\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 9\*B\*a\*e^(2\*I\*d\*x + 2\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 12\*I\*A\*a\*e^(4\*I\*d\*x + 4\*I\*c) + 18\*B\*a\*e^(4\*I\*d\*x + 4\*I\*c) + 18\*I\*A\*a\*e^(2\*I\*d\*x + 2\*I\*c) + 18\*B\*a\*e^(2\*I\*d\*x + 2\*I\*c) + 3\*I\*A\*a\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 3\*B\*a\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) + 6\*I\*A\*a + 8\*B\*a)/(d\*e^(6\*I\*d\*x + 6\*I\*c) + 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Mupad** [B]

time = 6.11, size = 82, normalized size = 0.90

$$\frac{\tan(c + dx)^2 \left(\frac{Ba}{2} + \frac{Aa li}{2}\right)}{d} - \frac{\ln(\tan(c + dx) + li) (Ba + Aa li)}{d} + \frac{\tan(c + dx) (Aa - Ba li)}{d} + \frac{Ba \tan(c + dx)^3 li}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] (tan(c + d\*x)^2\*((A\*a\*1i)/2 + (B\*a)/2))/d - (log(tan(c + d\*x) + 1i)\*(A\*a\*1i + B\*a))/d + (tan(c + d\*x)\*(A\*a - B\*a\*1i))/d + (B\*a\*tan(c + d\*x)^3\*1i)/(3\*d)

### 3.2 $\int \tan(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$

Optimal. Leaf size=69

$$-a(iA + B)x - \frac{a(A - iB) \log(\cos(c + dx))}{d} + \frac{a(iA + B) \tan(c + dx)}{d} + \frac{iaB \tan^2(c + dx)}{2d}$$

[Out]  $-a*(I*A+B)*x - a*(A-I*B)*\ln(\cos(d*x+c))/d + a*(I*A+B)*\tan(d*x+c)/d + 1/2*I*a*B*\tan(d*x+c)^2/d$

Rubi [A]

time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {3673, 3606, 3556}

$$\frac{a(B + iA) \tan(c + dx)}{d} - \frac{a(A - iB) \log(\cos(c + dx))}{d} - ax(B + iA) + \frac{iaB \tan^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(a*(I*A + B)*x) - (a*(A - I*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (a*(I*A + B)*\text{Tan}[c + d*x])/d + ((I/2)*a*B*\text{Tan}[c + d*x]^2)/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3606

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3673

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1))), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m)}*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \tan(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx &= \frac{iaB \tan^2(c+dx)}{2d} + \int \tan(c+dx)(a(A-iB) \\ &= -a(iA+B)x + \frac{a(iA+B) \tan(c+dx)}{d} + \frac{iaB \tan^2(c+dx)}{2d} \\ &= -a(iA+B)x - \frac{a(A-iB) \log(\cos(c+dx))}{d} + \end{aligned}$$

**Mathematica [A]**

time = 0.34, size = 70, normalized size = 1.01

$$\frac{a((-2iA-2B)\text{ArcTan}(\tan(c+dx)) - 2(A-iB) \log(\cos(c+dx)) + 2(iA+B) \tan(c+dx) + iB \tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

`[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]``[Out] (a*(((−2*I)*A − 2*B)*ArcTan[Tan[c + d*x]] − 2*(A − I*B)*Log[Cos[c + d*x]] + 2*(I*A + B)*Tan[c + d*x] + I*B*Tan[c + d*x]^2))/(2*d)`**Maple [A]**

time = 0.03, size = 72, normalized size = 1.04

method	result	size
derivativedivides	$\frac{a \left( \frac{iB \tan^2(dx+c)}{2} + iA \tan(dx+c) + B \tan(dx+c) + \frac{(-iB+A) \ln(1+\tan^2(dx+c))}{2} + (-iA-B) \arctan(\tan(dx+c)) \right)}{d}$	72
default	$\frac{a \left( \frac{iB \tan^2(dx+c)}{2} + iA \tan(dx+c) + B \tan(dx+c) + \frac{(-iB+A) \ln(1+\tan^2(dx+c))}{2} + (-iA-B) \arctan(\tan(dx+c)) \right)}{d}$	72
norman	$(-Aai - aB)x + \frac{(Aai+aB) \tan(dx+c)}{d} + \frac{iaB \tan^2(dx+c)}{2d} + \frac{(-iaB+aA) \ln(1+\tan^2(dx+c))}{2d}$	74
risch	$\frac{2aBc}{d} + \frac{2iaAc}{d} + \frac{2ia(iA e^{2i(dx+c)} + 2B e^{2i(dx+c)} + iA + B)}{d(e^{2i(dx+c)} + 1)^2} + \frac{ia \ln(e^{2i(dx+c)} + 1)B}{d} - \frac{a \ln(e^{2i(dx+c)} + 1)A}{d}$	109

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)``[Out] 1/d*a*(1/2*I*B*tan(d*x+c)^2+I*A*tan(d*x+c)+B*tan(d*x+c)+1/2*(A-I*B)*ln(1+tan(d*x+c)^2)+(-I*A-B)*arctan(tan(d*x+c)))`**Maxima [A]**

time = 0.69, size = 68, normalized size = 0.99

$$\frac{-iBa \tan(dx+c)^2 - 2(dx+c)(-iA-B)a - (A-iB)a \log(\tan(dx+c)^2 + 1) + 2(-iA-B)a \tan(dx+c)}{2d}$$





$1) + 2Aa e^{(2I dx + 2Ic)} - 4I B a e^{(2I dx + 2Ic)} + Aa \log(e^{(2I dx + 2Ic)} + 1) - I B a \log(e^{(2I dx + 2Ic)} + 1) + 2Aa - 2I B a$   
 $)/(d e^{(4I dx + 4Ic)} + 2d e^{(2I dx + 2Ic)} + d)$

**Mupad [B]**

time = 6.08, size = 59, normalized size = 0.86

$$\frac{\ln(\tan(c + dx) + 1i) (Aa - Ba 1i)}{d} + \frac{\tan(c + dx) (Ba + Aa 1i)}{d} + \frac{Ba \tan(c + dx)^2 1i}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] (log(tan(c + d\*x) + 1i)\*(A\*a - B\*a\*1i))/d + (tan(c + d\*x)\*(A\*a\*1i + B\*a))/d + (B\*a\*tan(c + d\*x)^2\*1i)/(2\*d)

### 3.3 $\int (a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$

Optimal. Leaf size=46

$$a(A - iB)x - \frac{a(iA + B) \log(\cos(c + dx))}{d} + \frac{iaB \tan(c + dx)}{d}$$

[Out]  $a*(A-I*B)*x-a*(I*A+B)*\ln(\cos(d*x+c))/d+I*a*B*\tan(d*x+c)/d$

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {3606, 3556}

$$-\frac{a(B + iA) \log(\cos(c + dx))}{d} + ax(A - iB) + \frac{iaB \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $a*(A - I*B)*x - (a*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d + (I*a*B*\text{Tan}[c + d*x])/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3606

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rubi steps

$$\begin{aligned} \int (a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= a(A - iB)x + \frac{iaB \tan(c + dx)}{d} + (a(iA + B)) \int \tan(c + dx) \\ &= a(A - iB)x - \frac{a(iA + B) \log(\cos(c + dx))}{d} + \frac{iaB \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 66, normalized size = 1.43

$$aAx - \frac{iaB \text{ArcTan}(\tan(c + dx))}{d} - \frac{iaA \log(\cos(c + dx))}{d} - \frac{aB \log(\cos(c + dx))}{d} + \frac{iaB \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $a*A*x - (I*a*B*ArcTan[Tan[c + d*x]])/d - (I*a*A*Log[Cos[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d + (I*a*B*Tan[c + d*x])/d$

**Maple [A]**

time = 0.02, size = 50, normalized size = 1.09

method	result	size
derivativedivides	$\frac{a \left( iB \tan(dx+c) + \frac{(iA+B) \ln(1+\tan^2(dx+c))}{2} + (-iB+A) \arctan(\tan(dx+c)) \right)}{d}$	50
default	$\frac{a \left( iB \tan(dx+c) + \frac{(iA+B) \ln(1+\tan^2(dx+c))}{2} + (-iB+A) \arctan(\tan(dx+c)) \right)}{d}$	50
norman	$(-iaB + aA)x + \frac{iaB \tan(dx+c)}{d} + \frac{(Aai+aB) \ln(1+\tan^2(dx+c))}{2d}$	52
risch	$\frac{2iaBc}{d} - \frac{2aAc}{d} - \frac{2aB}{d(e^{2i(dx+c)}+1)} - \frac{a \ln(e^{2i(dx+c)}+1)B}{d} - \frac{ia \ln(e^{2i(dx+c)}+1)A}{d}$	78

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $1/d*a*(I*B*tan(d*x+c)+1/2*(I*A+B)*ln(1+tan(d*x+c)^2)+(A-I*B)*arctan(tan(d*x+c)))$

**Maxima [A]**

time = 0.51, size = 50, normalized size = 1.09

$$\frac{2(dx+c)(A-iB)a - (-iA-B)a \log(\tan(dx+c)^2+1) + 2iBa \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $1/2*(2*(d*x+c)*(A-I*B)*a - (-I*A-B)*a*log(tan(d*x+c)^2+1) + 2*I*B*a*tan(d*x+c))/d$

**Fricas [A]**

time = 1.62, size = 64, normalized size = 1.39

$$\frac{2Ba - ((-iA-B)ae^{(2i dx+2i c)} + (-iA-B)a) \log(e^{(2i dx+2i c)} + 1)}{de^{(2i dx+2i c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-(2Ba - ((-IA - B)a * e^{(2I*d*x + 2I*c)} + (-IA - B)a) * \log(e^{(2I*d*x + 2I*c)} + 1)) / (d * e^{(2I*d*x + 2I*c)} + d)$

**Sympy [A]**

time = 0.21, size = 53, normalized size = 1.15

$$-\frac{2Ba}{de^{2ic}e^{2idx} + d} - \frac{ia(A - iB) \log(e^{2idx} + e^{-2ic})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

[Out]  $-2Ba / (d * \exp(2I*c) * \exp(2I*d*x) + d) - I*a*(A - I*B) * \log(\exp(2I*d*x) + \exp(-2I*c)) / d$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs.  $2(40) = 80$ .

time = 0.44, size = 103, normalized size = 2.24

$$\frac{-iAae^{(2i dx+2i c)} \log(e^{(2i dx+2i c)} + 1) - Bae^{(2i dx+2i c)} \log(e^{(2i dx+2i c)} + 1) - iAa \log(e^{(2i dx+2i c)} + 1) - Ba \log(e^{(2i dx+2i c)} + 1) - 2Ba}{de^{(2i dx+2i c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $(-IA*a*e^{(2I*d*x + 2I*c)} * \log(e^{(2I*d*x + 2I*c)} + 1) - B*a*e^{(2I*d*x + 2I*c)} * \log(e^{(2I*d*x + 2I*c)} + 1) - IA*a * \log(e^{(2I*d*x + 2I*c)} + 1) - B*a * \log(e^{(2I*d*x + 2I*c)} + 1) - 2*B*a) / (d * e^{(2I*d*x + 2I*c)} + d)$

**Mupad [B]**

time = 6.06, size = 38, normalized size = 0.83

$$\frac{\ln(\tan(c + dx) + 1i) (Ba + Aa 1i)}{d} + \frac{Ba \tan(c + dx) 1i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)`

[Out]  $(\log(\tan(c + d*x) + 1i) * (A*a*1i + B*a)) / d + (B*a*tan(c + d*x)*1i) / d$

### 3.4 $\int \cot(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$

Optimal. Leaf size=40

$$a(iA + B)x - \frac{iaB \log(\cos(c + dx))}{d} + \frac{aA \log(\sin(c + dx))}{d}$$

[Out] a\*(I\*A+B)\*x-I\*a\*B\*ln(cos(d\*x+c))/d+a\*A\*ln(sin(d\*x+c))/d

Rubi [A]

time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {3670, 3556, 3612}

$$ax(B + iA) + \frac{aA \log(\sin(c + dx))}{d} - \frac{iaB \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] a\*(I\*A + B)\*x - (I\*a\*B\*Log[Cos[c + d\*x]])/d + (a\*A\*Log[Sin[c + d\*x]])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3670

Int[((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

Rubi steps

$$\begin{aligned} \int \cot(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx &= (iaB) \int \tan(c+dx) dx + \int \cot(c+dx)(aA + \\ &= a(ia+B)x - \frac{iaB \log(\cos(c+dx))}{d} + (aA) \int \cot(c+dx) dx \\ &= a(ia+B)x - \frac{iaB \log(\cos(c+dx))}{d} + \frac{aA \log(\sin(c+dx))}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.07, size = 49, normalized size = 1.22

$$iaAx + aBx - \frac{iaB \log(\cos(c+dx))}{d} + \frac{aA(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

`[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]``[Out] I*a*A*x + a*B*x - (I*a*B*Log[Cos[c + d*x]])/d + (a*A*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d`**Maple [A]**

time = 0.10, size = 46, normalized size = 1.15

method	result	size
derivativedivides	$\frac{iaA(dx+c) - iaB \ln(\cos(dx+c)) + aA \ln(\sin(dx+c)) + aB(dx+c)}{d}$	46
default	$\frac{iaA(dx+c) - iaB \ln(\cos(dx+c)) + aA \ln(\sin(dx+c)) + aB(dx+c)}{d}$	46
norman	$(Aai + aB)x + \frac{aA \ln(\tan(dx+c))}{d} - \frac{(-iaB+aA) \ln(1+\tan^2(dx+c))}{2d}$	51
risch	$-\frac{2iaAc}{d} - \frac{2aBc}{d} + \frac{aA \ln(e^{2i(dx+c)} - 1)}{d} - \frac{ia \ln(e^{2i(dx+c)} + 1)B}{d}$	57

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)``[Out] 1/d*(I*a*A*(d*x+c)-I*a*B*ln(cos(d*x+c))+a*A*ln(sin(d*x+c))+a*B*(d*x+c))`**Maxima [A]**

time = 0.51, size = 49, normalized size = 1.22

$$\frac{2(dx+c)(iA+B)a - (A-iB)a \log(\tan(dx+c)^2 + 1) + 2Aa \log(\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(2*(d*x + c)*(I*A + B)*a - (A - I*B)*a*\log(\tan(d*x + c)^2 + 1) + 2*A*a*\log(\tan(d*x + c)))/d$

**Fricas** [A]

time = 1.60, size = 36, normalized size = 0.90

$$\frac{-iBa \log(e^{(2i dx + 2i c)} + 1) + Aa \log(e^{(2i dx + 2i c)} - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $(-I*B*a*\log(e^{(2*I*d*x + 2*I*c)} + 1) + A*a*\log(e^{(2*I*d*x + 2*I*c)} - 1))/d$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs.  $2(36) = 72$ .

time = 1.08, size = 94, normalized size = 2.35

$$\frac{Aa \log\left(\frac{-Aa - iBa}{Aae^{2ic} + iBae^{2ic}} + e^{2idx}\right)}{d} - \frac{iBa \log\left(\frac{Aa + iBa}{Aae^{2ic} + iBae^{2ic}} + e^{2idx}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out]  $A*a*\log\left(\frac{-A*a - I*B*a}{A*a*\exp(2*I*c) + I*B*a*\exp(2*I*c)} + \exp(2*I*d*x)\right)/d - I*B*a*\log\left(\frac{A*a + I*B*a}{A*a*\exp(2*I*c) + I*B*a*\exp(2*I*c)} + \exp(2*I*d*x)\right)/d$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs.  $2(36) = 72$ .

time = 0.55, size = 74, normalized size = 1.85

$$\frac{-iBa \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) + iBa \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right) - Aa \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) + 2(Aa - iBa) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $(-I*B*a*\log(\tan(1/2*d*x + 1/2*c) + 1) + I*B*a*\log(\tan(1/2*d*x + 1/2*c) - 1) - A*a*\log(\tan(1/2*d*x + 1/2*c)) + 2*(A*a - I*B*a)*\log(\tan(1/2*d*x + 1/2*c) + I))/d$

**Mupad [B]**

time = 6.23, size = 36, normalized size = 0.90

$$\frac{A a \ln(\tan(c + d x))}{d} - \frac{a \ln(\tan(c + d x) + 1i) (A - B 1i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] (A\*a\*log(tan(c + d\*x)))/d - (a\*log(tan(c + d\*x) + 1i)\*(A - B\*1i))/d



### 3.5 $\int \cot^2(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=44

$$-a(A-iB)x - \frac{aA \cot(c+dx)}{d} + \frac{a(iA+B) \log(\sin(c+dx))}{d}$$

[Out]  $-a*(A-I*B)*x - a*A*\cot(d*x+c)/d + a*(I*A+B)*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3672, 3612, 3556}

$$\frac{a(B+iA) \log(\sin(c+dx))}{d} - ax(A-iB) - \frac{aA \cot(c+dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(a*(A - I*B)*x) - (a*A*\text{Cot}[c + d*x])/d + (a*(I*A + B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ ; FreeQ}\{c, d\}, x]$

Rule 3612

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[a*c + b*d, 0]$

Rule 3672

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{m+1}/(b*f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x] \text{ ; FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx)}{d} + \int \cot(c + dx)(a(iA + B) \\ &= -a(A - iB)x - \frac{aA \cot(c + dx)}{d} + (a(iA + B)) \\ &= -a(A - iB)x - \frac{aA \cot(c + dx)}{d} + \frac{a(iA + B)}{d} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.21, size = 84, normalized size = 1.91

$$iaBx - \frac{aA \cot(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)\right)}{d} + \frac{iaA(\log(\cos(c + dx)) + \log(\tan(c + dx)))}{d} + \frac{aB(\log(\cos(c + dx)) + \log(\tan(c + dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] I\*a\*B\*x - (a\*A\*Cot[c + d\*x]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d\*x]^2])/d + (I\*a\*A\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]]))/d + (a\*B\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]]))/d

### Maple [A]

time = 0.07, size = 57, normalized size = 1.30

method	result	size
derivativedivides	$\frac{iaA \ln(\sin(dx+c)) + iaB(dx+c) + aA(-\cot(dx+c) - dx - c) + aB \ln(\sin(dx+c))}{d}$	57
default	$\frac{iaA \ln(\sin(dx+c)) + iaB(dx+c) + aA(-\cot(dx+c) - dx - c) + aB \ln(\sin(dx+c))}{d}$	57
risch	$-\frac{2iaBc}{d} + \frac{2aAc}{d} - \frac{2iaA}{d(e^{2i(dx+c)} - 1)} + \frac{a \ln(e^{2i(dx+c)} - 1)B}{d} + \frac{ia \ln(e^{2i(dx+c)} - 1)A}{d}$	78
norman	$\frac{(iaB - aA)x \tan(dx+c) - \frac{aA}{d}}{\tan(dx+c)} + \frac{(Aai + aB) \ln(\tan(dx+c))}{d} - \frac{(Aai + aB) \ln(1 + \tan^2(dx+c))}{2d}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(I\*a\*A\*ln(sin(d\*x+c))+I\*a\*B\*(d\*x+c)+a\*A\*(-cot(d\*x+c)-d\*x-c)+a\*B\*ln(sin(d\*x+c)))

### Maxima [A]

time = 0.53, size = 64, normalized size = 1.45

$$\frac{2(dx+c)(A-iB)a + (iA+B)a \log(\tan(dx+c)^2 + 1) - 2(iA+B)a \log(\tan(dx+c)) + \frac{2Aa}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/2*(2*(d*x + c)*(A - I*B)*a + (I*A + B)*a*\log(\tan(d*x + c)^2 + 1) - 2*(I*A + B)*a*\log(\tan(d*x + c)) + 2*A*a/\tan(d*x + c))/d$

**Fricas** [A]

time = 1.82, size = 62, normalized size = 1.41

$$\frac{-2i Aa + ((i A + B)ae^{(2i dx+2i c)} + (-i A - B)a) \log(e^{(2i dx+2i c)} - 1)}{de^{(2i dx+2i c)} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $(-2*I*A*a + ((I*A + B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a)*\log(e^{(2*I*d*x + 2*I*c)} - 1))/(d*e^{(2*I*d*x + 2*I*c)} - d)$

**Sympy** [A]

time = 0.24, size = 53, normalized size = 1.20

$$-\frac{2iAa}{de^{2ic}e^{2idx} - d} + \frac{ia(A - iB) \log(e^{2idx} - e^{-2ic})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out]  $-2*I*A*a/(d*\exp(2*I*c)*\exp(2*I*d*x) - d) + I*a*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs.  $2(40) = 80$ .

time = 0.63, size = 104, normalized size = 2.36

$$\frac{Aa \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 4(-i Aa - Ba) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i\right) + 2(i Aa + Ba) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{-2i Aa \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2Ba \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - Aa}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $1/2*(A*a*\tan(1/2*d*x + 1/2*c) + 4*(-I*A*a - B*a)*\log(\tan(1/2*d*x + 1/2*c) + I) + 2*(I*A*a + B*a)*\log(\tan(1/2*d*x + 1/2*c)) + (-2*I*A*a*\tan(1/2*d*x + 1/2*c) - 2*B*a*\tan(1/2*d*x + 1/2*c) - A*a)/\tan(1/2*d*x + 1/2*c))/d$

**Mupad [B]**

time = 6.21, size = 39, normalized size = 0.89

$$-\frac{A a \cot(c + d x)}{d} + \frac{a \operatorname{atan}(2 \tan(c + d x) + 1i) (B + A 1i) 2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)`

[Out] `(a*atan(2*tan(c + d*x) + 1i)*(A*1i + B)*2i)/d - (A*a*cot(c + d*x))/d`

### 3.6 $\int \cot^3(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=68

$$-a(iA+B)x - \frac{a(iA+B) \cot(c+dx)}{d} - \frac{aA \cot^2(c+dx)}{2d} - \frac{a(A-iB) \log(\sin(c+dx))}{d}$$

[Out]  $-a*(I*A+B)*x - a*(I*A+B)*\cot(d*x+c)/d - 1/2*a*A*\cot(d*x+c)^2/d - a*(A-I*B)*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3672, 3610, 3612, 3556}

$$-\frac{a(B+ia) \cot(c+dx)}{d} - \frac{a(A-iB) \log(\sin(c+dx))}{d} - ax(B+ia) - \frac{aA \cot^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(a*(I*A + B)*x) - (a*(I*A + B)*\text{Cot}[c + d*x])/d - (a*A*\text{Cot}[c + d*x]^2)/(2*d) - (a*(A - I*B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3610

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{m+1}/(f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3612

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)}{2d} + \int \cot^2(c + dx)(a(iA + B) \\ &= -\frac{a(iA + B) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} + \\ &= -a(iA + B)x - \frac{a(iA + B) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} \\ &= -a(iA + B)x - \frac{a(iA + B) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.39, size = 76, normalized size = 1.12

$$\frac{a(A \cot^2(c + dx) + 2(iA + B) \cot(c + dx) {}_2F_1(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)) + 2(A - iB)(\log(\cos(c + dx)) + \log(\tan(c + dx))))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] -1/2\*(a\*(A\*Cot[c + d\*x]^2 + 2\*(I\*A + B)\*Cot[c + d\*x]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d\*x]^2] + 2\*(A - I\*B)\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]])))/d

Maple [A]

time = 0.09, size = 81, normalized size = 1.19

method	result
derivativedivides	$\frac{iaA(-\cot(dx+c)-dx-c)+iaB \ln(\sin(dx+c))+aA \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) + aB(-\cot(dx+c)-dx-c)}{d}$

default	$\frac{iaA(-\cot(dx+c)-dx-c)+iaB\ln(\sin(dx+c))+aA\left(-\frac{(\cot^2(dx+c))}{2}-\ln(\sin(dx+c))\right)+aB(-\cot(dx+c)-dx-c)}{d}$
norman	$\frac{(-Aai-aB)x(\tan^2(dx+c))-\frac{aA}{2d}-\frac{(Aai+aB)\tan(dx+c)}{d}}{\tan(dx+c)^2} - \frac{(-iaB+aA)\ln(\tan(dx+c))}{d} + \frac{(-iaB+aA)\ln(1+\tan^2(dx+c))}{2d}$
risch	$\frac{2aBc}{d} + \frac{2iaAc}{d} - \frac{2ia(2iAe^{2i(dx+c)}+Be^{2i(dx+c)}-iA-B)}{d(e^{2i(dx+c)}-1)^2} + \frac{ia\ln(e^{2i(dx+c)}-1)B}{d} - \frac{aA\ln(e^{2i(dx+c)}-1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d}*(I*a*A*(-\cot(d*x+c)-d*x-c)+I*a*B*\ln(\sin(d*x+c))+a*A*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+a*B*(-\cot(d*x+c)-d*x-c))$

**Maxima** [A]

time = 0.51, size = 84, normalized size = 1.24

$$\frac{2(dx+c)(-iA-B)a+(A-iB)a\log(\tan(dx+c)^2+1)-2(A-iB)a\log(\tan(dx+c))+\frac{2(-iA-B)a\tan(dx+c)-Aa}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out]  $\frac{1}{2}*(2*(d*x+c)*(-I*A-B)*a+(A-I*B)*a*\log(\tan(d*x+c)^2+1)-2*(A-I*B)*a*\log(\tan(d*x+c))+\frac{2*(-I*A-B)*a*\tan(d*x+c)-A*a}{\tan(d*x+c)^2})/d$

**Fricas** [A]

time = 1.09, size = 111, normalized size = 1.63

$$\frac{2(2A-iB)ae^{(2i dx+2i c)}-2(A-iB)a-((A-iB)ae^{(4i dx+4i c)}-2(A-iB)ae^{(2i dx+2i c)}+(A-iB)a)\log(e^{(2i dx+2i c)}-1)}{de^{(4i dx+4i c)}-2de^{(2i dx+2i c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out]  $(2*(2*A-I*B)*a*e^{(2*I*d*x+2*I*c)}-2*(A-I*B)*a-((A-I*B)*a*e^{(4*I*d*x+4*I*c)}-2*(A-I*B)*a*e^{(2*I*d*x+2*I*c)}+(A-I*B)*a)*\log(e^{(2*I*d*x+2*I*c)}-1))/(d*e^{(4*I*d*x+4*I*c)}-2*d*e^{(2*I*d*x+2*I*c)}+d)$

**Sympy** [A]

time = 0.35, size = 109, normalized size = 1.60

$$-\frac{a(A-iB)\log(e^{2idx}-e^{-2ic})}{d} + \frac{-2Aa+2iBa+(4Aae^{2ic}-2iBae^{2ic})e^{2idx}}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out]  $-a*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (-2*A*a + 2*I*B*a + (4*A*a*\exp(2*I*c) - 2*I*B*a*\exp(2*I*c))*\exp(2*I*d*x))/(d*\exp(4*I*c)*\exp(4*I*d*x) - 2*d*\exp(2*I*c)*\exp(2*I*d*x) + d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs.  $2(60) = 120$ .

time = 0.73, size = 162, normalized size = 2.38

$$\frac{Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4i Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 16(Aa - iBa) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + i\right) + 8(Aa - iBa) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) - \frac{12Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 12i Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4i Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Aa}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/8*(A*a*\tan(1/2*d*x + 1/2*c)^2 - 4*I*A*a*\tan(1/2*d*x + 1/2*c) - 4*B*a*\tan(1/2*d*x + 1/2*c) - 16*(A*a - I*B*a)*\log(\tan(1/2*d*x + 1/2*c) + I) + 8*(A*a - I*B*a)*\log(\tan(1/2*d*x + 1/2*c)) - (12*A*a*\tan(1/2*d*x + 1/2*c)^2 - 12*I*B*a*\tan(1/2*d*x + 1/2*c)^2 - 4*I*A*a*\tan(1/2*d*x + 1/2*c) - 4*B*a*\tan(1/2*d*x + 1/2*c) - A*a)/\tan(1/2*d*x + 1/2*c)^2)/d$

**Mupad [B]**

time = 6.21, size = 60, normalized size = 0.88

$$\frac{\frac{Aa}{2} + \tan(c + dx) (Ba + Aa i)}{d \tan(c + dx)^2} - \frac{a \operatorname{atan}(2 \tan(c + dx) + i) (A - B i) 2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out]  $-((A*a)/2 + \tan(c + d*x)*(A*a*1i + B*a))/(d*\tan(c + d*x)^2) - (a*\operatorname{atan}(2*\tan(c + d*x) + 1i)*(A - B*1i)*2i)/d$



### 3.7 $\int \cot^4(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=89

$$a(A-iB)x + \frac{a(A-iB) \cot(c+dx)}{d} - \frac{a(iA+B) \cot^2(c+dx)}{2d} - \frac{aA \cot^3(c+dx)}{3d} - \frac{a(iA+B) \log(\sin(c+dx))}{d}$$

[Out]  $a*(A-I*B)*x + a*(A-I*B)*\cot(d*x+c)/d - 1/2*a*(I*A+B)*\cot(d*x+c)^2/d - 1/3*a*A*\cot(d*x+c)^3/d - a*(I*A+B)*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3672, 3610, 3612, 3556}

$$-\frac{a(B+iA) \cot^2(c+dx)}{2d} + \frac{a(A-iB) \cot(c+dx)}{d} - \frac{a(B+iA) \log(\sin(c+dx))}{d} + ax(A-iB) - \frac{aA \cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^4*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $a*(A - I*B)*x + (a*(A - I*B)*\text{Cot}[c + d*x])/d - (a*(I*A + B)*\text{Cot}[c + d*x]^2)/(2*d) - (a*A*\text{Cot}[c + d*x]^3)/(3*d) - (a*(I*A + B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3610

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m+1)}/(f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3612

$\text{Int}[(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx)}{3d} + \int \cot^3(c + dx)(a(iA + B) \\
&= -\frac{a(iA + B) \cot^2(c + dx)}{2d} - \frac{aA \cot^3(c + dx)}{3d} + \\
&= \frac{a(A - iB) \cot(c + dx)}{d} - \frac{a(iA + B) \cot^2(c + dx)}{2d} \\
&= a(A - iB)x + \frac{a(A - iB) \cot(c + dx)}{d} - \frac{a(iA + B)}{d} \\
&= a(A - iB)x + \frac{a(A - iB) \cot(c + dx)}{d} - \frac{a(iA + B)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.74, size = 102, normalized size = 1.15

$$\frac{a(2A \cot^3(c + dx) {}_2F_1(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c + dx)) + 6iB \cot(c + dx) {}_2F_1(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)) + 3(iA + B)(\cot^2(c + dx) + 2(\log(\cos(c + dx)) + \log(\tan(c + dx))))}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] -1/6\*(a\*(2\*A\*Cot[c + d\*x]^3\*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d\*x]^2] + (6\*I)\*B\*Cot[c + d\*x]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d\*x]^2] + 3\*(I\*A + B)\*(Cot[c + d\*x]^2 + 2\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]])))/d

### Maple [A]

time = 0.09, size = 99, normalized size = 1.11

method	result
derivativedivides	$\frac{iaA \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + iaB(-\cot(dx+c) - dx - c) + aA \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx + c \right) + aB \left( -\cot(dx+c) \right)}{d}$

default	$\frac{iaA\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)+iaB(-\cot(dx+c)-dx-c)+aA\left(-\frac{\cot^3(dx+c)}{3}+\cot(dx+c)+dx+c\right)+aB\left(-\frac{\cot^4(dx+c)}{4}+\cot^2(dx+c)+dx+c\right)}{d}$
norman	$\frac{(-iaB+aA)\frac{\tan^2(dx+c)}{d}+(-iaB+aA)x\frac{\tan^3(dx+c)}{\tan(dx+c)^3}-\frac{aA}{3d}-\frac{(Aai+aB)\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(Aai+aB)\ln(\tan(dx+c))}{d}+\frac{(Aai+aB)\ln(\tan(dx+c))}{d}+\frac{(Aai+aB)\ln(\tan(dx+c))}{d}$
risch	$\frac{2iaBc}{d}-\frac{2aAc}{d}+\frac{2a(9iAe^{4i(dx+c)}+6Be^{4i(dx+c)}-9iAe^{2i(dx+c)}-9Be^{2i(dx+c)}+4iA+3B)}{3d(e^{2i(dx+c)}-1)^3}-\frac{a\ln(e^{2i(dx+c)}-1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS E)`

[Out] `1/d*(I*a*A*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+I*a*B*(-cot(d*x+c)-d*x-c)+a*A*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a*B*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))`

**Maxima** [A]

time = 0.53, size = 103, normalized size = 1.16

$$\frac{6(dx+c)(A-iB)a-3(-iA-B)a\log(\tan(dx+c)^2+1)+6(-iA-B)a\log(\tan(dx+c))+\frac{6(A-iB)a\tan(dx+c)^2+3(-iA-B)a\tan(dx+c)-2Aa}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `1/6*(6*(d*x+c)*(A-I*B)*a-3*(-I*A-B)*a*log(tan(d*x+c)^2+1)+6*(-I*A-B)*a*log(tan(d*x+c))+6*(A-I*B)*a*tan(d*x+c)^2+3*(-I*A-B)*a*tan(d*x+c)-2*A*a)/tan(d*x+c)^3/d`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(77) = 154.

time = 0.95, size = 166, normalized size = 1.87

$$\frac{6(-3iA-2B)ae^{4i(dx+4i)c}+18(iA+B)ae^{2i(dx+2i)c}+2(-4iA-3B)a+3((iA+B)ae^{6i(dx+6i)c}+3(-iA-B)ae^{4i(dx+4i)c}+3(iA+B)ae^{2i(dx+2i)c})+(-iA-B)a\log(e^{2i(dx+2i)c}-1)}{3(de^{6i(dx+6i)c}-3de^{4i(dx+4i)c}+3de^{2i(dx+2i)c}-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `-1/3*(6*(-3*I*A-2*B)*a*e^(4*I*d*x+4*I*c)+18*(I*A+B)*a*e^(2*I*d*x+2*I*c)+2*(-4*I*A-3*B)*a+3*((I*A+B)*a*e^(6*I*d*x+6*I*c)+3*(-I*A-B)*a*e^(4*I*d*x+4*I*c)+3*(I*A+B)*a*e^(2*I*d*x+2*I*c)+(-I*A-B)*a*log(e^(2*I*d*x+2*I*c)-1))/(d*e^(6*I*d*x+6*I*c)-3*d*e^(4*I*d*x+4*I*c)+3*d*e^(2*I*d*x+2*I*c)-d)`

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(73) = 146.  
time = 0.35, size = 168, normalized size = 1.89

$$-\frac{ia(A - iB) \log(e^{2idx} - e^{-2ic})}{d} + \frac{8iAa + 6Ba + (-18iAae^{2ic} - 18Bae^{2ic})e^{2idx} + (18iAae^{4ic} + 12Bae^{4ic})e^{4idx}}{3de^{6ic}e^{6idx} - 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} - 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x)

[Out]  $-I*a*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (8*I*A*a + 6*B*a + (-18*I*A*a*\exp(2*I*c) - 18*B*a*\exp(2*I*c))*\exp(2*I*d*x) + (18*I*A*a*\exp(4*I*c) + 12*B*a*\exp(4*I*c))*\exp(4*I*d*x))/(3*d*\exp(6*I*c)*\exp(6*I*d*x) - 9*d*\exp(4*I*c)*\exp(4*I*d*x) + 9*d*\exp(2*I*c)*\exp(2*I*d*x) - 3*d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(77) = 154.  
time = 0.82, size = 221, normalized size = 2.48

$$\frac{Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 3I Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 15Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 12I Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 48(I Aa + Ba) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + I) - 24(I Aa + Ba) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c)) - \frac{-44I Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 44B a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 18Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 12I Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 30Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 30Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24I Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24I Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{24d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x, algorithm="giac")

[Out]  $\frac{1}{24}*(A*a*\tan(1/2*d*x + 1/2*c)^3 - 3*I*A*a*\tan(1/2*d*x + 1/2*c)^2 - 3*B*a*\tan(1/2*d*x + 1/2*c)^2 - 15*A*a*\tan(1/2*d*x + 1/2*c) + 12*I*B*a*\tan(1/2*d*x + 1/2*c) + 48*(I*A*a + B*a)*\log(\tan(1/2*d*x + 1/2*c) + I) - 24*(I*A*a + B*a)*\log(\tan(1/2*d*x + 1/2*c)) - (-44*I*A*a*\tan(1/2*d*x + 1/2*c)^3 - 44*B*a*\tan(1/2*d*x + 1/2*c)^3 - 15*A*a*\tan(1/2*d*x + 1/2*c)^2 + 12*I*B*a*\tan(1/2*d*x + 1/2*c)^2 + 3*I*A*a*\tan(1/2*d*x + 1/2*c) + 3*B*a*\tan(1/2*d*x + 1/2*c) + A*a)/\tan(1/2*d*x + 1/2*c)^3)/d$

**Mupad [B]**

time = 6.25, size = 80, normalized size = 0.90

$$-\frac{(-Aa + Ba li) \tan(c + dx)^2 + \left(\frac{Ba}{2} + \frac{Aa li}{2}\right) \tan(c + dx) + \frac{Aa}{3}}{d \tan(c + dx)^3} - \frac{a \operatorname{atan}(2 \tan(c + dx) + li) (B + A li) 2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*li), x)

[Out]  $-((A*a)/3 + \tan(c + d*x)*((A*a*li)/2 + (B*a)/2) - \tan(c + d*x)^2*(A*a - B*a*li))/(d*\tan(c + d*x)^3) - (a*\operatorname{atan}(2*\tan(c + d*x) + li)*(A*li + B)*2i)/d$

### 3.8 $\int \cot^5(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=111

$$a(iA+B)x + \frac{a(iA+B) \cot(c+dx)}{d} + \frac{a(A-iB) \cot^2(c+dx)}{2d} - \frac{a(iA+B) \cot^3(c+dx)}{3d} - \frac{aA \cot^4(c+dx)}{4d} + \dots$$

[Out]  $a*(I*A+B)*x+a*(I*A+B)*\cot(d*x+c)/d+1/2*a*(A-I*B)*\cot(d*x+c)^2/d-1/3*a*(I*A+B)*\cot(d*x+c)^3/d-1/4*a*A*\cot(d*x+c)^4/d+a*(A-I*B)*\ln(\sin(d*x+c))/d$

**Rubi [A]**

time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3672, 3610, 3612, 3556}

$$-\frac{a(B+iA) \cot^3(c+dx)}{3d} + \frac{a(A-iB) \cot^2(c+dx)}{2d} + \frac{a(B+iA) \cot(c+dx)}{d} + \frac{a(A-iB) \log(\sin(c+dx))}{d} + ax(B+iA) - \frac{aA \cot^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^5*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $a*(I*A + B)*x + (a*(I*A + B)*\text{Cot}[c + d*x])/d + (a*(A - I*B)*\text{Cot}[c + d*x]^2)/(2*d) - (a*(I*A + B)*\text{Cot}[c + d*x]^3)/(3*d) - (a*A*\text{Cot}[c + d*x]^4)/(4*d) + (a*(A - I*B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3610

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3612

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^5(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot^4(c + dx)}{4d} + \int \cot^4(c + dx)(a(iA + B) \\
&= -\frac{a(iA + B) \cot^3(c + dx)}{3d} - \frac{aA \cot^4(c + dx)}{4d} + \\
&= \frac{a(A - iB) \cot^2(c + dx)}{2d} - \frac{a(iA + B) \cot^3(c + dx)}{3d} \\
&= \frac{a(iA + B) \cot(c + dx)}{d} + \frac{a(A - iB) \cot^2(c + dx)}{2d} \\
&= a(iA + B)x + \frac{a(iA + B) \cot(c + dx)}{d} + \frac{a(A - iB) \cot^2(c + dx)}{2d} \\
&= a(iA + B)x + \frac{a(iA + B) \cot(c + dx)}{d} + \frac{a(A - iB) \cot^2(c + dx)}{2d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.88, size = 96, normalized size = 0.86

$$\frac{a(-6(A - iB) \cot^2(c + dx) + 3A \cot^4(c + dx) + 4(iA + B) \cot^3(c + dx) {}_2F_1(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c + dx)) - 12(A - iB)(\log(\cos(c + dx)) + \log(\tan(c + dx))))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] -1/12*(a*(-6*(A - I*B)*Cot[c + d*x]^2 + 3*A*Cot[c + d*x]^4 + 4*(I*A + B)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2] - 12*(A - I*B)*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/d
```

Maple [A]

time = 0.10, size = 112, normalized size = 1.01

method	result
derivativedivides	$\frac{iaA \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + iaB \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + aA \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} \right)}{d}$
default	$\frac{iaA \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + iaB \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + aA \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} \right)}{d}$
norman	$\frac{(Aai+aB) \frac{\tan^3(dx+c)}{d} + (Aai+aB)x \tan^4(dx+c) - \frac{aA}{4d} + \frac{(-iaB+aA) \tan^2(dx+c)}{2d} - \frac{(Aai+aB) \tan(dx+c)}{3d}}{\tan(dx+c)^4} + \frac{(-iaB+aA)}{3d}$
risch	$-\frac{2aBc}{d} - \frac{2iaAc}{d} + \frac{2ia(12iAe^{6i(dx+c)} + 9Be^{6i(dx+c)} - 18iAe^{4i(dx+c)} - 18Be^{4i(dx+c)} + 16iAe^{2i(dx+c)} + 13Be^{2i(dx+c)} + 1)}{3d(e^{2i(dx+c)} - 1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS E)`

[Out]  $\frac{1}{d} * (I * A * A * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + I * a * B * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))) + a * A * (-1/4 * \cot(d*x+c)^4 + 1/2 * \cot(d*x+c)^2 + \ln(\sin(d*x+c))) + a * B * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c))$

**Maxima [A]**

time = 0.52, size = 116, normalized size = 1.05

$$\frac{12(dx+c)(iA+B)a - 6(A-iB)a \log(\tan(dx+c)^2 + 1) + 12(A-iB)a \log(\tan(dx+c)) - \frac{12(-iA-B)a \tan(dx+c)^3 - 6(A-iB)a \tan(dx+c)^2 + 4(iA+B)a \tan(dx+c) + 3Aa}{\tan(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{12} * (12 * (d*x + c) * (I * A + B) * a - 6 * (A - I * B) * a * \log(\tan(d*x + c)^2 + 1) + 12 * (A - I * B) * a * \log(\tan(d*x + c)) - (12 * (-I * A - B) * a * \tan(d*x + c)^3 - 6 * (A - I * B) * a * \tan(d*x + c)^2 + 4 * (I * A + B) * a * \tan(d*x + c) + 3 * A * a) / \tan(d*x + c)^4) / d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs.  $2(95) = 190$ .

time = 0.85, size = 206, normalized size = 1.86

$$\frac{6(A-3iB)ae^{6i(dx+c)} - 36(A-iB)ae^{4i(dx+c)} + 2(16A-13iB)ae^{2i(dx+c)} - 8(A-iB)a - 3((A-iB)ae^{8i(dx+c)} - 4(A-iB)ae^{6i(dx+c)} + 6(A-iB)ae^{4i(dx+c)} - 4(A-iB)ae^{2i(dx+c)} + (A-iB)a) \log(e^{2i(dx+c)} - 1)}{3(de^{8i(dx+c)} - 4de^{6i(dx+c)} + 6de^{4i(dx+c)} - 4de^{2i(dx+c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/3*(6*(4*A - 3*I*B)*a*e^{(6*I*d*x + 6*I*c)} - 36*(A - I*B)*a*e^{(4*I*d*x + 4*I*c)} + 2*(16*A - 13*I*B)*a*e^{(2*I*d*x + 2*I*c)} - 8*(A - I*B)*a - 3*((A - I*B)*a*e^{(8*I*d*x + 8*I*c)} - 4*(A - I*B)*a*e^{(6*I*d*x + 6*I*c)} + 6*(A - I*B)*a*e^{(4*I*d*x + 4*I*c)} - 4*(A - I*B)*a*e^{(2*I*d*x + 2*I*c)} + (A - I*B)*a)*\log(e^{(2*I*d*x + 2*I*c)} - 1)/(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs.  $2(92) = 184$ .

time = 0.91, size = 218, normalized size = 1.96

$$\frac{a(A - iB) \log(e^{2idx} - e^{-2ic})}{d} + \frac{8Aa - 8iBa + (-32Aae^{2ic} + 26iBae^{2ic})e^{2idx} + (36Aae^{4ic} - 36iBae^{4ic})e^{4idx} + (-24Aae^{6ic} + 18iBae^{6ic})e^{6idx}}{3de^{8ic}e^{8idx} - 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} - 12de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

[Out]  $a*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (8*A*a - 8*I*B*a + (-32*A*a*\exp(2*I*c) + 26*I*B*a*\exp(2*I*c))*\exp(2*I*d*x) + (36*A*a*\exp(4*I*c) - 36*I*B*a*\exp(4*I*c))*\exp(4*I*d*x) + (-24*A*a*\exp(6*I*c) + 18*I*B*a*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) - 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) - 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 282 vs.  $2(95) = 190$ .

time = 0.98, size = 282, normalized size = 2.54

$$\frac{3 \operatorname{Arctan}\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 9 \operatorname{Arctan}\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 9 \operatorname{Arctan}\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3 \operatorname{Arctan}\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 240 \operatorname{Arctan}\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 120 \operatorname{Arctan}\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 384(Aa - I Ba) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + I\right) - 192(Aa - I Ba) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) + 400Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 400I Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 120I Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 120B Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 36Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24I B Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 8I Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8B Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Aa}{d \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $-1/192*(3*A*a*\tan(1/2*d*x + 1/2*c)^4 - 8*I*A*a*\tan(1/2*d*x + 1/2*c)^3 - 8*B*a*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*\tan(1/2*d*x + 1/2*c)^2 + 24*I*B*a*\tan(1/2*d*x + 1/2*c)^2 + 120*I*A*a*\tan(1/2*d*x + 1/2*c) + 120*B*a*\tan(1/2*d*x + 1/2*c) + 384*(A*a - I*B*a)*\log(\tan(1/2*d*x + 1/2*c) + I) - 192*(A*a - I*B*a)*\log(\tan(1/2*d*x + 1/2*c)) + (400*A*a*\tan(1/2*d*x + 1/2*c)^4 - 400*I*B*a*\tan(1/2*d*x + 1/2*c)^4 - 120*I*A*a*\tan(1/2*d*x + 1/2*c)^3 - 120*B*a*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*\tan(1/2*d*x + 1/2*c)^2 + 24*I*B*a*\tan(1/2*d*x + 1/2*c)^2 + 8*I*A*a*\tan(1/2*d*x + 1/2*c) + 8*B*a*\tan(1/2*d*x + 1/2*c) + 3*A*a)/\tan(1/2*d*x + 1/2*c)^4)/d$

**Mupad [B]**

time = 6.48, size = 100, normalized size = 0.90

$$\frac{(-B a - A a \operatorname{li}) \tan(c + dx)^3 + \left(-\frac{A a}{2} + \frac{B a \operatorname{li}}{2}\right) \tan(c + dx)^2 + \left(\frac{B a}{3} + \frac{A a \operatorname{li}}{3}\right) \tan(c + dx) + \frac{A a}{4}}{d \tan(c + dx)^4} + \frac{a \operatorname{atan}(2 \tan(c + dx) + \operatorname{li}) (A - B \operatorname{li}) 2i}{d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^5*(A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*1i),x)$

[Out]  $(a*\text{atan}(2*\tan(c + d*x) + 1i)*(A - B*1i)*2i)/d - ((A*a)/4 + \tan(c + d*x)*((A*a*1i)/3 + (B*a)/3) - \tan(c + d*x)^3*(A*a*1i + B*a) - \tan(c + d*x)^2*((A*a)/2 - (B*a*1i)/2))/(d*\tan(c + d*x)^4)$

### 3.9 $\int \tan^2(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=141

$$-2a^2(A-iB)x + \frac{2a^2(iA+B) \log(\cos(c+dx))}{d} + \frac{2a^2(A-iB) \tan(c+dx)}{d} + \frac{a^2(iA+B) \tan^2(c+dx)}{d} - \frac{a^2(4A-iB) \tan^3(c+dx)}{12d}$$

[Out]  $-2*a^2*(A-I*B)*x+2*a^2*(I*A+B)*\ln(\cos(d*x+c))/d+2*a^2*(A-I*B)*\tan(d*x+c)/d+a^2*(I*A+B)*\tan(d*x+c)^2/d-1/12*a^2*(4*A-5*I*B)*\tan(d*x+c)^3/d+1/4*I*B*\tan(d*x+c)^3*(a^2+I*a^2*\tan(d*x+c))/d$

Rubi [A]

time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3675, 3673, 3609, 3606, 3556}

$$-\frac{a^2(4A-5iB)\tan^3(c+dx)}{12d} + \frac{a^2(B+iA)\tan^2(c+dx)}{d} + \frac{2a^2(A-iB)\tan(c+dx)}{d} + \frac{2a^2(B+iA)\log(\cos(c+dx))}{d} - 2a^2x(A-iB) + \frac{iB \tan^3(c+dx)(a^2+ia^2 \tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

[In] `Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]), x]`

[Out]  $-2*a^2*(A - I*B)*x + (2*a^2*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d + (2*a^2*(A - I*B)*\text{Tan}[c + d*x])/d + (a^2*(I*A + B)*\text{Tan}[c + d*x]^2)/d - (a^2*(4*A - (5*I)*B)*\text{Tan}[c + d*x]^3)/(12*d) + ((I/4)*B*\text{Tan}[c + d*x]^3*(a^2 + I*a^2*\text{Tan}[c + d*x]))/d$

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3606

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]`

Rule 3609

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m-1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]`

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \tan^2(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{iB \tan^3(c + dx)(a^2 + ia^2 \tan(c + dx))}{4d} + \frac{1}{4} \\
&= -\frac{a^2(4A - 5iB) \tan^3(c + dx)}{12d} + \frac{iB \tan^3(c + dx)}{4d} \\
&= \frac{a^2(iA + B) \tan^2(c + dx)}{d} - \frac{a^2(4A - 5iB) \tan^3(c + dx)}{12d} \\
&= -2a^2(A - iB)x + \frac{2a^2(A - iB) \tan(c + dx)}{d} \\
&= -2a^2(A - iB)x + \frac{2a^2(iA + B) \log(\cos(c + dx))}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 305 vs. 2(141) = 282.

time = 5.29, size = 305, normalized size = 2.16

$$\frac{(-4(A - iB)d \cos^2(c + dx) \cos(2c) - \sin(2c)) + 2(A - iB) \operatorname{ArcTan}[\tan(c + dx)] \cos^2(c + dx) \cos(2c) - \sin(2c) + (A + B) \cos^2(c + dx) \log(\cos^2(c + dx)) \cos(2c) - \sin(2c) - \frac{1}{2} B \cos^2(c + dx) \cos(2c) - \frac{1}{2} (7A - 8B) \cos^2(c + dx) \sin^2(c) \cos(2c) - \sin(2c) \sin(d) + \frac{1}{4} (A - 2B) \cos^2(c) \sin(d) \cos^2(c) + \tan^2(c) - \frac{1}{2} \cos^2(c + dx) \cos(2c) - \sin(2c) (-4A - 5B + 2(A - 2B) \tan^2(c)) \cos^2(c + dx) + B \tan^2(c + dx)}{d(\cos(d) + \sin(d))(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
[Out] ((-4*(A - I*B)*d*x*Cos[c + d*x]^3*(Cos[2*c] - I*Sin[2*c]) + 2*(A - I*B)*Arc
Tan[Tan[3*c + d*x]]*Cos[c + d*x]^3*(Cos[2*c] - I*Sin[2*c]) + (I*A + B)*Cos[
```

$$c + d*x]^3 * \text{Log}[\text{Cos}[c + d*x]^2 * (\text{Cos}[2*c] - I * \text{Sin}[2*c]) - (B * \text{Sec}[c + d*x] * (\text{Cos}[2*c] - I * \text{Sin}[2*c])) / 4 + ((7*A - (8*I)*B) * \text{Cos}[c + d*x]^2 * \text{Sec}[c] * (\text{Cos}[2*c] - I * \text{Sin}[2*c]) * \text{Sin}[d*x]) / 3 + ((A - (2*I)*B) * \text{Cos}[c] * \text{Sin}[d*x] * (I + \text{Tan}[c]))^2 / 3 - (\text{Cos}[c + d*x] * (\text{Cos}[2*c] - I * \text{Sin}[2*c]) * ((-6*I)*A - 9*B + 2*(A - (2*I)*B) * \text{Tan}[c])) / 6 * (a + I*a*\text{Tan}[c + d*x])^2 * (A + B*\text{Tan}[c + d*x]) / (d * (\text{Cos}[d*x] + I * \text{Sin}[d*x])^2 * (A * \text{Cos}[c + d*x] + B * \text{Sin}[c + d*x]))$$

**Maple [A]**

time = 0.05, size = 121, normalized size = 0.86

method	result
derivativedivides	$\frac{a^2 \left( \frac{2iB(\tan^3(dx+c))}{3} - \frac{B(\tan^4(dx+c))}{4} + iA(\tan^2(dx+c)) - \frac{A(\tan^3(dx+c))}{3} - 2iB \tan(dx+c) + B(\tan^2(dx+c)) + 2A \tan(dx+c) \right)}{d}$
default	$\frac{a^2 \left( \frac{2iB(\tan^3(dx+c))}{3} - \frac{B(\tan^4(dx+c))}{4} + iA(\tan^2(dx+c)) - \frac{A(\tan^3(dx+c))}{3} - 2iB \tan(dx+c) + B(\tan^2(dx+c)) + 2A \tan(dx+c) \right)}{d}$
norman	$(2iB a^2 - 2a^2 A) x + \frac{(iA a^2 + a^2 B)(\tan^2(dx+c))}{d} - \frac{(-2iB a^2 + a^2 A)(\tan^3(dx+c))}{3d} + \frac{2(-iB a^2 + a^2 A) \tan(dx+c)}{d}$
risch	$-\frac{4ia^2 Bc}{d} + \frac{4a^2 Ac}{d} + \frac{2a^2(15iA e^{6i(dx+c)} + 21B e^{6i(dx+c)} + 33iA e^{4i(dx+c)} + 36B e^{4i(dx+c)} + 25iA e^{2i(dx+c)} + 29B e^{2i(dx+c)})}{3d(e^{2i(dx+c)} + 1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} a^2 \left( \frac{2}{3} I B \tan(d*x+c)^3 - \frac{1}{4} B \tan(d*x+c)^4 + I A \tan(d*x+c)^2 - \frac{1}{3} A \tan(d*x+c)^3 - 2 I B \tan(d*x+c) + B \tan(d*x+c)^2 + 2 A \tan(d*x+c) + \frac{1}{2} (-2 B - 2 I A) \ln(1 + \tan(d*x+c)^2) + (-2 A + 2 I B) \arctan(\tan(d*x+c)) \right)$

**Maxima [A]**

time = 0.55, size = 112, normalized size = 0.79

$$\frac{3 B a^2 \tan(dx+c)^4 + 4(A - 2iB) a^2 \tan(dx+c)^3 + 12(-iA - B) a^2 \tan(dx+c)^2 + 24(dx+c)(A - iB) a^2 - 12(-iA - B) a^2 \log(\tan(dx+c)^2 + 1) - 24(A - iB) a^2 \tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-\frac{1}{12} (3 B a^2 \tan(dx+c)^4 + 4(A - 2 I B) a^2 \tan(dx+c)^3 + 12(-I A - B) a^2 \tan(dx+c)^2 + 24(dx+c)(A - I B) a^2 - 12(-I A - B) a^2 \log(\tan(dx+c)^2 + 1) - 24(A - I B) a^2 \tan(dx+c)) / d$$

**Fricas [A]**

time = 0.69, size = 236, normalized size = 1.67

$$\frac{2(3(-5iA - 7B)a^2 e^{6i(dx+c)} + 3(-11iA - 12B)a^2 e^{4i(dx+c)} + (-25iA - 29B)a^2 e^{2i(dx+c)} + (-7iA - 8B)a^2 + 3((-iA - B)a^{2i(8i(dx+c))} + 4(-iA - B)a^{2i(6i(dx+c))} + 6(-iA - B)a^{2i(4i(dx+c))} + 4(-iA - B)a^{2i(2i(dx+c))} + (-iA - B)a^2) \log(e^{2i(dx+c)} + 1))}{3(d e^{6i(dx+c)} + 4 d e^{4i(dx+c)} + 6 d e^{2i(dx+c)} + 4 d e^{2i(dx+c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-2/3*(3*(-5*I*A - 7*B)*a^2*e^{(6*I*d*x + 6*I*c)} + 3*(-11*I*A - 12*B)*a^2*e^{(4*I*d*x + 4*I*c)} + (-25*I*A - 29*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-7*I*A - 8*B)*a^2 + 3*((-I*A - B)*a^2*e^{(8*I*d*x + 8*I*c)} + 4*(-I*A - B)*a^2*e^{(6*I*d*x + 6*I*c)} + 6*(-I*A - B)*a^2*e^{(4*I*d*x + 4*I*c)} + 4*(-I*A - B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^2)*\log(e^{(2*I*d*x + 2*I*c)} + 1)/(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy** [A]

time = 0.50, size = 236, normalized size = 1.67

$$\frac{2ia^2(A - iB)\log\left(\frac{e^{2idx} + e^{-2ic}}{d}\right) + \frac{14iAa^2 + 16Ba^2 + (50iAa^2e^{2ic} + 58Ba^2e^{2ic})e^{2idx} + (66iAa^2e^{4ic} + 72Ba^2e^{4ic})e^{4idx} + (30iAa^2e^{6ic} + 42Ba^2e^{6ic})e^{6idx}}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] 
$$2*I*a**2*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (14*I*A*a**2 + 16*B*a**2 + (50*I*A*a**2*\exp(2*I*c) + 58*B*a**2*\exp(2*I*c))*\exp(2*I*d*x) + (66*I*A*a**2*\exp(4*I*c) + 72*B*a**2*\exp(4*I*c))*\exp(4*I*d*x) + (30*I*A*a**2*\exp(6*I*c) + 42*B*a**2*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) + 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) + 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs.  $2(123) = 246$ .

time = 0.71, size = 408, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$-2/3*(-3*I*A*a^2*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 3*B*a^2*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 12*I*A*a^2*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 12*B*a^2*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 18*I*A*a^2*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 18*B*a^2*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 12*I*A*a^2*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 12*B*a^2*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 15*I*A*a^2*e^{(6*I*d*x + 6*I*c)} - 21*B*a^2*e^{(6*I*d*x + 6*I*c)} - 33*I*A*a^2*e^{(4*I*d*x + 4*I*c)} - 36*B*a^2*e^{(4*I*d*x + 4*I*c)})$$

$$\begin{aligned} & \int (4dx + 4c) - 25Aa^2e^{2dx+2c} - 29Ba^2e^{2dx+2c} - 3Aa^2\log(e^{2dx+2c}+1) - 3Ba^2\log(e^{2dx+2c}+1) \\ & - 7Aa^2 - 8Ba^2 / (de^{8dx+8c} + 4de^{6dx+6c} + 6de^{4dx+4c} + 4de^{2dx+2c} + d) \end{aligned}$$

**Mupad [B]**

time = 6.10, size = 153, normalized size = 1.09

$$\frac{\tan(c+dx)^3 \left( \frac{a^2(B+Ai)}{3} + \frac{Ba^2i}{3} \right)}{d} - \frac{\tan(c+dx) (-Aa^2 + a^2(B+Ai)i + Ba^2i)}{d} + \frac{\tan(c+dx)^2 \left( \frac{a^2(B+Ai)}{2} + \frac{Ba^2}{2} + \frac{Aa^2i}{2} \right)}{d} - \frac{\ln(\tan(c+dx)+1) (2Ba^2 + Aa^22i)}{d} - \frac{Ba^2 \tan(c+dx)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*i)^2,x)

[Out] (tan(c + d\*x)^3\*((a^2\*(A\*i + B)\*i)/3 + (B\*a^2\*i)/3))/d - (tan(c + d\*x)\*(a^2\*(A\*i + B)\*i - A\*a^2 + B\*a^2\*i))/d + (tan(c + d\*x)^2\*((A\*a^2\*i)/2 + (a^2\*(A\*i + B))/2 + (B\*a^2)/2))/d - (log(tan(c + d\*x) + i)\*(A\*a^2\*2i + 2\*B\*a^2))/d - (B\*a^2\*tan(c + d\*x)^4)/(4\*d)

### 3.10 $\int \tan(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=107

$$-2a^2(iA+B)x - \frac{2a^2(A-iB)\log(\cos(c+dx))}{d} + \frac{a^2(iA+B)\tan(c+dx)}{d} + \frac{A(a+ia \tan(c+dx))^2}{2d} - \frac{iB(a+ia \tan(c+dx))^3}{3ad}$$

[Out]  $-2*a^2*(I*A+B)*x - 2*a^2*(A-I*B)*\ln(\cos(d*x+c))/d + a^2*(I*A+B)*\tan(d*x+c)/d + 1/2*A*(a+I*a*\tan(d*x+c))^2/d - 1/3*I*B*(a+I*a*\tan(d*x+c))^3/a/d$

Rubi [A]

time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ ,

Rules used = {3673, 3608, 3558, 3556}

$$\frac{a^2(B+iA)\tan(c+dx)}{d} - \frac{2a^2(A-iB)\log(\cos(c+dx))}{d} - 2a^2x(B+iA) + \frac{A(a+ia \tan(c+dx))^2}{2d} - \frac{iB(a+ia \tan(c+dx))^3}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-2*a^2*(I*A + B)*x - (2*a^2*(A - I*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (a^2*(I*A + B)*\text{Tan}[c + d*x])/d + (A*(a + I*a*\text{Tan}[c + d*x])^2)/(2*d) - ((I/3)*B*(a + I*a*\text{Tan}[c + d*x])^3)/(a*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3558

Int[((a\_.) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)])^2, x\_Symbol] := Simp[(a^2 - b^2)\*x, x] + (Dist[2\*a\*b, Int[Tan[c + d\*x], x], x] + Simp[b^2\*(Tan[c + d\*x]/d), x]) /; FreeQ[{a, b, c, d}, x]

Rule 3608

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[B

```
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \tan(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{iB(a + ia \tan(c + dx))^3}{3ad} + \int (a + ia \tan(c + dx))^2 dx \\ &= \frac{A(a + ia \tan(c + dx))^2}{2d} - \frac{iB(a + ia \tan(c + dx))^3}{3ad} \\ &= -2a^2(iA + B)x + \frac{a^2(iA + B) \tan(c + dx)}{d} + \frac{a^2(iA + B) \tan^2(c + dx)}{2d} \\ &= -2a^2(iA + B)x - \frac{2a^2(A - iB) \log(\cos(c + dx))}{d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 273 vs. 2(107) = 214.  
time = 3.34, size = 273, normalized size = 2.55

$\frac{2i(A + B) \operatorname{ArcTan}[\tan(2c + dx)] \cos^2(c + dx) \cos(2c) - i \sin(2c) - (A - iB) \cos^2(c + dx) \log(\cos^2(c + dx)) \cos(2c) - i \sin(2c) + (A - iB) \cos^2(c + dx) (-4dx \cos(2c) - 4dx \sin(2c)) + \frac{1}{2}(6A - 7iB) \cos^2(c + dx) \operatorname{sech}(c) \cos(2c) + \sin(2c) \sin(dx) + \frac{1}{2}B \cos(c) \sin(dx) (1 + \tan(c)^2) - \frac{1}{2} \cos(c + dx) \cos(2c) - i \sin(2c) (3A - 6iB + 2B \tan(c)) (a + ia \tan(c + dx)) (A + B \tan(c + dx))}{d(\cos(dx) + i \sin(dx))^2 (A \cos(c + dx) + B \sin(c + dx))}$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
[Out] ((2*(I*A + B)*ArcTan[Tan[3*c + d*x]]*Cos[c + d*x]^3*(Cos[2*c] - I*Sin[2*c])
- (A - I*B)*Cos[c + d*x]^3*Log[Cos[c + d*x]^2]*(Cos[2*c] - I*Sin[2*c]) + (
A - I*B)*Cos[c + d*x]^3*(-4*I)*d*x*Cos[2*c] - 4*d*x*Sin[2*c]) + ((6*A - (7
*I)*B)*Cos[c + d*x]^2*Sec[c]*(I*Cos[2*c] + Sin[2*c])*Sin[d*x])/3 + (B*Cos[c
]*Sin[d*x]*(I + Tan[c])^2)/3 - (Cos[c + d*x]*(Cos[2*c] - I*Sin[2*c])*(3*A -
(6*I)*B + 2*B*Tan[c]))/6)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(
d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**  
time = 0.04, size = 99, normalized size = 0.93

method	result
derivativedivides	$\frac{a^2 \left( iB(\tan^2(dx+c)) - \frac{B(\tan^3(dx+c))}{3} + 2iA \tan(dx+c) - \frac{A(\tan^2(dx+c))}{2} + 2B \tan(dx+c) + \frac{(-2iB+2A) \ln(1+\tan^2(dx+c))}{2} \right)}{d} + \dots$
default	$\frac{a^2 \left( iB(\tan^2(dx+c)) - \frac{B(\tan^3(dx+c))}{3} + 2iA \tan(dx+c) - \frac{A(\tan^2(dx+c))}{2} + 2B \tan(dx+c) + \frac{(-2iB+2A) \ln(1+\tan^2(dx+c))}{2} \right)}{d} + \dots$



norman	$(-2iAa^2 - 2a^2B)x - \frac{(-2iBa^2 + a^2A)(\tan^2(dx+c))}{2d} + \frac{2(iAa^2 + a^2B)\tan(dx+c)}{d} - \frac{a^2B(\tan^3(dx+c))}{3d} +$
risch	$\frac{4a^2Bc}{d} + \frac{4ia^2Ac}{d} + \frac{2ia^2(9iAe^{4i(dx+c)} + 15Be^{4i(dx+c)} + 15iAe^{2i(dx+c)} + 18Be^{2i(dx+c)} + 6iA + 7B)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{2ia^2 \ln(e^{2i(dx+c)} + 1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d}a^2(I*B*\tan(dx+c)^2 - 1/3*B*\tan(dx+c)^3 + 2*I*A*\tan(dx+c) - 1/2*A*\tan(dx+c)^2 + 2*B*\tan(dx+c) + 1/2*(2*A - 2*I*B)*\ln(1 + \tan(dx+c)^2) + (-2*B - 2*I*A)*\arctan(\tan(dx+c)))$

**Maxima [A]**

time = 0.95, size = 92, normalized size = 0.86

$$\frac{2Ba^2 \tan(dx+c)^3 + 3(A-2iB)a^2 \tan(dx+c)^2 + 12(dx+c)(iA+B)a^2 - 6(A-iB)a^2 \log(\tan(dx+c)^2 + 1) + 12(-iA-B)a^2 \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{-1/6*(2*B*a^2*\tan(dx+c)^3 + 3*(A-2*I*B)*a^2*\tan(dx+c)^2 + 12*(dx+c)*(I*A+B)*a^2 - 6*(A-I*B)*a^2*\log(\tan(dx+c)^2 + 1) + 12*(-I*A-B)*a^2*\tan(dx+c))/d}$

**Fricas [A]**

time = 0.66, size = 175, normalized size = 1.64

$$\frac{2(3(3A-5iB)a^2e^{4i(dx+c)} + 3(5A-6iB)a^2e^{2i(dx+c)} + (6A-7iB)a^2 + 3((A-iB)a^2e^{6i(dx+c)} + 3(A-iB)a^2e^{4i(dx+c)} + 3(A-iB)a^2e^{2i(dx+c)} + (A-iB)a^2)\log(e^{2i(dx+c)} + 1))}{3(de^{6i(dx+c)} + 3de^{4i(dx+c)} + 3de^{2i(dx+c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{-2/3*(3*(3A-5*I*B)*a^2*e^{(4*I*d*x+4*I*c)} + 3*(5A-6*I*B)*a^2*e^{(2*I*d*x+2*I*c)} + (6A-7*I*B)*a^2 + 3*((A-I*B)*a^2*e^{(6*I*d*x+6*I*c)} + 3*(A-I*B)*a^2*e^{(4*I*d*x+4*I*c)} + 3*(A-I*B)*a^2*e^{(2*I*d*x+2*I*c)} + (A-I*B)*a^2)*\log(e^{(2*I*d*x+2*I*c)} + 1))/(d*e^{(6*I*d*x+6*I*c)} + 3*d*e^{(4*I*d*x+4*I*c)} + 3*d*e^{(2*I*d*x+2*I*c)} + d)}$

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs.  $2(88) = 176$ .

time = 0.35, size = 178, normalized size = 1.66

$$\frac{2a^2(A-iB)\log(e^{2idx} + e^{-2ic})}{d} + \frac{-12Aa^2 + 14iBa^2 + (-30Aa^2e^{2ic} + 36iBa^2e^{2ic})e^{2idx} + (-18Aa^2e^{4ic} + 30iBa^2e^{4ic})e^{4idx}}{3de^{6ic}e^{6idx} + 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x)

[Out]  $-2*a**2*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (-12*A*a**2 + 14*I*B*a**2 + (-30*A*a**2*\exp(2*I*c) + 36*I*B*a**2*\exp(2*I*c))*\exp(2*I*d*x) + (-18*A*a**2*\exp(4*I*c) + 30*I*B*a**2*\exp(4*I*c))*\exp(4*I*d*x))/(3*d*\exp(6*I*c)*\exp(6*I*d*x) + 9*d*\exp(4*I*c)*\exp(4*I*d*x) + 9*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs.  $2(91) = 182$ .  
time = 0.60, size = 312, normalized size = 2.92

$\frac{2(3A^2e^{2Ic}\log(e^{2Ic}+1) - 3B^2e^{2Ic}\log(e^{2Ic}+1) + 9A^2e^{2Ic}\log(e^{2Ic}+1) - 9B^2e^{2Ic}\log(e^{2Ic}+1) + 9A^2e^{2Ic}\log(e^{2Ic}+1) - 9B^2e^{2Ic}\log(e^{2Ic}+1) + 9A^2e^{2Ic}\log(e^{2Ic}+1) - 15B^2e^{2Ic}\log(e^{2Ic}+1) + 15A^2e^{2Ic}\log(e^{2Ic}+1) - 15B^2e^{2Ic}\log(e^{2Ic}+1) + 3A^2\log(e^{2Ic}+1) - 3B^2\log(e^{2Ic}+1) + 6A^2 - 7B^2)}{3(6e^{6Ic}+3d^2e^{4Ic}+3d^2e^{2Ic}+d)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-2/3*(3*A*a^2*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 3*I*B*a^2*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 9*A*a^2*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 9*I*B*a^2*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 9*A*a^2*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 9*I*B*a^2*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 9*A*a^2*e^{(4*I*d*x + 4*I*c)} - 15*I*B*a^2*e^{(4*I*d*x + 4*I*c)} + 15*A*a^2*e^{(2*I*d*x + 2*I*c)} - 18*I*B*a^2*e^{(2*I*d*x + 2*I*c)} + 3*A*a^2*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 3*I*B*a^2*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 6*A*a^2 - 7*I*B*a^2)/(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Mupad** [B]

time = 6.13, size = 111, normalized size = 1.04

$$\frac{\tan(c+dx)^2\left(\frac{a^2(B+Ai)1i}{2} + \frac{Ba^21i}{2}\right)}{d} + \frac{\tan(c+dx)(a^2(B+Ai) + Ba^2 + Aa^21i)}{d} + \frac{\ln(\tan(c+dx) + 1i)(2Aa^2 - Ba^22i)}{d} - \frac{Ba^2\tan(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out]  $(\tan(c + d*x)^2*((a^2*(A*1i + B)*1i)/2 + (B*a^2*1i)/2))/d + (\tan(c + d*x)*(A*a^2*1i + a^2*(A*1i + B) + B*a^2))/d + (\log(\tan(c + d*x) + 1i)*(2*A*a^2 - B*a^2*2i))/d - (B*a^2*\tan(c + d*x)^3)/(3*d)$

### 3.11 $\int (a + ia \tan(c + dx))^2 (A + B \tan(c + dx)) dx$

Optimal. Leaf size=80

$$2a^2(A - iB)x - \frac{2a^2(iA + B) \log(\cos(c + dx))}{d} - \frac{a^2(A - iB) \tan(c + dx)}{d} + \frac{B(a + ia \tan(c + dx))^2}{2d}$$

[Out]  $2*a^2*(A-I*B)*x-2*a^2*(I*A+B)*\ln(\cos(d*x+c))/d-a^2*(A-I*B)*\tan(d*x+c)/d+1/2*B*(a+I*a*\tan(d*x+c))^2/d$

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3608, 3558, 3556}

$$-\frac{a^2(A - iB) \tan(c + dx)}{d} - \frac{2a^2(B + iA) \log(\cos(c + dx))}{d} + 2a^2x(A - iB) + \frac{B(a + ia \tan(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $2*a^2*(A - I*B)*x - (2*a^2*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d - (a^2*(A - I*B)*\text{Tan}[c + d*x])/d + (B*(a + I*a*\text{Tan}[c + d*x])^2)/(2*d)$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3558

$\text{Int}[(a_. + (b_.)*\text{tan}[(c_.) + (d_.)*(x_.)])^2, x\_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[b^2*(\text{Tan}[c + d*x]/d), x]) /; \text{FreeQ}\{a, b, c, d\}, x]$

Rule 3608

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Dist}[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& !\text{LtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
\int (a + ia \tan(c + dx))^2 (A + B \tan(c + dx)) dx &= \frac{B(a + ia \tan(c + dx))^2}{2d} - (-A + iB) \int (a + ia \tan(c + dx)) dx \\
&= 2a^2(A - iB)x - \frac{a^2(A - iB) \tan(c + dx)}{d} + \frac{B(a + ia \tan(c + dx))^2}{2d} \\
&= 2a^2(A - iB)x - \frac{2a^2(iA + B) \log(\cos(c + dx))}{d} - \frac{a^2(A - iB)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 263 vs.  $2(80) = 160$ .  
time = 1.83, size = 263, normalized size = 3.29

$$\frac{e^{i \arctan(\frac{B \tan^2(dx+c)}{2} - A \tan(dx+c) + 2iB \tan(dx+c) + \frac{(2iA+2B) \ln(1+\tan^2(dx+c))}{2} + (-2iB+2A) \arctan(\tan(dx+c)))}}{d \cos(dx+c) + i \sin(dx+c)} (-A - iB) \text{ArcTan}(\tan(3c + dx)) \cos(c \cos^2(c + dx) - (4iA dx \cos(3c + 2dx) + 4B dx \cos(3c + 2dx) + (A + B) \cos(c + 2dx)) (4d - i \log(\cos^2(c + dx))) + A \cos(3c + 2dx) \log(\cos^2(c + dx)) - iB \cos(3c + 2dx) \log(\cos^2(c + dx)) + 2 \cos(c) (-iB + 4iA dx + 4B dx + (A - iB) \log(\cos^2(c + dx))) + 2A \sin(c) + 4B \sin(c) - 2iA \sin(c + 2dx) - 4B \sin(c + 2dx))}{d \cos(dx+c) + i \sin(dx+c)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(a^2 \text{Sec}[c] \text{Sec}[c + d*x]^2 (\text{Cos}[2*d*x] + I \text{Sin}[2*d*x]) * (-8*(A - I*B) \text{ArcTan}[\text{Tan}[3*c + d*x]] * \text{Cos}[c] * \text{Cos}[c + d*x]^2 - I*((4*I)*A*d*x * \text{Cos}[3*c + 2*d*x] + 4*B*d*x * \text{Cos}[3*c + 2*d*x] + (I*A + B) * \text{Cos}[c + 2*d*x] * (4*d*x - I * \text{Log}[\text{Cos}[c + d*x]^2]) + A * \text{Cos}[3*c + 2*d*x] * \text{Log}[\text{Cos}[c + d*x]^2] - I*B * \text{Cos}[3*c + 2*d*x] * \text{Log}[\text{Cos}[c + d*x]^2] + 2 * \text{Cos}[c] * ((-I)*B + (4*I)*A*d*x + 4*B*d*x + (A - I*B) * \text{Log}[\text{Cos}[c + d*x]^2]) + (2*I)*A * \text{Sin}[c] + 4*B * \text{Sin}[c] - (2*I)*A * \text{Sin}[c + 2*d*x] - 4*B * \text{Sin}[c + 2*d*x])) / (4*d * (\text{Cos}[d*x] + I * \text{Sin}[d*x])^2)$

**Maple [A]**

time = 0.04, size = 76, normalized size = 0.95

method	result
derivativedivides	$\frac{a^2 \left( -\frac{B(\tan^2(dx+c))}{2} - A \tan(dx+c) + 2iB \tan(dx+c) + \frac{(2iA+2B) \ln(1+\tan^2(dx+c))}{2} + (-2iB+2A) \arctan(\tan(dx+c)) \right)}{d}$
default	$\frac{a^2 \left( -\frac{B(\tan^2(dx+c))}{2} - A \tan(dx+c) + 2iB \tan(dx+c) + \frac{(2iA+2B) \ln(1+\tan^2(dx+c))}{2} + (-2iB+2A) \arctan(\tan(dx+c)) \right)}{d}$
norman	$(-2iB a^2 + 2a^2 A) x - \frac{(-2iB a^2 + a^2 A) \tan(dx+c)}{d} - \frac{a^2 B (\tan^2(dx+c))}{2d} + \frac{(iA a^2 + a^2 B) \ln(1+\tan^2(dx+c))}{d}$
risch	$\frac{4ia^2 Bc}{d} - \frac{4a^2 Ac}{d} - \frac{2a^2 (iA e^{2i(dx+c)} + 3B e^{2i(dx+c)} + iA + 2B)}{d(e^{2i(dx+c)} + 1)^2} - \frac{2a^2 \ln(e^{2i(dx+c)} + 1) B}{d} - \frac{2ia^2 \ln(e^{2i(dx+c)} + 1) A}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out]  $1/d*a^2*(-1/2*B*tan(d*x+c)^2-A*tan(d*x+c)+2*I*B*tan(d*x+c)+1/2*(2*B+2*I*A)*\ln(1+tan(d*x+c)^2)+(2*A-2*I*B)*arctan(tan(d*x+c)))$

**Maxima [A]**

time = 0.82, size = 71, normalized size = 0.89

$$\frac{Ba^2 \tan(dx+c)^2 - 4(dx+c)(A-iB)a^2 - 2(iA+B)a^2 \log(\tan(dx+c)^2+1) + 2(A-2iB)a^2 \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")**[Out]** -1/2\*(B\*a^2\*tan(d\*x+c)^2 - 4\*(d\*x+c)\*(A-I\*B)\*a^2 - 2\*(I\*A+B)\*a^2\*log(tan(d\*x+c)^2+1) + 2\*(A-2\*I\*B)\*a^2\*tan(d\*x+c))/d**Fricas [A]**

time = 0.85, size = 121, normalized size = 1.51

$$\frac{2((iA+3B)a^2e^{2i dx+2i c} + (iA+2B)a^2 + ((iA+B)a^2e^{4i dx+4i c} + 2(iA+B)a^2e^{2i dx+2i c} + (iA+B)a^2) \log(e^{2i dx+2i c} + 1))}{de^{4i dx+4i c} + 2de^{2i dx+2i c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")**[Out]** -2\*((I\*A+3\*B)\*a^2\*e^(2\*I\*d\*x+2\*I\*c) + (I\*A+2\*B)\*a^2 + ((I\*A+B)\*a^2\*e^(4\*I\*d\*x+4\*I\*c) + 2\*(I\*A+B)\*a^2\*e^(2\*I\*d\*x+2\*I\*c) + (I\*A+B)\*a^2)\*log(e^(2\*I\*d\*x+2\*I\*c)+1)/(d\*e^(4\*I\*d\*x+4\*I\*c) + 2\*d\*e^(2\*I\*d\*x+2\*I\*c) + d)**Sympy [A]**

time = 0.35, size = 122, normalized size = 1.52

$$-\frac{2ia^2(A-iB) \log(e^{2idx} + e^{-2ic})}{d} + \frac{-2iAa^2 - 4Ba^2 + (-2iAa^2e^{2ic} - 6Ba^2e^{2ic})e^{2idx}}{de^{4ic}e^{4idx} + 2de^{2ic}e^{2idx} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x)**[Out]** -2\*I\*a\*\*2\*(A-I\*B)\*log(exp(2\*I\*d\*x) + exp(-2\*I\*c))/d + (-2\*I\*A\*a\*\*2 - 4\*B\*a\*\*2 + (-2\*I\*A\*a\*\*2\*exp(2\*I\*c) - 6\*B\*a\*\*2\*exp(2\*I\*c))\*exp(2\*I\*d\*x))/(d\*exp(4\*I\*c)\*exp(4\*I\*d\*x) + 2\*d\*exp(2\*I\*c)\*exp(2\*I\*d\*x) + d)**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(70) = 140.

time = 0.54, size = 214, normalized size = 2.68

$$\frac{2(iAa^2e^{4i dx+4i c} \log(e^{2i dx+2i c} + 1) + Ba^2e^{4i dx+4i c} \log(e^{2i dx+2i c} + 1) + 2iAa^2e^{2i dx+2i c} \log(e^{2i dx+2i c} + 1) + 2Ba^2e^{2i dx+2i c} \log(e^{2i dx+2i c} + 1) + iAa^2e^{2i dx+2i c} + 3Ba^2e^{2i dx+2i c} + iAa^2 \log(e^{2i dx+2i c} + 1) + Ba^2 \log(e^{2i dx+2i c} + 1) + iAa^2 + 2Ba^2)}{de^{4i dx+4i c} + 2de^{2i dx+2i c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

```
[Out] -2*(I*A*a^2*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + B*a^2*e^(4*I
*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*A*a^2*e^(2*I*d*x + 2*I*c)*
log(e^(2*I*d*x + 2*I*c) + 1) + 2*B*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x +
2*I*c) + 1) + I*A*a^2*e^(2*I*d*x + 2*I*c) + 3*B*a^2*e^(2*I*d*x + 2*I*c) +
I*A*a^2*log(e^(2*I*d*x + 2*I*c) + 1) + B*a^2*log(e^(2*I*d*x + 2*I*c) + 1) +
I*A*a^2 + 2*B*a^2)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

**Mupad [B]**

time = 6.10, size = 76, normalized size = 0.95

$$\frac{\ln(\tan(c + dx) + 1i) (2B a^2 + A a^2 2i)}{d} + \frac{\tan(c + dx) (a^2 (B + A 1i) 1i + B a^2 1i)}{d} - \frac{B a^2 \tan(c + dx)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(A*a^2*2i + 2*B*a^2))/d + (tan(c + d*x)*(a^2*(A*1i
+ B)*1i + B*a^2*1i))/d - (B*a^2*tan(c + d*x)^2)/(2*d)
```

### 3.12 $\int \cot(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=75

$$2a^2(iA+B)x + \frac{a^2(A-2iB) \log(\cos(c+dx))}{d} + \frac{a^2A \log(\sin(c+dx))}{d} + \frac{iB(a^2 + ia^2 \tan(c+dx))}{d}$$

[Out]  $2*a^2*(I*A+B)*x+a^2*(A-2*I*B)*\ln(\cos(d*x+c))/d+a^2*A*\ln(\sin(d*x+c))/d+I*B*(a^2+I*a^2*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3675, 3670, 3556, 3612}

$$\frac{a^2(A-2iB) \log(\cos(c+dx))}{d} + 2a^2x(B+iA) + \frac{a^2A \log(\sin(c+dx))}{d} + \frac{iB(a^2 + ia^2 \tan(c+dx))}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]*(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $2*a^2*(I*A + B)*x + (a^2*(A - (2*I)*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (a^2*A*\text{Log}[\text{Sin}[c + d*x]])/d + (I*B*(a^2 + I*a^2*\text{Tan}[c + d*x]))/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3612

$\text{Int}[((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3670

$\text{Int}[(((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]))/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[B*(d/b), \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Dist}[1/b, \text{Int}[\text{Simp}[A*b*c + (A*b*d + B*(b*c - a*d))*\text{Tan}[e + f*x], x]/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

## Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{iB(a^2 + ia^2 \tan(c + dx))}{d} + \int \cot(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx \\ &= \frac{iB(a^2 + ia^2 \tan(c + dx))}{d} - (a^2(A - 2iB)) \int \cot(c + dx) dx \\ &= 2a^2(iA + B)x + \frac{a^2(A - 2iB) \log(\cos(c + dx))}{d} \\ &= 2a^2(iA + B)x + \frac{a^2(A - 2iB) \log(\cos(c + dx))}{d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 201 vs.  $2(75) = 150$ .

time = 2.19, size = 201, normalized size = 2.68

$$\frac{a^2(-8i(A - iB)\text{ArcTan}[\tan(3c + dx)]\cos(c + dx) + \sec(c)\cos(dx)(8i(A + B)dx + (A - 2iB)\log(\cos^2(c + dx)) + A\log(\sin^2(c + dx))) + \cos(2c + dx)(8i(A + B)dx + (A - 2iB)\log(\cos^2(c + dx)) + A\log(\sin^2(c + dx)) - 4B\sin(dx))(\cos(2dx) + i\sin(2dx))(A + B\tan(c + dx))}{4d(\cos(dx) + i\sin(dx))^2(A\cos(c + dx) + B\sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

```
[Out] (a^2*((-8*I)*(A - I*B)*ArcTan[Tan[3*c + d*x]]*Cos[c + d*x] + Sec[c]*(Cos[d*x]
*(8*(I*A + B)*d*x + (A - (2*I)*B)*Log[Cos[c + d*x]^2] + A*Log[Sin[c + d*x]
]^2)) + Cos[2*c + d*x]*(8*(I*A + B)*d*x + (A - (2*I)*B)*Log[Cos[c + d*x]^2]
+ A*Log[Sin[c + d*x]^2]) - 4*B*Sin[d*x]))*(Cos[2*d*x] + I*Sin[2*d*x])*(A +
B*Tan[c + d*x]))/(4*d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c
+ d*x]))
```

**Maple [A]**

time = 0.09, size = 86, normalized size = 1.15

method	result
--------	--------



norman	$(2iAa^2 + 2a^2B)x - \frac{a^2B \tan(dx+c)}{d} + \frac{a^2A \ln(\tan(dx+c))}{d} - \frac{(-iBa^2 + a^2A) \ln(1+\tan^2(dx+c))}{d}$
derivativdivides	$\frac{a^2A \ln(\cos(dx+c)) - a^2B(\tan(dx+c) - dx - c) + 2iAa^2(dx+c) - 2iBa^2 \ln(\cos(dx+c)) + a^2A \ln(\sin(dx+c)) + a^2B(dx+c)}{d}$
default	$\frac{a^2A \ln(\cos(dx+c)) - a^2B(\tan(dx+c) - dx - c) + 2iAa^2(dx+c) - 2iBa^2 \ln(\cos(dx+c)) + a^2A \ln(\sin(dx+c)) + a^2B(dx+c)}{d}$
risch	$-\frac{4ia^2Ac}{d} - \frac{4a^2Bc}{d} - \frac{2ia^2B}{d(e^{2i(dx+c)}+1)} + \frac{a^2A \ln(e^{2i(dx+c)}-1)}{d} - \frac{2ia^2 \ln(e^{2i(dx+c)}+1)B}{d} + \frac{a^2 \ln(e^{2i(dx+c)}+1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (a^2 * A * \ln(\cos(dx+c)) - a^2 * B * (\tan(dx+c) - dx - c) + 2 * I * A * a^2 * (dx+c) - 2 * I * B * a^2 * \ln(\cos(dx+c)) + a^2 * A * \ln(\sin(dx+c)) + a^2 * B * (dx+c))$

**Maxima** [A]

time = 0.50, size = 67, normalized size = 0.89

$$\frac{2(dx+c)(-iA-B)a^2 + (A-iB)a^2 \log(\tan(dx+c)^2+1) - Aa^2 \log(\tan(dx+c)) + Ba^2 \tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-(2*(dx+c)*(-iA-B)*a^2 + (A-iB)*a^2*\log(\tan(dx+c)^2+1) - A*a^2*\log(\tan(dx+c)) + B*a^2*\tan(dx+c))/d$

**Fricas** [A]

time = 0.91, size = 97, normalized size = 1.29

$$\frac{-2iBa^2 + ((A-2iB)a^2e^{2i(dx+2ic)} + (A-2iB)a^2) \log(e^{2i(dx+2ic)}+1) + (Aa^2e^{2i(dx+2ic)} + Aa^2) \log(e^{2i(dx+2ic)}-1)}{de^{2i(dx+2ic)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $(-2*I*B*a^2 + ((A-2*I*B)*a^2*e^{(2*I*d*x+2*I*c)} + (A-2*I*B)*a^2)*\log(e^{(2*I*d*x+2*I*c)}+1) + (A*a^2*e^{(2*I*d*x+2*I*c)} + A*a^2)*\log(e^{(2*I*d*x+2*I*c)}-1))/(d*e^{(2*I*d*x+2*I*c)}+d)$

**Sympy** [A]

time = 1.35, size = 109, normalized size = 1.45

$$\frac{Aa^2 \log(e^{2idx} - e^{-2ic})}{d} - \frac{2iBa^2}{de^{2ic}e^{2idx} + d} + \frac{a^2(A-2iB) \log\left(e^{2idx} + \frac{(-iAa^2 - Ba^2 + ia^2(A-2iB))e^{-2ic}}{Ba^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x)

[Out]  $A a^2 \log(\exp(2 I d x) - \exp(-2 I c)) / d - 2 I B a^2 / (d \exp(2 I c) \exp(2 I d x) + d) + a^2 (A - 2 I B) \log(\exp(2 I d x) + (-I A a^2 - B a^2 + I a^2 (A - 2 I B)) \exp(-2 I c) / (B a^2)) / d$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs.  $2(67) = 134$ .

time = 0.76, size = 174, normalized size = 2.32

$$\frac{A a^2 \log(\tan(\frac{1}{2} d x + \frac{1}{2} c)) + (A a^2 - 2 i B a^2) \log(\tan(\frac{1}{2} d x + \frac{1}{2} c) + 1) - 4 (A a^2 - i B a^2) \log(\tan(\frac{1}{2} d x + \frac{1}{2} c) + i) + (A a^2 - 2 i B a^2) \log(\tan(\frac{1}{2} d x + \frac{1}{2} c) - 1) - \frac{A a^2 \tan(\frac{1}{2} d x + \frac{1}{2} c)^2 - 2 i B a^2 \tan(\frac{1}{2} d x + \frac{1}{2} c) - A a^2 + 2 i B a^2}{\tan(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, algorithm="giac")

[Out]  $(A a^2 \log(\tan(1/2 d x + 1/2 c)) + (A a^2 - 2 I B a^2) \log(\tan(1/2 d x + 1/2 c) + 1) - 4 (A a^2 - I B a^2) \log(\tan(1/2 d x + 1/2 c) + I) + (A a^2 - 2 I B a^2) \log(\tan(1/2 d x + 1/2 c) - 1) - (A a^2 \tan(1/2 d x + 1/2 c)^2 - 2 I B a^2 \tan(1/2 d x + 1/2 c) - A a^2 + 2 I B a^2) / (\tan(1/2 d x + 1/2 c)^2 - 1)) / d$

**Mupad** [B]

time = 6.20, size = 70, normalized size = 0.93

$$\frac{A a^2 \ln(\tan(c + d x))}{d} - \frac{2 A a^2 \ln(\tan(c + d x) + 1 i)}{d} - \frac{B a^2 \tan(c + d x)}{d} + \frac{B a^2 \ln(\tan(c + d x) + 1 i) 2 i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2, x)

[Out]  $(A a^2 \log(\tan(c + d x))) / d - (2 A a^2 \log(\tan(c + d x) + 1 i)) / d + (B a^2 \log(\tan(c + d x) + 1 i) * 2 i) / d - (B a^2 \tan(c + d x)) / d$

### 3.13 $\int \cot^2(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=79

$$-2a^2(A-iB)x + \frac{a^2B \log(\cos(c+dx))}{d} + \frac{a^2(2iA+B) \log(\sin(c+dx))}{d} - \frac{A \cot(c+dx)(a^2+ia^2 \tan(c+dx))}{d}$$

[Out]  $-2*a^2*(A-I*B)*x+a^2*B*\ln(\cos(d*x+c))/d+a^2*(2*I*A+B)*\ln(\sin(d*x+c))/d-A*\cot(d*x+c)*(a^2+I*a^2*\tan(d*x+c))/d$

Rubi [A]

time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3674, 3670, 3556, 3612}

$$\frac{a^2(B+2iA) \log(\sin(c+dx))}{d} - 2a^2x(A-iB) - \frac{A \cot(c+dx)(a^2+ia^2 \tan(c+dx))}{d} + \frac{a^2B \log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-2*a^2*(A - I*B)*x + (a^2*B*\text{Log}[\text{Cos}[c + d*x]])/d + (a^2*((2*I)*A + B)*\text{Log}[\text{Sin}[c + d*x]])/d - (A*\text{Cot}[c + d*x]*(a^2 + I*a^2*\text{Tan}[c + d*x]))/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3612

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[a*c + b*d, 0]$

Rule 3670

$\text{Int}[(A_. + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[B*(d/b), \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Dist}[1/b, \text{Int}[\text{Simp}[A*b*c + (A*b*d + B*(b*c - a*d))*\text{Tan}[e + f*x], x]/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

## Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{A \cot(c + dx)(a^2 + ia^2 \tan(c + dx))}{d} + \int \cot^2(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx \\ &= -\frac{A \cot(c + dx)(a^2 + ia^2 \tan(c + dx))}{d} - (a^2 B \tan(c + dx)) \\ &= -2a^2(A - iB)x + \frac{a^2 B \log(\cos(c + dx))}{d} - \frac{A}{d} \\ &= -2a^2(A - iB)x + \frac{a^2 B \log(\cos(c + dx))}{d} + \frac{a^2 B \tan(c + dx)}{d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 202 vs. 2(79) = 158.

time = 2.31, size = 202, normalized size = 2.56

$$\frac{a^2(B + A \cot(c + dx))(\cos(2dx) + i \sin(2dx))(\csc(c) \cos(2c + dx) (8(A - iB)dx - B \log(\cos^2(c + dx)) + (-2(A - B) \log(\sin^2(c + dx))) + \cos(dx) (-8(A - iB)dx + B \log(\cos^2(c + dx)) + (2(A + B) \log(\sin^2(c + dx))) + 4A \sin(dx)) + 8(A - iB) \operatorname{ArcTan}(\tan(3c + dx)) \sin(c + dx))}{4d(\cos(dx) + i \sin(dx))^2(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

```
[Out] (a^2*(B + A*Cot[c + d*x])*(Cos[2*d*x] + I*Sin[2*d*x])*(Csc[c]*(Cos[2*c + d*
x]*(8*(A - I*B)*d*x - B*Log[Cos[c + d*x]^2] + ((-2*I)*A - B)*Log[Sin[c + d*
x]^2]) + Cos[d*x]*(-8*(A - I*B)*d*x + B*Log[Cos[c + d*x]^2] + ((2*I)*A + B)
*Log[Sin[c + d*x]^2]) + 4*A*Sin[d*x]) + 8*(A - I*B)*ArcTan[Tan[3*c + d*x]]*
Sin[c + d*x]))/(4*d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d
*x]))
```

**Maple [A]**

time = 0.09, size = 88, normalized size = 1.11

method	result
--------	--------

derivativedivides	$\frac{-a^2 A(dx+c) + a^2 B \ln(\cos(dx+c)) + 2iA a^2 \ln(\sin(dx+c)) + 2iB a^2(dx+c) + a^2 A(-\cot(dx+c) - dx - c) + a^2 B \ln(\sin(dx+c))}{d}$
default	$\frac{-a^2 A(dx+c) + a^2 B \ln(\cos(dx+c)) + 2iA a^2 \ln(\sin(dx+c)) + 2iB a^2(dx+c) + a^2 A(-\cot(dx+c) - dx - c) + a^2 B \ln(\sin(dx+c))}{d}$
norman	$\frac{(2iB a^2 - 2a^2 A)x \tan(dx+c) - \frac{a^2 A}{d}}{\tan(dx+c)} + \frac{(2iA a^2 + a^2 B) \ln(\tan(dx+c))}{d} - \frac{(iA a^2 + a^2 B) \ln(1 + \tan^2(dx+c))}{d}$
risch	$-\frac{4ia^2 Bc}{d} + \frac{4a^2 Ac}{d} - \frac{2ia^2 A}{d(e^{2i(dx+c)} - 1)} + \frac{a^2 \ln(e^{2i(dx+c)} - 1)B}{d} + \frac{2ia^2 \ln(e^{2i(dx+c)} - 1)A}{d} + \frac{a^2 \ln(e^{2i(dx+c)} + 1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*(-a^2*A*(d*x+c) + a^2*B*\ln(\cos(d*x+c)) + 2*I*A*a^2*\ln(\sin(d*x+c)) + 2*I*B*a^2*(d*x+c) + a^2*A*(-\cot(d*x+c) - d*x - c) + a^2*B*\ln(\sin(d*x+c)))$

**Maxima** [A]

time = 0.53, size = 74, normalized size = 0.94

$$\frac{2(dx+c)(A-iB)a^2 - (-iA-B)a^2 \log(\tan(dx+c)^2 + 1) - (2iA+B)a^2 \log(\tan(dx+c)) + \frac{Aa^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-(2*(d*x + c)*(A - I*B)*a^2 - (-I*A - B)*a^2*\log(\tan(d*x + c)^2 + 1) - (2*I*A + B)*a^2*\log(\tan(d*x + c)) + A*a^2/\tan(d*x + c))/d$

**Fricas** [A]

time = 0.57, size = 102, normalized size = 1.29

$$\frac{-2iAa^2 + (Ba^2e^{(2i dx+2i c)} - Ba^2) \log(e^{(2i dx+2i c)} + 1) + ((2iA + B)a^2e^{(2i dx+2i c)} + (-2iA - B)a^2) \log(e^{(2i dx+2i c)} - 1)}{de^{(2i dx+2i c)} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $(-2*I*A*a^2 + (B*a^2*e^{(2*I*d*x + 2*I*c)} - B*a^2)*\log(e^{(2*I*d*x + 2*I*c)} + 1) + ((2*I*A + B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-2*I*A - B)*a^2)*\log(e^{(2*I*d*x + 2*I*c)} - 1))/(d*e^{(2*I*d*x + 2*I*c)} - d)$

**Sympy** [A]

time = 1.37, size = 109, normalized size = 1.38

$$-\frac{2iAa^2}{de^{2ic}e^{2idx} - d} + \frac{Ba^2 \log(e^{2idx} + e^{-2ic})}{d} + \frac{ia^2 \cdot (2A - iB) \log\left(e^{2idx} + \frac{(Aa^2 - iBa^2 - a^2 \cdot (2A - iB))e^{-2ic}}{Aa^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out]  $-2*I*A*a^{**2}/(d*\exp(2*I*c)*\exp(2*I*d*x) - d) + B*a^{**2}*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + I*a^{**2}*(2*A - I*B)*\log(\exp(2*I*d*x) + (A*a^{**2} - I*B*a^{**2} - a^{**2}*(2*A - I*B))*\exp(-2*I*c)/(A*a^{**2}))/d$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs.  $2(73) = 146$ .

time = 0.94, size = 155, normalized size = 1.96

$$\frac{2Ba^2 \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) + 2Ba^2 \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) + Aa^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 8(iAa^2 + Ba^2) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + i) + 2(2iAa^2 + Ba^2) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c)) + \frac{-4iAa^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 2Ba^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - Aa^2}{\tan(\frac{1}{2}dx + \frac{1}{2}c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $1/2*(2*B*a^2*\log(\tan(1/2*d*x + 1/2*c) + 1) + 2*B*a^2*\log(\tan(1/2*d*x + 1/2*c) - 1) + A*a^2*\tan(1/2*d*x + 1/2*c) - 8*(I*A*a^2 + B*a^2)*\log(\tan(1/2*d*x + 1/2*c) + I) + 2*(2*I*A*a^2 + B*a^2)*\log(\tan(1/2*d*x + 1/2*c)) + (-4*I*A*a^2*\tan(1/2*d*x + 1/2*c) - 2*B*a^2*\tan(1/2*d*x + 1/2*c) - A*a^2)/\tan(1/2*d*x + 1/2*c))/d$

**Mupad** [B]

time = 6.32, size = 87, normalized size = 1.10

$$\frac{B a^2 \ln(\tan(c + dx))}{d} - \frac{2 B a^2 \ln(\tan(c + dx) + 1i)}{d} - \frac{A a^2 \cot(c + dx)}{d} + \frac{A a^2 \ln(\tan(c + dx)) 2i}{d} - \frac{A a^2 \ln(\tan(c + dx) + 1i) 2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out]  $(A*a^2*\log(\tan(c + d*x))*2i)/d + (B*a^2*\log(\tan(c + d*x)))/d - (A*a^2*\log(\tan(c + d*x) + 1i)*2i)/d - (2*B*a^2*\log(\tan(c + d*x) + 1i))/d - (A*a^2*\cot(c + d*x))/d$

### 3.14 $\int \cot^3(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=94

$$-2a^2(iA+B)x - \frac{a^2(3iA+2B)\cot(c+dx)}{2d} - \frac{2a^2(A-iB)\log(\sin(c+dx))}{d} - \frac{A\cot^2(c+dx)(a^2+ia^2\tan(c+dx))}{2d}$$

[Out]  $-2*a^2*(I*A+B)*x-1/2*a^2*(3*I*A+2*B)*\cot(d*x+c)/d-2*a^2*(A-I*B)*\ln(\sin(d*x+c))/d-1/2*A*\cot(d*x+c)^2*(a^2+I*a^2*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3674, 3672, 3612, 3556}

$$\frac{a^2(2B+3iA)\cot(c+dx)}{2d} - \frac{2a^2(A-iB)\log(\sin(c+dx))}{d} - 2a^2x(B+iA) - \frac{A\cot^2(c+dx)(a^2+ia^2\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-2*a^2*(I*A + B)*x - (a^2*((3*I)*A + 2*B)*\text{Cot}[c + d*x])/(2*d) - (2*a^2*(A - I*B)*\text{Log}[\text{Sin}[c + d*x]])/d - (A*\text{Cot}[c + d*x]^2*(a^2 + I*a^2*\text{Tan}[c + d*x]))/(2*d)$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3612**

$\text{Int}[(c_. + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

**Rule 3672**

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{m+1}/(b*f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m,$





**Maple [A]**

time = 0.10, size = 113, normalized size = 1.20

method	result
derivativedivides	$\frac{-a^2 A \ln(\sin(dx+c)) - a^2 B(dx+c) + 2iA a^2(-\cot(dx+c) - dx - c) + 2iB a^2 \ln(\sin(dx+c)) + a^2 A \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right)}{d}$
default	$\frac{-a^2 A \ln(\sin(dx+c)) - a^2 B(dx+c) + 2iA a^2(-\cot(dx+c) - dx - c) + 2iB a^2 \ln(\sin(dx+c)) + a^2 A \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right)}{d}$
risch	$\frac{4a^2 Bc}{d} + \frac{4ia^2 Ac}{d} - \frac{2ia^2(3iA e^{2i(dx+c)} + B e^{2i(dx+c)} - 2iA - B)}{d(e^{2i(dx+c)} - 1)^2} + \frac{2ia^2 \ln(e^{2i(dx+c)} - 1)B}{d} - \frac{2a^2 A \ln(e^{2i(dx+c)} - 1)}{d}$
norman	$\frac{(-2iA a^2 - 2a^2 B)x(\tan^2(dx+c)) - \frac{a^2 A}{2d} - \frac{(2iA a^2 + a^2 B)\tan(dx+c)}{d}}{\tan(dx+c)^2} + \frac{(-iB a^2 + a^2 A) \ln(1 + \tan^2(dx+c))}{d} - \frac{2(-iB a^2)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/d*(-a^2*A*ln(sin(d*x+c))-a^2*B*(d*x+c)+2*I*A*a^2*(-cot(d*x+c)-d*x-c)+2*I*
B*a^2*ln(sin(d*x+c))+a^2*A*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+a^2*B*(-cot(d
*x+c)-d*x-c))
```

**Maxima [A]**

time = 0.65, size = 94, normalized size = 1.00

$$\frac{4(dx+c)(iA+B)a^2 - 2(A-iB)a^2 \log(\tan(dx+c)^2 + 1) + 4(A-iB)a^2 \log(\tan(dx+c)) - \frac{2(-2iA-B)a^2 \tan(dx+c) - Aa^2}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="
maxima")
```

```
[Out] -1/2*(4*(d*x + c)*(I*A + B)*a^2 - 2*(A - I*B)*a^2*log(tan(d*x + c)^2 + 1) +
4*(A - I*B)*a^2*log(tan(d*x + c)) - (2*(-2*I*A - B)*a^2*tan(d*x + c) - A*a
^2)/tan(d*x + c)^2)/d
```

**Fricas [A]**

time = 0.58, size = 123, normalized size = 1.31

$$\frac{2((3A-iB)a^2 e^{2i dx+2i c} - (2A-iB)a^2 - ((A-iB)a^2 e^{4i dx+4i c} - 2(A-iB)a^2 e^{2i dx+2i c} + (A-iB)a^2) \log(e^{2i dx+2i c} - 1))}{de^{4i dx+4i c} - 2de^{2i dx+2i c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="
fricas")
```

[Out]  $2*((3A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} - (2*A - I*B)*a^2 - ((A - I*B)*a^2*e^{(4*I*d*x + 4*I*c)} - 2*(A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (A - I*B)*a^2)*\log(e^{(2*I*d*x + 2*I*c)} - 1)/(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [A]**

time = 0.36, size = 119, normalized size = 1.27

$$-\frac{2a^2(A - iB)\log(e^{2idx} - e^{-2ic})}{d} + \frac{-4Aa^2 + 2iBa^2 + (6Aa^2e^{2ic} - 2iBa^2e^{2ic})e^{2idx}}{de^{4ic}e^{4idx} - 2de^{2ic}e^{2idx} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)`

[Out]  $-2*a**2*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (-4*A*a**2 + 2*I*B*a**2 + (6*A*a**2*\exp(2*I*c) - 2*I*B*a**2*\exp(2*I*c))*\exp(2*I*d*x))/(d*\exp(4*I*c)*\exp(4*I*d*x) - 2*d*\exp(2*I*c)*\exp(2*I*d*x) + d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs.  $2(82) = 164$ .

time = 1.14, size = 186, normalized size = 1.98

$$\frac{Aa^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 8i Aa^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 4Ba^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 32(Aa^2 - iBa^2) \log(\tan(\frac{1}{2} dx + \frac{1}{2} c) + i) + 16(Aa^2 - iBa^2) \log(\tan(\frac{1}{2} dx + \frac{1}{2} c)) - \frac{24Aa^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 24iBa^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 8iAa^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 4Ba^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) - Aa^2}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $-1/8*(A*a^2*\tan(1/2*d*x + 1/2*c)^2 - 8*I*A*a^2*\tan(1/2*d*x + 1/2*c) - 4*B*a^2*\tan(1/2*d*x + 1/2*c) - 32*(A*a^2 - I*B*a^2)*\log(\tan(1/2*d*x + 1/2*c) + I) + 16*(A*a^2 - I*B*a^2)*\log(\tan(1/2*d*x + 1/2*c)) - (24*A*a^2*\tan(1/2*d*x + 1/2*c)^2 - 24*I*B*a^2*\tan(1/2*d*x + 1/2*c)^2 - 8*I*A*a^2*\tan(1/2*d*x + 1/2*c) - 4*B*a^2*\tan(1/2*d*x + 1/2*c) - A*a^2)/\tan(1/2*d*x + 1/2*c)^2)/d$

**Mupad [B]**

time = 6.20, size = 67, normalized size = 0.71

$$-\frac{\frac{Aa^2}{2} + \tan(c + dx)(Ba^2 + Aa^2 2i)}{d \tan(c + dx)^2} - \frac{4a^2 \operatorname{atan}(2 \tan(c + dx) + 1i)(B + A 1i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)`

[Out]  $-((A*a^2)/2 + \tan(c + d*x)*(A*a^2*2i + B*a^2))/(d*\tan(c + d*x)^2) - (4*a^2*\operatorname{atan}(2*\tan(c + d*x) + 1i)*(A*1i + B))/d$

### 3.15 $\int \cot^4(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=117

$$2a^2(A-iB)x + \frac{2a^2(A-iB)\cot(c+dx)}{d} - \frac{a^2(4iA+3B)\cot^2(c+dx)}{6d} - \frac{2a^2(iA+B)\log(\sin(c+dx))}{d} - \frac{A\cot^3(c+dx)}{3d}$$

[Out]  $2*a^2*(A-I*B)*x + 2*a^2*(A-I*B)*\cot(d*x+c)/d - 1/6*a^2*(4*I*A+3*B)*\cot(d*x+c)^2/d - 2*a^2*(I*A+B)*\ln(\sin(d*x+c))/d - 1/3*A*\cot(d*x+c)^3*(a^2+I*a^2*\tan(d*x+c))/d$

Rubi [A]

time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3672, 3610, 3612, 3556}

$$-\frac{a^2(3B+4iA)\cot^2(c+dx)}{6d} + \frac{2a^2(A-iB)\cot(c+dx)}{d} - \frac{2a^2(B+iA)\log(\sin(c+dx))}{d} + 2a^2x(A-iB) - \frac{A\cot^3(c+dx)(a^2+ia^2\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out]  $2*a^2*(A - I*B)*x + (2*a^2*(A - I*B)*\text{Cot}[c + d*x])/d - (a^2*((4*I)*A + 3*B)*\text{Cot}[c + d*x]^2)/(6*d) - (2*a^2*(I*A + B)*\text{Log}[\text{Sin}[c + d*x]])/d - (A*\text{Cot}[c + d*x]^3*(a^2 + I*a^2*\text{Tan}[c + d*x]))/(3*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rule 3674

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{A \cot^3(c + dx) (a^2 + ia^2 \tan(c + dx))}{3d} + \frac{1}{3} \int \\
&= -\frac{a^2(4iA + 3B) \cot^2(c + dx)}{6d} - \frac{A \cot^3(c + dx)}{3d} \\
&= \frac{2a^2(A - iB) \cot(c + dx)}{d} - \frac{a^2(4iA + 3B) \cot^2(c + dx)}{6d} \\
&= 2a^2(A - iB)x + \frac{2a^2(A - iB) \cot(c + dx)}{d} - \frac{a^2(4iA + 3B) \cot^2(c + dx)}{6d} \\
&= 2a^2(A - iB)x + \frac{2a^2(A - iB) \cot(c + dx)}{d} - \frac{a^2(4iA + 3B) \cot^2(c + dx)}{6d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 435 vs.  $2(117) = 234$ .  
time = 2.22, size = 435, normalized size = 3.72

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

```
[Out] (a^2*Csc[c]*Csc[c + d*x]^3*(Cos[2*d*x] + I*Sin[2*d*x])*((12*I)*A*Cos[2*c +
d*x] + 6*B*Cos[2*c + d*x] - 36*A*d*x*Cos[2*c + d*x] + (36*I)*B*d*x*Cos[2*c
+ d*x] - 12*A*d*x*Cos[2*c + 3*d*x] + (12*I)*B*d*x*Cos[2*c + 3*d*x] + 12*A*d
*x*Cos[4*c + 3*d*x] - (12*I)*B*d*x*Cos[4*c + 3*d*x] + (9*I)*A*Cos[2*c + d*x
]*Log[Sin[c + d*x]^2] + 9*B*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] + (3*I)*A*Co
s[2*c + 3*d*x]*Log[Sin[c + d*x]^2] + 3*B*Cos[2*c + 3*d*x]*Log[Sin[c + d*x]^
2] - (3*I)*A*Cos[4*c + 3*d*x]*Log[Sin[c + d*x]^2] - 3*B*Cos[4*c + 3*d*x]*Lo
g[Sin[c + d*x]^2] + 3*Cos[d*x]*(2*B*(-1 - (6*I)*d*x) + 4*A*(-I + 3*d*x) + (
(-3*I)*A - 3*B)*Log[Sin[c + d*x]^2]) - 24*A*Sin[d*x] + (24*I)*B*Sin[d*x] -
48*(A - I*B)*ArcTan[Tan[3*c + d*x]]*Sin[c]*Sin[c + d*x]^3 - 18*A*Sin[2*c +
d*x] + (12*I)*B*Sin[2*c + d*x] + 14*A*Sin[2*c + 3*d*x] - (12*I)*B*Sin[2*c +
3*d*x]))/(24*d*(Cos[d*x] + I*Sin[d*x])^2)
```

**Maple [A]**

time = 0.10, size = 142, normalized size = 1.21

method	result
derivativedivides	$\frac{-a^2 A(-\cot(dx+c)-dx-c)-a^2 B \ln(\sin(dx+c))+2i A a^2 \left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)+2i B a^2(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{-a^2 A(-\cot(dx+c)-dx-c)-a^2 B \ln(\sin(dx+c))+2i A a^2 \left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)+2i B a^2(-\cot(dx+c)-dx-c)}{d}$
risch	$\frac{4ia^2 Bc}{d} - \frac{4a^2 Ac}{d} + \frac{2a^2(15iA e^{4i(dx+c)}+9B e^{4i(dx+c)}-18iA e^{2i(dx+c)}-15B e^{2i(dx+c)}+7iA+6B)}{3d(e^{2i(dx+c)}-1)^3} - \frac{2a^2 \ln(e^{2i(dx+c)})}{d}$
norman	$\frac{(-2iB a^2+2a^2 A)x(\tan^3(dx+c))-\frac{a^2 A}{3d}+\frac{2(-iB a^2+a^2 A)(\tan^2(dx+c))}{d}-\frac{(2iA a^2+a^2 B)\tan(dx+c)}{2d}}{\tan(dx+c)^3} + \frac{(iA a^2+a^2 B)\ln(1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/d*(-a^2*A*(-cot(d*x+c)-d*x-c)-a^2*B*ln(sin(d*x+c))+2*I*A*a^2*(-1/2*cot(d*
x+c)^2-ln(sin(d*x+c)))+2*I*B*a^2*(-cot(d*x+c)-d*x-c)+a^2*A*(-1/3*cot(d*x+c)
^3+cot(d*x+c)+d*x+c)+a^2*B*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

**Maxima [A]**

time = 0.55, size = 111, normalized size = 0.95

$$\frac{12(dx+c)(A-iB)a^2+6(iA+B)a^2 \log(\tan(dx+c)^2+1)-12(iA+B)a^2 \log(\tan(dx+c))+\frac{12(A-iB)a^2 \tan(dx+c)^2+3(-2iA-B)a^2 \tan(dx+c)-2Aa^2}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="
maxima")
```

[Out]  $1/6*(12*(d*x + c)*(A - I*B)*a^2 + 6*(I*A + B)*a^2*\log(\tan(d*x + c)^2 + 1) - 12*(I*A + B)*a^2*\log(\tan(d*x + c)) + (12*(A - I*B)*a^2*\tan(d*x + c)^2 + 3*(-2*I*A - B)*a^2*\tan(d*x + c) - 2*A*a^2)/\tan(d*x + c)^3/d$

**Fricas** [A]

time = 0.47, size = 181, normalized size = 1.55

$$\frac{2(3(-5iA - 3B)a^2e^{4i dx + 4i c}) + 3(6iA + 5B)a^2e^{2i dx + 2i c} + (-7iA - 6B)a^2 + 3((iA + B)a^2e^{6i dx + 6i c}) + 3(-iA - B)a^2e^{4i dx + 4i c} + 3(iA + B)a^2e^{2i dx + 2i c} + (-iA - B)a^2 \log(e^{2i dx + 2i c} - 1)}{3(de^{6i dx + 6i c} - 3de^{4i dx + 4i c} + 3de^{2i dx + 2i c} - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-2/3*(3*(-5*I*A - 3*B)*a^2*e^{(4*I*d*x + 4*I*c)} + 3*(6*I*A + 5*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-7*I*A - 6*B)*a^2 + 3*((I*A + B)*a^2*e^{(6*I*d*x + 6*I*c)} + 3*(-I*A - B)*a^2*e^{(4*I*d*x + 4*I*c)} + 3*(I*A + B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^2)*\log(e^{(2*I*d*x + 2*I*c)} - 1)/(d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} - d)$

**Sympy** [A]

time = 0.71, size = 182, normalized size = 1.56

$$-\frac{2ia^2(A - iB) \log(e^{2idx} - e^{-2ic})}{d} + \frac{14iAa^2 + 12Ba^2 + (-36iAa^2e^{2ic} - 30Ba^2e^{2ic})e^{2idx} + (30iAa^2e^{4ic} + 18Ba^2e^{4ic})e^{4idx}}{3de^{6ic}e^{6idx} - 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} - 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)`

[Out]  $-2*I*a**2*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (14*I*A*a**2 + 12*B*a**2 + (-36*I*A*a**2*\exp(2*I*c) - 30*B*a**2*\exp(2*I*c))*\exp(2*I*d*x) + (30*I*A*a**2*\exp(4*I*c) + 18*B*a**2*\exp(4*I*c))*\exp(4*I*d*x))/(3*d*\exp(6*I*c)*\exp(6*I*d*x) - 9*d*\exp(4*I*c)*\exp(4*I*d*x) + 9*d*\exp(2*I*c)*\exp(2*I*d*x) - 3*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 255 vs.  $2(103) = 206$ .

time = 1.33, size = 255, normalized size = 2.18

$$\frac{Aa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 6iAa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 3Ba^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 27Aa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 24iBa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 96(-iAa^2 - Ba^2) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i\right) + 48(-iAa^2 - Ba^2) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) - \frac{36iAa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 36Ba^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 36iAa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 36Ba^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 36iAa^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 36Ba^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $1/24*(A*a^2*\tan(1/2*d*x + 1/2*c)^3 - 6*I*A*a^2*\tan(1/2*d*x + 1/2*c)^2 - 3*B*a^2*\tan(1/2*d*x + 1/2*c)^2 - 27*A*a^2*\tan(1/2*d*x + 1/2*c) + 24*I*B*a^2*\tan(1/2*d*x + 1/2*c) - 96*(-I*A*a^2 - B*a^2)*\log(\tan(1/2*d*x + 1/2*c) + I) +$

$$48*(-I*A*a^2 - B*a^2)*\log(\tan(1/2*d*x + 1/2*c)) - (-88*I*A*a^2*\tan(1/2*d*x + 1/2*c)^3 - 88*B*a^2*\tan(1/2*d*x + 1/2*c)^3 - 27*A*a^2*\tan(1/2*d*x + 1/2*c)^2 + 24*I*B*a^2*\tan(1/2*d*x + 1/2*c)^2 + 6*I*A*a^2*\tan(1/2*d*x + 1/2*c) + 3*B*a^2*\tan(1/2*d*x + 1/2*c) + A*a^2)/\tan(1/2*d*x + 1/2*c)^3)/d$$

**Mupad [B]**

time = 6.28, size = 93, normalized size = 0.79

$$\frac{\frac{Aa^2}{3} - \tan(c+dx)^2(2Aa^2 - Ba^2 2i) + \tan(c+dx)\left(\frac{Ba^2}{2} + Aa^2 1i\right)}{d \tan(c+dx)^3} - \frac{a^2 \operatorname{atan}(2 \tan(c+dx) + 1i) (B + A 1i) 4i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] - ((A\*a^2)/3 - tan(c + d\*x)^2\*(2\*A\*a^2 - B\*a^2\*2i) + tan(c + d\*x)\*(A\*a^2\*1i + (B\*a^2)/2))/(d\*tan(c + d\*x)^3) - (a^2\*atan(2\*tan(c + d\*x) + 1i)\*(A\*1i + B)\*4i)/d

### 3.16 $\int \cot^5(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=139

$$2a^2(iA+B)x + \frac{2a^2(iA+B) \cot(c+dx)}{d} + \frac{a^2(A-iB) \cot^2(c+dx)}{d} - \frac{a^2(5iA+4B) \cot^3(c+dx)}{12d} + \frac{2a^2(A-iB)}{d}$$

[Out]  $2*a^2*(I*A+B)*x+2*a^2*(I*A+B)*\cot(d*x+c)/d+a^2*(A-I*B)*\cot(d*x+c)^2/d-1/12*a^2*(5*I*A+4*B)*\cot(d*x+c)^3/d+2*a^2*(A-I*B)*\ln(\sin(d*x+c))/d-1/4*A*\cot(d*x+c)^4*(a^2+I*a^2*\tan(d*x+c))/d$

Rubi [A]

time = 0.23, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3672, 3610, 3612, 3556}

$$-\frac{a^2(4B+5iA)\cot^3(c+dx)}{12d} + \frac{a^2(A-iB)\cot^2(c+dx)}{d} + \frac{2a^2(B+iA)\cot(c+dx)}{d} + \frac{2a^2(A-iB)\log(\sin(c+dx))}{d} + 2a^2x(B+iA) - \frac{A\cot^4(c+dx)(a^2+ia^2\tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^5*(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $2*a^2*(I*A + B)*x + (2*a^2*(I*A + B)*\text{Cot}[c + d*x])/d + (a^2*(A - I*B)*\text{Cot}[c + d*x]^2)/d - (a^2*((5*I)*A + 4*B)*\text{Cot}[c + d*x]^3)/(12*d) + (2*a^2*(A - I*B)*\text{Log}[\text{Sin}[c + d*x]])/d - (A*\text{Cot}[c + d*x]^4*(a^2 + I*a^2*\text{Tan}[c + d*x]))/(4*d)$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]], x] / d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 3610

$\text{Int}[((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)/(f*(m + 1)*(a^2 + b^2)}), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3612

$\text{Int}(((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[m, -1]$



Q[a\*c + b\*d, 0]

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rule 3674

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \cot^5(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{A \cot^4(c + dx)(a^2 + ia^2 \tan(c + dx))}{4d} + \frac{1}{4} \\
 &= -\frac{a^2(5iA + 4B) \cot^3(c + dx)}{12d} - \frac{A \cot^4(c + dx)}{4d} \\
 &= \frac{a^2(A - iB) \cot^2(c + dx)}{d} - \frac{a^2(5iA + 4B) \cot^3(c + dx)}{12d} \\
 &= \frac{2a^2(iA + B) \cot(c + dx)}{d} + \frac{a^2(A - iB) \cot^2(c + dx)}{d} \\
 &= 2a^2(iA + B)x + \frac{2a^2(iA + B) \cot(c + dx)}{d} + \frac{a^2(A - iB) \cot^2(c + dx)}{d} \\
 &= 2a^2(iA + B)x + \frac{2a^2(iA + B) \cot(c + dx)}{d} + \frac{a^2(A - iB) \cot^2(c + dx)}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 902 vs. 2(139) = 278.  
time = 7.14, size = 902, normalized size = 6.49

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
[Out] a^2*(((I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*Csc[c + d*x]*(-1/4*(A*Cos[2*c]) + (I/4)*A*Sin[2*c]))/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*Csc[c]*(Cos[2*c]/3 - (I/3)*Sin[2*c])*((2*I)*A*Sin[d*x] + B*Sin[d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*Csc[c]*((-4*I)*A*Cos[c] - 2*B*Cos[c] + 9*A*Sin[c] - (6*I)*B*Sin[c])*(Cos[2*c]/6 - (I/6)*Sin[2*c])*Sin[c + d*x])/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*Csc[c]*(Cos[2*c]/3 - (I/3)*Sin[2*c])*((-8*I)*A*Sin[d*x] - 7*B*Sin[d*x])*Sin[c + d*x]^2)/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*(A*Cos[c] - I*B*Cos[c] - I*A*Sin[c] - B*Sin[c])*((-2*I)*ArcTan[Tan[3*c + d*x]]*Cos[c] - 2*ArcTan[Tan[3*c + d*x]]*Sin[c])*Sin[c + d*x]^3)/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*(A*Cos[c] - I*B*Cos[c] - I*A*Sin[c] - B*Sin[c])*(Cos[c]*Log[Sin[c + d*x]^2] - I*Log[Sin[c + d*x]^2]*Sin[c])*Sin[c + d*x]^3)/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (x*(I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*((6*I)*A*Cos[c]^2 + 6*B*Cos[c]^2 - 2*A*Cos[c]^2*Cot[c] + (2*I)*B*Cos[c]^2*Cot[c] + 6*A*Cos[c]*Sin[c] - (6*I)*B*Cos[c]*Sin[c] - (2*I)*A*Sin[c]^2 - 2*B*Sin[c]^2 + (A - I*B)*Cot[c]*(2*Cos[2*c] - (2*I)*Sin[2*c]))*Sin[c + d*x]^3)/((Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I*A + B)*(I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*(2*d*x*Cos[2*c] - (2*I)*d*x*Sin[2*c])*Sin[c + d*x]^3)/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])))
```

Maple [A]

time = 0.11, size = 168, normalized size = 1.21

method	result
derivativedivides	$-a^2A\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)-a^2B(-\cot(dx+c)-dx-c)+2iAa^2\left(-\frac{\cot^3(dx+c)}{3}+\cot(dx+c)+dx+c\right)+2iB$
default	$-a^2A\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)-a^2B(-\cot(dx+c)-dx-c)+2iAa^2\left(-\frac{\cot^3(dx+c)}{3}+\cot(dx+c)+dx+c\right)+2iB$
risch	$-\frac{4a^2Bc}{d}-\frac{4ia^2Ac}{d}+\frac{2ia^2(21iAe^{6i(dx+c)}+15Be^{6i(dx+c)}-36iAe^{4i(dx+c)}-33Be^{4i(dx+c)}+29iAe^{2i(dx+c)}+25Be^{2i(dx+c)})}{3d(e^{2i(dx+c)}-1)^4}$
norman	$\frac{(-iBa^2+a^2A)(\tan^2(dx+c))}{d}+(2iAa^2+2a^2B)x(\tan^4(dx+c))-\frac{a^2A}{4d}+\frac{2(iAa^2+a^2B)(\tan^3(dx+c))}{d}-\frac{(2iAa^2+a^2B)\tan(dx+c)}{3d}$ $\tan(dx+c)^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d}(-a^2 A(-\frac{1}{3}\cot(d*x+c)^2 - \ln(\sin(d*x+c))) - a^2 B(-\cot(d*x+c) - d*x - c) + 2 I A a^2(-\frac{1}{3}\cot(d*x+c)^3 + \cot(d*x+c) + d*x + c) + 2 I B a^2(-\frac{1}{2}\cot(d*x+c)^2 - \ln(\sin(d*x+c))) + a^2 A(-\frac{1}{4}\cot(d*x+c)^4 + \frac{1}{2}\cot(d*x+c)^2 + \ln(\sin(d*x+c))) + a^2 B(-\frac{1}{3}\cot(d*x+c)^3 + \cot(d*x+c) + d*x + c)$

**Maxima** [A]

time = 0.50, size = 132, normalized size = 0.95

$$\frac{24(dx+c)(-iA-B)a^2 + 12(A-iB)a^2 \log(\tan(dx+c)^2 + 1) - 24(A-iB)a^2 \log(\tan(dx+c)) - \frac{24(iA+B)a^2 \tan(dx+c)^3 + 12(A-iB)a^2 \tan(dx+c)^2 + 4(-2iA-B)a^2 \tan(dx+c) - 3Aa^2}{\tan(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-\frac{1}{12}(24(dx+c)(-IA-B)a^2 + 12(A-IB)a^2 \log(\tan(dx+c)^2 + 1) - 24(A-IB)a^2 \log(\tan(dx+c)) - (24(IA+B)a^2 \tan(dx+c)^3 + 12(A-IB)a^2 \tan(dx+c)^2 + 4(-2IA-B)a^2 \tan(dx+c) - 3Aa^2 \tan(dx+c)^4)/d)$

**Fricas** [A]

time = 0.61, size = 227, normalized size = 1.63

$$\frac{2(3(7A-5iB)a^2 e^{6i(dx+c)} - 3(12A-11iB)a^2 e^{4i(dx+c)} + (29A-25iB)a^2 e^{2i(dx+c)} - (8A-7iB)a^2 - 3((A-iB)a^2 e^{8i(dx+c)} - 4(A-iB)a^2 e^{6i(dx+c)} + 6(A-iB)a^2 e^{4i(dx+c)} - 4(A-iB)a^2 e^{2i(dx+c)} + (A-iB)a^2) \log(e^{2i(dx+c)} - 1))}{3(d e^{8i(dx+c)} - 4d e^{6i(dx+c)} + 6d e^{4i(dx+c)} - 4d e^{2i(dx+c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-\frac{2}{3}(3(7A-5iB)a^2 e^{6i(dx+c)} - 3(12A-11iB)a^2 e^{4i(dx+c)} + (29A-25iB)a^2 e^{2i(dx+c)} - (8A-7iB)a^2 - 3((A-iB)a^2 e^{8i(dx+c)} - 4(A-iB)a^2 e^{6i(dx+c)} + 6(A-iB)a^2 e^{4i(dx+c)} - 4(A-iB)a^2 e^{2i(dx+c)} + (A-iB)a^2) \log(e^{2i(dx+c)} - 1))/(d e^{8i(dx+c)} - 4d e^{6i(dx+c)} + 6d e^{4i(dx+c)} - 4d e^{2i(dx+c)} + d)$

**Sympy** [A]

time = 0.58, size = 235, normalized size = 1.69

$$\frac{2a^2(A-iB) \log(e^{2idx} - e^{-2ic})}{d} + \frac{16Aa^2 - 14iBa^2 + (-58Aa^2 e^{2ic} + 50iBa^2 e^{2ic}) e^{2idx} + (72Aa^2 e^{4ic} - 66iBa^2 e^{4ic}) e^{4idx} + (-42Aa^2 e^{6ic} + 30iBa^2 e^{6ic}) e^{6idx}}{3d e^{8ic} e^{8idx} - 12d e^{6ic} e^{6idx} + 18d e^{4ic} e^{4idx} - 12d e^{2ic} e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)`

[Out]  $2*a**2*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (16*A*a**2 - 14*I*B*a**2 + (-58*A*a**2*\exp(2*I*c) + 50*I*B*a**2*\exp(2*I*c))*\exp(2*I*d*x) + (72*A*a**2*\exp(4*I*c) - 66*I*B*a**2*\exp(4*I*c))*\exp(4*I*d*x) + (-42*A*a**2*\exp(6*I*c) + 30*I*B*a**2*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) - 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) - 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs.  $2(123) = 246$ .  
time = 1.26, size = 322, normalized size = 2.32

$\frac{3A^2 \tan^3(dx + c) - 10A^2 \tan^2(dx + c) + 8B^2 \tan(dx + c) + 60A^2 \tan(dx + c) + 60B^2 \tan(dx + c) + 240A^2 \tan(dx + c) + 216B^2 \tan(dx + c) + 768(A^2 - B^2) \log(\tan(dx + c)) - 384(A^2 - B^2) \log(\tan(dx + c)) + \frac{800A^2 \tan^2(dx + c) - 800I B a^2 \tan^2(dx + c) + 240A^2 \tan(dx + c) + 216B^2 \tan(dx + c) + 768(A^2 - B^2) \log(\tan(dx + c)) + 384(A^2 - B^2) \log(\tan(dx + c))}{d \tan^4(dx + c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^5*(a+I*a*tan(dx+c))^2*(A+B*tan(dx+c)),x, algorithm="giac")`

[Out]  $-1/192*(3*A*a^2*\tan(1/2*d*x + 1/2*c)^4 - 16*I*A*a^2*\tan(1/2*d*x + 1/2*c)^3 - 8*B*a^2*\tan(1/2*d*x + 1/2*c)^3 - 60*A*a^2*\tan(1/2*d*x + 1/2*c)^2 + 48*I*B*a^2*\tan(1/2*d*x + 1/2*c)^2 + 240*I*A*a^2*\tan(1/2*d*x + 1/2*c) + 216*B*a^2*\tan(1/2*d*x + 1/2*c) + 768*(A*a^2 - I*B*a^2)*\log(\tan(1/2*d*x + 1/2*c) + I) - 384*(A*a^2 - I*B*a^2)*\log(\tan(1/2*d*x + 1/2*c)) + (800*A*a^2*\tan(1/2*d*x + 1/2*c)^4 - 800*I*B*a^2*\tan(1/2*d*x + 1/2*c)^4 - 240*I*A*a^2*\tan(1/2*d*x + 1/2*c)^3 - 216*B*a^2*\tan(1/2*d*x + 1/2*c)^3 - 60*A*a^2*\tan(1/2*d*x + 1/2*c)^2 + 48*I*B*a^2*\tan(1/2*d*x + 1/2*c)^2 + 16*I*A*a^2*\tan(1/2*d*x + 1/2*c) + 8*B*a^2*\tan(1/2*d*x + 1/2*c) + 3*A*a^2)/\tan(1/2*d*x + 1/2*c)^4)/d$

**Mupad** [B]

time = 6.44, size = 113, normalized size = 0.81

$$\frac{\tan(c + dx)^2 (Aa^2 - Ba^2 li) + \tan(c + dx)^3 (2Ba^2 + Aa^2 2i) - \frac{Aa^2}{4} - \tan(c + dx) \left( \frac{Ba^2}{3} + \frac{Aa^2 2i}{3} \right)}{d \tan^4(c + dx)} + \frac{4a^2 \operatorname{atan}(2 \tan(c + dx) + li) (B + A li)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)`

[Out]  $(\tan(c + d*x)^2*(A*a^2 - B*a^2*1i) + \tan(c + d*x)^3*(A*a^2*2i + 2*B*a^2) - (A*a^2)/4 - \tan(c + d*x)*((A*a^2*2i)/3 + (B*a^2)/3))/(d*\tan(c + d*x)^4) + (4*a^2*\operatorname{atan}(2*\tan(c + d*x) + 1i)*(A*1i + B))/d$

### 3.17 $\int \tan^2(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

Optimal. Leaf size=182

$$-4a^3(A-iB)x + \frac{4a^3(iA+B)\log(\cos(c+dx))}{d} + \frac{4a^3(A-iB)\tan(c+dx)}{d} + \frac{2a^3(iA+B)\tan^2(c+dx)}{d} - \frac{a^3(45A-47iB)\tan^3(c+dx)}{60d} - \frac{(5A-7iB)\tan^3(c+dx)(a^3+ia^2\tan(c+dx))}{20d} + \frac{2a^3(B+iA)\tan^2(c+dx)}{d} + \frac{4a^3(A-iB)\tan(c+dx)}{d} + \frac{4a^3(B+iA)\log(\cos(c+dx))}{d} - 4a^3x(A-iB) + \frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

[Out]  $-4*a^3*(A-I*B)*x+4*a^3*(I*A+B)*\ln(\cos(d*x+c))/d+4*a^3*(A-I*B)*\tan(d*x+c)/d+2*a^3*(I*A+B)*\tan(d*x+c)^2/d-1/60*a^3*(45*A-47*I*B)*\tan(d*x+c)^3/d+1/5*I*a*B*\tan(d*x+c)^3*(a+I*a*\tan(d*x+c))^2/d-1/20*(5*A-7*I*B)*\tan(d*x+c)^3*(a^3+I*a^3*\tan(d*x+c))/d$

Rubi [A]

time = 0.30, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3675, 3673, 3609, 3606, 3556}

$$\frac{-a^3(45A-47iB)\tan^3(c+dx)}{60d} - \frac{(5A-7iB)\tan^3(c+dx)(a^3+ia^2\tan(c+dx))}{20d} + \frac{2a^3(B+iA)\tan^2(c+dx)}{d} + \frac{4a^3(A-iB)\tan(c+dx)}{d} + \frac{4a^3(B+iA)\log(\cos(c+dx))}{d} - 4a^3x(A-iB) + \frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-4*a^3*(A - I*B)*x + (4*a^3*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d + (4*a^3*(A - I*B)*\text{Tan}[c + d*x])/d + (2*a^3*(I*A + B)*\text{Tan}[c + d*x]^2)/d - (a^3*(45*A - (47*I)*B)*\text{Tan}[c + d*x]^3)/(60*d) + ((I/5)*a*B*\text{Tan}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])^2)/d - ((5*A - (7*I)*B)*\text{Tan}[c + d*x]^3*(a^3 + I*a^3*\text{Tan}[c + d*x]))/(20*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rule 3609

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x]

```
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \tan^2(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \frac{iaB \tan^3(c + dx)(a + ia \tan(c + dx))^2}{5d} + \frac{1}{5} \int \\
 &= \frac{iaB \tan^3(c + dx)(a + ia \tan(c + dx))^2}{5d} - \frac{(5A}{5d} \\
 &= -\frac{a^3(45A - 47iB) \tan^3(c + dx)}{60d} + \frac{iaB \tan^3(c}{60d} \\
 &= \frac{2a^3(iA + B) \tan^2(c + dx)}{d} - \frac{a^3(45A - 47iB)}{60a} \\
 &= -4a^3(A - iB)x + \frac{4a^3(A - iB) \tan(c + dx)}{d} + \\
 &= -4a^3(A - iB)x + \frac{4a^3(iA + B) \log(\cos(c + dx))}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 847 vs.  $2(182) = 364$ .  
time = 7.25, size = 847, normalized size = 4.65

---

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] (Cos[c + d*x]^4*(I*A*Cos[(3*c)/2] + B*Cos[(3*c)/2] + A*Sin[(3*c)/2] - I*B*Sin[(3*c)/2])*(2*Cos[(3*c)/2]*Log[Cos[c + d*x]^2] - (2*I)*Log[Cos[c + d*x]^2]*Sin[(3*c)/2])*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (Sec[c]*Sec[c + d*x]*(Cos[3*c]/240 - (I/240)*Sin[3*c]))*((195*I)*A*Cos[d*x] + 225*B*Cos[d*x] - 300*A*d*x*Cos[d*x] + (300*I)*B*d*x*Cos[d*x] + (195*I)*A*Cos[2*c + d*x] + 225*B*Cos[2*c + d*x] - 300*A*d*x*Cos[2*c + d*x] + (300*I)*B*d*x*Cos[2*c + d*x] + (75*I)*A*Cos[2*c + 3*d*x] + 105*B*Cos[2*c + 3*d*x] - 150*A*d*x*Cos[2*c + 3*d*x] + (150*I)*B*d*x*Cos[2*c + 3*d*x] + (75*I)*A*Cos[4*c + 3*d*x] + 105*B*Cos[4*c + 3*d*x] - 150*A*d*x*Cos[4*c + 3*d*x] + (150*I)*B*d*x*Cos[4*c + 3*d*x] - 30*A*d*x*Cos[4*c + 5*d*x] + (30*I)*B*d*x*Cos[4*c + 5*d*x] - 30*A*d*x*Cos[6*c + 5*d*x] + (30*I)*B*d*x*Cos[6*c + 5*d*x] + 420*A*Sin[d*x] - (470*I)*B*Sin[d*x] - 330*A*Sin[2*c + d*x] + (360*I)*B*Sin[2*c + d*x] + 270*A*Sin[2*c + 3*d*x] - (280*I)*B*Sin[2*c + 3*d*x] - 105*A*Sin[4*c + 3*d*x] + (135*I)*B*Sin[4*c + 3*d*x] + 75*A*Sin[4*c + 5*d*x] - (83*I)*B*Sin[4*c + 5*d*x]))*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (x*Cos[c + d*x]^4*(2*A*Cos[c] - (2*I)*B*Cos[c] - 2*A*Cos[c]^3 + (2*I)*B*Cos[c]^3 - (4*I)*A*Sin[c] - 4*B*Sin[c] + (8*I)*A*Cos[c]^2*Sin[c] + 8*B*Cos[c]^2*Sin[c] + 12*A*Cos[c]*Sin[c]^2 - (12*I)*B*Cos[c]*Sin[c]^2 - (8*I)*A*Sin[c]^3 - 8*B*Sin[c]^3 - 2*A*Sin[c]*Tan[c] + (2*I)*B*Sin[c]*Tan[c] - 2*A*Sin[c]^3*Tan[c] + (2*I)*B*Sin[c]^3*Tan[c] - I*(A - I*B)*(4*Cos[3*c] - (4*I)*Sin[3*c])*Tan[c]))*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/((Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

Maple [A]

time = 0.06, size = 146, normalized size = 0.80

method	result
derivativedivides	$a^3 \left( -\frac{iB(\tan^5(dx+c))}{5} - \frac{iA(\tan^4(dx+c))}{4} + \frac{4iB(\tan^3(dx+c))}{3} - \frac{3B(\tan^4(dx+c))}{4} + 2iA(\tan^2(dx+c)) - A(\tan^3(dx+c)) - 4iB \right) \frac{1}{d}$
default	$a^3 \left( -\frac{iB(\tan^5(dx+c))}{5} - \frac{iA(\tan^4(dx+c))}{4} + \frac{4iB(\tan^3(dx+c))}{3} - \frac{3B(\tan^4(dx+c))}{4} + 2iA(\tan^2(dx+c)) - A(\tan^3(dx+c)) - 4iB \right) \frac{1}{d}$
norman	$(4iB a^3 - 4A a^3) x - \frac{(iA a^3 + 3B a^3)(\tan^4(dx+c))}{4d} - \frac{(-4iB a^3 + 3A a^3)(\tan^3(dx+c))}{3d} + \frac{4(-iB a^3 + A a^3) \tan^2(dx+c)}{d}$
risch	$-\frac{8ia^3Bc}{d} + \frac{8a^3Ac}{d} + \frac{2a^3(180iA e^{8i(dx+c)} + 240B e^{8i(dx+c)} + 525iA e^{6i(dx+c)} + 585B e^{6i(dx+c)} + 615iA e^{4i(dx+c)} + 690B e^{4i(dx+c)} + 15d(e^{2i(dx+c)} + 1))^5}{15d(e^{2i(dx+c)} + 1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

[Out]  $1/d*a^3*(-1/5*I*B*\tan(d*x+c)^5-1/4*I*A*\tan(d*x+c)^4+4/3*I*B*\tan(d*x+c)^3-3/4*B*\tan(d*x+c)^4+2*I*A*\tan(d*x+c)^2-A*\tan(d*x+c)^3-4*I*B*\tan(d*x+c)+2*B*\tan(d*x+c)^2+4*A*\tan(d*x+c)+1/2*(-4*I*A-4*B)*\ln(1+\tan(d*x+c)^2)+(4*I*B-4*A)*\arctan(\tan(d*x+c))$

**Maxima** [A]

time = 0.51, size = 132, normalized size = 0.73

$$\frac{12iBa^3 \tan(dx+c)^5 + 15(iA+3B)a^3 \tan(dx+c)^4 + 20(3A-4iB)a^3 \tan(dx+c)^3 + 120(-iA-B)a^3 \tan(dx+c)^2 + 240(dx+c)(A-iB)a^3 + 120(iA+B)a^3 \log(\tan(dx+c)^2+1) - 240(A-iB)a^3 \tan(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/60*(12*I*B*a^3*\tan(d*x+c)^5 + 15*(I*A + 3*B)*a^3*\tan(d*x+c)^4 + 20*(3*A - 4*I*B)*a^3*\tan(d*x+c)^3 + 120*(-I*A - B)*a^3*\tan(d*x+c)^2 + 240*(d*x+c)*(A - I*B)*a^3 + 120*(I*A + B)*a^3*\log(\tan(d*x+c)^2 + 1) - 240*(A - I*B)*a^3*\tan(d*x+c))/d$

**Fricas** [A]

time = 0.43, size = 291, normalized size = 1.60

$$\frac{2(60(-3iA-4B)a^3e^{8i(dx+c)} + 15(-35iA-39B)a^3e^{6i(dx+c)} + 5(-123iA-139B)a^3e^{4i(dx+c)} + 5(-69iA-77B)a^3e^{2i(dx+c)} + (-75iA-83B)a^3 + 30((-iA-B)a^{10i(dx+c)} + 5(-iA-B)a^{8i(dx+c)} + 10(-iA-B)a^{6i(dx+c)} + 5(-iA-B)a^{4i(dx+c)} + (-iA-B)a^{2i(dx+c)} + (-iA-B)a^2 \log(e^{2i(dx+c)}+1)))}{15(d^{10i(dx+c)} + 5d^{8i(dx+c)} + 10d^{6i(dx+c)} + 10d^{4i(dx+c)} + 5d^{2i(dx+c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-2/15*(60*(-3*I*A - 4*B)*a^3*e^{(8*I*d*x + 8*I*c)} + 15*(-35*I*A - 39*B)*a^3*e^{(6*I*d*x + 6*I*c)} + 5*(-123*I*A - 139*B)*a^3*e^{(4*I*d*x + 4*I*c)} + 5*(-69*I*A - 77*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-75*I*A - 83*B)*a^3 + 30*((-I*A - B)*a^3*e^{(10*I*d*x + 10*I*c)} + 5*(-I*A - B)*a^3*e^{(8*I*d*x + 8*I*c)} + 10*(-I*A - B)*a^3*e^{(6*I*d*x + 6*I*c)} + 10*(-I*A - B)*a^3*e^{(4*I*d*x + 4*I*c)} + 5*(-I*A - B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} + 1))/(d*e^{(10*I*d*x + 10*I*c)} + 5*d*e^{(8*I*d*x + 8*I*c)} + 10*d*e^{(6*I*d*x + 6*I*c)} + 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy** [A]

time = 0.57, size = 292, normalized size = 1.60

$$\frac{4ia^3(A-iB)\log(e^{2idx}+e^{-2ic})}{d} + \frac{150iAa^3 + 166Ba^3 + (690iAa^3e^{2ic} + 770Ba^3e^{2ic})e^{2idx} + (1230iAa^3e^{4ic} + 1390Ba^3e^{4ic})e^{4idx} + (1050iAa^3e^{6ic} + 1170Ba^3e^{6ic})e^{6idx} + (360iAa^3e^{8ic} + 480Ba^3e^{8ic})e^{8idx}}{15de^{10ic}e^{10idx} + 75de^{8ic}e^{8idx} + 150de^{6ic}e^{6idx} + 150de^{4ic}e^{4idx} + 75de^{2ic}e^{2idx} + 15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)`

[Out]  $4*I*a**3*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (150*I*A*a**3 + 166*B*a**3 + (690*I*A*a**3*\exp(2*I*c) + 770*B*a**3*\exp(2*I*c))*\exp(2*I*d*x) + ($



$1230*I*A*a**3*exp(4*I*c) + 1390*B*a**3*exp(4*I*c))*exp(4*I*d*x) + (1050*I*A$   
 $*a**3*exp(6*I*c) + 1170*B*a**3*exp(6*I*c))*exp(6*I*d*x) + (360*I*A*a**3*exp$   
 $(8*I*c) + 480*B*a**3*exp(8*I*c))*exp(8*I*d*x))/(15*d*exp(10*I*c)*exp(10*I*d$   
 $*x) + 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x) + 150*d*$   
 $exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than  
 twice the leaf count of optimal. 504 vs.  $2(158) = 316$ .  
 time = 0.84, size = 504, normalized size = 2.77

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

**[Out]**  $-2/15*(-30*I*A*a^3*e^{(10*I*d*x + 10*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 30*$   
 $B*a^3*e^{(10*I*d*x + 10*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 150*I*A*a^3*e^{(8$   
 $*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 150*B*a^3*e^{(8*I*d*x + 8*I*c$   
 $)*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 300*I*A*a^3*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I$   
 $*d*x + 2*I*c)} + 1) - 300*B*a^3*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)}$   
 $+ 1) - 300*I*A*a^3*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 300*B$   
 $*a^3*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 150*I*A*a^3*e^{(2*I*$   
 $d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 150*B*a^3*e^{(2*I*d*x + 2*I*c)}*l$   
 $og(e^{(2*I*d*x + 2*I*c)} + 1) - 180*I*A*a^3*e^{(8*I*d*x + 8*I*c)} - 240*B*a^3*e$   
 $^{(8*I*d*x + 8*I*c)} - 525*I*A*a^3*e^{(6*I*d*x + 6*I*c)} - 585*B*a^3*e^{(6*I*d*x$   
 $+ 6*I*c)} - 615*I*A*a^3*e^{(4*I*d*x + 4*I*c)} - 695*B*a^3*e^{(4*I*d*x + 4*I*c)}$   
 $- 345*I*A*a^3*e^{(2*I*d*x + 2*I*c)} - 385*B*a^3*e^{(2*I*d*x + 2*I*c)} - 30*I*A$   
 $*a^3*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 30*B*a^3*\log(e^{(2*I*d*x + 2*I*c)} + 1) -$   
 $75*I*A*a^3 - 83*B*a^3)/(d*e^{(10*I*d*x + 10*I*c)} + 5*d*e^{(8*I*d*x + 8*I*c)}$   
 $+ 10*d*e^{(6*I*d*x + 6*I*c)} + 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*$   
 $I*c)} + d)$

**Mupad [B]**

time = 6.15, size = 230, normalized size = 1.26

$$\frac{\tan(c+dx)^3 \left( \frac{Bd^2 - d^2(2A-B)}{d} + \frac{d^2(2B+A)}{3} \right) + \frac{\tan(c+dx) \left( Aa^3 - Ba^3 + a^3(2A-B) - a^3(2B+A) \right)}{d} - \frac{\tan(c+dx)^4 \left( \frac{Bd^2 + d^2(2B+A)}{d} \right)}{d} - \frac{\ln(\tan(c+dx)+1) \left( 4Ba^3 + Aa^3 \right)}{d} + \frac{\tan(c+dx)^2 \left( \frac{Bd^2 + Bd^2 + d^2(2A-B)}{2} + \frac{d^2(2B+A)}{2} \right)}{d} - \frac{Ba^3 \tan(c+dx)^3}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3,x)

**[Out]**  $(\tan(c + d*x)^3*((B*a^3*1i)/3 - (a^3*(2*A - B*1i))/3 + (a^3*(A*1i + 2*B)*1i$   
 $)/3))/d + (\tan(c + d*x)*(A*a^3 - B*a^3*1i + a^3*(2*A - B*1i) - a^3*(A*1i +$   
 $2*B)*1i))/d - (\tan(c + d*x)^4*((B*a^3)/4 + (a^3*(A*1i + 2*B))/4))/d - (\log($   
 $\tan(c + d*x) + 1i)*(A*a^3*4i + 4*B*a^3))/d + (\tan(c + d*x)^2*((A*a^3*1i)/2$   
 $+ (B*a^3)/2 + (a^3*(2*A - B*1i)*1i)/2 + (a^3*(A*1i + 2*B))/2))/d - (B*a^3*t$   
 $\tan(c + d*x)^5*1i)/(5*d)$

### 3.18 $\int \tan(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

Optimal. Leaf size=138

$$-4a^3(iA+B)x - \frac{4a^3(A-iB) \log(\cos(c+dx))}{d} + \frac{2a^3(iA+B) \tan(c+dx)}{d} + \frac{a(A-iB)(a+ia \tan(c+dx))^2}{2d}$$

[Out]  $-4*a^3*(I*A+B)*x - 4*a^3*(A-I*B)*\ln(\cos(d*x+c))/d + 2*a^3*(I*A+B)*\tan(d*x+c)/d + 1/2*a*(A-I*B)*(a+I*a*\tan(d*x+c))^2/d + 1/3*A*(a+I*a*\tan(d*x+c))^3/d - 1/4*I*B*(a+I*a*\tan(d*x+c))^4/a/d$

Rubi [A]

time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ ,

Rules used = {3673, 3608, 3559, 3558, 3556}

$$\frac{2a^3(B+iA)\tan(c+dx)}{d} - \frac{4a^3(A-iB)\log(\cos(c+dx))}{d} - 4a^3x(B+iA) + \frac{a(A-iB)(a+ia \tan(c+dx))^2}{2d} + \frac{A(a+ia \tan(c+dx))^3}{3d} - \frac{iB(a+ia \tan(c+dx))^4}{4ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-4*a^3*(I*A + B)*x - (4*a^3*(A - I*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (2*a^3*(I*A + B)*\text{Tan}[c + d*x])/d + (a*(A - I*B)*(a + I*a*\text{Tan}[c + d*x])^2)/(2*d) + (A*(a + I*a*\text{Tan}[c + d*x])^3)/(3*d) - ((I/4)*B*(a + I*a*\text{Tan}[c + d*x])^4)/(a*d)$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3558

$\text{Int}[((a_.) + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^2, x\_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[b^2*(\text{Tan}[c + d*x]/d), x]) /; \text{FreeQ}[\{a, b, c, d\}, x]$

Rule 3559

$\text{Int}[((a_.) + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*((a + b*\text{Tan}[c + d*x])^{(n-1)}/(d*(n-1))), x] + \text{Dist}[2*a, \text{Int}[(a + b*\text{Tan}[c + d*x])^{(n-1)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[n, 1]$

Rule 3608

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{iB(a + ia \tan(c + dx))^4}{4ad} + \int (a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx \\
&= \frac{A(a + ia \tan(c + dx))^3}{3d} - \frac{iB(a + ia \tan(c + dx))^4}{4ad} \\
&= \frac{a(A - iB)(a + ia \tan(c + dx))^2}{2d} + \frac{A(a + ia \tan(c + dx))^3}{3d} \\
&= -4a^3(iA + B)x + \frac{2a^3(iA + B) \tan(c + dx)}{d} \\
&= -4a^3(iA + B)x - \frac{4a^3(A - iB) \log(\cos(c + dx))}{d}
\end{aligned}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 980 vs. 2(138) = 276.  
time = 7.01, size = 980, normalized size = 7.10

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] (Cos[c + d*x]^4*(A*Cos[(3*c)/2] - I*B*Cos[(3*c)/2] - I*A*Sin[(3*c)/2] - B*Sin[(3*c)/2])*(-2*Cos[(3*c)/2]*Log[Cos[c + d*x]^2] + (2*I)*Log[Cos[c + d*x]^2]*Sin[(3*c)/2])*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (Cos[c + d*x]^2*(-9*A*Cos[c] + (15*I)*B*Cos[c] - (2*I)*A*Sin[c] - 6*B*Sin[c])*(Cos[3*c]/6 - (I/6)*Sin[3*c])*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/(d*(Cos[c/2] -
```

$$\begin{aligned} & \sin[c/2]) * (\cos[c/2] + \sin[c/2]) * (\cos[d*x] + I * \sin[d*x])^3 * (A * \cos[c + d*x] + \\ & B * \sin[c + d*x]) + (((-1/4 * I) * B * \cos[3*c] - (B * \sin[3*c]) / 4) * (a + I * a * \tan[c \\ & + d*x])^3 * (A + B * \tan[c + d*x])) / (d * (\cos[d*x] + I * \sin[d*x])^3 * (A * \cos[c + d*x] \\ & + B * \sin[c + d*x])) + ((A - I * B) * \cos[c + d*x]^4 * ((-4 * I) * d * x * \cos[3*c] - 4 * d \\ & * x * \sin[3*c]) * (a + I * a * \tan[c + d*x])^3 * (A + B * \tan[c + d*x])) / (d * (\cos[d*x] + \\ & I * \sin[d*x])^3 * (A * \cos[c + d*x] + B * \sin[c + d*x])) + (\cos[c + d*x] * (\cos[3*c] / \\ & 3 - (I / 3) * \sin[3*c]) * ((-I) * A * \sin[d*x] - 3 * B * \sin[d*x]) * (a + I * a * \tan[c + d*x]) \\ & ^3 * (A + B * \tan[c + d*x])) / (d * (\cos[c/2] - \sin[c/2]) * (\cos[c/2] + \sin[c/2]) * (\cos \\ & [d*x] + I * \sin[d*x])^3 * (A * \cos[c + d*x] + B * \sin[c + d*x])) + (\cos[c + d*x]^3 \\ & * (\cos[3*c] / 3 - (I / 3) * \sin[3*c]) * ((13 * I) * A * \sin[d*x] + 15 * B * \sin[d*x]) * (a + I * a \\ & * \tan[c + d*x])^3 * (A + B * \tan[c + d*x])) / (d * (\cos[c/2] - \sin[c/2]) * (\cos[c/2] + \\ & \sin[c/2]) * (\cos[d*x] + I * \sin[d*x])^3 * (A * \cos[c + d*x] + B * \sin[c + d*x])) + ( \\ & x * \cos[c + d*x]^4 * ((2 * I) * A * \cos[c] + 2 * B * \cos[c] - (2 * I) * A * \cos[c]^3 - 2 * B * \cos[ \\ & c]^3 + 4 * A * \sin[c] - (4 * I) * B * \sin[c] - 8 * A * \cos[c]^2 * \sin[c] + (8 * I) * B * \cos[c]^2 \\ & * \sin[c] + (12 * I) * A * \cos[c] * \sin[c]^2 + 12 * B * \cos[c] * \sin[c]^2 + 8 * A * \sin[c]^3 - \\ & (8 * I) * B * \sin[c]^3 - (2 * I) * A * \sin[c] * \tan[c] - 2 * B * \sin[c] * \tan[c] - (2 * I) * A * \sin[ \\ & c]^3 * \tan[c] - 2 * B * \sin[c]^3 * \tan[c] + (A - I * B) * (4 * \cos[3*c] - (4 * I) * \sin[3*c]) \\ & * \tan[c]) * (a + I * a * \tan[c + d*x])^3 * (A + B * \tan[c + d*x])) / (((\cos[d*x] + I * \sin[ \\ & d*x])^3 * (A * \cos[c + d*x] + B * \sin[c + d*x])) \end{aligned}$$

**Maple [A]**

time = 0.05, size = 123, normalized size = 0.89

method	result
derivativedivides	$a^3 \left( -\frac{iB(\tan^4(dx+c))}{4} - \frac{iA(\tan^3(dx+c))}{3} + 2iB(\tan^2(dx+c)) - B(\tan^3(dx+c)) + 4iA \tan(dx+c) - \frac{3A(\tan^2(dx+c))}{2} + 4B \tan(dx+c) \right) / d$
default	$a^3 \left( -\frac{iB(\tan^4(dx+c))}{4} - \frac{iA(\tan^3(dx+c))}{3} + 2iB(\tan^2(dx+c)) - B(\tan^3(dx+c)) + 4iA \tan(dx+c) - \frac{3A(\tan^2(dx+c))}{2} + 4B \tan(dx+c) \right) / d$
norman	$\frac{(-4iAa^3 - 4Ba^3)x - \frac{(iAa^3 + 3Ba^3)(\tan^3(dx+c))}{3d} - \frac{(-4iBa^3 + 3Aa^3)(\tan^2(dx+c))}{2d} + \frac{4(iAa^3 + Ba^3)\tan(dx+c)}{d}}{d}$
risch	$\frac{8a^3Bc}{d} + \frac{8ia^3Ac}{d} + \frac{2ia^3(24iAe^{6i(dx+c)} + 36Be^{6i(dx+c)} + 57iAe^{4i(dx+c)} + 69Be^{4i(dx+c)} + 46iAe^{2i(dx+c)} + 54Be^{2i(dx+c)} + 3d(e^{2i(dx+c)} + 1)^4)}{3d(e^{2i(dx+c)} + 1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} a^3 \left( -\frac{1}{4} I B \tan(d*x+c)^4 - \frac{1}{3} I A \tan(d*x+c)^3 + 2 I B \tan(d*x+c)^2 - B \tan(d*x+c) \right) + \frac{4}{d} (i A a^3 + B a^3) \tan(d*x+c) - \frac{3}{2} A \tan(d*x+c)^2 + 4 B \tan(d*x+c) + \frac{1}{2} (-4 I B + 4 A) \ln(1 + \tan(d*x+c)^2) + (-4 I A - 4 B) \arctan(\tan(d*x+c))$$

**Maxima [A]**

time = 0.53, size = 114, normalized size = 0.83

$$\frac{3iBa^3 \tan(dx+c)^4 + 4(iA+3B)a^3 \tan(dx+c)^3 + 6(3A-4iB)a^3 \tan(dx+c)^2 + 48(dx+c)(iA+B)a^3 - 24(A-iB)a^3 \log(\tan(dx+c)^2+1) + 48(-iA-B)a^3 \tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/12*(3*I*B*a^3*\tan(d*x + c)^4 + 4*(I*A + 3*B)*a^3*\tan(d*x + c)^3 + 6*(3*A - 4*I*B)*a^3*\tan(d*x + c)^2 + 48*(d*x + c)*(I*A + B)*a^3 - 24*(A - I*B)*a^3*\log(\tan(d*x + c)^2 + 1) + 48*(-I*A - B)*a^3*\tan(d*x + c))/d$$

**Fricas** [A]

time = 0.46, size = 227, normalized size = 1.64

$$\frac{-2(12(2A-3iB)a^3e^{6i dx+6i c}+3(19A-23iB)a^3e^{4i dx+4i c}+2(23A-27iB)a^3e^{2i dx+2i c}+(13A-15iB)a^3+6((A-iB)a^3e^{8i dx+8i c}+4(A-iB)a^3e^{6i dx+6i c}+6(A-iB)a^3e^{4i dx+4i c}+4(A-iB)a^3e^{2i dx+2i c}+(A-iB)a^3)\log(e^{2i dx+2i c}+1))}{3(d e^{8i dx+8i c}+4 d e^{6i dx+6i c}+6 d e^{4i dx+4i c}+4 d e^{2i dx+2i c}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-2/3*(12*(2*A - 3*I*B)*a^3*e^{(6*I*d*x + 6*I*c)} + 3*(19*A - 23*I*B)*a^3*e^{(4*I*d*x + 4*I*c)} + 2*(23*A - 27*I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (13*A - 15*I*B)*a^3 + 6*((A - I*B)*a^3*e^{(8*I*d*x + 8*I*c)} + 4*(A - I*B)*a^3*e^{(6*I*d*x + 6*I*c)} + 6*(A - I*B)*a^3*e^{(4*I*d*x + 4*I*c)} + 4*(A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (A - I*B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} + 1))/(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(114) = 228.

time = 0.51, size = 235, normalized size = 1.70

$$-\frac{4a^3(A-iB)\log(e^{2idx}+e^{-2ic})}{d} + \frac{-26Aa^3+30iBa^3+(-92Aa^3e^{2ic}+108iBa^3e^{2ic})e^{2idx}+(-114Aa^3e^{4ic}+138iBa^3e^{4ic})e^{4idx}+(-48Aa^3e^{6ic}+72iBa^3e^{6ic})e^{6idx}}{3de^{8ic}e^{8idx}+12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}+12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x)

[Out] 
$$-4*a**3*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (-26*A*a**3 + 30*I*B*a**3 + (-92*A*a**3*\exp(2*I*c) + 108*I*B*a**3*\exp(2*I*c))*\exp(2*I*d*x) + (-114*A*a**3*\exp(4*I*c) + 138*I*B*a**3*\exp(4*I*c))*\exp(4*I*d*x) + (-48*A*a**3*\exp(6*I*c) + 72*I*B*a**3*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) + 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) + 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(116) = 232.

time = 0.69, size = 408, normalized size = 2.96

$$\frac{-4a^3(A-iB)\log(\exp(2I dx) + \exp(-2I c))}{d} + \frac{(-26Aa^3 + 30iBa^3 + (-92Aa^3 \exp(2I c) + 108iBa^3 \exp(2I c)) \exp(2I dx) + (-114Aa^3 \exp(4I c) + 138iBa^3 \exp(4I c)) \exp(4I dx) + (-48Aa^3 \exp(6I c) + 72iBa^3 \exp(6I c)) \exp(6I dx))}{3d \exp(8I c) \exp(8I dx) + 12d \exp(6I c) \exp(6I dx) + 18d \exp(4I c) \exp(4I dx) + 12d \exp(2I c) \exp(2I dx) + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$-2/3*(6*A*a^3*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 6*I*B*a^3*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*A*a^3*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 24*I*B*a^3*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 36*A*a^3*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 36*I*B*a^3*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*A*a^3*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 24*I*B*a^3*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*A*a^3*e^{(6*I*d*x + 6*I*c)} - 36*I*B*a^3*e^{(6*I*d*x + 6*I*c)} + 57*A*a^3*e^{(4*I*d*x + 4*I*c)} - 69*I*B*a^3*e^{(4*I*d*x + 4*I*c)} + 46*A*a^3*e^{(2*I*d*x + 2*I*c)} - 54*I*B*a^3*e^{(2*I*d*x + 2*I*c)} + 6*A*a^3*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 6*I*B*a^3*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 13*A*a^3 - 15*I*B*a^3)/(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Mupad [B]**

time = 6.04, size = 176, normalized size = 1.28

$$\frac{\tan(c+dx)^2 \left( \frac{B a^3 1i}{2} - \frac{a^3 (2A-B 1i)}{2} + \frac{a^3 (2B+A 1i) 1i}{2} \right) + \frac{\tan(c+dx) (A a^3 1i + B a^3 + a^3 (2A-B 1i) 1i + a^3 (2B+A 1i))}{d} - \frac{\tan(c+dx)^3 \left( \frac{B a^3}{3} + \frac{a^3 (2B+A 1i)}{3} \right) + \frac{\ln(\tan(c+dx)+1) (4A a^3 - B a^3 4i)}{d} - \frac{B a^3 \tan(c+dx)^4 1i}{4d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] 
$$(\tan(c + d*x)^2*((B*a^3*1i)/2 - (a^3*(2*A - B*1i))/2 + (a^3*(A*1i + 2*B)*1i)/2))/d + (\tan(c + d*x)*(A*a^3*1i + B*a^3 + a^3*(2*A - B*1i)*1i + a^3*(A*1i + 2*B)))/d - (\tan(c + d*x)^3*((B*a^3)/3 + (a^3*(A*1i + 2*B))/3))/d + (\log(\tan(c + d*x) + 1i)*(4*A*a^3 - B*a^3*4i))/d - (B*a^3*\tan(c + d*x)^4*1i)/(4*d)$$

### 3.19 $\int (a + ia \tan(c + dx))^3 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=110

$$4a^3(A-iB)x - \frac{4a^3(iA+B)\log(\cos(c+dx))}{d} - \frac{2a^3(A-iB)\tan(c+dx)}{d} + \frac{a(iA+B)(a+ia\tan(c+dx))^2}{2d}$$

[Out]  $4a^3(A-I*B)*x - 4a^3(I*A+B)*\ln(\cos(d*x+c))/d - 2a^3(A-I*B)*\tan(d*x+c)/d + 1/2*a*(I*A+B)*(a+I*a*\tan(d*x+c))^2/d + 1/3*B*(a+I*a*\tan(d*x+c))^3/d$

**Rubi [A]**

time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {3608, 3559, 3558, 3556}

$$-\frac{2a^3(A-iB)\tan(c+dx)}{d} - \frac{4a^3(B+iA)\log(\cos(c+dx))}{d} + 4a^3x(A-iB) + \frac{a(B+iA)(a+ia\tan(c+dx))^2}{2d} + \frac{B(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $4a^3(A - I*B)*x - (4a^3*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d - (2a^3*(A - I*B)*\text{Tan}[c + d*x])/d + (a*(I*A + B)*(a + I*a*\text{Tan}[c + d*x])^2)/(2*d) + (B*(a + I*a*\text{Tan}[c + d*x])^3)/(3*d)$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3558**

Int[((a\_.) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)])^2, x\_Symbol] := Simp[(a^2 - b^2)\*x, x] + (Dist[2\*a\*b, Int[Tan[c + d\*x], x], x] + Simp[b^2\*(Tan[c + d\*x]/d), x]) /; FreeQ[{a, b, c, d}, x]

**Rule 3559**

Int[((a\_.) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)])^(n\_), x\_Symbol] := Simp[b\*((a + b\*Tan[c + d\*x])^(n - 1)/(d\*(n - 1))), x] + Dist[2\*a, Int[(a + b\*Tan[c + d\*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

**Rule 3608**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e,





norman	$(-4iB a^3 + 4A a^3) x - \frac{(iA a^3 + 3B a^3)(\tan^2(dx+c))}{2d} - \frac{(-4iB a^3 + 3A a^3) \tan(dx+c)}{d} - \frac{iB a^3 (\tan^3(dx+c))}{3d}$
risch	$\frac{8ia^3Bc}{d} - \frac{8a^3Ac}{d} - \frac{2a^3(12iA e^{4i(dx+c)} + 24B e^{4i(dx+c)} + 21iA e^{2i(dx+c)} + 33B e^{2i(dx+c)} + 9iA + 13B)}{3d(e^{2i(dx+c)} + 1)^3} - \frac{4a^3 \ln(e^{2i(dx+c)} + 1)}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*a^3*(-1/3*I*B*tan(d*x+c)^3-1/2*I*A*tan(d*x+c)^2+4*I*B*tan(d*x+c)-3/2*B*tan(d*x+c)^2-3*A*tan(d*x+c)+1/2*(4*I*A+4*B)*\ln(1+\tan(d*x+c)^2)+(-4*I*B+4*A)*\arctan(\tan(d*x+c)))$

**Maxima** [A]

time = 0.61, size = 96, normalized size = 0.87

$$\frac{2iBa^3 \tan(dx+c)^3 + 3(iA+3B)a^3 \tan(dx+c)^2 - 24(dx+c)(A-iB)a^3 + 12(-iA-B)a^3 \log(\tan(dx+c)^2+1) + 6(3A-4iB)a^3 \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/6*(2*I*B*a^3*\tan(d*x+c)^3 + 3*(I*A + 3*B)*a^3*\tan(d*x+c)^2 - 24*(d*x+c)*(A - I*B)*a^3 + 12*(-I*A - B)*a^3*\log(\tan(d*x+c)^2 + 1) + 6*(3*A - 4*I*B)*a^3*\tan(d*x+c))/d$

**Fricas** [A]

time = 0.52, size = 175, normalized size = 1.59

$$\frac{2(12(iA+2B)a^3 e^{(4i dx+4i c)} + 3(7iA+11B)a^3 e^{(2i dx+2i c)} + (9iA+13B)a^3 + 6((iA+B)a^3 e^{(6i dx+6i c)} + 3(iA+B)a^3 e^{(4i dx+4i c)} + 3(iA+B)a^3 e^{(2i dx+2i c)} + (iA+B)a^3) \log(e^{(2i dx+2i c)} + 1))}{3(d e^{(6i dx+6i c)} + 3d e^{(4i dx+4i c)} + 3d e^{(2i dx+2i c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-2/3*(12*(I*A + 2*B)*a^3*e^{(4*I*d*x + 4*I*c)} + 3*(7*I*A + 11*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (9*I*A + 13*B)*a^3 + 6*((I*A + B)*a^3*e^{(6*I*d*x + 6*I*c)} + 3*(I*A + B)*a^3*e^{(4*I*d*x + 4*I*c)} + 3*(I*A + B)*a^3*e^{(2*I*d*x + 2*I*c)} + (I*A + B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} + 1)/(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy** [A]

time = 0.41, size = 184, normalized size = 1.67

$$-\frac{4ia^3(A-iB)\log(e^{2idx} + e^{-2ic})}{d} + \frac{-18iAa^3 - 26Ba^3 + (-42iAa^3e^{2ic} - 66Ba^3e^{2ic})e^{2idx} + (-24iAa^3e^{4ic} - 48Ba^3e^{4ic})e^{4idx}}{3de^{6ic}e^{6idx} + 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)`

[Out]  $-4*I*a**3*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (-18*I*A*a**3 - 26*B*a**3 + (-42*I*A*a**3*\exp(2*I*c) - 66*B*a**3*\exp(2*I*c))*\exp(2*I*d*x) + (-24*I*A*a**3*\exp(4*I*c) - 48*B*a**3*\exp(4*I*c))*\exp(4*I*d*x))/(3*d*\exp(6*I*c))*\exp(6*I*d*x) + 9*d*\exp(4*I*c)*\exp(4*I*d*x) + 9*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs.  $2(94) = 188$ .  
time = 0.59, size = 312, normalized size = 2.84

$$\frac{2(9IAd^2e^{2Icd}\log(e^{2Icd}+1) + 6Bd^2e^{2Icd}\log(e^{2Icd}+1) + 18Ad^2e^{2Icd}\log(e^{2Icd}+1) + 18Bd^2e^{2Icd}\log(e^{2Icd}+1) + 18Ad^2e^{2Icd}\log(e^{2Icd}+1) + 18Bd^2e^{2Icd}\log(e^{2Icd}+1) + 12Ad^2e^{2Icd} + 24Bd^2e^{2Icd} + 21Ad^2e^{2Icd} + 33Bd^2e^{2Icd} + 6Ad^2\log(e^{2Icd}+1) + 6Bd^2\log(e^{2Icd}+1) + 9Ad^2 + 13Bd^2)}{3(d^{6Icd} + 3d^{4Icd} + 3d^{2Icd} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $-2/3*(6*I*A*a^3*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 6*B*a^3*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 18*I*A*a^3*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 18*B*a^3*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 18*I*A*a^3*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 18*B*a^3*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 12*I*A*a^3*e^{(4*I*d*x + 4*I*c)} + 24*B*a^3*e^{(4*I*d*x + 4*I*c)} + 21*I*A*a^3*e^{(2*I*d*x + 2*I*c)} + 33*B*a^3*e^{(2*I*d*x + 2*I*c)} + 6*I*A*a^3*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 6*B*a^3*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 9*I*A*a^3 + 13*B*a^3)/(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Mupad** [B]

time = 6.13, size = 125, normalized size = 1.14

$$-\frac{\tan(c+dx)^2\left(\frac{Ba^3}{2} + \frac{a^3(2B+A1i)}{2}\right)}{d} + \frac{\ln(\tan(c+dx)+1i)(4Ba^3+Aa^34i)}{d} + \frac{\tan(c+dx)(Ba^31i - a^3(2A-B1i) + a^3(2B+A1i)1i)}{d} - \frac{Ba^3\tan(c+dx)^31i}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)`

[Out]  $(\log(\tan(c + d*x) + 1i)*(A*a^3*4i + 4*B*a^3))/d - (\tan(c + d*x)^2*((B*a^3)/2 + (a^3*(A*1i + 2*B))/2))/d + (\tan(c + d*x)*(B*a^3*1i - a^3*(2*A - B*1i) + a^3*(A*1i + 2*B)*1i))/d - (B*a^3*\tan(c + d*x)^3*1i)/(3*d)$

### 3.20 $\int \cot(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

Optimal. Leaf size=107

$$4a^3(iA+B)x + \frac{a^3(3A-4iB)\log(\cos(c+dx))}{d} + \frac{a^3A\log(\sin(c+dx))}{d} + \frac{iaB(a+ia \tan(c+dx))^2}{2d} - \frac{(A-2iB)}{d}$$

[Out]  $4*a^3*(I*A+B)*x+a^3*(3*A-4*I*B)*\ln(\cos(d*x+c))/d+a^3*A*\ln(\sin(d*x+c))/d+1/2*I*a*B*(a+I*a*\tan(d*x+c))^2/d-(A-2*I*B)*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.20, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3675, 3670, 3556, 3612}

$$-\frac{(A-2iB)(a^3+ia^3 \tan(c+dx))}{d} + \frac{a^3(3A-4iB)\log(\cos(c+dx))}{d} + 4a^3x(B+iA) + \frac{a^3A\log(\sin(c+dx))}{d} + \frac{iaB(a+ia \tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $4*a^3*(I*A + B)*x + (a^3*(3*A - (4*I)*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (a^3*A*\text{Log}[\text{Sin}[c + d*x]])/d + ((I/2)*a*B*(a + I*a*\text{Tan}[c + d*x])^2)/d - ((A - (2*I)*B)*(a^3 + I*a^3*\text{Tan}[c + d*x]))/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3612

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3670

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[B*(d/b), \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Dist}[1/b, \text{Int}[\text{Simp}[A*b*c + (A*b*d + B*(b*c - a*d))*\text{Tan}[e + f*x], x]/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$



**Maple [A]**

time = 0.10, size = 133, normalized size = 1.24

method	result
norman	$(4iAa^3 + 4Ba^3)x - \frac{(iAa^3 + 3Ba^3)\tan(dx+c)}{d} - \frac{iBa^3(\tan^2(dx+c))}{2d} + \frac{Aa^3\ln(\tan(dx+c))}{d} - \frac{2(-iBa^3)}{d}$
derivativedivides	$\frac{-iAa^3(\tan(dx+c)-dx-c) - iBa^3\left(\frac{(\tan^2(dx+c))}{2} + \ln(\cos(dx+c))\right) + 3Aa^3\ln(\cos(dx+c)) - 3Ba^3(\tan(dx+c)-dx-c)}{d}$
default	$\frac{-iAa^3(\tan(dx+c)-dx-c) - iBa^3\left(\frac{(\tan^2(dx+c))}{2} + \ln(\cos(dx+c))\right) + 3Aa^3\ln(\cos(dx+c)) - 3Ba^3(\tan(dx+c)-dx-c)}{d}$
risch	$-\frac{8a^3Bc}{d} - \frac{8ia^3Ac}{d} - \frac{2ia^3(iAe^{2i(dx+c)} + 4Be^{2i(dx+c)} + iA + 3B)}{d(e^{2i(dx+c)} + 1)^2} - \frac{4ia^3\ln(e^{2i(dx+c)} + 1)B}{d} + \frac{3a^3\ln(e^{2i(dx+c)} + 1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d}(-I*A*a^3*(\tan(dx+c)-dx-c) - I*B*a^3*(\frac{1}{2}\tan(dx+c)^2 + \ln(\cos(dx+c))) + 3*A*a^3*\ln(\cos(dx+c)) - 3*B*a^3*(\tan(dx+c)-dx-c) + 3*I*A*a^3*(dx+c) - 3*I*B*a^3*\ln(\cos(dx+c)) + A*a^3*\ln(\sin(dx+c)) + B*a^3*(dx+c))$

**Maxima [A]**

time = 0.52, size = 89, normalized size = 0.83

$$\frac{iBa^3\tan(dx+c)^2 + 8(dx+c)(-iA-B)a^3 + 4(A-iB)a^3\log(\tan(dx+c)^2+1) - 2Aa^3\log(\tan(dx+c)) + 2(iA+3B)a^3\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out]  $-\frac{1}{2}(I*B*a^3*\tan(dx+c)^2 + 8*(dx+c)*(-I*A-B)*a^3 + 4*(A-I*B)*a^3*\log(\tan(dx+c)^2+1) - 2*A*a^3*\log(\tan(dx+c)) + 2*(I*A+3*B)*a^3*\tan(dx+c))/d$

**Fricas [A]**

time = 0.57, size = 172, normalized size = 1.61

$$\frac{2(A-4iB)a^3e^{2i(dx+2i)c} + 2(A-3iB)a^3 + ((3A-4iB)a^3e^{4i(dx+4i)c} + 2(3A-4iB)a^3e^{2i(dx+2i)c} + (3A-4iB)a^3)\log(e^{2i(dx+2i)c}+1) + (Aa^3e^{4i(dx+4i)c} + 2Aa^3e^{2i(dx+2i)c} + Aa^3)\log(e^{2i(dx+2i)c}-1)}{de^{4i(dx+4i)c} + 2de^{2i(dx+2i)c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out]  $(2*(A-4*I*B)*a^3*e^{(2*I*d*x+2*I*c)} + 2*(A-3*I*B)*a^3 + ((3*A-4*I*B)*a^3*e^{(4*I*d*x+4*I*c)} + 2*(3*A-4*I*B)*a^3*e^{(2*I*d*x+2*I*c)} + (3*A-4*I*B)*a^3)$



[In] `int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)`

[Out]  $(A*a^3*\log(\tan(c + d*x)))/d - (\tan(c + d*x)*(B*a^3 + a^3*(A*1i + 2*B)))/d - (4*a^3*\log(\tan(c + d*x) + 1i)*(A - B*1i))/d - (B*a^3*\tan(c + d*x)^2*1i)/(2*d)$

### 3.21 $\int \cot^2(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=116

$$-4a^3(A-iB)x + \frac{a^3(iA+3B)\log(\cos(c+dx))}{d} + \frac{a^3(3iA+B)\log(\sin(c+dx))}{d} - \frac{aA \cot(c+dx)(a+ia \tan(c+dx))^2}{d}$$

[Out]  $-4*a^3*(A-I*B)*x + a^3*(I*A+3*B)*\ln(\cos(d*x+c))/d + a^3*(3*I*A+B)*\ln(\sin(d*x+c))/d - a*A*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^2/d + (I*A-B)*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3675, 3670, 3556, 3612}

$$\frac{(-B+iA)(a^3+ia^3 \tan(c+dx))}{d} + \frac{a^3(B+3iA)\log(\sin(c+dx))}{d} + \frac{a^3(3B+iA)\log(\cos(c+dx))}{d} - 4a^3x(A-iB) - \frac{aA \cot(c+dx)(a+ia \tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c+d*x]^2*(a+I*a*\text{Tan}[c+d*x])^3*(A+B*\text{Tan}[c+d*x]),x]$

[Out]  $-4*a^3*(A-I*B)*x + (a^3*(I*A+3*B)*\text{Log}[\text{Cos}[c+d*x]])/d + (a^3*((3*I)*A+B)*\text{Log}[\text{Sin}[c+d*x]])/d - (a*A*\text{Cot}[c+d*x]*(a+I*a*\text{Tan}[c+d*x])^2)/d + ((I*A-B)*(a^3+I*a^3*\text{Tan}[c+d*x]))/d$

**Rule 3556**

$\text{Int}[\text{tan}[(c_.)+(d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3612**

$\text{Int}[(c_.+(d_.)*\text{tan}(e_.+(f_.)*(x_.)))/((a_.)+(b_.)*\text{tan}(e_.+(f_.)*(x_.)))*(x_.)], x\_Symbol] \rightarrow \text{Simp}[(a*c+b*d)*(x/(a^2+b^2)), x] + \text{Dist}[(b*c-a*d)/(a^2+b^2), \text{Int}[(b-a*\text{Tan}[e+f*x])/(a+b*\text{Tan}[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[a^2+b^2, 0] \&\& \text{NeQ}[a*c+b*d, 0]$

**Rule 3670**

$\text{Int}[(A_.+(B_.)*\text{tan}(e_.+(f_.)*(x_.)))*((c_.)+(d_.)*\text{tan}(e_.+(f_.)*(x_.)))/((a_.)+(b_.)*\text{tan}(e_.+(f_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[B*(d/b), \text{Int}[\text{Tan}[e+f*x], x], x] + \text{Dist}[1/b, \text{Int}[\text{Simp}[A*b*c+(A*b*d+B*(b*c-a*d))*\text{Tan}[e+f*x], x]/(a+b*\text{Tan}[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c-a*d, 0]$



## Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

## Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \cot^2(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^2}{d} + \int \dots \\
&= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^2}{d} + \frac{(iA + 3B) \log(\cos(c + dx))}{d} \\
&= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^2}{d} + \frac{(iA + 3B) \log(\cos(c + dx))}{d} \\
&= -4a^3(A - iB)x + \frac{a^3(iA + 3B) \log(\cos(c + dx))}{d} \\
&= -4a^3(A - iB)x + \frac{a^3(iA + 3B) \log(\cos(c + dx))}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 291 vs. 2(116) = 232.  
time = 2.64, size = 291, normalized size = 2.51

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] (a^3*Csc[c]*Csc[c + d*x]*Sec[c]*Sec[c + d*x]*(14*A*d*x*Cos[4*c + 2*d*x] - (10*I)*B*d*x*Cos[4*c + 2*d*x] - I*A*Cos[4*c + 2*d*x]*Log[Cos[c + d*x]^2] - 3*B*Cos[4*c + 2*d*x]*Log[Cos[c + d*x]^2] - (3*I)*A*Cos[4*c + 2*d*x]*Log[Sin[c + d*x]^2] - B*Cos[4*c + 2*d*x]*Log[Sin[c + d*x]^2] + Cos[2*d*x]*(2*(-7*A + (5*I)*B)*d*x + (I*A + 3*B)*Log[Cos[c + d*x]^2] + ((3*I)*A + B)*Log[Sin[c + d*x]^2])) - 4*A*Sin[2*c] - (4*I)*B*Sin[2*c] + 4*A*Sin[2*d*x] - (4*I)*B*Sin[2*d*x] + 4*A*Sin[2*(c + d*x)] + (4*I)*B*Sin[2*(c + d*x)] + 4*(3*A - I*B)*ArcTan[Tan[4*c + d*x]]*Sin[2*c]*Sin[2*(c + d*x)]))/(16*d)
```

**Maple [A]**

time = 0.10, size = 124, normalized size = 1.07

method	result
norman	$\frac{(4iB a^3 - 4A a^3)x \tan(dx+c) - \frac{A a^3}{d} - \frac{iB a^3 (\tan^2(dx+c))}{d}}{\tan(dx+c)} + \frac{(3iA a^3 + B a^3) \ln(\tan(dx+c))}{d} - \frac{2(iA a^3 + B a^3) \ln(1 + \tan^2(dx+c))}{d}$
derivativedivides	$\frac{iA a^3 \ln(\cos(dx+c)) - iB a^3 (\tan(dx+c) - dx - c) - 3A a^3 (dx+c) + 3B a^3 \ln(\cos(dx+c)) + 3iA a^3 \ln(\sin(dx+c)) + 3iB a^3 (dx+c)}{d}$
default	$\frac{iA a^3 \ln(\cos(dx+c)) - iB a^3 (\tan(dx+c) - dx - c) - 3A a^3 (dx+c) + 3B a^3 \ln(\cos(dx+c)) + 3iA a^3 \ln(\sin(dx+c)) + 3iB a^3 (dx+c)}{d}$
risch	$-\frac{8ia^3 Bc}{d} + \frac{8a^3 Ac}{d} + \frac{2a^3 (-iA e^{2i(dx+c)} + B e^{2i(dx+c)} - iA - B)}{d(e^{2i(dx+c)} + 1)(e^{2i(dx+c)} - 1)} + \frac{3a^3 \ln(e^{2i(dx+c)} + 1)B}{d} + \frac{ia^3 \ln(e^{2i(dx+c)} + 1)A}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERB OSE)
```

```
[Out] 1/d*(I*A*a^3*ln(cos(d*x+c))-I*B*a^3*(tan(d*x+c)-d*x-c)-3*A*a^3*(d*x+c)+3*B*a^3*ln(cos(d*x+c))+3*I*A*a^3*ln(sin(d*x+c))+3*I*B*a^3*(d*x+c)+A*a^3*(-cot(d*x+c)-d*x-c)+B*a^3*ln(sin(d*x+c)))
```

**Maxima [A]**

time = 0.50, size = 84, normalized size = 0.72

$$\frac{4(dx+c)(A-iB)a^3 + 2(iA+B)a^3 \log(\tan(dx+c)^2 + 1) - (3iA+B)a^3 \log(\tan(dx+c)) + iBa^3 \tan(dx+c) + \frac{Aa^3}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(4*(d*x + c)*(A - I*B)*a^3 + 2*(I*A + B)*a^3*log(tan(d*x + c)^2 + 1) - (3*I*A + B)*a^3*log(tan(d*x + c)) + I*B*a^3*tan(d*x + c) + A*a^3/tan(d*x + c))/d
```

**Fricas [A]**

time = 0.68, size = 141, normalized size = 1.22

$$\frac{2(iA-B)a^3 e^{2i(dx+2ic)} + 2(iA+B)a^3 - ((iA+3B)a^3 e^{4i(dx+4ic)} + (-iA-3B)a^3) \log(e^{2i(dx+2ic)} + 1) - ((3iA+B)a^3 e^{4i(dx+4ic)} + (-3iA-B)a^3) \log(e^{2i(dx+2ic)} - 1)}{d e^{4i(dx+4ic)} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-(2*(I*A - B)*a^3*e^{(2*I*d*x + 2*I*c)} + 2*(I*A + B)*a^3 - ((I*A + 3*B)*a^3*e^{(4*I*d*x + 4*I*c)} + (-I*A - 3*B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} + 1) - ((3*I*A + B)*a^3*e^{(4*I*d*x + 4*I*c)} + (-3*I*A - B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} - 1))/(d*e^{(4*I*d*x + 4*I*c)} - d)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(99) = 198.

time = 1.12, size = 219, normalized size = 1.89

$$\frac{ia^3(A - 3iB) \log\left(e^{2idx} + \frac{2Aa^3 - 2iBa^3 - a^3(A - 3iB)}{Aa^3e^{2ic} + iBa^3e^{2ic}}\right)}{d} + \frac{ia^3 \cdot (3A - iB) \log\left(e^{2idx} + \frac{2Aa^3 - 2iBa^3 - a^3(3A - iB)}{Aa^3e^{2ic} + iBa^3e^{2ic}}\right)}{d} + \frac{-2iAa^3 - 2Ba^3 + (-2iAa^3e^{2ic} + 2Ba^3e^{2ic})e^{2idx}}{de^{4ic}e^{4idx} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out]  $I*a**3*(A - 3*I*B)*\log(\exp(2*I*d*x) + (2*A*a**3 - 2*I*B*a**3 - a**3*(A - 3*I*B)))/(A*a**3*\exp(2*I*c) + I*B*a**3*\exp(2*I*c))/d + I*a**3*(3*A - I*B)*\log(\exp(2*I*d*x) + (2*A*a**3 - 2*I*B*a**3 - a**3*(3*A - I*B)))/(A*a**3*\exp(2*I*c) + I*B*a**3*\exp(2*I*c))/d + (-2*I*A*a**3 - 2*B*a**3 + (-2*I*A*a**3*\exp(2*I*c) + 2*B*a**3*\exp(2*I*c))*\exp(2*I*d*x))/(d*\exp(4*I*c)*\exp(4*I*d*x) - d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(104) = 208.

time = 1.26, size = 257, normalized size = 2.22

$$\frac{3Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 6(iAa^2 + 3Ba^2) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) - 48(iAa^2 + Ba^2) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + i\right) + 6(iAa^2 + 3Ba^2) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right) - 6(-3iAa^2 - Ba^2) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - i\right) + \frac{-20Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 14Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 12Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 10Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 14Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 12Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 10Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $1/6*(3*A*a^3*\tan(1/2*d*x + 1/2*c) + 6*(I*A*a^3 + 3*B*a^3)*\log(\tan(1/2*d*x + 1/2*c) + 1) - 48*(I*A*a^3 + B*a^3)*\log(\tan(1/2*d*x + 1/2*c) + I) + 6*(I*A*a^3 + 3*B*a^3)*\log(\tan(1/2*d*x + 1/2*c) - 1) - 6*(-3*I*A*a^3 - B*a^3)*\log(\tan(1/2*d*x + 1/2*c)) + (-10*I*A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 14*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 3*A*a^3*\tan(1/2*d*x + 1/2*c)^2 + 12*I*B*a^3*\tan(1/2*d*x + 1/2*c)^2 + 10*I*A*a^3*\tan(1/2*d*x + 1/2*c) + 14*B*a^3*\tan(1/2*d*x + 1/2*c) + 3*A*a^3)/(\tan(1/2*d*x + 1/2*c)^3 - \tan(1/2*d*x + 1/2*c))/d$

**Mupad** [B]

time = 6.43, size = 76, normalized size = 0.66

$$\frac{a^3 \ln(\tan(c + dx)) (B + A3i)}{d} - \frac{4a^3 \ln(\tan(c + dx) + i) (B + A1i)}{d} - \frac{Aa^3 \cot(c + dx)}{d} - \frac{Ba^3 \tan(c + dx) \operatorname{li}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (a^3*log(tan(c + d*x))*(A*3i + B))/d - (4*a^3*log(tan(c + d*x) + 1i)*(A*1i + B))/d - (A*a^3*cot(c + d*x))/d - (B*a^3*tan(c + d*x)*1i)/d
```

### 3.22 $\int \cot^3(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=123

$$-4a^3(iA+B)x + \frac{ia^3B \log(\cos(c+dx))}{d} - \frac{a^3(4A-3iB) \log(\sin(c+dx))}{d} - \frac{aA \cot^2(c+dx)(a+ia \tan(c+dx))^2}{2d}$$

[Out]  $-4a^3(IA+B)x + I a^3 B \ln(\cos(dx+c))/d - a^3(4A-3IB) \ln(\sin(dx+c))/d - 1/2 a A \cot(dx+c)^2 (a+I a \tan(dx+c))^2 / d - (2IA+B) \cot(dx+c) (a^3 + I a^3 \tan(dx+c)) / d$

**Rubi [A]**

time = 0.22, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3674, 3670, 3556, 3612}

$$-\frac{a^3(4A-3iB) \log(\sin(c+dx))}{d} - \frac{(B+2iA) \cot(c+dx)(a^3+ia^3 \tan(c+dx))}{d} - 4a^3x(B+iA) + \frac{ia^3B \log(\cos(c+dx))}{d} - \frac{aA \cot^2(c+dx)(a+ia \tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-4a^3(IA+B)x + (Ia^3B \text{Log}[\text{Cos}[c+d*x]])/d - (a^3(4A-(3I)B) \text{Log}[\text{Sin}[c+d*x]])/d - (aA \text{Cot}[c+d*x]^2 (a+Ia \text{Tan}[c+d*x])^2)/(2d) - (((2I)A+B) \text{Cot}[c+d*x] (a^3+Ia^3 \text{Tan}[c+d*x]))/d$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

**Rule 3670**

Int[(((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

## Rule 3674

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

```

## Rubi steps

$$\begin{aligned}
\int \cot^3(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^2}{2d} + \frac{1}{2} \int \\
&= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^2}{2d} - \frac{(2i)}{2d} \\
&= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^2}{2d} - \frac{(2i)}{2d} \\
&= -4a^3(iA + B)x + \frac{ia^3 B \log(\cos(c + dx))}{d} - \frac{a^3}{d} \\
&= -4a^3(iA + B)x + \frac{ia^3 B \log(\cos(c + dx))}{d} - \frac{a^3}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1010 vs. 2(123) = 246.  
time = 6.96, size = 1010, normalized size = 8.21

Antiderivative was successfully verified.

```

[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] a^3*(((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(-1/2*(A*Cos[3*c]) + (I/2)*
A*Sin[3*c])*Sin[c + d*x]^2)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] +
B*Sin[c + d*x])) + ((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*Csc[c/2]*Sec[
c/2]*(Cos[3*c]/2 - (I/2)*Sin[3*c])*((3*I)*A*Sin[d*x] + B*Sin[d*x])*Sin[c +
d*x]^3)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (
(I/2)*B*Cos[3*c]*(I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*Log[Cos[c + d*x]
^2]*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c
+ d*x])) + ((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(4*A*Cos[(3*c)/2] - (
3*I)*B*Cos[(3*c)/2] - (4*I)*A*Sin[(3*c)/2] - 3*B*Sin[(3*c)/2])*(I*ArcTan[Ta

```

$$\begin{aligned} & n[4*c + d*x]]*Cos[(3*c)/2] + ArcTan[Tan[4*c + d*x]]*Sin[(3*c)/2])*Sin[c + d \\ & *x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (( \\ & I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(4*A*Cos[(3*c)/2] - (3*I)*B*Cos[(3 \\ & *c)/2] - (4*I)*A*Sin[(3*c)/2] - 3*B*Sin[(3*c)/2])*(-1/2*(Cos[(3*c)/2]*Log[S \\ & in[c + d*x]^2) + (I/2)*Log[Sin[c + d*x]^2]*Sin[(3*c)/2])*Sin[c + d*x]^4)/( \\ & d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (B*(I + Co \\ & t[c + d*x])^3*(B + A*Cot[c + d*x])*Log[Cos[c + d*x]^2]*Sin[3*c]*Sin[c + d*x \\ & ]^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (( \\ & A - I*B)*(I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*((-4*I)*d*x*Cos[3*c] - 4 \\ & *d*x*Sin[3*c])*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] \\ & + B*Sin[c + d*x])) + (x*(I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*Sin[c + \\ & d*x]^4*((B*Cos[c])/2 - (16*I)*A*Cos[c]^3 - (25*B*Cos[c]^3)/2 + 4*A*Cos[c]^3 \\ & *Cot[c] - (3*I)*B*Cos[c]^3*Cot[c] - I*B*Sin[c] - 24*A*Cos[c]^2*Sin[c] + (20 \\ & *I)*B*Cos[c]^2*Sin[c] + (16*I)*A*Cos[c]*Sin[c]^2 + 15*B*Cos[c]*Sin[c]^2 + 4 \\ & *A*Sin[c]^3 - (5*I)*B*Sin[c]^3 + (2*A - I*B + 2*A*Cos[2*c] - (2*I)*B*Cos[2* \\ & c])*Csc[c]*Sec[c]*(-Cos[3*c] + I*Sin[3*c]) - (B*Sin[c]*Tan[c])/2 - (B*Sin[c \\ & ]^3*Tan[c])/2))/((Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x] \\ & ))) \end{aligned}$$

**Maple [A]**

time = 0.11, size = 139, normalized size = 1.13

method	result
norman	$\frac{(-4iAa^3 - 4Ba^3)x(\tan^2(dx+c)) - \frac{Aa^3}{2d} - \frac{(3iAa^3 + Ba^3)\tan(dx+c)}{d}}{\tan(dx+c)^2} - \frac{(-3iBa^3 + 4Aa^3)\ln(\tan(dx+c))}{d} + \frac{2(-iBa^3)}{d}$
derivativedivides	$\frac{-iAa^3(dx+c) + iBa^3 \ln(\cos(dx+c)) - 3Aa^3 \ln(\sin(dx+c)) - 3Ba^3(dx+c) + 3iAa^3(-\cot(dx+c) - dx - c) + 3iBa^3 \ln(\sin(dx+c))}{d}$
default	$\frac{-iAa^3(dx+c) + iBa^3 \ln(\cos(dx+c)) - 3Aa^3 \ln(\sin(dx+c)) - 3Ba^3(dx+c) + 3iAa^3(-\cot(dx+c) - dx - c) + 3iBa^3 \ln(\sin(dx+c))}{d}$
risch	$\frac{8a^3Bc}{d} + \frac{8ia^3Ac}{d} - \frac{2ia^3(4iAe^{2i(dx+c)} + Be^{2i(dx+c)} - 3iA - B)}{d(e^{2i(dx+c)} - 1)^2} + \frac{3ia^3 \ln(e^{2i(dx+c)} - 1)B}{d} - \frac{4Aa^3 \ln(e^{2i(dx+c)} - 1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(-I\*A\*a^3\*(d\*x+c)+I\*B\*a^3\*ln(cos(d\*x+c))-3\*A\*a^3\*ln(sin(d\*x+c))-3\*B\*a^3\*(d\*x+c)+3\*I\*A\*a^3\*(-cot(d\*x+c)-d\*x-c)+3\*I\*B\*a^3\*ln(sin(d\*x+c))+A\*a^3\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+B\*a^3\*(-cot(d\*x+c)-d\*x-c))

**Maxima [A]**

time = 0.50, size = 96, normalized size = 0.78

$$\frac{8(dx+c)(iA+B)a^3 - 4(A-iB)a^3 \log(\tan(dx+c)^2 + 1) + 2(4A-3iB)a^3 \log(\tan(dx+c)) - \frac{2(-3iA-B)a^3 \tan(dx+c) - Aa^3}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/2*(8*(d*x + c)*(I*A + B)*a^3 - 4*(A - I*B)*a^3*\log(\tan(d*x + c)^2 + 1) + 2*(4*A - 3*I*B)*a^3*\log(\tan(d*x + c)) - (2*(-3*I*A - B)*a^3*\tan(d*x + c) - A*a^3)/\tan(d*x + c)^2)/d$

**Fricas** [A]

time = 0.63, size = 179, normalized size = 1.46

$$\frac{2(4A - iB)a^3 e^{2i dx + 2i c} - 2(3A - iB)a^3 + (iBa^3 e^{4i dx + 4i c} - 2iBa^3 e^{2i dx + 2i c} + iBa^3) \log(e^{2i dx + 2i c} + 1) - ((4A - 3iB)a^3 e^{4i dx + 4i c} - 2(4A - 3iB)a^3 e^{2i dx + 2i c} + (4A - 3iB)a^3) \log(e^{2i dx + 2i c} - 1)}{de^{4i dx + 4i c} - 2de^{2i dx + 2i c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $(2*(4*A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} - 2*(3*A - I*B)*a^3 + (I*B*a^3*e^{(4*I*d*x + 4*I*c)} - 2*I*B*a^3*e^{(2*I*d*x + 2*I*c)} + I*B*a^3)*\log(e^{(2*I*d*x + 2*I*c)} + 1) - ((4*A - 3*I*B)*a^3*e^{(4*I*d*x + 4*I*c)} - 2*(4*A - 3*I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (4*A - 3*I*B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} - 1))/(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs.  $2(109) = 218$ .

time = 1.18, size = 226, normalized size = 1.84

$$\frac{iBa^3 \log\left(\frac{2Aa^3 - iBa^3}{2Aa^3 e^{2ic} - iBa^3 e^{2ic}} + e^{2idx}\right)}{d} - \frac{a^3 \cdot (4A - 3iB) \log\left(e^{2idx} + \frac{2Aa^3 - 2iBa^3 - a^3 \cdot (4A - 3iB)}{2Aa^3 e^{2ic} - iBa^3 e^{2ic}}\right)}{d} + \frac{-6Aa^3 + 2iBa^3 + (8Aa^3 e^{2ic} - 2iBa^3 e^{2ic}) e^{2idx}}{de^{4ic} e^{4idx} - 2de^{2ic} e^{2idx} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out]  $I*B*a**3*\log((2*A*a**3 - I*B*a**3)/(2*A*a**3*\exp(2*I*c) - I*B*a**3*\exp(2*I*c)) + \exp(2*I*d*x))/d - a**3*(4*A - 3*I*B)*\log(\exp(2*I*d*x) + (2*A*a**3 - 2*I*B*a**3 - a**3*(4*A - 3*I*B)))/(2*A*a**3*\exp(2*I*c) - I*B*a**3*\exp(2*I*c))/d + (-6*A*a**3 + 2*I*B*a**3 + (8*A*a**3*\exp(2*I*c) - 2*I*B*a**3*\exp(2*I*c))*\exp(2*I*d*x))/(d*\exp(4*I*c)*\exp(4*I*d*x) - 2*d*\exp(2*I*c)*\exp(2*I*d*x) + d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs.  $2(109) = 218$ .

time = 1.25, size = 223, normalized size = 1.81

$$\frac{Aa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 8iBa^3 \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) - 8iBa^3 \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right) - 12iAa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 4Ba^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 64(Aa^3 - iBa^3) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i\right) + 8(4Aa^3 - 3iBa^3) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) - \frac{8d^2 \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 80d^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 4d^2 \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 4Ba^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 8iBa^3 \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{8d}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/8*(A*a^3*\tan(1/2*d*x + 1/2*c)^2 - 8*I*B*a^3*\log(\tan(1/2*d*x + 1/2*c) + 1) \\ & - 8*I*B*a^3*\log(\tan(1/2*d*x + 1/2*c) - 1) - 12*I*A*a^3*\tan(1/2*d*x + 1/2*c) \\ & - 4*B*a^3*\tan(1/2*d*x + 1/2*c) - 64*(A*a^3 - I*B*a^3)*\log(\tan(1/2*d*x + 1/2*c) + I) \\ & + 8*(4*A*a^3 - 3*I*B*a^3)*\log(\tan(1/2*d*x + 1/2*c)) - (48*A*a^3*\tan(1/2*d*x + 1/2*c)^2 \\ & - 36*I*B*a^3*\tan(1/2*d*x + 1/2*c)^2 - 12*I*A*a^3*\tan(1/2*d*x + 1/2*c) - 4*B*a^3*\tan(1/2*d*x + 1/2*c) \\ & - A*a^3)/\tan(1/2*d*x + 1/2*c)^2)/d \end{aligned}$$

**Mupad [B]**

time = 6.44, size = 88, normalized size = 0.72

$$\frac{\frac{Aa^3}{2} + \tan(c + dx) (Ba^3 + Aa^3 3i)}{d \tan(c + dx)^2} - \frac{a^3 \ln(\tan(c + dx)) (4A - B 3i)}{d} + \frac{4a^3 \ln(\tan(c + dx) + 1i) (A - B 1i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] 
$$\begin{aligned} & (4*a^3*\log(\tan(c + d*x) + 1i)*(A - B*1i))/d - (a^3*\log(\tan(c + d*x))*(4*A - \\ & B*3i))/d - ((A*a^3)/2 + \tan(c + d*x)*(A*a^3*3i + B*a^3))/(d*\tan(c + d*x)^2 \\ & ) \end{aligned}$$

### 3.23 $\int \cot^4(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

Optimal. Leaf size=134

$$4a^3(A-iB)x + \frac{a^3(17A-15iB)\cot(c+dx)}{6d} - \frac{4a^3(iA+B)\log(\sin(c+dx))}{d} - \frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

[Out]  $4*a^3*(A-I*B)*x+1/6*a^3*(17*A-15*I*B)*\cot(d*x+c)/d-4*a^3*(I*A+B)*\ln(\sin(d*x+c))/d-1/3*a*A*\cot(d*x+c)^3*(a+I*a*\tan(d*x+c))^2/d-1/6*(5*I*A+3*B)*\cot(d*x+c)^2*(a^3+I*a^3*\tan(d*x+c))/d$

Rubi [A]

time = 0.26, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3674, 3672, 3612, 3556}

$$\frac{a^3(17A-15iB)\cot(c+dx)}{6d} - \frac{4a^3(B+iA)\log(\sin(c+dx))}{d} - \frac{(3B+5iA)\cot^2(c+dx)(a^3+ia^3\tan(c+dx))}{6d} + 4a^3x(A-iB) - \frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c+d*x]^4*(a+I*a*\text{Tan}[c+d*x])^3*(A+B*\text{Tan}[c+d*x]),x]$

[Out]  $4*a^3*(A-I*B)*x + (a^3*(17*A - (15*I)*B)*\text{Cot}[c+d*x])/(6*d) - (4*a^3*(I*A+B)*\text{Log}[\text{Sin}[c+d*x]])/d - (a*A*\text{Cot}[c+d*x]^3*(a+I*a*\text{Tan}[c+d*x])^2)/(3*d) - (((5*I)*A+3*B)*\text{Cot}[c+d*x]^2*(a^3+I*a^3*\text{Tan}[c+d*x]))/(6*d)$

Rule 3556

$\text{Int}[\tan[(c_.)+(d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]], x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3612

$\text{Int}[(c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)]/((a_.)+(b_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c+b*d)*(x/(a^2+b^2)), x] + \text{Dist}[(b*c-a*d)/(a^2+b^2), \text{Int}[(b-a*\text{Tan}[e+f*x])/(a+b*\text{Tan}[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[a^2+b^2, 0] \&\& \text{NeQ}[a*c+b*d, 0]$

Rule 3672

$\text{Int}[(a_.)+(b_.)*\tan[(e_.)+(f_.)*(x_.)]^m*((A_.)+(B_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c-a*d)*(A*b-a*B)*((a+b*\text{Tan}[e+f*x])^{m+1}/(b*f*(m+1)*(a^2+b^2))), x] + \text{Dist}[1/(a^2+b^2), \text{Int}[(a+b*\text{Tan}[e+f*x])^{m+1}*\text{Simp}[a*A*c+b*B*c+A*b*d-a*B*d-(A*b*c-a*B*c-a*A*d-b*B*d)*\text{Tan}[e+f*x], x],$

`x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]`

### Rule 3674

`Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]`

### Rubi steps

$$\begin{aligned} \int \cot^4(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^2}{3d} + \frac{1}{3} \\ &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^2}{3d} - \frac{5}{3} \\ &= \frac{a^3(17A - 15iB) \cot(c + dx)}{6d} - \frac{aA \cot^3(c + dx)}{3d} \\ &= 4a^3(A - iB)x + \frac{a^3(17A - 15iB) \cot(c + dx)}{6d} \\ &= 4a^3(A - iB)x + \frac{a^3(17A - 15iB) \cot(c + dx)}{6d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 442 vs.  $2(134) = 268$ .

time = 3.60, size = 442, normalized size = 3.30

Antiderivative was successfully verified.

[In] `Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]), x]`

[Out] `(a^3*Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*(Cos[3*d*x] + I*Sin[3*d*x])*((9*I)*A*Cos[2*c + d*x] + 3*B*Cos[2*c + d*x] - 36*A*d*x*Cos[2*c + d*x] + (36*I)*B*d*x*Cos[2*c + d*x] - 12*A*d*x*Cos[2*c + 3*d*x] + (12*I)*B*d*x*Cos[2*c + 3*d*x] + 12*A*d*x*Cos[4*c + 3*d*x] - (12*I)*B*d*x*Cos[4*c + 3*d*x] + (9*I)*A*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] + 9*B*Cos[2*c + d*x]*Log[Sin[c + d*x]^2] +`

$$\frac{(3I)A\cos[2c + 3dx]\log[\sin[c + dx]^2] + 3B\cos[2c + 3dx]\log[\sin[c + dx]^2] - (3I)A\cos[4c + 3dx]\log[\sin[c + dx]^2] - 3B\cos[4c + 3dx]\log[\sin[c + dx]^2] + \cos[dx]\left((-9I)A - 3B + 36Adx - (36I)Bdx + ((-9I)A - 9B)\log[\sin[c + dx]^2]\right) - 24A\sin[dx] + (18I)B\sin[dx] - 48(A - IB)\operatorname{ArcTan}[\tan[4c + dx]]\sin[c]\sin[c + dx]^3 - 15A\sin[2c + dx] + (9I)B\sin[2c + dx] + 13A\sin[2c + 3dx] - (9I)B\sin[2c + 3dx])}{(24d(\cos[dx] + I\sin[dx])^3)}$$

**Maple [A]**

time = 0.10, size = 168, normalized size = 1.25

method	result
risch	$\frac{8ia^3Bc}{d} - \frac{8a^3Ac}{d} + \frac{2a^3(24iAe^{4i(dx+c)} + 12Be^{4i(dx+c)} - 33iAe^{2i(dx+c)} - 21Be^{2i(dx+c)} + 13iA + 9B)}{3d(e^{2i(dx+c)} - 1)^3} - \frac{4a^3 \ln(e^{2i(dx+c)})}{d}$
norman	$\frac{(-3iBa^3 + 4Aa^3)(\tan^2(dx+c))}{d} + \frac{(-4iBa^3 + 4Aa^3)x(\tan^3(dx+c)) - \frac{Aa^3}{3d} - \frac{(3iAa^3 + Ba^3)\tan(dx+c)}{2d}}{\tan(dx+c)^3} - \frac{4(iAa^3 + Ba^3)\ln(\sin(dx+c))}{d}$
derivativdivides	$-iAa^3 \ln(\sin(dx+c)) - iBa^3(dx+c) - 3Aa^3(-\cot(dx+c) - dx - c) - 3Ba^3 \ln(\sin(dx+c)) + 3iAa^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right)$
default	$-iAa^3 \ln(\sin(dx+c)) - iBa^3(dx+c) - 3Aa^3(-\cot(dx+c) - dx - c) - 3Ba^3 \ln(\sin(dx+c)) + 3iAa^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(dx+c)^4*(a+I*a*tan(dx+c))^3*(A+B*tan(dx+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d}(-Ia^3 \ln(\sin(dx+c)) - IBa^3(dx+c) - 3Aa^3(-\cot(dx+c) - dx - c) - 3Ba^3 \ln(\sin(dx+c)) + 3IAa^3(-\frac{1}{2}\cot(dx+c)^2 - \ln(\sin(dx+c))) + 3IBa^3(-\cot(dx+c) - dx - c) + Aa^3(-\frac{1}{3}\cot(dx+c)^3 + \cot(dx+c) + dx + c) + Ba^3(-\frac{1}{2}\cot(dx+c)^2 - \ln(\sin(dx+c))))$$

**Maxima [A]**

time = 0.51, size = 115, normalized size = 0.86

$$\frac{24(dx+c)(A - iB)a^3 - 12(-A - B)a^3 \log(\tan(dx+c)^2 + 1) - 24(iA + B)a^3 \log(\tan(dx+c)) + \frac{6(4A - 3iB)a^3 \tan(dx+c)^2 + 3(-3iA - B)a^3 \tan(dx+c) - 2Aa^3}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^4*(a+I*a*tan(dx+c))^3*(A+B*tan(dx+c)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{6} \frac{(24(dx+c)(A - IB)a^3 - 12(-IA - B)a^3 \log(\tan(dx+c)^2 + 1) - 24(IA + B)a^3 \log(\tan(dx+c)) + (6(4A - 3IB)a^3 \tan(dx+c)^2 + 3(-3IA - B)a^3 \tan(dx+c) - 2Aa^3)/\tan(dx+c)^3)/d}$$

**Fricas** [A]

time = 0.63, size = 181, normalized size = 1.35

$$\frac{-2(12(-2iA - B)a^3e^{4i dx + 4i c}) + 3(11iA + 7B)a^3e^{2i dx + 2i c} + (-13iA - 9B)a^3 + 6((iA + B)a^3e^{6i dx + 6i c}) + 3(-iA - B)a^3e^{4i dx + 4i c} + 3(iA + B)a^3e^{2i dx + 2i c} + (-iA - B)a^3 \log(e^{2i dx + 2i c} - 1))}{3(de^{6i dx + 6i c} - 3de^{4i dx + 4i c} + 3de^{2i dx + 2i c} - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-2/3*(12*(-2*I*A - B)*a^3*e^{(4*I*d*x + 4*I*c)} + 3*(11*I*A + 7*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-13*I*A - 9*B)*a^3 + 6*((I*A + B)*a^3*e^{(6*I*d*x + 6*I*c)} + 3*(-I*A - B)*a^3*e^{(4*I*d*x + 4*I*c)} + 3*(I*A + B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^3)*\log(e^{(2*I*d*x + 2*I*c)} - 1)/(d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} - d)$

**Sympy** [A]

time = 0.55, size = 182, normalized size = 1.36

$$\frac{4ia^3(A - iB) \log(e^{2idx} - e^{-2ic})}{d} + \frac{26iAa^3 + 18Ba^3 + (-66iAa^3e^{2ic} - 42Ba^3e^{2ic})e^{2idx} + (48iAa^3e^{4ic} + 24Ba^3e^{4ic})e^{4idx}}{3de^{6ic}e^{6idx} - 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} - 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out]  $-4*I*a**3*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (26*I*A*a**3 + 18*B*a**3 + (-66*I*A*a**3*\exp(2*I*c) - 42*B*a**3*\exp(2*I*c))*\exp(2*I*d*x) + (48*I*A*a**3*\exp(4*I*c) + 24*B*a**3*\exp(4*I*c))*\exp(4*I*d*x))/(3*d*\exp(6*I*c)*\exp(6*I*d*x) - 9*d*\exp(4*I*c)*\exp(4*I*d*x) + 9*d*\exp(2*I*c)*\exp(2*I*d*x) - 3*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(116) = 232.

time = 0.87, size = 254, normalized size = 1.90

$$\frac{Aa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 9iAa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 3Ba^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 51Aa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) + 36iBa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 192(-iAa^3 - Ba^3) \log(\tan(\frac{1}{2} dx + \frac{1}{2} c) + i) - 96(iAa^3 + Ba^3) \log(\tan(\frac{1}{2} dx + \frac{1}{2} c)) - \frac{-176iAa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 176Ba^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 51Aa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 9iBa^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 3Ba^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) \tan(\frac{1}{2} dx + \frac{1}{2} c)^2}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $1/24*(A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 9*I*A*a^3*\tan(1/2*d*x + 1/2*c)^2 - 3*B*a^3*\tan(1/2*d*x + 1/2*c)^2 - 51*A*a^3*\tan(1/2*d*x + 1/2*c) + 36*I*B*a^3*\tan(1/2*d*x + 1/2*c) - 192*(-I*A*a^3 - B*a^3)*\log(\tan(1/2*d*x + 1/2*c) + I) - 96*(I*A*a^3 + B*a^3)*\log(\tan(1/2*d*x + 1/2*c)) - (-176*I*A*a^3*\tan(1/2*d*x + 1/2*c)^2 - 176*B*a^3*\tan(1/2*d*x + 1/2*c)^2 - 51*A*a^3*\tan(1/2*d*x + 1/2*c) \tan(1/2*d*x + 1/2*c)^2 - 9iBa^3 \tan(1/2*d*x + 1/2*c) \tan(1/2*d*x + 1/2*c)^2 + 3Ba^3 \tan(1/2*d*x + 1/2*c) \tan(1/2*d*x + 1/2*c)^2)$

$*c)^2 + 36*I*B*a^3*\tan(1/2*d*x + 1/2*c)^2 + 9*I*A*a^3*\tan(1/2*d*x + 1/2*c) + 3*B*a^3*\tan(1/2*d*x + 1/2*c) + A*a^3)/\tan(1/2*d*x + 1/2*c)^3)/d$

**Mupad [B]**

time = 6.32, size = 93, normalized size = 0.69

$$\frac{\frac{Aa^3}{3} - \tan(c+dx)^2(4Aa^3 - Ba^33i) + \tan(c+dx)\left(\frac{Ba^3}{2} + \frac{Aa^33i}{2}\right)}{d \tan(c+dx)^3} - \frac{a^3 \operatorname{atan}(2 \tan(c+dx) + 1i) (B + A1i) 8i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)`

[Out]  $-\left(\frac{Aa^3}{3} - \tan(c+dx)^2(4Aa^3 - Ba^33i) + \tan(c+dx)\left(\frac{Aa^33i}{2} + \frac{Ba^3}{2}\right)\right)/(d \tan(c+dx)^3) - (a^3 \operatorname{atan}(2 \tan(c+dx) + 1i) * (A * 1i + B) * 8i) / d$

### 3.24 $\int \cot^5(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=157

$$4a^3(iA+B)x + \frac{4a^3(iA+B) \cot(c+dx)}{d} + \frac{a^3(15A-14iB) \cot^2(c+dx)}{12d} + \frac{4a^3(A-iB) \log(\sin(c+dx))}{d} - \frac{aA}{d}$$

[Out]  $4*a^3*(I*A+B)*x + 4*a^3*(I*A+B)*\cot(d*x+c)/d + 1/12*a^3*(15*A-14*I*B)*\cot(d*x+c)^2/d + 4*a^3*(A-I*B)*\ln(\sin(d*x+c))/d - 1/4*a*A*\cot(d*x+c)^4*(a+I*a*\tan(d*x+c))^2/d - 1/6*(3*I*A+2*B)*\cot(d*x+c)^3*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.29, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3672, 3610, 3612, 3556}

$$\frac{a^3(15A-14iB) \cot^2(c+dx)}{12d} + \frac{4a^3(B+iA) \cot(c+dx)}{d} + \frac{4a^3(A-iB) \log(\sin(c+dx))}{d} - \frac{(2B+3iA) \cot^3(c+dx)(a^3+ia^3 \tan(c+dx))}{6d} + \frac{4a^3x(B+iA)}{d} - \frac{aA \cot^4(c+dx)(a+ia \tan(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^5\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $4*a^3*(I*A + B)*x + (4*a^3*(I*A + B)*\cot[c + d*x])/d + (a^3*(15*A - (14*I)*B)*\cot[c + d*x]^2)/(12*d) + (4*a^3*(A - I*B)*\log[\sin[c + d*x]])/d - (a*A*\cot[c + d*x]^4*(a + I*a*\tan[c + d*x])^2)/(4*d) - (((3*I)*A + 2*B)*\cot[c + d*x]^3*(a^3 + I*a^3*\tan[c + d*x]))/(6*d)$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && Ne

$Q[a*c + b*d, 0]$

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rule 3674

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cot^5(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^4(c + dx)(a + ia \tan(c + dx))^2}{4d} + \frac{1}{4} \int \dots \\ &= -\frac{aA \cot^4(c + dx)(a + ia \tan(c + dx))^2}{4d} - \frac{(3i)}{4d} \int \dots \\ &= \frac{a^3(15A - 14iB) \cot^2(c + dx)}{12d} - \frac{aA \cot^4(c + dx)}{4d} \\ &= \frac{4a^3(iA + B) \cot(c + dx)}{d} + \frac{a^3(15A - 14iB) \cot^2(c + dx)}{12d} \\ &= 4a^3(iA + B)x + \frac{4a^3(iA + B) \cot(c + dx)}{d} + \frac{a^3(15A - 14iB) \cot^2(c + dx)}{12d} \\ &= 4a^3(iA + B)x + \frac{4a^3(iA + B) \cot(c + dx)}{d} + \frac{a^3(15A - 14iB) \cot^2(c + dx)}{12d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1007 vs.  $2(157) = 314$ .  
time = 6.96, size = 1007, normalized size = 6.41



Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] a^3*(((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(-1/4*(A*Cos[3*c]) + (I/4)*
A*Sin[3*c]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])
) + ((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*Csc[c/2]*Sec[c/2]*(Cos[3*c]/
6 - (I/6)*Sin[3*c])*((3*I)*A*Sin[d*x] + B*Sin[d*x])*Sin[c + d*x])/(d*(Cos[d
*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x]
)^3*(B + A*Cot[c + d*x])*Csc[c/2]*Sec[c/2]*((-6*I)*A*Cos[c] - 2*B*Cos[c] +
15*A*Sin[c] - (9*I)*B*Sin[c])*(Cos[3*c]/12 - (I/12)*Sin[3*c])*Sin[c + d*x]^
2)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I +
Cot[c + d*x])^3*(B + A*Cot[c + d*x])*Csc[c/2]*Sec[c/2]*(Cos[3*c]/6 - (I/6)*
Sin[3*c])*((-15*I)*A*Sin[d*x] - 13*B*Sin[d*x])*Sin[c + d*x]^3)/(d*(Cos[d*x]
+ I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^3
*(B + A*Cot[c + d*x])*(A*Cos[(3*c)/2] - I*B*Cos[(3*c)/2] - I*A*Sin[(3*c)/2]
- B*Sin[(3*c)/2])*((-4*I)*ArcTan[Tan[4*c + d*x]]*Cos[(3*c)/2] - 4*ArcTan[T
an[4*c + d*x]]*Sin[(3*c)/2])*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(
A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^3*(B + A*Cot[c + d*
x])*(A*Cos[(3*c)/2] - I*B*Cos[(3*c)/2] - I*A*Sin[(3*c)/2] - B*Sin[(3*c)/2])
*(2*Cos[(3*c)/2]*Log[Sin[c + d*x]^2] - (2*I)*Log[Sin[c + d*x]^2]*Sin[(3*c)/
2])*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c
+ d*x])) + (x*(I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*((16*I)*A*Cos[c]^3
+ 16*B*Cos[c]^3 - 4*A*Cos[c]^3*Cot[c] + (4*I)*B*Cos[c]^3*Cot[c] + 24*A*Cos[
c]^2*Sin[c] - (24*I)*B*Cos[c]^2*Sin[c] - (16*I)*A*Cos[c]*Sin[c]^2 - 16*B*Co
s[c]*Sin[c]^2 - 4*A*Sin[c]^3 + (4*I)*B*Sin[c]^3 + (A - I*B)*Cot[c]*(4*Cos[3
*c] - (4*I)*Sin[3*c]))*Sin[c + d*x]^4)/((Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c
+ d*x] + B*Sin[c + d*x])) + ((I*A + B)*(I + Cot[c + d*x])^3*(B + A*Cot[c +
d*x])*(4*d*x*Cos[3*c] - (4*I)*d*x*Sin[3*c])*Sin[c + d*x]^4)/(d*(Cos[d*x] +
I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])))
```

Maple [A]

time = 0.11, size = 205, normalized size = 1.31

method	result
risch	$-\frac{8a^3Bc}{d} - \frac{8ia^3Ac}{d} + \frac{2ia^3(36iAe^{6i(dx+c)} + 24Be^{6i(dx+c)} - 69iAe^{4i(dx+c)} - 57Be^{4i(dx+c)} + 54iAe^{2i(dx+c)} + 46Be^{2i(dx+c)})}{3d(e^{2i(dx+c)} - 1)^4}$
norman	$\frac{(4iAa^3 + 4Ba^3)x(\tan^4(dx+c)) - \frac{Aa^3}{4d} + \frac{(-3iBa^3 + 4Aa^3)(\tan^2(dx+c))}{2d} - \frac{(3iAa^3 + Ba^3)\tan(dx+c)}{3d} + \frac{4(iAa^3 + Ba^3)(\tan^3(dx+c))}{d}}{\tan(dx+c)^4}$
derivativedivides	$-iAa^3(-\cot(dx+c) - dx - c) - iBa^3 \ln(\sin(dx+c)) - 3Aa^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) - 3Ba^3(-\cot(dx+c) - dx - c)$
default	$-iAa^3(-\cot(dx+c) - dx - c) - iBa^3 \ln(\sin(dx+c)) - 3Aa^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) - 3Ba^3(-\cot(dx+c) - dx - c)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $\frac{1}{d}(-I A a^3(-\cot(dx+c)-dx-c)-I B a^3 \ln(\sin(dx+c))-3 A a^3(-\frac{1}{2} \cot(dx+c)^2-\ln(\sin(dx+c)))-3 B a^3(-\cot(dx+c)-dx-c)+3 I A a^3(-\frac{1}{3} \cot(dx+c)^3+\cot(dx+c)+dx+c)+3 I B a^3(-\frac{1}{2} \cot(dx+c)^2-\ln(\sin(dx+c)))+A a^3(-\frac{1}{4} \cot(dx+c)^4+\frac{1}{2} \cot(dx+c)^2+\ln(\sin(dx+c)))+B a^3(-\frac{1}{3} \cot(dx+c)^3+\cot(dx+c)+dx+c))$

**Maxima** [A]

time = 0.50, size = 134, normalized size = 0.85

$$\frac{48(dx+c)(-iA-B)a^3+24(A-iB)a^3 \log(\tan(dx+c)^2+1)-48(A-iB)a^3 \log(\tan(dx+c))-\frac{48(iA+B)a^3 \tan(dx+c)^3+6(4A-3iB)a^3 \tan(dx+c)^2+4(-3iA-B)a^3 \tan(dx+c)-3Aa^3}{\tan(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-\frac{1}{12}(48(dx+c)(-I A-B)a^3+24(A-I B)a^3 \log(\tan(dx+c)^2+1)-48(A-I B)a^3 \log(\tan(dx+c))-(48(I A+B)a^3 \tan(dx+c)^3+6(4 A-3 I B)a^3 \tan(dx+c)^2+4(-3 I A-B)a^3 \tan(dx+c)-3 A a^3)/\tan(dx+c)^4)/d$

**Fricas** [A]

time = 0.50, size = 228, normalized size = 1.45

$$\frac{2(12(3A-2iB)a^3e^{6i dx+6i c}-3(23A-19iB)a^3e^{4i dx+4i c}+2(27A-23iB)a^3e^{2i dx+2i c}-(15A-13iB)a^3-6((A-iB)a^2e^{8i dx+8i c}-4(A-iB)a^2e^{6i dx+6i c}+6(A-iB)a^2e^{4i dx+4i c}-4(A-iB)a^2e^{2i dx+2i c}+(A-iB)a^2 \log(e^{2i dx+2i c}-1))}{3(d e^{8i dx+8i c}-4 d e^{6i dx+6i c}+6 d e^{4i dx+4i c}-4 d e^{2i dx+2i c}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-\frac{2}{3}(12(3A-2I B)a^3e^{(6I dx+6I c)}-3(23A-19I B)a^3e^{(4I dx+4I c)}+2(27A-23I B)a^3e^{(2I dx+2I c)}-(15A-13I B)a^3-6((A-I B)a^3e^{(8I dx+8I c)}-4(A-I B)a^3e^{(6I dx+6I c)}+6(A-I B)a^3e^{(4I dx+4I c)}-4(A-I B)a^3e^{(2I dx+2I c)}+(A-I B)a^3 \log(e^{(2I dx+2I c)}-1)))/(d e^{(8I dx+8I c)}-4 d e^{(6I dx+6I c)}+6 d e^{(4I dx+4I c)}-4 d e^{(2I dx+2I c)}+d)$

**Sympy** [A]

time = 1.53, size = 235, normalized size = 1.50

$$\frac{4a^3(A-iB) \log(e^{2idx}-e^{-2ic})}{d} + \frac{30Aa^3-26iBa^3+(-108Aa^3e^{2ic}+92iBa^3e^{2ic})e^{2idx}+(138Aa^3e^{4ic}-114iBa^3e^{4ic})e^{4idx}+(-72Aa^3e^{6ic}+48iBa^3e^{6ic})e^{6idx}}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*5\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out]  $4*a**3*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (30*A*a**3 - 26*I*B*a**3 + (-108*A*a**3*\exp(2*I*c) + 92*I*B*a**3*\exp(2*I*c))*\exp(2*I*d*x) + (138*A*a**3*\exp(4*I*c) - 114*I*B*a**3*\exp(4*I*c))*\exp(4*I*d*x) + (-72*A*a**3*\exp(6*I*c) + 48*I*B*a**3*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) - 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) - 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs.  $2(137) = 274$ .

time = 0.95, size = 322, normalized size = 2.05

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/192*(3*A*a^3*\tan(1/2*d*x + 1/2*c)^4 - 24*I*A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 8*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 108*A*a^3*\tan(1/2*d*x + 1/2*c)^2 + 72*I*B*a^3*\tan(1/2*d*x + 1/2*c)^2 + 456*I*A*a^3*\tan(1/2*d*x + 1/2*c) + 408*B*a^3*\tan(1/2*d*x + 1/2*c) + 1536*(A*a^3 - I*B*a^3)*\log(\tan(1/2*d*x + 1/2*c) + I) - 768*(A*a^3 - I*B*a^3)*\log(\tan(1/2*d*x + 1/2*c)) + (1600*A*a^3*\tan(1/2*d*x + 1/2*c)^4 - 1600*I*B*a^3*\tan(1/2*d*x + 1/2*c)^4 - 456*I*A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 408*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 108*A*a^3*\tan(1/2*d*x + 1/2*c)^2 + 72*I*B*a^3*\tan(1/2*d*x + 1/2*c)^2 + 24*I*A*a^3*\tan(1/2*d*x + 1/2*c) + 8*B*a^3*\tan(1/2*d*x + 1/2*c) + 3*A*a^3)/\tan(1/2*d*x + 1/2*c)^4)/d$

**Mupad** [B]

time = 6.57, size = 114, normalized size = 0.73

$$\frac{\tan(c+dx)^2 \left( 2Aa^3 - \frac{Ba^3 3i}{2} \right) + \tan(c+dx)^3 (4Ba^3 + Aa^3 4i) - \frac{Aa^2}{4} - \tan(c+dx) \left( \frac{Ba^2}{3} + Aa^3 1i \right)}{d \tan(c+dx)^4} + \frac{8a^3 \operatorname{atan}(2 \tan(c+dx) + 1i) (B + A 1i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^5\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out]  $(\tan(c + d*x)^2*(2*A*a^3 - (B*a^3*3i)/2) + \tan(c + d*x)^3*(A*a^3*4i + 4*B*a^3) - (A*a^3)/4 - \tan(c + d*x)*(A*a^3*1i + (B*a^3)/3))/(d*\tan(c + d*x)^4) + (8*a^3*\operatorname{atan}(2*\tan(c + d*x) + 1i)*(A*1i + B))/d$

### 3.25 $\int \cot^6(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=180

$$-4a^3(A-iB)x - \frac{4a^3(A-iB)\cot(c+dx)}{d} + \frac{2a^3(iA+B)\cot^2(c+dx)}{d} + \frac{a^3(47A-45iB)\cot^3(c+dx)}{60d} + \frac{4a^3(iA+B)\cot^4(c+dx)}{60d}$$

[Out]  $-4*a^3*(A-I*B)*x - 4*a^3*(A-I*B)*\cot(d*x+c)/d + 2*a^3*(I*A+B)*\cot(d*x+c)^2/d + 1/60*a^3*(47*A-45*I*B)*\cot(d*x+c)^3/d + 4*a^3*(I*A+B)*\ln(\sin(d*x+c))/d - 1/5*a*A*\cot(d*x+c)^5*(a+I*a*\tan(d*x+c))^2/d - 1/20*(7*I*A+5*B)*\cot(d*x+c)^4*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.31, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3672, 3610, 3612, 3556}

$$\frac{a^3(47A-45iB)\cot^3(c+dx)}{60d} + \frac{2a^3(B+iA)\cot^2(c+dx)}{d} - \frac{4a^3(A-iB)\cot(c+dx)}{d} + \frac{4a^3(B+iA)\log(\sin(c+dx))}{d} - \frac{(5B+7iA)\cot^4(c+dx)(a^3+ia^3\tan(c+dx))}{20d} - 4a^3x(A-iB) - \frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^6\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-4*a^3*(A - I*B)*x - (4*a^3*(A - I*B)*\cot[c + d*x])/d + (2*a^3*(I*A + B)*\cot[c + d*x]^2)/d + (a^3*(47*A - (45*I)*B)*\cot[c + d*x]^3)/(60*d) + (4*a^3*(I*A + B)*\log[\sin[c + d*x]])/d - (a*A*\cot[c + d*x]^5*(a + I*a*\tan[c + d*x])^2)/(5*d) - (((7*I)*A + 5*B)*\cot[c + d*x]^4*(a^3 + I*a^3*\tan[c + d*x]))/(20*d)$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a

\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rule 3674

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \cot^6(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^5(c + dx)(a + ia \tan(c + dx))^2}{5d} + \frac{1}{5} \\
 &= -\frac{aA \cot^5(c + dx)(a + ia \tan(c + dx))^2}{5d} - \frac{7}{5} \\
 &= \frac{a^3(47A - 45iB) \cot^3(c + dx)}{60d} - \frac{aA \cot^5(c + dx)}{5d} \\
 &= \frac{2a^3(iA + B) \cot^2(c + dx)}{d} + \frac{a^3(47A - 45iB)}{60d} \\
 &= -\frac{4a^3(A - iB) \cot(c + dx)}{d} + \frac{2a^3(iA + B)}{d} \\
 &= -4a^3(A - iB)x - \frac{4a^3(A - iB) \cot(c + dx)}{d} \\
 &= -4a^3(A - iB)x - \frac{4a^3(A - iB) \cot(c + dx)}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 943 vs.  $2(180) = 360$ .  
time = 7.17, size = 943, normalized size = 5.24

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] a^3*(((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(I*A*Cos[(3*c)/2] + B*Cos[(3*c)/2] + A*Sin[(3*c)/2] - I*B*Sin[(3*c)/2])*((-4*I)*ArcTan[Tan[4*c + d*x]]*Cos[(3*c)/2] - 4*ArcTan[Tan[4*c + d*x]]*Sin[(3*c)/2])*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(I*A*Cos[(3*c)/2] + B*Cos[(3*c)/2] + A*Sin[(3*c)/2] - I*B*Sin[(3*c)/2])*(2*Cos[(3*c)/2]*Log[Sin[c + d*x]^2] - (2*I)*Log[Sin[c + d*x]^2]*Sin[(3*c)/2])*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (x*(I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*(-16*A*Cos[c]^3 + (16*I)*B*Cos[c]^3 - (4*I)*A*Cos[c]^3*Cot[c] - 4*B*Cos[c]^3*Cot[c] + (24*I)*A*Cos[c]^2*Sin[c] + 24*B*Cos[c]^2*Sin[c] + 16*A*Cos[c]*Sin[c]^2 - (16*I)*B*Cos[c]*Sin[c]^2 - (4*I)*A*Sin[c]^3 - 4*B*Sin[c]^3 + (I*A + B)*Cot[c]*(4*Cos[3*c] - (4*I)*Sin[3*c]))*Sin[c + d*x]^4)/((Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*Csc[c]*Csc[c + d*x]*(Cos[3*c]/240 - (I/240)*Sin[3*c])*((225*I)*A*Cos[d*x] + 195*B*Cos[d*x] - 300*A*d*x*Cos[d*x] + (300*I)*B*d*x*Cos[d*x] - (225*I)*A*Cos[2*c + d*x] - 195*B*Cos[2*c + d*x] + 300*A*d*x*Cos[2*c + d*x] - (300*I)*B*d*x*Cos[2*c + d*x] - (105*I)*A*Cos[2*c + 3*d*x] - 75*B*Cos[2*c + 3*d*x] + 150*A*d*x*Cos[2*c + 3*d*x] - (150*I)*B*d*x*Cos[2*c + 3*d*x] + (105*I)*A*Cos[4*c + 3*d*x] + 75*B*Cos[4*c + 3*d*x] - 150*A*d*x*Cos[4*c + 3*d*x] + (150*I)*B*d*x*Cos[4*c + 3*d*x] - 30*A*d*x*Cos[4*c + 5*d*x] + (30*I)*B*d*x*Cos[4*c + 5*d*x] + 30*A*d*x*Cos[6*c + 5*d*x] - (30*I)*B*d*x*Cos[6*c + 5*d*x] + 470*A*Sin[d*x] - (420*I)*B*Sin[d*x] + 360*A*Sin[2*c + d*x] - (330*I)*B*Sin[2*c + d*x] - 280*A*Sin[2*c + 3*d*x] + (270*I)*B*Sin[2*c + 3*d*x] - 135*A*Sin[4*c + 3*d*x] + (105*I)*B*Sin[4*c + 3*d*x] + 83*A*Sin[4*c + 5*d*x] - (75*I)*B*Sin[4*c + 5*d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.12, size = 246, normalized size = 1.37

method	result
risch	$-\frac{8ia^3Bc}{d} + \frac{8a^3Ac}{d} - \frac{2a^3(240iAe^{8i(dx+c)} + 180Be^{8i(dx+c)} - 585iAe^{6i(dx+c)} - 525Be^{6i(dx+c)} + 695iAe^{4i(dx+c)} + 615B)}{15d(e^{2i(dx+c)} - 1)^5}$
norman	$\frac{(4iBa^3 - 4Aa^3)x(\tan^5(dx+c)) - \frac{Aa^3}{5d} + \frac{(-3iBa^3 + 4Aa^3)(\tan^2(dx+c))}{3d} - \frac{(3iAa^3 + Ba^3)\tan(dx+c)}{4d} - \frac{4(-iBa^3 + Aa^3)(\tan^4(dx+c))}{d}}{\tan(dx+c)^5}$

derivativedivides	$\frac{-iA a^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) - iB a^3 (-\cot(dx+c) - dx - c) - 3A a^3 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right) - 3}{-iA a^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) - iB a^3 (-\cot(dx+c) - dx - c) - 3A a^3 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right) - 3}$
default	$\frac{-iA a^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) - iB a^3 (-\cot(dx+c) - dx - c) - 3A a^3 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right) - 3}{-iA a^3 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) - iB a^3 (-\cot(dx+c) - dx - c) - 3A a^3 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right) - 3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d * (-I * A * a^3 * (-1/2 * \cot(dx+c)^2 - \ln(\sin(dx+c))) - I * B * a^3 * (-\cot(dx+c) - dx - c) - 3 * A * a^3 * (-1/3 * \cot(dx+c)^3 + \cot(dx+c) + dx + c) - 3 * B * a^3 * (-1/2 * \cot(dx+c)^2 - \ln(\sin(dx+c))) + 3 * I * A * a^3 * (-1/4 * \cot(dx+c)^4 + 1/2 * \cot(dx+c)^2 + \ln(\sin(dx+c))) + 3 * I * B * a^3 * (-1/3 * \cot(dx+c)^3 + \cot(dx+c) + dx + c) + A * a^3 * (-1/5 * \cot(dx+c)^5 + 1/3 * \cot(dx+c)^3 - \cot(dx+c) - dx - c) + B * a^3 * (-1/4 * \cot(dx+c)^4 + 1/2 * \cot(dx+c)^2 + \ln(\sin(dx+c))))$

**Maxima** [A]

time = 0.49, size = 151, normalized size = 0.84

$$\frac{240(dx+c)(A-iB)a^3+120(iA+B)a^3\log(\tan(dx+c)^2+1)+240(-iA-B)a^3\log(\tan(dx+c))+\frac{240(A-iB)a^3\tan(dx+c)^4-120(iA+B)a^3\tan(dx+c)^2-20(4A-3iB)a^3\tan(dx+c)^2-15(-3iA-B)a^3\tan(dx+c)+12Aa^3}{\tan(dx+c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out]  $-1/60 * (240 * (dx + c) * (A - I * B) * a^3 + 120 * (I * A + B) * a^3 * \log(\tan(dx + c)^2 + 1) + 240 * (-I * A - B) * a^3 * \log(\tan(dx + c)) + (240 * (A - I * B) * a^3 * \tan(dx + c)^4 - 120 * (I * A + B) * a^3 * \tan(dx + c)^3 - 20 * (4 * A - 3 * I * B) * a^3 * \tan(dx + c)^2 - 15 * (-3 * I * A - B) * a^3 * \tan(dx + c) + 12 * A * a^3) / \tan(dx + c)^5) / d$

**Fricas** [A]

time = 0.48, size = 287, normalized size = 1.59

$$\frac{2(60(4iA+3B)a^3e^{6i(dx+c)}+15(-39iA-35B)a^3e^{6i(dx+c)}+5(139iA+123B)a^3e^{4i(dx+c)}+5(-77iA-69B)a^3e^{2i(dx+c)}+(83iA+75B)a^3+30((-iA-B)a^3e^{10i(dx+c)}+5(iA+B)a^3e^{8i(dx+c)}+10(-iA-B)a^3e^{6i(dx+c)}+10(iA+B)a^3e^{4i(dx+c)}+5(-iA-B)a^3e^{2i(dx+c)}+(iA+B)a^3\log(e^{2i(dx+c)}-1)))}{15(d\cot^5(dx+c)-5d\cot^4(dx+c)+10d\cot^3(dx+c)-10d\cot^2(dx+c)+5d\cot(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out]  $-2/15 * (60 * (4 * I * A + 3 * B) * a^3 * e^{(8 * I * dx + 8 * I * c)} + 15 * (-39 * I * A - 35 * B) * a^3 * e^{(6 * I * dx + 6 * I * c)} + 5 * (139 * I * A + 123 * B) * a^3 * e^{(4 * I * dx + 4 * I * c)} + 5 * (-77 * I * A - 69 * B) * a^3 * e^{(2 * I * dx + 2 * I * c)} + (83 * I * A + 75 * B) * a^3 + 30 * ((-I * A - B) * a^3 * e^{(10 * I * dx + 10 * I * c)} + 5 * (I * A + B) * a^3 * e^{(8 * I * dx + 8 * I * c)} + 10 * (-I * A - B) * a^3 * e^{(6 * I * dx + 6 * I * c)} + 10 * (I * A + B) * a^3 * e^{(4 * I * dx + 4 * I * c)} + 5 * (-I * A - B) * a^3 * e^{(2 * I * dx + 2 * I * c)} + (iA + B)a^3 \log(e^{2i(dx+c)} - 1))$





Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^6*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)`

[Out]  $(a^3 \operatorname{atan}(2 \tan(c + dx) + 1i) (A + B) 8i) / d - (\tan(c + dx)^4 (4Aa^3 - B a^3 4i) - \tan(c + dx)^2 ((4Aa^3) / 3 - B a^3 1i) - \tan(c + dx)^3 (A a^3 2i + 2B a^3) + (A a^3) / 5 + \tan(c + dx) ((A a^3 3i) / 4 + (B a^3) / 4)) / (d \tan(c + dx)^5)$

### 3.26 $\int \tan^2(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=225

$$-8a^4(A-iB)x + \frac{8a^4(iA+B) \log(\cos(c+dx))}{d} + \frac{8a^4(A-iB) \tan(c+dx)}{d} + \frac{4a^4(iA+B) \tan^2(c+dx)}{d} - \frac{a^4(92A-93iB) \tan^3(c+dx)}{60d} - \frac{(12A-13iB) \tan^3(c+dx)(a+ia \tan(c+dx))}{20d} + \frac{4a^4(B+iA) \tan^2(c+dx)}{d} + \frac{8a^4(A-iB) \log(\cos(c+dx))}{d} - \frac{8a^4(A-iB)}{10d} - \frac{(2A-3iB) \tan^3(c+dx)(a^2+ia^2 \tan(c+dx))^2}{10d} + \frac{iaB \tan^3(c+dx)(a+ia \tan(c+dx))^2}{6d}$$

[Out]  $-8*a^4*(A-I*B)*x+8*a^4*(I*A+B)*\ln(\cos(d*x+c))/d+8*a^4*(A-I*B)*\tan(d*x+c)/d+4*a^4*(I*A+B)*\tan(d*x+c)^2/d-1/60*a^4*(92*A-93*I*B)*\tan(d*x+c)^3/d+1/6*I*a*B*\tan(d*x+c)^3*(a+I*a*\tan(d*x+c))^3/d-1/10*(2*A-3*I*B)*\tan(d*x+c)^3*(a^2+I*a^2*\tan(d*x+c))^2/d-1/20*(12*A-13*I*B)*\tan(d*x+c)^3*(a^4+I*a^4*\tan(d*x+c))/d$

Rubi [A]

time = 0.43, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3675, 3673, 3609, 3606, 3556}

$$\frac{a^4(92A-93iB) \tan^3(c+dx)}{60d} - \frac{(12A-13iB) \tan^3(c+dx)(a+ia \tan(c+dx))}{20d} + \frac{4a^4(B+iA) \tan^2(c+dx)}{d} + \frac{8a^4(A-iB) \log(\cos(c+dx))}{d} + \frac{8a^4(B+iA) \log(\cos(c+dx))}{d} - \frac{8a^4(A-iB)}{10d} - \frac{(2A-3iB) \tan^3(c+dx)(a^2+ia^2 \tan(c+dx))^2}{10d} + \frac{iaB \tan^3(c+dx)(a+ia \tan(c+dx))^2}{6d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-8*a^4*(A - I*B)*x + (8*a^4*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d + (8*a^4*(A - I*B)*\text{Tan}[c + d*x])/d + (4*a^4*(I*A + B)*\text{Tan}[c + d*x]^2)/d - (a^4*(92*A - (93*I)*B)*\text{Tan}[c + d*x]^3)/(60*d) + ((I/6)*a*B*\text{Tan}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])^3)/d - ((2*A - (3*I)*B)*\text{Tan}[c + d*x]^3*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/(10*d) - ((12*A - (13*I)*B)*\text{Tan}[c + d*x]^3*(a^4 + I*a^4*\text{Tan}[c + d*x]))/(20*d)$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3606

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3609

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^m*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}$

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan^2(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{iaB \tan^3(c + dx)(a + ia \tan(c + dx))^3}{6d} + \frac{1}{6} \\
&= \frac{iaB \tan^3(c + dx)(a + ia \tan(c + dx))^3}{6d} - \frac{1}{6} \\
&= \frac{iaB \tan^3(c + dx)(a + ia \tan(c + dx))^3}{6d} - \frac{1}{6} \\
&= -\frac{a^4(92A - 93iB) \tan^3(c + dx)}{60d} + \frac{iaB \tan^3(c + dx)}{60d} \\
&= \frac{4a^4(iA + B) \tan^2(c + dx)}{d} - \frac{a^4(92A - 93iB)}{60d} \\
&= -8a^4(A - iB)x + \frac{8a^4(A - iB) \tan(c + dx)}{d} \\
&= -8a^4(A - iB)x + \frac{8a^4(iA + B) \log(\cos(c + dx))}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 951 vs.  $2(225) = 450$ .

time = 7.40, size = 951, normalized size = 4.23

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] (Cos[c + d*x]^5*(I*A*Cos[2*c] + B*Cos[2*c] + A*Sin[2*c] - I*B*Sin[2*c])*(4*
Cos[2*c]*Log[Cos[c + d*x]^2] - (4*I)*Log[Cos[c + d*x]^2]*Sin[2*c])*(a + I*a
*Tan[c + d*x])^4*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[
c + d*x] + B*Sin[c + d*x])) + (Sec[c]*Sec[c + d*x]*(Cos[4*c]/240 - (I/240)*
Sin[4*c])*((420*I)*A*Cos[c] + 490*B*Cos[c] - 600*A*d*x*Cos[c] + (600*I)*B*d
*x*Cos[c] + (300*I)*A*Cos[c + 2*d*x] + 345*B*Cos[c + 2*d*x] - 450*A*d*x*Cos
[c + 2*d*x] + (450*I)*B*d*x*Cos[c + 2*d*x] + (300*I)*A*Cos[3*c + 2*d*x] + 3
45*B*Cos[3*c + 2*d*x] - 450*A*d*x*Cos[3*c + 2*d*x] + (450*I)*B*d*x*Cos[3*c
+ 2*d*x] + (90*I)*A*Cos[3*c + 4*d*x] + 120*B*Cos[3*c + 4*d*x] - 180*A*d*x*C
os[3*c + 4*d*x] + (180*I)*B*d*x*Cos[3*c + 4*d*x] + (90*I)*A*Cos[5*c + 4*d*x
] + 120*B*Cos[5*c + 4*d*x] - 180*A*d*x*Cos[5*c + 4*d*x] + (180*I)*B*d*x*Cos
[5*c + 4*d*x] - 30*A*d*x*Cos[5*c + 6*d*x] + (30*I)*B*d*x*Cos[5*c + 6*d*x] -
30*A*d*x*Cos[7*c + 6*d*x] + (30*I)*B*d*x*Cos[7*c + 6*d*x] - 790*A*Sin[c] +
(860*I)*B*Sin[c] + 720*A*Sin[c + 2*d*x] - (780*I)*B*Sin[c + 2*d*x] - 465*A
*Sin[3*c + 2*d*x] + (510*I)*B*Sin[3*c + 2*d*x] + 354*A*Sin[3*c + 4*d*x] - (
366*I)*B*Sin[3*c + 4*d*x] - 120*A*Sin[5*c + 4*d*x] + (150*I)*B*Sin[5*c + 4*
d*x] + 79*A*Sin[5*c + 6*d*x] - (86*I)*B*Sin[5*c + 6*d*x])*(a + I*a*Tan[c +
d*x])^4*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x]
+ B*Sin[c + d*x])) + (x*Cos[c + d*x]^5*(4*A*Cos[c]^2 - (4*I)*B*Cos[c]^2 - 4
*A*Cos[c]^4 + (4*I)*B*Cos[c]^4 - (12*I)*A*Cos[c]*Sin[c] - 12*B*Cos[c]*Sin[c
] + (20*I)*A*Cos[c]^3*Sin[c] + 20*B*Cos[c]^3*Sin[c] - 12*A*Sin[c]^2 + (12*I
)*B*Sin[c]^2 + 40*A*Cos[c]^2*Sin[c]^2 - (40*I)*B*Cos[c]^2*Sin[c]^2 - (40*I)
*A*Cos[c]*Sin[c]^3 - 40*B*Cos[c]*Sin[c]^3 - 20*A*Sin[c]^4 + (20*I)*B*Sin[c]
^4 + (4*I)*A*Sin[c]^2*Tan[c] + 4*B*Sin[c]^2*Tan[c] + (4*I)*A*Sin[c]^4*Tan[c
] + 4*B*Sin[c]^4*Tan[c] - I*(A - I*B)*(8*Cos[4*c] - (8*I)*Sin[4*c])*Tan[c])
*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]))/((Cos[d*x] + I*Sin[d*x])^4*
(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.11, size = 168, normalized size = 0.75

method	result
derivativedivides	$a^4 \left( -\frac{4iB(\tan^5(dx+c))}{5} + \frac{B(\tan^6(dx+c))}{6} - iA(\tan^4(dx+c)) + \frac{A(\tan^5(dx+c))}{5} + \frac{8iB(\tan^3(dx+c))}{3} - \frac{7B(\tan^4(dx+c))}{4} + 4iA(\tan^2(dx+c)) \right)$
default	$a^4 \left( -\frac{4iB(\tan^5(dx+c))}{5} + \frac{B(\tan^6(dx+c))}{6} - iA(\tan^4(dx+c)) + \frac{A(\tan^5(dx+c))}{5} + \frac{8iB(\tan^3(dx+c))}{3} - \frac{7B(\tan^4(dx+c))}{4} + 4iA(\tan^2(dx+c)) \right)$

norman	$(8iB a^4 - 8A a^4) x - \frac{(4iA a^4 + 7B a^4)(\tan^4(dx+c))}{4d} - \frac{(-8iB a^4 + 7A a^4)(\tan^3(dx+c))}{3d} + \frac{8(-iB a^4 + A a^4)}{d}$
risch	$-\frac{16ia^4Bc}{d} + \frac{16a^4Ac}{d} + \frac{4a^4(210iA e^{10i(dx+c)} + 270B e^{10i(dx+c)} + 765iA e^{8i(dx+c)} + 855B e^{8i(dx+c)} + 1210iA e^{6i(dx+c)} + 1500B e^{6i(dx+c)} + 1210iA e^{4i(dx+c)} + 270B e^{4i(dx+c)} + 765iA e^{2i(dx+c)} + 855B e^{2i(dx+c)} + 1210iA e^{0i(dx+c)} + 270B e^{0i(dx+c)} + 765iA e^{-2i(dx+c)} + 855B e^{-2i(dx+c)} + 1210iA e^{-4i(dx+c)} + 270B e^{-4i(dx+c)} + 765iA e^{-6i(dx+c)} + 855B e^{-6i(dx+c)} + 1210iA e^{-8i(dx+c)} + 270B e^{-8i(dx+c)} + 765iA e^{-10i(dx+c)} + 855B e^{-10i(dx+c)})}{60d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERB OSE)`

[Out]  $\frac{1}{d} a^4 (-\frac{4}{5} I B \tan(dx+c)^5 + \frac{1}{6} B^2 \tan(dx+c)^6 - I A \tan(dx+c)^4 + \frac{1}{5} A^2 \tan(dx+c)^5 + \frac{8}{3} I B \tan(dx+c)^3 - \frac{7}{4} B^2 \tan(dx+c)^4 + 4 I A \tan(dx+c)^2 - \frac{7}{3} A^2 \tan(dx+c)^3 - 8 I B \tan(dx+c) + 4 B^2 \tan(dx+c)^2 + 8 A \tan(dx+c) + \frac{1}{2} (-8 B - 8 I A) \ln(1 + \tan(dx+c)^2) + (-8 A + 8 I B) \arctan(\tan(dx+c)))$

**Maxima** [A]

time = 0.66, size = 150, normalized size = 0.67

$\frac{10B a^4 \tan(dx+c)^6 + 12(A-4iB) a^4 \tan(dx+c)^5 - 15(4iA+7B) a^4 \tan(dx+c)^4 - 20(7A-8iB) a^4 \tan(dx+c)^3 - 240(-iA-B) a^4 \tan(dx+c)^2 - 480(dx+c)(A-iB) a^4 - 240(iA+B) a^4 \log(\tan(dx+c)^2+1) + 480(A-iB) a^4 \tan(dx+c)}{60d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{60} (10 B a^4 \tan(dx+c)^6 + 12 (A - 4 I B) a^4 \tan(dx+c)^5 - 15 (4 I A + 7 B) a^4 \tan(dx+c)^4 - 20 (7 A - 8 I B) a^4 \tan(dx+c)^3 - 240 (-I A - B) a^4 \tan(dx+c)^2 - 480 (d x + c) (A - I B) a^4 - 240 (I A + B) a^4 \log(\tan(dx+c)^2 + 1) + 480 (A - I B) a^4 \tan(dx+c)) / d$

**Fricas** [A]

time = 0.43, size = 345, normalized size = 1.53

$\frac{4(30(-7A-9B)a^{10i(dx+c)}+45(-17A-19B)a^{8i(dx+c)}+10(-121A-135B)a^{6i(dx+c)}+15(-68A-75B)a^{4i(dx+c)}+6(-74A-81B)a^{2i(dx+c)}+30(-IA-B)a^{0i(dx+c)}+6(-IA-B)a^{10i(dx+c)}+15(-IA-B)a^{8i(dx+c)}+20(-IA-B)a^{6i(dx+c)}+15(-IA-B)a^{4i(dx+c)}+6(-IA-B)a^{2i(dx+c)}+(-IA-B)\log(\tan^2(dx+c)+1))}{15(d^6 \tan^{10}(dx+c) + 6 d^5 \tan^8(dx+c) + 15 d^4 \tan^6(dx+c) + 20 d^3 \tan^4(dx+c) + 15 d^2 \tan^2(dx+c) + 6 d \tan(dx+c) + d)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-4/15 (30 (-7 I A - 9 B) a^4 e^{(10 I d x + 10 I c)} + 45 (-17 I A - 19 B) a^4 e^{(8 I d x + 8 I c)} + 10 (-121 I A - 135 B) a^4 e^{(6 I d x + 6 I c)} + 15 (-68 I A - 75 B) a^4 e^{(4 I d x + 4 I c)} + 6 (-74 I A - 81 B) a^4 e^{(2 I d x + 2 I c)} + (-79 I A - 86 B) a^4 + 30 ((-I A - B) a^4 e^{(12 I d x + 12 I c)} + 6 (-I A - B) a^4 e^{(10 I d x + 10 I c)} + 15 (-I A - B) a^4 e^{(8 I d x + 8 I c)} + 20 (-I A - B) a^4 e^{(6 I d x + 6 I c)} + 15 (-I A - B) a^4 e^{(4 I d x + 4 I c)} + 6 (-I A - B) a^4 e^{(2 I d x + 2 I c)} + (-I A - B) a^4) \log(e^{(2 I d x + 2 I c)} + 1)) / (d e^{(12 I d x + 12 I c)} + 6 d e^{(10 I d x + 10 I c)}$

c) + 15\*d\*e^(8\*I\*d\*x + 8\*I\*c) + 20\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 15\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 6\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy [A]**

time = 0.97, size = 348, normalized size = 1.55

$$\frac{8ia^4(A - iB) \log(e^{2idx} + e^{-2ic})}{d} + \frac{316iAa^4 + 344Ba^4 + (1776iAa^4e^{2ic} + 1944Ba^4e^{2ic})e^{2idx} + (4080iAa^4e^{4ic} + 4500Ba^4e^{4ic})e^{4idx} + (4840iAa^4e^{6ic} + 5400Ba^4e^{6ic})e^{6idx} + (3060iAa^4e^{8ic} + 3420Ba^4e^{8ic})e^{8idx} + (840iAa^4e^{10ic} + 1080Ba^4e^{10ic})e^{10idx}}{15de^{12ic}e^{12idx} + 90de^{10ic}e^{10idx} + 225de^{8ic}e^{8idx} + 300de^{6ic}e^{6idx} + 225de^{4ic}e^{4idx} + 90de^{2ic}e^{2idx} + 15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out] 8\*I\*a\*\*4\*(A - I\*B)\*log(exp(2\*I\*d\*x) + exp(-2\*I\*c))/d + (316\*I\*A\*a\*\*4 + 344\*B\*a\*\*4 + (1776\*I\*A\*a\*\*4\*exp(2\*I\*c) + 1944\*B\*a\*\*4\*exp(2\*I\*c))\*exp(2\*I\*d\*x) + (4080\*I\*A\*a\*\*4\*exp(4\*I\*c) + 4500\*B\*a\*\*4\*exp(4\*I\*c))\*exp(4\*I\*d\*x) + (4840\*I\*A\*a\*\*4\*exp(6\*I\*c) + 5400\*B\*a\*\*4\*exp(6\*I\*c))\*exp(6\*I\*d\*x) + (3060\*I\*A\*a\*\*4\*exp(8\*I\*c) + 3420\*B\*a\*\*4\*exp(8\*I\*c))\*exp(8\*I\*d\*x) + (840\*I\*A\*a\*\*4\*exp(10\*I\*c) + 1080\*B\*a\*\*4\*exp(10\*I\*c))\*exp(10\*I\*d\*x))/(15\*d\*exp(12\*I\*c)\*exp(12\*I\*d\*x) + 90\*d\*exp(10\*I\*c)\*exp(10\*I\*d\*x) + 225\*d\*exp(8\*I\*c)\*exp(8\*I\*d\*x) + 300\*d\*exp(6\*I\*c)\*exp(6\*I\*d\*x) + 225\*d\*exp(4\*I\*c)\*exp(4\*I\*d\*x) + 90\*d\*exp(2\*I\*c)\*exp(2\*I\*d\*x) + 15\*d)

**Giac [B]** Both result and optimal contain **B** complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(195) = 390.

time = 0.91, size = 600, normalized size = 2.67

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] -4/15\*(-30\*I\*A\*a^4\*e^(12\*I\*d\*x + 12\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 30\*B\*a^4\*e^(12\*I\*d\*x + 12\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 180\*I\*A\*a^4\*e^(10\*I\*d\*x + 10\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 180\*B\*a^4\*e^(10\*I\*d\*x + 10\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 450\*I\*A\*a^4\*e^(8\*I\*d\*x + 8\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 450\*B\*a^4\*e^(8\*I\*d\*x + 8\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 600\*I\*A\*a^4\*e^(6\*I\*d\*x + 6\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 600\*B\*a^4\*e^(6\*I\*d\*x + 6\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 450\*I\*A\*a^4\*e^(4\*I\*d\*x + 4\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 450\*B\*a^4\*e^(4\*I\*d\*x + 4\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 180\*I\*A\*a^4\*e^(2\*I\*d\*x + 2\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 180\*B\*a^4\*e^(2\*I\*d\*x + 2\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 210\*I\*A\*a^4\*e^(10\*I\*d\*x + 10\*I\*c) - 270\*B\*a^4\*e^(10\*I\*d\*x + 10\*I\*c) - 765\*I\*A\*a^4\*e^(8\*I\*d\*x + 8\*I\*c) - 855\*B\*a^4\*e^(8\*I\*d\*x + 8\*I\*c) - 1210\*I\*A\*a^4\*e^(6\*I\*d\*x + 6\*I\*c) - 1350\*B\*a^4\*e^(6\*I\*d\*x + 6\*I\*c) - 1020\*I\*A\*a^4\*e^(4\*I\*d\*x + 4\*I\*c) - 1125\*B\*a^4\*e^(4\*I\*d\*x + 4\*I\*c) - 444\*I\*A\*a^4\*e^(2\*I\*d\*x + 2\*I\*c) - 486\*B\*a^4\*e^(2\*I\*d\*x + 2\*I\*c) - 30\*I\*A\*a^4\*log(e^(2\*I\*d\*x + 2

\*I\*c) + 1) - 30\*B\*a^4\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 79\*I\*A\*a^4 - 86\*B\*a^4) / (d\*e^(12\*I\*d\*x + 12\*I\*c) + 6\*d\*e^(10\*I\*d\*x + 10\*I\*c) + 15\*d\*e^(8\*I\*d\*x + 8\*I\*c) + 20\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 15\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 6\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Mupad [B]**

time = 6.21, size = 308, normalized size = 1.37

$\frac{\tan(c+dx)^2(-a^4(A-B) + \frac{4A^2B}{d} + \frac{4AB^2}{d})}{d} - \frac{\tan(c+dx)(-A^4-3a^4(A-B) + a^4(B+AB) + B^4 + a^4(B+AB))}{d} - \frac{\tan(c+dx)^2(\frac{4A^2B}{d} + \frac{4AB^2}{d})}{d} - \frac{\ln(\tan(c+dx)+1)(8Ba^4+A^4)}{d} - \frac{\tan(c+dx)^2(\frac{4A^2B}{d} + \frac{4AB^2}{d} + \frac{4B^2}{d} + \frac{4AB^2}{d})}{d} - \frac{\tan(c+dx)^2(\frac{4A^2B}{d} + \frac{4AB^2}{d})}{d} - \frac{Ba^4 \tan(c+dx)^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] (tan(c + d\*x)^3\*((a^4\*(A\*3i + B)\*1i)/3 - a^4\*(A - B\*1i) + (B\*a^4\*1i)/3 + (a^4\*(A\*1i + 3\*B)\*1i)/3))/d - (tan(c + d\*x)\*(a^4\*(A\*3i + B)\*1i - 3\*a^4\*(A - B\*1i) - A\*a^4 + B\*a^4\*1i + a^4\*(A\*1i + 3\*B)\*1i))/d - (tan(c + d\*x)^5\*((B\*a^4\*1i)/5 + (a^4\*(A\*1i + 3\*B)\*1i)/5))/d - (log(tan(c + d\*x) + 1i)\*(A\*a^4\*8i + 8\*B\*a^4))/d + (tan(c + d\*x)^2\*((A\*a^4\*1i)/2 + (a^4\*(A - B\*1i)\*3i)/2 + (a^4\*(A\*3i + B))/2 + (B\*a^4)/2 + (a^4\*(A\*1i + 3\*B))/2))/d - (tan(c + d\*x)^4\*((a^4\*(A - B\*1i)\*3i)/4 + (B\*a^4)/4 + (a^4\*(A\*1i + 3\*B))/4))/d + (B\*a^4\*tan(c + d\*x)^6)/(6\*d)

### 3.27 $\int \tan(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=168

$$-8a^4(iA+B)x - \frac{8a^4(A-iB) \log(\cos(c+dx))}{d} + \frac{4a^4(iA+B) \tan(c+dx)}{d} + \frac{a(A-iB)(a+ia \tan(c+dx))^3}{3d}$$

[Out]  $-8*a^4*(I*A+B)*x - 8*a^4*(A-I*B)*\ln(\cos(d*x+c))/d + 4*a^4*(I*A+B)*\tan(d*x+c)/d + 1/3*a*(A-I*B)*(a+I*a*\tan(d*x+c))^3/d + 1/4*A*(a+I*a*\tan(d*x+c))^4/d - 1/5*I*B*(a+I*a*\tan(d*x+c))^5/a/d + (A-I*B)*(a^2+I*a^2*\tan(d*x+c))^2/d$

Rubi [A]

time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3673, 3608, 3559, 3558, 3556}

$$\frac{4a^4(B+IA)\tan(c+dx)}{d} - \frac{8a^4(A-iB)\log(\cos(c+dx))}{d} - 8a^4x(B+iA) + \frac{(A-iB)(a^2+ia^2\tan(c+dx))^2}{d} + \frac{a(A-iB)(a+ia\tan(c+dx))^2}{3d} + \frac{A(a+ia\tan(c+dx))^4}{4d} - \frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]*(a + I*a*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-8*a^4*(I*A + B)*x - (8*a^4*(A - I*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (4*a^4*(I*A + B)*\text{Tan}[c + d*x])/d + (a*(A - I*B)*(a + I*a*\text{Tan}[c + d*x])^3)/(3*d) + (A*(a + I*a*\text{Tan}[c + d*x])^4)/(4*d) - ((I/5)*B*(a + I*a*\text{Tan}[c + d*x])^5)/(a*d) + ((A - I*B)*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/d$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3558

$\text{Int}[(a_. + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^2, x\_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[b^2*(\text{Tan}[c + d*x]/d), x]) /; \text{FreeQ}\{a, b, c, d\}, x]$

Rule 3559

$\text{Int}[(a_. + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*((a + b*\text{Tan}[c + d*x])^{(n-1)}/(d*(n-1))), x] + \text{Dist}[2*a, \text{Int}[(a + b*\text{Tan}[c + d*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3608



```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{iB(a + ia \tan(c + dx))^5}{5ad} + \int (a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= \frac{A(a + ia \tan(c + dx))^4}{4d} - \frac{iB(a + ia \tan(c + dx))^5}{5ad} \\
&= \frac{a(A - iB)(a + ia \tan(c + dx))^3}{3d} + \frac{A(a + ia \tan(c + dx))^4}{4d} \\
&= \frac{a(A - iB)(a + ia \tan(c + dx))^3}{3d} + \frac{A(a + ia \tan(c + dx))^4}{4d} \\
&= -8a^4(iA + B)x + \frac{4a^4(iA + B) \tan(c + dx)}{d} \\
&= -8a^4(iA + B)x - \frac{8a^4(A - iB) \log(\cos(c + dx))}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 589 vs. 2(168) = 336.  
time = 3.80, size = 589, normalized size = 3.51

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

```
[Out] (a^4*Sec[c]*Sec[c + d*x]^5*(-60*A*Cos[2*c + 3*d*x] + (90*I)*B*Cos[2*c + 3*d*x] - (150*I)*A*d*x*Cos[2*c + 3*d*x] - 150*B*d*x*Cos[2*c + 3*d*x] - 60*A*Cos[4*c + 3*d*x] + (90*I)*B*Cos[4*c + 3*d*x] - (150*I)*A*d*x*Cos[4*c + 3*d*x] - 150*B*d*x*Cos[4*c + 3*d*x] - (30*I)*A*d*x*Cos[4*c + 5*d*x] - 30*B*d*x*Co
```

$$\begin{aligned} & s[4*c + 5*d*x] - (30*I)*A*d*x*\text{Cos}[6*c + 5*d*x] - 30*B*d*x*\text{Cos}[6*c + 5*d*x] \\ & - 75*A*\text{Cos}[2*c + 3*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] + (75*I)*B*\text{Cos}[2*c + 3*d*x]*\text{Log} \\ & [\text{Cos}[c + d*x]^2] - 75*A*\text{Cos}[4*c + 3*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] + (75*I)*B*\text{Cos} \\ & [4*c + 3*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - 15*A*\text{Cos}[4*c + 5*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] \\ & + (15*I)*B*\text{Cos}[4*c + 5*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - 15*A*\text{Cos}[6*c + 5*d*x]* \\ & \text{Log}[\text{Cos}[c + d*x]^2] + (15*I)*B*\text{Cos}[6*c + 5*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - (15*I) \\ & )*\text{Cos}[d*x]*((-11*I)*A - 14*B + 20*A*d*x - (20*I)*B*d*x - (10*I)*(A - I*B)*\text{L} \\ & \text{og}[\text{Cos}[c + d*x]^2]) - (15*I)*\text{Cos}[2*c + d*x]*((-11*I)*A - 14*B + 20*A*d*x - \\ & (20*I)*B*d*x - (10*I)*(A - I*B)*\text{Log}[\text{Cos}[c + d*x]^2]) + (400*I)*A*\text{Sin}[d*x] + \\ & 445*B*\text{Sin}[d*x] - (300*I)*A*\text{Sin}[2*c + d*x] - 345*B*\text{Sin}[2*c + d*x] + (260*I) \\ & *A*\text{Sin}[2*c + 3*d*x] + 275*B*\text{Sin}[2*c + 3*d*x] - (90*I)*A*\text{Sin}[4*c + 3*d*x] - \\ & 120*B*\text{Sin}[4*c + 3*d*x] + (70*I)*A*\text{Sin}[4*c + 5*d*x] + 79*B*\text{Sin}[4*c + 5*d*x] \\ & )/(120*d) \end{aligned}$$

**Maple [A]**

time = 0.09, size = 145, normalized size = 0.86

method	result
derivativedivides	$a^4 \left( -iB \tan^4(dx+c) + \frac{B \tan^5(dx+c)}{5} - \frac{4iA \tan^3(dx+c)}{3} + \frac{A \tan^4(dx+c)}{4} + 4iB \tan^2(dx+c) - \frac{7B \tan^3(dx+c)}{3} + 8iA \tan \right) \frac{d}{d}$
default	$a^4 \left( -iB \tan^4(dx+c) + \frac{B \tan^5(dx+c)}{5} - \frac{4iA \tan^3(dx+c)}{3} + \frac{A \tan^4(dx+c)}{4} + 4iB \tan^2(dx+c) - \frac{7B \tan^3(dx+c)}{3} + 8iA \tan \right) \frac{d}{d}$
norman	$(-8iA a^4 - 8B a^4) x - \frac{(4iA a^4 + 7B a^4) \tan^3(dx+c)}{3d} - \frac{(-8iB a^4 + 7A a^4) \tan^2(dx+c)}{2d} + \frac{8(iA a^4 + B a^4) \tan(dx+c)}{d}$
risch	$\frac{16a^4 Bc}{d} + \frac{16ia^4 Ac}{d} + \frac{4ia^4 (150iA e^{8i(dx+c)} + 210B e^{8i(dx+c)} + 465iA e^{6i(dx+c)} + 555B e^{6i(dx+c)} + 565iA e^{4i(dx+c)} + 655B e^{4i(dx+c)} + 150iA e^{2i(dx+c)} + 150B e^{2i(dx+c)} + 150iA)}{15d(e^{2i(dx+c)} + 1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS E)`

[Out]  $1/d*a^4*(-I*B*\text{tan}(d*x+c)^4+1/5*B*\text{tan}(d*x+c)^5-4/3*I*A*\text{tan}(d*x+c)^3+1/4*A*\text{tan}(d*x+c)^4+4*I*B*\text{tan}(d*x+c)^2-7/3*B*\text{tan}(d*x+c)^3+8*I*A*\text{tan}(d*x+c)-7/2*A*\text{tan}(d*x+c)^2+8*B*\text{tan}(d*x+c)+1/2*(8*A-8*I*B)*\ln(1+\text{tan}(d*x+c)^2)+(-8*B-8*I*A)*\arctan(\text{tan}(d*x+c))$

**Maxima [A]**

time = 0.93, size = 132, normalized size = 0.79

$$\frac{12Ba^4 \tan(dx+c)^5 + 15(A-4iB)a^4 \tan(dx+c)^4 - 20(4iA+7B)a^4 \tan(dx+c)^3 - 30(7A-8iB)a^4 \tan(dx+c)^2 - 480(dx+c)(iA+Ba^4) + 240(A-iB)a^4 \log(\tan(dx+c)^2+1) - 480(-iA-B)a^4 \tan(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{60}*(12*B*a^4*\tan(d*x + c)^5 + 15*(A - 4*I*B)*a^4*\tan(d*x + c)^4 - 20*(4*I*A + 7*B)*a^4*\tan(d*x + c)^3 - 30*(7*A - 8*I*B)*a^4*\tan(d*x + c)^2 - 480*(d*x + c)*(I*A + B)*a^4 + 240*(A - I*B)*a^4*\log(\tan(d*x + c)^2 + 1) - 480*(-I*A - B)*a^4*\tan(d*x + c))/d$

**Fricas** [A]

time = 0.48, size = 279, normalized size = 1.66

$\frac{4(30(5A-7iB)a^{10}e^{10dx+10c} + 15(31A-37iB)a^{10}e^{6dx+6c} + 5(113A-131iB)a^{10}e^{4dx+4c} + 5(64A-73iB)a^{10}e^{2dx+2c} + (70A-79iB)a^4 + 30((A-iB)a^{10}e^{10dx+10c} + 5(A-iB)a^{10}e^{6dx+6c} + 10(A-iB)a^{10}e^{4dx+4c} + 5(A-iB)a^{10}e^{2dx+2c} + (A-iB)a^4)\log(e^{2dx+2c} + 1))}{15(d e^{10dx+10c} + 5d e^{6dx+6c} + 10d e^{4dx+4c} + 10d e^{2dx+2c} + d)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-4/15*(30*(5*A - 7*I*B)*a^4*e^{(8*I*d*x + 8*I*c)} + 15*(31*A - 37*I*B)*a^4*e^{(6*I*d*x + 6*I*c)} + 5*(113*A - 131*I*B)*a^4*e^{(4*I*d*x + 4*I*c)} + 5*(64*A - 73*I*B)*a^4*e^{(2*I*d*x + 2*I*c)} + (70*A - 79*I*B)*a^4 + 30*((A - I*B)*a^4*e^{(10*I*d*x + 10*I*c)} + 5*(A - I*B)*a^4*e^{(8*I*d*x + 8*I*c)} + 10*(A - I*B)*a^4*e^{(6*I*d*x + 6*I*c)} + 10*(A - I*B)*a^4*e^{(4*I*d*x + 4*I*c)} + 5*(A - I*B)*a^4*e^{(2*I*d*x + 2*I*c)} + (A - I*B)*a^4)*\log(e^{(2*I*d*x + 2*I*c)} + 1))/(d*e^{(10*I*d*x + 10*I*c)} + 5*d*e^{(8*I*d*x + 8*I*c)} + 10*d*e^{(6*I*d*x + 6*I*c)} + 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(138) = 276.

time = 0.58, size = 291, normalized size = 1.73

$-\frac{8a^4(A-iB)\log(e^{2idx} + e^{-2ic})}{d} + \frac{-280Aa^4 + 316iBa^4 + (-1280Aa^4e^{2ic} + 1460iBa^4e^{2ic})e^{2idx} + (-2260Aa^4e^{4ic} + 2620iBa^4e^{4ic})e^{4idx} + (-1860Aa^4e^{6ic} + 2220iBa^4e^{6ic})e^{6idx} + (-600Aa^4e^{8ic} + 840iBa^4e^{8ic})e^{8idx}}{15de^{10ic}e^{10idx} + 75de^{8ic}e^{8idx} + 150de^{6ic}e^{6idx} + 150de^{4ic}e^{4idx} + 75de^{2ic}e^{2idx} + 15d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)`

[Out]  $-8*a**4*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (-280*A*a**4 + 316*I*B*a**4 + (-1280*A*a**4*\exp(2*I*c) + 1460*I*B*a**4*\exp(2*I*c))*\exp(2*I*d*x) + (-2260*A*a**4*\exp(4*I*c) + 2620*I*B*a**4*\exp(4*I*c))*\exp(4*I*d*x) + (-1860*A*a**4*\exp(6*I*c) + 2220*I*B*a**4*\exp(6*I*c))*\exp(6*I*d*x) + (-600*A*a**4*\exp(8*I*c) + 840*I*B*a**4*\exp(8*I*c))*\exp(8*I*d*x))/(15*d*\exp(10*I*c)*\exp(10*I*d*x) + 75*d*\exp(8*I*c)*\exp(8*I*d*x) + 150*d*\exp(6*I*c)*\exp(6*I*d*x) + 150*d*\exp(4*I*c)*\exp(4*I*d*x) + 75*d*\exp(2*I*c)*\exp(2*I*d*x) + 15*d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(142) = 284.

time = 0.75, size = 504, normalized size = 3.00

.....

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -4/15*(30*A*a^4*e^{(10*I*d*x + 10*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 30*I*B \\ & *a^4*e^{(10*I*d*x + 10*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 150*A*a^4*e^{(8*I*d*x + 8*I*c)} \\ & *\log(e^{(2*I*d*x + 2*I*c)} + 1) - 150*I*B*a^4*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) \\ & + 300*A*a^4*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 300*I*B*a^4*e^{(6*I*d*x + 6*I*c)} \\ & *\log(e^{(2*I*d*x + 2*I*c)} + 1) + 300*A*a^4*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 300*I*B \\ & *a^4*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 150*A*a^4*e^{(2*I*d*x + 2*I*c)} \\ & *\log(e^{(2*I*d*x + 2*I*c)} + 1) - 150*I*B*a^4*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) \\ & + 150*A*a^4*e^{(8*I*d*x + 8*I*c)} - 210*I*B*a^4*e^{(8*I*d*x + 8*I*c)} + 465*A*a^4*e^{(6*I*d*x + 6*I*c)} \\ & - 555*I*B*a^4*e^{(6*I*d*x + 6*I*c)} + 565*A*a^4*e^{(4*I*d*x + 4*I*c)} - 655*I*B*a^4*e^{(4*I*d*x + 4*I*c)} \\ & + 320*A*a^4*e^{(2*I*d*x + 2*I*c)} - 365*I*B*a^4*e^{(2*I*d*x + 2*I*c)} + 30*A*a^4 \\ & *4*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 30*I*B*a^4*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 70*A*a^4 \\ & - 79*I*B*a^4)/(d*e^{(10*I*d*x + 10*I*c)} + 5*d*e^{(8*I*d*x + 8*I*c)} + 10*d*e^{(6*I*d*x + 6*I*c)} \\ & + 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*I*c)} + d) \end{aligned}$$

**Mupad [B]**

time = 6.20, size = 240, normalized size = 1.43

$$\frac{\tan(c+dx)^2 \left( -\frac{3a^4(A-B)}{2} + \frac{a^4(B+A)}{2} + \frac{B^2}{2} + \frac{a^4(B+A)}{2} \right) + \tan(c+dx) (Aa^4 + a^4(A-B) + 3a^4(B+A) + Ba^4 + a^4(3B+A)) - \frac{\tan(c+dx)^2 \left( \frac{B^2}{2} + \frac{a^4(B+A)}{2} \right) + \ln(\tan(c+dx) + 1) (8Aa^4 - B^2)}{d} - \frac{\tan(c+dx)^2 \left( a^4(A-B) + \frac{B^2}{2} + \frac{a^4(B+A)}{2} \right) + Ba^4 \tan(c+dx)^2}{5d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] 
$$\begin{aligned} & (\tan(c + d*x)^2*((a^4*(A*3i + B)*1i)/2 - (3*a^4*(A - B*1i))/2 + (B*a^4*1i)/ \\ & 2 + (a^4*(A*1i + 3*B)*1i)/2))/d + (\tan(c + d*x)*(A*a^4*1i + a^4*(A - B*1i)* \\ & 3i + a^4*(A*3i + B) + B*a^4 + a^4*(A*1i + 3*B)))/d - (\tan(c + d*x)^4*((B*a^4 \\ & 4*1i)/4 + (a^4*(A*1i + 3*B)*1i)/4))/d + (\log(\tan(c + d*x) + 1i)*(8*A*a^4 - \\ & B*a^4*8i))/d - (\tan(c + d*x)^3*(a^4*(A - B*1i)*1i + (B*a^4)/3 + (a^4*(A*1i \\ & + 3*B))/3))/d + (B*a^4*tan(c + d*x)^5)/(5*d) \end{aligned}$$

### 3.28 $\int (a + ia \tan(c + dx))^4 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=140

$$8a^4(A-iB)x - \frac{8a^4(iA+B)\log(\cos(c+dx))}{d} - \frac{4a^4(A-iB)\tan(c+dx)}{d} + \frac{a(iA+B)(a+ia\tan(c+dx))^3}{3d} +$$

[Out]  $8*a^4*(A-I*B)*x - 8*a^4*(I*A+B)*\ln(\cos(d*x+c))/d - 4*a^4*(A-I*B)*\tan(d*x+c)/d + 1/3*a*(I*A+B)*(a+I*a*\tan(d*x+c))^3/d + 1/4*B*(a+I*a*\tan(d*x+c))^4/d + (I*A+B)*(a^2+I*a^2*\tan(d*x+c))^2/d$

**Rubi [A]**

time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {3608, 3559, 3558, 3556}

$$-\frac{4a^4(A-iB)\tan(c+dx)}{d} - \frac{8a^4(B+iA)\log(\cos(c+dx))}{d} + 8a^4x(A-iB) + \frac{(B+iA)(a^2+ia^2\tan(c+dx))^2}{d} + \frac{a(B+iA)(a+ia\tan(c+dx))^3}{3d} + \frac{B(a+ia\tan(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $8*a^4*(A - I*B)*x - (8*a^4*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/d - (4*a^4*(A - I*B)*\text{Tan}[c + d*x])/d + (a*(I*A + B)*(a + I*a*\text{Tan}[c + d*x])^3)/(3*d) + (B*(a + I*a*\text{Tan}[c + d*x])^4)/(4*d) + ((I*A + B)*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/d$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3558**

$\text{Int}[(a_. + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^2, x\_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[b^2*(\text{Tan}[c + d*x]/d), x]) /; \text{FreeQ}\{a, b, c, d\}, x]$

**Rule 3559**

$\text{Int}[(a_. + (b_.)*\tan[(c_.) + (d_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*((a + b*\text{Tan}[c + d*x])^{(n-1)}/(d*(n-1))), x] + \text{Dist}[2*a, \text{Int}[(a + b*\text{Tan}[c + d*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[n, 1]$

**Rule 3608**

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Dist}$

$[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ !\text{LtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int (a + ia \tan(c + dx))^4 (A + B \tan(c + dx)) dx &= \frac{B(a + ia \tan(c + dx))^4}{4d} - (-A + iB) \int (a + ia \tan(c + dx))^4 dx \\ &= \frac{a(iA + B)(a + ia \tan(c + dx))^3}{3d} + \frac{B(a + ia \tan(c + dx))^4}{4d} \\ &= \frac{a(iA + B)(a + ia \tan(c + dx))^3}{3d} + \frac{B(a + ia \tan(c + dx))^4}{4d} \\ &= 8a^4(A - iB)x - \frac{4a^4(A - iB) \tan(c + dx)}{d} + \frac{a(iA + B)(a + ia \tan(c + dx))^3}{3d} \\ &= 8a^4(A - iB)x - \frac{8a^4(iA + B) \log(\cos(c + dx))}{d} - \frac{4a^4(A - iB)}{d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 448 vs.  $2(140) = 280$ .  
time = 2.70, size = 448, normalized size = 3.20

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(a^4*\text{Sec}[c]*\text{Sec}[c + d*x]^4*((-6*I)*A*\text{Cos}[3*c + 2*d*x] - 12*B*\text{Cos}[3*c + 2*d*x] + 24*A*d*x*\text{Cos}[3*c + 2*d*x] - (24*I)*B*d*x*\text{Cos}[3*c + 2*d*x] + 6*A*d*x*\text{Cos}[3*c + 4*d*x] - (6*I)*B*d*x*\text{Cos}[3*c + 4*d*x] + 6*A*d*x*\text{Cos}[5*c + 4*d*x] - (6*I)*B*d*x*\text{Cos}[5*c + 4*d*x] - (12*I)*A*\text{Cos}[3*c + 2*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - 12*B*\text{Cos}[3*c + 2*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - (3*I)*A*\text{Cos}[3*c + 4*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - 3*B*\text{Cos}[3*c + 4*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - (3*I)*A*\text{Cos}[5*c + 4*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] - 3*B*\text{Cos}[5*c + 4*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] + 3*\text{Cos}[c]*((-4*I)*A - 7*B + 12*A*d*x - (12*I)*B*d*x + ((-6*I)*A - 6*B)*\text{Log}[\text{Cos}[c + d*x]^2]) + 6*\text{Cos}[c + 2*d*x]*((-I)*A - 2*B + 4*A*d*x - (4*I)*B*d*x + ((-2*I)*A - 2*B)*\text{Log}[\text{Cos}[c + d*x]^2]) + 33*A*\text{Sin}[c] - (42*I)*B*\text{Sin}[c] - 32*A*\text{Sin}[c + 2*d*x] + (38*I)*B*\text{Sin}[c + 2*d*x] + 12*A*\text{Sin}[3*c + 2*d*x] - (18*I)*B*\text{Sin}[3*c + 2*d*x] - 11*A*\text{Sin}[3*c + 4*d*x] + (14*I)*B*\text{Sin}[3*c + 4*d*x])/(12*d)$

**Maple [A]**

time = 0.08, size = 122, normalized size = 0.87

method	result
derivativedivides	$a^4 \left( -\frac{4iB(\tan^3(dx+c))}{3} + \frac{B(\tan^4(dx+c))}{4} - 2iA(\tan^2(dx+c)) + \frac{A(\tan^3(dx+c))}{3} + 8iB \tan(dx+c) - \frac{7B(\tan^2(dx+c))}{2} - 7A \tan(dx+c) \right) \frac{1}{d}$
default	$a^4 \left( -\frac{4iB(\tan^3(dx+c))}{3} + \frac{B(\tan^4(dx+c))}{4} - 2iA(\tan^2(dx+c)) + \frac{A(\tan^3(dx+c))}{3} + 8iB \tan(dx+c) - \frac{7B(\tan^2(dx+c))}{2} - 7A \tan(dx+c) \right) \frac{1}{d}$
norman	$(-8iB a^4 + 8A a^4) x - \frac{(4iA a^4 + 7B a^4)(\tan^2(dx+c))}{2d} - \frac{(-8iB a^4 + 7A a^4) \tan(dx+c)}{d} + \frac{(-4iB a^4 + A a^4) \ln(1 + \tan^2(dx+c))}{3d}$
risch	$\frac{16ia^4 Bc}{d} - \frac{16a^4 Ac}{d} - \frac{4a^4(18iA e^{6i(dx+c)} + 30B e^{6i(dx+c)} + 45iA e^{4i(dx+c)} + 63B e^{4i(dx+c)} + 38iA e^{2i(dx+c)} + 50B e^{2i(dx+c)})}{3d(e^{2i(dx+c)} + 1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^4*(-4/3*I*B*tan(d*x+c)^3+1/4*B*tan(d*x+c)^4-2*I*A*tan(d*x+c)^2+1/3*A*tan(d*x+c)^3+8*I*B*tan(d*x+c)-7/2*B*tan(d*x+c)^2-7*A*tan(d*x+c)+1/2*(8*B+8*I*A)*ln(1+tan(d*x+c)^2)+(8*A-8*I*B)*arctan(tan(d*x+c)))
```

**Maxima** [A]

time = 0.48, size = 114, normalized size = 0.81

$$\frac{3Ba^4 \tan(dx+c)^4 + 4(A-4iB)a^4 \tan(dx+c)^3 - 6(4iA+7B)a^4 \tan(dx+c)^2 + 96(dx+c)(A-iB)a^4 - 48(-iA-B)a^4 \log(\tan(dx+c)^2+1) - 12(7A-8iB)a^4 \tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*B*a^4*tan(d*x+c)^4 + 4*(A-4*I*B)*a^4*tan(d*x+c)^3 - 6*(4*I*A+7*B)*a^4*tan(d*x+c)^2 + 96*(d*x+c)*(A-I*B)*a^4 - 48*(-I*A-B)*a^4*log(tan(d*x+c)^2+1) - 12*(7*A-8*I*B)*a^4*tan(d*x+c))/d
```

**Fricas** [A]

time = 0.46, size = 227, normalized size = 1.62

$$\frac{4(6(3iA+5B)a^4 e^{6i(dx+c)} + 9(5iA+7B)a^4 e^{4i(dx+c)} + 2(19iA+25B)a^4 e^{2i(dx+c)} + (11iA+14B)a^4 + 6((iA+B)a^4 e^{8i(dx+c)} + 4(iA+B)a^4 e^{6i(dx+c)} + 6(iA+B)a^4 e^{4i(dx+c)} + 4(iA+B)a^4 e^{2i(dx+c)} + (iA+B)a^4 \log(e^{2i(dx+c)}+1)))}{3(d e^{8i(dx+c)} + 4d e^{6i(dx+c)} + 6d e^{4i(dx+c)} + 4d e^{2i(dx+c)} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4/3*(6*(3*I*A+5*B)*a^4*e^(6*I*d*x+6*I*c)+9*(5*I*A+7*B)*a^4*e^(4*I*d*x+4*I*c)+2*(19*I*A+25*B)*a^4*e^(2*I*d*x+2*I*c)+(11*I*A+14*B)*a^4+6*((I*A+B)*a^4*e^(8*I*d*x+8*I*c)+4*(I*A+B)*a^4*e^(6*I*d*x+6*I*c)+6*(I*A+B)*a^4*e^(4*I*d*x+4*I*c)+4*(I*A+B)*a^4*e^(2*I*d*x+2*I*c)+(I*A+B)*a^4)*log(e^(2*I*d*x+2*I*c)+1)/(d*e^(8*I*d*x+8*I*c))+4*d*e^(6*I*d*x+6*I*c)+6*d*e^(4*I*d*x+4*I*c)+4*d*e^(2*I*d*x+2*I*c)+d
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs.  $2(116) = 232$ .  
time = 0.51, size = 241, normalized size = 1.72

$$-\frac{8ia^4(A - iB)\log(e^{2idx} + e^{-2ic})}{d} + \frac{-44iAa^4 - 56Ba^4 + (-152iAa^4e^{2ic} - 200Ba^4e^{2ic})e^{2idx} + (-180iAa^4e^{4ic} - 252Ba^4e^{4ic})e^{4idx} + (-72iAa^4e^{6ic} - 120Ba^4e^{6ic})e^{6idx}}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out]  $-8*I*a**4*(A - I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/d + (-44*I*A*a**4 - 56*B*a**4 + (-152*I*A*a**4*\exp(2*I*c) - 200*B*a**4*\exp(2*I*c))*\exp(2*I*d*x) + (-180*I*A*a**4*\exp(4*I*c) - 252*B*a**4*\exp(4*I*c))*\exp(4*I*d*x) + (-72*I*A*a**4*\exp(6*I*c) - 120*B*a**4*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) + 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) + 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs.  $2(120) = 240$ .  
time = 0.69, size = 408, normalized size = 2.91

$$\frac{4*B*a^{8ic}*\log(\exp(8ic)) + 4*B*a^{6ic}*\log(\exp(6ic)) + 36*A*a^{8ic}*\log(\exp(8ic)) + 36*B*a^{6ic}*\log(\exp(6ic)) + 36*A*a^{4ic}*\log(\exp(4ic)) + 36*B*a^{2ic}*\log(\exp(2ic)) + 24*A*a^{8ic}*\log(\exp(8ic)) + 24*B*a^{6ic}*\log(\exp(6ic)) + 24*A*a^{4ic}*\log(\exp(4ic)) + 24*B*a^{2ic}*\log(\exp(2ic)) + 50*A*a^{8ic}*\log(\exp(8ic)) + 50*B*a^{6ic}*\log(\exp(6ic)) + 45*A*a^{4ic}*\log(\exp(4ic)) + 45*B*a^{2ic}*\log(\exp(2ic)) + 11*A*a^{8ic}*\log(\exp(8ic)) + 11*B*a^{6ic}*\log(\exp(6ic)) + 14*A*a^{4ic}*\log(\exp(4ic)) + 14*B*a^{2ic}*\log(\exp(2ic))}{3d*\exp(8ic)*\exp(8idx) + 12*d*\exp(6ic)*\exp(6idx) + 18*d*\exp(4ic)*\exp(4idx) + 12*d*\exp(2ic)*\exp(2idx) + 3*d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-4/3*(6*I*A*a^4*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 6*B*a^4*e^{(8*I*d*x + 8*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*I*A*a^4*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*B*a^4*e^{(6*I*d*x + 6*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 36*I*A*a^4*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 36*B*a^4*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*I*A*a^4*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 24*B*a^4*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 18*I*A*a^4*e^{(6*I*d*x + 6*I*c)} + 30*B*a^4*e^{(6*I*d*x + 6*I*c)} + 45*I*A*a^4*e^{(4*I*d*x + 4*I*c)} + 63*B*a^4*e^{(4*I*d*x + 4*I*c)} + 38*I*A*a^4*e^{(2*I*d*x + 2*I*c)} + 50*B*a^4*e^{(2*I*d*x + 2*I*c)} + 6*I*A*a^4*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 6*B*a^4*\log(e^{(2*I*d*x + 2*I*c)} + 1) + 11*I*A*a^4 + 14*B*a^4)/(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Mupad [B]**

time = 6.12, size = 181, normalized size = 1.29

$$\frac{B a^4 \tan(c + d x)^4}{4 d} + \frac{\ln(\tan(c + d x) + 1) (8 B a^4 + A a^4 8 i)}{d} - \frac{\tan(c + d x)^2 \left( \frac{a^4 (A - B 11) 2}{2} + \frac{B a^4}{2} + \frac{a^4 (3 B + A 11)}{2} \right)}{d} + \frac{\tan(c + d x) (-3 a^4 (A - B 11) + a^4 (B + A 3 i) 1 i + B a^4 1 i + a^4 (3 B + A 11) 1 i)}{d} - \frac{\tan(c + d x)^2 \left( \frac{B a^4 11}{3} + \frac{a^4 (3 B + A 11) 11}{3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)



```
[Out] (log(tan(c + d*x) + 1i)*(A*a^4*8i + 8*B*a^4))/d - (tan(c + d*x)^3*((B*a^4*1
i)/3 + (a^4*(A*1i + 3*B)*1i)/3))/d - (tan(c + d*x)^2*((a^4*(A - B*1i)*3i)/2
+ (B*a^4)/2 + (a^4*(A*1i + 3*B))/2))/d + (tan(c + d*x)*(a^4*(A*3i + B)*1i
- 3*a^4*(A - B*1i) + B*a^4*1i + a^4*(A*1i + 3*B)*1i))/d + (B*a^4*tan(c + d*
x)^4)/(4*d)
```

### 3.29 $\int \cot(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=142

$$8a^4(iA+B)x + \frac{a^4(7A-8iB)\log(\cos(c+dx))}{d} + \frac{a^4A\log(\sin(c+dx))}{d} + \frac{iaB(a+ia \tan(c+dx))^3}{3d} - \frac{(A-2iB)}{d}$$

[Out]  $8a^4(I*A+B)*x + a^4*(7*A-8*I*B)*\ln(\cos(d*x+c))/d + a^4*A*\ln(\sin(d*x+c))/d + 1/3*I*a*B*(a+I*a*\tan(d*x+c))^3/d - 1/2*(A-2*I*B)*(a^2+I*a^2*\tan(d*x+c))^2/d - (3*A-4*I*B)*(a^4+I*a^4*\tan(d*x+c))/d$

Rubi [A]

time = 0.29, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3675, 3670, 3556, 3612}

$$-\frac{(3A-4iB)(a^4+ia^4 \tan(c+dx))}{d} + \frac{a^4(7A-8iB)\log(\cos(c+dx))}{d} + 8a^4x(B+iA) + \frac{a^4A\log(\sin(c+dx))}{d} - \frac{(A-2iB)(a^2+ia^2 \tan(c+dx))^2}{2d} + \frac{iaB(a+ia \tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c+d*x]*(a+I*a*\text{Tan}[c+d*x])^4*(A+B*\text{Tan}[c+d*x]),x]$

[Out]  $8a^4*(I*A+B)*x + (a^4*(7*A-(8*I)*B)*\text{Log}[\text{Cos}[c+d*x]])/d + (a^4*A*\text{Log}[\text{Sin}[c+d*x]])/d + ((I/3)*a*B*(a+I*a*\text{Tan}[c+d*x])^3)/d - ((A-(2*I)*B)*(a^2+I*a^2*\text{Tan}[c+d*x])^2)/(2*d) - ((3*A-(4*I)*B)*(a^4+I*a^4*\text{Tan}[c+d*x]))/d$

Rule 3556

$\text{Int}[\tan[(c_.)+(d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]], x]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 3612

$\text{Int}[(c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)]/((a_.)+(b_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c+b*d)*(x/(a^2+b^2)), x] + \text{Dist}[(b*c-a*d)/(a^2+b^2), \text{Int}[(b-a*\text{Tan}[e+f*x])/(a+b*\text{Tan}[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[a^2+b^2, 0] \&\& \text{NeQ}[a*c+b*d, 0]$

Rule 3670

$\text{Int}[(A_.)+(B_.)*\tan[(e_.)+(f_.)*(x_.)]*((c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)])/((a_.)+(b_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[B*(d/b), \text{Int}[\text{Tan}[e+f*x], x], x] + \text{Dist}[1/b, \text{Int}[\text{Simp}[A*b*c+(A*b*d+B*(b*c-a*d))*\text{Tan}[e+f*x], x]/(a+b*\text{Tan}[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d,$

e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

### Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \cot(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{iaB(a + ia \tan(c + dx))^3}{3d} + \frac{1}{3} \int \cot(c + dx) \\
 &= \frac{iaB(a + ia \tan(c + dx))^3}{3d} - \frac{(A - 2iB)(a^2 + d^2)}{2d} \\
 &= \frac{iaB(a + ia \tan(c + dx))^3}{3d} - \frac{(A - 2iB)(a^2 + d^2)}{2d} \\
 &= \frac{iaB(a + ia \tan(c + dx))^3}{3d} - \frac{(A - 2iB)(a^2 + d^2)}{2d} \\
 &= 8a^4(iA + B)x + \frac{a^4(7A - 8iB) \log(\cos(c + dx))}{d} \\
 &= 8a^4(iA + B)x + \frac{a^4(7A - 8iB) \log(\cos(c + dx))}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 429 vs.  $2(142) = 284$ .  
time = 5.66, size = 429, normalized size = 3.02

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (a^4\*Sec[c]\*Sec[c + d\*x]^3\*(Cos[4\*d\*x] + I\*Sin[4\*d\*x])\*((48\*I)\*A\*d\*x\*Cos[2\*c + 3\*d\*x] + 48\*B\*d\*x\*Cos[2\*c + 3\*d\*x] + (48\*I)\*A\*d\*x\*Cos[4\*c + 3\*d\*x] + 48\*B\*d\*x\*Cos[4\*c + 3\*d\*x] + 21\*A\*Cos[2\*c + 3\*d\*x]\*Log[Cos[c + d\*x]^2] - (24\*I)\*B\*Cos[2\*c + 3\*d\*x]\*Log[Cos[c + d\*x]^2] + 21\*A\*Cos[4\*c + 3\*d\*x]\*Log[Cos[c

$$+ d*x]^2] - (24*I)*B*\text{Cos}[4*c + 3*d*x]*\text{Log}[\text{Cos}[c + d*x]^2] + 3*A*\text{Cos}[2*c + 3*d*x]*\text{Log}[\text{Sin}[c + d*x]^2] + 3*A*\text{Cos}[4*c + 3*d*x]*\text{Log}[\text{Sin}[c + d*x]^2] + 3*\text{Cos}[d*x]*(4*A - (16*I)*B + (48*I)*A*d*x + 48*B*d*x + 3*(7*A - (8*I)*B))*\text{Log}[\text{Cos}[c + d*x]^2] + 3*A*\text{Log}[\text{Sin}[c + d*x]^2]) + 3*\text{Cos}[2*c + d*x]*(4*A - (16*I)*B + (48*I)*A*d*x + 48*B*d*x + 3*(7*A - (8*I)*B))*\text{Log}[\text{Cos}[c + d*x]^2] + 3*A*\text{Log}[\text{Sin}[c + d*x]^2]) - (96*I)*A*\text{Sin}[d*x] - 168*B*\text{Sin}[d*x] + (48*I)*A*\text{Sin}[2*c + d*x] + 96*B*\text{Sin}[2*c + d*x] - (48*I)*A*\text{Sin}[2*c + 3*d*x] - 88*B*\text{Sin}[2*c + 3*d*x]))/(48*d*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4)$$

**Maple [A]**

time = 0.21, size = 184, normalized size = 1.30

method	result
norman	$(8iAa^4 + 8Ba^4)x - \frac{(4iAa^4 + 7Ba^4)\tan(dx+c)}{d} + \frac{(-4iBa^4 + Aa^4)(\tan^2(dx+c))}{2d} + \frac{Ba^4(\tan^3(dx+c))}{3d} + \dots$
risch	$-\frac{16a^4Bc}{d} - \frac{16ia^4Ac}{d} - \frac{2ia^4(15iAe^{4i(dx+c)} + 36Be^{4i(dx+c)} + 27iAe^{2i(dx+c)} + 54Be^{2i(dx+c)} + 12iA + 22B)}{3d(e^{2i(dx+c)} + 1)^3} - \frac{8ia^4 \ln(\dots)}{3d}$
derivativedivides	$Aa^4 \left( \frac{(\tan^2(dx+c))}{2} + \ln(\cos(dx+c)) \right) + Ba^4 \left( \frac{(\tan^3(dx+c))}{3} - \tan(dx+c) + dx+c \right) - 4iAa^4(\tan(dx+c) - dx - c) - 4iBa^4 \left( \dots \right)$
default	$Aa^4 \left( \frac{(\tan^2(dx+c))}{2} + \ln(\cos(dx+c)) \right) + Ba^4 \left( \frac{(\tan^3(dx+c))}{3} - \tan(dx+c) + dx+c \right) - 4iAa^4(\tan(dx+c) - dx - c) - 4iBa^4 \left( \dots \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+B*a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)-4*I*A*a^4*(tan(d*x+c)-d*x-c)-4*I*B*a^4*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+6*A*a^4*ln(cos(d*x+c))-6*B*a^4*(tan(d*x+c)-d*x-c)+4*I*A*a^4*(d*x+c)-4*I*B*a^4*ln(cos(d*x+c))+A*a^4*ln(sin(d*x+c))+B*a^4*(d*x+c))
```

**Maxima [A]**

time = 0.48, size = 107, normalized size = 0.75

$$\frac{2Ba^4 \tan(dx+c)^3 + 3(A-4iB)a^4 \tan(dx+c)^2 - 48(dx+c)(-iA-B)a^4 - 24(A-iB)a^4 \log(\tan(dx+c)^2 + 1) + 6Aa^4 \log(\tan(dx+c)) - 6(4iA+7B)a^4 \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*B*a^4*tan(d*x + c)^3 + 3*(A - 4*I*B)*a^4*tan(d*x + c)^2 - 48*(d*x + c)*(-I*A - B)*a^4 - 24*(A - I*B)*a^4*log(tan(d*x + c)^2 + 1) + 6*A*a^4*log(tan(d*x + c)) - 6*(4*I*A + 7*B)*a^4*tan(d*x + c))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs.  $2(122) = 244$ .  
time = 0.46, size = 246, normalized size = 1.73

$$\frac{6(5A - 12iB)a^6 e^{6i4d+6c} + 54(A - 2iB)a^6 e^{6i2d+2c} + 4(6A - 11iB)a^4 + 3((7A - 8iB)a^6 e^{6i4d+6c} + 3(7A - 8iB)a^6 e^{6i2d+2c} + (7A - 8iB)a^6) \log(e^{6i2d+2c} + 1) + 3(Aa^6 e^{6i4d+6c} + 3Aa^6 e^{6i2d+2c} + 3Aa^6 e^{6i2d+2c} + Aa^6) \log(e^{6i2d+2c} - 1)}{3(d e^{6i4d+6c} + 3d e^{6i2d+2c} + 3d e^{6i2d+2c} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{3} * (6 * (5 * A - 12 * I * B) * a^4 * e^{(4 * I * d * x + 4 * I * c)} + 54 * (A - 2 * I * B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} + 4 * (6 * A - 11 * I * B) * a^4 + 3 * ((7 * A - 8 * I * B) * a^4 * e^{(6 * I * d * x + 6 * I * c)} + 3 * (7 * A - 8 * I * B) * a^4 * e^{(4 * I * d * x + 4 * I * c)} + 3 * (7 * A - 8 * I * B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} + (7 * A - 8 * I * B) * a^4) * \log(e^{(2 * I * d * x + 2 * I * c)} + 1) + 3 * (A * a^4 * e^{(6 * I * d * x + 6 * I * c)} + 3 * A * a^4 * e^{(4 * I * d * x + 4 * I * c)} + 3 * A * a^4 * e^{(2 * I * d * x + 2 * I * c)} + A * a^4) * \log(e^{(2 * I * d * x + 2 * I * c)} - 1)) / (d * e^{(6 * I * d * x + 6 * I * c)} + 3 * d * e^{(4 * I * d * x + 4 * I * c)} + 3 * d * e^{(2 * I * d * x + 2 * I * c)} + d)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 289 vs.  $2(121) = 242$ .  
time = 2.14, size = 289, normalized size = 2.04

$$\frac{Aa^4 \log\left(\frac{-3Aa^4 + 4iBe^4}{3Aa^4 e^{2ic} - 4iBa^4 e^{2ic}} + e^{2idx}\right)}{d} + \frac{a^4 \cdot (7A - 8iB) \log\left(e^{2idx} + \frac{-4Aa^4 + 4iBa^4 + a^4(7A - 8iB)}{3Aa^4 e^{2ic} - 4iBa^4 e^{2ic}}\right)}{d} + \frac{24Aa^4 - 44iBa^4 + (54Aa^4 e^{2ic} - 108iBa^4 e^{2ic}) e^{2idx} + (30Aa^4 e^{4ic} - 72iBa^4 e^{4ic}) e^{4idx}}{3de^{6ic} e^{6idx} + 9de^{4ic} e^{4idx} + 9de^{2ic} e^{2idx} + 3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out]  $A * a^{**4} * \log((-3 * A * a^{**4} + 4 * I * B * a^{**4}) / (3 * A * a^{**4} * \exp(2 * I * c) - 4 * I * B * a^{**4} * \exp(2 * I * c)) + \exp(2 * I * d * x)) / d + a^{**4} * (7 * A - 8 * I * B) * \log(\exp(2 * I * d * x) + (-4 * A * a^{**4} + 4 * I * B * a^{**4} + a^{**4} * (7 * A - 8 * I * B)) / (3 * A * a^{**4} * \exp(2 * I * c) - 4 * I * B * a^{**4} * \exp(2 * I * c))) / d + (24 * A * a^{**4} - 44 * I * B * a^{**4} + (54 * A * a^{**4} * \exp(2 * I * c) - 108 * I * B * a^{**4} * \exp(2 * I * c)) * \exp(2 * I * d * x) + (30 * A * a^{**4} * \exp(4 * I * c) - 72 * I * B * a^{**4} * \exp(4 * I * c)) * \exp(4 * I * d * x)) / (3 * d * \exp(6 * I * c) * \exp(6 * I * d * x) + 9 * d * \exp(4 * I * c) * \exp(4 * I * d * x) + 9 * d * \exp(2 * I * c) * \exp(2 * I * d * x) + 3 * d)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs.  $2(122) = 244$ .  
time = 1.27, size = 332, normalized size = 2.34

$$\frac{6Aa^4 \log(\tan(\frac{1}{2}dx + \frac{1}{2}c)) + 6(7Aa^4 - 8iBa^4) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) - 96(Aa^4 - iBa^4) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) + 6(7Aa^4 - 8iBa^4) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

```
[Out] 1/6*(6*A*a^4*log(tan(1/2*d*x + 1/2*c)) + 6*(7*A*a^4 - 8*I*B*a^4)*log(tan(1/2*d*x + 1/2*c) + 1) - 96*(A*a^4 - I*B*a^4)*log(tan(1/2*d*x + 1/2*c) + I) + 6*(7*A*a^4 - 8*I*B*a^4)*log(tan(1/2*d*x + 1/2*c) - 1) - (77*A*a^4*tan(1/2*d*x + 1/2*c)^6 - 88*I*B*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*I*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 84*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 243*A*a^4*tan(1/2*d*x + 1/2*c)^4 + 312*I*B*a^4*tan(1/2*d*x + 1/2*c)^4 + 96*I*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 184*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 243*A*a^4*tan(1/2*d*x + 1/2*c)^2 - 312*I*B*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*I*A*a^4*tan(1/2*d*x + 1/2*c) - 84*B*a^4*tan(1/2*d*x + 1/2*c) - 77*A*a^4 + 88*I*B*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

**Mupad [B]**

time = 6.22, size = 133, normalized size = 0.94

$$\frac{A a^4 \ln(\tan(c + d x))}{d} - \frac{\tan(c + d x) (a^4 (A - B i) 3i + B a^4 + a^4 (3B + A i))}{d} - \frac{\tan(c + d x)^2 \left( \frac{B a^4 i}{2} + \frac{a^4 (3B + A i) i}{2} \right)}{d} - \frac{8 a^4 \ln(\tan(c + d x) + i) (A - B i)}{d} + \frac{B a^4 \tan(c + d x)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (A*a^4*log(tan(c + d*x)))/d - (tan(c + d*x)*(a^4*(A - B*1i)*3i + B*a^4 + a^4*(A*1i + 3*B)))/d - (tan(c + d*x)^2*((B*a^4*1i)/2 + (a^4*(A*1i + 3*B)*1i)/2))/d - (8*a^4*log(tan(c + d*x) + 1i)*(A - B*1i))/d + (B*a^4*tan(c + d*x)^3)/(3*d)
```

### 3.30 $\int \cot^2(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=144

$$-8a^4(A-iB)x + \frac{a^4(4iA+7B)\log(\cos(c+dx))}{d} + \frac{a^4(4iA+B)\log(\sin(c+dx))}{d} - \frac{aA \cot(c+dx)(a+ia \tan(c+dx))^3}{d}$$

[Out]  $-8a^4(A-iB)x + a^4(4iA+7B)\ln(\cos(dx+c))/d + a^4(4iA+B)\ln(\sin(dx+c))/d - aA \cot(dx+c)(a+i a \tan(dx+c))^3/d + 1/2(2iA-B)(a^2+i a^2 \tan(dx+c))^2/d - 3B(a^4+i a^4 \tan(dx+c))/d$

Rubi [A]

time = 0.29, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3675, 3670, 3556, 3612}

$$\frac{a^4(B+4iA)\log(\sin(c+dx))}{d} + \frac{a^4(7B+4iA)\log(\cos(c+dx))}{d} - 8a^4x(A-iB) - \frac{3B(a^4+ia^4 \tan(c+dx))}{d} + \frac{(-B+2iA)(a^2+ia^2 \tan(c+dx))^2}{2d} - \frac{aA \cot(c+dx)(a+ia \tan(c+dx))^3}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-8a^4(A-iB)x + (a^4((4i)A+7B)\text{Log}[\text{Cos}[c+d*x]])/d + (a^4((4i)A+B)\text{Log}[\text{Sin}[c+d*x]])/d - (aA \cot[c+d*x](a+i a \tan[c+d*x])^3)/d + (((2i)A-B)(a^2+i a^2 \tan[c+d*x])^2)/(2d) - (3B(a^4+i a^4 \tan[c+d*x]))/d$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3670

Int((((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d,

$e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

#### Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
 \int \cot^2(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^3}{d} + \int \cot^2(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
 &= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^3}{d} + \frac{(2iA - B)(a + ia \tan(c + dx))^4}{d} \\
 &= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^3}{d} + \frac{(2iA - B)(a + ia \tan(c + dx))^4}{d} \\
 &= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^3}{d} + \frac{(2iA - B)(a + ia \tan(c + dx))^4}{d} \\
 &= -8a^4(A - iB)x + \frac{a^4(4iA + 7B) \log(\cos(c + dx))}{d} \\
 &= -8a^4(A - iB)x + \frac{a^4(4iA + 7B) \log(\cos(c + dx))}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1122 vs.  $2(144) = 288$ .



time = 7.19, size = 1122, normalized size = 7.79

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] a^4*((A*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Csc[c]*(Cos[4*c] - I*Sin[4*c])*Sin[d*x]*Sin[c + d*x]^4)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*((4*I)*A*Cos[2*c] + B*Cos[2*c] + 4*A*Sin[2*c] - I*B*Sin[2*c])*((-I)*ArcTan[Tan[5*c + d*x]]*Cos[2*c] - ArcTan[Tan[5*c + d*x]]*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*((4*I)*A*Cos[2*c] + 7*B*Cos[2*c] + 4*A*Sin[2*c] - (7*I)*B*Sin[2*c])*((Cos[2*c]*Log[Cos[c + d*x]^2])/2 - (I/2)*Log[Cos[c + d*x]^2]*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*((4*I)*A*Cos[2*c] + B*Cos[2*c] + 4*A*Sin[2*c] - I*B*Sin[2*c])*((Cos[2*c]*Log[Sin[c + d*x]^2])/2 - (I/2)*Log[Sin[c + d*x]^2]*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((A - I*B)*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*(-8*d*x*Cos[4*c] + (8*I)*d*x*Sin[4*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (x*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Sin[c + d*x]^5*(2*A*Cos[c]^2 - ((7*I)/2)*B*Cos[c]^2 - 22*A*Cos[c]^4 + ((17*I)/2)*B*Cos[c]^4 - (4*I)*A*Cos[c]^4*Cot[c] - B*Cos[c]^4*Cot[c] - (6*I)*A*Cos[c]*Sin[c] - (21*B*Cos[c]*Sin[c])/2 + (50*I)*A*Cos[c]^3*Sin[c] + (55*B*Cos[c]^3*Sin[c])/2 - 6*A*Sin[c]^2 + ((21*I)/2)*B*Sin[c]^2 + 60*A*Cos[c]^2*Sin[c]^2 - (45*I)*B*Cos[c]^2*Sin[c]^2 - (40*I)*A*Cos[c]*Sin[c]^3 - 40*B*Cos[c]*Sin[c]^3 - 14*A*Sin[c]^4 + ((37*I)/2)*B*Sin[c]^4 + (-3*B + (4*I)*A*Cos[2*c] + 4*B*Cos[2*c])*Csc[c]*Sec[c]*(Cos[4*c] - I*Sin[4*c]) + (2*I)*A*Sin[c]^2*Tan[c] + (7*B*Sin[c]^2*Tan[c])/2 + (2*I)*A*Sin[c]^4*Tan[c] + (7*B*Sin[c]^4*Tan[c])/2))/((Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Sec[c]*(Cos[4*c] - I*Sin[4*c])*(A*Sin[d*x] - (4*I)*B*Sin[d*x])*Sin[c + d*x]^4*Tan[c + d*x])/((Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*((B*Cos[4*c])/2 - (I/2)*B*Sin[4*c])*Sin[c + d*x]^3*Tan[c + d*x]^2)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.20, size = 166, normalized size = 1.15

method	result
norman	$\frac{(8iB a^4 - 8A a^4)x \tan(dx+c) + \frac{(-4iB a^4 + A a^4)(\tan^2(dx+c))}{d} - \frac{A a^4}{d} + \frac{B a^4(\tan^3(dx+c))}{2d}}{\tan(dx+c)} + \frac{(4iA a^4 + B a^4) \ln(\tan(dx+c))}{d}$

derivativdivides	$\frac{A a^4 (\tan(dx+c) - dx - c) + B a^4 \left( \frac{\tan^2(dx+c)}{2} + \ln(\cos(dx+c)) \right) + 4i A a^4 \ln(\cos(dx+c)) - 4i B a^4 (\tan(dx+c) - dx - c) - 6A a^4}{d}$
default	$\frac{A a^4 (\tan(dx+c) - dx - c) + B a^4 \left( \frac{\tan^2(dx+c)}{2} + \ln(\cos(dx+c)) \right) + 4i A a^4 \ln(\cos(dx+c)) - 4i B a^4 (\tan(dx+c) - dx - c) - 6A a^4}{d}$
risch	$-\frac{16i a^4 B c}{d} + \frac{16a^4 A c}{d} + \frac{2a^4 (5B e^{4i(dx+c)} - 2iA e^{2i(dx+c)} - B e^{2i(dx+c)} - 2iA - 4B)}{d(e^{2i(dx+c)} + 1)^2 (e^{2i(dx+c)} - 1)} + \frac{a^4 \ln(e^{2i(dx+c)} - 1) B}{d} + \frac{4i a^4}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERB  
OSE)`

[Out]  $\frac{1}{d} * (A * a^4 * (\tan(dx+c) - dx - c) + B * a^4 * (\frac{1}{2} * \tan(dx+c)^2 + \ln(\cos(dx+c)))) + 4 * I * A * a^4 * \ln(\cos(dx+c)) - 4 * I * B * a^4 * (\tan(dx+c) - dx - c) - 6 * A * a^4 * (dx+c) + 6 * B * a^4 * \ln(\cos(dx+c)) + 4 * I * A * a^4 * \ln(\sin(dx+c)) + 4 * I * B * a^4 * (dx+c) + A * a^4 * (-\cot(dx+c) - dx - c) + B * a^4 * \ln(\sin(dx+c))$

**Maxima** [A]

time = 0.51, size = 102, normalized size = 0.71

$$\frac{B a^4 \tan(dx+c)^2 - 16(dx+c)(A - iB)a^4 - 8(iA + B)a^4 \log(\tan(dx+c)^2 + 1) + 2(4iA + B)a^4 \log(\tan(dx+c)) + 2(A - 4iB)a^4 \tan(dx+c) - \frac{2Aa^4}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (B * a^4 * \tan(dx+c)^2 - 16 * (dx+c) * (A - I * B) * a^4 - 8 * (I * A + B) * a^4 * \log(\tan(dx+c)^2 + 1) + 2 * (4 * I * A + B) * a^4 * \log(\tan(dx+c)) + 2 * (A - 4 * I * B) * a^4 * \tan(dx+c) - 2 * A * a^4 / \tan(dx+c)) / d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(126) = 252.

time = 0.53, size = 254, normalized size = 1.76

$$\frac{10 B a^4 e^{(4i dx + 4i c)} - 2(2i A + B) a^4 e^{(2i dx + 2i c)} - 4(i A + 2B) a^4 + ((4i A + 7B) a^4 e^{(6i dx + 6i c)} + (4i A + 7B) a^4 e^{(4i dx + 4i c)} + (-4i A - 7B) a^4 e^{(2i dx + 2i c)} + (-4i A - 7B) a^4) \log(e^{(2i dx + 2i c)} + 1) + ((4i A + B) a^4 e^{(6i dx + 6i c)} + (4i A + B) a^4 e^{(4i dx + 4i c)} + (-4i A - B) a^4 e^{(2i dx + 2i c)} + (-4i A - B) a^4) \log(e^{(2i dx + 2i c)} - 1)}{d e^{(6i dx + 6i c)} + d e^{(4i dx + 4i c)} - d e^{(2i dx + 2i c)} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $(10 * B * a^4 * e^{(4 * I * d * x + 4 * I * c)} - 2 * (2 * I * A + B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} - 4 * (I * A + 2 * B) * a^4 + ((4 * I * A + 7 * B) * a^4 * e^{(6 * I * d * x + 6 * I * c)} + (4 * I * A + 7 * B) * a^4 * e^{(4 * I * d * x + 4 * I * c)} + (-4 * I * A - 7 * B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} + (-4 * I * A - 7 * B) * a^4) * \log(e^{(2 * I * d * x + 2 * I * c)} + 1) + ((4 * I * A + B) * a^4 * e^{(6 * I * d * x + 6 * I * c)} + (4 * I * A + B) * a^4 * e^{(4 * I * d * x + 4 * I * c)} + (-4 * I * A - B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} + (-4 * I * A - B) * a^4) * \log(e^{(2 * I * d * x + 2 * I * c)} - 1)$



[In]  $\text{int}(\cot(c + d*x)^2*(A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*1i)^4,x)$

[Out]  $(a^4*\log(\tan(c + d*x))*(A*4i + B))/d - (\tan(c + d*x)*(B*a^4*1i + a^4*(A*1i + 3*B)*1i))/d - (8*a^4*\log(\tan(c + d*x) + 1i)*(A*1i + B))/d - (A*a^4*\cot(c + d*x))/d + (B*a^4*\tan(c + d*x)^2)/(2*d)$

### 3.31 $\int \cot^3(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=156

$$-8a^4(iA+B)x - \frac{a^4(A-4iB)\log(\cos(c+dx))}{d} - \frac{a^4(7A-4iB)\log(\sin(c+dx))}{d} - \frac{aA \cot^2(c+dx)(a+ia \tan(c+dx))}{2d}$$

[Out]  $-8*a^4*(I*A+B)*x - a^4*(A-4*I*B)*\ln(\cos(d*x+c))/d - a^4*(7*A-4*I*B)*\ln(\sin(d*x+c))/d - 1/2*a*A*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^3/d - 1/2*(5*I*A+2*B)*\cot(d*x+c)*(a^2+I*a^2*\tan(d*x+c))^2/d - 3*A*(a^4+I*a^4*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3675, 3670, 3556, 3612}

$$-\frac{a^4(7A-4iB)\log(\sin(c+dx))}{d} - \frac{a^4(A-4iB)\log(\cos(c+dx))}{d} - 8a^4x(B+iA) - \frac{3A(a^4+ia^4\tan(c+dx))}{d} - \frac{(2B+5iA)\cot(c+dx)(a^2+ia^2\tan(c+dx))^2}{2d} - \frac{aA \cot^2(c+dx)(a+ia \tan(c+dx))^3}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-8*a^4*(I*A + B)*x - (a^4*(A - (4*I)*B)*\text{Log}[\text{Cos}[c + d*x]])/d - (a^4*(7*A - (4*I)*B)*\text{Log}[\text{Sin}[c + d*x]])/d - (a*A*\text{Cot}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^3)/(2*d) - (((5*I)*A + 2*B)*\text{Cot}[c + d*x]*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/(2*d) - (3*A*(a^4 + I*a^4*\text{Tan}[c + d*x]))/d$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

**Rule 3670**

Int((((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d,

`e, f, A, B}, x] && NeQ[b*c - a*d, 0]`

#### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

#### Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
 \int \cot^3(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^3}{2d} + \frac{1}{2} \int \\
 &= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^3}{2d} - \frac{(5i)}{2d} \\
 &= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^3}{2d} - \frac{(5i)}{2d} \\
 &= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^3}{2d} - \frac{(5i)}{2d} \\
 &= -8a^4(iA + B)x - \frac{a^4(A - 4iB) \log(\cos(c + dx))}{d} \\
 &= -8a^4(iA + B)x - \frac{a^4(A - 4iB) \log(\cos(c + dx))}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1116 vs.  $2(156) = 312$ .

time = 7.26, size = 1116, normalized size = 7.15

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] a^4*(((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*(-1/2*(A*Cos[4*c]) + (I/2)*
A*Sin[4*c])*Sin[c + d*x]^3)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] +
B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Csc[c]*(Cos[4
*c] - I*Sin[4*c])*((4*I)*A*Sin[d*x] + B*Sin[d*x])*Sin[c + d*x]^4)/(d*(Cos[d
*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x]
)^4*(B + A*Cot[c + d*x])*(7*A*Cos[2*c] - (4*I)*B*Cos[2*c] - (7*I)*A*Sin[2*c
] - 4*B*Sin[2*c])*(I*ArcTan[Tan[5*c + d*x]]*Cos[2*c] + ArcTan[Tan[5*c + d*x
]])*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] +
B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*(A*Cos[2*c]
- (4*I)*B*Cos[2*c] - I*A*Sin[2*c] - 4*B*Sin[2*c])*(-1/2*(Cos[2*c]*Log[Cos[c
+ d*x]^2]) + (I/2)*Log[Cos[c + d*x]^2]*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d
*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x]
)^4*(B + A*Cot[c + d*x])*(7*A*Cos[2*c] - (4*I)*B*Cos[2*c] - (7*I)*A*Sin[2*c
] - 4*B*Sin[2*c])*(-1/2*(Cos[2*c]*Log[Sin[c + d*x]^2]) + (I/2)*Log[Sin[c +
d*x]^2]*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d
*x] + B*Sin[c + d*x])) + ((A - I*B)*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x
])*((-8*I)*d*x*Cos[4*c] - 8*d*x*Sin[4*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*
Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (x*(I + Cot[c + d*x])^4*(B
+ A*Cot[c + d*x])*Sin[c + d*x]^5*((I/2)*A*Cos[c]^2 + 2*B*Cos[c]^2 - ((71*I
)/2)*A*Cos[c]^4 - 22*B*Cos[c]^4 + 7*A*Cos[c]^4*Cot[c] - (4*I)*B*Cos[c]^4*Co
t[c] + (3*A*Cos[c]*Sin[c])/2 - (6*I)*B*Cos[c]*Sin[c] - (145*A*Cos[c]^3*Sin[
c])/2 + (50*I)*B*Cos[c]^3*Sin[c] - ((3*I)/2)*A*Sin[c]^2 - 6*B*Sin[c]^2 + (7
5*I)*A*Cos[c]^2*Sin[c]^2 + 60*B*Cos[c]^2*Sin[c]^2 + 40*A*Cos[c]*Sin[c]^3 -
(40*I)*B*Cos[c]*Sin[c]^3 - ((19*I)/2)*A*Sin[c]^4 - 14*B*Sin[c]^4 + (3*A + 4
*A*Cos[2*c] - (4*I)*B*Cos[2*c])*Csc[c]*Sec[c]*(-Cos[4*c] + I*Sin[4*c]) - (A
*Sin[c]^2*Tan[c])/2 + (2*I)*B*Sin[c]^2*Tan[c] - (A*Sin[c]^4*Tan[c])/2 + (2*
I)*B*Sin[c]^4*Tan[c]))/((Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c
+ d*x])) + (B*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Sec[c]*(Cos[4*c] -
I*Sin[4*c])*Sin[d*x]*Sin[c + d*x]^4*Tan[c + d*x])/(d*(Cos[d*x] + I*Sin[d*x
])^4*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.21, size = 171, normalized size = 1.10

method	result
norman	$\frac{(-8iAa^4 - 8Ba^4)x(\tan^2(dx+c)) + \frac{Ba^4(\tan^3(dx+c))}{d} - \frac{Aa^4}{2d} - \frac{(4iAa^4 + Ba^4)\tan(dx+c)}{d}}{\tan(dx+c)^2} - \frac{(-4iBa^4 + 7Aa^4)\ln(\tan(dx+c))}{d}$

derivativdivides	$-A a^4 \ln(\cos(dx+c)) + B a^4 (\tan(dx+c) - dx - c) - 4iA a^4 (dx+c) + 4iB a^4 \ln(\cos(dx+c)) - 6A a^4 \ln(\sin(dx+c)) - 6B a^4 (dx+c)$
default	$-A a^4 \ln(\cos(dx+c)) + B a^4 (\tan(dx+c) - dx - c) - 4iA a^4 (dx+c) + 4iB a^4 \ln(\cos(dx+c)) - 6A a^4 \ln(\sin(dx+c)) - 6B a^4 (dx+c)$
risch	$\frac{16a^4 Bc}{d} + \frac{16ia^4 Ac}{d} - \frac{2ia^4 (5iA e^{4i(dx+c)} + iA e^{2i(dx+c)} + 2B e^{2i(dx+c)} - 4iA - 2B)}{d(e^{2i(dx+c)} + 1)(e^{2i(dx+c)} - 1)^2} + \frac{4ia^4 \ln(e^{2i(dx+c)} - 1)B}{d} - \frac{7A a^4}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} * (-A a^4 \ln(\cos(dx+c)) + B a^4 (\tan(dx+c) - dx - c) - 4iA a^4 (dx+c) + 4iB a^4 \ln(\cos(dx+c)) - 6A a^4 \ln(\sin(dx+c)) - 6B a^4 (dx+c) + 4iA a^4 (-\cot(dx+c) - dx - c) + 4iB a^4 \ln(\sin(dx+c)) + A a^4 (-1/2 \cot(dx+c)^2 - \ln(\sin(dx+c))) + B a^4 (-\cot(dx+c) - dx - c))$$

**Maxima** [A]

time = 0.51, size = 108, normalized size = 0.69

$$\frac{16(dx+c)(iA+B)a^4 - 8(A-iB)a^4 \log(\tan(dx+c)^2+1) + 2(7A-4iB)a^4 \log(\tan(dx+c)) - 2Ba^4 \tan(dx+c) - \frac{2(-4iA-B)a^4 \tan(dx+c) - Aa^4}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out] 
$$-1/2 * (16 * (d * x + c) * (I * A + B) * a^4 - 8 * (A - I * B) * a^4 * \log(\tan(d * x + c)^2 + 1) + 2 * (7 * A - 4 * I * B) * a^4 * \log(\tan(d * x + c)) - 2 * B * a^4 * \tan(d * x + c) - (2 * (-4 * I * A - B) * a^4 * \tan(d * x + c) - A * a^4) / \tan(d * x + c)^2) / d$$

**Fricas** [A]

time = 0.54, size = 255, normalized size = 1.63

$$\frac{10Aa^4e^{(4+4i)c} + 2(A-2iB)a^4e^{(2+2i)c} - 4(2A-iB)a^4 - ((A-4iB)a^4e^{(6+6i)c} - (A-4iB)a^4e^{(4+4i)c} - (A-4iB)a^4e^{(2+2i)c} + (A-4iB)a^4) \log(e^{(2+2i)c} + 1) - ((7A-4iB)a^4e^{(6+6i)c} - (7A-4iB)a^4e^{(4+4i)c} - (7A-4iB)a^4e^{(2+2i)c} + (7A-4iB)a^4) \log(e^{(2+2i)c} - 1)}{d e^{(4+4i)c} - d e^{(2+2i)c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out] 
$$(10 * A * a^4 * e^{(4 * I * d * x + 4 * I * c)} + 2 * (A - 2 * I * B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} - 4 * (2 * A - I * B) * a^4 - ((A - 4 * I * B) * a^4 * e^{(6 * I * d * x + 6 * I * c)} - (A - 4 * I * B) * a^4 * e^{(4 * I * d * x + 4 * I * c)} - (A - 4 * I * B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} + (A - 4 * I * B) * a^4) * \log(e^{(2 * I * d * x + 2 * I * c)} + 1) - ((7 * A - 4 * I * B) * a^4 * e^{(6 * I * d * x + 6 * I * c)} - (7 * A - 4 * I * B) * a^4 * e^{(4 * I * d * x + 4 * I * c)} - (7 * A - 4 * I * B) * a^4 * e^{(2 * I * d * x + 2 * I * c)} + (7 * A - 4 * I * B) * a^4) * \log(e^{(2 * I * d * x + 2 * I * c)} - 1)) / d$$



$(7*A - 4*I*B)*a^4*\log(e^{(2*I*d*x + 2*I*c)} - 1)/(d*e^{(6*I*d*x + 6*I*c)} - d*e^{(4*I*d*x + 4*I*c)} - d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [A]**

time = 1.35, size = 252, normalized size = 1.62

$$\frac{a^4(A - 4iB)\log\left(e^{2idx} + \frac{(4Aa^4 - 4iBa^4 - a^4(A - 4iB))e^{-2ic}}{3Aa^4}\right)}{d} - \frac{a^4 \cdot (7A - 4iB)\log\left(e^{2idx} + \frac{(4Aa^4 - 4iBa^4 - a^4(7A - 4iB))e^{-2ic}}{3Aa^4}\right)}{d} + \frac{10Aa^4e^{4ic}e^{4idx} - 8Aa^4 + 4iBa^4 + (2Aa^4e^{2ic} - 4iBa^4e^{2ic})e^{2idx}}{de^{6ic}e^{6idx} - de^{4ic}e^{4idx} - de^{2ic}e^{2idx} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out]  $-a**4*(A - 4*I*B)*\log(\exp(2*I*d*x) + (4*A*a**4 - 4*I*B*a**4 - a**4*(A - 4*I*B))*\exp(-2*I*c)/(3*A*a**4))/d - a**4*(7*A - 4*I*B)*\log(\exp(2*I*d*x) + (4*A*a**4 - 4*I*B*a**4 - a**4*(7*A - 4*I*B))*\exp(-2*I*c)/(3*A*a**4))/d + (10*A*a**4*\exp(4*I*c)*\exp(4*I*d*x) - 8*A*a**4 + 4*I*B*a**4 + (2*A*a**4*\exp(2*I*c) - 4*I*B*a**4*\exp(2*I*c))*\exp(2*I*d*x))/(d*\exp(6*I*c)*\exp(6*I*d*x) - d*\exp(4*I*c)*\exp(4*I*d*x) - d*\exp(2*I*c)*\exp(2*I*d*x) + d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs.  $2(138) = 276$ .

time = 0.98, size = 317, normalized size = 2.03

$$\frac{A^4 \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 16A^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 4B^4 \tan^3\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8(A^4 - 4iB^4) \log(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1) - 128(A^4 - iB^4) \log(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + i) + 8(A^4 - 4iB^4) \log(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1) + 8(7A^4 - 4iB^4) \log(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)) - 8(A^4 \tan^4\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 4iB^4 \tan^4\left(\frac{1}{2}dx + \frac{1}{2}c\right) + (A^4 + 4iB^4) \tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1) - (84A^4 \tan^4\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 48iB^4 \tan^4\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 16iA^4 \tan^4\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4B^4 \tan^4\left(\frac{1}{2}dx + \frac{1}{2}c\right) - A^4)/\tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/8*(A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 16*I*A*a^4*\tan(1/2*d*x + 1/2*c) - 4*B*a^4*\tan(1/2*d*x + 1/2*c) + 8*(A*a^4 - 4*I*B*a^4)*\log(\tan(1/2*d*x + 1/2*c) + 1) - 128*(A*a^4 - I*B*a^4)*\log(\tan(1/2*d*x + 1/2*c) + I) + 8*(A*a^4 - 4*I*B*a^4)*\log(\tan(1/2*d*x + 1/2*c) - 1) + 8*(7*A*a^4 - 4*I*B*a^4)*\log(\tan(1/2*d*x + 1/2*c)) - 8*(A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 4*I*B*a^4*\tan(1/2*d*x + 1/2*c)^2 - 2*B*a^4*\tan(1/2*d*x + 1/2*c) - A*a^4 + 4*I*B*a^4)/(\tan(1/2*d*x + 1/2*c)^2 - 1) - (84*A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 48*I*B*a^4*\tan(1/2*d*x + 1/2*c)^2 - 16*I*A*a^4*\tan(1/2*d*x + 1/2*c) - 4*B*a^4*\tan(1/2*d*x + 1/2*c) - A*a^4)/\tan(1/2*d*x + 1/2*c)^2)/d$

**Mupad [B]**

time = 6.79, size = 102, normalized size = 0.65

$$\frac{B a^4 \tan(c + dx)}{d} - \frac{a^4 \ln(\tan(c + dx)) (7A - 4Bi)}{d} + \frac{8 a^4 \ln(\tan(c + dx) + 1i) (A - B 1i)}{d} - \frac{\cot(c + dx)^2 \left(\frac{A a^4}{2} + \tan(c + dx) (B a^4 + A a^4 4i)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out]  $(8*a^4*\log(\tan(c + d*x) + 1i)*(A - B*1i))/d - (a^4*\log(\tan(c + d*x))*(7*A - B*4i))/d - (\cot(c + d*x)^2*((A*a^4)/2 + \tan(c + d*x)*(A*a^4*4i + B*a^4)))/d + (B*a^4*\tan(c + d*x))/d$

### 3.32 $\int \cot^4(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=163

$$8a^4(A-iB)x - \frac{a^4 B \log(\cos(c+dx))}{d} - \frac{a^4(8iA+7B) \log(\sin(c+dx))}{d} - \frac{aA \cot^3(c+dx)(a+ia \tan(c+dx))^3}{3d}$$

[Out]  $8a^4(A-iB)x - a^4 B \ln(\cos(dx+c))/d - a^4(8iA+7B) \ln(\sin(dx+c))/d - 1/3 aA \cot(dx+c)^3 (a+i a \tan(dx+c))^3 / d - 1/2 (2iA+B) \cot(dx+c)^2 (a^2+i a^2 \tan(dx+c))^2 / d + (4A-3iB) \cot(dx+c) (a^4+i a^4 \tan(dx+c)) / d$

Rubi [A]

time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3674, 3670, 3556, 3612}

$$-\frac{a^4(7B+8iA) \log(\sin(c+dx))}{d} + \frac{(4A-3iB) \cot(c+dx)(a^4+ia^4 \tan(c+dx))}{d} + 8a^4x(A-iB) - \frac{a^4 B \log(\cos(c+dx))}{d} - \frac{(B+2iA) \cot^2(c+dx)(a^2+ia^2 \tan(c+dx))^2}{2d} - \frac{aA \cot^3(c+dx)(a+ia \tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c+d*x]^4*(a+I*a*\text{Tan}[c+d*x])^4*(A+B*\text{Tan}[c+d*x]),x]$

[Out]  $8a^4(A-iB)x - (a^4 B \text{Log}[\text{Cos}[c+d*x]])/d - (a^4((8i)A+7B) \text{Log}[\text{Sin}[c+d*x]])/d - (aA \text{Cot}[c+d*x]^3 (a+I a \text{Tan}[c+d*x])^3)/(3d) - ((2i)A+B) \text{Cot}[c+d*x]^2 (a^2+I a^2 \text{Tan}[c+d*x])^2/(2d) + ((4A-(3i)B) \text{Cot}[c+d*x] (a^4+I a^4 \text{Tan}[c+d*x]))/d$

Rule 3556

$\text{Int}[\tan[(c_.)+(d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 3612

$\text{Int}[(c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)]/((a_.)+(b_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c+b*d)*(x/(a^2+b^2)), x] + \text{Dist}[(b*c-a*d)/(a^2+b^2), \text{Int}[(b-a*\tan[e+f*x])/(a+b*\tan[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[a^2+b^2, 0] \&\& \text{NeQ}[a*c+b*d, 0]$

Rule 3670

$\text{Int}[(A_.)+(B_.)*\tan[(e_.)+(f_.)*(x_.)]*((c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[B*(d/b), \text{Int}[\text{Tan}[e+f*x], x], x] + \text{Dist}[1/b, \text{Int}[\text{Simp}[A*b*c+(A*b*d+B*(b*c-a*d))*\text{Tan}[e+f*x], x]/(a+b*\tan[e+f*x]), x], x] /; \text{FreeQ}\{a, b, c, d,$

e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned} \int \cot^4(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^3}{3d} + \frac{1}{3} \\ &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^3}{3d} - \frac{1}{3} \\ &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^3}{3d} - \frac{1}{3} \\ &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^3}{3d} - \frac{1}{3} \\ &= 8a^4(A - iB)x - \frac{a^4B \log(\cos(c + dx))}{d} - \frac{aA}{3} \\ &= 8a^4(A - iB)x - \frac{a^4B \log(\cos(c + dx))}{d} - \frac{aA}{3} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1138 vs.  $2(163) = 326$ .  
time = 7.11, size = 1138, normalized size = 6.98

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] a^4*((A*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Csc[c]*(Cos[4*c]/3 - (I/3)
)*Sin[4*c])*Sin[d*x]*Sin[c + d*x]^2)/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c
+ d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*Csc[
c]*(-2*A*Cos[c] - (12*I)*A*Sin[c] - 3*B*Sin[c])*(Cos[4*c]/6 - (I/6)*Sin[4*c
```

$$\begin{aligned} & ]*\sin[c + d*x]^3)/(d*(\cos[d*x] + I*\sin[d*x])^4*(A*\cos[c + d*x] + B*\sin[c + \\ & d*x])) + ((I + \cot[c + d*x])^4*(B + A*\cot[c + d*x])*Csc[c]*((-2*\cos[4*c])/ \\ & 3 + ((2*I)/3)*\sin[4*c])*(11*A*\sin[d*x] - (6*I)*B*\sin[d*x])*Sin[c + d*x]^4)/ \\ & (d*(\cos[d*x] + I*\sin[d*x])^4*(A*\cos[c + d*x] + B*\sin[c + d*x])) - (B*\cos[4* \\ & c]*(I + \cot[c + d*x])^4*(B + A*\cot[c + d*x])*Log[\cos[c + d*x]^2]*\sin[c + d* \\ & x]^5)/(2*d*(\cos[d*x] + I*\sin[d*x])^4*(A*\cos[c + d*x] + B*\sin[c + d*x])) + ( \\ & (I + \cot[c + d*x])^4*(B + A*\cot[c + d*x])*(8*A*\cos[2*c] - (7*I)*B*\cos[2*c] \\ & - (8*I)*A*\sin[2*c] - 7*B*\sin[2*c])*(-(\text{ArcTan}[\tan[5*c + d*x]]*\cos[2*c]) + I* \\ & \text{ArcTan}[\tan[5*c + d*x]]*\sin[2*c])*Sin[c + d*x]^5)/(d*(\cos[d*x] + I*\sin[d*x]) \\ & ^4*(A*\cos[c + d*x] + B*\sin[c + d*x])) + ((I + \cot[c + d*x])^4*(B + A*\cot[c \\ & + d*x])*(8*A*\cos[2*c] - (7*I)*B*\cos[2*c] - (8*I)*A*\sin[2*c] - 7*B*\sin[2*c]) \\ & *((-1/2*I)*\cos[2*c]*Log[\sin[c + d*x]^2] - (Log[\sin[c + d*x]^2]*\sin[2*c])/2) \\ & *Sin[c + d*x]^5)/(d*(\cos[d*x] + I*\sin[d*x])^4*(A*\cos[c + d*x] + B*\sin[c + d* \\ & x])) + ((I/2)*B*(I + \cot[c + d*x])^4*(B + A*\cot[c + d*x])*Log[\cos[c + d*x] \\ & ^2]*\sin[4*c]*\sin[c + d*x]^5)/(d*(\cos[d*x] + I*\sin[d*x])^4*(A*\cos[c + d*x] + \\ & B*\sin[c + d*x])) + ((A - I*B)*(I + \cot[c + d*x])^4*(B + A*\cot[c + d*x])*(8 \\ & *d*x*\cos[4*c] - (8*I)*d*x*\sin[4*c])*Sin[c + d*x]^5)/(d*(\cos[d*x] + I*\sin[d* \\ & x])^4*(A*\cos[c + d*x] + B*\sin[c + d*x])) + (x*(I + \cot[c + d*x])^4*(B + A*C \\ & ot[c + d*x])*Sin[c + d*x]^5*((I/2)*B*\cos[c]^2 + 40*A*\cos[c]^4 - ((71*I)/2)* \\ & B*\cos[c]^4 + (8*I)*A*\cos[c]^4*\cot[c] + 7*B*\cos[c]^4*\cot[c] + (3*B*\cos[c]*\sin \\ & [c])/2 - (80*I)*A*\cos[c]^3*\sin[c] - (145*B*\cos[c]^3*\sin[c])/2 - ((3*I)/2)* \\ & B*\sin[c]^2 - 80*A*\cos[c]^2*\sin[c]^2 + (75*I)*B*\cos[c]^2*\sin[c]^2 + (40*I)*A \\ & *Cos[c]*Sin[c]^3 + 40*B*\cos[c]*Sin[c]^3 + 8*A*\sin[c]^4 - ((19*I)/2)*B*\sin[c \\ & ]^4 - I*(4*A - (3*I)*B + 4*A*\cos[2*c] - (4*I)*B*\cos[2*c])*Csc[c]*Sec[c]*(Co \\ & s[4*c] - I*\sin[4*c]) - (B*\sin[c]^2*\tan[c])/2 - (B*\sin[c]^4*\tan[c])/2))/((Co \\ & s[d*x] + I*\sin[d*x])^4*(A*\cos[c + d*x] + B*\sin[c + d*x])) \end{aligned}$$

Maple [A]

time = 0.21, size = 191, normalized size = 1.17

method	result
norman	$\frac{(-4iB a^4 + 7A a^4) \frac{(\tan^2(dx+c))}{d} + (-8iB a^4 + 8A a^4) x \tan^3(dx+c) - \frac{A a^4}{3d} - \frac{(4iA a^4 + B a^4) \tan(dx+c)}{2d}}{\tan(dx+c)^3} + \frac{4(iA a^4 + B a^4) \ln \frac{d}{d}}{d}$
risch	$\frac{16ia^4Bc}{d} - \frac{16a^4Ac}{d} + \frac{2a^4(36iA e^{4i(dx+c)} + 15B e^{4i(dx+c)} - 54iA e^{2i(dx+c)} - 27B e^{2i(dx+c)} + 22iA + 12B)}{3d(e^{2i(dx+c)} - 1)^3} - \frac{7a^4 \ln(e^{2i(dx+c)})}{d}$
derivativedivides	$A a^4(dx+c) - B a^4 \ln(\cos(dx+c)) - 4iA a^4 \ln(\sin(dx+c)) - 4iB a^4(dx+c) - 6A a^4(-\cot(dx+c) - dx - c) - 6B a^4 \ln(\sin(dx+c))$
default	$A a^4(dx+c) - B a^4 \ln(\cos(dx+c)) - 4iA a^4 \ln(\sin(dx+c)) - 4iB a^4(dx+c) - 6A a^4(-\cot(dx+c) - dx - c) - 6B a^4 \ln(\sin(dx+c))$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $1/d*(A*a^4*(d*x+c)-B*a^4*\ln(\cos(d*x+c))-4*I*A*a^4*\ln(\sin(d*x+c))-4*I*B*a^4*(d*x+c)-6*A*a^4*(-\cot(d*x+c)-d*x-c)-6*B*a^4*\ln(\sin(d*x+c))+4*I*A*a^4*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+4*I*B*a^4*(-\cot(d*x+c)-d*x-c)+A*a^4*(-1/3*\cot(d*x+c)^3+\cot(d*x+c)+d*x+c)+B*a^4*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c))))$

**Maxima** [A]

time = 0.62, size = 117, normalized size = 0.72

$$\frac{48(dx+c)(A-iB)a^4-24(-iA-B)a^4\log(\tan(dx+c)^2+1)+6(-8iA-7B)a^4\log(\tan(dx+c))+\frac{6(7A-4iB)a^4\tan(dx+c)^2+3(-4iA-B)a^4\tan(dx+c)-2Aa^4}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/6*(48*(d*x+c)*(A-I*B)*a^4-24*(-I*A-B)*a^4*\log(\tan(d*x+c)^2+1)+6*(-8*I*A-7*B)*a^4*\log(\tan(d*x+c))+6*(7*A-4*I*B)*a^4*\tan(d*x+c)^2+3*(-4*I*A-B)*a^4*\tan(d*x+c)-2*A*a^4)/\tan(d*x+c)^3/d$

**Fricas** [A]

time = 0.50, size = 249, normalized size = 1.53

$$\frac{6(-12iA-5B)a^4e^{4i(dx+c)}+54(2iA+B)a^4e^{2i(dx+c)}+4(-11iA-6B)a^4+3(Ba^4e^{6i(dx+c)}-3Ba^4e^{4i(dx+c)}+3Ba^4e^{2i(dx+c)}-Ba^4)\log(e^{2i(dx+c)}+1)+3((8iA+7B)a^4e^{6i(dx+c)}+3(-8iA-7B)a^4e^{4i(dx+c)}+3(8iA+7B)a^4e^{2i(dx+c)}+(-8iA-7B)a^4)\log(e^{2i(dx+c)}-1)}{3(de^{6i(dx+c)}-3de^{4i(dx+c)}+3de^{2i(dx+c)}-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/3*(6*(-12*I*A-5*B)*a^4*e^{(4*I*d*x+4*I*c)}+54*(2*I*A+B)*a^4*e^{(2*I*d*x+2*I*c)}+4*(-11*I*A-6*B)*a^4+3*(B*a^4*e^{(6*I*d*x+6*I*c)}-3*B*a^4*e^{(4*I*d*x+4*I*c)}+3*B*a^4*e^{(2*I*d*x+2*I*c)}-B*a^4)*\log(e^{(2*I*d*x+2*I*c)}+1)+3*((8*I*A+7*B)*a^4*e^{(6*I*d*x+6*I*c)}+3*(-8*I*A-7*B)*a^4*e^{(4*I*d*x+4*I*c)}+3*(8*I*A+7*B)*a^4*e^{(2*I*d*x+2*I*c)}+(-8*I*A-7*B)*a^4)*\log(e^{(2*I*d*x+2*I*c)}-1))/(d*e^{(6*I*d*x+6*I*c)}-3*d*e^{(4*I*d*x+4*I*c)}+3*d*e^{(2*I*d*x+2*I*c)}-d)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs.  $2(143) = 286$ .

time = 2.95, size = 292, normalized size = 1.79

$$-\frac{Ba^4\log\left(\frac{4Aa^4-3iBa^4}{4Aa^4e^{2ic}-3iBa^4e^{2ic}}+e^{2idx}\right)}{d}-\frac{ia^4\cdot(8A-7iB)\log\left(e^{2idx}+\frac{4Aa^4-4iBa^4-a^4(8A-7iB)}{4Aa^4e^{2ic}-3iBa^4e^{2ic}}\right)}{d}+\frac{44iAa^4+24Ba^4+(-108iAa^4e^{2ic}-54Ba^4e^{2ic})e^{2idx}+(72iAa^4e^{4ic}+30Ba^4e^{4ic})e^{4idx}}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)`

[Out]  $-B*a**4*\log((4*A*a**4-3*I*B*a**4)/(4*A*a**4*\exp(2*I*c)-3*I*B*a**4*\exp(2*I*c))+\exp(2*I*d*x))/d-I*a**4*(8*A-7*I*B)*\log(\exp(2*I*d*x)+(4*A*a**4-3*I*B*a**4)/\exp(2*I*c))$

$$\frac{4 - 4I*B*a**4 - a**4*(8*A - 7*I*B))/(4*A*a**4*exp(2*I*c) - 3*I*B*a**4*exp(2*I*c)))/d + (44*I*A*a**4 + 24*B*a**4 + (-108*I*A*a**4*exp(2*I*c) - 54*B*a**4*exp(2*I*c))*exp(2*I*d*x) + (72*I*A*a**4*exp(4*I*c) + 30*B*a**4*exp(4*I*c)))*exp(4*I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) - 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d)$$

**Giac [A]**

time = 1.06, size = 291, normalized size = 1.79

$$\frac{A^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 12A^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 3B^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 24B^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 24B^4 \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) - 24B^4 \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right) - 87A^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 48B^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 384(-IA^4 - B^4) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) - 24(8A^4 + 7B^4) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) - (-352IA^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 308B^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 87A^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 48IB^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 12IA^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3B^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + A^4)/\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/24\*(A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^3 - 12\*I\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^2 - 3\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c)^2 - 24\*B\*a^4\*log(tan(1/2\*d\*x + 1/2\*c) + 1) - 24\*B\*a^4\*log(tan(1/2\*d\*x + 1/2\*c) - 1) - 87\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c) + 48\*I\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c) - 384\*(-I\*A\*a^4 - B\*a^4)\*log(tan(1/2\*d\*x + 1/2\*c) + 1) - 24\*(8\*I\*A\*a^4 + 7\*B\*a^4)\*log(tan(1/2\*d\*x + 1/2\*c)) - (-352\*I\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^3 - 308\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c)^3 - 87\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^2 + 48\*I\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c)^2 + 12\*I\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c) + 3\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c) + A\*a^4)/tan(1/2\*d\*x + 1/2\*c)^3)/d

**Mupad [B]**

time = 6.71, size = 113, normalized size = 0.69

$$-\frac{\frac{Aa^4}{3} - \tan(c+dx)^2(7Aa^4 - Ba^44i) + \tan(c+dx)\left(\frac{Ba^4}{2} + Aa^42i\right)}{d \tan(c+dx)^3} - \frac{a^4 \ln(\tan(c+dx))(7B + A8i)}{d} + \frac{8a^4 \ln(\tan(c+dx) + 1i)(B + A1i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] (8\*a^4\*log(tan(c + d\*x) + 1i)\*(A\*1i + B))/d - (a^4\*log(tan(c + d\*x))\*(A\*8i + 7\*B))/d - ((A\*a^4)/3 - tan(c + d\*x)^2\*(7\*A\*a^4 - B\*a^4\*4i) + tan(c + d\*x)\*(A\*a^4\*2i + (B\*a^4)/2))/(d\*tan(c + d\*x)^3)

### 3.33 $\int \cot^5(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=177

$$8a^4(iA+B)x + \frac{a^4(67iA + 64B) \cot(c + dx)}{12d} + \frac{8a^4(A - iB) \log(\sin(c + dx))}{d} - \frac{aA \cot^4(c + dx)(a + ia \tan(c + dx))}{4d}$$

[Out]  $8*a^4*(I*A+B)*x+1/12*a^4*(67*I*A+64*B)*\cot(d*x+c)/d+8*a^4*(A-I*B)*\ln(\sin(d*x+c))/d-1/4*a*A*\cot(d*x+c)^4*(a+I*a*\tan(d*x+c))^3/d-1/12*(7*I*A+4*B)*\cot(d*x+c)^3*(a^2+I*a^2*\tan(d*x+c))^2/d+1/12*(19*A-16*I*B)*\cot(d*x+c)^2*(a^4+I*a^4*\tan(d*x+c))/d$

Rubi [A]

time = 0.36, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3674, 3672, 3612, 3556}

$$\frac{a^4(64B + 67iA) \cot(c + dx)}{12d} + \frac{8a^4(A - iB) \log(\sin(c + dx))}{d} + \frac{(19A - 16iB) \cot^2(c + dx) (a^4 + ia^4 \tan(c + dx))}{12d} + 8a^4x(B + iA) - \frac{(4B + 7iA) \cot^3(c + dx) (a^2 + ia^2 \tan(c + dx))^2}{12d} - \frac{aA \cot^4(c + dx) (a + ia \tan(c + dx))^2}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^5\*(a + I\*a\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out]  $8*a^4*(I*A + B)*x + (a^4*((67*I)*A + 64*B)*\text{Cot}[c + d*x])/(12*d) + (8*a^4*(A - I*B)*\text{Log}[\text{Sin}[c + d*x]])/d - (a*A*\text{Cot}[c + d*x]^4*(a + I*a*\text{Tan}[c + d*x])^3)/(4*d) - (((7*I)*A + 4*B)*\text{Cot}[c + d*x]^3*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/(12*d) + ((19*A - (16*I)*B)*\text{Cot}[c + d*x]^2*(a^4 + I*a^4*\text{Tan}[c + d*x]))/(12*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2)





```
]*Sin[c + d*x] + 4*(12*A - (6*I)*B + ((-4*I)*A - B)*Cot[c])*(Cos[4*c] - I*Sin[4*c])*Sin[c + d*x]^2 - (8*I)*(14*A - (11*I)*B)*Csc[c]*(Cos[4*c] - I*Sin[4*c])*Sin[d*x]*Sin[c + d*x]^3 - (96*I)*(A - I*B)*ArcTan[Tan[5*c + d*x]]*(Cos[4*c] - I*Sin[4*c])*Sin[c + d*x]^4 + 48*(A - I*B)*Log[Sin[c + d*x]^2]*(Cos[4*c] - I*Sin[4*c])*Sin[c + d*x]^4 + 192*(A - I*B)*d*x*(I*Cos[4*c] + Sin[4*c])*Sin[c + d*x]^4)/(12*d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.22, size = 227, normalized size = 1.28

method	result
risch	$-\frac{16a^4 Bc}{d} - \frac{16ia^4 Ac}{d} + \frac{4ia^4(30iAe^{6i(dx+c)} + 18Be^{6i(dx+c)} - 63iAe^{4i(dx+c)} - 45Be^{4i(dx+c)} + 50iAe^{2i(dx+c)} + 38B)}{3d(e^{2i(dx+c)} - 1)^4}$
norman	$\frac{(8iAa^4 + 7Ba^4)(\tan^3(dx+c))}{d} + (8iAa^4 + 8Ba^4)x(\tan^4(dx+c)) - \frac{Aa^4}{4d} + \frac{(-4iBa^4 + 7Aa^4)(\tan^2(dx+c))}{2d} - \frac{(4iAa^4 + Ba^4)\tan(dx+c)}{3d}$
derivativedivides	$Aa^4 \ln(\sin(dx+c)) + Ba^4(dx+c) - 4iAa^4(-\cot(dx+c) - dx - c) - 4iBa^4 \ln(\sin(dx+c)) - 6Aa^4 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)$
default	$Aa^4 \ln(\sin(dx+c)) + Ba^4(dx+c) - 4iAa^4(-\cot(dx+c) - dx - c) - 4iBa^4 \ln(\sin(dx+c)) - 6Aa^4 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(A\*a^4\*ln(sin(d\*x+c))+B\*a^4\*(d\*x+c)-4\*I\*A\*a^4\*(-cot(d\*x+c)-d\*x-c)-4\*I\*B\*a^4\*ln(sin(d\*x+c))-6\*A\*a^4\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))-6\*B\*a^4\*(-cot(d\*x+c)-d\*x-c)+4\*I\*A\*a^4\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)+4\*I\*B\*a^4\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+A\*a^4\*(-1/4\*cot(d\*x+c)^4+1/2\*cot(d\*x+c)^2+ln(sin(d\*x+c)))+B\*a^4\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c))

**Maxima [A]**

time = 0.64, size = 136, normalized size = 0.77

$$\frac{96(dx+c)(-iA-B)a^4 + 48(A-iB)a^4 \log(\tan(dx+c)^2 + 1) - 96(A-iB)a^4 \log(\tan(dx+c)) - \frac{12(8iA+7B)a^4 \tan(dx+c)^3 + 6(7A-4iB)a^4 \tan(dx+c)^2 + 4(-4iA-B)a^4 \tan(dx+c) - 3Aa^4}{\tan(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] -1/12\*(96\*(d\*x + c)\*(-I\*A - B)\*a^4 + 48\*(A - I\*B)\*a^4\*log(tan(d\*x + c)^2 + 1) - 96\*(A - I\*B)\*a^4\*log(tan(d\*x + c)) - (12\*(8\*I\*A + 7\*B)\*a^4\*tan(d\*x + c)^3 + 6\*(7\*A - 4\*I\*B)\*a^4\*tan(d\*x + c)^2 + 4\*(-4\*I\*A - B)\*a^4\*tan(d\*x + c) - 3\*A\*a^4)/tan(d\*x + c)^4/d

**Fricas [A]**

time = 0.51, size = 228, normalized size = 1.29

$$\frac{4(6(5A-3iB)a^4e^{6i dx+6i c}-9(7A-5iB)a^4e^{4i dx+4i c}+2(25A-19iB)a^4e^{2i dx+2i c})-(14A-11iB)a^4-6((A-iB)a^4e^{8i dx+8i c}-4(A-iB)a^4e^{6i dx+6i c}+6(A-iB)a^4e^{4i dx+4i c}-4(A-iB)a^4e^{2i dx+2i c})+(A-iB)a^4\log(e^{2i dx+2i c}-1)}{3(de^{8i dx+8i c}-4de^{6i dx+6i c}+6de^{4i dx+4i c}-4de^{2i dx+2i c}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-4/3*(6*(5*A - 3*I*B)*a^4*e^{(6*I*d*x + 6*I*c)} - 9*(7*A - 5*I*B)*a^4*e^{(4*I*d*x + 4*I*c)} + 2*(25*A - 19*I*B)*a^4*e^{(2*I*d*x + 2*I*c)} - (14*A - 11*I*B)*a^4 - 6*((A - I*B)*a^4*e^{(8*I*d*x + 8*I*c)} - 4*(A - I*B)*a^4*e^{(6*I*d*x + 6*I*c)} + 6*(A - I*B)*a^4*e^{(4*I*d*x + 4*I*c)} - 4*(A - I*B)*a^4*e^{(2*I*d*x + 2*I*c)} + (A - I*B)*a^4*\log(e^{(2*I*d*x + 2*I*c)} - 1))/(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [A]**

time = 0.76, size = 235, normalized size = 1.33

$$\frac{8a^4(A-iB)\log(e^{2idx}-e^{-2ic})}{d} + \frac{56Aa^4-44iBa^4+(-200Aa^4e^{2ic}+152iBa^4e^{2ic})e^{2idx}+(252Aa^4e^{4ic}-180iBa^4e^{4ic})e^{4idx}+(-120Aa^4e^{6ic}+72iBa^4e^{6ic})e^{6idx}}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*5\*(a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out]  $8*a**4*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (56*A*a**4 - 44*I*B*a**4 + (-200*A*a**4*\exp(2*I*c) + 152*I*B*a**4*\exp(2*I*c))*\exp(2*I*d*x) + (252*A*a**4*\exp(4*I*c) - 180*I*B*a**4*\exp(4*I*c))*\exp(4*I*d*x) + (-120*A*a**4*\exp(6*I*c) + 72*I*B*a**4*\exp(6*I*c))*\exp(6*I*d*x))/(3*d*\exp(8*I*c)*\exp(8*I*d*x) - 12*d*\exp(6*I*c)*\exp(6*I*d*x) + 18*d*\exp(4*I*c)*\exp(4*I*d*x) - 12*d*\exp(2*I*c)*\exp(2*I*d*x) + 3*d)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs.  $2(153) = 306$ .

time = 1.20, size = 322, normalized size = 1.82

$$\frac{8Aa^4\log(\frac{1}{2}dx+\frac{1}{2}c)-32Aa^4\log(\frac{1}{2}dx+\frac{1}{2}c)-96B^2\log(\frac{1}{2}dx+\frac{1}{2}c)-180Aa^4\log(\frac{1}{2}dx+\frac{1}{2}c)+360Aa^4\log(\frac{1}{2}dx+\frac{1}{2}c)+696B^2\log(\frac{1}{2}dx+\frac{1}{2}c)+3072Aa^4\log(\frac{1}{2}dx+\frac{1}{2}c)-1328(A^4-iB^2)\log(\frac{1}{2}dx+\frac{1}{2}c)-1328(A^4-iB^2)\log(\frac{1}{2}dx+\frac{1}{2}c)}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/192*(3*A*a^4*\tan(1/2*d*x + 1/2*c)^4 - 32*I*A*a^4*\tan(1/2*d*x + 1/2*c)^3 - 8*B*a^4*\tan(1/2*d*x + 1/2*c)^3 - 180*A*a^4*\tan(1/2*d*x + 1/2*c)^2 + 96*I*B*a^4*\tan(1/2*d*x + 1/2*c)^2 + 864*I*A*a^4*\tan(1/2*d*x + 1/2*c) + 696*B*a^4$

\*tan(1/2\*d\*x + 1/2\*c) + 3072\*(A\*a^4 - I\*B\*a^4)\*log(tan(1/2\*d\*x + 1/2\*c) + I) - 1536\*(A\*a^4 - I\*B\*a^4)\*log(tan(1/2\*d\*x + 1/2\*c)) + (3200\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^4 - 3200\*I\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c)^4 - 864\*I\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^3 - 696\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c)^3 - 180\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c)^2 + 96\*I\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c)^2 + 32\*I\*A\*a^4\*tan(1/2\*d\*x + 1/2\*c) + 8\*B\*a^4\*tan(1/2\*d\*x + 1/2\*c) + 3\*A\*a^4)/tan(1/2\*d\*x + 1/2\*c)^4)/d

**Mupad [B]**

time = 6.57, size = 114, normalized size = 0.64

$$\frac{\tan(c+dx)^2 \left( \frac{7Aa^4}{2} - Ba^4 2i \right) + \tan(c+dx)^3 (7Ba^4 + Aa^4 8i) - \frac{Aa^4}{4} - \tan(c+dx) \left( \frac{Ba^4}{3} + \frac{Aa^4 4i}{3} \right)}{d \tan(c+dx)^4} + \frac{16a^4 \operatorname{atan}(2 \tan(c+dx) + 1i) (B + A 1i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^5\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] (tan(c + d\*x)^2\*((7\*A\*a^4)/2 - B\*a^4\*2i) + tan(c + d\*x)^3\*(A\*a^4\*8i + 7\*B\*a^4) - (A\*a^4)/4 - tan(c + d\*x)\*((A\*a^4\*4i)/3 + (B\*a^4)/3))/(d\*tan(c + d\*x)^4) + (16\*a^4\*atan(2\*tan(c + d\*x) + 1i)\*(A\*1i + B))/d

### 3.34 $\int \cot^6(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=200

$$-8a^4(A-iB)x - \frac{8a^4(A-iB)\cot(c+dx)}{d} + \frac{a^4(148iA+145B)\cot^2(c+dx)}{60d} + \frac{8a^4(iA+B)\log(\sin(c+dx))}{d}$$

[Out]  $-8*a^4*(A-I*B)*x - 8*a^4*(A-I*B)*\cot(d*x+c)/d + 1/60*a^4*(148*I*A+145*B)*\cot(d*x+c)^2/d + 8*a^4*(I*A+B)*\ln(\sin(d*x+c))/d - 1/5*a*A*\cot(d*x+c)^5*(a+I*a*\tan(d*x+c))^3/d - 1/20*(8*I*A+5*B)*\cot(d*x+c)^4*(a^2+I*a^2*\tan(d*x+c))^2/d + 1/30*(28*A-25*I*B)*\cot(d*x+c)^3*(a^4+I*a^4*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.40, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3672, 3610, 3612, 3556}

$$\frac{a^4(145B+148iA)\cot^2(c+dx)}{60d} - \frac{8a^4(A-iB)\cot(c+dx)}{d} + \frac{8a^4(B+iA)\log(\sin(c+dx))}{d} + \frac{(28A-25iB)\cot^3(c+dx)(a^4+ia^4\tan(c+dx))}{30d} - 8a^4x(A-iB) - \frac{(5B+8iA)\cot^4(c+dx)(a^2+ia^2\tan(c+dx))^2}{20d} - \frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^6*(a + I*a*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-8*a^4*(A - I*B)*x - (8*a^4*(A - I*B)*\text{Cot}[c + d*x])/d + (a^4*((148*I)*A + 145*B)*\text{Cot}[c + d*x]^2)/(60*d) + (8*a^4*(I*A + B)*\text{Log}[\text{Sin}[c + d*x]])/d - (a*A*\text{Cot}[c + d*x]^5*(a + I*a*\text{Tan}[c + d*x])^3)/(5*d) - (((8*I)*A + 5*B)*\text{Cot}[c + d*x]^4*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/(20*d) + ((28*A - (25*I)*B)*\text{Cot}[c + d*x]^3*(a^4 + I*a^4*\text{Tan}[c + d*x]))/(30*d)$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3610**

$\text{Int}[((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

**Rule 3612**

$\text{Int}(((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a$

\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rule 3674

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \cot^6(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot^5(c + dx)(a + ia \tan(c + dx))^3}{5d} + \frac{1}{5} \\
 &= -\frac{aA \cot^5(c + dx)(a + ia \tan(c + dx))^3}{5d} - \frac{8a^4(A - iB) \cot(c + dx)}{d} + \frac{a^4(148iA + 145B) \cot^2(c + dx)}{60d} \\
 &= -\frac{aA \cot^5(c + dx)(a + ia \tan(c + dx))^3}{5d} - \frac{a^4(148iA + 145B) \cot^2(c + dx)}{60d} \\
 &= -\frac{a^4(148iA + 145B) \cot^2(c + dx)}{60d} - \frac{aA \cot^5(c + dx)(a + ia \tan(c + dx))^3}{5d} \\
 &= -\frac{8a^4(A - iB) \cot(c + dx)}{d} + \frac{a^4(148iA + 145B) \cot^2(c + dx)}{60d} \\
 &= -8a^4(A - iB)x - \frac{8a^4(A - iB) \cot(c + dx)}{d} \\
 &= -8a^4(A - iB)x - \frac{8a^4(A - iB) \cot(c + dx)}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 542 vs.  $2(200) = 400$ .  
time = 6.49, size = 542, normalized size = 2.71

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] (a^4*(I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*(-8*(A - I*B)*d*x*(Cos[4*c]
- I*Sin[4*c])*Sin[c + d*x]^5 + 8*(A - I*B)*ArcTan[Tan[5*c + d*x]]*(Cos[4*c]
- I*Sin[4*c])*Sin[c + d*x]^5 + 4*(A - I*B)*Log[Sin[c + d*x]^2]*(I*Cos[4*c]
+ Sin[4*c])*Sin[c + d*x]^5 + (Csc[c]*(Cos[4*c] - I*Sin[4*c])*(15*(A*(14*I
- 20*d*x) + B*(11 + (20*I)*d*x))*Cos[d*x] + 15*((-14*I)*A - 11*B + 20*A*d*x
- (20*I)*B*d*x)*Cos[2*c + d*x] - (90*I)*A*Cos[2*c + 3*d*x] - 60*B*Cos[2*c
+ 3*d*x] + 150*A*d*x*Cos[2*c + 3*d*x] - (150*I)*B*d*x*Cos[2*c + 3*d*x] + (9
0*I)*A*Cos[4*c + 3*d*x] + 60*B*Cos[4*c + 3*d*x] - 150*A*d*x*Cos[4*c + 3*d*x
] + (150*I)*B*d*x*Cos[4*c + 3*d*x] - 30*A*d*x*Cos[4*c + 5*d*x] + (30*I)*B*d
*x*Cos[4*c + 5*d*x] + 30*A*d*x*Cos[6*c + 5*d*x] - (30*I)*B*d*x*Cos[6*c + 5*
d*x] + 445*A*Sin[d*x] - (400*I)*B*Sin[d*x] + 345*A*Sin[2*c + d*x] - (300*I)
*B*Sin[2*c + d*x] - 275*A*Sin[2*c + 3*d*x] + (260*I)*B*Sin[2*c + 3*d*x] - 1
20*A*Sin[4*c + 3*d*x] + (90*I)*B*Sin[4*c + 3*d*x] + 79*A*Sin[4*c + 5*d*x] -
(70*I)*B*Sin[4*c + 5*d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c +
d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.23, size = 279, normalized size = 1.40

method	result
risch	$-\frac{16ia^4Bc}{d} + \frac{16a^4Ac}{d} - \frac{4a^4(210iAe^{8i(dx+c)} + 150Be^{8i(dx+c)} - 555iAe^{6i(dx+c)} - 465Be^{6i(dx+c)} + 655iAe^{4i(dx+c)} + 505Be^{4i(dx+c)} - 15d(e^{2i(dx+c)} - 1)^5)}{15d(e^{2i(dx+c)} - 1)^5}$
norman	$\frac{(8iBa^4 - 8Aa^4)x(\tan^5(dx+c)) - \frac{Aa^4}{5d} + \frac{(-4iBa^4 + 7Aa^4)(\tan^2(dx+c))}{3d} - \frac{(4iAa^4 + Ba^4)\tan(dx+c)}{4d} - \frac{8(-iBa^4 + Aa^4)(\tan^4(dx+c))}{d}}{\tan(dx+c)^5}$
derivativedivides	$Aa^4(-\cot(dx+c) - dx - c) + Ba^4 \ln(\sin(dx+c)) - 4iAa^4 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) - 4iBa^4(-\cot(dx+c) - dx - c)$
default	$Aa^4(-\cot(dx+c) - dx - c) + Ba^4 \ln(\sin(dx+c)) - 4iAa^4 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) - 4iBa^4(-\cot(dx+c) - dx - c)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/d*(A*a^4*(-cot(d*x+c)-d*x-c)+B*a^4*ln(sin(d*x+c))-4*I*A*a^4*(-1/2*cot(d*x
+c)^2-ln(sin(d*x+c)))-4*I*B*a^4*(-cot(d*x+c)-d*x-c)-6*A*a^4*(-1/3*cot(d*x+c
```

$$\begin{aligned} &)^3 + \cot(dx+c) + dx+c - 6Ba^4(-1/2\cot(dx+c)^2 - \ln(\sin(dx+c))) + 4Ia^4 \\ &(-1/4\cot(dx+c)^4 + 1/2\cot(dx+c)^2 + \ln(\sin(dx+c))) + 4IBa^4(-1/3\cot(dx+c) \\ &+ c)^3 + \cot(dx+c) + dx+c + Aa^4(-1/5\cot(dx+c)^5 + 1/3\cot(dx+c)^3 - \cot(dx+c) \\ &- dx-c) + Ba^4(-1/4\cot(dx+c)^4 + 1/2\cot(dx+c)^2 + \ln(\sin(dx+c))) \end{aligned}$$

**Maxima** [A]

time = 0.56, size = 153, normalized size = 0.76

$$\frac{480(dx+c)(A-iB)a^4 + 240(iA+B)a^4 \log(\tan(dx+c)^2 + 1) + 480(-iA-B)a^4 \log(\tan(dx+c)) + \frac{480(A-iB)a^4 \tan(dx+c)^4 - 30(8iA+7B)a^4 \tan(dx+c)^3 - 20(7A-4iB)a^4 \tan(dx+c)^2 - 15(-4iA-B)a^4 \tan(dx+c) + 12Aa^4}{\tan(dx+c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^6\*(a+I\*a\*tan(dx+c))^4\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} &-1/60*(480*(dx+c)*(A-I*B)*a^4 + 240*(I*A+B)*a^4*\log(\tan(dx+c)^2 + \\ &1) + 480*(-I*A-B)*a^4*\log(\tan(dx+c)) + (480*(A-I*B)*a^4*\tan(dx+c) \\ &)^4 - 30*(8*I*A+7*B)*a^4*\tan(dx+c)^3 - 20*(7*A-4*I*B)*a^4*\tan(dx+c) \\ &^2 - 15*(-4*I*A-B)*a^4*\tan(dx+c) + 12*A*a^4)/\tan(dx+c)^5)/d \end{aligned}$$

**Fricas** [A]

time = 0.47, size = 287, normalized size = 1.44

$$\frac{4(30(7iA+5B)a^{6d+6i} + 15(-37iA-31B)a^{6d+6i} + 5(131iA+113B)a^{6d+6i} + 5(-73iA-64B)a^{6d+6i} + (79iA+70B)a^4 + 30((-iA-B)a^{10d+10i} + 5(iA+B)a^{10d+10i} + 10(-iA-B)a^{10d+10i} + 10(iA+B)a^{10d+10i} + 5(-iA-B)a^{10d+10i} + (iA+B)a^4 \log(e^{2d+2i}-1))}{15(d^{10d+10i}-5d^{10d+10i}+10d^{10d+10i}-10d^{10d+10i}+5d^{10d+10i}-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^6\*(a+I\*a\*tan(dx+c))^4\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} &-4/15*(30*(7iA+5B)*a^4*e^{(8i*d*x+8i*c)} + 15*(-37iA-31B)*a^4*e^{(6i*d*x+6i*c)} \\ &+ 5*(131iA+113B)*a^4*e^{(4i*d*x+4i*c)} + 5*(-73iA-64B)*a^4*e^{(2i*d*x+2i*c)} \\ &+ (79iA+70B)*a^4 + 30*((-iA-B)*a^4*e^{(10i*d*x+10i*c)} + 5*(iA+B)*a^4*e^{(8i*d*x+8i*c)} \\ &+ 10*(-iA-B)*a^4*e^{(6i*d*x+6i*c)} + 10*(iA+B)*a^4*e^{(4i*d*x+4i*c)} + 5*(-iA-B)*a^4*e^{(2i*d*x+2i*c)} \\ &+ (iA+B)*a^4)*\log(e^{(2i*d*x+2i*c)} - 1))/d + e^{(10i*d*x+10i*c)} - 5*d*e^{(8i*d*x+8i*c)} + 10*d*e^{(6i*d*x+6i*c)} - 10*d*e^{(4i*d*x+4i*c)} + 5*d*e^{(2i*d*x+2i*c)} - d \end{aligned}$$

**Sympy** [A]

time = 3.44, size = 296, normalized size = 1.48

$$\frac{8iA^4(A-iB)\log(e^{2idx}-e^{-2ic})}{d} + \frac{-316iA^4-280B^4+(1460iA^4e^{2ic}+1280B^4e^{2ic})e^{2idx}+(-2620iA^4e^{4ic}-2260B^4e^{4ic})e^{4idx}+(2220iA^4e^{6ic}+1860B^4e^{6ic})e^{6idx}+(-840iA^4e^{8ic}-600B^4e^{8ic})e^{8idx}}{15de^{10ic}e^{10idx}-75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}-150de^{4ic}e^{4idx}+75de^{2ic}e^{2idx}-15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*6\*(a+I\*a\*tan(dx+c))\*\*4\*(A+B\*tan(dx+c)),x)

[Out] 
$$\begin{aligned} &8*I*a**4*(A-I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (-316*I*A*a**4 - 280 \\ &*B*a**4 + (1460*I*A*a**4*\exp(2*I*c) + 1280*B*a**4*\exp(2*I*c))*\exp(2*I*d*x) \end{aligned}$$

+ (-2620\*I\*A\*a\*\*4\*exp(4\*I\*c) - 2260\*B\*a\*\*4\*exp(4\*I\*c))\*exp(4\*I\*d\*x) + (2220\*I\*A\*a\*\*4\*exp(6\*I\*c) + 1860\*B\*a\*\*4\*exp(6\*I\*c))\*exp(6\*I\*d\*x) + (-840\*I\*A\*a\*\*4\*exp(8\*I\*c) - 600\*B\*a\*\*4\*exp(8\*I\*c))\*exp(8\*I\*d\*x))/(15\*d\*exp(10\*I\*c)\*exp(10\*I\*d\*x) - 75\*d\*exp(8\*I\*c)\*exp(8\*I\*d\*x) + 150\*d\*exp(6\*I\*c)\*exp(6\*I\*d\*x) - 150\*d\*exp(4\*I\*c)\*exp(4\*I\*d\*x) + 75\*d\*exp(2\*I\*c)\*exp(2\*I\*d\*x) - 15\*d)

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs.  $2(174) = 348$ .

time = 1.28, size = 391, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^6\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{960}*(6*A*a^4*\tan(1/2*d*x + 1/2*c)^5 - 60*I*A*a^4*\tan(1/2*d*x + 1/2*c)^4 - 15*B*a^4*\tan(1/2*d*x + 1/2*c)^4 - 310*A*a^4*\tan(1/2*d*x + 1/2*c)^3 + 160*I*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 1200*I*A*a^4*\tan(1/2*d*x + 1/2*c)^2 + 900*B*a^4*\tan(1/2*d*x + 1/2*c)^2 + 4740*A*a^4*\tan(1/2*d*x + 1/2*c) - 4320*I*B*a^4*\tan(1/2*d*x + 1/2*c) - 15360*(I*A*a^4 + B*a^4)*\log(\tan(1/2*d*x + 1/2*c) + I) - 7680*(-I*A*a^4 - B*a^4)*\log(\tan(1/2*d*x + 1/2*c)) + (-17536*I*A*a^4*\tan(1/2*d*x + 1/2*c)^5 - 17536*B*a^4*\tan(1/2*d*x + 1/2*c)^5 - 4740*A*a^4*\tan(1/2*d*x + 1/2*c)^4 + 4320*I*B*a^4*\tan(1/2*d*x + 1/2*c)^4 + 1200*I*A*a^4*\tan(1/2*d*x + 1/2*c)^3 + 900*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 310*A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 160*I*B*a^4*\tan(1/2*d*x + 1/2*c)^2 - 60*I*A*a^4*\tan(1/2*d*x + 1/2*c) - 15*B*a^4*\tan(1/2*d*x + 1/2*c) - 6*A*a^4)/\tan(1/2*d*x + 1/2*c)^5)/d$

**Mupad [B]**

time = 6.85, size = 140, normalized size = 0.70

$$\frac{\frac{Aa^4}{5} - \tan(c+dx)^2 \left( \frac{7Aa^4}{3} - \frac{Ba^4 4i}{3} \right) + \tan(c+dx)^4 (8Aa^4 - Ba^4 8i) - \tan(c+dx)^3 \left( \frac{7Ba^4}{2} + Aa^4 4i \right) + \tan(c+dx) \left( \frac{Ba^4}{4} + Aa^4 1i \right)}{d \tan(c+dx)^5} + \frac{a^4 \operatorname{atan}(2 \tan(c+dx) + 1i) (B + A 1i) 16i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^6\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out]  $(a^4*\operatorname{atan}(2*\tan(c + d*x) + 1i)*(A*1i + B)*16i)/d - (\tan(c + d*x)^4*(8*A*a^4 - B*a^4*8i) - \tan(c + d*x)^2*((7*A*a^4)/3 - (B*a^4*4i)/3) - \tan(c + d*x)^3*(A*a^4*4i + (7*B*a^4)/2) + (A*a^4)/5 + \tan(c + d*x)*(A*a^4*1i + (B*a^4)/4))/(d*\tan(c + d*x)^5)$



### 3.35 $\int \cot^7(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=223

$$-8a^4(iA+B)x - \frac{8a^4(iA+B) \cot(c+dx)}{d} - \frac{4a^4(A-iB) \cot^2(c+dx)}{d} + \frac{a^4(93iA+92B) \cot^3(c+dx)}{60d} - \frac{8a^4($$

[Out]  $-8*a^4*(I*A+B)*x - 8*a^4*(I*A+B)*\cot(d*x+c)/d - 4*a^4*(A-I*B)*\cot(d*x+c)^2/d + 1/60*a^4*(93*I*A+92*B)*\cot(d*x+c)^3/d - 8*a^4*(A-I*B)*\ln(\sin(d*x+c))/d - 1/6*a*A*\cot(d*x+c)^6*(a+I*a*\tan(d*x+c))^3/d - 1/10*(3*I*A+2*B)*\cot(d*x+c)^5*(a^2+I*a^2*\tan(d*x+c))^2/d + 1/20*(13*A-12*I*B)*\cot(d*x+c)^4*(a^4+I*a^4*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.43, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3674, 3672, 3610, 3612, 3556}

$$\frac{a^4(92B+93iA)\cot^3(c+dx)}{60d} - \frac{4a^4(A-iB)\cot^2(c+dx)}{d} - \frac{8a^4(B+iA)\cot(c+dx)}{d} - \frac{8a^4(A-iB)\log(\sin(c+dx))}{d} + \frac{(13A-12iB)\cot^4(c+dx)(a^4+ia^4\tan(c+dx))}{20d} - \frac{8a^4x(B+iA)}{d} - \frac{(2B+3iA)\cot^5(c+dx)(a^2+ia^2\tan(c+dx))^2}{10d} - \frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^7*(a + I*a*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-8*a^4*(I*A + B)*x - (8*a^4*(I*A + B)*\text{Cot}[c + d*x])/d - (4*a^4*(A - I*B)*\text{Cot}[c + d*x]^2)/d + (a^4*((93*I)*A + 92*B)*\text{Cot}[c + d*x]^3)/(60*d) - (8*a^4*(A - I*B)*\text{Log}[\text{Sin}[c + d*x]])/d - (a*A*\text{Cot}[c + d*x]^6*(a + I*a*\text{Tan}[c + d*x])^3)/(6*d) - (((3*I)*A + 2*B)*\text{Cot}[c + d*x]^5*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/(10*d) + ((13*A - (12*I)*B)*\text{Cot}[c + d*x]^4*(a^4 + I*a^4*\text{Tan}[c + d*x]))/(20*d)$

**Rule 3556**

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3610**

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

**Rule 3612**

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a$

$*d)/(a^2 + b^2)$ , Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

#### Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

#### Rule 3674

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] :> Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned}
\int \cot^7(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx &= -\frac{aA \cot^6(c+dx)(a+ia \tan(c+dx))^3}{6d} + \frac{1}{6} \\
&= -\frac{aA \cot^6(c+dx)(a+ia \tan(c+dx))^3}{6d} - \frac{3}{6} \\
&= -\frac{aA \cot^6(c+dx)(a+ia \tan(c+dx))^3}{6d} - \frac{3}{6} \\
&= \frac{a^4(93iA+92B) \cot^3(c+dx)}{60d} - \frac{aA \cot^6(c+dx)}{6d} \\
&= -\frac{4a^4(A-iB) \cot^2(c+dx)}{d} + \frac{a^4(93iA+92B)}{6d} \\
&= -\frac{8a^4(iA+B) \cot(c+dx)}{d} - \frac{4a^4(A-iB) \cot(c+dx)}{d} \\
&= -8a^4(iA+B)x - \frac{8a^4(iA+B) \cot(c+dx)}{d} \\
&= -8a^4(iA+B)x - \frac{8a^4(iA+B) \cot(c+dx)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1009 vs. 2(223) = 446.  
time = 7.30, size = 1009, normalized size = 4.52

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] a^4*(((I + Cot[c + d*x])^4*(B + A*Cot[c + d*x])*(A*Cos[2*c] - I*B*Cos[2*c]
- I*A*Sin[2*c] - B*Sin[2*c])*((8*I)*ArcTan[Tan[5*c + d*x]]*Cos[2*c] + 8*Arc
Tan[Tan[5*c + d*x]]*Sin[2*c])*Sin[c + d*x]^5)/(d*(Cos[d*x] + I*Sin[d*x])^4*
(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c + d*x])^4*(B + A*Cot[c + d
*x])*(A*Cos[2*c] - I*B*Cos[2*c] - I*A*Sin[2*c] - B*Sin[2*c])*(-4*Cos[2*c]*L
og[Sin[c + d*x]^2] + (4*I)*Log[Sin[c + d*x]^2]*Sin[2*c])*Sin[c + d*x]^5)/(d
*(Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (x*(I + Cot
[c + d*x])^4*(B + A*Cot[c + d*x])*((-40*I)*A*Cos[c]^4 - 40*B*Cos[c]^4 + 8*A
*Cos[c]^4*Cot[c] - (8*I)*B*Cos[c]^4*Cot[c] - 80*A*Cos[c]^3*Sin[c] + (80*I)*
B*Cos[c]^3*Sin[c] + (80*I)*A*Cos[c]^2*Sin[c]^2 + 80*B*Cos[c]^2*Sin[c]^2 + 4
0*A*Cos[c]*Sin[c]^3 - (40*I)*B*Cos[c]*Sin[c]^3 - (8*I)*A*Sin[c]^4 - 8*B*Sin
[c]^4 + (A - I*B)*Cot[c]*(-8*Cos[4*c] + (8*I)*Sin[4*c]))*Sin[c + d*x]^5)/((
Cos[d*x] + I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) + ((I + Cot[c +
d*x])^4*(B + A*Cot[c + d*x])*Csc[c]*Csc[c + d*x]*(Cos[4*c]/240 - (I/240)*S
```

```

in[4*c])*((860*I)*A*Cos[c] + 790*B*Cos[c] - (780*I)*A*Cos[c + 2*d*x] - 720*
B*Cos[c + 2*d*x] - (510*I)*A*Cos[3*c + 2*d*x] - 465*B*Cos[3*c + 2*d*x] + (3
66*I)*A*Cos[3*c + 4*d*x] + 354*B*Cos[3*c + 4*d*x] + (150*I)*A*Cos[5*c + 4*d
*x] + 120*B*Cos[5*c + 4*d*x] - (86*I)*A*Cos[5*c + 6*d*x] - 79*B*Cos[5*c + 6
*d*x] - 490*A*Sin[c] + (420*I)*B*Sin[c] - (600*I)*A*d*x*Sin[c] - 600*B*d*x*
Sin[c] - 345*A*Sin[c + 2*d*x] + (300*I)*B*Sin[c + 2*d*x] - (450*I)*A*d*x*Si
n[c + 2*d*x] - 450*B*d*x*Sin[c + 2*d*x] + 345*A*Sin[3*c + 2*d*x] - (300*I)*
B*Sin[3*c + 2*d*x] + (450*I)*A*d*x*Sin[3*c + 2*d*x] + 450*B*d*x*Sin[3*c + 2
*d*x] + 120*A*Sin[3*c + 4*d*x] - (90*I)*B*Sin[3*c + 4*d*x] + (180*I)*A*d*x*
Sin[3*c + 4*d*x] + 180*B*d*x*Sin[3*c + 4*d*x] - 120*A*Sin[5*c + 4*d*x] + (9
0*I)*B*Sin[5*c + 4*d*x] - (180*I)*A*d*x*Sin[5*c + 4*d*x] - 180*B*d*x*Sin[5*
c + 4*d*x] - (30*I)*A*d*x*Sin[5*c + 6*d*x] - 30*B*d*x*Sin[5*c + 6*d*x] + (3
0*I)*A*d*x*Sin[7*c + 6*d*x] + 30*B*d*x*Sin[7*c + 6*d*x]))/(d*(Cos[d*x] + I*
Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x]))

```

**Maple [A]**

time = 0.25, size = 332, normalized size = 1.49

method	result
risch	$\frac{16a^4 Bc}{d} + \frac{16ia^4 Ac}{d} - \frac{4ia^4 (270iA e^{10i(dx+c)} + 210B e^{10i(dx+c)} - 855iA e^{8i(dx+c)} - 765B e^{8i(dx+c)} + 1350iA e^{6i(dx+c)} + 15d(e^{2i(dx+c)} - 1))}{15d(e^{2i(dx+c)} - 1)}$
norman	$\frac{(-8iA a^4 - 8B a^4)x(\tan^6(dx+c)) - \frac{A a^4}{6d} + \frac{(-4iB a^4 + 7A a^4)(\tan^2(dx+c))}{4d} - \frac{(4iA a^4 + B a^4)\tan(dx+c)}{5d} - \frac{4(-iB a^4 + A a^4)(\tan^4(dx+c))}{d}}{\tan(dx+c)^6}$
derivativedivides	$A a^4 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) + B a^4 (-\cot(dx+c) - dx - c) - 4iA a^4 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right) - 4iB a^4 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right)$
default	$A a^4 \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) + B a^4 (-\cot(dx+c) - dx - c) - 4iA a^4 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right) - 4iB a^4 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx + c \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^7\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(A\*a^4\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+B\*a^4\*(-cot(d\*x+c)-d\*x-c)-4\*I\*A\*a^4\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)-4\*I\*B\*a^4\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))-6\*A\*a^4\*(-1/4\*cot(d\*x+c)^4+1/2\*cot(d\*x+c)^2+ln(sin(d\*x+c)))-6\*B\*a^4\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)+4\*I\*A\*a^4\*(-1/5\*cot(d\*x+c)^5+1/3\*cot(d\*x+c)^3-cot(d\*x+c)-d\*x-c)+4\*I\*B\*a^4\*(-1/4\*cot(d\*x+c)^4+1/2\*cot(d\*x+c)^2+ln(sin(d\*x+c)))+A\*a^4\*(-1/6\*cot(d\*x+c)^6+1/4\*cot(d\*x+c)^4-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+B\*a^4\*(-1/5\*cot(d\*x+c)^5+1/3\*cot(d\*x+c)^3-cot(d\*x+c)-d\*x-c))

**Maxima [A]**

time = 0.66, size = 172, normalized size = 0.77

$\frac{480(dx+c)(iA+B)a^4 - 240(A-iB)a^4 \log(\tan(dx+c)^2+1) + 480(A-iB)a^4 \log(\tan(dx+c)) - \frac{480(-iA-B)a^4 \tan(dx+c)^5 - 240(A-iB)a^4 \tan(dx+c)^4 + 20(8iA+7B)a^4 \tan(dx+c)^3 + 15(7A-4iB)a^4 \tan(dx+c)^2 + 12(-4iA-B)a^4 \tan(dx+c) - 10Aa^4}{\tan(dx+c)^6}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^7\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/60*(480*(d*x + c)*(I*A + B)*a^4 - 240*(A - I*B)*a^4*\log(\tan(d*x + c))^2 + 1) + 480*(A - I*B)*a^4*\log(\tan(d*x + c)) - (480*(-I*A - B)*a^4*\tan(d*x + c)^5 - 240*(A - I*B)*a^4*\tan(d*x + c)^4 + 20*(8*I*A + 7*B)*a^4*\tan(d*x + c)^3 + 15*(7*A - 4*I*B)*a^4*\tan(d*x + c)^2 + 12*(-4*I*A - B)*a^4*\tan(d*x + c) - 10*A*a^4)/\tan(d*x + c)^6)/d$$

**Fricas** [A]

time = 0.45, size = 332, normalized size = 1.49

$$\frac{4(30(A - I*B)a^{2n+2m} - 45(19A - 17I*B)a^{2n+2m} + 10(135A - 121I*B)a^{2n+2m} - 15(75A - 69I*B)a^{2n+2m} + 6(81A - 74I*B)a^{2n+2m} - (86A - 79I*B)a^{2n+2m} - 30(A - I*B)a^{2n+2m} - 6(A - I*B)a^{2n+2m} + 15(A - I*B)a^{2n+2m} - 20(A - I*B)a^{2n+2m} + 15(A - I*B)a^{2n+2m} - 6(A - I*B)a^{2n+2m} + (A - I*B)a^2)\log(e^{2n+2m} - 1)}{15(d^{2n+2m} - 6d^{2n+2m} + 15d^{2n+2m} - 20d^{2n+2m} + 15d^{2n+2m} - 6d^{2n+2m} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^7\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$4/15*(30*(9*A - 7*I*B)*a^4*e^{(10*I*d*x + 10*I*c)} - 45*(19*A - 17*I*B)*a^4*e^{(8*I*d*x + 8*I*c)} + 10*(135*A - 121*I*B)*a^4*e^{(6*I*d*x + 6*I*c)} - 15*(75*A - 68*I*B)*a^4*e^{(4*I*d*x + 4*I*c)} + 6*(81*A - 74*I*B)*a^4*e^{(2*I*d*x + 2*I*c)} - (86*A - 79*I*B)*a^4 - 30*((A - I*B)*a^4*e^{(12*I*d*x + 12*I*c)} - 6*(A - I*B)*a^4*e^{(10*I*d*x + 10*I*c)} + 15*(A - I*B)*a^4*e^{(8*I*d*x + 8*I*c)} - 20*(A - I*B)*a^4*e^{(6*I*d*x + 6*I*c)} + 15*(A - I*B)*a^4*e^{(4*I*d*x + 4*I*c)} - 6*(A - I*B)*a^4*e^{(2*I*d*x + 2*I*c)} + (A - I*B)*a^4)*\log(e^{(2*I*d*x + 2*I*c)} - 1))/(d*e^{(12*I*d*x + 12*I*c)} - 6*d*e^{(10*I*d*x + 10*I*c)} + 15*d*e^{(8*I*d*x + 8*I*c)} - 20*d*e^{(6*I*d*x + 6*I*c)} + 15*d*e^{(4*I*d*x + 4*I*c)} - 6*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy** [A]

time = 1.42, size = 347, normalized size = 1.56

$$\frac{8a^4(A - iB)\log(e^{2idx} - e^{-2ic})}{d} + \frac{-344Aa^4 + 316iBa^4 + (1944Aa^4e^{2ic} - 1776iBa^4e^{2ic})e^{2idx} + (-4500Aa^4e^{4ic} + 4080iBa^4e^{4ic})e^{4idx} + (5400Aa^4e^{6ic} - 4840iBa^4e^{6ic})e^{6idx} + (-3420Aa^4e^{8ic} + 3060iBa^4e^{8ic})e^{8idx} + (1080Aa^4e^{10ic} - 840iBa^4e^{10ic})e^{10idx}}{15de^{12ic}e^{12idx} - 90de^{10ic}e^{10idx} + 225de^{8ic}e^{8idx} - 300de^{6ic}e^{6idx} + 225de^{4ic}e^{4idx} - 90de^{2ic}e^{2idx} + 15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*7\*(a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out] 
$$-8*a**4*(A - I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/d + (-344*A*a**4 + 316*I*B*a**4 + (1944*A*a**4*\exp(2*I*c) - 1776*I*B*a**4*\exp(2*I*c))*\exp(2*I*d*x) + (-4500*A*a**4*\exp(4*I*c) + 4080*I*B*a**4*\exp(4*I*c))*\exp(4*I*d*x) + (5400*A*a**4*\exp(6*I*c) - 4840*I*B*a**4*\exp(6*I*c))*\exp(6*I*d*x) + (-3420*A*a**4*\exp(8*I*c) + 3060*I*B*a**4*\exp(8*I*c))*\exp(8*I*d*x) + (1080*A*a**4*\exp(10*I*c) - 840*I*B*a**4*\exp(10*I*c))*\exp(10*I*d*x))/(15*d*\exp(12*I*c)*\exp(12*I*d*x) - 90*d*\exp(10*I*c)*\exp(10*I*d*x) + 225*d*\exp(8*I*c)*\exp(8*I*d*x) - 300*$$

$d*\exp(6*I*c)*\exp(6*I*d*x) + 225*d*\exp(4*I*c)*\exp(4*I*d*x) - 90*d*\exp(2*I*c)*\exp(2*I*d*x) + 15*d$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs.  $2(195) = 390$ .

time = 1.35, size = 459, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^7\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/1920*(5*A*a^4*\tan(1/2*d*x + 1/2*c)^6 - 48*I*A*a^4*\tan(1/2*d*x + 1/2*c)^5 \\ & - 12*B*a^4*\tan(1/2*d*x + 1/2*c)^5 - 240*A*a^4*\tan(1/2*d*x + 1/2*c)^4 + 120 \\ & *I*B*a^4*\tan(1/2*d*x + 1/2*c)^4 + 880*I*A*a^4*\tan(1/2*d*x + 1/2*c)^3 + 620* \\ & B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 2835*A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 2400*I*B \\ & *a^4*\tan(1/2*d*x + 1/2*c)^2 - 10080*I*A*a^4*\tan(1/2*d*x + 1/2*c) - 9480*B*a \\ & ^4*\tan(1/2*d*x + 1/2*c) - 30720*(A*a^4 - I*B*a^4)*\log(\tan(1/2*d*x + 1/2*c) \\ & + I) + 15360*(A*a^4 - I*B*a^4)*\log(\tan(1/2*d*x + 1/2*c)) - (37632*A*a^4*\tan \\ & (1/2*d*x + 1/2*c)^6 - 37632*I*B*a^4*\tan(1/2*d*x + 1/2*c)^6 - 10080*I*A*a^4* \\ & \tan(1/2*d*x + 1/2*c)^5 - 9480*B*a^4*\tan(1/2*d*x + 1/2*c)^5 - 2835*A*a^4*\tan \\ & (1/2*d*x + 1/2*c)^4 + 2400*I*B*a^4*\tan(1/2*d*x + 1/2*c)^4 + 880*I*A*a^4*\tan \\ & (1/2*d*x + 1/2*c)^3 + 620*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 240*A*a^4*\tan(1/2* \\ & d*x + 1/2*c)^2 - 120*I*B*a^4*\tan(1/2*d*x + 1/2*c)^2 - 48*I*A*a^4*\tan(1/2*d* \\ & x + 1/2*c) - 12*B*a^4*\tan(1/2*d*x + 1/2*c) - 5*A*a^4)/\tan(1/2*d*x + 1/2*c)^6 \\ & )/d \end{aligned}$$

**Mupad [B]**

time = 7.59, size = 162, normalized size = 0.73

$$\frac{\tan(c+d*x)^4(4Aa^4 - Ba^4i) - \tan(c+d*x)^2\left(\frac{7Aa^4}{4} - Ba^4i\right) + \tan(c+d*x)^5(8Ba^4 + Aa^48i) - \tan(c+d*x)^3\left(\frac{7Ba^4}{3} + \frac{Aa^48i}{3}\right) + \frac{Aa^4}{6} + \tan(c+d*x)\left(\frac{Ba^4}{5} + \frac{Aa^48i}{5}\right) - \frac{16a^4 \operatorname{atan}(2 \tan(c+d*x) + i)(B + Ai)}{d}}{d \tan(c+d*x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^7\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] 
$$\begin{aligned} & -(\tan(c + d*x)^4*(4*A*a^4 - B*a^4*4i) - \tan(c + d*x)^2*((7*A*a^4)/4 - B*a^4 \\ & *4i) + \tan(c + d*x)^5*(A*a^4*8i + 8*B*a^4) - \tan(c + d*x)^3*((A*a^4*8i)/3 \\ & + (7*B*a^4)/3) + (A*a^4)/6 + \tan(c + d*x)*((A*a^4*4i)/5 + (B*a^4)/5))/ (d*\tan \\ & (c + d*x)^6) - (16*a^4*\operatorname{atan}(2*\tan(c + d*x) + 1i)*(A*1i + B))/d \end{aligned}$$

$$3.36 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=129

$$\frac{3(iA - B)x}{2a} - \frac{(A + 2iB) \log(\cos(c + dx))}{ad} - \frac{3(iA - B) \tan(c + dx)}{2ad} - \frac{(A + 2iB) \tan^2(c + dx)}{2ad} + \frac{(iA - B) \tan(c + dx)}{2d(a + ia \tan(c + dx))}$$

[Out] 3/2\*(I\*A-B)\*x/a-(A+2\*I\*B)\*ln(cos(d\*x+c))/a/d-3/2\*(I\*A-B)\*tan(d\*x+c)/a/d-1/2\*(A+2\*I\*B)\*tan(d\*x+c)^2/a/d+1/2\*(I\*A-B)\*tan(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3676, 3609, 3606, 3556}

$$\frac{(-B + iA) \tan^3(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{(A + 2iB) \tan^2(c + dx)}{2ad} - \frac{3(-B + iA) \tan(c + dx)}{2ad} - \frac{(A + 2iB) \log(\cos(c + dx))}{ad} + \frac{3x(-B + iA)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] (3\*(I\*A - B)\*x)/(2\*a) - ((A + (2\*I)\*B)\*Log[Cos[c + d\*x]])/(a\*d) - (3\*(I\*A - B)\*Tan[c + d\*x])/(2\*a\*d) - ((A + (2\*I)\*B)\*Tan[c + d\*x]^2)/(2\*a\*d) + ((I\*A - B)\*Tan[c + d\*x]^3)/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rule 3609

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3676

```

Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \frac{(iA - B) \tan^3(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{\int \tan^2(c + dx)(3a(iA - B) + 2a(A + 2iB) \tan(c + dx))}{2a^2} \\
&= -\frac{(A + 2iB) \tan^2(c + dx)}{2ad} + \frac{(iA - B) \tan^3(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{\int \tan(c + dx)(A + B \tan(c + dx))}{2a} \\
&= \frac{3(iA - B)x}{2a} - \frac{3(iA - B) \tan(c + dx)}{2ad} - \frac{(A + 2iB) \tan^2(c + dx)}{2ad} \\
&= \frac{3(iA - B)x}{2a} - \frac{(A + 2iB) \log(\cos(c + dx))}{ad} - \frac{3(iA - B) \tan(c + dx)}{2ad}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 898 vs.  $2(129) = 258$ .  
time = 6.81, size = 898, normalized size = 6.96

Antiderivative was successfully verified.

```

[In] Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
[Out] ((A*Cos[c/2] + (2*I)*B*Cos[c/2] + I*A*Sin[c/2] - 2*B*Sin[c/2])*(I*ArcTan[Tan[d*x]]*Cos[c/2] - ArcTan[Tan[d*x]]*Sin[c/2])*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + ((A*Cos[c/2] + (2*I)*B*Cos[c/2] + I*A*Sin[c/2] - 2*B*Sin[c/2])*(-1/2*(Cos[c/2]*Log[Cos[c + d*x]^2]) - (I/2)*Log[Cos[c + d*x]^2]*Sin[c/2])*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + ((A + I*B)*Cos[2*d*x]*(Cos[c]/4 - (I/4)*Sin[c])*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + (Sec[c + d*x]^2*((-1/2*I)*B*Cos[c] + (B*Sin[c])/2)*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + ((A + I*B)*((3*I)/2)*d*x*Cos[c] - (3*d*x*Sin[c])/2)*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + (((-I)*A + B)*(Cos[

```



$$\begin{aligned} & c]/4 - (I/4)*\text{Sin}[c])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])*\text{Sin}[2*d*x]*(A + B*\text{Tan}[c + d*x] \\ & ))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])) + (\text{Sec}[c + \\ & d*x]*(\text{Cos}[d*x] + I*\text{Sin}[d*x])*(A*\text{Cos}[c - d*x] + I*B*\text{Cos}[c - d*x] - A*\text{Cos}[c + \\ & d*x] - I*B*\text{Cos}[c + d*x] + I*A*\text{Sin}[c - d*x] - B*\text{Sin}[c - d*x] - I*A*\text{Sin}[c + \\ & d*x] + B*\text{Sin}[c + d*x])*(A + B*\text{Tan}[c + d*x]))/(2*d*(\text{Cos}[c/2] - \text{Sin}[c/2])*(\text{Co} \\ & s[c/2] + \text{Sin}[c/2])*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x]) \\ & ) + (x*(\text{Cos}[d*x] + I*\text{Sin}[d*x])*((-I)*A*\text{Sec}[c] + 2*B*\text{Sec}[c] + (A + (2*I)*B)* \\ & (\text{Cos}[c] + I*\text{Sin}[c])*\text{Tan}[c])*(A + B*\text{Tan}[c + d*x]))/((A*\text{Cos}[c + d*x] + B*\text{Sin}[ \\ & c + d*x])*(a + I*a*\text{Tan}[c + d*x])) \end{aligned}$$

**Maple [A]**

time = 0.13, size = 98, normalized size = 0.76

method	result
derivativedivides	$\frac{B \tan(dx+c) - \frac{iB(\tan^2(dx+c))}{2} - iA \tan(dx+c) + \left(\frac{5A}{4} + \frac{7iB}{4}\right) \ln(\tan(dx+c)-i) - \frac{\frac{iA}{2} - \frac{B}{2}}{\tan(dx+c)-i} + \frac{i(iA+B) \ln(\tan(dx+c)+i)}{4}}{da}$
default	$\frac{B \tan(dx+c) - \frac{iB(\tan^2(dx+c))}{2} - iA \tan(dx+c) + \left(\frac{5A}{4} + \frac{7iB}{4}\right) \ln(\tan(dx+c)-i) - \frac{\frac{iA}{2} - \frac{B}{2}}{\tan(dx+c)-i} + \frac{i(iA+B) \ln(\tan(dx+c)+i)}{4}}{da}$
norman	$\frac{\frac{(-iA+B)(\tan^3(dx+c))}{ad} - \frac{3(-iA+B)x}{2a} + \frac{2iB+A}{2ad} + \frac{3(-iA+B)\tan(dx+c)}{2ad} - \frac{3(-iA+B)x(\tan^2(dx+c))}{2a} - \frac{iB(\tan^4(dx+c))}{2ad}}{1+\tan^2(dx+c)} + \frac{(2iB}{ad}$
risch	$-\frac{7xB}{2a} + \frac{5ixA}{2a} + \frac{ie^{-2i(dx+c)}B}{4ad} + \frac{e^{-2i(dx+c)}A}{4ad} - \frac{4BC}{ad} + \frac{2iAc}{ad} + \frac{2i(-iAe^{2i(dx+c)}-iA+B)}{da(e^{2i(dx+c)}+1)^2} - \frac{2i \ln(e^{2i(dx+c)}+1)}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d/a*(B*\text{tan}(d*x+c)-1/2*I*B*\text{tan}(d*x+c)^2-I*A*\text{tan}(d*x+c)+(5/4*A+7/4*I*B)*\ln(\text{tan}(d*x+c)-I)-(1/2*I*A-1/2*B)/(\text{tan}(d*x+c)-I)+1/4*I*(I*A+B)*\ln(\text{tan}(d*x+c)+I))$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 0.47, size = 186, normalized size = 1.44

$$\frac{2(-5iA+7B)dx e^{(6i dx+6i c)} + (4(-5iA+7B)dx - 9A - iB) e^{(4i dx+4i c)} + 2((-5iA+7B)dx - 5A - 5iB) e^{(2i dx+2i c)} + 4((A+2iB) e^{(6i dx+6i c)} + 2(A+2iB) e^{(4i dx+4i c)} + (A+2iB) e^{(2i dx+2i c)}) \log(e^{(2i dx+2i c)} + 1) - A - iB}{4(ad e^{(6i dx+6i c)} + 2ad e^{(4i dx+4i c)} + ad e^{(2i dx+2i c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-1/4*(2*(-5*I*A + 7*B)*d*x*e^{(6*I*d*x + 6*I*c)} + (4*(-5*I*A + 7*B)*d*x - 9*A - I*B)*e^{(4*I*d*x + 4*I*c)} + 2*((-5*I*A + 7*B)*d*x - 5*A - 5*I*B)*e^{(2*I*d*x + 2*I*c)} + 4*((A + 2*I*B)*e^{(6*I*d*x + 6*I*c)} + 2*(A + 2*I*B)*e^{(4*I*d*x + 4*I*c)} + (A + 2*I*B)*e^{(2*I*d*x + 2*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} + 1) - A - I*B)/(a*d*e^{(6*I*d*x + 6*I*c)} + 2*a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)})$

**Sympy [A]**

time = 0.46, size = 196, normalized size = 1.52

$$\frac{2Ae^{2ic}e^{2idx} + 2A + 2iB}{ade^{4ic}e^{4idx} + 2ade^{2ic}e^{2idx} + ad} + \begin{cases} \frac{(A+iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{5iA-7B}{2a} + \frac{(5iAe^{2ic}-iA-7Be^{2ic}+B)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(5iA-7B)}{2a} - \frac{(A+2iB)\log(e^{2idx} + e^{-2ic})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out]  $(2*A*\exp(2*I*c)*\exp(2*I*d*x) + 2*A + 2*I*B)/(a*d*\exp(4*I*c)*\exp(4*I*d*x) + 2*a*d*\exp(2*I*c)*\exp(2*I*d*x) + a*d) + \text{Piecewise}(((A + I*B)*\exp(-2*I*c)*\exp(-2*I*d*x)/(4*a*d), \text{Ne}(a*d*\exp(2*I*c), 0)), (x*(-(5*I*A - 7*B)/(2*a) + (5*I*A*\exp(2*I*c) - I*A - 7*B*\exp(2*I*c) + B)*\exp(-2*I*c)/(2*a)), \text{True})) + x*(5*I*A - 7*B)/(2*a) - (A + 2*I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/(a*d)$

**Giac [A]**

time = 0.77, size = 125, normalized size = 0.97

$$\frac{(5A+7iB)\log(\tan(dx+c)-i)}{a} - \frac{(A-iB)\log(-i\tan(dx+c)+1)}{a} - \frac{2(iBa\tan(dx+c)^2+2iAa\tan(dx+c)-2Ba\tan(dx+c))}{a^2} - \frac{5A\tan(dx+c)+7iB\tan(dx+c)-3iA+5B}{a(\tan(dx+c)-i)}$$

$4d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $1/4*((5*A + 7*I*B)*\log(\tan(d*x + c) - I)/a - (A - I*B)*\log(-I*\tan(d*x + c) + 1)/a - 2*(I*B*a*\tan(d*x + c)^2 + 2*I*A*a*\tan(d*x + c) - 2*B*a*\tan(d*x + c))/a^2 - (5*A*\tan(d*x + c) + 7*I*B*\tan(d*x + c) - 3*I*A + 5*B)/(a*(\tan(d*x + c) - I)))/d$

**Mupad [B]**

time = 6.38, size = 141, normalized size = 1.09

$$\frac{\frac{A}{2a} - \frac{A+B1i}{2a} + \frac{A+B2i}{2a}}{d(1 + \tan(c+dx)1i)} - \frac{\tan(c+dx)\left(-\frac{B}{a} + \frac{A1i}{a}\right)}{d} + \frac{\ln(\tan(c+dx)+1i)(B+A1i)1i}{4ad} + \frac{\ln(\tan(c+dx)-i)(5A+B7i)}{4ad} - \frac{B\tan(c+dx)^21i}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)`

[Out] 
$$\frac{A}{2a} - \frac{(A + B1i)}{2a} + \frac{(A + B2i)}{2a} \Big/ (d(\tan(c + d*x)1i + 1)) -$$

$$\frac{(\tan(c + d*x)((A1i)/a - B/a))/d + (\log(\tan(c + d*x) + 1i)(A1i + B)1i)}{4ad} + \frac{(\log(\tan(c + d*x) - 1i)(5A + B7i))}{4ad} - \frac{(B\tan(c + d*x)^21i)}{2ad}$$

$$3.37 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=101

$$\frac{(A+3iB)x}{2a} + \frac{(iA-B) \log(\cos(c+dx))}{ad} - \frac{(A+3iB) \tan(c+dx)}{2ad} + \frac{(iA-B) \tan^2(c+dx)}{2d(a+ia \tan(c+dx))}$$

[Out] 1/2\*(A+3\*I\*B)\*x/a+(I\*A-B)\*ln(cos(d\*x+c))/a/d-1/2\*(A+3\*I\*B)\*tan(d\*x+c)/a/d+1/2\*(I\*A-B)\*tan(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {3676, 3606, 3556}

$$\frac{(-B+iA) \tan^2(c+dx)}{2d(a+ia \tan(c+dx))} - \frac{(A+3iB) \tan(c+dx)}{2ad} + \frac{(-B+iA) \log(\cos(c+dx))}{ad} + \frac{x(A+3iB)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] ((A + (3\*I)\*B)\*x)/(2\*a) + ((I\*A - B)\*Log[Cos[c + d\*x]])/(a\*d) - ((A + (3\*I)\*B)\*Tan[c + d\*x])/(2\*a\*d) + ((I\*A - B)\*Tan[c + d\*x]^2)/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3606**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

**Rule 3676**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m+1)\*(c + d\*Tan[e + f\*x])^(n-1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m-n) - a\*A\*(m+n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)} dx &= \frac{(iA-B)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))} - \frac{\int \tan(c+dx)(2a(iA-B)+a(A+3iB))}{2a^2} \\ &= \frac{(A+3iB)x}{2a} - \frac{(A+3iB)\tan(c+dx)}{2ad} + \frac{(iA-B)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))} \\ &= \frac{(A+3iB)x}{2a} + \frac{(iA-B)\log(\cos(c+dx))}{ad} - \frac{(A+3iB)\tan(c+dx)}{2ad} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. 2(101) = 202.  
time = 4.64, size = 240, normalized size = 2.38

$$\frac{(\cos(dx) + i \sin(dx))(-4Adx \sec(c) - 4Bdx \sec(c) + (-A + B)\cos(2dx)(\cos(c) - i \sin(c)) + 2(A + 3iB)dx(\cos(c) + i \sin(c)) + 4(A + iB)\text{ArcTan}(\tan(dx))(\cos(c) + i \sin(c)) + 2(A + iB)\log(\cos^2(c + dx))(\cos(c) + i \sin(c)) + (A + iB)(-\cos(c) + i \sin(c))\sin(2dx) + 4(A + iB)dx(-i \cos(c) + \sin(c))\tan(c) + 4B \sec(c + dx)\sin(dx)(-i + \tan(c)))(A + B \tan(c + dx))}{4d(A \cos(c + dx) + B \sin(c + dx))(a + ia \tan(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
[Out] ((Cos[d*x] + I*Sin[d*x])*(-4*A*d*x*Sec[c] - (4*I)*B*d*x*Sec[c] + ((-I)*A + B)*Cos[2*d*x]*(Cos[c] - I*Sin[c]) + 2*(A + (3*I)*B)*d*x*(Cos[c] + I*Sin[c]) + 4*(A + I*B)*ArcTan[Tan[d*x]]*(Cos[c] + I*Sin[c]) + (2*I)*(A + I*B)*Log[Cos[c + d*x]^2]*(Cos[c] + I*Sin[c]) + (A + I*B)*(-Cos[c] + I*Sin[c])*Sin[2*d*x] + 4*(A + I*B)*d*x*((-I)*Cos[c] + Sin[c])*Tan[c] + 4*B*Sec[c + d*x]*Sin[d*x]*(-I + Tan[c]))*(A + B*Tan[c + d*x]))/(4*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x]))
```

**Maple [A]**

time = 0.10, size = 78, normalized size = 0.77

method	result
derivativedivides	$\frac{-iB \tan(dx+c) - \frac{\frac{A}{2} + \frac{iB}{2}}{\tan(dx+c)-i} + \left(-\frac{3iA}{4} + \frac{5B}{4}\right) \ln(\tan(dx+c)-i) - \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{4}}{da}$
default	$\frac{-iB \tan(dx+c) - \frac{\frac{A}{2} + \frac{iB}{2}}{\tan(dx+c)-i} + \left(-\frac{3iA}{4} + \frac{5B}{4}\right) \ln(\tan(dx+c)-i) - \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{4}}{da}$
norman	$\frac{\frac{(3iB+A)x}{2a} + \frac{-iA+B}{2ad} - \frac{(3iB+A)\tan(dx+c)}{2ad} + \frac{(3iB+A)x(\tan^2(dx+c))}{2a} - \frac{iB(\tan^3(dx+c))}{ad}}{1+\tan^2(dx+c)} + \frac{(-iA+B)\ln(1+\tan^2(dx+c))}{2ad}$
risch	$\frac{5ixB}{2a} + \frac{3xA}{2a} + \frac{e^{-2i(dx+c)}B}{4ad} - \frac{ie^{-2i(dx+c)}A}{4ad} + \frac{2iBc}{ad} + \frac{2Ac}{ad} + \frac{2B}{da(e^{2i(dx+c)}+1)} - \frac{\ln(e^{2i(dx+c)}+1)B}{ad} + i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[E]
```

[Out]  $1/d/a*(-I*B*\tan(d*x+c)-(1/2*A+1/2*I*B)/(\tan(d*x+c)-I)+(-3/4*I*A+5/4*B)*\ln(\tan(d*x+c)-I)-1/4*I*(A-I*B)*\ln(\tan(d*x+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.48, size = 127, normalized size = 1.26

$$\frac{2(3A+5iB)dx e^{4i dx+4i c} + (2(3A+5iB)dx - iA+9B)e^{2i dx+2i c} - 4((-iA+B)e^{4i dx+4i c} + (-iA+B)e^{2i dx+2i c}) \log(e^{2i dx+2i c} + 1) - iA+B}{4(ad e^{4i dx+4i c} + ad e^{2i dx+2i c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/4*(2*(3A+5I*B)*d*x*e^{(4I*d*x+4I*c)} + (2*(3A+5I*B)*d*x - I*A+9*B)*e^{(2I*d*x+2I*c)} - 4*((-I*A+B)*e^{(4I*d*x+4I*c)} + (-I*A+B)*e^{(2I*d*x+2I*c)})*\log(e^{(2I*d*x+2I*c)}+1) - I*A+B)/(a*d*e^{(4I*d*x+4I*c)} + a*d*e^{(2I*d*x+2I*c)})$

**Sympy** [A]

time = 0.32, size = 151, normalized size = 1.50

$$\frac{2B}{ade^{2ic}e^{2idx} + ad} + \begin{cases} \frac{(-iA+B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{3A+5iB}{2a} + \frac{(3Ae^{2ic}-A+5iBe^{2ic}-iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(3A+5iB)}{2a} + \frac{i(A+iB)\log(e^{2idx}+e^{-2ic})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)`

[Out]  $2*B/(a*d*\exp(2*I*c)*\exp(2*I*d*x) + a*d) + \text{Piecewise}((( -I*A + B)*\exp(-2*I*c)*\exp(-2*I*d*x)/(4*a*d), \text{Ne}(a*d*\exp(2*I*c), 0)), (x*(-(3*A + 5*I*B)/(2*a) + (3*A*\exp(2*I*c) - A + 5*I*B*\exp(2*I*c) - I*B)*\exp(-2*I*c)/(2*a)), \text{True})) + x*(3*A + 5*I*B)/(2*a) + I*(A + I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/(a*d)$

**Giac** [A]

time = 0.63, size = 101, normalized size = 1.00

$$\frac{(-iA-B)\log(\tan(dx+c)+i)}{a} - \frac{(3iA-5B)\log(-i\tan(dx+c)-1)}{a} - \frac{4iB\tan(dx+c)}{a} - \frac{-3iA\tan(dx+c)+5B\tan(dx+c)-A-3iB}{a(\tan(dx+c)-i)}$$


---


$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*((-I*A - B)*log(tan(d*x + c) + I)/a - (3*I*A - 5*B)*log(-I*tan(d*x + c) - 1)/a - 4*I*B*tan(d*x + c)/a - (-3*I*A*tan(d*x + c) + 5*B*tan(d*x + c) - A - 3*I*B)/(a*(tan(d*x + c) - I)))/d
```

**Mupad [B]**

time = 6.30, size = 95, normalized size = 0.94

$$-\frac{\ln(\tan(c+dx)+1i)(B+A1i)}{4ad} - \frac{B \tan(c+dx) 1i}{ad} - \frac{(A+B 1i) 1i}{2ad(1+\tan(c+dx) 1i)} - \frac{\ln(\tan(c+dx)-i)(-5B+A3i)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] - (log(tan(c + d*x) + 1i)*(A*1i + B))/(4*a*d) - (B*tan(c + d*x)*1i)/(a*d) - ((A + B*1i)*1i)/(2*a*d*(tan(c + d*x)*1i + 1)) - (log(tan(c + d*x) - 1i)*(A*3i - 5*B))/(4*a*d)
```

$$3.38 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

Optimal. Leaf size=67

$$-\frac{(iA - B)x}{2a} + \frac{iB \log(\cos(c + dx))}{ad} - \frac{A + iB}{2ad(1 + i \tan(c + dx))}$$

[Out] -1/2\*(I\*A-B)\*x/a+I\*B\*ln(cos(d\*x+c))/a/d+1/2\*(-A-I\*B)/a/d/(1+I\*tan(d\*x+c))

**Rubi [A]**

time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3670, 3556, 12, 3607, 8}

$$-\frac{A + iB}{2ad(1 + i \tan(c + dx))} - \frac{x(-B + iA)}{2a} + \frac{iB \log(\cos(c + dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] -1/2\*((I\*A - B)\*x)/a + (I\*B\*Log[Cos[c + d\*x]])/(a\*d) - (A + I\*B)/(2\*a\*d\*(1 + I\*Tan[c + d\*x]))

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3607

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(- (b\*c - a\*d))\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3670



```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)} dx &= -\frac{i \int \frac{a(iA-B)\tan(c+dx)}{a+ia\tan(c+dx)} dx}{a} - \frac{(iB) \int \tan(c+dx) dx}{a} \\ &= \frac{iB \log(\cos(c+dx))}{ad} - (-A-iB) \int \frac{\tan(c+dx)}{a+ia\tan(c+dx)} dx \\ &= \frac{iB \log(\cos(c+dx))}{ad} - \frac{A+iB}{2d(a+ia\tan(c+dx))} - \frac{(iA-B) \int 1 dx}{2a} \\ &= -\frac{(iA-B)x}{2a} + \frac{iB \log(\cos(c+dx))}{ad} - \frac{A+iB}{2d(a+ia\tan(c+dx))} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 148 vs.  $2(67) = 134$ .  
time = 0.81, size = 148, normalized size = 2.21

$$\frac{\cos(c+dx)(A+B\tan(c+dx))(iA-B-2Adx+2iBdx+2B\log(\cos^2(c+dx))+(A+iB-2iAdx-2Bdx+2iB\log(\cos^2(c+dx)))\tan(c+dx)+4BArcTan(\tan(dx))(-i+\tan(c+dx)))}{4ad(A\cos(c+dx)+B\sin(c+dx))(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] (Cos[c + d\*x]\*(A + B\*Tan[c + d\*x])\*(I\*A - B - 2\*A\*d\*x + (2\*I)\*B\*d\*x + 2\*B\*Log[Cos[c + d\*x]^2] + (A + I\*B - (2\*I)\*A\*d\*x - 2\*B\*d\*x + (2\*I)\*B\*Log[Cos[c + d\*x]^2])\*Tan[c + d\*x] + 4\*B\*ArcTan[Tan[d\*x]]\*(-I + Tan[c + d\*x]))/(4\*a\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(-I + Tan[c + d\*x]))

**Maple [A]**

time = 0.09, size = 68, normalized size = 1.01

method	result	size
derivativedivides	$\frac{\left(-\frac{A}{4}-\frac{3iB}{4}\right)\ln(\tan(dx+c)-i)-\frac{-\frac{iA}{2}+\frac{B}{2}}{\tan(dx+c)-i}-\frac{i(iA+B)\ln(\tan(dx+c)+i)}{4}}{da}$	68
default	$\frac{\left(-\frac{A}{4}-\frac{3iB}{4}\right)\ln(\tan(dx+c)-i)-\frac{-\frac{iA}{2}+\frac{B}{2}}{\tan(dx+c)-i}-\frac{i(iA+B)\ln(\tan(dx+c)+i)}{4}}{da}$	68
risch	$\frac{3xB}{2a} - \frac{ixA}{2a} - \frac{ie^{-2i(dx+c)}B}{4ad} - \frac{e^{-2i(dx+c)}A}{4ad} + \frac{2Bc}{ad} + \frac{i\ln(e^{2i(dx+c)}+1)B}{ad}$	86

norman	$\frac{\frac{(-iA+B)x}{2a} - \frac{iB+A}{2ad} - \frac{(-iA+B)\tan(dx+c)}{2ad} + \frac{(-iA+B)x(\tan^2(dx+c))}{2a}}{1+\tan^2(dx+c)} - \frac{iB \ln(1+\tan^2(dx+c))}{2ad}$	103
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d/a*((-1/4*A-3/4*I*B)*\ln(\tan(d*x+c)-I)-(-1/2*I*A+1/2*B)/(\tan(d*x+c)-I)-1/4*I*(I*A+B)*\ln(\tan(d*x+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.44, size = 66, normalized size = 0.99

$$\frac{(2(iA - 3B)dx e^{(2i dx + 2i c)} - 4i B e^{(2i dx + 2i c)} \log(e^{(2i dx + 2i c)} + 1) + A + i B) e^{(-2i dx - 2i c)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/4*(2*(I*A - 3*B)*d*x*e^{(2*I*d*x + 2*I*c)} - 4*I*B*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} + 1) + A + I*B)*e^{(-2*I*d*x - 2*I*c)}/(a*d)$

**Sympy** [A]

time = 0.24, size = 119, normalized size = 1.78

$$\frac{iB \log(e^{2idx} + e^{-2ic})}{ad} + \begin{cases} \frac{(-A-iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{-iA+3B}{2a} + \frac{(-iAe^{2ic}+iA+3Be^{2ic}-B)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(-iA+3B)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)`

[Out]  $I*B*\log(\exp(2*I*d*x) + \exp(-2*I*c))/(a*d) + \text{Piecewise}((( -A - I*B)*\exp(-2*I*c)*\exp(-2*I*d*x)/(4*a*d), \text{Ne}(a*d*\exp(2*I*c), 0)), (x*(-(-I*A + 3*B)/(2*a) +$

$(-I*A*\exp(2*I*c) + I*A + 3*B*\exp(2*I*c) - B)*\exp(-2*I*c)/(2*a)), \text{True})) + x*(-I*A + 3*B)/(2*a)$

**Giac [A]**

time = 0.49, size = 82, normalized size = 1.22

$$\frac{\frac{(A+3iB)\log(\tan(dx+c)-i)}{a} - \frac{(A-iB)\log(i\tan(dx+c)-1)}{a} - \frac{A\tan(dx+c)+3iB\tan(dx+c)+iA+B}{a(\tan(dx+c)-i)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/4*((A + 3*I*B)*\log(\tan(d*x + c) - I)/a - (A - I*B)*\log(I*\tan(d*x + c) - 1)/a - (A*\tan(d*x + c) + 3*I*B*\tan(d*x + c) + I*A + B)/(a*(\tan(d*x + c) - I)))/d$

**Mupad [B]**

time = 6.24, size = 81, normalized size = 1.21

$$-\frac{\frac{A}{2a} + \frac{B1i}{2a}}{d(1 + \tan(c + dx)1i)} + \frac{\ln(\tan(c + dx) + 1i)(A - B1i)}{4ad} - \frac{\ln(\tan(c + dx) - i)(A + B3i)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out]  $(\log(\tan(c + d*x) + 1i)*(A - B*1i))/(4*a*d) - (A/(2*a) + (B*1i)/(2*a))/(d*(\tan(c + d*x)*1i + 1)) - (\log(\tan(c + d*x) - 1i)*(A + B*3i))/(4*a*d)$

$$3.39 \quad \int \frac{A+B \tan(c+dx)}{a+ia \tan(c+dx)} dx$$

Optimal. Leaf size=47

$$\frac{(A-iB)x}{2a} + \frac{iA-B}{2d(a+ia \tan(c+dx))}$$

[Out] 1/2\*(A-I\*B)\*x/a+1/2\*(I\*A-B)/d/(a+I\*a\*tan(d\*x+c))

Rubi [A]

time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3607, 8}

$$\frac{-B+iA}{2d(a+ia \tan(c+dx))} + \frac{x(A-iB)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x]),x]

[Out] ((A - I\*B)\*x)/(2\*a) + (I\*A - B)/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A+B \tan(c+dx)}{a+ia \tan(c+dx)} dx &= \frac{iA-B}{2d(a+ia \tan(c+dx))} + \frac{(A-iB) \int 1 dx}{2a} \\ &= \frac{(A-iB)x}{2a} + \frac{iA-B}{2d(a+ia \tan(c+dx))} \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 102 vs.  $2(47) = 94$ .

time = 0.41, size = 102, normalized size = 2.17

$$\frac{\cos(c + dx)(A + B \tan(c + dx))(A - 2iAdx + B(i - 2dx) + (B - 2iBdx + A(-i + 2dx)) \tan(c + dx))}{4ad(A \cos(c + dx) + B \sin(c + dx))(-i + \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x]),x]

[Out] (Cos[c + d\*x]\*(A + B\*Tan[c + d\*x])\*(A - (2\*I)\*A\*d\*x + B\*(I - 2\*d\*x) + (B - (2\*I)\*B\*d\*x + A\*(-I + 2\*d\*x))\*Tan[c + d\*x]))/(4\*a\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))\*(-I + Tan[c + d\*x]))

Maple [A]

time = 0.08, size = 68, normalized size = 1.45

method	result	size
risch	$-\frac{ixB}{2a} + \frac{xA}{2a} - \frac{e^{-2i(dx+c)}B}{4ad} + \frac{ie^{-2i(dx+c)}A}{4ad}$	54
derivativedivides	$-\frac{-\frac{A}{2} - \frac{iB}{2}}{\tan(dx+c)-i} + \left(-\frac{iA}{4} - \frac{B}{4}\right) \ln(\tan(dx+c)-i) + \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{4}$ da	68
default	$-\frac{-\frac{A}{2} - \frac{iB}{2}}{\tan(dx+c)-i} + \left(-\frac{iA}{4} - \frac{B}{4}\right) \ln(\tan(dx+c)-i) + \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{4}$ da	68
norman	$\frac{(-iB+A)x}{2a} - \frac{-iA+B}{2ad} + \frac{(-iB+A)x(\tan^2(dx+c))}{2a} + \frac{(iB+A)\tan(dx+c)}{2ad}$ 1+tan <sup>2</sup> (dx+c)	81

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d/a\*(-(-1/2\*A-1/2\*I\*B)/(tan(d\*x+c)-I)+(-1/4\*I\*A-1/4\*B)\*ln(tan(d\*x+c)-I)+1/4\*I\*(A-I\*B)\*ln(tan(d\*x+c)+I))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]

time = 0.45, size = 42, normalized size = 0.89

$$\frac{(2(A - iB)dx e^{(2i dx + 2i c)} + iA - B) e^{(-2i dx - 2i c)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $1/4*(2*(A - I*B)*d*x*e^{(2*I*d*x + 2*I*c)} + I*A - B)*e^{(-2*I*d*x - 2*I*c)/(a*d)}$

**Sympy [A]**

time = 0.12, size = 87, normalized size = 1.85

$$\begin{cases} \frac{(iA-B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{A-iB}{2a} + \frac{(Ae^{2ic}+A-iBe^{2ic}+iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out] Piecewise(((I\*A - B)\*exp(-2\*I\*c)\*exp(-2\*I\*d\*x)/(4\*a\*d), Ne(a\*d\*exp(2\*I\*c), 0)), (x\*(-(A - I\*B)/(2\*a) + (A\*exp(2\*I\*c) + A - I\*B\*exp(2\*I\*c) + I\*B)\*exp(-2\*I\*c)/(2\*a)), True)) + x\*(A - I\*B)/(2\*a)

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs.  $2(35) = 70$ .

time = 0.48, size = 85, normalized size = 1.81

$$-\frac{\frac{(iA+B)\log(\tan(dx+c)-i)}{a} + \frac{(-iA-B)\log(-i\tan(dx+c)+1)}{a} + \frac{-iA\tan(dx+c)-B\tan(dx+c)-3A-iB}{a(\tan(dx+c)-i)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/4*((I*A + B)*\log(\tan(d*x + c) - I)/a + (-I*A - B)*\log(-I*\tan(d*x + c) + 1)/a + (-I*A*\tan(d*x + c) - B*\tan(d*x + c) - 3*A - I*B)/(a*(\tan(d*x + c) - I)))/d$

**Mupad [B]**

time = 6.17, size = 45, normalized size = 0.96

$$\frac{-\frac{B}{2a} + \frac{A \operatorname{li}}{2a}}{d(1 + \tan(c + dx) \operatorname{li})} - \frac{x(B + A \operatorname{li}) \operatorname{li}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + a\*tan(c + d\*x)\*1i),x)

[Out]  $((A*1i)/(2*a) - B/(2*a))/(d*(\tan(c + d*x)*1i + 1)) - (x*(A*1i + B)*1i)/(2*a)$

$$3.40 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

Optimal. Leaf size=62

$$-\frac{(iA - B)x}{2a} + \frac{A \log(\sin(c + dx))}{ad} + \frac{A + iB}{2d(a + ia \tan(c + dx))}$$

[Out]  $-1/2*(I*A-B)*x/a+A*\ln(\sin(d*x+c))/a/d+1/2*(A+I*B)/d/(a+I*a*\tan(d*x+c))$

Rubi [A]

time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3677, 3612, 3556}

$$\frac{A + iB}{2d(a + ia \tan(c + dx))} - \frac{x(-B + iA)}{2a} + \frac{A \log(\sin(c + dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out]  $-1/2*((I*A - B)*x)/a + (A*\text{Log}[\text{Sin}[c + d*x]])/(a*d) + (A + I*B)/(2*d*(a + I*a*\text{Tan}[c + d*x]))$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3677

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

## Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx &= \frac{A+iB}{2d(a+ia \tan(c+dx))} + \frac{\int \cot(c+dx)(2aA-a(iA-B) \tan(c+dx))}{2a^2} \\ &= -\frac{(iA-B)x}{2a} + \frac{A+iB}{2d(a+ia \tan(c+dx))} + \frac{A \int \cot(c+dx) dx}{a} \\ &= -\frac{(iA-B)x}{2a} + \frac{A \log(\sin(c+dx))}{ad} + \frac{A+iB}{2d(a+ia \tan(c+dx))} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs.  $2(62) = 124$ .  
time = 0.89, size = 150, normalized size = 2.42

$$\frac{\cos(c+dx)(A+B \tan(c+dx))(-iA+B+2Adx-2iBdx-2iA \log(\sin^2(c+dx)))+(-A-iB+2iAdx+2Bdx+2A \log(\sin^2(c+dx))) \tan(c+dx)-4iA \operatorname{ArcTan}(\tan(dx))(-i+\tan(c+dx))}{4ad(A \cos(c+dx)+B \sin(c+dx))(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]), x]

[Out] (Cos[c + d\*x]\*(A + B\*Tan[c + d\*x])\*((-I)\*A + B + 2\*A\*d\*x - (2\*I)\*B\*d\*x - (2\*I)\*A\*Log[Sin[c + d\*x]^2] + (-A - I\*B + (2\*I)\*A\*d\*x + 2\*B\*d\*x + 2\*A\*Log[Sin[c + d\*x]^2])\*Tan[c + d\*x] - (4\*I)\*A\*ArcTan[Tan[d\*x]]\*(-I + Tan[c + d\*x]))/(4\*a\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(-I + Tan[c + d\*x]))

**Maple [A]**

time = 0.23, size = 77, normalized size = 1.24

method	result	size
derivativedivides	$\frac{A \ln(\tan(dx+c)) - \frac{\frac{iA}{2} - \frac{B}{2}}{\tan(dx+c)-i} + \left(-\frac{3A}{4} - \frac{iB}{4}\right) \ln(\tan(dx+c)-i) + \left(-\frac{A}{4} + \frac{iB}{4}\right) \ln(\tan(dx+c)+i)}{da}$	77
default	$\frac{A \ln(\tan(dx+c)) - \frac{\frac{iA}{2} - \frac{B}{2}}{\tan(dx+c)-i} + \left(-\frac{3A}{4} - \frac{iB}{4}\right) \ln(\tan(dx+c)-i) + \left(-\frac{A}{4} + \frac{iB}{4}\right) \ln(\tan(dx+c)+i)}{da}$	77
risch	$\frac{xB}{2a} - \frac{3ixA}{2a} + \frac{ie^{-2i(dx+c)}B}{4ad} + \frac{e^{-2i(dx+c)}A}{4ad} - \frac{2iAc}{ad} + \frac{A \ln(e^{2i(dx+c)}-1)}{ad}$	85
norman	$\frac{\frac{iB+A}{2ad} + \frac{(-iA+B)x}{2a} + \frac{(-iA+B) \tan(dx+c)}{2ad} + \frac{(-iA+B)x \tan^2(dx+c)}{2a}}{1+\tan^2(dx+c)} + \frac{A \ln(\tan(dx+c))}{ad} - \frac{A \ln(1+\tan^2(dx+c))}{2ad}$	117

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d/a\*(A\*ln(tan(d\*x+c))-(1/2\*I\*A-1/2\*B)/(tan(d\*x+c)-I)+(-3/4\*A-1/4\*I\*B)\*ln(tan(d\*x+c)-I)+(-1/4\*A+1/4\*I\*B)\*ln(tan(d\*x+c)+I))



**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError &gt;&gt; ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 0.44, size = 68, normalized size = 1.10

$$\frac{(2(3iA - B)dx e^{(2i dx + 2i c)} - 4Ae^{(2i dx + 2i c)} \log(e^{(2i dx + 2i c)} - 1) - A - iB)e^{(-2i dx - 2i c)}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-1/4*(2*(3*I*A - B)*d*x*e^{(2*I*d*x + 2*I*c)} - 4*A*e^{(2*I*d*x + 2*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} - 1) - A - I*B)*e^{(-2*I*d*x - 2*I*c)}/(a*d)$ **Sympy [A]**

time = 0.21, size = 116, normalized size = 1.87

$$\frac{A \log(e^{2idx} - e^{-2ic})}{ad} + \begin{cases} \frac{(A+iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{-3iA+B}{2a} + \frac{(-3iAe^{2ic}-iA+Be^{2ic}+B)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(-3iA+B)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out]  $A*\log(\exp(2*I*d*x) - \exp(-2*I*c))/(a*d) + \text{Piecewise}(((A + I*B)*\exp(-2*I*c)*\exp(-2*I*d*x)/(4*a*d), \text{Ne}(a*d*\exp(2*I*c), 0)), (x*(-(-3*I*A + B)/(2*a) + (-3*I*A*\exp(2*I*c) - I*A + B*\exp(2*I*c) + B)*\exp(-2*I*c)/(2*a)), \text{True})) + x*(-3*I*A + B)/(2*a)$ **Giac [A]**

time = 0.55, size = 99, normalized size = 1.60

$$\frac{(3A+iB)\log(\tan(dx+c)-i)}{a} + \frac{(A-iB)\log(-i\tan(dx+c)+1)}{a} - \frac{4A\log(\tan(dx+c))}{a} - \frac{3A\tan(dx+c)+iB\tan(dx+c)-5iA+3B}{a(\tan(dx+c)-i)}$$


---


$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$-1/4*((3A + I*B)*\log(\tan(dx + c) - I)/a + (A - I*B)*\log(-I*\tan(dx + c) + 1)/a - 4*A*\log(\tan(dx + c))/a - (3*A*\tan(dx + c) + I*B*\tan(dx + c) - 5*I*A + 3*B)/(a*(\tan(dx + c) - I)))/d$$

**Mupad [B]**

time = 6.26, size = 98, normalized size = 1.58

$$\frac{\frac{A}{2a} + \frac{B i}{2a}}{d(1 + \tan(c + dx) i)} + \frac{A \ln(\tan(c + dx))}{ad} + \frac{\ln(\tan(c + dx) + i) (B + A i) i}{4ad} - \frac{\ln(\tan(c + dx) - i) (3A + B i)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] 
$$(A/(2*a) + (B*1i)/(2*a))/(d*(\tan(c + d*x)*1i + 1)) + (A*\log(\tan(c + d*x)))/(a*d) + (\log(\tan(c + d*x) + 1i)*(A*1i + B)*1i)/(4*a*d) - (\log(\tan(c + d*x) - 1i)*(3*A + B*1i))/(4*a*d)$$

$$3.41 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=102

$$\frac{(3A+iB)x}{2a} - \frac{(3A+iB) \cot(c+dx)}{2ad} - \frac{(iA-B) \log(\sin(c+dx))}{ad} + \frac{(A+iB) \cot(c+dx)}{2d(a+ia \tan(c+dx))}$$

[Out] -1/2\*(3\*A+I\*B)\*x/a-1/2\*(3\*A+I\*B)\*cot(d\*x+c)/a/d-(I\*A-B)\*ln(sin(d\*x+c))/a/d+1/2\*(A+I\*B)\*cot(d\*x+c)/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$\frac{(3A+iB) \cot(c+dx)}{2ad} - \frac{(-B+iA) \log(\sin(c+dx))}{ad} + \frac{(A+iB) \cot(c+dx)}{2d(a+ia \tan(c+dx))} - \frac{x(3A+iB)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] -1/2\*((3\*A + I\*B)\*x)/a - ((3\*A + I\*B)\*Cot[c + d\*x])/(2\*a\*d) - ((I\*A - B)\*Log[Sin[c + d\*x]])/(a\*d) + ((A + I\*B)\*Cot[c + d\*x])/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

## Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

## Rubi steps

$$\begin{aligned} \int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \frac{(A + iB) \cot(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \cot^2(c + dx)(a(3A + iB) - 2a(iA - B))}{2a^2} \\ &= -\frac{(3A + iB) \cot(c + dx)}{2ad} + \frac{(A + iB) \cot(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \cot(c + dx)}{2a^2} \\ &= -\frac{(3A + iB)x}{2a} - \frac{(3A + iB) \cot(c + dx)}{2ad} + \frac{(A + iB) \cot(c + dx)}{2d(a + ia \tan(c + dx))} \\ &= -\frac{(3A + iB)x}{2a} - \frac{(3A + iB) \cot(c + dx)}{2ad} - \frac{(iA - B) \log(\sin(c + dx))}{ad} \end{aligned}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 225 vs.  $2(102) = 204$ .

time = 2.70, size = 225, normalized size = 2.21

$$\frac{(\cos(dx) + i \sin(dx)) \{ (-A + B) \cos(2dx) \cos(c) - i \sin(c) + 2(A + iB) dx \cos(c) + i \sin(c) - (3A + iB) dx \cos(c) + i \sin(c) - 2(A + iB) \operatorname{ArcTan}(\tan(dx)) \cos(c) + i \sin(c) + (-iA + B) \log(\sin^2(c + dx)) \cos(c) + i \sin(c) + 2A(i + \cot(c)) \cos(c + dx) \sin(dx) - \frac{1}{2} (A + iB) \cos(c) - i \sin(c) \sin(2dx) \} (A + B \tan(c + dx))}{2d(A \cos(c + dx) + B \sin(c + dx))(a + ia \tan(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
[Out] ((Cos[d*x] + I*Sin[d*x])*((( -I)*A + B)*Cos[2*d*x]*(Cos[c] - I*Sin[c])/2 +
2*(A + I*B)*d*x*(Cos[c] + I*Sin[c]) - (3*A + I*B)*d*x*(Cos[c] + I*Sin[c])
- 2*(A + I*B)*ArcTan[Tan[d*x]]*(Cos[c] + I*Sin[c]) + (( -I)*A + B)*Log[Sin[c
+ d*x]^2]*(Cos[c] + I*Sin[c]) + 2*A*(I + Cot[c])*Csc[c + d*x]*Sin[d*x] - (
(A + I*B)*(Cos[c] - I*Sin[c])*Sin[2*d*x])/2)*(A + B*Tan[c + d*x]))/(2*d*(A*
Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x]))
```

**Maple** [A]

time = 0.21, size = 93, normalized size = 0.91

method	result
derivativedivides	$\frac{(-iA+B)\ln(\tan(dx+c)) - \frac{A}{\tan(dx+c)} + \left(\frac{5iA}{4} - \frac{3B}{4}\right)\ln(\tan(dx+c)-i) - \frac{\frac{A}{2} + \frac{iB}{2}}{\tan(dx+c)-i} - \frac{i(-iB+A)\ln(\tan(dx+c)+i)}{4}}{da}$
default	$\frac{(-iA+B)\ln(\tan(dx+c)) - \frac{A}{\tan(dx+c)} + \left(\frac{5iA}{4} - \frac{3B}{4}\right)\ln(\tan(dx+c)-i) - \frac{\frac{A}{2} + \frac{iB}{2}}{\tan(dx+c)-i} - \frac{i(-iB+A)\ln(\tan(dx+c)+i)}{4}}{da}$
risch	$-\frac{3ixB}{2a} - \frac{5xA}{2a} + \frac{e^{-2i(dx+c)}B}{4ad} - \frac{ie^{-2i(dx+c)}A}{4ad} - \frac{2iBc}{ad} - \frac{2Ac}{ad} - \frac{2iA}{ad(e^{2i(dx+c)}-1)} + \frac{\ln(e^{2i(dx+c)}-1)B}{ad}$
norman	$\frac{-\frac{A}{ad} - \frac{(iB+3A)x \tan(dx+c)}{2a} - \frac{(iB+3A)x(\tan^3(dx+c))}{2a} - \frac{(iB+3A)(\tan^2(dx+c))}{2da} + \frac{(-iA+B)\tan(dx+c)}{2ad}}{\tan(dx+c)(1+\tan^2(dx+c))} + \frac{(-iA+B)\ln(\tan(dx+c))}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a} * ((-I*A+B) * \ln(\tan(d*x+c)) - A/\tan(d*x+c) + (5/4*I*A - 3/4*B) * \ln(\tan(d*x+c) - I) - (1/2*A + 1/2*I*B) / (\tan(d*x+c) - I) - 1/4*I*(A - I*B) * \ln(\tan(d*x+c) + I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.51, size = 129, normalized size = 1.26

$$\frac{2(5A + 3iB)dx e^{4i dx + 4i c} - (2(5A + 3iB)dx - 9iA + B)e^{2i dx + 2i c} + 4((iA - B)e^{4i dx + 4i c} + (-iA + B)e^{2i dx + 2i c}) \log(e^{2i dx + 2i c} - 1) - iA + B}{4(ade^{4i dx + 4i c} - ade^{2i dx + 2i c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/4*(2*(5*A + 3*I*B)*d*x*e^{(4*I*d*x + 4*I*c)} - (2*(5*A + 3*I*B)*d*x - 9*I*A + B)*e^{(2*I*d*x + 2*I*c)} + 4*((I*A - B)*e^{(4*I*d*x + 4*I*c)} + (-I*A + B)*e^{(2*I*d*x + 2*I*c)}) * \log(e^{(2*I*d*x + 2*I*c)} - 1) - I*A + B) / (a*d*e^{(4*I*d*x + 4*I*c)} - a*d*e^{(2*I*d*x + 2*I*c)})$

**Sympy [A]**

time = 0.33, size = 158, normalized size = 1.55

$$-\frac{2iA}{ade^{2ic}e^{2idx} - ad} + \begin{cases} \frac{(-iA+B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x \left( -\frac{-5A-3iB}{2a} + \frac{(-5Ae^{2ic}-A-3iBe^{2ic}-iB)e^{-2ic}}{2a} \right) & \text{otherwise} \end{cases} + \frac{x(-5A-3iB)}{2a} - \frac{i(A+iB)\log(e^{2idx} - e^{-2ic})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

**[Out]**  $-2*I*A/(a*d*\exp(2*I*c)*\exp(2*I*d*x) - a*d) + \text{Piecewise}((( -I*A + B)*\exp(-2*I*c)*\exp(-2*I*d*x)/(4*a*d), \text{Ne}(a*d*\exp(2*I*c), 0)), (x*(-(-5*A - 3*I*B)/(2*a) + (-5*A*\exp(2*I*c) - A - 3*I*B*\exp(2*I*c) - I*B)*\exp(-2*I*c)/(2*a)), \text{True})) + x*(-5*A - 3*I*B)/(2*a) - I*(A + I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/(a*d)$

**Giac [A]**

time = 0.68, size = 135, normalized size = 1.32

$$\frac{\frac{2(-5iA+3B)\log(\tan(dx+c)-i)}{a} + \frac{2(iA+B)\log(-i\tan(dx+c)+1)}{a} + \frac{8(iA-B)\log(\tan(dx+c))}{a} + \frac{A\tan(dx+c)^2 - iB\tan(dx+c)^2 - 13iA\tan(dx+c) + 3B\tan(dx+c) - 8A}{(-i\tan(dx+c)^2 - \tan(dx+c))a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

**[Out]**  $-1/8*(2*(-5*I*A + 3*B)*\log(\tan(d*x + c) - I)/a + 2*(I*A + B)*\log(-I*\tan(d*x + c) + 1)/a + 8*(I*A - B)*\log(\tan(d*x + c))/a + (A*\tan(d*x + c)^2 - I*B*\tan(d*x + c)^2 - 13*I*A*\tan(d*x + c) + 3*B*\tan(d*x + c) - 8*A)/((-I*\tan(d*x + c)^2 - \tan(d*x + c))*a))/d$

**Mupad [B]**

time = 6.46, size = 126, normalized size = 1.24

$$-\frac{\frac{A}{a} + \tan(c+dx) \left( -\frac{B}{2a} + \frac{A3i}{2a} \right)}{d(\tan(c+dx)^2 li + \tan(c+dx))} - \frac{\ln(\tan(c+dx))(-B+Ali)}{ad} - \frac{\ln(\tan(c+dx)+1)(B+Ali)}{4ad} + \frac{\ln(\tan(c+dx)-1)(-3B+A5i)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

**[Out]**  $(\log(\tan(c + d*x) - 1i)*(A*5i - 3*B))/(4*a*d) - (\log(\tan(c + d*x))*(A*1i - B))/(a*d) - (\log(\tan(c + d*x) + 1i)*(A*1i + B))/(4*a*d) - (A/a + \tan(c + d*x))*((A*3i)/(2*a) - B/(2*a))/(d*(\tan(c + d*x) + \tan(c + d*x)^2*1i))$

$$3.42 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=131

$$\frac{3(iA - B)x}{2a} + \frac{3(iA - B) \cot(c + dx)}{2ad} - \frac{(2A + iB) \cot^2(c + dx)}{2ad} - \frac{(2A + iB) \log(\sin(c + dx))}{ad} + \frac{(A + iB) \cot^2(c + dx)}{2d(a + ia \tan(c + dx))}$$

[Out] 3/2\*(I\*A-B)\*x/a+3/2\*(I\*A-B)\*cot(d\*x+c)/a/d-1/2\*(2\*A+I\*B)\*cot(d\*x+c)^2/a/d-(2\*A+I\*B)\*ln(sin(d\*x+c))/a/d+1/2\*(A+I\*B)\*cot(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{(2A + iB) \cot^2(c + dx)}{2ad} + \frac{3(-B + iA) \cot(c + dx)}{2ad} - \frac{(2A + iB) \log(\sin(c + dx))}{ad} + \frac{(A + iB) \cot^2(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{3x(-B + iA)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]), x]

[Out] (3\*(I\*A - B)\*x)/(2\*a) + (3\*(I\*A - B)\*Cot[c + d\*x])/(2\*a\*d) - ((2\*A + I\*B)\*Cot[c + d\*x]^2)/(2\*a\*d) - ((2\*A + I\*B)\*Log[Sin[c + d\*x]])/(a\*d) + ((A + I\*B)\*Cot[c + d\*x]^2)/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

## Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \frac{(A + iB) \cot^2(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \cot^3(c + dx)(2a(2A + iB) - 3a(iA - B))}{2a^2} \\
&= -\frac{(2A + iB) \cot^2(c + dx)}{2ad} + \frac{(A + iB) \cot^2(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \cot^2(c + dx)}{2a} \\
&= \frac{3(iA - B) \cot(c + dx)}{2ad} - \frac{(2A + iB) \cot^2(c + dx)}{2ad} + \frac{(A + iB) \cot^2(c + dx)}{2d(a + ia \tan(c + dx))} \\
&= \frac{3(iA - B)x}{2a} + \frac{3(iA - B) \cot(c + dx)}{2ad} - \frac{(2A + iB) \cot^2(c + dx)}{2ad} + \frac{\int \cot^2(c + dx)}{2a} \\
&= \frac{3(iA - B)x}{2a} + \frac{3(iA - B) \cot(c + dx)}{2ad} - \frac{(2A + iB) \cot^2(c + dx)}{2ad} + \frac{\int \cot^2(c + dx)}{2a}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 902 vs. 2(131) = 262.  
time = 6.96, size = 902, normalized size = 6.89

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*A*Cos[c/2] + I*B*Cos[c/2] + (2*I)*A*Sin[c/2] - B*Sin[c/2])*(I*ArcTan[Tan[d*x]]*Cos[c/2] - ArcTan[Tan[d*x]]*Sin[c/2])*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + ((2*A*Cos[c/2] + I*B*Cos[c/2] + (2*I)*A*Sin[c/2] - B*Sin[c/2])*(-1/2*(Cos[c/2]*Log[Sin[c + d*x]^2]) - (I/2)*Log[Sin[c + d*x]^2]*Sin[c/2])*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + (x*(2*A*Csc[c] + I*B*Csc[c] + (2*A + I*B)*Cot[c]*(-Cos[c] - I*Sin[c]))*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/((A*Co
```



$$\begin{aligned} & s[c + d*x] + B*\sin[c + d*x])*(a + I*a*\tan[c + d*x])) + ((A + I*B)*\cos[2*d*x] \\ & ]*(-1/4*\cos[c] + (I/4)*\sin[c])*(\cos[d*x] + I*\sin[d*x])*(A + B*\tan[c + d*x]) \\ & )/(d*(A*\cos[c + d*x] + B*\sin[c + d*x])*(a + I*a*\tan[c + d*x])) + (\csc[c + d \\ & *x]^2*(-1/2*(A*\cos[c]) - (I/2)*A*\sin[c])*(\cos[d*x] + I*\sin[d*x])*(A + B*\tan \\ & [c + d*x]))/(d*(A*\cos[c + d*x] + B*\sin[c + d*x])*(a + I*a*\tan[c + d*x])) + \\ & ((A + I*B)*(((3*I)/2)*d*x*\cos[c] - (3*d*x*\sin[c])/2)*(\cos[d*x] + I*\sin[d*x] \\ & )*(A + B*\tan[c + d*x]))/(d*(A*\cos[c + d*x] + B*\sin[c + d*x])*(a + I*a*\tan[c \\ & + d*x])) + ((A + I*B)*((I/4)*\cos[c] + \sin[c]/4)*(\cos[d*x] + I*\sin[d*x])*Si \\ & n[2*d*x]*(A + B*\tan[c + d*x]))/(d*(A*\cos[c + d*x] + B*\sin[c + d*x])*(a + I* \\ & a*\tan[c + d*x])) + (\csc[c/2]*\csc[c + d*x]*\sec[c/2]*(\cos[d*x] + I*\sin[d*x])* \\ & ((A*\cos[c - d*x])/2 + (I/2)*B*\cos[c - d*x] - (A*\cos[c + d*x])/2 - (I/2)*B*\cos \\ & [c + d*x] + (I/2)*A*\sin[c - d*x] - (B*\sin[c - d*x])/2 - (I/2)*A*\sin[c + d \\ & *x] + (B*\sin[c + d*x])/2)*(A + B*\tan[c + d*x]))/(2*d*(A*\cos[c + d*x] + B*Si \\ & n[c + d*x])*(a + I*a*\tan[c + d*x])) \end{aligned}$$

Maple [A]

time = 0.24, size = 111, normalized size = 0.85

method	result
derivativedivides	$\frac{(-iB-2A)\ln(\tan(dx+c)) - \frac{-iA+B}{\tan(dx+c)} - \frac{A}{2\tan(dx+c)^2} - \frac{-\frac{iA}{2} + \frac{B}{2}}{\tan(dx+c)-i} + \left(\frac{7A}{4} + \frac{5iB}{4}\right)\ln(\tan(dx+c)-i) + \left(\frac{A}{4} - \frac{iB}{4}\right)\ln(\tan(dx+c))}{da}$
default	$\frac{(-iB-2A)\ln(\tan(dx+c)) - \frac{-iA+B}{\tan(dx+c)} - \frac{A}{2\tan(dx+c)^2} - \frac{-\frac{iA}{2} + \frac{B}{2}}{\tan(dx+c)-i} + \left(\frac{7A}{4} + \frac{5iB}{4}\right)\ln(\tan(dx+c)-i) + \left(\frac{A}{4} - \frac{iB}{4}\right)\ln(\tan(dx+c))}{da}$
risch	$-\frac{5xB}{2a} + \frac{7ixA}{2a} - \frac{ie^{-2i(dx+c)}B}{4ad} - \frac{e^{-2i(dx+c)}A}{4ad} - \frac{2Bc}{ad} + \frac{4iAc}{ad} - \frac{2i(Be^{2i(dx+c)}+iA-B)}{ad(e^{2i(dx+c)}-1)^2} - \frac{i\ln(e^{2i(dx+c)})}{ad}$
norman	$\frac{-\frac{A}{2ad} - \frac{(-iA+B)\tan(dx+c)}{ad} - \frac{3(-iA+B)(\tan^3(dx+c))}{2ad} - \frac{3(-iA+B)x(\tan^2(dx+c))}{2a} - \frac{3(-iA+B)x(\tan^4(dx+c))}{2a} - \frac{(iB+2A)(\tan^2(dx+c))}{2ad}}{\tan(dx+c)^2(1+\tan^2(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d/a*((-2*A-I*B)*\ln(\tan(d*x+c))-(-I*A+B)/\tan(d*x+c)-1/2*A/\tan(d*x+c)^2-(-1/2*I*A+1/2*B)/(\tan(d*x+c)-I)+(7/4*A+5/4*I*B)*\ln(\tan(d*x+c)-I)+(1/4*A-1/4*I*B)*\ln(\tan(d*x+c)+I))$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.48, size = 188, normalized size = 1.44

$$\frac{2(-7iA+5B)dx e^{6i dx+6i c} + (4(7iA-5B)dx + A + 9iB)e^{4i dx+4i c} + 2((-7iA+5B)dx - 5A - 5iB)e^{2i dx+2i c} + 4((2A+iB)e^{6i dx+6i c} - 2(2A+iB)e^{4i dx+4i c} + (2A+iB)e^{2i dx+2i c}) \log(e^{2i dx+2i c} - 1) + A + iB}{4(ad e^{6i dx+6i c} - 2ade^{4i dx+4i c} + ad e^{2i dx+2i c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-1/4*(2*(-7*I*A + 5*B)*d*x*e^{(6*I*d*x + 6*I*c)} + (4*(7*I*A - 5*B)*d*x + A + 9*I*B)*e^{(4*I*d*x + 4*I*c)} + 2*((-7*I*A + 5*B)*d*x - 5*A - 5*I*B)*e^{(2*I*d*x + 2*I*c)} + 4*((2*A + I*B)*e^{(6*I*d*x + 6*I*c)} - 2*(2*A + I*B)*e^{(4*I*d*x + 4*I*c)} + (2*A + I*B)*e^{(2*I*d*x + 2*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} - 1) + A + I*B)/(a*d*e^{(6*I*d*x + 6*I*c)} - 2*a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)})$

**Sympy** [A]

time = 0.50, size = 199, normalized size = 1.52

$$\frac{2A - 2iB e^{2ic} e^{2id x} + 2iB}{ade^{4ic} e^{4id x} - 2ade^{2ic} e^{2id x} + ad} + \begin{cases} \frac{(-A-iB)e^{-2ic}e^{-2id x}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x \left( -\frac{7iA-5B}{2a} + \frac{(7iAe^{2ic}+iA-5Be^{2ic}-B)e^{-2ic}}{2a} \right) & \text{otherwise} \end{cases} + \frac{x(7iA-5B)}{2a} - \frac{(2A+iB)\log(e^{2id x} - e^{-2ic})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out]  $(2*A - 2*I*B*\exp(2*I*c)*\exp(2*I*d*x) + 2*I*B)/(a*d*\exp(4*I*c)*\exp(4*I*d*x) - 2*a*d*\exp(2*I*c)*\exp(2*I*d*x) + a*d) + \text{Piecewise}((( -A - I*B)*\exp(-2*I*c)*\exp(-2*I*d*x)/(4*a*d), \text{Ne}(a*d*\exp(2*I*c), 0)), (x*(-(7*I*A - 5*B)/(2*a) + (7*I*A*\exp(2*I*c) + I*A - 5*B*\exp(2*I*c) - B)*\exp(-2*I*c)/(2*a)), \text{True})) + x*(7*I*A - 5*B)/(2*a) - (2*A + I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/(a*d)$

**Giac** [A]

time = 0.81, size = 165, normalized size = 1.26

$$\frac{\frac{4(2A+iB)\log(-i \tan(dx+c))}{a} - \frac{(7A+5iB)\log(\tan(dx+c)-i)}{a} - \frac{(A-iB)\log(-i \tan(dx+c)+1)}{a} + \frac{7A \tan(dx+c)+5iB \tan(dx+c)-9iA+7B}{a(\tan(dx+c)-i)} - \frac{2(6A \tan(dx+c)^2+3iB \tan(dx+c)^2+2iA \tan(dx+c)-2B \tan(dx+c)-A)}{a \tan(dx+c)^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/4*(4*(2*A + I*B)*\log(-I*\tan(d*x + c))/a - (7*A + 5*I*B)*\log(\tan(d*x + c) - I)/a - (A - I*B)*\log(-I*\tan(d*x + c) + 1)/a + (7*A*\tan(d*x + c) + 5*I*B*$

$$\frac{\tan(dx + c) - 9IA + 7B}{a(\tan(dx + c) - I)} - \frac{2(6A\tan(dx + c)^2 + 3IB\tan(dx + c)^2 + 2IA\tan(dx + c) - 2B\tan(dx + c) - A)}{a\tan(dx + c)^2} / d$$

**Mupad [B]**

time = 6.51, size = 153, normalized size = 1.17

$$-\frac{\tan(c+dx)^2 \left(\frac{3A}{2a} + \frac{B3i}{2a}\right) + \frac{A}{2a} - \tan(c+dx) \left(-\frac{B}{a} + \frac{A1i}{2a}\right)}{d (\tan(c+dx)^3 1i + \tan(c+dx)^2)} - \frac{\ln(\tan(c+dx)) (2A+B1i)}{ad} + \frac{\ln(\tan(c+dx)+1i) (A-B1i)}{4ad} + \frac{\ln(\tan(c+dx)-i) (7A+B5i)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(4\*a\*d) - (log(tan(c + d\*x))\*(2\*A + B\*1i))/(a\*d) - (tan(c + d\*x)^2\*((3\*A)/(2\*a) + (B\*3i)/(2\*a)) + A/(2\*a) - tan(c + d\*x)\*((A\*1i)/(2\*a) - B/a))/(d\*(tan(c + d\*x)^2 + tan(c + d\*x)^3\*1i)) + (log(tan(c + d\*x) - 1i)\*(7\*A + B\*5i))/(4\*a\*d)

$$3.43 \quad \int \frac{\cot^4(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=155

$$\frac{(5A + 3iB)x}{2a} + \frac{(5A + 3iB) \cot(c + dx)}{2ad} + \frac{(iA - B) \cot^2(c + dx)}{ad} - \frac{(5A + 3iB) \cot^3(c + dx)}{6ad} + \frac{2(iA - B) \log(\sin(c + dx))}{ad}$$

[Out] 1/2\*(5\*A+3\*I\*B)\*x/a+1/2\*(5\*A+3\*I\*B)\*cot(d\*x+c)/a/d+(I\*A-B)\*cot(d\*x+c)^2/a/d-1/6\*(5\*A+3\*I\*B)\*cot(d\*x+c)^3/a/d+2\*(I\*A-B)\*ln(sin(d\*x+c))/a/d+1/2\*(A+I\*B)\*cot(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{(5A + 3iB) \cot^3(c + dx)}{6ad} + \frac{(-B + iA) \cot^2(c + dx)}{ad} + \frac{(5A + 3iB) \cot(c + dx)}{2ad} + \frac{2(-B + iA) \log(\sin(c + dx))}{ad} + \frac{(A + iB) \cot^3(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{x(5A + 3iB)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^4\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] ((5\*A + (3\*I)\*B)\*x)/(2\*a) + ((5\*A + (3\*I)\*B)\*Cot[c + d\*x])/(2\*a\*d) + ((I\*A - B)\*Cot[c + d\*x]^2)/(a\*d) - ((5\*A + (3\*I)\*B)\*Cot[c + d\*x]^3)/(6\*a\*d) + (2\*(I\*A - B)\*Log[Sin[c + d\*x]])/(a\*d) + ((A + I\*B)\*Cot[c + d\*x]^3)/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[b, 0]

Q[a\*c + b\*d, 0]

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cot^4(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \frac{(A + iB) \cot^3(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \cot^4(c + dx)(a(5A + 3iB) - 4a(iA - B) \tan(c + dx))}{2a^2} \\ &= -\frac{(5A + 3iB) \cot^3(c + dx)}{6ad} + \frac{(A + iB) \cot^3(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \cot^3(c + dx)(A + B \tan(c + dx))}{2a} \\ &= \frac{(iA - B) \cot^2(c + dx)}{ad} - \frac{(5A + 3iB) \cot^3(c + dx)}{6ad} + \frac{(A + iB) \cot^3(c + dx)}{2d(a + ia \tan(c + dx))} \\ &= \frac{(5A + 3iB) \cot(c + dx)}{2ad} + \frac{(iA - B) \cot^2(c + dx)}{ad} - \frac{(5A + 3iB) \cot^3(c + dx)}{6ad} \\ &= \frac{(5A + 3iB)x}{2a} + \frac{(5A + 3iB) \cot(c + dx)}{2ad} + \frac{(iA - B) \cot^2(c + dx)}{ad} \\ &= \frac{(5A + 3iB)x}{2a} + \frac{(5A + 3iB) \cot(c + dx)}{2ad} + \frac{(iA - B) \cot^2(c + dx)}{ad} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1062 vs. 2(155) = 310.  
time = 7.05, size = 1062, normalized size = 6.85

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
[Out] ((A*Cos[c/2] + I*B*Cos[c/2] + I*A*Sin[c/2] - B*Sin[c/2])*(2*ArcTan[Tan[d*x]]*Cos[c/2] + (2*I)*ArcTan[Tan[d*x]]*Sin[c/2])*(Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])) + ((A*Cos[c/2] + I*B*Cos[c/2] + I*A*Sin[c/2] - B*Sin[c/2])*(I*Cos[c/2]*L
```

$$\begin{aligned} & \log[\sin[c + d*x]^2] - \log[\sin[c + d*x]^2 * \sin[c/2]] * (\cos[d*x] + i * \sin[d*x]) * \\ & (A + B * \tan[c + d*x]) / (d * (A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + \\ & d*x])) + (x * ((-2 * I) * A * \csc[c] + 2 * B * \csc[c] + I * (A + I * B) * \cot[c] * (2 * \cos[c] + \\ & (2 * I) * \sin[c])) * (\cos[d*x] + i * \sin[d*x]) * (A + B * \tan[c + d*x]) / ((A * \cos[c + d \\ & *x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) + ((A + I * B) * \cos[2 * d*x] * ((I/4 \\ & ) * \cos[c] + \sin[c]/4) * (\cos[d*x] + i * \sin[d*x]) * (A + B * \tan[c + d*x]) / (d * (A * \cos \\ & [c + d*x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) + (\csc[c/2] * \csc[c + d \\ & x]^2 * \sec[c/2] * (-1/12 * \cos[c] - (I/12) * \sin[c]) * (2 * A * \cos[c] - (3 * I) * A * \sin[c] + \\ & 3 * B * \sin[c]) * (\cos[d*x] + i * \sin[d*x]) * (A + B * \tan[c + d*x]) / (d * (A * \cos[c + d \\ & x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) + ((5 * A + (3 * I) * B) * ((d*x * \cos[c \\ & ])/2 + (I/2) * d*x * \sin[c]) * (\cos[d*x] + i * \sin[d*x]) * (A + B * \tan[c + d*x]) / (d * ( \\ & A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) + ((A + I * B) * (\cos[c \\ & ]/4 - (I/4) * \sin[c]) * (\cos[d*x] + i * \sin[d*x]) * \sin[2 * d*x] * (A + B * \tan[c + d*x] \\ & )) / (d * (A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) + (\csc[c/2] \\ & * \csc[c + d*x]^3 * \sec[c/2] * (\cos[d*x] + i * \sin[d*x]) * ((I/2) * A * \cos[c - d*x] - (I \\ & /2) * A * \cos[c + d*x] - (A * \sin[c - d*x])/2 + (A * \sin[c + d*x])/2) * (A + B * \tan[c \\ & + d*x])) / (6 * d * (A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) + ( \\ & \csc[c/2] * \csc[c + d*x] * \sec[c/2] * (\cos[d*x] + i * \sin[d*x]) * (((-7 * I)/2) * A * \cos[c \\ & - d*x] + (3 * B * \cos[c - d*x])/2 + ((7 * I)/2) * A * \cos[c + d*x] - (3 * B * \cos[c + d*x] \\ & ))/2 + (7 * A * \sin[c - d*x])/2 + ((3 * I)/2) * B * \sin[c - d*x] - (7 * A * \sin[c + d*x] \\ & )/2 - ((3 * I)/2) * B * \sin[c + d*x]) * (A + B * \tan[c + d*x]) / (6 * d * (A * \cos[c + d*x] + \\ & B * \sin[c + d*x]) * (a + i * a * \tan[c + d*x])) \end{aligned}$$

Maple [A]

time = 0.24, size = 129, normalized size = 0.83

method	result
derivativedivides	$\frac{(2iA-2B) \ln(\tan(dx+c)) - \frac{-iB-2A}{\tan(dx+c)} - \frac{-iA+B}{2 \tan(dx+c)^2} - \frac{A}{3 \tan(dx+c)^3} + \left(-\frac{9iA}{4} + \frac{7B}{4}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{A}{2} - \frac{iB}{2}}{\tan(dx+c)-i} + \frac{i(-iB-2A)}{\tan(dx+c)-i}}{da}$
default	$\frac{(2iA-2B) \ln(\tan(dx+c)) - \frac{-iB-2A}{\tan(dx+c)} - \frac{-iA+B}{2 \tan(dx+c)^2} - \frac{A}{3 \tan(dx+c)^3} + \left(-\frac{9iA}{4} + \frac{7B}{4}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{A}{2} - \frac{iB}{2}}{\tan(dx+c)-i} + \frac{i(-iB-2A)}{\tan(dx+c)-i}}{da}$
risch	$\frac{7ixB}{2a} + \frac{9xA}{2a} - \frac{e^{-2i(dx+c)}B}{4ad} + \frac{ie^{-2i(dx+c)}A}{4ad} + \frac{4iBc}{ad} + \frac{4Ac}{ad} + \frac{4iA e^{4i(dx+c)} - 6iA e^{2i(dx+c)} + 2B e^{2i(dx+c)} + \frac{14i}{3}}{ad(e^{2i(dx+c)}-1)^3}$
norman	$\frac{-\frac{A}{3ad} - \frac{(-iA+B) \tan(dx+c)}{2ad} + \frac{(3iB+5A) \tan^2(dx+c)}{3ad} + \frac{(3iB+5A) \tan^4(dx+c)}{2ad} + \frac{(3iB+5A)x \tan^3(dx+c)}{2a} + \frac{(3iB+5A)x \tan^5(dx+c)}{2a}}{\tan(dx+c)^3(1+\tan^2(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d/a\*((2\*I\*A-2\*B)\*ln(tan(d\*x+c))-(-2\*A-I\*B)/tan(d\*x+c)-1/2\*(-I\*A+B)/tan(d\*x+c)^2-1/3\*A/tan(d\*x+c)^3+(-9/4\*I\*A+7/4\*B)\*ln(tan(d\*x+c)-I)-(-1/2\*A-1/2\*I\*B)/(tan(d\*x+c)-I)+1/4\*I\*(A-I\*B)\*ln(tan(d\*x+c)+I))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.47, size = 249, normalized size = 1.61

$$\frac{6(9A+7iB)dx e^{6i dx+6ic} - 3(6(9A+7iB)dx - 17iA+B)e^{6i dx+6ic} + 3(6(9A+7iB)dx - 27iA+11B)e^{6i dx+6ic} - (6(9A+7iB)dx - 65iA+33B)e^{2i dx+2ic} - 24((-iA+B)e^{6i dx+6ic} + 3(iA-B)e^{6i dx+6ic}) + 3(-iA+B)e^{4i dx+4ic} + (iA-B)e^{2i dx+2ic} \log(e^{2i dx+2ic} - 1) - 3iA+3B}{12(ad e^{6i dx+6ic} - 3ad e^{6i dx+6ic} + 3ad e^{4i dx+4ic} - ad e^{2i dx+2ic})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{12} * (6 * (9 * A + 7 * I * B) * d * x * e^{(8 * I * d * x + 8 * I * c)} - 3 * (6 * (9 * A + 7 * I * B) * d * x - 17 * I * A + B) * e^{(6 * I * d * x + 6 * I * c)} + 3 * (6 * (9 * A + 7 * I * B) * d * x - 27 * I * A + 11 * B) * e^{(4 * I * d * x + 4 * I * c)} - (6 * (9 * A + 7 * I * B) * d * x - 65 * I * A + 33 * B) * e^{(2 * I * d * x + 2 * I * c)} - 24 * ((-I * A + B) * e^{(8 * I * d * x + 8 * I * c)} + 3 * (I * A - B) * e^{(6 * I * d * x + 6 * I * c)} + 3 * (-I * A + B) * e^{(4 * I * d * x + 4 * I * c)} + (I * A - B) * e^{(2 * I * d * x + 2 * I * c)}) * \log(e^{(2 * I * d * x + 2 * I * c)} - 1) - 3 * I * A + 3 * B) / (a * d * e^{(8 * I * d * x + 8 * I * c)} - 3 * a * d * e^{(6 * I * d * x + 6 * I * c)} + 3 * a * d * e^{(4 * I * d * x + 4 * I * c)} - a * d * e^{(2 * I * d * x + 2 * I * c)})$

**Sympy** [A]

time = 0.43, size = 253, normalized size = 1.63

$$\frac{12iAe^{4ic}e^{4idx} + 14iA - 6B + (-18iAe^{2ic} + 6Be^{2ic})e^{2idx}}{3ade^{6ic}e^{6idx} - 9ade^{4ic}e^{4idx} + 9ade^{2ic}e^{2idx} - 3ad} + \begin{cases} \frac{(iA-B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{9A+7iB}{2a} + \frac{(9Ae^{2ic}+A+7iBe^{2ic}+iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(9A+7iB)}{2a} + \frac{2i(A+iB)\log(e^{2idx} - e^{-2ic})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out]  $(12 * I * A * \exp(4 * I * c) * \exp(4 * I * d * x) + 14 * I * A - 6 * B + (-18 * I * A * \exp(2 * I * c) + 6 * B * \exp(2 * I * c)) * \exp(2 * I * d * x)) / (3 * a * d * \exp(6 * I * c) * \exp(6 * I * d * x) - 9 * a * d * \exp(4 * I * c) * \exp(4 * I * d * x) + 9 * a * d * \exp(2 * I * c) * \exp(2 * I * d * x) - 3 * a * d) + \text{Piecewise}(((I * A - B) * \exp(-2 * I * c) * \exp(-2 * I * d * x)) / (4 * a * d), \text{Ne}(a * d * \exp(2 * I * c), 0)), (x * (-(9 * A + 7 * I * B) / (2 * a) + (9 * A * \exp(2 * I * c) + A + 7 * I * B * \exp(2 * I * c) + I * B) * \exp(-2 * I * c) / (2 * a)), \text{True})) + x * (9 * A + 7 * I * B) / (2 * a) + 2 * I * (A + I * B) * \log(\exp(2 * I * d * x) - \exp(-2 * I * c)) / (a * d)$

**Giac [A]**

time = 0.99, size = 186, normalized size = 1.20

$$\frac{\frac{3(9A-7B)\log(\tan(dx+c)-1)}{a} + \frac{3(-1A-B)\log(-1\tan(dx+c)+1)}{a} + \frac{24(-1A+B)\log(\tan(dx+c))}{a} + \frac{3(-9A\tan(dx+c)+7B\tan(dx+c)-11A-9iB)}{a(\tan(dx+c)-1)} + \frac{2i(22A\tan(dx+c)^3+22iB\tan(dx+c)^2+12iA\tan(dx+c)^2-6B\tan(dx+c)^2-3A\tan(dx+c)-3iB\tan(dx+c)-2iA)}{a\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] -1/12\*(3\*(9\*I\*A - 7\*B)\*log(tan(d\*x + c) - I)/a + 3\*(-I\*A - B)\*log(-I\*tan(d\*x + c) + 1)/a + 24\*(-I\*A + B)\*log(tan(d\*x + c))/a + 3\*(-9\*I\*A\*tan(d\*x + c) + 7\*B\*tan(d\*x + c) - 11\*A - 9\*I\*B)/(a\*(tan(d\*x + c) - I)) + 2\*I\*(22\*A\*tan(d\*x + c)^3 + 22\*I\*B\*tan(d\*x + c)^3 + 12\*I\*A\*tan(d\*x + c)^2 - 6\*B\*tan(d\*x + c)^2 - 3\*A\*tan(d\*x + c) - 3\*I\*B\*tan(d\*x + c) - 2\*I\*A)/(a\*tan(d\*x + c)^3))/d

**Mupad [B]**

time = 6.55, size = 174, normalized size = 1.12

$$\frac{\tan(c+dx)^2\left(\frac{3A}{2a} + \frac{B1i}{2a}\right) + \tan(c+dx)^3\left(-\frac{3B}{2a} + \frac{A5i}{2a}\right) - \frac{A}{3a} + \tan(c+dx)\left(-\frac{B}{2a} + \frac{A1i}{2a}\right)}{d(\tan(c+dx)^41i + \tan(c+dx)^3)} + \frac{2\ln(\tan(c+dx))(-B+A1i)}{ad} + \frac{\ln(\tan(c+dx)+1i)(B+A1i)}{4ad} - \frac{\ln(\tan(c+dx)-1i)(-7B+A9i)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^4\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] (tan(c + d\*x)^2\*((3\*A)/(2\*a) + (B\*1i)/(2\*a)) + tan(c + d\*x)^3\*((A\*5i)/(2\*a) - (3\*B)/(2\*a)) - A/(3\*a) + tan(c + d\*x)\*((A\*1i)/(6\*a) - B/(2\*a)))/(d\*(tan(c + d\*x)^3 + tan(c + d\*x)^4\*1i)) + (2\*log(tan(c + d\*x))\*(A\*1i - B))/(a\*d) + (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(4\*a\*d) - (log(tan(c + d\*x) - 1i)\*(A\*9i - 7\*B))/(4\*a\*d)



$$3.44 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=142

$$-\frac{3(iA-3B)x}{4a^2} + \frac{(A+2iB)\log(\cos(c+dx))}{a^2d} + \frac{3(iA-3B)\tan(c+dx)}{4a^2d} + \frac{(A+2iB)\tan^2(c+dx)}{2a^2d(1+i\tan(c+dx))} + \frac{(iA-B)}{4d(a+ia\tan(c+dx))}$$

[Out]  $-3/4*(I*A-3*B)*x/a^2+(A+2*I*B)*\ln(\cos(d*x+c))/a^2/d+3/4*(I*A-3*B)*\tan(d*x+c)/a^2/d+1/2*(A+2*I*B)*\tan(d*x+c)^2/a^2/d/(1+I*\tan(d*x+c))+1/4*(I*A-B)*\tan(d*x+c)^3/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ ,

Rules used = {3676, 3606, 3556}

$$\frac{(A+2iB)\tan^2(c+dx)}{2a^2d(1+i\tan(c+dx))} + \frac{3(-3B+iA)\tan(c+dx)}{4a^2d} + \frac{(A+2iB)\log(\cos(c+dx))}{a^2d} - \frac{3x(-3B+iA)}{4a^2} + \frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]^3*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^2,x]$

[Out]  $(-3*(I*A-3*B)*x)/(4*a^2) + ((A+(2*I)*B)*\text{Log}[\text{Cos}[c+d*x]])/(a^2*d) + (3*(I*A-3*B)*\text{Tan}[c+d*x])/(4*a^2*d) + ((A+(2*I)*B)*\text{Tan}[c+d*x]^2)/(2*a^2*d*(1+I*\text{Tan}[c+d*x])) + ((I*A-B)*\text{Tan}[c+d*x]^3)/(4*d*(a+I*a*\text{Tan}[c+d*x])^2)$

**Rule 3556**

$\text{Int}[\tan[(c_.)+(d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

**Rule 3606**

$\text{Int}[(a_.+(b_.)*\tan[(e_.)+(f_.)*(x_.)])*((c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c-b*d)*x, x] + (\text{Dist}[b*c+a*d, \text{Int}[\text{Tan}[e+f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e+f*x]/f), x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[b*c+a*d, 0]$

**Rule 3676**

$\text{Int}[(a_.+(b_.)*\tan[(e_.)+(f_.)*(x_.)])^{(m_.)}*((A_.)+(B_.)*\tan[(e_.)+(f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(-A*b-a*B)*(a+b*\text{Tan}[e+f*x])^m*((c+d*\text{Tan}[e+f*x])^n/(2*a*f*m)), x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a+b*\text{Tan}[e+f*x])^{(m+1)}*(c+d*\text{Tan}[e+f*x])^{(n-1)}*\text{Simp}[A*(a*c*m+b*d*n)-B*(b*c*m+a*d*n)-d*(b*B*(m-n)-a*A*(m+n))*\text{Tan}[e+f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&$

& NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2} dx &= \frac{(iA-B)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\int \frac{\tan^2(c+dx)(3a(iA-B)+a(A+5iB)\tan(c+dx))}{a+ia\tan(c+dx)} dx}{4a^2} \\
 &= \frac{(A+2iB)\tan^2(c+dx)}{2a^2d(1+i\tan(c+dx))} + \frac{(iA-B)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2} + \frac{\int \tan(c+dx)}{4a^2} \\
 &= -\frac{3(iA-3B)x}{4a^2} + \frac{3(iA-3B)\tan(c+dx)}{4a^2d} + \frac{(A+2iB)\tan^2(c+dx)}{2a^2d(1+i\tan(c+dx))} \\
 &= -\frac{3(iA-3B)x}{4a^2} + \frac{(A+2iB)\log(\cos(c+dx))}{a^2d} + \frac{3(iA-3B)\tan(c+dx)}{4a^2d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 956 vs.  $2(142) = 284$ .  
time = 6.75, size = 956, normalized size = 6.73

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
[Out] -1/4*((2*A + (3*I)*B)*Cos[2*d*x]*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + (Sec[c + d*x]*(A*Cos[c] + (2*I)*B*Cos[c] + I*A*Sin[c] - 2*B*Sin[c]))*((-I)*ArcTan[Tan[d*x]]*Cos[c] + ArcTan[Tan[d*x]]*Sin[c])*(Cos[d*x] + I*Sin[d*x])^2*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + (Sec[c + d*x]*(A*Cos[c] + (2*I)*B*Cos[c] + I*A*Sin[c] - 2*B*Sin[c]))*((Cos[c]*Log[Cos[c + d*x]^2])/2 + (I/2)*Log[Cos[c + d*x]^2]*Sin[c])*(Cos[d*x] + I*Sin[d*x])^2*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + ((A + I*B)*Cos[4*d*x]*Sec[c + d*x]*(Cos[2*c]/16 - (I/16)*Sin[2*c])*(Cos[d*x] + I*Sin[d*x])^2*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + (((-I)*A + 3*B)*Sec[c + d*x]*((3*d*x*Cos[2*c])/4 + ((3*I)/4)*d*x*Sin[2*c]))*(Cos[d*x] + I*Sin[d*x])^2*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + ((I/4)*(2*A + (3*I)*B)*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*Sin[2*d*x]*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + (((-I)*A + B)*Sec[c + d*x]*(Cos[2*c]/16 - (I/16)*Sin[2*c])*(Cos[d*x] + I*Sin[d*x])^2*Sin[4*d*x]*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2) + ((I/2)*Sec[c]*Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^2*(-(B*Cos[2*c]
```

$$-d*x]) + B*\text{Cos}[2*c + d*x] - I*B*\text{Sin}[2*c - d*x] + I*B*\text{Sin}[2*c + d*x])*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) + (x*\text{Sec}[c + d*x]*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2*(I*A - 2*B - A*\text{Tan}[c] - (2*I)*B*\text{Tan}[c] + (A + (2*I)*B)*(-\text{Cos}[2*c] - I*\text{Sin}[2*c])*\text{Tan}[c])*(A + B*\text{Tan}[c + d*x])))/((A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2)$$

**Maple [A]**

time = 0.16, size = 98, normalized size = 0.69

method	result
derivativedivides	$\frac{-B \tan(dx+c) - \frac{-\frac{5iA}{4} + \frac{7B}{4}}{\tan(dx+c)-i} - \frac{\frac{A}{2} + \frac{iB}{2}}{2(\tan(dx+c)-i)^2} + \left(-\frac{7A}{8} - \frac{17iB}{8}\right) \ln(\tan(dx+c)-i) + \frac{i(iA+B) \ln(\tan(dx+c)+i)}{8}}{da^2}$
default	$\frac{-B \tan(dx+c) - \frac{-\frac{5iA}{4} + \frac{7B}{4}}{\tan(dx+c)-i} - \frac{\frac{A}{2} + \frac{iB}{2}}{2(\tan(dx+c)-i)^2} + \left(-\frac{7A}{8} - \frac{17iB}{8}\right) \ln(\tan(dx+c)-i) + \frac{i(iA+B) \ln(\tan(dx+c)+i)}{8}}{da^2}$
risch	$\frac{17xB}{4a^2} - \frac{7ixA}{4a^2} - \frac{3ie^{-2i(dx+c)}B}{4a^2d} - \frac{e^{-2i(dx+c)}A}{2a^2d} + \frac{ie^{-4i(dx+c)}B}{16a^2d} + \frac{e^{-4i(dx+c)}A}{16a^2d} + \frac{4Bc}{da^2} - \frac{2iAc}{da^2} - \frac{2}{da^2(e^{2i(dx+c)} + 1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERB OSE)`

[Out]  $\frac{1}{d/a^2} * (-B*\text{tan}(d*x+c) - (-5/4*I*A + 7/4*B) / (\text{tan}(d*x+c) - I) - 1/2*(1/2*A + 1/2*I*B) / (\text{tan}(d*x+c) - I)^2 + (-7/8*A - 17/8*I*B) * \ln(\text{tan}(d*x+c) - I) + 1/8*I*(I*A + B) * \ln(\text{tan}(d*x+c) + I))$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 0.49, size = 150, normalized size = 1.06

$$\frac{-4(7iA - 17B)dx e^{6i dx + 6i c} + 4((7iA - 17B)dx + 2A + 11iB)e^{4i dx + 4i c} + (7A + 11iB)e^{2i dx + 2i c} - 16((A + 2iB)e^{6i dx + 6i c} + (A + 2iB)e^{4i dx + 4i c}) \log(e^{2i dx + 2i c} + 1) - A - iB}{16(a^2 d e^{6i dx + 6i c} + a^2 d e^{4i dx + 4i c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $-1/16*(4*(7*I*A - 17*B)*d*x*e^{(6*I*d*x + 6*I*c)} + 4*((7*I*A - 17*B)*d*x + 2*A + 11*I*B)*e^{(4*I*d*x + 4*I*c)} + (7*A + 11*I*B)*e^{(2*I*d*x + 2*I*c)} - 16*((A + 2*I*B)*e^{(6*I*d*x + 6*I*c)} + (A + 2*I*B)*e^{(4*I*d*x + 4*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} + 1) - A - I*B)/(a^2*d*e^{(6*I*d*x + 6*I*c)} + a^2*d*e^{(4*I*d*x + 4*I*c)})$

**Sympy [A]**

time = 0.46, size = 262, normalized size = 1.85

$$-\frac{2iB}{a^2 d e^{2ic} e^{2idx} + a^2 d} + \begin{cases} \frac{((4Aa^2 d e^{2ic} + 4iBa^2 d e^{2ic})e^{-4idx} + (-32Aa^2 d e^{4ic} - 48iBa^2 d e^{4ic})e^{-2idx})e^{-6ic}}{64a^4 d^2} & \text{for } a^4 d^2 e^{6ic} \neq 0 \\ x \left( -\frac{7iA+17B}{4a^2} + \frac{(-7iAe^{4ic} + 4iAe^{2ic} - iA + 17Be^{4ic} - 6Be^{2ic} + B)e^{-4ic}}{4a^2} \right) & \text{otherwise} \end{cases} + \frac{x(-7iA + 17B)}{4a^2} + \frac{(A + 2iB) \log(e^{2idx} + e^{-2ic})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)`

[Out]  $-2*I*B/(a**2*d*\exp(2*I*c)*\exp(2*I*d*x) + a**2*d) + \text{Piecewise}(\left(\left(\left(4*A*a**2*d*\exp(2*I*c) + 4*I*B*a**2*d*\exp(2*I*c)\right)*\exp(-4*I*d*x) + (-32*A*a**2*d*\exp(4*I*c) - 48*I*B*a**2*d*\exp(4*I*c))*\exp(-2*I*d*x)\right)*\exp(-6*I*c)/(64*a**4*d**2), \text{Ne}(a**4*d**2*\exp(6*I*c), 0)), (x*(-(-7*I*A + 17*B)/(4*a**2) + (-7*I*A*\exp(4*I*c) + 4*I*A*\exp(2*I*c) - I*A + 17*B*\exp(4*I*c) - 6*B*\exp(2*I*c) + B)*\exp(-4*I*c)/(4*a**2)), \text{True})) + x*(-7*I*A + 17*B)/(4*a**2) + (A + 2*I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/(a**2*d)$

**Giac [A]**

time = 0.83, size = 120, normalized size = 0.85

$$\frac{\frac{2(A-iB)\log(\tan(dx+c)+i)}{a^2} + \frac{2(7A+17iB)\log(\tan(dx+c)-i)}{a^2} + \frac{16B\tan(dx+c)}{a^2} - \frac{21A\tan(dx+c)^2 + 51iB\tan(dx+c)^2 - 22iA\tan(dx+c) + 74B\tan(dx+c) - 5A - 27iB}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

[Out]  $-1/16*(2*(A - I*B)*\log(\tan(d*x + c) + I)/a^2 + 2*(7*A + 17*I*B)*\log(\tan(d*x + c) - I)/a^2 + 16*B*\tan(d*x + c)/a^2 - (21*A*\tan(d*x + c)^2 + 51*I*B*\tan(d*x + c)^2 - 22*I*A*\tan(d*x + c) + 74*B*\tan(d*x + c) - 5*A - 27*I*B)/(a^2*(\tan(d*x + c) - I)^2))/d$

**Mupad [B]**

time = 6.41, size = 141, normalized size = 0.99

$$\frac{\frac{(A+B2i)li}{a^2} + \frac{B}{2a^2} - \tan(c+dx) \left( \frac{5(A+B2i)}{4a^2} - \frac{B3i}{4a^2} \right)}{d(\tan(c+dx)^2 li + 2\tan(c+dx) - i)} + \frac{\ln(\tan(c+dx) + li)(B + A li) li}{8a^2 d} - \frac{B \tan(c+dx)}{a^2 d} - \frac{\ln(\tan(c+dx) - i)(7A + B 17i)}{8a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*li)^2,x)`

[Out]  $((\left(\left(\left(A + B*2i\right)*li\right)/a^2 + B/\left(2*a^2\right) - \tan\left(c + d*x\right)*\left(\left(5*\left(A + B*2i\right)\right)/\left(4*a^2\right) - \left(B*3i\right)/\left(4*a^2\right)\right)\right)/\left(d*\left(2*\tan\left(c + d*x\right) + \tan\left(c + d*x\right)^2*li - li\right)\right) + \left(\log\left(\tan\left(c + d*x\right) + li\right)*\left(A*li + B\right)*li\right)/\left(8*a^2*d\right) - \left(B*\tan\left(c + d*x\right)\right)/\left(a^2*d\right) - \left(\log\left(\tan\left(c + d*x\right) - li\right)*\left(7*A + B*17i\right)\right)/\left(8*a^2*d\right)$

$$3.45 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=103

$$-\frac{(A+3iB)x}{4a^2} + \frac{B \log(\cos(c+dx))}{a^2d} + \frac{iA-3B}{4a^2d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^2(c+dx)}{4d(a+ia \tan(c+dx))^2}$$

[Out]  $-1/4*(A+3*I*B)*x/a^2+B*\ln(\cos(d*x+c))/a^2/d+1/4*(I*A-3*B)/a^2/d/(1+I*\tan(d*x+c))+1/4*(I*A-B)*\tan(d*x+c)^2/d/(a+I*a*\tan(d*x+c))^2$

**Rubi** [A]

time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ ,

Rules used = {3676, 3670, 3556, 12, 3607, 8}

$$\frac{-3B+iA}{4a^2d(1+i \tan(c+dx))} - \frac{x(A+3iB)}{4a^2} + \frac{B \log(\cos(c+dx))}{a^2d} + \frac{(-B+iA) \tan^2(c+dx)}{4d(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x])^2*(A+B*\text{Tan}[c+d*x])]/(a+I*a*\text{Tan}[c+d*x])^2,x]$

[Out]  $-1/4*((A+(3*I)*B)*x)/a^2+(B*\text{Log}[\text{Cos}[c+d*x]])/(a^2*d)+(I*A-3*B)/(4*a^2*d*(1+I*\text{Tan}[c+d*x]))+((I*A-B)*\text{Tan}[c+d*x]^2)/(4*d*(a+I*a*\text{Tan}[c+d*x])^2)$

Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 3556

$\text{Int}[\tan[(c_.)+(d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3607

$\text{Int}[(a_)+(b_.)*\tan[(e_.)+(f_.)*(x_.)]^(m_)*((c_.)+(d_.)*\tan[(e_.)+(f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(-b*c-a*d)*((a+b*\text{Tan}[e+f*x])^m/(2*a*f*m)), x] + \text{Dist}[(b*c+a*d)/(2*a*b), \text{Int}[(a+b*\text{Tan}[e+f*x])^(m+1), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{EqQ}[a^2+b^2, 0] \&\& \text{LtQ}[m, 0]$

## Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B*(d/b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

## Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

## Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx &= \frac{(iA - B) \tan^2(c + dx)}{4d(a + ia \tan(c + dx))^2} - \frac{\int \frac{\tan(c + dx)(2a(iA - B) + 4iaB \tan(c + dx))}{a + ia \tan(c + dx)} dx}{4a^2} \\ &= \frac{(iA - B) \tan^2(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{i \int -\frac{2a^2(A + 3iB) \tan(c + dx)}{a + ia \tan(c + dx)} dx}{4a^3} - \frac{B \int \tan(c + dx)}{a} \\ &= \frac{B \log(\cos(c + dx))}{a^2 d} + \frac{(iA - B) \tan^2(c + dx)}{4d(a + ia \tan(c + dx))^2} - \frac{(iA - 3B) \int \frac{\tan(c + dx)}{a + ia \tan(c + dx)} dx}{2a} \\ &= \frac{B \log(\cos(c + dx))}{a^2 d} + \frac{(iA - B) \tan^2(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{iA - 3B}{4d(a^2 + ia^2 \tan^2(c + dx))} \\ &= -\frac{(A + 3iB)x}{4a^2} + \frac{B \log(\cos(c + dx))}{a^2 d} + \frac{(iA - B) \tan^2(c + dx)}{4d(a + ia \tan(c + dx))^2} + \end{aligned}$$

**Mathematica [A]**

time = 0.88, size = 185, normalized size = 1.80

$$\frac{\sec^2(c + dx)(-4iA + 8B + \cos(2(c + dx))(iA - B + 4Adx - 4iBdx - 8B \log(\cos^2(c + dx))) + 16iBArcTan(\tan(dx))(\cos(2(c + dx)) + i \sin(2(c + dx))) + A \sin(2(c + dx)) + iB \sin(2(c + dx)) + 4iAdx \sin(2(c + dx)) + 4Bdx \sin(2(c + dx)) - 8iB \log(\cos^2(c + dx)) \sin(2(c + dx)))}{16a^2 d(-i + \tan(c + dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*((-4*I)*A + 8*B + Cos[2*(c + d*x)]*(I*A - B + 4*A*d*x - (4*I)*B*d*x - 8*B*Log[Cos[c + d*x]^2])) + (16*I)*B*ArcTan[Tan[d*x]]*(Cos[2*(c +
```

$$d*x]] + I*\text{Sin}[2*(c + d*x)] + A*\text{Sin}[2*(c + d*x)] + I*B*\text{Sin}[2*(c + d*x)] + (4*I)*A*d*x*\text{Sin}[2*(c + d*x)] + 4*B*d*x*\text{Sin}[2*(c + d*x)] - (8*I)*B*\text{Log}[\text{Cos}[c + d*x]^2]*\text{Sin}[2*(c + d*x)])/(16*a^2*d*(-I + \text{Tan}[c + d*x])^2)$$

**Maple [A]**

time = 0.13, size = 89, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{5iB}{4} - \frac{3A}{4} + \left(\frac{iA}{8} - \frac{7B}{8}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{iA}{2} + \frac{B}{2}}{2(\tan(dx+c)-i)^2} - \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{8}}{d a^2}$
default	$\frac{-\frac{5iB}{4} - \frac{3A}{4} + \left(\frac{iA}{8} - \frac{7B}{8}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{iA}{2} + \frac{B}{2}}{2(\tan(dx+c)-i)^2} - \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{8}}{d a^2}$
risch	$-\frac{7ixB}{4a^2} - \frac{xA}{4a^2} - \frac{e^{-2i(dx+c)}B}{2a^2d} + \frac{ie^{-2i(dx+c)}A}{4a^2d} + \frac{e^{-4i(dx+c)}B}{16a^2d} - \frac{ie^{-4i(dx+c)}A}{16a^2d} - \frac{2iBc}{a^2d} + \frac{B \ln(e^{2i(dx+c)}+1)}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out]  $1/d/a^2*(-(-5/4*I*B-3/4*A)/(\tan(d*x+c)-I)+(1/8*I*A-7/8*B)*\ln(\tan(d*x+c)-I)-1/2*(-1/2*I*A+1/2*B)/(\tan(d*x+c)-I)^2-1/8*I*(A-I*B)*\ln(\tan(d*x+c)+I))$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 0.46, size = 84, normalized size = 0.82

$$\frac{(4(A+7iB)dx e^{4i dx+4i c} - 16 B e^{4i dx+4i c} \log(e^{2i dx+2i c} + 1) + 4(-iA+2B)e^{2i dx+2i c} + iA-B)e^{-4i dx-4i c}}{16 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $-1/16*(4*(A+7*I*B)*d*x*e^{(4*I*d*x+4*I*c)} - 16*B*e^{(4*I*d*x+4*I*c)}*\log(e^{(2*I*d*x+2*I*c)}+1) + 4*(-I*A+2*B)*e^{(2*I*d*x+2*I*c)} + I*A-B)*e^{(-4*I*d*x-4*I*c)}/(a^2*d)$

**Sympy [A]**

time = 0.36, size = 223, normalized size = 2.17

$$\frac{B \log(e^{2idx} + e^{-2ic})}{a^2 d} + \begin{cases} \frac{((-4iAa^2 de^{2ic} + 4Ba^2 de^{2ic})e^{-4idx} + (16iAa^2 de^{4ic} - 32Ba^2 de^{4ic})e^{-2idx})e^{-6ic}}{64a^4 d^2} & \text{for } a^4 d^2 e^{6ic} \neq 0 \\ x \left( -\frac{A-7iB}{4a^2} + \frac{(-Ae^{4ic} + 2Ae^{2ic} - A - 7iBe^{4ic} + 4iBe^{2ic} - iB)e^{-4ic}}{4a^2} \right) & \text{otherwise} \end{cases} + \frac{x(-A - 7iB)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

**[Out]** B\*log(exp(2\*I\*d\*x) + exp(-2\*I\*c))/(a\*\*2\*d) + Piecewise(((((-4\*I\*A\*a\*\*2\*d\*exp(2\*I\*c) + 4\*B\*a\*\*2\*d\*exp(2\*I\*c))\*exp(-4\*I\*d\*x) + (16\*I\*A\*a\*\*2\*d\*exp(4\*I\*c) - 32\*B\*a\*\*2\*d\*exp(4\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-6\*I\*c)/(64\*a\*\*4\*d\*\*2), Ne(a\*\*4\*d\*\*2\*exp(6\*I\*c), 0)), (x\*(-(-A - 7\*I\*B)/(4\*a\*\*2) + (-A\*exp(4\*I\*c) + 2\*A\*exp(2\*I\*c) - A - 7\*I\*B\*exp(4\*I\*c) + 4\*I\*B\*exp(2\*I\*c) - I\*B)\*exp(-4\*I\*c)/(4\*a\*\*2)), True)) + x\*(-A - 7\*I\*B)/(4\*a\*\*2)

**Giac [A]**

time = 0.68, size = 107, normalized size = 1.04

$$\frac{\frac{2(iA+B)\log(\tan(dx+c)+i)}{a^2} + \frac{2(-iA+7B)\log(\tan(dx+c)-i)}{a^2} + \frac{3iA \tan(dx+c)^2 - 21B \tan(dx+c)^2 - 6A \tan(dx+c) + 22iB \tan(dx+c) + 5iA + 5B}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

**[Out]** -1/16\*(2\*(I\*A + B)\*log(tan(d\*x + c) + I)/a^2 + 2\*(-I\*A + 7\*B)\*log(tan(d\*x + c) - I)/a^2 + (3\*I\*A\*tan(d\*x + c)^2 - 21\*B\*tan(d\*x + c)^2 - 6\*A\*tan(d\*x + c) + 22\*I\*B\*tan(d\*x + c) + 5\*I\*A + 5\*B)/(a^2\*(tan(d\*x + c) - I)^2))/d

**Mupad [B]**

time = 6.29, size = 114, normalized size = 1.11

$$\frac{\frac{A}{2a^2} + \frac{B1i}{a^2} + \tan(c + dx) \left( -\frac{5B}{4a^2} + \frac{A3i}{4a^2} \right)}{d (\tan(c + dx)^2 1i + 2 \tan(c + dx) - i)} - \frac{\ln(\tan(c + dx) + 1i) (B + A 1i)}{8a^2 d} + \frac{\ln(\tan(c + dx) - i) (-7B + A 1i)}{8a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

**[Out]** (A/(2\*a^2) + (B\*1i)/a^2 + tan(c + d\*x)\*((A\*3i)/(4\*a^2) - (5\*B)/(4\*a^2)))/(d\*(2\*tan(c + d\*x) + tan(c + d\*x)^2\*1i - 1i)) - (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(8\*a^2\*d) + (log(tan(c + d\*x) - 1i)\*(A\*1i - 7\*B))/(8\*a^2\*d)



$$3.46 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

Optimal. Leaf size=76

$$-\frac{(iA+B)x}{4a^2} + \frac{A+3iB}{4a^2d(1+i \tan(c+dx))} - \frac{A+iB}{4d(a+ia \tan(c+dx))^2}$$

[Out]  $-1/4*(I*A+B)*x/a^2+1/4*(A+3*I*B)/a^2/d/(1+I*\tan(d*x+c))+1/4*(-A-I*B)/d/(a+I*a*\tan(d*x+c))^2$

Rubi [A]

time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3671, 3607, 8}

$$\frac{A+3iB}{4a^2d(1+i \tan(c+dx))} - \frac{x(B+iA)}{4a^2} - \frac{A+iB}{4d(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^2,x]$

[Out]  $-1/4*((I*A+B)*x)/a^2+(A+(3*I)*B)/(4*a^2*d*(1+I*\text{Tan}[c+d*x]))-(A+I*B)/(4*d*(a+I*a*\text{Tan}[c+d*x])^2)$

Rule 8

$\text{Int}[a_, x\_Symbol] \text{ :> Simp}[a*x, x] \text{ /; FreeQ}[a, x]$

Rule 3607

$\text{Int}[(a_ + (b_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\text{tan}[(e_ + (f_)*(x_))]), x\_Symbol] \text{ :> Simp}[(-(b*c - a*d))*((a + b*\text{Tan}[e + f*x])^m/(2*a*f*m)), x] + \text{Dist}[(b*c + a*d)/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, 0]$

Rule 3671

$\text{Int}[(a_ + (b_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}*((A_ + (B_)*\text{tan}[(e_ + (f_)*(x_))]), x\_Symbol] \text{ :> Simp}[(-(A*b - a*B))*(a*c + b*d)*((a + b*\text{Tan}[e + f*x])^m/(2*a^2*f*m)), x] + \text{Dist}[1/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*\text{Simp}[A*b*c + a*B*c + a*A*d + b*B*d + 2*a*B*d*\text{Tan}[e + f*x], x], x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, A, B\}, x] \ \&\ \& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2} dx = -\frac{A+iB}{4d(a+ia\tan(c+dx))^2} - \frac{i \int \frac{a(A+iB)+2aB\tan(c+dx)}{a+ia\tan(c+dx)} dx}{2a^2}$$

$$= \frac{A+3iB}{4a^2d(1+i\tan(c+dx))} - \frac{A+iB}{4d(a+ia\tan(c+dx))^2} - \frac{(iA+B) \int 1}{4a^2}$$

$$= -\frac{(iA+B)x}{4a^2} + \frac{A+3iB}{4a^2d(1+i\tan(c+dx))} - \frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

**Mathematica [A]**

time = 0.56, size = 92, normalized size = 1.21

$$\frac{\sec^2(c+dx)(-4iB+(A+4iAdx+B(i+4dx))\cos(2(c+dx))+(-iA+B-4Adx+4iBdx)\sin(2(c+dx)))}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out] (Sec[c + d\*x]^2\*((-4\*I)\*B + (A + (4\*I)\*A\*d\*x + B\*(I + 4\*d\*x))\*Cos[2\*(c + d\*x)] + ((-I)\*A + B - 4\*A\*d\*x + (4\*I)\*B\*d\*x)\*Sin[2\*(c + d\*x)])/(16\*a^2\*d\*(-I + Tan[c + d\*x])^2)

**Maple [A]**

time = 0.10, size = 89, normalized size = 1.17

method	result	size
risch	$-\frac{xB}{4a^2} - \frac{ixA}{4a^2} + \frac{ie^{-2i(dx+c)}B}{4a^2d} - \frac{ie^{-4i(dx+c)}B}{16a^2d} - \frac{e^{-4i(dx+c)}A}{16a^2d}$	73
derivativdivides	$\frac{\frac{iA}{4} - \frac{3B}{4}}{\tan(dx+c)-i} + \left(-\frac{A}{8} + \frac{iB}{8}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{A}{2} - \frac{iB}{2}}{2(\tan(dx+c)-i)^2} - \frac{i(iA+B) \ln(\tan(dx+c)+i)}{8}}{da^2}$	89
default	$\frac{\frac{iA}{4} - \frac{3B}{4}}{\tan(dx+c)-i} + \left(-\frac{A}{8} + \frac{iB}{8}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{A}{2} - \frac{iB}{2}}{2(\tan(dx+c)-i)^2} - \frac{i(iA+B) \ln(\tan(dx+c)+i)}{8}}{da^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] 1/d/a^2\*(-(1/4\*I\*A-3/4\*B)/(tan(d\*x+c)-I)+(-1/8\*A+1/8\*I\*B)\*ln(tan(d\*x+c)-I)-1/2\*(-1/2\*A-1/2\*I\*B)/(tan(d\*x+c)-I)^2-1/8\*I\*(I\*A+B)\*ln(tan(d\*x+c)+I))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.47, size = 52, normalized size = 0.68

$$\frac{(4(iA+B)dx e^{4i dx+4i c} - 4i B e^{(2i dx+2i c)} + A + i B) e^{(-4i dx-4i c)}}{16 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out]  $-1/16*(4*(I*A + B)*d*x*e^{(4*I*d*x + 4*I*c)} - 4*I*B*e^{(2*I*d*x + 2*I*c)} + A + I*B)*e^{(-4*I*d*x - 4*I*c)}/(a^2*d)$

**Sympy** [A]

time = 0.20, size = 167, normalized size = 2.20

$$\begin{cases} \frac{(16iBa^2de^{4ic}e^{-2idx} + (-4Aa^2de^{2ic} - 4iBa^2de^{2ic})e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(-\frac{-iA-B}{4a^2} + \frac{(-iAe^{4ic} + iA - Be^{4ic} + 2Be^{2ic} - B)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-iA - B)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise(((16\*I\*B\*a\*\*2\*d\*exp(4\*I\*c)\*exp(-2\*I\*d\*x) + (-4\*A\*a\*\*2\*d\*exp(2\*I\*c) - 4\*I\*B\*a\*\*2\*d\*exp(2\*I\*c))\*exp(-4\*I\*d\*x))\*exp(-6\*I\*c)/(64\*a\*\*4\*d\*\*2), Ne(a\*\*4\*d\*\*2\*exp(6\*I\*c), 0)), (x\*(-(-I\*A - B)/(4\*a\*\*2) + (-I\*A\*exp(4\*I\*c) + I\*A - B\*exp(4\*I\*c) + 2\*B\*exp(2\*I\*c) - B)\*exp(-4\*I\*c)/(4\*a\*\*2)), True)) + x\*(-I\*A - B)/(4\*a\*\*2)

**Giac** [A]

time = 0.55, size = 109, normalized size = 1.43

$$\frac{\frac{2(A-iB)\log(-i\tan(dx+c)+1)}{a^2} - \frac{2(A-iB)\log(-i\tan(dx+c)-1)}{a^2} + \frac{3A\tan(dx+c)^2 - 3iB\tan(dx+c)^2 - 10iA\tan(dx+c) + 6B\tan(dx+c) - 3A - 5iB}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $1/16*(2*(A - I*B)*\log(-I*\tan(d*x + c) + 1)/a^2 - 2*(A - I*B)*\log(-I*\tan(d*x + c) - 1)/a^2 + (3*A*\tan(d*x + c)^2 - 3*I*B*\tan(d*x + c)^2 - 10*I*A*\tan(d*x + c) + 6*B*\tan(d*x + c) - 3*A - 5*I*B)/(a^2*(\tan(d*x + c) - I)^2))/d$

**Mupad [B]**

time = 6.17, size = 106, normalized size = 1.39

$$\frac{\frac{B}{2a^2} + \tan(c + dx) \left( \frac{A}{4a^2} + \frac{B3i}{4a^2} \right)}{d (\tan(c + dx)^2 i + 2 \tan(c + dx) - i)} + \frac{\ln(\tan(c + dx) - i) (B + A i) i}{8 a^2 d} + \frac{\ln(\tan(c + dx) + i) (A - B i)}{8 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x)*(A + B*\tan(c + d*x)))/(a + a*\tan(c + d*x)*1i)^2, x)$

[Out]  $(B/(2*a^2) + \tan(c + d*x)*(A/(4*a^2) + (B*3i)/(4*a^2)))/(d*(2*\tan(c + d*x) + \tan(c + d*x)^2*1i - 1i)) + (\log(\tan(c + d*x) - 1i)*(A*1i + B)*1i)/(8*a^2*d) + (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(8*a^2*d)$

$$3.47 \quad \int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^2} dx$$

Optimal. Leaf size=80

$$\frac{(A-iB)x}{4a^2} + \frac{iA-B}{4d(a+ia \tan(c+dx))^2} + \frac{iA+B}{4d(a^2+ia^2 \tan(c+dx))}$$

[Out] 1/4\*(A-I\*B)\*x/a^2+1/4\*(I\*A-B)/d/(a+I\*a\*tan(d\*x+c))^2+1/4\*(I\*A+B)/d/(a^2+I\*a^2\*tan(d\*x+c))

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3607, 3560, 8}

$$\frac{B+iA}{4d(a^2+ia^2 \tan(c+dx))} + \frac{x(A-iB)}{4a^2} + \frac{-B+iA}{4d(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out] ((A - I\*B)\*x)/(4\*a^2) + (I\*A - B)/(4\*d\*(a + I\*a\*Tan[c + d\*x])^2) + (I\*A + B)/(4\*d\*(a^2 + I\*a^2\*Tan[c + d\*x]))

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[a\*((a + b\*Tan[c + d\*x])^n/(2\*b\*d\*n), x] + Dist[1/(2\*a), Int[(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^2} dx &= \frac{iA - B}{4d(a + ia \tan(c + dx))^2} + \frac{(A - iB) \int \frac{1}{a + ia \tan(c + dx)} dx}{2a} \\ &= \frac{iA - B}{4d(a + ia \tan(c + dx))^2} + \frac{iA + B}{4d(a^2 + ia^2 \tan(c + dx))} + \frac{(A - iB) \int 1 dx}{4a^2} \\ &= \frac{(A - iB)x}{4a^2} + \frac{iA - B}{4d(a + ia \tan(c + dx))^2} + \frac{iA + B}{4d(a^2 + ia^2 \tan(c + dx))} \end{aligned}$$

**Mathematica [A]**

time = 0.53, size = 94, normalized size = 1.18

$$\frac{\sec^2(c + dx)(4iA + (B(-1 - 4idx) + A(i + 4dx)) \cos(2(c + dx)) + (A + iB + 4iAdx + 4Bdx) \sin(2(c + dx)))}{16a^2 d(-i + \tan(c + dx))^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^2, x]`

```
[Out] -1/16*(Sec[c + d*x]^2*((4*I)*A + (B*(-1 - (4*I)*d*x) + A*(I + 4*d*x))*Cos[2*(c + d*x)] + (A + I*B + (4*I)*A*d*x + 4*B*d*x)*Sin[2*(c + d*x)])/(a^2*d*(-I + Tan[c + d*x])^2)
```

**Maple [A]**

time = 0.10, size = 89, normalized size = 1.11

method	result	size
risch	$-\frac{ixB}{4a^2} + \frac{xA}{4a^2} + \frac{ie^{-2i(dx+c)}A}{4a^2d} - \frac{e^{-4i(dx+c)}B}{16a^2d} + \frac{ie^{-4i(dx+c)}A}{16a^2d}$	73
derivativedivides	$-\frac{-\frac{A}{4} + \frac{iB}{4}}{\tan(dx+c)-i} - \frac{\frac{iA}{2} - \frac{B}{2}}{2(\tan(dx+c)-i)^2} + \left(-\frac{iA}{8} - \frac{B}{8}\right) \ln(\tan(dx+c)-i) + \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{8}$	89
default	$-\frac{-\frac{A}{4} + \frac{iB}{4}}{\tan(dx+c)-i} - \frac{\frac{iA}{2} - \frac{B}{2}}{2(\tan(dx+c)-i)^2} + \left(-\frac{iA}{8} - \frac{B}{8}\right) \ln(\tan(dx+c)-i) + \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{8}$	89

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2, x, method=_RETURNVERBOSE)`

```
[Out] 1/d/a^2*(-(-1/4*A+1/4*I*B)/(tan(d*x+c)-I)-1/2*(1/2*I*A-1/2*B)/(tan(d*x+c)-I)^2+(-1/8*I*A-1/8*B)*ln(tan(d*x+c)-I)+1/8*I*(A-I*B)*ln(tan(d*x+c)+I))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.40, size = 54, normalized size = 0.68

$$\frac{(4(A-iB)dx e^{(4i dx+4i c)} + 4i A e^{(2i dx+2i c)} + i A - B) e^{(-4i dx-4i c)}}{16 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{16} * (4 * (A - I * B) * d * x * e^{(4 * I * d * x + 4 * I * c)} + 4 * I * A * e^{(2 * I * d * x + 2 * I * c)} + I * A - B) * e^{(-4 * I * d * x - 4 * I * c)} / (a^2 * d)$

**Sympy** [A]

time = 0.18, size = 162, normalized size = 2.02

$$\begin{cases} \frac{(16iAa^2de^{4ic}e^{-2idx} + (4iAa^2de^{2ic} - 4Ba^2de^{2ic})e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x \left( -\frac{A-iB}{4a^2} + \frac{(Ae^{4ic} + 2Ae^{2ic} + A - iBe^{4ic} + iB)e^{-4ic}}{4a^2} \right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise(((16\*I\*A\*a\*\*2\*d\*exp(4\*I\*c))\*exp(-2\*I\*d\*x) + (4\*I\*A\*a\*\*2\*d\*exp(2\*I\*c) - 4\*B\*a\*\*2\*d\*exp(2\*I\*c))\*exp(-4\*I\*d\*x))\*exp(-6\*I\*c)/(64\*a\*\*4\*d\*\*2), Ne(a\*\*4\*d\*\*2\*exp(6\*I\*c), 0)), (x\*(-(A - I\*B)/(4\*a\*\*2) + (A\*exp(4\*I\*c) + 2\*A\*exp(2\*I\*c) + A - I\*B\*exp(4\*I\*c) + I\*B)\*exp(-4\*I\*c)/(4\*a\*\*2)), True)) + x\*(A - I\*B)/(4\*a\*\*2)

**Giac** [A]

time = 0.57, size = 110, normalized size = 1.38

$$\frac{\frac{2(-iA-B)\log(\tan(dx+c)+i)}{a^2} - \frac{2(-iA-B)\log(\tan(dx+c)-i)}{a^2} - \frac{3iA\tan(dx+c)^2+3B\tan(dx+c)^2+10A\tan(dx+c)-10iB\tan(dx+c)-11iA-3B}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $-1/16 * (2 * (-I * A - B) * \log(\tan(dx + c) + I) / a^2 - 2 * (-I * A - B) * \log(\tan(dx + c) - I) / a^2 - (3 * I * A * \tan(dx + c)^2 + 3 * B * \tan(dx + c)^2 + 10 * A * \tan(dx + c) - 10 * I * B * \tan(dx + c) - 11 * I * A - 3 * B) / (a^2 * (\tan(dx + c) - I)^2)) / d$

**Mupad [B]**

time = 6.17, size = 70, normalized size = 0.88

$$\frac{\frac{A}{2a^2} + \tan(c + dx) \left( \frac{B}{4a^2} + \frac{A1i}{4a^2} \right)}{d (\tan(c + dx)^2 1i + 2 \tan(c + dx) - i)} - \frac{x (B + A 1i) 1i}{4 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] (A/(2\*a^2) + tan(c + d\*x)\*((A\*1i)/(4\*a^2) + B/(4\*a^2)))/(d\*(2\*tan(c + d\*x) + tan(c + d\*x)^2\*1i - 1i)) - (x\*(A\*1i + B)\*1i)/(4\*a^2)



$$3.48 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=95

$$-\frac{(3iA - B)x}{4a^2} + \frac{A \log(\sin(c + dx))}{a^2 d} + \frac{3A + iB}{4a^2 d(1 + i \tan(c + dx))} + \frac{A + iB}{4d(a + ia \tan(c + dx))^2}$$

[Out]  $-1/4*(3*I*A-B)*x/a^2+A*\ln(\sin(d*x+c))/a^2/d+1/4*(3*A+I*B)/a^2/d/(1+I*\tan(d*x+c))+1/4*(A+I*B)/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3677, 3612, 3556}

$$\frac{3A + iB}{4a^2 d(1 + i \tan(c + dx))} - \frac{x(-B + 3iA)}{4a^2} + \frac{A \log(\sin(c + dx))}{a^2 d} + \frac{A + iB}{4d(a + ia \tan(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out]  $-1/4*(((3*I)*A - B)*x)/a^2 + (A*\text{Log}[\text{Sin}[c + d*x]])/(a^2*d) + (3*A + I*B)/(4*a^2*d*(1 + I*\text{Tan}[c + d*x])) + (A + I*B)/(4*d*(a + I*a*\text{Tan}[c + d*x])^2)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3677

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(n\_)), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0]

&& LtQ[m, 0] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx &= \frac{A+iB}{4d(a+ia \tan(c+dx))^2} + \frac{\int \frac{\cot(c+dx)(4aA-2a(iA-B) \tan(c+dx))}{a+ia \tan(c+dx)} dx}{4a^2} \\
 &= \frac{3A+iB}{4a^2d(1+i \tan(c+dx))} + \frac{A+iB}{4d(a+ia \tan(c+dx))^2} + \frac{\int \cot(c+dx)}{4a^2d(1+i \tan(c+dx))} \\
 &= -\frac{(3iA-B)x}{4a^2} + \frac{3A+iB}{4a^2d(1+i \tan(c+dx))} + \frac{A+iB}{4d(a+ia \tan(c+dx))^2} \\
 &= -\frac{(3iA-B)x}{4a^2} + \frac{A \log(\sin(c+dx))}{a^2d} + \frac{3A+iB}{4a^2d(1+i \tan(c+dx))} + \frac{A+iB}{4d(a+ia \tan(c+dx))^2}
 \end{aligned}$$

**Mathematica** [A]

time = 1.02, size = 184, normalized size = 1.94

$$\frac{i \sec^2(c+dx) (-8iA+4B+\cos(2(c+dx))(-A+B+4Adx-4iBdx-8iA \log(\sin^2(c+dx))) - 16iA \operatorname{ArcTan}(\tan(dx))(\cos(2(c+dx))+i \sin(2(c+dx))) - A \sin(2(c+dx)) - iB \sin(2(c+dx)) + 4iAdx \sin(2(c+dx)) + 4Bdx \sin(2(c+dx)) + 8A \log(\sin^2(c+dx)) \sin(2(c+dx)))}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out] ((-1/16\*I)\*Sec[c + d\*x]^2\*((-8\*I)\*A + 4\*B + Cos[2\*(c + d\*x)]\*((-I)\*A + B + 4\*A\*d\*x - (4\*I)\*B\*d\*x - (8\*I)\*A\*Log[Sin[c + d\*x]^2]) - 16\*A\*ArcTan[Tan[d\*x]]\*(Cos[2\*(c + d\*x)] + I\*Sin[2\*(c + d\*x)]) - A\*Sin[2\*(c + d\*x)] - I\*B\*Sin[2\*(c + d\*x)] + (4\*I)\*A\*d\*x\*Sin[2\*(c + d\*x)] + 4\*B\*d\*x\*Sin[2\*(c + d\*x)] + 8\*A\*Log[Sin[c + d\*x]^2]\*Sin[2\*(c + d\*x)]))/(a^2\*d\*(-I + Tan[c + d\*x])^2)

**Maple** [A]

time = 0.25, size = 98, normalized size = 1.03

method	result
derivativedivides	$\frac{A \ln(\tan(dx+c)) + \left(-\frac{7A}{8} - \frac{iB}{8}\right) \ln(\tan(dx+c)-i) - \frac{\frac{3iA}{4} - \frac{B}{4}}{\tan(dx+c)-i} - \frac{\frac{A}{2} + \frac{iB}{2}}{2(\tan(dx+c)-i)^2} + \left(-\frac{A}{8} + \frac{iB}{8}\right) \ln(\tan(dx+c)+i)}{d a^2}$
default	$\frac{A \ln(\tan(dx+c)) + \left(-\frac{7A}{8} - \frac{iB}{8}\right) \ln(\tan(dx+c)-i) - \frac{\frac{3iA}{4} - \frac{B}{4}}{\tan(dx+c)-i} - \frac{\frac{A}{2} + \frac{iB}{2}}{2(\tan(dx+c)-i)^2} + \left(-\frac{A}{8} + \frac{iB}{8}\right) \ln(\tan(dx+c)+i)}{d a^2}$
risch	$\frac{xB}{4a^2} - \frac{7ixA}{4a^2} + \frac{ie^{-2i(dx+c)}B}{4a^2d} + \frac{e^{-2i(dx+c)}A}{2a^2d} + \frac{ie^{-4i(dx+c)}B}{16a^2d} + \frac{e^{-4i(dx+c)}A}{16a^2d} - \frac{2iAc}{da^2} + \frac{A \ln(e^{2i(dx+c)}-1)}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out]  $1/d/a^2*(A*\ln(\tan(d*x+c))+(-7/8*A-1/8*I*B)*\ln(\tan(d*x+c)-I)-(3/4*I*A-1/4*B)/(\tan(d*x+c)-I)-1/2*(1/2*A+1/2*I*B)/(\tan(d*x+c)-I)^2+(-1/8*A+1/8*I*B)*\ln(\tan(d*x+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.47, size = 86, normalized size = 0.91

$$\frac{(4(7iA - B)dx e^{(4i dx + 4i c)} - 16Ae^{(4i dx + 4i c)} \log(e^{(2i dx + 2i c)} - 1) - 4(2A + iB)e^{(2i dx + 2i c)} - A - iB)e^{(-4i dx - 4i c)}}{16a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $-1/16*(4*(7*I*A - B)*d*x*e^{(4*I*d*x + 4*I*c)} - 16*A*e^{(4*I*d*x + 4*I*c)}*\log(e^{(2*I*d*x + 2*I*c)} - 1) - 4*(2*A + I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*e^{(-4*I*d*x - 4*I*c)}/(a^2*d)$

**Sympy** [A]

time = 0.29, size = 219, normalized size = 2.31

$$\frac{A \log(e^{2idx} - e^{-2ic})}{a^2d} + \begin{cases} \frac{((4Aa^2de^{2ic} + 4iBa^2de^{2ic})e^{-4idx} + (32Aa^2de^{4ic} + 16iBa^2de^{4ic})e^{-2idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(-\frac{7iA+B}{4a^2} + \frac{(-7iAe^{4ic} - 4iAe^{2ic} - iA + Be^{4ic} + 2Be^{2ic} + B)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-7iA + B)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)`

[Out]  $A*\log(\exp(2*I*d*x) - \exp(-2*I*c))/(a**2*d) + \text{Piecewise}(\left(\left(\left(4*A*a**2*d*\exp(2*I*c) + 4*I*B*a**2*d*\exp(2*I*c)\right)*\exp(-4*I*d*x) + (32*A*a**2*d*\exp(4*I*c) + 16*I*B*a**2*d*\exp(4*I*c))*\exp(-2*I*d*x)\right)*\exp(-6*I*c)/(64*a**4*d**2), \text{Ne}(a**4*d**2*\exp(6*I*c), 0)), (x*(-(-7*I*A + B)/(4*a**2) + (-7*I*A*\exp(4*I*c) - 4*I*A*\exp(2*I*c) - I*A + B*\exp(4*I*c) + 2*B*\exp(2*I*c) + B)*\exp(-4*I*c)/(4*a**2)), \text{True})) + x*(-7*I*A + B)/(4*a**2)$

**Giac [A]**

time = 0.74, size = 121, normalized size = 1.27

$$\frac{\frac{2(A-iB)\log(\tan(dx+c)+i)}{a^2} + \frac{2(7A+iB)\log(\tan(dx+c)-i)}{a^2} - \frac{16A\log(\tan(dx+c))}{a^2} - \frac{21A\tan(dx+c)^2+3iB\tan(dx+c)^2-54iA\tan(dx+c)+10B\tan(dx+c)-37A-11iB}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] -1/16\*(2\*(A - I\*B)\*log(tan(d\*x + c) + I)/a^2 + 2\*(7\*A + I\*B)\*log(tan(d\*x + c) - I)/a^2 - 16\*A\*log(tan(d\*x + c))/a^2 - (21\*A\*tan(d\*x + c)^2 + 3\*I\*B\*tan(d\*x + c)^2 - 54\*I\*A\*tan(d\*x + c) + 10\*B\*tan(d\*x + c) - 37\*A - 11\*I\*B)/(a^2\*(tan(d\*x + c) - I)^2))/d

**Mupad [B]**

time = 6.31, size = 129, normalized size = 1.36

$$\frac{\frac{B}{2a^2} - \frac{4i}{a^2} + \tan(c+dx)\left(\frac{3A}{4a^2} + \frac{B1i}{4a^2}\right)}{d(\tan(c+dx)^2 1i + 2\tan(c+dx) - i)} + \frac{A \ln(\tan(c+dx))}{a^2 d} - \frac{\ln(\tan(c+dx) + 1i)(A - B 1i)}{8a^2 d} - \frac{\ln(\tan(c+dx) - i)(7A + B 1i)}{8a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] (B/(2\*a^2) - (A\*1i)/a^2 + tan(c + d\*x)\*((3\*A)/(4\*a^2) + (B\*1i)/(4\*a^2)))/(d\*(2\*tan(c + d\*x) + tan(c + d\*x)^2\*1i - 1i)) + (A\*log(tan(c + d\*x)))/(a^2\*d) - (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(8\*a^2\*d) - (log(tan(c + d\*x) - 1i)\*(7\*A + B\*1i))/(8\*a^2\*d)

$$3.49 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=141

$$-\frac{3(3A+iB)x}{4a^2} - \frac{3(3A+iB) \cot(c+dx)}{4a^2d} - \frac{(2iA-B) \log(\sin(c+dx))}{a^2d} + \frac{(2A+iB) \cot(c+dx)}{2a^2d(1+i \tan(c+dx))} + \frac{(A+iB)}{4d(a+ia \tan(c+dx))}$$

[Out]  $-3/4*(3*A+I*B)*x/a^2-3/4*(3*A+I*B)*\cot(d*x+c)/a^2/d-(2*I*A-B)*\ln(\sin(d*x+c))/a^2/d+1/2*(2*A+I*B)*\cot(d*x+c)/a^2/d/(1+I*\tan(d*x+c))+1/4*(A+I*B)*\cot(d*x+c)/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.24, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{3(3A+iB) \cot(c+dx)}{4a^2d} - \frac{(-B+2iA) \log(\sin(c+dx))}{a^2d} + \frac{(2A+iB) \cot(c+dx)}{2a^2d(1+i \tan(c+dx))} - \frac{3x(3A+iB)}{4a^2} + \frac{(A+iB) \cot(c+dx)}{4d(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c+d*x]^2*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^2,x]$

[Out]  $(-3*(3*A+I*B)*x)/(4*a^2) - (3*(3*A+I*B)*\text{Cot}[c+d*x])/(4*a^2*d) - ((2*I*A-B)*\text{Log}[\text{Sin}[c+d*x]])/(a^2*d) + ((2*A+I*B)*\text{Cot}[c+d*x])/(2*a^2*d*(1+I*\text{Tan}[c+d*x])) + ((A+I*B)*\text{Cot}[c+d*x])/(4*d*(a+I*a*\text{Tan}[c+d*x])^2)$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3610**

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m+1)})/(f*(m+1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

**Rule 3612**

$\text{Int}[(c_. + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{Ne}$

Q[a\*c + b\*d, 0]

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx &= \frac{(A + iB) \cot(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{\int \frac{\cot^2(c + dx)(a(5A + iB) - 3a(iA - B) \tan(c + dx))}{a + ia \tan(c + dx)} dx}{4a^2} \\ &= \frac{(2A + iB) \cot(c + dx)}{2a^2d(1 + i \tan(c + dx))} + \frac{(A + iB) \cot(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{\int \cot^2(c + dx)}{4a^2} \\ &= -\frac{3(3A + iB) \cot(c + dx)}{4a^2d} + \frac{(2A + iB) \cot(c + dx)}{2a^2d(1 + i \tan(c + dx))} + \frac{(A + iB)}{4d(a + ia \tan(c + dx))} \\ &= -\frac{3(3A + iB)x}{4a^2} - \frac{3(3A + iB) \cot(c + dx)}{4a^2d} + \frac{(2A + iB) \cot(c + dx)}{2a^2d(1 + i \tan(c + dx))} \\ &= -\frac{3(3A + iB)x}{4a^2} - \frac{3(3A + iB) \cot(c + dx)}{4a^2d} - \frac{(2iA - B) \log(\sin(c + dx))}{a^2d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 960 vs. 2(141) = 282.  
time = 6.79, size = 960, normalized size = 6.81

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
[Out] (((-3*I)*A + 2*B)*Cos[2*d*x]*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(A + B*
Tan[c + d*x]))/(4*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x]
)^2) + (Sec[c + d*x]*((-2*I)*A*Cos[c] + B*Cos[c] + 2*A*Sin[c] + I*B*Sin[c])
*((-I)*ArcTan[Tan[d*x]]*Cos[c] + ArcTan[Tan[d*x]]*Sin[c])*(Cos[d*x] + I*Sin
[d*x])^2*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*
a*Tan[c + d*x])^2) + (Sec[c + d*x]*((-2*I)*A*Cos[c] + B*Cos[c] + 2*A*Sin[c]
```

$$\begin{aligned}
& + I*B*\sin[c]*((\cos[c]*\log[\sin[c + dx]^2])/2 + (I/2)*\log[\sin[c + dx]^2]* \\
& \sin[c])*(\cos[dx] + I*\sin[dx])^2*(A + B*\tan[c + dx])/(d*(A*\cos[c + dx] \\
& + B*\sin[c + dx])*(a + I*a*\tan[c + dx])^2 + (x*\sec[c + dx]*(-2*A - I*B + \\
& (2*I)*A*\cot[c] - B*\cot[c] + ((-2*I)*A + B)*\cot[c]*(\cos[2*c] + I*\sin[2*c])) \\
& *(\cos[dx] + I*\sin[dx])^2*(A + B*\tan[c + dx])/((A*\cos[c + dx] + B*\sin[c \\
& + dx])*(a + I*a*\tan[c + dx])^2 + (((-I)*A + B)*\cos[4*dx]*\sec[c + dx]* \\
& (\cos[2*c]/16 - (I/16)*\sin[2*c])*(\cos[dx] + I*\sin[dx])^2*(A + B*\tan[c + dx \\
& x]))/(d*(A*\cos[c + dx] + B*\sin[c + dx])*(a + I*a*\tan[c + dx])^2 + ((3*A \\
& + I*B)*\sec[c + dx]*((-3*dx*\cos[2*c])/4 - ((3*I)/4)*dx*\sin[2*c])*(\cos[dx \\
& x] + I*\sin[dx])^2*(A + B*\tan[c + dx]))/(d*(A*\cos[c + dx] + B*\sin[c + dx \\
& ])*(a + I*a*\tan[c + dx])^2 - ((3*A + (2*I)*B)*\sec[c + dx]*(\cos[dx] + I* \\
& \sin[dx])^2*\sin[2*dx]*(A + B*\tan[c + dx]))/(4*d*(A*\cos[c + dx] + B*\sin[c \\
& + dx])*(a + I*a*\tan[c + dx])^2 + ((A + I*B)*\sec[c + dx]*(-1/16*\cos[2*c \\
& ] + (I/16)*\sin[2*c])*(\cos[dx] + I*\sin[dx])^2*\sin[4*dx]*(A + B*\tan[c + dx \\
& x]))/(d*(A*\cos[c + dx] + B*\sin[c + dx])*(a + I*a*\tan[c + dx])^2 + (Csc[ \\
& c]*Csc[c + dx]*\sec[c + dx]*(\cos[dx] + I*\sin[dx])^2*((I/2)*A*\cos[2*c - d \\
& *x] - (I/2)*A*\cos[2*c + dx] - (A*\sin[2*c - dx])/2 + (A*\sin[2*c + dx])/2) \\
& *(A + B*\tan[c + dx]))/(d*(A*\cos[c + dx] + B*\sin[c + dx])*(a + I*a*\tan[c \\
& + dx])^2)
\end{aligned}$$

**Maple [A]**

time = 0.26, size = 114, normalized size = 0.81

method	result
derivativedivides	$-\frac{A}{\tan(dx+c)} + (-2iA+B) \ln(\tan(dx+c)) + \left(-\frac{7B}{8} + \frac{17iA}{8}\right) \ln(\tan(dx+c)-i) - \frac{\frac{5A}{4} + \frac{3iB}{4}}{\tan(dx+c)-i} - \frac{-\frac{iA}{2} + \frac{B}{2}}{2(\tan(dx+c)-i)^2} - \frac{i(-iB+A) \ln(\tan(dx+c)-i)}{d a^2}$
default	$-\frac{A}{\tan(dx+c)} + (-2iA+B) \ln(\tan(dx+c)) + \left(-\frac{7B}{8} + \frac{17iA}{8}\right) \ln(\tan(dx+c)-i) - \frac{\frac{5A}{4} + \frac{3iB}{4}}{\tan(dx+c)-i} - \frac{-\frac{iA}{2} + \frac{B}{2}}{2(\tan(dx+c)-i)^2} - \frac{i(-iB+A) \ln(\tan(dx+c)-i)}{d a^2}$
risch	$-\frac{7ixB}{4a^2} - \frac{17xA}{4a^2} + \frac{e^{-2i(dx+c)}B}{2a^2d} - \frac{3ie^{-2i(dx+c)}A}{4a^2d} + \frac{e^{-4i(dx+c)}B}{16a^2d} - \frac{ie^{-4i(dx+c)}A}{16a^2d} - \frac{2iBc}{a^2d} - \frac{4Ac}{da^2} - \frac{1}{a^2d(e^{2i(dx+c)} + 1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(dx+c)^2\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^2,x,method=\_RETURNVERB  
OSE)

[Out] 1/d/a^2\*(-A/tan(dx+c)+(-2\*I\*A+B)\*ln(tan(dx+c))+(-7/8\*B+17/8\*I\*A)\*ln(tan(dx+c)-I)-(5/4\*A+3/4\*I\*B)/(tan(dx+c)-I)-1/2\*(-1/2\*I\*A+1/2\*B)/(tan(dx+c)-I)^2-1/8\*I\*(A-I\*B)\*ln(tan(dx+c)+I))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 0.48, size = 152, normalized size = 1.08

$$\frac{4(17A + 7iB)dx e^{6i dx + 6i c} - 4((17A + 7iB)dx - 11iA + 2B)e^{4i dx + 4i c} - (11iA - 7B)e^{2i dx + 2i c} + 16((2iA - B)e^{6i dx + 6i c} + (-2iA + B)e^{4i dx + 4i c}) \log(e^{2i dx + 2i c} - 1) - iA + B}{16(a^2 d e^{6i dx + 6i c} - a^2 d e^{4i dx + 4i c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$-1/16*(4*(17*A + 7*I*B)*d*x*e^{(6*I*d*x + 6*I*c)} - 4*((17*A + 7*I*B)*d*x - 11*I*A + 2*B)*e^{(4*I*d*x + 4*I*c)} - (11*I*A - 7*B)*e^{(2*I*d*x + 2*I*c)} + 16*((2*I*A - B)*e^{(6*I*d*x + 6*I*c)} + (-2*I*A + B)*e^{(4*I*d*x + 4*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} - 1) - I*A + B)/(a^2*d*e^{(6*I*d*x + 6*I*c)} - a^2*d*e^{(4*I*d*x + 4*I*c)})$$

**Sympy [A]**

time = 0.54, size = 267, normalized size = 1.89

$$-\frac{2iA}{a^2 d e^{2ic} e^{2idx} - a^2 d} + \begin{cases} \frac{((-4iAa^2 d e^{2ic} + 4Ba^2 d e^{2ic})e^{-4idx} + (-48iAa^2 d e^{4ic} + 32Ba^2 d e^{4ic})e^{-2idx})e^{-6ic}}{64a^4 d^2} & \text{for } a^4 d^2 e^{6ic} \neq 0 \\ x\left(-\frac{-17A-7iB}{4a^2} + \frac{(-17Ae^{4ic}-6Ae^{2ic}-A-7iBe^{4ic}-4iBe^{2ic}-iB)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-17A-7iB)}{4a^2} - \frac{i(2A+iB)\log(e^{2idx} - e^{-2ic})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] 
$$-2*I*A/(a**2*d*\exp(2*I*c)*\exp(2*I*d*x) - a**2*d) + \text{Piecewise}(\left(\left(\left(-4*I*A*a**2*d*\exp(2*I*c) + 4*B*a**2*d*\exp(2*I*c)\right)*\exp(-4*I*d*x) + (-48*I*A*a**2*d*\exp(4*I*c) + 32*B*a**2*d*\exp(4*I*c))*\exp(-2*I*d*x)\right)*\exp(-6*I*c)/(64*a**4*d**2), \text{Ne}(a**4*d**2*\exp(6*I*c), 0)), \left(x*(-(-17*A - 7*I*B)/(4*a**2) + (-17*A*\exp(4*I*c) - 6*A*\exp(2*I*c) - A - 7*I*B*\exp(4*I*c) - 4*I*B*\exp(2*I*c) - I*B)*\exp(-4*I*c)/(4*a**2)\right), \text{True})) + x*(-17*A - 7*I*B)/(4*a**2) - I*(2*A + I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/(a**2*d)$$

**Giac [A]**

time = 0.92, size = 161, normalized size = 1.14

$$\frac{2(-iA-B)\log(\tan(dx+c)+i) - 2(-17iA+7B)\log(\tan(dx+c)-i) - 16(2iA-B)\log(\tan(dx+c)) - \frac{16(-2iA\tan(dx+c)+B\tan(dx+c)+A)}{a^2\tan(dx+c)} - \frac{51iA\tan(dx+c)^2-21B\tan(dx+c)^2+122A\tan(dx+c)+54iB\tan(dx+c)-75iA+37B}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")



[Out]  $\frac{1}{16} \cdot (2 \cdot (-I \cdot A - B) \cdot \log(\tan(dx + c) + I) / a^2 - 2 \cdot (-17 \cdot I \cdot A + 7 \cdot B) \cdot \log(\tan(dx + c) - I) / a^2 - 16 \cdot (2 \cdot I \cdot A - B) \cdot \log(\tan(dx + c)) / a^2 - 16 \cdot (-2 \cdot I \cdot A \cdot \tan(dx + c) + B \cdot \tan(dx + c) + A) / (a^2 \cdot \tan(dx + c)) - (51 \cdot I \cdot A \cdot \tan(dx + c)^2 - 2 \cdot 1 \cdot B \cdot \tan(dx + c)^2 + 122 \cdot A \cdot \tan(dx + c) + 54 \cdot I \cdot B \cdot \tan(dx + c) - 75 \cdot I \cdot A + 37 \cdot B) / (a^2 \cdot (\tan(dx + c) - I)^2)) / d$

**Mupad [B]**

time = 6.53, size = 164, normalized size = 1.16

$$\frac{\tan(c+dx)^2 \left(-\frac{3B}{4a^2} + \frac{A9i}{4a^2}\right) - \frac{A1i}{a^2} + \tan(c+dx) \left(\frac{7A}{2a^2} + \frac{B1i}{a^2}\right)}{d \left(\tan(c+dx)^3 1i + 2 \tan(c+dx)^2 - \tan(c+dx) 1i\right)} - \frac{\ln(\tan(c+dx)) (-B + A2i)}{a^2 d} - \frac{\ln(\tan(c+dx) + 1i) (B + A1i)}{8 a^2 d} + \frac{\ln(\tan(c+dx) - i) (-7B + A17i)}{8 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\cot(c + dx))^2 \cdot (A + B \cdot \tan(c + dx))) / (a + a \cdot \tan(c + dx) \cdot 1i)^2, x)$

[Out]  $(\log(\tan(c + dx) - 1i) \cdot (A \cdot 17i - 7 \cdot B)) / (8 \cdot a^2 \cdot d) - (\log(\tan(c + dx))) \cdot (A \cdot 2i - B) / (a^2 \cdot d) - (\log(\tan(c + dx) + 1i) \cdot (A \cdot 1i + B)) / (8 \cdot a^2 \cdot d) - (\tan(c + dx))^2 \cdot ((A \cdot 9i) / (4 \cdot a^2) - (3 \cdot B) / (4 \cdot a^2)) - (A \cdot 1i) / a^2 + \tan(c + dx) \cdot ((7 \cdot A) / (2 \cdot a^2) + (B \cdot 1i) / a^2) / (d \cdot (2 \cdot \tan(c + dx))^2 - \tan(c + dx) \cdot 1i + \tan(c + dx) \cdot 3 \cdot 1i))$

$$3.50 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=170

$$\frac{3(5iA - 3B)x}{4a^2} + \frac{3(5iA - 3B) \cot(c + dx)}{4a^2d} - \frac{(2A + iB) \cot^2(c + dx)}{a^2d} - \frac{2(2A + iB) \log(\sin(c + dx))}{a^2d} + \frac{(5A + 3iB)}{4a^2d(1 + i \tan(c + dx))}$$

[Out]  $\frac{3}{4}*(5*I*A-3*B)*x/a^2 + \frac{3}{4}*(5*I*A-3*B)*\cot(d*x+c)/a^2/d - (2*A+I*B)*\cot(d*x+c)^2/a^2/d - 2*(2*A+I*B)*\ln(\sin(d*x+c))/a^2/d + 1/4*(5*A+3*I*B)*\cot(d*x+c)^2/a^2/d / (1+I*\tan(d*x+c)) + 1/4*(A+I*B)*\cot(d*x+c)^2/d / (a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.27, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{(2A+iB)\cot^2(c+dx)}{a^2d} + \frac{3(-3B+5iA)\cot(c+dx)}{4a^2d} - \frac{2(2A+iB)\log(\sin(c+dx))}{a^2d} + \frac{(5A+3iB)\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))} + \frac{3x(-3B+5iA)}{4a^2} + \frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2, x]

[Out]  $\frac{3*((5*I)*A - 3*B)*x}{(4*a^2)} + \frac{3*((5*I)*A - 3*B)*\text{Cot}[c + d*x]}{(4*a^2*d)} - \frac{((2*A + I*B)*\text{Cot}[c + d*x]^2)}{(a^2*d)} - \frac{2*(2*A + I*B)*\text{Log}[\text{Sin}[c + d*x]]}{(a^2*d)} + \frac{((5*A + (3*I)*B)*\text{Cot}[c + d*x]^2)}{(4*a^2*d*(1 + I*\text{Tan}[c + d*x]))} + \frac{((A + I*B)*\text{Cot}[c + d*x]^2)}{(4*d*(a + I*a*\text{Tan}[c + d*x])^2)}$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[b, 0]

Q[a\*c + b\*d, 0]

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx &= \frac{(A+iB) \cot^2(c+dx)}{4d(a+ia \tan(c+dx))^2} + \frac{\int \frac{\cot^3(c+dx)(2a(3A+iB)-4a(iA-B) \tan(c+dx))}{a+ia \tan(c+dx)}}{4a^2} \\
 &= \frac{(5A+3iB) \cot^2(c+dx)}{4a^2d(1+i \tan(c+dx))} + \frac{(A+iB) \cot^2(c+dx)}{4d(a+ia \tan(c+dx))^2} + \frac{\int \cot^3(c+dx)}{4d(a+ia \tan(c+dx))} \\
 &= -\frac{(2A+iB) \cot^2(c+dx)}{a^2d} + \frac{(5A+3iB) \cot^2(c+dx)}{4a^2d(1+i \tan(c+dx))} + \frac{(A+iB) \cot^2(c+dx)}{4d(a+ia \tan(c+dx))} \\
 &= \frac{3(5iA-3B) \cot(c+dx)}{4a^2d} - \frac{(2A+iB) \cot^2(c+dx)}{a^2d} + \frac{(5A+3iB) \cot^2(c+dx)}{4a^2d(1+i \tan(c+dx))} \\
 &= \frac{3(5iA-3B)x}{4a^2} + \frac{3(5iA-3B) \cot(c+dx)}{4a^2d} - \frac{(2A+iB) \cot^2(c+dx)}{a^2d} \\
 &= \frac{3(5iA-3B)x}{4a^2} + \frac{3(5iA-3B) \cot(c+dx)}{4a^2d} - \frac{(2A+iB) \cot^2(c+dx)}{a^2d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1112 vs. 2(170) = 340.  
time = 7.06, size = 1112, normalized size = 6.54

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out] -1/4\*((4\*A + (3\*I)\*B)\*Cos[2\*d\*x]\*Sec[c + d\*x]\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^2) + (Sec[c + d\*x]\*(2\*A\*Cos[c] + I\*B\*Cos[c] + (2\*I)\*A\*Sin[c] - B\*Sin[c]

$$\begin{aligned} & )*((2*I)*\text{ArcTan}[\text{Tan}[d*x]]*\text{Cos}[c] - 2*\text{ArcTan}[\text{Tan}[d*x]]*\text{Sin}[c])*(\text{Cos}[d*x] + I \\ & * \text{Sin}[d*x])^2*(A + B*\text{Tan}[c + d*x])/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a \\ & + I*a*\text{Tan}[c + d*x])^2) + (\text{Sec}[c + d*x]*(2*A*\text{Cos}[c] + I*B*\text{Cos}[c] + (2*I)*A*\text{S} \\ & \text{in}[c] - B*\text{Sin}[c])*(-(\text{Cos}[c]*\text{Log}[\text{Sin}[c + d*x]^2]) - I*\text{Log}[\text{Sin}[c + d*x]^2]*\text{S} \\ & \text{in}[c])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2*(A + B*\text{Tan}[c + d*x])/(d*(A*\text{Cos}[c + d*x] + \\ & B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) + (x*\text{Sec}[c + d*x]*((4*I)*A - 2*B \\ & + 4*A*\text{Cot}[c] + (2*I)*B*\text{Cot}[c] + (2*A + I*B)*\text{Cot}[c]*(-2*\text{Cos}[2*c] - (2*I)*\text{Sin} \\ & [2*c]))*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2*(A + B*\text{Tan}[c + d*x]))/((A*\text{Cos}[c + d*x] + \\ & B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) + ((A + I*B)*\text{Cos}[4*d*x]*\text{Sec}[c + d \\ & *x]*(-1/16*\text{Cos}[2*c] + (I/16)*\text{Sin}[2*c])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2*(A + B*\text{Tan} \\ & [c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) \\ & + (\text{Csc}[c + d*x]^2*\text{Sec}[c + d*x]*(-1/2*(A*\text{Cos}[2*c]) - (I/2)*A*\text{Sin}[2*c])*(\text{Cos}[ \\ & d*x] + I*\text{Sin}[d*x])^2*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d \\ & *x])*(a + I*a*\text{Tan}[c + d*x])^2) + ((5*A + (3*I)*B)*\text{Sec}[c + d*x]*(((3*I)/4)*d \\ & *x*\text{Cos}[2*c] - (3*d*x*\text{Sin}[2*c])/4)*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2*(A + B*\text{Tan}[c + \\ & d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) + ((I \\ & /4)*(4*A + (3*I)*B)*\text{Sec}[c + d*x]*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2*\text{Sin}[2*d*x]*(A + \\ & B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x] \\ & )^2) + ((A + I*B)*\text{Sec}[c + d*x]*((I/16)*\text{Cos}[2*c] + \text{Sin}[2*c]/16)*(\text{Cos}[d*x] + \\ & I*\text{Sin}[d*x])^2*\text{Sin}[4*d*x]*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c \\ & + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) + (\text{Csc}[c]*\text{Csc}[c + d*x]*\text{Sec}[c + d*x]*(\text{Cos} \\ & [d*x] + I*\text{Sin}[d*x])^2*(A*\text{Cos}[2*c - d*x] + (I/2)*B*\text{Cos}[2*c - d*x] - A*\text{Cos}[2* \\ & c + d*x] - (I/2)*B*\text{Cos}[2*c + d*x] + I*A*\text{Sin}[2*c - d*x] - (B*\text{Sin}[2*c - d*x]) \\ & /2 - I*A*\text{Sin}[2*c + d*x] + (B*\text{Sin}[2*c + d*x])/2)*(A + B*\text{Tan}[c + d*x]))/(d*(A \\ & * \text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2) \end{aligned}$$

**Maple [A]**

time = 0.30, size = 132, normalized size = 0.78

method	result
derivativedivides	$-\frac{A}{2 \tan(dx+c)^2} + (-2iB-4A) \ln(\tan(dx+c)) - \frac{-2iA+B}{\tan(dx+c)} - \frac{-\frac{A}{2} - \frac{iB}{2}}{2(\tan(dx+c)-i)^2} - \frac{\frac{5B}{4} - \frac{7iA}{4}}{\tan(dx+c)-i} + \left(\frac{31A}{8} + \frac{17iB}{8}\right) \ln(\tan(dx+c)-i) - \frac{1}{da^2}$
default	$-\frac{A}{2 \tan(dx+c)^2} + (-2iB-4A) \ln(\tan(dx+c)) - \frac{-2iA+B}{\tan(dx+c)} - \frac{-\frac{A}{2} - \frac{iB}{2}}{2(\tan(dx+c)-i)^2} - \frac{\frac{5B}{4} - \frac{7iA}{4}}{\tan(dx+c)-i} + \left(\frac{31A}{8} + \frac{17iB}{8}\right) \ln(\tan(dx+c)-i) - \frac{1}{da^2}$
risch	$-\frac{17xB}{4a^2} + \frac{31ixA}{4a^2} - \frac{3ie^{-2i(dx+c)}B}{4a^2d} - \frac{e^{-2i(dx+c)}A}{a^2d} - \frac{ie^{-4i(dx+c)}B}{16a^2d} - \frac{e^{-4i(dx+c)}A}{16a^2d} - \frac{4Bc}{da^2} + \frac{8iAc}{da^2} - \frac{2i(-i)}{da^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d/a^2} * (-1/2*A/\tan(d*x+c)^2 + (-4*A-2*I*B)*\ln(\tan(d*x+c)) - (-2*I*A+B)/\tan(d*x+c) - 1/2*(-1/2*A-1/2*I*B)/(\tan(d*x+c)-I)^2 - (5/4*B-7/4*I*A)/(\tan(d*x+c)-I) + (31/8*A+17/8*I*B)*\ln(\tan(d*x+c)-I) + (1/8*A-1/8*I*B)*\ln(\tan(d*x+c)+I))$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
      expr: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 0.48, size = 215, normalized size = 1.26

$$\frac{4(-31iA + 17B)dx e^{(8iA + 17B)x} + 4(2(31iA - 17B)dx + 12A + 11iB)e^{(6iA + 11B)x} + (4(-31iA + 17B)dx - 95A - 55iB)e^{(4iA + 7B)x} + 2(7A + 5iB)e^{(2iA + 5B)x} + 32((2A + iB)e^{(8iA + 8B)x} - 2(2A + iB)e^{(6iA + 6B)x} + (2A + iB)e^{(4iA + 4B)x}) \log(e^{(2iA + 2B)x} - 1) + A + iB}{16(a^2 d e^{(8iA + 8B)x} - 2a^2 d e^{(6iA + 6B)x} + a^2 d e^{(4iA + 4B)x})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(-31*I*A + 17*B)*d*x*e^(8*I*d*x + 8*I*c) + 4*(2*(31*I*A - 17*B)*d*x + 12*A + 11*I*B)*e^(6*I*d*x + 6*I*c) + (4*(-31*I*A + 17*B)*d*x - 95*A - 5*5*I*B)*e^(4*I*d*x + 4*I*c) + 2*(7*A + 5*I*B)*e^(2*I*d*x + 2*I*c) + 32*((2*A + I*B)*e^(8*I*d*x + 8*I*c) - 2*(2*A + I*B)*e^(6*I*d*x + 6*I*c) + (2*A + I*B)*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) + A + I*B)/(a^2*d*e^(8*I*d*x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))
```

**Sympy [A]**

time = 0.53, size = 323, normalized size = 1.90

$$\frac{4A + 2iB + (-2Ae^{2ic} - 2iBe^{2ic})e^{2idx}}{a^2 d e^{4ic} e^{4idx} - 2a^2 d e^{2ic} e^{2idx} + a^2 d} + \begin{cases} \left( \frac{((-4Aa^2 d e^{2ic} - 4iBa^2 d e^{2ic})e^{-4idx} + (-64Aa^2 d e^{4ic} - 48iBa^2 d e^{4ic})e^{-2idx})e^{-6ic}}{64a^4 d^2} \right) & \text{for } a^4 d^2 e^{6ic} \neq 0 \\ x \left( -\frac{31iA - 17B}{4a^2} + \frac{(31iA e^{4ic} + 8iA e^{2ic} + A - 17B e^{4ic} - 6B e^{2ic} - B)e^{-4ic}}{4a^2} \right) & \text{otherwise} \end{cases} + \frac{x(31iA - 17B)}{4a^2} - \frac{2 \cdot (2A + iB) \log(e^{2idx} - e^{-2ic})}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] (4*A + 2*I*B + (-2*A*exp(2*I*c) - 2*I*B*exp(2*I*c))*exp(2*I*d*x))/(a**2*d*exp(4*I*c)*exp(4*I*d*x) - 2*a**2*d*exp(2*I*c)*exp(2*I*d*x) + a**2*d) + Piecewise(((((-4*A*a**2*d*exp(2*I*c) - 4*I*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x) + (-64*A*a**2*d*exp(4*I*c) - 48*I*B*a**2*d*exp(4*I*c))*exp(-2*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(-(31*I*A - 17*B)/(4*a**2) + (31*I*A*exp(4*I*c) + 8*I*A*exp(2*I*c) + I*A - 17*B*exp(4*I*c) - 6*B*exp(2*I*c) - B)*exp(-4*I*c)/(4*a**2)), True)) + x*(31*I*A - 17*B)/(4*a**2) - 2*(2*A + I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a**2*d)
```

**Giac [A]**

time = 1.25, size = 176, normalized size = 1.04

$$\frac{4(A-iB)\log(\tan(dx+c)+i)}{a^2} + \frac{4(31A+17iB)\log(\tan(dx+c)-i)}{a^2} - \frac{64(2A+iB)\log(\tan(dx+c))}{a^2} + \frac{3A\tan(dx+c)^4 - 3iB\tan(dx+c)^4 + 114iA\tan(dx+c)^3 - 78B\tan(dx+c)^3 + 173A\tan(dx+c)^2 + 115iB\tan(dx+c)^2 - 32iA\tan(dx+c) + 32B\tan(dx+c) + 16A}{(\tan(dx+c)^2 - i\tan(dx+c))^2 a^2}$$


---


$$32d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 1/32\*(4\*(A - I\*B)\*log(tan(d\*x + c) + I)/a^2 + 4\*(31\*A + 17\*I\*B)\*log(tan(d\*x + c) - I)/a^2 - 64\*(2\*A + I\*B)\*log(tan(d\*x + c))/a^2 + (3\*A\*tan(d\*x + c)^4 - 3\*I\*B\*tan(d\*x + c)^4 + 114\*I\*A\*tan(d\*x + c)^3 - 78\*B\*tan(d\*x + c)^3 + 173\*A\*tan(d\*x + c)^2 + 115\*I\*B\*tan(d\*x + c)^2 - 32\*I\*A\*tan(d\*x + c) + 32\*B\*tan(d\*x + c) + 16\*A)/((tan(d\*x + c)^2 - I\*tan(d\*x + c))^2\*a^2)/d

**Mupad [B]**

time = 6.62, size = 188, normalized size = 1.11

$$\frac{\tan(c+dx)^2 \left(-\frac{7B}{2a^2} + \frac{A11i}{2a^2}\right) - \tan(c+dx)^3 \left(\frac{15A}{4a^2} + \frac{B9i}{4a^2}\right) + \frac{A11i}{2a^2} + \tan(c+dx) \left(\frac{A}{a^2} + \frac{B11i}{a^2}\right) - \frac{2 \ln(\tan(c+dx)) (2A+B11i)}{a^2 d} + \frac{\ln(\tan(c+dx)+1i) (A-B11i)}{8a^2 d} + \frac{\ln(\tan(c+dx)-1i) (31A+B17i)}{8a^2 d}}{d (\tan(c+dx)^4 1i + 2 \tan(c+dx)^3 - \tan(c+dx)^2 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] (tan(c + d\*x)^2\*((A\*11i)/(2\*a^2) - (7\*B)/(2\*a^2)) - tan(c + d\*x)^3\*((15\*A)/(4\*a^2) + (B\*9i)/(4\*a^2)) + (A\*1i)/(2\*a^2) + tan(c + d\*x)\*(A/a^2 + (B\*1i)/a^2))/(d\*(2\*tan(c + d\*x)^3 - tan(c + d\*x)^2\*1i + tan(c + d\*x)^4\*1i)) - (2\*log(tan(c + d\*x))\*(2\*A + B\*1i))/(a^2\*d) + (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(8\*a^2\*d) + (log(tan(c + d\*x) - 1i)\*(31\*A + B\*17i))/(8\*a^2\*d)

$$3.51 \quad \int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=191

$$-\frac{(7A+25iB)x}{8a^3} - \frac{(iA-3B)\log(\cos(c+dx))}{a^3d} + \frac{(7A+25iB)\tan(c+dx)}{8a^3d} + \frac{(iA-B)\tan^4(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(5A+11iB)\tan^3(c+dx)}{24ad(a+ia \tan(c+dx))^2}$$

[Out]  $-1/8*(7*A+25*I*B)*x/a^3 - (I*A-3*B)*\ln(\cos(d*x+c))/a^3/d + 1/8*(7*A+25*I*B)*\tan(d*x+c)/a^3/d + 1/6*(I*A-B)*\tan(d*x+c)^4/d/(a+I*a*\tan(d*x+c))^3 + 1/24*(5*A+11*I*B)*\tan(d*x+c)^3/a/d/(a+I*a*\tan(d*x+c))^2 - 1/2*(I*A-3*B)*\tan(d*x+c)^2/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi [A]**

time = 0.32, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ ,

Rules used = {3676, 3606, 3556}

$$-\frac{(-3B+iA)\tan^2(c+dx)}{2d(a^3+ia^3\tan(c+dx))} + \frac{(7A+25iB)\tan(c+dx)}{8a^3d} - \frac{(-3B+iA)\log(\cos(c+dx))}{a^3d} - \frac{x(7A+25iB)}{8a^3} + \frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(5A+11iB)\tan^3(c+dx)}{24ad(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]^4*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^3, x]$

[Out]  $-1/8*((7*A+(25*I)*B)*x)/a^3 - ((I*A-3*B)*\text{Log}[\text{Cos}[c+d*x]])/(a^3*d) + ((7*A+(25*I)*B)*\text{Tan}[c+d*x])/(8*a^3*d) + ((I*A-B)*\text{Tan}[c+d*x]^4)/(6*d*(a+I*a*\text{Tan}[c+d*x])^3) + ((5*A+(11*I)*B)*\text{Tan}[c+d*x]^3)/(24*a*d*(a+I*a*\text{Tan}[c+d*x])^2) - ((I*A-3*B)*\text{Tan}[c+d*x]^2)/(2*d*(a^3+I*a^3*\text{Tan}[c+d*x]))$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3606**

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

**Rule 3676**

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[-(A*b - a*B)*(a + b*\text{Tan}[e + f*x])^m * ((c + d*\text{Tan}[e + f*x])^n / (2*a*f*m)), x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^n, x], x]$

])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] & & NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^4(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx &= \frac{(iA - B) \tan^4(c + dx)}{6d(a + ia \tan(c + dx))^3} - \frac{\int \frac{\tan^3(c+dx)(4a(iA-B)+a(A+7iB) \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx}{6a^2} \\
 &= \frac{(iA - B) \tan^4(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(5A + 11iB) \tan^3(c + dx)}{24ad(a + ia \tan(c + dx))^2} + \frac{\int \frac{\tan^2(c+dx)}{a+ia \tan(c+dx)} dx}{6a^2} \\
 &= \frac{(iA - B) \tan^4(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(5A + 11iB) \tan^3(c + dx)}{24ad(a + ia \tan(c + dx))^2} - \frac{(iA - 3B) \tan^2(c + dx)}{2d(a^3 + ia^2 dx)} \\
 &= -\frac{(7A + 25iB)x}{8a^3} + \frac{(7A + 25iB) \tan(c + dx)}{8a^3 d} + \frac{(iA - B) \tan^4(c + dx)}{6d(a + ia \tan(c + dx))^3} \\
 &= -\frac{(7A + 25iB)x}{8a^3} - \frac{(iA - 3B) \log(\cos(c + dx))}{a^3 d} + \frac{(7A + 25iB) \tan(c + dx)}{8a^3 d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1251 vs. 2(191) = 382.  
time = 6.86, size = 1251, normalized size = 6.55

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((11*A + (23*I)*B)*Cos[2*d*x]*Sec[c + d*x]^2*((I/16)*Cos[c] - Sin[c]/16)*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3 + (((-5*I)*A + 7*B)*Cos[4*d*x]*Sec[c + d*x]^2*(Cos[c]/32 - (I/32)*Sin[c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3 + (Sec[c + d*x]^2*((-I)*A*Cos[(3*c)/2] + 3*B*Cos[(3*c)/2] + A*Sin[(3*c)/2] + (3*I)*B*Sin[(3*c)/2])*(Cos[(3*c)/2]*Log[Cos[c + d*x]] + I*Log[Cos[c + d*x]]*Sin[(3*c)/2])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3 + ((A + I*B)*Cos[6*d*x]*Sec[c + d*x]^2*((I/48)*Cos[3*c] + Sin[3*c]/48)*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3 + ((7*A + (25*I)*B)*Sec[c + d*x]^2*(-1/8*(d*x*Cos[3*c]) - (I/8)*d*x*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3 + ((11*A + (23*I)*B)*Sec[c + d*x]^2*(Cos[c] - Sin[c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3
```



$$\begin{aligned} & ]^2 * (\cos[c]/16 + (I/16) * \sin[c]) * (\cos[d*x] + I * \sin[d*x])^3 * \sin[2*d*x] * (A + B \\ & * \tan[c + d*x]) / (d * (A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + I * a * \tan[c + d*x]) \\ & ^3) + ((5 * A + (7 * I) * B) * \sec[c + d*x]^2 * (-1/32 * \cos[c] + (I/32) * \sin[c]) * (\cos[d \\ & * x] + I * \sin[d*x])^3 * \sin[4*d*x] * (A + B * \tan[c + d*x])) / (d * (A * \cos[c + d*x] + B \\ & * \sin[c + d*x]) * (a + I * a * \tan[c + d*x])^3) + ((A + I * B) * \sec[c + d*x]^2 * (\cos[3 \\ & * c]/48 - (I/48) * \sin[3*c]) * (\cos[d*x] + I * \sin[d*x])^3 * \sin[6*d*x] * (A + B * \tan[c \\ & + d*x])) / (d * (A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + I * a * \tan[c + d*x])^3) + \\ & (\sec[c + d*x]^3 * (\cos[d*x] + I * \sin[d*x])^3 * (-(B * \cos[3*c - d*x]) + B * \cos[3*c \\ & + d*x] - I * B * \sin[3*c - d*x] + I * B * \sin[3*c + d*x]) * (A + B * \tan[c + d*x])) / (2 * \\ & d * (\cos[c/2] - \sin[c/2]) * (\cos[c/2] + \sin[c/2]) * (A * \cos[c + d*x] + B * \sin[c + d \\ & * x]) * (a + I * a * \tan[c + d*x])^3) + (x * \sec[c + d*x]^2 * (\cos[d*x] + I * \sin[d*x])^ \\ & 3 * ((A * \cos[c])/2 + ((3 * I)/2) * B * \cos[c] - (A * \cos[c]^3)/2 - ((3 * I)/2) * B * \cos[c]^ \\ & 3 + I * A * \sin[c] - 3 * B * \sin[c] - (2 * I) * A * \cos[c]^2 * \sin[c] + 6 * B * \cos[c]^2 * \sin[c] \\ & + 3 * A * \cos[c] * \sin[c]^2 + (9 * I) * B * \cos[c] * \sin[c]^2 + (2 * I) * A * \sin[c]^3 - 6 * B * S \\ & \sin[c]^3 - (A * \sin[c] * \tan[c])/2 - ((3 * I)/2) * B * \sin[c] * \tan[c] - (A * \sin[c]^3 * \tan \\ & [c])/2 - ((3 * I)/2) * B * \sin[c]^3 * \tan[c] + I * (A + (3 * I) * B) * (\cos[3*c] + I * \sin[3* \\ & c]) * \tan[c]) * (A + B * \tan[c + d*x])) / ((A * \cos[c + d*x] + B * \sin[c + d*x]) * (a + I \\ & * a * \tan[c + d*x])^3) \end{aligned}$$

**Maple [A]**

time = 0.18, size = 120, normalized size = 0.63

method	result
derivativedivides	$\frac{iB \tan(dx+c) - \frac{\frac{A}{2} + \frac{iB}{2}}{3(\tan(dx+c)-i)^3} + \left(-\frac{49B}{16} + \frac{15iA}{16}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{17A}{8} - \frac{31iB}{8}}{\tan(dx+c)-i} - \frac{-\frac{7iA}{4} + \frac{9B}{4}}{2(\tan(dx+c)-i)^2} + \frac{i(-iB+A) \ln(\tan(dx+c)-i)}{16}}{d a^3}$
default	$\frac{iB \tan(dx+c) - \frac{\frac{A}{2} + \frac{iB}{2}}{3(\tan(dx+c)-i)^3} + \left(-\frac{49B}{16} + \frac{15iA}{16}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{17A}{8} - \frac{31iB}{8}}{\tan(dx+c)-i} - \frac{-\frac{7iA}{4} + \frac{9B}{4}}{2(\tan(dx+c)-i)^2} + \frac{i(-iB+A) \ln(\tan(dx+c)-i)}{16}}{d a^3}$
risch	$-\frac{49ixB}{8a^3} - \frac{15xA}{8a^3} - \frac{23e^{-2i(dx+c)}B}{16a^3d} + \frac{11ie^{-2i(dx+c)}A}{16a^3d} + \frac{7e^{-4i(dx+c)}B}{32a^3d} - \frac{5ie^{-4i(dx+c)}A}{32a^3d} - \frac{e^{-6i(dx+c)}B}{48a^3d} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURNVERB  
OSE)

[Out] 1/d/a^3\*(I\*B\*tan(d\*x+c)-1/3\*(1/2\*A+1/2\*I\*B)/(tan(d\*x+c)-I)^3+(-49/16\*B+15/1  
6\*I\*A)\*ln(tan(d\*x+c)-I)-(-17/8\*A-31/8\*I\*B)/(tan(d\*x+c)-I)-1/2\*(-7/4\*I\*A+9/4  
\*B)/(tan(d\*x+c)-I)^2+1/16\*I\*(A-I\*B)\*ln(tan(d\*x+c)+I))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.47, size = 174, normalized size = 0.91

$$\frac{-12(15A + 49iB)dx e^{(8i dx + 8i c)} + 6(2(15A + 49iB)dx - 11iA + 55B)e^{(6i dx + 6i c)} + 3(-17iA + 39B)e^{(4i dx + 4i c)} - (-13iA + 19B)e^{(2i dx + 2i c)} + 96((iA - 3B)e^{(8i dx + 8i c)} + (iA - 3B)e^{(6i dx + 6i c)}) \log(e^{(2i dx + 2i c)} + 1) - 2iA + 2B}{96(a^3 d e^{(8i dx + 8i c)} + a^3 d e^{(6i dx + 6i c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out]  $-1/96*(12*(15*A + 49*I*B)*d*x*e^{(8*I*d*x + 8*I*c)} + 6*(2*(15*A + 49*I*B)*d*x - 11*I*A + 55*B)*e^{(6*I*d*x + 6*I*c)} + 3*(-17*I*A + 39*B)*e^{(4*I*d*x + 4*I*c)} - (-13*I*A + 19*B)*e^{(2*I*d*x + 2*I*c)} + 96*((I*A - 3*B)*e^{(8*I*d*x + 8*I*c)} + (I*A - 3*B)*e^{(6*I*d*x + 6*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 2*I*A + 2*B)/(a^3*d*e^{(8*I*d*x + 8*I*c)} + a^3*d*e^{(6*I*d*x + 6*I*c)})$

**Sympy** [A]

time = 0.65, size = 337, normalized size = 1.76

$$\frac{2B}{a^3 d e^{2i c} e^{2i d x} + a^3 d} + \begin{cases} \frac{((512Aa^6 d^2 e^{6i c} - 512Ba^6 d^2 e^{6i c})e^{-6i d x} + (-3840Aa^6 d^2 e^{6i c} + 5376Ba^6 d^2 e^{6i c})e^{-4i d x} + (16896Aa^6 d^2 e^{10i c} - 35328Ba^6 d^2 e^{10i c})e^{-2i d x})e^{-12i c}}{24576a^9 d^3} & \text{for } a^9 d^3 e^{12i c} \neq 0 \\ x\left(-\frac{15A-49iB}{8a^3} + \frac{(-15Ae^{6i c} + 11Ae^{4i c} - 5Ae^{2i c} + A - 49iBe^{6i c} + 23iBe^{4i c} - 7iBe^{2i c} + iB)e^{-6i c}}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x(-15A - 49iB)}{8a^3} - \frac{i(A + 3iB) \log(e^{2i d x} + e^{-2i c})}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out]  $-2*B/(a**3*d*\exp(2*I*c)*\exp(2*I*d*x) + a**3*d) + \text{Piecewise}(\left(\left(\left(512*I*A*a**6*d**2*\exp(6*I*c) - 512*B*a**6*d**2*\exp(6*I*c)\right)*\exp(-6*I*d*x) + (-3840*I*A*a**6*d**2*\exp(8*I*c) + 5376*B*a**6*d**2*\exp(8*I*c))*\exp(-4*I*d*x) + (16896*I*A*a**6*d**2*\exp(10*I*c) - 35328*B*a**6*d**2*\exp(10*I*c))*\exp(-2*I*d*x)\right)*\exp(-12*I*c)/(24576*a**9*d**3), \text{Ne}(a**9*d**3*\exp(12*I*c), 0)), (x*(-15*A - 49*I*B)/(8*a**3) + (-15*A*\exp(6*I*c) + 11*A*\exp(4*I*c) - 5*A*\exp(2*I*c) + A - 49*I*B*\exp(6*I*c) + 23*I*B*\exp(4*I*c) - 7*I*B*\exp(2*I*c) + I*B)*\exp(-6*I*c)/(8*a**3)), \text{True})) + x*(-15*A - 49*I*B)/(8*a**3) - I*(A + 3*I*B)*\log(\exp(2*I*d*x) + \exp(-2*I*c))/(a**3*d)$

**Giac** [A]

time = 1.22, size = 144, normalized size = 0.75

$$\frac{6(iA+B)\log(\tan(dx+c)+1)}{a^3} - \frac{6(-15iA+49B)\log(i\tan(dx+c)+1)}{a^3} + \frac{96iB\tan(dx+c)}{a^3} - \frac{165iA\tan(dx+c)^3 - 539B\tan(dx+c)^3 + 291A\tan(dx+c)^2 + 1245iB\tan(dx+c)^2 - 171iA\tan(dx+c) + 981B\tan(dx+c) - 29A - 259iB}{a^3(\tan(dx+c)-i)^3}$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $\frac{1}{96} \cdot (6 \cdot (I \cdot A + B) \cdot \log(\tan(dx + c) + I) / a^3 - 6 \cdot (-15 \cdot I \cdot A + 49 \cdot B) \cdot \log(I \cdot \tan(dx + c) + 1) / a^3 + 96 \cdot I \cdot B \cdot \tan(dx + c) / a^3 - (165 \cdot I \cdot A \cdot \tan(dx + c)^3 - 539 \cdot B \cdot \tan(dx + c)^3 + 291 \cdot A \cdot \tan(dx + c)^2 + 1245 \cdot I \cdot B \cdot \tan(dx + c)^2 - 171 \cdot I \cdot A \cdot \tan(dx + c) + 981 \cdot B \cdot \tan(dx + c) - 29 \cdot A - 259 \cdot I \cdot B) / (a^3 \cdot (\tan(dx + c) - I)^3)) / d$

**Mupad [B]**

time = 6.76, size = 184, normalized size = 0.96

$$\frac{\tan(c+dx) \left( \frac{B7i}{2a^3} + \frac{(-3B+A1)27i}{8a^3} \right) + \frac{4B}{3a^3} - \tan(c+dx)^2 \left( \frac{5B}{2a^3} + \frac{17(-3B+A1i)}{8a^3} \right) + \frac{17(-3B+A1i)}{12a^3}}{d \left( -\tan(c+dx)^3 \operatorname{Li} - 3 \tan(c+dx)^2 + \tan(c+dx) 3i + 1 \right)} + \frac{\ln(\tan(c+dx) + 1i) (B + A1i)}{16a^3 d} + \frac{B \tan(c+dx) \operatorname{Li}}{a^3 d} + \frac{\ln(\tan(c+dx) - i) (-49B + A15i)}{16a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((\tan(c + dx))^4 \cdot (A + B \cdot \tan(c + dx))) / (a + a \cdot \tan(c + dx) \cdot 1i)^3, x)$

[Out]  $(\tan(c + dx) \cdot ((B \cdot 7i) / (2 \cdot a^3) + ((A \cdot 1i - 3 \cdot B) \cdot 27i) / (8 \cdot a^3)) + (4 \cdot B) / (3 \cdot a^3) - \tan(c + dx)^2 \cdot ((5 \cdot B) / (2 \cdot a^3) + (17 \cdot (A \cdot 1i - 3 \cdot B)) / (8 \cdot a^3)) + (17 \cdot (A \cdot 1i - 3 \cdot B)) / (12 \cdot a^3)) / (d \cdot (\tan(c + dx) \cdot 3i - 3 \cdot \tan(c + dx)^2 - \tan(c + dx)^3 \cdot 1i + 1)) + (\log(\tan(c + dx) + 1i) \cdot (A \cdot 1i + B)) / (16 \cdot a^3 \cdot d) + (B \cdot \tan(c + dx) \cdot 1i) / (a^3 \cdot d) + (\log(\tan(c + dx) - 1i) \cdot (A \cdot 15i - 49 \cdot B)) / (16 \cdot a^3 \cdot d)$

$$3.52 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=148

$$\frac{(iA - 7B)x}{8a^3} - \frac{iB \log(\cos(c + dx))}{a^3 d} + \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2} + \frac{A + 7iB}{8d(a^3 + ia^3 \tan(c + dx))}$$

[Out] 1/8\*(I\*A-7\*B)\*x/a^3-I\*B\*ln(cos(d\*x+c))/a^3/d+1/6\*(I\*A-B)\*tan(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))^3+1/8\*(A+3\*I\*B)\*tan(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^2+1/8\*(A+7\*I\*B)/d/(a^3+I\*a^3\*tan(d\*x+c))

**Rubi [A]**

time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {3676, 3670, 3556, 12, 3607, 8}

$$\frac{A + 7iB}{8d(a^3 + ia^3 \tan(c + dx))} + \frac{x(-7B + iA)}{8a^3} - \frac{iB \log(\cos(c + dx))}{a^3 d} + \frac{(-B + iA) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] ((I\*A - 7\*B)\*x)/(8\*a^3) - (I\*B\*Log[Cos[c + d\*x]])/(a^3\*d) + ((I\*A - B)\*Tan[c + d\*x]^3)/(6\*d\*(a + I\*a\*Tan[c + d\*x])^3) + ((A + (3\*I)\*B)\*Tan[c + d\*x]^2)/(8\*a\*d\*(a + I\*a\*Tan[c + d\*x])^2) + (A + (7\*I)\*B)/(8\*d\*(a^3 + I\*a^3\*Tan[c + d\*x]))

**Rule 8**

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3607**

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2,

0] && LtQ[m, 0]

### Rule 3670

Int[(((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

### Rule 3676

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^3(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx &= \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} - \frac{\int \frac{\tan^2(c + dx)(3a(iA - B) + 6iaB \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx}{6a^2} \\
 &= \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2} + \frac{\int \frac{\tan(c + dx)}{a} dx}{8ad(a + ia \tan(c + dx))^2} \\
 &= \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2} - \frac{i \int -\frac{6a^3(iA - B)}{a} dx}{8ad(a + ia \tan(c + dx))^2} \\
 &= -\frac{iB \log(\cos(c + dx))}{a^3d} + \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2} \\
 &= -\frac{iB \log(\cos(c + dx))}{a^3d} + \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2} \\
 &= \frac{(iA - 7B)x}{8a^3} - \frac{iB \log(\cos(c + dx))}{a^3d} + \frac{(iA - B) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(A + 3iB) \tan^2(c + dx)}{8ad(a + ia \tan(c + dx))^2}
 \end{aligned}$$

### Mathematica [A]

time = 1.26, size = 178, normalized size = 1.20

$\frac{\sec^2(c + dx)((9iA - 51B) \cos(c + dx) - 2 \cos(3(c + dx))(-iA + B + 6Adx + 42iBdx - 48B \log(\cos(c + dx))) - 27A \sin(c + dx) - 81iB \sin(c + dx) + 2A \sin(3(c + dx)) + 2iB \sin(3(c + dx)) - 12iAdx \sin(3(c + dx)) + 84Bdx \sin(3(c + dx)) + 96iB \log(\cos(c + dx)) \sin(3(c + dx)))}{96a^3d(-i + \tan(c + dx))^3}$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] (Sec[c + d\*x]^3\*(((9\*I)\*A - 51\*B)\*Cos[c + d\*x] - 2\*Cos[3\*(c + d\*x)]\*((-I)\*A + B + 6\*A\*d\*x + (42\*I)\*B\*d\*x - 48\*B\*Log[Cos[c + d\*x]]) - 27\*A\*Sin[c + d\*x] - (81\*I)\*B\*Sin[c + d\*x] + 2\*A\*Sin[3\*(c + d\*x)] + (2\*I)\*B\*Sin[3\*(c + d\*x)] - (12\*I)\*A\*d\*x\*Sin[3\*(c + d\*x)] + 84\*B\*d\*x\*Sin[3\*(c + d\*x)] + (96\*I)\*B\*Log[Cos[c + d\*x]]\*Sin[3\*(c + d\*x)]))/(96\*a^3\*d\*(-I + Tan[c + d\*x])^3)

**Maple** [A]

time = 0.15, size = 110, normalized size = 0.74

method	result
derivativedivides	$\frac{\left(\frac{A}{16} + \frac{15iB}{16}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{17B}{8} + \frac{7iA}{8}}{\tan(dx+c)-i} - \frac{-\frac{5A}{4} - \frac{7iB}{4}}{2(\tan(dx+c)-i)^2} - \frac{-\frac{iA}{2} + \frac{B}{2}}{3(\tan(dx+c)-i)^3} + \frac{i(iA+B) \ln(\tan(dx+c)+i)}{16}}{da^3}$
default	$\frac{\left(\frac{A}{16} + \frac{15iB}{16}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{17B}{8} + \frac{7iA}{8}}{\tan(dx+c)-i} - \frac{-\frac{5A}{4} - \frac{7iB}{4}}{2(\tan(dx+c)-i)^2} - \frac{-\frac{iA}{2} + \frac{B}{2}}{3(\tan(dx+c)-i)^3} + \frac{i(iA+B) \ln(\tan(dx+c)+i)}{16}}{da^3}$
risch	$-\frac{15xB}{8a^3} + \frac{ixA}{8a^3} + \frac{11ie^{-2i(dx+c)}B}{16a^3d} + \frac{3e^{-2i(dx+c)}A}{16a^3d} - \frac{5ie^{-4i(dx+c)}B}{32a^3d} - \frac{3e^{-4i(dx+c)}A}{32a^3d} + \frac{ie^{-6i(dx+c)}B}{48a^3d} + \frac{e^{-6i(dx+c)}A}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURNVERB  
OSE)

[Out] 1/d/a^3\*((1/16\*A+15/16\*I\*B)\*ln(tan(d\*x+c)-I)-(-17/8\*B+7/8\*I\*A)/(tan(d\*x+c)-I)-1/2\*(-5/4\*A-7/4\*I\*B)/(tan(d\*x+c)-I)^2-1/3\*(-1/2\*I\*A+1/2\*B)/(tan(d\*x+c)-I)^3+1/16\*I\*(I\*A+B)\*ln(tan(d\*x+c)+I))

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.51, size = 104, normalized size = 0.70

$$\frac{(12(-iA + 15B)dx e^{6i dx + 6i c} + 96i B e^{6i dx + 6i c} \log(e^{2i dx + 2i c} + 1) - 6(3A + 11iB)e^{4i dx + 4i c} + 3(3A + 5iB)e^{2i dx + 2i c} - 2A - 2iB)e^{(-6i dx - 6i c)}}{96 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out]  $-1/96*(12*(-I*A + 15*B)*d*x*e^{(6*I*d*x + 6*I*c)} + 96*I*B*e^{(6*I*d*x + 6*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} + 1) - 6*(3*A + 11*I*B)*e^{(4*I*d*x + 4*I*c)} + 3*(3*A + 5*I*B)*e^{(2*I*d*x + 2*I*c)} - 2*A - 2*I*B)*e^{(-6*I*d*x - 6*I*c)}/(a^3*d)$

**Sympy** [A]

time = 0.71, size = 296, normalized size = 2.00

$$-\frac{iB \log(e^{2idx} + e^{-2ic})}{a^3 d} + \begin{cases} \frac{((512Aa^6 d^2 e^{6ic} + 512iBa^6 d^2 e^{6ic})e^{-6idx} + (-2304Aa^6 d^2 e^{8ic} - 3840iBa^6 d^2 e^{8ic})e^{-4idx} + (4608Aa^6 d^2 e^{10ic} + 16896iBa^6 d^2 e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9 d^3} & \text{for } a^3 d^3 e^{12ic} \neq 0 \\ x \left( -\frac{iA-15B}{8a^3} + \frac{(iAe^{6ic} - 3iAe^{4ic} + 3iAe^{2ic} - iA - 15Be^{6ic} + 11Be^{4ic} - 5Be^{2ic} + B)e^{-6ic}}{8a^3} \right) & \text{otherwise} \end{cases} + \frac{x(iA - 15B)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out]  $-I*B*\log(\exp(2*I*d*x) + \exp(-2*I*c))/(a**3*d) + \text{Piecewise}(((512*A*a**6*d**2*\exp(6*I*c) + 512*I*B*a**6*d**2*\exp(6*I*c))*\exp(-6*I*d*x) + (-2304*A*a**6*d**2*\exp(8*I*c) - 3840*I*B*a**6*d**2*\exp(8*I*c))*\exp(-4*I*d*x) + (4608*A*a**6*d**2*\exp(10*I*c) + 16896*I*B*a**6*d**2*\exp(10*I*c))*\exp(-2*I*d*x))*\exp(-12*I*c)/(24576*a**9*d**3), \text{Ne}(a**9*d**3*\exp(12*I*c), 0)), (x*(-(I*A - 15*B)/(8*a**3) + (I*A*\exp(6*I*c) - 3*I*A*\exp(4*I*c) + 3*I*A*\exp(2*I*c) - I*A - 15*B*\exp(6*I*c) + 11*B*\exp(4*I*c) - 5*B*\exp(2*I*c) + B)*\exp(-6*I*c)/(8*a**3)), \text{True})) + x*(I*A - 15*B)/(8*a**3)$

**Giac** [A]

time = 0.96, size = 130, normalized size = 0.88

$$\frac{6(A+15iB)\log(\tan(dx+c)-i)}{a^3} - \frac{6(A-iB)\log(-i\tan(dx+c)+1)}{a^3} - \frac{11A\tan(dx+c)^3+165iB\tan(dx+c)^3+51iA\tan(dx+c)^2+291B\tan(dx+c)^2+75A\tan(dx+c)-171iB\tan(dx+c)-29iA-29B}{a^3(\tan(dx+c)-i)^3}$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $1/96*(6*(A + 15*I*B)*\log(\tan(d*x + c) - I)/a^3 - 6*(A - I*B)*\log(-I*\tan(d*x + c) + 1)/a^3 - (11*A*\tan(d*x + c)^3 + 165*I*B*\tan(d*x + c)^3 + 51*I*A*\tan(d*x + c)^2 + 291*B*\tan(d*x + c)^2 + 75*A*\tan(d*x + c) - 171*I*B*\tan(d*x + c) - 29*I*A - 29*B)/(a^3*(\tan(d*x + c) - I)^3))/d$

**Mupad** [B]

time = 6.64, size = 146, normalized size = 0.99

$$\frac{5A}{12a^3} - \tan(c+dx)^2 \left( \frac{7A}{8a^3} + \frac{B17i}{8a^3} \right) + \frac{B17i}{12a^3} + \tan(c+dx) \left( -\frac{27B}{8a^3} + \frac{A9i}{8a^3} \right) - \frac{\ln(\tan(c+dx)+1i)(A-B1i)}{16a^3 d} + \frac{\ln(\tan(c+dx)-i)(A+B15i)}{16a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x)^3*(A + B*\tan(c + d*x)))/(a + a*\tan(c + d*x)*1i)^3,x)$

[Out]  $((5*A)/(12*a^3) - \tan(c + d*x)^2*((7*A)/(8*a^3) + (B*17i)/(8*a^3)) + (B*17i)/(12*a^3) + \tan(c + d*x)*((A*9i)/(8*a^3) - (27*B)/(8*a^3)))/(d*(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1)) - (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(16*a^3*d) + (\log(\tan(c + d*x) - 1i)*(A + B*15i))/(16*a^3*d)$



$$3.53 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=124

$$-\frac{(A-iB)x}{8a^3} + \frac{(iA-B) \tan^2(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iA-7B}{24ad(a+ia \tan(c+dx))^2} + \frac{iA+17B}{24d(a^3+ia^3 \tan(c+dx))}$$

[Out]  $-1/8*(A-I*B)*x/a^3+1/6*(I*A-B)*\tan(d*x+c)^2/d/(a+I*a*\tan(d*x+c))^3+1/24*(I*A-7*B)/a/d/(a+I*a*\tan(d*x+c))^2+1/24*(I*A+17*B)/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi [A]**

time = 0.34, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3676, 3671, 3607, 8}

$$\frac{17B+iA}{24d(a^3+ia^3 \tan(c+dx))} - \frac{x(A-iB)}{8a^3} + \frac{(-B+iA) \tan^2(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{-7B+iA}{24ad(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]^2*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^3,x]$

[Out]  $-1/8*((A-I*B)*x)/a^3 + ((I*A-B)*\text{Tan}[c+d*x]^2)/(6*d*(a+I*a*\text{Tan}[c+d*x])^3) + (I*A-7*B)/(24*a*d*(a+I*a*\text{Tan}[c+d*x])^2) + (I*A+17*B)/(24*d*(a^3+I*a^3*\text{Tan}[c+d*x]))$

Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3607

$\text{Int}(((a_) + (b_)*\text{tan}[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)])}, x\_Symbol) \rightarrow \text{Simp}[(-(b*c - a*d))*((a + b*\text{Tan}[e + f*x])^m/(2*a*f*m)), x] + \text{Dist}[(b*c + a*d)/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0]$

Rule 3671

$\text{Int}(((a_) + (b_)*\text{tan}[(e_) + (f_)*(x_)])^{(m_)*((A_) + (B_)*\text{tan}[(e_) + (f_)*(x_)])}, x\_Symbol) \rightarrow \text{Simp}[(-(A*b - a*B))*((a*c + b*d))*((a + b*\text{Tan}[e + f*x])^m/(2*a^2*f*m)), x] + \text{Dist}[1/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*\text{Simp}[A*b*c + a*B*c + a*A*d + b*B*d + 2*a*B*d*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m, -1] \&\& \text{EqQ}[a^2 + b^2, 0]$

## Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

## Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx &= \frac{(iA - B) \tan^2(c + dx)}{6d(a + ia \tan(c + dx))^3} - \frac{\int \frac{\tan(c + dx)(2a(iA - B) - a(A - 5iB) \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx}{6a^2} \\ &= \frac{(iA - B) \tan^2(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{iA - 7B}{24ad(a + ia \tan(c + dx))^2} + \frac{i \int \frac{a^2(iA - 7B)}{a^2} dx}{24d(a^3 + ia^2 \tan(c + dx))} \\ &= \frac{(iA - B) \tan^2(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{iA - 7B}{24ad(a + ia \tan(c + dx))^2} + \frac{i}{24d(a^3 + ia^2 \tan(c + dx))} \\ &= -\frac{(A - iB)x}{8a^3} + \frac{(iA - B) \tan^2(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{iA - 7B}{24ad(a + ia \tan(c + dx))^2} \end{aligned}$$

## Mathematica [A]

time = 1.09, size = 147, normalized size = 1.19

$$\frac{\sec^3(c + dx)(-9(A - iB) \cos(c + dx) + 2(A + iB - 6iAdx - 6Bdx) \cos(3(c + dx)) - 3iA \sin(c + dx) - 27B \sin(c + dx) - 2iA \sin(3(c + dx)) + 2B \sin(3(c + dx)) + 12Adx \sin(3(c + dx)) - 12iBdx \sin(3(c + dx)))}{96a^3 d(-i + \tan(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] (Sec[c + d\*x]^3\*(-9\*(A - I\*B)\*Cos[c + d\*x] + 2\*(A + I\*B - (6\*I)\*A\*d\*x - 6\*B\*d\*x)\*Cos[3\*(c + d\*x)] - (3\*I)\*A\*Sin[c + d\*x] - 27\*B\*Sin[c + d\*x] - (2\*I)\*A\*Sin[3\*(c + d\*x)] + 2\*B\*Sin[3\*(c + d\*x)] + 12\*A\*d\*x\*Sin[3\*(c + d\*x)] - (12\*I)\*B\*d\*x\*Sin[3\*(c + d\*x)])/(96\*a^3\*d\*(-I + Tan[c + d\*x])^3)

## Maple [A]

time = 0.12, size = 110, normalized size = 0.89

method	result
derivativedivides	$-\frac{-\frac{A}{2} - \frac{iB}{2}}{3(\tan(dx+c)-i)^3} - \frac{\frac{A}{8} + \frac{7iB}{8}}{\tan(dx+c)-i} + \left(\frac{iA}{16} + \frac{B}{16}\right) \ln(\tan(dx+c)-i) - \frac{\frac{3iA}{4} - \frac{5B}{4}}{2(\tan(dx+c)-i)^2} - \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{16}$ $d a^3$

default	$\frac{-\frac{A}{2} - \frac{iB}{2}}{3(\tan(dx+c)-i)^3} - \frac{\frac{A}{8} + \frac{7iB}{8}}{\tan(dx+c)-i} + \left(\frac{iA}{16} + \frac{B}{16}\right) \ln(\tan(dx+c)-i) - \frac{\frac{3iA}{4} - \frac{5B}{4}}{2(\tan(dx+c)-i)^2} - \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{16}$
risch	$\frac{ixB}{8a^3} - \frac{xA}{8a^3} + \frac{3e^{-2i(dx+c)}B}{16a^3d} + \frac{ie^{-2i(dx+c)}A}{16a^3d} - \frac{3e^{-4i(dx+c)}B}{32a^3d} + \frac{ie^{-4i(dx+c)}A}{32a^3d} + \frac{e^{-6i(dx+c)}B}{48a^3d} - \frac{ie^{-6i(dx+c)}A}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^3} \left( -\frac{1}{3} \left( -\frac{1}{2}A - \frac{1}{2}I*B \right) (\tan(dx+c)-I)^{-3} - \left( \frac{1}{8}A + \frac{7}{8}I*B \right) (\tan(dx+c)-I) + \left( \frac{1}{16}I*A + \frac{1}{16}B \right) \ln(\tan(dx+c)-I) - \frac{1}{2} \left( \frac{3}{4}I*A - \frac{5}{4}B \right) (\tan(dx+c)-I)^{-2} - \frac{1}{16}I*(A-I*B) \ln(\tan(dx+c)+I) \right)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.46, size = 78, normalized size = 0.63

$$\frac{(12(A-iB)dx e^{6i dx+6i c}) + 6(-iA-3B)e^{4i dx+4i c} + 3(-iA+3B)e^{2i dx+2i c} + 2iA-2B)e^{-6i dx-6i c}}{96a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

[Out]  $\frac{-1/96 * (12 * (A - I*B) * d*x * e^{(6*I*d*x + 6*I*c)} + 6 * (-I*A - 3*B) * e^{(4*I*d*x + 4*I*c)} + 3 * (-I*A + 3*B) * e^{(2*I*d*x + 2*I*c)} + 2*I*A - 2*B) * e^{(-6*I*d*x - 6*I*c)}}{a^3*d}$

**Sympy** [A]

time = 0.30, size = 258, normalized size = 2.08

$$\begin{cases} \frac{((-512iAa^6d^2e^{6ic}+512Ba^6d^2e^{6ic})e^{-6idx}+(768iAa^6d^2e^{8ic}-2304Ba^6d^2e^{8ic})e^{-4idx}+(1536iAa^6d^2e^{10ic}+4608Ba^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic} \neq 0 \\ x\left(-\frac{-A+iB}{8a^3} + \frac{(-Ae^{6ic}+Ae^{4ic}+Ae^{2ic}-A+iBe^{6ic}-3iBe^{4ic}+3iBe^{2ic}-iB)e^{-6ic}}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x(-A+iB)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] Piecewise(((((-512\*I\*A\*a\*\*6\*d\*\*2\*exp(6\*I\*c) + 512\*B\*a\*\*6\*d\*\*2\*exp(6\*I\*c))\*exp(-6\*I\*d\*x) + (768\*I\*A\*a\*\*6\*d\*\*2\*exp(8\*I\*c) - 2304\*B\*a\*\*6\*d\*\*2\*exp(8\*I\*c))\*exp(-4\*I\*d\*x) + (1536\*I\*A\*a\*\*6\*d\*\*2\*exp(10\*I\*c) + 4608\*B\*a\*\*6\*d\*\*2\*exp(10\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-12\*I\*c)/(24576\*a\*\*9\*d\*\*3), Ne(a\*\*9\*d\*\*3\*exp(12\*I\*c), 0)), (x\*(-(-A + I\*B)/(8\*a\*\*3) + (-A\*exp(6\*I\*c) + A\*exp(4\*I\*c) + A\*exp(2\*I\*c) - A + I\*B\*exp(6\*I\*c) - 3\*I\*B\*exp(4\*I\*c) + 3\*I\*B\*exp(2\*I\*c) - I\*B)\*exp(-6\*I\*c)/(8\*a\*\*3)), True)) + x\*(-A + I\*B)/(8\*a\*\*3)

**Giac** [A]

time = 0.81, size = 131, normalized size = 1.06

$$\frac{6(-iA-B)\log(\tan(dx+c)-i)}{a^3} + \frac{6(iA+B)\log(i\tan(dx+c)-1)}{a^3} + \frac{11iA\tan(dx+c)^3+11B\tan(dx+c)^3+45A\tan(dx+c)^2+51iB\tan(dx+c)^2-21iA\tan(dx+c)+75B\tan(dx+c)-3A-29iB}{a^3(\tan(dx+c)-i)^3}$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] -1/96\*(6\*(-I\*A - B)\*log(tan(d\*x + c) - I)/a^3 + 6\*(I\*A + B)\*log(I\*tan(d\*x + c) - 1)/a^3 + (11\*I\*A\*tan(d\*x + c)^3 + 11\*B\*tan(d\*x + c)^3 + 45\*A\*tan(d\*x + c)^2 + 51\*I\*B\*tan(d\*x + c)^2 - 21\*I\*A\*tan(d\*x + c) + 75\*B\*tan(d\*x + c) - 3\*A - 29\*I\*B)/(a^3\*(tan(d\*x + c) - I)^3))/d

**Mupad** [B]

time = 6.52, size = 111, normalized size = 0.90

$$\frac{\tan(c+dx)^2\left(-\frac{7B}{8a^3} + \frac{A1i}{8a^3}\right) + \frac{A1i}{12a^3} + \frac{5B}{12a^3} - \tan(c+dx)\left(\frac{A}{8a^3} - \frac{B9i}{8a^3}\right) + \frac{x(B+A1i)1i}{8a^3}}{d\left(-\tan(c+dx)^31i - 3\tan(c+dx)^2 + \tan(c+dx)3i + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] (tan(c + d\*x)^2\*((A\*1i)/(8\*a^3) - (7\*B)/(8\*a^3)) + (A\*1i)/(12\*a^3) + (5\*B)/(12\*a^3) - tan(c + d\*x)\*(A/(8\*a^3) - (B\*9i)/(8\*a^3)))/(d\*(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1)) + (x\*(A\*1i + B)\*1i)/(8\*a^3)

$$3.54 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=110

$$-\frac{(iA+B)x}{8a^3} - \frac{A+iB}{6d(a+ia \tan(c+dx))^3} + \frac{A+3iB}{8ad(a+ia \tan(c+dx))^2} + \frac{A-iB}{8d(a^3+ia^3 \tan(c+dx))}$$

[Out]  $-1/8*(I*A+B)*x/a^3+1/6*(-A-I*B)/d/(a+I*a*\tan(d*x+c))^3+1/8*(A+3*I*B)/a/d/(a+I*a*\tan(d*x+c))^2+1/8*(A-I*B)/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi** [A]

time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3671, 3607, 3560, 8}

$$\frac{A-iB}{8d(a^3+ia^3 \tan(c+dx))} - \frac{x(B+iA)}{8a^3} + \frac{A+3iB}{8ad(a+ia \tan(c+dx))^2} - \frac{A+iB}{6d(a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^3,x]$

[Out]  $-1/8*((I*A+B)*x)/a^3 - (A+I*B)/(6*d*(a+I*a*\text{Tan}[c+d*x])^3) + (A+(3*I)*B)/(8*a*d*(a+I*a*\text{Tan}[c+d*x])^2) + (A-I*B)/(8*d*(a^3+I*a^3*\text{Tan}[c+d*x]))$

**Rule 8**

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

**Rule 3560**

$\text{Int}[(a_ + (b_)*\text{tan}[(c_ + (d_)*(x_))])^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[a*((a + b*\text{Tan}[c + d*x])^n/(2*b*d*n)), x] + \text{Dist}[1/(2*a), \text{Int}[(a + b*\text{Tan}[c + d*x])^{(n+1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[n, 0]$

**Rule 3607**

$\text{Int}[(a_ + (b_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}), x\_Symbol] \rightarrow \text{Simp}[(-(b*c - a*d))*((a + b*\text{Tan}[e + f*x])^m/(2*a*f*m)), x] + \text{Dist}[(b*c + a*d)/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, 0]$

**Rule 3671**

$\text{Int}[(a_ + (b_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}*((A_ + (B_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\text{tan}[(e_ + (f_)*(x_))])^{(m_)}), x\_Symbol] \rightarrow \text{Simp}[(-($

```
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Dist[1/(
2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*d
+ 2*a*B*d*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3} dx &= -\frac{A+iB}{6d(a+ia\tan(c+dx))^3} - \frac{i \int \frac{a(A+iB)+2aB\tan(c+dx)}{(a+ia\tan(c+dx))^2} dx}{2a^2} \\ &= -\frac{A+iB}{6d(a+ia\tan(c+dx))^3} + \frac{A+3iB}{8ad(a+ia\tan(c+dx))^2} - \frac{(iA+B)}{8d(a^3+ia^2\tan(c+dx))} \\ &= -\frac{A+iB}{6d(a+ia\tan(c+dx))^3} + \frac{A+3iB}{8ad(a+ia\tan(c+dx))^2} + \frac{A}{8d(a^3+ia^2\tan(c+dx))} \\ &= -\frac{(iA+B)x}{8a^3} - \frac{A+iB}{6d(a+ia\tan(c+dx))^3} + \frac{A+3iB}{8ad(a+ia\tan(c+dx))^2} \end{aligned}$$

Mathematica [A]

time = 1.41, size = 148, normalized size = 1.35

$(\cos(3(c+dx)) - i\sin(3(c+dx)))(3(A+3iB)\cos(c+dx) - 2(A+6iAdx+B(i+6dx))\cos(3(c+dx)) + 9iA\sin(c+dx) - 3B\sin(c+dx) + 2iA\sin(3(c+dx)) - 2B\sin(3(c+dx)) + 12Adx\sin(3(c+dx)) - 12iBdx\sin(3(c+dx)))/96a^3d$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] ((Cos[3\*(c + d\*x)] - I\*Sin[3\*(c + d\*x)])\*(3\*(A + (3\*I)\*B)\*Cos[c + d\*x] - 2\*(A + (6\*I)\*A\*d\*x + B\*(I + 6\*d\*x))\*Cos[3\*(c + d\*x)] + (9\*I)\*A\*Sin[c + d\*x] - 3\*B\*Sin[c + d\*x] + (2\*I)\*A\*Sin[3\*(c + d\*x)] - 2\*B\*Sin[3\*(c + d\*x)] + 12\*A\*d\*x\*Sin[3\*(c + d\*x)] - (12\*I)\*B\*d\*x\*Sin[3\*(c + d\*x)]))/(96\*a^3\*d)

Maple [A]

time = 0.12, size = 110, normalized size = 1.00

method	result
derivativedivides	$-\frac{\frac{A}{4} + \frac{3iB}{4}}{2(\tan(dx+c)-i)^2} - \frac{\frac{iA}{8} + \frac{B}{8}}{\tan(dx+c)-i} - \frac{\frac{iA}{2} - \frac{B}{2}}{3(\tan(dx+c)-i)^3} + \left(-\frac{A}{16} + \frac{iB}{16}\right) \ln(\tan(dx+c)-i) - \frac{i(iA+B) \ln(\tan(dx+c)+i)}{16}$
default	$-\frac{\frac{A}{4} + \frac{3iB}{4}}{2(\tan(dx+c)-i)^2} - \frac{\frac{iA}{8} + \frac{B}{8}}{\tan(dx+c)-i} - \frac{\frac{iA}{2} - \frac{B}{2}}{3(\tan(dx+c)-i)^3} + \left(-\frac{A}{16} + \frac{iB}{16}\right) \ln(\tan(dx+c)-i) - \frac{i(iA+B) \ln(\tan(dx+c)+i)}{16}$
risch	$-\frac{xB}{8a^3} - \frac{ixA}{8a^3} + \frac{ie^{-2i(dx+c)}B}{16a^3d} + \frac{e^{-2i(dx+c)}A}{16a^3d} + \frac{ie^{-4i(dx+c)}B}{32a^3d} - \frac{e^{-4i(dx+c)}A}{32a^3d} - \frac{ie^{-6i(dx+c)}B}{48a^3d} - \frac{e^{-6i(dx+c)}A}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^3} \left( -\frac{1}{2} \frac{(1/4 A + 3/4 I B)}{(\tan(d*x+c) - I)^2} - \frac{(1/8 I A + 1/8 B)}{(\tan(d*x+c) - I)} - \frac{1}{3} \frac{(1/2 I A - 1/2 B)}{(\tan(d*x+c) - I)^3} + \frac{(-1/16 A + 1/16 I B) \ln(\tan(d*x+c) - I) - 1/16 I (I A + B) \ln(\tan(d*x+c) + I)}{1} \right)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.46, size = 74, normalized size = 0.67

$$\frac{(12(iA + B)dx e^{(6i dx + 6i c)} - 6(A + iB)e^{(4i dx + 4i c)} + 3(A - iB)e^{(2i dx + 2i c)} + 2A + 2iB)e^{(-6i dx - 6i c)}}{96 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

[Out]  $-\frac{1}{96} \frac{(12(I A + B) d x x e^{(6 I d x + 6 I c)} - 6(A + I B) e^{(4 I d x + 4 I c)} + 3(A - I B) e^{(2 I d x + 2 I c)} + 2A + 2 I B) e^{(-6 I d x - 6 I c)}}{(a^3 d)}$

**Sympy** [A]

time = 0.32, size = 260, normalized size = 2.36

$$\begin{cases} \frac{((-512Aa^6d^2e^{6ic} - 512iBa^6d^2e^{6ic})e^{-6idx} + (-768Aa^6d^2e^{8ic} + 768iBa^6d^2e^{8ic})e^{-4idx} + (1536Aa^6d^2e^{10ic} + 1536iBa^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic} \neq 0 \\ x \left( -\frac{iA-B}{8a^3} + \frac{(-iAe^{6ic} - iAe^{4ic} + iAe^{2ic} + iA - Be^{6ic} + Be^{4ic} + Be^{2ic} - B)e^{-6ic}}{8a^3} \right) & \text{otherwise} \end{cases} + \frac{x(-iA - B)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)`

[Out] `Piecewise(((((-512*A*a**6*d**2*exp(6*I*c) - 512*I*B*a**6*d**2*exp(6*I*c))*exp(-6*I*d*x) + (-768*A*a**6*d**2*exp(8*I*c) + 768*I*B*a**6*d**2*exp(8*I*c))*exp(-4*I*d*x) + (1536*A*a**6*d**2*exp(10*I*c) + 1536*I*B*a**6*d**2*exp(10*I*c))*exp(-2*I*d*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*(-(-I*A - B)/(8*a**3) + (-I*A*exp(6*I*c) - I*A*exp(4*I*c) + I*A*`

$\exp(2*I*c) + I*A - B*\exp(6*I*c) + B*\exp(4*I*c) + B*\exp(2*I*c) - B*\exp(-6*I*c)/(8*a**3)), True)) + x*(-I*A - B)/(8*a**3)$

**Giac [A]**

time = 0.66, size = 130, normalized size = 1.18

$$\frac{\frac{6(A-iB)\log(\tan(dx+c)-i)}{a^3} - \frac{6(A-iB)\log(i\tan(dx+c)-1)}{a^3} - \frac{11A\tan(dx+c)^3 - 11iB\tan(dx+c)^3 - 45iA\tan(dx+c)^2 - 45B\tan(dx+c)^2 - 69A\tan(dx+c) + 21iB\tan(dx+c) + 19iA + 3B}{a^3(\tan(dx+c)-i)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $-1/96*(6*(A - I*B)*\log(\tan(d*x + c) - I)/a^3 - 6*(A - I*B)*\log(I*\tan(d*x + c) - 1)/a^3 - (11*A*\tan(d*x + c)^3 - 11*I*B*\tan(d*x + c)^3 - 45*I*A*\tan(d*x + c)^2 - 45*B*\tan(d*x + c)^2 - 69*A*\tan(d*x + c) + 21*I*B*\tan(d*x + c) + 19*I*A + 3*B)/(a^3*(\tan(d*x + c) - I)^3))/d$

**Mupad [B]**

time = 6.50, size = 147, normalized size = 1.34

$$\frac{\frac{A}{12a^3} - \tan(c+dx)^2\left(\frac{A}{8a^3} - \frac{B1i}{8a^3}\right) + \frac{B1i}{12a^3} + \tan(c+dx)\left(-\frac{B}{8a^3} + \frac{A3i}{8a^3}\right)}{d(-\tan(c+dx)^31i - 3\tan(c+dx)^2 + \tan(c+dx)3i + 1)} + \frac{\ln(\tan(c+dx)-i)(B+A1i)1i}{16a^3d} + \frac{\ln(\tan(c+dx)+1i)(A-B1i)}{16a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out]  $(A/(12*a^3) - \tan(c + d*x)^2*(A/(8*a^3) - (B*1i)/(8*a^3)) + (B*1i)/(12*a^3) + \tan(c + d*x)*((A*3i)/(8*a^3) - B/(8*a^3)))/(d*(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1)) + (\log(\tan(c + d*x) - 1i)*(A*1i + B)*1i)/(16*a^3*d) + (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(16*a^3*d)$



### 3.55 $\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^3} dx$

**Optimal.** Leaf size=112

$$\frac{(A-iB)x}{8a^3} + \frac{iA-B}{6d(a+ia \tan(c+dx))^3} + \frac{iA+B}{8ad(a+ia \tan(c+dx))^2} + \frac{iA+B}{8d(a^3+ia^3 \tan(c+dx))}$$

[Out] 1/8\*(A-I\*B)\*x/a^3+1/6\*(I\*A-B)/d/(a+I\*a\*tan(d\*x+c))^3+1/8\*(I\*A+B)/a/d/(a+I\*a\*tan(d\*x+c))^2+1/8\*(I\*A+B)/d/(a^3+I\*a^3\*tan(d\*x+c))

**Rubi [A]**

time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3607, 3560, 8}

$$\frac{B+iA}{8d(a^3+ia^3 \tan(c+dx))} + \frac{x(A-iB)}{8a^3} + \frac{-B+iA}{6d(a+ia \tan(c+dx))^3} + \frac{B+iA}{8ad(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] ((A - I\*B)\*x)/(8\*a^3) + (I\*A - B)/(6\*d\*(a + I\*a\*Tan[c + d\*x])^3) + (I\*A + B)/(8\*a\*d\*(a + I\*a\*Tan[c + d\*x])^2) + (I\*A + B)/(8\*d\*(a^3 + I\*a^3\*Tan[c + d\*x]))

**Rule 8**

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

**Rule 3560**

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[a\*((a + b\*Tan[c + d\*x])^n/(2\*b\*d\*n)), x] + Dist[1/(2\*a), Int[(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

**Rule 3607**

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

**Rubi steps**

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^3} dx &= \frac{iA - B}{6d(a + ia \tan(c + dx))^3} + \frac{(A - iB) \int \frac{1}{(a + ia \tan(c + dx))^2} dx}{2a} \\
&= \frac{iA - B}{6d(a + ia \tan(c + dx))^3} + \frac{iA + B}{8ad(a + ia \tan(c + dx))^2} + \frac{(A - iB) \int \frac{1}{a + ia \tan(c + dx)}}{4a^2} \\
&= \frac{iA - B}{6d(a + ia \tan(c + dx))^3} + \frac{iA + B}{8ad(a + ia \tan(c + dx))^2} + \frac{iA + B}{8d(a^3 + ia^3 \tan(c + dx))} \\
&= \frac{(A - iB)x}{8a^3} + \frac{iA - B}{6d(a + ia \tan(c + dx))^3} + \frac{iA + B}{8ad(a + ia \tan(c + dx))^2} + \frac{iA + B}{8d(a^3 + ia^3 \tan(c + dx))}
\end{aligned}$$

**Mathematica [A]**

time = 0.78, size = 150, normalized size = 1.34

$$\frac{\sec^3(c + dx)((-27A + 3iB) \cos(c + dx) + 2(-A - iB + 6iAdx + 6Bdx) \cos(3(c + dx)) - 9iA \sin(c + dx) - 9B \sin(c + dx) + 2iA \sin(3(c + dx)) - 2B \sin(3(c + dx)) - 12Adx \sin(3(c + dx)) + 12iBdx \sin(3(c + dx)))}{96a^3d(-i + \tan(c + dx))^3}$$

Antiderivative was successfully verified.

`[In] Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^3, x]`

```
[Out] (Sec[c + d*x]^3*((-27*A + (3*I)*B)*Cos[c + d*x] + 2*(-A - I*B + (6*I)*A*d*x
+ 6*B*d*x)*Cos[3*(c + d*x)] - (9*I)*A*Sin[c + d*x] - 9*B*Sin[c + d*x] + (2
*I)*A*Sin[3*(c + d*x)] - 2*B*Sin[3*(c + d*x)] - 12*A*d*x*Sin[3*(c + d*x)] +
(12*I)*B*d*x*Sin[3*(c + d*x)]))/(96*a^3*d*(-I + Tan[c + d*x])^3)
```

**Maple [A]**

time = 0.12, size = 110, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{\frac{iA}{4} + \frac{B}{4}}{2(\tan(dx+c)-i)^2} - \frac{\frac{A}{2} + \frac{iB}{2}}{3(\tan(dx+c)-i)^3} - \frac{-\frac{A}{8} + \frac{iB}{8}}{\tan(dx+c)-i} + \left(-\frac{iA}{16} - \frac{B}{16}\right) \ln(\tan(dx+c)-i) + \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{16}}{da^3}$
default	$\frac{-\frac{\frac{iA}{4} + \frac{B}{4}}{2(\tan(dx+c)-i)^2} - \frac{\frac{A}{2} + \frac{iB}{2}}{3(\tan(dx+c)-i)^3} - \frac{-\frac{A}{8} + \frac{iB}{8}}{\tan(dx+c)-i} + \left(-\frac{iA}{16} - \frac{B}{16}\right) \ln(\tan(dx+c)-i) + \frac{i(-iB+A) \ln(\tan(dx+c)+i)}{16}}{da^3}$
risch	$-\frac{ixB}{8a^3} + \frac{xA}{8a^3} + \frac{e^{-2i(dx+c)}B}{16a^3d} + \frac{3ie^{-2i(dx+c)}A}{16a^3d} - \frac{e^{-4i(dx+c)}B}{32a^3d} + \frac{3ie^{-4i(dx+c)}A}{32a^3d} - \frac{e^{-6i(dx+c)}B}{48a^3d} + \frac{ie^{-6i(dx+c)}A}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3, x, method=_RETURNVERBOSE)`

```
[Out] 1/d/a^3*(-1/2*(1/4*I*A+1/4*B)/(tan(d*x+c)-I)^2-1/3*(1/2*A+1/2*I*B)/(tan(d*x
+c)-I)^3-(-1/8*A+1/8*I*B)/(tan(d*x+c)-I)+(-1/16*I*A-1/16*B)*ln(tan(d*x+c)-I
)+1/16*I*(A-I*B)*ln(tan(d*x+c)+I))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")**[Out]** Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.**Fricas [A]**

time = 0.49, size = 76, normalized size = 0.68

$$\frac{(12(A-iB)dx e^{(6i dx+6i c)} - 6(-3i A - B)e^{(4i dx+4i c)} - 3(-3i A + B)e^{(2i dx+2i c)} + 2i A - 2B)e^{(-6i dx-6i c)}}{96 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")**[Out]** 1/96\*(12\*(A - I\*B)\*d\*x\*e^(6\*I\*d\*x + 6\*I\*c) - 6\*(-3\*I\*A - B)\*e^(4\*I\*d\*x + 4\*I\*c) - 3\*(-3\*I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + 2\*I\*A - 2\*B)\*e^(-6\*I\*d\*x - 6\*I\*c)/(a^3\*d)**Sympy [A]**

time = 0.27, size = 258, normalized size = 2.30

$$\begin{cases} \frac{((512iAa^6d^2e^{6ic}-512Ba^6d^2e^{6ic})e^{-6idx}+(2304iAa^6d^2e^{8ic}-768Ba^6d^2e^{8ic})e^{-4idx}+(4608iAa^6d^2e^{10ic}+1536Ba^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic} \neq 0 \\ x\left(-\frac{A-iB}{8a^3} + \frac{(Ae^{6ic}+3Ae^{4ic}+3Ae^{2ic}+A-iB)e^{6ic}-iBe^{4ic}+iBe^{2ic}+iB)e^{-6ic}}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)**[Out]** Piecewise((((512\*I\*A\*a\*\*6\*d\*\*2\*exp(6\*I\*c) - 512\*B\*a\*\*6\*d\*\*2\*exp(6\*I\*c))\*exp(-6\*I\*d\*x) + (2304\*I\*A\*a\*\*6\*d\*\*2\*exp(8\*I\*c) - 768\*B\*a\*\*6\*d\*\*2\*exp(8\*I\*c))\*exp(-4\*I\*d\*x) + (4608\*I\*A\*a\*\*6\*d\*\*2\*exp(10\*I\*c) + 1536\*B\*a\*\*6\*d\*\*2\*exp(10\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-12\*I\*c)/(24576\*a\*\*9\*d\*\*3), Ne(a\*\*9\*d\*\*3\*exp(12\*I\*c), 0)), (x\*(-(A - I\*B)/(8\*a\*\*3) + (A\*exp(6\*I\*c) + 3\*A\*exp(4\*I\*c) + 3\*A\*exp(2\*I\*c) + A - I\*B\*exp(6\*I\*c) - I\*B\*exp(4\*I\*c) + I\*B\*exp(2\*I\*c) + I\*B)\*exp(-6\*I\*c)/(8\*a\*\*3)), True)) + x\*(A - I\*B)/(8\*a\*\*3)**Giac [A]**

time = 0.68, size = 131, normalized size = 1.17

$$\frac{6(iA+B)\log(\tan(dx+c)-i)}{a^3} + \frac{6(-iA-B)\log(i\tan(dx+c)-1)}{a^3} + \frac{-11iA\tan(dx+c)^3-11B\tan(dx+c)^3-45A\tan(dx+c)^2+45iB\tan(dx+c)^2+69iA\tan(dx+c)+69B\tan(dx+c)+51A-19iB}{a^3(\tan(dx+c)-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] 
$$-1/96*(6*(I*A + B)*\log(\tan(d*x + c) - I)/a^3 + 6*(-I*A - B)*\log(I*\tan(d*x + c) - 1)/a^3 + (-11*I*A*\tan(d*x + c)^3 - 11*B*\tan(d*x + c)^3 - 45*A*\tan(d*x + c)^2 + 45*I*B*\tan(d*x + c)^2 + 69*I*A*\tan(d*x + c) + 69*B*\tan(d*x + c) + 51*A - 19*I*B)/(a^3*(\tan(d*x + c) - I)^3)/d$$

**Mupad [B]**

time = 6.46, size = 111, normalized size = 0.99

$$\frac{\tan(c + dx)^2 \left( \frac{B}{8a^3} + \frac{A1i}{8a^3} \right) - \frac{A5i}{12a^3} - \frac{B}{12a^3} + \tan(c + dx) \left( \frac{3A}{8a^3} - \frac{B3i}{8a^3} \right)}{d \left( -\tan(c + dx)^3 1i - 3 \tan(c + dx)^2 + \tan(c + dx) 3i + 1 \right)} - \frac{x (B + A 1i) 1i}{8 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] 
$$-(\tan(c + d*x)^2*((A*1i)/(8*a^3) + B/(8*a^3)) - (A*5i)/(12*a^3) - B/(12*a^3) + \tan(c + d*x)*((3*A)/(8*a^3) - (B*3i)/(8*a^3)))/(d*(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1)) - (x*(A*1i + B)*1i)/(8*a^3)$$

$$3.56 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=131

$$-\frac{(7iA - B)x}{8a^3} + \frac{A \log(\sin(c + dx))}{a^3 d} + \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{3A + iB}{8ad(a + ia \tan(c + dx))^2} + \frac{7A + iB}{8d(a^3 + ia^3 \tan(c + dx))}$$

[Out] -1/8\*(7\*I\*A-B)\*x/a^3+A\*ln(sin(d\*x+c))/a^3/d+1/6\*(A+I\*B)/d/(a+I\*a\*tan(d\*x+c))^3+1/8\*(3\*A+I\*B)/a/d/(a+I\*a\*tan(d\*x+c))^2+1/8\*(7\*A+I\*B)/d/(a^3+I\*a^3\*tan(d\*x+c))

**Rubi [A]**

time = 0.25, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3677, 3612, 3556}

$$\frac{7A + iB}{8d(a^3 + ia^3 \tan(c + dx))} - \frac{x(-B + 7iA)}{8a^3} + \frac{A \log(\sin(c + dx))}{a^3 d} + \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{3A + iB}{8ad(a + ia \tan(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] -1/8\*(((7\*I)\*A - B)\*x)/a^3 + (A\*Log[Sin[c + d\*x]])/(a^3\*d) + (A + I\*B)/(6\*d\*(a + I\*a\*Tan[c + d\*x])^3) + (3\*A + I\*B)/(8\*a\*d\*(a + I\*a\*Tan[c + d\*x])^2) + (7\*A + I\*B)/(8\*d\*(a^3 + I\*a^3\*Tan[c + d\*x]))

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

**Rule 3677**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(n\_), x\_Symbol] :> Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ

[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

Rubi steps

$$\int \frac{\cot(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx = \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{\int \frac{\cot(c+dx)(6aA-3a(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx}{6a^2}$$

$$= \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{3A + iB}{8ad(a + ia \tan(c + dx))^2} + \frac{\int \frac{\cot(c+dx)(24a^2A-12a^2(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx}{8a^2}$$

$$= \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{3A + iB}{8ad(a + ia \tan(c + dx))^2} + \frac{7A - iB}{8d(a^3 + ia^3)}$$

$$= -\frac{(7iA - B)x}{8a^3} + \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{3A + iB}{8ad(a + ia \tan(c + dx))^2}$$

$$= -\frac{(7iA - B)x}{8a^3} + \frac{A \log(\sin(c + dx))}{a^3 d} + \frac{A + iB}{6d(a + ia \tan(c + dx))^3} + \frac{3A + iB}{8ad(a + ia \tan(c + dx))^2}$$

Mathematica [A]

time = 1.14, size = 180, normalized size = 1.37

$\frac{\sec^3(c + dx)((81A - 27B) \cos(c + dx) + 2 \cos(3(c + dx))(iA - B + 42Adx + 6iBdx + 48iA \log(\sin(c + dx))) - 51A \sin(c + dx) - 9iB \sin(c + dx) + 2A \sin(3(c + dx)) + 2iB \sin(3(c + dx)) + 84iAdx \sin(3(c + dx)) - 12Bdx \sin(3(c + dx)) - 96A \log(\sin(c + dx)) \sin(3(c + dx)))}{96a^3 d(-i + \tan(c + dx))^3}$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] (Sec[c + d\*x]^3(((81\*I)\*A - 27\*B)\*Cos[c + d\*x] + 2\*Cos[3\*(c + d\*x)]\*(I\*A - B + 42\*A\*d\*x + (6\*I)\*B\*d\*x + (48\*I)\*A\*Log[Sin[c + d\*x]]) - 51\*A\*Sin[c + d\*x] - (9\*I)\*B\*Sin[c + d\*x] + 2\*A\*Sin[3\*(c + d\*x)] + (2\*I)\*B\*Sin[3\*(c + d\*x)] + (84\*I)\*A\*d\*x\*Sin[3\*(c + d\*x)] - 12\*B\*d\*x\*Sin[3\*(c + d\*x)] - 96\*A\*Log[Sin[c + d\*x]]\*Sin[3\*(c + d\*x)]))/(96\*a^3\*d\*(-I + Tan[c + d\*x])^3)

Maple [A]

time = 0.28, size = 119, normalized size = 0.91

method	result
derivativedivides	$\frac{A \ln(\tan(dx+c)) - \frac{3A + iB}{4 + \frac{iB}{4}} + \left(-\frac{iB}{16} - \frac{15A}{16}\right) \ln(\tan(dx+c)-i) - \frac{\frac{7iA}{8} - \frac{B}{8}}{\tan(dx+c)-i} - \frac{-\frac{iA}{2} + \frac{B}{2}}{3(\tan(dx+c)-i)^3} + \left(-\frac{A}{16} + \frac{iB}{16}\right) \ln(\tan(dx+c)+i)}{d a^3}$
default	$\frac{A \ln(\tan(dx+c)) - \frac{3A + iB}{2(\tan(dx+c)-i)^2} + \left(-\frac{iB}{16} - \frac{15A}{16}\right) \ln(\tan(dx+c)-i) - \frac{\frac{7iA}{8} - \frac{B}{8}}{\tan(dx+c)-i} - \frac{-\frac{iA}{2} + \frac{B}{2}}{3(\tan(dx+c)-i)^3} + \left(-\frac{A}{16} + \frac{iB}{16}\right) \ln(\tan(dx+c)+i)}{d a^3}$
risch	$\frac{x B}{8a^3} - \frac{15ixA}{8a^3} + \frac{3ie^{-2i(dx+c)}B}{16a^3d} + \frac{11e^{-2i(dx+c)}A}{16a^3d} + \frac{3ie^{-4i(dx+c)}B}{32a^3d} + \frac{5e^{-4i(dx+c)}A}{32a^3d} + \frac{ie^{-6i(dx+c)}B}{48a^3d} + \frac{e^{-6i(dx+c)}A}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^3} \left( A \ln(\tan(dx+c)) - \frac{1}{2} \left( \frac{3}{4}A + \frac{1}{4}I*B \right) (\tan(dx+c) - I)^{-2} + \left( -\frac{1}{16}I*B - \frac{5}{16}A \right) \ln(\tan(dx+c) - I) - \frac{7}{8}I*A - \frac{1}{8}B \right) (\tan(dx+c) - I)^{-1} - \frac{1}{3} \left( -\frac{1}{2}I*A + \frac{1}{2}B \right) (\tan(dx+c) - I)^{-3} + \left( -\frac{1}{16}A + \frac{1}{16}I*B \right) \ln(\tan(dx+c) + I) \right)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.87, size = 104, normalized size = 0.79

$$\frac{(12(15iA - B)dx e^{(6i dx + 6i c)} - 96Ae^{(6i dx + 6i c)} \log(e^{(2i dx + 2i c)} - 1) - 6(11A + 3iB)e^{(4i dx + 4i c)} - 3(5A + 3iB)e^{(2i dx + 2i c)} - 2A - 2iB)e^{(-6i dx - 6i c)}}{96a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

[Out] 
$$-\frac{1}{96} \left( 12(15iA - B)dx e^{(6i dx + 6i c)} - 96Ae^{(6i dx + 6i c)} \log(e^{(2i dx + 2i c)} - 1) - 6(11A + 3iB)e^{(4i dx + 4i c)} - 3(5A + 3iB)e^{(2i dx + 2i c)} - 2A - 2iB \right) e^{(-6i dx - 6i c)} / (a^3d)$$

**Sympy** [A]

time = 0.44, size = 292, normalized size = 2.23

$$\frac{A \log(e^{2idx} - e^{-2ic})}{a^3d} + \begin{cases} \frac{((512Aa^6d^2e^{6ic} + 512iBa^6d^2e^{6ic})e^{-6idx} + (3840Aa^6d^2e^{8ic} + 2304iBa^6d^2e^{8ic})e^{-4idx} + (16896Aa^6d^2e^{10ic} + 4608iBa^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic} \neq 0 \\ x \left( -\frac{15iA+B}{8a^3} + \frac{(-15iAe^{6ic} - 11iAe^{4ic} - 5iAe^{2ic} - iA + Be^{6ic} + 3Be^{4ic} + 3Be^{2ic} + B)e^{-6ic}}{8a^3} \right) & \text{otherwise} \end{cases} + \frac{x(-15iA + B)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)`

[Out] 
$$A \log(\exp(2I*d*x) - \exp(-2I*c)) / (a**3*d) + \text{Piecewise}(\left( (512*A*a**6*d**2*\exp(6*I*c) + 512*I*B*a**6*d**2*\exp(6*I*c)) * \exp(-6*I*d*x) + (3840*A*a**6*d**2 \right)$$

\*exp(8\*I\*c) + 2304\*I\*B\*a\*\*6\*d\*\*2\*exp(8\*I\*c))\*exp(-4\*I\*d\*x) + (16896\*A\*a\*\*6\*d\*\*2\*exp(10\*I\*c) + 4608\*I\*B\*a\*\*6\*d\*\*2\*exp(10\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-12\*I\*c)/(24576\*a\*\*9\*d\*\*3), Ne(a\*\*9\*d\*\*3\*exp(12\*I\*c), 0)), (x\*(-(-15\*I\*A + B)/(8\*a\*\*3) + (-15\*I\*A\*exp(6\*I\*c) - 11\*I\*A\*exp(4\*I\*c) - 5\*I\*A\*exp(2\*I\*c) - I\*A + B\*exp(6\*I\*c) + 3\*B\*exp(4\*I\*c) + 3\*B\*exp(2\*I\*c) + B)\*exp(-6\*I\*c)/(8\*a\*\*3)), True)) + x\*(-15\*I\*A + B)/(8\*a\*\*3)

**Giac [A]**

time = 1.09, size = 145, normalized size = 1.11

$$\frac{\frac{6(15A+iB)\log(\tan(dx+c)-i)}{a^3} + \frac{6(A-iB)\log(i\tan(dx+c)-1)}{a^3} - \frac{96A\log(\tan(dx+c))}{a^3} - \frac{165A\tan(dx+c)^3+11iB\tan(dx+c)^3-579iA\tan(dx+c)^2+45B\tan(dx+c)^2-699A\tan(dx+c)-69iB\tan(dx+c)+301iA-51B}{a^3(\tan(dx+c)-i)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] -1/96\*(6\*(15\*A + I\*B)\*log(tan(d\*x + c) - I)/a^3 + 6\*(A - I\*B)\*log(I\*tan(d\*x + c) - 1)/a^3 - 96\*A\*log(tan(d\*x + c))/a^3 - (165\*A\*tan(d\*x + c)^3 + 11\*I\*B\*tan(d\*x + c)^3 - 579\*I\*A\*tan(d\*x + c)^2 + 45\*B\*tan(d\*x + c)^2 - 699\*A\*tan(d\*x + c) - 69\*I\*B\*tan(d\*x + c) + 301\*I\*A - 51\*B)/(a^3\*(tan(d\*x + c) - I)^3))/d

**Mupad [B]**

time = 6.58, size = 164, normalized size = 1.25

$$\frac{\frac{17A}{12a^3} - \tan(c+dx)^2\left(\frac{7A}{8a^2} + \frac{B1i}{8a^2}\right) + \frac{B5i}{12a^3} + \tan(c+dx)\left(-\frac{3B}{8a^2} + \frac{A17i}{8a^2}\right)}{d(-\tan(c+dx)^31i - 3\tan(c+dx)^2 + \tan(c+dx)3i + 1)} + \frac{A\ln(\tan(c+dx))}{a^3d} + \frac{\ln(\tan(c+dx)+1i)(B+A1i)1i}{16a^3d} - \frac{\ln(\tan(c+dx)-i)(15A+B1i)}{16a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] ((17\*A)/(12\*a^3) - tan(c + d\*x)^2\*((7\*A)/(8\*a^3) + (B\*1i)/(8\*a^3)) + (B\*5i)/(12\*a^3) + tan(c + d\*x)\*((A\*17i)/(8\*a^3) - (3\*B)/(8\*a^3)))/(d\*(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1)) + (A\*log(tan(c + d\*x)))/(a^3\*d) + (log(tan(c + d\*x) + 1i)\*(A\*1i + B)\*1i)/(16\*a^3\*d) - (log(tan(c + d\*x) - 1i)\*(15\*A + B\*1i))/(16\*a^3\*d)



$$3.57 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=183

$$\frac{(25A+7iB)x}{8a^3} - \frac{(25A+7iB)\cot(c+dx)}{8a^3d} - \frac{(3iA-B)\log(\sin(c+dx))}{a^3d} + \frac{(A+iB)\cot(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(11A-5iB)}{24ad(a+ia \tan(c+dx))^2}$$

[Out]  $-1/8*(25*A+7*I*B)*x/a^3-1/8*(25*A+7*I*B)*\cot(d*x+c)/a^3/d-(3*I*A-B)*\ln(\sin(d*x+c))/a^3/d+1/6*(A+I*B)*\cot(d*x+c)/d/(a+I*a*\tan(d*x+c))^3+1/24*(11*A+5*I*B)*\cot(d*x+c)/a/d/(a+I*a*\tan(d*x+c))^2+1/2*(3*A+I*B)*\cot(d*x+c)/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi [A]**

time = 0.36, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$\frac{(25A+7iB)\cot(c+dx)}{8a^3d} - \frac{(-B+3iA)\log(\sin(c+dx))}{a^3d} + \frac{(3A+iB)\cot(c+dx)}{2d(a^3+ia^3 \tan(c+dx))} - \frac{x(25A+7iB)}{8a^3} + \frac{(11A+5iB)\cot(c+dx)}{24ad(a+ia \tan(c+dx))^2} + \frac{(A+iB)\cot(c+dx)}{6d(a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out]  $-1/8*((25*A+(7*I)*B)*x)/a^3 - ((25*A+(7*I)*B)*\text{Cot}[c+d*x])/(8*a^3*d) - (((3*I)*A-B)*\text{Log}[\text{Sin}[c+d*x]])/(a^3*d) + ((A+I*B)*\text{Cot}[c+d*x])/(6*d*(a+I*a*\text{Tan}[c+d*x])^3) + ((11*A+(5*I)*B)*\text{Cot}[c+d*x])/(24*a*d*(a+I*a*\text{Tan}[c+d*x])^2) + ((3*A+I*B)*\text{Cot}[c+d*x])/(2*d*(a^3+I*a^3*\text{Tan}[c+d*x]))$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx &= \frac{(A + iB) \cot(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{\int \frac{\cot^2(c + dx)(a(7A + iB) - 4a(iA - B) \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx}{6a^2} \\ &= \frac{(A + iB) \cot(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(11A + 5iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^2} + \frac{\int \frac{\cot^2(c + dx)}{a} dx}{24ad(a + ia \tan(c + dx))^2} \\ &= \frac{(A + iB) \cot(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(11A + 5iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^2} + \frac{(3A + iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^2} \\ &= -\frac{(25A + 7iB) \cot(c + dx)}{8a^3d} + \frac{(A + iB) \cot(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(11A + 5iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^2} \\ &= -\frac{(25A + 7iB)x}{8a^3} - \frac{(25A + 7iB) \cot(c + dx)}{8a^3d} + \frac{(A + iB) \cot(c + dx)}{6d(a + ia \tan(c + dx))^3} \\ &= -\frac{(25A + 7iB)x}{8a^3} - \frac{(25A + 7iB) \cot(c + dx)}{8a^3d} - \frac{(3iA - B) \log(\sin(c + dx))}{a^3d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1282 vs.  $2(183) = 366$ .  
time = 6.95, size = 1282, normalized size = 7.01

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3, x]
```

```
[Out] (((-7*I)*A + 5*B)*Cos[4*d*x]*Sec[c + d*x]^2*(Cos[c]/32 - (I/32)*Sin[c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

$$\begin{aligned}
& d*x))*(a + I*a*Tan[c + d*x])^3) + (((-23*I)*A + 11*B)*Cos[2*d*x]*Sec[c + d \\
& *x]^2*(Cos[c]/16 + (I/16)*Sin[c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + \\
& d*x)))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + (Se \\
& c[c + d*x]^2*((-3*I)*A*Cos[(3*c)/2] + B*Cos[(3*c)/2] + 3*A*Sin[(3*c)/2] + I \\
& *B*Sin[(3*c)/2])*((-I)*ArcTan[Tan[d*x]]*Cos[(3*c)/2] + ArcTan[Tan[d*x]]*Sin \\
& [(3*c)/2])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d* \\
& x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + (Sec[c + d*x]^2*((-3*I)*A* \\
& Cos[(3*c)/2] + B*Cos[(3*c)/2] + 3*A*Sin[(3*c)/2] + I*B*Sin[(3*c)/2])*((Cos[ \\
& (3*c)/2]*Log[Sin[c + d*x]^2])/2 + (I/2)*Log[Sin[c + d*x]^2]*Sin[(3*c)/2])*( \\
& Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d*x] + B*Sin[c \\
& + d*x])*(a + I*a*Tan[c + d*x])^3) + (x*Sec[c + d*x]^2*(-6*A*Cos[c] - (2*I) \\
& *B*Cos[c] + (3*I)*A*Cos[c]*Cot[c] - B*Cos[c]*Cot[c] - (3*I)*A*Sin[c] + B*Si \\
& n[c] + ((-3*I)*A + B)*Cot[c]*(Cos[3*c] + I*Sin[3*c]))*(Cos[d*x] + I*Sin[d*x \\
& ])^3*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[ \\
& c + d*x])^3) + (((-I)*A + B)*Cos[6*d*x]*Sec[c + d*x]^2*(Cos[3*c]/48 - (I/48) \\
& *Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d \\
& *x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + ((25*A + (7*I)*B)*Sec[c + \\
& d*x]^2*(-1/8*(d*x*Cos[3*c]) - (I/8)*d*x*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^ \\
& 3*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c \\
& + d*x])^3) + ((23*A + (11*I)*B)*Sec[c + d*x]^2*(-1/16*Cos[c] - (I/16)*Sin[ \\
& c])*(Cos[d*x] + I*Sin[d*x])^3*Sin[2*d*x]*(A + B*Tan[c + d*x])/(d*(A*Cos[c \\
& + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + ((7*A + (5*I)*B)*Sec[c \\
& + d*x]^2*(-1/32*Cos[c] + (I/32)*Sin[c])*(Cos[d*x] + I*Sin[d*x])^3*Sin[4*d* \\
& x]*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[ \\
& c + d*x])^3) + ((A + I*B)*Sec[c + d*x]^2*(-1/48*Cos[3*c] + (I/48)*Sin[3*c]) \\
& *(Cos[d*x] + I*Sin[d*x])^3*Sin[6*d*x]*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d \\
& *x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + (Csc[c/2]*Csc[c + d*x]*Se \\
& c[c/2]*Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^3*((I/2)*A*Cos[3*c - d*x] - ( \\
& I/2)*A*Cos[3*c + d*x] - (A*Sin[3*c - d*x])/2 + (A*Sin[3*c + d*x])/2)*(A + B \\
& *Tan[c + d*x])/(2*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x \\
& ])^3)
\end{aligned}$$

Maple [A]

time = 0.30, size = 135, normalized size = 0.74

method	result
derivativedivides	$\frac{(-3iA+B) \ln(\tan(dx+c)) - \frac{A}{\tan(dx+c)} + \left(\frac{49iA}{16} - \frac{15B}{16}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{5iA}{4} + \frac{3B}{4}}{2(\tan(dx+c)-i)^2} - \frac{\frac{17A}{8} + \frac{7iB}{8}}{\tan(dx+c)-i} - \frac{-\frac{A}{2} - \frac{iB}{2}}{3(\tan(dx+c)-i)}}{da^3}$
default	$\frac{(-3iA+B) \ln(\tan(dx+c)) - \frac{A}{\tan(dx+c)} + \left(\frac{49iA}{16} - \frac{15B}{16}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{5iA}{4} + \frac{3B}{4}}{2(\tan(dx+c)-i)^2} - \frac{\frac{17A}{8} + \frac{7iB}{8}}{\tan(dx+c)-i} - \frac{-\frac{A}{2} - \frac{iB}{2}}{3(\tan(dx+c)-i)}}{da^3}$
risch	$-\frac{15ixB}{8a^3} - \frac{49xA}{8a^3} + \frac{11e^{-2i(dx+c)}B}{16a^3d} - \frac{23ie^{-2i(dx+c)}A}{16a^3d} + \frac{5e^{-4i(dx+c)}B}{32a^3d} - \frac{7ie^{-4i(dx+c)}A}{32a^3d} + \frac{e^{-6i(dx+c)}B}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^3} \left( (-3IA+B) \ln(\tan(dx+c)) - A/\tan(dx+c) + (49/16IA - 15/16B) \ln(\tan(dx+c)-I) - 1/2 * (-5/4IA + 3/4B) / (\tan(dx+c)-I)^2 - (17/8A + 7/8IB) / (\tan(dx+c)-I) - 1/3 * (-1/2A - 1/2IB) / (\tan(dx+c)-I)^3 - 1/16I * (A-IB) \ln(\tan(dx+c)+I) \right)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 0.61, size = 173, normalized size = 0.95

$$\frac{12(49A + 15iB)dx e^{(8i dx + 8i c)} - 6(2(49A + 15iB)dx - 55iA + 11B)e^{(6i dx + 6i c)} + 3(-39iA + 17B)e^{(4i dx + 4i c)} - (19iA - 13B)e^{(2i dx + 2i c)} + 96((3iA - B)e^{(8i dx + 8i c)} + (-3iA + B)e^{(6i dx + 6i c)}) \log(e^{(2i dx + 2i c)} - 1) - 2iA + 2B}{96(a^3 d e^{(8i dx + 8i c)} - a^3 d e^{(6i dx + 6i c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

[Out]  $-1/96 * (12 * (49A + 15IB) * dx * e^{(8I * dx + 8I * c)} - 6 * (2 * (49A + 15IB) * dx - 55I * A + 11B) * e^{(6I * dx + 6I * c)} + 3 * (-39I * A + 17B) * e^{(4I * dx + 4I * c)} - (19I * A - 13B) * e^{(2I * dx + 2I * c)} + 96 * ((3I * A - B) * e^{(8I * dx + 8I * c)} + (-3I * A + B) * e^{(6I * dx + 6I * c)}) * \log(e^{(2I * dx + 2I * c)} - 1) - 2I * A + 2B) / (a^3 * d * e^{(8I * dx + 8I * c)} - a^3 * d * e^{(6I * dx + 6I * c)})$

**Sympy** [A]

time = 0.56, size = 340, normalized size = 1.86

$$\frac{2iA}{a^3 d e^{2i c} e^{2i dx} - a^3 d} + \begin{cases} \frac{\left( \frac{(-512iAa^6 d^2 e^{6ic} + 512Ba^6 d^2 e^{6ic})e^{-6idx} + (-5376iAa^6 d^2 e^{8ic} + 3840Ba^6 d^2 e^{8ic})e^{-4idx} + (-35328iAa^6 d^2 e^{10ic} + 16896Ba^6 d^2 e^{10ic})e^{-2idx}}{24576a^9 d^3} \right)}{8a^3} & \text{for } a^3 d^3 e^{2ic} \neq 0 \\ x \left( -\frac{49A - 15iB}{8a^3} + \frac{(-49Ae^{6ic} - 23Ae^{4ic} - 7Ae^{2ic} - A - 15iB e^{6ic} - 11iB e^{4ic} - 5iB e^{2ic} - iB) e^{-6ic}}{8a^3} \right) & \text{otherwise} \end{cases} + \frac{x(-49A - 15iB)}{8a^3} - \frac{i(3A + iB) \log(e^{2idx} - e^{-2ic})}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)`

[Out]  $-2IA/(a**3*d*exp(2I*c)*exp(2I*dx) - a**3*d) + \text{Piecewise}(\left( (-512IA**6*d**2*exp(6I*c) + 512B*a**6*d**2*exp(6I*c)) * exp(-6I*dx) + (-5376IA**6*d**2*exp(8I*c) + 3840B*a**6*d**2*exp(8I*c)) * exp(-4I*dx) + (-35328I*A**6*d**2*exp(10I*c) + 16896B*a**6*d**2*exp(10I*c)) * exp(-2I*dx) \right))$

```
*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*(-(-49*A
- 15*I*B)/(8*a**3) + (-49*A*exp(6*I*c) - 23*A*exp(4*I*c) - 7*A*exp(2*I*c)
- A - 15*I*B*exp(6*I*c) - 11*I*B*exp(4*I*c) - 5*I*B*exp(2*I*c) - I*B)*exp(-
6*I*c)/(8*a**3)), True)) + x*(-49*A - 15*I*B)/(8*a**3) - I*(3*A + I*B)*log(
exp(2*I*d*x) - exp(-2*I*c))/(a**3*d)
```

**Giac [A]**

time = 1.02, size = 186, normalized size = 1.02

$$\frac{6(-49A+15B)\log(i\tan(dx+c)+1)}{a^3} + \frac{6(iA+B)\log(i\tan(dx+c)-1)}{a^3} + \frac{96(3iA-B)\log(\tan(dx+c))}{a^3} + \frac{96(-3iA\tan(dx+c)+B\tan(dx+c)+A)}{a^3\tan(dx+c)} + \frac{539A\tan(dx+c)^3+165iB\tan(dx+c)^2-1821iA\tan(dx+c)^2+579B\tan(dx+c)^2-2085A\tan(dx+c)-699iB\tan(dx+c)+819iA-301B}{a^3(i\tan(dx+c)+1)^3}$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/96*(6*(-49*I*A + 15*B)*log(I*tan(d*x + c) + 1)/a^3 + 6*(I*A + B)*log(I*t
an(d*x + c) - 1)/a^3 + 96*(3*I*A - B)*log(tan(d*x + c))/a^3 + 96*(-3*I*A*tan
n(d*x + c) + B*tan(d*x + c) + A)/(a^3*tan(d*x + c)) + (539*A*tan(d*x + c)^3
+ 165*I*B*tan(d*x + c)^3 - 1821*I*A*tan(d*x + c)^2 + 579*B*tan(d*x + c)^2
- 2085*A*tan(d*x + c) - 699*I*B*tan(d*x + c) + 819*I*A - 301*B)/(a^3*(I*tan
(d*x + c) + 1)^3))/d
```

**Mupad [B]**

time = 6.88, size = 197, normalized size = 1.08

$$\frac{\tan(c+dx)^3\left(\frac{25A}{8a^3} + \frac{B7i}{8a^3}\right) - \tan(c+dx)^2\left(-\frac{17B}{8a^3} + \frac{A63i}{8a^3}\right) + \frac{A11}{a^3} - \tan(c+dx)\left(\frac{71A}{12a^3} + \frac{B17i}{12a^3}\right)}{d(\tan(c+dx)^4 - \tan(c+dx)^3 3i - 3\tan(c+dx)^2 + \tan(c+dx) 1i)} - \frac{\ln(\tan(c+dx))(-B+A3i)}{a^3 d} - \frac{\ln(\tan(c+dx)+1)(B+A1i)}{16a^3 d} + \frac{\ln(\tan(c+dx)-1)(-15B+A49i)}{16a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(A*49i - 15*B))/(16*a^3*d) - (log(tan(c + d*x))*(A*
3i - B))/(a^3*d) - (log(tan(c + d*x) + 1i)*(A*1i + B))/(16*a^3*d) - (tan(c
+ d*x)^3*((25*A)/(8*a^3) + (B*7i)/(8*a^3)) - tan(c + d*x)^2*((A*63i)/(8*a^3
) - (17*B)/(8*a^3)) + (A*1i)/a^3 - tan(c + d*x)*((71*A)/(12*a^3) + (B*17i)/
(12*a^3)))/(d*(tan(c + d*x)*1i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*3i + tan
(c + d*x)^4))
```

$$3.58 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=216

$$\frac{5(11iA - 5B)x}{8a^3} + \frac{5(11iA - 5B) \cot(c+dx)}{8a^3d} - \frac{(7A + 3iB) \cot^2(c+dx)}{2a^3d} - \frac{(7A + 3iB) \log(\sin(c+dx))}{a^3d} + \frac{(A + iB) \cot^2(c+dx)}{6d(a+ia \tan(c+dx))^3}$$

[Out]  $5/8*(11*I*A-5*B)*x/a^3+5/8*(11*I*A-5*B)*\cot(d*x+c)/a^3/d-1/2*(7*A+3*I*B)*\cot(d*x+c)^2/a^3/d-(7*A+3*I*B)*\ln(\sin(d*x+c))/a^3/d+1/6*(A+I*B)*\cot(d*x+c)^2/d/(a+I*a*\tan(d*x+c))^3+1/24*(13*A+7*I*B)*\cot(d*x+c)^2/a/d/(a+I*a*\tan(d*x+c))^2+5/24*(11*A+5*I*B)*\cot(d*x+c)^2/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi [A]**

time = 0.41, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{(7A+3iB)\cot^2(c+dx)}{2a^3d} + \frac{5(-5B+11iA)\cot(c+dx)}{8a^3d} - \frac{(7A+3iB)\log(\sin(c+dx))}{a^3d} + \frac{5(11A+5iB)\cot^2(c+dx)}{24d(a^3+ia^3\tan(c+dx))} + \frac{5x(-5B+11iA)}{8a^3} + \frac{(13A+7iB)\cot^2(c+dx)}{24ad(a+ia\tan(c+dx))^2} + \frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out]  $(5*((11*I)*A - 5*B)*x)/(8*a^3) + (5*((11*I)*A - 5*B)*\text{Cot}[c + d*x])/(8*a^3*d) - ((7*A + (3*I)*B)*\text{Cot}[c + d*x]^2)/(2*a^3*d) - ((7*A + (3*I)*B)*\text{Log}[\text{Sin}[c + d*x]])/(a^3*d) + ((A + I*B)*\text{Cot}[c + d*x]^2)/(6*d*(a + I*a*\text{Tan}[c + d*x])^3) + ((13*A + (7*I)*B)*\text{Cot}[c + d*x]^2)/(24*a*d*(a + I*a*\text{Tan}[c + d*x])^2) + (5*(11*A + (5*I)*B)*\text{Cot}[c + d*x]^2)/(24*d*(a^3 + I*a^3*\text{Tan}[c + d*x]))$

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \frac{(A+iB) \cot^2(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{\int \frac{\cot^3(c+dx)(2a(4A+iB)-5a(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx}{6a^2} \\ &= \frac{(A+iB) \cot^2(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(13A+7iB) \cot^2(c+dx)}{24ad(a+ia \tan(c+dx))^2} + \frac{\int \frac{\cot^3(c+dx)}{a+ia \tan(c+dx)} dx}{24ad} \\ &= \frac{(A+iB) \cot^2(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(13A+7iB) \cot^2(c+dx)}{24ad(a+ia \tan(c+dx))^2} + \frac{5(11A+5iB) \cot(c+dx)}{24ad} \\ &= -\frac{(7A+3iB) \cot^2(c+dx)}{2a^3d} + \frac{(A+iB) \cot^2(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(13A+7iB) \cot^2(c+dx)}{24ad} \\ &= \frac{5(11iA-5B) \cot(c+dx)}{8a^3d} - \frac{(7A+3iB) \cot^2(c+dx)}{2a^3d} + \frac{(A+iB) \cot^2(c+dx)}{6d(a+ia \tan(c+dx))^3} \\ &= \frac{5(11iA-5B)x}{8a^3} + \frac{5(11iA-5B) \cot(c+dx)}{8a^3d} - \frac{(7A+3iB) \cot^2(c+dx)}{2a^3d} \\ &= \frac{5(11iA-5B)x}{8a^3} + \frac{5(11iA-5B) \cot(c+dx)}{8a^3d} - \frac{(7A+3iB) \cot^2(c+dx)}{2a^3d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1448 vs. 2(216) = 432.

time = 7.16, size = 1448, normalized size = 6.70

Warning: Unable to verify antiderivative.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

```
[Out] ((9*A + (7*I)*B)*Cos[4*d*x]*Sec[c + d*x]^2*(-1/32*Cos[c] + (I/32)*Sin[c])*
Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[
c + d*x])*(a + I*a*Tan[c + d*x])^3) + ((39*A + (23*I)*B)*Cos[2*d*x]*Sec[c +
d*x]^2*(-1/16*Cos[c] - (I/16)*Sin[c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[
c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) +
(Sec[c + d*x]^2*(7*A*Cos[(3*c)/2] + (3*I)*B*Cos[(3*c)/2] + (7*I)*A*Sin[(3*
c)/2] - 3*B*Sin[(3*c)/2])*(I*ArcTan[Tan[d*x]]*Cos[(3*c)/2] - ArcTan[Tan[d*x
]]*Sin[(3*c)/2])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[
c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + (Sec[c + d*x]^2*(7*A
*Cos[(3*c)/2] + (3*I)*B*Cos[(3*c)/2] + (7*I)*A*Sin[(3*c)/2] - 3*B*Sin[(3*c)
/2])*(-1/2*(Cos[(3*c)/2]*Log[Sin[c + d*x]^2]) - (I/2)*Log[Sin[c + d*x]^2]*S
in[(3*c)/2])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c +
d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + (x*Sec[c + d*x]^2*((14*I
)*A*Cos[c] - 6*B*Cos[c] + 7*A*Cos[c]*Cot[c] + (3*I)*B*Cos[c]*Cot[c] - 7*A*S
in[c] - (3*I)*B*Sin[c] + (7*A + (3*I)*B)*Cot[c]*(-Cos[3*c] - I*Sin[3*c]))*(
Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/((A*Cos[c + d*x] + B*Sin[c +
d*x])*(a + I*a*Tan[c + d*x])^3) + ((A + I*B)*Cos[6*d*x]*Sec[c + d*x]^2*(-1
/48*Cos[3*c] + (I/48)*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*
x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + (Csc[
c + d*x]^2*Sec[c + d*x]^2*(-1/2*(A*Cos[3*c]) - (I/2)*A*Sin[3*c])*(Cos[d*x]
+ I*Sin[d*x])^3*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*
(a + I*a*Tan[c + d*x])^3) + ((11*A + (5*I)*B)*Sec[c + d*x]^2*((5*I)/8)*d*x
*Cos[3*c] - (5*d*x*Sin[3*c])/8)*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*
x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + ((39*
A + (23*I)*B)*Sec[c + d*x]^2*((I/16)*Cos[c] - Sin[c]/16)*(Cos[d*x] + I*Sin[
d*x])^3*Sin[2*d*x]*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x
])*(a + I*a*Tan[c + d*x])^3) + ((9*A + (7*I)*B)*Sec[c + d*x]^2*((I/32)*Cos[
c] + Sin[c]/32)*(Cos[d*x] + I*Sin[d*x])^3*Sin[4*d*x]*(A + B*Tan[c + d*x]))/
(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^3) + ((A + I*B)
*Sec[c + d*x]^2*((I/48)*Cos[3*c] + Sin[3*c]/48)*(Cos[d*x] + I*Sin[d*x])^3*S
in[6*d*x]*(A + B*Tan[c + d*x]))/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I
*a*Tan[c + d*x])^3) + (Csc[c/2]*Csc[c + d*x]*Sec[c/2]*Sec[c + d*x]^2*(Cos[d
*x] + I*Sin[d*x])^3*((3*A*Cos[3*c - d*x])/2 + (I/2)*B*Cos[3*c - d*x] - (3*A
*Cos[3*c + d*x])/2 - (I/2)*B*Cos[3*c + d*x] + ((3*I)/2)*A*Sin[3*c - d*x] -
(B*Sin[3*c - d*x])/2 - ((3*I)/2)*A*Sin[3*c + d*x] + (B*Sin[3*c + d*x])/2)*(
A + B*Tan[c + d*x]))/(2*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c
+ d*x])^3)
```

Maple [A]

time = 0.34, size = 153, normalized size = 0.71

method	result
derivativedivides	$\frac{(-3iB-7A)\ln(\tan(dx+c))-\frac{-3iA+B}{\tan(dx+c)}-\frac{A}{2\tan(dx+c)^2}-\frac{-7A-5iB}{2(\tan(dx+c)-i)^2}+\left(\frac{49iB}{16}+\frac{111A}{16}\right)\ln(\tan(dx+c)-i)-\frac{-3iA+17B}{\tan(dx+c)-i}}{da^3}$



default	$\frac{(-3iB-7A)\ln(\tan(dx+c)) - \frac{-3iA+B}{\tan(dx+c)} - \frac{A}{2\tan(dx+c)^2} - \frac{-7A-5iB}{2(\tan(dx+c)-i)^2} + \left(\frac{49iB}{16} + \frac{111A}{16}\right)\ln(\tan(dx+c)-i) - \frac{-31iA+17B}{8\tan(dx+c)} - \frac{17B}{8}}{da^3}$
risch	$-\frac{49xB}{8a^3} + \frac{111ixA}{8a^3} - \frac{23ie^{-2i(dx+c)}B}{16a^3d} - \frac{39e^{-2i(dx+c)}A}{16a^3d} - \frac{7ie^{-4i(dx+c)}B}{32a^3d} - \frac{9e^{-4i(dx+c)}A}{32a^3d} - \frac{ie^{-6i(dx+c)}B}{48a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $1/d/a^3*((-7*A-3*I*B)*\ln(\tan(d*x+c))-(-3*I*A+B)/\tan(d*x+c)-1/2*A/\tan(d*x+c)^2-1/2*(-7/4*A-5/4*I*B)/(\tan(d*x+c)-I)^2+(49/16*I*B+111/16*A)*\ln(\tan(d*x+c)-I)-(-31/8*I*A+17/8*B)/(\tan(d*x+c)-I)-1/3*(1/2*I*A-1/2*B)/(\tan(d*x+c)-I)^3+(1/16*A-1/16*I*B)*\ln(\tan(d*x+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: `expt: undefined: 0 to a negative exponent.`

**Fricas** [A]

time = 0.51, size = 235, normalized size = 1.09

$$\frac{12(-111A+49B)\operatorname{dx}e^{10i(dx+c)}+6(4(111A-49B)\operatorname{dx}+103A+55iB)e^{8i(dx+c)}+3(4(-111A+49B)\operatorname{dx}-339A-149iB)e^{6i(dx+c)}+14(13A+7iB)e^{4i(dx+c)}+(23A+17iB)e^{2i(dx+c)}+96((7A+3iB)e^{10i(dx+c)}-2(7A+3iB)e^{8i(dx+c)}+(7A+3iB)e^{6i(dx+c)})\log(e^{2i(dx+c)}-1)+2A+2iB)}{96(a^3de^{10i(dx+c)}-2a^3de^{8i(dx+c)}+a^3de^{6i(dx+c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

[Out]  $-1/96*(12*(-111*I*A+49*B)*d*x*e^{(10*I*d*x+10*I*c)}+6*(4*(111*I*A-49*B)*d*x+103*A+55*I*B)*e^{(8*I*d*x+8*I*c)}+3*(4*(-111*I*A+49*B)*d*x-339*A-149*I*B)*e^{(6*I*d*x+6*I*c)}+14*(13*A+7*I*B)*e^{(4*I*d*x+4*I*c)}+(23*A+17*I*B)*e^{(2*I*d*x+2*I*c)}+96*((7*A+3*I*B)*e^{(10*I*d*x+10*I*c)}-2*(7*A+3*I*B)*e^{(8*I*d*x+8*I*c)}+(7*A+3*I*B)*e^{(6*I*d*x+6*I*c)})*\log(e^{(2*I*d*x+2*I*c)}-1)+2*A+2*I*B)/(a^3*d*e^{(10*I*d*x+10*I*c)}-2*a^3*d*e^{(8*I*d*x+8*I*c)}+a^3*d*e^{(6*I*d*x+6*I*c)})$

**Sympy** [A]

time = 1.21, size = 398, normalized size = 1.84

$$\frac{6A+2iB+(-4Ae^{2ic}-2iBe^{2ic})e^{2idx}}{a^3de^{6ic}e^{3idx}-2a^3de^{2ic}e^{2idx}+a^3d} + \begin{cases} \frac{((-512Aa^6d^6e^{6ic}-512iBa^6d^6e^{6ic})e^{-6idx}+(-6912Aa^6d^6e^{6ic}-5376iBa^6d^6e^{6ic})e^{-4idx}+(-59904Aa^6d^6e^{6ic}-35328iBa^6d^6e^{6ic})e^{-2idx})e^{-12ic}}{243768d^6} & \text{for } a^9d^6e^{12ic} \neq 0 \\ x\left(-\frac{111iA-49B}{8a^2} + \frac{(111iAe^{6ic}+39iAe^{4ic}+9iAe^{2ic}+iA-49Be^{6ic}-23Be^{4ic}-7Be^{2ic}-B)e^{-6ic}}{8a^2}\right) & \text{otherwise} \end{cases} + \frac{x(111iA-49B)}{8a^2} - \frac{(7A+3iB)\log(e^{2idx}-e^{-2ic})}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] (6\*A + 2\*I\*B + (-4\*A\*exp(2\*I\*c) - 2\*I\*B\*exp(2\*I\*c))\*exp(2\*I\*d\*x))/(a\*\*3\*d\*exp(4\*I\*c)\*exp(4\*I\*d\*x) - 2\*a\*\*3\*d\*exp(2\*I\*c)\*exp(2\*I\*d\*x) + a\*\*3\*d) + Piecewise(((((-512\*A\*a\*\*6\*d\*\*2\*exp(6\*I\*c) - 512\*I\*B\*a\*\*6\*d\*\*2\*exp(6\*I\*c))\*exp(-6\*I\*d\*x) + (-6912\*A\*a\*\*6\*d\*\*2\*exp(8\*I\*c) - 5376\*I\*B\*a\*\*6\*d\*\*2\*exp(8\*I\*c))\*exp(-4\*I\*d\*x) + (-59904\*A\*a\*\*6\*d\*\*2\*exp(10\*I\*c) - 35328\*I\*B\*a\*\*6\*d\*\*2\*exp(10\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-12\*I\*c)/(24576\*a\*\*9\*d\*\*3), Ne(a\*\*9\*d\*\*3\*exp(12\*I\*c), 0)), (x\*(-(111\*I\*A - 49\*B)/(8\*a\*\*3) + (111\*I\*A\*exp(6\*I\*c) + 39\*I\*A\*exp(4\*I\*c) + 9\*I\*A\*exp(2\*I\*c) + I\*A - 49\*B\*exp(6\*I\*c) - 23\*B\*exp(4\*I\*c) - 7\*B\*exp(2\*I\*c) - B)\*exp(-6\*I\*c)/(8\*a\*\*3)), True)) + x\*(111\*I\*A - 49\*B)/(8\*a\*\*3) - (7\*A + 3\*I\*B)\*log(exp(2\*I\*d\*x) - exp(-2\*I\*c))/(a\*\*3\*d)

**Giac** [A]

time = 1.36, size = 211, normalized size = 0.98

$$\frac{6(111A+49B)\log(i\tan(dx+c)+1) + 6(A-B)\log(i\tan(dx+c)-1) - 96(7A+3B)\log(\tan(dx+c)) + 48(21A\tan(dx+c)^2+9B\tan(dx+c)^2+6iA\tan(dx+c)-2B\tan(dx+c)-A) + 1221iA\tan(dx+c)^3-539B\tan(dx+c)^3+4035A\tan(dx+c)^2+1821iB\tan(dx+c)^2-4491iA\tan(dx+c)+2085B\tan(dx+c)-1693A-819iB}{a^3\tan(dx+c)^2} + \frac{1221iA\tan(dx+c)^3-539B\tan(dx+c)^3+4035A\tan(dx+c)^2+1821iB\tan(dx+c)^2-4491iA\tan(dx+c)+2085B\tan(dx+c)-1693A-819iB}{a^3\tan(dx+c)^2}$$

96d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] 1/96\*(6\*(111\*A + 49\*I\*B)\*log(I\*tan(d\*x + c) + 1)/a^3 + 6\*(A - I\*B)\*log(I\*tan(d\*x + c) - 1)/a^3 - 96\*(7\*A + 3\*I\*B)\*log(tan(d\*x + c))/a^3 + 48\*(21\*A\*tan(d\*x + c)^2 + 9\*I\*B\*tan(d\*x + c)^2 + 6\*I\*A\*tan(d\*x + c) - 2\*B\*tan(d\*x + c) - A)/(a^3\*tan(d\*x + c)^2) + (1221\*I\*A\*tan(d\*x + c)^3 - 539\*B\*tan(d\*x + c)^3 + 4035\*A\*tan(d\*x + c)^2 + 1821\*I\*B\*tan(d\*x + c)^2 - 4491\*I\*A\*tan(d\*x + c) + 2085\*B\*tan(d\*x + c) - 1693\*A - 819\*I\*B)/(a^3\*(I\*tan(d\*x + c) + 1)^3)/d

**Mupad** [B]

time = 7.14, size = 221, normalized size = 1.02

$$\frac{\frac{A}{\sqrt{a}} + \tan(c+dx)^3 \left(-\frac{63B}{8a} + \frac{A137i}{8a}\right) + \tan(c+dx)^2 \left(\frac{149A}{12a} + \frac{B71i}{12a}\right) - \tan(c+dx)^4 \left(\frac{55A}{8a} + \frac{B25i}{8a}\right) - \tan(c+dx) \left(-\frac{B}{a} + \frac{A39i}{a}\right) - \frac{\ln(\tan(c+dx)) (7A+B3i)}{a^3 d} + \frac{\ln(\tan(c+dx)+1) (A-B1i)}{16a^3 d} + \frac{\ln(\tan(c+dx)-1) (111A+B49i)}{16a^3 d}}{d \left(-\tan(c+dx)^3 1i - 3 \tan(c+dx)^1 + \tan(c+dx)^3 3i + \tan(c+dx)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(16\*a^3\*d) - (log(tan(c + d\*x))\*(7\*A + B\*3i))/(a^3\*d) - (tan(c + d\*x)^3\*((A\*137i)/(8\*a^3) - (63\*B)/(8\*a^3)) - tan(c + d\*x)^4\*((55\*A)/(8\*a^3) + (B\*25i)/(8\*a^3)) + tan(c + d\*x)^2\*((149\*A)/(12\*a^3) + (B\*71i)/(12\*a^3)) + A/(2\*a^3) - tan(c + d\*x)\*((A\*3i)/(2\*a^3) - B/a^3))/(d\*(tan(c + d\*x)^2 + tan(c + d\*x)^3\*3i - 3\*tan(c + d\*x)^4 - tan(c + d\*x)^5\*1i)) + (log(tan(c + d\*x) - 1i)\*(111\*A + B\*49i))/(16\*a^3\*d)

$$3.59 \quad \int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=185

$$\frac{(A+15iB)x}{16a^4} - \frac{B \log(\cos(c+dx))}{a^4 d} - \frac{iA-15B}{16a^4 d(1+i \tan(c+dx))} - \frac{(iA-7B) \tan^2(c+dx)}{16a^4 d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^4(c+dx)}{8d(a+ia \tan(c+dx))}$$

[Out] 1/16\*(A+15\*I\*B)\*x/a^4-B\*ln(cos(d\*x+c))/a^4/d+1/16\*(-I\*A+15\*B)/a^4/d/(1+I\*tan(d\*x+c))-1/16\*(I\*A-7\*B)\*tan(d\*x+c)^2/a^4/d/(1+I\*tan(d\*x+c))^2+1/8\*(I\*A-B)\*tan(d\*x+c)^4/d/(a+I\*a\*tan(d\*x+c))^4+1/12\*(A+3\*I\*B)\*tan(d\*x+c)^3/a/d/(a+I\*a\*tan(d\*x+c))^3

**Rubi [A]**

time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ ,

Rules used = {3676, 3670, 3556, 12, 3607, 8}

$$\frac{(-7B+iA) \tan^2(c+dx)}{16a^4 d(1+i \tan(c+dx))^2} - \frac{-15B+iA}{16a^4 d(1+i \tan(c+dx))} + \frac{x(A+15iB)}{16a^4} - \frac{B \log(\cos(c+dx))}{a^4 d} + \frac{(-B+iA) \tan^4(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(A+3iB) \tan^3(c+dx)}{12ad(a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^4\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] ((A + (15\*I)\*B)\*x)/(16\*a^4) - (B\*Log[Cos[c + d\*x]])/(a^4\*d) - (I\*A - 15\*B)/(16\*a^4\*d\*(1 + I\*Tan[c + d\*x])) - ((I\*A - 7\*B)\*Tan[c + d\*x]^2)/(16\*a^4\*d\*(1 + I\*Tan[c + d\*x])^2) + ((I\*A - B)\*Tan[c + d\*x]^4)/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) + ((A + (3\*I)\*B)\*Tan[c + d\*x]^3)/(12\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3)

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3607

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x]

, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

### Rule 3670

Int[(((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

### Rule 3676

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^4(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^4} dx &= \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4} - \frac{\int \frac{\tan^3(c + dx)(4a(iA - B) + 8iaB \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx}{8a^2} \\
 &= \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(A + 3iB) \tan^3(c + dx)}{12ad(a + ia \tan(c + dx))^3} + \frac{\int \frac{\tan^2(c + dx)}{a + ia \tan(c + dx)} dx}{12ad(a + ia \tan(c + dx))^3} \\
 &= -\frac{(iA - 7B) \tan^2(c + dx)}{16a^4d(1 + i \tan(c + dx))^2} + \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(A + 3iB) \tan^3(c + dx)}{12ad(a + ia \tan(c + dx))^3} \\
 &= -\frac{(iA - 7B) \tan^2(c + dx)}{16a^4d(1 + i \tan(c + dx))^2} + \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(A + 3iB) \tan^3(c + dx)}{12ad(a + ia \tan(c + dx))^3} \\
 &= -\frac{B \log(\cos(c + dx))}{a^4d} - \frac{(iA - 7B) \tan^2(c + dx)}{16a^4d(1 + i \tan(c + dx))^2} + \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
 &= -\frac{B \log(\cos(c + dx))}{a^4d} - \frac{(iA - 7B) \tan^2(c + dx)}{16a^4d(1 + i \tan(c + dx))^2} + \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
 &= \frac{(A + 15iB)x}{16a^4} - \frac{B \log(\cos(c + dx))}{a^4d} - \frac{(iA - 7B) \tan^2(c + dx)}{16a^4d(1 + i \tan(c + dx))^2} + \frac{(iA - B) \tan^4(c + dx)}{8d(a + ia \tan(c + dx))^4}
 \end{aligned}$$

**Mathematica [A]**

time = 1.23, size = 195, normalized size = 1.05

$$\frac{\sec^2(c+dx)(36A-96B+16(-4A+21B)\cos(2(c+dx))+3\cos(4(c+dx))(A-B+8Adx+120Bdx-128B\log(\cos(c+dx))+32A\sin(2(c+dx))+288B\sin(2(c+dx))+3A\sin(4(c+dx))+3iB\sin(4(c+dx))+24iAdx\sin(4(c+dx))-360Bdx\sin(4(c+dx))-384B\log(\cos(c+dx))\sin(4(c+dx)))}{384a^4(-1+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^4\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] (Sec[c + d\*x]^4\*((36\*I)\*A - 96\*B + 16\*((-4\*I)\*A + 21\*B)\*Cos[2\*(c + d\*x)] + 3\*Cos[4\*(c + d\*x)]\*(I\*A - B + 8\*A\*d\*x + (120\*I)\*B\*d\*x - 128\*B\*Log[Cos[c + d\*x]]) + 32\*A\*Sin[2\*(c + d\*x)] + (288\*I)\*B\*Sin[2\*(c + d\*x)] + 3\*A\*Sin[4\*(c + d\*x)] + (3\*I)\*B\*Sin[4\*(c + d\*x)] + (24\*I)\*A\*d\*x\*Sin[4\*(c + d\*x)] - 360\*B\*d\*x\*Sin[4\*(c + d\*x)] - (384\*I)\*B\*Log[Cos[c + d\*x]]\*Sin[4\*(c + d\*x)])/(384\*a^4\*d\*(-I + Tan[c + d\*x])^4)

**Maple [A]**

time = 0.20, size = 131, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{9iB}{4}-\frac{7A}{4}}{3(\tan(dx+c)-i)^3}-\frac{-\frac{31B}{8}+\frac{17iA}{8}}{2(\tan(dx+c)-i)^2}+\left(-\frac{iA}{32}+\frac{31B}{32}\right)\ln(\tan(dx+c)-i)-\frac{-\frac{iA}{2}+\frac{B}{2}}{4(\tan(dx+c)-i)^4}-\frac{\frac{49iB}{16}+\frac{15A}{16}}{\tan(dx+c)-i}+\frac{i(-iB+A)\ln(\tan(dx+c)-i)}{32}}{da^4}$
default	$\frac{-\frac{9iB}{4}-\frac{7A}{4}}{3(\tan(dx+c)-i)^3}-\frac{-\frac{31B}{8}+\frac{17iA}{8}}{2(\tan(dx+c)-i)^2}+\left(-\frac{iA}{32}+\frac{31B}{32}\right)\ln(\tan(dx+c)-i)-\frac{-\frac{iA}{2}+\frac{B}{2}}{4(\tan(dx+c)-i)^4}-\frac{\frac{49iB}{16}+\frac{15A}{16}}{\tan(dx+c)-i}+\frac{i(-iB+A)\ln(\tan(dx+c)-i)}{32}}{da^4}$
risch	$\frac{31ixB}{16a^4}+\frac{xA}{16a^4}+\frac{13e^{-2i(dx+c)}B}{16da^4}-\frac{ie^{-2i(dx+c)}A}{8da^4}-\frac{e^{-4i(dx+c)}B}{4da^4}+\frac{3ie^{-4i(dx+c)}A}{32da^4}+\frac{e^{-6i(dx+c)}B}{16da^4}-\frac{ie^{-6i(dx+c)}A}{24da^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x,method=\_RETURNVERB OSE)

[Out] 1/d/a^4\*(-1/3\*(-9/4\*I\*B-7/4\*A)/(tan(d\*x+c)-I)^3-1/2\*(-31/8\*B+17/8\*I\*A)/(tan(d\*x+c)-I)^2+(-1/32\*I\*A+31/32\*B)\*ln(tan(d\*x+c)-I)-1/4\*(-1/2\*I\*A+1/2\*B)/(tan(d\*x+c)-I)^4-(49/16\*I\*B+15/16\*A)/(tan(d\*x+c)-I)+1/32\*I\*(A-I\*B)\*ln(tan(d\*x+c)+I))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 2.30, size = 120, normalized size = 0.65

$$\frac{(24(A + 31iB)dx e^{(8i dx + 8i c)} - 384B e^{(8i dx + 8i c)} \log(e^{(2i dx + 2i c)} + 1) - 24(2iA - 13B)e^{(6i dx + 6i c)} - 12(-3iA + 8B)e^{(4i dx + 4i c)} - 8(2iA - 3B)e^{(2i dx + 2i c)} + 3iA - 3B)e^{(-8i dx - 8i c)}}{384 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] 1/384\*(24\*(A + 31\*I\*B)\*d\*x\*e^(8\*I\*d\*x + 8\*I\*c) - 384\*B\*e^(8\*I\*d\*x + 8\*I\*c)\*log(e^(2\*I\*d\*x + 2\*I\*c) + 1) - 24\*(2\*I\*A - 13\*B)\*e^(6\*I\*d\*x + 6\*I\*c) - 12\*(-3\*I\*A + 8\*B)\*e^(4\*I\*d\*x + 4\*I\*c) - 8\*(2\*I\*A - 3\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + 3\*I\*A - 3\*B)\*e^(-8\*I\*d\*x - 8\*I\*c)/(a^4\*d)

**Sympy** [A]

time = 1.56, size = 359, normalized size = 1.94

$$\frac{B \log(e^{2idx} + e^{-2ic})}{a^4 d} + \begin{cases} \left\{ \frac{(24576iAa^{12}d^3e^{12ic} - 24576Ba^{12}d^3e^{12ic})e^{-8idx} + (-131072iAa^{12}d^3e^{14ic} + 196608Ba^{12}d^3e^{14ic})e^{-6idx} + (294912iAa^{12}d^3e^{16ic} - 786432Ba^{12}d^3e^{16ic})e^{-4idx} + (-393216iAa^{12}d^3e^{18ic} + 2555904Ba^{12}d^3e^{18ic})e^{-2idx} - 20ic}{3145728a^{16}d^4} \right\} & \text{for } a^{16}d^4e^{20ic} \neq 0 \\ x \left( -\frac{A+31iB}{16a^4} + \frac{(Ae^{8ic} - 4Ae^{6ic} + 6Ae^{4ic} - 4Ae^{2ic} + A + 31iB)e^{8ic} - 26iBe^{6ic} + 16iBe^{4ic} - 6iBe^{2ic} + iB}{16a^4} \right) & \text{otherwise} \end{cases} + \frac{x(A + 31iB)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

[Out] -B\*log(exp(2\*I\*d\*x) + exp(-2\*I\*c))/(a\*\*4\*d) + Piecewise((((24576\*I\*A\*a\*\*12\*d\*\*3\*exp(12\*I\*c) - 24576\*B\*a\*\*12\*d\*\*3\*exp(12\*I\*c))\*exp(-8\*I\*d\*x) + (-131072\*I\*A\*a\*\*12\*d\*\*3\*exp(14\*I\*c) + 196608\*B\*a\*\*12\*d\*\*3\*exp(14\*I\*c))\*exp(-6\*I\*d\*x) + (294912\*I\*A\*a\*\*12\*d\*\*3\*exp(16\*I\*c) - 786432\*B\*a\*\*12\*d\*\*3\*exp(16\*I\*c))\*exp(-4\*I\*d\*x) + (-393216\*I\*A\*a\*\*12\*d\*\*3\*exp(18\*I\*c) + 2555904\*B\*a\*\*12\*d\*\*3\*exp(18\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-20\*I\*c)/(3145728\*a\*\*16\*d\*\*4), Ne(a\*\*16\*d\*\*4\*exp(20\*I\*c), 0)), (x\*(-(A + 31\*I\*B)/(16\*a\*\*4) + (A\*exp(8\*I\*c) - 4\*A\*exp(6\*I\*c) + 6\*A\*exp(4\*I\*c) - 4\*A\*exp(2\*I\*c) + A + 31\*I\*B\*exp(8\*I\*c) - 26\*I\*B\*exp(6\*I\*c) + 16\*I\*B\*exp(4\*I\*c) - 6\*I\*B\*exp(2\*I\*c) + I\*B)\*exp(-8\*I\*c)/(16\*a\*\*4)), True)) + x\*(A + 31\*I\*B)/(16\*a\*\*4)

**Giac** [A]

time = 1.31, size = 154, normalized size = 0.83

$$\frac{12(-iA-B)\log(\tan(dx+c)+i) - 12(-iA+31B)\log(\tan(dx+c)-i) - 25iA\tan(dx+c)^4 - 775B\tan(dx+c)^4 - 260A\tan(dx+c)^3 + 1924iB\tan(dx+c)^3 + 522iA\tan(dx+c)^2 + 1866B\tan(dx+c)^2 + 388A\tan(dx+c) - 772iB\tan(dx+c) - 103iA - 103B}{384 d \alpha^4 (\tan(dx+c)-i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] -1/384\*(12\*(-I\*A - B)\*log(tan(d\*x + c) + I)/a^4 - 12\*(-I\*A + 31\*B)\*log(tan(d\*x + c) - I)/a^4 - (25\*I\*A\*tan(d\*x + c)^4 - 775\*B\*tan(d\*x + c)^4 - 260\*A\*tan(d\*x + c)^3 + 1924\*I\*B\*tan(d\*x + c)^3 + 522\*I\*A\*tan(d\*x + c)^2 + 1866\*B\*tan(d\*x + c)^2 + 388\*A\*tan(d\*x + c) - 772\*I\*B\*tan(d\*x + c) - 103iA - 103iB)/384 d

$$\frac{\tan(dx + c)^2 + 388A \tan(dx + c) - 772I B \tan(dx + c) - 103I A - 103B}{(a^4 (\tan(dx + c) - I)^4) / d}$$

**Mupad [B]**

time = 6.77, size = 178, normalized size = 0.96

$$\frac{\tan(c+dx)^2 \left(-\frac{29B}{4a^4} + \frac{A7i}{4a^4}\right) - \tan(c+dx)^3 \left(\frac{15A}{16a^4} + \frac{B49i}{16a^4}\right) - \frac{A1i}{3a^4} + \frac{7B}{4a^4} + \tan(c+dx) \left(\frac{61A}{48a^4} + \frac{B97i}{16a^4}\right) + \frac{\ln(\tan(c+dx)+1)(B+A1i)}{32a^4d} - \frac{\ln(\tan(c+dx)-1)(A+B31i)1i}{32a^4d}}{d(\tan(c+dx)^4 - \tan(c+dx)^3 4i - 6 \tan(c+dx)^2 + \tan(c+dx) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^4\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] (tan(c + d\*x)^2\*((A\*7i)/(4\*a^4) - (29\*B)/(4\*a^4)) - tan(c + d\*x)^3\*((15\*A)/(16\*a^4) + (B\*49i)/(16\*a^4)) - (A\*1i)/(3\*a^4) + (7\*B)/(4\*a^4) + tan(c + d\*x)\*((61\*A)/(48\*a^4) + (B\*97i)/(16\*a^4)))/(d\*(tan(c + d\*x)\*4i - 6\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*4i + tan(c + d\*x)^4 + 1)) + (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(32\*a^4\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*31i)\*1i)/(32\*a^4\*d)

$$3.60 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=159

$$\frac{(iA+B)x}{16a^4} - \frac{A-13iB}{48a^4d(1+i \tan(c+dx))^2} + \frac{5A-29iB}{48a^4d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(A+5iB) \tan^2(c+dx)}{24ad(a+ia \tan(c+dx))^3}$$

[Out] 1/16\*(I\*A+B)\*x/a^4+1/48\*(-A+13\*I\*B)/a^4/d/(1+I\*tan(d\*x+c))^2+1/48\*(5\*A-29\*I\*B)/a^4/d/(1+I\*tan(d\*x+c))+1/8\*(I\*A-B)\*tan(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))^4+1/24\*(A+5\*I\*B)\*tan(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^3

**Rubi [A]**

time = 0.31, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3676, 3671, 3607, 8}

$$\frac{5A-29iB}{48a^4d(1+i \tan(c+dx))} - \frac{A-13iB}{48a^4d(1+i \tan(c+dx))^2} + \frac{x(B+iA)}{16a^4} + \frac{(-B+iA) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(A+5iB) \tan^2(c+dx)}{24ad(a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] ((I\*A + B)\*x)/(16\*a^4) - (A - (13\*I)\*B)/(48\*a^4\*d\*(1 + I\*Tan[c + d\*x])^2) + (5\*A - (29\*I)\*B)/(48\*a^4\*d\*(1 + I\*Tan[c + d\*x])) + ((I\*A - B)\*Tan[c + d\*x]^3)/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) + ((A + (5\*I)\*B)\*Tan[c + d\*x]^2)/(24\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3)

**Rule 8**

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

**Rule 3607**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

**Rule 3671**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-(A\*b - a\*B))\*((a\*c + b\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a^2\*f\*m)), x] + Dist[1/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[A\*b\*c + a\*B\*c + a\*A\*d + b\*B\*d + 2\*a\*B\*d\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &



& NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]

### Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(- (A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx &= \frac{(iA-B) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{\int \frac{\tan^2(c+dx)(3a(iA-B)-a(A-7iB) \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx}{8a^2} \\ &= \frac{(iA-B) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(A+5iB) \tan^2(c+dx)}{24ad(a+ia \tan(c+dx))^3} + \frac{\int \frac{\tan(c+dx)}{a+ia \tan(c+dx)} dx}{24ad(a+ia \tan(c+dx))^2} \\ &= -\frac{A-13iB}{48a^4d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(A+5iB) \tan^2(c+dx)}{24ad(a+ia \tan(c+dx))^3} \\ &= -\frac{A-13iB}{48a^4d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(A+5iB) \tan^2(c+dx)}{24ad(a+ia \tan(c+dx))^3} \\ &= \frac{(iA+B)x}{16a^4} - \frac{A-13iB}{48a^4d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^3(c+dx)}{8d(a+ia \tan(c+dx))^4} \end{aligned}$$

### Mathematica [A]

time = 1.26, size = 158, normalized size = 0.99

$$\frac{\sec^4(c+dx)(36iB+16(A-4iB)\cos(2(c+dx))+3(A+iB+8iAdx+8Bdx)\cos(4(c+dx))+32iA\sin(2(c+dx))+32B\sin(2(c+dx))-3iA\sin(4(c+dx))+3B\sin(4(c+dx))-24Adx\sin(4(c+dx))+24iBdx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4, x]

[Out] (Sec[c + d\*x]^4\*((36\*I)\*B + 16\*(A - (4\*I)\*B)\*Cos[2\*(c + d\*x)] + 3\*(A + I\*B + (8\*I)\*A\*d\*x + 8\*B\*d\*x)\*Cos[4\*(c + d\*x)] + (32\*I)\*A\*Sin[2\*(c + d\*x)] + 32\*B\*Sin[2\*(c + d\*x)] - (3\*I)\*A\*Sin[4\*(c + d\*x)] + 3\*B\*Sin[4\*(c + d\*x)] - 24\*A\*d\*x\*Sin[4\*(c + d\*x)] + (24\*I)\*B\*d\*x\*Sin[4\*(c + d\*x)])/(384\*a^4\*d\*(-I + Tan[c + d\*x])^4)

### Maple [A]

time = 0.16, size = 131, normalized size = 0.82

method	result
derivativedivides	$-\frac{-\frac{iA}{16} + \frac{15B}{16}}{\tan(dx+c)-i} - \frac{-\frac{A}{2} - \frac{iB}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{7A}{8} + \frac{17iB}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{5iA}{4} - \frac{7B}{4}}{3(\tan(dx+c)-i)^3} + \left(\frac{A}{32} - \frac{iB}{32}\right) \ln(\tan(dx+c)-i) + \frac{i(iA+B) \ln(\tan(dx+c)+1)}{32}$
default	$-\frac{-\frac{iA}{16} + \frac{15B}{16}}{\tan(dx+c)-i} - \frac{-\frac{A}{2} - \frac{iB}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{7A}{8} + \frac{17iB}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{5iA}{4} - \frac{7B}{4}}{3(\tan(dx+c)-i)^3} + \left(\frac{A}{32} - \frac{iB}{32}\right) \ln(\tan(dx+c)-i) + \frac{i(iA+B) \ln(\tan(dx+c)+1)}{32}$
risch	$\frac{xB}{16a^4} + \frac{ixA}{16a^4} - \frac{ie^{-2i(dx+c)}B}{8da^4} + \frac{e^{-2i(dx+c)}A}{16da^4} + \frac{3iBe^{-4i(dx+c)}}{32da^4} - \frac{ie^{-6i(dx+c)}B}{24da^4} - \frac{e^{-6i(dx+c)}A}{48da^4} + \frac{ie^{-8i(dx+c)}}{128da^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^4} * (-(-1/16*I*A+15/16*B)/(\tan(d*x+c)-I) - 1/4 * (-1/2*A-1/2*I*B)/(\tan(d*x+c)-I)^4 - 1/2 * (7/8*A+17/8*I*B)/(\tan(d*x+c)-I)^2 - 1/3 * (5/4*I*A-7/4*B)/(\tan(d*x+c)-I)^3 + (1/32*A-1/32*I*B) * \ln(\tan(d*x+c)-I) + 1/32 * I * (I*A+B) * \ln(\tan(d*x+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 2.59, size = 88, normalized size = 0.55

$$\frac{(24(-iA - B)dx e^{(8i dx + 8i c)} - 24(A - 2iB)e^{(6i dx + 6i c)} - 36i B e^{(4i dx + 4i c)} + 8(A + 2iB)e^{(2i dx + 2i c)} - 3A - 3iB)e^{(-8i dx - 8i c)}}{384 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

[Out]  $-1/384 * (24 * (-I * A - B) * d * x * e^{(8 * I * d * x + 8 * I * c)} - 24 * (A - 2 * I * B) * e^{(6 * I * d * x + 6 * I * c)} - 36 * I * B * e^{(4 * I * d * x + 4 * I * c)} + 8 * (A + 2 * I * B) * e^{(2 * I * d * x + 2 * I * c)} - 3 * A - 3 * I * B) * e^{(-8 * I * d * x - 8 * I * c)} / (a^4 * d)$

**Sympy** [A]

time = 0.45, size = 301, normalized size = 1.89

$$\begin{cases} \frac{(294912iBa^{12}d^3e^{16ic}e^{-4idx} + (24576Aa^{12}d^3e^{12ic} + 24576iBa^{12}d^3e^{12ic})e^{-8idx} + (-65536Aa^{12}d^3e^{14ic} - 131072iBa^{12}d^3e^{14ic})e^{-6idx} + (196608Aa^{12}d^3e^{18ic} - 393216iBa^{12}d^3e^{18ic})e^{-2idx})e^{-20ic}}{3145728a^{16}d^4} & \text{for } a^{16}d^4e^{20ic} \neq 0 \\ x \left( -\frac{iA+B}{16a^4} + \frac{(iAe^{8ic} - 2iAe^{6ic} + 2iAe^{2ic} - iA + Be^{8ic} - 4Be^{6ic} + 6Be^{4ic} - 4Be^{2ic} + B)e^{-8ic}}{16a^4} \right) & \text{otherwise} \end{cases} + \frac{x(iA+B)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

[Out] Piecewise(((294912\*I\*B\*a\*\*12\*d\*\*3\*exp(16\*I\*c)\*exp(-4\*I\*d\*x) + (24576\*A\*a\*\*12\*d\*\*3\*exp(12\*I\*c) + 24576\*I\*B\*a\*\*12\*d\*\*3\*exp(12\*I\*c))\*exp(-8\*I\*d\*x) + (-65536\*A\*a\*\*12\*d\*\*3\*exp(14\*I\*c) - 131072\*I\*B\*a\*\*12\*d\*\*3\*exp(14\*I\*c))\*exp(-6\*I\*d\*x) + (196608\*A\*a\*\*12\*d\*\*3\*exp(18\*I\*c) - 393216\*I\*B\*a\*\*12\*d\*\*3\*exp(18\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-20\*I\*c)/(3145728\*a\*\*16\*d\*\*4), Ne(a\*\*16\*d\*\*4\*exp(20\*I\*c), 0)), (x\*(-(I\*A + B)/(16\*a\*\*4) + (I\*A\*exp(8\*I\*c) - 2\*I\*A\*exp(6\*I\*c) + 2\*I\*A\*exp(2\*I\*c) - I\*A + B\*exp(8\*I\*c) - 4\*B\*exp(6\*I\*c) + 6\*B\*exp(4\*I\*c) - 4\*B\*exp(2\*I\*c) + B)\*exp(-8\*I\*c)/(16\*a\*\*4)), True)) + x\*(I\*A + B)/(16\*a\*\*4)

**Giac** [A]

time = 1.05, size = 153, normalized size = 0.96

$$\frac{12(A-iB)\log(-i\tan(dx+c)+1)}{a^4} - \frac{12(A-iB)\log(-i\tan(dx+c)-1)}{a^4} + \frac{25A\tan(dx+c)^4 - 25iB\tan(dx+c)^4 - 124iA\tan(dx+c)^3 + 260B\tan(dx+c)^3 - 54A\tan(dx+c)^2 - 522iB\tan(dx+c)^2 - 4iA\tan(dx+c) - 388B\tan(dx+c) - 7A + 103iB}{a^4(\tan(dx+c)-i)^4}$$

384 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] -1/384\*(12\*(A - I\*B)\*log(-I\*tan(d\*x + c) + 1)/a^4 - 12\*(A - I\*B)\*log(-I\*tan(d\*x + c) - 1)/a^4 + (25\*A\*tan(d\*x + c)^4 - 25\*I\*B\*tan(d\*x + c)^4 - 124\*I\*A\*tan(d\*x + c)^3 + 260\*B\*tan(d\*x + c)^3 - 54\*A\*tan(d\*x + c)^2 - 522\*I\*B\*tan(d\*x + c)^2 - 4\*I\*A\*tan(d\*x + c) - 388\*B\*tan(d\*x + c) - 7\*A + 103\*I\*B)/(a^4\*(tan(d\*x + c) - I)^4))/d

**Mupad** [B]

time = 6.62, size = 178, normalized size = 1.12

$$\frac{\frac{A}{12a^4} + \tan(c+dx)^3 \left(-\frac{15B}{16a^4} + \frac{A1i}{16a^4}\right) - \tan(c+dx)^2 \left(\frac{A}{4a^4} - \frac{B7i}{4a^4}\right) - \frac{B1i}{3a^4} + \tan(c+dx) \left(\frac{61B}{48a^4} + \frac{A13i}{48a^4}\right) + \frac{\ln(\tan(c+dx)-i)(A-B1i)}{32a^4d} + \frac{\ln(\tan(c+dx)+i)(B+A1i)1i}{32a^4d}}{d(\tan(c+dx)^4 - \tan(c+dx)^3 4i - 6\tan(c+dx)^2 + \tan(c+dx) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] (tan(c + d\*x)^3\*((A\*1i)/(16\*a^4) - (15\*B)/(16\*a^4)) - tan(c + d\*x)^2\*(A/(4\*a^4) - (B\*7i)/(4\*a^4)) + A/(12\*a^4) - (B\*1i)/(3\*a^4) + tan(c + d\*x)\*((A\*13i)/(48\*a^4) + (61\*B)/(48\*a^4)))/(d\*(tan(c + d\*x)\*4i - 6\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*4i + tan(c + d\*x)^4 + 1)) + (log(tan(c + d\*x) - 1i)\*(A - B\*1i))/(32\*a^4\*d) + (log(tan(c + d\*x) + 1i)\*(A\*1i + B)\*1i)/(32\*a^4\*d)

$$3.61 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=145

$$-\frac{(A-iB)x}{16a^4} + \frac{iA+5B}{16a^4d(1+i \tan(c+dx))^2} - \frac{iA+B}{16a^4d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{B}{6ad(a+ia \tan(c+dx))}$$

[Out] -1/16\*(A-I\*B)\*x/a^4+1/16\*(I\*A+5\*B)/a^4/d/(1+I\*tan(d\*x+c))^2+1/16\*(-I\*A-B)/a^4/d/(1+I\*tan(d\*x+c))+1/8\*(I\*A-B)\*tan(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^4-1/6\*B/a/d/(a+I\*a\*tan(d\*x+c))^3

**Rubi [A]**

time = 0.20, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3676, 3671, 3607, 3560, 8}

$$-\frac{B+iA}{16a^4d(1+i \tan(c+dx))} + \frac{5B+iA}{16a^4d(1+i \tan(c+dx))^2} - \frac{x(A-iB)}{16a^4} + \frac{(-B+iA) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{B}{6ad(a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] -1/16\*((A - I\*B)\*x)/a^4 + (I\*A + 5\*B)/(16\*a^4\*d\*(1 + I\*Tan[c + d\*x])^2) - (I\*A + B)/(16\*a^4\*d\*(1 + I\*Tan[c + d\*x])) + ((I\*A - B)\*Tan[c + d\*x]^2)/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) - B/(6\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3)

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a\_) + (b\_.)\*tan[(c\_) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[a\*((a + b\*Tan[c + d\*x])^n/(2\*b\*d\*n)), x] + Dist[1/(2\*a), Int[(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3607

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3671

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Dist[1/(
2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*d
+ 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

### Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx &= \frac{(iA-B) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{\int \frac{\tan(c+dx)(2a(iA-B)-2a(A-3iB) \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx}{8a^2} \\ &= \frac{(iA-B) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{B}{6ad(a+ia \tan(c+dx))^3} + \frac{i \int \frac{-8a^2B}{(a+ia \tan(c+dx))^2} dx}{6ad(a+ia \tan(c+dx))^3} \\ &= \frac{iA+5B}{16a^4d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{B}{6ad(a+ia \tan(c+dx))^3} \\ &= \frac{iA+5B}{16a^4d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{B}{6ad(a+ia \tan(c+dx))^3} \\ &= -\frac{(A-iB)x}{16a^4} + \frac{iA+5B}{16a^4d(1+i \tan(c+dx))^2} + \frac{(iA-B) \tan^2(c+dx)}{8d(a+ia \tan(c+dx))^4} - \frac{B}{6ad(a+ia \tan(c+dx))^3} \end{aligned}$$

### Mathematica [A]

time = 1.50, size = 144, normalized size = 0.99

$$\frac{(\cos(4(c+dx)) - i \sin(4(c+dx)))(-12iA - 16B \cos(2(c+dx))) + 3(iA - B + 8Adx - 8iBdx) \cos(4(c+dx)) - 32iB \sin(2(c+dx)) + 3A \sin(4(c+dx)) + 3iB \sin(4(c+dx)) + 24iAdx \sin(4(c+dx)) + 24Bdx \sin(4(c+dx))}{384a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
[Out] -1/384*((Cos[4*(c + d*x)] - I*Sin[4*(c + d*x)])*((-12*I)*A - 16*B*Cos[2*(c +
d*x)] + 3*(I*A - B + 8*A*d*x - (8*I)*B*d*x)*Cos[4*(c + d*x)] - (32*I)*B*S
```

$\text{in}[2*(c + d*x)] + 3*A*\text{Sin}[4*(c + d*x)] + (3*I)*B*\text{Sin}[4*(c + d*x)] + (24*I)*A*d*x*\text{Sin}[4*(c + d*x)] + 24*B*d*x*\text{Sin}[4*(c + d*x)])))/(a^4*d)$

**Maple [A]**

time = 0.16, size = 131, normalized size = 0.90

method	result
risch	$\frac{ixB}{16a^4} - \frac{xA}{16a^4} + \frac{e^{-2i(dx+c)}B}{16da^4} + \frac{ie^{-4i(dx+c)}A}{32da^4} - \frac{e^{-6i(dx+c)}B}{48da^4} + \frac{e^{-8i(dx+c)}B}{128da^4} - \frac{ie^{-8i(dx+c)}A}{128da^4}$
derivativdivides	$-\frac{\frac{A}{16} - \frac{iB}{16}}{\tan(dx+c)-i} - \frac{\frac{iA}{2} - \frac{B}{2}}{4(\tan(dx+c)-i)^4} + \left(\frac{iA}{32} + \frac{B}{32}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{iA}{8} + \frac{7B}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{5iB}{4} + \frac{3A}{4}}{3(\tan(dx+c)-i)^3} - \frac{i(-iB+A) \ln(\tan(dx+c))}{32}$
default	$-\frac{\frac{A}{16} - \frac{iB}{16}}{\tan(dx+c)-i} - \frac{\frac{iA}{2} - \frac{B}{2}}{4(\tan(dx+c)-i)^4} + \left(\frac{iA}{32} + \frac{B}{32}\right) \ln(\tan(dx+c)-i) - \frac{-\frac{iA}{8} + \frac{7B}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{5iB}{4} + \frac{3A}{4}}{3(\tan(dx+c)-i)^3} - \frac{i(-iB+A) \ln(\tan(dx+c))}{32}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^4} \left( -\frac{1}{16}A - \frac{1}{16}I*B \right) / (\tan(d*x+c)-I) - \frac{1}{4} \left( \frac{1}{2}I*A - \frac{1}{2}B \right) / (\tan(d*x+c)-I)^4 + \frac{1}{32}I*A + \frac{1}{32}B * \ln(\tan(d*x+c)-I) - \frac{1}{2} \left( -\frac{1}{8}I*A + \frac{7}{8}B \right) / (\tan(d*x+c)-I)^2 - \frac{1}{3} \left( \frac{5}{4}I*B + \frac{3}{4}A \right) / (\tan(d*x+c)-I)^3 - \frac{1}{32}I*(A-I*B) * \ln(\tan(d*x+c)+I)$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: `expt: undefined: 0 to a negative exponent.`

**Fricas [A]**

time = 2.86, size = 78, normalized size = 0.54

$$\frac{(24(A - iB)dx e^{(8i dx + 8i c)} - 24 B e^{(6i dx + 6i c)} - 12i A e^{(4i dx + 4i c)} + 8 B e^{(2i dx + 2i c)} + 3i A - 3 B) e^{(-8i dx - 8i c)}}{384 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

[Out]  $-\frac{1}{384} \left( 24(A - I*B) * d*x * e^{(8*I*d*x + 8*I*c)} - 24*B * e^{(6*I*d*x + 6*I*c)} - 12*I*A * e^{(4*I*d*x + 4*I*c)} + 8*B * e^{(2*I*d*x + 2*I*c)} + 3*I*A - 3*B \right) * e^{(-8*I*d*x - 8*I*c)} / (a^4*d)$

**Sympy [A]**

time = 0.57, size = 241, normalized size = 1.66

$$\left\{ \begin{array}{ll} \frac{(98304iAa^{12}d^3e^{16ic}e^{-4idx} + 196608Ba^{12}d^3e^{18ic}e^{-2idx} - 65536Ba^{12}d^3e^{14ic}e^{-6idx} + (-24576iAa^{12}d^3e^{12ic} + 24576Ba^{12}d^3e^{12ic})e^{-8idx})e^{-20ic}}{3145728a^{16}d^4} & \text{for } d^{16}d^4e^{20ic} \neq 0 \\ x\left(-\frac{A+iB}{16a^4} + \frac{(-Ae^{8ic} + 2Ae^{4ic} - A + iBe^{8ic} - 2iBe^{6ic} + 2iBe^{2ic} - iB)e^{-8ic}}{16a^4}\right) & \text{otherwise} \end{array} \right. + \frac{x(-A + iB)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

**[Out]** Piecewise(((98304\*I\*A\*a\*\*12\*d\*\*3\*exp(16\*I\*c)\*exp(-4\*I\*d\*x) + 196608\*B\*a\*\*12\*d\*\*3\*exp(18\*I\*c)\*exp(-2\*I\*d\*x) - 65536\*B\*a\*\*12\*d\*\*3\*exp(14\*I\*c)\*exp(-6\*I\*d\*x) + (-24576\*I\*A\*a\*\*12\*d\*\*3\*exp(12\*I\*c) + 24576\*B\*a\*\*12\*d\*\*3\*exp(12\*I\*c))\*exp(-8\*I\*d\*x)\*exp(-20\*I\*c)/(3145728\*a\*\*16\*d\*\*4), Ne(a\*\*16\*d\*\*4\*exp(20\*I\*c), 0)), (x\*(-(-A + I\*B)/(16\*a\*\*4) + (-A\*exp(8\*I\*c) + 2\*A\*exp(4\*I\*c) - A + I\*B\*exp(8\*I\*c) - 2\*I\*B\*exp(6\*I\*c) + 2\*I\*B\*exp(2\*I\*c) - I\*B)\*exp(-8\*I\*c)/(16\*a\*\*4)), True)) + x\*(-A + I\*B)/(16\*a\*\*4)

**Giac [A]**

time = 0.89, size = 151, normalized size = 1.04

$$\frac{\frac{12(iA+B)\log(\tan(dx+c)+i)}{a^4} + \frac{12(-iA-B)\log(\tan(dx+c)-i)}{a^4} + \frac{25iA\tan(dx+c)^4 + 25B\tan(dx+c)^4 + 124A\tan(dx+c)^3 - 124iB\tan(dx+c)^3 - 246iA\tan(dx+c)^2 - 54B\tan(dx+c)^2 - 124A\tan(dx+c) - 4iB\tan(dx+c) + 25iA - 7B}{a^4(\tan(dx+c)-i)^4}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

**[Out]**  $-1/384*(12*(I*A + B)*\log(\tan(d*x + c) + I)/a^4 + 12*(-I*A - B)*\log(\tan(d*x + c) - I)/a^4 + (25*I*A*\tan(d*x + c)^4 + 25*B*\tan(d*x + c)^4 + 124*A*\tan(d*x + c)^3 - 124*I*B*\tan(d*x + c)^3 - 246*I*A*\tan(d*x + c)^2 - 54*B*\tan(d*x + c)^2 - 124*A*\tan(d*x + c) - 4*I*B*\tan(d*x + c) + 25*I*A - 7*B)/(a^4*(\tan(d*x + c) - I)^4))/d$

**Mupad [B]**

time = 6.40, size = 135, normalized size = 0.93

$$\frac{\tan(c + dx)^2 \left(-\frac{B}{4a^4} + \frac{A1i}{4a^4}\right) - \tan(c + dx)^3 \left(\frac{A}{16a^4} - \frac{B1i}{16a^4}\right) + \frac{B}{12a^4} + \tan(c + dx) \left(\frac{A}{16a^4} + \frac{B13i}{48a^4}\right) + \frac{x(B + A1i) \operatorname{li}}{16a^4}}{d(\tan(c + dx)^4 - \tan(c + dx)^3 4i - 6 \tan(c + dx)^2 + \tan(c + dx) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4,x)

**[Out]**  $(\tan(c + d*x)^2*((A*1i)/(4*a^4) - B/(4*a^4)) - \tan(c + d*x)^3*(A/(16*a^4) - (B*1i)/(16*a^4)) + B/(12*a^4) + \tan(c + d*x)*(A/(16*a^4) + (B*13i)/(48*a^4)))/(d*(\tan(c + d*x)*4i - 6*\tan(c + d*x)^2 - \tan(c + d*x)^3*4i + \tan(c + d*x)^4 + 1)) + (x*(A*1i + B)*1i)/(16*a^4)$

$$3.62 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=143

$$-\frac{(iA+B)x}{16a^4} - \frac{A+iB}{8d(a+ia \tan(c+dx))^4} + \frac{A+3iB}{12ad(a+ia \tan(c+dx))^3} + \frac{A-iB}{16d(a^2+ia^2 \tan(c+dx))^2} + \frac{A}{16d(a^4+ia^4 \tan(c+dx))}$$

[Out]  $-1/16*(I*A+B)*x/a^4+1/8*(-A-I*B)/d/(a+I*a*\tan(d*x+c))^4+1/12*(A+3*I*B)/a/d/(a+I*a*\tan(d*x+c))^3+1/16*(A-I*B)/d/(a^2+I*a^2*\tan(d*x+c))^2+1/16*(A-I*B)/d/(a^4+I*a^4*\tan(d*x+c))$

**Rubi [A]**

time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ ,

Rules used = {3671, 3607, 3560, 8}

$$\frac{A-iB}{16d(a^4+ia^4 \tan(c+dx))} - \frac{x(B+ia)}{16a^4} + \frac{A-iB}{16d(a^2+ia^2 \tan(c+dx))^2} + \frac{A+3iB}{12ad(a+ia \tan(c+dx))^3} - \frac{A+iB}{8d(a+ia \tan(c+dx))^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^4, x]$

[Out]  $-1/16*((I*A+B)*x)/a^4 - (A+I*B)/(8*d*(a+I*a*\text{Tan}[c+d*x])^4) + (A+(3*I)*B)/(12*a*d*(a+I*a*\text{Tan}[c+d*x])^3) + (A-I*B)/(16*d*(a^2+I*a^2*\text{Tan}[c+d*x])^2) + (A-I*B)/(16*d*(a^4+I*a^4*\text{Tan}[c+d*x]))$

**Rule 8**

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

**Rule 3560**

$\text{Int}[(a_ + (b_)*\tan[(c_ + (d_)*(x_))])^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[a*((a + b*\text{Tan}[c + d*x])^n/(2*b*d*n)), x] + \text{Dist}[1/(2*a), \text{Int}[(a + b*\text{Tan}[c + d*x])^{(n+1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[n, 0]$

**Rule 3607**

$\text{Int}[(a_ + (b_)*\tan[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\tan[(e_ + (f_)*(x_))])^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(-b*c - a*d)*((a + b*\text{Tan}[e + f*x])^m/(2*a*f*m)), x] + \text{Dist}[(b*c + a*d)/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, 0]$

**Rule 3671**

$\text{Int}[(a_ + (b_)*\tan[(e_ + (f_)*(x_))])^{(m_)}*((A_ + (B_)*\tan[(e_ + (f_)*(x_))])^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(-$



$A*b - a*B))*(a*c + b*d)*((a + b*\text{Tan}[e + f*x])^m/(2*a^2*f*m)), x] + \text{Dist}[1/(2*a*b), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[A*b*c + a*B*c + a*A*d + b*B*d + 2*a*B*d*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B}, x] & NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4} dx &= -\frac{A+iB}{8d(a+ia\tan(c+dx))^4} - \frac{i \int \frac{a(A+iB)+2aB\tan(c+dx)}{(a+ia\tan(c+dx))^3} dx}{2a^2} \\ &= -\frac{A+iB}{8d(a+ia\tan(c+dx))^4} + \frac{A+3iB}{12ad(a+ia\tan(c+dx))^3} - \frac{(iA+B)x}{16d(a+ia\tan(c+dx))^2} \\ &= -\frac{A+iB}{8d(a+ia\tan(c+dx))^4} + \frac{A+3iB}{12ad(a+ia\tan(c+dx))^3} + \frac{(iA+B)x}{16d(a+ia\tan(c+dx))^2} \\ &= -\frac{A+iB}{8d(a+ia\tan(c+dx))^4} + \frac{A+3iB}{12ad(a+ia\tan(c+dx))^3} + \frac{(iA+B)x}{16d(a+ia\tan(c+dx))^2} \\ &= -\frac{(iA+B)x}{16a^4} - \frac{A+iB}{8d(a+ia\tan(c+dx))^4} + \frac{A+3iB}{12ad(a+ia\tan(c+dx))^3} \end{aligned}$$

Mathematica [A]

time = 1.28, size = 141, normalized size = 0.99

$$\frac{\sec^4(c+dx)(12iB+16A\cos(2(c+dx))-3(A+8iAdx+B(i+8dx))\cos(4(c+dx))+32iA\sin(2(c+dx))+3iA\sin(4(c+dx))-3B\sin(4(c+dx))+24Adx\sin(4(c+dx))-24iBdx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4, x]

[Out] (Sec[c + d\*x]^4\*((12\*I)\*B + 16\*A\*Cos[2\*(c + d\*x)] - 3\*(A + (8\*I)\*A\*d\*x + B\*(I + 8\*d\*x))\*Cos[4\*(c + d\*x)] + (32\*I)\*A\*Sin[2\*(c + d\*x)] + (3\*I)\*A\*Sin[4\*(c + d\*x)] - 3\*B\*Sin[4\*(c + d\*x)] + 24\*A\*d\*x\*Sin[4\*(c + d\*x)] - (24\*I)\*B\*d\*x\*Sin[4\*(c + d\*x)])/(384\*a^4\*d\*(-I + Tan[c + d\*x])^4)

Maple [A]

time = 0.15, size = 131, normalized size = 0.92

method	result
risch	$-\frac{xB}{16a^4} - \frac{ixA}{16a^4} + \frac{e^{-2i(dx+c)}A}{16da^4} + \frac{iBe^{-4i(dx+c)}}{32da^4} - \frac{e^{-6i(dx+c)}A}{48da^4} - \frac{ie^{-8i(dx+c)}B}{128da^4} - \frac{e^{-8i(dx+c)}A}{128da^4}$
derivativedivides	$-\frac{\frac{A}{8} - \frac{iB}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{3B}{4} - \frac{iA}{4}}{3(\tan(dx+c)-i)^3} - \frac{\frac{A}{2} + \frac{iB}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{iA}{16} + \frac{B}{16}}{\tan(dx+c)-i} + \left(-\frac{A}{32} + \frac{iB}{32}\right) \ln(\tan(dx+c)-i) - \frac{i(iA+B)\ln(\tan(dx+c)-i)}{32da^4}$

default	$\frac{\frac{A}{8} - \frac{iB}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{3B}{4} - \frac{iA}{4}}{3(\tan(dx+c)-i)^3} - \frac{\frac{A}{2} + \frac{iB}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{iA}{16} + \frac{B}{16}}{\tan(dx+c)-i} + \left(-\frac{A}{32} + \frac{iB}{32}\right) \ln(\tan(dx+c)-i) - \frac{i(iA+B) \ln(\tan(dx+c))}{32}$ $d a^4$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out]  $1/d/a^4*(-1/2*(1/8*A-1/8*I*B)/(\tan(d*x+c)-I)^2-1/3*(3/4*B-1/4*I*A)/(\tan(d*x+c)-I)^3-1/4*(1/2*A+1/2*I*B)/(\tan(d*x+c)-I)^4-(1/16*I*A+1/16*B)/(\tan(d*x+c)-I)+(-1/32*A+1/32*I*B)*\ln(\tan(d*x+c)-I)-1/32*I*(I*A+B)*\ln(\tan(d*x+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 3.51, size = 78, normalized size = 0.55

$$\frac{(24(iA+B)dx e^{(8i dx+8i c)} - 24 A e^{(6i dx+6i c)} - 12i B e^{(4i dx+4i c)} + 8 A e^{(2i dx+2i c)} + 3A + 3i B) e^{(-8i dx-8i c)}}{384 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

[Out]  $-1/384*(24*(I*A + B)*d*x*e^{(8*I*d*x + 8*I*c)} - 24*A*e^{(6*I*d*x + 6*I*c)} - 12*I*B*e^{(4*I*d*x + 4*I*c)} + 8*A*e^{(2*I*d*x + 2*I*c)} + 3*A + 3*I*B)*e^{(-8*I*d*x - 8*I*c)}/(a^4*d)$

**Sympy** [A]

time = 0.33, size = 246, normalized size = 1.72

$$\begin{cases} \frac{(196608Aa^{12}d^3e^{18ic}e^{-2idx} - 65536Aa^{12}d^3e^{14ic}e^{-6idx} + 98304iBa^{12}d^3e^{16ic}e^{-4idx} + (-24576Aa^{12}d^3e^{12ic} - 24576iBa^{12}d^3e^{12ic})e^{-8idx})e^{-20ic}}{3145728a^{16}d^4} & \text{for } a^{16}d^4e^{20ic} \neq 0 \\ x\left(-\frac{iA-B}{16a^4} + \frac{(-iAe^{8ic} - 2iAe^{6ic} + 2iAe^{2ic} + iA - Be^{8ic} + 2Be^{4ic} - B)e^{-8ic}}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{x(-iA-B)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)`

[Out] Piecewise(((196608\*A\*a\*\*12\*d\*\*3\*exp(18\*I\*c)\*exp(-2\*I\*d\*x) - 65536\*A\*a\*\*12\*d\*\*3\*exp(14\*I\*c)\*exp(-6\*I\*d\*x) + 98304\*I\*B\*a\*\*12\*d\*\*3\*exp(16\*I\*c)\*exp(-4\*I\*d\*x) + (-24576\*A\*a\*\*12\*d\*\*3\*exp(12\*I\*c) - 24576\*I\*B\*a\*\*12\*d\*\*3\*exp(12\*I\*c))\*exp(-8\*I\*d\*x))\*exp(-20\*I\*c)/(3145728\*a\*\*16\*d\*\*4), Ne(a\*\*16\*d\*\*4\*exp(20\*I\*c), 0)), (x\*(-(-I\*A - B)/(16\*a\*\*4) + (-I\*A\*exp(8\*I\*c) - 2\*I\*A\*exp(6\*I\*c) + 2\*I\*A\*exp(2\*I\*c) + I\*A - B\*exp(8\*I\*c) + 2\*B\*exp(4\*I\*c) - B)\*exp(-8\*I\*c)/(16\*a\*\*4)), True)) + x\*(-I\*A - B)/(16\*a\*\*4)

**Giac [A]**

time = 0.79, size = 154, normalized size = 1.08

$$\frac{\frac{12(A-iB)\log(i\tan(dx+c)+1)}{a^4} - \frac{12(A-iB)\log(i\tan(dx+c)-1)}{a^4} - \frac{25A\tan(dx+c)^4 - 25iB\tan(dx+c)^4 - 124iA\tan(dx+c)^3 - 124B\tan(dx+c)^3 - 246A\tan(dx+c)^2 + 246iB\tan(dx+c)^2 + 252iA\tan(dx+c) + 124B\tan(dx+c) + 57A - 25iB}{a^4(\tan(dx+c)-i)^4}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] 
$$-1/384*(12*(A - I*B)*\log(I*\tan(d*x + c) + 1)/a^4 - 12*(A - I*B)*\log(I*\tan(d*x + c) - 1)/a^4 - (25*A*\tan(d*x + c)^4 - 25*I*B*\tan(d*x + c)^4 - 124*I*A*\tan(d*x + c)^3 - 124*B*\tan(d*x + c)^3 - 246*A*\tan(d*x + c)^2 + 246*I*B*\tan(d*x + c)^2 + 252*I*A*\tan(d*x + c) + 124*B*\tan(d*x + c) + 57*A - 25*I*B)/(a^4*(\tan(d*x + c) - I)^4))/d$$

**Mupad [B]**

time = 6.51, size = 172, normalized size = 1.20

$$-\frac{\tan(c+dx)^2\left(\frac{A}{4a^4} - \frac{B1i}{4a^4}\right) + \tan(c+dx)^3\left(\frac{B}{16a^4} + \frac{A1i}{16a^4}\right) - \frac{A}{12a^4} - \tan(c+dx)\left(\frac{B}{16a^4} + \frac{A19i}{48a^4}\right)}{d(\tan(c+dx)^4 - \tan(c+dx)^3 4i - 6\tan(c+dx)^2 + \tan(c+dx) 4i + 1)} + \frac{\ln(\tan(c+dx) - i)(B + A1i)1i}{32a^4 d} + \frac{\ln(\tan(c+dx) + 1i)(A - B1i)}{32a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] 
$$\left(\log(\tan(c + d*x) - 1i)*(A*1i + B)*1i)/(32*a^4*d) - (\tan(c + d*x)^2*(A/(4*a^4) - (B*1i)/(4*a^4)) + \tan(c + d*x)^3*((A*1i)/(16*a^4) + B/(16*a^4)) - A/(12*a^4) - \tan(c + d*x)*((A*19i)/(48*a^4) + B/(16*a^4)))/(d*(\tan(c + d*x)*4i - 6*\tan(c + d*x)^2 - \tan(c + d*x)^3*4i + \tan(c + d*x)^4 + 1)) + (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(32*a^4*d)$$

### 3.63 $\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^4} dx$

**Optimal.** Leaf size=145

$$\frac{(A-iB)x}{16a^4} + \frac{iA-B}{8d(a+ia \tan(c+dx))^4} + \frac{iA+B}{12ad(a+ia \tan(c+dx))^3} + \frac{iA+B}{16d(a^2+ia^2 \tan(c+dx))^2} + \frac{iA}{16d(a^4+ia^4 \tan(c+dx))}$$

[Out] 1/16\*(A-I\*B)\*x/a^4+1/8\*(I\*A-B)/d/(a+I\*a\*tan(d\*x+c))^4+1/12\*(I\*A+B)/a/d/(a+I\*a\*tan(d\*x+c))^3+1/16\*(I\*A+B)/d/(a^2+I\*a^2\*tan(d\*x+c))^2+1/16\*(I\*A+B)/d/(a^4+I\*a^4\*tan(d\*x+c))

**Rubi [A]**

time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3607, 3560, 8}

$$\frac{B+iA}{16d(a^4+ia^4 \tan(c+dx))} + \frac{x(A-iB)}{16a^4} + \frac{B+iA}{16d(a^2+ia^2 \tan(c+dx))^2} + \frac{-B+iA}{8d(a+ia \tan(c+dx))^4} + \frac{B+iA}{12ad(a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^4, x]

[Out] ((A - I\*B)\*x)/(16\*a^4) + (I\*A - B)/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) + (I\*A + B)/(12\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3) + (I\*A + B)/(16\*d\*(a^2 + I\*a^2\*Tan[c + d\*x])^2) + (I\*A + B)/(16\*d\*(a^4 + I\*a^4\*Tan[c + d\*x]))

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a\_) + (b\_.)\*tan[(c\_) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[a\*((a + b\*Tan[c + d\*x])^n/(2\*b\*d\*n)), x] + Dist[1/(2\*a), Int[(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3607

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^4} dx &= \frac{iA - B}{8d(a + ia \tan(c + dx))^4} + \frac{(A - iB) \int \frac{1}{(a + ia \tan(c + dx))^3} dx}{2a} \\
&= \frac{iA - B}{8d(a + ia \tan(c + dx))^4} + \frac{iA + B}{12ad(a + ia \tan(c + dx))^3} + \frac{(A - iB) \int \frac{1}{(a + ia \tan(c + dx))}}{4a^2} \\
&= \frac{iA - B}{8d(a + ia \tan(c + dx))^4} + \frac{iA + B}{12ad(a + ia \tan(c + dx))^3} + \frac{iA + B}{16d(a^2 + ia^2 \tan(c + dx))} \\
&= \frac{iA - B}{8d(a + ia \tan(c + dx))^4} + \frac{iA + B}{12ad(a + ia \tan(c + dx))^3} + \frac{iA + B}{16d(a^2 + ia^2 \tan(c + dx))} \\
&= \frac{(A - iB)x}{16a^4} + \frac{iA - B}{8d(a + ia \tan(c + dx))^4} + \frac{iA + B}{12ad(a + ia \tan(c + dx))^3} + \frac{iA + B}{16d(a^2 + ia^2 \tan(c + dx))}
\end{aligned}$$

**Mathematica [A]**

time = 0.85, size = 160, normalized size = 1.10

$$\frac{\sec^4(c + dx)(36iA + 16(4iA + B) \cos(2(c + dx)) + 3(iA - B + 8Adx - 8iBdx) \cos(4(c + dx)) - 32A \sin(2(c + dx)) + 32iB \sin(2(c + dx)) + 3A \sin(4(c + dx)) + 3iB \sin(4(c + dx)) + 24iAdx \sin(4(c + dx)) + 24Bdx \sin(4(c + dx)))}{384a^4(-i + \tan(c + dx))^4}$$

Antiderivative was successfully verified.

**[In]** Integrate[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^4,x]

**[Out]** (Sec[c + d\*x]^4\*((36\*I)\*A + 16\*((4\*I)\*A + B)\*Cos[2\*(c + d\*x)] + 3\*(I\*A - B + 8\*A\*d\*x - (8\*I)\*B\*d\*x)\*Cos[4\*(c + d\*x)] - 32\*A\*Sin[2\*(c + d\*x)] + (32\*I)\*B\*Sin[2\*(c + d\*x)] + 3\*A\*Sin[4\*(c + d\*x)] + (3\*I)\*B\*Sin[4\*(c + d\*x)] + (24\*I)\*A\*d\*x\*Sin[4\*(c + d\*x)] + 24\*B\*d\*x\*Sin[4\*(c + d\*x)])/(384\*a^4\*d\*(-I + Tan[c + d\*x])^4)

**Maple [A]**

time = 0.15, size = 131, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{A}{16} + \frac{iB}{16}}{\tan(dx+c)-i} - \frac{-\frac{iA}{2} + \frac{B}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{A}{4} - \frac{iB}{4}}{3(\tan(dx+c)-i)^3} + \left(-\frac{iA}{32} - \frac{B}{32}\right) \ln(\tan(dx+c)-i) - \frac{\frac{iA}{8} + \frac{B}{8}}{2(\tan(dx+c)-i)^2} + \frac{i(-iB+A) \ln(\tan(dx+c)-i)}{32}$
default	$\frac{-\frac{A}{16} + \frac{iB}{16}}{\tan(dx+c)-i} - \frac{-\frac{iA}{2} + \frac{B}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{A}{4} - \frac{iB}{4}}{3(\tan(dx+c)-i)^3} + \left(-\frac{iA}{32} - \frac{B}{32}\right) \ln(\tan(dx+c)-i) - \frac{\frac{iA}{8} + \frac{B}{8}}{2(\tan(dx+c)-i)^2} + \frac{i(-iB+A) \ln(\tan(dx+c)-i)}{32}$
risch	$-\frac{ixB}{16a^4} + \frac{xA}{16a^4} + \frac{e^{-2i(dx+c)}B}{16da^4} + \frac{ie^{-2i(dx+c)}A}{8da^4} + \frac{3ie^{-4i(dx+c)}A}{32da^4} - \frac{e^{-6i(dx+c)}B}{48da^4} + \frac{ie^{-6i(dx+c)}A}{24da^4} - \frac{e^{-8i(dx+c)}A}{128da^4}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x,method=\_RETURNVERBOSE)

[Out]  $1/d/a^4*(-(-1/16*A+1/16*I*B)/(\tan(d*x+c)-I)-1/4*(-1/2*I*A+1/2*B)/(\tan(d*x+c)-I)^4-1/3*(1/4*A-1/4*I*B)/(\tan(d*x+c)-I)^3+(-1/32*I*A-1/32*B)*\ln(\tan(d*x+c)-I)-1/2*(1/8*I*A+1/8*B)/(\tan(d*x+c)-I)^2+1/32*I*(A-I*B)*\ln(\tan(d*x+c)+I)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 1.89, size = 88, normalized size = 0.61

$$\frac{(24(A-iB)dx e^{(8i dx+8i c)} - 24(-2iA-B)e^{(6i dx+6i c)} + 36i A e^{(4i dx+4i c)} - 8(-2iA+B)e^{(2i dx+2i c)} + 3iA-3B)e^{(-8i dx-8i c)}}{384 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

[Out]  $1/384*(24*(A-I*B)*d*x*e^{(8*I*d*x+8*I*c)} - 24*(-2*I*A-B)*e^{(6*I*d*x+6*I*c)} + 36*I*A*e^{(4*I*d*x+4*I*c)} - 8*(-2*I*A+B)*e^{(2*I*d*x+2*I*c)} + 3*I*A-3*B)*e^{(-8*I*d*x-8*I*c)}/(a^4*d)$

**Sympy** [A]

time = 0.36, size = 299, normalized size = 2.06

$$\begin{cases} \frac{(294912iAa^{12}d^3e^{16ic}e^{-4idx} + (24576iAa^{12}d^3e^{12ic} - 24576Ba^{12}d^3e^{12ic})e^{-8idx} + (131072iAa^{12}d^3e^{14ic} - 65536Ba^{12}d^3e^{14ic})e^{-6idx} + (393216iAa^{12}d^3e^{18ic} + 196608Ba^{12}d^3e^{18ic})e^{-2idx})e^{-20ic}}{3145728a^{16}d^4} & \text{for } a^{16}d^4e^{20ic} \neq 0 \\ x \left( -\frac{A-iB}{16a^4} + \frac{(Ae^{8ic} + 4Ae^{6ic} + 6Ae^{4ic} + 4Ae^{2ic} + A - iBe^{8ic} - 2iBe^{6ic} + 2iBe^{2ic} + B)e^{-8ic}}{16a^4} \right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)`

[Out] `Piecewise((((294912*I*A*a**12*d**3*exp(16*I*c))*exp(-4*I*d*x) + (24576*I*A*a**12*d**3*exp(12*I*c) - 24576*B*a**12*d**3*exp(12*I*c))*exp(-8*I*d*x) + (131072*I*A*a**12*d**3*exp(14*I*c) - 65536*B*a**12*d**3*exp(14*I*c))*exp(-6*I*d*x) + (393216*I*A*a**12*d**3*exp(18*I*c) + 196608*B*a**12*d**3*exp(18*I*c))*exp(-2*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*(-(A-I*B)/(16*a**4) + (A*exp(8*I*c) + 4*A*exp(6*I*c) + 6*A*exp(4*I*c) + 4*A*exp(2*I*c) + A - I*B*exp(8*I*c) - 2*I*B*exp(6*I*c) + 2*I*B*exp(2*I*c) + I*B)*exp(-8*I*c)/(16*a**4)), True)) + x*(A-I*B)/(16*a**4)`

**Giac** [A]

time = 0.72, size = 154, normalized size = 1.06

$$\frac{12(-iA-B)\log(\tan(dx+c)+i)}{a^4} - \frac{12(-iA-B)\log(\tan(dx+c)-i)}{a^4} - \frac{25iA \tan(dx+c)^4 + 25B \tan(dx+c)^4 + 124A \tan(dx+c)^3 - 124iB \tan(dx+c)^3 - 246iA \tan(dx+c)^2 - 246B \tan(dx+c)^2 - 252A \tan(dx+c) + 252iB \tan(dx+c) + 153iA + 57B}{a^4(\tan(dx+c)-i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] 
$$\frac{-1/384*(12*(-I*A - B)*\log(\tan(dx + c) + I)/a^4 - 12*(-I*A - B)*\log(\tan(dx + c) - I)/a^4 - (25*I*A*\tan(dx + c)^4 + 25*B*\tan(dx + c)^4 + 124*A*\tan(dx + c)^3 - 124*I*B*\tan(dx + c)^3 - 246*I*A*\tan(dx + c)^2 - 246*B*\tan(dx + c)^2 - 252*A*\tan(dx + c) + 252*I*B*\tan(dx + c) + 153*I*A + 57*B)/(a^4*(\tan(dx + c) - I)^4))/d$$

**Mupad [B]**

time = 6.68, size = 143, normalized size = 0.99

$$\frac{\frac{B}{12a^4} + \tan(c+dx)^3 \left( \frac{A}{16a^4} - \frac{B \operatorname{li}}{16a^4} \right) + \frac{A \operatorname{li}}{3a^4} - \tan(c+dx)^2 \left( \frac{B}{4a^4} + \frac{A \operatorname{li}}{4a^4} \right) - \tan(c+dx) \left( \frac{19A}{48a^4} - \frac{B \operatorname{li}}{48a^4} \right)}{d \left( \tan(c+dx)^4 - \tan(c+dx)^3 4i - 6 \tan(c+dx)^2 + \tan(c+dx) 4i + 1 \right)} - \frac{x(B + A \operatorname{li}) \operatorname{li}}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] 
$$\frac{\tan(c+dx)^3 \left( \frac{A}{16a^4} - \frac{B \operatorname{li}}{16a^4} \right) - \tan(c+dx)^2 \left( \frac{A \operatorname{li}}{4a^4} + \frac{B}{4a^4} \right) + \frac{A \operatorname{li}}{3a^4} + \frac{B}{12a^4} - \tan(c+dx) \left( \frac{19A}{48a^4} - \frac{B \operatorname{li}}{48a^4} \right)}{d \left( \tan(c+dx) 4i - 6 \tan(c+dx)^2 - \tan(c+dx)^3 4i + \tan(c+dx)^4 + 1 \right)} - \frac{x(A \operatorname{li} + B) \operatorname{li}}{16a^4}$$

$$3.64 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=162

$$-\frac{(15iA - B)x}{16a^4} + \frac{A \log(\sin(c + dx))}{a^4 d} + \frac{7A + iB}{16a^4 d(1 + i \tan(c + dx))^2} + \frac{15A + iB}{16a^4 d(1 + i \tan(c + dx))} + \frac{A + iB}{8d(a + ia \tan(c + dx))}$$

[Out] -1/16\*(15\*I\*A-B)\*x/a^4+A\*ln(sin(d\*x+c))/a^4/d+1/16\*(7\*A+I\*B)/a^4/d/(1+I\*tan(d\*x+c))^2+1/16\*(15\*A+I\*B)/a^4/d/(1+I\*tan(d\*x+c))+1/8\*(A+I\*B)/d/(a+I\*a\*tan(d\*x+c))^4+1/12\*(3\*A+I\*B)/a/d/(a+I\*a\*tan(d\*x+c))^3

**Rubi [A]**

time = 0.34, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3677, 3612, 3556}

$$\frac{15A + iB}{16a^4 d(1 + i \tan(c + dx))} + \frac{7A + iB}{16a^4 d(1 + i \tan(c + dx))^2} - \frac{x(-B + 15iA)}{16a^4} + \frac{A \log(\sin(c + dx))}{a^4 d} + \frac{A + iB}{8d(a + ia \tan(c + dx))^4} + \frac{3A + iB}{12ad(a + ia \tan(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] -1/16\*(((15\*I)\*A - B)\*x)/a^4 + (A\*Log[Sin[c + d\*x]])/(a^4\*d) + (7\*A + I\*B)/(16\*a^4\*d\*(1 + I\*Tan[c + d\*x])^2) + (15\*A + I\*B)/(16\*a^4\*d\*(1 + I\*Tan[c + d\*x])) + (A + I\*B)/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) + (3\*A + I\*B)/(12\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(x\_), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3677

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^n, x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m -



$b*d*(n + 1) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ  
 $[{a, b, c, d, e, f, A, B, n}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$   
 $\&\& \text{LtQ}[m, 0] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx &= \frac{A+iB}{8d(a+ia \tan(c+dx))^4} + \frac{\int \frac{\cot(c+dx)(8aA-4a(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx}{8a^2} \\ &= \frac{A+iB}{8d(a+ia \tan(c+dx))^4} + \frac{3A+iB}{12ad(a+ia \tan(c+dx))^3} + \frac{\int \frac{\cot(c+dx)}{(a+ia \tan(c+dx))^2} dx}{12ad} \\ &= \frac{7A+iB}{16a^4d(1+i \tan(c+dx))^2} + \frac{A+iB}{8d(a+ia \tan(c+dx))^4} + \frac{3}{12ad(a+ia \tan(c+dx))^2} \\ &= \frac{7A+iB}{16a^4d(1+i \tan(c+dx))^2} + \frac{A+iB}{8d(a+ia \tan(c+dx))^4} + \frac{3}{12ad(a+ia \tan(c+dx))^2} \\ &= -\frac{(15iA-B)x}{16a^4} + \frac{7A+iB}{16a^4d(1+i \tan(c+dx))^2} + \frac{A+iB}{8d(a+ia \tan(c+dx))^4} \\ &= -\frac{(15iA-B)x}{16a^4} + \frac{A \log(\sin(c+dx))}{a^4d} + \frac{7A+iB}{16a^4d(1+i \tan(c+dx))^2} \end{aligned}$$

**Mathematica [A]**

time = 1.23, size = 193, normalized size = 1.19

$\frac{\sec^4(c+dx)(96A+36iB+16(21A+4iB)\cos(2(c+dx))+3\cos(4(c+dx))(A+iB-120iAdx+8Bdx+128A\log(\sin(c+dx)))+288iA\sin(2(c+dx))-32B\sin(2(c+dx))-3iA\sin(4(c+dx))+3B\sin(4(c+dx))+360Adx\sin(4(c+dx))+24iBdx\sin(4(c+dx))+384iA\log(\sin(c+dx))\sin(4(c+dx)))}{384a^4d(-1+i \tan(c+dx))^4}$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] (Sec[c + d\*x]^4\*(96\*A + (36\*I)\*B + 16\*(21\*A + (4\*I)\*B)\*Cos[2\*(c + d\*x)] + 3 \*Cos[4\*(c + d\*x)]\*(A + I\*B - (120\*I)\*A\*d\*x + 8\*B\*d\*x + 128\*A\*Log[Sin[c + d\*x]]) + (288\*I)\*A\*Sin[2\*(c + d\*x)] - 32\*B\*Sin[2\*(c + d\*x)] - (3\*I)\*A\*Sin[4\*(c + d\*x)] + 3\*B\*Sin[4\*(c + d\*x)] + 360\*A\*d\*x\*Sin[4\*(c + d\*x)] + (24\*I)\*B\*d\*x\*Sin[4\*(c + d\*x)] + (384\*I)\*A\*Log[Sin[c + d\*x]]\*Sin[4\*(c + d\*x)]))/(384\*a^4\*d\*(-I + Tan[c + d\*x])^4)

**Maple [A]**

time = 0.33, size = 140, normalized size = 0.86

method	result
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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

[Out]  $A \cdot \log(\exp(2 \cdot I \cdot d \cdot x) - \exp(-2 \cdot I \cdot c)) / (a^{**4} \cdot d) + \text{Piecewise}(\left( (24576 \cdot A \cdot a^{**12} \cdot d^{**3} \cdot \exp(12 \cdot I \cdot c) + 24576 \cdot I \cdot B \cdot a^{**12} \cdot d^{**3} \cdot \exp(12 \cdot I \cdot c)) \cdot \exp(-8 \cdot I \cdot d \cdot x) + (196608 \cdot A \cdot a^{**12} \cdot d^{**3} \cdot \exp(14 \cdot I \cdot c) + 131072 \cdot I \cdot B \cdot a^{**12} \cdot d^{**3} \cdot \exp(14 \cdot I \cdot c)) \cdot \exp(-6 \cdot I \cdot d \cdot x) + (786432 \cdot A \cdot a^{**12} \cdot d^{**3} \cdot \exp(16 \cdot I \cdot c) + 294912 \cdot I \cdot B \cdot a^{**12} \cdot d^{**3} \cdot \exp(16 \cdot I \cdot c)) \cdot \exp(-4 \cdot I \cdot d \cdot x) + (2555904 \cdot A \cdot a^{**12} \cdot d^{**3} \cdot \exp(18 \cdot I \cdot c) + 393216 \cdot I \cdot B \cdot a^{**12} \cdot d^{**3} \cdot \exp(18 \cdot I \cdot c)) \cdot \exp(-2 \cdot I \cdot d \cdot x) \right) \cdot \exp(-20 \cdot I \cdot c) / (3145728 \cdot a^{**16} \cdot d^{**4}), \text{Ne}(a^{**16} \cdot d^{**4} \cdot \exp(20 \cdot I \cdot c), 0)), (x \cdot (-31 \cdot I \cdot A + B) / (16 \cdot a^{**4}) + (-31 \cdot I \cdot A \cdot \exp(8 \cdot I \cdot c) - 26 \cdot I \cdot A \cdot \exp(6 \cdot I \cdot c) - 16 \cdot I \cdot A \cdot \exp(4 \cdot I \cdot c) - 6 \cdot I \cdot A \cdot \exp(2 \cdot I \cdot c) - I \cdot A + B \cdot \exp(8 \cdot I \cdot c) + 4 \cdot B \cdot \exp(6 \cdot I \cdot c) + 6 \cdot B \cdot \exp(4 \cdot I \cdot c) + 4 \cdot B \cdot \exp(2 \cdot I \cdot c) + B) \cdot \exp(-8 \cdot I \cdot c) / (16 \cdot a^{**4})), \text{True})) + x \cdot (-31 \cdot I \cdot A + B) / (16 \cdot a^{**4})$

**Giac** [A]

time = 1.03, size = 165, normalized size = 1.02

$$\frac{\frac{12(A-I) \log(\tan(dx+c)+1)}{a^4} + \frac{12(31A+I) \log(\tan(dx+c)-1)}{a^4} - \frac{384A \log(\tan(dx+c))}{a^4} - \frac{775A \tan(dx+c)^4 + 25I B \tan(dx+c)^3 - 3460I A \tan(dx+c)^2 + 124B \tan(dx+c)^2 - 5898A \tan(dx+c)^2 - 246I B \tan(dx+c)^2 + 4612I A \tan(dx+c) - 252B \tan(dx+c) + 1447A + 153I B}{a^4 (\tan(dx+c)-1)^4}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out]  $-1/384 \cdot (12 \cdot (A - I \cdot B) \cdot \log(\tan(d \cdot x + c) + I) / a^4 + 12 \cdot (31 \cdot A + I \cdot B) \cdot \log(\tan(d \cdot x + c) - I) / a^4 - 384 \cdot A \cdot \log(\tan(d \cdot x + c)) / a^4 - (775 \cdot A \cdot \tan(d \cdot x + c)^4 + 25 \cdot I \cdot B \cdot \tan(d \cdot x + c)^4 - 3460 \cdot I \cdot A \cdot \tan(d \cdot x + c)^3 + 124 \cdot B \cdot \tan(d \cdot x + c)^3 - 5898 \cdot A \cdot \tan(d \cdot x + c)^2 - 246 \cdot I \cdot B \cdot \tan(d \cdot x + c)^2 + 4612 \cdot I \cdot A \cdot \tan(d \cdot x + c) - 252 \cdot B \cdot \tan(d \cdot x + c) + 1447 \cdot A + 153 \cdot I \cdot B) / (a^4 \cdot (\tan(d \cdot x + c) - I)^4)) / d$

**Mupad** [B]

time = 6.67, size = 196, normalized size = 1.21

$$\frac{\frac{7A}{4a^4} - \tan(c+dx)^3 \left( -\frac{B}{16a^4} + \frac{A15i}{16a^4} \right) - \tan(c+dx)^2 \left( \frac{13A}{4a^4} + \frac{B1i}{4a^4} \right) + \frac{B1i}{3a^4} + \tan(c+dx) \left( -\frac{19B}{48a^4} + \frac{A63i}{16a^4} \right) + \frac{A \ln(\tan(c+dx))}{a^4 d} + \frac{\ln(\tan(c+dx)+1) (B+A1i) i}{32a^4 d} - \frac{\ln(\tan(c+dx)-1) (31A+B1i)}{32a^4 d}}{d (\tan(c+dx)^4 - \tan(c+dx)^3 4i - 6 \tan(c+dx)^2 + \tan(c+dx) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c+d\*x)\*(A+B\*tan(c+d\*x)))/(a+a\*tan(c+d\*x)\*1i)^4,x)

[Out]  $((7 \cdot A) / (4 \cdot a^4) - \tan(c + d \cdot x)^3 \cdot ((A \cdot 15i) / (16 \cdot a^4) - B / (16 \cdot a^4)) - \tan(c + d \cdot x)^2 \cdot ((13 \cdot A) / (4 \cdot a^4) + (B \cdot 1i) / (4 \cdot a^4)) + (B \cdot 1i) / (3 \cdot a^4) + \tan(c + d \cdot x) \cdot ((A \cdot 63i) / (16 \cdot a^4) - (19 \cdot B) / (48 \cdot a^4))) / (d \cdot (\tan(c + d \cdot x)^4 i - 6 \cdot \tan(c + d \cdot x)^2 - \tan(c + d \cdot x)^3 \cdot 4i + \tan(c + d \cdot x)^4 + 1)) + (A \cdot \log(\tan(c + d \cdot x))) / (a^4 \cdot d) + (\log(\tan(c + d \cdot x) + 1i) \cdot (A \cdot 1i + B) \cdot 1i) / (32 \cdot a^4 \cdot d) - (\log(\tan(c + d \cdot x) - 1i) \cdot (31 \cdot A + B \cdot 1i)) / (32 \cdot a^4 \cdot d))$

$$3.65 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=220

$$\frac{5(13A + 3iB)x}{16a^4} - \frac{5(13A + 3iB) \cot(c + dx)}{16a^4 d} - \frac{(4iA - B) \log(\sin(c + dx))}{a^4 d} + \frac{(31A + 9iB) \cot(c + dx)}{48a^4 d(1 + i \tan(c + dx))^2} + \frac{(4A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4}$$

[Out]  $-5/16*(13*A+3*I*B)*x/a^4-5/16*(13*A+3*I*B)*\cot(d*x+c)/a^4/d-(4*I*A-B)*\ln(\sin(d*x+c))/a^4/d+1/48*(31*A+9*I*B)*\cot(d*x+c)/a^4/d/(1+I*\tan(d*x+c))^2+1/2*(4*A+I*B)*\cot(d*x+c)/a^4/d/(1+I*\tan(d*x+c))+1/8*(A+I*B)*\cot(d*x+c)/d/(a+I*a*\tan(d*x+c))^4+1/24*(7*A+3*I*B)*\cot(d*x+c)/a/d/(a+I*a*\tan(d*x+c))^3$

**Rubi [A]**

time = 0.49, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{5(13A + 3iB) \cot(c + dx)}{16a^4 d} - \frac{(-B + 4iA) \log(\sin(c + dx))}{a^4 d} + \frac{(4A + iB) \cot(c + dx)}{2a^4 d(1 + i \tan(c + dx))} + \frac{(31A + 9iB) \cot(c + dx)}{48a^4 d(1 + i \tan(c + dx))^2} - \frac{5x(13A + 3iB)}{16a^4} + \frac{(7A + 3iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^3} + \frac{(A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]^4,x]

[Out]  $(-5*(13*A + (3*I)*B)*x)/(16*a^4) - (5*(13*A + (3*I)*B)*\text{Cot}[c + d*x])/(16*a^4*d) - (((4*I)*A - B)*\text{Log}[\text{Sin}[c + d*x]])/(a^4*d) + ((31*A + (9*I)*B)*\text{Cot}[c + d*x])/(48*a^4*d*(1 + I*\text{Tan}[c + d*x])^2) + ((4*A + I*B)*\text{Cot}[c + d*x])/(2*a^4*d*(1 + I*\text{Tan}[c + d*x])) + ((A + I*B)*\text{Cot}[c + d*x])/(8*d*(a + I*a*\text{Tan}[c + d*x])^4) + ((7*A + (3*I)*B)*\text{Cot}[c + d*x])/(24*a*d*(a + I*a*\text{Tan}[c + d*x])^3)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a

\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^4} dx &= \frac{(A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{\int \frac{\cot^2(c + dx)(a(9A + iB) - 5a(iA - B) \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx}{8a^2} \\
 &= \frac{(A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(7A + 3iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^3} + \frac{\int \frac{\cot^2(c + dx)}{(a + ia \tan(c + dx))^2} dx}{24ad} \\
 &= \frac{(31A + 9iB) \cot(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{(A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(7A + 3iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^3} \\
 &= \frac{(31A + 9iB) \cot(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{(A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(7A + 3iB) \cot(c + dx)}{24ad(a + ia \tan(c + dx))^3} \\
 &= -\frac{5(13A + 3iB) \cot(c + dx)}{16a^4d} + \frac{(31A + 9iB) \cot(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{(A + iB) \cot(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
 &= -\frac{5(13A + 3iB)x}{16a^4} - \frac{5(13A + 3iB) \cot(c + dx)}{16a^4d} + \frac{(31A + 9iB) \cot(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} \\
 &= -\frac{5(13A + 3iB)x}{16a^4} - \frac{5(13A + 3iB) \cot(c + dx)}{16a^4d} - \frac{(4iA - B) \log(a + ia \tan(c + dx))}{a^4d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1466 vs. 2(220) = 440.  
time = 7.04, size = 1466, normalized size = 6.66

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Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] (((-15\*I)\*A + 8\*B)\*Cos[4\*d\*x]\*Sec[c + d\*x]^3\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (((-4\*I)\*A + 3\*B)\*Cos[6\*d\*x]\*Sec[c + d\*x]^3\*(Cos[2\*c]/48 - (I/48)\*Sin[2\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (((-36\*I)\*A + 13\*B)\*Cos[2\*d\*x]\*Sec[c + d\*x]^3\*(Cos[2\*c]/16 + (I/16)\*Sin[2\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (Sec[c + d\*x]^3\*((-4\*I)\*A\*Cos[2\*c] + B\*Cos[2\*c] + 4\*A\*Sin[2\*c] + I\*B\*Sin[2\*c]))\*((-I)\*ArcTan[Tan[d\*x]]\*Cos[2\*c] + ArcTan[Tan[d\*x]]\*Sin[2\*c])\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (Sec[c + d\*x]^3\*((-4\*I)\*A\*Cos[2\*c] + B\*Cos[2\*c] + 4\*A\*Sin[2\*c] + I\*B\*Sin[2\*c]))\*((Cos[2\*c]\*Log[Sin[c + d\*x]^2])/2 + (I/2)\*Log[Sin[c + d\*x]^2]\*Sin[2\*c])\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (x\*Sec[c + d\*x]^3\*(-12\*A\*Cos[c]^2 - (3\*I)\*B\*Cos[c]^2 + (4\*I)\*A\*Cos[c]^2\*Cot[c] - B\*Cos[c]^2\*Cot[c] - (12\*I)\*A\*Cos[c]\*Sin[c] + 3\*B\*Cos[c]\*Sin[c] + 4\*A\*Sin[c]^2 + I\*B\*Sin[c]^2 + ((-4\*I)\*A + B)\*Cot[c]\*(Cos[4\*c] + I\*Sin[4\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/((A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (((-I)\*A + B)\*Cos[8\*d\*x]\*Sec[c + d\*x]^3\*(Cos[4\*c]/128 - (I/128)\*Sin[4\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + ((13\*A + (3\*I)\*B)\*Sec[c + d\*x]^3\*((-5\*d\*x\*Cos[4\*c])/16 - ((5\*I)/16)\*d\*x\*Sin[4\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + ((36\*A + (13\*I)\*B)\*Sec[c + d\*x]^3\*(-1/16\*Cos[2\*c] - (I/16)\*Sin[2\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*Sin[2\*d\*x]\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) - ((15\*A + (8\*I)\*B)\*Sec[c + d\*x]^3\*(Cos[d\*x] + I\*Sin[d\*x])^4\*Sin[4\*d\*x]\*(A + B\*Tan[c + d\*x]))/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + ((4\*A + (3\*I)\*B)\*Sec[c + d\*x]^3\*(-1/48\*Cos[2\*c] + (I/48)\*Sin[2\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*Sin[6\*d\*x]\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + ((A + I\*B)\*Sec[c + d\*x]^3\*(-1/128\*Cos[4\*c] + (I/128)\*Sin[4\*c]))\*(Cos[d\*x] + I\*Sin[d\*x])^4\*Sin[8\*d\*x]\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4) + (Csc[c]\*Csc[c + d\*x]\*Sec[c + d\*x]^3\*(Cos[d\*x] + I\*Sin[d\*x])^4\*((I/2)\*A\*Cos[4\*c - d\*x] - (I/2)\*A\*Cos[4\*c + d\*x] - (A\*Sin[4\*c - d\*x])/2 + (A\*Sin[4\*c + d\*x])/2)\*(A + B\*Tan[c + d\*x]))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^4)

Maple [A]

time = 0.36, size = 156, normalized size = 0.71

method	result
derivativedivides	$-\frac{A}{\tan(dx+c)} + (-4iA+B) \ln(\tan(dx+c)) - \frac{\frac{7B}{8} - \frac{17iA}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{iA}{2} - \frac{B}{2}}{4(\tan(dx+c)-i)^4} + \left(-\frac{31B}{32} + \frac{129iA}{32}\right) \ln(\tan(dx+c)-i) - \frac{\frac{49A}{16} + \frac{1}{16}}{\tan(dx+c)}$ $da^4$

default	$-\frac{A}{\tan(dx+c)} + (-4iA+B) \ln(\tan(dx+c)) - \frac{\frac{7B}{8} - \frac{17iA}{8}}{2(\tan(dx+c)-i)^2} - \frac{\frac{iA}{2} - \frac{B}{2}}{4(\tan(dx+c)-i)^4} + \left(-\frac{31B}{32} + \frac{129iA}{32}\right) \ln(\tan(dx+c)-i) - \frac{\frac{49A}{16} + \frac{13B}{16}}{\tan(dx+c)}$
risch	$-\frac{2iA}{a^4 d (e^{2i(dx+c)} - 1)} - \frac{129xA}{16a^4} + \frac{13e^{-2i(dx+c)}B}{16da^4} - \frac{2iBc}{da^4} + \frac{e^{-4i(dx+c)}B}{4da^4} - \frac{31ixB}{16a^4} + \frac{e^{-6i(dx+c)}B}{16da^4} - \frac{9ie^{-2i(dx+c)}B}{4da^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out]  $1/d/a^4*(-A/\tan(dx+c)+(-4*I*A+B)*\ln(\tan(dx+c))-1/2*(7/8*B-17/8*I*A)/(\tan(dx+c)-I)^2-1/4*(1/2*I*A-1/2*B)/(\tan(dx+c)-I)^4+(-31/32*B+129/32*I*A)*\ln(\tan(dx+c)-I)-(49/16*A+15/16*I*B)/(\tan(dx+c)-I)-1/3*(-5/4*A-3/4*I*B)/(\tan(dx+c)-I)^3-1/32*I*(A-I*B)*\ln(\tan(dx+c)+I))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 2.08, size = 190, normalized size = 0.86

$$\frac{24(129A+31iB)dx e^{10i(dx+c)} - 24((129A+31iB)dx - 68iA + 13B)e^{8i(dx+c)} + 36(-19iA + 6B)e^{6i(dx+c)} + 4(-37iA + 18B)e^{4i(dx+c)} - (29iA - 21B)e^{2i(dx+c)} + 384((4iA - B)e^{10i(dx+c)} + (-4iA + B)e^{8i(dx+c)}) \log(e^{2i(dx+c)} - 1) - 3iA + 3B}{384(a^4 d e^{10i(dx+c)} - a^4 d e^{8i(dx+c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")`

[Out]  $-1/384*(24*(129*A + 31*I*B)*d*x*e^{(10*I*d*x + 10*I*c)} - 24*((129*A + 31*I*B)*d*x - 68*I*A + 13*B)*e^{(8*I*d*x + 8*I*c)} + 36*(-19*I*A + 6*B)*e^{(6*I*d*x + 6*I*c)} + 4*(-37*I*A + 18*B)*e^{(4*I*d*x + 4*I*c)} - (29*I*A - 21*B)*e^{(2*I*d*x + 2*I*c)} + 384*((4*I*A - B)*e^{(10*I*d*x + 10*I*c)} + (-4*I*A + B)*e^{(8*I*d*x + 8*I*c)})*\log(e^{(2*I*d*x + 2*I*c)} - 1) - 3*I*A + 3*B)/(a^4*d*e^{(10*I*d*x + 10*I*c)} - a^4*d*e^{(8*I*d*x + 8*I*c)})$

**Sympy** [A]

time = 1.16, size = 406, normalized size = 1.85

$$-\frac{2iA}{a^4 d e^{2i(dx+c)} - a^4 d} + \begin{cases} \left( \frac{(-24576iA^2 d^2 e^{12i(dx+c)} + 24576B d^2 e^{12i(dx+c)} - 262144iA d^2 e^{12i(dx+c)} + 196608B d^2 e^{12i(dx+c)}) e^{-6i(dx+c)} + (-1474560iA^2 d^2 e^{10i(dx+c)} + 786432B d^2 e^{10i(dx+c)}) e^{-4i(dx+c)} + (-707888iA^2 d^2 e^{8i(dx+c)} + 255504B d^2 e^{8i(dx+c)}) e^{-2i(dx+c)}}{3145728d^2} \right) & \text{for } a^{16} d^{20} e^{20ic} \neq 0 \\ x \left( -\frac{129A-31iB}{16a^4} + \frac{(-129Ae^{2i(dx+c)} - 72Ae^{4i(dx+c)} - 30Ae^{6i(dx+c)} - 8Ae^{8i(dx+c)} - A - 31iB e^{10i(dx+c)} - 20B e^{12i(dx+c)} - 16iB e^{14i(dx+c)} - 6iB e^{16i(dx+c)} - iB) e^{-2i(dx+c)}}{16a^4} \right) & \text{otherwise} \end{cases} + \frac{x(-129A-31iB)}{16a^4} - \frac{i(4A+iB) \log(e^{2i(dx+c)} - e^{-2i(dx+c)})}{a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

[Out]  $-2*I*A/(a**4*d*\exp(2*I*c)*\exp(2*I*d*x) - a**4*d) + \text{Piecewise}(\left(\left(\left(-24576*I*A*a**12*d**3*\exp(12*I*c) + 24576*B*a**12*d**3*\exp(12*I*c)\right)*\exp(-8*I*d*x) + (-262144*I*A*a**12*d**3*\exp(14*I*c) + 196608*B*a**12*d**3*\exp(14*I*c))*\exp(-6*I*d*x) + (-1474560*I*A*a**12*d**3*\exp(16*I*c) + 786432*B*a**12*d**3*\exp(16*I*c))*\exp(-4*I*d*x) + (-7077888*I*A*a**12*d**3*\exp(18*I*c) + 2555904*B*a**12*d**3*\exp(18*I*c))*\exp(-2*I*d*x)\right)*\exp(-20*I*c)/(3145728*a**16*d**4), \text{Ne}(a**16*d**4*\exp(20*I*c), 0)), (x*(-(-129*A - 31*I*B)/(16*a**4) + (-129*A*\exp(8*I*c) - 72*A*\exp(6*I*c) - 30*A*\exp(4*I*c) - 8*A*\exp(2*I*c) - A - 31*I*B*\exp(8*I*c) - 26*I*B*\exp(6*I*c) - 16*I*B*\exp(4*I*c) - 6*I*B*\exp(2*I*c) - I*B)*\exp(-8*I*c)/(16*a**4)), \text{True})) + x*(-129*A - 31*I*B)/(16*a**4) - I*(4*A + I*B)*\log(\exp(2*I*d*x) - \exp(-2*I*c))/(a**4*d)$

**Giac** [A]

time = 1.33, size = 205, normalized size = 0.93

$$\frac{12(-A-B)\log(\tan(dx+c)+1) - 12(-129A+31B)\log(\tan(dx+c)-1) - 384(4A-B)\log(\tan(dx+c)) - 384(-4A\tan(dx+c)+B\tan(dx+c)+A) - 3225A\tan(dx+c)^4 - 775B\tan(dx+c)^4 + 14076A\tan(dx+c)^3 + 3460B\tan(dx+c)^3 - 23286A\tan(dx+c)^2 + 5898B\tan(dx+c)^2 - 17404A\tan(dx+c) - 4612B\tan(dx+c) + 5017A - 1447B}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out]  $1/384*(12*(-I*A - B)*\log(\tan(dx + c) + I)/a^4 - 12*(-129*I*A + 31*B)*\log(\tan(dx + c) - I)/a^4 - 384*(4*I*A - B)*\log(\tan(dx + c))/a^4 - 384*(-4*I*A*\tan(dx + c) + B*\tan(dx + c) + A)/(a^4*\tan(dx + c)) - (3225*I*A*\tan(dx + c)^4 - 775*B*\tan(dx + c)^4 + 14076*A*\tan(dx + c)^3 + 3460*I*B*\tan(dx + c)^3 - 23286*I*A*\tan(dx + c)^2 + 5898*B*\tan(dx + c)^2 - 17404*A*\tan(dx + c) - 4612*I*B*\tan(dx + c) + 5017*I*A - 1447*B)/(a^4*(\tan(dx + c) - I)^4)/d$

**Mupad** [B]

time = 7.05, size = 226, normalized size = 1.03

$$\frac{\frac{A}{a^2} + \tan(c+dx) \left( \frac{65A}{16a^2} + \frac{B15i}{16a^2} \right) - \tan(c+dx)^2 \left( \frac{351A}{32a^2} + \frac{B63i}{16a^2} \right) - \tan(c+dx)^3 \left( -\frac{13B}{4a^2} + \frac{457i}{32a^2} \right) + \tan(c+dx) \left( -\frac{7B}{4a^2} + \frac{426i}{32a^2} \right) - \frac{\ln(\tan(c+dx))(-B+4A)}{a^4 d} - \frac{\ln(\tan(c+dx)+1)(B+A1i)}{32a^4 d} + \frac{\ln(\tan(c+dx)-1)(-31B+A129i)}{32a^4 d}}{d(\tan(c+dx)^5 - \tan(c+dx)^3 - 6\tan(c+dx)^2 + \tan(c+dx)^2 + \tan(c+dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out]  $(\log(\tan(c + d*x) - 1i)*(A*129i - 31*B))/(32*a^4*d) - (\log(\tan(c + d*x))*(A*4i - B))/(a^4*d) - (\log(\tan(c + d*x) + 1i)*(A*1i + B))/(32*a^4*d) - (\tan(c + d*x)^4*((65*A)/(16*a^4) + (B*15i)/(16*a^4)) - \tan(c + d*x)^3*((A*57i)/(4*a^4) - (13*B)/(4*a^4)) - \tan(c + d*x)^2*((851*A)/(48*a^4) + (B*63i)/(16*a^4)) + A/a^4 + \tan(c + d*x)*((A*26i)/(3*a^4) - (7*B)/(4*a^4)))/(d*(\tan(c + d*x) + \tan(c + d*x)^2*4i - 6*\tan(c + d*x)^3 - \tan(c + d*x)^4*4i + \tan(c + d*x)^5))$



$$3.66 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=255

$$\frac{5(35iA - 13B)x}{16a^4} + \frac{5(35iA - 13B) \cot(c + dx)}{16a^4d} - \frac{(11A + 4iB) \cot^2(c + dx)}{2a^4d} - \frac{(11A + 4iB) \log(\sin(c + dx))}{a^4d} +$$

[Out] 5/16\*(35\*I\*A-13\*B)\*x/a^4+5/16\*(35\*I\*A-13\*B)\*cot(d\*x+c)/a^4/d-1/2\*(11\*A+4\*I\*B)\*cot(d\*x+c)^2/a^4/d-(11\*A+4\*I\*B)\*ln(sin(d\*x+c))/a^4/d+1/48\*(43\*A+17\*I\*B)\*cot(d\*x+c)^2/a^4/d/(1+I\*tan(d\*x+c))^2+5/48\*(35\*A+13\*I\*B)\*cot(d\*x+c)^2/a^4/d/(1+I\*tan(d\*x+c))+1/8\*(A+I\*B)\*cot(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^4+1/6\*(2\*A+I\*B)\*cot(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^3

**Rubi** [A]

time = 0.54, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3610, 3612, 3556}

$$-\frac{(11A + 4iB) \cot^2(c + dx)}{2a^4d} + \frac{5(-13B + 35iA) \cot(c + dx)}{16a^4d} - \frac{(11A + 4iB) \log(\sin(c + dx))}{a^4d} + \frac{5(35A + 13iB) \cot^2(c + dx)}{48a^4d(1 + i \tan(c + dx))} + \frac{(43A + 17iB) \cot^2(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{5x(-13B + 35iA)}{16a^4} + \frac{(2A + iB) \cot^2(c + dx)}{6ad(a + ia \tan(c + dx))^3} + \frac{(A + iB) \cot^2(c + dx)}{8d(a + ia \tan(c + dx))^4}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] (5\*((35\*I)\*A - 13\*B)\*x)/(16\*a^4) + (5\*((35\*I)\*A - 13\*B)\*Cot[c + d\*x])/(16\*a^4\*d) - ((11\*A + (4\*I)\*B)\*Cot[c + d\*x]^2)/(2\*a^4\*d) - ((11\*A + (4\*I)\*B)\*Log[Sin[c + d\*x]])/(a^4\*d) + ((43\*A + (17\*I)\*B)\*Cot[c + d\*x]^2)/(48\*a^4\*d\*(1 + I\*Tan[c + d\*x])^2) + (5\*(35\*A + (13\*I)\*B)\*Cot[c + d\*x]^2)/(48\*a^4\*d\*(1 + I\*Tan[c + d\*x])) + ((A + I\*B)\*Cot[c + d\*x]^2)/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) + ((2\*A + I\*B)\*Cot[c + d\*x]^2)/(6\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a

\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx &= \frac{(A+iB) \cot^2(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{\int \frac{\cot^3(c+dx)(2a(5A+iB)-6a(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx}{8a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(2A+iB) \cot^2(c+dx)}{6ad(a+ia \tan(c+dx))^3} + \frac{\int \frac{\cot^3(c+dx)}{a+ia \tan(c+dx)} dx}{6ad(a+ia \tan(c+dx))^3} \\
 &= \frac{(43A+17iB) \cot^2(c+dx)}{48a^4d(1+i \tan(c+dx))^2} + \frac{(A+iB) \cot^2(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(2A+iB) \cot^2(c+dx)}{6ad(a+ia \tan(c+dx))^3} \\
 &= \frac{(43A+17iB) \cot^2(c+dx)}{48a^4d(1+i \tan(c+dx))^2} + \frac{(A+iB) \cot^2(c+dx)}{8d(a+ia \tan(c+dx))^4} + \frac{(2A+iB) \cot^2(c+dx)}{6ad(a+ia \tan(c+dx))^3} \\
 &= -\frac{(11A+4iB) \cot^2(c+dx)}{2a^4d} + \frac{(43A+17iB) \cot^2(c+dx)}{48a^4d(1+i \tan(c+dx))^2} + \frac{(A+iB) \cot^2(c+dx)}{8d(a+ia \tan(c+dx))^4} \\
 &= \frac{5(35iA-13B) \cot(c+dx)}{16a^4d} - \frac{(11A+4iB) \cot^2(c+dx)}{2a^4d} + \frac{(43A+17iB) \cot^2(c+dx)}{48a^4d(1+i \tan(c+dx))^2} \\
 &= \frac{5(35iA-13B)x}{16a^4} + \frac{5(35iA-13B) \cot(c+dx)}{16a^4d} - \frac{(11A+4iB) \cot^2(c+dx)}{2a^4d} \\
 &= \frac{5(35iA-13B)x}{16a^4} + \frac{5(35iA-13B) \cot(c+dx)}{16a^4d} - \frac{(11A+4iB) \cot^2(c+dx)}{2a^4d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1625 vs.  $2(255) = 510$ .  
time = 7.18, size = 1625, normalized size = 6.37

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] 
$$\begin{aligned} & (-3*(8*A + (5*I)*B)*\text{Cos}[4*d*x]*\text{Sec}[c + d*x]^3*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A \\ & + B*\text{Tan}[c + d*x]))/(32*d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + \\ & d*x])^4) + ((5*A + (4*I)*B)*\text{Cos}[6*d*x]*\text{Sec}[c + d*x]^3*(-1/48*\text{Cos}[2*c] + (I \\ & /48)*\text{Sin}[2*c])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c \\ & + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + ((25*A + (12*I)*B)*\text{Cos} \\ & [2*d*x]*\text{Sec}[c + d*x]^3*((-3*\text{Cos}[2*c])/16 - ((3*I)/16)*\text{Sin}[2*c])*(\text{Cos}[d*x] + \\ & I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*( \\ & a + I*a*\text{Tan}[c + d*x])^4) + (\text{Sec}[c + d*x]^3*(11*A*\text{Cos}[2*c] + (4*I)*B*\text{Cos}[2*c] \\ & ] + (11*I)*A*\text{Sin}[2*c] - 4*B*\text{Sin}[2*c])*(I*\text{ArcTan}[\text{Tan}[d*x]]*\text{Cos}[2*c] - \text{ArcTan} \\ & [\text{Tan}[d*x]]*\text{Sin}[2*c])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c + d*x]))/(d*(A* \\ & \text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + (\text{Sec}[c + d*x]^3* \\ & (11*A*\text{Cos}[2*c] + (4*I)*B*\text{Cos}[2*c] + (11*I)*A*\text{Sin}[2*c] - 4*B*\text{Sin}[2*c])*(-1/2 \\ & *(\text{Cos}[2*c]*\text{Log}[\text{Sin}[c + d*x]^2]) - (I/2)*\text{Log}[\text{Sin}[c + d*x]^2]*\text{Sin}[2*c])*(\text{Cos}[ \\ & d*x] + I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d \\ & *x])*(a + I*a*\text{Tan}[c + d*x])^4) + (x*\text{Sec}[c + d*x]^3*((33*I)*A*\text{Cos}[c]^2 - 12* \\ & B*\text{Cos}[c]^2 + 11*A*\text{Cos}[c]^2*\text{Cot}[c] + (4*I)*B*\text{Cos}[c]^2*\text{Cot}[c] - 33*A*\text{Cos}[c]*\text{S} \\ & \text{in}[c] - (12*I)*B*\text{Cos}[c]*\text{Sin}[c] - (11*I)*A*\text{Sin}[c]^2 + 4*B*\text{Sin}[c]^2 + (11*A + \\ & (4*I)*B)*\text{Cot}[c]*(-\text{Cos}[4*c] - I*\text{Sin}[4*c]))*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A + B \\ & * \text{Tan}[c + d*x]))/((A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4 \\ & ) + ((A + I*B)*\text{Cos}[8*d*x]*\text{Sec}[c + d*x]^3*(-1/128*\text{Cos}[4*c] + (I/128)*\text{Sin}[4*c] \\ & ]*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{S} \\ & \text{in}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + (\text{Csc}[c + d*x]^2*\text{Sec}[c + d*x]^3*(-1 \\ & /2*(A*\text{Cos}[4*c] - (I/2)*A*\text{Sin}[4*c])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c \\ & + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + ( \\ & (35*A + (13*I)*B)*\text{Sec}[c + d*x]^3*((5*I)/16)*d*x*\text{Cos}[4*c] - (5*d*x*\text{Sin}[4*c] \\ & )/16*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + \\ & B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + ((25*A + (12*I)*B)*\text{Sec}[c + d*x] \\ & ^3*((3*I)/16)*\text{Cos}[2*c] - (3*\text{Sin}[2*c])/16*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*\text{Sin}[2* \\ & d*x]*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Ta} \\ & n[c + d*x])^4) + (((3*I)/32)*(8*A + (5*I)*B)*\text{Sec}[c + d*x]^3*(\text{Cos}[d*x] + I*\text{S} \\ & \text{in}[d*x])^4*\text{Sin}[4*d*x]*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + \\ & d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + ((5*A + (4*I)*B)*\text{Sec}[c + d*x]^3*((I/48)*\text{C} \\ & \text{os}[2*c] + \text{Sin}[2*c]/48)*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^4*\text{Sin}[6*d*x]*(A + B*\text{Tan}[c + \\ & d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) + ((A \\ & + I*B)*\text{Sec}[c + d*x]^3*((I/128)*\text{Cos}[4*c] + \text{Sin}[4*c]/128)*(\text{Cos}[d*x] + I*\text{Sin}[ \\ & d*x])^4*\text{Sin}[8*d*x]*(A + B*\text{Tan}[c + d*x]))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x] \\ & ]*(a + I*a*\text{Tan}[c + d*x])^4) + (\text{Csc}[c]*\text{Csc}[c + d*x]*\text{Sec}[c + d*x]^3*(\text{Cos}[d*x] \\ & + I*\text{Sin}[d*x])^4*(2*A*\text{Cos}[4*c - d*x] + (I/2)*B*\text{Cos}[4*c - d*x] - 2*A*\text{Cos}[4* \\ & c + d*x] - (I/2)*B*\text{Cos}[4*c + d*x] + (2*I)*A*\text{Sin}[4*c - d*x] - (B*\text{Sin}[4*c - d \\ & *x])/2 - (2*I)*A*\text{Sin}[4*c + d*x] + (B*\text{Sin}[4*c + d*x])/2)*(A + B*\text{Tan}[c + d*x] \\ & ))/(d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^4) \end{aligned}$$

**Maple [A]**

time = 0.41, size = 174, normalized size = 0.68

method	result
derivativedivides	$-\frac{A}{2 \tan(dx+c)^2} + (-4iB - 11A) \ln(\tan(dx+c)) - \frac{-4iA+B}{\tan(dx+c)} - \frac{\frac{A}{2} + \frac{iB}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{49B}{16} - \frac{111iA}{16}}{\tan(dx+c)-i} - \frac{-\frac{31A}{8} - \frac{17iB}{8}}{2(\tan(dx+c)-i)^2} - \frac{-\frac{5B}{4} + \frac{7iA}{4}}{3(\tan(dx+c)-i)^3} - \frac{1}{d a^4}$
default	$-\frac{A}{2 \tan(dx+c)^2} + (-4iB - 11A) \ln(\tan(dx+c)) - \frac{-4iA+B}{\tan(dx+c)} - \frac{\frac{A}{2} + \frac{iB}{2}}{4(\tan(dx+c)-i)^4} - \frac{\frac{49B}{16} - \frac{111iA}{16}}{\tan(dx+c)-i} - \frac{-\frac{31A}{8} - \frac{17iB}{8}}{2(\tan(dx+c)-i)^2} - \frac{-\frac{5B}{4} + \frac{7iA}{4}}{3(\tan(dx+c)-i)^3} - \frac{1}{d a^4}$
risch	$-\frac{129xB}{16a^4} - \frac{ie^{-8i(dx+c)}B}{128da^4} + \frac{351ixA}{16a^4} - \frac{75e^{-2i(dx+c)}A}{16da^4} - \frac{ie^{-6i(dx+c)}B}{12da^4} - \frac{3e^{-4i(dx+c)}A}{4da^4} - \frac{15iBe^{-4i(dx+c)}}{32da^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/2*A/tan(d*x+c)^2+(-11*A-4*I*B)*ln(tan(d*x+c))-(-4*I*A+B)/tan(d*x+c)-1/4*(1/2*A+1/2*I*B)/(tan(d*x+c)-I)^4-(49/16*B-111/16*I*A)/(tan(d*x+c)-I)-1/2*(-31/8*A-17/8*I*B)/(tan(d*x+c)-I)^2-1/3*(-5/4*B+7/4*I*A)/(tan(d*x+c)-I)^3+(351/32*A+129/32*I*B)*ln(tan(d*x+c)-I)+(1/32*A-1/32*I*B)*ln(tan(d*x+c)+I))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 1.98, size = 253, normalized size = 0.99

$$\frac{72(-117A + 43B)dx^{22}e^{22ic} + 24(6(117A - 43B)dx + 171A + 68B)dx^{20}e^{20ic} + 12(6(-117A + 43B)dx - 532A - 193B)dx^{18}e^{18ic} + 8(158A + 67B)dx^{16}e^{16ic} + (211A + 119B)dx^{14}e^{14ic} + 2(17A + 13B)dx^{12}e^{12ic} + 384((11A + 4B)dx^{10}e^{10ic} - 2(11A + 4B)dx^{8}e^{8ic} + (11A + 4B)dx^{6}e^{6ic}) \ln\left(\frac{e^{2i(dx+c)} - 1}{e^{2i(dx+c)} + 1}\right) + 3A + 3iB}{384(e^{2i(dx+c)} - 1)^2 a^4 d e^{2i(dx+c)} + a^4 d e^{2i(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/384*(72*(-117*I*A + 43*B)*d*x*e^(12*I*d*x + 12*I*c) + 24*(6*(117*I*A - 43*B)*d*x + 171*A + 68*I*B)*e^(10*I*d*x + 10*I*c) + 12*(6*(-117*I*A + 43*B)*d*x - 532*A - 193*I*B)*e^(8*I*d*x + 8*I*c) + 8*(158*A + 67*I*B)*e^(6*I*d*x
```

+ 6\*I\*c) + (211\*A + 119\*I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*(17\*A + 13\*I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + 384\*((11\*A + 4\*I\*B)\*e^(12\*I\*d\*x + 12\*I\*c) - 2\*(11\*A + 4\*I\*B)\*e^(10\*I\*d\*x + 10\*I\*c) + (11\*A + 4\*I\*B)\*e^(8\*I\*d\*x + 8\*I\*c))\*log(e^(2\*I\*d\*x + 2\*I\*c) - 1) + 3\*A + 3\*I\*B)/(a^4\*d\*e^(12\*I\*d\*x + 12\*I\*c) - 2\*a^4\*d\*e^(10\*I\*d\*x + 10\*I\*c) + a^4\*d\*e^(8\*I\*d\*x + 8\*I\*c))

**Sympy** [A]

time = 0.83, size = 466, normalized size = 1.83

$$\frac{8A + 2iB + (-6Ae^{2ic} - 2iBe^{2ic})e^{2idx}}{a^4 d e^{4ix} e^{4idx} - 2a^4 d e^{2ix} e^{2idx} + a^4 d} + \begin{cases} \left( \frac{(-24576A^2 d^{12} e^{12ic} - 24576iA B d^{12} e^{12ic})e^{-4idx} + (-327680A^2 d^{12} e^{12ic} - 262144iA B d^{12} e^{12ic})e^{-6idx} + (-2359296A^2 d^{12} e^{12ic} - 1474560iA B d^{12} e^{12ic})e^{-8idx} + (-14745600A^2 d^{12} e^{12ic} - 7077888iA B d^{12} e^{12ic})e^{-10idx}}{3145728d^{16}} \right) & \text{for } a^{16} d^{16} e^{20ic} \neq 0 \\ x \left( -\frac{351iA - 129B}{16a^4} + \frac{1851iA^{16} + 150iA^{14}B + 48iA^{12}B^2 + 10iA^{10}B^3 + 1iA^{129}B^{16} - 72B^{16} - 30B^{14} - 8B^{12} - B}{16a^4} \right) & \text{otherwise} \end{cases} + \frac{x(351iA - 129B)}{16a^4} - \frac{(11A + 4iB) \log(e^{2idx} - e^{-2ic})}{a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

[Out] (8\*A + 2\*I\*B + (-6\*A\*exp(2\*I\*c) - 2\*I\*B\*exp(2\*I\*c))\*exp(2\*I\*d\*x))/(a\*\*4\*d\*exp(4\*I\*c)\*exp(4\*I\*d\*x) - 2\*a\*\*4\*d\*exp(2\*I\*c)\*exp(2\*I\*d\*x) + a\*\*4\*d) + Piecewise(((((-24576\*A\*a\*\*12\*d\*\*3\*exp(12\*I\*c) - 24576\*I\*B\*a\*\*12\*d\*\*3\*exp(12\*I\*c))\*exp(-8\*I\*d\*x) + (-327680\*A\*a\*\*12\*d\*\*3\*exp(14\*I\*c) - 262144\*I\*B\*a\*\*12\*d\*\*3\*exp(14\*I\*c))\*exp(-6\*I\*d\*x) + (-2359296\*A\*a\*\*12\*d\*\*3\*exp(16\*I\*c) - 1474560\*I\*B\*a\*\*12\*d\*\*3\*exp(16\*I\*c))\*exp(-4\*I\*d\*x) + (-14745600\*A\*a\*\*12\*d\*\*3\*exp(18\*I\*c) - 7077888\*I\*B\*a\*\*12\*d\*\*3\*exp(18\*I\*c))\*exp(-2\*I\*d\*x))\*exp(-20\*I\*c)/(3145728\*a\*\*16\*d\*\*4), Ne(a\*\*16\*d\*\*4\*exp(20\*I\*c), 0)), (x\*(-(351\*I\*A - 129\*B)/(16\*a\*\*4) + (351\*I\*A\*exp(8\*I\*c) + 150\*I\*A\*exp(6\*I\*c) + 48\*I\*A\*exp(4\*I\*c) + 10\*I\*A\*exp(2\*I\*c) + I\*A - 129\*B\*exp(8\*I\*c) - 72\*B\*exp(6\*I\*c) - 30\*B\*exp(4\*I\*c) - 8\*B\*exp(2\*I\*c) - B)\*exp(-8\*I\*c)/(16\*a\*\*4)), True)) + x\*(351\*I\*A - 129\*B)/(16\*a\*\*4) - (11\*A + 4\*I\*B)\*log(exp(2\*I\*d\*x) - exp(-2\*I\*c))/(a\*\*4\*d)

**Giac** [A]

time = 1.18, size = 228, normalized size = 0.89

$$\frac{\frac{12(A-B) \log(\tan(dx+c)) + 36(117A+43B) \log(\tan(dx+c)) - 384(11A+4iB) \log(\tan(dx+c))}{a^4} + \frac{192(83A \tan(dx+c)^2 + 12iB \tan(dx+c)^2 + 96A \tan(dx+c) - 2iB \tan(dx+c) - A)}{a^4 \tan(dx+c)^2} - \frac{8775A \tan(dx+c)^4 + 3225iB \tan(dx+c)^4 - 37764A \tan(dx+c)^3 + 14076iB \tan(dx+c)^3 - 61386A \tan(dx+c)^2 - 23286iB \tan(dx+c)^2 + 44804iA \tan(dx+c) - 17404B \tan(dx+c) + 12455A + 5017iB}{a^4 \tan(dx+c)^4}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] 1/384\*(12\*(A - I\*B)\*log(tan(d\*x + c) + I)/a^4 + 36\*(117\*A + 43\*I\*B)\*log(tan(d\*x + c) - I)/a^4 - 384\*(11\*A + 4\*I\*B)\*log(tan(d\*x + c))/a^4 + 192\*(33\*A\*tan(d\*x + c)^2 + 12\*I\*B\*tan(d\*x + c)^2 + 8\*I\*A\*tan(d\*x + c) - 2\*B\*tan(d\*x + c) - A)/(a^4\*tan(d\*x + c)^2) - (8775\*A\*tan(d\*x + c)^4 + 3225\*I\*B\*tan(d\*x + c)^4 - 37764\*I\*A\*tan(d\*x + c)^3 + 14076\*B\*tan(d\*x + c)^3 - 61386\*A\*tan(d\*x + c)^2 - 23286\*I\*B\*tan(d\*x + c)^2 + 44804\*I\*A\*tan(d\*x + c) - 17404\*B\*tan(d\*x + c) + 12455\*A + 5017\*I\*B)/(a^4\*(tan(d\*x + c) - I)^4)/d

**Mupad** [B]

time = 7.64, size = 251, normalized size = 0.98

$$\frac{\tan(c+dx)^4 \left( \frac{153A}{32a^4} + \frac{B}{4a^4} \right) + \tan(c+dx)^3 \left( -\frac{69B}{16a^4} + \frac{417iA}{32a^4} \right) - \tan(c+dx)^2 \left( \frac{373A}{128a^4} + \frac{B}{32a^4} \right) - \tan(c+dx) \left( \frac{851B}{32a^4} + \frac{4299iA}{32a^4} \right) - \frac{1}{2a^4} + \tan(c+dx) \left( -\frac{B}{2a^4} + \frac{42B}{32a^4} \right) - \frac{\ln(\tan(c+dx)) (11A+B+4i)}{a^4 d} + \frac{\ln(\tan(c+dx)+1) (A-B+1i)}{32a^4 d} + \frac{\ln(\tan(c+dx)-1) (351A+B+129B)}{32a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^3*(A + B*\tan(c + d*x))/(a + a*\tan(c + d*x)*1i)^4, x)$

[Out]  $(\tan(c + d*x)^4*((153*A)/(4*a^4) + (B*57i)/(4*a^4)) + \tan(c + d*x)^5*((A*175i)/(16*a^4) - (65*B)/(16*a^4)) - \tan(c + d*x)^2*((271*A)/(12*a^4) + (B*26i)/(3*a^4)) - \tan(c + d*x)^3*((A*2269i)/(48*a^4) - (851*B)/(48*a^4)) - A/(2*a^4) + \tan(c + d*x)*((A*2i)/a^4 - B/a^4))/(d*(\tan(c + d*x)^2 + \tan(c + d*x)^3*4i - 6*\tan(c + d*x)^4 - \tan(c + d*x)^5*4i + \tan(c + d*x)^6)) - (\log(\tan(c + d*x))*(11*A + B*4i))/(a^4*d) + (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(32*a^4*d) + (\log(\tan(c + d*x) - 1i)*(351*A + B*129i))/(32*a^4*d)$

### 3.67 $\int \tan^3(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=194

$$\frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - \frac{8(7A - iB) \sqrt{a + ia \tan(c + dx)}}{35d} + \frac{2(7A - iB) \tan^2(c + dx)}{7d}$$

[Out] (A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))\*2^(1/2)\*a^(1/2)/d-8/35\*(7\*A-I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d+2/35\*(7\*A-I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^2/d+2/7\*B\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3/d-2/105\*(7\*A-31\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(3/2)/a/d

**Rubi** [A]

time = 0.34, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3678, 3673, 3608, 3561, 212}

$$\frac{2(7A - iB) \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{35d} - \frac{2(7A - 31iB)(a + ia \tan(c + dx))^{3/2}}{105ad} - \frac{8(7A - iB) \sqrt{a + ia \tan(c + dx)}}{35d} + \frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2B \tan^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{7d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^3\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[2]\*Sqrt[a]\*(A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/d - (8\*(7\*A - I\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(35\*d) + (2\*(7\*A - I\*B)\*Tan[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(35\*d) + (2\*B\*Tan[c + d\*x]^3\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(7\*d) - (2\*(7\*A - (31\*I)\*B)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(105\*a\*d)

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist

$[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ !\text{LtQ}[m, 0]$

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)]))*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)]), x\_Symbol] :> \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(b*f*(m + 1)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{LeQ}[m, -1]$

### Rule 3678

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)]))*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] :> \text{Simp}[B*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^n/(f*(m + n))), x] + \text{Dist}[1/(a*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n - 1)}*\text{Simp}[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \tan^3(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{2B \tan^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{7d} + \frac{2 \int}{d} \\ &= \frac{2(7A - iB) \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{35d} \\ &= \frac{2(7A - iB) \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{35d} \\ &= -\frac{8(7A - iB) \sqrt{a + ia \tan(c + dx)}}{35d} + \frac{2(7A - iB)}{d} \\ &= -\frac{8(7A - iB) \sqrt{a + ia \tan(c + dx)}}{35d} + \frac{2(7A - iB)}{d} \\ &= \frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} \end{aligned}$$

**Mathematica [A]**



time = 2.95, size = 201, normalized size = 1.04

$$\frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) \left( \frac{\sqrt{2} (A - iB) \sinh^{-1}(e^{i(c+dx)})}{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \sqrt{1 + e^{2i(c+dx)}}} + \frac{2}{105} \sqrt{\sec(c + dx)} (-112A + 46iB + (-7iA - 46B) \tan(c + dx) + 3 \sec^2(c + dx)(7A - iB + 5B \tan(c + dx))) \right)}{d \sec^{\frac{3}{2}}(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
[Out] (sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x])*((sqrt[2]*(A - I*B)*ArcSin
h[E^(I*(c + d*x))])/(sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1
+ E^((2*I)*(c + d*x))]) + (2*sqrt[Sec[c + d*x]]*(-112*A + (46*I)*B + ((-7*
I)*A - 46*B)*Tan[c + d*x] + 3*Sec[c + d*x]^2*(7*A - I*B + 5*B*Tan[c + d*x]
))/105))/(d*Sec[c + d*x]^(3/2)*(A*cos[c + d*x] + B*sin[c + d*x]))
```

**Maple [A]**

time = 0.25, size = 162, normalized size = 0.84

method	result
derivativedivides	$2 \left( -\frac{iB(a+ia \tan(dx+c))^{7/2}}{7} + \frac{2iBa(a+ia \tan(dx+c))^{5/2}}{5} + \frac{Aa(a+ia \tan(dx+c))^{5/2}}{5} - \frac{2iB a^2(a+ia \tan(dx+c))^{3/2}}{3} - \frac{A a^2(a+ia \tan(dx+c))^{3/2}}{3} \right) / d a^3$
default	$2 \left( -\frac{iB(a+ia \tan(dx+c))^{7/2}}{7} + \frac{2iBa(a+ia \tan(dx+c))^{5/2}}{5} + \frac{Aa(a+ia \tan(dx+c))^{5/2}}{5} - \frac{2iB a^2(a+ia \tan(dx+c))^{3/2}}{3} - \frac{A a^2(a+ia \tan(dx+c))^{3/2}}{3} \right) / d a^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3*(A+B*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2/d/a^3*(-1/7*I*B*(a+I*a*tan(d*x+c))^(7/2)+2/5*I*B*a*(a+I*a*tan(d*x+c))^(5
/2)+1/5*A*a*(a+I*a*tan(d*x+c))^(5/2)-2/3*I*B*a^2*(a+I*a*tan(d*x+c))^(3/2)-1
/3*A*a^2*(a+I*a*tan(d*x+c))^(3/2)+A*a^3*(a+I*a*tan(d*x+c))^(1/2)-1/2*a^(7/2
)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

**Maxima [A]**

time = 0.49, size = 153, normalized size = 0.79

$$\frac{105 \sqrt{2} (A - iB) a^{\frac{3}{2}} \log \left( \frac{-\sqrt{2} \sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{ia \tan(dx+c) + a}} \right) - 60i (ia \tan(dx+c) + a)^{\frac{5}{2}} Ba + 84 (ia \tan(dx+c) + a)^{\frac{5}{2}} (A + 2iB) a^2 - 140 (ia \tan(dx+c) + a)^{\frac{3}{2}} (A + 2iB) a^3 + 420 \sqrt{ia \tan(dx+c) + a} A a^4}{210 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/210*(105*\sqrt{2}*(A - I*B)*a^{9/2}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a})/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a})) - 60*I*(I*a*\tan(d*x + c) + a)^{7/2}*B*a + 84*(I*a*\tan(d*x + c) + a)^{5/2}*(A + 2*I*B)*a^2 - 140*(I*a*\tan(d*x + c) + a)^{3/2}*(A + 2*I*B)*a^3 + 420*\sqrt{I*a*\tan(d*x + c) + a}*A*a^4)/(a^4*d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs.  $2(151) = 302$ .  
time = 1.20, size = 440, normalized size = 2.27

$$\frac{105\sqrt{2}(A^2-2IA^2B-B^2)\sqrt{a}\log\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{Ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{Ia\tan(dx+c)+a}}\right)-60I(Ia\tan(dx+c)+a)^{7/2}B+84(Ia\tan(dx+c)+a)^{5/2}(A+2IB)a^2-140(Ia\tan(dx+c)+a)^{3/2}(A+2IB)a^3+420\sqrt{Ia\tan(dx+c)+a}Aa^4}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $1/210*(105*\sqrt{2}*(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(A^2 - 2*I*A*B - B^2)*a/d^2}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} - (I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{(A^2 - 2*I*A*B - B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)/(I*A + B)} - 105*\sqrt{2}*(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(A^2 - 2*I*A*B - B^2)*a/d^2}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} - (-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{(A^2 - 2*I*A*B - B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)/(I*A + B)} - 4*\sqrt{2}*((119*A - 92*I*B)*e^{(7*I*d*x + 7*I*c)} + 7*(37*A - 16*I*B)*e^{(5*I*d*x + 5*I*c)} + 35*(7*A - 4*I*B)*e^{(3*I*d*x + 3*I*c)} + 105*A*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c+dx)-i)}(A+B\tan(c+dx))\tan^3(c+dx)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(1/2)\*tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3, x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

time = 1.27, size = 216, normalized size = 1.11

$$\frac{2A\sqrt{a+a\tan(c+dx)}\operatorname{li}}{d} + \frac{2A(a+a\tan(c+dx))^{3/2}}{3ad} - \frac{2A(a+a\tan(c+dx))^{5/2}}{5a^2d} + \frac{B(a+a\tan(c+dx))^{3/2}4i}{3ad} - \frac{B(a+a\tan(c+dx))^{5/2}4i}{5a^2d} + \frac{B(a+a\tan(c+dx))^{7/2}2i}{7a^3d} - \frac{\sqrt{2}B\sqrt{-a}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{-a}}\right)\operatorname{li}}{d} - \frac{\sqrt{2}A\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{a}}\right)\operatorname{li}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2*A*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a*d) - (2*A*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2*A*(a + a*tan(c + d*x)*1i)^(5/2))/(5*a^2*d) + (B*(a + a*tan(c + d*x)*1i)^(3/2)*4i)/(3*a*d) - (B*(a + a*tan(c + d*x)*1i)^(5/2)*4i)/(5*a^2*d) + (B*(a + a*tan(c + d*x)*1i)^(7/2)*2i)/(7*a^3*d) - (2^(1/2)*B*(-a)^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/d - (2^(1/2)*A*a^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/d
```

### 3.68 $\int \tan^2(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=143

$$\frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - \frac{8B \sqrt{a + ia \tan(c + dx)}}{5d} + \frac{2B \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{5d}$$

[Out] (I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))\*2^(1/2)\*a^(1/2)/d-8/5\*B\*(a+I\*a\*tan(d\*x+c))^(1/2)/d+2/5\*B\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^2/d-2/15\*(5\*I\*A+B)\*(a+I\*a\*tan(d\*x+c))^(3/2)/a/d

**Rubi [A]**

time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3678, 3673, 3608, 3561, 212}

$$-\frac{2(B+5iA)(a+ia \tan(c+dx))^{3/2}}{15ad} + \frac{\sqrt{2} \sqrt{a} (B+iA) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2B \tan^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{5d} - \frac{8B \sqrt{a+ia \tan(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[2]\*Sqrt[a]\*(I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/d - (8\*B\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(5\*d) + (2\*B\*Tan[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(5\*d) - (2\*((5\*I)\*A + B)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(15\*a\*d)

**Rule 212**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 3561**

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

**Rule 3608**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e,

$f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{!LtQ}[m, 0]$

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \rightarrow \text{Simp}[B*d*(a + b*\tan[e + f*x])^{(m + 1)/(b*f*(m + 1))}, x] + \text{Int}[(a + b*\tan[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\tan[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!LeQ}[m, -1]$

### Rule 3678

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[B*(a + b*\tan[e + f*x])^m*((c + d*\tan[e + f*x])^n/(f*(m + n))), x] + \text{Dist}[1/(a*(m + n)), \text{Int}[(a + b*\tan[e + f*x])^m*(c + d*\tan[e + f*x])^{(n - 1)}*\text{Simp}[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*\tan[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{2B \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{5d} + 2 \dots \\ &= \frac{2B \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{5d} - 2 \dots \\ &= -\frac{8B \sqrt{a + ia \tan(c + dx)}}{5d} + \frac{2B \tan^2(c + dx)}{5d} \\ &= -\frac{8B \sqrt{a + ia \tan(c + dx)}}{5d} + \frac{2B \tan^2(c + dx)}{5d} \\ &= \frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} \end{aligned}$$

### Mathematica [A]

time = 2.31, size = 184, normalized size = 1.29

$$\frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) \left( \frac{\sqrt{2} (iA + B) \sinh^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{1 + e^{2i(c + dx)}}} \right)}{\sqrt{1 + e^{2i(c + dx)}}} + \frac{2}{15} \sqrt{\sec(c + dx)} (-5iA - 16B + 3B \sec^2(c + dx) + (5A - iB) \tan(c + dx)) \right)}{d \sec^3(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

```
[Out] (Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x])*((Sqrt[2]*(I*A + B)*ArcSin
h[E^(I*(c + d*x))])/(Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))])*Sqrt[1
+ E^((2*I)*(c + d*x))]) + (2*Sqrt[Sec[c + d*x]]*((-5*I)*A - 16*B + 3*B*Sec
[c + d*x]^2 + (5*A - I*B)*Tan[c + d*x]))/15))/(d*Sec[c + d*x]^(3/2)*(A*Cos[
c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.11, size = 124, normalized size = 0.87

method	result
derivativedivides	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{iBa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - ia^2 B \sqrt{a + ia \tan(dx+c)} - \frac{a^{\frac{5}{2}}(-i)}{d a^2} \right)$
default	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{iBa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - ia^2 B \sqrt{a + ia \tan(dx+c)} - \frac{a^{\frac{5}{2}}(-i)}{d a^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2*(A+B*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -2*I/d/a^2*(-1/5*I*B*(a+I*a*tan(d*x+c))^(5/2)+1/3*I*B*a*(a+I*a*tan(d*x+c))^(
3/2)+1/3*A*a*(a+I*a*tan(d*x+c))^(3/2)-I*a^2*B*(a+I*a*tan(d*x+c))^(1/2)-1/2
*a^(5/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/
2)))
```

**Maxima [A]**

time = 0.53, size = 130, normalized size = 0.91

$$\frac{i \left( 15 \sqrt{2} (A - iB) a^{\frac{7}{2}} \log \left( \frac{-\sqrt{2} \sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{ia \tan(dx+c) + a}} \right) - 12i (ia \tan(dx+c) + a)^{\frac{5}{2}} Ba + 20 (ia \tan(dx+c) + a)^{\frac{3}{2}} (A + iB) a^2 - 60i \sqrt{ia \tan(dx+c) + a} Ba^3 \right)}{30 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^2*(A+B*tan(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] -1/30*I*(15*sqrt(2)*(A - I*B)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 12*I*(I*a*
```

$\tan(dx + c) + a)^{5/2} B a + 20(I a \tan(dx + c) + a)^{3/2} (A + I B) a^2 - 60 I \sqrt{I a \tan(dx + c) + a} B a^3 / (a^3 d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs.  $2(112) = 224$ .

time = 1.69, size = 383, normalized size = 2.68

$$\frac{15\sqrt{2}\sqrt{d^2(a^2+2d^2b^2)+d}\sqrt{\frac{d^2-2AB-B^2}{d^2}}\log\left(\frac{(-1+2d^2b^2)\sqrt{\frac{d^2-2AB-B^2}{d^2}}\sqrt{\frac{a}{2d^2b^2+1}}}{1+2d^2b^2}\right)-15\sqrt{2}\sqrt{d^2(a^2+2d^2b^2)+d}\sqrt{\frac{d^2-2AB-B^2}{d^2}}\log\left(\frac{(-1+2d^2b^2)\sqrt{\frac{d^2-2AB-B^2}{d^2}}\sqrt{\frac{a}{2d^2b^2+1}}}{1+2d^2b^2}\right)+4\sqrt{2}(10A+17B)\sqrt{d^2(a^2+2d^2b^2)+d}\sqrt{\frac{a}{2d^2b^2+1}}}{30(d^2(a^2+2d^2b^2)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(dx+c))^(1/2)\*tan(dx+c)^2\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out]  $-1/30*(15*\sqrt{2}*(d*e^{4*I*d*x + 4*I*c} + 2*d*e^{2*I*d*x + 2*I*c} + d)*\sqrt{- (A^2 - 2*I*A*B - B^2)*a/d^2}*\log(-4*((-I*A - B)*a*e^{I*d*x + I*c} + (d*e^{2*I*d*x + 2*I*c} + d)*\sqrt{- (A^2 - 2*I*A*B - B^2)*a/d^2}*\sqrt{a/(e^{2*I*d*x + 2*I*c} + 1)}))e^{-I*d*x - I*c}/(I*A + B) - 15*\sqrt{2}*(d*e^{4*I*d*x + 4*I*c} + 2*d*e^{2*I*d*x + 2*I*c} + d)*\sqrt{- (A^2 - 2*I*A*B - B^2)*a/d^2}*\log(-4*((-I*A - B)*a*e^{I*d*x + I*c} - (d*e^{2*I*d*x + 2*I*c} + d)*\sqrt{- (A^2 - 2*I*A*B - B^2)*a/d^2}*\sqrt{a/(e^{2*I*d*x + 2*I*c} + 1)}))e^{-I*d*x - I*c}/(I*A + B) + 4*\sqrt{2}*((10*I*A + 17*B)*e^{5*I*d*x + 5*I*c} + 10*(I*A + 2*B)*e^{3*I*d*x + 3*I*c} + 15*B*e^{I*d*x + I*c})*\sqrt{a/(e^{2*I*d*x + 2*I*c} + 1))}/(d*e^{4*I*d*x + 4*I*c} + 2*d*e^{2*I*d*x + 2*I*c} + d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(dx+c))^(1/2)\*tan(dx+c)^2\*(A+B\*tan(dx+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(dx+c))^(1/2)\*tan(dx+c)^2\*(A+B\*tan(dx+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 7.12, size = 168, normalized size = 1.17

$$\frac{2B\sqrt{a+a\tan(c+dx)}\operatorname{li}}{d} - \frac{A(a+a\tan(c+dx)\operatorname{li})^{3/2}2i}{3ad} + \frac{2B(a+a\tan(c+dx)\operatorname{li})^{3/2}}{3ad} - \frac{2B(a+a\tan(c+dx)\operatorname{li})^{3/2}}{5a^2d} + \frac{\sqrt{2}A\sqrt{-a}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{-a}}\right)\operatorname{li}}{d} - \frac{\sqrt{2}B\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{a}}\right)\operatorname{li}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*a\*d) - (A\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*2i)/(3\*a\*d) - (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d - (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/(5\*a^2\*d) + (2^(1/2)\*A\*(-a)^(1/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/d - (2^(1/2)\*B\*a^(1/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*1i)/(2\*a^(1/2)))\*1i)/d



### 3.69 $\int \tan(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=105

$$\frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2A \sqrt{a + ia \tan(c + dx)}}{d} - \frac{2iB(a + ia \tan(c + dx))^{3/2}}{3ad}$$

[Out]  $-(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(dx+c))^{1/2}*2^{1/2}/a^{1/2})*2^{1/2}*a^{1/2}/d+2*A*(a+I*a*\tan(dx+c))^{1/2}/d-2/3*I*B*(a+I*a*\tan(dx+c))^{3/2}/a/d$

**Rubi [A]**

time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3673, 3608, 3561, 212}

$$\frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2A \sqrt{a + ia \tan(c + dx)}}{d} - \frac{2iB(a + ia \tan(c + dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-\left(\frac{\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]}{d} + \frac{2*A*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}{d} - \frac{((2*I)/3)*B*(a + I*a*\operatorname{Tan}[c + d*x])^{3/2}}{a*d}\right)$

**Rule 212**

$\operatorname{Int}[(a + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a + (b_*)*\operatorname{tan}[(c_*) + (d_*)*(x_)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

**Rule 3608**

$\operatorname{Int}[(a + (b_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)])^{(m_*)}*((c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)])], x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Dist}[(b*c + a*d)/b, \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^m, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned} \int \tan(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= -\frac{2iB(a + ia \tan(c + dx))^{3/2}}{3ad} + \int \sqrt{a + ia \tan(c + dx)} dx \\ &= \frac{2A \sqrt{a + ia \tan(c + dx)}}{d} - \frac{2iB(a + ia \tan(c + dx))^{3/2}}{3ad} \\ &= \frac{2A \sqrt{a + ia \tan(c + dx)}}{d} - \frac{2iB(a + ia \tan(c + dx))^{3/2}}{3ad} \\ &= -\frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} \end{aligned}$$

## Mathematica [A]

time = 1.18, size = 132, normalized size = 1.26

$$\frac{e^{-i(c+dx)} \left( -4iB e^{3i(c+dx)} + 6A e^{i(c+dx)} (1 + e^{2i(c+dx)}) - 3(A - iB) (1 + e^{2i(c+dx)})^{3/2} \sinh^{-1}(e^{i(c+dx)}) \right) \sqrt{a + ia \tan(c + dx)}}{3d(1 + e^{2i(c+dx)})}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

```
[Out] (((-4*I)*B*E^((3*I)*(c + d*x)) + 6*A*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))) - 3*(A - I*B)*(1 + E^((2*I)*(c + d*x)))^(3/2)*ArcSinh[E^(I*(c + d*x))]) *Sqrt[a + I*a*Tan[c + d*x]]/(3*d*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x))))
```

## Maple [A]

time = 0.09, size = 82, normalized size = 0.78

method	result
derivativedivides	$-\frac{2iB(a+ia \tan(dx+c))^{3/2}}{3} + 2aA \sqrt{a + ia \tan(dx+c)} - a^{3/2}(-iB+A) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{a + ia \tan(dx+c)}}{2\sqrt{a}} \right)$

default	$\frac{-\frac{2iB(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + 2aA \sqrt{a + ia \tan(dx+c)} - a^{\frac{3}{2}}(-iB+A)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx+c)}}{2\sqrt{a}}\right)}{ad}$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] `2/d/a*(-1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)+a*A*(a+I*a*tan(d*x+c))^(1/2)-1/2*a  
^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)  
))`

**Maxima** [A]

time = 0.49, size = 107, normalized size = 1.02

$$\frac{3\sqrt{2}(A-iB)a^{\frac{5}{2}} \log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}}\right) - 4i(ia \tan(dx+c)+a)^{\frac{3}{2}}Ba + 12\sqrt{ia \tan(dx+c)+a}Aa^2}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] `1/6*(3*sqrt(2)*(A - I*B)*a^(5/2)*log(-sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +  
c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)) - 4*I*(I*a*tan(d*  
x + c) + a)^(3/2)*B*a + 12*sqrt(I*a*tan(d*x + c) + a)*A*a^2)/(a^2*d)`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(80) = 160.

time = 1.57, size = 332, normalized size = 3.16

$$\frac{3\sqrt{2}(a^{2iB+3iC+d})\sqrt{\frac{(A^2-2AB-B^2)a}{d^2}} \log\left(\frac{e^{\frac{(-1+iA-B)\sqrt{a^{2iB+3iC+d}}\sqrt{\frac{(A^2-2AB-B^2)a}{d^2}}\sqrt{\frac{a}{2iB+3iC+1}}}}}{iAB}\right) - 3\sqrt{2}(a^{2iB+3iC+d})\sqrt{\frac{(A^2-2AB-B^2)a}{d^2}} \log\left(\frac{e^{\frac{(-1-iA-B)\sqrt{a^{2iB+3iC+d}}\sqrt{\frac{(A^2-2AB-B^2)a}{d^2}}\sqrt{\frac{a}{2iB+3iC+1}}}}}{iAB}\right) - 4\sqrt{2}((3A-2B)a^{2iB+3iC} + 3Aa^{2iB+3iC})\sqrt{\frac{a}{2iB+3iC+1}}}{6(d^{2iB+3iC+d})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] `-1/6*(3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)  
*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (I*d*e^(2*I*d*x + 2*I*c) + I*d)*s  
qrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-I*d  
*x - I*c)/(I*A + B) - 3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*  
I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x  
+ 2*I*c) - I*d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I  
*c) + 1))))*e^(-I*d*x - I*c)/(I*A + B) - 4*sqrt(2)*((3*A - 2*I*B)*e^(3*I*d*  
x + 3*I*c) + 3*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(  
2*I*d*x + 2*I*c) + d)`

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c+dx) - i)} (A + B \tan(c+dx)) \tan(c+dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))\*\*(1/2)\*tan(d\*x+c)\*(A+B\*tan(d\*x+c)),x)**[Out]** Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*tan(c + d\*x), x)**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")**[Out]** Timed out**Mupad [B]**

time = 6.93, size = 120, normalized size = 1.14

$$\frac{2A\sqrt{a+a\tan(c+dx)}\operatorname{li}}{d} - \frac{B(a+a\tan(c+dx)\operatorname{li})^{3/2}2i}{3ad} + \frac{\sqrt{2}B\sqrt{-a}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{-a}}\right)\operatorname{li}}{d} - \frac{\sqrt{2}A\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

**[Out]** (2\*A\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d - (B\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*2i)/(3\*a\*d) + (2^(1/2)\*B\*(-a)^(1/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)))/(2\*(-a)^(1/2))\*1i)/d - (2^(1/2)\*A\*a^(1/2)\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/d

### 3.70 $\int \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2B \sqrt{a + ia \tan(c + dx)}}{d}$$

[Out]  $-(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*a^{(1/2)}/d+2*B*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

Rubi [A]

time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {3608, 3561, 212}

$$\frac{2B \sqrt{a + ia \tan(c + dx)}}{d} - \frac{\sqrt{2} \sqrt{a} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-\left(\left(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]]/\left(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]\right)\right)/d + \left(2*B*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]\right)/d$

Rule 212

$\operatorname{Int}[\left((a_) + (b_)*(x_)^2\right)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\left(1/\left(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]\right)\right)*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3561

$\operatorname{Int}[\operatorname{Sqrt}[(a_) + (b_)*\operatorname{tan}[(c_) + (d_)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

Rule 3608

$\operatorname{Int}[\left((a_) + (b_)*\operatorname{tan}[(e_) + (f_)*(x_)]\right)^{(m_)*\left((c_) + (d_)*\operatorname{tan}[(e_) + (f_)*(x_)]\right)}, x\_Symbol] \rightarrow \operatorname{Simp}[d*\left((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)\right), x] + \operatorname{Dist}[(b*c + a*d)/b, \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^m, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 + b^2, 0] \ \&\& \ !\operatorname{LtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{2B \sqrt{a + ia \tan(c + dx)}}{d} - (-A + iB) \int \sqrt{a + ia \tan(c + dx)} dx \\ &= \frac{2B \sqrt{a + ia \tan(c + dx)}}{d} - \frac{(2a(iA + B)) \text{Subst}\left(\int \frac{1}{2a-x^2} dx\right)}{a} \\ &= -\frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} + \frac{2}{d} \end{aligned}$$

**Mathematica [A]**

time = 1.12, size = 87, normalized size = 1.16

$$\frac{e^{-i(c+dx)} \left( 2B e^{i(c+dx)} - i(A - iB) \sqrt{1 + e^{2i(c+dx)}} \sinh^{-1} \left( e^{i(c+dx)} \right) \right) \sqrt{a + ia \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((2*B*E^(I*(c + d*x)) - I*(A - I*B)*Sqrt[1 + E^((2*I)*(c + d*x))])*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]]/(d*E^(I*(c + d*x)))
```

**Maple [A]**

time = 0.09, size = 63, normalized size = 0.84

method	result	size
derivativedivides	$2i \left( -iB \sqrt{a + ia \tan(dx + c)} - \frac{\sqrt{a}^{(-iB+A)} \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{2} \right) / d$	63
default	$2i \left( -iB \sqrt{a + ia \tan(dx + c)} - \frac{\sqrt{a}^{(-iB+A)} \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{2} \right) / d$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*(-I*B*(a+I*a*tan(d*x+c))^(1/2)-1/2*a^(1/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

**Maxima [A]**

time = 0.56, size = 87, normalized size = 1.16

$$\frac{i \left( \sqrt{2} (A - iB) a^{\frac{3}{2}} \log \left( -\frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}} \right) - 4i \sqrt{i a \tan(dx+c) + a} B a \right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

**[Out]** 1/2\*I\*(sqrt(2)\*(A - I\*B)\*a^(3/2)\*log(-(sqrt(2)\*sqrt(a) - sqrt(I\*a\*tan(d\*x + c) + a))/(sqrt(2)\*sqrt(a) + sqrt(I\*a\*tan(d\*x + c) + a))) - 4\*I\*sqrt(I\*a\*tan(d\*x + c) + a)\*B\*a)/(a\*d)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(59) = 118.

time = 1.50, size = 272, normalized size = 3.63

$$\frac{4\sqrt{2}B\sqrt{\frac{a}{d^{2d+2d+1}}}e^{i(dx+c)} + \sqrt{2}d\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}} \log \left( \frac{4^{(-1-A-B)d^{2d+1}}(a^{(2d+2d+1)d}\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}}\sqrt{\frac{a}{d^{2d+2d+1}+1}})^{d^{2d+1}}}{1+iB} \right) - \sqrt{2}d\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}} \log \left( \frac{4^{(-1-A-B)d^{2d+1}}(a^{(2d+2d+1)d}\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}}\sqrt{\frac{a}{d^{2d+2d+1}+1}})^{d^{2d+1}}}{1+iB} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

**[Out]** 1/2\*(4\*sqrt(2)\*B\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*e^(I\*d\*x + I\*c) + sqrt(2)\*d\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*log(-4\*((-I\*A - B)\*a\*e^(I\*d\*x + I\*c) + (d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/(I\*A + B) - sqrt(2)\*d\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*log(-4\*((-I\*A - B)\*a\*e^(I\*d\*x + I\*c) - (d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/(I\*A + B))/d

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c+dx) - i)} (A + B \tan(c+dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x)**[Out]** Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x)), x)**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 0.53, size = 96, normalized size = 1.28

$$\frac{2B\sqrt{a+a\tan(c+dx)}\operatorname{li}}{d} - \frac{\sqrt{2}A\sqrt{-a}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{-a}}\right)\operatorname{li}}{d} - \frac{\sqrt{2}B\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d - (2^(1/2)\*A\*(-a)^(1/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/d - (2^(1/2)\*B\*a^(1/2)\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/d



### 3.71 $\int \cot(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=86

$$-\frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d}$$

[Out]  $-2*A*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})*a^{(1/2)}/d+(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*a^{(1/2)}/d$

**Rubi** [A]

time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {3681, 3561, 212, 3680, 65, 214}

$$\frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} - \frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]),x]$

[Out]  $(-2*\operatorname{Sqrt}[a]*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/d + (\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{A \int \cot(c + dx) (a - ia \tan(c + dx)) \sqrt{a + ia \tan(c + dx)} dx}{a} \\
&= \frac{(aA) \text{Subst} \left( \int \frac{1}{x \sqrt{a + iax}} dx, x, \tan(c + dx) \right)}{d} \\
&= \frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} \\
&= -\frac{2\sqrt{a} A \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{d} + \dots
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 192 vs.  $2(86) = 172$ .  
time = 2.03, size = 192, normalized size = 2.23

$$\frac{e^{-(c+dx)} \sqrt{1 + e^{2(c+dx)}} \left( 2(A - iB) \sinh^{-1} \left( \frac{e^{(c+dx)}}{\sqrt{2}} \right) + \sqrt{2} A \left( \log(1 - e^{i(c+dx)}) - \log(1 + e^{i(c+dx)}) \right) + \log \left( 1 - e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2(c+dx)}} \right) - \log \left( 1 + e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2(c+dx)}} \right) \right)}{2d} \sqrt{a + ia \tan(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
[Out] (Sqrt[1 + E^((2*I)*(c + d*x))]*(2*(A - I*B)*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*A*(Log[1 - E^(I*(c + d*x))] - Log[1 + E^(I*(c + d*x))] + Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))] - Log[1 + E^(I*(c + d*x))] + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]))*Sqrt[a + I*a*Tan[c + d*x]])/(2*d*E^(I*(c + d*x)))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(68) = 136.

time = 0.89, size = 312, normalized size = 3.63

method	result
default	$\frac{\sqrt{\frac{(i \sin(dx+c) + \cos(dx+c))a}{\cos(dx+c)}} \sqrt{-\frac{2 \cos(dx+c)}{\cos(dx+c)+1}} \left( iA \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-\frac{2 \cos(dx+c)}{\cos(dx+c)+1}}}{2}\right) + iB \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}}{\dots}\right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/d*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(I*A*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+I*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+I*A*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+B*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-A*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c)))*sin(d*x+c)/(I*sin(d*x+c)+cos(d*x+c)-1)
```

**Maxima [A]**

time = 0.49, size = 113, normalized size = 1.31

$$\frac{\sqrt{2}(A - iB)\sqrt{a} \log\left(\frac{\sqrt{2}\sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2}\sqrt{a} + \sqrt{ia \tan(dx+c) + a}}\right) - 2A\sqrt{a} \log\left(\frac{\sqrt{ia \tan(dx+c) + a} - \sqrt{a}}{\sqrt{ia \tan(dx+c) + a} + \sqrt{a}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] -1/2*(sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 2*A*sqrt(a)*log(
```

$(\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a})))/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs.  $2(65) = 130$ .  
time = 1.42, size = 447, normalized size = 5.20

$$\frac{1}{2}\sqrt{\frac{A^2 - 2AB - B^2}{a}} \log\left(\frac{A(-1+A-2B\sqrt{A^2-2AB-B^2}) - (A^2-2AB-B^2)\sqrt{\frac{A^2-2AB-B^2}{a}}}{A+B}\right) - \frac{1}{2}\sqrt{\frac{A^2 - 2AB - B^2}{a}} \log\left(\frac{A(-1+A-2B\sqrt{A^2-2AB-B^2}) - (A^2-2AB-B^2)\sqrt{\frac{A^2-2AB-B^2}{a}}}{A+B}\right) - \frac{1}{2}\sqrt{\frac{A^2 - 2AB - B^2}{a}} \log\left(\frac{A(-1+A-2B\sqrt{A^2-2AB-B^2}) - (A^2-2AB-B^2)\sqrt{\frac{A^2-2AB-B^2}{a}}}{A+B}\right) - \frac{1}{2}\sqrt{\frac{A^2 - 2AB - B^2}{a}} \log\left(\frac{A(-1+A-2B\sqrt{A^2-2AB-B^2}) - (A^2-2AB-B^2)\sqrt{\frac{A^2-2AB-B^2}{a}}}{A+B}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2}\sqrt{2}\sqrt{(A^2 - 2IAB - B^2)a/d^2} \log(-4*((-IA - B)a e^{I d x + I c} - (I d e^{2I d x + 2I c} + I d) \sqrt{(A^2 - 2IAB - B^2)a/d^2}) \sqrt{a/(e^{2I d x + 2I c} + 1)}) e^{-I d x - I c} / (IA + B) - \frac{1}{2}\sqrt{2}\sqrt{(A^2 - 2IAB - B^2)a/d^2} \log(-4*((-IA - B)a e^{I d x + I c} - (I d e^{2I d x + 2I c} - I d) \sqrt{(A^2 - 2IAB - B^2)a/d^2}) \sqrt{a/(e^{2I d x + 2I c} + 1)}) e^{-I d x - I c} / (IA + B) - \frac{1}{2}\sqrt{A^2 a/d^2} \log(16*(3A a^2 e^{2I d x + 2I c} + A a^2 + 2\sqrt{2})(a d e^{3I d x + 3I c} + a d e^{I d x + I c})) \sqrt{A^2 a/d^2} \sqrt{a/(e^{2I d x + 2I c} + 1)}) e^{-2I d x - 2I c} / A + \frac{1}{2}\sqrt{A^2 a/d^2} \log(16*(3A a^2 e^{2I d x + 2I c} + A a^2 - 2\sqrt{2})(a d e^{3I d x + 3I c} + a d e^{I d x + I c})) \sqrt{A^2 a/d^2} \sqrt{a/(e^{2I d x + 2I c} + 1)}) e^{-2I d x - 2I c} / A$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*cot(c + d\*x), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")



### 3.72 $\int \cot^2(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=123

$$\frac{\sqrt{a} (iA + 2B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{d} + \frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - A \cot(c + dx) \sqrt{a + ia \tan(c + dx)}$$

[Out]  $-(I*A+2*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})*a^{(1/2)}/d+(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)*2^{(1/2)}/a^{(1/2)})*2^{(1/2)*a^{(1/2)}/d}-A*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.25, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3679, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{\sqrt{a} (2B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{d} + \frac{\sqrt{2} \sqrt{a} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - \frac{A \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]),x]$

[Out]  $-\left(\left(\operatorname{Sqrt}[a]*(I*A + 2*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]]\right)/d\right) + \left(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]\right)/d - \left(A*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]\right)/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_)^m)*((c_.) + (d_.)*(x_)^n), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= -\frac{A \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{\int \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)} dx}{d} \\
&= -\frac{A \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{d} + (-A) \frac{\int \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)} dx}{d} \\
&= -\frac{A \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{(2a) \int \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)} dx}{d} \\
&= \frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} \\
&= -\frac{\sqrt{a} (iA + 2B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 293 vs. 2(123) = 246.  
time = 4.18, size = 293, normalized size = 2.38

$$\frac{(-8A \cot(c + dx) + e^{-i(c + dx)} \sqrt{1 + e^{2i(c + dx)}}) (8(iA + B) \sinh^{-1}(e^{i(c + dx)}) + \sqrt{2} (iA + 2B) (\log((-1 + e^{i(c + dx)})^2) - \log((1 + e^{i(c + dx)})^2) + \log(3 + 3e^{2i(c + dx)} + 2\sqrt{2} \sqrt{1 + e^{2i(c + dx)}} - 2e^{i(c + dx)} (1 + \sqrt{2} \sqrt{1 + e^{2i(c + dx)}})) - \log(3 + 3e^{2i(c + dx)} + 2\sqrt{2} \sqrt{1 + e^{2i(c + dx)}} + 2e^{i(c + dx)} (1 + \sqrt{2} \sqrt{1 + e^{2i(c + dx)}}))))}{8d} \sqrt{a + ia \tan(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
[Out] ((-8*A*Cot[c + d*x] + (Sqrt[1 + E^((2*I)*(c + d*x))])*(8*(I*A + B)*ArcSinh[E^
(I*(c + d*x))] + Sqrt[2]*(I*A + 2*B)*(Log[(-1 + E^(I*(c + d*x)))^2] - Log[
(1 + E^(I*(c + d*x)))^2] + Log[3 + 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*Sqrt[1
+ E^((2*I)*(c + d*x))] - 2*E^(I*(c + d*x))*(1 + Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))]]) - Log[3 + 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*Sqrt[1 + E^((2*I)
*(c + d*x))] + 2*E^(I*(c + d*x))*(1 + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))
]])))/E^(I*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])/(8*d)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1178 vs. 2(101) = 202.  
time = 0.71, size = 1179, normalized size = 9.59

method	result	size
default	Expression too large to display	1179

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```



```
[Out] 1/2/d*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c))^(1/2)*(2*I*A*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)-2*I*B*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos
(d*x+c)*sin(d*x+c)-2*I*A*cos(d*x+c)^2+I*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d
*x+c))*sin(d*x+c)+2*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-2*I*
B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*sin(d*x+c)-2*I*A*cos(d*x+c)+2*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)+2*I*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+A
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)*sin(d*x+c)+I*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln
((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))
*cos(d*x+c)*sin(d*x+c)+2*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)+2*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)*sin(d*x
+c)-2*I*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+A*(-2*cos(d*x+c)/(cos(d*x+c)
)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+2*B*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-2*I*B*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)*sin(d*x+c)+2*A*cos(d*x+c)*sin(d*x+c))/(I*sin(d*x+c)+cos(
d*x+c)-1)/(cos(d*x+c)+1)
```

**Maxima [A]**

time = 0.58, size = 145, normalized size = 1.18

$$\frac{i \left( \frac{\sqrt{2}^{(A-iB) \log \left( -\frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}} \right)}}{\sqrt{a}} - \frac{(A-2iB) \log \left( \frac{\sqrt{i a \tan(dx+c) + a} - \sqrt{a}}{\sqrt{i a \tan(dx+c) + a} + \sqrt{a}} \right)}{\sqrt{a}} - \frac{2i \sqrt{i a \tan(dx+c) + a} A}{a \tan(dx+c)} \right) a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] -1/2*I*(sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
)))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - (A - 2*I*B)*lo
```

$g((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a}))/\sqrt{a} - 2*I*\sqrt{I*a*\tan(d*x + c) + a}*A/(a*\tan(d*x + c))*a/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 649 vs.  $2(95) = 190$ .

time = 1.75, size = 649, normalized size = 5.28



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-1/4*(2*\sqrt{2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{-(A^2 - 2*I*A*B - B^2)*a/d^2}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} + (d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(A^2 - 2*I*A*B - B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)/(I*A + B)} - 2*\sqrt{2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{-(A^2 - 2*I*A*B - B^2)*a/d^2}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} - (d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(A^2 - 2*I*A*B - B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)/(I*A + B)} - (d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{-(A^2 - 4*I*A*B - 4*B^2)*a/d^2}*\log(-16*(3*(-I*A - 2*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - 2*B)*a^2 + 2*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)}))\sqrt{-(A^2 - 4*I*A*B - 4*B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)/(I*A + 2*B)} + (d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{-(A^2 - 4*I*A*B - 4*B^2)*a/d^2}*\log(-16*(3*(-I*A - 2*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - 2*B)*a^2 - 2*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)}))\sqrt{-(A^2 - 4*I*A*B - 4*B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)/(I*A + 2*B)} + 4*\sqrt{2}*(I*A*e^{(3*I*d*x + 3*I*c)} + I*A*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(2*I*d*x + 2*I*c)} - d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^2, x)
```

**Mupad [B]**

time = 7.06, size = 168, normalized size = 1.37

$$\frac{\cot(c+dx) \left( A \sqrt{a+a \tan(c+dx)} \operatorname{II} + A \sqrt{a} \tan(c+dx) \operatorname{atanh}\left(\frac{\sqrt{a+a \tan(c+dx)} \operatorname{II}}{\sqrt{a}}\right) \operatorname{II} + 2 B \sqrt{a} \tan(c+dx) \operatorname{atanh}\left(\frac{\sqrt{a+a \tan(c+dx)} \operatorname{II}}{\sqrt{a}}\right) - \sqrt{2} A \sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx)} \operatorname{II}}{2 \sqrt{a}}\right) \tan(c+dx) \operatorname{II} - \sqrt{2} B \sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx)} \operatorname{II}}{2 \sqrt{a}}\right) \tan(c+dx) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] -(cot(c + d*x)*(A*(a + a*tan(c + d*x)*1i)^(1/2) + A*a^(1/2)*tan(c + d*x)*atanh((a + a*tan(c + d*x)*1i)^(1/2)/a^(1/2))*1i + 2*B*a^(1/2)*tan(c + d*x)*atanh((a + a*tan(c + d*x)*1i)^(1/2)/a^(1/2)) - 2^(1/2)*A*a^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2)))*tan(c + d*x)*1i - 2^(1/2)*B*a^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2)))*tan(c + d*x)))/d
```

### 3.73 $\int \cot^3(c+dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=169

$$\frac{\sqrt{a} (7A - 4iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{4d} - \frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - (iA$$

[Out] 1/4\*(7\*A-4\*I\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))\*a^(1/2)/d-(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))\*2^(1/2)\*a^(1/2)/d-1/4\*(I\*A+4\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-1/2\*A\*cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 0.38, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{\sqrt{a} (7A - 4iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{4d} - \frac{\sqrt{2} \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - \frac{(4B + iA) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} - \frac{A \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^3\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[a]\*(7\*A - (4\*I)\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]])/(4\*d) - (Sqrt[2]\*Sqrt[a]\*(A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/d - ((I\*A + 4\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(4\*d) - (A\*Cot[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(2\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 214**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[(((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)) / ((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
\int \cot^3(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx &= -\frac{A \cot^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{2d} + \frac{\int \cot^2(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx}{2d} \\
&= -\frac{(iA+4B) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4d} \\
&= -\frac{(iA+4B) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4d} \\
&= -\frac{(iA+4B) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4d} \\
&= -\frac{\sqrt{2} \sqrt{a} (A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\
&= \frac{\sqrt{a} (7A-4iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{4d}
\end{aligned}$$

**Mathematica [A]**

time = 3.26, size = 230, normalized size = 1.36

$$\frac{\left( \frac{-8\sqrt{2} (A-iB) \sinh^{-1}(e^{i(c+dx)}) + 2(7A-4iB) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}}} - \frac{2 \csc(c+dx) (2A \csc(c+dx) + (iA+4B) \sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)} \right) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx))}{8d \sec^{\frac{3}{2}}(c+dx) (A \cos(c+dx) + B \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^3\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

```
[Out] (((-8*Sqrt[2]*(A - I*B)*ArcSinh[E^(I*(c + d*x))] + 2*(7*A - (4*I)*B)*ArcTan
h[(Sqrt[2]*E^(I*(c + d*x))]/Sqrt[1 + E^((2*I)*(c + d*x))])]/(Sqrt[E^(I*(c +
d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))] - (2*Csc[c
+ d*x]*(2*A*Csc[c + d*x] + (I*A + 4*B)*Sec[c + d*x])/Sec[c + d*x]^(3/2))*
Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(8*d*Sec[c + d*x]^(3/2)*(A
*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2239 vs. 2(137) = 274.

time = 0.64, size = 2240, normalized size = 13.25

method	result	size
default	Expression too large to display	2240

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out] 
$$\begin{aligned} & -1/8/d*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*(-6*A*\cos(d*x+c)^2*\sin(d*x+c) \\ & -2*A*\cos(d*x+c)*\sin(d*x+c)+8*B*\cos(d*x+c)^3-8*B*\cos(d*x+c)-8*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)-8*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c) \\ & +8*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3+8 \\ & *I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^3+8 \\ & *I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2+8*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^2+6*I*A*\cos(d*x+c)^3+7*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-4*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})+4*I*A*\cos(d*x+c)^2-8*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^3+8*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3-8*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^2+8*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2+8*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)+7*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^3+4*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^3+7*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2+4*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2-8*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}-7*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)-8*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}-4*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)-8*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)-2*I*A*\cos(d*x+c)+8*I*B \end{aligned}$$

\*cos(d\*x+c)^2\*sin(d\*x+c)-7\*I\*A\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*arctan(1/(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2))-4\*I\*B\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*ln((sin(d\*x+c)\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)+1)/sin(d\*x+c))+8\*I\*B\*cos(d\*x+c)\*sin(d\*x+c)-7\*A\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*ln((sin(d\*x+c)\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)+1)/sin(d\*x+c))\*cos(d\*x+c)^3+4\*B\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*arctan(1/(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2))\*cos(d\*x+c)^3-7\*A\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*ln((sin(d\*x+c)\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)+1)/sin(d\*x+c))\*cos(d\*x+c)^2+4\*B\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*arctan(1/(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2))\*cos(d\*x+c)^2+8\*A\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*arctanh(1/2\*2^(1/2)\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2))\*sin(d\*x+c)/cos(d\*x+c))\*2^(1/2)+7\*A\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*ln((sin(d\*x+c)\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)+1)/sin(d\*x+c))\*cos(d\*x+c)-8\*B\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*arctan(1/2\*2^(1/2)\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2))\*2^(1/2)-4\*B\*(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*arctan(1/(-2\*cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2))\*cos(d\*x+c)/(I\*sin(d\*x+c)+cos(d\*x+c)-1)/sin(d\*x+c)/(cos(d\*x+c)+1)

**Maxima [A]**

time = 0.51, size = 202, normalized size = 1.20

$$a^2 \left( \frac{4\sqrt{2}^{(A-iB)\log\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}}{a^{\frac{3}{2}}} - \frac{(7A-4iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left((ia\tan(dx+c)+a)^{\frac{3}{2}(A-4iB)}+\sqrt{ia\tan(dx+c)+a}^{(A+4iB)a}\right)}{(ia\tan(dx+c)+a)^2a-2(ia\tan(dx+c)+a)a^2+a^3} \right)$$

8d

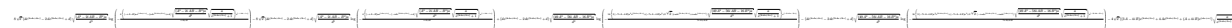
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/8\*a^2\*(4\*sqrt(2)\*(A - I\*B)\*log(-(sqrt(2)\*sqrt(a) - sqrt(I\*a\*tan(d\*x + c) + a))/(sqrt(2)\*sqrt(a) + sqrt(I\*a\*tan(d\*x + c) + a)))/a^(3/2) - (7\*A - 4\*I\*B)\*log((sqrt(I\*a\*tan(d\*x + c) + a) - sqrt(a))/(sqrt(I\*a\*tan(d\*x + c) + a) + sqrt(a)))/a^(3/2) + 2\*((I\*a\*tan(d\*x + c) + a)^(3/2)\*(A - 4\*I\*B) + sqrt(I\*a\*tan(d\*x + c) + a)\*(A + 4\*I\*B)\*a)/((I\*a\*tan(d\*x + c) + a)^2\*a - 2\*(I\*a\*tan(d\*x + c) + a)\*a^2 + a^3))/d

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(130) = 260.

time = 1.75, size = 730, normalized size = 4.32



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")



```
[Out] -1/16*(8*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt
((A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (I*d*e
^(2*I*d*x + 2*I*c) + I*d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/(I*A + B)) - 8*sqrt(2)*(d*e^(4*I*d*x +
4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*lo
g(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(
(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x -
I*c)/(I*A + B)) + (d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sq
rt((49*A^2 - 56*I*A*B - 16*B^2)*a/d^2)*log(-16*(3*(-7*I*A - 4*B)*a^2*e^(2*I
*d*x + 2*I*c) + (-7*I*A - 4*B)*a^2 + 2*sqrt(2)*(I*a*d*e^(3*I*d*x + 3*I*c) +
I*a*d*e^(I*d*x + I*c))*sqrt((49*A^2 - 56*I*A*B - 16*B^2)*a/d^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)))e^(-2*I*d*x - 2*I*c)/(7*I*A + 4*B)) - (d*e^(4*I*d*
x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt((49*A^2 - 56*I*A*B - 16*B^2)
*a/d^2)*log(-16*(3*(-7*I*A - 4*B)*a^2*e^(2*I*d*x + 2*I*c) + (-7*I*A - 4*B)*
a^2 + 2*sqrt(2)*(-I*a*d*e^(3*I*d*x + 3*I*c) - I*a*d*e^(I*d*x + I*c))*sqrt((
49*A^2 - 56*I*A*B - 16*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))e^(-2
*I*d*x - 2*I*c)/(7*I*A + 4*B)) - 4*sqrt(2)*((3*A - 4*I*B)*e^(5*I*d*x + 5*I*
c) + 4*A*e^(3*I*d*x + 3*I*c) + (A + 4*I*B)*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*cot(c + d*x)**3,
x)
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^3, x
)
```

**Mupad** [B]

time = 6.91, size = 702, normalized size = 4.15

$$\frac{\sqrt{ia} \sqrt{\tan(c + dx) - i} (A + B \tan(c + dx)) \cot^3(c + dx)}{\sqrt{ia} \sqrt{\tan(c + dx) - i} (A + B \tan(c + dx)) \cot^3(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^3*(A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*1i)^{(1/2)}, x)$

[Out] 
$$\begin{aligned} &(((A*a^2 + B*a^2*4i)*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(4*d) + ((A*a - B*a*4i) \\ &*(a + a*\tan(c + d*x)*1i)^{(3/2)})/(4*d))/((a + a*\tan(c + d*x)*1i)^2 - 2*a*(a \\ &+ a*\tan(c + d*x)*1i) + a^2) - (\text{atan}((17*A^3*a^4*d*(-a/2)^{(1/2)}*(a + a*\tan(c \\ &+ d*x)*1i)^{(1/2)})/(17*A^3*a^5*d - B^3*a^5*d*16i + 24*A*B^2*a^5*d - A^2*B*a \\ &^5*d*9i) - (B^3*a^4*d*(-a/2)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)}*16i)/(17*A \\ &^3*a^5*d - B^3*a^5*d*16i + 24*A*B^2*a^5*d - A^2*B*a^5*d*9i) + (24*A*B^2*a^4 \\ &*d*(-a/2)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(17*A^3*a^5*d - B^3*a^5*d*16 \\ &i + 24*A*B^2*a^5*d - A^2*B*a^5*d*9i) - (A^2*B*a^4*d*(-a/2)^{(1/2)}*(a + a*\tan \\ &(c + d*x)*1i)^{(1/2)}*9i)/(17*A^3*a^5*d - B^3*a^5*d*16i + 24*A*B^2*a^5*d - A^ \\ &2*B*a^5*d*9i))*(A*1i + B)*(-a/2)^{(1/2)}*2i)/d + ((-a)^{(1/2)}*\text{atan}((119*A^3*(- \\ &a)^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(4*((119*A^3*a^5*d)/4 - B^3*a^5*d \\ &*16i + 36*A*B^2*a^5*d - A^2*B*a^5*d*3i)) - (B^3*(-a)^{(9/2)}*d*(a + a*\tan(c + \\ &d*x)*1i)^{(1/2)}*16i)/((119*A^3*a^5*d)/4 - B^3*a^5*d*16i + 36*A*B^2*a^5*d - \\ &A^2*B*a^5*d*3i) + (36*A*B^2*(-a)^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)})/((1 \\ &19*A^3*a^5*d)/4 - B^3*a^5*d*16i + 36*A*B^2*a^5*d - A^2*B*a^5*d*3i) - (A^2*B \\ &*(-a)^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)}*3i)/((119*A^3*a^5*d)/4 - B^3*a^ \\ &5*d*16i + 36*A*B^2*a^5*d - A^2*B*a^5*d*3i))*(A*7i + 4*B)*1i)/(4*d) \end{aligned}$$

$$3.74 \quad \int \cot^4(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=210

$$\frac{\sqrt{a} (9iA + 14B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{8d} - \frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \dots$$

[Out] 1/8\*(9\*I\*A+14\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))\*a^(1/2)/d-(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))\*2^(1/2)\*a^(1/2)/d+1/8\*(7\*A-2\*I\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-1/12\*(I\*A+6\*B)\*cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-1/3\*A\*cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 0.50, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{\sqrt{a} (14B + 9iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{8d} - \frac{\sqrt{2} \sqrt{a} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} - \frac{(6B + iA) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{12d} + \frac{(7A - 2iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} - \frac{A \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^4\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[a]\*((9\*I)\*A + 14\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]])/(8\*d) - (Sqrt[2]\*Sqrt[a]\*(I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/d + ((7\*A - (2\*I)\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(8\*d) - ((I\*A + 6\*B)\*Cot[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(12\*d) - (A\*Cot[c + d\*x]^3\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= -\frac{A \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d} + \int \\
&= -\frac{(iA + 6B) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{12d} \\
&= \frac{(7A - 2iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
&= \frac{(7A - 2iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
&= \frac{(7A - 2iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
&= -\frac{\sqrt{2} \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} \\
&= \frac{\sqrt{a} (9iA + 14B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{8d}
\end{aligned}$$

**Mathematica [A]**

time = 4.41, size = 414, normalized size = 1.97

$$\frac{\left( \frac{2 \left( (iA + B) \operatorname{arcsinh}^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right) + (iA - 14B) \left( \operatorname{arcsinh}^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right) \right) - \operatorname{arcsinh}^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right) \right) + \operatorname{arcsinh}^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right) \right)}{\sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}}}} \frac{4 \operatorname{arcsinh}^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right) - 2(A + B) \operatorname{arcsinh}^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{64d \sec^3(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[Cot[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
[Out] ((((-2*I)*(32*Sqrt[2]*(A - I*B)*ArcSinh[E^(I*(c + d*x))] + (9*A - (14*I)*B)
*(Log[(-1 + E^(I*(c + d*x)))^2] - Log[(1 + E^(I*(c + d*x)))^2] + Log[3 + 3*
E^((2*I)*(c + d*x)) + 2*Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))] - 2*E^(I*(c +
d*x))*(1 + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) - Log[3 + 3*E^((2*I)*(c
+ d*x)) + 2*Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(1 +
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])))/(Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) - (4*Csc[c + d*x]^3*(-13*A
+ (6*I)*B + (29*A - (6*I)*B)*Cos[2*(c + d*x)] + 2*(I*A + 6*B)*Sin[2*(c + d
*x)])))/(3*Sqrt[Sec[c + d*x]])*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*
x]))/(64*d*Sec[c + d*x]^(3/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))

```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1782 vs. 2(172) = 344.

time = 0.51, size = 1783, normalized size = 8.49

method	result	size
default	Expression too large to display	1783

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/48/d*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c))^(1/2)*(42*A*cos(d*x+c)-12*
B*cos(d*x+c)*sin(d*x+c)+62*A*cos(d*x+c)^4-46*A*cos(d*x+c)^2+36*B*cos(d*x+c)
^3*sin(d*x+c)+27*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+42*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((s
in(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))-36
*I*B*cos(d*x+c)^4+12*I*B*cos(d*x+c)^3+36*I*B*cos(d*x+c)^2-12*I*B*cos(d*x+c)
-24*B*cos(d*x+c)^2*sin(d*x+c)-58*A*cos(d*x+c)^3-48*I*B*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))+48*B*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-
96*A*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-96*B*2^(1/2)*cos(d*x+c)^2*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+48*A*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))+27*I*A*cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*
x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))-42*I*B*
cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))-54*I*A*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d
*x+c))+84*I*B*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+48*I*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))+62*I*A*cos(d*x+c)^3*sin(d*x+c)-4*I*A*cos(d*x+c)^2*sin(d*x
+c)-42*I*A*cos(d*x+c)*sin(d*x+c)+27*I*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x
+c))-42*I*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))-54*A*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-84*B*cos(d*x+c)^2*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)-cos(d*x+c)+1)/sin(d*x+c))+48*A*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+48*B*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+27*A*cos(d*x+c)^4*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+42*B*
cos(d*x+c)^4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*
```

$$\begin{aligned} & x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))+48*I*A*2^{(1/2)}*\cos(d*x \\ & +c)^4*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+ \\ & c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))-48*I*B*2^{(1/2)}*\cos(d*x+c)^4 \\ & *(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)})-96*I*A*2^{(1/2)}*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+ \\ & 1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+ \\ & c)/\cos(d*x+c))+96*I*B*2^{(1/2)}*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{( \\ & 1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}))/ (I*\sin(d*x+c \\ & )+\cos(d*x+c)-1)/\sin(d*x+c)^3 \end{aligned}$$

**Maxima [A]**

time = 0.58, size = 249, normalized size = 1.19

$$i a^3 \left( \frac{2 \left( 3 (i a \tan(dx+c)+a)^{\frac{5}{2}} (7A-2iB) - 40 (i a \tan(dx+c)+a)^{\frac{3}{2}} A a + 3 \sqrt{i a \tan(dx+c)+a} (9A+2iB)a^2 \right)}{(i a \tan(dx+c)+a)^3 a^2 - 3 (i a \tan(dx+c)+a)^2 a^3 + 3 (i a \tan(dx+c)+a) a^4 - a^5} + \frac{24 \sqrt{2}^{(A-iB)} \log \left( -\frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c)+a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c)+a}} \right)}{a^{\frac{5}{2}}} - \frac{3^{(9A-14iB)} \log \left( \frac{\sqrt{i a \tan(dx+c)+a} - \sqrt{a}}{\sqrt{i a \tan(dx+c)+a} + \sqrt{a}} \right)}{a^{\frac{5}{2}}} \right) / 48 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/48\*I\*a^3\*(2\*(3\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*(7\*A - 2\*I\*B) - 40\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*A\*a + 3\*sqrt(I\*a\*tan(d\*x + c) + a)\*(9\*A + 2\*I\*B)\*a^2)/((I\*a\*tan(d\*x + c) + a)^3\*a^2 - 3\*(I\*a\*tan(d\*x + c) + a)^2\*a^3 + 3\*(I\*a\*tan(d\*x + c) + a)\*a^4 - a^5) + 24\*sqrt(2)\*(A - I\*B)\*log(-(sqrt(2)\*sqrt(a) - sqrt(I\*a\*tan(d\*x + c) + a))/(sqrt(2)\*sqrt(a) + sqrt(I\*a\*tan(d\*x + c) + a)))/a^(5/2) - 3\*(9\*A - 14\*I\*B)\*log((sqrt(I\*a\*tan(d\*x + c) + a) - sqrt(a))/(sqrt(I\*a\*tan(d\*x + c) + a) + sqrt(a)))/a^(5/2))/d

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 823 vs. 2(163) = 326.

time = 1.91, size = 823, normalized size = 3.92

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/96\*(48\*sqrt(2)\*(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*log(-4\*((-I\*A - B)\*a\*e^(I\*d\*x + I\*c) + (d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/(I\*A + B) - 4\*8\*sqrt(2)\*(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2)\*log(-4\*((-I\*A - B)\*a\*e^(I\*d\*x + I\*c) - (d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(A^2 - 2\*I\*A\*B - B^2)\*a/d^2

```
) * sqrt(a / (e^(2*I*d*x + 2*I*c) + 1))) * e^(-I*d*x - I*c) / (I*A + B)) - 3*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d) * sqrt(- (81*A^2 - 252*I*A*B - 196*B^2) * a / d^2) * log(-16*(3*(-9*I*A - 14*B) * a^2 * e^(2*I*d*x + 2*I*c) + (-9*I*A - 14*B) * a^2 + 2*sqrt(2) * (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))) * sqrt(- (81*A^2 - 252*I*A*B - 196*B^2) * a / d^2) * sqrt(a / (e^(2*I*d*x + 2*I*c) + 1))) * e^(-2*I*d*x - 2*I*c) / (9*I*A + 14*B)) + 3*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d) * sqrt(- (81*A^2 - 252*I*A*B - 196*B^2) * a / d^2) * log(-16*(3*(-9*I*A - 14*B) * a^2 * e^(2*I*d*x + 2*I*c) + (-9*I*A - 14*B) * a^2 - 2*sqrt(2) * (a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))) * sqrt(- (81*A^2 - 252*I*A*B - 196*B^2) * a / d^2) * sqrt(a / (e^(2*I*d*x + 2*I*c) + 1))) * e^(-2*I*d*x - 2*I*c) / (9*I*A + 14*B)) + 4*sqrt(2) * ((31*I*A + 18*B) * e^(7*I*d*x + 7*I*c) + (5*I*A + 6*B) * e^(5*I*d*x + 5*I*c) + (I*A - 18*B) * e^(3*I*d*x + 3*I*c) - 3*(-9*I*A + 2*B) * e^(I*d*x + I*c)) * sqrt(a / (e^(2*I*d*x + 2*I*c) + 1))) / (d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c+dx) - i)} (A + B \tan(c+dx)) \cot^4(c+dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*cot(c + d*x)**4, x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^4, x)
```

**Mupad [B]**

time = 7.09, size = 735, normalized size = 3.50

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}(\cot(c + d*x)^4*(A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*1i)^{(1/2)},x)$

[Out]  $(a^{(1/2)}*\text{atan}((423*A^3*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(8*((A^3*a^5*d*423i)/8 + 119*B^3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4)) - (B^3*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)}*119i)/((A^3*a^5*d*423i)/8 + 119*B^3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4) + (139*A*B^2*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(2*((A^3*a^5*d*423i)/8 + 119*B^3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4)) - (A^2*B*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)}*347i)/(4*((A^3*a^5*d*423i)/8 + 119*B^3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4)))*(A*9i + 14*B)*1i)/(8*d) - (\text{atan}((47*32^{(1/2)}*A^3*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(8*(A^3*a^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*a^5*d)) - (32^{(1/2)}*B^3*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)}*17i)/(2*(A^3*a^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*a^5*d)) + (8*32^{(1/2)}*A*B^2*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)})/(A^3*a^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*a^5*d) - (32^{(1/2)}*A^2*B*a^{(9/2)}*d*(a + a*\tan(c + d*x)*1i)^{(1/2)}*51i)/(8*(A^3*a^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*a^5*d)))*(A*1i + B)*(a/32)^{(1/2)}*8i)/d - (((9*A*a^3 + B*a^3*2i)*(a + a*\tan(c + d*x)*1i)^{(1/2)}*1i)/(8*d) + ((7*A*a - B*a*2i)*(a + a*\tan(c + d*x)*1i)^{(5/2)}*1i)/(8*d) - (A*a^2*(a + a*\tan(c + d*x)*1i)^{(3/2)}*5i)/(3*d))/(3*a*(a + a*\tan(c + d*x)*1i)^2 - 3*a^2*(a + a*\tan(c + d*x)*1i) - (a + a*\tan(c + d*x)*1i)^3 + a^3)$

### 3.75 $\int \tan^2(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=197

$$\frac{2\sqrt{2} a^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} - \frac{8a(7iA + 8B) \sqrt{a + ia \tan(c + dx)}}{35d} + \frac{2a(7iA + 8B) \tan^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{7d}$$

[Out]  $2a^{3/2}(IA+B)\operatorname{arctanh}(1/2(a+Ia*\tan(dx+c))^{1/2}*2^{1/2}/a^{1/2})*2^{1/2}/d-8/35*a*(7IA+8B)*(a+Ia*\tan(dx+c))^{1/2}/d+2/35*a*(7IA+8B)*(a+Ia*\tan(dx+c))^{1/2}*\tan(dx+c)^2/d+2/7*IA*B*(a+Ia*\tan(dx+c))^{1/2}*\tan(dx+c)^3/d-4/105*(21IA+19B)*(a+Ia*\tan(dx+c))^{3/2}/d$

**Rubi [A]**

time = 0.36, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3675, 3678, 3673, 3608, 3561, 212}

$$\frac{2\sqrt{2} a^{3/2}(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} + \frac{2a(8B+7iA) \tan^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{35d} - \frac{4(19B+21iA)(a+ia \tan(c+dx))^{3/2}}{105d} - \frac{8a(8B+7iA) \sqrt{a+ia \tan(c+dx)}}{35d} + \frac{2iaB \tan^3(c+dx) \sqrt{a+ia \tan(c+dx)}}{7d}$$

Antiderivative was successfully verified.

[In] `Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]`

[Out]  $(2*\text{Sqrt}[2]*a^{3/2}*(IA + B)*\text{ArcTanh}[\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]]/(\text{Sqrt}[2]*\text{Sqrt}[a]))/d - (8*a*((7*I)*A + 8*B)*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]/(35*d) + (2*a*((7*I)*A + 8*B)*\text{Tan}[c + d*x]^2*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]/(35*d) + ((2*I)/7)*a*B*\text{Tan}[c + d*x]^3*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]/d - (4*((21*I)*A + 19*B)*(a + I*a*\text{Tan}[c + d*x])^{3/2})/(105*d)$

**Rule 212**

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

**Rule 3561**

`Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2*(b/d), Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]`

**Rule 3608**

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist`

$[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)]), x\_Symbol] := \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1))), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rule 3675

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] := \text{Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{(m - 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)}/(d*f*(m + n))), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3678

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] := \text{Simp}[B*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^n/(f*(m + n))), x] + \text{Dist}[1/(a*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n - 1)}*\text{Simp}[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
\int \tan^2(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \frac{2iaB \tan^3(c+dx) \sqrt{a+ia \tan(c+dx)}}{7d} + \dots \\
&= \frac{2a(7iA+8B) \tan^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{35d} + \dots \\
&= \frac{2a(7iA+8B) \tan^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{35d} + \dots \\
&= -\frac{8a(7iA+8B) \sqrt{a+ia \tan(c+dx)}}{35d} + \frac{2a}{d} \\
&= -\frac{8a(7iA+8B) \sqrt{a+ia \tan(c+dx)}}{35d} + \frac{2a}{d} \\
&= \frac{2\sqrt{2} a^{3/2} (iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 3.80, size = 239, normalized size = 1.21

$$\frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) \left( \frac{-2\sqrt{2}(A+B) \tanh^{-1}\left(\frac{e^{i(c+dx)}}{1+i \tan(c+dx)}\right) - \frac{1}{210} \sec^5(c+dx) (21(17A-18iB) \cos(c+dx) + (147A-158iB) \cos(3(c+dx)) + 42iA \sin(c+dx) - 7B \sin(c+dx) + 42iA \sin(3(c+dx)) + 53B \sin(3(c+dx))) (i + \tan(c+dx))}{d \sec^5(c+dx) (A \cos(c+dx) + B \sin(c+dx))} \right)}{d \sec^5(c+dx) (A \cos(c+dx) + B \sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] ((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])*((2*Sqrt[2]*(I*A + B)*ArcSinh[E^(I*(c + d*x))])/(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(3/2)*(1 + E^((2*I)*(c + d*x)))^(3/2)) - (Sec[c + d*x]^(5/2)*(21*(17*A - (18*I)*B)*Cos[c + d*x] + (147*A - (158*I)*B)*Cos[3*(c + d*x)] + (42*I)*A*Sin[c + d*x] - 7*B*Sin[c + d*x] + (42*I)*A*Sin[3*(c + d*x)] + 53*B*Sin[3*(c + d*x)])*(I + Tan[c + d*x])/210))/(d*Sec[c + d*x]^(5/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.11, size = 164, normalized size = 0.83

method	result
derivativedivides	$ 2i \left( -\frac{iB(a+ia \tan(dx+c))^{7/2}}{7} + \frac{iBa(a+ia \tan(dx+c))^{5/2}}{5} + \frac{Aa(a+ia \tan(dx+c))^{5/2}}{5} - \frac{iB a^2(a+ia \tan(dx+c))^{3/2}}{3} - ia^3 B \sqrt{a+ia \tan(dx+c)} \right) $



$I*A - B)*a^2*e^{(I*d*x + I*c)} - \text{sqrt}(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\text{sqrt}(a/(e^{(2*I*d*x + 2*I*c)} + 1)))*e^{(-I*d*x - I*c)}/((-I*A - B)*a) + 2*\text{sqrt}(2)*((189*I*A + 211*B)*a*e^{(7*I*d*x + 7*I*c)} + 7*(57*I*A + 53*B)*a*e^{(5*I*d*x + 5*I*c)} + 35*(9*I*A + 11*B)*a*e^{(3*I*d*x + 3*I*c)} + 105*(I*A + B)*a*e^{(I*d*x + I*c)})*\text{sqrt}(a/(e^{(2*I*d*x + 2*I*c)} + 1)))/(d*e^{(6*I*d*x + 6*I*c)} + 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{3}{2}} (A + B \tan(c + dx)) \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 7.42, size = 211, normalized size = 1.07

$$\frac{2B(a + a \tan(c + dx))^{3/2}}{3d} - \frac{Aa \sqrt{a + a \tan(c + dx)}}{d} - \frac{2Ba \sqrt{a + a \tan(c + dx)}}{d} - \frac{A(a + a \tan(c + dx))^{5/2}}{5ad} + \frac{2B(a + a \tan(c + dx))^{3/2}}{5ad} - \frac{2B(a + a \tan(c + dx))^{1/2}}{7a^2d} - \frac{\sqrt{2} A(-a)^{3/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)}}{1 + \sqrt{-a}}\right)}{d} - \frac{\sqrt{2} B a^{3/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)}}{1 + \sqrt{-a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/(5\*a\*d) - (A\*a\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*2i)/d - (2\*B\*a\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d - (A\*(a + a\*tan(c + d\*x)\*1i)^(5/2)\*2i)/(5\*a\*d) - (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*d) - (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(7/2))/(7\*a^2\*d) - (2^(1/2)\*A\*(-a)^(3/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2))))\*2i)/d - (2^(1/2)\*B\*a^(3/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*1i)/(2\*a^(1/2))))\*2i)/d

### 3.76 $\int \tan(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=137

$$\frac{2\sqrt{2} a^{3/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a(A-iB)\sqrt{a+ia \tan(c+dx)}}{d} + \frac{2A(a+ia \tan(c+dx))^{3/2}}{3d}$$

[Out]  $-2*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+2*a*(A-I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d+2/3*A*(a+I*a*\tan(d*x+c))^{(3/2)}/d-2/5*I*B*(a+I*a*\tan(d*x+c))^{(5/2)}/a/d$

**Rubi [A]**

time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3673, 3608, 3559, 3561, 212}

$$\frac{2\sqrt{2} a^{3/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a(A-iB)\sqrt{a+ia \tan(c+dx)}}{d} + \frac{2A(a+ia \tan(c+dx))^{3/2}}{3d} - \frac{2iB(a+ia \tan(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c+d*x]*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-2*\operatorname{Sqrt}[2]*a^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]))/d+(2*a*(A-I*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d+(2*A*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})/(3*d)-(((2*I)/5)*B*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)})/(a*d)$

**Rule 212**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 3559**

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[b*((a+b*\operatorname{Tan}[c+d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[2*a, \operatorname{Int}[(a+b*\operatorname{Tan}[c+d*x])^{(n-1)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{EqQ}[a^2+b^2, 0] \&\& \operatorname{GtQ}[n, 1]$

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a-x^2), x], x, \operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]], x] /; \operatorname{FreeQ}[\{a,$

b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

### Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \tan(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= -\frac{2iB(a + ia \tan(c + dx))^{5/2}}{5ad} + \int (a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx \\ &= \frac{2A(a + ia \tan(c + dx))^{3/2}}{3d} - \frac{2iB(a + ia \tan(c + dx))^{5/2}}{5ad} \\ &= \frac{2a(A - iB) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2A(a + ia \tan(c + dx))^{3/2}}{3d} \\ &= \frac{2a(A - iB) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2A(a + ia \tan(c + dx))^{3/2}}{3d} \\ &= -\frac{2\sqrt{2} a^{3/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \end{aligned}$$

### Mathematica [A]

time = 3.69, size = 204, normalized size = 1.49

$$\frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) \left( -\frac{2\sqrt{2}(A - iB) \sinh^{-1}\left(\frac{e^{i(c + dx)}}{1 + e^{2i(c + dx)}}\right)}{(1 + e^{2i(c + dx)})^{3/2}} + \frac{1}{15} \sec^3(c + dx)(-20iA - 15B + (-20iA - 21B) \cos(2(c + dx)) + (5A - 6iB) \sin(2(c + dx))) \right)}{d \sec^{3/2}(c + dx)(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x])\*((-2\*Sqrt[2]\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))]]/(E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x))))^(3/2)



$(1 + E^{((2*I)*(c + d*x))}^{(3/2)}) + (\text{Sec}[c + d*x]^{(3/2)} * ((-20*I)*A - 15*B + ((-20*I)*A - 21*B)*\text{Cos}[2*(c + d*x)] + (5*A - (6*I)*B)*\text{Sin}[2*(c + d*x)]) * (I + \text{Tan}[c + d*x]) / 15) / (d * \text{Sec}[c + d*x]^{(5/2)} * (A * \text{Cos}[c + d*x] + B * \text{Sin}[c + d*x]))$

**Maple [A]**

time = 0.09, size = 123, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{2iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 2ia^2B\sqrt{a+ia \tan(dx+c)} + 2a^2A\sqrt{a+ia \tan(dx+c)}}{ad}$
default	$\frac{-\frac{2iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 2ia^2B\sqrt{a+ia \tan(dx+c)} + 2a^2A\sqrt{a+ia \tan(dx+c)}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE RBOSE)`

[Out]  $2/d/a * (-1/5 * I * B * (a + I * a * \tan(dx + c))^{(5/2)} + 1/3 * A * a * (a + I * a * \tan(dx + c))^{(3/2)} - I * a^2 * B * (a + I * a * \tan(dx + c))^{(1/2)} + a^2 * A * (a + I * a * \tan(dx + c))^{(1/2)} - a^{(5/2)} * (A - I * B) * 2^{(1/2)} * \text{arctanh}(1/2 * (a + I * a * \tan(dx + c))^{(1/2)} * 2^{(1/2)} / a^{(1/2)}))$

**Maxima [A]**

time = 0.50, size = 130, normalized size = 0.95

$$\frac{15\sqrt{2}(A-iB)a^{\frac{7}{2}}\log\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-6i(i\tan(dx+c)+a)^{\frac{5}{2}}Ba+10(i\tan(dx+c)+a)^{\frac{3}{2}}Aa^2+30\sqrt{ia\tan(dx+c)+a}(A-iB)a^3}{15a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/15 * (15 * \text{sqrt}(2) * (A - I * B) * a^{(7/2)} * \log(-(\text{sqrt}(2) * \text{sqrt}(a) - \text{sqrt}(I * a * \tan(dx + c) + a)) / (\text{sqrt}(2) * \text{sqrt}(a) + \text{sqrt}(I * a * \tan(dx + c) + a))) - 6 * I * (I * a * \tan(dx + c) + a)^{(5/2)} * B * a + 10 * (I * a * \tan(dx + c) + a)^{(3/2)} * A * a^2 + 30 * \text{sqrt}(I * a * \tan(dx + c) + a) * (A - I * B) * a^3) / (a^2 * d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 415 vs.  $2(104) = 208$ .

time = 3.12, size = 415, normalized size = 3.03

$$\frac{15\sqrt{2}\sqrt{\frac{A^2-2iAB-8B^2}{d^2}}(d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d)\log\left(\frac{(-1+2i\sqrt{2}\sqrt{\frac{A^2-2iAB-8B^2}{d^2}})\sqrt{\frac{A^2-2iAB-8B^2}{d^2}}\sqrt{\frac{a}{2d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d}}}{(-1+2i\sqrt{2}\sqrt{\frac{A^2-2iAB-8B^2}{d^2}})\sqrt{\frac{A^2-2iAB-8B^2}{d^2}}\sqrt{\frac{a}{2d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d}}}\right)-15\sqrt{2}\sqrt{\frac{A^2-2iAB-8B^2}{d^2}}(d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d)\log\left(\frac{(-1+2i\sqrt{2}\sqrt{\frac{A^2-2iAB-8B^2}{d^2}})\sqrt{\frac{A^2-2iAB-8B^2}{d^2}}\sqrt{\frac{a}{2d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d}}}{(-1+2i\sqrt{2}\sqrt{\frac{A^2-2iAB-8B^2}{d^2}})\sqrt{\frac{A^2-2iAB-8B^2}{d^2}}\sqrt{\frac{a}{2d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d}}}\right)-2\sqrt{2}(25A-2iB)\text{sqrt}(i^2e^{2i(d^2x+cd)}+1)+10(4A-3iB)\text{sqrt}(i^2e^{2i(d^2x+cd)}+1)+15(A-iB)\text{sqrt}(i^2e^{2i(d^2x+cd)}+1)\sqrt{\frac{a}{2d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d}}}{15(d^2i^2e^{2i(d^2x+cd)}+2id^2e^{i(d^2x+cd)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/15*(15*\sqrt{2}*\sqrt{(A^2 - 2*I*A*B - B^2)*a^3/d^2}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^2*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^3/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a)) - 15*\sqrt{2}*\sqrt{(A^2 - 2*I*A*B - B^2)*a^3/d^2}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^2*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^3/d^2}*(-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a)) - 2*\sqrt{2}*((25*A - 27*I*B)*a*e^{(5*I*d*x + 5*I*c)} + 10*(4*A - 3*I*B)*a*e^{(3*I*d*x + 3*I*c)} + 15*(A - I*B)*a*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{3}{2}} (A + B \tan(c + dx)) \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))^(3/2)\*(A + B\*tan(c + d\*x))\*tan(c + d\*x), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*tan(d\*x + c), x)

**Mupad [B]**

time = 6.93, size = 163, normalized size = 1.19

$$\frac{2A(a+a\tan(c+dx)\operatorname{li})^{3/2}}{3d} + \frac{2Aa\sqrt{a+a\tan(c+dx)\operatorname{li}}}{d} - \frac{Ba\sqrt{a+a\tan(c+dx)\operatorname{li}}}{d} \operatorname{li} 2i - \frac{B(a+a\tan(c+dx)\operatorname{li})^{3/2}}{5ad} \operatorname{li} 2i - \frac{\sqrt{2}B(-a)^{3/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)\operatorname{li}}}{2\sqrt{-a}}\right)}{d} \operatorname{li} 2i - \frac{2\sqrt{2}Aa^{3/2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)\operatorname{li}}}{2\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*li)^(3/2),x)

```
[Out] (2*A*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) + (2*A*a*(a + a*tan(c + d*x)*1i)^(1/2))/d - (B*a*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/d - (B*(a + a*tan(c + d*x)*1i)^(5/2)*2i)/(5*a*d) - (2^(1/2)*B*(-a)^(3/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*2i)/d - (2*2^(1/2)*A*a^(3/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/d
```

### 3.77 $\int (a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=107

$$\frac{2\sqrt{2} a^{3/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2a(iA + B) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2B(a + ia \tan(c + dx))^{3/2}}{3d}$$

[Out]  $-2*a^{(3/2)}*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+2*a*(I*A+B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d+2/3*B*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3608, 3559, 3561, 212}

$$\frac{2\sqrt{2} a^{3/2} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{2a(B + iA) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2B(a + ia \tan(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(-2*\operatorname{Sqrt}[2]*a^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + (2*a*(I*A + B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d + (2*B*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d)$

**Rule 212**

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

**Rule 3559**

$\operatorname{Int}[(a_) + (b_)*\operatorname{tan}[(c_) + (d_)*(x_)])^{(n_)}, x\_Symbol] \rightarrow \operatorname{Simp}[b*((a + b*\operatorname{Tan}[c + d*x])^{(n-1)}/(d*(n-1))), x] + \operatorname{Dist}[2*a, \operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^{(n-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 + b^2, 0] \ \&\& \operatorname{GtQ}[n, 1]$

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a_) + (b_)*\operatorname{tan}[(c_) + (d_)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

## Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

## Rubi steps

$$\begin{aligned}
 \int (a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx)) dx &= \frac{2B(a + ia \tan(c + dx))^{3/2}}{3d} - (-A + iB) \int (a + ia \tan(c + dx))^{3/2} dx \\
 &= \frac{2a(iA + B) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2B(a + ia \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{2a(iA + B) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2B(a + ia \tan(c + dx))^{3/2}}{3d} \\
 &= -\frac{2\sqrt{2} a^{3/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d}
 \end{aligned}$$

## Mathematica [A]

time = 2.61, size = 190, normalized size = 1.78

$$\frac{(a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx)) \left( -\frac{2i\sqrt{2} (A - iB) \sinh^{-1} \left( \frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)}{\left( \frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{3/2} (1 + e^{2i(c+dx)})^{3/2}} + \frac{2}{3} \sqrt{\sec(c + dx)} (\cos(c) - i \sin(c)) (i \cos(dx) + \sin(dx)) (3A - 4iB + B \tan(c + dx)) \right)}{d \sec^{3/2}(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x])\*(((2\*I)\*Sqrt[2]\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))])/(E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x))))^(3/2)\*(1 + E^((2\*I)\*(c + d\*x)))^(3/2)) + (2\*Sqrt[Sec[c + d\*x]]\*(Cos[c] - I\*Sin[c])\*(I\*Cos[d\*x] + Sin[d\*x])\*(3\*A - (4\*I)\*B + B\*Tan[c + d\*x]))/3)/(d\*Sec[c + d\*x]^(5/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

## Maple [A]

time = 0.09, size = 99, normalized size = 0.93

method	result
derivativedivides	$  \frac{2i \left( -\frac{iB(a + ia \tan(dx + c))^{3/2}}{3} - iaB \sqrt{a + ia \tan(dx + c)} + aA \sqrt{a + ia \tan(dx + c)} - a^{3/2} (-iB + A) \sqrt{2} \right)}{d}  $

default	$\frac{2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - iaB \sqrt{a+ia \tan(dx+c)} + aA \sqrt{a+ia \tan(dx+c)} - a^{\frac{3}{2}}(-iB+A) \sqrt{2} \right)}{d}$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $2*I/d*(-1/3*I*B*(a+I*a*\tan(d*x+c))^{3/2}-I*a*B*(a+I*a*\tan(d*x+c))^{1/2}+a*A*(a+I*a*\tan(d*x+c))^{1/2}-a^{3/2}*(A-I*B)*2^{1/2}*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{1/2}*2^{1/2}/a^{1/2}))$

**Maxima** [A]

time = 0.51, size = 111, normalized size = 1.04

$$\frac{i \left( 3\sqrt{2}(A-iB)a^{\frac{5}{2}} \log \left( -\frac{\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}} \right) - 2i(ia \tan(dx+c)+a)^{\frac{3}{2}}Ba + 6\sqrt{ia \tan(dx+c)+a}(A-iB)a^2 \right)}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/3*I*(3*\sqrt{2}*(A-I*B)*a^{5/2}*\log(-(\sqrt{2}*\sqrt{a}-\sqrt{I*a*\tan(d*x+c)+a})/(\sqrt{2}*\sqrt{a}+\sqrt{I*a*\tan(d*x+c)+a}))-2*I*(I*a*\tan(d*x+c)+a)^{3/2}*B*a+6*\sqrt{I*a*\tan(d*x+c)+a}*(A-I*B)*a^2)/(a*d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 359 vs.  $2(82) = 164$ .

time = 2.63, size = 359, normalized size = 3.36

$$\frac{3\sqrt{2}\sqrt{\frac{A^2-2AB-B^2}{d^2}}(d^{2d+2d+2d}+d)\log\left(\frac{\left(\frac{(-1+A-B)a^{2d+2d+2d}\sqrt{\frac{(A-2AB-B^2)a^2}{d^2}}(d^{2d+2d+2d}+d)\sqrt{\frac{a}{d^{2d+2d+2d}+1}}\right)^{d^{2d+2d+2d}}}{(-1+A-B)}}{-3\sqrt{2}\sqrt{\frac{(A-2AB-B^2)a^2}{d^2}}(d^{2d+2d+2d}+d)\log\left(\frac{\left(\frac{(-1+A-B)a^{2d+2d+2d}\sqrt{\frac{(A-2AB-B^2)a^2}{d^2}}(d^{2d+2d+2d}+d)\sqrt{\frac{a}{d^{2d+2d+2d}+1}}\right)^{d^{2d+2d+2d}}}{(-1+A-B)}}{-2\sqrt{2}((-3A-5B)a^{2d+2d}+3(-1+A-B)a^{d^{2d+2d}})\sqrt{\frac{a}{d^{2d+2d+2d}+1}}}\right)}{3(d^{2d+2d+2d}+d)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/3*(3*\sqrt{2}*\sqrt{-(A^2-2I*AB-B^2)}*a^3/d^2)*(d*e^{(2*I*d*x+2*I*c)}+d)*\log(4*((-I*A-B)*a^2*e^{(I*d*x+I*c)}+\sqrt{-(A^2-2I*AB-B^2)}*a^{3/d^2}*(d*e^{(2*I*d*x+2*I*c)}+d)*\sqrt{a/(e^{(2*I*d*x+2*I*c)}+1)}))e^{(-I*d*x-I*c)/((-I*A-B)*a)}-3*\sqrt{2}*\sqrt{-(A^2-2I*AB-B^2)}*a^3/d^2*(d*e^{(2*I*d*x+2*I*c)}+d)*\log(4*((-I*A-B)*a^2*e^{(I*d*x+I*c)}-\sqrt{-(A^2-2I*AB-B^2)}*a^3/d^2)*(d*e^{(2*I*d*x+2*I*c)}+d)*\sqrt{a/(e^{(2*I*d*x+2*I*c)}+1)}))e^{(-I*d*x-I*c)/((-I*A-B)*a)}-2*\sqrt{2}*((-3*I*A-5*B)*a*e^{(3*I*d*x+3*I*c)}+3*(-I*A-B)*a*e^{(I*d*x+I*c)})*\sqrt{a/(e^{(2*I*d*x+2*I*c)}+1)})/(d*e^{(2*I*d*x+2*I*c)}+d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B \tan(c+dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*(A + B\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 6.64, size = 139, normalized size = 1.30

$$\frac{2B(a+a\tan(c+dx))^{3/2}}{3d} + \frac{Aa\sqrt{a+a\tan(c+dx)}\operatorname{li}^2i}{d} + \frac{2Ba\sqrt{a+a\tan(c+dx)}\operatorname{li}}{d} + \frac{\sqrt{2}A(-a)^{3/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{-a}}\right)^{2i}}{d} - \frac{2\sqrt{2}Ba^{3/2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}\operatorname{li}}{2\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*d) + (A\*a\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*2i)/d + (2\*B\*a\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d + (2^(1/2)\*A\*(-a)^(3/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2))))\*2i)/d - (2\*2^(1/2)\*B\*a^(3/2)\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/d

### 3.78 $\int \cot(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=113

$$\frac{2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{2}a^{3/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2iaB\sqrt{a+ia \tan(c+dx)}}{d}$$

[Out]  $-2a^{(3/2)}A \operatorname{arctanh}\left(\frac{(a+Ia \tan(dx+c))^{(1/2)}/a^{(1/2)}}{d}\right) + 2a^{(3/2)}(A-I*B) \operatorname{arctanh}\left(\frac{1/2*(a+Ia \tan(dx+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)}}{d}\right) + 2Ia*B \operatorname{arctanh}\left(\frac{(a+Ia \tan(dx+c))^{(1/2)}}{d}\right)$

**Rubi [A]**

time = 0.25, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3675, 3681, 3561, 212, 3680, 65, 214}

$$\frac{2\sqrt{2}a^{3/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2iaB\sqrt{a+ia \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-2a^{(3/2)}A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/d + (2*\operatorname{Sqrt}[2]*a^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + ((2*I)*a*B*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$



Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
  n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
  e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
  a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
  b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
  A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
  d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
  [e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \frac{2iaB \sqrt{a+ia \tan(c+dx)}}{d} + 2 \int \cot(c+dx) \sqrt{a+ia \tan(c+dx)} dx \\
&= \frac{2iaB \sqrt{a+ia \tan(c+dx)}}{d} + A \int \cot(c+dx) \sqrt{a+ia \tan(c+dx)} dx \\
&= \frac{2iaB \sqrt{a+ia \tan(c+dx)}}{d} + \frac{(a^2 A) \operatorname{Subst}\left(\int \cot(u) \sqrt{a+ia \tan(u)} du\right)}{d} \\
&= \frac{2\sqrt{2} a^{3/2} (A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\
&= -\frac{2a^{3/2} A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \dots
\end{aligned}$$

**Mathematica [A]**

time = 2.05, size = 157, normalized size = 1.39

$$\frac{\sqrt{2} a e^{-i(c+dx)} \left( i\sqrt{2} B e^{i(c+dx)} + \sqrt{2} (A-iB) \sqrt{1+e^{2i(c+dx)}} \sinh^{-1}\left(e^{i(c+dx)}\right) - A \sqrt{1+e^{2i(c+dx)}} \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) \right) \sqrt{a+ia \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[2]\*a\*(I\*Sqrt[2]\*B\*E^(I\*(c + d\*x)) + Sqrt[2]\*(A - I\*B)\*Sqrt[1 + E^((2\*I)\*(c + d\*x))])\*ArcSinh[E^(I\*(c + d\*x))] - A\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcTanH[(Sqrt[2]\*E^(I\*(c + d\*x)))/Sqrt[1 + E^((2\*I)\*(c + d\*x))]])\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(d\*E^(I\*(c + d\*x)))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(91) = 182.

time = 0.47, size = 467, normalized size = 4.13

method	result
default	$ -\frac{a \sqrt{\frac{(i \sin(dx+c) + \cos(dx+c))a}{\cos(dx+c)}}}{d} \left( 2iA \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-\frac{2 \cos(dx+c)}{\cos(dx+c)+1}}}{2}\right) \sqrt{-\frac{2 \cos(dx+c)}{\cos(dx+c)+1}} \sin(dx+c) + 2iB \sqrt{2} \arcsin\left(\frac{\sqrt{2} \sqrt{-\frac{2 \cos(dx+c)}{\cos(dx+c)+1}}}{2}\right) \right) $

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVE  
RBOSE)

[Out]  $-1/d*a*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*(2*I*A*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)+2*I*B*2^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)+I*A*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)-2*A*2^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)+2*B*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)-A*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)-2*I*B*\cos(d*x+c)+2*I*B+2*B*\sin(d*x+c))/(I*\sin(d*x+c)+\cos(d*x+c)-1)$

**Maxima [A]**

time = 0.50, size = 130, normalized size = 1.15

$$\frac{\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-Aa^{\frac{3}{2}}\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)-2i\sqrt{ia\tan(dx+c)+a}Ba}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-(\sqrt{2}*(A-I*B)*a^{(3/2)}*\log(-(\sqrt{2}*\sqrt{a}-\sqrt{I*a*\tan(d*x+c)+a})/(\sqrt{2}*\sqrt{a}+\sqrt{I*a*\tan(d*x+c)+a}))-A*a^{(3/2)}*\log((\sqrt{I*a*\tan(d*x+c)+a}-\sqrt{a})/(\sqrt{I*a*\tan(d*x+c)+a}+\sqrt{a}))-2*I*\sqrt{I*a*\tan(d*x+c)+a}*B*a)/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 514 vs.  $2(86) = 172$ .

time = 3.06, size = 514, normalized size = 4.55

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/2*(-4*I*\sqrt{2}*B*a*\sqrt{a/(e^{(2*I*d*x+2*I*c)}+1)}*e^{(I*d*x+I*c)}-2*\sqrt{2}*\sqrt{(A^2-2*I*A*B-B^2)*a^3/d^2}*d*\log(4*((-I*A-B)*a^2*e^{(I*d*x+I*c)}-\sqrt{(A^2-2*I*A*B-B^2)*a^3/d^2}*(I*d*e^{(2*I*d*x+2*I*c)}+I*d)*\sqrt{a/(e^{(2*I*d*x+2*I*c)}+1)})*e^{(-I*d*x-I*c)}/((-I*A-B)*a))+2*\sqrt{2}*\sqrt{(A^2-2*I*A*B-B^2)*a^3/d^2}*d*\log(4*((-I*A-B)*a^2*e^{(I*d*x+I*c)}-\sqrt{(A^2-2*I*A*B-B^2)*a^3/d^2}*(-I*d*e^{(2*I*d*x+2*I*c)}-I*d)*\sqrt{a/(e^{(2*I*d*x+2*I*c)}+1)})*e^{(-I*d*x-I*c)}/((-I*A-B)*a))$



$$\begin{aligned} & *A*B^2*a^8*d - A^2*B*a^8*d*160i) - (32*2^{(1/2)}*B^3*a^6*d*(-a^3)^{(1/2)}*(a + \\ & a*\tan(c + d*x)*1i)^{(1/2)})/(32*A^3*a^8*d + B^3*a^8*d*64i - 192*A*B^2*a^8*d - \\ & A^2*B*a^8*d*160i) - (2^{(1/2)}*A*B^2*a^6*d*(-a^3)^{(1/2)}*(a + a*\tan(c + d*x)* \\ & 1i)^{(1/2)}*96i)/(32*A^3*a^8*d + B^3*a^8*d*64i - 192*A*B^2*a^8*d - A^2*B*a^8* \\ & d*160i) + (80*2^{(1/2)}*A^2*B*a^6*d*(-a^3)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} \\ & )/(32*A^3*a^8*d + B^3*a^8*d*64i - 192*A*B^2*a^8*d - A^2*B*a^8*d*160i))*(A* \\ & 1i + B)*(-a^3)^{(1/2))/d \end{aligned}$$

### 3.79 $\int \cot^2(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=125

$$\frac{a^{3/2}(3iA + 2B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{2} a^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d}$$

[Out]  $-a^{(3/2)}*(3*I*A+2*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d+2*a^{(3/2)}*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d-a*A*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

Rubi [A]

time = 0.26, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3674, 3681, 3561, 212, 3680, 65, 214}

$$\frac{a^{3/2}(2B + 3iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{2} a^{3/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} - \frac{aA \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^2*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-((a^{(3/2)}*((3*I)*A + 2*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/d + (2*\operatorname{Sqrt}[2]*a^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d - (a*A*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 212

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= -\frac{aA \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{d} + \int \\
&= -\frac{aA \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{d} - \frac{1}{2} \left( \frac{aA \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{d} + \frac{2\sqrt{2} a^{3/2}(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} \right) \\
&= -\frac{aA \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{d} + \frac{2\sqrt{2} a^{3/2}(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} \\
&= -\frac{a^{3/2}(3iA+2B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.89, size = 201, normalized size = 1.61

$$\frac{ae^{-\frac{1}{2}i(4c+5dx)} \sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}} (1+e^{2i(c+dx)})^{3/2} \left( (-4iA-4B) \sinh^{-1}(e^{i(c+dx)}) + \sqrt{2} (3iA+2B) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) + A \sqrt{1+e^{2i(c+dx)}} \csc(c+dx) \sec(c+dx) \left( \cos\left(\frac{dx}{2}\right) + i \sin\left(\frac{dx}{2}\right) \right) \right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] -1/2*(a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))^(3/2)*((-4*I)*A - 4*B)*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*(3*I)*A + 2*B)*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]] + A*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c + d*x]*Sec[c + d*x]*(Cos[(d*x)/2] + I*Sin[(d*x)/2])/(Sqrt[2]*d*E^((I/2)*(4*c + 5*d*x)))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1116 vs.  $2(103) = 206$ .

time = 0.56, size = 1117, normalized size = 8.94

method	result	size
default	Expression too large to display	1117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)), x, method=_RETURN VERBOSE)
```



```
[Out] 1/2/d*a*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c))^(1/2)*sin(d*x+c)*(3*I*A*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)
)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)^2-4*I*B*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*2^(1/2)*cos(d*x+c)^2+4*A*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*I*B*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*2^(1/2)+4*B*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))-4*I*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-3*I
*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))+3*A*cos(d*x+c)^2*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*I*B*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))+2*B*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+
c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))-4*A*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))+2*I*A*cos(d*x+c)*sin(d*x+c)-2*I*B*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
^2-4*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+4*I*A*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2-3*A*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*A*cos(d*x
+c)^2-2*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))-2*A*cos(d*x+c))/(cos(d*x+
c)+1)/(I*sin(d*x+c)+cos(d*x+c)-1)/(-1+cos(d*x+c))
```

**Maxima [A]**

time = 0.51, size = 145, normalized size = 1.16

$$\frac{i \left( 2\sqrt{2} (A - iB) \sqrt{a} \log \left( -\frac{\sqrt{2} \sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{ia \tan(dx+c) + a}} \right) - (3A - 2iB) \sqrt{a} \log \left( \frac{\sqrt{ia \tan(dx+c) + a} - \sqrt{a}}{\sqrt{ia \tan(dx+c) + a} + \sqrt{a}} \right) - \frac{2i \sqrt{ia \tan(dx+c) + a} A}{\tan(dx+c)} \right) a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] -1/2*I*(2*sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*
x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - (3*A - 2*I*B
)*sqrt(a)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c)
+ a) + sqrt(a))) - 2*I*sqrt(I*a*tan(d*x + c) + a)*A/tan(d*x + c))*a/d
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs.  $2(98) = 196$ .

time = 3.39, size = 685, normalized size = 5.48

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*(4*\sqrt{2}*\sqrt{-(A^2 - 2*I*A*B - B^2)}*a^3/d^2)*(d*e^{(2*I*d*x + 2*I*c)} \\ & - d)*\log(4*((-I*A - B)*a^2*e^{(I*d*x + I*c)} + \sqrt{-(A^2 - 2*I*A*B - B^2)}*a \\ & ^3/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(- \\ & I*d*x - I*c)/((-I*A - B)*a)} - 4*\sqrt{2}*\sqrt{-(A^2 - 2*I*A*B - B^2)}*a^3/d^2 \\ & *(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(4*((-I*A - B)*a^2*e^{(I*d*x + I*c)} - \sqrt{ \\ & -(A^2 - 2*I*A*B - B^2)}*a^3/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{a/(e^{(2*I \\ & *d*x + 2*I*c)} + 1)})*e^{(-I*d*x - I*c)/((-I*A - B)*a)} - \sqrt{-(9*A^2 - 12*I \\ & *A*B - 4*B^2)}*a^3/d^2*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-16*(3*(-3*I*A - 2*B \\ & )*a^2*e^{(2*I*d*x + 2*I*c)} + (-3*I*A - 2*B)*a^2 + 2*\sqrt{2}*\sqrt{-(9*A^2 - 1 \\ & 2*I*A*B - 4*B^2)}*a^3/d^2)*(d*e^{(3*I*d*x + 3*I*c)} + d*e^{(I*d*x + I*c)})*\sqrt{ \\ & a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)/(3*I*A + 2*B)} + \sqrt{-( \\ & 9*A^2 - 12*I*A*B - 4*B^2)}*a^3/d^2*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-16*(3*( \\ & -3*I*A - 2*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-3*I*A - 2*B)*a^2 - 2*\sqrt{2}*\sqrt{ \\ & -(9*A^2 - 12*I*A*B - 4*B^2)}*a^3/d^2)*(d*e^{(3*I*d*x + 3*I*c)} + d*e^{(I*d*x + \\ & I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)/(3*I*A + 2*B \\ & )} + 4*\sqrt{2}*(I*A*a*e^{(3*I*d*x + 3*I*c)} + I*A*a*e^{(I*d*x + I*c)})*\sqrt{a/( \\ & e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(2*I*d*x + 2*I*c)} - d) \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{3/2} (A + B \tan(c + dx)) \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^2, x)

**Mupad [B]**

time = 8.08, size = 2338, normalized size = 18.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] 
$$- 2*\operatorname{atanh}\left(\frac{6*d^4*(a + a*\tan(c + d*x)*1i)^{1/2}*((3*B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}}{(8*a^6) + (A*B*a^3*7i)/(2*d^2)}\right)^{1/2} / ((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2} / (A^3*a^{11}*d*10i + 32*B^3*a^{11}*d + A*B^2*a^{11}*d*72i - 32*A^2*B*a^{11}*d + A*a^2*d^3*((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}*2i) + (2*A^2*a^6*d^2*(a + a*\tan(c + d*x)*1i)^{1/2}*((3*B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}) / (8*a^6) + (A*B*a^3*7i)/(2*d^2))^{1/2} / (A^3*a^8*d*10i + 32*B^3*a^8*d + A*B^2*a^8*d*72i - 32*A^2*B*a^8*d + (A*d^3*((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}*2i) / a) + (8*B^2*a^6*d^2*(a + a*\tan(c + d*x)*1i)^{1/2}*((3*B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}) / (8*a^6) + (A*B*a^3*7i)/(2*d^2))^{1/2} / (A^3*a^8*d*10i + 32*B^3*a^8*d + A*B^2*a^8*d*72i - 32*A^2*B*a^8*d + (A*d^3*((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}*2i) / a) + (A*B*a^6*d^2*(a + a*\tan(c + d*x)*1i)^{1/2}*((3*B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}) / (8*a^6) + (A*B*a^3*7i)/(2*d^2))^{1/2} / (A^3*a^8*d*10i + 32*B^3*a^8*d + A*B^2*a^8*d*72i - 32*A^2*B*a^8*d + (A*d^3*((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}*2i) / a) * ((3*B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2} / (8*a^6) + (A*B*a^3*7i)/(2*d^2))^{1/2} - 2*\operatorname{atanh}\left(\frac{2*A^2*a^6*d^2*(a + a*\tan(c + d*x)*1i)^{1/2}*((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}}{(8*a^6) - (17*A^2*a^3)/(8*d^2) + (3*B^2*a^3)/(2*d^2) + (A*B*a^3*7i)/(2*d^2)}\right)^{1/2} / (A^3*a^8*d*10i + 32*B^3*a^8*d + A*B^2*a^8*d*72i - 32*A^2*B*a^8*d - (A*d^3*((A^4*a^{18})/d^4 + (16*B^4*a^{18})/d^4 - (8*A^2*B^2*a^{18})/d^4 + (A*B^3*a^{18}*32i)/d^4 + (A^3*B*a^{18}*8i)/d^4)^{1/2}*2i) / a) - (6*d^4*(a + a*\tan(c + d*x)*1i)^{1/2}*((A^4*a^{18})/d^4$$

$$\begin{aligned}
& + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B \\
& *a^18*8i)/d^4)^{(1/2)}/(8*a^6) - (17*A^2*a^3)/(8*d^2) + (3*B^2*a^3)/(2*d^2) + \\
& (A*B*a^3*7i)/(2*d^2))^{(1/2)}*((A^4*a^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B \\
& ^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^{(1/2)})/(A^3*a^11 \\
& *d*10i + 32*B^3*a^11*d + A*B^2*a^11*d*72i - 32*A^2*B*a^11*d - A*a^2*d^3*((A \\
& ^4*a^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/ \\
& d^4 + (A^3*B*a^18*8i)/d^4)^{(1/2)}*2i) + (8*B^2*a^6*d^2*(a + a*tan(c + d*x)* \\
& i)^{(1/2)}*((A^4*a^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B \\
& ^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^{(1/2)}/(8*a^6) - (17*A^2*a^3)/(8*d^2 \\
& ) + (3*B^2*a^3)/(2*d^2) + (A*B*a^3*7i)/(2*d^2))^{(1/2)})/(A^3*a^8*d*10i + 32* \\
& B^3*a^8*d + A*B^2*a^8*d*72i - 32*A^2*B*a^8*d - (A*d^3*((A^4*a^18)/d^4 + (16 \\
& *B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18* \\
& 8i)/d^4)^{(1/2)}*2i)/a) + (A*B*a^6*d^2*(a + a*tan(c + d*x)*i)^{(1/2)}*((A^4*a \\
& ^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 \\
& + (A^3*B*a^18*8i)/d^4)^{(1/2)}/(8*a^6) - (17*A^2*a^3)/(8*d^2) + (3*B^2*a^3)/( \\
& 2*d^2) + (A*B*a^3*7i)/(2*d^2))^{(1/2)}*8i)/(A^3*a^8*d*10i + 32*B^3*a^8*d + A* \\
& B^2*a^8*d*72i - 32*A^2*B*a^8*d - (A*d^3*((A^4*a^18)/d^4 + (16*B^4*a^18)/d^4 \\
& - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^{(1/2)} \\
& *2i)/a))*(((A^4*a^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B \\
& ^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^{(1/2)}/(8*a^6) - (17*A^2*a^3)/(8*d^2 \\
& ) + (3*B^2*a^3)/(2*d^2) + (A*B*a^3*7i)/(2*d^2))^{(1/2)} - (A*a*cot(c + d*x)*( \\
& a + a*tan(c + d*x)*i)^{(1/2)})/d
\end{aligned}$$

### 3.80 $\int \cot^3(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=171

$$\frac{a^{3/2}(11A - 12iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4d} - \frac{2\sqrt{2} a^{3/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

[Out]  $1/4*a^{(3/2)}*(11*A-12*I*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d-2*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d-1/4*a*(5*I*A+4*B)*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/2*a*A*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.39, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3674, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{a^{3/2}(11A - 12iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4d} - \frac{2\sqrt{2} a^{3/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a(4B + 5iA) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} - \frac{aA \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^3*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(a^{(3/2)}*(11*A - (12*I)*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(4*d) - (2*\operatorname{Sqrt}[2]*a^{(3/2)}*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d - (a*((5*I)*A + 4*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(4*d) - (a*A*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(2*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 212

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3561

Int[Sqrt[(a\_) + (b\_.)\*tan[(c\_) + (d\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3674

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rule 3679

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[(((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)]))/((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a

\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \cot^3(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{2d} + \\
 &= -\frac{a(5iA + 4B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
 &= -\frac{a(5iA + 4B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
 &= -\frac{a(5iA + 4B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
 &= -\frac{2\sqrt{2} a^{3/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\
 &= \frac{a^{3/2}(11A - 12iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 400 vs. 2(171) = 342.  
time = 5.85, size = 400, normalized size = 2.34

$$\frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) \left( \frac{-i\sqrt{2}(A - iB) \operatorname{sech}^{-1}(e^{i(c + dx)}) - 2(11A - 12iB) \left( \log(-1 + e^{i(c + dx)})^2 - \log((1 + e^{i(c + dx)})^2) + \log\left(\frac{1 + 2\sqrt{2}e^{i(c + dx)} + 2\sqrt{2}\sqrt{1 + e^{2i(c + dx)}}}{1 + e^{2i(c + dx)}}\right) - \log\left(\frac{1 + \sqrt{2}\sqrt{1 + e^{2i(c + dx)}}}{1 + e^{2i(c + dx)}}\right)\right)}{\left(\frac{e^{i(c + dx)}}{1 + e^{2i(c + dx)}}\right)^{3/2}} \right) + \frac{\operatorname{ArcSinh}\left(\frac{e^{i(c + dx)} + 2\sqrt{2}A + 2\sqrt{2}B \tan(c + dx)}{e^{i(c + dx)}}\right)}{e^{i(c + dx)}}}{32d \operatorname{sech}^3(c + dx)(A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x])\*((-64\*sqrt[2]\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))] - 2\*(11\*A - (12\*I)\*B)\*(Log[(-1 + E^(I\*(c + d\*x)))^2] - Log[(1 + E^(I\*(c + d\*x)))^2] + Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]] - 2\*E^(I\*(c + d\*x))\*(1 + sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]])] - Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]] + 2\*E^(I\*(c + d\*x))\*(1 + sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]])/((E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x))))^(3/2)\*(1 + E^((2\*I)\*(c + d\*x)))^(3/2)) + ((8\*I)\*Csc[c + d\*x]\*(2\*A\*Csc[c + d\*x] + ((5\*I)\*A + 4\*B)\*Sec[c + d\*x])\*(I + Tan[c + d\*x])/Sec[c + d\*x]^(5/2)))/(32\*d\*Sec[c + d\*x]^(5/2)\*(A\*cos[c + d\*x] + B\*sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal.  $1289$  vs.  $2(139) = 278$ .  
time = 0.59, size = 1290, normalized size = 7.54

method	result	size
default	Expression too large to display	1290

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out]  $\frac{1}{8}d*a*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{1/2}*(-10*A*\cos(d*x+c)-8*B*\cos(d*x+c)*\sin(d*x+c)+16*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^2*\sin(d*x+c)*2^{1/2}+16*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c)*2^{1/2}+11*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^2*\sin(d*x+c)-4*A*\cos(d*x+c)^2+16*A*2^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-16*B*2^{1/2}*a*\operatorname{rctan}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-8*I*B*\cos(d*x+c)^3+8*I*B*\cos(d*x+c)+12*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c)-16*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)*2^{1/2}-16*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c))*\sin(d*x+c)*2^{1/2}-16*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c)*2^{1/2}+16*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^2*\sin(d*x+c)*2^{1/2}+8*B*\cos(d*x+c)^2*\sin(d*x+c)-11*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c)+12*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^2*\sin(d*x+c)-11*I*A*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-12*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\sin(d*x+c)+14*A*\cos(d*x+c)^3+14*I*A*\cos(d*x+c)^2*\sin(d*x+c)-10*I*A*\cos(d*x+c)*\sin(d*x+c)-12*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)+11*A*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/(-1+\cos(d*x+c))/((I*\sin(d*x+c)+\cos(d*x+c)-1)/(\cos(d*x+c)+1))$

**Maxima [A]**



time = 0.49, size = 203, normalized size = 1.19

$$\frac{\left( \frac{8\sqrt{2}^{(A-iB)} \log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}}\right)}{\sqrt{a}} - \frac{(11A-12iB) \log\left(\frac{\sqrt{ia \tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia \tan(dx+c)+a}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\left((ia \tan(dx+c)+a)^{\frac{3}{2}}(5A-4iB)-\sqrt{ia \tan(dx+c)+a}(3A-4iB)a\right)}{(ia \tan(dx+c)+a)^2-2(ia \tan(dx+c)+a)a+a^2} \right) a^2}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/8\*(8\*sqrt(2)\*(A - I\*B)\*log(-(sqrt(2)\*sqrt(a) - sqrt(I\*a\*tan(d\*x + c) + a))/(sqrt(2)\*sqrt(a) + sqrt(I\*a\*tan(d\*x + c) + a)))/sqrt(a) - (11\*A - 12\*I\*B)\*log((sqrt(I\*a\*tan(d\*x + c) + a) - sqrt(a))/(sqrt(I\*a\*tan(d\*x + c) + a) + sqrt(a)))/sqrt(a) + 2\*((I\*a\*tan(d\*x + c) + a)^(3/2)\*(5\*A - 4\*I\*B) - sqrt(I\*a\*tan(d\*x + c) + a)\*(3\*A - 4\*I\*B)\*a)/((I\*a\*tan(d\*x + c) + a)^2 - 2\*(I\*a\*tan(d\*x + c) + a)\*a + a^2))\*a^2/d

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(132) = 264.

time = 2.75, size = 762, normalized size = 4.46



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] -1/16\*(16\*sqrt(2)\*sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^3/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(4\*((-I\*A - B)\*a^2\*e^(I\*d\*x + I\*c) - sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^3/d^2)\*(I\*d\*e^(2\*I\*d\*x + 2\*I\*c) + I\*d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a) - 16\*sqrt(2)\*sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^3/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(4\*((-I\*A - B)\*a^2\*e^(I\*d\*x + I\*c) - sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^3/d^2)\*(-I\*d\*e^(2\*I\*d\*x + 2\*I\*c) - I\*d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a) + sqrt((121\*A^2 - 264\*I\*A\*B - 144\*B^2)\*a^3/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(-16\*(3\*(-11\*I\*A - 12\*B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) + (-11\*I\*A - 12\*B)\*a^2 + 2\*sqrt(2)\*sqrt((121\*A^2 - 264\*I\*A\*B - 144\*B^2)\*a^3/d^2)\*(I\*d\*e^(3\*I\*d\*x + 3\*I\*c) + I\*d\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-2\*I\*d\*x - 2\*I\*c)/(11\*I\*A + 12\*B) - sqrt((121\*A^2 - 264\*I\*A\*B - 144\*B^2)\*a^3/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(-16\*(3\*(-11\*I\*A - 12\*B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) + (-11\*I\*A - 12\*B)\*a^2 + 2\*sqrt(2)\*sqrt((121\*A^2 - 264\*I\*A\*B - 144\*B^2)\*a^3/d^2)\*(-I\*d\*e^(3\*I\*d\*x + 3\*I\*c) - I\*d\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-2\*I\*d\*x - 2\*I\*c)/(11\*I\*A + 12\*B) - 4\*sqrt(2)\*((7\*A - 4\*I\*B)\*a\*e^(5\*I\*d\*x + 5\*I\*c) + 4\*A\*a\*e^(3\*I\*d\*x + 3\*I\*c))

\*c) - (3\*A - 4\*I\*B)\*a\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))/(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{3}{2}} (A + B \tan(c + dx)) \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^3, x)

**Mupad [B]**

time = 7.82, size = 2500, normalized size = 14.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] 2\*atanh((3\*d^4\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((249\*A^2\*a^3)/(128\*d^2) - ((49\*A^4\*a^18)/(4\*d^4) + (64\*B^4\*a^18)/d^4 + (40\*A^2\*B^2\*a^18)/d^4 + (A\*B^3\*a^18\*64i)/d^4 + (A^3\*B\*a^18\*28i)/d^4)^(1/2)/(64\*a^6) - (17\*B^2\*a^3)/(8\*d^2) - (A\*B\*a^3\*65i)/(16\*d^2))^(1/2)\*((49\*A^4\*a^18)/(4\*d^4) + (64\*B^4\*a^18)/d^4 + (40\*A^2\*B^2\*a^18)/d^4 + (A\*B^3\*a^18\*64i)/d^4 + (A^3\*B\*a^18\*28i)/d^4)^(1/2))/(2\*((133\*A^3\*a^11\*d)/16 - B^3\*a^11\*d\*20i + 29\*A\*B^2\*a^11\*d + (A^2\*B\*a^11\*d\*3i)/4 + (3\*A\*a^2\*d^3\*((49\*A^4\*a^18)/(4\*d^4) + (64\*B^4\*a^18)/d^4 + (40\*A^2\*B^2\*a^18)/d^4 + (A\*B^3\*a^18\*64i)/d^4 + (A^3\*B\*a^18\*28i)/d^4)^(1/2))/8 - (B\*a^2\*d^3\*((49\*A^4\*a^18)/(4\*d^4) + (64\*B^4\*a^18)/d^4 + (40\*A^2\*B^2\*a^18)/d^4 + (A\*B^3\*a^18\*64i)/d^4 + (A^3\*B\*a^18\*28i)/d^4)^(1/2)\*1i)/2)) + (7\*A^2\*a^6\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((249\*A^2\*a^3)/(128\*d^2) - ((49\*A^4\*a^18)

$$\begin{aligned}
& / (4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 \\
& + (A^3*B*a^18*28i)/d^4)^{(1/2)} / (64*a^6) - (17*B^2*a^3)/(8*d^2) - (A*B*a^3*6 \\
& 5i)/(16*d^2))^{(1/2)} / (4*((133*A^3*a^8*d)/16 - B^3*a^8*d*20i + 29*A*B^2*a^8* \\
& d + (A^2*B*a^8*d*3i)/4 + (3*A*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^ \\
& 4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1 \\
& /2)} / (8*a) - (B*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^ \\
& 2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} * i) / (2*a) \\
& ) + (4*B^2*a^6*d^2*(a + a*tan(c + d*x)*i))^{(1/2)} * ((249*A^2*a^3)/(128*d^2) - \\
& ((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^ \\
& 3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} / (64*a^6) - (17*B^2*a^3)/(8*d^ \\
& 2) - (A*B*a^3*65i)/(16*d^2))^{(1/2)} / ((133*A^3*a^8*d)/16 - B^3*a^8*d*20i + 2 \\
& 9*A*B^2*a^8*d + (A^2*B*a^8*d*3i)/4 + (3*A*d^3*((49*A^4*a^18)/(4*d^4) + (64* \\
& B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18* \\
& 28i)/d^4)^{(1/2)} / (8*a) - (B*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 \\
& + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2} \\
& ) * i) / (2*a) + (A*B*a^6*d^2*(a + a*tan(c + d*x)*i))^{(1/2)} * ((249*A^2*a^3)/(1 \\
& 28*d^2) - ((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^ \\
& 4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} / (64*a^6) - (17*B^2*a \\
& ^3)/(8*d^2) - (A*B*a^3*65i)/(16*d^2))^{(1/2)} * 2i) / ((133*A^3*a^8*d)/16 - B^3*a \\
& ^8*d*20i + 29*A*B^2*a^8*d + (A^2*B*a^8*d*3i)/4 + (3*A*d^3*((49*A^4*a^18)/(4 \\
& *d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + \\
& (A^3*B*a^18*28i)/d^4)^{(1/2)} / (8*a) - (B*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^ \\
& 4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28 \\
& i)/d^4)^{(1/2)} * i) / (2*a)) * ((249*A^2*a^3)/(128*d^2) - ((49*A^4*a^18)/(4*d^4) \\
& + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3* \\
& B*a^18*28i)/d^4)^{(1/2)} / (64*a^6) - (17*B^2*a^3)/(8*d^2) - (A*B*a^3*65i)/(16* \\
& d^2))^{(1/2)} - (((3*A*a^3 - B*a^3*4i)*(a + a*tan(c + d*x)*i))^{(1/2)} / (4*d) - \\
& ((5*A*a^2 - B*a^2*4i)*(a + a*tan(c + d*x)*i))^{(3/2)} / (4*d)) / ((a + a*tan(c \\
& + d*x)*i)^2 - 2*a*(a + a*tan(c + d*x)*i) + a^2) + 2*atanh((7*A^2*a^6*d^2* \\
& (a + a*tan(c + d*x)*i))^{(1/2)} * ((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + \\
& (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} \\
& / (64*a^6) + (249*A^2*a^3)/(128*d^2) - (17*B^2*a^3)/(8*d^2) - (A*B*a^3*65i)/ \\
& (16*d^2))^{(1/2)} / (4*((133*A^3*a^8*d)/16 - B^3*a^8*d*20i + 29*A*B^2*a^8*d + \\
& (A^2*B*a^8*d*3i)/4 - (3*A*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + \\
& (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} \\
& / (8*a) + (B*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^ \\
& 18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} * i) / (2*a)) - \\
& (3*d^4*(a + a*tan(c + d*x)*i))^{(1/2)} * (((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18 \\
& )/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4 \\
& )^{(1/2)} / (64*a^6) + (249*A^2*a^3)/(128*d^2) - (17*B^2*a^3)/(8*d^2) - (A*B*a^ \\
& 3*65i)/(16*d^2))^{(1/2)} * ((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2 \\
& *B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} / (2*((1 \\
& 33*A^3*a^11*d)/16 - B^3*a^11*d*20i + 29*A*B^2*a^11*d + (A^2*B*a^11*d*3i)/4 \\
& - (3*A*a^2*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^1 \\
& 8)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^{(1/2)} / 8 + (B*a^2*d^3
\end{aligned}$$



### 3.81 $\int \cot^4(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=213

$$\frac{a^{3/2}(23iA + 22B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{8d} - \frac{2\sqrt{2} a^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

[Out]  $1/8*a^{(3/2)}*(23*I*A+22*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d-2*a^{(3/2)}*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+1/8*a*(9*A-10*I*B)*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/12*a*(7*I*A+6*B)*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/3*a*A*\cot(d*x+c)^3*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.50, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3674, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{a^{3/2}(22B + 23iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{8d} - \frac{2\sqrt{2} a^{3/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a(6B + 7iA) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{12d} + \frac{a(9A - 10iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} - \frac{aA \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^4*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(a^{(3/2)}*((23*I)*A + 22*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(8*d) - (2*\operatorname{Sqrt}[2]*a^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + (a*(9*A - (10*I)*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(8*d) - (a*((7*I)*A + 6*B)*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(12*d) - (a*A*\operatorname{Cot}[c + d*x]^3*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(3*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3681

Int((((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]))/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*

d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \cot^4(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d} + \\
 &= -\frac{a(7iA + 6B) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{12d} \\
 &= \frac{a(9A - 10iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{a(9A - 10iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{a(9A - 10iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= -\frac{2\sqrt{2} a^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\
 &= \frac{a^{3/2}(23iA + 22B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{8d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 439 vs. 2(213) = 426.  
time = 6.28, size = 439, normalized size = 2.06

$$\frac{\left( -\frac{2\left(4\sqrt{2}(A-10iB)\sqrt{a+ia\tan(c+dx)}+23A-22B\right)\left(\frac{a+ia\tan(c+dx)}{\sqrt{2}\sqrt{a}}\right)^2-\log\left(\frac{1+i\sqrt{2}\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\left(\frac{a+ia\tan(c+dx)}{\sqrt{2}\sqrt{a}}\right)^2}+\frac{2\sqrt{2}a^{3/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{a^{3/2}(23iA+22B)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d} \right)}{8d\sin^2(c+dx)(A\cos(c+dx)+B\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((((-2\*I)\*(64\*sqrt[2]\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))] + (23\*A - (22\*I)\*B)\*(Log[-1 + E^(I\*(c + d\*x))]^2 - Log[(1 + E^(I\*(c + d\*x))]^2) + Log[3 + 3\*I\*E^((2\*I)\*(c + d\*x)) + 2\*sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))] - 2\*I\*E^(I\*(c + d\*x))\*(1 + sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]]) - Log[3 + 3\*I\*E^((2\*I)\*(c + d\*x)) + 2\*sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))] + 2\*I\*E^(I\*(c + d\*x))\*(1 + sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]])))/((E^(I\*(c + d\*x))/(1 + E^((2\*I

$$\begin{aligned} & )*(c + d*x)))^{(3/2)}*(1 + E^{((2*I)*(c + d*x))})^{(3/2)} - (4*Csc[c + d*x]^3*( \\ & \text{Cos}[c] - I*\text{Sin}[c])*(-19*A + (30*I)*B + 5*(7*A - (6*I)*B)*\text{Cos}[2*(c + d*x)] + \\ & 2*((7*I)*A + 6*B)*\text{Sin}[2*(c + d*x)]))/(\text{Sqrt}[\text{Sec}[c + d*x]]*(3*\text{Cos}[d*x] + (3* \\ & I)*\text{Sin}[d*x]))*(a + I*a*\text{Tan}[c + d*x])^{(3/2)}*(A + B*\text{Tan}[c + d*x])/(64*d*\text{Sec} \\ & [c + d*x]^{(5/2)}*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])) \end{aligned}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1803 vs.  $2(175) = 350$ .  
time = 0.69, size = 1804, normalized size = 8.47

method	result	size
default	Expression too large to display	1804

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURN VERBOSE)`

[Out] 
$$\begin{aligned} & 1/48/d*a*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*(54*A*\cos(d*x+c)-60 \\ & *B*\cos(d*x+c)*\sin(d*x+c)+98*A*\cos(d*x+c)^4+96*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c) \\ & +1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d* \\ & x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^4-82*A*\cos(d*x+c)^2+69*I*A*(-2*\cos(d*x+c) \\ & /(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & )-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^4-66*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+ \\ & 1))^{(1/2)}*\operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^4-138*I \\ & *A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2+132*I*B*(-2*\cos(d*x \\ & +c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos \\ & (d*x+c)^2+96*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}* \\ & (-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}-96*I*B* \\ & (-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)})*2^{(1/2)}+98*I*A*\cos(d*x+c)^3*\sin(d*x+c)-28*I*A*\cos(d*x+c) \\ & ^2*\sin(d*x+c)+69*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(- \\ & 2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-54*I*A*\cos(d*x \\ & +c)*\sin(d*x+c)-66*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctan}(1/(-2*\cos \\ & (d*x+c)/(\cos(d*x+c)+1))^{(1/2)}+84*I*B*\cos(d*x+c)^2-60*I*B*\cos(d*x+c)-84*I*B \\ & *\cos(d*x+c)^4+60*I*B*\cos(d*x+c)^3+84*B*\cos(d*x+c)^3*\sin(d*x+c)+69*A*(-2*\cos \\ & (d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & )+66*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/ \\ & (\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-24*B*\cos(d*x+c)^2*\sin(d*x+c) \\ & -70*A*\cos(d*x+c)^3+96*B*2^{(1/2)}*\cos(d*x+c)^4*(-2*\cos(d*x+c)/(\cos(d*x+c)+1)) \\ & ^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/ \\ & \cos(d*x+c))-192*A*2^{(1/2)}*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & *\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}))-192*B*2^{(1/2)}*\cos \\ & (d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos \\ & (d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))+96*A*2^{(1/2)}*\cos(d*x+c) \end{aligned}$$



$$\begin{aligned} &^4*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})-138*A*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})-132*B*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))+96*A*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})+96*B*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))+69*A*\cos(d*x+c)^4*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})+66*B*\cos(d*x+c)^4*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-96*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^4-192*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^2+192*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2)/(-1+\cos(d*x+c))/(I*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c)/(\cos(d*x+c)+1) \end{aligned}$$

**Maxima [A]**

time = 0.51, size = 253, normalized size = 1.19

$$i a^3 \left( \frac{48 \sqrt{2}^{(A-B)} \log\left(\frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}}\right)}{a^{\frac{3}{2}}} - \frac{3(23A-22iB) \log\left(\frac{\sqrt{i a \tan(dx+c) + a} - \sqrt{a}}{\sqrt{i a \tan(dx+c) + a} + \sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2 \left( 3(i a \tan(dx+c) + a)^{\frac{5}{2}} (9A-10iB) - 8(i a \tan(dx+c) + a)^{\frac{3}{2}} (5A-6iB)a + 3 \sqrt{i a \tan(dx+c) + a} (7A-6iB)a^2 \right)}{(i a \tan(dx+c) + a)^3 a - 3(i a \tan(dx+c) + a)^2 a^2 + 3(i a \tan(dx+c) + a) a^3 - a^4} \right) / 48d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{48} I a^3 (48 \sqrt{2} (A - I B) \log(-\sqrt{2} \sqrt{a} - \sqrt{I a \tan(dx+c) + a}) / (\sqrt{2} \sqrt{a} + \sqrt{I a \tan(dx+c) + a})) / a^{3/2} - 3(23A - 22 I B) \log((\sqrt{I a \tan(dx+c) + a} - \sqrt{a}) / (\sqrt{I a \tan(dx+c) + a} + \sqrt{a})) / a^{3/2} + 2(3(I a \tan(dx+c) + a)^{5/2} (9A - 10 I B) - 8(I a \tan(dx+c) + a)^{3/2} (5A - 6 I B) a + 3 \sqrt{I a \tan(dx+c) + a} (7A - 6 I B) a^2) / ((I a \tan(dx+c) + a)^3 a - 3(I a \tan(dx+c) + a)^2 a^2 + 3(I a \tan(dx+c) + a) a^3 - a^4)) / d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs.  $2(166) = 332$ .

time = 3.28, size = 856, normalized size = 4.02

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

```
[Out] 1/96*(96*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6*I*c)
) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A - B
)*a^2*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d*x
+ 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A -
B)*a)) - 96*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6*
I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A
- B)*a^2*e^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d
*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A
- B)*a)) - 3*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^2)*(d*e^(6*I*d*x
+ 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(-16*
(3*(-23*I*A - 22*B)*a^2*e^(2*I*d*x + 2*I*c) + (-23*I*A - 22*B)*a^2 + 2*sqrt
(2)*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^2)*(d*e^(3*I*d*x + 3*I*c)
+ d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))e^(-2*I*d*x - 2*I*c
)/(23*I*A + 22*B)) + 3*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^2)*(d*e
^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
*log(-16*(3*(-23*I*A - 22*B)*a^2*e^(2*I*d*x + 2*I*c) + (-23*I*A - 22*B)*a^2
- 2*sqrt(2)*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^2)*(d*e^(3*I*d*x
+ 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))e^(-2*I*d*
x - 2*I*c)/(23*I*A + 22*B)) - 4*sqrt(2)*(7*(-7*I*A - 6*B)*a*e^(7*I*d*x + 7*
I*c) - (11*I*A - 18*B)*a*e^(5*I*d*x + 5*I*c) - (-17*I*A - 42*B)*a*e^(3*I*d*
x + 3*I*c) + 3*(-7*I*A - 6*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x
+ 2*I*c) - d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{3/2} (A + B \tan(c + dx)) \cot^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*cot(c + d*x)*
*4, x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^4,
x)
```



$$\begin{aligned}
& B^4 a^{18} / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + \\
& (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} / (2 a)) * ((249 B^2 a^3) / (128 d^2) - (1041 A^2 a^3) / (512 d^2) - ((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} / (64 a^6) + (A B a^3 509 i) / (128 d^2))^{(1/2)} + 2 * \operatorname{atanh}((17 A^2 a^6 d^2 * (a + a * \tan(c + d * x) * i))^{(1/2)} * (((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} / (64 a^6) - (1041 A^2 a^3) / (512 d^2) + (249 B^2 a^3) / (128 d^2) + (A B a^3 509 i) / (128 d^2))^{(1/2)}) / (4 * ((A^3 a^8 d 663 i) / 32 + (133 B^3 a^8 d) / 4 + (A B^2 a^8 d 387 i) / 8 + (89 A^2 B a^8 d) / 16 - (A d^3 * ((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} * 7 i) / (4 a) - (3 B d^3 * ((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2))} / (2 a)) - (6 d^4 * (a + a * \tan(c + d * x) * i))^{(1/2)} * (((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} / (2 a)) - (1041 A^2 a^3) / (512 d^2) + (249 B^2 a^3) / (128 d^2) + (A B a^3 509 i) / (128 d^2))^{(1/2)} * ((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2))} / ((A^3 a^{11} d 663 i) / 32 + (133 B^3 a^{11} d) / 4 + (A B^2 a^{11} d 387 i) / 8 + (89 A^2 B a^{11} d) / 16 - (A a^2 d^3 * ((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} * 7 i) / 4 - (3 B a^2 d^3 * ((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2))} / 2) + (7 B^2 a^6 d^2 * (a + a * \tan(c + d * x) * i))^{(1/2)} * (((289 A^4 a^{18}) / (64 d^4) + (49 B^4 a^{18}) / (4 d^4) + (101 A^2 B^2 a^{18}) / (8 d^4) + (A B^3 a^{18} 21 i) / (2 d^4) + (A^3 B a^{18} 51 i) / (8 d^4)^{(1/2)} / (64 a^6) - (1041 A^2 a^3) / (512 d^2) + (249 B^2 a^3) / (128 d^2) + (A B a^3 509 i) / (128 d^2))^{(1/2)} / ((A^3 a^8 d 663 i) / 32 + (133 B^3 a^8 d) / 4 + (A B^2 a^8 d 387 i) / 8 + \dots
\end{aligned}$$



```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

### Rule 3678

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \tan^2(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{2iaB \tan^3(c + dx)(a + ia \tan(c + dx))^{3/2}}{9d} \\
&= -\frac{2a^2(3A - 4iB) \tan^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{21d} \\
&= \frac{2a^2(45iA + 46B) \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{105d} \\
&= \frac{2a^2(45iA + 46B) \tan^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{105d} \\
&= -\frac{8a^2(45iA + 46B) \sqrt{a + ia \tan(c + dx)}}{105d} \\
&= -\frac{8a^2(45iA + 46B) \sqrt{a + ia \tan(c + dx)}}{105d} \\
&= \frac{4\sqrt{2} a^{5/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 5.25, size = 284, normalized size = 1.15

$$\frac{\left(4\sqrt{2}(iA + B)e^{-3i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) - \frac{i\sec^2(c+dx)(\cos(2c)-i\sin(2c))(2205A-2331iB+12(260A-251i)B)\cos(2(c+dx))+915A-961iB)\cos(4(c+dx))+390iA\sin(2(c+dx))+282iB\sin(2(c+dx))+285iA\sin(4(c+dx))+331iB\sin(4(c+dx))}{1260(\cos(dx)+i\sin(dx))^2}\right)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{d\sec^3(c+dx)(A\cos(c+dx)+B\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] (((4*sqrt(2)*(I*A + B)*sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/E^((3*I)*(c + d*x)) - (I/1260)*Sec[c + d*x]^(9/2)*(Cos[2*c] - I*Sin[2*c])*(2205*A - (2331*I)*B + 12*(260*A - (251*I)*B)*Cos[2*(c + d*x)] + (915*A - (961*I)*B)*Cos[4*(c + d*x)] + (390*I)*A*Sin[2*(c + d*x)] + 282*B*Sin[2*(c + d*x)] + (285*I)*A*Sin[4*(c + d*x)] + 331*B*Sin[4*(c + d*x)]))/(Cos[d*x] + I*Sin[d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/(d*Sec[c + d*x]^(7/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.11, size = 206, normalized size = 0.84

method	result
--------	--------

derivativedivides	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{9/2}}{9} + \frac{iBa(a+ia \tan(dx+c))^{7/2}}{7} + \frac{Aa(a+ia \tan(dx+c))^{7/2}}{7} - \frac{ia^2B(a+ia \tan(dx+c))^{5/2}}{5} - \frac{iBa^3(a+ia \tan(dx+c))^{5/2}}{3} \right)$
default	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{9/2}}{9} + \frac{iBa(a+ia \tan(dx+c))^{7/2}}{7} + \frac{Aa(a+ia \tan(dx+c))^{7/2}}{7} - \frac{ia^2B(a+ia \tan(dx+c))^{5/2}}{5} - \frac{iBa^3(a+ia \tan(dx+c))^{5/2}}{3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out] 
$$-2*I/d/a^2*(-1/9*I*B*(a+I*a*\tan(d*x+c))^{(9/2)}+1/7*I*B*a*(a+I*a*\tan(d*x+c))^{(7/2)}+1/7*A*a*(a+I*a*\tan(d*x+c))^{(7/2)}-1/5*I*a^2*B*(a+I*a*\tan(d*x+c))^{(5/2)}-1/3*I*B*a^3*(a+I*a*\tan(d*x+c))^{(3/2)}+1/3*A*a^3*(a+I*a*\tan(d*x+c))^{(3/2)}-2*I*B*a^4*(a+I*a*\tan(d*x+c))^{(1/2)}+2*A*a^4*(a+I*a*\tan(d*x+c))^{(1/2)}-2*a^{(9/2)}*(A-I*B)*2^{(1/2)}*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)}))$$

**Maxima** [A]

time = 0.51, size = 176, normalized size = 0.72

$$\frac{2i \left( 315\sqrt{2} (A - iB)a^{\frac{11}{2}} \log\left(-\frac{\sqrt{2}\sqrt{a-\sqrt{ia \tan(dx+c)+a}}}{\sqrt{2}\sqrt{a+\sqrt{ia \tan(dx+c)+a}}}\right) - 35i (ia \tan(dx+c) + a)^{\frac{9}{2}}Ba + 45 (ia \tan(dx+c) + a)^{\frac{7}{2}}(A + iB)a^2 - 63i (ia \tan(dx+c) + a)^{\frac{5}{2}}Ba^3 + 105 (ia \tan(dx+c) + a)^{\frac{3}{2}}(A - iB)a^4 + 630\sqrt{ia \tan(dx+c) + a} (A - iB)a^5 \right)}{315a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorit  
hm="maxima")`

[Out] 
$$-2/315*I*(315*\sqrt{2}*(A - I*B)*a^{(11/2)}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(dx+c) + a})/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(dx+c) + a})) - 35*I*(I*a*\tan(dx+c) + a)^{(9/2)}*B*a + 45*(I*a*\tan(dx+c) + a)^{(7/2)}*(A + I*B)*a^2 - 63*I*(I*a*\tan(dx+c) + a)^{(5/2)}*B*a^3 + 105*(I*a*\tan(dx+c) + a)^{(3/2)}*(A - I*B)*a^4 + 630*\sqrt{I*a*\tan(dx+c) + a}*(A - I*B)*a^5)/(a^3*d)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs.  $2(193) = 386$ .

time = 3.53, size = 531, normalized size = 2.16

$$\frac{\left( 315\sqrt{2} (A - iB)a^{\frac{11}{2}} \log\left(-\frac{\sqrt{2}\sqrt{a-\sqrt{ia \tan(dx+c)+a}}}{\sqrt{2}\sqrt{a+\sqrt{ia \tan(dx+c)+a}}}\right) - 35i (ia \tan(dx+c) + a)^{\frac{9}{2}}Ba + 45 (ia \tan(dx+c) + a)^{\frac{7}{2}}(A + iB)a^2 - 63i (ia \tan(dx+c) + a)^{\frac{5}{2}}Ba^3 + 105 (ia \tan(dx+c) + a)^{\frac{3}{2}}(A - iB)a^4 + 630\sqrt{ia \tan(dx+c) + a} (A - iB)a^5 \right)}{315a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorit  
hm="fricas")`

[Out] 
$$-2/315*(315*\sqrt{2}*\sqrt{-(A^2 - 2*I*A*B - B^2)}*a^{5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + 4*d*e^{(0*I*d*x + 0*I*c)}))$$



$2*I*c) + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} + \sqrt{-(A^2 - 2*I*A*B - B^2)}*a^5/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-I*d*x - I*c)/((-I*A - B)*a^2)} - 315*\sqrt{2}*\sqrt{-(A^2 - 2*I*A*B - B^2)}*a^5/d^2)*(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} - \sqrt{-(A^2 - 2*I*A*B - B^2)}*a^5/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))*e^{(-I*d*x - I*c)/((-I*A - B)*a^2)} + 2*\sqrt{2}*(2*(300*I*A + 323*B)*a^2*e^{(9*I*d*x + 9*I*c)} + 27*(65*I*A + 61*B)*a^2*e^{(7*I*d*x + 7*I*c)} + 63*(35*I*A + 37*B)*a^2*e^{(5*I*d*x + 5*I*c)} + 1365*(I*A + B)*a^2*e^{(3*I*d*x + 3*I*c)} + 315*(I*A + B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{5/2} (A + B \tan(c + dx)) \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(5/2)\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 7.57, size = 258, normalized size = 1.05

$$\frac{2B(a + a \tan(c + dx))^{5/2} - A(a + a \tan(c + dx))^{3/2} - 2B(a + a \tan(c + dx))^{1/2} - A^2 \sqrt{a + a \tan(c + dx)} \operatorname{arctan}\left(\frac{a + a \tan(c + dx)}{1}\right) - A(a + a \tan(c + dx))^{1/2} - 4B^2 \sqrt{a + a \tan(c + dx)} \operatorname{arctan}\left(\frac{a + a \tan(c + dx)}{1}\right) - 2B(a + a \tan(c + dx))^{1/2} - 2B(a + a \tan(c + dx))^{1/2} + \sqrt{2} A (-a)^{5/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)}}{1 - \sqrt{a}}\right) a - \sqrt{2} B a^{5/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)}}{1 - \sqrt{a}}\right) a}{9ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(7/2))/(7\*a\*d) - (A\*a\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*2i)/(3\*d) - (2\*B\*a\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*d) - (A\*a^2\*(a +

$$\begin{aligned}
& a \tan(c + d x) i^{1/2} / d - (A (a + a \tan(c + d x) i)^{7/2} i^{2i}) / (7 a \\
& * d) - (4 B a^2 (a + a \tan(c + d x) i)^{1/2}) / d - (2 B (a + a \tan(c + d x) \\
& i)^{5/2}) / (5 d) - (2 B (a + a \tan(c + d x) i)^{9/2}) / (9 a^2 d) + (2^{1/2} \\
& * A (-a)^{5/2} \operatorname{atan}((2^{1/2} (a + a \tan(c + d x) i)^{1/2}) / (2 (-a)^{1/2})) * \\
& 4 i) / d - (2^{1/2} * B a^{5/2} \operatorname{atan}((2^{1/2} (a + a \tan(c + d x) i)^{1/2} i) / \\
& (2 a^{1/2})) * 4 i) / d
\end{aligned}$$

### 3.83 $\int \tan(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=171

$$\frac{4\sqrt{2} a^{5/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{4a^2(A-iB)\sqrt{a+ia \tan(c+dx)}}{d} + \frac{2a(A-iB)(a+ia \tan(c+dx))^{5/2}}{d} - \frac{2iB(a+ia \tan(c+dx))^{7/2}}{7ad}$$

[Out]  $-4*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+4*a^2*(A-I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d+2/3*a*(A-I*B)*(a+I*a*\tan(d*x+c))^{(3/2)}/d+2/5*A*(a+I*a*\tan(d*x+c))^{(5/2)}/d-2/7*I*B*(a+I*a*\tan(d*x+c))^{(7/2)}/a/d$

**Rubi** [A]

time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3673, 3608, 3559, 3561, 212}

$$\frac{4\sqrt{2} a^{5/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{4a^2(A-iB)\sqrt{a+ia \tan(c+dx)}}{d} + \frac{2a(A-iB)(a+ia \tan(c+dx))^{3/2}}{3d} + \frac{2A(a+ia \tan(c+dx))^{5/2}}{5d} - \frac{2iB(a+ia \tan(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c+d*x]*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-4*\operatorname{Sqrt}[2]*a^{(5/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]))/d+(4*a^2*(A-I*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d+(2*a*(A-I*B)*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})/(3*d)+(2*A*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)})/(5*d)-(((2*I)/7)*B*(a+I*a*\operatorname{Tan}[c+d*x])^{(7/2)})/(a*d)$

**Rule 212**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \operatorname{LtQ}[b, 0])$

**Rule 3559**

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[b*((a+b*\operatorname{Tan}[c+d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[2*a, \operatorname{Int}[(a+b*\operatorname{Tan}[c+d*x])^{(n-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2+b^2, 0] \ \&\& \operatorname{GtQ}[n, 1]$

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a-x^2), x], x, \operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]], x] /; \operatorname{FreeQ}\{a,$

b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

### Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \tan(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= -\frac{2iB(a + ia \tan(c + dx))^{7/2}}{7ad} + \int (a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx \\
 &= \frac{2A(a + ia \tan(c + dx))^{5/2}}{5d} - \frac{2iB(a + ia \tan(c + dx))^{7/2}}{7ad} \\
 &= \frac{2a(A - iB)(a + ia \tan(c + dx))^{3/2}}{3d} + \frac{2A(a + ia \tan(c + dx))^{5/2}}{5d} \\
 &= \frac{4a^2(A - iB)\sqrt{a + ia \tan(c + dx)}}{d} + \frac{2a(A - iB)(a + ia \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{4a^2(A - iB)\sqrt{a + ia \tan(c + dx)}}{d} + \frac{2a(A - iB)(a + ia \tan(c + dx))^{3/2}}{3d} \\
 &= -\frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}
 \end{aligned}$$

### Mathematica [A]

time = 3.70, size = 268, normalized size = 1.57

$$\frac{\left(-4\sqrt{2}(A - iB)e^{-3i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right) + \frac{\sec^2(c+dx)(\cos(2c) - i\sin(2c))(21(37A - 35iB)\cos(c+dx) + (287A - 305iB)\cos(3(c+dx)) + 77(A\sin(c+dx) + 35B\sin(c+dx)) + 77(A\sin(3(c+dx)) + 95B\sin(3(c+dx)))}{210(\cos(d) + i\sin(d))^2}\right)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{d \sec^3(c + dx)(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

```
[Out] (((-4*Sqrt[2]*(A - I*B)*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqr
t[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])/E^((3*I)*(c + d*x)) +
(Sec[c + d*x]^(7/2)*(Cos[2*c] - I*Sin[2*c])*(21*(37*A - (35*I)*B)*Cos[c + d
*x] + (287*A - (305*I)*B)*Cos[3*(c + d*x)] + (77*I)*A*Sin[c + d*x] + 35*B*S
in[c + d*x] + (77*I)*A*Sin[3*(c + d*x)] + 95*B*Sin[3*(c + d*x)]))/(210*(Cos
[d*x] + I*Sin[d*x])^2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])/
(d*Sec[c + d*x]^(7/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [A]**

time = 0.09, size = 165, normalized size = 0.96

method	result
derivativedivides	$-\frac{2iB(a+ia \tan(dx+c))^{\frac{7}{2}}}{7} + \frac{2Aa(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2iB a^2(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{2A a^2(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 4ia^3 B \sqrt{a + ia}$
default	$-\frac{2iB(a+ia \tan(dx+c))^{\frac{7}{2}}}{7} + \frac{2Aa(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2iB a^2(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{2A a^2(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 4ia^3 B \sqrt{a + ia}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/d/a*(-1/7*I*B*(a+I*a*tan(d*x+c))^(7/2)+1/5*A*a*(a+I*a*tan(d*x+c))^(5/2)-1
/3*I*a^2*B*(a+I*a*tan(d*x+c))^(3/2)+1/3*A*a^2*(a+I*a*tan(d*x+c))^(3/2)-2*I*
B*a^3*(a+I*a*tan(d*x+c))^(1/2)+2*A*a^3*(a+I*a*tan(d*x+c))^(1/2)-2*a^(7/2)*(
A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 153, normalized size = 0.89

$$\frac{2 \left( 105 \sqrt{2} (A - i B) a^{\frac{3}{2}} \log \left( \frac{\sqrt{2} \sqrt{a - i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}} \right) - 15i (i a \tan(dx+c) + a)^{\frac{3}{2}} B a + 21 (i a \tan(dx+c) + a)^{\frac{5}{2}} A a^2 + 35 (i a \tan(dx+c) + a)^{\frac{3}{2}} (A - i B) a^3 + 210 \sqrt{i a \tan(dx+c) + a} (A - i B) a^4 \right)}{105 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 2/105*(105*sqrt(2)*(A - I*B)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 15*I*(I*a*t
an(d*x + c) + a)^(7/2)*B*a + 21*(I*a*tan(d*x + c) + a)^(5/2)*A*a^2 + 35*(I*
a*tan(d*x + c) + a)^(3/2)*(A - I*B)*a^3 + 210*sqrt(I*a*tan(d*x + c) + a)*(A
- I*B)*a^4)/(a^2*d)
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs.  $2(130) = 260$ .



**Mupad [B]**

time = 7.32, size = 212, normalized size = 1.24

$$\frac{2A(a+a\tan(c+dx))^{5/2}}{5d} + \frac{2Aa(a+a\tan(c+dx))^{3/2}}{3d} - \frac{Ba(a+a\tan(c+dx))^{3/2}2i}{3d} + \frac{4Aa^2\sqrt{a+a\tan(c+dx)}i}{d} - \frac{Ba^2\sqrt{a+a\tan(c+dx)}i}{d}4i - \frac{B(a+a\tan(c+dx))^{3/2}2i}{7ad} + \frac{\sqrt{2}B(-a)^{5/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}i}{z\sqrt{-a}}\right)4i}{d} + \frac{\sqrt{2}Aa^{5/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)}i}{z\sqrt{a}}\right)4i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] (2\*A\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/(5\*d) + (2\*A\*a\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*d) - (B\*a\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*2i)/(3\*d) + (4\*A\*a^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d - (B\*a^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*4i)/d - (B\*(a + a\*tan(c + d\*x)\*1i)^(7/2)\*2i)/(7\*a\*d) + (2^(1/2)\*B\*(-a)^(5/2)\*atan(2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*4i)/d + (2^(1/2)\*A\*a^(5/2)\*atan(2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*1i)/(2\*a^(1/2)))\*4i)/d

### 3.84 $\int (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=141

$$\frac{4\sqrt{2} a^{5/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{4a^2 (iA + B) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2a(iA + B)(a + ia \tan(c + dx))^{3/2}}{3d} + \frac{2B(a + ia \tan(c + dx))^{5/2}}{5d}$$

[Out]  $-4*a^{(5/2)}*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+4*a^2*(I*A+B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d+2/3*a*(I*A+B)*(a+I*a*\tan(d*x+c))^{(3/2)}/d+2/5*B*(a+I*a*\tan(d*x+c))^{(5/2)}/d$

**Rubi [A]**

time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ ,

Rules used = {3608, 3559, 3561, 212}

$$\frac{4\sqrt{2} a^{5/2} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d} + \frac{4a^2 (B + iA) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2a(B + iA)(a + ia \tan(c + dx))^{3/2}}{3d} + \frac{2B(a + ia \tan(c + dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(-4*\operatorname{Sqrt}[2]*a^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + (4*a^2*(I*A + B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/d + (2*a*(I*A + B)*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d) + (2*B*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)})/(5*d)$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3559

$\operatorname{Int}[(a_ + (b_)*\operatorname{tan}[(c_ + (d_)*(x_))])^{(n_)}, x\_Symbol] \rightarrow \operatorname{Simp}[b*((a + b*\operatorname{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[2*a, \operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^{(n-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{EqQ}[a^2 + b^2, 0] \ \&\& \operatorname{GtQ}[n, 1]$

Rule 3561

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\operatorname{tan}[(c_ + (d_)*(x_))])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$



Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx &= \frac{2B(a + ia \tan(c + dx))^{5/2}}{5d} - (-A + iB) \int (a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx)) dx \\
 &= \frac{2a(iA + B)(a + ia \tan(c + dx))^{3/2}}{3d} + \frac{2B(a + ia \tan(c + dx))^{5/2}}{5d} - (-A + iB) \int (a + ia \tan(c + dx))^{1/2} (A + B \tan(c + dx)) dx \\
 &= \frac{4a^2(iA + B) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2a(iA + B)(a + ia \tan(c + dx))^{3/2}}{3d} - (-A + iB) \int (a + ia \tan(c + dx))^{1/2} (A + B \tan(c + dx)) dx \\
 &= \frac{4a^2(iA + B) \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2a(iA + B)(a + ia \tan(c + dx))^{3/2}}{3d} - \frac{4\sqrt{2} a^{5/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{d}
 \end{aligned}$$

Mathematica [A]

time = 3.06, size = 236, normalized size = 1.67

$$\frac{\left( -4i\sqrt{2} (A - iB) e^{-3i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{1 + e^{2i(c+dx)}} \sinh^{-1} \left( e^{i(c+dx)} \right) + \frac{\sec^{\frac{5}{2}}(c+dx) \cos(2c) - i \sin(2c) (35(A+B) + (35A+41B) \cos(2(c+dx)) + (-5A+11iB) \sin(2(c+dx)))}{15(\cos(dx) + i \sin(dx))^2} \right) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{d \sec^{\frac{7}{2}}(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((((-4\*I)\*Sqrt[2]\*(A - I\*B)\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcSinh[E^(I\*(c + d\*x))])/E^((3\*I)\*(c + d\*x)) + (Sec[c + d\*x]^(5/2)\*(Cos[2\*c] - I\*Sin[2\*c])\*(35\*(I\*A + B) + ((35\*I)\*A + 41\*B)\*Cos[2\*(c + d\*x)] + (-5\*A + (11\*I)\*B)\*Sin[2\*(c + d\*x)]))/(15\*(Cos[d\*x] + I\*Sin[d\*x])^2))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

Maple [A]

time = 0.09, size = 141, normalized size = 1.00

method	result
--------	--------

derivativedivides	$2i \left( \frac{-iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{iBa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 2ia^2B \sqrt{a+ia \tan(dx+c)} + 2a^2A \sqrt{\dots} \right)$
default	$2i \left( \frac{-iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{iBa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 2ia^2B \sqrt{a+ia \tan(dx+c)} + 2a^2A \sqrt{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*(-1/5*I*B*(a+I*a*tan(d*x+c))^(5/2)-1/3*I*B*a*(a+I*a*tan(d*x+c))^(3/2)+1/3*A*a*(a+I*a*tan(d*x+c))^(3/2)-2*I*B*a^2*(a+I*a*tan(d*x+c))^(1/2)+2*a^2*A*(a+I*a*tan(d*x+c))^(1/2)-2*a^(5/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

**Maxima [A]**

time = 0.49, size = 134, normalized size = 0.95

$$2i \left( 15 \sqrt{2} (A - iB)a^{\frac{5}{2}} \log \left( -\frac{\sqrt{2} \sqrt{a - \sqrt{ia \tan(dx+c) + a}}}{\sqrt{2} \sqrt{a + \sqrt{ia \tan(dx+c) + a}}} \right) - 3i (ia \tan(dx+c) + a)^{\frac{3}{2}} Ba + 5 (ia \tan(dx+c) + a)^{\frac{3}{2}} (A - iB)a^2 + 30 \sqrt{ia \tan(dx+c) + a} (A - iB)a^3 \right)$$

15 ad

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*I*(15*sqrt(2)*(A - I*B)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x+c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x+c) + a))) - 3*I*(I*a*tan(d*x+c) + a)^(5/2)*B*a + 5*(I*a*tan(d*x+c) + a)^(3/2)*(A - I*B)*a^2 + 30*sqrt(I*a*tan(d*x+c) + a)*(A - I*B)*a^3)/(a*d)
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(108) = 216.

time = 2.82, size = 421, normalized size = 2.99

$$2 \left( 15 \sqrt{2} \sqrt{\frac{A^2 - 2AB - B^2}{d}} \sqrt{a + \sqrt{ia \tan(dx+c) + a}} \log \left( \frac{\sqrt{\frac{A^2 - 2AB - B^2}{d}} \sqrt{a + \sqrt{ia \tan(dx+c) + a}}}{\sqrt{\frac{A^2 - 2AB - B^2}{d}} \sqrt{a - \sqrt{ia \tan(dx+c) + a}}} \right) - 15 \sqrt{2} \sqrt{\frac{A^2 - 2AB - B^2}{d}} \sqrt{a + \sqrt{ia \tan(dx+c) + a}} \log \left( \frac{\sqrt{\frac{A^2 - 2AB - B^2}{d}} \sqrt{a - \sqrt{ia \tan(dx+c) + a}}}{\sqrt{\frac{A^2 - 2AB - B^2}{d}} \sqrt{a + \sqrt{ia \tan(dx+c) + a}}} \right) - 2 \sqrt{2} (2 - 15A - 13B) a^{\frac{5}{2}} \sqrt{a + \sqrt{ia \tan(dx+c) + a}} + 3(-1A - B) a^{\frac{3}{2}} \sqrt{a + \sqrt{ia \tan(dx+c) + a}} + 15(-1A - B) a^{\frac{3}{2}} \sqrt{a - \sqrt{ia \tan(dx+c) + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/15*(15*sqrt(2)*sqrt(-(A^2 - 2I*A*B - B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) + sqrt(-(A^2 - 2I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-I*d*x - I*c)/((-I*A - B)*a^2) - 15*sqrt(2)*sqrt(-(A^2 - 2I*A*B - B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt(-(A^2 - 2I*A*B -
```

$$B^2 * a^5 / d^2 * (d * e^{(2 * I * d * x + 2 * I * c)} + d) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)} * e^{(-I * d * x - I * c)} / ((-I * A - B) * a^2) - 2 * \sqrt{2} * (2 * (-10 * I * A - 13 * B) * a^2 * e^{(5 * I * d * x + 5 * I * c)} + 35 * (-I * A - B) * a^2 * e^{(3 * I * d * x + 3 * I * c)} + 15 * (-I * A - B) * a^2 * e^{(I * d * x + I * c)}) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)} / (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{5}{2}} (A + B \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(5/2)\*(A + B\*tan(c + d\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad** [B]

time = 0.96, size = 188, normalized size = 1.33

$$\frac{2B(a + a \tan(c + dx))^{5/2}}{5d} + \frac{Aa(a + a \tan(c + dx))^{3/2}}{3d} + \frac{2Ba(a + a \tan(c + dx))^{1/2}}{3d} + \frac{Aa^2 \sqrt{a + a \tan(c + dx)}}{d} + \frac{4Ba^2 \sqrt{a + a \tan(c + dx)}}{d} - \frac{\sqrt{2} A (-a)^{3/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)}}{2\sqrt{-a}}\right)}{d} + \frac{\sqrt{2} B a^{3/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)}}{2\sqrt{-a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/(5\*d) + (A\*a\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*2i)/(3\*d) + (2\*B\*a\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*d) + (A\*a^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*4i)/d + (4\*B\*a^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/d - (2^(1/2)\*A\*(-a)^(5/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)))/(2\*(-a)^(1/2)))\*4i)/d + (2^(1/2)\*B\*a^(5/2)\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*1i)/(2\*a^(1/2)))\*4i)/d

### 3.85 $\int \cot(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=147

$$\frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{4\sqrt{2}a^{5/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2a^2(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

[Out]  $-2a^{5/2}A \operatorname{arctanh}\left(\frac{(a+Ia \tan(dx+c))^{1/2}}{a^{1/2}}\right)/d + 4a^{5/2}(A-I B) \operatorname{arctanh}\left(\frac{1/2(a+Ia \tan(dx+c))^{1/2} \cdot 2^{1/2}}{a^{1/2}}\right) \cdot 2^{1/2}/d - 2a^2(A-I B) \operatorname{arctanh}\left(\frac{(a+Ia \tan(dx+c))^{1/2}}{d}\right) + 2/3 I a B (a+Ia \tan(dx+c))^{3/2}/d$

**Rubi [A]**

time = 0.35, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3675, 3681, 3561, 212, 3680, 65, 214}

$$\frac{4\sqrt{2}a^{5/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a^2(A-2iB) \sqrt{a+ia \tan(c+dx)}}{d} + \frac{2iaB(a+ia \tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c+d*x]*(a+I*a*\text{Tan}[c+d*x])^{5/2}*(A+B*\text{Tan}[c+d*x]),x]$

[Out]  $(-2a^{5/2}A \operatorname{ArcTanh}[\text{Sqrt}[a+Ia*\text{Tan}[c+d*x]]/\text{Sqrt}[a]])/d + (4*\text{Sqrt}[2]*a^{5/2}(A-I B) \operatorname{ArcTanh}[\text{Sqrt}[a+Ia*\text{Tan}[c+d*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])])/d - (2a^2(A-(2I)B) \text{Sqrt}[a+Ia*\text{Tan}[c+d*x]])/d + (((2I)/3)*a*B(a+Ia*\text{Tan}[c+d*x])^{3/2})/d$

**Rule 65**

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b}))^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

**Rule 214**

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
  b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x]))], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{2iaB(a + ia \tan(c + dx))^{3/2}}{3d} + \frac{2}{3} \int \cot(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx \\
&= -\frac{2a^2(A - 2iB)\sqrt{a + ia \tan(c + dx)}}{d} + \frac{2iaB(a + ia \tan(c + dx))^{3/2}}{3d} \\
&= -\frac{2a^2(A - 2iB)\sqrt{a + ia \tan(c + dx)}}{d} + \frac{2iaB(a + ia \tan(c + dx))^{3/2}}{3d} \\
&= -\frac{2a^2(A - 2iB)\sqrt{a + ia \tan(c + dx)}}{d} + \frac{2iaB(a + ia \tan(c + dx))^{3/2}}{3d} \\
&= \frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} \\
&= -\frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{2iaB(a + ia \tan(c + dx))^{3/2}}{3d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 429 vs.  $2(147) = 294$ .  
time = 8.01, size = 429, normalized size = 2.92

$$\frac{e^{-3ia\sqrt{a^2}}(8(A-iB)\sinh^{-1}(e^{i(c+dx)}) + \sqrt{2}A(\log(1-e^{i(c+dx)}) - \log(1+e^{i(c+dx)})) + \log(1-e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}) - \log(1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}})))(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{2}A\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)\sqrt{1+e^{2i(c+dx)}}\sec^2(c+dx)\cos(dx)+i\sin(dx)^2(A\cos(c+dx)+B\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[E^(I\*d\*x)]\*(8\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))] + Sqrt[2]\*A\*(Log[1 - E^(I\*(c + d\*x))] - Log[1 + E^(I\*(c + d\*x))] + Log[1 - E^(I\*(c + d\*x)) + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]] - Log[1 + E^(I\*(c + d\*x)) + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]]))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(Sqrt[2]\*d\*E^((2\*I)\*c)\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*Sec[c + d\*x]^(7/2)\*(Cos[d\*x] + I\*Sin[d\*x])^(5/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])) + (Cos[c + d\*x]^3\*((3\*A - (8\*I)\*B)\*((-2\*Cos[2\*c])/3 + ((2\*I)/3)\*Sin[2\*c]) + Sec[c + d\*x]\*(((2\*I)/3)\*B\*Cos[3\*c + d\*x] - (2\*B\*Sin[3\*c + d\*x])/3))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(d\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 964 vs.  $2(119) = 238$ .  
time = 0.55, size = 965, normalized size = 6.56

method	result	size
default	Expression too large to display	965

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $\frac{1}{6}d^2a^2 \left( (I \sin(dx+c) + \cos(dx+c)) \frac{a}{\cos(dx+c)} \right)^{1/2} (-28IB \cos(dx+c) + 12IA (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} 2^{1/2} \arctan(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) - 12IA \cos(dx+c) \sin(dx+c) - 12A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \operatorname{arctanh}(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) / \cos(dx+c)) 2^{1/2} \cos(dx+c) \sin(dx+c) + 12IB (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} 2^{1/2} \operatorname{arctanh}(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) / \cos(dx+c)) \sin(dx+c) + 12B (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \arctan(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) 2^{1/2} \cos(dx+c) \sin(dx+c) + 3IA (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) - 12A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} 2^{1/2} \operatorname{arctanh}(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) / \cos(dx+c)) \sin(dx+c) + 3IA (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cos(dx+c) \sin(dx+c) - 3A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \ln((\sin(dx+c) (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1) / \sin(dx+c)) \cos(dx+c) \sin(dx+c) + 12B (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} 2^{1/2} \arctan(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) - 3A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \ln((\sin(dx+c) (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1) / \sin(dx+c)) \sin(dx+c) + 32IB \cos(dx+c)^2 - 4IB - 12A \cos(dx+c)^2 + 12IA (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \arctan(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) 2^{1/2} \cos(dx+c) \sin(dx+c) - 32B \cos(dx+c) \sin(dx+c) + 12A \cos(dx+c) + 12IB (-2 \cos(dx+c) / (\cos(dx+c)+1))^{3/2} \operatorname{arctanh}(1/2 2^{1/2} (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \sin(dx+c) / \cos(dx+c)) 2^{1/2} \cos(dx+c) \sin(dx+c) + 4B \sin(dx+c)) / (I \sin(dx+c) + \cos(dx+c) - 1) / \cos(dx+c)$

**Maxima [A]**

time = 0.52, size = 154, normalized size = 1.05

$$\frac{6\sqrt{2}(A-iB)a^{\frac{5}{2}} \log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}}\right) - 3Aa^{\frac{5}{2}} \log\left(\frac{\sqrt{ia \tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia \tan(dx+c)+a}+\sqrt{a}}\right) - 2i(ia \tan(dx+c)+a)^{\frac{3}{2}}Ba + 6\sqrt{ia \tan(dx+c)+a}(A-2iB)a^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out]  $-1/3(6\sqrt{2})(A - IB)a^{5/2} \log(-(\sqrt{2})\sqrt{a} - \sqrt{Ia \tan(dx+c) + a}) / (\sqrt{2})\sqrt{a} + \sqrt{Ia \tan(dx+c) + a}) - 3Aa^{5/2} \log$

$$g((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a})) - 2*I*(I*a*\tan(d*x + c) + a)^{(3/2)}*B*a + 6*\sqrt{I*a*\tan(d*x + c) + a}*(A - 2*I*B)*a^2)/d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(112) = 224.

time = 2.78, size = 610, normalized size = 4.15

$$\frac{\frac{1}{6} \left( \frac{12 \sqrt{2} \sqrt{(A^2 - 2IA*B - B^2)a^5/d^2} (d e^{(2I*d*x + 2I*c)} + d) \log(4 * ((-I*A - B) a^3 e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2IA*B - B^2)a^5/d^2} (I*d e^{(2I*d*x + 2I*c)} + I*d) \sqrt{a/(e^{(2I*d*x + 2I*c)} + 1)})) e^{(-I*d*x - I*c)} / ((-I*A - B) a^2)} - 12 \sqrt{2} \sqrt{(A^2 - 2IA*B - B^2)a^5/d^2} (d e^{(2I*d*x + 2I*c)} + d) \log(4 * ((-I*A - B) a^3 e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2IA*B - B^2)a^5/d^2} (-I*d e^{(2I*d*x + 2I*c)} - I*d) \sqrt{a/(e^{(2I*d*x + 2I*c)} + 1)})) e^{(-I*d*x - I*c)} / ((-I*A - B) a^2)} - 3 \sqrt{A^2 a^5/d^2} (d e^{(2I*d*x + 2I*c)} + d) \log(16 * (3A a^3 e^{(2I*d*x + 2I*c)} + A a^3 + 2 \sqrt{2} \sqrt{A^2 a^5/d^2} (d e^{(3I*d*x + 3I*c)} + d e^{(I*d*x + I*c)}) \sqrt{a/(e^{(2I*d*x + 2I*c)} + 1)}) e^{(-2I*d*x - 2I*c)} / (A a)) + 3 \sqrt{A^2 a^5/d^2} (d e^{(2I*d*x + 2I*c)} + d) \log(16 * (3A a^3 e^{(2I*d*x + 2I*c)} + A a^3 - 2 \sqrt{2} \sqrt{A^2 a^5/d^2} (d e^{(3I*d*x + 3I*c)} + d e^{(I*d*x + I*c)}) \sqrt{a/(e^{(2I*d*x + 2I*c)} + 1)}) e^{(-2I*d*x - 2I*c)} / (A a)) - 4 \sqrt{2} * ((3A - 8I*B) a^2 e^{(3I*d*x + 3I*c)} + 3(A - 2I*B) a^2 e^{(I*d*x + I*c)}) \sqrt{a/(e^{(2I*d*x + 2I*c)} + 1)} / (d e^{(2I*d*x + 2I*c)} + d) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(12\*sqrt(2)\*sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^5/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(4\*((-I\*A - B)\*a^3\*e^(I\*d\*x + I\*c) - sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^5/d^2)\*(I\*d\*e^(2\*I\*d\*x + 2\*I\*c) + I\*d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a^2) - 12\*sqrt(2)\*sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^5/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(4\*((-I\*A - B)\*a^3\*e^(I\*d\*x + I\*c) - sqrt((A^2 - 2\*I\*A\*B - B^2)\*a^5/d^2)\*(-I\*d\*e^(2\*I\*d\*x + 2\*I\*c) - I\*d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a^2) - 3\*sqrt(A^2\*a^5/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(16\*(3\*A\*a^3\*e^(2\*I\*d\*x + 2\*I\*c) + A\*a^3 + 2\*sqrt(2)\*sqrt(A^2\*a^5/d^2)\*(d\*e^(3\*I\*d\*x + 3\*I\*c) + d\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-2\*I\*d\*x - 2\*I\*c)/(A\*a)) + 3\*sqrt(A^2\*a^5/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(16\*(3\*A\*a^3\*e^(2\*I\*d\*x + 2\*I\*c) + A\*a^3 - 2\*sqrt(2)\*sqrt(A^2\*a^5/d^2)\*(d\*e^(3\*I\*d\*x + 3\*I\*c) + d\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-2\*I\*d\*x - 2\*I\*c)/(A\*a)) - 4\*sqrt(2)\*((3\*A - 8\*I\*B)\*a^2\*e^(3\*I\*d\*x + 3\*I\*c) + 3\*(A - 2\*I\*B)\*a^2\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))/(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.





### 3.86 $\int \cot^2(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=158

$$-\frac{a^{5/2}(5iA + 2B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{4\sqrt{2} a^{5/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + a$$

[Out]  $-a^{(5/2)}*(5*I*A+2*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d+4*a^{(5/2)}*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+a^{(5/2)}*(I*A-2*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d-a*A*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

Rubi [A]

time = 0.36, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3674, 3675, 3681, 3561, 212, 3680, 65, 214}

$$-\frac{a^{5/2}(2B + 5iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{4\sqrt{2} a^{5/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{a^2(-2B + iA) \sqrt{a + ia \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)(a + ia \tan(c + dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^2*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-((a^{(5/2)}*((5*I)*A + 2*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/d) + (4*\operatorname{Sqrt}[2]*a^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + (a^2*(I*A - 2*B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d - (a*A*\operatorname{Cot}[c + d*x]*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/d$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b}))^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 212

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

#### Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

#### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

#### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

#### Rule 3681

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a

\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \cot^2(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx)(a + ia \tan(c + dx))^{3/2}}{d} + \int \frac{a^2(iA - 2B)\sqrt{a + ia \tan(c + dx)}}{d} dx - \frac{aA \cot(c + dx)(a + ia \tan(c + dx))^{3/2}}{d} \\
 &= \frac{a^2(iA - 2B)\sqrt{a + ia \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)(a + ia \tan(c + dx))^{3/2}}{d} \\
 &= \frac{a^2(iA - 2B)\sqrt{a + ia \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)(a + ia \tan(c + dx))^{3/2}}{d} \\
 &= \frac{a^2(iA - 2B)\sqrt{a + ia \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)(a + ia \tan(c + dx))^{3/2}}{d} \\
 &= \frac{4\sqrt{2} a^{5/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} \\
 &= -\frac{a^{5/2}(5iA + 2B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 413 vs. 2(158) = 316.

time = 7.22, size = 413, normalized size = 2.61

$$\frac{\left( e^{-30i \arctan\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)} \sqrt{1+e^{2i(c+dx)}} (32\sqrt{2}(iA+B) \operatorname{arcsinh}\left(e^{i(c+dx)}\right) + 20(iA+2B) \left(\log\left((-1+e^{i(c+dx)})^2\right) - \log\left(1+e^{i(c+dx)}\right)\right) + \log\left(1+3e^{2i(c+dx)}+2\sqrt{2}\sqrt{1+e^{2i(c+dx)}}-2e^{i(c+dx)}(1+\sqrt{2}\sqrt{1+e^{2i(c+dx)}})\right) - \log\left(1+3e^{2i(c+dx)}+2\sqrt{2}\sqrt{1+e^{2i(c+dx)}}+2e^{i(c+dx)}(1+\sqrt{2}\sqrt{1+e^{2i(c+dx)}})\right)) - \frac{2A \operatorname{arcsinh}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right) \operatorname{arcsinh}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)}{\sqrt{200(1+4i) \operatorname{Im}(i+e^{2i(c+dx)})}} \right) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{d \operatorname{Im}(1+4i) \operatorname{Im}(i+e^{2i(c+dx)})}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((Sqrt[E^(I\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x))))\*Sqrt[1 + E^((2\*I)\*(c + d\*x))])\*(32\*Sqrt[2]\*(I\*A + B)\*ArcSinh[E^(I\*(c + d\*x))] + 2\*((5\*I)\*A + 2\*B)\*(Log[(-1 + E^(I\*(c + d\*x)))^2] - Log[(1 + E^(I\*(c + d\*x)))^2] + Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))] - 2\*E^(I\*(c + d\*x))\*(1 + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]]) - Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))] + 2\*E^(I\*(c + d\*x))\*(1 + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]])))/E^((3\*I)\*(c + d\*x)) - (8\*(A\*Csc[c + d\*x] + 2\*B\*Sec[c + d\*x])\*(Cos[2\*c] - I\*Sin[2\*c]))/(Sqrt[Sec[c + d\*x]]\*(Cos[d\*x] + I\*Sin[d\*x])^2))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x])/(8\*d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1140 vs.  $2(132) = 264$ .

time = 0.56, size = 1141, normalized size = 7.22

method	result	size
default	Expression too large to display	1141

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/2/d*a^2*(5*I*A*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))-8*I*B*2^{1/2}*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})+8*A*2^{1/2}*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})+8*I*B*2^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+8*B*2^{1/2}*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c)-8*I*A*2^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-5*I*A*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+5*A*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})+2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})+2*B*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))-8*A*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})+2*I*A*\cos(d*x+c)*\sin(d*x+c)-2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}))*\cos(d*x+c)^2-8*B*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c)+4*I*B+8*I*A*2^{1/2}*\cos(d*x+c)^2*\operatorname{arctanh}(1/2*2^{1/2}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)/\cos(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+2*A*\cos(d*x+c)^2-5*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})+4*B*\cos(d*x+c)*\sin(d*x+c)-2*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))-2*A*\cos(d*x+c)-4*I*B*\cos(d*x+c)^2-4*B*\sin(d*x+c))*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{1/2}/(I*\sin(d*x+c)+\cos(d*x+c)-1)/\sin(d*x+c) \end{aligned}$$

**Maxima [A]**

time = 0.49, size = 163, normalized size = 1.03

$$\frac{i \left( 4\sqrt{2}(A-iB)a^{\frac{3}{2}} \log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}}\right) - (5A-2iB)a^{\frac{3}{2}} \log\left(\frac{\sqrt{ia \tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia \tan(dx+c)+a}+\sqrt{a}}\right) - 4i\sqrt{ia \tan(dx+c)+a}Ba - \frac{2i\sqrt{ia \tan(dx+c)+a}Aa}{\tan(dx+c)} \right) a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*I*(4*sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - (5*A - 2*I*B)*a^(3/2)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a))) - 4*I*sqrt(I*a*tan(d*x + c) + a)*B*a - 2*I*sqrt(I*a*tan(d*x + c) + a)*A*a/tan(d*x + c))*a/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 705 vs.  $2(125) = 250$ .

time = 2.79, size = 705, normalized size = 4.46



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 8*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - sqrt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-16*(3*(-5*I*A - 2*B)*a^3*e^(2*I*d*x + 2*I*c) + (-5*I*A - 2*B)*a^3 + 2*sqrt(2)*sqrt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c)))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-2*I*d*x - 2*I*c)/((5*I*A + 2*B)*a) + sqrt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-16*(3*(-5*I*A - 2*B)*a^3*e^(2*I*d*x + 2*I*c) + (-5*I*A - 2*B)*a^3 - 2*sqrt(2)*sqrt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c)))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-2*I*d*x - 2*I*c)/((5*I*A + 2*B)*a) + 4*sqrt(2)*((I*A + 2*B)*a^2*e^(3*I*d*x + 3*I*c) + (I*A - 2*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^2, x)

**Mupad [B]**

time = 8.38, size = 2500, normalized size = 15.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] atan((d^4\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*(((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)/(8\*a^6) - (57\*A^2\*a^5)/(8\*d^2) + (9\*B^2\*a^5)/(2\*d^2) + (A\*B\*a^5\*21i)/(2\*d^2))^(1/2)\*((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)\*6i)/(A^3\*a^14\*d\*126i - 336\*B^3\*a^14\*d - A\*B^2\*a^14\*d\*1032i + 876\*A^2\*B\*a^14\*d + A\*a^3\*d^3\*((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)\*2i - 4\*B\*a^3\*d^3\*((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)) + (A^2\*a^8\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*(((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)/(8\*a^6) - (57\*A^2\*a^5)/(8\*d^2) + (9\*B^2\*a^5)/(2\*d^2) + (A\*B\*a^5\*21i)/(2\*d^2))^(1/2)\*14i)/(A^3\*a^11\*d\*126i - 336\*B^3\*a^11\*d + A\*d^3\*((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)\*2i - 4\*B\*d^3\*((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2) - A\*B^2\*a^11\*d\*1032i + 876\*A^2\*B\*a^11\*d) - (B^2\*a^8\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*(((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2)/(8\*a^6) - (57\*A^2\*a^5)/(8\*d^2) + (9\*B^2\*a^5)/(2\*d^2) + (A\*B\*a^5\*21i)/(2\*d^2))^(1/2)\*56i)/(A^3\*a^11\*d\*126i - 336\*B^3\*a^11\*d + A\*d^3\*((49\*A^4\*a^22)/d^4 + (784\*B^4\*a^22)/d^4 - (2328\*A^2\*B^2\*a^22)/d^4 + (A\*B^3\*a^22\*2464i)/d^4 - (A^3\*B\*a^22\*616i)/d^4)^(1/2))

$$\begin{aligned}
& 2)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616 \\
& i)/d^4)^{(1/2)*2i - 4*B*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22) \\
& A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2) - \\
& A*B^2*a^11*d*1032i + 876*A^2*B*a^11*d) + (88*A*B*a^8*d^2*(a + a*tan(c + d* \\
& x)*1i)^{(1/2)*(((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22) \\
& /d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)/(8*a^6) - (57* \\
& A^2*a^5)/(8*d^2) + (9*B^2*a^5)/(2*d^2) + (A*B*a^5*21i)/(2*d^2))^{(1/2)))/(A^3 \\
& *a^11*d*126i - 336*B^3*a^11*d + A*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d \\
& ^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d \\
& ^4)^{(1/2)*2i - 4*B*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2* \\
& B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2) - A*B \\
& ^2*a^11*d*1032i + 876*A^2*B*a^11*d)*(((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d \\
& ^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d \\
& ^4)^{(1/2)/(8*a^6) - (57*A^2*a^5)/(8*d^2) + (9*B^2*a^5)/(2*d^2) + (A*B*a^5*2 \\
& 1i)/(2*d^2))^{(1/2)*2i - atan((d^4*(a + a*tan(c + d*x)*1i)^{(1/2)*((9*B^2*a^5) \\
& )/(2*d^2) - (57*A^2*a^5)/(8*d^2) - ((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 \\
& - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4) \\
& ^{(1/2)/(8*a^6) + (A*B*a^5*21i)/(2*d^2))^{(1/2)*((49*A^4*a^22)/d^4 + (784*B^4 \\
& *a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22 \\
& *616i)/d^4)^{(1/2)*6i)/(A^3*a^14*d*126i - 336*B^3*a^14*d - A*B^2*a^14*d*1032 \\
& i + 876*A^2*B*a^14*d - A*a^3*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - \\
& (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{( \\
& 1/2)*2i + 4*B*a^3*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B \\
& ^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)) - (A^ \\
& 2*a^8*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)*((9*B^2*a^5)/(2*d^2) - (57*A^2*a^5) \\
& )/(8*d^2) - ((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^ \\
& 4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)/(8*a^6) + (A*B*a^ \\
& 5*21i)/(2*d^2))^{(1/2)*14i)/(A^3*a^11*d*126i - 336*B^3*a^11*d - A*d^3*((49*A \\
& ^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2 \\
& 464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)*2i + 4*B*d^3*((49*A^4*a^22)/d^4 + \\
& (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^ \\
& 3*B*a^22*616i)/d^4)^{(1/2) - A*B^2*a^11*d*1032i + 876*A^2*B*a^11*d) + (B^2*a \\
& ^8*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)*((9*B^2*a^5)/(2*d^2) - (57*A^2*a^5)/(8 \\
& *d^2) - ((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + \\
& (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)/(8*a^6) + (A*B*a^5*2 \\
& 1i)/(2*d^2))^{(1/2)*56i)/(A^3*a^11*d*126i - 336*B^3*a^11*d - A*d^3*((49*A^4* \\
& a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464 \\
& i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)*2i + 4*B*d^3*((49*A^4*a^22)/d^4 + (78 \\
& 4*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B \\
& *a^22*616i)/d^4)^{(1/2) - A*B^2*a^11*d*1032i + 876*A^2*B*a^11*d) - (88*A*B*a \\
& ^8*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)*((9*B^2*a^5)/(2*d^2) - (57*A^2*a^5)/(8 \\
& *d^2) - ((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + \\
& (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^{(1/2)/(8*a^6) + (A*B*a^5*2 \\
& 1i)/(2*d^2))^{(1/2)))/(A^3*a^11*d*126i - 336*B^3*a^11*d - A*d^3*((49*A^4*a^22 \\
& )/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)...
\end{aligned}$$



### 3.87 $\int \cot^3(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=173

$$\frac{a^{5/2}(23A - 20iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4d} - \frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

[Out]  $1/4*a^{(5/2)}*(23*A-20*I*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d-4*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d-1/4*a^2*(7*I*A+4*B)*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/2*a*A*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.40, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3674, 3681, 3561, 212, 3680, 65, 214}

$$\frac{a^{5/2}(23A - 20iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4d} - \frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a^2(4B + 7iA) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} - \frac{aA \cot^2(c + dx) (a + ia \tan(c + dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^3*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(a^{(5/2)}*(23*A - (20*I)*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(4*d) - (4*\operatorname{Sqrt}[2]*a^{(5/2)}*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d - (a^2*((7*I)*A + 4*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(4*d) - (a*A*\operatorname{Cot}[c + d*x]^2*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3561

Int[Sqrt[(a\_) + (b\_.)\*tan[(c\_) + (d\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3674

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[(((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)]))/((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
\int \cot^3(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)(a + ia \tan(c + dx))^{3/2}}{2d} + \\
&= -\frac{a^2(7iA + 4B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
&= -\frac{a^2(7iA + 4B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
&= -\frac{a^2(7iA + 4B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
&= -\frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\
&= -\frac{a^{5/2}(23A - 20iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 427 vs.  $2(173) = 346$ .  
time = 8.72, size = 427, normalized size = 2.47

$$\frac{(-2a^{5/2+5iA} \sqrt{\frac{a^{5/2+5iA}}{1+a^{5/2+5iA}}}) \sqrt{1+a^{5/2+5iA}} (4\sqrt{2}(A-iB) \operatorname{ArcSinh}\left(\frac{e^{i(c+dx)}}{\sqrt{1+a^{5/2+5iA}}}\right) + (23A-20iB) (\log((-1+e^{i(c+dx)})^2) - \log((1+e^{i(c+dx)})^2) + \log(3+3e^{i(c+dx)} + 2\sqrt{2}\sqrt{1+a^{5/2+5iA}} - 2a^{i(c+dx)}(1+\sqrt{2}\sqrt{1+a^{5/2+5iA}})) - \log(3+3e^{i(c+dx)} + 2\sqrt{2}\sqrt{1+a^{5/2+5iA}} + 2a^{i(c+dx)}(1+\sqrt{2}\sqrt{1+a^{5/2+5iA}}))) - \frac{\operatorname{Im}\left(\frac{a^{5/2+5iA} \sqrt{1+a^{5/2+5iA}}}{\sqrt{1+a^{5/2+5iA}}}\right)}{4d \sqrt{2} \sqrt{1+a^{5/2+5iA}}})}{32d \sec^2(c+dx)(A \cos(c+dx) + B \sin(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((((-2\*sqrt[E^(I\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x)))]\*sqrt[1 + E^((2\*I)\*(c + d\*x))])\*(64\*sqrt[2]\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))] + (23\*A - (20\*I)\*B) \* (Log[(-1 + E^(I\*(c + d\*x)))^2] - Log[(1 + E^(I\*(c + d\*x)))^2] + Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]] - 2\*E^(I\*(c + d\*x))\*(1 + sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]]) - Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]] + 2\*E^(I\*(c + d\*x))\*(1 + sqrt[2]\*sqrt[1 + E^((2\*I)\*(c + d\*x))]])))/E^((3\*I)\*(c + d\*x)) - (8\*Csc[c + d\*x]\*(2\*A\*Csc[c + d\*x] + ((9\*I)\*A + 4\*B)\*Sec[c + d\*x])\*(Cos[2\*c] - I\*Sin[2\*c]))/(Sec[c + d\*x]^(3/2)\*(Cos[d\*x] + I\*Sin[d\*x])^2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(32\*d\*Sec[c + d\*x]^(7/2)\*(A\*cos[c + d\*x] + B\*sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1291 vs.  $2(141) = 282$ .  
time = 0.55, size = 1292, normalized size = 7.47

method	result	size
default	Expression too large to display	1292

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{8}d^2a^2(-18A\cos(dx+c)-8B\cos(dx+c)\sin(dx+c)+23IA(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\arctan(1/(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})\cos(dx+c)^2\sin(dx+c)-4A\cos(dx+c)^2+20IB(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\ln((\sin(dx+c)*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}-\cos(dx+c)+1)/\sin(dx+c))\cos(dx+c)^2\sin(dx+c)-32IA(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\arctan(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*2^{1/2}\sin(dx+c)-32IB(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\operatorname{arctanh}(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*2^{1/2}\sin(dx+c)/\cos(dx+c)+32A*2^{1/2}\operatorname{arctanh}(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*2^{1/2}\sin(dx+c)/\cos(dx+c))*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\sin(dx+c)-32B*2^{1/2}\operatorname{arctan}(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\sin(dx+c)-8IB\cos(dx+c)^3+8IB\cos(dx+c)-23IA\arctan(1/(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\sin(dx+c)-20IB(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\ln((\sin(dx+c)*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}-\cos(dx+c)+1)/\sin(dx+c))*\sin(dx+c)+32IA(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\arctan(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*2^{1/2}\cos(dx+c)^2\sin(dx+c)+32IB(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\operatorname{arctanh}(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*2^{1/2}\cos(dx+c)^2\sin(dx+c)-32A(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\operatorname{arctanh}(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*2^{1/2}\cos(dx+c)^2\sin(dx+c)+32B(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\arctan(1/2*2^{1/2}*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*\cos(dx+c)^2\sin(dx+c)*2^{1/2}+8B\cos(dx+c)^2\sin(dx+c)-23A(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\ln((\sin(dx+c)*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^2\sin(dx+c)+20B(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\arctan(1/(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*\cos(dx+c)^2\sin(dx+c)+22A\cos(dx+c)^3-20B(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\arctan(1/(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2})*\sin(dx+c)+23A\ln((\sin(dx+c)*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}-\cos(dx+c)+1)/\sin(dx+c))*(-2\cos(dx+c)/(\cos(dx+c)+1))^{1/2}\sin(dx+c)+22IA\cos(dx+c)^2\sin(dx+c)-18IA\cos(dx+c)\sin(dx+c))*((I\sin(dx+c)+\cos(dx+c))*a/\cos(dx+c))^{1/2}/(-1+\cos(dx+c))/(I\sin(dx+c)+\cos(dx+c)-1)/(\cos(dx+c)+1)$$

**Maxima [A]**

time = 0.56, size = 206, normalized size = 1.19

$$\left( 16\sqrt{2}(A-iB)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-ia\tan(dx+c)+a}{\sqrt{2}\sqrt{a}+ia\tan(dx+c)+a}\right)-(23A-20iB)\sqrt{a}\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)+\frac{2((ia\tan(dx+c)+a)^2(9A-4iB)a-\sqrt{ia\tan(dx+c)+a}(7A-4iB)a^2)}{(ia\tan(dx+c)+a)^2-2(ia\tan(dx+c)+a)a^2} \right) a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{8}*(16*\sqrt{2}*(A - I*B)*\sqrt{a}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a}))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a})) - (23*A - 20*I*B)*\sqrt{a}*\log((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a})) + 2*((I*a*\tan(d*x + c) + a)^{(3/2)}*(9*A - 4*I*B)*a - \sqrt{I*a*\tan(d*x + c) + a}*(7*A - 4*I*B)*a^2)/((I*a*\tan(d*x + c) + a)^2 - 2*(I*a*\tan(d*x + c) + a)*a + a^2))/a^2/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs.  $2(134) = 268$ .  
time = 2.12, size = 774, normalized size = 4.47

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/16*(32*\sqrt{2}*\sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) - 32*\sqrt{2}*\sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) + \sqrt{(529*A^2 - 920*I*A*B - 400*B^2)*a^5/d^2}*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-16*(3*(-23*I*A - 20*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-23*I*A - 20*B)*a^3 + 2*\sqrt{2}*\sqrt{(529*A^2 - 920*I*A*B - 400*B^2)*a^5/d^2}*(I*d*e^{(3*I*d*x + 3*I*c)} + I*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((23*I*A + 20*B)*a)) - \sqrt{(529*A^2 - 920*I*A*B - 400*B^2)*a^5/d^2}*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-16*(3*(-23*I*A - 20*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-23*I*A - 20*B)*a^3 + 2*\sqrt{2}*\sqrt{(529*A^2 - 920*I*A*B - 400*B^2)*a^5/d^2}*(-I*d*e^{(3*I*d*x + 3*I*c)} - I*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((23*I*A + 20*B)*a)) - 4*\sqrt{2}*((11*A - 4*I*B)*a^2*e^{(5*I*d*x + 5*I*c)} + 4*A*a^2*e^{(3*I*d*x + 3*I*c)} - (7*A - 4*I*B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d) \end{aligned}$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)`

**Mupad** [B]

time = 8.47, size = 2500, normalized size = 14.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)`

[Out] 
$$2*\operatorname{atanh}\left(\frac{17*A^2*a^8*d^2*(a + a*\tan(c + d*x)*1i)^{1/2}*((1041*A^2*a^5)/(128*d^2) - ((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2}/(64*a^6) - (57*B^2*a^5)/(8*d^2) - (A*B*a^5*243i)/(16*d^2)}{4*((663*A^3*a^11*d)/16 - B^3*a^11*d*252i - (7*A*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2})/8 + (B*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2}*1i)/2 + 507*A*B^2*a^11*d + (A^2*B*a^11*d*861i)/4}\right) - (3*d^4*(a + a*\tan(c + d*x)*1i)^{1/2}*((1041*A^2*a^5)/(128*d^2) - ((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2}/(64*a^6) - (57*B^2*a^5)/(8*d^2) - (A*B*a^5*243i)/(16*d^2))^{1/2}*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2})/(2*((663*A^3*a^14*d)/16 - B^3*a^14*d*252i + 507*A*B^2*a^14*d + (A^2*B*a^14*d*861i)/4 - (7*A*a^3*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2})/8 + (B*a^3*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^{1/2}*1i)/2) + (28*B^2*a^8*d^2*(a + a*\tan(c + d*x)*1i)^{1/2}*((1041*A^2*a^5)/(128*d^2) - ((289*A$$



$$\begin{aligned} & *a^{22}5824i)/d^4 + (A^3B*a^{22}884i)/d^4)^{(1/2)*1i)/2 + 507*A*B^2*a^{11}*d + \\ & (A^2*B*a^{11}*d*861i)/4) + (A*B*a^8*d^2*(a + a*ta\dots \end{aligned}$$



$$3.88 \quad \int \cot^4(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=217

$$\frac{a^{5/2}(45iA + 46B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{8d} - \frac{4\sqrt{2} a^{5/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

[Out]  $1/8*a^{(5/2)}*(45*I*A+46*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d-4*a^{(5/2)}*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+1/8*a^2*(19*A-18*I*B)*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/4*a^2*(3*I*A+2*B)*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/3*a*A*\cot(d*x+c)^3*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.53, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3674, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{a^{5/2}(46B + 45iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{8d} - \frac{4\sqrt{2} a^{5/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a^2(2B + 3iA) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} + \frac{a^2(19A - 18iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} - \frac{aA \cot^3(c + dx) (a + ia \tan(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^4*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(a^{(5/2)}*((45*I)*A + 46*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(8*d) - (4*\operatorname{Sqrt}[2]*a^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + (a^2*(19*A - (18*I)*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(8*d) - (a^2*((3*I)*A + 2*B)*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(4*d) - (a*A*\operatorname{Cot}[c + d*x]^3*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3561

Int[Sqrt[(a\_) + (b\_.)\*tan[(c\_) + (d\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3674

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3679

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3681

Int((((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)]))/((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*

d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \cot^4(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx)(a + ia \tan(c + dx))^{3/2}}{3d} + \\
 &= -\frac{a^2(3iA + 2B) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} \\
 &= \frac{a^2(19A - 18iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{a^2(19A - 18iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{a^2(19A - 18iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= -\frac{4\sqrt{2} a^{5/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\
 &= \frac{a^{5/2}(45iA + 46B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{8d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 634 vs. 2(217) = 434.

time = 8.67, size = 634, normalized size = 2.92

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((-1/32\*I)\*Sqrt[E^(I\*d\*x)]\*(256\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))] + Sqrt[2]\*(45\*A - (46\*I)\*B)\*(Log[(-1 + E^(I\*(c + d\*x)))^2] - Log[(1 + E^(I\*(c + d\*x)))^2] + Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]] - 2\*E^(I\*(c + d\*x))\*(1 + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]]) - Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]] + 2\*E^(I\*(c + d\*x))\*(1 + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]]))\*(a + I\*a\*Tan[c + d\*x])

$$\begin{aligned} & (c + dx)^{5/2} (A + B \tan[c + dx]) / (\sqrt{2} d E^{(2I)c} \sqrt{E^{I(c + dx)}} / (1 + E^{(2I)(c + dx)})) \sqrt{1 + E^{(2I)(c + dx)}} \sec[c + dx] \\ & ^{7/2} (\cos[dx] + I \sin[dx])^{5/2} (A \cos[c + dx] + B \sin[c + dx]) + \\ & (\cos[c + dx]^3 (\csc[c] (65A \cos[c] - (54I) B \cos[c] + (26I) A \sin[c] + \\ & 12B \sin[c]) (\cos[2c]/24 - (I/24) \sin[2c]) + \csc[c] \csc[c + dx]^2 (-4A \cos[c] - \\ & (13I) A \sin[c] - 6B \sin[c]) (\cos[2c]/12 - (I/12) \sin[2c]) + A \csc[c] \csc[c + dx]^3 \\ & (\cos[2c]/3 - (I/3) \sin[2c]) \sin[dx] + \csc[c] \csc[c + dx] (\cos[2c]/24 - (I/24) \sin[2c]) \\ & (-65A \sin[dx] + (54I) B \sin[dx])) (a + I a \tan[c + dx])^{5/2} (A + B \tan[c + dx]) / (d (\cos[dx] + I \sin[dx])^2 (A \cos[c + dx] + B \sin[c + dx])) \end{aligned}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2505 vs.  $2(179) = 358$ .  
time = 0.63, size = 2506, normalized size = 11.55

method	result	size
default	Expression too large to display	2506

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(dx+c)^4*(a+I*a*tan(dx+c))^(5/2)*(A+B*tan(dx+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/48/d*a^2*(52*A*\cos(dx+c)^2*\sin(dx+c)-114*A*\cos(dx+c)*\sin(dx+c)-192*I \\ & *A*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos \\ & (dx+c)/(\cos(dx+c)+1))^{(1/2)}*\sin(dx+c)/\cos(dx+c))*\sin(dx+c)-108*B*\cos(d \\ & *x+c)^3-132*B*\cos(dx+c)^4+132*B*\cos(dx+c)^2+108*B*\cos(dx+c)-192*A*2^{(1/2)} \\ & )*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(c \\ & os(dx+c)+1))^{(1/2)})*\cos(dx+c)*\sin(dx+c)-192*B*2^{(1/2)}*(-2*\cos(dx+c)/(co \\ & s(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & )*\sin(dx+c)/\cos(dx+c))*\cos(dx+c)*\sin(dx+c)-132*I*B*\cos(dx+c)^3*\sin(dx+x \\ & c)-24*I*B*\cos(dx+c)^2*\sin(dx+c)+108*I*B*\cos(dx+c)*\sin(dx+c)+114*I*A*\cos \\ & (dx+c)+166*I*A*\cos(dx+c)^2-130*I*A*\cos(dx+c)^3-182*I*A*\cos(dx+c)^4+192* \\ & I*A*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*co \\ & s(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\sin(dx+c)/\cos(dx+c))*\cos(dx+c)^3*\sin(dx+x \\ & c)-192*I*B*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}* \\ & (-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)^3*\sin(dx+c)+192*I*A*2^{(1/2)} \\ & )*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/ \\ & (\cos(dx+c)+1))^{(1/2)}*\sin(dx+c)/\cos(dx+c))*\cos(dx+c)^2*\sin(dx+c)-192*I* \\ & B*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d \\ & *x+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)^2*\sin(dx+c)-192*I*A*2^{(1/2)}*(-2*co \\ & s(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+ \\ & c)+1))^{(1/2)}*\sin(dx+c)/\cos(dx+c))*\cos(dx+c)*\sin(dx+c)+192*I*B*2^{(1/2)}*( \\ & -2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos \\ & (dx+c)+1))^{(1/2)})*\cos(dx+c)*\sin(dx+c)+192*I*B*2^{(1/2)}*(-2*\cos(dx+c)/(\cos \\ & (dx+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})* \end{aligned}$$

$$\begin{aligned} & \sin(dx+c)+192*A*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)} \\ & (1/2)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)^3*\sin(dx+c)+192*B*2^{(1/2)} \\ & (-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \sin(dx+c)/\cos(dx+c))*\cos(dx+c)^3*\sin(dx+c)+192*A*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \arctan(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)^2*\sin(dx+c)+192*B*2^{(1/2)} \\ & (-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \sin(dx+c)/\cos(dx+c))*\cos(dx+c)^2*\sin(dx+c)+135*I*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^3*\sin(dx+c) \\ & -138*I*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}) \\ & *\cos(dx+c)^3*\sin(dx+c)+135*I*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & -\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^2*\sin(dx+c)-138*I*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)^2*\sin(dx+c)-135*I*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)*\sin(dx+c) \\ & +138*I*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}) \\ & *\cos(dx+c)*\sin(dx+c)-192*A*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)} \\ & (-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\sin(dx+c)-135*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)*\sin(dx+c)-192*B*2^{(1/2)} \\ & (-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \sin(dx+c)/\cos(dx+c))*\sin(dx+c)-138*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c) \\ & *(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)*\sin(dx+c) \\ & -135*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}) \\ & *\sin(dx+c)-138*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & -\cos(dx+c)+1)/\sin(dx+c))*\sin(dx+c)+182*A*\cos(dx+c)^3*\sin(dx+c)+135*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)^3*\sin(dx+c)+138*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^2*\sin(dx+c) \\ & -135*I*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & -\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^2*\sin(dx+c)-135*I*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)} \\ & \ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\sin(dx+c) \\ & +138*I*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}) \\ & *\sin(dx+c))*((I*\sin(dx+c)+\cos(dx+c))*a/\cos(dx+c))^{(1/2)}/(-1+\cos(dx+c))/ \\ & (I*\sin(dx+c)+\cos(dx+c)-1)/(\cos(dx+c)+1)^2 \end{aligned}$$

Maxima [A]



$$+ 1)))e^{(-2I*d*x - 2I*c)/((45I*A + 46*B)*a)} + 4\sqrt{2}*((91I*A + 66*B)*a^2e^{(7I*d*x + 7I*c)} - 7*(I*A + 6*B)*a^2e^{(5I*d*x + 5I*c)} + (-59I*A - 66*B)*a^2e^{(3I*d*x + 3I*c)} - 3*(-13I*A - 14*B)*a^2e^{(I*d*x + I*c)})\sqrt{a/(e^{(2I*d*x + 2I*c)} + 1)})/(d*e^{(6I*d*x + 6I*c)} - 3*d*e^{(4I*d*x + 4I*c)} + 3*d*e^{(2I*d*x + 2I*c)} - d)$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^4, x)

**Mupad** [B]

time = 8.47, size = 2500, normalized size = 11.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out]  $2*\operatorname{atanh}((23A^2a^8d^2(a + a*\tan(c + d*x)*1i)^{(1/2)}*((1041B^2a^5)/(128d^2) - (4073A^2a^5)/(512d^2) - ((529A^4a^{22})/(64d^4) + (289B^4a^{22})/(4d^4) + (149A^2B^2a^{22})/(8d^4) + (A*B^3a^{22}*187i)/(2d^4) + (A^3B*a^{22}*253i)/(8d^4))^{(1/2)})/(64a^6) + (A*B*a^5*2059i)/(128d^2))^{(1/2)})/(4*((A^3a^{11}d*1771i)/32 + (663B^3a^{11}d)/4 - (A*d^3*((529A^4a^{22})/(64d^4) + (289B^4a^{22})/(4d^4) + (149A^2B^2a^{22})/(8d^4) + (A*B^3a^{22}*187i)/(2d^4) + (A^3B*a^{22}*253i)/(8d^4))^{(1/2)}*13i)/4 - (7B*d^3*((529A^4a^{22})/(64d^4) + (289B^4a^{22})/(4d^4) + (149A^2B^2a^{22})/(8d^4) + (A*B^3a^{22}*187i)/(2d^4) + (A^3B*a^{22}*253i)/(8d^4))^{(1/2)})/2 + (A*B^2a^{11}d*2167i)/8 - (797A^2B*a^{11}d)/16)) - (6d^4*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((1$

$$\begin{aligned}
& 041*B^2*a^5)/(128*d^2) - (4073*A^2*a^5)/(512*d^2) - ((529*A^4*a^22)/(64*d^4) \\
& ) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i) \\
& /(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)}/(64*a^6) + (A*B*a^5*2059i)/(128 \\
& *d^2))^{(1/2)}*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B \\
& ^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{( \\
& 1/2)}/((A^3*a^14*d*1771i)/32 + (663*B^3*a^14*d)/4 + (A*B^2*a^14*d*2167i)/8 \\
& - (797*A^2*B*a^14*d)/16 - (A*a^3*d^3*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^ \\
& 22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3 \\
& *B*a^22*253i)/(8*d^4))^{(1/2)}*13i)/4 - (7*B*a^3*d^3*((529*A^4*a^22)/(64*d^4) \\
& + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/ \\
& (2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)})/2) + (17*B^2*a^8*d^2*(a + a*tan \\
& (c + d*x)*1i)^{(1/2)}*((1041*B^2*a^5)/(128*d^2) - (4073*A^2*a^5)/(512*d^2) - \\
& ((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d \\
& ^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)}/(64*a^6) \\
& + (A*B*a^5*2059i)/(128*d^2))^{(1/2)}/((A^3*a^11*d*1771i)/32 + (663*B^3*a^11 \\
& *d)/4 - (A*d^3*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2 \\
& *B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4)) \\
& ^{(1/2)}*13i)/4 - (7*B*d^3*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) \\
& + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i \\
& )/(8*d^4))^{(1/2)})/2 + (A*B^2*a^11*d*2167i)/8 - (797*A^2*B*a^11*d)/16) + (A* \\
& B*a^8*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)}*((1041*B^2*a^5)/(128*d^2) - (4073*A \\
& ^2*a^5)/(512*d^2) - ((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (14 \\
& 9*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8* \\
& d^4))^{(1/2)}/(64*a^6) + (A*B*a^5*2059i)/(128*d^2))^{(1/2)}*11i)/((A^3*a^11*d*1 \\
& 771i)/32 + (663*B^3*a^11*d)/4 - (A*d^3*((529*A^4*a^22)/(64*d^4) + (289*B^4*a \\
& ^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A \\
& ^3*B*a^22*253i)/(8*d^4))^{(1/2)}*13i)/4 - (7*B*d^3*((529*A^4*a^22)/(64*d^4) + \\
& (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2 \\
& *d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)})/2 + (A*B^2*a^11*d*2167i)/8 - (797 \\
& *A^2*B*a^11*d)/16))*((1041*B^2*a^5)/(128*d^2) - (4073*A^2*a^5)/(512*d^2) - \\
& ((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d \\
& ^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)}/(64*a^6) \\
& + (A*B*a^5*2059i)/(128*d^2))^{(1/2)} + 2*atanh((6*d^4*(a + a*tan(c + d*x)*1i \\
& )^{(1/2)}*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a \\
& ^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2) \\
& }/(64*a^6) - (4073*A^2*a^5)/(512*d^2) + (1041*B^2*a^5)/(128*d^2) + (A*B*a^5* \\
& 2059i)/(128*d^2))^{(1/2)}*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + \\
& (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i) \\
& /((A^3*a^14*d*1771i)/32 + (663*B^3*a^14*d)/4 + (A*B^2*a^14*d*2167i)/8 - (797*A^2*B*a^14*d)/16 + (A*a^3*d^3*((529*A^4*a^22)/(64*d^4) + \\
& (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2* \\
& d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)}*13i)/4 + (7*B*a^3*d^3*((529*A^4*a^2 \\
& 2)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3* \\
& a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^{(1/2)})/2) + (23*A^2*a^8*d^2 \\
& *(a + a*tan(c + d*x)*1i)^{(1/2)}*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(
\end{aligned}$$





$$3.89 \quad \int \cot^5(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=261

$$\frac{3a^{5/2}(121A - 120iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{64d} + \frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

[Out]  $-3/64*a^{(5/2)}*(121*A-120*I*B)*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d+4*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}/d+1/64*a^2*(149*I*A+152*B)*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(1/2)}/d+1/96*a^2*(107*A-104*I*B)*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/24*a^2*(11*I*A+8*B)*\cot(d*x+c)^3*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/4*a*A*\cot(d*x+c)^4*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

Rubi [A]

time = 0.66, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3674, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{3a^{5/2}(121A - 120iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{64d} + \frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a^2(8B + 11iA) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{24d} + \frac{a^2(107A - 104iB) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{96d} + \frac{a^2(152B + 149iA) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{64d} - \frac{aA \cot^4(c + dx) (a + ia \tan(c + dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^5*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(-3*a^{(5/2)}*(121*A - (120*I)*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(64*d) + (4*\operatorname{Sqrt}[2]*a^{(5/2)}*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/d + (a^2*((149*I)*A + 152*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(64*d) + (a^2*(107*A - (104*I)*B)*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(96*d) - (a^2*((11*I)*A + 8*B)*\operatorname{Cot}[c + d*x]^3*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(24*d) - (a*A*\operatorname{Cot}[c + d*x]^4*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(4*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 212

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{NegQ}[a/b] \&\& \operatorname{Gt}$

$Q[a, 0] \parallel LtQ[b, 0]$

#### Rule 214

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[\{a, b\}, x] \&\& NegQ[a/b]$

#### Rule 3561

$Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x\_Symbol] := Dist[-2*(b/d), Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[\{a, b, c, d\}, x] \&\& EqQ[a^2 + b^2, 0]$

#### Rule 3674

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^{(n_)}, x\_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^{(m-1)}*((c + d*Tan[e + f*x])^{(n+1)})/(d*f*(b*c + a*d)*(n+1)), x] - Dist[a/(d*(b*c + a*d)*(n+1)), Int[(a + b*Tan[e + f*x])^{(m-1)}*(c + d*Tan[e + f*x])^{(n+1)}*Simp[A*b*d*(m-n-2) - B*(b*c*(m-1) + a*d*(n+1)) + (a*A*d*(m+n) - B*(a*c*(m-1) + b*d*(n+1)))*Tan[e + f*x], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B\}, x] \&\& NeQ[b*c - a*d, 0] \&\& EqQ[a^2 + b^2, 0] \&\& GtQ[m, 1] \&\& LtQ[n, -1]$

#### Rule 3679

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^{(n_)}, x\_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^{(n+1)})/(f*(n+1)*(c^2 + d^2)), x] - Dist[1/(a*(n+1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^{(n+1)}*Simp[A*(b*d*m - a*c*(n+1)) - B*(b*c*m + a*d*(n+1)) - a*(B*c - A*d)*(m+n+1)*Tan[e + f*x], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, m\}, x] \&\& NeQ[b*c - a*d, 0] \&\& EqQ[a^2 + b^2, 0] \&\& LtQ[n, -1]$

#### Rule 3680

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^{(n_)}, x\_Symbol] := Dist[b*(B/f), Subst[Int[(a + b*x)^{(m-1)}*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& NeQ[b*c - a*d, 0] \&\& EqQ[a^2 + b^2, 0] \&\& EqQ[A*b + a*B, 0]$

#### Rule 3681

$Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^{(n_)})/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x\_Symbol] := Dist[$

$A*b + a*B)/(b*c + a*d)$ ,  $\text{Int}[(a + b*\text{Tan}[e + f*x])^m, x]$ ,  $x] - \text{Dist}[(B*c - A*d)/(b*c + a*d)$ ,  $\text{Int}[(a + b*\text{Tan}[e + f*x])^m*((a - b*\text{Tan}[e + f*x])/(c + d*\text{Tan}[e + f*x]))]$ ,  $x]$  /;  $\text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[A*b + a*B, 0]$

Rubi steps

$$\begin{aligned} \int \cot^5(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^4(c + dx)(a + ia \tan(c + dx))^{3/2}}{4d} + \\ &= -\frac{a^2(11iA + 8B) \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{24d} \\ &= \frac{a^2(107A - 104iB) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{96d} \\ &= \frac{a^2(149iA + 152B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{64d} \\ &= \frac{a^2(149iA + 152B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{64d} \\ &= \frac{a^2(149iA + 152B) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{64d} \\ &= \frac{4\sqrt{2} a^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{d} \\ &= -\frac{3a^{5/2}(121A - 120iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{64d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 698 vs.  $2(261) = 522$ .  
time = 8.92, size = 698, normalized size = 2.67

Warning: Unable to verify antiderivative.

[In]  $\text{Integrate}[\text{Cot}[c + d*x]^5*(a + I*a*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(\text{Sqrt}[E^{(I*d*x)}]*(2048*(A - I*B)*\text{ArcSinh}[E^{(I*(c + d*x))}] + 3*\text{Sqrt}[2]*(121*A - (120*I)*B)*(\text{Log}[(-1 + E^{(I*(c + d*x))})^2] - \text{Log}[(1 + E^{(I*(c + d*x))})^2]) + \text{Log}[3 + 3*E^{((2*I)*(c + d*x))} + 2*\text{Sqrt}[2]*\text{Sqrt}[1 + E^{((2*I)*(c + d*x))}])$



$$\begin{aligned}
& x+c)/(\cos(d*x+c)+1))^{(1/2)}*\cos(d*x+c)^3-3072*I*B*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\
& )*\sin(d*x+c)/\cos(d*x+c))*\cos(d*x+c)^3-3072*I*A*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2-3072*I*B*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*\cos(d*x+c)^2+1536*I*A*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)+1536*I*B*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*\cos(d*x+c)-1089*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))+1080*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})+3072*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^3-3072*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3+3072*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)^2-3072*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2-1536*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}*\cos(d*x+c)+1536*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)+1089*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})+1080*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-912*I*B*\cos(d*x+c)*\sin(d*x+c)+1040*I*B*\cos(d*x+c)^3*\sin(d*x+c)-1328*I*B* \\
& \cos(d*x+c)^2*\sin(d*x+c)-1166*A*\cos(d*x+c)^3*\sin(d*x+c)+2178*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^3-2160*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^3+2178*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2-2160*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2-1536*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*2^{(1/2)}-1089*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)+1536*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}+1080*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)-1690*A*\cos(d*x+c)^4*\sin(d*x+c)+894*I*A*\cos(d*x+c)+1690*I*A*\cos(d*x+c)^5+524*I*A*\cos(d*x+c)^4-2488*I*A*\cos(d*x+c)^3-428*I*A*\cos(d*x+c)^2+1089*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \operatorname{arctan}(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)+1080*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}* \\
& \ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos
\end{aligned}$$

$(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)-1536*A*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))*\cos(d*x+c)^5+1536*B*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\operatorname{arctan}(1/2*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)/\cos(d*x+c))$

**Maxima [A]**

time = 0.55, size = 293, normalized size = 1.12

$$a^4 \left( \frac{768 \sqrt{2} (A-I*B) \log\left(\frac{-\sqrt{2} \sqrt{a} - \sqrt{I*a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{I*a \tan(dx+c) + a}}\right)}{a^3} - \frac{9(121 A - 120 I*B) \log\left(\frac{\sqrt{I*a \tan(dx+c) + a} - \sqrt{a}}{\sqrt{I*a \tan(dx+c) + a} + \sqrt{a}}\right)}{a^3} + \frac{2(3(I*a \tan(dx+c) + a)^2(149 A - 152 I*B) - (I*a \tan(dx+c) + a)^2(1127 A - 1160 I*B) + (I*a \tan(dx+c) + a)^2(1049 A - 1016 I*B) - 3 \sqrt{I*a \tan(dx+c) + a} (107 A - 104 I*B) a^2)}{(I*a \tan(dx+c) + a)^3 a^4 - 4(I*a \tan(dx+c) + a)^2 a^3 + 6(I*a \tan(dx+c) + a) a^2 - 4(I*a \tan(dx+c) + a) a + a^2} \right) / 384 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/384*a^4*(768*\sqrt{2}*(A - I*B)*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a}))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a}))/a^{3/2} - 9*(121*A - 120*I*B)*\log((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a}))/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a}))/a^{3/2} + 2*(3*(I*a*\tan(d*x + c) + a)^{7/2}*(149*A - 152*I*B) - (I*a*\tan(d*x + c) + a)^{5/2}*(1127*A - 1160*I*B)*a + (I*a*\tan(d*x + c) + a)^{3/2}*(1049*A - 1016*I*B)*a^2 - 3*\sqrt{I*a*\tan(d*x + c) + a}*(107*A - 104*I*B)*a^3)/((I*a*\tan(d*x + c) + a)^4*a - 4*(I*a*\tan(d*x + c) + a)^3*a^2 + 6*(I*a*\tan(d*x + c) + a)^2*a^3 - 4*(I*a*\tan(d*x + c) + a)*a^4 + a^5))/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 944 vs. 2(206) = 412.

time = 1.98, size = 944, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $1/768*(1536*\sqrt{2}*\sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2) - 1536*\sqrt{2}*\sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2) + 9*\sqrt{(14641*A^2 - 29040*I*A*B - 14400*B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((-I*A - B)*a^3*e^{(I*d*x + I*c)} - \sqrt{(A^2 - 2*I*A*B - B^2)*a^5/d^2}*(-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)$

$$\begin{aligned}
& 2*I*c) + d)*\log(16*(3*(-121*I*A - 120*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-121*I \\
& *A - 120*B)*a^3 + 2*\sqrt{2}*\sqrt{(14641*A^2 - 29040*I*A*B - 14400*B^2)*a^5/d^2}*(I*d*e^{(3*I*d*x + 3*I*c)} + I*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2 \\
& *I*c)} + 1)))*e^{(-2*I*d*x - 2*I*c)/((-121*I*A - 120*B)*a)} - 9*\sqrt{(14641*A \\
& ^2 - 29040*I*A*B - 14400*B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I* \\
& d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(1 \\
& 6*(3*(-121*I*A - 120*B)*a^3*e^{(2*I*d*x + 2*I*c)} + (-121*I*A - 120*B)*a^3 + \\
& 2*\sqrt{2}*\sqrt{(14641*A^2 - 29040*I*A*B - 14400*B^2)*a^5/d^2}*(-I*d*e^{(3*I* \\
& d*x + 3*I*c)} - I*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)))*e^{(- \\
& 2*I*d*x - 2*I*c)/((-121*I*A - 120*B)*a)} - 4*\sqrt{2}*(13*(65*A - 56*I*B)*a^ \\
& 2*e^{(9*I*d*x + 9*I*c)} - 2*(215*A - 392*I*B)*a^2*e^{(7*I*d*x + 7*I*c)} - 4*(35 \\
& *A - 104*I*B)*a^2*e^{(5*I*d*x + 5*I*c)} + 2*(407*A - 392*I*B)*a^2*e^{(3*I*d*x \\
& + 3*I*c)} - 3*(107*A - 104*I*B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2* \\
& I*c)} + 1)))/(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d \\
& *x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)
\end{aligned}$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*5\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^5, x)

**Mupad [B]**

time = 8.59, size = 2500, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^5\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)





$$\begin{aligned}
& 3*B*a^{22}*34153i)/(8192*d^4)^{(1/2)} - B*d^3*((485809*A^4*a^{22})/(262144*d^4) \\
& + (529*B^4*a^{22})/(64*d^4) + (11229*A^2*B^2*a^{22})/(2048*d^4) + (A*B^3*a^{22}*1 \\
& 127i)/(128*d^4) + (A^3*B*a^{22}*34153i)/(8192*d^4)^{(1/2)}*208i + (21783*A*B^2 \\
& *a^{11*d})/4 + (A^2*B*a^{11*d}*6993i)/32))*(((485809*A^4*a^{22})/(262144*d^4) + ( \\
& 529*B^4*a^{22})/(64*d^4) + (11229*A^2*B^2*a^{22})/(2048*d^4) + (A*B^3*a^{22}*1127 \\
& i)/(128*d^4) + (A^3*B*a^{22}*34153i)/(8192*d^4)^{(1/2)})/(64*a^6) + (262841*A^2 \\
& *a^5)/(32768*d^2) - (4073*B^2*a^5)/(512*d^2) - (A*B*a^5*32719i)/(2048*d^2)) \\
& ^{(1/2)} - 2*atanh((697*A^2*a^8*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)}*((262841*A^ \\
& 2*a^5)/(32768*d^2) - ((485809*A^4*a^{22})/(262144*d^4) + (529*B^4*a^{22})/(64*d \\
& ^4) + (11229*A^2*B^2*a^{22})/(2048*d^4) + (A*B^3*a^{22}*1127i)/(128*d^4) + (A^3 \\
& *B*a^{22}*34153i)/(8192*d^4)^{(1/2)})/(64*a^6) - (4073*B^2*a^5)/(512*d^2) - (A \\
& B*a^5*32719i)/(2048*d^2))^{(1/2)})/(4*((431443*A^3*a^{11*d})/256 - B^3*a^{11*d}*3 \\
& 542i - 214*A*d^3*((485809*A^4*a^{22})/(262144*d^4) + (529*B^4*a^{22})/(64*d^4) \\
& + (11229*A^2*B^2*a^{22})/(2048*d^4) + (A*B^3*a^{22}*1127i)/(128*d^4) + (A^3*B*a \\
& ^{22}*34153i)/(8192*d^4)^{(1/2)} + B*d^3*((485809*A^4*a^{22})/(262144*d^4) + (52 \\
& 9*B^4*a^{22})/(64*d^4) + (11229*A^2*B^2*a^{22})/(2048*d^4) + (A*B^3*a^{22}*1127i) \\
& /((128*d^4) + (A^3*B*a^{22}*34153i)/(8192*d^4)^{(1/2)}*208i + (21783*A*B^2*a^{11 \\
& *d})/4 + (A^2*B*a^{11*d}*6993i)/32)) - (384*d^4*(a + a*tan(c + d*x)*1i)^{(1/2)}* \\
& ((262841*A^2*a^5)/(32768*d^2) - ((485809*A^4*a^{22})/(262144*d^4) + (529*B^4*a \\
& a^{22})/(64*d^4) + (11229*A^2*B^2*a^{22})/(2048*d^4) + (A*B^3*a^{22}*1127i)/(128* \\
& d^4) + (A^3*B*a^{22}*34153i)/(8192*d^4)^{(1/2)})/(64*a^6) - (4073*B^2*a^5)/(512 \\
& *d^2) - (A*B*a^5*32719i)/(2048*d^2))^{(1/2)}*((48...
\end{aligned}$$

$$3.90 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=205

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(iA-B) \tan^3(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} + \frac{4(5A+7iB)\sqrt{a+ia \tan(c+dx)}}{5ad}$$

[Out] 1/2\*(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/d\*2^(1/2)/a^(1/2)+4/5\*(5\*A+7\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d-1/5\*(5\*A+7\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^2/a/d+(I\*A-B)\*tan(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))^(1/2)-1/15\*(25\*A+23\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(3/2)/a^2/d

**Rubi** [A]

time = 0.34, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3676, 3678, 3673, 3608, 3561, 212}

$$-\frac{(25A+23iB)(a+ia \tan(c+dx))^{3/2}}{15a^2d} + \frac{(-B+iA) \tan^3(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(5A+7iB) \tan^2(c+dx)\sqrt{a+ia \tan(c+dx)}}{5ad} + \frac{4(5A+7iB)\sqrt{a+ia \tan(c+dx)}}{5ad} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/(Sqrt[2]\*Sqrt[a]\*d) + ((I\*A - B)\*Tan[c + d\*x]^3)/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (4\*(5\*A + (7\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(5\*a\*d) - ((5\*A + (7\*I)\*B)\*Tan[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(5\*a\*d) - ((25\*A + (23\*I)\*B)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(15\*a^2\*d)

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3561

Int[Sqrt[(a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3608

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist

$[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ !\text{LtQ}[m, 0]$

### Rule 3673

$\text{Int}[(a + b*\text{tan}[(e + f*x)])^m * (A + B*\text{tan}[(e + f*x)]) * (c + d*\text{tan}[(e + f*x)]), x\_Symbol] \rightarrow \text{Simp}[B*d*(a + b*\text{Tan}[e + f*x])^{m+1} / (b*f*(m+1)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m * \text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{LeQ}[m, -1]$

### Rule 3676

$\text{Int}[(a + b*\text{tan}[(e + f*x)])^m * (A + B*\text{tan}[(e + f*x)]) * (c + d*\text{tan}[(e + f*x)])^n, x\_Symbol] \rightarrow \text{Simp}[(-A*b - a*B)*(a + b*\text{Tan}[e + f*x])^m * (c + d*\text{Tan}[e + f*x])^n / (2*a*f*m), x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1} * (c + d*\text{Tan}[e + f*x])^{n-1} * \text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m-n) - a*A*(m+n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\ \& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, 0] \ \&\& \ \text{GtQ}[n, 0]$

### Rule 3678

$\text{Int}[(a + b*\text{tan}[(e + f*x)])^m * (A + B*\text{tan}[(e + f*x)]) * (c + d*\text{tan}[(e + f*x)])^n, x\_Symbol] \rightarrow \text{Simp}[B*(a + b*\text{Tan}[e + f*x])^m * (c + d*\text{Tan}[e + f*x])^n / (f*(m+n)), x] + \text{Dist}[1/(a*(m+n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m * (c + d*\text{Tan}[e + f*x])^{n-1} * \text{Simp}[a*A*c*(m+n) - B*(b*c*m + a*d*n) + (a*A*d*(m+n) - B*(b*d*m - a*c*n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}} dx &= \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)} dx}{5ad} \\
&= \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5ad} \\
&= \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5ad} \\
&= \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{4(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{5ad} \\
&= \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{4(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{5ad} \\
&= \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

**Mathematica [A]**

time = 3.17, size = 176, normalized size = 0.86

$$\frac{((A-iB)\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{2}\sqrt{a}}\right) + \frac{1}{30}\sec^2(c+dx)(5(23A+37iB)\cos(c+dx) + (25A+59iB)\cos(3(c+dx)) + 4i(5A+16iB) + (5A+22iB)\cos(2(c+dx)))\sin(c+dx))(A+B\tan(c+dx))}{2d(A\cos(c+dx)+B\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]], x]
```

```
[Out] (((A - I*B)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + (Sec[c + d*x]^2*(5*(23*A + (37*I)*B)*Cos[c + d*x] + (25*A + (59*I)*B)*Cos[3*(c + d*x)] + (4*I)*(5*A + (16*I)*B + (5*A + (22*I)*B)*Cos[2*(c + d*x)])*Sin[c + d*x]))/30)*(A + B*Tan[c + d*x]))/(2*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [A]**

time = 0.11, size = 168, normalized size = 0.82

method	result
derivativedivides	$2 \left( -\frac{iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3} + \frac{Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3} - 2ia^2B\sqrt{a+ia\tan(dx+c)} - a^2 \right)$

default	$2 \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2iBa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{Aa(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} - 2ia^2B \sqrt{a+ia \tan(dx+c)} - a^2A \right)$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-2/d/a^3*(-1/5*I*B*(a+I*a*\tan(dx+c))^{5/2}+2/3*I*B*a*(a+I*a*\tan(dx+c))^{3/2}+1/3*A*a*(a+I*a*\tan(dx+c))^{3/2}-2*I*B*a^2*(a+I*a*\tan(dx+c))^{1/2}-a^2*A*(a+I*a*\tan(dx+c))^{1/2}-1/4*a^{5/2}*(A-I*B)*2^{1/2}*\operatorname{arctanh}(1/2*(a+I*a*\tan(dx+c))^{1/2}*2^{1/2}/a^{1/2})-1/2*a^3*(A+I*B)/(a+I*a*\tan(dx+c))^{1/2})$$

**Maxima** [A]

time = 0.50, size = 157, normalized size = 0.77

$$\frac{15\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-24i(i\tan(dx+c)+a)^{\frac{5}{2}}Ba+40(i\tan(dx+c)+a)^{\frac{3}{2}}(A+2iB)a^2-120\sqrt{ia\tan(dx+c)+a}(A+2iB)a^3-\frac{60(A+iB)a^4}{\sqrt{ia\tan(dx+c)+a}}}{60a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] 
$$-1/60*(15*\sqrt{2}*(A-I*B)*a^{7/2}*\log(-(\sqrt{2}*\sqrt{a}-\sqrt{I*a*\tan(dx+c)+a})/(\sqrt{2}*\sqrt{a}+\sqrt{I*a*\tan(dx+c)+a}))-24*I*(I*a*\tan(dx+c)+a)^{5/2}*B*a+40*(I*a*\tan(dx+c)+a)^{3/2}*(A+2*I*B)*a^2-120*\sqrt{I*a*\tan(dx+c)+a}*(A+2*I*B)*a^3-60*(A+I*B)*a^4/\sqrt{I*a*\tan(dx+c)+a})/(a^4*d)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs.  $2(163) = 326$ .

time = 2.27, size = 447, normalized size = 2.18

$$\frac{15\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-24i(i\tan(dx+c)+a)^{\frac{5}{2}}Ba+40(i\tan(dx+c)+a)^{\frac{3}{2}}(A+2iB)a^2-120\sqrt{ia\tan(dx+c)+a}(A+2iB)a^3-\frac{60(A+iB)a^4}{\sqrt{ia\tan(dx+c)+a}}}{60a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] 
$$-1/60*(15*\sqrt{2}*(a*d*e^{5*I*d*x+5*I*c}+2*a*d*e^{3*I*d*x+3*I*c}+a*d*e^{I*d*x+I*c})*\sqrt{(A^2-2*I*A*B-B^2)/(a*d^2)}*\log(-4*((-I*A-B)*a*e^{I*d*x+I*c}+(I*a*d*e^{2*I*d*x+2*I*c}+I*a*d)*\sqrt{a/(e^{2*I*d*x+2*I*c})}))$$

$2*I*c) + 1))\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2))} * e^{(-I*d*x - I*c)/(I*A + B)} - 15*\sqrt{2}*(a*d*e^{(5*I*d*x + 5*I*c)} + 2*a*d*e^{(3*I*d*x + 3*I*c)} + a*d * e^{(I*d*x + I*c)})\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2)} * \log(-4*((-I*A - B)*a * e^{(I*d*x + I*c)} + (-I*a*d*e^{(2*I*d*x + 2*I*c)} - I*a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1))}\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2))} * e^{(-I*d*x - I*c)/(I*A + B)} - 2*\sqrt{2}*((35*A + 103*I*B)*e^{(6*I*d*x + 6*I*c)} + 5*(25*A + 41*I*B)*e^{(4*I*d*x + 4*I*c)} + 15*(7*A + 11*I*B)*e^{(2*I*d*x + 2*I*c)} + 15*A + 15*I*B) * \sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)))/(a*d*e^{(5*I*d*x + 5*I*c)} + 2*a*d*e^{(3*I * d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^3(c + dx)}{\sqrt{ia} (\tan(c + dx) - i)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^3/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad [B]**

time = 7.95, size = 236, normalized size = 1.15

$$\frac{A}{d\sqrt{a+\tan(c+dx)}\sqrt{1}} + \frac{B\sqrt{1}}{d\sqrt{a+\tan(c+dx)}\sqrt{1}} + \frac{2A\sqrt{a+\tan(c+dx)}\sqrt{1}}{ad} - \frac{2A(a+\tan(c+dx))\sqrt{1}}{3a^2d} + \frac{B\sqrt{a+\tan(c+dx)}\sqrt{1}}{ad} - \frac{B(a+\tan(c+dx))\sqrt{1}}{3a^2d} + \frac{B(a+\tan(c+dx))\sqrt{1}}{5a^2d} + \frac{\sqrt{2}B\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+\tan(c+dx)}\sqrt{1}}{2\sqrt{-a}}\right)\sqrt{1}}{2\sqrt{-a}d} - \frac{\sqrt{2}A\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+\tan(c+dx)}\sqrt{1}}{2\sqrt{-a}}\right)\sqrt{1}}{2\sqrt{-a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] A/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) + (B\*1i)/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) + (2\*A\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(a\*d) - (2\*A\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*a^2\*d) + (B\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*4i)/(a\*d) - (B\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*4i)/(3\*a^2\*d) + (B\*(a + a\*tan(c + d\*x)\*1i)^(5/2)\*2i)/(5\*a^3\*d) + (2^(1/2)\*B\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)))/(2\*(-a)^(1/2)))\*1i)/(2\*(-a)^(1/2)\*d) - (2^(1/2)\*A\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*1i)/(2\*a^(1/2)))\*1i)/(2\*a^(1/2)\*d)

$$3.91 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=159

$$\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(iA-B) \tan^2(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{4(iA-B)\sqrt{a+ia \tan(c+dx)}}{ad} + (3iA)$$

[Out] 1/2\*(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/d\*2^(1/2)/a^(1/2)-4\*(I\*A-B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d+(I\*A-B)\*tan(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/3\*(3\*I\*A-5\*B)\*(a+I\*a\*tan(d\*x+c))^(3/2)/a^2/d

**Rubi [A]**

time = 0.22, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3676, 3673, 3608, 3561, 212}

$$\frac{(-5B+3iA)(a+ia \tan(c+dx))^{3/2}}{3a^2d} + \frac{(-B+iA) \tan^2(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{4(-B+iA)\sqrt{a+ia \tan(c+dx)}}{ad} + \frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])]/(Sqrt[2]\*Sqrt[a]\*d) + ((I\*A - B)\*Tan[c + d\*x]^2)/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - (4\*(I\*A - B)\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(a\*d) + (((3\*I)\*A - 5\*B)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(3\*a^2\*d)

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 3561**

Int[Sqrt[(a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

**Rule 3608**

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e,



$f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& !\text{LtQ}[m, 0]$

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\tan[e + f*x])^{(m + 1)/(b*f*(m + 1)}), x] + \text{Int}[(a + b*\tan[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\tan[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

### Rule 3676

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(-A*b - a*B)*(a + b*\tan[e + f*x])^m*((c + d*\tan[e + f*x])^n/(2*a*f*m)), x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\tan[e + f*x])^{(m + 1)}*(c + d*\tan[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*\tan[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} dx &= \frac{(iA - B) \tan^2(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} - \frac{\int \tan(c + dx) \sqrt{a + ia \tan(c + dx)}}{2} \\ &= \frac{(iA - B) \tan^2(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} + \frac{(3iA - 5B)(a + ia \tan(c + dx))^{3/2}}{3a^2 d} \\ &= \frac{(iA - B) \tan^2(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} - \frac{4(iA - B) \sqrt{a + ia \tan(c + dx)}}{ad} + \frac{(3iA - 5B)(a + ia \tan(c + dx))^{3/2}}{3a^2 d} \\ &= \frac{(iA - B) \tan^2(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} - \frac{4(iA - B) \sqrt{a + ia \tan(c + dx)}}{ad} + \frac{(3iA - 5B)(a + ia \tan(c + dx))^{3/2}}{3a^2 d} \\ &= \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{\sqrt{2} \sqrt{a} d} + \frac{(iA - B) \tan^2(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} \end{aligned}$$

### Mathematica [A]

time = 2.23, size = 147, normalized size = 0.92

$$\frac{\left( (iA + B) \sqrt{1 + e^{2i(c+dx)}} \sinh^{-1} \left( e^{i(c+dx)} \right) + \frac{1}{3} \sec(c + dx) (9(-iA + B) + (-9iA + 5B) \cos(2(c + dx)) + (6A + 2iB) \sin(2(c + dx))) \right) (A + B \tan(c + dx))}{2d(A \cos(c + dx) + B \sin(c + dx)) \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],
x]
```

```
[Out] (((I*A + B)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + (Sec[c
+ d*x]*(9*((-I)*A + B) + ((-9*I)*A + 5*B)*Cos[2*(c + d*x)] + (6*A + (2*I)*
B)*Sin[2*(c + d*x)]))/3)*(A + B*Tan[c + d*x]))/(2*d*(A*Cos[c + d*x] + B*Sine
[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [A]**

time = 0.11, size = 127, normalized size = 0.80

method	result
derivativedivides	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + iaB \sqrt{a + ia \tan(dx+c)} + aA \sqrt{a + ia \tan(dx+c)} - \frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}}{da^2} \right)$
default	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + iaB \sqrt{a + ia \tan(dx+c)} + aA \sqrt{a + ia \tan(dx+c)} - \frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}}{da^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -2*I/d/a^2*(-1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)+I*a*B*(a+I*a*tan(d*x+c))^(1/2)
)+a*A*(a+I*a*tan(d*x+c))^(1/2)-1/4*a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+1/2*a^2*(A+I*B)/(a+I*a*tan(d*x+c))^(1
/2))
```

**Maxima [A]**

time = 0.51, size = 134, normalized size = 0.84

$$\frac{i \left( 3\sqrt{2}(A-iB)a^{\frac{5}{2}} \log\left( \frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}} \right) - 8i(ia \tan(dx+c)+a)^{\frac{3}{2}}Ba + 24\sqrt{ia \tan(dx+c)+a}(A+iB)a^2 + \frac{12(A+iB)a^3}{\sqrt{ia \tan(dx+c)+a}} \right)}{12a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/12*I*(3*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*I*(I*a*ta
```

$n(dx + c) + a)^{3/2} * B * a + 24 * \sqrt{I * a * \tan(dx + c) + a} * (A + I * B) * a^2 + 1$   
 $2 * (A + I * B) * a^3 / \sqrt{I * a * \tan(dx + c) + a} / (a^3 * d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs.  $2(127) = 254$ .

time = 1.83, size = 391, normalized size = 2.46

$$\frac{3\sqrt{2}\sqrt{a^2(d^2x^2+c^2)+ad^2}\sqrt{\frac{d^2-2AB-B^2}{ad^2}}\log\left(\frac{(-1+2Bd^{2x+c})\sqrt{a^2(d^2x^2+c^2)+ad^2}\sqrt{\frac{d^2-2AB-B^2}{ad^2}}}{2d^{2x+c}+1}\right)-3\sqrt{2}\sqrt{a^2(d^2x^2+c^2)+ad^2}\sqrt{\frac{d^2-2AB-B^2}{ad^2}}\log\left(\frac{(-1+2Bd^{2x+c})\sqrt{a^2(d^2x^2+c^2)+ad^2}\sqrt{\frac{d^2-2AB-B^2}{ad^2}}}{2d^{2x+c}+1}\right)+2\sqrt{2}(15A-7B)d^{4x+4c}+18(IA-B)d^{2x+2c}+3(A-3B)\sqrt{\frac{a}{d^{2x+c}+1}}}{12(a^2(d^2x^2+c^2)+ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^(1/2),x, algorithm="fricas")

[Out]  $-1/12*(3*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} + (a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}))*e^{(-I*d*x - I*c)/(I*A + B)} - 3*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} - (a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}))*e^{(-I*d*x - I*c)/(I*A + B)} + 2*\sqrt{2}*((15*I*A - 7*B)*e^{(4*I*d*x + 4*I*c)} + 18*(I*A - B)*e^{(2*I*d*x + 2*I*c)} + 3*I*A - 3*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^2(c + dx)}{\sqrt{ia(\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*2\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + dx))\*tan(c + dx)\*\*2/sqrt(I\*a\*(tan(c + dx) - I)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^2/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad [B]**

time = 7.67, size = 188, normalized size = 1.18

$$\frac{A \operatorname{li}}{d \sqrt{a + a \tan(c + dx) \operatorname{li}}} + \frac{B}{d \sqrt{a + a \tan(c + dx) \operatorname{li}}} - \frac{A \sqrt{a + a \tan(c + dx) \operatorname{li}}}{ad} + \frac{2B \sqrt{a + a \tan(c + dx) \operatorname{li}}}{ad} - \frac{2B(a + a \tan(c + dx) \operatorname{li})^{3/2}}{3a^2 d} - \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2\sqrt{-a}}\right) \operatorname{li}}{2\sqrt{-a} d} - \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2\sqrt{a}}\right) \operatorname{li}}{2\sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] B/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) - (A\*1i)/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) - (A\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*2i)/(a\*d) + (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(a\*d) - (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/(3\*a^2\*d) - (2^(1/2)\*A\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/(2\*(-a)^(1/2)\*d) - (2^(1/2)\*B\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*1i)/(2\*a^(1/2)))\*1i)/(2\*a^(1/2)\*d)

$$3.92 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=109

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d} - \frac{A+iB}{d \sqrt{a+ia \tan(c+dx)}} - \frac{2iB \sqrt{a+ia \tan(c+dx)}}{ad}$$

[Out]  $-1/2*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{1/2}*2^{1/2}/a^{1/2})/d*2^{1/2}/a^{1/2}+(-A-I*B)/d/(a+I*a*\tan(d*x+c))^{1/2}-2*I*B*(a+I*a*\tan(d*x+c))^{1/2}/a/d$

**Rubi [A]**

time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3673, 3607, 3561, 212}

$$\frac{A+iB}{d \sqrt{a+ia \tan(c+dx)}} - \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d} - \frac{2iB \sqrt{a+ia \tan(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]],x]$

[Out]  $-(((A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*d))-(A+I*B)/(d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])-((2*I)*B*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(a*d)$

**Rule 212**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\tan[(c_+) + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

**Rule 3607**

$\operatorname{Int}[(a_+ + (b_+)*\tan[(e_+) + (f_+)*(x_+)])^{m_+}*((c_+) + (d_+)*\tan[(e_+) + (f_+)*(x_+)]), x\_Symbol] \rightarrow \operatorname{Simp}[(-(b*c - a*d))*((a + b*\operatorname{Tan}[e + f*x])^m/(2*a*f*m)), x] + \operatorname{Dist}[(b*c + a*d)/(2*a*b), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{m+1}, x]$

, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}} dx &= -\frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad} + \int \frac{-B+A\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}} dx \\ &= -\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(iA+B)}{ad} \\ &= -\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(A-iB)}{ad} \\ &= -\frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}} \end{aligned}$$

### Mathematica [A]

time = 1.43, size = 140, normalized size = 1.28

$$\frac{e^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(A(1+e^{2i(c+dx)})+iB(1+5e^{2i(c+dx)})+(A-iB)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}(e^{i(c+dx)})\right)}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] -((Sqrt[(a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]\*(A\*(1 + E^((2\*I)\*(c + d\*x)))) + I\*B\*(1 + 5\*E^((2\*I)\*(c + d\*x)))) + (A - I\*B)\*E^(I\*(c + d\*x))\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcSinh[E^(I\*(c + d\*x))])/(Sqrt[2]\*a\*d\*E^((2\*I)\*(c + d\*x)))

### Maple [A]

time = 0.10, size = 88, normalized size = 0.81

method	result
derivativedivides	$\frac{-2iB\sqrt{a+ia\tan(dx+c)} - \frac{\sqrt{a}(-iB+A)\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{2}}{ad} - \frac{1}{\sqrt{a+i}}$
default	$\frac{-2iB\sqrt{a+ia\tan(dx+c)} - \frac{\sqrt{a}(-iB+A)\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{2}}{ad} - \frac{1}{\sqrt{a+i}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $2/d/a*(-I*B*(a+I*a*\tan(dx+c))^{1/2}-1/4*a^{1/2}*(A-I*B)*2^{1/2}*\operatorname{arctanh}(1/2*(a+I*a*\tan(dx+c))^{1/2}*2^{1/2}/a^{1/2})-1/2*a*(A+I*B)/(a+I*a*\tan(dx+c))^{1/2})$

**Maxima [A]**

time = 0.54, size = 110, normalized size = 1.01

$$\frac{\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-8i\sqrt{ia\tan(dx+c)+a}Ba-\frac{4(A+iB)a^2}{\sqrt{ia\tan(dx+c)+a}}}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm  
="maxima")`

[Out]  $1/4*(\sqrt{2}*(A-I*B)*a^{3/2}*\log(-(\sqrt{2}*\sqrt{a}-\sqrt{I*a*\tan(dx+c)+a}))/(\sqrt{2}*\sqrt{a}+\sqrt{I*a*\tan(dx+c)+a}))-8*I*\sqrt{I*a*\tan(dx+c)+a}*B*a-4*(A+I*B)*a^2/\sqrt{I*a*\tan(dx+c)+a}/(a^2*d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 327 vs.  $2(84) = 168$ .

time = 2.72, size = 327, normalized size = 3.00

$$\frac{\left(\sqrt{2}ad\sqrt{\frac{A^2-2iAB-B^2}{a^2}}e^{i(A+iC)}\log\left(\frac{e^{i(A-B)\tan(dx+c)}+i\sqrt{\frac{a}{2iA^2+1}}\sqrt{\frac{A^2-2iAB-B^2}{a^2}}}{e^{i(A+B)\tan(dx+c)}+i\sqrt{\frac{a}{2iA^2+1}}\sqrt{\frac{A^2-2iAB-B^2}{a^2}}}\right)-\sqrt{2}ad\sqrt{\frac{A^2-2iAB-B^2}{a^2}}e^{i(A+iC)}\log\left(\frac{e^{i(A-B)\tan(dx+c)}-i\sqrt{\frac{a}{2iA^2+1}}\sqrt{\frac{A^2-2iAB-B^2}{a^2}}}{e^{i(A+B)\tan(dx+c)}-i\sqrt{\frac{a}{2iA^2+1}}\sqrt{\frac{A^2-2iAB-B^2}{a^2}}}\right)-2\sqrt{2}((A+5iB)e^{2i(A+iC)}+A+iB)\sqrt{\frac{a}{2iA^2+1}}\right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm  
="fricas")`

[Out]  $1/4*(\sqrt{2}*a*d*\sqrt{(A^2-2iA*B-B^2)/(a*d^2)}*e^{i(A*d*x+I*c)}*\log(-4*((-I*A-B)*a*e^{i(A*d*x+I*c)}+(I*a*d*e^{2i(A*d*x+2i*c)}+I*a*d)*\sqrt{a$

$$\frac{1}{(e^{2Ix+2Ic} + 1)} \sqrt{\frac{A^2 - 2IAB - B^2}{a^2d^2}} e^{-Ix - Ic} / (IA + B) - \sqrt{2} a d \sqrt{\frac{A^2 - 2IAB - B^2}{a^2d^2}} e^{Ix + Ic} \log(-4((-IA - B)a e^{Ix + Ic} + (-Ia d e^{2Ix + 2Ic} - I a d) \sqrt{a/(e^{2Ix + 2Ic} + 1)} \sqrt{\frac{A^2 - 2IAB - B^2}{a^2d^2}}) e^{-Ix - Ic} / (IA + B) - 2\sqrt{2}((A + 5IB) e^{2Ix + 2Ic} + A + IB) \sqrt{a/(e^{2Ix + 2Ic} + 1)}) e^{-Ix - Ic} / (a d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan(c + dx)}{\sqrt{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad [B]**

time = 7.38, size = 141, normalized size = 1.29

$$-\frac{A}{d \sqrt{a + a \tan(c + dx)} \operatorname{li}} - \frac{B \operatorname{li}}{d \sqrt{a + a \tan(c + dx)} \operatorname{li}} - \frac{B \sqrt{a + a \tan(c + dx)} \operatorname{li}^2}{ad} - \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)} \operatorname{li}}{2\sqrt{-a}}\right) \operatorname{li}}{2\sqrt{-a} d} - \frac{\sqrt{2} A \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx)} \operatorname{li}}{2\sqrt{a}}\right)}{2\sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] - A/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) - (B\*1i)/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) - (B\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*2i)/(a\*d) - (2^(1/2)\*B\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/(2\*(-a)^(1/2)\*d) - (2^(1/2)\*A\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2)))/(2\*a^(1/2)\*d)



$$3.93 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=82

$$-\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{iA-B}{d\sqrt{a+ia \tan(c+dx)}}$$

[Out]  $-1/2*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)*2^{(1/2)}/a^{(1/2)}}/d*2^{(1/2)}/a^{(1/2)}+(I*A-B)/d/(a+I*a*\tan(d*x+c))^{(1/2)}$

**Rubi** [A]

time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {3607, 3561, 212}

$$\frac{-B+iA}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/Sqrt[a+I*a*\operatorname{Tan}[c+d*x]],x]$

[Out]  $-(((I*A+B)*\operatorname{ArcTanh}[Sqrt[a+I*a*\operatorname{Tan}[c+d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[a]*d))+(I*A-B)/(d*Sqrt[a+I*a*\operatorname{Tan}[c+d*x]])$

**Rule 212**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

**Rule 3561**

$\operatorname{Int}[Sqrt[(a_+ + (b_+)*\tan[(c_+) + (d_+)*(x_+) ]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, Sqrt[a + b*\operatorname{Tan}[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

**Rule 3607**

$\operatorname{Int}[(a_+ + (b_+)*\tan[(e_+) + (f_+)*(x_+) ])^{(m_+)*((c_+) + (d_+)*\tan[(e_+) + (f_+)*(x_+) ])], x\_Symbol] \rightarrow \operatorname{Simp}[(-b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^m/(2*a*f*m)), x] + \operatorname{Dist}[(b*c + a*d)/(2*a*b), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 + b^2,$

0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{a + ia \tan(c + dx)}} dx &= \frac{iA - B}{d\sqrt{a + ia \tan(c + dx)}} + \frac{(A - iB) \int \sqrt{a + ia \tan(c + dx)} dx}{2a} \\ &= \frac{iA - B}{d\sqrt{a + ia \tan(c + dx)}} - \frac{(iA + B) \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \sqrt{a + ia \tan(c + dx)}\right)}{d} \\ &= -\frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d} + \frac{iA - B}{d\sqrt{a + ia \tan(c + dx)}} \end{aligned}$$

Mathematica [A]

time = 1.11, size = 129, normalized size = 1.57

$$\frac{ie^{-2i(c+dx)} \sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \left( (A+iB)(1+e^{2i(c+dx)}) - (A-iB)e^{i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \sinh^{-1}(e^{i(c+dx)}) \right)}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] (I\*Sqrt[(a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]\*((A + I\*B)\*(1 + E^((2\*I)\*(c + d\*x))) - (A - I\*B)\*E^(I\*(c + d\*x))\*Sqrt[1 + E^((2\*I)\*(c + d\*x))])\*ArcSinh[E^(I\*(c + d\*x))])/(Sqrt[2]\*a\*d\*E^((2\*I)\*(c + d\*x)))

Maple [A]

time = 0.10, size = 71, normalized size = 0.87

method	result	size
derivativedivides	$2i \frac{\left( \frac{(-\frac{iB}{2} + \frac{A}{2}) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}} - \frac{-\frac{A}{2} - \frac{iB}{2}}{\sqrt{a + ia \tan(dx + c)}} \right)}{d}$	71
default	$2i \frac{\left( \frac{(-\frac{iB}{2} + \frac{A}{2}) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}} - \frac{-\frac{A}{2} - \frac{iB}{2}}{\sqrt{a + ia \tan(dx + c)}} \right)}{d}$	71

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $2*I/d*(-1/2*(-1/2*I*B+1/2*A)*2^(1/2)/a^(1/2)*\operatorname{arctanh}(1/2*(a+I*a*\tan(dx+c))^(1/2)*2^(1/2)/a^(1/2))-(-1/2*A-1/2*I*B)/(a+I*a*\tan(dx+c))^(1/2))$

**Maxima** [A]

time = 0.54, size = 91, normalized size = 1.11

$$\frac{i \left( \sqrt{2} (A - iB) \sqrt{a} \log \left( -\frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}} \right) + \frac{4(A+iB)a}{\sqrt{i a \tan(dx+c) + a}} \right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")`

[Out]  $1/4*I*(\sqrt{2}*(A - I*B)*\sqrt{a}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(dx+c) + a}))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(dx+c) + a})) + 4*(A + I*B)*a/\sqrt{I*a*\tan(dx+c) + a}/(a*d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs.  $2(63) = 126$ .

time = 2.35, size = 328, normalized size = 4.00

$$\frac{\left( \sqrt{2}ad\sqrt{\frac{A^2-2AB-B^2}{a^2}}e^{i(dx+c)}\log\left(\frac{-((-1+iB)e^{2i(dx+c)}+1)\sqrt{\frac{a}{2^{2i(dx+c)}+1}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}e^{i(dx+c)}}{(-1+iB)e^{2i(dx+c)}+1}\right) - \sqrt{2}ad\sqrt{\frac{A^2-2AB-B^2}{a^2}}e^{i(dx+c)}\log\left(\frac{((-1+iB)e^{2i(dx+c)}-1)\sqrt{\frac{a}{2^{2i(dx+c)}+1}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}e^{i(dx+c)}}{(-1+iB)e^{2i(dx+c)}-1}\right) - 2\sqrt{2}((-1+iB)e^{2i(dx+c)}-iA+B)\sqrt{\frac{a}{2^{2i(dx+c)}+1}}e^{i(dx+c)} \right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")`

[Out]  $1/4*(\sqrt{2}*a*d*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}*e^{(I*d*x + I*c)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} + (a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}))e^{(-I*d*x - I*c)/(I*A + B)} - \sqrt{2}*a*d*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}*e^{(I*d*x + I*c)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} - (a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}))e^{(-I*d*x - I*c)/(I*A + B)} - 2*\sqrt{2}*((-I*A + B)*e^{(2*I*d*x + 2*I*c)} - I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*e^{(-I*d*x - I*c)}/(a*d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{ia(\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad** [B]

time = 0.76, size = 117, normalized size = 1.43

$$\frac{A \operatorname{li}}{d \sqrt{a + a \tan(c + dx) \operatorname{li}}} - \frac{B}{d \sqrt{a + a \tan(c + dx) \operatorname{li}}} + \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2 \sqrt{-a}}\right) \operatorname{li}}{2 \sqrt{-a} d} - \frac{\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2 \sqrt{a}}\right)}{2 \sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] (A\*1i)/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) - B/(d\*(a + a\*tan(c + d\*x)\*1i)^(1/2)) + (2^(1/2)\*A\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/(2\*(-a)^(1/2)\*d) - (2^(1/2)\*B\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/(2\*a^(1/2)\*d)

$$3.94 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=114

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d} + \frac{A+iB}{d \sqrt{a+ia \tan(c+dx)}}$$

[Out]  $-2*A*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{1/2}/a^{1/2})/d/a^{1/2}+1/2*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{1/2}*2^{1/2}/a^{1/2})/d*2^{1/2}/a^{1/2}+(A+I*B)/d/(a+I*a*\tan(d*x+c))^{1/2}$

**Rubi** [A]

time = 0.24, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3677, 3681, 3561, 212, 3680, 65, 214}

$$\frac{A+iB}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]],x]$

[Out]  $(-2*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a]*d) + ((A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*d) + (A+I*B)/(d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
  b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
  + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
  b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
  (f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
  A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
  d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
  [e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx &= \frac{A+iB}{d\sqrt{a+ia \tan(c+dx)}} + \frac{\int \cot(c+dx) \sqrt{a+ia \tan(c+dx)} (aA)}{a^2} \\
&= \frac{A+iB}{d\sqrt{a+ia \tan(c+dx)}} + \frac{A \int \cot(c+dx)(a-ia \tan(c+dx)) \sqrt{a}}{a^2} \\
&= \frac{A+iB}{d\sqrt{a+ia \tan(c+dx)}} + \frac{A \text{Subst}\left(\int \frac{1}{x\sqrt{a+iax}} dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{A+iB}{d\sqrt{a+ia \tan(c+dx)}} \\
&= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}}
\end{aligned}$$

**Mathematica [A]**

time = 2.49, size = 208, normalized size = 1.82

$$\frac{\left( (A+iB)\sqrt{1+e^{2i(c+dx)}} + (A-iB)e^{i(c+dx)} \sinh^{-1}\left(e^{i(c+dx)}\right) - 2\sqrt{2}Ae^{i(c+dx)} \tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) \right) \sqrt{\sec(c+dx)}}{2d\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

```

[Out] (((A + I*B)*Sqrt[1 + E^((2*I)*(c + d*x))] + (A - I*B)*E^(I*(c + d*x))*ArcSi
nh[E^(I*(c + d*x))] - 2*Sqrt[2]*A*E^(I*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c
+ d*x))]/Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[Sec[c + d*x]]/(2*d*Sqrt[E^(I
*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))])

```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 947 vs. 2(93) = 186.

time = 0.94, size = 948, normalized size = 8.32

method	result	size
default	Expression too large to display	948

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x,method=\_RETURNVE  
RBOSE)

[Out] 
$$-1/4/d/a*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*(I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\sin(d*x+c)-I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)+2*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)+2*I*A*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)+A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)-I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\sin(d*x+c)+2*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-2*A*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)+2*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)+A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}+2*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})-4*I*B*\cos(d*x+c)-4*A*\cos(d*x+c))/(I*\sin(d*x+c)+\cos(d*x+c))$$

**Maxima** [A]

time = 0.56, size = 133, normalized size = 1.17

$$\frac{\sqrt{2}^{(A-iB)\log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)} - 4A\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{4d\sqrt{a}} - \frac{4(A+iB)}{\sqrt{ia\tan(dx+c)+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm  
="maxima")

[Out] 
$$-1/4*(\sqrt{2}*(A - I*B)*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a}))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a}))/\sqrt{a} - 4*A*\log((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a}))/\sqrt{a} - 4*(A + I*B)/\sqrt{I*a*\tan(d*x + c) + a})/d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(88) = 176.

time = 2.09, size = 575, normalized size = 5.04

$$\left(\frac{\sqrt{2}^{(A-iB)\log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)} - 4A\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{4d\sqrt{a}} - \frac{4(A+iB)}{\sqrt{ia\tan(dx+c)+a}}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$-1/4*(\sqrt{2})a*d*\sqrt{(A^2 - 2IA*B - B^2)/(a*d^2)}*e^{I*d*x + I*c}*\log(-4*((-IA - B)*a*e^{I*d*x + I*c} + (IA*d*e^{2I*d*x + 2I*c} + IA*d)*\sqrt{a/(e^{2I*d*x + 2I*c} + 1))*\sqrt{(A^2 - 2IA*B - B^2)/(a*d^2)})*e^{-I*d*x - I*c}/(IA + B)) - \sqrt{2}a*d*\sqrt{(A^2 - 2IA*B - B^2)/(a*d^2)}*e^{I*d*x + I*c}*\log(-4*((-IA - B)*a*e^{I*d*x + I*c} + (-IA*d*e^{2I*d*x + 2I*c} - IA*d)*\sqrt{a/(e^{2I*d*x + 2I*c} + 1))*\sqrt{(A^2 - 2IA*B - B^2)/(a*d^2)})*e^{-I*d*x - I*c}/(IA + B)) + 2a*d*\sqrt{A^2/(a*d^2)}*e^{I*d*x + I*c}*\log(16*(3A*a^2*e^{2I*d*x + 2I*c} + A*a^2 + 2*\sqrt{2}*(a^2*d*e^{3I*d*x + 3I*c} + a^2*d*e^{I*d*x + I*c}))*\sqrt{a/(e^{2I*d*x + 2I*c} + 1))*\sqrt{A^2/(a*d^2)})*e^{-2I*d*x - 2I*c}/A) - 2a*d*\sqrt{A^2/(a*d^2)}*e^{I*d*x + I*c}*\log(16*(3A*a^2*e^{2I*d*x + 2I*c} + A*a^2 - 2*\sqrt{2}*(a^2*d*e^{3I*d*x + 3I*c} + a^2*d*e^{I*d*x + I*c}))*\sqrt{a/(e^{2I*d*x + 2I*c} + 1))*\sqrt{A^2/(a*d^2)})*e^{-2I*d*x - 2I*c}/A) - 2*\sqrt{2}*(A + IB)*e^{2I*d*x + 2I*c} + A + IB)*\sqrt{a/(e^{2I*d*x + 2I*c} + 1)))*e^{-I*d*x - I*c}/(a*d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot(c + dx)}{\sqrt{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad [B]**

time = 7.14, size = 515, normalized size = 4.52

$$\frac{A + B i}{d \sqrt{a + i \tan(c + dx)}} - \frac{2 A \operatorname{atanh}\left(\frac{a e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}}\right) + \frac{1 + i e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}} + \frac{d e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}}}{\sqrt{a d}} + \sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}}\right) + \frac{\sqrt{2} e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}} + \frac{\sqrt{2} a e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}} + \frac{\sqrt{2} a^2 e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}} + \frac{1 + \sqrt{2} e^{i d x} \sqrt{a + i \tan(c + dx)}}{\sqrt{a^2 + i a \tan(c + dx)}}\right) (B + A i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + dx) \cdot (A + B \tan(c + dx)) / (a + a \tan(c + dx) \cdot i)^{1/2}, x)$

[Out]  $(A + B \cdot i) / (d \cdot (a + a \tan(c + dx) \cdot i)^{1/2}) - (2 \cdot A \cdot \text{atanh}((28 \cdot A^3 \cdot a^{3/2}) \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2}) / (28 \cdot A^3 \cdot a^2 \cdot d + 4 \cdot A \cdot B^2 \cdot a^2 \cdot d + A^2 \cdot B \cdot a^2 \cdot d \cdot 8i) + (4 \cdot A \cdot B^2 \cdot a^{3/2} \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2}) / (28 \cdot A^3 \cdot a^2 \cdot d + 4 \cdot A \cdot B^2 \cdot a^2 \cdot d + A^2 \cdot B \cdot a^2 \cdot d \cdot 8i) + (A^2 \cdot B \cdot a^{3/2} \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2} \cdot 8i) / (28 \cdot A^3 \cdot a^2 \cdot d + 4 \cdot A \cdot B^2 \cdot a^2 \cdot d + A^2 \cdot B \cdot a^2 \cdot d \cdot 8i)) / (a^{1/2} \cdot d) + (2^{1/2} \cdot \text{atanh}((2^{1/2} \cdot A^3 \cdot (-a)^{3/2}) \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2} \cdot 7i) / (2 \cdot (7 \cdot A^3 \cdot a^2 \cdot d - B^3 \cdot a^2 \cdot d \cdot i + 3 \cdot A \cdot B^2 \cdot a^2 \cdot d - A^2 \cdot B \cdot a^2 \cdot d \cdot 5i)) + (2^{1/2} \cdot B^3 \cdot (-a)^{3/2} \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2}) / (2 \cdot (7 \cdot A^3 \cdot a^2 \cdot d - B^3 \cdot a^2 \cdot d \cdot i + 3 \cdot A \cdot B^2 \cdot a^2 \cdot d - A^2 \cdot B \cdot a^2 \cdot d \cdot 5i)) + (2^{1/2} \cdot A \cdot B^2 \cdot (-a)^{3/2} \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2} \cdot 3i) / (2 \cdot (7 \cdot A^3 \cdot a^2 \cdot d - B^3 \cdot a^2 \cdot d \cdot i + 3 \cdot A \cdot B^2 \cdot a^2 \cdot d - A^2 \cdot B \cdot a^2 \cdot d \cdot 5i)) + (5 \cdot 2^{1/2} \cdot A^2 \cdot B \cdot (-a)^{3/2} \cdot d \cdot (a + a \tan(c + dx) \cdot i)^{1/2}) / (2 \cdot (7 \cdot A^3 \cdot a^2 \cdot d - B^3 \cdot a^2 \cdot d \cdot i + 3 \cdot A \cdot B^2 \cdot a^2 \cdot d - A^2 \cdot B \cdot a^2 \cdot d \cdot 5i))) \cdot (A \cdot i + B) / (2 \cdot (-a)^{1/2} \cdot d)$

$$3.95 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=167

$$\frac{(iA - 2B) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}} \right)}{\sqrt{a} d} + \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}} \right)}{\sqrt{2} \sqrt{a} d} + \frac{(A + iB) \cot(c)}{d \sqrt{a+ia \tan(c+dx)}}$$

[Out] (I\*A-2\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2\*(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/d\*2^(1/2)/a^(1/2)+(A+I\*B)\*cot(d\*x+c)/d/(a+I\*a\*tan(d\*x+c))^(1/2)-(2\*A+I\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d

Rubi [A]

time = 0.38, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(-2B + iA) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}} \right)}{\sqrt{a} d} + \frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}} \right)}{\sqrt{2} \sqrt{a} d} - \frac{(2A + iB) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{ad} + \frac{(A + iB) \cot(c+dx)}{d \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((I\*A - 2\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]])/(Sqrt[a]\*d) + ((I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/(Sqrt[2]\*Sqrt[a]\*d) + ((A + I\*B)\*Cot[c + d\*x])/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((2\*A + I\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(a\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a

\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx &= \frac{(A+iB) \cot(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} + \frac{\int \cot^2(c+dx) \sqrt{a+ia \tan(c+dx)} (a+ia \tan(c+dx))}{d\sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \cot(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(2A+iB) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{ad} \\
 &= \frac{(A+iB) \cot(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(2A+iB) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{ad} \\
 &= \frac{(A+iB) \cot(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(2A+iB) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{ad} \\
 &= \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(A+iB) \cot(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(iA-2B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}
 \end{aligned}$$

**Mathematica [A]**

time = 4.01, size = 224, normalized size = 1.34

$$\frac{(B+A \cot(c+dx)) \left( -2A \cos(c+dx) + \frac{(-1+e^{2i(c+dx)}) \left( (A-iB) \sinh^{-1}(e^{i(c+dx)}) + \sqrt{2} (A+2iB) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) \right) \sqrt{\sec(c+dx)}}{\sqrt{2} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}}} + 2(-2iA+B) \sin(c+dx) \right)}{2d(A \cos(c+dx) + B \sin(c+dx)) \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]], x]

[Out] ((B + A\*Cot[c + d\*x])\*(-2\*A\*Cos[c + d\*x] + ((-1 + E^((2\*I)\*(c + d\*x))))\*((A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))] + Sqrt[2]\*(A + (2\*I)\*B)\*ArcTanh[(Sqrt[2]\*E^(I\*(c + d\*x))]/Sqrt[1 + E^((2\*I)\*(c + d\*x))]])\*Sqrt[Sec[c + d\*x]])/(Sqrt[2]\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]) + 2\*((-2\*I)\*A + B)\*Sin[c + d\*x]))/(2\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))\*Sqrt[a + I\*a\*Tan[c + d\*x]]

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal.  $2726$  vs.  $2(141) = 282$ .  
time = 0.71, size = 2727, normalized size = 16.33

method	result	size
default	Expression too large to display	2727

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURN  
VERBOSE)`

[Out] 
$$-1/4/d/a*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{1/2}*(-4*A*\cos(d*x+c)^2*\sin(d*x+c)+I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*2^{1/2}*\cos(d*x+c)^2-A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*2^{1/2}*\cos(d*x+c)^2*\sin(d*x+c)+I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c)-I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*2^{1/2}*\sin(d*x+c)+2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^2*\sin(d*x+c)-I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*2^{1/2}*\cos(d*x+c)+I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*2^{1/2}*\cos(d*x+c)^3-4*B*\cos(d*x+c)^3+4*B*\cos(d*x+c)-A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))-2*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}))+I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/2*2^{1/2}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*2^{1/2}*\cos(d*x+c)^2*\sin(d*x+c)-2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)-A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)+2*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^3+2*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^3+A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2+2*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\cos(d*x+c)^2-A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)-2*B*(-2*\cos(d$$

$$\begin{aligned} & *x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})* \\ & \cos(d*x+c)+A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos \\ & (d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1 \\ & /2)}*\sin(d*x+c)-I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2* \\ & \cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\sin(d*x+c)-I*A*( \\ & -2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1)) \\ & ^{(1/2)})*\cos(d*x+c)^3+2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x \\ & +c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+ \\ & c)^3-2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+ \\ & c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)+B*(-2*\cos(d*x \\ & +c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\si \\ & n(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3+B*(-2*c \\ & \cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+ \\ & c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2+I \\ & *A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c) \\ & +1))^{(1/2)})*\cos(d*x+c)-B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{( \\ & 1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1)) \\ & ^{(1/2)})*2^{(1/2)}*\cos(d*x+c)-I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan( \\ & 1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2+2*I*B*(-2*\cos(d*x+c)/ \\ & \cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-co \\ & s(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2-I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/ \\ & 2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c) \\ & /(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}+I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ar \\ & ctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})-B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1 \\ & ))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos( \\ & d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}-2*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{( \\ & 1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin \\ & (d*x+c))-8*I*A*\cos(d*x+c)+8*I*A*\cos(d*x+c)^3+A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1 \\ & ))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2*\sin(d* \\ & x+c)-2*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c) \\ & /(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c))/ \\ & (-1+\cos(d*x+c))/(\sin(d*x+c)+\cos(d*x+c))/(\cos(d*x+c)+1) \end{aligned}$$

**Maxima [A]**

time = 0.60, size = 184, normalized size = 1.10

$$i a \left( \frac{\sqrt{2}^{(A-i B)} \log \left( \frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}} \right)}{a^{\frac{3}{2}}} + \frac{2^{(A+2i B)} \log \left( \frac{\sqrt{i a \tan(dx+c) + a} - \sqrt{a}}{\sqrt{i a \tan(dx+c) + a} + \sqrt{a}} \right)}{a^{\frac{3}{2}}} + \frac{4((i a \tan(dx+c) + a)(2 A + i B) - (A + i B) a)}{(i a \tan(dx+c) + a)^{\frac{3}{2}} a - \sqrt{i a \tan(dx+c) + a} a^2} \right) / 4 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/4\*I\*a\*(sqrt(2)\*(A - I\*B)\*log(-(sqrt(2)\*sqrt(a) - sqrt(I\*a\*tan(d\*x + c) + a))/(sqrt(2)\*sqrt(a) + sqrt(I\*a\*tan(d\*x + c) + a)))/a^(3/2) + 2\*(A + 2\*I\*B

) $\log((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a}))/a^{3/2} + 4*((I*a*\tan(d*x + c) + a)*(2*A + I*B) - (A + I*B)*a)/((I*a*\tan(d*x + c) + a)^{3/2}*a - \sqrt{I*a*\tan(d*x + c) + a}*a^2))/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 742 vs.  $2(135) = 270$ .

time = 1.66, size = 742, normalized size = 4.44



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] 
$$-1/4*(\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} + (a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}))e^{(-I*d*x - I*c)/(I*A + B)} - \sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} - (a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a*d^2)}))e^{(-I*d*x - I*c)/(I*A + B)} - (a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{-(A^2 + 4*I*A*B - 4*B^2)/(a*d^2)}*\log(-16*(3*(I*A - 2*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (I*A - 2*B)*a^2 + 2*\sqrt{2}*(a^2*d*e^{(3*I*d*x + 3*I*c)} + a^2*d*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{-(A^2 + 4*I*A*B - 4*B^2)/(a*d^2)})e^{(-2*I*d*x - 2*I*c)/(-I*A + 2*B)} + (a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{-(A^2 + 4*I*A*B - 4*B^2)/(a*d^2)}*\log(-16*(3*(I*A - 2*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (I*A - 2*B)*a^2 - 2*\sqrt{2}*(a^2*d*e^{(3*I*d*x + 3*I*c)} + a^2*d*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{-(A^2 + 4*I*A*B - 4*B^2)/(a*d^2)})e^{(-2*I*d*x - 2*I*c)/(-I*A + 2*B)} + 2*\sqrt{2}*((3*I*A - B)*e^{(4*I*d*x + 4*I*c)} + 2*I*A*e^{(2*I*d*x + 2*I*c)} - I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^2(c + dx)}{\sqrt{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)`



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

**Mupad [B]**

time = 8.62, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*i)^(1/2),x)
```

```
[Out] 2*atanh((3*d^4*(a + a*tan(c + d*x)*i)^(1/2)*((9*B^2)/(16*a*d^2) - (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)/(16*a^6) - (A*B*3i)/(8*a*d^2))^(1/2)*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2))/((A^3*a^5*d*i)/2 + (35*B^3*a^5*d)/2 + (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)*3i)/2 - (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2))/2 - (A*B^2*a^5*d*57i)/2 - (15*A^2*B*a^5*d)/2) + (A^2*a^2*d^2*(a + a*tan(c + d*x)*i)^(1/2)*((9*B^2)/(16*a*d^2) - (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)/(16*a^6) - (A*B*3i)/(8*a*d^2))^(1/2))/((A^3*a^2*d*i)/2 + (35*B^3*a^2*d)/2 - (A*B^2*a^2*d*57i)/2 - (15*A^2*B*a^2*d)/2 + (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)*3i)/(2*a^3) - (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2))/(2*a^3)) - (7*B^2*a^2*d^2*(a + a*tan(c + d*x)*i)^(1/2)*((9*B^2)/(16*a*d^2) - (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)/(16*a^6) - (A*B*3i)/(8*a*d^2))^(1/2))/((A^3*a^2*d*i)/2 + (35*B^3*a^2*d)/2 - (A*B^2*a^2*d*57i)/2 - (15*A^2*B*a^2*d)/2 + (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)*3i)/(2*a^3) - (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2))/(2*a^3))
```

$$\begin{aligned}
& (2*a^3)) + (A*B*a^2*d^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((9*B^2)/(16*a*d^2) - \\
& (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/(16*a^6) - (A \\
& *B*3i)/(8*a*d^2))^{(1/2)}*10i)/((A^3*a^2*d*1i)/2 + (35*B^3*a^2*d)/2 - (A*B^2* \\
& a^2*d*57i)/2 - (15*A^2*B*a^2*d)/2 + (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/ \\
& d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4 \\
& )^{(1/2)}*3i)/(2*a^3) - (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A \\
& ^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/(2* \\
& a^3)))*((9*B^2)/(16*a*d^2) - (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4 \\
& *a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*2 \\
& 0i)/d^4)^{(1/2)}/(16*a^6) - (A*B*3i)/(8*a*d^2))^{(1/2)} - 2*atanh((3*d^4*(a + a \\
& *tan(c + d*x)*1i)^{(1/2)}*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2 \\
& *a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/(16*a^6) - \\
& (3*A^2)/(16*a*d^2) + (9*B^2)/(16*a*d^2) - (A*B*3i)/(8*a*d^2))^{(1/2)}*((A^4* \\
& a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/ \\
& d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/((A^3*a^5*d*1i)/2 + (35*B^3*a^5*d)/2 - ( \\
& A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3 \\
& *a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}*3i)/2 + (3*B*d^3*((A^4*a^10)/ \\
& d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + \\
& (A^3*B*a^10*20i)/d^4)^{(1/2)}/2 - (A*B^2*a^5*d*57i)/2 - (15*A^2*B*a^5*d)/2) \\
& - (A^2*a^2*d^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((A^4*a^10)/d^4 + (49*B^4*a^1 \\
& 0)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/ \\
& d^4)^{(1/2)}/(16*a^6) - (3*A^2)/(16*a*d^2) + (9*B^2)/(16*a*d^2) - (A*B*3i)/(8 \\
& *a*d^2))^{(1/2)}/((A^3*a^2*d*1i)/2 + (35*B^3*a^2*d)/2 - (A*B^2*a^2*d*57i)/2 \\
& - (15*A^2*B*a^2*d)/2 - (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^ \\
& 2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}*3i)/( \\
& 2*a^3) + (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/ \\
& d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/(2*a^3)) + (7*B^ \\
& 2*a^2*d^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^ \\
& 4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^ \\
& (1/2)}/(16*a^6) - (3*A^2)/(16*a*d^2) + (9*B^2)/(16*a*d^2) - (A*B*3i)/(8*a*d^ \\
& 2))^{(1/2)}/((A^3*a^2*d*1i)/2 + (35*B^3*a^2*d)/2 - (A*B^2*a^2*d*57i)/2 - (15 \\
& *A^2*B*a^2*d)/2 - (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2 \\
& *a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}*3i)/(2*a^3 \\
& ) + (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - \\
& (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/(2*a^3)) - (A*B*a^2*d \\
& ^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (11 \\
& 4*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^{(1/2)}/( \\
& 16*a^6) - (3*A^2)/(16*a*d^2) + (9*B^2)/(16*a*d^2) - (A*B*3i)/(8*a*d^2))^{(1/ \\
& 2)}*10i)/((A^3*a^2*d*1i)/2 + (35*B^3*a^2*d)/2 - (A*B^2*a^2*d*57i)/2 - (15*A^ \\
& 2*B*a^2*d)/2 - (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^ \\
& 10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*2...
\end{aligned}$$

$$3.96 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=219

$$\frac{(11A + 4iB) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}} \right)}{4\sqrt{a} d} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}} \right)}{\sqrt{2}\sqrt{a} d} + \frac{(A + iB) \cot(c+dx)}{d\sqrt{a+ia \tan(c+dx)}}$$

[Out] 1/4\*(11\*A+4\*I\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2\*(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/d\*2^(1/2)/a^(1/2)+(A+I\*B)\*cot(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/4\*(7\*I\*A-8\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d-1/2\*(3\*A+2\*I\*B)\*cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d

**Rubi** [A]

time = 0.50, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(11A + 4iB) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}} \right)}{4\sqrt{a} d} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}} \right)}{\sqrt{2}\sqrt{a} d} - \frac{(3A + 2iB) \cot^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{2ad} + \frac{(A + iB) \cot^2(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} + \frac{(-8B + 7iA) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((11\*A + (4\*I)\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]]/(4\*Sqrt[a]\*d) - ((A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])]/(Sqrt[2]\*Sqrt[a]\*d) + ((A + I\*B)\*Cot[c + d\*x]^2)/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (((7\*I)\*A - 8\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(4\*a\*d) - ((3\*A + (2\*I)\*B)\*Cot[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(2\*a\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3681

Int((((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]))/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*

d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx &= \frac{(A+iB) \cot^2(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{\int \cot^3(c+dx) \sqrt{a+ia \tan(c+dx)} (a+ia \tan(c+dx))}{d \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} - \frac{(3A+2iB) \cot^2(c+dx) \sqrt{a+ia \tan(c+dx)}}{2ad} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(7iA-8B) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4ad} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(7iA-8B) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4ad} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(7iA-8B) \cot(c+dx) \sqrt{a+ia \tan(c+dx)}}{4ad} \\
 &= -\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d} + \frac{(A+iB) \cot^2(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(11A+4iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a} d} - \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{\sqrt{2} \sqrt{a} d}
 \end{aligned}$$

**Mathematica [A]**

time = 4.39, size = 363, normalized size = 1.66

$$\frac{(-\sqrt{1+e^{2I(c+dx)}} (16(A-iB) \operatorname{ArcSinh}[e^{I(c+dx)}] + \sqrt{2} (11A+4iB) (\log((-1+e^{I(c+dx)})^2) - \log(1+e^{I(c+dx)})^2) + \log(1+3e^{2I(c+dx)} + 2\sqrt{2} \sqrt{1+e^{2I(c+dx)}} - 2e^{I(c+dx)} (1+\sqrt{2} \sqrt{1+e^{2I(c+dx)}})) - \log(1+3e^{2I(c+dx)} + 2\sqrt{2} \sqrt{1+e^{2I(c+dx)}} + 2e^{I(c+dx)} (1+\sqrt{2} \sqrt{1+e^{2I(c+dx)}}))) + 4 \cot(c+dx) \cos(c+dx) - 9A - 8iB + (5A+8iB) \cos(2(c+dx)) + (A+iB) \sin(2(c+dx)) (A+B \tan(c+dx))}{32(A \cos(c+dx) + B \sin(c+dx)) \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]], x]

[Out] ((-Sqrt[1 + E^((2\*I)\*(c + d\*x))])\*(16\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))]) + Sqrt[2]\*(11\*A + (4\*I)\*B)\*(Log[(-1 + E^(I\*(c + d\*x)))^2] - Log[(1 + E^(I\*(c + d\*x)))^2] + Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]] - 2\*E^(I\*(c + d\*x))\*(1 + Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]]) - Log[3 + 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]])



$$\begin{aligned} & \sin(dx+c+1)^{1/2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}}\right) / \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}} \cos(dx+c)^2 - 4 I B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \cos(dx+c)^2 \sin(dx+c) \\ & + 4 I B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}}\right) / \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}} \cos(dx+c) - 11 A \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \cos(dx+c) - 4 A \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}}\right) / \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}} \cos(dx+c)^3 - 4 B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \cos(dx+c)^3 \\ & + 4 B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \\ & \cos(dx+c) - 11 A \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \\ & - 4 I B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \\ & - 11 A \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \\ & + 4 B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \\ & + 16 B \cos(dx+c)^2 \sin(dx+c) - 11 A \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \cos(dx+c)^2 \sin(dx+c) \\ & - 4 B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \\ & \cos(dx+c)^2 \sin(dx+c) - 11 I A \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \\ & \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \sin(dx+c) - 20 A \cos(dx+c)^3 - 4 I A \cos(dx+c)^2 \sin(dx+c) \\ & + 11 A \cos(dx+c)^2 \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \\ & - 4 B \cos(dx+c)^2 \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \\ & + 4 B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \arctan\left(\frac{1}{-2 \cos(dx+c) / (\cos(dx+c)+1)}\right)^{1/2} \sin(dx+c) \\ & + 11 A \ln\left(\frac{\sin(dx+c) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} - \cos(dx+c)+1}{\sin(dx+c)}\right) \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \sin(dx+c) - 32 I B \cos(dx+c)^3 + 32 I B \cos(dx+c) - 4 A \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}}\right) / \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}} \cos(dx+c) - 4 B \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}}\right) / \left(-2 \cos(dx+c) / (\cos(dx+c)+1)\right)^{1/2} \\ & \sqrt{2} \sqrt{\frac{I \cos(dx+c) - I - \sin(dx+c)}{\sin(dx+c)}} \sin(dx+c) / (-1 + \cos(dx+c)) / (I \sin(dx+c) + \cos(dx+c)) / (\cos(dx+c)+1) \end{aligned}$$

**Maxima [A]**

time = 0.51, size = 232, normalized size = 1.06

$$a^2 \left( \frac{2 \left( (i a \tan(dx+c)+a)^2 (7 A+8 i B) - (i a \tan(dx+c)+a) (13 A+12 i B) a + 4 (A+i B) a^2 \right)}{(i a \tan(dx+c)+a)^2 a^2 - 2 (i a \tan(dx+c)+a)^2 a^3 + \sqrt{i a \tan(dx+c)+a} a^4} - \frac{2 \sqrt{2}^{(A-i B) \log\left(\frac{-\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c)+a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c)+a}}\right)}}{a^2} + \frac{(11 A+4 i B) \log\left(\frac{\sqrt{i a \tan(dx+c)+a} - \sqrt{a}}{\sqrt{i a \tan(dx+c)+a} + \sqrt{a}}\right)}}{a^2} \right)$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 
$$-1/8*a^2*(2*((I*a*\tan(d*x + c) + a)^2*(7*A + 8*I*B) - (I*a*\tan(d*x + c) + a)*(13*A + 12*I*B)*a + 4*(A + I*B)*a^2)/((I*a*\tan(d*x + c) + a)^{(5/2)}*a^2 - 2*(I*a*\tan(d*x + c) + a)^{(3/2)}*a^3 + \sqrt{I*a*\tan(d*x + c) + a}*a^4) - 2*\sqrt{2}*(A - I*B)*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a})/(\sqrt{2})*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a}))/a^{(5/2)} + (11*A + 4*I*B)*\log((\sqrt{I*a*\tan(d*x + c) + a} - \sqrt{a})/(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a}))/a^{(5/2)}/d$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(172) = 344.  
time = 1.90, size = 835, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/16*(4*\sqrt{2}*(a*d*e^{(5*I*d*x + 5*I*c)} - 2*a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})*\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} + (I*a*d*e^{(2*I*d*x + 2*I*c)} + I*a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2)}))e^{(-I*d*x - I*c)/(I*A + B)} \\ & - 4*\sqrt{2}*(a*d*e^{(5*I*d*x + 5*I*c)} - 2*a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})*\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2)}*\log(-4*((-I*A - B)*a*e^{(I*d*x + I*c)} + (-I*a*d*e^{(2*I*d*x + 2*I*c)} - I*a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(A^2 - 2*I*A*B - B^2)/(a*d^2)}))e^{(-I*d*x - I*c)/(I*A + B)} \\ & + (a*d*e^{(5*I*d*x + 5*I*c)} - 2*a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})*\sqrt{(121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2)}*\log(-16*(3*(11*I*A - 4*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (11*I*A - 4*B)*a^2 + 2*\sqrt{2}*(I*a^2*d*e^{(3*I*d*x + 3*I*c)} + I*a^2*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2)}))e^{(-2*I*d*x - 2*I*c)/(-11*I*A + 4*B)} \\ & - (a*d*e^{(5*I*d*x + 5*I*c)} - 2*a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)})*\sqrt{(121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2)}*\log(-16*(3*(11*I*A - 4*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (11*I*A - 4*B)*a^2 + 2*\sqrt{2}*(-I*a^2*d*e^{(3*I*d*x + 3*I*c)} - I*a^2*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2)}))e^{(-2*I*d*x - 2*I*c)/(-11*I*A + 4*B)} \\ & - 4*\sqrt{2}*(3*(A + 2*I*B)*e^{(6*I*d*x + 6*I*c)} - 2*(3*A + I*B)*e^{(4*I*d*x + 4*I*c)} - (7*A + 6*I*B)*e^{(2*I*d*x + 2*I*c)} + 2*A + 2*I*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})/(a*d*e^{(5*I*d*x + 5*I*c)} - 2*a*d*e^{(3*I*d*x + 3*I*c)} + a*d*e^{(I*d*x + I*c)}) \end{aligned}$$

**Sympy [F]**



time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^3(c + dx)}{\sqrt{ia(\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*3/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^3/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad [B]**

time = 8.65, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] 2\*atanh((12\*d^4\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((129\*A^2)/(128\*a\*d^2) - ((12769\*A^4\*a^10)/(4\*d^4) + (16\*B^4\*a^10)/d^4 - (3156\*A^2\*B^2\*a^10)/d^4 - (A\*B^3\*a^10\*416i)/d^4 + (A^3\*B\*a^10\*5876i)/d^4)^(1/2)/(64\*a^6) - (3\*B^2)/(16\*a\*d^2) + (A\*B\*9i)/(16\*a\*d^2))^(1/2)\*((12769\*A^4\*a^10)/(4\*d^4) + (16\*B^4\*a^10)/d^4 - (3156\*A^2\*B^2\*a^10)/d^4 - (A\*B^3\*a^10\*416i)/d^4 + (A^3\*B\*a^10\*5876i)/d^4)^(1/2))/(B^3\*a^5\*d\*8i - (1469\*A^3\*a^5\*d)/2 + 9\*A\*d^3\*((12769\*A^4\*a^10)/(4\*d^4) + (16\*B^4\*a^10)/d^4 - (3156\*A^2\*B^2\*a^10)/d^4 - (A\*B^3\*a^10\*416i)/d^4 + (A^3\*B\*a^10\*5876i)/d^4)^(1/2) + B\*d^3\*((12769\*A^4\*a^10)/(4\*d^4) + (16\*B^4\*a^10)/d^4 - (3156\*A^2\*B^2\*a^10)/d^4 - (A\*B^3\*a^10\*416i)/d^4 + (A^3\*B\*a^10\*5876i)/d^4)^(1/2)\*6i + 156\*A\*B^2\*a^5\*d - A^2\*B\*a^5\*d\*789i) - (226\*A^2\*a^2\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((129\*A^2)/(128\*a\*d^2) - ((12769\*A^4\*a^10)/(4\*d^4) + (16\*B^4\*a^10)/d^4 - (3156\*A^2\*B^2\*a^10)/d^4 - (A\*B^3\*a^10\*416i)/d^4 + (A^3\*B\*a^10\*5876i)/d^4)^(1/2)/(64\*a^6) - (3\*B^2)/(16\*a\*d^2) + (A\*B\*9i)/(16\*a\*d^2))^(1/2))/(B^3\*a^2\*d\*8i - (1469\*A^3\*a^2\*d)/2 + 156\*A\*B^2\*a^2\*

$$\begin{aligned}
& d - A^2 B a^2 d^* 789i + (9 A d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / a^3 + (B d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) * 6i) / a^3 + (16 B^2 a^2 d^2 (a + a \tan(c + d x) * i)^{(1/2)}) * ((129 A^2) / (128 a d^2) - ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (64 a^6) - (3 B^2) / (16 a d^2) + (A B * 9i) / (16 a d^2))^{(1/2)}) / (B^3 a^2 d^* 8i - (1469 A^3 a^2 d) / 2 + 156 A B^2 a^2 d - A^2 B a^2 d^* 789i + (9 A d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / a^3 + (B d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) * 6i) / a^3 - (A B a^2 d^2 (a + a \tan(c + d x) * i)^{(1/2)}) * ((129 A^2) / (128 a d^2) - ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (64 a^6) - (3 B^2) / (16 a d^2) + (A B * 9i) / (16 a d^2))^{(1/2)}) * 208i) / (B^3 a^2 d^* 8i - (1469 A^3 a^2 d) / 2 + 156 A B^2 a^2 d - A^2 B a^2 d^* 789i + (9 A d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / a^3 + (B d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) * 6i) / a^3)) * ((129 A^2) / (128 a d^2) - ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (64 a^6) - (3 B^2) / (16 a d^2) + (A B * 9i) / (16 a d^2))^{(1/2)}) + 2 * \operatorname{atanh}((12 d^4 (a + a \tan(c + d x) * i)^{(1/2)}) * (((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (64 a^6) + (129 A^2) / (128 a d^2) - (3 B^2) / (16 a d^2) + (A B * 9i) / (16 a d^2))^{(1/2)}) * ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (((1469 A^3 a^5 d) / 2 - B^3 a^5 d^* 8i + 9 A d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) + B d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) * 6i - 156 A B^2 a^5 d + A^2 B a^5 d^* 789i) + (226 A^2 a^2 d^2 (a + a \tan(c + d x) * i)^{(1/2)}) * (((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (64 a^6) + (129 A^2) / (128 a d^2) - (3 B^2) / (16 a d^2) + (A B * 9i) / (16 a d^2))^{(1/2)}) / ((1469 A^3 a^2 d) / 2 - B^3 a^2 d^* 8i - 156 A B^2 a^2 d + A^2 B a^2 d^* 789i + (9 A d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / a^3 + (B d^3 ((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) * 6i) / a^3 - (16 B^2 a^2 d^2 (a + a \tan(c + d x) * i)^{(1/2)}) * (((12769 A^4 a^{10}) / (4 d^4) + (16 B^4 a^{10}) / d^4 - (3156 A^2 B^2 a^{10}) / d^4 - (A B^3 a^{10} 416i) / d^4 + (A^3 B a^{10} 5876i) / d^4)^{(1/2)}) / (64 a^6) + (129 A^2) / (128 a d^2) - (3 B^2) / (16 a d^2) + (A B * 9i)
\end{aligned}$$

$$\begin{aligned}
& )/(16*a*d^2)^{(1/2)})/((1469*A^3*a^2*d)/2 - B^3*a^2*d*8i - 156*A*B^2*a^2*d + \\
& A^2*B*a^2*d*789i + (9*A*d^3*((12769*A^4*a^10)/(4*d^4) + (16*B^4*a^10)/d^4 \\
& - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 + (A^3*B*a^10*5876i)/d^4) \\
& ^{(1/2)})/a^3 + (B*d^3*((12769*A^4*a^10)/(4*d^4) + (16*B^4*a^10)/d^4 - (3156* \\
& A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 + (A^3*B*a^10*5876i)/d^4)^{(1/2)}*6 \\
& i)/a^3 + (A*B*a^2*d^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*(((12769*A^4*a^10)/(4* \\
& d^4) + (16*B^4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 \\
& + (A^3*B*a^10*5876i)/d^4)^{(1/2)})/(64*a^6) + (129\dots
\end{aligned}$$

$$3.97 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=209

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2}d} + \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(3A + 5iB) \tan^2(c + dx)}{2ad\sqrt{a + ia \tan(c + dx)}} - \frac{2(3A + 5iB) \tan(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}}$$

[Out] 1/4\*(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(3/2)/d \*2^(1/2)-2\*(3\*A+5\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d+1/2\*(3\*A+5\*I\*B)\*tan(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/3\*(I\*A-B)\*tan(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))^(3/2)+1/6\*(11\*A+21\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(3/2)/a^3/d

**Rubi [A]**

time = 0.36, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3676, 3673, 3608, 3561, 212}

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2}d} + \frac{(11A + 21iB)(a + ia \tan(c + dx))^{3/2}}{6a^3d} - \frac{2(3A + 5iB)\sqrt{a + ia \tan(c + dx)}}{a^2d} + \frac{(-B + iA) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(3A + 5iB) \tan^2(c + dx)}{2ad\sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2),x]

[Out] ((A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/(2\*Sqrt[2]\*a^(3/2)\*d) + ((I\*A - B)\*Tan[c + d\*x]^3)/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((3\*A + (5\*I)\*B)\*Tan[c + d\*x]^2)/(2\*a\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - (2\*(3\*A + (5\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(a^2\*d) + ((11\*A + (21\*I)\*B)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(6\*a^3\*d)

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist

$[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& !\text{LtQ}[m, 0]$

### Rule 3673

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] := \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1))), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

### Rule 3676

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] := \text{Simp}[(-A*b - a*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^n/(2*a*f*m)), x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\tan^3(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx &= \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} - \frac{\int \frac{\tan^2(c + dx)(3a(iA - B) + \frac{3}{2}a(A + 3iB) \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}}}{3a^2} \\ &= \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(3A + 5iB) \tan^2(c + dx)}{2ad \sqrt{a + ia \tan(c + dx)}} + \frac{\int \tan(c + dx)}{\sqrt{a + ia \tan(c + dx)}} \\ &= \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(3A + 5iB) \tan^2(c + dx)}{2ad \sqrt{a + ia \tan(c + dx)}} + \frac{(11A + 5iB) \tan(c + dx)}{2ad \sqrt{a + ia \tan(c + dx)}} \\ &= \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(3A + 5iB) \tan^2(c + dx)}{2ad \sqrt{a + ia \tan(c + dx)}} - \frac{2(3A + 5iB)}{2ad \sqrt{a + ia \tan(c + dx)}} \\ &= \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(3A + 5iB) \tan^2(c + dx)}{2ad \sqrt{a + ia \tan(c + dx)}} - \frac{2(3A + 5iB)}{2ad \sqrt{a + ia \tan(c + dx)}} \\ &= \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2} d} + \frac{(iA - B) \tan^3(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} \end{aligned}$$

**Mathematica [A]**

time = 4.03, size = 176, normalized size = 0.84

$$\frac{-\frac{24i(A-iB)e^{3i(c+dx)}\sinh^{-1}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}}{(1+e^{2i(c+dx)})^{3/2}} + i \sec^3(c+dx)(21(3A+5iB)\cos(c+dx) + (37A+51iB)\cos(3(c+dx)) + 2i(39A+61iB+(39A+53iB)\cos(2(c+dx)))\sin(c+dx))}{24ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (((-24\*I)\*(A - I\*B)\*E^((3\*I)\*(c + d\*x))\*ArcSinh[E^(I\*(c + d\*x))])/(1 + E^(2\*I\*(c + d\*x)))^(3/2) + I\*Sec[c + d\*x]^3\*(21\*(3\*A + (5\*I)\*B)\*Cos[c + d\*x] + (37\*A + (51\*I)\*B)\*Cos[3\*(c + d\*x)] + (2\*I)\*(39\*A + (61\*I)\*B + (39\*A + (53\*I)\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x]))/(24\*a\*d\*(-I + Tan[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [A]**

time = 0.11, size = 153, normalized size = 0.73

method	result
derivativedivides	$2 \left( -\frac{iB(a+ia\tan(dx+c))^{3/2}}{3} + 2iaB\sqrt{a+ia\tan(dx+c)} + aA\sqrt{a+ia\tan(dx+c)} - \frac{a^{3/2}(-iB+A)\sqrt{2}}{da^3} \right)$
default	$2 \left( -\frac{iB(a+ia\tan(dx+c))^{3/2}}{3} + 2iaB\sqrt{a+ia\tan(dx+c)} + aA\sqrt{a+ia\tan(dx+c)} - \frac{a^{3/2}(-iB+A)\sqrt{2}}{da^3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2), x, method=\_RETURNVERBOSE)

[Out] -2/d/a^3\*(-1/3\*I\*B\*(a+I\*a\*tan(d\*x+c))^(3/2)+2\*I\*B\*a\*(a+I\*a\*tan(d\*x+c))^(1/2))+a\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)-1/8\*a^(3/2)\*(A-I\*B)\*2^(1/2)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))+1/4\*a^2\*(5\*A+7\*I\*B)/(a+I\*a\*tan(d\*x+c))^(1/2)-1/6\*a^3\*(A+I\*B)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Maxima [A]**

time = 0.52, size = 160, normalized size = 0.77

$$\frac{3\sqrt{2}(A-iB)a^{5/2}\log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right) - 16i(ia\tan(dx+c)+a)^{3/2}Ba + 48\sqrt{ia\tan(dx+c)+a}(A+2iB)a^2 + \frac{4(3(ia\tan(dx+c)+a)(5A+7iB)a^3-2(A+iB)a^4)}{(ia\tan(dx+c)+a)^2}}{24a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] 
$$-1/24*(3*\sqrt{2}*(A - I*B)*a^{5/2}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(dx + c) + a})/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(dx + c) + a})) - 16*I*(I*a*\tan(dx + c) + a)^{3/2}*B*a + 48*\sqrt{I*a*\tan(dx + c) + a}*(A + 2*I*B)*a^2 + 4*(3*(I*a*\tan(dx + c) + a)*(5*A + 7*I*B)*a^3 - 2*(A + I*B)*a^4)/(I*a*\tan(dx + c) + a)^{3/2})/(a^4*d)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs.  $2(162) = 324$ .  
time = 1.79, size = 440, normalized size = 2.11

$$\frac{3\sqrt{\frac{1}{2}}\sqrt{a^{2d}\tan^{2}(c+dx)+a^{2d}}\sqrt{\frac{A-2IAB-B^2}{2a^{2d}\tan^{2}(c+dx)+a^{2d}}}\sqrt{\frac{A-2IAB-B^2}{2a^{2d}\tan^{2}(c+dx)+a^{2d}}}}{12(a^{2d}\tan^{2}(c+dx)+a^{2d})^{3/2}} - \frac{3\sqrt{\frac{1}{2}}\sqrt{a^{2d}\tan^{2}(c+dx)+a^{2d}}\sqrt{\frac{A-2IAB-B^2}{2a^{2d}\tan^{2}(c+dx)+a^{2d}}}\sqrt{\frac{A-2IAB-B^2}{2a^{2d}\tan^{2}(c+dx)+a^{2d}}}}{12(a^{2d}\tan^{2}(c+dx)+a^{2d})^{3/2}} + \sqrt{2}(19A+26I*B)a^{6d}\tan^{6}(c+dx)+317A+26I*B)a^{4d}\tan^{4}(c+dx)+4(2A+3I*B)a^{2d}\tan^{2}(c+dx)-A-I*B)\sqrt{\frac{a}{2a^{2d}\tan^{2}(c+dx)+a^{2d}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$-1/12*(3*\sqrt{1/2}*(a^{2*d}*e^{(5*I*d*x + 5*I*c)} + a^{2*d}*e^{(3*I*d*x + 3*I*c)})*\sqrt{((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*\log(-4*(\sqrt{2}*\sqrt{1/2}*(I*a^{2*d}*e^{(2*I*d*x + 2*I*c)} + I*a^{2*d})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((A^2 - 2*I*A*B - B^2)/(a^3*d^2))} + (-I*A - B)*a*e^{(I*d*x + I*c)})*e^{(-I*d*x - I*c)}/(I*A + B)) - 3*\sqrt{1/2}*(a^{2*d}*e^{(5*I*d*x + 5*I*c)} + a^{2*d}*e^{(3*I*d*x + 3*I*c)})*\sqrt{((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*\log(-4*(\sqrt{2}*\sqrt{1/2}*(-I*a^{2*d}*e^{(2*I*d*x + 2*I*c)} - I*a^{2*d})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((A^2 - 2*I*A*B - B^2)/(a^3*d^2))} + (-I*A - B)*a*e^{(I*d*x + I*c)})*e^{(-I*d*x - I*c)}/(I*A + B))} + \sqrt{2}*(2*(19*A + 26*I*B)*e^{(6*I*d*x + 6*I*c)} + 3*(17*A + 29*I*B)*e^{(4*I*d*x + 4*I*c)} + 6*(2*A + 3*I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))/(a^{2*d}*e^{(5*I*d*x + 5*I*c)} + a^{2*d}*e^{(3*I*d*x + 3*I*c)})$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^3(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3/(I\*a\*(tan(c + d\*x) - I))\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^3/(I\*a\*tan(d\*x + c) + a)^(3/2), x)

**Mupad [B]**

time = 7.44, size = 233, normalized size = 1.11

$$\frac{B i}{3 d} - \frac{B(a + a \tan(c + d x) i) i}{3 a d} + \frac{A a - 5 A(a + a \tan(c + d x) i)}{3 a d(a + a \tan(c + d x) i)^{3/2}} - \frac{2 A \sqrt{a + a \tan(c + d x) i}}{a^2 d} - \frac{B \sqrt{a + a \tan(c + d x) i} i}{a^2 d} + \frac{B(a + a \tan(c + d x) i)^{3/2} i}{3 a^3 d} - \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + d x) i}}{2 \sqrt{-a}}\right) i}{4(-a)^{3/2} d} + \frac{\sqrt{2} A \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + d x) i}}{2 \sqrt{a}}\right)}{4 a^{3/2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] ((B\*1i)/(3\*d) - (B\*(a + a\*tan(c + d\*x)\*1i)\*7i)/(2\*a\*d))/(a + a\*tan(c + d\*x)\*1i)^(3/2) + ((A\*a)/3 - (5\*A\*(a + a\*tan(c + d\*x)\*1i))/2)/(a\*d\*(a + a\*tan(c + d\*x)\*1i)^(3/2)) - (2\*A\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(a^2\*d) - (B\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*4i)/(a^2\*d) + (B\*(a + a\*tan(c + d\*x)\*1i)^(3/2)\*2i)/(3\*a^3\*d) - (2^(1/2)\*B\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)))/(2\*(-a)^(1/2)))\*1i)/(4\*(-a)^(3/2)\*d) + (2^(1/2)\*A\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)))/(2\*a^(1/2)))/(4\*a^(3/2)\*d)



$$3.98 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=167

$$\frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2} d} + \frac{(iA - B) \tan^2(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{5iA - 11B}{6ad\sqrt{a + ia \tan(c + dx)}} + \frac{(iA - 7B) \sqrt{a + ia \tan(c + dx)}}{3a^2 d}$$

[Out] 1/4\*(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(3/2)/d \*2^(1/2)+1/6\*(5\*I\*A-11\*B)/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/3\*(I\*A-7\*B)\*(a+I\*a \*tan(d\*x+c))^(1/2)/a^2/d+1/3\*(I\*A-B)\*tan(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^(3/2 )

**Rubi [A]**

time = 0.24, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3676, 3673, 3607, 3561, 212}

$$\frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2} d} + \frac{(-7B + iA) \sqrt{a + ia \tan(c + dx)}}{3a^2 d} + \frac{(-B + iA) \tan^2(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{-11B + 5iA}{6ad\sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2),x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/(2\*Sqrt[2 ]\*a^(3/2)\*d) + ((I\*A - B)\*Tan[c + d\*x]^2)/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2) ) + ((5\*I)\*A - 11\*B)/(6\*a\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + ((I\*A - 7\*B)\*Sqrt [a + I\*a\*Tan[c + d\*x]])/(3\*a^2\*d)

**Rule 212**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\* ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

**Rule 3561**

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

**Rule 3607**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-(b\*c - a\*d))\*((a + b\*Tan[e + f\*x])^m/(2\*a

```
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[m, 0]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_.)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3676

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_.)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \frac{(iA-B) \tan^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} - \frac{\int \frac{\tan(c+dx)(2a(iA-B)+\frac{1}{2}a(A+7iB) \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}}}{3a^2} \\
&= \frac{(iA-B) \tan^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(iA-7B) \sqrt{a+ia \tan(c+dx)}}{3a^2d} - \frac{\int \frac{\tan(c+dx)(2a(iA-B)+\frac{1}{2}a(A+7iB) \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}}}{3a^2} \\
&= \frac{(iA-B) \tan^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{5iA-11B}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{(iA-7B) \sqrt{a+ia \tan(c+dx)}}{3a^2d} \\
&= \frac{(iA-B) \tan^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{5iA-11B}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{(iA-7B) \sqrt{a+ia \tan(c+dx)}}{3a^2d} \\
&= \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(iA-B) \tan^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]**

time = 2.83, size = 167, normalized size = 1.00

$$\frac{A(-1 + 7e^{2i(c+dx)} + 8e^{4i(c+dx)}) + iB(-1 + 13e^{2i(c+dx)} + 38e^{4i(c+dx)}) + 3(A - iB)e^{3i(c+dx)}\sqrt{1 + e^{2i(c+dx)}} \sinh^{-1}(e^{i(c+dx)})}{3ad(1 + e^{2i(c+dx)})^2(-i + \tan(c + dx))\sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (A\*(-1 + 7\*E^((2\*I)\*(c + d\*x)) + 8\*E^((4\*I)\*(c + d\*x))) + I\*B\*(-1 + 13\*E^((2\*I)\*(c + d\*x)) + 38\*E^((4\*I)\*(c + d\*x))) + 3\*(A - I\*B)\*E^((3\*I)\*(c + d\*x)))\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcSinh[E^(I\*(c + d\*x))]/(3\*a\*d\*(1 + E^((2\*I)\*(c + d\*x)))^2\*(-I + Tan[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [A]**

time = 0.11, size = 116, normalized size = 0.69

method	result
derivativedivides	$2i \left( -iB \sqrt{a + ia \tan(dx + c)} - \frac{\sqrt{a} (-iB+A) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{8} \right) - \frac{\sqrt{a} (-iB+A) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}$
default	$2i \left( -iB \sqrt{a + ia \tan(dx + c)} - \frac{\sqrt{a} (-iB+A) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{8} \right) - \frac{\sqrt{a} (-iB+A) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2), x, method=\_RETURNVERBOSE)

[Out] -2\*I/d/a^2\*(-I\*B\*(a+I\*a\*tan(d\*x+c))^(1/2)-1/8\*a^(1/2)\*(A-I\*B)\*2^(1/2)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))-1/4\*a\*(3\*A+5\*I\*B)/(a+I\*a\*tan(d\*x+c))^(1/2)+1/6\*a^2\*(A+I\*B)/(a+I\*a\*tan(d\*x+c))^(3/2))

**Maxima [A]**

time = 0.57, size = 137, normalized size = 0.82

$$i \left( 3\sqrt{2}(A - iB)a^{\frac{3}{2}} \log\left(-\frac{\sqrt{2}\sqrt{a} - \sqrt{ia \tan(dx + c) + a}}{\sqrt{2}\sqrt{a} + \sqrt{ia \tan(dx + c) + a}}\right) - 48i \sqrt{ia \tan(dx + c) + a} Ba - \frac{4(3(ia \tan(dx + c) + a)(3A + 5iB)a^2 - 2(A + iB)a^3)}{(ia \tan(dx + c) + a)^{\frac{3}{2}}} \right) / 24a^3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2), x, algorithm="maxima")

[Out]  $-1/24*I*(3*\sqrt{2}*(A - I*B)*a^{(3/2)}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(dx + c)}) + a))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(dx + c)}) - 48*I*\sqrt{I*a*\tan(dx + c)} + a)*B*a - 4*(3*(I*a*\tan(dx + c) + a)*(3*A + 5*I*B)*a^2 - 2*(A + I*B)*a^3)/(I*a*\tan(dx + c) + a)^{(3/2)}/(a^3*d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs.  $2(128) = 256$ .  
time = 1.91, size = 375, normalized size = 2.25

$$\left( \frac{3\sqrt{2}a^2\sqrt{\frac{A-2AB-B^2}{a^2}}e^{(2I*d*x+2I*c)}\log\left(\frac{1+\sqrt{2}\sqrt{\frac{A-2AB-B^2}{a^2}}\sqrt{\frac{A-2AB-B^2}{a^2}}\sqrt{\frac{A-2AB-B^2}{a^2}}}{\sqrt{2I*d*x+2I*c}}\right)}{12a^2} - 3\sqrt{2}a^2\sqrt{\frac{A-2AB-B^2}{a^2}}e^{(2I*d*x+2I*c)}\log\left(\frac{1+\sqrt{2}\sqrt{\frac{A-2AB-B^2}{a^2}}\sqrt{\frac{A-2AB-B^2}{a^2}}\sqrt{\frac{A-2AB-B^2}{a^2}}}{\sqrt{2I*d*x+2I*c}}\right)}{12a^2} + \sqrt{2}(2(-4A+19B)e^{(4I*d*x+4I*c)} - (7A-13B)e^{(2I*d*x+2I*c)} + (A-B)\sqrt{\frac{A-2AB-B^2}{a^2}})e^{(-3I*d*x-3I*c)} \right) / (a^2*d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(dx+c)^2*(A+B*tan(dx+c))/(a+I*a*tan(dx+c))^(3/2),x, algorithm="fricas")`

[Out]  $-1/12*(3*\sqrt{1/2}*a^2*d*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)})*e^{(3*I*d*x + 3*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)})) + (-I*A - B)*a*e^{(I*d*x + I*c)})*e^{(-I*d*x - I*c)/(I*A + B)} - 3*\sqrt{1/2}*a^2*d*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)})*e^{(3*I*d*x + 3*I*c)}*\log(4*(\sqrt{2}*\sqrt{1/2}*(a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)})) - (-I*A - B)*a*e^{(I*d*x + I*c)})*e^{(-I*d*x - I*c)/(I*A + B)} + \sqrt{2}*(2*(-4*I*A + 19*B)*e^{(4*I*d*x + 4*I*c)} - (7*I*A - 13*B)*e^{(2*I*d*x + 2*I*c)} + I*A - B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-3*I*d*x - 3*I*c)/(a^2*d)}$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^2(c + dx)}{(i a (\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(dx+c)**2*(A+B*tan(dx+c))/(a+I*a*tan(dx+c))**(3/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(dx+c)^2*(A+B*tan(dx+c))/(a+I*a*tan(dx+c))^(3/2),x, algorithm="giac")`

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^2/(I\*a\*tan(d\*x + c) + a)^(3/2), x)

**Mupad [B]**

time = 0.70, size = 186, normalized size = 1.11

$$-\frac{\frac{A1i}{3d} - \frac{A(a + a \tan(c + dx))1i3i}{2ad}}{(a + a \tan(c + dx)1i)^{3/2}} + \frac{\frac{Ba}{3} - \frac{5B(a + a \tan(c + dx))1i}{2}}{ad(a + a \tan(c + dx)1i)^{3/2}} - \frac{2B\sqrt{a + a \tan(c + dx)1i}}{a^2d} + \frac{\sqrt{2}A \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a + a \tan(c + dx)1i}}{2\sqrt{-a}}\right)1i}{4(-a)^{3/2}d} + \frac{\sqrt{2}B \operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{a + a \tan(c + dx)1i}}{2\sqrt{a}}\right)}{4a^{3/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2), x)

[Out] ((B\*a)/3 - (5\*B\*(a + a\*tan(c + d\*x)\*1i))/2)/(a\*d\*(a + a\*tan(c + d\*x)\*1i)^(3/2)) - ((A\*1i)/(3\*d) - (A\*(a + a\*tan(c + d\*x)\*1i)\*3i)/(2\*a\*d))/(a + a\*tan(c + d\*x)\*1i)^(3/2) - (2\*B\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(a^2\*d) + (2^(1/2)\*A\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/(4\*(-a)^(3/2)\*d) + (2^(1/2)\*B\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/(4\*a^(3/2)\*d)

$$3.99 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=119

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{A + iB}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{A + 3iB}{2ad\sqrt{a + ia \tan(c + dx)}}$$

[Out]  $-1/4*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+1/2*(A+3*I*B)/a/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/3*(-A-I*B)/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3671, 3607, 3561, 212}

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2} d} - \frac{A + iB}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{A + 3iB}{2ad\sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x]*(A + B*\operatorname{Tan}[c + d*x]))/(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $-1/2*((A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(\operatorname{Sqrt}[2]*a^{(3/2)}*d) - (A + I*B)/(3*d*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}) + (A + (3*I)*B)/(2*a*d*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3561

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\operatorname{tan}[(c_ + (d_)*(x_)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

Rule 3607

$\operatorname{Int}[(a_ + (b_)*\operatorname{tan}[(e_ + (f_)*(x_)]])^{(m_)*((c_ + (d_)*\operatorname{tan}[(e_ + (f_)*(x_)]))}, x\_Symbol] \rightarrow \operatorname{Simp}[(-b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^m/(2*a*f*m)), x] + \operatorname{Dist}[(b*c + a*d)/(2*a*b), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 + b^2,$

0] && LtQ[m, 0]

### Rule 3671

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(-(A\*b - a\*B))\*(a\*c + b\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a^2\*f\*m)), x] + Dist[1/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[A\*b\*c + a\*B\*c + a\*A\*d + b\*B\*d + 2\*a\*B\*d\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] & NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}} dx &= -\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{i \int \frac{a(A+iB)+2aB\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}} dx}{2a^2} \\ &= -\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{A+3iB}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{(iA+)}{2ad\sqrt{a+ia\tan(c+dx)}} \\ &= -\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{A+3iB}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{(A-i)}{2ad\sqrt{a+ia\tan(c+dx)}} \\ &= -\frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}} \end{aligned}$$

### Mathematica [A]

time = 2.35, size = 145, normalized size = 1.22

$$\frac{\sqrt{1+e^{2i(c+dx)}}(-iA(-1+2e^{2i(c+dx)})+B(-1+8e^{2i(c+dx)}))+3(iA+B)e^{3i(c+dx)}\sinh^{-1}(e^{i(c+dx)})}{3ad(1+e^{2i(c+dx)})^{3/2}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*((-I)\*A\*(-1 + 2\*E^((2\*I)\*(c + d\*x)))) + B\*(-1 + 8\*E^((2\*I)\*(c + d\*x)))) + 3\*(I\*A + B)\*E^((3\*I)\*(c + d\*x))\*ArcSinh[E^(I\*(c + d\*x))]/(3\*a\*d\*(1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*(-I + Tan[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

### Maple [A]

time = 0.10, size = 96, normalized size = 0.81

method	result
derivativedivides	$\frac{\left(\frac{A}{4} - \frac{iB}{4}\right)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx+c)} \sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(-\frac{A}{4} - \frac{3iB}{4}\right)}{\sqrt{a + ia \tan(dx+c)}} - \frac{a(iB+A)}{3(a + ia \tan(dx+c))^{\frac{3}{2}}}$
default	$\frac{\left(\frac{A}{4} - \frac{iB}{4}\right)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx+c)} \sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(-\frac{A}{4} - \frac{3iB}{4}\right)}{\sqrt{a + ia \tan(dx+c)}} - \frac{a(iB+A)}{3(a + ia \tan(dx+c))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $2/d/a*(-1/2*(1/4*A-1/4*I*B)*2^(1/2)/a^(1/2)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(-1/4*A-3/4*I*B)/(a+I*a*\tan(d*x+c))^(1/2)-1/6*a*(A+I*B)/(a+I*a*\tan(d*x+c))^(3/2)$

**Maxima** [A]

time = 0.50, size = 116, normalized size = 0.97

$$\frac{3\sqrt{2}(A-iB)\sqrt{a} \log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}}\right) + \frac{4(3(ia \tan(dx+c)+a)(A+3iB)a-2(A+iB)a^2)}{(ia \tan(dx+c)+a)^{\frac{3}{2}}}}{24a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm  
="maxima")`

[Out]  $1/24*(3*\sqrt{2}*(A - I*B)*\sqrt{a}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a})) + 4*(3*(I*a*\tan(d*x + c) + a)*(A + 3*I*B)*a - 2*(A + I*B)*a^2)/(I*a*\tan(d*x + c) + a)^(3/2)/(a^2*d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs.  $2(88) = 176$ .

time = 2.07, size = 369, normalized size = 3.10

$$\frac{\left(3\sqrt{\frac{a}{2}}\sqrt{\frac{B^2-3AB-B^2}{a^2}}e^{iB \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{a+ia \tan(dx+c)}}\right)} - \sqrt{\frac{a}{2}}\sqrt{\frac{B^2-3AB-B^2}{a^2}}e^{iB \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{a+ia \tan(dx+c)}}\right)}\right) - 3\sqrt{\frac{a}{2}}\sqrt{\frac{B^2-3AB-B^2}{a^2}}e^{iB \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{a+ia \tan(dx+c)}}\right)} - \sqrt{\frac{a}{2}}\sqrt{\frac{B^2-3AB-B^2}{a^2}}e^{iB \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{a+ia \tan(dx+c)}}\right)} + \sqrt{2}(2(A+4iB)e^{iB \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{a+ia \tan(dx+c)}}\right)} + (A+7iB)e^{iB \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{a+ia \tan(dx+c)}}\right)} - A - iB)\sqrt{\frac{a}{2(a+ia \tan(dx+c))}}\right)}{12a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm  
="fricas")`



```
[Out] 1/12*(3*sqrt(1/2)*a^2*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x +
3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I
*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*d*
sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*
c))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*(2*(A + 4*I*B)*e^(4*I*d*x + 4*I*c
) + (A + 7*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2),
x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*tan(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x
)
```

**Mupad [B]**

time = 7.07, size = 163, normalized size = 1.37

$$\frac{\frac{B \operatorname{li}}{3d} - \frac{B(a+a \tan(c+dx) \operatorname{li}) 3i}{2ad}}{(a+a \tan(c+dx) \operatorname{li})^{3/2}} - \frac{\frac{A}{3} - \frac{A(a+a \tan(c+dx) \operatorname{li})}{2a}}{d(a+a \tan(c+dx) \operatorname{li})^{3/2}} + \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx) \operatorname{li}}}{2\sqrt{-a}}\right) \operatorname{li}}{4(-a)^{3/2}d}}{4(-a)^{3/2}d} - \frac{\sqrt{2} A \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx) \operatorname{li}}}{2\sqrt{a}}\right)}{4a^{3/2}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] (2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)
/(4*(-a)^(3/2)*d) - (A/3 - (A*(a + a*tan(c + d*x)*1i))/(2*a))/(d*(a + a*tan
(c + d*x)*1i)^(3/2)) - ((B*1i)/(3*d) - (B*(a + a*tan(c + d*x)*1i)*3i)/(2*a*
d))/(a + a*tan(c + d*x)*1i)^(3/2) - (2^(1/2)*A*atanh((2^(1/2)*(a + a*tan(c
+ d*x)*1i)^(1/2))/(2*a^(1/2)))/(4*a^(3/2)*d)
```

$$3.100 \quad \int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=121

$$-\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{iA-B}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{iA+B}{2ad\sqrt{a+ia \tan(c+dx)}}$$

[Out]  $-1/4*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+1/2*(I*A+B)/a/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/3*(I*A-B)/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3607, 3560, 3561, 212}

$$-\frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{-B+iA}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{B+iA}{2ad\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)},x]$

[Out]  $-1/2*((I*A+B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(\operatorname{Sqrt}[2]*a^{(3/2)}*d)+(I*A-B)/(3*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})+(I*A+B)/(2*a*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3560

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[a*((a+b*\operatorname{Tan}[c+d*x])^n/(2*b*d*n)), x] + \operatorname{Dist}[1/(2*a), \operatorname{Int}[(a+b*\operatorname{Tan}[c+d*x])^{(n+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2+b^2, 0] \ \&\& \operatorname{LtQ}[n, 0]$

Rule 3561

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a-x^2), x], x, \operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2+b^2, 0]$

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-(b\*c - a\*d))\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^{3/2}} dx &= \frac{iA - B}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(A - iB) \int \frac{1}{\sqrt{a + ia \tan(c + dx)}} dx}{2a} \\ &= \frac{iA - B}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{iA + B}{2ad \sqrt{a + ia \tan(c + dx)}} + \frac{(A - iB) \int \sqrt{a + ia \tan(c + dx)}}{4ad} \\ &= \frac{iA - B}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{iA + B}{2ad \sqrt{a + ia \tan(c + dx)}} - \frac{(iA + B) \text{Subst}\left(\int \sqrt{a + ia \tan(c + dx)}\right)}{4ad} \\ &= -\frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{iA - B}{3d(a + ia \tan(c + dx))^{3/2}} \end{aligned}$$

**Mathematica** [A]

time = 2.31, size = 143, normalized size = 1.18

$$\frac{\sqrt{1 + e^{2i(c+dx)}} (A + 4Ae^{2i(c+dx)} - iB(-1 + 2e^{2i(c+dx)})) - 3(A - iB)e^{3i(c+dx)} \sinh^{-1}(e^{i(c+dx)})}{3ad(1 + e^{2i(c+dx)})^{3/2} (-i + \tan(c + dx)) \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*(A + 4\*A\*E^((2\*I)\*(c + d\*x)) - I\*B\*(-1 + 2\*E^((2\*I)\*(c + d\*x)))) - 3\*(A - I\*B)\*E^((3\*I)\*(c + d\*x))\*ArcSinh[E^(I\*(c + d\*x))])/(3\*a\*d\*(1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*(-I + Tan[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple** [A]

time = 0.10, size = 96, normalized size = 0.79

method	result
--------	--------



```
[Out] 1/12*(3*sqrt(1/2)*a^2*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x +
3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A
- B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*d*sq
rt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*sq
rt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))*e^
(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*(2*(-2*I*A - B)*e^(4*I*d*x + 4*I*c) - (
5*I*A + B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

**Mupad [B]**

time = 6.99, size = 162, normalized size = 1.34

$$\frac{\frac{A \operatorname{li}}{3d} + \frac{A(a + a \tan(c + dx) \operatorname{li}) \operatorname{li}}{2ad}}{(a + a \tan(c + dx) \operatorname{li})^{3/2}} - \frac{\frac{B}{3} - \frac{B(a + a \tan(c + dx) \operatorname{li})}{2a}}{d(a + a \tan(c + dx) \operatorname{li})^{3/2}} - \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2\sqrt{-a}}\right) \operatorname{li}}{4(-a)^{3/2} d}}{4a^{3/2} d} - \frac{\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2\sqrt{a}}\right)}{4a^{3/2} d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] ((A*1i)/(3*d) + (A*(a + a*tan(c + d*x)*1i)*1i)/(2*a*d))/(a + a*tan(c + d*x)
*1i)^(3/2) - (B/3 - (B*(a + a*tan(c + d*x)*1i))/(2*a))/(d*(a + a*tan(c + d*
x)*1i)^(3/2)) - (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*
(-a)^(1/2)))*1i)/(4*(-a)^(3/2)*d) - (2^(1/2)*B*atanh((2^(1/2)*(a + a*tan(c
+ d*x)*1i)^(1/2))/(2*a^(1/2)))/(4*a^(3/2)*d)
```

$$3.101 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=156

$$\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}}$$

[Out]  $-2*A*\operatorname{arctanh}\left(\frac{(a+I*a*\tan(d*x+c))^{1/2}}{a^{1/2}}\right)/a^{3/2}/d+1/4*(A-I*B)*\operatorname{arctanh}\left(\frac{(1/2*(a+I*a*\tan(d*x+c))^{1/2}*2^{1/2}}{a^{1/2}}\right)/a^{3/2}/d*2^{1/2}+1/2*(3*A+I*B)/a/d/(a+I*a*\tan(d*x+c))^{1/2}+1/3*(A+I*B)/d/(a+I*a*\tan(d*x+c))^{3/2}$

**Rubi [A]**

time = 0.35, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3677, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{3A+iB}{2ad\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(\cot[c+d*x]*(A+B*\tan[c+d*x]))}{(a+I*a*\tan[c+d*x])^{3/2}}, x\right]$

[Out]  $(-2*A*\operatorname{ArcTanh}\left[\frac{\sqrt{a+I*a*\tan[c+d*x]}}{\sqrt{a}}\right])/a^{3/2}*d + ((A-I*B)*\operatorname{ArcTanh}\left[\frac{\sqrt{a+I*a*\tan[c+d*x]}}{\sqrt{2}*\sqrt{a}}\right])/(2*\sqrt{2}*a^{3/2}*d) + (A+I*B)/(3*d*(a+I*a*\tan[c+d*x])^{3/2}) + (3*A+I*B)/(2*a*d*\sqrt{a+I*a*\tan[c+d*x]})$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_)^m)*((c_.) + (d_.)*(x_)^n), x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^n], x], x, (a + b*x)^{1/p}], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b]$

Rule 3561

```
Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2*(b/d),
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3681

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x]))], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)(3aA-\frac{3}{2}a(iA-B) \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx}{3a^2} \\
&= \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{3A+iB}{2ad\sqrt{a+ia \tan(c+dx)}} + \frac{\int \cot(c+dx)}{3a^2} \\
&= \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{3A+iB}{2ad\sqrt{a+ia \tan(c+dx)}} + \frac{A \int \cot(c+dx)}{3a^2} \\
&= \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{3A+iB}{2ad\sqrt{a+ia \tan(c+dx)}} + \frac{A \operatorname{Subst}\left(\int \cot(c+dx)}{3a^2}\right)}{3a^2} \\
&= \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{A+iB}{3d(a+ia \tan(c+dx))^{3/2}} \\
&= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 4.12, size = 192, normalized size = 1.23

$$\frac{-22iA + 10B - \frac{12i(A-iB)e^{3i(c+dx)} \sinh^{-1}(e^{i(c+dx)})}{(1+e^{2i(c+dx)})^{3/2}} + \frac{48i\sqrt{2}Ae^{3i(c+dx)} \tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right)}{(1+e^{2i(c+dx)})^{3/2}} + 18A \tan(c+dx) + 6iB \tan(c+dx)}{12ad(-i + \tan(c+dx))\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2), x]
```

```
[Out] ((-22*I)*A + 10*B - ((12*I)*(A - I*B)*E^((3*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(3/2) + ((48*I)*Sqrt[2]*A*E^((3*I)*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x))]/Sqrt[1 + E^((2*I)*(c + d*x))]])/(1 + E^((2*I)*(c + d*x)))^(3/2) + 18*A*Tan[c + d*x] + (6*I)*B*Tan[c + d*x])/(12*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(124) = 248.

time = 0.48, size = 1026, normalized size = 6.58

method	result	size
--------	--------	------



default	Expression too large to display	1026
---------	---------------------------------	------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$\frac{1}{24} \frac{d}{a^2} \left( (I \sin(dx+c) + \cos(dx+c)) \frac{a}{\cos(dx+c)} \right)^{1/2} \left( -16 I A \cos(dx+c)^3 \sin(dx+c) + 4 I B \cos(dx+c)^2 + 3 I B^2 \right)^{1/2} \cos(dx+c) \frac{(-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) - 12 I A \cos(dx+c) / (\cos(dx+c)+1)}{(\cos(dx+c)+1)^{1/2}} \ln\left(\frac{\sin(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1}{\sin(dx+c)}\right) - 36 I A \cos(dx+c) \sin(dx+c) + 12 I A \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \sin(dx+c) - 3 A \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} \cos(dx+c) + 16 A \cos(dx+c)^4 - 12 I A \cos(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \ln\left(\frac{\sin(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1}{\sin(dx+c)}\right) + 3 B \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} \sin(dx+c) + 16 B \cos(dx+c)^3 \sin(dx+c) + 3 I B^2 \right)^{1/2} \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) + 16 I B \cos(dx+c)^4 - 3 A \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} - 12 A \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot \cos(dx+c) - 12 A \ln\left(\frac{\sin(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1}{\sin(dx+c)}\right) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \sin(dx+c) + 3 I A \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} \sin(dx+c) + 28 A \cos(dx+c)^2 - 12 A \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) + 12 B \cos(dx+c) \sin(dx+c)$$

**Maxima [A]**

time = 0.50, size = 161, normalized size = 1.03

$$\frac{{}_3\sqrt{2}^{(A-iB)} \log\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{ia \tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}} - \frac{{}_{24}A \log\left(\frac{\sqrt{ia \tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia \tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{4(3ia \tan(dx+c)+a)(3A+iB)+2(A+iB)a}{(ia \tan(dx+c)+a)^{\frac{3}{2}} a}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm  
="maxima")`

[Out] 
$$-1/24 \cdot (3 \cdot \sqrt{2}) \cdot (A - I \cdot B) \cdot \log\left(\frac{(\sqrt{2} \cdot \sqrt{a}) - \sqrt{I \cdot a \cdot \tan(dx+c) + a}}{(\sqrt{2} \cdot \sqrt{a}) + \sqrt{I \cdot a \cdot \tan(dx+c) + a}}\right) / a^{3/2} - 24 \cdot A \cdot \log\left(\frac{\sqrt{I \cdot a \cdot \tan(dx+c) + a} - \sqrt{a}}{\sqrt{I \cdot a \cdot \tan(dx+c) + a} + \sqrt{a}}\right) / a^{3/2}$$

$$\frac{t(I*a*\tan(d*x + c) + a) - \sqrt{a}}{(\sqrt{I*a*\tan(d*x + c) + a} + \sqrt{a})} / a^{(3/2)} - 4*(3*(I*a*\tan(d*x + c) + a)*(3*A + I*B) + 2*(A + I*B)*a) / ((I*a*\tan(d*x + c) + a)^{(3/2)*a)} / d$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(117) = 234.

time = 1.73, size = 624, normalized size = 4.00



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/12*(3*\sqrt{1/2}*a^2*d*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^3*d^2)})*e^{(3*I*d*x + 3*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(I*a^2*d*e^{(2*I*d*x + 2*I*c)} + I*a^2*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^3*d^2)}) + (-I*A - B)*a*e^{(I*d*x + I*c)})*e^{(-I*d*x - I*c)/(I*A + B)} - 3*\sqrt{1/2}*a^2*d*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^3*d^2)}*e^{(3*I*d*x + 3*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(-I*a^2*d*e^{(2*I*d*x + 2*I*c)} - I*a^2*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^3*d^2)}) + (-I*A - B)*a*e^{(I*d*x + I*c)})*e^{(-I*d*x - I*c)/(I*A + B)} + 6*a^2*d*\sqrt{A^2/(a^3*d^2)}*e^{(3*I*d*x + 3*I*c)}*\log(16*(3*A*a^2*e^{(2*I*d*x + 2*I*c)} + A*a^2 + 2*\sqrt{2}*(a^3*d*e^{(3*I*d*x + 3*I*c)} + a^3*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{A^2/(a^3*d^2)}))e^{(-2*I*d*x - 2*I*c)/A} - 6*a^2*d*\sqrt{A^2/(a^3*d^2)}*e^{(3*I*d*x + 3*I*c)}*\log(16*(3*A*a^2*e^{(2*I*d*x + 2*I*c)} + A*a^2 - 2*\sqrt{2}*(a^3*d*e^{(3*I*d*x + 3*I*c)} + a^3*d*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{A^2/(a^3*d^2)}))e^{(-2*I*d*x - 2*I*c)/A} - \sqrt{2}*(2*(5*A + 2*I*B)*e^{(4*I*d*x + 4*I*c)} + (11*A + 5*I*B)*e^{(2*I*d*x + 2*I*c)} + A + I*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-3*I*d*x - 3*I*c)/(a^2*d)} \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)/(I\*a\*(tan(c + d\*x) - I))^(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x
)
```

**Mupad [B]**

time = 6.70, size = 563, normalized size = 3.61

$$\frac{A \sqrt{a} + (2AB \operatorname{atanh}(\frac{d \sqrt{a} \sqrt{a + \tan(c + dx)})}{\sqrt{a^2 + d^2(a + \tan(c + dx))}}) - 2A \operatorname{atanh}(\frac{d \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}}) + \frac{A^2 \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}} + \frac{d^2 \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}}}{(a + a \tan(c + dx))^{3/2}} + \frac{\sqrt{2} \operatorname{atanh}(\frac{\sqrt{2} d \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}}) - \frac{\sqrt{2} d \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}} + \frac{\sqrt{2} A \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}} + \frac{2 \sqrt{2} d \sqrt{a} \sqrt{a + \tan(c + dx)}}{\sqrt{a^2 + d^2(a + \tan(c + dx))}})}{4 a^2 d} (B + A i) \sqrt{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] ((A + B*1i)/(3*d) + ((3*A + B*1i)*(a + a*tan(c + d*x)*1i))/(2*a*d))/(a + a*
tan(c + d*x)*1i)^(3/2) - (2*A*atanh((31*A^3*d*(a + a*tan(c + d*x)*1i)^(1/2)
))/(a^3)^(1/2)*(31*A^3*d)/a + (A*B^2*d)/a + (A^2*B*d*2i)/a) + (A*B^2*d*(a
+ a*tan(c + d*x)*1i)^(1/2))/(a^3)^(1/2)*((31*A^3*d)/a + (A*B^2*d)/a + (A^
2*B*d*2i)/a) + (A^2*B*d*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/((a^3)^(1/2)*((3
1*A^3*d)/a + (A*B^2*d)/a + (A^2*B*d*2i)/a)))/(d*(a^3)^(1/2)) + (2^(1/2)*at
anh((2^(1/2)*A^3*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*31i)/(16*((31
*A^3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*d*29i)/8)
) + (2^(1/2)*B^3*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(16*((31*A^3
*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*d*29i)/8)) +
(2^(1/2)*A*B^2*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*3i)/(16*((31*A^
3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*d*29i)/8)) +
(29*2^(1/2)*A^2*B*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(16*((31*A
^3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*d*29i)/8)))
*(A*1i + B)*(-a^3)^(1/2)/(4*a^3*d)
```

$$3.102 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=217

$$\frac{(3iA - 2B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2}d} + \frac{(A + iB) \cot(c + dx)}{3d(a + ia \tan(c + dx))}$$

[Out] (3\*I\*A-2\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4\*(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(3/2)/d\*2^(1/2)+1/6\*(13\*A+7\*I\*B)\*cot(d\*x+c)/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)-1/2\*(7\*A+3\*I\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d+1/3\*(A+I\*B)\*cot(d\*x+c)/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.53, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(-2B + 3iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2}d} - \frac{(7A + 3iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{2a^2d} + \frac{(13A + 7iB) \cot(c + dx)}{6ad \sqrt{a + ia \tan(c + dx)}} + \frac{(A + iB) \cot(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (((3\*I)\*A - 2\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]]/(a^(3/2)\*d) + ((I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])]/(2\*Sqrt[2]\*a^(3/2)\*d) + ((A + I\*B)\*Cot[c + d\*x])/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((13\*A + (7\*I)\*B)\*Cot[c + d\*x])/(6\*a\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((7\*A + (3\*I)\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(2\*a^2\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 214**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a

\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot^2(c+dx)(a(4A+iB)-\frac{5}{2}a(iA-B) \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}}}{3a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(13A+7iB) \cot(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} + \frac{\int \cot^2(c+dx)}{3a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(13A+7iB) \cot(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} - \frac{(7A+3iB)}{3a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(13A+7iB) \cot(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} - \frac{(7A+3iB)}{3a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(13A+7iB) \cot(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} - \frac{(7A+3iB)}{3a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(13A+7iB) \cot(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} - \frac{(7A+3iB)}{3a^2} \\
 &= \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} \\
 &= \frac{(3iA-2B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A+iB) \cot(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A]

time = 4.76, size = 259, normalized size = 1.19

$$\frac{\sqrt{\sec(c+dx)} \left( \sqrt{2} \left( \frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{3/2} (1+e^{2i(c+dx)})^{3/2} \left( (iA+B) \sinh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}} \right) + 2\sqrt{2} (3iA-2B) \tanh^{-1} \left( \frac{-\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}} \right) \right) - \frac{\cos(c+dx)(-3(5A+3iB)+9(3A+iB) \cos(2(c+dx))+29iA-11B) \sin(2(c+dx))}{3\sqrt{\sec(c+dx)}} \right) (A+B \tan(c+dx))}{4d(A \cos(c+dx)+B \sin(c+dx))(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (Sqrt[Sec[c + d\*x]]\*(Sqrt[2]\*(E^(I\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x))))^(3/2)\*(1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*((I\*A + B)\*ArcSinh[E^(I\*(c + d\*x))] + 2\*Sqrt[2]\*((3\*I)\*A - 2\*B)\*ArcTanh[(Sqrt[2]\*E^(I\*(c + d\*x))]/Sqrt[1 + E^((2\*I)\*(c + d\*x))]]) - (Csc[c + d\*x]\*(-3\*(5\*A + (3\*I)\*B) + 9\*(3\*A + I\*B)\*Cos[2\*(c + d\*x)] + ((29\*I)\*A - 11\*B)\*Sin[2\*(c + d\*x)]))/(3\*Sqrt[Sec[c + d\*x]])\*(A + B\*Tan[c + d\*x]))/(4\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^(3/2))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2817 vs. 2(178) = 356.  
time = 0.54, size = 2818, normalized size = 12.99

method	result	size
default	Expression too large to display	2818

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURN  
VERBOSE)`

[Out] 
$$\begin{aligned} & -1/24/d/a^2*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*(-84*A*\cos(d*x+c) \\ & )*\sin(d*x+c)+3*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I \\ & * \cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})* \\ & 2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-12*B*\cos(d*x+c)^4+28*B*\cos(d*x+c)^2+16*A*\cos \\ & (d*x+c)^5*\sin(d*x+c)+16*I*A*\cos(d*x+c)^6+36*I*A*\cos(d*x+c)^4-52*I*A*\cos(d* \\ & x+c)^2-18*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x \\ & +c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-12*B*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})-16*B*\cos( \\ & d*x+c)^6-18*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos \\ & (d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2*\sin(d* \\ & x+c)-12*I*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2*\sin(d*x+c)+3*I*B*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x \\ & +c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\sin(d*x+c)+3*I*A*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x \\ & +c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2+3*I*A*(-2*\cos \\ & (d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c) \\ & ))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3-3* \\ & I*A*2^{(1/2)}*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos( \\ & d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d \\ & *x+c)+18*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)})*\sin(d*x+c)-12*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln \\ & ((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c)) \\ & )*\sin(d*x+c)+44*A*\cos(d*x+c)^3*\sin(d*x+c)+18*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1) \\ & )^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin \\ & (d*x+c))*\cos(d*x+c)^3+12*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/ \\ & (-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^3+18*A*(-2*\cos(d*x+c)/(\cos \\ & (d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d \\ & *x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2+12*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & )*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)^2-18*A*(-2*\cos(d \\ & *x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ & )-\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)-12*B*(-2*\cos(d*x+c)/(\cos(d*x+c)+1) \\ & )^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)-3*A*(-2* \end{aligned}$$





```
[Out] -1/24*I*a*(4*(3*(I*a*tan(d*x + c) + a)^2*(7*A + 3*I*B) - (I*a*tan(d*x + c) + a)*(13*A + 7*I*B)*a - 2*(A + I*B)*a^2)/((I*a*tan(d*x + c) + a)^(5/2)*a^2 - (I*a*tan(d*x + c) + a)^(3/2)*a^3) + 3*sqrt(2)*(A - I*B)*log(-sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) + 12*(3*A + 2*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(5/2))/d
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 814 vs.  $2(170) = 340$ .  
time = 6.07, size = 814, normalized size = 3.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-(9*A^2 + 12*I*A*B - 4*B^2)/(a^3*d^2))*log(-16*(3*(3*I*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (3*I*A - 2*B)*a^2 + 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(9*A^2 + 12*I*A*B - 4*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-3*I*A + 2*B)) + 3*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-(9*A^2 + 12*I*A*B - 4*B^2)/(a^3*d^2))*log(-16*(3*(3*I*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (3*I*A - 2*B)*a^2 - 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(9*A^2 + 12*I*A*B - 4*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-3*I*A + 2*B)) + sqrt(2)*(2*(14*I*A - 5*B)*e^(6*I*d*x + 6*I*c) - (-13*I*A + B)*e^(4*I*d*x + 4*I*c) + 2*(-8*I*A + 5*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^2(c + dx)}{(ia (\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*2/(I\*a\*(tan(c + d\*x) - I))\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^2/(I\*a\*tan(d\*x + c) + a)^(3/2), x)

**Mupad** [B]

time = 8.23, size = 2500, normalized size = 11.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] 2\*atanh((3\*d^4\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((33\*B^2)/(64\*a^3\*d^2) - (73\*A^2)/(64\*a^3\*d^2) - ((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2)/(64\*a^6) - (A\*B\*47i)/(32\*a^3\*d^2))^(1/2)\*((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2))/(A^3\*a^2\*d\*781i)/4 + (279\*B^3\*a^2\*d)/4 - (A\*B^2\*a^2\*d\*1223i)/4 - (1717\*A^2\*B\*a^2\*d)/4 + (A\*d^3\*((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2)\*13i)/(4\*a) - (7\*B\*d^3\*((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2))/(4\*a) + (71\*A^2\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((33\*B^2)/(64\*a^3\*d^2) - (73\*A^2)/(64\*a^3\*d^2) - ((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2)/(64\*a^6) - (A\*B\*47i)/(32\*a^3\*d^2))^(1/2))/(A^3\*d\*781i)/(4\*a) + (279\*B^3\*d)/(4\*a) - (A\*B^2\*d\*1223i)/(4\*a) - (1717\*A^2\*B\*d)/(4\*a) + (A\*d^3\*((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2)\*13i)/(4\*a^4) - (7\*B\*d^3\*((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2))/(4\*a^4) - (31\*B^2\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((33\*B^2)/(64\*a^3\*d^2) - (73\*A^2)/(64\*a^3\*d^2) - ((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2))/(4\*a^4) - (31\*B^2\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((33\*B^2)/(64\*a^3\*d^2) - (73\*A^2)/(64\*a^3\*d^2) - ((5041\*A^4\*a^6)/d^4 + (961\*B^4\*a^6)/d^4 - (14006\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*6076i)/d^4 + (A^3\*B\*a^6\*13916i)/d^4)^(1/2))/(4\*a^4)

$$\begin{aligned}
& 6*13916i)/d^4)^{(1/2)}/(64*a^6) - (A*B*47i)/(32*a^3*d^2))^{(1/2)}/((A^3*d*781i) \\
& )/(4*a) + (279*B^3*d)/(4*a) - (A*B^2*d*1223i)/(4*a) - (1717*A^2*B*d)/(4*a) \\
& + (A*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 \\
& - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}*13i)/(4*a^4) - (7*B \\
& *d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A \\
& *B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}/(4*a^4)) + (A*B*d^2*(a \\
& + a*tan(c + d*x)*1i)^{(1/2)}*((33*B^2)/(64*a^3*d^2) - (73*A^2)/(64*a^3*d^2) \\
& - ((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^ \\
& 3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}/(64*a^6) - (A*B*47i)/(32*a \\
& ^3*d^2))^{(1/2)}*98i)/((A^3*d*781i)/(4*a) + (279*B^3*d)/(4*a) - (A*B^2*d*1223 \\
& i)/(4*a) - (1717*A^2*B*d)/(4*a) + (A*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6 \\
& )/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i \\
& )/d^4)^{(1/2)}*13i)/(4*a^4) - (7*B*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^ \\
& 4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^ \\
& 4)^{(1/2)}/(4*a^4)))*(-(d^2*((73*A^2*a^3 - 33*B^2*a^3)/d^2 + (A*B*a^3*94i)/ \\
& d^2)^2 + 128*a^6*((A*B^3 + (3*A^3*B)/2)*1i)/d^4 - ((9*A^4)/4 + (11*A^2*B^2 \\
& )/4 + B^4)/d^4))^{(1/2)} + 73*A^2*a^3 - 33*B^2*a^3 + A*B*a^3*94i)/(64*a^6*d^2 \\
& ))^{(1/2)} - (((A*a + B*a*1i)*1i)/(3*d) + ((13*A + B*7i)*(a + a*tan(c + d*x)* \\
& 1i)*1i)/(6*d) - ((7*A + B*3i)*(a + a*tan(c + d*x)*1i)^2*1i)/(2*a*d))/(a*(a \\
& + a*tan(c + d*x)*1i)^{(3/2)} - (a + a*tan(c + d*x)*1i)^{(5/2)}) - 2*atanh((3*d^ \\
& 4*(a + a*tan(c + d*x)*1i)^{(1/2)}*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - \\
& (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{( \\
& 1/2)}/(64*a^6) - (73*A^2)/(64*a^3*d^2) + (33*B^2)/(64*a^3*d^2) - (A*B*47i)/( \\
& 32*a^3*d^2))^{(1/2)}*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2 \\
& *a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}/((A^3*a^ \\
& 2*d*781i)/4 + (279*B^3*a^2*d)/4 - (A*B^2*a^2*d*1223i)/4 - (1717*A^2*B*a^2*d \\
& )/4 - (A*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/ \\
& d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}*13i)/(4*a) + (7 \\
& *B*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - \\
& (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}/(4*a)) - (71*A^2*d^2 \\
& *(a + a*tan(c + d*x)*1i)^{(1/2)}*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - ( \\
& 14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1 \\
& /2)}/(64*a^6) - (73*A^2)/(64*a^3*d^2) + (33*B^2)/(64*a^3*d^2) - (A*B*47i)/(3 \\
& 2*a^3*d^2))^{(1/2)}/((A^3*d*781i)/(4*a) + (279*B^3*d)/(4*a) - (A*B^2*d*1223i \\
& )/(4*a) - (1717*A^2*B*d)/(4*a) - (A*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6) \\
& )/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i) \\
& )/d^4)^{(1/2)}*13i)/(4*a^4) + (7*B*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 \\
& - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4 \\
& )^{(1/2)}/(4*a^4)) + (31*B^2*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)}*((5041*A^4*a \\
& ^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d \\
& ^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}/(64*a^6) - (73*A^2)/(64*a^3*d^2) + (33*B \\
& ^2)/(64*a^3*d^2) - (A*B*47i)/(32*a^3*d^2))^{(1/2)}/((A^3*d*781i)/(4*a) + (27 \\
& 9*B^3*d)/(4*a) - (A*B^2*d*1223i)/(4*a) - (1717*A^2*B*d)/(4*a) - (A*d^3*((50 \\
& 41*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6* \\
& 6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^{(1/2)}*13i)/(4*a^4) + (7*B*d^3*((5041*A
\end{aligned}$$

$$^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^...$$

$$3.103 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=268

$$\frac{(23A + 12iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{4a^{3/2}d} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2}d} + \frac{(A + iB)c}{3d(a + ia \tan(c + dx))^{3/2}}$$

[Out] 1/4\*(23\*A+12\*I\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/4\*(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(3/2)/d-1/4\*(1/2)+1/6\*(17\*A+11\*I\*B)\*cot(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)+7/4\*(3\*I\*A-2\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d-1/6\*(22\*A+13\*I\*B)\*cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d+1/3\*(A+I\*B)\*cot(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi** [A]

time = 0.65, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(23A + 12iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}} \right)}{4a^{3/2}d} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2}d} - \frac{(22A + 13iB) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{6a^2d} + \frac{7(-2B + 3iA) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4a^2d} + \frac{(17A + 11iB) \cot^2(c + dx)}{6ad \sqrt{a + ia \tan(c + dx)}} + \frac{(A + iB) \cot^2(c + dx)}{3d(a + ia \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] ((23\*A + (12\*I)\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]]/(4\*a^(3/2)\*d) - ((A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])]/(2\*Sqrt[2]\*a^(3/2)\*d) + ((A + I\*B)\*Cot[c + d\*x]^2)/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((17\*A + (11\*I)\*B)\*Cot[c + d\*x]^2)/(6\*a\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (7\*((3\*I)\*A - 2\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(4\*a^2\*d) - ((22\*A + (13\*I)\*B)\*Cot[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(6\*a^2\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]]] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3681

Int((((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]))/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*

d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot^3(c+dx)(a(5A+2iB)-\frac{7}{2}a(iA-B) \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx}{3a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+11iB) \cot^2(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{\int \cot^3(c+dx)}{3a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+11iB) \cot^2(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} - \frac{(22A+11iB)}{3a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+11iB) \cot^2(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{7(3iA+2B)}{3a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+11iB) \cot^2(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{7(3iA+2B)}{3a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+11iB) \cot^2(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{7(3iA+2B)}{3a^2} \\
 &= \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+11iB) \cot^2(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{7(3iA+2B)}{3a^2} \\
 &= -\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} \\
 &= \frac{(23A+12iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2} d} - \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2} d} + \frac{(A+iB) \cot^2(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}}
 \end{aligned}$$

**Mathematica [A]**

time = 5.34, size = 283, normalized size = 1.06

$$\frac{\sqrt{\sec(c+dx)} \left( \sqrt{2} \left( \frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{3/2} (1+e^{2i(c+dx)})^{3/2} \left( -2(A-iB) \sinh^{-1}(e^{i(c+dx)}) + \sqrt{2} (23A+12iB) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) \right) + \frac{\cos^2(c+dx) \left( -(30A+29iB) \cos(c+dx) + (38A+29iB) \cos(2(c+dx)) + 6(-3iA+5B+12iA-9B) \cos(2(c+dx)) \sin(c+dx) \right)}{2\sqrt{\sec(c+dx)}} \right) (A+B \tan(c+dx))}{8d(A \cos(c+dx) + B \sin(c+dx))(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x])^3\*(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] (Sqrt[Sec[c + d\*x]]\*(Sqrt[2]\*(E^(I\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x))))^(3/2)\*(1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*(-2\*(A - I\*B)\*ArcSinh[E^(I\*(c + d\*x))])

$$+ \text{Sqrt}[2] * (23*A + (12*I)*B) * \text{ArcTanh}[(\text{Sqrt}[2] * E^{(I*(c + d*x))}) / \text{Sqrt}[1 + E^{((2*I)*(c + d*x))}]] + (\text{Csc}[c + d*x]^2 * (-((50*A + (29*I)*B) * \text{Cos}[c + d*x]) + (38*A + (29*I)*B) * \text{Cos}[3*(c + d*x)] + 6*((-9*I)*A + 5*B + ((12*I)*A - 9*B) * \text{Cos}[2*(c + d*x)]) * \text{Sin}[c + d*x])) / (3 * \text{Sqrt}[\text{Sec}[c + d*x]]) * (A + B * \text{Tan}[c + d*x]) / (8 * d * (A * \text{Cos}[c + d*x] + B * \text{Sin}[c + d*x]) * (a + I * a * \text{Tan}[c + d*x])^{3/2})$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2817 vs.  $2(221) = 442$ .  
time = 0.57, size = 2818, normalized size = 10.51

method	result	size
default	Expression too large to display	2818

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/48/d/a^2 * ((I * \sin(d*x+c) + \cos(d*x+c)) * a / \cos(d*x+c))^{1/2} * (-168*B * \cos(d*x+c) * \sin(d*x+c) - 6*A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1/2 * 2^{1/2} * (I * \cos(d*x+c) - I - \sin(d*x+c)) / \sin(d*x+c) / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & * 2^{1/2} * \cos(d*x+c)^3 - 6*A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1/2 * 2^{1/2} * (I * \cos(d*x+c) - I - \sin(d*x+c)) / \sin(d*x+c) / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & )^{1/2} * 2^{1/2} * \cos(d*x+c)^2 + 120*A * \cos(d*x+c)^4 - 176*A * \cos(d*x+c)^2 + 6*B * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1/2 * 2^{1/2} * (I * \cos(d*x+c) - I - \sin(d*x+c)) / \sin(d*x+c) / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & * 2^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) + 252*I * A * \cos(d*x+c) * \sin(d*x+c) + 36*I * B * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1 / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & - 32*I * A * \cos(d*x+c)^5 * \sin(d*x+c) - 136*I * A * \cos(d*x+c)^3 * \sin(d*x+c) + 6*I * A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1/2 * 2^{1/2} * (I * \cos(d*x+c) - I - \sin(d*x+c)) / \sin(d*x+c) / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & * 2^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) + 32*B * \cos(d*x+c)^5 * \sin(d*x+c) + 32*I * B * \cos(d*x+c)^6 + 72*I * B * \cos(d*x+c)^4 - 104*I * B * \cos(d*x+c)^2 + 69*A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1 / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & * \cos(d*x+c) + 6*A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1/2 * 2^{1/2} * (I * \cos(d*x+c) - I - \sin(d*x+c)) / \sin(d*x+c) / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & * 2^{1/2} - 6*I * B * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1/2 * 2^{1/2} * (I * \cos(d*x+c) - I - \sin(d*x+c)) / \sin(d*x+c) / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) \\ & * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \sin(d*x+c) + 36*I * B * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \ln((\sin(d*x+c) * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} - \cos(d*x+c) + 1) / \sin(d*x+c)) * \sin(d*x+c) \\ & - 69*I * A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \ln((\sin(d*x+c) * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} - \cos(d*x+c) + 1) / \sin(d*x+c)) * \cos(d*x+c)^3 \\ & - 36*I * B * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \arctan(1 / (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2}) * \cos(d*x+c)^3 - 69*I * A * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} * \ln((\sin(d*x+c) * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} - \cos(d*x+c) + 1) / \sin(d*x+c)) * \cos(d*x+c)^2 \\ & - 36*I * B * (-2 * \cos(d*x+c) / (\cos(d*x+c)+1))^{1/2} \end{aligned}$$





Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*a^2*(2*(21*(I*a*tan(d*x + c) + a)^3*(3*A + 2*I*B) - (I*a*tan(d*x + c) + a)^2*(107*A + 68*I*B)*a + 2*(I*a*tan(d*x + c) + a)*(17*A + 11*I*B)*a^2 + 4*(A + I*B)*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a^3 - 2*(I*a*tan(d*x + c) + a)^(5/2)*a^4 + (I*a*tan(d*x + c) + a)^(3/2)*a^5) - 3*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(7/2) + 3*(23*A + 12*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(7/2))/d
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(209) = 418.  
time = 6.58, size = 903, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*(12*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 12*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) + 3*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((529*A^2 + 552*I*A*B - 144*B^2)/(a^3*d^2))*log(-16*(3*(23*I*A - 12*B)*a^2*e^(2*I*d*x + 2*I*c) + (23*I*A - 12*B)*a^2 + 2*sqrt(2)*(I*a^3*d*e^(3*I*d*x + 3*I*c) + I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((529*A^2 + 552*I*A*B - 144*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-23*I*A + 12*B)) - 3*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((529*A^2 + 552*I*A*B - 144*B^2)/(a^3*d^2))*log(-16*(3*(23*I*A - 12*B)*a^2*e^(2*I*d*x + 2*I*c) + (23*I*A - 12*B)*a^2 + 2*sqrt(2)*(-I*a^3*d*e^(3*I*d*x + 3*I*c) - I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((529*A^2 + 552*I*A*B - 144*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-23*I*A + 12*B)) - 4*sqrt(2)*((37*A + 28*I*B)*e^(8*I*d*x + 8*I*c) - 3*(11*A + 5*I*B)*e^(6*I*d*x + 6*I*c) - (50*A + 29*I*B)*e^(4*I*d*x + 4*I*c) + 3*(7*A + 5*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^3(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*3/(I\*a\*(tan(c + d\*x) - I))\*\*(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^3/(I\*a\*tan(d\*x + c) + a)^(3/2), x)

**Mupad [B]**

time = 8.41, size = 2500, normalized size = 9.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] 2\*atanh((48\*d^4\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((531\*A^2)/(128\*a^3\*d^2) - ((277729\*A^4\*a^6)/(4\*d^4) + (5041\*B^4\*a^6)/d^4 - (114701\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*39476i)/d^4 + (A^3\*B\*a^6\*146506i)/d^4)^(1/2))/(64\*a^6) - (73\*B^2)/(64\*a^3\*d^2) + (A\*B\*137i)/(32\*a^3\*d^2))^(1/2)\*((277729\*A^4\*a^6)/(4\*d^4) + (5041\*B^4\*a^6)/d^4 - (114701\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*39476i)/d^4 + (A^3\*B\*a^6\*146506i)/d^4)^(1/2))/(B^3\*a^2\*d\*3124i - 25296\*A^3\*a^2\*d + 19048\*A\*B^2\*a^2\*d - A^2\*B\*a^2\*d\*38282i + (88\*A\*d^3\*((277729\*A^4\*a^6)/(4\*d^4) + (5041\*B^4\*a^6)/d^4 - (114701\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*39476i)/d^4 + (A^3\*B\*a^6\*146506i)/d^4)^(1/2))/a + (B\*d^3\*((277729\*A^4\*a^6)/(4\*d^4) + (5041\*B^4\*a^6)/d^4 - (114701\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*39476i)/d^4 + (A^3\*B\*a^6\*146506i)/d^4)^(1/2)\*52i)/a) - (4216\*A^2\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((531\*A^2)/(128\*a^3\*d^2) - ((277729\*A^4\*a^6)/(4\*d^4) + (5041\*B^4\*a^6)/d^4 - (114701\*A^2\*B^2\*a^6)/d^4 - (A\*B^3\*a^6\*39476i)/d^4 + (A^3\*B\*a^6\*146506i)/d^4)^(1/2))

$$\begin{aligned}
& \left( \frac{1}{2} \right) / (64a^6) - (73B^2) / (64a^3d^2) + (AB*137i) / (32a^3d^2)^{(1/2)} / ((B^3*d*3124i)/a - (25296*A^3*d)/a + (19048*AB^2*d)/a - (A^2*B*d*38282i)/a + \\
& (88*A*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/a^4 + \\
& (B*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/a^4 + \\
& (1136*B^2*d^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((531*A^2)/(128*a^3*d^2) - ((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (A \\
& *B^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/(64*a^6) - (73*B^2)/(64*a^3*d^2) + (AB*137i)/(32*a^3*d^2)^{(1/2)})/((B^3*d*3124i)/a - (25296*A^3 \\
& *d)/a + (19048*AB^2*d)/a - (A^2*B*d*38282i)/a + (88*A*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*3947 \\
& 6i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/a^4 + (B*d^3*((277729*A^4*a^6)/(4 \\
& *d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/ \\
& d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)}*52i)/a^4) - (AB*d^2*(a + a*\tan(c + d* \\
& x)*1i)^{(1/2)}*((531*A^2)/(128*a^3*d^2) - ((277729*A^4*a^6)/(4*d^4) + (5041*B \\
& ^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^ \\
& 6*146506i)/d^4)^{(1/2)})/(64*a^6) - (73*B^2)/(64*a^3*d^2) + (AB*137i)/(32*a^3 \\
& *d^2)^{(1/2)}*4448i)/((B^3*d*3124i)/a - (25296*A^3*d)/a + (19048*AB^2*d)/a \\
& - (A^2*B*d*38282i)/a + (88*A*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6) \\
& /d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*14650 \\
& 6i)/d^4)^{(1/2)})/a^4 + (B*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 \\
& - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/ \\
& d^4)^{(1/2)}*52i)/a^4)*(-2*d^2*(((531*A^2*a^3)/2 - 73*B^2*a^3)/d^2 + (AB* \\
& a^3*274i)/d^2)^2 + 128*a^6*(((33*AB^3)/8 + (253*A^3*B)/32)*1i)/d^4 - ((52 \\
& 9*A^4)/64 + (431*A^2*B^2)/64 + (9*B^4)/4)/d^4)^{(1/2)} - 531*A^2*a^3 + 146*B \\
& ^2*a^3 - AB*a^3*548i)/(128*a^6*d^2)^{(1/2)} + 2*atanh((48*d^4*(a + a*\tan(c \\
& + d*x)*1i)^{(1/2)}*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701* \\
& A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/ \\
& (64*a^6) + (531*A^2)/(128*a^3*d^2) - (73*B^2)/(64*a^3*d^2) + (AB*137i)/(32 \\
& *a^3*d^2)^{(1/2)}*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A \\
& ^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/ \\
& (25296*A^3*a^2*d - B^3*a^2*d*3124i - 19048*AB^2*a^2*d + A^2*B*a^2*d*38282i \\
& + (88*A*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B \\
& ^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/a + \\
& (B*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6 \\
& )/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)}*52i)/a) + ( \\
& 4216*A^2*d^2*(a + a*\tan(c + d*x)*1i)^{(1/2)}*((277729*A^4*a^6)/(4*d^4) + (50 \\
& 41*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^4 + (A^3* \\
& B*a^6*146506i)/d^4)^{(1/2)})/(64*a^6) + (531*A^2)/(128*a^3*d^2) - (73*B^2)/(64 \\
& *a^3*d^2) + (AB*137i)/(32*a^3*d^2)^{(1/2)})/((25296*A^3*d)/a - (B^3*d*3124i \\
& )/a - (19048*AB^2*d)/a + (A^2*B*d*38282i)/a + (88*A*d^3*((277729*A^4*a^6)/ \\
& (4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i \\
& )/d^4 + (A^3*B*a^6*146506i)/d^4)^{(1/2)})/a^4 + (B*d^3*((277729*A^4*a^6)/(4*d \\
& ^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (AB^3*a^6*39476i)/d^
\end{aligned}$$

$$\begin{aligned}
& 4 + (A^3 B a^6 * 146506i) / d^4)^{(1/2)} * 52i) / a^4 - (1136 * B^2 * d^2 * (a + a * \tan(c + \\
& d * x) * i)^{(1/2)} * (((277729 * A^4 * a^6) / (4 * d^4) + (5041 * B^4 * a^6) / d^4 - (114701 * A \\
& ^2 * B^2 * a^6) / d^4 - (A * B^3 * a^6 * 39476i) / d^4 + (A^3 * B * a^6 * 146506i) / d^4)^{(1/2)} / ( \\
& 64 * a^6) + (531 * A^2) / (128 * a^3 * d^2) - (73 * B^2) / (64 * a^3 * d^2) + (A * B * 137i) / (32 * \\
& a^3 * d^2))^{(1/2)} / ((25296 * A^3 * d) / a - (B^3 * d * 3124i) / a - (19048 * A * B^2 * d) / a + ( \\
& A^2 * B * d * 38282i) / a + (88 * A * d^3 * ((277729 * A^4 * a^6) / (4 * d^4) + (5041 * B^4 * a^6) / d^ \\
& 4 - (114701 * A^2 * B^2 * a^6) / d^4 - (A * B^3 * a^6 * 39476i) / d^4 + (A^3 * B * a^6 * 146506i) \\
& / d^4)^{(1/2)} / a^4 + (B * d^3 * ((277729 * A^4 * a^6) / (4 * d^4) + (5041 * B^4 * a^6) / d^4 - \\
& (114701 * A^2 * B^2 * a^6) / d^4 - (A * B^3 * a^6 * 39476i) / d^4) / d \dots
\end{aligned}$$

$$3.104 \quad \int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=255

$$\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(iA-B) \tan^4(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(11A+21iB) \tan^3(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} - \frac{(39iA-89B) \tan^2(c+dx)}{20a^2d(a+ia \tan(c+dx))^{3/2}}$$

[Out]  $-1/8*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}+1/5*(39*I*A-89*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/a^3/d-1/20*(39*I*A-89*B)*\tan(d*x+c)^2/a^2/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(I*A-B)*\tan(d*x+c)^4/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/30*(11*A+21*I*B)*\tan(d*x+c)^3/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}-1/60*(151*I*A-361*B)*(a+I*a*\tan(d*x+c))^{(3/2)}/a^4/d$

**Rubi [A]**

time = 0.50, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3676, 3673, 3608, 3561, 212}

$$\frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{(-361B+151iA)(a+ia \tan(c+dx))^{3/2}}{60a^4d} + \frac{(-89B+39iA)\sqrt{a+ia \tan(c+dx)}}{5a^3d} - \frac{(-89B+39iA)\tan^2(c+dx)}{20a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x]^4*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $-1/4*((I*A+B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/( \operatorname{Sqrt}[2]*a^{(5/2)}*d) + ((I*A-B)*\operatorname{Tan}[c+d*x]^4)/(5*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}) + ((11*A+(21*I)*B)*\operatorname{Tan}[c+d*x]^3)/(30*a*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}) - (((39*I)*A-89*B)*\operatorname{Tan}[c+d*x]^2)/(20*a^2*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]) + (((39*I)*A-89*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(5*a^3*d) - (((151*I)*A-361*B)*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})/(60*a^4*d)$

**Rule 212**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\operatorname{tan}[(c_+ + (d_+)*(x_+))]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

**Rule 3608**

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}} dx &= \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\tan^3(c+dx)(4a(iA-B)+\frac{1}{2}a(3A+13iB)\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}}{5a^2} \\
&= \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\tan^2(c+dx)(11A+21iB)\tan(c+dx)}{(a+ia\tan(c+dx))^{1/2}}}{30ad} \\
&= \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(39iA)}{20a^2d} \\
&= \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(39iA)}{20a^2d} \\
&= \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(39iA)}{20a^2d} \\
&= \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(39iA)}{20a^2d} \\
&= -\frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}
\end{aligned}$$

**Mathematica [A]**

time = 5.12, size = 191, normalized size = 0.75

$$\frac{120(iA+B)e^{5i(c+dx)}\operatorname{arcsinh}^{-1}(e^{i(c+dx)})}{(1+e^{2i(c+dx)})^{5/2}} + \sec^2(c+dx)(-84iA+174B+(-317iA+747B)\cos(2(c+dx))+(-233iA+493B)\cos(4(c+dx))+340A\sin(2(c+dx))+780iB\sin(2(c+dx))+230A\sin(4(c+dx))+490iB\sin(4(c+dx)))}{120a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] ((120*(I*A + B)*E^((5*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))^(5/2) + Sec[c + d*x]^4*((-84*I)*A + 174*B + ((-317*I)*A + 747*B)*Cos[2*(c + d*x)] + ((-233*I)*A + 493*B)*Cos[4*(c + d*x)] + 340*A*Sin[2*(c + d*x)] + (780*I)*B*Sin[2*(c + d*x)] + 230*A*Sin[4*(c + d*x)] + (490*I)*B*Sin[4*(c + d*x)))/(120*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [A]**

time = 0.11, size = 181, normalized size = 0.71

method	result
--------	--------



derivativedivides	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + 3iaB \sqrt{a+ia \tan(dx+c)} + aA \sqrt{a+ia \tan(dx+c)} - \frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}}{\dots} \right)$
default	$2i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{3}{2}}}{3} + 3iaB \sqrt{a+ia \tan(dx+c)} + aA \sqrt{a+ia \tan(dx+c)} - \frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURN  
VERBOSE)`

[Out]  $2*I/d/a^4*(-1/3*I*B*(a+I*a*\tan(d*x+c))^{3/2}+3*I*B*a*(a+I*a*\tan(d*x+c))^{1/2}+a*A*(a+I*a*\tan(d*x+c))^{1/2}-1/16*a^{3/2}*(A-I*B)*2^{1/2}*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{1/2})*2^{1/2}/a^{1/2})+1/8*a^2*(31*I*B+17*A)/(a+I*a*\tan(d*x+c))^{1/2}-1/12*a^3*(9*I*B+7*A)/(a+I*a*\tan(d*x+c))^{3/2}+1/10*a^4*(A+I*B)/(a+I*a*\tan(d*x+c))^{5/2}$

**Maxima [A]**

time = 0.54, size = 185, normalized size = 0.73

$$i \left( 15 \sqrt{2} (A - iB) a^{\frac{3}{2}} \log \left( \frac{-\sqrt{2} \sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{ia \tan(dx+c) + a}} \right) - 160i (ia \tan(dx+c) + a)^{\frac{3}{2}} Ba + 480 \sqrt{ia \tan(dx+c) + a} (A + 3iB) a^2 + \frac{4 \left( 15 (ia \tan(dx+c) + a)^2 (17A + 31iB) a^3 - 10 (ia \tan(dx+c) + a) (7A + 9iB) a^4 + 12 (A + iB) a^5 \right)}{(ia \tan(dx+c) + a)^{\frac{5}{2}}} \right) / 240 a^5 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")`

[Out]  $1/240*I*(15*\sqrt{2}*(A - I*B)*a^{5/2}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x+c) + a})/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x+c) + a})) - 160*I*(I*a*\tan(d*x+c) + a)^{3/2}*B*a + 480*\sqrt{I*a*\tan(d*x+c) + a}*(A + 3*I*B)*a^2 + 4*(15*(I*a*\tan(d*x+c) + a)^2*(17*A + 31*I*B)*a^3 - 10*(I*a*\tan(d*x+c) + a)*(7*A + 9*I*B)*a^4 + 12*(A + I*B)*a^5)/(I*a*\tan(d*x+c) + a)^{5/2}/(a^5*d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs.  $2(198) = 396$ .

time = 5.86, size = 457, normalized size = 1.79

$$i \left( 15 \sqrt{2} (A - iB) a^{\frac{3}{2}} \log \left( \frac{-\sqrt{2} \sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{ia \tan(dx+c) + a}} \right) - 160i (ia \tan(dx+c) + a)^{\frac{3}{2}} Ba + 480 \sqrt{ia \tan(dx+c) + a} (A + 3iB) a^2 + \frac{4 \left( 15 (ia \tan(dx+c) + a)^2 (17A + 31iB) a^3 - 10 (ia \tan(dx+c) + a) (7A + 9iB) a^4 + 12 (A + iB) a^5 \right)}{(ia \tan(dx+c) + a)^{\frac{5}{2}}} \right) / 240 a^5 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{120} * (15 * \sqrt{1/2} * (a^3 * d * e^{(7 * I * d * x + 7 * I * c)} + a^3 * d * e^{(5 * I * d * x + 5 * I * c)}) * \sqrt{-(A^2 - 2 * I * A * B - B^2) / (a^5 * d^2)} * \log(-4 * (\sqrt{2} * \sqrt{1/2} * (a^3 * d * e^{(2 * I * d * x + 2 * I * c)} + a^3 * d) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{-(A^2 - 2 * I * A * B - B^2) / (a^5 * d^2)} + (-I * A - B) * a * e^{(I * d * x + I * c)} * e^{(-I * d * x - I * c)} / (I * A + B)) - 15 * \sqrt{1/2} * (a^3 * d * e^{(7 * I * d * x + 7 * I * c)} + a^3 * d * e^{(5 * I * d * x + 5 * I * c)}) * \sqrt{-(A^2 - 2 * I * A * B - B^2) / (a^5 * d^2)} * \log(4 * (\sqrt{2} * \sqrt{1/2} * (a^3 * d * e^{(2 * I * d * x + 2 * I * c)} + a^3 * d) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{-(A^2 - 2 * I * A * B - B^2) / (a^5 * d^2)} - (-I * A - B) * a * e^{(I * d * x + I * c)} * e^{(-I * d * x - I * c)} / (I * A + B)) + \sqrt{2} * ((463 * I * A - 983 * B) * e^{(8 * I * d * x + 8 * I * c)} - 3 * (-219 * I * A + 509 * B) * e^{(6 * I * d * x + 6 * I * c)} - 12 * (-14 * I * A + 29 * B) * e^{(4 * I * d * x + 4 * I * c)} + (-23 * I * A + 33 * B) * e^{(2 * I * d * x + 2 * I * c)} + 3 * I * A - 3 * B) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) / (a^3 * d * e^{(7 * I * d * x + 7 * I * c)} + a^3 * d * e^{(5 * I * d * x + 5 * I * c)})$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^4(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*4/(I\*a\*(tan(c + d\*x) - I))\*\*(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^4/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [B]**

time = 6.96, size = 279, normalized size = 1.09

$$\frac{A \operatorname{li} - \frac{d \operatorname{atan}(c+d x) \operatorname{li}}{4 a d} + \frac{d \operatorname{atan}(c+d x) \operatorname{li}^2 \operatorname{li}}{4 a^2 d}}{(a + a \tan(c + d x) \operatorname{li})^{5/2}} + \frac{A \sqrt{a + a \tan(c + d x) \operatorname{li}} \operatorname{li}}{a^2 d} - \frac{6 B \sqrt{a + a \tan(c + d x) \operatorname{li}}}{a^2 d} + \frac{2 B (a + a \tan(c + d x) \operatorname{li})^{3/2}}{3 a^2 d} - \frac{B a^2 + 11 B (a + a \tan(c + d x) \operatorname{li})^2 - 11 B (a + a \tan(c + d x) \operatorname{li})}{a^2 d (a + a \tan(c + d x) \operatorname{li})^{5/2}} + \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + d x) \operatorname{li}}}{2 \sqrt{-a}}\right) \operatorname{li}}{8 (-a)^{5/2} d} + \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + d x) \operatorname{li}}}{2 \sqrt{-a}}\right) \operatorname{li}}{8 a^{5/2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] ((A*1i)/(5*d) - (A*(a + a*tan(c + d*x)*1i)*7i)/(6*a*d) + (A*(a + a*tan(c +
d*x)*1i)^2*17i)/(4*a^2*d))/(a + a*tan(c + d*x)*1i)^(5/2) + (A*(a + a*tan(c
+ d*x)*1i)^(1/2)*2i)/(a^3*d) - (6*B*(a + a*tan(c + d*x)*1i)^(1/2))/(a^3*d)
+ (2*B*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a^4*d) - ((B*a^2)/5 + (31*B*(a + a
*tan(c + d*x)*1i)^2)/4 - (3*B*a*(a + a*tan(c + d*x)*1i))/2)/(a^2*d*(a + a*t
an(c + d*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(
1/2)))/(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d) + (2^(1/2)*B*atan((2^(1/2)*(a +
a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(8*a^(5/2)*d)
```

$$3.105 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=211

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{4\sqrt{2} a^{5/2} d} + \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(7A + 17iB) \tan^2(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{1}{60a^2 d}$$

[Out] 1/8\*(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(5/2)/d \*2^(1/2)+1/60\*(41\*A+151\*I\*B)/a^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/30\*(13\*A+83\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d+1/5\*(I\*A-B)\*tan(d\*x+c)^3/d/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(7\*A+17\*I\*B)\*tan(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.37, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3676, 3673, 3607, 3561, 212}

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{4\sqrt{2} a^{5/2} d} + \frac{(13A + 83iB) \sqrt{a + ia \tan(c + dx)}}{30a^3 d} + \frac{41A + 151iB}{60a^2 d \sqrt{a + ia \tan(c + dx)}} + \frac{(-B + iA) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(7A + 17iB) \tan^2(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] ((A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])]/(4\*Sqrt[2]\*a^(5/2)\*d) + ((I\*A - B)\*Tan[c + d\*x]^3)/(5\*d\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + ((7\*A + (17\*I)\*B)\*Tan[c + d\*x]^2)/(30\*a\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (41\*A + (151\*I)\*B)/(60\*a^2\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + ((13\*A + (83\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(30\*a^3\*d)

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3561

Int[Sqrt[(a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)]]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3607

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])], x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a

\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rule 3676

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(-A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^3(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{5/2}} dx &= \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} - \frac{\int \frac{\tan^2(c + dx)(3a(iA - B) + \frac{1}{2}a(A + 11iB) \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx}{5a^2} \\
 &= \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(7A + 17iB) \tan^2(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{\int \frac{\tan(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx}{60a^2} \\
 &= \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(7A + 17iB) \tan^2(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{(13A + 17iB) \tan(c + dx)}{60a^2} \\
 &= \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(7A + 17iB) \tan^2(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{(13A + 17iB) \tan(c + dx)}{60a^2} \\
 &= \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(7A + 17iB) \tan^2(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{(13A + 17iB) \tan(c + dx)}{60a^2} \\
 &= \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{4\sqrt{2} a^{5/2} d} + \frac{(iA - B) \tan^3(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}}
 \end{aligned}$$

**Mathematica [A]**

time = 4.22, size = 193, normalized size = 0.91

$$\frac{-A(3 - 16e^{2i(c+dx)} + 64e^{4i(c+dx)} + 83e^{6i(c+dx)}) + iB(3 - 26e^{2i(c+dx)} + 194e^{4i(c+dx)} + 463e^{6i(c+dx)}) + 15(A - iB)e^{5i(c+dx)}\sqrt{1 + e^{2i(c+dx)}}\sinh^{-1}(e^{i(c+dx)})}{15a^2d(1 + e^{2i(c+dx)})^3(-i + \tan(c + dx))^2\sqrt{a + ia\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] -1/15\*(A\*(3 - 16\*E^((2\*I)\*(c + d\*x)) + 64\*E^((4\*I)\*(c + d\*x)) + 83\*E^((6\*I)\*(c + d\*x))) + I\*B\*(3 - 26\*E^((2\*I)\*(c + d\*x)) + 194\*E^((4\*I)\*(c + d\*x)) + 463\*E^((6\*I)\*(c + d\*x))) + 15\*(A - I\*B)\*E^((5\*I)\*(c + d\*x))\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcSinh[E^(I\*(c + d\*x))])/(a^2\*d\*(1 + E^((2\*I)\*(c + d\*x)))^3\*(-I + Tan[c + d\*x])^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [A]**

time = 0.11, size = 142, normalized size = 0.67

method	result
derivativedivides	$\frac{2 \left( -iB \sqrt{a + ia \tan(dx + c)} - \frac{\sqrt{a}^{(-iB+A)} \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{16} \right)}{8\sqrt{a}}$
default	$\frac{2 \left( -iB \sqrt{a + ia \tan(dx + c)} - \frac{\sqrt{a}^{(-iB+A)} \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a + ia \tan(dx + c)} \sqrt{2}}{2\sqrt{a}}\right)}{16} \right)}{da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2), x, method=\_RETURNVERBOSE)

[Out] -2/d/a^3\*(-I\*B\*(a+I\*a\*tan(d\*x+c))^(1/2)-1/16\*a^(1/2)\*(A-I\*B)\*2^(1/2)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))-1/8\*a\*(7\*A+17\*I\*B)/(a+I\*a\*tan(d\*x+c))^(1/2)+1/12\*a^2\*(5\*A+7\*I\*B)/(a+I\*a\*tan(d\*x+c))^(3/2)-1/10\*a^3\*(A+I\*B)/(a+I\*a\*tan(d\*x+c))^(5/2))

**Maxima [A]**

time = 0.49, size = 162, normalized size = 0.77

$$\frac{15\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(\frac{-\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-480i\sqrt{ia\tan(dx+c)+a}Ba-\frac{4(15(ia\tan(dx+c)+a)^2(7A+17iB)a^2-10(ia\tan(dx+c)+a)(5A+7iB)a^3+12(A+iB)a^4)}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}}{240a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 
$$-1/240*(15*\sqrt{2}*(A - I*B)*a^{3/2}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(dx + c) + a})/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(dx + c) + a})) - 480*I*\sqrt{2}*(I*a*\tan(dx + c) + a)*B*a - 4*(15*(I*a*\tan(dx + c) + a)^2*(7*A + 17*I*B)*a^2 - 10*(I*a*\tan(dx + c) + a)*(5*A + 7*I*B)*a^3 + 12*(A + I*B)*a^4)/(I*a*\tan(dx + c) + a)^{5/2})/(a^4*d)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs.  $2(162) = 324$ .  
time = 2.17, size = 392, normalized size = 1.86

$$\left( \frac{15 \sqrt{2} a^2 \sqrt{\frac{A^2 - 2AB - B^2}{a^2}} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{\frac{A^2 - 2AB - B^2}{a^2}} \sqrt{\frac{a}{a + I a \tan(dx + c)}}}{\frac{A^2 - 2AB - B^2}{a^2}}\right) - 15 \sqrt{2} a^2 \sqrt{\frac{A^2 - 2AB - B^2}{a^2}} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{\frac{A^2 - 2AB - B^2}{a^2}} \sqrt{\frac{a}{a + I a \tan(dx + c)}}}{\frac{A^2 - 2AB - B^2}{a^2}}\right)}{120 a^4} - \sqrt{2} (83A + 463IB) e^{2I dx + 2Ic} + 2(83A + 97IB) e^{4I dx + 4Ic} - 2(8A + 13IB) e^{6I dx + 6Ic} + 3A + 3IB \sqrt{\frac{a}{a + I a \tan(dx + c)}} \right) / (a^4 d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$-1/120*(15*\sqrt{1/2}*a^3*d*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^5*d^2)}*e^{5*I*d*x + 5*I*c}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(I*a^3*d*e^{2*I*d*x + 2*I*c} + I*a^3*d)*\sqrt{a/(e^{2*I*d*x + 2*I*c} + 1))*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^5*d^2)} + (-I*A - B)*a*e^{I*d*x + I*c})*e^{-I*d*x - I*c}/(I*A + B)) - 15*\sqrt{1/2}*a^3*d*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^5*d^2)}*e^{5*I*d*x + 5*I*c}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(-I*a^3*d*e^{2*I*d*x + 2*I*c} - I*a^3*d)*\sqrt{a/(e^{2*I*d*x + 2*I*c} + 1))*\sqrt{(A^2 - 2*I*A*B - B^2)/(a^5*d^2)} + (-I*A - B)*a*e^{I*d*x + I*c})*e^{-I*d*x - I*c}/(I*A + B)) - \sqrt{2}*((83*A + 463*I*B)*e^{6*I*d*x + 6*I*c} + 2*(32*A + 97*I*B)*e^{4*I*d*x + 4*I*c} - 2*(8*A + 13*I*B)*e^{2*I*d*x + 2*I*c} + 3*A + 3*I*B)*\sqrt{a/(e^{2*I*d*x + 2*I*c} + 1)})*e^{-5*I*d*x - 5*I*c}/(a^3*d)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^3(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3/(I\*a\*(tan(c + d\*x) - I))\*\*(5/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^3/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [B]**

time = 6.89, size = 230, normalized size = 1.09

$$\frac{B \operatorname{li}\left(\frac{B(a+\tan(c+dx))^{17}}{6ad} + \frac{B(a+\tan(c+dx))^{17}}{4a^2d}\right)}{(a+a \tan(c+dx))^{5/2}} + \frac{B \sqrt{a+a \tan(c+dx)} \operatorname{li}(2)}{a^3d} + \frac{\frac{Aa^2}{8} + \frac{7A(a+\tan(c+dx))^{17}}{4} - \frac{5Aa(a+\tan(c+dx))^{17}}{6}}{a^2d(a+a \tan(c+dx))^{5/2}} + \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx)} \operatorname{li}}{2\sqrt{-a}}\right) \operatorname{li}}{8(-a)^{5/2}d} + \frac{\sqrt{2} A \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx)} \operatorname{li}}{2\sqrt{a}}\right)}{8a^{5/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] ((B\*1i)/(5\*d) - (B\*(a + a\*tan(c + d\*x)\*1i)\*7i)/(6\*a\*d) + (B\*(a + a\*tan(c + d\*x)\*1i)^2\*17i)/(4\*a^2\*d))/(a + a\*tan(c + d\*x)\*1i)^(5/2) + (B\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*2i)/(a^3\*d) + ((A\*a^2)/5 + (7\*A\*(a + a\*tan(c + d\*x)\*1i)^2)/4 - (5\*A\*a\*(a + a\*tan(c + d\*x)\*1i))/6)/(a^2\*d\*(a + a\*tan(c + d\*x)\*1i)^(5/2)) + (2^(1/2)\*B\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/(8\*(-a)^(5/2)\*d) + (2^(1/2)\*A\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/(8\*a^(5/2)\*d)



$$3.106 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=167

$$\frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{4\sqrt{2} a^{5/2} d} + \frac{(iA - B) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{3iA - 13B}{30ad(a + ia \tan(c + dx))^{3/2}} - \frac{1}{20a^2}$$

[Out] 1/8\*(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(5/2)/d \*2^(1/2)+1/20\*(-I\*A+31\*B)/a^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/5\*(I\*A-B)\*tan(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(3\*I\*A-13\*B)/a/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3676, 3671, 3607, 3561, 212}

$$\frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{4\sqrt{2} a^{5/2} d} - \frac{-31B + iA}{20a^2 d \sqrt{a + ia \tan(c + dx)}} + \frac{(-B + iA) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{-13B + 3iA}{30ad(a + ia \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2),x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/(4\*Sqrt[2]\*a^(5/2)\*d) + ((I\*A - B)\*Tan[c + d\*x]^2)/(5\*d\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + ((3\*I)\*A - 13\*B)/(30\*a\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) - (I\*A - 31\*B)/(20\*a^2\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a

```
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]
```

### Rule 3671

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(-(A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Dist[1/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

### Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)], x_Symbol] := Simp[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{5/2}} dx &= \frac{(iA - B) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} - \frac{\int \frac{\tan(c + dx)(2a(iA - B) - \frac{1}{2}a(A - 9iB) \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}}}{5a^2} \\
 &= \frac{(iA - B) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{3iA - 13B}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{i \int \frac{1}{2}a^2}{20a^2d} \\
 &= \frac{(iA - B) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{3iA - 13B}{30ad(a + ia \tan(c + dx))^{3/2}} - \frac{1}{20a^2d} \\
 &= \frac{(iA - B) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{3iA - 13B}{30ad(a + ia \tan(c + dx))^{3/2}} - \frac{1}{20a^2d} \\
 &= \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{4\sqrt{2} a^{5/2}d} + \frac{(iA - B) \tan^2(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}}
 \end{aligned}$$

**Mathematica** [A]

time = 3.18, size = 176, normalized size = 1.05

$$\frac{e^{-6i(c+dx)}(1+e^{2i(c+dx)})^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}(-3iA(1-3e^{2i(c+dx)}+e^{4i(c+dx)})+B(3-19e^{2i(c+dx)}+83e^{4i(c+dx)}))\right)+15(iA+B)e^{5i(c+dx)}\sinh^{-1}(e^{i(c+dx)})\sec^2(c+dx)}{240a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] ((1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*(Sqrt[1 + E^((2\*I)\*(c + d\*x))])\*((-3\*I)\*A\*(1 - 3\*E^((2\*I)\*(c + d\*x)) + E^((4\*I)\*(c + d\*x))) + B\*(3 - 19\*E^((2\*I)\*(c + d\*x)) + 83\*E^((4\*I)\*(c + d\*x)))) + 15\*(I\*A + B)\*E^((5\*I)\*(c + d\*x))\*ArcSinh[E^(I\*(c + d\*x))])\*Sec[c + d\*x]^2)/(240\*a^2\*d\*E^((6\*I)\*(c + d\*x))\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [A]**

time = 0.11, size = 124, normalized size = 0.74

method	result
derivativedivides	$2i \left( \frac{\left(\frac{A}{8} - \frac{iB}{8}\right)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}} - \frac{-\frac{7iB}{8} - \frac{A}{8}}{\sqrt{a+ia\tan(dx+c)}} - \frac{a(5iB+3)}{12(a+ia\tan(dx+c))} \right) \frac{1}{da^2}$
default	$2i \left( \frac{\left(\frac{A}{8} - \frac{iB}{8}\right)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}} - \frac{-\frac{7iB}{8} - \frac{A}{8}}{\sqrt{a+ia\tan(dx+c)}} - \frac{a(5iB+3)}{12(a+ia\tan(dx+c))} \right) \frac{1}{da^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2), x, method=\_RETURN VERBOSE)

[Out] -2\*I/d/a^2\*(-1/2\*(1/8\*A-1/8\*I\*B)\*2^(1/2)/a^(1/2)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))-(-7/8\*I\*B-1/8\*A)/(a+I\*a\*tan(d\*x+c))^(1/2)-1/12\*a\*(3\*A+5\*I\*B)/(a+I\*a\*tan(d\*x+c))^(3/2)+1/10\*a^2\*(A+I\*B)/(a+I\*a\*tan(d\*x+c))^(5/2))

**Maxima [A]**

time = 0.49, size = 141, normalized size = 0.84

$$\frac{i\left(15\sqrt{2}(A-iB)\sqrt{a}\log\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(15(ia\tan(dx+c)+a)^2(A+7iB)a-10(ia\tan(dx+c)+a)(3A+5iB)a^2+12(A+iB)a^3\right)}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}\right)}{240a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 
$$-1/240*I*(15*\sqrt{2}*(A - I*B)*\sqrt{a}*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan(d*x + c) + a})/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(d*x + c) + a})) + 4*(15*(I*a*\tan(d*x + c) + a)^2*(A + 7*I*B)*a - 10*(I*a*\tan(d*x + c) + a)*(3*A + 5*I*B)*a^2 + 12*(A + I*B)*a^3)/(I*a*\tan(d*x + c) + a)^{(5/2)}/(a^3*d)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs.  $2(128) = 256$ .  
time = 4.95, size = 393, normalized size = 2.35

$$\left( \frac{15 \sqrt{2} a \sqrt{\frac{A^2 - 2 I A B - B^2}{a^5 d^2}} \log\left(\frac{\sqrt{2} \sqrt{\frac{A^2 - 2 I A B - B^2}{a^5 d^2}} \sqrt{\frac{a}{a^2 + I a \tan(d x + c)}}}{\sqrt{2} \sqrt{\frac{A^2 - 2 I A B - B^2}{a^5 d^2}} \sqrt{\frac{a}{a^2 + I a \tan(d x + c)}}}\right) - 15 \sqrt{2} a \sqrt{\frac{A^2 - 2 I A B - B^2}{a^5 d^2}} \log\left(\frac{\sqrt{2} \sqrt{\frac{A^2 - 2 I A B - B^2}{a^5 d^2}} \sqrt{\frac{a}{a^2 + I a \tan(d x + c)}}}{\sqrt{2} \sqrt{\frac{A^2 - 2 I A B - B^2}{a^5 d^2}} \sqrt{\frac{a}{a^2 + I a \tan(d x + c)}}}\right) - \sqrt{2} (-3 A + 83 B) e^{6 I d x + 6 I c} - 2 (-3 A - 32 B) e^{4 I d x + 4 I c} - 2 (-3 A + 8 B) e^{2 I d x + 2 I c} - 3 A + 3 B \sqrt{\frac{a}{a^2 + I a \tan(d x + c)}}}{120 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$-1/120*(15*\sqrt{1/2}*a^3*d*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)}*e^{(5*I*d*x + 5*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)}) + (-I*A - B)*a*e^{(I*d*x + I*c)}*e^{(-I*d*x - I*c)/(I*A + B)} - 15*\sqrt{1/2}*a^3*d*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)}*e^{(5*I*d*x + 5*I*c)}*\log(4*(\sqrt{2}*\sqrt{1/2}*(a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)}) - (-I*A - B)*a*e^{(I*d*x + I*c)}*e^{(-I*d*x - I*c)/(I*A + B)} - \sqrt{2}*((-3*I*A + 83*B)*e^{(6*I*d*x + 6*I*c)} - 2*(-3*I*A - 32*B)*e^{(4*I*d*x + 4*I*c)} - 2*(-3*I*A + 8*B)*e^{(2*I*d*x + 2*I*c)} - 3*I*A + 3*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-5*I*d*x - 5*I*c)}/(a^3*d)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^2(c + dx)}{(ia (\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2/(I\*a\*(tan(c + d\*x) - I))\*\*(5/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^2/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [B]**

time = 6.74, size = 187, normalized size = 1.12

$$\frac{A \operatorname{li}}{20 d (a + a \tan (c + d x) \operatorname{li})^{5/2}} + \frac{A \tan (c + d x)^2 \operatorname{li}}{4 d (a + a \tan (c + d x) \operatorname{li})^{5/2}} + \frac{\frac{B a^2}{5} + \frac{7 B (a + a \tan (c + d x) \operatorname{li})^2}{4} - \frac{5 B a (a + a \tan (c + d x) \operatorname{li})}{6}}{a^2 d (a + a \tan (c + d x) \operatorname{li})^{5/2}} - \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan (c + d x) \operatorname{li}}}{2 \sqrt{-a}}\right) \operatorname{li}}{8 (-a)^{3/2} d} + \frac{\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a + a \tan (c + d x) \operatorname{li}}}{2 \sqrt{a}}\right)}{8 a^{5/2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] (A\*1i)/(20\*d\*(a + a\*tan(c + d\*x)\*1i)^(5/2)) + (A\*tan(c + d\*x)^2\*1i)/(4\*d\*(a + a\*tan(c + d\*x)\*1i)^(5/2)) + ((B\*a^2)/5 + (7\*B\*(a + a\*tan(c + d\*x)\*1i)^2)/4 - (5\*B\*a\*(a + a\*tan(c + d\*x)\*1i))/6)/(a^2\*d\*(a + a\*tan(c + d\*x)\*1i)^(5/2)) - (2^(1/2)\*A\*atan((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*(-a)^(1/2)))\*1i)/(8\*(-a)^(5/2)\*d) + (2^(1/2)\*B\*atanh((2^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/(2\*a^(1/2))))/(8\*a^(5/2)\*d)

$$3.107 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=153

$$-\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{A+3iB}{6ad(a+ia \tan(c+dx))^{3/2}} + \frac{A+iB}{4a^2d}$$

[Out]  $-1/8*(A-I*B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}+1/4*(A-I*B)/a^2/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(-A-I*B)/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/6*(A+3*I*B)/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ ,

Rules used = {3671, 3607, 3560, 3561, 212}

$$-\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{A-iB}{4a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{A+3iB}{6ad(a+ia \tan(c+dx))^{3/2}} - \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $-1/4*((A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/( \operatorname{Sqrt}[2]*a^{(5/2)*d} - (A+I*B)/(5*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}) + (A+(3*I)*B)/(6*a*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}) + (A-I*B)/(4*a^2*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3560

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[a*((a+b*\operatorname{Tan}[c+d*x])^n/(2*b*d*n)), x] + \operatorname{Dist}[1/(2*a), \operatorname{Int}[(a+b*\operatorname{Tan}[c+d*x])^{(n+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2+b^2, 0] \ \&\& \operatorname{LtQ}[n, 0]$

Rule 3561

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a-x^2), x], x, \operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2+b^2, 0]$

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(-(b\*c - a\*d))\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3671

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(-(A\*b - a\*B))\*(a\*c + b\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a^2\*f\*m)), x] + Dist[1/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[A\*b\*c + a\*B\*c + a\*A\*d + b\*B\*d + 2\*a\*B\*d\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}} dx &= -\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{i \int \frac{a(A+iB)+2aB\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx}{2a^2} \\
 &= -\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{A+3iB}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{(iA+B)}{4a^2d} \\
 &= -\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{A+3iB}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{(iA+B)}{4a^2d} \\
 &= -\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{A+3iB}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{(iA+B)}{4a^2d} \\
 &= -\frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}
 \end{aligned}$$

Mathematica [A]

time = 2.64, size = 176, normalized size = 1.15

$$\frac{e^{-6i(c+dx)}(1+e^{2i(c+dx)})^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}(-3iB(1-3e^{2i(c+dx)}+e^{4i(c+dx)})+A(-3-e^{2i(c+dx)}+17e^{4i(c+dx)}))-15(A-iB)e^{5i(c+dx)}\sinh^{-1}(e^{i(c+dx)})\right)\sec^2(c+dx)}{240a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out]  $((1 + E^{((2I)(c + dx))})^{3/2} * (\text{Sqrt}[1 + E^{((2I)(c + dx))}] * ((-3I) * B * (1 - 3 * E^{((2I)(c + dx))} + E^{((4I)(c + dx))}) + A * (-3 - E^{((2I)(c + dx))} + 17 * E^{((4I)(c + dx))}) - 15 * (A - I * B) * E^{((5I)(c + dx))} * \text{ArcSinh}[E^{(I(c + dx))}] * \text{Sec}[c + dx]^2) / (240 * a^2 * d * E^{((6I)(c + dx))} * \text{Sqrt}[a + I * a * \text{Tan}[c + dx]]))$

**Maple [A]**

time = 0.10, size = 121, normalized size = 0.79

method	result
derivativedivides	$\frac{(-iB+A)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a+ia \tan(dx+c)} \sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}} - \frac{2\left(-\frac{A}{4} - \frac{3iB}{4}\right)}{3(a+ia \tan(dx+c))^{\frac{3}{2}}} - \frac{a(iB+A)}{5(a+ia \tan(dx+c))^{\frac{5}{2}}} - \frac{1}{4a\sqrt{a+ia \tan(dx+c)}}$
default	$\frac{(-iB+A)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a+ia \tan(dx+c)} \sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}} - \frac{2\left(-\frac{A}{4} - \frac{3iB}{4}\right)}{3(a+ia \tan(dx+c))^{\frac{3}{2}}} - \frac{a(iB+A)}{5(a+ia \tan(dx+c))^{\frac{5}{2}}} - \frac{1}{4a\sqrt{a+ia \tan(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(dx+c)*(A+B*tan(dx+c))/(a+I*a*tan(dx+c))^(5/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $2/d/a * (-1/16 * (A - I * B) / a^{3/2} * 2^{1/2} * \operatorname{arctanh}(1/2 * (a + I * a * \tan(dx+c))^{1/2} * 2^{1/2} / a^{1/2}) - 1/3 * (-1/4 * A - 3/4 * I * B) / (a + I * a * \tan(dx+c))^{3/2} - 1/10 * a * (A + I * B) / (a + I * a * \tan(dx+c))^{5/2} - 1/8 * a * (-A + I * B) / (a + I * a * \tan(dx+c))^{1/2})$

**Maxima [A]**

time = 0.52, size = 136, normalized size = 0.89

$$\frac{15\sqrt{2}^{(A-iB)} \log\left(\frac{\sqrt{2}\sqrt{a} - \sqrt{ia \tan(dx+c) + a}}{\sqrt{2}\sqrt{a} + \sqrt{ia \tan(dx+c) + a}}\right)}{\sqrt{a}} + \frac{4(15(ia \tan(dx+c) + a)^2(A-iB) + 10(ia \tan(dx+c) + a)(A+3iB)a - 12(A+iB)a^2)}{(ia \tan(dx+c) + a)^{\frac{5}{2}}}$$


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$240 a^2 d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(dx+c)*(A+B*tan(dx+c))/(a+I*a*tan(dx+c))^(5/2),x, algorithm  
="maxima")`

[Out]  $1/240 * (15 * \sqrt{2} * (A - I * B) * \log(-(\sqrt{2} * \sqrt{a} - \sqrt{I * a * \tan(dx+c) + a}) / (\sqrt{2} * \sqrt{a} + \sqrt{I * a * \tan(dx+c) + a})) / \sqrt{a} + 4 * (15 * (I * a * \tan(dx+c) + a)^2 * (A - I * B) + 10 * (I * a * \tan(dx+c) + a) * (A + 3 * I * B) * a - 12 * (A + I * B) * a^2) / (I * a * \tan(dx+c) + a)^{5/2}) / (a^2 * d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs.  $2(114) = 228$ .



time = 2.55, size = 391, normalized size = 2.56

$$\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}e^{i(c+dx)}\log\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}}{\sqrt{\frac{A^2-2AB-B^2}{a^2}}}\right)-15\sqrt{\frac{1}{2}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}e^{i(c+dx)}\log\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}\sqrt{\frac{A^2-2AB-B^2}{a^2}}}{\sqrt{\frac{A^2-2AB-B^2}{a^2}}}\right)+\sqrt{2}\left((7A-3B)e^{i(c+dx)}+2(8A+3B)e^{i(c+dx)}-2(2A-3B)e^{i(c+dx)}-3A-3B\right)\sqrt{\frac{1}{2}}\right)}{120a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/120\*(15\*sqrt(1/2)\*a^3\*d\*sqrt((A^2 - 2\*I\*A\*B - B^2)/(a^5\*d^2))\*e^(5\*I\*d\*x + 5\*I\*c)\*log(-4\*(sqrt(2)\*sqrt(1/2)\*(I\*a^3\*d\*e^(2\*I\*d\*x + 2\*I\*c) + I\*a^3\*d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((A^2 - 2\*I\*A\*B - B^2)/(a^5\*d^2)) + (-I\*A - B)\*a\*e^(I\*d\*x + I\*c))\*e^(-I\*d\*x - I\*c)/(I\*A + B)) - 15\*sqrt(1/2)\*a^3\*d\*sqrt((A^2 - 2\*I\*A\*B - B^2)/(a^5\*d^2))\*e^(5\*I\*d\*x + 5\*I\*c)\*log(-4\*(sqrt(2)\*sqrt(1/2)\*(-I\*a^3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - I\*a^3\*d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((A^2 - 2\*I\*A\*B - B^2)/(a^5\*d^2)) + (-I\*A - B)\*a\*e^(I\*d\*x + I\*c))\*e^(-I\*d\*x - I\*c)/(I\*A + B)) + sqrt(2)\*((17\*A - 3\*I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) + 2\*(8\*A + 3\*I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*(2\*A - 3\*I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) - 3\*A - 3\*I\*B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*e^(-5\*I\*d\*x - 5\*I\*c)/(a^3\*d)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)/(I\*a\*(tan(c + d\*x) - I))^(5/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [B]**

time = 6.73, size = 186, normalized size = 1.22

$$\frac{-\frac{A}{5} + \frac{A(a + a \tan(c + dx) \operatorname{li})}{6a} + \frac{A(a + a \tan(c + dx) \operatorname{li})^2}{4a^2}}{d(a + a \tan(c + dx) \operatorname{li})^{5/2}} + \frac{B \operatorname{li}}{20d(a + a \tan(c + dx) \operatorname{li})^{5/2}} + \frac{B \tan(c + dx)^2 \operatorname{li}}{4d(a + a \tan(c + dx) \operatorname{li})^{5/2}} - \frac{\sqrt{2} B \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2\sqrt{-a}}\right) \operatorname{li}}{8(-a)^{5/2}d}} - \frac{\sqrt{2} A \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a + a \tan(c + dx) \operatorname{li}}}{2\sqrt{a}}\right)}{8a^{5/2}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] ((A*(a + a*tan(c + d*x)*1i))/(6*a) - A/5 + (A*(a + a*tan(c + d*x)*1i)^2)/(4
*a^2))/(d*(a + a*tan(c + d*x)*1i)^(5/2)) + (B*1i)/(20*d*(a + a*tan(c + d*x)
*1i)^(5/2)) + (B*tan(c + d*x)^2*1i)/(4*d*(a + a*tan(c + d*x)*1i)^(5/2)) - (
2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/
(8*(-a)^(5/2)*d) - (2^(1/2)*A*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))
/(2*a^(1/2))))/(8*a^(5/2)*d)
```

$$3.108 \quad \int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=155

$$-\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{iA-B}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{iA+B}{6ad(a+ia \tan(c+dx))^{3/2}} + \frac{1}{4a^2d}$$

[Out]  $-1/8*(I*A+B)*\operatorname{arctanh}(1/2*(a+I*a*\tan(d*x+c))^{(1/2)*2^{(1/2)}/a^{(1/2)}/a^{(5/2)}/d*2^{(1/2)+1/4*(I*A+B)/a^2/d/(a+I*a*\tan(d*x+c))^{(1/2)+1/5*(I*A-B)/d/(a+I*a*\tan(d*x+c))^{(5/2)+1/6*(I*A+B)/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}}$

**Rubi [A]**

time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3607, 3560, 3561, 212}

$$-\frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{B+iA}{4a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{-B+iA}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{B+iA}{6ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $-1/4*((I*A+B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(\operatorname{Sqrt}[2]*a^{(5/2)*d}+(I*A-B)/(5*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)})+(I*A+B)/(6*a*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})+(I*A+B)/(4*a^2*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]])$

**Rule 212**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

**Rule 3560**

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[a*((a+b*\operatorname{Tan}[c+d*x])^n/(2*b*d*n)), x] + \operatorname{Dist}[1/(2*a), \operatorname{Int}[(a+b*\operatorname{Tan}[c+d*x])^{(n+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2+b^2, 0] \ \&\& \operatorname{LtQ}[n, 0]$

**Rule 3561**

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\operatorname{tan}[(c_+) + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a-x^2), x], x, \operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2+b^2, 0]$

## Rule 3607

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[(-b*c - a*d)*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[m, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^{5/2}} dx &= \frac{iA - B}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(A - iB) \int \frac{1}{(a + ia \tan(c + dx))^{3/2}} dx}{2a} \\
&= \frac{iA - B}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{iA + B}{6ad(a + ia \tan(c + dx))^{3/2}} + \frac{(A - iB) \int \frac{1}{\sqrt{a + ia \tan(c + dx)}} dx}{4a} \\
&= \frac{iA - B}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{iA + B}{6ad(a + ia \tan(c + dx))^{3/2}} + \frac{iA + B}{4a^2 d \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{iA - B}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{iA + B}{6ad(a + ia \tan(c + dx))^{3/2}} + \frac{iA + B}{4a^2 d \sqrt{a + ia \tan(c + dx)}} \\
&= -\frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}} \right)}{4\sqrt{2} a^{5/2} d} + \frac{iA - B}{5d(a + ia \tan(c + dx))^{5/2}} +
\end{aligned}$$

**Mathematica** [A]

time = 2.44, size = 176, normalized size = 1.14

$$\frac{e^{-6i(c+dx)}(1 + e^{2i(c+dx)})^{3/2} \left( \sqrt{1 + e^{2i(c+dx)}} (B(3 + e^{2i(c+dx)} - 17e^{4i(c+dx)}) - iA(3 + 11e^{2i(c+dx)} + 23e^{4i(c+dx)})) + 15(iA + B)e^{5i(c+dx)} \sinh^{-1}(e^{i(c+dx)}) \right) \sec^2(c + dx)}{240a^2 d \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] -1/240*((1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))])*(B*(
3 + E^((2*I)*(c + d*x)) - 17*E^((4*I)*(c + d*x))) - I*A*(3 + 11*E^((2*I)*(c
+ d*x)) + 23*E^((4*I)*(c + d*x)))) + 15*(I*A + B)*E^((5*I)*(c + d*x))*ArcS
inh[E^(I*(c + d*x))])*Sec[c + d*x]^2)/(a^2*d*E^((6*I)*(c + d*x))*Sqrt[a + I
*a*Tan[c + d*x]])
```

**Maple** [A]

time = 0.09, size = 123, normalized size = 0.79



```
[Out] 1/120*(15*sqrt(1/2)*a^3*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x
+ 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I
*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d
*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqrt(2)*
sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))
*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*((23*I*A + 17*B)*e^(6*I*d*x + 6*I*c)
- 2*(-17*I*A - 8*B)*e^(4*I*d*x + 4*I*c) - 2*(-7*I*A + 2*B)*e^(2*I*d*x + 2
I*c) + 3*I*A - 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d*x - 5*I*c)
/(a^3*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2), x)
```

```
[Out] Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2), x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

**Mupad [B]**

time = 6.58, size = 205, normalized size = 1.32

$$\frac{\frac{A}{5d} + \frac{A(a+a \tan(c+dx)) \operatorname{li}}{6ad} + \frac{A(a+a \tan(c+dx)) \operatorname{li}^2}{4a^2d}}{(a+a \tan(c+dx))^{5/2}} + \frac{-\frac{B}{5} + \frac{B(a+a \tan(c+dx)) \operatorname{li}}{6a} + \frac{B(a+a \tan(c+dx)) \operatorname{li}^2}{4a^2}}{d(a+a \tan(c+dx))^{5/2}} + \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx)} \operatorname{li}}{2\sqrt{-a}}\right) \operatorname{li}}{8(-a)^{5/2}d}}{\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{a+a \tan(c+dx)} \operatorname{li}}{2\sqrt{a}}\right)}{8a^{5/2}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

```
[Out] ((A*1i)/(5*d) + (A*(a + a*tan(c + d*x)*1i)*1i)/(6*a*d) + (A*(a + a*tan(c +
d*x)*1i)^2*1i)/(4*a^2*d))/(a + a*tan(c + d*x)*1i)^(5/2) + ((B*(a + a*tan(c
+ d*x)*1i))/(6*a) - B/5 + (B*(a + a*tan(c + d*x)*1i)^2)/(4*a^2))/(d*(a + a
tan(c + d*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(
1/2)))/(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d) - (2^(1/2)*B*atanh((2^(1/2)*(a
+ a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

$$3.109 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=192

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{A+iB}{5d(a+ia \tan(c+dx))^{3/2}}$$

[Out]  $-2*A*\operatorname{arctanh}((a+I*a*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/a^{(5/2)}/d+1/8*(A-I*B)*\operatorname{arctan}h(1/2*(a+I*a*\tan(d*x+c))^{(1/2)}*2^{(1/2)}/a^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}+1/4*(7*A+I*B)/a^2/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(A+I*B)/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/6*(3*A+I*B)/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.45, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3677, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{7A+iB}{4a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{3A+iB}{6ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $(-2*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(a^{(5/2)*d}) + ((A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(4*\operatorname{Sqrt}[2]*a^{(5/2)*d}) + (A+I*B)/(5*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}) + (3*A+I*B)/(6*a*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}) + (7*A+I*B)/(4*a^2*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)}, x], x, (a+bx)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 212**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 214**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[(((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]))/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx &= \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{\int \frac{\cot(c+dx)(5aA-\frac{5}{2}a(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx}{5a^2} \\
&= \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{3A+iB}{6ad(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} dx}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{3A+iB}{6ad(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} dx}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{3A+iB}{6ad(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} dx}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{3A+iB}{6ad(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} dx}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{A+iB}{5d(a+ia \tan(c+dx))^{5/2}} \\
&= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 4.38, size = 233, normalized size = 1.21

$$\frac{e^{-6i(c+dx)}(1+e^{2i(c+dx)})^{3/2} \left( \sqrt{1+e^{2i(c+dx)}} (iB(3+11e^{2i(c+dx)}+23e^{4i(c+dx)})+3A(1+7e^{2i(c+dx)}+41e^{4i(c+dx)})) + 15(A-iB)e^{5i(c+dx)} \sinh^{-1}(e^{i(c+dx)}) - 120\sqrt{2}Ae^{5i(c+dx)} \tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) \right) \sec^2(c+dx)}{240a^2d\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] ((1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))])*(I*B*(3 + 11*E^((2*I)*(c + d*x)) + 23*E^((4*I)*(c + d*x))) + 3*A*(1 + 7*E^((2*I)*(c + d*x)) + 41*E^((4*I)*(c + d*x)))) + 15*(A - I*B)*E^((5*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))] - 120*Sqrt[2]*A*E^((5*I)*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^2)/(240*a^2*d*E^((6*I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1083 vs.  $2(154) = 308$ .

time = 0.51, size = 1084, normalized size = 5.65

method	result	size
default	Expression too large to display	1084

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $\frac{1}{240} \frac{1}{d} \frac{1}{a^3} \left( (I \sin(dx+c) + \cos(dx+c)) \frac{a}{\cos(dx+c)} \right)^{1/2} \left( -192 I A \cos(dx+c)^3 \sin(dx+c) + 20 I B \cos(dx+c)^2 + 192 A \cos(dx+c)^6 + 192 B \cos(dx+c)^5 \sin(dx+c) + 15 I B (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \right) \cos(dx+c) \cdot 2^{1/2} + 192 I B \cos(dx+c)^6 - 120 I A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \ln((\sin(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1) / \sin(dx+c)) - 420 I A \cos(dx+c) \sin(dx+c) + 120 I A \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \sin(dx+c) - 120 I A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \ln((\sin(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1) / \sin(dx+c)) \cdot \cos(dx+c) - 15 A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} \cos(dx+c) + 96 A \cos(dx+c)^4 + 15 I B (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} \sin(dx+c) + 32 B \cos(dx+c)^3 \sin(dx+c) - 192 I A \cos(dx+c)^5 \sin(dx+c) - 64 I B \cos(dx+c)^4 - 15 A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} - 120 A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot \cos(dx+c) - 120 A \ln((\sin(dx+c) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} - \cos(dx+c) + 1) / \sin(dx+c)) \cdot (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \sin(dx+c) + 15 I A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1/2 \cdot 2^{1/2} (I \cos(dx+c) - I \sin(dx+c)) / \sin(dx+c) / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) \cdot 2^{1/2} \sin(dx+c) - 120 A (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2} \arctan(1 / (-2 \cos(dx+c) / (\cos(dx+c)+1))^{1/2}) + 300 A \cos(dx+c)^2 + 60 B \cos(dx+c) \sin(dx+c) \right)$

**Maxima [A]**

time = 0.50, size = 186, normalized size = 0.97

$$\frac{15 \sqrt{2}^{(A-iB)} \log\left(\frac{\sqrt{2} \sqrt{a} - \sqrt{i a \tan(dx+c) + a}}{\sqrt{2} \sqrt{a} + \sqrt{i a \tan(dx+c) + a}}\right)}{a^{\frac{5}{2}}} - \frac{240 A \log\left(\frac{\sqrt{i a \tan(dx+c) + a} - \sqrt{a}}{\sqrt{i a \tan(dx+c) + a} + \sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{4 (15 (i a \tan(dx+c) + a)^2 (7 A + i B) + 10 (i a \tan(dx+c) + a) (3 A + i B) a + 12 (A + i B) a^2)}{(i a \tan(dx+c) + a)^{\frac{5}{2}} a^2}}{240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm  
="maxima")`

```
[Out] -1/240*(15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) - 240*A*log((
sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)
))/a^(5/2) - 4*(15*(I*a*tan(d*x + c) + a)^2*(7*A + I*B) + 10*(I*a*tan(d*x +
c) + a)*(3*A + I*B)*a + 12*(A + I*B)*a^2)/((I*a*tan(d*x + c) + a)^(5/2)*a^
2))/d
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 644 vs.  $2(145) = 290$ .  
time = 3.44, size = 644, normalized size = 3.35



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x
+ 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2)) +
(-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^
3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(
2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^(I*d*x
+ I*c))*e^(-I*d*x - I*c)/(I*A + B)) + 60*a^3*d*sqrt(A^2/(a^5*d^2))*e^(5*I*d
*x + 5*I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 + 2*sqrt(2)*(a^4*d*
e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(A^2/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - 60*a^3*d*sqrt(A^2/(a^5*d
^2))*e^(5*I*d*x + 5*I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 - 2*sq
rt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(A^2/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - sqrt(2)*((1
23*A + 23*I*B)*e^(6*I*d*x + 6*I*c) + 2*(72*A + 17*I*B)*e^(4*I*d*x + 4*I*c)
+ 2*(12*A + 7*I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot(c + dx)}{(ia (\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2),
x)
```



$$3.110 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=259

$$\frac{(5iA - 2B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{4\sqrt{2} a^{5/2}d} + \frac{(A + iB) \cot(c + dx)}{5d(a + ia \tan(c + dx))}$$

[Out] (5\*I\*A-2\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+1/8\*(I\*A+B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(5/2)/d\*2^(1/2)+1/12\*(41\*A+15\*I\*B)\*cot(d\*x+c)/a^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)-7/4\*(3\*A+I\*B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d+1/5\*(A+I\*B)\*cot(d\*x+c)/d/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(19\*A+9\*I\*B)\*cot(d\*x+c)/a/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.69, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(-2B + 5iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{4\sqrt{2} a^{5/2}d} - \frac{7(3A + iB) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4a^2d} + \frac{(41A + 15iB) \cot(c + dx)}{12a^2d \sqrt{a + ia \tan(c + dx)}} + \frac{(19A + 9iB) \cot(c + dx)}{30ad(a + ia \tan(c + dx))^{3/2}} + \frac{(A + iB) \cot(c + dx)}{5d(a + ia \tan(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] (((5\*I)\*A - 2\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]]/(a^(5/2)\*d) + ((I\*A + B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])]/(4\*Sqrt[2]\*a^(5/2)\*d) + ((A + I\*B)\*Cot[c + d\*x])/(5\*d\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + ((19\*A + (9\*I)\*B)\*Cot[c + d\*x])/(30\*a\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((41\*A + (15\*I)\*B)\*Cot[c + d\*x])/(12\*a^2\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - (7\*(3\*A + I\*B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(4\*a^3\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3681

Int((((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]))/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*

d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{\int \frac{\cot^2(c+dx)(a(6A+iB)-\frac{7}{2}a(iA-B) \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx}{5a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(19A+9iB) \cot(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot^2}{(a+ia \tan(c+dx))^{3/2}} dx}{12a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(19A+9iB) \cot(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{(41A+21iB) \cot(c+dx)}{12a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(19A+9iB) \cot(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{(41A+21iB) \cot(c+dx)}{12a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(19A+9iB) \cot(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{(41A+21iB) \cot(c+dx)}{12a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(19A+9iB) \cot(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{(41A+21iB) \cot(c+dx)}{12a^2} \\
 &= \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(19A+9iB) \cot(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{(41A+21iB) \cot(c+dx)}{12a^2} \\
 &= \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} \\
 &= \frac{(5iA-2B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(A+iB) \cot(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}}
 \end{aligned}$$

**Mathematica [A]**

time = 7.70, size = 287, normalized size = 1.11

$$\frac{\sec^3(c+dx) \left( \sqrt{2} e^{2i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} (iA+B) \sinh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) + 4\sqrt{2} (5iA-2B) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{1+e^{2i(c+dx)}}}\right) + \frac{-80(-17A-6iB+(20A+6iB) \cos(2(c+dx))) \cos(c+dx) + 14(-13iA+3B+2(-29iA+9B) \cos(2(c+dx))) \sec(c+dx)}{15 \sec^2(c+dx)} \right)}{8d(A \cos(c+dx) + B \sin(c+dx))(a+ia \tan(c+dx))^{5/2}} (A+B \tan(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x])^2\*(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] (Sec[c + d\*x])^(3/2)\*(Sqrt[2]\*E^((2\*I)\*(c + d\*x))\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))])\*((I\*A + B)\*ArcSinh[E^(I

$$\begin{aligned} &*(c + d*x))] + 4*\text{Sqrt}[2]*((5*I)*A - 2*B)*\text{ArcTanh}[(\text{Sqrt}[2]*E^{(I*(c + d*x))})/ \\ &\text{Sqrt}[1 + E^{((2*I)*(c + d*x))}]] + (-40*(-17*A - (6*I)*B + (20*A + (6*I)*B)* \\ &\text{Cos}[2*(c + d*x)])*\text{Csc}[c + d*x] + 14*((-13*I)*A + 3*B + 2*((-29*I)*A + 9*B)* \\ &\text{Cos}[2*(c + d*x)])*\text{Sec}[c + d*x])/(15*\text{Sec}[c + d*x]^{(3/2)})*(A + B*\text{Tan}[c + d*x \\ &)))/(8*d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^{(5/2)}) \end{aligned}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2857 vs.  $2(214) = 428$ .  
time = 0.59, size = 2858, normalized size = 11.03

method	result	size
default	Expression too large to display	2858

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURN VERBOSE)`

[Out] 
$$\begin{aligned} &-1/240/d/a^3*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*(-1260*A*\cos(d* \\ &x+c)*\sin(d*x+c)+15*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)} \\ &)*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ &)*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-204*B*\cos(d*x+c)^4+300*B*\cos(d*x+c)^2-3 \\ &00*I*A*\cos(d*x+c)^3*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d \\ &*x+c)/(\cos(d*x+c)+1))^{(1/2)}+120*I*B*\cos(d*x+c)^3*(-2*\cos(d*x+c)/(\cos(d*x+c \\ &)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+ \\ &1)/\sin(d*x+c))-300*I*A*\cos(d*x+c)^2*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ar \\ &ctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}+120*I*B*\cos(d*x+c)^2*(-2*\cos(d \\ &*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{( \\ &1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))+300*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)} \\ &)*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)})*\cos(d*x+c)-120*I*B*\cos(d*x+ \\ &c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos( \\ &d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1)/\sin(d*x+c))-15*I*A*2^{(1/2)}*(-2*\cos(d*x+c)/( \\ &\cos(d*x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+ \\ &c)/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}+300*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c) \\ &+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-\cos(d*x+c)+1 \\ &)/\sin(d*x+c))*\sin(d*x+c)+120*I*B*\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{( \\ &1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}-300*I*A*(-2*\cos(d*x+c) \\ &/(\cos(d*x+c)+1))^{(1/2)}*\ln((\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}- \\ &\cos(d*x+c)+1)/\sin(d*x+c))*\cos(d*x+c)^2*\sin(d*x+c)-120*I*B*\cos(d*x+c)^2*\sin( \\ &d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(d*x+c)/(\cos(d* \\ &x+c)+1))^{(1/2)}+15*I*B*2^{(1/2)}*\sin(d*x+c)*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1 \\ &/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c) \\ &/(\cos(d*x+c)+1))^{(1/2)}+15*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\arctan \\ &(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+c)/(\cos(d*x \\ &+c)+1))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^3+15*I*A*(-2*\cos(d*x+c)/(\cos(d*x+c)+1))^{( \\ &1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(d*x+c)-I-\sin(d*x+c))/\sin(d*x+c)/(-2*\cos(d*x+ \end{aligned}$$



$c)/(\cos(dx+c)+1))^{(1/2)}*2^{(1/2)}*\cos(dx+c)^2-15*I*A*2^{(1/2)}*\cos(dx+c)*(-$   
 $2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(d$   
 $*x+c))/\sin(dx+c)/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}+192*I*B*\cos(dx+c)^$   
 $7*\sin(dx+c)+228*I*B*\cos(dx+c)^3*\sin(dx+c)-420*I*B*\cos(dx+c)*\sin(dx+c)+$   
 $300*I*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(d$   
 $*x+c)+1))^{(1/2)}-120*I*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c$   
 $)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))+160*A*\cos($   
 $dx+c)^5*\sin(dx+c)-300*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+x$   
 $c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))-120*B*(-2$   
 $*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{($   
 $1/2)}+96*B*\cos(dx+c)^6+192*A*\cos(dx+c)^7*\sin(dx+c)+192*I*A*\cos(dx+c)^8+$   
 $64*I*A*\cos(dx+c)^6+564*I*A*\cos(dx+c)^4-820*I*A*\cos(dx+c)^2-15*I*B*(-2*co$   
 $s(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(dx+c$   
 $))/\sin(dx+c)/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*2^{(1/2)}*\cos(dx+c)^2*si$   
 $n(dx+c)-192*B*\cos(dx+c)^8+300*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*arct$   
 $an(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\sin(dx+c)-120*B*(-2*\cos(dx+c)/$   
 $(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-c$   
 $os(dx+c)+1)/\sin(dx+c))*\sin(dx+c)+668*A*\cos(dx+c)^3*\sin(dx+c)+300*A*(-2$   
 $*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-2*\cos(dx+c)/(\cos(dx+c)$   
 $+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^3+120*B*(-2*\cos(dx+c)/(\cos$   
 $(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)})*\cos(dx+c)$   
 $^3+300*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-2*\cos(dx+c)$   
 $/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)^2+120*B*(-2*\cos$   
 $(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)$   
 $)*\cos(dx+c)^2-300*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\ln((\sin(dx+c)*(-$   
 $2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}-\cos(dx+c)+1)/\sin(dx+c))*\cos(dx+c)-120$   
 $*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos(dx+c)$   
 $+1))^{(1/2)})*\cos(dx+c)-15*A*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*\arctan(1/2$   
 $*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(dx+c))/\sin(dx+c)/(-2*\cos(dx+c)/(\cos(dx+c)+$   
 $1))^{(1/2)}*2^{(1/2)}*\sin(dx+c)+15*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*arc$   
 $tan(1/2*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(dx+c))/\sin(dx+c)/(-2*\cos(dx+c)/(\cos($   
 $dx+c)+1))^{(1/2)}*2^{(1/2)}*\cos(dx+c)^3+15*B*(-2*\cos(dx+c)/(\cos(dx+c)+1))^{($   
 $1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(dx+c))/\sin(dx+c)/(-2*\cos(dx$   
 $+c)/(\cos(dx+c)+1))^{(1/2)}*2^{(1/2)}*\cos(dx+c)^2-15*B*(-2*\cos(dx+c)/(\cos(d$   
 $x+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(dx+c))/\sin(dx+c)/(-$   
 $2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*2^{(1/2)}*\cos(dx+c)-15*B*(-2*\cos(dx+c)/$   
 $(\cos(dx+c)+1))^{(1/2)}*\arctan(1/2*2^{(1/2)}*(I*\cos(dx+c)-I-\sin(dx+c))/\sin(d$   
 $x+c)/(-2*\cos(dx+c)/(\cos(dx+c)+1))^{(1/2)}*2^{(1/2)}-300*A*(-2*\cos(dx+c)/(co$   
 $s(dx+c)+1))^{(1/2)}*\arctan(1/(-2*\cos(dx+c)/(\cos...$

**Maxima [A]**

time = 0.50, size = 240, normalized size = 0.93

$$ia \left( \frac{4 \left( 105 (i a \tan(dx+c)+a)^3 (3A+B) - 5 (i a \tan(dx+c)+a)^2 (41A+15B)a - 2 (i a \tan(dx+c)+a) (19A+9B)a^2 - 12(A+B)a^3 \right)}{(i a \tan(dx+c)+a)^2 a^3 - (i a \tan(dx+c)+a)^2 a^4} + \frac{15 \sqrt{2}^{(A-iB)} \log \left( \frac{\sqrt{2} \sqrt{a} - i a \tan(dx+c) + a}{\sqrt{2} \sqrt{a} + i a \tan(dx+c) + a} \right)}{a^{\frac{5}{2}}} + \frac{120 (5A+2iB) \log \left( \frac{\sqrt{i a \tan(dx+c) + a} - \sqrt{a}}{\sqrt{i a \tan(dx+c) + a} + \sqrt{a}} \right)}{a^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/240*I*a*(4*(105*(I*a*tan(d*x + c) + a)^3*(3*A + I*B) - 5*(I*a*tan(d*x + c) + a)^2*(41*A + 15*I*B)*a - 2*(I*a*tan(d*x + c) + a)*(19*A + 9*I*B)*a^2 - 12*(A + I*B)*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a^3 - (I*a*tan(d*x + c) + a)^(5/2)*a^4) + 15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(7/2) + 120*(5*A + 2*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(7/2))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 834 vs.  $2(204) = 408$ .

time = 2.77, size = 834, normalized size = 3.22

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2))*log(-16*(3*(5*I*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (5*I*A - 2*B)*a^2 + 2*sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*sqrt(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-5*I*A + 2*B)) + 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2))*log(-16*(3*(5*I*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (5*I*A - 2*B)*a^2 - 2*sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*sqrt(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-5*I*A + 2*B)) - sqrt(2)*((-403*I*A + 123*B)*e^(8*I*d*x + 8*I*c) + (-151*I*A + 21*B)*e^(6*I*d*x + 6*I*c) - 40*(-7*I*A + 3*B)*e^(4*I*d*x + 4*I*c) + (31*I*A - 21*B)*e^(2*I*d*x + 2*I*c) + 3*I*A - 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^2(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*2/(I\*a\*(tan(c + d\*x) - I))\*\*(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^2/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [B]**

time = 8.25, size = 2500, normalized size = 9.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] 2\*atanh((12\*a\*d^4\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((129\*B^2)/(256\*a^5\*d^2) - (801\*A^2)/(256\*a^5\*d^2) - ((638401\*A^4\*a^2)/(16\*d^4) + (16129\*B^4\*a^2)/(16\*d^4) - (307555\*A^2\*B^2\*a^2)/(8\*d^4) - (A\*B^3\*a^2\*40767i)/(4\*d^4) + (A^3\*B\*a^2\*256479i)/(4\*d^4))^(1/2)/(64\*a^6) - (A\*B\*319i)/(128\*a^5\*d^2))^(1/2)\*((638401\*A^4\*a^2)/(16\*d^4) + (16129\*B^4\*a^2)/(16\*d^4) - (307555\*A^2\*B^2\*a^2)/(8\*d^4) - (A\*B^3\*a^2\*40767i)/(4\*d^4) + (A^3\*B\*a^2\*256479i)/(4\*d^4))^(1/2))/((A^3\*d\*31161i)/8 + (2159\*B^3\*d)/8 - (A\*B^2\*d\*15867i)/8 - (38621\*A^2\*B\*d)/8 + (A\*d^3\*((638401\*A^4\*a^2)/(16\*d^4) + (16129\*B^4\*a^2)/(16\*d^4) - (307555\*A^2\*B^2\*a^2)/(8\*d^4) - (A\*B^3\*a^2\*40767i)/(4\*d^4) + (A^3\*B\*a^2\*256479i)/(4\*d^4))^(1/2)\*41i)/(2\*a) - (15\*B\*d^3\*((638401\*A^4\*a^2)/(16\*d^4) + (16129\*B^4\*a^2)/(16\*d^4) - (307555\*A^2\*B^2\*a^2)/(8\*d^4) - (A\*B^3\*a^2\*40767i)/(4\*d^4) + (A^3\*B\*a^2\*256479i)/(4\*d^4))^(1/2))/(2\*a)) + (799\*A^2\*a^2\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/2)\*((129\*B^2)/(256\*a^5\*d^2) - (801\*A^2)/(256\*a^5\*d^2) - ((638401

$$\begin{aligned}
& *A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) \\
& ) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)}/(64*a^6) \\
& ) - (A*B*319i)/(128*a^5*d^2))^{(1/2)}/((A^3*d*31161i)/8 + (2159*B^3*d)/8 - ( \\
& A*B^2*d*15867i)/8 - (38621*A^2*B*d)/8 + (A*d^3*((638401*A^4*a^2)/(16*d^4) + \\
& (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767 \\
& i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)}*41i)/(2*a) - (15*B*d^3*((63 \\
& 8401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8 \\
& *d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)})/(2 \\
& *a)) - (127*B^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^{(1/2)}*((129*B^2)/(256*a^5*d \\
& ^2) - (801*A^2)/(256*a^5*d^2) - ((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2 \\
& )/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A \\
& ^3*B*a^2*256479i)/(4*d^4))^{(1/2)}/(64*a^6) - (A*B*319i)/(128*a^5*d^2))^{(1/2) \\
& ))/((A^3*d*31161i)/8 + (2159*B^3*d)/8 - (A*B^2*d*15867i)/8 - (38621*A^2*B*d) \\
& /8 + (A*d^3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555 \\
& *A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4 \\
& *d^4))^{(1/2)}*41i)/(2*a) - (15*B*d^3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4 \\
& *a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) \\
& + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)})/(2*a)) + (A*B*a^2*d^2*(a + a*tan(c + \\
& d*x)*1i)^{(1/2)}*((129*B^2)/(256*a^5*d^2) - (801*A^2)/(256*a^5*d^2) - ((63840 \\
& 1*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^ \\
& 4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)}/(64*a^ \\
& 6) - (A*B*319i)/(128*a^5*d^2))^{(1/2)}*642i)/((A^3*d*31161i)/8 + (2159*B^3*d) \\
& /8 - (A*B^2*d*15867i)/8 - (38621*A^2*B*d)/8 + (A*d^3*((638401*A^4*a^2)/(16* \\
& d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2 \\
& *40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)}*41i)/(2*a) - (15*B*d^ \\
& 3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a \\
& ^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/ \\
& 2)})/(2*a)))*(-(4*d^2*(((801*A^2*a)/4 - (129*B^2*a)/4)/d^2 + (A*B*a*319i)/( \\
& 2*d^2))^{(1/2)} + 128*a^6*(((3*A*B^3)/4 + (15*A^3*B)/8)*1i)/(a^4*d^4) - ((25*A^4 \\
& )/16 + (11*A^2*B^2)/16 + B^4/4)/(a^4*d^4))^{(1/2)} + 801*A^2*a - 129*B^2*a + \\
& A*B*a*638i)/(256*a^6*d^2))^{(1/2)} - (((A*a + B*a*1i)*1i)/(5*d) + ((19*A + B \\
& *9i)*(a + a*tan(c + d*x)*1i)*1i)/(30*d) - ((3*A + B*1i)*(a + a*tan(c + d*x) \\
& *1i)^3*7i)/(4*a^2*d) + ((41*A + B*15i)*(a + a*tan(c + d*x)*1i)^2*1i)/(12*a* \\
& d))/(a*(a + a*tan(c + d*x)*1i)^{(5/2)} - (a + a*tan(c + d*x)*1i)^{(7/2)}) - 2*a \\
& tanh((12*a*d^4*(a + a*tan(c + d*x)*1i)^{(1/2)}*((638401*A^4*a^2)/(16*d^4) + \\
& (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i \\
& )/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)}/(64*a^6) - (801*A^2)/(256*a^ \\
& 5*d^2) + (129*B^2)/(256*a^5*d^2) - (A*B*319i)/(128*a^5*d^2))^{(1/2)}*((638401 \\
& *A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4 \\
& ) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)})/((A^3* \\
& d*31161i)/8 + (2159*B^3*d)/8 - (A*B^2*d*15867i)/8 - (38621*A^2*B*d)/8 - (A \\
& d^3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2 \\
& *a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{( \\
& 1/2)}*41i)/(2*a) + (15*B*d^3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(1 \\
& 6*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B
\end{aligned}$$

$$\begin{aligned}
& *a^2*256479i)/(4*d^4))^{(1/2)} / (2*a)) - (799*A^2*a^2*d^2*(a + a*\tan(c + d*x) \\
& *1i)^{(1/2)} * (((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555 \\
& *A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4 \\
& *d^4))^{(1/2)} / (64*a^6) - (801*A^2)/(256*a^5*d^2) + (129*B^2)/(256*a^5*d^2) - \\
& (A*B*319i)/(128*a^5*d^2))^{(1/2)} / ((A^3*d*31161i)/8 + (2159*B^3*d)/8 - (A*B \\
& ^2*d*15867i)/8 - (38621*A^2*B*d)/8 - (A*d^3*((638401*A^4*a^2)/(16*d^4) + (1 \\
& 6129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/ \\
& (4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^{(1/2)} *41i) / (2*a) + (15*B*d^3*((63840 \\
& 1*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) \dots
\end{aligned}$$

$$3.111 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=312

$$\frac{(43A + 20iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{4\sqrt{2} a^{5/2}d} + \frac{(A + iB) \cot(c + dx)}{5d(a + ia \tan(c + dx))^{3/2}}$$

[Out] 1/4\*(43\*A+20\*I\*B)\*arctanh((a+I\*a\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-1/8\*(A-I\*B)\*arctanh(1/2\*(a+I\*a\*tan(d\*x+c))^(1/2)\*2^(1/2)/a^(1/2))/a^(5/2)/d\*2^(1/2)+1/60\*(337\*A+167\*I\*B)\*cot(d\*x+c)^2/a^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)+21/4\*(2\*I\*A-B)\*cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d-1/12\*(85\*A+41\*I\*B)\*cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d+1/5\*(A+I\*B)\*cot(d\*x+c)^2/d/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(23\*A+13\*I\*B)\*cot(d\*x+c)^2/a/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.81, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3679, 3681, 3561, 212, 3680, 65, 214}

$$\frac{(43A + 20iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{2} \sqrt{a}}\right)}{4\sqrt{2} a^{5/2}d} - \frac{(85A + 41iB) \cot^2(c + dx) \sqrt{a + ia \tan(c + dx)}}{12a^3d} + \frac{21(-B + 2iA) \cot(c + dx) \sqrt{a + ia \tan(c + dx)}}{4a^3d} + \frac{(337A + 167iB) \cot^2(c + dx)}{60a^2d \sqrt{a + ia \tan(c + dx)}} + \frac{(23A + 13iB) \cot^2(c + dx)}{30a^2d (a + ia \tan(c + dx))^{3/2}} + \frac{(A + iB) \cot(c + dx)}{5d(a + ia \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] ((43\*A + (20\*I)\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/Sqrt[a]])/(4\*a^(5/2)\*d) - ((A - I\*B)\*ArcTanh[Sqrt[a + I\*a\*Tan[c + d\*x]]/(Sqrt[2]\*Sqrt[a])])/(4\*Sqrt[2]\*a^(5/2)\*d) + ((A + I\*B)\*Cot[c + d\*x]^2)/(5\*d\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + ((23\*A + (13\*I)\*B)\*Cot[c + d\*x]^2)/(30\*a\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((337\*A + (167\*I)\*B)\*Cot[c + d\*x]^2)/(60\*a^2\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (21\*((2\*I)\*A - B)\*Cot[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(4\*a^3\*d) - ((85\*A + (41\*I)\*B)\*Cot[c + d\*x]^2\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(12\*a^3\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 212**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 3561

Int[Sqrt[(a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/d), Subst[Int[1/(2\*a - x^2), x], x, Sqrt[a + b\*Tan[c + d\*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

#### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

#### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

#### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

#### Rule 3681





```
[In] Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(Sqrt[2]*E^((2*I)*(c + d*x))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-(A - I*B)*ArcSinh[E^(I*(c + d*x))]) + Sqrt[2]*(43*A + (20*I)*B)*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[1 + E^((2*I)*(c + d*x))]]) + (Csc[c + d*x]^2*(212*A + (112*I)*B - 15*(44*A + (21*I)*B)*Cos[2*(c + d*x)] + (388*A + (203*I)*B)*Cos[4*(c + d*x)] - (695*I)*A*Sin[2*(c + d*x)] + 340*B*Sin[2*(c + d*x)] + (385*I)*A*Sin[4*(c + d*x)] - 200*B*Sin[4*(c + d*x)])/(15*Sqrt[Sec[c + d*x]])*(A + B*Tan[c + d*x]))/(8*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2875 vs.  $2(259) = 518$ .

time = 0.63, size = 2876, normalized size = 9.22

method	result	size
default	Expression too large to display	2876

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240/d/a^3*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c))^(1/2)*(-1260*B*cos(d*x+c)*sin(d*x+c)-15*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)^3-15*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)^2+1164*A*cos(d*x+c)^4-1700*A*cos(d*x+c)^2+15*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+192*B*cos(d*x+c)^7*sin(d*x+c)+192*I*B*cos(d*x+c)^8+64*I*B*cos(d*x+c)^6+564*I*B*cos(d*x+c)^4-820*I*B*cos(d*x+c)^2-192*I*A*cos(d*x+c)^7*sin(d*x+c)-320*I*A*cos(d*x+c)^5*sin(d*x+c)-1348*I*A*cos(d*x+c)^3*sin(d*x+c)+2520*I*A*cos(d*x+c)*sin(d*x+c)+645*I*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))+300*I*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+192*A*cos(d*x+c)^8+160*B*cos(d*x+c)^5*sin(d*x+c)+645*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+15*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)-645*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+300*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln((sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)^3-300*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```





```
*A*B - 400*B^2)/(a^5*d^2))*log(-16*(3*(43*I*A - 20*B)*a^2*e^(2*I*d*x + 2*I*c) + (43*I*A - 20*B)*a^2 + 2*sqrt(2)*(-I*a^4*d*e^(3*I*d*x + 3*I*c) - I*a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((1849*A^2 + 1720*I*A*B - 400*B^2)/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-43*I*A + 20*B)) - 2*sqrt(2)*((773*A + 403*I*B)*e^(10*I*d*x + 10*I*c) - 6*(97*A + 42*I*B)*e^(8*I*d*x + 8*I*c) - (931*A + 431*I*B)*e^(6*I*d*x + 6*I*c) + 3*(153*A + 83*I*B)*e^(4*I*d*x + 4*I*c) + 2*(19*A + 14*I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^3(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

**Mupad [B]**

time = 8.39, size = 2500, normalized size = 8.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*i)^(5/2),x)
```

```
[Out] 2*atanh((192*a*d^4*(a + a*tan(c + d*x)*i)^(1/2)*((3699*A^2)/(256*a^5*d^2) - ((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2)/(64*a^6) - (801*B^2)/(256*a^5*d^2) + (A*B*1719i)/(128*a^5*d^2))^(
```

$$\begin{aligned}
& 1/2)*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/(B^3*d*62322i - 643278*A^3*d + 407502*A*B^2*d - A^2*B*d*887274i + (680*A*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/a + (B*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2)*328i)/a - (59152*A^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((3699*A^2)/(256*a^5*d^2) - ((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/(64*a^6) - (801*B^2)/(256*a^5*d^2) + (A*B*1719i)/(128*a^5*d^2))^(1/2))/(B^3*d*62322i - 643278*A^3*d + 407502*A*B^2*d - A^2*B*d*887274i + (680*A*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/a + (B*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2)*328i)/a + (12784*B^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((3699*A^2)/(256*a^5*d^2) - ((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/(64*a^6) - (801*B^2)/(256*a^5*d^2) + (A*B*1719i)/(128*a^5*d^2))^(1/2))/(B^3*d*62322i - 643278*A^3*d + 407502*A*B^2*d - A^2*B*d*887274i + (680*A*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/a + (B*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2)*328i)/a - (A*B*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((3699*A^2)/(256*a^5*d^2) - ((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/(64*a^6) - (801*B^2)/(256*a^5*d^2) + (A*B*1719i)/(128*a^5*d^2))^(1/2)*55072i)/(B^3*d*62322i - 643278*A^3*d + 407502*A*B^2*d - A^2*B*d*887274i + (680*A*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2))/a + (B*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2)*328i)/a)*(-(4*d^2*(((3699*A^2*a)/4 - (801*B^2*a)/4)/d^2 + (A*B*a*1719i)/(2*d^2))^2 + 128*a^6*(((115*A*B^3)/32 + (989*A^3*B)/128)*1i)/(a^4*d^4) - ((1849*A^4)/256 + (1191*A^2*B^2)/256 + (25*B^4)/16)/(a^4*d^4))^(1/2) - 3699*A^2*a + 801*B^2*a - A*B*a*3438i)/(256*a^6*d^2))^(1/2) - ((A*a^2 + B*a^2*1i)/(5*d) + ((337*A + B*167i)*(a + a*tan(c + d*x)*1i)^2)/(60*d) + ((23*A*a + B*a*13i)*(a + a*tan(c + d*x)*1i))/(30*d) + (21*(2*A + B*1i)*(a + a*tan(c + d*x)*1i)^4)/(4*a^2*d) - ((211*A + B*104i)*(a + a*tan(c + d*x)*1i)^3)/(12*a*d))/((a + a*tan(c + d*x)*1i)^(9/2) - 2*a*(a + a*tan(c + d*x)*1i)^(7/2) + a^2*(a + a*tan(c + d*x)*1i)^(5/2)) + 2*ata
\end{aligned}$$



$$3.112 \quad \int \tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=130

$$\frac{2\sqrt[4]{-1} a(iA + B)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{2a(iA + B)\sqrt{\tan(c + dx)}}{d} + \frac{2a(A - iB)\tan^{\frac{3}{2}}(c + dx)}{3d}$$

[Out]  $-2*(-1)^{(1/4)}*a*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-2*a*(I*A+B)*\tan(d*x+c)^{(1/2)}/d+2/3*a*(A-I*B)*\tan(d*x+c)^{(3/2)}/d+2/5*a*(I*A+B)*\tan(d*x+c)^{(5/2)}/d+2/7*I*a*B*\tan(d*x+c)^{(7/2)}/d$

**Rubi [A]**

time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3673, 3609, 3614, 211}

$$\frac{2\sqrt[4]{-1} a(B + iA)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} + \frac{2a(B + iA)\tan^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a(A - iB)\tan^{\frac{3}{2}}(c + dx)}{3d} - \frac{2a(B + iA)\sqrt{\tan(c + dx)}}{d} + \frac{2iaB \tan^{\frac{7}{2}}(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(-2*(-1)^{(1/4)}*a*(I*A + B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d - (2*a*(I*A + B)*\text{Sqrt}[\text{Tan}[c + d*x]])/d + (2*a*(A - I*B)*\text{Tan}[c + d*x]^{(3/2)})/(3*d) + (2*a*(I*A + B)*\text{Tan}[c + d*x]^{(5/2)})/(5*d) + (((2*I)/7)*a*B*\text{Tan}[c + d*x]^{(7/2)})/d$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3614

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*c

Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{2iaB \tan^{\frac{7}{2}}(c + dx)}{7d} + \int \tan^{\frac{5}{2}}(c + dx)(a(A - iB) \\
 &= \frac{2a(iA + B) \tan^{\frac{5}{2}}(c + dx)}{5d} + \frac{2iaB \tan^{\frac{7}{2}}(c + dx)}{7d} \\
 &= \frac{2a(A - iB) \tan^{\frac{3}{2}}(c + dx)}{3d} + \frac{2a(iA + B) \tan^{\frac{5}{2}}(c + dx)}{5d} \\
 &= -\frac{2a(iA + B) \sqrt{\tan(c + dx)}}{d} + \frac{2a(A - iB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2a(iA + B) \sqrt{\tan(c + dx)}}{d} + \frac{2a(A - iB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2^4 \sqrt{-1} a(iA + B) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 280 vs. 2(130) = 260.

time = 4.06, size = 280, normalized size = 2.15

$$\frac{\cos^2(c + dx)(\cos(dx) - i \sin(dx))(a + ia \tan(c + dx))(A + B \tan(c + dx)) \left( \frac{2i(A + B) \sqrt{-1} \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \right) - \frac{1}{10} \cos(c + dx)(i + \tan(c)) \sqrt{\tan(c + dx)} (8i(A - iB) + 5(7iA + 4B) \tan(c + dx) + \cos(2(c + dx))(12i(A - iB) + 5(7iA + 10B) \tan(c + dx))) \right)}{d(A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]), x]

[Out] (Cos[c + d\*x]^2\*(Cos[d\*x] - I\*Sin[d\*x])\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x])\*((2\*(I\*A + B)\*Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]/(E^(I\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]



) - (Cos[c]\*Sec[c + d\*x]^2\*(I + Tan[c])\*Sqrt[Tan[c + d\*x]]\*(84\*(A - I\*B) + 5\*((7\*I)\*A + 4\*B)\*Tan[c + d\*x] + Cos[2\*(c + d\*x)]\*(126\*(A - I\*B) + 5\*((7\*I)\*A + 10\*B)\*Tan[c + d\*x]))/(105))/(d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(105) = 210.

time = 0.05, size = 272, normalized size = 2.09

method	result
derivativedivides	$a \left( \frac{2iB \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{2iA \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{2A \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2iA \left( \sqrt{\tan(dx+c)} \right) \right)$
default	$a \left( \frac{2iB \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{2iA \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{2A \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2iA \left( \sqrt{\tan(dx+c)} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*a\*(2/7\*I\*B\*tan(d\*x+c)^(7/2)+2/5\*I\*A\*tan(d\*x+c)^(5/2)+2/5\*B\*tan(d\*x+c)^(5/2)-2/3\*I\*B\*tan(d\*x+c)^(3/2)+2/3\*A\*tan(d\*x+c)^(3/2)-2\*I\*A\*tan(d\*x+c)^(1/2)-2\*B\*tan(d\*x+c)^(1/2)+1/4\*(I\*A+B)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))+1/4\*(-A+I\*B)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))))

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(100) = 200.

time = 0.61, size = 202, normalized size = 1.55

-120 B tan(dx+c)^3 + 160 (-A - B) tan(dx+c)^2 - 200 (A - B) tan(dx+c) + 400 (A + B) sqrt(tan(dx+c)) - 160 (2 sqrt((1 - (A + B + 1) B) arctan(sqrt(2) sqrt(2 + sqrt(tan(dx+c)))) + 2 sqrt((1 - (A + B + 1) B) arctan(-1/2 sqrt(2) sqrt(2 + sqrt(tan(dx+c)))) - sqrt(-(1 + (A + B - 1) B) log(sqrt(2) sqrt(tan(dx+c)) + tan(dx+c + 1))) + sqrt(-(1 + (A + B - 1) B) log(-sqrt(2) sqrt(tan(dx+c)) + tan(dx+c + 1))))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

```
[Out] -1/420*(-120*I*B*a*tan(d*x + c)^(7/2) + 168*(-I*A - B)*a*tan(d*x + c)^(5/2)
- 280*(A - I*B)*a*tan(d*x + c)^(3/2) + 840*(I*A + B)*a*sqrt(tan(d*x + c))
- 105*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(2)*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 480 vs.  $2(100) = 200$ .

time = 2.19, size = 480, normalized size = 3.69

$$\frac{105(d^2\sqrt{-I^2A^2 - 2AB + IB^2} + 2d\sqrt{-I^2A^2 - 2AB + IB^2})\left(\frac{1}{2}\sqrt{\frac{2A - 2B + 2d}{d}}\arctan\left(\frac{1}{2}\sqrt{\frac{2A - 2B + 2d}{d}}\sqrt{\frac{2A - 2B + 2d}{d}} + 2\sqrt{\frac{2A - 2B + 2d}{d}}\sqrt{\tan(dx + c)}\right) + \sqrt{\frac{2A - 2B + 2d}{d}}\arctan\left(-\frac{1}{2}\sqrt{\frac{2A - 2B + 2d}{d}}\sqrt{\frac{2A - 2B + 2d}{d}} - 2\sqrt{\frac{2A - 2B + 2d}{d}}\sqrt{\tan(dx + c)}\right)\right) - \sqrt{2}\left(-\frac{1}{2}(I + 1)A + \frac{1}{2}(I - 1)B\right)\log\left(\sqrt{2}\sqrt{\tan(dx + c)} + \tan(dx + c) + 1\right) + \sqrt{2}\left(-\frac{1}{2}(I + 1)A + \frac{1}{2}(I - 1)B\right)\log\left(-\sqrt{2}\sqrt{\tan(dx + c)} + \tan(dx + c) + 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(105*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a) - 105*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a) - 4*((161*I*A + 176*B)*a*e^(6*I*d*x + 6*I*c) + (329*I*A + 284*B)*a*e^(4*I*d*x + 4*I*c) + (259*I*A + 304*B)*a*e^(2*I*d*x + 2*I*c) + (91*I*A + 76*B)*a)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int A \tan^{\frac{7}{2}}(c + dx) dx + \int B \tan^{\frac{9}{2}}(c + dx) dx + \int (-iA \tan^{\frac{5}{2}}(c + dx)) dx + \int (-iB \tan^{\frac{7}{2}}(c + dx)) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(A*tan(c + d*x)**(7/2), x) + Integral(B*tan(c + d*x)**(9/2), x) + Integral(-I*A*tan(c + d*x)**(5/2), x) + Integral(-I*B*tan(c + d*x)**(7/2), x))
```

**Giac [A]**

time = 0.79, size = 142, normalized size = 1.09

$$\frac{(i-1)\sqrt{2}(Aa-iBa)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(-15iBad^6\tan(dx+c)^{\frac{7}{2}} - 21iAad^6\tan(dx+c)^{\frac{5}{2}} - 21Bad^6\tan(dx+c)^{\frac{3}{2}} - 35Aad^6\tan(dx+c)^{\frac{1}{2}} + 35iBad^6\tan(dx+c)^{\frac{3}{2}} + 105iAad^6\sqrt{\tan(dx+c)} + 105Bad^6\sqrt{\tan(dx+c)}\right)}{105d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] (I - 1)\*sqrt(2)\*(A\*a - I\*B\*a)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d - 2/105\*(-15\*I\*B\*a\*d^6\*tan(d\*x + c)^(7/2) - 21\*I\*A\*a\*d^6\*tan(d\*x + c)^(5/2) - 21\*B\*a\*d^6\*tan(d\*x + c)^(3/2) - 35\*A\*a\*d^6\*tan(d\*x + c)^(1/2) + 35\*I\*B\*a\*d^6\*tan(d\*x + c)^(3/2) + 105\*I\*A\*a\*d^6\*sqrt(tan(d\*x + c)) + 105\*B\*a\*d^6\*sqrt(tan(d\*x + c)))/d^7

**Mupad [B]**

time = 10.12, size = 161, normalized size = 1.24

$$\frac{2Aa\tan(c+dx)^{3/2}}{3d} - \frac{Aa\sqrt{\tan(c+dx)}2i}{d} + \frac{Aa\tan(c+dx)^{5/2}2i}{5d} - \frac{2Ba\sqrt{\tan(c+dx)}}{d} - \frac{Ba\tan(c+dx)^{3/2}2i}{3d} + \frac{2Ba\tan(c+dx)^{5/2}}{5d} + \frac{Ba\tan(c+dx)^{7/2}2i}{7d} - \frac{(-1)^{1/4}Aa\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{\tan(c+dx)}}{i}\right)2i}{d} + \frac{\sqrt{2}Ba\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(c+dx)}\left(\frac{1}{2}-\frac{1}{2}i\right)\right)(1+i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] (2\*A\*a\*tan(c + d\*x)^(3/2))/(3\*d) - (A\*a\*tan(c + d\*x)^(1/2)\*2i)/d + (A\*a\*tan(c + d\*x)^(5/2)\*2i)/(5\*d) - (2\*B\*a\*tan(c + d\*x)^(1/2))/d - (B\*a\*tan(c + d\*x)^(3/2)\*2i)/(3\*d) + (2\*B\*a\*tan(c + d\*x)^(5/2))/(5\*d) + (B\*a\*tan(c + d\*x)^(7/2)\*2i)/(7\*d) - ((-1)^(1/4)\*A\*a\*atan((-1)^(1/4)\*tan(c + d\*x)^(1/2)\*1i)\*2i)/d + (2^(1/2)\*B\*a\*atan(2^(1/2)\*tan(c + d\*x)^(1/2)\*(1/2 - 1i/2))\*(1 + 1i))/d

### 3.113 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=105

$$\frac{2\sqrt[4]{-1} a(A-iB)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d} + \frac{2a(iA+B)\tan^{\frac{3}{2}}(c+dx)}{3d}$$

[Out]  $2*(-1)^{(1/4)}*a*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+2*a*(A-I*B)*\tan(d*x+c)^{(1/2)}/d+2/3*a*(I*A+B)*\tan(d*x+c)^{(3/2)}/d+2/5*I*a*B*\tan(d*x+c)^{(5/2)}/d$

**Rubi [A]**

time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3673, 3609, 3614, 211}

$$\frac{2\sqrt[4]{-1} a(A-iB)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d} + \frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^{(3/2)}*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(2*(-1)^{(1/4)}*a*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]])/d + (2*a*(A - I*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/d + (2*a*(I*A + B)*\text{Tan}[c + d*x]^{(3/2)})/(3*d) + (((2*I)/5)*a*B*\text{Tan}[c + d*x]^{(5/2)})/d$

Rule 211

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2])/a]*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3609

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)])^{(m_)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)])}, x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3614

$\text{Int}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)])/ \text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /;$  FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned} \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{2iaB \tan^{\frac{5}{2}}(c + dx)}{5d} + \int \tan^{\frac{3}{2}}(c + dx)(a(A - \\ &= \frac{2a(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} + \frac{2iaB \tan^{\frac{5}{2}}(c + dx)}{5d} \\ &= \frac{2a(A - iB) \sqrt{\tan(c + dx)}}{d} + \frac{2a(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} \\ &= \frac{2a(A - iB) \sqrt{\tan(c + dx)}}{d} + \frac{2a(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} \\ &= \frac{2\sqrt[4]{-1} a(A - iB) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 266 vs.  $2(105) = 210$ .

time = 2.55, size = 266, normalized size = 2.53

$$\frac{\cos^2(c + dx)(\cos(dx) - i \sin(dx)) \left( \frac{2(A + B)e^{-ix} \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}\right)}{\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}} \right) + \frac{1}{15} \sec^2(c + dx)(\cos(c) - i \sin(c))(3(5A - 4iB) + 3(5A - 6iB) \cos(2(c + dx)) + 5(iA + B) \sin(2(c + dx))) \sqrt{\tan(c + dx)}}{d(A \cos(c + dx) + B \sin(c + dx))} (a + ia \tan(c + dx))(A + B \tan(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
[Out] (Cos[c + d*x]^2*(Cos[d*x] - I*Sin[d*x])*((2*(I*A + B)*Sqrt[(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x))))*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))])/(E^(I*c)*Sqrt[((-1)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))])) + (Sec[c + d*x]^2*(Cos[c] - I*Sin[c])*(3*(5*A - (4*I)*B) + 3*(5*A - (6*I)*B)*Cos[2*(c + d*x)] + 5*(I*A + B)*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*x]]/15)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs.  $2(85) = 170$ .

time = 0.04, size = 251, normalized size = 2.39

method	result
derivativedivides	$a \left( \frac{2iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{2B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2iB \left( \sqrt{\tan}(dx+c) \right) + 2A \left( \sqrt{\tan}(dx+c) \right) + \dots \right) \quad (iB-A) \sqrt{\tan}(dx+c)$
default	$a \left( \frac{2iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{2B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2iB \left( \sqrt{\tan}(dx+c) \right) + 2A \left( \sqrt{\tan}(dx+c) \right) + \dots \right) \quad (iB-A) \sqrt{\tan}(dx+c)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*a*(2/5*I*B*tan(d*x+c)^{(5/2)}+2/3*I*A*tan(d*x+c)^{(3/2)}+2/3*B*tan(d*x+c)^{(3/2)}-2*I*B*tan(d*x+c)^{(1/2)}+2*A*tan(d*x+c)^{(1/2)}+1/4*(-A+I*B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c))/(1-2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c)))$   
 $+2*arctan(1+2^{(1/2)}*tan(d*x+c)^{(1/2)})+2*arctan(-1+2^{(1/2)}*tan(d*x+c)^{(1/2)}))$   
 $+1/4*(-I*A-B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c))/(1+2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c)))$   
 $+2*arctan(1+2^{(1/2)}*tan(d*x+c)^{(1/2)})+2*arctan(-1+2^{(1/2)}*tan(d*x+c)^{(1/2)}))$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs.  $2(81) = 162$ .  
time = 0.55, size = 186, normalized size = 1.77

$$\frac{-24B \tan(dx+c)^4 + 40(-A-B) \tan(dx+c)^3 - 120(A-B) \sqrt{\tan(dx+c)} - 15(2\sqrt{2}(-i+1)A+(i-1)B) \arctan\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-2\sqrt{\tan(dx+c)}}\right) + 2\sqrt{2}(-i+1)A+(i-1)B \arctan\left(\frac{-1+\sqrt{2}\sqrt{\tan(dx+c)}}{1+2\sqrt{\tan(dx+c)}}\right) + \sqrt{2}((i-1)A+(i+1)B) \log(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1) - \sqrt{2}((i-1)A+(i+1)B) \log(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out]  $-1/60*(-24*I*B*a*tan(d*x+c)^{(5/2)}+40*(-I*A-B)*a*tan(d*x+c)^{(3/2)}-120*(A-I*B)*a*\sqrt{\tan(d*x+c)}-15*(2*\sqrt{2})*(-(I+1)*A+(I-1)*B)*$   
 $*arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(d*x+c)}))$   
 $+2*\sqrt{2}*(-(I+1)*A+(I-1)*B)*arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(d*x+c)}))$   
 $+sqrt(2)*((I-1)*A+(I+1)*B)*log(sqrt(2)*sqrt(tan(d*x+c))+tan(d*x+c)+1)$   
 $-sqrt(2)*((I-1)*A+(I+1)*B)*log(-sqrt(2)*sqrt(tan(d*x+c))+tan(d*x+c)+1))*a/d$



[Out]  $(I - 1)\sqrt{2}*(I*A*a + B*a)*\arctan(-(1/2*I - 1/2)\sqrt{2})\sqrt{\tan(dx + c)}/d - 2/15*(-3*I*B*a*d^4*\tan(dx + c)^{(5/2)} - 5*I*A*a*d^4*\tan(dx + c)^{(3/2)} - 5*B*a*d^4*\tan(dx + c)^{(3/2)} - 15*A*a*d^4*\sqrt{\tan(dx + c)} + 15*I*B*a*d^4*\sqrt{\tan(dx + c)})/d^5$

Mupad [B]

time = 8.40, size = 130, normalized size = 1.24

$$\frac{2Aa\sqrt{\tan(c+dx)}}{d} + \frac{Aa\tan(c+dx)^{3/2}2i}{3d} - \frac{Ba\sqrt{\tan(c+dx)}2i}{d} + \frac{2Ba\tan(c+dx)^{3/2}}{3d} + \frac{Ba\tan(c+dx)^{5/2}2i}{5d} + \frac{\sqrt{2}Aa\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(c+dx)}\left(\frac{1}{2}-\frac{1}{2}i\right)\right)(-1-i)}{d} - \frac{(-1)^{1/4}Ba\operatorname{atan}\left((-1)^{1/4}\sqrt{\tan(c+dx)}i\right)2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)`

[Out]  $(2*A*a*\tan(c + d*x)^{(1/2)})/d + (A*a*\tan(c + d*x)^{(3/2)*2i)/(3*d) - (B*a*\tan(c + d*x)^{(1/2)*2i)/d + (2*B*a*\tan(c + d*x)^{(3/2)})/(3*d) + (B*a*\tan(c + d*x)^{(5/2)*2i)/(5*d) - (2^{(1/2)}*A*a*\operatorname{atan}(2^{(1/2)}*\tan(c + d*x)^{(1/2)}*(1/2 - 1i/2))*(1 + 1i))/d - ((-1)^{(1/4)}*B*a*\operatorname{atan}((-1)^{(1/4)}*\tan(c + d*x)^{(1/2)*1i}*2i)/d$



$$3.114 \quad \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=80

$$\frac{2\sqrt[4]{-1} a(iA + B) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} + \frac{2a(iA + B) \sqrt{\tan(c + dx)}}{d} + \frac{2iaB \tan^{3/2}(c + dx)}{3d}$$

[Out]  $2*(-1)^{(1/4)}*a*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+2*a*(I*A+B)*\tan(d*x+c)^{(1/2)}/d+2/3*I*a*B*\tan(d*x+c)^{(3/2)}/d$

**Rubi [A]**

time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3673, 3609, 3614, 211}

$$\frac{2\sqrt[4]{-1} a(B + iA) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} + \frac{2a(B + iA) \sqrt{\tan(c + dx)}}{d} + \frac{2iaB \tan^{3/2}(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(2*(-1)^{(1/4)}*a*(I*A + B)*\operatorname{ArcTan}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/d + (2*a*(I*A + B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d + (((2*I)/3)*a*B*\operatorname{Tan}[c + d*x]^{(3/2)})/d$

Rule 211

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{PosQ}[a/b]$

Rule 3609

$\operatorname{Int}[(a_ + (b_)*\operatorname{tan}[(e_ + (f_)*(x_)]))^m*((c_ + (d_)*\operatorname{tan}[(e_ + (f_)*(x_)])), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{m-1}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\operatorname{Tan}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{GtQ}[m, 0]$

Rule 3614

$\operatorname{Int}[(c_ + (d_)*\operatorname{tan}[(e_ + (f_)*(x_)]))/\operatorname{Sqrt}[(b_)*\operatorname{tan}[(e_ + (f_)*(x_)])], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))(A+B \tan(c+dx)) dx &= \frac{2iaB \tan^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\tan(c+dx)} (a(A \\
&= \frac{2a(iA+B)\sqrt{\tan(c+dx)}}{d} + \frac{2iaB \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= \frac{2a(iA+B)\sqrt{\tan(c+dx)}}{d} + \frac{2iaB \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= \frac{2\sqrt[4]{-1} a(iA+B) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 1.65, size = 112, normalized size = 1.40

$$\frac{2a\sqrt{\tan(c+dx)} \left( (-3iA-3B) \tanh^{-1} \left( \sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \right) + \sqrt{i \tan(c+dx)} (3iA+3B+iB \tan(c+dx)) \right)}{3d\sqrt{i \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Tan[c + d*x]]*(((3*I)*A - 3*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c +
d*x)))]/(1 + E^((2*I)*(c + d*x)))]]) + Sqrt[I*Tan[c + d*x]]*(((3*I)*A + 3*B +
I*B*Tan[c + d*x]))/(3*d*Sqrt[I*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(65) = 130.

time = 0.04, size = 226, normalized size = 2.82

method	result
--------	--------

derivativedivides	$a \left( \frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2iA \left( \sqrt{\tan(dx+c)} \right) + 2B \left( \sqrt{\tan(dx+c)} \right) + \frac{(-iA-B) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)} \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)} \right) \right)$
default	$a \left( \frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2iA \left( \sqrt{\tan(dx+c)} \right) + 2B \left( \sqrt{\tan(dx+c)} \right) + \frac{(-iA-B) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)} \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*a*(2/3*I*B*\tan(d*x+c)^{(3/2)}+2*I*A*\tan(d*x+c)^{(1/2)}+2*B*\tan(d*x+c)^{(1/2)}+1/4*(-I*A-B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(A-I*B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs.  $2(62) = 124$ .

time = 0.60, size = 170, normalized size = 2.12

$$\frac{-8iBd \tan(dx+c)^2 + 24(-iA-B)d \sqrt{\tan(dx+c)} + 3(2\sqrt{2}(i-1)A+(i+1)B) \arctan\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right) + 2\sqrt{2}(i-1)A+(i+1)B \arctan\left(\frac{-1+\sqrt{2}\sqrt{\tan(dx+c)}}{1+\sqrt{2}\sqrt{\tan(dx+c)}}\right) - \sqrt{2}(i-1)A+(i+1)B \log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1}{\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c)+1}\right) + \sqrt{2}(i-1)A+(i+1)B \log\left(\frac{-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c)+1}\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out]  $-1/12*(-8*I*B*a*\tan(d*x+c)^{(3/2)}+24*(-I*A-B)*a*\sqrt{\tan(d*x+c)}+3*(2*\sqrt{2}*((I-1)*A+(I+1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(d*x+c)}))+2*\sqrt{2}*((I-1)*A+(I+1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(d*x+c)}))- \sqrt{2}*(-(I+1)*A+(I-1)*B)*\log(\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1)+\sqrt{2}*(-(I+1)*A+(I-1)*B)*\log(-\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1))*a/d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs.  $2(62) = 124$ .

time = 1.80, size = 372, normalized size = 4.65

$$\frac{3(d\sqrt{\tan(dx+c)}+d)\sqrt{\frac{(-1A^2-2AB+1B^2)d}{d^2}} \log\left(\frac{1+(A+B)\sqrt{\tan(dx+c)}+1}{1+\sqrt{\tan(dx+c)}}\sqrt{\frac{(-1A^2-2AB+1B^2)d}{d^2}}\sqrt{\frac{-1+2\sqrt{\tan(dx+c)}+1}{2\sqrt{\tan(dx+c)}+1}}\right) - 3(d\sqrt{\tan(dx+c)}+d)\sqrt{\frac{(-1A^2-2AB+1B^2)d}{d^2}} \log\left(\frac{1+(A-B)\sqrt{\tan(dx+c)}+1}{1+\sqrt{\tan(dx+c)}}\sqrt{\frac{(-1A^2-2AB+1B^2)d}{d^2}}\sqrt{\frac{-1+2\sqrt{\tan(dx+c)}+1}{2\sqrt{\tan(dx+c)}+1}}\right) + 4(-3A-4B)\sqrt{\tan(dx+c)}+(-3A-2B)\sqrt{\frac{-1+2\sqrt{\tan(dx+c)}+1}{2\sqrt{\tan(dx+c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/6*(3*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2} * \log(2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} + (d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2} * \sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})) * e^{(-2*I*d*x - 2*I*c)/((I*A + B)*a)} - 3*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2} * \log(2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} - (d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2} * \sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})) * e^{(-2*I*d*x - 2*I*c)/((I*A + B)*a)} + 4*((-3*I*A - 4*B)*a*e^{(2*I*d*x + 2*I*c)} + (-3*I*A - 2*B)*a)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int A \tan^{\frac{3}{2}}(c + dx) dx + \int B \tan^{\frac{5}{2}}(c + dx) dx + \int \left( -iA \sqrt{\tan(c + dx)} \right) dx + \int \left( -iB \tan^{\frac{3}{2}}(c + dx) \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out]  $I*a*(\text{Integral}(A*\tan(c + d*x)**(3/2), x) + \text{Integral}(B*\tan(c + d*x)**(5/2), x) + \text{Integral}(-I*A*\sqrt{\tan(c + d*x)}, x) + \text{Integral}(-I*B*\tan(c + d*x)**(3/2), x))$

**Giac [A]**

time = 0.60, size = 82, normalized size = 1.02

$$\frac{(i-1)\sqrt{2}(Aa-iBa)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(-iBad^2\tan(dx+c)^{\frac{3}{2}}-3iAad^2\sqrt{\tan(dx+c)}-3Bad^2\sqrt{\tan(dx+c)}\right)}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-(I-1)*\sqrt{2}*(A*a-I*B*a)*\arctan\left(-\left(\frac{1}{2}I-\frac{1}{2}\right)*\sqrt{2}*\sqrt{\tan(d*x+c)}\right)/d - 2/3*(-I*B*a*d^2*\tan(d*x+c)^{(3/2)} - 3*I*A*a*d^2*\sqrt{\tan(d*x+c)} - 3*B*a*d^2*\sqrt{\tan(d*x+c)})/d^3$

**Mupad [B]**

time = 7.71, size = 99, normalized size = 1.24

$$\frac{Aa\sqrt{\tan(c+dx)}\operatorname{atanh}\left(\frac{1}{2}\sqrt{2}\sqrt{\tan(c+dx)}\right)}{d} + \frac{2Ba\sqrt{\tan(c+dx)}}{d} + \frac{Ba\tan(c+dx)^{3/2}\operatorname{atanh}\left(\frac{1}{2}\sqrt{2}\sqrt{\tan(c+dx)}\right)}{3d} - \frac{2(-1)^{1/4}Aa\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{\tan(c+dx)}}{d}\right)}{d} + \frac{\sqrt{2}Ba\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(c+dx)}\left(\frac{1}{2}-\frac{1}{2}i\right)\right)(-1-i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)`

[Out]  $(A*a*\tan(c + d*x)^{(1/2)*2i}/d + (2*B*a*\tan(c + d*x)^{(1/2)})/d + (B*a*\tan(c + d*x)^{(3/2)*2i}/(3*d) - (2*(-1)^{(1/4)}*A*a*\operatorname{atanh}((-1)^{(1/4)}*\tan(c + d*x)^{(1/2)}))/d - (2^{(1/2)}*B*a*\operatorname{atan}(2^{(1/2)}*\tan(c + d*x)^{(1/2)}*(1/2 - 1i/2))*(1 + 1i))/d$

$$3.115 \quad \int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

Optimal. Leaf size=55

$$-\frac{2\sqrt[4]{-1} a(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{2iaB\sqrt{\tan(c+dx)}}{d}$$

[Out]  $-2*(-1)^{(1/4)}*a*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+2*I*a*B*\tan(d*x+c)^{(1/2)}/d$

Rubi [A]

time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {3673, 3614, 211}

$$\frac{2iaB\sqrt{\tan(c+dx)}}{d} - \frac{2\sqrt[4]{-1} a(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]],x]

[Out]  $(-2*(-1)^{(1/4)}*a*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d + ((2*I)*a*B*\text{Sqrt}[\text{Tan}[c + d*x]])/d$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3614

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{2iaB \sqrt{\tan(c + dx)}}{d} + \int \frac{a(A - iB) + a(iA + B) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx \\ &= \frac{2iaB \sqrt{\tan(c + dx)}}{d} + \frac{(2a^2(A - iB)^2) \text{Subst}\left(\int \frac{1}{\sqrt{a(A - iB) - \tan(x)}} dx\right)}{d} \\ &= -\frac{2\sqrt[4]{-1} a(A - iB) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{1}\right)}{d} + \frac{2iaB \sqrt{\tan(c + dx)}}{d} \end{aligned}$$

**Mathematica [A]**

time = 1.38, size = 92, normalized size = 1.67

$$\frac{2a \left( (A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) + iB \sqrt{i \tan(c + dx)} \right) \sqrt{\tan(c + dx)}}{d \sqrt{i \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
```

```
[Out] (2*a*((A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))] + I*B*Sqrt[I*Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(d*Sqrt[I*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs.  $2(45) = 90$ .

time = 0.04, size = 201, normalized size = 3.65

method	result
derivativedivides	$a \left( 2iB \left( \sqrt{\tan(dx+c)} \right) + \frac{(-iB+A) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right)$
default	$a \left( 2iB \left( \sqrt{\tan(dx+c)} \right) + \frac{(-iB+A) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*a*(2*I*B*tan(d*x+c)^{(1/2)}+1/4*(A-I*B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c))/(1-2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c))))+2*arctan(1+2^{(1/2)}*tan(d*x+c)^{(1/2)})+2*arctan(-1+2^{(1/2)}*tan(d*x+c)^{(1/2)}))+1/4*(I*A+B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c))/(1+2^{(1/2)}*tan(d*x+c)^{(1/2)}+tan(d*x+c))))+2*arctan(1+2^{(1/2)}*tan(d*x+c)^{(1/2)})+2*arctan(-1+2^{(1/2)}*tan(d*x+c)^{(1/2)}))$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs.  $2(43) = 86$ .  
time = 0.51, size = 151, normalized size = 2.75

$$-Si\ Ba\sqrt{\tan(dx+c)} + (2\sqrt{2}(-(i+1)A+(i-1)B)\arctan(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})) + 2\sqrt{2}(-(i+1)A+(i-1)B)\arctan(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})) + \sqrt{2}((i-1)A+(i+1)B)\log(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1) - \sqrt{2}((i-1)A+(i+1)B)\log(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1)))/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm  
="maxima")`

[Out]  $-1/4*(-8*I*B*a*\sqrt{\tan(dx+c)} + (2*\sqrt{2}*(-(I+1)*A+(I-1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)})) + 2*\sqrt{2}*(-(I+1)*A+(I-1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)})) + \sqrt{2}*((I-1)*A+(I+1)*B)*\log(\sqrt{2}*\sqrt{\tan(dx+c)} + \tan(dx+c)+1) - \sqrt{2}*((I-1)*A+(I+1)*B)*\log(-\sqrt{2}*\sqrt{\tan(dx+c)} + \tan(dx+c)+1)))/d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs.  $2(43) = 86$ .  
time = 2.04, size = 314, normalized size = 5.71

$$-d\ i\ B\ a\sqrt{\frac{-1-e^{(2i dx+2c)}+1}{e^{(2i dx+2c)}+1}} - \sqrt{\frac{(A^2+2AB-iB^2)a^2}{d^2}} d \log\left(\frac{2\left(\frac{(A-iB)\sqrt{2i dx+2c}+(-i)d(2i dx+2c)+d}{(A+iB)a}\sqrt{\frac{(A^2+2AB-iB^2)a^2}{d^2}}\sqrt{\frac{-1-e^{(2i dx+2c)}+1}{e^{(2i dx+2c)}+1}}\right)^{(1-2i dx-2c)}}{(A+iB)a}\right) + \sqrt{\frac{(A^2+2AB-iB^2)a^2}{d^2}} d \log\left(\frac{2\left(\frac{(A-iB)\sqrt{2i dx+2c}+(-i)d(2i dx+2c)+d}{(A+iB)a}\sqrt{\frac{(A^2+2AB-iB^2)a^2}{d^2}}\sqrt{\frac{-1-e^{(2i dx+2c)}+1}{e^{(2i dx+2c)}+1}}\right)^{(1-2i dx-2c)}}{(A+iB)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm  
="fricas")`

[Out]  $-1/2*(-4*I*B*a*\sqrt{(-I*e^{(2I*d*x+2I*c)}+I)/(e^{(2I*d*x+2I*c)}+1)} - \sqrt{-(I*A^2+2*A*B-I*B^2)*a^2/d^2}*d*\log(2*((A-I*B)*a*e^{(2I*d*x+2I*c)}+(I*d*e^{(2I*d*x+2I*c)}+I*d)*\sqrt{-(I*A^2+2*A*B-I*B^2)*a^2/d^2}*\sqrt{(-I*e^{(2I*d*x+2I*c)}+I)/(e^{(2I*d*x+2I*c)}+1)})))*e^{(-2I$



$*dx - 2*I*c)/((I*A + B)*a)) + \sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2}*d*\log$   
 $(2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{$   
 $(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*$   
 $I*d*x + 2*I*c)} + 1)))*e^{(-2*I*d*x - 2*I*c)}/((I*A + B)*a)))/d$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int A \sqrt{\tan(c+dx)} dx + \int B \tan^{\frac{3}{2}}(c+dx) dx + \int \left( -\frac{iA}{\sqrt{\tan(c+dx)}} \right) dx + \int \left( -iB \sqrt{\tan(c+dx)} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2), x)

[Out] I\*a\*(Integral(A\*sqrt(tan(c + d\*x)), x) + Integral(B\*tan(c + d\*x)\*\*(3/2), x)  
 + Integral(-I\*A/sqrt(tan(c + d\*x)), x) + Integral(-I\*B\*sqrt(tan(c + d\*x)),  
 x))

**Giac [A]**

time = 0.61, size = 47, normalized size = 0.85

$$\frac{(i-1)\sqrt{2}(-iAa - Ba)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} + \frac{2iBa\sqrt{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2), x, algorithm="giac")

[Out] (I - 1)\*sqrt(2)\*(-I\*A\*a - B\*a)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x +  
 c)))/d + 2\*I\*B\*a\*sqrt(tan(d\*x + c))/d

**Mupad [B]**

time = 7.08, size = 68, normalized size = 1.24

$$-\frac{2(-1)^{1/4}Ba\operatorname{atanh}\left((-1)^{1/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{Ba\sqrt{\tan(c+dx)}2i}{d} + \frac{\sqrt{2}Aa\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(c+dx)}\left(\frac{1}{2}-\frac{1}{2}i\right)\right)(1+i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i))/tan(c + d\*x)^(1/2), x)

[Out] (B\*a\*tan(c + d\*x)^(1/2)\*2i)/d + (2^(1/2)\*A\*a\*atan(2^(1/2)\*tan(c + d\*x)^(1/2)  
 )\*(1/2 - 1i/2))\*(1 + 1i)/d - (2\*(-1)^(1/4)\*B\*a\*atanh((-1)^(1/4)\*tan(c + d\*  
 x)^(1/2))/d

$$3.116 \quad \int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=53

$$-\frac{2\sqrt[4]{-1} a(iA + B)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{2aA}{d\sqrt{\tan(c + dx)}}$$

[Out]  $-2*(-1)^{(1/4)}*a*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-2*a*A/d/\tan(d*x+c)^{(1/2)}$

Rubi [A]

time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {3672, 3614, 211}

$$-\frac{2aA}{d\sqrt{\tan(c + dx)}} - \frac{2\sqrt[4]{-1} a(B + iA)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x])/(\text{Tan}[c + d*x]^{(3/2)}), x]$

[Out]  $(-2*(-1)^{(1/4)}*a*(I*A + B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]])/d - (2*a*A)/(d*\text{Sqrt}[\text{Tan}[c + d*x]])$

Rule 211

$\text{Int}[(a_) + (b_)*(x_)^2]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 3614

$\text{Int}[(c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)])/(\text{Sqrt}[(b_)*\text{tan}[(e_) + (f_)*(x_)])], x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

Rule 3672

$\text{Int}[(a_) + (b_)*\text{tan}[(e_) + (f_)*(x_)]]^{(m_)}*((A_) + (B_)*\text{tan}[(e_) + (f_)*(x_)])*((c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2aA}{d\sqrt{\tan(c + dx)}} + \int \frac{a(iA + B) - a(A - iB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} \\ &= -\frac{2aA}{d\sqrt{\tan(c + dx)}} + \frac{(2a^2(iA + B)^2) \text{Subst}\left(\int \frac{1}{a(iA+B)+a(\cdot)}\right)}{d} \\ &= -\frac{2\sqrt[4]{-1} a(iA + B) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{2aA}{d\sqrt{\tan(c + dx)}} \end{aligned}$$

**Mathematica [A]**

time = 2.03, size = 76, normalized size = 1.43

$$\frac{2a \left( -A + (A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) \sqrt{i \tan(c + dx)} \right)}{d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2), x]
```

```
[Out] (2*a*(-A + (A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))]])*Sqrt[I*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(44) = 88.

time = 0.04, size = 202, normalized size = 3.81

method	result
derivativedivides	$a \left( -\frac{2A}{\sqrt{\tan(dx + c)}} + \frac{(iA+B)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right) \right)}{4} \right)$

default	$a \left( -\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(iA+B)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right) \right)}{4} \right)$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $\frac{1}{d} a \left( -2 \frac{A}{\tan(dx+c)^{1/2}} + \frac{1}{4} (iA+B) 2^{1/2} \left( \ln \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{2 \tan(dx+c)^{1/2}} \right) + 2 \arctan \left( \frac{-1+2^{1/2} \tan(dx+c)^{1/2}}{2 \tan(dx+c)^{1/2}} \right) \right) + \frac{1}{4} (-A+iB) 2^{1/2} \left( \ln \left( \frac{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{2 \tan(dx+c)^{1/2}} \right) + 2 \arctan \left( \frac{-1+2^{1/2} \tan(dx+c)^{1/2}}{2 \tan(dx+c)^{1/2}} \right) \right) \right)$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs.  $2(43) = 86$ .  
time = 0.51, size = 151, normalized size = 2.85

$$\frac{(2\sqrt{2}((i-1)A+(i+1)B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{\tan(dx+c)}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((i-1)A+(i+1)B)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(-(i+1)A+(i-1)B)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(-(i+1)A+(i-1)B)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right))a-\frac{4A}{\sqrt{\tan(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm  
="maxima")`

[Out]  $\frac{1}{4} \left( (2\sqrt{2}((i-1)A+(i+1)B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{\tan(dx+c)}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((i-1)A+(i+1)B)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}(-(i+1)A+(i-1)B)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}(-(i+1)A+(i-1)B)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right))a-8Aa/\sqrt{\tan(dx+c)} \right) / d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs.  $2(43) = 86$ .  
time = 1.64, size = 367, normalized size = 6.92

$$\frac{(d^{(2i+1)(A+B)} - d) \sqrt{\frac{-1-A^2-2AB+1B^2}{d^2}} \log\left(\frac{2 \left( (A+iB)\sqrt{d^{2i+1}+1} (d^{2i+1}+d) \sqrt{\frac{-1-A^2-2AB+1B^2}{d^2}} \sqrt{\frac{-1+d^{2i+1}+1}{d^{2i+1}+1}} \right) e^{-2i(A+iB)x}}{(A+iB)^2}\right) - (d^{2i+1} - d) \sqrt{\frac{-1-A^2-2AB+1B^2}{d^2}} \log\left(\frac{2 \left( (A-iB)\sqrt{d^{2i+1}+1} (d^{2i+1}+d) \sqrt{\frac{-1-A^2-2AB+1B^2}{d^2}} \sqrt{\frac{-1+d^{2i+1}+1}{d^{2i+1}+1}} \right) e^{2i(A-iB)x}}{(A-iB)^2}\right) - 2(iA)d^{2i+1} + iAa}{2(d^{2i+1} - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm  
="fricas")`

```
[Out] 1/2*((d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a) - 4*(I*A*a*e^(2*I*d*x + 2*I*c) + I*A*a)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int \frac{A}{\sqrt{\tan(c+dx)}} dx + \int B \sqrt{\tan(c+dx)} dx + \int \left( -\frac{iA}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \left( -\frac{iB}{\sqrt{\tan(c+dx)}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

```
[Out] I*a*(Integral(A/sqrt(tan(c + d*x)), x) + Integral(B*sqrt(tan(c + d*x)), x) + Integral(-I*A/tan(c + d*x)**(3/2), x) + Integral(-I*B/sqrt(tan(c + d*x)), x))
```

**Giac [A]**

time = 0.71, size = 47, normalized size = 0.89

$$-\frac{(i+1)\sqrt{2}(-iAa - Ba)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2Aa}{d\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(I + 1)*sqrt(2)*(-I*A*a - B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c)))/d - 2*A*a/(d*sqrt(tan(d*x + c)))
```

**Mupad [B]**

time = 6.97, size = 67, normalized size = 1.26

$$\frac{2(-1)^{1/4}Aa\operatorname{atanh}\left((-1)^{1/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2Aa}{d\sqrt{\tan(c+dx)}} + \frac{\sqrt{2}Ba\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(c+dx)}\left(\frac{1}{2} - \frac{1}{2}i\right)\right)(1+li)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/tan(c + d*x)^(3/2),x)
```

```
[Out] (2*(-1)^(1/4)*A*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - (2*A*a)/(d*tan(c + d*x)^(1/2)) + (2^(1/2)*B*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/2))*(1 + 1i))/d
```

$$3.117 \quad \int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=78

$$\frac{2\sqrt[4]{-1} a(A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2aA}{3d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a(iA + B)}{d \sqrt{\tan(c+dx)}}$$

[Out]  $2*(-1)^{(1/4)}*a*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-2*a*(I*A+B)/d/\tan(d*x+c)^{(1/2)}-2/3*a*A/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3672, 3610, 3614, 211}

$$\frac{2\sqrt[4]{-1} a(A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2a(B + iA)}{d \sqrt{\tan(c+dx)}} - \frac{2aA}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{5/2}}, x]$

[Out]  $(2*(-1)^{(1/4)}*a*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]])/d - (2*a*A)/(3*d*\text{Tan}[c + d*x]^{3/2}) - (2*a*(I*A + B))/(d*\text{Sqrt}[\text{Tan}[c + d*x]])$

Rule 211

$\text{Int}[\frac{(a_) + (b_)*(x_)^2}{(x_)^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[a/b, 2]}{a}*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rule 3610

$\text{Int}[\frac{(a_ + (b_)*\text{tan}[e_ + (f_)*(x_)])^{(m_)}*((c_ + (d_)*\text{tan}[e_ + (f_)*(x_)]) + (f_)*(x_))}{(f_)*(x_)}, x\_Symbol] \rightarrow \text{Simp}[\frac{(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})}{(f*(m + 1)*(a^2 + b^2))}, x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3614

$\text{Int}[\frac{(c_ + (d_)*\text{tan}[e_ + (f_)*(x_)])}{\text{Sqrt}[(b_)*\text{tan}[e_ + (f_)*(x_) + (f_)*(x_)]}], x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

Rule 3672

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} + \int \frac{a(iA + B) - a(A - iB) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2a(iA + B)}{d \sqrt{\tan(c + dx)}} + \int \frac{-a(A - iB)}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2a(iA + B)}{d \sqrt{\tan(c + dx)}} + \frac{(2a^2(A - iB)^2)}{3d \sqrt{\tan(c + dx)}} \\
&= \frac{2\sqrt[4]{-1} a(A - iB) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

**Mathematica** [A]

time = 1.43, size = 94, normalized size = 1.21

$$\frac{2a \left( 3iA + 3B + A \cot(c + dx) - 3i(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) \sqrt{i \tan(c + dx)} \right)}{3d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),
x]

```

```

[Out] (-2*a*((3*I)*A + 3*B + A*Cot[c + d*x] - (3*I)*(A - I*B)*ArcTanh[Sqrt[(-1 +
E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))]]*Sqrt[I*Tan[c + d*x]])/(3*
d*Sqrt[Tan[c + d*x]])

```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(64) = 128.

time = 0.04, size = 220, normalized size = 2.82





time = 1.47, size = 427, normalized size = 5.47

$$\frac{3(d^{2I+1}e^{2Ic} - 2d^{2I}e^{2Ic} + d^2)\sqrt{\frac{(A^2+2AB-1B^2)a^2}{d^2}} \log\left(\frac{2\left(\frac{(A-I*B)a^2e^{2Ic}}{d^2} + \sqrt{\frac{(A^2+2AB-1B^2)a^2}{d^2}}\right)}{2A^2}\right) - 3(d^{2I+1}e^{2Ic} - 2d^{2I}e^{2Ic} + d^2)\sqrt{\frac{(A^2+2AB-1B^2)a^2}{d^2}} \log\left(\frac{2\left(\frac{(A+I*B)a^2e^{2Ic}}{d^2} + \sqrt{\frac{(A^2+2AB-1B^2)a^2}{d^2}}\right)}{2A^2}\right)}{d(d^{2I+1}e^{2Ic} - 2d^{2I}e^{2Ic} + d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/6*(3*(d*e^{4*I*d*x + 4*I*c} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2}*\log(2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} + (I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((I*A + B)*a)) - 3*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2}*\log(2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((I*A + B)*a)) - 4*((4*A - 3*I*B)*a*e^{(4*I*d*x + 4*I*c)} + 2*A*a*e^{(2*I*d*x + 2*I*c)} - (2*A - 3*I*B)*a)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}}/(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d) \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int \frac{A}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \frac{B}{\sqrt{\tan(c+dx)}} dx + \int \left( -\frac{iA}{\tan^{\frac{5}{2}}(c+dx)} \right) dx + \int \left( -\frac{iB}{\tan^{\frac{3}{2}}(c+dx)} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2),x)

[Out] 
$$I*a*(\text{Integral}(A/\tan(c + d*x)**(3/2), x) + \text{Integral}(B/\sqrt{\tan(c + d*x)}), x) + \text{Integral}(-I*A/\tan(c + d*x)**(5/2), x) + \text{Integral}(-I*B/\tan(c + d*x)**(3/2), x)$$

**Giac** [A]

time = 0.74, size = 69, normalized size = 0.88

$$\frac{(i-1)\sqrt{2}(-iAa - Ba)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2(3iAa\tan(dx+c) + 3Ba\tan(dx+c) + Aa)}{3d\tan(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] 
$$-(I - 1)*\sqrt{2}*(-I*A*a - B*a)*\arctan(-\left(\frac{1}{2}I - \frac{1}{2}\right)*\sqrt{2}*\sqrt{\tan(d*x + c)})/d - 2/3*(3*I*A*a*\tan(d*x + c) + 3*B*a*\tan(d*x + c) + A*a)/(d*\tan(d*x + c)^{(3/2)})$$

**Mupad [B]**

time = 7.53, size = 99, normalized size = 1.27

$$\frac{2(-1)^{1/4} B a \operatorname{atanh}\left((-1)^{1/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2 B a}{d \sqrt{\tan(c+dx)}} - \frac{\frac{2 A a}{3 d} + \frac{A a \tan(c+dx) 2i}{d}}{\tan(c+dx)^{3/2}} + \frac{\sqrt{2} A a \operatorname{atan}\left(\sqrt{2} \sqrt{\tan(c+dx)} \left(\frac{1}{2} - \frac{1}{2}i\right)\right) (-1-i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/tan(c + d*x)^(5/2),x)
```

```
[Out] (2*(-1)^(1/4)*B*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - (2*B*a)/(d*tan(c + d*x)^(1/2)) - (2^(1/2)*A*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/2))*(1 + 1i))/d - ((2*A*a)/(3*d) + (A*a*tan(c + d*x)*2i)/d)/tan(c + d*x)^(3/2)
```

$$3.118 \quad \int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt[4]{-1} a(iA + B) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2aA}{5d \tan^{\frac{5}{2}}(c+dx)} - \frac{2a(iA + B)}{3d \tan^{\frac{3}{2}}(c+dx)} + \frac{2a(A - iB)}{d \sqrt{\tan(c+dx)}}$$

[Out] 2\*(-1)^(1/4)\*a\*(I\*A+B)\*arctan((-1)^(3/4)\*tan(d\*x+c)^(1/2))/d+2\*a\*(A-I\*B)/d/  
tan(d\*x+c)^(1/2)-2/5\*a\*A/d/tan(d\*x+c)^(5/2)-2/3\*a\*(I\*A+B)/d/tan(d\*x+c)^(3/2)  
)

**Rubi [A]**

time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3672, 3610, 3614, 211}

$$\frac{2\sqrt[4]{-1} a(B + iA) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2a(B + iA)}{3d \tan^{\frac{3}{2}}(c+dx)} + \frac{2a(A - iB)}{d \sqrt{\tan(c+dx)}} - \frac{2aA}{5d \tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2),x]

[Out] (2\*(-1)^(1/4)\*a\*(I\*A + B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]/d - (2\*a\*A)/(5\*d\*Tan[c + d\*x]^(5/2)) - (2\*a\*(I\*A + B))/(3\*d\*Tan[c + d\*x]^(3/2)) + (2\*a\*(A - I\*B))/(d\*Sqrt[Tan[c + d\*x]])

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3610**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3614**

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

## Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

## Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} + \int \frac{a(iA + B) - a(A - iB) \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)} dx \\ &= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(iA + B)}{3d \tan^{\frac{3}{2}}(c + dx)} + \int \frac{-a(A - iB) - a(iA + B) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(iA + B)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2a(A - iB)}{d \sqrt{\tan(c + dx)}} + \int \frac{-a(iA + B) - a(A - iB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx \\ &= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(iA + B)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2a(A - iB)}{d \sqrt{\tan(c + dx)}} - \frac{2\sqrt{-1} a(iA + B) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{1}\right)}{d} \\ &= \frac{2\sqrt{-1} a(iA + B) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{1}\right)}{d} - \frac{2a(A - iB)}{5d \tan^{\frac{5}{2}}(c + dx)} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 265 vs.  $2(103) = 206$ .  
time = 3.43, size = 265, normalized size = 2.57

$$\frac{\cos^2(c + dx)(\cos(dx) - i \sin(dx)) \left( -\frac{2i(A - iB)e^{-ix} \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right)}{\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}} - \frac{\csc^2(c+dx)(\cos(c) - i \sin(c))(-12A + 15iB + 3(6A - 5iB)\cos(2(c+dx)) + 5(iA + B)\sin(2(c+dx)))}{15\sqrt{\tan(c+dx)}} \right)}{d(A \cos(c + dx) + B \sin(c + dx))} (a + ia \tan(c + dx))(A + B \tan(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),
x]
```

```
[Out] (Cos[c + d*x]^2*(Cos[d*x] - I*Sin[d*x])*((( -2*I)*(A - I*B)*Sqrt[((-I)*(-1 +
E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))]*ArcTanh[Sqrt[(-1 + E^((2*
I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))]])/(E^(I*c)*Sqrt[(-1 + E^((2*I)*(c
+ d*x)))]/(1 + E^((2*I)*(c + d*x)))])) - (Csc[c + d*x]^2*(Cos[c] - I*Sin[c])
*(-12*A + (15*I)*B + 3*(6*A - (5*I)*B)*Cos[2*(c + d*x)] + 5*(I*A + B)*Sin[2
```

$(c + dx)))/(15\sqrt{\tan(c + dx)})(a + I a \tan(c + dx))(A + B \tan(c + dx))/(d(A \cos(c + dx) + B \sin(c + dx)))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs.  $2(84) = 168$ .

time = 0.04, size = 236, normalized size = 2.29

method	result
derivativedivides	$a \left( -\frac{2A}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2(iB-A)}{\sqrt{\tan(dx+c)}} - \frac{2(iA+B)}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{(-iA-B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)} \right)$
default	$a \left( -\frac{2A}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2(iB-A)}{\sqrt{\tan(dx+c)}} - \frac{2(iA+B)}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{(-iA-B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*a*(-2/5*A/\tan(d*x+c)^{(5/2)}-2*(-A+I*B)/\tan(d*x+c)^{(1/2)}-2/3*(I*A+B)/\tan(d*x+c)^{(3/2)}+1/4*(-I*A-B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+1/4*(A-I*B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs.  $2(81) = 162$ .

time = 0.60, size = 187, normalized size = 1.82

$\frac{15(2\sqrt{2}^{(i-1)A+(i+1)B})\arctan\left(\frac{1+\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}{1-\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}\right)+2\sqrt{2}^{(i-1)A+(i+1)B}\arctan\left(\frac{-1+\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}{1+\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}\right)-\sqrt{2}^{-(i+1)A+(i-1)B}\log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}{\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}\right)+\sqrt{2}^{-(i+1)A+(i-1)B}\log\left(\frac{-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}\right)}{60d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm  
="maxima")`

[Out]  $-1/60*(15*(2*\sqrt{2})*((I-1)*A+(I+1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)}))+2*\sqrt{2}*((I-1)*A+(I+1)*B)*\arctan(-1/2*\sqrt{2}$

(2)\*(sqrt(2) - 2\*sqrt(tan(d\*x + c))) - sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*log(sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) + sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1))\*a - 8\*(15\*(A - I\*B)\*a\*tan(d\*x + c)^2 + 5\*(-I\*A - B)\*a\*tan(d\*x + c) - 3\*A\*a)/tan(d\*x + c)^(5/2))/d

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(81) = 162.  
time = 2.17, size = 486, normalized size = 4.72

$$\frac{15(d^{6I+1} - 2d^{6I} + 2d^{6I-1} - d^{\frac{1}{2}} \sqrt{-15A^2 - 20AB + 10B^2}) \log\left(\frac{1 + \sqrt{-15A^2 - 20AB + 10B^2} \sqrt{\tan(dx+c)}}{2\sqrt{\tan(dx+c)+1}}\right) - 15(d^{6I+1} - 2d^{6I} + 2d^{6I-1} - d^{\frac{1}{2}} \sqrt{-15A^2 - 20AB + 10B^2}) \log\left(\frac{1 - \sqrt{-15A^2 - 20AB + 10B^2} \sqrt{\tan(dx+c)}}{2\sqrt{\tan(dx+c)+1}}\right) + 4(-23A - 20B)a^2 e^{6I dx + 6Ic} + (11A + 20B)a^2 e^{2I dx + 2Ic} + (-13A - 10B)a \sqrt{(-Ie^{2I dx + 2Ic} + I)/(e^{2I dx + 2Ic} + 1)}}{15d \tan(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] -1/30\*(15\*(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*log(2\*((A - I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) + (d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))) \* e^(-2\*I\*d\*x - 2\*I\*c)/((I\*A + B)\*a) - 15\*(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*log(2\*((A - I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) - (d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))) \* e^(-2\*I\*d\*x - 2\*I\*c)/((I\*A + B)\*a) + 4\*((-23\*I\*A - 20\*B)\*a\*e^(6\*I\*d\*x + 6\*I\*c) + (I\*A + 10\*B)\*a\*e^(4\*I\*d\*x + 4\*I\*c) + (11\*I\*A + 20\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) + (-13\*I\*A - 10\*B)\*a)\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))/(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int \frac{A}{\tan^{\frac{5}{2}}(c+dx)} dx + \int \frac{B}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{iA}{\tan^{\frac{7}{2}}(c+dx)} \right) dx + \int \left( -\frac{iB}{\tan^{\frac{5}{2}}(c+dx)} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out] I\*a\*(Integral(A/tan(c + d\*x)\*\*(5/2), x) + Integral(B/tan(c + d\*x)\*\*(3/2), x) + Integral(-I\*A/tan(c + d\*x)\*\*(7/2), x) + Integral(-I\*B/tan(c + d\*x)\*\*(5/2), x))

**Giac [A]**

time = 0.92, size = 93, normalized size = 0.90

$$\frac{(i+1)\sqrt{2}(iAa+Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} + \frac{2(15Aa\tan(dx+c)^2-15iBa\tan(dx+c)^2-5iAa\tan(dx+c)-5Ba\tan(dx+c)-3Aa)}{15d\tan(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out]  $-(I + 1)*\sqrt{2}*(I*A*a + B*a)*\arctan(-\frac{1}{2}I - \frac{1}{2})*\sqrt{2}*\sqrt{\tan(d*x + c)}/d + \frac{2}{15}*(15*A*a*\tan(d*x + c)^2 - 15*I*B*a*\tan(d*x + c)^2 - 5*I*A*a*\tan(d*x + c) - 5*B*a*\tan(d*x + c) - 3*A*a)/(d*\tan(d*x + c)^{(5/2)})$

**Mupad [B]**

time = 8.74, size = 123, normalized size = 1.19

$$-\frac{\frac{2Ba}{3d} + \frac{Ba \tan(c+dx) 2i}{d}}{\tan(c+dx)^{3/2}} + \frac{\sqrt{2} Ba \operatorname{atan}\left(\sqrt{2} \sqrt{\tan(c+dx)} \left(\frac{1}{2} - \frac{1}{2}i\right)\right) (-1-i)}{d} - \frac{2Aa \left(15(-1)^{1/4} \tan(c+dx)^{5/2} \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{\tan(c+dx)}}{15d \tan(c+dx)^{5/2}}\right) - 15 \tan(c+dx)^2 + 3 + \tan(c+dx) 5i\right)}{15d \tan(c+dx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i))/tan(c + d\*x)^(7/2),x)

[Out]  $-\left(\frac{2Ba}{3d} + \frac{Ba*a*\tan(c + d*x)*2i}{d}\right)/\tan(c + d*x)^{(3/2)} - \frac{2^{(1/2)}*B*a*\operatorname{atan}\left(2^{(1/2)}*\tan(c + d*x)^{(1/2)}*\left(\frac{1}{2} - \frac{1}{2}i\right)\right)*(1 + 1i)}{d} - \frac{2*A*a*(\tan(c + d*x)*5i - 15*\tan(c + d*x)^2 + 15*(-1)^{(1/4)}*\tan(c + d*x)^{(5/2)}*\operatorname{atanh}\left(\frac{(-1)^{(1/4)}*\tan(c + d*x)^{(1/2)}}{15*d*\tan(c + d*x)^{(5/2)}\right) + 3)}{(15*d*\tan(c + d*x)^{(5/2)})}$

$$3.119 \quad \int \tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=183

$$\frac{4\sqrt{-1} a^2(iA + B)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{4a^2(iA + B)\sqrt{\tan(c + dx)}}{d} + \frac{4a^2(A - iB)\tan^{\frac{3}{2}}(c + dx)}{3d}$$

[Out]  $-4*(-1)^{(1/4)}*a^2*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-4*a^2*(I*A+B)*\tan(d*x+c)^{(1/2)}/d+4/3*a^2*(A-I*B)*\tan(d*x+c)^{(3/2)}/d+4/5*a^2*(I*A+B)*\tan(d*x+c)^{(5/2)}/d-2/63*a^2*(9*A-11*I*B)*\tan(d*x+c)^{(7/2)}/d+2/9*I*B*\tan(d*x+c)^{(7/2)}*(a^2+I*a^2*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3675, 3673, 3609, 3614, 211}

$$\frac{4\sqrt{-1} a^2(B + iA)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{2a^2(9A - 11iB)\tan^{\frac{1}{2}}(c + dx)}{63d} + \frac{4a^2(B + iA)\tan^{\frac{3}{2}}(c + dx)}{5d} + \frac{4a^2(A - iB)\tan^{\frac{3}{2}}(c + dx)}{3d} - \frac{4a^2(B + iA)\sqrt{\tan(c + dx)}}{d} + \frac{2iB \tan^{\frac{7}{2}}(c + dx)(a^2 + ia^2 \tan(c + dx))}{9d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(-4*(-1)^{(1/4)}*a^2*(I*A + B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d - (4*a^2*(I*A + B)*\text{Sqrt}[\text{Tan}[c + d*x]])/d + (4*a^2*(A - I*B)*\text{Tan}[c + d*x]^{(3/2)})/(3*d) + (4*a^2*(I*A + B)*\text{Tan}[c + d*x]^{(5/2)})/(5*d) - (2*a^2*(9*A - (11*I)*B)*\text{Tan}[c + d*x]^{(7/2)})/(63*d) + (((2*I)/9)*B*\text{Tan}[c + d*x]^{(7/2)}*(a^2 + I*a^2*\text{Tan}[c + d*x]))/d$

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3609**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

**Rule 3614**

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*



$\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)/(b*f*(m + 1))}), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{LeQ}[m, -1]$

### Rule 3675

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{(m - 1)*((c + d*\text{Tan}[e + f*x])^{(n + 1)/(d*f*(m + n))}), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ !\text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned} \int \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{2iB \tan^{\frac{7}{2}}(c + dx)(a^2 + ia^2 \tan(c + dx))}{9d} + \dots \\ &= -\frac{2a^2(9A - 11iB) \tan^{\frac{7}{2}}(c + dx)}{63d} + \frac{2iB \tan^{\frac{7}{2}}(c + dx)}{63d} \\ &= \frac{4a^2(iA + B) \tan^{\frac{5}{2}}(c + dx)}{5d} - \frac{2a^2(9A - 11iB) \tan^{\frac{7}{2}}(c + dx)}{63d} \\ &= \frac{4a^2(A - iB) \tan^{\frac{3}{2}}(c + dx)}{3d} + \frac{4a^2(iA + B) \tan^{\frac{5}{2}}(c + dx)}{5d} \\ &= -\frac{4a^2(iA + B) \sqrt{\tan(c + dx)}}{d} + \frac{4a^2(A - iB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\ &= -\frac{4a^2(iA + B) \sqrt{\tan(c + dx)}}{d} + \frac{4a^2(A - iB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\ &= -\frac{4\sqrt{-1} a^2(iA + B) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} \end{aligned}$$

**Mathematica [A]**

time = 4.99, size = 315, normalized size = 1.72

$$\frac{\cos^2(c+dx) \left( \frac{4(A+B)e^{-2c} \sqrt{\frac{-1+e^{2(c+dx)}}{1+e^{2(c+dx)}}} \operatorname{tanh}^{-1} \left( \sqrt{\frac{-1+e^{2(c+dx)}}{1+e^{2(c+dx)}}} \right) - \frac{1 \operatorname{sech}^2(c+dx) (\cos(2c) - \sin(2c)) (21(84A - 89B) + 140(18A - 17B) \cos(2(c+dx)) + (756A - 791B) \cos(4(c+dx)) + 30(11A + 8B) \sin(2(c+dx)) + 15(17A + 20B) \sin(4(c+dx))) \sqrt{\tan(c+dx)}}{1260} \right)}{\sqrt{\frac{-1+e^{2(c+dx)}}{1+e^{2(c+dx)}}}} \frac{(a + ia \tan(c+dx))^2 (A + B \tan(c+dx))}{d(\cos(dx) + i \sin(dx))^2 (A \cos(c+dx) + B \sin(c+dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (Cos[c + d\*x]^3\*((4\*(I\*A + B)\*Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x)))]])/(E^((2\*I)\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x)))] - (I/1260)\*Sec[c + d\*x]^4\*(Cos[2\*c] - I\*Sin[2\*c])\*(21\*(84\*A - (89\*I)\*B) + 140\*(18\*A - (17\*I)\*B)\*Cos[2\*(c + d\*x)] + (756\*A - (791\*I)\*B)\*Cos[4\*(c + d\*x)] + 30\*((11\*I)\*A + 8\*B)\*Sin[2\*(c + d\*x)] + 15\*((17\*I)\*A + 20\*B)\*Sin[4\*(c + d\*x)])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/(d\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

Maple [A]

time = 0.04, size = 298, normalized size = 1.63

method	result
derivativedivides	$a^2 \left( -\frac{2B \left( \tan^{\frac{9}{2}}(dx+c) \right)}{9} + \frac{4iB \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} - \frac{2A \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{4iA \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{4B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{4iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} \right)$
default	$a^2 \left( -\frac{2B \left( \tan^{\frac{9}{2}}(dx+c) \right)}{9} + \frac{4iB \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} - \frac{2A \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{4iA \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{4B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{4iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, method=\_RETURN VERBOSE)

[Out] 1/d\*a^2\*(-2/9\*B\*tan(d\*x+c)^(9/2)+4/7\*I\*B\*tan(d\*x+c)^(7/2)-2/7\*A\*tan(d\*x+c)^(7/2)+4/5\*I\*A\*tan(d\*x+c)^(5/2)+4/5\*B\*tan(d\*x+c)^(5/2)-4/3\*I\*B\*tan(d\*x+c)^(3/2)+4/3\*A\*tan(d\*x+c)^(3/2)-4\*I\*A\*tan(d\*x+c)^(1/2)-4\*B\*tan(d\*x+c)^(1/2)+1/4\*(2\*B+2\*I\*A)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan

$$(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(-2*A+2*I*B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$$

**Maxima** [A]

time = 0.50, size = 230, normalized size = 1.26

$$\frac{140 B^2 \tan(d x+c)^2 + 180(A-2 I B) \tan(d x+c) + 504(-A-B) \tan(d x+c) - 840(A-I B) \tan(d x+c) + 2520(A+B) \tan(d x+c) + 315(2 \sqrt{2} \arctan(\sqrt{2} \sqrt{\tan(d x+c)}) + 2 \sqrt{2} \arctan(\sqrt{2} \sqrt{\tan(d x+c)})) + 2 \sqrt{2} \arctan(\sqrt{2} \sqrt{\tan(d x+c)}) - 315(2 \sqrt{2} \arctan(\sqrt{2} \sqrt{\tan(d x+c)}) + 2 \sqrt{2} \arctan(\sqrt{2} \sqrt{\tan(d x+c)})) - \sqrt{2} \log(\sqrt{2} \sqrt{\tan(d x+c)} + 1) - \sqrt{2} \log(\sqrt{2} \sqrt{\tan(d x+c)} - 1) + \sqrt{2} \log(\sqrt{2} \sqrt{\tan(d x+c)} + 1) + \sqrt{2} \log(\sqrt{2} \sqrt{\tan(d x+c)} - 1)}{630}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/630*(140*B*a^2*\tan(d*x + c)^{(9/2)} + 180*(A - 2*I*B)*a^2*\tan(d*x + c)^{(7/2)} + 504*(-I*A - B)*a^2*\tan(d*x + c)^{(5/2)} - 840*(A - I*B)*a^2*\tan(d*x + c)^{(3/2)} + 2520*(I*A + B)*a^2*\sqrt{\tan(d*x + c)} - 315*(2*\sqrt{2}*((I - 1)*A + (I + 1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(d*x + c)})) + 2*\sqrt{2}*(2*((I - 1)*A + (I + 1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(d*x + c)}))) - \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1) + \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(-\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1))*a^2)/d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs.  $2(145) = 290$ .

time = 2.45, size = 555, normalized size = 3.03

$$\frac{1}{315} \sqrt{-(-I A^2 - 2 A B + I B^2)} a^4 / d^2 \left( (d e^{(8 I d x + 8 I c)} + 4 d e^{(6 I d x + 6 I c)} + 6 d e^{(4 I d x + 4 I c)} + 4 d e^{(2 I d x + 2 I c)} + d) \log(-2((A - I B) a^2 e^{(2 I d x + 2 I c)} + \sqrt{-(-I A^2 - 2 A B + I B^2)} a^4 / d^2) (d e^{(2 I d x + 2 I c)} + d) \sqrt{((-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1))) e^{(-2 I d x - 2 I c)} / ((-I A - B) a^2) - 315 \sqrt{-(-I A^2 - 2 A B + I B^2)} a^4 / d^2 (d e^{(8 I d x + 8 I c)} + 4 d e^{(6 I d x + 6 I c)} + 6 d e^{(4 I d x + 4 I c)} + 4 d e^{(2 I d x + 2 I c)} + d) \log(-2((A - I B) a^2 e^{(2 I d x + 2 I c)} - \sqrt{-(-I A^2 - 2 A B + I B^2)} a^4 / d^2) (d e^{(2 I d x + 2 I c)} + d) \sqrt{((-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1))) e^{(-2 I d x - 2 I c)} / ((-I A - B) a^2) - 2((1011 I A + 1091 B) a^2 e^{(8 I d x + 8 I c)} + 10(285 I A + 262 B) a^2 e^{(6 I d x + 6 I c)} + 42(84 I A + 89 B) a^2 e^{(4 I d x + 4 I c)} + 10(219 I A + 214 B) a^2 e^{(2 I d x + 2 I c)} + (501 I A + 491 B) a^2) \sqrt{((-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1))) e^{(-2 I d x - 2 I c)} / ((-I A - B) a^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$1/315*(315*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} + \sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2)) - 315*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2*(d*e^{(8*I*d*x + 8*I*c)} + 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} + 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} - \sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2)) - 2*((1011*I*A + 1091*B)*a^2*e^{(8*I*d*x + 8*I*c)} + 10*(285*I*A + 262*B)*a^2*e^{(6*I*d*x + 6*I*c)} + 42*(84*I*A + 89*B)*a^2*e^{(4*I*d*x + 4*I*c)} + 10*(219*I*A + 214*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (501*I*A + 491*B)*a^2)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2)$$



$$3.120 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=156

$$\frac{4\sqrt{-1} a^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{4a^2(A-iB)\sqrt{\tan(c+dx)}}{d} + \frac{4a^2(iA+B)\tan^{\frac{3}{2}}(c+dx)}{3d}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+4*a^2*(A-I*B)*\tan(d*x+c)^{(1/2)}/d+4/3*a^2*(I*A+B)*\tan(d*x+c)^{(3/2)}/d-2/35*a^2*(7*A-9*I*B)*\tan(d*x+c)^{(5/2)}/d+2/7*I*B*\tan(d*x+c)^{(5/2)}*(a^2+I*a^2*\tan(d*x+c))/d$

**Rubi** [A]

time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3675, 3673, 3609, 3614, 211}

$$\frac{4\sqrt{-1} a^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(7A-9iB)\tan^{\frac{3}{2}}(c+dx)}{35d} + \frac{4a^2(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4a^2(A-iB)\sqrt{\tan(c+dx)}}{d} + \frac{2iB\tan^{\frac{3}{2}}(c+dx)(a^2+ia^2\tan(c+dx))}{7d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(4*(-1)^{(1/4)}*a^2*(A-I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c+d*x]]])/d + (4*a^2*(A-I*B)*\text{Sqrt}[\text{Tan}[c+d*x]])/d + (4*a^2*(I*A+B)*\text{Tan}[c+d*x]^{(3/2)})/(3*d) - (2*a^2*(7*A-(9*I)*B)*\text{Tan}[c+d*x]^{(5/2)})/(35*d) + (((2*I)/7)*B*\text{Tan}[c+d*x]^{(5/2)}*(a^2+I*a^2*\text{Tan}[c+d*x]))/d$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m-1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3614

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{2iB \tan^{\frac{5}{2}}(c + dx)(a^2 + ia^2 \tan(c + dx))}{7d} + \frac{2}{7} \\
 &= -\frac{2a^2(7A - 9iB) \tan^{\frac{5}{2}}(c + dx)}{35d} + \frac{2iB \tan^{\frac{5}{2}}(c + dx)}{35d} \\
 &= \frac{4a^2(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} - \frac{2a^2(7A - 9iB)}{35d} \\
 &= \frac{4a^2(A - iB) \sqrt{\tan(c + dx)}}{d} + \frac{4a^2(iA + B) \tan^{\frac{5}{2}}(c + dx)}{35d} \\
 &= \frac{4a^2(A - iB) \sqrt{\tan(c + dx)}}{d} + \frac{4a^2(iA + B) \tan^{\frac{5}{2}}(c + dx)}{35d} \\
 &= \frac{4\sqrt[4]{-1} a^2(A - iB) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d}
 \end{aligned}$$

Mathematica [A]

time = 4.12, size = 307, normalized size = 1.97

$$\cos^3(c + dx) \left( \frac{4i(A + B)e^{-2i\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}} \operatorname{tanh}^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right)}{\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}} + \frac{1}{35} \sec^2(c + dx) (\cos(2c) - i \sin(2c)) (21(29A - 28iB) \cos(c + dx) + 21(11A - 12iB) \cos(3(c + dx)) + 70iA \sin(c + dx) + 25B \sin(c + dx) + 70iA \sin(3(c + dx)) + 85B \sin(3(c + dx))) \sqrt{\tan(c + dx)} \right) (a + ia \tan(c + dx))^2 (A + B \tan(c + dx))$$


---

$d(\cos(dx) + i \sin(dx))(A \cos(c + dx) + B \sin(c + dx))$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (Cos[c + d\*x]^3\*((4\*(I\*A + B)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))])/(E^((2\*I)\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))) + (Sec[c + d\*x]^3\*(Cos[2\*c] - I\*Sin[2\*c])\*(21\*(29\*A - (28\*I)\*B)\*Cos[c + d\*x] + 21\*(11\*A - (12\*I)\*B)\*Cos[3\*(c + d\*x)] + (70\*I)\*A\*Sin[c + d\*x] + 25\*B\*Sin[c + d\*x] + (70\*I)\*A\*Sin[3\*(c + d\*x)] + 85\*B\*Sin[3\*(c + d\*x)])\*Sqrt[Tan[c + d\*x]])/210\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/(d\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(130) = 260.  
time = 0.04, size = 275, normalized size = 1.76

method	result
derivativedivides	$a^2 \left( -\frac{2B \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{4iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{2A \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{4iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{4B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 4iB \left( \sqrt{\tan(dx+c)} \right) \right)$
default	$a^2 \left( -\frac{2B \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{4iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{2A \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{4iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{4B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 4iB \left( \sqrt{\tan(dx+c)} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, method=\_RETURN VERBOSE)

[Out] 1/d\*a^2\*(-2/7\*B\*tan(d\*x+c)^(7/2)+4/5\*I\*B\*tan(d\*x+c)^(5/2)-2/5\*A\*tan(d\*x+c)^(5/2)+4/3\*I\*A\*tan(d\*x+c)^(3/2)+4/3\*B\*tan(d\*x+c)^(3/2)-4\*I\*B\*tan(d\*x+c)^(1/2)+4\*A\*tan(d\*x+c)^(1/2)+1/4\*(-2\*A+2\*I\*B)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(-2\*B-2\*I\*A)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))

**Maxima [A]**

time = 0.50, size = 212, normalized size = 1.36





```
[Out] -a**2*(Integral(-A*tan(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)**(7/2), x) + Integral(-B*tan(c + d*x)**(5/2), x) + Integral(B*tan(c + d*x)**(9/2), x) + Integral(-2*I*A*tan(c + d*x)**(5/2), x) + Integral(-2*I*B*tan(c + d*x)**(7/2), x))
```

**Giac** [A]

time = 0.99, size = 160, normalized size = 1.03

$$\frac{(2i-2)\sqrt{2}(iAa^2+Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2(15Ba^2d^6\tan(dx+c)^3+21Aa^2d^6\tan(dx+c)^3-42iBa^2d^6\tan(dx+c)^3-70iAa^2d^6\tan(dx+c)^3-70Ba^2d^6\tan(dx+c)^3-210Aa^2d^6\sqrt{\tan(dx+c)}+210iBa^2d^6\sqrt{\tan(dx+c)})}{105d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*I - 2)*sqrt(2)*(I*A*a^2 + B*a^2)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c)))/d - 2/105*(15*B*a^2*d^6*tan(d*x + c)^(7/2) + 21*A*a^2*d^6*tan(d*x + c)^(5/2) - 42*I*B*a^2*d^6*tan(d*x + c)^(5/2) - 70*I*A*a^2*d^6*tan(d*x + c)^(3/2) - 70*B*a^2*d^6*tan(d*x + c)^(3/2) - 210*A*a^2*d^6*sqrt(tan(d*x + c)) + 210*I*B*a^2*d^6*sqrt(tan(d*x + c)))/d^7
```

**Mupad** [B]

time = 9.28, size = 291, normalized size = 1.87

$$\frac{1Aa^2\sqrt{\tan(dx+c)} + 2A^2\sqrt{\tan(dx+c)}^{3/2} + 2A^2\sqrt{\tan(dx+c)}^{5/2} + Ba^2\sqrt{\tan(dx+c)} + 2B^2\sqrt{\tan(dx+c)}^{3/2} + 2B^2\sqrt{\tan(dx+c)}^{5/2} + 2B^2\sqrt{\tan(dx+c)}^{7/2} + \sqrt{2}A^2b(-A^2d^6 + \sqrt{2}A^2d^6\sqrt{\tan(dx+c)})(-2+2i) + \sqrt{2}A^2b(-A^2d^6 + \sqrt{2}A^2d^6\sqrt{\tan(dx+c)})(-2-2i) + \sqrt{2}B^2b(-4B^2d^6 + \sqrt{2}B^2d^6\sqrt{\tan(dx+c)})(-2+2i) + \sqrt{2}B^2b(-4B^2d^6 + \sqrt{2}B^2d^6\sqrt{\tan(dx+c)})(-2-2i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (4*A*a^2*tan(c + d*x)^(1/2))/d + (A*a^2*tan(c + d*x)^(3/2)*4i)/(3*d) - (2*A*a^2*tan(c + d*x)^(5/2))/(5*d) - (B*a^2*tan(c + d*x)^(1/2)*4i)/d + (4*B*a^2*tan(c + d*x)^(3/2))/(3*d) + (B*a^2*tan(c + d*x)^(5/2)*4i)/(5*d) - (2*B*a^2*tan(c + d*x)^(7/2))/(7*d) + (2^(1/2)*A*a^2*log(-A*a^2*d*4i - 2^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4i)^(1/2)*A*a^2*log(2*(-4i)^(1/2)*A*a^2*d*tan(c + d*x)^(1/2) - A*a^2*d*4i))/d + (2^(1/2)*B*a^2*log(-4*B*a^2*d - 2^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/d - (4i)^(1/2)*B*a^2*log(2*4i^(1/2)*B*a^2*d*tan(c + d*x)^(1/2) - 4*B*a^2*d))/d
```

### 3.121 $\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=129

$$\frac{4\sqrt[4]{-1} a^2 (iA + B) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} + \frac{4a^2 (iA + B) \sqrt{\tan(c + dx)}}{d} - \frac{2a^2 (5A - 7iB) \tan^{3/2}(c + dx)}{15d}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+4*a^2*(I*A+B)*\tan(d*x+c)^{(1/2)}/d-2/15*a^2*(5*A-7*I*B)*\tan(d*x+c)^{(3/2)}/d+2/5*I*B*\tan(d*x+c)^{(3/2)}*(a^2+I*a^2*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3675, 3673, 3609, 3614, 211}

$$\frac{4\sqrt[4]{-1} a^2 (B + iA) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{2a^2 (5A - 7iB) \tan^{3/2}(c + dx)}{15d} + \frac{4a^2 (B + iA) \sqrt{\tan(c + dx)}}{d} + \frac{2iB \tan^{3/2}(c + dx) (a^2 + ia^2 \tan(c + dx))}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(4*(-1)^{(1/4)}*a^2*(I*A + B)*\operatorname{ArcTan}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/d + (4*a^2*(I*A + B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d - (2*a^2*(5*A - (7*I)*B)*\operatorname{Tan}[c + d*x]^{(3/2)})/(15*d) + (((2*I)/5)*B*\operatorname{Tan}[c + d*x]^{(3/2)}*(a^2 + I*a^2*\operatorname{Tan}[c + d*x]))/d$

Rule 211

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2])/a]*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b]$

Rule 3609

$\operatorname{Int}[(a_) + (b_)*\operatorname{tan}[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*\operatorname{tan}[(e_) + (f_)*(x_)])}, x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m-1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\operatorname{Tan}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 + b^2, 0] \&\& \operatorname{GtQ}[m, 0]$

Rule 3614

$\operatorname{Int}[(c_) + (d_)*\operatorname{tan}[(e_) + (f_)*(x_)])/ \operatorname{Sqrt}[(b_)*\operatorname{tan}[(e_) + (f_)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f, x\} \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2 (A + B \tan(c + dx)) dx &= \frac{2iB \tan^{\frac{3}{2}}(c + dx) (a^2 + ia^2 \tan(c + dx))}{5d} \\
&= -\frac{2a^2(5A - 7iB) \tan^{\frac{3}{2}}(c + dx)}{15d} + \frac{2iB \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{4a^2(iA + B) \sqrt{\tan(c + dx)}}{d} - \frac{2a^2(5A - 7iB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{4a^2(iA + B) \sqrt{\tan(c + dx)}}{d} - \frac{2a^2(5A - 7iB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{4\sqrt{-1} a^2 (iA + B) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 272 vs. 2(129) = 258.

time = 4.18, size = 272, normalized size = 2.11

$$\frac{\cos^2(c + dx) \left( \frac{4i(A + B)e^{-2ix} \sqrt{\frac{i(-1 + e^{2i(c + dx)})}{1 + e^{2i(c + dx)}}} \operatorname{tanh}^{-1} \left( \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \right)}{\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}} + \frac{1}{15} \sec^2(c + dx) (\cos(2c) - i \sin(2c)) (30iA + 27B + (30iA + 33B) \cos(2(c + dx)) - 5(A - 2iB) \sin(2(c + dx))) \sqrt{\tan(c + dx)} \right) (a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{d(\cos(dx) + i \sin(dx))^2 (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (Cos[c + d\*x]^3\*(((−4\*I)\*(A − I\*B)\*Sqrt[((−I)\*(−1 + E^((2\*I)\*(c + d\*x))))]/(1 + E^((2\*I)\*(c + d\*x)))]\*ArcTanh[Sqrt[(−1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]])/(E^((2\*I)\*c)\*Sqrt[(−1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))])) + (Sec[c + d\*x]^2\*(Cos[2\*c] − I\*Sin[2\*c])\*((30\*I)\*A + 27\*B + ((30\*I)\*A + 33\*B)\*Cos[2\*(c + d\*x)] − 5\*(A − (2\*I)\*B)\*Sin[2\*(c + d\*x)])\*Sqrt[Tan[c + d\*x]]/15)\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/(d\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(108) = 216.

time = 0.04, size = 252, normalized size = 1.95

method	result
derivativedivides	$a^2 \left( -\frac{2B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{4iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - \frac{2A \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 4iA \left( \sqrt{\tan(dx+c)} \right) + 4B \left( \sqrt{\tan(dx+c)} \right) + \dots \right)$
default	$a^2 \left( -\frac{2B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{4iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - \frac{2A \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 4iA \left( \sqrt{\tan(dx+c)} \right) + 4B \left( \sqrt{\tan(dx+c)} \right) + \dots \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x,method=\_RETURN VERBOSE)

[Out] 1/d\*a^2\*(-2/5\*B\*tan(d\*x+c)^(5/2)+4/3\*I\*B\*tan(d\*x+c)^(3/2)-2/3\*A\*tan(d\*x+c)^(3/2)+4\*I\*A\*tan(d\*x+c)^(1/2)+4\*B\*tan(d\*x+c)^(1/2)+1/4\*(-2\*B-2\*I\*A)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(2\*A-2\*I\*B)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))

**Maxima [A]**

time = 0.58, size = 194, normalized size = 1.50

$\frac{12 B^2 \tan(dx+c)^3 + 20(A-2B)^2 \tan(dx+c)^2 + 120(-1A-B)^2 \sqrt{\tan(dx+c)} + 15(2\sqrt{2}(-1)A+(1)B) \arctan\left(\frac{1}{\sqrt{2}}(\sqrt{2}+2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}(-1)A+(1)B) \arctan\left(-\frac{1}{\sqrt{2}}(\sqrt{2}-2\sqrt{\tan(dx+c)})\right) - \sqrt{2}(-1)A+(1)B) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)\right) + \sqrt{2}(-1)A+(1)B) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)\right)}{30d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/30*(12*B*a^2*\tan(dx + c)^{5/2} + 20*(A - 2*I*B)*a^2*\tan(dx + c)^{3/2} + 120*(-I*A - B)*a^2*\sqrt{\tan(dx + c)} + 15*(2*\sqrt{2}*((I - 1)*A + (I + 1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(dx + c)}))) + 2*\sqrt{2}*((I - 1)*A + (I + 1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(dx + c)}))) - \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1) + \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(-\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1))*a^2/d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 441 vs.  $2(103) = 206$ .  
time = 3.72, size = 441, normalized size = 3.42

$$\frac{15\sqrt{-1}\sqrt{-2AB+I^2B^2}\sqrt{d}\sqrt{\tan(dx+c)}\sqrt{\frac{1-2AB+I^2B^2}{d^2}}\sqrt{\frac{1+\sqrt{1-2AB+I^2B^2}}{d^2}}}{15(d^2\sqrt{\tan(dx+c)}+2d\sqrt{\tan(dx+c)+d})\log\left(\frac{1-2AB+I^2B^2}{d^2}\sqrt{\frac{1+\sqrt{1-2AB+I^2B^2}}{d^2}}\right)} - 15\sqrt{-1}\sqrt{-2AB+I^2B^2}\sqrt{d}\sqrt{\tan(dx+c)}\sqrt{\frac{1-2AB+I^2B^2}{d^2}}\sqrt{\frac{1-\sqrt{1-2AB+I^2B^2}}{d^2}}}{15(d^2\sqrt{\tan(dx+c)}+2d\sqrt{\tan(dx+c)+d})\log\left(\frac{1-2AB+I^2B^2}{d^2}\sqrt{\frac{1-\sqrt{1-2AB+I^2B^2}}{d^2}}\right)} + 2\sqrt{2}\sqrt{-1}\sqrt{-2AB+I^2B^2}\sqrt{d}\sqrt{\tan(dx+c)}\sqrt{\frac{1-2AB+I^2B^2}{d^2}}\sqrt{\frac{1+\sqrt{1-2AB+I^2B^2}}{d^2}}}{15(d^2\sqrt{\tan(dx+c)}+2d\sqrt{\tan(dx+c)+d})} + 2\sqrt{2}\sqrt{-1}\sqrt{-2AB+I^2B^2}\sqrt{d}\sqrt{\tan(dx+c)}\sqrt{\frac{1-2AB+I^2B^2}{d^2}}\sqrt{\frac{1-\sqrt{1-2AB+I^2B^2}}{d^2}}}{15(d^2\sqrt{\tan(dx+c)}+2d\sqrt{\tan(dx+c)+d})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/15*(15*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} + \sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2)) - 15*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} - \sqrt{-(-I*A^2 - 2*A*B + I*B^2)}*a^4/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2)) + 2*((-35*I*A - 43*B)*a^2*e^{(4*I*d*x + 4*I*c)} + 6*(-10*I*A - 9*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-25*I*A - 23*B)*a^2)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}}/(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int(-A\sqrt{\tan(c+dx)})dx + \int A\tan^{\frac{3}{2}}(c+dx)dx + \int(-B\tan^{\frac{3}{2}}(c+dx))dx + \int B\tan^{\frac{3}{2}}(c+dx)dx + \int(-2iA\tan^{\frac{3}{2}}(c+dx))dx + \int(-2iB\tan^{\frac{3}{2}}(c+dx))dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] 
$$-a**2*(\text{Integral}(-A*\sqrt{\tan(c + dx)}), x) + \text{Integral}(A*\tan(c + dx)**(5/2), x) + \text{Integral}(-B*\tan(c + dx)**(3/2), x) + \text{Integral}(B*\tan(c + dx)**(7/2), x) + \text{Integral}(-2*I*A*\tan(c + dx)**(3/2), x) + \text{Integral}(-2*I*B*\tan(c + dx)**(5/2), x)$$

**Giac [A]**

time = 0.87, size = 126, normalized size = 0.98

$$\frac{(2i-2)\sqrt{2}(Aa^2 - iBa^2)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(3Ba^2d^4\tan(dx+c)^{\frac{5}{2}} + 5Aa^2d^4\tan(dx+c)^{\frac{3}{2}} - 10iBa^2d^4\tan(dx+c)^{\frac{3}{2}} - 30iAa^2d^4\sqrt{\tan(dx+c)} - 30Ba^2d^4\sqrt{\tan(dx+c)}\right)}{15d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-(2*I - 2)*\sqrt{2}*(A*a^2 - I*B*a^2)*\arctan(-\frac{1}{2}*I - \frac{1}{2})*\sqrt{2}*\sqrt{\tan(dx+c)}/d - \frac{2}{15}*(3*B*a^2*d^4*\tan(dx+c)^{\frac{5}{2}} + 5*A*a^2*d^4*\tan(dx+c)^{\frac{3}{2}} - 10*I*B*a^2*d^4*\tan(dx+c)^{\frac{3}{2}} - 30*I*A*a^2*d^4*\sqrt{\tan(dx+c)} - 30*B*a^2*d^4*\sqrt{\tan(dx+c)})/d^5$

**Mupad [B]**

time = 7.73, size = 256, normalized size = 1.98

$$\frac{Aa^2\sqrt{\tan(c+dx)}i}{d} - \frac{2Aa^2\tan(c+dx)^{3/2}}{3d} + \frac{4Ba^2\sqrt{\tan(c+dx)}}{d} - \frac{B^2\tan(c+dx)^{3/2}i}{3d} - \frac{2Ba^2\tan(c+dx)^{5/2}}{5d} + \frac{\sqrt{2}Aa^2\ln(4Aa^2d+\sqrt{2}Aa^2d\sqrt{\tan(c+dx)}(-2-2i)(1+i))}{d} - \frac{\sqrt{2}Aa^2\ln(4Aa^2d+2\sqrt{2}Aa^2d\sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^2\ln(-Ba^2d+\sqrt{2}Ba^2d\sqrt{\tan(c+dx)}(-2+2i)(1-i))}{d} - \frac{\sqrt{2}Ba^2\ln(-Ba^2d+2\sqrt{2}Ba^2d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out]  $(A*a^2*\tan(c + d*x)^{\frac{1}{2}}*4i)/d - (2*A*a^2*\tan(c + d*x)^{\frac{3}{2}})/(3*d) + (4*B*a^2*\tan(c + d*x)^{\frac{1}{2}})/d + (B*a^2*\tan(c + d*x)^{\frac{3}{2}}*4i)/(3*d) - (2*B*a^2*\tan(c + d*x)^{\frac{5}{2}})/(5*d) + (2^{\frac{1}{2}}*A*a^2*\log(4*A*a^2*d - 2^{\frac{1}{2}}*A*a^2*d*\tan(c + d*x)^{\frac{1}{2}}*(2 + 2i))*(1 + 1i))/d - (4i^{\frac{1}{2}}*A*a^2*\log(4*A*a^2*d + 2*4i^{\frac{1}{2}}*A*a^2*d*\tan(c + d*x)^{\frac{1}{2}}))/d + (2^{\frac{1}{2}}*B*a^2*\log(-B*a^2*d*4i - 2^{\frac{1}{2}}*B*a^2*d*\tan(c + d*x)^{\frac{1}{2}}*(2 - 2i))*(1 - 1i))/d - ((-4i)^{\frac{1}{2}}*B*a^2*\log(2*(-4i)^{\frac{1}{2}}*B*a^2*d*\tan(c + d*x)^{\frac{1}{2}} - B*a^2*d*4i))/d$

$$3.122 \quad \int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=104

$$\frac{4\sqrt[4]{-1} a^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{3d} + \frac{2iB\sqrt{\tan(c+dx)}}{3d}$$

[Out]  $-4*(-1)^{(1/4)}*a^2*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-2/3*a^2*(3*A-5*I*B)*\tan(d*x+c)^{(1/2)}/d+2/3*I*B*\tan(d*x+c)^{(1/2)}*(a^2+I*a^2*\tan(d*x+c))/d$

**Rubi** [A]

time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3675, 3673, 3614, 211}

$$\frac{4\sqrt[4]{-1} a^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{3d} + \frac{2iB\sqrt{\tan(c+dx)}(a^2+ia^2 \tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]],x]

[Out]  $(-4*(-1)^{(1/4)}*a^2*(A-I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c+d*x]]])/d - (2*a^2*(3*A-(5*I)*B)*\text{Sqrt}[\text{Tan}[c+d*x]])/(3*d) + (((2*I)/3)*B*\text{Sqrt}[\text{Tan}[c+d*x]]*(a^2+I*a^2*\text{Tan}[c+d*x]))/d$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3614

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

## Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{2iB \sqrt{\tan(c + dx)} (a^2 + ia^2 \tan(c + dx))}{3d} + \frac{2}{3} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\ &= -\frac{2a^2(3A - 5iB) \sqrt{\tan(c + dx)}}{3d} + \frac{2iB \sqrt{\tan(c + dx)} (a^2 + ia^2 \tan(c + dx))}{3d} \\ &= -\frac{2a^2(3A - 5iB) \sqrt{\tan(c + dx)}}{3d} + \frac{2iB \sqrt{\tan(c + dx)} (a^2 + ia^2 \tan(c + dx))}{3d} \\ &= -\frac{4\sqrt{-1} a^2 (A - iB) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} - \frac{2}{3} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \end{aligned}$$

**Mathematica [A]**

time = 2.61, size = 110, normalized size = 1.06

$$\frac{2a^2 \sqrt{\tan(c + dx)} \left( -6(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) + \sqrt{i \tan(c + dx)} (3A - 6iB + B \tan(c + dx)) \right)}{3d \sqrt{i \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]
],x]
```

```
[Out] (-2*a^2*Sqrt[Tan[c + d*x]]*(-6*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d
*x)))/(1 + E^((2*I)*(c + d*x))]]) + Sqrt[I*Tan[c + d*x]]*(3*A - (6*I)*B + B
*Tan[c + d*x]))/(3*d*Sqrt[I*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs.  $2(86) = 172$ .

time = 0.04, size = 229, normalized size = 2.20



method	result
derivativedivides	$a^2 \left( -\frac{2B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2A \left( \sqrt{\tan(dx+c)} \right) + 4iB \left( \sqrt{\tan(dx+c)} \right) + \frac{(-2iB+2A)\sqrt{2}}{\ln \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}} \right) \right)} \right)$
default	$a^2 \left( -\frac{2B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2A \left( \sqrt{\tan(dx+c)} \right) + 4iB \left( \sqrt{\tan(dx+c)} \right) + \frac{(-2iB+2A)\sqrt{2}}{\ln \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}} \right) \right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURN  
VERBOSE)`

[Out]  $1/d*a^2*(-2/3*B*\tan(d*x+c)^{(3/2)}-2*A*\tan(d*x+c)^{(1/2)}+4*I*B*\tan(d*x+c)^{(1/2)}+1/4*(2*A-2*I*B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+1/4*(2*B+2*I*A)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs.  $2(82) = 164$ .  
time = 0.62, size = 174, normalized size = 1.67

$$\frac{4B^2 \tan(dx+c)^2 + 12(A-2iB)a^2 \sqrt{\tan(dx+c)} + 2(2\sqrt{2}(-i+1)A+(i-1)B) \arctan\left(\frac{1}{\sqrt{2}}\sqrt{\tan(dx+c)}\right) + 2\sqrt{2}(-i+1)A+(i-1)B \arctan\left(-\frac{1}{\sqrt{2}}\sqrt{\tan(dx+c)}\right) + \sqrt{2}((i-1)A+(i+1)B) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - \sqrt{2}((i-1)A+(i+1)B) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $-1/6*(4*B*a^2*\tan(d*x+c)^{(3/2)}+12*(A-2*I*B)*a^2*\sqrt{\tan(d*x+c)}+3*(2*\sqrt{2})*(-(I+1)*A+(I-1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(d*x+c)}))+2*\sqrt{2}*(-(I+1)*A+(I-1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(d*x+c)}))+\sqrt{2}*((I-1)*A+(I+1)*B)*\log(\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1)-\sqrt{2}*((I-1)*A+(I+1)*B)*\log(-\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1))*a^2/d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs.  $2(82) = 164$ .



**Mupad [B]**

time = 6.84, size = 221, normalized size = 2.12

$$\frac{2Aa^2\sqrt{\tan(c+dx)}}{d} + \frac{Ba^2\sqrt{\tan(c+dx)}4i}{d} - \frac{2Ba^2\tan(c+dx)^{3/2}}{3d} + \frac{\sqrt{2}Aa^2\ln(Aa^2d4i + \sqrt{2}Aa^2d\sqrt{\tan(c+dx)}(-2+2i))(1-i)}{d} - \frac{\sqrt{-4i}Aa^2\ln(Aa^2d4i + 2\sqrt{-4i}Aa^2d\sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^2\ln(4Ba^2d + \sqrt{2}Ba^2d\sqrt{\tan(c+dx)}(-2-2i))(1+i)}{d} - \frac{\sqrt{4i}Ba^2\ln(4Ba^2d + 2\sqrt{4i}Ba^2d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2)/tan(c + d\*x)^(1/2),x)

[Out] (B\*a^2\*tan(c + d\*x)^(1/2)\*4i)/d - (2\*A\*a^2\*tan(c + d\*x)^(1/2))/d - (2\*B\*a^2\*tan(c + d\*x)^(3/2))/(3\*d) + (2^(1/2)\*A\*a^2\*log(A\*a^2\*d\*4i - 2^(1/2)\*A\*a^2\*d\*tan(c + d\*x)^(1/2)\*(2 - 2i))\*(1 - 1i))/d - ((-4i)^(1/2)\*A\*a^2\*log(A\*a^2\*d\*4i + 2\*(-4i)^(1/2)\*A\*a^2\*d\*tan(c + d\*x)^(1/2)))/d + (2^(1/2)\*B\*a^2\*log(4\*B\*a^2\*d - 2^(1/2)\*B\*a^2\*d\*tan(c + d\*x)^(1/2)\*(2 + 2i))\*(1 + 1i))/d - (4i^(1/2)\*B\*a^2\*log(4\*B\*a^2\*d + 2\*4i^(1/2)\*B\*a^2\*d\*tan(c + d\*x)^(1/2)))/d

$$3.123 \quad \int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=98

$$-\frac{4\sqrt[4]{-1} a^2(iA+B)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{2a^2(iA-B)\sqrt{\tan(c+dx)}}{d} - \frac{2A(a^2+ia^2 \tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

[Out]  $-4*(-1)^{(1/4)}*a^2*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+2*a^2*(I*A-B)*\tan(d*x+c)^{(1/2)}/d-2*A*(a^2+I*a^2*\tan(d*x+c))/d/\tan(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3674, 3673, 3614, 211}

$$-\frac{4\sqrt[4]{-1} a^2(B+IA)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{2a^2(-B+IA)\sqrt{\tan(c+dx)}}{d} - \frac{2A(a^2+ia^2 \tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{3/2}}, x]$

[Out]  $(-4*(-1)^{(1/4)}*a^2*(I*A + B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d + (2*a^2*(I*A - B)*\text{Sqrt}[\text{Tan}[c + d*x]])/d - (2*A*(a^2 + I*a^2*\text{Tan}[c + d*x]))/(d*\text{Sqrt}[\text{Tan}[c + d*x]])$

**Rule 211**

$\text{Int}[\frac{(a_) + (b_)*(x_)^2}{(x_)^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[a/b, 2]}{a}*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3614**

$\text{Int}[\frac{(c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)]}{\text{Sqrt}[(b_)*\text{tan}[(e_) + (f_)*(x_)]]}, x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /;$  FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

**Rule 3673**

$\text{Int}[\frac{(a_) + (b_)*\text{tan}[(e_) + (f_)*(x_)]^{(m_)}*((A_) + (B_)*\text{tan}[(e_) + (f_)*(x_)]}{(c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)]}, x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m)}*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

## Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2A(a^2 + ia^2 \tan(c + dx))}{d\sqrt{\tan(c + dx)}} + 2 \int \frac{(a + ia \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2a^2(iA - B)\sqrt{\tan(c + dx)}}{d} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{d\sqrt{\tan(c + dx)}} \\ &= \frac{2a^2(iA - B)\sqrt{\tan(c + dx)}}{d} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{d\sqrt{\tan(c + dx)}} \\ &= -\frac{4\sqrt[4]{-1} a^2(iA + B) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} + \end{aligned}$$

**Mathematica [A]**

time = 2.52, size = 85, normalized size = 0.87

$$\frac{2a^2 \left( A - 2(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) \sqrt{i \tan(c + dx)} + B \tan(c + dx) \right)}{d\sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2), x]
```

```
[Out] (-2*a^2*(A - 2*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[I*Tan[c + d*x]] + B*Tan[c + d*x]))/(d*Sqrt[Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(85) = 170.

time = 0.04, size = 217, normalized size = 2.21

method	result
derivativedivides	$a^2 \left( -2B \left( \sqrt{\tan(dx+c)} \right) - \frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(2iA+2B)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right)}{\right)}$
default	$a^2 \left( -2B \left( \sqrt{\tan(dx+c)} \right) - \frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(2iA+2B)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right)}{\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} a^2 \left( -2B \tan(dx+c)^{1/2} - 2A \tan(dx+c)^{-1/2} + \frac{1}{4} (2B+2IA) \tan(dx+c)^{1/2} \left( \ln \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) \right) + \frac{1}{4} (-2A+2IB) \tan(dx+c)^{1/2} \left( \ln \left( \frac{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) \right) \right)$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs.  $2(82) = 164$ .  
time = 0.51, size = 170, normalized size = 1.73

$$\frac{4Bd^2\sqrt{\tan(dx+c)} - (2\sqrt{2}(i-1)A+(i+1)B)\arctan\left(\frac{1+\sqrt{2}(\sqrt{\tan(dx+c)})}{1-\sqrt{2}(\sqrt{\tan(dx+c)})}\right) + 2\sqrt{2}(i-1)A+(i+1)B\arctan\left(\frac{1-\sqrt{2}(\sqrt{\tan(dx+c)})}{1+\sqrt{2}(\sqrt{\tan(dx+c)})}\right) - \sqrt{2}(i+1)A+(i-1)B\log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1}{\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c)+1}\right) + \sqrt{2}(i+1)A+(i-1)B\log\left(\frac{-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c)+1}\right)}{2d} a^2 + \frac{4Aa^2}{\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,algorithm="maxima")`

[Out]  $-\frac{1}{2} (4B a^2 \sqrt{\tan(dx+c)} - (2\sqrt{2}((I-1)A+(I+1)B) \arctan\left(\frac{1/2\sqrt{2}(\sqrt{2}+\sqrt{\tan(dx+c)})}{\sqrt{2}-2\sqrt{\tan(dx+c)}}\right) + 2\sqrt{2}((I-1)A+(I+1)B) \arctan\left(\frac{-1/2\sqrt{2}(\sqrt{2}-\sqrt{\tan(dx+c)})}{\sqrt{2}+2\sqrt{\tan(dx+c)}}\right) - \sqrt{2}((-I+1)A+(I-1)B) \log(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2}((-I+1)A+(I-1)B) \log(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1)) a^2 + 4A a^2 / \sqrt{\tan(dx+c)}) / d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs.  $2(82) = 164$ .

time = 1.65, size = 386, normalized size = 3.94

$$\frac{\sqrt{-1A^2-2AB+1B^2} (d^{2A+2B+d} - d) \log\left(\frac{1 + \frac{1}{d} \sqrt{-1A^2-2AB+1B^2} (d^{2A+2B+d} - d) \sqrt{\frac{d^{2A+2B+d} + 1}{d^{2A+2B+d} + 1}}}{1 + \frac{1}{d} \sqrt{-1A^2-2AB+1B^2} (d^{2A+2B+d} - d)}\right) - \sqrt{-1A^2-2AB+1B^2} (d^{2A+2B+d} - d) \log\left(\frac{1 + \frac{1}{d} \sqrt{-1A^2-2AB+1B^2} (d^{2A+2B+d} - d) \sqrt{\frac{d^{2A+2B+d} + 1}{d^{2A+2B+d} + 1}}}{1 + \frac{1}{d} \sqrt{-1A^2-2AB+1B^2} (d^{2A+2B+d} - d)}\right) - 2((A+B)d^{2A+2B} + (A-B)d^A) \sqrt{\frac{d^{2A+2B+d} + 1}{d^{2A+2B+d} + 1}}}{d^{2A+2B+d} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] (sqrt(-1\*A^2 - 2\*A\*B + I\*B^2)\*a^4/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*log(-2\*((A - I\*B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) + sqrt(-1\*A^2 - 2\*A\*B + I\*B^2)\*a^4/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))\*e^(-2\*I\*d\*x - 2\*I\*c)/((-I\*A - B)\*a^2) - sqrt(-1\*A^2 - 2\*A\*B + I\*B^2)\*a^4/d^2\*(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*log(-2\*((A - I\*B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) - sqrt(-1\*A^2 - 2\*A\*B + I\*B^2)\*a^4/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))\*e^(-2\*I\*d\*x - 2\*I\*c)/((-I\*A - B)\*a^2) - 2\*((I\*A + B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) + (I\*A - B)\*a^2)\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))/(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int \left( -\frac{A}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int A \sqrt{\tan(c+dx)} dx + \int \left( -\frac{B}{\sqrt{\tan(c+dx)}} \right) dx + \int B \tan^{\frac{3}{2}}(c+dx) dx + \int \left( -\frac{2iA}{\sqrt{\tan(c+dx)}} \right) dx + \int (-2iB \sqrt{\tan(c+dx)}) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x)

[Out] -a\*\*2\*(Integral(-A/tan(c + d\*x)\*\*(3/2), x) + Integral(A\*sqrt(tan(c + d\*x)), x) + Integral(-B/sqrt(tan(c + d\*x)), x) + Integral(B\*tan(c + d\*x)\*\*(3/2), x) + Integral(-2\*I\*A/sqrt(tan(c + d\*x)), x) + Integral(-2\*I\*B\*sqrt(tan(c + d\*x)), x))

**Giac** [A]

time = 0.89, size = 70, normalized size = 0.71

$$\frac{2Ba^2\sqrt{\tan(dx+c)}}{d} - \frac{(2i+2)\sqrt{2}(-iAa^2 - Ba^2)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2Aa^2}{d\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] -2\*B\*a^2\*sqrt(tan(d\*x + c))/d - (2\*I + 2)\*sqrt(2)\*(-I\*A\*a^2 - B\*a^2)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d - 2\*A\*a^2/(d\*sqrt(tan(d\*x + c)))

**Mupad [B]**

time = 6.72, size = 203, normalized size = 2.07

$$\frac{2Aa^2}{d\sqrt{\tan(c+dx)}} - \frac{2Ba^2\sqrt{\tan(c+dx)}}{d} + \frac{\sqrt{2}Aa^2\ln(-4Aa^2d + \sqrt{2}Aa^2d\sqrt{\tan(c+dx)}(-2-2i))(1+i)}{d} - \frac{\sqrt{4i}Aa^2\ln(-4Aa^2d + 2\sqrt{4i}Aa^2d\sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^2\ln(Ba^2d4i + \sqrt{2}Ba^2d\sqrt{\tan(c+dx)}(-2+2i))(1-i)}{d} - \frac{\sqrt{-4i}Ba^2\ln(Ba^2d4i + 2\sqrt{-4i}Ba^2d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2)/tan(c + d\*x)^(3/2),x)

[Out]  $(2^{(1/2)}*A*a^2*\log(-4*A*a^2*d - 2^{(1/2)}*A*a^2*d*\tan(c + d*x)^{(1/2)}*(2 + 2i))*(1 + 1i))/d - (2*B*a^2*\tan(c + d*x)^{(1/2)})/d - (2*A*a^2)/(d*\tan(c + d*x)^{(1/2)}) - (4i^{(1/2)}*A*a^2*\log(2*4i^{(1/2)}*A*a^2*d*\tan(c + d*x)^{(1/2)} - 4*A*a^2*d))/d + (2^{(1/2)}*B*a^2*\log(B*a^2*d*4i - 2^{(1/2)}*B*a^2*d*\tan(c + d*x)^{(1/2)}*(2 - 2i))*(1 - 1i))/d - ((-4i)^{(1/2)}*B*a^2*\log(B*a^2*d*4i + 2*(-4i)^{(1/2)}*B*a^2*d*\tan(c + d*x)^{(1/2)}))/d$



$$3.124 \quad \int \frac{(a+ia \tan(c+dx))^2 (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=102

$$\frac{4\sqrt[4]{-1} a^2 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2 (5iA + 3B)}{3d \sqrt{\tan(c+dx)}} - \frac{2A(a^2 + ia^2 \tan(c+dx))}{3d \tan^{\frac{3}{2}}(c+dx)}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-2/3*a^2*(5*I*A+3*B)/d/\tan(d*x+c)^{(1/2)}-2/3*A*(a^2+I*a^2*\tan(d*x+c))/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3674, 3672, 3614, 211}

$$\frac{4\sqrt[4]{-1} a^2 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2 (3B + 5iA)}{3d \sqrt{\tan(c+dx)}} - \frac{2A(a^2 + ia^2 \tan(c+dx))}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{(5/2)}}, x]$

[Out]  $(4*(-1)^{(1/4)}*a^2*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d - (2*a^2*((5*I)*A + 3*B))/(3*d*\text{Sqrt}[\text{Tan}[c + d*x]]) - (2*A*(a^2 + I*a^2*\text{Tan}[c + d*x]))/(3*d*\text{Tan}[c + d*x]^{(3/2)})$

Rule 211

$\text{Int}[\frac{(a_) + (b_)*(x_)^2}{(x_)^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[a/b, 2]}{a}*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 3614

$\text{Int}[\frac{(c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)]}{\text{Sqrt}[(b_)*\text{tan}[(e_) + (f_)*(x_)])}], x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

Rule 3672

$\text{Int}[\frac{(a_) + (b_)*\text{tan}[(e_) + (f_)*(x_) ]^m * ((A_) + (B_)*\text{tan}[(e_) + (f_)*(x_)])}{(c_) + (d_)*\text{tan}[(e_) + (f_)*(x_)]}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{m+1}/(b*f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[m,$

-1] && NeQ[a^2 + b^2, 0]

### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2A(a^2 + ia^2 \tan(c + dx))}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{(a + ia \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= -\frac{2a^2(5iA + 3B)}{3d \sqrt{\tan(c + dx)}} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{1}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= -\frac{2a^2(5iA + 3B)}{3d \sqrt{\tan(c + dx)}} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{(12a^4)}{3d} \int \frac{1}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{4\sqrt{-1} a^2 (A - iB) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} - \frac{2a^2}{3d} \int \frac{1}{\tan^{\frac{3}{2}}(c + dx)} dx \end{aligned}$$

### Mathematica [A]

time = 2.13, size = 96, normalized size = 0.94

$$\frac{2a^2 \left( 6iA + 3B + A \cot(c + dx) - 6i(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) \sqrt{i \tan(c + dx)} \right)}{3d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2), x]
```

```
[Out] (-2*a^2*((6*I)*A + 3*B + A*Cot[c + d*x] - (6*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[I*Tan[c + d*x]])/(3*d*Sqrt[Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs.  $2(85) = 170$ .  
time = 0.04, size = 222, normalized size = 2.18

method	result
derivativedivides	$a^2 \left( -\frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(2iA+B)}{\sqrt{\tan(dx+c)}} + \frac{(2iB-2A)\sqrt{2}}{4} \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2 \arctan \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) \right)$
default	$a^2 \left( -\frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(2iA+B)}{\sqrt{\tan(dx+c)}} + \frac{(2iB-2A)\sqrt{2}}{4} \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2 \arctan \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURN  
VERBOSE)`

[Out]  $1/d*a^2*(-2/3*A/\tan(dx+c)^{(3/2)}-2*(2*I*A+B)/\tan(dx+c)^{(1/2)}+1/4*(-2*A+2*I*B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c))/(1-2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c)))+2*\arctan(1+2^{(1/2)}*\tan(dx+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(dx+c)^{(1/2)}))+1/4*(-2*B-2*I*A)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c))/(1+2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c)))+2*\arctan(1+2^{(1/2)}*\tan(dx+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(dx+c)^{(1/2)}))$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs.  $2(82) = 164$ .  
time = 0.55, size = 177, normalized size = 1.74

$$\frac{3(2\sqrt{2}(-i+1)A+(i-1)B)\arctan\left(\frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}\right)+2\sqrt{2}(-i+1)A+(i-1)B\arctan\left(\frac{-1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}\right)+\sqrt{2}(i-1)A+(i+1)B\log\left(\frac{\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)+1)}{\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)-1)}\right)-\sqrt{2}(i-1)A+(i+1)B\log\left(\frac{-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)+1)}{\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)-1)}\right)+\frac{4(2i-2A-2B)\sqrt{\tan(dx+c)-a^2}}{\tan(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")`

[Out]  $1/6*(3*(2*\sqrt{2})*(-(I+1)*A+(I-1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)}))+2*\sqrt{2})*(-(I+1)*A+(I-1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)}))+\sqrt{2})*((I-1)*A+(I+1)*B)*\log(\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)-\sqrt{2})*((I-1)*A+(I+1)*B)*\log(-\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)*a^2+4*(3*(-2*I*A-B)*a^2*\tan(dx+c)-A*a^2)/\tan(dx+c)^{(3/2)}/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 441 vs.  $2(82) = 164$ .  
time = 1.85, size = 441, normalized size = 4.32

$$\frac{3\sqrt{\frac{U^2+2AB-B^2}{d^2}} \log\left(\frac{\sqrt{\frac{U^2+2AB-B^2}{d^2}} \sqrt{\frac{-1+e^{2I*d*x+2I*c}}{2d^{2I*d*x+2I*c}+1}}}{\sqrt{\frac{U^2+2AB-B^2}{d^2}}}\right) - 3\sqrt{\frac{U^2+2AB-B^2}{d^2}} \log\left(\frac{\sqrt{\frac{U^2+2AB-B^2}{d^2}} \sqrt{\frac{-1+e^{2I*d*x+2I*c}}{2d^{2I*d*x+2I*c}+1}}}{-1+e^{2I*d*x+2I*c}}\right) - 2\left(\frac{7A-3I*B}{d}\sqrt{\frac{U^2+2AB-B^2}{d^2}} + 2A\sqrt{\frac{U^2+2AB-B^2}{d^2}} - (5A-3I*B)\sqrt{\frac{U^2+2AB-B^2}{d^2}}\right) \sqrt{\frac{-1+e^{2I*d*x+2I*c}}{2d^{2I*d*x+2I*c}+1}}}{3(d^{2I*d*x+2I*c}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="fricas")

[Out]  $-1/3*(3*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2}*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} + \sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2) - 3*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2}*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^2*e^{(2*I*d*x + 2*I*c)} + \sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2}*(-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^2) - 2*((7*A - 3*I*B)*a^2*e^{(4*I*d*x + 4*I*c)} + 2*A*a^2*e^{(2*I*d*x + 2*I*c)} - (5*A - 3*I*B)*a^2)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int \left( -\frac{A}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{A}{\sqrt{\tan(c+dx)}} dx + \int \left( -\frac{B}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int B \sqrt{\tan(c+dx)} dx + \int \left( -\frac{2iA}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \left( -\frac{2iB}{\sqrt{\tan(c+dx)}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x)

[Out]  $-a**2*(Integral(-A/tan(c + d*x)**(5/2), x) + Integral(A/sqrt(tan(c + d*x)), x) + Integral(-B/tan(c + d*x)**(3/2), x) + Integral(B*sqrt(tan(c + d*x)), x) + Integral(-2*I*A/tan(c + d*x)**(3/2), x) + Integral(-2*I*B/sqrt(tan(c + d*x)), x))$

**Giac [A]**

time = 0.98, size = 79, normalized size = 0.77

$$\frac{(2i-2)\sqrt{2}(-iAa^2 - Ba^2)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2(6iAa^2 \tan(dx+c) + 3Ba^2 \tan(dx+c) + Aa^2)}{3d \tan(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out]  $-(2*I - 2)*\sqrt{2}*(-I*A*a^2 - B*a^2)*\arctan(-(1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(dx + c)})/d - 2/3*(6*I*A*a^2*\tan(dx + c) + 3*B*a^2*\tan(dx + c) + A*a^2)/(d*\tan(dx + c)^{(3/2)})$

**Mupad [B]**

time = 7.12, size = 222, normalized size = 2.18

$$\frac{2Aa^2 + \frac{A^2 \tan(c+dx)}{\tan(c+dx)^{3/2}}}{d \sqrt{\tan(c+dx)}} + \frac{2Ba^2}{d \sqrt{\tan(c+dx)}} + \frac{\sqrt{2} A a^2 \ln(-A a^2 d 4i + \sqrt{2} A a^2 d \sqrt{\tan(c+dx)} (-2+2i)) (1-i)}{d} - \frac{\sqrt{-4} A a^2 \ln(-A a^2 d 4i + 2\sqrt{-4} A a^2 d \sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2} B a^2 \ln(-4 B a^2 d + \sqrt{2} B a^2 d \sqrt{\tan(c+dx)} (-2-2i)) (1+i)}{d} - \frac{\sqrt{4} B a^2 \ln(-4 B a^2 d + 2\sqrt{4} B a^2 d \sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*1i)^2)/\tan(c + d*x)^{(5/2)},x)$

[Out]  $(2^{(1/2)}*A*a^2*\log(-A*a^2*d*4i - 2^{(1/2)}*A*a^2*d*\tan(c + d*x)^{(1/2)}*(2 - 2i))*(1 - 1i))/d - (2*B*a^2)/(d*\tan(c + d*x)^{(1/2)}) - ((2*A*a^2)/(3*d) + (A*a^2*\tan(c + d*x)*4i)/d)/\tan(c + d*x)^{(3/2)} - ((-4i)^{(1/2)}*A*a^2*\log(2*(-4i)^{(1/2)}*A*a^2*d*\tan(c + d*x)^{(1/2)} - A*a^2*d*4i))/d + (2^{(1/2)}*B*a^2*\log(-4*B*a^2*d - 2^{(1/2)}*B*a^2*d*\tan(c + d*x)^{(1/2)}*(2 + 2i))*(1 + 1i))/d - (4i^{(1/2)}*B*a^2*\log(2*4i^{(1/2)}*B*a^2*d*\tan(c + d*x)^{(1/2)} - 4*B*a^2*d))/d$

$$3.125 \quad \int \frac{(a+ia \tan(c+dx))^2 (A+B \tan(c+dx))}{\tan^7(c+dx)} dx$$

**Optimal.** Leaf size=127

$$\frac{4\sqrt[4]{-1} a^2 (iA + B) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2 (7iA + 5B)}{15d \tan^{3/2}(c+dx)} + \frac{4a^2 (A - iB)}{d \sqrt{\tan(c+dx)}} - \frac{2A(a^2 + ia^2 \tan(c+dx))}{5d \tan^{5/2}(c+dx)}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+4*a^2*(A-I*B)/d/\tan(d*x+c)^{(1/2)}-2/15*a^2*(7*I*A+5*B)/d/\tan(d*x+c)^{(3/2)}-2/5*A*(a^2+I*a^2*\tan(d*x+c))/d/\tan(d*x+c)^{(5/2)}$

**Rubi [A]**

time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ ,

Rules used = {3674, 3672, 3610, 3614, 211}

$$\frac{4\sqrt[4]{-1} a^2 (B + iA) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2 (5B + 7iA)}{15d \tan^{3/2}(c+dx)} + \frac{4a^2 (A - iB)}{d \sqrt{\tan(c+dx)}} - \frac{2A(a^2 + ia^2 \tan(c+dx))}{5d \tan^{5/2}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x])]/\operatorname{Tan}[c + d*x]^{(7/2)}, x]$

[Out]  $(4*(-1)^{(1/4)}*a^2*(I*A + B)*\operatorname{ArcTan}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/d - (2*a^2*((7*I)*A + 5*B))/(15*d*\operatorname{Tan}[c + d*x]^{(3/2)}) + (4*a^2*(A - I*B))/(d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) - (2*A*(a^2 + I*a^2*\operatorname{Tan}[c + d*x]))/(5*d*\operatorname{Tan}[c + d*x]^{(5/2)})$

Rule 211

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2])/a]*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 3610

$\operatorname{Int}[(a_) + (b_)*\operatorname{tan}[e_] + (f_)*(x_)]^{(m_)*((c_) + (d_)*\operatorname{tan}[e_] + (f_)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 + b^2))), x] + \operatorname{Dist}[1/(a^2 + b^2), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m + 1)}*\operatorname{Simp}[a*c + b*d - (b*c - a*d)*\operatorname{Tan}[e + f*x], x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \ \operatorname{LtQ}[m, -1]$

Rule 3614

$\operatorname{Int}[(c_) + (d_)*\operatorname{tan}[e_] + (f_)*(x_)]/\operatorname{Sqrt}[(b_)*\operatorname{tan}[e_] + (f_)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \ \operatorname{EqQ}[c^2 + d^2, 0]$

## Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

## Rule 3674

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2A(a^2 + ia^2 \tan(c + dx))}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + ia \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx \\ &= -\frac{2a^2(7iA + 5B)}{15d \tan^{\frac{3}{2}}(c + dx)} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + ia \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= -\frac{2a^2(7iA + 5B)}{15d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2(A - iB)}{d \sqrt{\tan(c + dx)}} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{5d \tan^{\frac{5}{2}}(c + dx)} \\ &= -\frac{2a^2(7iA + 5B)}{15d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2(A - iB)}{d \sqrt{\tan(c + dx)}} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{5d \tan^{\frac{5}{2}}(c + dx)} \\ &= \frac{4\sqrt{-1} a^2 (iA + B) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{5d \tan^{\frac{5}{2}}(c + dx)} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 272 vs.  $2(127) = 254$ .

time = 3.98, size = 272, normalized size = 2.14

$$\cos^3(c + dx) \left( \frac{4i(A-iB)e^{-2ic} \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) - \frac{\cos^2(c+dx)(\cos(2c) - i \sin(2c))(-27A+30iB+(33A-30iB)\cos(2(c+dx))+5(2iA+B)\sin(2(c+dx)))}{15\sqrt{\tan(c+dx)}}}{\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}} \right) (a + ia \tan(c + dx))^2 (A + B \tan(c + dx))$$


---


$$d(\cos(dx) + i \sin(dx))^2 (A \cos(c + dx) + B \sin(c + dx))$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] (Cos[c + d\*x]^3\*(((4\*I)\*(A - I\*B)\*Sqrt[(-1)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))])/(E^((2\*I)\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))) - (Csc[c + d\*x]^2\*(Cos[2\*c] - I\*Sin[2\*c])\*(-27\*A + (30\*I)\*B + (33\*A - (30\*I)\*B)\*Cos[2\*(c + d\*x)] + 5\*((2\*I)\*A + B)\*Sin[2\*(c + d\*x)]))/(15\*Sqrt[Tan[c + d\*x]])\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x))/(d\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(107) = 214.  
time = 0.05, size = 240, normalized size = 1.89

method	result
derivativedivides	$a^2 \left( -\frac{2A}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2(2iB-2A)}{\sqrt{\tan(dx+c)}} - \frac{2(2iA+B)}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{(-2iA-2B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)\right)} \right)$
default	$a^2 \left( -\frac{2A}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2(2iB-2A)}{\sqrt{\tan(dx+c)}} - \frac{2(2iA+B)}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{(-2iA-2B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)\right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2), x, method=\_RETURN VERBOSE)

[Out] 1/d\*a^2\*(-2/5\*A/tan(d\*x+c)^(5/2)-2\*(-2\*A+2\*I\*B)/tan(d\*x+c)^(1/2)-2/3\*(2\*I\*A+B)/tan(d\*x+c)^(3/2)+1/4\*(-2\*B-2\*I\*A)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(2\*A-2\*I\*B)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))

**Maxima [A]**

time = 0.51, size = 195, normalized size = 1.54

$\frac{15(2\sqrt{x}(-1)A+(i+1)B)\arctan\left(\frac{1}{2}\sqrt{x}(\sqrt{x}+2\sqrt{\tan(dx+c)})\right)+2\sqrt{x}((-1)A+(i+1)B)\arctan\left(-\frac{1}{2}\sqrt{x}(\sqrt{x}-2\sqrt{\tan(dx+c)})\right)-\sqrt{x}((-i+1)A+(i-1)B)\log\left(\sqrt{x}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{x}((-i+1)A+(i-1)B)\log\left(-\sqrt{x}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{30d} + \frac{1}{30d} \frac{(-2iA-2B)\sqrt{2}\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)\right)}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)\right)}$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] 
$$-1/30*(15*(2*\sqrt{2})*((I-1)*A+(I+1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)})) + 2*\sqrt{2}*((I-1)*A+(I+1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)})) - \sqrt{2}*(-(I+1)*A+(I-1)*B)*\log(\sqrt{2}*\sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2}*(-(I+1)*A+(I-1)*B)*\log(-\sqrt{2}*\sqrt{\tan(dx+c)} + \tan(dx+c) + 1))*a^2 - 4*(30*(A-I*B)*a^2*\tan(dx+c)^2 + 5*(-2*I*A-B)*a^2*\tan(dx+c) - 3*A*a^2)/\tan(dx+c)^{(5/2)}/d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs.  $2(103) = 206$ .

time = 2.69, size = 502, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 
$$-1/15*(15*\sqrt{-(-I*A^2-2*A*B+I*B^2)}*a^4/d^2)*(d*e^{(6*I*d*x+6*I*c)} - 3*d*e^{(4*I*d*x+4*I*c)} + 3*d*e^{(2*I*d*x+2*I*c)} - d)*\log(-2*((A-I*B)*a^2*e^{(2*I*d*x+2*I*c)} + \sqrt{-(-I*A^2-2*A*B+I*B^2)}*a^4/d^2)*(d*e^{(2*I*d*x+2*I*c)} + d)*\sqrt{(-I*e^{(2*I*d*x+2*I*c)} + I)/(e^{(2*I*d*x+2*I*c)} + 1)}))e^{(-2*I*d*x-2*I*c)}/((-I*A-B)*a^2) - 15*\sqrt{-(-I*A^2-2*A*B+I*B^2)}*a^4/d^2*(d*e^{(6*I*d*x+6*I*c)} - 3*d*e^{(4*I*d*x+4*I*c)} + 3*d*e^{(2*I*d*x+2*I*c)} - d)*\log(-2*((A-I*B)*a^2*e^{(2*I*d*x+2*I*c)} - \sqrt{-(-I*A^2-2*A*B+I*B^2)}*a^4/d^2)*(d*e^{(2*I*d*x+2*I*c)} + d)*\sqrt{(-I*e^{(2*I*d*x+2*I*c)} + I)/(e^{(2*I*d*x+2*I*c)} + 1)}))e^{(-2*I*d*x-2*I*c)}/((-I*A-B)*a^2) + 2*((-43*I*A-35*B)*a^2*e^{(6*I*d*x+6*I*c)} + (11*I*A+25*B)*a^2*e^{(4*I*d*x+4*I*c)} + (31*I*A+35*B)*a^2*e^{(2*I*d*x+2*I*c)} + (-23*I*A-25*B)*a^2)*\sqrt{(-I*e^{(2*I*d*x+2*I*c)} + I)/(e^{(2*I*d*x+2*I*c)} + 1)}}/(d*e^{(6*I*d*x+6*I*c)} - 3*d*e^{(4*I*d*x+4*I*c)} + 3*d*e^{(2*I*d*x+2*I*c)} - d)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int \left( -\frac{A}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{A}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{B}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{B}{\sqrt{\tan(c+dx)}} dx + \int \left( -\frac{2iA}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \left( -\frac{2iB}{\tan^{\frac{3}{2}}(c+dx)} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out]  $-a^{**2}*(Integral(-A/\tan(c + d*x)**(7/2), x) + Integral(A/\tan(c + d*x)**(3/2), x) + Integral(-B/\tan(c + d*x)**(5/2), x) + Integral(B/\sqrt{\tan(c + d*x)}, x) + Integral(-2*I*A/\tan(c + d*x)**(5/2), x) + Integral(-2*I*B/\tan(c + d*x)**(3/2), x))$

**Giac** [A]

time = 1.21, size = 108, normalized size = 0.85

$$-\frac{(2i+2)\sqrt{2}(-iAa^2 - Ba^2)\arctan\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} + \frac{2(30Aa^2\tan(dx+c)^2 - 30iBa^2\tan(dx+c)^2 - 10iAa^2\tan(dx+c) - 5Ba^2\tan(dx+c) - 3Aa^2)}{15d\tan(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="giac")`

[Out]  $-(2*I + 2)*\sqrt{2}*(-I*A*a^2 - B*a^2)*\arctan((1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(dx+c)})/d + 2/15*(30*A*a^2*\tan(dx+c)^2 - 30*I*B*a^2*\tan(dx+c)^2 - 10*I*A*a^2*\tan(dx+c) - 5*B*a^2*\tan(dx+c) - 3*A*a^2)/(d*\tan(dx+c)^(5/2))$

**Mupad** [B]

time = 7.99, size = 258, normalized size = 2.03

$$\frac{4Aa^2 - 4A^2\log(\tan(c+dx)) + 2A^2\log(\tan(c+dx))}{\tan(c+dx)^{7/2}} - \frac{4Aa^2 + 2A^2\log(\tan(c+dx))}{\tan(c+dx)^{5/2}} + \frac{\sqrt{2}Aa^2\ln(4Aa^2d + \sqrt{2}Aa^2d\sqrt{\tan(c+dx)}(-2-2i))(1+i)}{d} + \frac{\sqrt{2}Aa^2\ln(4Aa^2d + 2\sqrt{2}Aa^2d\sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^2\ln(-Ba^2d + \sqrt{2}Ba^2d\sqrt{\tan(c+dx)}(-2+2i))(1-i)}{d} - \frac{\sqrt{-4}Ba^2\ln(-Ba^2d + 2\sqrt{-4}Ba^2d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/tan(c + d*x)^(7/2),x)`

[Out]  $(2^{(1/2)}*A*a^2*\log(4*A*a^2*d - 2^{(1/2)}*A*a^2*d*\tan(c + d*x)^{(1/2)}*(2 + 2i))*(1 + 1i))/d - ((2*B*a^2)/(3*d) + (B*a^2*\tan(c + d*x)*4i)/d)/\tan(c + d*x)^{(3/2)} - ((2*A*a^2)/(5*d) + (A*a^2*\tan(c + d*x)*4i)/(3*d) - (4*A*a^2*\tan(c + d*x)^2)/d)/\tan(c + d*x)^{(5/2)} - (4i^{(1/2)}*A*a^2*\log(4*A*a^2*d + 2*4i^{(1/2)}*A*a^2*d*\tan(c + d*x)^{(1/2)}))/d + (2^{(1/2)}*B*a^2*\log(-B*a^2*d*4i - 2^{(1/2)}*B*a^2*d*\tan(c + d*x)^{(1/2)}*(2 - 2i))*(1 - 1i))/d - ((-4i)^{(1/2)}*B*a^2*\log(2*(-4i)^{(1/2)}*B*a^2*d*\tan(c + d*x)^{(1/2)} - B*a^2*d*4i))/d$

$$3.126 \quad \int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=154

$$-\frac{4\sqrt[4]{-1} a^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(9iA+7B)}{35d \tan^{\frac{5}{2}}(c+dx)} + \frac{4a^2(A-iB)}{3d \tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(iA+B)}{d\sqrt{\tan(c+dx)}}$$

[Out]  $-4*(-1)^{(1/4)}*a^2*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+4*a^2*(I*A+B)/d/\tan(d*x+c)^{(1/2)}-2/35*a^2*(9*I*A+7*B)/d/\tan(d*x+c)^{(5/2)}+4/3*a^2*(A-I*B)/d/\tan(d*x+c)^{(3/2)}-2/7*A*(a^2+I*a^2*\tan(d*x+c))/d/\tan(d*x+c)^{(7/2)}$

**Rubi** [A]

time = 0.21, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3674, 3672, 3610, 3614, 211}

$$-\frac{4\sqrt[4]{-1} a^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{4a^2(A-iB)}{3d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2(7B+9iA)}{35d \tan^{\frac{5}{2}}(c+dx)} + \frac{4a^2(B+iA)}{d\sqrt{\tan(c+dx)}} - \frac{2A(a^2+ia^2 \tan(c+dx))}{7d \tan^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\left((a + I*a*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x])\right)/\text{Tan}[c + d*x]^{(9/2)}, x]$

[Out]  $(-4*(-1)^{(1/4)}*a^2*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d - (2*a^2*((9*I)*A + 7*B))/(35*d*\text{Tan}[c + d*x]^{(5/2)}) + (4*a^2*(A - I*B))/(3*d*\text{Tan}[c + d*x]^{(3/2)}) + (4*a^2*(I*A + B))/(d*\text{Sqrt}[\text{Tan}[c + d*x]]) - (2*A*(a^2 + I*a^2*\text{Tan}[c + d*x]))/(7*d*\text{Tan}[c + d*x]^{(7/2)})$

Rule 211

$\text{Int}[\left((a_) + (b_)*(x_)^2\right)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Rt}[a/b, 2]/a*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rule 3610

$\text{Int}[\left((a_) + (b_)*\tan[(e_) + (f_)*(x_)]\right)^{(m_)*\left((c_) + (d_)*\tan[(e_) + (f_)*(x_)]\right)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\left((a + b*\text{Tan}[e + f*x])^{(m+1)}/(f*(m+1)*(a^2 + b^2))\right), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3614

$\text{Int}[\left((c_) + (d_)*\tan[(e_) + (f_)*(x_)]\right)/\text{Sqrt}[(b_)*\tan[(e_) + (f_)*(x_)]], x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*c$

$\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3672

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] :> \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 + b^2))], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 3674

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(n_.)}), x\_Symbol] :> \text{Simp}[(-a^2)*(B*c - A*d)*(a + b*\text{Tan}[e + f*x])^{(m - 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(d*f*(b*c + a*d)*(n + 1))], x] - \text{Dist}[a/(d*(b*c + a*d)*(n + 1)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx &= -\frac{2A(a^2 + ia^2 \tan(c + dx))}{7d \tan^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{(a + ia \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx \\ &= -\frac{2a^2(9iA + 7B)}{35d \tan^{\frac{5}{2}}(c + dx)} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{7d \tan^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{(a + ia \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx \\ &= -\frac{2a^2(9iA + 7B)}{35d \tan^{\frac{5}{2}}(c + dx)} + \frac{4a^2(A - iB)}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{7d \tan^{\frac{7}{2}}(c + dx)} \\ &= -\frac{2a^2(9iA + 7B)}{35d \tan^{\frac{5}{2}}(c + dx)} + \frac{4a^2(A - iB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2(iA + B)}{d \sqrt{\tan(c + dx)}} \\ &= -\frac{2a^2(9iA + 7B)}{35d \tan^{\frac{5}{2}}(c + dx)} + \frac{4a^2(A - iB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2(iA + B)}{d \sqrt{\tan(c + dx)}} \\ &= -\frac{4\sqrt{-1} a^2 (A - iB) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} - \frac{2A(a^2 + ia^2 \tan(c + dx))}{7d \tan^{\frac{7}{2}}(c + dx)} \end{aligned}$$

**Mathematica [A]**

time = 5.48, size = 296, normalized size = 1.92

$$\frac{\cos^3(c+dx) \left( \frac{4(A-1B)e^{-2ix} \sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{tanh}^{-1} \left( \sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \right) - \frac{\operatorname{sech}^2(c+dx) \cos(2c) - i \sin(2c) ((-25A+70iB) \cos(c+dx) + (85A-70iB) \cos(3c+dx)) + 42(-8iA-9B+(12iA+11iB) \cos(2c+dx)) \sin(c+dx)}{210 \sqrt{\tan(c+dx)}}}{\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}} \right) (a+ia \tan(c+dx))^2 (A+B \tan(c+dx))}{d(\cos(dx)+i \sin(dx))^2 (A \cos(c+dx)+B \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] (Cos[c + d\*x]^3\*((4\*(A - I\*B)\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))]\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]])/(E^((2\*I)\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))])) - (Csc[c + d\*x]^3\*(Cos[2\*c] - I\*Sin[2\*c])\*((-25\*A + (70\*I)\*B)\*Cos[c + d\*x] + (85\*A - (70\*I)\*B)\*Cos[3\*(c + d\*x)] + 42\*((-8\*I)\*A - 9\*B + ((12\*I)\*A + 11\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x]))/(210\*Sqrt[Tan[c + d\*x]]))\* (a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/(d\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

Maple [A]

time = 0.04, size = 258, normalized size = 1.68

method	result
derivativedivides	$a^2 \left( -\frac{2A}{7 \tan(dx+c)^{\frac{7}{2}}} - \frac{2(2iB-2A)}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(-2iA-2B)}{\sqrt{\tan(dx+c)}} - \frac{2(2iA+B)}{5 \tan(dx+c)^{\frac{5}{2}}} + \frac{(-2iB+2A)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}} \left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right)\right)} \right)$
default	$a^2 \left( -\frac{2A}{7 \tan(dx+c)^{\frac{7}{2}}} - \frac{2(2iB-2A)}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(-2iA-2B)}{\sqrt{\tan(dx+c)}} - \frac{2(2iA+B)}{5 \tan(dx+c)^{\frac{5}{2}}} + \frac{(-2iB+2A)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}} \left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right)\right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2), x, method=\_RETURN VERBOSE)

[Out] 1/d\*a^2\*(-2/7\*A/tan(d\*x+c)^(7/2)-2/3\*(-2\*A+2\*I\*B)/tan(d\*x+c)^(3/2)-2\*(-2\*B-2\*I\*A)/tan(d\*x+c)^(1/2)-2/5\*(2\*I\*A+B)/tan(d\*x+c)^(5/2)+1/4\*(2\*A-2\*I\*B)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(2\*B+2\*I\*A)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)^(1/2)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))



**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int \left( -\frac{A}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{A}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{B}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{B}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{2iA}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \left( -\frac{2iB}{\tan^{\frac{3}{2}}(c+dx)} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(9/2),x)

**[Out]** -a\*\*2\*(Integral(-A/tan(c + d\*x)\*\*(9/2), x) + Integral(A/tan(c + d\*x)\*\*(5/2), x) + Integral(-B/tan(c + d\*x)\*\*(7/2), x) + Integral(B/tan(c + d\*x)\*\*(3/2), x) + Integral(-2\*I\*A/tan(c + d\*x)\*\*(7/2), x) + Integral(-2\*I\*B/tan(c + d\*x)\*\*(5/2), x))

**Giac [A]**

time = 1.19, size = 136, normalized size = 0.88

$$\frac{(2i-2)\sqrt{2}(-iAa^2 - Ba^2)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2(-210iAa^2\tan(dx+c)^3 - 210Ba^2\tan(dx+c)^3 - 70Aa^2\tan(dx+c)^2 + 70iBa^2\tan(dx+c)^2 + 42iAa^2\tan(dx+c) + 21Ba^2\tan(dx+c) + 15Aa^2)}{105d\tan(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="giac")

**[Out]** (2\*I - 2)\*sqrt(2)\*(-I\*A\*a^2 - B\*a^2)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d - 2/105\*(-210\*I\*A\*a^2\*tan(d\*x + c)^3 - 210\*B\*a^2\*tan(d\*x + c)^3 - 70\*A\*a^2\*tan(d\*x + c)^2 + 70\*I\*B\*a^2\*tan(d\*x + c)^2 + 42\*I\*A\*a^2\*tan(d\*x + c) + 21\*B\*a^2\*tan(d\*x + c) + 15\*A\*a^2)/(d\*tan(d\*x + c)^(7/2))

**Mupad [B]**

time = 9.52, size = 293, normalized size = 1.90

$$\frac{\frac{1}{\tan(c+dx)^{\frac{9}{2}}} + \frac{A^2 \operatorname{Im}(\ln(\frac{1+i}{1-i} \sqrt{\tan(dx+c)}))}{\tan(c+dx)^{\frac{9}{2}}} - \frac{A^2 \operatorname{Im}(\ln(\frac{1+i}{1-i} \sqrt{\tan(dx+c)}))}{\tan(c+dx)^{\frac{9}{2}}}}{\tan(c+dx)^{\frac{9}{2}}} + \frac{\sqrt{2} A^2 \ln(A^2 d d + \sqrt{2} A^2 d \sqrt{\tan(c+dx)} (-2+2i)) (1-i)}{d} - \frac{\sqrt{-2} A^2 \ln(A^2 d d + 2\sqrt{-2} A^2 d \sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2} B^2 \ln(4 B^2 d + \sqrt{2} B^2 d \sqrt{\tan(c+dx)} (-2-2i)) (1+1i)}{d} + \frac{\sqrt{-2} B^2 \ln(4 B^2 d + 2\sqrt{-2} B^2 d \sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2)/tan(c + d\*x)^(9/2),x)

**[Out]** (2^(1/2)\*A\*a^2\*log(A\*a^2\*d\*4i - 2^(1/2)\*A\*a^2\*d\*tan(c + d\*x)^(1/2)\*(2 - 2i))\*(1 - 1i))/d - ((2\*B\*a^2)/(5\*d) + (B\*a^2\*tan(c + d\*x)\*4i)/(3\*d) - (4\*B\*a^2\*tan(c + d\*x)^2)/d)/tan(c + d\*x)^(5/2) - ((2\*A\*a^2)/(7\*d) + (A\*a^2\*tan(c + d\*x)\*4i)/(5\*d) - (4\*A\*a^2\*tan(c + d\*x)^2)/(3\*d) - (A\*a^2\*tan(c + d\*x)^3\*4i)/d)/tan(c + d\*x)^(7/2) - ((-4i)^(1/2)\*A\*a^2\*log(A\*a^2\*d\*4i + 2\*(-4i)^(1/2)\*A\*a^2\*d\*tan(c + d\*x)^(1/2)))/d + (2^(1/2)\*B\*a^2\*log(4\*B\*a^2\*d - 2^(1/2)\*B\*a^2\*d\*tan(c + d\*x)^(1/2)\*(2 + 2i))\*(1 + 1i))/d - (4i^(1/2)\*B\*a^2\*log(4\*B\*a^2\*d + 2\*4i^(1/2)\*B\*a^2\*d\*tan(c + d\*x)^(1/2)))/d

$$3.127 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=198

$$\frac{8\sqrt[4]{-1} a^3(A-iB)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{8a^3(A-iB)\sqrt{\tan(c+dx)}}{d} + \frac{8a^3(iA+B)\tan^{\frac{3}{2}}(c+dx)}{3d}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+8*a^3*(A-I*B)*\tan(d*x+c)^{(1/2)}/d+8/3*a^3*(I*A+B)*\tan(d*x+c)^{(3/2)}/d-16/315*a^3*(18*A-19*I*B)*\tan(d*x+c)^{(5/2)}/d+2/9*I*a*B*\tan(d*x+c)^{(5/2)}*(a+I*a*\tan(d*x+c))^2/d-2/63*(9*A-13*I*B)*\tan(d*x+c)^{(5/2)}*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.32, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3675, 3673, 3609, 3614, 211}

$$\frac{8\sqrt[4]{-1} a^3(A-iB)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3(18A-19IB)\tan^{\frac{5}{2}}(c+dx)}{315d} + \frac{8a^3(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d} - \frac{2(9A-13IB)\tan^{\frac{5}{2}}(c+dx)(a^3+ia^3\tan(c+dx))}{63d} + \frac{8a^3(A-iB)\sqrt{\tan(c+dx)}}{d} + \frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(8*(-1)^{(1/4)}*a^3*(A-I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c+d*x]]])/d + (8*a^3*(A-I*B)*\text{Sqrt}[\text{Tan}[c+d*x]])/d + (8*a^3*(I*A+B)*\text{Tan}[c+d*x]^{(3/2)})/(3*d) - (16*a^3*(18*A-(19*I)*B)*\text{Tan}[c+d*x]^{(5/2)})/(315*d) + (((2*I)/9)*a*B*\text{Tan}[c+d*x]^{(5/2)}*(a+I*a*\text{Tan}[c+d*x])^2)/d - (2*(9*A-(13*I)*B)*\text{Tan}[c+d*x]^{(5/2)}*(a^3+I*a^3*\text{Tan}[c+d*x]))/(63*d)$

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3609**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m-1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

**Rule 3614**

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*



$\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1))), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

### Rule 3675

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{(m - 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)}/(d*f*(m + n))), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 1] \&\& !\text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned} \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \frac{2iaB \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2}{9d} + \dots \\ &= \frac{2iaB \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2}{9d} - \dots \\ &= -\frac{16a^3(18A - 19iB) \tan^{\frac{5}{2}}(c + dx)}{315d} + \frac{2iaB \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2}{9d} \\ &= \frac{8a^3(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} - \frac{16a^3(18A - 19iB) \tan^{\frac{5}{2}}(c + dx)}{315d} \\ &= \frac{8a^3(A - iB) \sqrt{\tan(c + dx)}}{d} + \frac{8a^3(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} \\ &= \frac{8a^3(A - iB) \sqrt{\tan(c + dx)}}{d} + \frac{8a^3(iA + B) \tan^{\frac{3}{2}}(c + dx)}{3d} \\ &= \frac{8\sqrt{-1} a^3(A - iB) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 496 vs.  $2(198) = 396$ .

time = 8.04, size = 496, normalized size = 2.51

MA - (dx)^n / (sqrt(a+bx+cx^2)) \* (sqrt(a+bx+cx^2)) / (sqrt(a+bx+cx^2)) ...

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (-8\*(A - I\*B)\*Sqrt[(-1)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))) \* ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))] \* Cos[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]) / (d \* E^((3\*I)\*c) \* Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))] / (1 + E^((2\*I)\*(c + d\*x))) \* (Cos[d\*x] + I\*Sin[d\*x])^3 \* (A \* Cos[c + d\*x] + B \* Sin[c + d\*x])) + (Cos[c + d\*x]^4 \* (Sec[c] \* (144 \* A \* Cos[c] - (1547 \* I) \* B \* Cos[c] + (465 \* I) \* A \* Sin[c] + 555 \* B \* Sin[c]) \* ((2 \* Cos[3 \* c]) / 315 - ((2 \* I) / 315) \* Sin[3 \* c]) + Sec[c] \* Sec[c + d\*x]^2 \* (189 \* A \* Cos[c] - (322 \* I) \* B \* Cos[c] + (45 \* I) \* A \* Sin[c] + 135 \* B \* Sin[c]) \* ((-2 \* Cos[3 \* c]) / 315 + ((2 \* I) / 315) \* Sin[3 \* c]) + Sec[c + d\*x]^4 \* (((-2 \* I) / 9) \* B \* Cos[3 \* c] - (2 \* B \* Sin[3 \* c]) / 9) + Sec[c] \* Sec[c + d\*x]^3 \* ((2 \* Cos[3 \* c]) / 7 - ((2 \* I) / 7) \* Sin[3 \* c]) \* ((-I) \* A \* Sin[d\*x] - 3 \* B \* Sin[d\*x]) + Sec[c] \* Sec[c + d\*x] \* ((2 \* Cos[3 \* c]) / 21 - ((2 \* I) / 21) \* Sin[3 \* c]) \* ((31 \* I) \* A \* Sin[d\*x] + 37 \* B \* Sin[d\*x])) \* Sqrt[Tan[c + d\*x]] \* (a + I\*a\*Tan[c + d\*x])^3 \* (A + B\*Tan[c + d\*x]) / (d \* (Cos[d\*x] + I\*Sin[d\*x])^3 \* (A \* Cos[c + d\*x] + B \* Sin[c + d\*x]))

Maple [A]

time = 0.04, size = 299, normalized size = 1.51

method	result
derivativedivides	$a^3 \left( -\frac{2iB \left( \tan^{\frac{9}{2}}(dx+c) \right)}{9} - \frac{2iA \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} - \frac{6B \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{8iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{6A \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{8iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} \right)$
default	$a^3 \left( -\frac{2iB \left( \tan^{\frac{9}{2}}(dx+c) \right)}{9} - \frac{2iA \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} - \frac{6B \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{8iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{6A \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{8iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

```
[Out] 1/d*a^3*(-2/9*I*B*tan(d*x+c)^(9/2)-2/7*I*A*tan(d*x+c)^(7/2)-6/7*B*tan(d*x+c)^(7/2)+8/5*I*B*tan(d*x+c)^(5/2)-6/5*A*tan(d*x+c)^(5/2)+8/3*I*A*tan(d*x+c)^(3/2)+8/3*B*tan(d*x+c)^(3/2)-8*I*B*tan(d*x+c)^(1/2)+8*A*tan(d*x+c)^(1/2)+1/4*(4*I*B-4*A)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))+1/4*(-4*I*A-4*B)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 234, normalized size = 1.18

$\frac{70 B^2 \tan(d x+c)^3 + 90 I B A + 3 B^2 \tan(d x+c)^3 + 126 I A - 4 B^2 \tan(d x+c)^3 + 84 I(-A-B) \sqrt{\tan(d x+c)^2+1} - 2520(A-I B) \sqrt{\tan(d x+c)^2+1} + 315(2 \sqrt{2}(I+1) A-(I-1) B) \arctan\left(\frac{1+\sqrt{2} \sqrt{\tan(d x+c)^2+1}}{\sqrt{2}}\right)+2 \sqrt{2}(I+1) A-(I-1) B) \arctan\left(\frac{-1+\sqrt{2} \sqrt{\tan(d x+c)^2+1}}{\sqrt{2}}\right)-\sqrt{2}(I-1) A+(I+1) B) \log\left(\frac{\sqrt{2} \sqrt{\tan(d x+c)^2+1}+\tan(d x+c)+1}{\sqrt{2} \sqrt{\tan(d x+c)^2+1}-\tan(d x+c)+1}\right)+\sqrt{2}(I-1) A+(I+1) B) \log\left(\frac{-\sqrt{2} \sqrt{\tan(d x+c)^2+1}+\tan(d x+c)+1}{-\sqrt{2} \sqrt{\tan(d x+c)^2+1}-\tan(d x+c)+1}\right)}{315 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/315*(70*I*B*a^3*tan(d*x + c)^(9/2) + 90*(I*A + 3*B)*a^3*tan(d*x + c)^(7/2) + 126*(3*A - 4*I*B)*a^3*tan(d*x + c)^(5/2) + 840*(-I*A - B)*a^3*tan(d*x + c)^(3/2) - 2520*(A - I*B)*a^3*sqrt(tan(d*x + c)) + 315*(2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^3)/d
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs.  $2(158) = 316$ .

time = 2.98, size = 561, normalized size = 2.83

$\frac{1}{315 d} \left( \frac{70 B^2 \tan(d x+c)^3 + 90 I B A + 3 B^2 \tan(d x+c)^3 + 126 I A - 4 B^2 \tan(d x+c)^3 + 84 I(-A-B) \sqrt{\tan(d x+c)^2+1} - 2520(A-I B) \sqrt{\tan(d x+c)^2+1} + 315(2 \sqrt{2}(I+1) A-(I-1) B) \arctan\left(\frac{1+\sqrt{2} \sqrt{\tan(d x+c)^2+1}}{\sqrt{2}}\right)+2 \sqrt{2}(I+1) A-(I-1) B) \arctan\left(\frac{-1+\sqrt{2} \sqrt{\tan(d x+c)^2+1}}{\sqrt{2}}\right)-\sqrt{2}(I-1) A+(I+1) B) \log\left(\frac{\sqrt{2} \sqrt{\tan(d x+c)^2+1}+\tan(d x+c)+1}{\sqrt{2} \sqrt{\tan(d x+c)^2+1}-\tan(d x+c)+1}\right)+\sqrt{2}(I-1) A+(I+1) B) \log\left(\frac{-\sqrt{2} \sqrt{\tan(d x+c)^2+1}+\tan(d x+c)+1}{-\sqrt{2} \sqrt{\tan(d x+c)^2+1}-\tan(d x+c)+1}\right)}{315 d} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/315*(315*sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3) - 315*sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)
```



[Out]  $(8Aa^3 \tan(c + dx)^{1/2})/d + (Aa^3 \tan(c + dx)^{3/2} * 8i)/(3d) - (6Aa^3 \tan(c + dx)^{5/2})/(5d) - (Aa^3 \tan(c + dx)^{7/2} * 2i)/(7d) - (Ba^3 \tan(c + dx)^{1/2} * 8i)/d + (8Ba^3 \tan(c + dx)^{3/2})/(3d) + (Ba^3 \tan(c + dx)^{5/2} * 8i)/(5d) - (6Ba^3 \tan(c + dx)^{7/2})/(7d) - (Ba^3 \tan(c + dx)^{9/2} * 2i)/(9d) + (2^{1/2} * Aa^3 * \log(-Aa^3 * d * 8i - 2^{1/2} * Aa^3 * d * \tan(c + dx)^{1/2} * (4 - 4i)) * (2 - 2i))/d - ((-16i)^{1/2} * Aa^3 * \log(2 * (-16i)^{1/2} * Aa^3 * d * \tan(c + dx)^{1/2} - Aa^3 * d * 8i))/d + (2^{1/2} * Ba^3 * \log(-8 * Ba^3 * d - 2^{1/2} * Ba^3 * d * \tan(c + dx)^{1/2} * (4 + 4i)) * (2 + 2i))/d - (16i^{1/2} * Ba^3 * \log(2 * 16i^{1/2} * Ba^3 * d * \tan(c + dx)^{1/2} - 8 * Ba^3 * d))/d$

$$3.128 \quad \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3 (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=171

$$\frac{8\sqrt[4]{-1} a^3 (iA + B) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} + \frac{8a^3 (iA + B) \sqrt{\tan(c + dx)}}{d} - \frac{8a^3 (21A - 23iB) \tan^{3/2}(c + dx)}{105d}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+8*a^3*(I*A+B)*\tan(d*x+c)^{(1/2)}/d-8/105*a^3*(21*A-23*I*B)*\tan(d*x+c)^{(3/2)}/d+2/7*I*a*B*\tan(d*x+c)^{(3/2)}*(a+I*a*\tan(d*x+c))^2/d-2/35*(7*A-11*I*B)*\tan(d*x+c)^{(3/2)}*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.29, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3675, 3673, 3609, 3614, 211}

$$\frac{8\sqrt{-1} a^3 (B + iA) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{8a^3 (21A - 23iB) \tan^{3/2}(c + dx)}{105d} - \frac{2(7A - 11iB) \tan^{3/2}(c + dx) (a^3 + ia^3 \tan(c + dx))}{35d} + \frac{8a^3 (B + iA) \sqrt{\tan(c + dx)}}{d} + \frac{2iaB \tan^{3/2}(c + dx) (a + ia \tan(c + dx))^2}{7d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(8*(-1)^{(1/4)}*a^3*(I*A + B)*\operatorname{ArcTan}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/d + (8*a^3*(I*A + B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d - (8*a^3*(21*A - (23*I)*B)*\operatorname{Tan}[c + d*x]^{(3/2)})/(105*d) + (((2*I)/7)*a*B*\operatorname{Tan}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^2)/d - (2*(7*A - (11*I)*B)*\operatorname{Tan}[c + d*x]^{(3/2)}*(a^3 + I*a^3*\operatorname{Tan}[c + d*x]))/(35*d)$

**Rule 211**

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2])/a]*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b]$

**Rule 3609**

$\operatorname{Int}[(a + (b*x)*\tan[e + (f*x)])^{(m)}*((c + (d*x)*\tan[e + (f*x)])), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\tan[e + f*x])^m/(f*m)), x] + \operatorname{Int}[(a + b*\tan[e + f*x])^{(m-1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\tan[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{GtQ}[m, 0]$

**Rule 3614**

$\operatorname{Int}[(c + (d*x)*\tan[e + (f*x)])/(\operatorname{Sqrt}[(b*x)*\tan[e + (f*x)]]), x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*$



Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $((-8*I)*(A - I*B)*\text{Sqrt}[((-I)*(-1 + E^{((2*I)*(c + d*x))})/(1 + E^{((2*I)*(c + d*x))})])*\text{ArcTanh}[\text{Sqrt}[(-1 + E^{((2*I)*(c + d*x))})/(1 + E^{((2*I)*(c + d*x))})])]*\text{Cos}[c + d*x]^4*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x])/(d*E^{((3*I)*c)}*\text{Sqrt}[(-1 + E^{((2*I)*(c + d*x))})/(1 + E^{((2*I)*(c + d*x))})])*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^3*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])) + (\text{Cos}[c + d*x]^4*(\text{Sec}[c]*\text{Sec}[c + d*x]^2*(7*A*\text{Cos}[c] - (21*I)*B*\text{Cos}[c] + 5*B*\text{Sin}[c]))*(((-2*I)/35)*\text{Cos}[3*c] - (2*\text{Sin}[3*c])/35) + \text{Sec}[c]*((441*I)*A*\text{Cos}[c] + 483*B*\text{Cos}[c] - 105*A*\text{Sin}[c] + (155*I)*B*\text{Sin}[c]))*((2*\text{Cos}[3*c])/105 - ((2*I)/105)*\text{Sin}[3*c]) - I*B*\text{Sec}[c]*\text{Sec}[c + d*x]^3*((2*\text{Cos}[3*c])/7 - ((2*I)/7)*\text{Sin}[3*c])*\text{Sin}[d*x] + \text{Sec}[c]*\text{Sec}[c + d*x]*((-2*\text{Cos}[3*c])/21 + ((2*I)/21)*\text{Sin}[3*c])*(21*A*\text{Sin}[d*x] - (31*I)*B*\text{Sin}[d*x]))*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x]))/(d*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^3*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))$

**Maple [A]**

time = 0.04, size = 276, normalized size = 1.61

method	result
derivativedivides	$a^3 \left( -\frac{2iB \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} - \frac{2iA \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{6B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{8iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2A \left( \tan^{\frac{3}{2}}(dx+c) \right) + 8iA \left( \sqrt{\tan(dx+c)} \right) \right)$
default	$a^3 \left( -\frac{2iB \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} - \frac{2iA \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{6B \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{8iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2A \left( \tan^{\frac{3}{2}}(dx+c) \right) + 8iA \left( \sqrt{\tan(dx+c)} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURN VERBOSE)

[Out]  $1/d*a^3*(-2/7*I*B*\text{tan}(d*x+c)^{(7/2)}-2/5*I*A*\text{tan}(d*x+c)^{(5/2)}-6/5*B*\text{tan}(d*x+c)^{(5/2)}+8/3*I*B*\text{tan}(d*x+c)^{(3/2)}-2*A*\text{tan}(d*x+c)^{(3/2)}+8*I*A*\text{tan}(d*x+c)^{(1/2)}+8*B*\text{tan}(d*x+c)^{(1/2)}+1/4*(-4*I*A-4*B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))/(1-2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))+2*\arctan(1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}))+1/4*(-4*I*B+4*A)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))/(1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}-\text{tan}(d*x+c)))$





**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-ia^3 \left( \int (-3A \tan^3(c+dx)) dx + \int A \tan^2(c+dx) dx + \int (-3B \tan^3(c+dx)) dx + \int B \tan^2(c+dx) dx + \int iA \sqrt{\tan(c+dx)} dx + \int (-3iA \tan^{\frac{5}{2}}(c+dx)) dx + \int iB \tan^{\frac{3}{2}}(c+dx) dx + \int (-3iB \tan^{\frac{5}{2}}(c+dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

**[Out]**  $-I*a**3*(Integral(-3*A*tan(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)**(7/2), x) + Integral(-3*B*tan(c + d*x)**(5/2), x) + Integral(B*tan(c + d*x)**(9/2), x) + Integral(I*A*sqrt(tan(c + d*x)), x) + Integral(-3*I*A*tan(c + d*x)**(5/2), x) + Integral(I*B*tan(c + d*x)**(3/2), x) + Integral(-3*I*B*tan(c + d*x)**(7/2), x))$

**Giac [A]**

time = 1.00, size = 160, normalized size = 0.94

$$\frac{(4i-4)\sqrt{2}(Aa^3 - iBa^3)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right) - 2(15iBa^3d^6\tan(dx+c)^{\frac{7}{2}} + 21iAa^3d^6\tan(dx+c)^{\frac{5}{2}} + 63Ba^3d^6\tan(dx+c)^{\frac{3}{2}} + 105Aa^3d^6\tan(dx+c)^{\frac{1}{2}} - 140iBd^6\tan(dx+c)^{\frac{7}{2}} - 420iAd^6\sqrt{\tan(dx+c)} - 420Bd^6\sqrt{\tan(dx+c)})}{105d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

**[Out]**  $-(4*I - 4)*sqrt(2)*(A*a^3 - I*B*a^3)*arctan(-((1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c)))/d - 2/105*(15*I*B*a^3*d^6*tan(d*x + c)^(7/2) + 21*I*A*a^3*d^6*tan(d*x + c)^(5/2) + 63*B*a^3*d^6*tan(d*x + c)^(3/2) + 105*A*a^3*d^6*tan(d*x + c)^(1/2) - 140*I*B*a^3*d^6*tan(d*x + c)^(7/2) - 420*I*A*a^3*d^6*sqrt(tan(d*x + c)) - 420*B*a^3*d^6*sqrt(tan(d*x + c)))/d^7$

**Mupad [B]**

time = 8.92, size = 292, normalized size = 1.71

$$\frac{Aa^3\sqrt{\tan(c+dx)}\ln\left(\frac{2Aa^3\tan(c+dx)^{1/2} - Aa^3\tan(c+dx)^{3/2}}{2Aa^3\sqrt{\tan(c+dx)}}\right) - Ba^3\sqrt{\tan(c+dx)}\ln\left(\frac{2Ba^3\tan(c+dx)^{1/2} - Ba^3\tan(c+dx)^{3/2}}{2Ba^3\sqrt{\tan(c+dx)}}\right) + \sqrt{2}Aa^3\ln\left(\frac{Aa^3d + \sqrt{2}Aa^3d\sqrt{\tan(c+dx)}}{(-1-4i)(2-2i)}\right) + \sqrt{2}Aa^3\ln\left(\frac{Aa^3d + 2\sqrt{2}Aa^3d\sqrt{\tan(c+dx)}}{2}\right) + \sqrt{2}Ba^3\ln\left(\frac{-Ba^3d + \sqrt{2}Ba^3d\sqrt{\tan(c+dx)}}{(-1+4i)(2-2i)}\right) + \sqrt{2}Ba^3\ln\left(\frac{-Ba^3d + 2\sqrt{2}Ba^3d\sqrt{\tan(c+dx)}}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3,x)

**[Out]**  $(A*a^3*tan(c + d*x)^(1/2)*8i)/d - (2*A*a^3*tan(c + d*x)^(3/2))/d - (A*a^3*tan(c + d*x)^(5/2)*2i)/(5*d) + (8*B*a^3*tan(c + d*x)^(1/2))/d + (B*a^3*tan(c + d*x)^(3/2)*8i)/(3*d) - (6*B*a^3*tan(c + d*x)^(5/2))/(5*d) - (B*a^3*tan(c + d*x)^(7/2)*2i)/(7*d) + (2^(1/2)*A*a^3*log(8*A*a^3*d - 2^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)*(4 + 4i))*(2 + 2i))/d - (16i^(1/2)*A*a^3*log(8*A*a^3*d + 2*16i^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^3*log(-B*a^3*d*8i - 2^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)*(4 - 4i))*(2 - 2i))/d - ((-16i)^(1/2)*B*a^3*log(2*(-16i)^(1/2)*B*a^3*d*tan(c + d*x)^(1/2) - B*a^3*d*8i))/d$

$$3.129 \quad \int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=146

$$\frac{8\sqrt{-1} a^3 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3 (5A - 6iB) \sqrt{\tan(c+dx)}}{15d} + \frac{2iaB \sqrt{\tan(c+dx)}}{5d}$$

[Out]  $-8*(-1)^{(1/4)}*a^3*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-16/15*a^3*(5*A-6*I*B)*\tan(d*x+c)^{(1/2)}/d+2/5*I*a*B*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^2/d-2/15*(5*A-9*I*B)*\tan(d*x+c)^{(1/2)}*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3675, 3673, 3614, 211}

$$\frac{8\sqrt{-1} a^3 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3 (5A - 6iB) \sqrt{\tan(c+dx)}}{15d} - \frac{2(5A - 9iB) \sqrt{\tan(c+dx)} (a^3 + ia^3 \tan(c+dx))}{15d} + \frac{2iaB \sqrt{\tan(c+dx)} (a + ia \tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]],x]

[Out]  $(-8*(-1)^{(1/4)}*a^3*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d - (16*a^3*(5*A - (6*I)*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(15*d) + (((2*I)/5)*a*B*\text{Sqrt}[\text{Tan}[c + d*x]])*(a + I*a*\text{Tan}[c + d*x])^2/d - (2*(5*A - (9*I)*B)*\text{Sqrt}[\text{Tan}[c + d*x]])*(a^3 + I*a^3*\text{Tan}[c + d*x])/(15*d)$

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3614**

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

**Rule 3673**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

## Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{2iaB \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2}{5d} + \frac{2}{5} \int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\ &= \frac{2iaB \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2}{5d} - \frac{2(5A - 9iB)}{5} \int \frac{(a + ia \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\ &= -\frac{16a^3(5A - 6iB) \sqrt{\tan(c + dx)}}{15d} + \frac{2iaB \sqrt{\tan(c + dx)}}{5} \int \frac{(a + ia \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\ &= -\frac{16a^3(5A - 6iB) \sqrt{\tan(c + dx)}}{15d} + \frac{2iaB \sqrt{\tan(c + dx)}}{5} \int \frac{(a + ia \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\ &= -\frac{8\sqrt{-1} a^3 (A - iB) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} - \frac{2(5A - 9iB)}{5} \int \frac{(a + ia \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \end{aligned}$$

## Mathematica [A]

time = 5.84, size = 273, normalized size = 1.87

$$\cos^4(c + dx) \left( \frac{8(A - iB) e^{-3i} \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \right)}{\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}} - \frac{1}{15} \sec^2(c + dx) (\cos(3c) - i \sin(3c)) (45A - 57iB + 9(5A - 7iB) \cos(2(c + dx)) + 5(iA + 3B) \sin(2(c + dx))) \sqrt{\tan(c + dx)} \right) (a + ia \tan(c + dx))^3 (A + B \tan(c + dx))$$


---


$$d(\cos(dx) + i \sin(dx))^3 (A \cos(c + dx) + B \sin(c + dx))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]
], x]
```

```
[Out] (Cos[c + d*x]^4*((8*(A - I*B)*Sqrt[((-1)*(-1 + E^((2*I)*(c + d*x))))]/(1 + E
^((2*I)*(c + d*x))))*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))]])/(E^((3*I)*c)*Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))] - (Sec[c + d*x]^2*(Cos[3*c] - I*Sin[3*c])*(45*A - (57*I)*B + 9
*(5*A - (7*I)*B)*Cos[2*(c + d*x)] + 5*(I*A + 3*B)*Sin[2*(c + d*x)])*Sqrt[Ta
```

$n[c + d*x]])/15)*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x]))/(d*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^3*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs.  $2(122) = 244$ .

time = 0.04, size = 253, normalized size = 1.73

method	result
derivativedivides	$a^3 \left( -\frac{2iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{2iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2B \left( \tan^{\frac{3}{2}}(dx+c) \right) + 8iB \left( \sqrt{\tan}(dx+c) \right) - 6A \left( \sqrt{\tan}(dx+c) \right) + \dots \right)$
default	$a^3 \left( -\frac{2iB \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} - \frac{2iA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2B \left( \tan^{\frac{3}{2}}(dx+c) \right) + 8iB \left( \sqrt{\tan}(dx+c) \right) - 6A \left( \sqrt{\tan}(dx+c) \right) + \dots \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $1/d*a^3*(-2/5*I*B*\text{tan}(d*x+c)^{(5/2)}-2/3*I*A*\text{tan}(d*x+c)^{(3/2)}-2*B*\text{tan}(d*x+c)^{(3/2)}+8*I*B*\text{tan}(d*x+c)^{(1/2)}-6*A*\text{tan}(d*x+c)^{(1/2)}+1/4*(-4*I*B+4*A)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))/(1-2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))+2*\arctan(1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}))+1/4*(4*I*A+4*B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))/(1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}+\text{tan}(d*x+c)))+2*\arctan(1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\text{tan}(d*x+c)^{(1/2)}))$

**Maxima [A]**

time = 0.52, size = 196, normalized size = 1.34

$\frac{6iB^2 \tan(dx+c)^2 + 10i(A+3B)a^3 \tan(dx+c)^2 + 30(A-4iB)a^3 \sqrt{\tan(dx+c)} - 15(2\sqrt{2}(i+1)A-(i-1)B) \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}(i+1)A-(i-1)B) \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})\right) - \sqrt{2}(i-1)A+(i+1)B) \log(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)) + \sqrt{2}(i-1)A+(i+1)B) \log(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c))}{15d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $-1/15*(6*I*B*a^3*\text{tan}(d*x + c)^{(5/2)} + 10*(I*A + 3*B)*a^3*\text{tan}(d*x + c)^{(3/2)} + 30*(3*A - 4*I*B)*a^3*\text{sqrt}(\text{tan}(d*x + c)) - 15*(2*\text{sqrt}(2))*((I + 1)*A - (I - 1)*B)*\arctan(1/2*\text{sqrt}(2))*(\text{sqrt}(2) + 2*\text{sqrt}(\text{tan}(d*x + c)))) + 2*\text{sqrt}(2)*(($

$(I + 1)*A - (I - 1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(dx + c)}))$   
 $- \sqrt{2}*((I - 1)*A + (I + 1)*B)*\log(\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx$   
 $+ c) + 1) + \sqrt{2}*((I - 1)*A + (I + 1)*B)*\log(-\sqrt{2}*\sqrt{\tan(dx + c)}$   
 $) + \tan(dx + c) + 1))*a^3)/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs.  $2(116) = 232$ .  
time = 2.35, size = 447, normalized size = 3.06

$$\frac{2 \left( \frac{15 \sqrt{-1A^2 + 2AB - B^2} (d^{2I+1} + 2d^{2I} + d) \log \left( \frac{1A^2 + 2AB - B^2}{d^{2I+1} + 2d^{2I} + d} \right) - 15 \sqrt{-1A^2 + 2AB - B^2} (d^{2I+1} + 2d^{2I} + d) \log \left( \frac{1A^2 + 2AB - B^2}{d^{2I+1} + 2d^{2I} + d} \right) \right)}{15(d^{2I+1} + 2d^{2I} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(dx+c))^3\*(A+B\*tan(dx+c))/tan(dx+c)^(1/2),x, algorithm="fricas")

[Out]  $2/15*(15*\sqrt{-(I*A^2 + 2*A*B - I*B^2)}*a^6/d^2)*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + \sqrt{-(I*A^2 + 2*A*B - I*B^2)}*a^6/d^2)*(I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{-(2*I*d*x - 2*I*c)}/(((-I*A - B)*a^3)) - 15*\sqrt{-(I*A^2 + 2*A*B - I*B^2)}*a^6/d^2)*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + \sqrt{-(I*A^2 + 2*A*B - I*B^2)}*a^6/d^2)*(-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{-(2*I*d*x - 2*I*c)}/(((-I*A - B)*a^3)) - 2*((25*A - 39*I*B)*a^3*e^{(4*I*d*x + 4*I*c)} + 3*(15*A - 19*I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + 4*(5*A - 6*I*B)*a^3)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-ia^3 \left( \int (-3A\sqrt{\tan(c+dx)}) dx + \int A \tan^3(c+dx) dx + \int (-3B \tan^3(c+dx)) dx + \int B \tan^3(c+dx) dx + \int \frac{iA}{\sqrt{\tan(c+dx)}} dx + \int (-3A \tan^3(c+dx)) dx + \int iB \sqrt{\tan(c+dx)} dx + \int (-3iB \tan^3(c+dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(dx+c))\*\*3\*(A+B\*tan(dx+c))/tan(dx+c)\*\*(1/2),x)

[Out]  $-I*a**3*(\text{Integral}(-3*A*\sqrt{\tan(c + d*x)}, x) + \text{Integral}(A*\tan(c + d*x)**(5/2), x) + \text{Integral}(-3*B*\tan(c + d*x)**(3/2), x) + \text{Integral}(B*\tan(c + d*x)**(7/2), x) + \text{Integral}(I*A/\sqrt{\tan(c + d*x)}, x) + \text{Integral}(-3*I*A*\tan(c + d*x)**(3/2), x) + \text{Integral}(I*B*\sqrt{\tan(c + d*x)}, x) + \text{Integral}(-3*I*B*\tan(c + d*x)**(5/2), x))$

**Giac [A]**

time = 0.94, size = 127, normalized size = 0.87

$$\frac{(4i - 4) \sqrt{2} (-iAa^3 - Ba^3) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right) - 2 \left( 3iBa^3d^4 \tan(dx + c)^{\frac{5}{2}} + 5iAa^3d^4 \tan(dx + c)^{\frac{3}{2}} + 15Ba^3d^4 \tan(dx + c)^{\frac{3}{2}} + 45Aa^3d^4 \sqrt{\tan(dx + c)} - 60iBa^3d^4 \sqrt{\tan(dx + c)} \right)}{15d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] (4\*I - 4)\*sqrt(2)\*(-I\*A\*a^3 - B\*a^3)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d - 2/15\*(3\*I\*B\*a^3\*d^4\*tan(d\*x + c)^(5/2) + 5\*I\*A\*a^3\*d^4\*tan(d\*x + c)^(3/2) + 15\*B\*a^3\*d^4\*tan(d\*x + c)^(3/2) + 45\*A\*a^3\*d^4\*sqrt(tan(d\*x + c)) - 60\*I\*B\*a^3\*d^4\*sqrt(tan(d\*x + c)))/d^5

**Mupad [B]**

time = 7.16, size = 257, normalized size = 1.76

$$\frac{6Aa^3\sqrt{\tan(c+dx)}}{d} - \frac{Aa^3\tan(c+dx)^{3/2}}{3d} + \frac{Ba^3\sqrt{\tan(c+dx)}}{d} - \frac{2Ba^3\tan(c+dx)^{3/2}}{d} - \frac{Ba^3\tan(c+dx)^{5/2}}{5d} + \frac{\sqrt{2}Aa^3\ln(Aa^3d\sqrt{\tan(c+dx)}(-4+4i)(2-2i))}{d} - \frac{\sqrt{-10}Aa^3\ln(Aa^3d\sqrt{\tan(c+dx)}(-4+4i)(2+2i))}{d} + \frac{\sqrt{2}Ba^3\ln(8Ba^3d+2\sqrt{10}Ba^3d\sqrt{\tan(c+dx)}(-4-4i)(2+2i))}{d} - \frac{\sqrt{10}Ba^3\ln(8Ba^3d+2\sqrt{10}Ba^3d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/tan(c + d\*x)^(1/2),x)

[Out] (B\*a^3\*tan(c + d\*x)^(1/2)\*8i)/d - (A\*a^3\*tan(c + d\*x)^(3/2)\*2i)/(3\*d) - (6\*A\*a^3\*tan(c + d\*x)^(1/2))/d - (2\*B\*a^3\*tan(c + d\*x)^(3/2))/d - (B\*a^3\*tan(c + d\*x)^(5/2)\*2i)/(5\*d) + (2^(1/2)\*A\*a^3\*log(A\*a^3\*d\*8i - 2^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 - 4i))\*(2 - 2i))/d - ((-16i)^(1/2)\*A\*a^3\*log(A\*a^3\*d\*8i + 2\*(-16i)^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)))/d + (2^(1/2)\*B\*a^3\*log(8\*B\*a^3\*d - 2^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 + 4i))\*(2 + 2i))/d - (16i^(1/2)\*B\*a^3\*log(8\*B\*a^3\*d + 2\*16i^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2)))/d

$$3.130 \quad \int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=134

$$\frac{8\sqrt{-1} a^3 (iA + B) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3 B \sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+ia \tan(c+dx))^2}{d\sqrt{\tan(c+dx)}} + \dots$$

[Out]  $-8*(-1)^{(1/4)}*a^3*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d-16/3*a^3*B*\tan(d*x+c)^{(1/2)}/d-2*a*A*(a+I*a*\tan(d*x+c))^2/d/\tan(d*x+c)^{(1/2)}+2/3*(3*I*A-B)*\tan(d*x+c)^{(1/2)}*(a^3+I*a^3*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3674, 3675, 3673, 3614, 211}

$$\frac{8\sqrt{-1} a^3 (B + iA) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{2(-B + 3iA) \sqrt{\tan(c+dx)} (a^3 + ia^3 \tan(c+dx))}{3d} - \frac{16a^3 B \sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+ia \tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{(3/2)}}, x]$

[Out]  $(-8*(-1)^{(1/4)}*a^3*(I*A + B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d - (16*a^3*B*\text{Sqrt}[\text{Tan}[c + d*x]])/(3*d) - (2*a*A*(a + I*a*\text{Tan}[c + d*x])^2)/(d*\text{Sqrt}[\text{Tan}[c + d*x]]) + (2*((3*I)*A - B)*\text{Sqrt}[\text{Tan}[c + d*x]]*(a^3 + I*a^3*\text{Tan}[c + d*x]))/(3*d)$

**Rule 211**

$\text{Int}[(a + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

**Rule 3614**

$\text{Int}[(c + (d_*)*\text{tan}[(e_*) + (f_*)*(x_)])/(\text{Sqrt}[(b_*)*\text{tan}[(e_*) + (f_*)*(x_)])], x\_Symbol] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

**Rule 3673**

$\text{Int}[(a + (b_*)*\text{tan}[(e_*) + (f_*)*(x_)])^{(m_*)}*((A_*) + (B_*)*\text{tan}[(e_*) + (f_*)*(x_)])^{(c_*) + (d_*)*\text{tan}[(e_*) + (f_*)*(x_)]}, x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m+1)}/(b*f*(m+1))), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m)*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{LeQ}[m, -1]$



## Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

## Rule 3675

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^2}{d\sqrt{\tan(c + dx)}} + 2 \int \frac{(a + ia \tan(c + dx))^2}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2aA(a + ia \tan(c + dx))^2}{d\sqrt{\tan(c + dx)}} + \frac{2(3iA - B)\sqrt{\tan(c + dx)}}{3d} \\
&= -\frac{16a^3B\sqrt{\tan(c + dx)}}{3d} - \frac{2aA(a + ia \tan(c + dx))^2}{d\sqrt{\tan(c + dx)}} + \frac{2(3iA - B)\sqrt{\tan(c + dx)}}{3d} \\
&= -\frac{16a^3B\sqrt{\tan(c + dx)}}{3d} - \frac{2aA(a + ia \tan(c + dx))^2}{d\sqrt{\tan(c + dx)}} + \frac{2(3iA - B)\sqrt{\tan(c + dx)}}{3d} \\
&= -\frac{8\sqrt{-1} a^3 (iA + B) \tan^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right)}{d}
\end{aligned}$$

## Mathematica [A]

time = 5.20, size = 151, normalized size = 1.13

$$\frac{a^3 \csc^2(c + dx) \left( -12(A - iB) \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right) \sin(2(c + dx)) + (-3iA + B + (-3iA - B) \cos(2(c + dx)) + 3(A - 3iB) \sin(2(c + dx))) \sqrt{i \tan(c + dx)} \right) \sqrt{i \tan(c + dx)} \sqrt{\tan(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out] 
$$-1/3*(a^3*\text{Csc}[c + d*x]^2*(-12*(A - I*B)*\text{ArcTanh}[\text{Sqrt}[(-1 + E^{((2*I)*(c + d*x)})]/(1 + E^{((2*I)*(c + d*x)})])]*\text{Sin}[2*(c + d*x)] + ((-3*I)*A + B + ((-3*I)*A - B)*\text{Cos}[2*(c + d*x)] + 3*(A - (3*I)*B)*\text{Sin}[2*(c + d*x)])*\text{Sqrt}[I*\text{Tan}[c + d*x]]*\text{Sqrt}[I*\text{Tan}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])/d$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs.  $2(114) = 228$ .

time = 0.04, size = 241, normalized size = 1.80

method	result
derivativedivides	$a^3 \left( -\frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2iA \left( \sqrt{\tan(dx+c)} \right) - 6B \left( \sqrt{\tan(dx+c)} \right) - \frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(4iA+4B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)} \right)$
default	$a^3 \left( -\frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2iA \left( \sqrt{\tan(dx+c)} \right) - 6B \left( \sqrt{\tan(dx+c)} \right) - \frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(4iA+4B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x, method=\_RETURNVERBOSE)

[Out] 
$$1/d*a^3*(-2/3*I*B*\tan(d*x+c)^{(3/2)}-2*I*A*\tan(d*x+c)^{(1/2)}-6*B*\tan(d*x+c)^{(1/2)}-2*A/\tan(d*x+c)^{(1/2)}+1/4*(4*I*A+4*B)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(4*I*B-4*A)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$$

**Maxima [A]**

time = 0.51, size = 190, normalized size = 1.42

$$\frac{2iB^2 \tan(dx+c)^3 + 6i(A+3B)^2 \sqrt{\tan(dx+c)} - 3(2\sqrt{2}(i-1)A+(i+1)B) \arctan\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right) + 2\sqrt{2}(i-1)A+(i+1)B) \arctan\left(\frac{-1+\sqrt{2}\sqrt{\tan(dx+c)}}{1+\sqrt{2}\sqrt{\tan(dx+c)}}\right) - \sqrt{2}(i-1)A+(i-1)B) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1\right) + \sqrt{2}(i-1)A+(i-1)B) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1\right)}{3d} + \frac{4iA^2 + 4B^2}{\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] 
$$-1/3*(2*I*B*a^3*\tan(d*x + c)^{(3/2)} + 6*(I*A + 3*B)*a^3*\sqrt{\tan(d*x + c)} - 3*(2*\sqrt{2}*((I - 1)*A + (I + 1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(d*x + c)}))) + 2*\sqrt{2}*((I - 1)*A + (I + 1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(d*x + c)}))) - \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1) + \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(-\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1))*a^3 + 6*A*a^3/\sqrt{\tan(d*x + c)})/d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs.  $2(108) = 216$ .

time = 2.21, size = 405, normalized size = 3.02

$$2 \left( \frac{\sqrt{-1A^2 - 2AB + B^2}}{d} \log \left( \frac{-1A^2 - 2AB + B^2}{d^2} \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \right) - 3 \sqrt{-1A^2 - 2AB + B^2} \log \left( \frac{-1A^2 - 2AB + B^2}{d^2} \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \right) - 2(3(A + 5B)a^{3+3i} + (3A - B)a^{3+3i} - 4B)a^3 \sqrt{\frac{-1A^2 - 2AB + B^2}{d^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] 
$$2/3*(3*\sqrt{-1A^2 - 2AB + B^2}*a^6/d^2)*(d*e^{(4*I*d*x + 4*I*c)} - d)*\log(-2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + \sqrt{-1A^2 - 2AB + B^2}*a^6/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^3) - 3*\sqrt{-1A^2 - 2AB + B^2}*a^6/d^2*(d*e^{(4*I*d*x + 4*I*c)} - d)*\log(-2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} - \sqrt{-1A^2 - 2AB + B^2}*a^6/d^2)*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^3) - 2*((3*I*A + 5*B)*a^3*e^{(4*I*d*x + 4*I*c)} + (3*I*A - B)*a^3*e^{(2*I*d*x + 2*I*c)} - 4*B*a^3)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}}/(d*e^{(4*I*d*x + 4*I*c)} - d)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-ia^3 \left( \int \left( -\frac{3A}{\sqrt{\tan(c+dx)}} \right) dx + \int A \tan^3(c+dx) dx + \int (-3B\sqrt{\tan(c+dx)}) dx + \int B \tan^3(c+dx) dx + \int \frac{iA}{\tan^3(c+dx)} dx + \int (-3iA\sqrt{\tan(c+dx)}) dx + \int \frac{iB}{\sqrt{\tan(c+dx)}} dx + \int (-3iB \tan^3(c+dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2),x)

[Out] 
$$-I*a**3*(\text{Integral}(-3*A/\sqrt{\tan(c + d*x)}, x) + \text{Integral}(A*\tan(c + d*x)**(3/2), x) + \text{Integral}(-3*B*\sqrt{\tan(c + d*x)}, x) + \text{Integral}(B*\tan(c + d*x)**(5/2), x) + \text{Integral}(I*A/\tan(c + d*x)**(3/2), x) + \text{Integral}(-3*I*A*\sqrt{\tan(c + d*x)}, x) + \text{Integral}(I*B/\sqrt{\tan(c + d*x)}, x) + \text{Integral}(-3*I*B*\tan(c + d*x)**(3/2), x))$$

**Giac [A]**

time = 1.08, size = 110, normalized size = 0.82

$$\frac{2Aa^3}{d\sqrt{\tan(dx+c)}} - \frac{(4i+4)\sqrt{2}(-iAa^3 - Ba^3)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(iBa^3d^2\tan(dx+c)^{\frac{3}{2}} + 3iAa^3d^2\sqrt{\tan(dx+c)} + 9Ba^3d^2\sqrt{\tan(dx+c)}\right)}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] -2\*A\*a^3/(d\*sqrt(tan(d\*x + c))) - (4\*I + 4)\*sqrt(2)\*(-I\*A\*a^3 - B\*a^3)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d - 2/3\*(I\*B\*a^3\*d^2\*tan(d\*x + c)^(3/2) + 3\*I\*A\*a^3\*d^2\*sqrt(tan(d\*x + c)) + 9\*B\*a^3\*d^2\*sqrt(tan(d\*x + c)))/d^3

**Mupad [B]**

time = 6.90, size = 239, normalized size = 1.78

$$\frac{2Aa^3}{d\sqrt{\tan(c+dx)}} - \frac{Aa^3\sqrt{\tan(c+dx)}}{d} - \frac{6Ba^3\sqrt{\tan(c+dx)}}{d} - \frac{Ba^3\tan(c+dx)^{3/2}}{3d} + \frac{\sqrt{2}Aa^3\ln(-8Aa^3d + \sqrt{2}Aa^3d\sqrt{\tan(c+dx)}(-4-4i))(2+2i)}{d} - \frac{\sqrt{16}Aa^3\ln(-8Aa^3d + 2\sqrt{16}Aa^3d\sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^3\ln(Ba^3d8i + \sqrt{2}Ba^3d\sqrt{\tan(c+dx)}(-4+4i))(2-2i)}{d} - \frac{\sqrt{-16}Ba^3\ln(Ba^3d8i + 2\sqrt{-16}Ba^3d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/tan(c + d\*x)^(3/2),x)

[Out] (2^(1/2)\*A\*a^3\*log(-8\*A\*a^3\*d - 2^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 + 4i))\*(2 + 2i))/d - (A\*a^3\*tan(c + d\*x)^(1/2)\*2i)/d - (6\*B\*a^3\*tan(c + d\*x)^(1/2))/d - (B\*a^3\*tan(c + d\*x)^(3/2)\*2i)/(3\*d) - (2\*A\*a^3)/(d\*tan(c + d\*x)^(1/2)) - (16i^(1/2)\*A\*a^3\*log(2\*16i^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2) - 8\*A\*a^3\*d))/d + (2^(1/2)\*B\*a^3\*log(B\*a^3\*d\*8i - 2^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2))\*(4 - 4i))\*(2 - 2i))/d - ((-16i)^(1/2)\*B\*a^3\*log(B\*a^3\*d\*8i + 2\*(-16i)^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2)))/d

$$3.131 \quad \int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=136

$$\frac{8\sqrt[4]{-1} a^3 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3 A \sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+ia \tan(c+dx))^2}{3d \tan^{\frac{3}{2}}(c+dx)}$$

[Out] 8\*(-1)^(1/4)\*a^3\*(A-I\*B)\*arctan((-1)^(3/4)\*tan(d\*x+c)^(1/2))/d-16/3\*a^3\*A\*tan(d\*x+c)^(1/2)/d-2/3\*a\*A\*(a+I\*a\*tan(d\*x+c))^2/d/tan(d\*x+c)^(3/2)-2/3\*(7\*I\*A+3\*B)\*(a^3+I\*a^3\*tan(d\*x+c))/d/tan(d\*x+c)^(1/2)

**Rubi [A]**

time = 0.24, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3674, 3673, 3614, 211}

$$\frac{8\sqrt[4]{-1} a^3 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2(3B + 7iA)(a^3 + ia^3 \tan(c+dx))}{3d \sqrt{\tan(c+dx)}} - \frac{16a^3 A \sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+ia \tan(c+dx))^2}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2),x]

[Out] (8\*(-1)^(1/4)\*a^3\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]/d - (16\*a^3\*A\*Sqrt[Tan[c + d\*x]]/(3\*d) - (2\*a\*A\*(a + I\*a\*Tan[c + d\*x])^2)/(3\*d\*Tan[c + d\*x]^(3/2)) - (2\*((7\*I)\*A + 3\*B)\*(a^3 + I\*a^3\*Tan[c + d\*x]))/(3\*d\*Sqrt[Tan[c + d\*x]]))

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3614**

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

**Rule 3673**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

## Rule 3674

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

```

## Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^2}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{(a + ia \tan(c + dx))^2}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2aA(a + ia \tan(c + dx))^2}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(7iA + 3B)(a^3 + ia^3 \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\
&= -\frac{16a^3 A \sqrt{\tan(c + dx)}}{3d} - \frac{2aA(a + ia \tan(c + dx))^2}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(7iA + 3B)(a^3 + ia^3 \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\
&= -\frac{16a^3 A \sqrt{\tan(c + dx)}}{3d} - \frac{2aA(a + ia \tan(c + dx))^2}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(7iA + 3B)(a^3 + ia^3 \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\
&= \frac{8\sqrt{-1} a^3 (A - iB) \tan^{-1}\left(\left(-1\right)^{3/4} \sqrt{\tan(c + dx)}\right)}{d} - \frac{16a^3 A \sqrt{\tan(c + dx)}}{3d}
\end{aligned}$$

## Mathematica [A]

time = 5.08, size = 266, normalized size = 1.96

$$\frac{\cos^4(c + dx) \left( \frac{8(A - iB)e^{-3ic} \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right)}{\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}} - \frac{1}{2} \csc^2(c + dx) (\cos(3c) - i \sin(3c)) (A + 3iB + (A - 3iB) \cos(2(c + dx)) + 3(3A + B) \sin(2(c + dx))) \sqrt{\tan(c + dx)} \right)}{d(\cos(dx) + i \sin(dx))^3 (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (Cos[c + d\*x]^4\*((-8\*(A - I\*B)\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))]/(1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]])/(E^((3\*I)\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))])) - (Csc[c + d\*x]^2\*(Cos[3\*c] - I\*Sin[3\*c])\*(A + (3\*I)\*B + (A - (3\*I)\*B)\*Cos[2\*(c + d\*x)] + 3\*((3\*I)\*A + B)\*Sin[2\*(c + d\*x)])\*Sqrt[Tan[c +

$d*x]]/3)*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x])/(d*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^3*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs.  $2(114) = 228$ .

time = 0.05, size = 234, normalized size = 1.72

method	result
derivativedivides	$a^3 \left( -\frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(3iA+B)}{\sqrt{\tan(dx+c)}} - 2iB \left( \sqrt{\tan(dx+c)} \right) + \frac{(4iB-4A)\sqrt{2}}{\ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) +}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) +} \right)}$
default	$a^3 \left( -\frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(3iA+B)}{\sqrt{\tan(dx+c)}} - 2iB \left( \sqrt{\tan(dx+c)} \right) + \frac{(4iB-4A)\sqrt{2}}{\ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) +}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) +} \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $1/d*a^3*(-2/3*A/\tan(d*x+c)^{(3/2)}-2*(3*I*A+B)/\tan(d*x+c)^{(1/2)}-2*I*B*\tan(d*x+c)^{(1/2)}+1/4*(4*I*B-4*A)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(-4*I*A-4*B)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima [A]**

time = 0.53, size = 191, normalized size = 1.40

$\frac{6iBa^3\sqrt{\tan(dx+c)}+3(2\sqrt{2}(i+1)A-(i-1)B)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\sqrt{2}(i+1)A-(i-1)B\arctan\left(-\frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)-\sqrt{2}((i-1)A+(i+1)B)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}((i-1)A+(i+1)B)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{3d}a^2-\frac{2(3iA-B)\tan(dx+c)}{\tan(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")`

[Out]  $-1/3*(6*I*B*a^3*\text{sqrt}(\tan(d*x + c)) + 3*(2*\text{sqrt}(2)*((I + 1)*A - (I - 1)*B)*\text{arctan}(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2*\text{sqrt}(\tan(d*x + c)))) + 2*\text{sqrt}(2)*((I + 1)*A - (I - 1)*B)*\text{arctan}(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2*\text{sqrt}(\tan(d*x + c)))) - \text{sqrt}(2)$

)\*((I - 1)\*A + (I + 1)\*B)\*log(sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) + sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1))\*a^3 - 2\*(3\*(-3\*I\*A - B)\*a^3\*tan(d\*x + c) - A\*a^3)/tan(d\*x + c)^(3/2))/d

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 438 vs.  $2(110) = 220$ .  
time = 1.81, size = 438, normalized size = 3.22

$$2 \left( \frac{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}}{(d \sqrt{\tan(d x + c)} - 2 d \sqrt{\tan(d x + c)} + d) \log\left(\frac{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}}{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}}\right)} \right) - 3 \left( \frac{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}}{(d \sqrt{\tan(d x + c)} - 2 d \sqrt{\tan(d x + c)} + d) \log\left(\frac{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}}{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}}\right)} \right) - 2 \left( \frac{0 \cdot A - 3 \cdot B}{\sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}} \right) \sqrt{\frac{0 \cdot A^2 + 2 \cdot A \cdot B - 1 \cdot B^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="fricas")

[Out]  $-2/3*(3*\sqrt{-1*A^2 + 2*A*B - 1*B^2}*a^6/d^2)*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + \sqrt{-1*A^2 + 2*A*B - 1*B^2}*a^6/d^2)*(I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^3)) - 3*\sqrt{-1*A^2 + 2*A*B - 1*B^2}*a^6/d^2*(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} + \sqrt{-1*A^2 + 2*A*B - 1*B^2}*a^6/d^2)*(-I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)}/((-I*A - B)*a^3)) - 2*((5*A - 3*I*B)*a^3*e^{(4*I*d*x + 4*I*c)} + (A + 3*I*B)*a^3*e^{(2*I*d*x + 2*I*c)} - 4*A*a^3)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(4*I*d*x + 4*I*c)} - 2*d*e^{(2*I*d*x + 2*I*c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-i a^3 \left( \int \left( -\frac{3A}{\tan^3(c+dx)} \right) dx + \int A \sqrt{\tan(c+dx)} dx + \int \left( -\frac{3B}{\sqrt{\tan(c+dx)}} \right) dx + \int B \tan^3(c+dx) dx + \int \frac{iA}{\tan^3(c+dx)} dx + \int \left( -\frac{3iA}{\sqrt{\tan(c+dx)}} \right) dx + \int \frac{iB}{\tan^3(c+dx)} dx + \int (-3iB \sqrt{\tan(c+dx)}) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x)

[Out]  $-I*a**3*(Integral(-3*A/tan(c + d*x)**(3/2), x) + Integral(A*\sqrt{\tan(c + d*x)}, x) + Integral(-3*B/\sqrt{\tan(c + d*x)}, x) + Integral(B*\tan(c + d*x)**(3/2), x) + Integral(I*A/tan(c + d*x)**(5/2), x) + Integral(-3*I*A/\sqrt{\tan(c + d*x)}, x) + Integral(I*B/tan(c + d*x)**(3/2), x) + Integral(-3*I*B*\sqrt{\tan(c + d*x)}, x))$

**Giac [A]**

time = 1.10, size = 97, normalized size = 0.71

$$-\frac{2iBa^3\sqrt{\tan(dx+c)}}{d} - \frac{(4i-4)\sqrt{2}(-iAa^3 - Ba^3)\arctan\left(-\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}}{d} + \frac{2(-9iAa^3\tan(dx+c) - 3Ba^3\tan(dx+c) - Aa^3)}{3d\tan(dx+c)^{\frac{3}{2}}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out]  $-2*I*B*a^3*\sqrt{\tan(d*x+c)}/d - (4*I-4)*\sqrt{2}*(-I*A*a^3-B*a^3)*\arctan(-\frac{1}{2}I-\frac{1}{2}\sqrt{2}*\sqrt{\tan(d*x+c)})/d + \frac{2}{3}*(-9*I*A*a^3*\tan(d*x+c) - 3*B*a^3*\tan(d*x+c) - A*a^3)/(d*\tan(d*x+c)^{(3/2)})$

**Mupad [B]**

time = 7.06, size = 240, normalized size = 1.76

$$\frac{\frac{1}{2} \frac{A^2 + A^2 \tan^2(c+dx)}{\tan(c+dx)^2} - \frac{2 B a^3}{d \sqrt{\tan(c+dx)}} - \frac{B a^3 \sqrt{\tan(c+dx)}}{d} + \frac{\sqrt{2} A^2 \ln(-A^2 d^2 + \sqrt{2} A^2 d \sqrt{\tan(c+dx)} (-4+4i)) (2-2i)}{d} - \frac{\sqrt{-10} A^2 \ln(-A^2 d^2 + 2 \sqrt{-10} A^2 d \sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2} B a^3 \ln(-8 B^2 d + \sqrt{2} B a^3 d \sqrt{\tan(c+dx)} (-4-4i)) (2+2i)}{d} - \frac{\sqrt{10} B a^3 \ln(-8 B^2 d + 2 \sqrt{10} B a^3 d \sqrt{\tan(c+dx)})}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/tan(c + d\*x)^(5/2),x)

[Out]  $(2^{(1/2)}*A*a^3*\log(-A*a^3*d*8i - 2^{(1/2)}*A*a^3*d*\tan(c+d*x)^{(1/2)}*(4-4i))*(2-2i))/d - (2*B*a^3)/(d*\tan(c+d*x)^{(1/2)}) - (B*a^3*\tan(c+d*x)^{(1/2)}*2i)/d - ((2*A*a^3)/(3*d) + (A*a^3*\tan(c+d*x)*6i)/d)/\tan(c+d*x)^{(3/2)} - ((-16i)^{(1/2)}*A*a^3*\log(2*(-16i)^{(1/2)}*A*a^3*d*\tan(c+d*x)^{(1/2)} - A*a^3*d*8i))/d + (2^{(1/2)}*B*a^3*\log(-8*B*a^3*d - 2^{(1/2)}*B*a^3*d*\tan(c+d*x)^{(1/2)}*(4+4i))*(2+2i))/d - (16i)^{(1/2)}*B*a^3*\log(2*16i^{(1/2)}*B*a^3*d*\tan(c+d*x)^{(1/2)} - 8*B*a^3*d))/d$

$$3.132 \quad \int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=144

$$\frac{8\sqrt[4]{-1} a^3(iA+B)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{16a^3(6A-5iB)}{15d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^2}{5d \tan^{\frac{5}{2}}(c+dx)} - \frac{2(9iA}{d}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(I*A+B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+16/15*a^3*(6*A-5*I*B)/d/\tan(d*x+c)^{(1/2)}-2/5*a*A*(a+I*a*\tan(d*x+c))^2/d/\tan(d*x+c)^{(5/2)}-2/15*(9*I*A+5*B)*(a^3+I*a^3*\tan(d*x+c))/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.25, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3674, 3672, 3614, 211}

$$\frac{8\sqrt[4]{-1} a^3(B+iA)\text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} - \frac{2(5B+9iA)(a^3+ia^3 \tan(c+dx))}{15d \tan^{\frac{5}{2}}(c+dx)} + \frac{16a^3(6A-5iB)}{15d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^2}{5d \tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out]  $(8*(-1)^{(1/4)}*a^3*(I*A+B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c+d*x]]])/d+(16*a^3*(6*A-(5*I)*B))/(15*d*\text{Sqrt}[\text{Tan}[c+d*x]])-(2*a*A*(a+I*a*\text{Tan}[c+d*x])^2)/(5*d*\text{Tan}[c+d*x]^{(5/2)})-(2*((9*I)*A+5*B)*(a^3+I*a^3*\text{Tan}[c+d*x]))/(15*d*\text{Tan}[c+d*x]^{(3/2)})$

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3614

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m+1)/(b\*f\*(m+1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m+1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x],

x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rubi steps

$$\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx = -\frac{2aA(a + ia \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + ia \tan(c + dx))^2}{\tan^{\frac{5}{2}}(c + dx)} dx$$

$$= -\frac{2aA(a + ia \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(9iA + 5B)(a^3 + ia^3 \tan(c + dx))}{15d \tan^{\frac{3}{2}}(c + dx)}$$

$$= \frac{16a^3(6A - 5iB)}{15d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(9iA + 5B)(a^3 + ia^3 \tan(c + dx))}{15d \tan^{\frac{3}{2}}(c + dx)}$$

$$= \frac{16a^3(6A - 5iB)}{15d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(9iA + 5B)(a^3 + ia^3 \tan(c + dx))}{15d \tan^{\frac{3}{2}}(c + dx)}$$

$$= \frac{8\sqrt{-1} a^3 (iA + B) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} + \frac{1}{15} \int \frac{(a + ia \tan(c + dx))^2}{\tan^{\frac{5}{2}}(c + dx)} dx$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 449 vs. 2(144) = 288.  
time = 7.91, size = 449, normalized size = 3.12

$$\frac{8(A - iB) \sqrt{-1} \sqrt{\tan(c + dx)} \operatorname{arctanh}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{\sqrt{-1}}\right) + \frac{2aA(a + ia \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(9iA + 5B)(a^3 + ia^3 \tan(c + dx))}{15d \tan^{\frac{3}{2}}(c + dx)}}{15d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(9iA + 5B)(a^3 + ia^3 \tan(c + dx))}{15d \tan^{\frac{3}{2}}(c + dx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] ((-8\*I)\*(A - I\*B)\*Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]

$$\begin{aligned} & ]*\text{Cos}[c + d*x]^4*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x))/(d*E^{((3*I} \\ & *c)*\text{Sqrt}[(-1 + E^{((2*I)*(c + d*x))})/(1 + E^{((2*I)*(c + d*x))})]*(\text{Cos}[d*x] + \\ & I*\text{Sin}[d*x])^3*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])) + (\text{Cos}[c + d*x]^4*(\text{Csc}[c]* \\ & (63*A*\text{Cos}[c] - (45*I)*B*\text{Cos}[c] + (15*I)*A*\text{Sin}[c] + 5*B*\text{Sin}[c])*((2*\text{Cos}[3*c] \\ & )/15 - ((2*I)/15)*\text{Sin}[3*c]) + \text{Csc}[c]*\text{Csc}[c + d*x]^2*(3*A*\text{Cos}[c] + (15*I)*A* \\ & \text{Sin}[c] + 5*B*\text{Sin}[c])*((-2*\text{Cos}[3*c])/15 + ((2*I)/15)*\text{Sin}[3*c]) + A*\text{Csc}[c]*\text{Cs} \\ & c[c + d*x]^3*((2*\text{Cos}[3*c])/5 - ((2*I)/5)*\text{Sin}[3*c])*\text{Sin}[d*x] + \text{Csc}[c]*\text{Csc}[c \\ & + d*x]*((-6*\text{Cos}[3*c])/5 + ((6*I)/5)*\text{Sin}[3*c])*(7*A*\text{Sin}[d*x] - (5*I)*B*\text{Sin}[d \\ & *x]))*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x))/(d* \\ & (\text{Cos}[d*x] + I*\text{Sin}[d*x])^3*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])) \end{aligned}$$

**Maple [A]**

time = 0.05, size = 240, normalized size = 1.67

method	result
derivativedivides	$a^3 \left( -\frac{2A}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2(3iA+B)}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(3iB-4A)}{\sqrt{\tan(dx+c)}} + \frac{(-4iA-4B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)} \right)$
default	$a^3 \left( -\frac{2A}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2(3iA+B)}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(3iB-4A)}{\sqrt{\tan(dx+c)}} + \frac{(-4iA-4B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x,method=\_RETURN VERBOSE)

[Out] 1/d\*a^3\*(-2/5\*A/tan(d\*x+c)^(5/2)-2/3\*(3\*I\*A+B)/tan(d\*x+c)^(3/2)-2\*(3\*I\*B-4\*A)/tan(d\*x+c)^(1/2)+1/4\*(-4\*I\*A-4\*B)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(-4\*I\*B+4\*A)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))

**Maxima [A]**

time = 0.50, size = 197, normalized size = 1.37

$$\frac{1}{15} \left( 2\sqrt{2}^{(i-1)A+(i+1)B} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}^{(i-1)A+(i+1)B} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})\right) - \sqrt{2}^{-(i+1)A+(i-1)B} \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \sqrt{2}^{-(i+1)A+(i-1)B} \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) \right) a^3 - \frac{2(15iA-8iB)\sqrt{\tan(dx+c)^2-1} + (15iA-8iB)\sqrt{\tan(dx+c)-3A^2}}{\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.



\*x)), x) + Integral(I\*A/tan(c + d\*x)\*\*(7/2), x) + Integral(-3\*I\*A/tan(c + d\*x)\*\*(3/2), x) + Integral(I\*B/tan(c + d\*x)\*\*(5/2), x) + Integral(-3\*I\*B/sqrt(tan(c + d\*x)), x)

**Giac [A]**

time = 1.38, size = 107, normalized size = 0.74

$$-\frac{(4i+4)\sqrt{2}(iAa^3+Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} + \frac{2(60Aa^3\tan(dx+c)^2-45iBa^3\tan(dx+c)^2-15iAa^3\tan(dx+c)-5Ba^3\tan(dx+c)-3Aa^3)}{15d\tan(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] -(4\*I + 4)\*sqrt(2)\*(I\*A\*a^3 + B\*a^3)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d + 2/15\*(60\*A\*a^3\*tan(d\*x + c)^2 - 45\*I\*B\*a^3\*tan(d\*x + c)^2 - 15\*I\*A\*a^3\*tan(d\*x + c) - 5\*B\*a^3\*tan(d\*x + c) - 3\*A\*a^3)/(d\*tan(d\*x + c)^(5/2))

**Mupad [B]**

time = 7.39, size = 258, normalized size = 1.79

$$-\frac{2Aa^3}{d} - \frac{8A^2a^3\tan^2(dx+c) + 8A^2a^3\tan(dx+c)}{d} - \frac{2Ba^3}{d} - \frac{8A^2a^3\tan^2(dx+c)}{d} + \frac{\sqrt{2}Aa^3\ln(8Aa^3d + \sqrt{2}Aa^3d\sqrt{\tan(c+dx)})(-4-4i)(2+2i)}{d} - \frac{\sqrt{16}Aa^3\ln(8Aa^3d + 2\sqrt{16}Aa^3d\sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^3\ln(-Ba^3d8i + \sqrt{2}Ba^3d\sqrt{\tan(c+dx)})(-4+4i)(2-2i)}{d} - \frac{\sqrt{-16}Ba^3\ln(-Ba^3d8i + 2\sqrt{-16}Ba^3d\sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/tan(c + d\*x)^(7/2),x)

[Out] (2^(1/2)\*A\*a^3\*log(8\*A\*a^3\*d - 2^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 + 4i))\*(2 + 2i))/d - ((2\*B\*a^3)/(3\*d) + (B\*a^3\*tan(c + d\*x)\*6i)/d)/tan(c + d\*x)^(3/2) - ((2\*A\*a^3)/(5\*d) + (A\*a^3\*tan(c + d\*x)\*2i)/d - (8\*A\*a^3\*tan(c + d\*x)^2)/d)/tan(c + d\*x)^(5/2) - (16i^(1/2)\*A\*a^3\*log(8\*A\*a^3\*d + 2\*16i^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)))/d + (2^(1/2)\*B\*a^3\*log(-B\*a^3\*d\*8i - 2^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 - 4i))\*(2 - 2i))/d - ((-16i)^(1/2)\*B\*a^3\*log(2\*(-16i)^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2) - B\*a^3\*d\*8i))/d

$$3.133 \quad \int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=169

$$\frac{8\sqrt{-1} a^3 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{8a^3(23A - 21iB)}{105d \tan^{\frac{3}{2}}(c+dx)} + \frac{8a^3(iA + B)}{d\sqrt{\tan(c+dx)}} - \frac{2aA(a + ia)}{7d \tan^{\frac{7}{2}}(c+dx)}$$

[Out]  $-8*(-1)^{(1/4)}*a^3*(A-I*B)*\arctan((-1)^{(3/4)}*\tan(d*x+c)^{(1/2)})/d+8*a^3*(I*A+B)/d/\tan(d*x+c)^{(1/2)}+8/105*a^3*(23*A-21*I*B)/d/\tan(d*x+c)^{(3/2)}-2/7*a*A*(a+I*a*\tan(d*x+c))^2/d/\tan(d*x+c)^{(7/2)}-2/35*(11*I*A+7*B)*(a^3+I*a^3*\tan(d*x+c))/d/\tan(d*x+c)^{(5/2)}$

**Rubi [A]**

time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3674, 3672, 3610, 3614, 211}

$$\frac{8\sqrt{-1} a^3 (A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c+dx)}\right)}{d} + \frac{8a^3(23A - 21iB)}{105d \tan^{\frac{3}{2}}(c+dx)} - \frac{2(7B + 11iA)(a^3 + ia^3 \tan(c+dx))}{35d \tan^{\frac{5}{2}}(c+dx)} + \frac{8a^3(B + iA)}{d\sqrt{\tan(c+dx)}} - \frac{2aA(a + ia \tan(c+dx))^2}{7d \tan^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{(9/2)}}, x]$

[Out]  $(-8*(-1)^{(1/4)}*a^3*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])/d + (8*a^3*(23*A - (21*I)*B))/(105*d*\text{Tan}[c + d*x]^{(3/2)}) + (8*a^3*(I*A + B))/(d*\text{Sqrt}[\text{Tan}[c + d*x]]) - (2*a*A*(a + I*a*\text{Tan}[c + d*x])^2)/(7*d*\text{Tan}[c + d*x]^{(7/2)}) - (2*((11*I)*A + 7*B)*(a^3 + I*a^3*\text{Tan}[c + d*x]))/(35*d*\text{Tan}[c + d*x]^{(5/2)})$

**Rule 211**

$\text{Int}[\frac{(a_1 + (b_1)*(x_1)^2)^{-1}}{x_1}, x_1] \rightarrow \text{Simp}[\frac{\text{Rt}[a/b, 2]}{a}*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

**Rule 3610**

$\text{Int}[\frac{(a_1 + (b_1)*\tan[(e_1) + (f_1)*(x_1)])^{(m_1)}*((c_1) + (d_1)*\tan[(e_1) + (f_1)*(x_1)])}{x_1}, x_1] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m+1)})/(f*(m+1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

**Rule 3614**

$\text{Int}[\frac{(c_1 + (d_1)*\tan[(e_1) + (f_1)*(x_1)])}{\text{Sqrt}[(b_1)*\tan[(e_1) + (f_1)*(x_1)]]}, x_1] \rightarrow \text{Dist}[2*(c^2/f), \text{Subst}[\text{Int}[1/(b*c - d*x^2), x], x, \text{Sqrt}[b*$

Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

#### Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

#### Rule 3674

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^2}{7d \tan^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{(a + ia \tan(c + dx))^2}{\tan^{\frac{7}{2}}(c + dx)} dx \\
 &= -\frac{2aA(a + ia \tan(c + dx))^2}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(11iA + 7B)(a^3 + ia^3 \tan(c + dx))}{35d \tan^{\frac{5}{2}}(c + dx)} \\
 &= \frac{8a^3(23A - 21iB)}{105d \tan^{\frac{3}{2}}(c + dx)} - \frac{2aA(a + ia \tan(c + dx))^2}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(11iA + 7B)(a^3 + ia^3 \tan(c + dx))}{35d \tan^{\frac{5}{2}}(c + dx)} \\
 &= \frac{8a^3(23A - 21iB)}{105d \tan^{\frac{3}{2}}(c + dx)} + \frac{8a^3(iA + B)}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^2}{7d \tan^{\frac{7}{2}}(c + dx)} \\
 &= \frac{8a^3(23A - 21iB)}{105d \tan^{\frac{3}{2}}(c + dx)} + \frac{8a^3(iA + B)}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^2}{7d \tan^{\frac{7}{2}}(c + dx)} \\
 &= -\frac{8\sqrt{-1} a^3 (A - iB) \tan^{-1} \left( (-1)^{3/4} \sqrt{\tan(c + dx)} \right)}{d} + \frac{8}{105d} \int \frac{(a + ia \tan(c + dx))^2}{\tan^{\frac{7}{2}}(c + dx)} dx
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 495 vs.  $2(169) = 338$ .



time = 8.88, size = 495, normalized size = 2.93

$$\frac{(A + I B \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan(c + dx)^{9/2}}, x$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] (8\*(A - I\*B)\*Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))) \* ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))] \* Cos[c + d\*x]^4\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x])/(d\*E^((3\*I)\*c)\*Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]\*(Cos[d\*x] + I\*Sin[d\*x])^3\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])) + (Cos[c + d\*x]^4\*(Csc[c]\*Csc[c + d\*x]^2\*((-63\*I)\*A\*Cos[c] - 21\*B\*Cos[c] + 170\*A\*Sin[c] - (105\*I)\*B\*Sin[c])\*((2\*Cos[3\*c])/105 - ((2\*I)/105)\*Sin[3\*c]) + Csc[c]\*((483\*I)\*A\*Cos[c] + 441\*B\*Cos[c] - 155\*A\*Sin[c] + (105\*I)\*B\*Sin[c])\*((2\*Cos[3\*c])/105 - ((2\*I)/105)\*Sin[3\*c]) + Csc[c + d\*x]^4\*((-2\*A\*Cos[3\*c])/7 + ((2\*I)/7)\*A\*Sin[3\*c]) + Csc[c]\*Csc[c + d\*x]\*((2\*Cos[3\*c])/5 - ((2\*I)/5)\*Sin[3\*c])\*((-23\*I)\*A\*Sin[d\*x] - 21\*B\*Sin[d\*x]) + Csc[c]\*Csc[c + d\*x]^3\*((2\*Cos[3\*c])/5 - ((2\*I)/5)\*Sin[3\*c])\*((3\*I)\*A\*Sin[d\*x] + B\*Sin[d\*x]))\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x])/(d\*(Cos[d\*x] + I\*Sin[d\*x])^3\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

Maple [A]

time = 0.05, size = 258, normalized size = 1.53

method	result
derivativedivides	$a^3 \left( \frac{2A}{7 \tan(dx+c)^{7/2}} - \frac{2(-4iA-4B)}{\sqrt{\tan(dx+c)}} - \frac{2(3iA+B)}{5 \tan(dx+c)^{5/2}} - \frac{2(3iB-4A)}{3 \tan(dx+c)^{3/2}} + \frac{(-4iB+4A)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right)\right)} \right)$
default	$a^3 \left( \frac{2A}{7 \tan(dx+c)^{7/2}} - \frac{2(-4iA-4B)}{\sqrt{\tan(dx+c)}} - \frac{2(3iA+B)}{5 \tan(dx+c)^{5/2}} - \frac{2(3iB-4A)}{3 \tan(dx+c)^{3/2}} + \frac{(-4iB+4A)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right)\right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2), x, method=\_RETURNVERBOSE)

```
[Out] 1/d*a^3*(-2/7*A/tan(d*x+c)^(7/2)-2*(-4*I*A-4*B)/tan(d*x+c)^(1/2)-2/5*(3*I*A+B)/tan(d*x+c)^(5/2)-2/3*(3*I*B-4*A)/tan(d*x+c)^(3/2)+1/4*(-4*I*B+4*A)*2^(1/2)*(ln(((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(4*I*A+4*B)*2^(1/2)*(ln(((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

**Maxima [A]**

time = 0.51, size = 215, normalized size = 1.27

$$\frac{105(2\sqrt{2}(i+1)A-(i-1)B)\operatorname{arctan}\left(\frac{1+\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}\right)+2\sqrt{2}(i+1)A-(i-1)B\operatorname{arctan}\left(\frac{-1+\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}\right)-\sqrt{2}(i-1)A+(i+1)B\log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}{\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}\right)+\sqrt{2}(i-1)A+(i+1)B\log\left(\frac{-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}\right)}{105d} + \frac{1(40iA+40iB)\operatorname{arctan}\left(\frac{1+\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}\right)+1(40iA-40iB)\operatorname{arctan}\left(\frac{-1+\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}\right)+1(40iA+40iB)\operatorname{arctan}\left(\frac{1+\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})}\right)+1(40iA-40iB)\operatorname{arctan}\left(\frac{-1+\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*(2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^3 + 2*(420*(I*A + B)*a^3*tan(d*x + c)^3 + 35*(4*A - 3*I*B)*a^3*tan(d*x + c)^2 + 21*(-3*I*A - B)*a^3*tan(d*x + c) - 15*A*a^3)/tan(d*x + c)^(7/2))/d
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs.  $2(137) = 274$ .

time = 2.67, size = 561, normalized size = 3.32

$$\frac{1}{105} \left( \frac{105 \sqrt{2} \left( (I+1)A - (I-1)B \right) \operatorname{arctan} \left( \frac{1 + \sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})} \right) + 2 \sqrt{2} \left( (I+1)A - (I-1)B \right) \operatorname{arctan} \left( \frac{-1 + \sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})}{\sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})} \right) - \sqrt{2} \left( (I-1)A + (I+1)B \right) \log \left( \frac{\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1}{\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1} \right) + \sqrt{2} \left( (I-1)A + (I+1)B \right) \log \left( \frac{-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1}{-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1} \right)}{105d} + \frac{2 \left( 420 (IA + B) a^3 \tan(dx+c)^3 + 35 (4A - 3IB) a^3 \tan(dx+c)^2 + 21 (-3IA - B) a^3 \tan(dx+c) - 15 A a^3 \right)}{105d} \right) \frac{1}{\tan(dx+c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(105*sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 105*sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2))*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((319*
```

$$A - 273*I*B)*a^3*e^{(8*I*d*x + 8*I*c)} - 3*(109*A - 133*I*B)*a^3*e^{(6*I*d*x + 6*I*c)} - 5*(19*A - 21*I*B)*a^3*e^{(4*I*d*x + 4*I*c)} + 3*(129*A - 133*I*B)*a^3*e^{(2*I*d*x + 2*I*c)} - 4*(41*A - 42*I*B)*a^3*\sqrt{( -I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)))/(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-ia^3 \left( \int \left( -\frac{3A}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{A}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{3B}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{B}{\sqrt{\tan(c+dx)}} dx + \int \frac{iA}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{3iA}{\tan^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{iB}{\tan^{\frac{3}{2}}(c+dx)} dx + \int \left( -\frac{3iB}{\tan^{\frac{3}{2}}(c+dx)} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(9/2), x)

[Out] -I\*a\*\*3\*(Integral(-3\*A/tan(c + d\*x)\*\*(7/2), x) + Integral(A/tan(c + d\*x)\*\*(3/2), x) + Integral(-3\*B/tan(c + d\*x)\*\*(5/2), x) + Integral(B/sqrt(tan(c + d\*x)), x) + Integral(I\*A/tan(c + d\*x)\*\*(9/2), x) + Integral(-3\*I\*A/tan(c + d\*x)\*\*(5/2), x) + Integral(I\*B/tan(c + d\*x)\*\*(7/2), x) + Integral(-3\*I\*B/tan(c + d\*x)\*\*(3/2), x))

**Giac [A]**

time = 1.46, size = 136, normalized size = 0.80

$$\frac{(4i-4)\sqrt{2}(-iAa^3 - Ba^3)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2(-420iAa^3 \tan(dx+c)^3 - 420Ba^3 \tan(dx+c)^3 - 140Aa^3 \tan(dx+c)^2 + 105iBa^3 \tan(dx+c)^2 + 63iAa^3 \tan(dx+c) + 21Ba^3 \tan(dx+c) + 15Aa^3)}{105d \tan(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2), x, algorithm="giac")

[Out] (4\*I - 4)\*sqrt(2)\*(-I\*A\*a^3 - B\*a^3)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/d - 2/105\*(-420\*I\*A\*a^3\*tan(d\*x + c)^3 - 420\*B\*a^3\*tan(d\*x + c)^3 - 140\*A\*a^3\*tan(d\*x + c)^2 + 105\*I\*B\*a^3\*tan(d\*x + c)^2 + 63\*I\*A\*a^3\*tan(d\*x + c) + 21\*B\*a^3\*tan(d\*x + c) + 15\*A\*a^3)/(d\*tan(d\*x + c)^(7/2))

**Mupad [B]**

time = 9.52, size = 293, normalized size = 1.73

$$\frac{\frac{144i^2 - 4a^2 \operatorname{Im}(\sqrt{2})}{\tan(c+dx)^{\frac{7}{2}}} - \frac{4a^2 \operatorname{Im}(\sqrt{2})}{\tan(c+dx)^{\frac{5}{2}}} - \frac{4a^2 \operatorname{Im}(\sqrt{2})}{\tan(c+dx)^{\frac{3}{2}}} + \frac{4a^2 \operatorname{Im}(\sqrt{2})}{\tan(c+dx)^{\frac{1}{2}}}}{d} + \frac{\sqrt{2}Aa^3 \ln(Aa^3 d \sqrt{\tan(c+dx)} - 4 + 4i)}{d} - \frac{\sqrt{2}Aa^3 \ln(Aa^3 d \sqrt{\tan(c+dx)} + 2\sqrt{10}Aa^3 d \sqrt{\tan(c+dx)})}{d} + \frac{\sqrt{2}Ba^3 \ln(8Ba^3 d + \sqrt{2}Ba^3 d \sqrt{\tan(c+dx)} - 4 - 4i)}{d} - \frac{\sqrt{10}Ba^3 \ln(8Ba^3 d + 2\sqrt{10}Ba^3 d \sqrt{\tan(c+dx)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/tan(c + d\*x)^(9/2), x)

[Out] (2^(1/2)\*A\*a^3\*log(A\*a^3\*d\*8i - 2^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 - 4i))\*(2 - 2i))/d - ((2\*B\*a^3)/(5\*d) + (B\*a^3\*tan(c + d\*x)\*2i)/d - (8\*B\*a^3\*tan(c + d\*x)^2)/d)/tan(c + d\*x)^(5/2) - ((2\*A\*a^3)/(7\*d) + (A\*a^3\*tan(c + d\*x)\*6i)/(5\*d) - (8\*A\*a^3\*tan(c + d\*x)^2)/(3\*d) - (A\*a^3\*tan(c + d\*x)^3\*8i)/d)/tan(c + d\*x)^(7/2) - ((-16i)^(1/2)\*A\*a^3\*log(A\*a^3\*d\*8i + 2\*(-16i)^(1/2)\*A\*a^3\*d\*tan(c + d\*x)^(1/2)))/d + (2^(1/2)\*B\*a^3\*log(8\*B\*a^3\*d - 2^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2)\*(4 + 4i))\*(2 + 2i))/d - (16i^(1/2)\*B\*a^3\*log(8\*B\*a^3\*d + 2\*16i^(1/2)\*B\*a^3\*d\*tan(c + d\*x)^(1/2)))/d

$$3.134 \quad \int \frac{\tan^5(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=306

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad}$$

[Out] (1/8+1/8\*I)\*((4+I)\*A+(1+6\*I)\*B)\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)+(1/8+1/8\*I)\*((4+I)\*A+(1+6\*I)\*B)\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)-(1/16+1/16\*I)\*((1+4\*I)\*A-(6+I)\*B)\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)-1/16\*((3-5\*I)\*A+(5+7\*I)\*B)\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)-5/2\*(I\*A-B)\*tan(d\*x+c)^(1/2)/a/d-1/6\*(3\*A+7\*I\*B)\*tan(d\*x+c)^(3/2)/a/d+1/2\*(I\*A-B)\*tan(d\*x+c)^(5/2)/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.29, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} ad} + \frac{(-B+iA) \tan^3(c+dx)}{2d(a+ia \tan(c+dx))} + \frac{(3A+7iB) \tan^2(c+dx)}{6ad} - \frac{5(-B+iA) \tan(c+dx)}{2ad} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((1+4i)A - (6+i)B\right) \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} ad} - \frac{((3-5i)A + (5+7i)B) \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{8\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] ((-1/4 - I/4)\*((4 + I)\*A + (1 + 6\*I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*a\*d) + ((1/4 + I/4)\*((4 + I)\*A + (1 + 6\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*a\*d) - ((1/8 + I/8)\*((1 + 4\*I)\*A - (6 + I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(Sqrt[2]\*a\*d) - ((3 - 5\*I)\*A + (5 + 7\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(8\*Sqrt[2]\*a\*d) - (5\*(I\*A - B)\*Sqrt[Tan[c + d\*x]]/(2\*a\*d) - ((3\*A + (7\*I)\*B)\*Tan[c + d\*x]^(3/2))/(6\*a\*d) + ((I\*A - B)\*Tan[c + d\*x]^(5/2))/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \ \text{Dist}[(d*q + a*e)/(2*a*c), \ \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \ \text{Dist}[(d*q - a*e)/(2*a*c), \ \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3609

$\text{Int}[\frac{((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \ :> \ \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \ \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \ \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \ \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3676

```

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx &= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))} - \frac{\int \tan^{\frac{3}{2}}(c+dx) \left(\frac{5}{2}a(iA-B) + \frac{1}{2}a(3A+B)\right)}{2a^2} \\
&= -\frac{(3A+7iB) \tan^{\frac{3}{2}}(c+dx)}{6ad} + \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))} - \frac{\int \sqrt{\tan(c+dx)}}{2a^2} \\
&= -\frac{5(iA-B) \sqrt{\tan(c+dx)}}{2ad} - \frac{(3A+7iB) \tan^{\frac{3}{2}}(c+dx)}{6ad} + \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))} \\
&= -\frac{5(iA-B) \sqrt{\tan(c+dx)}}{2ad} - \frac{(3A+7iB) \tan^{\frac{3}{2}}(c+dx)}{6ad} + \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))} \\
&= -\frac{5(iA-B) \sqrt{\tan(c+dx)}}{2ad} - \frac{(3A+7iB) \tan^{\frac{3}{2}}(c+dx)}{6ad} + \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))} \\
&= -\frac{5(iA-B) \sqrt{\tan(c+dx)}}{2ad} - \frac{(3A+7iB) \tan^{\frac{3}{2}}(c+dx)}{6ad} + \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))} \\
&= \frac{((3-5i)A + (5+7i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{8\sqrt{2} ad} \\
&= -\frac{((3+5i)A - (5-7i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{4\sqrt{2} ad} + \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{2d(a+ia \tan(c+dx))}
\end{aligned}$$

**Mathematica [A]**

time = 2.52, size = 248, normalized size = 0.81

$$\frac{(\cos(dx) + i \sin(dx))(A + B \tan(c + dx)) \left( (-1 - i) \left( (i + i)A + (1 + 6i)B \right) \operatorname{ArcSinh}(\cos(c + dx) - \sin(c + dx)) + (-1 - 4i)A + (6 + i)B \right) \log\left(\cos(c + dx) + \sin(c + dx) + \sqrt{4 \tan^2(c + dx) + 1}\right) + \frac{5}{2} \sec(c + dx) (\cos(dx) - i \sin(dx)) \sqrt{4 \tan^2(c + dx) + 1} + \frac{5}{2} \sec(c + dx) (\cos(dx) - i \sin(dx)) (-15iA + 19B + (-15A + 11B) \cos(2(c + dx)) + 4(3A + 2iB) \sin(2(c + dx))) \tan(c + dx)}{8i(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),
x]

```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x])*((-1 - I)*((4 + I)*A + (1 + 6*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + ((-1 - 4*I)*A + (6 + I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sec[c + d*x]*(Cos[c] + I*Sin[c])*Sqrt[Sin[2*(c + d*x)]] + (2*Sec[c + d*x]*(Cos[d*x] - I*Sin[d*x]))*((-15*I)*A + 19*B + ((-15*I)*A + 11*B)*Cos[2*(c + d*x)] + 4*(3*A + (2*I)*B)*Sin[2*(c + d*x)]*Tan[c + d*x])/3)/(8*d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

**Maple [A]**

time = 0.12, size = 166, normalized size = 0.54

method	result
derivativedivides	$-\frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2B \left( \sqrt{\tan(dx+c)} \right) - 2iA \left( \sqrt{\tan(dx+c)} \right) - \frac{i \left( \frac{i(iB+A) \left( \sqrt{\tan(dx+c)} \right)^{4(2iA-3B) \arctan(\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)-i})}}{\tan(dx+c)-i} + \frac{4(2iA-3B) \arctan(\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)-i})}{2} \right)}{2}$
default	$-\frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2B \left( \sqrt{\tan(dx+c)} \right) - 2iA \left( \sqrt{\tan(dx+c)} \right) - \frac{i \left( \frac{i(iB+A) \left( \sqrt{\tan(dx+c)} \right)^{4(2iA-3B) \arctan(\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)-i})}}{\tan(dx+c)-i} + \frac{4(2iA-3B) \arctan(\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)-i})}{2} \right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/d/a*(-2/3*I*B*tan(d*x+c)^(3/2)+2*B*tan(d*x+c)^(1/2)-2*I*A*tan(d*x+c)^(1/2)
)-1/2*I*(-I*(A+I*B)*tan(d*x+c)^(1/2)/(tan(d*x+c)-I)+4*(2*I*A-3*B)/(2^(1/2)-
I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4*(-1/4*A+1/4*I*
B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm
="maxima")
```

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
 expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(221) = 442.  
 time = 2.67, size = 707, normalized size = 2.31



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{24} * (3 * (a * d * e^{(4 * I * d * x + 4 * I * c)} + a * d * e^{(2 * I * d * x + 2 * I * c)}) * \sqrt{(I * A^2 + 2 * A * B - I * B^2)} / (a^2 * d^2) * \log(2 * ((a * d * e^{(2 * I * d * x + 2 * I * c)} + a * d) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(I * A^2 + 2 * A * B - I * B^2)} / (a^2 * d^2) + (A - I * B) * e^{(2 * I * d * x + 2 * I * c)} * e^{(-2 * I * d * x - 2 * I * c)} / (I * A + B)) - 3 * (a * d * e^{(4 * I * d * x + 4 * I * c)} + a * d * e^{(2 * I * d * x + 2 * I * c)}) * \sqrt{(I * A^2 + 2 * A * B - I * B^2)} / (a^2 * d^2) * \log(-2 * ((a * d * e^{(2 * I * d * x + 2 * I * c)} + a * d) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(I * A^2 + 2 * A * B - I * B^2)} / (a^2 * d^2) - (A - I * B) * e^{(2 * I * d * x + 2 * I * c)} * e^{(-2 * I * d * x - 2 * I * c)} / (I * A + B)) - 6 * (a * d * e^{(4 * I * d * x + 4 * I * c)} + a * d * e^{(2 * I * d * x + 2 * I * c)}) * \sqrt{(-4 * I * A^2 + 12 * A * B + 9 * I * B^2)} / (a^2 * d^2) * \log(-((a * d * e^{(2 * I * d * x + 2 * I * c)} + a * d) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-4 * I * A^2 + 12 * A * B + 9 * I * B^2)} / (a^2 * d^2) + 2 * A + 3 * I * B) * e^{(-2 * I * d * x - 2 * I * c)} / (a * d) + 6 * (a * d * e^{(4 * I * d * x + 4 * I * c)} + a * d * e^{(2 * I * d * x + 2 * I * c)}) * \sqrt{(-4 * I * A^2 + 12 * A * B + 9 * I * B^2)} / (a^2 * d^2) * \log(((a * d * e^{(2 * I * d * x + 2 * I * c)} + a * d) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-4 * I * A^2 + 12 * A * B + 9 * I * B^2)} / (a^2 * d^2) - 2 * A - 3 * I * B) * e^{(-2 * I * d * x - 2 * I * c)} / (a * d) + 2 * ((-27 * I * A + 19 * B) * e^{(4 * I * d * x + 4 * I * c)} - 2 * (15 * I * A - 19 * B) * e^{(2 * I * d * x + 2 * I * c)} - 3 * I * A + 3 * B) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) / (a * d * e^{(4 * I * d * x + 4 * I * c)} + a * d * e^{(2 * I * d * x + 2 * I * c)})$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \tan^{\frac{5}{2}}(c+dx)}{\tan(c+dx)-i} dx + \int \frac{B \tan^{\frac{7}{2}}(c+dx)}{\tan(c+dx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out] 
$$-I * (\text{Integral}(A * \tan(c + d * x) ** (5/2) / (\tan(c + d * x) - I), x) + \text{Integral}(B * \tan(c + d * x) ** (7/2) / (\tan(c + d * x) - I), x)) / a$$



**Giac [A]**

time = 0.67, size = 165, normalized size = 0.54

$$\frac{(i-1)\sqrt{2}(-2iA+3B)\arctan\left(\frac{1}{2}\left(i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} + \frac{(i+1)\sqrt{2}(-iA-B)\arctan\left(\frac{1}{2}\left(i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} - \frac{A\sqrt{\tan(dx+c)}+iB\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)} - \frac{2(iBa^2d^2\tan(dx+c)^2+3iAa^2d^2\sqrt{\tan(dx+c)}-3Ba^2d^2\sqrt{\tan(dx+c)})}{3a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] (1/2\*I - 1/2)\*sqrt(2)\*(-2\*I\*A + 3\*B)\*arctan((1/2\*I + 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a\*d) + (1/4\*I + 1/4)\*sqrt(2)\*(-I\*A - B)\*arctan((1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a\*d) - 1/2\*(A\*sqrt(tan(d\*x + c)) + I\*B\*sqrt(tan(d\*x + c)))/(a\*d\*(tan(d\*x + c) - I)) - 2/3\*(I\*B\*a^2\*d^2\*tan(d\*x + c)^(3/2) + 3\*I\*A\*a^2\*d^2\*sqrt(tan(d\*x + c)) - 3\*B\*a^2\*d^2\*sqrt(tan(d\*x + c)))/(a^3\*d^3)

**Mupad [B]**

time = 11.33, size = 305, normalized size = 1.00

$$\operatorname{atan}\left(\frac{ad\sqrt{\tan(c+dx)}\sqrt{\frac{d^2+1}{2d^2}}}{A}\right)\sqrt{\frac{A^2+1}{a^2d^2}}2i - \operatorname{atan}\left(\frac{ad\sqrt{\tan(c+dx)}\sqrt{\frac{d^2+1}{2d^2}}}{A}\right)\sqrt{\frac{A^2+1}{16a^2d^2}}2i + \operatorname{atan}\left(\frac{2ad\sqrt{\tan(c+dx)}\sqrt{\frac{d^2+1}{2d^2}}}{3B}\right)\sqrt{\frac{B^2+9}{4a^2d^2}}2i + \operatorname{atan}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{d^2+1}{16a^2d^2}}}{B}\right)\sqrt{\frac{B^2+1}{16a^2d^2}}2i - \frac{A\sqrt{\tan(c+dx)}2i}{ad} + \frac{2B\sqrt{\tan(c+dx)}}{ad} - \frac{B\tan(c+dx)^{3/2}2i}{3ad} - \frac{A\sqrt{\tan(c+dx)}2i}{2ad(1+\tan(c+dx))} + \frac{B\sqrt{\tan(c+dx)}}{2ad(1+\tan(c+dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] atan((a\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(a^2\*d^2))^(1/2)\*1i)/A)\*(-(A^2\*1i)/(a^2\*d^2))^(1/2)\*2i - atan((a\*d\*tan(c + d\*x)^(1/2)\*((A^2\*1i)/(16\*a^2\*d^2))^(1/2)\*4i)/A)\*((A^2\*1i)/(16\*a^2\*d^2))^(1/2)\*2i + atan((2\*a\*d\*tan(c + d\*x)^(1/2)\*((B^2\*9i)/(4\*a^2\*d^2))^(1/2))/(3\*B))\*((B^2\*9i)/(4\*a^2\*d^2))^(1/2)\*2i + atan((4\*a\*d\*tan(c + d\*x)^(1/2)\*(-(B^2\*1i)/(16\*a^2\*d^2))^(1/2))/B)\*(-(B^2\*1i)/(16\*a^2\*d^2))^(1/2)\*2i - (A\*tan(c + d\*x)^(1/2)\*2i)/(a\*d) + (2\*B\*tan(c + d\*x)^(1/2))/(a\*d) - (B\*tan(c + d\*x)^(3/2)\*2i)/(3\*a\*d) - (A\*tan(c + d\*x)^(1/2)\*1i)/(2\*a\*d\*(tan(c + d\*x)\*1i + 1)) + (B\*tan(c + d\*x)^(1/2))/(2\*a\*d\*(tan(c + d\*x)\*1i + 1))

$$3.135 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=275

$$\frac{((1-3i)A+(3+5i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((1+2i)A-(4+i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad}$$

[Out] 1/8\*((1-3\*I)\*A+(3+5\*I)\*B)\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)-(1/8+1/8\*I)\*((1+2\*I)\*A-(4+I)\*B)\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)-(1/16+1/16\*I)\*((2+I)\*A+(1+4\*I)\*B)\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)+(1/16+1/16\*I)\*((2+I)\*A+(1+4\*I)\*B)\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)-1/2\*(A+5\*I\*B)\*tan(d\*x+c)^(1/2)/a/d+1/2\*(I\*A-B)\*tan(d\*x+c)^(3/2)/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.24, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((1-3i)A+(3+5i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((1+2i)A-(4+i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} + \frac{(A+B+iA)\tan^3(c+dx)}{2i(a+ia\tan(c+dx))} - \frac{(A+5iB)\sqrt{\tan(c+dx)}}{2ad} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((2+i)A+(1+4i)B)\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}}{1+i}\right)}{\sqrt{2}ad} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((2+i)A+(1+4i)B)\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}}{1+i}\right)}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] -1/4\*(((1-3\*I)\*A+(3+5\*I)\*B)\*ArcTan[1-Sqrt[2]\*Sqrt[Tan[c+d\*x]]])/(Sqrt[2]\*a\*d)-((1/4+I/4)\*((1+2\*I)\*A-(4+I)\*B)\*ArcTan[1+Sqrt[2]\*Sqrt[Tan[c+d\*x]]])/(Sqrt[2]\*a\*d)-((1/8+I/8)\*((2+I)\*A+(1+4\*I)\*B)\*Log[1-Sqrt[2]\*Sqrt[Tan[c+d\*x]]+Tan[c+d\*x]]/(Sqrt[2]\*a\*d)+((1/8+I/8)\*((2+I)\*A+(1+4\*I)\*B)\*Log[1+Sqrt[2]\*Sqrt[Tan[c+d\*x]]+Tan[c+d\*x]]/(Sqrt[2]\*a\*d)-((A+(5\*I)\*B)\*Sqrt[Tan[c+d\*x]]/(2\*a\*d)+((I\*A-B)\*Tan[c+d\*x]^(3/2))/(2\*d\*(a+I\*a\*Tan[c+d\*x])))

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x]]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x]]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^n\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^n, x\_Symbol] := Sim

```
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])
]^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \frac{(iA - B) \tan^{\frac{3}{2}}(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{\int \sqrt{\tan(c + dx)} \left( \frac{3}{2}a(iA - B) + \frac{1}{2}a(A - B) \right)}{2a^2} \\
 &= -\frac{(A + 5iB) \sqrt{\tan(c + dx)}}{2ad} + \frac{(iA - B) \tan^{\frac{3}{2}}(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{\int \frac{-\frac{1}{2}a(A + B)}{a + ia \tan(c + dx)} dx}{2a^2} \\
 &= -\frac{(A + 5iB) \sqrt{\tan(c + dx)}}{2ad} + \frac{(iA - B) \tan^{\frac{3}{2}}(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(A + B)}{a + ia \tan(c + dx)} dx\right)}{2a^2} \\
 &= -\frac{(A + 5iB) \sqrt{\tan(c + dx)}}{2ad} + \frac{(iA - B) \tan^{\frac{3}{2}}(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{((1 + 3i)A - (3 - 5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad} \\
 &= -\frac{(A + 5iB) \sqrt{\tan(c + dx)}}{2ad} + \frac{(iA - B) \tan^{\frac{3}{2}}(c + dx)}{2d(a + ia \tan(c + dx))} - \frac{((1 + 3i)A - (3 - 5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad} \\
 &= -\frac{((1 - 3i)A + (3 + 5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{4\sqrt{2} ad} + \frac{((1 + 3i)A - (3 - 5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad}
 \end{aligned}$$

**Mathematica** [A]

time = 1.03, size = 220, normalized size = 0.80

$$\frac{(\cos(dx) + i \sin(dx))(A + B \tan(c + dx)) \left( -\left( (1 - 3i)A + (3 + 5i)B \right) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i)(2 + i)A + (1 + 4i)B \right) \log\left(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}\right) + (-4 \cos(dx) + 4i \sin(dx))(A + 5iB) \cos(c + dx) - 4B \sin(c + dx) \tan(c + dx)}{8i(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),
 x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*(A + B*Tan[c + d*x])*(-(((1 - 3*I)*A + (3 + 5*I)*
B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] - (1 + I)*((2 + I)*A + (1 + 4*I)*B)*
Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)])])*Sec[c + d*x]*(Co
s[c] + I*Sin[c])*Sqrt[Sin[2*(c + d*x)]) + (-4*Cos[d*x] + (4*I)*Sin[d*x])*
```

$(A + (5*I)*B)*\text{Cos}[c + d*x] - 4*B*\text{Sin}[c + d*x])* \text{Tan}[c + d*x]) / (8*d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])* \text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x]))$

**Maple [A]**

time = 0.09, size = 143, normalized size = 0.52

method	result
derivativedivides	$-2iB \left( \sqrt{\tan(dx+c)} \right) + \frac{i \left( \frac{i(iA-B) \left( \sqrt{\tan(dx+c)} \right)}{\tan(dx+c)-i} - \frac{4(2iB+A) \arctan \left( \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{2} + \frac{4 \left( -\frac{iA}{4} - \frac{B}{4} \right) \arctan \left( \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{da}$
default	$-2iB \left( \sqrt{\tan(dx+c)} \right) + \frac{i \left( \frac{i(iA-B) \left( \sqrt{\tan(dx+c)} \right)}{\tan(dx+c)-i} - \frac{4(2iB+A) \arctan \left( \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{2} + \frac{4 \left( -\frac{iA}{4} - \frac{B}{4} \right) \arctan \left( \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{da}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVE RBOSE)`

[Out]  $1/d/a*(-2*I*B*\text{tan}(d*x+c)^{(1/2)}+1/2*I*(-I*(I*A-B)*\text{tan}(d*x+c)^{(1/2)}/(\text{tan}(d*x+c)-I)-4*(A+2*I*B)/(2^{(1/2)}-I*2^{(1/2)})*\text{arctan}(2*\text{tan}(d*x+c)^{(1/2)}/(2^{(1/2)}-I*2^{(1/2)})))+4*(-1/4*I*A-1/4*B)/(2^{(1/2)}+I*2^{(1/2)})*\text{arctan}(2*\text{tan}(d*x+c)^{(1/2)}/(2^{(1/2)}+I*2^{(1/2)}))$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(198) = 396.

time = 1.06, size = 621, normalized size = 2.26



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/8*(a*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-
2*((I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*
B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((-I*A^2
- 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((-I*a*d*e^(2*I*d*x
+ 2*I*c) - I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c
))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 2*a*d*sqrt((I*A^2 - 4*A*B - 4*I*B^2)/(
a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 - 4*A*B -
4*I*B^2)/(a^2*d^2)) + I*A - 2*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*a*d*sqrt((
I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((a*d*e^(2*I*d
*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2)) - I*A + 2*B)*e^(-2*I*d*x - 2
*I*c)/(a*d)) - 2*((A + 9*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \tan^{\frac{3}{2}}(c+dx)}{\tan(c+dx)-i} dx + \int \frac{B \tan^{\frac{5}{2}}(c+dx)}{\tan(c+dx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*(Integral(A*tan(c + d*x)**(3/2)/(tan(c + d*x) - I), x) + Integral(B*tan(
c + d*x)**(5/2)/(tan(c + d*x) - I), x))/a
```

**Giac [A]**

time = 0.61, size = 118, normalized size = 0.43

$$\frac{(i+1)\sqrt{2}(A-iB)\arctan\left(-\frac{1}{2}(i-\frac{1}{2})\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} + \frac{(i-1)\sqrt{2}(A+2iB)\arctan\left(-\frac{1}{2}(i+\frac{1}{2})\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} - \frac{2iB\sqrt{\tan(dx+c)}}{ad} - \frac{-iA\sqrt{\tan(dx+c)} + B\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*(A - I*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x
+ c)))/(a*d) + (1/2*I - 1/2)*sqrt(2)*(A + 2*I*B)*arctan(-(1/2*I + 1/2)*sqr
```

$t(2)*\sqrt{\tan(dx + c)}/(a*d) - 2*I*B*\sqrt{\tan(dx + c)}/(a*d) - 1/2*(-I*A*\sqrt{\tan(dx + c)} + B*\sqrt{\tan(dx + c)})/(a*d*(\tan(dx + c) - I))$

**Mupad [B]**

time = 10.61, size = 270, normalized size = 0.98

$$-\operatorname{atan}\left(\frac{2ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2 11}{4a^2 d^2}}}{A}\right)\sqrt{\frac{A^2 11}{4a^2 d^2}} 2i - \operatorname{atan}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{-\frac{A^2 11}{16a^2 d^2}}}{A}\right)\sqrt{-\frac{A^2 11}{16a^2 d^2}} 2i + \operatorname{atan}\left(\frac{ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2 11}{a^2 d^2}} 1i}{B}\right)\sqrt{\frac{B^2 11}{a^2 d^2}} 2i - \operatorname{atan}\left(\frac{ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2 11}{16a^2 d^2}} 4i}{B}\right)\sqrt{\frac{B^2 11}{16a^2 d^2}} 2i - \frac{B\sqrt{\tan(c+dx)} 2i}{ad} - \frac{A\sqrt{\tan(c+dx)}}{2ad(1+\tan(c+dx) 1i)} - \frac{B\sqrt{\tan(c+dx)} 1i}{2ad(1+\tan(c+dx) 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((\tan(c + dx))^{3/2}*(A + B*\tan(c + dx)))/(a + a*\tan(c + dx)*1i), x$

[Out]  $\operatorname{atan}((a*d*\tan(c + dx))^{1/2}*(-(B^2*1i)/(a^2*d^2))^{1/2}*1i)/B)*(-(B^2*1i)/(a^2*d^2))^{1/2}*2i - \operatorname{atan}((4*a*d*\tan(c + dx))^{1/2}*(-(A^2*1i)/(16*a^2*d^2))^{1/2})/A)*(-(A^2*1i)/(16*a^2*d^2))^{1/2}*2i - \operatorname{atan}((2*a*d*\tan(c + dx))^{1/2}*((A^2*1i)/(4*a^2*d^2))^{1/2})/A)*((A^2*1i)/(4*a^2*d^2))^{1/2}*2i - \operatorname{atan}((a*d*\tan(c + dx))^{1/2}*((B^2*1i)/(16*a^2*d^2))^{1/2}*4i)/B)*((B^2*1i)/(16*a^2*d^2))^{1/2}*2i - (B*\tan(c + dx))^{1/2}*2i/(a*d) - (A*\tan(c + dx))^{1/2}/(2*a*d*(\tan(c + dx)*1i + 1)) - (B*\tan(c + dx))^{1/2}*1i/(2*a*d*(\tan(c + dx)*1i + 1))$

$$3.136 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=236

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad}$$

[Out] (1/8-1/8\*I)\*(A+(2-I)\*B)\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)+(1/8-1/8\*I)\*(A+(2-I)\*B)\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)+(1/16+1/16\*I)\*(A-(2+I)\*B)\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)-(1/16+1/16\*I)\*(A-(2+I)\*B)\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)+1/2\*(I\*A-B)\*tan(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.19, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} ad} + \frac{(-B + iA) \sqrt{\tan(c+dx)}}{2i(a + ia \tan(c+dx))} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - (2 + i)B) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} ad}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - (2 + i)B) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} ad}\right)}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] ((-1/4 + I/4)\*(A + (2 - I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*a\*d) + ((1/4 - I/4)\*(A + (2 - I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*a\*d) + ((1/8 + I/8)\*(A - (2 + I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(Sqrt[2]\*a\*d) - ((1/8 + I/8)\*(A - (2 + I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(Sqrt[2]\*a\*d) + ((I\*A - B)\*Sqrt[Tan[c + d\*x]]/(2\*d\*(a + I\*a\*Tan[c + d\*x])))

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 642**



Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{2d(a+ia \tan(c+dx))} - \frac{\int \frac{\frac{1}{2}a(iA-B) - \frac{1}{2}a(A-3iB) \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{2a^2} \\
&= \frac{(iA-B) \sqrt{\tan(c+dx)}}{2d(a+ia \tan(c+dx))} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(iA-B) - \frac{1}{2}a(A-3iB)x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{a^2 d} \\
&= \frac{(iA-B) \sqrt{\tan(c+dx)}}{2d(a+ia \tan(c+dx))} - \frac{\left(\left(\frac{1}{4} + \frac{i}{4}\right) (A - (2+i)B)\right) \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{ad} \\
&= \frac{(iA-B) \sqrt{\tan(c+dx)}}{2d(a+ia \tan(c+dx))} - \frac{\left(\left(\frac{1}{8} + \frac{i}{8}\right) (A - (2+i)B)\right) \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{ad} \\
&= \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - (2+i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{\sqrt{2} ad} \\
&= -\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2-i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2-i)B)}{\sqrt{2} ad}
\end{aligned}$$

**Mathematica [A]**

time = 0.82, size = 198, normalized size = 0.84

$$\frac{(\cos(dx) + i \sin(dx)) \left(4(A + iB)(i \cos(dx) + \sin(dx)) \sin(c + dx) + (1 + i) \left((A + (2 - i)B) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) + i(A - (2 + i)B) \log\left(\frac{\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}}{\cos(c + dx) - \sin(c + dx)}\right)\right) \sec(c + dx) (i \cos(c) - \sin(c)) \sqrt{\sin(2(c + dx))}\right) (A + B \tan(c + dx))}{8d(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]), x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*(4*(A + I*B)*(I*Cos[d*x] + Sin[d*x])*Sin[c + d*x] + (1 + I)*((A + (2 - I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + I*(A - (2 + I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)])])*Sec[c + d*x]*(I*Cos[c] - Sin[c])*Sqrt[Sin[2*(c + d*x)])*(A + B*Tan[c + d*x]))/(8*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

**Maple [A]**

time = 0.17, size = 124, normalized size = 0.53

method	result	size
--------	--------	------

derivativedivides	$\frac{i \left( \frac{i(B+A) \sqrt{\tan(dx+c)}}{\tan(dx+c)-i} - \frac{4B \arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} \right)}{2} + \frac{4\left(\frac{A}{4}-\frac{iB}{4}\right) \arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}$	124
default	$\frac{i \left( \frac{i(B+A) \sqrt{\tan(dx+c)}}{\tan(dx+c)-i} - \frac{4B \arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} \right)}{2} + \frac{4\left(\frac{A}{4}-\frac{iB}{4}\right) \arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}$	124

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d/a*(1/2*I*(-I*(A+I*B)*\tan(d*x+c)^{(1/2)}/(\tan(d*x+c)-I)-4*B/(2^{(1/2)}-I*2^{(1/2)}))*\arctan(2*\tan(d*x+c)^{(1/2)}/(2^{(1/2)}-I*2^{(1/2)})))+4*(1/4*A-1/4*I*B)/(2^{(1/2)}+I*2^{(1/2)})*\arctan(2*\tan(d*x+c)^{(1/2)}/(2^{(1/2)}+I*2^{(1/2)}))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs.  $2(177) = 354$ .

time = 1.35, size = 570, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm  
="fricas")`

```
[Out] -1/8*(a*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(2
*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*e^(
2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((I*A^2 + 2*A*B
- I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((a*d*e^(2*I*d*x + 2*I*c) +
a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*
A^2 + 2*A*B - I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*
x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log
(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(I*B^2/(a^2*d^2)) + I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)
) + 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((a*d*e^(2*I*d*x +
2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(I*B^2/(a^2*d^2)) - I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*((I*A - B)*e^
(2*I*d*x + 2*I*c) + I*A - B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)))e^(-2*I*d*x - 2*I*c)/(a*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \sqrt{\tan(c+dx)}}{\tan(c+dx)-i} dx + \int \frac{B \tan^{\frac{3}{2}}(c+dx)}{\tan(c+dx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)), x)
```

```
[Out] -I*(Integral(A*sqrt(tan(c + d*x))/(tan(c + d*x) - I), x) + Integral(B*tan(c
+ d*x)**(3/2)/(tan(c + d*x) - I), x))/a
```

**Giac [A]**

time = 0.56, size = 97, normalized size = 0.41

$$\frac{(i-1)\sqrt{2}B \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} - \frac{(i-1)\sqrt{2}(A-iB) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} + \frac{A\sqrt{\tan(dx+c)} + iB\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)), x, algorithm
="giac")
```

```
[Out] -(1/2*I - 1/2)*sqrt(2)*B*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c)))/(
a*d) - (1/4*I - 1/4)*sqrt(2)*(A - I*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(t
an(d*x + c)))/(a*d) + 1/2*(A*sqrt(tan(d*x + c)) + I*B*sqrt(tan(d*x + c)))/(
a*d*(tan(d*x + c) - I))
```

**Mupad [B]**

time = 8.51, size = 184, normalized size = 0.78

$$-\operatorname{atan}\left(\frac{2ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2 1i}{4a^2 d^2}}}{B}\right)\sqrt{\frac{B^2 1i}{4a^2 d^2}} - \operatorname{atan}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2 1i}{16a^2 d^2}}}{B}\right)\sqrt{\frac{B^2 1i}{16a^2 d^2}} - \frac{2\sqrt{\frac{1}{16}i} \operatorname{Aatanh}\left(4\sqrt{\frac{1}{16}i}\sqrt{\tan(c+dx)}\right)}{ad} + \frac{A\sqrt{\tan(c+dx)} 1i}{2ad(1+\tan(c+dx) 1i)} - \frac{B\sqrt{\tan(c+dx)}}{2ad(1+\tan(c+dx) 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x)^{(1/2)}*(A + B*\tan(c + d*x)))/(a + a*\tan(c + d*x)*1i),x)$

[Out]  $(A*\tan(c + d*x)^{(1/2)}*1i)/(2*a*d*(\tan(c + d*x)*1i + 1)) - \text{atan}((4*a*d*\tan(c + d*x)^{(1/2)}*(-(B^2*1i)/(16*a^2*d^2))^{(1/2)})/B)*(-(B^2*1i)/(16*a^2*d^2))^{(1/2)}*2i - (2*(1i/16)^{(1/2)}*A*\text{atanh}(4*(1i/16)^{(1/2)}*\tan(c + d*x)^{(1/2)}))/(a*d) - \text{atan}((2*a*d*\tan(c + d*x)^{(1/2)}*((B^2*1i)/(4*a^2*d^2))^{(1/2)})/B)*((B^2*1i)/(4*a^2*d^2))^{(1/2)}*2i - (B*\tan(c + d*x)^{(1/2)})/(2*a*d*(\tan(c + d*x)*1i + 1))$

$$3.137 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))} dx$$

**Optimal.** Leaf size=234

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2+i)A+B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2+i)A+B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad}$$

[Out] (1/8-1/8\*I)\*((2+I)\*A+B)\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)+(1/8-1/8\*I)\*((2+I)\*A+B)\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/a/d\*2^(1/2)-1/16\*((3+I)\*A-(1+I)\*B)\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)+1/16\*((3+I)\*A-(1+I)\*B)\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a/d\*2^(1/2)+1/2\*(A+I\*B)\*tan(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.19, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (B + (2+i)A) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (B + (2+i)A) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} ad} + \frac{(A+iB) \sqrt{\tan(c+dx)}}{2d(a+ia \tan(c+dx))} - \frac{((3+i)A - (1+i)B) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} ad}\right)}{8\sqrt{2} ad} + \frac{((3+i)A - (1+i)B) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} ad}\right)}{8\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])),x]

[Out] ((-1/4 + I/4)\*((2 + I)\*A + B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]])/(Sqrt[2]\*a\*d) + ((1/4 - I/4)\*((2 + I)\*A + B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]])/(Sqrt[2]\*a\*d) - (((3 + I)\*A - (1 + I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(8\*Sqrt[2]\*a\*d) + (((3 + I)\*A - (1 + I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(8\*Sqrt[2]\*a\*d) + ((A + I\*B)\*Sqrt[Tan[c + d\*x]])/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 642**

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

#### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} dx &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{2d(a + ia \tan(c + dx))} + \frac{\int \frac{\frac{1}{2}a(3A-iB) - \frac{1}{2}a(iA-B) \tan(c+dx)}{\sqrt{\tan(c + dx)}} dx}{2a^2} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{2d(a + ia \tan(c + dx))} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3A-iB) - \frac{1}{2}a(iA-B)x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{a^2 d} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{2d(a + ia \tan(c + dx))} + \frac{((3 + i)A - (1 + i)B) \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{4ad} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{2d(a + ia \tan(c + dx))} - \frac{((3 + i)A - (1 + i)B) \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{-1-x^2} dx, x, \sqrt{\tan(c + dx)}\right)}{8\sqrt{2} a} \\
 &= -\frac{((3 + i)A - (1 + i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad} \\
 &= -\frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2 + i)A + B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} ad} + \frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2 + i)A + B) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} ad}
 \end{aligned}$$

**Mathematica [A]**

time = 0.94, size = 199, normalized size = 0.85

$$\frac{(\cos(dx) + i \sin(dx)) \left( 4(A + iB)(\cos(dx) - i \sin(dx)) \sin(c + dx) + (1 + i) \left( (2 + i)A + B \right) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) + ((-1 - 2i)A + iB) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}) \right) \sec(c + dx) (i \cos(c) - \sin(c)) \sqrt{\sin(2(c + dx))}}{8d(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} (A + B \tan(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])), x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*(4*(A + I*B)*(Cos[d*x] - I*Sin[d*x])*Sin[c + d*x] + (1 + I)*(((2 + I)*A + B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + ((-1 - 2*I)*A + I*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)])])*Sec[c + d*x]*(I*Cos[c] - Sin[c])*Sqrt[Sin[2*(c + d*x)])*(A + B*Tan[c + d*x]))/(8*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

**Maple [A]**

time = 0.12, size = 121, normalized size = 0.52

method	result	size
--------	--------	------



derivativedivides	$\frac{\frac{i(iB+A)\left(\sqrt{\tan(dx+c)}\right)}{2(\tan(dx+c)-i)} - \frac{2iA \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} + \frac{4\left(\frac{iA}{4}+\frac{B}{4}\right) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$	121
default	$\frac{\frac{i(iB+A)\left(\sqrt{\tan(dx+c)}\right)}{2(\tan(dx+c)-i)} - \frac{2iA \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} + \frac{4\left(\frac{iA}{4}+\frac{B}{4}\right) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$	121

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] `1/d/a*(-1/2*I*(A+I*B)*tan(d*x+c)^(1/2)/(tan(d*x+c)-I)-2*I*A/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(1/4*I*A+1/4*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))`

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(177) = 354.

time = 2.17, size = 571, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] `-1/8*(a*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B) - a*d*sqrt((-I*A^2`

$$\begin{aligned}
& - 2*A*B + I*B^2)/(a^2*d^2))*e^{(2*I*d*x + 2*I*c)}*\log(-2*((-I*a*d*e^{(2*I*d*x + 2*I*c)} - I*a*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))} - (A - I*B)*e^{(2*I*d*x + 2*I*c)})*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 2*a*d*\sqrt{I*A^2/(a^2*d^2)})*e^{(2*I*d*x + 2*I*c)}*\log(((a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{I*A^2/(a^2*d^2)} + I*A)*e^{(-2*I*d*x - 2*I*c)/(a*d)} + 2*a*d*\sqrt{I*A^2/(a^2*d^2)})*e^{(2*I*d*x + 2*I*c)}*\log(-((a*d*e^{(2*I*d*x + 2*I*c)} + a*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{I*A^2/(a^2*d^2)} - I*A)*e^{(-2*I*d*x - 2*I*c)/(a*d)} - 2*((A + I*B)*e^{(2*I*d*x + 2*I*c)} + A + I*B)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-2*I*d*x - 2*I*c)/(a*d)}
\end{aligned}$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c)),x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real -I

**Giac [A]**

time = 0.71, size = 98, normalized size = 0.42

$$-\frac{(i-1)\sqrt{2}A\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} - \frac{(i-1)\sqrt{2}(iA+B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} - \frac{iA\sqrt{\tan(dx+c)} - B\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-(1/2*I - 1/2)*\sqrt{2}*A*\arctan((1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)})/(a*d) - (1/4*I - 1/4)*\sqrt{2}*(I*A + B)*\arctan(-(1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)})/(a*d) - 1/2*(I*A*\sqrt{\tan(d*x + c)} - B*\sqrt{\tan(d*x + c)})/(a*d*(\tan(d*x + c) - I))$

**Mupad [B]**

time = 7.20, size = 184, normalized size = 0.79

$$-\operatorname{atan}\left(\frac{2ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2 1i}{4a^2 d^2}}}{A}\right)\sqrt{\frac{A^2 1i}{4a^2 d^2}} + \operatorname{atan}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2 1i}{16a^2 d^2}}}{A}\right)\sqrt{\frac{A^2 1i}{16a^2 d^2}} - \frac{2\sqrt{\frac{1}{16}i}B\operatorname{atanh}\left(4\sqrt{\frac{1}{16}i}\sqrt{\tan(c+dx)}\right)}{ad} + \frac{A\sqrt{\tan(c+dx)}}{2ad(1+\tan(c+dx)1i)} + \frac{B\sqrt{\tan(c+dx)}1i}{2ad(1+\tan(c+dx)1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)),x)

[Out]  $\operatorname{atan}((4*a*d*\tan(c + d*x)^(1/2)*(-(A^2*1i)/(16*a^2*d^2))^(1/2))/A)*(-(A^2*1i)/(16*a^2*d^2))^(1/2)*2i - \operatorname{atan}((2*a*d*\tan(c + d*x)^(1/2)*((A^2*1i)/(4*a^2*$

$$\begin{aligned}
& d^2)^{(1/2))/A)*((A^2*1i)/(4*a^2*d^2))^{(1/2)*2i - (2*(1i/16)^{(1/2)*B*atanh(} \\
& 4*(1i/16)^{(1/2)*tan(c + d*x)^{(1/2)))/(a*d) + (A*tan(c + d*x)^{(1/2)))/(2*a*d*} \\
& (tan(c + d*x)*1i + 1)) + (B*tan(c + d*x)^{(1/2)*1i)/(2*a*d*(tan(c + d*x)*1i} \\
& + 1))
\end{aligned}$$

$$3.138 \quad \int \frac{A+B \tan(c+dx)}{\tan^2(c+dx)(a+ia \tan(c+dx))} dx$$

**Optimal.** Leaf size=267

$$\frac{((5+3i)A - (3-i)B)\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{4\sqrt{2} ad} + \frac{((-5-3i)A + (3-i)B)\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{4\sqrt{2} ad}$$

[Out]  $-1/8*((5+3*I)*A+(-3+I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+1/8*((-5-3*I)*A+(3-I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(-1/16+1/16*I)*((4+I)*A+(1+2*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a/d*2^{(1/2)}+1/16*((5-3*I)*A+(3+I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a/d*2^{(1/2)}+1/2*(-5*A-I*B)/a/d/\tan(d*x+c)^{(1/2)}+1/2*(A+I*B)/d/\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))$

**Rubi [A]**

time = 0.26, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((5+3i)A - (3-i)B)\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{4\sqrt{2} ad} + \frac{((-5-3i)A + (3-i)B)\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{4\sqrt{2} ad} + \frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))} - \frac{5A+iB}{2ad\sqrt{\tan(c+dx)}} \cdot \frac{\left(\frac{1-i}{2}\right)\left((4+i)A+(1+2i)B\right)\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{\sqrt{2}ad}\right) + \frac{(5-3i)A+(3+i)B}{8\sqrt{2}ad}\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}{\sqrt{2}ad}\right)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])),x]

[Out]  $((5+3*I)*A - (3-I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]/(4*\text{Sqrt}[2]*a*d) + (((-5-3*I)*A + (3-I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(4*\text{Sqrt}[2]*a*d) - ((1/8 - I/8)*((4+I)*A + (1+2*I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]/(8*\text{Sqrt}[2]*a*d) + (((5-3*I)*A + (3+I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]/(8*\text{Sqrt}[2]*a*d) - (5*A + I*B)/(2*a*d*\text{Sqrt}[\text{Tan}[c + d*x]]) + (A + I*B)/(2*d*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x]))$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3610

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^n\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^n, x\_Symbol] := Sim

```
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} dx = \frac{A + iB}{2d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} + \frac{\int \frac{\frac{1}{2}a(5A+iB) - \frac{3}{2}a(iA-B) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} dx}{2a^2}$$

$$= -\frac{5A + iB}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} + \dots$$

$$= -\frac{5A + iB}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} + \dots$$

$$= -\frac{5A + iB}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} - \dots$$

$$= -\frac{5A + iB}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} - \dots$$

$$= -\frac{((5 - 3i)A + (3 + i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad}$$

$$= \frac{((5 + 3i)A - (3 - i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{4\sqrt{2} ad} - \dots$$

**Mathematica [A]**

time = 1.08, size = 217, normalized size = 0.81

$$\frac{(\cos(dx) + i \sin(dx)) \left( (-4 \cos(dx) + 4i \sin(dx))(4A \cos(c + dx) + (5A - B) \sin(c + dx)) + ((3 - 5i)A + (1 + 3i)B) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i)((4 + i)A + (1 + 2i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}) \right) \sec(c + dx) (i \cos(c) - \sin(c)) \sqrt{\sin(2(c + dx))}}{8i(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} (A + B \tan(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])), x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*((-4*Cos[d*x] + (4*I)*Sin[d*x])*(4*A*Cos[c + d*x] + ((5*I)*A - B)*Sin[c + d*x]) + ((3 - 5*I)*A + (1 + 3*I)*B)*ArcSin[Cos[c +
```

$$d*x] - \text{Sin}[c + d*x]] - (1 + I)*((4 + I)*A + (1 + 2*I)*B)*\text{Log}[\text{Cos}[c + d*x] + \text{Sin}[c + d*x] + \text{Sqrt}[\text{Sin}[2*(c + d*x)]]]*\text{Sec}[c + d*x]*(I*\text{Cos}[c] - \text{Sin}[c])* \text{Sqrt}[\text{Sin}[2*(c + d*x)]]*(A + B*\text{Tan}[c + d*x])]/(8*d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x]))$$

**Maple [A]**

time = 0.10, size = 140, normalized size = 0.52

method	result
derivativedivides	$-\frac{\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{i(iA-B)\left(\sqrt{\tan(dx+c)}\right)}{2\tan(dx+c)-2i}}{da} - \frac{2(iB+2A)\arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} + \frac{4\left(-\frac{A}{4}+\frac{iB}{4}\right)\arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}$
default	$-\frac{\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{i(iA-B)\left(\sqrt{\tan(dx+c)}\right)}{2\tan(dx+c)-2i}}{da} - \frac{2(iB+2A)\arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} + \frac{4\left(-\frac{A}{4}+\frac{iB}{4}\right)\arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c)),x,method=\_RETURNVE RBOSE)

[Out] 1/d/a\*(-2\*A/tan(d\*x+c)^(1/2)+1/2\*I\*(I\*A-B)\*tan(d\*x+c)^(1/2)/(tan(d\*x+c)-I)- 2\*(2\*A+I\*B)/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2))) + 4\*(-1/4\*A+1/4\*I\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm ="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(200) = 400.

time = 2.64, size = 703, normalized size = 2.63



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/8*((a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 + 2*A*
B - I*B^2)/(a^2*d^2))*log(2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)
/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)
) - (a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 + 2*A*B
- I*B^2)/(a^2*d^2))*log(-2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)
/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)
) + 2*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((-4*I*A^2 +
4*A*B + I*B^2)/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-4*I*A^2 + 4*A*B + I
*B^2)/(a^2*d^2)) + 2*A + I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*(a*d*e^(4*I*d
*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a^2
*d^2))*log(-((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a^2*d^2)) -
2*A - I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2*((-9*I*A + B)*e^(4*I*d*x + 4*I*
c) - 8*I*A*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*
I*c))
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

**Giac** [A]

time = 0.86, size = 111, normalized size = 0.42

$$\frac{(i+1)\sqrt{2}(iA+B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} + \frac{(i-1)\sqrt{2}(-2iA+B)\arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} + \frac{-5iA\tan(dx+c)+B\tan(dx+c)-4A}{2\left(i\tan(dx+c)^{\frac{3}{2}}+\sqrt{\tan(dx+c)}\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] (1/4*I + 1/4)*sqrt(2)*(I*A + B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x
+ c)))/(a*d) + (1/2*I - 1/2)*sqrt(2)*(-2*I*A + B)*arctan(-(1/2*I + 1/2)*sqr
```



$$t(2)*\sqrt{\tan(dx + c)} / (a*d) + 1/2*(-5*I*A*\tan(dx + c) + B*\tan(dx + c) - 4*A) / ((I*\tan(dx + c)^{(3/2)} + \sqrt{\tan(dx + c)})*a*d)$$

Mupad [B]

time = 7.37, size = 266, normalized size = 1.00

$$2 \operatorname{atanh}\left(\frac{ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2-1}{a^2d^2}}}{A}\right)\sqrt{\frac{A^2-1}{a^2d^2}} + 2 \operatorname{atanh}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2-1}{16a^2d^2}}}{A}\right)\sqrt{\frac{A^2-1}{16a^2d^2}} - \operatorname{atan}\left(\frac{2ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2-1}{4a^2d^2}}}{B}\right)\sqrt{\frac{B^2-1}{4a^2d^2}} + \operatorname{atan}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2-1}{16a^2d^2}}}{B}\right)\sqrt{\frac{B^2-1}{16a^2d^2}} - \frac{2d + \frac{A \operatorname{atan}(ad)}{ax}}{\sqrt{\tan(c+dx)^2 + \tan(c+dx)^{3/2} + 1}} + \frac{B\sqrt{\tan(c+dx)}}{2ad(1 + \tan(c+dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)),x)

[Out] 2\*atanh((a\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(a^2\*d^2))^(1/2))/A)\*(-(A^2\*1i)/(a^2\*d^2))^(1/2) + 2\*atanh((4\*a\*d\*tan(c + d\*x)^(1/2)\*((A^2\*1i)/(16\*a^2\*d^2))^(1/2))/A)\*((A^2\*1i)/(16\*a^2\*d^2))^(1/2) - atan((2\*a\*d\*tan(c + d\*x)^(1/2)\*((B^2\*1i)/(4\*a^2\*d^2))^(1/2))/B)\*((B^2\*1i)/(4\*a^2\*d^2))^(1/2)\*2i + atan((4\*a\*d\*tan(c + d\*x)^(1/2)\*(-(B^2\*1i)/(16\*a^2\*d^2))^(1/2))/B)\*(-(B^2\*1i)/(16\*a^2\*d^2))^(1/2)\*2i - ((2\*A)/(a\*d) + (A\*tan(c + d\*x)\*5i)/(2\*a\*d))/(tan(c + d\*x)^(1/2) + tan(c + d\*x)^(3/2)\*1i) + (B\*tan(c + d\*x)^(1/2))/(2\*a\*d\*(tan(c + d\*x)\*1i + 1))

$$3.139 \quad \int \frac{A+B \tan(c+dx)}{\tan^2(c+dx)(a+ia \tan(c+dx))} dx$$

**Optimal.** Leaf size=296

$$\frac{((7-5i)A+(5+3i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} - \frac{\left(\frac{1}{4}-\frac{i}{4}\right)((6+i)A+(1+4i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad}$$

[Out]  $-1/8*((7-5*I)*A+(5+3*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(-1/8+1/8*I)*((6+I)*A+(1+4*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+1/16*((7+5*I)*A+(-5+3*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a/d*2^{(1/2)}+1/16*((-7-5*I)*A+(5-3*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a/d*2^{(1/2)}+5/2*(I*A-B)/a/d/\tan(d*x+c)^{(1/2)}+1/6*(-7*A-3*I*B)/a/d/\tan(d*x+c)^{(3/2)}+1/2*(A+I*B)/d/\tan(d*x+c)^{(3/2)}/(a+I*a*\tan(d*x+c))$

**Rubi [A]**

time = 0.28, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(7-5i)A+(5+3i)B\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} - \frac{(i-i)(6+i)A+(1+4i)B\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad} + \frac{A+iB}{2a\tan^2(c+dx)(a+ia\tan(c+dx))} - \frac{7A+3iB}{4a^2\tan^2(c+dx)} + \frac{5(-B+iA)}{2a^2\sqrt{\tan(c+dx)}} + \frac{(7+5i)A-(5-3i)B\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{8\sqrt{2}ad} + \frac{(5-3i)B-(7+5i)A\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}{\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{8\sqrt{2}ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])),x]

[Out]  $((((7-5*I)*A+(5+3*I)*B)*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])]/(4*\text{Sqrt}[2]*a*d) - ((1/4-I/4)*((6+I)*A+(1+4*I)*B)*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])]/(\text{Sqrt}[2]*a*d) + (((7+5*I)*A-(5-3*I)*B)*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(8*\text{Sqrt}[2]*a*d) + (((-7-5*I)*A+(5-3*I)*B)*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(8*\text{Sqrt}[2]*a*d) - (7*A+(3*I)*B)/(6*a*d*\text{Tan}[c+d*x]^{(3/2)}) + (5*(I*A-B))/(2*a*d*\text{Sqrt}[\text{Tan}[c+d*x]]) + (A+I*B)/(2*d*\text{Tan}[c+d*x]^{(3/2)}*(a+I*a*\text{Tan}[c+d*x]))$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

#### Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

#### Rule 3610

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(f_.)*(x_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

#### Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)])}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))} dx = \frac{A + iB}{2d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} + \frac{\int \frac{\frac{1}{2}a(7A+3iB) - \frac{5}{2}a(iA-B) \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)} dx}{2a^2}$$

$$= -\frac{7A + 3iB}{6ad \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{2d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} + \int \frac{-\frac{5}{2}a(iA-B) \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)} dx$$

$$= -\frac{7A + 3iB}{6ad \tan^{\frac{3}{2}}(c + dx)} + \frac{5(iA - B)}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))}$$

$$= -\frac{7A + 3iB}{6ad \tan^{\frac{3}{2}}(c + dx)} + \frac{5(iA - B)}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))}$$

$$= -\frac{7A + 3iB}{6ad \tan^{\frac{3}{2}}(c + dx)} + \frac{5(iA - B)}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))}$$

$$= -\frac{7A + 3iB}{6ad \tan^{\frac{3}{2}}(c + dx)} + \frac{5(iA - B)}{2ad \sqrt{\tan(c + dx)}} + \frac{A + iB}{2d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))}$$

$$= \frac{((7 + 5i)A - (5 - 3i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad}$$

$$= \frac{((7 - 5i)A + (5 + 3i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{4\sqrt{2} ad} - \frac{((7 - 5i)A - (5 + 3i)B) \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{8\sqrt{2} ad}$$

**Mathematica [A]**

time = 1.41, size = 241, normalized size = 0.81

$\frac{(\cos(dx) + i \sin(dx)) \left( (1 - i) \left( ((6 + i)A + (1 + 4i)B) \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) + ((-1 - 6i)A + (4 + i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx)))} \right) \sec(c + dx) \left( \cos(c) + i \sin(c) \sqrt{\sin(2(c + dx))} + \frac{1}{2} \cos(c + dx) (\cos(dx) - i \sin(dx)) (-19A - 15iB + (11A + 15iB) \cos(2(c + dx)) + (8A - 12iB) \sin(2(c + dx))) \right) \right) (A + B \tan(c + dx))}{8i(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))}$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])), x]

[Out] ((Cos[d\*x] + I\*Sin[d\*x])\*((1 - I)\*((6 + I)\*A + (1 + 4\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] + ((-1 - 6\*I)\*A + (4 + I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*Sec[c + d\*x]\*(Cos[c] + I\*Sin[c])\*Sqrt[Sin[2\*(c + d\*x)]] + (2\*Csc[c + d\*x]\*(Cos[d\*x] - I\*Sin[d\*x]\*(-19\*A - (15\*I)\*B + (11\*A + (15\*I)\*B)\*Cos[2\*(c + d\*x)] + ((8\*I)\*A - 12\*B)\*Sin[2\*(c + d\*x)])))/3\*(A + B\*Tan[c + d\*x])/(8\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x]))

**Maple [A]**

time = 0.09, size = 160, normalized size = 0.54

method	result
derivativedivides	$\frac{\frac{2(-iA+B)}{\sqrt{\tan(dx+c)}} - \frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}}}{da} \frac{i \left( \frac{i(iA-B) \left( \sqrt{\tan(dx+c)} \right)^{4(2iB+3A) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}}{\tan(dx+c)-i} - \frac{4(2iB+3A) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} \right)}{2}}{4(-)}$
default	$\frac{\frac{2(-iA+B)}{\sqrt{\tan(dx+c)}} - \frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}}}{da} \frac{i \left( \frac{i(iA-B) \left( \sqrt{\tan(dx+c)} \right)^{4(2iB+3A) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}}{\tan(dx+c)-i} - \frac{4(2iB+3A) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} \right)}{2}}{4(-)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c)),x,method=\_RETURNVE RBOSE)

[Out] 1/d/a\*(-2\*(-I\*A+B)/tan(d\*x+c)^(1/2)-2/3\*A/tan(d\*x+c)^(3/2)-1/2\*I\*(I\*(I\*A-B)\*tan(d\*x+c)^(1/2)/(tan(d\*x+c)-I)-4\*(3\*A+2\*I\*B)/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2))))+4\*(-1/4\*I\*A-1/4\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2))))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(221) = 442.  
time = 2.26, size = 795, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{24} \left( 3(a^d e^{(6I d x + 6I c)} - 2a^d e^{(4I d x + 4I c)} + a^d e^{(2I d x + 2I c)}) \sqrt{\frac{-I A^2 - 2A B + I B^2}{a^2 d^2}} \log\left(\frac{-2(I a^d e^{(2I d x + 2I c)} + I a^d) \sqrt{\frac{-I e^{(2I d x + 2I c)} + I}{e^{(2I d x + 2I c)} + 1}}}{(A - I B) e^{(2I d x + 2I c)}}\right) - (A - I B) e^{(2I d x + 2I c)} e^{(-2I d x - 2I c)} / (I A + B) - 3(a^d e^{(6I d x + 6I c)} - 2a^d e^{(4I d x + 4I c)} + a^d e^{(2I d x + 2I c)}) \sqrt{\frac{-I A^2 - 2A B + I B^2}{a^2 d^2}} \log\left(\frac{-2(-I a^d e^{(2I d x + 2I c)} - I a^d) \sqrt{\frac{-I e^{(2I d x + 2I c)} + I}{e^{(2I d x + 2I c)} + 1}}}{(A - I B) e^{(2I d x + 2I c)}}\right) e^{(-2I d x - 2I c)} / (I A + B) - 6(a^d e^{(6I d x + 6I c)} - 2a^d e^{(4I d x + 4I c)} + a^d e^{(2I d x + 2I c)}) \sqrt{\frac{(9I A^2 - 12A B - 4I B^2)}{a^2 d^2}} \log\left(\frac{(a^d e^{(2I d x + 2I c)} + a^d) \sqrt{\frac{-I e^{(2I d x + 2I c)} + I}{e^{(2I d x + 2I c)} + 1}}}{(9I A^2 - 12A B - 4I B^2) / (a^2 d^2)}\right) + 3I A - 2B) e^{(-2I d x - 2I c)} / (a^d) + 6(a^d e^{(6I d x + 6I c)} - 2a^d e^{(4I d x + 4I c)} + a^d e^{(2I d x + 2I c)}) \sqrt{\frac{(9I A^2 - 12A B - 4I B^2)}{a^2 d^2}} \log\left(\frac{(a^d e^{(2I d x + 2I c)} + a^d) \sqrt{\frac{-I e^{(2I d x + 2I c)} + I}{e^{(2I d x + 2I c)} + 1}}}{(9I A^2 - 12A B - 4I B^2) / (a^2 d^2)}\right) - 3I A + 2B) e^{(-2I d x - 2I c)} / (a^d) - 2((19A + 27I B) e^{(6I d x + 6I c)} - (19A + 3I B) e^{(4I d x + 4I c)} - (35A + 27I B) e^{(2I d x + 2I c)} + 3A + 3I B) \sqrt{\frac{-I e^{(2I d x + 2I c)} + I}{e^{(2I d x + 2I c)} + 1}}) / (a^d e^{(6I d x + 6I c)} - 2a^d e^{(4I d x + 4I c)} + a^d e^{(2I d x + 2I c)}) \right)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A}{\tan^{\frac{7}{2}}(c+dx) - i \tan^{\frac{5}{2}}(c+dx)} dx + \int \frac{B \tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx) - i \tan^{\frac{5}{2}}(c+dx)} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2)/(a+I\*a\*tan(d\*x+c)),x)

[Out] -I\*(Integral(A/(tan(c + d\*x)\*\*(7/2) - I\*tan(c + d\*x)\*\*(5/2)), x) + Integral(B\*tan(c + d\*x)/(tan(c + d\*x)\*\*(7/2) - I\*tan(c + d\*x)\*\*(5/2)), x))/a

**Giac** [A]

time = 1.01, size = 141, normalized size = 0.48

$$\frac{(i-1)\sqrt{2}(iA+B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} - \frac{(i-1)\sqrt{2}(3A+2iB)\arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} - \frac{-iA\sqrt{\tan(dx+c)}+B\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)} + \frac{2i(3A\tan(dx+c)+3iB\tan(dx+c)+iA)}{3ad\tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] (1/4\*I - 1/4)\*sqrt(2)\*(I\*A + B)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a\*d) - (1/2\*I - 1/2)\*sqrt(2)\*(3\*A + 2\*I\*B)\*arctan(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a\*d) - 1/2\*(-I\*A\*sqrt(tan(d\*x + c)) + B\*sqrt(tan(d\*x + c)))/(a\*d\*(tan(d\*x + c) - I)) + 2/3\*I\*(3\*A\*tan(d\*x + c) + 3\*I\*B\*tan(d\*x + c) + I\*A)/(a\*d\*tan(d\*x + c)^(3/2))

**Mupad** [B]

time = 9.82, size = 303, normalized size = 1.02

$$\operatorname{atan}\left(\frac{2ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{3A}}}{3A}\right)\sqrt{\frac{A^2B}{4a^2d^2}} - \operatorname{atan}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{16a^2d^2}}}{A}\right)\sqrt{\frac{A^2B}{16a^2d^2}} + 2\operatorname{atanh}\left(\frac{ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2B}{a^2d^2}}}{B}\right)\sqrt{\frac{B^2B}{a^2d^2}} + 2\operatorname{atanh}\left(\frac{4ad\sqrt{\tan(c+dx)}\sqrt{\frac{B^2B}{16a^2d^2}}}{B}\right)\sqrt{\frac{B^2B}{16a^2d^2}} - \frac{2A + 2A\operatorname{atanh}^2 - A\operatorname{atanh}^2}{\tan(c+dx)^{3/2} + \tan(c+dx)^{5/2}} - \frac{2B + 2B\operatorname{atanh}^2}{\sqrt{\tan(c+dx)} + \tan(c+dx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)),x)

[Out] atan((2\*a\*d\*tan(c + d\*x)^(1/2)\*((A^2\*9i)/(4\*a^2\*d^2))^(1/2))/(3\*A))\*((A^2\*9i)/(4\*a^2\*d^2))^(1/2)\*2i - atan((4\*a\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(16\*a^2\*d^2))^(1/2))/A)\*(-(A^2\*1i)/(16\*a^2\*d^2))^(1/2)\*2i + 2\*atanh((a\*d\*tan(c + d\*x)^(1/2)\*(-(B^2\*1i)/(a^2\*d^2))^(1/2))/B)\*(-(B^2\*1i)/(a^2\*d^2))^(1/2) + 2\*atanh((4\*a\*d\*tan(c + d\*x)^(1/2)\*((B^2\*1i)/(16\*a^2\*d^2))^(1/2))/B)\*((B^2\*1i)/(16\*a^2\*d^2))^(1/2) - ((2\*A)/(3\*a\*d) - (A\*tan(c + d\*x)\*4i)/(3\*a\*d) + (5\*A\*tan(c + d\*x)^2)/(2\*a\*d))/(tan(c + d\*x)^(3/2) + tan(c + d\*x)^(5/2)\*1i) - ((2\*B)/(a\*d) + (B\*tan(c + d\*x)\*5i)/(2\*a\*d))/(tan(c + d\*x)^(1/2) + tan(c + d\*x)^(3/2)\*1i)

$$3.140 \quad \int \frac{\tan^5(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=316

$$\frac{((9+5i)A - (25-21i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d} - \frac{((9+5i)A - (25-21i)B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d}$$

[Out]  $-1/32*((9+5*I)*A+(-25+21*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}-1/32*((9+5*I)*A+(-25+21*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}+(-1/64+1/64*I)*((7+2*I)*A+(2+23*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}+(1/64-1/64*I)*((7+2*I)*A+(2+23*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}+5/8*(I*A-5*B)*\tan(d*x+c)^{(1/2)}/a^2/d+1/8*(3*A+7*I*B)*\tan(d*x+c)^{(3/2)}/a^2/d/(1+I*\tan(d*x+c))+1/4*(I*A-B)*\tan(d*x+c)^{(5/2)}/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.39, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(9+5i)A - (25-21i)B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d} - \frac{(9+5i)A - (25-21i)B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{16\sqrt{2} a^2 d} + \frac{(3A+7iB)\tan^2(c+dx)}{8a^2 d(1+\tan(c+dx))} + \frac{5(-5B+iA)\sqrt{\tan(c+dx)}}{8a^2 d} - \frac{(\frac{1}{2}-\frac{1}{2}i)(7+2i)A+(2+23i)B \log(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1)}{\sqrt{2} a^2 d} + \frac{(\frac{1}{2}-\frac{1}{2}i)(7+2i)A+(2+23i)B \log(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1)}{\sqrt{2} a^2 d} + \frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x]^{(5/2)}*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x])^2,x]$

[Out]  $((9+5*I)*A - (25-21*I)*B)*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]/(16*\operatorname{Sqrt}[2]*a^2*d) - ((9+5*I)*A - (25-21*I)*B)*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]/(16*\operatorname{Sqrt}[2]*a^2*d) - ((1/32 - I/32)*((7+2*I)*A + (2+23*I)*B)*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]] + \operatorname{Tan}[c+d*x]]/( \operatorname{Sqrt}[2]*a^2*d) + ((1/32 - I/32)*((7+2*I)*A + (2+23*I)*B)*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]] + \operatorname{Tan}[c+d*x]]/( \operatorname{Sqrt}[2]*a^2*d) + (5*(I*A - 5*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(8*a^2*d) + ((3*A + (7*I)*B)*\operatorname{Tan}[c+d*x]^{(3/2)})/(8*a^2*d*(1 + I*\operatorname{Tan}[c+d*x])) + ((I*A - B)*\operatorname{Tan}[c+d*x]^{(5/2)})/(4*d*(a + I*a*\operatorname{Tan}[c+d*x])^2)$

**Rule 210**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

$\operatorname{Int}[(a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{q = 1 - 4*S\operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \operatorname{RationalQ}[q] \&\& (\operatorname{EqQ}[q^2, 1] \parallel \operatorname{!RationalQ}[b^2 - 4*a*c]) /; \operatorname{FreeQ}[a, b, c, x]$



$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \ \text{Dist}[(d*q + a*e)/(2*a*c), \ \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \ \text{Dist}[(d*q - a*e)/(2*a*c), \ \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3609

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \ :> \ \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \ \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \ \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \ \text{Sqrt}[b*\text{Tan}[e + f*x]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{\frac{5}{2}}(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx = \frac{(iA - B) \tan^{\frac{5}{2}}(c + dx)}{4d(a + ia \tan(c + dx))^2} - \frac{\int \frac{\tan^{\frac{3}{2}}(c + dx)(\frac{5}{2}a(iA - B) + \frac{1}{2}a(A + 9iB) \tan(c + dx))}{a + ia \tan(c + dx)}}{4a^2}$$

$$= \frac{(3A + 7iB) \tan^{\frac{3}{2}}(c + dx)}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \tan^{\frac{5}{2}}(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{\int \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))}$$

$$= \frac{5(iA - 5B) \sqrt{\tan(c + dx)}}{8a^2d} + \frac{(3A + 7iB) \tan^{\frac{3}{2}}(c + dx)}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \tan^{\frac{5}{2}}(c + dx)}{4d(a + ia \tan(c + dx))^2}$$

$$= \frac{5(iA - 5B) \sqrt{\tan(c + dx)}}{8a^2d} + \frac{(3A + 7iB) \tan^{\frac{3}{2}}(c + dx)}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \tan^{\frac{5}{2}}(c + dx)}{4d(a + ia \tan(c + dx))^2}$$

$$= \frac{5(iA - 5B) \sqrt{\tan(c + dx)}}{8a^2d} + \frac{(3A + 7iB) \tan^{\frac{3}{2}}(c + dx)}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \tan^{\frac{5}{2}}(c + dx)}{4d(a + ia \tan(c + dx))^2}$$

$$= \frac{5(iA - 5B) \sqrt{\tan(c + dx)}}{8a^2d} + \frac{(3A + 7iB) \tan^{\frac{3}{2}}(c + dx)}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \tan^{\frac{5}{2}}(c + dx)}{4d(a + ia \tan(c + dx))^2}$$

$$= \frac{((9 - 5i)A + (25 + 21i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) + \tan(c + dx)}{32\sqrt{2} a^2d} - \frac{((9 + 5i)A - (25 - 21i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^2d}$$

**Mathematica [A]**

time = 1.29, size = 255, normalized size = 0.81

$\frac{\tan(c + dx)(\cos(dx) + i \sin(dx))^{5/2}(A + B \tan(c + dx)) \left( \frac{((9 - 9i)A + (25 + 25i)B) \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i)((7 + 5i)A + (2 + 23i)B) \log\left(\frac{\cos(c + dx) + \sin(c + dx) + \sqrt{\tan(2c + dx)}}{\cos(2c) - \sin(2c)}\right) + 2(\cos(2dx) + \sin(2dx)) \sqrt{\tan(2c + dx)}}{32iA \cos(c + dx) + B \sin(c + dx)} \sqrt{\tan(c + dx)}^2 (c + i \tan(c + dx)) \right)}{32iA \cos(c + dx) + B \sin(c + dx)} \sqrt{\tan(c + dx)}^2 (c + i \tan(c + dx)) \right)}$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2, x]
```

```
[Out] (Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(A + B*Tan[c + d*x])*(((5 - 9*I)*A
+ (21 + 25*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] - (1 + I)*((7 + 2*I)*
A + (2 + 23*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]
)*Sec[c + d*x]*(I*Cos[2*c] - Sin[2*c])*Sqrt[Sin[2*(c + d*x)]] + 2*(I*Cos[2*
d*x] + Sin[2*d*x])*(5*A + (9*I)*B + (5*A + (41*I)*B)*Cos[2*(c + d*x)] + ((7
*I)*A - 43*B)*Sin[2*(c + d*x)]*Tan[c + d*x]))/(32*d*(A*Cos[c + d*x] + B*Si
n[c + d*x])*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2)
```

**Maple [A]**

time = 0.10, size = 159, normalized size = 0.50

method	result
derivativedivides	$-2B\left(\sqrt{\tan(dx+c)}\right) + \frac{i \left( \frac{(-7\frac{i}{2}A + 11\frac{B}{2}) \left(\tan^{\frac{3}{2}}(dx+c)\right) + (-5\frac{A}{2} - 9\frac{iB}{2}) \left(\sqrt{\tan(dx+c)}\right)}{(\tan(dx+c)-i)^2} + \frac{(7iA-23B) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2-i}\sqrt{2-i}}\right)}{\sqrt{2-i}\sqrt{2-i}} \right)}{4 da^2}$
default	$-2B\left(\sqrt{\tan(dx+c)}\right) + \frac{i \left( \frac{(-7\frac{i}{2}A + 11\frac{B}{2}) \left(\tan^{\frac{3}{2}}(dx+c)\right) + (-5\frac{A}{2} - 9\frac{iB}{2}) \left(\sqrt{\tan(dx+c)}\right)}{(\tan(dx+c)-i)^2} + \frac{(7iA-23B) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2-i}\sqrt{2-i}}\right)}{\sqrt{2-i}\sqrt{2-i}} \right)}{4 da^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d/a^2*(-2*B*tan(d*x+c)^(1/2)+1/4*I*(((7/2*I*A+11/2*B)*tan(d*x+c)^(3/2)+(
-5/2*A-9/2*I*B)*tan(d*x+c)^(1/2))/(tan(d*x+c)-I)^2+(7*I*A-23*B)/(2^(1/2)-I*
2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+1/2*I*(I*A+B)/(2^(
1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorit
hm="maxima")
```

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs.  $2(235) = 470$ .  
time = 2.44, size = 664, normalized size = 2.10



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")`

[Out] 
$$\frac{1}{32} \left( 2a^2 d \sqrt{\frac{IA^2 + 2AB - IB^2}{a^4 d^2}} e^{(4Ix + 4Ic)} \log\left( 2 \left( a^2 d e^{(2Ix + 2Ic)} + a^2 d \right) \sqrt{\frac{-Ie^{(2Ix + 2Ic)} + I}{e^{(2Ix + 2Ic)} + 1}} \sqrt{\frac{IA^2 + 2AB - IB^2}{a^4 d^2}} + (A - IB) e^{(2Ix + 2Ic)} e^{-(2Ix - 2Ic)} / (IA + B) \right) - 2a^2 d \sqrt{\frac{IA^2 + 2AB - IB^2}{a^4 d^2}} e^{(4Ix + 4Ic)} \log\left( -2 \left( a^2 d e^{(2Ix + 2Ic)} + a^2 d \right) \sqrt{\frac{-Ie^{(2Ix + 2Ic)} + I}{e^{(2Ix + 2Ic)} + 1}} \sqrt{\frac{IA^2 + 2AB - IB^2}{a^4 d^2}} - (A - IB) e^{(2Ix + 2Ic)} e^{-(2Ix - 2Ic)} / (IA + B) \right) + a^2 d \sqrt{\frac{-49IA^2 + 322AB + 529IB^2}{a^4 d^2}} e^{(4Ix + 4Ic)} \log\left( \frac{1}{8} \left( a^2 d e^{(2Ix + 2Ic)} + a^2 d \right) \sqrt{\frac{-Ie^{(2Ix + 2Ic)} + I}{e^{(2Ix + 2Ic)} + 1}} \sqrt{\frac{-49IA^2 + 322AB + 529IB^2}{a^4 d^2}} + 7A + 23IB \right) e^{-(2Ix - 2Ic)} / (a^2 d) - a^2 d \sqrt{\frac{-49IA^2 + 322AB + 529IB^2}{a^4 d^2}} e^{(4Ix + 4Ic)} \log\left( -\frac{1}{8} \left( a^2 d e^{(2Ix + 2Ic)} + a^2 d \right) \sqrt{\frac{-Ie^{(2Ix + 2Ic)} + I}{e^{(2Ix + 2Ic)} + 1}} \sqrt{\frac{-49IA^2 + 322AB + 529IB^2}{a^4 d^2}} - 7A - 23IB \right) e^{-(2Ix - 2Ic)} / (a^2 d) - 2(6(-IA + 7B) e^{(4Ix + 4Ic)} - (5IA - 9B) e^{(2Ix + 2Ic)} + IA - B) \sqrt{\frac{-Ie^{(2Ix + 2Ic)} + I}{e^{(2Ix + 2Ic)} + 1}} e^{-(4Ix - 4Ic)} / (a^2 d) \right)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \tan^{\frac{5}{2}}(c+dx)}{\tan^2(c+dx) - 2i \tan(c+dx) - 1} dx + \int \frac{B \tan^{\frac{7}{2}}(c+dx)}{\tan^2(c+dx) - 2i \tan(c+dx) - 1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)`

[Out] 
$$-\left( \text{Integral}\left( \frac{A \tan(c + dx)^{5/2}}{\tan(c + dx)^2 - 2I \tan(c + dx) - 1}, x \right) + \text{Integral}\left( \frac{B \tan(c + dx)^{7/2}}{\tan(c + dx)^2 - 2I \tan(c + dx) - 1}, x \right) \right) / a^2$$

**Giac [A]**

time = 0.85, size = 145, normalized size = 0.46

$$\frac{(i+1)\sqrt{2}(-iA-B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} + \frac{(i-1)\sqrt{2}(-7iA+23B)\arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} - \frac{2B\sqrt{\tan(dx+c)}}{a^2d} + \frac{7A\tan(dx+c)^3+11iB\tan(dx+c)^3-5iA\sqrt{\tan(dx+c)}+9B\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $-(1/8*I + 1/8)*\sqrt{2}*(-I*A - B)*\arctan(-(1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(dx+c)})/(a^2*d) + (1/16*I - 1/16)*\sqrt{2}*(-7*I*A + 23*B)*\arctan(-(1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(dx+c)})/(a^2*d) - 2*B*\sqrt{\tan(dx+c)}/(a^2*d) + 1/8*(7*A*\tan(dx+c)^{(3/2)} + 11*I*B*\tan(dx+c)^{(3/2)} - 5*I*A*\sqrt{\tan(dx+c)} + 9*B*\sqrt{\tan(dx+c)})/(a^2*d*(\tan(dx+c) - I)^2)$

**Mupad [B]**

time = 9.78, size = 334, normalized size = 1.06

$$\frac{5A\sqrt{\tan(c+dx)} + \frac{d\arctan(c+dx)}{2a^2d} + \frac{11B\arctan(c+dx)}{8a^2d} + \frac{5\sqrt{\tan(c+dx)}}{8a^2d}}{\tan(c+dx)^2+1+2\tan(c+dx)-1} + 2\operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2+1}{64a^4d^2}}}{A}\right)\sqrt{\frac{A^2+1}{64a^4d^2}} + 2\operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2+49i}{256a^4d^2}}}{7A}\right)\sqrt{\frac{A^2+49i}{256a^4d^2}} + \frac{2B\sqrt{\tan(c+dx)}}{a^2d} + \operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2+1}{64a^4d^2}}}{B}\right)\sqrt{\frac{B^2+1}{64a^4d^2}} - 2\operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2+529i}{256a^4d^2}}}{23B}\right)\sqrt{\frac{B^2+529i}{256a^4d^2}} - 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out]  $((5*A*\tan(c+d*x)^{(1/2)})/(8*a^2*d) + (A*\tan(c+d*x)^{(3/2)}*7i)/(8*a^2*d))/(2*\tan(c+d*x) + \tan(c+d*x)^2*1i - 1i) + ((B*\tan(c+d*x)^{(1/2)}*9i)/(8*a^2*d) - (11*B*\tan(c+d*x)^{(3/2)})/(8*a^2*d))/(2*\tan(c+d*x) + \tan(c+d*x)^2*1i - 1i) + 2*\operatorname{atanh}((8*a^2*d*\tan(c+d*x)^{(1/2)}*((A^2*1i)/(64*a^4*d^2))^{(1/2)})/A)*((A^2*1i)/(64*a^4*d^2))^{(1/2)} + 2*\operatorname{atanh}((16*a^2*d*\tan(c+d*x)^{(1/2)}*(-(A^2*49i)/(256*a^4*d^2))^{(1/2)})/(7*A))*(-(A^2*49i)/(256*a^4*d^2))^{(1/2)} + \operatorname{atan}((8*a^2*d*\tan(c+d*x)^{(1/2)}*(-(B^2*1i)/(64*a^4*d^2))^{(1/2)})/B)*(-(B^2*1i)/(64*a^4*d^2))^{(1/2)}*2i - \operatorname{atan}((16*a^2*d*\tan(c+d*x)^{(1/2)}*((B^2*529i)/(256*a^4*d^2))^{(1/2)})/(23*B))*((B^2*529i)/(256*a^4*d^2))^{(1/2)}*2i - (2*B*\tan(c+d*x)^{(1/2)})/(a^2*d)$

$$3.141 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=277

$$\frac{((1+3i)A+(9+5i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} - \frac{((1+3i)A+(9+5i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d}$$

[Out]  $-1/32*((1+3*I)*A+(9+5*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)} - 1/32*((1+3*I)*A+(9+5*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)} + 1/64*((1-3*I)*A+(-9+5*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)} - 1/64*((1-3*I)*A+(-9+5*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)} + 1/8*(A+5*I*B)*\tan(d*x+c)^{(1/2)}/a^2/d/(1+I*\tan(d*x+c)) + 1/4*(I*A-B)*\tan(d*x+c)^{(3/2)}/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.34, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((1+3i)A+(9+5i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} - \frac{((1+3i)A+(9+5i)B)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{16\sqrt{2}a^2d} + \frac{(A+5iB)\sqrt{\tan(c+dx)}}{8a^2(1+i\tan(c+dx))} + \frac{((1-3i)A-(9-5i)B)\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{32\sqrt{2}a^2d}\right)}{32\sqrt{2}a^2d} - \frac{((1-3i)A-(9-5i)B)\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}{32\sqrt{2}a^2d}\right)}{32\sqrt{2}a^2d} + \frac{(-B+iA)\tan^3(c+dx)}{4d(c+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]^{(3/2)}*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^2,x]$

[Out]  $((((1+3*I)*A+(9+5*I)*B)*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])]/(16*\text{Sqrt}[2]*a^2*d) - (((1+3*I)*A+(9+5*I)*B)*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])]/(16*\text{Sqrt}[2]*a^2*d) + (((1-3*I)*A-(9-5*I)*B)*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(32*\text{Sqrt}[2]*a^2*d) - (((1-3*I)*A-(9-5*I)*B)*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(32*\text{Sqrt}[2]*a^2*d) + ((A+(5*I)*B)*\text{Sqrt}[\text{Tan}[c+d*x]])/(8*a^2*d*(1+I*\text{Tan}[c+d*x])) + ((I*A-B)*\text{Tan}[c+d*x]^{(3/2)})/(4*d*(a+I*a*\text{Tan}[c+d*x])^2)$

**Rule 210**

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

$\text{Int}[(a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /;$  RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx &= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4d(a+ia \tan(c+dx))^2} - \int \frac{\sqrt{\tan(c+dx)} \left(\frac{3}{2}a(iA-B) - \frac{1}{2}a(A-7iB) \tan(c+dx)\right)}{4a^2} \\
&= \frac{(A+5iB) \sqrt{\tan(c+dx)}}{8a^2d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4d(a+ia \tan(c+dx))^2} + \int \frac{-\frac{1}{2}a^2(A+5iB) \sqrt{\tan(c+dx)}}{4a^2} \\
&= \frac{(A+5iB) \sqrt{\tan(c+dx)}}{8a^2d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4d(a+ia \tan(c+dx))^2} + \text{Subst}\left(\int \frac{-\frac{1}{2}a^2(A+5iB) \sqrt{t}}{4a^2} dt\right) \\
&= \frac{(A+5iB) \sqrt{\tan(c+dx)}}{8a^2d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4d(a+ia \tan(c+dx))^2} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) \sqrt{\tan(c+dx)}}{d} \\
&= \frac{(A+5iB) \sqrt{\tan(c+dx)}}{8a^2d(1+i \tan(c+dx))} + \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4d(a+ia \tan(c+dx))^2} + \frac{((1-3i)A - (9-5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^2d} \\
&= \frac{((1+3i)A + (9+5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2d} - \frac{((1+3i)A - (9-5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^2d}
\end{aligned}$$

### Mathematica [A]

time = 1.24, size = 243, normalized size = 0.88

$\frac{\sec(c+dx)(\cos(dx)+i \sin(dx))^2 \left(4i(\cos(2dx)+\sin(2dx)) \sin(c+dx) + (-iA+5B)\cos(c+dx) + (3A+7iB)\sin(c+dx) - (1+i) \left(\frac{(-1+2i)A+(2+7i)B}{\sqrt{\tan(c+dx)}} \operatorname{ArcSin}(\cos(c+dx)-\sin(c+dx)) + ((-2+i)A+(7+2i)B) \log(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx)))}\right)\right)}{32i(A\cos(c+dx)+B\sin(c+dx))\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^2} \sec(c+dx)(\cos(2c)-\sin(2c))\sqrt{\sin(2(c+dx))} (A+B \tan(c+dx))\right)}{32\sqrt{2} a^2d}$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2, x]

[Out] (Sec[c + d\*x]\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(4\*(I\*Cos[2\*d\*x] + Sin[2\*d\*x])\*Sin[c + d\*x]\*(((−I)\*A + 5\*B)\*Cos[c + d\*x] + (3\*A + (7\*I)\*B)\*Sin[c + d\*x]) − (1 + I)\*(((−1 + 2\*I)\*A + (2 + 7\*I)\*B)\*ArcSin[Cos[c + d\*x] − Sin[c + d\*x]] + ((−2 + I)\*A + (7 + 2\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]]))\*Sec[c + d\*x]\*(I\*Cos[2\*c] − Sin[2\*c])\*Sqrt[Sin[2\*(c + d\*x)]]\*(A + B\*Tan[c + d\*x]))/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2)

### Maple [A]

time = 0.13, size = 147, normalized size = 0.53



method	result
derivativedivides	$i \left( \frac{\left( -\frac{7iB}{2} - \frac{3A}{2} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + \left( -\frac{5B}{2} + \frac{iA}{2} \right) \left( \sqrt{\tan(dx+c)} \right)}{(\tan(dx+c)-i)^2} - \frac{(-7iB+A) \arctan\left( \frac{2\left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{\sqrt{2}-i\sqrt{2}} \right)$
default	$i \left( \frac{\left( -\frac{7iB}{2} - \frac{3A}{2} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + \left( -\frac{5B}{2} + \frac{iA}{2} \right) \left( \sqrt{\tan(dx+c)} \right)}{(\tan(dx+c)-i)^2} - \frac{(-7iB+A) \arctan\left( \frac{2\left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{\sqrt{2}-i\sqrt{2}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d/a^2*(1/4*I*((( -7/2*I*B-3/2*A)*tan(d*x+c)^(3/2)+(-5/2*B+1/2*I*A)*tan(d*x
+c)^(1/2))/(tan(d*x+c)-I)^2-(-7*I*B+A)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x
+c)^(1/2)/(2^(1/2)-I*2^(1/2))))-1/2*I*(A-I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*
tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(210) = 420.

time = 1.98, size = 664, normalized size = 2.40



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{32}*(2*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)})*e^{(4*I*d*x + 4*I*c)}*\log(-2*((I*a^2*d*e^{(2*I*d*x + 2*I*c)} + I*a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)} - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 2*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)})*e^{(4*I*d*x + 4*I*c)}*\log(-2*((-I*a^2*d*e^{(2*I*d*x + 2*I*c)} - I*a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)} - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} + a^2*d*\sqrt{(I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2)})*e^{(4*I*d*x + 4*I*c)}*\log(1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{((I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2)) + I*A + 7*B}*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} - a^2*d*\sqrt{(I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2)})*e^{(4*I*d*x + 4*I*c)}*\log(-1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{((I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2)) - I*A - 7*B}*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} + 2*(2*(A + 3*I*B)*e^{(4*I*d*x + 4*I*c)} + (A + 5*I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))*e^{(-4*I*d*x - 4*I*c)/(a^2*d)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \tan^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx + \int \frac{B \tan^{\frac{5}{2}}(c+dx)}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out]  $-(\text{Integral}(A*\tan(c + d*x)**(3/2)/(\tan(c + d*x)**2 - 2*I*\tan(c + d*x) - 1), x) + \text{Integral}(B*\tan(c + d*x)**(5/2)/(\tan(c + d*x)**2 - 2*I*\tan(c + d*x) - 1), x))/a**2$

**Giac [A]**

time = 0.73, size = 123, normalized size = 0.44

$$\frac{(i+1)\sqrt{2}(A-iB)\arctan\left(-\left(\frac{i}{2}-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} + \frac{(i-1)\sqrt{2}(A-7iB)\arctan\left(-\left(\frac{i}{2}+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} - \frac{3iA\tan(dx+c)^{\frac{3}{2}} - 7B\tan(dx+c)^{\frac{3}{2}} + A\sqrt{\tan(dx+c)} + 5iB\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $-(1/8*I + 1/8)*\sqrt{2}*(A - I*B)*\arctan(-1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)))/(a^2*d) + (1/16*I - 1/16)*\sqrt{2}*(A - 7*I*B)*\arctan(-1/2*I + 1/2)$

\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^2\*d) - 1/8\*(3\*I\*A\*tan(d\*x + c)^(3/2) - 7\*B\*tan(d\*x + c)^(3/2) + A\*sqrt(tan(d\*x + c)) + 5\*I\*B\*sqrt(tan(d\*x + c)))/(a^2\*d\*(tan(d\*x + c) - I)^2)

**Mupad [B]**

time = 9.81, size = 318, normalized size = 1.15

$$\frac{-\frac{1}{2} \frac{\operatorname{atan}\left(\frac{d \sqrt{\tan(c+dx)}}{A}\right) \sqrt{\frac{A^2-1}{64a^4d^2}}}{\tan(c+dx)^2+2\tan(c+dx)-1} + \frac{1}{2} \frac{\sqrt{\tan(c+dx)}}{\tan(c+dx)^2+2\tan(c+dx)-1} + \frac{1}{2} \frac{\sqrt{\tan(c+dx)}}{\tan(c+dx)^2+2\tan(c+dx)-1} + \frac{\operatorname{atan}\left(\frac{d \sqrt{\tan(c+dx)}}{A}\right) \sqrt{\frac{A^2-1}{64a^4d^2}}}{\tan(c+dx)^2+2\tan(c+dx)-1}}{\tan(c+dx)^2+2\tan(c+dx)-1} - \operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}}{A}\sqrt{\frac{A^2-1}{64a^4d^2}}\right) \sqrt{\frac{A^2-1}{64a^4d^2}} - 2i - \operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}}{A}\sqrt{\frac{A^2-1}{256a^4d^2}}\right) \sqrt{\frac{A^2-1}{256a^4d^2}} - 2i + 2 \operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}}{B}\sqrt{\frac{B^2-1}{64a^4d^2}}\right) \sqrt{\frac{B^2-1}{64a^4d^2}} + 2 \operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}}{7B}\sqrt{\frac{B^2-49}{256a^4d^2}}\right) \sqrt{\frac{B^2-49}{256a^4d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] ((5\*B\*tan(c + d\*x)^(1/2))/(8\*a^2\*d) + (B\*tan(c + d\*x)^(3/2)\*7i)/(8\*a^2\*d))/(2\*tan(c + d\*x) + tan(c + d\*x)^2\*1i - 1i) - ((A\*tan(c + d\*x)^(1/2)\*1i)/(8\*a^2\*d) - (3\*A\*tan(c + d\*x)^(3/2))/(8\*a^2\*d))/(2\*tan(c + d\*x) + tan(c + d\*x)^2\*1i - 1i) - atan((8\*a^2\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(64\*a^4\*d^2))^(1/2))/A)\*(-(A^2\*1i)/(64\*a^4\*d^2))^(1/2)\*2i - atan((16\*a^2\*d\*tan(c + d\*x)^(1/2)\*((A^2\*1i)/(256\*a^4\*d^2))^(1/2))/A)\*((A^2\*1i)/(256\*a^4\*d^2))^(1/2)\*2i + 2\*a\*tanh((8\*a^2\*d\*tan(c + d\*x)^(1/2)\*((B^2\*1i)/(64\*a^4\*d^2))^(1/2))/B)\*((B^2\*1i)/(64\*a^4\*d^2))^(1/2) + 2\*atanh((16\*a^2\*d\*tan(c + d\*x)^(1/2)\*(-(B^2\*49i)/(256\*a^4\*d^2))^(1/2))/(7\*B))\*(-(B^2\*49i)/(256\*a^4\*d^2))^(1/2)

$$3.142 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=279

$$\frac{((-1+3i)A+(1+3i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} - \frac{((-1+3i)A+(1+3i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d}$$

[Out]  $-1/32*((-1+3*I)*A+(1+3*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}-1/32*((-1+3*I)*A+(1+3*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}+1/64*((1+3*I)*A+(1-3*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}-1/64*((1+3*I)*A+(1-3*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}+1/8*(I*A+3*B)*\tan(d*x+c)^{(1/2)}/a^2/d/(1+I*\tan(d*x+c))+1/4*(I*A-B)*\tan(d*x+c)^{(1/2)}/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.31, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((1+3i)B-(1-3i)A)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} - \frac{((1+3i)B-(1-3i)A)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{16\sqrt{2}a^2d} + \frac{(3B+iA)\sqrt{\tan(c+dx)}}{8a^2d(1+\tan(c+dx))} + \frac{((1+3i)A+(1-3i)B)\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\right)}{32\sqrt{2}a^2d} - \frac{((1+3i)A+(1-3i)B)\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\right)}{32\sqrt{2}a^2d} + \frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[\text{Tan}[c+d*x]]*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^2,x]$

[Out]  $(((-1+3*I)*A+(1+3*I)*B)*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])/(16*\text{Sqrt}[2]*a^2*d) - (((-1+3*I)*A+(1+3*I)*B)*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])/(16*\text{Sqrt}[2]*a^2*d) + (((1+3*I)*A+(1-3*I)*B)*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(32*\text{Sqrt}[2]*a^2*d) - (((1+3*I)*A+(1-3*I)*B)*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(32*\text{Sqrt}[2]*a^2*d) + ((I*A+3*B)*\text{Sqrt}[\text{Tan}[c+d*x]])/(8*a^2*d*(1+I*\text{Tan}[c+d*x])) + ((I*A-B)*\text{Sqrt}[\text{Tan}[c+d*x]])/(4*d*(a+I*a*\text{Tan}[c+d*x])^2)$

Rule 210

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

$\text{Int}[(a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q-x^2), x], x, 1+2*c*(x/b)], x] /;$  RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2-4\*a\*c]) /;

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx = \frac{(iA - B) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} - \frac{\int \frac{\frac{1}{2}a(iA - B) - \frac{1}{2}a(3A - 5iB) \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} dx}{4a^2}$$

$$= \frac{(iA + 3B) \sqrt{\tan(c + dx)}}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} - \frac{\int \frac{1}{2}a^2(3A - 5iB) \tan(c + dx)}{4a^2}$$

$$= \frac{(iA + 3B) \sqrt{\tan(c + dx)}}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} - \text{Subst}\left(\frac{\int \frac{1}{2}a^2(3A - 5iB) \tan(c + dx)}{4a^2}, \sqrt{\tan(c + dx)}, c + dx\right)$$

$$= \frac{(iA + 3B) \sqrt{\tan(c + dx)}}{8a^2d(1 + i \tan(c + dx))} + \frac{(iA - B) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} - \frac{((1 + 3i)A + (1 - 3i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^2d}$$

$$= \frac{((1 + 3i)A + (1 - 3i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^2d} + \frac{((1 + 3i)A + (1 - 3i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^2d}$$

**Mathematica [A]**

time = 1.12, size = 241, normalized size = 0.86

```
sec(c + dx)cos(dx) + i sin(dx)^2 (-4cos(2dx) - i sin(2dx)) sin(c + dx)((-3iA - B) cos(c + dx) + (A - 3iB) sin(c + dx)) + (1 - i) (((1 + 2i)A + (2 + i)B) ArcSin(cos(c + dx) - sin(c + dx)) + ((-2 - i)A + (1 + 2i)B) log(cos(c + dx) + sin(c + dx) + sqrt(sin(2c + 2dx)))) sec(c + dx)(cos(2c) - sin(2c)) sqrt(sin(2c + 2dx)) (A + B tan(c + dx))
32d(A cos(c + dx) + B sin(c + dx)) sqrt(tan(c + dx)) (a + ia tan(c + dx))^2
```

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^
2,x]
```

```
[Out] (Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(-4*(Cos[2*d*x] - I*Sin[2*d*x])*Sin
[c + d*x]*((-3*I)*A - B)*Cos[c + d*x] + (A - (3*I)*B)*Sin[c + d*x]) + (1 -
I)*(((1 + 2*I)*A + (2 + I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + ((-2 -
I)*A + (1 + 2*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)
]]])*Sec[c + d*x]*(I*Cos[2*c] - Sin[2*c])*Sqrt[Sin[2*(c + d*x)]]*(A + B*Tan
[c + d*x]))/(32*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[Tan[c + d*x]]*(a
+ I*a*Tan[c + d*x])^2)
```

**Maple [A]**

time = 0.12, size = 148, normalized size = 0.53

method	result
derivativedivides	$i \frac{\left( \frac{(-\frac{iA}{2} - \frac{3B}{2}) \left( \tan^{\frac{3}{2}}(dx+c) \right) + (-\frac{3A}{2} + \frac{iB}{2}) \left( \sqrt{\tan(dx+c)} \right)}{(\tan(dx+c)-i)^2} + \frac{(iA-B) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} \right)}{4} - \frac{i(iA+B) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2\left(\sqrt{2}-i\sqrt{2}\right)}$
default	$i \frac{\left( \frac{(-\frac{iA}{2} - \frac{3B}{2}) \left( \tan^{\frac{3}{2}}(dx+c) \right) + (-\frac{3A}{2} + \frac{iB}{2}) \left( \sqrt{\tan(dx+c)} \right)}{(\tan(dx+c)-i)^2} + \frac{(iA-B) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}} \right)}{4} - \frac{i(iA+B) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2\left(\sqrt{2}-i\sqrt{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d/a^2*(1/4*I*(((1/2*I*A-3/2*B)*tan(d*x+c)^(3/2)+(-3/2*A+1/2*I*B)*tan(d*x
+c)^(1/2))/(tan(d*x+c)-I)^2+(I*A-B)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)
^(1/2)/(2^(1/2)-I*2^(1/2))))-1/2*I*(I*A+B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan
(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorit
hm="maxima")
```

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 658 vs.  $2(212) = 424$ .  
time = 2.92, size = 658, normalized size = 2.36

$$\frac{\int \frac{A \sqrt{\tan(c+dx)}}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx + \int \frac{B \tan^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$-1/32*(2*a^2*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(2*((a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 2*a^2*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(-2*((a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)} - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - a^2*d*\sqrt{(-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)} + A + I*B)*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} + a^2*d*\sqrt{(-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(-1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} + a^2*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)} - A - I*B)*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} + 2*(2*(-I*A - B)*e^{(4*I*d*x + 4*I*c)} - (3*I*A + B)*e^{(2*I*d*x + 2*I*c)} - I*A + B)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-4*I*d*x - 4*I*c)/(a^2*d)}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sqrt{\tan(c+dx)}}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx + \int \frac{B \tan^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] 
$$-(\text{Integral}(A*\sqrt{\tan(c+dx)})/(\tan(c+dx)**2 - 2*I*\tan(c+dx) - 1), x) + \text{Integral}(B*\tan(c+dx)**(3/2)/(\tan(c+dx)**2 - 2*I*\tan(c+dx) - 1), x)/a**2$$

**Giac** [A]

time = 0.67, size = 127, normalized size = 0.46

$$\frac{(i-1)\sqrt{2}(iA-B)\arctan\left(\left(\frac{i}{2}+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} - \frac{(i+1)\sqrt{2}(-iA-B)\arctan\left(\left(\frac{i}{2}-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} + \frac{A \tan(dx+c)^{\frac{3}{2}} - 3iB \tan(dx+c)^{\frac{3}{2}} - 3iA \sqrt{\tan(dx+c)} - B \sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $(1/16*I - 1/16)*\sqrt{2}*(I*A - B)*\arctan((1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)})/(a^2*d) - (1/8*I + 1/8)*\sqrt{2}*(-I*A - B)*\arctan((1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)})/(a^2*d) + 1/8*(A*\tan(d*x + c)^{(3/2)} - 3*I*B*\tan(d*x + c)^{(3/2)} - 3*I*A*\sqrt{\tan(d*x + c)} - B*\sqrt{\tan(d*x + c)})/(a^2*d*(\tan(d*x + c) - I)^2)$

**Mupad [B]**

time = 9.75, size = 318, normalized size = 1.14

$$\frac{\frac{3A\sqrt{\tan(c+dx)}}{2\sqrt{a^2d^2+21\tan(c+dx)}-1} - \frac{A\arctan(\frac{\sqrt{a^2d^2+21\tan(c+dx)}}{21})}{21\sqrt{a^2d^2+21\tan(c+dx)}-1} - \frac{3B\sqrt{\tan(c+dx)}}{2\sqrt{a^2d^2+21\tan(c+dx)}-1} + \frac{B\sqrt{\tan(c+dx)}}{21\sqrt{a^2d^2+21\tan(c+dx)}-1}}{\tan(c+dx)^2-1} - 2\operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2+1}{64a^2d^2}}}{A}\right)\sqrt{\frac{A^2+1}{64a^2d^2}} + 2\operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2+1}{256a^2d^2}}}{A}\right)\sqrt{\frac{A^2+1}{256a^2d^2}} - \operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2+1}{64a^2d^2}}}{B}\right)\sqrt{\frac{B^2+1}{64a^2d^2}} - 2\operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2+1}{256a^2d^2}}}{B}\right)\sqrt{\frac{B^2+1}{256a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out]  $((3*A*\tan(c + d*x)^{(1/2)})/(8*a^2*d) + (A*\tan(c + d*x)^{(3/2)}*1i)/(8*a^2*d))/(2*\tan(c + d*x) + \tan(c + d*x)^2*1i - 1i) - ((B*\tan(c + d*x)^{(1/2)}*1i)/(8*a^2*d) - (3*B*\tan(c + d*x)^{(3/2)})/(8*a^2*d))/(2*\tan(c + d*x) + \tan(c + d*x)^2*1i - 1i) - 2*\operatorname{atanh}((8*a^2*d*\tan(c + d*x)^{(1/2)}*((A^2*1i)/(64*a^4*d^2))^{(1/2)})/A)*((A^2*1i)/(64*a^4*d^2))^{(1/2)} + 2*\operatorname{atanh}((16*a^2*d*\tan(c + d*x)^{(1/2)}*(-A^2*1i)/(256*a^4*d^2))^{(1/2)})/A)*((-A^2*1i)/(256*a^4*d^2))^{(1/2)} - \operatorname{atan}((8*a^2*d*\tan(c + d*x)^{(1/2)}*(-B^2*1i)/(64*a^4*d^2))^{(1/2)})/B)*((-B^2*1i)/(64*a^4*d^2))^{(1/2)}*2i - \operatorname{atan}((16*a^2*d*\tan(c + d*x)^{(1/2)}*(B^2*1i)/(256*a^4*d^2))^{(1/2)})/B)*((B^2*1i)/(256*a^4*d^2))^{(1/2)}*2i$

$$3.143 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=285

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((-2 + 7i)A + (1 + 2i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{\left((9 - 5i)A + (1 - 3i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d}$$

[Out]  $(-1/32-1/32*I)*((-2+7*I)*A+(1+2*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^{2/d}*2^{(1/2)}+1/32*((9-5*I)*A+(1-3*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^{2/d}*2^{(1/2)}+(1/64+1/64*I)*((-7+2*I)*A+(2+I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^{2/d}*2^{(1/2)}+1/64*((9+5*I)*A-(1+3*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^{2/d}*2^{(1/2)}+1/8*(5*A+I*B)*\tan(d*x+c)^{(1/2)}/a^{2/d}/(1+I*\tan(d*x+c))+1/4*(A+I*B)*\tan(d*x+c)^{(1/2)}/d/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.34, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((-2 + 7i)A + (1 + 2i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{\left((9 - 5i)A + (1 - 3i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d} + \frac{\left(\frac{5A + I B}{32} \sqrt{\tan(c+dx)}\right)}{32\sqrt{2} \left(1 + \tan(c+dx)\right)} + \frac{\left(\frac{1}{8} + \frac{i}{8}\right) \left((2 + i)B - (7 - 2i)A\right) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} a^2 d}\right)}{\sqrt{2} a^2 d} + \frac{\left((9 + 5i)A - (1 + 3i)B\right) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{32\sqrt{2} a^2 d}\right)}{32\sqrt{2} a^2 d} + \frac{\left(\frac{A + I B}{4} \sqrt{\tan(c+dx)}\right)}{4(a + i a \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2),x]

[Out]  $\left(\left(\frac{1}{16} + \frac{I}{16}\right) \left((-2 + 7I)A + (1 + 2I)B\right) \text{ArcTan}\left[1 - \text{Sqrt}[2] \text{Sqrt}[\text{Tan}[c + d*x]]\right]\right) / \left(\text{Sqrt}[2] * a^2 * d\right) + \left(\left((9 - 5I)A + (1 - 3I)B\right) \text{ArcTan}\left[1 + \text{Sqrt}[2] \text{Sqrt}[\text{Tan}[c + d*x]]\right]\right) / \left(16 * \text{Sqrt}[2] * a^2 * d\right) + \left(\left(\frac{1}{32} + \frac{I}{32}\right) \left((-7 + 2I)A + (2 + I)B\right) \text{Log}\left[1 - \text{Sqrt}[2] \text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]\right]\right) / \left(\text{Sqrt}[2] * a^2 * d\right) + \left(\left((9 + 5I)A - (1 + 3I)B\right) \text{Log}\left[1 + \text{Sqrt}[2] \text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]\right]\right) / \left(32 * \text{Sqrt}[2] * a^2 * d\right) + \left((5A + I B) \text{Sqrt}[\text{Tan}[c + d*x]]\right) / \left(8 * a^2 * d * (1 + I * \text{Tan}[c + d*x])\right) + \left((A + I B) \text{Sqrt}[\text{Tan}[c + d*x]]\right) / \left(4 * d * (a + I * a * \text{Tan}[c + d*x])^2\right)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

#### Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

#### Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\text{tan}[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3677

$\text{Int}[\frac{((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}}{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \ :> \ \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

&& LtQ[m, 0] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2} dx &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A - iB) - \frac{3}{2}a(iA - B) \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} dx}{4a^2} \\
 &= \frac{(5A + iB) \sqrt{\tan(c + dx)}}{8a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} + \int \frac{\frac{3}{2}a^2(\dots)}{\dots} \\
 &= \frac{(5A + iB) \sqrt{\tan(c + dx)}}{8a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} + \text{Subst} \\
 &= \frac{(5A + iB) \sqrt{\tan(c + dx)}}{8a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} + \frac{((9 + \dots)}{\dots} \\
 &= \frac{(5A + iB) \sqrt{\tan(c + dx)}}{8a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \sqrt{\tan(c + dx)}}{4d(a + ia \tan(c + dx))^2} - \frac{((9 + \dots)}{\dots} \\
 &= -\frac{((9 + 5i)A - (1 + 3i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^2 d} \\
 &= -\frac{((9 - 5i)A + (1 - 3i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^2 d} + \dots
 \end{aligned}$$

**Mathematica [A]**

time = 1.26, size = 243, normalized size = 0.85

$\frac{\sec(c + dx)(\cos(dx) + i \sin(dx))^2 \left( 4(\cos(2dx) + \sin(2dx)) \sin(c + dx)((-7A + 3B) \cos(c + dx) + (5A + iB) \sin(c + dx)) + ((5 + 9i)A + (3 + i)B) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i)((2 + 7i)A + (1 - 2i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}) \right)}{32d(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2}$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2), x]

[Out] (Sec[c + d\*x]\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(4\*(I\*Cos[2\*d\*x] + Sin[2\*d\*x])\*Sin[c + d\*x]\*((( -7\*I)\*A + 3\*B)\*Cos[c + d\*x] + (5\*A + I\*B)\*Sin[c + d\*x]) + (((5 + 9\*I)\*A + (3 + I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] - (1 + I)\*((2 + 7\*I)\*A + (1 - 2\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*Sec[c + d\*x]\*(I\*Cos[2\*c] - Sin[2\*c])\*Sqrt[Sin[2\*(c + d\*x)]]\*(A + B\*Tan[c + d\*x]))/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2)

**Maple [A]**

time = 0.12, size = 144, normalized size = 0.51

method	result
derivativedivides	$-\frac{\left(\frac{5iA}{2} - \frac{B}{2}\right) \left(\tan^{\frac{3}{2}}(dx+c)\right) + \left(\frac{7A}{2} + \frac{3iB}{2}\right) \left(\sqrt{\tan}(dx+c)\right)}{4(\tan(dx+c)-i)^2} - \frac{(7iA+B) \arctan\left(\frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)} + \frac{i(-iB+A) \arctan\left(\frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2\sqrt{2}+}$
default	$-\frac{\left(\frac{5iA}{2} - \frac{B}{2}\right) \left(\tan^{\frac{3}{2}}(dx+c)\right) + \left(\frac{7A}{2} + \frac{3iB}{2}\right) \left(\sqrt{\tan}(dx+c)\right)}{4(\tan(dx+c)-i)^2} - \frac{(7iA+B) \arctan\left(\frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)} + \frac{i(-iB+A) \arctan\left(\frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2\sqrt{2}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d/a^2*(-1/4*((5/2*I*A-1/2*B)*tan(d*x+c)^(3/2)+(7/2*A+3/2*I*B)*tan(d*x+c)^(1/2))/
(tan(d*x+c)-I)^2-1/4*(7*I*A+B)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))
+1/2*I*(A-I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs.  $2(212) = 424$ .

time = 2.01, size = 664, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)
*log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))
- (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*d
*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*((-I*a
^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A - I*B)*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a^2*d*sqrt((49*I*A^2
+ 14*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*e^(2*I*d*
x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((49*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2)) + 7*I*A + B)*e^(-2*I*d*x
- 2*I*c)/(a^2*d)) + a^2*d*sqrt((49*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2))*e^(4*
I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((49*I*A^2 + 14*A*B - I*
B^2)/(a^4*d^2)) - 7*I*A - B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - 2*(2*(3*A + I*
B)*e^(4*I*d*x + 4*I*c) + (7*A + 3*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(-4*I*d*x - 4*I*
c)/(a^2*d)
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

**Giac** [A]

time = 0.96, size = 126, normalized size = 0.44

$$\frac{(i+1)\sqrt{2}(A-iB)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} + \frac{(i-1)\sqrt{2}(7A-iB)\arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} - \frac{5iA\tan(dx+c)^{\frac{3}{2}} - B\tan(dx+c)^{\frac{3}{2}} + 7A\sqrt{\tan(dx+c)} + 3iB\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] (1/8*I + 1/8)*sqrt(2)*(A - I*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x
+ c)))/(a^2*d) + (1/16*I - 1/16)*sqrt(2)*(7*A - I*B)*arctan(-(1/2*I + 1/2)*
sqrt(2)*sqrt(tan(d*x + c)))/(a^2*d) - 1/8*(5*I*A*tan(d*x + c)^(3/2) - B*tan
(d*x + c)^(3/2) + 7*A*sqrt(tan(d*x + c)) + 3*I*B*sqrt(tan(d*x + c)))/(a^2*d
*(tan(d*x + c) - I)^2)
```

**Mupad** [B]

time = 9.68, size = 318, normalized size = 1.12

$$\frac{-\frac{1}{2}\frac{A\sqrt{\tan(dx+c)}}{\tan(dx+c)+2} + \frac{1}{2}\frac{A\sqrt{\tan(dx+c)}}{\tan(dx+c)-2}}{\tan(dx+c)+2} + \frac{1}{2}\frac{B\sqrt{\tan(dx+c)}}{\tan(dx+c)+2} + \frac{1}{2}\frac{B\sqrt{\tan(dx+c)}}{\tan(dx+c)-2} + \operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(dx+c)}\sqrt{\frac{A^2-11}{64a^2d^2}}}{A}\right) + \operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(dx+c)}\sqrt{\frac{A^2-49}{256a^2d^2}}}{7A}\right) - 2\operatorname{atan}\left(\frac{A^2-49}{256a^2d^2}\right) - 2\operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(dx+c)}\sqrt{\frac{B^2-11}{64a^2d^2}}}{B}\right) + 2\operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(dx+c)}\sqrt{\frac{B^2-11}{256a^2d^2}}}{B}\right) + 2\operatorname{atanh}\left(\frac{B^2-11}{256a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] ((3*B*tan(c + d*x)^(1/2))/(8*a^2*d) + (B*tan(c + d*x)^(3/2)*1i)/(8*a^2*d))/
(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) - ((A*tan(c + d*x)^(1/2)*7i)/(8*a
^2*d) - (5*A*tan(c + d*x)^(3/2))/(8*a^2*d))/(2*tan(c + d*x) + tan(c + d*x)^
2*1i - 1i) + atan((8*a^2*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(64*a^4*d^2))^(1/2
))/A)*(-(A^2*1i)/(64*a^4*d^2))^(1/2)*2i - atan((16*a^2*d*tan(c + d*x)^(1/2)
*((A^2*49i)/(256*a^4*d^2))^(1/2))/(7*A))*((A^2*49i)/(256*a^4*d^2))^(1/2)*2i
- 2*atanh((8*a^2*d*tan(c + d*x)^(1/2)*((B^2*1i)/(64*a^4*d^2))^(1/2))/B)*((
B^2*1i)/(64*a^4*d^2))^(1/2) + 2*atanh((16*a^2*d*tan(c + d*x)^(1/2)*(-(B^2*1
i)/(256*a^4*d^2))^(1/2))/B)*(-(B^2*1i)/(256*a^4*d^2))^(1/2)
```

$$3.144 \quad \int \frac{A+B \tan(c+dx)}{\tan^2(c+dx)(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=318

$$\frac{((25+21i)A - (9-5i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((2+23i)A - (7+2i)B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^2 d}$$

[Out]  $-1/32*((25+21*I)*A+(-9+5*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}+(-1/32+1/32*I)*((2+23*I)*A-(7+2*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}+(-1/64+1/64*I)*((23+2*I)*A+(2+7*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}+(1/64-1/64*I)*((23+2*I)*A+(2+7*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}-5/8*(5*A+I*B)/a^2/d/\tan(d*x+c)^{(1/2)}+1/8*(7*A+3*I*B)/a^2/d/\tan(d*x+c)^{(1/2)}/(1+I*\tan(d*x+c))+1/4*(A+I*B)/d/\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.39, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((25+21i)A - (9-5i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^2 d} - \frac{(h-h)(2+23i)A - (7+2i)B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} a^2 d} - \frac{5(A+IB)}{8a^2 \sqrt{\tan(c+dx)}} + \frac{7A+3iB}{8a^2 d(1+i \tan(c+dx)) \sqrt{\tan(c+dx)}} - \frac{(h-h)((2+23i)A + (2+7i)B) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} a^2 d}\right)}{\sqrt{2} a^2 d} - \frac{(h-h)((2+23i)A + (2+7i)B) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} a^2 d}\right)}{\sqrt{2} a^2 d} - \frac{A+IB}{4d \sqrt{\tan(c+dx)} (a+i \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])]/(\operatorname{Tan}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^2), x]$

[Out]  $((25+21*I)*A - (9-5*I)*B)*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(16*\operatorname{Sqrt}[2]*a^2*d) - ((1/16 - I/16)*((2+23*I)*A - (7+2*I)*B)*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^2*d) - ((1/32 - I/32)*((23+2*I)*A + (2+7*I)*B)*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^2*d) + ((1/32 - I/32)*((23+2*I)*A + (2+7*I)*B)*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^2*d) - (5*(5*A + I*B))/(8*a^2*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) + (7*A + (3*I)*B)/(8*a^2*d*(1 + I*\operatorname{Tan}[c + d*x])*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) + (A + I*B)/(4*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^2)$

**Rule 210**

$\operatorname{Int}[(a_0 + (b_0)*(x_0)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

$\operatorname{Int}[(a_0 + (b_0)*(x_0) + (c_0)*(x_0)^2)^{-1}, x\_Symbol] := \operatorname{With}\{q = 1 - 4*S\operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \operatorname{RationalQ}[q] \&\& (\operatorname{EqQ}[q^2, 1] \parallel \operatorname{!RationalQ}[b^2 - 4*a*c]) /; \operatorname{Free}$



$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

Rule 3610

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(f_.)*(x_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)])}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx &= \frac{A + iB}{4d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(9A+iB) - \frac{5}{2}a(iA-B) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx}{4a^2} \\
&= \frac{7A + 3iB}{8a^2 d (1 + i \tan(c + dx)) \sqrt{\tan(c + dx)}} + \frac{A + iB}{4d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2} \\
&= -\frac{5(5A + iB)}{8a^2 d \sqrt{\tan(c + dx)}} + \frac{7A + 3iB}{8a^2 d (1 + i \tan(c + dx)) \sqrt{\tan(c + dx)}} \\
&= -\frac{5(5A + iB)}{8a^2 d \sqrt{\tan(c + dx)}} + \frac{7A + 3iB}{8a^2 d (1 + i \tan(c + dx)) \sqrt{\tan(c + dx)}} \\
&= -\frac{5(5A + iB)}{8a^2 d \sqrt{\tan(c + dx)}} + \frac{7A + 3iB}{8a^2 d (1 + i \tan(c + dx)) \sqrt{\tan(c + dx)}} \\
&= -\frac{5(5A + iB)}{8a^2 d \sqrt{\tan(c + dx)}} + \frac{7A + 3iB}{8a^2 d (1 + i \tan(c + dx)) \sqrt{\tan(c + dx)}} \\
&= -\frac{5(5A + iB)}{8a^2 d \sqrt{\tan(c + dx)}} + \frac{7A + 3iB}{8a^2 d (1 + i \tan(c + dx)) \sqrt{\tan(c + dx)}} \\
&= -\frac{((25 - 21i)A + (9 + 5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^2 d} \\
&= \frac{((25 + 21i)A - (9 - 5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^2 d}
\end{aligned}$$

**Mathematica** [A]

time = 1.42, size = 250, normalized size = 0.79

$$\frac{\operatorname{arctan}\left(\frac{a \tan(c + dx) + \sqrt{\tan(c + dx)}}{1 - a \tan(c + dx)}\right) \sqrt{\tan(c + dx)}}{32(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)}} + \frac{((25 - 21i)A + (9 + 5i)B) \operatorname{ArcSin}\left(\frac{\sqrt{\tan(c + dx)}}{1 + i \tan(c + dx)}\right) - (1 + i)((25 + 21i)A + (9 - 5i)B) \log\left(\frac{1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)}{1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)}\right) + (-2 \cos(2dx) + 2i \sin(2dx))(-9A - 5iB + (41A + 5iB) \cos(2c + dx) + (43A - 7iB) \sin(2c + dx))}{16\sqrt{2} a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^2),x]

[Out] (Sec[c + d\*x]\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(((21 - 25\*I)\*A + (5 + 9\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] - (1 + I)\*((23 + 2\*I)\*A + (2 + 7\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*Sec[c + d\*x]\*(I\*Cos[2\*c] - Sin[2\*c])\*Sqrt[Sin[2\*(c + d\*x)]] + (-2\*Cos[2\*d\*x] + (2\*I)\*Sin[2\*d\*x])\*(-9\*A - (5\*I)\*B + (41\*A + (5\*I)\*B)\*Cos[2\*(c + d\*x)] + ((43\*I)\*A - 7\*B)\*Sin[2\*(c + d\*x)])\*(A + B\*Tan[c + d\*x])/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2)

Maple [A]

time = 0.10, size = 157, normalized size = 0.49

method	result
derivativedivides	$-\frac{\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(-\frac{9A}{2} - \frac{5iB}{2})\left(\tan^{\frac{3}{2}}(dx+c)\right) + \left(\frac{11iA}{2} - \frac{7B}{2}\right)\left(\sqrt{\tan(dx+c)}\right)}{4(\tan(dx+c)-i)^2}}{da^2} - \frac{(7iB+23A) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2-i}\sqrt{2-i}}\right)}{4\left(\sqrt{2-i}\sqrt{2-i}\right)}$
default	$-\frac{\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{(-\frac{9A}{2} - \frac{5iB}{2})\left(\tan^{\frac{3}{2}}(dx+c)\right) + \left(\frac{11iA}{2} - \frac{7B}{2}\right)\left(\sqrt{\tan(dx+c)}\right)}{4(\tan(dx+c)-i)^2}}{da^2} - \frac{(7iB+23A) \arctan\left(\frac{2\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{2-i}\sqrt{2-i}}\right)}{4\left(\sqrt{2-i}\sqrt{2-i}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] 1/d/a^2\*(-2\*A/tan(d\*x+c)^(1/2)+1/4\*((-9/2\*A-5/2\*I\*B)\*tan(d\*x+c)^(3/2)+(11/2\*I\*A-7/2\*B)\*tan(d\*x+c)^(1/2))/(tan(d\*x+c)-I)^2-1/4\*(23\*A+7\*I\*B)/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2)))+1/2\*I\*(I\*A+B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2)))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs.  $2(235) = 470$ .  
time = 0.90, size = 763, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(2*(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*log(2*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B) - 2*(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*log(-2*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B) + (a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2))*log(1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)) + 23*A + 7*I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d) - (a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2))*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)) - 23*A - 7*I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d) - 2*(6*(7*I*A - B)*e^(6*I*d*x + 6*I*c) - (-33*I*A + B)*e^(4*I*d*x + 4*I*c) + 2*(-5*I*A + 3*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

**Giac [A]**

time = 1.25, size = 143, normalized size = 0.45

$$\frac{(i-1)\sqrt{2}(A-iB)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} + \frac{(i-1)\sqrt{2}(-23iA+7B)\arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} - \frac{2A}{a^2d\sqrt{\tan(dx+c)}} - \frac{9A\tan(dx+c)^3+5iB\tan(dx+c)^3-11iA\sqrt{\tan(dx+c)}+7B\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] (1/8\*I - 1/8)\*sqrt(2)\*(A - I\*B)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^2\*d) + (1/16\*I - 1/16)\*sqrt(2)\*(-23\*I\*A + 7\*B)\*arctan(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^2\*d) - 2\*A/(a^2\*d\*sqrt(tan(d\*x + c))) - 1/8\*(9\*A\*tan(d\*x + c)^(3/2) + 5\*I\*B\*tan(d\*x + c)^(3/2) - 11\*I\*A\*sqrt(tan(d\*x + c)) + 7\*B\*sqrt(tan(d\*x + c)))/(a^2\*d\*(tan(d\*x + c) - I)^2)

**Mupad [B]**

time = 9.83, size = 338, normalized size = 1.06

$$\frac{\frac{16a^2d\sqrt{\tan(c+dx)}}{\tan(c+dx)^2+2\tan(c+dx)-1} + 2\operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{64a^2d^2}}}{A}\right)\sqrt{\frac{A^2B}{64a^2d^2}} + 2\operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{256a^2d^2}}}{23A}\right)\sqrt{\frac{A^2B}{256a^2d^2}} + \operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2B}{64a^2d^2}}}{B}\right)\sqrt{\frac{B^2B}{64a^2d^2}} - \operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2B}{256a^2d^2}}}{7B}\right)\sqrt{\frac{B^2B}{256a^2d^2}} - \frac{\frac{0.430304d}{256a^2d^2} - \frac{0.430304d}{256a^2d^2}}{2\tan(c+dx)^2 + \sqrt{\tan(c+dx)} - 1 + \tan(c+dx)^2} - 11}{\tan(c+dx)^2 + 2\tan(c+dx) - 1} + 2\operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{64a^2d^2}}}{A}\right)\sqrt{\frac{A^2B}{64a^2d^2}} + 2\operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{256a^2d^2}}}{23A}\right)\sqrt{\frac{A^2B}{256a^2d^2}} + \operatorname{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2B}{64a^2d^2}}}{B}\right)\sqrt{\frac{B^2B}{64a^2d^2}} - \operatorname{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2B}{256a^2d^2}}}{7B}\right)\sqrt{\frac{B^2B}{256a^2d^2}} - \frac{\frac{0.430304d}{256a^2d^2} - \frac{0.430304d}{256a^2d^2}}{2\tan(c+dx)^2 + \sqrt{\tan(c+dx)} - 1 + \tan(c+dx)^2} - 11$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^2),x)

[Out] 2\*atanh((8\*a^2\*d\*tan(c + d\*x)^(1/2)\*((A^2\*1i)/(64\*a^4\*d^2))^(1/2))/A)\*((A^2\*1i)/(64\*a^4\*d^2))^(1/2) - ((B\*tan(c + d\*x)^(1/2)\*7i)/(8\*a^2\*d) - (5\*B\*tan(c + d\*x)^(3/2))/(8\*a^2\*d))/(2\*tan(c + d\*x) + tan(c + d\*x)^2\*1i - 1i) + 2\*atanh((16\*a^2\*d\*tan(c + d\*x)^(1/2)\*(-A^2\*529i)/(256\*a^4\*d^2))^(1/2))/(23\*A))\*(-A^2\*529i)/(256\*a^4\*d^2))^(1/2) + atan((8\*a^2\*d\*tan(c + d\*x)^(1/2)\*(-B^2\*1i)/(64\*a^4\*d^2))^(1/2))/B)\*(-B^2\*1i)/(64\*a^4\*d^2))^(1/2)\*2i - atan((16\*a^2\*d\*tan(c + d\*x)^(1/2)\*((B^2\*49i)/(256\*a^4\*d^2))^(1/2))/(7\*B))\*((B^2\*49i)/(256\*a^4\*d^2))^(1/2)\*2i - ((43\*A\*tan(c + d\*x))/(8\*a^2\*d) - (A\*2i)/(a^2\*d) + (A\*tan(c + d\*x)^2\*25i)/(8\*a^2\*d))/(2\*tan(c + d\*x)^(3/2) - tan(c + d\*x)^(1/2)\*1i + tan(c + d\*x)^(5/2)\*1i)

$$3.145 \quad \int \frac{A+B \tan(c+dx)}{\tan^2(c+dx)(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=347

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((47+2i)A + (2+23i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^2 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((47+2i)A + (2+23i)B\right)}{\sqrt{2}}$$

[Out]  $(-1/32+1/32*I)*((47+2*I)*A+(2+23*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}+(-1/32+1/32*I)*((47+2*I)*A+(2+23*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^2/d*2^{(1/2)}+1/64*((49+45*I)*A+(-25+21*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}+(-1/64+1/64*I)*((2+47*I)*A-(23+2*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2/d*2^{(1/2)}+5/8*(9*I*A-5*B)/a^2/d/\tan(d*x+c)^{(1/2)}-7/24*(7*A+3*I*B)/a^2/d/\tan(d*x+c)^{(3/2)}+1/8*(9*A+5*I*B)/a^2/d/(1+I*\tan(d*x+c))/\tan(d*x+c)^{(3/2)}+1/4*(A+I*B)/d/\tan(d*x+c)^{(3/2)}/(a+I*a*\tan(d*x+c))^2$

**Rubi [A]**

time = 0.43, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((47+2i)A + (2+23i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^2 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((47+2i)A + (2+23i)B\right) \text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} a^2 d} - \frac{1/4 + 5/8 I}{32 a^2 \tan(c+dx)} - \frac{9/4 + 5/8 I}{8 a^2 (1 + i \tan(c+dx)) \tan(c+dx)} - \frac{5/8 - 5/8 I + 9/4}{8 a^2 \sqrt{\tan(c+dx)}} - \frac{(49+45i)A - (25-21i)B \log(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1)}{32 \sqrt{2} a^2 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((2+47i)A - (23+2i)B\right) \log(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1)}{\sqrt{2} a^2 d} - \frac{A + iB}{4 d \tan^2(c+dx) (a + i a \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^2), x]

[Out]  $((1/16 - I/16)*((47 + 2*I)*A + (2 + 23*I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]/(\text{Sqrt}[2]*a^2*d) - ((1/16 - I/16)*((47 + 2*I)*A + (2 + 23*I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]/(\text{Sqrt}[2]*a^2*d) + (((49 + 45*I)*A - (25 - 21*I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]/(32*\text{Sqrt}[2]*a^2*d) - ((1/32 - I/32)*((2 + 47*I)*A - (23 + 2*I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]/(32*\text{Sqrt}[2]*a^2*d) - (7*(7*A + (3*I)*B))/(24*a^2*d*\text{Tan}[c + d*x]^{(3/2)}) + (9*A + (5*I)*B)/(8*a^2*d*(1 + I*\text{Tan}[c + d*x])*\text{Tan}[c + d*x]^{(3/2)}) + (5*((9*I)*A - 5*B))/(8*a^2*d*\text{Sqrt}[\text{Tan}[c + d*x]]) + (A + I*B)/(4*d*\text{Tan}[c + d*x]^{(3/2)}*(a + I*a*\text{Tan}[c + d*x])^2)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

NeQ[c^2 + d^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx &= \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(11A+3iB) - \frac{7}{2}a(iA-B) \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx}{4a^2} \\
 &= \frac{9A + 5iB}{8a^2d(1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} \\
 &= -\frac{7(7A + 3iB)}{24a^2d \tan^{\frac{3}{2}}(c + dx)} + \frac{9A + 5iB}{8a^2d(1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} \\
 &= -\frac{7(7A + 3iB)}{24a^2d \tan^{\frac{3}{2}}(c + dx)} + \frac{9A + 5iB}{8a^2d(1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} \\
 &= -\frac{7(7A + 3iB)}{24a^2d \tan^{\frac{3}{2}}(c + dx)} + \frac{9A + 5iB}{8a^2d(1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} \\
 &= -\frac{7(7A + 3iB)}{24a^2d \tan^{\frac{3}{2}}(c + dx)} + \frac{9A + 5iB}{8a^2d(1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} \\
 &= -\frac{7(7A + 3iB)}{24a^2d \tan^{\frac{3}{2}}(c + dx)} + \frac{9A + 5iB}{8a^2d(1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{4d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} \\
 &= \frac{((49 + 45i)A - (25 - 21i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^2d} \\
 &= \frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((47 + 2i)A + (2 + 23i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} a^2d}
 \end{aligned}$$



**Mathematica [A]**

time = 1.91, size = 282, normalized size = 0.81

$$\frac{i \cos(c+dx) \cos(dx) + \sin(dx)^2 \left( \frac{1}{2} \cos(c+dx) \cos(2dx) - i \sin(2dx) \right) (-205A + 129B) \cos(c+dx) + (205A - 129B) \cos(3c+3dx) - 2(-71A - 27B + (199A + 123B) \cos(2c+2dx)) \sin(c+dx) + (1+i) \left( (47+2i)A + (2+23i)B \right) \operatorname{ArcSin}(\cos(c+dx) - \sin(c+dx)) + (-2-47i)A + (23+2i)B \log(\cos(c+dx) + \sin(c+dx) + \sqrt{\tan(2c+2d)})}{32(A \cos(c+dx) + B \sin(c+dx)) \sqrt{\tan(c+dx)}} (A + B \tan(c+dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2), x]
```

```
[Out] ((-1/32*I)*Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*((Csc[c + d*x]*(Cos[2*d*x] - I*Sin[2*d*x])*((-269*I)*A + 129*B)*Cos[c + d*x] + ((205*I)*A - 129*B)*Cos[3*(c + d*x)] - 2*(-71*A - (27*I)*B + (199*A + (123*I)*B)*Cos[2*(c + d*x)])*Sin[c + d*x]))/3 + (1 + I)*((47 + 2*I)*A + (2 + 23*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + ((-2 - 47*I)*A + (23 + 2*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sec[c + d*x]*(Cos[2*c] + I*Sin[2*c])*Sqrt[Sin[2*(c + d*x)]]*(A + B*Tan[c + d*x])/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2)
```

**Maple [A]**

time = 0.10, size = 176, normalized size = 0.51

method	result
derivativedivides	$\frac{\frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(-2iA+B)}{\sqrt{\tan(dx+c)}}}{d a^2} \left( \frac{i \left( \left( -\frac{13A}{2} - \frac{9iB}{2} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + \left( \frac{15iA}{2} - \frac{11B}{2} \right) \left( \sqrt{\tan(dx+c)} \right) \right)}{(\tan(dx+c)-i)^2} \right) \frac{(23iB+47A) a}{4}$
default	$\frac{\frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}} - \frac{2(-2iA+B)}{\sqrt{\tan(dx+c)}}}{d a^2} \left( \frac{i \left( \left( -\frac{13A}{2} - \frac{9iB}{2} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + \left( \frac{15iA}{2} - \frac{11B}{2} \right) \left( \sqrt{\tan(dx+c)} \right) \right)}{(\tan(dx+c)-i)^2} \right) \frac{(23iB+47A) a}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURN VERBOSE)
```

```
[Out] 1/d/a^2*(-2/3*A/tan(d*x+c)^(3/2)-2*(-2*I*A+B)/tan(d*x+c)^(1/2)-1/4*I*((-13/2*A-9/2*I*B)*tan(d*x+c)^(3/2)+(15/2*I*A-11/2*B)*tan(d*x+c)^(1/2))/(tan(d*x+c)-I)^2-(47*A+23*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))
```

2)-I\*2^(1/2))))-1/2\*I\*(A-I\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)  
/(2^(1/2)+I\*2^(1/2))))

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorit  
hm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than  
twice the leaf count of optimal. 861 vs. 2(258) = 516.

time = 0.86, size = 861, normalized size = 2.48

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorit  
hm="fricas")

[Out] 
$$\frac{1}{96} \left( 6(a^2 d e^{(8 I d x + 8 I c)} - 2 a^2 d e^{(6 I d x + 6 I c)} + a^2 d e^{(4 I d x + 4 I c)}) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^4 d^2)} \log(-2((I a^2 d e^{(2 I d x + 2 I c)} + I a^2 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}) / (e^{(2 I d x + 2 I c)} + 1)) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^4 d^2)} - (A - I B) e^{(2 I d x + 2 I c)} e^{(-2 I d x - 2 I c)} / (I A + B) - 6(a^2 d e^{(8 I d x + 8 I c)} - 2 a^2 d e^{(6 I d x + 6 I c)} + a^2 d e^{(4 I d x + 4 I c)}) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^4 d^2)} \log(-2((-I a^2 d e^{(2 I d x + 2 I c)} - I a^2 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}) / (e^{(2 I d x + 2 I c)} + 1)) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^4 d^2)} - (A - I B) e^{(2 I d x + 2 I c)} e^{(-2 I d x - 2 I c)} / (I A + B) - 3(a^2 d e^{(8 I d x + 8 I c)} - 2 a^2 d e^{(6 I d x + 6 I c)} + a^2 d e^{(4 I d x + 4 I c)}) \sqrt{(2209 I A^2 - 2162 A B - 529 I B^2)/(a^4 d^2)} \log(-1/8((a^2 d e^{(2 I d x + 2 I c)} + a^2 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}) / (e^{(2 I d x + 2 I c)} + 1)) \sqrt{(2209 I A^2 - 2162 A B - 529 I B^2)/(a^4 d^2)} + 47 I A - 23 B) e^{(-2 I d x - 2 I c)} / (a^2 d) + 3(a^2 d e^{(8 I d x + 8 I c)} - 2 a^2 d e^{(6 I d x + 6 I c)} + a^2 d e^{(4 I d x + 4 I c)}) \sqrt{(2209 I A^2 - 2162 A B - 529 I B^2)/(a^4 d^2)} \log(1/8((a^2 d e^{(2 I d x + 2 I c)} + a^2 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}) / (e^{(2 I d x + 2 I c)} + 1)) \sqrt{(2209 I A^2 - 2162 A B - 529 I B^2)/(a^4 d^2)} - 47 I A + 23 B) e^{(-2 I d x - 2 I c)} / (a^2 d) - 2(2(101 A + 63 I B) e^{(8 I d x + 8 I c)} - (103 A + 27 I B) e^{(6 I d x + 6 I c)} - (269 A + 129 I B) e^{(4 I d x + 4 I c)} + 3(13 A + 9 I B) e^{(2 I d x + 2 I c)} + 3 A + 3 I B) \sqrt{(($$

$$-Ie^{(2I*d*x + 2I*c) + I}/(e^{(2I*d*x + 2I*c) + 1}))/((a^2*d*e^{(8I*d*x + 8I*c)} - 2*a^2*d*e^{(6I*d*x + 6I*c)} + a^2*d*e^{(4I*d*x + 4I*c)})$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2)/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac** [A]

time = 1.41, size = 162, normalized size = 0.47

$$\frac{(i-1)\sqrt{2}(47A+23iB)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} + \frac{(i+1)\sqrt{2}(A-iB)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} - \frac{2(-6iA\tan(dx+c)+3B\tan(dx+c)+A)}{3a^2d\tan(dx+c)^3} - \frac{-13iA\tan(dx+c)^3+9B\tan(dx+c)^3-15A\sqrt{\tan(dx+c)}-11iB\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] (1/16\*I - 1/16)\*sqrt(2)\*(47\*A + 23\*I\*B)\*arctan((1/2\*I + 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^2\*d) + (1/8\*I + 1/8)\*sqrt(2)\*(A - I\*B)\*arctan((1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^2\*d) - 2/3\*(-6\*I\*A\*tan(d\*x + c) + 3\*B\*tan(d\*x + c) + A)/(a^2\*d\*tan(d\*x + c)^(3/2)) - 1/8\*(-13\*I\*A\*tan(d\*x + c)^(3/2) + 9\*B\*tan(d\*x + c)^(3/2) - 15\*A\*sqrt(tan(d\*x + c)) - 11\*I\*B\*sqrt(tan(d\*x + c)))/(a^2\*d\*(tan(d\*x + c) - I)^2)

**Mupad** [B]

time = 10.48, size = 373, normalized size = 1.07

$$-\operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2+1}{64a^4d^2}}}{A}\right) - \frac{\sqrt{-A^2+1}}{64a^4d^2} \operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2+2209}{256a^4d^2}}}{47A}\right) + \frac{\sqrt{A^2+2209}}{256a^4d^2} \operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2+1}{64a^4d^2}}}{B}\right) + \frac{\sqrt{B^2+1}}{64a^4d^2} \operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2+529}{256a^4d^2}}}{23B}\right) + \frac{\sqrt{B^2+529}}{256a^4d^2} + \frac{44a^2d^2d^2 - 64a^2d^2d^2 + 44a^2d^2 + 44a^2d^2d^2d^2}{2\tan(c+dx)^{11} - \tan(c+dx)^{10} + \tan(c+dx)^{9} - 2\tan(c+dx)^8 - \sqrt{\tan(c+dx)}^{11} + \tan(c+dx)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)^2),x)

[Out] atan((16\*a^2\*d\*tan(c + d\*x)^(1/2)\*((A^2\*2209i)/(256\*a^4\*d^2))^(1/2))/(47\*A)) \* ((A^2\*2209i)/(256\*a^4\*d^2))^(1/2)\*2i - atan((8\*a^2\*d\*tan(c + d\*x)^(1/2)\*((A^2\*1i)/(64\*a^4\*d^2))^(1/2))/A) \* ((A^2\*1i)/(64\*a^4\*d^2))^(1/2)\*2i + 2\*atanh((8\*a^2\*d\*tan(c + d\*x)^(1/2)\*((B^2\*1i)/(64\*a^4\*d^2))^(1/2))/B) \* ((B^2\*1i)/(64\*a^4\*d^2))^(1/2) + 2\*atanh((16\*a^2\*d\*tan(c + d\*x)^(1/2)\*((B^2\*529i)/(256\*a^4\*d^2))^(1/2))/(23\*B)) \* ((B^2\*529i)/(256\*a^4\*d^2))^(1/2) + ((A\*2i)/(3\*a^2\*d) + (8\*A\*tan(c + d\*x))/(3\*a^2\*d) + (A\*tan(c + d\*x)^2\*221i)/(24\*a^2\*d) - (45\*A\*tan(c + d\*x)^3)/(8\*a^2\*d))/(2\*tan(c + d\*x)^(5/2) - tan(c + d\*x)^(3/2))\*1i + tan(c + d\*x)^(7/2)\*1i - ((43\*B\*tan(c + d\*x))/(8\*a^2\*d) - (B\*2i)/(a^2\*d) + (B\*tan(c + d\*x)^2\*25i)/(8\*a^2\*d))/(2\*tan(c + d\*x)^(3/2) - tan(c + d\*x)^(1/2))\*1i + tan(c + d\*x)^(5/2)\*1i

$$3.146 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=393

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((29+i)A + (1+76i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) - \left(\frac{1}{16} + \frac{i}{16}\right) ((29+i)A + (1+76i)B)A}{\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((29+i)A + (1+76i)B)A}{\sqrt{2} a^3 d}$$

[Out]  $(-1/32-1/32*I)*((29+I)*A+(1+76*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}-(1/32+1/32*I)*((29+I)*A+(1+76*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}-1/64*((28-30*I)*A+(75+77*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}-(1/64+1/64*I)*((1+29*I)*A-(76+I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}+15/8*(2*I*A-5*B)*\tan(d*x+c)^{(1/2)}/a^3/d+7/24*(4*A+11*I*B)*\tan(d*x+c)^{(3/2)}/a^3/d+1/6*(I*A-B)*\tan(d*x+c)^{(9/2)}/d/(a+I*a*\tan(d*x+c))^3+1/4*(A+2*I*B)*\tan(d*x+c)^{(7/2)}/a/d/(a+I*a*\tan(d*x+c))^2-3/8*(2*I*A-5*B)*\tan(d*x+c)^{(5/2)}/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi [A]**

time = 0.57, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((29+i)A + (1+76i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) - \left(\frac{1}{16} + \frac{i}{16}\right) ((29+i)A + (1+76i)B)A}{\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((29+i)A + (1+76i)B)A}{\sqrt{2} a^3 d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x])^{(9/2)}*(A + B*\operatorname{Tan}[c + d*x])]/(a + I*a*\operatorname{Tan}[c + d*x])^3, x]$

[Out]  $((1/16 + I/16)*((29 + I)*A + (1 + 76*I)*B)*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^3*d) - ((1/16 + I/16)*((29 + I)*A + (1 + 76*I)*B)*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^3*d) - (((28 - 30*I)*A + (75 + 77*I)*B)*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(32*\operatorname{Sqrt}[2]*a^3*d) - ((1/32 + I/32)*((1 + 29*I)*A - (76 + I)*B)*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^3*d) + (15*((2*I)*A - 5*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(8*a^3*d) + (7*(4*A + (11*I)*B)*\operatorname{Tan}[c + d*x]^{(3/2)})/(24*a^3*d) + ((I*A - B)*\operatorname{Tan}[c + d*x]^{(9/2)})/(6*d*(a + I*a*\operatorname{Tan}[c + d*x])^3) + ((A + (2*I)*B)*\operatorname{Tan}[c + d*x]^{(7/2)})/(4*a*d*(a + I*a*\operatorname{Tan}[c + d*x])^2) - (3*((2*I)*A - 5*B)*\operatorname{Tan}[c + d*x]^{(5/2)})/(8*d*(a^3 + I*a^3*\operatorname{Tan}[c + d*x]))$

**Rule 210**

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

#### Rule 3609

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

NeQ[c^2 + d^2, 0]

Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] & NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} - \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)(\frac{9}{2}a(iA-B)+\frac{3}{2}a(A+5iB) \tan(c+dx))}{(a+ia \tan(c+dx))^2}}{6a^2} \\
 &= \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+2iB) \tan^{\frac{7}{2}}(c+dx)}{4ad(a+ia \tan(c+dx))^2} + \frac{\int \tan^{\frac{5}{2}}(c+dx)}{6a^2} \\
 &= \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+2iB) \tan^{\frac{7}{2}}(c+dx)}{4ad(a+ia \tan(c+dx))^2} - \frac{3(2iA-5B)}{8d(a^3+ia^2d)} \\
 &= \frac{7(4A+11iB) \tan^{\frac{3}{2}}(c+dx)}{24a^3d} + \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+2iB) \tan^{\frac{7}{2}}(c+dx)}{4ad(a+ia \tan(c+dx))^2} \\
 &= \frac{15(2iA-5B) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{7(4A+11iB) \tan^{\frac{3}{2}}(c+dx)}{24a^3d} + \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} \\
 &= \frac{15(2iA-5B) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{7(4A+11iB) \tan^{\frac{3}{2}}(c+dx)}{24a^3d} + \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} \\
 &= \frac{15(2iA-5B) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{7(4A+11iB) \tan^{\frac{3}{2}}(c+dx)}{24a^3d} + \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} \\
 &= \frac{15(2iA-5B) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{7(4A+11iB) \tan^{\frac{3}{2}}(c+dx)}{24a^3d} + \frac{(iA-B) \tan^{\frac{9}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} \\
 &= -\frac{((28-30i)A+(75+77i)B) \log\left(1-\sqrt{2} \sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{32\sqrt{2} a^3d} \\
 &= \frac{\left(\frac{1}{16}+\frac{i}{16}\right) ((29+i)A+(1+76i)B) \tan^{-1}\left(1-\sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3d}
 \end{aligned}$$

**Mathematica [A]**

time = 2.63, size = 300, normalized size = 0.76

$$\frac{a^2 c^2 + d^2 \cos^2(d x) + \cos(d x) (7 A + B \sin(d x)) \left[ \left( (30 - 29) A + (77 + 75 I) B \operatorname{ArcSin}(\cos(c + d x) - \sin(c + d x)) + (1 + I) (-29 + (1 + (-76 I) B) \log(\cos(c + d x) + \sin(c + d x) + \sqrt{\cos^2(c + d x)})) \right) \sqrt{\cos^2(c + d x)} + (\cos(3 c) - \sin(3 c)) \sqrt{\cos^2(c + d x)} + (\cos(3 d x) + \sin(3 d x)) (23 A + 69 B + 2(30 A + 241 I) \cos(2(c + d x)) + (147 A + 349 I) \cos(c + d x)) + 194 A \sin(2(c + d x)) - 502 B \sin(2(c + d x)) + 145 A \sin(c + d x) - 347 B \sin(c + d x)) \tan(c + d x) \right]}{96 (A \cos(c + d x) + B \sin(c + d x)) \sqrt{\tan^2(c + d x)} (a + I a \tan(c + d x))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(9/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3, x]

[Out] (Sec[c + d\*x]^3\*(Cos[d\*x] + I\*Sin[d\*x])^3\*(A + B\*Tan[c + d\*x])\*(3\*((30 - 28\*I)\*A + (77 + 75\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] + (1 + I)\*((-29 + I)\*A + (1 - 76\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*(I\*Cos[3\*c] - Sin[3\*c])\*Sqrt[Sin[2\*(c + d\*x)]] + (I\*Cos[3\*d\*x] + Sin[3\*d\*x])\*(33\*A + (69\*I)\*B + 2\*(90\*A + (241\*I)\*B)\*Cos[2\*(c + d\*x)] + (147\*A + (349\*I)\*B)\*Cos[4\*(c + d\*x)] + (194\*I)\*A\*Sin[2\*(c + d\*x)] - 502\*B\*Sin[2\*(c + d\*x)] + (145\*I)\*A\*Sin[4\*(c + d\*x)] - 347\*B\*Sin[4\*(c + d\*x)])\*Tan[c + d\*x])/(96\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3)

**Maple [A]**

time = 0.11, size = 204, normalized size = 0.52

method	result
derivativedivides	$\frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 6B \left( \sqrt{\tan(dx+c)} \right) + 2iA \left( \sqrt{\tan(dx+c)} \right) + \frac{-5i(7iB+4A) \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( -\frac{182iB}{3} - \frac{98A}{3} \right) \left( \tan(dx+c) \right)}{\tan(dx+c)}$
default	$\frac{2iB \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 6B \left( \sqrt{\tan(dx+c)} \right) + 2iA \left( \sqrt{\tan(dx+c)} \right) + \frac{-5i(7iB+4A) \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( -\frac{182iB}{3} - \frac{98A}{3} \right) \left( \tan(dx+c) \right)}{\tan(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(9/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3, x, method=\_RETURN VERBOSE)

[Out] 1/d/a^3\*(2/3\*I\*B\*tan(d\*x+c)^(3/2)-6\*B\*tan(d\*x+c)^(1/2)+2\*I\*A\*tan(d\*x+c)^(1/2)+1/8\*I\*((-5\*I\*(7\*I\*B+4\*A)\*tan(d\*x+c)^(5/2)+(-182/3\*I\*B-98/3\*A)\*tan(d\*x+c)^(3/2)+(-27\*B+14\*I\*A)\*tan(d\*x+c)^(1/2)))/(tan(d\*x+c)-I)^3+2\*(-76\*B+29\*I\*A)/(

$$2^{(1/2)} - I \cdot 2^{(1/2)}) \cdot \arctan(2 \cdot \tan(dx+c)^{(1/2)} / (2^{(1/2)} - I \cdot 2^{(1/2)})) + 4 \cdot (1/16 \cdot A - 1/16 \cdot I \cdot B) / (2^{(1/2)} + I \cdot 2^{(1/2)}) \cdot \arctan(2 \cdot \tan(dx+c)^{(1/2)} / (2^{(1/2)} + I \cdot 2^{(1/2)}))$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(9/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 779 vs.  $2(294) = 588$ .

time = 0.74, size = 779, normalized size = 1.98



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(9/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/96 \cdot (3 \cdot (a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \cdot \sqrt{(I A^2 + 2 A B - I B^2) / (a^6 d^2)}) \cdot \log(2 \cdot ((a^3 d e^{(2 I d x + 2 I c)} + a^3 d) \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)}) \cdot \sqrt{(I A^2 + 2 A B - I B^2) / (a^6 d^2)}) \\ & + (A - I B) \cdot e^{(2 I d x + 2 I c)} \cdot e^{(-2 I d x - 2 I c)} / (I A + B) - 3 \cdot (a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \cdot \sqrt{(I A^2 + 2 A B - I B^2) / (a^6 d^2)} \\ & \cdot \log(-2 \cdot ((a^3 d e^{(2 I d x + 2 I c)} + a^3 d) \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)}) \cdot \sqrt{(I A^2 + 2 A B - I B^2) / (a^6 d^2)}) \\ & - (A - I B) \cdot e^{(2 I d x + 2 I c)} \cdot e^{(-2 I d x - 2 I c)} / (I A + B) - 3 \cdot (a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \cdot \sqrt{((-841 I A^2 + 4408 A B + 5776 I B^2) / (a^6 d^2))} \\ & \cdot \log(1/8 \cdot ((a^3 d e^{(2 I d x + 2 I c)} + a^3 d) \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)}) \cdot \sqrt{((-841 I A^2 + 4408 A B + 5776 I B^2) / (a^6 d^2))} \\ & + 29 \cdot A + 76 \cdot I B) \cdot e^{(-2 I d x - 2 I c)} / (a^3 d) + 3 \cdot (a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \cdot \sqrt{((-841 I A^2 + 4408 A B + 5776 I B^2) / (a^6 d^2))} \\ & \cdot \log(-1/8 \cdot ((a^3 d e^{(2 I d x + 2 I c)} + a^3 d) \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)}) \cdot \sqrt{((-841 I A^2 + 4408 A B + 5776 I B^2) / (a^6 d^2))} \\ & - 29 \cdot A - 76 \cdot I B) \cdot e^{(-2 I d x - 2 I c)} / (a^3 d) + 2 \cdot (2 \cdot (-73 I A + 174 B) \cdot e^{(8 I d x + 8 I c)} - (187 I A - 492 B) \cdot e^{(6 I d x + 6 I c)} + 3 \cdot (-11 I A + 23 B) \cdot e^{(4 I d x + 4 I c)} - (-7 I A + 10 B) \cdot e^{(2 I d x + 2 I c)} \end{aligned}$$





$$\begin{aligned}
& + d*x)^{3i+1} - ((27*B*\tan(c + d*x)^{(1/2)})/(8*a^{3*d}) + (B*\tan(c + d*x)^{(3/2)*91i})/(12*a^{3*d}) - (35*B*\tan(c + d*x)^{(5/2)})/(8*a^{3*d}))/(\tan(c + d*x)*3 \\
& i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^{3i+1}) + (A*\tan(c + d*x)^{(1/2)*2i})/( \\
& a^{3*d}) - (6*B*\tan(c + d*x)^{(1/2)})/(a^{3*d}) + (B*\tan(c + d*x)^{(3/2)*2i})/(3*a^{ \\
& 3*d})
\end{aligned}$$

$$3.147 \quad \int \frac{\tan^7(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=364

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((1+6i)A - (29+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{\left((5-7i)A + (28+30i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^3 d}$$

[Out] (1/32+1/32\*I)\*((1+6\*I)\*A-(29+I)\*B)\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-1/32\*((5-7\*I)\*A+(28+30\*I)\*B)\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+(1/64+1/64\*I)\*((6+I)\*A+(1+29\*I)\*B)\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a^3/d\*2^(1/2)-(1/64+1/64\*I)\*((6+I)\*A+(1+29\*I)\*B)\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a^3/d\*2^(1/2)+5/8\*(A+6\*I\*B)\*tan(d\*x+c)^(1/2)/a^3/d+1/6\*(I\*A-B)\*tan(d\*x+c)^(7/2)/d/(a+I\*a\*tan(d\*x+c))^3+1/12\*(2\*A+5\*I\*B)\*tan(d\*x+c)^(5/2)/a/d/(a+I\*a\*tan(d\*x+c))^2-7/24\*(I\*A-4\*B)\*tan(d\*x+c)^(3/2)/d/(a^3+I\*a^3\*tan(d\*x+c))

**Rubi [A]**

time = 0.51, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((1+6i)A - (29+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{\left((5-7i)A + (28+30i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^3 d} + \frac{7(-4B + 4A) \tan^2(c+dx)}{24(a^3 + ia^3 \tan(c+dx))} + \frac{5(A + 6iB) \sqrt{\tan(c+dx)}}{8a^3 d} + \frac{(1+i) \left((6+i)A + (1+29i)B\right) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} a^3 d}\right)}{\sqrt{2} a^3 d} + \frac{(1+i) \left((6+i)A + (1+29i)B\right) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} a^3 d}\right)}{\sqrt{2} a^3 d} + \frac{(-B + 4A) \tan^2(c+dx)}{8(a^3 + ia^3 \tan(c+dx))} + \frac{(2A + 5iB) \tan^2(c+dx)}{12(a^3 + ia^3 \tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(7/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] ((-1/16 - I/16)\*((1 + 6\*I)\*A - (29 + I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*a^3\*d) - (((5 - 7\*I)\*A + (28 + 30\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(16\*Sqrt[2]\*a^3\*d) + ((1/32 + I/32)\*((6 + I)\*A + (1 + 29\*I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(Sqrt[2]\*a^3\*d) - ((1/32 + I/32)\*((6 + I)\*A + (1 + 29\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(Sqrt[2]\*a^3\*d) + (5\*(A + (6\*I)\*B)\*Sqrt[Tan[c + d\*x]])/(8\*a^3\*d) + ((I\*A - B)\*Tan[c + d\*x]^(7/2))/(6\*d\*(a + I\*a\*Tan[c + d\*x])^3) + ((2\*A + (5\*I)\*B)\*Tan[c + d\*x]^(5/2))/(12\*a\*d\*(a + I\*a\*Tan[c + d\*x])^2) - (7\*(I\*A - 4\*B)\*Tan[c + d\*x]^(3/2))/(24\*d\*(a^3 + I\*a^3\*Tan[c + d\*x]))

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

## Rule 3676

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} - \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)(\frac{7}{2}a(iA-B)+\frac{1}{2}a(A+13iB) \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx}{6a^2} \\
&= \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(2A+5iB) \tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{\int \tan^{\frac{3}{2}}(c+dx)}{6a^2} \\
&= \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(2A+5iB) \tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} - \frac{7(iA-B)}{24d(a+ia \tan(c+dx))^3} \\
&= \frac{5(A+6iB) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(2A+5iB) \tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} \\
&= \frac{5(A+6iB) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(2A+5iB) \tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} \\
&= \frac{5(A+6iB) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(2A+5iB) \tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} \\
&= \frac{5(A+6iB) \sqrt{\tan(c+dx)}}{8a^3d} + \frac{(iA-B) \tan^{\frac{7}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(2A+5iB) \tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} \\
&= \frac{((5+7i)A - (28-30i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^3d} \\
&= \frac{((5-7i)A + (28+30i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^3d}
\end{aligned}$$

## Mathematica [A]

time = 1.93, size = 286, normalized size = 0.79

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(7/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] (Sec[c + d\*x]^2\*(Cos[d\*x] + I\*Sin[d\*x])^3\*(A + B\*Tan[c + d\*x])\*((-I)\*(((7 + 5\*I)\*A - (30 - 28\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] + (1 - I)\*((6 + I)\*A + (1 + 29\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*Sec[c + d\*x]\*(Cos[3\*c] + I\*Sin[3\*c])\*Sqrt[Sin[2\*(c + d\*x)]] + (2\*(Cos[3\*d\*x] - I\*Sin[3\*d\*x])\*((9\*A + (33\*I)\*B)\*Cos[c + d\*x] + 21\*(A + (7\*I)\*B)\*Cos[3\*(c + d\*x)] + (2\*I)\*(19\*A + (97\*I)\*B + (19\*A + (145\*I)\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x])\*Tan[c + d\*x])/3)/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3)

**Maple [A]**

time = 0.11, size = 181, normalized size = 0.50

method	result
derivativedivides	$2iB \left( \sqrt{\tan(dx+c)} \right) - \frac{i \left( \frac{-i(9iA-20B) \left( \tan \frac{5}{2}(dx+c) \right) + \left( -\frac{38iA}{3} + \frac{98B}{3} \right) \left( \tan \frac{3}{2}(dx+c) \right) + (-14iB-5A) \left( \sqrt{\tan(dx+c)} \right)}{(\tan(dx+c)-i)^3} \right)}{8 da^3}$
default	$2iB \left( \sqrt{\tan(dx+c)} \right) - \frac{i \left( \frac{-i(9iA-20B) \left( \tan \frac{5}{2}(dx+c) \right) + \left( -\frac{38iA}{3} + \frac{98B}{3} \right) \left( \tan \frac{3}{2}(dx+c) \right) + (-14iB-5A) \left( \sqrt{\tan(dx+c)} \right)}{(\tan(dx+c)-i)^3} \right)}{8 da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(7/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURN VERBOSE)

[Out] 1/d/a^3\*(2\*I\*B\*tan(d\*x+c)^(1/2)-1/8\*I\*((-I\*(9\*I\*A-20\*B)\*tan(d\*x+c)^(5/2)+(-38/3\*I\*A+98/3\*B)\*tan(d\*x+c)^(3/2)+(-5\*A-14\*I\*B)\*tan(d\*x+c)^(1/2))/(tan(d\*x+c)-I)^3-2\*(29\*I\*B+6\*A)/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2))))+4\*(1/16\*I\*A+1/16\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2))))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

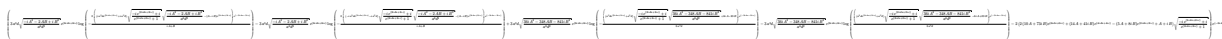
Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs.  $2(271) = 542$ .  
time = 0.73, size = 685, normalized size = 1.88



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(3*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))
- (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d
*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((-I*a
^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*B)*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 3*a^3*d*sqrt((36*I*A
^2 - 348*A*B - 841*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*((a^3*d*e
^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2)) + 6*I*A - 29
*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 3*a^3*d*sqrt((36*I*A^2 - 348*A*B - 841*
I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*d*e^(2*I*d*x + 2*I*c) +
a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2)) - 6*I*A + 29*B)*e^(-2*I*d*x - 2*
I*c)/(a^3*d)) - 2*(2*(10*A + 73*I*B)*e^(6*I*d*x + 6*I*c) + (14*A + 41*I*B)*
e^(4*I*d*x + 4*I*c) - (5*A + 8*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(-6*I*d*x - 6*I*c)/
(a^3*d)
```

**Sympy** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

**Giac [A]**

time = 1.01, size = 165, normalized size = 0.45

$$\frac{(i-1)\sqrt{2}(6A+29iB)\arctan\left(\frac{(i+\frac{1}{2})\sqrt{2}\sqrt{\tan(dx+c)}}{16a^3d}\right) + (i+1)\sqrt{2}(A-iB)\arctan\left(\frac{-(i-\frac{1}{2})\sqrt{2}\sqrt{\tan(dx+c)}}{16a^3d}\right) + \frac{2iB\sqrt{\tan(dx+c)}}{a^3d} + \frac{-27iA\tan(dx+c)^2 + 60B\tan(dx+c)^2 - 38A\tan(dx+c)^2 - 98iB\tan(dx+c)^2 + 15iA\sqrt{\tan(dx+c)} - 42B\sqrt{\tan(dx+c)}}{24a^3d(\tan(dx+c)-i)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(7/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] (1/16\*I - 1/16)\*sqrt(2)\*(6\*A + 29\*I\*B)\*arctan((1/2\*I + 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^3\*d) + (1/16\*I + 1/16)\*sqrt(2)\*(A - I\*B)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^3\*d) + 2\*I\*B\*sqrt(tan(d\*x + c))/(a^3\*d) + 1/24\*(-27\*I\*A\*tan(d\*x + c)^(5/2) + 60\*B\*tan(d\*x + c)^(5/2) - 38\*A\*tan(d\*x + c)^(3/2) - 98\*I\*B\*tan(d\*x + c)^(3/2) + 15\*I\*A\*sqrt(tan(d\*x + c)) - 42\*B\*sqrt(tan(d\*x + c)))/(a^3\*d\*(tan(d\*x + c) - I)^3)

**Mupad [B]**

time = 9.97, size = 395, normalized size = 1.09

$$\frac{\arctan\left(\frac{8a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{14}\right)\sqrt{\frac{29iB}{32a^3d}} + i\operatorname{atan}\left(\frac{16a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{A}\right)\sqrt{\frac{29iB}{32a^3d}} + i\operatorname{atan}\left(\frac{a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{B}\right)\sqrt{\frac{29iB}{32a^3d}} + \frac{29iB}{256a^3d}}{\sqrt{\frac{29iB}{32a^3d}} + i\operatorname{atan}\left(\frac{a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{29B}\right)\sqrt{\frac{29iB}{32a^3d}} + \frac{29iB}{256a^3d}} + \frac{14\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}} + \frac{14\operatorname{atan}\left(\frac{a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{B}\right)\sqrt{\frac{29iB}{32a^3d}}}{-\tan(c+dx)^2 - 3\tan(c+dx) + \tan(c+dx)^2 + 1}}{\sqrt{\frac{29iB}{32a^3d}} + i\operatorname{atan}\left(\frac{a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{29B}\right)\sqrt{\frac{29iB}{32a^3d}} + \frac{29iB}{256a^3d}} + \frac{B\sqrt{\tan(c+dx)}\sqrt{\frac{29iB}{32a^3d}}}{a^3d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*i)^3,x)

[Out] atan((8\*a^3\*d\*tan(c + d\*x)^(1/2)\*((A^2\*9i)/(64\*a^6\*d^2))^(1/2))/(3\*A))\*((A^2\*9i)/(64\*a^6\*d^2))^(1/2)\*2i + atan((16\*a^3\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(256\*a^6\*d^2))^(1/2))/A)\*(-(A^2\*1i)/(256\*a^6\*d^2))^(1/2)\*2i + atan((a^3\*d\*tan(c + d\*x)^(1/2)\*((B^2\*1i)/(256\*a^6\*d^2))^(1/2)\*16i)/B)\*((B^2\*1i)/(256\*a^6\*d^2))^(1/2)\*2i - atan((a^3\*d\*tan(c + d\*x)^(1/2)\*(-(B^2\*841i)/(256\*a^6\*d^2))^(1/2)\*16i)/(29\*B))\*(-(B^2\*841i)/(256\*a^6\*d^2))^(1/2)\*2i + ((5\*A\*tan(c + d\*x)^(1/2))/(8\*a^3\*d) + (A\*tan(c + d\*x)^(3/2)\*19i)/(12\*a^3\*d) - (9\*A\*tan(c + d\*x)^(5/2))/(8\*a^3\*d))/(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1) - ((49\*B\*tan(c + d\*x)^(3/2))/(12\*a^3\*d) - (B\*tan(c + d\*x)^(1/2)\*7i)/(4\*a^3\*d) + (B\*tan(c + d\*x)^(5/2)\*5i)/(2\*a^3\*d))/(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1) + (B\*tan(c + d\*x)^(1/2)\*2i)/(a^3\*d)



$$3.148 \quad \int \frac{\tan^5(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=307

$$\frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^3 d} - \frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^3 d}$$

```
[Out] -1/32*(2*A+(5-7*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^3/d*2^(1/2)-1/3
2*(2*A+(5-7*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a^3/d*2^(1/2)-1/64*(2*
A-(5+7*I)*B)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^3/d*2^(1/2)+1/64*(
2*A-(5+7*I)*B)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^3/d*2^(1/2)+1/6*
(I*A-B)*tan(d*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^3+1/12*(A+4*I*B)*tan(d*x+c)^(
3/2)/a/d/(a+I*a*tan(d*x+c))^2+5/8*B*tan(d*x+c)^(1/2)/d/(a^3+I*a^3*tan(d*x+c
))
```

**Rubi** [A]

time = 0.42, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^3 d} - \frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c + dx)} + 1\right)}{16\sqrt{2} a^3 d} - \frac{(2A - (5 + 7i)B) \log\left(\tan(c + dx) - \sqrt{2} \sqrt{\tan(c + dx)} + 1\right)}{32\sqrt{2} a^3 d} + \frac{(2A - (5 + 7i)B) \log\left(\tan(c + dx) + \sqrt{2} \sqrt{\tan(c + dx)} + 1\right)}{32\sqrt{2} a^3 d} + \frac{5B \sqrt{\tan(c + dx)}}{8d(a^2 + ia^2 \tan(c + dx))} + \frac{(-B + iA) \tan^3(c + dx)}{6d(a + ia \tan(c + dx))^2} + \frac{(A + 4iB) \tan^3(c + dx)}{12ad(a + ia \tan(c + dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((2*A + (5 - 7*I)*B)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(16*Sqrt[2]*a^
3*d) - ((2*A + (5 - 7*I)*B)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(16*Sqr
t[2]*a^3*d) - ((2*A - (5 + 7*I)*B)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]]/(32*Sqrt[2]*a^3*d) + ((2*A - (5 + 7*I)*B)*Log[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]]/(32*Sqrt[2]*a^3*d) + ((I*A - B)*Tan[c + d*x]
^(5/2))/(6*d*(a + I*a*Tan[c + d*x])^3) + ((A + (4*I)*B)*Tan[c + d*x]^(3/2)
)/(12*a*d*(a + I*a*Tan[c + d*x])^2) + (5*B*Sqrt[Tan[c + d*x]])/(8*d*(a^3 + I
*a^3*Tan[c + d*x]))
```

**Rule 210**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]/b), x] \ /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[2(d/e), 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2(d/e), 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NeQ}[c^2d^2 + a^2e^2, 0] \ \&\& \ \text{NeQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[(-a)c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\text{Sqrt}[(b_.)\tan[(e_.) + (f_.)x]}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(bc + dx^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \cdot \text{Tan}[e + fx]]], x] \ /; \text{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3676

$\text{Int}[\frac{(a_.) + (b_.)\tan[(e_.) + (f_.)x]^m \cdot ((A_.) + (B_.)\tan[(e_.) + (f_.)x])^n}{(c_.) + (d_.)\tan[(e_.) + (f_.)x]^n}, x\_Symbol] \rightarrow \text{Simp}[(-A^2b - a^2B) \cdot (a + b \cdot \text{Tan}[e + fx])^m \cdot (c + d \cdot \text{Tan}[e + fx])^n / (2af^2m), x] + \text{Dist}[1/(2a^2m), \text{Int}[(a + b \cdot \text{Tan}[e + fx])^{m+1} \cdot (c + d \cdot \text{Tan}[e + fx])^{n-1} \cdot \text{Simp}[A(a^2cm + b^2dn) - B(b^2cm + a^2dn) - d(bB(m-n) - aA(m+n)) \cdot \text{Tan}[e + fx], x], x], x] \ /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \ \&\& \ \text{NeQ}[b^2c - a^2d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, 0] \ \&\& \ \text{GtQ}[n, 0]$

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} - \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)(\frac{5}{2}a(iA-B)-\frac{1}{2}a(A-11iB) \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx}{6a^2} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+4iB) \tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{\int \sqrt{\tan(c+dx)} dx}{8d(a^3+ia^2 \tan(c+dx))} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+4iB) \tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{5B \sqrt{\tan(c+dx)}}{8d(a^3+ia^2 \tan(c+dx))} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+4iB) \tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{5B \sqrt{\tan(c+dx)}}{8d(a^3+ia^2 \tan(c+dx))} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+4iB) \tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{5B \sqrt{\tan(c+dx)}}{8d(a^3+ia^2 \tan(c+dx))} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+4iB) \tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{5B \sqrt{\tan(c+dx)}}{8d(a^3+ia^2 \tan(c+dx))} \\
&\quad - \frac{(2A-(5+7i)B) \log\left(1-\sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^3 d} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{(A+4iB) \tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia \tan(c+dx))^2} + \frac{5B \sqrt{\tan(c+dx)}}{8d(a^3+ia^2 \tan(c+dx))} \\
&\quad - \frac{(2A+(5-7i)B) \tan^{-1}\left(1-\sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^3 d} - \frac{(2A+(5-7i)B) \tan^{-1}\left(1+\sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^3 d}
\end{aligned}$$

**Mathematica [A]**

time = 1.41, size = 254, normalized size = 0.83

$$\frac{\sec^2(c+dx)(\cos(dx)+i \sin(dx))^2 \left( (2A+(5-7i)B) \operatorname{ArcSin}(\cos(c+dx)-\sin(c+dx)) + (1-i)((1+i)A+(1-6i)B) \log(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))}) \right) \sec(c+dx)(\cos(3c)+i \sin(3c)) \sqrt{\sin(2(c+dx))} + \frac{1}{2}(\cos(3dx)-i \sin(3dx)) \sin(c+dx)(3A-6B+3(-A+7i)B) \cos(2(c+dx)) + (A+19iB) \sin(2(c+dx)) \right) (A+B \tan(c+dx))}{32d(A \cos(c+dx)+B \sin(c+dx)) \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3, x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^3*(((2*A + (5 - 7*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + (1 - I)*((1 + I)*A + (1 - 6*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)])]))*Sec[c + d*x]*(Cos[3*c] + I*Sin[3*c])*Sqrt[Sin[2*(c + d*x)]] + (4*(Cos[3*d*x] - I*Sin[3*d*x])*Sin[c + d*x]*((3*I)*A - 6*B + 3*(-I)*A + 7*B)*Cos[2*(c + d*x)] + (A + (19*I)*B)*Sin[2*(c + d*x)]))/3*(A + B*Tan[c + d*x]))/(32*d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3
```

**Maple [A]**

time = 0.14, size = 163, normalized size = 0.53

method	result
derivativedivides	$i \frac{-i(9iB+2A) \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( -\frac{38iB}{3} - \frac{2A}{3} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) - 5B \left( \sqrt{\tan}(dx+c) \right)}{(\tan(dx+c)-i)^3} - \frac{2^{(iA+6B)} \arctan \left( \frac{2 \left( \sqrt{\tan}(dx+c) \right)}{\sqrt{2}-i\sqrt{2}} \right)}{\sqrt{2}-i\sqrt{2}}$ <hr/> $d a^3$
default	$i \frac{-i(9iB+2A) \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( -\frac{38iB}{3} - \frac{2A}{3} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) - 5B \left( \sqrt{\tan}(dx+c) \right)}{(\tan(dx+c)-i)^3} - \frac{2^{(iA+6B)} \arctan \left( \frac{2 \left( \sqrt{\tan}(dx+c) \right)}{\sqrt{2}-i\sqrt{2}} \right)}{\sqrt{2}-i\sqrt{2}}$ <hr/> $d a^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d/a^3} \left( -\frac{1}{8} I \left( (-I(9IB+2A) \tan^{\frac{5}{2}}(dx+c) + (-\frac{38}{3}IB - \frac{2}{3}A) \tan^{\frac{3}{2}}(dx+c) - 5B \sqrt{\tan}(dx+c)) / (\tan(dx+c) - I)^3 - 2^{(IA+6B)} / (2^{\frac{1}{2}} - I 2^{\frac{1}{2}}) \arctan(2 \tan^{\frac{1}{2}}(dx+c) / (2^{\frac{1}{2}} - I 2^{\frac{1}{2}})) \right) + 4 \left( -\frac{1}{16}A + \frac{1}{16}IB \right) / (2^{\frac{1}{2}} + I 2^{\frac{1}{2}}) \arctan(2 \tan^{\frac{1}{2}}(dx+c) / (2^{\frac{1}{2}} + I 2^{\frac{1}{2}})) \right)$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

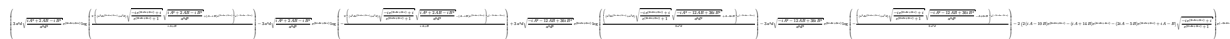
Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs.  $2(242) = 484$ .

time = 0.63, size = 681, normalized size = 2.22



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{96} \cdot (3a^3 d \sqrt{(IA^2 + 2AB - IB^2)/(a^6 d^2)}) e^{(6I dx + 6Ic)} \log(2((a^3 d e^{(2I dx + 2Ic)} + a^3 d) \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(IA^2 + 2AB - IB^2)/(a^6 d^2)} + (A - IB) e^{(2I dx + 2Ic)}) e^{(-2I dx - 2Ic)/(IA + B)} - 3a^3 d \sqrt{(IA^2 + 2AB - IB^2)/(a^6 d^2)}) e^{(6I dx + 6Ic)} \log(-2((a^3 d e^{(2I dx + 2Ic)} + a^3 d) \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(IA^2 + 2AB - IB^2)/(a^6 d^2)} - (A - IB) e^{(2I dx + 2Ic)}) e^{(-2I dx - 2Ic)/(IA + B)} + 3a^3 d \sqrt{(-IA^2 - 12AB + 36IB^2)/(a^6 d^2)}) e^{(6I dx + 6Ic)} \log(1/8((a^3 d e^{(2I dx + 2Ic)} + a^3 d) \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(-IA^2 - 12AB + 36IB^2)/(a^6 d^2)} + A - 6IB) e^{(-2I dx - 2Ic)/(a^3 d)}) - 3a^3 d \sqrt{(-IA^2 - 12AB + 36IB^2)/(a^6 d^2)}) e^{(6I dx + 6Ic)} \log(-1/8((a^3 d e^{(2I dx + 2Ic)} + a^3 d) \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(-IA^2 - 12AB + 36IB^2)/(a^6 d^2)} - A + 6IB) e^{(-2I dx - 2Ic)/(a^3 d)}) - 2(2(IA - 10B) e^{(6I dx + 6Ic)} - (IA + 14B) e^{(4I dx + 4Ic)} - (2IA - 5B) e^{(2I dx + 2Ic)} + IA - B) \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) e^{(-6I dx - 6Ic)/(a^3 d)}$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [A]

time = 0.95, size = 135, normalized size = 0.44

$$\frac{(i+1)\sqrt{2}(A-6iB)\arctan\left(\frac{i}{2} + \frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} + \frac{(i-1)\sqrt{2}(A-iB)\arctan\left(-\frac{i}{2} - \frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} - \frac{6A\tan(dx+c)^3 + 27iB\tan(dx+c)^3 - 2iA\tan(dx+c)^3 + 38B\tan(dx+c)^3 - 15iB\sqrt{\tan(dx+c)}}{24a^3d(\tan(dx+c)-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $-(1/16I + 1/16)\sqrt{2}(A - 6IB)\arctan((1/2I + 1/2)\sqrt{2}\sqrt{\tan(dx+c)})/(a^3d) + (1/16I - 1/16)\sqrt{2}(A - IB)\arctan(-(1/2I - 1/2)\sqrt{2}\sqrt{\tan(dx+c)})/(a^3d) - 1/24(6A\tan(dx+c)^{5/2} + 27IB$

$$*B*\tan(d*x + c)^{(5/2)} - 2*I*A*\tan(d*x + c)^{(3/2)} + 38*B*\tan(d*x + c)^{(3/2)} - 15*I*B*\sqrt{\tan(d*x + c)})/(a^3*d*(\tan(d*x + c) - I)^3)$$

**Mupad [B]**

time = 8.17, size = 308, normalized size = 1.00

$$\frac{\frac{1}{2} \sqrt{\tan(c+dx)} - \frac{2 \operatorname{atanh}\left(\frac{1}{\sqrt{2}}\right) + \operatorname{atanh}\left(\frac{1}{\sqrt{2}}\right) \ln}{2 \sqrt{2}}}{-\tan(c+dx)^2 - 3 \tan(c+dx) + \tan(c+dx) + 1} + \frac{\frac{2 \operatorname{atanh}\left(\frac{1}{\sqrt{2}}\right) + \operatorname{atanh}\left(\frac{1}{\sqrt{2}}\right) \ln}{2 \sqrt{2}}}{-\tan(c+dx)^2 - 3 \tan(c+dx) + \tan(c+dx) + 1} - \frac{(-1)^{1/4} \operatorname{atanh}\left((-1)^{1/4} \sqrt{\tan(c+dx)}\right)}{8 a^3 d} + \frac{(-1)^{1/4} \operatorname{atanh}\left((-1)^{1/4} \sqrt{\tan(c+dx)}\right)}{8 a^3 d} + \operatorname{atan}\left(\frac{8 a^3 d \sqrt{\tan(c+dx)} \sqrt{\frac{B^2 9 i}{64 a^6 d^2}}}{3 B}\right) \sqrt{\frac{B^2 9 i}{64 a^6 d^2}} + \operatorname{atan}\left(\frac{16 a^3 d \sqrt{\tan(c+dx)} \sqrt{-\frac{B^2 1 i}{256 a^6 d^2}}}{B}\right) \sqrt{-\frac{B^2 1 i}{256 a^6 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] atan((8\*a^3\*d\*tan(c + d\*x)^(1/2)\*((B^2\*9i)/(64\*a^6\*d^2))^(1/2))/(3\*B))\*((B^2\*9i)/(64\*a^6\*d^2))^(1/2)\*2i + atan((16\*a^3\*d\*tan(c + d\*x)^(1/2)\*(-B^2\*1i)/(256\*a^6\*d^2))^(1/2)/B)\*(-B^2\*1i)/(256\*a^6\*d^2))^(1/2)\*2i + ((5\*B\*tan(c + d\*x)^(1/2))/(8\*a^3\*d) + (B\*tan(c + d\*x)^(3/2)\*19i)/(12\*a^3\*d) - (9\*B\*tan(c + d\*x)^(5/2))/(8\*a^3\*d))/(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1) + ((A\*tan(c + d\*x)^(3/2))/(12\*a^3\*d) + (A\*tan(c + d\*x)^(5/2)\*1i)/(4\*a^3\*d))/(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1) - ((-1)^(1/4)\*A\*atanh((-1)^(1/4)\*tan(c + d\*x)^(1/2)))/(8\*a^3\*d) + ((-1)^(1/4)\*A\*atanh((-1)^(1/4)\*tan(c + d\*x)^(1/2)))/(8\*a^3\*d)

$$3.149 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=309

$$\frac{((1+i)A+2B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} - \frac{((1+i)A+2B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} + \dots$$

[Out]  $-1/32*((1+I)*A+2*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}-1/32*((1+I)*A+2*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}-1/64*((-1+I)*A+2*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}+1/64*((-1+I)*A+2*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}+1/6*(I*A-B)*\tan(d*x+c)^{(3/2)}/d/(a+I*a*\tan(d*x+c))^3+1/4*I*B*\tan(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^2+1/8*(A-2*I*B)*\tan(d*x+c)^{(1/2)}/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi** [A]

time = 0.42, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(2B+(1+i)A)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} - \frac{(2B+(1+i)A)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{16\sqrt{2}a^3d} + \frac{(A-2iB)\sqrt{\tan(c+dx)}}{8d(a^3+ia^3\tan(c+dx))} - \frac{(2B-(1-i)A)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{32\sqrt{2}a^3d} + \frac{(2B-(1-i)A)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{32\sqrt{2}a^3d} + \frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^2} + \frac{iB\sqrt{\tan(c+dx)}}{4a^2d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c+d*x]^{(3/2)}*(A+B*\text{Tan}[c+d*x]))/(a+I*a*\text{Tan}[c+d*x])^3,x]$

[Out]  $((((1+I)*A+2*B)*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]])/(16*\text{Sqrt}[2]*a^3*d) - (((1+I)*A+2*B)*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]])/(16*\text{Sqrt}[2]*a^3*d) - (((-1+I)*A+2*B)*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(32*\text{Sqrt}[2]*a^3*d) + (((-1+I)*A+2*B)*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(32*\text{Sqrt}[2]*a^3*d) + ((I*A-B)*\text{Tan}[c+d*x]^{(3/2)})/(6*d*(a+I*a*\text{Tan}[c+d*x])^3) + ((I/4)*B*\text{Sqrt}[\text{Tan}[c+d*x]])/(a*d*(a+I*a*\text{Tan}[c+d*x])^2) + ((A-(2*I)*B)*\text{Sqrt}[\text{Tan}[c+d*x]])/(8*d*(a^3+I*a^3*\text{Tan}[c+d*x]))$

**Rule 210**

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

**Rule 631**

$\text{Int}[(a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{Free}$

$Q\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \ :> \ \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]/b), x] \ ; \ \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \ :> \ \text{With}\{q = \text{Rt}[2(d/e), 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x] \ ; \ \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \ :> \ \text{With}\{q = \text{Rt}[-2(d/e), 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x] \ ; \ \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \ :> \ \text{With}\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x] \ ; \ \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[(-a)c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\text{Sqrt}[(b_.)\tan[(e_.) + (f_.)x]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(bc + dx^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \cdot \tan[e + fx]]], x] \ ; \ \text{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3676

$\text{Int}[\frac{(a_.) + (b_.)\tan[(e_.) + (f_.)x]^m \cdot ((A_.) + (B_.)\tan[(e_.) + (f_.)x]) \cdot ((c_.) + (d_.)\tan[(e_.) + (f_.)x])^n}{(a + b \cdot \tan[e + fx])^m \cdot (c + d \cdot \tan[e + fx])^n}, x\_Symbol] \ :> \ \text{Simp}[(-A \cdot b - a \cdot B) \cdot (a + b \cdot \tan[e + fx])^m \cdot (c + d \cdot \tan[e + fx])^n / (2 \cdot a \cdot f \cdot m), x] + \text{Dist}[1/(2 \cdot a^2 \cdot m), \text{Int}[(a + b \cdot \tan[e + fx])^{m+1} \cdot (c + d \cdot \tan[e + fx])^{n-1} \cdot \text{Simp}[A \cdot (a \cdot c \cdot m + b \cdot d \cdot n) - B \cdot (b \cdot c \cdot m + a \cdot d \cdot n) - d \cdot (b \cdot B \cdot (m - n) - a \cdot A \cdot (m + n)) \cdot \tan[e + fx], x], x], x] \ ; \ \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \ \&\& \ \text{NeQ}[bc - ad, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, 0] \ \&\& \ \text{GtQ}[n, 0]$



## Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} - \frac{\int \frac{\sqrt{\tan(c+dx)} \left(\frac{3}{2}a(iA-B) - \frac{3}{2}a(A-3iB)\right)}{(a+ia \tan(c+dx))^2} dx}{6a^2} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iB \sqrt{\tan(c+dx)}}{4ad(a+ia \tan(c+dx))^2} + \frac{\int \frac{-3ia^2E}{\sqrt{\tan(c+dx)}} dx}{6a^2} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iB \sqrt{\tan(c+dx)}}{4ad(a+ia \tan(c+dx))^2} + \frac{(A-2iE)}{8d(a^3+ia^2dx)} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iB \sqrt{\tan(c+dx)}}{4ad(a+ia \tan(c+dx))^2} + \frac{(A-2iE)}{8d(a^3+ia^2dx)} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iB \sqrt{\tan(c+dx)}}{4ad(a+ia \tan(c+dx))^2} + \frac{(A-2iE)}{8d(a^3+ia^2dx)} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iB \sqrt{\tan(c+dx)}}{4ad(a+ia \tan(c+dx))^2} + \frac{(A-2iE)}{8d(a^3+ia^2dx)} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{6d(a+ia \tan(c+dx))^3} + \frac{iB \sqrt{\tan(c+dx)}}{4ad(a+ia \tan(c+dx))^2} + \frac{(A-2iE)}{8d(a^3+ia^2dx)} \\
&\quad + \frac{((-1+i)A+2B) \log\left(1-\sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^3 d} \\
&= \frac{((1+i)A+2B) \tan^{-1}\left(1-\sqrt{2} \sqrt{\tan(c+dx)}\right)}{16\sqrt{2} a^3 d} - \frac{((1+i)A+2B) \log\left(1-\sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^3 d}
\end{aligned}$$

## Mathematica [A]

time = 2.15, size = 274, normalized size = 0.89

$$\frac{e^{-4i(c+dx)} \left( (1-2e^{2i(c+dx)} - e^{4i(c+dx)} + 2e^{6i(c+dx)}) (B - Be^{2i(c+dx)} - iA(1+2e^{2i(c+dx)})) - 3Be^{4i(c+dx)} \sqrt{-1+e^{4i(c+dx)}} \operatorname{ArcTan}\left(\sqrt{-1+e^{4i(c+dx)}}\right) + 6((A+B)e^{4i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \sqrt{1+e^{2i(c+dx)}} \tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right) \right) \operatorname{csc}(c+dx) (\cos(3(c+dx)) - i \sin(3(c+dx))) \sqrt{\tan(c+dx)}}{96a^3 d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] (((1 - 2\*E^((2\*I)\*(c + d\*x)) - E^((4\*I)\*(c + d\*x)) + 2\*E^((6\*I)\*(c + d\*x))) \* (B - B\*E^((2\*I)\*(c + d\*x)) - I\*A\*(1 + 2\*E^((2\*I)\*(c + d\*x)))) - 3\*B\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((4\*I)\*(c + d\*x))]\*ArcTan[Sqrt[-1 + E^((4\*I)\*(c + d\*x))]] + 6\*(I\*A + B)\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcTanH[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x))]])\*Csc[c + d\*x]\*(Cos[3\*(c + d\*x)] - I\*Sin[3\*(c + d\*x)])\*Sqrt[Tan[c + d\*x]])/(96\*a^3\*d\*E^((4\*I)\*(c + d\*x)))

**Maple [A]**

time = 0.13, size = 152, normalized size = 0.49

method	result
derivativedivides	$\frac{-i(-2iB+A)\left(\tan^{\frac{5}{2}}(dx+c)\right) + \left(-\frac{10A}{3} + \frac{2iB}{3}\right)\left(\tan^{\frac{3}{2}}(dx+c)\right) + iA\left(\sqrt{\tan}(dx+c)\right)}{8(\tan(dx+c)-i)^3} - \frac{B \arctan\left(\frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)} + \frac{4\left(-\frac{iA}{16}\right)}{da^3}$
default	$\frac{-i(-2iB+A)\left(\tan^{\frac{5}{2}}(dx+c)\right) + \left(-\frac{10A}{3} + \frac{2iB}{3}\right)\left(\tan^{\frac{3}{2}}(dx+c)\right) + iA\left(\sqrt{\tan}(dx+c)\right)}{8(\tan(dx+c)-i)^3} - \frac{B \arctan\left(\frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)} + \frac{4\left(-\frac{iA}{16}\right)}{da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURN VERBOSE)

[Out] 1/d/a^3\*(1/8\*(-I\*(A-2\*I\*B)\*tan(d\*x+c)^(5/2)+(-10/3\*A+2/3\*I\*B)\*tan(d\*x+c)^(3/2)+I\*A\*tan(d\*x+c)^(1/2))/(tan(d\*x+c)-I)^3-1/4\*B/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2)))+4\*(-1/16\*I\*A-1/16\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
 expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs.  $2(242) = 484$ .

time = 0.64, size = 635, normalized size = 2.06

$$\frac{\frac{1}{96} \left( 3a^3 d \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(-2((Ia^3 d e^{(2I dx + 2Ic)} + Ia^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} + I) / (e^{(2I dx + 2Ic)} + 1) \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} - (A - IB) e^{(2I dx + 2Ic)} e^{(-2I dx - 2Ic)/(IA + B)} - 3a^3 d \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(-2((-Ia^3 d e^{(2I dx + 2Ic)} - Ia^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} - (A - IB) e^{(2I dx + 2Ic)} e^{(-2I dx - 2Ic)/(IA + B)} + 24a^3 d \sqrt{-1/64 IB^2/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(1/8(8(a^3 d e^{(2I dx + 2Ic)} + a^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{-1/64 IB^2/(a^6 d^2)} + B) e^{(-2I dx - 2Ic)/(a^3 d)} - 24a^3 d \sqrt{-1/64 IB^2/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(-1/8(8(a^3 d e^{(2I dx + 2Ic)} + 2Ic) + a^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{-1/64 IB^2/(a^6 d^2)} - B) e^{(-2I dx - 2Ic)/(a^3 d)} + 2(2(A - IB) e^{(6I dx + 6Ic)} + (4A + IB) e^{(4I dx + 4Ic)} - (A - 2IB) e^{(2I dx + 2Ic)} - A - IB) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) e^{(-6I dx - 6Ic)/(a^3 d)} \right)}{a^6 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(3/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{96} (3a^3 d \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(-2((Ia^3 d e^{(2I dx + 2Ic)} + Ia^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} + I) / (e^{(2I dx + 2Ic)} + 1) \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} - (A - IB) e^{(2I dx + 2Ic)} e^{(-2I dx - 2Ic)/(IA + B)} - 3a^3 d \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(-2((-Ia^3 d e^{(2I dx + 2Ic)} - Ia^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(-Ia^2 - 2AB + IB^2)/(a^6 d^2)} - (A - IB) e^{(2I dx + 2Ic)} e^{(-2I dx - 2Ic)/(IA + B)} + 24a^3 d \sqrt{-1/64 IB^2/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(1/8(8(a^3 d e^{(2I dx + 2Ic)} + a^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{-1/64 IB^2/(a^6 d^2)} + B) e^{(-2I dx - 2Ic)/(a^3 d)} - 24a^3 d \sqrt{-1/64 IB^2/(a^6 d^2)} e^{(6I dx + 6Ic)} \log(-1/8(8(a^3 d e^{(2I dx + 2Ic)} + 2Ic) + a^3 d) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{-1/64 IB^2/(a^6 d^2)} - B) e^{(-2I dx - 2Ic)/(a^3 d)} + 2(2(A - IB) e^{(6I dx + 6Ic)} + (4A + IB) e^{(4I dx + 4Ic)} - (A - 2IB) e^{(2I dx + 2Ic)} - A - IB) \sqrt{(-Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}) e^{(-6I dx - 6Ic)/(a^3 d)}$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*(3/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))\*\*3,x)

[Out] Timed out

**Giac** [A]

time = 0.86, size = 131, normalized size = 0.42

$$\frac{(i+1) \sqrt{2} B \arctan\left(\frac{\frac{1}{2}i + \frac{1}{2}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right) + (i-1) \sqrt{2} (iA+B) \arctan\left(\frac{-\frac{1}{2}i - \frac{1}{2}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right)}{16 a^3 d} - \frac{3i A \tan(dx+c)^2 + 6 B \tan(dx+c)^2 + 10 A \tan(dx+c)^2 - 2i B \tan(dx+c)^2 - 3i A \sqrt{\tan(dx+c)}}{24 a^3 d (\tan(dx+c) - i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $-(1/16*I + 1/16)*\sqrt{2}*B*\arctan((1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)}) / (a^3*d) + (1/16*I - 1/16)*\sqrt{2}*(I*A + B)*\arctan(-(1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)}) / (a^3*d) - 1/24*(3*I*A*\tan(d*x + c)^{(5/2)} + 6*B*\tan(d*x + c)^{(5/2)} + 10*A*\tan(d*x + c)^{(3/2)} - 2*I*B*\tan(d*x + c)^{(3/2)} - 3*I*A*\sqrt{\tan(d*x + c)}) / (a^3*d*(\tan(d*x + c) - I)^3)$

**Mupad [B]**

time = 6.69, size = 239, normalized size = 0.77

$$\frac{\frac{A\sqrt{\tan(c+dx)}}{\tan^2} - \frac{A\tan(c+dx)^{3/2}}{8*d} + \frac{A\tan(c+dx)^{5/2}}{12*d}}{-\tan(c+dx)^3 \operatorname{li} - 3\tan(c+dx)^2 + \tan(c+dx) \operatorname{li} + 1} + \frac{\frac{B\tan(c+dx)^{3/2}}{12*d} + \frac{B\tan(c+dx)^{5/2}}{4*d}}{-\tan(c+dx)^3 \operatorname{li} - 3\tan(c+dx)^2 + \tan(c+dx) \operatorname{li} + 1} - \frac{(-1)^{1/4} B \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{\tan(c+dx)}}{8a^3 d}\right)}{8a^3 d} + \frac{(-1)^{1/4} B \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{\tan(c+dx)}}{8a^3 d}\right)}{8a^3 d} - \frac{\sqrt{-\frac{1}{256}} A \operatorname{atan}\left(16\sqrt{-\frac{1}{256}} \sqrt{\tan(c+dx)}\right) 2i}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out]  $((A*\tan(c + d*x)^{(1/2)})/(8*a^3*d) + (A*\tan(c + d*x)^{(3/2)}*5i)/(12*a^3*d) - (A*\tan(c + d*x)^{(5/2)})/(8*a^3*d))/(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1) + ((B*\tan(c + d*x)^{(3/2)})/(12*a^3*d) + (B*\tan(c + d*x)^{(5/2)}*1i)/(4*a^3*d))/(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1) - ((-1i/256)^{(1/2)}*A*\operatorname{atan}(16*(-1i/256)^{(1/2)}*\tan(c + d*x)^{(1/2)})*2i)/(a^3*d) - ((-1)^{(1/4)}*B*\operatorname{atan}((-1)^{(1/4)}*\tan(c + d*x)^{(1/2)}))/(8*a^3*d) + ((-1)^{(1/4)}*B*\operatorname{atanh}((-1)^{(1/4)}*\tan(c + d*x)^{(1/2)}))/(8*a^3*d)$

$$3.150 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=317

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((1+i)A+B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((1+i)A+B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d}$$

[Out]  $(-1/32-1/32*I)*((1+I)*A+B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}$   
 $)-(1/32+1/32*I)*((1+I)*A+B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}$   
 $+1/64*(2*I*A+(1-I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}$   
 $-1/64*(2*I*A+(1-I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}$   
 $+1/6*(I*A-B)*\tan(d*x+c)^{(1/2)}/d/(a+I*a*\tan(d*x+c))^3+1/12*(I*A+2*B)*\tan(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^2+1/8*B*\tan(d*x+c)^{(1/2)}/d/(a^3+I*a^3*\tan(d*x+c))$

**Rubi [A]**

time = 0.42, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3676, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) (B + (1+i)A) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} + \frac{i}{16}\right) (B + (1+i)A) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} a^3 d} + \frac{(2iA + (1-i)B) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{32\sqrt{2} a^3 d}\right)}{32\sqrt{2} a^3 d} - \frac{(2iA + (1-i)B) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{32\sqrt{2} a^3 d}\right)}{32\sqrt{2} a^3 d} + \frac{B \sqrt{\tan(c+dx)}}{8d(a^2 + ia \tan(c+dx))} + \frac{(-B + iA) \sqrt{\tan(c+dx)}}{6d(a + ia \tan(c+dx))^2} + \frac{(2B + iA) \sqrt{\tan(c+dx)}}{12ad(a + ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{\sqrt{\tan[c+dx]}(A+B \tan[c+dx])}{(a+I*a*\tan[c+dx])^3}, x\right]$

[Out]  $\left(\frac{1}{16} + \frac{I}{16}\right) ((1+I)A+B) \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[c+dx]}\right] / (\sqrt{2} a^3 d) - \left(\frac{1}{16} + \frac{I}{16}\right) ((1+I)A+B) \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[c+dx]}\right] / (\sqrt{2} a^3 d) + \left(\frac{(2I)A + (1-I)B}{32\sqrt{2} a^3 d}\right) \log\left[\frac{1 - \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx]}{32\sqrt{2} a^3 d}\right] - \left(\frac{(2I)A + (1-I)B}{32\sqrt{2} a^3 d}\right) \log\left[\frac{1 + \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx]}{32\sqrt{2} a^3 d}\right] + \frac{(I*A - B) \sqrt{\tan[c+dx]}}{(6*d*(a + I*a*\tan[c+dx])^3} + \frac{(I*A + 2*B) \sqrt{\tan[c+dx]}}{(12*a*d*(a + I*a*\tan[c+dx])^2} + \frac{(B*\sqrt{\tan[c+dx]})}{(8*d*(a^3 + I*a^3*\tan[c+dx]))}$

**Rule 210**

$\operatorname{Int}\left[\frac{(a_+ + (b_-)*(x_-)^2)^{-1}}{x\_Symbol}\right] := \operatorname{Simp}\left[\frac{-(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1}}{(x/\operatorname{Rt}[-a, 2])}\right], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \& \& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

$\operatorname{Int}\left[\frac{(a_+ + (b_-)*(x_-) + (c_-)*(x_-)^2)^{-1}}{x\_Symbol}\right] := \operatorname{With}\left[\{q = 1 - 4*S \operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}\left[-2/b, \operatorname{Subst}\left[\operatorname{Int}\left[1/(q - x^2), x\right], x, 1 + 2*c*(x/b)\right]\right]\right]$

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &

& NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} - \frac{\int \frac{\frac{1}{2}a(iA-B) - \frac{1}{2}a(5A-7iB) \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^2} dx}{6a^2} \\
 &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} + \frac{(iA+2B) \sqrt{\tan(c+dx)}}{12ad(a+ia \tan(c+dx))^2} - \frac{\int \sqrt{\tan(c+dx)}}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^2} dx \\
 &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} + \frac{(iA+2B) \sqrt{\tan(c+dx)}}{12ad(a+ia \tan(c+dx))^2} + \frac{B}{8d(a+ia \tan(c+dx))} \\
 &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} + \frac{(iA+2B) \sqrt{\tan(c+dx)}}{12ad(a+ia \tan(c+dx))^2} + \frac{B}{8d(a+ia \tan(c+dx))} \\
 &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} + \frac{(iA+2B) \sqrt{\tan(c+dx)}}{12ad(a+ia \tan(c+dx))^2} + \frac{B}{8d(a+ia \tan(c+dx))} \\
 &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} + \frac{(iA+2B) \sqrt{\tan(c+dx)}}{12ad(a+ia \tan(c+dx))^2} + \frac{B}{8d(a+ia \tan(c+dx))} \\
 &= \frac{(iA-B) \sqrt{\tan(c+dx)}}{6d(a+ia \tan(c+dx))^3} + \frac{(iA+2B) \sqrt{\tan(c+dx)}}{12ad(a+ia \tan(c+dx))^2} + \frac{B}{8d(a+ia \tan(c+dx))} \\
 &= \frac{(2iA + (1-i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{32\sqrt{2} a^3 d} \\
 &= \frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((1+i)A + B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{B}{8d(a+ia \tan(c+dx))}
 \end{aligned}$$

**Mathematica** [A]

time = 2.01, size = 272, normalized size = 0.86

$$\frac{e^{-4i(dx)} \left( (A + iB + Ae^{2i(dx)} - 2iBe^{2i(dx)}) (-1 - 2e^{2i(dx)} + e^{4i(dx)} + 2e^{6i(dx)}) - 3Ae^{6i(dx)} \sqrt{-1 + e^{4i(dx)}} \operatorname{ArcTan} \left( \frac{\sqrt{-1 + e^{4i(dx)}}}{\sqrt{1 + e^{2i(dx)}}} \right) - 6(A - iB)e^{6i(dx)} \sqrt{-1 + e^{2i(dx)}} \sqrt{1 + e^{2i(dx)}} \tanh^{-1} \left( \frac{-1 + e^{2i(dx)}}{1 + e^{2i(dx)}} \right) \right) \sec(c + dx) (\cos(3(c + dx)) - i \sin(3(c + dx)))}{96a^3 d \sqrt{\tan(c + dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3, x]

[Out] (((A + I\*B + A\*E^((2\*I)\*(c + d\*x)) - (2\*I)\*B\*E^((2\*I)\*(c + d\*x)))\*(-1 - 2\*E^((2\*I)\*(c + d\*x)) + E^((4\*I)\*(c + d\*x)) + 2\*E^((6\*I)\*(c + d\*x))) - 3\*A\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((4\*I)\*(c + d\*x))]\*ArcTan[Sqrt[-1 + E^((4\*I)\*(c + d\*x))]] - 6\*(A - I\*B)\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]])\*Sec[c + d\*x]\*(Cos[3\*(c + d\*x)] - I\*Sin[3\*(c + d\*x)])))/(96\*a^3\*d\*E^((4\*I)\*(c + d\*x))\*Sqrt[Tan[c + d\*x]])

Maple [A]

time = 0.13, size = 152, normalized size = 0.48

method	result
derivativedivides	$\frac{-iB \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( -\frac{10B}{3} - \frac{2iA}{3} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + (iB-2A) \left( \sqrt{\tan(dx+c)} \right)}{8(\tan(dx+c)-i)^3} - \frac{A \arctan \left( \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{4 \left( \sqrt{2}-i\sqrt{2} \right)} + \frac{4 \left( \frac{A}{16} - \frac{iB}{16} \right)}{da^3}$
default	$\frac{-iB \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( -\frac{10B}{3} - \frac{2iA}{3} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + (iB-2A) \left( \sqrt{\tan(dx+c)} \right)}{8(\tan(dx+c)-i)^3} - \frac{A \arctan \left( \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2}-i\sqrt{2}} \right)}{4 \left( \sqrt{2}-i\sqrt{2} \right)} + \frac{4 \left( \frac{A}{16} - \frac{iB}{16} \right)}{da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURN VERBOSE)

[Out] 1/d/a^3\*(1/8\*(-I\*B\*tan(d\*x+c)^(5/2)+(-10/3\*B-2/3\*I\*A)\*tan(d\*x+c)^(3/2)+(-2\*A+I\*B)\*tan(d\*x+c)^(1/2))/(tan(d\*x+c)-I)^3-1/4\*A/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2)))+4\*(1/16\*A-1/16\*I\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2)))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs.  $2(244) = 488$ .  
time = 0.59, size = 631, normalized size = 1.99

$$\frac{\frac{\sqrt{a^3 d \sqrt{I^2 A^2 + 2 A B - I^2 B^2}} e^{6 I d x + 6 I c} \log(2((a^3 d e^{2 I d x + 2 I c} + a^3 d) \sqrt{(-I e^{2 I d x + 2 I c} + I)/(e^{2 I d x + 2 I c} + 1)) \sqrt{(I^2 A^2 + 2 A B - I^2 B^2)/(a^6 d^2)} + (A - I B) e^{2 I d x + 2 I c}) e^{-2 I d x - 2 I c} / (I A + B)) - 3 a^3 d \sqrt{(I^2 A^2 + 2 A B - I^2 B^2)/(a^6 d^2)} e^{6 I d x + 6 I c} \log(-2((a^3 d e^{2 I d x + 2 I c} + a^3 d) \sqrt{(-I e^{2 I d x + 2 I c} + I)/(e^{2 I d x + 2 I c} + 1)) \sqrt{(I^2 A^2 + 2 A B - I^2 B^2)/(a^6 d^2)} - (A - I B) e^{2 I d x + 2 I c}) e^{-2 I d x - 2 I c} / (I A + B)) - 24 a^3 d \sqrt{-1/64 I A^2 / (a^6 d^2)} e^{6 I d x + 6 I c} \log(1/8(8(a^3 d e^{2 I d x + 2 I c} + a^3 d) \sqrt{(-I e^{2 I d x + 2 I c} + I)/(e^{2 I d x + 2 I c} + 1)) \sqrt{-1/64 I A^2 / (a^6 d^2)} + A) e^{-2 I d x - 2 I c} / (a^3 d)) + 24 a^3 d \sqrt{-1/64 I A^2 / (a^6 d^2)} e^{6 I d x + 6 I c} \log(-1/8(8(a^3 d e^{2 I d x + 2 I c} + a^3 d) \sqrt{(-I e^{2 I d x + 2 I c} + I)/(e^{2 I d x + 2 I c} + 1)) \sqrt{-1/64 I A^2 / (a^6 d^2)} - A) e^{-2 I d x - 2 I c} / (a^3 d)) + 2*(2*(-I A - 2 B) e^{6 I d x + 6 I c} - (5 I A + 4 B) e^{4 I d x + 4 I c} - (4 I A - B) e^{2 I d x + 2 I c} - I A + B) \sqrt{(-I e^{2 I d x + 2 I c} + I)/(e^{2 I d x + 2 I c} + 1))} e^{-6 I d x - 6 I c} / (a^3 d)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/96*(3*a^3*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)})*e^{(6*I*d*x + 6*I*c)}* \\ & \log(2*((a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I) / (e^{(2*I*d*x + 2*I*c)} + 1))*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)} + (A - \\ & I*B)*e^{(2*I*d*x + 2*I*c)})*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 3*a^3*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}* \\ & \log(-2*((a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I) / (e^{(2*I*d*x + 2*I*c)} + 1))*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)} - (A - \\ & I*B)*e^{(2*I*d*x + 2*I*c)})*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 24*a^3*d*\sqrt{-1/64*I*A^2/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}* \\ & \log(1/8*(8*(a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I) / (e^{(2*I*d*x + 2*I*c)} + 1))*\sqrt{-1/64*I*A^2/(a^6*d^2)} + A)*e^{(-2*I*d*x - 2*I*c)/(a^3*d)} + \\ & 24*a^3*d*\sqrt{-1/64*I*A^2/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}* \log(-1/8*(8*(a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d) \\ & *\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I) / (e^{(2*I*d*x + 2*I*c)} + 1))*\sqrt{-1/64*I*A^2/(a^6*d^2)} - A)*e^{(-2*I*d*x - 2*I*c)/(a^3*d)} + \\ & 2*(2*(-I*A - 2*B)*e^{(6*I*d*x + 6*I*c)} - (5*I*A + 4*B)*e^{(4*I*d*x + 4*I*c)} - (4*I*A - B)*e^{(2*I*d*x + 2*I*c)} - I*A + B) \\ & *\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I) / (e^{(2*I*d*x + 2*I*c)} + 1))} * e^{(-6*I*d*x - 6*I*c)/(a^3*d)} \end{aligned}$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [A]

time = 0.83, size = 131, normalized size = 0.41

$$\frac{(i+1)\sqrt{2}A\arctan\left(\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\tan(dx+c)}\right) - (i-1)\sqrt{2}(A-iB)\arctan\left(-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\tan(dx+c)}\right) - 3iB\tan(dx+c)^{\frac{3}{2}} + 2iA\tan(dx+c)^{\frac{3}{2}} + 10B\tan(dx+c)^{\frac{3}{2}} + 6A\sqrt{\tan(dx+c)} - 3iB\sqrt{\tan(dx+c)}}{16a^3d} - \frac{3iB\tan(dx+c)^{\frac{3}{2}} + 2iA\tan(dx+c)^{\frac{3}{2}} + 10B\tan(dx+c)^{\frac{3}{2}} + 6A\sqrt{\tan(dx+c)} - 3iB\sqrt{\tan(dx+c)}}{24a^3d(\tan(dx+c)-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $-(1/16*I + 1/16)*\sqrt{2}*A*\arctan((1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)}) / (a^3*d) - (1/16*I - 1/16)*\sqrt{2}*(A - I*B)*\arctan(-(1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)}) / (a^3*d) - 1/24*(3*I*B*\tan(d*x + c)^{(5/2)} + 2*I*A*\tan(d*x + c)^{(3/2)} + 10*B*\tan(d*x + c)^{(3/2)} + 6*A*\sqrt{\tan(d*x + c)} - 3*I*B*\sqrt{\tan(d*x + c)}) / (a^3*d*(\tan(d*x + c) - I)^3)$

**Mupad [B]**

time = 6.59, size = 239, normalized size = 0.75

$$\frac{\frac{B\sqrt{\tan(c+dx)}}{8a^3d} - \frac{B\tan(c+dx)^{3/2}}{8a^3d} + \frac{B\tan(c+dx)^{5/2}}{12a^3d}}{-\tan(c+dx)^3 \operatorname{li} - 3\tan(c+dx)^2 + \tan(c+dx) 3i + 1} + \frac{-\frac{A\tan(c+dx)^{3/2}}{12a^3d} + \frac{A\sqrt{\tan(c+dx)}}{4a^3d}}{-\tan(c+dx)^3 \operatorname{li} - 3\tan(c+dx)^2 + \tan(c+dx) 3i + 1} - \frac{(-1)^{1/4} A \operatorname{atan}\left((-1)^{1/4} \sqrt{\tan(c+dx)}\right)}{8a^3d} - \frac{(-1)^{1/4} A \operatorname{atanh}\left((-1)^{1/4} \sqrt{\tan(c+dx)}\right)}{8a^3d} - \frac{\sqrt{-\frac{1}{256}i} B \operatorname{atan}\left(16\sqrt{-\frac{1}{256}i} \sqrt{\tan(c+dx)}\right)}{a^3d} 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out]  $((B*\tan(c + d*x)^{(1/2)})/(8*a^3*d) + (B*\tan(c + d*x)^{(3/2)}*5i)/(12*a^3*d) - (B*\tan(c + d*x)^{(5/2)})/(8*a^3*d))/(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1) + ((A*\tan(c + d*x)^{(1/2)}*1i)/(4*a^3*d) - (A*\tan(c + d*x)^{(3/2)})/(12*a^3*d))/(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1) - ((-1)^{(1/4)}*A*\operatorname{atan}\left((-1)^{(1/4)}*\tan(c + d*x)^{(1/2)}\right))/(8*a^3*d) - ((-1)^{(1/4)}*A*\operatorname{atanh}\left((-1)^{(1/4)}*\tan(c + d*x)^{(1/2)}\right))/(8*a^3*d) - ((-1i/256)^{(1/2)}*B*\operatorname{atan}\left(16*(-1i/256)^{(1/2)}*\tan(c + d*x)^{(1/2)}\right)*2i)/(a^3*d)$

$$3.151 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=315

$$-\frac{((7-5i)A-2iB)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((7-5i)A-2iB)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d}$$

[Out] 1/32\*((7-5\*I)\*A-2\*I\*B)\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/32\*((7-5\*I)\*A-2\*I\*B)\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-1/64\*((7+5\*I)\*A-2\*I\*B)\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a^3/d\*2^(1/2)+1/64\*((7+5\*I)\*A-2\*I\*B)\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/a^3/d\*2^(1/2)+1/6\*(A+I\*B)\*tan(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))^3+1/12\*(4\*A+I\*B)\*tan(d\*x+c)^(1/2)/a/d/(a+I\*a\*tan(d\*x+c))^2+5/8\*A\*tan(d\*x+c)^(1/2)/d/(a^3+I\*a^3\*tan(d\*x+c))

**Rubi** [A]

time = 0.43, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((7-5i)A-2iB)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((7-5i)A-2iB)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{16\sqrt{2}a^3d} - \frac{((7+5i)A-2iB)\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{2}\right)}{32\sqrt{2}a^3d} + \frac{((7+5i)A-2iB)\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}{2}\right)}{32\sqrt{2}a^3d} + \frac{5A\sqrt{\tan(c+dx)}}{8d(a^2+ia^2\tan(c+dx))} - \frac{(4A+iB)\sqrt{\tan(c+dx)}}{12ad(a+ia\tan(c+dx))^2} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out] -1/16\*(((7-5\*I)\*A-(2\*I)\*B)\*ArcTan[1-Sqrt[2]\*Sqrt[Tan[c+d\*x]]])/(Sqrt[2]\*a^3\*d) + (((7-5\*I)\*A-(2\*I)\*B)\*ArcTan[1+Sqrt[2]\*Sqrt[Tan[c+d\*x]]])/(16\*Sqrt[2]\*a^3\*d) - (((7+5\*I)\*A-(2\*I)\*B)\*Log[1-Sqrt[2]\*Sqrt[Tan[c+d\*x]]+Tan[c+d\*x]])/(32\*Sqrt[2]\*a^3\*d) + (((7+5\*I)\*A-(2\*I)\*B)\*Log[1+Sqrt[2]\*Sqrt[Tan[c+d\*x]]+Tan[c+d\*x]])/(32\*Sqrt[2]\*a^3\*d) + ((A+I\*B)\*Sqrt[Tan[c+d\*x]])/(6\*d\*(a+I\*a\*Tan[c+d\*x])^3) + ((4\*A+I\*B)\*Sqrt[Tan[c+d\*x]])/(12\*a\*d\*(a+I\*a\*Tan[c+d\*x])^2) + (5\*A\*Sqrt[Tan[c+d\*x]])/(8\*d\*(a^3+I\*a^3\*Tan[c+d\*x]))

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]/b), x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2(d/e), 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2(d/e), 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[(-a)c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\text{Sqrt}[(b_.)\tan[(e_.) + (f_.)x] + (c_.) + (d_.)\tan[(e_.) + (f_.)x]}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(bc + dx^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \cdot \text{Tan}[e + fx]]], x] \ /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3677

$\text{Int}[\frac{((a_.) + (b_.)\tan[(e_.) + (f_.)x])^m \cdot ((A_.) + (B_.)\tan[(e_.) + (f_.)x]) \cdot ((c_.) + (d_.)\tan[(e_.) + (f_.)x])^n}{(a + b \cdot \text{Tan}[e + fx])^m \cdot (c + d \cdot \text{Tan}[e + fx])^{n+1} / (2fm \cdot (bc - ad))}, x] + \text{Dist}[1/(2am \cdot (bc - ad)), \text{Int}[(a + b \cdot \text{Tan}[e + fx])^{m+1} \cdot (c + d \cdot \text{Tan}[e + fx])^n \cdot \text{Simp}[A \cdot (bcm - ad(2m + n + 1)) + B \cdot (acm - bd(n + 1)) + d \cdot (Ab - aB) \cdot (m + n + 1) \cdot \text{Tan}[e + fx], x], x], x] \ /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[bc - ad, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

&& LtQ[m, 0] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} dx &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A - iB) - \frac{5}{2}a(iA - B) \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2} da}{6a^2} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{(4A + iB) \sqrt{\tan(c + dx)}}{12ad(a + ia \tan(c + dx))^2} + \frac{\int \frac{3a^2}{\sqrt{\tan(c + dx)}} da}{12ad} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{(4A + iB) \sqrt{\tan(c + dx)}}{12ad(a + ia \tan(c + dx))^2} + \frac{5}{8d} \frac{a^2}{\sqrt{\tan(c + dx)}} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{(4A + iB) \sqrt{\tan(c + dx)}}{12ad(a + ia \tan(c + dx))^2} + \frac{5}{8d} \frac{a^2}{\sqrt{\tan(c + dx)}} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{(4A + iB) \sqrt{\tan(c + dx)}}{12ad(a + ia \tan(c + dx))^2} + \frac{5}{8d} \frac{a^2}{\sqrt{\tan(c + dx)}} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{(4A + iB) \sqrt{\tan(c + dx)}}{12ad(a + ia \tan(c + dx))^2} + \frac{5}{8d} \frac{a^2}{\sqrt{\tan(c + dx)}} \\
 &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{6d(a + ia \tan(c + dx))^3} + \frac{(4A + iB) \sqrt{\tan(c + dx)}}{12ad(a + ia \tan(c + dx))^2} + \frac{5}{8d} \frac{a^2}{\sqrt{\tan(c + dx)}} \\
 &= \frac{((7 + 5i)A - 2iB) \log \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx) \right)}{32\sqrt{2} a^3 d} \\
 &= -\frac{((7 - 5i)A - 2iB) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} \right)}{16\sqrt{2} a^3 d} + \frac{((7 - 5i)A - 2iB) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} \right)}{16\sqrt{2} a^3 d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.70, size = 258, normalized size = 0.82

$$\frac{ac^2(c+dx)\cos(dx) + i\sin(dx)^2 \left( (5+7i)A + 2B \right) \text{ArcSin}(\cos(c+dx) - \sin(c+dx)) - (1+i)((1+6i)A + (1-i)B) \log(\cos(c+dx) + \sin(c+dx) + \sqrt{\tan(2(c+dx))})}{32iA\cos(c+dx) + B\sin(c+dx)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3} \frac{ac(c+dx)(\cos(3c) - \sin(3c))\sqrt{\tan(2(c+dx))} + \frac{1}{2}(\cos(3dx) - i\sin(3dx))\sin(c+dx)(6A + 3iB + 3i7A + iB)\cos(2(c+dx)) + (19A - B)\sin(2(c+dx))}{(A + B \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out] (Sec[c + d\*x]^2\*(Cos[d\*x] + I\*Sin[d\*x])^3\*(((5 + 7\*I)\*A + 2\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] - (1 + I)\*((1 + 6\*I)\*A + (1 - I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*Sec[c + d\*x]\*(I\*Cos[3\*c] - Sin[3\*c])\*Sqrt[Sin[2\*(c + d\*x)]] + (4\*(Cos[3\*d\*x] - I\*Sin[3\*d\*x])\*Sin[c + d\*x])

$$\frac{(6A + (3I)B + 3(7A + I)B)\cos[2(c + dx)] + ((19I)A - B)\sin[2(c + dx)]}{3} \cdot (A + B\tan[c + dx]) / (32d \cdot (A\cos[c + dx] + B\sin[c + dx]) \cdot \sqrt{\tan[c + dx]} \cdot (a + I \cdot a \cdot \tan[c + dx])^3)$$

**Maple [A]**

time = 0.13, size = 157, normalized size = 0.50

method	result
derivativedivides	$-\frac{5iA \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( \frac{38A}{3} + \frac{2iB}{3} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + (-9iA+2B) \left( \sqrt{\tan}(dx+c) \right)}{8(\tan(dx+c)-i)^3} - \frac{(6iA+B) \arctan\left( \frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}} \right)}{4\left(\sqrt{2}-i\sqrt{2}\right)} + \frac{1}{da^3}$
default	$-\frac{5iA \left( \tan^{\frac{5}{2}}(dx+c) \right) + \left( \frac{38A}{3} + \frac{2iB}{3} \right) \left( \tan^{\frac{3}{2}}(dx+c) \right) + (-9iA+2B) \left( \sqrt{\tan}(dx+c) \right)}{8(\tan(dx+c)-i)^3} - \frac{(6iA+B) \arctan\left( \frac{2\left(\sqrt{\tan}(dx+c)\right)}{\sqrt{2}-i\sqrt{2}} \right)}{4\left(\sqrt{2}-i\sqrt{2}\right)} + \frac{1}{da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURN VERBOSE)

[Out] 1/d/a^3\*(-1/8\*(5\*I\*A\*tan(d\*x+c)^(5/2)+(38/3\*A+2/3\*I\*B)\*tan(d\*x+c)^(3/2)+(-9\*I\*A+2\*B)\*tan(d\*x+c)^(1/2))/(tan(d\*x+c)-I)^3-1/4\*(6\*I\*A+B)/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2)))+4\*(1/16\*I\*A+1/16\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(244) = 488.

time = 0.57, size = 682, normalized size = 2.17



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/96*(3*a^3*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)} \\ & * \log(-2*((I*a^3*d*e^{(2*I*d*x + 2*I*c)} + I*a^3*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)} \\ & - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)}) - 3*a^3*d \\ & * \sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(-2*((-I*a^3*d*e^{(2*I*d*x + 2*I*c)} - I*a^3*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)} \\ & - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)}) - 3*a^3*d*\sqrt{(36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(1/8*((a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2)} + 6*I*A + B)*e^{(-2*I*d*x - 2*I*c)/(a^3*d)} + 3*a^3*d*\sqrt{(36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(-1/8*((a^3*d*e^{(2*I*d*x + 2*I*c)} + a^3*d)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2)} - 6*I*A - B)*e^{(-2*I*d*x - 2*I*c)/(a^3*d)} - 2*(2*(10*A + I*B)*e^{(6*I*d*x + 6*I*c)} + (26*A + 5*I*B)*e^{(4*I*d*x + 4*I*c)} + (7*A + 4*I*B)*e^{(2*I*d*x + 2*I*c)} + A + I*B)*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-6*I*d*x - 6*I*c)/(a^3*d)} \end{aligned}$$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real -I

Giac [A]

time = 1.36, size = 137, normalized size = 0.43

$$\frac{(i+1)\sqrt{2}(6iA+B)\arctan\left(\frac{i+1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} + \frac{(i-1)\sqrt{2}(-iA-B)\arctan\left(-\frac{i-1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} - \frac{15iA\tan(dx+c)^{\frac{3}{2}} + 38A\tan(dx+c)^{\frac{5}{2}} + 2iB\tan(dx+c)^{\frac{3}{2}} - 27iA\sqrt{\tan(dx+c)} + 6B\sqrt{\tan(dx+c)}}{24a^3d(\tan(dx+c)-i)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -(1/16*I + 1/16)*\sqrt{2}*(6*I*A + B)*\arctan((1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)})/(a^3*d) + (1/16*I - 1/16)*\sqrt{2}*(-I*A - B)*\arctan(-(1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(d*x + c)})/(a^3*d) \\ & - 1/24*(15*I*A*\tan(d*x + c)^{(5/2)} + 38*A*\tan(d*x + c)^{(3/2)} + 2*I*B*\tan(d*x + c)^{(3/2)} - 27*I*A*\sqrt{\tan(d*x + c)} + 6*B*\sqrt{\tan(d*x + c)})/(a^3*d*(\tan(d*x + c) - I)^3) \end{aligned}$$

**Mupad [B]**

time = 6.67, size = 308, normalized size = 0.98

$$\frac{\frac{A\sqrt{\tan(c+dx)}}{-\tan(c+dx)^2-3\tan(c+dx)+\tan(c+dx)^3+1} - \frac{5A\operatorname{atanh}(\sqrt{\tan(c+dx)})}{-3A} + \frac{\frac{5A\operatorname{atanh}(\sqrt{\tan(c+dx)})}{-3A} + \frac{A\sqrt{\tan(c+dx)}}{-\tan(c+dx)^2-3\tan(c+dx)+\tan(c+dx)^3+1}}{-\tan(c+dx)^2-3\tan(c+dx)+\tan(c+dx)^3+1} - \frac{(-1)^{1/4}B\operatorname{atanh}((-1)^{1/4}\sqrt{\tan(c+dx)})}{8a^2d} - \frac{(-1)^{1/4}B\operatorname{atanh}((-1)^{1/4}\sqrt{\tan(c+dx)})}{8a^2d} - \operatorname{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}}{3A}\sqrt{\frac{A^2B}{64a^2d^2}}\right)\sqrt{\frac{A^2B}{64a^2d^2}}^{2i} + \operatorname{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}}{A}\sqrt{\frac{-A^2B}{256a^2d^2}}\right)\sqrt{\frac{A^2B}{256a^2d^2}}^{2i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^3),x)

[Out] atan((16\*a^3\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(256\*a^6\*d^2))^(1/2))/A)\*(-(A^2\*1i)/(256\*a^6\*d^2))^(1/2)\*2i - atan((8\*a^3\*d\*tan(c + d\*x)^(1/2)\*((A^2\*9i)/(64\*a^6\*d^2))^(1/2))/(3\*A))\*((A^2\*9i)/(64\*a^6\*d^2))^(1/2)\*2i + ((9\*A\*tan(c + d\*x)^(1/2))/(8\*a^3\*d) + (A\*tan(c + d\*x)^(3/2)\*19i)/(12\*a^3\*d) - (5\*A\*tan(c + d\*x)^(5/2))/(8\*a^3\*d))/(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1) + ((B\*tan(c + d\*x)^(1/2)\*1i)/(4\*a^3\*d) - (B\*tan(c + d\*x)^(3/2))/(12\*a^3\*d))/(tan(c + d\*x)\*3i - 3\*tan(c + d\*x)^2 - tan(c + d\*x)^3\*1i + 1) - ((-1)^(1/4)\*B\*atan((-1)^(1/4)\*tan(c + d\*x)^(1/2)))/(8\*a^3\*d) - ((-1)^(1/4)\*B\*atanh((-1)^(1/4)\*tan(c + d\*x)^(1/2)))/(8\*a^3\*d)



$$3.152 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=364

$$\frac{((30 + 28i)A - (7 - 5i)B)\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((1 + 29i)A - (6 + i)B)\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} a^3 d}$$

[Out]  $-1/32*((30+28*I)*A+(-7+5*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+(-1/32+1/32*I)*((1+29*I)*A-(6+I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+(-1/64+1/64*I)*((29+I)*A+(1+6*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}+(1/64-1/64*I)*((29+I)*A+(1+6*I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}-5/8*(6*A+I*B)/a^3/d/\tan(d*x+c)^{(1/2)}+1/6*(A+I*B)/d/\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^3+1/12*(5*A+2*I*B)/a/d/\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^2+7/24*(4*A+I*B)/d/\tan(d*x+c)^{(1/2)}/(a^3+I*a^3*\tan(d*x+c))$

**Rubi** [A]

time = 0.53, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(30+28i)A-(7-5i)B \operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} - \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((1+29i)A-(6+i)B) \operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} - \frac{5iA+2B}{8a^3d \tan(c+dx)^{1/2}} + \frac{7iA+1B}{24a^3d \tan(c+dx)^{1/2}} + \frac{(i-1)(29+3A+(1+6i)B) \ln\left(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{2\sqrt{2}a^3d} + \frac{(i-1)(29+3A+(1+6i)B) \ln\left(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{2\sqrt{2}a^3d} + \frac{A+1B}{a^3d \tan(c+dx)^{1/2}} + \frac{5A+2B}{12a^3d \tan(c+dx)^{1/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out]  $((30 + 28i)A - (7 - 5i)B)\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]/(16*\text{Sqrt}[2]*a^3*d) - ((1/16 - I/16)*((1 + 29i)A - (6 + i)B)\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]/(\text{Sqrt}[2]*a^3*d) - ((1/32 - I/32)*((29 + i)A + (1 + 6i)B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]/(\text{Sqrt}[2]*a^3*d) + ((1/32 - I/32)*((29 + i)A + (1 + 6i)B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]/(\text{Sqrt}[2]*a^3*d) - (5*(6*A + I*B))/(8*a^3*d*\text{Sqrt}[\text{Tan}[c + d*x]]) + (A + I*B)/(6*d*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^3) + (5*A + (2*I)*B)/(12*a*d*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^2) + (7*(4*A + I*B))/(24*d*\text{Sqrt}[\text{Tan}[c + d*x]]*(a^3 + I*a^3*\text{Tan}[c + d*x]))$

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

NeQ[c^2 + d^2, 0]

Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} dx &= \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A + iB) - \frac{7}{2}a(iA - B)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} dx}{6a^2} \\
 &= \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} + \frac{5A + iB}{12ad \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} \\
 &= \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} + \frac{5A + iB}{12ad \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} \\
 &= -\frac{5(6A + iB)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} \\
 &= -\frac{5(6A + iB)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} \\
 &= -\frac{5(6A + iB)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} \\
 &= -\frac{5(6A + iB)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} \\
 &= -\frac{((30 - 28i)A + (7 + 5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^3d} \\
 &= \frac{((30 + 28i)A - (7 - 5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{16\sqrt{2} a^3d} - \frac{((30 - 28i)A + (7 + 5i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^3d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.84, size = 278, normalized size = 0.76

$\frac{a^2(c+dx)\cos(dx) + i\sin(dx)^2 \left( \frac{1}{3}\cos(3dx) - i\sin(3dx) \right) (49A + 19B)\cos(c+dx) - (145A + 19B)\cos(3(c+dx)) + 6(-19A + 2B + 7(-7A + B)\cos(2(c+dx)))\sin(c+dx) + \left( (29 - 30A) + (5 + 7B)\text{ArcSin}(\cos(c+dx) - \sin(c+dx)) - (1 + i)(29 + 3A + (1 + 6i)B)\log(\cos(c+dx) + \sin(c+dx) + \sqrt{\sin(2(c+dx))}) \right) \cos(c+dx) + \cos(3c) - \sin(3c) + \sqrt{\sin(2(c+dx))}}{32iA\cos(c+dx) + 32B\sin(c+dx) + 32i\sin(c+dx)(a + i\sqrt{\tan(c+dx)})^2}$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3),x]

[Out] (Sec[c + d\*x]^2\*(Cos[d\*x] + I\*Sin[d\*x])^3\*((2\*(Cos[3\*d\*x] - I\*Sin[3\*d\*x])\*(49\*A + (19\*I)\*B)\*Cos[c + d\*x] - (145\*A + (19\*I)\*B)\*Cos[3\*(c + d\*x)] + 6\*((-19\*I)\*A + 2\*B + 7\*((-7\*I)\*A + B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x])/3 + (((28 - 30\*I)\*A + (5 + 7\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] - (1 + I)\*((29 + I)\*A + (1 + 6\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*Sec[c + d\*x]\*(I\*Cos[3\*c] - Sin[3\*c])\*Sqrt[Sin[2\*(c + d\*x)]]\*(A + B\*Tan[c + d\*x]))/(32\*d\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3)

**Maple [A]**

time = 0.11, size = 177, normalized size = 0.49

method	result
derivativedivides	$-\frac{\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{i(14iA-5B)\left(\tan^{\frac{5}{2}}(dx+c)\right) + \left(\frac{98iA}{3} - \frac{38B}{3}\right)\left(\tan^{\frac{3}{2}}(dx+c)\right) + (9iB+20A)\left(\sqrt{\tan(dx+c)}\right)}{8(\tan(dx+c)-i)^3} \frac{(6iB+29A)}{da^3}$
default	$-\frac{\frac{2A}{\sqrt{\tan(dx+c)}} + \frac{i(14iA-5B)\left(\tan^{\frac{5}{2}}(dx+c)\right) + \left(\frac{98iA}{3} - \frac{38B}{3}\right)\left(\tan^{\frac{3}{2}}(dx+c)\right) + (9iB+20A)\left(\sqrt{\tan(dx+c)}\right)}{8(\tan(dx+c)-i)^3} \frac{(6iB+29A)}{da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^3,x,method=\_RETURN VERBOSE)

[Out] 1/d/a^3\*(-2\*A/tan(d\*x+c)^(1/2)+1/8\*(I\*(14\*I\*A-5\*B)\*tan(d\*x+c)^(5/2)+(98/3\*I\*A-38/3\*B)\*tan(d\*x+c)^(3/2)+(20\*A+9\*I\*B)\*tan(d\*x+c)^(1/2))/(tan(d\*x+c)-I)^3 -1/4\*(6\*I\*B+29\*A)/(2^(1/2)-I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)-I\*2^(1/2)))+4\*(-1/16\*A+1/16\*I\*B)/(2^(1/2)+I\*2^(1/2))\*arctan(2\*tan(d\*x+c)^(1/2)/(2^(1/2)+I\*2^(1/2)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 784 vs.  $2(273) = 546$ .

time = 0.67, size = 784, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*log(2*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*log(-2*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B) + 3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2))*log(1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)) + 29*A + 6*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2))*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)) - 29*A - 6*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d) - 2*(2*(73*I*A - 10*B)*e^(8*I*d*x + 8*I*c) + 3*(35*I*A - 2*B)*e^(6*I*d*x + 6*I*c) - (49*I*A - 19*B)*e^(4*I*d*x + 4*I*c) + 3*(-3*I*A + 2*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real -I

**Giac [A]**

time = 1.71, size = 167, normalized size = 0.46

$$\frac{(i+1)\sqrt{2}(29A+6iB)\arctan\left(\frac{1}{2}\left(i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} - \frac{(i+1)\sqrt{2}(iA+B)\arctan\left(\frac{1}{2}\left(i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} - \frac{2A}{a^3d\sqrt{\tan(dx+c)}} - \frac{42iA\tan(dx+c)^{\frac{3}{2}} - 15B\tan(dx+c)^{\frac{3}{2}} + 98A\tan(dx+c)^{\frac{3}{2}} + 38iB\tan(dx+c)^{\frac{3}{2}} - 60iA\sqrt{\tan(dx+c)} + 27B\sqrt{\tan(dx+c)}}{24a^3d(-1-\tan(dx+c)-1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $-(1/16*I + 1/16)*\sqrt{2}*(29*A + 6*I*B)*\arctan((1/2*I + 1/2)*\sqrt{2}*\sqrt{\tan(dx+c)})/(a^3*d) - (1/16*I + 1/16)*\sqrt{2}*(I*A + B)*\arctan((1/2*I - 1/2)*\sqrt{2}*\sqrt{\tan(dx+c)})/(a^3*d) - 2*A/(a^3*d*\sqrt{\tan(dx+c)}) - 1/24*(42*I*A*\tan(dx+c)^{(5/2)} - 15*B*\tan(dx+c)^{(5/2)} + 98*A*\tan(dx+c)^{(3/2)} + 38*I*B*\tan(dx+c)^{(3/2)} - 60*I*A*\sqrt{\tan(dx+c)} + 27*B*\sqrt{\tan(dx+c)})/(a^3*d*(-I*\tan(dx+c) - 1)^3)$

**Mupad [B]**

time = 6.83, size = 389, normalized size = 1.07

$$2\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{41i}{256a^6d^2}}}{A}\right)\sqrt{\frac{41i}{256a^6d^2}} + 2\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{841i}{256a^6d^2}}}{29A}\right)\sqrt{\frac{841i}{256a^6d^2}} - \operatorname{atan}\left(\frac{8a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{9i}{64a^6d^2}}}{3B}\right)\sqrt{\frac{9i}{64a^6d^2}} + \operatorname{atan}\left(\frac{16a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{9i}{256a^6d^2}}}{B}\right)\sqrt{\frac{9i}{256a^6d^2}} - \frac{2A}{\sqrt{\tan(c+dx)} + \tan(c+dx)^2} - \frac{42iA + 38iB\tan(c+dx)^{\frac{3}{2}} - 60iA\sqrt{\tan(c+dx)} + 27B\sqrt{\tan(c+dx)}}{\sqrt{\tan(c+dx)} + \tan(c+dx)^2 - 3\tan(c+dx)^2 - \tan(c+dx)^3} + \frac{12A\sqrt{\tan(c+dx)} + 27B\sqrt{\tan(c+dx)}}{-\tan(c+dx)^2 - 3\tan(c+dx) + \tan(c+dx)^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*i)^3),x)

[Out]  $2*\operatorname{atanh}((16*a^3*d*\tan(c + d*x)^{(1/2)}*((A^2*i)/(256*a^6*d^2))^{(1/2)})/A)*((A^2*i)/(256*a^6*d^2))^{(1/2)} + 2*\operatorname{atanh}((16*a^3*d*\tan(c + d*x)^{(1/2)}*(-A^2*841i)/(256*a^6*d^2))^{(1/2)})/(29*A))*(-(A^2*841i)/(256*a^6*d^2))^{(1/2)} - \operatorname{atan}((8*a^3*d*\tan(c + d*x)^{(1/2)}*((B^2*9i)/(64*a^6*d^2))^{(1/2)})/(3*B))*((B^2*9i)/(64*a^6*d^2))^{(1/2)}*2i + \operatorname{atan}((16*a^3*d*\tan(c + d*x)^{(1/2)}*(-B^2*1i)/(256*a^6*d^2))^{(1/2)})/B)*(-(B^2*1i)/(256*a^6*d^2))^{(1/2)}*2i - ((2*A)/(a^3*d) + (A*\tan(c + d*x)*17i)/(2*a^3*d) - (121*A*\tan(c + d*x)^2)/(12*a^3*d) - (A*\tan(c + d*x)^3*15i)/(4*a^3*d))/(\tan(c + d*x)^{(1/2)} + \tan(c + d*x)^{(3/2)}*3i - 3*\tan(c + d*x)^{(5/2)} - \tan(c + d*x)^{(7/2)}*1i) + ((9*B*\tan(c + d*x)^{(1/2)})/(8*a^3*d) + (B*\tan(c + d*x)^{(3/2)}*19i)/(12*a^3*d) - (5*B*\tan(c + d*x)^{(5/2)})/(8*a^3*d))/(\tan(c + d*x)*3i - 3*\tan(c + d*x)^2 - \tan(c + d*x)^3*1i + 1)$

$$3.153 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=393

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((76+i)A + (1+29i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((76+i)A + (1+29i)B)}{\sqrt{2} a^3 d}$$

[Out]  $(-1/32+1/32*I)*((76+I)*A+(1+29*I)*B)*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+(-1/32+1/32*I)*((76+I)*A+(1+29*I)*B)*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+1/64*((77+75*I)*A+(-30+28*I)*B)*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}+(-1/64+1/64*I)*((1+76*I)*A-(29+I)*B)*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^3/d*2^{(1/2)}+15/8*(5*I*A-2*B)/a^3/d/\tan(d*x+c)^{(1/2)}-7/24*(11*A+4*I*B)/a^3/d/\tan(d*x+c)^{(3/2)}+1/6*(A+I*B)/d/\tan(d*x+c)^{(3/2)}/(a+I*a*\tan(d*x+c))^3+1/4*(2*A+I*B)/a/d/\tan(d*x+c)^{(3/2)}/(a+I*a*\tan(d*x+c))^2+3/8*(5*A+2*I*B)/d/\tan(d*x+c)^{(3/2)}/(a^3+I*a^3*\tan(d*x+c))$

**Rubi** [A]

time = 0.57, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((76+i)A + (1+29i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} a^3 d} - \frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((76+i)A + (1+29i)B)}{\sqrt{2} a^3 d} + \frac{35A + 20B}{8 \sqrt{2} a^3 d \sqrt{\tan(c+dx)}} - \frac{311A + 68B}{256 \sqrt{2} a^3 d \sqrt{\tan(c+dx)}} - \frac{35(-29+54A)}{8 \sqrt{2} a^3 d \sqrt{\tan(c+dx)}} - \frac{(77+75IA - (30-28IB) \log(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1))}{32 \sqrt{2} a^3 d} - \frac{(1-i) \left( (1+76IA - (29+IB) \log(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1)) \right)}{\sqrt{2} a^3 d} + \frac{2A+IB}{8 \sqrt{2} a^3 d \sqrt{\tan(c+dx)}} + \frac{A+IB}{8 \sqrt{2} a^3 d \sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out]  $((1/16 - I/16)*((76 + I)*A + (1 + 29*I)*B)*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^3*d) - ((1/16 - I/16)*((76 + I)*A + (1 + 29*I)*B)*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*a^3*d) + (((77 + 75*I)*A - (30 - 28*I)*B)*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(32*\operatorname{Sqrt}[2]*a^3*d) - ((1/32 - I/32)*((1 + 76*I)*A - (29 + I)*B)*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(32*\operatorname{Sqrt}[2]*a^3*d) - (7*(11*A + (4*I)*B))/(24*a^3*d*\operatorname{Tan}[c + d*x]^(3/2)) + (15*((5*I)*A - 2*B))/(8*a^3*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) + (A + I*B)/(6*d*\operatorname{Tan}[c + d*x]^(3/2)*(a + I*a*\operatorname{Tan}[c + d*x])^3) + (2*A + I*B)/(4*a*d*\operatorname{Tan}[c + d*x]^(3/2)*(a + I*a*\operatorname{Tan}[c + d*x])^2) + (3*(5*A + (2*I)*B))/(8*d*\operatorname{Tan}[c + d*x]^(3/2)*(a^3 + I*a^3*\operatorname{Tan}[c + d*x]))$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```



NeQ[c^2 + d^2, 0]

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^3} dx &= \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} + \frac{\int \frac{\frac{3}{2}a(5A+iB) - \frac{9}{2}a(iA-B) \tan(c + dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^3} dx}{6a^2} \\
&= \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} + \frac{2A + iB}{4ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} + \frac{2A + iB}{4ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= -\frac{7(11A + 4iB)}{24a^3d \tan^{\frac{3}{2}}(c + dx)} + \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} + \frac{A + iB}{4ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= -\frac{7(11A + 4iB)}{24a^3d \tan^{\frac{3}{2}}(c + dx)} + \frac{15(5iA - 2B)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= -\frac{7(11A + 4iB)}{24a^3d \tan^{\frac{3}{2}}(c + dx)} + \frac{15(5iA - 2B)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= -\frac{7(11A + 4iB)}{24a^3d \tan^{\frac{3}{2}}(c + dx)} + \frac{15(5iA - 2B)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= -\frac{7(11A + 4iB)}{24a^3d \tan^{\frac{3}{2}}(c + dx)} + \frac{15(5iA - 2B)}{8a^3d \sqrt{\tan(c + dx)}} + \frac{A + iB}{6d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} \\
&= \frac{((77 + 75i)A - (30 - 28i)B) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{32\sqrt{2} a^3d} \\
&= \frac{\left(\frac{1}{16} - \frac{i}{16}\right) ((76 + i)A + (1 + 29i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} a^3d}
\end{aligned}$$

**Mathematica [A]**

time = 2.30, size = 306, normalized size = 0.78

$\frac{a^2(c+dx)\cos(dx) + \sin(dx)^2 \left( (1-i) \left( (29+3A+(1+29i)B)\operatorname{ArcSinh}(\cos(c+dx) - \sin(c+dx)) + (-1-76iA+(29+13i)B)\log(\cos(c+dx) + \sin(c+dx) + \sqrt{2}\sqrt{\tan(c+dx)}) \right) \sin(c+dx) \cos(3c) + \frac{15(5iA-2B)}{8a^3d} \sqrt{\tan(c+dx)} - \frac{15(5iA-2B)(30i-33B-23iA+90iB)\cos(2c+dx)}{8a^3d} + (30i+147iB)\cos(c+dx) - 30iA\sin(2c+dx) + 34iB\sin(c+dx) + 347iA\sin(c+dx) - 147iB\sin(c+dx) \right) (A+iB)\sin(c+dx)}{32iA\sin(c+dx) + B\sin(c+dx) \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^3}$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out] (Sec[c + d\*x]^2\*(Cos[d\*x] + I\*Sin[d\*x])^3\*((1 - I)\*(((76 + I)\*A + (1 + 29\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] + ((-1 - 76\*I)\*A + (29 + I)\*B)\*Log

$$[\text{Cos}[c + d*x] + \text{Sin}[c + d*x] + \text{Sqrt}[\text{Sin}[2*(c + d*x)]]] * \text{Sec}[c + d*x] * (\text{Cos}[3*c] + I * \text{Sin}[3*c]) * \text{Sqrt}[\text{Sin}[2*(c + d*x)]] + (\text{Csc}[c + d*x] * (\text{Cos}[3*d*x] - I * \text{Sin}[3*d*x]) * (69*A + (33*I)*B - 2*(241*A + (90*I)*B) * \text{Cos}[2*(c + d*x)] + (349*A + (147*I)*B) * \text{Cos}[4*(c + d*x)] - (502*I)*A * \text{Sin}[2*(c + d*x)] + 194*B * \text{Sin}[2*(c + d*x)] + (347*I)*A * \text{Sin}[4*(c + d*x)] - 145*B * \text{Sin}[4*(c + d*x)])) / (32*d * (A * \text{Cos}[c + d*x] + B * \text{Sin}[c + d*x]) * \text{Sqrt}[\text{Tan}[c + d*x]] * (a + I*a * \text{Tan}[c + d*x])^3)$$

**Maple [A]**

time = 0.11, size = 196, normalized size = 0.50

method	result
derivativedivides	$\frac{\frac{2(-3iA+B)}{\sqrt{\tan(dx+c)}} - \frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}}}{8} \cdot \frac{i \frac{i(27iA-14B) \left(\tan^{\frac{5}{2}}(dx+c)\right) + \left(\frac{182iA}{3} - \frac{98B}{3}\right) \left(\tan^{\frac{3}{2}}(dx+c)\right) + (20iB+35A) \left(\sqrt{\tan(dx+c)}\right)}{(\tan(dx+c)-i)^3}}{da^3}$
default	$\frac{\frac{2(-3iA+B)}{\sqrt{\tan(dx+c)}} - \frac{2A}{3 \tan(dx+c)^{\frac{3}{2}}}}{8} \cdot \frac{i \frac{i(27iA-14B) \left(\tan^{\frac{5}{2}}(dx+c)\right) + \left(\frac{182iA}{3} - \frac{98B}{3}\right) \left(\tan^{\frac{3}{2}}(dx+c)\right) + (20iB+35A) \left(\sqrt{\tan(dx+c)}\right)}{(\tan(dx+c)-i)^3}}{da^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out]  $1/d/a^3 * (-2 * (-3*I*A+B) / \tan(d*x+c)^{(1/2)} - 2/3*A / \tan(d*x+c)^{(3/2)} - 1/8*I * ((I * (27*I*A-14*B) * \tan(d*x+c)^{(5/2)} + (182/3*I*A-98/3*B) * \tan(d*x+c)^{(3/2)} + (35*A+20*I*B) * \tan(d*x+c)^{(1/2)}) / (\tan(d*x+c)-I)^3 - 2 * (29*I*B+76*A) / (2^{(1/2)}-I*2^{(1/2)}) * \arctan(2*\tan(d*x+c)^{(1/2)} / (2^{(1/2)}-I*2^{(1/2)}))) + 4 * (-1/16*I*A-1/16*B) / (2^{(1/2)}+I*2^{(1/2)}) * \arctan(2*\tan(d*x+c)^{(1/2)} / (2^{(1/2)}+I*2^{(1/2)})))$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
 expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 875 vs.  $2(296) = 592$ .  
 time = 0.56, size = 875, normalized size = 2.23

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")`

[Out] 
$$\frac{1}{96} \left( 3(a^3 d e^{(10 I d x + 10 I c)} - 2 a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^6 d^2)} \log(-2((I a^3 d e^{(2 I d x + 2 I c)} + I a^3 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^6 d^2)} - (A - I B) e^{(2 I d x + 2 I c)}) e^{(-2 I d x - 2 I c)/(I A + B)} - 3(a^3 d e^{(10 I d x + 10 I c)} - 2 a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^6 d^2)} \log(-2((-I a^3 d e^{(2 I d x + 2 I c)} - I a^3 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(-I A^2 - 2 A B + I B^2)/(a^6 d^2)} - (A - I B) e^{(2 I d x + 2 I c)}) e^{(-2 I d x - 2 I c)/(I A + B)} - 3(a^3 d e^{(10 I d x + 10 I c)} - 2 a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \sqrt{(5776 I A^2 - 4408 A B - 841 I B^2)/(a^6 d^2)} \log(-1/8((a^3 d e^{(2 I d x + 2 I c)} + a^3 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(5776 I A^2 - 4408 A B - 841 I B^2)/(a^6 d^2)} + 76 I A - 29 B) e^{(-2 I d x - 2 I c)/(a^3 d)} + 3(a^3 d e^{(10 I d x + 10 I c)} - 2 a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \sqrt{(5776 I A^2 - 4408 A B - 841 I B^2)/(a^6 d^2)} \log(1/8((a^3 d e^{(2 I d x + 2 I c)} + a^3 d) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(5776 I A^2 - 4408 A B - 841 I B^2)/(a^6 d^2)} - 76 I A + 29 B) e^{(-2 I d x - 2 I c)/(a^3 d)} - 2(2(174 A + 73 I B) e^{(10 I d x + 10 I c)} - (144 A + 41 I B) e^{(8 I d x + 8 I c)} - (423 A + 154 I B) e^{(6 I d x + 6 I c)} + (79 A + 40 I B) e^{(4 I d x + 4 I c)} + (11 A + 8 I B) e^{(2 I d x + 2 I c)} + A + I B) \sqrt{(-I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} + 1)}) / (a^3 d e^{(10 I d x + 10 I c)} - 2 a^3 d e^{(8 I d x + 8 I c)} + a^3 d e^{(6 I d x + 6 I c)}) \right)$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**3,x)`

[Out] Timed out

**Giac** [A]

time = 0.80, size = 182, normalized size = 0.46

$$\frac{(i-1)\sqrt{2}(iA+B)\arctan\left(\frac{-\frac{1}{2}i-1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} - \frac{(i-1)\sqrt{2}(76A+29iB)\arctan\left(-\frac{1}{2}i+\frac{1}{2}\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} - \frac{225A\tan(dx+c)^4+90iB\tan(dx+c)^4-598iA\tan(dx+c)^3+242B\tan(dx+c)^3-489A\tan(dx+c)^2-204iB\tan(dx+c)^2+96iA\tan(dx+c)-48B\tan(dx+c)-16A}{2i(-i\tan(dx+c)^2-\sqrt{\tan(dx+c)})^2}a^3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] (1/16\*I - 1/16)\*sqrt(2)\*(I\*A + B)\*arctan(-(1/2\*I - 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^3\*d) - (1/16\*I - 1/16)\*sqrt(2)\*(76\*A + 29\*I\*B)\*arctan(-(1/2\*I + 1/2)\*sqrt(2)\*sqrt(tan(d\*x + c)))/(a^3\*d) - 1/24\*(225\*A\*tan(d\*x + c)^4 + 90\*I\*B\*tan(d\*x + c)^4 - 598\*I\*A\*tan(d\*x + c)^3 + 242\*B\*tan(d\*x + c)^3 - 489\*A\*tan(d\*x + c)^2 - 204\*I\*B\*tan(d\*x + c)^2 + 96\*I\*A\*tan(d\*x + c) - 48\*B\*tan(d\*x + c) - 16\*A)/((-I\*tan(d\*x + c)^(3/2) - sqrt(tan(d\*x + c)))^3\*a^3\*d)

**Mupad** [B]

time = 8.53, size = 425, normalized size = 1.08

$$-\frac{\left(\frac{16a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{256a^6d^2}}}{A}\right)\sqrt{\frac{A^2B}{256a^6d^2}}}{2} + \frac{\left(\frac{4a^3d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2B}{256a^6d^2}}}{2A}\right)\sqrt{\frac{A^2B}{256a^6d^2}}}{2} - \frac{24b + 4ab\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right) - 4ab\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right) + 2\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right)}{\tan(c+dx)^{3/2} - 3\tan(c+dx)^{5/2} + \tan(c+dx)^{7/2} - \tan(c+dx)^{9/2}} + \frac{\left(\frac{A^2B}{256a^6d^2}\right)\sqrt{\frac{A^2B}{256a^6d^2}}}{2} + 2\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right)\sqrt{\frac{A^2B}{256a^6d^2}} - \frac{24b + 4ab\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right) - 4ab\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right) + 2\operatorname{atanh}\left(\frac{16a^3d\sqrt{\tan(c+dx)}}{256a^6d^2}\right)}{\sqrt{\tan(c+dx) + \tan(c+dx)^3 - 3\tan(c+dx)^5 - \tan(c+dx)^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)^3),x)

[Out] atan((4\*a^3\*d\*tan(c + d\*x)^(1/2)\*((A^2\*361i)/(16\*a^6\*d^2))^(1/2))/(19\*A))\*((A^2\*361i)/(16\*a^6\*d^2))^(1/2)\*2i - atan((16\*a^3\*d\*tan(c + d\*x)^(1/2)\*(-(A^2\*1i)/(256\*a^6\*d^2))^(1/2))/A)\*(-(A^2\*1i)/(256\*a^6\*d^2))^(1/2)\*2i - ((A\*2i)/(3\*a^3\*d) + (4\*A\*tan(c + d\*x))/(a^3\*d) + (A\*tan(c + d\*x)^2\*163i)/(8\*a^3\*d) - (299\*A\*tan(c + d\*x)^3)/(12\*a^3\*d) - (A\*tan(c + d\*x)^4\*75i)/(8\*a^3\*d))/(tan(c + d\*x)^(3/2)\*1i - 3\*tan(c + d\*x)^(5/2) - tan(c + d\*x)^(7/2)\*3i + tan(c + d\*x)^(9/2)) + 2\*atanh((16\*a^3\*d\*tan(c + d\*x)^(1/2)\*((B^2\*1i)/(256\*a^6\*d^2))^(1/2))/B)\*((B^2\*1i)/(256\*a^6\*d^2))^(1/2) + 2\*atanh((16\*a^3\*d\*tan(c + d\*x)^(1/2)\*(-(B^2\*841i)/(256\*a^6\*d^2))^(1/2))/(29\*B))\*(-(B^2\*841i)/(256\*a^6\*d^2))^(1/2) - ((2\*B)/(a^3\*d) + (B\*tan(c + d\*x)\*17i)/(2\*a^3\*d) - (121\*B\*tan(c + d\*x)^2)/(12\*a^3\*d) - (B\*tan(c + d\*x)^3\*15i)/(4\*a^3\*d))/(tan(c + d\*x)^(1/2) + tan(c + d\*x)^(3/2)\*3i - 3\*tan(c + d\*x)^(5/2) - tan(c + d\*x)^(7/2)\*1i)

$$3.154 \quad \int \tan^{\frac{3}{2}}(c+dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=200

$$\frac{(-1)^{3/4} \sqrt{a} (4iA + 7B) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{4d} + \frac{(1 + i) \sqrt{a} (iA + B) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}$$

[Out]  $\frac{1}{4}(-1)^{3/4}(4IA+7B)\operatorname{arctan}\left(\frac{(-1)^{3/4}a^{1/2}\tan(dx+c)^{1/2}}{a+Ia\tan(dx+c)^{1/2}}\right)+\frac{1}{d}(1+I)(IA+B)\operatorname{arctanh}\left(\frac{(1+I)a^{1/2}\tan(dx+c)^{1/2}}{a+Ia\tan(dx+c)^{1/2}}\right)+\frac{1}{d+1/4}(4A-I B)\tan(dx+c)^{1/2}(a+Ia\tan(dx+c)^{1/2})^{1/2}+\frac{1}{d+1/2}B(a+Ia\tan(dx+c)^{1/2})\tan(dx+c)^{3/2}/d$

**Rubi [A]**

time = 0.45, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} \sqrt{a} (7B + 4iA) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{4d} + \frac{(4A - iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{4d} + \frac{(1 + i) \sqrt{a} (B + iA) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} + \frac{B \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c + d*x]^{3/2} \operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((-1)^{3/4} \operatorname{Sqrt}[a] * ((4*I)*A + 7*B) \operatorname{ArcTan}[\frac{(-1)^{3/4} \operatorname{Sqrt}[a] \operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}]) / (4*d) + ((1 + I) \operatorname{Sqrt}[a] * (I*A + B) \operatorname{ArcTanh}[\frac{(1 + I) \operatorname{Sqrt}[a] \operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}]) / d + ((4*A - I*B) \operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] \operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]) / (4*d) + (B*\operatorname{Tan}[c + d*x]^{3/2} \operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]) / (2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_)^m)*((c_.) + (d_.)*(x_)^n), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p*(m+1)-1}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{1/p}], x]] /;$   $\operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
\int \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx &= \frac{B \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{2d} + \frac{\int \sqrt{a+ia \tan(c+dx)} dx}{2d} \\
&= \frac{(4A-iB) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} \\
&= \frac{(4A-iB) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} \\
&= \frac{(4A-iB) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} \\
&= \frac{(1+i) \sqrt{a} (iA+B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d} \\
&= \frac{(1+i) \sqrt{a} (iA+B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d} \\
&= \frac{\sqrt[4]{-1} \sqrt{a} (4A-7iB) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{4d}
\end{aligned}$$

**Mathematica [F]**

time = 4.66, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]), x]

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 837 vs.  $2(159) = 318$ .

time = 0.30, size = 838, normalized size = 4.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c)),x,method=\_RE  
TURNVERBOSE)



```
[Out] -1/8/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*(4*I*A*(I*a)^(1/2)*ln(-
-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)-4*I*B*(I*a)^(1/2)*ln(-(-2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(
tan(d*x+c)+I))*2^(1/2)*a-6*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(
1/2)*(-I*a)^(1/2)*tan(d*x+c)-7*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*
x+c)+4*B*(I*a)^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)+4*B*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2))*(-I*a)^(1/2)*tan(d*x+c)^2-
8*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2))*(-I*a)^(1/2)-4*I*A*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2
)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+4*A*(I*a)^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*2^(1/2)*a+8*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2))*(-I*a)^(1
/2)*tan(d*x+c)+4*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)
)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-2*B*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2))*(-I*a)^(1/2)-7*B*ln(1/2*(2*I*a*ta
n(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2)
))*(-I*a)^(1/2)*a/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-tan(d
*x+c)+I)/(-I*a)^(1/2)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(3/2
), x)
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs.  $2(149) = 298$ .

time = 0.48, size = 771, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")
```

```
[Out] -1/8*(4*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a
/d^2)*log((I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d*x + I*c)
```

```

+ sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e
^(-I*d*x - I*c)/(I*A + B)) - 4*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I
*A^2 + 2*A*B - I*B^2)*a/d^2)*log((-I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B - I*B^2
)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 2*sqrt(2)*((4*A - 3*I*
B)*e^(3*I*d*x + 3*I*c) + (4*A + I*B)*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) -
(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-16*I*A^2 - 56*A*B + 49*I*B^2)*a/d^2)*lo
g((sqrt(2)*((4*I*A + 7*B)*e^(2*I*d*x + 2*I*c) + 4*I*A + 7*B)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1)) + 2*I*d*sqrt((-16*I*A^2 - 56*A*B + 49*I*B^2)*a/d^2)*e^(I*d*x + I*c))*
e^(-I*d*x - I*c)/(4*I*A + 7*B)) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-16*I*A
^2 - 56*A*B + 49*I*B^2)*a/d^2)*log((sqrt(2)*((4*I*A + 7*B)*e^(2*I*d*x + 2*I
*c) + 4*I*A + 7*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*d*sqrt((-16*I*A^2 - 56*A*B + 4
9*I*B^2)*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(4*I*A + 7*B)))/(d*e^(2*I
*d*x + 2*I*c) + d)

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c+dx) - i)} (A + B \tan(c+dx)) \tan^{\frac{3}{2}}(c+dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*tan(d*x+c)**(3/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*tan(c + d*x)**(3
/2), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);OUTPUT:Evaluation time: 0.41Unable to divide
, perhaps due to rounding error%%{%{%%{%{poly1[-8,0]:[1,0,-2]%%},[0]%%
%},0):
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

### 3.155 $\int \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=152

$$\frac{(-1)^{3/4} \sqrt{a} (2A - iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{(1+i) \sqrt{a} (A - iB) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}$$

[Out]  $-(-1)^{(3/4)}*(2*A-I*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}/d-(1+I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}/d+B*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.32, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} \sqrt{a} (2A - iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{(1+i) \sqrt{a} (A - iB) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{B \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $-((( -1)^{(3/4)}*\sqrt{a}*(2*A - I*B)*\operatorname{ArcTan}[((-1)^{(3/4)}*\sqrt{a}*\sqrt{\tan[c + d*x]})/\sqrt{a + I*a*\tan[c + d*x]})]/d) - ((1 + I)*\sqrt{a}*(A - I*B)*\operatorname{ArcTanh}[(1 + I)*\sqrt{a}*\sqrt{\tan[c + d*x]})/\sqrt{a + I*a*\tan[c + d*x]})/d + (B*\sqrt{\tan[c + d*x]}*\sqrt{a + I*a*\tan[c + d*x]})/d$

**Rule 65**

`Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

**Rule 209**

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} + \\
&= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} + \\
&= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} - \\
&= -\frac{(1+i)\sqrt{a} (A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} \\
&= -\frac{(1+i)\sqrt{a} (A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt[4]{-1} \sqrt{a} (2iA+B) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 560 vs.  $2(152) = 304$ .  
time = 2.70, size = 560, normalized size = 3.68

$$\frac{B \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} + \frac{B \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} - \frac{(1+i)\sqrt{a} (A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{(1+i)\sqrt{a} (A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{\sqrt[4]{-1} \sqrt{a} (2iA+B) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-1/4*(\text{Sqrt}[E^{(I*(c + d*x))}]/(1 + E^{((2*I)*(c + d*x))}))*\text{Sqrt}[((-I)*(-1 + E^{((2*I)*(c + d*x))})/(1 + E^{((2*I)*(c + d*x))}))*(-8*B*E^{(I*(c + d*x))}*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}] + 8*(I*A + B)*(1 + E^{((2*I)*(c + d*x))}))*\text{Log}[E^{(I*(c + d*x))} + \text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] - I*\text{Sqrt}[2]*(2*A - I*B)*(1 + E^{((2*I)*(c + d*x))}))*\text{Log}[1 - 3*E^{((2*I)*(c + d*x))} - 2*\text{Sqrt}[2]*E^{(I*(c + d*x))}*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] + (2*I)*\text{Sqrt}[2]*A*\text{Log}[1 - 3*E^{((2*I)*(c + d*x))} + 2*\text{Sqrt}[2]*E^{(I*(c + d*x))}*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] + \text{Sqrt}[2]*B*\text{Log}[1 - 3*E^{((2*I)*(c + d*x))} + 2*\text{Sqrt}[2]*E^{(I*(c + d*x))}*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] + (2*I)*\text{Sqrt}[2]*A*E^{((2*I)*(c + d*x))}*\text{Log}[1 - 3*E^{((2*I)*(c + d*x))} + 2*\text{Sqrt}[2]*E^{(I*(c + d*x))}*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] + \text{Sqr}$

$$t[2]*B*E^{((2*I)*(c+d*x))*\text{Log}[1-3*E^{((2*I)*(c+d*x))}+2*\text{Sqrt}[2]*E^{(I*(c+d*x))*\text{Sqrt}[-1+E^{((2*I)*(c+d*x))}]]]*\text{Sqrt}[a+I*a*\text{Tan}[c+d*x]]]/(\text{Sqrt}[2]*d*E^{(I*(c+d*x))*\text{Sqrt}[-1+E^{((2*I)*(c+d*x))}]}*\text{Sqrt}[\text{Sec}[c+d*x]])$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 712 vs.  $2(123) = 246$ .

time = 0.12, size = 713, normalized size = 4.69

method	result
derivativedivides	$\frac{\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{ia}\sqrt{2}\ln\left(\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}}{\tan(dx+c)}\right)\right)}{\dots}$
default	$\frac{\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{ia}\sqrt{2}\ln\left(\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}}{\tan(dx+c)}\right)\right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/2/d*\tan(d*x+c)^{(1/2)}*(a*(1+I*\tan(d*x+c)))^{(1/2)}*(I*B*(I*a)^{(1/2)}*2^{(1/2)} \\ & * \ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}+I*a-3*a \\ & * \tan(d*x+c))/(\tan(d*x+c)+I)*a*\tan(d*x+c)+2*I*A*\ln(1/2*(2*I*a*\tan(d*x+c)+2* \\ & (a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)} \\ & *a*\tan(d*x+c)+I*A*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan \\ & (d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)*a-A*(I \\ & *a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c) \\ & ))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)*a*\tan(d*x+c)-I*B*\ln(1/2*(2*I* \\ & a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)} \\ & *(-I*a)^{(1/2)}*a-2*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d \\ & *x+c)))^{(1/2)}+B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))) \\ & ^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\tan(d*x+c)+B*(I*a)^{(1/2)}* \\ & 2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+ \\ & I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)*a+2*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d \\ & *x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)+2*A*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a* \\ & \tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)} \\ & *a)/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}/(I*a)^{(1/2)}/(-\tan(d*x+c)+I)/(-I*a \\ & )^{(1/2)} \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(I\*a\*tan(d\*x + c) + a)\*sqrt(tan(d\*x + c)), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(116) = 232.  
time = 0.52, size = 673, normalized size = 4.43



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(2\*sqrt(2)\*B\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*e^(I\*d\*x + I\*c) - sqrt(2)\*d\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*log((sqrt(2)\*d\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*e^(I\*d\*x + I\*c) + sqrt(2)\*((I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A + B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))\*e^(-I\*d\*x - I\*c)/(I\*A + B)) + sqrt(2)\*d\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*log(-sqrt(2)\*d\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*e^(I\*d\*x + I\*c) - sqrt(2)\*((I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A + B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))\*e^(-I\*d\*x - I\*c)/(I\*A + B)) + d\*sqrt((4\*I\*A^2 + 4\*A\*B - I\*B^2)\*a/d^2)\*log((sqrt(2)\*((2\*I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + 2\*I\*A + B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)) + 2\*d\*sqrt((4\*I\*A^2 + 4\*A\*B - I\*B^2)\*a/d^2)\*e^(I\*d\*x + I\*c))\*e^(-I\*d\*x - I\*c)/(2\*I\*A + B)) - d\*sqrt((4\*I\*A^2 + 4\*A\*B - I\*B^2)\*a/d^2)\*log((sqrt(2)\*((2\*I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + 2\*I\*A + B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)) - 2\*d\*sqrt((4\*I\*A^2 + 4\*A\*B - I\*B^2)\*a/d^2)\*e^(I\*d\*x + I\*c))\*e^(-I\*d\*x - I\*c)/(2\*I\*A + B))/d

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c+dx) - i)} (A + B \tan(c+dx)) \sqrt{\tan(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x)), x)



**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to rounding error%%{%%{[%%{%%{poly1[-2*i,0]:[1,0,-2]%%},[0]%%},0]:[1,0,%%{-1,[1]%%}}
```

**Mupad [B]**

time = 24.35, size = 2225, normalized size = 14.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] - ((B*tan(c + d*x)^(3/2)*2i)/(d*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)))^3) + (2*B*tan(c + d*x)^(1/2))/(a*d*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)))/((tan(c + d*x)^2/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^4 - 1/a^2 + (tan(c + d*x)*2i)/(a*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) - ((-a)^(1/2)*atan((A^4*(-a)^(21/2)*tan(c + d*x)^(1/2)*(7168 - 7168i))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))*(A^4*a^10*3584i + B^4*a^10*512i - 4096*A*B^3*a^10 + 10240*A^3*B*a^10 - A^2*B^2*a^10*10240i - (3584*A^4*a^11*tan(c + d*x)))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (512*B^4*a^11*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (10240*A^2*B^2*a^11*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (A*B^3*a^11*tan(c + d*x)*4096i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (A^3*B*a^11*tan(c + d*x)*10240i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) + (B^4*(-a)^(21/2))*tan(c + d*x)^(1/2)*(1024 - 1024i)/(((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))*(A^4*a^10*3584i + B^4*a^10*512i - 4096*A*B^3*a^10 + 10240*A^3*B*a^10 - A^2*B^2*a^10*10240i - (3584*A^4*a^11*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (512*B^4*a^11*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (10240*A^2*B^2*a^11*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (A*B^3*a^11*tan(c + d*x)*4096i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (A^3*B*a^11*tan(c + d*x)*10240i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) + (A*B^3*(-a)^(21/2)*tan(c + d*x)^(1/2)*(8192 + 8192i))/(((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))*(A^4*a^10*3584i + B^4*a^10*512i - 4096*A*B^3*a^10 + 10240*A^3*B*a^10 - A^2*B^2*a^10*10240i -
```

$$\begin{aligned}
& (3584*A^4*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 - \\
& (512*B^4*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + \\
& (10240*A^2*B^2*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 - \\
& (A*B^3*a^{11}*\tan(c + d*x)*4096i)/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + \\
& (A^3*B*a^{11}*\tan(c + d*x)*10240i)/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2)) - \\
& (A^3*B*(-a)^{(21/2)}*\tan(c + d*x)^{(1/2)}*(20480 + 20480i))/(((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*(A^4*a^{10}*3584i + B^4*a^{10}*512i - 4096*A*B^3*a^{10} + 10240*A^3*B*a^{10} - A^2*B^2*a^{10}*10240i - (3584*A^4*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 - (512*B^4*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + (10240*A^2*B^2*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 - (A*B^3*a^{11}*\tan(c + d*x)*4096i)/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + (A^3*B*a^{11}*\tan(c + d*x)*10240i)/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2)) - (A^2*B^2*(-a)^{(21/2)}*\tan(c + d*x)^{(1/2)}*(20480 - 20480i))/(((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*(A^4*a^{10}*3584i + B^4*a^{10}*512i - 4096*A*B^3*a^{10} + 10240*A^3*B*a^{10} - A^2*B^2*a^{10}*10240i - (3584*A^4*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 - (512*B^4*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + (10240*A^2*B^2*a^{11}*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 - (A*B^3*a^{11}*\tan(c + d*x)*4096i)/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + (A^3*B*a^{11}*\tan(c + d*x)*10240i)/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2)))*(A*1i + B)*(1 - 1i))/d - ((-1)^{(1/4)}*a^{(1/2)}*atan((-1)^{(1/4)}*A^5*\tan(c + d*x)^{(1/2)}*25690112i)/(a^{(15/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*((25690112*A^5)/a^8 - (B^5*262144i)/a^8 + (3670016*A*B^4)/a^8 - (A^4*B*56885248i)/a^8 + (A^2*B^3*19398656i)/a^8 - (48234496*A^3*B^2)/a^8)) + (262144*(-1)^{(1/4)}*B^5*\tan(c + d*x)^{(1/2)})/(a^{(15/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*((25690112*A^5)/a^8 - (B^5*262144i)/a^8 + (3670016*A*B^4)/a^8 - (A^4*B*56885248i)/a^8 + (A^2*B^3*19398656i)/a^8 - (48234496*A^3*B^2)/a^8)) + ((-1)^{(1/4)}*A*B^4*\tan(c + d*x)^{(1/2)}*3670016i)/(a^{(15/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*((25690112*A^5)/a^8 - (B^5*262144i)/a^8 + (3670016*A*B^4)/a^8 - (A^4*B*56885248i)/a^8 + (A^2*B^3*19398656i)/a^8 - (48234496*A^3*B^2)/a^8)) + (56885248*(-1)^{(1/4)}*A^4*B*\tan(c + d*x)^{(1/2)})/(a^{(15/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*((25690112*A^5)/a^8 - (B^5*262144i)/a^8 + (3670016*A*B^4)/a^8 - (A^4*B*56885248i)/a^8 + (A^2*B^3*19398656i)/a^8 - (48234496*A^3*B^2)/a^8)) - (19398656*(-1)^{(1/4)}*A^2*B^3*\tan(c + d*x)^{(1/2)})/(a^{(15/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*((25690112*A^5)/a^8 - (B^5*262144i)/a^8 + (3670016*A*B^4)/a^8 - (A^4*B*56885248i)/a^8 + (A^2*B^3*19398656i)/a^8 - (48234496*A^3*B^2)/a^8)) - ((-1)^{(1/4)}*A^3*B^2*\tan(c + d*x)^{(1/2)}*48234496i)/(a^{(15/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})*((25690112*A^5)/a^8 - (B^5*262144i)/a^8 + (3670016*A*B^4)/a^8 - (A^4*B*56885248i)/a^8 + (A^2*B^3*19398656i)/a^8 - (48234496*A^3*B^2)/a^8)))*(2*A - B*1i)*2i)/d
\end{aligned}$$

$$3.156 \quad \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Optimal. Leaf size=112

$$\frac{2(-1)^{3/4} \sqrt{a} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} - \frac{(1 + i) \sqrt{a} (iA + B) \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}$$

[Out]  $-2*(-1)^{(3/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}/d-(1+I)*(I*A+B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}/d$

Rubi [A]

time = 0.21, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$ , Rules used = {3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(1 + i) \sqrt{a} (B + iA) \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} - \frac{2(-1)^{3/4} \sqrt{a} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]], x]$

[Out]  $(-2*(-1)^{(3/4)}*\operatorname{Sqrt}[a]*B*\operatorname{ArcTan}[((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d - ((1 + I)*\operatorname{Sqrt}[a]*(I*A + B)*\operatorname{ArcTanh}[((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= - \left( (-A + iB) \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx \right) + \frac{(iB) \int \dots}{\dots} \\
&= \frac{(iaB) \text{Subst} \left( \int \frac{1}{\sqrt{x} \sqrt{a + iax}} dx, x, \tan(c + dx) \right)}{d} \\
&= - \frac{(1 + i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= - \frac{(1 + i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= - \frac{2(-1)^{3/4} \sqrt{a} B \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 238 vs.  $2(112) = 224$ .  
time = 2.32, size = 238, normalized size = 2.12

$$\frac{\sqrt{-\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}} \cos(c + dx) \left( 4(A - iB) \log \left( e^{i(c+dx)} + \sqrt{-1 + e^{2i(c+dx)}} \right) + i\sqrt{2} B \left( \log \left( 1 - 3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \right) - \log \left( 1 - 3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \right) \right) \right) \sqrt{a + ia \tan(c + dx)}}{2d \sqrt{-1 + e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

[Out] (Sqrt[((-1)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))]\*Cos[c + d\*x]\*(4\*(A - I\*B)\*Log[E^(I\*(c + d\*x)) + Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] + I\*Sqrt[2]\*B\*(Log[1 - 3\*E^((2\*I)\*(c + d\*x)) - 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - Log[1 - 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]))\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(2\*d\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 501 vs.  $2(89) = 178$ .  
time = 0.12, size = 502, normalized size = 4.48

method	result
--------	--------

derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^a \left(iA\sqrt{ia}\sqrt{2}\ln\left(\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}}{\tan(dx+c)}\right)\right)}{\tan(dx+c)}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^a \left(iA\sqrt{ia}\sqrt{2}\ln\left(\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}}{\tan(dx+c)}\right)\right)}{\tan(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RE  
TURNVERBOSE)`

[Out]  $\frac{1}{2}d*(a*(1+I*\tan(d*x+c)))^{1/2}*\tan(d*x+c)^{1/2}*a*(I*A*(I*a)^{1/2}*2^{1/2})*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*\tan(d*x+c)-2*I*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*(2*\tan(d*x+c)-I*B*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))+B*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*\tan(d*x+c)+A*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))-2*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*(2*\tan(d*x+c)-I*B*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)))/(-\tan(d*x+c)+I)/(-I*a)^{1/2}$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)/sqrt(tan(d*x + c)), x)`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs.  $2(84) = 168$ .

time = 0.46, size = 539, normalized size = 4.81

$$\frac{1}{2}d*(a*(1+I*\tan(d*x+c)))^{1/2}*\tan(d*x+c)^{1/2}*a*(I*A*(I*a)^{1/2}*2^{1/2})*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*\tan(d*x+c)-2*I*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*(2*\tan(d*x+c)-I*B*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))+B*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*\tan(d*x+c)+A*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))-2*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*(2*\tan(d*x+c)-I*B*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2})*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)))/(-\tan(d*x+c)+I)/(-I*a)^{1/2}$$

Verification of antiderivative is not currently implemented for this CAS.



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*1i)^{(1/2)})/\tan(c + d*x)^{(1/2)}, x)$

[Out]  $(B*a^{(1/2)}*\log((a^{(1/2)}*\tan(c + d*x)^{(1/2)}*(2 - 2i))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})) - (a*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + 1i*(1/2 + 1i/2))/d + (2^{(1/2)}*B*a^{(1/2)}*\log(2^{(1/2)}*(1 - 1i) + (2*a^{(1/2)}*\tan(c + d*x)^{(1/2)}))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)}))*(1 + 1i))/d - ((1i/2)^{(1/2)}*B*a^{(1/2)}*\log((2*(-1)^{(3/4)}*2^{(1/2)}*a^{(1/2)}*\tan(c + d*x)^{(1/2)}))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)}) - (a*\tan(c + d*x))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})^2 + 1i))/d - (4i^{(1/2)}*B*a^{(1/2)}*\log((-1)^{(3/4)} + (a^{(1/2)}*\tan(c + d*x)^{(1/2)}))/((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)}))/d + (2*(1i/2)^{(1/2)}*A*(-a)^{(1/2)}*\operatorname{atanh}((2*(1i/2)^{(1/2)}*(-a)^{(1/2)}*\tan(c + d*x)^{(1/2)}*((a + a*\tan(c + d*x)*1i)^{(1/2)} - a^{(1/2)})))/(a*\tan(c + d*x) - a*1i + a^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)*1i}))/d$



$$3.157 \quad \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

**Optimal.** Leaf size=90

$$\frac{(1+i)\sqrt{a} (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} - \frac{2A\sqrt{a + ia \tan(c + dx)}}{d\sqrt{\tan(c + dx)}}$$

[Out] (1+I)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*a^(1/2)/d-2\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)

**Rubi** [A]

time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ ,

Rules used = {3679, 12, 3625, 211}

$$\frac{(1+i)\sqrt{a} (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} - \frac{2A\sqrt{a + ia \tan(c + dx)}}{d\sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2),x]

[Out] (((1 + I)\*Sqrt[a]\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])]/d - (2\*A\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*Sqrt[Tan[c + d\*x]]))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

## Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]

```

## Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{2 \int \frac{a(iA+B) \sqrt{a + ia \tan(c + dx)}}{2 \sqrt{\tan(c + dx)}}}{a} \\
&= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + (iA + B) \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} \\
&= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{(2a^2(A - iB)) \text{Subst}\left(\int \frac{1}{\sqrt{-ia}}}{\sqrt{-ia}}\right)}{d} \\
&= \frac{(1 + i) \sqrt{a} (A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}
\end{aligned}$$

**Mathematica** [A]

time = 3.37, size = 156, normalized size = 1.73

$$\frac{(A - iB)e^{-i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}}} - \frac{2A \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3
/2),x]

```

```

[Out] ((A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 +
E^((2*I)*(c + d*x))]]*Sqrt[a + I*a*Tan[c + d*x]]/(d*E^(I*(c + d*x))*Sqrt[
((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))] - (2*A*Sqrt[a
+ I*a*Tan[c + d*x]))/(d*Sqrt[Tan[c + d*x]])

```



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{2} \sqrt{2} (d e^{2 I d x + 2 I c} - d) \sqrt{-(-I A^2 - 2 A B + I B^2) a / d^2} \log((\sqrt{2} d \sqrt{-(-I A^2 - 2 A B + I B^2) a / d^2} e^{(I d x + I c)} + \sqrt{2} ((I A + B) e^{2 I d x + 2 I c} + I A + B) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(-I e^{2 I d x + 2 I c} + I) / (e^{(2 I d x + 2 I c)} + 1)}) e^{(-I d x - I c) / (I A + B)} - \sqrt{2} (d e^{2 I d x + 2 I c} - d) \sqrt{-(-I A^2 - 2 A B + I B^2) a / d^2} \log(-(\sqrt{2} d \sqrt{-(-I A^2 - 2 A B + I B^2) a / d^2} e^{(I d x + I c)} - \sqrt{2} ((I A + B) e^{2 I d x + 2 I c} + I A + B) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(-I e^{2 I d x + 2 I c} + I) / (e^{(2 I d x + 2 I c)} + 1)}) e^{(-I d x - I c) / (I A + B)} - 4 \sqrt{2} (I A e^{3 I d x + 3 I c} + I A e^{(I d x + I c)}) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)}) / (d e^{(2 I d x + 2 I c)} - d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia(\tan(c+dx)-i)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))/tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li}}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(3/2), x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(3/2), x)
```

$$3.158 \quad \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Optimal. Leaf size=135

$$\frac{(1+i)\sqrt{a} (iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia \tan(c+dx)}}{3d \tan^{\frac{3}{2}}(c+dx)} - \frac{2(iA+3B)\sqrt{a+ia \tan(c+dx)}}{3d \sqrt{\tan(c+dx)}}$$

[Out] (1+I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*a^(1/2)/d-2/3\*(I\*A+3\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/3\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3679, 12, 3625, 211}

$$-\frac{2(3B+iA)\sqrt{a+ia \tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} + \frac{(1+i)\sqrt{a} (B+iA) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia \tan(c+dx)}}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] ((1 + I)\*Sqrt[a]\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/d - (2\*A\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d\*Tan[c + d\*x]^(3/2)) - (2\*(I\*A + 3\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d\*Sqrt[Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && Ne

$Q[c^2 + d^2, 0]$

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2 \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{\frac{3}{2}}(c + dx)} dx}{3} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(iA + 3B) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(iA + 3B) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(iA + 3B) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= \frac{(1 + i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \end{aligned}$$

### Mathematica [A]

time = 3.60, size = 174, normalized size = 1.29

$$\frac{(iA + B)e^{-i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{-\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}}} - \frac{2(iA + 3B + A \cot(c + dx)) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out]  $((I*A + B)*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}])*\text{ArcTanh}[E^{(I*(c + d*x))}/\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]]*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]/(d*E^{(I*(c + d*x))}*\text{Sqrt}[( -I)*(-1 + E^{((2*I)*(c + d*x))})]/(1 + E^{((2*I)*(c + d*x))})) - (2*(I*A + 3*B + A*\text{Cot}[c + d*x])*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(3*d*\text{Sqrt}[\text{Tan}[c + d*x]])$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs.  $2(110) = 220$ .  
time = 0.12, size = 553, normalized size = 4.10

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} \left( 3iA\sqrt{2} \ln\left( -\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))}{\tan(dx+c)+i} \right) \right)}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))} \left( 3iA\sqrt{2} \ln\left( -\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))}{\tan(dx+c)+i} \right) \right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/6/d*(a*(1+I*\text{tan}(d*x+c)))^{1/2}/\text{tan}(d*x+c)^{3/2}*(3*I*A^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c)))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^3-12*B*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)^2-3*I*B*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c)))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^2+3*B*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c)))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^3-8*A*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)-4*I*A*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)^2+3*A*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c)))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^2+12*I*B*(-I*a)^{1/2}*\text{tan}(d*x+c)*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+4*I*A*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2})/(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}/(-\text{tan}(d*x+c)+I)/(-I*a)^{1/2} \end{aligned}$$

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,algorithm="maxima")`





[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/tan(c + d\*x)^(5/2), x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/tan(c + d\*x)^(5/2), x)

$$3.159 \quad \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

**Optimal.** Leaf size=178

$$\frac{(1+i)\sqrt{a}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia\tan(c+dx)}}{5d \tan^{\frac{5}{2}}(c+dx)} - \frac{2(iA+5B)\sqrt{a+ia\tan(c+dx)}}{15d \tan^{\frac{3}{2}}(c+dx)}$$

[Out]  $(-1-I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}/d+2/15*(13*A-5*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/5*A*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(5/2)}-2/15*(I*A+5*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.36, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3679, 12, 3625, 211}

$$-\frac{2(5B+iA)\sqrt{a+ia\tan(c+dx)}}{15d \tan^{\frac{3}{2}}(c+dx)} + \frac{2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{15d \sqrt{\tan(c+dx)}} - \frac{(1+i)\sqrt{a}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia\tan(c+dx)}}{5d \tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/\operatorname{Tan}[c + d*x]^{(7/2)}, x]$

[Out]  $((-1 - I)*\operatorname{Sqrt}[a]*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]))/d - (2*A*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(5*d*\operatorname{Tan}[c + d*x]^{(5/2)}) - (2*(I*A + 5*B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d*\operatorname{Tan}[c + d*x]^{(3/2)}) + (2*(13*A - (5*I)*B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

**Rule 211**

$\operatorname{Int}[(a_*) + (b_*)(x_)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{PosQ}[a/b]$

**Rule 3625**

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\operatorname{tan}[(e_*) + (f_*)(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

$\wedge 2 * x^2), x], x, \text{Sqrt}[c + d * \text{Tan}[e + f * x]] / \text{Sqrt}[a + b * \text{Tan}[e + f * x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3679

$\text{Int}[\{(a\_.) + (b\_.) * \text{tan}[(e\_.) + (f\_.) * (x\_.)]\}^m * \{(A\_.) + (B\_.) * \text{tan}[(e\_.) + (f\_.) * (x\_.)]\} * \{(c\_.) + (d\_.) * \text{tan}[(e\_.) + (f\_.) * (x\_.)]\}^n, x\_Symbol] :> \text{Simp}[(A * d - B * c) * (a + b * \text{Tan}[e + f * x])^m * (c + d * \text{Tan}[e + f * x])^{n + 1} / (f * (n + 1) * (c^2 + d^2)), x] - \text{Dist}[1 / (a * (n + 1) * (c^2 + d^2)), \text{Int}[(a + b * \text{Tan}[e + f * x])^m * (c + d * \text{Tan}[e + f * x])^{n + 1} * \text{Simp}[A * (b * d * m - a * c * (n + 1)) - B * (b * c * m + a * d * (n + 1)) - a * (B * c - A * d) * (m + n + 1) * \text{Tan}[e + f * x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2 \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{\frac{5}{2}}(c + dx)} dx}{5a} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\ &= -\frac{(1 + i) \sqrt{a} (A - iB) \tanh^{-1} \left( \frac{(1 + i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \end{aligned}$$

### Mathematica [A]

time = 3.85, size = 211, normalized size = 1.19

$$\frac{(A - iB) e^{-i(c + dx)} \sqrt{-1 + e^{2i(c + dx)}} \tanh^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{-1 + e^{2i(c + dx)}}} \right) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\frac{i(-1 + e^{2i(c + dx)})}{1 + e^{2i(c + dx)}}}} - \frac{\csc^2(c + dx) (-10A + 5iB + (16A - 5iB) \cos(2(c + dx)) + (iA + 5B) \sin(2(c + dx))) \sqrt{a + ia \tan(c + dx)}}{15d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2),x]

[Out] -(((A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*E^(I\*(c + d\*x))\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))])) - (Csc[c + d\*x]^2\*(-10\*A + (5\*I)\*B + (16\*A - (5\*I)\*B)\*Cos[2\*(c + d\*x)] + (I\*A + 5\*B)\*Sin[2\*(c + d\*x)])\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(15\*d\*Sqrt[Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs.  $2(145) = 290$ .  
time = 0.12, size = 630, normalized size = 3.54

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}}{\dots} \left( 15iB\sqrt{2} \ln\left( -\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))}{\tan(dx+c)+i} \right) \right)$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}}{\dots} \left( 15iB\sqrt{2} \ln\left( -\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))}{\tan(dx+c)+i} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x,method=\_RETURNVERBOSE)

[Out] -1/30/d\*(a\*(1+I\*tan(d\*x+c)))^(1/2)\*(15\*I\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^4+15\*I\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^3-15\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^4-20\*I\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^3+15\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^3-56\*I\*A\*(-I\*a)^(1/2)\*tan(d\*x+c)^2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+52\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^3+20\*I\*B\*(-I\*a)^(1/2)\*tan(d\*x+c)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)-40\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^2+12\*I\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)-16\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)/(-tan(d\*x+c)+I)/(-I\*a)^(1/2)

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(135) = 270.  
time = 1.57, size = 543, normalized size = 3.05

$$\frac{\sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a/d^2} \log(\sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a/d^2} e^{(I dx + Ic)} + \sqrt{2} (IA + B) e^{(2I dx + 2Ic)} + IA + B) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}}{e^{(-I dx - Ic)}/(IA + B)} - 15 \sqrt{2} (d e^{(6I dx + 6Ic)} - 3 d e^{(4I dx + 4Ic)} + 3 d e^{(2I dx + 2Ic)} - d) \sqrt{-(-IA^2 - 2AB + IB^2)a/d^2} \log(-\sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a/d^2} e^{(I dx + Ic)} - \sqrt{2} (IA + B) e^{(2I dx + 2Ic)} + IA + B) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}}{e^{(-I dx - Ic)}/(IA + B)} + 4 \sqrt{2} ((-17IA - 10B) e^{(7I dx + 7Ic)} + 3IA e^{(5I dx + 5Ic)} + 5(IA + 2B) e^{(3I dx + 3Ic)} - 15IA e^{(I dx + Ic)}) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)} \sqrt{(-I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} + 1)}}{(d e^{(6I dx + 6Ic)} - 3 d e^{(4I dx + 4Ic)} + 3 d e^{(2I dx + 2Ic)} - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*log((sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-I*d*x - I*c)/(I*A + B) - 15*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*log(-sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))))*e^(-I*d*x - I*c)/(I*A + B) + 4*sqrt(2)*((-17*I*A - 10*B)*e^(7*I*d*x + 7*I*c) + 3*I*A*e^(5*I*d*x + 5*I*c) + 5*(I*A + 2*B)*e^(3*I*d*x + 3*I*c) - 15*I*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li}}{\tan(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(7/2)
,x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(7/2)
, x)
```

$$3.160 \quad \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx$$

Optimal. Leaf size=221

$$\frac{(1-i)\sqrt{a}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{2(iA+7B)\sqrt{a+ia\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)}$$

[Out] (1-I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*a^(1/2)/d+2/105\*(43\*I\*A+91\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/7\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2)-2/35\*(I\*A+7\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)+2/105\*(31\*A-7\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.46, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3679, 12, 3625, 211}

$$\frac{2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(7B+iA)\sqrt{a+ia\tan(c+dx)}}{35d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(91B+43iA)\sqrt{a+ia\tan(c+dx)}}{105d\sqrt{\tan(c+dx)}} + \frac{(1-i)\sqrt{a}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] ((1 - I)\*Sqrt[a]\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/d - (2\*A\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(7\*d\*Tan[c + d\*x]^(7/2)) - (2\*(I\*A + 7\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(35\*d\*Tan[c + d\*x]^(5/2)) + (2\*(31\*A - (7\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(3/2)) + (2\*((43\*I)\*A + 91\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Sqrt[Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a



$^2*x^2), x], x, \text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3679

$\text{Int}[\{(a\_.) + (b\_.)*\text{tan}[(e\_.) + (f\_.)*(x\_)]\}^{(m\_)}*\{(A\_.) + (B\_.)*\text{tan}[(e\_.) + (f\_.)*(x\_)]\}^{(n\_)}, x\_Symbol] \rightarrow \text{Simp}[(A*d - B*c)*(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n+1}/(f*(n+1)*(c^2 + d^2)), x] - \text{Dist}[1/(a*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n+1}*\text{Simp}[A*(b*d*m - a*c*(n+1)) - B*(b*c*m + a*d*(n+1)) - a*(B*c - A*d)*(m+n+1)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} + \frac{2 \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{\frac{7}{2}}(c + dx)} dx}{7} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{\frac{5}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{\frac{5}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{\frac{5}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{\frac{5}{2}}(c + dx)} \\ &= -\frac{2A \sqrt{a + ia \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{\frac{5}{2}}(c + dx)} \\ &= -\frac{(1+i)\sqrt{a} (iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \end{aligned}$$

**Mathematica** [A]

time = 5.73, size = 239, normalized size = 1.08

$$\frac{i(A-iB)e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}} - \frac{\csc^3(c+dx)(7(2A+iB)\cos(c+dx)+(46A-7iB)\cos(3(c+dx))+4(-20iA-35B+(23iA+56B)\cos(2(c+dx)))\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{210d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] ((-I)\*(A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*E^(I\*(c + d\*x))\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))] - (Csc[c + d\*x]^3\*(7\*(2\*A + I\*B)\*Cos[c + d\*x] + (46\*A - (7\*I)\*B)\*Cos[3\*(c + d\*x)] + 4\*((-20\*I)\*A - 35\*B + ((23\*I)\*A + 56\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(210\*d\*Sqrt[Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(180) = 360.

time = 0.12, size = 707, normalized size = 3.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2), x, method=\_RETURNVERBOSE)

[Out] 1/210/d\*(a\*(1+I\*tan(d\*x+c)))^(1/2)/tan(d\*x+c)^(7/2)\*(-172\*I\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^4-364\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^4+392\*I\*B\*(-I\*a)^(1/2)\*tan(d\*x+c)^3\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+105\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^5-296\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^3-60\*I\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+105\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^4+112\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^2+105\*I\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^5+72\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)+136\*I\*A\*(-I\*a)^(1/2)\*tan(d\*x+c)^2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)-105\*I\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^4-84\*I\*B\*(-I\*a)^(1/2)\*tan(d\*x+c)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2))/(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)/(-tan(d\*x+c)+I)/(-I\*a)^(1/2)

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(168) = 336.  
time = 0.91, size = 593, normalized size = 2.68

$$\frac{\sqrt{2} \left( \frac{1}{2} (105 \sqrt{2} (d e^{8 I d x + 8 I c} - 4 d e^{6 I d x + 6 I c} + 6 d e^{4 I d x + 4 I c} - 4 d e^{2 I d x + 2 I c} + d) \sqrt{-(I A^2 + 2 A B - I B^2) a / d^2} \log((I \sqrt{2} d \sqrt{-(I A^2 + 2 A B - I B^2) a / d^2} e^{I d x + I c} + \sqrt{2} ((I A + B) e^{2 I d x + 2 I c} + I A + B) \sqrt{a / (e^{2 I d x + 2 I c} + 1)}) \sqrt{(-I e^{2 I d x + 2 I c} + I) / (e^{2 I d x + 2 I c} + 1)}) e^{-I d x - I c} / (I A + B) - 105 \sqrt{2} (d e^{8 I d x + 8 I c} - 4 d e^{6 I d x + 6 I c} + 6 d e^{4 I d x + 4 I c} - 4 d e^{2 I d x + 2 I c} + d) \sqrt{-(I A^2 + 2 A B - I B^2) a / d^2} \log((-I \sqrt{2} d \sqrt{-(I A^2 + 2 A B - I B^2) a / d^2} e^{I d x + I c} + \sqrt{2} ((I A + B) e^{2 I d x + 2 I c} + I A + B) \sqrt{a / (e^{2 I d x + 2 I c} + 1)}) \sqrt{(-I e^{2 I d x + 2 I c} + I) / (e^{2 I d x + 2 I c} + 1)}) e^{-I d x - I c} / (I A + B) - 4 \sqrt{2} ((92 A - 119 I B) e^{9 I d x + 9 I c} - 20 (A - 7 I B) e^{7 I d x + 7 I c} + 14 (2 A + I B) e^{5 I d x + 5 I c} + 140 (A - I B) e^{3 I d x + 3 I c} + 105 I B e^{I d x + I c}) \sqrt{a / (e^{2 I d x + 2 I c} + 1)} \sqrt{(-I e^{2 I d x + 2 I c} + I) / (e^{2 I d x + 2 I c} + 1)}) / (d e^{8 I d x + 8 I c} - 4 d e^{6 I d x + 6 I c} + 6 d e^{4 I d x + 4 I c} - 4 d e^{2 I d x + 2 I c} + d) \right)}{d e^{8 I d x + 8 I c} - 4 d e^{6 I d x + 6 I c} + 6 d e^{4 I d x + 4 I c} - 4 d e^{2 I d x + 2 I c} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="fricas")

[Out] 1/210\*(105\*sqrt(2)\*(d\*e^(8\*I\*d\*x + 8\*I\*c) - 4\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*d\*e^(4\*I\*d\*x + 4\*I\*c) - 4\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*log((I\*sqrt(2)\*d\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*e^(I\*d\*x + I\*c) + sqrt(2)\*((I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A + B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))e^(-I\*d\*x - I\*c)/(I\*A + B) - 105\*sqrt(2)\*(d\*e^(8\*I\*d\*x + 8\*I\*c) - 4\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*d\*e^(4\*I\*d\*x + 4\*I\*c) - 4\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*log((-I\*sqrt(2)\*d\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*e^(I\*d\*x + I\*c) + sqrt(2)\*((I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A + B)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))e^(-I\*d\*x - I\*c)/(I\*A + B) - 4\*sqrt(2)\*((92\*A - 119\*I\*B)\*e^(9\*I\*d\*x + 9\*I\*c) - 20\*(A - 7\*I\*B)\*e^(7\*I\*d\*x + 7\*I\*c) + 14\*(2\*A + I\*B)\*e^(5\*I\*d\*x + 5\*I\*c) + 140\*(A - I\*B)\*e^(3\*I\*d\*x + 3\*I\*c) + 105\*I\*B\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))/(d\*e^(8\*I\*d\*x + 8\*I\*c) - 4\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*d\*e^(4\*I\*d\*x + 4\*I\*c) - 4\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(9/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li}}{\tan(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/tan(c + d\*x)^(9/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/tan(c + d\*x)^(9/2), x)

### 3.161 $\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=248

$$\frac{(-1)^{3/4} a^{3/2} (22iA + 23B) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{8d} + \frac{(2 + 2i) a^{3/2} (iA + B) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}$$

[Out]  $1/8*(-1)^{(3/4)}*a^{(3/2)}*(22*I*A+23*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+(2+2*I)*a^{(3/2)}*(I*A+B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+1/8*a*(10*A-9*I*B)*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d+1/12*a*(6*I*A+7*B)*\tan(d*x+c)^{(1/2)}*\tan(d*x+c)^{(3/2)}/d+1/3*I*a*B*(a+I*a*\tan(d*x+c))^{(1/2)}*\tan(d*x+c)^{(5/2)}/d$

**Rubi [A]**

time = 0.60, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {3675, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} a^{3/2} (23B + 22iA) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{8d} + \frac{(2 + 2i) a^{3/2} (B + iA) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} + \frac{a(7B + 6iA) \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{12d} + \frac{a(10A - 9iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} + \frac{iaB \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((-1)^{(3/4)}*a^{(3/2)}*((22*I)*A + 23*B)*\operatorname{ArcTan}[((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]]/(8*d) + ((2 + 2*I)*a^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]])/d + (a*(10*A - (9*I)*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(8*d) + (a*((6*I)*A + 7*B)*\operatorname{Tan}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(12*d) + ((I/3)*a*B*\operatorname{Tan}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^{p/b})^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*A \operatorname{rcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a$

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x

] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= \frac{iaB \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d} + \\
 &= \frac{a(6iA + 7B) \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{12d} \\
 &= \frac{a(10A - 9iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{a(10A - 9iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{a(10A - 9iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} \\
 &= \frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
 &= \frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
 &= -\frac{\sqrt{-1} a^{3/2}(22A - 23iB) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a + ia \tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{8d}
 \end{aligned}$$

Mathematica [A]

time = 4.77, size = 420, normalized size = 1.69

$$\frac{\left( \frac{\sqrt{2} e^{i(c+dx)} \sqrt{\frac{i[-1 + e^{2i(c+dx)}]}{1 + e^{2i(c+dx)}}}}{\sqrt{-1 + e^{2i(c+dx)}}} \left( -12(A-B) \operatorname{Im}\left( e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \right) + \sqrt{2} (22A - 23iB) \left( \operatorname{Im}\left( (1 - 3a^{3/2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) - \operatorname{Im}\left( (1 - 3a^{3/2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) \right) \right) \right) + \frac{4 \sqrt{-1} (c+dx) \cos(c - dx) (20A - 19iB + 5iA - 7iB) \cos(2c+dx) + 20(A + B) \sin(2c+dx)}{4 \sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\tan(c + dx)}}{6id \sec^3(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] (((Sqrt[2]*Sqrt[(-I)*(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))
)*(-128*(A - I*B)*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] + S
qrt[2]*(22*A - (23*I)*B)*(Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c
+ d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] - Log[1 - 3*E^((2*I)*(c + d*x)) +
2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(E^(I*(c + d*x
))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))])) + (4*Sec[c + d*x]^(5/2)*(Cos[c] - I*Sin[c])*(30*A - (19*I)*B + 5*(6
*A - (7*I)*B)*Cos[2*(c + d*x)] + 2*((6*I)*A + 7*B)*Sin[2*(c + d*x)]*Sqrt[T
an[c + d*x]])/(3*Cos[d*x] + (3*I)*Sin[d*x]))*(a + I*a*Tan[c + d*x])^(3/2)*(
A + B*Tan[c + d*x]))/(64*d*Sec[c + d*x]^(5/2)*(A*Cos[c + d*x] + B*Sin[c + d
*x])))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(198) = 396.  
time = 0.15, size = 652, normalized size = 2.63

method	result
derivativedivides	$\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}{}_a\left(16iB\sqrt{ia}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))\right)$
default	$\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}{}_a\left(16iB\sqrt{ia}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)), x, method=_RE
TURNVERBOSE)
```

```
[Out] 1/48/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(16*I*B*(I*a)^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+24*I*A*(I*a)^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+27*I*B*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(1/2
)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-54*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+28*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*tan(d*x+c)-24*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d
*x+c)+I))*a-30*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+60*A*(I*a)^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-48*I*ln(1/2*(2*I*a*tan(d*x+c)+2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1
```



$$\frac{1}{2} * a + 24 * (I * a)^{(1/2)} * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I * a)^{(1/2)} * (a * \tan(d * x + c)) * (1 + I * \tan(d * x + c)))^{(1/2)} + I * a - 3 * a * \tan(d * x + c)) / (\tan(d * x + c) + I) * a - 48 * \ln(1/2 * (2 * I * a * \tan(d * x + c) + 2 * (a * \tan(d * x + c)) * (1 + I * \tan(d * x + c)))^{(1/2)} * (I * a)^{(1/2)} + a) / (I * a)^{(1/2)} * (-I * a)^{(1/2)} * a) / (I * a)^{(1/2)} / (-I * a)^{(1/2)} / (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*tan(d\*x + c)^(3/2), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 911 vs. 2(184) = 368.

time = 1.39, size = 911, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{48} * (48 * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^3 / d^2) * (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((I * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^3 / d^2) * d * e^{(I * d * x + I * c)} + \sqrt{2} * ((-I * A - B) * a * e^{(2 * I * d * x + 2 * I * c)} + (-I * A - B) * a) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * e^{(-I * d * x - I * c)} / ((-I * A - B) * a)) - 48 * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^3 / d^2) * (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((-I * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^3 / d^2) * d * e^{(I * d * x + I * c)} + \sqrt{2} * ((-I * A - B) * a * e^{(2 * I * d * x + 2 * I * c)} + (-I * A - B) * a) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * e^{(-I * d * x - I * c)} / ((-I * A - B) * a)) + 2 * \sqrt{2} * (7 * (6 * A - 7 * I * B) * a * e^{(5 * I * d * x + 5 * I * c)} + 2 * (30 * A - 19 * I * B) * a * e^{(3 * I * d * x + 3 * I * c)} + 3 * (6 * A - 7 * I * B) * a * e^{(I * d * x + I * c)}) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) + 3 * \sqrt{(-484 * I * A^2 - 1012 * A * B + 529 * I * B^2)} * a^3 / d^2) * (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((\sqrt{2} * ((22 * I * A + 23 * B) * a * e^{(2 * I * d * x + 2 * I * c)} + (22 * I * A + 23 * B) * a) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) + 2 * I * \sqrt{(-484 * I * A^2 - 1012 * A * B + 529 * I * B^2)} * a^3 / d^2) * d * e^{(I * d * x + I * c)}) * e^{(-I * d * x - I * c)} / ((22 * I * A + 23 * B) * a$

) - 3\*sqrt((-484\*I\*A^2 - 1012\*A\*B + 529\*I\*B^2)\*a^3/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log((sqrt(2)\*((22\*I\*A + 23\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) + (22\*I\*A + 23\*B)\*a)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)) - 2\*I\*sqrt((-484\*I\*A^2 - 1012\*A\*B + 529\*I\*B^2)\*a^3/d^2)\*d\*e^(I\*d\*x + I\*c))\*e^(-I\*d\*x - I\*c)/((22\*I\*A + 23\*B)\*a))/(d\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx);OUTPUT:Evaluation time: 1.32Unable to divide , perhaps due to rounding error%%{%%{%%{poly1[-16,0]:[1,0,-2]%%},[0]%%},0]

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) li)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*li)^(3/2),x)

[Out] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*li)^(3/2),x)

$$3.162 \quad \int \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=204

$$\frac{(-1)^{3/4} a^{3/2} (12A - 11iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + (2+2i) a^{3/2} (A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{4d}$$

[Out]  $-1/4*(-1)^{(3/4)}*a^{(3/2)}*(12*A-11*I*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d-(2+2*I)*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+1/4*a*(4*I*A+5*B)*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d+1/2*I*a*B*(a+I*a*\tan(d*x+c))^{(1/2)}*\tan(d*x+c)^{(3/2)}/d$

**Rubi** [A]

time = 0.46, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {3675, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} a^{3/2} (12A - 11iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + (2+2i) a^{3/2} (A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + \frac{a(5B+4iA) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} + \frac{iaB \tan^3(c+dx) \sqrt{a+ia \tan(c+dx)}}{2d}}{4d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $-1/4*((-1)^{(3/4)}*a^{(3/2)}*(12*A-(11*I)*B)*\operatorname{ArcTan}(((1+i)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d-((2+2*I)*a^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}(((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]))/d+(a*((4*I)*A+5*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(4*d)+((I/2)*a*B*\operatorname{Tan}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d$

**Rule 65**

$\operatorname{Int}(((a_.)+(b_.)*(x_.))^{(m_.)}*((c_.)+(d_.)*(x_.))^{(n_.)},x\_Symbol) := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)},x],x,(a+b*x)^{(1/p)}],x]] /; \operatorname{FreeQ}\{a,b,c,d,x\} \&\& \operatorname{NeQ}[b*c-a*d,0] \&\& \operatorname{LtQ}[-1,m,0] \&\& \operatorname{LeQ}[-1,n,0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n],\operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a,b,c,d,m,n,x]$

**Rule 209**

$\operatorname{Int}(((a_.)+(b_.)*(x_.)^2)^{-1},x\_Symbol) := \operatorname{Simp}[(1/(\operatorname{Rt}[a,2]*\operatorname{Rt}[b,2]))* \operatorname{ArcTan}[\operatorname{Rt}[b,2]*(x/\operatorname{Rt}[a,2])],x] /; \operatorname{FreeQ}\{a,b,x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a,0] \mid \mid \operatorname{GtQ}[b,0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

## Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rubi steps

$$\begin{aligned}
 \int \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx &= \frac{iaB \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{2d} \\
 &= \frac{a(4iA+5B) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} \\
 &= \frac{a(4iA+5B) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} \\
 &= \frac{a(4iA+5B) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{4d} \\
 &= -\frac{(2+2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a+ia \tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} \\
 &= -\frac{(2+2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a+ia \tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} \\
 &= -\frac{\sqrt[4]{-1} a^{3/2}(12iA+11B) \tan^{-1}\left(\frac{(-1)^{3/4}}{\sqrt{a+ia \tan(c+dx)}}\right)}{4d}
 \end{aligned}$$

## Mathematica [A]

time = 3.78, size = 389, normalized size = 1.91

$$\frac{(a+ia \tan(c+dx))^{3/2} (A+B \tan(c+dx)) \left( \frac{\sqrt{2} e^{-i \operatorname{atan}\left(\frac{1(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)} (-4664-103) \operatorname{Im}\left(e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}\right) + \sqrt{2} (1034+110) \left( \operatorname{Im}\left(1-3e^{2i(c+dx)}-2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}\right) - \operatorname{Im}\left(1-3e^{2i(c+dx)}+2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}\right)\right)}{\sqrt{-1+e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}} + 8 \sqrt{\sec(c+dx)} (i \cos(c) + \sin(c)) \cos(dx) - i \sin(dx) \sqrt{\tan(c+dx)} (4A-5iB+2B \tan(c+dx)) \right)}{32d \sec^3(c+dx) (A \cos(c+dx) + B \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

```
[Out] ((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])*((Sqrt[2]*Sqrt[((-I)*(-1
+ E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x))))*(-64*I)*(A - I*B)*Log[
E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt[2]*((12*I)*A + 11*
B)*(Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^
(2*I)*(c + d*x))] - Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c + d*
x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*
(c + d*x))]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))])) + 8*Sqrt[Sec[
c + d*x]]*(I*Cos[c] + Sin[c])*(Cos[d*x] - I*Sin[d*x])*Sqrt[Tan[c + d*x]]*(4*
A - (5*I)*B + 2*B*Tan[c + d*x]))/(32*d*Sec[c + d*x]^(5/2)*(A*Cos[c + d*x]
+ B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs.  $2(162) = 324$ .  
time = 0.13, size = 565, normalized size = 2.77

method	result
derivativedivides	$\frac{\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}a\left(-4iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\sqrt{ia}\right)}{-}$
default	$\frac{\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}a\left(-4iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\sqrt{ia}\right)}{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/8/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(-4*I*B*(I*a)^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+4*I*A*ln(1/2*(2
*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a
)^(1/2))*(-I*a)^(1/2)*a-8*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(
1/2)*(-I*a)^(1/2)-4*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+5*B
*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-10*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(I*a)^(1/2)*(-I*a)^(1/2)+8*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-4*(I*a)^(1/
2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-8*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a
)/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x +
c)), x)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 829 vs.  $2(150) = 300$ .

time = 1.23, size = 829, normalized size = 4.06



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")
```

```
[Out] 1/8*(8*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*
c) + d)*log((sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I
*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))e^(-I*d*x - I*c)/((-I*A - B)*a) - 8*sqrt(2)*sqrt(-(-I*A^2 - 2*A*
B + I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-(sqrt(2)*sqrt(-(-I*A^2
- 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*a*e^(2*I
*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A
- B)*a) + 2*sqrt(2)*((4*I*A + 7*B)*a*e^(3*I*d*x + 3*I*c) + (4*I*A + 3*B)*a
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt((144*I*A^2 + 264*A*B - 121*I*B
^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((12*I*A + 11*B)*a*e^
(2*I*d*x + 2*I*c) + (12*I*A + 11*B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 2*sqrt((144*I*
A^2 + 264*A*B - 121*I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c))e^(-I*d*x - I*c)/((1
2*I*A + 11*B)*a) - sqrt((144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2)*(d*e^(2
*I*d*x + 2*I*c) + d)*log((sqrt(2)*((12*I*A + 11*B)*a*e^(2*I*d*x + 2*I*c) +
(12*I*A + 11*B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*sqrt((144*I*A^2 + 264*A*B - 121*
I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c))e^(-I*d*x - I*c)/((12*I*A + 11*B)*a))/((
d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{3}{2}} (A + B \tan(c + dx)) \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*sqrt(tan(c + d*x)), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);OUTPUT:Evaluation time: 0.53Unable to divide
, perhaps due to rounding error%%{%}{%%{%}{poly1[-8*i,0]:[1,0,-2]%%},[0]
%%},0
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\tan(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```



$$3.163 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=156

$$\frac{(-1)^{3/4}a^{3/2}(2iA+3B)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(2+2i)a^{3/2}(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

[Out]  $-(1)^{3/4}a^{3/2}(2iA+3B)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)/d - (2+2i)a^{3/2}(iA+B)\text{ArcTanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)/d + iAaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}/d$

**Rubi [A]**

time = 0.33, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3675, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4}a^{3/2}(3B+2iA)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(2+2i)a^{3/2}(B+iA)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^{3/2}*(A + B*\text{Tan}[c + d*x])}{\text{Sqrt}[\text{Tan}[c + d*x]]}, x]$

[Out]  $-\frac{((1)^{3/4}a^{3/2}((2i)A + 3B)\text{ArcTan}[\frac{(-1)^{3/4}\sqrt{a}\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + I*a*\text{Tan}[c + d*x]}}])}{d} - \frac{((2 + 2i)a^{3/2}(iA + B)\text{ArcTanh}[\frac{(1 + i)\sqrt{a}\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + I*a*\text{Tan}[c + d*x]}}])}{d} + \frac{(I*a*B*\text{Sqrt}[\text{Tan}[c + d*x] ]*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x] ])}{d}$

**Rule 65**

$\text{Int}[\frac{(a + b*x)^m}{(c + d*x)^n}, x] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{1/p}], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\text{Int}[\frac{(a + b*x)^m}{(c + d*x)^n}, x] := \text{Simp}[\frac{1}{\text{Rt}[a, 2]*\text{Rt}[b, 2]}\text{ArcTan}[\frac{\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])}{\text{Rt}[a, 2]}], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \text{ || } \text{GtQ}[b, 0])$

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{iaB \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} + \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx \\
&= \frac{iaB \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} + (2a(A - iB) \sqrt{\tan(c + dx)}) \\
&= \frac{iaB \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} - \frac{(a^2(2A - 3iB) \sqrt{\tan(c + dx)})}{d} \\
&= -\frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= -\frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= \frac{\sqrt[4]{-1} a^{3/2} (2A - 3iB) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.09, size = 221, normalized size = 1.42

$$\frac{ae^{-i(c+dx)} \left( 2\sqrt{2} (A - iB) (1 + e^{2i(c+dx)}) \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) + i \left( \sqrt{2} B e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} + (2iA + 3B) (1 + e^{2i(c+dx)}) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) \right) \right) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{\sqrt{2} d \sqrt{-1 + e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
```

```
[Out] (a*(2*Sqrt[2]*(A - I*B)*(1 + E^((2*I)*(c + d*x)))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + I*(Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))] + ((2*I)*A + 3*B)*(1 + E^((2*I)*(c + d*x)))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x))]/Sqrt[-1 + E^((2*I)*(c + d*x))]))*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 485 vs.  $2(126) = 252$ .

time = 0.12, size = 486, normalized size = 3.12

method	result
--------	--------

derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^a \left(-iB \ln\left(\frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}}\right)\right)}{\sqrt{a(1+i \tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^a \left(-iB \ln\left(\frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}}\right)\right)}$
default	$\frac{\sqrt{a(1+i \tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^a \left(-iB \ln\left(\frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}}\right)\right)}{\sqrt{a(1+i \tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^a \left(-iB \ln\left(\frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\frac{1}{2}d^*(a*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^{(1/2)}*a*(-I*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)})$$
  

$$*(-I*a)^{(1/2)}*a+2*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a+2*A*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)})$$
  

$$*(-I*a)^{(1/2)}*a+2*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2))*a*(I*a)^{(1/2)}-I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a+2*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2))*(-I*a)^{(1/2)}*a)/(I*a)^{(1/2)}/(-I*a)^{(1/2)}/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)/sqrt(tan(d*x + c)), x)`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 743 vs.  $2(116) = 232$ .

time = 1.02, size = 743, normalized size = 4.76



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{2} \cdot (2 \cdot I \cdot \sqrt{2}) \cdot B \cdot a \cdot \sqrt{a / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)} \cdot e^{(I \cdot d \cdot x + I \cdot c)} - 2 \cdot \sqrt{2} \cdot \sqrt{-(I \cdot A^2 + 2 \cdot A \cdot B - I \cdot B^2)} \cdot a^3 / d^2 \cdot \log((I \cdot \sqrt{2}) \cdot \sqrt{-(I \cdot A^2 + 2 \cdot A \cdot B - I \cdot B^2)} \cdot a^3 / d^2 \cdot d \cdot e^{(I \cdot d \cdot x + I \cdot c)} + \sqrt{2} \cdot ((-I \cdot A - B) \cdot a \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + (-I \cdot A - B) \cdot a) \cdot \sqrt{a / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)}) \cdot e^{(-I \cdot d \cdot x - I \cdot c)} / ((-I \cdot A - B) \cdot a)) + 2 \cdot \sqrt{2} \cdot \sqrt{-(I \cdot A^2 + 2 \cdot A \cdot B - I \cdot B^2)} \cdot a^3 / d^2 \cdot \log((-I \cdot \sqrt{2}) \cdot \sqrt{-(I \cdot A^2 + 2 \cdot A \cdot B - I \cdot B^2)} \cdot a^3 / d^2 \cdot d \cdot e^{(I \cdot d \cdot x + I \cdot c)} + \sqrt{2} \cdot ((-I \cdot A - B) \cdot a \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + (-I \cdot A - B) \cdot a) \cdot \sqrt{a / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)}) \cdot e^{(-I \cdot d \cdot x - I \cdot c)} / ((-I \cdot A - B) \cdot a)) - \sqrt{(-4 \cdot I \cdot A^2 - 12 \cdot A \cdot B + 9 \cdot I \cdot B^2)} \cdot a^3 / d^2 \cdot \log((\sqrt{2}) \cdot ((2 \cdot I \cdot A + 3 \cdot B) \cdot a \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + (2 \cdot I \cdot A + 3 \cdot B) \cdot a) \cdot \sqrt{a / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)}) + 2 \cdot I \cdot \sqrt{(-4 \cdot I \cdot A^2 - 12 \cdot A \cdot B + 9 \cdot I \cdot B^2)} \cdot a^3 / d^2 \cdot d \cdot e^{(I \cdot d \cdot x + I \cdot c)} \cdot e^{(-I \cdot d \cdot x - I \cdot c)} / ((2 \cdot I \cdot A + 3 \cdot B) \cdot a)) + \sqrt{(-4 \cdot I \cdot A^2 - 12 \cdot A \cdot B + 9 \cdot I \cdot B^2)} \cdot a^3 / d^2 \cdot \log((\sqrt{2}) \cdot ((2 \cdot I \cdot A + 3 \cdot B) \cdot a \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + (2 \cdot I \cdot A + 3 \cdot B) \cdot a) \cdot \sqrt{a / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)}) - 2 \cdot I \cdot \sqrt{(-4 \cdot I \cdot A^2 - 12 \cdot A \cdot B + 9 \cdot I \cdot B^2)} \cdot a^3 / d^2 \cdot d \cdot e^{(I \cdot d \cdot x + I \cdot c)} \cdot e^{(-I \cdot d \cdot x - I \cdot c)} / ((2 \cdot I \cdot A + 3 \cdot B) \cdot a))) / d$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^{\frac{3}{2}} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))^(3/2)\*(A + B\*tan(c + d\*x))/sqrt(tan(c + d\*x)), x)

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for

the root of a polynomial with parameters. This might be wrong. Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(1/2), x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(1/2), x)

$$3.164 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^3(c+dx)} dx$$

**Optimal.** Leaf size=146

$$\frac{2\sqrt[4]{-1} a^{3/2} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{(2+2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}$$

[Out]  $2*(-1)^{(1/4)}*a^{(3/2)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+(2+2*I)*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d-2*a*A*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.31, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3674, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(2+2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{2\sqrt[4]{-1} a^{3/2} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x])]/\operatorname{Tan}[c+d*x]^{(3/2)}, x]$

[Out]  $(2*(-1)^{(1/4)}*a^{(3/2)}*B*\operatorname{ArcTan}[( (-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d + ((2+2*I)*a^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}[( (1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/d - (2*a*A*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(d*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]



Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + 2 \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx \\
 &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - B \int \frac{(a - ia \tan(c + dx))^{3/2}}{\sqrt{\tan(c + dx)}} dx \\
 &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{(4a^3(A - iB)) \text{Subst}\left(\int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx\right)}{d} \\
 &= \frac{(2 + 2i)a^{3/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
 &= \frac{(2 + 2i)a^{3/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
 &= \frac{2\sqrt[4]{-1} a^{3/2} B \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} + \dots
 \end{aligned}$$

**Mathematica [A]**

time = 2.49, size = 234, normalized size = 1.60

$$\frac{ae^{-\frac{1}{2}i(4c+5dx)} \sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}} (1+e^{2i(c+dx)})^2 \left(2(iA+B) \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) - \sqrt{2} B \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) - A \sqrt{-1+e^{2i(c+dx)}} \csc(c+dx)\right) \sec(c+dx) \left(\cos\left(\frac{dx}{2}\right) + i \sin\left(\frac{dx}{2}\right)\right) \sqrt{\tan(c+dx)}}{\sqrt{2} d \sqrt{-1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out] (a\*sqrt[(a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]\*(1 + E^((2\*I)\*(c + d\*x)))^2\*(2\*(I\*A + B)\*ArcTanh[E^(I\*(c + d\*x))/sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - sqrt[2]\*B\*ArcTanh[(sqrt[2]\*E^(I\*(c + d\*x)))/sqrt[-1 + E^((2\*I)\*(c + d\*x))]]) - A\*sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Csc[c + d\*x]\*Sec[c + d\*x]\*(Cos[(d\*x)/2] + I\*Sin[(d\*x)/2])\*sqrt[Tan[c + d\*x]]/(sqrt[2]\*d\*E^((I/2)\*(4\*c + 5\*d\*x))\*sqrt[-1 + E^((2\*I)\*(c + d\*x))])]

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(118) = 236.

time = 0.12, size = 521, normalized size = 3.57

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} a \left( 4iA \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{ia} + a \right)}{\sqrt{a(1+i \tan(dx+c))} a \left( 4iA \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{ia} + a \right)}$
default	$\sqrt{a(1+i \tan(dx+c))} a \left( 4iA \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{ia} + a \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(4*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan
(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*
tan(d*x+c)-I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c))))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+2
*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(
1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+2*I*ln(1/2*(2*I*a*tan(d*x+c)
+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)
^(1/2)*a*tan(d*x+c)-(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c))))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d
*x+c)-4*A*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)-2*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(1/2
)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c))/tan(d*x+c)^(1/2)/(I*a)^(1/2)/(-
I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)
```

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 748 vs.  $2(110) = 220$ .

time = 1.17, size = 748, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] 
$$-1/2*(2*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*d*e^{(I*d*x + I*c)} + \sqrt{2}*((-I*A - B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a)) - 2*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-(\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*d*e^{(I*d*x + I*c)} - \sqrt{2}*((-I*A - B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a)) + 4*\sqrt{2}*(I*A*a*e^{(3*I*d*x + 3*I*c)} + I*A*a*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)} + \sqrt{-4*I*B^2*a^3/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*(B*a*e^{(2*I*d*x + 2*I*c)} + B*a)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)} + \sqrt{-4*I*B^2*a^3/d^2}*d*e^{(I*d*x + I*c)})e^{(-I*d*x - I*c)}/(B*a)) - \sqrt{-4*I*B^2*a^3/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*(B*a*e^{(2*I*d*x + 2*I*c)} + B*a)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)} - \sqrt{-4*I*B^2*a^3/d^2}*d*e^{(I*d*x + I*c)})e^{(-I*d*x - I*c)}/(B*a)))/(d*e^{(2*I*d*x + 2*I*c)} - d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^{\frac{3}{2}}(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))^(3/2)\*(A + B\*tan(c + d\*x))/tan(c + d\*x)^(3/2), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for

the root of a polynomial with parameters. This might be wrong. Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2}}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(3/2), x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(3/2), x)

$$3.165 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=137

$$\frac{(2+2i)a^{3/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2aA \sqrt{a+ia \tan(c+dx)}}{3d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a(4iA+3B) \sqrt{a+ia \tan(c+dx)}}{3d \sqrt{\tan(c+dx)}}$$

[Out] (2+2\*I)\*a^(3/2)\*(I\*A+B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d-2/3\*a\*(4\*I\*A+3\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2))-2/3\*a\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.24, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(2+2i)a^{3/2}(B+ia) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a(3B+4iA) \sqrt{a+ia \tan(c+dx)}}{3d \sqrt{\tan(c+dx)}} - \frac{2aA \sqrt{a+ia \tan(c+dx)}}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] ((2 + 2\*I)\*a^(3/2)\*(I\*A + B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])]/d - (2\*a\*A\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d\*Tan[c + d\*x]^(3/2)) - (2\*a\*((4\*I)\*A + 3\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d\*Sqrt[Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{5/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{3d \tan^{3/2}(c + dx)} + \frac{2}{3} \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{3/2}(c + dx)} dx \\
 &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{3d \tan^{3/2}(c + dx)} - \frac{2a(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{3d \tan^{3/2}(c + dx)} - \frac{2a(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{3d \tan^{3/2}(c + dx)} - \frac{2a(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\
 &= \frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d}
 \end{aligned}$$

**Mathematica [A]**

time = 3.60, size = 221, normalized size = 1.61

$$\frac{ae^{-3i(c+dx)}\sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}(1+e^{2i(c+dx)})^2\left(e^{i(c+dx)}\sqrt{1-e^{2i(c+dx)}}(3B(-1+e^{2i(c+dx)})+iA(-3+5e^{2i(c+dx)}))+3(iA+B)(-1+e^{2i(c+dx)})^2\text{ArcSin}(e^{i(c+dx)})\right)(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d(1-e^{2i(c+dx)})^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (a\*sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*(1 + E^((2\*I)\*(c + d\*x)))^2\*(E^(I\*(c + d\*x))\*sqrt[1 - E^((2\*I)\*(c + d\*x))])\*(3\*B\*(-1 + E^((2\*I)\*(c + d\*x))) + I\*A\*(-3 + 5\*E^((2\*I)\*(c + d\*x)))) + 3\*(I\*A + B)\*(-1 + E^((2\*I)\*(c + d\*x)))^2\*ArcSin[E^(I\*(c + d\*x))]\*(-I + Tan[c + d\*x])\*sqrt[a + I\*a\*Tan[c + d\*x]]/(3\*d\*E^((3\*I)\*(c + d\*x))\*(1 - E^((2\*I)\*(c + d\*x))))^(5/2))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(112) = 224.

time = 0.12, size = 618, normalized size = 4.51

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} a \left( -12iB \ln \left( \frac{2ia \tan(dx+c)+2 \sqrt{a \tan(dx+c)} (1+i \tan(dx+c))}{2\sqrt{ia}} \right) \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))} a \left( -12iB \ln \left( \frac{2ia \tan(dx+c)+2 \sqrt{a \tan(dx+c)} (1+i \tan(dx+c))}{2\sqrt{ia}} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2), x, method=\_RE TURNVERBOSE)

[Out] -1/6/d\*(a\*(1+I\*tan(d\*x+c)))^(1/2)\*a/tan(d\*x+c)^(3/2)\*(-12\*I\*B\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c)^2+3\*I\*(I\*a)^(1/2)\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I)\*a\*tan(d\*x+c)^2+16\*I\*A\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*tan(d\*x+c)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+12\*A\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c)^2+6\*I\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c)^2-3\*(I\*a)^(1/2)\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I)\*a\*tan(d\*x+c)^2+12\*B\*(I\*a)^(1/2)\*(-I\*a)^(1/2)





Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))/tan(c + d*x)*
*(5/2), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^{3/2}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(5/2)
,x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(5/2)
, x)
```

$$3.166 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=181

$$\frac{(2+2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{5d \tan^{\frac{5}{2}}(c+dx)} - \frac{2a(6iA+5B)\sqrt{a+ia \tan(c+dx)}}{15d \tan^{\frac{3}{2}}(c+dx)}$$

[Out]  $(-2-2*I)*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+4/15*a*(9*A-10*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/5*a*A*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(5/2)}-2/15*a*(6*I*A+5*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(3/2)}$

Rubi [A]

time = 0.37, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(2+2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a(5B+6iA)\sqrt{a+ia \tan(c+dx)}}{15d \tan^{\frac{3}{2}}(c+dx)} + \frac{4a(9A-10iB)\sqrt{a+ia \tan(c+dx)}}{15d\sqrt{\tan(c+dx)}} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{5d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x])]/\operatorname{Tan}[c+d*x]^{(7/2)}, x]$

[Out]  $((-2-2*I)*a^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}(((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]))/d - (2*a*A*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(5*d*\operatorname{Tan}[c+d*x]^{(5/2)}) - (2*a*((6*I)*A+5*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(15*d*\operatorname{Tan}[c+d*x]^{(3/2)}) + (4*a*(9*A-(10*I)*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(15*d*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}[\operatorname{Q}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]]$

Rule 211

$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\operatorname{tan}[(e_*) + (f_*)(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)(x_)]], x\_Symbol] := \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

$^2*x^2)$ ,  $x]$ ,  $x$ ,  $\text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]]$ ,  $x]$  /;  $\text{FreeQ}[\{a, b, c, d, e, f\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{NeQ}[c^2 + d^2, 0]$

#### Rule 3674

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(-a^2)*(B*c - A*d)*(a + b*\text{Tan}[e + f*x])^{(m - 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(d*f*(b*c + a*d)*(n + 1)), x] - \text{Dist}[a/(d*(b*c + a*d)*(n + 1)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1))]*\text{Tan}[e + f*x], x], x], x] /;  $\text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{GtQ}[m, 1]$  &&  $\text{LtQ}[n, -1]$$

#### Rule 3679

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(A*d - B*c)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(f*(n + 1)*(c^2 + d^2)), x] - \text{Dist}[1/(a*(n + 1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;  $\text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{LtQ}[n, -1]$$

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(6iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(6iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(6iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(6iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{(2 + 2i)a^{3/2}(A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 5.60, size = 237, normalized size = 1.31

$$-\frac{a \csc^2(c + dx)(-15A + 20iB + (21A - 20iB) \cos(2(c + dx)) + (6iA + 5B) \sin(2(c + dx))) \sqrt{a + ia \tan(c + dx)}}{15d \sqrt{\tan(c + dx)}} + \frac{a(A - iB) e^{-3i(c + dx)} \sqrt{-\frac{i(-1 + e^{2i(c + dx)})}{1 + e^{2i(c + dx)}}} (1 + e^{2i(c + dx)})^2 \tanh^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{-1 + e^{2i(c + dx)}}} \right) (-i + \tan(c + dx)) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{-1 + e^{2i(c + dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2), x]
```

```
[Out] -1/15*(a*Csc[c + d*x]^2*(-15*A + (20*I)*B + (21*A - (20*I)*B)*Cos[2*(c + d*x)] + ((6*I)*A + 5*B)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]]/(d*Sqrt[Tan[c + d*x]]) + (a*(A - I*B)*Sqrt[(-I)*(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x))))*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 706 vs.  $2(148) = 296$ .

time = 0.12, size = 707, normalized size = 3.91

method	result
--------	--------

derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} a \left( -72A\sqrt{ia} \sqrt{-ia} (\tan^2(dx+c)) \sqrt{a\tan(dx+c)} (1+i\tan(dx+c)) \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))} a \left( -72A\sqrt{ia} \sqrt{-ia} (\tan^2(dx+c)) \sqrt{a\tan(dx+c)} (1+i\tan(dx+c)) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/30/d*(a*(1+I*\tan(d*x+c)))^{(1/2)}*a/\tan(d*x+c)^{(5/2)}*(-72*A*(I*a)^{(1/2)}*(- \\ & I*a)^{(1/2)}*\tan(d*x+c)^2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+60*I*A*\ln(1/2 \\ & *(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/( \\ & I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\tan(d*x+c)^3-15*I*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{ \\ & (1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c) \\ & )/(\tan(d*x+c)+I))*a*\tan(d*x+c)^3+80*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*\tan(d*x+c) \\ & ^2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+60*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a \\ & * \tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2) \\ & )*a*\tan(d*x+c)^3+30*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x \\ & +c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\tan(d*x+c)^3-15*(I*a \\ & )^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)) \\ & )^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^3+24*I*A*(I*a)^{(1/ \\ & 2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)-30*\ln(1/2* \\ & (2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I \\ & *a)^{(1/2)}*(-I*a)^{(1/2)}*a*\tan(d*x+c)^3+20*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan \\ & (d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)+12*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a \\ & * \tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)})/(I*a)^{(1/2)}/(-I*a)^{(1/2)}/(a*\tan(d*x+c) \\ & *(1+I*\tan(d*x+c)))^{(1/2)} \end{aligned}$$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, alg  
orithm="maxima")`

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs.  $2(137) = 274$ .

time = 0.74, size = 593, normalized size = 3.28

$$\frac{\int \frac{(a + I a \tan(dx + c))^{3/2} (A + B \tan(dx + c))}{\tan(dx + c)^{7/2}} dx}{\int \frac{(a + I a \tan(dx + c))^{3/2} (A + B \tan(dx + c))}{\tan(dx + c)^{7/2}} dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6*
I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*
sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A
- B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
I*c)/((-I*A - B)*a) - 15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*
(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
- d)*log(-(sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c)
) - sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a) - 2*sqrt(2)*((-27*I*A - 25*B)*a*e^
(7*I*d*x + 7*I*c) + 3*(I*A + 5*B)*a*e^(5*I*d*x + 5*I*c) + 5*(3*I*A + 5*B)*a
*e^(3*I*d*x + 3*I*c) + 15*(-I*A - B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
)/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
) - d)
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);;OUTPUT:Warning, need to choose a branch for
```

the root of a polynomial with parameters. This might be wrong. Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2}}{\tan(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(7/2), x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(7/2), x)

$$3.167 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=225

$$\frac{(2-2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{7d \tan^{\frac{7}{2}}(c+dx)} - \frac{2a(8iA+7B)\sqrt{a+ia \tan(c+dx)}}{35d \tan^{\frac{5}{2}}(c+dx)}$$

[Out] (2-2\*I)\*a^(3/2)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d+4/105\*a\*(67\*I\*A+63\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/7\*a\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2)-2/35\*a\*(8\*I\*A+7\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)+4/105\*a\*(19\*A-21\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

**Rubi [A]**

time = 0.48, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(2-2i)a^{3/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{4a(19A-21iB)\sqrt{a+ia \tan(c+dx)}}{105d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a(7B+8iA)\sqrt{a+ia \tan(c+dx)}}{35d \tan^{\frac{3}{2}}(c+dx)} + \frac{4a(63B+67iA)\sqrt{a+ia \tan(c+dx)}}{105d \sqrt{\tan(c+dx)}} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{7d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] ((2 - 2\*I)\*a^(3/2)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]]/Sqrt[a + I\*a\*Tan[c + d\*x]])]/d - (2\*a\*A\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(7\*d\*Tan[c + d\*x]^(7/2)) - (2\*a\*((8\*I)\*A + 7\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(35\*d\*Tan[c + d\*x]^(5/2)) + (4\*a\*(19\*A - (21\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(3/2)) + (4\*a\*((67\*I)\*A + 63\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Sqrt[Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**



```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

#### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

#### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{9/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} + \frac{2}{7} \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{5/2}(c + dx)} dx \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2a(8iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2a(8iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2a(8iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2a(8iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2a(8iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2a(8iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 7.14, size = 261, normalized size = 1.16

$$\frac{a \csc^2(c + dx) (7(A + 6iB) \cos(c + dx) + (53A - 42iB) \cos(3(c + dx)) + 2(-110iA - 105B + (158iA + 147B) \cos(2(c + dx))) \sin(c + dx)) \sqrt{a + ia \tan(c + dx)}}{210d \sqrt{\tan(c + dx)}} + \frac{a(iA + B)e^{-2i(c + dx)} \sqrt{\frac{1(-1 + e^{2i(c + dx)})}{1 + e^{2i(c + dx)}}} (1 + e^{2i(c + dx)})^2 \tanh^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{-1 + e^{2i(c + dx)}}} \right) (-1 + \tan(c + dx)) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{-1 + e^{2i(c + dx)}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] -1/210\*(a\*Csc[c + d\*x]^3\*(7\*(A + (6\*I)\*B)\*Cos[c + d\*x] + (53\*A - (42\*I)\*B)\*Cos[3\*(c + d\*x)] + 2\*((-110\*I)\*A - 105\*B + ((158\*I)\*A + 147\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*Sqrt[Tan[c + d\*x]]) + (a\*(I\*A + B)\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))]\*(1 + E^((2\*I)\*(c + d\*x)))^2\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]\*(-I + Tan[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*E^((3\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(184) = 368.

time = 0.12, size = 796, normalized size = 3.54 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_RE  
TURNVERBOSE)`

[Out]  $\frac{1}{210}d \cdot (a(1+I \tan(dx+c)))^{1/2} \cdot a / \tan(dx+c)^{7/2} \cdot (504B(Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot \tan(dx+c)^3 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} - 96IA \cdot (Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot \tan(dx+c) \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} - 420IB \cdot \ln(1/2 \cdot (2Ia \tan(dx+c) + 2 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} \cdot (Ia)^{1/2} + a) / (Ia)^{1/2}) \cdot (-Ia)^{1/2} \cdot a \cdot \tan(dx+c)^4 + 152A \cdot (Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot \tan(dx+c)^2 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} + 536IA \cdot (Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot \tan(dx+c)^3 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} + 420A \cdot \ln(1/2 \cdot (2Ia \tan(dx+c) + 2 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} \cdot (Ia)^{1/2} + a) / (Ia)^{1/2}) \cdot (-Ia)^{1/2} \cdot a \cdot \tan(dx+c)^4 + 105I \cdot (Ia)^{1/2} \cdot 2^{1/2} \cdot \ln(-(-2 \cdot 2^{1/2} \cdot (-Ia)^{1/2} \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} + Ia - 3a \cdot \tan(dx+c))) / (\tan(dx+c) + I) \cdot a \cdot \tan(dx+c)^4 - 105 \cdot (Ia)^{1/2} \cdot 2^{1/2} \cdot \ln(-(-2 \cdot 2^{1/2} \cdot (-Ia)^{1/2} \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} + Ia - 3a \cdot \tan(dx+c))) / (\tan(dx+c) + I) \cdot a \cdot \tan(dx+c)^4 + 210I \cdot \ln(1/2 \cdot (2Ia \tan(dx+c) + 2 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} \cdot (Ia)^{1/2} + a) / (Ia)^{1/2}) \cdot (-Ia)^{1/2} \cdot a \cdot \tan(dx+c)^4 + 210 \cdot \ln(1/2 \cdot (2Ia \tan(dx+c) + 2 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} \cdot (Ia)^{1/2} + a) / (Ia)^{1/2}) \cdot (-Ia)^{1/2} \cdot a \cdot \tan(dx+c)^4 - 168IB \cdot (Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot \tan(dx+c)^2 \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} - 84B \cdot (Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2} \cdot \tan(dx+c) - 60A \cdot (Ia)^{1/2} \cdot (-Ia)^{1/2} \cdot (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2}) / (Ia)^{1/2} / (-Ia)^{1/2} / (a \tan(dx+c) \cdot (1+I \tan(dx+c)))^{1/2}$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, alg  
orithm="maxima")`

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs.  $2(171) = 342$ .

time = 0.94, size = 638, normalized size = 2.84

---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, alg  
orithm="fricas")`

```
[Out] -1/105*(105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(8*I*d*x +
8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x
+ 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*e^(I
*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A - B)*a)) - 105*sqrt(2)*sqrt(-(I*A
^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*
c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(2)
*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A
- B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x -
I*c)/((-I*A - B)*a)) + 2*sqrt(2)*((211*A - 189*I*B)*a*e^(9*I*d*x + 9*I*c)
- 10*(16*A - 21*I*B)*a*e^(7*I*d*x + 7*I*c) + 14*(A + 6*I*B)*a*e^(5*I*d*x +
5*I*c) + 70*(4*A - 3*I*B)*a*e^(3*I*d*x + 3*I*c) - 105*(A - I*B)*a*e^(I*d*x
+ I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*
c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^{3/2}}{\tan(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(9/2), x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(9/2), x)
```

$$3.168 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=269

$$\frac{(2+2i)a^{3/2}(A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{9d \tan^{\frac{9}{2}}(c+dx)} - \frac{2a(10iA+9B)\sqrt{a+ia \tan(c+dx)}}{63d \tan^{\frac{7}{2}}(c+dx)}$$

[Out] (2+2\*I)\*a^(3/2)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d-4/315\*a\*(193\*A-201\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/9\*a\*A\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(9/2)-2/63\*a\*(10\*I\*A+9\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2)+4/105\*a\*(11\*A-12\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)+4/315\*a\*(61\*I\*A+57\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.62, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(2+2i)a^{3/2}(A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{4a(57B+61iA)\sqrt{a+ia \tan(c+dx)}}{315d \tan^{\frac{5}{2}}(c+dx)} + \frac{4a(11A-12iB)\sqrt{a+ia \tan(c+dx)}}{105d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a(9B+10iA)\sqrt{a+ia \tan(c+dx)}}{63d \tan^{\frac{1}{2}}(c+dx)} - \frac{4a(193A-201iB)\sqrt{a+ia \tan(c+dx)}}{315d \sqrt{\tan(c+dx)}} - \frac{2aA\sqrt{a+ia \tan(c+dx)}}{9d \tan^{\frac{1}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] ((2 + 2\*I)\*a^(3/2)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/d - (2\*a\*A\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(9\*d\*Tan[c + d\*x]^(9/2)) - (2\*a\*((10\*I)\*A + 9\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(63\*d\*Tan[c + d\*x]^(7/2)) + (4\*a\*(11\*A - (12\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(5/2)) + (4\*a\*((61\*I)\*A + 57\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(315\*d\*Tan[c + d\*x]^(3/2)) - (4\*a\*(193\*A - (201\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(315\*d\*Sqrt[Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{11/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} + \frac{2}{9} \int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan^{7/2}(c + dx)} dx \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + ia \tan(c + dx)}}{9d \tan^{9/2}(c + dx)} - \frac{2a(10iA + 9B) \sqrt{a + ia \tan(c + dx)}}{63d \tan^{7/2}(c + dx)} \\
&= \frac{(2 + 2i)a^{3/2}(A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 8.77, size = 242, normalized size = 0.90

$$\frac{\left( \frac{2520(A-iB)e^{-i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) + \csc^{11}(c+dx) (-1197A + 1134iB + 12(117A - 134iB) \cos(2(c+dx)) + (-487A + 474iB) \cos(4(c+dx)) + 144A \sin(2(c+dx)) + 138B \sin(2(c+dx)) - 172iA \sin(4(c+dx)) - 159B \sin(4(c+dx)))}{\sqrt{-\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}}} \right) \sqrt{a + ia \tan(c + dx)}}{1260d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] (a\*((2520\*(A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/(E^(I\*(c + d\*x))\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))]]) + (Csc[c + d\*x]^4\*(-1197\*A + (1134\*I)\*B + 12\*(117\*A - (134\*I)\*B)\*Cos[2\*(c + d\*x)] + (-487\*A + (474\*I)\*B)\*Cos



$$\frac{[4*(c + d*x)] + (144*I)*A*\sin[2*(c + d*x)] + 138*B*\sin[2*(c + d*x)] - (172*I)*A*\sin[4*(c + d*x)] - 159*B*\sin[4*(c + d*x)]}{\sqrt{\tan[c + d*x]}}*\sqrt{a + I*a*\tan[c + d*x]}/(1260*d)$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs.  $2(220) = 440$ .

time = 0.12, size = 885, normalized size = 3.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{630}d*(a*(1+I*\tan(d*x+c)))^{1/2}*a/\tan(d*x+c)^{9/2}*(-1544*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^4*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+1608*I*B*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^4*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+1260*I*A*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2})*(-I*a)^{1/2}*a*\tan(d*x+c)^5+456*B*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^3*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+630*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2})*(-I*a)^{1/2}*a*\tan(d*x+c)^5+1260*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2})*(-I*a)^{1/2}*a*\tan(d*x+c)^5-288*I*B*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}-315*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)*a*\tan(d*x+c)^5+264*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}-315*I*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I)*a*\tan(d*x+c)^5-630*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2})*(-I*a)^{1/2}*a*\tan(d*x+c)^5-200*I*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+488*I*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^3*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}-180*B*(I*a)^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*\tan(d*x+c)-140*A*(I*a)^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2})/(I*a)^{1/2}/(-I*a)^{1/2}/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}$

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 701 vs.  $2(205) = 410$ .  
time = 1.04, size = 701, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/315*(315*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(d*e^{(10*I*d*x + 10*I*c)} - 5*d*e^{(8*I*d*x + 8*I*c)} + 10*d*e^{(6*I*d*x + 6*I*c)} - 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*d*e^{(I*d*x + I*c)} + \sqrt{2}*((-I*A - B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-I*d*x - I*c)}/((-I*A - B)*a)) - 315*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(d*e^{(10*I*d*x + 10*I*c)} - 5*d*e^{(8*I*d*x + 8*I*c)} + 10*d*e^{(6*I*d*x + 6*I*c)} - 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-(\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*d*e^{(I*d*x + I*c)} - \sqrt{2}*((-I*A - B)*a*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})*e^{(-I*d*x - I*c)}/((-I*A - B)*a)) + 2*\sqrt{2}*((659*I*A + 633*B)*a*e^{(11*I*d*x + 11*I*c)} + 7*(-127*I*A - 159*B)*a*e^{(9*I*d*x + 9*I*c)} + 18*(47*I*A + 29*B)*a*e^{(7*I*d*x + 7*I*c)} + 42*(27*I*A + 19*B)*a*e^{(5*I*d*x + 5*I*c)} + 105*(-9*I*A - 11*B)*a*e^{(3*I*d*x + 3*I*c)} + 315*(I*A + B)*a*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)})/(d*e^{(10*I*d*x + 10*I*c)} - 5*d*e^{(8*I*d*x + 8*I*c)} + 10*d*e^{(6*I*d*x + 6*I*c)} - 10*d*e^{(4*I*d*x + 4*I*c)} + 5*d*e^{(2*I*d*x + 2*I*c)} - d) \end{aligned}$$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)`

[Out] Timed out

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(11/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for  
 the root of a polynomial with parameters. This might be wrong.Non regular v  
 alue [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2}}{\tan(c + dx)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(11/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/tan(c + d\*x)^(11/2), x)

$$3.169 \quad \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=298

$$\frac{3(-1)^{3/4}a^{5/2}(120iA + 121B)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d} + \frac{(4+4i)a^{5/2}(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

[Out]  $\frac{3(-1)^{3/4}a^{5/2}(120iA + 121B)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d} + \frac{(4+4i)a^{5/2}(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$

**Rubi [A]**

time = 0.75, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {3675, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{3(-1)^{3/4}a^{5/2}(121B + 120iA)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d} + \frac{(4+4i)a^{5/2}(B + iA)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{a^2(8A - 11iB)\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24d} + \frac{a^2(107B + 104iA)\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{96d} + \frac{a^2(152A - 149iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{64d} + \frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^{3/2}*(a + I*a*\text{Tan}[c + d*x])^{5/2}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(3(-1)^{3/4}a^{5/2}((120*I)*A + 121*B)*\text{ArcTan}[\frac{(-1)^{3/4}\sqrt{a}\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + I*a*\text{Tan}[c + d*x]}}])/(64*d) + ((4 + 4*I)*a^{5/2}*(I*A + B)*\text{ArcTanh}[\frac{(1 + I)\sqrt{a}\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + I*a*\text{Tan}[c + d*x]}}])/d + (a^2*(152*A - (149*I)*B)*\sqrt{\text{Tan}[c + d*x]}\sqrt{a + I*a*\text{Tan}[c + d*x]})/(64*d) + (a^2*((104*I)*A + 107*B)*\text{Tan}[c + d*x]^{3/2}\sqrt{a + I*a*\text{Tan}[c + d*x]})/(96*d) - (a^2*(8*A - (11*I)*B)*\text{Tan}[c + d*x]^{5/2}\sqrt{a + I*a*\text{Tan}[c + d*x]})/(24*d) + ((I/4)*a*B*\text{Tan}[c + d*x]^{5/2}*(a + I*a*\text{Tan}[c + d*x])^{3/2})/d$

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{n, x}], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

#### Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

#### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dis

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rubi steps

$$\begin{aligned}
\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{iaB \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}}{4d} + \\
&= -\frac{a^2(8A - 11iB) \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{24d} \\
&= \frac{a^2(104iA + 107B) \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{96d} \\
&= \frac{a^2(152A - 149iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{64d} \\
&= \frac{a^2(152A - 149iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{64d} \\
&= \frac{a^2(152A - 149iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{64d} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= -\frac{3\sqrt[4]{-1} a^{5/2}(120A - 121iB) \tan^{-1}\left(\frac{(-1)^{3/4}}{\sqrt{a + ia \tan(c + dx)}}\right)}{64d}
\end{aligned}$$

**Mathematica [A]**

time = 7.45, size = 459, normalized size = 1.54

$$\frac{\left(\frac{\sqrt{2}e^{i(c+dx)}\sqrt{1-\frac{1-i+e^{2i(c+dx)}}{2}}}{1+e^{2i(c+dx)}} - \frac{e^{i(c+dx)}\sqrt{1-i+e^{2i(c+dx)}}}{\sqrt{2}} + \frac{e^{i(c+dx)}\sqrt{1-i+e^{2i(c+dx)}}}{\sqrt{2}} - \frac{e^{i(c+dx)}\sqrt{1-i+e^{2i(c+dx)}}}{\sqrt{2}}\right)}{512a^2(c+dx)(A\cos(c+dx)+B\sin(c+dx))} (a + i\tan(c+dx))^{5/2}(A+B\tan(c+dx))$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((Sqrt[2]\*Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*(-2048\*(A - I\*B)\*Log[E^(I\*(c + d\*x)) + Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] + 3\*Sqrt[2]\*(120\*A - (121\*I)\*B)\*(Log[1 - 3\*E^((2\*I)\*(c + d\*x)) - 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - Log[1 - 3\*E^((2\*I)\*(c + d\*x))] + 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])))/(E^((2\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))] + (2\*Sec[c + d\*x]^(7/2)\*(Cos[2\*c] - I\*Sin[2\*c])\*((1304\*A - (1205\*I)\*B)\*Cos[c + d\*x] + (520\*A - (583\*I)\*B)\*Cos[3\*(c + d\*x)] + (208\*I)\*A\*Sin[c + d\*x] + 70\*B\*Sin[c + d\*x] + (208\*I)\*A\*Sin[3\*(c + d\*x)] + 262\*B\*Sin[3\*(c + d\*x)]\*Sqrt[Tan[c + d\*x]])/(3\*Cos[2\*d\*x] + (3\*I)\*Sin[2\*d\*x]))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(512\*d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. 2(240) = 480.

time = 0.13, size = 742, normalized size = 2.49

method	result
derivativedivides	$\frac{\left(\sqrt{\tan(dx+c)}\sqrt{a(1+i\tan(dx+c))}\right)a^2\left(-96B\sqrt{ia}\sqrt{-ia}(\tan^3(dx+c))\sqrt{a\tan(dx+c)}\right)}{\dots}$
default	$\left(\sqrt{\tan(dx+c)}\sqrt{a(1+i\tan(dx+c))}\right)a^2\left(-96B\sqrt{ia}\sqrt{-ia}(\tan^3(dx+c))\sqrt{a\tan(dx+c)}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/384/d\*tan(d\*x+c)^(1/2)\*(a\*(1+I\*tan(d\*x+c)))^(1/2)\*a^2\*(-96\*B\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*tan(d\*x+c)^3\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)-128\*A\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*tan(d\*x+c)^2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+272\*I\*B\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*tan(d\*x+c)^2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+416\*I\*A\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)+447\*I\*B\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))

$$\left. \right)^{(1/2)} * (I*a)^{(1/2)} + a / (I*a)^{(1/2)} * (-I*a)^{(1/2)} * a - 894 * I * B * (I*a)^{(1/2)} * (-I*a)^{(1/2)} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} + 428 * B * (I*a)^{(1/2)} * (-I*a)^{(1/2)} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} * \tan(d*x+c) - 384 * I * (I*a)^{(1/2)} * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I*a)^{(1/2)} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} + I * a - 3 * a * \tan(d*x+c)) / (\tan(d*x+c) + I)) * a - 456 * A * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} * (I*a)^{(1/2)} + a) / (I*a)^{(1/2)} * (-I*a)^{(1/2)} * a + 912 * A * (I*a)^{(1/2)} * (-I*a)^{(1/2)} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} - 768 * I * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} * (I*a)^{(1/2)} + a) / (I*a)^{(1/2)} * (-I*a)^{(1/2)} * a + 384 * (I*a)^{(1/2)} * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I*a)^{(1/2)} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} + I * a - 3 * a * \tan(d*x+c)) / (\tan(d*x+c) + I)) * a - 768 * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)} * (I*a)^{(1/2)} + a) / (I*a)^{(1/2)} * (-I*a)^{(1/2)} * a) / (I*a)^{(1/2)} / (-I*a)^{(1/2)} / (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{(1/2)}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*tan(d\*x + c)^(3/2), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1017 vs.  $2(224) = 448$ .

time = 1.00, size = 1017, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{384} * (768 * \sqrt{2}) * \sqrt{-(I*A^2 + 2*A*B - I*B^2)} * a^5 / d^2 * (d * e^{(6*I*d*x + 6*I*c)} + 3*d * e^{(4*I*d*x + 4*I*c)} + 3*d * e^{(2*I*d*x + 2*I*c)} + d) * \log\left(\frac{(I * \sqrt{2}) * \sqrt{-(I*A^2 + 2*A*B - I*B^2)} * a^5 / d^2 * d * e^{(I*d*x + I*c)} + \sqrt{2} * ((-I*A - B) * a^2 * e^{(2*I*d*x + 2*I*c)} + (-I*A - B) * a^2) * \sqrt{a / (e^{(2*I*d*x + 2*I*c)} + 1)}}{(e^{(2*I*d*x + 2*I*c)} + 1)} * e^{(-I*d*x - I*c)} / ((-I*A - B) * a^2)\right) - 768 * \sqrt{2} * \sqrt{-(I*A^2 + 2*A*B - I*B^2)} * a^5 / d^2 * (d * e^{(6*I*d*x + 6*I*c)} + 3*d * e^{(4*I*d*x + 4*I*c)} + 3*d * e^{(2*I*d*x + 2*I*c)} + d) * \log\left(\frac{(-I * \sqrt{2}) * \sqrt{-(I*A^2 + 2*A*B - I*B^2)} * a^5 / d^2 * d * e^{(I*d*x + I*c)} + \sqrt{2} * ((-I*A - B) * a^2 * e^{(2*I*d*x + 2*I*c)} + (-I*A - B) * a^2) * \sqrt{a / (e^{(2*I*d*x + 2*I*c)} + 1)}}{(e^{(2*I*d*x + 2*I*c)} + 1)} * e^{(2*I*d*x + 2*I*c)} + I) / (e^{(2*I*d*x + 2*I*c)} + 1)\right)$$



```

*d*x + 2*I*c) + 1))) * e^(-I*d*x - I*c) / ((-I*A - B)*a^2)) + 2*sqrt(2)*(13*(56
*A - 65*I*B)*a^2*e^(7*I*d*x + 7*I*c) + 3*(504*A - 425*I*B)*a^2*e^(5*I*d*x +
5*I*c) + (1096*A - 1135*I*B)*a^2*e^(3*I*d*x + 3*I*c) + 3*(104*A - 107*I*B)
*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 9*sqrt(-(14400*I*A^2 + 29040*A*
B - 14641*I*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c)
+ 3*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((-120*I*A - 121*B)*a^2*e^(2*I*
d*x + 2*I*c) + (-120*I*A - 121*B)*a^2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 2*I*sqrt(-(144
00*I*A^2 + 29040*A*B - 14641*I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x -
I*c) / ((-120*I*A - 121*B)*a^2)) + 9*sqrt(-(14400*I*A^2 + 29040*A*B - 14641*
I*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2
*I*d*x + 2*I*c) + d)*log((sqrt(2)*((-120*I*A - 121*B)*a^2*e^(2*I*d*x + 2*I*
c) + (-120*I*A - 121*B)*a^2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*sqrt(-(14400*I*A^2 +
29040*A*B - 14641*I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c) / ((-1
20*I*A - 121*B)*a^2)) / (d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3
*d*e^(2*I*d*x + 2*I*c) + d)

```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.56Unable to divide
, perhaps due to rounding error%%{%}{[%%{poly1[-128,0]:[1,0,-2]}], [0
%%], 0
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x  
)
```

```
[Out] int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),  
x)
```

### 3.170 $\int \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=252

$$\frac{(-1)^{3/4} a^{5/2} (46A - 45iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + (4+4i) a^{5/2} (A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{8d}$$

[Out]  $-1/8*(-1)^{(3/4)}*a^{(5/2)}*(46*A-45*I*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d-(4+4*I)*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+1/8*a^2*(18*I*A+19*B)*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d-1/4*a^2*(2*A-3*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}*\tan(d*x+c)^{(3/2)}/d+1/3*I*a*B*\tan(d*x+c)^{(3/2)}*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

**Rubi** [A]

time = 0.60, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {3675, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} a^{5/2} (46A - 45iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + (4+4i) a^{5/2} (A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + \frac{a^2(2A-3iB) \tan^3(c+dx) \sqrt{a+ia \tan(c+dx)}}{4d} + \frac{a^2(19B+18iA) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{8d} + \frac{iaB \tan^3(c+dx) (a+ia \tan(c+dx))^{3/2}}{3d}}{8d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(a+I*a*\operatorname{Tan}[c+d*x])^{5/2}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $-1/8*((-1)^{(3/4)}*a^{(5/2)}*(46*A-(45*I)*B)*\operatorname{ArcTan}(((1+i)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]))/d-(((4+4*I)*a^{(5/2)}*(A-I*B)*\operatorname{ArcTanh}(((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]))/d+(a^2*((18*I)*A+19*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(8*d)-(a^2*(2*A-(3*I)*B)*\operatorname{Tan}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(4*d)+((I/3)*a*B*\operatorname{Tan}[c+d*x]^{(3/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a$

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x

] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx = \frac{iaB \tan^{3/2}(c + dx)(a + ia \tan(c + dx))^{3/2}}{3d} = -\frac{a^2(2A - 3iB) \tan^{3/2}(c + dx) \sqrt{a + ia \tan(c + dx)}}{4d} = \frac{a^2(18iA + 19B) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} = \frac{a^2(18iA + 19B) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} = \frac{a^2(18iA + 19B) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{8d} = -\frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a + ia \tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} = -\frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a + ia \tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} = -\frac{\sqrt{-1} a^{5/2}(46iA + 45B) \tan^{-1}\left(\frac{(-1)^{3/4}}{\sqrt{a + ia \tan(c + dx)}}\right)}{8d}$$

Mathematica [A]

time = 6.51, size = 426, normalized size = 1.69

$$\frac{\left(\frac{\sqrt{2} e^{-i \pi (1+n)}}{\sqrt{1+e^{2i(c+dx)}}} \left(\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^{(2B(A-iB)) \operatorname{Im}\left(e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}\right) + \sqrt{2} (18iA+19B) \left(\operatorname{Im}\left(-1-3e^{i(c+dx)} \sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}\right) - \operatorname{Im}\left(-1-3e^{i(c+dx)} \sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}\right)\right)}{\sqrt{-1+e^{2i(c+dx)}}} \frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right) + \frac{1 \operatorname{Im}\left(e^{i(c+dx)} \cos(2c) - \sin(2c)\right) (5iA+45B+19iA+45B) \cos(2c+2dx) + (-12i+26B) \operatorname{Im}\left(e^{i(c+dx)}\right) \sqrt{\tan(c+dx)}}{4 \cos(2c) + 4 \cos(2dx)} (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{64d \operatorname{sech}^2(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

```
[Out] (((Sqrt[2]*Sqrt[(-I)*(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x))))*
[*(-256*I)*(A - I*B)*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]]
+ Sqrt[2]*((46*I)*A + 45*B)*(Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(
I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] - Log[1 - 3*E^((2*I)*(c + d*x)
) + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(E^((2*I)*
(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
)*(c + d*x))]) + (4*Sec[c + d*x]^(5/2)*(Cos[2*c] - I*Sin[2*c])*((54*I)*A +
49*B + ((54*I)*A + 65*B)*Cos[2*(c + d*x)] + (-12*A + (26*I)*B)*Sin[2*(c +
d*x)])*Sqrt[Tan[c + d*x]]/(3*Cos[2*d*x] + (3*I)*Sin[2*d*x]))*(a + I*a*Tan[
c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/(64*d*Sec[c + d*x]^(7/2)*(A*Cos[c + d
*x] + B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(202) = 404.  
time = 0.12, size = 653, normalized size = 2.59

method	result
derivativedivides	$\frac{\left(\sqrt{\tan(dx+c)}\right) \sqrt{a(1+i \tan(dx+c))} a^2 \left(16B \sqrt{a \tan(dx+c)(1+i \tan(dx+c))} \sqrt{ia}\right)}{\dots}$
default	$\frac{\left(\sqrt{\tan(dx+c)}\right) \sqrt{a(1+i \tan(dx+c))} a^2 \left(16B \sqrt{a \tan(dx+c)(1+i \tan(dx+c))} \sqrt{ia}\right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/48/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(16*B*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-52*I*B*(I*a)
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+54*I*A
*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(1/
2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-108*I*A*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)+24*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I
*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-48*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)+I*a-3*a*tan(d*x+c))/(tan
(d*x+c)+I)*a+57*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)
)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-114*B*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+96*I*ln(1/2*(2*I*a*tan(d*x+c)
+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)
```

$$\begin{aligned} & \sqrt{a-48(Ia)^{1/2} \cdot 2^{1/2} \cdot \ln(-(-2 \cdot 2^{1/2}) \cdot (-Ia)^{1/2} \cdot (a \cdot \tan(dx+c) \cdot \\ & (1+I \cdot \tan(dx+c)))^{1/2} + Ia - 3 \cdot a \cdot \tan(dx+c)) / (\tan(dx+c) + I)} \cdot a - 96 \cdot \ln(1/2 \cdot (2 \cdot \\ & Ia \cdot \tan(dx+c) + 2 \cdot (a \cdot \tan(dx+c) \cdot (1+I \cdot \tan(dx+c))))^{1/2} \cdot (Ia)^{1/2} + a) / (Ia)^{1/2} \\ & \cdot (-Ia)^{1/2} \cdot a / (Ia)^{1/2} / (-Ia)^{1/2} / (a \cdot \tan(dx+c) \cdot (1+I \cdot \tan(dx+c)))^{1/2} \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(1/2)\*(a+I\*a\*tan(dx+c))^(5/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(dx + c) + A)\*(I\*a\*tan(dx + c) + a)^(5/2)\*sqrt(tan(dx + c)), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(188) = 376.

time = 0.79, size = 932, normalized size = 3.70

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(1/2)\*(a+I\*a\*tan(dx+c))^(5/2)\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \frac{1}{48} \cdot (96 \cdot \sqrt{2}) \cdot \sqrt{-(-IA^2 - 2AB + IB^2) \cdot a^5/d^2} \cdot (d \cdot e^{(4I \cdot dx + 4I \cdot c)} + 2 \cdot d \cdot e^{(2I \cdot dx + 2I \cdot c)} + d) \cdot \log((\sqrt{2}) \cdot \sqrt{-(-IA^2 - 2AB + IB^2) \cdot a^5/d^2} \cdot d \cdot e^{(I \cdot dx + I \cdot c)} + \sqrt{2}) \cdot ((-IA - B) \cdot a^2 \cdot e^{(2I \cdot dx + 2I \cdot c)} + (-IA - B) \cdot a^2) \cdot \sqrt{a/(e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I)/(e^{(2I \cdot dx + 2I \cdot c)} + 1)}) \cdot e^{(-I \cdot dx - I \cdot c)} / ((-IA - B) \cdot a^2) \\ & - 96 \cdot \sqrt{2} \cdot \sqrt{-(-IA^2 - 2AB + IB^2) \cdot a^5/d^2} \cdot (d \cdot e^{(4I \cdot dx + 4I \cdot c)} + 2 \cdot d \cdot e^{(2I \cdot dx + 2I \cdot c)} + d) \cdot \log(-(\sqrt{2}) \cdot \sqrt{-(-IA^2 - 2AB + IB^2) \cdot a^5/d^2} \cdot d \cdot e^{(I \cdot dx + I \cdot c)} - \sqrt{2}) \cdot ((-IA - B) \cdot a^2 \cdot e^{(2I \cdot dx + 2I \cdot c)} + (-IA - B) \cdot a^2) \cdot \sqrt{a/(e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I)/(e^{(2I \cdot dx + 2I \cdot c)} + 1)}) \cdot e^{(-I \cdot dx - I \cdot c)} / ((-IA - B) \cdot a^2) \\ & + 2 \cdot \sqrt{2} \cdot ((66 \cdot IA + 91 \cdot B) \cdot a^2 \cdot e^{(5I \cdot dx + 5I \cdot c)} - 2 \cdot (-54 \cdot IA - 49 \cdot B) \cdot a^2 \cdot e^{(3I \cdot dx + 3I \cdot c)} - 3 \cdot (-14 \cdot IA - 13 \cdot B) \cdot a^2 \cdot e^{(I \cdot dx + I \cdot c)}) \cdot \sqrt{a/(e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I)/(e^{(2I \cdot dx + 2I \cdot c)} + 1)} + 3 \cdot \sqrt{(2116 \cdot IA^2 + 4140 \cdot AB - 2025 \cdot IB^2) \cdot a^5/d^2} \cdot (d \cdot e^{(4I \cdot dx + 4I \cdot c)} + 2 \cdot d \cdot e^{(2I \cdot dx + 2I \cdot c)} + d) \cdot \log((\sqrt{2}) \cdot ((46 \cdot IA + 45 \cdot B) \cdot a^2 \cdot e^{(2I \cdot dx + 2I \cdot c)} + (46 \cdot IA + 45 \cdot B) \cdot a^2) \cdot \sqrt{a/(e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(-I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I)/(e^{(2I \cdot dx + 2I \cdot c)} + 1)} + 2 \cdot \sqrt{(2116 \cdot IA^2 + 4140 \cdot AB - 2025 \cdot IB^2) \cdot a^5/d^2} \cdot d \cdot e^{(I \cdot dx + I \cdot c)}) \cdot e^{(-I \cdot dx} \end{aligned}$$

$$- I*c)/((46*I*A + 45*B)*a^2)) - 3*\sqrt{(2116*I*A^2 + 4140*A*B - 2025*I*B^2)}*a^5/d^2*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*\log((\sqrt{2})*((46*I*A + 45*B)*a^2*e^(2*I*d*x + 2*I*c) + (46*I*A + 45*B)*a^2)*\sqrt{a/(e^(2*I*d*x + 2*I*c) + 1)}*\sqrt{(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)}) - 2*\sqrt{(2116*I*A^2 + 4140*A*B - 2025*I*B^2)}*a^5/d^2*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/((46*I*A + 45*B)*a^2)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:Evaluation time: 1.8Unable to divide, perhaps due to rounding error%%{%%{poly1[-16\*i,0]:[1,0,-2]%%},[0]%%},0

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\tan(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)



$$3.171 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=206

$$\frac{(-1)^{3/4}a^{5/2}(20iA+23B)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{4d} + \frac{(4-4i)a^{5/2}(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}$$

[Out]  $-1/4*(-1)^{(3/4)}*a^{(5/2)}*(20*I*A+23*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+(4-4*I)*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d-1/4*a^2*(4*A-7*I*B)*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d+1/2*I*a*B*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.46, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ ,

Rules used = {3675, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4}a^{5/2}(23B+20iA)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{4d} + \frac{(4-4i)a^{5/2}(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{a^2(4A-7iB)\sqrt{\tan(c+dx)}\sqrt{a+ia \tan(c+dx)}}{4d} + \frac{iaB\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x])}{\text{Sqrt}[\text{Tan}[c + d*x]]}, x]$

[Out]  $-1/4*((-1)^{(3/4)}*a^{(5/2)}*((20*I)*A + 23*B)*\text{ArcTan}[\frac{((-1)^{(3/4)}*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])}{\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]}])/d + ((4 - 4*I)*a^{(5/2)}*(A - I*B)*\text{ArcTanh}[\frac{((1 + I)*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])}{\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]}])/d - (a^2*(4*A - (7*I)*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(4*d) + ((I/2)*a*B*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^{(3/2)})/d$

**Rule 65**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))^{(m_.)}*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\text{Int}[\frac{(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \text{Simp}[\frac{1}{(\text{Rt}[a, 2]*\text{Rt}[b, 2])}] * \text{ArcTan}[\frac{\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])}{\text{Rt}[a, 2]}], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \|\| \text{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{iaB \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}}{2d} + \frac{1}{2} \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{a^2(4A - 7iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{4d} \\
&= -\frac{a^2(4A - 7iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{4d} \\
&= -\frac{a^2(4A - 7iB) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}{4d} \\
&= -\frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= -\frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= \frac{\sqrt{-1} a^{5/2}(20A - 23iB) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{4d}
\end{aligned}$$

**Mathematica [A]**

time = 6.53, size = 394, normalized size = 1.91

$$\frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx)) \left( \frac{\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}} \left( 128(A-iB) \log(e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) - \sqrt{2} (20A - 23iB) \left( \log(1 - 3e^{i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) - \log(1 - 3e^{i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) \right) - 4\sqrt{\sec(c + dx)} \cos(c) - i \sin(c) \right) \sqrt{\tan(c + dx)}}{(14A - 9iB - 2B \tan(c + dx))} \right)}{32d \sec^2(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
```

```
[Out] ((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])*((Sqrt[2]*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x))))*(128*(A - I*B)*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] - Sqrt[2]*(20*A - (23*I)*B)*(Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] - Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/(E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))] - (8*Sqrt[Sec[c + d*x]]*(Cos[2*c] - I*Sin[2*c])*Sqrt[Tan[c + d*x]]*(4*A - (9*I)*B + 2*B*T
```

$\text{an}[c + d*x]))/(\text{Cos}[d*x] + I*\text{Sin}[d*x]^2))/(\text{32}*d*\text{Sec}[c + d*x]^{(7/2)}*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(164) = 328.

time = 0.12, size = 566, normalized size = 2.75

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^{a^2} \left(-9iB \ln\left(\frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}}\right)\right)}{\dots}$
default	$\frac{\sqrt{a(1+i \tan(dx+c))} \left(\sqrt{\tan(dx+c)}\right)^{a^2} \left(-9iB \ln\left(\frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}}\right)\right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\frac{1}{8}d*(a*(1+I*\tan(dx+c)))^{(1/2)}*\tan(dx+c)^{(1/2)}*a^2*(-9*I*B*\ln(1/2*(2*I*a*\tan(dx+c)+2*(a*\tan(dx+c)*(1+I*\tan(dx+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a+18*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(dx+c)*(1+I*\tan(dx+c)))^{(1/2)}-4*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(dx+c)*(1+I*\tan(dx+c)))^{(1/2)}*\tan(dx+c)+8*I*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(dx+c)*(1+I*\tan(dx+c))))^{(1/2)}+I*a-3*a*\tan(dx+c))/(\tan(dx+c)+I))*a+12*A*\ln(1/2*(2*I*a*\tan(dx+c)+2*(a*\tan(dx+c)*(1+I*\tan(dx+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a-8*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(dx+c)*(1+I*\tan(dx+c)))^{(1/2)}+16*I*\ln(1/2*(2*I*a*\tan(dx+c)+2*(a*\tan(dx+c)*(1+I*\tan(dx+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*a*(-I*a)^{(1/2)}-8*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(dx+c)*(1+I*\tan(dx+c))))^{(1/2)}+I*a-3*a*\tan(dx+c))/(\tan(dx+c)+I))*a+16*\ln(1/2*(2*I*a*\tan(dx+c)+2*(a*\tan(dx+c)*(1+I*\tan(dx+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a)/(I*a)^{(1/2)}/(-I*a)^{(1/2)}/(a*\tan(dx+c)*(1+I*\tan(dx+c)))^{(1/2)}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, alg  
orithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)/sqrt(tan(d*x +  
c)), x)`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 849 vs.  $2(152) = 304$ .  
time = 0.76, size = 849, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(16*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A - B)*a^2) - 16*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A - B)*a^2) + 2*sqrt(2)*((4*A - 11*I*B)*a^2*e^(3*I*d*x + 3*I*c) + (4*A - 7*I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((20*I*A + 23*B)*a^2*e^(2*I*d*x + 2*I*c) + (20*I*A + 23*B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 2*I*sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c))e^(-I*d*x - I*c)/((20*I*A + 23*B)*a^2) - sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((20*I*A + 23*B)*a^2*e^(2*I*d*x + 2*I*c) + (20*I*A + 23*B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c))e^(-I*d*x - I*c)/((20*I*A + 23*B)*a^2))/(d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^{\frac{5}{2}} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x))/sqrt(tan(c + d*x)), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(1/2), x)

$$3.172 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=196

$$\frac{(-1)^{3/4}a^{5/2}(2A-5iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{(4+4i)a^{5/2}(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}$$

[Out]  $(-1)^{3/4}a^{5/2}(2A-5iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)/d + (4+4i)a^{5/2}(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)/d + a^2(2I)A-B)\sqrt{\tan(c+dx)}\sqrt{a+ia \tan(c+dx)}/d - (2aA(a+ia \tan(c+dx))^{3/2})/d - (2aA(a+ia \tan(c+dx))^{3/2})/d - (2aA(a+ia \tan(c+dx))^{3/2})/d - (2aA(a+ia \tan(c+dx))^{3/2})/d$

**Rubi [A]**

time = 0.45, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {3674, 3675, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4}a^{5/2}(2A-5iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{(4+4i)a^{5/2}(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia \tan(c+dx)}}{d} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^{5/2}*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{3/2}}, x]$

[Out]  $((-1)^{3/4}a^{5/2}(2A - (5I)B)\text{ArcTan}[\frac{(-1)^{3/4}\sqrt{a}\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + I*a*\text{Tan}[c + d*x]}}])/d + ((4 + 4I)a^{5/2}(A - I*B)\text{ArcTan}[\frac{(1 + I)\sqrt{a}\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + I*a*\text{Tan}[c + d*x]}}])/d + (a^2*((2I)A - B)\sqrt{\text{Tan}[c + d*x]}\sqrt{a + I*a*\text{Tan}[c + d*x]})/d - (2aA*(a + I*a*\text{Tan}[c + d*x])^{3/2})/(d*\sqrt{\text{Tan}[c + d*x]})$

**Rule 65**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b})^n), x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\text{Int}[\frac{(a_. + (b_.)*(x_.)^2)^{-1}}{x\_Symbol], x\_Symbol] := \text{Simp}[\frac{1}{(\text{Rt}[a, 2]*\text{Rt}[b, 2])}] * \text{ArcTan}[\frac{\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])}{\text{Rt}[a, 2]}], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a



$a^2 + b^2, 0]$  && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^{3/2}}{d\sqrt{\tan(c + dx)}} + 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx \\
 &= \frac{a^2(2iA - B)\sqrt{\tan(c + dx)}\sqrt{a + ia \tan(c + dx)}}{d} - 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx \\
 &= \frac{a^2(2iA - B)\sqrt{\tan(c + dx)}\sqrt{a + ia \tan(c + dx)}}{d} - 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx \\
 &= \frac{a^2(2iA - B)\sqrt{\tan(c + dx)}\sqrt{a + ia \tan(c + dx)}}{d} - 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx \\
 &= \frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} - 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx \\
 &= \frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} - 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx \\
 &= \frac{4\sqrt{-1} a^{5/2}(2iA + 5B) \tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} - 2 \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx
 \end{aligned}$$

### Mathematica [A]

time = 6.89, size = 389, normalized size = 1.98

$$\frac{\left(\frac{\sqrt{2} e^{-2i(c+dx)} \sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}} \left(\frac{8i(A+B)\log(e^{i(c+dx)} + \sqrt{-1+e^{2i(c+dx)}})}{(2i(A+B)\log(e^{i(c+dx)} + \sqrt{-1+e^{2i(c+dx)}})) - \sqrt{2}(2A-3iB)\left(\log\left(\frac{1-3e^{2i(c+dx)}-2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}}{1-3e^{2i(c+dx)}+2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}}\right) - \log\left(\frac{1-3e^{2i(c+dx)}+2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}}{1-3e^{2i(c+dx)}-2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}}\right)}\right)}{\sqrt{-1+e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}} - \frac{8(B+2A\cos(c+dx))\sqrt{\sec(c+dx)} \left(\frac{\cos(2c)}{\cos(dx)+1\sin(2c)}\right) \sqrt{\tan(c+dx)}}{(\cos(dx)+1\sin(2c))} \right) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{8d \sec^3(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2),x]

[Out] (((Sqrt[2]\*Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))])\*(32\*(I\*A + B)\*Log[E^(I\*(c + d\*x)) + Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - I\*Sqrt[2]\*(2\*A - (5\*I)\*B)\*(Log[1 - 3\*E^((2\*I)\*(c + d\*x)) - 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - Log[1 - 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])))/(E^((2\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))])) - (8\*(B + 2\*A\*Cot[c + d\*x])\*Sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c])\*Sqrt[Tan[c + d\*x]])/(Cos[d\*x] + I\*Sin[d\*x])^2\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(8\*d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs.  $2(161) = 322$ .  
time = 0.12, size = 565, normalized size = 2.88

method	result
derivativedivides	$\sqrt{a(1+i\tan(dx+c))} a^2 \left( \frac{6iA \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{ia} + \dots}{\dots} \right)$
default	$\sqrt{a(1+i \tan(dx+c))} a^2 \left( \frac{6iA \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{ia} + \dots}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x,method=\_RETURNVERBOSE)

[Out] 1/2/d\*(a\*(1+I\*tan(d\*x+c)))^(1/2)\*a^2\*(6\*I\*A\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c)-2\*I\*(I\*a)^(1/2)\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I)\*a\*tan(d\*x+c)+3\*B\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c)-2\*B\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)+4\*I\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c)-2\*(I\*a)^(1/2)\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)+I\*a-3\*a\*tan(d\*x+c))/(tan(d\*x+c)+I)\*a\*tan(d\*x+c)-4\*A\*(I\*a)^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)-4\*ln(1/2\*(2\*I\*a\*tan(d\*x+c)+2\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c))))^(1/2)\*(I\*a)^(1/2)+a)/(I\*a)^(1/2))\*(-I\*a)^(1/2)\*a\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)/(I\*a)^(1/2)/(-I\*a)^(1/2)

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(150) = 300.

time = 0.70, size = 857, normalized size = 4.37

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] 
$$-1/2*(4*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*d*e^{(I*d*x + I*c)} + \sqrt{2}*((-I*A - B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^2)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) - 4*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-(\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*d*e^{(I*d*x + I*c)} - \sqrt{2}*((-I*A - B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^2)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)}))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) + 2*\sqrt{2}*((2*I*A + B)*a^2*e^{(3*I*d*x + 3*I*c)} + (2*I*A - B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)} + \sqrt{(4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*((2*I*A + 5*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (2*I*A + 5*B)*a^2)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)} + 2*\sqrt{(4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2}*d*e^{(I*d*x + I*c)}))e^{(-I*d*x - I*c)}/((2*I*A + 5*B)*a^2)) - \sqrt{(4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log((\sqrt{2}*((2*I*A + 5*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (2*I*A + 5*B)*a^2)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)} - 2*\sqrt{(4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2}*d*e^{(I*d*x + I*c)}))e^{(-I*d*x - I*c)}/((2*I*A + 5*B)*a^2)))/(d*e^{(2*I*d*x + 2*I*c)} - d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^{\frac{5}{2}} (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2), x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(5/2)\*(A + B\*tan(c + d\*x))/tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(3/2), x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(3/2), x)

$$3.173 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=190

$$\frac{2(-1)^{3/4}a^{5/2}B \operatorname{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{(4+4i)a^{5/2}(iA+B) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}$$

[Out]  $2*(-1)^{(3/4)}*a^{(5/2)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))}^{(1/2)})/d+(4+4*I)*a^{(5/2)}*(I*A+B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))}^{(1/2)})/d-2*a^2*(2*I*A+B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/3*a*A*(a+I*a*\tan(d*x+c))^{(3/2)}/d/\tan(d*x+c)^{(3/2)}$

**Rubi** [A]

time = 0.45, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3674, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(4+4i)a^{5/2}(B+iA) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{2(-1)^{3/4}a^{5/2}B \operatorname{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a^2(B+2iA)\sqrt{a+ia \tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{((a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]))}{\operatorname{Tan}[c + d*x]^{(5/2)}}, x]$

[Out]  $(2*(-1)^{(3/4)}*a^{(5/2)}*B*\operatorname{ArcTan}[\frac{((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}])/d + ((4 + 4*I)*a^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\frac{((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}])/d - (2*a^2*((2*I)*A + B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) - (2*a*A*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d*\operatorname{Tan}[c + d*x]^{(3/2)})$

**Rule 65**

$\operatorname{Int}[\frac{((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}}{x\_Symbol}] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[\frac{((a_.) + (b_.)*(x_.)^2)^{-1}}{x\_Symbol}] := \operatorname{Simp}[\frac{1}{(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2])}] * \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \|\| \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{5/2}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} + \frac{2}{3} \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{5/2}(c + dx)} dx \\
&= -\frac{2a^2(2iA + B) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a^2(2iA + B) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a^2(2iA + B) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
&= \frac{2(-1)^{3/4}a^{5/2}B \tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} + \dots
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 456 vs. 2(190) = 380.  
time = 7.18, size = 456, normalized size = 2.40

$$\frac{\left( \frac{\sqrt{2} e^{-i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}}}{\sqrt{2} B \log\left(\frac{e^{i(c+dx)} (\sqrt{2} - \sqrt{2} e^{i(c+dx)}) \sqrt{-1 + e^{2i(c+dx)}}}{e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}}\right)} + B(A + B) \log\left(e^{-i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}\right) - \sqrt{2} B \log\left(\frac{e^{i(c+dx)} (\sqrt{2} + \sqrt{2} e^{i(c+dx)}) \sqrt{-1 + e^{2i(c+dx)}}}{e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}}\right)}{\sqrt{\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}}} - \frac{4i(A + B) \sqrt{A \cos(c + dx)} \sqrt{\frac{\sin(c + dx)}{\cos(c + dx)}} \operatorname{atan2}(\sin(2c), -\cos(2c))}{8i(A + B) \sqrt{A \cos(c + dx)} \sqrt{\tan(c + dx)}} \right) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{2d \sec^2(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (((Sqrt[2]\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*(Sqrt[2]\*B\*Log[(2\*E^(((7\*I)/2)\*c)\*(Sqrt[2] - I\*Sqrt[2]\*E^(I\*(c + d\*x)) + (2\*I)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/(B\*(-I + E^(I\*(c + d\*x)))))) + 8\*(I\*A + B)\*Log[(E^(I\*(c + d\*x)) + Sqrt[-1 + E^((2\*I)\*(c + d\*x))])/E^(I\*c)] - Sqrt[2]\*B\*Log[((-2\*I)\*E^(((7\*I)/2)\*c)\*((-I)\*Sqrt[2] + Sqrt[2]\*E^(I\*(c + d\*x)) + 2\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/(B\*(I + E^(I\*(c + d\*x))))]

)]/(E^((3\*I)\*(c + d\*x))\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x)))] - (4\*((7\*I)\*A + 3\*B + A\*Cot[c + d\*x])\*Sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c]))/(3\*(Cos[d\*x] + I\*Sin[d\*x])^2\*Sqrt[Tan[c + d\*x]]))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(2\*d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 619 vs.  $2(154) = 308$ .

time = 0.12, size = 620, normalized size = 3.26

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} a^2 \left( -9iB \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{a(1+i \tan(dx+c))} \right)}{\dots}$
default	$\frac{\sqrt{a(1+i \tan(dx+c))} a^2 \left( -9iB \ln \left( \frac{2ia \tan(dx+c)+2\sqrt{a \tan(dx+c)}(1+i \tan(dx+c))}{2\sqrt{ia}} \right) \sqrt{a(1+i \tan(dx+c))} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x,method=\_RE  
TURNVERBOSE)

[Out] 
$$-1/3/d*(a*(1+I*\tan(d*x+c)))^{1/2}*a^2/\tan(d*x+c)^{3/2}*(-9*I*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^2+3*I*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^2+14*I*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+12*A*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^2+6*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^2-3*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^2+6*B*(I*a)^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*\tan(d*x+c)+6*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^2+2*A*(I*a)^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2})/(I*a)^{1/2}/(-I*a)^{1/2}/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)/tan(d\*x + c)^(5/2), x)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs.  $2(144) = 288$ .  
time = 0.69, size = 846, normalized size = 4.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \frac{1}{6} * (12 * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^{5/d^2} * (d * e^{(4 * I * d * x + 4 * I * c)} - 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((I * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^{5/d^2} * d * e^{(I * d * x + I * c)} + \sqrt{2} * ((-I * A - B) * a^2 * e^{(2 * I * d * x + 2 * I * c)} + (-I * A - B) * a^2) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{((-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1))) * e^{(-I * d * x - I * c)} / ((-I * A - B) * a^2)) \\ & - 12 * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^{5/d^2} * (d * e^{(4 * I * d * x + 4 * I * c)} - 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((-I * \sqrt{2} * \sqrt{-(I * A^2 + 2 * A * B - I * B^2)} * a^{5/d^2} * d * e^{(I * d * x + I * c)} + \sqrt{2} * ((-I * A - B) * a^2 * e^{(2 * I * d * x + 2 * I * c)} + (-I * A - B) * a^2) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{((-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1))) * e^{(-I * d * x - I * c)} / ((-I * A - B) * a^2)) \\ & + 4 * \sqrt{2} * ((8 * A - 3 * I * B) * a^2 * e^{(5 * I * d * x + 5 * I * c)} + 2 * A * a^2 * e^{(3 * I * d * x + 3 * I * c)} - 3 * (2 * A - I * B) * a^2 * e^{(I * d * x + I * c)}) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)} * \sqrt{((-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)) + 3 * \sqrt{4 * I * B^2 * a^5 / d^2} * (d * e^{(4 * I * d * x + 4 * I * c)} - 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((\sqrt{2} * (B * a^2 * e^{(2 * I * d * x + 2 * I * c)} + B * a^2) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{((-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)) + I * \sqrt{4 * I * B^2 * a^5 / d^2} * d * e^{(I * d * x + I * c)}) * e^{(-I * d * x - I * c)} / (B * a^2)) - 3 * \sqrt{4 * I * B^2 * a^5 / d^2} * (d * e^{(4 * I * d * x + 4 * I * c)} - 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \log((\sqrt{2} * (B * a^2 * e^{(2 * I * d * x + 2 * I * c)} + B * a^2) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{((-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)) - I * \sqrt{4 * I * B^2 * a^5 / d^2} * d * e^{(I * d * x + I * c)}) * e^{(-I * d * x - I * c)} / (B * a^2)) / (d * e^{(4 * I * d * x + 4 * I * c)} - 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^{\frac{5}{2}} (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x))/tan(c + d*x)*
*(5/2), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(5/2)
,x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(5/2)
, x)
```

$$3.174 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=185

$$\frac{(4+4i)a^{5/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a^2(8iA+5B)\sqrt{a+ia \tan(c+dx)}}{15d \tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2(38A-35iB)\sqrt{a+ia \tan(c+dx)}}{15d \sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{5d \tan^{\frac{5}{2}}(c+dx)}$$

[Out]  $(-4-4*I)*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d+2/15*a^2*(38*A-35*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/15*a^2*(8*I*A+5*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(3/2)}-2/5*a*A*(a+I*a*\tan(d*x+c))^{(3/2)}/d/\tan(d*x+c)^{(5/2)}$

**Rubi** [A]

time = 0.38, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(4+4i)a^{5/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a^2(5B+8iA)\sqrt{a+ia \tan(c+dx)}}{15d \tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2(38A-35iB)\sqrt{a+ia \tan(c+dx)}}{15d \sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{5d \tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}*(A+B*\operatorname{Tan}[c+d*x])]/\operatorname{Tan}[c+d*x]^{(7/2)}, x]$

[Out]  $((-4-4*I)*a^{(5/2)}*(A-I*B)*\operatorname{ArcTanH}(((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]))/d - (2*a^2*((8*I)*A+5*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(15*d*\operatorname{Tan}[c+d*x]^{(3/2)}) + (2*a^2*(38*A-(35*I)*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(15*d*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]) - (2*a*A*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)})/(5*d*\operatorname{Tan}[c+d*x]^{(5/2)})$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

$\operatorname{Int}[(a_*) + (b_*)(x_)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\tan[(e_*) + (f_*)(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\tan[(e_*) + (f_*)(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

#### Rule 3674

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

```

#### Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{7/2}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^{3/2}}{5d \tan^{5/2}(c + dx)} + \frac{2}{5} \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{7/2}(c + dx)} dx \\
&= -\frac{2a^2(8iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{5d \tan^{5/2}(c + dx)} \\
&= -\frac{2a^2(8iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2a^2(38A - 3B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
&= -\frac{2a^2(8iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2a^2(38A - 3B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
&= -\frac{2a^2(8iA + 5B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2a^2(38A - 3B) \sqrt{a + ia \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
&= -\frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 8.32, size = 323, normalized size = 1.75

$$\frac{4\sqrt{2} e^{-2ic} \sqrt{e^{idx}} \sqrt{\frac{-i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}} (e^{i(c+dx)} \sqrt{1 - e^{2i(c+dx)}} (5B(3 - 7e^{2i(c+dx)} + 4e^{4i(c+dx)}) + iA(15 - 35e^{2i(c+dx)} + 26e^{4i(c+dx)})) + 15(iA + B) (-1 + e^{2i(c+dx)})^3 \text{ArcSin}(e^{i(c+dx)})) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{15d (1 - e^{2i(c+dx)})^{7/2} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sec^2(c + dx) (\cos(dx) + i \sin(dx))^{5/2} (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] (-4\*sqrt[2]\*sqrt[E^(I\*d\*x)]\*sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]\*(E^(I\*(c + d\*x))\*sqrt[1 - E^((2\*I)\*(c + d\*x))]\*(5\*B\*(3 - 7\*E^((2\*I)\*(c + d\*x)) + 4\*E^((4\*I)\*(c + d\*x))) + I\*A\*(15 - 35\*E^((2\*I)\*(c + d\*x)) + 26\*E^((4\*I)\*(c + d\*x)))) + 15\*(I\*A + B)\*(-1 + E^((2\*I)\*(c + d\*x)))^3\*ArcSin[E^(I\*(c + d\*x))]\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(15\*d\*E^((2\*I)\*c)\*(1 - E^((2\*I)\*(c + d\*x)))^(7/2)\*sqrt[E^(I\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x)))]\*Sec[c + d\*x]^(7/2)\*(Cos[d\*x] + I\*Sin[d\*x])^(5/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(152) = 304.

time = 0.12, size = 709, normalized size = 3.83

method	result
derivativedivides	$\frac{\sqrt{a(1+i \tan(dx+c))} a^2 \left( -76A\sqrt{ia} \sqrt{-ia} (\tan^2(dx+c)) \sqrt{a \tan(dx+c)(1+i \tan(dx+c))} \right)}{-}$
default	$\frac{\sqrt{a(1+i \tan(dx+c))} a^2 \left( -76A\sqrt{ia} \sqrt{-ia} (\tan^2(dx+c)) \sqrt{a \tan(dx+c)(1+i \tan(dx+c))} \right)}{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/15/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(5/2)*(-76*A*(I*a)^(1/2)*
(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*I*A*ln(1
/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)
/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-15*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*
2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+
c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+70*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+
c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*B*ln(1/2*(2*I*a*tan(d*x+c)+2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1
/2)*a*tan(d*x+c)^3+30*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-15*(I
*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+22*I*A*(I*a)^(
1/2)*(-I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-30*ln(1/
2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/
(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3+10*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+6*A*(I*a)^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)
```

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 608 vs.  $2(141) = 282$ .

time = 0.61, size = 608, normalized size = 3.29

$$\left( \frac{\sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a^5/d^2} (d e^{(6Ix + 6Ic)} - 3d e^{(4Ix + 4Ic)} + 3d e^{(2Ix + 2Ic)} - d) \log(\sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a^5/d^2} d e^{(Ix + Ic)} + \sqrt{2}((-IA - B)a^2 e^{(2Ix + 2Ic)} + (-IA - B)a^2) \sqrt{a/(e^{(2Ix + 2Ic)} + 1)}) \sqrt{(-I e^{(2Ix + 2Ic)} + I)/(e^{(2Ix + 2Ic)} + 1)}) e^{(-Ix - Ic)} / ((-IA - B)a^2)} - 15 \sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a^5/d^2} (d e^{(6Ix + 6Ic)} - 3d e^{(4Ix + 4Ic)} + 3d e^{(2Ix + 2Ic)} - d) \log(-\sqrt{2} \sqrt{-(-IA^2 - 2AB + IB^2)a^5/d^2} d e^{(Ix + Ic)} - \sqrt{2}((-IA - B)a^2 e^{(2Ix + 2Ic)} + (-IA - B)a^2) \sqrt{a/(e^{(2Ix + 2Ic)} + 1)}) \sqrt{(-I e^{(2Ix + 2Ic)} + I)/(e^{(2Ix + 2Ic)} + 1)}) e^{(-Ix - Ic)} / ((-IA - B)a^2)} - 2 \sqrt{2} (2(-13IA - 10B)a^2 e^{(7Ix + 7Ic)} + 3(3IA + 5B)a^2 e^{(5Ix + 5Ic)} + 20(IA + B)a^2 e^{(3Ix + 3Ic)} + 15(-IA - B)a^2 e^{(Ix + Ic)}) \sqrt{a/(e^{(2Ix + 2Ic)} + 1)} \sqrt{(-I e^{(2Ix + 2Ic)} + I)/(e^{(2Ix + 2Ic)} + 1)}) / (d e^{(6Ix + 6Ic)} - 3d e^{(4Ix + 4Ic)} + 3d e^{(2Ix + 2Ic)} - d) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/15\*(15\*sqrt(2)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^5/d^2)\*(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*log((sqrt(2)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^5/d^2)\*d\*e^(I\*d\*x + I\*c) + sqrt(2)\*((-I\*A - B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) + (-I\*A - B)\*a^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a^2) - 15\*sqrt(2)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^5/d^2)\*(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*log(-sqrt(2)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^5/d^2)\*d\*e^(I\*d\*x + I\*c) - sqrt(2)\*((-I\*A - B)\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) + (-I\*A - B)\*a^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))))\*e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a^2) - 2\*sqrt(2)\*(2\*(-13\*I\*A - 10\*B)\*a^2\*e^(7\*I\*d\*x + 7\*I\*c) + 3\*(3\*I\*A + 5\*B)\*a^2\*e^(5\*I\*d\*x + 5\*I\*c) + 20\*(I\*A + B)\*a^2\*e^(3\*I\*d\*x + 3\*I\*c) + 15\*(-I\*A - B)\*a^2\*e^(I\*d\*x + I\*c))\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1)))/(d\*e^(6\*I\*d\*x + 6\*I\*c) - 3\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - d)

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x)

[Out] Timed out

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for

the root of a polynomial with parameters. This might be wrong. Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\tan(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(7/2), x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(7/2), x)



$$3.175 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=231

$$\frac{(4-4i)a^{5/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a^2(10iA+7B)\sqrt{a+ia \tan(c+dx)}}{35d \tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2(80A-77iB)\sqrt{a+ia \tan(c+dx)}}{105d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2(7B+10iA)\sqrt{a+ia \tan(c+dx)}}{35d \tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(133B+130iA)\sqrt{a+ia \tan(c+dx)}}{105d \sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{7d \tan^{\frac{7}{2}}(c+dx)}$$

[Out] (4-4\*I)\*a^(5/2)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d+4/105\*a^2\*(130\*I\*A+133\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/35\*a^2\*(10\*I\*A+7\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)+2/105\*a^2\*(80\*A-77\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)-2/7\*a\*A\*(a+I\*a\*tan(d\*x+c))^(3/2)/d/tan(d\*x+c)^(7/2)

**Rubi [A]**

time = 0.51, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(4-4i)a^{5/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{2a^2(80A-77iB)\sqrt{a+ia \tan(c+dx)}}{105d \tan^{\frac{3}{2}}(c+dx)} - \frac{2a^2(7B+10iA)\sqrt{a+ia \tan(c+dx)}}{35d \tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(133B+130iA)\sqrt{a+ia \tan(c+dx)}}{105d \sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{7d \tan^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] ((4 - 4\*I)\*a^(5/2)\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])]/d - (2\*a^2\*((10\*I)\*A + 7\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(35\*d\*Tan[c + d\*x]^(5/2)) + (2\*a^2\*(80\*A - (77\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(3/2)) + (4\*a^2\*((130\*I)\*A + 133\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Sqrt[Tan[c + d\*x]]) - (2\*a\*A\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(7\*d\*Tan[c + d\*x]^(7/2))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_)
+ (f_)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

#### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

#### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{9/2}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^{3/2}}{7d \tan^{7/2}(c + dx)} + \frac{2}{7} \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{9/2}(c + dx)} dx \\
&= -\frac{2a^2(10iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{7d \tan^{7/2}(c + dx)} \\
&= -\frac{2a^2(10iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2a^2(80A - 7B)}{7d \tan^{7/2}(c + dx)} \\
&= -\frac{2a^2(10iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2a^2(80A - 7B)}{7d \tan^{7/2}(c + dx)} \\
&= -\frac{2a^2(10iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2a^2(80A - 7B)}{7d \tan^{7/2}(c + dx)} \\
&= -\frac{2a^2(10iA + 7B) \sqrt{a + ia \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2a^2(80A - 7B)}{7d \tan^{7/2}(c + dx)} \\
&= -\frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 9.47, size = 363, normalized size = 1.57

$$\frac{4\sqrt{2} e^{-2ic} \sqrt{e^{2dx}} \sqrt{\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}} \left( e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} (71B(-15 + 50e^{2i(c+dx)} - 61e^{4i(c+dx)} + 26e^{6i(c+dx)}) - 5A(-21 + 70e^{2i(c+dx)} - 77e^{4i(c+dx)} + 40e^{6i(c+dx)})) + 105(A - iB)(-1 + e^{2i(c+dx)})^4 \log \left( \frac{e^{i(c+dx)} + \sqrt{-1 + e^{2i(c+dx)}}}{\sqrt{a + ia \tan(c + dx)}} \right) \right)}{105d(-1 + e^{2i(c+dx)})^{9/2} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{sech}^3(c + dx) (\cos(dx) + i \sin(dx))^{5/2} (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] (4\*sqrt[2]\*sqrt[E^(I\*d\*x)]\*sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x))))\*(E^(I\*(c + d\*x))\*sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*((7\*I)\*B\*(-15 + 50\*E^((2\*I)\*(c + d\*x)) - 61\*E^((4\*I)\*(c + d\*x)) + 26\*E^((6\*I)\*(c + d\*x))) - 5\*A\*(-21 + 70\*E^((2\*I)\*(c + d\*x)) - 77\*E^((4\*I)\*(c + d\*x)) + 40\*E^((6\*I)\*(c + d\*x)))) + 105\*(A - I\*B)\*(-1 + E^((2\*I)\*(c + d\*x)))^4\*Log[E^(I\*(c + d\*x)) + sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x])/(105\*d\*E^((2\*I)\*c)\*(-1 + E^((2\*I)\*(c + d\*x)))^(9/2)\*sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sec[c + d\*x]^(7/2)\*(Cos[d\*x] + I\*Sin[d\*x])^(5/2)\*(A\*cos[c + d\*x] + B\*sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs.  $2(190) = 380$ .

time = 0.12, size = 798, normalized size = 3.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{105}d*(a*(1+I*\tan(d*x+c)))^{1/2}*a^2/\tan(d*x+c)^{7/2}*(532*B*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^3*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+520*I*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^3*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}-420*I*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^4+160*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}-154*I*B*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)^2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}+420*A*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^4+210*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^4-105*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^4+105*I*(I*a)^{1/2}*2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^4+210*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{1/2}*(I*a)^{1/2}+a)/(I*a)^{1/2}*(-I*a)^{1/2}*a*\tan(d*x+c)^4-90*I*A*(I*a)^{1/2}*(-I*a)^{1/2}*\tan(d*x+c)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}-42*B*(I*a)^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}*\tan(d*x+c)-30*A*(I*a)^{1/2}*(-I*a)^{1/2}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2})/(I*a)^{1/2}/(-I*a)^{1/2}/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{1/2}$

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 657 vs.  $2(177) = 354$ .

time = 0.65, size = 657, normalized size = 2.84

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="fricas")

[Out] 
$$-2/105*(105*\sqrt{2}*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log((I*\sqrt{2}*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*d*e^{(I*d*x + I*c)} + \sqrt{2}*((-I*A - B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^2)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) - 105*\sqrt{2}*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log((-I*\sqrt{2}*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*d*e^{(I*d*x + I*c)} + \sqrt{2}*((-I*A - B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-I*A - B)*a^2)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) + 2*\sqrt{2}*(2*(100*A - 91*I*B)*a^2*e^{(9*I*d*x + 9*I*c)} - 5*(37*A - 49*I*B)*a^2*e^{(7*I*d*x + 7*I*c)} - 7*(5*A - 11*I*B)*a^2*e^{(5*I*d*x + 5*I*c)} + 245*(A - I*B)*a^2*e^{(3*I*d*x + 3*I*c)} - 105*(A - I*B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}/(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)$$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(9/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular v alue [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^{5/2}}{\tan(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^(5/2))/tan(c + d*x)^(9/2), x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^(5/2))/tan(c + d*x)^(9/2), x)
```

$$3.176 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=277

$$\frac{(4+4i)a^{5/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a^2(4iA+3B)\sqrt{a+ia \tan(c+dx)}}{21d \tan^{\frac{7}{2}}(c+dx)} + \frac{2a^2(46A-45iB)\sqrt{a+ia \tan(c+dx)}}{9d \tan^{\frac{9}{2}}(c+dx)}$$

[Out] (4+4\*I)\*a^(5/2)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d-8/315\*a^2\*(197\*A-195\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/21\*a^2\*(4\*I\*A+3\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2)+2/105\*a^2\*(46\*A-45\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)+8/315\*a^2\*(59\*I\*A+60\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)-2/9\*a\*A\*(a+I\*a\*tan(d\*x+c))^(3/2)/d/tan(d\*x+c)^(9/2)

**Rubi [A]**

time = 0.64, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(4+4i)a^{5/2}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{8a^2(60B+59iA)\sqrt{a+ia \tan(c+dx)}}{315d \tan^{\frac{9}{2}}(c+dx)} + \frac{2a^2(46A-45iB)\sqrt{a+ia \tan(c+dx)}}{105d \tan^{\frac{5}{2}}(c+dx)} - \frac{2a^2(3B+4iA)\sqrt{a+ia \tan(c+dx)}}{21d \tan^{\frac{7}{2}}(c+dx)} - \frac{8a^2(197A-195iB)\sqrt{a+ia \tan(c+dx)}}{315d \sqrt{\tan(c+dx)}} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{9d \tan^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] ((4 + 4\*I)\*a^(5/2)\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])]/d - (2\*a^2\*((4\*I)\*A + 3\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(21\*d\*Tan[c + d\*x]^(7/2)) + (2\*a^2\*(46\*A - (45\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(5/2)) + (8\*a^2\*((59\*I)\*A + 60\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(315\*d\*Tan[c + d\*x]^(3/2)) - (8\*a^2\*(197\*A - (195\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(315\*d\*Sqrt[Tan[c + d\*x]]) - (2\*a\*A\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(9\*d\*Tan[c + d\*x]^(9/2))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*tan[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3674

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rubi steps



$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^{3/2}}{9d \tan^{\frac{9}{2}}(c + dx)} + \frac{2}{9} \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)} dx \\
&= -\frac{2a^2(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2a^2(46A - 4B)}{10d \tan^{\frac{5}{2}}(c + dx)} \\
&= -\frac{2a^2(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2a^2(46A - 4B)}{10d \tan^{\frac{5}{2}}(c + dx)} \\
&= -\frac{2a^2(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2a^2(46A - 4B)}{10d \tan^{\frac{5}{2}}(c + dx)} \\
&= -\frac{2a^2(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2a^2(46A - 4B)}{10d \tan^{\frac{5}{2}}(c + dx)} \\
&= -\frac{2a^2(4iA + 3B) \sqrt{a + ia \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2a^2(46A - 4B)}{10d \tan^{\frac{5}{2}}(c + dx)} \\
&= \frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 9.25, size = 246, normalized size = 0.89

$$a^2 \left( \frac{1260(A - iB) \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right)}{\sqrt{\frac{i(-1 + e^{2i(c+dx)})}{1 + e^{2i(c+dx)}}}} + \frac{\csc^2(2(c+dx))(-2331A + 2205iB + 12(251A - 260iB) \cos(2(c+dx)) + (-961A + 915iB) \cos(4(c+dx)) + 282iA \sin(2(c+dx)) + 390B \sin(2(c+dx)) - 331iA \sin(4(c+dx)) - 285B \sin(4(c+dx)))}{\tan^{\frac{7}{2}}(c+dx)} \right) \sqrt{a + ia \tan(c + dx)}$$

315d

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] (a^2\*((1260\*(A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/(E^(I\*(c + d\*x))\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x))])) + (Csc[2\*(c + d\*x)]^2\*(-2331\*A + (2205\*I)\*B + 12\*(251\*A - (260\*I)\*B)\*Cos[2\*(c + d\*x)] + (-961\*A + (915\*I)\*B)\*Cos[4\*(c + d\*x)] + (282\*I)\*A\*Sin[2\*(c + d\*x)] + 390\*B\*Sin[2\*(c + d\*x)] -

$(331*I)*A*\sin[4*(c + d*x)] - 285*B*\sin[4*(c + d*x)])/\tan[c + d*x]^{(5/2)}* \sqrt{a + I*a*\tan[c + d*x]})/(315*d)$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 886 vs.  $2(228) = 456$ .

time = 0.12, size = 887, normalized size = 3.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x,method=_R ETURNVERBOSE)`

[Out]  $\frac{1}{315} \frac{d}{dx} (a(1+I\tan(dx+c)))^{1/2} a^2 / \tan(dx+c)^{9/2} (-1576 A (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c)^4 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} - 315 I (Ia)^{1/2} 2^{1/2} \ln(-(-2 \cdot 2^{1/2} (-Ia)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + Ia - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^5 - 190 I A (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c) (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + 480 B (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c)^3 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + 472 I A (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c)^3 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + 1260 B \ln(1/2 (2 I a \tan(dx+c) + 2 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} (Ia)^{1/2} + a) / (Ia)^{1/2}) (-Ia)^{1/2} a \tan(dx+c)^5 + 630 I \ln(1/2 (2 I a \tan(dx+c) + 2 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} (Ia)^{1/2} + a) / (Ia)^{1/2}) (-Ia)^{1/2} a \tan(dx+c)^5 - 315 (Ia)^{1/2} 2^{1/2} \ln(-(-2 \cdot 2^{1/2} (-Ia)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + Ia - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^5 + 276 A (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c)^2 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} - 270 I B (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c)^2 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} - 630 \ln(1/2 (2 I a \tan(dx+c) + 2 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} (Ia)^{1/2} + a) / (Ia)^{1/2}) (-Ia)^{1/2} a \tan(dx+c)^5 + 1260 I A \ln(1/2 (2 I a \tan(dx+c) + 2 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} (Ia)^{1/2} + a) / (Ia)^{1/2}) (-Ia)^{1/2} a \tan(dx+c)^5 + 1560 I B (Ia)^{1/2} (-Ia)^{1/2} \tan(dx+c)^4 (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} - 90 B (Ia)^{1/2} (-Ia)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c) - 70 A (Ia)^{1/2} (-Ia)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} / (Ia)^{1/2} / (-Ia)^{1/2} / (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2}$

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs.  $2(213) = 426$ .

time = 0.58, size = 722, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A - B)*a^2) - 315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)*log(-(sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))e^(-I*d*x - I*c)/((-I*A - B)*a^2) + 2*sqrt(2)*(2*(323*I*A + 300*B)*a^2*e^(11*I*d*x + 11*I*c) + 77*(-13*I*A - 15*B)*a^2*e^(9*I*d*x + 9*I*c) + 18*(38*I*A + 25*B)*a^2*e^(7*I*d*x + 7*I*c) + 42*(23*I*A + 20*B)*a^2*e^(5*I*d*x + 5*I*c) + 1050*(-I*A - B)*a^2*e^(3*I*d*x + 3*I*c) + 315*(I*A + B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for  
 the root of a polynomial with parameters. This might be wrong.Non regular v  
 alue [

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\tan(c + dx)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(11/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(11/2), x)

$$3.177 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=323

$$\frac{(4+4i)a^{5/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2a^2(14iA+11B)\sqrt{a+ia \tan(c+dx)}}{99d \tan^{\frac{9}{2}}(c+dx)} + \frac{2a^2(212A-209iB)\sqrt{a+ia \tan(c+dx)}}{693d \tan^{\frac{7}{2}}(c+dx)} + \frac{4a^2(250iA+253B)\sqrt{a+ia \tan(c+dx)}}{1155d \tan^{\frac{5}{2}}(c+dx)} - \frac{8a^2(655A-649iB)\sqrt{a+ia \tan(c+dx)}}{3465d \tan^{\frac{3}{2}}(c+dx)} - \frac{8a^2(2155iA+2167B)\sqrt{a+ia \tan(c+dx)}}{3465d \tan^{\frac{1}{2}}(c+dx)}$$

[Out] (4+4\*I)\*a^(5/2)\*(I\*A+B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d-8/3465\*a^2\*(2155\*I\*A+2167\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(1/2)-2/99\*a^2\*(14\*I\*A+11\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(9/2)+2/693\*a^2\*(212\*A-209\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2)+4/1155\*a^2\*(250\*I\*A+253\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)-8/3465\*a^2\*(655\*A-649\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)-2/11\*a\*A\*(a+I\*a\*tan(d\*x+c))^(3/2)/d/tan(d\*x+c)^(11/2)

**Rubi** [A]

time = 0.78, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3674, 3679, 12, 3625, 211}

$$\frac{(4+4i)a^{5/2}(B+iA) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{8a^2(655A-649iB)\sqrt{a+ia \tan(c+dx)}}{3465d \tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(253B+250iA)\sqrt{a+ia \tan(c+dx)}}{1155d \tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2(212A-209iB)\sqrt{a+ia \tan(c+dx)}}{693d \tan^{\frac{7}{2}}(c+dx)} - \frac{2a^2(14iB+11iA)\sqrt{a+ia \tan(c+dx)}}{99d \tan^{\frac{9}{2}}(c+dx)} - \frac{8a^2(2167B+2155iA)\sqrt{a+ia \tan(c+dx)}}{3465d \tan^{\frac{11}{2}}(c+dx)} - \frac{2aA(a+ia \tan(c+dx))^{3/2}}{11d \tan^{\frac{11}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(13/2)), x]

[Out] ((4 + 4\*I)\*a^(5/2)\*(I\*A + B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])]/d - (2\*a^2\*((14\*I)\*A + 11\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(99\*d\*Tan[c + d\*x]^(9/2)) + (2\*a^2\*(212\*A - (209\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(693\*d\*Tan[c + d\*x]^(7/2)) + (4\*a^2\*((250\*I)\*A + 253\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(1155\*d\*Tan[c + d\*x]^(5/2)) - (8\*a^2\*(655\*A - (649\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3465\*d\*Tan[c + d\*x]^(3/2)) - (8\*a^2\*((2155\*I)\*A + 2167\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3465\*d\*Sqrt[Tan[c + d\*x]]) - (2\*a\*A\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(11\*d\*Tan[c + d\*x]^(11/2))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{\frac{13}{2}}(c + dx)} dx &= -\frac{2aA(a + ia \tan(c + dx))^{3/2}}{11d \tan^{\frac{11}{2}}(c + dx)} + \frac{2}{11} \int \frac{(a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{\frac{13}{2}}(c + dx)} dx \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} - \frac{2aA(a + ia \tan(c + dx))^{3/2}}{11d \tan^{\frac{11}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a^2(14iA + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2a^2(212A + 11B) \sqrt{a + ia \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

### Mathematica [A]

time = 11.35, size = 328, normalized size = 1.02

$$\frac{4\sqrt{2}a^2(14iA + 11B)\sqrt{-1 + e^{2i(c + dx)}} \sqrt{\frac{a + ia \tan(c + dx)}{1 + e^{2i(c + dx)}}} \operatorname{tanh}^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{-1 + e^{2i(c + dx)}}} \right) + a^2 \cos^2(c + dx) \sin^2(c + dx) (69i95A - 47iB) \cos(c + dx) + (-3225A + 6743iB) \cos^3(c + dx) + 3995A \cos^5(c + dx) - 3641iB \cos^3(c + dx) + 8493iA \sin(c + dx) + 8441iB \sin(c + dx) - 62185iA \sin^3(c + dx) - 63703iB \sin^3(c + dx) + 10925iA \sin^5(c + dx) + 10671iB \sin^5(c + dx)}{\sqrt{a + ia \tan(c + dx)}}}{99d \tan^{\frac{9}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(13/2), x]

[Out] (4\*sqrt(2)\*a^2\*(I\*A + B)\*sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*sqrt[(a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]\*ArcTanh[E^(I\*(c + d\*x))/sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/(d\*E^(I\*(c + d\*x))\*sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]

$$\frac{1}{(1 + E^{((2*I)*(c + d*x)))})} - (a^2 * Csc[c + d*x]^3 * Sec[c + d*x]^2 * (66 * (95 * A - (47 * I) * B) * Cos[c + d*x] + (-5225 * A + (6743 * I) * B) * Cos[3 * (c + d*x)] + 3995 * A * Cos[5 * (c + d*x)] - (3641 * I) * B * Cos[5 * (c + d*x)] + (84810 * I) * A * Sin[c + d*x] + 84414 * B * Sin[c + d*x] - (42185 * I) * A * Sin[3 * (c + d*x)] - 43703 * B * Sin[3 * (c + d*x)] + (10925 * I) * A * Sin[5 * (c + d*x)] + 10571 * B * Sin[5 * (c + d*x)]) * Sqrt[a + I * a * Tan[c + d*x]]) / (27720 * d * Tan[c + d*x]^(5/2))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 975 vs.  $2(266) = 532$ .

time = 0.12, size = 976, normalized size = 3.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(13/2),x,method=\_RETURVERBOSE)

[Out] 
$$\begin{aligned} & -1/3465/d * (a * (1 + I * \tan(d*x+c)))^{1/2} * a^2 / \tan(d*x+c)^{11/2} * (17336 * B * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^5 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 6930 * I * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c))))^{1/2} * (I * a)^{1/2} + a) / (I * a)^{1/2}) * (-I * a)^{1/2} * a * \tan(d*x+c)^6 + 3465 * I * (I * a)^{1/2} * 2^{1/2} * \ln(-(-2 * 2^{1/2} * (-I * a)^{1/2} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c))))^{1/2} + I * a - 3 * a * \tan(d*x+c)) / (\tan(d*x+c) + I)) * a * \tan(d*x+c)^6 + 5240 * A * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^4 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 17240 * I * A * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^5 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 13860 * A * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c))))^{1/2} * (I * a)^{1/2} + a) / (I * a)^{1/2}) * (-I * a)^{1/2} * a * \tan(d*x+c)^6 - 3000 * I * A * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^3 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} - 3465 * (I * a)^{1/2} * 2^{1/2} * \ln(-(-2 * 2^{1/2} * (-I * a)^{1/2} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c))))^{1/2} + I * a - 3 * a * \tan(d*x+c)) / (\tan(d*x+c) + I)) * a * \tan(d*x+c)^6 - 3036 * B * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^3 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 2090 * I * B * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 6930 * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c))))^{1/2} * (I * a)^{1/2} + a) / (I * a)^{1/2}) * (-I * a)^{1/2} * a * \tan(d*x+c)^6 - 2120 * A * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 1610 * I * A * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c) * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} - 13860 * I * B * \ln(1/2 * (2 * I * a * \tan(d*x+c) + 2 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c))))^{1/2} * (I * a)^{1/2} + a) / (I * a)^{1/2}) * (-I * a)^{1/2} * a * \tan(d*x+c)^6 - 5192 * I * B * (I * a)^{1/2} * (-I * a)^{1/2} * \tan(d*x+c)^4 * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} + 770 * B * (I * a)^{1/2} * (-I * a)^{1/2} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} * \tan(d*x+c) + 630 * A * (I * a)^{1/2} * (-I * a)^{1/2} * (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2}) / (I * a)^{1/2} / (-I * a)^{1/2} / (a * \tan(d*x+c) * (1 + I * \tan(d*x+c)))^{1/2} \end{aligned}$$

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(13/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs.  $2(249) = 498$ .  
time = 0.54, size = 771, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(13/2),x, algorithm="fricas")

[Out] 
$$\frac{2}{3465} \cdot (3465 \sqrt{2}) \sqrt{-(I^2 A^2 + 2AB - I^2 B^2)} a^{5/d^2} (d e^{(12I dx + 12I c)} - 6d e^{(10I dx + 10I c)} + 15d e^{(8I dx + 8I c)} - 20d e^{(6I dx + 6I c)} + 15d e^{(4I dx + 4I c)} - 6d e^{(2I dx + 2I c)} + d) \log((I \sqrt{2}) \sqrt{-(I^2 A^2 + 2AB - I^2 B^2)} a^{5/d^2} d e^{(I dx + I c)} + \sqrt{2} ((-IA - B) a^2 e^{(2I dx + 2I c)} + (-IA - B) a^2) \sqrt{a/(e^{(2I dx + 2I c)} + 1)}) \sqrt{(-I e^{(2I dx + 2I c)} + I)/(e^{(2I dx + 2I c)} + 1)}) e^{(-I dx - I c)} / ((-IA - B) a^2) - 3465 \sqrt{2} \sqrt{-(I^2 A^2 + 2AB - I^2 B^2)} a^{5/d^2} (d e^{(12I dx + 12I c)} - 6d e^{(10I dx + 10I c)} + 15d e^{(8I dx + 8I c)} - 20d e^{(6I dx + 6I c)} + 15d e^{(4I dx + 4I c)} - 6d e^{(2I dx + 2I c)} + d) \log((-I \sqrt{2}) \sqrt{-(I^2 A^2 + 2AB - I^2 B^2)} a^{5/d^2} d e^{(I dx + I c)} + \sqrt{2} ((-IA - B) a^2 e^{(2I dx + 2I c)} + (-IA - B) a^2) \sqrt{a/(e^{(2I dx + 2I c)} + 1)}) \sqrt{(-I e^{(2I dx + 2I c)} + I)/(e^{(2I dx + 2I c)} + 1)}) e^{(-I dx - I c)} / ((-IA - B) a^2) + 2 \sqrt{2} (2(3730A - 3553IB) a^2 e^{(13I dx + 13I c)} - 9(1805A - 2013IB) a^2 e^{(11I dx + 11I c)} + 55(397A - 337IB) a^2 e^{(9I dx + 9I c)} + 66(95A - 47IB) a^2 e^{(7I dx + 7I c)} - 1386(15A - 16IB) a^2 e^{(5I dx + 5I c)} + 15015(A - IB) a^2 e^{(3I dx + 3I c)} - 3465(A - IB) a^2 e^{(I dx + I c)}) \sqrt{a/(e^{(2I dx + 2I c)} + 1)}) \sqrt{(-I e^{(2I dx + 2I c)} + I)/(e^{(2I dx + 2I c)} + 1)}) / (d e^{(12I dx + 12I c)} - 6d e^{(10I dx + 10I c)} + 15d e^{(8I dx + 8I c)} - 20d e^{(6I dx + 6I c)} + 15d e^{(4I dx + 4I c)} - 6d e^{(2I dx + 2I c)} + d)$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(13/2),x)

[Out] Timed out

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(13/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\tan(c + dx)^{13/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(13/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/tan(c + d\*x)^(13/2), x)

$$3.178 \quad \int \frac{(a+ia \tan(c+dx))^{5/2} \left( \frac{3bB}{2a} + B \tan(c+dx) \right)}{\tan^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=190

$$\frac{2(-1)^{3/4} a^{5/2} B \operatorname{ArcTan} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d} + \frac{(2+2i) a^{3/2} (2a+3ib) B \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d}$$

[Out] 2\*(-1)^(3/4)\*a^(5/2)\*B\*arctan((-1)^(3/4)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c)^(1/2))/d+(2+2\*I)\*a^(3/2)\*(2\*a+3\*I\*b)\*B\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c)^(1/2))/d-2\*a\*(a+3\*I\*b)\*B\*(a+I\*a\*tan(d\*x+c)^(1/2)/d/tan(d\*x+c)^(1/2)-b\*B\*(a+I\*a\*tan(d\*x+c)^(3/2)/d/tan(d\*x+c)^(3/2))

Rubi [A]

time = 0.48, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {3674, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{2(-1)^{3/4} a^{5/2} B \operatorname{ArcTan} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d} + \frac{(2+2i) a^{3/2} B (2a+3ib) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d} - \frac{bB(a+ia \tan(c+dx))^{3/2}}{d \tan^{\frac{3}{2}}(c+dx)} - \frac{2aB(a+3ib) \sqrt{a+ia \tan(c+dx)}}{d \sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^(5/2)\*((3\*b\*B)/(2\*a) + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (2\*(-1)^(3/4)\*a^(5/2)\*B\*ArcTan[(-1)^(3/4)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + I\*a\*Tan[c + d\*x]]/d + ((2 + 2\*I)\*a^(3/2)\*(2\*a + (3\*I)\*b)\*B\*ArcTanh[(1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + I\*a\*Tan[c + d\*x]]/d - (2\*a\*(a + (3\*I)\*b)\*B\*Sqrt[a + I\*a\*Tan[c + d\*x]]/(d\*Sqrt[Tan[c + d\*x]]) - (b\*B\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(d\*Tan[c + d\*x]^(3/2))

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)]]/Sqrt[(c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3674

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^{5/2} \left(\frac{3bB}{2a} + B \tan(c + dx)\right)}{\tan^{5/2}(c + dx)} dx &= -\frac{bB(a + ia \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} + \frac{2}{3} \int \frac{(a + ia \tan(c + dx))^{5/2}}{\tan^{5/2}(c + dx)} dx \\
 &= -\frac{2a(a + 3ib)B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + ia \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
 &= -\frac{2a(a + 3ib)B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + ia \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
 &= -\frac{2a(a + 3ib)B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + ia \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
 &= \frac{(2 + 2i)a^{3/2}(2a + 3ib)B \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
 &= \frac{(2 + 2i)a^{3/2}(2a + 3ib)B \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} \\
 &= \frac{2(-1)^{3/4}a^{5/2}B \tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{d} + \dots
 \end{aligned}$$

**Mathematica [A]**

time = 6.68, size = 326, normalized size = 1.72

$$\frac{\left( \frac{2i\sqrt{2}e^{-2i(c+dx)}\sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(-2(2a+3ib)\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\sqrt{2}a\tanh^{-1}\left(\frac{\sqrt{2}e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}} - \frac{2(2a+7ib+b\cot(c+dx))\sqrt{\sec(c+dx)}\sqrt{\cos(2c)-i\sin(2c)}}{(\cos(dx)+i\sin(dx))^2\sqrt{\tan(c+dx)}} \right) (a + ia \tan(c + dx))^{5/2} \left(\frac{3bB}{2a} + B \tan(c + dx)\right)}{d \sec^{3/2}(c + dx)(3b \cos(c + dx) + 2a \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^(5/2)\*((3\*b\*B)/(2\*a) + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2),x]

[Out] ((((-2\*I)\*Sqrt[2]\*Sqrt[((-I)\*(-1 + E^((2\*I)\*(c + d\*x))))]/(1 + E^((2\*I)\*(c + d\*x))))\*(-2\*(2\*a + (3\*I)\*b)\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] + Sqrt[2]\*a\*ArcTanh[(Sqrt[2]\*E^(I\*(c + d\*x))]/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]))/E^((2\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))] - (2\*(2\*a + (7\*I)\*b + b\*Cot[c + d\*x])\*Sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c]))/((Cos[d\*x] + I\*Sin[d\*x])^2

$\text{Sqrt}[\text{Tan}[c + d*x]])*(a + I*a*\text{Tan}[c + d*x])^{(5/2)}*((3*b*B)/(2*a) + B*\text{Tan}[c + d*x]))/(d*\text{Sec}[c + d*x]^{(7/2)}*(3*b*\text{Cos}[c + d*x] + 2*a*\text{Sin}[c + d*x]))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 619 vs.  $2(156) = 312$ .

time = 0.18, size = 620, normalized size = 3.26

method	result
derivativedivides	$Ba \sqrt{a(1+i \tan(dx+c))} \left( \frac{6i \ln \left( \frac{2ia \tan(dx+c)+2 \sqrt{a \tan(dx+c)} (1+i \tan(dx+c))}{2 \sqrt{ia}} \right) \sqrt{ia} + a}{2 \sqrt{ia}} \right)$
default	$Ba \sqrt{a(1+i \tan(dx+c))} \left( \frac{6i \ln \left( \frac{2ia \tan(dx+c)+2 \sqrt{a \tan(dx+c)} (1+i \tan(dx+c))}{2 \sqrt{ia}} \right) \sqrt{ia} + a}{2 \sqrt{ia}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{2}d*B*a*(a*(1+I*\tan(d*x+c)))^{(1/2)}*(6*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a^{2*\tan(d*x+c)^2-I*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^2+2*I*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\tan(d*x+c)^2-14*I*\tan(d*x+c)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*b-12*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*b*\tan(d*x+c)^2-(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^2-2*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c))))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\tan(d*x+c)^2-4*a*\tan(d*x+c)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}*(-I*a)^{(1/2)}-2*b*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}*(-I*a)^{(1/2)}/\tan(d*x+c)^{(3/2)}/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}/(I*a)^{(1/2)}/(-I*a)^{(1/2)}$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,algorithm="maxima")`

[Out]  $\frac{1}{2} \int \frac{(2B \tan(dx + c) + 3Bb/a)(Ia \tan(dx + c) + a)^{5/2}}{\tan(dx + c)^{5/2}} dx$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 900 vs.  $2(146) = 292$ .  
time = 0.60, size = 900, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")`

[Out]  $\frac{1}{2} (2\sqrt{2})(d e^{4Ix + 4Ic} - 2d e^{2Ix + 2Ic} + d) \sqrt{(-4IB^2a^5 + 12B^2a^4b + 9IB^2a^3b^2)/d^2} \log(\sqrt{2} d \sqrt{(-4IB^2a^5 + 12B^2a^4b + 9IB^2a^3b^2)/d^2} e^{Ix + Ic} + \sqrt{2} (2IBa^2 - 3Bab + (2IBa^2 - 3Bab) e^{2Ix + 2Ic})) \sqrt{a/(e^{2Ix + 2Ic} + 1)} \sqrt{(-I e^{2Ix + 2Ic} + I)/(e^{2Ix + 2Ic} + 1))} e^{-Ix - Ic} / (2IBa^2 - 3Bab) - 2\sqrt{2} (d e^{4Ix + 4Ic} - 2d e^{2Ix + 2Ic} + d) \sqrt{(-4IB^2a^5 + 12B^2a^4b + 9IB^2a^3b^2)/d^2} \log(-\sqrt{2} d \sqrt{(-4IB^2a^5 + 12B^2a^4b + 9IB^2a^3b^2)/d^2} e^{Ix + Ic} - \sqrt{2} (2IBa^2 - 3Bab) e^{2Ix + 2Ic}) \sqrt{a/(e^{2Ix + 2Ic} + 1)} \sqrt{(-I e^{2Ix + 2Ic} + I)/(e^{2Ix + 2Ic} + 1))} e^{-Ix - Ic} / (2IBa^2 - 3Bab) + 4\sqrt{2} (Bab e^{3Ix + 3Ic} - (IBa^2 - 4Bab) e^{5Ix + 5Ic} - (-IBa^2 + 3Bab) e^{Ix + Ic}) \sqrt{a/(e^{2Ix + 2Ic} + 1)} \sqrt{(-I e^{2Ix + 2Ic} + I)/(e^{2Ix + 2Ic} + 1))} + \sqrt{4IB^2a^5/d^2} (d e^{4Ix + 4Ic} - 2d e^{2Ix + 2Ic} + d) \log(\sqrt{2} (Ba^2 e^{2Ix + 2Ic} + Ba^2) \sqrt{a/(e^{2Ix + 2Ic} + 1)} \sqrt{(-I e^{2Ix + 2Ic} + I)/(e^{2Ix + 2Ic} + 1))} + I \sqrt{4IB^2a^5/d^2} d e^{Ix + Ic}) e^{-Ix - Ic} / (Ba^2) - \sqrt{4IB^2a^5/d^2} (d e^{4Ix + 4Ic} - 2d e^{2Ix + 2Ic} + d) \log(\sqrt{2} (Ba^2 e^{2Ix + 2Ic} + Ba^2) \sqrt{a/(e^{2Ix + 2Ic} + 1)} \sqrt{(-I e^{2Ix + 2Ic} + I)/(e^{2Ix + 2Ic} + 1))} - I \sqrt{4IB^2a^5/d^2} d e^{Ix + Ic}) e^{-Ix - Ic} / (Ba^2)) / (d e^{4Ix + 4Ic} - 2d e^{2Ix + 2Ic} + d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \left( \int \frac{2a^3 \sqrt{ia \tan(c+dx) + a}}{\tan^2(c+dx)} dx + \int \frac{-2a^3 \sqrt{ia \tan(c+dx) + a}}{\sqrt{\tan(c+dx)}} dx + \int \frac{4ia^2 \sqrt{ia \tan(c+dx) + a}}{\sqrt{\tan(c+dx)}} dx + \int \frac{3a^2 \sqrt{ia \tan(c+dx) + a}}{\tan^2(c+dx)} dx + \int \frac{-3a^2 \sqrt{ia \tan(c+dx) + a}}{\sqrt{\tan(c+dx)}} dx + \int \frac{6a^2 \sqrt{ia \tan(c+dx) + a}}{\tan^2(c+dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))**(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)`

```
[Out] B*(Integral(2*a**3*sqrt(I*a*tan(c + d*x) + a)/tan(c + d*x)**(3/2), x) + Integral(-2*a**3*sqrt(I*a*tan(c + d*x) + a)*sqrt(tan(c + d*x)), x) + Integral(4*I*a**3*sqrt(I*a*tan(c + d*x) + a)/sqrt(tan(c + d*x)), x) + Integral(3*a**2*b*sqrt(I*a*tan(c + d*x) + a)/tan(c + d*x)**(5/2), x) + Integral(-3*a**2*b*sqrt(I*a*tan(c + d*x) + a)/sqrt(tan(c + d*x)), x) + Integral(6*I*a**2*b*sqrt(I*a*tan(c + d*x) + a)/tan(c + d*x)**(3/2), x))/(2*a)
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B \tan(c + dx) + \frac{3Bb}{2a}) (a + a \tan(c + dx) i)^{5/2}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(5/2),x)
```

```
[Out] int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(5/2), x)
```



$$3.179 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=205

$$\frac{(-1)^{3/4}(2iA - B)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a}d} - \frac{(\frac{1}{2} - \frac{i}{2})(A - iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a}d}$$

[Out]  $(-1)^{3/4}*(2*I*A-B)*\arctan((-1)^{3/4}*a^{1/2}*\tan(d*x+c)^{1/2}/(a+I*a*\tan(d*x+c))^{1/2})/d/a^{1/2}+(-1/2+1/2*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{1/2}*\tan(d*x+c)^{1/2}/(a+I*a*\tan(d*x+c))^{1/2})/d/a^{1/2}-(A+2*I*B)*\tan(d*x+c)^{1/2}*(a+I*a*\tan(d*x+c))^{1/2}/a/d+(I*A-B)*\tan(d*x+c)^{3/2}/d/(a+I*a*\tan(d*x+c))^{1/2}$

**Rubi [A]**

time = 0.45, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {3676, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4}(-B+2iA)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a}d} + \frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia \tan(c+dx)}}{ad} - \frac{(\frac{1}{2}-\frac{i}{2})(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out]  $((-1)^{3/4}*((2*I)*A - B)*\text{ArcTan}[((-1)^{3/4}*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(\text{Sqrt}[a]*d) - ((1/2 - I/2)*(A - I*B)*\text{ArcTanh}[(1 + I)*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(\text{Sqrt}[a]*d) + ((I*A - B)*\text{Tan}[c + d*x]^{3/2})/(d*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]) - ((A + (2*I)*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(a*d)$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 209**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

## Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx &= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{\int \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d\sqrt{a+ia \tan(c+dx)}} dx \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(A+2iB) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{ad} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(A+2iB) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{ad} \\
&= \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(A+2iB) \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}}{ad} \\
&= \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} + \frac{(iA-B)}{d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} + \frac{(iA-B)}{d\sqrt{a+ia \tan(c+dx)}} \\
&= -\frac{\sqrt[4]{-1} (2A+iB) \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA+B)}{d\sqrt{a+ia \tan(c+dx)}}
\end{aligned}$$

## Mathematica [A]

time = 3.05, size = 277, normalized size = 1.35

$$\frac{\left(\frac{\sqrt{2} \sqrt{-1+e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left((iA+B) \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) + \sqrt{2} (-2iA+B) \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)}{\sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}} \sqrt{\sec(c+dx)}}} - 2((A+2iB) \cos(c+dx) - B \sin(c+dx)) \sqrt{\tan(c+dx)}\right) (A+B \tan(c+dx))}{2d(A \cos(c+dx) + B \sin(c+dx)) \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((Sqrt[2]*Sqrt[-1 + E^((2*I)*(c + d*x))])*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))])*((I*A + B)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]) + Sqrt[2]*((-2*I)*A + B)*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/(Sqrt[(-I)*(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[Sec[c + d*x]]) - 2*((A + (2*I)*B)*Cos[c + d*x] - B*Sin[c + d*x])*Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x))/(2*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs.  $2(165) = 330$ .

time = 0.15, size = 1135, normalized size = 5.54

method	result	size
derivativedivides	Expression too large to display	1135
default	Expression too large to display	1135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(-I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a-2*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)+B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)^2+8*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-8*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+2*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+2*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)-4*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)^2+4*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a+4*I*A*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-2*I*B*ln(1/2*(2*I*a*tan(d*x+c)
```



$$\begin{aligned} & \frac{(e^{d^2x + c}) \sqrt{a/(e^{2d^2x + 2c}) + 1}}{(e^{2d^2x + 2c}) + 1} \sqrt{(-Ie^{2d^2x + 2c} + I)/(e^{2d^2x + 2c}) + 1} \\ & + (-3Ia^2d^2e^{2d^2x + 2c} + Iad^2) \sqrt{(-4I^2A^2 + 4A^2B + I^2B^2)/(a^2d^2)} \\ & / ((2IA - B)e^{2d^2x + 2c} + 2IA - B) + 2\sqrt{2}((A + 3IB)e^{2d^2x + 2c} + A + IB) \sqrt{a/(e^{2d^2x + 2c}) + 1} \\ & \sqrt{(-Ie^{2d^2x + 2c} + I)/(e^{2d^2x + 2c}) + 1}} e^{-I^2d^2x - I^2c} / (a^2d) \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{\sqrt{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.Non regular value [

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{3/2} (A + B \tan(c + dx))}{\sqrt{a + a \tan(c + dx)} \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2), x)

$$3.180 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=156

$$\frac{2\sqrt[4]{-1} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} + \frac{(i...)}{d}$$

[Out]  $-2*(-1)^{1/4}*B*\arctan((-1)^{3/4}*a^{1/2}*\tan(d*x+c)^{1/2}/(a+I*a*\tan(d*x+c))^{1/2})/d/a^{1/2}-(1/2+1/2*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{1/2}*\tan(d*x+c)^{1/2}/(a+I*a*\tan(d*x+c))^{1/2})/d/a^{1/2}+(I*A-B)*\tan(d*x+c)^{1/2}/d/(a+I*a*\tan(d*x+c))^{1/2}$

**Rubi** [A]

time = 0.31, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia \tan(c+dx)}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} - \frac{2\sqrt[4]{-1} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(A+B*\operatorname{Tan}[c+d*x]))/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]],x]$

[Out]  $(-2*(-1)^{1/4}*B*\operatorname{ArcTan}((-1)^{3/4}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(\operatorname{Sqrt}[a]*d) - ((1/2+I/2)*(A-I*B)*\operatorname{ArcTanh}(((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]))/(\operatorname{Sqrt}[a]*d) + ((I*A-B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)} (A + B \tan(c+dx))}{\sqrt{a + ia \tan(c+dx)}} dx &= \frac{(iA - B) \sqrt{\tan(c+dx)}}{d \sqrt{a + ia \tan(c+dx)}} - \frac{\int \frac{\sqrt{a + ia \tan(c+dx)} (\frac{1}{2}a(iA-B)+)}{\sqrt{\tan(c+dx)}}}{a^2} \\
&= \frac{(iA - B) \sqrt{\tan(c+dx)}}{d \sqrt{a + ia \tan(c+dx)}} + \frac{B \int \frac{(a-ia \tan(c+dx)) \sqrt{a + ia \tan(c+dx)}}{\sqrt{\tan(c+dx)}}}{a^2} \\
&= \frac{(iA - B) \sqrt{\tan(c+dx)}}{d \sqrt{a + ia \tan(c+dx)}} - \frac{(a(A - iB)) \text{Subst} \left( \int \frac{1}{-ia-2a^2x^2} dx \right)}{d} \\
&= -\frac{(\frac{1}{2} + \frac{i}{2}) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + ia \tan(c+dx)}} \right)}{\sqrt{a} d} + \frac{(iA)}{d \sqrt{a}} \\
&= -\frac{(\frac{1}{2} + \frac{i}{2}) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + ia \tan(c+dx)}} \right)}{\sqrt{a} d} + \frac{(iA)}{d \sqrt{a}} \\
&= -\frac{2\sqrt[4]{-1} B \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + ia \tan(c+dx)}} \right)}{\sqrt{a} d} - \frac{(\frac{1}{2} + \frac{i}{2}) (A - iB)}{\sqrt{a} d}
\end{aligned}$$

**Mathematica [A]**

time = 2.28, size = 183, normalized size = 1.17

$$\frac{\left( i(A + iB) \sqrt{-1 + e^{2i(c+dx)}} - i(A - iB) e^{i(c+dx)} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) + 2\sqrt{2} B e^{i(c+dx)} \tanh^{-1} \left( \frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sqrt{\tan(c+dx)}}{d \sqrt{-1 + e^{2i(c+dx)}} \sqrt{a + ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]], x]
```

```
[Out] ((I*(A + I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))] - I*(A - I*B)*E^(I*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] + 2*Sqrt[2]*B*E^(I*(c + d*x))*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]]/(d*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 893 vs.  $2(123) = 246$ .

time = 0.13, size = 894, normalized size = 5.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(I*B*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+2*I*A*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-A*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+2*B*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-8*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+4*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+4*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+A*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-4*I*A*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+4*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-4*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+4*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^2/(I*a)^(1/2)/(-I*a)^(1/2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 718 vs. 2(116) = 232.

time = 0.78, size = 718, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$-1/4*(\sqrt{2})a*d*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)}*e^{(I*d*x + I*c)}*\log((\sqrt{2})a*d*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)}*e^{(I*d*x + I*c)} + \sqrt{2}*((I*A + B)*e^{(2*I*d*x + 2*I*c)} + I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}/(4*I*A + 4*B)) - \sqrt{2})a*d*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)}*e^{(I*d*x + I*c)}*\log(-(\sqrt{2})a*d*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)}*e^{(I*d*x + I*c)} - \sqrt{2}*((I*A + B)*e^{(2*I*d*x + 2*I*c)} + I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}/(4*I*A + 4*B)) - a*d*\sqrt{-4*I*B^2/(a*d^2)}*e^{(I*d*x + I*c)}*\log(52/605*(4*\sqrt{2}*(B*e^{(3*I*d*x + 3*I*c)} + B*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))} + (3*a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{-4*I*B^2/(a*d^2)})/(B*e^{(2*I*d*x + 2*I*c)} + B)) + a*d*\sqrt{-4*I*B^2/(a*d^2)}*e^{(I*d*x + I*c)}*\log(52/605*(4*\sqrt{2}*(B*e^{(3*I*d*x + 3*I*c)} + B*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))} - (3*a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{-4*I*B^2/(a*d^2)})/(B*e^{(2*I*d*x + 2*I*c)} + B)) + 2*\sqrt{2}*((-I*A + B)*e^{(2*I*d*x + 2*I*c)} - I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}))\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-I*d*x - I*c)}/(a*d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\tan(c + dx)}}{\sqrt{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x))/sqrt(I\*a\*(tan(c + d\*x) - I)), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for

the root of a polynomial with parameters. This might be wrong. Non regular value [

**Mupad [B]**

time = 24.21, size = 2500, normalized size = 16.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x)^{(1/2)}*(A + B*\tan(c + d*x)))/(a + a*\tan(c + d*x)*i)^{(1/2)}, x)$

[Out] 
$$-(A*a^{(5/2)}*\tan(c + d*x)^{(1/2)}*4i + 4*A*a^{(5/2)}*\tan(c + d*x)^{(3/2)} - 4*B*a^{(5/2)}*\tan(c + d*x)^{(1/2)} + B*a^{(5/2)}*\tan(c + d*x)^{(3/2)}*4i + A*a^{(3/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)*4i - 4*B*a^{(3/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i) + (1i/8)^{(1/2)}*A*(-a)^{(5/2)}*\text{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2)*8i + 8*(1i/8)^{(1/2)}*B*(-a)^{(5/2)}*\text{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2) - A*a^2*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}*8i - 4*A*a^2*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} + 8*B*a^2*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} - B*a^2*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}*4i + 4i^{(1/2)}*B*(-a)^{(5/2)}*\text{atan}((4i^{(1/2)}*a^{12}*\tan(c + d*x)^{(1/2)})/(2*(-a)^{(23/2)}*a^{(1/2)} - 2*(-a)^{(23/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}))*8i + (1i/8)^{(1/2)}*A*(-a)^{(5/2)}*\text{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2)*\tan(c + d*x)^2*8i + 8*(1i/8)^{(1/2)}*B*(-a)^{(5/2)}*\text{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c +$$

$$\begin{aligned}
& d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*1i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2))*\tan(c + d*x)^2 - (1i/8)^{(1/2)}*A*(-a)^{(3/2)}*\operatorname{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)})*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*1i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2))**(a + a*\tan(c + d*x)*1i)*8i + 4i^{(1/2)}*B*(-a)^{(5/2)}*\tan(c + d*x)^2*\operatorname{atan}((4i^{(1/2)}*a^{12}*\tan(c + d*x)^{(1/2)}))/(2*(-a)^{(23/2)}*a^{(1/2)} - 2*(-a)^{(23/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2}))*8i - 8*(1i/8)^{(1/2)}*B*(-a)^{(3/2)}*\operatorname{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)})*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*1i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2))**(a + a*\tan(c + d*x)*1i) - 4i^{(1/2)}*B*(-a)^{(3/2)}*\operatorname{atan}((4i^{(1/2)}*a^{12}*\tan(c + d*x)^{(1/2)}))/(2*(-a)^{(23/2)}*a^{(1/2)} - 2*(-a)^{(23/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2}))**(a + a*\tan(c + d*x)*1i)*8i - (1i/8)^{(1/2)}*A*(-a)^{(1/2)}*a^{(3/2)}*\operatorname{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)})*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*1i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} + a^{16} + a^{16}*\tan(c + d*x)^2))**(a + a*\tan(c + d*x)*1i)^{(1/2)}*12i + (1i/8)^{(1/2)}*A*(-a)^{(3/2)}*a^{(1/2)}*\operatorname{atanh}(((1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)})*\tan(c + d*x)^{(1/2)}*4i - 4*(1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(3/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} - (1i/8)^{(1/2)}*(-a)^{(31/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)}*8i + 4*(1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(17/2)}*\tan(c + d*x)^{(3/2)} + (1i/8)^{(1/2)}*(-a)^{(15/2)}*a^{(15/2)}*\tan(c + d*x)^{(1/2)}*(a + a*\tan(c + d*x)*1i)*4i)/(a^{15}*(a + a*\tan(c + d*x)*1i) - 2*a^{(31/2)}*(a + a*\tan(c + d*x)*1i)^{(1/2)} + a^{16} + \dots
\end{aligned}$$

$$3.181 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=99

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d} + \frac{(A+iB) \sqrt{\tan(c+dx)}}{d \sqrt{a+ia \tan(c+dx)}}$$

[Out] (1/2-1/2\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d/a^(1/2)+(A+I\*B)\*tan(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3677, 12, 3625, 211}

$$\frac{(A+iB) \sqrt{\tan(c+dx)}}{d \sqrt{a+ia \tan(c+dx)}} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]]),x]

[Out] ((1/2 - I/2)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(Sqrt[a]\*d) + ((A + I\*B)\*Sqrt[Tan[c + d\*x]])/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

## Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} dx &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{d \sqrt{a + ia \tan(c + dx)}} + \frac{\int \frac{a(A - iB) \sqrt{a + ia \tan(c + dx)}}{2 \sqrt{\tan(c + dx)}} dx}{a^2} \\
&= \frac{(A + iB) \sqrt{\tan(c + dx)}}{d \sqrt{a + ia \tan(c + dx)}} + \frac{(A - iB) \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx}{2a} \\
&= \frac{(A + iB) \sqrt{\tan(c + dx)}}{d \sqrt{a + ia \tan(c + dx)}} - \frac{(a(iA + B)) \text{Subst}\left(\int \frac{1}{-ia - 2a^2 x^2} dx\right)}{d} \\
&= -\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{\sqrt{a} d} + \frac{(A - iB) \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx}{2a}
\end{aligned}$$

## Mathematica [A]

time = 1.90, size = 123, normalized size = 1.24

$$\frac{\left( (A + iB) \sqrt{-1 + e^{2i(c+dx)}} + (A - iB) e^{i(c+dx)} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) \right) \sqrt{\tan(c + dx)}}{d \sqrt{-1 + e^{2i(c+dx)}} \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]), x]
```

```
[Out] (((A + I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))] + (A - I*B)*E^(I*(c + d*x))*ArcT
anh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Tan[c + d*x]]/(d
*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(79) = 158.  
time = 0.13, size = 639, normalized size = 6.45

method	result
derivativedivides	$\frac{\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}\left(iA\sqrt{2}\ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+\tan(dx+c))}{\tan(dx+c)}\right)\right)}{-}$
default	$\frac{\left(\sqrt{\tan(dx+c)}\right)\sqrt{a(1+i\tan(dx+c))}\left(iA\sqrt{2}\ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+\tan(dx+c))}{\tan(dx+c)}\right)\right)}{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(I*A*2^(1/2)*ln(-(-2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))
/(tan(d*x+c)+I))*a*tan(d*x+c)^2-2*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
*tan(d*x+c)+B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-I*A*2^(1/2
)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*
a*tan(d*x+c))/(tan(d*x+c)+I))*a+4*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*tan(d*x+c)+2*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+
c)+4*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-B*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a-4*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*tan(d*x+c)+4*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/a/(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^2/(-I*a)^(1/2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```



**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 416 vs.  $2(73) = 146$ .  
time = 0.85, size = 416, normalized size = 4.20

$$\left( \sqrt{3} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}}}{\sqrt{\frac{1A^2+2AB-1B^2}{a^2}}} \right) \log \left( \frac{\sqrt{2} \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}}}{\sqrt{\frac{1A^2+2AB-1B^2}{a^2}}} \right) \right) - \sqrt{2} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}}}{\sqrt{\frac{1A^2+2AB-1B^2}{a^2}}} \right) \log \left( \frac{\sqrt{2} \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}}}{\sqrt{\frac{1A^2+2AB-1B^2}{a^2}}} \right) + 2 \sqrt{2} (A+iB) e^{2Ic} \sqrt{\frac{a}{e^{2Ic}+1}} \sqrt{\frac{-1+e^{2Ic}+1}{e^{2Ic}+1}} e^{-Ic}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{2} a d \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}} e^{Ic} \log \left( \frac{\sqrt{2} \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}}}{\sqrt{\frac{1A^2+2AB-1B^2}{a^2}}} \right) e^{Ic} + \sqrt{2} \left( (IA+B) e^{2Ic} + IA+B \right) \sqrt{\frac{a}{e^{2Ic}+1}} \sqrt{\frac{-1+e^{2Ic}+1}{e^{2Ic}+1}} + \sqrt{2} \left( -I e^{2Ic} + I \right) \sqrt{\frac{a}{e^{2Ic}+1}} \sqrt{\frac{-1+e^{2Ic}+1}{e^{2Ic}+1}} \right) / (4IA+4B) - \sqrt{2} a d \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}} e^{Ic} \log \left( \frac{\sqrt{2} \sqrt{-\frac{1A^2+2AB-1B^2}{a^2}}}{\sqrt{\frac{1A^2+2AB-1B^2}{a^2}}} \right) e^{Ic} + \sqrt{2} \left( (IA+B) e^{2Ic} + IA+B \right) \sqrt{\frac{a}{e^{2Ic}+1}} \sqrt{\frac{-1+e^{2Ic}+1}{e^{2Ic}+1}} \right) / (4IA+4B) + 2 \sqrt{2} (A+IB) e^{2Ic} \sqrt{\frac{a}{e^{2Ic}+1}} \sqrt{\frac{-1+e^{2Ic}+1}{e^{2Ic}+1}} e^{-Ic} / (a d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{ia(\tan(c + dx) - i)} \sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/(sqrt(I\*a\*(tan(c + d\*x) - I))\*sqrt(tan(c + d\*x))), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Factor: Only one algebraic extension allowed Error: Bad Argument Value

Mupad [B]

time = 11.62, size = 426, normalized size = 4.30

$$\frac{B \ln \left( \frac{\sqrt{a} \sqrt{\tan(c+dx)} - \frac{a \tan(c+dx)}{\sqrt{a + a \tan(c+dx) \sqrt{a}}}}{\sqrt{a}} \right) (1+b)}{\left( \sqrt{a + a \tan(c+dx) \sqrt{a}} \right)^2} + \frac{A \sqrt{\tan(c+dx)} b}{\left( \sqrt{a + a \tan(c+dx) \sqrt{a}} \right)^2} + \frac{2B \sqrt{\tan(c+dx)}}{\left( \sqrt{a + a \tan(c+dx) \sqrt{a}} \right)^2} + \frac{\sqrt{a} B \ln \left( \frac{a \tan(c+dx)}{\sqrt{a + a \tan(c+dx) \sqrt{a}}} + \frac{1 - \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + a \tan(c+dx) \sqrt{a}}} \right)}{\left( \sqrt{a + a \tan(c+dx) \sqrt{a}} \right)^2} + \frac{2 \sqrt{a} \operatorname{atanh} \left( \frac{\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + a \tan(c+dx) \sqrt{a}}} \right)}{\left( \sqrt{a + a \tan(c+dx) \sqrt{a}} \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)),x)

[Out] (B\*log((a^(1/2)\*tan(c + d\*x)^(1/2)\*(2 - 2i))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2)) - (a\*tan(c + d\*x))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))^2 + 1i)\*(1/4 + 1i/4))/(a^(1/2)\*d) + (A\*tan(c + d\*x)^(1/2)\*2i)/(((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))\*(d\*1i - (a\*d\*tan(c + d\*x))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))^2)) - (2\*B\*tan(c + d\*x)^(1/2))/(((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))\*(d\*1i - (a\*d\*tan(c + d\*x))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))^2)) - ((1i/8)^(1/2)\*B\*log((2\*(-1)^(3/4)\*2^(1/2)\*a^(1/2)\*tan(c + d\*x)^(1/2))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2)) - (a\*tan(c + d\*x))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))^2 + 1i))/(a^(1/2)\*d) + (2\*(1i/8)^(1/2)\*A\*atanh((32\*(1i/8)^(1/2)\*A^2\*(-a)^(9/2)\*tan(c + d\*x)^(1/2))/((A^2\*a^4\*4i - (4\*A^2\*a^5\*tan(c + d\*x))/((a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))^2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2) - a^(1/2))))/((-a)^(1/2)\*d)

$$3.182 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=143

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d} + \frac{A+iB}{d \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} - \frac{(3A+iB)\sqrt{a+ia \tan(c+dx)}}{ad \sqrt{\tan(c+dx)}}$$

[Out] (1/2+1/2\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d/a^(1/2)+(A+I\*B)/d/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)-(3\*A+I\*B)\*sqrt(a+I\*a\*tan(d\*x+c))/a/d/tan(d\*x+c)^(1/2)

**Rubi** [A]

time = 0.24, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$\frac{A+iB}{d \sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} - \frac{(3A+iB)\sqrt{a+ia \tan(c+dx)}}{ad \sqrt{\tan(c+dx)}} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]),x]

[Out] ((1/2 + I/2)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(Sqrt[a]\*d) + (A + I\*B)/(d\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((3\*A + I\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(a\*d\*Sqrt[Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && Ne

$Q[c^2 + d^2, 0]$

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} dx &= \frac{A + iB}{d \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} + \frac{\int \sqrt{a + ia \tan(c + dx)} dx}{\tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} \\ &= \frac{A + iB}{d \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} - \frac{(3A + iB) \sqrt{a + ia \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} \\ &= \frac{A + iB}{d \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} - \frac{(3A + iB) \sqrt{a + ia \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} \\ &= \frac{A + iB}{d \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} - \frac{(3A + iB) \sqrt{a + ia \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} \\ &= \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{\sqrt{a} d} + \frac{\int \sqrt{a + ia \tan(c + dx)} dx}{d \sqrt{\tan(c + dx)}} \end{aligned}$$

**Mathematica [A]**

time = 2.04, size = 181, normalized size = 1.27

$$\frac{\left( \frac{(A-iB)\sqrt{-1+e^{2i(c+dx)}} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) + \frac{-4A\cos(c+dx)+2(-3iA+B)\sin(c+dx)}{\sqrt{\tan(c+dx)}}}{\sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}} \right) (A+B\tan(c+dx))}{2d(A\cos(c+dx)+B\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]), x]
```

```
[Out] (((((A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))])*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))] + (-4*A*Cos[c + d*x] + 2*((-3*I)*A + B)*Sin[c + d*x])/Sqrt[Tan[c + d*x]])*(A + B*Tan[c + d*x])/(2*d*(A*Cos[c + d*x] + B*Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(117) = 234.

time = 0.14, size = 701, normalized size = 4.90

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))} \left( iB\sqrt{2} \ln\left( -\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))}{\tan(dx+c)+i} \right) \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))} \left( iB\sqrt{2} \ln\left( -\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)}(1+i\tan(dx+c))}{\tan(dx+c)+i} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2), x, method=_RE  
TURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2))  
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*  
a*tan(d*x+c)^3+2*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+  
I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-A*2  
^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I  
*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-I*B*2^(1/2)*ln(-(-2*2^(1/  
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(  
tan(d*x+c)+I))*a*tan(d*x+c)+4*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
```



**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{ia(\tan(c + dx) - i)} \tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(1/2)/tan(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/(sqrt(I\*a\*(tan(c + d\*x) - I))\*tan(c + d\*x)\*\*(3/2)), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2)/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError &gt;&gt; An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.59Factor: Only one algebraic extension allowed Error: Bad Argument Value

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{3/2} \sqrt{a + a \tan(c + dx) 1i}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^(1/2)), x)

$$3.183 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=191

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d} + \frac{A + iB}{d \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} - \frac{(5A + 3iB) \sqrt{a}}{3ad \tan^{\frac{3}{2}}(c+dx)}$$

[Out] (1/2+1/2\*I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/d/a^(1/2)+1/3\*(7\*I\*A-9\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(1/2)+(A+I\*B)/d/(a+I\*a\*tan(d\*x+c))^(1/2)/tan(d\*x+c)^(3/2)-1/3\*(5\*A+3\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$-\frac{(5A+3iB)\sqrt{a+ia \tan(c+dx)}}{3ad \tan^{\frac{3}{2}}(c+dx)} + \frac{A+iB}{d \tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} + \frac{(-9B+7iA)\sqrt{a+ia \tan(c+dx)}}{3ad \sqrt{\tan(c+dx)}} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (B + iA) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]),x]

[Out] ((1/2 + I/2)\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(Sqrt[a]\*d) + (A + I\*B)/(d\*Tan[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((5\*A + (3\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*a\*d\*Tan[c + d\*x]^(3/2)) + (((7\*I)\*A - 9\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*a\*d\*Sqrt[Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; F



FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} dx &= \frac{A + iB}{d \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} + \frac{\int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan} dx}{d \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{A + iB}{d \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(5A + 3iB) \sqrt{a + ia \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(5A + 3iB) \sqrt{a + ia \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(5A + 3iB) \sqrt{a + ia \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(5A + 3iB) \sqrt{a + ia \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} \\
&= \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{\sqrt{a} d} + \frac{1}{d \tan^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

**Mathematica [A]**

time = 2.40, size = 221, normalized size = 1.16

$$\frac{e^{-i(c+dx)} \left( -3B(1 - 6e^{2i(c+dx)} + 5e^{4i(c+dx)}) + iA(3 - 18e^{2i(c+dx)} + 7e^{4i(c+dx)}) + 3(iA + B)e^{i(c+dx)}(-1 + e^{2i(c+dx)})^{3/2} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) (A + B \tan(c + dx))}{6d(-1 + e^{2i(c+dx)})(A \cos(c + dx) + B \sin(c + dx)) \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]),x]

[Out] ((-3\*B\*(1 - 6\*E^((2\*I)\*(c + d\*x)) + 5\*E^((4\*I)\*(c + d\*x))) + I\*A\*(3 - 18\*E^((2\*I)\*(c + d\*x)) + 7\*E^((4\*I)\*(c + d\*x))) + 3\*(I\*A + B)\*E^(I\*(c + d\*x))\*(-1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*(A + B\*Tan[c + d\*x])/(6\*d\*E^(I\*(c + d\*x))\*(-1 + E^((2\*I)\*(c + d\*x))))\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 745 vs.  $2(155) = 310$ .

time = 0.14, size = 746, normalized size = 3.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/12/d*(a*(1+I*tan(d*x+c)))^(1/2)/a/tan(d*x+c)^(3/2)*(-6*I*B*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-36*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*tan(d*x+c)^3+28*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*tan(d*x+c)^3+3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*
x+c)^4+36*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2
-3*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+60*I*B*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+6*A*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+3*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a*tan(d*x+c)^4-3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+24*B
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+8*A*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)/(-tan(d*x+c)+I)^2/(-I*a)^(1/2)
```

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs.  $2(145) = 290$ .

time = 1.46, size = 534, normalized size = 2.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d
*e^(I*d*x + I*c))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*log((I*sqrt(2)*a*d
*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B
)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 3*sq
rt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x +
I*c))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*log((-I*sqrt(2)*a*d*sqrt(-(I*
A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d
*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + 2*sqrt(2)*((7*
A + 15*I*B)*e^(6*I*d*x + 6*I*c) - (11*A + 3*I*B)*e^(4*I*d*x + 4*I*c) - 15*(
A + I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(5*
I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{ia(\tan(c + dx) - i)} \tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2)/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**(
5/2)), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.69Factor: Only one
algebraic extension allowed Error: Bad Argument Value
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{5/2} \sqrt{a + a \tan(c + dx)} \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2))  
, x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2))  
, x)
```

$$3.184 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=237

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d} + \frac{A + iB}{d \tan^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} - \frac{(7A + 5iB) \sqrt{a+ia \tan(c+dx)}}{5ad \tan^{\frac{3}{2}}(c+dx)}$$

[Out]  $(-1/2-1/2*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/d/a^{(1/2)}+1/15*(61*A+35*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(1/2)}+(A+I*B)/d/(a+I*a*\tan(d*x+c))^{(1/2)}/\tan(d*x+c)^{(5/2)}-1/5*(7*A+5*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(5/2)}+1/15*(23*I*A-25*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(3/2)}$

Rubi [A]

time = 0.50, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$\frac{(-25B + 23iA)\sqrt{a+ia \tan(c+dx)}}{15ad \tan^{\frac{3}{2}}(c+dx)} - \frac{(7A + 5iB)\sqrt{a+ia \tan(c+dx)}}{5ad \tan^{\frac{3}{2}}(c+dx)} + \frac{A + iB}{d \tan^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} + \frac{(61A + 35iB)\sqrt{a+ia \tan(c+dx)}}{15ad \sqrt{\tan(c+dx)}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Tan}[c + d*x]^{(7/2)}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]), x]$

[Out]  $((-1/2 - I/2)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[a]*d) + (A + I*B)/(d*\operatorname{Tan}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]) - ((7*A + (5*I)*B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(5*a*d*\operatorname{Tan}[c + d*x]^{(5/2)}) + (((23*I)*A - 25*B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*a*d*\operatorname{Tan}[c + d*x]^{(3/2)}) + ((61*A + (35*I)*B)*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*a*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 211

$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\operatorname{tan}[(e_*) + (f_*)(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)(x_)]], x\_Symbol] := \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

$^2*x^2), x], x, \text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 3679

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(A*d - B*c)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(f*(n + 1)*(c^2 + d^2)), x] - \text{Dist}[1/(a*(n + 1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} dx &= \frac{A + iB}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} + \frac{\int \frac{\sqrt{a + ia \tan(c + dx)}}{\tan} dx}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{A + iB}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(7A + 5iB) \sqrt{a + ia \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(7A + 5iB) \sqrt{a + ia \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(7A + 5iB) \sqrt{a + ia \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(7A + 5iB) \sqrt{a + ia \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} \\
&= \frac{A + iB}{d \tan^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}} - \frac{(7A + 5iB) \sqrt{a + ia \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} \\
&= -\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{\sqrt{a} d} + \frac{1}{d \tan^{\frac{5}{2}}(c + dx)}
\end{aligned}$$

**Mathematica [A]**

time = 3.16, size = 241, normalized size = 1.02

$$\frac{\left( -\frac{(A-iB)\sqrt{-1+e^{2i(c+dx)}} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) - \frac{\csc^2(c+dx)(-5(2A+iB)\cos(c+dx)+(22A+5iB)\cos(3(c+dx))+(9(-7iA+5B)+(59iA-25B)\cos(2(c+dx)))\sin(c+dx)}{15\sqrt{\tan(c+dx)}}}{\sqrt{\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}} \right) (A+B \tan(c+dx))}{2d(A \cos(c+dx) + B \sin(c+dx)) \sqrt{a + ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(7/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]),x]

[Out] (((-(((A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/Sqrt[(-I)\*(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))) - (Csc[c + d\*x]^2\*(-5\*(2\*A + I\*B)\*Cos[c + d\*x] + (22\*A + (5\*I)\*B)\*Cos[3\*(c + d\*x)] + (9\*((-7\*I)\*A + 5\*B) + ((59\*I)\*A - 25\*B)\*Cos[2



$(c + dx) \sin(c + dx) / (15 \sqrt{\tan(c + dx)}) (A + B \tan(c + dx)) / (2d(A \cos(c + dx) + B \sin(c + dx)) \sqrt{a + I a \tan(c + dx)})$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs.  $2(193) = 386$ .

time = 0.14, size = 821, normalized size = 3.46 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{60} d (a(1+I \tan(dx+c))^{1/2} (15 I B 2^{1/2} \ln(-(-2)^{1/2} (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + I a - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^5 + 30 I A 2^{1/2} \ln(-(-2)^{1/2} (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + I a - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^4 - 15 A 2^{1/2} \ln(-(-2)^{1/2} (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + I a - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^5 - 15 I B 2^{1/2} \ln(-(-2)^{1/2} (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + I a - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^3 + 140 I B (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c)^4 + 30 B 2^{1/2} \ln(-(-2)^{1/2} (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + I a - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^4 - 396 I A (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c)^3 + 15 A 2^{1/2} \ln(-(-2)^{1/2} (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} + I a - 3 a \tan(dx+c)) / (\tan(dx+c) + I)) a \tan(dx+c)^3 + 244 A (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c)^4 + 180 B (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c)^3 + 16 I A (-I a)^{1/2} \tan(dx+c) (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} - 144 A (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c)^2 + 40 B (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} \tan(dx+c) + 24 A (-I a)^{1/2} (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} / a \tan(dx+c)^{5/2} / (a \tan(dx+c) (1+I \tan(dx+c)))^{1/2} / (-\tan(dx+c) + I)^2 / (-I a)^{1/2}$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs.  $2(181) = 362$ .

time = 1.40, size = 596, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(2)*(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I*d*x + 5*I*c) + 3*
a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(-I*A^2 - 2*A*B + I*B^
2)/(a*d^2))*log((sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d
*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1)))/(4*I*A + 4*B)) - 15*sqrt(2)*(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I
*d*x + 5*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(-I*
A^2 - 2*A*B + I*B^2)/(a*d^2))*log(-sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B
^2)/(a*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A
+ B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + 2*sqrt(2)*((-103*I*A + 35*B)*e^
(8*I*d*x + 8*I*c) + 6*(17*I*A - 15*B)*e^(6*I*d*x + 6*I*c) + 20*(2*I*A - B)*
e^(4*I*d*x + 4*I*c) + 30*(-5*I*A + 3*B)*e^(2*I*d*x + 2*I*c) + 15*I*A - 15*B
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1)))/(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I*d*x + 5*I*c)
+ 3*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);OUTPUT:Evaluation time: 1.01Factor: Only one
algebraic extension allowed Error: Bad Argument Value
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{7/2} \sqrt{a + a \tan(c + dx) i}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(7/2)\*(a + a\*tan(c + d\*x)\*i)^(1/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(7/2)\*(a + a\*tan(c + d\*x)\*i)^(1/2)), x)

$$3.185 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=203

$$\frac{2(-1)^{3/4} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{3/2} d} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A - iB) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{3/2} d} + \frac{(iA - B) \tan^3(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A+3iB) \sqrt{\tan(c+dx)}}{2ad \sqrt{a+ia \tan(c+dx)}}$$

[Out]  $2*(-1)^{(3/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))}^{(1/2)})/a^{(3/2)}/d+(-1/4+1/4*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))}^{(1/2)})/a^{(3/2)}/d+1/2*(A+3*I*B)*\tan(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/3*(I*A-B)*\tan(d*x+c)^{(3/2)}/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.44, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$-\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A - iB) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{3/2} d} + \frac{2(-1)^{3/4} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{3/2} d} + \frac{(-B+iA) \tan^3(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A+3iB) \sqrt{\tan(c+dx)}}{2ad \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x]^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x]^{(3/2)}), x]$

[Out]  $(2*(-1)^{(3/4)}*B*\operatorname{ArcTan}[\left((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]\right)]/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(a^{(3/2)}*d) - ((1/4 - I/4)*(A - I*B)*\operatorname{ArcTanh}[\left((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]\right)]/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(a^{(3/2)}*d) + ((I*A - B)*\operatorname{Tan}[c+d*x]^{(3/2)})/(3*d*(a+I*a*\operatorname{Tan}[c+d*x]^{(3/2)})) + ((A+(3*I)*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(2*a*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a,$

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}} dx &= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\tan(c+dx)} \left(\frac{3}{2}a(iA-B)+3iaB\tan(c+dx)\right)}{\sqrt{a+ia\tan(c+dx)}}}{3a^2} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(A+3iB)\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} + \frac{\int \sqrt{a+ia\tan(c+dx)}}{3a^2} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(A+3iB)\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{(A-iB)\sqrt{a+ia\tan(c+dx)}}{3a^2} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(A+3iB)\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{(iB)\sqrt{a+ia\tan(c+dx)}}{3a^2} \\
&= \frac{\left(\frac{1}{4} + \frac{i}{4}\right)(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{(iA-B)\sqrt{a+ia\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} \\
&= \frac{\left(\frac{1}{4} + \frac{i}{4}\right)(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{(iA-B)\sqrt{a+ia\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} \\
&= \frac{2(-1)^{3/4}B\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right)(iA+B)\sqrt{a+ia\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]**

time = 3.89, size = 285, normalized size = 1.40

$$\frac{e^{-i(c+dx)} \left( \sqrt{-1 + e^{2i(c+dx)}} (iA - B - 4iAe^{2i(c+dx)} + 10Be^{2i(c+dx)}) + 3(iA + B)e^{3i(c+dx)} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) - 12\sqrt{2} Be^{3i(c+dx)} \tanh^{-1} \left( \frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sec^{\frac{3}{2}}(c+dx) \sqrt{\tan(c+dx)}}{6\sqrt{2} ad \sqrt{-1 + e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} (-i + \tan(c+dx)) \sqrt{a + ia \tan(c+dx)}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] ((Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*(I\*A - B - (4\*I)\*A\*E^((2\*I)\*(c + d\*x)) + 10\*B\*E^((2\*I)\*(c + d\*x))) + 3\*(I\*A + B)\*E^((3\*I)\*(c + d\*x))\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]) - 12\*Sqrt[2]\*B\*E^((3\*I)\*(c + d\*x))\*ArcTanh[(Sqrt[2]\*E^(I\*(c + d\*x)))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Sec[c +

$$d*x]^{(3/2)}*\text{Sqrt}[\text{Tan}[c + d*x]]/(6*\text{Sqrt}[2]*a*d*\text{E}^{(I*(c + d*x))}*\text{Sqrt}[-1 + \text{E}^{(2*I)*(c + d*x)}])*\text{Sqrt}[\text{E}^{(I*(c + d*x))}/(1 + \text{E}^{((2*I)*(c + d*x))})]*(-1 + \text{Tan}[c + d*x])*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]]$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs.  $2(160) = 320$ .

time = 0.14, size = 1223, normalized size = 6.02

method	result	size
derivativedivides	Expression too large to display	1223
default	Expression too large to display	1223

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & 1/24*I/d*\text{tan}(d*x+c)^{(1/2)}*(a*(1+I*\text{tan}(d*x+c)))^{(1/2)}/a^2*(-3*I*A*(I*a)^{(1/2)} \\ & )*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)} \\ & )+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a+20*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*\text{tan}(d \\ & *x+c)^2*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}-36*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)} \\ & )*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}-3*A*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)} \\ & )*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}+I*a-3*a*\text{tan}(d*x+c) \\ & )/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^3-72*I*B*\ln(1/2*(2*I*a*\text{tan}(d*x+c)+2*(a*\text{tan}(d \\ & *x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\text{tan} \\ & n(d*x+c)^2+3*I*B*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d* \\ & x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+ \\ & c)^3-9*I*B*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*( \\ & 1+I*\text{tan}(d*x+c)))^{(1/2)}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)+9*B \\ & *2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)} \\ & )+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*(I*a)^{(1/2)}*a*\text{tan}(d*x+c)^2+24*B*\ln(1/2 \\ & *(2*I*a*\text{tan}(d*x+c)+2*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/( \\ & I*a)^{(1/2)}*(-I*a)^{(1/2)}*a*\text{tan}(d*x+c)^3+9*I*A*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)} \\ & )*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}+I*a-3*a*\text{tan}(d*x+c) \\ & )/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^2+44*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d* \\ & x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}*\text{tan}(d*x+c)^2+9*A*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)} \\ & )*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}+I*a-3*a*\text{tan}(d*x+c) \\ & )/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)-32*I*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+ \\ & c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}*\text{tan}(d*x+c)+24*I*B*\ln(1/2*(2*I*a*\text{tan}(d*x+c)+2*(a* \\ & \text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}*(-I*a)^{(1/2)} \\ & )*a-3*B*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)*(1+I* \\ & \text{tan}(d*x+c)))^{(1/2)}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a-72*B*\ln(1/2*(2*I*a \\ & *\text{tan}(d*x+c)+2*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1 \\ & /2)}*(-I*a)^{(1/2)}*a*\text{tan}(d*x+c)+80*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x+c)* \\ & (1+I*\text{tan}(d*x+c)))^{(1/2)}*\text{tan}(d*x+c)-12*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\text{tan}(d*x \end{aligned}$$

$$+c)*(1+I*\tan(d*x+c))^{(1/2)})/(a*\tan(d*x+c)*(1+I*\tan(d*x+c))^{(1/2)})/(-\tan(d*x+c)+I)^3/(I*a)^{(1/2)}/(-I*a)^{(1/2)}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 758 vs. 2(149) = 298.

time = 0.87, size = 758, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/12*(3*\sqrt{1/2}*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)})*e^{(3*I*d*x + 3*I*c)}*\log((2*I*\sqrt{1/2}*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)}) \\ & )*e^{(I*d*x + I*c)} + \sqrt{2}*((I*A + B)*e^{(2*I*d*x + 2*I*c)} + I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}/(4*I*A + 4*B)) - 3*\sqrt{1/2}*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)} \\ & )*e^{(3*I*d*x + 3*I*c)}*\log((-2*I*\sqrt{1/2}*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)})*e^{(I*d*x + I*c)} + \sqrt{2}*((I*A + B)*e^{(2*I*d*x + 2*I*c)} + I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}/(4*I*A + 4*B)) + 3*a^2*d*\sqrt{4*I*B^2/(a^3*d^2)} \\ & )*e^{(3*I*d*x + 3*I*c)}*\log(52/605*(4*\sqrt{2})*(B*e^{(3*I*d*x + 3*I*c)} + B*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)) - (3*I*a^2*d*e^{(2*I*d*x + 2*I*c)} - I*a^2*d)*\sqrt{4*I*B^2/(a^3*d^2))}/(B*e^{(2*I*d*x + 2*I*c)} + B)) - 3*a^2*d*\sqrt{4*I*B^2/(a^3*d^2)} \\ & )*e^{(3*I*d*x + 3*I*c)}*\log(52/605*(4*\sqrt{2})*(B*e^{(3*I*d*x + 3*I*c)} + B*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)) - (-3*I*a^2*d*e^{(2*I*d*x + 2*I*c)} + I*a^2*d)*\sqrt{4*I*B^2/(a^3*d^2))}/(B*e^{(2*I*d*x + 2*I*c)} + B)) - \sqrt{2}*(2*(2*A + 5*I*B)*e^{(4*I*d*x + 4*I*c)} + 3*(A + 3*I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1))}*e^{(-3*I*d*x - 3*I*c)}/(a^2*d) \end{aligned}$$



**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/(I\*a\*(tan(c + d\*x) - I))\*  
\*(3/2), x)**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, alg  
orithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2)  
, x)[Out] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2)  
, x)

$$3.186 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=150

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(iA + 5B) \sqrt{\tan(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}}$$

[Out]  $(-1/4-1/4*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/a^{(3/2)}/d+1/6*(I*A+5*B)*\tan(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/3*(I*A-B)*\tan(d*x+c)^{(1/2)}/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3676, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{(-B + iA) \sqrt{\tan(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(5B + iA) \sqrt{\tan(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $((-1/4 - I/4)*(A - I*B)*\operatorname{ArcTanH}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]))/(a^{(3/2)}*d) + ((I*A - B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(3*d*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}) + ((I*A + 5*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(6*a*d*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

$\operatorname{Int}(((a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol) := \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]], x\_Symbol] := \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a^2*x^2), x], x, \operatorname{Sqrt}[c + d*\operatorname{Tan}[e + f*x]]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[e + f*x]]], x] /;$  F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\tan(c+dx)} (A + B \tan(c+dx))}{(a + ia \tan(c+dx))^{3/2}} dx &= \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{2}a(iA-B) - a(A-2iB) \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a + ia \tan(c+dx)}}}{3a^2} \\
 &= \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}} + \frac{(iA + 5B) \sqrt{\tan(c+dx)}}{6ad \sqrt{a + ia \tan(c+dx)}} - \frac{\int \frac{3a^2}{\sqrt{a + ia \tan(c+dx)}}}{3a^2} \\
 &= \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}} + \frac{(iA + 5B) \sqrt{\tan(c+dx)}}{6ad \sqrt{a + ia \tan(c+dx)}} - \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}} \\
 &= \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}} + \frac{(iA + 5B) \sqrt{\tan(c+dx)}}{6ad \sqrt{a + ia \tan(c+dx)}} - \frac{(A - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}} \\
 &= -\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{(iA - B) \sqrt{\tan(c+dx)}}{3d(a + ia \tan(c+dx))^{3/2}}
 \end{aligned}$$

**Mathematica [A]**

time = 3.06, size = 228, normalized size = 1.52

$$\frac{e^{-i(c+dx)} \left( \sqrt{-1 + e^{2i(c+dx)}} (A + 2Ae^{2i(c+dx)} - iB(-1 + 4e^{2i(c+dx)})) - 3(A - iB)e^{3i(c+dx)} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sec^{\frac{3}{2}}(c + dx) \sqrt{\tan(c + dx)}}{6\sqrt{2} ad \sqrt{-1 + e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} (-i + \tan(c + dx)) \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2), x]
```

```
[Out] ((Sqrt[-1 + E^((2*I)*(c + d*x))]*(A + 2*A*E^((2*I)*(c + d*x)) - I*B*(-1 + 4*E^((2*I)*(c + d*x)))) - 3*(A - I*B)*E^((3*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(3/2)*Sqrt[Tan[c + d*x]])/(6*Sqrt[2]*a*d*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs.  $2(120) = 240$ .

time = 0.13, size = 868, normalized size = 5.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2), x, method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(9*I*A^2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+4*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-9*I*B*2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-3*A^2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-16*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+12*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+9*B*2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+3*I*B*2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-3*I*A^2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+9*A^2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-20*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-3*B*2^(1/2)*ln((-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-32*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-12*A*(-I*a)^(1/2)
```



[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x))/(I\*a\*(tan(c + d\*x) - I))\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2), x)

$$3.187 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=148

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(7A+iB)\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia \tan(c+dx)}}$$

[Out] (1/4-1/4\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/a^(3/2)/d+1/6\*(7\*A+I\*B)\*tan(d\*x+c)^(1/2)/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/3\*(A+I\*B)\*tan(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi** [A]

time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(7A+iB)\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)), x]

[Out] (((1/4 - I/4)\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])]/(a^(3/2)\*d) + ((A + I\*B)\*Sqrt[Tan[c + d\*x]])/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((7\*A + I\*B)\*Sqrt[Tan[c + d\*x]])/(6\*a\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}} dx &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A - iB) - a(iA - B) \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} dx}{3a^2} \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(7A + iB) \sqrt{\tan(c + dx)}}{6ad \sqrt{a + ia \tan(c + dx)}} + \frac{\int \frac{3a}{\sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} dx}{3a^2} \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(7A + iB) \sqrt{\tan(c + dx)}}{6ad \sqrt{a + ia \tan(c + dx)}} + \frac{(A + B) \sqrt{\tan(c + dx)}}{3a^2} \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{3d(a + ia \tan(c + dx))^{3/2}} + \frac{(7A + iB) \sqrt{\tan(c + dx)}}{6ad \sqrt{a + ia \tan(c + dx)}} - \frac{(iA + B) \sqrt{\tan(c + dx)}}{3a^2} \\ &= -\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{a^{3/2}d} + \frac{(A + B) \sqrt{\tan(c + dx)}}{3d} \end{aligned}$$

### Mathematica [A]

time = 3.25, size = 230, normalized size = 1.55

$$\frac{e^{-i(c+dx)} \left( \sqrt{-1 + e^{2i(c+dx)}} (B + 2Be^{2i(c+dx)} - iA(1 + 8e^{2i(c+dx)})) - 3i(A - iB)e^{3i(c+dx)} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sec^{\frac{3}{2}}(c + dx) \sqrt{\tan(c + dx)}}{6\sqrt{2} ad \sqrt{-1 + e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} (-i + \tan(c + dx)) \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.



[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)),x]

[Out] ((Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*(B + 2\*B\*E^((2\*I)\*(c + d\*x)) - I\*A\*(1 + 8\*E^((2\*I)\*(c + d\*x)))) - (3\*I)\*(A - I\*B)\*E^((3\*I)\*(c + d\*x))\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Sec[c + d\*x]^(3/2)\*Sqrt[Tan[c + d\*x]])/(6\*Sqrt[2]\*a\*d\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*(-I + Tan[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(118) = 236.

time = 0.13, size = 868, normalized size = 5.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(3/2),x,method=\_RE  
TURNVERBOSE)

[Out] 1/24/d\*tan(d\*x+c)^(1/2)\*(a\*(1+I\*tan(d\*x+c)))^(1/2)\*(3\*I\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^3-9\*I\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^2+3\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)^3-9\*I\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)+28\*I\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^2+9\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)+16\*I\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)-9\*B\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a\*tan(d\*x+c)-4\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)^2-36\*I\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)-3\*A\*2^(1/2)\*ln(-(-2\*2^(1/2)\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)+I\*a-3\*a\*tan(d\*x+c)))/(tan(d\*x+c)+I))\*a+64\*A\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)\*tan(d\*x+c)+12\*B\*(-I\*a)^(1/2)\*(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2))/a^2/(a\*tan(d\*x+c)\*(1+I\*tan(d\*x+c)))^(1/2)/(-tan(d\*x+c)+I)^3/(-I\*a)^(1/2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.9Factor: Only one
algebraic extension allowed Error: Bad Argument Value
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2))
,x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2))
, x)
```

$$3.188 \quad \int \frac{A+B \tan(c+dx)}{\tan^2(c+dx)(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=194

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{3/2}} + \frac{1}{6ad\sqrt{\tan(c+dx)}}$$

[Out] (1/4+1/4\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/a^(3/2)/d+1/6\*(11\*A+5\*I\*B)/a/d/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)-1/6\*(25\*A+7\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(1/2)+1/3\*(A+I\*B)/d/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.38, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} - \frac{(25A+7iB)\sqrt{a+ia \tan(c+dx)}}{6a^2d\sqrt{\tan(c+dx)}} + \frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{3/2}} + \frac{11A+5iB}{6ad\sqrt{\tan(c+dx)}\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)), x]

[Out] ((1/4 + I/4)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(a^(3/2)\*d) + (A + I\*B)/(3\*d\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (11\*A + (5\*I)\*B)/(6\*a\*d\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((25\*A + (7\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(6\*a^2\*d\*Sqrt[Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

$^2*x^2), x], x, \text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 3679

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(A*d - B*c)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(f*(n + 1)*(c^2 + d^2)), x] - \text{Dist}[1/(a*(n + 1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}} dx &= \frac{A + iB}{3d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A+iB)-2a(iA}{\tan^{\frac{3}{2}}(c+dx)} \sqrt{a + ia}}{3a^2}} \\
&= \frac{A + iB}{3d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{11A}{6ad \sqrt{\tan(c + dx)}} \\
&= \frac{A + iB}{3d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{11A}{6ad \sqrt{\tan(c + dx)}} \\
&= \frac{A + iB}{3d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{11A}{6ad \sqrt{\tan(c + dx)}} \\
&= \frac{A + iB}{3d \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{11A}{6ad \sqrt{\tan(c + dx)}} \\
&= \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{a^{3/2}d} + \frac{11A}{3d \sqrt{\tan(c + dx)}}
\end{aligned}$$

**Mathematica [A]**

time = 3.01, size = 237, normalized size = 1.22

$$\frac{ie^{-2i(c+dx)} \left( \sqrt{-1 + e^{2i(c+dx)}} (iB(-1 - 7e^{2i(c+dx)} + 8e^{4i(c+dx)}) + A(-1 - 13e^{2i(c+dx)} + 38e^{4i(c+dx)})) - 3(A - iB)e^{3i(c+dx)}(-1 + e^{2i(c+dx)}) \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \csc(c + dx) \sec(c + dx) \sqrt{\tan(c + dx)}}{12ad \sqrt{-1 + e^{2i(c+dx)}} (-i + \tan(c + dx)) \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)), x]
```

```
[Out] ((I/12)*(Sqrt[-1 + E^((2*I)*(c + d*x))])*(I*B*(-1 - 7*E^((2*I)*(c + d*x)) + 8*E^((4*I)*(c + d*x))) + A*(-1 - 13*E^((2*I)*(c + d*x)) + 38*E^((4*I)*(c + d*x)))) - 3*(A - I*B)*E^((3*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))*ArcTan[h[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]]*Csc[c + d*x]*Sec[c + d*x])*Sqrt[Tan[c + d*x]]/(a*d*E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 930 vs.  $2(156) = 312$ .

time = 0.14, size = 931, normalized size = 4.80 Too large to display



[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} \cdot (3 \sqrt{\frac{1}{2}} \cdot (a^2 d e^{(5 I d x + 5 I c)} - a^2 d e^{(3 I d x + 3 I c)}) \cdot \sqrt{\frac{(I A^2 + 2 A B - I B^2)}{a^3 d^2}} \cdot \log\left(\frac{2 \sqrt{\frac{1}{2}} a^2 d \sqrt{(I A^2 + 2 A B - I B^2)} e^{(I d x + I c)} + \sqrt{2} \cdot ((I A + B) e^{(2 I d x + 2 I c)} + I A + B) \sqrt{\frac{a}{(e^{(2 I d x + 2 I c)} + 1)}} \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}}{e^{(2 I d x + 2 I c)} + 1}\right) / (4 I A + 4 B) - 3 \sqrt{\frac{1}{2}} \cdot (a^2 d e^{(5 I d x + 5 I c)} - a^2 d e^{(3 I d x + 3 I c)}) \cdot \sqrt{\frac{(I A^2 + 2 A B - I B^2)}{a^3 d^2}} \cdot \log\left(-\frac{2 \sqrt{\frac{1}{2}} a^2 d \sqrt{(I A^2 + 2 A B - I B^2)} e^{(I d x + I c)} - \sqrt{2} \cdot ((I A + B) e^{(2 I d x + 2 I c)} + I A + B) \sqrt{\frac{a}{(e^{(2 I d x + 2 I c)} + 1)}} \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}}{e^{(2 I d x + 2 I c)} + 1}\right) / (4 I A + 4 B) - \sqrt{2} \cdot (2 \cdot (19 I A - 4 B) e^{(6 I d x + 6 I c)} - (-25 I A + B) e^{(4 I d x + 4 I c)} + 2 \cdot (-7 I A + 4 B) e^{(2 I d x + 2 I c)} - I A + B) \sqrt{\frac{a}{(e^{(2 I d x + 2 I c)} + 1)}} \cdot \sqrt{(-I e^{(2 I d x + 2 I c)} + I)}}{e^{(2 I d x + 2 I c)} + 1} / (a^2 d e^{(5 I d x + 5 I c)} - a^2 d e^{(3 I d x + 3 I c)})$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}} \tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*tan(c + d\*x)\*\*(3/2)), x)

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:Evaluation time: 1.21Factor: Only one algebraic extension allowed Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{3/2} (a + a \tan(c + dx) i)^{3/2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2))  
, x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2))  
, x)
```

$$3.189 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{\frac{3}{2}}} dx$$

**Optimal.** Leaf size=240

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{A + iB}{3d \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{\frac{3}{2}}} + \frac{A + iB}{2ad \tan^{\frac{3}{2}}(c+dx)}$$

[Out] (1/4+1/4\*I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/a^(3/2)/d+1/6\*(39\*I\*A-25\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(1/2)+1/2\*(5\*A+3\*I\*B)/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)/tan(d\*x+c)^(3/2)-1/6\*(21\*A+11\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(3/2)+1/3\*(A+I\*B)/d/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.51, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (B + iA) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} - \frac{(21A + 11iB)\sqrt{a+ia \tan(c+dx)}}{6a^2d \tan^{\frac{3}{2}}(c+dx)} + \frac{(-25B + 39iA)\sqrt{a+ia \tan(c+dx)}}{6a^2d \sqrt{\tan(c+dx)}} + \frac{5A + 3iB}{2ad \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia \tan(c+dx)}} + \frac{A + iB}{3d \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)), x]

[Out] ((1/4 + I/4)\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(a^(3/2)\*d) + (A + I\*B)/(3\*d\*Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (5\*A + (3\*I)\*B)/(2\*a\*d\*Tan[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((21\*A + (11\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(6\*a^2\*d\*Tan[c + d\*x]^(3/2)) + (((39\*I)\*A - 25\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(6\*a^2\*d\*Sqrt[Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

$^2*x^2), x], x, \text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3677

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)])^{(n_)}, x\_Symbol] :> \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& !\text{GtQ}[n, 0]$

#### Rule 3679

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)])^{(n_)}, x\_Symbol] :> \text{Simp}[(A*d - B*c)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(f*(n + 1)*(c^2 + d^2)), x] - \text{Dist}[1/(a*(n + 1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n+1)}*\text{Simp}[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[n, -1]$

#### Rubi steps



**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs.  $2(194) = 388$ .  
time = 0.13, size = 1012, normalized size = 4.22

method	result	size
derivativedivides	Expression too large to display	1012
default	Expression too large to display	1012

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/24/d*(a*(1+I*\tan(d*x+c)))^{(1/2)}/a^2/\tan(d*x+c)^{(3/2)}*(-9*I*A^2^{(1/2)}*\ln(- \\ & (-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan \\ & (d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^3-100*B*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1 \\ & +I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^4+3*I*B*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)} \\ & *(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I) \\ & )*a*\tan(d*x+c)^2+3*B*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+ \\ & I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^5+384 \\ & *A*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^3+156*I*A* \\ & (-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^4-48*I*B*(-I* \\ & a)^{(1/2)}*\tan(d*x+c)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+9*A*2^{(1/2)}*\ln(- \\ & (-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d \\ & *x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^4+204*B*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I \\ & *\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^2-16*I*A*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan \\ & (d*x+c)))^{(1/2)}-9*I*B*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1 \\ & +I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^4-9* \\ & B*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)} \\ & )+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^3-32*A*(-I*a)^{(1/2)}*(a* \\ & \tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)+256*I*B*(-I*a)^{(1/2)}*(a*\tan(d* \\ & x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^3-3*A*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a \\ & )^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+ \\ & c)+I))*a*\tan(d*x+c)^2+3*I*A*2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x \\ & +c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c \\ & )^5-276*I*A*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^2 \\ & )/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}/(-\tan(d*x+c)+I)^3/(-I*a)^{(1/2)} \end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, alg  
orithm="maxima")`



```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx)::OUTPUT:Evaluation time: 1.6Factor: Only one
algebraic extension allowed Error: Bad Argument Value
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{5/2} (a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2))
,x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2))
, x)
```

$$3.190 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=249

$$\frac{2\sqrt{-1} \operatorname{BArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{\left(\frac{1}{8} + \frac{i}{8}\right)(A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{iA}{5d(a+ia \tan(c+dx))^{5/2}}$$

[Out]  $2*(-1)^{(1/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))}^{(1/2)})/a^{(5/2)}/d+(1/8+1/8*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))}^{(1/2)})/a^{(5/2)}/d-1/4*(I*A-7*B)*\tan(d*x+c)^{(1/2)}/a^{2/d}/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(I*A-B)*\tan(d*x+c)^{(5/2)}/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/6*(A+3*I*B)*\tan(d*x+c)^{(3/2)}/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.57, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right)(A-iB) \operatorname{tanh}^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{2\sqrt{-1} \operatorname{BArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} - \frac{(-7B+iA)\sqrt{\tan(c+dx)}}{4a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x]^{(5/2)}*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x]^{(5/2)}), x]$

[Out]  $(2*(-1)^{(1/4)}*B*\operatorname{ArcTan}[\frac{(-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]}{\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]}])/a^{(5/2)*d} + ((1/8 + I/8)*(A - I*B)*\operatorname{ArcTanh}[\frac{(1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}])/a^{(5/2)*d} + ((I*A - B)*\operatorname{Tan}[c + d*x]^{(5/2)})/(5*d*(a + I*a*\operatorname{Tan}[c + d*x]^{(5/2)})) + ((A + (3*I)*B)*\operatorname{Tan}[c + d*x]^{(3/2)})/(6*a*d*(a + I*a*\operatorname{Tan}[c + d*x]^{(3/2)})) - ((I*A - 7*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(4*a^2*d*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a$



, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{\frac{5}{2}}} dx &= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{5}{2}}} - \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)(\frac{5}{2}a(iA-B)+5iaB \tan(c+dx))}{(a+ia \tan(c+dx))^{\frac{3}{2}}} dx}{5a^2} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{5}{2}}} + \frac{(A+3iB) \tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia \tan(c+dx))^{\frac{3}{2}}} + \frac{\int \sqrt{\tan(c+dx)}}{6ad} dx \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{5}{2}}} + \frac{(A+3iB) \tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia \tan(c+dx))^{\frac{3}{2}}} - \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{5}{2}}} + \frac{(A+3iB) \tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia \tan(c+dx))^{\frac{3}{2}}} - \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(iA-B) \tan^{\frac{5}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{5}{2}}} + \frac{(A+3iB) \tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia \tan(c+dx))^{\frac{3}{2}}} - \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{4a^2d\sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(\frac{1}{8} + \frac{i}{8})(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{3}{2}}} \\
&= \frac{(\frac{1}{8} + \frac{i}{8})(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B) \tan^{\frac{3}{2}}(c+dx)}{5d(a+ia \tan(c+dx))^{\frac{3}{2}}} \\
&= \frac{2\sqrt{-1} B \tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(\frac{1}{8} + \frac{i}{8})(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 4.48, size = 275, normalized size = 1.10

$$\frac{e^{-2i(c+dx)} \left( \sqrt{-1 + e^{2i(c+dx)}} (iA(3 - 11e^{2i(c+dx)} + 23e^{4i(c+dx)}) - 3B(1 - 7e^{2i(c+dx)} + 41e^{4i(c+dx)}) - 15i(A - iB)e^{5i(c+dx)} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) + 120\sqrt{2} B e^{5i(c+dx)} \tanh^{-1}\left(\frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) \right) \sec^2(c+dx) \sqrt{\tan(c+dx)}}{60a^2d\sqrt{-1 + e^{2i(c+dx)}} (-i + \tan(c+dx))^2 \sqrt{a + ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] ((Sqrt[-1 + E^((2*I)*(c + d*x))])*(I*A*(3 - 11*E^((2*I)*(c + d*x))) + 23*E^((4*I)*(c + d*x)))) - 3*B*(1 - 7*E^((2*I)*(c + d*x)) + 41*E^((4*I)*(c + d*x)))
```

$$) - (15*I)*(A - I*B)*E^{((5*I)*(c + d*x))*ArcTanh[E^{(I*(c + d*x))/Sqrt[-1 + E^{((2*I)*(c + d*x))}] + 120*Sqrt[2]*B*E^{((5*I)*(c + d*x))*ArcTanh[(Sqrt[2]*E^{(I*(c + d*x))/Sqrt[-1 + E^{((2*I)*(c + d*x))}]])*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]]]/(60*a^2*d*E^{((2*I)*(c + d*x))*Sqrt[-1 + E^{((2*I)*(c + d*x))}]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1541 vs.  $2(198) = 396$ .

time = 0.14, size = 1542, normalized size = 6.19

method	result	size
derivativdivides	Expression too large to display	1542
default	Expression too large to display	1542

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/240/d*\tan(d*x+c)^{(1/2)}*(a*(1+I*\tan(d*x+c)))^{(1/2)}/a^3*(-15*A*(I*a)^{(1/2)} \\ & *2^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)} \\ & +I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^4+60*B*(I*a)^{(1/2)}*2^{(1/2)} \\ & )*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3* \\ & a*\tan(d*x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^3+90*A*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(- \\ & -2*2^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d \\ & *x+c))/(\tan(d*x+c)+I))*a*\tan(d*x+c)^2-60*B*(I*a)^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I* \\ & a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x \\ & +c)+I))*2^{(1/2)}*a*\tan(d*x+c)+588*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c) \\ & *(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^3+60*I*A*(I*a)^{(1/2)}*2^{(1/2)}*\ln(-(-2*2^{(1/2)} \\ & (1/2)*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c) \\ & )/(\tan(d*x+c)+I))*a*\tan(d*x+c)^3-1380*I*B*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d \\ & *x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)-308*I*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*( \\ & a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*\tan(d*x+c)^2-960*I*B*\ln(1/2*(2*I*a*\tan \\ & (d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}) \\ & *(-I*a)^{(1/2)}*a*\tan(d*x+c)^3+60*I*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)* \\ & (1+I*\tan(d*x+c)))^{(1/2)}+148*A*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*(a*\tan(d*x+c)*(1+I*t \\ & an(d*x+c)))^{(1/2)}*\tan(d*x+c)^3+1548*B*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)} \\ & *(I*a)^{(1/2)}*(-I*a)^{(1/2)}*\tan(d*x+c)^2-220*A*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)) \\ & )^{(1/2)}*(I*a)^{(1/2)}*(-I*a)^{(1/2)}*\tan(d*x+c)+240*B*\ln(1/2*(2*I*a*\tan(d*x+c)+ \\ & 2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)})*(-I*a)^{(1/2)} \\ & *a-420*B*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}*(-I*a)^{(1/2)} \\ & )+240*B*\ln(1/2*(2*I*a*\tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I \\ & *a)^{(1/2)}+a)/(I*a)^{(1/2)})*(-I*a)^{(1/2)}*a*\tan(d*x+c)^4-1440*B*\ln(1/2*(2*I*a* \\ & tan(d*x+c)+2*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}*(I*a)^{(1/2)}+a)/(I*a)^{(1/2)}) \\ & )*(-I*a)^{(1/2)}*a*\tan(d*x+c)^2-15*A*(I*a)^{(1/2)}*\ln(-(-2*2^{(1/2)}*(-I*a)^{(1/2)} \\ & (1/2)*(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}+I*a-3*a*\tan(d*x+c))/(\tan(d*x+c)+I) \end{aligned}$$

$$\begin{aligned} & ) * 2^{(1/2)} * a - 60 * I * A * (I * a)^{(1/2)} * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I * a)^{(1/2)} * (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)} + I * a - 3 * a * \tan(d * x + c)) / (\tan(d * x + c) + I)) * a * \tan(d * x + c) \\ & - 90 * I * B * (I * a)^{(1/2)} * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I * a)^{(1/2)} * (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)} + I * a - 3 * a * \tan(d * x + c)) / (\tan(d * x + c) + I)) * a * \tan(d * x + c)^2 \\ & + 15 * I * B * (I * a)^{(1/2)} * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I * a)^{(1/2)} * (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)} + I * a - 3 * a * \tan(d * x + c)) / (\tan(d * x + c) + I)) * a + 15 * I * B * (I * a)^{(1/2)} \\ & * 2^{(1/2)} * \ln(-(-2 * 2^{(1/2)} * (-I * a)^{(1/2)} * (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)} + I * a - 3 * a * \tan(d * x + c)) / (\tan(d * x + c) + I)) * a * \tan(d * x + c)^4 \\ & + 960 * I * B * \ln(1/2 * (2 * I * a * \tan(d * x + c) + 2 * (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)} * (I * a)^{(1/2)} + a) / (I * a)^{(1/2)}) * (-I * a)^{(1/2)} * a * \tan(d * x + c) \\ & / (a * \tan(d * x + c) * (1 + I * \tan(d * x + c)))^{(1/2)} / (-\tan(d * x + c) + I)^4 / (I * a)^{(1/2)} / (-I * a)^{(1/2)} \end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(185) = 370.

time = 0.98, size = 777, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/120 * (15 * \sqrt{1/2} * a^3 * d * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^5 * d^2)}) * e^{(5 * I * d * x + 5 * I * c)} * \log((2 * \sqrt{1/2} * a^3 * d * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^5 * d^2)}) * e^{(I * d * x + I * c)} + \sqrt{2} * ((I * A + B) * e^{(2 * I * d * x + 2 * I * c)} + I * A + B) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) / (4 * I * A + 4 * B)) - 15 * \sqrt{1/2} * a^3 * d * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^5 * d^2)} * e^{(5 * I * d * x + 5 * I * c)} * \log(- (2 * \sqrt{1/2} * a^3 * d * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^5 * d^2)}) * e^{(I * d * x + I * c)} - \sqrt{2} * ((I * A + B) * e^{(2 * I * d * x + 2 * I * c)} + I * A + B) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) / (4 * I * A + 4 * B)) - 30 * a^3 * d * \sqrt{-4 * I * B^2 / (a^5 * d^2)} * e^{(5 * I * d * x + 5 * I * c)} * \log(52 / 605 * (4 * \sqrt{2}) * (B * e^{(3 * I * d * x + 3 * I * c)} + B * e^{(I * d * x + I * c)}) * \sqrt{a / (e^{(2 * I * d * x + 2 * I * c)} + 1)}) * \sqrt{(-I * e^{(2 * I * d * x + 2 * I * c)} * x + 2 * I * c) + I} / (e^{(2 * I * d * x + 2 * I * c)} + 1)) + (3 * a^3 * d * e^{(2 * I * d * x + 2 * I * c)} \end{aligned}$$

$$\begin{aligned}
& - a^3 d \sqrt{-4 I B^2 / (a^5 d^2)} / (B e^{(2 I d x + 2 I c)} + B) + 30 a^3 d \sqrt{-4 I B^2 / (a^5 d^2)} e^{(5 I d x + 5 I c)} \log(52 / 605 (4 \sqrt{2} (B e^{(3 I d x + 3 I c)} + B e^{(I d x + I c)}) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)} - (3 a^3 d e^{(2 I d x + 2 I c)} - a^3 d) \sqrt{-4 I B^2 / (a^5 d^2)} / (B e^{(2 I d x + 2 I c)} + B) \\
& + \sqrt{2} ((-23 I A + 123 B) e^{(6 I d x + 6 I c)} - 6 (2 I A - 17 B) e^{(4 I d x + 4 I c)} - 2 (-4 I A + 9 B) e^{(2 I d x + 2 I c)} - 3 I A + 3 B) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(-I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} + 1)} e^{(-5 I d x - 5 I c)} / (a^3 d)
\end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{5/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

$$3.191 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=194

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right)(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA - B) \tan^3(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(A + 11iB) \sqrt{\tan(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}}$$

[Out]  $(-1/8+1/8*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/a^{(5/2)}/d+1/60*(13*A-37*I*B)*\tan(d*x+c)^{(1/2)}/a^2/d/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(I*A-B)*\tan(d*x+c)^{(3/2)}/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/30*(A+11*I*B)*\tan(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.40, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3676, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right)(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(13A - 37iB)\sqrt{\tan(c+dx)}}{60a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{(-B + iA) \tan^3(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(A + 11iB)\sqrt{\tan(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x]^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]))/(a + I*a*\operatorname{Tan}[c + d*x]^{(5/2)}, x]$

[Out]  $((-1/8 + I/8)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]))/a^{(5/2)*d} + ((I*A - B)*\operatorname{Tan}[c + d*x]^{(3/2)})/(5*d*(a + I*a*\operatorname{Tan}[c + d*x]^{(5/2)}) + ((A + (11*I)*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(30*a*d*(a + I*a*\operatorname{Tan}[c + d*x]^{(3/2)}) + ((13*A - (37*I)*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(60*a^2*d*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]))$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 211

$\operatorname{Int}(((a_*) + (b_.)*(x_)^2)^{-1}, x\_Symbol) \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_.)*\tan[(e_.) + (f_.)*(x_)]]/\operatorname{Sqrt}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

$^2*x^2), x], x, \text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3676

$\text{Int}[\{(a\_.) + (b\_.)*\text{tan}[(e\_.) + (f\_.)*(x\_)]\}^{(m\_)}*\{(A\_.) + (B\_.)*\text{tan}[(e\_.) + (f\_.)*(x\_)]\}^{(n\_)}, x\_Symbol] :> \text{Simp}[\{(-A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{m*}\}*(c + d*\text{Tan}[e + f*x])^{n/(2*a*f*m)}, x] + \text{Dist}[1/(2*a^{2*m}), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^{(n-1)}*\text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m-n) - a*A*(m+n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]$

### Rule 3677

$\text{Int}[\{(a\_.) + (b\_.)*\text{tan}[(e\_.) + (f\_.)*(x\_)]\}^{(m\_)}*\{(A\_.) + (B\_.)*\text{tan}[(e\_.) + (f\_.)*(x\_)]\}^{(n\_)}, x\_Symbol] :> \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^{m*}\{(c + d*\text{Tan}[e + f*x])^{(n+1)}/(2*f*m*(b*c - a*d))\}, x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& !\text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}} dx &= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\tan(c+dx)} \left(\frac{3}{2}a(iA-B)-a(A-4iB)\tan(c+dx)\right)}{(a+ia\tan(c+dx))^{3/2}} dx}{5a^2} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(A+11iB)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}} dx}{60a^2d} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(A+11iB)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{(13A-11B)\sqrt{\tan(c+dx)}}{60a^2d} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(A+11iB)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{(13A-11B)\sqrt{\tan(c+dx)}}{60a^2d} \\
&= \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(A+11iB)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{(13A-11B)\sqrt{\tan(c+dx)}}{60a^2d} \\
&= \frac{\left(\frac{1}{8} + \frac{i}{8}\right)(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}
\end{aligned}$$

**Mathematica [A]**

time = 4.43, size = 214, normalized size = 1.10

$$\frac{e^{-3i(c+dx)} \left( (-1 + e^{2i(c+dx)}) (iA(-3 + e^{2i(c+dx)} + 17e^{4i(c+dx)}) + B(3 - 11e^{2i(c+dx)} + 23e^{4i(c+dx)})) - 15i(A - iB)e^{5i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sec^3(c+dx)}{120a^2d\sqrt{\tan(c+dx)}(-i + \tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] (((-1 + E^((2*I)*(c + d*x)))*(I*A*(-3 + E^((2*I)*(c + d*x)) + 17*E^((4*I)*(c + d*x))) + B*(3 - 11*E^((2*I)*(c + d*x)) + 23*E^((4*I)*(c + d*x)))) - (15*I)*(A - I*B)*E^((5*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^3)/(120*a^2*d*E^((3*I)*(c + d*x))*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs.  $2(156) = 312$ .

time = 0.13, size = 1096, normalized size = 5.65



method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(-52*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3-148*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+308*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+15*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-212*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+15*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-60*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+220*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-90*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-90*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+60*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+15*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-60*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+15*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+220*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+60*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```



[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

$$3.192 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=196

$$-\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} + \frac{(iA - B) \sqrt{\tan(c+dx)}}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(3iA + 7B) \sqrt{\tan(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}}$$

[Out]  $(-1/8-1/8*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})/a^{(5/2)}/d-1/60*(3*I*A-13*B)*\tan(d*x+c)^{(1/2)}/a^{(2/2)}/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(I*A-B)*\tan(d*x+c)^{(1/2)}/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/30*(3*I*A+7*B)*\tan(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.40, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3676, 3677, 12, 3625, 211}

$$-\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} - \frac{(-13B + 3iA) \sqrt{\tan(c+dx)}}{60a^2d \sqrt{a+ia \tan(c+dx)}} + \frac{(7B + 3iA) \sqrt{\tan(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}} + \frac{(-B + iA) \sqrt{\tan(c+dx)}}{5d(a+ia \tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(A+B*\operatorname{Tan}[c+d*x]))/(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}, x]$

[Out]  $((-1/8 - I/8)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]))/(a^{(5/2)*d} + ((I*A - B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(5*d*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}) + (((3*I)*A + 7*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(30*a*d*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}) - (((3*I)*A - 13*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(60*a^{(2)*d}* \operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 211

$\operatorname{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\tan[(e_*) + (f_*)*(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

$^2*x^2)$ ,  $x$ ,  $\text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]]$ ,  $x$  /;  $\text{FreeQ}[\{a, b, c, d, e, f\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{NeQ}[c^2 + d^2, 0]$

### Rule 3676

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] :> \text{Simp}[\{(-A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n-1} + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n-1}]*\text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{LtQ}[m, 0]$  &&  $\text{GtQ}[n, 0]$

### Rule 3677

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n-1}/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n-1}]*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{LtQ}[m, 0]$  &&  $\text{!GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}} dx &= \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\frac{1}{2}a(iA-B)-a(2A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}} dx}{5a^2} \\
&= \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}} dx}{5a^2} \\
&= \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{60a} \\
&= \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{60a} \\
&= \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{60a} \\
&= \frac{\left(\frac{1}{8} + \frac{i}{8}\right)(A-iB)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}
\end{aligned}$$

**Mathematica [A]**

time = 3.52, size = 215, normalized size = 1.10

$$\frac{e^{-2i(c+dx)}\left(\sqrt{-1+e^{2i(c+dx)}}(-3iA(1+3e^{2i(c+dx)}+e^{4i(c+dx)})-B(-3+e^{2i(c+dx)}+17e^{4i(c+dx)}))+15(iA+B)e^{5i(c+dx)}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sec^2(c+dx)\sqrt{\tan(c+dx)}}{60a^2d\sqrt{-1+e^{2i(c+dx)}}(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] ((Sqrt[-1 + E^((2*I)*(c + d*x))]*((-3*I)*A*(1 + 3*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x))) - B*(-3 + E^((2*I)*(c + d*x)) + 17*E^((4*I)*(c + d*x)))) + 15*(I*A + B)*E^((5*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]]/(60*a^2*d*E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs. 2(158) = 316.

time = 0.13, size = 1096, normalized size = 5.59

method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(60*I*A*2^(1/2)*ln(
-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan
(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-12*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3-60*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)-15*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-212*
B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-90*I*B*2^(
1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*
a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+220*I*B*(-I*a)^(1/2)*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+60*B*2^(1/2)*ln(-(-2*2^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d
*x+c)+I))*a*tan(d*x+c)^3+15*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+60*I*
A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+90*A*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+15*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)^4-52*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*
tan(d*x+c)^3-60*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+12*I*A*(
-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-15*A*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a-60*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x
+c)))^(1/2)*tan(d*x+c)+60*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-tan(d*x+c)+I)^4/(-I*a)^(1/2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, alg
orithm="maxima")
```





[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+a \tan(c+dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

$$3.193 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=194

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(13A+3iB)\sqrt{\tan(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}}$$

[Out] (1/8-1/8\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/a^(5/2)/d+1/60\*(67\*A-3\*I\*B)\*tan(d\*x+c)^(1/2)/a^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/5\*(A+I\*B)\*tan(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(13\*A+3\*I\*B)\*tan(d\*x+c)^(1/2)/a/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.39, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} + \frac{(67A-3iB)\sqrt{\tan(c+dx)}}{60a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(13A+3iB)\sqrt{\tan(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out] ((1/8 - I/8)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(a^(5/2)\*d) + ((A + I\*B)\*Sqrt[Tan[c + d\*x]])/(5\*d\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + ((13\*A + (3\*I)\*B)\*Sqrt[Tan[c + d\*x]])/(30\*a\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((67\*A - (3\*I)\*B)\*Sqrt[Tan[c + d\*x]])/(60\*a^2\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

$\sqrt{2x^2}$ ,  $x$ ,  $\text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]]$ ,  $x$  /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^{5/2}} dx &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A - iB) - 2a(iA - B) \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} dx}{5a^2} \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(13A + 3iB) \sqrt{\tan(c + dx)}}{30ad(a + ia \tan(c + dx))^{3/2}} + \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(13A + 3iB) \sqrt{\tan(c + dx)}}{30ad(a + ia \tan(c + dx))^{3/2}} + \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(13A + 3iB) \sqrt{\tan(c + dx)}}{30ad(a + ia \tan(c + dx))^{3/2}} + \\ &= \frac{(A + iB) \sqrt{\tan(c + dx)}}{5d(a + ia \tan(c + dx))^{5/2}} + \frac{(13A + 3iB) \sqrt{\tan(c + dx)}}{30ad(a + ia \tan(c + dx))^{3/2}} + \\ &= -\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right)}{a^{5/2}d} + \frac{(A + iB) \sqrt{\tan(c + dx)}}{5d(a + ia \tan(c + dx))^{5/2}} \end{aligned}$$

### Mathematica [A]

time = 3.81, size = 216, normalized size = 1.11

$$\frac{e^{-2i(c+dx)} \left( \sqrt{-1 + e^{2i(c+dx)}} (3iB(1 + 3e^{2i(c+dx)} + e^{4i(c+dx)}) + A(3 + 19e^{2i(c+dx)} + 83e^{4i(c+dx)})) + 15(A - iB)e^{5i(c+dx)} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}}\right) \right) \sec^2(c + dx) \sqrt{\tan(c + dx)}}{60a^2d \sqrt{-1 + e^{2i(c+dx)}} (-i + \tan(c + dx))^2 \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] -1/60*((Sqrt[-1 + E^((2*I)*(c + d*x))]*((3*I)*B*(1 + 3*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x))) + A*(3 + 19*E^((2*I)*(c + d*x)) + 83*E^((4*I)*(c + d*x)))) + 15*(A - I*B)*E^((5*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]]/(a^2*d*E^((2*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs.  $2(156) = 312$ .

time = 0.13, size = 1096, normalized size = 5.65

method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(268*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3-12*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+15*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+15*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-60*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-1060*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-90*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-90*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+60*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+15*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+908*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c
```



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);;OUTPUT:Evaluation time: 1.52Factor: Only one
algebraic extension allowed Error: Bad Argument Value
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2))
,x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2))
, x)
```

$$3.194 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=240

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} + \frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{5/2}} + \frac{1}{30ad\sqrt{\tan(c+dx)}}$$

[Out] (1/8+1/8\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/a^(5/2)/d+1/60\*(151\*A+41\*I\*B)/a^2/d/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)-1/60\*(317\*A+67\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d/tan(d\*x+c)^(1/2)+1/5\*(A+I\*B)/d/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(17\*A+7\*I\*B)/a/d/tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.54, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} - \frac{(317A+67iB)\sqrt{a+ia \tan(c+dx)}}{60a^2d\sqrt{\tan(c+dx)}} + \frac{151A+41iB}{60a^2d\sqrt{\tan(c+dx)}\sqrt{a+ia \tan(c+dx)}} + \frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{5/2}} + \frac{17A+7iB}{30ad\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out] ((1/8 + I/8)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(a^(5/2)\*d) + (A + I\*B)/(5\*d\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + (17\*A + (7\*I)\*B)/(30\*a\*d\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (151\*A + (41\*I)\*B)/(60\*a^2\*d\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((317\*A + (67\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(60\*a^3\*d\*Sqrt[Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

```

### Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

```

### Rubi steps





$$(4*I)*B) + ((-466*I)*A + 86*B)*\text{Tan}[c + d*x]))/(15*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])))/(8*d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^(5/2))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs.  $2(194) = 388$ .

time = 0.13, size = 1158, normalized size = 4.82

method	result	size
derivativedivides	Expression too large to display	1158
default	Expression too large to display	1158

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/240/d*(a*(1+I*\text{tan}(d*x+c)))^{1/2}/a^3*(-4468*I*A*(-I*a)^{1/2}*(a*\text{tan}(d*x+c) \\ & *(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)^3+1268*A*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I \\ & *\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)^4+268*I*B*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I \\ & *\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)^4-15*A^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c) \\ & *(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a \\ & *\text{tan}(d*x+c)^5+908*B*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}( \\ & d*x+c)^3-1060*I*B*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d* \\ & x+c)^2+15*I*B^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d \\ & *x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^5+60*B^2^{1/2} \\ & *\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3 \\ & *a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^4-5660*A*(-I*a)^{1/2}*(a*\text{tan}(d* \\ & x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)^2+60*I*A^2^{1/2}*\ln(-(-2*2^{1/2}*(-I \\ & *a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d \\ & *x+c)+I))*a*\text{tan}(d*x+c)^4-90*I*B^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan} \\ & (d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d \\ & *x+c)^3+90*A^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d* \\ & x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^3+2940*I*A*(- \\ & I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)-60*I*A^2^{1/2} \\ & *\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a \\ & *\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)^2-60*B^2^{1/2}*\ln(-(-2*2^{1/2}*(-I \\ & *a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d* \\ & x+c)+I))*a*\text{tan}(d*x+c)^2+15*I*B^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}( \\ & d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d* \\ & x+c)-15*A^2^{1/2}*\ln(-(-2*2^{1/2}*(-I*a)^{1/2}*(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c) \\ & )))^{1/2}+I*a-3*a*\text{tan}(d*x+c))/(\text{tan}(d*x+c)+I))*a*\text{tan}(d*x+c)-420*B*(-I*a)^{1/2} \\ & *(a*\text{tan}(d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2}*\text{tan}(d*x+c)+480*A*(-I*a)^{1/2}*(a*\text{tan} \\ & (d*x+c)*(1+I*\text{tan}(d*x+c)))^{1/2})/\text{tan}(d*x+c)^{1/2}/(a*\text{tan}(d*x+c)*(1+I*\text{tan}( \\ & d*x+c)))^{1/2}/(-\text{tan}(d*x+c)+I)^4/(-I*a)^{1/2} \end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs.  $2(182) = 364$ .

time = 0.88, size = 529, normalized size = 2.20

$$\frac{1}{120} \sqrt{\frac{1}{2}} (15 \sqrt{2} (a^3 d e^{7 I d x + 7 I c} - a^3 d e^{5 I d x + 5 I c}) \sqrt{\frac{I A^2 + 2 A B - I B^2}{a^5 d^2}} \log\left(\frac{2 \sqrt{2} a^3 d \sqrt{\frac{I A^2 + 2 A B - I B^2}{a^5 d^2}} e^{I d x + I c} + \sqrt{2} (I A + B) e^{2 I d x + 2 I c} + I A + B}{e^{2 I d x + 2 I c} + 1}\right) \sqrt{\frac{-I e^{2 I d x + 2 I c} + I}{e^{2 I d x + 2 I c} + 1}} \sqrt{\frac{-I e^{2 I d x + 2 I c} + I}{e^{2 I d x + 2 I c} + 1}}}{(4 I A + 4 B)} - 15 \sqrt{2} (a^3 d e^{7 I d x + 7 I c} - a^3 d e^{5 I d x + 5 I c}) \sqrt{\frac{I A^2 + 2 A B - I B^2}{a^5 d^2}} \log\left(\frac{-2 \sqrt{2} a^3 d \sqrt{\frac{I A^2 + 2 A B - I B^2}{a^5 d^2}} e^{I d x + I c} - \sqrt{2} (I A + B) e^{2 I d x + 2 I c} + I A + B}{e^{2 I d x + 2 I c} + 1}\right) \sqrt{\frac{-I e^{2 I d x + 2 I c} + I}{e^{2 I d x + 2 I c} + 1}} \sqrt{\frac{-I e^{2 I d x + 2 I c} + I}{e^{2 I d x + 2 I c} + 1}}}{(4 I A + 4 B)} + \sqrt{2} ((-463 I A + 83 B) e^{8 I d x + 8 I c} + (-269 I A + 19 B) e^{6 I d x + 6 I c} - 20 (-11 I A + 4 B) e^{4 I d x + 4 I c} + (29 I A - 19 B) e^{2 I d x + 2 I c} + 3 I A - 3 B) \sqrt{\frac{a}{e^{2 I d x + 2 I c} + 1}} \sqrt{\frac{-I e^{2 I d x + 2 I c} + I}{e^{2 I d x + 2 I c} + 1}} \sqrt{\frac{-I e^{2 I d x + 2 I c} + I}{e^{2 I d x + 2 I c} + 1}}}{(a^3 d e^{7 I d x + 7 I c} - a^3 d e^{5 I d x + 5 I c})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))
*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*log((2*sqrt(1/2)*a^3*d*sqrt((I*A^2
+ 2*A*B - I*B^2)/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x
+ 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/2)*(a
^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((I*A^2 + 2*A*B -
I*B^2)/(a^5*d^2))*log((-2*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^
5*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt(2)*((-463*I*A + 83*B)*e^(8*I*d*
x + 8*I*c) + (-269*I*A + 19*B)*e^(6*I*d*x + 6*I*c) - 20*(-11*I*A + 4*B)*e^(
4*I*d*x + 4*I*c) + (29*I*A - 19*B)*e^(2*I*d*x + 2*I*c) + 3*I*A - 3*B)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)))/(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

[Out] Timed out

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.32Factor: Only one  
algebraic extension allowed Error: Bad Argument Value

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{3/2} (a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)), x)

$$3.195 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=286

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{A+iB}{5d \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}} + \frac{1}{30ad \tan^{\frac{3}{2}}(c+dx)}$$

[Out] (1/8+1/8\*I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))/a^(5/2)/d+1/60\*(707\*I\*A-317\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d/tan(d\*x+c)^(1/2)+1/20\*(89\*A+39\*I\*B)/a^2/d/(a+I\*a\*tan(d\*x+c))^(1/2)/tan(d\*x+c)^(3/2)-1/60\*(361\*A+151\*I\*B)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^3/d/tan(d\*x+c)^(3/2)+1/5\*(A+I\*B)/d/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(21\*A+11\*I\*B)/a/d/tan(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi** [A]

time = 0.68, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (B + iA) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} - \frac{(361A + 151iB)\sqrt{a+ia \tan(c+dx)}}{60a^3d \tan^3(c+dx)} + \frac{(-317B + 707iA)\sqrt{a+ia \tan(c+dx)}}{60a^3d \sqrt{\tan(c+dx)}} + \frac{89A + 39iB}{20a^2d \tan^3(c+dx)\sqrt{a+ia \tan(c+dx)}} + \frac{21A + 11iB}{30ad \tan^3(c+dx)(a+ia \tan(c+dx))^{3/2}} + \frac{A+iB}{5d \tan^3(c+dx)(a+ia \tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out] ((1/8 + I/8)\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])/(a^(5/2)\*d) + (A + I\*B)/(5\*d\*Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + (21\*A + (11\*I)\*B)/(30\*a\*d\*Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (89\*A + (39\*I)\*B)/(20\*a^2\*d\*Tan[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((361\*A + (151\*I)\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(60\*a^3\*d\*Tan[c + d\*x]^(3/2)) + (((707\*I)\*A - 317\*B)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(60\*a^3\*d\*Sqrt[Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_)
+ (f_)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} dx &= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(13A+3iB)-4a(iA-)}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))} dx}{5a^2} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{A + iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{21A - 21iB}{30ad \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{a^{5/2}d} + \frac{21A - 21iB}{5d \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}}
\end{aligned}$$

### Mathematica [A]

time = 4.75, size = 268, normalized size = 0.94

$$\frac{e^{-3i(c+dx)} \left( B(3 + 23e^{2i(c+dx)} + 168e^{4i(c+dx)} - 657e^{6i(c+dx)} + 463e^{8i(c+dx)}) - iA(3 + 33e^{2i(c+dx)} + 348e^{4i(c+dx)} - 1527e^{6i(c+dx)} + 983e^{8i(c+dx)}) - 15i(A - iB)e^{5i(c+dx)}(-1 + e^{2i(c+dx)})^{3/2} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sec^3(c + dx)}{120a^2d(-1 + e^{2i(c+dx)}) \sqrt{\tan(c + dx)} (-i + \tan(c + dx))^2 \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out] ((B\*(3 + 23\*E^((2\*I)\*(c + d\*x))) + 168\*E^((4\*I)\*(c + d\*x))) - 657\*E^((6\*I)\*(c + d\*x)) + 463\*E^((8\*I)\*(c + d\*x))) - I\*A\*(3 + 33\*E^((2\*I)\*(c + d\*x))) + 348\*E^((4\*I)\*(c + d\*x)) - 1527\*E^((6\*I)\*(c + d\*x)) + 983\*E^((8\*I)\*(c + d\*x))) - (15\*I)\*(A - I\*B)\*E^((5\*I)\*(c + d\*x))\*(-1 + E^((2\*I)\*(c + d\*x)))^(3/2)\*Arc





$(x+c)*(1+I*\tan(d*x+c))^{(1/2)}/a^3/\tan(d*x+c)^{(3/2)}/(a*\tan(d*x+c)*(1+I*\tan(d*x+c)))^{(1/2)}/(-\tan(d*x+c)+I)^4/(-I*a)^{(1/2)}$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(218) = 436.

time = 0.82, size = 588, normalized size = 2.06

$$\frac{1}{120} \sqrt{\frac{1}{2}} (15 \sqrt{\frac{1}{2}} (a^3 d e^{(9 I d x + 9 I c)} - 2 a^3 d e^{(7 I d x + 7 I c)} + a^3 d e^{(5 I d x + 5 I c)}) \sqrt{\frac{-I A^2 - 2 A B + I B^2}{a^5 d^2}} \log\left(\frac{2 I \sqrt{\frac{1}{2}} a^3 d \sqrt{\frac{-I A^2 - 2 A B + I B^2}{a^5 d^2}} e^{(I d x + I c)} + \sqrt{2} (I A + B) e^{(2 I d x + 2 I c)} + I A + B}{\sqrt{a} (e^{(2 I d x + 2 I c)} + 1)}\right) \sqrt{\frac{-I e^{(2 I d x + 2 I c)} + I}{e^{(2 I d x + 2 I c)} + 1}}) / (4 I A + 4 B) - 15 \sqrt{\frac{1}{2}} (a^3 d e^{(9 I d x + 9 I c)} - 2 a^3 d e^{(7 I d x + 7 I c)} + a^3 d e^{(5 I d x + 5 I c)}) \sqrt{\frac{-I A^2 - 2 A B + I B^2}{a^5 d^2}} \log\left(\frac{-2 I \sqrt{\frac{1}{2}} a^3 d \sqrt{\frac{-I A^2 - 2 A B + I B^2}{a^5 d^2}} e^{(I d x + I c)} + \sqrt{2} (I A + B) e^{(2 I d x + 2 I c)} + I A + B}{\sqrt{a} (e^{(2 I d x + 2 I c)} + 1)}\right) \sqrt{\frac{-I e^{(2 I d x + 2 I c)} + I}{e^{(2 I d x + 2 I c)} + 1}}) / (4 I A + 4 B) + \sqrt{2} ((983 A + 463 I B) e^{(10 I d x + 10 I c)} - 2 (272 A + 97 I B) e^{(8 I d x + 8 I c)} - 3 (393 A + 163 I B) e^{(6 I d x + 6 I c)} + (381 A + 191 I B) e^{(4 I d x + 4 I c)} + 2 (18 A + 13 I B) e^{(2 I d x + 2 I c)} + 3 A + 3 I B) \sqrt{\frac{a}{e^{(2 I d x + 2 I c)} + 1}} \sqrt{\frac{-I e^{(2 I d x + 2 I c)} + I}{e^{(2 I d x + 2 I c)} + 1}}) / (a^3 d e^{(9 I d x + 9 I c)} - 2 a^3 d e^{(7 I d x + 7 I c)} + a^3 d e^{(5 I d x + 5 I c)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out]  $-1/120*(15*\sqrt{1/2}*(a^3*d*e^{(9*I*d*x + 9*I*c)} - 2*a^3*d*e^{(7*I*d*x + 7*I*c)} + a^3*d*e^{(5*I*d*x + 5*I*c)})*\sqrt{((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*\log((2*I*\sqrt{1/2}*a^3*d*\sqrt{((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))}*e^{(I*d*x + I*c)} + \sqrt{2}*(I*A + B)*e^{(2*I*d*x + 2*I*c)} + I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)))/(4*I*A + 4*B)} - 15*\sqrt{1/2}*(a^3*d*e^{(9*I*d*x + 9*I*c)} - 2*a^3*d*e^{(7*I*d*x + 7*I*c)} + a^3*d*e^{(5*I*d*x + 5*I*c)})*\sqrt{((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*\log((-2*I*\sqrt{1/2}*a^3*d*\sqrt{((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))}*e^{(I*d*x + I*c)} + \sqrt{2}*(I*A + B)*e^{(2*I*d*x + 2*I*c)} + I*A + B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)))/(4*I*A + 4*B)} + \sqrt{2}*((983*A + 463*I*B)*e^{(10*I*d*x + 10*I*c)} - 2*(272*A + 97*I*B)*e^{(8*I*d*x + 8*I*c)} - 3*(393*A + 163*I*B)*e^{(6*I*d*x + 6*I*c)} + (381*A + 191*I*B)*e^{(4*I*d*x + 4*I*c)} + 2*(18*A + 13*I*B)*e^{(2*I*d*x + 2*I*c)} + 3*A + 3*I*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((-I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + 1)))/(a^3*d*e^{(9*I*d*x + 9*I*c)} - 2*a^3*d*e^{(7*I*d*x + 7*I*c)} + a^3*d*e^{(5*I*d*x + 5*I*c)})$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2)/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{5/2} (a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)), x)

### 3.196 $\int \sqrt[3]{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$

Optimal. Leaf size=201

$$\frac{\sqrt[3]{a} (A - iB)x}{2^{2/3}} - \frac{\sqrt{3} \sqrt[3]{a} (iA + B) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{2^{2/3}d} + \frac{\sqrt[3]{a} (iA + B) \log(\cos(c + dx))}{2^{2/3}d}$$

[Out]  $-1/4*a^{(1/3)}*(A-I*B)*x*2^{(1/3)}+1/4*a^{(1/3)}*(I*A+B)*\ln(\cos(d*x+c))*2^{(1/3)}/d$   
 $+3/4*a^{(1/3)}*(I*A+B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(1/3)}/d$   
 $-1/2*a^{(1/3)}*(I*A+B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/$   
 $a^{(1/3)}*3^{(1/2)})*3^{(1/2)}*2^{(1/3)}/d+3*B*(a+I*a*\tan(d*x+c))^{(1/3)}/d$

**Rubi** [A]

time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3608, 3562, 59, 631, 210, 31}

$$-\frac{\sqrt{3} \sqrt[3]{a} (B + iA) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{2^{2/3}d} + \frac{3 \sqrt[3]{a} (B + iA) \log\left(\frac{\sqrt{2} \sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)}}{2^{2/3}d}\right)}{2^{2/3}d} + \frac{\sqrt[3]{a} (B + iA) \log(\cos(c + dx))}{2^{2/3}d} - \frac{\sqrt[3]{a} x (A - iB)}{2^{2/3}} + \frac{3B \sqrt[3]{a + ia \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-1/2*(a^{(1/3)}*(A - I*B)*x)/2^{(2/3)} - (\operatorname{Sqrt}[3]*a^{(1/3)}*(I*A + B)*\operatorname{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)})/(\operatorname{Sqrt}[3]*a^{(1/3)})])/(2^{(2/3)}*d)$   
 $+ (a^{(1/3)}*(I*A + B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(2*2^{(2/3)}*d) + (3*a^{(1/3)}*(I*A + B)*\operatorname{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)}])/(2*2^{(2/3)}*d) + (3*B*(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)})/d$

Rule 31

$\operatorname{Int}[(a + (b_*)*(x_*)^{(-1)}, x\_Symbol] := \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 59

$\operatorname{Int}[1/(((a_*) + (b_*)*(x_*)*((c_*) + (d_*)*(x_*)^{(2/3)})), x\_Symbol] := \operatorname{With}\{q = \operatorname{Rt}[(b*c - a*d)/b, 3]\}, \operatorname{Simp}[-\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/(2*b*q^2), x] + (-\operatorname{Dist}[3/(2*b*q), \operatorname{Subst}[\operatorname{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{(1/3)}], x] - \operatorname{Dist}[3/(2*b*q^2), \operatorname{Subst}[\operatorname{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}], x]) /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{PosQ}[(b*c - a*d)/b]$

Rule 210

$\operatorname{Int}[(a + (b_*)*(x_*)^2)^{(-1)}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

### Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \sqrt[3]{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{3B \sqrt[3]{a + ia \tan(c + dx)}}{d} - (-A + iB) \int \sqrt[3]{a + ia \tan(c + dx)} dx \\
 &= \frac{3B \sqrt[3]{a + ia \tan(c + dx)}}{d} - \frac{(a(iA + B)) \text{Subst}\left(\int \frac{1}{(a-x)(a+x)} dx, x, \sqrt[3]{a + ia \tan(c + dx)}\right)}{d} \\
 &= -\frac{\sqrt[3]{a} (A - iB)x}{2 \cdot 2^{2/3}} + \frac{\sqrt[3]{a} (iA + B) \log(\cos(c + dx))}{2 \cdot 2^{2/3} d} + \frac{3B \sqrt[3]{a}}{2 \cdot 2^{2/3} d} \\
 &= -\frac{\sqrt[3]{a} (A - iB)x}{2 \cdot 2^{2/3}} + \frac{\sqrt[3]{a} (iA + B) \log(\cos(c + dx))}{2 \cdot 2^{2/3} d} + \frac{3\sqrt[3]{a}}{2 \cdot 2^{2/3} d} \\
 &= -\frac{\sqrt[3]{a} (A - iB)x}{2 \cdot 2^{2/3}} - \frac{\sqrt{3} \sqrt[3]{a} (iA + B) \tan^{-1}\left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a} + \sqrt{3} \sqrt[3]{a}}{2 \cdot 2^{2/3} d}}{1 - \frac{2^{2/3} \sqrt[3]{a} - \sqrt{3} \sqrt[3]{a}}{2 \cdot 2^{2/3} d}}\right)}{2^{2/3} d}
 \end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^(1/3)\*(A + B\*Tan[c + d\*x]),x]

[Out] \$Aborted

Maple [A]

time = 0.06, size = 159, normalized size = 0.79

method	result
derivativedivides	$3i \left( -iB(a+ia \tan(dx+c))^{\frac{1}{3}} + \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}} - \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}}(a+ia \tan(dx+c))^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}} \right) \frac{1}{d}$
default	$3i \left( -iB(a+ia \tan(dx+c))^{\frac{1}{3}} + \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}} - \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}}(a+ia \tan(dx+c))^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}} \right) \frac{1}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(1/3)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 3\*I/d\*(-I\*B\*(a+I\*a\*tan(d\*x+c))^(1/3)+(1/6\*2^(1/3)/a^(2/3)\*ln((a+I\*a\*tan(d\*x+c))^(1/3)-2^(1/3)\*a^(1/3))-1/12\*2^(1/3)/a^(2/3)\*ln((a+I\*a\*tan(d\*x+c))^(2/3)+2^(1/3)\*a^(1/3)\*(a+I\*a\*tan(d\*x+c))^(1/3)+2^(2/3)\*a^(2/3))-1/6\*2^(1/3)/a^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2^(2/3)/a^(1/3)\*(a+I\*a\*tan(d\*x+c))^(1/3)+1)))\*a\*(A-I\*B)

Maxima [A]

time = 0.57, size = 167, normalized size = 0.83

$$i \left( 2\sqrt{3} 2^{\frac{1}{3}} (A - iB) a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} 2^{\frac{1}{3}} (2^{\frac{1}{3}} a^{\frac{1}{3}} + 2(i a \tan(dx+c) + a)^{\frac{1}{3}})}{6a^{\frac{2}{3}}}\right) + 2^{\frac{1}{3}} (A - iB) a^{\frac{1}{3}} \log\left(2^{\frac{1}{3}} a^{\frac{1}{3}} + 2^{\frac{1}{3}} (i a \tan(dx+c) + a)^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}}\right) - 2^{\frac{1}{3}} (A - iB) a^{\frac{1}{3}} \log\left(-2^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{1}{3}}\right) + 12i (i a \tan(dx+c) + a)^{\frac{1}{3}} B a \right) \frac{1}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/3)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/4*I*(2*\sqrt{3}*2^{(1/3)}*(A - I*B)*a^{(4/3)}*\arctan(1/6*\sqrt{3}*2^{(2/3)}*(2^{(1/3)}*a^{(1/3)} + 2*(I*a*\tan(dx + c) + a)^{(1/3)})/a^{(1/3)}) + 2^{(1/3)}*(A - I*B)*a^{(4/3)}*\log(2^{(2/3)}*a^{(2/3)} + 2^{(1/3)}*(I*a*\tan(dx + c) + a)^{(1/3)}*a^{(1/3)} + (I*a*\tan(dx + c) + a)^{(2/3)}) - 2*2^{(1/3)}*(A - I*B)*a^{(4/3)}*\log(-2^{(1/3)}*a^{(1/3)} + (I*a*\tan(dx + c) + a)^{(1/3)}) + 12*I*(I*a*\tan(dx + c) + a)^{(1/3)}*B*a)/(a*d)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs.  $2(146) = 292$ .  
time = 0.68, size = 405, normalized size = 2.01

$$0 \cdot 2^{1/3} \left( \frac{(a + I a \tan(dx + c))^{1/3} + 1}{(a + I a \tan(dx + c))^{1/3}} \right) \log \left( \frac{(a + I a \tan(dx + c))^{1/3} + 1}{(a + I a \tan(dx + c))^{1/3}} \right) + 2^{1/3} \left( \frac{(a + I a \tan(dx + c))^{1/3} + 1}{(a + I a \tan(dx + c))^{1/3}} \right) \log \left( \frac{(a + I a \tan(dx + c))^{1/3} + 1}{(a + I a \tan(dx + c))^{1/3}} \right) + 2^{1/3} \left( \frac{(a + I a \tan(dx + c))^{1/3} + 1}{(a + I a \tan(dx + c))^{1/3}} \right) \log \left( \frac{(a + I a \tan(dx + c))^{1/3} + 1}{(a + I a \tan(dx + c))^{1/3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/2*(6*2^{(1/3)}*B*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} + (1/4)^{(1/3)}*(-I*\sqrt{3}*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^{(1/3)}*\log((2^{(1/3)}*(I*A + B)*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} + (1/4)^{(1/3)}*(I*\sqrt{3}*d + d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^{(1/3)})/(I*A + B) + (1/4)^{(1/3)}*(I*\sqrt{3}*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^{(1/3)}*\log((2^{(1/3)}*(I*A + B)*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} + (1/4)^{(1/3)}*(-I*\sqrt{3}*d + d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^{(1/3)})/(I*A + B) + 2*(1/4)^{(1/3)}*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^{(1/3)}*\log((2^{(1/3)}*(I*A + B)*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} - 2*(1/4)^{(1/3)}*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^{(1/3)})/(I*A + B)))/d$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))**(1/3)*(A+B*tan(d*x+c)),x)`

[Out] `Integral((I*a*(tan(c + d*x) - I))**(1/3)*(A + B*tan(c + d*x)), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.



### 3.197 $\int \tan^2(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=270

$$\frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} - \frac{\sqrt{3} a^{2/3}(iA + B)\text{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3}\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2} d} - \frac{a^{2/3}(iA + B) \log(\cos(c + dx))}{2\sqrt[3]{2} d}$$

[Out]  $1/4*a^{(2/3)}*(A-I*B)*x*2^{(2/3)}-1/4*a^{(2/3)}*(I*A+B)*\ln(\cos(d*x+c))*2^{(2/3)}/d-3/4*a^{(2/3)}*(I*A+B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(2/3)}/d-1/2*a^{(2/3)}*(I*A+B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)}*3^{(1/2)})*3^{(1/2)}*2^{(2/3)}/d-9/8*B*(a+I*a*\tan(d*x+c))^{(2/3)}/d+3/8*B*\tan(d*x+c)^2*(a+I*a*\tan(d*x+c))^{(2/3)}/d-3/20*(4*I*A+B)*(a+I*a*\tan(d*x+c))^{(5/3)}/a/d$

**Rubi [A]**

time = 0.31, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3678, 3673, 3608, 3562, 57, 631, 210, 31}

$$\frac{\sqrt{3} a^{2/3}(B + iA)\text{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3}\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2} d} - \frac{3a^{2/3}(B + iA) \log\left(\frac{\sqrt{2}\sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)}}{2\sqrt[3]{2} d}\right)}{2\sqrt[3]{2} d} + \frac{a^{2/3}x(A - iB)}{2\sqrt[3]{2}} - \frac{3(B + 4iA)(a + ia \tan(c + dx))^{5/3}}{20ad} + \frac{3B \tan^2(c + dx)(a + ia \tan(c + dx))^{2/3}}{8d} - \frac{9B(a + ia \tan(c + dx))^{2/3}}{8d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^{(2/3)}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(a^{(2/3)}*(A - I*B)*x)/(2*2^{(1/3)}) - (\text{Sqrt}[3]*a^{(2/3)}*(I*A + B)*\text{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\text{Tan}[c + d*x])^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(2^{(1/3)}*d) - (a^{(2/3)}*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/(2*2^{(1/3)}*d) - (3*a^{(2/3)}*(I*A + B)*\text{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\text{Tan}[c + d*x])^{(1/3)}])/(2*2^{(1/3)}*d) - (9*B*(a + I*a*\text{Tan}[c + d*x])^{(2/3)})/(8*d) + (3*B*\text{Tan}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^{(2/3)})/(8*d) - (3*((4*I)*A + B)*(a + I*a*\text{Tan}[c + d*x])^{(5/3)})/(20*a*d)$

**Rule 31**

$\text{Int}[(a + (b*x))^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$   $\text{FreeQ}\{a, b, x\}$

**Rule 57**

$\text{Int}[1/(((a + (b*x))^{(1/3)})*((c + (d*x))^{(1/3)})), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(b*c - a*d)/b, 3]\}, \text{Simp}[-\text{Log}[\text{RemoveContent}[a + b*x, x]]/(2*b*q), x] + (\text{Dist}[3/(2*b), \text{Subst}[\text{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{(1/3)}], x] - \text{Dist}[3/(2*b*q), \text{Subst}[\text{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}], x]]) /;$



FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 3562

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

### Rule 3608

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

### Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_)), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \tan^2(c+dx)(a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx &= \frac{3B \tan^2(c+dx)(a+ia \tan(c+dx))^{2/3}}{8d} + \frac{3}{8d} \\
&= \frac{3B \tan^2(c+dx)(a+ia \tan(c+dx))^{2/3}}{8d} - \frac{3}{8d} \\
&= -\frac{9B(a+ia \tan(c+dx))^{2/3}}{8d} + \frac{3B \tan^2(c+dx)}{8d} \\
&= -\frac{9B(a+ia \tan(c+dx))^{2/3}}{8d} + \frac{3B \tan^2(c+dx)}{8d} \\
&= \frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} - \frac{a^{2/3}(iA+B) \log(\cos(c+dx))}{2\sqrt[3]{2}d} \\
&= \frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} - \frac{a^{2/3}(iA+B) \log(\cos(c+dx))}{2\sqrt[3]{2}d} \\
&= \frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} - \frac{\sqrt{3} a^{2/3}(iA+B) \tan^{-1}\left(\frac{\cos(c+dx)}{\sin(c+dx)}\right)}{2\sqrt[3]{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.53, size = 104, normalized size = 0.39

$$\frac{3(a+ia \tan(c+dx))^{2/3} \left( -8iA - 22B + 10(iA+B) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right) \right) + 5B \sec^2(c+dx) + (8A-2iB) \tan(c+dx)}{40d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^(2/3)\*(A + B\*Tan[c + d\*x]), x]

[Out] (3\*(a + I\*a\*Tan[c + d\*x])^(2/3)\*((-8\*I)\*A - 22\*B + 10\*(I\*A + B)\*Hypergeometric2F1[2/3, 1, 5/3, E^((2\*I)\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))] + 5\*B\*Sec[c + d\*x]^2 + (8\*A - (2\*I)\*B)\*Tan[c + d\*x]))/(40\*d)

**Maple [A]**

time = 0.07, size = 222, normalized size = 0.82

method	result
derivativedivides	$3i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{8}{3}}}{8} + \frac{iBa(a+ia \tan(dx+c))^{\frac{5}{3}}}{5} + \frac{Aa(a+ia \tan(dx+c))^{\frac{5}{3}}}{5} - \frac{ia^2B(a+ia \tan(dx+c))^{\frac{2}{3}}}{2} + \frac{2^{\frac{2}{3}} \ln((a+ia \tan(dx+c)))}{\dots} \right)$
default	$3i \left( -\frac{iB(a+ia \tan(dx+c))^{\frac{8}{3}}}{8} + \frac{iBa(a+ia \tan(dx+c))^{\frac{5}{3}}}{5} + \frac{Aa(a+ia \tan(dx+c))^{\frac{5}{3}}}{5} - \frac{ia^2B(a+ia \tan(dx+c))^{\frac{2}{3}}}{2} + \frac{2^{\frac{2}{3}} \ln((a+ia \tan(dx+c)))}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out]  $-3*I/d/a^2*(-1/8*I*B*(a+I*a*\tan(dx+c))^{8/3}+1/5*I*B*a*(a+I*a*\tan(dx+c))^{5/3}+1/5*A*a*(a+I*a*\tan(dx+c))^{5/3}-1/2*I*a^2*B*(a+I*a*\tan(dx+c))^{2/3}+(1/6*2^{2/3}/a^{1/3}*\ln((a+I*a*\tan(dx+c))^{1/3}-2^{1/3}*a^{1/3})-1/12*2^{2/3}/a^{1/3}*\ln((a+I*a*\tan(dx+c))^{2/3}+2^{1/3}*a^{1/3}*(a+I*a*\tan(dx+c))^{1/3}+2^{2/3}*a^{2/3}))+1/6*3^{1/2}*2^{2/3}/a^{1/3}*\arctan(1/3*3^{1/2}*(2^{2/3}/a^{1/3}*(a+I*a*\tan(dx+c))^{1/3}+1)))*a^3*(A-I*B)$

**Maxima [A]**

time = 0.53, size = 210, normalized size = 0.78

$$\frac{i \left( 20 \sqrt{3} 2^{\frac{2}{3}} (A - i B) a^{\frac{2}{3}} \arctan \left( \frac{\sqrt{3} a^{\frac{1}{3}} (a^{\frac{2}{3}} + i a \tan(dx+c) + a^{\frac{2}{3}})}{a^{\frac{1}{3}}} \right) - 10 \cdot 2^{\frac{2}{3}} (A - i B) a^{\frac{2}{3}} \log \left( 2^{\frac{2}{3}} a^{\frac{2}{3}} + 2^{\frac{1}{3}} (i a \tan(dx+c) + a)^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}} \right) + 20 \cdot 2^{\frac{2}{3}} (A - i B) a^{\frac{2}{3}} \log \left( -2^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{1}{3}} \right) - 15i (i a \tan(dx+c) + a)^{\frac{2}{3}} B a + 24 (i a \tan(dx+c) + a)^{\frac{1}{3}} (A + i B) a^{\frac{2}{3}} - 60i (i a \tan(dx+c) + a)^{\frac{1}{3}} B a^{\frac{2}{3}} \right)}{40 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/40*I*(20*\sqrt{3})*2^{2/3}*(A - I*B)*a^{11/3}*\arctan(1/6*\sqrt{3})*2^{2/3}*(2^{1/3}*a^{1/3} + 2*(I*a*\tan(dx+c) + a)^{1/3})/a^{1/3} - 10*2^{2/3}*(A - I*B)*a^{11/3}*\log(2^{2/3}*a^{2/3} + 2^{1/3}*(I*a*\tan(dx+c) + a)^{1/3})*a^{1/3} + (I*a*\tan(dx+c) + a)^{2/3} + 20*2^{2/3}*(A - I*B)*a^{11/3}*\log(-2^{1/3}*a^{1/3} + (I*a*\tan(dx+c) + a)^{1/3}) - 15*I*(I*a*\tan(dx+c) + a)^{8/3}*B*a + 24*(I*a*\tan(dx+c) + a)^{5/3}*(A + I*B)*a^2 - 60*I*(I*a*\tan(dx+c) + a)^{2/3}*B*a^3)/(a^3*d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(201) = 402.  
time = 0.69, size = 660, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(2/3)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/10*(3*2^{(2/3)}*(2*(2*I*A + 3*B)*e^{(4*I*d*x + 4*I*c)} + 2*(2*I*A + 3*B)*e^{(2*I*d*x + 2*I*c)} + 5*B)*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(2/3)}*e^{(4/3*I*d*x + 4/3*I*c)} \\ & - 10*(1/2)^{(1/3)}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^{(1/3)}*\log((2^{(1/3)}*(A^2 - 2*I*A*B - B^2)*a*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} + 2*(1/2)^{(2/3)}*d^2*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^{(2/3}))/((A^2 - 2*I*A*B - B^2)*a)) + 5*(1/2)^{(1/3)}*((-I*\sqrt{3})*d + d)*e^{(4*I*d*x + 4*I*c)} + 2*(-I*\sqrt{3})*d + d)*e^{(2*I*d*x + 2*I*c)} - I*\sqrt{3})*d + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^{(1/3)}*\log((2^{(1/3)}*(A^2 - 2*I*A*B - B^2)*a*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} - (1/2)^{(2/3)}*(I*\sqrt{3})*d^2 + d^2)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^{(2/3}))/((A^2 - 2*I*A*B - B^2)*a)) + 5*(1/2)^{(1/3)}*((I*\sqrt{3})*d + d)*e^{(4*I*d*x + 4*I*c)} + 2*(I*\sqrt{3})*d + d)*e^{(2*I*d*x + 2*I*c)} + I*\sqrt{3})*d + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^{(1/3)}*\log((2^{(1/3)}*(A^2 - 2*I*A*B - B^2)*a*(a/(e^{(2*I*d*x + 2*I*c)} + 1))^{(1/3)}*e^{(2/3*I*d*x + 2/3*I*c)} - (1/2)^{(2/3)}*(-I*\sqrt{3})*d^2 + d^2)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^{(2/3}))/((A^2 - 2*I*A*B - B^2)*a)))/(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d) \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{2/3} (A + B \tan(c + dx)) \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(2/3)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(2/3)\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(2/3)*tan(d*x + c)^2, x)
```

**Mupad [B]**

time = 7.75, size = 436, normalized size = 1.61

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(2/3),x)
```

```
[Out] (3*B*(a + a*tan(c + d*x)*1i)^(5/3))/(5*a*d) - (A*(a + a*tan(c + d*x)*1i)^(5/3)*3i)/(5*a*d) - (3*B*(a + a*tan(c + d*x)*1i)^(2/3))/(2*d) - (3*B*(a + a*tan(c + d*x)*1i)^(8/3))/(8*a^2*d) - (2^(2/3)*B*a^(2/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) - 2^(1/3)*a^(1/3)))/(2*d) + ((1i/2)^(1/3)*A*a^(2/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) + (-1)^(1/3)*2^(1/3)*a^(1/3)))/d + ((1i/2)^(1/3)*A*a^(2/3)*log((-1)^(1/3)*2^(1/3)*a^(1/3))/2 - (a*(tan(c + d*x)*1i + 1))^(1/3) + ((-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 - 1/2))/d - (2^(2/3)*B*a^(2/3)*log((9*B^2*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d^2 - (9*2^(1/3)*B^2*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/d^2)*((3^(1/2)*1i)/2 - 1/2))/(2*d) + (2^(2/3)*B*a^(2/3)*log((9*B^2*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d^2 - (9*2^(1/3)*B^2*a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2))/(2*d) - ((1i/2)^(1/3)*A*a^(2/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) - ((-1)^(1/3)*2^(1/3)*a^(1/3))/2 + ((-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/d
```

### 3.198 $\int \tan(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx$

Optimal. Leaf size=232

$$\frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} + \frac{\sqrt{3} a^{2/3}(A - iB) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{2} d} + \frac{a^{2/3}(A - iB) \log(\cos(c + dx))}{2\sqrt[3]{2} d}$$

[Out]  $1/4*a^{(2/3)}*(I*A+B)*x*2^{(2/3)}+1/4*a^{(2/3)}*(A-I*B)*\ln(\cos(d*x+c))*2^{(2/3)}/d+3/4*a^{(2/3)}*(A-I*B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(2/3)}/d+1/2*a^{(2/3)}*(A-I*B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)}*3^{(1/2)})*3^{(1/2)}*2^{(2/3)}/d+3/2*A*(a+I*a*\tan(d*x+c))^{(2/3)}/d-3/5*I*B*(a+I*a*\tan(d*x+c))^{(5/3)}/a/d$

**Rubi [A]**

time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3673, 3608, 3562, 57, 631, 210, 31}

$$\frac{\sqrt{3} a^{2/3}(A - iB) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{2} d} + \frac{3a^{2/3}(A - iB) \log\left(\frac{\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)}}{2\sqrt[3]{2} d}\right)}{2\sqrt[3]{2} d} + \frac{a^{2/3}(A - iB) \log(\cos(c + dx))}{2\sqrt[3]{2} d} + \frac{a^{2/3}x(B + iA)}{2\sqrt[3]{2}} + \frac{3A(a + ia \tan(c + dx))^{2/3}}{2d} - \frac{3iB(a + ia \tan(c + dx))^{5/3}}{5ad}$$

Antiderivative was successfully verified.

[In] `Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]`

[Out]  $(a^{(2/3)}*(I*A + B)*x)/(2*2^{(1/3)}) + (\operatorname{Sqrt}[3]*a^{(2/3)}*(A - I*B)*\operatorname{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\tan[c + d*x])^{(1/3)})/(\operatorname{Sqrt}[3]*a^{(1/3)})])/(2^{(1/3)}*d) + (a^{(2/3)}*(A - I*B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(2*2^{(1/3)}*d) + (3*a^{(2/3)}*(A - I*B)*\operatorname{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\tan[c + d*x])^{(1/3)}])/(2*2^{(1/3)}*d) + (3*A*(a + I*a*\tan[c + d*x])^{(2/3)})/(2*d) - (((3*I)/5)*B*(a + I*a*\tan[c + d*x])^{(5/3)})/(a*d)$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 57

`Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]`

Rule 210

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \tan(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx &= -\frac{3iB(a + ia \tan(c + dx))^{5/3}}{5ad} + \int (a + ia \tan(c + dx))^{2/3} dx \\
&= \frac{3A(a + ia \tan(c + dx))^{2/3}}{2d} - \frac{3iB(a + ia \tan(c + dx))^{5/3}}{5ad} \\
&= \frac{3A(a + ia \tan(c + dx))^{2/3}}{2d} - \frac{3iB(a + ia \tan(c + dx))^{5/3}}{5ad} \\
&= \frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} + \frac{a^{2/3}(A - iB) \log(\cos(c + dx))}{2\sqrt[3]{2} d} \\
&= \frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} + \frac{a^{2/3}(A - iB) \log(\cos(c + dx))}{2\sqrt[3]{2} d} \\
&= \frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} + \frac{\sqrt{3} a^{2/3}(A - iB) \tan^{-1}\left(\frac{1 + \tan(c + dx)}{1 - \tan(c + dx)}\right)}{2\sqrt[3]{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.92, size = 115, normalized size = 0.50

$$\frac{3(e^{idx})^{2/3} (a + ia \tan(c + dx))^{2/3} \left(10A - 4iB - 5(A - iB) {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right) + 4B \tan(c + dx)\right)}{20d(\cos(dx) + i \sin(dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^(2/3)\*(A + B\*Tan[c + d\*x]),x]

[Out] (3\*(E^(I\*d\*x))^(2/3)\*(a + I\*a\*Tan[c + d\*x])^(2/3)\*(10\*A - (4\*I)\*B - 5\*(A - I\*B)\*Hypergeometric2F1[2/3, 1, 5/3, E^((2\*I)\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))] + 4\*B\*Tan[c + d\*x]))/(20\*d\*(Cos[d\*x] + I\*Sin[d\*x])^(2/3))

**Maple [A]**

time = 0.05, size = 181, normalized size = 0.78

method	result
--------	--------



derivativedivides	$-\frac{3iB(a+ia \tan(dx+c))^{\frac{5}{3}}}{5} + \frac{3aA(a+ia \tan(dx+c))^{\frac{2}{3}}}{2} + 3 \left( \frac{2^{\frac{2}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}} - \frac{2^{\frac{2}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{12a^{\frac{1}{3}}} \right)$
default	$-\frac{3iB(a+ia \tan(dx+c))^{\frac{5}{3}}}{5} + \frac{3aA(a+ia \tan(dx+c))^{\frac{2}{3}}}{2} + 3 \left( \frac{2^{\frac{2}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}} - \frac{2^{\frac{2}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{12a^{\frac{1}{3}}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $3/d/a*(-1/5*I*B*(a+I*a*\tan(dx+c))^{5/3}+1/2*a*A*(a+I*a*\tan(dx+c))^{2/3}+(1/6*2^{2/3}/a^{1/3}*\ln((a+I*a*\tan(dx+c))^{1/3}-2^{1/3}*a^{1/3})-1/12*2^{2/3}/a^{1/3}*\ln((a+I*a*\tan(dx+c))^{2/3}+2^{1/3}*a^{1/3})*(a+I*a*\tan(dx+c))^{1/3}+2^{2/3}*a^{2/3})+1/6*3^{1/2}*2^{2/3}/a^{1/3}*\arctan(1/3*3^{1/2}*(2^{2/3}/a^{1/3}*(a+I*a*\tan(dx+c))^{1/3}+1)))*a^2*(A-I*B)$

**Maxima [A]**

time = 0.54, size = 187, normalized size = 0.81

$$\frac{10\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(a^{\frac{1}{3}}+2^{\frac{1}{3}}(i a \tan(dx+c)+a)^{\frac{1}{3}})}{6a^{\frac{1}{3}}}\right) - 5 \cdot 2^{\frac{2}{3}}(A-iB)a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i a \tan(dx+c)+a)^{\frac{1}{3}}+(i a \tan(dx+c)+a)^{\frac{1}{3}}\right) + 10 \cdot 2^{\frac{2}{3}}(A-iB)a^{\frac{1}{3}}\log\left(-2^{\frac{2}{3}}a^{\frac{2}{3}}+(i a \tan(dx+c)+a)^{\frac{1}{3}}\right) - 12i(i a \tan(dx+c)+a)^{\frac{1}{3}}Ba + 30(i a \tan(dx+c)+a)^{\frac{1}{3}}Aa^2}{20a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out]  $1/20*(10*\sqrt{3}*2^{2/3}*(A-I*B)*a^{8/3}*\arctan(1/6*\sqrt{3}*2^{2/3}*(2^{1/3}*a^{1/3}+2*(I*a*\tan(dx+c)+a)^{1/3})/a^{1/3})-5*2^{2/3}*(A-I*B)*a^{8/3}*\log(2^{2/3}*a^{2/3}+2^{1/3}*(I*a*\tan(dx+c)+a)^{1/3}*(I*a*\tan(dx+c)+a)^{1/3})+(I*a*\tan(dx+c)+a)^{2/3})+10*2^{2/3}*(A-I*B)*a^{8/3}*\log(-2^{1/3}*a^{1/3}+(I*a*\tan(dx+c)+a)^{1/3})-12*I*(I*a*\tan(dx+c)+a)^{5/3}*B*a+30*(I*a*\tan(dx+c)+a)^{2/3}*A*a^2)/(a^2*d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs.  $2(168) = 336$ .

time = 0.65, size = 571, normalized size = 2.46

$$\frac{10\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(a^{\frac{1}{3}}+2^{\frac{1}{3}}(i a \tan(dx+c)+a)^{\frac{1}{3}})}{6a^{\frac{1}{3}}}\right) - 5 \cdot 2^{\frac{2}{3}}(A-iB)a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i a \tan(dx+c)+a)^{\frac{1}{3}}+(i a \tan(dx+c)+a)^{\frac{1}{3}}\right) + 10 \cdot 2^{\frac{2}{3}}(A-iB)a^{\frac{1}{3}}\log\left(-2^{\frac{2}{3}}a^{\frac{2}{3}}+(i a \tan(dx+c)+a)^{\frac{1}{3}}\right) - 12i(i a \tan(dx+c)+a)^{\frac{1}{3}}Ba + 30(i a \tan(dx+c)+a)^{\frac{1}{3}}Aa^2}{20a^2d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\tan(c + d*x)*(A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*i)^{(2/3)}, x)$

[Out]  $(3*A*(a + a*\tan(c + d*x)*i)^{(2/3)})/(2*d) - (B*(a + a*\tan(c + d*x)*i)^{(5/3)}*i)/(5*a*d) + (2^{(2/3)}*A*a^{(2/3)}*\log((a*(\tan(c + d*x)*i + 1))^{(1/3)} - 2^{(1/3)}*a^{(1/3)}))/(2*d) + ((i/2)^{(1/3)}*B*a^{(2/3)}*\log((a*(\tan(c + d*x)*i + 1))^{(1/3)} + (-1)^{(1/3)}*2^{(1/3)}*a^{(1/3)}))/d + ((i/2)^{(1/3)}*B*a^{(2/3)}*\log((-1)^{(1/3)}*2^{(1/3)}*a^{(1/3)})/2 - (a*(\tan(c + d*x)*i + 1))^{(1/3)} + ((-1)^{(5/6)}*2^{(1/3)}*3^{(1/2)}*a^{(1/3)})/2)*((3^{(1/2)}*i)/2 - 1/2))/d + (2^{(2/3)}*A*a^{(2/3)}*\log((9*A^2*a^2*(a + a*\tan(c + d*x)*i)^{(1/3)})/d^2 - (9*2^{(1/3)}*A^2*a^{(7/3)}*((3^{(1/2)}*i)/2 - 1/2)^2)/d^2)*((3^{(1/2)}*i)/2 - 1/2))/(2*d) - (2^{(2/3)}*A*a^{(2/3)}*\log((9*A^2*a^2*(a + a*\tan(c + d*x)*i)^{(1/3)})/d^2 - (9*2^{(1/3)}*A^2*a^{(7/3)}*((3^{(1/2)}*i)/2 + 1/2)^2)/d^2)*((3^{(1/2)}*i)/2 + 1/2))/(2*d) - ((i/2)^{(1/3)}*B*a^{(2/3)}*\log((a*(\tan(c + d*x)*i + 1))^{(1/3)} - ((-1)^{(1/3)}*2^{(1/3)}*a^{(1/3)})/2 + ((-1)^{(5/6)}*2^{(1/3)}*3^{(1/2)}*a^{(1/3)})/2)*((3^{(1/2)}*i)/2 + 1/2))/d$

### 3.199 $\int (a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=202

$$-\frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} + \frac{\sqrt{3} a^{2/3}(iA+B) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3} \sqrt[3]{a+ia \tan(c+dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{2} d} + \frac{a^{2/3}(iA+B) \log(\cos(c+dx))}{2\sqrt[3]{2} d}$$

[Out]  $-1/4*a^{(2/3)}*(A-I*B)*x*2^{(2/3)}+1/4*a^{(2/3)}*(I*A+B)*\ln(\cos(d*x+c))*2^{(2/3)}/d$   
 $+3/4*a^{(2/3)}*(I*A+B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(2/3)}/d$   
 $+1/2*a^{(2/3)}*(I*A+B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/$   
 $a^{(1/3)*3^{(1/2)})*3^{(1/2)}*2^{(2/3)}/d+3/2*B*(a+I*a*\tan(d*x+c))^{(2/3)}/d$

**Rubi [A]**

time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3608, 3562, 57, 631, 210, 31}

$$\frac{\sqrt{3} a^{2/3}(B+iA) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2^{2/3} \sqrt[3]{a+ia \tan(c+dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt[3]{2} d} + \frac{3a^{2/3}(B+iA) \log\left(\frac{\sqrt{2} \sqrt[3]{a} - \sqrt[3]{a+ia \tan(c+dx)}}{2\sqrt[3]{2} d}\right)}{2\sqrt[3]{2} d} + \frac{a^{2/3}(B+iA) \log(\cos(c+dx))}{2\sqrt[3]{2} d} - \frac{a^{2/3}x(A-iB)}{2\sqrt[3]{2}} + \frac{3B(a+ia \tan(c+dx))^{2/3}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^{(2/3)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-1/2*(a^{(2/3)}*(A - I*B)*x)/2^{(1/3)} + (\operatorname{Sqrt}[3]*a^{(2/3)}*(I*A + B)*\operatorname{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)})/(\operatorname{Sqrt}[3]*a^{(1/3)})])/(2^{(1/3)}*d)$   
 $+ (a^{(2/3)}*(I*A + B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(2*2^{(1/3)}*d) + (3*a^{(2/3)}*(I*A + B)*\operatorname{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)}])/(2*2^{(1/3)}*d) + (3*B*(a + I*a*\operatorname{Tan}[c + d*x])^{(2/3)})/(2*d)$

**Rule 31**

$\operatorname{Int}[(a_ + (b_)*(x_))^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}[\{a, b\}, x]$

**Rule 57**

$\operatorname{Int}[1/(((a_ + (b_)*(x_))*((c_ + (d_)*(x_))^{(1/3)})), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Rt}[(b*c - a*d)/b, 3]\}, \operatorname{Simp}[-\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/(2*b*q), x] + (\operatorname{Dist}[3/(2*b), \operatorname{Subst}[\operatorname{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{(1/3)}], x] - \operatorname{Dist}[3/(2*b*q), \operatorname{Subst}[\operatorname{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}], x])] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{PosQ}[(b*c - a*d)/b]$

**Rule 210**

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3562

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

### Rule 3608

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
 \int (a + ia \tan(c + dx))^{2/3} (A + B \tan(c + dx)) dx &= \frac{3B(a + ia \tan(c + dx))^{2/3}}{2d} - (-A + iB) \int (a + ia \tan(c + dx))^{2/3} dx \\
 &= \frac{3B(a + ia \tan(c + dx))^{2/3}}{2d} - \frac{(a(iA + B)) \text{Subst}\left(\int \frac{1}{(a-x)} dx, x, a + ia \tan(c + dx)\right)}{2d} \\
 &= -\frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} + \frac{a^{2/3}(iA + B) \log(\cos(c + dx))}{2\sqrt[3]{2} d} + \frac{3B}{2d} \\
 &= -\frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} + \frac{a^{2/3}(iA + B) \log(\cos(c + dx))}{2\sqrt[3]{2} d} + \frac{3a}{2d} \\
 &= -\frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} + \frac{\sqrt{3} a^{2/3}(iA + B) \tan^{-1}\left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a}}{\sqrt{3}}}{\sqrt{2} d}\right)}{\sqrt[3]{2} d}
 \end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.68, size = 91, normalized size = 0.45

$$\frac{3 \left( \frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{2/3} \left( -2B + (iA + B) {}_2F_1 \left( \frac{2}{3}, 1; \frac{5}{3}; \frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right) \right)}{2\sqrt[3]{2} d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^(2/3)\*(A + B\*Tan[c + d\*x]),x]

[Out] (-3\*((a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x))))^(2/3)\*(-2\*B + (I\*A + B)\*Hypergeometric2F1[2/3, 1, 5/3, E^((2\*I)\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]))/(2\*2^(1/3)\*d)

**Maple [A]**

time = 0.05, size = 159, normalized size = 0.79

method	result
derivativedivides	$3i \left( -\frac{iB(a+ia \tan(dx+c))^{2/3}}{2} + \frac{2^{2/3} \ln \left( (a+ia \tan(dx+c))^{1/3} - 2^{1/3} a^{1/3} \right)}{6a^{1/3}} - \frac{2^{2/3} \ln \left( (a+ia \tan(dx+c))^{2/3} + 2^{1/3} a^{1/3} (a+ia \tan(dx+c))^{1/3} + 2^{2/3} a^{2/3} \right)}{12a^{1/3}} \right) d$
default	$3i \left( -\frac{iB(a+ia \tan(dx+c))^{2/3}}{2} + \frac{2^{2/3} \ln \left( (a+ia \tan(dx+c))^{1/3} - 2^{1/3} a^{1/3} \right)}{6a^{1/3}} - \frac{2^{2/3} \ln \left( (a+ia \tan(dx+c))^{2/3} + 2^{1/3} a^{1/3} (a+ia \tan(dx+c))^{1/3} + 2^{2/3} a^{2/3} \right)}{12a^{1/3}} \right) d$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(2/3)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 3\*I/d\*(-1/2\*I\*B\*(a+I\*a\*tan(d\*x+c))^(2/3)+(1/6\*2^(2/3)/a^(1/3)\*ln((a+I\*a\*tan(d\*x+c))^(1/3)-2^(1/3)\*a^(1/3))-1/12\*2^(2/3)/a^(1/3)\*ln((a+I\*a\*tan(d\*x+c))^(2/3)+2^(1/3)\*a^(1/3)\*(a+I\*a\*tan(d\*x+c))^(1/3)+2^(2/3)\*a^(2/3))+1/6\*3^(1/2)\*2^(2/3)/a^(1/3)\*arctan(1/3\*3^(1/2)\*(2^(2/3)/a^(1/3)\*(a+I\*a\*tan(d\*x+c))^(1/3)+1)))\*a\*(A-I\*B)

**Maxima [A]**

time = 0.51, size = 168, normalized size = 0.83

$$\frac{i \left( 2\sqrt{3} 2^{2/3} (A - iB) a^{5/3} \arctan \left( \frac{\sqrt{3} 2^{1/3} (2^{1/3} a^{1/3} + 2(i a \tan(dx+c) + a)^{1/3})}{6a^{1/3}} \right) - 2^{2/3} (A - iB) a^{5/3} \log \left( 2^{2/3} a^{1/3} + 2^{1/3} (i a \tan(dx+c) + a)^{1/3} + (i a \tan(dx+c) + a)^{2/3} \right) + 2 \cdot 2^{2/3} (A - iB) a^{5/3} \log \left( -2^{1/3} a^{1/3} + (i a \tan(dx+c) + a)^{1/3} \right) - 6i (i a \tan(dx+c) + a)^{5/3} B a \right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/4*I*(2*sqrt(3)*2^(2/3)*(A - I*B)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*(A - I*B)*a^(5/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 2*2^(2/3)*(A - I*B)*a^(5/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 6*I*(I*a*tan(d*x + c) + a)^(2/3)*B*a)/(a*d)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 489 vs.  $2(146) = 292$ .

time = 0.66, size = 489, normalized size = 2.42

$$\frac{3 \sqrt{3} \left( \frac{2^{1/3} a^{1/3} (A - I B) \arctan\left(\frac{2^{1/3} a^{1/3} (A - I B) \sqrt{3}}{2 \sqrt{3} a^{1/3} (A - I B) + 2^{2/3} (I a \tan(dx + c) + a)^{1/3}}\right) - 2^{2/3} (A - I B) a^{5/3} \log\left(\frac{2^{2/3} a^{2/3} + 2^{1/3} (I a \tan(dx + c) + a)^{1/3} a^{1/3}}{2^{2/3} a^{2/3} + 2^{1/3} (I a \tan(dx + c) + a)^{1/3} a^{1/3}}\right) + 2 \sqrt{3} a^{5/3} \log\left(\frac{-2^{1/3} a^{1/3} + (I a \tan(dx + c) + a)^{1/3}}{2^{1/3} a^{1/3} + (I a \tan(dx + c) + a)^{1/3}}\right) - 6 I (I a \tan(dx + c) + a)^{2/3} B a}{a^2} \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(3*2^(2/3)*B*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x + 4/3*I*c) + 2*(1/2)^(1/3)*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2*(1/2)^(2/3)*d^2*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + (1/2)^(1/3)*(I*sqrt(3)*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 + d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + (1/2)^(1/3)*(-I*sqrt(3)*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(-I*sqrt(3)*d^2 + d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)))/d
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{2/3} (A + B \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
[Out] Integral((I*a*(tan(c + d*x) - I))^(2/3)*(A + B*tan(c + d*x)), x)
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(2/3)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(2/3), x)

**Mupad [B]**

time = 7.30, size = 367, normalized size = 1.82

$\frac{3B(a + a \tan(c + dx))^{\frac{2}{3}}}{2d} + \frac{2^{\frac{2}{3}} B a^{\frac{2}{3}} \log(a(\tan(c + dx) + 1)^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}})}{2d} - \frac{(1i/2)^{\frac{1}{3}} A a^{\frac{2}{3}} \log(a(\tan(c + dx) + 1)^{\frac{1}{3}} + (-1)^{\frac{1}{3}} 2^{\frac{1}{3}} a^{\frac{1}{3}})}{d} - \frac{(1i/2)^{\frac{1}{3}} A a^{\frac{2}{3}} \log((-1)^{\frac{1}{3}} 2^{\frac{1}{3}} a^{\frac{1}{3}})}{2} - \frac{a(\tan(c + dx) + 1)^{\frac{1}{3}} + (-1)^{\frac{5}{6}} 2^{\frac{1}{3}} 3^{\frac{1}{2}} a^{\frac{1}{3}}}{2} \left( \frac{3^{\frac{1}{2}} 1i}{2} - \frac{1}{2} \right) / d + \frac{2^{\frac{2}{3}} B a^{\frac{2}{3}} \log(9 B^2 a^2 (a + a \tan(c + dx) + 1)^{\frac{1}{3}})}{d^2} - \frac{9 \cdot 2^{\frac{1}{3}} B^2 a^{\frac{7}{3}} \left( \frac{3^{\frac{1}{2}} 1i}{2} - \frac{1}{2} \right)^2 / d^2 \left( \frac{3^{\frac{1}{2}} 1i}{2} - \frac{1}{2} \right)}{2d} - \frac{2^{\frac{2}{3}} B a^{\frac{2}{3}} \log(9 B^2 a^2 (a + a \tan(c + dx) + 1)^{\frac{1}{3}})}{d^2} - \frac{9 \cdot 2^{\frac{1}{3}} B^2 a^{\frac{7}{3}} \left( \frac{3^{\frac{1}{2}} 1i}{2} + \frac{1}{2} \right)^2 / d^2 \left( \frac{3^{\frac{1}{2}} 1i}{2} + \frac{1}{2} \right)}{2d} + \frac{(1i/2)^{\frac{1}{3}} A a^{\frac{2}{3}} \log(a(\tan(c + dx) + 1)^{\frac{1}{3}} - (-1)^{\frac{1}{3}} 2^{\frac{1}{3}} a^{\frac{1}{3}})}{2} + \frac{(-1)^{\frac{5}{6}} 2^{\frac{1}{3}} 3^{\frac{1}{2}} a^{\frac{1}{3}}}{2} \left( \frac{3^{\frac{1}{2}} 1i}{2} + \frac{1}{2} \right) / d$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(2/3),x)

[Out]  $(3*B*(a + a*\tan(c + d*x)*1i)^{(2/3)})/(2*d) + (2^{(2/3)}*B*a^{(2/3)}*\log((a*(\tan(c + d*x)*1i + 1))^{(1/3)} - 2^{(1/3)}*a^{(1/3)}))/(2*d) - ((1i/2)^{(1/3)}*A*a^{(2/3)}*\log((a*(\tan(c + d*x)*1i + 1))^{(1/3)} + (-1)^{(1/3)}*2^{(1/3)}*a^{(1/3)}))/d - ((1i/2)^{(1/3)}*A*a^{(2/3)}*\log((-1)^{(1/3)}*2^{(1/3)}*a^{(1/3)}))/2 - (a*(\tan(c + d*x)*1i + 1))^{(1/3)} + ((-1)^{(5/6)}*2^{(1/3)}*3^{(1/2)}*a^{(1/3)})/2 * ((3^{(1/2)}*1i)/2 - 1/2))/d + (2^{(2/3)}*B*a^{(2/3)}*\log((9*B^2*a^2*(a + a*\tan(c + d*x)*1i)^{(1/3)}))/d^2 - (9*2^{(1/3)}*B^2*a^{(7/3)}*((3^{(1/2)}*1i)/2 - 1/2)^2/d^2*((3^{(1/2)}*1i)/2 - 1/2)))/(2*d) - (2^{(2/3)}*B*a^{(2/3)}*\log((9*B^2*a^2*(a + a*\tan(c + d*x)*1i)^{(1/3)}))/d^2 - (9*2^{(1/3)}*B^2*a^{(7/3)}*((3^{(1/2)}*1i)/2 + 1/2)^2/d^2*((3^{(1/2)}*1i)/2 + 1/2)))/(2*d) + ((1i/2)^{(1/3)}*A*a^{(2/3)}*\log((a*(\tan(c + d*x)*1i + 1))^{(1/3)} - (-1)^{(1/3)}*2^{(1/3)}*a^{(1/3)}))/2 + ((-1)^{(5/6)}*2^{(1/3)}*3^{(1/2)}*a^{(1/3)})/2 * ((3^{(1/2)}*1i)/2 + 1/2))/d$



### 3.200 $\int \cot(c+dx)(a+ia \tan(c+dx))^{2/3}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=289

$$-\frac{a^{2/3}(iA+B)x}{2\sqrt[3]{2}} + \frac{\sqrt{3} a^{2/3} A \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d} - \frac{\sqrt{3} a^{2/3}(A-iB) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2} d}$$

[Out]  $-1/4*a^{(2/3)}*(I*A+B)*x*2^{(2/3)}-1/4*a^{(2/3)}*(A-I*B)*\ln(\cos(d*x+c))*2^{(2/3)}/d$   
 $-1/2*a^{(2/3)}*A*\ln(\tan(d*x+c))/d+3/2*a^{(2/3)}*A*\ln(a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})/d-3/4*a^{(2/3)}*(A-I*B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(2/3)}/d+a^{(2/3)}*A*\arctan(1/3*(a^{(1/3)}+2*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)}*3^{(1/2)})*3^{(1/2)}/d-1/2*a^{(2/3)}*(A-I*B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)}*3^{(1/2)})*3^{(1/2)}*2^{(2/3)}/d$

Rubi [A]

time = 0.26, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3681, 3562, 57, 631, 210, 31, 3680}

$$-\frac{\sqrt{3} a^{2/3}(A-iB) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2} d} + \frac{\sqrt{3} a^{2/3} A \operatorname{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d} - \frac{3a^{2/3}(A-iB) \log\left(\frac{\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a + ia \tan(c+dx)}}{2\sqrt[3]{2} d}\right)}{2\sqrt[3]{2} d} - \frac{a^{2/3}(A-iB) \log(\cos(c+dx))}{2\sqrt[3]{2} d} - \frac{a^{2/3} A \log(\tan(c+dx))}{2d} + \frac{3a^{2/3} A \log\left(\frac{\sqrt[3]{a} - \sqrt[3]{a + ia \tan(c+dx)}}{2d}\right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]*(a + I*a*\operatorname{Tan}[c + d*x])^{(2/3)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-1/2*(a^{(2/3)}*(I*A + B)*x)/2^{(1/3)} + (\operatorname{Sqrt}[3]*a^{(2/3)}*A*\operatorname{ArcTan}[(a^{(1/3)} + 2*(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)})/(\operatorname{Sqrt}[3]*a^{(1/3)})])/d - (\operatorname{Sqrt}[3]*a^{(2/3)}*(A - I*B)*\operatorname{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)})/(\operatorname{Sqrt}[3]*a^{(1/3)})])/(2^{(1/3)}*d) - (a^{(2/3)}*(A - I*B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(2*2^{(1/3)}*d) - (a^{(2/3)}*A*\operatorname{Log}[\operatorname{Tan}[c + d*x]])/(2*d) + (3*a^{(2/3)}*A*\operatorname{Log}[a^{(1/3)} - (a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)}])/(2*d) - (3*a^{(2/3)}*(A - I*B)*\operatorname{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)}])/(2*2^{(1/3)}*d)$

Rule 31

$\operatorname{Int}[(a + (b*x)^{-1}), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 57

$\operatorname{Int}[1/((a + (b*x)^{-1})*((c + (d*x)^{1/3})), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Rt}[(b*c - a*d)/b, 3]\}, \operatorname{Simp}[-\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/(2*b*q), x] + (\operatorname{Dist}[3/(2*b), \operatorname{Subst}[\operatorname{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{1/3}], x] - \operatorname{Dist}[3/(2*b*q), \operatorname{Subst}[\operatorname{Int}[1/(q - x), x], x, (c + d*x)^{1/3}], x]) /;$

FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 3562

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3681

Int[(((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]))/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b + a\*B)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m, x], x] - Dist[(B\*c - A\*d)/(b\*c + a\*d), Int[(a + b\*Tan[e + f\*x])^m\*((a - b\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x]))], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned}
\int \cot(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx &= \frac{A \int \cot(c + dx)(a - ia \tan(c + dx))(a + ia \tan(c + dx)) dx}{a} \\
&= \frac{(aA) \text{Subst} \left( \int \frac{1}{x^3 \sqrt[3]{a + iax}} dx, x, \tan(c + dx) \right)}{d} \\
&= -\frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} - \frac{a^{2/3}(A - iB) \log(\cos(c + dx))}{2\sqrt[3]{2} d} \\
&= -\frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} - \frac{a^{2/3}(A - iB) \log(\cos(c + dx))}{2\sqrt[3]{2} d} \\
&= -\frac{a^{2/3}(iA + B)x}{2\sqrt[3]{2}} + \frac{\sqrt{3} a^{2/3} A \tan^{-1} \left( \frac{1 + \sqrt[3]{2} \tan(c + dx)}{\sqrt{3}} \right)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.05, size = 127, normalized size = 0.44

$$\frac{3 \left( \frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{2/3} \left( (A - iB) {}_2F_1 \left( \frac{2}{3}, 1; \frac{5}{3}; \frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right) - 2A {}_2F_1 \left( \frac{2}{3}, 1; \frac{5}{3}; \frac{2e^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right) \right)}{2\sqrt[3]{2} d}$$

Antiderivative was successfully verified.

```

[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
[Out] (3*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*((A - I*B)*Hypergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))] - 2*A*Hypergeometric2F1[2/3, 1, 5/3, (2*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]))/(2*2^(1/3)*d)

```

**Maple [F]**

time = 0.25, size = 0, normalized size = 0.00

$$\int \cot(dx + c)(a + ia \tan(dx + c))^{2/3}(A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)

```

[Out]  $\int (\cot(dx+c) * (a + I*a*\tan(dx+c))^{2/3} * (A+B*\tan(dx+c)), x)$

**Maxima [A]**

time = 0.53, size = 252, normalized size = 0.87

$$\frac{2\sqrt{3}^{\frac{1}{3}}(A - iB)a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}^{\frac{1}{3}}(A^2 - 3iAB - 3A^2B^2 + I^2B^3)}{a^{\frac{2}{3}}}\right) - 2^{\frac{1}{3}}(A - iB)a^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}a^{\frac{2}{3}} + 2^{\frac{1}{3}}(i a \tan(dx+c) + a)^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}}\right) + 2^{\frac{1}{3}}(A - iB)a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{2}{3}} + 2^{\frac{1}{3}}(i a \tan(dx+c) + a)^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}}\right) - 4\sqrt{3}^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}^{\frac{1}{3}}(A^2 - 3iAB - 3A^2B^2 + I^2B^3)}{a^{\frac{2}{3}}}\right) + 2Aa^{\frac{1}{3}}\log\left((i a \tan(dx+c) + a)^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}} + a^{\frac{1}{3}}\right) - 4Aa^{\frac{1}{3}}\log\left((i a \tan(dx+c) + a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/4*(2*\sqrt{3})^{2/3}*(A - I*B)*a^{2/3}*\arctan(1/6*\sqrt{3})^{2/3}*(2^{1/3})*a^{1/3} + 2*(I*a*\tan(dx+c) + a)^{1/3}/a^{1/3} - 2^{2/3}*(A - I*B)*a^{2/3}*\log(2^{2/3}*a^{2/3} + 2^{1/3}*(I*a*\tan(dx+c) + a)^{1/3}*a^{1/3} + (I*a*\tan(dx+c) + a)^{2/3}) + 2*2^{2/3}*(A - I*B)*a^{2/3}*\log(-2^{1/3}*a^{1/3} + (I*a*\tan(dx+c) + a)^{1/3}) - 4*\sqrt{3}*A*a^{2/3}*\arctan(1/3*\sqrt{3})*(2*(I*a*\tan(dx+c) + a)^{1/3} + a^{1/3})/a^{1/3} + 2*A*a^{2/3}*\log((I*a*\tan(dx+c) + a)^{2/3} + (I*a*\tan(dx+c) + a)^{1/3}*a^{1/3} + a^{2/3}) - 4*A*a^{2/3}*\log((I*a*\tan(dx+c) + a)^{1/3} - a^{1/3})/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 711 vs.  $2(211) = 422$ .

time = 0.78, size = 711, normalized size = 2.46

$$\frac{1}{4} \left( \frac{2^{1/3} (A^2 - 2IA^*AB - B^2) a^{2/3} \log\left(\frac{a}{e^{2I^*d^*x} + 2I^*c} + 1\right) e^{2/3 I^*d^*x + 2/3 I^*c} - (1/2)^{2/3} (I^* \sqrt{3} d^2 - d^2) (-(A^3 - 3I^*A^2B - 3A^*B^2 + I^*B^3) a^2/d^3)^{2/3} / ((A^2 - 2I^*A^*AB - B^2) a)}{1} + \frac{2^{1/3} (A^2 - 2IA^*AB - B^2) a^{2/3} \log\left(\frac{a}{e^{2I^*d^*x} + 2I^*c} + 1\right) e^{2/3 I^*d^*x + 2/3 I^*c} - (1/2)^{2/3} (-I^* \sqrt{3} d^2 - d^2) (-(A^3 - 3I^*A^2B - 3A^*B^2 + I^*B^3) a^2/d^3)^{2/3} / ((A^2 - 2I^*A^*AB - B^2) a)}{1} + \frac{1/2 (A^3 a^2/d^3)^{1/3} (I^* \sqrt{3} - 1) \log\left(\frac{1}{2} (2^{2/3} A^2 a^{2/3} (a/(e^{2I^*d^*x} + 2I^*c) + 1))^{1/3} e^{2/3 I^*d^*x + 2/3 I^*c} + (I^* \sqrt{3} d^2 + d^2) (A^3 a^2/d^3)^{2/3} / (A^2 a)\right)}{1} + \frac{1/2 (A^3 a^2/d^3)^{1/3} (-I^* \sqrt{3} - 1) \log\left(\frac{1}{2} (2^{2/3} A^2 a^{2/3} (a/(e^{2I^*d^*x} + 2I^*c) + 1))^{1/3} e^{2/3 I^*d^*x + 2/3 I^*c} + (-I^* \sqrt{3} d^2 + d^2) (A^3 a^2/d^3)^{2/3} / (A^2 a)\right)}{1} + \frac{(1/2)^{1/3} (-(A^3 - 3I^*A^2B - 3A^*B^2 + I^*B^3) a^2/d^3)^{1/3} \log\left(\frac{2^{1/3} (A^2 - 2I^*A^*AB - B^2) a^{2/3} \log\left(\frac{a}{e^{2I^*d^*x} + 2I^*c} + 1\right) e^{2/3 I^*d^*x + 2/3 I^*c} - 2(1/2)^{2/3} d^2 (-(A^3 - 3I^*A^2B - 3A^*B^2 + I^*B^3) a^2/d^3)^{2/3}}{2^{1/3} (A^2 - 2I^*A^*AB - B^2) a^{2/3} \log\left(\frac{a}{e^{2I^*d^*x} + 2I^*c} + 1\right) e^{2/3 I^*d^*x + 2/3 I^*c} - (1/2)^{2/3} (I^* \sqrt{3} d^2 - d^2) (-(A^3 - 3I^*A^2B - 3A^*B^2 + I^*B^3) a^2/d^3)^{2/3} / ((A^2 - 2I^*A^*AB - B^2) a)}\right)}{1} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/2*(1/2)^{1/3}*(-I*\sqrt{3} - 1)*(-(A^3 - 3I^*A^2*B - 3A^*B^2 + I^*B^3)*a^2/d^3)^{1/3}*\log((2^{1/3}*(A^2 - 2I^*A^*B - B^2)*a*(a/(e^{2I^*d*x} + 2I^*c) + 1))^{1/3}*e^{2/3*I^*d*x + 2/3*I^*c} - (1/2)^{2/3}*(I*\sqrt{3}*d^2 - d^2)*(-(A^3 - 3I^*A^2*B - 3A^*B^2 + I^*B^3)*a^2/d^3)^{2/3})/((A^2 - 2I^*A^*B - B^2)*a) + 1/2*(1/2)^{1/3}*(I*\sqrt{3} - 1)*(-(A^3 - 3I^*A^2*B - 3A^*B^2 + I^*B^3)*a^2/d^3)^{1/3}*\log((2^{1/3}*(A^2 - 2I^*A^*B - B^2)*a*(a/(e^{2I^*d*x} + 2I^*c) + 1))^{1/3}*e^{2/3*I^*d*x + 2/3*I^*c} - (1/2)^{2/3}*(-I*\sqrt{3}*d^2 - d^2)*(-(A^3 - 3I^*A^2*B - 3A^*B^2 + I^*B^3)*a^2/d^3)^{2/3})/((A^2 - 2I^*A^*B - B^2)*a) + 1/2*(A^3*a^2/d^3)^{1/3}*(I*\sqrt{3} - 1)*\log(1/2*(2^{2/3})*A^2*a*(a/(e^{2I^*d*x} + 2I^*c) + 1))^{1/3}*e^{2/3*I^*d*x + 2/3*I^*c} + (I*\sqrt{3}*d^2 + d^2)*(A^3*a^2/d^3)^{2/3})/(A^2*a) + 1/2*(A^3*a^2/d^3)^{1/3}*(-I*\sqrt{3} - 1)*\log(1/2*(2^{2/3})*A^2*a*(a/(e^{2I^*d*x} + 2I^*c) + 1))^{1/3}*e^{2/3*I^*d*x + 2/3*I^*c} + (-I*\sqrt{3}*d^2 + d^2)*(A^3*a^2/d^3)^{2/3})/(A^2*a) + (1/2)^{1/3}*(-(A^3 - 3I^*A^2*B - 3A^*B^2 + I^*B^3)*a^2/d^3)^{1/3}*\log((2^{1/3}*(A^2 - 2I^*A^*B - B^2)*a*(a/(e^{2I^*d*x} + 2I^*c) + 1))^{1/3}*e^{2/3*I^*d*x + 2/3*I^*c} - 2*(1/2)^{2/3}*d^2*(-(A^3 - 3I^*A^2*B - 3A^*B^2 + I^*B^3)*a^2/d^3)^{2/3})/((A^2 - 2I^*A^*B - B^2)*a)$

3))/((A^2 - 2\*I\*A\*B - B^2)\*a)) + (A^3\*a^2/d^3)^(1/3)\*log((2^(1/3)\*A^2\*a\*(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^(1/3)\*e^(2/3\*I\*d\*x + 2/3\*I\*c) - (A^3\*a^2/d^3)^(2/3)\*d^2)/(A^2\*a))

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{2/3} (A + B \tan(c + dx)) \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))\*\*(2/3)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(2/3)\*(A + B\*tan(c + d\*x))\*cot(c + d\*x), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+I\*a\*tan(d\*x+c))^(2/3)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(2/3)\*cot(d\*x + c), x)

**Mupad [B]**

time = 6.75, size = 1761, normalized size = 6.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(2/3),x)

[Out] log(- (486\*d^3\*(3\*A\*B^2\*a^9 - B^3\*a^9\*1i + A^2\*B\*a^9\*3i) - (1458\*a^7\*d^6\*((A^3\*a^2)/d^3)^(2/3) + 243\*d\*(a + a\*tan(c + d\*x)\*1i)^(1/3)\*(B^2\*a^8\*d^3 - 5\*A^2\*a^8\*d^3 + A\*B\*a^8\*d^3\*2i))\*((A^3\*a^2)/d^3)^(1/3))\*((A^3\*a^2)/d^3)^(2/3) - 243\*d\*(a + a\*tan(c + d\*x)\*1i)^(1/3)\*(A^5\*a^10 - A^4\*B\*a^10\*4i + A^2\*B^3\*a^10\*2i - 5\*A^3\*B^2\*a^10))\*((A^3\*a^2)/d^3)^(1/3) + log(- (486\*d^3\*(3\*A\*B^2\*a^9 - B^3\*a^9\*1i + A^2\*B\*a^9\*3i) - (1458\*a^7\*d^6\*(-(A^3\*a^2 + B^3\*a^2\*1i - 3\*A\*B^2\*a^2 - A^2\*B\*a^2\*3i)/(2\*d^3)))^(2/3) + 243\*d\*(a + a\*tan(c + d\*x)\*1i)^(1/3)\*(B^2\*a^8\*d^3 - 5\*A^2\*a^8\*d^3 + A\*B\*a^8\*d^3\*2i))\*(-(A^3\*a^2 + B^3\*a^2\*1i - 3\*A\*B^2\*a^2 - A^2\*B\*a^2\*3i)/(2\*d^3))^(1/3))\*(-(A^3\*a^2 + B^3\*a^2\*1i - 3\*A\*B^2\*a^2 - A^2\*B\*a^2\*3i)/(2\*d^3))^(2/3) - 243\*d\*(a + a\*tan(c + d\*x)\*1i)^(1/3)

$$\begin{aligned}
& \left(\frac{1}{3}\right) * (A^5 * a^{10} - A^4 * B * a^{10} * 4i + A^2 * B^3 * a^{10} * 2i - 5 * A^3 * B^2 * a^{10}) * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{1}{3}} + \left(\log\left(-\left(\left(3^{\frac{1}{2}} * 1i - 1\right)^2 * \left(486 * d^3 * \left(3 * A * B^2 * a^9 - B^3 * a^9 * 1i + A^2 * B * a^9 * 3i\right) - \left(\left(3^{\frac{1}{2}} * 1i - 1\right) * \left(243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(B^2 * a^8 * d^3 - 5 * A^2 * a^8 * d^3 + A * B * a^8 * d^3 * 2i\right) + \left(729 * a^7 * d^6 * \left(3^{\frac{1}{2}} * 1i - 1\right)^2 * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{2}{3}}\right) / 2\right) * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{1}{3}}\right) / 2 * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{2}{3}}\right) / 4 - 243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(A^5 * a^{10} - A^4 * B * a^{10} * 4i + A^2 * B^3 * a^{10} * 2i - 5 * A^3 * B^2 * a^{10}\right) * \left(3^{\frac{1}{2}} * 1i - 1\right) * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{1}{3}}\right) / 2 - \left(\log\left(-\left(\left(3^{\frac{1}{2}} * 1i + 1\right)^2 * \left(486 * d^3 * \left(3 * A * B^2 * a^9 - B^3 * a^9 * 1i + A^2 * B * a^9 * 3i\right) + \left(\left(3^{\frac{1}{2}} * 1i + 1\right) * \left(243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(B^2 * a^8 * d^3 - 5 * A^2 * a^8 * d^3 + A * B * a^8 * d^3 * 2i\right) + \left(729 * a^7 * d^6 * \left(3^{\frac{1}{2}} * 1i + 1\right)^2 * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{2}{3}}\right) / 2\right) * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{1}{3}}\right) / 2 * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{2}{3}}\right) / 4 - 243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(A^5 * a^{10} - A^4 * B * a^{10} * 4i + A^2 * B^3 * a^{10} * 2i - 5 * A^3 * B^2 * a^{10}\right) * \left(3^{\frac{1}{2}} * 1i + 1\right) * \left(\left(A^3 * a^2\right) / d^3\right)^{\frac{1}{3}}\right) / 2 - \log\left(-\left(\left(3^{\frac{1}{2}} * 1i\right) / 2 + 1 / 2\right)^2 * \left(486 * d^3 * \left(3 * A * B^2 * a^9 - B^3 * a^9 * 1i + A^2 * B * a^9 * 3i\right) + \left(243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(B^2 * a^8 * d^3 - 5 * A^2 * a^8 * d^3 + A * B * a^8 * d^3 * 2i\right) + 1458 * a^7 * d^6 * \left(\left(3^{\frac{1}{2}} * 1i\right) / 2 + 1 / 2\right)^2 * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{2}{3}}\right) * \left(\left(3^{\frac{1}{2}} * 1i\right) / 2 + 1 / 2\right) * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{1}{3}} * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{2}{3}} - 243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(A^5 * a^{10} - A^4 * B * a^{10} * 4i + A^2 * B^3 * a^{10} * 2i - 5 * A^3 * B^2 * a^{10}\right) * \left(\left(3^{\frac{1}{2}} * 1i\right) / 2 + 1 / 2\right) * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{1}{3}} + \log\left(-\left(\left(3^{\frac{1}{2}} * 1i\right) / 2 - 1 / 2\right)^2 * \left(486 * d^3 * \left(3 * A * B^2 * a^9 - B^3 * a^9 * 1i + A^2 * B * a^9 * 3i\right) - \left(243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(B^2 * a^8 * d^3 - 5 * A^2 * a^8 * d^3 + A * B * a^8 * d^3 * 2i\right) + 1458 * a^7 * d^6 * \left(\left(3^{\frac{1}{2}} * 1i\right) / 2 - 1 / 2\right)^2 * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{2}{3}}\right) * \left(\left(3^{\frac{1}{2}} * 1i\right) / 2 - 1 / 2\right) * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{1}{3}} * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{2}{3}} - 243 * d * \left(a + a * \tan(c + d * x) * 1i\right)^{\frac{1}{3}} * \left(A^5 * a^{10} - A^4 * B * a^{10} * 4i + A^2 * B^3 * a^{10} * 2i - 5 * A^3 * B^2 * a^{10}\right) * \left(\left(3^{\frac{1}{2}} * 1i\right) / 2 - 1 / 2\right) * \left(-\left(A^3 * a^2 + B^3 * a^2 * 1i - 3 * A * B^2 * a^2 - A^2 * B * a^2 * 3i\right) / \left(2 * d^3\right)\right)^{\frac{1}{3}}\right)
\end{aligned}$$

### 3.201 $\int \cot^2(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=342

$$\frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} + \frac{a^{2/3}(2iA + 3B)\text{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d} - \frac{\sqrt{3}a^{2/3}(iA + B)\text{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2}d}$$

[Out]  $1/4*a^{(2/3)}*(A-I*B)*x*2^{(2/3)}-1/4*a^{(2/3)}*(I*A+B)*\ln(\cos(d*x+c))*2^{(2/3)}/d-1/6*a^{(2/3)}*(2*I*A+3*B)*\ln(\tan(d*x+c))/d+1/2*a^{(2/3)}*(2*I*A+3*B)*\ln(a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})/d-3/4*a^{(2/3)}*(I*A+B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(2/3)}/d+1/3*a^{(2/3)}*(2*I*A+3*B)*\arctan(1/3*(a^{(1/3)}+2*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)*3^{(1/2)}})/d*3^{(1/2)}-1/2*a^{(2/3)}*(I*A+B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)*3^{(1/2)}})*3^{(1/2)}*2^{(2/3)}/d-A*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^{(2/3)}/d$

**Rubi** [A]

time = 0.42, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3679, 3681, 3562, 57, 631, 210, 31, 3680}

$$\frac{a^{2/3}(3B + 2iA)\text{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d} - \frac{\sqrt{3}a^{2/3}(B + iA)\text{ArcTan}\left(\frac{\sqrt[3]{a} + 2\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d} - \frac{a^{2/3}(3B + 2iA)\log(\tan(c + dx))}{6d} + \frac{a^{2/3}(3B + 2iA)\log(\sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)})}{2d} - \frac{3a^{2/3}(B + iA)\log(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)})}{2\sqrt[3]{2}d} - \frac{a^{2/3}(B + iA)\log(\cos(c + dx))}{2\sqrt[3]{2}d} + \frac{a^{2/3}(A - iB)}{2\sqrt[3]{2}} - \frac{\text{ArcTan}(c + dx)(a + ia \tan(c + dx))^{2/3}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^(2/3)\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(a^{(2/3)}*(A - I*B)*x)/(2*2^{(1/3)}) + (a^{(2/3)}*((2*I)*A + 3*B)*\text{ArcTan}[(a^{(1/3)} + 2*(a + I*a*\tan[c + d*x])^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(\text{Sqrt}[3]*d) - (\text{Sqrt}[3]*a^{(2/3)}*(I*A + B)*\text{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\tan[c + d*x])^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/((2^{(1/3)}*d) - (a^{(2/3)}*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]])/(2*2^{(1/3)}*d) - (a^{(2/3)}*((2*I)*A + 3*B)*\text{Log}[\text{Tan}[c + d*x]])/(6*d) + (a^{(2/3)}*((2*I)*A + 3*B)*\text{Log}[a^{(1/3)} - (a + I*a*\tan[c + d*x])^{(1/3)}])/(2*d) - (3*a^{(2/3)}*(I*A + B)*\text{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\tan[c + d*x])^{(1/3)}])/(2*2^{(1/3)}*d) - (A*\text{Cot}[c + d*x]*(a + I*a*\tan[c + d*x])^{(2/3)})/d$

**Rule 31**

Int[((a\_) + (b\_)\*(x\_))^(n-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 57**

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(1/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, Simp[-Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x]

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x]) /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

### Rule 210

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3562

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

### Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3681

```
Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
```



$A*b + a*B)/(b*c + a*d)$ ,  $\text{Int}[(a + b*\text{Tan}[e + f*x])^m, x]$ ,  $x] - \text{Dist}[(B*c - A*d)/(b*c + a*d)$ ,  $\text{Int}[(a + b*\text{Tan}[e + f*x])^m*((a - b*\text{Tan}[e + f*x])/(c + d*\text{Tan}[e + f*x]))$ ,  $x]$ ,  $x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\}$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{NeQ}[A*b + a*B, 0]$

Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^{2/3}}{d} + \dots \\ &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^{2/3}}{d} + \dots \\ &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^{2/3}}{d} + \dots \\ &= \frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} - \frac{a^{2/3}(iA + B) \log(\cos(c + dx))}{2\sqrt[3]{2} d} \\ &= \frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} - \frac{a^{2/3}(iA + B) \log(\cos(c + dx))}{2\sqrt[3]{2} d} \\ &= \frac{a^{2/3}(A - iB)x}{2\sqrt[3]{2}} + \frac{a^{2/3}(2iA + 3B) \tan^{-1} \left( \frac{\tan(c + dx)}{\sqrt{2}} \right)}{2\sqrt[3]{2} d} \end{aligned}$$

**Mathematica** [F]

time = 4.19, size = 0, normalized size = 0.00

$$\int \cot^2(c + dx)(a + ia \tan(c + dx))^{2/3}(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In]  $\text{Integrate}[\text{Cot}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^{2/3}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $\text{Integrate}[\text{Cot}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^{2/3}*(A + B*\text{Tan}[c + d*x]), x]$

**Maple** [F]

time = 0.23, size = 0, normalized size = 0.00

$$\int (\cot^2(dx + c))(a + ia \tan(dx + c))^{2/3}(A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)`

[Out] `int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)`

**Maxima** [A]

time = 0.55, size = 298, normalized size = 0.87

$$\left( \frac{6\sqrt{3}d^{3/2}(A+B)\arctan\left(\frac{\sqrt{3}d^{1/2}(a^2+2^2(a+Ia\tan(dx+c))^2+(a\tan(dx+c))^2)}{a^2}\right)}{d^2} - \frac{3a^2(A-B)\log\left(\frac{a^2+2^2(a+Ia\tan(dx+c))^2+(a\tan(dx+c))^2}{a^2}\right)}{d^2} - \frac{6a^2(A-B)\log\left(\frac{-2^2a^2+(a+Ia\tan(dx+c))^2}{a^2}\right)}{d^2} - \frac{4\sqrt{3}(A-B)\arctan\left(\frac{\sqrt{3}(a+Ia\tan(dx+c))^2+a^2}{a^2}\right)}{d^2} + \frac{2(A-B)\log\left(\frac{(a\tan(dx+c))^2+(a\tan(dx+c))^2+(a\tan(dx+c))^2}{a^2}\right)}{d^2} - \frac{4(A-B)\log\left(\frac{(a\tan(dx+c))^2+a^2}{a^2}\right)}{d^2} - \frac{12(a\tan(dx+c))^2}{d^2} \right) a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `-1/12*I*(6*sqrt(3)*2^(2/3)*(A - I*B)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(1/3) - 3*2^(2/3)*(A - I*B)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3))/a^(1/3) + 6*2^(2/3)*(A - I*B)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/a^(1/3) - 4*sqrt(3)*(2*A - 3*I*B)*arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) + 2*(2*A - 3*I*B)*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - 4*(2*A - 3*I*B)*log((I*a*tan(d*x + c) + a)^(1/3) - a^(1/3))/a^(1/3) - 12*I*(I*a*tan(d*x + c) + a)^(2/3)*A/(a*tan(d*x + c)))*a/d`

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1096 vs.  $2(255) = 510$ .

time = 0.64, size = 1096, normalized size = 3.20

---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `-1/6*(6*2^(2/3)*(I*A*e^(2*I*d*x + 2*I*c) + I*A)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x + 4/3*I*c) - 6*(1/2)^(1/3)*(d*e^(2*I*d*x + 2*I*c) - d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + 2*(1/2)^(2/3)*d^2*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(2/3))/(A^2 - 2*I*A*B - B^2)*a) - 3*(1/2)^(1/3)*((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) - I*sqrt(3)*d + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 + d^2))*((I*A^3 + 3*A^2`

```

*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) - 3*(1/2)^
(1/3)*((-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)*((I*A^3 +
3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2
)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/
3)*(-I*sqrt(3)*d^2 + d^2)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(2/
3)))/((A^2 - 2*I*A*B - B^2)*a)) - ((-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) +
I*sqrt(3)*d + d)*((-8*I*A^3 - 36*A^2*B + 54*I*A*B^2 + 27*B^3)*a^2/d^3)^(1/3
)*log(1/2*(2*2^(1/3)*(4*A^2 - 12*I*A*B - 9*B^2)*a*(a/(e^(2*I*d*x + 2*I*c) +
1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (I*sqrt(3)*d^2 - d^2)*((-8*I*A^3 - 36*
A^2*B + 54*I*A*B^2 + 27*B^3)*a^2/d^3)^(2/3)))/((4*A^2 - 12*I*A*B - 9*B^2)*a
) - ((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) - I*sqrt(3)*d + d)*((-8*I*A^3 -
36*A^2*B + 54*I*A*B^2 + 27*B^3)*a^2/d^3)^(1/3)*log(1/2*(2*2^(1/3)*(4*A^2 -
12*I*A*B - 9*B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c) + (-I*sqrt(3)*d^2 - d^2)*((-8*I*A^3 - 36*A^2*B + 54*I*A*B^2 + 27*B^3)*
a^2/d^3)^(2/3)))/((4*A^2 - 12*I*A*B - 9*B^2)*a)) - 2*(d*e^(2*I*d*x + 2*I*c)
- d)*((-8*I*A^3 - 36*A^2*B + 54*I*A*B^2 + 27*B^3)*a^2/d^3)^(1/3)*log((2^(1/
3)*(4*A^2 - 12*I*A*B - 9*B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*
I*d*x + 2/3*I*c) + d^2*(-8*I*A^3 - 36*A^2*B + 54*I*A*B^2 + 27*B^3)*a^2/d^3
)^(2/3)))/((4*A^2 - 12*I*A*B - 9*B^2)*a)))/(d*e^(2*I*d*x + 2*I*c) - d)

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{2}{3}} (A + B \tan(c + dx)) \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*(2/3)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(2/3)\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*  
\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^(2/3)\*(A+B\*tan(d\*x+c)),x, algorit  
hm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(2/3)\*cot(d\*x + c)^2,  
x)

**Mupad [B]**

time = 8.07, size = 2500, normalized size = 7.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^2*(A + B*\tan(c + d*x))*(a + a*\tan(c + d*x)*i)^{(2/3)}, x)$

[Out]  $\log\left(\frac{(18*d^3*(A^3*a^9*19i - A*B^2*a^9*27i + 45*A^2*B*a^9) - (1458*a^7*d^6*((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i - 594*A^3*a^{12} + B^3*a^{12}*1458i - 1458*A*B^2*a^{12} + A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(2/3)} - 9*d*(a + a*\tan(c + d*x)*i)^{(1/3)}*(135*B^2*a^8*d^3 - 75*A^2*a^8*d^3 + A*B*a^8*d^3*198i))*\left(-((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i - 594*A^3*a^{12} + B^3*a^{12}*1458i - 1458*A*B^2*a^{12} + A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(1/3)}\right)*\left(-((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i - 594*A^3*a^{12} + B^3*a^{12}*1458i - 1458*A*B^2*a^{12} + A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(2/3)} + 9*d*(a + a*\tan(c + d*x)*i)^{(1/3)}*(A^5*a^{10}*16i + 27*B^5*a^{10} + A*B^4*a^{10}*126i + 92*A^4*B*a^{10} - 231*A^2*B^3*a^{10} - A^3*B^2*a^{10}*208i))*\left(-((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i - 594*A^3*a^{12} + B^3*a^{12}*1458i - 1458*A*B^2*a^{12} + A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(1/3)} + \log\left(\frac{(18*d^3*(A^3*a^9*19i - A*B^2*a^9*27i + 45*A^2*B*a^9) - (1458*a^7*d^6*((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i + 594*A^3*a^{12} - B^3*a^{12}*1458i + 1458*A*B^2*a^{12} - A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(2/3)} - 9*d*(a + a*\tan(c + d*x)*i)^{(1/3)}*(135*B^2*a^8*d^3 - 75*A^2*a^8*d^3 + A*B*a^8*d^3*198i))*\left((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i + 594*A^3*a^{12} - B^3*a^{12}*1458i + 1458*A*B^2*a^{12} - A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(1/3)}\right)*\left((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3))^2 - 11664*a^{10}*((432*A^6*a^{14} - 1458*B^6*a^{14} + 15066*A^2*B^4*a^{14} - 10044*A^4*B^2*a^{14})/d^6 - ((7290*A*B^5*a^{14} + 3240*A^5*B*a^{14} - 16470*A^3*B^3*a^{14})*i)/d^6))^{(1/2)}*i + 594*A^3*a^{12} - B^3*a^{12}*1458i + 1458*A*B^2*a^{12} - A^2*B*a^{12}*486i)*i)/(5832*a^{10}*d^3)}\right)^{(2/3)} + 9*d*(a + a*\tan(c + d*x)*i)^{(1/3)}*(A^5*a^{10}*16i + 27*B^5*a^{10} + A*B^4*a^{10}*126i + 92*$

$$\begin{aligned}
& A^4 B a^{10} - 231 A^2 B^3 a^{10} - A^3 B^2 a^{10} \cdot 208i) * (((d^3 * (((594 A^3 a^{12} \\
& + 1458 A B^2 a^{12}) * i) / d^3 + (1458 B^3 a^{12} + 486 A^2 B a^{12}) / d^3)^2 - 116 \\
& 64 a^{10} * ((432 A^6 a^{14} - 1458 B^6 a^{14} + 15066 A^2 B^4 a^{14} - 10044 A^4 B^2 \\
& a^{14}) / d^6 - ((7290 A B^5 a^{14} + 3240 A^5 B a^{14} - 16470 A^3 B^3 a^{14}) * i) / \\
& d^6))^{(1/2)} * i + 594 A^3 a^{12} - B^3 a^{12} * 1458i + 1458 A B^2 a^{12} - A^2 B a^{12} \\
& * 486i) * i) / (5832 a^{10} d^3)^{(1/3)} + \log(((3^{(1/2)} * i) / 2 - 1/2)^2 * (18 d^3 * \\
& (A^3 a^9 * 19i - A B^2 a^9 * 27i + 45 A^2 B a^9) + ((3^{(1/2)} * i) / 2 - 1/2) * (9 d * \\
& (a + a \tan(c + d x) * i))^{(1/3)} * (135 B^2 a^8 d^3 - 75 A^2 a^8 d^3 + A B a^8 d \\
& ^3 * 198i) - 1458 a^7 d^6 * ((3^{(1/2)} * i) / 2 - 1/2)^2 * (-((d^3 * (((594 A^3 a^{12} + \\
& 1458 A B^2 a^{12}) * i) / d^3 + (1458 B^3 a^{12} + 486 A^2 B a^{12}) / d^3)^2 - 11664 \\
& a^{10} * ((432 A^6 a^{14} - 1458 B^6 a^{14} + 15066 A^2 B^4 a^{14} - 10044 A^4 B^2 a^{14}) / d^6 - ((7290 A B^5 a^{14} + \\
& 3240 A^5 B a^{14} - 16470 A^3 B^3 a^{14}) * i) / d^6))^{(1/2)} * i - 594 A^3 a^{12} + B^3 a^{12} * 1458i - 1458 A B^2 a^{12} + A^2 B a^{12} \\
& * 486i) * i) / (5832 a^{10} d^3)^{(2/3)}) * (-((d^3 * (((594 A^3 a^{12} + 1458 A B^2 a^{12}) * i) / d^3 + (1458 B^3 a^{12} + \\
& 486 A^2 B a^{12}) / d^3)^2 - 11664 a^{10} * ((432 A^6 a^{14} - 1458 B^6 a^{14} + 15066 A^2 B^4 a^{14} - 10044 A^4 B^2 a^{14}) / d^6 - ((7 \\
& 290 A B^5 a^{14} + 3240 A^5 B a^{14} - 16470 A^3 B^3 a^{14}) * i) / d^6))^{(1/2)} * i - \\
& 594 A^3 a^{12} + B^3 a^{12} * 1458i - 1458 A B^2 a^{12} + A^2 B a^{12} * 486i) * i) / (58 \\
& 32 a^{10} d^3)^{(1/3)}) * (-((d^3 * (((594 A^3 a^{12} + 1458 A B^2 a^{12}) * i) / d^3 + \\
& (1458 B^3 a^{12} + 486 A^2 B a^{12}) / d^3)^2 - 11664 a^{10} * ((432 A^6 a^{14} - 1458 B^6 a^{14} + 15066 A^2 B^4 a^{14} - 10044 A^4 B^2 a^{14}) / d^6 - ((7290 A B^5 a^{14} \\
& + 3240 A^5 B a^{14} - 16470 A^3 B^3 a^{14}) * i) / d^6))^{(1/2)} * i - 594 A^3 a^{12} \\
& + B^3 a^{12} * 1458i - 1458 A B^2 a^{12} + A^2 B a^{12} * 486i) * i) / (5832 a^{10} d^3)^{(2/3)} + 9 d * (a + a \tan(c + d x) * i))^{(1/3)} * (A^5 a^{10} * 16i + 27 B^5 a^{10} + A B \\
& ^4 a^{10} * 126i + 92 A^4 B a^{10} - 231 A^2 B^3 a^{10} - A^3 B^2 a^{10} * 208i) * ((3^{(1/2)} * i) / 2 - 1/2) * (-((d^3 * (((594 A^3 a^{12} + 14...
\end{aligned}$$

$$3.202 \quad \int \frac{A+B \tan(c+dx)}{\sqrt[3]{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=213

$$-\frac{(A-iB)x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{\sqrt{3}(iA+B)\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia \tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}d} + \frac{(iA+B)\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}d} + \frac{3(iA+B)}{4\sqrt[3]{2}\sqrt[3]{a}}$$

[Out] -1/8\*(A-I\*B)\*x\*2^(2/3)/a^(1/3)+1/8\*(I\*A+B)\*ln(cos(d\*x+c))\*2^(2/3)/a^(1/3)/d  
+3/8\*(I\*A+B)\*ln(2^(1/3)\*a^(1/3)-(a+I\*a\*tan(d\*x+c))^(1/3))\*2^(2/3)/a^(1/3)/d  
+1/4\*(I\*A+B)\*arctan(1/3\*(a^(1/3)+2^(2/3)\*(a+I\*a\*tan(d\*x+c))^(1/3))/a^(1/3)\*  
3^(1/2))\*3^(1/2)\*2^(2/3)/a^(1/3)/d+3/2\*(I\*A-B)/d/(a+I\*a\*tan(d\*x+c))^(1/3)

Rubi [A]

time = 0.12, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3607, 3562, 57, 631, 210, 31}

$$\frac{\sqrt{3}(B+iA)\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia \tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}d} + \frac{3(-B+iA)}{2d\sqrt[3]{a+ia \tan(c+dx)}} + \frac{3(B+iA)\log(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia \tan(c+dx)})}{4\sqrt[3]{2}\sqrt[3]{a}d} + \frac{(B+iA)\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}d} - \frac{x(A-iB)}{4\sqrt[3]{2}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^(1/3), x]

[Out] -1/4\*((A - I\*B)\*x)/(2^(1/3)\*a^(1/3)) + (Sqrt[3]\*(I\*A + B)\*ArcTan[(a^(1/3) + 2^(2/3)\*(a + I\*a\*Tan[c + d\*x])^(1/3))/(Sqrt[3]\*a^(1/3))])/(2\*2^(1/3)\*a^(1/3)\*d) + ((I\*A + B)\*Log[Cos[c + d\*x]])/(4\*2^(1/3)\*a^(1/3)\*d) + (3\*(I\*A + B)\*Log[2^(1/3)\*a^(1/3) - (a + I\*a\*Tan[c + d\*x])^(1/3)])/(4\*2^(1/3)\*a^(1/3)\*d) + (3\*(I\*A - B))/(2\*d\*(a + I\*a\*Tan[c + d\*x])^(1/3))

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(1/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, Simp[-Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x] + (Dist[3/(2\*b), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

Rule 210

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3562

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

### Rule 3607

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt[3]{a + ia \tan(c + dx)}} dx &= \frac{3(iA - B)}{2d \sqrt[3]{a + ia \tan(c + dx)}} + \frac{(A - iB) \int (a + ia \tan(c + dx))^{2/3} dx}{2a} \\
&= \frac{3(iA - B)}{2d \sqrt[3]{a + ia \tan(c + dx)}} - \frac{(iA + B) \text{Subst} \left( \int \frac{1}{(a-x) \sqrt[3]{a+x}} dx, x, ia \tan(c + dx) \right)}{2d} \\
&= -\frac{(A - iB)x}{4\sqrt[3]{2} \sqrt[3]{a}} + \frac{(iA + B) \log(\cos(c + dx))}{4\sqrt[3]{2} \sqrt[3]{a} d} + \frac{3(iA - B)}{2d \sqrt[3]{a + ia \tan(c + dx)}} + \frac{(3(iA + B) \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)))}{4\sqrt[3]{2} \sqrt[3]{a} d} \\
&= -\frac{(A - iB)x}{4\sqrt[3]{2} \sqrt[3]{a}} + \frac{(iA + B) \log(\cos(c + dx))}{4\sqrt[3]{2} \sqrt[3]{a} d} + \frac{3(iA + B) \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)})}{4\sqrt[3]{2} \sqrt[3]{a} d} \\
&\quad + \frac{\sqrt{3} (iA + B) \tan^{-1} \left( \frac{1 + \frac{2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{2\sqrt[3]{2} \sqrt[3]{a} d} \\
&= -\frac{(A - iB)x}{4\sqrt[3]{2} \sqrt[3]{a}} + \frac{\sqrt{3} (iA + B) \tan^{-1} \left( \frac{1 + \frac{2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{2\sqrt[3]{2} \sqrt[3]{a} d} + \frac{(iA + B) \log(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)})}{4\sqrt[3]{2} \sqrt[3]{a} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.73, size = 137, normalized size = 0.64

$$\frac{3ie^{-2i(c+dx)} \left( \frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{2/3} \left( -2(A+iB)(1+e^{2i(c+dx)}) + (A-iB)e^{2i(c+dx)} {}_2F_1 \left( \frac{2}{3}, 1; \frac{5}{3}; \frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}} \right) \right)}{4\sqrt[3]{2} ad}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^(1/3), x]

[Out] (((-3\*I)/4)\*((a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x))))^(2/3)\*(-2\*(A + I\*B)\*(1 + E^((2\*I)\*(c + d\*x)))) + (A - I\*B)\*E^((2\*I)\*(c + d\*x))\*Hypergeometric2F1[2/3, 1, 5/3, E^((2\*I)\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))])/(2^(1/3)\*a\*d\*E^((2\*I)\*(c + d\*x)))

**Maple [A]**

time = 0.05, size = 166, normalized size = 0.78

method	result
--------	--------



derivativedivides	$3i \left( \frac{-\frac{A}{2} - \frac{iB}{2}}{(a+ia \tan(dx+c))^{\frac{1}{3}}} + \frac{2^{\frac{2}{3}} \ln \left( (a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}} \right)}{6a^{\frac{1}{3}}} - \frac{2^{\frac{2}{3}} \ln \left( (a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}} (a+ia \tan(dx+c))^{\frac{1}{3}} + 2^{\frac{2}{3}} a^{\frac{2}{3}} \right)}{12a^{\frac{1}{3}}} \right) d$
default	$3i \left( \frac{-\frac{A}{2} - \frac{iB}{2}}{(a+ia \tan(dx+c))^{\frac{1}{3}}} + \frac{2^{\frac{2}{3}} \ln \left( (a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}} \right)}{6a^{\frac{1}{3}}} - \frac{2^{\frac{2}{3}} \ln \left( (a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}} (a+ia \tan(dx+c))^{\frac{1}{3}} + 2^{\frac{2}{3}} a^{\frac{2}{3}} \right)}{12a^{\frac{1}{3}}} \right) d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)`

[Out]  $3I/d * (-(-1/2*A - 1/2*I*B) / (a+I*a*tan(d*x+c))^{1/3} + (1/6*2^{2/3}/a^{1/3}) * \ln((a+I*a*tan(d*x+c))^{1/3} - 2^{1/3}*a^{1/3}) - 1/12*2^{2/3}/a^{1/3} * \ln((a+I*a*tan(d*x+c))^{2/3} + 2^{1/3}*a^{1/3}*(a+I*a*tan(d*x+c))^{1/3} + 2^{2/3}*a^{2/3})) + 1/6*3^{1/2}*2^{2/3}/a^{1/3} * \arctan(1/3*3^{1/2}*(2^{2/3}/a^{1/3}*(a+I*a*tan(d*x+c))^{1/3} + 1)) * (-1/2*I*B + 1/2*A)$

**Maxima [A]**

time = 0.55, size = 172, normalized size = 0.81

$$\frac{i \left( 2\sqrt{3} 2^{\frac{2}{3}} (A - iB) a^{\frac{2}{3}} \arctan \left( \frac{\sqrt{3} 2^{\frac{2}{3}} (2^{\frac{1}{3}} a^{\frac{1}{3}} + 2(i a \tan(dx+c) + a)^{\frac{1}{3}})}{a a^{\frac{1}{3}}} \right) - 2^{\frac{2}{3}} (A - iB) a^{\frac{2}{3}} \log \left( 2^{\frac{2}{3}} a^{\frac{2}{3}} + 2^{\frac{1}{3}} (i a \tan(dx+c) + a)^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}} \right) + 2 \cdot 2^{\frac{2}{3}} (A - iB) a^{\frac{2}{3}} \log \left( -2^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{1}{3}} \right) + \frac{12(A+iB)a}{(i a \tan(dx+c) + a)^{\frac{2}{3}}} \right)}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")`

[Out]  $1/8*I*(2*\sqrt{3}*2^{2/3}*(A - I*B)*a^{2/3}*\arctan(1/6*\sqrt{3}*2^{2/3}*(2^{1/3}*a^{1/3} + 2*(I*a*\tan(d*x + c) + a)^{1/3})/a^{1/3}) - 2^{2/3}*(A - I*B)*a^{2/3}*\log(2^{2/3}*a^{2/3} + 2^{1/3}*(I*a*\tan(d*x + c) + a)^{1/3}*a^{1/3} + (I*a*\tan(d*x + c) + a)^{2/3}) + 2*2^{2/3}*(A - I*B)*a^{2/3}*\log(-2^{1/3}*a^{1/3} + (I*a*\tan(d*x + c) + a)^{1/3}) + 12*(A + I*B)*a/(I*a*\tan(d*x + c) + a)^{1/3})/(a*d)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs.  $2(156) = 312$ .

time = 0.90, size = 547, normalized size = 2.57

$$\frac{i \left( 2\sqrt{3} 2^{\frac{2}{3}} (A - iB) a^{\frac{2}{3}} \arctan \left( \frac{\sqrt{3} 2^{\frac{2}{3}} (2^{\frac{1}{3}} a^{\frac{1}{3}} + 2(i a \tan(dx+c) + a)^{\frac{1}{3}})}{a a^{\frac{1}{3}}} \right) - 2^{\frac{2}{3}} (A - iB) a^{\frac{2}{3}} \log \left( 2^{\frac{2}{3}} a^{\frac{2}{3}} + 2^{\frac{1}{3}} (i a \tan(dx+c) + a)^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{2}{3}} \right) + 2 \cdot 2^{\frac{2}{3}} (A - iB) a^{\frac{2}{3}} \log \left( -2^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{1}{3}} \right) + \frac{12(A+iB)a}{(i a \tan(dx+c) + a)^{\frac{2}{3}}} \right)}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
[Out] 1/4*(2*(1/2)^(1/3)*a*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^3))^(1/3)
*e^(2*I*d*x + 2*I*c)*log((2*(1/2)^(2/3)*a*d^2*((-I*A^3 - 3*A^2*B + 3*I*A*B^2
+ B^3)/(a*d^3))^(2/3) + 2^(1/3)*(A^2 - 2*I*A*B - B^2)*(a/(e^(2*I*d*x + 2*
I*c) + 1)))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))/(A^2 - 2*I*A*B - B^2) - (1/2)^(1
/3)*(I*sqrt(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^3))^(1
/3)*e^(2*I*d*x + 2*I*c)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*(a/(e^(2*I*d*x +
2*I*c) + 1)))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (1/2)^(2/3)*(I*sqrt(3)*a*d^2
- a*d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^3))^(2/3))/(A^2 - 2*I*A
*B - B^2) - (1/2)^(1/3)*(-I*sqrt(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*
B^2 + B^3)/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log((2^(1/3)*(A^2 - 2*I*A*B -
B^2)*(a/(e^(2*I*d*x + 2*I*c) + 1)))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (1/2)^(
2/3)*(-I*sqrt(3)*a*d^2 - a*d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^
3))^(2/3))/(A^2 - 2*I*A*B - B^2) - 3*2^(2/3)*((-I*A + B)*e^(2*I*d*x + 2*I*
c) - I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x + 4/3*I*c)*
e^(-2*I*d*x - 2*I*c)/(a*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt[3]{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x)
[Out] Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))^(1/3), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((B*tan(d*x + c) + A)/(I*a*tan(d*x + c) + a)^(1/3), x)
```

**Mupad [B]**

time = 6.99, size = 383, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B \cdot \tan(c + d \cdot x)) / (a + a \cdot \tan(c + d \cdot x) \cdot i)^{1/3}, x)$

[Out]  $(A \cdot 3i) / (2 \cdot d \cdot (a + a \cdot \tan(c + d \cdot x) \cdot i)^{1/3}) - (3 \cdot B) / (2 \cdot d \cdot (a + a \cdot \tan(c + d \cdot x) \cdot i)^{1/3}) - ((i/16)^{1/3} \cdot A \cdot \log((a \cdot (\tan(c + d \cdot x) \cdot i + 1))^{1/3} + (-1)^{1/3} \cdot 2^{1/3} \cdot a^{1/3})) / (a^{1/3} \cdot d) + (4^{1/3} \cdot B \cdot \log(18 \cdot B^2 \cdot d \cdot (a + a \cdot \tan(c + d \cdot x) \cdot i)^{1/3} - 9 \cdot 4^{2/3} \cdot B^2 \cdot a^{1/3} \cdot d)) / (4 \cdot a^{1/3} \cdot d) + (4^{1/3} \cdot B \cdot \log(18 \cdot B^2 \cdot d \cdot (a + a \cdot \tan(c + d \cdot x) \cdot i)^{1/3} - 9 \cdot 4^{2/3} \cdot B^2 \cdot a^{1/3} \cdot d \cdot ((3^{1/2} \cdot i) / 2 - 1/2)^2) \cdot ((3^{1/2} \cdot i) / 2 - 1/2)) / (4 \cdot a^{1/3} \cdot d) - (4^{1/3} \cdot B \cdot \log(18 \cdot B^2 \cdot d \cdot (a + a \cdot \tan(c + d \cdot x) \cdot i)^{1/3} - 9 \cdot 4^{2/3} \cdot B^2 \cdot a^{1/3} \cdot d \cdot ((3^{1/2} \cdot i) / 2 + 1/2)^2) \cdot ((3^{1/2} \cdot i) / 2 + 1/2)) / (4 \cdot a^{1/3} \cdot d) - ((i/16)^{1/3} \cdot A \cdot \log((( -1)^{1/3} \cdot 2^{1/3} \cdot a^{1/3}) / 2 - (a \cdot (\tan(c + d \cdot x) \cdot i + 1))^{1/3} + ((-1)^{5/6}) \cdot 2^{1/3} \cdot 3^{1/2} \cdot a^{1/3}) / 2) \cdot ((3^{1/2} \cdot i) / 2 - 1/2)) / (a^{1/3} \cdot d) + ((i/16)^{1/3} \cdot A \cdot \log((a \cdot (\tan(c + d \cdot x) \cdot i + 1))^{1/3} - ((-1)^{1/3} \cdot 2^{1/3} \cdot a^{1/3}) / 2 + ((-1)^{5/6}) \cdot 2^{1/3} \cdot 3^{1/2} \cdot a^{1/3}) / 2) \cdot ((3^{1/2} \cdot i) / 2 + 1/2)) / (a^{1/3} \cdot d)$

### 3.203 $\int \frac{A+B \tan(c+dx)}{(a+ia \tan(c+dx))^{2/3}} dx$

**Optimal.** Leaf size=213

$$-\frac{(A-iB)x}{4 \cdot 2^{2/3} a^{2/3}} - \frac{\sqrt{3} (iA+B) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a+2^{2/3}} \sqrt[3]{a+ia \tan(c+dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{2 \cdot 2^{2/3} a^{2/3} d} + \frac{(iA+B) \log(\cos(c+dx))}{4 \cdot 2^{2/3} a^{2/3} d} + \frac{3(iA+B)}{4 \cdot 2^{2/3} a^{2/3} d}$$

[Out]  $-1/8*(A-I*B)*x*2^{(1/3)}/a^{(2/3)}+1/8*(I*A+B)*\ln(\cos(d*x+c))*2^{(1/3)}/a^{(2/3)}/d$   
 $+3/8*(I*A+B)*\ln(2^{(1/3)}*a^{(1/3)}-(a+I*a*\tan(d*x+c))^{(1/3)})*2^{(1/3)}/a^{(2/3)}/d$   
 $-1/4*(I*A+B)*\arctan(1/3*(a^{(1/3)}+2^{(2/3)}*(a+I*a*\tan(d*x+c))^{(1/3)})/a^{(1/3)}*3^{(1/2)})*3^{(1/2)}*2^{(1/3)}/a^{(2/3)}/d+3/4*(I*A-B)/d/(a+I*a*\tan(d*x+c))^{(2/3)}$

**Rubi [A]**

time = 0.12, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3607, 3562, 59, 631, 210, 31}

$$-\frac{\sqrt{3} (B+iA) \operatorname{ArcTan}\left(\frac{\sqrt[3]{a+2^{2/3}} \sqrt[3]{a+ia \tan(c+dx)}}{\sqrt{3} \sqrt[3]{a}}\right)}{2 \cdot 2^{2/3} a^{2/3} d} + \frac{3(B+iA) \log\left(\frac{\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a+ia \tan(c+dx)}}{4 \cdot 2^{2/3} a^{2/3} d}\right)}{4 \cdot 2^{2/3} a^{2/3} d} + \frac{(B+iA) \log(\cos(c+dx))}{4 \cdot 2^{2/3} a^{2/3} d} - \frac{x(A-iB)}{4 \cdot 2^{2/3} a^{2/3}} + \frac{3(-B+iA)}{4d(a+ia \tan(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B \operatorname{Tan}[c + d*x])/(a + I*a*\operatorname{Tan}[c + d*x])^{(2/3)}, x]$

[Out]  $-1/4*((A - I*B)*x)/(2^{(2/3)}*a^{(2/3)}) - (\operatorname{Sqrt}[3]*(I*A + B)*\operatorname{ArcTan}[(a^{(1/3)} + 2^{(2/3)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)})/(\operatorname{Sqrt}[3]*a^{(1/3)})])/(2*2^{(2/3)}*a^{(2/3)}*d) + ((I*A + B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(4*2^{(2/3)}*a^{(2/3)}*d) + (3*(I*A + B)*\operatorname{Log}[2^{(1/3)}*a^{(1/3)} - (a + I*a*\operatorname{Tan}[c + d*x])^{(1/3)}])/(4*2^{(2/3)}*a^{(2/3)}*d) + (3*(I*A - B))/(4*d*(a + I*a*\operatorname{Tan}[c + d*x])^{(2/3)})$

Rule 31

$\operatorname{Int}[(a + (b*x))^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 59

$\operatorname{Int}[1/(((a + (b*x))^{(2/3)} + (c + (d*x))^{(2/3)})), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[(b*c - a*d)/b, 3]\}, \operatorname{Simp}[-\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/(2*b*q^2), x] + (-\operatorname{Dist}[3/(2*b*q), \operatorname{Subst}[\operatorname{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{(1/3)}], x] - \operatorname{Dist}[3/(2*b*q^2), \operatorname{Subst}[\operatorname{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}], x]) /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{PosQ}[(b*c - a*d)/b]$

Rule 210

$\operatorname{Int}[(a + (b*x)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 3562

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

### Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^{2/3}} dx &= \frac{3(iA - B)}{4d(a + ia \tan(c + dx))^{2/3}} + \frac{(A - iB) \int \sqrt[3]{a + ia \tan(c + dx)} dx}{2a} \\
 &= \frac{3(iA - B)}{4d(a + ia \tan(c + dx))^{2/3}} - \frac{(iA + B) \text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}} dx, x, ia \tan(c + dx)\right)}{2d} \\
 &= -\frac{(A - iB)x}{4 \cdot 2^{2/3} a^{2/3}} + \frac{(iA + B) \log(\cos(c + dx))}{4 \cdot 2^{2/3} a^{2/3} d} + \frac{3(iA - B)}{4d(a + ia \tan(c + dx))^{2/3}} - \frac{(iA + B) \log\left(\frac{\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt[3]{a}}\right)}{4 \cdot 2^{2/3} a^{2/3} d} \\
 &= -\frac{(A - iB)x}{4 \cdot 2^{2/3} a^{2/3}} + \frac{(iA + B) \log(\cos(c + dx))}{4 \cdot 2^{2/3} a^{2/3} d} + \frac{3(iA + B) \log\left(\sqrt[3]{2} \sqrt[3]{a} - \sqrt[3]{a + ia \tan(c + dx)}\right)}{4 \cdot 2^{2/3} a^{2/3} d} \\
 &= -\frac{(A - iB)x}{4 \cdot 2^{2/3} a^{2/3}} - \frac{\sqrt{3} (iA + B) \tan^{-1}\left(\frac{1 + \frac{2^{2/3} \sqrt[3]{a + ia \tan(c + dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3} a^{2/3} d} + \frac{(iA + B) \log\left(\frac{\sqrt[3]{a + ia \tan(c + dx)}}{\sqrt[3]{a}}\right)}{4 \cdot 2^{2/3} a^{2/3} d}
 \end{aligned}$$

**Mathematica [F]**

time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + ia \tan(c + dx))^{2/3}} dx$$

Verification is not applicable to the result.

`[In] Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(2/3), x]``[Out] Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(2/3), x]`**Maple [A]**

time = 0.05, size = 166, normalized size = 0.78

method	result
derivativedivides	$3i \left( -\frac{-\frac{A}{2} - \frac{iB}{2}}{2(a+ia \tan(dx+c))^{\frac{2}{3}}} + \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}} - \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}}(a+ia \tan(dx+c))^{\frac{1}{3}} + 2^{\frac{2}{3}} a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}} \right) d$
default	$3i \left( -\frac{-\frac{A}{2} - \frac{iB}{2}}{2(a+ia \tan(dx+c))^{\frac{2}{3}}} + \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{1}{3}} - 2^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}} - \frac{2^{\frac{1}{3}} \ln\left((a+ia \tan(dx+c))^{\frac{2}{3}} + 2^{\frac{1}{3}} a^{\frac{1}{3}}(a+ia \tan(dx+c))^{\frac{1}{3}} + 2^{\frac{2}{3}} a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}} \right) d$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3), x, method=_RETURNVERBOSE)`

```
[Out] 3*I/d*(-1/2*(-1/2*A-1/2*I*B)/(a+I*a*tan(d*x+c))^(2/3)+(1/6*2^(1/3)/a^(2/3)*
ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a
*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3)
)-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*ta
n(d*x+c))^(1/3)+1)))*(-1/2*I*B+1/2*A))
```

**Maxima [A]**

time = 0.52, size = 171, normalized size = 0.80

$$i \left( 2\sqrt{3} 2^{\frac{1}{3}} (A - iB) a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} 2^{\frac{1}{3}} (2^{\frac{1}{3}} a^{\frac{1}{3}} + 2(i a \tan(dx+c) + a)^{\frac{1}{3}})}{6a^{\frac{2}{3}}}\right) + 2^{\frac{1}{3}} (A - iB) a^{\frac{1}{3}} \log\left(2^{\frac{1}{3}} a^{\frac{1}{3}} + 2^{\frac{1}{3}} (i a \tan(dx+c) + a)^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{1}{3}}\right) - 2 \cdot 2^{\frac{1}{3}} (A - iB) a^{\frac{1}{3}} \log\left(-2^{\frac{1}{3}} a^{\frac{1}{3}} + (i a \tan(dx+c) + a)^{\frac{1}{3}}\right) - \frac{6(A+iB)d}{(i a \tan(dx+c) + a)^{\frac{2}{3}}} \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="maxima")
[Out] -1/8*I*(2*sqrt(3)*2^(1/3)*(A - I*B)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*(A - I*B)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 2*2^(1/3)*(A - I*B)*a^(1/3)*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 6*(A + I*B)*a/(I*a*tan(d*x + c) + a)^(2/3))/(a*d)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs.  $2(156) = 312$ .  
time = 1.81, size = 493, normalized size = 2.31

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="fricas")
[Out] 1/8*(4*(1/4)^(1/3)*a*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log(-(2*(1/4)^(1/3)*a*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3) - 2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))/(I*A + B)) - 2*(1/4)^(1/3)*(-I*sqrt(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log((2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/4)^(1/3)*(I*sqrt(3)*a*d - a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3))/(I*A + B)) - 2*(1/4)^(1/3)*(I*sqrt(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log((2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/4)^(1/3)*(-I*sqrt(3)*a*d - a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3))/(I*A + B)) - 3*2^(1/3)*((-I*A + B)*e^(2*I*d*x + 2*I*c) - I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))*e^(-2*I*d*x - 2*I*c))/(a*d)
```

**Sympy** [F]  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(ia(\tan(c + dx) - i))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x)
[Out] Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))^(2/3), x)
```

**Giac** [F]  
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/(I\*a\*tan(d\*x + c) + a)^(2/3), x)

**Mupad [B]**

time = 0.67, size = 390, normalized size = 1.83

---

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + a\*tan(c + d\*x)\*1i)^(2/3),x)

[Out] (A\*3i)/(4\*d\*(a + a\*tan(c + d\*x)\*1i)^(2/3)) - (3\*B)/(4\*d\*(a + a\*tan(c + d\*x)\*1i)^(2/3)) - ((1i/32)^(1/3)\*A\*log(A\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/3)\*36i + 144\*(1i/32)^(1/3)\*A\*a^(1/3)\*d^2))/(a^(2/3)\*d) + (2^(1/3)\*B\*log(36\*B\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/3) - 36\*2^(1/3)\*B\*a^(1/3)\*d^2))/(4\*a^(2/3)\*d) - ((1i/32)^(1/3)\*A\*log(A\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/3)\*36i + 144\*(1i/32)^(1/3)\*A\*a^(1/3)\*d^2\*((3^(1/2)\*1i)/2 - 1/2))\*((3^(1/2)\*1i)/2 - 1/2))/(a^(2/3)\*d) + ((1i/32)^(1/3)\*A\*log(A\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/3)\*36i - 144\*(1i/32)^(1/3)\*A\*a^(1/3)\*d^2\*((3^(1/2)\*1i)/2 + 1/2))\*((3^(1/2)\*1i)/2 + 1/2))/(a^(2/3)\*d) + (2^(1/3)\*B\*log(36\*B\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/3) - 36\*2^(1/3)\*B\*a^(1/3)\*d^2\*((3^(1/2)\*1i)/2 - 1/2))\*((3^(1/2)\*1i)/2 - 1/2))/(4\*a^(2/3)\*d) - (2^(1/3)\*B\*log(36\*B\*d^2\*(a + a\*tan(c + d\*x)\*1i)^(1/3) + 36\*2^(1/3)\*B\*a^(1/3)\*d^2\*((3^(1/2)\*1i)/2 + 1/2))\*((3^(1/2)\*1i)/2 + 1/2))/(4\*a^(2/3)\*d)



### 3.204 $\int \tan^m(c+dx)(a+ia \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=290

$$\frac{2a^4(A(64 + 60m + 19m^2 + 2m^3) - iB(67 + 60m + 19m^2 + 2m^3)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)(3 + m)(4 + m)} + \frac{8a^4(A - iB) {}_2F_1(1, 1, 1, 1, 1, 1)}{d(m + 1)}$$

[Out]  $-2*a^4*(A*(2*m^3+19*m^2+60*m+64)-I*B*(2*m^3+19*m^2+60*m+67))*\tan(d*x+c)^{(1+m)}/d/(3+m)/(4+m)/(m^2+3*m+2)+8*a^4*(A-I*B)*\text{hypergeom}([1, 1+m], [2+m], I*\tan(d*x+c))*\tan(d*x+c)^{(1+m)}/d/(1+m)+I*a*B*\tan(d*x+c)^{(1+m)}*(a+I*a*\tan(d*x+c))^3/d/(4+m)-(A*(4+m)-I*B*(7+m))*\tan(d*x+c)^{(1+m)}*(a^2+I*a^2*\tan(d*x+c))^2/d/(3+m)/(4+m)-2*(A*(4+m)^2-I*B*(m^2+8*m+19))*\tan(d*x+c)^{(1+m)}*(a^4+I*a^4*\tan(d*x+c))/d/(4+m)/(m^2+5*m+6)$

**Rubi [A]**

time = 0.75, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3675, 3673, 3618, 12, 66}

$$\frac{8a^4(A-iB)\tan^{m+1}(c+dx) {}_2F_1(1, m+2, 1, \tan(c+dx))}{d(m+1)} - \frac{2(A(m+4)^2 - iB(m^2+8m+19))(a^2+ia^2\tan(c+dx))\tan^{m+1}(c+dx)}{d(m+2)(m+3)(m+4)} - \frac{2a^4(A(2m^3+19m^2+60m+64) - iB(2m^3+19m^2+60m+67))\tan^{m+1}(c+dx)}{d(m+1)(m+2)(m+3)(m+4)} - \frac{(A(m+4) - iB(m+7))(a^2+ia^2\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)(m+4)} + \frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^m*(a + I*a*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(-2*a^4*(A*(64 + 60*m + 19*m^2 + 2*m^3) - I*B*(67 + 60*m + 19*m^2 + 2*m^3))*\text{Tan}[c + d*x]^{(1 + m)})/(d*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + (8*a^4*(A - I*B)*\text{Hypergeometric2F1}[1, 1 + m, 2 + m, I*\text{Tan}[c + d*x]]*\text{Tan}[c + d*x]^{(1 + m)})/(d*(1 + m)) + (I*a*B*\text{Tan}[c + d*x]^{(1 + m)}*(a + I*a*\text{Tan}[c + d*x])^3)/(d*(4 + m)) - ((A*(4 + m) - I*B*(7 + m))*\text{Tan}[c + d*x]^{(1 + m)}*(a^2 + I*a^2*\text{Tan}[c + d*x])^2)/(d*(3 + m)*(4 + m)) - (2*(A*(4 + m)^2 - I*B*(19 + 8*m + m^2))*\text{Tan}[c + d*x]^{(1 + m)}*(a^4 + I*a^4*\text{Tan}[c + d*x]))/(d*(2 + m)*(3 + m)*(4 + m))$

**Rule 12**

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 66**

$\text{Int}[(b_*)(x_)^m*((c_) + (d_*)(x_))^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[c^n*((b*x)^{(m+1)})/(b*(m+1))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x] /;$  FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \tan^m(c + dx)(a + ia \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{iaB \tan^{1+m}(c + dx)(a + ia \tan(c + dx))^3}{d(4 + m)} + \\
&= \frac{iaB \tan^{1+m}(c + dx)(a + ia \tan(c + dx))^3}{d(4 + m)} \\
&= \frac{iaB \tan^{1+m}(c + dx)(a + ia \tan(c + dx))^3}{d(4 + m)} \\
&= -\frac{2a^4(A(64 + 60m + 19m^2 + 2m^3) - iB(64 + 60m + 19m^2 + 2m^3))}{d(1 + m)(24 + 2m)} \\
&= -\frac{2a^4(A(64 + 60m + 19m^2 + 2m^3) - iB(64 + 60m + 19m^2 + 2m^3))}{d(1 + m)(24 + 2m)} \\
&= -\frac{2a^4(A(64 + 60m + 19m^2 + 2m^3) - iB(64 + 60m + 19m^2 + 2m^3))}{d(1 + m)(24 + 2m)} \\
&= -\frac{2a^4(A(64 + 60m + 19m^2 + 2m^3) - iB(64 + 60m + 19m^2 + 2m^3))}{d(1 + m)(24 + 2m)}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1805 vs. 2(290) = 580.  
time = 9.97, size = 1805, normalized size = 6.22

Warning: Unable to verify antiderivative.

```

[In] Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
[Out] (2^(3 - m)*(I*A + B)*((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
d*x))))^m*Cos[c + d*x]^5*(2^m*Hypergeometric2F1[1, m, 1 + m, -((-1 + E^((2*
I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] - (1 + E^((2*I)*(c + d*x)))^m*Hy
pergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(c + d*x)))/2])*(a + I*a*Tan[c +
d*x])^4*(A + B*Tan[c + d*x]))/(d*E^((2*I)*c)*(1 + E^((2*I)*c))^m*(Cos[d*x]
+ I*Sin[d*x])^4*(A*Cos[c + d*x] + B*Sin[c + d*x])) - ((8*I)*(A - I*B)*(-1
+ E^((2*I)*(c + d*x)))^m*(((I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x))))^m*Cos[c + d*x]^5*(-(Hypergeometric2F1[1, m, 1 + m, (1 - E^((2*I)
)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]/((1 + E^((2*I)*(c + d*x)))^m*m)) -
((1 + E^((2*I)*c))*(-1 + E^((2*I)*(c + d*x))))*(1 + E^((2*I)*(c + d*x)))^(-
1 - m)*Hypergeometric2F1[1, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*x)))/(1 + E^
((2*I)*(c + d*x)))]/(1 + m) + Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)
)*(c + d*x)))/2]/(2^m*m))*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]))/(d*

```

$$\begin{aligned}
& E^{\left((4I)c\right)} \left(1 + E^{\left((2I)c\right)}\right) \left(-1 + E^{\left((2I)(c + dx)\right)}\right) / \left(1 + E^{\left((2I)(c + dx)\right)}\right)^m \left(\cos[dx] + I \sin[dx]\right)^4 \left(A \cos[c + dx] + B \sin[c + dx]\right) + \\
& \left(\cos[c + dx]^5 \left(\left(A - (2I)B\right) \sec[c + dx]^2 \left(-4I \cos[4c] - 4 \sin[4c]\right)\right)\right) / \left(2 + m\right) + \left(\left(A - (2I)B\right) \left(-3 - 2m + \cos[2c]\right) \sec[c]^2 \left(-2I \cos[4c] - 2 \sin[4c]\right)\right) / \left(\left(1 + m\right) \left(2 + m\right)\right) + \left(\left(I A \cos[c - dx] + 2 B \cos[c - dx] - I A \cos[c + dx] - 2 B \cos[c + dx]\right) \sec[c]^2 \sec[c + dx] \left(2 \cos[4c] - (2I) \sin[4c]\right)\right) / \left(\left(1 + m\right) \tan[c + dx]^m \left(a + I a \tan[c + dx]\right)^4 \left(A + B \tan[c + dx]\right)\right) / \left(d \left(\cos[dx] + I \sin[dx]\right)^4 \left(A \cos[c + dx] + B \sin[c + dx]\right)\right) + \left(\cos[c + dx]^5 \left(\left(-8B - 9Bm - 2Bm^2 + 8B \cos[2c] + 3Bm \cos[2c]\right) \sec[c]^2 \sec[c + dx]^2 \left(\cos[4c]/2 - (I/2) \sin[4c]\right)\right)\right) / \left(\left(2 + m\right) \left(12 + 7m + m^2\right)\right) + \left(\left(-B \cos[c - dx]\right) + B \cos[c + dx]\right) \sec[c]^2 \sec[c + dx]^3 \left(\cos[4c]/2 - (I/2) \sin[4c]\right) / \left(3 + m\right) + \left(\left(-B \cos[c - dx]\right) + B \cos[c + dx]\right) \sec[c]^2 \sec[c + dx] \left(\cos[4c] - I \sin[4c]\right) / \left(\left(1 + m\right) \left(3 + m\right)\right) + \left(\left(-1 - 10m - 2m^2 + 5 \cos[2c] + 2m \cos[2c]\right) \sec[c]^2 \left(B \cos[4c] - I B \sin[4c]\right)\right) / \left(\left(1 + m\right) \left(2 + m\right) \left(3 + m\right) \left(4 + m\right)\right) + \left(\sec[c + dx]^4 \left(B \cos[4c] - I B \sin[4c]\right)\right) / \left(4 + m\right) \tan[c + dx]^m \left(a + I a \tan[c + dx]\right)^4 \left(A + B \tan[c + dx]\right) / \left(d \left(\cos[dx] + I \sin[dx]\right)^4 \left(A \cos[c + dx] + B \sin[c + dx]\right)\right) + \left(\cos[c + dx]^5 \left(\sec[c]^2 \sec[c + dx]^2 \left(B - B \cos[2c] + A \sin[2c] - (4I) B \sin[2c]\right) \left(\cos[4c]/2 - (I/2) \sin[4c]\right)\right)\right) / \left(3 + m\right) + \left(\sec[c]^2 \sec[c + dx]^3 \left(\cos[4c]/2 - (I/2) \sin[4c]\right) \left(B \cos[c - dx] - B \cos[c + dx] - A \sin[c - dx] + (4I) B \sin[c - dx] + A \sin[c + dx] - (4I) B \sin[c + dx]\right)\right) / \left(3 + m\right) + \left(\sec[c]^2 \sec[c + dx] \left(\cos[4c] - I \sin[4c]\right) \left(B \cos[c - dx] - B \cos[c + dx] - A \sin[c - dx] + (4I) B \sin[c - dx] + A \sin[c + dx] - (4I) B \sin[c + dx]\right)\right) / \left(\left(1 + m\right) \left(3 + m\right)\right) + \left(\sec[c] \left(A \cos[c] - (4I) B \cos[c] + B \sin[c]\right) \left(2 \cos[4c] - (2I) \sin[4c]\right) \tan[c]\right) / \left(\left(1 + m\right) \left(3 + m\right)\right) \tan[c + dx]^m \left(a + I a \tan[c + dx]\right)^4 \left(A + B \tan[c + dx]\right) / \left(d \left(\cos[dx] + I \sin[dx]\right)^4 \left(A \cos[c + dx] + B \sin[c + dx]\right)\right) + \left(\cos[c + dx]^5 \left(\sec[c]^2 \sec[c + dx] \left(2 \cos[4c] - (2I) \sin[4c]\right) \left(-I A \cos[c - dx] - 2 B \cos[c - dx] + I A \cos[c + dx] + 2 B \cos[c + dx] + 2 A \sin[c - dx] - (3I) B \sin[c - dx] - 2 A \sin[c + dx] + (3I) B \sin[c + dx]\right)\right)\right) / \left(1 + m\right) + \left(\sec[c] \left(2 A \cos[c] - (3I) B \cos[c] + I A \sin[c] + 2 B \sin[c]\right) \left(-4 \cos[4c] + (4I) \sin[4c]\right) \tan[c]\right) / \left(1 + m\right) \tan[c + dx]^m \left(a + I a \tan[c + dx]\right)^4 \left(A + B \tan[c + dx]\right) / \left(d \left(\cos[dx] + I \sin[dx]\right)^4 \left(A \cos[c + dx] + B \sin[c + dx]\right)\right)
\end{aligned}$$

**Maple [F]**

time = 0.52, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + ia \tan(dx + c))^4 (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(dx+c)^m\*(a+I\*a\*tan(dx+c))^4\*(A+B\*tan(dx+c)),x)

[Out] int(tan(dx+c)^m\*(a+I\*a\*tan(dx+c))^4\*(A+B\*tan(dx+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^4\*tan(d\*x + c)^m, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(16\*((A - I\*B)\*a^4\*e^(10\*I\*d\*x + 10\*I\*c) + (A + I\*B)\*a^4\*e^(8\*I\*d\*x + 8\*I\*c))\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m/(e^(10\*I\*d\*x + 10\*I\*c) + 5\*e^(8\*I\*d\*x + 8\*I\*c) + 10\*e^(6\*I\*d\*x + 6\*I\*c) + 10\*e^(4\*I\*d\*x + 4\*I\*c) + 5\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left( \int A \tan^2(c+dx) dx + \int (-6A \tan^2(c+dx) \tan^m(c+dx)) dx + \int A \tan^3(c+dx) \tan^m(c+dx) dx + \int B \tan(c+dx) \tan^m(c+dx) dx + \int (-6B \tan^2(c+dx) \tan^m(c+dx)) dx + \int B \tan^3(c+dx) \tan^m(c+dx) dx + \int 6A \tan(c+dx) \tan^m(c+dx) dx + \int (-6A \tan^2(c+dx) \tan^m(c+dx)) dx + \int 6B \tan^2(c+dx) \tan^m(c+dx) dx + \int (-6B \tan^3(c+dx) \tan^m(c+dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+I\*a\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out] a\*\*4\*(Integral(A\*tan(c + d\*x)\*\*m, x) + Integral(-6\*A\*tan(c + d\*x)\*\*2\*tan(c + d\*x)\*\*m, x) + Integral(A\*tan(c + d\*x)\*\*4\*tan(c + d\*x)\*\*m, x) + Integral(B\*tan(c + d\*x)\*tan(c + d\*x)\*\*m, x) + Integral(-6\*B\*tan(c + d\*x)\*\*3\*tan(c + d\*x)\*\*m, x) + Integral(B\*tan(c + d\*x)\*\*5\*tan(c + d\*x)\*\*m, x) + Integral(4\*I\*A\*tan(c + d\*x)\*tan(c + d\*x)\*\*m, x) + Integral(-4\*I\*A\*tan(c + d\*x)\*\*3\*tan(c + d\*x)\*\*m, x) + Integral(4\*I\*B\*tan(c + d\*x)\*\*2\*tan(c + d\*x)\*\*m, x) + Integral(-4\*I\*B\*tan(c + d\*x)\*\*4\*tan(c + d\*x)\*\*m, x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^4\*tan(d\*x + c)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4,x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^4, x)

### 3.205 $\int \tan^m(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=205

$$\frac{a^3(A(15+11m+2m^2)-iB(17+11m+2m^2))\tan^{1+m}(c+dx)}{d(1+m)(2+m)(3+m)} + \frac{4a^3(A-iB){}_2F_1(1,1+m;2+m;i \tan(c+dx))}{d(1+m)}$$

[Out]  $-a^3*(A*(2*m^2+11*m+15)-I*B*(2*m^2+11*m+17))*\tan(d*x+c)^{(1+m)}/d/(3+m)/(m^2+3*m+2)+4*a^3*(A-I*B)*\text{hypergeom}([1, 1+m], [2+m], I*\tan(d*x+c))*\tan(d*x+c)^{(1+m)}/d/(1+m)+I*a*B*\tan(d*x+c)^{(1+m)}*(a+I*a*\tan(d*x+c))^2/d/(3+m)-(A*(3+m)-I*B*(5+m))*\tan(d*x+c)^{(1+m)}*(a^3+I*a^3*\tan(d*x+c))/d/(2+m)/(3+m)$

**Rubi** [A]

time = 0.46, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3675, 3673, 3618, 12, 66}

$$\frac{4a^3(A-iB)\tan^{m+1}(c+dx){}_2F_1(1,m+1;m+2;i \tan(c+dx))}{d(m+1)} - \frac{a^3(A(2m^2+11m+15)-iB(2m^2+11m+17))\tan^{m+1}(c+dx)}{d(m+1)(m+2)(m+3)} - \frac{(A(m+3)-iB(m+5))(a^3+ia^3 \tan(c+dx))\tan^{m+1}(c+dx)}{d(m+2)(m+3)} + \frac{iaB(a+ia \tan(c+dx))^2 \tan^{m+1}(c+dx)}{d(m+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^m*(a + I*a*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-((a^3*(A*(15+11*m+2*m^2)-I*B*(17+11*m+2*m^2))*\text{Tan}[c+d*x]^{(1+m)})/(d*(1+m)*(2+m)*(3+m))) + (4*a^3*(A-I*B)*\text{Hypergeometric2F1}[1, 1+m, 2+m, I*\text{Tan}[c+d*x]]*\text{Tan}[c+d*x]^{(1+m)})/(d*(1+m)) + (I*a*B*\text{Tan}[c+d*x]^{(1+m)}*(a+I*a*\text{Tan}[c+d*x])^2)/(d*(3+m)) - ((A*(3+m)-I*B*(5+m))*\text{Tan}[c+d*x]^{(1+m)}*(a^3+I*a^3*\text{Tan}[c+d*x]))/(d*(2+m)*(3+m))$

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 66

$\text{Int}[(b_)*(x_)^m*((c_)+(d_)*(x_)^n), x\_Symbol] \rightarrow \text{Simp}[c^n*((b*x)^(m+1)/(b*(m+1)))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x] /; \text{FreeQ}[\{b, c, d, m, n\}, x] \&\& \text{!IntegerQ}[m] \&\& (\text{IntegerQ}[n] \mid\mid (\text{GtQ}[c, 0] \&\& \text{!EqQ}[n, -2^{(-1)}] \&\& \text{EqQ}[c^2-d^2, 0] \&\& \text{GtQ}[-d/(b*c), 0]))$

Rule 3618

$\text{Int}[(a_)+(b_)*\text{tan}[(e_)+(f_)*(x_)])^m*((c_)+(d_)*\text{tan}[(e_)+(f_)*(x_)]), x\_Symbol] \rightarrow \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a+(b/d)*x)^m/(d^2+c$

\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rule 3675

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \tan^m(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \frac{iaB \tan^{1+m}(c + dx)(a + ia \tan(c + dx))^2}{d(3 + m)} + \dots \\
 &= \frac{iaB \tan^{1+m}(c + dx)(a + ia \tan(c + dx))^2}{d(3 + m)} - \dots \\
 &= -\frac{a^3(A(15 + 11m + 2m^2) - iB(17 + 11m + m^2))}{d(1 + m)(6 + 5m + m^2)} \\
 &= -\frac{a^3(A(15 + 11m + 2m^2) - iB(17 + 11m + m^2))}{d(1 + m)(6 + 5m + m^2)} \\
 &= -\frac{a^3(A(15 + 11m + 2m^2) - iB(17 + 11m + m^2))}{d(1 + m)(6 + 5m + m^2)} \\
 &= -\frac{a^3(A(15 + 11m + 2m^2) - iB(17 + 11m + m^2))}{d(1 + m)(6 + 5m + m^2)}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1305 vs.  $2(205) = 410$ .



time = 8.73, size = 1305, normalized size = 6.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
[Out] (2^(2 - m)*(I*A + B)*((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
d*x))))^m*Cos[c + d*x]^4*(2^m*Hypergeometric2F1[1, m, 1 + m, -((-1 + E^((2*
I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))] - (1 + E^((2*I)*(c + d*x)))^m*Hy
pergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(c + d*x)))/2])*(a + I*a*Tan[c +
d*x])^3*(A + B*Tan[c + d*x]))/(d*E^(I*c)*(1 + E^((2*I)*c))^m*(Cos[d*x] + I
*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) - ((4*I)*(A - I*B)*(-1 + E^
((2*I)*(c + d*x)))^m*((-I)*(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c +
d*x))))^m*Cos[c + d*x]^4*(-(Hypergeometric2F1[1, m, 1 + m, (1 - E^((2*I)*(c
+ d*x)))/(1 + E^((2*I)*(c + d*x)))]/((1 + E^((2*I)*(c + d*x)))^m*m)) - ((1
+ E^((2*I)*c))*(-1 + E^((2*I)*(c + d*x)))*(1 + E^((2*I)*(c + d*x)))^(-1 -
m)*Hypergeometric2F1[1, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]/(1 + m) + Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(c
+ d*x)))/2])/(2^m*m))*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/(d*E^((
3*I)*c)*(1 + E^((2*I)*c))*((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*
x))))^m*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (Cos
[c + d*x]^4*((A - (3*I)*B)*Sec[c + d*x]^2*((-I)*Cos[3*c] - Sin[3*c]))/(2 +
m) + ((A - (3*I)*B)*(-3 - 2*m + Cos[2*c])*Sec[c]^2*((-1/2*I)*Cos[3*c] - Si
n[3*c]/2))/((1 + m)*(2 + m)) + ((I*A*Cos[c - d*x] + 3*B*Cos[c - d*x] - I*A*
Cos[c + d*x] - 3*B*Cos[c + d*x])*Sec[c]^2*Sec[c + d*x]*(Cos[3*c]/2 - (I/2)*
Sin[3*c]))/(1 + m)*Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c +
d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (C
os[c + d*x]^4*(Sec[c]^2*Sec[c + d*x]*(Cos[3*c]/2 - (I/2)*Sin[3*c])*((-I)*A
*Cos[c - d*x] - 3*B*Cos[c - d*x] + I*A*Cos[c + d*x] + 3*B*Cos[c + d*x] + 3*
A*Sin[c - d*x] - (5*I)*B*Sin[c - d*x] - 3*A*Sin[c + d*x] + (5*I)*B*Sin[c +
d*x]))/(1 + m) + (Sec[c]*(3*A*Cos[c] - (5*I)*B*Cos[c] + I*A*Sin[c] + 3*B*Si
n[c])*(-Cos[3*c] + I*Sin[3*c])*Tan[c])/((1 + m)*Tan[c + d*x]^m*(a + I*a*Tan
[c + d*x])^3*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c +
d*x] + B*Sin[c + d*x])) + (Cos[c + d*x]^4*((-I)*B*Sec[c]*Sec[c + d*x]^3*(C
os[3*c] - I*Sin[3*c])*Sin[d*x])/(3 + m) - (I*B*Sec[c]*Sec[c + d*x]*(2*Cos[3
*c] - (2*I)*Sin[3*c])*Sin[d*x])/((1 + m)*(3 + m)) - (I*B*Sec[c + d*x]^2*(Co
s[3*c] - I*Sin[3*c])*Tan[c])/(3 + m) - (I*(2*B*Cos[3*c] - (2*I)*B*Sin[3*c])
*Tan[c])/((1 + m)*(3 + m))*Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^3*(A + B*
Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x
]))
```

**Maple [F]**

time = 0.33, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + ia \tan(dx + c))^3 (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)`

[Out] `int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^3*tan(d*x + c)^m, x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral(8*((A - I*B)*a^3*e^(8*I*d*x + 8*I*c) + (A + I*B)*a^3*e^(6*I*d*x + 6*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m/(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*I*c) + 1), x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-ia^3 \left( \int iA \tan^m(c+dx) dx + \int (-3A \tan(c+dx) \tan^m(c+dx)) dx + \int A \tan^2(c+dx) \tan^m(c+dx) dx + \int (-3B \tan^2(c+dx) \tan^m(c+dx)) dx + \int B \tan^3(c+dx) \tan^m(c+dx) dx + \int (-3iA \tan^2(c+dx) \tan^m(c+dx)) dx + \int iB \tan(c+dx) \tan^m(c+dx) dx + \int (-3iB \tan^3(c+dx) \tan^m(c+dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)`

[Out] `-I*a**3*(Integral(I*A*tan(c + d*x)**m, x) + Integral(-3*A*tan(c + d*x)*tan(c + d*x)**m, x) + Integral(A*tan(c + d*x)**3*tan(c + d*x)**m, x) + Integral(-3*B*tan(c + d*x)**2*tan(c + d*x)**m, x) + Integral(B*tan(c + d*x)**4*tan(c + d*x)**m, x) + Integral(-3*I*A*tan(c + d*x)**2*tan(c + d*x)**m, x) + Integral(I*B*tan(c + d*x)*tan(c + d*x)**m, x) + Integral(-3*I*B*tan(c + d*x)**3*tan(c + d*x)**m, x))`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^3*tan(d*x + c)^m, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

### 3.206 $\int \tan^m(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=132

$$\frac{ia^2(B + (iA + B)(2 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} + \frac{2a^2(A - iB) {}_2F_1(1, 1 + m; 2 + m; i \tan(c + dx)) \tan^{1+m}(c + dx)}{d(1 + m)}$$

[Out] I\*a^2\*(B+(I\*A+B)\*(2+m))\*tan(d\*x+c)^(1+m)/d/(1+m)/(2+m)+2\*a^2\*(A-I\*B)\*hypergeom([1, 1+m], [2+m], I\*tan(d\*x+c))\*tan(d\*x+c)^(1+m)/d/(1+m)+I\*B\*tan(d\*x+c)^(1+m)\*(a^2+I\*a^2\*tan(d\*x+c))/d/(2+m)

Rubi [A]

time = 0.26, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3675, 3673, 3618, 12, 66}

$$\frac{2a^2(A - iB) \tan^{m+1}(c + dx) {}_2F_1(1, m + 1; m + 2; i \tan(c + dx))}{d(m + 1)} + \frac{ia^2(B + (m + 2)(B + iA)) \tan^{m+1}(c + dx)}{d(m + 1)(m + 2)} + \frac{iB(a^2 + ia^2 \tan(c + dx)) \tan^{m+1}(c + dx)}{d(m + 2)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (I\*a^2\*(B + (I\*A + B)\*(2 + m))\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)\*(2 + m)) + (2\*a^2\*(A - I\*B)\*Hypergeometric2F1[1, 1 + m, 2 + m, I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)) + (I\*B\*Tan[c + d\*x]^(1 + m)\*(a^2 + I\*a^2\*Tan[c + d\*x]))/(d\*(2 + m))

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 66

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[c^n\*((b\*x)^(m + 1)/(b\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0]))

Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \tan^m(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{iB \tan^{1+m}(c + dx)(a^2 + ia^2 \tan(c + dx))}{d(2 + m)} + \\ &= \frac{ia^2(B + (iA + B)(2 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} \\ &= \frac{ia^2(B + (iA + B)(2 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} \\ &= \frac{ia^2(B + (iA + B)(2 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} \\ &= \frac{ia^2(B + (iA + B)(2 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1587 vs.  $2(132) = 264$ .  
time = 8.14, size = 1587, normalized size = 12.02

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
[Out] (I*A*(((−I)*(−1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c + d*x]^3*(2^m*Hypergeometric2F1[1, m, 1 + m, −((−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] − (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1 − E^((2*I)*(c + d*x)))/2])*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(2^m*d*m*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (B*(((−I)*(−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c + d*x]^3*(2^m*Hypergeometric2F1[1, m, 1 + m, −((−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] − (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1 − E^((2*I)*(c + d*x)))/2])*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(2^m*d*m*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) − ((2*I)*(A − I*B)*(−1 + E^((2*I)*(c + d*x)))^m*(((−I)*(−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c + d*x]^3*(−(Hypergeometric2F1[1, m, 1 + m, (1 − E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]/((1 + E^((2*I)*(c + d*x)))^m*m)) − ((1 + E^((2*I)*c))*(-1 + E^((2*I)*(c + d*x)))*(1 + E^((2*I)*(c + d*x)))^(-1 − m)*Hypergeometric2F1[1, 1 + m, 2 + m, (1 − E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]/(1 + m) + Hypergeometric2F1[m, m, 1 + m, (1 − E^((2*I)*(c + d*x)))/2]/(2^m*m))*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(d*E^((2*I)*c)*(1 + E^((2*I)*c))*((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (A*(((−I)*(−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c + d*x]^3*(2^m*Hypergeometric2F1[1, m, 1 + m, −((−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))] − (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1 − E^((2*I)*(c + d*x)))/2])*Tan[c]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(2^m*d*m*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) − (I*B*(((−I)*(−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c + d*x]^3*(2^m*Hypergeometric2F1[1, m, 1 + m, −((−1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))] − (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1 − E^((2*I)*(c + d*x)))/2])*Tan[c]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(2^m*d*m*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (Cos[c + d*x]^3*(((B*Cos[c − d*x] − B*Cos[c + d*x])*Sec[c]^2*Sec[c + d*x]*(Cos[2*c]/2 − (I/2)*Sin[2*c]))/(1 + m) + ((−3 − 2*m + Cos[2*c])*Sec[c]^2*(-1/2*(B*Cos[2*c]) + (I/2)*B*Sin[2*c]))/((1 + m)*(2 + m)) + (Sec[c + d*x]^2*(-(B*Cos[2*c]) + I*B*Sin[2*c]))/(2 + m))*Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])) + (Cos[c + d*x]^3*(((Sec[c]^2*Sec[c + d*x]*(Cos[2*c]/2 − (I/2)*Sin[2*c])*(-(B*Cos[c − d*x]) + B*Cos[c + d*x] + A*Sin[c − d*x] − (2*I)*B*Sin[c − d*x] − A*Sin[c + d*x] + (2*I)*B*Sin[c + d*x]))/(1 + m) + (Sec[c]*(A*Cos[c] − (2*I)*B*Cos[c] + B*Sin[c])*(-Cos[2*c] + I*Sin[2*c])*Tan[c])/((1 + m))*Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

Maple [F]

time = 0.33, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + ia \tan(dx + c))^2 (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^2\*tan(d\*x + c)^m, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(4\*((A - I\*B)\*a^2\*e^(6\*I\*d\*x + 6\*I\*c) + (A + I\*B)\*a^2\*e^(4\*I\*d\*x + 4\*I\*c))\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m/(e^(6\*I\*d\*x + 6\*I\*c) + 3\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int (-A \tan^m(c + dx)) dx + \int A \tan^2(c + dx) \tan^m(c + dx) dx + \int (-B \tan(c + dx) \tan^m(c + dx)) dx + \int B \tan^3(c + dx) \tan^m(c + dx) dx + \int (-2iA \tan(c + dx) \tan^m(c + dx)) dx + \int (-2iB \tan^2(c + dx) \tan^m(c + dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] -a\*\*2\*(Integral(-A\*tan(c + d\*x)\*\*m, x) + Integral(A\*tan(c + d\*x)\*\*2\*tan(c + d\*x)\*\*m, x) + Integral(-B\*tan(c + d\*x)\*tan(c + d\*x)\*\*m, x) + Integral(B\*tan(c + d\*x)\*\*3\*tan(c + d\*x)\*\*m, x) + Integral(-2\*I\*A\*tan(c + d\*x)\*tan(c + d\*x)\*\*m, x) + Integral(-2\*I\*B\*tan(c + d\*x)\*\*2\*tan(c + d\*x)\*\*m, x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^2\*tan(d\*x + c)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2, x)



### 3.207 $\int \tan^m(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=70

$$\frac{iaB \tan^{1+m}(c+dx)}{d(1+m)} + \frac{a(A-iB) {}_2F_1(1, 1+m; 2+m; i \tan(c+dx)) \tan^{1+m}(c+dx)}{d(1+m)}$$

[Out]  $I*a*B*\tan(d*x+c)^{(1+m)}/d/(1+m)+a*(A-I*B)*\text{hypergeom}([1, 1+m], [2+m], I*\tan(d*x+c))*\tan(d*x+c)^{(1+m)}/d/(1+m)$

Rubi [A]

time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {3673, 3618, 66}

$$\frac{a(A-iB) \tan^{m+1}(c+dx) {}_2F_1(1, m+1; m+2; i \tan(c+dx))}{d(m+1)} + \frac{iaB \tan^{m+1}(c+dx)}{d(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^m*(a + I*a*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(I*a*B*\text{Tan}[c + d*x]^{(1+m)})/(d*(1+m)) + (a*(A - I*B)*\text{Hypergeometric2F1}[1, 1+m, 2+m, I*\text{Tan}[c + d*x]]*\text{Tan}[c + d*x]^{(1+m)})/(d*(1+m))$

Rule 66

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[c^n*((b*x)^{(m+1})/(b*(m+1)))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{IntegerQ}[n] \ || \ (\text{GtQ}[c, 0] \ \&\& \ !(\text{EqQ}[n, -2^{(-1)}] \ \&\& \ \text{EqQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[-d/(b*c), 0])))$

Rule 3618

$\text{Int}[(a_*) + (b_*)*\text{tan}[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\text{tan}[(e_*) + (f_*)*(x_*)]), x\_Symbol] \rightarrow \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

Rule 3673

$\text{Int}[(a_*) + (b_*)*\text{tan}[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\text{tan}[(e_*) + (f_*)*(x_*)]), x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c,$

$d, e, f, A, B, m, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \tan^m(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{iaB \tan^{1+m}(c + dx)}{d(1 + m)} + \int \tan^m(c + dx)(a(A - \\ &= \frac{iaB \tan^{1+m}(c + dx)}{d(1 + m)} + \frac{(ia^2(A - iB)^2) \text{Subst}}{d(1 + m)} \\ &= \frac{iaB \tan^{1+m}(c + dx)}{d(1 + m)} + \frac{a(A - iB) {}_2F_1(1, 1 + m)}{d(1 + m)} \end{aligned}$$

**Mathematica** [F]

time = 2.02, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.24, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c))(a + ia \tan(dx + c))(A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x)

[Out] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(2\*((A - I\*B)\*a\*e^(4\*I\*d\*x + 4\*I\*c) + (A + I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c))\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m/(e^(4\*I\*d\*x + 4\*I\*c) + 2\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$ia \left( \int (-iA \tan^m(c + dx)) dx + \int A \tan(c + dx) \tan^m(c + dx) dx + \int B \tan^2(c + dx) \tan^m(c + dx) dx + \int (-iB \tan(c + dx) \tan^m(c + dx)) dx \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] I\*a\*(Integral(-I\*A\*tan(c + d\*x)\*\*m, x) + Integral(A\*tan(c + d\*x)\*tan(c + d\*x)\*\*m, x) + Integral(B\*tan(c + d\*x)\*\*2\*tan(c + d\*x)\*\*m, x) + Integral(-I\*B\*tan(c + d\*x)\*tan(c + d\*x)\*\*m, x))

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i), x)

$$3.208 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=168

$$\frac{(A(1-m) - iB(1+m)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{2ad(1+m)} + \frac{(iA - B)m {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; -\tan^2(c+dx)\right) \tan^{2+m}(c+dx)}{2ad(2+m)}$$

[Out] 1/2\*(A\*(1-m)-I\*B\*(1+m))\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/a/d/(1+m)+1/2\*(I\*A-B)\*m\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/a/d/(2+m)+1/2\*(A+I\*B)\*tan(d\*x+c)^(1+m)/d/(a+I\*a\*tan(d\*x+c))

**Rubi [A]**

time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3619, 3557, 371}

$$\frac{(A(1-m) - iB(m+1)) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\tan^2(c+dx)\right)}{2ad(m+1)} + \frac{m(-B+iA) \tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\tan^2(c+dx)\right)}{2ad(m+2)} + \frac{(A+iB) \tan^{m+1}(c+dx)}{2d(a+ia \tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] ((A\*(1 - m) - I\*B\*(1 + m))\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/(2\*a\*d\*(1 + m)) + ((I\*A - B)\*m\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/(2\*a\*d\*(2 + m)) + ((A + I\*B)\*Tan[c + d\*x]^(1 + m))/(2\*d\*(a + I\*a\*Tan[c + d\*x]))

**Rule 371**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*((c\*x)^(m + 1)/(c\*(m + 1)))\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

**Rule 3557**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

**Rule 3619**

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2

+ d^2, 0] && !IntegerQ[2\*m]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \frac{(A + iB) \tan^{1+m}(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{\int \tan^m(c + dx)(a(A(1 - m) - iB)}{2a} \\ &= \frac{(A + iB) \tan^{1+m}(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{((iA - B)m) \int \tan^{1+m}(c + dx) dx}{2a} \\ &= \frac{(A + iB) \tan^{1+m}(c + dx)}{2d(a + ia \tan(c + dx))} + \frac{((iA - B)m) \text{Subst}\left(\int \frac{x^{1+m}}{1+x^2} dx, x, \tan(c + dx)\right)}{2ad} \\ &= \frac{(A - Am - iB(1 + m)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c + dx)\right) \tan^{1+m}}{2ad(1 + m)} \end{aligned}$$

### Mathematica [F]

time = 4.26, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx$$

Verification is not applicable to the result.

[In] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]), x]

### Maple [F]

time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{a + ia \tan(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)`

[Out] `int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)`

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral(1/2*((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(-2*I*d*x - 2*I*c)/a, x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \tan^m(c+dx)}{\tan(c+dx)-i} dx + \int \frac{B \tan(c+dx) \tan^m(c+dx)}{\tan(c+dx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)`

[Out] `-I*(Integral(A*tan(c + d*x)**m/(tan(c + d*x) - I), x) + Integral(B*tan(c + d*x)*tan(c + d*x)**m/(tan(c + d*x) - I), x))/a`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(I\*a\*tan(d\*x + c) + a), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{a + a \tan(c + dx) \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i), x)

$$3.209 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=226

$$\frac{(1-m)(A(1-m) - iB(1+m)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{4a^2d(1+m)} + \frac{(A(2-m) - iBm) \tan(c+dx)}{4a^2d(1+i \tan(c+dx))}$$

[Out] 1/4\*(1-m)\*(A\*(1-m)-I\*B\*(1+m))\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/a^2/d/(1+m)+1/4\*(A\*(2-m)-I\*B\*m)\*tan(d\*x+c)^(1+m)/a^2/d/(1+I\*tan(d\*x+c))+1/4\*m\*(I\*A\*(2-m)+B\*m)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/a^2/d/(2+m)+1/4\*(A+I\*B)\*tan(d\*x+c)^(1+m)/d/(a+I\*a\*tan(d\*x+c))^2

**Rubi [A]**

time = 0.34, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3619, 3557, 371}

$$\frac{(1-m)(A(1-m) - iB(1+m)) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\tan^2(c+dx)\right)}{4a^2d(m+1)} + \frac{m(Bm + iA(2-m)) \tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\tan^2(c+dx)\right)}{4a^2d(m+2)} + \frac{(A(2-m) - iBm) \tan^{m+1}(c+dx)}{4a^2d(1+i \tan(c+dx))} + \frac{(A+iB) \tan^{m+1}(c+dx)}{4d(a+ia \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out] ((1 - m)\*(A\*(1 - m) - I\*B\*(1 + m))\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/(4\*a^2\*d\*(1 + m)) + ((A\*(2 - m) - I\*B\*m)\*Tan[c + d\*x]^(1 + m))/(4\*a^2\*d\*(1 + I\*Tan[c + d\*x])) + (m\*(I\*A\*(2 - m) + B\*m))\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/(4\*a^2\*d\*(2 + m)) + ((A + I\*B)\*Tan[c + d\*x]^(1 + m))/(4\*d\*(a + I\*a\*Tan[c + d\*x])^2)

Rule 371

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*((c\*x)^(m+1)/(c\*(m+1)))\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3619

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[(



$b*\text{Tan}[e + f*x]^{(m + 1)}, x]$ ,  $x]$  /;  $\text{FreeQ}[\{b, c, d, e, f, m\}, x]$  &&  $\text{NeQ}[c^2 + d^2, 0]$  && !IntegerQ[2\*m]

### Rule 3677

$\text{Int}[(a + b*\text{Tan}[e + f*x])^{(m)}*((A + B*\text{Tan}[e + f*x]) + (f_*)*(x_))^{(n)}, x\_Symbol] := \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^{(m)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)}/(2*f*m*(b*c - a*d))), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^{(n)}*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x]$  /;  $\text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{LtQ}[m, 0]$  && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^2} dx &= \frac{(A + iB) \tan^{1+m}(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{\int \frac{\tan^m(c + dx)(a(A(3-m) - iB(1+m)) - a(iA - B))}{a + ia \tan(c + dx)} dx}{4a^2} \\ &= \frac{(A(2 - m) - iBm) \tan^{1+m}(c + dx)}{4a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \tan^{1+m}(c + dx)}{4d(a + ia \tan(c + dx))^2} \\ &= \frac{(A(2 - m) - iBm) \tan^{1+m}(c + dx)}{4a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \tan^{1+m}(c + dx)}{4d(a + ia \tan(c + dx))^2} \\ &= \frac{(A(2 - m) - iBm) \tan^{1+m}(c + dx)}{4a^2 d(1 + i \tan(c + dx))} + \frac{(A + iB) \tan^{1+m}(c + dx)}{4d(a + ia \tan(c + dx))^2} \\ &= \frac{(1 - m)(A(1 - m) - iB(1 + m)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c + dx)\right)}{4a^2 d(1 + m)} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 565 vs. 2(226) = 452.  
time = 4.40, size = 565, normalized size = 2.50

$$\frac{(A + iB) \tan^{1+m}(c + dx)}{4d(a + ia \tan(c + dx))^2} + \frac{\int \frac{\tan^m(c + dx)(a(A(3-m) - iB(1+m)) - a(iA - B))}{a + ia \tan(c + dx)} dx}{4a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Tan[c + d\*x])^m\*(A + B\*Tan[c + d\*x])/(a + I\*a\*Tan[c + d\*x])^2,x]  
[Out] ((-1/16\*I)\*((-1)\*(-1 + E^((2\*I)\*(c + d\*x))))/(1 + E^((2\*I)\*(c + d\*x))))^m\*(((A + I\*B)\*(-1 + E^((2\*I)\*(c + d\*x))))^(1 + m)\*(1 + E^((2\*I)\*(c + d\*x))))^(2 - m))/E^((4\*I)\*d\*x) + E^((2\*I)\*(c - d\*x))\*(-1 + E^((2\*I)\*(c + d\*x)))^(1 + m)\*(1 + E^((2\*I)\*(c + d\*x)))^(2 - m)\*(A\*(3 - 2\*m) - I\*(B + 2\*B\*m)) + (2^(2

$$\begin{aligned}
& - m) * E^{((4*I)*c)} * (-1 + E^{((2*I)*(c + d*x))})^{(1 + m)} * (A * (-3 + 2*m) + I * (B + \\
& 2*B*m)) * \text{Hypergeometric2F1}[-1 + m, 1 + m, 2 + m, (1 - E^{((2*I)*(c + d*x))})/2 \\
& ] / (1 + m) + (2^{(1 - m)} * E^{((4*I)*c)} * ((-1 + E^{((2*I)*(c + d*x))}) / (1 + E^{((2*I)* \\
& I)*(c + d*x)}))^{m} * (I * B * (-1 + 2*m^2) + A * (1 - 4*m + 2*m^2)) * (-2^{m} * (1 + m) * \text{H} \\
& ypergeometric2F1[1, m, 1 + m, (1 - E^{((2*I)*(c + d*x))}) / (1 + E^{((2*I)*(c + \\
& d*x)}))]) + (1 + E^{((2*I)*(c + d*x))})^{m} * ((1 + m) * \text{Hypergeometric2F1}[m, m, 1 + \\
& m, (1 - E^{((2*I)*(c + d*x))}) / 2] + (-1 + E^{((2*I)*(c + d*x))}) * m * \text{Hypergeomet} \\
& ric2F1[m, 1 + m, 2 + m, (1 - E^{((2*I)*(c + d*x))}) / 2])) / (m * (1 + m)) * \text{Sec}[c \\
& + d*x] * (\text{Cos}[d*x] + I * \text{Sin}[d*x])^{2 * (A + B * \text{Tan}[c + d*x])} / (d * E^{((2*I)*c)} * ((-1 \\
& + E^{((2*I)*(c + d*x))}) / (1 + E^{((2*I)*(c + d*x))}))^{m} * (A * \text{Cos}[c + d*x] + B * \text{Sin} \\
& [c + d*x]) * (a + I * a * \text{Tan}[c + d*x])^{2}
\end{aligned}$$

**Maple [F]**

time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + ia \tan(dx + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] integral(1/4\*((A - I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*A\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m\*e^(-4\*I\*d\*x - 4\*I\*c)/a^2, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \tan^m(c+dx)}{\tan^2(c+dx) - 2i \tan(c+dx) - 1} dx + \int \frac{B \tan(c+dx) \tan^m(c+dx)}{\tan^2(c+dx) - 2i \tan(c+dx) - 1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] -(Integral(A\*tan(c + d\*x)\*\*m/(tan(c + d\*x)\*\*2 - 2\*I\*tan(c + d\*x) - 1), x) + Integral(B\*tan(c + d\*x)\*tan(c + d\*x)\*\*m/(tan(c + d\*x)\*\*2 - 2\*I\*tan(c + d\*x) - 1), x))/a\*\*2

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(I\*a\*tan(d\*x + c) + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + a \tan(c + dx) 1i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2, x)

$$3.210 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=308

$$\frac{(1-m)(iB(3+m-2m^2) - A(3-7m+2m^2)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{24a^3d(1+m)} + \frac{(2-m)(iB(3+m-2m^2) - A(3-7m+2m^2)) \tan^{1+m}(c+dx)}{24a^3d(1+m)}$$

[Out] -1/24\*(1-m)\*(I\*B\*(-2\*m^2+m+3)-A\*(2\*m^2-7\*m+3))\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/a^3/d/(1+m)+1/24\*(2-m)\*m\*(B+I\*A\*(5-2\*m)+2\*B\*m)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/a^3/d/(2+m)+1/6\*(A+I\*B)\*tan(d\*x+c)^(1+m)/d/(a+I\*a\*tan(d\*x+c))^3+1/24\*(I\*B\*(1-2\*m)+A\*(7-2\*m))\*tan(d\*x+c)^(1+m)/a/d/(a+I\*a\*tan(d\*x+c))^2+1/24\*(2-m)\*(A\*(5-2\*m)-I\*(2\*B\*m+B))\*tan(d\*x+c)^(1+m)/d/(a^3+I\*a^3\*tan(d\*x+c))

**Rubi [A]**

time = 0.60, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3619, 3557, 371}

$$\frac{(1-m)(-A(2m^2-7m+3)+iB(-2m^2+m+3))\tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)}{24a^3d(m+1)} + \frac{(2-m)m(iA(5-2m)+2Bm+B)\tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)}{24a^3d(m+2)} + \frac{(2-m)(A(5-2m)-i(2Bm+B))\tan^{m+1}(c+dx)}{24d(a^3+ia^3\tan(c+dx))} + \frac{(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{24d(a+ia\tan(c+dx))^2} + \frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3, x]

[Out] -1/24\*((1-m)\*(I\*B\*(3+m-2\*m^2) - A\*(3-7\*m+2\*m^2))\*Hypergeometric2F1[1, (1+m)/2, (3+m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1+m))/(a^3\*d\*(1+m)) + ((2-m)\*m\*(B + I\*A\*(5-2\*m) + 2\*B\*m)\*Hypergeometric2F1[1, (2+m)/2, (4+m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2+m))/(24\*a^3\*d\*(2+m)) + ((A + I\*B)\*Tan[c + d\*x]^(1+m))/(6\*d\*(a + I\*a\*Tan[c + d\*x])^3) + ((I\*B\*(1-2\*m) + A\*(7-2\*m))\*Tan[c + d\*x]^(1+m))/(24\*a\*d\*(a + I\*a\*Tan[c + d\*x])^2) + ((2-m)\*(A\*(5-2\*m) - I\*(B + 2\*B\*m))\*Tan[c + d\*x]^(1+m))/(24\*d\*(a^3 + I\*a^3\*Tan[c + d\*x]))

**Rule 371**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*((c\*x)^(m+1)/(c\*(m+1)))\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

**Rule 3557**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3619

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2 + d^2, 0] && !IntegerQ[2\*m]

Rule 3677

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^3} dx &= \frac{(A + iB) \tan^{1+m}(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{\int \frac{\tan^m(c + dx)(a(A(5-m) - iB(1+m)) - a(ia - (a + ia \tan(c + dx))^2))}{6a^2} dx}{6a^2} \\ &= \frac{(A + iB) \tan^{1+m}(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(iB(1 - 2m) + A(7 - 2m)) \tan^{1+m}}{24ad(a + ia \tan(c + dx))^2} \\ &= \frac{(A + iB) \tan^{1+m}(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(iB(1 - 2m) + A(7 - 2m)) \tan^{1+m}}{24ad(a + ia \tan(c + dx))^2} \\ &= \frac{(A + iB) \tan^{1+m}(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(iB(1 - 2m) + A(7 - 2m)) \tan^{1+m}}{24ad(a + ia \tan(c + dx))^2} \\ &= \frac{(A + iB) \tan^{1+m}(c + dx)}{6d(a + ia \tan(c + dx))^3} + \frac{(iB(1 - 2m) + A(7 - 2m)) \tan^{1+m}}{24ad(a + ia \tan(c + dx))^2} \\ &= \frac{(1 - m)(iB(3 + m - 2m^2) - A(3 - 7m + 2m^2)) {}_2F_1\left(1, \frac{1+m}{2}; \right)}{24a^3d(1 + m)} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 712 vs. 2(308) = 616.  
time = 49.53, size = 712, normalized size = 2.31

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-1/96*I)*((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*
((2*(A + I*B)*(-1 + E^((2*I)*(c + d*x)))^(1 + m)*(1 + E^((2*I)*(c + d*x)))^
(3 - m))/E^((6*I)*d*x) + E^((2*I)*(c - 2*d*x))*(-1 + E^((2*I)*(c + d*x)))^(
1 + m)*(1 + E^((2*I)*(c + d*x)))^(3 - m)*(A*(5 - 2*m) - I*(B + 2*B*m)) - (2
*(-1 + E^((2*I)*(c + d*x)))^(1 + m)*(1 + E^((2*I)*(c + d*x)))^(3 - m)*(I*B*
(2 + m - 2*m^2) + A*(-4 + 7*m - 2*m^2)))/E^((2*I)*(-2*c + d*x)) + (3*2^(3 -
m)*E^((6*I)*c)*(-1 + E^((2*I)*(c + d*x)))^(1 + m)*(I*B*(2 + m - 2*m^2) + A
*(-4 + 7*m - 2*m^2))*Hypergeometric2F1[-2 + m, 1 + m, 2 + m, (1 - E^((2*I)*
(c + d*x)))/2])/2)/(1 + m) + (2^(1 - m)*E^((6*I)*c)*((-1 + E^((2*I)*(c + d*x)
))/2)/(1 + E^((2*I)*(c + d*x))))^m*(A*(-3 + 20*m - 18*m^2 + 4*m^3) + I*B*(3 - 4
*m - 6*m^2 + 4*m^3))*(2^m*(1 + m)*Hypergeometric2F1[1, m, 1 + m, (1 - E^((2
*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))] - (1 + E^((2*I)*(c + d*x)))^m*(2
*(-1 + E^((2*I)*(c + d*x))))*m*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (1 -
E^((2*I)*(c + d*x)))/2] + (1 + m)*Hypergeometric2F1[m, m, 1 + m, (1 - E^((2
*I)*(c + d*x)))/2] + (-1 + E^((2*I)*(c + d*x))))*m*Hypergeometric2F1[m, 1 +
m, 2 + m, (1 - E^((2*I)*(c + d*x)))/2])))/(m*(1 + m))*Sec[c + d*x]^2*(Cos[
d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x))/(d*E^((3*I)*c)*((-1 + E^((2*I)*
(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*(A*Cos[c + d*x] + B*Sin[c + d*x])*(
a + I*a*Tan[c + d*x])^3)
```

**Maple [F]**

time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + ia \tan(dx + c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(1/8\*((A - I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) + (3\*A - I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) + (3\*A + I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m\*e^(-6\*I\*d\*x - 6\*I\*c)/a^3, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \tan^m(c+dx)}{\tan^3(c+dx) - 3i \tan^2(c+dx) - 3 \tan(c+dx) + i} dx + \int \frac{B \tan(c+dx) \tan^m(c+dx)}{\tan^3(c+dx) - 3i \tan^2(c+dx) - 3 \tan(c+dx) + i} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] I\*(Integral(A\*tan(c + d\*x)\*\*m/(tan(c + d\*x)\*\*3 - 3\*I\*tan(c + d\*x)\*\*2 - 3\*tan(c + d\*x) + I), x) + Integral(B\*tan(c + d\*x)\*tan(c + d\*x)\*\*m/(tan(c + d\*x)\*\*3 - 3\*I\*tan(c + d\*x)\*\*2 - 3\*tan(c + d\*x) + I), x))/a\*\*3

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(I\*a\*tan(d\*x + c) + a)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3, x)

$$3.211 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=386

$$\frac{(3 - 4m + m^2) (iB(1 - m^2) - A(1 - 4m + m^2)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{48a^4d(1+m)} \quad (iB(1 - m^2) - A(1 - 4m + m^2))$$

[Out] -1/48\*(m^2-4\*m+3)\*(I\*B\*(-m^2+1)-A\*(m^2-4\*m+1))\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/a^4/d/(1+m)-1/48\*(I\*B\*(-m^2+3\*m+1)-A\*(m^2-7\*m+13))\*tan(d\*x+c)^(1+m)/a^4/d/(1+I\*tan(d\*x+c))^2-1/48\*(2-m)\*(I\*B\*(-m^2+2\*m+2)-A\*(m^2-6\*m+8))\*tan(d\*x+c)^(1+m)/a^4/d/(1+I\*tan(d\*x+c))+1/48\*(2-m)\*m\*(B\*(-m^2+2\*m+2)+I\*A\*(m^2-6\*m+8))\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/a^4/d/(2+m)+1/8\*(A+I\*B)\*tan(d\*x+c)^(1+m)/d/(a+I\*a\*tan(d\*x+c))^4+1/24\*(I\*B\*(1-m)+A\*(5-m))\*tan(d\*x+c)^(1+m)/a/d/(a+I\*a\*tan(d\*x+c))^3

**Rubi [A]**

time = 0.86, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3677, 3619, 3557, 371}

$\frac{(m^2 - 4m + 3)(-A(m^2 - 4m + 1) + iB(1 - m^2)) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)}{48a^4d(1+m)} - \frac{(2-m)m(B(-m^2+2m+2) + A(m^2-6m+8)) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)}{48a^4d(1+m)} - \frac{(2-m)(-A(m^2-6m+8) + iB(-m^2+2m+2)) \tan^{m+1}(c+dx)}{48a^4d(1+m)} - \frac{(A(5-m) + iB(1-m)) \tan^{m+1}(c+dx)}{48a^4d(1+m)} + \frac{(A+iB) \tan^{m+1}(c+dx)}{8a^4d(1+m)}$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^4,x]

[Out] -1/48\*((3 - 4\*m + m^2)\*(I\*B\*(1 - m^2) - A\*(1 - 4\*m + m^2))\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/(a^4\*d\*(1 + m)) - ((I\*B\*(1 + 3\*m - m^2) - A\*(13 - 7\*m + m^2))\*Tan[c + d\*x]^(1 + m))/(48\*a^4\*d\*(1 + I\*Tan[c + d\*x])^2) - ((2 - m)\*(I\*B\*(2 + 2\*m - m^2) - A\*(8 - 6\*m + m^2))\*Tan[c + d\*x]^(1 + m))/(48\*a^4\*d\*(1 + I\*Tan[c + d\*x])) + ((2 - m)\*m\*(B\*(2 + 2\*m - m^2) + I\*A\*(8 - 6\*m + m^2))\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/(48\*a^4\*d\*(2 + m)) + ((A + I\*B)\*Tan[c + d\*x]^(1 + m))/(8\*d\*(a + I\*a\*Tan[c + d\*x])^4) + ((I\*B\*(1 - m) + A\*(5 - m))\*Tan[c + d\*x]^(1 + m))/(24\*a\*d\*(a + I\*a\*Tan[c + d\*x])^3)

**Rule 371**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*((c\*x)^(m+1)/(c\*(m+1))\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

**Rule 3557**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !



IntegerQ[n]

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2 + d^2, 0] && !IntegerQ[2*m]
```

Rule 3677

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^4} dx &= \frac{(A + iB) \tan^{1+m}(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{\int \frac{\tan^m(c + dx)(a(A(7-m) - iB(1+m)) - a(ia \tan(c + dx)))}{(a + ia \tan(c + dx))^3} dx}{8a^2} \\
&= \frac{(A + iB) \tan^{1+m}(c + dx)}{8d(a + ia \tan(c + dx))^4} + \frac{(iB(1 - m) + A(5 - m)) \tan^{1+m}(c + dx)}{24ad(a + ia \tan(c + dx))^3} \\
&= -\frac{(iB(1 + 3m - m^2) - A(13 - 7m + m^2)) \tan^{1+m}(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{A \tan^{1+m}(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
&= -\frac{(iB(1 + 3m - m^2) - A(13 - 7m + m^2)) \tan^{1+m}(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{A \tan^{1+m}(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
&= -\frac{(iB(1 + 3m - m^2) - A(13 - 7m + m^2)) \tan^{1+m}(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{A \tan^{1+m}(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
&= -\frac{(iB(1 + 3m - m^2) - A(13 - 7m + m^2)) \tan^{1+m}(c + dx)}{48a^4d(1 + i \tan(c + dx))^2} + \frac{A \tan^{1+m}(c + dx)}{8d(a + ia \tan(c + dx))^4} \\
&= -\frac{(3 - 4m + m^2)(iB(1 - m^2) - A(1 - 4m + m^2)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{1+m}{2}, \frac{1 + i \tan(c + dx)}{a + ia \tan(c + dx)}\right)}{48a^4d(1 + m)}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 921 vs.  $2(386) = 772$ .

time = 50.77, size = 921, normalized size = 2.39

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((-1/384*I)*((( -I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m
*((3*(A + I*B)*(-1 + E^((2*I)*(c + d*x)))^(1 + m)*(1 + E^((2*I)*(c + d*x)))
^(4 - m))/E^((8*I)*d*x) + E^((2*I)*(c - 3*d*x))*(-1 + E^((2*I)*(c + d*x)))^
(1 + m)*(1 + E^((2*I)*(c + d*x)))^(4 - m)*(A*(7 - 2*m) - I*(B + 2*B*m)) - E
^((4*I)*(c - d*x))*(-1 + E^((2*I)*(c + d*x)))^(1 + m)*(1 + E^((2*I)*(c + d*
x)))^(4 - m)*(I*B*(3 + 2*m - 2*m^2) + A*(-9 + 10*m - 2*m^2)) + (((-1 + E^((
2*I)*(c + d*x)))^(1 + m)*(1 + E^((2*I)*(c + d*x)))^(4 - m)*m*(1 + m)*(A*(13
- 44*m + 26*m^2 - 4*m^3) - I*B*(7 - 4*m - 10*m^2 + 4*m^3)))/E^((2*I)*(-3*c
+ d*x)) - 2^(5 - m)*E^((8*I)*c)*(-1 + E^((2*I)*(c + d*x)))^(1 + m)*m*(A*(1
3 - 44*m + 26*m^2 - 4*m^3) - I*B*(7 - 4*m - 10*m^2 + 4*m^3))*Hypergeometric
2F1[-3 + m, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*x)))/2] + 2^(2 - m)*E^((8*I)
*c)*((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^m*(A*(3 - 32*m +
40*m^2 - 16*m^3 + 2*m^4) + I*B*(-3 + 8*m + 4*m^2 - 8*m^3 + 2*m^4))*(-(2^m*
(1 + m)*Hypergeometric2F1[1, m, 1 + m, (1 - E^((2*I)*(c + d*x)))/(1 + E^((2
*I)*(c + d*x)))] + (1 + E^((2*I)*(c + d*x)))^m*(4*(-1 + E^((2*I)*(c + d*x)
))*m*Hypergeometric2F1[-2 + m, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*x)))/2] +
2*(-1 + E^((2*I)*(c + d*x)))*m*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (1
- E^((2*I)*(c + d*x)))/2] + Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(c
+ d*x)))/2] + m*Hypergeometric2F1[m, m, 1 + m, (1 - E^((2*I)*(c + d*x)))/2
] - m*Hypergeometric2F1[m, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*x)))/2] + E^
((2*I)*(c + d*x))*m*Hypergeometric2F1[m, 1 + m, 2 + m, (1 - E^((2*I)*(c + d*
x)))/2]))/(m*(1 + m))*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^4*(A + B*Tan
[c + d*x))/(d*E^((4*I)*c)*((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d
*x))))^m*(A*Cos[c + d*x] + B*Sin[c + d*x))*(a + I*a*Tan[c + d*x])^4)
```

**Maple [F]**

time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + ia \tan(dx + c))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] integral(1/16\*((A - I\*B)\*e^(8\*I\*d\*x + 8\*I\*c) + 2\*(2\*A - I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*A\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*(2\*A + I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m\*e^(-8\*I\*d\*x - 8\*I\*c)/a^4, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A \tan^m(c+dx)}{\tan^4(c+dx) - 4i \tan^3(c+dx) - 6 \tan^2(c+dx) + 4i \tan(c+dx) + 1} dx + \int \frac{B \tan(c+dx) \tan^m(c+dx)}{\tan^4(c+dx) - 4i \tan^3(c+dx) - 6 \tan^2(c+dx) + 4i \tan(c+dx) + 1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*4,x)

[Out] (Integral(A\*tan(c + d\*x)\*\*m/(tan(c + d\*x)\*\*4 - 4\*I\*tan(c + d\*x)\*\*3 - 6\*tan(c + d\*x)\*\*2 + 4\*I\*tan(c + d\*x) + 1), x) + Integral(B\*tan(c + d\*x)\*tan(c + d\*x)\*\*m/(tan(c + d\*x)\*\*4 - 4\*I\*tan(c + d\*x)\*\*3 - 6\*tan(c + d\*x)\*\*2 + 4\*I\*tan(c + d\*x) + 1), x))/a\*\*4

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(I\*a\*tan(d\*x + c) + a)^4, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4, x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^4, x)

### 3.212 $\int \tan^m(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=316

$$\frac{4a^3(A - iB)F_1\left(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx), i \tan(c + dx)\right) \sqrt{1 + i \tan(c + dx)} \tan^{1+m}(c + dx) + 2a^2}{d(1 + m) \sqrt{a + ia \tan(c + dx)}}$$

```
[Out] 2*a^2*(2*B*(4*m^2+17*m+19)+I*A*(8*m^2+34*m+35))*hypergeom([1/2, -m], [3/2], 1
+I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^m/d/(3+2*m)/(5+2*m)/((-I
*tan(d*x+c))^m)+4*a^3*(A-I*B)*AppellF1(1+m, 1/2, 1, 2+m, -I*tan(d*x+c), I*tan(d*
x+c))*(1+I*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(1+m)/(a+I*a*tan(d*x+c))^(1
/2)+2*a^2*(2*I*B*(4+m)-A*(5+2*m))*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)
/d/(3+2*m)/(5+2*m)+2*I*a*B*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c))^(3/2)/d/(5+2
*m)
```

**Rubi [A]**

time = 0.67, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3675, 3682, 3645, 140, 138, 3680, 69, 67}

$$\frac{4a^3(A - iB)\sqrt{1 + i \tan(c + dx)} \tan^{1+m}(c + dx) F_1\left(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx), i \tan(c + dx)\right) + 2a^2(2B(4m^2 + 17m + 19) + 4A(8m^2 + 34m + 35))\sqrt{a + ia \tan(c + dx)} \tan^m(c + dx) (-i \tan(c + dx))^{-m} F_1\left(\frac{1}{2}, -m; \frac{3}{2}; 1 + i \tan(c + dx)\right) + 2a^2(-A(2m + 5) + 2iB(m + 4))\sqrt{a + ia \tan(c + dx)} \tan^{m+1}(c + dx) + 2iaB(a + ia \tan(c + dx))^{3/2} \tan^{m+1}(c + dx)}{d(2m + 3)d(2m + 5)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] (4*a^3*(A - I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c
+ d*x]]*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[a +
I*a*Tan[c + d*x]]) + (2*a^2*(2*B*(19 + 17*m + 4*m^2) + I*A*(35 + 34*m + 8*m
^2))*Hypergeometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]]*Tan[c + d*x]^m*Sqr
t[a + I*a*Tan[c + d*x]])/(d*(3 + 2*m)*(5 + 2*m)*((-I)*Tan[c + d*x])^m) + (2
*a^2*((2*I)*B*(4 + m) - A*(5 + 2*m))*Tan[c + d*x]^(1 + m)*Sqrt[a + I*a*Tan[
c + d*x]])/(d*(3 + 2*m)*(5 + 2*m)) + ((2*I)*a*B*Tan[c + d*x]^(1 + m)*(a + I
*a*Tan[c + d*x])^(3/2))/(d*(5 + 2*m))
```

**Rule 67**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m)*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
|| GtQ[-d/(b*c), 0])
```

**Rule 69**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(-b)*(c/
d)]^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[(-d)*(x/c)
```

)^m\*(c + d\*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&  
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0]

### Rule 138

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_  
Symbol] := Simp[c^n\*e^p\*((b\*x)^(m + 1)/(b\*(m + 1)))\*AppellF1[m + 1, -n, -p,  
m + 2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &  
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

### Rule 140

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_  
Symbol] := Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[  
n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e,  
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

### Rule 3645

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) +  
(f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(b/f), Subst[Int[(a + x)^(m - 1)\*((c  
+ (d/b)\*x)^n/(b^2 + a\*x)], x], x, b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d  
, e, f, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^  
2, 0]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) +  
(f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Sim  
p[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m +  
n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[  
e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c -  
a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a,  
b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && G  
tQ[m, 1] && !LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) +  
(f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dis  
t[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x  
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a  
^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^m(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{2iaB \tan^{1+m}(c + dx)(a + ia \tan(c + dx))}{d(5 + 2m)} \\
&= \frac{2a^2(2iB(4 + m) - A(5 + 2m)) \tan^{1+m}(c + dx)}{d(3 + 2m)(5 + 2m)} \\
&= \frac{2a^2(2iB(4 + m) - A(5 + 2m)) \tan^{1+m}(c + dx)}{d(3 + 2m)(5 + 2m)} \\
&= \frac{2a^2(2iB(4 + m) - A(5 + 2m)) \tan^{1+m}(c + dx)}{d(3 + 2m)(5 + 2m)} \\
&= \frac{2a^2(2iB(4 + m) - A(5 + 2m)) \tan^{1+m}(c + dx)}{d(3 + 2m)(5 + 2m)} \\
&= \frac{4a^3(A - iB)F_1\left(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx)\right)}{d(1 + \dots)}
\end{aligned}$$

**Mathematica [F]**

time = 4.31, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),
x]
```

```
[Out] Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),
x]
```

**Maple [F]**

time = 0.46, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + ia \tan(dx + c))^{\frac{5}{2}} (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*tan(d\*x + c)^m, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(4\*sqrt(2)\*((A - I\*B)\*a^2\*e^(7\*I\*d\*x + 7\*I\*c) + (A + I\*B)\*a^2\*e^(5\*I\*d\*x + 5\*I\*c))\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))/(e^(6\*I\*d\*x + 6\*I\*c) + 3\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^m, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

### 3.213 $\int \tan^m(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=227

$$\frac{2a^2(A - iB)F_1\left(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx), i \tan(c + dx)\right) \sqrt{1 + i \tan(c + dx)} \tan^{1+m}(c + dx)}{d(1 + m)\sqrt{a + ia \tan(c + dx)}} + \frac{2a(I}{d(2m + 3)}$$

[Out] 2\*a\*(B+(I\*A+B)\*(3+2\*m))\*hypergeom([1/2, -m], [3/2], 1+I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^m/d/(3+2\*m)/((-I\*tan(d\*x+c))^m)+2\*a^2\*(A-I\*B)\*AppellF1(1+m, 1/2, 1, 2+m, -I\*tan(d\*x+c), I\*tan(d\*x+c))\*(1+I\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(a+I\*a\*tan(d\*x+c))^(1/2)+2\*I\*a\*B\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(3+2\*m)

**Rubi [A]**

time = 0.45, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3675, 3682, 3645, 140, 138, 3680, 69, 67}

$$\frac{2a^2(A - iB)\sqrt{1 + i \tan(c + dx)} \tan^{m+1}(c + dx)F_1\left(m + 1; \frac{1}{2}, 1; m + 2; -i \tan(c + dx), i \tan(c + dx)\right)}{d(m + 1)\sqrt{a + ia \tan(c + dx)}} + \frac{2a(B + (2m + 3)(B + iA))\sqrt{a + ia \tan(c + dx)} \tan^m(c + dx)(-i \tan(c + dx))^{-m} {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; i \tan(c + dx) + 1\right)}{d(2m + 3)} + \frac{2iaB\sqrt{a + ia \tan(c + dx)} \tan^{m+1}(c + dx)}{d(2m + 3)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (2\*a^2\*(A - I\*B)\*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x]]\*Sqrt[1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (2\*a\*(B + (I\*A + B)\*(3 + 2\*m))\*Hypergeometric2F1[1/2, -m, 3/2, 1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*(3 + 2\*m)\*((-I)\*Tan[c + d\*x])^m) + ((2\*I)\*a\*B\*Tan[c + d\*x]^(1 + m)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*(3 + 2\*m))

**Rule 67**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(n + 1)/(d\*(n + 1)\*(-d/(b\*c))^m))\*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b\*c), 0])

**Rule 69**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[(-b)\*(c/d)^IntPart[m]\*((b\*x)^FracPart[m]/((-d)\*(x/c))^FracPart[m]), Int[(-d)\*(x/c)]^m\*(c + d\*x)^n, x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0]

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
```

- Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \tan^m(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= \frac{2iaB \tan^{1+m}(c + dx) \sqrt{a + ia \tan(c + dx)}}{d(3 + 2m)} \\
 &= \frac{2iaB \tan^{1+m}(c + dx) \sqrt{a + ia \tan(c + dx)}}{d(3 + 2m)} \\
 &= \frac{2iaB \tan^{1+m}(c + dx) \sqrt{a + ia \tan(c + dx)}}{d(3 + 2m)} \\
 &= \frac{2iaB \tan^{1+m}(c + dx) \sqrt{a + ia \tan(c + dx)}}{d(3 + 2m)} \\
 &= \frac{2a^2(A - iB)F_1(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx))}{d(1 + m)}
 \end{aligned}$$

Mathematica [F]

time = 2.98, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

Maple [F]

time = 0.38, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c))(a + ia \tan(dx + c))^{\frac{3}{2}}(A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)`

[Out] `int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m, x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral(2*sqrt(2)*((A - I*B)*a*e^(5*I*d*x + 5*I*c) + (A + I*B)*a*e^(3*I*d*x + 3*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^{\frac{3}{2}} (A + B \tan(c + dx)) \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] `Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*tan(c + d*x)**m, x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*tan(d\*x + c)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2), x)

$$3.214 \quad \int \tan^m(c+dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=159

$$\frac{a(A - iB)F_1\left(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx), i \tan(c + dx)\right) \sqrt{1 + i \tan(c + dx)} \tan^{1+m}(c + dx)}{d(1 + m) \sqrt{a + ia \tan(c + dx)}} + \frac{2B}{2}$$

[Out] 2\*B\*hypergeom([1/2, -m], [3/2], 1+I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^m/d/((-I\*tan(d\*x+c))^m)+a\*(A-I\*B)\*AppellF1(1+m, 1/2, 1, 2+m, -I\*tan(d\*x+c), I\*tan(d\*x+c))\*(1+I\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(a+I\*a\*tan(d\*x+c))^(1/2)

**Rubi [A]**

time = 0.23, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3682, 3645, 140, 138, 3680, 69, 67}

$$\frac{a(A - iB) \sqrt{1 + i \tan(c + dx)} \tan^{m+1}(c + dx) F_1\left(m + 1; \frac{1}{2}, 1; m + 2; -i \tan(c + dx), i \tan(c + dx)\right)}{d(m + 1) \sqrt{a + ia \tan(c + dx)}} + \frac{2B \sqrt{a + ia \tan(c + dx)} \tan^m(c + dx) (-i \tan(c + dx))^{-m} {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; i \tan(c + dx) + 1\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (a\*(A - I\*B)\*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x]]\*Sqrt[1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (2\*B\*Hypergeometric2F1[1/2, -m, 3/2, 1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(d\*((-I)\*Tan[c + d\*x])^m)

Rule 67

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(n + 1)/(d\*(n + 1)\*(-d/(b\*c))^(m))\*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b\*c), 0])

Rule 69

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[((-b)\*(c/d))^IntPart[m]\*((b\*x)^FracPart[m]/((-d)\*(x/c))^FracPart[m]), Int[((-d)\*(x/c))^m\*(c + d\*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0]

Rule 138

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m + 1)/(b\*(m + 1)))\*AppellF1[m + 1, -n, -p,

$m + 2, (-d)*(x/c), (-f)*(x/e), x] /; \text{FreeQ}\{b, c, d, e, f, m, n, p\}, x] \&$   
 $\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{GtQ}[c, 0] \&\& (\text{IntegerQ}[p] \parallel \text{GtQ}[e, 0])$

#### Rule 140

$\text{Int}[(b \cdot x)^m \cdot (c + d \cdot x)^n \cdot (e + f \cdot x)^p, x\_Symbol] \rightarrow \text{Dist}[c^{\text{IntPart}[n]} \cdot (c + d \cdot x)^{\text{FracPart}[n]} / (1 + d \cdot (x/c))^{\text{FracPart}[n]}, \text{Int}[(b \cdot x)^m \cdot (1 + d \cdot (x/c))^n \cdot (e + f \cdot x)^p, x], x] /; \text{FreeQ}\{b, c, d, e, f, m, n, p\}, x] \&\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{!GtQ}[c, 0]$

#### Rule 3645

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (c + d \cdot \tan[e + f \cdot x])^n, x\_Symbol] \rightarrow \text{Dist}[a \cdot (b/f), \text{Subst}[\text{Int}[(a + x)^{m-1} \cdot (c + (d/b) \cdot x)^n / (b^2 + a \cdot x), x], x, b \cdot \tan[e + f \cdot x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3680

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (A + B \cdot \tan[e + f \cdot x])^n, x\_Symbol] \rightarrow \text{Dist}[b \cdot (B/f), \text{Subst}[\text{Int}[(a + b \cdot x)^{m-1} \cdot (c + d \cdot x)^n, x], x, \tan[e + f \cdot x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{EqQ}[A \cdot b + a \cdot B, 0]$

#### Rule 3682

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (A + B \cdot \tan[e + f \cdot x])^n, x\_Symbol] \rightarrow \text{Dist}[(A \cdot b + a \cdot B)/b, \text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (c + d \cdot \tan[e + f \cdot x])^n, x], x] - \text{Dist}[B/b, \text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (c + d \cdot \tan[e + f \cdot x])^n \cdot (a - b \cdot \tan[e + f \cdot x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[A \cdot b + a \cdot B, 0]$

#### Rubi steps



$$\int \tan^m(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx = - \left( (-A + iB) \int \tan^m(c + dx) \sqrt{a + ia \tan(c + dx)} dx \right)$$

$$= \frac{(iaB) \text{Subst} \left( \int \frac{x^m}{\sqrt{a + iax}} dx, x, \tan(c + dx) \right)}{d}$$

$$= \frac{(iaB(-i \tan(c + dx))^{-m} \tan^m(c + dx)) \text{Subst} \left( \int \frac{x^m}{\sqrt{a + iax}} dx, x, \tan(c + dx) \right)}{d}$$

$$= \frac{a(A - iB) F_1 \left( 1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx) \right)}{d(1 + m)}$$

**Mathematica [F]**

time = 2.29, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.46, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) \sqrt{a + ia \tan(dx + c)} (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(I\*a\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(sqrt(2)\*((A - I\*B)\*e^(3\*I\*d\*x + 3\*I\*c) + (A + I\*B)\*e^(I\*d\*x + I\*c))\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))/(e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(I\*a\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx) 1i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2), x)

$$3.215 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=214

$$\frac{(A+iB) \tan^{1+m}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(A-iB) F_1\left(1+m; \frac{1}{2}, 1; 2+m; -i \tan(c+dx), i \tan(c+dx)\right) \sqrt{1+i \tan(c+dx)}}{2d(1+m) \sqrt{a+ia \tan(c+dx)}}$$

[Out] (I\*A-B)\*(1+2\*m)\*hypergeom([1/2, -m], [3/2], 1+I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^m/a/d/((-I\*tan(d\*x+c))^m)+(A+I\*B)\*tan(d\*x+c)^(1+m)/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/2\*(A-I\*B)\*AppellF1(1+m, 1/2, 1, 2+m, -I\*tan(d\*x+c), I\*tan(d\*x+c))\*(1+I\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(a+I\*a\*tan(d\*x+c))^(1/2)

**Rubi** [A]

time = 0.40, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3682, 3645, 140, 138, 3680, 69, 67}

$$\frac{(A-iB) \sqrt{1+i \tan(c+dx)} \tan^{m+1}(c+dx) F_1\left(m+1; \frac{1}{2}, 1; m+2; -i \tan(c+dx), i \tan(c+dx)\right)}{2d(m+1) \sqrt{a+ia \tan(c+dx)}} + \frac{(2m+1)(-B+iA) \sqrt{a+ia \tan(c+dx)} \tan^m(c+dx) (-i \tan(c+dx))^{-m} {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; i \tan(c+dx)+1\right)}{ad} + \frac{(A+iB) \tan^{m+1}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((A + I\*B)\*Tan[c + d\*x]^(1 + m))/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + ((A - I\*B)\*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x]]\*Sqrt[1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m))/(2\*d\*(1 + m)\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + ((I\*A - B)\*(1 + 2\*m)\*Hypergeometric2F1[1/2, -m, 3/2, 1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(a\*d\*((-I)\*Tan[c + d\*x])^m)

**Rule 67**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(n + 1)/(d\*(n + 1)\*(-d/(b\*c))^m)\*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b\*c), 0])

**Rule 69**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[(-b)\*(c/d)^IntPart[m]\*((b\*x)^FracPart[m]/((-d)\*(x/c))^FracPart[m]), Int[((-d)\*(x/c))^m\*(c + d\*x)^n, x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0]

**Rule 138**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

#### Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

#### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

#### Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
```

d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} dx &= \frac{(A + iB) \tan^{1+m}(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} + \frac{\int \tan^m(c + dx) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{a + ia \tan(c + dx)}} \\
 &= \frac{(A + iB) \tan^{1+m}(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} + \frac{(A - iB) \int \tan^m(c + dx) \sqrt{a + ia \tan(c + dx)}}{2a} \\
 &= \frac{(A + iB) \tan^{1+m}(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} + \frac{(a(iA + B)) \text{Subst}\left(\int \frac{(-\frac{ix}{a})^m}{\sqrt{a + x(-a^2 + x^2)}} dx\right)}{2d} \\
 &= \frac{(A + iB) \tan^{1+m}(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} - \frac{((A + iB)(1 + 2m)(-i \tan(c + dx))}{d \sqrt{a + ia \tan(c + dx)}} \\
 &= \frac{(A + iB) \tan^{1+m}(c + dx)}{d \sqrt{a + ia \tan(c + dx)}} + \frac{(A - iB) F_1\left(1 + m; \frac{1}{2}, 1; 2 + m; -i \tan(c + dx)\right)}{2a}
 \end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]], x]

[Out] \$Aborted

**Maple [F]**

time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{\sqrt{a + ia \tan(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2), x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2), x)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x - I*c)/a, x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{\sqrt{ia (\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/sqrt(I\*a\*tan(d\*x + c) + a), x  
)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{\sqrt{a + a \tan(c + dx)} \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(1/2), x)

$$3.216 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=285

$$\frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m) - i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} + \frac{(A-iB)F_1(1+m; \frac{1}{2}, 1; 2+m; -i \tan(c+dx))}{4d\sqrt{a+ia \tan(c+dx)}}$$

[Out] 1/6\*(1+2\*m)\*(B+I\*A\*(5-4\*m)+4\*B\*m)\*hypergeom([1/2, -m], [3/2], 1+I\*tan(d\*x+c))  
 \*(a+I\*a\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^m/a^2/d/((-I\*tan(d\*x+c))^m)+1/6\*(A\*(5-  
 4\*m)-I\*(4\*B\*m+B))\*tan(d\*x+c)^(1+m)/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)+1/4\*(A-I\*B)  
 \*AppellF1(1+m, 1/2, 1, 2+m, -I\*tan(d\*x+c), I\*tan(d\*x+c))\*(1+I\*tan(d\*x+c))^(1/2)\*  
 tan(d\*x+c)^(1+m)/a/d/(1+m)/(a+I\*a\*tan(d\*x+c))^(1/2)+1/3\*(A+I\*B)\*tan(d\*x+c)^(  
 1+m)/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.63, antiderivative size = 285, normalized size of antiderivative = 1.00, number of  
 steps used = 9, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ ,  
 Rules used = {3677, 3682, 3645, 140, 138, 3680, 69, 67}

$$\frac{(2m+1)(A(5-4m)+4Bm+B)\sqrt{a+ia \tan(c+dx)} \tan^m(c+dx) (-i \tan(c+dx))^{-m} {}_2F_1(\frac{1}{2}, -m; \frac{3}{2}; i \tan(c+dx)+1)}{6a^2d} + \frac{(A-iB)\sqrt{1+i \tan(c+dx)} \tan^{m+1}(c+dx) F_1(m+1; \frac{1}{2}, 1; m+2; -i \tan(c+dx), i \tan(c+dx))}{4ad(m+1)\sqrt{a+ia \tan(c+dx)}} + \frac{(A(5-4m)-i(4Bm+B)) \tan^{m+1}(c+dx)}{6ad\sqrt{a+ia \tan(c+dx)}} + \frac{(A+iB) \tan^{m+1}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]  
 [Out] ((A + I\*B)\*Tan[c + d\*x]^(1 + m))/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((A\*(  
 5 - 4\*m) - I\*(B + 4\*B\*m))\*Tan[c + d\*x]^(1 + m))/(6\*a\*d\*Sqrt[a + I\*a\*Tan[c +  
 d\*x]]) + ((A - I\*B)\*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)\*Tan[c + d\*x], I\*Ta  
 n[c + d\*x]]\*Sqrt[1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m))/(4\*a\*d\*(1 + m)\*S  
 qrt[a + I\*a\*Tan[c + d\*x]]) + ((1 + 2\*m)\*(B + I\*A\*(5 - 4\*m) + 4\*B\*m)\*Hyperge  
 ometric2F1[1/2, -m, 3/2, 1 + I\*Tan[c + d\*x]]\*Tan[c + d\*x]^m\*Sqrt[a + I\*a\*Ta  
 n[c + d\*x]])/(6\*a^2\*d\*(-I)\*Tan[c + d\*x]^m)

**Rule 67**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((c + d\*x)  
 )^(n + 1)/(d\*(n + 1)\*(-d/(b\*c))^m)\*Hypergeometric2F1[-m, n + 1, n + 2, 1 +  
 d\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]  
 || GtQ[-d/(b\*c), 0])

**Rule 69**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[(-b)\*(c/  
 d)]^IntPart[m]\*((b\*x)^FracPart[m]/((-d)\*(x/c))^FracPart[m]), Int[(-d)\*(x/c)  
 )^m\*(c + d\*x)^n, x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&  
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0]



Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
```

- Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{\int \frac{\tan^m(c+dx)(a(A(2-m)-iB(1+m))-\frac{1}{2}a(iA-))}{\sqrt{a+ia \tan(c+dx)}} dx}{3a^2} \\
 &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m)-i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m)-i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m)-i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m)-i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m)-i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}} \\
 &= \frac{(A+iB) \tan^{1+m}(c+dx)}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(A(5-4m)-i(B+4Bm)) \tan^{1+m}(c+dx)}{6ad \sqrt{a+ia \tan(c+dx)}}
 \end{aligned}$$

Mathematica [F]

time = 10.27, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

Verification is not applicable to the result.

[In] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

Maple [F]

time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx+c))(A+B \tan(dx+c))}{(a+ia \tan(dx+c))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)`

[Out] `int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)`

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(1/4*sqrt(2)*((A - I*B)*e^(4*I*d*x + 4*I*c) + 2*A*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/a^2, x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(I*a*(tan(c + d*x) - I))**(3/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

$$3.217 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=363

$$\frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m) + A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} - \frac{(iB(13+12m-16m^2) - A(37-52m+16m^2)) \tan^{1+m}(c+dx)}{60a^2d \sqrt{a+ia \tan(c+dx)}} + \frac{(A-I*B)*\text{AppellF1}(1+m, 1/2, 1, 2+m, -I*\tan(c+dx), I*\tan(c+dx))}{(60*a^2*d*\text{Sqrt}[a+I*a*\tan(c+dx)])} + \frac{((A-I*B)*\text{AppellF1}[1+m, 1/2, 1, 2+m, (-I)*\tan(c+dx), I*\tan(c+dx)]*\text{Sqrt}[1+I*\tan(c+dx)]*\tan(c+dx)^{(1+m)})}{(8*a^2*d*(1+m)*\text{Sqrt}[a+I*a*\tan(c+dx)])} + \frac{((1+2*m)*(B*(13+12*m-16*m^2) + I*A*(37-52*m+16*m^2))*\text{Hypergeometric2F1}[1/2, -m, 3/2, 1+I*\tan(c+dx)]*\tan(c+dx)^m*\text{Sqrt}[a+I*a*\tan(c+dx)])}{(60*a^3*d*((-I)*\tan(c+dx))^{(m)})}$$

```
[Out] 1/60*(1+2*m)*(B*(-16*m^2+12*m+13)+I*A*(16*m^2-52*m+37))*hypergeom([1/2, -m], [3/2], 1+I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(1/2)*tan(d*x+c)^m/a^3/d/((-I*tan(d*x+c))^m)-1/60*(I*B*(-16*m^2+12*m+13)-A*(16*m^2-52*m+37))*tan(d*x+c)^(1+m)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/8*(A-I*B)*AppellF1(1+m, 1/2, 1, 2+m, -I*tan(d*x+c), I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/a^2/d/(1+m)/(a+I*a*tan(d*x+c))^(1/2)+1/5*(A+I*B)*tan(d*x+c)^(1+m)/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(I*B*(1-4*m)+A*(11-4*m))*tan(d*x+c)^(1+m)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

**Rubi [A]**

time = 0.91, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3677, 3682, 3645, 140, 138, 3680, 69, 67}

$\frac{(2m+1)(B(-16m^2+12m+13)+A(16m^2-52m+37))\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)/(-i\tan(c+dx))^{m+1} + (A-I*B)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)F_1(m+1, \frac{1}{2}, 1+m+2, -i\tan(c+dx), i\tan(c+dx))}{60a^3d} - \frac{(A-I*B)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)}{60a^2d} - \frac{(B(13+12m-16m^2)-A(37-52m+16m^2))\tan^{m+1}(c+dx)}{60a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{(A+I*B)\tan^{m+1}(c+dx)}{8a^2d} + \frac{(A+I*B)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] ((A + I*B)*Tan[c + d*x]^(1 + m))/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((I*B*(1 - 4*m) + A*(11 - 4*m))*Tan[c + d*x]^(1 + m))/(30*a*d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I*B*(13 + 12*m - 16*m^2) - A*(37 - 52*m + 16*m^2))*Tan[c + d*x]^(1 + m))/(60*a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((A - I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(8*a^2*d*(1 + m)*Sqrt[a + I*a*Tan[c + d*x]]) + ((1 + 2*m)*(B*(13 + 12*m - 16*m^2) + I*A*(37 - 52*m + 16*m^2))*Hypergeometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]]*Tan[c + d*x]^m*Sqrt[a + I*a*Tan[c + d*x]])/(60*a^3*d*((-I)*Tan[c + d*x])^m)
```

**Rule 67**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^(m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])
```

**Rule 69**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[((-b)*(c/d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

### Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] := Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

### Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] := Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c + (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]
```

### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

## Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx &= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{\int \frac{\tan^m(c+dx)(a(A(4-m)-iB(1+m))-\frac{1}{2}a(iA+B))}{(a+ia \tan(c+dx))^{3/2}} dx}{5a^2} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{1+m}(c+dx)}{5d(a+ia \tan(c+dx))^{5/2}} + \frac{(iB(1-4m)+A(11-4m)) \tan^{1+m}(c+dx)}{30ad(a+ia \tan(c+dx))^{3/2}}
\end{aligned}$$

## Mathematica [F]

time = 45.80, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

[Out] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

**Maple [F]**

time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + ia \tan(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(1/8\*sqrt(2)\*((A - I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) + (3\*A - I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) + (3\*A + I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^m\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*e^(-5\*I\*d\*x - 5\*I\*c)/a^3, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(ia (\tan(c + dx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/(I\*a\*(tan(c + d\*x) - I))\*\*(5/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*i)^(5/2),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*i)^(5/2), x)

### 3.218 $\int \tan^m(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=167

$$\frac{(A - iB)F_1(1 + m; 1 - n, 1; 2 + m; -i \tan(c + dx), i \tan(c + dx))(1 + i \tan(c + dx))^{-n} \tan^{1+m}(c + dx)(a + i \tan(c + dx))^n}{d(1 + m)}$$

```
[Out] (A-I*B)*AppellF1(1+m,1-n,1,2+m,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1+m)
*(a+I*a*tan(d*x+c))^n/d/(1+m)/((1+I*tan(d*x+c))^n)+I*B*hypergeom([1+m, 1-n],
[2+m],-I*tan(d*x+c))*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c))^n/d/(1+m)/((1+I*t
an(d*x+c))^n)
```

**Rubi [A]**

time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3682, 3645, 140, 138, 3680, 68, 66}

$$\frac{(A - iB) \tan^{m+1}(c + dx)(1 + i \tan(c + dx))^{-n}(a + ia \tan(c + dx))^n F_1(m + 1; 1 - n, 1; m + 2; -i \tan(c + dx), i \tan(c + dx))}{d(m + 1)} + \frac{iB \tan^{m+1}(c + dx)(1 + i \tan(c + dx))^{-n}(a + ia \tan(c + dx))^n {}_2F_1(m + 1, 1 - n; m + 2; -i \tan(c + dx))}{d(m + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((A - I*B)*AppellF1[1 + m, 1 - n, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*
x])*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^n/(d*(1 + m)*(1 + I*Tan[c
+ d*x])^n) + (I*B*Hypergeometric2F1[1 + m, 1 - n, 2 + m, (-I)*Tan[c + d*x]]
*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^n)/(d*(1 + m)*(1 + I*Tan[c + d
*x])^n)
```

**Rule 66**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[c^n*((b*x)
)^(m + 1)/(b*(m + 1))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
&& !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

**Rule 68**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

**Rule 138**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

#### Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

#### Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \tan^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx &= -\left((-A+iB) \int \tan^m(c+dx)(a+ia \tan(c+dx))^n dx\right) \\
&= \frac{(iaB) \text{Subst}\left(\int x^m(a+iax)^{-1+n} dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{(iB(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx)))}{d} \\
&= \frac{(A-iB)F_1(1+m; 1-n, 1; 2+m; -i \tan(c+dx))}{d}
\end{aligned}$$

**Mathematica [F]**

time = 12.17, size = 0, normalized size = 0.00

$$\int \tan^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Tan[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.29, size = 0, normalized size = 0.00

$$\int (\tan^m(dx+c))(a+ia \tan(dx+c))^n(A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^m, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c)
/(e^(2*I*d*x + 2*I*c) + 1))^n*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))^m/(e^(2*I*d*x + 2*I*c) + 1), x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*tan(c + d*x)**m,
x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^m, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

### 3.219 $\int \tan^3(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=245

$$\frac{2(iBn - A(3+n))(a + ia \tan(c + dx))^n}{dn(2+n)(3+n)} + \frac{(A - iB) {}_2F_1(1, n; 1+n; \frac{1}{2}(1 + i \tan(c + dx)))}{2dn} (a + ia \tan(c + dx))^n$$

[Out]  $2*(I*B*n-A*(3+n))*(a+I*a*\tan(d*x+c))^n/d/n/(2+n)/(3+n)+1/2*(A-I*B)*\text{hypergeom}([1, n], [1+n], 1/2+1/2*I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/n-(I*B*n-A*(3+n))*\tan(d*x+c)^2*(a+I*a*\tan(d*x+c))^n/d/(2+n)/(3+n)+B*\tan(d*x+c)^3*(a+I*a*\tan(d*x+c))^n/d/(3+n)-(A*n*(3+n)-I*B*(n^2+3*n+6))*(a+I*a*\tan(d*x+c))^{(1+n)}/a/d/(3+n)/(n^2+3*n+2)$

**Rubi [A]**

time = 0.45, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3678, 3673, 3608, 3562, 70}

$$\frac{(A-iB)(a+ia \tan(c+dx))^n {}_2F_1(1, n; n+1; \frac{1}{2}(i \tan(c+dx)+1))}{2dn} - \frac{(An+3-iB(n^2+3n+6))(a+ia \tan(c+dx))^{n+1}}{ad(n+1)(n+2)(n+3)} - \frac{(-A(n+3)+iBn) \tan^2(c+dx)(a+ia \tan(c+dx))^n}{d(n+2)(n+3)} + \frac{2(-A(n+3)+iBn)(a+ia \tan(c+dx))^n}{dn(n+2)(n+3)} + \frac{B \tan^3(c+dx)(a+ia \tan(c+dx))^n}{d(n+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(2*(I*B*n - A*(3+n))*(a + I*a*\text{Tan}[c + d*x])^n)/(d*n*(2+n)*(3+n)) + ((A - I*B)*\text{Hypergeometric2F1}[1, n, 1+n, (1 + I*\text{Tan}[c + d*x])/2]*(a + I*a*\text{Tan}[c + d*x])^n)/(2*d*n) - ((I*B*n - A*(3+n))*\text{Tan}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^n)/(d*(2+n)*(3+n)) + (B*\text{Tan}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])^n)/(d*(3+n)) - ((A*n*(3+n) - I*B*(6 + 3*n + n^2))*(a + I*a*\text{Tan}[c + d*x])^{(1+n)})/(a*d*(1+n)*(2+n)*(3+n))$

**Rule 70**

$\text{Int}[(a + b*x)^m*((c + d*x)^n), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^n*(a + b*x)^{(m+1)}/(b^{(n+1)}*(m+1))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(a + b*x)/(b*c - a*d)], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& !\text{IntegerQ}\{m\} \&\& \text{IntegerQ}\{n\}$

**Rule 3562**

$\text{Int}[(a + b*\tan[(c + d*x)])^n, x\_Symbol] \rightarrow \text{Dist}[-b/d, \text{Subst}[\text{Int}[(a + x)^{(n-1)}/(a - x), x], x, b*\text{Tan}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \&\& \text{EqQ}[a^2 + b^2, 0]$

**Rule 3608**

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]
```

### Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3678

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \tan^3(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx &= \frac{B \tan^3(c + dx)(a + ia \tan(c + dx))^n}{d(3 + n)} + \frac{\int \tan^2(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx}{d(3 + n)} \\
&= -\frac{(iBn - A(3 + n)) \tan^2(c + dx)(a + ia \tan(c + dx))^n}{d(2 + n)(3 + n)} + \frac{\int \tan(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx}{d(2 + n)(3 + n)} \\
&= -\frac{(iBn - A(3 + n)) \tan^2(c + dx)(a + ia \tan(c + dx))^n}{d(2 + n)(3 + n)} + \frac{\int (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx}{dn(2 + n)(3 + n)} \\
&= \frac{2(iBn - A(3 + n))(a + ia \tan(c + dx))^n}{dn(2 + n)(3 + n)} - \frac{\int (a + ia \tan(c + dx))^n dx}{dn(2 + n)(3 + n)} \\
&= \frac{2(iBn - A(3 + n))(a + ia \tan(c + dx))^n}{dn(2 + n)(3 + n)} + \frac{\int (a + ia \tan(c + dx))^n dx}{dn(2 + n)(3 + n)}
\end{aligned}$$

### Mathematica [F]

time = 40.09, size = 0, normalized size = 0.00

$$\int \tan^3(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Tan[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.26, size = 0, normalized size = 0.00

$$\int (\tan^3(dx + c)) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(tan(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^3, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="fricas")

[Out] integral(((I\*A + B)\*e^(8\*I\*d\*x + 8\*I\*c) - 2\*(I\*A + 2\*B)\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*B\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*(-I\*A + 2\*B)\*e^(2\*I\*d\*x + 2\*I\*c) - I\*A + B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n/(e^(8\*I\*d\*x + 8\*I\*c) + 4\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*e^(4\*I\*d\*x + 4\*I\*c) + 4\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)



**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \tan^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*n\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^3 (A + B \tan(c + dx)) (a + a \tan(c + dx) li)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(tan(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

### 3.220 $\int \tan^2(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

Optimal. Leaf size=164

$$\frac{2B(a+ia \tan(c+dx))^n}{dn(2+n)} + \frac{(iA+B) {}_2F_1(1, n; 1+n; \frac{1}{2}(1+i \tan(c+dx))) (a+ia \tan(c+dx))^n}{2dn} + \frac{B \tan^2(c+dx)}{dn(2+n)}$$

[Out]  $-2*B*(a+I*a*\tan(d*x+c))^n/d/n/(2+n)+1/2*(I*A+B)*\text{hypergeom}([1, n], [1+n], 1/2+1/2*I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/n+B*\tan(d*x+c)^2*(a+I*a*\tan(d*x+c))^n/d/(2+n)-(B*n+I*A*(2+n))*(a+I*a*\tan(d*x+c))^{(1+n)}/a/d/(1+n)/(2+n)$

Rubi [A]

time = 0.22, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3678, 3673, 3608, 3562, 70}

$$\frac{(B+iA)(a+ia \tan(c+dx))^n {}_2F_1(1, n; n+1; \frac{1}{2}(i \tan(c+dx)+1))}{2dn} - \frac{(Bn+iA(n+2))(a+ia \tan(c+dx))^{n+1}}{ad(n+1)(n+2)} + \frac{B \tan^2(c+dx)(a+ia \tan(c+dx))^n}{d(n+2)} - \frac{2B(a+ia \tan(c+dx))^n}{dn(n+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c+d*x]^2*(a+I*a*\text{Tan}[c+d*x])^n*(A+B*\text{Tan}[c+d*x]), x]$

[Out]  $(-2*B*(a+I*a*\text{Tan}[c+d*x])^n)/(d*n*(2+n)) + ((I*A+B)*\text{Hypergeometric2F1}[1, n, 1+n, (1+I*\text{Tan}[c+d*x])/2]*(a+I*a*\text{Tan}[c+d*x])^n)/(2*d*n) + (B*\text{Tan}[c+d*x]^2*(a+I*a*\text{Tan}[c+d*x])^n)/(d*(2+n)) - ((B*n+I*A*(2+n))*(a+I*a*\text{Tan}[c+d*x])^{(1+n)})/(a*d*(1+n)*(2+n))$

Rule 70

$\text{Int}[(a_+ + (b_+)*(x_+))^{(m_+)}*((c_+ + (d_+)*(x_+))^{(n_+)}, x\_Symbol] := \text{Simp}[(b_*c - a_*d)^n*(a + b*x)^{(m+1)}/(b^{(n+1)}*(m+1))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*((a+b*x)/(b*c-a*d))], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& !\text{IntegerQ}\{m\} \&\& \text{IntegerQ}\{n\}$

Rule 3562

$\text{Int}[(a_+ + (b_+)*\tan[(c_+ + (d_+)*(x_+)])^{(n_+)}, x\_Symbol] := \text{Dist}[-b/d, \text{Subst}[\text{Int}[(a+x)^{(n-1)}/(a-x), x], x, b*\text{Tan}[c+d*x]], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}\{a^2 + b^2, 0\}$

Rule 3608

$\text{Int}[(a_+ + (b_+)*\tan[(e_+ + (f_+)*(x_+)])^{(m_+)}*((c_+ + (d_+)*\tan[(e_+ + (f_+)*(x_+)])^{(n_+)}, x\_Symbol] := \text{Simp}[d*((a+b*\text{Tan}[e+f*x])^m/(f*m)), x] + \text{Dist}[(b*c+a*d)/b, \text{Int}[(a+b*\text{Tan}[e+f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e,$

$f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{!LtQ}[m, 0]$

### Rule 3673

$\text{Int}[\left((a_{.}) + (b_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(m_{.})}\left((A_{.}) + (B_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)\left((c_{.}) + (d_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right), x\_Symbol] \rightarrow \text{Simp}[B*d*\left((a + b*\text{Tan}[e + f*x])^{(m + 1)} / (b*f*(m + 1))\right), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!LeQ}[m, -1]$

### Rule 3678

$\text{Int}[\left((a_{.}) + (b_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(m_{.})}\left((A_{.}) + (B_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(n_{.})}, x\_Symbol] \rightarrow \text{Simp}[B*(a + b*\text{Tan}[e + f*x])^m*\left((c + d*\text{Tan}[e + f*x])^n / (f*(m + n))\right), x] + \text{Dist}[1/(a*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n - 1)}*\text{Simp}[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \tan^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx &= \frac{B \tan^2(c + dx)(a + ia \tan(c + dx))^n}{d(2 + n)} + \frac{\int \tan^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{d(2 + n)} \\ &= \frac{B \tan^2(c + dx)(a + ia \tan(c + dx))^n}{d(2 + n)} - \frac{(Bn)}{d(2 + n)} \\ &= -\frac{2B(a + ia \tan(c + dx))^n}{dn(2 + n)} + \frac{B \tan^2(c + dx)}{dn(2 + n)} \\ &= -\frac{2B(a + ia \tan(c + dx))^n}{dn(2 + n)} + \frac{B \tan^2(c + dx)}{dn(2 + n)} \\ &= -\frac{2B(a + ia \tan(c + dx))^n}{dn(2 + n)} + \frac{(iA + B) {}_2F_1}{dn(2 + n)} \end{aligned}$$

### Mathematica [F]

time = 14.31, size = 0, normalized size = 0.00

$$\int \tan^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.44, size = 0, normalized size = 0.00

$$\int (\tan^2(dx + c)) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^2, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(-((A - I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) - (A - 3\*I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) - (A + 3\*I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n/(e^(6\*I\*d\*x + 6\*I\*c) + 3\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I)\*\*n\*(A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^2 (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

### 3.221 $\int \tan(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

Optimal. Leaf size=111

$$\frac{A(a+ia \tan(c+dx))^n}{dn} - \frac{(A-iB) {}_2F_1(1, n; 1+n; \frac{1}{2}(1+i \tan(c+dx)))}{2dn} (a+ia \tan(c+dx))^n - \frac{iB(a+ia \tan(c+dx))^{n+1}}{ad(1+n)}$$

[Out]  $A*(a+I*a*\tan(d*x+c))^n/d/n-1/2*(A-I*B)*\text{hypergeom}([1, n], [1+n], 1/2+1/2*I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/n-I*B*(a+I*a*\tan(d*x+c))^{(1+n)}/a/d/(1+n)$

**Rubi [A]**

time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3673, 3608, 3562, 70}

$$\frac{(A-iB)(a+ia \tan(c+dx))^n {}_2F_1(1, n; n+1; \frac{1}{2}(i \tan(c+dx)+1))}{2dn} + \frac{A(a+ia \tan(c+dx))^n}{dn} - \frac{iB(a+ia \tan(c+dx))^{n+1}}{ad(n+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c+d*x]*(a+I*a*\text{Tan}[c+d*x])^n*(A+B*\text{Tan}[c+d*x]), x]$

[Out]  $(A*(a+I*a*\text{Tan}[c+d*x])^n)/(d*n) - ((A-I*B)*\text{Hypergeometric2F1}[1, n, 1+n, (1+I*\text{Tan}[c+d*x])/2]*(a+I*a*\text{Tan}[c+d*x])^n)/(2*d*n) - (I*B*(a+I*a*\text{Tan}[c+d*x])^{(1+n)})/(a*d*(1+n))$

Rule 70

$\text{Int}[((a_) + (b_.)*(x_))^{(m_)}*((c_) + (d_.)*(x_))^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^n*((a + b*x)^{(m+1)})/(b^{(n+1)}*(m+1))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*((a + b*x)/(b*c - a*d))], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3562

$\text{Int}(((a_) + (b_.)*\tan[(c_) + (d_.)*(x_)])^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[-b/d, \text{Subst}[\text{Int}[(a+x)^{(n-1)}/(a-x), x], x, b*\text{Tan}[c+d*x]], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \&\& \text{EqQ}[a^2 + b^2, 0]$

Rule 3608

$\text{Int}(((a_) + (b_.)*\tan[(e_) + (f_.)*(x_)])^{(m_)}*((c_) + (d_.)*\tan[(e_) + (f_.)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m)/(f*m), x] + \text{Dist}[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& !\text{LtQ}[m, 0]$

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned} \int \tan(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx &= -\frac{iB(a + ia \tan(c + dx))^{1+n}}{ad(1+n)} + \int (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx \\ &= \frac{A(a + ia \tan(c + dx))^n}{dn} - \frac{iB(a + ia \tan(c + dx))^{1+n}}{ad(1+n)} \\ &= \frac{A(a + ia \tan(c + dx))^n}{dn} - \frac{iB(a + ia \tan(c + dx))^{1+n}}{ad(1+n)} \\ &= \frac{A(a + ia \tan(c + dx))^n}{dn} - \frac{(A - iB) {}_2F_1(1, n; 2+n, -e^{2i(c+dx)})}{dn} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 270 vs. 2(111) = 222.  
time = 21.32, size = 270, normalized size = 2.43

$$2^{-1+n} e^{-2idax} (e^{idax})^n \left( \frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^n \left( \frac{(A + iB)e^{2idax} (1 + e^{2i(c+dx)})^n {}_2F_1(n, 2+n; 1+n; -e^{2i(c+dx)})}{dn} + \frac{e^{2ic} \left( -\frac{2iBd^{2d(1+n)x}}{(1 + e^{2i(c+dx)})^{1+n}} - \frac{(A - iB)e^{2i(c+dx)(2+n)x} (1 + e^{2i(c+dx)})^n {}_2F_1(2+n, 2+n; 3+n; -e^{2i(c+dx)})}{2+n} \right)}{d} \right) \sec^{-n}(c + dx) (\cos(dx) + i \sin(dx))^{-n} (a + ia \tan(c + dx))^n$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] (2^(-1 + n)\*(E^(I\*d\*x))^n\*(E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x))))^n\*(((A + I\*B)\*E^((2\*I)\*d\*n\*x)\*(1 + E^((2\*I)\*(c + d\*x))))^n\*Hypergeometric2F1[n, 2 + n, 1 + n, -E^((2\*I)\*(c + d\*x))])/(d\*n) + (E^((2\*I)\*c)\*((-2\*I)\*B\*E^((2\*I)\*d\*(1 + n)\*x))/((1 + E^((2\*I)\*(c + d\*x)))\*(1 + n)) - ((A - I\*B)\*E^((2\*I)\*(c + d\*(2 + n)\*x))\*(1 + E^((2\*I)\*(c + d\*x))))^n\*Hypergeometric2F1[2 + n, 2 + n, 3 + n, -E^((2\*I)\*(c + d\*x))])/(2 + n))/d\*(a + I\*a\*Tan[c + d\*x])^n/(E^((2\*I)\*d\*n\*x)\*Sec[c + d\*x]^n\*(Cos[d\*x] + I\*Sin[d\*x])^n)

**Maple [F]**

time = 0.43, size = 0, normalized size = 0.00

$$\int \tan(dx + c) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)`

[Out] `int(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c), x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral((( -I*A - B)*e^(4*I*d*x + 4*I*c) + 2*B*e^(2*I*d*x + 2*I*c) + I*A - B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)`

[Out] `Integral((I*a*(tan(c + d*x) - I))^n*(A + B*tan(c + d*x))*tan(c + d*x), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")`



[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx) (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

### 3.222 $\int (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx$

Optimal. Leaf size=78

$$\frac{B(a + ia \tan(c + dx))^n}{dn} - \frac{(iA + B) {}_2F_1(1, n; 1 + n; \frac{1}{2}(1 + i \tan(c + dx))) (a + ia \tan(c + dx))^n}{2dn}$$

[Out] B\*(a+I\*a\*tan(d\*x+c))^n/d/n-1/2\*(I\*A+B)\*hypergeom([1, n], [1+n], 1/2+1/2\*I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/n

Rubi [A]

time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3608, 3562, 70}

$$\frac{B(a + ia \tan(c + dx))^n}{dn} - \frac{(B + iA)(a + ia \tan(c + dx))^n {}_2F_1(1, n; n + 1; \frac{1}{2}(i \tan(c + dx) + 1))}{2dn}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] (B\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*n) - ((I\*A + B)\*Hypergeometric2F1[1, n, 1 + n, (1 + I\*Tan[c + d\*x])/2]\*(a + I\*a\*Tan[c + d\*x])^n)/(2\*d\*n)

Rule 70

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 3562

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rule 3608

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx &= \frac{B(a + ia \tan(c + dx))^n}{dn} - (-A + iB) \int (a + ia \tan(c + dx))^n dx \\ &= \frac{B(a + ia \tan(c + dx))^n}{dn} - \frac{(a(iA + B)) \text{Subst}\left(\int \frac{(a+x)^{-1+n}}{a-x} dx\right)}{d} \\ &= \frac{B(a + ia \tan(c + dx))^n}{dn} - \frac{(iA + B) {}_2F_1\left(1, n; 1 + n; \frac{1}{2}(1 - \dots)\right)}{dn} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 171 vs. 2(78) = 156.  
time = 4.17, size = 171, normalized size = 2.19

$$\frac{2^{-1+n} (e^{idx})^n \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n ((-iA+B)(1+n) - i(A-iB)e^{2i(c+dx)}(1+e^{2i(c+dx)})^n) {}_2F_1(1+n, 1+n; 2+n; -e^{2i(c+dx)}) \sec^{-n}(c+dx) (\cos(dx) + i \sin(dx))^{-n} (a + ia \tan(c+dx))^n}{dn(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] (2^(-1 + n)\*(E^(I\*d\*x))^n\*(E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x))))^n\*((( -I)\*A + B)\*(1 + n) - I\*(A - I\*B)\*E^((2\*I)\*(c + d\*x))\*(1 + E^((2\*I)\*(c + d\*x))))^n\*n\*Hypergeometric2F1[1 + n, 1 + n, 2 + n, -E^((2\*I)\*(c + d\*x))]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*n\*(1 + n)\*Sec[c + d\*x]^n\*(Cos[d\*x] + I\*Sin[d\*x])^n)

**Maple [F]**

time = 0.25, size = 0, normalized size = 0.00

$$\int (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(((A - I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c) / (e^(2\*I\*d\*x + 2\*I\*c) + 1))^n / (e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))^n\*(A + B\*tan(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \tan(c + dx)) (a + a \tan(c + dx) li)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*li)^n,x)

[Out] int((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*li)^n, x)

### 3.223 $\int \cot(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=97

$$\frac{(A - iB) {}_2F_1(1, n; 1 + n; \frac{1}{2}(1 + i \tan(c + dx))) (a + ia \tan(c + dx))^n}{2dn} - \frac{A {}_2F_1(1, n; 1 + n; 1 + i \tan(c + dx))}{dn}$$

[Out] 1/2\*(A-I\*B)\*hypergeom([1, n], [1+n], 1/2+1/2\*I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/n-A\*hypergeom([1, n], [1+n], 1+I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/n

**Rubi [A]**

time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3681, 3562, 70, 3680, 67}

$$\frac{(A - iB)(a + ia \tan(c + dx))^n {}_2F_1(1, n; n + 1; \frac{1}{2}(i \tan(c + dx) + 1))}{2dn} - \frac{A(a + ia \tan(c + dx))^n {}_2F_1(1, n; n + 1; i \tan(c + dx) + 1)}{dn}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] ((A - I\*B)\*Hypergeometric2F1[1, n, 1 + n, (1 + I\*Tan[c + d\*x])/2]\*(a + I\*a\*Tan[c + d\*x])^n)/(2\*d\*n) - (A\*Hypergeometric2F1[1, n, 1 + n, 1 + I\*Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*n)

**Rule 67**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[((c + d\*x)^(n + 1)/(d\*(n + 1)\*(-d/(b\*c))^m)\*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b\*c), 0])

**Rule 70**

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*(a + b\*x)/(b\*c - a\*d)], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

**Rule 3562**

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[-b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

**Rule 3680**

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3681

```
Int((((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x]))], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx &= \frac{A \int \cot(c + dx)(a - ia \tan(c + dx))(a + ia \tan(c + dx))^n dx}{a} \\ &= \frac{(aA) \text{Subst}\left(\int \frac{(a+iax)^{-1+n}}{x} dx, x, \tan(c + dx)\right)}{d} + \dots \\ &= \frac{(A - iB) {}_2F_1\left(1, n; 1 + n; \frac{1}{2}(1 + i \tan(c + dx))\right)}{2dn} \end{aligned}$$

### Mathematica [F]

time = 15.33, size = 0, normalized size = 0.00

$$\int \cot(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

```
[Out] Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

### Maple [F]

time = 0.27, size = 0, normalized size = 0.00

$$\int \cot(dx + c)(a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)`

[Out] `int(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c), x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral(((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A - B)*(2*a*e^(2*I*d*x + 2*I*c) / (e^(2*I*d*x + 2*I*c) + 1))^n / (e^(2*I*d*x + 2*I*c) - 1), x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)`

[Out] `Integral((I*a*(tan(c + d*x) - I))^n*(A + B*tan(c + d*x))*cot(c + d*x), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx) (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)



### 3.224 $\int \cot^2(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=131

$$\frac{A \cot(c+dx)(a+ia \tan(c+dx))^n}{d} + \frac{(iA+B) {}_2F_1\left(1, n; 1+n; \frac{1}{2}(1+i \tan(c+dx))\right) (a+ia \tan(c+dx))^n}{2dn}$$

[Out]  $-A \cot(dx+c) (a+I*a*\tan(dx+c))^{n/d+1/2} (I*A+B) \operatorname{hypergeom}([1, n], [1+n], 1/2+1/2*I*\tan(dx+c)) (a+I*a*\tan(dx+c))^{n/d-n} (B+I*A*n) \operatorname{hypergeom}([1, n], [1+n], 1+I*\tan(dx+c)) (a+I*a*\tan(dx+c))^{n/d/n}$

**Rubi [A]**

time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {3679, 3681, 3562, 70, 3680, 67}

$$\frac{(B+iA)(a+ia \tan(c+dx))^n {}_2F_1\left(1, n; n+1; \frac{1}{2}(i \tan(c+dx)+1)\right)}{2dn} - \frac{(B+iAn)(a+ia \tan(c+dx))^n {}_2F_1\left(1, n; n+1; i \tan(c+dx)+1\right)}{dn} - \frac{A \cot(c+dx)(a+ia \tan(c+dx))^n}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^2*(a+I*a*\operatorname{Tan}[c+d*x])^n*(A+B*\operatorname{Tan}[c+d*x]), x]$

[Out]  $-((A*\operatorname{Cot}[c+d*x]*(a+I*a*\operatorname{Tan}[c+d*x])^n)/d) + ((I*A+B)*\operatorname{Hypergeometric2F1}[1, n, 1+n, (1+I*\operatorname{Tan}[c+d*x])/2]*(a+I*a*\operatorname{Tan}[c+d*x])^n)/(2*d*n) - ((B+I*A*n)*\operatorname{Hypergeometric2F1}[1, n, 1+n, 1+I*\operatorname{Tan}[c+d*x]]*(a+I*a*\operatorname{Tan}[c+d*x])^n)/(d*n)$

Rule 67

$\operatorname{Int}[(c_+ + d_+ * x_+)^{m_+} * ((c_-) + (d_-) * (x_-))^{n_-}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{n+1} / (d*(n+1)*(-d/(b*c))^{m_+}) * \operatorname{Hypergeometric2F1}[-m, n+1, n+2, 1+d*(x/c)], x] /;$  FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b\*c), 0])

Rule 70

$\operatorname{Int}[(a_+ + b_+ * x_+)^{m_+} * ((c_-) + (d_-) * (x_-))^{n_-}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)^{n+1} * (a + b*x)^{m+1} / (b^{n+1} * (m+1)) * \operatorname{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(a + b*x)/(b*c - a*d)], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 3562

$\operatorname{Int}[(a_+ + b_+ * \tan[(c_-) + (d_-) * (x_-)])^{n_-}, x\_Symbol] \rightarrow \operatorname{Dist}[-b/d, \operatorname{Subst}[\operatorname{Int}[(a + x)^{n-1} / (a - x), x], x, b*\operatorname{Tan}[c + d*x]], x] /;$  FreeQ[{a, b,

c, d, n}, x] && EqQ[a^2 + b^2, 0]

### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

### Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3681

```
Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \cot^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^n}{d} + \frac{\int \cot}{d} \\
 &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^n}{d} + (-A) \\
 &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^n}{d} + \frac{(a + ia)}{d} \\
 &= -\frac{A \cot(c + dx)(a + ia \tan(c + dx))^n}{d} + \frac{(ia + a)}{d}
 \end{aligned}$$

**Mathematica [F]**

time = 27.58, size = 0, normalized size = 0.00

$$\int \cot^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Cot[c + d\*x]^2\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.37, size = 0, normalized size = 0.00

$$\int (\cot^2(dx + c))(a + ia \tan(dx + c))^n(A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^2, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(-((A - I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*A\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n/(e^(4\*I\*d\*x + 4\*I\*c) - 2\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*n\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^2 (A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

### 3.225 $\int \cot^3(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=185

$$\frac{(2B + iAn) \cot(c + dx)(a + ia \tan(c + dx))^n}{2d} - \frac{A \cot^2(c + dx)(a + ia \tan(c + dx))^n}{2d} - \frac{(A - iB) {}_2F_1(1, n; 1 + n; i \tan(c + dx))}{2d}$$

[Out]  $-1/2*(2*B+I*A*n)*\cot(d*x+c)*(a+I*a*\tan(d*x+c))^n/d-1/2*A*\cot(d*x+c)^2*(a+I*a*\tan(d*x+c))^n/d-1/2*(A-I*B)*\text{hypergeom}([1, n], [1+n], 1/2+1/2*I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/n-1/2*(2*I*B*n-A*(n^2-n+2))*\text{hypergeom}([1, n], [1+n], 1+I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/n$

**Rubi** [A]

time = 0.40, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {3679, 3681, 3562, 70, 3680, 67}

$$\frac{-A(n^2-n+2)+2iBn(a+ia \tan(c+dx))^n {}_2F_1(1, n; n+1; i \tan(c+dx)+1)}{2dn} - \frac{(A-iB)(a+ia \tan(c+dx))^n {}_2F_1(1, n; n+1; \frac{1}{2}(i \tan(c+dx)+1))}{2dn} - \frac{(2B+iAn) \cot(c+dx)(a+ia \tan(c+dx))^n}{2d} - \frac{A \cot^2(c+dx)(a+ia \tan(c+dx))^n}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^3*(a + I*a*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-1/2*((2*B + I*A*n)*\text{Cot}[c + d*x]*(a + I*a*\text{Tan}[c + d*x])^n)/d - (A*\text{Cot}[c + d*x]^2*(a + I*a*\text{Tan}[c + d*x])^n)/(2*d) - ((A - I*B)*\text{Hypergeometric2F1}[1, n, 1 + n, (1 + I*\text{Tan}[c + d*x])/2]*(a + I*a*\text{Tan}[c + d*x])^n)/(2*d*n) - (((2*I)*B*n - A*(2 - n + n^2))*\text{Hypergeometric2F1}[1, n, 1 + n, 1 + I*\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(2*d*n)$

**Rule 67**

$\text{Int}[(b_.*(x_))^{(m_)}*((c_.) + (d_.*(x_))^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}/(d*(n+1)*(-d/(b*c))^{(m)})*\text{Hypergeometric2F1}[-m, n+1, n+2, 1+d*(x/c)], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& (\text{IntegerQ}[m] \ || \ \text{GtQ}[-d/(b*c), 0])$

**Rule 70**

$\text{Int}[(a_.) + (b_.*(x_))^{(m_)}*((c_.) + (d_.*(x_))^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^n*((a + b*x)^{(m+1)}/(b^{(n+1)}*(m+1)))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*((a + b*x)/(b*c - a*d))], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

**Rule 3562**

$\text{Int}[(a_.) + (b_.*\tan[(c_.) + (d_.*(x_))])^{(n_)}), x\_Symbol] \rightarrow \text{Dist}[-b/d, \text{Subst}[\text{Int}[(a + x)^{(n-1)}/(a - x), x], x, b*\text{Tan}[c + d*x]], x] /; \text{FreeQ}\{a, b,$

c, d, n}, x] && EqQ[a^2 + b^2, 0]

### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

### Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3681

```
Int[(((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]))/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(
A*b + a*B)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m, x], x] - Dist[(B*c - A*
d)/(b*c + a*d), Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*Tan
[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a
*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \cot^3(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx &= -\frac{A \cot^2(c + dx)(a + ia \tan(c + dx))^n}{2d} + \frac{\int \cot^2(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{2d} \\
 &= -\frac{(2B + iAn) \cot(c + dx)(a + ia \tan(c + dx))^n}{2d} + \frac{\int \cot(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{2d} \\
 &= -\frac{(2B + iAn) \cot(c + dx)(a + ia \tan(c + dx))^n}{2d} + \frac{\int \cot(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{2d} \\
 &= -\frac{(2B + iAn) \cot(c + dx)(a + ia \tan(c + dx))^n}{2d} + \frac{\int \cot(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{2d} \\
 &= -\frac{(2B + iAn) \cot(c + dx)(a + ia \tan(c + dx))^n}{2d} + \frac{\int \cot(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{2d}
 \end{aligned}$$

**Mathematica [F]**

time = 39.62, size = 0, normalized size = 0.00

$$\int \cot^3(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Cot[c + d\*x]^3\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.58, size = 0, normalized size = 0.00

$$\int (\cot^3(dx + c))(a + ia \tan(dx + c))^n(A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^3, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="fricas")

[Out] integral((( -I\*A - B)\*e^(6\*I\*d\*x + 6\*I\*c) + (-3\*I\*A - B)\*e^(4\*I\*d\*x + 4\*I\*c) + (-3\*I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) - I\*A + B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n/(e^(6\*I\*d\*x + 6\*I\*c) - 3\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*e^(2\*I\*d\*x + 2\*I\*c) - 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*n\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^3 (A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)





```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

#### Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

#### Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q},
x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

#### Rule 3680



**Mathematica [F]**

time = 9.22, size = 0, normalized size = 0.00

$$\int \tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

```
[Out] Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

**Maple [F]**

time = 0.32, size = 0, normalized size = 0.00

$$\int \left( \tan^{\frac{5}{2}}(dx + c) \right) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

```
[Out] int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^(5/2),
x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-((A - I*B)*e^(6*I*d*x + 6*I*c) - (A - 3*I*B)*e^(4*I*d*x + 4*I*c)
- (A + 3*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2
```

$(I dx + 2Ic) + 1)^n \sqrt{(-I e^{(2I dx + 2Ic)} + I) / (e^{(2I dx + 2Ic)} + 1)} / (e^{(6I dx + 6Ic)} + 3e^{(4I dx + 4Ic)} + 3e^{(2I dx + 2Ic)} + 1), x$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^(5/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

$$3.227 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=291

$$\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)} - \frac{2(A - iB)F_1\left(\frac{1}{2}; 1 - n, 1; \frac{3}{2}; -i \tan(c + dx), i \tan(c + dx)\right)}{d(1 + 2n)(3 + 2n)}$$

[Out]  $-2*(2*I*B*n-A*(3+2*n))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d/(4*n^2+8*n+3) - 2*(A-I*B)*\text{AppellF1}(1/2, 1-n, 1, 3/2, -I*\tan(d*x+c), I*\tan(d*x+c))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d/((1+I*\tan(d*x+c))^n)+2*(2*A*n*(3+2*n)-I*B*(4*n^2+6*n+3))*\text{hypergeom}([1/2, 1-n], [3/2], -I*\tan(d*x+c))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d/(4*n^2+8*n+3)/((1+I*\tan(d*x+c))^n)+2*B*\tan(d*x+c)^{(3/2)}*(a+I*a*\tan(d*x+c))^n/d/(3+2*n)$

**Rubi [A]**

time = 0.55, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3678, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(A-iB)\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i\tan(c+dx), i\tan(c+dx)\right) + 2(2An(2n+3)-iB(4n^2+6n+3))\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n; \frac{3}{2}; -i\tan(c+dx)\right) - 2(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n + 2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+1)(2n+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^{(3/2)}*(a + I*a*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(-2*((2*I)*B*n - A*(3 + 2*n))*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(d*(1 + 2*n)*(3 + 2*n)) - (2*(A - I*B)*\text{AppellF1}[1/2, 1 - n, 1, 3/2, (-I)*\text{Tan}[c + d*x], I*\text{Tan}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(d*(1 + I*\text{Tan}[c + d*x])^n) + (2*(2*A*n*(3 + 2*n) - I*B*(3 + 6*n + 4*n^2))*\text{Hypergeometric2F1}[1/2, 1 - n, 3/2, (-I)*\text{Tan}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(d*(1 + 2*n)*(3 + 2*n)*(1 + I*\text{Tan}[c + d*x])^n) + (2*B*\text{Tan}[c + d*x]^{(3/2)}*(a + I*a*\text{Tan}[c + d*x])^n)/(d*(3 + 2*n))$

**Rule 66**

$\text{Int}[(b*x)^m*((c + d*x)^n), x\_Symbol] \rightarrow \text{Simp}[c^n*((b*x)^{m+1}/(b*(m+1)))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ (\text{IntegerQ}[n] \ || \ (\text{GtQ}[c, 0] \ \&\& \ !(\text{EqQ}[n, -2^{(-1)}] \ \&\& \ \text{EqQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[-d/(b*c), 0])))$

**Rule 68**

$\text{Int}[(b*x)^m*((c + d*x)^n), x\_Symbol] \rightarrow \text{Dist}[c^n*\text{IntPart}[n]*((c + d*x)^{\text{FracPart}[n]}/(1 + d*(x/c))^{\text{FracPart}[n]}), \text{Int}[(b*x)^m*(1 + d*(x/c))^n, x], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}$

[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)]) && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

### Rule 129

Int[((e\_.)\*(x\_))^(p\_)\*((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

### Rule 440

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

### Rule 441

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[a^IntPart[p]\*((a + b\*x^n)^FracPart[p]/(1 + b\*(x^n/a))^FracPart[p]), Int[(1 + b\*(x^n/a))^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

### Rule 3645

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(b/f), Subst[Int[(a + x)^(m - 1)\*((c + (d/b)\*x)^n/(b^2 + a\*x)), x], x, b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3678

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dis

`t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A*b + a*B, 0]`

### Rule 3682

`Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x] - Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]`

### Rubi steps

$$\begin{aligned}
 \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx &= \frac{2B \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n}{d(3 + 2n)} + \frac{2 \int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx}{d(3 + 2n)} \\
 &= -\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)} \\
 &= -\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)} \\
 &= -\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)} \\
 &= -\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)} \\
 &= -\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)} \\
 &= -\frac{2(2iBn - A(3 + 2n)) \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n)}
 \end{aligned}$$

### Mathematica [F]

time = 10.54, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$



Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Tan[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.33, size = 0, normalized size = 0.00

$$\int \left( \tan^{\frac{3}{2}}(dx + c) \right) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^(3/2), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="fricas")

[Out] integral((( -I\*A - B)\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*B\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A - B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))/(e^(4\*I\*d\*x + 4\*I\*c) + 2\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

### 3.228 $\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=215

$$\frac{2B\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n}{d(1 + 2n)} - \frac{2(iA + B)F_1\left(\frac{1}{2}; 1 - n, 1; \frac{3}{2}; -i \tan(c + dx), i \tan(c + dx)\right) (1 + i \tan(c + dx))^n}{d}$$

```
[Out] 2*B*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/(1+2*n)-2*(I*A+B)*AppellF1(1/2,
1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n
/d/((1+I*tan(d*x+c))^n)+2*(2*B*n+I*A*(1+2*n))*hypergeom([1/2, 1-n],[3/2],-I
*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/((1+I*tan(d*x+
c))^n)
```

**Rubi [A]**

time = 0.33, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3678, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(B+iA)\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i\tan(c+dx), i\tan(c+dx)\right)}{d} + \frac{2(2Bn+iA(2n+1))\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i\tan(c+dx)\right)}{d(2n+1)} + \frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)) - (2*(I*A +
B)*AppellF1[1/2, 1 - n, 1, 3/2, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[Ta
n[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) + (2*(2*B*
n + I*A*(1 + 2*n))*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sq
rt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)*(1 + I*Tan[c + d*x]
)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[c^n*((b*x)
)^(m + 1)/(b*(m + 1))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
&& !(EqQ[n, -2^(-1)]) && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0]))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)]) && EqQ[c^2 - d^2, 0])) || !RationalQ[n]
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n (A+B \tan(c+dx)) dx &= \frac{2B \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n}{d(1+2n)} \\
&= \frac{2B \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n}{d(1+2n)} \\
&= \frac{2B \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n}{d(1+2n)} \\
&= \frac{2B \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n}{d(1+2n)} \\
&= \frac{2B \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n}{d(1+2n)} \\
&= \frac{2B \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n}{d(1+2n)}
\end{aligned}$$

Mathematica [F]

time = 11.90, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^n (A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

[Out] Integrate[Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.34, size = 0, normalized size = 0.00

$$\int \left( \sqrt{\tan(dx + c)} \right) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*sqrt(tan(d\*x + c)), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="fricas")

[Out] integral(((A - I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c) / (e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I) / (e^(2\*I\*d\*x + 2\*I\*c) + 1)) / (e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)), x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I)\*\*n\*(A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*sqrt(tan(d\*x + c)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\tan(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

$$3.229 \quad \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=158

$$\frac{2(A-iB)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right) (1+i \tan(c+dx))^{-n} \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))}{d}$$

[Out] 2\*(A-I\*B)\*AppellF1(1/2,1-n,1,3/2,-I\*tan(d\*x+c),I\*tan(d\*x+c))\*tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n/d/((1+I\*tan(d\*x+c))^n)+2\*I\*B\*hypergeom([1/2, 1-n],[3/2],-I\*tan(d\*x+c))\*tan(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^n/d/((1+I\*tan(d\*x+c))^n)

**Rubi [A]**

time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(A-iB)\sqrt{\tan(c+dx)}(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d} + \frac{2iB\sqrt{\tan(c+dx)}(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]],x]

[Out] (2\*(A - I\*B)\*AppellF1[1/2, 1 - n, 1, 3/2, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*(1 + I\*Tan[c + d\*x])^n) + ((2\*I)\*B\*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)\*Tan[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*(1 + I\*Tan[c + d\*x])^n)

**Rule 66**

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Simp[c^n\*((b\*x)^(m+1)/(b\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

**Rule 68**

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

**Rule 129**

Int[((e\_)\*(x\_))^(p\_)\*((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p+1)-1)



$(a + b(x^k/e))^m (c + d(x^k/e))^n, x, (e*x)^{1/k}, x]$  /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

#### Rule 440

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 441

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[a^IntPart[p]\*((a + b\*x^n)^FracPart[p]/(1 + b\*(x^n/a))^FracPart[p]), Int[(1 + b\*(x^n/a))^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 3645

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[a\*(b/f), Subst[Int[(a + x)^(m - 1)\*((c + (d/b)\*x)^n/(b^2 + a\*x)], x], x, b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

#### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= - \left( (-A + iB) \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\tan(c + dx)}} dx \right) + \frac{(iB) \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\tan(c + dx)}} dx}{d} \\
&= \frac{(iaB) \text{Subst} \left( \int \frac{(a+iax)^{-1+n}}{\sqrt{x}} dx, x, \tan(c + dx) \right)}{d} + \frac{(a^2(iA + iB)) \text{Subst} \left( \int \frac{(a+iax)^{-1+n}}{\sqrt{x}} dx, x, \tan(c + dx) \right)}{d} \\
&= - \frac{(2a^3(A - iB)) \text{Subst} \left( \int \frac{(a+iax^2)^{-1+n}}{-a^2+ia^2x^2} dx, x, \sqrt{\tan(c + dx)} \right)}{d} \\
&= \frac{2iB {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; -i \tan(c + dx)\right) (1 + i \tan(c + dx))^{-n}}{d} \\
&= \frac{2(A - iB) F_1\left(\frac{1}{2}; 1 - n, 1; \frac{3}{2}; -i \tan(c + dx), i \tan(c + dx)\right)}{d}
\end{aligned}$$

**Mathematica [F]**

time = 13.07, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
```

```
[Out] Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
```

**Maple [F]**

time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(dx + c))^n (A + B \tan(dx + c))}{\sqrt{\tan(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2), x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2), x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A - B)*(2*a*e^(2*I*d*x + 2*I*c) / (e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))/(e^(2*I*d*x + 2*I*c) - 1), x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^n (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))^n*(A + B*tan(c + d*x))/sqrt(tan(c + d*x)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="giac")
```

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/sqrt(tan(d\*x + c)),  
x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/tan(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/tan(c + d\*x)^(1/2), x)

$$3.230 \quad \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=194

$$-\frac{2A(a+ia \tan(c+dx))^n}{d\sqrt{\tan(c+dx)}} + \frac{2(iA+B)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right) (1+i \tan(c+dx))^{-n}}{d}$$

[Out]  $-2*A*(a+I*a*\tan(d*x+c))^n/d/\tan(d*x+c)^{(1/2)}+2*(I*A+B)*\text{AppellF1}(1/2, 1-n, 1, 3/2, -I*\tan(d*x+c), I*\tan(d*x+c))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d/((1+I*\tan(d*x+c))^n)-2*I*A*(1-2*n)*\text{hypergeom}([1/2, 1-n], [3/2], -I*\tan(d*x+c))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d/((1+I*\tan(d*x+c))^n)$

**Rubi [A]**

time = 0.33, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3679, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(B+iA)\sqrt{\tan(c+dx)}(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d} - \frac{2iA(1-2n)\sqrt{\tan(c+dx)}(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right)}{d} - \frac{2A(a+ia \tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x])/ \text{Tan}[c + d*x]^{(3/2)}, x]$

[Out]  $(-2*A*(a + I*a*\text{Tan}[c + d*x])^n)/(d*\text{Sqrt}[\text{Tan}[c + d*x]]) + (2*(I*A + B)*\text{AppellF1}[1/2, 1 - n, 1, 3/2, (-I)*\text{Tan}[c + d*x], I*\text{Tan}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])*(a + I*a*\text{Tan}[c + d*x])^n/(d*(1 + I*\text{Tan}[c + d*x])^n) - ((2*I)*A*(1 - 2*n))*\text{Hypergeometric2F1}[1/2, 1 - n, 3/2, (-I)*\text{Tan}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n/(d*(1 + I*\text{Tan}[c + d*x])^n)$

**Rule 66**

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[c^n*((b*x)^{(m+1})/(b*(m+1)))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ (\text{IntegerQ}[n] \ || \ (\text{GtQ}[c, 0] \ \&\& \ !(\text{EqQ}[n, -2^{(-1)}] \ \&\& \ \text{EqQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[-d/(b*c), 0])))$

**Rule 68**

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[c^{\text{IntPart}[n]}*((c + d*x)^{\text{FracPart}[n]}/(1 + d*(x/c))^{\text{FracPart}[n]}), \text{Int}[(b*x)^m*(1 + d*(x/c))^n, x], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ !\text{GtQ}[c, 0] \ \&\& \ !\text{GtQ}[-d/(b*c), 0] \ \&\& \ ((\text{RationalQ}[m] \ \&\& \ !(\text{EqQ}[n, -2^{(-1)}] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]))) \ || \ !\text{RationalQ}[n]$

**Rule 129**

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

#### Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

#### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

## Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2A(a + ia \tan(c + dx))^n}{d \sqrt{\tan(c + dx)}} + \frac{2 \int \frac{(a + ia \tan(c + dx))^n (\frac{1}{2}a(B + 2iA))}{\sqrt{\tan(c + dx)}} dx}{a} \\
 &= -\frac{2A(a + ia \tan(c + dx))^n}{d \sqrt{\tan(c + dx)}} + (iA + B) \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\tan(c + dx)}} dx \\
 &= -\frac{2A(a + ia \tan(c + dx))^n}{d \sqrt{\tan(c + dx)}} - \frac{(a^2(A - iB)) \text{Subst}\left(\int \frac{1}{\sqrt{1 - u^2}} du, \frac{a + ia \tan(c + dx)}{\sqrt{\tan(c + dx)}}\right)}{\sqrt{\tan(c + dx)}} \\
 &= -\frac{2A(a + ia \tan(c + dx))^n}{d \sqrt{\tan(c + dx)}} - \frac{(2a^3(iA + B)) \text{Subst}\left(\int \frac{1}{\sqrt{1 - u^2}} du, \frac{a + ia \tan(c + dx)}{\sqrt{\tan(c + dx)}}\right)}{\sqrt{\tan(c + dx)}} \\
 &= -\frac{2A(a + ia \tan(c + dx))^n}{d \sqrt{\tan(c + dx)}} - \frac{2iA(1 - 2n) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; \frac{a + ia \tan(c + dx)}{\sqrt{\tan(c + dx)}}\right)}{\sqrt{\tan(c + dx)}} \\
 &= -\frac{2A(a + ia \tan(c + dx))^n}{d \sqrt{\tan(c + dx)}} + \frac{2(iA + B)F_1\left(\frac{1}{2}; 1 - n, 1; \frac{3}{2}; \frac{a + ia \tan(c + dx)}{\sqrt{\tan(c + dx)}}\right)}{\sqrt{\tan(c + dx)}}
 \end{aligned}$$

**Mathematica** [F]

time = 5.99, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification is not applicable to the result.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

**Maple [F]**

time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(dx + c))^n (A + B \tan(dx + c))}{\tan(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x)

[Out] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x)

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] integral(-((A - I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*A\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))/(e^(4\*I\*d\*x + 4\*I\*c) - 2\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^n (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))^n\*(A + B\*tan(c + d\*x))/tan(c + d\*x)\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/tan(d\*x + c)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^n}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/tan(c + d\*x)^(3/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/tan(c + d\*x)^(3/2), x)

$$3.231 \quad \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=247

$$\frac{2A(a+ia \tan(c+dx))^n}{3d \tan^{\frac{3}{2}}(c+dx)} - \frac{2(3B+2iAn)(a+ia \tan(c+dx))^n}{3d \sqrt{\tan(c+dx)}} - \frac{2(A-iB)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx)\right)}{3d \tan^{\frac{3}{2}}(c+dx)}$$

[Out]  $-2/3*(3*B+2*I*A*n)*(a+I*a*\tan(d*x+c))^n/d/\tan(d*x+c)^{(1/2)}-2*(A-I*B)*\text{Appell}$   
 $F_1(1/2, 1-n, 1, 3/2, -I*\tan(d*x+c), I*\tan(d*x+c))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*$   
 $x+c))^n/d/((1+I*\tan(d*x+c))^n)-2/3*(1-2*n)*(3*I*B-2*A*n)*\text{hypergeom}([1/2, 1-$   
 $n], [3/2], -I*\tan(d*x+c))*\tan(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d/((1+I*\tan(d$   
 $*x+c))^n)-2/3*A*(a+I*a*\tan(d*x+c))^n/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.48, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3679, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(A-iB)\sqrt{\tan(c+dx)}(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{3d} - \frac{2(1-2n)(-2An+3iB)\sqrt{\tan(c+dx)}(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n; \frac{3}{2}; -i \tan(c+dx)\right)}{3d} - \frac{2(3B+2iAn)(a+ia \tan(c+dx))^n}{3d \sqrt{\tan(c+dx)}} - \frac{2A(a+ia \tan(c+dx))^n}{3d \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out]  $(-2*A*(a+I*a*\text{Tan}[c+d*x])^n)/(3*d*\text{Tan}[c+d*x]^{(3/2)}) - (2*(3*B+(2*I)*$   
 $A*n)*(a+I*a*\text{Tan}[c+d*x])^n)/(3*d*\text{Sqrt}[\text{Tan}[c+d*x]]) - (2*(A-I*B)*\text{Appell}$   
 $F_1[1/2, 1-n, 1, 3/2, (-I)*\text{Tan}[c+d*x], I*\text{Tan}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*$   
 $x]]*(a+I*a*\text{Tan}[c+d*x])^n)/(d*(1+I*\text{Tan}[c+d*x])^n) - (2*(1-2*n)*((3$   
 $*I)*B-2*A*n)*\text{Hypergeometric2F1}[1/2, 1-n, 3/2, (-I)*\text{Tan}[c+d*x]]*\text{Sqrt}[\text{T}$   
 $\text{an}[c+d*x]]*(a+I*a*\text{Tan}[c+d*x])^n)/(3*d*(1+I*\text{Tan}[c+d*x])^n)$

**Rule 66**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[c^n\*((b\*x)^(m+1)/(b\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

**Rule 68**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[c^IntPart[n]\*((c+d\*x)^FracPart[n]/(1+d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1+d\*(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

$a^2 + b^2, 0]$  && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2 \int \frac{(a + ia \tan(c + dx))^n (\frac{1}{2}a(3B + 2iA) \tan^{\frac{3}{2}}(c + dx))}{3a} dx}{3a} \\ &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(3B + 2iAn)(a + ia \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(3B + 2iAn)(a + ia \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(3B + 2iAn)(a + ia \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(3B + 2iAn)(a + ia \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(3B + 2iAn)(a + ia \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2A(a + ia \tan(c + dx))^n}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(3B + 2iAn)(a + ia \tan(c + dx))}{3d \sqrt{\tan(c + dx)}} \end{aligned}$$

### Mathematica [F]

time = 7.67, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification is not applicable to the result.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

**Maple** [F]

time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(dx + c))^n (A + B \tan(dx + c))}{\tan(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2), x)

[Out] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/tan(d\*x + c)^(5/2), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2), x, algorithm="fricas")

[Out] integral((( -I\*A - B)\*e^(6\*I\*d\*x + 6\*I\*c) + (-3\*I\*A - B)\*e^(4\*I\*d\*x + 4\*I\*c) + (-3\*I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) - I\*A + B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((-I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))/(e^(6\*I\*d\*x + 6\*I\*c) - 3\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*e^(2\*I\*d\*x + 2\*I\*c) - 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^n (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*n\*(A + B\*tan(c + d\*x))/tan(c + d\*x)\*\*(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))~n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/tan(d\*x + c)^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^n}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/tan(c + d\*x)^(5/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/tan(c + d\*x)^(5/2), x)

### 3.232 $\int \tan^2(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=87

$$-((aA-bB)x) + \frac{(Ab+aB)\log(\cos(c+dx))}{d} + \frac{(aA-bB)\tan(c+dx)}{d} + \frac{(Ab+aB)\tan^2(c+dx)}{2d} + \frac{bB\tan^3(c+dx)}{3d}$$

[Out]  $-(A*a-B*b)*x+(A*b+B*a)*\ln(\cos(d*x+c))/d+(A*a-B*b)*\tan(d*x+c)/d+1/2*(A*b+B*a)*\tan(d*x+c)^2/d+1/3*b*B*\tan(d*x+c)^3/d$

Rubi [A]

time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3673, 3609, 3606, 3556}

$$\frac{(aB+Ab)\tan^2(c+dx)}{2d} + \frac{(aA-bB)\tan(c+dx)}{d} + \frac{(aB+Ab)\log(\cos(c+dx))}{d} - x(aA-bB) + \frac{bB\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left((a*A - b*B)*x\right) + \left((A*b + a*B)*\text{Log}[\text{Cos}[c + d*x]]\right)/d + \left((a*A - b*B)*\text{Tan}[c + d*x]\right)/d + \left((A*b + a*B)*\text{Tan}[c + d*x]^2\right)/(2*d) + \left(b*B*\text{Tan}[c + d*x]^3\right)/(3*d)$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3606

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)*\left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3609

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)}*\left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \tan^2(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{bB \tan^3(c + dx)}{3d} + \int \tan^2(c + dx)(aA - bB + \\ &= \frac{(Ab + aB) \tan^2(c + dx)}{2d} + \frac{bB \tan^3(c + dx)}{3d} + \\ &= -(aA - bB)x + \frac{(aA - bB) \tan(c + dx)}{d} + \frac{(Ab + aB) \log(\cos(c + dx))}{d} \\ &= -(aA - bB)x + \frac{(Ab + aB) \log(\cos(c + dx))}{d} + \end{aligned}$$

### Mathematica [A]

time = 0.40, size = 86, normalized size = 0.99

$$\frac{(-6aA + 6bB)\text{ArcTan}(\tan(c + dx)) + 6(Ab + aB) \log(\cos(c + dx)) + 6(aA - bB) \tan(c + dx) + 3(Ab + aB) \tan^2(c + dx) + 2bB \tan^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((-6*a*A + 6*b*B)*ArcTan[Tan[c + d*x]] + 6*(A*b + a*B)*Log[Cos[c + d*x]] +
6*(a*A - b*B)*Tan[c + d*x] + 3*(A*b + a*B)*Tan[c + d*x]^2 + 2*b*B*Tan[c + d
*x]^3)/(6*d)
```

### Maple [A]

time = 0.06, size = 99, normalized size = 1.14

method	result
norman	$(-aA + Bb)x + \frac{(aA - Bb) \tan(dx + c)}{d} + \frac{(Ab + aB) \tan^2(dx + c)}{2d} + \frac{bB \tan^3(dx + c)}{3d} - \frac{(Ab + aB) \ln(1 + \tan^2(dx + c))}{2d}$
derivativedivides	$\frac{bB \tan^3(dx + c)}{3} + \frac{Ab \tan^2(dx + c)}{2} + \frac{Ba \tan^2(dx + c)}{2} + aA \tan(dx + c) - bB \tan(dx + c) + \frac{(-Ab - aB) \ln(1 + \tan^2(dx + c))}{2} + (-aA)$
default	$\frac{bB \tan^3(dx + c)}{3} + \frac{Ab \tan^2(dx + c)}{2} + \frac{Ba \tan^2(dx + c)}{2} + aA \tan(dx + c) - bB \tan(dx + c) + \frac{(-Ab - aB) \ln(1 + \tan^2(dx + c))}{2} + (-aA)$
risch	$-iAbx - iBax - Aax + Bbx - \frac{2iAbc}{d} - \frac{2iaBc}{d} + \frac{2i(-3iAb e^{4i(dx+c)} - 3iBa e^{4i(dx+c)} + 3Aa e^{4i(dx+c)} - 6$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*B*b*tan(d*x+c)^3+1/2*A*b*tan(d*x+c)^2+1/2*B*a*tan(d*x+c)^2+a*A*tan
(d*x+c)-b*B*tan(d*x+c)+1/2*(-A*b-B*a)*ln(1+tan(d*x+c)^2)+(-A*a+B*b)*arctan(
tan(d*x+c)))
```

**Maxima** [A]

time = 0.57, size = 86, normalized size = 0.99

$$\frac{2Bb \tan(dx+c)^3 + 3(Ba+Ab) \tan(dx+c)^2 - 6(Aa-Bb)(dx+c) - 3(Ba+Ab) \log(\tan(dx+c)^2+1) + 6(Aa-Bb) \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(2*B*b*tan(d*x + c)^3 + 3*(B*a + A*b)*tan(d*x + c)^2 - 6*(A*a - B*b)*(d
*x + c) - 3*(B*a + A*b)*log(tan(d*x + c)^2 + 1) + 6*(A*a - B*b)*tan(d*x + c
))/d
```

**Fricas** [A]

time = 0.71, size = 85, normalized size = 0.98

$$\frac{2Bb \tan(dx+c)^3 - 6(Aa-Bb)dx + 3(Ba+Ab) \tan(dx+c)^2 + 3(Ba+Ab) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 6(Aa-Bb) \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*B*b*tan(d*x + c)^3 - 6*(A*a - B*b)*d*x + 3*(B*a + A*b)*tan(d*x + c)^
2 + 3*(B*a + A*b)*log(1/(tan(d*x + c)^2 + 1)) + 6*(A*a - B*b)*tan(d*x + c)
)/d
```

**Sympy** [A]

time = 0.11, size = 136, normalized size = 1.56

$$\begin{cases} -Aax + \frac{Aa \tan(c+dx)}{d} - \frac{Ab \log(\tan^2(c+dx)+1)}{2d} + \frac{Ab \tan^2(c+dx)}{2d} - \frac{Ba \log(\tan^2(c+dx)+1)}{2d} + \frac{Ba \tan^2(c+dx)}{2d} + Bbx + \frac{Bb \tan^3(c+dx)}{3d} - \frac{Bb \tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(A+B \tan(c))(a+b \tan(c)) \tan^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

```
[Out] Piecewise((-A*a*x + A*a*tan(c + d*x)/d - A*b*log(tan(c + d*x)**2 + 1)/(2*d)
+ A*b*tan(c + d*x)**2/(2*d) - B*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*tan
```

$(c + dx)**2/(2*d) + B*b*x + B*b*tan(c + dx)**3/(3*d) - B*b*tan(c + dx)/d$   
, Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*tan(c)\*\*2, True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 1017 vs.  $2(83) = 166$ .

time = 0.97, size = 1017, normalized size = 11.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(a+b\*tan(dx+c))\*(A+B\*tan(dx+c)),x, algorithm="giac")

[Out] 
$$-1/6*(6*A*a*d*x*tan(dx)^3*tan(c)^3 - 6*B*b*d*x*tan(dx)^3*tan(c)^3 - 3*B*a*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1))*tan(dx)^3*tan(c)^3 - 3*A*b*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1))*tan(dx)^3*tan(c)^3 - 18*A*a*d*x*tan(dx)^2*tan(c)^2 + 18*B*b*d*x*tan(dx)^2*tan(c)^2 - 3*B*a*tan(dx)^3*tan(c)^3 - 3*A*b*tan(dx)^3*tan(c)^3 + 9*B*a*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1))*tan(dx)^2*tan(c)^2 + 9*A*b*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1))*tan(dx)^2*tan(c)^2 + 6*A*a*tan(dx)^3*tan(c)^2 - 6*B*b*tan(dx)^3*tan(c)^2 + 6*A*a*tan(dx)^2*tan(c)^3 - 6*B*b*tan(dx)^2*tan(c)^3 + 18*A*a*d*x*tan(dx)*tan(c) - 18*B*b*d*x*tan(dx)*tan(c) - 3*B*a*tan(dx)^3*tan(c) - 3*A*b*tan(dx)^3*tan(c) + 3*B*a*tan(dx)^2*tan(c)^2 + 3*A*b*tan(dx)^2*tan(c)^2 - 3*B*a*tan(dx)*tan(c)^3 - 3*A*b*tan(dx)*tan(c)^3 + 2*B*b*tan(dx)^3 - 9*B*a*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1))*tan(dx)*tan(c) - 9*A*b*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1))*tan(dx)*tan(c) - 12*A*a*tan(dx)^2*tan(c) + 18*B*b*tan(dx)^2*tan(c) - 12*A*a*tan(dx)*tan(c)^2 + 18*B*b*tan(dx)*tan(c)^2 + 2*B*b*tan(c)^3 - 6*A*a*d*x + 6*B*b*d*x + 3*B*a*tan(dx)^2 + 3*A*b*tan(dx)^2 - 3*B*a*tan(dx)*tan(c) - 3*A*b*tan(dx)*tan(c) + 3*B*a*tan(c)^2 + 3*A*b*tan(c)^2 + 3*B*a*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1)) + 3*A*b*log(4*(tan(dx)^4*tan(c)^2 - 2*tan(dx)^3*tan(c) + tan(dx)^2*tan(c)^2 + tan(dx)^2 - 2*tan(dx)*tan(c) + 1)/(tan(c)^2 + 1)) + 6*A*a*tan(dx) - 6*B*b*tan(dx) + 6*A*a*tan(c) - 6*B*b*tan(c) + 3*B*a + 3*A*b)/(d*tan(dx)^3*tan(c)^3 - 3*d*tan(dx)^2*tan(c)^2 + 3*d*tan(dx)*tan(c) - d)$$

**Mupad [B]**

time = 6.20, size = 84, normalized size = 0.97

$$\frac{\tan(c + dx) (Aa - Bb) - \ln(\tan(c + dx)^2 + 1) \left(\frac{Ab}{2} + \frac{Ba}{2}\right) + \tan(c + dx)^2 \left(\frac{Ab}{2} + \frac{Ba}{2}\right) - dx (Aa - Bb) + \frac{Bb \tan(c + dx)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

```
[Out] (tan(c + d*x)*(A*a - B*b) - log(tan(c + d*x)^2 + 1)*((A*b)/2 + (B*a)/2) + t  
an(c + d*x)^2*((A*b)/2 + (B*a)/2) - d*x*(A*a - B*b) + (B*b*tan(c + d*x)^3)/  
3)/d
```

### 3.233 $\int \tan(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=65

$$-((Ab + aB)x) - \frac{(aA - bB) \log(\cos(c + dx))}{d} + \frac{(Ab + aB) \tan(c + dx)}{d} + \frac{bB \tan^2(c + dx)}{2d}$$

[Out]  $-(A*b+B*a)*x-(A*a-B*b)*\ln(\cos(d*x+c))/d+(A*b+B*a)*\tan(d*x+c)/d+1/2*b*B*\tan(d*x+c)^2/d$

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3673, 3606, 3556}

$$\frac{(aB + Ab) \tan(c + dx)}{d} - \frac{(aA - bB) \log(\cos(c + dx))}{d} - x(aB + Ab) + \frac{bB \tan^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]*(a + b*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left(\left(A*b + a*B\right)*x\right) - \left(\left(a*A - b*B\right)*\text{Log}[\text{Cos}[c + d*x]]\right)/d + \left(\left(A*b + a*B\right)*\text{Tan}[c + d*x]\right)/d + \left(b*B*\text{Tan}[c + d*x]^2\right)/\left(2*d\right)$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3606

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)*\left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3673

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)}*\left((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Simp}[B*d*\left((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1))\right), x] + \text{Int}[\left(a + b*\text{Tan}[e + f*x]\right)^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \tan(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx &= \frac{bB \tan^2(c+dx)}{2d} + \int \tan(c+dx)(aA-bB + \\ &= -(Ab+aB)x + \frac{(Ab+aB) \tan(c+dx)}{d} + \frac{bB \tan^2(c+dx)}{2d} \\ &= -(Ab+aB)x - \frac{(aA-bB) \log(\cos(c+dx))}{d} + \end{aligned}$$

**Mathematica [A]**

time = 0.21, size = 67, normalized size = 1.03

$$\frac{-2(Ab+aB)\text{ArcTan}(\tan(c+dx)) + 2(-aA+bB)\log(\cos(c+dx)) + 2(Ab+aB)\tan(c+dx) + bB \tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

`[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]), x]``[Out] (-2*(A*b + a*B)*ArcTan[Tan[c + d*x]] + 2*(-(a*A) + b*B)*Log[Cos[c + d*x]] + 2*(A*b + a*B)*Tan[c + d*x] + b*B*Tan[c + d*x]^2)/(2*d)`**Maple [A]**

time = 0.05, size = 74, normalized size = 1.14

method	result
norman	$(-Ab - aB)x + \frac{(Ab+aB)\tan(dx+c)}{d} + \frac{bB(\tan^2(dx+c))}{2d} + \frac{(aA-Bb)\ln(1+\tan^2(dx+c))}{2d}$
derivativedivides	$\frac{\frac{bB(\tan^2(dx+c))}{2} + A \tan(dx+c)b + B \tan(dx+c)a + \frac{(aA-Bb)\ln(1+\tan^2(dx+c))}{2}}{d} + (-Ab-aB) \arctan(\tan(dx+c))$
default	$\frac{\frac{bB(\tan^2(dx+c))}{2} + A \tan(dx+c)b + B \tan(dx+c)a + \frac{(aA-Bb)\ln(1+\tan^2(dx+c))}{2}}{d} + (-Ab-aB) \arctan(\tan(dx+c))$
risch	$-Abx - Bax + iAax - iBbx + \frac{2iaAc}{d} - \frac{2iBbc}{d} + \frac{2i(-iBbe^{2i(dx+c)} + be^{2i(dx+c)}A + aBe^{2i(dx+c)} + Ab)}{d(e^{2i(dx+c)} + 1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)), x, method=_RETURNVERBOSE)``[Out] 1/d*(1/2*B*tan(d*x+c)^2*b+A*tan(d*x+c)*b+B*tan(d*x+c)*a+1/2*(A*a-B*b)*ln(1+tan(d*x+c)^2)+(-A*b-B*a)*arctan(tan(d*x+c)))`**Maxima [A]**

time = 0.53, size = 66, normalized size = 1.02

$$\frac{Bb \tan(dx+c)^2 - 2(Ba+Ab)(dx+c) + (Aa-Bb) \log(\tan(dx+c)^2 + 1) + 2(Ba+Ab) \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(B*b*\tan(d*x + c)^2 - 2*(B*a + A*b)*(d*x + c) + (A*a - B*b)*\log(\tan(d*x + c)^2 + 1) + 2*(B*a + A*b)*\tan(d*x + c))/d$

**Fricas** [A]

time = 0.65, size = 66, normalized size = 1.02

$$\frac{Bb \tan(dx + c)^2 - 2(Ba + Ab)dx - (Aa - Bb) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2(Ba + Ab) \tan(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(B*b*\tan(d*x + c)^2 - 2*(B*a + A*b)*d*x - (A*a - B*b)*\log(1/(\tan(d*x + c)^2 + 1)) + 2*(B*a + A*b)*\tan(d*x + c))/d$

**Sympy** [A]

time = 0.09, size = 104, normalized size = 1.60

$$\begin{cases} \frac{Aa \log(\tan^2(c+dx)+1)}{2d} - Abx + \frac{Ab \tan(c+dx)}{d} - Bax + \frac{Ba \tan(c+dx)}{d} - \frac{Bb \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb \tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A + B \tan(c))(a + b \tan(c)) \tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise(((A\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*b\*x + A\*b\*tan(c + d\*x)/d - B\*a\*x + B\*a\*tan(c + d\*x)/d - B\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*tan(c + d\*x)\*\*2/(2\*d), Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*tan(c), True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(63) = 126.

time = 0.63, size = 616, normalized size = 9.48

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/2*(2*B*a*d*x*\tan(d*x)^2*\tan(c)^2 + 2*A*b*d*x*\tan(d*x)^2*\tan(c)^2 + A*a*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan$

```

(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - B*b*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 4*B
*a*d*x*tan(d*x)*tan(c) - 4*A*b*d*x*tan(d*x)*tan(c) - B*b*tan(d*x)^2*tan(c)^
2 - 2*A*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)
+ 2*B*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)
+ 2*B*a*tan(d*x)^2*tan(c) + 2*A*b*tan(d*x)^2*tan(c) + 2*B*a*tan(d*x)*tan(c)
^2 + 2*A*b*tan(d*x)*tan(c)^2 + 2*B*a*d*x + 2*A*b*d*x - B*b*tan(d*x)^2 - B*b
*tan(c)^2 + A*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - B*b*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 2*B*a*tan(d*x) - 2*A*b*tan
(d*x) - 2*B*a*tan(c) - 2*A*b*tan(c) - B*b)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan
(d*x)*tan(c) + d)

```

**Mupad [B]**

time = 6.17, size = 63, normalized size = 0.97

$$\frac{\tan(c + dx) (Ab + Ba) + \ln(\tan(c + dx)^2 + 1) \left(\frac{Aa}{2} - \frac{Bb}{2}\right) - dx (Ab + Ba) + \frac{Bb \tan(c + dx)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] (tan(c + d\*x)\*(A\*b + B\*a) + log(tan(c + d\*x)^2 + 1)\*((A\*a)/2 - (B\*b)/2) - d\*x\*(A\*b + B\*a) + (B\*b\*tan(c + d\*x)^2)/2)/d

### 3.234 $\int (a + b \tan(c + dx))(A + B \tan(c + dx)) dx$

Optimal. Leaf size=42

$$(aA - bB)x - \frac{(Ab + aB) \log(\cos(c + dx))}{d} + \frac{bB \tan(c + dx)}{d}$$

[Out] (A\*a-B\*b)\*x-(A\*b+B\*a)\*ln(cos(d\*x+c))/d+b\*B\*tan(d\*x+c)/d

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {3606, 3556}

$$-\frac{(aB + Ab) \log(\cos(c + dx))}{d} + x(aA - bB) + \frac{bB \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] (a\*A - b\*B)\*x - ((A\*b + a\*B)\*Log[Cos[c + d\*x]])/d + (b\*B\*Tan[c + d\*x])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rubi steps

$$\begin{aligned} \int (a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= (aA - bB)x + \frac{bB \tan(c + dx)}{d} + (Ab + aB) \int \tan(c + dx) dx \\ &= (aA - bB)x - \frac{(Ab + aB) \log(\cos(c + dx))}{d} + \frac{bB \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 59, normalized size = 1.40

$$aAx - \frac{bBArcTan(\tan(c + dx))}{d} - \frac{Ab \log(\cos(c + dx))}{d} - \frac{aB \log(\cos(c + dx))}{d} + \frac{bB \tan(c + dx)}{d}$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] a\*A\*x - (b\*B\*ArcTan[Tan[c + d\*x]])/d - (A\*b\*Log[Cos[c + d\*x]])/d - (a\*B\*Log[Cos[c + d\*x]])/d + (b\*B\*Tan[c + d\*x])/d

**Maple** [A]

time = 0.03, size = 51, normalized size = 1.21

method	result
norman	$(aA - Bb)x + \frac{bB \tan(dx+c)}{d} + \frac{(Ab+aB) \ln(1+\tan^2(dx+c))}{2d}$
derivativedivides	$\frac{bB \tan(dx+c) + \frac{(Ab+aB) \ln(1+\tan^2(dx+c))}{2} + (aA-Bb) \arctan(\tan(dx+c))}{d}$
default	$\frac{bB \tan(dx+c) + \frac{(Ab+aB) \ln(1+\tan^2(dx+c))}{2} + (aA-Bb) \arctan(\tan(dx+c))}{d}$
risch	$iAbx + iBax + Aax - Bbx + \frac{2iAbc}{d} + \frac{2iaBc}{d} + \frac{2iBb}{d(e^{2i(dx+c)}+1)} - \frac{\ln(e^{2i(dx+c)}+1)Ab}{d} - \frac{a \ln(e^{2i(dx+c)}+1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(b\*B\*tan(d\*x+c)+1/2\*(A\*b+B\*a)\*ln(1+tan(d\*x+c)^2)+(A\*a-B\*b)\*arctan(tan(d\*x+c)))

**Maxima** [A]

time = 0.56, size = 50, normalized size = 1.19

$$\frac{2Bb \tan(dx+c) + 2(Aa - Bb)(dx+c) + (Ba + Ab) \log(\tan(dx+c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*(2\*B\*b\*tan(d\*x + c) + 2\*(A\*a - B\*b)\*(d\*x + c) + (B\*a + A\*b)\*log(tan(d\*x + c)^2 + 1))/d

**Fricas** [A]

time = 0.52, size = 50, normalized size = 1.19

$$\frac{2(Aa - Bb)dx + 2Bb \tan(dx+c) - (Ba + Ab) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $1/2*(2*(A*a - B*b)*d*x + 2*B*b*\tan(d*x + c) - (B*a + A*b)*\log(1/(\tan(d*x + c)^2 + 1)))/d$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs.  $2(36) = 72$ .

time = 0.07, size = 73, normalized size = 1.74

$$\begin{cases} Aax + \frac{Ab \log(\tan^2(c+dx)+1)}{2d} + \frac{Ba \log(\tan^2(c+dx)+1)}{2d} - Bbx + \frac{Bb \tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B \tan(c))(a + b \tan(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((A*a*x + A*b*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x)**2 + 1)/(2*d) - B*b*x + B*b*tan(c + d*x)/d, Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c)), True))`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs.  $2(42) = 84$ .

time = 0.52, size = 329, normalized size = 7.83

$$\frac{2Aa \tan(d) \tan(c) - 2Bb \tan(d) \tan(c) - B \log\left(\frac{(\tan(d) \tan(c) - 1) \sqrt{1 + \tan^2(d) \tan^2(c)}}{\tan(d) \tan(c) - 1}\right) \tan(d) \tan(c) - Ab \log\left(\frac{(\tan(d) \tan(c) - 1) \sqrt{1 + \tan^2(d) \tan^2(c)}}{\tan(d) \tan(c) - 1}\right) \tan(d) \tan(c) - 2Aa + 2Bb + B \log\left(\frac{(\tan(d) \tan(c) - 1) \sqrt{1 + \tan^2(d) \tan^2(c)}}{\tan(d) \tan(c) - 1}\right) + Ab \log\left(\frac{(\tan(d) \tan(c) - 1) \sqrt{1 + \tan^2(d) \tan^2(c)}}{\tan(d) \tan(c) - 1}\right) - 2Bb \tan(d) - 2Bb \tan(c)}{2d \tan(d) \tan(c) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $1/2*(2*A*a*d*x*\tan(d*x)*\tan(c) - 2*B*b*d*x*\tan(d*x)*\tan(c) - B*a*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)*\tan(c) - A*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)*\tan(c) - 2*A*a*d*x + 2*B*b*d*x + B*a*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1)) + A*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1)) - 2*B*b*\tan(d*x) - 2*B*b*\tan(c))/(d*\tan(d*x)*\tan(c) - d)$

**Mupad** [B]

time = 6.32, size = 55, normalized size = 1.31

$$\frac{Bb \tan(c + dx) + \frac{Ab \ln(\tan(c+dx)^2+1)}{2} + \frac{Ba \ln(\tan(c+dx)^2+1)}{2} + Aadx - Bbdx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)`

[Out]  $(B*b*\tan(c + d*x) + (A*b*\log(\tan(c + d*x)^2 + 1))/2 + (B*a*\log(\tan(c + d*x)^2 + 1))/2 + A*a*d*x - B*b*d*x)/d$

$$3.235 \quad \int \cot(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=37

$$(Ab + aB)x - \frac{bB \log(\cos(c + dx))}{d} + \frac{aA \log(\sin(c + dx))}{d}$$

[Out] (A\*b+B\*a)\*x-b\*B\*ln(cos(d\*x+c))/d+a\*A\*ln(sin(d\*x+c))/d

Rubi [A]

time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3670, 3556, 3612}

$$x(aB + Ab) + \frac{aA \log(\sin(c + dx))}{d} - \frac{bB \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] (A\*b + a\*B)\*x - (b\*B\*Log[Cos[c + d\*x]])/d + (a\*A\*Log[Sin[c + d\*x]])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3670

Int[((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

Rubi steps

$$\begin{aligned} \int \cot(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx &= (bB) \int \tan(c+dx) dx + \int \cot(c+dx)(aA+(Ab+aB)x) dx \\ &= (Ab+aB)x - \frac{bB \log(\cos(c+dx))}{d} + (aA) \int \cot(c+dx) dx \\ &= (Ab+aB)x - \frac{bB \log(\cos(c+dx))}{d} + \frac{aA \log(\sin(c+dx))}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 44, normalized size = 1.19

$$Abx + aBx - \frac{bB \log(\cos(c+dx))}{d} + \frac{aA(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] A\*b\*x + a\*B\*x - (b\*B\*Log[Cos[c + d\*x]])/d + (a\*A\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]]))/d

**Maple [A]**

time = 0.13, size = 43, normalized size = 1.16

method	result	size
derivativedivides	$\frac{aA \ln(\sin(dx+c)) + aB(dx+c) + Ab(dx+c) - Bb \ln(\cos(dx+c))}{d}$	43
default	$\frac{aA \ln(\sin(dx+c)) + aB(dx+c) + Ab(dx+c) - Bb \ln(\cos(dx+c))}{d}$	43
norman	$(Ab + aB)x + \frac{aA \ln(\tan(dx+c))}{d} - \frac{(aA - Bb) \ln(1 + \tan^2(dx+c))}{2d}$	48
risch	$Abx + Bax - iAax + iBbx - \frac{2iaAc}{d} + \frac{2iBbc}{d} + \frac{aA \ln(e^{2i(dx+c)} - 1)}{d} - \frac{\ln(e^{2i(dx+c)} + 1) Bb}{d}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(a\*A\*ln(sin(d\*x+c))+a\*B\*(d\*x+c)+A\*b\*(d\*x+c)-B\*b\*ln(cos(d\*x+c)))

**Maxima [A]**

time = 0.55, size = 52, normalized size = 1.41

$$\frac{2Aa \log(\tan(dx+c)) + 2(Ba + Ab)(dx+c) - (Aa - Bb) \log(\tan(dx+c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(2*A*a*\log(\tan(d*x + c)) + 2*(B*a + A*b)*(d*x + c) - (A*a - B*b)*\log(\tan(d*x + c)^2 + 1))/d$

**Fricas** [A]

time = 0.61, size = 59, normalized size = 1.59

$$\frac{2(Ba + Ab)dx + Aa \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) - Bb \log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(2*(B*a + A*b)*d*x + A*a*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)) - B*b*\log(1/(\tan(d*x + c)^2 + 1)))/d$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs.  $2(34) = 68$ .

time = 0.21, size = 78, normalized size = 2.11

$$\begin{cases} -\frac{Aa \log(\tan^2(c+dx)+1)}{2d} + \frac{Aa \log(\tan(c+dx))}{d} + Abx + Bax + \frac{Bb \log(\tan^2(c+dx)+1)}{2d} & \text{for } d \neq 0 \\ x(A + B \tan(c))(a + b \tan(c)) \cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((-A\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*a\*log(tan(c + d\*x))/d + A\*b\*x + B\*a\*x + B\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d), Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*cot(c), True))

**Giac** [A]

time = 0.53, size = 53, normalized size = 1.43

$$\frac{2 Aa \log(|\tan(dx + c)|) + 2(Ba + Ab)(dx + c) - (Aa - Bb) \log(\tan(dx + c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{2}*(2*A*a*\log(\text{abs}(\tan(d*x + c))) + 2*(B*a + A*b)*(d*x + c) - (A*a - B*b)*\log(\tan(d*x + c)^2 + 1))/d$

**Mupad [B]**

time = 6.47, size = 69, normalized size = 1.86

$$\frac{A a \ln(\tan(c + dx))}{d} - \frac{\ln(\tan(c + dx) - i) (A + B i) (a + b i)}{2d} + \frac{\ln(\tan(c + dx) + i) (A - B i) (b + a i) i}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b)\*1i)/(2\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a + b\*1i))/(2\*d) + (A\*a\*log(tan(c + d\*x)))/d

### 3.236 $\int \cot^2(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=43

$$-((aA - bB)x) - \frac{aA \cot(c + dx)}{d} + \frac{(Ab + aB) \log(\sin(c + dx))}{d}$$

[Out]  $-(A*a-B*b)*x-a*A*\cot(d*x+c)/d+(A*b+B*a)*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3672, 3612, 3556}

$$\frac{(aB + Ab) \log(\sin(c + dx))}{d} - (x(aA - bB)) - \frac{aA \cot(c + dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]),x]$

[Out]  $-\frac{((a*A - b*B)*x) - (a*A*\text{Cot}[c + d*x])}{d} + \frac{((A*b + a*B)*\text{Log}[\text{Sin}[c + d*x]])}{d}$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> } \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ /; } \text{FreeQ}\{c, d\}, x]$

Rule 3612

$\text{Int}[\frac{((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}{((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \text{ :> } \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[a*c + b*d, 0]$

Rule 3672

$\text{Int}[\frac{((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}{((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \text{ :> } \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx)) dx &= -\frac{aA \cot(c+dx)}{d} + \int \cot(c+dx)(Ab+aB - \\ &= -(aA-bB)x - \frac{aA \cot(c+dx)}{d} + (Ab+aB) \int \\ &= -(aA-bB)x - \frac{aA \cot(c+dx)}{d} + \frac{(Ab+aB) \log(\cos(c+dx) \tan(c+dx))}{d} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.12, size = 78, normalized size = 1.81

$$bBx - \frac{aA \cot(c+dx) {}_2F_1(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c+dx))}{d} + \frac{Ab(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d} + \frac{aB(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^2\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]), x]

[Out] b\*B\*x - (a\*A\*Cot[c + d\*x]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d\*x]^2])/d + (A\*b\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]]))/d + (a\*B\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]]))/d

**Maple [A]**

time = 0.11, size = 53, normalized size = 1.23

method	result
derivativdivides	$\frac{aA(-\cot(dx+c)-dx-c)+aB \ln(\sin(dx+c))+Ab \ln(\sin(dx+c))+Bb(dx+c)}{d}$
default	$\frac{aA(-\cot(dx+c)-dx-c)+aB \ln(\sin(dx+c))+Ab \ln(\sin(dx+c))+Bb(dx+c)}{d}$
norman	$\frac{(-aA+Bb)x \tan(dx+c) - \frac{aA}{d}}{\tan(dx+c)} + \frac{(Ab+aB) \ln(\tan(dx+c))}{d} - \frac{(Ab+aB) \ln(1+\tan^2(dx+c))}{2d}$
risch	$-iAbx - iBax - Aax + Bbx - \frac{2iAbc}{d} - \frac{2iaBc}{d} - \frac{2iaA}{d(e^{2i(dx+c)}-1)} + \frac{\ln(e^{2i(dx+c)}-1)Ab}{d} + \frac{a \ln(e^{2i(dx+c)}-1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(a\*A\*(-cot(d\*x+c)-d\*x-c)+a\*B\*ln(sin(d\*x+c))+A\*b\*ln(sin(d\*x+c))+B\*b\*(d\*x+c))

**Maxima [A]**

time = 0.56, size = 68, normalized size = 1.58

$$\frac{2(Aa - Bb)(dx + c) + (Ba + Ab) \log(\tan(dx + c)^2 + 1) - 2(Ba + Ab) \log(\tan(dx + c)) + \frac{2Aa}{\tan(dx + c)}}{2d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-1/2*(2*(A*a - B*b)*(d*x + c) + (B*a + A*b)*\log(\tan(d*x + c)^2 + 1) - 2*(B*a + A*b)*\log(\tan(d*x + c)) + 2*A*a/\tan(d*x + c))/d$$

**Fricas** [A]

time = 0.56, size = 73, normalized size = 1.70

$$\frac{2(Aa - Bb)dx \tan(dx + c) - (Ba + Ab) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx + c) + 2Aa}{2d \tan(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] 
$$-1/2*(2*(A*a - B*b)*d*x*\tan(d*x + c) - (B*a + A*b)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*x + c) + 2*A*a)/(d*\tan(d*x + c))$$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(36) = 72.

time = 0.46, size = 122, normalized size = 2.84

$$\begin{cases} \infty Aax & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c)) \cot^2(c) & \text{for } d = 0 \\ -Aax - \frac{Aa}{d \tan(c+dx)} - \frac{Ab \log(\tan^2(c+dx)+1)}{2d} + \frac{Ab \log(\tan(c+dx))}{d} - \frac{Ba \log(\tan^2(c+dx)+1)}{2d} + \frac{Ba \log(\tan(c+dx))}{d} + Bbx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*a*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(A + B*tan(c))*(a + b*tan(c))*cot(c)**2, Eq(d, 0)), (-A*a*x - A*a/(d*tan(c + d*x)) - A*b*log(tan(c + d*x)**2 + 1)/(2*d) + A*b*log(tan(c + d*x))/d - B*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d + B*b*x, True))`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(43) = 86.

time = 0.63, size = 119, normalized size = 2.77

$$\frac{Aa \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2(Aa - Bb)(dx + c) - 2(Ba + Ab) \log\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right) + 2(Ba + Ab) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - \frac{2Ba \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2Ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + Aa}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $\frac{1}{2}*(A*a*\tan(1/2*d*x + 1/2*c) - 2*(A*a - B*b)*(d*x + c) - 2*(B*a + A*b)*\log(\tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(B*a + A*b)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) - (2*B*a*\tan(1/2*d*x + 1/2*c) + 2*A*b*\tan(1/2*d*x + 1/2*c) + A*a)/\tan(1/2*d*x + 1/2*c))/d$

**Mupad [B]**

time = 6.21, size = 87, normalized size = 2.02

$$\frac{\ln(\tan(c+dx))(Ab+Ba)}{d} - \frac{\ln(\tan(c+dx)+1i)(A-B1i)(b+a1i)}{2d} - \frac{Aa \cot(c+dx)}{d} + \frac{\ln(\tan(c+dx)-i)(A+B1i)(a+b1i)1i}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)`

[Out]  $(\log(\tan(c + d*x))*(A*b + B*a))/d + (\log(\tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i)*1i)/(2*d) - (\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b))/(2*d) - (A*a*\cot(c + d*x))/d$

### 3.237 $\int \cot^3(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$

Optimal. Leaf size=66

$$-((Ab + aB)x) - \frac{(Ab + aB) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} - \frac{(aA - bB) \log(\sin(c + dx))}{d}$$

[Out]  $-(A*b+B*a)*x - (A*b+B*a)*\cot(d*x+c)/d - 1/2*a*A*\cot(d*x+c)^2/d - (A*a-B*b)*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3672, 3610, 3612, 3556}

$$-\frac{(aB + Ab) \cot(c + dx)}{d} - \frac{(aA - bB) \log(\sin(c + dx))}{d} - x(aB + Ab) - \frac{aA \cot^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^3*(a + b*\text{Tan}[c + d*x])*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\frac{((A*b + a*B)*x) - ((A*b + a*B)*\text{Cot}[c + d*x])/d - (a*A*\text{Cot}[c + d*x]^2)/(2*d)}{d} - \frac{((a*A - b*B)*\text{Log}[\text{Sin}[c + d*x]])}{d}$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3610

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3612

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

## Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

## Rubi steps

$$\begin{aligned}
\int \cot^3(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)}{2d} + \int \cot^2(c + dx)(Ab + aB - \\
&= -\frac{(Ab + aB) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} + \\
&= -(Ab + aB)x - \frac{(Ab + aB) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} \\
&= -(Ab + aB)x - \frac{(Ab + aB) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.33, size = 77, normalized size = 1.17

$$\frac{aA \cot^2(c + dx) + 2(Ab + aB) \cot(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)\right) + 2(aA - bB)(\log(\cos(c + dx)) + \log(\tan(c + dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] -1/2*(a*A*Cot[c + d*x]^2 + 2*(A*b + a*B)*Cot[c + d*x]*Hypergeometric2F1[-1/
2, 1, 1/2, -Tan[c + d*x]^2] + 2*(a*A - b*B)*(Log[Cos[c + d*x]] + Log[Tan[c
+ d*x]]))/d
```

## Maple [A]

time = 0.14, size = 77, normalized size = 1.17

method	result
derivativedivides	$\frac{aA \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) + aB(-\cot(dx+c)-dx-c) + Ab(-\cot(dx+c)-dx-c) + Bb \ln(\sin(dx+c))}{d}$

default	$\frac{aA \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) + aB(-\cot(dx+c)-dx-c) + Ab(-\cot(dx+c)-dx-c) + Bb \ln(\sin(dx+c))}{d}$
norman	$\frac{(-Ab-aB)x(\tan^2(dx+c)) - \frac{aA}{2d} - \frac{(Ab+aB)\tan(dx+c)}{d}}{\tan(dx+c)^2} - \frac{(aA-Bb)\ln(\tan(dx+c))}{d} + \frac{(aA-Bb)\ln(1+\tan^2(dx+c))}{2d}$
risch	$-Abx - Bax + iAax - iBbx + \frac{2iaAc}{d} - \frac{2iBbc}{d} - \frac{2i(iAa e^{2i(dx+c)} + b e^{2i(dx+c)} A + aB e^{2i(dx+c)} - Ab - a)}{d(e^{2i(dx+c)} - 1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (a * A * (-1/2 * \cot(d * x + c)^2 - \ln(\sin(d * x + c))) + a * B * (-\cot(d * x + c) - d * x - c) + A * b * (-\cot(d * x + c) - d * x - c) + B * b * \ln(\sin(d * x + c)))$

**Maxima** [A]

time = 0.56, size = 86, normalized size = 1.30

$$\frac{2(Ba + Ab)(dx + c) - (Aa - Bb) \log(\tan(dx + c)^2 + 1) + 2(Aa - Bb) \log(\tan(dx + c)) + \frac{Aa + 2(Ba + Ab)\tan(dx + c)}{\tan(dx + c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{-1/2 * (2 * (B * a + A * b) * (d * x + c) - (A * a - B * b) * \log(\tan(d * x + c)^2 + 1) + 2 * (A * a - B * b) * \log(\tan(d * x + c)) + (A * a + 2 * (B * a + A * b) * \tan(d * x + c)) / \tan(d * x + c)^2) / d}{d}$

**Fricas** [A]

time = 0.59, size = 95, normalized size = 1.44

$$\frac{(Aa - Bb) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^2 + (2(Ba + Ab)dx + Aa) \tan(dx+c)^2 + Aa + 2(Ba + Ab) \tan(dx+c)}{2d \tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{-1/2 * ((A * a - B * b) * \log(\tan(d * x + c)^2 / (\tan(d * x + c)^2 + 1)) * \tan(d * x + c)^2 + (2 * (B * a + A * b) * d * x + A * a) * \tan(d * x + c)^2 + A * a + 2 * (B * a + A * b) * \tan(d * x + c)) / (d * \tan(d * x + c)^2)}{d}$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(56) = 112.

time = 0.67, size = 150, normalized size = 2.27

$$\begin{cases} \infty Aax & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c)) (a + b \tan(c)) \cot^3(c) & \text{for } d = 0 \\ \frac{Aa \log(\tan^2(c+dx)+1)}{2d} - \frac{Aa \log(\tan(c+dx))}{d} - \frac{Aa}{2d \tan^2(c+dx)} - Abx - \frac{Ab}{d \tan(c+dx)} - Bax - \frac{Ba}{d \tan(c+dx)} - \frac{Bb \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb \log(\tan(c+dx))}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*A\*a\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))),  
 (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*cot(c)\*\*3, Eq(d, 0)), (A\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*a\*log(tan(c + d\*x))/d - A\*a/(2\*d\*tan(c + d\*x)\*\*2) - A\*b\*x - A\*b/(d\*tan(c + d\*x)) - B\*a\*x - B\*a/(d\*tan(c + d\*x)) - B\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*log(tan(c + d\*x))/d, True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(64) = 128.

time = 0.74, size = 179, normalized size = 2.71

$$\frac{Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8(Ba + Ab)(dx + c) - 8(Aa - Bb) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) + 8(Aa - Bb) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - \frac{12Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Aa}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] -1/8\*(A\*a\*tan(1/2\*d\*x + 1/2\*c)^2 - 4\*B\*a\*tan(1/2\*d\*x + 1/2\*c) - 4\*A\*b\*tan(1/2\*d\*x + 1/2\*c) + 8\*(B\*a + A\*b)\*(d\*x + c) - 8\*(A\*a - B\*b)\*log(tan(1/2\*d\*x + 1/2\*c)^2 + 1) + 8\*(A\*a - B\*b)\*log(abs(tan(1/2\*d\*x + 1/2\*c))) - (12\*A\*a\*tan(1/2\*d\*x + 1/2\*c)^2 - 12\*B\*b\*tan(1/2\*d\*x + 1/2\*c)^2 - 4\*B\*a\*tan(1/2\*d\*x + 1/2\*c) - 4\*A\*b\*tan(1/2\*d\*x + 1/2\*c) - A\*a)/tan(1/2\*d\*x + 1/2\*c)^2/d

**Mupad** [B]

time = 6.25, size = 108, normalized size = 1.64

$$\frac{\ln(\tan(c + dx)) (Aa - Bb)}{d} - \frac{\cot(c + dx)^2 \left(\frac{Aa}{2} + \tan(c + dx) (Ab + Ba)\right)}{d} + \frac{\ln(\tan(c + dx) - i) (A + B i) (a + b i)}{2d} - \frac{\ln(\tan(c + dx) + i) (A - B i) (b + a i) i}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a + b\*1i))/(2\*d) - (cot(c + d\*x)^2\*((A\*a)/2 + tan(c + d\*x)\*(A\*b + B\*a)))/d - (log(tan(c + d\*x))\*(A\*a - B\*b))/d - (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b)\*1i)/(2\*d)

$$3.238 \quad \int \cot^4(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=87

$$(aA-bB)x + \frac{(aA-bB) \cot(c+dx)}{d} - \frac{(Ab+aB) \cot^2(c+dx)}{2d} - \frac{aA \cot^3(c+dx)}{3d} - \frac{(Ab+aB) \log(\sin(c+dx))}{d}$$

[Out] (A\*a-B\*b)\*x+(A\*a-B\*b)\*cot(d\*x+c)/d-1/2\*(A\*b+B\*a)\*cot(d\*x+c)^2/d-1/3\*a\*A\*cot(d\*x+c)^3/d-(A\*b+B\*a)\*ln(sin(d\*x+c))/d

Rubi [A]

time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3672, 3610, 3612, 3556}

$$-\frac{(aB+Ab) \cot^2(c+dx)}{2d} + \frac{(aA-bB) \cot(c+dx)}{d} - \frac{(aB+Ab) \log(\sin(c+dx))}{d} + x(aA-bB) - \frac{aA \cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]), x]

[Out] (a\*A - b\*B)\*x + ((a\*A - b\*B)\*Cot[c + d\*x])/d - ((A\*b + a\*B)\*Cot[c + d\*x]^2)/(2\*d) - (a\*A\*Cot[c + d\*x]^3)/(3\*d) - ((A\*b + a\*B)\*Log[Sin[c + d\*x]])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx)}{3d} + \int \cot^3(c + dx)(Ab + aB - \\
&= -\frac{(Ab + aB) \cot^2(c + dx)}{2d} - \frac{aA \cot^3(c + dx)}{3d} + \\
&= \frac{(aA - bB) \cot(c + dx)}{d} - \frac{(Ab + aB) \cot^2(c + dx)}{2d} \\
&= (aA - bB)x + \frac{(aA - bB) \cot(c + dx)}{d} - \frac{(Ab + aB)}{d} \\
&= (aA - bB)x + \frac{(aA - bB) \cot(c + dx)}{d} - \frac{(Ab + aB)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.72, size = 101, normalized size = 1.16

$$\frac{2aA \cot^3(c + dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c + dx)\right) + 6bB \cot(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)\right) + 3(Ab + aB) (\cot^2(c + dx) + 2(\log(\cos(c + dx)) + \log(\tan(c + dx))))}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] -1/6\*(2\*a\*A\*Cot[c + d\*x]^3\*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d\*x]^2] + 6\*b\*B\*Cot[c + d\*x]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d\*x]^2] + 3\*(A\*b + a\*B)\*(Cot[c + d\*x]^2 + 2\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]])))/d

**Maple [A]**

time = 0.14, size = 95, normalized size = 1.09

method	result
derivativedivides	$ \frac{aA \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx+c \right) + aB \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right) + Ab \left( -\frac{(\cot^2(dx+c))}{2} - \ln(\sin(dx+c)) \right)}{d} $



default	$\frac{aA \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + aB \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + Ab \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)}{d}$
norman	$\frac{(aA-Bb) \frac{\tan^2(dx+c)}{d} + (aA-Bb)x \frac{\tan^3(dx+c)}{\tan(dx+c)^3} - \frac{aA}{3d} - \frac{(Ab+aB) \tan(dx+c)}{2d}}{\tan(dx+c)^3} - \frac{(Ab+aB) \ln(\tan(dx+c))}{d} + \frac{(Ab+aB) \ln(\tan(dx+c))}{d}$
risch	$iAbx + iBax + Aax - Bbx + \frac{2iAbc}{d} + \frac{2iaBc}{d} - \frac{2i(3iAb e^{4i(dx+c)} + 3iBa e^{4i(dx+c)} - 6Aa e^{4i(dx+c)} + 3Bb e^{4i(dx+c)})}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (aA * (-1/3 * \cot(dx+c)^3 + \cot(dx+c) + dx+c) + aB * (-1/2 * \cot(dx+c)^2 - \ln(\sin(dx+c))) + Ab * (-1/2 * \cot(dx+c)^2 - \ln(\sin(dx+c))) + B * b * (-\cot(dx+c) - dx - c))$

**Maxima** [A]

time = 0.56, size = 104, normalized size = 1.20

$$\frac{6(Aa - Bb)(dx + c) + 3(Ba + Ab) \log(\tan(dx + c)^2 + 1) - 6(Ba + Ab) \log(\tan(dx + c)) + \frac{6(Aa - Bb) \tan(dx + c)^2 - 2Aa - 3(Ba + Ab) \tan(dx + c)}{\tan(dx + c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{6} * (6 * (A * a - B * b) * (d * x + c) + 3 * (B * a + A * b) * \log(\tan(d * x + c)^2 + 1) - 6 * (B * a + A * b) * \log(\tan(d * x + c)) + (6 * (A * a - B * b) * \tan(d * x + c)^2 - 2 * A * a - 3 * (B * a + A * b) * \tan(d * x + c)) / \tan(d * x + c)^3) / d$

**Fricas** [A]

time = 0.53, size = 121, normalized size = 1.39

$$\frac{3(Ba + Ab) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 - 3(2(Aa - Bb)dx - Ba - Ab) \tan(dx+c)^3 - 6(Aa - Bb) \tan(dx+c)^2 + 2Aa + 3(Ba + Ab) \tan(dx+c)}{6d \tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/6 * (3 * (B * a + A * b) * \log(\tan(d * x + c)^2 / (\tan(d * x + c)^2 + 1)) * \tan(d * x + c)^3 - 3 * (2 * (A * a - B * b) * d * x - B * a - A * b) * \tan(d * x + c)^3 - 6 * (A * a - B * b) * \tan(d * x + c)^2 + 2 * A * a + 3 * (B * a + A * b) * \tan(d * x + c)) / (d * \tan(d * x + c)^3)$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(75) = 150.

time = 0.98, size = 180, normalized size = 2.07

$$\begin{cases} \infty Aax & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c)) \cot^4(c) & \text{for } d = 0 \\ Aax + \frac{Aa}{d \tan(c+dx)} - \frac{Aa}{3d \tan^3(c+dx)} + \frac{Ab \log(\tan^2(c+dx)+1)}{2d} - \frac{Ab \log(\tan(c+dx))}{d} - \frac{Ab}{2d \tan^2(c+dx)} + \frac{Ba \log(\tan^2(c+dx)+1)}{2d} - \frac{Ba \log(\tan(c+dx))}{d} - \frac{Ba}{2d \tan^2(c+dx)} - Bbx - \frac{Bb}{d \tan(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*A\*a\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*cot(c)\*\*4, Eq(d, 0)), (A\*a\*x + A\*a/(d\*tan(c + d\*x)) - A\*a/(3\*d\*tan(c + d\*x)\*\*3) + A\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*b\*log(tan(c + d\*x))/d - A\*b/(2\*d\*tan(c + d\*x)\*\*2) + B\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*a\*log(tan(c + d\*x))/d - B\*a/(2\*d\*tan(c + d\*x)\*\*2) - B\*b\*x - B\*b/(d\*tan(c + d\*x)), True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(83) = 166.

time = 0.83, size = 237, normalized size = 2.72

$$\frac{Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 3Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 15Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24(Aa - Bb)(dx + c) + 24(Ba + Ab) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) - 24(Ba + Ab) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + \frac{44Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 44Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 12Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 12Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 3Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/24\*(A\*a\*tan(1/2\*d\*x + 1/2\*c)^3 - 3\*B\*a\*tan(1/2\*d\*x + 1/2\*c)^2 - 3\*A\*b\*tan(1/2\*d\*x + 1/2\*c)^2 - 15\*A\*a\*tan(1/2\*d\*x + 1/2\*c) + 12\*B\*b\*tan(1/2\*d\*x + 1/2\*c) + 24\*(A\*a - B\*b)\*(d\*x + c) + 24\*(B\*a + A\*b)\*log(tan(1/2\*d\*x + 1/2\*c)^2 + 1) - 24\*(B\*a + A\*b)\*log(abs(tan(1/2\*d\*x + 1/2\*c))) + (44\*B\*a\*tan(1/2\*d\*x + 1/2\*c)^3 + 44\*A\*b\*tan(1/2\*d\*x + 1/2\*c)^3 + 15\*A\*a\*tan(1/2\*d\*x + 1/2\*c)^2 - 12\*B\*b\*tan(1/2\*d\*x + 1/2\*c)^2 - 3\*B\*a\*tan(1/2\*d\*x + 1/2\*c) - 3\*A\*b\*tan(1/2\*d\*x + 1/2\*c) - A\*a)/tan(1/2\*d\*x + 1/2\*c)^3/d

**Mupad** [B]

time = 6.41, size = 127, normalized size = 1.46

$$\frac{\cot(c+dx)^3 \left( (Bb - Aa) \tan(c+dx)^2 + \left(\frac{4b}{2} + \frac{Bb}{2}\right) \tan(c+dx) + \frac{4a}{3} \right) - \frac{\ln(\tan(c+dx)) (Ab + Ba)}{d} - \frac{\ln(\tan(c+dx) - i) (A + B1i) (a + b1i) 1i}{2d} + \frac{\ln(\tan(c+dx) + 1i) (A - B1i) (b + a1i)}{2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b))/(2\*d) - (log(tan(c + d\*x))\*(A\*b + B\*a))/d - (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a + b\*1i)\*1i)/(2\*d) - (cot(c + d\*x)^3\*((A\*a)/3 + tan(c + d\*x)\*((A\*b)/2 + (B\*a)/2) - tan(c + d\*x)^2\*(A\*a - B\*b)))/d

$$3.239 \quad \int \cot^5(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=108

$$(Ab+aB)x + \frac{(Ab+aB) \cot(c+dx)}{d} + \frac{(aA-bB) \cot^2(c+dx)}{2d} - \frac{(Ab+aB) \cot^3(c+dx)}{3d} - \frac{aA \cot^4(c+dx)}{4d} +$$

[Out] (A\*b+B\*a)\*x+(A\*b+B\*a)\*cot(d\*x+c)/d+1/2\*(A\*a-B\*b)\*cot(d\*x+c)^2/d-1/3\*(A\*b+B\*a)\*cot(d\*x+c)^3/d-1/4\*a\*A\*cot(d\*x+c)^4/d+(A\*a-B\*b)\*ln(sin(d\*x+c))/d

Rubi [A]

time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3672, 3610, 3612, 3556}

$$-\frac{(aB+Ab) \cot^3(c+dx)}{3d} + \frac{(aA-bB) \cot^2(c+dx)}{2d} + \frac{(aB+Ab) \cot(c+dx)}{d} + \frac{(aA-bB) \log(\sin(c+dx))}{d} + x(aB+Ab) - \frac{aA \cot^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^5\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] (A\*b + a\*B)\*x + ((A\*b + a\*B)\*Cot[c + d\*x])/d + ((a\*A - b\*B)\*Cot[c + d\*x]^2)/(2\*d) - ((A\*b + a\*B)\*Cot[c + d\*x]^3)/(3\*d) - (a\*A\*Cot[c + d\*x]^4)/(4\*d) + ((a\*A - b\*B)\*Log[Sin[c + d\*x]])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^5(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= -\frac{aA \cot^4(c + dx)}{4d} + \int \cot^4(c + dx)(Ab + aB - \\
&= -\frac{(Ab + aB) \cot^3(c + dx)}{3d} - \frac{aA \cot^4(c + dx)}{4d} + \\
&= \frac{(aA - bB) \cot^2(c + dx)}{2d} - \frac{(Ab + aB) \cot^3(c + dx)}{3d} \\
&= \frac{(Ab + aB) \cot(c + dx)}{d} + \frac{(aA - bB) \cot^2(c + dx)}{2d} \\
&= (Ab + aB)x + \frac{(Ab + aB) \cot(c + dx)}{d} + \frac{(aA - bB) \cot^2(c + dx)}{2d} \\
&= (Ab + aB)x + \frac{(Ab + aB) \cot(c + dx)}{d} + \frac{(aA - bB) \cot^2(c + dx)}{2d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.82, size = 100, normalized size = 0.93

$$\frac{4(Ab + aB) \cot^3(c + dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c + dx)\right) + 3((-2aA + 2bB) \cot^2(c + dx) + aA \cot^4(c + dx) - 4(aA - bB)(\log(\cos(c + dx)) + \log(\tan(c + dx))))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] -1/12*(4*(A*b + a*B)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c
+ d*x]^2] + 3*((-2*a*A + 2*b*B)*Cot[c + d*x]^2 + a*A*Cot[c + d*x]^4 - 4*(a
*A - b*B)*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/d
```

Maple [A]

time = 0.16, size = 108, normalized size = 1.00

method	result
derivativedivides	$\frac{aA \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} + \ln(\sin(dx+c)) \right) + aB \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + Ab \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right)}{d}$
default	$\frac{aA \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} + \ln(\sin(dx+c)) \right) + aB \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + Ab \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right)}{d}$
norman	$\frac{\frac{(Ab+aB)(\tan^3(dx+c))}{d} + (Ab+aB)x(\tan^4(dx+c)) - \frac{aA}{4d} - \frac{(Ab+aB)\tan(dx+c)}{3d} + \frac{(aA-Bb)(\tan^2(dx+c))}{2d}}{\tan(dx+c)^4} + \frac{(aA-Bb)\ln(\tan(dx+c))}{d}$
risch	$Abx + Bax - iAax + iBbx - \frac{2iaAc}{d} + \frac{2iBbc}{d} - \frac{2(-6iAb e^{6i(dx+c)} - 6iBa e^{6i(dx+c)} + 6Aa e^{6i(dx+c)} - 3B^2)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (a * A * (-1/4 * \cot(d*x+c)^4 + 1/2 * \cot(d*x+c)^2 + \ln(\sin(d*x+c))) + a * B * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + A * b * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + B * b * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))))$

**Maxima [A]**

time = 0.51, size = 122, normalized size = 1.13

$$\frac{12(Ba + Ab)(dx + c) - 6(Aa - Bb) \log(\tan(dx + c)^2 + 1) + 12(Aa - Bb) \log(\tan(dx + c)) + \frac{12(Ba + Ab)\tan(dx + c)^3 + 6(Aa - Bb)\tan(dx + c)^2 - 3Aa - 4(Ba + Ab)\tan(dx + c)}{\tan(dx + c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{12} * (12 * (B * a + A * b) * (d * x + c) - 6 * (A * a - B * b) * \log(\tan(d * x + c)^2 + 1) + 12 * (A * a - B * b) * \log(\tan(d * x + c)) + (12 * (B * a + A * b) * \tan(d * x + c)^3 + 6 * (A * a - B * b) * \tan(d * x + c)^2 - 3 * A * a - 4 * (B * a + A * b) * \tan(d * x + c)) / \tan(d * x + c)^4) / d$

**Fricas [A]**

time = 0.54, size = 138, normalized size = 1.28

$$\frac{6(Aa - Bb) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^4 + 3(4(Ba + Ab)dx + 3Aa - 2Bb) \tan(dx+c)^4 + 12(Ba + Ab) \tan(dx+c)^3 + 6(Aa - Bb) \tan(dx+c)^2 - 3Aa - 4(Ba + Ab) \tan(dx+c)}{12d \tan(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{12} * (6 * (A * a - B * b) * \log(\tan(d * x + c)^2 / (\tan(d * x + c)^2 + 1)) * \tan(d * x + c)^4 + 3 * (4 * (B * a + A * b) * d * x + 3 * A * a - 2 * B * b) * \tan(d * x + c)^4 + 12 * (B * a + A * b) * \tan(d * x + c)^3 + 6 * (A * a - B * b) * \tan(d * x + c)^2 - 3 * A * a - 4 * (B * a + A * b) * \tan(d * x + c)) / (d * \tan(d * x + c)^4)$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 211 vs.  $2(95) = 190$ .

time = 1.49, size = 211, normalized size = 1.95

$$\begin{cases} \infty Aax & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c)) (a + b \tan(c)) \cot^5(c) & \text{for } d = 0 \\ -\frac{Aa \log(\tan^2(c+dx)+1)}{2d} + \frac{Aa \log(\tan(c+dx))}{d} + \frac{Aa}{2d \tan^2(c+dx)} - \frac{Aa}{4d \tan^4(c+dx)} + Abx + \frac{Ab}{d \tan(c+dx)} - \frac{Ab}{3d \tan^3(c+dx)} + Bax + \frac{Ba}{d \tan(c+dx)} - \frac{Ba}{3d \tan^3(c+dx)} + \frac{Bb \log(\tan^2(c+dx)+1)}{2d} - \frac{Bb \log(\tan(c+dx))}{d} - \frac{Bb}{2d \tan^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*5\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x)

[Out] Piecewise((zoo\*A\*a\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*cot(c)\*\*5, Eq(d, 0)), (-A\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*a\*log(tan(c + d\*x))/d + A\*a/(2\*d\*tan(c + d\*x)\*\*2) - A\*a/(4\*d\*tan(c + d\*x)\*\*4) + A\*b\*x + A\*b/(d\*tan(c + d\*x)) - A\*b/(3\*d\*tan(c + d\*x)\*\*3) + B\*a\*x + B\*a/(d\*tan(c + d\*x)) - B\*a/(3\*d\*tan(c + d\*x)\*\*3) + B\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*b\*log(tan(c + d\*x))/d - B\*b/(2\*d\*tan(c + d\*x)\*\*2), True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 299 vs.  $2(102) = 204$ .

time = 0.99, size = 299, normalized size = 2.77

$$\frac{3Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 8Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 8Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 36Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 120Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 120Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 192(Aa - Bb) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) - 192(Aa - Bb) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 + \frac{400Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 400Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 120Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 120Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 36Aa \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24Bb \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 8Ba \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8Ab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 3Aa}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)), x, algorithm="giac")

[Out]  $-1/192*(3*A*a*\tan(1/2*d*x + 1/2*c)^4 - 8*B*a*\tan(1/2*d*x + 1/2*c)^3 - 8*A*b*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*\tan(1/2*d*x + 1/2*c)^2 + 24*B*b*\tan(1/2*d*x + 1/2*c)^2 + 120*B*a*\tan(1/2*d*x + 1/2*c) + 120*A*b*\tan(1/2*d*x + 1/2*c) - 192*(B*a + A*b)*(d*x + c) + 192*(A*a - B*b)*\log(\tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(A*a - B*b)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + (400*A*a*\tan(1/2*d*x + 1/2*c)^4 - 400*B*b*\tan(1/2*d*x + 1/2*c)^4 - 120*B*a*\tan(1/2*d*x + 1/2*c)^3 - 120*A*b*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a*\tan(1/2*d*x + 1/2*c)^2 + 24*B*b*\tan(1/2*d*x + 1/2*c)^2 + 8*B*a*\tan(1/2*d*x + 1/2*c) + 8*A*b*\tan(1/2*d*x + 1/2*c) + 3*A*a)/\tan(1/2*d*x + 1/2*c)^4/d$

**Mupad [B]**

time = 6.39, size = 145, normalized size = 1.34

$$\frac{\ln(\tan(c+dx)) (Aa - Bb) - \cot(c+dx)^4 ((-Ab - Ba) \tan(c+dx)^3 + (\frac{Bb}{2} - \frac{Aa}{2}) \tan(c+dx)^2 + (\frac{Aa}{3} + \frac{Bb}{3}) \tan(c+dx) + \frac{Aa}{3}) - \ln(\tan(c+dx) - 1) (A + B \operatorname{li}) (a + b \operatorname{li}) + \ln(\tan(c+dx) + 1) (A - B \operatorname{li}) (b + a \operatorname{li}) \operatorname{li}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^5\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)), x)

```
[Out] (log(tan(c + d*x))*(A*a - B*b))/d - (cot(c + d*x)^4*((A*a)/4 + tan(c + d*x)
*((A*b)/3 + (B*a)/3) - tan(c + d*x)^3*(A*b + B*a) - tan(c + d*x)^2*((A*a)/2
- (B*b)/2))/d - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i))/(2*d) + (l
og(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)*1i)/(2*d)
```

### 3.240 $\int \tan^2(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=148

$$-((a^2A - Ab^2 - 2abB)x) + \frac{(2aAb + a^2B - b^2B) \log(\cos(c + dx))}{d} - \frac{b(Ab + aB) \tan(c + dx)}{d} - \frac{B(a + b \tan(c + dx))^2}{2d}$$

[Out]  $-(A*a^2 - A*b^2 - 2*B*a*b)*x + (2*A*a*b + B*a^2 - B*b^2)*\ln(\cos(d*x+c))/d - b*(A*b + B*a)*\tan(d*x+c)/d - 1/2*B*(a+b*\tan(d*x+c))^2/d + 1/12*(4*A*b - B*a)*(a+b*\tan(d*x+c))^3/b^2/d + 1/4*B*\tan(d*x+c)*(a+b*\tan(d*x+c))^3/b/d$

Rubi [A]

time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3688, 3711, 3609, 3606, 3556}

$$\frac{(a^2B + 2aAb - b^2B) \log(\cos(c + dx))}{d} - x(a^2A - 2abB - Ab^2) + \frac{(4Ab - aB)(a + b \tan(c + dx))^2}{12b^2d} - \frac{b(aB + Ab) \tan(c + dx)}{d} + \frac{B \tan(c + dx)(a + b \tan(c + dx))^3}{4bd} - \frac{B(a + b \tan(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(a^2*A - A*b^2 - 2*a*b*B)*x + ((2*a*A*b + a^2*B - b^2*B)*\text{Log}[\text{Cos}[c + d*x]])/d - (b*(A*b + a*B)*\text{Tan}[c + d*x])/d - (B*(a + b*\text{Tan}[c + d*x])^2)/(2*d) + ((4*A*b - a*B)*(a + b*\text{Tan}[c + d*x])^3)/(12*b^2*d) + (B*\text{Tan}[c + d*x]*(a + b*\text{Tan}[c + d*x])^3)/(4*b*d)$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3606

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3609

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{m-1}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$



## Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

## Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned}
\int \tan^2(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{B \tan(c + dx)(a + b \tan(c + dx))^3}{4bd} + \frac{\int (a + b \tan(c + dx))^3 dx}{4bd} \\
&= \frac{(4Ab - aB)(a + b \tan(c + dx))^3}{12b^2d} + \frac{B \tan(c + dx)(a + b \tan(c + dx))^3}{4bd} \\
&= -\frac{B(a + b \tan(c + dx))^2}{2d} + \frac{(4Ab - aB)(a + b \tan(c + dx))^3}{12b^2d} \\
&= -(a^2A - Ab^2 - 2abB)x - \frac{b(Ab + aB) \tan(c + dx)}{d} \\
&= -(a^2A - Ab^2 - 2abB)x + \frac{(2aAb + a^2B - b^3)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 6.14, size = 221, normalized size = 1.49

$$\frac{B \tan(c + dx)(a + b \tan(c + dx))^3}{4bd} + \frac{(4Ab - aB)(a + b \tan(c + dx))^3}{12b^2d} + \frac{2((Ab - aB)(i(a + ib)^2 \log(i - \tan(c + dx)) - i(a - ib)^2 \log(i + \tan(c + dx)) - 2b^2 \tan(c + dx)) - B((ia - b)^3 \log(i - \tan(c + dx)) - (ia + b)^3 \log(i + \tan(c + dx)) + 6a^2 \tan(c + dx) + b^3 \tan^2(c + dx)))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

```
[Out] (B*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) + (((4*A*b - a*B)*(a + b*Tan[c + d*x])^3)/(3*b*d) + (2*((A*b - a*B)*(I*(a + I*b)^2*Log[I - Tan[c + d*x]]) - I*(a - I*b)^2*Log[I + Tan[c + d*x]]) - 2*b^2*Tan[c + d*x]) - B*((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]]) + 6*a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/d)/(4*b)
```

**Maple [A]**

time = 0.07, size = 176, normalized size = 1.19

method	result
norman	$(-a^2 A + A b^2 + 2 B a b) x + \frac{(a^2 A - A b^2 - 2 B a b) \tan(dx+c)}{d} + \frac{(2 A a b + a^2 B - b^2 B) (\tan^2(dx+c))}{2d} + \frac{b(A b + 2 a^2 A)}{2d}$
derivativdivides	$\frac{b^2 B (\tan^4(dx+c))}{4} + \frac{A b^2 (\tan^3(dx+c))}{3} + \frac{2 B a b (\tan^3(dx+c))}{3} + A a b (\tan^2(dx+c)) + \frac{B a^2 (\tan^2(dx+c))}{2} - \frac{b^2 B (\tan^2(dx+c))}{2} + a^2 A$
default	$\frac{b^2 B (\tan^4(dx+c))}{4} + \frac{A b^2 (\tan^3(dx+c))}{3} + \frac{2 B a b (\tan^3(dx+c))}{3} + A a b (\tan^2(dx+c)) + \frac{B a^2 (\tan^2(dx+c))}{2} - \frac{b^2 B (\tan^2(dx+c))}{2} + a^2 A$
risch	$\frac{2i B b^2 c}{d} - \frac{2ia^2 Bc}{d} - \frac{4i A a b c}{d} - A a^2 x + A b^2 x + 2 B a b x - 2i A a b x - i B a^2 x + i B b^2 x + \frac{2i A a^2 e^{\dots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^2*B*tan(d*x+c)^4+1/3*A*b^2*tan(d*x+c)^3+2/3*B*a*b*tan(d*x+c)^3+A*a*b*tan(d*x+c)^2+1/2*B*a^2*tan(d*x+c)^2-1/2*b^2*B*tan(d*x+c)^2+a^2*A*tan(d*x+c)-A*b^2*tan(d*x+c)-2*B*a*b*tan(d*x+c)+1/2*(-2*A*a*b-B*a^2+B*b^2)*ln(1+tan(d*x+c)^2)+(-A*a^2+A*b^2+2*B*a*b)*arctan(tan(d*x+c)))
```

**Maxima [A]**

time = 0.51, size = 147, normalized size = 0.99

$$\frac{3 B b^2 \tan(dx+c)^4 + 4(2 B a b + A b^2) \tan(dx+c)^3 + 6(B a^2 + 2 A a b - B b^2) \tan(dx+c)^2 - 12(A a^2 - 2 B a b - A b^2)(dx+c) - 6(B a^2 + 2 A a b - B b^2) \log(\tan(dx+c)^2 + 1) + 12(A a^2 - 2 B a b - A b^2) \tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*B*b^2*tan(d*x + c)^4 + 4*(2*B*a*b + A*b^2)*tan(d*x + c)^3 + 6*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^2 - 12*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) - 6*(B*a^2 + 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1) + 12*(A*a^2 - 2*B*a*b - A*b^2)*tan(d*x + c))/d
```

**Fricas [A]**

time = 0.53, size = 146, normalized size = 0.99

$$\frac{3 B b^2 \tan(dx+c)^4 + 4(2 B a b + A b^2) \tan(dx+c)^3 - 12(A a^2 - 2 B a b - A b^2) d x + 6(B a^2 + 2 A a b - B b^2) \tan(dx+c)^2 + 6(B a^2 + 2 A a b - B b^2) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 12(A a^2 - 2 B a b - A b^2) \tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{12}*(3*B*b^2*\tan(d*x + c)^4 + 4*(2*B*a*b + A*b^2)*\tan(d*x + c)^3 - 12*(A*a^2 - 2*B*a*b - A*b^2)*d*x + 6*(B*a^2 + 2*A*a*b - B*b^2)*\tan(d*x + c)^2 + 6*(B*a^2 + 2*A*a*b - B*b^2)*\log(1/(\tan(d*x + c)^2 + 1)) + 12*(A*a^2 - 2*B*a*b - A*b^2)*\tan(d*x + c))/d$

**Sympy** [A]

time = 0.15, size = 246, normalized size = 1.66

$$\begin{cases} -Aa^2x + \frac{Aa^2 \tan(c+dx)}{d} - \frac{Ab \log(\tan^2(c+dx)+1)}{d} + \frac{dab \tan^2(c+dx)}{d} + Ab^2x + \frac{Ab^2 \tan^3(c+dx)}{3d} - \frac{Ab^2 \tan(c+dx)}{d} - \frac{Ba^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Ba^2 \tan^2(c+dx)}{2d} + 2Babx + \frac{2Bab \tan^3(c+dx)}{3d} - \frac{2Bab \tan(c+dx)}{d} + \frac{Bb^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^2 \tan^4(c+dx)}{2d} - \frac{Bb^2 \tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A+B \tan(c))(a+b \tan(c))^2 \tan^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((-A\*a\*\*2\*x + A\*a\*\*2\*tan(c + d\*x)/d - A\*a\*b\*log(tan(c + d\*x)\*\*2 + 1)/d + A\*a\*b\*tan(c + d\*x)\*\*2/d + A\*b\*\*2\*x + A\*b\*\*2\*tan(c + d\*x)\*\*3/(3\*d) - A\*b\*\*2\*tan(c + d\*x)/d - B\*a\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*a\*\*2\*tan(c + d\*x)\*\*2/(2\*d) + 2\*B\*a\*b\*x + 2\*B\*a\*b\*tan(c + d\*x)\*\*3/(3\*d) - 2\*B\*a\*b\*tan(c + d\*x)/d + B\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*\*2\*tan(c + d\*x)\*\*4/(4\*d) - B\*b\*\*2\*tan(c + d\*x)\*\*2/(2\*d), Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*2\*tan(c)\*\*2, True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. 2(141) = 282.

time = 1.94, size = 2228, normalized size = 15.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/12*(12*A*a^2*d*x*\tan(d*x)^4*\tan(c)^4 - 24*B*a*b*d*x*\tan(d*x)^4*\tan(c)^4 - 12*A*b^2*d*x*\tan(d*x)^4*\tan(c)^4 - 6*B*a^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 - 12*A*a*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 + 6*B*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 - 48*A*a^2*d*x*\tan(d*x)^3*\tan(c)^3 + 96*B*a*b*d*x*\tan(d*x)^3*\tan(c)^3 + 48*A*b^2*d*x*\tan(d*x)^3*\tan(c)^3 - 6*B*a^2*\tan(d*x)^4*\tan(c)^4 - 12*A*a*b*\tan(d*x)^4*\tan(c)^4 + 9*B*b^2*$

$$\begin{aligned}
& 2*\tan(d*x)^4*\tan(c)^4 + 24*B*a^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3* \\
& \tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 + 48*A*a*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d \\
& *x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(t \\
& \tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 - 24*B*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2 \\
& *\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + \\
& 1)/(\tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 + 12*A*a^2*\tan(d*x)^4*\tan(c)^3 - 24 \\
& *B*a*b*\tan(d*x)^4*\tan(c)^3 - 12*A*b^2*\tan(d*x)^4*\tan(c)^3 + 12*A*a^2*\tan(d* \\
& x)^3*\tan(c)^4 - 24*B*a*b*\tan(d*x)^3*\tan(c)^4 - 12*A*b^2*\tan(d*x)^3*\tan(c)^4 \\
& + 72*A*a^2*d*x*\tan(d*x)^2*\tan(c)^2 - 144*B*a*b*d*x*\tan(d*x)^2*\tan(c)^2 - 7 \\
& 2*A*b^2*d*x*\tan(d*x)^2*\tan(c)^2 - 6*B*a^2*\tan(d*x)^4*\tan(c)^2 - 12*A*a*b*\tan \\
& (d*x)^4*\tan(c)^2 + 6*B*b^2*\tan(d*x)^4*\tan(c)^2 + 12*B*a^2*\tan(d*x)^3*\tan(c \\
& )^3 + 24*A*a*b*\tan(d*x)^3*\tan(c)^3 - 24*B*b^2*\tan(d*x)^3*\tan(c)^3 - 6*B*a^2 \\
& *\tan(d*x)^2*\tan(c)^4 - 12*A*a*b*\tan(d*x)^2*\tan(c)^4 + 6*B*b^2*\tan(d*x)^2*\tan \\
& (c)^4 + 8*B*a*b*\tan(d*x)^4*\tan(c) + 4*A*b^2*\tan(d*x)^4*\tan(c) - 36*B*a^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan \\
& (d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^2*\tan(c)^2 - 72*A \\
& *a*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 \\
& + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^2*\tan(c)^2 \\
& + 36*B*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan \\
& (c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^2*\tan \\
& (c)^2 - 36*A*a^2*\tan(d*x)^3*\tan(c)^2 + 96*B*a*b*\tan(d*x)^3*\tan(c)^2 + 48*A* \\
& b^2*\tan(d*x)^3*\tan(c)^2 - 36*A*a^2*\tan(d*x)^2*\tan(c)^3 + 96*B*a*b*\tan(d*x)^ \\
& 2*\tan(c)^3 + 48*A*b^2*\tan(d*x)^2*\tan(c)^3 + 8*B*a*b*\tan(d*x)*\tan(c)^4 + 4*A \\
& *b^2*\tan(d*x)*\tan(c)^4 - 3*B*b^2*\tan(d*x)^4 - 48*A*a^2*d*x*\tan(d*x)*\tan(c) \\
& + 96*B*a*b*d*x*\tan(d*x)*\tan(c) + 48*A*b^2*d*x*\tan(d*x)*\tan(c) + 12*B*a^2*\tan \\
& (d*x)^3*\tan(c) + 24*A*a*b*\tan(d*x)^3*\tan(c) - 24*B*b^2*\tan(d*x)^3*\tan(c) - \\
& 12*B*a^2*\tan(d*x)^2*\tan(c)^2 - 24*A*a*b*\tan(d*x)^2*\tan(c)^2 + 12*B*b^2*\tan \\
& (d*x)^2*\tan(c)^2 + 12*B*a^2*\tan(d*x)*\tan(c)^3 + 24*A*a*b*\tan(d*x)*\tan(c)^3 \\
& - 24*B*b^2*\tan(d*x)*\tan(c)^3 - 3*B*b^2*\tan(c)^4 - 8*B*a*b*\tan(d*x)^3 - 4*A* \\
& b^2*\tan(d*x)^3 + 24*B*a^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) \\
& + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1)) \\
& *\tan(d*x)*\tan(c) + 48*A*a*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) \\
& ) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1 \\
& ))*\tan(d*x)*\tan(c) - 24*B*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan \\
& (c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + \\
& 1))*\tan(d*x)*\tan(c) + 36*A*a^2*\tan(d*x)^2*\tan(c) - 96*B*a*b*\tan(d*x)^2*\tan \\
& (c) - 48*A*b^2*\tan(d*x)^2*\tan(c) + 36*A*a^2*\tan(d*x)*\tan(c)^2 - 96*B*a*b*\tan \\
& (d*x)*\tan(c)^2 - 48*A*b^2*\tan(d*x)*\tan(c)^2 - 8*B*a*b*\tan(c)^3 - 4*A*b^2*\tan \\
& (c)^3 + 12*A*a^2*d*x - 24*B*a*b*d*x - 12*A*b^2*d*x - 6*B*a^2*\tan(d*x)^2 - \\
& 12*A*a*b*\tan(d*x)^2 + 6*B*b^2*\tan(d*x)^2 + 12*B*a^2*\tan(d*x)*\tan(c) + 24*A \\
& *a*b*\tan(d*x)*\tan(c) - 24*B*b^2*\tan(d*x)*\tan(c) - 6*B*a^2*\tan(c)^2 - 12*A*a \\
& *b*\tan(c)^2 + 6*B*b^2*\tan(c)^2 - 6*B*a^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan \\
& (d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/ \\
& (\tan(c)^2 + 1)) - 12*A*a*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c)
\end{aligned}$$

$$+ \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1) / (\tan(c)^2 + 1) \\ + 6Bb^2 \log(4(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1) / (\tan(c)^2 + 1)) - 12Aa^2 \tan(dx) \\ + 24Bab \tan(dx) + 12Ab^2 \tan(dx) - 12Aa^2 \tan(c) + 24Bab \tan(c) + 12Ab^2 \tan(c) - 6Ba^2 - 12Aab + 9Bb^2) / (d \tan(dx)^4 \tan(c)^4 - 4d \tan(dx)^3 \tan(c)^3 + 6d \tan(dx)^2 \tan(c)^2 - 4d \tan(dx) \tan(c) + d)$$

**Mupad [B]**

time = 6.20, size = 151, normalized size = 1.02

$$x(-Aa^2 + 2Bab + Ab^2) + \frac{\tan(c+dx)^3 \left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)}{d} - \frac{\tan(c+dx)(-Aa^2 + 2Bab + Ab^2)}{d} - \frac{\ln(\tan(c+dx)^2 + 1) \left(\frac{Ba^2}{2} + Aab - \frac{Bb^2}{2}\right)}{d} + \frac{\tan(c+dx)^2 \left(\frac{Ba^2}{2} + Aab - \frac{Bb^2}{2}\right)}{d} + \frac{Bb^2 \tan(c+dx)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + dx)^2\*(A + B\*tan(c + dx))\*(a + b\*tan(c + dx))^2,x)

[Out] x\*(Ab^2 - Aa^2 + 2Bab) + (tan(c + dx)^3\*((Ab^2)/3 + (2Bab)/3))/d - (tan(c + dx)\*(Ab^2 - Aa^2 + 2Bab))/d - (log(tan(c + dx)^2 + 1)\*((Ba^2)/2 - (Bb^2)/2 + Aab))/d + (tan(c + dx)^2\*((Ba^2)/2 - (Bb^2)/2 + Aab))/d + (Bb^2\*tan(c + dx)^4)/(4\*d)

### 3.241 $\int \tan(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=112

$$-((2aAb + a^2B - b^2B)x) - \frac{(a^2A - Ab^2 - 2abB) \log(\cos(c + dx))}{d} + \frac{b(aA - bB) \tan(c + dx)}{d} + \frac{A(a + b \tan(c + dx))^3}{3bd}$$

[Out]  $-(2Aa^2b + B^2a^2 - B^2b^2)x - (A^2a^2 - Ab^2 - 2B^2ab) \ln(\cos(dx+c))/d + b(A^2a - B^2b) \tan(dx+c)/d + 1/2A^2(a+b \tan(dx+c))^2/d + 1/3B^2(a+b \tan(dx+c))^3/b/d$

Rubi [A]

time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3673, 3609, 3606, 3556}

$$-\frac{(a^2A - 2abB - Ab^2) \log(\cos(c + dx))}{d} - x(a^2B + 2aAb - b^2B) + \frac{b(aA - bB) \tan(c + dx)}{d} + \frac{A(a + b \tan(c + dx))^2}{2d} + \frac{B(a + b \tan(c + dx))^3}{3bd}$$

Antiderivative was successfully verified.

[In] `Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]), x]`

[Out]  $-\frac{((2a^2A^2b + a^2B^2 - b^2B^2)x) - ((a^2A^2 - Ab^2 - 2a^2bB) \text{Log}[\text{Cos}[c + d*x]])}{d} + \frac{b(a^2A - b^2B) \text{Tan}[c + d*x]}{d} + \frac{A(a + b \text{Tan}[c + d*x])^2}{2d} + \frac{B(a + b \text{Tan}[c + d*x])^3}{3b^2d}$

Rule 3556

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3606

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]`

Rule 3609

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]`

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \tan(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^3}{3bd} + \int (-B + A \tan(c + dx)) (a + b \tan(c + dx))^2 dx \\ &= \frac{A(a + b \tan(c + dx))^2}{2d} + \frac{B(a + b \tan(c + dx))}{3bd} \\ &= -(2aAb + a^2B - b^2B)x + \frac{b(aA - bB) \tan(c + dx)}{d} \\ &= -(2aAb + a^2B - b^2B)x - \frac{(a^2A - Ab^2 - 2ab^2) \tan(c + dx)}{d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.22, size = 172, normalized size = 1.54

$$\frac{2B(a + b \tan(c + dx))^3 + 3(aA + bB)(i((a + ib)^2 \log(i - \tan(c + dx)) - (a - ib)^2 \log(i + \tan(c + dx))) - 2b^2 \tan(c + dx) + 3A((ia - b)^3 \log(i - \tan(c + dx)) - (ia + b)^3 \log(i + \tan(c + dx)) + 6ab^2 \tan(c + dx) + b^3 \tan^2(c + dx))}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

```
[Out] (2*B*(a + b*Tan[c + d*x])^3 + 3*(a*A + b*B)*(I*((a + I*b)^2*Log[I - Tan[c +
d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]]) - 2*b^2*Tan[c + d*x]) + 3*A*((I
*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b
^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(6*b*d)
```

**Maple [A]**

time = 0.06, size = 135, normalized size = 1.21

method	result
norman	$(-2Aab - a^2B + b^2B)x + \frac{(2Aab + a^2B - b^2B) \tan(dx+c)}{d} + \frac{b(Ab + 2aB) \tan^2(dx+c)}{2d} + \frac{b^2B \tan^3(dx+c)}{3d}$
derivativedivides	$\frac{\frac{b^2B \tan^3(dx+c)}{3} + \frac{Ab^2 \tan^2(dx+c)}{2} + Bab \tan^2(dx+c) + 2Aab \tan(dx+c) + B a^2 \tan(dx+c) - b^2B \tan(dx+c) + \frac{(a^2A - Ab^2 - 2ab^2) \tan(dx+c)}{d}}{d}$
default	$\frac{\frac{b^2B \tan^3(dx+c)}{3} + \frac{Ab^2 \tan^2(dx+c)}{2} + Bab \tan^2(dx+c) + 2Aab \tan(dx+c) + B a^2 \tan(dx+c) - b^2B \tan(dx+c) + \frac{(a^2A - Ab^2 - 2ab^2) \tan(dx+c)}{d}}{d}$

risch

$$-2Aabx - B a^2 x + B b^2 x - 2iBabx + \frac{2ia^2 Ac}{d} - \frac{2iAb^2 c}{d} - \frac{4iBabc}{d} + \frac{2i(-3iAb^2 e^{4i(dx+c)} - 6iBab e^{4i(dx+c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (1/3 * b^2 * B * \tan(d*x+c)^3 + 1/2 * A * b^2 * \tan(d*x+c)^2 + B * a * b * \tan(d*x+c)^2 + 2 * A * a * b * \tan(d*x+c) + B * a^2 * \tan(d*x+c) - b^2 * B * \tan(d*x+c) + 1/2 * (A * a^2 - A * b^2 - 2 * B * a * b) * \ln(1 + \tan(d*x+c)^2) + (-2 * A * a * b - B * a^2 + B * b^2) * \arctan(\tan(d*x+c)))$

**Maxima [A]**

time = 0.57, size = 120, normalized size = 1.07

$$\frac{2 B b^2 \tan(dx+c)^3 + 3(2 Bab + Ab^2) \tan(dx+c)^2 - 6(Ba^2 + 2 Aab - Bb^2)(dx+c) + 3(Aa^2 - 2 Bab - Ab^2) \log(\tan(dx+c)^2 + 1) + 6(Ba^2 + 2 Aab - Bb^2) \tan(dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{6} * (2 * B * b^2 * \tan(dx+c)^3 + 3 * (2 * B * a * b + A * b^2) * \tan(dx+c)^2 - 6 * (B * a^2 + 2 * A * a * b - B * b^2) * (dx+c) + 3 * (A * a^2 - 2 * B * a * b - A * b^2) * \log(\tan(dx+c)^2 + 1) + 6 * (B * a^2 + 2 * A * a * b - B * b^2) * \tan(dx+c)) / d$

**Fricas [A]**

time = 0.52, size = 119, normalized size = 1.06

$$\frac{2 B b^2 \tan(dx+c)^3 - 6(Ba^2 + 2 Aab - Bb^2)dx + 3(2 Bab + Ab^2) \tan(dx+c)^2 - 3(Aa^2 - 2 Bab - Ab^2) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) + 6(Ba^2 + 2 Aab - Bb^2) \tan(dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{6} * (2 * B * b^2 * \tan(dx+c)^3 - 6 * (B * a^2 + 2 * A * a * b - B * b^2) * dx + 3 * (2 * B * a * b + A * b^2) * \tan(dx+c)^2 - 3 * (A * a^2 - 2 * B * a * b - A * b^2) * \log(1 / (\tan(dx+c)^2 + 1)) + 6 * (B * a^2 + 2 * A * a * b - B * b^2) * \tan(dx+c)) / d$

**Sympy [A]**

time = 0.12, size = 192, normalized size = 1.71

$$\begin{cases} \frac{Aa^2 \log(\tan^2(c+dx)+1) - 2Aabx + \frac{2Aab \tan(c+dx)}{d} - \frac{Ab^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Ab^2 \tan^2(c+dx)}{2d} - Ba^2 x + \frac{Ba^2 \tan(c+dx)}{d} - \frac{Bab \log(\tan^2(c+dx)+1)}{d} + \frac{Bab \tan^2(c+dx)}{d} + Bb^2 x + \frac{Bb^2 \tan^3(c+dx)}{3d} - \frac{Bb^2 \tan(c+dx)}{d}}{x(A+B \tan(c))(a+b \tan(c))^2 \tan(c)} & \text{for } d \neq 0 \\ x(A+B \tan(c))(a+b \tan(c))^2 \tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((A*a**2*log(tan(c+d*x)**2+1)/(2*d) - 2*A*a*b*x + 2*A*a*b*tan(c+d*x)/d - A*b**2*log(tan(c+d*x)**2+1)/(2*d) + A*b**2*tan(c+d*x)**2`



$$\frac{1}{(2d) - B a^2 x + B a^2 \tan(c + dx)/d - B a b \log(\tan(c + dx)^2 + 1)/d + B a b \tan(c + dx)^2/d + B b^2 x + B b^2 \tan(c + dx)^3/(3d) - B b^2 \tan(c + dx)/d, \text{Ne}(d, 0)), (x(A + B \tan(c))(a + b \tan(c))^2 \tan(c), \text{True}))$$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. 2(108) = 216.

time = 1.12, size = 1509, normalized size = 13.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(a+b\*tan(dx+c))^2\*(A+B\*tan(dx+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/6*(6*B*a^2*d*x*\tan(dx)^3*\tan(c)^3 + 12*A*a*b*d*x*\tan(dx)^3*\tan(c)^3 - \\ & 6*B*b^2*d*x*\tan(dx)^3*\tan(c)^3 + 3*A*a^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1) \\ & /(\tan(c)^2 + 1))*\tan(dx)^3*\tan(c)^3 - 6*B*a*b*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) \\ & + 1)/(\tan(c)^2 + 1))*\tan(dx)^3*\tan(c)^3 - 3*A*b^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) \\ & + 1)/(\tan(c)^2 + 1))*\tan(dx)^3*\tan(c)^3 - 18*B*a^2*d*x*\tan(dx)^2*\tan(c)^2 - 36*A*a*b*d*x*\tan(dx)^2*\tan(c)^2 + 18*B*b^2*d*x*\tan(dx)^2*\tan(c)^2 \\ & - 6*B*a*b*\tan(dx)^3*\tan(c)^3 - 3*A*b^2*\tan(dx)^3*\tan(c)^3 - 9*A*a^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^2*\tan(c)^2 + 18*B*a \\ & *b*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^2*\tan(c)^2 + \\ & 9*A*b^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^2*\tan(c)^2 + \\ & 6*B*a^2*\tan(dx)^3*\tan(c)^2 + 12*A*a*b*\tan(dx)^3*\tan(c)^2 - 6*B*b^2*\tan(dx)^3*\tan(c)^2 + 6*B*a^2*\tan(dx)^2*\tan(c)^3 + 12*A*a*b*\tan(dx)^2*\tan(c)^3 \\ & - 6*B*b^2*\tan(dx)^2*\tan(c)^3 + 18*B*a^2*d*x*\tan(dx)*\tan(c) + 36*A*a*b*d*x*\tan(dx)*\tan(c) - 18*B*b^2*d*x*\tan(dx)*\tan(c) - 6*B*a*b*\tan(dx)^3*\tan(c) \\ & - 3*A*b^2*\tan(dx)^3*\tan(c) + 6*B*a*b*\tan(dx)^2*\tan(c)^2 + 3*A*b^2*\tan(dx)^2*\tan(c)^2 - 6*B*a*b*\tan(dx)*\tan(c)^3 - 3*A*b^2*\tan(dx)*\tan(c)^3 \\ & + 2*B*b^2*\tan(dx)^3 + 9*A*a^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)*\tan(c) \\ & - 18*B*a*b*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)*\tan(c) \\ & - 9*A*b^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)*\tan(c) \\ & - 12*B*a^2*\tan(dx)^2*\tan(c) - 24*A*a*b*\tan(dx)^2*\tan(c) + 18*B*b^2*\tan(dx)^2*\tan(c) - 12*B*a^2*\tan(dx)*\tan(c)^2 - 24*A*a \end{aligned}$$

```

b*tan(d*x)*tan(c)^2 + 18*B*b^2*tan(d*x)*tan(c)^2 + 2*B*b^2*tan(c)^3 - 6*B*a
^2*d*x - 12*A*a*b*d*x + 6*B*b^2*d*x + 6*B*a*b*tan(d*x)^2 + 3*A*b^2*tan(d*x)
^2 - 6*B*a*b*tan(d*x)*tan(c) - 3*A*b^2*tan(d*x)*tan(c) + 6*B*a*b*tan(c)^2 +
3*A*b^2*tan(c)^2 - 3*A*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
)) + 6*B*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 3*A*b^2*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 6*B*a^2*tan(d*x) + 12*A*a
*b*tan(d*x) - 6*B*b^2*tan(d*x) + 6*B*a^2*tan(c) + 12*A*a*b*tan(c) - 6*B*b^2
*tan(c) + 6*B*a*b + 3*A*b^2)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)
^2 + 3*d*tan(d*x)*tan(c) - d)

```

**Mupad [B]**

time = 6.21, size = 121, normalized size = 1.08

$$\frac{\tan(c+dx)^2 \left(\frac{Ab^2}{2} + Bab\right)}{d} - x(Ba^2 + 2Aab - Bb^2) + \frac{\tan(c+dx)(Ba^2 + 2Aab - Bb^2)}{d} - \frac{\ln(\tan(c+dx)^2 + 1) \left(-\frac{Aa^2}{2} + Bab + \frac{Ab^2}{2}\right)}{d} + \frac{Bb^2 \tan(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (tan(c + d*x)^2*((A*b^2)/2 + B*a*b))/d - x*(B*a^2 - B*b^2 + 2*A*a*b) + (tan
(c + d*x)*(B*a^2 - B*b^2 + 2*A*a*b))/d - (log(tan(c + d*x)^2 + 1)*((A*b^2)/
2 - (A*a^2)/2 + B*a*b))/d + (B*b^2*tan(c + d*x)^3)/(3*d)
```

### 3.242 $\int (a + b \tan(c + dx))^2 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=87

$$(a^2A - Ab^2 - 2abB)x - \frac{(2aAb + a^2B - b^2B) \log(\cos(c + dx))}{d} + \frac{b(Ab + aB) \tan(c + dx)}{d} + \frac{B(a + b \tan(c + dx))^2}{2d}$$

[Out] (A\*a^2-A\*b^2-2\*B\*a\*b)\*x-(2\*A\*a\*b+B\*a^2-B\*b^2)\*ln(cos(d\*x+c))/d+b\*(A\*b+B\*a)\*tan(d\*x+c)/d+1/2\*B\*(a+b\*tan(d\*x+c))^2/d

**Rubi [A]**

time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3609, 3606, 3556}

$$-\frac{(a^2B + 2aAb - b^2B) \log(\cos(c + dx))}{d} + x(a^2A - 2abB - Ab^2) + \frac{b(aB + Ab) \tan(c + dx)}{d} + \frac{B(a + b \tan(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (a^2\*A - A\*b^2 - 2\*a\*b\*B)\*x - ((2\*a\*A\*b + a^2\*B - b^2\*B)\*Log[Cos[c + d\*x]])/d + (b\*(A\*b + a\*B)\*Tan[c + d\*x])/d + (B\*(a + b\*Tan[c + d\*x])^2)/(2\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rule 3609

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int (a + b \tan(c + dx))^2 (A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^2}{2d} + \int (a + b \tan(c + dx))(aA - bB + \\ &= (a^2 A - Ab^2 - 2abB) x + \frac{b(Ab + aB) \tan(c + dx)}{d} + \frac{B(a + b \tan(c + dx))^2}{2d} \\ &= (a^2 A - Ab^2 - 2abB) x - \frac{(2aAb + a^2 B - b^2 B) \log(\cos(c + dx))}{d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.30, size = 96, normalized size = 1.10

$$\frac{(a + ib)^2(-iA + B) \log(i - \tan(c + dx)) + (a - ib)^2(iA + B) \log(i + \tan(c + dx)) + 2b(Ab + 2aB) \tan(c + dx) + b^2 B \tan^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] ((a + I\*b)^2\*((-I)\*A + B)\*Log[I - Tan[c + d\*x]] + (a - I\*b)^2\*(I\*A + B)\*Log[I + Tan[c + d\*x]] + 2\*b\*(A\*b + 2\*a\*B)\*Tan[c + d\*x] + b^2\*B\*Tan[c + d\*x]^2)/(2\*d)

**Maple [A]**

time = 0.05, size = 97, normalized size = 1.11

method	result
norman	$(a^2 A - Ab^2 - 2Bab) x + \frac{b(Ab+2aB) \tan(dx+c)}{d} + \frac{b^2 B (\tan^2(dx+c))}{2d} + \frac{(2Aab+a^2 B-b^2 B) \ln(1+\tan^2(dx+c))}{2d}$
derivativedivides	$\frac{\frac{b^2 B (\tan^2(dx+c))}{2} + Ab^2 \tan(dx+c) + 2Bab \tan(dx+c) + \frac{(2Aab+a^2 B-b^2 B) \ln(1+\tan^2(dx+c))}{2}}{d} + (a^2 A - Ab^2 - 2Bab) \arctan(\tan(dx+c))$
default	$\frac{\frac{b^2 B (\tan^2(dx+c))}{2} + Ab^2 \tan(dx+c) + 2Bab \tan(dx+c) + \frac{(2Aab+a^2 B-b^2 B) \ln(1+\tan^2(dx+c))}{2}}{d} + (a^2 A - Ab^2 - 2Bab) \arctan(\tan(dx+c))$
risch	$\frac{4iAabc}{d} - \frac{2iBb^2c}{d} - iBb^2x + Aa^2x - Ab^2x - 2Babx + \frac{2ia^2Bc}{d} + iBa^2x + \frac{2ib(bee^{2i(dx+c)}A+2aB)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(1/2\*b^2\*B\*tan(d\*x+c)^2+A\*b^2\*tan(d\*x+c)+2\*B\*a\*b\*tan(d\*x+c)+1/2\*(2\*A\*a\*b+B\*a^2-B\*b^2)\*ln(1+tan(d\*x+c)^2)+(A\*a^2-A\*b^2-2\*B\*a\*b)\*arctan(tan(d\*x+c)))

**Maxima [A]**

time = 0.51, size = 91, normalized size = 1.05

$$\frac{Bb^2 \tan(dx+c)^2 + 2(Aa^2 - 2Bab - Ab^2)(dx+c) + (Ba^2 + 2Aab - Bb^2) \log(\tan(dx+c)^2 + 1) + 2(2Bab + Ab^2) \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(B*b^2*\tan(d*x + c)^2 + 2*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b - B*b^2)*\log(\tan(d*x + c)^2 + 1) + 2*(2*B*a*b + A*b^2)*\tan(d*x + c))/d$

**Fricas** [A]

time = 0.44, size = 91, normalized size = 1.05

$$\frac{Bb^2 \tan(dx + c)^2 + 2(Aa^2 - 2Bab - Ab^2)dx - (Ba^2 + 2Aab - Bb^2) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2(2Bab + Ab^2) \tan(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{2}*(B*b^2*\tan(d*x + c)^2 + 2*(A*a^2 - 2*B*a*b - A*b^2)*d*x - (B*a^2 + 2*A*a*b - B*b^2)*\log(1/(\tan(d*x + c)^2 + 1)) + 2*(2*B*a*b + A*b^2)*\tan(d*x + c))/d$

**Sympy** [A]

time = 0.10, size = 143, normalized size = 1.64

$$\begin{cases} Aa^2x + \frac{Aab \log(\tan^2(c+dx)+1)}{d} - Ab^2x + \frac{Ab^2 \tan(c+dx)}{d} + \frac{Ba^2 \log(\tan^2(c+dx)+1)}{2d} - 2Babx + \frac{2Bab \tan(c+dx)}{d} - \frac{Bb^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^2 \tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A + B \tan(c))(a + b \tan(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((A*a**2*x + A*a*b*log(tan(c + d*x)**2 + 1)/d - A*b**2*x + A*b**2*tan(c + d*x)/d + B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*B*a*b*x + 2*B*a*b*tan(c + d*x)/d - B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**2, True))`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(85) = 170.

time = 0.81, size = 901, normalized size = 10.36

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out]  $\frac{1}{2}*(2*A*a^2*d*x*\tan(d*x)^2*\tan(c)^2 - 4*B*a*b*d*x*\tan(d*x)^2*\tan(c)^2 - 2*A*b^2*d*x*\tan(d*x)^2*\tan(c)^2 - B*a^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))/(ta$

```

n(c)^2 + 1)) * tan(d*x)^2 * tan(c)^2 - 2*A*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1)) * tan(d*x)^2 * tan(c)^2 + B*b^2*log(4*(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(c)^2 + 1)) * tan(d*x)^2 * tan(c)^2 - 4*A*a^2*d*x*tan(d*x)*tan(c) + 8*
B*a*b*d*x*tan(d*x)*tan(c) + 4*A*b^2*d*x*tan(d*x)*tan(c) + B*b^2*tan(d*x)^2*
tan(c)^2 + 2*B*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) * tan(d
*x)*tan(c) + 4*A*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) * tan(d
*x)*tan(c) - 2*B*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) * tan(
d*x)*tan(c) - 4*B*a*b*tan(d*x)^2*tan(c) - 2*A*b^2*tan(d*x)^2*tan(c) - 4*B*a
*b*tan(d*x)*tan(c)^2 - 2*A*b^2*tan(d*x)*tan(c)^2 + 2*A*a^2*d*x - 4*B*a*b*d*
x - 2*A*b^2*d*x + B*b^2*tan(d*x)^2 + B*b^2*tan(c)^2 - B*a^2*log(4*(tan(d*x)
^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 2*A*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1)) + B*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
)) + 4*B*a*b*tan(d*x) + 2*A*b^2*tan(d*x) + 4*B*a*b*tan(c) + 2*A*b^2*tan(c)
+ B*b^2)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c) + d)

```

**Mupad [B]**

time = 6.23, size = 91, normalized size = 1.05

$$\frac{\ln(\tan(c+dx)^2+1) \left( \frac{Ba^2}{2} + Aab - \frac{Bb^2}{2} \right)}{d} - x(-Aa^2 + 2Bab + Ab^2) + \frac{\tan(c+dx)(Ab^2 + 2Bab)}{d} + \frac{Bb^2 \tan(c+dx)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] (log(tan(c + d\*x)^2 + 1)\*((B\*a^2)/2 - (B\*b^2)/2 + A\*a\*b))/d - x\*(A\*b^2 - A\*a^2 + 2\*B\*a\*b) + (tan(c + d\*x)\*(A\*b^2 + 2\*B\*a\*b))/d + (B\*b^2\*tan(c + d\*x)^2)/(2\*d)

### 3.243 $\int \cot(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=70

$$(2aAb + a^2B - b^2B) x - \frac{b(Ab + 2aB) \log(\cos(c + dx))}{d} + \frac{a^2A \log(\sin(c + dx))}{d} + \frac{b^2B \tan(c + dx)}{d}$$

[Out] (2\*A\*a\*b+B\*a^2-B\*b^2)\*x-b\*(A\*b+2\*B\*a)\*ln(cos(d\*x+c))/d+a^2\*A\*ln(sin(d\*x+c))/d+b^2\*B\*tan(d\*x+c)/d

**Rubi** [A]

time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3687, 3705, 3556}

$$x(a^2B + 2aAb - b^2B) + \frac{a^2A \log(\sin(c + dx))}{d} - \frac{b(2aB + Ab) \log(\cos(c + dx))}{d} + \frac{b^2B \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (2\*a\*A\*b + a^2\*B - b^2\*B)\*x - (b\*(A\*b + 2\*a\*B)\*Log[Cos[c + d\*x]])/d + (a^2\*A\*Log[Sin[c + d\*x]])/d + (b^2\*B\*Tan[c + d\*x])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3687

Int[(((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]))/((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[b^2\*B\*(Tan[e + f\*x]/(d\*f)), x] + Dist[1/d, Int[(a^2\*A\*d - b^2\*B\*c + (2\*a\*A\*b + B\*(a^2 - b^2))\*d\*Tan[e + f\*x] + (A\*b^2\*d - b\*B\*(b\*c - 2\*a\*d))\*Tan[e + f\*x]^2)/(c + d\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3705

Int[((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2/tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[B\*x, x] + (Dist[A, Int[1/Tan[e + f\*x], x], x] + Dist[C, Int[Tan[e + f\*x], x], x]) /; FreeQ[{e, f, A, B, C}, x] && NeQ[A, C]

Rubi steps

$$\begin{aligned} \int \cot(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx &= \frac{b^2 B \tan(c+dx)}{d} + \int \cot(c+dx)(a^2 A + (2aAb \\ &= (2aAb + a^2 B - b^2 B) x + \frac{b^2 B \tan(c+dx)}{d} + (a \\ &= (2aAb + a^2 B - b^2 B) x - \frac{b(Ab + 2aB) \log(\cos(c+dx))}{d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.20, size = 93, normalized size = 1.33

$$\frac{(a+ib)^2(A+iB) \log(i-\tan(c+dx)) - 2a^2 A \log(\tan(c+dx)) + (a-ib)^2(A-iB) \log(i+\tan(c+dx)) - 2bB(a+b \tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] -1/2\*((a + I\*b)^2\*(A + I\*B)\*Log[I - Tan[c + d\*x]] - 2\*a^2\*A\*Log[Tan[c + d\*x]] + (a - I\*b)^2\*(A - I\*B)\*Log[I + Tan[c + d\*x]] - 2\*b\*B\*(a + b\*Tan[c + d\*x]))/d

**Maple [A]**

time = 0.15, size = 82, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2 A \ln(\sin(dx+c)) + a^2 B(dx+c) + 2Aab(dx+c) - 2Bab \ln(\cos(dx+c)) - A b^2 \ln(\cos(dx+c)) + b^2 B(\tan(dx+c) - dx - c)}{d}$
default	$\frac{a^2 A \ln(\sin(dx+c)) + a^2 B(dx+c) + 2Aab(dx+c) - 2Bab \ln(\cos(dx+c)) - A b^2 \ln(\cos(dx+c)) + b^2 B(\tan(dx+c) - dx - c)}{d}$
norman	$(2Aab + a^2 B - b^2 B) x + \frac{b^2 B \tan(dx+c)}{d} + \frac{a^2 A \ln(\tan(dx+c))}{d} - \frac{(a^2 A - A b^2 - 2Bab) \ln(1 + \tan^2(dx+c))}{2d}$
risch	$2Aabx + B a^2 x - B b^2 x + 2iBabx - \frac{2ia^2 Ac}{d} + \frac{4iBabc}{d} + \frac{2ib^2 B}{d(e^{2i(dx+c)} + 1)} + \frac{2iA b^2 c}{d} - iA a^2 x + i$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(a^2\*A\*ln(sin(d\*x+c))+a^2\*B\*(d\*x+c)+2\*A\*a\*b\*(d\*x+c)-2\*B\*a\*b\*ln(cos(d\*x+c))-A\*b^2\*ln(cos(d\*x+c))+b^2\*B\*(tan(d\*x+c)-d\*x-c))

**Maxima [A]**

time = 0.53, size = 85, normalized size = 1.21

$$\frac{2Aa^2 \log(\tan(dx+c)) + 2Bb^2 \tan(dx+c) + 2(Ba^2 + 2Aab - Bb^2)(dx+c) - (Aa^2 - 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{2d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(2*A*a^2*\log(\tan(dx + c)) + 2*B*b^2*\tan(dx + c) + 2*(B*a^2 + 2*A*a*b - B*b^2)*(dx + c) - (A*a^2 - 2*B*a*b - A*b^2)*\log(\tan(dx + c)^2 + 1))/d$

**Fricas** [A]

time = 0.46, size = 92, normalized size = 1.31

$$\frac{Aa^2 \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) + 2Bb^2 \tan(dx+c) + 2(Ba^2 + 2Aab - Bb^2)dx - (2Bab + Ab^2) \log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(A*a^2*\log(\tan(dx + c)^2/(\tan(dx + c)^2 + 1)) + 2*B*b^2*\tan(dx + c) + 2*(B*a^2 + 2*A*a*b - B*b^2)*dx - (2*B*a*b + A*b^2)*\log(1/(\tan(dx + c)^2 + 1)))/d$

**Sympy** [A]

time = 0.36, size = 129, normalized size = 1.84

$$\begin{cases} -\frac{Aa^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Aa^2 \log(\tan(c+dx))}{d} + 2Aabx + \frac{Ab^2 \log(\tan^2(c+dx)+1)}{2d} + Ba^2x + \frac{Bab \log(\tan^2(c+dx)+1)}{d} - Bb^2x + \frac{Bb^2 \tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + B \tan(c)) (a + b \tan(c))^2 \cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((-A\*a\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*a\*\*2\*log(tan(c + d\*x)))/d + 2\*A\*a\*b\*x + A\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*a\*\*2\*x + B\*a\*b\*log(tan(c + d\*x)\*\*2 + 1)/d - B\*b\*\*2\*x + B\*b\*\*2\*tan(c + d\*x)/d, Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*2\*cot(c), True))

**Giac** [A]

time = 0.80, size = 86, normalized size = 1.23

$$\frac{2Aa^2 \log(|\tan(dx+c)|) + 2Bb^2 \tan(dx+c) + 2(Ba^2 + 2Aab - Bb^2)(dx+c) - (Aa^2 - 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{2}*(2*A*a^2*\log(\text{abs}(\tan(dx + c))) + 2*B*b^2*\tan(dx + c) + 2*(B*a^2 + 2*A*a*b - B*b^2)*(dx + c) - (A*a^2 - 2*B*a*b - A*b^2)*\log(\tan(dx + c)^2 + 1))/d$

**Mupad [B]**

time = 6.37, size = 90, normalized size = 1.29

$$\frac{A a^2 \ln(\tan(c + dx))}{d} + \frac{\ln(\tan(c + dx) + 1i) (A - B 1i) (b + a 1i)^2}{2d} + \frac{B b^2 \tan(c + dx)}{d} + \frac{\ln(\tan(c + dx) - 1i) (A + B 1i) (-b + a 1i)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] (A\*a^2\*log(tan(c + d\*x)))/d + (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b)^2)/(2\*d) + (B\*b^2\*tan(c + d\*x))/d + (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a\*1i - b)^2)/(2\*d)

### 3.244 $\int \cot^2(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=72

$$-((a^2A - Ab^2 - 2abB)x) - \frac{a^2A \cot(c+dx)}{d} - \frac{b^2B \log(\cos(c+dx))}{d} + \frac{a(2Ab + aB) \log(\sin(c+dx))}{d}$$

[Out]  $-(A*a^2-A*b^2-2*B*a*b)*x-a^2*A*\cot(d*x+c)/d-b^2*B*\ln(\cos(d*x+c))/d+a*(2*A*b+B*a)*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {3685, 3705, 3556}

$$-x(a^2A - 2abB - Ab^2) - \frac{a^2A \cot(c+dx)}{d} + \frac{a(aB + 2Ab) \log(\sin(c+dx))}{d} - \frac{b^2B \log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(a^2*A - A*b^2 - 2*a*b*B)*x - (a^2*A*\text{Cot}[c + d*x])/d - (b^2*B*\text{Log}[\text{Cos}[c + d*x]])/d + (a*(2*A*b + a*B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3685

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^n, x\_Symbol] \rightarrow \text{Simp}[-(B*c - A*d)*(b*c - a*d)^2*((c + d*\text{Tan}[e + f*x])^{n+1}/(f*d^2*(n+1)*(c^2 + d^2))), x] + \text{Dist}[1/(d*(c^2 + d^2)), \text{Int}[(c + d*\text{Tan}[e + f*x])^{n+1}]*\text{Simp}[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*\text{Tan}[e + f*x] + b^2*B*(c^2 + d^2)*\text{Tan}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[n, -1]$

Rule 3705

$\text{Int}[(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2)/\text{tan}[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[B*x, x] + (\text{Dist}[A, \text{Int}[1/\text{Tan}[e + f*x], x], x] + \text{Dist}[C, \text{Int}[\text{Tan}[e + f*x], x], x]) /; \text{FreeQ}\{e, f, A, B, C$

}, x] && NeQ[A, C]

Rubi steps

$$\begin{aligned} \int \cot^2(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx &= -\frac{a^2 A \cot(c+dx)}{d} + \int \cot(c+dx) (a(2Ab+a^2) + b^2) dx \\ &= -(a^2 A - Ab^2 - 2abB) x - \frac{a^2 A \cot(c+dx)}{d} + \int \cot(c+dx) (a(2Ab+a^2) + b^2) dx \\ &= -(a^2 A - Ab^2 - 2abB) x - \frac{a^2 A \cot(c+dx)}{d} - \int \cot(c+dx) (a(2Ab+a^2) + b^2) dx \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.18, size = 100, normalized size = 1.39

$$\frac{-2a^2 A \cot(c+dx) + i(a+ib)^2(A+iB) \log(i - \tan(c+dx)) + 2a(2Ab+aB) \log(\tan(c+dx)) - (a-ib)^2(iA+B) \log(i + \tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (-2\*a^2\*A\*Cot[c + d\*x] + I\*(a + I\*b)^2\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + 2\*a\*(2\*A\*b + a\*B)\*Log[Tan[c + d\*x]] - (a - I\*b)^2\*(I\*A + B)\*Log[I + Tan[c + d\*x]])/(2\*d)

**Maple [A]**

time = 0.14, size = 84, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2 A(-\cot(dx+c)-dx-c)+a^2 B \ln(\sin(dx+c))+2Aab \ln(\sin(dx+c))+2Bab(dx+c)+A b^2(dx+c)-b^2 B \ln(\cos(dx+c))}{d}$
default	$\frac{a^2 A(-\cot(dx+c)-dx-c)+a^2 B \ln(\sin(dx+c))+2Aab \ln(\sin(dx+c))+2Bab(dx+c)+A b^2(dx+c)-b^2 B \ln(\cos(dx+c))}{d}$
norman	$\frac{(-a^2 A+A b^2+2Bab) x \tan(dx+c)-\frac{a^2 A}{d}}{\tan(dx+c)} + \frac{a(2Ab+aB) \ln(\tan(dx+c))}{d} - \frac{(2Aab+a^2 B-b^2 B) \ln(1+\tan^2(dx+c))}{2d}$
risch	$iB b^2 x - \frac{4iAabc}{d} - \frac{2ia^2 Bc}{d} - A a^2 x + A b^2 x + 2Babx + \frac{2iB b^2 c}{d} - iB a^2 x - 2iAabx - \frac{2ia}{d(e^{2i(dx+c)})}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(a^2\*A\*(-cot(d\*x+c)-d\*x-c)+a^2\*B\*ln(sin(d\*x+c))+2\*A\*a\*b\*ln(sin(d\*x+c))+2\*B\*a\*b\*(d\*x+c)+A\*b^2\*(d\*x+c)-b^2\*B\*ln(cos(d\*x+c)))

**Maxima [A]**

time = 0.51, size = 93, normalized size = 1.29

$$\frac{2(Aa^2 - 2Bab - Ab^2)(dx + c) + (Ba^2 + 2Aab - Bb^2) \log(\tan(dx + c)^2 + 1) - 2(Ba^2 + 2Aab) \log(\tan(dx + c)) + \frac{2Aa^2}{\tan(dx + c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1) - 2*(B*a^2 + 2*A*a*b)*log(tan(d*x + c)) + 2*A*a^2/tan(d*x + c))/d
```

**Fricas [A]**

time = 0.48, size = 112, normalized size = 1.56

$$\frac{Bb^2 \log\left(\frac{1}{\tan(dx+c)^2+1}\right) \tan(dx+c) + 2(Aa^2 - 2Bab - Ab^2)dx \tan(dx+c) + 2Aa^2 - (Ba^2 + 2Aab) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)}{2d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(B*b^2*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(A*a^2 - 2*B*a*b - A*b^2)*d*x*tan(d*x + c) + 2*A*a^2 - (B*a^2 + 2*A*a*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c))/(d*tan(d*x + c))
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 167 vs.  $2(66) = 132$ .

time = 0.65, size = 167, normalized size = 2.32

$$\begin{cases} \infty Aa^2 x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c))^2 \cot^2(c) & \text{for } d = 0 \\ -Aa^2 x - \frac{Aa^2}{d \tan(c+dx)} - \frac{Aab \log(\tan^2(c+dx)+1)}{d} + \frac{2Aab \log(\tan(c+dx))}{d} + Ab^2 x - \frac{Ba^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Ba^2 \log(\tan(c+dx))}{d} + 2Babx + \frac{Bb^2 \log(\tan^2(c+dx)+1)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*A*a**2*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(A + B*tan(c))*(a + b*tan(c))**2*cot(c)**2, Eq(d, 0)), (-A*a**2*x - A*a**2/(d*tan(c + d*x)) - A*a*b*log(tan(c + d*x)**2 + 1)/d + 2*A*a*b*log(tan(c + d*x))/d + A*b**2*x - B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x))/d + 2*B*a*b*x + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d), True))
```

**Giac [A]**

time = 0.96, size = 118, normalized size = 1.64

$$\frac{2(Aa^2 - 2Bab - Ab^2)(dx + c) + (Ba^2 + 2Aab - Bb^2) \log(\tan(dx + c)^2 + 1) - 2(Ba^2 + 2Aab) \log(|\tan(dx + c)|) + \frac{2(Ba^2 \tan(dx+c) + 2Aab \tan(dx+c) + Aa^2)}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/2*(2*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b - B*b^2)*\log(\tan(d*x + c)^2 + 1) - 2*(B*a^2 + 2*A*a*b)*\log(\text{abs}(\tan(d*x + c))) + 2*(B*a^2*\tan(d*x + c) + 2*A*a*b*\tan(d*x + c) + A*a^2)/\tan(d*x + c))/d$

**Mupad [B]**

time = 6.37, size = 100, normalized size = 1.39

$$\frac{\ln(\tan(c + dx)) (B a^2 + 2 A b a)}{d} - \frac{\ln(\tan(c + dx) - i) (-B + A i) (-b + a i)^2}{2 d} + \frac{\ln(\tan(c + dx) + i) (B + A i) (b + a i)^2}{2 d} - \frac{A a^2 \cot(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out]  $(\log(\tan(c + d*x))*(B*a^2 + 2*A*a*b))/d - (\log(\tan(c + d*x) - i)*(A*i - B)*(a*i - b)^2)/(2*d) + (\log(\tan(c + d*x) + i)*(A*i + B)*(a*i + b)^2)/(2*d) - (A*a^2*\cot(c + d*x))/d$

$$3.245 \quad \int \cot^3(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=88

$$(b^2B - a(2Ab + aB))x - \frac{a(2Ab + aB) \cot(c + dx)}{d} - \frac{a^2A \cot^2(c + dx)}{2d} - \frac{(a^2A - Ab^2 - 2abB) \log(\sin(c + dx))}{d}$$

[Out]  $(b^2*B - a*(2*A*b + B*a))*x - a*(2*A*b + B*a)*\cot(d*x + c)/d - 1/2*a^2*A*\cot(d*x + c)^2/d - (A*a^2 - A*b^2 - 2*B*a*b)*\ln(\sin(d*x + c))/d$

**Rubi [A]**

time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {3685, 3709, 3612, 3556}

$$-\frac{(a^2A - 2abB - Ab^2) \log(\sin(c + dx))}{d} - \frac{a^2A \cot^2(c + dx)}{2d} + x(b^2B - a(aB + 2Ab)) - \frac{a(aB + 2Ab) \cot(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(b^2*B - a*(2*A*b + a*B))*x - (a*(2*A*b + a*B)*\cot[c + d*x])/d - (a^2*A*\cot[c + d*x]^2)/(2*d) - ((a^2*A - A*b^2 - 2*a*b*B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3685

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^2\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(- (B\*c - A\*d)\*(b\*c - a\*d)^2\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*d^2\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[B\*(b\*c - a\*d)^2 + A\*d\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + d\*(B\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + A\*(2\*a\*b\*c - a^2\*d + b^2\*d))\*Tan[e + f\*x] + b^2\*B\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*

$c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[n, -1]$

### Rule 3709

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)] + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \text{ :> } \text{Simp}[(A*b^2 - a*b*B + a^2*C)*((a + b*\tan[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\tan[e + f*x])^{(m + 1)}*\text{Simp}[b*B + a*(A - C) - (A*b - a*B - b*C)*\tan[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{a^2 A \cot^2(c + dx)}{2d} + \int \cot^2(c + dx) (a(2Ab + aB) + b^2 \tan(c + dx)) dx \\ &= -\frac{a(2Ab + aB) \cot(c + dx)}{d} - \frac{a^2 A \cot^2(c + dx)}{2d} \\ &= (b^2 B - a(2Ab + aB)) x - \frac{a(2Ab + aB) \cot(c + dx)}{d} \\ &= (b^2 B - a(2Ab + aB)) x - \frac{a(2Ab + aB) \cot(c + dx)}{d} \end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.24, size = 123, normalized size = 1.40

$$\frac{-2a(2Ab + aB) \cot(c + dx) - a^2 A \cot^2(c + dx) + (a + ib)^2(A + iB) \log(i - \tan(c + dx)) - 2(a^2 A - Ab^2 - 2abB) \log(\tan(c + dx)) + (a - ib)^2(A - iB) \log(i + \tan(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (-2\*a\*(2\*A\*b + a\*B)\*Cot[c + d\*x] - a^2\*A\*Cot[c + d\*x]^2 + (a + I\*b)^2\*(A + I\*B)\*Log[I - Tan[c + d\*x]] - 2\*(a^2\*A - A\*b^2 - 2\*a\*b\*B)\*Log[Tan[c + d\*x]] + (a - I\*b)^2\*(A - I\*B)\*Log[I + Tan[c + d\*x]])/(2\*d)

**Maple** [A]

time = 0.17, size = 107, normalized size = 1.22

method	result
derivativedivides	$\frac{a^2 A \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + a^2 B (-\cot(dx+c) - dx - c) + 2Aab(-\cot(dx+c) - dx - c) + 2Bab \ln(\sin(dx+c)) + A}{d}$



default	$\frac{a^2 A \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + a^2 B(-\cot(dx+c) - dx - c) + 2Aab(-\cot(dx+c) - dx - c) + 2Bab \ln(\sin(dx+c))}{d}$
norman	$\frac{(-2Aab - a^2 B + b^2 B)x(\tan^2(dx+c)) - \frac{a^2 A}{2d} - \frac{a(2Ab + aB)\tan(dx+c)}{d}}{\tan(dx+c)^2} - \frac{(a^2 A - Ab^2 - 2Bab) \ln(\tan(dx+c))}{d} + \frac{(a^2 A - Ab^2 - 2Bab) \ln(\sin(dx+c))}{d}$
risch	$-2Aabx - B a^2 x + B b^2 x - 2iBabx + \frac{2ia^2 Ac}{d} - \frac{2ia(iAa e^{2i(dx+c)} + 2b e^{2i(dx+c)} A + aB e^{2i(dx+c)} - 2Aa)}{d(e^{2i(dx+c)} - 1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (a^2 * A * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))) + a^2 * B * (-\cot(d*x+c) - d*x - c) + 2 * A * a * b * (-\cot(d*x+c) - d*x - c) + 2 * B * a * b * \ln(\sin(d*x+c)) + A * b^2 * \ln(\sin(d*x+c)) + b^2 * B * (d*x+c))$

**Maxima [A]**

time = 0.51, size = 120, normalized size = 1.36

$$\frac{2(Ba^2 + 2Aab - Bb^2)(dx+c) - (Aa^2 - 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1) + 2(Aa^2 - 2Bab - Ab^2) \log(\tan(dx+c)) + \frac{Aa^2 + 2(Ba^2 + 2Aab) \tan(dx+c)}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out]  $-1/2 * (2 * (B * a^2 + 2 * A * a * b - B * b^2) * (d * x + c) - (A * a^2 - 2 * B * a * b - A * b^2) * \log(\tan(d * x + c)^2 + 1) + 2 * (A * a^2 - 2 * B * a * b - A * b^2) * \log(\tan(d * x + c)) + (A * a^2 + 2 * (B * a^2 + 2 * A * a * b) * \tan(d * x + c)) / \tan(d * x + c)^2) / d$

**Fricas [A]**

time = 0.50, size = 122, normalized size = 1.39

$$\frac{(Aa^2 - 2Bab - Ab^2) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2 + 1}\right) \tan(dx+c)^2 + Aa^2 + (Aa^2 + 2(Ba^2 + 2Aab - Bb^2)dx) \tan(dx+c)^2 + 2(Ba^2 + 2Aab) \tan(dx+c)}{2d \tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out]  $-1/2 * ((A * a^2 - 2 * B * a * b - A * b^2) * \log(\tan(d * x + c)^2 / (\tan(d * x + c)^2 + 1)) * \tan(d * x + c)^2 + A * a^2 + (A * a^2 + 2 * (B * a^2 + 2 * A * a * b - B * b^2) * d * x) * \tan(d * x + c)^2 + 2 * (B * a^2 + 2 * A * a * b) * \tan(d * x + c)) / (d * \tan(d * x + c)^2)$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(78) = 156.

time = 0.96, size = 214, normalized size = 2.43

$$\begin{cases} \infty A a^2 x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c))^2 \cot^3(c) & \text{for } d = 0 \\ \frac{A a^2 \log(\tan^2(c+dx)+1) - A a^2 \log(\tan(c+dx)) - \frac{A a^2}{2d \tan^2(c+dx)} - 2Aabx - \frac{2Aab}{d \tan(c+dx)} - \frac{A b^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{A b^2 \log(\tan(c+dx))}{d} - B a^2 x - \frac{B a^2}{d \tan(c+dx)} - \frac{B ab \log(\tan^2(c+dx)+1)}{d} + \frac{2B ab \log(\tan(c+dx))}{d} + B b^2 x}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*A\*a\*\*2\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*2\*cot(c)\*\*3, Eq(d, 0)), (A\*a\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*a\*\*2\*log(tan(c + d\*x))/d - A\*a\*\*2/(2\*d\*tan(c + d\*x)\*\*2) - 2\*A\*a\*b\*x - 2\*A\*a\*b/(d\*tan(c + d\*x)) - A\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*b\*\*2\*log(tan(c + d\*x))/d - B\*a\*\*2\*x - B\*a\*\*2/(d\*tan(c + d\*x)) - B\*a\*b\*log(tan(c + d\*x)\*\*2 + 1)/d + 2\*B\*a\*b\*log(tan(c + d\*x))/d + B\*b\*\*2\*x, True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(86) = 172.

time = 1.13, size = 237, normalized size = 2.69

$$\frac{Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 8Aab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8(Ba^2 + 2Aab - Bb^2)(dx + c) - 8(Aa^2 - 2Bab - Ab^2) \log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right) + 8(Aa^2 - 2Bab - Ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - \frac{12Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 24Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 12Aa^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Ba^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 8Aab \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Aa^2}{8d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] -1/8\*(A\*a^2\*tan(1/2\*d\*x + 1/2\*c)^2 - 4\*B\*a^2\*tan(1/2\*d\*x + 1/2\*c) - 8\*A\*a\*b\*tan(1/2\*d\*x + 1/2\*c) + 8\*(B\*a^2 + 2\*A\*a\*b - B\*b^2)\*(d\*x + c) - 8\*(A\*a^2 - 2\*B\*a\*b - A\*b^2)\*log(tan(1/2\*d\*x + 1/2\*c)^2 + 1) + 8\*(A\*a^2 - 2\*B\*a\*b - A\*b^2)\*log(abs(tan(1/2\*d\*x + 1/2\*c))) - (12\*A\*a^2\*tan(1/2\*d\*x + 1/2\*c)^2 - 24\*B\*a\*b\*tan(1/2\*d\*x + 1/2\*c)^2 - 12\*A\*b^2\*tan(1/2\*d\*x + 1/2\*c)^2 - 4\*B\*a^2\*tan(1/2\*d\*x + 1/2\*c) - 8\*A\*a\*b\*tan(1/2\*d\*x + 1/2\*c) - A\*a^2)/tan(1/2\*d\*x + 1/2\*c)^2/d

**Mupad** [B]

time = 6.40, size = 127, normalized size = 1.44

$$\frac{\ln(\tan(c + dx))(-Aa^2 + 2Baba + Ab^2)}{d} - \frac{\cot(c + dx)^2 \left(\frac{Aa^2}{2} + \tan(c + dx)(Ba^2 + 2Aba)\right)}{d} - \frac{\ln(\tan(c + dx) + 1i)(A - B1i)(b + a1i)^2}{2d} - \frac{\ln(\tan(c + dx) - 1i)(A + B1i)(-b + a1i)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] (log(tan(c + d\*x))\*(A\*b^2 - A\*a^2 + 2\*B\*a\*b))/d - (cot(c + d\*x)^2\*((A\*a^2)/2 + tan(c + d\*x)\*(B\*a^2 + 2\*A\*a\*b)))/d - (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b)^2)/(2\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a\*1i - b)^2)/(2\*d)

$$3.246 \quad \int \cot^4(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=118

$$(a^2A - Ab^2 - 2abB)x + \frac{(a^2A - Ab^2 - 2abB) \cot(c+dx)}{d} - \frac{a(2Ab + aB) \cot^2(c+dx)}{2d} - \frac{a^2A \cot^3(c+dx)}{3d} +$$

[Out] (A\*a^2-A\*b^2-2\*B\*a\*b)\*x+(A\*a^2-A\*b^2-2\*B\*a\*b)\*cot(d\*x+c)/d-1/2\*a\*(2\*A\*b+B\*a)\*cot(d\*x+c)^2/d-1/3\*a^2\*A\*cot(d\*x+c)^3/d+(b^2\*B-a\*(2\*A\*b+B\*a))\*ln(sin(d\*x+c))/d

Rubi [A]

time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3685, 3709, 3610, 3612, 3556}

$$\frac{(a^2A - 2abB - Ab^2) \cot(c+dx)}{d} + x(a^2A - 2abB - Ab^2) - \frac{a^2A \cot^3(c+dx)}{3d} + \frac{(b^2B - a(aB + 2Ab)) \log(\sin(c+dx))}{d} - \frac{a(aB + 2Ab) \cot^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (a^2\*A - A\*b^2 - 2\*a\*b\*B)\*x + ((a^2\*A - A\*b^2 - 2\*a\*b\*B)\*Cot[c + d\*x])/d - (a\*(2\*A\*b + a\*B)\*Cot[c + d\*x]^2)/(2\*d) - (a^2\*A\*Cot[c + d\*x]^3)/(3\*d) + ((b^2\*B - a\*(2\*A\*b + a\*B))\*Log[Sin[c + d\*x]])/d

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

## Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

## Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A - C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \cot^4(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{a^2 A \cot^3(c + dx)}{3d} + \int \cot^3(c + dx)(a(2Ab + aB) + b(A + B \tan(c + dx))) dx \\ &= -\frac{a(2Ab + aB) \cot^2(c + dx)}{2d} - \frac{a^2 A \cot^3(c + dx)}{3d} \\ &= \frac{(a^2 A - Ab^2 - 2abB) \cot(c + dx)}{d} - \frac{a(2Ab + aB)}{d} \\ &= (a^2 A - Ab^2 - 2abB) x + \frac{(a^2 A - Ab^2 - 2abB)}{d} \\ &= (a^2 A - Ab^2 - 2abB) x + \frac{(a^2 A - Ab^2 - 2abB)}{d} \end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.92, size = 152, normalized size = 1.29

$$\frac{6(a^2 A - Ab^2 - 2abB) \cot(c + dx) - 3a(2Ab + aB) \cot^2(c + dx) - 2a^2 A \cot^3(c + dx) + 3(a + ib)^2(-iA + B) \log(i - \tan(c + dx)) - 6(2aAb + a^2 B - b^2 B) \log(\tan(c + dx)) + 3(a - ib)^2(iA + B) \log(i + \tan(c + dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]), x]
```

```
[Out] (6*(a^2*A - A*b^2 - 2*a*b*B)*Cot[c + d*x] - 3*a*(2*A*b + a*B)*Cot[c + d*x]^2 - 2*a^2*A*Cot[c + d*x]^3 + 3*(a + I*b)^2*((-I)*A + B)*Log[I - Tan[c + d*x]
```

]] - 6\*(2\*a\*A\*b + a^2\*B - b^2\*B)\*Log[Tan[c + d\*x]] + 3\*(a - I\*b)^2\*(I\*A + B)\*Log[I + Tan[c + d\*x]]/(6\*d)

**Maple [A]**

time = 0.16, size = 136, normalized size = 1.15

method	result
derivativedivides	$\frac{a^2 A \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + a^2 B \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + 2Aab \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)}{d}$
default	$\frac{a^2 A \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + a^2 B \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + 2Aab \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)}{d}$
norman	$\frac{(a^2 A - A b^2 - 2Bab) (\tan^2(dx+c))}{d} + \frac{(a^2 A - A b^2 - 2Bab) x (\tan^3(dx+c)) - \frac{a^2 A}{3d} - \frac{a(2Ab+aB) \tan(dx+c)}{2d}}{\tan(dx+c)^3} - \frac{(2Aab+a^2 B - b^2 B)}{d}$
risch	$-i B b^2 x + \frac{2ia^2 Bc}{d} + i B a^2 x + A a^2 x - A b^2 x - 2Babx - \frac{2i(6iAab e^{4i(dx+c)} + 3iB a^2 e^{4i(dx+c)} - 6A)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(a^2\*A\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)+a^2\*B\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+2\*A\*a\*b\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+2\*B\*a\*b\*(-cot(d\*x+c)-d\*x-c)+A\*b^2\*(-cot(d\*x+c)-d\*x-c)+b^2\*B\*ln(sin(d\*x+c)))

**Maxima [A]**

time = 0.52, size = 149, normalized size = 1.26

$$\frac{6(Aa^2 - 2Bab - Ab^2)(dx+c) + 3(Ba^2 + 2Aab - Bb^2) \log(\tan(dx+c)^2 + 1) - 6(Ba^2 + 2Aab - Bb^2) \log(\tan(dx+c)) - \frac{2Aa^2 - 6(Aa^2 - 2Bab - Ab^2) \tan(dx+c)^2 + 3(Ba^2 + 2Aab) \tan(dx+c)}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/6\*(6\*(A\*a^2 - 2\*B\*a\*b - A\*b^2)\*(d\*x + c) + 3\*(B\*a^2 + 2\*A\*a\*b - B\*b^2)\*log(tan(d\*x + c)^2 + 1) - 6\*(B\*a^2 + 2\*A\*a\*b - B\*b^2)\*log(tan(d\*x + c)) - (2\*A\*a^2 - 6\*(A\*a^2 - 2\*B\*a\*b - A\*b^2)\*tan(d\*x + c)^2 + 3\*(B\*a^2 + 2\*A\*a\*b)\*tan(d\*x + c))/tan(d\*x + c)^3)/d

**Fricas [A]**

time = 0.45, size = 157, normalized size = 1.33

$$\frac{3(Ba^2 + 2Aab - Bb^2) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 + 3(Ba^2 + 2Aab - 2(Aa^2 - 2Bab - Ab^2)dx) \tan(dx+c)^3 + 2Aa^2 - 6(Aa^2 - 2Bab - Ab^2) \tan(dx+c)^2 + 3(Ba^2 + 2Aab) \tan(dx+c)}{6d \tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/6*(3*(B*a^2 + 2*A*a*b - B*b^2)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^3 + 3*(B*a^2 + 2*A*a*b - 2*(A*a^2 - 2*B*a*b - A*b^2)*d*x)*\tan(d*x + c)^3 + 2*A*a^2 - 6*(A*a^2 - 2*B*a*b - A*b^2)*\tan(d*x + c)^2 + 3*(B*a^2 + 2*A*a*b)*\tan(d*x + c))/(d*\tan(d*x + c)^3)$$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs.  $2(107) = 214$ .

time = 1.41, size = 260, normalized size = 2.20

$$\left\{ \begin{array}{l} \int \frac{Aa^2x}{x(A+B\tan(c))(a+b\tan(c))^2\cot^4(c)} \\ Aa^2x + \frac{Aa^2}{d\tan(c+dx)} - \frac{Aa^2}{3d\tan^3(c+dx)} + \frac{Aab\log(\tan^2(c+dx)+1)}{d} - \frac{2Aab\log(\tan(c+dx))}{d} - \frac{Aab}{d\tan^2(c+dx)} - A^2x - \frac{Aa^2}{d\tan(c+dx)} + \frac{Ba^2\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba^2\log(\tan(c+dx))}{d} - \frac{Ba^2}{2d\tan^2(c+dx)} - 2Babx - \frac{2Aab}{d\tan(c+dx)} - \frac{Bb^2\log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^2\log(\tan(c+dx))}{d} \end{array} \right. \begin{array}{l} \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ \text{for } d=0 \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*A\*\*2\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*2\*cot(c)\*\*4, Eq(d, 0)), (A\*\*2\*x + A\*\*2/(d\*tan(c + d\*x)) - A\*\*2/(3\*d\*tan(c + d\*x)\*\*3) + A\*a\*b\*log(tan(c + d\*x)\*\*2 + 1)/d - 2\*A\*a\*b\*log(tan(c + d\*x))/d - A\*a\*b/(d\*tan(c + d\*x)\*\*2) - A\*b\*\*2\*x - A\*b\*\*2/(d\*tan(c + d\*x)) + B\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*\*2\*log(tan(c + d\*x))/d - B\*\*2/(2\*d\*tan(c + d\*x)\*\*2) - 2\*B\*a\*b\*x - 2\*B\*a\*b/(d\*tan(c + d\*x)) - B\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*\*2\*log(tan(c + d\*x))/d, True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs.  $2(114) = 228$ .

time = 1.46, size = 334, normalized size = 2.83

$$\frac{A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 3B^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 6A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 24B^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 12A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 24B^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 24(A^2 - 2Bab - B^2)(dx + c) + 24(B^2 + 2Aab - B^2)\log(\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1) - 24(B^2 + 2Aab - B^2)\log(\tan(\frac{1}{2}dx + \frac{1}{2}c)) + \frac{44A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 88A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 44A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 15A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 24A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 6A^2 \tan(\frac{1}{2}dx + \frac{1}{2}c) - A^2}{\tan(\frac{1}{2}dx + \frac{1}{2}c)^3} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$\frac{1}{24}(A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c)^3 - 3B^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 6A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) + 24B^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) + 12A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) + 24B^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) + 24(A^2 - 2B*a*b - B^2)(d*x + c) + 24(B^2 + 2A*a*b - B^2)\log(\tan(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1) - 24(B^2 + 2A*a*b - B^2)\log(\tan(\frac{1}{2}d*x + \frac{1}{2}c))) + (44B^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c)^3 + 88A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 44A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) + 15A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 24B^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) - 6A^2 \tan(\frac{1}{2}d*x + \frac{1}{2}c) - A^2)/\tan(\frac{1}{2}d*x + \frac{1}{2}c)^3 / d$$

**Mupad [B]**

time = 6.37, size = 156, normalized size = 1.32

$$\frac{\cot(c+dx)^3 \left( \frac{Aa^2}{3} + \tan(c+dx)^2 (-Aa^2 + 2Bab + Ab^2) + \tan(c+dx) \left( \frac{Ba^2}{2} + Aba \right) \right)}{d} - \frac{\ln(\tan(c+dx)) (Ba^2 + 2Aab - Bb^2)}{d} + \frac{\ln(\tan(c+dx) - i) (-B + A1i) (-b + a1i)^2}{2d} - \frac{\ln(\tan(c+dx) + 1i) (B + A1i) (b + a1i)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] (log(tan(c + d\*x) - 1i)\*(A\*1i - B)\*(a\*1i - b)^2)/(2\*d) - (log(tan(c + d\*x)) \* (B\*a^2 - B\*b^2 + 2\*A\*a\*b))/d - (cot(c + d\*x)^3\*((A\*a^2)/3 + tan(c + d\*x)^2 \* (A\*b^2 - A\*a^2 + 2\*B\*a\*b) + tan(c + d\*x)\*((B\*a^2)/2 + A\*a\*b)))/d - (log(tan(c + d\*x) + 1i)\*(A\*1i + B)\*(a\*1i + b)^2)/(2\*d)

### 3.247 $\int \cot^5(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$

Optimal. Leaf size=151

$$(2aAb + a^2B - b^2B)x - \frac{(b^2B - a(2Ab + aB)) \cot(c + dx)}{d} + \frac{(a^2A - Ab^2 - 2abB) \cot^2(c + dx)}{2d} - \frac{a(2Ab + aB)}{2d}$$

[Out]  $(2A^2a^2b + B^2a^2 - B^2b^2)x - (b^2B - a(2Ab + aB)) \cot(dx+c)/d + 1/2(A^2a^2 - A^2b^2 - 2B^2a^2b) \cot(dx+c)^2/d - 1/3a^2(2Ab + aB) \cot(dx+c)^3/d - 1/4a^2A \cot(dx+c)^4/d + (A^2a^2 - A^2b^2 - 2B^2a^2b) \ln(\sin(dx+c))/d$

Rubi [A]

time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3685, 3709, 3610, 3612, 3556}

$$\frac{(a^2A - 2abB - Ab^2) \cot^2(c + dx)}{2d} + \frac{(a^2A - 2abB - Ab^2) \log(\sin(c + dx))}{d} + x(a^2B + 2aAb - b^2B) - \frac{a^2A \cot^4(c + dx)}{4d} - \frac{(b^2B - a(aB + 2Ab)) \cot(c + dx)}{d} - \frac{a(aB + 2Ab) \cot^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^5*(a + b*\text{Tan}[c + d*x])^2*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(2*a*A*b + a^2*B - b^2*B)*x - ((b^2*B - a*(2*A*b + a*B))*\text{Cot}[c + d*x])/d + ((a^2*A - A*b^2 - 2*a*b*B)*\text{Cot}[c + d*x]^2)/(2*d) - (a*(2*A*b + a*B)*\text{Cot}[c + d*x]^3)/(3*d) - (a^2*A*\text{Cot}[c + d*x]^4)/(4*d) + ((a^2*A - A*b^2 - 2*a*b*B)*\text{Log}[\text{Sin}[c + d*x]])/d$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 3610

$\text{Int}[((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3612

$\text{Int}(((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c + b*d)*(x/(a^2 + b^2)), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[m, -1]$



$Q[a*c + b*d, 0]$

### Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

### Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A - C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \cot^5(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= -\frac{a^2 A \cot^4(c + dx)}{4d} + \int \cot^4(c + dx) (a(2Ab + aB) + b^2 B) dx \\
 &= -\frac{a(2Ab + aB) \cot^3(c + dx)}{3d} - \frac{a^2 A \cot^4(c + dx)}{4d} + \int \cot^3(c + dx) (a(2Ab + aB) + b^2 B) dx \\
 &= \frac{(a^2 A - Ab^2 - 2abB) \cot^2(c + dx)}{2d} - \frac{a(2Ab + aB) \cot(c + dx)}{d} + \int \cot^2(c + dx) (a(2Ab + aB) + b^2 B) dx \\
 &= \frac{(b^2 B - a(2Ab + aB)) \cot(c + dx)}{d} + \frac{(a^2 A - Ab^2 - 2abB) \cot(c + dx)}{2d} - \frac{a(2Ab + aB) \cot(c + dx)}{d} + \int \cot(c + dx) (a(2Ab + aB) + b^2 B) dx \\
 &= (2aAb + a^2 B - b^2 B) x - \frac{(b^2 B - a(2Ab + aB)) \cot(c + dx)}{d} + \frac{(a^2 A - Ab^2 - 2abB) \cot(c + dx)}{2d} - \frac{a(2Ab + aB) \cot(c + dx)}{d} \\
 &= (2aAb + a^2 B - b^2 B) x - \frac{(b^2 B - a(2Ab + aB)) \cot(c + dx)}{d} + \frac{(a^2 A - Ab^2 - 2abB) \cot(c + dx)}{2d} - \frac{a(2Ab + aB) \cot(c + dx)}{d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.89, size = 180, normalized size = 1.19

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^5\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(12*(2*a*A*b + a^2*B - b^2*B)*\text{Cot}[c + d*x] + 6*(a^2*A - A*b^2 - 2*a*b*B)*\text{Cot}[c + d*x]^2 - 4*a*(2*A*b + a*B)*\text{Cot}[c + d*x]^3 - 3*a^2*A*\text{Cot}[c + d*x]^4 - 6*((a + I*b)^2*(A + I*B)*\text{Log}[I - \text{Tan}[c + d*x]] + (-2*a^2*A + 2*A*b^2 + 4*a*b*B)*\text{Log}[\text{Tan}[c + d*x]] + (a - I*b)^2*(A - I*B)*\text{Log}[I + \text{Tan}[c + d*x]]))/ (12*d)$

Maple [A]

time = 0.18, size = 162, normalized size = 1.07

method	result
derivativedivides	$a^2 A \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} + \ln(\sin(dx+c)) \right) + a^2 B \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + 2Aab \left( -\frac{\cot^3(dx+c)}{3} \right)$
default	$a^2 A \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} + \ln(\sin(dx+c)) \right) + a^2 B \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + 2Aab \left( -\frac{\cot^3(dx+c)}{3} \right)$
norman	$\frac{(2Aab+a^2B-b^2B)(\tan^3(dx+c))}{d} + (2Aab+a^2B-b^2B)x \frac{(\tan^4(dx+c)) - \frac{a^2A}{4d} + \frac{(a^2A-Ab^2-2Bab)(\tan^2(dx+c))}{2d} - \frac{a(2Ab+aB)\tan(dx+c)}{3d}}{\tan(dx+c)^4}$
risch	$2Aabx + B a^2 x - B b^2 x + 2iBabx - \frac{2ia^2Ac}{d} + \frac{4iBabc}{d} - iA a^2 x + \frac{2iA b^2 c}{d} + iA b^2 x - \frac{2i(3iAb^2c)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^5\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out]  $1/d*(a^2*A*(-1/4*\cot(d*x+c)^4+1/2*\cot(d*x+c)^2+\ln(\sin(d*x+c)))+a^2*B*(-1/3*\cot(d*x+c)^3+\cot(d*x+c)+d*x+c)+2*A*a*b*(-1/3*\cot(d*x+c)^3+\cot(d*x+c)+d*x+c)+2*B*a*b*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+A*b^2*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+b^2*B*(-\cot(d*x+c)-d*x-c))$

Maxima [A]

time = 0.51, size = 175, normalized size = 1.16

$$\frac{12(Ba^2 + 2Aab - Bb^2)(dx+c) - 6(Aa^2 - 2Bab - Ab^2)\log(\tan(dx+c)^2 + 1) + 12(Aa^2 - 2Bab - Ab^2)\log(\tan(dx+c)) + \frac{12(Ba^2 + 2Aab - Bb^2)\tan(dx+c)^3 - 3Aa^2 + 6(Aa^2 - 2Bab - Ab^2)\tan(dx+c)^2 - 4(Ba^2 + 2Aab)\tan(dx+c)}{\tan(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out]  $1/12*(12*(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c) - 6*(A*a^2 - 2*B*a*b - A*b^2)*\log(\tan(d*x + c)^2 + 1) + 12*(A*a^2 - 2*B*a*b - A*b^2)*\log(\tan(d*x + c)) + (12*(B*a^2 + 2*A*a*b - B*b^2)*\tan(d*x + c)^3 - 3*A*a^2 + 6*(A*a^2 - 2*B*a*b$

- A\*b^2)\*tan(d\*x + c)^2 - 4\*(B\*a^2 + 2\*A\*a\*b)\*tan(d\*x + c))/tan(d\*x + c)^4  
)/d

**Fricas** [A]

time = 0.48, size = 191, normalized size = 1.26

$$\frac{6(Aa^2 - 2Bab - Ab^2) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)+1}\right) \tan(dx+c)^4 + 3(3Aa^2 - 4Bab - 2Ab^2 + 4(Ba^2 + 2Aab - Bb^2)dx) \tan(dx+c)^4 + 12(Ba^2 + 2Aab - Bb^2) \tan(dx+c)^3 - 3Aa^2 + 6(Aa^2 - 2Bab - Ab^2) \tan(dx+c)^2 - 4(Ba^2 + 2Aab) \tan(dx+c) - 2Ab^2}{12d \tan(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/12\*(6\*(A\*a^2 - 2\*B\*a\*b - A\*b^2)\*log(tan(d\*x + c)^2/(tan(d\*x + c)^2 + 1))\*tan(d\*x + c)^4 + 3\*(3\*A\*a^2 - 4\*B\*a\*b - 2\*A\*b^2 + 4\*(B\*a^2 + 2\*A\*a\*b - B\*b^2)\*d\*x)\*tan(d\*x + c)^4 + 12\*(B\*a^2 + 2\*A\*a\*b - B\*b^2)\*tan(d\*x + c)^3 - 3\*A\*a^2 + 6\*(A\*a^2 - 2\*B\*a\*b - A\*b^2)\*tan(d\*x + c)^2 - 4\*(B\*a^2 + 2\*A\*a\*b)\*tan(d\*x + c))/(d\*tan(d\*x + c)^4)

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(136) = 272.

time = 2.25, size = 313, normalized size = 2.07

$$\begin{cases} \frac{5Aa^2x}{x(A+B\tan(c))(a+b\tan(c))^2\cos^2(c)} & \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ -\frac{Aa^2\log(\tan^2(c+d*x))+d^2\log(\tan(c+d*x))}{4} + \frac{Aa^2}{2d\cos^2(c)} + \frac{2Aab}{2d\cos^2(c)} + 2Aabx + \frac{2Ab^2}{2d\cos^2(c)} - \frac{2Aab}{2d\cos^2(c)} + \frac{Aa^2\log(\tan^2(c+d*x))+1}{4} - \frac{Aa^2\log(\tan(c+d*x))}{2d\cos^2(c)} + Ba^2x + \frac{Ba^2}{2d\cos^2(c)} - \frac{Ba^2}{2d\cos^2(c)} + \frac{Bab\log(\tan^2(c+d*x))+1}{4} - \frac{2Bab\log(\tan(c+d*x))}{2d\cos^2(c)} - \frac{Bab}{2d\cos^2(c)} - Bb^2x - \frac{Bb^2}{2d\cos^2(c)} & \text{for } d=0 \\ \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*5\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*A\*a\*\*2\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*2\*cot(c)\*\*5, Eq(d, 0)), (-A\*a\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*a\*\*2\*log(tan(c + d\*x))/d + A\*a\*\*2/(2\*d\*tan(c + d\*x)\*\*2) - A\*a\*\*2/(4\*d\*tan(c + d\*x)\*\*4) + 2\*A\*a\*b\*x + 2\*A\*a\*b/(d\*tan(c + d\*x)) - 2\*A\*a\*b/(3\*d\*tan(c + d\*x)\*\*3) + A\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*b\*\*2\*log(tan(c + d\*x))/d - A\*b\*\*2/(2\*d\*tan(c + d\*x)\*\*2) + B\*a\*\*2\*x + B\*a\*\*2/(d\*tan(c + d\*x)) - B\*a\*\*2/(3\*d\*tan(c + d\*x)\*\*3) + B\*a\*b\*log(tan(c + d\*x)\*\*2 + 1)/d - 2\*B\*a\*b\*log(tan(c + d\*x))/d - B\*a\*b/(d\*tan(c + d\*x)\*\*2) - B\*b\*\*2\*x - B\*b\*\*2/(d\*tan(c + d\*x)), True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(145) = 290.

time = 1.27, size = 435, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^5\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

```
[Out] -1/192*(3*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 8*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 1
6*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*B*a*b
*tan(1/2*d*x + 1/2*c)^2 + 24*A*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*B*a^2*tan(1
/2*d*x + 1/2*c) + 240*A*a*b*tan(1/2*d*x + 1/2*c) - 96*B*b^2*tan(1/2*d*x + 1
/2*c) - 192*(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c) + 192*(A*a^2 - 2*B*a*b - A*
b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(A*a^2 - 2*B*a*b - A*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c))) + (400*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 800*B*a*b*ta
n(1/2*d*x + 1/2*c)^4 - 400*A*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*B*a^2*tan(1/2
*d*x + 1/2*c)^3 - 240*A*a*b*tan(1/2*d*x + 1/2*c)^3 + 96*B*b^2*tan(1/2*d*x +
1/2*c)^3 - 36*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*B*a*b*tan(1/2*d*x + 1/2*c)
^2 + 24*A*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*B*a^2*tan(1/2*d*x + 1/2*c) + 16*A*
a*b*tan(1/2*d*x + 1/2*c) + 3*A*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

**Mupad [B]**

time = 6.34, size = 182, normalized size = 1.21

$$\frac{\cot(c+dx)^4 \left( \frac{d^2}{4} + \tan(c+dx)^2 \left( -\frac{d^2}{4} + B a b + \frac{d^2}{4} \right) - \tan(c+dx)^3 (B a^2 + 2 A a b - B b^2) + \tan(c+dx) \left( \frac{B a^2}{4} + \frac{3 A d a}{4} \right) \right)}{d} - \frac{\ln(\tan(c+dx)) (-A a^2 + 2 B a b + A b^2)}{d} + \frac{\ln(\tan(c+dx) + 1) (A - B b) (b + a)^2}{2d} + \frac{\ln(\tan(c+dx) - 1) (A + B b) (-b + a)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x))
*(A*b^2 - A*a^2 + 2*B*a*b))/d - (cot(c + d*x)^4*((A*a^2)/4 + tan(c + d*x)^2
*((A*b^2)/2 - (A*a^2)/2 + B*a*b) - tan(c + d*x)^3*(B*a^2 - B*b^2 + 2*A*a*b)
+ tan(c + d*x)*((B*a^2)/3 + (2*A*a*b)/3))/d + (log(tan(c + d*x) - 1i)*(A
+ B*1i)*(a*1i - b)^2)/(2*d)
```

### 3.248 $\int \tan^2(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=201

$$-\left((a^3A - 3aAb^2 - 3a^2bB + b^3B)x\right) + \frac{(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \log(\cos(c+dx))}{d} - \frac{b(2aAb + a^2B - b^2B)}{d}$$

[Out]  $-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x+(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*\ln(\cos(d*x+c))/d-b*(2*A*a*b+B*a^2-B*b^2)*\tan(d*x+c)/d-1/2*(A*b+B*a)*(a+b*\tan(d*x+c))^2/d-1/3*B*(a+b*\tan(d*x+c))^3/d+1/20*(5*A*b-B*a)*(a+b*\tan(d*x+c))^4/b^2/d+1/5*B*\tan(d*x+c)*(a+b*\tan(d*x+c))^4/b/d$

**Rubi** [A]

time = 0.26, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3688, 3711, 3609, 3606, 3556}

$$\frac{b(a^2B + 2aAb - b^2B) \tan(c+dx)}{d} + \frac{(a^2B + 3a^2Ab - 3ab^2B - Ab^2) \log(\cos(c+dx))}{d} - \frac{x(a^3A - 3a^2bB - 3aAb^2 + b^3B)}{d} + \frac{(5Ab - aB)(a + b \tan(c+dx))^4}{20b^2d} - \frac{(aB + Ab)(a + b \tan(c+dx))^2}{2d} + \frac{B \tan(c+dx)(a + b \tan(c+dx))^4}{5bd} - \frac{B(a + b \tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-\left((a^3A - 3a^2Ab^2 - 3a^2b^2B + b^3B)x\right) + \left(\left(3a^2Ab - Ab^3 + a^3B - 3a^2b^2B\right) \log[\cos[c + d*x]]\right)/d - \left(b(2aAb + a^2B - b^2B) \tan[c + d*x]\right)/d - \left(\left(Ab + a^2B\right) \left(a + b \tan[c + d*x]\right)^2\right)/(2d) - \left(B(a + b \tan[c + d*x])^3\right)/(3d) + \left(\left(5Ab - aB\right) \left(a + b \tan[c + d*x]\right)^4\right)/(20b^2d) + \left(B \tan[c + d*x] \left(a + b \tan[c + d*x]\right)^4\right)/(5b^2d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rule 3609

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x]

, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

### Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \tan^2(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \frac{B \tan(c + dx)(a + b \tan(c + dx))^4}{5bd} + \frac{\int (a + b \tan(c + dx))^4 dx}{5bd} \\
 &= \frac{(5Ab - aB)(a + b \tan(c + dx))^4}{20b^2d} + \frac{B \tan(c + dx)(a + b \tan(c + dx))^4}{5bd} \\
 &= -\frac{B(a + b \tan(c + dx))^3}{3d} + \frac{(5Ab - aB)(a + b \tan(c + dx))^4}{20b^2d} \\
 &= -\frac{(Ab + aB)(a + b \tan(c + dx))^2}{2d} - \frac{B(a + b \tan(c + dx))^4}{5bd} \\
 &= -(a^3A - 3aAb^2 - 3a^2bB + b^3B)x - \frac{b(2aAb - 3a^2A - 3a^2Ab^2 - 3a^2bB + b^3B)}{5bd} \\
 &= -(a^3A - 3aAb^2 - 3a^2bB + b^3B)x + \frac{(3a^2Ab - 3a^2A - 3a^2Ab^2 - 3a^2bB + b^3B)b}{5bd}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.45, size = 241, normalized size = 1.20

$\frac{35Ab - aB}{5bd} \int (a + b \tan(c + dx))^4 dx + 12B \tan(c + dx)(a + b \tan(c + dx))^3 - 30(Ab - aB) \int (a - b)^2 \log(i - \tan(c + dx)) - (a + b)^2 \log(i + \tan(c + dx)) + 6ab^2 \tan(c + dx) + b^2 \tan^2(c + dx) + 10B(3i(a + b)^2 \log(i - \tan(c + dx)) - 3i(a - b)^2 \log(i + \tan(c + dx)) + 6b^2(-6a^2 + b^2) \tan(c + dx) - 12ab^2 \tan^2(c + dx) - 2b^4 \tan^4(c + dx)) dx$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((3*(5*A*b - a*B)*(a + b*\tan[c + d*x])^4)/b + 12*B*\tan[c + d*x]*(a + b*\tan[c + d*x])^4 - 30*(A*b - a*B)*((I*a - b)^3*\log[I - \tan[c + d*x]] - (I*a + b)^3*\log[I + \tan[c + d*x]] + 6*a*b^2*\tan[c + d*x] + b^3*\tan[c + d*x]^2) + 10*B*((3*I)*(a + I*b)^4*\log[I - \tan[c + d*x]] - (3*I)*(a - I*b)^4*\log[I + \tan[c + d*x]] + 6*b^2*(-6*a^2 + b^2)*\tan[c + d*x] - 12*a*b^3*\tan[c + d*x]^2 - 2*b^4*\tan[c + d*x]^3))/(60*b*d)$

**Maple [A]**

time = 0.10, size = 271, normalized size = 1.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $1/d*(1/5*B*b^3*\tan(d*x+c)^5+1/4*A*b^3*\tan(d*x+c)^4+3/4*B*a*b^2*\tan(d*x+c)^4+A*a*b^2*\tan(d*x+c)^3+B*a^2*b*\tan(d*x+c)^3-1/3*B*b^3*\tan(d*x+c)^3+3/2*A*a^2*b*\tan(d*x+c)^2-1/2*A*b^3*\tan(d*x+c)^2+1/2*B*a^3*\tan(d*x+c)^2-3/2*B*a*b^2*\tan(d*x+c)^2+A*a^3*\tan(d*x+c)-3*A*a*b^2*\tan(d*x+c)-3*B*a^2*b*\tan(d*x+c)+B*b^3*\tan(d*x+c)+1/2*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*\ln(1+\tan(d*x+c)^2)+(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*\arctan(\tan(d*x+c))$

**Maxima [A]**

time = 0.54, size = 214, normalized size = 1.06

$\frac{12 B b^3 \tan(d x+c)^5+15(3 B a b^2+A b^3) \tan(d x+c)^4+20(3 B a^2 b+3 A a b^2-B b^3) \tan(d x+c)^3+30(B a^3+3 A a^2 b-B b^3) \tan(d x+c)^2-60(A a^3-3 B a^2 b-3 A a b^2+B b^3) \tan(d x+c)-30(B a^2+3 A a b^2-3 B a b^2-A b^3) \log(\tan(d x+c)^2+1)+60(A a^2-3 B a b^2-3 A a b^2+B b^3) \tan(d x+c)}{60 d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $1/60*(12*B*b^3*\tan(d*x + c)^5 + 15*(3*B*a*b^2 + A*b^3)*\tan(d*x + c)^4 + 20*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*\tan(d*x + c)^3 + 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\tan(d*x + c)^2 - 60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c) - 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\log(\tan(d*x + c)^2 + 1) + 60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\tan(d*x + c))/d$

**Fricas [A]**

time = 0.53, size = 213, normalized size = 1.06

$\frac{12 B b^3 \tan(d x+c)^5+15(3 B a b^2+A b^3) \tan(d x+c)^4+20(3 B a^2 b+3 A a b^2-B b^3) \tan(d x+c)^3-60(A a^3-3 B a^2 b-3 A a b^2+B b^3) \tan(d x+c)+30(B a^2+3 A a b^2-3 B a b^2-A b^3) \log\left(\frac{1}{\tan(d x+c)^2+1}\right)+60(A a^2-3 B a b^2-3 A a b^2+B b^3) \tan(d x+c)}{60 d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")





$$\begin{aligned}
&^2 + 1)) * \tan(dx)^5 * \tan(c)^5 - 300 * A * a^3 * dx * \tan(dx)^4 * \tan(c)^4 + 900 * B * a^2 * b * dx * \tan(dx)^4 * \tan(c)^4 + 900 * A * a * b^2 * dx * \tan(dx)^4 * \tan(c)^4 - 300 * B * b^3 * dx * \tan(dx)^4 * \tan(c)^4 - 30 * B * a^3 * \tan(dx)^5 * \tan(c)^5 - 90 * A * a^2 * b * \tan(dx)^5 * \tan(c)^5 + 135 * B * a * b^2 * \tan(dx)^5 * \tan(c)^5 + 45 * A * b^3 * \tan(dx)^5 * \tan(c)^5 + 150 * B * a^3 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan(c)^4 + 450 * A * a^2 * b * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan(c)^4 - 450 * B * a * b^2 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan(c)^4 - 150 * A * b^3 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan(c)^4 + 60 * A * a^3 * \tan(dx)^5 * \tan(c)^4 - 180 * B * a^2 * b * \tan(dx)^5 * \tan(c)^4 - 180 * A * a * b^2 * \tan(dx)^5 * \tan(c)^4 + 60 * B * b^3 * \tan(dx)^5 * \tan(c)^4 + 60 * A * a^3 * \tan(dx)^4 * \tan(c)^5 - 180 * B * a^2 * b * \tan(dx)^4 * \tan(c)^5 - 180 * A * a * b^2 * \tan(dx)^4 * \tan(c)^5 + 60 * B * b^3 * \tan(dx)^4 * \tan(c)^5 + 600 * A * a^3 * dx * \tan(dx)^3 * \tan(c)^3 - 1800 * B * a^2 * b * dx * \tan(dx)^3 * \tan(c)^3 - 1800 * A * a * b^2 * dx * \tan(dx)^3 * \tan(c)^3 + 600 * B * b^3 * dx * \tan(dx)^3 * \tan(c)^3 - 30 * B * a^3 * \tan(dx)^5 * \tan(c)^3 - 90 * A * a^2 * b * \tan(dx)^5 * \tan(c)^3 + 90 * B * a * b^2 * \tan(dx)^5 * \tan(c)^3 + 30 * A * b^3 * \tan(dx)^5 * \tan(c)^3 + 90 * B * a^3 * \tan(dx)^4 * \tan(c)^4 + 270 * A * a^2 * b * \tan(dx)^4 * \tan(c)^4 - 495 * B * a * b^2 * \tan(dx)^4 * \tan(c)^4 - 165 * A * b^3 * \tan(dx)^4 * \tan(c)^4 - 30 * B * a^3 * \tan(dx)^3 * \tan(c)^5 - 90 * A * a^2 * b * \tan(dx)^3 * \tan(c)^5 + 90 * B * a * b^2 * \tan(dx)^3 * \tan(c)^5 + 30 * A * b^3 * \tan(dx)^3 * \tan(c)^5 + 60 * B * a^2 * b * \tan(dx)^5 * \tan(c)^2 + 60 * A * a * b^2 * \tan(dx)^5 * \tan(c)^2 - 20 * B * b^3 * \tan(dx)^5 * \tan(c)^2 - 300 * B * a^3 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^3 * \tan(c)^3 - 900 * A * a^2 * b * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^3 * \tan(c)^3 + 900 * B * a * b^2 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^3 * \tan(c)^3 + 300 * A * b^3 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^3 * \tan(c)^3 - 240 * A * a^3 * \tan(dx)^4 * \tan(c)^3 + 900 * B * a^2 * b * \tan(dx)^4 * \tan(c)^3 + 900 * A * a * b^2 * \tan(dx)^4 * \tan(c)^3 - 300 * B * b^3 * \tan(dx)^4 * \tan(c)^3 - 240 * A * a^3 * \tan(dx)^3 * \tan(c)^4 + 900 * B * a^2 * b * \tan(dx)^3 * \tan(c)^4 + 900 * A * a * b^2 * \tan(dx)^3 * \tan(c)^4 - 300 * B * b^3 * \tan(dx)^3 * \tan(c)^4 + 60 * B * a^2 * b * \tan(dx)^2 * \tan(c)^5 + 60 * A * a * b^2 * \tan(dx)^2 * \tan(c)^5 - 20 * B * b^3 * \tan(dx)^2 * \tan(c)^5 - 45 * B * a * b^2 * \tan(dx)^5 * \tan(c) - 15 * A * b^3 * \tan(dx)^5 * \tan(c) - 600 * A * a^3 * dx * \tan(dx)^2 * \tan(c)^2 + 1800 * B * a^2 * b * dx * \tan(dx)^2 * \tan(c)^2 + 1800 * A * a * b^2 * dx * \tan(dx)^2 * \tan(c)^2 - 600 * B * b^3 * dx * \tan(dx)^2 * \tan(c)^2 + 90 * B * a^3 * \tan(dx)^4 * \tan(c)^2 + 270 * A * a^2 * b * \tan(dx)^4 * \tan(c)^2 - 450 * B * a * b^2 * \tan(dx)^4 * \tan(c)^2 - 150 * A * b^3 * \tan(dx)^4 * \tan(c)^2 - 120 * B * a^3 * \tan(dx)^3 * \tan(c)^3 - 360 * A * a^2 * b * \tan(dx)^3 * \tan(c)^3 + 540 * B * a * b^2 * \tan(dx)^3 * \tan(c)^3 + 180 * A * b^3 * \tan(dx)^3 * \tan(c)^3 + 90 * B * a^3 * \tan(dx)^2 * \tan(c)^4 + 270 * A * a^2 * b * \tan(dx)^2 * \tan(c)^4 - 450 * B * a * b^
\end{aligned}$$

$$\begin{aligned}
& 2*\tan(d*x)^2*\tan(c)^4 - 150*A*b^3*\tan(d*x)^2*\tan(c)^4 - 45*B*a*b^2*\tan(d*x) \\
& * \tan(c)^5 - 15*A*b^3*\tan(d*x)*\tan(c)^5 + 12*B*b^3*\tan(d*x)^5 - 120*B*a^2*b* \\
& \tan(d*x)^4*\tan(c) - 120*A*a*b^2*\tan(d*x)^4*\tan(c) + 100*B*b^3*\tan(d*x)^4*\tan \\
& (c) + 300*B*a^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x) \\
& )^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x) \\
& ^2*\tan(c)^2 + 900*A*a^2*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) \\
& + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1)) \\
& *\tan(d*x)^2*\tan(c)^2 - 900*B*a*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3 \\
& *\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c) \\
& )^2 + 1))*\tan(d*x)^2*\tan(c)^2 - 300*A*b^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan \\
& (d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1) \\
& /(\tan(c)^2 + 1))*\tan(d*x)^2*\tan(c)^2 + 360*A*a^...
\end{aligned}$$

**Mupad [B]**

time = 6.39, size = 217, normalized size = 1.08

$$\frac{\tan(c+dx)(Aa^3+Bb^3-3ab(Ab+Ba))}{d} - \frac{\tan(c+dx)^2 \left( \frac{Bb^2}{d} - ab(Ab+Ba) \right)}{d} - x(Aa^3-3Ba^2b-3Aa^2b+Bb^3) + \frac{\ln(\tan(c+dx)^2+1) \left( -\frac{Bb^2}{d} - \frac{3Ab^2a}{d} + \frac{3Bb^2a}{d} + \frac{Aa^2}{d} \right)}{d} + \frac{\tan(c+dx)^4 \left( \frac{Ab^2}{d} + \frac{3Bb^2a}{d} \right)}{d} - \frac{\tan(c+dx)^2 \left( -\frac{Bb^2}{d} - \frac{3Ab^2a}{d} + \frac{3Bb^2a}{d} + \frac{Aa^2}{d} \right)}{d} + \frac{Bb^3 \tan(c+dx)^5}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out] (tan(c + d\*x)\*(A\*a^3 + B\*b^3 - 3\*a\*b\*(A\*b + B\*a)))/d - (tan(c + d\*x)^3\*((B\*b^3)/3 - a\*b\*(A\*b + B\*a)))/d - x\*(A\*a^3 + B\*b^3 - 3\*A\*a\*b^2 - 3\*B\*a^2\*b) + (log(tan(c + d\*x)^2 + 1)\*((A\*b^3)/2 - (B\*a^3)/2 - (3\*A\*a^2\*b)/2 + (3\*B\*a\*b^2)/2))/d + (tan(c + d\*x)^4\*((A\*b^3)/4 + (3\*B\*a\*b^2)/4))/d - (tan(c + d\*x)^2\*((A\*b^3)/2 - (B\*a^3)/2 - (3\*A\*a^2\*b)/2 + (3\*B\*a\*b^2)/2))/d + (B\*b^3\*tan(c + d\*x)^5)/(5\*d)

$$3.249 \quad \int \tan(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=165

$$-((3a^2Ab - Ab^3 + a^3B - 3ab^2B)x) - \frac{(a^3A - 3aAb^2 - 3a^2bB + b^3B) \log(\cos(c+dx))}{d} + \frac{b(a^2A - Ab^2 - 2abB)}{d}$$

[Out]  $-(3Aa^2b - Ab^3 + Ba^3 - 3Bab^2)x - (Aa^3 - 3AAb^2 - 3Bba^2 + Bb^3) \ln(\cos(dx+c)) / d + b(Aa^2 - Ab^2 - 2AbB) \tan(dx+c) / d + 1/2(Aa - Bb)(a + b \tan(dx+c))^2 / d + 1/3A(a + b \tan(dx+c))^3 / d + 1/4B(a + b \tan(dx+c))^4 / b/d$

Rubi [A]

time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3673, 3609, 3606, 3556}

$$\frac{b(a^2A - 2abB - Ab^2) \tan(c+dx)}{d} - \frac{(a^3A - 3a^2bB - 3aAb^2 + b^3B) \log(\cos(c+dx))}{d} - x(a^3B + 3a^2Ab - 3ab^2B - Ab^3) + \frac{(aA - bB)(a + b \tan(c+dx))^2}{2d} + \frac{A(a + b \tan(c+dx))^3}{3d} + \frac{B(a + b \tan(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-((3a^2Ab - Ab^3 + a^3B - 3a^2bB)x) - ((a^3A - 3a^2bB - 3aAb^2 + b^3B) \log[\cos[c + d*x]]) / d + (b(a^2A - Ab^2 - 2aAbB) \tan[c + d*x]) / d + ((aA - bB)(a + b \tan[c + d*x])^2) / (2*d) + (A(a + b \tan[c + d*x])^3) / (3*d) + (B(a + b \tan[c + d*x])^4) / (4*b*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rule 3609

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m-1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned}
\int \tan(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^4}{4bd} + \int (-B + A \tan(c + dx)) (a + b \tan(c + dx))^3 dx \\
&= \frac{A(a + b \tan(c + dx))^3}{3d} + \frac{B(a + b \tan(c + dx))^4}{4bd} \\
&= \frac{(aA - bB)(a + b \tan(c + dx))^2}{2d} + \frac{A(a + b \tan(c + dx))^3}{3d} \\
&= -(3a^2Ab - Ab^3 + a^3B - 3ab^2B)x + \frac{b(a^2A - Ab^2)}{3d} \\
&= -(3a^2Ab - Ab^3 + a^3B - 3ab^2B)x - \frac{(a^3A - 3ab^2B)}{3d}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.99, size = 209, normalized size = 1.27

$$\frac{-6iA(a+ib)^4 \log(i - \tan(c+dx)) + 6iA(a-ib)^4 \log(i + \tan(c+dx)) - 12A^2(-6a^2 + b^2) \tan(c+dx) + 24iAb^3 \tan^2(c+dx) + 4A^3 \tan^3(c+dx) + 3B(a+b \tan(c+dx))^4 - 6(aA+bB)((ia-b)^3 \log(i - \tan(c+dx)) - (ia+b)^3 \log(i + \tan(c+dx)) + 6ab^2 \tan(c+dx) + b^3 \tan^2(c+dx))}{12bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] ((-6\*I)\*A\*(a + I\*b)^4\*Log[I - Tan[c + d\*x]] + (6\*I)\*A\*(a - I\*b)^4\*Log[I + Tan[c + d\*x]] - 12\*A\*b^2\*(-6\*a^2 + b^2)\*Tan[c + d\*x] + 24\*a\*A\*b^3\*Tan[c + d\*x]^2 + 4\*A\*b^4\*Tan[c + d\*x]^3 + 3\*B\*(a + b\*Tan[c + d\*x])^4 - 6\*(a\*A + b\*B)\*((I\*a - b)^3\*Log[I - Tan[c + d\*x]] - (I\*a + b)^3\*Log[I + Tan[c + d\*x]] + 6\*a\*b^2\*Tan[c + d\*x] + b^3\*Tan[c + d\*x]^2))/(12\*b\*d)

**Maple** [A]

time = 0.08, size = 213, normalized size = 1.29

method	result
norman	$(-3Aa^2b + Ab^3 - Ba^3 + 3Bab^2)x + \frac{(3Aa^2b - Ab^3 + Ba^3 - 3Bab^2) \tan(dx+c)}{d} + \frac{Bb^3(\tan^4(dx+c))}{4d} + \dots$

derivativedivides	$\frac{B b^3 (\tan^4(dx+c))}{4} + \frac{A b^3 (\tan^3(dx+c))}{3} + B a b^2 (\tan^3(dx+c)) + \frac{3 A a b^2 (\tan^2(dx+c))}{2} + \frac{3 B a^2 b (\tan^2(dx+c))}{2} - \frac{B b^3 (\tan^2(dx+c))}{2}$
default	$\frac{B b^3 (\tan^4(dx+c))}{4} + \frac{A b^3 (\tan^3(dx+c))}{3} + B a b^2 (\tan^3(dx+c)) + \frac{3 A a b^2 (\tan^2(dx+c))}{2} + \frac{3 B a^2 b (\tan^2(dx+c))}{2} - \frac{B b^3 (\tan^2(dx+c))}{2}$
risch	$\frac{2i B b^3 c}{d} - \frac{6i B a^2 b c}{d} - \frac{6i A a b^2 c}{d} + \frac{2i a^3 A c}{d} - 3 A a^2 b x + A b^3 x - B a^3 x + 3 B a b^2 x + \frac{2i(9 A a^2 b - 12 A a b^2 + 3 B a^3)}{d} \arctan(\tan(dx+c))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} \left( \frac{1}{4} B b^3 \tan^4(dx+c) + \frac{1}{3} A b^3 \tan^3(dx+c) + B a b^2 \tan^3(dx+c) + \frac{3}{2} A a b^2 \tan^2(dx+c) + \frac{3}{2} B a^2 b \tan^2(dx+c) - \frac{1}{2} B b^3 \tan^2(dx+c) + 3 A a^2 b \tan(dx+c) - A b^3 \tan(dx+c) + B a^3 \tan(dx+c) - 3 B a b^2 \tan(dx+c) + \frac{1}{2} (A a^3 - 3 A a^2 b - 3 B a^2 b + B b^3) \ln(1 + \tan^2(dx+c)) + (-3 A a^2 b + A b^3 - B a^3 + 3 B a b^2) \arctan(\tan(dx+c)) \right)$$

**Maxima** [A]

time = 0.57, size = 179, normalized size = 1.08

$$\frac{3 B b^3 \tan(dx+c)^4 + 4(3 B a b^2 + A b^3) \tan(dx+c)^3 + 6(3 B a^2 b + 3 A a b^2 - B b^3) \tan(dx+c)^2 - 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \tan(dx+c) + 6(A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3) \log(\tan(dx+c)^2 + 1) + 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \tan(dx+c)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{12} (3 B b^3 \tan^4(dx+c) + 4(3 B a b^2 + A b^3) \tan^3(dx+c) + 6(3 B a^2 b + 3 A a b^2 - B b^3) \tan^2(dx+c) - 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \tan(dx+c) + 6(A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3) \log(\tan^2(dx+c) + 1) + 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \tan(dx+c)) / d$$

**Fricas** [A]

time = 1.21, size = 178, normalized size = 1.08

$$\frac{3 B b^3 \tan(dx+c)^4 + 4(3 B a b^2 + A b^3) \tan(dx+c)^3 - 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) dx + 6(3 B a^2 b + 3 A a b^2 - B b^3) \tan(dx+c)^2 - 6(A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) + 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \tan(dx+c)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{12} (3 B b^3 \tan^4(dx+c) + 4(3 B a b^2 + A b^3) \tan^3(dx+c) - 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) dx + 6(3 B a^2 b + 3 A a b^2 - B b^3) \tan^2(dx+c) - 6(A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3) \log(1/(\tan^2(dx+c) + 1)) + 12(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \tan(dx+c)) / d$$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 311 vs.  $2(151) = 302$ .

time = 0.17, size = 311, normalized size = 1.88

$$\left\{ \frac{A^2 \log(\tan^2(c+dx)+1) - 3A^2bx + \frac{3A^2b^2 \tan(c+dx)}{2d} - \frac{3A^2b^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{3A^2b^2 \tan^2(c+dx)}{2d} + Ab^2x + \frac{Ab^2 \tan^2(c+dx)}{2d} - \frac{Ab^2 \tan(c+dx)}{d} - Bb^2x + \frac{Bb^2 \tan(c+dx)}{d} - \frac{3Bb^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{3Bb^2 \tan^2(c+dx)}{2d} + 3Bb^2x + \frac{Bb^2 \tan^3(c+dx)}{d} - \frac{3Bb^2 \tan(c+dx)}{d} + \frac{Bb^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^2 \tan^2(c+dx)}{2d} - \frac{Bb^2 \tan(c+dx)}{d} \right\} \text{ for } d \neq 0$$

otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((A\*a\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - 3\*A\*a\*\*2\*b\*x + 3\*A\*a\*\*2\*b\*tan(c + d\*x)/d - 3\*A\*a\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + 3\*A\*a\*b\*\*2\*tan(c + d\*x)\*\*2/(2\*d) + A\*b\*\*3\*x + A\*b\*\*3\*tan(c + d\*x)\*\*3/(3\*d) - A\*b\*\*3\*tan(c + d\*x)/d - B\*a\*\*3\*x + B\*a\*\*3\*tan(c + d\*x)/d - 3\*B\*a\*\*2\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + 3\*B\*a\*\*2\*b\*tan(c + d\*x)\*\*2/(2\*d) + 3\*B\*a\*b\*\*2\*x + B\*a\*b\*\*2\*tan(c + d\*x)\*\*3/d - 3\*B\*a\*b\*\*2\*tan(c + d\*x)/d + B\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*\*3\*tan(c + d\*x)\*\*4/(4\*d) - B\*b\*\*3\*tan(c + d\*x)\*\*2/(2\*d), Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*3\*tan(c), True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 2870 vs.  $2(159) = 318$ .

time = 2.28, size = 2870, normalized size = 17.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/12*(12*B*a^3*d*x*\tan(d*x)^4*\tan(c)^4 + 36*A*a^2*b*d*x*\tan(d*x)^4*\tan(c)^4 - 36*B*a*b^2*d*x*\tan(d*x)^4*\tan(c)^4 - 12*A*b^3*d*x*\tan(d*x)^4*\tan(c)^4 + 6*A*a^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 - 18*B*a^2*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 - 18*A*a*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 + 6*B*b^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^4*\tan(c)^4 - 48*B*a^3*d*x*\tan(d*x)^3*\tan(c)^3 - 144*A*a^2*b*d*x*\tan(d*x)^3*\tan(c)^3 + 144*B*a*b^2*d*x*\tan(d*x)^3*\tan(c)^3 + 48*A*b^3*d*x*\tan(d*x)^3*\tan(c)^3 - 18*B*a^2*b*\tan(d*x)^4*\tan(c)^4 - 18*A*a*b^2*\tan(d*x)^4*\tan(c)^4 + 9*B*b^3*\tan(d*x)^4*\tan(c)^4 - 24*A*a^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 + 72*B*a^2*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*$

$$\begin{aligned}
& \text{an}(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 + 72*A*a*b^2*\log(4* \\
& (\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x) \\
& ^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 - 24*B*b^3* \\
& \log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan \\
& (d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^3*\tan(c)^3 + 12* \\
& B*a^3*\tan(d*x)^4*\tan(c)^3 + 36*A*a^2*b*\tan(d*x)^4*\tan(c)^3 - 36*B*a*b^2*\tan \\
& (d*x)^4*\tan(c)^3 - 12*A*b^3*\tan(d*x)^4*\tan(c)^3 + 12*B*a^3*\tan(d*x)^3*\tan(c) \\
& )^4 + 36*A*a^2*b*\tan(d*x)^3*\tan(c)^4 - 36*B*a*b^2*\tan(d*x)^3*\tan(c)^4 - 12* \\
& A*b^3*\tan(d*x)^3*\tan(c)^4 + 72*B*a^3*d*x*\tan(d*x)^2*\tan(c)^2 + 216*A*a^2*b* \\
& d*x*\tan(d*x)^2*\tan(c)^2 - 216*B*a*b^2*d*x*\tan(d*x)^2*\tan(c)^2 - 72*A*b^3*d* \\
& x*\tan(d*x)^2*\tan(c)^2 - 18*B*a^2*b*\tan(d*x)^4*\tan(c)^2 - 18*A*a*b^2*\tan(d*x) \\
& )^4*\tan(c)^2 + 6*B*b^3*\tan(d*x)^4*\tan(c)^2 + 36*B*a^2*b*\tan(d*x)^3*\tan(c)^3 \\
& + 36*A*a*b^2*\tan(d*x)^3*\tan(c)^3 - 24*B*b^3*\tan(d*x)^3*\tan(c)^3 - 18*B*a^2 \\
& *b*\tan(d*x)^2*\tan(c)^4 - 18*A*a*b^2*\tan(d*x)^2*\tan(c)^4 + 6*B*b^3*\tan(d*x)^ \\
& 2*\tan(c)^4 + 12*B*a*b^2*\tan(d*x)^4*\tan(c) + 4*A*b^3*\tan(d*x)^4*\tan(c) + 36* \\
& A*a^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^ \\
& 2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^2*\tan(c)^2 \\
& - 108*B*a^2*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^ \\
& 2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(d*x)^2 \\
& *\tan(c)^2 - 108*A*a*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \\
& \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan \\
& (d*x)^2*\tan(c)^2 + 36*B*b^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan \\
& (c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + \\
& 1))*\tan(d*x)^2*\tan(c)^2 - 36*B*a^3*\tan(d*x)^3*\tan(c)^2 - 108*A*a^2*b*\tan(d \\
& *x)^3*\tan(c)^2 + 144*B*a*b^2*\tan(d*x)^3*\tan(c)^2 + 48*A*b^3*\tan(d*x)^3*\tan \\
& (c)^2 - 36*B*a^3*\tan(d*x)^2*\tan(c)^3 - 108*A*a^2*b*\tan(d*x)^2*\tan(c)^3 + 144 \\
& *B*a*b^2*\tan(d*x)^2*\tan(c)^3 + 48*A*b^3*\tan(d*x)^2*\tan(c)^3 + 12*B*a*b^2*\tan \\
& (d*x)*\tan(c)^4 + 4*A*b^3*\tan(d*x)*\tan(c)^4 - 3*B*b^3*\tan(d*x)^4 - 48*B*a^3 \\
& *d*x*\tan(d*x)*\tan(c) - 144*A*a^2*b*d*x*\tan(d*x)*\tan(c) + 144*B*a*b^2*d*x*\tan \\
& (d*x)*\tan(c) + 48*A*b^3*d*x*\tan(d*x)*\tan(c) + 36*B*a^2*b*\tan(d*x)^3*\tan(c) \\
& + 36*A*a*b^2*\tan(d*x)^3*\tan(c) - 24*B*b^3*\tan(d*x)^3*\tan(c) - 36*B*a^2*b*\tan \\
& (d*x)^2*\tan(c)^2 - 36*A*a*b^2*\tan(d*x)^2*\tan(c)^2 + 12*B*b^3*\tan(d*x)^2*\tan \\
& (c)^2 + 36*B*a^2*b*\tan(d*x)*\tan(c)^3 + 36*A*a*b^2*\tan(d*x)*\tan(c)^3 - 24* \\
& B*b^3*\tan(d*x)*\tan(c)^3 - 3*B*b^3*\tan(c)^4 - 12*B*a*b^2*\tan(d*x)^3 - 4*A*b^ \\
& 3*\tan(d*x)^3 - 24*A*a^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \\
& \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan \\
& (d*x)*\tan(c) + 72*B*a^2*b*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) \\
& ) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 + 1) \\
& )*\tan(d*x)*\tan(c) + 72*A*a*b^2*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan \\
& (c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c)^2 \\
& + 1))*\tan(d*x)*\tan(c) - 24*B*b^3*\log(4*(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3* \\
& *\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)/(\tan(c) \\
& ^2 + 1))*\tan(d*x)*\tan(c) + 36*B*a^3*\tan(d*x)^2*\tan(c) + 108*A*a^2*b*\tan(d*x) \\
& )^2*\tan(c) - 144*B*a*b^2*\tan(d*x)^2*\tan(c) - 48*A*b^3*\tan(d*x)^2*\tan(c) + 3 \\
& 6*B*a^3*\tan(d*x)*\tan(c)^2 + 108*A*a^2*b*\tan(d*x)*\tan(c)^2 - 144*B*a*b^2*\tan
\end{aligned}$$

$(d*x)*\tan(c)^2 - 48*A*b^3*\tan(d*x)*\tan(c)^2 - 12*B*a*b^2*\tan(c)^3 - 4*A*b^3*$   
 $*\tan(c)^3 + 12*B*a^3*d*x + 36*A*a^2*b*d*x - 36*B*a*b^2*d*x - 12*A*b^3*d*x -$   
 $18*B*a^2*b*\tan(d*x)^2 - 18*A*a*b^2*\tan(d*x)^2 + 6*B*b^3*\tan(d*x)^2 + 36*B*$   
 $a^2*b*\tan(d*x)*\tan(c) + 36*A*a*b^2*\tan(d*x)*\tan(c) - 24*B*b^3*\tan(d*x)*\tan(c)$   
 $- 18*B*a^2*b*\tan(c)^2 - 18*A*a*b^2*\tan(c)^2 \dots$

**Mupad [B]**

time = 6.33, size = 181, normalized size = 1.10

$$x(-B a^3 - 3 A a^2 b + 3 B a b^2 + A b^3) - \frac{\tan(c + dx)^2 \left( \frac{B b^2}{2} - \frac{3 a b (A b + B a)}{2} \right)}{d} - \frac{\tan(c + dx) (-B a^3 - 3 A a^2 b + 3 B a b^2 + A b^3)}{d} + \frac{\ln(\tan(c + dx)^2 + 1) \left( \frac{A c^2}{2} - \frac{3 B a^2 b}{2} - \frac{3 A a b^2}{2} + \frac{B b^2}{2} \right)}{d} + \frac{\tan(c + dx)^3 \left( \frac{A b^2}{2} + B a b \right)}{d} + \frac{B b^3 \tan(c + dx)^4}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out]  $x*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2) - (\tan(c + d*x)^2*((B*b^3)/2 - (3$   
 $*a*b*(A*b + B*a))/2))/d - (\tan(c + d*x)*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*$   
 $b^2))/d + (\log(\tan(c + d*x)^2 + 1)*((A*a^3)/2 + (B*b^3)/2 - (3*A*a*b^2)/2 -$   
 $(3*B*a^2*b)/2))/d + (\tan(c + d*x)^3*((A*b^3)/3 + B*a*b^2))/d + (B*b^3*\tan(c$   
 $+ d*x)^4)/(4*d)$



### 3.250 $\int (a + b \tan(c + dx))^3 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=140

$$(a^3 A - 3aAb^2 - 3a^2 bB + b^3 B) x - \frac{(3a^2 Ab - Ab^3 + a^3 B - 3ab^2 B) \log(\cos(c + dx))}{d} + \frac{b(2aAb + a^2 B - b^2 B)}{d}$$

[Out]  $(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x-(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*\ln(\cos(dx+c))/d+b*(2*A*a*b+B*a^2-B*b^2)*\tan(dx+c)/d+1/2*(A*b+B*a)*(a+b*\tan(dx+c))^2/d+1/3*B*(a+b*\tan(dx+c))^3/d$

**Rubi [A]**

time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3609, 3606, 3556}

$$\frac{b(a^2 B + 2aAb - b^2 B) \tan(c + dx)}{d} - \frac{(a^3 B + 3a^2 Ab - 3ab^2 B - Ab^3) \log(\cos(c + dx))}{d} + x(a^3 A - 3a^2 bB - 3aAb^2 + b^3 B) + \frac{(aB + Ab)(a + b \tan(c + dx))^2}{2d} + \frac{B(a + b \tan(c + dx))^3}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])^3*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*x - ((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*\text{Log}[\text{Cos}[c + d*x]])/d + (b*(2*a*A*b + a^2*B - b^2*B)*\text{Tan}[c + d*x])/d + ((A*b + a*B)*(a + b*\text{Tan}[c + d*x])^2)/(2*d) + (B*(a + b*\text{Tan}[c + d*x])^3)/(3*d)$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3606**

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

**Rule 3609**

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
\int (a + b \tan(c + dx))^3 (A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^3}{3d} + \int (a + b \tan(c + dx))^2 (aA - bB + \\
&= \frac{(Ab + aB)(a + b \tan(c + dx))^2}{2d} + \frac{B(a + b \tan(c + dx))^3}{3d} + \\
&= (a^3 A - 3aAb^2 - 3a^2 bB + b^3 B) x + \frac{b(2aAb + a^2 B - b^2 B) \tan(c + dx)}{d} \\
&= (a^3 A - 3aAb^2 - 3a^2 bB + b^3 B) x - \frac{(3a^2 Ab - Ab^3 + a^3 B - b^3 B) \tan^3(c + dx)}{6d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.65, size = 130, normalized size = 0.93

$$\frac{3(a + ib)^3(-iA + B) \log(i - \tan(c + dx)) + 3(a - ib)^3(iA + B) \log(i + \tan(c + dx)) + 6b(3aAb + 3a^2 B - b^2 B) \tan(c + dx) + 3b^2(Ab + 3aB) \tan^2(c + dx) + 2b^3 B \tan^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (3\*(a + I\*b)^3\*((-I)\*A + B)\*Log[I - Tan[c + d\*x]] + 3\*(a - I\*b)^3\*(I\*A + B)\*Log[I + Tan[c + d\*x]] + 6\*b\*(3\*a\*A\*b + 3\*a^2\*B - b^2\*B)\*Tan[c + d\*x] + 3\*b^2\*(A\*b + 3\*a\*B)\*Tan[c + d\*x]^2 + 2\*b^3\*B\*Tan[c + d\*x]^3)/(6\*d)

**Maple [A]**

time = 0.06, size = 159, normalized size = 1.14

method	result
norman	$(A a^3 - 3A a b^2 - 3B a^2 b + B b^3) x + \frac{b(3A a b + 3a^2 B - b^2 B) \tan(dx+c)}{d} + \frac{B b^3 (\tan^3(dx+c))}{3d} + \frac{b^2 (Ab + 3aB) \tan^2(dx+c)}{d}$
derivativedivides	$\frac{B b^3 (\tan^3(dx+c))}{3} + \frac{A b^3 (\tan^2(dx+c))}{2} + \frac{3B a b^2 (\tan^2(dx+c))}{2} + 3A a b^2 \tan(dx+c) + 3B a^2 b \tan(dx+c) - B b^3 \tan(dx+c) + \frac{b^2 (Ab + 3aB) \tan^2(dx+c)}{d}$
default	$\frac{B b^3 (\tan^3(dx+c))}{3} + \frac{A b^3 (\tan^2(dx+c))}{2} + \frac{3B a b^2 (\tan^2(dx+c))}{2} + 3A a b^2 \tan(dx+c) + 3B a^2 b \tan(dx+c) - B b^3 \tan(dx+c) + \frac{b^2 (Ab + 3aB) \tan^2(dx+c)}{d}$
risch	$A a^3 x - 3A a b^2 x - 3B a^2 b x + B b^3 x + \frac{2iB a^3 c}{d} - 3iB a b^2 x + 3iA a^2 b x - iA b^3 x + iB a^3 x$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(1/3\*B\*b^3\*tan(d\*x+c)^3+1/2\*A\*b^3\*tan(d\*x+c)^2+3/2\*B\*a\*b^2\*tan(d\*x+c)^2+3\*A\*a\*b^2\*tan(d\*x+c)+3\*B\*a^2\*b\*tan(d\*x+c)-B\*b^3\*tan(d\*x+c)+1/2\*(3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*ln(1+tan(d\*x+c)^2)+(A\*a^3-3\*A\*a\*b^2-3\*B\*a^2\*b+B\*b^3)\*arctan(tan(d\*x+c)))

**Maxima [A]**

time = 0.53, size = 143, normalized size = 1.02

$$\frac{2 B b^3 \tan(dx+c)^3 + 3(3 B a b^2 + A b^3) \tan(dx+c)^2 + 6(A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3)(dx+c) + 3(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \log(\tan(dx+c)^2 + 1) + 6(3 B a^2 b + 3 A a b^2 - B b^3) \tan(dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{6} * (2 * B * b^3 * \tan(d * x + c)^3 + 3 * (3 * B * a * b^2 + A * b^3) * \tan(d * x + c)^2 + 6 * (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2 + B * b^3) * (d * x + c) + 3 * (B * a^3 + 3 * A * a^2 * b - 3 * B * a * b^2 - A * b^3) * \log(\tan(d * x + c)^2 + 1) + 6 * (3 * B * a^2 * b + 3 * A * a * b^2 - B * b^3) * \tan(d * x + c)) / d$

**Fricas [A]**

time = 0.79, size = 142, normalized size = 1.01

$$\frac{2 B b^3 \tan(dx+c)^3 + 6(A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3) dx + 3(3 B a b^2 + A b^3) \tan(dx+c)^2 - 3(B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 6(3 B a^2 b + 3 A a b^2 - B b^3) \tan(dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{6} * (2 * B * b^3 * \tan(d * x + c)^3 + 6 * (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2 + B * b^3) * d * x + 3 * (3 * B * a * b^2 + A * b^3) * \tan(d * x + c)^2 - 3 * (B * a^3 + 3 * A * a^2 * b - 3 * B * a * b^2 - A * b^3) * \log(1 / (\tan(d * x + c)^2 + 1)) + 6 * (3 * B * a^2 * b + 3 * A * a * b^2 - B * b^3) * \tan(d * x + c)) / d$

**Sympy [A]**

time = 0.13, size = 240, normalized size = 1.71

$$\begin{cases} A a^3 x + \frac{3 A a^2 b \log(\tan^2(c+d x)+1)}{2 d} - 3 A a b^2 x + \frac{3 A a b^2 \tan(c+d x)}{d} - \frac{A b^3 \log(\tan^2(c+d x)+1)}{2 d} + \frac{A b^3 \tan^2(c+d x)}{2 d} + \frac{B a^3 \log(\tan^2(c+d x)+1)}{2 d} - 3 B a^2 b x + \frac{3 B a^2 b \tan(c+d x)}{d} - \frac{3 B a b^2 \log(\tan^2(c+d x)+1)}{2 d} + \frac{3 B a b^2 \tan^2(c+d x)}{2 d} + B b^3 x + \frac{B b^3 \tan^2(c+d x)}{2 d} - \frac{B b^3 \tan(c+d x)}{d} & \text{for } d \neq 0 \\ x(A+B \tan(c))(a+b \tan(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((A\*a\*\*3\*x + 3\*A\*a\*\*2\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - 3\*A\*a\*b\*\*2\*x + 3\*A\*a\*b\*\*2\*tan(c + d\*x)/d - A\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*b\*\*3\*tan(c + d\*x)\*\*2/(2\*d) + B\*a\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - 3\*B\*a\*\*2\*b\*x + 3\*B\*a\*\*2\*b\*tan(c + d\*x)/d - 3\*B\*a\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + 3\*B\*a\*b\*\*2\*tan(c + d\*x)\*\*2/(2\*d) + B\*b\*\*3\*x + B\*b\*\*3\*tan(c + d\*x)\*\*3/(3\*d) - B\*b\*\*3\*tan(c + d\*x)/d, Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*3, True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 1911 vs. 2(136) = 272.

time = 1.50, size = 1911, normalized size = 13.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{6} * (6 * A * a^3 * d * x * \tan(d * x)^3 * \tan(c)^3 - 18 * B * a^2 * b * d * x * \tan(d * x)^3 * \tan(c)^3 - 18 * A * a * b^2 * d * x * \tan(d * x)^3 * \tan(c)^3 + 6 * B * b^3 * d * x * \tan(d * x)^3 * \tan(c)^3 - 3 * B * a^3 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^3 * \tan(c)^3 - 9 * A * a^2 * b * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^3 * \tan(c)^3 + 9 * B * a * b^2 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^3 * \tan(c)^3 + 3 * A * b^3 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^3 * \tan(c)^3 - 18 * A * a^3 * d * x * \tan(d * x)^2 * \tan(c)^2 + 54 * B * a^2 * b * d * x * \tan(d * x)^2 * \tan(c)^2 + 54 * A * a * b^2 * d * x * \tan(d * x)^2 * \tan(c)^2 - 18 * B * b^3 * d * x * \tan(d * x)^2 * \tan(c)^2 + 9 * B * a * b^2 * \tan(d * x)^3 * \tan(c)^3 + 3 * A * b^3 * \tan(d * x)^3 * \tan(c)^3 + 9 * B * a^3 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^2 * \tan(c)^2 + 27 * A * a^2 * b * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^2 * \tan(c)^2 - 27 * B * a * b^2 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^2 * \tan(c)^2 - 9 * A * b^3 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x)^2 * \tan(c)^2 - 18 * B * a^2 * b * \tan(d * x)^3 * \tan(c)^2 - 18 * A * a * b^2 * \tan(d * x)^3 * \tan(c)^2 + 6 * B * b^3 * \tan(d * x)^3 * \tan(c)^2 - 18 * B * a^2 * b * \tan(d * x)^2 * \tan(c)^3 - 18 * A * a * b^2 * \tan(d * x)^2 * \tan(c)^3 + 6 * B * b^3 * \tan(d * x)^2 * \tan(c)^3 + 18 * A * a^3 * d * x * \tan(d * x) * \tan(c) - 54 * B * a^2 * b * d * x * \tan(d * x) * \tan(c) - 54 * A * a * b^2 * d * x * \tan(d * x) * \tan(c) + 18 * B * b^3 * d * x * \tan(d * x) * \tan(c) + 9 * B * a * b^2 * \tan(d * x)^3 * \tan(c) + 3 * A * b^3 * \tan(d * x)^3 * \tan(c) - 9 * B * a * b^2 * \tan(d * x)^2 * \tan(c)^2 - 3 * A * b^3 * \tan(d * x)^2 * \tan(c)^2 + 9 * B * a * b^2 * \tan(d * x) * \tan(c)^3 + 3 * A * b^3 * \tan(d * x) * \tan(c)^3 - 2 * B * b^3 * \tan(d * x)^3 - 9 * B * a^3 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x) * \tan(c) - 27 * A * a^2 * b * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x) * \tan(c) + 27 * B * a * b^2 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x) * \tan(c) + 9 * A * b^3 * \log(4 * (\tan(d * x)^4 * \tan(c)^2 - 2 * \tan(d * x)^3 * \tan(c) + \tan(d * x)^2 * \tan(c)^2 + \tan(d * x)^2 - 2 * \tan(d * x) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(d * x) * \tan(c) + 36 * B * a^2 * b * \tan(d * x)^2 * \tan(c) + 36 * A * a * b^2 * \tan(d * x)^2 * \tan(c) - 18 * B * b^3 * \tan(d * x)^2 * \tan(c) + 36 * B * a^2 * b * \tan(d * x) * \tan(c)^2 + 36 * A * a * b^2 * \tan(d * x) * \tan(c)^2 - 18 * B * b^3 * \tan(d * x) * \tan(c)^2 - 2 * B * b^3 * \tan(c)^3 - 6 * A * a^3 * d * x + 18 * B * a^2 * b * d * x + 18 * A * a * b^2 * d * x - 6 * B * b^3 * d * x - 9 * B * a * b^2 * \tan(d * x)^2 - 3 * A * b^3 * \tan(d * x)^2 + 9 * B * a * b^2 * \tan(d * x) * \tan(c) + 3 * A * b^3 * \tan(d * x) * \tan(c)$

$\tan(dx) \cdot \tan(c) - 9B \cdot a \cdot b^2 \cdot \tan(c)^2 - 3A \cdot b^3 \cdot \tan(c)^2 + 3B \cdot a^3 \cdot \log(4(\tan(dx)^4 \cdot \tan(c)^2 - 2 \cdot \tan(dx)^3 \cdot \tan(c) + \tan(dx)^2 \cdot \tan(c)^2 + \tan(dx)^2 - 2 \cdot \tan(dx) \cdot \tan(c) + 1)/(\tan(c)^2 + 1)) + 9A \cdot a^2 \cdot b \cdot \log(4(\tan(dx)^4 \cdot \tan(c)^2 - 2 \cdot \tan(dx)^3 \cdot \tan(c) + \tan(dx)^2 \cdot \tan(c)^2 + \tan(dx)^2 - 2 \cdot \tan(dx) \cdot \tan(c) + 1)/(\tan(c)^2 + 1)) - 9B \cdot a \cdot b^2 \cdot \log(4(\tan(dx)^4 \cdot \tan(c)^2 - 2 \cdot \tan(dx)^3 \cdot \tan(c) + \tan(dx)^2 \cdot \tan(c)^2 + \tan(dx)^2 - 2 \cdot \tan(dx) \cdot \tan(c) + 1)/(\tan(c)^2 + 1)) - 3A \cdot b^3 \cdot \log(4(\tan(dx)^4 \cdot \tan(c)^2 - 2 \cdot \tan(dx)^3 \cdot \tan(c) + \tan(dx)^2 \cdot \tan(c)^2 + \tan(dx)^2 - 2 \cdot \tan(dx) \cdot \tan(c) + 1)/(\tan(c)^2 + 1)) - 18B \cdot a^2 \cdot b \cdot \tan(dx) - 18A \cdot a \cdot b^2 \cdot \tan(dx) + 6B \cdot b^3 \cdot \tan(dx) - 18B \cdot a^2 \cdot b \cdot \tan(c) - 18A \cdot a \cdot b^2 \cdot \tan(c) + 6B \cdot b^3 \cdot \tan(c) - 9B \cdot a \cdot b^2 - 3A \cdot b^3)/(d \cdot \tan(dx)^3 \cdot \tan(c)^3 - 3d \cdot \tan(dx)^2 \cdot \tan(c)^2 + 3d \cdot \tan(dx) \cdot \tan(c) - d)$

**Mupad [B]**

time = 6.24, size = 142, normalized size = 1.01

$$x(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) - \frac{\ln(\tan(c+dx)^2 + 1) \left(-\frac{Bb^3}{2} - \frac{3Aa^2b}{2} + \frac{3Bab^2}{2} + \frac{Ab^3}{2}\right)}{d} + \frac{\tan(c+dx)^2 \left(\frac{Ab^3}{2} + \frac{3Bab^2}{2}\right)}{d} - \frac{\tan(c+dx)(Bb^3 - 3ab(Ab+Ba))}{d} + \frac{Bb^3 \tan(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out]  $x \cdot (A \cdot a^3 + B \cdot b^3 - 3A \cdot a \cdot b^2 - 3B \cdot a^2 \cdot b) - (\log(\tan(c + d \cdot x)^2 + 1) \cdot ((A \cdot b^3)/2 - (B \cdot a^3)/2 - (3A \cdot a^2 \cdot b)/2 + (3B \cdot a \cdot b^2)/2))/d + (\tan(c + d \cdot x)^2 \cdot ((A \cdot b^3)/2 + (3B \cdot a \cdot b^2)/2))/d - (\tan(c + d \cdot x) \cdot (B \cdot b^3 - 3a \cdot b \cdot (A \cdot b + B \cdot a)))/d + (B \cdot b^3 \cdot \tan(c + d \cdot x)^3)/(3 \cdot d)$

### 3.251 $\int \cot(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$

Optimal. Leaf size=117

$$(3a^2Ab - Ab^3 + a^3B - 3ab^2B)x - \frac{b(3aAb + 3a^2B - b^2B) \log(\cos(c + dx))}{d} + \frac{a^3A \log(\sin(c + dx))}{d} + \frac{b^2(Ab + B^2)}{2d}$$

[Out] (3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*x-b\*(3\*A\*a\*b+3\*B\*a^2-B\*b^2)\*ln(cos(d\*x+c))/d+a^3\*A\*ln(sin(d\*x+c))/d+b^2\*(A\*b+2\*B\*a)\*tan(d\*x+c)/d+1/2\*b\*B\*(a+b\*tan(d\*x+c))^2/d

Rubi [A]

time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3688, 3718, 3705, 3556}

$$\frac{a^3A \log(\sin(c + dx))}{d} - \frac{b(3a^2B + 3aAb - b^2B) \log(\cos(c + dx))}{d} + x(a^3B + 3a^2Ab - 3ab^2B - Ab^3) + \frac{b^2(2aB + Ab) \tan(c + dx)}{d} + \frac{bB(a + b \tan(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*x - (b\*(3\*a\*A\*b + 3\*a^2\*B - b^2\*B)\*Log[Cos[c + d\*x]])/d + (a^3\*A\*Log[Sin[c + d\*x]])/d + (b^2\*(A\*b + 2\*a\*B)\*Tan[c + d\*x])/d + (b\*B\*(a + b\*Tan[c + d\*x])^2)/(2\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(p\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3705

```
Int[((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2
)/tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[B*x, x] + (Dist[A, Int[1/Tan[e
+ f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

### Rule 3718

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_
)*(x_)]^(n_))*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f
_)*(x_)]^2), x_Symbol] := Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \cot(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \frac{bB(a + b \tan(c + dx))^2}{2d} + \frac{1}{2} \int \cot(c + dx)(a + b \tan(c + dx))^2 dx \\
&= \frac{b^2(Ab + 2aB) \tan(c + dx)}{d} + \frac{bB(a + b \tan(c + dx))}{2d} \\
&= (3a^2 Ab - Ab^3 + a^3 B - 3ab^2 B) x + \frac{b^2(Ab + 2aB)}{2d} \\
&= (3a^2 Ab - Ab^3 + a^3 B - 3ab^2 B) x - \frac{b(3aAb + 2a^2 B)}{2d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.40, size = 115, normalized size = 0.98

$$\frac{-(a + ib)^3(A + iB) \log(i - \tan(c + dx)) + 2a^3 A \log(\tan(c + dx)) - (a - ib)^3(A - iB) \log(i + \tan(c + dx)) + 2b^2(Ab + 2aB) \tan(c + dx) + bB(a + b \tan(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

```
[Out] (-((a + I*b)^3*(A + I*B)*Log[I - Tan[c + d*x]]) + 2*a^3*A*Log[Tan[c + d*x]]
- (a - I*b)^3*(A - I*B)*Log[I + Tan[c + d*x]] + 2*b^2*(A*b + 2*a*B)*Tan[c
+ d*x] + b*B*(a + b*Tan[c + d*x])^2)/(2*d)
```

**Maple [A]**

time = 0.18, size = 131, normalized size = 1.12

method	result
norman	$(3Aa^2b - Ab^3 + Ba^3 - 3Ba^2b^2)x + \frac{b^2(Ab+3aB)\tan(dx+c)}{d} + \frac{Bb^3(\tan^2(dx+c))}{2d} + \frac{Aa^3\ln(\tan(dx+c))}{d}$
derivativedivides	$\frac{Aa^3\ln(\sin(dx+c))+Ba^3(dx+c)+3Aa^2b(dx+c)-3Ba^2b\ln(\cos(dx+c))-3Aa^2b^2\ln(\cos(dx+c))+3Ba^2b^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{Aa^3\ln(\sin(dx+c))+Ba^3(dx+c)+3Aa^2b(dx+c)-3Ba^2b\ln(\cos(dx+c))-3Aa^2b^2\ln(\cos(dx+c))+3Ba^2b^2(\tan(dx+c)-dx-c)}{d}$
risch	$\frac{2ib^2(-iBbe^{2i(dx+c)}+be^{2i(dx+c)}A+3aBe^{2i(dx+c)}+Ab+3aB)}{d(e^{2i(dx+c)}+1)^2} - \frac{2ia^3Ac}{d} + \frac{6iBa^2bc}{d} + \frac{6iAab^2c}{d} + 3Aa^2bx - A$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*(A*a^3*\ln(\sin(d*x+c))+B*a^3*(d*x+c)+3*A*a^2*b*(d*x+c)-3*B*a^2*b*\ln(\cos(d*x+c))-3*A*a*b^2*\ln(\cos(d*x+c))+3*B*a*b^2*(\tan(d*x+c)-d*x-c)+A*b^3*(\tan(d*x+c)-d*x-c)+B*b^3*(1/2*\tan(d*x+c)^2+\ln(\cos(d*x+c))))$

**Maxima [A]**

time = 0.51, size = 124, normalized size = 1.06

$$\frac{Bb^3 \tan(dx+c)^2 + 2Aa^3 \log(\tan(dx+c)) + 2(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)(dx+c) - (Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log(\tan(dx+c)^2 + 1) + 2(3Bab^2 + Ab^3) \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/2*(B*b^3*\tan(d*x+c)^2 + 2*A*a^3*\log(\tan(d*x+c)) + 2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x+c) - (A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\log(\tan(d*x+c)^2 + 1) + 2*(3*B*a*b^2 + A*b^3)*\tan(d*x+c))/d$

**Fricas [A]**

time = 0.78, size = 133, normalized size = 1.14

$$\frac{Bb^3 \tan(dx+c)^2 + Aa^3 \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) + 2(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)dx - (3Ba^2b + 3Aab^2 - Bb^3) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2(3Bab^2 + Ab^3) \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/2*(B*b^3*\tan(d*x+c)^2 + A*a^3*\log(\tan(d*x+c)^2/(\tan(d*x+c)^2+1)) + 2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*d*x - (3*B*a^2*b + 3*A*a*b^2 - B*b^3)*\log(1/(\tan(d*x+c)^2+1)) + 2*(3*B*a*b^2 + A*b^3)*\tan(d*x+c))/d$



**Sympy [A]**

time = 0.57, size = 204, normalized size = 1.74

$$\begin{cases} -\frac{Aa^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{Aa^3 \log(\tan(c+dx))}{d} + 3Aa^2bx + \frac{3Aa^2 \log(\tan^2(c+dx)+1)}{2d} - Ab^3x + \frac{Ab^3 \tan(c+dx)}{d} + Ba^3x + \frac{3Ba^2b \log(\tan^2(c+dx)+1)}{2d} - 3Bab^2x + \frac{3Ba^2 \tan(c+dx)}{d} - \frac{Bb^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^3 \tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A+B \tan(c))(a+b \tan(c))^3 \cot(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

**[Out]** Piecewise((-A\*a\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*a\*\*3\*log(tan(c + d\*x))/d + 3\*A\*a\*\*2\*b\*x + 3\*A\*a\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*b\*\*3\*x + A\*b\*\*3\*tan(c + d\*x)/d + B\*a\*\*3\*x + 3\*B\*a\*\*2\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - 3\*B\*a\*b\*\*2\*x + 3\*B\*a\*b\*\*2\*tan(c + d\*x)/d - B\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*\*3\*tan(c + d\*x)\*\*2/(2\*d), Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*3\*cot(c), True))

**Giac [A]**

time = 1.19, size = 129, normalized size = 1.10

$$\frac{Bb^3 \tan(dx+c)^2 + 2Aa^3 \log(|\tan(dx+c)|) + 6Bab^2 \tan(dx+c) + 2Ab^3 \tan(dx+c) + 2(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)(dx+c) - (Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log(\tan(dx+c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

**[Out]** 1/2\*(B\*b^3\*tan(d\*x + c)^2 + 2\*A\*a^3\*log(abs(tan(d\*x + c)))) + 6\*B\*a\*b^2\*tan(d\*x + c) + 2\*A\*b^3\*tan(d\*x + c) + 2\*(B\*a^3 + 3\*A\*a^2\*b - 3\*B\*a\*b^2 - A\*b^3)\*(d\*x + c) - (A\*a^3 - 3\*B\*a^2\*b - 3\*A\*a\*b^2 + B\*b^3)\*log(tan(d\*x + c)^2 + 1))/d

**Mupad [B]**

time = 6.42, size = 118, normalized size = 1.01

$$\frac{\tan(c+dx) (Ab^3 + 3Bab^2)}{d} + \frac{Aa^3 \ln(\tan(c+dx))}{d} + \frac{Bb^3 \tan(c+dx)^2}{2d} - \frac{\ln(\tan(c+dx) + 1i) (A - B1i) (b + a1i)^3 1i}{2d} - \frac{\ln(\tan(c+dx) - 1i) (A + B1i) (-b + a1i)^3 1i}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

**[Out]** (tan(c + d\*x)\*(A\*b^3 + 3\*B\*a\*b^2))/d + (A\*a^3\*log(tan(c + d\*x)))/d - (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b)^3\*1i)/(2\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a\*1i - b)^3\*1i)/(2\*d) + (B\*b^3\*tan(c + d\*x)^2)/(2\*d)



```
Int[((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2
)/tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[B*x, x] + (Dist[A, Int[1/Tan[e
+ f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

### Rule 3718

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_
)*(x_)]^(n_))*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f
_)*(x_)]^2), x_Symbol] := Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Simp
[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned} \int \cot^2(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx)(a + b \tan(c + dx))^2}{d} + \int \cot \\ &= \frac{b^2(aA + bB) \tan(c + dx)}{d} - \frac{aA \cot(c + dx)(a + b \tan(c + dx))^2}{d} \\ &= -(a^3A - 3aAb^2 - 3a^2bB + b^3B)x + \frac{b^2(aA + bB) \tan(c + dx)}{d} \\ &= -(a^3A - 3aAb^2 - 3a^2bB + b^3B)x - \frac{b^2(Ab + a^2)}{d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.36, size = 113, normalized size = 0.95

$$\frac{-2a^3A \cot(c + dx) + i(a + ib)^3(A + iB) \log(i - \tan(c + dx)) + 2a^2(3Ab + aB) \log(\tan(c + dx)) + (ia + b)^3(A - iB) \log(i + \tan(c + dx)) + 2b^3B \tan(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

```
[Out] (-2*a^3*A*Cot[c + d*x] + I*(a + I*b)^3*(A + I*B)*Log[I - Tan[c + d*x]] + 2*
a^2*(3*A*b + a*B)*Log[Tan[c + d*x]] + (I*a + b)^3*(A - I*B)*Log[I + Tan[c +
d*x]] + 2*b^3*B*Tan[c + d*x])/(2*d)
```

**Maple [A]**

time = 0.16, size = 123, normalized size = 1.03

method	result
derivativedivides	$\frac{A a^3(-\cot(dx+c)-dx-c)+B a^3 \ln(\sin(dx+c))+3A a^2 b \ln(\sin(dx+c))+3B a^2 b(dx+c)+3A a b^2(dx+c)-3B a b^2 \ln(\cos(dx+c))}{d}$
default	$\frac{A a^3(-\cot(dx+c)-dx-c)+B a^3 \ln(\sin(dx+c))+3A a^2 b \ln(\sin(dx+c))+3B a^2 b(dx+c)+3A a b^2(dx+c)-3B a b^2 \ln(\cos(dx+c))}{d}$
norman	$\frac{(-A a^3+3A a b^2+3B a^2 b-B b^3)x \tan(dx+c)+\frac{B b^3(\tan^2(dx+c))}{d}-\frac{A a^3}{d}}{\tan(dx+c)} + \frac{a^2(3A b+a B) \ln(\tan(dx+c))}{d} - \frac{(3A a^2 b-A b^3) \ln(\cos(dx+c))}{d}$
risch	$-A a^3 x + 3A a b^2 x + 3B a^2 b x - B b^3 x - \frac{2i B a^3 c}{d} + 3i B a b^2 x - 3i A a^2 b x - \frac{2i(A a^3 e^{2i(dx+c)} - E)}{d(e^{2i(dx+c)})}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (A a^3 (-\cot(dx+c) - dx - c) + B a^3 \ln(\sin(dx+c)) + 3 A a^2 b \ln(\sin(dx+c)) + 3 B a^2 b (dx+c) + 3 A a b^2 (dx+c) - 3 B a b^2 \ln(\cos(dx+c)) - A b^3 \ln(\cos(dx+c)) + B b^3 (\tan(dx+c) - dx - c))$

**Maxima** [A]

time = 0.51, size = 125, normalized size = 1.05

$$\frac{2 B b^3 \tan(dx+c) - \frac{2 A a^3}{\tan(dx+c)} - 2 (A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3)(dx+c) - (B a^3 + 3 A a^2 b - 3 B a b^2 - A b^3) \log(\tan(dx+c)^2 + 1) + 2 (B a^3 + 3 A a^2 b) \log(\tan(dx+c))}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (2 * B * b^3 * \tan(dx+c) - 2 * A * a^3 / \tan(dx+c) - 2 * (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2 + B * b^3) * (dx+c) - (B * a^3 + 3 * A * a^2 * b - 3 * B * a * b^2 - A * b^3) * \log(\tan(dx+c)^2 + 1) + 2 * (B * a^3 + 3 * A * a^2 * b) * \log(\tan(dx+c))) / d$

**Fricas** [A]

time = 0.68, size = 145, normalized size = 1.22

$$\frac{2 B b^3 \tan(dx+c)^2 - 2 A a^3 - 2 (A a^3 - 3 B a^2 b - 3 A a b^2 + B b^3) dx \tan(dx+c) + (B a^3 + 3 A a^2 b) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c) - (3 B a b^2 + A b^3) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) \tan(dx+c)}{2 d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{2} * (2 * B * b^3 * \tan(dx+c)^2 - 2 * A * a^3 - 2 * (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2 + B * b^3) * dx * \tan(dx+c) + (B * a^3 + 3 * A * a^2 * b) * \log(\tan(dx+c)^2 / (\tan(dx+c)^2 + 1)) * \tan(dx+c) - (3 * B * a * b^2 + A * b^3) * \log(1 / (\tan(dx+c)^2 + 1)) * \tan(dx+c)) / (d * \tan(dx+c))$

**Sympy [A]**

time = 0.96, size = 223, normalized size = 1.87

$$\begin{cases} \infty Aa^3x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c))^3 \cot^2(c) & \text{for } d = 0 \\ -Aa^3x - \frac{Aa^3}{2 \tan(c+dx)} - \frac{3Aa^2b \log(\tan^2(c+dx)+1)}{2d} + \frac{3Aa^2b \log(\tan(c+dx))}{d} + 3Aab^2x + \frac{Ab^3 \log(\tan^2(c+dx)+1)}{2d} - \frac{Ba^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{Ba^3 \log(\tan(c+dx))}{d} + 3Ba^2bx + \frac{3Bab^2 \log(\tan^2(c+dx)+1)}{2d} - Bb^3x + \frac{Bb^3 \tan(c+dx)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)\*\*2\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

**[Out]** Piecewise((zoo\*A\*a\*\*3\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*3\*cot(c)\*\*2, Eq(d, 0)), (-A\*a\*\*3\*x - A\*a\*\*3/(d\*tan(c + d\*x)) - 3\*A\*a\*\*2\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + 3\*A\*a\*\*2\*b\*log(tan(c + d\*x))/d + 3\*A\*a\*b\*\*2\*x + A\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*a\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*a\*\*3\*log(tan(c + d\*x))/d + 3\*B\*a\*\*2\*b\*x + 3\*B\*a\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*b\*\*3\*x + B\*b\*\*3\*tan(c + d\*x)/d, True))

**Giac [A]**

time = 1.44, size = 152, normalized size = 1.28

$$\frac{2Bb^3 \tan(dx+c) - 2(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3)(dx+c) - (Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3) \log(\tan(dx+c)^2 + 1) + 2(Ba^3 + 3Aa^2b) \log(|\tan(dx+c)|) - \frac{2(Ba^3 \tan(dx+c) + 3Aa^2b \tan(dx+c) + Aa^3)}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

**[Out]** 1/2\*(2\*B\*b^3\*tan(d\*x + c) - 2\*(A\*a^3 - 3\*B\*a^2\*b - 3\*A\*a\*b^2 + B\*b^3)\*(d\*x + c) - (B\*a^3 + 3\*A\*a^2\*b - 3\*B\*a\*b^2 - A\*b^3)\*log(tan(d\*x + c)^2 + 1) + 2\*(B\*a^3 + 3\*A\*a^2\*b)\*log(abs(tan(d\*x + c)))) - 2\*(B\*a^3\*tan(d\*x + c) + 3\*A\*a^2\*b\*tan(d\*x + c) + A\*a^3)/tan(d\*x + c))/d

**Mupad [B]**

time = 6.41, size = 114, normalized size = 0.96

$$\frac{\ln(\tan(c+dx))(Ba^3+3Aba^2)}{d} - \frac{Aa^3 \cot(c+dx)}{d} + \frac{Bb^3 \tan(c+dx)}{d} + \frac{\ln(\tan(c+dx)-i)(A+Bl)(a+bl)^3 \operatorname{li}}{2d} - \frac{\ln(\tan(c+dx)+li)(A-Bli)(a-bl)^3 \operatorname{li}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

**[Out]** (log(tan(c + d\*x))\*(B\*a^3 + 3\*A\*a^2\*b))/d + (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a + b\*1i)^3\*1i)/(2\*d) - (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a - b\*1i)^3\*1i)/(2\*d) - (A\*a^3\*cot(c + d\*x))/d + (B\*b^3\*tan(c + d\*x))/d

### 3.253 $\int \cot^3(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=127

$$-\left((3a^2Ab - Ab^3 + a^3B - 3ab^2B)x - \frac{a^2(2Ab + aB)\cot(c + dx)}{d} - \frac{b^3B \log(\cos(c + dx))}{d} - \frac{a(a^2A - 3Ab^2 - 3B^2)}{d}\right)$$

[Out]  $-(3Aa^2b - Ab^3 + Ba^3 - 3Bab^2)x - a^2(2Ab + Ba)\cot(dx + c)/d - b^3B \ln(\cos(dx + c))/d - a(Aa^2 - 3Ab^2 - 3B^2)\ln(\sin(dx + c))/d - 1/2aA\cot(dx + c)^2(a + b\tan(dx + c))^2/d$

**Rubi [A]**

time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {3686, 3716, 3705, 3556}

$$-\frac{a(a^2A - 3abB - 3Ab^2)\log(\sin(c + dx))}{d} - \frac{a^2(aB + 2Ab)\cot(c + dx)}{d} - x(a^3B + 3a^2Ab - 3ab^2B - Ab^3) - \frac{aA\cot^2(c + dx)(a + b\tan(c + dx))^2}{2d} - \frac{b^3B \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left(\left(3a^2Ab - Ab^3 + a^3B - 3a^2b^2B\right)x - \left(a^2(2Ab + aB)\cot[c + d*x]\right)/d - \left(b^3B \log[\cos[c + d*x]]\right)/d - \left(a(a^2A - 3Ab^2 - 3a^2b^2B)\log[\sin[c + d*x]]\right)/d - \left(aA\cot[c + d*x]^2(a + b\tan[c + d*x])^2\right)/(2d)\right)$

Rule 3556

Int[tan[(c.) + (d.)\*(x.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3686

Int[((a.) + (b.)\*tan[(e.) + (f.)\*(x.)])^(m.)\*((A.) + (B.)\*tan[(e.) + (f.)\*(x.)])^(n.), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

Rule 3705

```
Int[((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2
)/tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[B*x, x] + (Dist[A, Int[1/Tan[e
+ f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

### Rule 3716

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)]*(c_) + (d_)*tan[(e_) + (f_
)*(x_)]^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f
_)*(x_)]^2), x_Symbol] := Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]
```

### Rubi steps

$$\begin{aligned} \int \cot^3(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)(a + b \tan(c + dx))^2}{2d} + \frac{1}{2} \int \\ &= -\frac{a^2(2Ab + aB) \cot(c + dx)}{d} - \frac{aA \cot^2(c + dx)}{2d} \\ &= -(3a^2Ab - Ab^3 + a^3B - 3ab^2B) x - \frac{a^2(2Ab)}{2d} \\ &= -(3a^2Ab - Ab^3 + a^3B - 3ab^2B) x - \frac{a^2(2Ab)}{2d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.29, size = 126, normalized size = 0.99

$$\frac{-2a^2(3Ab + aB) \cot(c + dx) - a^3A \cot^2(c + dx) + (a + ib)^3(A + iB) \log(i - \tan(c + dx)) - 2a(a^2A - 3Ab^2 - 3abB) \log(\tan(c + dx)) + (a - ib)^3(A - iB) \log(i + \tan(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] (-2\*a^2\*(3\*A\*b + a\*B)\*Cot[c + d\*x] - a^3\*A\*Cot[c + d\*x]^2 + (a + I\*b)^3\*(A + I\*B)\*Log[I - Tan[c + d\*x]] - 2\*a\*(a^2\*A - 3\*A\*b^2 - 3\*a\*b\*B)\*Log[Tan[c + d\*x]] + (a - I\*b)^3\*(A - I\*B)\*Log[I + Tan[c + d\*x]])/(2\*d)

### Maple [A]

time = 0.20, size = 138, normalized size = 1.09

method	result
derivativedivides	$\frac{A a^3 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + B a^3 (-\cot(dx+c) - dx - c) + 3A a^2 b (-\cot(dx+c) - dx - c) + 3B a^2 b \ln(\sin(dx+c))}{d}$
default	$\frac{A a^3 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + B a^3 (-\cot(dx+c) - dx - c) + 3A a^2 b (-\cot(dx+c) - dx - c) + 3B a^2 b \ln(\sin(dx+c))}{d}$
norman	$\frac{(-3A a^2 b + A b^3 - B a^3 + 3B a b^2) x (\tan^2(dx+c)) - \frac{A a^3}{2d} - \frac{a^2(3Ab+aB)}{d} \frac{\tan(dx+c)}{\tan(dx+c)^2}}{\tan(dx+c)^2} + \frac{(A a^3 - 3A a b^2 - 3B a^2 b + B b^3) \ln(1+\tan(dx+c))}{2d}$
risch	$-\frac{6iB a^2 b c}{d} + \frac{2iB b^3 c}{d} - \frac{6iA a b^2 c}{d} + \frac{2ia^3 A c}{d} - 3A a^2 b x + A b^3 x - B a^3 x + 3B a b^2 x - 3iA a b^2 x -$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS E)`

[Out]  $\frac{1}{d} * (A * a^3 * (-1/2 * \cot(dx+c)^2 - \ln(\sin(dx+c))) + B * a^3 * (-\cot(dx+c) - dx - c) + 3 * A * a^2 * b * (-\cot(dx+c) - dx - c) + 3 * B * a^2 * b * \ln(\sin(dx+c)) + 3 * A * a * b^2 * \ln(\sin(dx+c)) + 3 * B * a * b^2 * (dx+c) + A * b^3 * (dx+c) - B * b^3 * \ln(\cos(dx+c)))$

**Maxima** [A]

time = 0.51, size = 142, normalized size = 1.12

$$\frac{2(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)(dx+c) - (Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log(\tan(dx+c)^2 + 1) + 2(Aa^3 - 3Ba^2b - 3Aab^2) \log(\tan(dx+c)) + \frac{Aa^3 + 2(Ba^3 + 3Aa^2b) \tan(dx+c)}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-\frac{1}{2} * (2 * (B * a^3 + 3 * A * a^2 * b - 3 * B * a * b^2 - A * b^3) * (dx + c) - (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2 + B * b^3) * \log(\tan(dx + c)^2 + 1) + 2 * (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2) * \log(\tan(dx + c)) + (A * a^3 + 2 * (B * a^3 + 3 * A * a^2 * b) * \tan(dx + c))) / \tan(dx + c)^2 / d$

**Fricas** [A]

time = 0.65, size = 162, normalized size = 1.28

$$\frac{Bb^3 \log\left(\frac{1}{\tan(dx+c)^2+1}\right) \tan(dx+c)^2 + Aa^3 + (Aa^3 - 3Ba^2b - 3Aab^2) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^2 + (Aa^3 + 2(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)dx) \tan(dx+c)^2 + 2(Ba^3 + 3Aa^2b) \tan(dx+c)}{2d \tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-\frac{1}{2} * (B * b^3 * \log(1/(\tan(dx + c)^2 + 1)) * \tan(dx + c)^2 + A * a^3 + (A * a^3 - 3 * B * a^2 * b - 3 * A * a * b^2) * \log(\tan(dx + c)^2 / (\tan(dx + c)^2 + 1)) * \tan(dx + c))$



$$\begin{aligned} &^2 + (A*a^3 + 2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*d*x)*\tan(d*x + c)^2 \\ &+ 2*(B*a^3 + 3*A*a^2*b)*\tan(d*x + c))/(d*\tan(d*x + c)^2) \end{aligned}$$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(121) = 242.

time = 1.35, size = 262, normalized size = 2.06

$$\begin{cases} \infty A^2 x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c))^3 \cot^3(c) & \text{for } d = 0 \\ \frac{Aa^3 \log(\tan^2(c+dx)+1) - Aa^2 b \log(\tan(c+dx)) - \frac{Aa^2 b^2}{2 \tan^2(c+dx)} - 3Aa^2 b^2 x - \frac{3Aa^2 b^2}{2 \tan(c+dx)} - \frac{3Aa^2 b^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{3Aa^2 b \log(\tan(c+dx))}{d} + Ab^3 x - Ba^3 x - \frac{Ba^3}{2 \tan(c+dx)} - \frac{3Ba^2 b \log(\tan^2(c+dx)+1)}{2d} + \frac{3Ba^2 b \log(\tan(c+dx))}{d} + 3Bab^2 x + \frac{3B^2 \log(\tan^2(c+dx)+1)}{2d} + 3Bab^2 x + \frac{3B^2 \log(\tan^2(c+dx)+1)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*A\*a\*\*3\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*3\*cot(c)\*\*3, Eq(d, 0)), (A\*a\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*a\*\*3\*log(tan(c + d\*x))/d - A\*a\*\*3/(2\*d\*tan(c + d\*x)\*\*2) - 3\*A\*a\*\*2\*b\*x - 3\*A\*a\*\*2\*b/(d\*tan(c + d\*x)) - 3\*A\*a\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + 3\*A\*a\*b\*\*2\*log(tan(c + d\*x))/d + A\*b\*\*3\*x - B\*a\*\*3\*x - B\*a\*\*3/(d\*tan(c + d\*x)) - 3\*B\*a\*\*2\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + 3\*B\*a\*\*2\*b\*log(tan(c + d\*x))/d + 3\*B\*a\*b\*\*2\*x + B\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d), True))

**Giac [A]**

time = 1.43, size = 193, normalized size = 1.52

$$\frac{2(Ba^3 + 3Aa^2b - 3Ba^2b - Ab^3)(dx + c) - (Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log(\tan(dx + c)^2 + 1) + 2(Aa^3 - 3Ba^2b - 3Aab^2) \log(|\tan(dx + c)|) - \frac{3Aa^2 \tan(dx + c)^2 - 9Ba^2b \tan(dx + c)^2 - 9Aa^2 \tan(dx + c)^2 - 2Ba^2 \tan(dx + c) - 6Aa^2b \tan(dx + c) - Aa^2}{\tan(dx + c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/2*(2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c) - (A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\log(\tan(d*x + c)^2 + 1) + 2*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)*\log(\tan(d*x + c))) - (3*A*a^3*\tan(d*x + c)^2 - 9*B*a^2*b*\tan(d*x + c)^2 - 9*A*a*b^2*\tan(d*x + c)^2 - 2*B*a^3*\tan(d*x + c) - 6*A*a^2*b*\tan(d*x + c) - A*a^3)/\tan(d*x + c)^2)/d$

**Mupad [B]**

time = 6.39, size = 135, normalized size = 1.06

$$\frac{\ln(\tan(c + dx))(-Aa^3 + 3Ba^2b + 3Aab^2) - \cot(c + dx)^2(\tan(c + dx)(Ba^3 + 3Aab^2) + \frac{Aa^2}{2})}{d} + \frac{\ln(\tan(c + dx) + 1i)(A - B1i)(b + a1i)^3 1i}{2d} + \frac{\ln(\tan(c + dx) - 1i)(A + B1i)(-b + a1i)^3 1i}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out]  $(\log(\tan(c + d*x))*(3*A*a*b^2 - A*a^3 + 3*B*a^2*b))/d - (\cot(c + d*x)^2*(\tan(c + d*x)*(B*a^3 + 3*A*a^2*b) + (A*a^3)/2))/d + (\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^3*1i)/(2*d) + (\log(\tan(c + d*x) - 1i)*(A + B*1i)*(a*1i - b)^3*1i)/(2*d)$

### 3.254 $\int \cot^4(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$

Optimal. Leaf size=154

$$(a^3A - 3aAb^2 - 3a^2bB + b^3B)x + \frac{a(3a^2A - 8Ab^2 - 9abB) \cot(c+dx)}{3d} - \frac{a^2(5Ab + 3aB) \cot^2(c+dx)}{6d} - \frac{(3a^2A - 3aAb^2 - 3a^2bB + b^3B) \cot^3(c+dx)}{3d}$$

[Out] (A\*a^3-3\*A\*a\*b^2-3\*B\*a^2\*b+B\*b^3)\*x+1/3\*a\*(3\*A\*a^2-8\*A\*b^2-9\*B\*a\*b)\*cot(d\*x+c)/d-1/6\*a^2\*(5\*A\*b+3\*B\*a)\*cot(d\*x+c)^2/d-(3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*ln(sin(d\*x+c))/d-1/3\*a\*A\*cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^2/d

Rubi [A]

time = 0.25, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3686, 3716, 3709, 3612, 3556}

$$\frac{a(3a^2A - 9abB - 8Ab^2) \cot(c+dx)}{3d} - \frac{a^2(3aB + 5Ab) \cot^2(c+dx)}{6d} - \frac{(a^3B + 3a^2Ab - 3ab^2B - Ab^3) \log(\sin(c+dx))}{d} + x(a^3A - 3a^2bB - 3aAb^2 + b^3B) - \frac{aA \cot^3(c+dx)(a+b \tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] (a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*x + (a\*(3\*a^2\*A - 8\*A\*b^2 - 9\*a\*b\*B)\*Cot[c + d\*x])/(3\*d) - (a^2\*(5\*A\*b + 3\*a\*B)\*Cot[c + d\*x]^2)/(6\*d) - ((3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Log[Sin[c + d\*x]])/d - (a\*A\*Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^2)/(3\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m-1)\*((c + d\*Tan[e + f\*x])^(n+1)/(d\*f\*(n+1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n+1)\*(c^2 + d^2)),

```

Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

```

### Rule 3709

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

```

### Rule 3716

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]

```

### Rubi steps

$$\begin{aligned}
\int \cot^4(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx)(a + b \tan(c + dx))^2}{3d} + \frac{1}{3} \int \\
&= -\frac{a^2(5Ab + 3aB) \cot^2(c + dx)}{6d} - \frac{aA \cot^3(c + dx)}{3d} \\
&= \frac{a(3a^2A - 8Ab^2 - 9abB) \cot(c + dx)}{3d} - \frac{a^2(5Ab + 3aB) \cot^2(c + dx)}{6d} \\
&= (a^3A - 3aAb^2 - 3a^2bB + b^3B) x + \frac{a(3a^2A - 8a^2bB - 3a^2B^2)}{3d} \\
&= (a^3A - 3aAb^2 - 3a^2bB + b^3B) x + \frac{a(3a^2A - 8a^2bB - 3a^2B^2)}{3d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.80, size = 164, normalized size = 1.06

$$\frac{6(a^2A - 3Ab^2 - 3abB)\cot(c+dx) - 3a^2(3Ab + aB)\cot^2(c+dx) - 2a^3A\cot^3(c+dx) + 3(a+ib)^3(-iA+B)\log(i - \tan(c+dx)) - 6(3a^2Ab - Ab^3 + a^3B - 3ab^2B)\log(\tan(c+dx)) + 3(a-ib)^3(iA+B)\log(i + \tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (6\*a\*(a^2\*A - 3\*A\*b^2 - 3\*a\*b\*B)\*Cot[c + d\*x] - 3\*a^2\*(3\*A\*b + a\*B)\*Cot[c + d\*x]^2 - 2\*a^3\*A\*Cot[c + d\*x]^3 + 3\*(a + I\*b)^3\*((-I)\*A + B)\*Log[I - Tan[c + d\*x]] - 6\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Log[Tan[c + d\*x]] + 3\*(a - I\*b)^3\*(I\*A + B)\*Log[I + Tan[c + d\*x]])/(6\*d)

**Maple [A]**

time = 0.17, size = 166, normalized size = 1.08

method	result
derivativedivides	$Aa^3\left(-\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c\right) + Ba^3\left(-\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c))\right) + 3Aa^2b\left(-\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c))\right)$
default	$Aa^3\left(-\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c\right) + Ba^3\left(-\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c))\right) + 3Aa^2b\left(-\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c))\right)$
norman	$\frac{(Aa^3 - 3Aab^2 - 3Ba^2b + Bb^3)x(\tan^3(dx+c)) + \frac{a(a^2A - 3Ab^2 - 3Bab)(\tan^2(dx+c))}{d} - \frac{Aa^3}{3d} - \frac{a^2(3Ab + aB)\tan(dx+c)}{2d}}{\tan(dx+c)^3} - (3Aa^2b - 3Aab^2 - 3Ba^2b + Bb^3)\ln(\sin(dx+c))$
risch	$Aa^3x - 3Aa^2bx - 3Ba^2bx + Bb^3x - \frac{2iAb^3c}{d} + \frac{2iBa^3c}{d} - 3iBa^2bx + 3iAa^2bx - \frac{2ia(-6Aa^2b - 3Aab^2 - 3Ba^2b + Bb^3)\ln(\sin(dx+c))}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(A\*a^3\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)+B\*a^3\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+3\*A\*a^2\*b\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))+3\*B\*a^2\*b\*(-cot(d\*x+c)-d\*x-c)+3\*A\*a\*b^2\*(-cot(d\*x+c)-d\*x-c)+3\*B\*a\*b^2\*ln(sin(d\*x+c))+A\*b^3\*ln(sin(d\*x+c))+B\*b^3\*(d\*x+c))

**Maxima [A]**

time = 0.50, size = 180, normalized size = 1.17

$$\frac{6(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3)(dx+c) + 3(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)\log(\tan(dx+c)^2 + 1) - 6(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3)\log(\tan(dx+c)) - \frac{2Aa^3 - 6(Aa^3 - 3Ba^2b - 3Aab^2)\tan(dx+c)^2 + 3(Ba^3 + 3Aa^2b)\tan(dx+c)}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")



[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{24}*(A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 3*B*a^3*\tan(1/2*d*x + 1/2*c)^2 - 9*A*a^2*b*\tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*\tan(1/2*d*x + 1/2*c) + 36*B*a^2*b*\tan(1/2*d*x + 1/2*c) + 36*A*a*b^2*\tan(1/2*d*x + 1/2*c) + 24*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c) + 24*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\log(\tan(1/2*d*x + 1/2*c)^2 + 1) - 24*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + (44*B*a^3*\tan(1/2*d*x + 1/2*c)^3 + 132*A*a^2*b*\tan(1/2*d*x + 1/2*c)^3 - 132*B*a*b^2*\tan(1/2*d*x + 1/2*c)^3 - 44*A*b^3*\tan(1/2*d*x + 1/2*c)^3 + 15*A*a^3*\tan(1/2*d*x + 1/2*c)^2 - 36*B*a^2*b*\tan(1/2*d*x + 1/2*c)^2 - 36*A*a*b^2*\tan(1/2*d*x + 1/2*c)^2 - 3*B*a^3*\tan(1/2*d*x + 1/2*c) - 9*A*a^2*b*\tan(1/2*d*x + 1/2*c) - A*a^3)/\tan(1/2*d*x + 1/2*c)^3)/d$

**Mupad [B]**

time = 6.46, size = 169, normalized size = 1.10

$$\frac{\ln(\tan(c+dx))(-B^3-3A^2b+3Bab^2+Ab^3)}{d} - \frac{\cot(c+dx)^3\left(\tan(c+dx)\left(\frac{Bx^2}{2} + \frac{3Abx}{2}\right) + \frac{Ax^3}{3} + \tan(c+dx)^2(-A^3+3Bab^2+3Aab^2)\right)}{d} - \frac{\ln(\tan(c+dx)-i)\frac{(A+B1i)(a+b1i)^31i}{2d}}{2d} + \frac{\ln(\tan(c+dx)+1i)\frac{(A-B1i)(a-b1i)^31i}{2d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out]  $(\log(\tan(c + d*x))*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2))/d - (\cot(c + d*x)^3*(\tan(c + d*x)*((B*a^3)/2 + (3*A*a^2*b)/2) + (A*a^3)/3 + \tan(c + d*x)^2*(3*A*a*b^2 - A*a^3 + 3*B*a^2*b))/d - (\log(\tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i)^3*1i)/(2*d) + (\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a - b*1i)^3*1i)/(2*d)$

$$3.255 \quad \int \cot^5(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=191

$$(3a^2Ab - Ab^3 + a^3B - 3ab^2B)x + \frac{(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \cot(c+dx)}{d} + \frac{a(2a^2A - 5Ab^2 - 6abB) \cot(c+dx)}{4d}$$

[Out] (3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*x+(3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*cot(d\*x+c)/d+1/4\*a\*(2\*A\*a^2-5\*A\*b^2-6\*B\*a\*b)\*cot(d\*x+c)^2/d-1/6\*a^2\*(3\*A\*b+2\*B\*a)\*cot(d\*x+c)^3/d+(A\*a^3-3\*A\*a\*b^2-3\*B\*a^2\*b+B\*b^3)\*ln(sin(d\*x+c))/d-1/4\*a\*A\*cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^2/d

Rubi [A]

time = 0.31, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3686, 3716, 3709, 3610, 3612, 3556}

$$\frac{a(2a^2A - 6abB - 5Ab^2) \cot^2(c+dx)}{4d} - \frac{a^2(2aB + 3Ab) \cot^2(c+dx)}{6d} + \frac{(a^2B + 3a^2Ab - 3ab^2B - Ab^3) \cot(c+dx)}{d} + \frac{(a^3A - 3a^2bB - 3aAb^2 + b^2B) \log(\sin(c+dx))}{d} + x(a^2B + 3a^2Ab - 3ab^2B - Ab^3) - \frac{aA \cot^4(c+dx)(a+b \tan(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^5\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*x + ((3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Cot[c + d\*x])/d + (a\*(2\*a^2\*A - 5\*A\*b^2 - 6\*a\*b\*B)\*Cot[c + d\*x]^2)/(4\*d) - (a^2\*(3\*A\*b + 2\*a\*B)\*Cot[c + d\*x]^3)/(6\*d) + ((a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*Log[Sin[c + d\*x]])/d - (a\*A\*Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^2)/(4\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a

\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(n\_), x\_Symbol] :> Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3709

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] :> Simp[(A\*b^2 - a\*b\*B + a^2\*C)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rule 3716

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] :> Simp[(-b\*c - a\*d)\*(c^2\*C - B\*c\*d + A\*d^2)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d^2\*f\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*d\*(A\*c - c\*C + B\*d) + b\*(c^2\*C - B\*c\*d + A\*d^2) + d\*(A\*b\*c + a\*B\*c - b\*c\*C - a\*A\*d + b\*B\*d + a\*C\*d)\*Tan[e + f\*x] + b\*C\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

### Rubi steps



$$\begin{aligned}
 \int \cot^5(c + dx)(a + b \tan(c + dx))^3(A + B \tan(c + dx)) dx &= -\frac{aA \cot^4(c + dx)(a + b \tan(c + dx))^2}{4d} + \frac{1}{4} \int \\
 &= -\frac{a^2(3Ab + 2aB) \cot^3(c + dx)}{6d} - \frac{aA \cot^4(c + dx)}{4d} \\
 &= \frac{a(2a^2A - 5Ab^2 - 6abB) \cot^2(c + dx)}{4d} - \frac{a^2(3Ab + 2aB) \cot^3(c + dx)}{6d} \\
 &= \frac{(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \cot(c + dx)}{d} + \frac{a^2(3Ab + 2aB) \cot^2(c + dx)}{6d} \\
 &= (3a^2Ab - Ab^3 + a^3B - 3ab^2B) x + \frac{(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \cot(c + dx)}{d} \\
 &= (3a^2Ab - Ab^3 + a^3B - 3ab^2B) x + \frac{(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \cot(c + dx)}{d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 0.50, size = 199, normalized size = 1.04

$$\frac{12(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \cot(c + dx) + 6(a^2A - 3Ab^2 - 3abB) \cot^2(c + dx) - 4a^2(3Ab + aB) \cot^3(c + dx) - 3a^3A \cot^4(c + dx) - 6(a + ib)^3(A + iB) \log(i - \tan(c + dx)) + 12(a^3A - 3aAb^2 - 3a^2bB + b^3B) \log(\tan(c + dx)) - 6(a - ib)^3(A - iB) \log(i + \tan(c + dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^5\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] (12\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Cot[c + d\*x] + 6\*a\*(a^2\*A - 3\*A\*b^2 - 3\*a\*b\*B)\*Cot[c + d\*x]^2 - 4\*a^2\*(3\*A\*b + a\*B)\*Cot[c + d\*x]^3 - 3\*a^3\*A\*Cot[c + d\*x]^4 - 6\*(a + I\*b)^3\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + 12\*(a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*Log[Tan[c + d\*x]] - 6\*(a - I\*b)^3\*(A - I\*B)\*Log[I + Tan[c + d\*x]])/(12\*d)

**Maple [A]**

time = 0.21, size = 203, normalized size = 1.06

method	result
derivativedivides	$A a^3 \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} + \ln(\sin(dx+c)) \right) + B a^3 \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + 3A a^2 b \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right)$
default	$A a^3 \left( -\frac{\cot^4(dx+c)}{4} + \frac{\cot^2(dx+c)}{2} + \ln(\sin(dx+c)) \right) + B a^3 \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + 3A a^2 b \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right)$
norman	$\frac{(3A a^2 b - A b^3 + B a^3 - 3B a b^2)(\tan^3(dx+c))}{d} + (3A a^2 b - A b^3 + B a^3 - 3B a b^2)x(\tan^4(dx+c)) - \frac{A a^3}{4d} + \frac{a(a^2 A - 3A b^2 - 3B a b)}{2d}(\tan^2(dx+c))$
risch	$-\frac{2i(12A a^2 b - 9B a b^2 + 9B a b^2 e^{6i(dx+c)} - 27B a b^2 e^{4i(dx+c)} + 27B a b^2 e^{2i(dx+c)} + 36a^2 b e^{4i(dx+c)} A - 30a^2 b e^{2i(dx+c)} A - 12a^3 A)}{12d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (A * a^3 * (-1/4 * \cot(d*x+c)^4 + 1/2 * \cot(d*x+c)^2 + \ln(\sin(d*x+c))) + B * a^3 * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + 3 * A * a^2 * b * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + 3 * B * a^2 * b * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))) + 3 * A * a * b^2 * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))) + 3 * B * a * b^2 * (-\cot(d*x+c) - d*x-c) + A * b^3 * (-\cot(d*x+c) - d*x-c) + B * b^3 * \ln(\sin(d*x+c)))$

**Maxima** [A]

time = 0.50, size = 215, normalized size = 1.13

$$\frac{12(Ba^3 + 3Aa^2b - 3Ba^2b - Ab^3)(dx + c) - 6(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log(\tan(dx + c)^2 + 1) + 12(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log(\tan(dx + c)) - \frac{3Aa^3 - 12(Ba^3 + 3Aa^2b - 3Ba^2b - Ab^3) \tan(dx + c)^3 - 6(Aa^3 - 3Ba^2b - 3Aab^2) \tan(dx + c)^2 + 4(Ba^3 + 3Aa^2b) \tan(dx + c)}{\tan(dx + c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{12} * (12 * (Ba^3 + 3Aa^2b - 3B * a * b^2 - Ab^3) * (d*x + c) - 6 * (Aa^3 - 3B * a^2 * b - 3A * a * b^2 + B * b^3) * \log(\tan(d*x + c)^2 + 1) + 12 * (Aa^3 - 3B * a^2 * b - 3A * a * b^2 + B * b^3) * \log(\tan(d*x + c)) - (3A * a^3 - 12 * (Ba^3 + 3Aa^2b - 3B * a * b^2 - Ab^3) * \tan(d*x + c)^3 - 6 * (Aa^3 - 3B * a^2 * b - 3A * a * b^2) * \tan(d*x + c)^2 + 4 * (Ba^3 + 3Aa^2b) * \tan(d*x + c)) / \tan(d*x + c)^4) / d$

**Fricas** [A]

time = 0.82, size = 225, normalized size = 1.18

$$\frac{6(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \log\left(\frac{\tan(dx+c)}{\tan(dx+c)^2+1}\right) \tan(dx+c)^4 + 3(3Aa^3 - 6Ba^2b - 6Aab^2 + 4(Ba^3 + 3Aa^2b - 3Ba^2b - Ab^3)dx) \tan(dx+c)^4 - 3Aa^3 + 12(Ba^3 + 3Aa^2b - 3Ba^2b - Ab^3) \tan(dx+c)^3 + 6(Aa^3 - 3Ba^2b - 3Aab^2) \tan(dx+c)^2 - 4(Ba^3 + 3Aa^2b) \tan(dx+c)}{12d \tan(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{12} * (6 * (Aa^3 - 3B * a^2 * b - 3A * a * b^2 + B * b^3) * \log(\tan(d*x + c)^2 / (\tan(d*x + c)^2 + 1)) * \tan(d*x + c)^4 + 3 * (3A * a^3 - 6B * a^2 * b - 6A * a * b^2 + 4 * (Ba^3 + 3A * a^2 * b - 3B * a * b^2 - A * b^3) * d * x) * \tan(d*x + c)^4 - 3A * a^3 + 12 * (Ba^3 + 3A * a^2 * b - 3B * a * b^2 - A * b^3) * \tan(d*x + c)^3 + 6 * (Aa^3 - 3B * a^2 * b - 3A * a * b^2) * \tan(d*x + c)^2 - 4 * (Ba^3 + 3A * a^2 * b) * \tan(d*x + c)) / (d * \tan(d*x + c)^4)$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs.  $2(187) = 374$ .

time = 2.97, size = 400, normalized size = 2.09

$$\frac{\left\{ \begin{array}{l} \frac{6Aa^3}{d} \\ \frac{d(A + B \tan(c)) (a + b \tan(c))^2 \cos^2(c)}{d^2 \tan^2(c+1)} + \frac{6A^2 \log(\tan^2(c+1))}{2d \tan^2(c+1)} + \frac{6A^2}{2d \tan^2(c+1)} + 3Aa^2x + \frac{3A^2c}{2d \tan^2(c+1)} - \frac{6A^2}{2d \tan^2(c+1)} + \frac{3Aa^2 \tan^2(c+1)}{2d} + \frac{3Aa^2 \log(\tan^2(c+1))}{2d \tan^2(c+1)} - \frac{3Aa^2}{2d \tan^2(c+1)} - A^2x - \frac{A^2}{2d \tan^2(c+1)} + Bx^2 + \frac{Bx}{2d \tan^2(c+1)} - \frac{B^2}{2d \tan^2(c+1)} + \frac{3B^2 \log(\tan^2(c+1))}{2d} - \frac{3B^2 \log(\tan^2(c+1))}{2d \tan^2(c+1)} - \frac{3B^2}{2d \tan^2(c+1)} - 3B^2x - \frac{3B^2}{2d \tan^2(c+1)} + \frac{3B^2 \log(\tan^2(c+1))}{2d} + \frac{3B^2 \log(\tan^2(c+1))}{2d \tan^2(c+1)} \end{array} \right\}}{\text{for } c = 0 \vee c = -d \wedge (c = -d \vee d = 0) \text{ for } d = 0 \text{ otherwise}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*a**3*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(A + B*tan(c))*(a + b*tan(c))**3*cot(c)**5, Eq(d, 0)), (-A*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + A*a**3*log(tan(c + d*x))/d + A*a**3/(2*d*tan(c + d*x)**2) - A*a**3/(4*d*tan(c + d*x)**4) + 3*A*a**2*b*x + 3*A*a**2*b/(d*tan(c + d*x)) - A*a**2*b/(d*tan(c + d*x)**3) + 3*A*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - 3*A*a*b**2*log(tan(c + d*x))/d - 3*A*a*b**2/(2*d*tan(c + d*x)**2) - A*b**3*x - A*b**3/(d*tan(c + d*x)) + B*a**3*x + B*a**3/(d*tan(c + d*x)) - B*a**3/(3*d*tan(c + d*x)**3) + 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a**2*b*log(tan(c + d*x))/d - 3*B*a**2*b/(2*d*tan(c + d*x)**2) - 3*B*a*b**2*x - 3*B*a*b**2/(d*tan(c + d*x)) - B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**3*log(tan(c + d*x))/d, True))`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(185) = 370.

time = 1.12, size = 528, normalized size = 2.76

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] 
$$\begin{aligned} & -1/192*(3*A*a^3*\tan(1/2*d*x + 1/2*c)^4 - 8*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 24*A*a^2*b*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a^3*\tan(1/2*d*x + 1/2*c)^2 + 72*B*a^2*b*\tan(1/2*d*x + 1/2*c)^2 + 72*A*a*b^2*\tan(1/2*d*x + 1/2*c)^2 + 120*B*a^3*\tan(1/2*d*x + 1/2*c) + 360*A*a^2*b*\tan(1/2*d*x + 1/2*c) - 288*B*a*b^2*\tan(1/2*d*x + 1/2*c) - 96*A*b^3*\tan(1/2*d*x + 1/2*c) - 192*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c) + 192*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\log(\tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + (400*A*a^3*\tan(1/2*d*x + 1/2*c)^4 - 1200*B*a^2*b*\tan(1/2*d*x + 1/2*c)^4 - 1200*A*a*b^2*\tan(1/2*d*x + 1/2*c)^4 + 400*B*b^3*\tan(1/2*d*x + 1/2*c)^4 - 120*B*a^3*\tan(1/2*d*x + 1/2*c)^3 - 360*A*a^2*b*\tan(1/2*d*x + 1/2*c)^3 + 288*B*a*b^2*\tan(1/2*d*x + 1/2*c)^3 + 96*A*b^3*\tan(1/2*d*x + 1/2*c)^3 - 36*A*a^3*\tan(1/2*d*x + 1/2*c)^2 + 72*B*a^2*b*\tan(1/2*d*x + 1/2*c)^2 + 72*A*a*b^2*\tan(1/2*d*x + 1/2*c)^2 + 8*B*a^3*\tan(1/2*d*x + 1/2*c) + 24*A*a^2*b*\tan(1/2*d*x + 1/2*c) + 3*A*a^3)/\tan(1/2*d*x + 1/2*c)^4/d \end{aligned}$$

**Mupad** [B]

time = 6.53, size = 204, normalized size = 1.07

$$\frac{\ln(\tan(c+d*x)) (Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3)}{d} - \frac{\cot(c+d*x) (\tan(c+d*x) (\frac{8b^2}{3d} + Ab^2) + \frac{4b^2}{d} + \tan(c+d*x)^2 (-\frac{4b^2}{3d} + \frac{3Ab^2}{d} + \frac{3Ab^2}{d}) + \tan(c+d*x) (-Ba^3 - 3Aa^2b + 3Ba^2 + Ab^3))}{d} - \frac{\ln(\tan(c+d*x) + 1) (A - B) (b + a) \sqrt{1}}{2d} - \frac{\ln(\tan(c+d*x) - 1) (A + B) (-b + a) \sqrt{1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^5*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x))^3,x)$

[Out]  $(\log(\tan(c + d*x))*(A*a^3 + B*b^3 - 3*A*a*b^2 - 3*B*a^2*b))/d - (\cot(c + d*x))^4*(\tan(c + d*x)*((B*a^3)/3 + A*a^2*b) + (A*a^3)/4 + \tan(c + d*x)^2*((3*A*a*b^2)/2 - (A*a^3)/2 + (3*B*a^2*b)/2) + \tan(c + d*x)^3*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2))/d - (\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^3*1i)/(2*d) - (\log(\tan(c + d*x) - 1i)*(A + B*1i)*(a*1i - b)^3*1i)/(2*d)$

$$3.256 \quad \int \cot^6(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=233

$$-\left((a^3A - 3aAb^2 - 3a^2bB + b^3B)x\right) - \frac{(a^3A - 3aAb^2 - 3a^2bB + b^3B) \cot(c+dx)}{d} + \frac{(3a^2Ab - Ab^3 + a^3B - 3a^2bB + b^3B) \cot^2(c+dx)}{2d}$$

[Out]  $-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x - (A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*\cot(d*x+c)/d + 1/2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*\cot(d*x+c)^2/d + 1/15*a*(5*A*a^2-12*A*b^2-15*B*a*b)*\cot(d*x+c)^3/d - 1/20*a^2*(7*A*b+5*B*a)*\cot(d*x+c)^4/d + (3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*\ln(\sin(d*x+c))/d - 1/5*a*A*\cot(d*x+c)^5*(a+b*\tan(d*x+c))^2/d$

Rubi [A]

time = 0.34, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3686, 3716, 3709, 3610, 3612, 3556}

$$\frac{a(5a^2A - 15abB - 12A^2) \cot^2(c+dx)}{15d} - \frac{a^2(5aB + 7Ab) \cot^4(c+dx)}{20d} + \frac{(a^3B + 3a^2Ab - 3a^2bB - Ab^3) \cot^2(c+dx)}{2d} - \frac{(a^3A - 3a^2bB - 3aAb^2 + b^3B) \cot(c+dx)}{d} + \frac{(a^3B + 3a^2Ab - 3a^2bB - Ab^3) \log(\sin(c+dx))}{d} - \frac{x(a^3A - 3a^2bB - 3aAb^2 + b^3B) - aA \cot^5(c+dx)(a+b \tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^6\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-\left((a^3A - 3a^2Ab + b^3B)x\right) - \left((a^3A - 3a^2Ab + b^3B) \cot[c + d*x]\right)/d + \left((3a^2Ab - Ab^3 + a^3B - 3a^2bB) \cot[c + d*x]^2\right)/(2*d) + \left(a*(5a^2A - 12Ab^2 - 15a*b*B) \cot[c + d*x]^3\right)/(15*d) - \left(a^2*(7Ab + 5a*B) \cot[c + d*x]^4\right)/(20*d) + \left((3a^2Ab - Ab^3 + a^3B - 3a^2bB) \log[\sin[c + d*x]]\right)/d - \left(a*A*\cot[c + d*x]^5*(a + b*\tan[c + d*x])^2\right)/(5*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)
)*(x_)], x_Symbol] := Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f
_.)*(x_)])^2, x_Symbol] := Simp[(-b*c - a*d)*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]
```

### Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx &= -\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{1}{5} \int \\
&= -\frac{a^2(7Ab+5aB)\cot^4(c+dx)}{20d} - \frac{aA\cot^5(c+dx)}{5d} \\
&= \frac{a(5a^2A-12Ab^2-15abB)\cot^3(c+dx)}{15d} - \frac{a^2}{15d} \\
&= \frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot^2(c+dx)}{2d} \\
&= -\frac{(a^3A-3aAb^2-3a^2bB+b^3B)\cot(c+dx)}{d} \\
&= -(a^3A-3aAb^2-3a^2bB+b^3B)x - \frac{(a^3A-3aAb^2-3a^2bB+b^3B)}{d} \\
&= -(a^3A-3aAb^2-3a^2bB+b^3B)x - \frac{(a^3A-3aAb^2-3a^2bB+b^3B)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 0.75, size = 237, normalized size = 1.02

$$\frac{-60(a^3A-3aAb^2-3a^2bB+b^3B)\cot(c+dx)+30(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot^2(c+dx)+20a(a^2A-3aAb^2-3a^2bB)\cot^3(c+dx)-15a^2(3Ab+aB)\cot^4(c+dx)-12a^3A\cot^5(c+dx)+30(a+b)^2(A+iB)\log(i-\tan(c+dx))+60(3a^2Ab-Ab^3+a^3B-3ab^2B)\log(\tan(c+dx))+30(a+b)^2(A-iB)\log(i+\tan(c+dx))}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^6\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] (-60\*(a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*Cot[c + d\*x] + 30\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Cot[c + d\*x]^2 + 20\*a\*(a^2\*A - 3\*aAb^2 - 3\*a\*b\*B)\*Cot[c + d\*x]^3 - 15\*a^2\*(3\*A\*b + a\*B)\*Cot[c + d\*x]^4 - 12\*a^3\*A\*Cot[c + d\*x]^5 + (30\*I)\*(a + I\*b)^3\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + 60\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Log[Tan[c + d\*x]] + 30\*(I\*a + b)^3\*(A - I\*B)\*Log[I + Tan[c + d\*x]])/(60\*d)

**Maple [A]**

time = 0.22, size = 244, normalized size = 1.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^6\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(A\*a^3\*(-1/5\*cot(d\*x+c)^5+1/3\*cot(d\*x+c)^3-cot(d\*x+c)-d\*x-c)+B\*a^3\*(-1/4\*cot(d\*x+c)^4+1/2\*cot(d\*x+c)^2+ln(sin(d\*x+c)))+3\*A\*a^2\*b\*(-1/4\*cot(d\*x+c)^4+1/2\*cot(d\*x+c)^2+ln(sin(d\*x+c)))+3\*B\*a^2\*b\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)+3\*A\*a\*b^2\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)+3\*B\*a\*b^2\*(-1/2\*cot(d

$*x+c)^{-2} \ln(\sin(dx+c)) + A*b^3*(-1/2*\cot(dx+c)^{-2} \ln(\sin(dx+c))) + B*b^3*(-\cot(dx+c) - dx - c)$

**Maxima [A]**

time = 0.51, size = 250, normalized size = 1.07

$$\frac{60(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3)(dx + c) + 30(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3) \log(\tan(dx + c)^2 + 1) - 60(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3) \log(\tan(dx + c)) + \frac{60(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \tan(dx + c)^4 + 12Aa^3 - 30(Ba^3 + 3Aa^2b - 3Bab^2 - Ab^3) \tan(dx + c)^3 - 20(Aa^3 - 3Ba^2b - 3Aab^2) \tan(dx + c)^2 + 15(Ba^3 + 3Aa^2b) \tan(dx + c)}{\tan(dx + c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^6\*(a+b\*tan(dx+c))^3\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out]  $-1/60*(60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(dx + c) + 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\log(\tan(dx + c)^2 + 1) - 60*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\log(\tan(dx + c)) + (60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\tan(dx + c)^4 + 12*A*a^3 - 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\tan(dx + c)^3 - 20*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)*\tan(dx + c)^2 + 15*(B*a^3 + 3*A*a^2*b)*\tan(dx + c))/\tan(dx + c)^5)/d$

**Fricas [A]**

time = 0.77, size = 266, normalized size = 1.14

$$\frac{30(Ba^3 + 3Aa^2b - 3Aab^2 - Ab^3) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)}\right) \tan(dx+c)^5 + 15(3Ba^3 + 9Aa^2b - 6Bab^2 - 2Ab^3 - 4(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3)dx) \tan(dx+c)^4 - 60(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \tan(dx+c)^3 - 12Aa^3 + 30(Ba^3 + 3Aa^2b - 3Aab^2 - Ab^3) \tan(dx+c)^2 + 20(Aa^3 - 3Ba^2b - 3Aab^2) \tan(dx+c) - 15(Ba^3 + 3Aa^2b) \tan(dx+c)}{60d \tan(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^6\*(a+b\*tan(dx+c))^3\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out]  $1/60*(30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\log(\tan(dx + c)^2/(\tan(dx + c)^2 + 1))*\tan(dx + c)^5 + 15*(3*B*a^3 + 9*A*a^2*b - 6*B*a*b^2 - 2*A*b^3 - 4*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*dx)*\tan(dx + c)^5 - 60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*\tan(dx + c)^4 - 12*A*a^3 + 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*\tan(dx + c)^3 + 20*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)*\tan(dx + c)^2 - 15*(B*a^3 + 3*A*a^2*b)*\tan(dx + c))/(d*\tan(dx + c)^5)$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 471 vs.  $2(231) = 462$ .

time = 5.08, size = 471, normalized size = 2.02

$$\frac{30Aa^3}{(Aa^3 - 3Ba^2b - 3Aab^2 - Ab^3) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)}\right) \tan(dx+c)^5 + 15(3Ba^3 + 9Aa^2b - 6Bab^2 - 2Ab^3 - 4(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3)dx) \tan(dx+c)^4 - 60(Aa^3 - 3Ba^2b - 3Aab^2 + Bb^3) \tan(dx+c)^3 - 12Aa^3 + 30(Ba^3 + 3Aa^2b - 3Aab^2 - Ab^3) \tan(dx+c)^2 + 20(Aa^3 - 3Ba^2b - 3Aab^2) \tan(dx+c) - 15(Ba^3 + 3Aa^2b) \tan(dx+c)}{60d \tan(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*6\*(a+b\*tan(dx+c))\*\*3\*(A+B\*tan(dx+c)),x)

[Out] Piecewise((zoo\*A\*a\*\*3\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*3\*cot(c)\*\*6, Eq(d, 0)), (-A\*a\*\*3\*x -





Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^6*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x))^3,x)$

[Out]  $(\log(\tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i)^3*1i)/(2*d) - (\log(\tan(c + d*x))*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2))/d - (\cot(c + d*x)^5*(\tan(c + d*x)*((B*a^3)/4 + (3*A*a^2*b)/4) + (A*a^3)/5 + \tan(c + d*x)^2*(A*a*b^2 - (A*a^3)/3 + B*a^2*b) + \tan(c + d*x)^4*(A*a^3 + B*b^3 - 3*A*a*b^2 - 3*B*a^2*b) + \tan(c + d*x)^3*((A*b^3)/2 - (B*a^3)/2 - (3*A*a^2*b)/2 + (3*B*a*b^2)/2)))/d - (\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a - b*1i)^3*1i)/(2*d)$

$$3.257 \quad \int \tan^2(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=263

$$-((a^4 A - 6a^2 A b^2 + A b^4 - 4a^3 b B + 4a b^3 B) x) + \frac{(4a^3 A b - 4a A b^3 + a^4 B - 6a^2 b^2 B + b^4 B) \log(\cos(c + dx))}{d}$$

[Out]  $-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x+(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*\ln(\cos(d*x+c))/d-b*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*\tan(d*x+c)/d-1/2*(2*A*a*b+B*a^2-B*b^2)*(a+b*\tan(d*x+c))^2/d-1/3*(A*b+B*a)*(a+b*\tan(d*x+c))^3/d-1/4*B*(a+b*\tan(d*x+c))^4/d+1/30*(6*A*b-B*a)*(a+b*\tan(d*x+c))^5/b^2/d+1/6*B*\tan(d*x+c)*(a+b*\tan(d*x+c))^5/b/d$

Rubi [A]

time = 0.30, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3688, 3711, 3609, 3606, 3556}

$$\frac{(a^2 B + 2aAb - b^2 B)(a + b \tan(c + dx))^2}{2d} - \frac{b(a^2 B + 3a^2 Ab - 3ab^2 B - Ab^3) \tan(c + dx)}{d} + \frac{(a^4 B + 4a^3 Ab - 6a^2 b^2 B - 4aAb^3 + b^4 B) \log(\cos(c + dx))}{d} - x(a^4 A - 4a^3 b B - 6a^2 A b^2 + 4a b^3 B + Ab^4) + \frac{(6Ab - aB)(a + b \tan(c + dx))^2}{30b^2 d} - \frac{(aB + Ab)(a + b \tan(c + dx))^2}{3d} + \frac{B \tan(c + dx)(a + b \tan(c + dx))^2}{6bd} - \frac{B(a + b \tan(c + dx))^4}{4d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-(a^4 A - 6a^2 A b^2 + A b^4 - 4a^3 b B + 4a b^3 B) x + ((4a^3 A b - 4a^2 A b^3 + a^4 B - 6a^2 b^2 B + b^4 B) \text{Log}[\text{Cos}[c + d*x]])/d - (b*(3a^2 A b - A b^3 + a^3 B - 3a b^2 B) \text{Tan}[c + d*x])/d - ((2a^2 A b + a^2 B - b^2 B) * (a + b \text{Tan}[c + d*x])^2)/(2*d) - ((A b + a B) * (a + b \text{Tan}[c + d*x])^3)/(3*d) - (B * (a + b \text{Tan}[c + d*x])^4)/(4*d) + ((6A b - a B) * (a + b \text{Tan}[c + d*x])^5)/(30*b^2*d) + (B \text{Tan}[c + d*x] * (a + b \text{Tan}[c + d*x])^5)/(6*b*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rule 3609

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^m\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \tan^2(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{B \tan(c + dx)(a + b \tan(c + dx))^5}{6bd} + \frac{\int (a + b \tan(c + dx))^5 dx}{6bd} \\
 &= \frac{(6Ab - aB)(a + b \tan(c + dx))^5}{30b^2d} + \frac{B \tan(c + dx)(a + b \tan(c + dx))^5}{6bd} \\
 &= -\frac{B(a + b \tan(c + dx))^4}{4d} + \frac{(6Ab - aB)(a + b \tan(c + dx))^5}{30b^2d} \\
 &= -\frac{(Ab + aB)(a + b \tan(c + dx))^3}{3d} - \frac{B(a + b \tan(c + dx))^4}{6bd} \\
 &= -\frac{(2aAb + a^2B - b^2B)(a + b \tan(c + dx))^2}{2d} - \frac{B(a + b \tan(c + dx))^4}{6bd} \\
 &= -(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B)x - \frac{B(a + b \tan(c + dx))^4}{6bd} \\
 &= -(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B)x - \frac{B(a + b \tan(c + dx))^4}{6bd}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 3.76, size = 290, normalized size = 1.10

$\frac{10B^2 \tan(dx+c) + 12(4Ba^2 + AB^2) \tan(dx+c)^2 + 15(6Ba^2 + 4AB^2 - B^3) \tan(dx+c)^3 + 20(4Ba^2 + 6AB^2 - 4Ba^3 - AB^3) \tan(dx+c)^4 + 30(Ba^2 + 4Aa^2 - 6Ba^3 - 4Aa^4 + B^2) \tan(dx+c)^5 + 60(Aa^2 - 4Ba^3 - 6Aa^4 + 4Ba^5 + AB^2) \tan(dx+c)^6 + 30(Ba^2 + 4Aa^2 - 6Ba^3 - 4Aa^4 + B^2) \log(\tan(dx+c)^2 + 1) + 10(Aa^2 - 4Ba^3 - 6Aa^4 + 4Ba^5 + AB^2) \tan(dx+c) \log(\tan(dx+c)^2 + 1) + 10(AB - AB^2) \log(\tan(dx+c)^2 + 1) - 30(A - B) \log(\tan(dx+c)^2 + 1) + 6(A - B) \log(\tan(dx+c)^2 + 1) - 6(A + B) \log(\tan(dx+c)^2 + 1) - 60a^2 b^2 - b^3 \tan(dx+c) + 6b^3(-10a^2 + b^2) \tan^2(dx+c) - 20a^2 b^2 \tan^3(dx+c) - 3b^3 \tan^4(dx+c) - 60a^2 b^2 - b^3 \tan(dx+c) + 6b^3(-10a^2 + b^2) \tan^2(dx+c) - 20a^2 b^2 \tan^3(dx+c) - 3b^3 \tan^4(dx+c)$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((2*(6*A*b - a*B)*(a + b*\text{Tan}[c + d*x])^5)/b + 10*B*\text{Tan}[c + d*x]*(a + b*\text{Tan}[c + d*x])^5 + 10*(A*b - a*B)*((3*I)*(a + I*b)^4*\text{Log}[I - \text{Tan}[c + d*x]] - (3*I)*(a - I*b)^4*\text{Log}[I + \text{Tan}[c + d*x]] + 6*b^2*(-6*a^2 + b^2)*\text{Tan}[c + d*x] - 12*a*b^3*\text{Tan}[c + d*x]^2 - 2*b^4*\text{Tan}[c + d*x]^3) + 5*B*((6*I)*(a + I*b)^5*\text{Log}[I - \text{Tan}[c + d*x]] - 6*(I*a + b)^5*\text{Log}[I + \text{Tan}[c + d*x]] - 60*a*b^2*(2*a^2 - b^2)*\text{Tan}[c + d*x] + 6*b^3*(-10*a^2 + b^2)*\text{Tan}[c + d*x]^2 - 20*a*b^4*\text{Tan}[c + d*x]^3 - 3*b^5*\text{Tan}[c + d*x]^4))/(60*b*d)$

**Maple [A]**

time = 0.12, size = 386, normalized size = 1.47

method	result
norman	$(-A a^4 + 6A a^2 b^2 - A b^4 + 4B a^3 b - 4B a b^3) x + \frac{(A a^4 - 6A a^2 b^2 + A b^4 - 4B a^3 b + 4B a b^3) \tan(dx+c)}{d}$
derivativedivides	$\frac{B b^4 (\tan^6(dx+c))}{6} + \frac{A b^4 (\tan^5(dx+c))}{5} + \frac{4B a b^3 (\tan^5(dx+c))}{5} + A a b^3 (\tan^4(dx+c)) + \frac{3B a^2 b^2 (\tan^4(dx+c))}{2} - \frac{B b^4 (\tan^4(dx+c))}{4}$
default	$\frac{B b^4 (\tan^6(dx+c))}{6} + \frac{A b^4 (\tan^5(dx+c))}{5} + \frac{4B a b^3 (\tan^5(dx+c))}{5} + A a b^3 (\tan^4(dx+c)) + \frac{3B a^2 b^2 (\tan^4(dx+c))}{2} - \frac{B b^4 (\tan^4(dx+c))}{4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $1/d*(1/6*B*b^4*\text{tan}(d*x+c)^6+1/5*A*b^4*\text{tan}(d*x+c)^5+4/5*B*a*b^3*\text{tan}(d*x+c)^5+A*a*b^3*\text{tan}(d*x+c)^4+3/2*B*a^2*b^2*\text{tan}(d*x+c)^4-1/4*B*b^4*\text{tan}(d*x+c)^4+2*A*a^2*b^2*\text{tan}(d*x+c)^3-1/3*A*b^4*\text{tan}(d*x+c)^3+4/3*B*a^3*b*\text{tan}(d*x+c)^3-4/3*B*a*b^3*\text{tan}(d*x+c)^3+2*A*a^3*b*\text{tan}(d*x+c)^2-2*A*a*b^3*\text{tan}(d*x+c)^2+1/2*B*a^4*\text{tan}(d*x+c)^2-3*B*a^2*b^2*\text{tan}(d*x+c)^2+1/2*B*b^4*\text{tan}(d*x+c)^2+A*a^4*\text{tan}(d*x+c)-6*A*a^2*b^2*\text{tan}(d*x+c)+A*b^4*\text{tan}(d*x+c)-4*B*a^3*b*\text{tan}(d*x+c)+4*B*a*b^3*\text{tan}(d*x+c)+1/2*(-4*A*a^3*b+4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*\ln(1+\text{tan}(d*x+c)^2)+(-A*a^4+6*A*a^2*b^2-A*b^4+4*B*a^3*b-4*B*a*b^3)*\arctan(\text{tan}(d*x+c))$

**Maxima [A]**

time = 0.51, size = 290, normalized size = 1.10

$\frac{10B^2 \tan(dx+c)^2 + 12(4Ba^2 + AB^2) \tan(dx+c)^3 + 15(6Ba^2 + 4AB^2 - B^3) \tan(dx+c)^4 + 20(4Ba^2 + 6AB^2 - 4Ba^3 - AB^3) \tan(dx+c)^5 + 30(Ba^2 + 4Aa^2 - 6Ba^3 - 4Aa^4 + B^2) \tan(dx+c)^6 + 60(Aa^2 - 4Ba^3 - 6Aa^4 + 4Ba^5 + AB^2) \tan(dx+c)^7 + 30(Ba^2 + 4Aa^2 - 6Ba^3 - 4Aa^4 + B^2) \log(\tan(dx+c)^2 + 1) + 10(Aa^2 - 4Ba^3 - 6Aa^4 + 4Ba^5 + AB^2) \tan(dx+c) \log(\tan(dx+c)^2 + 1) + 10(AB - AB^2) \log(\tan(dx+c)^2 + 1) - 30(A - B) \log(\tan(dx+c)^2 + 1) + 6(A - B) \log(\tan(dx+c)^2 + 1) - 6(A + B) \log(\tan(dx+c)^2 + 1) - 60a^2 b^2 - b^3 \tan(dx+c) + 6b^3(-10a^2 + b^2) \tan^2(dx+c) - 20a^2 b^2 \tan^3(dx+c) - 3b^3 \tan^4(dx+c) - 60a^2 b^2 - b^3 \tan(dx+c) + 6b^3(-10a^2 + b^2) \tan^2(dx+c) - 20a^2 b^2 \tan^3(dx+c) - 3b^3 \tan^4(dx+c)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(10*B*b^4*tan(d*x + c)^6 + 12*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^5 + 15*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^4 + 20*(4*B*a^3*b + 6*A*a^2*b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c)^3 + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c)^2 - 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1) + 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*tan(d*x + c))/d
```

**Fricas** [A]

time = 0.77, size = 289, normalized size = 1.10

$\frac{10 B^5 \tan(dx + c)^5 + 12(4 B a^3 b + 6 A a^2 b^2 - 4 B a b^3 - A b^4) \tan(dx + c)^4 + 15(6 B a^2 b^2 + 4 A a b^3 - B b^4) \tan(dx + c)^3 + 30(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) \tan(dx + c)^2 - 60(A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4) \tan(dx + c) - 30(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) \log(\tan(dx + c)^2 + 1) + 60(A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4) \tan(dx + c)}{60 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*B*b^4*tan(d*x + c)^6 + 12*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^5 + 15*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^4 + 20*(4*B*a^3*b + 6*A*a^2*b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c)^3 - 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c)^2 + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*tan(d*x + c))/d
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 536 vs.  $2(248) = 496$ .

time = 0.27, size = 536, normalized size = 2.04

$\frac{(-A^4 x^4 + 4 A^3 x^3 \tan(c + dx) - 6 A^2 x^2 \tan^2(c + dx) + 4 A x \tan^3(c + dx) - \tan^4(c + dx)) \tan^2(c + dx) + 2 A^3 x^3 \tan(c + dx) - 6 A^2 x^2 \tan^2(c + dx) + 4 A x \tan^3(c + dx) - \tan^4(c + dx)}{(1 + \tan^2(c + dx))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

```
[Out] Piecewise((-A*a**4*x + A*a**4*tan(c + d*x)/d - 2*A*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*A*a**3*b*tan(c + d*x)**2/d + 6*A*a**2*b**2*x + 2*A*a**2*b**2*tan(c + d*x)**3/d - 6*A*a**2*b**2*tan(c + d*x)/d + 2*A*a*b**3*log(tan(c + d*x)**2 + 1)/d + A*a*b**3*tan(c + d*x)**4/d - 2*A*a*b**3*tan(c + d*x)**2/d - A*b**4*x + A*b**4*tan(c + d*x)**5/(5*d) - A*b**4*tan(c + d*x)**3/(3*d) + A*b**4*tan(c + d*x)/d - B*a**4*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**4*tan(c + d*x)**2/(2*d) + 4*B*a**3*b*x + 4*B*a**3*b*tan(c + d*x)**3/(3*d) - 4*B*a
```

```
*3*b*tan(c + d*x)/d + 3*B*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 3*B*a**2*b
**2*tan(c + d*x)**4/(2*d) - 3*B*a**2*b**2*tan(c + d*x)**2/d - 4*B*a*b**3*x
+ 4*B*a*b**3*tan(c + d*x)**5/(5*d) - 4*B*a*b**3*tan(c + d*x)**3/(3*d) + 4*B
*a*b**3*tan(c + d*x)/d - B*b**4*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**4*tan
(c + d*x)**6/(6*d) - B*b**4*tan(c + d*x)**4/(4*d) + B*b**4*tan(c + d*x)**2/
(2*d), Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**4*tan(c)**2, True))
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 6392 vs.  $2(252) = 504$ .

time = 7.37, size = 6392, normalized size = 24.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*A*a^4*d*x*tan(d*x)^6*tan(c)^6 - 240*B*a^3*b*d*x*tan(d*x)^6*tan(c)
^6 - 360*A*a^2*b^2*d*x*tan(d*x)^6*tan(c)^6 + 240*B*a*b^3*d*x*tan(d*x)^6*tan
(c)^6 + 60*A*b^4*d*x*tan(d*x)^6*tan(c)^6 - 30*B*a^4*log(4*(tan(d*x)^4*tan(c)
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 - 120*A*a^3*b*log(4*(tan(d*x)
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 + 180*B*a^2*b^2*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 + 120*A
a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6
- 30*B*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*ta
n(c)^6 - 360*A*a^4*d*x*tan(d*x)^5*tan(c)^5 + 1440*B*a^3*b*d*x*tan(d*x)^5*ta
n(c)^5 + 2160*A*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 - 1440*B*a*b^3*d*x*tan(d*x)
^5*tan(c)^5 - 360*A*b^4*d*x*tan(d*x)^5*tan(c)^5 - 30*B*a^4*tan(d*x)^6*tan(c)
)^6 - 120*A*a^3*b*tan(d*x)^6*tan(c)^6 + 270*B*a^2*b^2*tan(d*x)^6*tan(c)^6 +
180*A*a*b^3*tan(d*x)^6*tan(c)^6 - 55*B*b^4*tan(d*x)^6*tan(c)^6 + 180*B*a^4
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 72
0*A*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(
c)^5 - 1080*B*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
(d*x)^5*tan(c)^5 - 720*A*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^5*tan(c)^5 + 180*B*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
```

$$\begin{aligned}
& n(c)^2 + 1)) * \tan(dx)^5 * \tan(c)^5 + 60 * A * a^4 * \tan(dx)^6 * \tan(c)^5 - 240 * B * a^3 \\
& * b * \tan(dx)^6 * \tan(c)^5 - 360 * A * a^2 * b^2 * \tan(dx)^6 * \tan(c)^5 + 240 * B * a * b^3 * \tan \\
& (dx)^6 * \tan(c)^5 + 60 * A * b^4 * \tan(dx)^6 * \tan(c)^5 + 60 * A * a^4 * \tan(dx)^5 * \tan \\
& (c)^6 - 240 * B * a^3 * b * \tan(dx)^5 * \tan(c)^6 - 360 * A * a^2 * b^2 * \tan(dx)^5 * \tan(c)^6 \\
& + 240 * B * a * b^3 * \tan(dx)^5 * \tan(c)^6 + 60 * A * b^4 * \tan(dx)^5 * \tan(c)^6 + 900 * A * a^4 \\
& * d * x * \tan(dx)^4 * \tan(c)^4 - 3600 * B * a^3 * b * d * x * \tan(dx)^4 * \tan(c)^4 - 5400 * A * a^2 \\
& * b^2 * d * x * \tan(dx)^4 * \tan(c)^4 + 3600 * B * a * b^3 * d * x * \tan(dx)^4 * \tan(c)^4 + 900 \\
& * A * b^4 * d * x * \tan(dx)^4 * \tan(c)^4 - 30 * B * a^4 * \tan(dx)^6 * \tan(c)^4 - 120 * A * a^3 * b \\
& * \tan(dx)^6 * \tan(c)^4 + 180 * B * a^2 * b^2 * \tan(dx)^6 * \tan(c)^4 + 120 * A * a * b^3 * \tan \\
& (dx)^6 * \tan(c)^4 - 30 * B * b^4 * \tan(dx)^6 * \tan(c)^4 + 120 * B * a^4 * \tan(dx)^5 * \tan(c) \\
& ^5 + 480 * A * a^3 * b * \tan(dx)^5 * \tan(c)^5 - 1260 * B * a^2 * b^2 * \tan(dx)^5 * \tan(c)^5 \\
& - 840 * A * a * b^3 * \tan(dx)^5 * \tan(c)^5 + 270 * B * b^4 * \tan(dx)^5 * \tan(c)^5 - 30 * B * a^4 \\
& * \tan(dx)^4 * \tan(c)^6 - 120 * A * a^3 * b * \tan(dx)^4 * \tan(c)^6 + 180 * B * a^2 * b^2 * \tan \\
& (dx)^4 * \tan(c)^6 + 120 * A * a * b^3 * \tan(dx)^4 * \tan(c)^6 - 30 * B * b^4 * \tan(dx)^4 * \tan \\
& (c)^6 + 80 * B * a^3 * b * \tan(dx)^6 * \tan(c)^3 + 120 * A * a^2 * b^2 * \tan(dx)^6 * \tan(c)^3 \\
& - 80 * B * a * b^3 * \tan(dx)^6 * \tan(c)^3 - 20 * A * b^4 * \tan(dx)^6 * \tan(c)^3 - 450 * B * a^4 \\
& * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \\
& \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan(c)^4 - 1 \\
& 800 * A * a^3 * b * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \tan(dx)^2 * \tan \\
& (c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan \\
& (c)^4 + 2700 * B * a^2 * b^2 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 * \tan(c) + \\
& \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c)^2 + 1)) * \tan \\
& (dx)^4 * \tan(c)^4 + 1800 * A * a * b^3 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan(dx)^3 \\
& * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / (\tan(c) \\
& ^2 + 1)) * \tan(dx)^4 * \tan(c)^4 - 450 * B * b^4 * \log(4 * (\tan(dx)^4 * \tan(c)^2 - 2 * \tan \\
& (dx)^3 * \tan(c) + \tan(dx)^2 * \tan(c)^2 + \tan(dx)^2 - 2 * \tan(dx) * \tan(c) + 1) / \\
& (\tan(c)^2 + 1)) * \tan(dx)^4 * \tan(c)^4 - 300 * A * a^4 * \tan(dx)^5 * \tan(c)^4 + 1440 * \\
& B * a^3 * b * \tan(dx)^5 * \tan(c)^4 + 2160 * A * a^2 * b^2 * \tan(dx)^5 * \tan(c)^4 - 1440 * B * a \\
& * b^3 * \tan(dx)^5 * \tan(c)^4 - 360 * A * b^4 * \tan(dx)^5 * \tan(c)^4 - 300 * A * a^4 * \tan(dx) \\
& ^4 * \tan(c)^5 + 1440 * B * a^3 * b * \tan(dx)^4 * \tan(c)^5 + 2160 * A * a^2 * b^2 * \tan(dx)^4 \\
& * \tan(c)^5 - 1440 * B * a * b^3 * \tan(dx)^4 * \tan(c)^5 - 360 * A * b^4 * \tan(dx)^4 * \tan(c) \\
& ^5 + 80 * B * a^3 * b * \tan(dx)^3 * \tan(c)^6 + 120 * A * a^2 * b^2 * \tan(dx)^3 * \tan(c)^6 - 8 \\
& 0 * B * a * b^3 * \tan(dx)^3 * \tan(c)^6 - 20 * A * b^4 * \tan(dx)^3 * \tan(c)^6 - 90 * B * a^2 * b^2 \\
& * \tan(dx)^6 * \tan(c)^2 - 60 * A * a * b^3 * \tan(dx)^6 * \tan(c)^2 + 15 * B * b^4 * \tan(dx)^6 \\
& * \tan(c)^2 - 1200 * A * a^4 * d * x * \tan(dx)^3 * \tan(c)^3 + 4800 * B * a^3 * b * d * x * \tan(dx) \\
& ^3 * \tan(c)^3 + 7200 * A * a^2 * b^2 * d * x * \tan(dx)^3 * \tan(c)^3 - 4800 * B * a * b^3 * d * x * \tan \\
& (dx)^3 * \tan(c)^3 - 1200 * A * b^4 * d * x * \tan(dx)^3 * \tan(c)^3 + 120 * B * a^4 * \tan(dx)^5 \\
& * \tan(c)^3 + 480 * A * a^3 * b * \tan(dx)^5 * \tan(c)^3 - 1080 * B * a^2 * b^2 * \tan(dx)^5 * \tan \\
& (c)^3 - 720 * A * a * b^3 * \tan(dx)^5 * \tan(c)^3 + 180 * B \dots
\end{aligned}$$

**Mupad [B]**

time = 6.37, size = 300, normalized size = 1.14

$$\frac{\tan(c+dx)(A^4+AB^3+4B^2b(2Ab+2Ba))}{d} - \frac{\tan(c+dx)(4d^2+4B^2b^2-2A^2b(2Ab+2Ba))}{d} - x(A^4-4B^2b^2-6A^2b^2+4B^2b^2+AB^3) + \frac{\tan(c+dx)(4d^2+4B^2b^2)}{d} - \ln(\tan(c+dx)^2+1) \frac{(4d^2+2A^2b^2-3B^2b^2-2A^2b^2+4d^2)}{d} - \frac{\tan(c+dx)(4d^2-2A^2b^2+2Ba)}{d} + \frac{\tan(c+dx)(4d^2+4B^2b^2-2A^2b^2-2A^2b^2+4d^2)}{d} + \frac{B^4 \tan(c+dx)^2}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}(\tan(c + d*x)^2*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x))^4,x)$

[Out]  $(\tan(c + d*x)*(A*a^4 + A*b^4 + 4*B*a*b^3 - 2*a^2*b*(3*A*b + 2*B*a)))/d - (\tan(c + d*x)^3*((A*b^4)/3 + (4*B*a*b^3)/3 - (2*a^2*b*(3*A*b + 2*B*a))/3))/d - x*(A*a^4 + A*b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b) + (\tan(c + d*x)^5*((A*b^4)/5 + (4*B*a*b^3)/5))/d - (\log(\tan(c + d*x)^2 + 1)*((B*a^4)/2 + (B*b^4)/2 - 3*B*a^2*b^2 - 2*A*a*b^3 + 2*A*a^3*b))/d - (\tan(c + d*x)^4*((B*b^4)/4 - (a*b^2*(2*A*b + 3*B*a))/2))/d + (\tan(c + d*x)^2*((B*a^4)/2 + (B*b^4)/2 + 2*A*a^3*b - a*b^2*(2*A*b + 3*B*a)))/d + (B*b^4*\tan(c + d*x)^6)/(6*d)$

### 3.258 $\int \tan(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=226

$$-\left((4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x\right) - \frac{(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) \log(\cos(c + dx))}{d} +$$

[Out]  $-(4Aa^3b - 4AaAb^3 + Ba^4 - 6Bb^2a^2 + Bb^4)x - (Aa^4 - 6Aa^2b^2 + Ab^4 - 4a^3bB + 4ab^3B) \ln(\cos(dx+c)) / d + b(Aa^3 - 3AaAb^2 - 3Ba^2b + Bb^3) \tan(dx+c) / d + 1/2(Aa^2 - Ab^2 - 2Bab) (a+b \tan(dx+c))^2 / d + 1/3(Aa - Bb) (a+b \tan(dx+c))^3 / d + 1/4A (a+b \tan(dx+c))^4 / d + 1/5B (a+b \tan(dx+c))^5 / b/d$

**Rubi [A]**

time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3673, 3609, 3606, 3556}

$$\frac{(a^2A - 2abB - Ab^2)(a + b \tan(c + dx))^2}{2d} + \frac{b(a^3A - 3a^2bB - 3aAb^2 + b^3B) \tan(c + dx)}{d} - \frac{(a^4A - 4a^2bB - 6a^2Ab^2 + 4ab^3B + Ab^4) \log(\cos(c + dx))}{d} - x(a^4B + 4a^3Ab - 6a^2b^2B - 4aAb^3 + b^4B) + \frac{(aA - bB)(a + b \tan(c + dx))^2}{3d} + \frac{A(a + b \tan(c + dx))^4}{4d} + \frac{B(a + b \tan(c + dx))^5}{5d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x] * (a + b*\text{Tan}[c + d*x])^4 * (A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left((4a^3A^3b - 4a^2A^2b^2 + a^4B - 6a^2b^2B + b^4B)x\right) - \left((a^4A - 6a^2A^2b^2 + Ab^4 - 4a^3bB + 4a^2b^3B) \text{Log}[\text{Cos}[c + d*x]]\right) / d + (b(a^3A - 3a^2A^2b^2 - 3a^2b^2B + b^3B) \text{Tan}[c + d*x]) / d + ((a^2A - Ab^2 - 2a^2bB) * (a + b \text{Tan}[c + d*x])^2) / (2*d) + ((aA - bB) * (a + b \text{Tan}[c + d*x])^3) / (3*d) + (A * (a + b \text{Tan}[c + d*x])^4) / (4*d) + (B * (a + b \text{Tan}[c + d*x])^5) / (5*b*d)$

**Rule 3556**

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]] / d, x] /;$   $\text{FreeQ}\{c, d\}, x]$

**Rule 3606**

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)] * ((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[b*d*(\text{Tan}[e + f*x]/f), x]) /;$   $\text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

**Rule 3609**

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^m * ((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m / (f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{m-1} * \text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x]$

, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \tan(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^5}{5bd} + \int (-B + A \tan(c + dx)) (a + b \tan(c + dx))^4 dx \\
 &= \frac{A(a + b \tan(c + dx))^4}{4d} + \frac{B(a + b \tan(c + dx))^5}{5bd} \\
 &= \frac{(aA - bB)(a + b \tan(c + dx))^3}{3d} + \frac{A(a + b \tan(c + dx))^4}{4d} \\
 &= \frac{(a^2A - Ab^2 - 2abB)(a + b \tan(c + dx))^2}{2d} + \frac{(aA - bB)(a + b \tan(c + dx))^3}{3d} \\
 &= -(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x \\
 &= -(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 2.38, size = 257, normalized size = 1.14

$\frac{12B(a + b \tan(c + dx))^5 + 10(aA + bB)(3(a + b) \log(-\tan(c + dx)) - 3(a - b) \log(1 + \tan(c + dx)) + 6F^2(-6a^2 + F^2) \tan(c + dx) - 12ab^2 \tan^2(c + dx) - 2b^4 \tan^3(c + dx)) - 5A(6(a + b)^2 \log(-\tan(c + dx)) - 6(a + b) \log(1 + \tan(c + dx)) - 60ab^2(2a^2 - F^2) \tan(c + dx) + 6F^2(-10a^2 + F^2) \tan^2(c + dx) - 20ab^2 \tan^3(c + dx) - 3b^4 \tan^4(c + dx))}{60bd}$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (12\*B\*(a + b\*Tan[c + d\*x])^5 + 10\*(a\*A + b\*B)\*((3\*I)\*(a + I\*b)^4\*Log[I - Tan[c + d\*x]] - (3\*I)\*(a - I\*b)^4\*Log[I + Tan[c + d\*x]] + 6\*b^2\*(-6\*a^2 + b^2)\*Tan[c + d\*x] - 12\*a\*b^3\*Tan[c + d\*x]^2 - 2\*b^4\*Tan[c + d\*x]^3) - 5\*A\*((6\*I)\*(a + I\*b)^5\*Log[I - Tan[c + d\*x]] - 6\*(I\*a + b)^5\*Log[I + Tan[c + d\*x]] - 60\*a\*b^2\*(2\*a^2 - b^2)\*Tan[c + d\*x] + 6\*b^3\*(-10\*a^2 + b^2)\*Tan[c + d\*x]^2 - 20\*a\*b^4\*Tan[c + d\*x]^3 - 3\*b^5\*Tan[c + d\*x]^4))/(60\*b\*d)

**Maple [A]**

time = 0.10, size = 309, normalized size = 1.37 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/5*B*b^4*tan(d*x+c)^5+1/4*A*b^4*tan(d*x+c)^4+B*a*b^3*tan(d*x+c)^4+4/3
*A*a*b^3*tan(d*x+c)^3+2*B*a^2*b^2*tan(d*x+c)^3-1/3*B*b^4*tan(d*x+c)^3+3*A*a
^2*b^2*tan(d*x+c)^2-1/2*A*b^4*tan(d*x+c)^2+2*B*a^3*b*tan(d*x+c)^2-2*B*a*b^3
*tan(d*x+c)^2+4*A*a^3*b*tan(d*x+c)-4*A*a*b^3*tan(d*x+c)+B*a^4*tan(d*x+c)-6*
B*a^2*b^2*tan(d*x+c)+B*b^4*tan(d*x+c)+1/2*(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*
b+4*B*a*b^3)*ln(1+tan(d*x+c)^2)+(-4*A*a^3*b+4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b
^4)*arctan(tan(d*x+c))
```

**Maxima** [A]

time = 0.52, size = 246, normalized size = 1.09

$\frac{12 B b^4 \tan(dx+c)^5 + 15(4 B a b^3 + A b^4) \tan(dx+c)^4 + 20(6 B a^2 b^2 + 4 A a^2 b^3 - B b^4) \tan(dx+c)^3 + 30(4 B a^3 b + 6 A a^2 b^2 - 4 B a^2 b^3 - A b^4) \tan(dx+c)^2 - 60(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a^2 b^3 + B b^4)(dx+c) + 30(A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a^2 b^3 + A b^4) \log(\tan(dx+c)^2 + 1) + 60(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a^2 b^3 + B b^4) \tan(dx+c)}{60 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*(12*B*b^4*tan(d*x + c)^5 + 15*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 + 20*
(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^3 + 30*(4*B*a^3*b + 6*A*a^2*
b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c)^2 - 60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b
^2 - 4*A*a*b^3 + B*b^4)*(d*x + c) + 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4
*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1) + 60*(B*a^4 + 4*A*a^3*b - 6*B*a^2
*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c))/d
```

**Fricas** [A]

time = 0.82, size = 245, normalized size = 1.08

$\frac{12 B b^4 \tan(dx+c)^5 + 15(4 B a b^3 + A b^4) \tan(dx+c)^4 + 20(6 B a^2 b^2 + 4 A a^2 b^3 - B b^4) \tan(dx+c)^3 - 60(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a^2 b^3 + B b^4) dx + 30(A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a^2 b^3 + A b^4) \log\left(\frac{\tan(dx+c)^2 + 1}{\cos(dx+c)^2}\right) + 60(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a^2 b^3 + B b^4) \tan(dx+c)}{60 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*B*b^4*tan(d*x + c)^5 + 15*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 + 20*
(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^3 - 60*(B*a^4 + 4*A*a^3*b -
6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*d*x + 30*(4*B*a^3*b + 6*A*a^2*b^2 - 4*B*a*
b^3 - A*b^4)*tan(d*x + c)^2 - 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b
^3 + A*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b
^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c))/d
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs.  $2(214) = 428$ .

time = 0.22, size = 437, normalized size = 1.93

$\frac{1}{d} \left( \frac{12 B b^4 \tan(dx+c)^5 + 15(4 B a b^3 + A b^4) \tan(dx+c)^4 + 20(6 B a^2 b^2 + 4 A a^2 b^3 - B b^4) \tan(dx+c)^3 - 60(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a^2 b^3 + B b^4) dx + 30(A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a^2 b^3 + A b^4) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) + 60(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a^2 b^3 + B b^4) \tan(dx+c)}{60} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

```
[Out] Piecewise((A**4*log(tan(c + d*x)**2 + 1)/(2*d) - 4*A**3*b*x + 4*A**3*
b*tan(c + d*x)/d - 3*A**2*b**2*log(tan(c + d*x)**2 + 1)/d + 3*A**2*b**2
*tan(c + d*x)**2/d + 4*A*a*b**3*x + 4*A*a*b**3*tan(c + d*x)**3/(3*d) - 4*A*
a*b**3*tan(c + d*x)/d + A*b**4*log(tan(c + d*x)**2 + 1)/(2*d) + A*b**4*tan(
c + d*x)**4/(4*d) - A*b**4*tan(c + d*x)**2/(2*d) - B*a**4*x + B*a**4*tan(
c + d*x)/d - 2*B*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*B*a**3*b*tan(c + d*x)*
**2/d + 6*B*a**2*b**2*x + 2*B*a**2*b**2*tan(c + d*x)**3/d - 6*B*a**2*b**2*ta
n(c + d*x)/d + 2*B*a*b**3*log(tan(c + d*x)**2 + 1)/d + B*a*b**3*tan(c + d*x)
)**4/d - 2*B*a*b**3*tan(c + d*x)**2/d - B*b**4*x + B*b**4*tan(c + d*x)**5/(
5*d) - B*b**4*tan(c + d*x)**3/(3*d) + B*b**4*tan(c + d*x)/d, Ne(d, 0)), (x*
(A + B*tan(c))*(a + b*tan(c))**4*tan(c), True))
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 4789 vs.  $2(218) = 436$ .

time = 4.62, size = 4789, normalized size = 21.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(60*B*a^4*d*x*tan(d*x)^5*tan(c)^5 + 240*A*a^3*b*d*x*tan(d*x)^5*tan(c)
^5 - 360*B*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 - 240*A*a*b^3*d*x*tan(d*x)^5*tan
(c)^5 + 60*B*b^4*d*x*tan(d*x)^5*tan(c)^5 + 30*A*a^4*log(4*(tan(d*x)^4*tan(c)
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 - 120*B*a^3*b*log(4*(tan(d*x)
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 - 180*A*a^2*b^2*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 120*B*
a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5
+ 30*A*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*ta
n(c)^5 - 300*B*a^4*d*x*tan(d*x)^4*tan(c)^4 - 1200*A*a^3*b*d*x*tan(d*x)^4*ta
n(c)^4 + 1800*B*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 + 1200*A*a*b^3*d*x*tan(d*x)
^4*tan(c)^4 - 300*B*b^4*d*x*tan(d*x)^4*tan(c)^4 - 120*B*a^3*b*tan(d*x)^5*ta
n(c)^5 - 180*A*a^2*b^2*tan(d*x)^5*tan(c)^5 + 180*B*a*b^3*tan(d*x)^5*tan(c)^
5 + 45*A*b^4*tan(d*x)^5*tan(c)^5 - 150*A*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 600*B*a^3*b*log(4*(tan(d*x)^4*tan
```

$$\begin{aligned}
& (c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx) \\
& * \tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^4*\tan(c)^4 + 900*A*a^2*b^2*\log(4*(\tan \\
& (dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - \\
& 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^4*\tan(c)^4 - 600*B*a*b^3*1 \\
& \log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan \\
& (dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^4*\tan(c)^4 - 150* \\
& A*b^4*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 \\
& + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^4*\tan(c)^4 \\
& + 60*B*a^4*\tan(dx)^5*\tan(c)^4 + 240*A*a^3*b*\tan(dx)^5*\tan(c)^4 - 360*B*a \\
& ^2*b^2*\tan(dx)^5*\tan(c)^4 - 240*A*a*b^3*\tan(dx)^5*\tan(c)^4 + 60*B*b^4*\tan \\
& (dx)^5*\tan(c)^4 + 60*B*a^4*\tan(dx)^4*\tan(c)^5 + 240*A*a^3*b*\tan(dx)^4*\tan \\
& (c)^5 - 360*B*a^2*b^2*\tan(dx)^4*\tan(c)^5 - 240*A*a*b^3*\tan(dx)^4*\tan(c)^5 \\
& + 60*B*b^4*\tan(dx)^4*\tan(c)^5 + 600*B*a^4*d*x*\tan(dx)^3*\tan(c)^3 + 2400 \\
& *A*a^3*b*d*x*\tan(dx)^3*\tan(c)^3 - 3600*B*a^2*b^2*d*x*\tan(dx)^3*\tan(c)^3 - \\
& 2400*A*a*b^3*d*x*\tan(dx)^3*\tan(c)^3 + 600*B*b^4*d*x*\tan(dx)^3*\tan(c)^3 - \\
& 120*B*a^3*b*\tan(dx)^5*\tan(c)^3 - 180*A*a^2*b^2*\tan(dx)^5*\tan(c)^3 + 120* \\
& B*a*b^3*\tan(dx)^5*\tan(c)^3 + 30*A*b^4*\tan(dx)^5*\tan(c)^3 + 360*B*a^3*b*\tan \\
& (dx)^4*\tan(c)^4 + 540*A*a^2*b^2*\tan(dx)^4*\tan(c)^4 - 660*B*a*b^3*\tan(dx) \\
& )^4*\tan(c)^4 - 165*A*b^4*\tan(dx)^4*\tan(c)^4 - 120*B*a^3*b*\tan(dx)^3*\tan(c) \\
& )^5 - 180*A*a^2*b^2*\tan(dx)^3*\tan(c)^5 + 120*B*a*b^3*\tan(dx)^3*\tan(c)^5 + \\
& 30*A*b^4*\tan(dx)^3*\tan(c)^5 + 120*B*a^2*b^2*\tan(dx)^5*\tan(c)^2 + 80*A*a* \\
& b^3*\tan(dx)^5*\tan(c)^2 - 20*B*b^4*\tan(dx)^5*\tan(c)^2 + 300*A*a^4*\log(4*(\tan \\
& (dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 \\
& - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^3*\tan(c)^3 - 1200*B*a^3* \\
& b*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \\
& \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^3*\tan(c)^3 - 1 \\
& 800*A*a^2*b^2*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2 \\
& *\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan(dx)^3* \\
& \tan(c)^3 + 1200*B*a*b^3*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \\
& \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 + 1))*\tan \\
& (dx)^3*\tan(c)^3 + 300*A*b^4*\log(4*(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan \\
& (c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)/(\tan(c)^2 \\
& + 1))*\tan(dx)^3*\tan(c)^3 - 240*B*a^4*\tan(dx)^4*\tan(c)^3 - 960*A*a^3*b*\tan \\
& (dx)^4*\tan(c)^3 + 1800*B*a^2*b^2*\tan(dx)^4*\tan(c)^3 + 1200*A*a*b^3*\tan(dx) \\
& )^4*\tan(c)^3 - 300*B*b^4*\tan(dx)^4*\tan(c)^3 - 240*B*a^4*\tan(dx)^3*\tan(c) \\
& ^4 - 960*A*a^3*b*\tan(dx)^3*\tan(c)^4 + 1800*B*a^2*b^2*\tan(dx)^3*\tan(c)^4 + \\
& 1200*A*a*b^3*\tan(dx)^3*\tan(c)^4 - 300*B*b^4*\tan(dx)^3*\tan(c)^4 + 120*B*a \\
& ^2*b^2*\tan(dx)^2*\tan(c)^5 + 80*A*a*b^3*\tan(dx)^2*\tan(c)^5 - 20*B*b^4*\tan \\
& (dx)^2*\tan(c)^5 - 60*B*a*b^3*\tan(dx)^5*\tan(c) - 15*A*b^4*\tan(dx)^5*\tan(c) \\
& - 600*B*a^4*d*x*\tan(dx)^2*\tan(c)^2 - 2400*A*a^3*b*d*x*\tan(dx)^2*\tan(c)^2 \\
& + 3600*B*a^2*b^2*d*x*\tan(dx)^2*\tan(c)^2 + 2400*A*a*b^3*d*x*\tan(dx)^2*\tan \\
& (c)^2 - 600*B*b^4*d*x*\tan(dx)^2*\tan(c)^2 + 360*B*a^3*b*\tan(dx)^4*\tan(c)^2 \\
& + 540*A*a^2*b^2*\tan(dx)^4*\tan(c)^2 - 600*B*a*b^3*\tan(dx)^4*\tan(c)^2 - 15 \\
& 0*A*b^4*\tan(dx)^4*\tan(c)^2 - 480*B*a^3*b*\tan(dx)^3*\tan(c)^3 - 720*A*a^2*b \\
& ^2*\tan(dx)^3*\tan(c)^3 + 720*B*a*b^3*\tan(dx)^3*\tan(c)^3 + 180*A*b^4*\tan(dx)
\end{aligned}$$

$x^3 \tan(c)^3 + 360 B^3 a^3 b \tan(dx)^2 \tan(c)^4 + 540 A a^2 b^2 \tan(dx)^2 \tan(c)^4 - 600 B^3 a b^3 \tan(dx)^2 \tan(c)^4 - 15 \dots$

**Mupad [B]**

time = 6.32, size = 251, normalized size = 1.11

$$\frac{\tan(c+dx) (B^3 a^3 b^3 - 2 a^3 b^3 (2 A b + 3 B a))}{d} - \frac{\tan(c+dx)^2 \left( \frac{4 a^4}{d} + 2 B a^3 b - a^2 b (3 A b + 2 B a) \right)}{d} - x (B^3 a^3 + 4 A a^2 b - 6 B^2 a^2 b^2 - 4 A a b^3 + B^3 b^3) + \frac{\tan(c+dx)^2 \left( \frac{4 a^4}{d} + B a^3 b \right)}{d} + \frac{\ln(\tan(c+dx)^2 + 1) \left( \frac{4 a^4}{d} - 2 B a^3 b - 3 A a^2 b^2 + 2 B a b^3 + \frac{4 a^4}{d} \right)}{d} - \frac{\tan(c+dx)^2 \left( \frac{4 a^4}{d} - \frac{2 a^2 b (3 A b + 2 B a)}{d} \right)}{d} + \frac{B^3 \tan(c+dx)^2}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)`

[Out]  $(\tan(c + dx) * (B^3 a^4 + B^3 b^4 + 4 A a^3 b - 2 a^3 b^2 * (2 A b + 3 B a))) / d - (\tan(c + dx)^2 * ((A b^4) / 2 + 2 B^2 a b^3 - a^2 b * (3 A b + 2 B a))) / d - x * (B^3 a^4 + B^3 b^4 - 6 B^2 a^2 b^2 - 4 A a^3 b^3 + 4 A a^2 b^3) + (\tan(c + dx)^4 * ((A b^4) / 4 + B^2 a b^3)) / d + (\log(\tan(c + dx)^2 + 1) * ((A a^4) / 2 + (A b^4) / 2 - 3 A a^2 b^2 + 2 B^2 a b^3 - 2 B^2 a^3 b)) / d - (\tan(c + dx)^3 * ((B b^4) / 3 - (2 a^3 b^2 * (2 A b + 3 B a)) / 3)) / d + (B^3 b^4 * \tan(c + dx)^5) / (5 d)$

### 3.259 $\int (a + b \tan(c + dx))^4 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=202

$$(a^4 A - 6a^2 A b^2 + A b^4 - 4a^3 b B + 4a b^3 B) x - \frac{(4a^3 A b - 4a A b^3 + a^4 B - 6a^2 b^2 B + b^4 B) \log(\cos(c + dx))}{d} + \frac{b(3a^4 A - 6a^2 A b^2 + A b^4 - 4a^3 b B + 4a b^3 B)}{d}$$

[Out] (A\*a^4-6\*A\*a^2\*b^2+A\*b^4-4\*B\*a^3\*b+4\*B\*a\*b^3)\*x-(4\*A\*a^3\*b-4\*A\*a\*b^3+B\*a^4-6\*B\*a^2\*b^2+B\*b^4)\*ln(cos(d\*x+c))/d+b\*(3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*tan(d\*x+c)/d+1/2\*(2\*A\*a\*b+B\*a^2-B\*b^2)\*(a+b\*tan(d\*x+c))^2/d+1/3\*(A\*b+B\*a)\*(a+b\*tan(d\*x+c))^3/d+1/4\*B\*(a+b\*tan(d\*x+c))^4/d

**Rubi [A]**

time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3609, 3606, 3556}

$$\frac{(a^2 B + 2aAb - b^2 B)(a + b \tan(c + dx))^2}{2d} + \frac{b(a^2 B + 3a^2 Ab - 3ab^2 B - Ab^3) \tan(c + dx)}{d} - \frac{(a^4 B + 4a^3 Ab - 6a^2 b^2 B - 4aAb^3 + b^4 B) \log(\cos(c + dx))}{d} + x(a^4 A - 4a^3 b B - 6a^2 A b^2 + 4a b^3 B + A b^4) + \frac{(aB + Ab)(a + b \tan(c + dx))^2}{3d} + \frac{B(a + b \tan(c + dx))^4}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out] (a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*x - ((4\*a^3\*A\*b - 4\*a\*A\*b^3 + a^4\*B - 6\*a^2\*b^2\*B + b^4\*B)\*Log[Cos[c + d\*x]])/d + (b\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Tan[c + d\*x])/d + ((2\*a\*A\*b + a^2\*B - b^2\*B)\*(a + b\*Tan[c + d\*x])^2)/(2\*d) + ((A\*b + a\*B)\*(a + b\*Tan[c + d\*x])^3)/(3\*d) + (B\*(a + b\*Tan[c + d\*x])^4)/(4\*d)

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3606**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

**Rule 3609**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2,



0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int (a + b \tan(c + dx))^4 (A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^4}{4d} + \int (a + b \tan(c + dx))^3 (aA - bB \\
 &= \frac{(Ab + aB)(a + b \tan(c + dx))^3}{3d} + \frac{B(a + b \tan(c + dx))^4}{4d} + \\
 &= \frac{(2aAb + a^2B - b^2B)(a + b \tan(c + dx))^2}{2d} + \frac{(Ab + aB)(a + b \tan(c + dx))^3}{3d} + \\
 &= (a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B)x + \frac{b(3a^2Ab - 4a^3b^2)}{2d} \\
 &= (a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B)x - \frac{(4a^3Ab - 4a^4b^2)}{2d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 2.21, size = 240, normalized size = 1.19

$$\frac{-2(Ab - aB)(3i(a + b)\log(i - \tan(c + dx)) - 3i(a - b)\log(i + \tan(c + dx)) + 6i^2(-6a^2 + b^2)\tan(c + dx) - 12ab^2 \tan^2(c + dx) - 2i^3 \tan^3(c + dx)) + B(6(-ia + b)^2 \log(i - \tan(c + dx)) + 6(ia + b)^2 \log(i + \tan(c + dx)) + 60ab^2(2a^2 - b^2)\tan(c + dx) - 6i^2(-10a^2 + b^2)\tan^2(c + dx) + 20ab^2 \tan^3(c + dx) + 3i^3 \tan^4(c + dx))}{12ad}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (-2\*(A\*b - a\*B)\*((3\*I)\*(a + I\*b)^4\*Log[I - Tan[c + d\*x]] - (3\*I)\*(a - I\*b)^4\*Log[I + Tan[c + d\*x]] + 6\*b^2\*(-6\*a^2 + b^2)\*Tan[c + d\*x] - 12\*a\*b^3\*Tan[c + d\*x]^2 - 2\*b^4\*Tan[c + d\*x]^3) + B\*(6\*((-I)\*a + b)^5\*Log[I - Tan[c + d\*x]] + 6\*(I\*a + b)^5\*Log[I + Tan[c + d\*x]] + 60\*a\*b^2\*(2\*a^2 - b^2)\*Tan[c + d\*x] - 6\*b^3\*(-10\*a^2 + b^2)\*Tan[c + d\*x]^2 + 20\*a\*b^4\*Tan[c + d\*x]^3 + 3\*b^5\*Tan[c + d\*x]^4)/(12\*b\*d)

**Maple [A]**

time = 0.08, size = 237, normalized size = 1.17

method	result
norman	$(Aa^4 - 6Aa^2b^2 + Ab^4 - 4Ba^3b + 4Ba^2b^3)x + \frac{b(6Aa^2b - Ab^3 + 4Ba^3 - 4Ba^2b^2)\tan(dx+c)}{d} + \frac{Bb^4}{d}$
derivativedivides	$\frac{Bb^4(\tan^4(dx+c))}{4} + \frac{Ab^4(\tan^3(dx+c))}{3} + \frac{4Ba^2b^3(\tan^3(dx+c))}{3} + 2Aa^2b^3(\tan^2(dx+c)) + 3Ba^2b^2(\tan^2(dx+c)) - \frac{Bb^4(\tan^2(dx+c))}{2}$
default	$\frac{Bb^4(\tan^4(dx+c))}{4} + \frac{Ab^4(\tan^3(dx+c))}{3} + \frac{4Ba^2b^3(\tan^3(dx+c))}{3} + 2Aa^2b^3(\tan^2(dx+c)) + 3Ba^2b^2(\tan^2(dx+c)) - \frac{Bb^4(\tan^2(dx+c))}{2}$

risch	$-\frac{\ln(e^{2i(dx+c)}+1)Bb^4}{d} - 6Aa^2b^2x - 4Ba^3bx + 4Bab^3x + 4iAa^3bx + iBb^4x - \frac{a^4 \ln(e^{2i(dx+c)}+1)}{d}$
-------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} \left( \frac{1}{4} B^2 b^4 \tan^4(dx+c) + \frac{1}{3} A b^4 \tan^3(dx+c) + \frac{4}{3} B a b^3 \tan^2(dx+c) + 2 A^2 a b^3 \tan(dx+c) + \frac{1}{2} B^2 b^4 \tan^2(dx+c) + 6 A^2 a^2 b^2 \tan(dx+c) - A b^4 \tan(dx+c) + 4 B a^3 b \tan(dx+c) - 4 B^2 a b^3 \tan(dx+c) + \frac{1}{2} (4 A^2 a^3 b - 4 A^2 a b^3 + B a^4 - 6 B^2 a^2 b^2 + B b^4) \ln(1 + \tan^2(dx+c)) + (A^2 a^4 - 6 A^2 a^2 b^2 + A b^4 - 4 B a^3 b + 4 B^2 a b^3) \arctan(\tan(dx+c)) \right)$$

**Maxima** [A]

time = 0.51, size = 202, normalized size = 1.00

$$\frac{3 B b^4 \tan(dx+c)^4 + 4 (4 B a b^3 + A b^4) \tan(dx+c)^3 + 6 (6 B a^2 b^2 + 4 A a b^3 - B b^4) \tan(dx+c)^2 + 12 (A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4) dx + 6 (B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) \log(\tan(dx+c)^2 + 1) + 12 (4 B a^3 b + 6 A a^2 b^2 - 4 B a b^3 - A b^4) \tan(dx+c)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{12} (3 B^2 b^4 \tan^4(dx+c) + 4 (4 B^2 a b^3 + A b^4) \tan^3(dx+c) + 6 (6 B^2 a^2 b^2 + 4 A^2 a b^3 - B b^4) \tan^2(dx+c) + 12 (A^2 a^4 - 4 B^2 a^3 b - 6 A^2 a^2 b^2 + 4 B^2 a b^3 + A b^4) (dx+c) + 6 (B^2 a^4 + 4 A^2 a^3 b - 6 B^2 a^2 b^2 - 4 A^2 a b^3 + B b^4) \log(\tan^2(dx+c) + 1) + 12 (4 B^2 a^3 b + 6 A^2 a^2 b^2 - 4 B^2 a b^3 - A b^4) \tan(dx+c)) / d$$

**Fricas** [A]

time = 0.72, size = 201, normalized size = 1.00

$$\frac{3 B b^4 \tan(dx+c)^4 + 4 (4 B a b^3 + A b^4) \tan(dx+c)^3 + 12 (A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4) dx + 6 (6 B a^2 b^2 + 4 A a b^3 - B b^4) \tan(dx+c)^2 - 6 (B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) \log\left(\frac{1}{\tan^2(dx+c)+1}\right) + 12 (4 B a^3 b + 6 A a^2 b^2 - 4 B a b^3 - A b^4) \tan(dx+c)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{12} (3 B^2 b^4 \tan^4(dx+c) + 4 (4 B^2 a b^3 + A b^4) \tan^3(dx+c) + 12 (A^2 a^4 - 4 B^2 a^3 b - 6 A^2 a^2 b^2 + 4 B^2 a b^3 + A b^4) dx + 6 (6 B^2 a^2 b^2 + 4 A^2 a b^3 - B b^4) \tan^2(dx+c) - 6 (B^2 a^4 + 4 A^2 a^3 b - 6 B^2 a^2 b^2 - 4 A^2 a b^3 + B b^4) \log(1/(\tan^2(dx+c) + 1)) + 12 (4 B^2 a^3 b + 6 A^2 a^2 b^2 - 4 B^2 a b^3 - A b^4) \tan(dx+c)) / d$$

**Sympy** [A]

time = 0.16, size = 347, normalized size = 1.72

$$\frac{\left\{ \begin{array}{l} A a^2 x + \frac{2 A a b \tan(dx+c)}{d} - 6 A a^2 b^2 x + \frac{4 A b^3 \tan(dx+c)}{d} - \frac{2 A b^4 \tan^2(dx+c)}{d} + A b^4 x + \frac{d^2 \tan^2(dx+c)}{d} - \frac{d^2 \tan^2(dx+c)}{d} + \frac{B a^4 \tan^2(dx+c)}{d} - 4 B a^3 b x + \frac{4 B a^2 b^2 \tan(dx+c)}{d} - \frac{4 B a b^3 \tan^2(dx+c)}{d} + \frac{4 B b^4 \tan^3(dx+c)}{d} + \frac{6 A^2 a^2 b^2 \tan^2(dx+c)}{d} + 4 B a b^3 x + \frac{4 B a^2 b^3 \tan(dx+c)}{d} - \frac{4 B a b^4 \tan^2(dx+c)}{d} + \frac{6 A^2 a^2 b^2 \tan^2(dx+c)}{d} - \frac{6 A^2 a b^3 \tan^3(dx+c)}{d} \end{array} \right\}}{d(A+B \tan(c))(a+b \tan(c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((A\*a\*\*4\*x + 2\*A\*a\*\*3\*b\*log(tan(c + d\*x)\*\*2 + 1)/d - 6\*A\*a\*\*2\*b\*\*2\*x + 6\*A\*a\*\*2\*b\*\*2\*tan(c + d\*x)/d - 2\*A\*a\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/d + 2\*A\*a\*b\*\*3\*tan(c + d\*x)\*\*2/d + A\*b\*\*4\*x + A\*b\*\*4\*tan(c + d\*x)\*\*3/(3\*d) - A\*b\*\*4\*tan(c + d\*x)/d + B\*a\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - 4\*B\*a\*\*3\*b\*x + 4\*B\*a\*\*3\*b\*tan(c + d\*x)/d - 3\*B\*a\*\*2\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/d + 3\*B\*a\*\*2\*b\*\*2\*tan(c + d\*x)\*\*2/d + 4\*B\*a\*b\*\*3\*x + 4\*B\*a\*b\*\*3\*tan(c + d\*x)\*\*3/(3\*d) - 4\*B\*a\*b\*\*3\*tan(c + d\*x)/d + B\*b\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*\*4\*tan(c + d\*x)\*\*4/(4\*d) - B\*b\*\*4\*tan(c + d\*x)\*\*2/(2\*d), Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*4, True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 3383 vs. 2(196) = 392.

time = 2.87, size = 3383, normalized size = 16.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/12*(12*A*a^4*d*x*tan(d*x)^4*tan(c)^4 - 48*B*a^3*b*d*x*tan(d*x)^4*tan(c)^4 \\ & - 72*A*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 + 48*B*a*b^3*d*x*tan(d*x)^4*tan(c)^4 \\ & + 12*A*b^4*d*x*tan(d*x)^4*tan(c)^4 - 6*B*a^4*log(4*(tan(d*x)^4*tan(c)^2 - \\ & 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) \\ & + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 24*A*a^3*b*log(4*(tan(d*x)^4*tan \\ & (c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x) \\ & )*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 36*B*a^2*b^2*log(4*(tan \\ & (d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - \\ & 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 24*A*a*b^3*lo \\ & g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan \\ & (d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 6*B*b \\ & ^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + \\ & tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - \\ & 48*A*a^4*d*x*tan(d*x)^3*tan(c)^3 + 192*B*a^3*b*d*x*tan(d*x)^3*tan(c)^3 + 28 \\ & 8*A*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 - 192*B*a*b^3*d*x*tan(d*x)^3*tan(c)^3 - \\ & 48*A*b^4*d*x*tan(d*x)^3*tan(c)^3 + 36*B*a^2*b^2*tan(d*x)^4*tan(c)^4 + 24*A \\ & *a*b^3*tan(d*x)^4*tan(c)^4 - 9*B*b^4*tan(d*x)^4*tan(c)^4 + 24*B*a^4*log(4*( \\ & tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^ \\ & 2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 96*A*a^3*b \\ & *log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t \\ & an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 14 \\ & 4*B*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t \\ & an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*ta \\ & n(c)^3 - 96*A*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan \\ & (d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d \end{aligned}$$

```

*x)^3*tan(c)^3 + 24*B*b^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))
*tan(d*x)^3*tan(c)^3 - 48*B*a^3*b*tan(d*x)^4*tan(c)^3 - 72*A*a^2*b^2*tan(d*
x)^4*tan(c)^3 + 48*B*a*b^3*tan(d*x)^4*tan(c)^3 + 12*A*b^4*tan(d*x)^4*tan(c)
^3 - 48*B*a^3*b*tan(d*x)^3*tan(c)^4 - 72*A*a^2*b^2*tan(d*x)^3*tan(c)^4 + 48
*B*a*b^3*tan(d*x)^3*tan(c)^4 + 12*A*b^4*tan(d*x)^3*tan(c)^4 + 72*A*a^4*d*x*
tan(d*x)^2*tan(c)^2 - 288*B*a^3*b*d*x*tan(d*x)^2*tan(c)^2 - 432*A*a^2*b^2*d
*x*tan(d*x)^2*tan(c)^2 + 288*B*a*b^3*d*x*tan(d*x)^2*tan(c)^2 + 72*A*b^4*d*x
*tan(d*x)^2*tan(c)^2 + 36*B*a^2*b^2*tan(d*x)^4*tan(c)^2 + 24*A*a*b^3*tan(d*
x)^4*tan(c)^2 - 6*B*b^4*tan(d*x)^4*tan(c)^2 - 72*B*a^2*b^2*tan(d*x)^3*tan(c
)^3 - 48*A*a*b^3*tan(d*x)^3*tan(c)^3 + 24*B*b^4*tan(d*x)^3*tan(c)^3 + 36*B*
a^2*b^2*tan(d*x)^2*tan(c)^4 + 24*A*a*b^3*tan(d*x)^2*tan(c)^4 - 6*B*b^4*tan(
d*x)^2*tan(c)^4 - 16*B*a*b^3*tan(d*x)^4*tan(c) - 4*A*b^4*tan(d*x)^4*tan(c)
- 36*B*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan
(c)^2 - 144*A*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)^2*tan(c)^2 + 216*B*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^2*tan(c)^2 + 144*A*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 36*B*b^4*log(4*(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 144*B*a^3*b*tan(d*x)^3*tan(c)^2
+ 216*A*a^2*b^2*tan(d*x)^3*tan(c)^2 - 192*B*a*b^3*tan(d*x)^3*tan(c)^2 - 48*
A*b^4*tan(d*x)^3*tan(c)^2 + 144*B*a^3*b*tan(d*x)^2*tan(c)^3 + 216*A*a^2*b^2
*tan(d*x)^2*tan(c)^3 - 192*B*a*b^3*tan(d*x)^2*tan(c)^3 - 48*A*b^4*tan(d*x)^
2*tan(c)^3 - 16*B*a*b^3*tan(d*x)*tan(c)^4 - 4*A*b^4*tan(d*x)*tan(c)^4 + 3*B
*b^4*tan(d*x)^4 - 48*A*a^4*d*x*tan(d*x)*tan(c) + 192*B*a^3*b*d*x*tan(d*x)*t
an(c) + 288*A*a^2*b^2*d*x*tan(d*x)*tan(c) - 192*B*a*b^3*d*x*tan(d*x)*tan(c)
- 48*A*b^4*d*x*tan(d*x)*tan(c) - 72*B*a^2*b^2*tan(d*x)^3*tan(c) - 48*A*a*b
^3*tan(d*x)^3*tan(c) + 24*B*b^4*tan(d*x)^3*tan(c) + 72*B*a^2*b^2*tan(d*x)^2
*tan(c)^2 + 48*A*a*b^3*tan(d*x)^2*tan(c)^2 - 12*B*b^4*tan(d*x)^2*tan(c)^2 -
72*B*a^2*b^2*tan(d*x)*tan(c)^3 - 48*A*a*b^3*tan(d*x)*tan(c)^3 + 24*B*b^4*t
an(d*x)*tan(c)^3 + 3*B*b^4*tan(c)^4 + 16*B*a*b^3*tan(d*x)^3 + 4*A*b^4*tan(d
*x)^3 + 24*B*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)
*tan(c) + 96*A*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)*tan(c) - 144*B*a^2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)*tan(c) - 96*A*a*b^3*log(4*(tan(d*x)...

```

Mupad [B]

time = 6.27, size = 205, normalized size = 1.01

$$x(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Ba^2b^2 + Ab^4) - \frac{\tan(c+dx)(Ab^4 + 4Ba^3b - 2a^2b(3Ab + 2Ba))}{d} + \frac{\tan(c+dx)^3\left(\frac{Ab^4}{3} + \frac{4Ba^3b}{3}\right)}{d} + \frac{\ln(\tan(c+dx)^2 + 1)\left(\frac{Bb^4}{2} + 2Aa^3b - 3Ba^2b^2 - 2Aa^2b + \frac{Bb^4}{2}\right)}{d} - \frac{\tan(c+dx)^2\left(\frac{Bb^4}{2} - a^2(2Ab + 3Ba)\right)}{d} + \frac{Bb^4 \tan(c+dx)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^4,x)

[Out] x\*(A\*a^4 + A\*b^4 - 6\*A\*a^2\*b^2 + 4\*B\*a\*b^3 - 4\*B\*a^3\*b) - (tan(c + d\*x)\*(A\*b^4 + 4\*B\*a\*b^3 - 2\*a^2\*b\*(3\*A\*b + 2\*B\*a)))/d + (tan(c + d\*x)^3\*((A\*b^4)/3 + (4\*B\*a\*b^3)/3))/d + (log(tan(c + d\*x)^2 + 1)\*((B\*a^4)/2 + (B\*b^4)/2 - 3\*B\*a^2\*b^2 - 2\*A\*a\*b^3 + 2\*A\*a^3\*b))/d - (tan(c + d\*x)^2\*((B\*b^4)/2 - a\*b^2\*(2\*A\*b + 3\*B\*a)))/d + (B\*b^4\*tan(c + d\*x)^4)/(4\*d)

### 3.260 $\int \cot(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=172

$$(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x - \frac{b(6a^2Ab - Ab^3 + 4a^3B - 4ab^2B) \log(\cos(c+dx))}{d} + \frac{a^4A \log(\sin(c+dx))}{d}$$

[Out]  $(4Aa^3b - 4Aab^3 + B^2a^4 - 6B^2ab^2 + B^2b^4)x - b(6Aa^2b - Ab^3 + 4A^3B - 4Aab^2B) \ln(\cos(dx+c))/d + a^4A \ln(\sin(dx+c))/d + b^2(3Aa^2b + 3A^2B - B^2b^2) \tan(dx+c)/d + 1/2b(Ab + 2A^2B)(a+b \tan(dx+c))^2/d + 1/3b^2B(a+b \tan(dx+c))^3/d$

Rubi [A]

time = 0.32, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {3688, 3728, 3718, 3705, 3556}

$$\frac{a^4A \log(\sin(c+dx))}{d} + \frac{b^2(3a^2B + 3aAb - b^2B) \tan(c+dx)}{d} - \frac{b(4a^3B + 6a^2Ab - 4ab^2B - Ab^3) \log(\cos(c+dx))}{d} + x(a^4B + 4a^3Ab - 6a^2b^2B - 4aAb^3 + b^4B) + \frac{b(2aB + Ab)(a+b \tan(c+dx))^2}{2d} + \frac{bB(a+b \tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x - (b(6a^2Ab - Ab^3 + 4a^3B - 4ab^2B) \log(\cos[c + d*x]))/d + (a^4A \log(\sin[c + d*x]))/d + (b^2(3a^2Ab + 3a^2B - b^2B) \tan[c + d*x])/d + (b(Ab + 2a^2B)(a + b \tan[c + d*x])^2)/(2d) + (b^2B(a + b \tan[c + d*x])^3)/(3d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m-1)\*((c + d\*Tan[e + f\*x])^(n+1)/(d\*f\*(m+n))), x] + Dist[1/(d\*(m+n)), Int[(a + b\*Tan[e + f\*x])^(m-2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m+n) - b\*B\*(b\*c\*(m-1) + a\*d\*(n+1)) + d\*(m+n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m-1) - b\*(A\*b + a\*B)\*d\*(m+n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3705

```
Int[((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2
)/tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[B*x, x] + (Dist[A, Int[1/Tan[e
+ f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3718

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_
)*(x_)]^(n_))*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f
_)*(x_)]^2), x_Symbol] := Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]
```

Rule 3728

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)]^(m_))*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)]^(n_))*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_
) + (f_)*(x_)]^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && (!IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\int \cot(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{bB(a + b \tan(c + dx))^3}{3d} + \frac{1}{3} \int \cot(c + dx)(a + b \tan(c + dx))^4 dx \\
&= \frac{b(Ab + 2aB)(a + b \tan(c + dx))^2}{2d} + \frac{bB(a + b \tan(c + dx))^4}{d} \\
&= \frac{b^2(3aAb + 3a^2B - b^2B) \tan(c + dx)}{d} + \frac{b(Ab + b^2B)}{d} \\
&= (4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x + \frac{b(Ab + b^2B)}{d} \\
&= (4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x - \frac{b(Ab + b^2B)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.93, size = 149, normalized size = 0.87

$$\frac{-3(a+ib)^4(A+ib)\log(i-\tan(c+dx))+6a^4A\log(\tan(c+dx))-3(a-ib)^4(A-ib)\log(i+\tan(c+dx))+6b^2(3aAb+3a^2B-b^2B)\tan(c+dx)+3b(Ab+2aB)(a+b\tan(c+dx))^2+2bB(a+b\tan(c+dx))^3}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(-3*(a + I*b)^4*(A + I*B)*\text{Log}[I - \text{Tan}[c + d*x]] + 6*a^4*A*\text{Log}[\text{Tan}[c + d*x]] - 3*(a - I*b)^4*(A - I*B)*\text{Log}[I + \text{Tan}[c + d*x]] + 6*b^2*(3*a*A*b + 3*a^2*B - b^2*B)*\text{Tan}[c + d*x] + 3*b*(A*b + 2*a*B)*(a + b*\text{Tan}[c + d*x])^2 + 2*b*B*(a + b*\text{Tan}[c + d*x])^3)/(6*d)$

**Maple [A]**

time = 0.23, size = 190, normalized size = 1.10

method	result
norman	$(4Aa^3b - 4Aab^3 + Ba^4 - 6Ba^2b^2 + Bb^4)x + \frac{b^2(4Aab+6a^2B-b^2B)\tan(dx+c)}{d} + \frac{Bb^4(\tan^3(dx+c)-\tan(dx+c))}{3d}$
derivativedivides	$Aa^4\ln(\sin(dx+c))+Ba^4(dx+c)+4Aa^3b(dx+c)-4Ba^3b\ln(\cos(dx+c))-6Aa^2b^2\ln(\cos(dx+c))+6Ba^2b^2(\tan(dx+c)-dx+c)$
default	$Aa^4\ln(\sin(dx+c))+Ba^4(dx+c)+4Aa^3b(dx+c)-4Ba^3b\ln(\cos(dx+c))-6Aa^2b^2\ln(\cos(dx+c))+6Ba^2b^2(\tan(dx+c)-dx+c)$
risch	$\frac{Aa^4\ln(e^{2i(dx+c)}-1)}{d} + \frac{8iBa^3bc}{d} - \frac{2ia^4Ac}{d} - 4iBab^3x + \frac{\ln(e^{2i(dx+c)}+1)Ab^4}{d} + 4Aa^3bx - 4Aab^3x -$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out]  $1/d*(A*a^4*\ln(\sin(d*x+c))+B*a^4*(d*x+c)+4*A*a^3*b*(d*x+c)-4*B*a^3*b*\ln(\cos(d*x+c))-6*A*a^2*b^2*\ln(\cos(d*x+c))+6*B*a^2*b^2*(\tan(d*x+c)-d*x-c)+4*A*a*b^3*(\tan(d*x+c)-d*x-c)+4*B*a*b^3*(1/2*\tan(d*x+c)^2+\ln(\cos(d*x+c)))+A*b^4*(1/2*\tan(d*x+c)^2+\ln(\cos(d*x+c)))+B*b^4*(1/3*\tan(d*x+c)^3-\tan(d*x+c)+d*x+c))$

**Maxima [A]**

time = 0.56, size = 175, normalized size = 1.02

$$\frac{2Bb^4\tan(dx+c)^3+6Aa^4\log(\tan(dx+c))+3(4Bab^3+Ab^4)\tan(dx+c)^2+6(Ba^4+4Aa^3b-6Ba^2b^2-4Aab^3+Bb^4)(dx+c)-3(Aa^4-4Ba^3b-6Aa^2b^2+4Bab^3+Ab^4)\log(\tan(dx+c)^2+1)+6(6Ba^2b^2+4Aab^3-Bb^4)\tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out]  $1/6*(2*B*b^4*\tan(d*x + c)^3 + 6*A*a^4*\log(\tan(d*x + c)) + 3*(4*B*a*b^3 + A*b^4)*\tan(d*x + c)^2 + 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)$



4)\*(d\*x + c) - 3\*(A\*a^4 - 4\*B\*a^3\*b - 6\*A\*a^2\*b^2 + 4\*B\*a\*b^3 + A\*b^4)\*log(tan(d\*x + c)^2 + 1) + 6\*(6\*B\*a^2\*b^2 + 4\*A\*a\*b^3 - B\*b^4)\*tan(d\*x + c))/d

**Fricas** [A]

time = 1.26, size = 185, normalized size = 1.08

$$\frac{2 B b^4 \tan (d x+c)^3+3 A a^4 \log \left(\frac{\tan (d x+c)^2}{\tan (d x+c)^2+1}\right)+6\left(B a^4+4 A a^3 b-6 B a^2 b^2-4 A a b^3+B b^4\right) d x+3\left(4 B a b^3+A b^4\right) \tan (d x+c)^2-3\left(4 B a^3 b+6 A a^2 b^2-4 B a b^3-A b^4\right) \log \left(\frac{1}{\tan (d x+c)^2+1}\right)+6\left(6 B a^2 b^2+4 A a b^3-B b^4\right) \tan (d x+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(2\*B\*b^4\*tan(d\*x + c)^3 + 3\*A\*a^4\*log(tan(d\*x + c)^2/(tan(d\*x + c)^2 + 1)) + 6\*(B\*a^4 + 4\*A\*a^3\*b - 6\*B\*a^2\*b^2 - 4\*A\*a\*b^3 + B\*b^4)\*d\*x + 3\*(4\*B\*a\*b^3 + A\*b^4)\*tan(d\*x + c)^2 - 3\*(4\*B\*a^3\*b + 6\*A\*a^2\*b^2 - 4\*B\*a\*b^3 - A\*b^4)\*log(1/(tan(d\*x + c)^2 + 1)) + 6\*(6\*B\*a^2\*b^2 + 4\*A\*a\*b^3 - B\*b^4)\*tan(d\*x + c))/d

**Sympy** [A]

time = 0.90, size = 291, normalized size = 1.69

$$\begin{cases} \frac{-A a^4 \log (\tan (c+d x)+1)+A a^4 \log (\tan (c+d x))}{d}+4 A a^2 b x+\frac{3 A a^3 b \log (\tan (c+d x)+1)-4 A a b^3 x+5 A a b^3 \log (c+d x)-A b^4 \log (\tan (c+d x)+1)+A b^4 \log (c+d x)+B a^4 x+2 B a^3 b \log (\tan (c+d x)+1)-6 B a^2 b^2 x+\frac{6 B a^2 b \log (c+d x)-2 B a b^3 \log (\tan (c+d x)+1)+2 B a b^3 \log (c+d x)+B b^4 x+\frac{B b^4 \tan (c+d x)-B b^4 \tan (c+d x)}{d}}{d} & \text{for } d \neq 0 \\ x(A+B \tan (c))(a+b \tan (c))^4 \cot (c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out] Piecewise((-A\*a\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*a\*\*4\*log(tan(c + d\*x)))/d + 4\*A\*a\*\*3\*b\*x + 3\*A\*a\*\*2\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/d - 4\*A\*a\*b\*\*3\*x + 4\*A\*a\*b\*\*3\*tan(c + d\*x)/d - A\*b\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*b\*\*4\*tan(c + d\*x)\*\*2/(2\*d) + B\*a\*\*4\*x + 2\*B\*a\*\*3\*b\*log(tan(c + d\*x)\*\*2 + 1)/d - 6\*B\*a\*\*2\*b\*\*2\*x + 6\*B\*a\*\*2\*b\*\*2\*tan(c + d\*x)/d - 2\*B\*a\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/d + 2\*B\*a\*b\*\*3\*tan(c + d\*x)\*\*2/d + B\*b\*\*4\*x + B\*b\*\*4\*tan(c + d\*x)\*\*3/(3\*d) - B\*b\*\*4\*tan(c + d\*x)/d, Ne(d, 0)), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*4\*cot(c), True))

**Giac** [A]

time = 1.75, size = 191, normalized size = 1.11

$$\frac{2 B b^4 \tan (d x+c)^3+12 B a b^3 \tan (d x+c)^2+3 A a^4 \tan (d x+c)^2+6 A a^4 \log (|\tan (d x+c)|)+36 B a^2 b^2 \tan (d x+c)+24 A a b^3 \tan (d x+c)-6 B b^4 \tan (d x+c)+6\left(B a^4+4 A a^3 b-6 B a^2 b^2-4 A a b^3+B b^4\right)(d x+c)-3\left(A a^4-4 B a^3 b-6 A a^2 b^2+4 B a b^3+A b^4\right) \log (\tan (d x+c)^2+1)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/6\*(2\*B\*b^4\*tan(d\*x + c)^3 + 12\*B\*a\*b^3\*tan(d\*x + c)^2 + 3\*A\*b^4\*tan(d\*x + c)^2 + 6\*A\*a^4\*log(abs(tan(d\*x + c))) + 36\*B\*a^2\*b^2\*tan(d\*x + c) + 24\*A\*a

$*b^3*\tan(dx + c) - 6*B*b^4*\tan(dx + c) + 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(dx + c) - 3*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*\log(\tan(dx + c)^2 + 1))/d$

**Mupad [B]**

time = 6.49, size = 151, normalized size = 0.88

$$\frac{\tan(c+dx)^2 \left(\frac{A b^4}{2} + 2 B a b^3\right)}{d} - \frac{\tan(c+dx) (B b^4 - 2 a b^2 (2 A b + 3 B a))}{d} + \frac{A a^4 \ln(\tan(c+dx))}{d} - \frac{\ln(\tan(c+dx) + 1i) (A - B 1i) (b + a 1i)^4}{2d} - \frac{\ln(\tan(c+dx) - 1i) (A + B 1i) (-b + a 1i)^4}{2d} + \frac{B b^4 \tan(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^4,x)

[Out]  $(\tan(c + d*x)^2*((A*b^4)/2 + 2*B*a*b^3))/d - (\tan(c + d*x)*(B*b^4 - 2*a*b^2*(2*A*b + 3*B*a)))/d + (A*a^4*\log(\tan(c + d*x)))/d - (\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^4)/(2*d) - (\log(\tan(c + d*x) - 1i)*(A + B*1i)*(a*1i - b)^4)/(2*d) + (B*b^4*\tan(c + d*x)^3)/(3*d)$

### 3.261 $\int \cot^2(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=175

$$-\left((a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B)x\right) - \frac{b^2(4aAb + 6a^2B - b^2B) \log(\cos(c + dx))}{d} + \frac{a^3(4Ab + aB)}{d}$$

[Out]  $-(A*a^4-6*A*a^2*b^2+A*b^4-4*A*b^3*B+4*A*b^3*B)*x-b^2*(4*A*a*b+6*B*a^2-B*b^2)*\ln(\cos(d*x+c))/d+a^3*(4*A*b+B*a)*\ln(\sin(d*x+c))/d+b^2*(A*a^2+A*b^2+3*B*a*b)*\tan(d*x+c)/d+1/2*b*(2*A*a+B*b)*(a+b*\tan(d*x+c))^2/d-a*A*\cot(d*x+c)*(a+b*\tan(d*x+c))^3/d$

Rubi [A]

time = 0.32, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3686, 3728, 3718, 3705, 3556}

$$\frac{a^3(aB + 4Ab) \log(\sin(c + dx))}{d} + \frac{b^2(a^2A + 3abB + Ab^2) \tan(c + dx)}{d} - \frac{b^2(6a^2B + 4aAb - b^2B) \log(\cos(c + dx))}{d} - x(a^4A - 4a^3bB - 6a^2Ab^2 + 4ab^3B + Ab^4) + \frac{b(2aA + bB)(a + b \tan(c + dx))^2}{2d} - \frac{aA \cot(c + dx)(a + b \tan(c + dx))^3}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left((a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4a^2b^3B)x\right) - (b^2*(4aAb + 6a^2B - b^2B)*\text{Log}[\text{Cos}[c + d*x]])/d + (a^3*(4Ab + aB)*\text{Log}[\text{Sin}[c + d*x]])/d + (b^2*(a^2A + A*b^2 + 3a*b*B)*\text{Tan}[c + d*x])/d + (b*(2aA + bB)*(a + b*\text{Tan}[c + d*x])^2)/(2*d) - (a*A*\text{Cot}[c + d*x]*(a + b*\text{Tan}[c + d*x])^3)/d$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3686

$\text{Int}[\left((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]\right)^{(m_.)}*\left((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(B*c - A*d)*(a + b*\text{Tan}[e + f*x])^{(m-1)}*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(d*f*(n+1)*(c^2 + d^2)), x] - \text{Dist}[1/(d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-2)}*(c + d*\text{Tan}[e + f*x])^{(n+1)}*\text{Simp}[a*A*d*(b*d*(m-1) - a*c*(n+1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m-1) + a*d*(n+1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n+1)*\text{Tan}[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m+n) - b*B*(c^2*(m-1) - d^2*(n+1)))*\text{Tan}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\&$

`LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])`

### Rule 3705

```
Int[((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2
)/tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[B*x, x] + (Dist[A, Int[1/Tan[e
+ f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

### Rule 3718

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)
*(x_)]^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f
_)*(x_)]^2), x_Symbol] := Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]
```

### Rule 3728

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)]^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)]^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)]^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rubi steps

$$\begin{aligned}
\int \cot^2(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx)(a + b \tan(c + dx))^3}{d} + \int \cot \\
&= \frac{b(2aA + bB)(a + b \tan(c + dx))^2}{2d} - \frac{aA \cot(c + dx)}{d} \\
&= \frac{b^2(a^2A + Ab^2 + 3abB) \tan(c + dx)}{d} + \frac{b(2aA + bB)}{d} \\
&= -(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) x \\
&= -(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) x
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 0.68, size = 134, normalized size = 0.77

$$\frac{-2a^4A \cot(c + dx) + i(a + ib)^4(A + iB) \log(i - \tan(c + dx)) + 2a^3(4Ab + aB) \log(\tan(c + dx)) - (a - ib)^4(iA + B) \log(i + \tan(c + dx)) + 2b^3(Ab + 4aB) \tan(c + dx) + b^4B \tan^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out] (-2\*a^4\*A\*Cot[c + d\*x] + I\*(a + I\*b)^4\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + 2\*a^3\*(4\*A\*b + a\*B)\*Log[Tan[c + d\*x]] - (a - I\*b)^4\*(I\*A + B)\*Log[I + Tan[c + d\*x]] + 2\*b^3\*(A\*b + 4\*a\*B)\*Tan[c + d\*x] + b^4\*B\*Tan[c + d\*x]^2)/(2\*d)

**Maple [A]**

time = 0.21, size = 172, normalized size = 0.98

method	result
derivativedivides	$A a^4(-\cot(dx+c)-dx-c)+B a^4 \ln(\sin(dx+c))+4A a^3 b \ln(\sin(dx+c))+4B a^3 b(dx+c)+6A a^2 b^2(dx+c)-6B a^2 b^2 \ln(\cos(dx+c))$
default	$A a^4(-\cot(dx+c)-dx-c)+B a^4 \ln(\sin(dx+c))+4A a^3 b \ln(\sin(dx+c))+4B a^3 b(dx+c)+6A a^2 b^2(dx+c)-6B a^2 b^2 \ln(\cos(dx+c))$
norman	$\frac{(-A a^4+6A a^2 b^2-A b^4+4B a^3 b-4B a b^3)x \tan(dx+c)+\frac{b^3(A b+4 a B)\left(\tan^2(dx+c)\right)}{d}-\frac{A a^4}{d}+\frac{B b^4\left(\tan^3(dx+c)\right)}{2 d}}{\tan(dx+c)}+\frac{a^3(4 A b^3+3 a^2 B)}{2 d}$
risch	$\frac{\ln\left(e^{2 i(dx+c)}+1\right) B b^4}{d}-4 i A a^3 b x-i B b^4 x+6 A a^2 b^2 x+4 B a^3 b x-4 B a b^3 x+\frac{a^4 \ln\left(e^{2 i(dx+c)}-1\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)  
E)

[Out]  $1/d*(A*a^4*(-cot(d*x+c)-d*x-c)+B*a^4*ln(sin(d*x+c))+4*A*a^3*b*ln(sin(d*x+c))+4*B*a^3*b*(d*x+c)+6*A*a^2*b^2*(d*x+c)-6*B*a^2*b^2*ln(cos(d*x+c))-4*A*a*b^3*ln(cos(d*x+c))+4*B*a*b^3*(tan(d*x+c)-d*x-c)+A*b^4*(tan(d*x+c)-d*x-c)+B*b^4*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))$

**Maxima** [A]

time = 0.53, size = 164, normalized size = 0.94

$$\frac{Bb^4 \tan(dx+c)^2 - \frac{2Aa^4}{\tan(dx+c)} - 2(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)(dx+c) - (Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3 + Bb^4) \log(\tan(dx+c)^2 + 1) + 2(Ba^4 + 4Aa^3b) \log(\tan(dx+c)) + 2(4Bab^3 + Ab^4) \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/2*(B*b^4*tan(d*x + c)^2 - 2*A*a^4/tan(d*x + c) - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - (B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1) + 2*(B*a^4 + 4*A*a^3*b)*log(tan(d*x + c)) + 2*(4*B*a*b^3 + A*b^4)*tan(d*x + c))/d$

**Fricas** [A]

time = 1.06, size = 193, normalized size = 1.10

$$\frac{Bb^4 \tan(dx+c)^3 - 2Aa^4 + (Ba^4 + 4Aa^3b) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c) - (6Ba^2b^2 + 4Aab^3 - Bb^4) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) \tan(dx+c) + 2(4Bab^3 + Ab^4) \tan(dx+c)^2 + (Bb^4 - 2(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)dx) \tan(dx+c)}{2d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/2*(B*b^4*tan(d*x + c)^3 - 2*A*a^4 + (B*a^4 + 4*A*a^3*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c) - (6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^2 + (B*b^4 - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x)*tan(d*x + c))/(d*tan(d*x + c))$

**Sympy** [A]

time = 1.39, size = 289, normalized size = 1.65

$$\begin{cases} \infty Aa^4 x & \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(A+B \tan(c))(a+b \tan(c))^4 \cos^2(c) & \text{for } d=0 \\ -Aa^4 x - \frac{Aa^4}{\tan(c)} - \frac{2Aa^3 b}{\tan(c)^2} - \frac{2Aa^2 b^2}{\tan(c)^3} + \frac{4Aa b^3}{\tan(c)^4} + \frac{4Aa^2 b^2 x}{d} + \frac{2Aa^3 b \log(\tan^2(c+dx)+1)}{d} - Ab^4 x + \frac{4B^4 \tan(c+dx)}{d} - \frac{Bb^4 \log(\tan^2(c+dx)+1)}{2d} + \frac{2B^4 \log(\tan(c+dx))}{d} + 4Ba^3 b x + \frac{4Ba^2 b^2 x}{d} - 4Ba b^3 x + \frac{4Ba^2 b \log(c+dx)}{d} - \frac{Bb^4 \log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^4 \tan^2(c+dx)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(A + B*tan(c))*(a + b*tan(c))**4*cot(c)**2, Eq(d, 0)), (-A*a**4*x - A*a**4/(d*tan(c + d*x)) - 2*A*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*A*a**3*b*log(tan(c + d*x))/d + 6*A*a**2*b**2*x + 2*A*a*b**3*log(tan(c + d*x)**2 +`

1)/d - A\*b\*\*4\*x + A\*b\*\*4\*tan(c + d\*x)/d - B\*a\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*a\*\*4\*log(tan(c + d\*x))/d + 4\*B\*a\*\*3\*b\*x + 3\*B\*a\*\*2\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/d - 4\*B\*a\*b\*\*3\*x + 4\*B\*a\*b\*\*3\*tan(c + d\*x)/d - B\*b\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*b\*\*4\*tan(c + d\*x)\*\*2/(2\*d), True))

**Giac** [A]

time = 1.33, size = 195, normalized size = 1.11

$$\frac{B^4 \tan(dx+c)^2 + 8 B a b^3 \tan(dx+c) + 2 A b^4 \tan(dx+c) - 2 (A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4)(dx+c) - (B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) \log(\tan(dx+c)^2 + 1) + 2 (B a^4 + 4 A a^3 b) \log(|\tan(dx+c)|) - \frac{2 (B a^4 \tan(dx+c) + 4 A a^3 b \tan(dx+c) + A a^4)}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(B\*b^4\*tan(d\*x + c)^2 + 8\*B\*a\*b^3\*tan(d\*x + c) + 2\*A\*b^4\*tan(d\*x + c) - 2\*(A\*a^4 - 4\*B\*a^3\*b - 6\*A\*a^2\*b^2 + 4\*B\*a\*b^3 + A\*b^4)\*(d\*x + c) - (B\*a^4 + 4\*A\*a^3\*b - 6\*B\*a^2\*b^2 - 4\*A\*a\*b^3 + B\*b^4)\*log(tan(d\*x + c)^2 + 1) + 2\*(B\*a^4 + 4\*A\*a^3\*b)\*log(abs(tan(d\*x + c))) - 2\*(B\*a^4\*tan(d\*x + c) + 4\*A\*a^3\*b\*tan(d\*x + c) + A\*a^4)/tan(d\*x + c))/d

**Mupad** [B]

time = 6.44, size = 142, normalized size = 0.81

$$\frac{\tan(c+dx) (A b^4 + 4 B a b^3)}{d} + \frac{\ln(\tan(c+dx)) (B a^4 + 4 A b a^3)}{d} + \frac{\ln(\tan(c+dx) - i) (-B + A i) (-b + a i)^4}{2d} - \frac{\ln(\tan(c+dx) + i) (B + A i) (b + a i)^4}{2d} - \frac{A a^4 \cot(c+dx)}{d} + \frac{B b^4 \tan(c+dx)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^4,x)

[Out] (tan(c + d\*x)\*(A\*b^4 + 4\*B\*a\*b^3))/d + (log(tan(c + d\*x))\*(B\*a^4 + 4\*A\*a^3\*b))/d + (log(tan(c + d\*x) - 1i)\*(A\*1i - B)\*(a\*1i - b)^4)/(2\*d) - (log(tan(c + d\*x) + 1i)\*(A\*1i + B)\*(a\*1i + b)^4)/(2\*d) - (A\*a^4\*cot(c + d\*x))/d + (B\*b^4\*tan(c + d\*x)^2)/(2\*d)

### 3.262 $\int \cot^3(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=186

$$-\left((4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x\right) - \frac{b^3(Ab + 4aB) \log(\cos(c + dx))}{d} - \frac{a^2(a^2A - 6Ab^2 - 4abB) \log(\sin(c + dx))}{d}$$

[Out]  $-(4Aa^3b - 4Aab^3 + B^4 - 6B^2a^2b^2 + B^4b^4)x - b^3(Ab + 4Ba) \ln(\cos(dx + c)) / d - a^2(Aa^2 - 6Aab^2 - 4a^2B) \ln(\sin(dx + c)) / d + b^2(3Aab + B^2a^2 + B^2b^2) \tan(dx + c) / d - 1/2a(5Ab + 2Ba) \cot(dx + c) (a + b \tan(dx + c))^2 / d - 1/2aA \cot(dx + c)^2 (a + b \tan(dx + c))^3 / d$

Rubi [A]

time = 0.35, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3686, 3726, 3718, 3705, 3556}

$$\frac{b^2(a^2B + 3aAb + b^2B) \tan(c + dx)}{d} - \frac{a^2(a^2A - 4aAb - 6Ab^2) \log(\sin(c + dx))}{d} - x(a^2B + 4a^2Ab - 6a^2b^2B - 4aAb^3 + b^4B) - \frac{b^3(4aB + Ab) \log(\cos(c + dx))}{d} - \frac{a(2aB + 5Ab) \cot(c + dx)(a + b \tan(c + dx))^2}{2d} - \frac{aA \cot^2(c + dx)(a + b \tan(c + dx))^3}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^3(a + b*\text{Tan}[c + d*x])^4(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left((4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x\right) - (b^3(Ab + 4aB) \text{Log}[\text{Cos}[c + d*x]]) / d - (a^2(a^2A - 6Aab^2 - 4a^2B) \text{Log}[\text{Sin}[c + d*x]]) / d + (b^2(3aAb + a^2B + b^2B) \text{Tan}[c + d*x]) / d - (a(5Ab + 2aB) \text{Cot}[c + d*x] (a + b \text{Tan}[c + d*x])^2) / (2*d) - (aA \text{Cot}[c + d*x]^2 (a + b \text{Tan}[c + d*x])^3) / (2*d)$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]] / d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 3686

$\text{Int}[(a_.) + (b_.) \tan[(e_.) + (f_.)(x_.)]^{(m_.)} ((A_.) + (B_.) \tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d) (B*c - A*d) (a + b \text{Tan}[e + f*x])^{(m-1)} ((c + d \text{Tan}[e + f*x])^{(n+1)} / (d*f*(n+1)*(c^2 + d^2))), x] - \text{Dist}[1/(d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b \text{Tan}[e + f*x])^{(m-2)} (c + d \text{Tan}[e + f*x])^{(n+1)} \text{Simp}[a*A*d*(b*d*(m-1) - a*c*(n+1)) + (b*B*c - (A*b + a*B)*d) (b*c*(m-1) + a*d*(n+1)) - d*((a*A - b*B) (b*c - a*d) + (A*b + a*B) (a*c + b*d)) (n+1) \text{Tan}[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d) (m+n) - b*B*(c^2*(m-1) - d^2*(n+1))) \text{Tan}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\&$



$\text{LtQ}[n, -1] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegersQ}[2*m, 2*n])$

### Rule 3705

$\text{Int}[\frac{((A_) + (B_)*\tan[(e_) + (f_)*(x_)]) + (C_)*\tan[(e_) + (f_)*(x_)]^2}{\tan[(e_) + (f_)*(x_)]}, x\_Symbol] \ :> \ \text{Simp}[B*x, x] + (\text{Dist}[A, \text{Int}[1/\tan[e + f*x], x], x] + \text{Dist}[C, \text{Int}[\tan[e + f*x], x], x]) \ /; \ \text{FreeQ}\{e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[A, C]$

### Rule 3718

$\text{Int}[\frac{((a_) + (b_)*\tan[(e_) + (f_)*(x_)])*((c_) + (d_)*\tan[(e_) + (f_)*(x_)]^n * ((A_) + (B_)*\tan[(e_) + (f_)*(x_)] + (C_)*\tan[(e_) + (f_)*(x_)]^2)}{x\_Symbol} \ :> \ \text{Simp}[b*c*\tan[e + f*x]*((c + d*\tan[e + f*x])^{n+1}/(d*f*(n+2))), x] - \text{Dist}[1/(d*(n+2)), \text{Int}[(c + d*\tan[e + f*x])^n * \text{Simp}[b*c*C - a*A*d*(n+2) - (A*b + a*B - b*C)*d*(n+2)*\tan[e + f*x] - (a*C*d*(n+2) - b*(c*C - B*d*(n+2)))*\tan[e + f*x]^2, x], x] \ /; \ \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ !\text{LtQ}[n, -1]$

### Rule 3726

$\text{Int}[\frac{((a_) + (b_)*\tan[(e_) + (f_)*(x_)])^{(m)}*((c_) + (d_)*\tan[(e_) + (f_)*(x_)]^n * ((A_) + (B_)*\tan[(e_) + (f_)*(x_)] + (C_)*\tan[(e_) + (f_)*(x_)]^2)}{x\_Symbol} \ :> \ \text{Simp}[(A*d^2 + c*(c*C - B*d))*(a + b*\tan[e + f*x])^m * ((c + d*\tan[e + f*x])^{n+1}/(d*f*(n+1)*(c^2 + d^2))), x] - \text{Dist}[1/(d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\tan[e + f*x])^{m-1} * (c + d*\tan[e + f*x])^{n+1} * \text{Simp}[A*d*(b*d*m - a*c*(n+1)) + (c*C - B*d)*(b*c*m + a*d*(n+1)) - d*(n+1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*\tan[e + f*x] - b*(d*(B*c - A*d)*(m+n+1) - C*(c^2*m - d^2*(n+1)))*\tan[e + f*x]^2, x], x] \ /; \ \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))dx &= -\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d} + \frac{1}{2} \int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))dx \\
&= -\frac{a(5Ab+2aB)\cot(c+dx)(a+b\tan(c+dx))^3}{2d} \\
&= \frac{b^2(3aAb+a^2B+b^2B)\tan(c+dx)}{d} - \frac{a(5Ab+2aB)\cot(c+dx)(a+b\tan(c+dx))^3}{2d} \\
&= -(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)x + \frac{b^2(3aAb+a^2B+b^2B)\tan(c+dx)}{d} \\
&= -(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)x + \frac{b^2(3aAb+a^2B+b^2B)\tan(c+dx)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.46, size = 140, normalized size = 0.75

$$\frac{-2a^3(4Ab+aB)\cot(c+dx) - a^4A\cot^2(c+dx) + (a+ib)^4(A+ib)\log(i-\tan(c+dx)) - 2a^2(a^2A-6Ab^2-4abB)\log(\tan(c+dx)) + (a-ib)^4(A-ib)\log(i+\tan(c+dx)) + 2b^4B\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out] (-2\*a^3\*(4\*A\*b + a\*B)\*Cot[c + d\*x] - a^4\*A\*Cot[c + d\*x]^2 + (a + I\*b)^4\*(A + I\*B)\*Log[I - Tan[c + d\*x]] - 2\*a^2\*(a^2\*A - 6\*A\*b^2 - 4\*a\*b\*B)\*Log[Tan[c + d\*x]] + (a - I\*b)^4\*(A - I\*B)\*Log[I + Tan[c + d\*x]] + 2\*b^4\*B\*Tan[c + d\*x])/ (2\*d)

**Maple [A]**

time = 0.24, size = 177, normalized size = 0.95

method	result
derivativedivides	$\frac{Aa^4\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)+Ba^4(-\cot(dx+c)-dx-c)+4Aa^3b(-\cot(dx+c)-dx-c)+4Ba^3b\ln(\sin(dx+c))}{1}$
default	$Aa^4\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)+Ba^4(-\cot(dx+c)-dx-c)+4Aa^3b(-\cot(dx+c)-dx-c)+4Ba^3b\ln(\sin(dx+c))$
norman	$\frac{(-4Aa^3b+4Aab^3-Ba^4+6Ba^2b^2-Bb^4)x(\tan^2(dx+c))+\frac{Bb^4(\tan^3(dx+c))}{d}-\frac{Aa^4}{2d}-\frac{a^3(4Ab+aB)\tan(dx+c)}{d}}{\tan(dx+c)^2} + \frac{(Aa^4-6Ba^3b)}{2d}$
risch	$-\frac{Aa^4\ln(e^{2i(dx+c)}-1)}{d} - \frac{\ln(e^{2i(dx+c)}+1)Ab^4}{d} - 4Aa^3bx + 4Aab^3x + 6Ba^2b^2x + \frac{2i(-iAa^4e^{4i(dx+c)})}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out]  $1/d*(A*a^4*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+B*a^4*(-\cot(d*x+c)-d*x-c)+4*A*a^3*b*(-\cot(d*x+c)-d*x-c)+4*B*a^3*b*\ln(\sin(d*x+c))+6*A*a^2*b^2*\ln(\sin(d*x+c))+6*B*a^2*b^2*(d*x+c)+4*A*a*b^3*(d*x+c)-4*B*a*b^3*\ln(\cos(d*x+c))-A*b^4*\ln(\cos(d*x+c))+B*b^4*(\tan(d*x+c)-d*x-c))$

**Maxima** [A]

time = 0.49, size = 173, normalized size = 0.93

$$\frac{2 B b^4 \tan (d x+c)-2\left(B a^4+4 A a^3 b-6 B a^2 b^2-4 A a b^3+B b^4\right)(d x+c)+\left(A a^4-4 B a^3 b-6 A a^2 b^2+4 B a b^3+A b^4\right) \log (\tan (d x+c)^2+1)-2\left(A a^4-4 B a^3 b-6 A a^2 b^2\right) \log (\tan (d x+c))-A a^4+2\left(B a^4+4 A a^3 b\right) \frac{\tan (d x+c)}{\tan (d x+c)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/2*(2*B*b^4*\tan(d*x + c) - 2*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c) + (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*\log(\tan(d*x + c)^2 + 1) - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*\log(\tan(d*x + c)) - (A*a^4 + 2*(B*a^4 + 4*A*a^3*b)*\tan(d*x + c))/\tan(d*x + c)^2)/d$

**Fricas** [A]

time = 0.80, size = 199, normalized size = 1.07

$$\frac{2 B b^4 \tan (d x+c)^3-A a^4-\left(A a^4-4 B a^3 b-6 A a^2 b^2\right) \log \left(\frac{\tan (d x+c)^2}{\tan (d x+c)^2+1}\right) \tan (d x+c)^2-\left(4 B a b^3+A b^4\right) \log \left(\frac{1}{\tan (d x+c)^2+1}\right) \tan (d x+c)^2-\left(A a^4+2\left(B a^4+4 A a^3 b-6 B a^2 b^2-4 A a b^3+B b^4\right) d x\right) \tan (d x+c)^2-2\left(B a^4+4 A a^3 b\right) \tan (d x+c)}{2 d \tan (d x+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $1/2*(2*B*b^4*\tan(d*x + c)^3 - A*a^4 - (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^2 - (4*B*a*b^3 + A*b^4)*\log(1/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^2 - (A*a^4 + 2*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*d*x)*\tan(d*x + c)^2 - 2*(B*a^4 + 4*A*a^3*b)*\tan(d*x + c))/d*\tan(d*x + c)^2$

**Sympy** [A]

time = 2.16, size = 309, normalized size = 1.66

$$\begin{cases} \infty A a^4 x & \text{for } (c = 0 \vee c = -d x) \wedge (c = -d x \vee d = 0) \\ x(A + B \tan(c))(a + b \tan(c))^4 \cot^3(c) & \text{for } d = 0 \\ \frac{2 B b^4 \tan^3(d x+c) - A a^4 - (A a^4 - 4 B a^3 b - 6 A a^2 b^2) \log\left(\frac{\tan^2(d x+c)}{\tan^2(d x+c)+1}\right) \tan(d x+c)^2 - (4 B a b^3 + A b^4) \log\left(\frac{1}{\tan^2(d x+c)+1}\right) \tan(d x+c)^2 - (A a^4 + 2(B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) d x) \tan(d x+c)^2 - 2(B a^4 + 4 A a^3 b) \tan(d x+c)}{2 d \tan^2(d x+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(A + B*tan(c))*(a + b*tan(c))**4*cot(c)**3, Eq(d, 0)), (A*a**4*log(tan(c + d*x)**2 + 1)/(2*d) - A*a**4*log(tan(c + d*x))/d - A*a**4/(2*d*tan(c + d*x)**2) - 4*A*a**3*b*x - 4*A*a**3*b/(d*tan(c + d*x)) - 3*A*a**2*b**2*log`

```
(tan(c + d*x)**2 + 1)/d + 6*A*a**2*b**2*log(tan(c + d*x))/d + 4*A*a*b**3*x
+ A*b**4*log(tan(c + d*x)**2 + 1)/(2*d) - B*a**4*x - B*a**4/(d*tan(c + d*x))
) - 2*B*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*B*a**3*b*log(tan(c + d*x))/d
+ 6*B*a**2*b**2*x + 2*B*a*b**3*log(tan(c + d*x)**2 + 1)/d - B*b**4*x + B*b*
*4*tan(c + d*x)/d, True))
```

**Giac [A]**

time = 1.46, size = 224, normalized size = 1.20

$$\frac{2 B b^4 \tan(dx+c) - 2 (B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4)(dx+c) + (A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4) \log(\tan(dx+c)^2 + 1) - 2 (A a^4 - 4 B a^3 b - 6 A a^2 b^2) \log(|\tan(dx+c)|) + \frac{3 A a^4 \tan(dx+c)^2 - 12 B a^3 b \tan(dx+c)^2 - 18 A a^2 b^2 \tan(dx+c)^2 - 2 B a^4 \tan(dx+c) - 8 A a^3 b \tan(dx+c) - A a^4}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*B*b^4*tan(d*x + c) - 2*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3
+ B*b^4)*(d*x + c) + (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*
log(tan(d*x + c)^2 + 1) - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*log(abs(tan(d
*x + c))) + (3*A*a^4*tan(d*x + c)^2 - 12*B*a^3*b*tan(d*x + c)^2 - 18*A*a^2*
b^2*tan(d*x + c)^2 - 2*B*a^4*tan(d*x + c) - 8*A*a^3*b*tan(d*x + c) - A*a^4)
/tan(d*x + c)^2)/d
```

**Mupad [B]**

time = 6.45, size = 149, normalized size = 0.80

$$\frac{\ln(\tan(c+dx))(-Aa^4+4Ba^3b+6Aa^2b^2)}{d} - \frac{\cot(c+dx)^2(\tan(c+dx)(Ba^4+4Ab^3a^3+\frac{Aa^4}{2})}{d} + \frac{\ln(\tan(c+dx)+1i)(A-B1i)(b+a1i)^4}{2d} + \frac{Bb^4\tan(c+dx)}{d} + \frac{\ln(\tan(c+dx)-i)(A+B1i)(-b+a1i)^4}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x))*(6*A*a^2*b^2 - A*a^4 + 4*B*a^3*b))/d - (cot(c + d*x)^2*(
tan(c + d*x)*(B*a^4 + 4*A*a^3*b) + (A*a^4)/2))/d + (log(tan(c + d*x) + 1i)*
(A - B*1i)*(a*1i + b)^4)/(2*d) + (B*b^4*tan(c + d*x))/d + (log(tan(c + d*x)
- 1i)*(A + B*1i)*(a*1i - b)^4)/(2*d)
```

### 3.263 $\int \cot^4(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=187

$$(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) x + \frac{a^2(a^2A - 3Ab^2 - 3abB) \cot(c+dx)}{d} - \frac{b^4B \log(\cos(c+dx))}{d} - \frac{a}{d}$$

[Out] (A\*a^4-6\*A\*a^2\*b^2+A\*b^4-4\*B\*a^3\*b+4\*B\*a\*b^3)\*x+a^2\*(A\*a^2-3\*A\*b^2-3\*B\*a\*b)\*cot(d\*x+c)/d-b^4\*B\*ln(cos(d\*x+c))/d-a\*(4\*A\*a^2\*b-4\*A\*b^3+B\*a^3-6\*B\*a\*b^2)\*ln(sin(d\*x+c))/d-1/2\*a\*(2\*A\*b+B\*a)\*cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^2/d-1/3\*a\*A\*cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^3/d

Rubi [A]

time = 0.36, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3686, 3726, 3716, 3705, 3556}

$$\frac{a^2(a^2A - 3abB - 3Ab^2) \cot(c+dx)}{d} - \frac{a(a^3B + 4a^2Ab - 6ab^2B - 4Ab^3) \log(\sin(c+dx))}{d} + x(a^4A - 4a^3bB - 6a^2Ab^2 + 4ab^3B + Ab^4) - \frac{a(aB + 2Ab) \cot^2(c+dx)(a+b \tan(c+dx))^2}{2d} - \frac{aA \cot^3(c+dx)(a+b \tan(c+dx))^3}{3d} - \frac{b^4B \log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out] (a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*x + (a^2\*(a^2\*A - 3\*A\*b^2 - 3\*a\*b\*B)\*Cot[c + d\*x])/d - (b^4\*B\*Log[Cos[c + d\*x]])/d - (a\*(4\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B - 6\*a\*b^2\*B)\*Log[Sin[c + d\*x]])/d - (a\*(2\*A\*b + a\*B)\*Cot[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^2)/(2\*d) - (a\*A\*Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^3)/(3\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&

LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3705

Int[((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2)/tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[B\*x, x] + (Dist[A, Int[1/Tan[e + f\*x], x], x] + Dist[C, Int[Tan[e + f\*x], x], x]) /; FreeQ[{e, f, A, B, C}, x] && NeQ[A, C]

### Rule 3716

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^2)^n\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(-b\*c - a\*d)\*(c^2\*C - B\*c\*d + A\*d^2)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d^2\*f\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*d\*(A\*c - c\*C + B\*d) + b\*(c^2\*C - B\*c\*d + A\*d^2) + d\*(A\*b\*c + a\*B\*c - b\*c\*C - a\*A\*d + b\*B\*d + a\*C\*d)\*Tan[e + f\*x] + b\*C\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

### Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^n\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*d\*(b\*d\*m - a\*c\*(n + 1)) + (c\*C - B\*d)\*(b\*c\*m + a\*d\*(n + 1)) - d\*(n + 1)\*((A - C)\*(b\*c - a\*d) + B\*(a\*c + b\*d))\*Tan[e + f\*x] - b\*(d\*(B\*c - A\*d)\*(m + n + 1) - C\*(c^2\*m - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))dx &= -\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d} + \frac{1}{3} \int \\
&= -\frac{a(2Ab+aB)\cot^2(c+dx)(a+b\tan(c+dx))}{2d} \\
&= \frac{a^2(a^2A-3Ab^2-3abB)\cot(c+dx)}{d} - \frac{a(2Ab)}{d} \\
&= (a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)x + \\
&= (a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)x +
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.72, size = 167, normalized size = 0.89

$$\frac{6a^2(a^2A-6Ab^2-4abB)\cot(c+dx)-3a^3(4Ab+aB)\cot^2(c+dx)-2a^4A\cot^3(c+dx)+3(a+ib)^4(-iA+B)\log(i-\tan(c+dx))-6a(4a^2Ab-4Ab^2+a^2B-6ab^2B)\log(\tan(c+dx))+3(a-ib)^4(iA+B)\log(i+\tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (6\*a^2\*(a^2\*A - 6\*A\*b^2 - 4\*a\*b\*B)\*Cot[c + d\*x] - 3\*a^3\*(4\*A\*b + a\*B)\*Cot[c + d\*x]^2 - 2\*a^4\*A\*Cot[c + d\*x]^3 + 3\*(a + I\*b)^4\*((-I)\*A + B)\*Log[I - Tan[c + d\*x]] - 6\*a\*(4\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B - 6\*a\*b^2\*B)\*Log[Tan[c + d\*x]] + 3\*(a - I\*b)^4\*(I\*A + B)\*Log[I + Tan[c + d\*x]])/(6\*d)

**Maple [A]**

time = 0.23, size = 197, normalized size = 1.05

method	result
derivativedivides	$A a^4 \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + B a^4 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + 4A a^3 b \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)$
default	$A a^4 \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right) + B a^4 \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + 4A a^3 b \left( -\frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right)$
norman	$\frac{(A a^4 - 6A a^2 b^2 + A b^4 - 4B a^3 b + 4B a b^3)x(\tan^3(dx+c)) + \frac{a^2(a^2A - 6A b^2 - 4B ab)(\tan^2(dx+c))}{d} - \frac{A a^4}{3d} - \frac{a^3(4Ab + aB)\tan(dx+c)}{2d}}{\tan(dx+c)^3}$
risch	$-\frac{\ln(e^{2i(dx+c)}+1)B b^4}{d} - 6A a^2 b^2 x - 4B a^3 b x + 4B a b^3 x - \frac{a^4 \ln(e^{2i(dx+c)}-1)B}{d} + 4iA a^3 b x + iE$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOS E)

[Out]  $1/d*(A*a^4*(-1/3*\cot(d*x+c)^3+\cot(d*x+c)+d*x+c)+B*a^4*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+4*A*a^3*b*(-1/2*\cot(d*x+c)^2-\ln(\sin(d*x+c)))+4*B*a^3*b*(-\cot(d*x+c)-d*x-c)+6*A*a^2*b^2*(-\cot(d*x+c)-d*x-c)+6*B*a^2*b^2*\ln(\sin(d*x+c))+4*A*a*b^3*\ln(\sin(d*x+c))+4*B*a*b^3*(d*x+c)+A*b^4*(d*x+c)-B*b^4*\ln(\cos(d*x+c))$

**Maxima [A]**

time = 0.52, size = 202, normalized size = 1.08

$$\frac{6(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)(dx + c) + 3(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3 + Bb^4) \log(\tan(dx + c)^2 + 1) - 6(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3) \log(\tan(dx + c)) - \frac{2Aa^4 - 6(Aa^4 - 4Ba^3b - 6Aa^2b^2) \tan(dx + c)^2 + 3(Ba^4 + 4Aa^3b) \tan(dx + c)}{\tan(dx + c)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/6*(6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) + 3*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*\log(\tan(d*x + c)^2 + 1) - 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*\log(\tan(d*x + c)) - (2*A*a^4 - 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*\tan(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*\tan(d*x + c))/\tan(d*x + c)^3)/d$

**Fricas [A]**

time = 0.68, size = 222, normalized size = 1.19

$$\frac{3Bb^4 \log\left(\frac{1}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 + 2Aa^4 + 3(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 + 3(Ba^4 + 4Aa^3b - 2(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)dx) \tan(dx+c)^3 - 6(Aa^4 - 4Ba^3b - 6Aa^2b^2) \tan(dx+c)^2 + 3(Ba^4 + 4Aa^3b) \tan(dx+c)}{6d \tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/6*(3*B*b^4*\log(1/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^3 + 2*A*a^4 + 3*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^3 + 3*(B*a^4 + 4*A*a^3*b - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x)*\tan(d*x + c)^3 - 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*\tan(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*\tan(d*x + c))/(d*\tan(d*x + c)^3)$

**Sympy [A]**

time = 2.97, size = 369, normalized size = 1.97

$$\frac{\begin{cases} 8Aa^4x \\ x(A+B\tan(c))(a+b\tan(c))^4\cot^4(c) \\ Aa^4x + \frac{3Bb^4 \log\left(\frac{1}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 + 2Aa^4 + 3(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 + 3(Ba^4 + 4Aa^3b - 2(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)dx) \tan(dx+c)^3 - 6(Aa^4 - 4Ba^3b - 6Aa^2b^2) \tan(dx+c)^2 + 3(Ba^4 + 4Aa^3b) \tan(dx+c)}{6d \tan(dx+c)^3} \end{cases}}{\begin{cases} \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ \text{for } d=0 \\ \text{otherwise} \end{cases}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*a**4*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(A + B*tan(c))*(a + b*tan(c))**4*cot(c)**4, Eq(d, 0)), (A*a**4*x + A`



```
*a**4/(d*tan(c + d*x)) - A*a**4/(3*d*tan(c + d*x)**3) + 2*A*a**3*b*log(tan(
c + d*x)**2 + 1)/d - 4*A*a**3*b*log(tan(c + d*x))/d - 2*A*a**3*b/(d*tan(c +
d*x)**2) - 6*A*a**2*b**2*x - 6*A*a**2*b**2/(d*tan(c + d*x)) - 2*A*a*b**3*1
og(tan(c + d*x)**2 + 1)/d + 4*A*a*b**3*log(tan(c + d*x))/d + A*b**4*x + B*a
**4*log(tan(c + d*x)**2 + 1)/(2*d) - B*a**4*log(tan(c + d*x))/d - B*a**4/(2
*d*tan(c + d*x)**2) - 4*B*a**3*b*x - 4*B*a**3*b/(d*tan(c + d*x)) - 3*B*a**2
*b**2*log(tan(c + d*x)**2 + 1)/d + 6*B*a**2*b**2*log(tan(c + d*x))/d + 4*B*
a*b**3*x + B*b**4*log(tan(c + d*x)**2 + 1)/(2*d), True))
```

**Giac** [A]

time = 1.59, size = 281, normalized size = 1.50

$$\frac{6(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Ba^2b^2 - 4Aa^2b^2) + 3(Ba^4 + 4Aa^3b - 6Aa^2b^2 - 4Aa^2b^2) \log(\tan(dx+c)^2+1) - 6(Ba^4 + 4Aa^3b - 6Aa^2b^2 - 4Aa^2b^2) \log(\tan(dx+c)) + \frac{11Ba^4 \tan(dx+c)^4 + 44Aa^3b \tan(dx+c)^3 - 66Aa^2b^2 \tan(dx+c)^2 - 44Aa^2b^2 \tan(dx+c) + 11Aa^2b^2 \tan(dx+c)^2 - 6Aa^2b^2 \tan(dx+c) - 3Ba^4 \tan(dx+c) - 12Aa^3b \tan(dx+c) - 2Aa^4}{6d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) + 3*
(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 +
1) - 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*log(abs(tan(d*x + c)))
+ (11*B*a^4*tan(d*x + c)^3 + 44*A*a^3*b*tan(d*x + c)^3 - 66*B*a^2*b^2*tan(
d*x + c)^3 - 44*A*a*b^3*tan(d*x + c)^3 + 6*A*a^4*tan(d*x + c)^2 - 24*B*a^3*
b*tan(d*x + c)^2 - 36*A*a^2*b^2*tan(d*x + c)^2 - 3*B*a^4*tan(d*x + c) - 12*
A*a^3*b*tan(d*x + c) - 2*A*a^4)/tan(d*x + c)^3)/d
```

**Mupad** [B]

time = 6.55, size = 177, normalized size = 0.95

$$\frac{\ln(\tan(c+dx)) (Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aa^2b^2) - \cot(c+dx)^3 (\tan(c+dx) (\frac{Ba^4}{2} + 2Aba^3) + \frac{4a^4}{3} + \tan(c+dx)^2 (-Aa^4 + 4Ba^3b + 6Aa^2b^2)) - \ln(\tan(c+dx) - 1) (-B + A11) (-b + a11)^4 + \ln(\tan(c+dx) + 1) (B + A11) (b + a11)^4}{2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(A*1i + B)*(a*1i + b)^4)/(2*d) - (cot(c + d*x))^3*(t
an(c + d*x)*((B*a^4)/2 + 2*A*a^3*b) + (A*a^4)/3 + tan(c + d*x)^2*(6*A*a^2*b
^2 - A*a^4 + 4*B*a^3*b))/d - (log(tan(c + d*x) - 1i)*(A*1i - B)*(a*1i - b)
^4)/(2*d) - (log(tan(c + d*x))*(B*a^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b
))/d
```

$$3.264 \quad \int \cot^5(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=225

$$(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x + \frac{a(24a^2Ab - 19Ab^3 + 6a^3B - 34ab^2B) \cot(c+dx)}{6d} + \frac{a^2(6a^2A - 19aAb + 6a^2B - 34ab^2B) \cot^2(c+dx)}{12d} + \frac{a^3(6a^2A - 19aAb + 6a^2B - 34ab^2B) \cot^3(c+dx)}{12d} + \frac{a^4(6a^2A - 19aAb + 6a^2B - 34ab^2B) \cot^4(c+dx)}{12d} + \frac{a^5(6a^2A - 19aAb + 6a^2B - 34ab^2B) \cot^5(c+dx)}{12d}$$

[Out] (4\*A\*a^3\*b-4\*A\*a\*b^3+B\*a^4-6\*B\*a^2\*b^2+B\*b^4)\*x+1/6\*a\*(24\*A\*a^2\*b-19\*A\*b^3+6\*B\*a^3-34\*B\*a\*b^2)\*cot(d\*x+c)/d+1/12\*a^2\*(6\*A\*a^2-13\*A\*b^2-16\*B\*a\*b)\*cot(d\*x+c)^2/d+(A\*a^4-6\*A\*a^2\*b^2+A\*b^4-4\*B\*a^3\*b+4\*B\*a\*b^3)\*ln(sin(d\*x+c))/d-1/12\*a\*(7\*A\*b+4\*B\*a)\*cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^2/d-1/4\*a\*A\*cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^3/d

Rubi [A]

time = 0.43, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3686, 3726, 3716, 3709, 3612, 3556}

$$\frac{a^2(6a^2A - 19aAb - 13a^2B) \cot^2(c+dx)}{12d} + \frac{a(6a^3B + 24a^2Ab - 34a^2b^2B - 19aAb^3) \cot(c+dx)}{6d} + \frac{(a^4A - 4a^3bB - 6a^2A^2 + 4a^2b^2B + Ab^4) \log(\sin(c+dx))}{d} + \frac{x(a^5B + 4a^4Ab - 6a^3b^2B - 4aAb^3 + b^4B)}{6d} - \frac{a(4aB + 7Ab) \cot^2(c+dx)(a+b \tan(c+dx))^2}{12d} - \frac{aA \cot^4(c+dx)(a+b \tan(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^5\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (4\*a^3\*A\*b - 4\*a\*A\*b^3 + a^4\*B - 6\*a^2\*b^2\*B + b^4\*B)\*x + (a\*(24\*a^2\*A\*b - 19\*A\*b^3 + 6\*a^3\*B - 34\*a\*b^2\*B)\*Cot[c + d\*x])/(6\*d) + (a^2\*(6\*a^2\*A - 13\*A\*b^2 - 16\*a\*b\*B)\*Cot[c + d\*x]^2)/(12\*d) + ((a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*Log[Sin[c + d\*x]])/d - (a\*(7\*A\*b + 4\*a\*B)\*Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x]^2)/(12\*d) - (a\*A\*Cot[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^3)/(4\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3686

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

```

### Rule 3709

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

```

### Rule 3716

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[(-b*c - a*d)*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]

```

### Rule 3726

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
 \int \cot^5(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot^4(c + dx)(a + b \tan(c + dx))^3}{4d} + \frac{1}{4} \int \dots \\
 &= -\frac{a(7Ab + 4aB) \cot^3(c + dx)(a + b \tan(c + dx))}{12d} \\
 &= \frac{a^2(6a^2A - 13Ab^2 - 16abB) \cot^2(c + dx)}{12d} - \frac{a(\dots)}{12d} \\
 &= \frac{a(24a^2Ab - 19Ab^3 + 6a^3B - 34ab^2B) \cot(c + dx)}{6d} \\
 &= (4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x + \dots \\
 &= (4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x + \dots
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
 time = 0.61, size = 211, normalized size = 0.94

$\frac{12a(4a^2Ab - 4Ab^3 + a^2B - 6ab^2B) \cot(c + dx) + 6a^2(a^2A - 6Ab^2 - 4abB) \cot^2(c + dx) - 4a^3(4Ab + aB) \cot^3(c + dx) - 3a^4A \cot^4(c + dx) - 6(a + ib)^4(A + iB) \log(i - \tan(c + dx)) + 12(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^2B) \log(\tan(c + dx)) - 6(a - ib)^4(A - iB) \log(i + \tan(c + dx))}{12d}$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^5\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (12\*a\*(4\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B - 6\*a\*b^2\*B)\*Cot[c + d\*x] + 6\*a^2\*(a^2\*A - 6\*A\*b^2 - 4\*a\*b\*B)\*Cot[c + d\*x]^2 - 4\*a^3\*(4\*A\*b + a\*B)\*Cot[c + d\*x]^3 - 3\*a^4\*A\*Cot[c + d\*x]^4 - 6\*(a + I\*b)^4\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + 12\*(a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*Log[Tan[c + d\*x]] - 6\*(a - I\*b)^4\*(A - I\*B)\*Log[I + Tan[c + d\*x]])/(12\*d)

**Maple [A]**

time = 0.23, size = 233, normalized size = 1.04

method	result
derivativedivides	$A a^4 \left( -\frac{(\cot^4(dx+c))}{4} + \frac{(\cot^2(dx+c))}{2} + \ln(\sin(dx+c)) \right) + B a^4 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx+c \right) + 4A a^3 b \left( -\frac{(\cot^3(dx+c))}{3} \right)$
default	$A a^4 \left( -\frac{(\cot^4(dx+c))}{4} + \frac{(\cot^2(dx+c))}{2} + \ln(\sin(dx+c)) \right) + B a^4 \left( -\frac{(\cot^3(dx+c))}{3} + \cot(dx+c) + dx+c \right) + 4A a^3 b \left( -\frac{(\cot^3(dx+c))}{3} \right)$
norman	$\frac{(4A a^3 b - 4A a b^3 + B a^4 - 6B a^2 b^2 + B b^4) x (\tan^4(dx+c)) + \frac{a(4A a^2 b - 4A b^3 + B a^3 - 6B a b^2) (\tan^3(dx+c))}{d} - \frac{A a^4}{4d} + \frac{a^2(a^2 A - 6A a b^2 + A b^4)}{4d}}{\tan(dx+c)^4}$

risch	$\frac{\ln(e^{2i(dx+c)}-1)Ab^4}{d} + \frac{Aa^4 \ln(e^{2i(dx+c)}-1)}{d} + \frac{12iAa^2b^2c}{d} - \frac{2ia^4Ac}{d} - 4iBab^3x + 4Aa^3bx - 4Aab^3$
-------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS E)`

[Out] 
$$\frac{1}{d} * (A * a^4 * (-1/4 * \cot(d*x+c)^4 + 1/2 * \cot(d*x+c)^2 + \ln(\sin(d*x+c))) + B * a^4 * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + 4 * A * a^3 * b * (-1/3 * \cot(d*x+c)^3 + \cot(d*x+c) + d*x+c) + 4 * B * a^3 * b * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))) + 6 * A * a^2 * b^2 * (-1/2 * \cot(d*x+c)^2 - \ln(\sin(d*x+c))) + 6 * B * a^2 * b^2 * (-\cot(d*x+c) - d*x-c) + 4 * A * a * b^3 * (-\cot(d*x+c) - d*x-c) + 4 * B * a * b^3 * \ln(\sin(d*x+c)) + A * b^4 * \ln(\sin(d*x+c)) + B * b^4 * (d*x+c))$$

**Maxima** [A]

time = 0.57, size = 246, normalized size = 1.09

$$\frac{12(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3 + Bb^4)(dx+c) - 6(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log(\tan(dx+c)^2 + 1) + 12(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log(\tan(dx+c)) - \frac{3Aa^4 - 12(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3 + Bb^4) \tan(dx+c)^2 - 6(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \tan(dx+c)^2 + 4(Ba^4 + 4Aa^3b) \tan(dx+c)}{\tan(dx+c)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{12} * (12 * (B * a^4 + 4 * A * a^3 * b - 6 * B * a^2 * b^2 - 4 * A * a * b^3 + B * b^4) * (d * x + c) - 6 * (A * a^4 - 4 * B * a^3 * b - 6 * A * a^2 * b^2 + 4 * B * a * b^3 + A * b^4) * \log(\tan(d * x + c)^2 + 1) + 12 * (A * a^4 - 4 * B * a^3 * b - 6 * A * a^2 * b^2 + 4 * B * a * b^3 + A * b^4) * \log(\tan(d * x + c)) - (3 * A * a^4 - 12 * (B * a^4 + 4 * A * a^3 * b - 6 * B * a^2 * b^2 - 4 * A * a * b^3) * \tan(d * x + c)^3 - 6 * (A * a^4 - 4 * B * a^3 * b - 6 * A * a^2 * b^2) * \tan(d * x + c)^2 + 4 * (B * a^4 + 4 * A * a^3 * b) * \tan(d * x + c)) / \tan(d * x + c)^4) / d$$

**Fricas** [A]

time = 0.78, size = 249, normalized size = 1.11

$$\frac{6(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^3 - 3Aa^4 + 3(3Aa^4 - 8Ba^3b - 12Aa^2b^2 + 4(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3 + Bb^4) \tan(dx+c)^2 + 12(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3) \tan(dx+c)^2 + 6(Aa^4 - 4Ba^3b - 6Aa^2b^2) \tan(dx+c)^2 - 4(Ba^4 + 4Aa^3b) \tan(dx+c))}{12d \tan(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{12} * (6 * (A * a^4 - 4 * B * a^3 * b - 6 * A * a^2 * b^2 + 4 * B * a * b^3 + A * b^4) * \log(\tan(d * x + c)^2 / (\tan(d * x + c)^2 + 1)) * \tan(d * x + c)^4 - 3 * A * a^4 + 3 * (3 * A * a^4 - 8 * B * a^3 * b - 12 * A * a^2 * b^2 + 4 * (B * a^4 + 4 * A * a^3 * b - 6 * B * a^2 * b^2 - 4 * A * a * b^3 + B * b^4) * d * x) * \tan(d * x + c)^4 + 12 * (B * a^4 + 4 * A * a^3 * b - 6 * B * a^2 * b^2 - 4 * A * a * b^3) * \tan(d * x + c)^3 + 6 * (A * a^4 - 4 * B * a^3 * b - 6 * A * a^2 * b^2) * \tan(d * x + c)^2 - 4 * (B * a^4 + 4 * A * a^3 * b) * \tan(d * x + c)) / (d * \tan(d * x + c)^4)$$



$$\begin{aligned} &)^2 + 96*B*a^3*b*\tan(1/2*d*x + 1/2*c)^2 + 144*A*a^2*b^2*\tan(1/2*d*x + 1/2*c) \\ &)^2 + 8*B*a^4*\tan(1/2*d*x + 1/2*c) + 32*A*a^3*b*\tan(1/2*d*x + 1/2*c) + 3*A* \\ &a^4)/\tan(1/2*d*x + 1/2*c)^4)/d \end{aligned}$$

**Mupad [B]**

time = 6.47, size = 218, normalized size = 0.97

$$\frac{\ln(\tan(c+dx)) (Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Ba^2b^2 + Ab^4)}{d} - \frac{\cot(c+dx) (\tan(c+dx) (\frac{Aa^4}{3} + \frac{4Aa^3b}{3}) + \frac{Aa^4}{4} - \tan(c+dx) (Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aa^2b^2) + \tan(c+dx) (\frac{-4a^4}{3} + 2Ba^3b + 3Aa^2b^2))}{d} - \frac{\ln(\tan(c+dx) + 1) (A - B1) (b + a1)^4}{2d} - \frac{\ln(\tan(c+dx) - 1) (A + B1) (-b + a1)^4}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^5\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^4,x)

[Out] (log(tan(c + d\*x))\*(A\*a^4 + A\*b^4 - 6\*A\*a^2\*b^2 + 4\*B\*a\*b^3 - 4\*B\*a^3\*b))/d - (cot(c + d\*x)^4\*(tan(c + d\*x)\*((B\*a^4)/3 + (4\*A\*a^3\*b)/3) + (A\*a^4)/4 - tan(c + d\*x)^3\*(B\*a^4 - 6\*B\*a^2\*b^2 - 4\*A\*a\*b^3 + 4\*A\*a^3\*b) + tan(c + d\*x)^2\*(3\*A\*a^2\*b^2 - (A\*a^4)/2 + 2\*B\*a^3\*b))/d - (log(tan(c + d\*x) + 1i)\*(A - B\*1i)\*(a\*1i + b)^4)/(2\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i)\*(a\*1i - b)^4)/(2\*d)

### 3.265 $\int \cot^6(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

Optimal. Leaf size=273

$$-\left((a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B)x\right) - \frac{(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) \cot(c+dx)}{d} + \frac{a(40$$

[Out]  $-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*\cot(d*x+c)/d+1/20*a*(40*A*a^2*b-28*A*b^3+10*B*a^3-55*B*a*b^2)*\cot(d*x+c)^2/d+1/30*a^2*(10*A*a^2-18*A*b^2-25*B*a*b)*\cot(d*x+c)^3/d+(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*\ln(\sin(d*x+c))/d-1/20*a*(8*A*b+5*B*a)*\cot(d*x+c)^4*(a+b*\tan(d*x+c))^2/d-1/5*a*A*\cot(d*x+c)^5*(a+b*\tan(d*x+c))^3/d$

Rubi [A]

time = 0.49, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3686, 3726, 3716, 3709, 3610, 3612, 3556}

$$\frac{a^2(10a^2A - 25abB - 18Ab^2)\cot^2(c+dx)}{30d} + \frac{a(10a^2B + 40a^2Ab - 55a^2B^2 - 28Ab^3)\cot^2(c+dx)}{20d} - \frac{(a^4A - 4a^2bB - 6a^2Ab^2 + 4ab^3B + Ab^4)\cot(c+dx)}{d} + \frac{(a^4B + 4a^2Ab - 6a^2B^2 - 4aAb^2 + b^4B)\log(\sin(c+dx))}{d} - \frac{a^2(a^4A - 4a^2bB - 6a^2Ab^2 + 4ab^3B + Ab^4)}{20d} - \frac{a(5abB + 8Ab^2)\cot^2(c+dx)(a+b\tan(c+dx))^2}{20d} - \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^6\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left((a^4A - 6a^2A*b^2 + A*b^4 - 4a^3*b*B + 4a*b^3*B)*x\right) - \left((a^4A - 6a^2A*b^2 + A*b^4 - 4a^3*b*B + 4a*b^3*B)*\cot[c + d*x]\right)/d + (a*(40a^2A*b - 28A*b^3 + 10a^3*B - 55a*b^2*B)*\cot[c + d*x]^2)/(20*d) + (a^2*(10a^2A - 18A*b^2 - 25a*b*B)*\cot[c + d*x]^3)/(30*d) + \left((4a^3A*b - 4aA*b^3 + a^4*B - 6a^2*b^2*B + b^4*B)*\log[\sin[c + d*x]]\right)/d - (a*(8A*b + 5a*B)*\cot[c + d*x]^4*(a + b*\tan[c + d*x])^2)/(20*d) - (aA*\cot[c + d*x]^5*(a + b*\tan[c + d*x])^3)/(5*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]



Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/(a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3709

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[(A\*b^2 - a\*b\*B + a^2\*C)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

Rule 3716

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[(-b\*c - a\*d)\*(c^2\*C - B\*c\*d + A\*d^2)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d^2\*f\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*d\*(A\*c - c\*C + B\*d) + b\*(c^2\*C - B\*c\*d + A\*d^2) + d\*(A\*b\*c + a\*B\*c - b\*c\*C - a\*A\*d + b\*B\*d + a\*C\*d)\*Tan[e + f\*x] + b\*C\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

Rule 3726

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.)

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \cot^6(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot^5(c + dx)(a + b \tan(c + dx))^3}{5d} + \frac{1}{5} \int \cot^5(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= -\frac{a(8Ab + 5aB) \cot^4(c + dx)(a + b \tan(c + dx))^3}{20d} + \frac{1}{5} \int \cot^4(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= \frac{a^2(10a^2A - 18Ab^2 - 25abB) \cot^3(c + dx)}{30d} - \frac{a(4a^2Ab - 28Ab^3 + 10a^3B - 55ab^2B) \cot^2(c + dx)}{20d} + \frac{1}{5} \int \cot^3(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= -\frac{(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) \cot(c + dx)}{d} + \frac{1}{5} \int \cot^2(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= -(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) x + \frac{1}{5} \int \cot(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= -(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) x - \frac{1}{5} \int \cot(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 0.99, size = 257, normalized size = 0.94

$$\frac{-60(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4ab^3B) \cot(c + dx) + 30a(4a^2Ab - 4a^3B + a^2B - 6ab^2B) \cot^2(c + dx) + 20a^2(4a^2A - 6a^2Ab^2 - 4ab^2B) \cot^3(c + dx) - 15a^3(4a^2A + a^2B) \cot^4(c + dx) - 12a^4A \cot^5(c + dx) + 30(a + b)^2(A + B) \log(-\tan(c + dx)) + 60(4a^3Ab - 4a^2B^2 - 6a^2B + b^2B) \log(\tan(c + dx)) - 30(a - b)^2(A + B) \log(1 + \tan(c + dx))}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^6\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out] (-60\*(a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*Cot[c + d\*x] + 30\*a\*(4\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B - 6\*a\*b^2\*B)\*Cot[c + d\*x]^2 + 20\*a^2\*(a^2\*A - 6\*A\*b^2 - 4\*a\*b\*B)\*Cot[c + d\*x]^3 - 15\*a^3\*(4\*A\*b + a\*B)\*Cot[c + d\*x]^4 - 12\*a^4\*A\*Cot[c + d\*x]^5 + (30\*I)\*(a + I\*b)^4\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + 60\*(4\*a^3\*A\*b - 4\*a\*A\*b^3 + a^4\*B - 6\*a^2\*b^2\*B + b^4\*B)\*Log[Tan[c + d\*x]] - 30\*(a - I\*b)^4\*(I\*A + B)\*Log[I + Tan[c + d\*x]])/(60\*d)

**Maple [A]**

time = 0.26, size = 285, normalized size = 1.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+B*a^4*(-1/
4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+4*A*a^3*b*(-1/4*cot(d*x+c)^
4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+4*B*a^3*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+
d*x+c)+6*A*a^2*b^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+6*B*a^2*b^2*(-1/2*c
ot(d*x+c)^2-ln(sin(d*x+c)))+4*A*a*b^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+4*
B*a*b^3*(-cot(d*x+c)-d*x-c)+A*b^4*(-cot(d*x+c)-d*x-c)+B*b^4*ln(sin(d*x+c)))
```

**Maxima [A]**

time = 0.51, size = 289, normalized size = 1.06

$$\frac{60(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Aab^3 + A^2b^4) \log(\tan(dx+c)) + 30(Ba^4 + 4Aa^3b - 6Aa^2b^2 - 4Aab^3 + B^2b^4) \log(\tan(dx+c)^2 + 1) - 60(Ba^4 + 4Aa^3b - 6Aa^2b^2 - 4Aab^3 + B^2b^4) \log(\tan(dx+c)) + 12Aa^4 - 60Aa^3b - 6Aa^2b^2 + 4Aab^3 + A^2b^4 \tan(dx+c)^5 - 20(Aa^4 - 4Aa^3b - 6Aa^2b^2 + 4Aab^3) \tan(dx+c)^4 + 20(Aa^4 - 4Aa^3b - 6Aa^2b^2 + 4Aab^3) \tan(dx+c)^3 - 15(Ba^4 + 4Aa^3b) \tan(dx+c)^2 + 15(Ba^4 + 4Aa^3b) \tan(dx+c)}{60d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/60*(60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) +
30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^
2 + 1) - 60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d
*x + c)) + (12*A*a^4 + 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*
b^4)*tan(d*x + c)^4 - 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*tan(
d*x + c)^3 - 20*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 + 15*(B*a^
4 + 4*A*a^3*b)*tan(d*x + c))/tan(d*x + c)^5)/d
```

**Fricas [A]**

time = 0.68, size = 300, normalized size = 1.10

$$\frac{30(Ba^4 + 4Aa^3b - 6Aa^2b^2 - 4Aab^3 + A^2b^4) \log\left(\frac{\tan(dx+c)^2 + 1}{\tan(dx+c)}\right) \tan(dx+c)^5 + 15(3Ba^4 + 12Aa^3b - 12Aa^2b^2 - 8Aab^3 - 4(Aa^4 - 4Aa^3b - 6Aa^2b^2 + 4Aab^3 + A^2b^4) \tan(dx+c)) \tan(dx+c)^5 - 12Aa^4 - 60Aa^3b - 6Aa^2b^2 + 4Aab^3 + A^2b^4 \tan(dx+c)^5 + 30(Ba^4 + 4Aa^3b - 6Aa^2b^2 - 4Aab^3) \tan(dx+c)^4 + 20(Aa^4 - 4Aa^3b - 6Aa^2b^2 + 4Aab^3) \tan(dx+c)^3 - 15(Ba^4 + 4Aa^3b) \tan(dx+c)}{60d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/60*(30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x
+ c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^5 + 15*(3*B*a^4 + 12*A*a^3*b - 12
*B*a^2*b^2 - 8*A*a*b^3 - 4*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A
*b^4)*d*x)*tan(d*x + c)^5 - 12*A*a^4 - 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2
+ 4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 -
```



```

*b^3 + B*b^4)*log(abs(tan(1/2*d*x + 1/2*c))) - (2192*B*a^4*tan(1/2*d*x + 1/
2*c)^5 + 8768*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 13152*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 - 8768*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 2192*B*b^4*tan(1/2*d*x +
1/2*c)^5 + 660*A*a^4*tan(1/2*d*x + 1/2*c)^4 - 2400*B*a^3*b*tan(1/2*d*x + 1
/2*c)^4 - 3600*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1920*B*a*b^3*tan(1/2*d*x
+ 1/2*c)^4 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^4 - 180*B*a^4*tan(1/2*d*x + 1/2
*c)^3 - 720*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 720*B*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 70*A*a^4*tan(1/2*d*x + 1/2*c)
^2 + 160*B*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 240*A*a^2*b^2*tan(1/2*d*x + 1/2*c
)^2 + 15*B*a^4*tan(1/2*d*x + 1/2*c) + 60*A*a^3*b*tan(1/2*d*x + 1/2*c) + 6*A
*a^4)/tan(1/2*d*x + 1/2*c)^5)/d

```

**Mupad [B]**

time = 6.68, size = 263, normalized size = 0.96

$$\frac{\ln(\tan(c+dx)) (B a^4 + 4 A a^3 b - 6 B a^2 b^2 - 4 A a b^3 + B b^4) - \cot(c+dx)^2 (\tan(c+dx) (\frac{A^2}{4} + A b^2) + 4 b^2 - \tan(c+dx)^2 (\frac{A^2}{4} + 2 A a^2 b - 3 B a^2 b^2 - 2 A a b^3) + \tan(c+dx)^2 (-\frac{A^2}{4} + \frac{4 A a^2 b}{4} + 2 A a^2 b^2) + \tan(c+dx)^4 (A a^4 - 4 B a^3 b - 6 A a^2 b^2 + 4 B a b^3 + A b^4))}{d} + \frac{\ln(\tan(c+dx) - 1) (-B + A b) (-b + a b)}{2d} - \frac{\ln(\tan(c+dx) + 1) (B + A b) (b + a b)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(tan(c + d*x))*(B*a^4 + B*b^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b))/d
- (cot(c + d*x)^5*(tan(c + d*x)*((B*a^4)/4 + A*a^3*b) + (A*a^4)/5 - tan(c
+ d*x)^3*((B*a^4)/2 - 3*B*a^2*b^2 - 2*A*a*b^3 + 2*A*a^3*b) + tan(c + d*x)^2
*(2*A*a^2*b^2 - (A*a^4)/3 + (4*B*a^3*b)/3) + tan(c + d*x)^4*(A*a^4 + A*b^4
- 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b))/d + (log(tan(c + d*x) - 1i)*(A*1i
- B)*(a*1i - b)^4)/(2*d) - (log(tan(c + d*x) + 1i)*(A*1i + B)*(a*1i + b)^4)
/(2*d)
```

### 3.266 $\int \cot^7(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=323

$$-\left((4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B)x\right) - \frac{(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) \cot(c+dx)}{d} - \frac{(a^4A - 4a^3Ab + 4a^2Ab^2 - 4aAb^3 + b^4B) \cot^2(c+dx)}{d} - \frac{(a^4A - 4a^3Ab + 4a^2Ab^2 - 4aAb^3 + b^4B) \cot^3(c+dx)}{d} - \frac{(a^4A - 4a^3Ab + 4a^2Ab^2 - 4aAb^3 + b^4B) \cot^4(c+dx)}{d} - \frac{(a^4A - 4a^3Ab + 4a^2Ab^2 - 4aAb^3 + b^4B) \cot^5(c+dx)}{d} - \frac{(a^4A - 4a^3Ab + 4a^2Ab^2 - 4aAb^3 + b^4B) \cot^6(c+dx)}{d} - \frac{(a^4A - 4a^3Ab + 4a^2Ab^2 - 4aAb^3 + b^4B) \cot^7(c+dx)}{d}$$

[Out]  $-(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*x-(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*\cot(d*x+c)/d-1/2*(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*\cot(d*x+c)^2/d+1/15*a*(20*A*a^2*b-13*A*b^3+5*B*a^3-27*B*a*b^2)*\cot(d*x+c)^3/d+1/20*a^2*(5*A*a^2-8*A*b^2-12*B*a*b)*\cot(d*x+c)^4/d-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*\ln(\sin(d*x+c))/d-1/10*a*(3*A*b+2*B*a)*\cot(d*x+c)^5*(a+b*\tan(d*x+c))^2/d-1/6*a*A*\cot(d*x+c)^6*(a+b*\tan(d*x+c))^3/d$

**Rubi [A]**

time = 0.58, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3686, 3726, 3716, 3709, 3610, 3612, 3556}

$$\frac{d^2(c^2A - 12aB - 8a^2B^2 \cot^2(c+dx))}{2d} - \frac{d(5a^2B + 2a^2Ab - 27a^2B^2 - 13a^2B^2 \cot^2(c+dx))}{10d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{2d} - \frac{(a^2B + 4a^2Ab - 6a^2B^2 - 4a^2B^2 \cot^2(c+dx))}{d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{d} - \frac{(c^2A - 4a^2B - 6a^2Ab^2 + 4a^2B^2 + a^2B^2 \cot^2(c+dx))}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^7*(a + b*\text{Tan}[c + d*x])^4*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left(\left(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B\right)x\right) - \left(\left(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B\right)\text{Cot}[c + d*x]\right)/d - \left(\left(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4a^2b^3B\right)\text{Cot}[c + d*x]^2\right)/(2*d) + \left(a*(20a^2Ab - 13Ab^3 + 5a^3B - 27a^2b^2B)\text{Cot}[c + d*x]^3\right)/(15*d) + \left(a^2*(5a^2A - 8Ab^2 - 12a^2bB)\text{Cot}[c + d*x]^4\right)/(20*d) - \left(\left(a^4A - 6a^2Ab^2 + Ab^4 - 4a^3bB + 4a^2b^3B\right)\text{Log}[\text{Sin}[c + d*x]]\right)/d - \left(a*(3Ab + 2aB)\text{Cot}[c + d*x]^5*(a + b*\text{Tan}[c + d*x])^2\right)/(10*d) - \left(a*A*\text{Cot}[c + d*x]^6*(a + b*\text{Tan}[c + d*x])^3\right)/(6*d)$

**Rule 3556**

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] := \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

**Rule 3610**

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)}*\left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] := \text{Simp}[(b*c - a*d)*\left((a + b*\text{Tan}[e + f*x])^{(m + 1)} / (f*(m + 1)*(a^2 + b^2)\right), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a,$

b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

### Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3709

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] :> Simp[(A\*b^2 - a\*b\*B + a^2\*C)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rule 3716

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] :> Simp[(-b\*c - a\*d)\*(c^2\*C - B\*c\*d + A\*d^2)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d^2\*f\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*d\*(A\*c - c\*C + B\*d) + b\*(c^2\*C - B\*c\*d + A\*d^2) + d\*(A\*b\*c + a\*B\*c - b\*c\*C - a\*A\*d + b\*B\*d + a\*C\*d)\*Tan[e + f\*x] + b\*C\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

### Rule 3726

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \cot^7(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= -\frac{aA \cot^6(c + dx)(a + b \tan(c + dx))^3}{6d} + \frac{1}{6} \int \cot^6(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= -\frac{a(3Ab + 2aB) \cot^5(c + dx)(a + b \tan(c + dx))^4}{10d} + \frac{1}{6} \int \cot^5(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx \\
&= \frac{a^2(5a^2A - 8Ab^2 - 12abB) \cot^4(c + dx)(a + b \tan(c + dx))^4}{20d} - \frac{a(3Ab + 2aB) \cot^3(c + dx)(a + b \tan(c + dx))^4}{15d} \\
&= \frac{a(20a^2Ab - 13Ab^3 + 5a^3B - 27ab^2B) \cot^3(c + dx)(a + b \tan(c + dx))^4}{15d} - \frac{a^2(5a^2A - 8Ab^2 - 12abB) \cot^2(c + dx)(a + b \tan(c + dx))^4}{2d} \\
&= -\frac{(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) \cot^2(c + dx)(a + b \tan(c + dx))^4}{d} \\
&= -(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x \cot^2(c + dx)(a + b \tan(c + dx))^4 \\
&= -(4a^3Ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) x \cot^2(c + dx)(a + b \tan(c + dx))^4
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 0.83, size = 299, normalized size = 0.93

$-\frac{60(a^4b - 6a^3b^2 + a^2b^3 - 6a^2b^2B + 6a^2b^3B) \cot^2(c + dx) - 30(a^4A - 6a^3Ab^2 + 6a^3B - 6a^2b^2B + 6a^2b^3B) \cot^2(c + dx) + 20a^4Ab - 4a^4B + 6a^3B - 6a^2b^2B + 6a^2b^3B}{10d} - \frac{12a^2(4Ab + aB) \cot^2(c + dx) - 12a^2A \cot^2(c + dx) + 30(a + b)(A + B) \log(-\tan(c + dx)) - 60(a^4A - 6a^3Ab^2 + 6a^3B - 6a^2b^2B + 6a^2b^3B) \log(\tan(c + dx)) + 30(a - b)(A - B) \log(1 + \tan(c + dx))}{10d}$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^7\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]), x]

[Out] (-60\*(4\*a^3\*A\*b - 4\*a\*A\*b^3 + a^4\*B - 6\*a^2\*b^2\*B + b^4\*B)\*Cot[c + d\*x] - 30\*(a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*Cot[c + d\*x]^2 + 20\*a\*(4\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B - 6\*a\*b^2\*B)\*Cot[c + d\*x]^3 + 15\*a^2\*(a^2\*A



$$\begin{aligned}
& - 6A^2b - 4a^2b^2 + 4a^2b^2B) \cot[c + dx]^4 - 12a^3(4Ab + a^2B) \cot[c + dx]^5 \\
& - 10a^4A \cot[c + dx]^6 + 30(a + Ib)^4(A + IB) \log[I - \tan[c + dx]] \\
& - 60(a^4A - 6a^2A^2b^2 + A^2b^4 - 4a^3b^2B + 4a^2b^3B) \log[\tan[c + dx]] \\
& ] + 30(a - Ib)^4(A - IB) \log[I + \tan[c + dx]] / (60d)
\end{aligned}$$

**Maple [A]**

time = 0.29, size = 338, normalized size = 1.05

method	result
derivativedivides	$A a^4 \left( -\frac{\cot^6(dx+c)}{6} + \frac{\cot^4(dx+c)}{4} - \frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + B a^4 \left( -\frac{\cot^5(dx+c)}{5} + \frac{\cot^3(dx+c)}{3} - \cot(dx+c) \right)$
default	$A a^4 \left( -\frac{\cot^6(dx+c)}{6} + \frac{\cot^4(dx+c)}{4} - \frac{\cot^2(dx+c)}{2} - \ln(\sin(dx+c)) \right) + B a^4 \left( -\frac{\cot^5(dx+c)}{5} + \frac{\cot^3(dx+c)}{3} - \cot(dx+c) \right)$
norman	$(-4A a^3b + 4A a b^3 - B a^4 + 6B a^2b^2 - B b^4) x (\tan^6(dx+c)) - \frac{A a^4}{6d} - \frac{(4A a^3b - 4A a b^3 + B a^4 - 6B a^2b^2 + B b^4) (\tan^5(dx+c))}{d} - \dots$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} (A a^4 (-\frac{1}{6} \cot(dx+c)^6 + \frac{1}{4} \cot(dx+c)^4 - \frac{1}{2} \cot(dx+c)^2 - \ln(\sin(dx+c))) + B a^4 (-\frac{1}{5} \cot(dx+c)^5 + \frac{1}{3} \cot(dx+c)^3 - \cot(dx+c) - dx - c) + 4A a^3 b (-\frac{1}{5} \cot(dx+c)^5 + \frac{1}{3} \cot(dx+c)^3 - \cot(dx+c) - dx - c) + 4B a^3 b (-\frac{1}{4} \cot(dx+c)^4 + \frac{1}{2} \cot(dx+c)^2 + \ln(\sin(dx+c))) + 6A a^2 b^2 (-\frac{1}{4} \cot(dx+c)^4 + \frac{1}{2} \cot(dx+c)^2 + \ln(\sin(dx+c))) + 6B a^2 b^2 (-\frac{1}{3} \cot(dx+c)^3 + \cot(dx+c) + dx + c) + 4A a b^3 (-\frac{1}{3} \cot(dx+c)^3 + \cot(dx+c) + dx + c) + 4B a b^3 (-\frac{1}{2} \cot(dx+c)^2 - \ln(\sin(dx+c))) + A b^4 (-\frac{1}{2} \cot(dx+c)^2 - \ln(\sin(dx+c))) + B b^4 (-\cot(dx+c) - dx - c))$

**Maxima [A]**

time = 0.51, size = 333, normalized size = 1.03

$$\frac{60(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aa^2b^3 + Bb^4)(dx+c) - 30(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log(\tan(dx+c)^2 + 1) + 60(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log(\tan(dx+c)) + \dots}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-\frac{1}{60} (60(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aa^2b^3 + Bb^4)(dx+c) - 30(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log(\tan(dx+c)^2 + 1) + 60(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \log(\tan(dx+c)) + (60(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aa^2b^3 + Bb^4) \tan(dx+c)^5 + 10Aa^4 + 30(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4) \dots)$

$$\begin{aligned} &^4) \cdot \tan(dx + c)^4 - 20 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b - 6 \cdot B \cdot a^2 \cdot b^2 - 4 \cdot A \cdot a \cdot b^3) \cdot \tan(dx \\ & \cdot x + c)^3 - 15 \cdot (A \cdot a^4 - 4 \cdot B \cdot a^3 \cdot b - 6 \cdot A \cdot a^2 \cdot b^2) \cdot \tan(dx + c)^2 + 12 \cdot (B \cdot a^4 \\ & + 4 \cdot A \cdot a^3 \cdot b) \cdot \tan(dx + c) / \tan(dx + c)^6 / d \end{aligned}$$

**Fricas** [A]

time = 0.92, size = 350, normalized size = 1.08

$\frac{30(A^4 - 4B^3 - 6A^2b + 4Bb^2 - 6A^2b^2) \tan(dx + c)^5 + 11(A^4 - 30B^3 - 54A^2b + 24Bb^2 - 6A^2b^2 - 4A^2b^2) \tan(dx + c)^4 + 60(B^4 + 4A^3b - 6B^3 - 4A^2b^2) \tan(dx + c)^3 + 30(A^4 - 4B^3 - 6A^2b + 4Bb^2 - 6A^2b^2) \tan(dx + c)^2 - 15(A^4 - 4B^3 - 6A^2b + 4Bb^2 - 6A^2b^2) \tan(dx + c) + 12(B^4 + 4A^3b) \tan(dx + c)}{60 \tan(dx + c)^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^7\*(a+b\*tan(dx+c))^4\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out]  $-1/60 \cdot (30 \cdot (A \cdot a^4 - 4 \cdot B \cdot a^3 \cdot b - 6 \cdot A \cdot a^2 \cdot b^2 + 4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \log(\tan(dx + c)^2 / (\tan(dx + c)^2 + 1)) \cdot \tan(dx + c)^6 + 5 \cdot (11 \cdot A \cdot a^4 - 36 \cdot B \cdot a^3 \cdot b - 5 \cdot 4 \cdot A \cdot a^2 \cdot b^2 + 24 \cdot B \cdot a \cdot b^3 + 6 \cdot A \cdot b^4 + 12 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b - 6 \cdot B \cdot a^2 \cdot b^2 - 4 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot dx) \cdot \tan(dx + c)^6 + 60 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b - 6 \cdot B \cdot a^2 \cdot b^2 - 4 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot \tan(dx + c)^5 + 10 \cdot A \cdot a^4 + 30 \cdot (A \cdot a^4 - 4 \cdot B \cdot a^3 \cdot b - 6 \cdot A \cdot a^2 \cdot b^2 + 4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \tan(dx + c)^4 - 20 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b - 6 \cdot B \cdot a^2 \cdot b^2 - 4 \cdot A \cdot a \cdot b^3) \cdot \tan(dx + c)^3 - 15 \cdot (A \cdot a^4 - 4 \cdot B \cdot a^3 \cdot b - 6 \cdot A \cdot a^2 \cdot b^2) \cdot \tan(dx + c)^2 + 12 \cdot (B \cdot a^4 + 4 \cdot A \cdot a^3 \cdot b) \cdot \tan(dx + c) / (d \cdot \tan(dx + c)^6)$

**Sympy** [A]

time = 10.00, size = 643, normalized size = 1.99

$\frac{30(A^4 - 4B^3 - 6A^2b + 4Bb^2 - 6A^2b^2) \tan(dx + c)^5 + 11(A^4 - 30B^3 - 54A^2b + 24Bb^2 - 6A^2b^2 - 4A^2b^2) \tan(dx + c)^4 + 60(B^4 + 4A^3b - 6B^3 - 4A^2b^2) \tan(dx + c)^3 + 30(A^4 - 4B^3 - 6A^2b + 4Bb^2 - 6A^2b^2) \tan(dx + c)^2 - 15(A^4 - 4B^3 - 6A^2b + 4Bb^2 - 6A^2b^2) \tan(dx + c) + 12(B^4 + 4A^3b) \tan(dx + c)}{60 \tan(dx + c)^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*7\*(a+b\*tan(dx+c))\*\*4\*(A+B\*tan(dx+c)),x)

[Out] Piecewise((zoo\*A\*\*4\*x, (Eq(c, 0) | Eq(c, -d\*x)) & (Eq(d, 0) | Eq(c, -d\*x))), (x\*(A + B\*tan(c))\*(a + b\*tan(c))\*\*4\*cot(c)\*\*7, Eq(d, 0)), (A\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*\*4\*log(tan(c + d\*x))/d - A\*\*4/(2\*d\*tan(c + d\*x)\*\*2) + A\*\*4/(4\*d\*tan(c + d\*x)\*\*4) - A\*\*4/(6\*d\*tan(c + d\*x)\*\*6) - 4\*A\*\*3\*b\*x - 4\*A\*\*3\*b/(d\*tan(c + d\*x)) + 4\*A\*\*3\*b/(3\*d\*tan(c + d\*x)\*\*3) - 4\*A\*\*3\*b/(5\*d\*tan(c + d\*x)\*\*5) - 3\*A\*\*2\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/d + 6\*A\*\*2\*b\*\*2\*log(tan(c + d\*x))/d + 3\*A\*\*2\*b\*\*2/(d\*tan(c + d\*x)\*\*2) - 3\*A\*\*2\*b\*\*2/(2\*d\*tan(c + d\*x)\*\*4) + 4\*A\*\*2\*b\*\*3\*x + 4\*A\*\*2\*b\*\*3/(d\*tan(c + d\*x)) - 4\*A\*\*2\*b\*\*3/(3\*d\*tan(c + d\*x)\*\*3) + A\*\*2\*b\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*\*2\*b\*\*4\*log(tan(c + d\*x))/d - A\*\*2\*b\*\*4/(2\*d\*tan(c + d\*x)\*\*2) - B\*\*4\*x - B\*\*4/(d\*tan(c + d\*x)) + B\*\*4/(3\*d\*tan(c + d\*x)\*\*3) - B\*\*4/(5\*d\*tan(c + d\*x)\*\*5) - 2\*B\*\*3\*b\*log(tan(c + d\*x)\*\*2 + 1)/d + 4\*B\*\*3\*b\*log(tan(c + d\*x))/d + 2\*B\*\*3\*b/(d\*tan(c + d\*x)\*\*2) - B\*\*3\*b/(d\*tan(c + d\*x)\*\*4) + 6\*B\*\*2\*b\*\*2\*x + 6\*B\*\*2\*b\*\*2/(d\*tan(c + d\*x)) - 2\*B\*\*2\*b\*\*2/(d\*tan(c + d\*x)\*\*3) + 2\*B\*\*2\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/d - 4\*B\*\*2\*b\*\*3\*log

$(\tan(c + d*x))/d - 2*B*a*b**3/(d*\tan(c + d*x)**2) - B*b**4*x - B*b**4/(d*\tan(c + d*x)), True))$

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 943 vs. 2(313) = 626.

time = 1.89, size = 943, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] 
$$-1/1920*(5*A*a^4*\tan(1/2*d*x + 1/2*c)^6 - 12*B*a^4*\tan(1/2*d*x + 1/2*c)^5 - 48*A*a^3*b*\tan(1/2*d*x + 1/2*c)^5 - 60*A*a^4*\tan(1/2*d*x + 1/2*c)^4 + 120*B*a^3*b*\tan(1/2*d*x + 1/2*c)^4 + 180*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^4 + 140*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 560*A*a^3*b*\tan(1/2*d*x + 1/2*c)^3 - 480*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 - 320*A*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 435*A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 1440*B*a^3*b*\tan(1/2*d*x + 1/2*c)^2 - 2160*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^2 + 960*B*a*b^3*\tan(1/2*d*x + 1/2*c)^2 + 240*A*b^4*\tan(1/2*d*x + 1/2*c)^2 - 1320*B*a^4*\tan(1/2*d*x + 1/2*c) - 5280*A*a^3*b*\tan(1/2*d*x + 1/2*c) + 7200*B*a^2*b^2*\tan(1/2*d*x + 1/2*c) + 4800*A*a*b^3*\tan(1/2*d*x + 1/2*c) - 960*B*b^4*\tan(1/2*d*x + 1/2*c) + 1920*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c) - 1920*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*\log(\tan(1/2*d*x + 1/2*c)^2 + 1) + 1920*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*\log(\tan(1/2*d*x + 1/2*c)) - (4704*A*a^4*\tan(1/2*d*x + 1/2*c)^6 - 18816*B*a^3*b*\tan(1/2*d*x + 1/2*c)^6 - 28224*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^6 + 18816*B*a*b^3*\tan(1/2*d*x + 1/2*c)^6 + 4704*A*b^4*\tan(1/2*d*x + 1/2*c)^6 - 1320*B*a^4*\tan(1/2*d*x + 1/2*c)^5 - 5280*A*a^3*b*\tan(1/2*d*x + 1/2*c)^5 + 7200*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^5 + 4800*A*a*b^3*\tan(1/2*d*x + 1/2*c)^5 - 960*B*b^4*\tan(1/2*d*x + 1/2*c)^5 - 435*A*a^4*\tan(1/2*d*x + 1/2*c)^4 + 1440*B*a^3*b*\tan(1/2*d*x + 1/2*c)^4 + 2160*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^4 - 960*B*a*b^3*\tan(1/2*d*x + 1/2*c)^4 - 240*A*b^4*\tan(1/2*d*x + 1/2*c)^4 + 140*B*a^4*\tan(1/2*d*x + 1/2*c)^3 + 560*A*a^3*b*\tan(1/2*d*x + 1/2*c)^3 - 480*B*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 - 320*A*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 60*A*a^4*\tan(1/2*d*x + 1/2*c)^2 - 120*B*a^3*b*\tan(1/2*d*x + 1/2*c)^2 - 180*A*a^2*b^2*\tan(1/2*d*x + 1/2*c)^2 - 12*B*a^4*\tan(1/2*d*x + 1/2*c) - 48*A*a^3*b*\tan(1/2*d*x + 1/2*c) - 5*A*a^4)/\tan(1/2*d*x + 1/2*c)^6/d$$

**Mupad [B]**

time = 6.96, size = 307, normalized size = 0.95

$\cot(c + dx)^7 \left( \tan(c + dx) \left( \frac{5d^6}{4} + \frac{3d^4}{2} \right) + \frac{5d^5}{2} - \tan(c + dx)^2 \left( \frac{5d^5}{2} + \frac{3d^3}{2} - 2Bd^4P - \frac{3d^3}{2} \right) + \tan(c + dx)^3 \left( -\frac{5d^4}{2} + Bd^3 + \frac{3d^2}{2} \right) + \tan(c + dx)^4 \left( \frac{5d^3}{2} - 2Bd^2P - 3Ad^3P + 2Bd^2P + \frac{5d^2}{2} \right) + \tan(c + dx)^5 (Bd^2 + 4A^2b^2 - 6Bd^2P - 4Ad^2P + B^2P) \right) \ln(\tan(c + dx)) (Ad^6 - 4Bd^5P - 6Ad^4P^2 + 4Bd^4P + 4A^2) \ln(\tan(c + dx) + 1) (A - B) (b + a)^2 \ln(\tan(c + dx) - 1) (A + B) (-b + a)^2$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^7*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x))^4,x)$

[Out]  $(\log(\tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^4)/(2*d) - (\log(\tan(c + d*x))$   
 $*(A*a^4 + A*b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b))/d - (\cot(c + d*x))^6$   
 $*(\tan(c + d*x)*((B*a^4)/5 + (4*A*a^3*b)/5) + (A*a^4)/6 - \tan(c + d*x)^3*((B$   
 $*a^4)/3 - 2*B*a^2*b^2 - (4*A*a*b^3)/3 + (4*A*a^3*b)/3) + \tan(c + d*x)^2*((3$   
 $*A*a^2*b^2)/2 - (A*a^4)/4 + B*a^3*b) + \tan(c + d*x)^4*((A*a^4)/2 + (A*b^4)/$   
 $2 - 3*A*a^2*b^2 + 2*B*a*b^3 - 2*B*a^3*b) + \tan(c + d*x)^5*(B*a^4 + B*b^4 -$   
 $6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b))/d + (\log(\tan(c + d*x) - 1i)*(A + B*1$   
 $i)*(a*1i - b)^4)/(2*d)$

$$3.267 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=127

$$-\frac{(Ab - aB)x}{a^2 + b^2} + \frac{(aA + bB) \log(\cos(c + dx))}{(a^2 + b^2)d} - \frac{a^3(Ab - aB) \log(a + b \tan(c + dx))}{b^3(a^2 + b^2)d} + \frac{(Ab - aB) \tan(c + dx)}{b^2d}$$

[Out]  $-(A*b-B*a)*x/(a^2+b^2)+(A*a+B*b)*\ln(\cos(d*x+c))/(a^2+b^2)/d-a^3*(A*b-B*a)*\ln(a+b*\tan(d*x+c))/b^3/(a^2+b^2)/d+(A*b-B*a)*\tan(d*x+c)/b^2/d+1/2*B*\tan(d*x+c)^2/b/d$

**Rubi [A]**

time = 0.27, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3688, 3728, 3707, 3698, 31, 3556}

$$\frac{(aA + bB) \log(\cos(c + dx))}{d(a^2 + b^2)} - \frac{x(Ab - aB)}{a^2 + b^2} - \frac{a^3(Ab - aB) \log(a + b \tan(c + dx))}{b^3d(a^2 + b^2)} + \frac{(Ab - aB) \tan(c + dx)}{b^2d} + \frac{B \tan^2(c + dx)}{2bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c + d*x]^3*(A + B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x]), x]$

[Out]  $-(((A*b - a*B)*x)/(a^2 + b^2)) + ((a*A + b*B)*\text{Log}[\text{Cos}[c + d*x]])/((a^2 + b^2)*d) - (a^3*(A*b - a*B)*\text{Log}[a + b*\text{Tan}[c + d*x]])/(b^3*(a^2 + b^2)*d) + ((A*b - a*B)*\text{Tan}[c + d*x])/(b^2*d) + (B*\text{Tan}[c + d*x]^2)/(2*b*d)$

Rule 31

$\text{Int}[(a_.) + (b_.)*(x_.)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3688

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]]^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{(m-1)}*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(d*f*(m+n)), x] + \text{Dist}[1/(d*(m+n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-2)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a^2*A*d*(m+n) - b*B*(b*c*(m-1) + a*d*(n+1)) + d*(m+n)*(2*a*A*b + B*(a^2 - b^2))*\text{Tan}[e + f*x] - (b*B*(b*c - a*d)*(m-1) - b*(A*b + a*B)*d*(m+n))*\text{Tan}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\& (\text{IntegerQ}[m] \|\ \text{IntegersQ}[2*m, 2*n]) \&\& !(\text{IGtQ}[n, 1] \&$

& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))

### Rule 3698

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2), x\_Symbol] := Dist[A/(b\*f), Subst[Int[(a + x)^m, x], x, b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

### Rule 3707

Int[((A\_) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2)/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*A + b\*B - a\*C)\*(x/(a^2 + b^2)), x] + (Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x]), x], x] - Dist[(A\*b - a\*B - b\*C)/(a^2 + b^2), Int[Tan[e + f\*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A\*b - a\*B - b\*C, 0]

### Rule 3728

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2), x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*(b\*c\*m + a\*d\*(n + 1)) + d\*(A\*b + a\*B - b\*C)\*(m + n + 1)\*Tan[e + f\*x] - (C\*m\*(b\*c - a\*d) - b\*B\*d\*(m + n + 1))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx &= \frac{B\tan^2(c+dx)}{2bd} + \frac{\int \frac{\tan(c+dx)(-2aB-2bB\tan(c+dx)+2(Ab-aB)\tan^2(c+dx))}{a+b\tan(c+dx)}}{2b} \\
&= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\tan^2(c+dx)}{2bd} + \frac{\int \frac{-2a(Ab-aB)-2Ab^2\tan(c+dx)}{a+b\tan(c+dx)}}{2b} \\
&= -\frac{(Ab-aB)x}{a^2+b^2} + \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\tan^2(c+dx)}{2bd} - \frac{a^3(Ab-aB)\log(\cos(c+dx))}{b^3(a^2+b^2)} \\
&= -\frac{(Ab-aB)x}{a^2+b^2} + \frac{(aA+bB)\log(\cos(c+dx))}{(a^2+b^2)d} + \frac{(Ab-aB)\tan(c+dx)}{b^2d} \\
&= -\frac{(Ab-aB)x}{a^2+b^2} + \frac{(aA+bB)\log(\cos(c+dx))}{(a^2+b^2)d} - \frac{a^3(Ab-aB)\log(\cos(c+dx))}{b^3(a^2+b^2)}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.01, size = 138, normalized size = 1.09

$$\frac{-\frac{b(A+iB)\log(i-\tan(c+dx))}{a+ib} - \frac{b(A-iB)\log(i+\tan(c+dx))}{a-ib} + \frac{2a^3(-Ab+aB)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)} + \frac{2(Ab-aB)\tan(c+dx)}{b} + B\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out]  $(-((b*(A + I*B)*\text{Log}[I - \text{Tan}[c + d*x]])/(a + I*b)) - (b*(A - I*B)*\text{Log}[I + \text{Tan}[c + d*x]])/(a - I*b) + (2*a^3*(-(A*b) + a*B)*\text{Log}[a + b*\text{Tan}[c + d*x]])/(b^2*(a^2 + b^2)) + (2*(A*b - a*B)*\text{Tan}[c + d*x])/b + B*\text{Tan}[c + d*x]^2)/(2*b*d)$

**Maple [A]**

time = 0.20, size = 127, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{bB(\tan^2(dx+c))}{2} + A\tan(dx+c)b - B\tan(dx+c)a}{b^2} + \frac{(-aA-Bb)\ln(1+\tan^2(dx+c))}{2} + \frac{(-Ab+aB)\arctan(\tan(dx+c))}{a^2+b^2} - \frac{a^3(Ab-aB)\ln(a+b\tan(dx+c))}{b^3(a^2+b^2)}$
default	$\frac{\frac{bB(\tan^2(dx+c))}{2} + A\tan(dx+c)b - B\tan(dx+c)a}{b^2} + \frac{(-aA-Bb)\ln(1+\tan^2(dx+c))}{2} + \frac{(-Ab+aB)\arctan(\tan(dx+c))}{a^2+b^2} - \frac{a^3(Ab-aB)\ln(a+b\tan(dx+c))}{b^3(a^2+b^2)}$
norman	$\frac{(Ab-aB)\tan(dx+c)}{b^2d} - \frac{(Ab-aB)x}{a^2+b^2} + \frac{B(\tan^2(dx+c))}{2bd} - \frac{(aA+Bb)\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{a^3(Ab-aB)\ln(a+b\tan(dx+c))}{b^3(a^2+b^2)d}$
risch	$-\frac{xB}{ib-a} - \frac{2ia^4Bx}{b^3(a^2+b^2)} + \frac{2ia^2Bc}{b^3d} + \frac{2ia^2Bx}{b^3} + \frac{2ia^3Ax}{b^2(a^2+b^2)} - \frac{2iAac}{b^2d} - \frac{ixA}{ib-a} + \frac{2ia^3Ac}{b^2d(a^2+b^2)} + \frac{2i(-iBbe^{2i(dx+c)})}{b^3(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/b^2*(1/2*B*tan(d*x+c)^2*b+A*tan(d*x+c)*b-B*tan(d*x+c)*a)+1/(a^2+b^2)
*(1/2*(-A*a-B*b)*ln(1+tan(d*x+c)^2)+(-A*b+B*a)*arctan(tan(d*x+c)))-1/b^3*a^
3*(A*b-B*a)/(a^2+b^2)*ln(a+b*tan(d*x+c))
```

**Maxima [A]**

time = 0.58, size = 130, normalized size = 1.02

$$\frac{\frac{2(Ba - Ab)(dx+c)}{a^2+b^2} + \frac{2(Ba^4 - Aa^3b) \log(b \tan(dx+c)+a)}{a^2b^3+b^5} - \frac{(Aa+Bb) \log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{Bb \tan(dx+c)^2 - 2(Ba - Ab) \tan(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a^4 - A*a^3*b)*log(b*tan(d*
x + c) + a)/(a^2*b^3 + b^5) - (A*a + B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^
2) + (B*b*tan(d*x + c)^2 - 2*(B*a - A*b)*tan(d*x + c))/b^2)/d
```

**Fricas [A]**

time = 1.51, size = 190, normalized size = 1.50

$$\frac{2(Ba^3 - Ab^4)dx + (Ba^2b^2 + Bb^4) \tan(dx+c)^2 + (Ba^4 - Aa^3b) \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - (Ba^4 - Aa^3b - Aab^3 - Bb^4) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) - 2(Ba^3b - Aa^2b^2 + Bab^3 - Ab^4) \tan(dx+c)}{2(a^2b^3 + b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*(B*a*b^3 - A*b^4)*d*x + (B*a^2*b^2 + B*b^4)*tan(d*x + c)^2 + (B*a^4
- A*a^3*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c
)^2 + 1)) - (B*a^4 - A*a^3*b - A*a*b^3 - B*b^4)*log(1/(tan(d*x + c)^2 + 1))
- 2*(B*a^3*b - A*a^2*b^2 + B*a*b^3 - A*b^4)*tan(d*x + c))/((a^2*b^3 + b^5)
*d)
```

**Sympy [C] Result contains complex when optimal does not.**

time = 0.77, size = 1297, normalized size = 10.21

$$\frac{\int dx(A + B \tan(c)) \tan^2(c)}{\dots}$$

for a = 0 ∧ b = 0 ∧ d = 0  
for b = 0  
for a = -ib  
for a = ib  
for d = 0  
otherwise

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
[Out] Piecewise((zoo*x*(A + B*tan(c))*tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
((-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*tan(c + d*x)**2/(2*d) + B*x + B*ta
```



```

n(c + d*x)**3/(3*d) - B*tan(c + d*x)/d)/a, Eq(b, 0)), (-3*A*d*x*tan(c + d*x)
)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*A*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d)
+ I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d)
+ A*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*A*tan(c +
d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*A/(2*b*d*tan(c + d*x) - 2*I*b*d)
- 3*I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*B*d*x/(2*b*d*t
an(c + d*x) - 2*I*b*d) - 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*t
an(c + d*x) - 2*I*b*d) + 2*I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x)
- 2*I*b*d) + B*tan(c + d*x)**3/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B*tan(c
+ d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*B/(2*b*d*tan(c + d*x) - 2*I*
b*d), Eq(a, -I*b)), (-3*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) -
3*I*A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*A*log(tan(c + d*x)**2 + 1)*ta
n(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + A*log(tan(c + d*x)**2 + 1)/(2*b
*d*tan(c + d*x) + 2*I*b*d) + 2*A*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*
b*d) + 3*A/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*I*B*d*x*tan(c + d*x)/(2*b*d*t
an(c + d*x) + 2*I*b*d) - 3*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*B*log(t
an(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*B*log
(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*tan(c + d*x)**3/(2
*b*d*tan(c + d*x) + 2*I*b*d) - I*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*
I*b*d) - 3*I*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(A + B*tan(c
))*tan(c)**3/(a + b*tan(c)), Eq(d, 0)), (-2*A*a**3*b*log(a/b + tan(c + d*x)
)/(2*a**2*b**3*d + 2*b**5*d) + 2*A*a**2*b**2*tan(c + d*x)/(2*a**2*b**3*d +
2*b**5*d) - A*a*b**3*log(tan(c + d*x)**2 + 1)/(2*a**2*b**3*d + 2*b**5*d) -
2*A*b**4*d*x/(2*a**2*b**3*d + 2*b**5*d) + 2*A*b**4*tan(c + d*x)/(2*a**2*b**
3*d + 2*b**5*d) + 2*B*a**4*log(a/b + tan(c + d*x))/(2*a**2*b**3*d + 2*b**5*
d) - 2*B*a**3*b*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) + B*a**2*b**2*tan(c
+ d*x)**2/(2*a**2*b**3*d + 2*b**5*d) + 2*B*a*b**3*d*x/(2*a**2*b**3*d + 2*b
**5*d) - 2*B*a*b**3*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) - B*b**4*log(ta
n(c + d*x)**2 + 1)/(2*a**2*b**3*d + 2*b**5*d) + B*b**4*tan(c + d*x)**2/(2*a
**2*b**3*d + 2*b**5*d), True))

```

**Giac** [A]

time = 0.75, size = 135, normalized size = 1.06

$$\frac{\frac{2(Ba - Ab)(dx + c)}{a^2 + b^2} - \frac{(Aa + Bb) \log(\tan(dx + c)^2 + 1)}{a^2 + b^2} + \frac{2(Ba^4 - Aa^3b) \log(|b \tan(dx + c) + a|)}{a^2 b^3 + b^5} + \frac{Bb \tan(dx + c)^2 - 2Ba \tan(dx + c) + 2Ab \tan(dx + c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*(B\*a - A\*b)\*(d\*x + c)/(a^2 + b^2) - (A\*a + B\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + 2\*(B\*a^4 - A\*a^3\*b)\*log(abs(b\*tan(d\*x + c) + a))/(a^2\*b^3 + b^5) + (B\*b\*tan(d\*x + c)^2 - 2\*B\*a\*tan(d\*x + c) + 2\*A\*b\*tan(d\*x + c))/b^2)/d

**Mupad [B]**

time = 6.52, size = 144, normalized size = 1.13

$$\frac{\tan(c+dx) \left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{d} - \frac{\ln(\tan(c+dx) - i) (-B + A1i)}{2d(-b + a1i)} + \frac{\ln(a + b\tan(c+dx)) (Ba^4 - Aa^3b)}{d(a^2b^3 + b^5)} - \frac{\ln(\tan(c+dx) + 1i) (A - B1i)}{2d(a - b1i)} + \frac{B\tan(c+dx)^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

```
[Out] (tan(c + d*x)*(A/b - (B*a)/b^2))/d - (log(tan(c + d*x) - 1i)*(A*1i - B))/(2
*d*(a*1i - b)) + (log(a + b*tan(c + d*x))*(B*a^4 - A*a^3*b))/(d*(b^5 + a^2*
b^3)) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a - b*1i)) + (B*tan(c + d
*x)^2)/(2*b*d)
```

$$3.268 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=101

$$-\frac{(aA + bB)x}{a^2 + b^2} - \frac{(Ab - aB) \log(\cos(c + dx))}{(a^2 + b^2)d} + \frac{a^2(Ab - aB) \log(a + b \tan(c + dx))}{b^2(a^2 + b^2)d} + \frac{B \tan(c + dx)}{bd}$$

[Out]  $-(A*a+B*b)*x/(a^2+b^2)-(A*b-B*a)*\ln(\cos(d*x+c))/(a^2+b^2)/d+a^2*(A*b-B*a)*\ln(a+b*\tan(d*x+c))/b^2/(a^2+b^2)/d+B*\tan(d*x+c)/b/d$

**Rubi [A]**

time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3687, 3707, 3698, 31, 3556}

$$\frac{a^2(Ab - aB) \log(a + b \tan(c + dx))}{b^2 d (a^2 + b^2)} - \frac{(Ab - aB) \log(\cos(c + dx))}{d (a^2 + b^2)} - \frac{x(aA + bB)}{a^2 + b^2} + \frac{B \tan(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out]  $-(((a*A + b*B)*x)/(a^2 + b^2)) - ((A*b - a*B)*\text{Log}[\text{Cos}[c + d*x]])/((a^2 + b^2)*d) + (a^2*(A*b - a*B)*\text{Log}[a + b*\text{Tan}[c + d*x]])/(b^2*(a^2 + b^2)*d) + (B*\text{Tan}[c + d*x])/(b*d)$

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(−1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3687

Int[(((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]))/((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[b^2\*B\*(Tan[e + f\*x]/(d\*f)), x] + Dist[1/d, Int[(a^2\*A\*d - b^2\*B\*c + (2\*a\*A\*b + B\*(a^2 - b^2))\*d\*Tan[e + f\*x] + (A\*b^2\*d - b\*B\*(b\*c - 2\*a\*d))\*Tan[e + f\*x]^2)/(c + d\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
(f_.)*(x_)]^2), x_Symbol] := Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

### Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
+ Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx &= \frac{B\tan(c+dx)}{bd} + \frac{\int \frac{-aB-bB\tan(c+dx)+(Ab-aB)\tan^2(c+dx)}{a+b\tan(c+dx)} dx}{b} \\ &= -\frac{(aA+bB)x}{a^2+b^2} + \frac{B\tan(c+dx)}{bd} + \frac{(Ab-aB)\int \tan(c+dx) dx}{a^2+b^2} + \frac{B\tan(c+dx)}{bd} \\ &= -\frac{(aA+bB)x}{a^2+b^2} - \frac{(Ab-aB)\log(\cos(c+dx))}{(a^2+b^2)d} + \frac{B\tan(c+dx)}{bd} + \frac{B\tan(c+dx)}{bd} \\ &= -\frac{(aA+bB)x}{a^2+b^2} - \frac{(Ab-aB)\log(\cos(c+dx))}{(a^2+b^2)d} + \frac{a^2(Ab-aB)\log(\cos(c+dx))}{b^2(a^2+b^2)} + \frac{2B\tan(c+dx)}{bd} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.41, size = 118, normalized size = 1.17

$$\frac{\frac{i(A+iB)\log(i-\tan(c+dx))}{a+ib} - \frac{(iA+B)\log(i+\tan(c+dx))}{a-ib} + \frac{2a^2(Ab-aB)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)} + \frac{2B\tan(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]), x]
```

```
[Out] ((I*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b) - ((I*A + B)*Log[I + Tan[c +
d*x]])/(a - I*b) + (2*a^2*(A*b - a*B)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 +
b^2)) + (2*B*Tan[c + d*x])/b)/(2*d)
```

### Maple [A]

time = 0.13, size = 101, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{B \tan(dx+c)}{b} + \frac{(Ab-aB) \ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (-aA-Bb) \arctan(\tan(dx+c))}{a^2+b^2} + \frac{a^2(Ab-aB) \ln(a+b \tan(dx+c))}{b^2(a^2+b^2)}}{d}$
default	$\frac{\frac{B \tan(dx+c)}{b} + \frac{(Ab-aB) \ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (-aA-Bb) \arctan(\tan(dx+c))}{a^2+b^2} + \frac{a^2(Ab-aB) \ln(a+b \tan(dx+c))}{b^2(a^2+b^2)}}{d}$
norman	$\frac{B \tan(dx+c)}{bd} - \frac{(aA+Bb)x}{a^2+b^2} + \frac{a^2(Ab-aB) \ln(a+b \tan(dx+c))}{b^2(a^2+b^2)d} + \frac{(Ab-aB) \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$
risch	$-\frac{ixB}{ib-a} + \frac{xA}{ib-a} + \frac{2iAx}{b} + \frac{2iAc}{bd} - \frac{2iaBx}{b^2} - \frac{2iaBc}{b^2d} - \frac{2ia^2Ax}{(a^2+b^2)b} - \frac{2ia^2Ac}{d(a^2+b^2)b} + \frac{2ia^3Bx}{b^2(a^2+b^2)} + \frac{2ia^3Bc}{d(a^2+b^2)b^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*(B/b*\tan(dx+c)+1/(a^2+b^2)*(1/2*(A*b-B*a)*\ln(1+\tan(dx+c)^2)+(-A*a-B*b)*\arctan(\tan(dx+c)))+1/b^2*a^2*(A*b-B*a)/(a^2+b^2)*\ln(a+b*\tan(dx+c))$

**Maxima** [A]

time = 0.54, size = 109, normalized size = 1.08

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{2(Ba^3-Aa^2b) \log(b \tan(dx+c)+a)}{a^2b^2+b^4} + \frac{(Ba-Ab) \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2B \tan(dx+c)}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a^3 - A*a^2*b)*\log(b*\tan(d*x + c) + a)/(a^2*b^2 + b^4) + (B*a - A*b)*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*B*\tan(d*x + c)/b)/d$

**Fricas** [A]

time = 0.97, size = 149, normalized size = 1.48

$$\frac{2(Aab^2 + Bb^3)dx + (Ba^3 - Aa^2b) \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - (Ba^3 - Aa^2b + Bab^2 - Ab^3) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) - 2(Ba^2b + Bb^3) \tan(dx+c)}{2(a^2b^2 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/2*(2*(A*a*b^2 + B*b^3)*d*x + (B*a^3 - A*a^2*b)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - (B*a^3 - A*a^2*b + B*a*b$

$$\sqrt{2 - A*b^3} * \log(1/(\tan(dx + c)^2 + 1)) - 2*(B*a^2*b + B*b^3)*\tan(dx + c) / ((a^2*b^2 + b^4)*d)$$

**Sympy [C]** Result contains complex when optimal does not.  
 time = 0.60, size = 1015, normalized size = 10.05

$$\frac{\int \sqrt{2 - A \tan^2(c) \tan^2(dx + c)} \tan(c) dx}{\dots}$$

for a = 0 ∧ b = 0 ∧ d = 0  
 for a = -ib  
 for a = ib  
 for b = 0  
 for d = 0  
 otherwise

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(dx+c)**2*(A+B*tan(dx+c))/(a+b*tan(dx+c)), x)
```

```
[Out] Piecewise((zoo*x*(A + B*tan(c))*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (I
*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + A*d*x/(2*b*d*tan(c + d
*x) - 2*I*b*d) + A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)
) - 2*I*b*d) - I*A*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d)
- I*A/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*B*d*x*tan(c + d*x)/(2*b*d*tan(c +
d*x) - 2*I*b*d) + 3*I*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B*log(tan(c
+ d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*log(tan(c +
d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*B*tan(c + d*x)**2/(2*b*d*tan
(c + d*x) - 2*I*b*d) + 3*B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I
*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + A*d*x/(2*b*d*tan(c +
d*x) + 2*I*b*d) + A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d
*x) + 2*I*b*d) + I*A*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d)
) + I*A/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*B*d*x*tan(c + d*x)/(2*b*d*tan(c
+ d*x) + 2*I*b*d) - 3*I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*log(tan(
c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c
+ d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B*tan(c + d*x)**2/(2*b*d*
tan(c + d*x) + 2*I*b*d) + 3*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), ((
-A*x + A*tan(c + d*x)/d - B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)
**2/(2*d))/a, Eq(b, 0)), (x*(A + B*tan(c))*tan(c)**2/(a + b*tan(c)), Eq(d
, 0)), (2*A*a**2*b*log(a/b + tan(c + d*x))/(2*a**2*b**2*d + 2*b**4*d) - 2*A
*a*b**2*d*x/(2*a**2*b**2*d + 2*b**4*d) + A*b**3*log(tan(c + d*x)**2 + 1)/(2
*a**2*b**2*d + 2*b**4*d) - 2*B*a**3*log(a/b + tan(c + d*x))/(2*a**2*b**2*d
+ 2*b**4*d) + 2*B*a**2*b*tan(c + d*x)/(2*a**2*b**2*d + 2*b**4*d) - B*a*b**2
*log(tan(c + d*x)**2 + 1)/(2*a**2*b**2*d + 2*b**4*d) - 2*B*b**3*d*x/(2*a**2
*b**2*d + 2*b**4*d) + 2*B*b**3*tan(c + d*x)/(2*a**2*b**2*d + 2*b**4*d), Tru
e))
```

**Giac [A]**  
 time = 0.59, size = 110, normalized size = 1.09

$$\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{(Ba-Ab) \log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(Ba^3-Aa^2b) \log(|b \tan(dx+c)+a|)}{a^2b^2+b^4} - \frac{2B \tan(dx+c)}{b}$$

2 d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 2*(B*a^3 - A*a^2*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b^2 + b^4) - 2*B*tan(d*x + c)/b)/d
```

**Mupad [B]**

time = 6.41, size = 117, normalized size = 1.16

$$\frac{B \tan(c + dx)}{bd} + \frac{\ln(\tan(c + dx) + 1i)(A - B1i)}{2d(b + a1i)} - \frac{\ln(a + b \tan(c + dx))(Ba^3 - Aa^2b)}{d(a^2b^2 + b^4)} + \frac{\ln(\tan(c + dx) - i)(-B + A1i)}{2d(a + b1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*1i + b)) - (log(a + b*tan(c + d*x))*(B*a^3 - A*a^2*b))/(d*(b^4 + a^2*b^2)) + (B*tan(c + d*x))/(b*d) + (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a + b*1i))
```

$$3.269 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=80

$$\frac{(Ab - aB)x}{a^2 + b^2} - \frac{B \log(\cos(c + dx))}{bd} - \frac{a(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{b(a^2 + b^2)d}$$

[Out] (A\*b-B\*a)\*x/(a^2+b^2)-B\*ln(cos(d\*x+c))/b/d-a\*(A\*b-B\*a)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/b/(a^2+b^2)/d

**Rubi [A]**

time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {3670, 3556, 12, 3612, 3611}

$$-\frac{a(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{bd(a^2 + b^2)} + \frac{x(Ab - aB)}{a^2 + b^2} - \frac{B \log(\cos(c + dx))}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((A\*b - a\*B)\*x)/(a^2 + b^2) - (B\*Log[Cos[c + d\*x]])/(b\*d) - (a\*(A\*b - a\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/(b\*(a^2 + b^2)\*d)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; F



reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3670

Int[(((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[B\*(d/b), Int[Tan[e + f\*x], x], x] + Dist[1/b, Int[Simp[A\*b\*c + (A\*b\*d + B\*(b\*c - a\*d))\*Tan[e + f\*x], x]/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx &= \frac{\int \frac{(Ab-aB)\tan(c+dx)}{a+b\tan(c+dx)} dx}{b} + \frac{B \int \tan(c+dx) dx}{b} \\ &= -\frac{B \log(\cos(c+dx))}{bd} + \frac{(Ab-aB) \int \frac{\tan(c+dx)}{a+b\tan(c+dx)} dx}{b} \\ &= \frac{(Ab-aB)x}{a^2+b^2} - \frac{B \log(\cos(c+dx))}{bd} - \frac{(a(Ab-aB)) \int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)} dx}{b(a^2+b^2)} \\ &= \frac{(Ab-aB)x}{a^2+b^2} - \frac{B \log(\cos(c+dx))}{bd} - \frac{a(Ab-aB) \log(a \cos(c+dx))}{b(a^2+b^2)} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.13, size = 98, normalized size = 1.22

$$\frac{(a-ib)b(A+iB)\log(i-\tan(c+dx)) + (a+ib)b(A-iB)\log(i+\tan(c+dx)) + 2a(-Ab+aB)\log(a+b\tan(c+dx))}{2b(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((a - I\*b)\*b\*(A + I\*B)\*Log[I - Tan[c + d\*x]] + (a + I\*b)\*b\*(A - I\*B)\*Log[I + Tan[c + d\*x]] + 2\*a\*(-(A\*b) + a\*B)\*Log[a + b\*Tan[c + d\*x]])/(2\*b\*(a^2 + b^2)\*d)

### Maple [A]

time = 0.13, size = 87, normalized size = 1.09

method	result
--------	--------

derivativedivides	$\frac{\frac{(aA+Bb)\ln(1+\tan^2(dx+c))}{2} + (Ab-aB)\arctan(\tan(dx+c)) - \frac{a(Ab-aB)\ln(a+b\tan(dx+c))}{(a^2+b^2)b}}{d}$
default	$\frac{\frac{(aA+Bb)\ln(1+\tan^2(dx+c))}{2} + (Ab-aB)\arctan(\tan(dx+c)) - \frac{a(Ab-aB)\ln(a+b\tan(dx+c))}{(a^2+b^2)b}}{d}$
norman	$\frac{(Ab-aB)x}{a^2+b^2} + \frac{(aA+Bb)\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{a(Ab-aB)\ln(a+b\tan(dx+c))}{(a^2+b^2)bd}$
risch	$\frac{xB}{ib-a} + \frac{ixA}{ib-a} + \frac{2iaAx}{a^2+b^2} + \frac{2iaAc}{d(a^2+b^2)} - \frac{2ia^2Bx}{b(a^2+b^2)} - \frac{2ia^2Bc}{bd(a^2+b^2)} + \frac{2iBx}{b} + \frac{2iBc}{bd} - \frac{a\ln\left(e^{2i(dx+c)} - \frac{ib+a}{ib-a}\right)A}{d(a^2+b^2)} +$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*(1/(a^2+b^2)*(1/2*(A*a+B*b)*\ln(1+\tan(d*x+c)^2)+(A*b-B*a)*\arctan(\tan(d*x+c)))-a*(A*b-B*a)/(a^2+b^2)/b*\ln(a+b*\tan(d*x+c)))$

**Maxima** [A]

time = 0.50, size = 94, normalized size = 1.18

$$-\frac{\frac{2(Ba-Ab)(dx+c)}{a^2+b^2} - \frac{2(Ba^2-Aab)\log(b\tan(dx+c)+a)}{a^2b+b^3} - \frac{(Aa+Bb)\log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) - 2*(B*a^2 - A*a*b)*\log(b*\tan(d*x + c) + a)/(a^2*b + b^3) - (A*a + B*b)*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2))/d$

**Fricas** [A]

time = 1.02, size = 110, normalized size = 1.38

$$\frac{2(Bab - Ab^2)dx - (Ba^2 - Aab)\log\left(\frac{b^2\tan(dx+c)^2 + 2ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) + (Ba^2 + Bb^2)\log\left(\frac{1}{\tan(dx+c)^2 + 1}\right)}{2(a^2b + b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/2*(2*(B*a*b - A*b^2)*d*x - (B*a^2 - A*a*b)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) + (B*a^2 + B*b^2)*\log(1/(\tan(d*x + c)^2 + 1)))/((a^2*b + b^3)*d)$

**Sympy [C]** Result contains complex when optimal does not.

time = 0.52, size = 700, normalized size = 8.75

$$\left\{ \begin{array}{l} \frac{\sqrt{c}x(A+B\tan(c))}{2bd\tan(c+dx)-2ibd} - \frac{iAdx}{2bd\tan(c+dx)-2ibd} - \frac{A}{2bd\tan(c+dx)-2ibd} + \frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd} + \frac{Bdx}{2bd\tan(c+dx)-2ibd} + \frac{B\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)-2ibd} - \frac{iB\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)-2ibd} - \frac{iB}{2bd\tan(c+dx)-2ibd} \\ \frac{Adx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{iAdx}{2bd\tan(c+dx)+2ibd} - \frac{A}{2bd\tan(c+dx)+2ibd} - \frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{Bdx}{2bd\tan(c+dx)+2ibd} + \frac{B\log(\tan^2(c+dx)+1)\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{iB\log(\tan^2(c+dx)+1)}{2bd\tan(c+dx)+2ibd} + \frac{iB}{2bd\tan(c+dx)+2ibd} \\ \frac{A\log(\tan^2(c+dx)+1)}{2d} - Bx + \frac{B\tan(c+dx)}{d} \\ \frac{x(A+B\tan(c))\tan(c)}{a+b\tan(c)} \\ -\frac{2Aab\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2bd+2b^3d} + \frac{Aab\log(\tan^2(c+dx)+1)}{2a^2bd+2b^3d} + \frac{2Aa^2dx}{2a^2bd+2b^3d} + \frac{2Ba^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2bd+2b^3d} - \frac{2Babdx}{2a^2bd+2b^3d} + \frac{Bb^2\log(\tan^2(c+dx)+1)}{2a^2bd+2b^3d} \end{array} \right. \begin{array}{l} \text{for } a=0 \wedge b=0 \wedge d=0 \\ \text{for } a=-ib \\ \text{for } a=ib \\ \text{for } b=0 \\ \text{for } d=0 \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*x\*(A + B\*tan(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*A\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - A/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (A\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*A\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - A/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), ((A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*x + B\*tan(c + d\*x)/d)/a, Eq(b, 0)), (x\*(A + B\*tan(c))\*tan(c)/(a + b\*tan(c)), Eq(d, 0)), (-2\*A\*a\*b\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) + A\*a\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) + 2\*A\*b\*\*2\*d\*x/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) + 2\*B\*a\*\*2\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) - 2\*B\*a\*b\*d\*x/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) + B\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d), True))

**Giac [A]**

time = 0.50, size = 95, normalized size = 1.19

$$\frac{\frac{2(Ba-Ab)(dx+c)}{a^2+b^2} - \frac{(Aa+Bb)\log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2(Ba^2-Aab)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] -1/2\*(2\*(B\*a - A\*b)\*(d\*x + c)/(a^2 + b^2) - (A\*a + B\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) - 2\*(B\*a^2 - A\*a\*b)\*log(abs(b\*tan(d\*x + c) + a))/(a^2\*b + b^3))/d

**Mupad [B]**

time = 6.66, size = 100, normalized size = 1.25

$$\frac{\ln(\tan(c+dx) - i)(-B + A1i)}{2d(-b + a1i)} + \frac{\ln(\tan(c+dx) + 1i)(A - B1i)}{2d(a - b1i)} - \frac{a \ln(a + b \tan(c+dx))(Ab - Ba)}{bd(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(a\*1i - b)) + (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(2\*d\*(a - b\*1i)) - (a\*log(a + b\*tan(c + d\*x))\*(A\*b - B\*a))/(b\*d\*(a^2 + b^2))

$$3.270 \quad \int \frac{A+B \tan(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=58

$$\frac{(aA + bB)x}{a^2 + b^2} + \frac{(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2) d}$$

[Out] (A\*a+B\*b)\*x/(a^2+b^2)+(A\*b-B\*a)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)/d

**Rubi [A]**

time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {3612, 3611}

$$\frac{(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)} + \frac{x(aA + bB)}{a^2 + b^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]), x]

[Out] ((a\*A + b\*B)\*x)/(a^2 + b^2) + ((A\*b - a\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)\*d)

Rule 3611

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{a + b \tan(c + dx)} dx &= \frac{(aA + bB)x}{a^2 + b^2} + \frac{(Ab - aB) \int \frac{b - a \tan(c + dx)}{a + b \tan(c + dx)} dx}{a^2 + b^2} \\ &= \frac{(aA + bB)x}{a^2 + b^2} + \frac{(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2) d} \end{aligned}$$

**Mathematica [A]**

time = 0.07, size = 66, normalized size = 1.14

$$\frac{2(aA + bB)\text{ArcTan}(\tan(c + dx)) - (Ab - aB)(\log(\sec^2(c + dx)) - 2\log(a + b\tan(c + dx)))}{2(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]),x]

[Out] (2\*(a\*A + b\*B)\*ArcTan[Tan[c + d\*x]] - (A\*b - a\*B)\*(Log[Sec[c + d\*x]^2] - 2\*Log[a + b\*Tan[c + d\*x]]))/(2\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.08, size = 82, normalized size = 1.41

method	result
derivativdivides	$\frac{\frac{(-Ab+aB)\ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (aA+Bb)\arctan(\tan(dx+c))}{a^2+b^2} + \frac{(Ab-aB)\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{(-Ab+aB)\ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (aA+Bb)\arctan(\tan(dx+c))}{a^2+b^2} + \frac{(Ab-aB)\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{(aA+Bb)x}{a^2+b^2} + \frac{(Ab-aB)\ln(a+b\tan(dx+c))}{d(a^2+b^2)} - \frac{(Ab-aB)\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$
risch	$\frac{ixB}{ib-a} - \frac{xA}{ib-a} - \frac{2iAbx}{a^2+b^2} + \frac{2iBax}{a^2+b^2} - \frac{2iAbc}{d(a^2+b^2)} + \frac{2iBac}{d(a^2+b^2)} + \frac{\ln\left(e^{2i(dx+c)} - \frac{ib+a}{ib-a}\right)Ab}{d(a^2+b^2)} - \frac{\ln\left(e^{2i(dx+c)} - \frac{ib+a}{ib-a}\right)a}{d(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(1/(a^2+b^2)\*(1/2\*(-A\*b+B\*a)\*ln(1+tan(d\*x+c)^2)+(A\*a+B\*b)\*arctan(tan(d\*x+c)))+(A\*b-B\*a)/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.50, size = 88, normalized size = 1.52

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{2(Ba-Ab)\log(b\tan(dx+c)+a)}{a^2+b^2} + \frac{(Ba-Ab)\log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*(2\*(A\*a + B\*b)\*(d\*x + c)/(a^2 + b^2) - 2\*(B\*a - A\*b)\*log(b\*tan(d\*x + c) + a)/(a^2 + b^2) + (B\*a - A\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2))/d

**Fricas [A]**

time = 1.36, size = 76, normalized size = 1.31

$$\frac{2(Aa + Bb)dx - (Ba - Ab) \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right)}{2(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")**[Out]** 1/2\*(2\*(A\*a + B\*b)\*d\*x - (B\*a - A\*b)\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)))/(a^2 + b^2)\*d**Sympy [C]** Result contains complex when optimal does not.

time = 0.46, size = 524, normalized size = 9.03

$$\left\{ \begin{array}{ll} \frac{\infty x(A+B \tan (c))}{\tan (c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{i A d x \tan (c+d x)}{2 b d \tan (c+d x)-2 i b d} + \frac{A d x}{2 b d \tan (c+d x)-2 i b d} + \frac{i A}{2 b d \tan (c+d x)-2 i b d} + \frac{B d x \tan (c+d x)}{2 b d \tan (c+d x)-2 i b d} - \frac{i B d x}{2 b d \tan (c+d x)-2 i b d} - \frac{B}{2 b d \tan (c+d x)-2 i b d} & \text{for } a=-i b \\ -\frac{i A d x \tan (c+d x)}{2 b d \tan (c+d x)+2 i b d} + \frac{A d x}{2 b d \tan (c+d x)+2 i b d} - \frac{i A}{2 b d \tan (c+d x)+2 i b d} + \frac{B d x \tan (c+d x)}{2 b d \tan (c+d x)+2 i b d} + \frac{i B d x}{2 b d \tan (c+d x)+2 i b d} - \frac{B}{2 b d \tan (c+d x)+2 i b d} & \text{for } a=i b \\ \frac{x(A+B \tan (c))}{a+b \tan (c)} & \text{for } d=0 \\ A x + \frac{B \log (\tan ^2(c+d x)+1)}{2 d} & \text{for } b=0 \\ \frac{2 A a d x}{2 a^2 d+2 b^2 d} + \frac{2 A b \log (\frac{a}{b}+\tan (c+d x))}{2 a^2 d+2 b^2 d} - \frac{A b \log (\tan ^2(c+d x)+1)}{2 a^2 d+2 b^2 d} - \frac{2 B a \log (\frac{a}{b}+\tan (c+d x))}{2 a^2 d+2 b^2 d} + \frac{B a \log (\tan ^2(c+d x)+1)}{2 a^2 d+2 b^2 d} + \frac{2 B b d x}{2 a^2 d+2 b^2 d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

**[Out]** Piecewise((zoo\*x\*(A + B\*tan(c))/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (I\*A\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + A\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + I\*A/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (-I\*A\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + A\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - I\*A/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), (x\*(A + B\*tan(c))/(a + b\*tan(c)), Eq(d, 0)), ((A\*x + B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d))/a, Eq(b, 0)), (2\*A\*a\*d\*x/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + 2\*A\*b\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*d + 2\*b\*\*2\*d) - A\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*d + 2\*b\*\*2\*d) - 2\*B\*a\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + B\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + 2\*B\*b\*d\*x/(2\*a\*\*2\*d + 2\*b\*\*2\*d), True))

**Giac [A]**

time = 0.50, size = 94, normalized size = 1.62

$$\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{(Ba-Ab) \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2(Bab-Ab^2) \log(|b \tan(dx+c)+a|)}{a^2 b+b^3}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{2} * (2 * (A * a + B * b) * (d * x + c) / (a^2 + b^2) + (B * a - A * b) * \log(\tan(d * x + c)^2 + 1) / (a^2 + b^2) - 2 * (B * a * b - A * b^2) * \log(\text{abs}(b * \tan(d * x + c) + a)) / (a^2 * b + b^3)) / d$

**Mupad [B]**

time = 6.67, size = 93, normalized size = 1.60

$$\frac{\ln(a + b \tan(c + dx)) (Ab - Ba)}{d (a^2 + b^2)} - \frac{\ln(\tan(c + dx) + 1i) (A - B 1i)}{2 d (b + a 1i)} - \frac{\ln(\tan(c + dx) - i) (-B + A 1i)}{2 d (a + b 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x)),x)

[Out]  $(\log(a + b * \tan(c + d * x)) * (A * b - B * a)) / (d * (a^2 + b^2)) - (\log(\tan(c + d * x) + 1i) * (A - B * 1i)) / (2 * d * (a * 1i + b)) - (\log(\tan(c + d * x) - 1i) * (A * 1i - B)) / (2 * d * (a + b * 1i))$



$$3.271 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=80

$$-\frac{(Ab - aB)x}{a^2 + b^2} + \frac{A \log(\sin(c + dx))}{ad} - \frac{b(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{a(a^2 + b^2)d}$$

[Out]  $-(A*b-B*a)*x/(a^2+b^2)+A*\ln(\sin(d*x+c))/a/d-b*(A*b-B*a)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/a/(a^2+b^2)/d$

Rubi [A]

time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3692, 3611, 3556}

$$-\frac{b(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{ad(a^2 + b^2)} - \frac{x(Ab - aB)}{a^2 + b^2} + \frac{A \log(\sin(c + dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out]  $-(((A*b - a*B)*x)/(a^2 + b^2)) + (A*\text{Log}[\text{Sin}[c + d*x]])/(a*d) - (b*(A*b - a*B)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(a*(a^2 + b^2)*d)$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3692

Int[((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/(((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(B\*(b\*c + a\*d) + A\*(a\*c - b\*d))\*(x/((a^2 + b^2)\*(c^2 + d^2))), x] + (Dist[b\*(A\*b - a\*B)/((b\*c - a\*d)\*(a^2 + b^2)), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] + Dist[d\*((B\*c - A\*d)/((b\*c - a\*d)\*(c^2 + d^2))), Int[(d - c\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rubi steps

$$\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx = -\frac{(Ab-aB)x}{a^2+b^2} + \frac{A \int \cot(c+dx) dx}{a} - \frac{(b(Ab-aB)) \int \frac{b-a \tan(c+dx)}{a+b \tan(c+dx)}}{a(a^2+b^2)}$$

$$= -\frac{(Ab-aB)x}{a^2+b^2} + \frac{A \log(\sin(c+dx))}{ad} - \frac{b(Ab-aB) \log(a \cos(c+dx))}{a(a^2+b^2)}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.26, size = 113, normalized size = 1.41

$$\frac{\frac{(A+iB) \log(i-\tan(c+dx))}{a+ib} - \frac{2A \log(\tan(c+dx))}{a} + \frac{(A-iB) \log(i+\tan(c+dx))}{a-ib} + \frac{2b(Ab-aB) \log(a+b \tan(c+dx))}{a(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -1/2\*(((A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*b) - (2\*A\*Log[Tan[c + d\*x]])/a + ((A - I\*B)\*Log[I + Tan[c + d\*x]])/(a - I\*b) + (2\*b\*(A\*b - a\*B)\*Log[a + b\*Tan[c + d\*x]])/(a\*(a^2 + b^2)))/d

**Maple [A]**

time = 0.24, size = 101, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{A \ln(\tan(dx+c))}{a} + \frac{(-aA-Bb) \ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (-Ab+aB) \arctan(\tan(dx+c)) - (Ab-aB)b \ln(a+b \tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{A \ln(\tan(dx+c))}{a} + \frac{(-aA-Bb) \ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (-Ab+aB) \arctan(\tan(dx+c)) - (Ab-aB)b \ln(a+b \tan(dx+c))}{a^2+b^2}}{d}$
norman	$-\frac{(Ab-aB)x}{a^2+b^2} + \frac{A \ln(\tan(dx+c))}{ad} - \frac{(aA+Bb) \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{(Ab-aB)b \ln(a+b \tan(dx+c))}{(a^2+b^2)ad}$
risch	$-\frac{xB}{ib-a} - \frac{ixA}{ib-a} - \frac{2iAx}{a} - \frac{2iAc}{ad} + \frac{2ib^2Ax}{a(a^2+b^2)} + \frac{2ib^2Ac}{ad(a^2+b^2)} - \frac{2ibBx}{a^2+b^2} - \frac{2ibBc}{d(a^2+b^2)} + \frac{A \ln(e^{2i(dx+c)}-1)}{ad} - \frac{b^2}{a^2+b^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(1/a\*A\*ln(tan(d\*x+c))+1/(a^2+b^2)\*(1/2\*(-A\*a-B\*b)\*ln(1+tan(d\*x+c)^2)+(-A\*b+B\*a)\*arctan(tan(d\*x+c)))-(A\*b-B\*a)\*b/a/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.50, size = 107, normalized size = 1.34

$$\frac{\frac{2(Ba-Ab)(dx+c)}{a^2+b^2} + \frac{2(Bab-Ab^2)\log(b\tan(dx+c)+a)}{a^3+ab^2} - \frac{(Aa+Bb)\log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2A\log(\tan(dx+c))}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*(2\*(B\*a - A\*b)\*(d\*x + c)/(a^2 + b^2) + 2\*(B\*a\*b - A\*b^2)\*log(b\*tan(d\*x + c) + a)/(a^3 + a\*b^2) - (A\*a + B\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + 2\*A\*log(tan(d\*x + c))/a)/d

**Fricas [A]**

time = 1.15, size = 118, normalized size = 1.48

$$\frac{2(Ba^2 - Aab)dx + (Aa^2 + Ab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) + (Bab - Ab^2)\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2(a^3 + ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(2\*(B\*a^2 - A\*a\*b)\*d\*x + (A\*a^2 + A\*b^2)\*log(tan(d\*x + c)^2/(tan(d\*x + c)^2 + 1)) + (B\*a\*b - A\*b^2)\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)))/(a^3 + a\*b^2)\*d

**Sympy [C] Result contains complex when optimal does not.**

time = 1.13, size = 952, normalized size = 11.90

$$\left\{ \begin{array}{ll} \frac{\int x(A+B\tan(c))\cot(c)}{\tan(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{A\log(\tan^2(c+dx)+1)}{a} + \frac{A\log(\tan(c+dx)) + Bx}{a} & \text{for } b=0 \\ -A\int \frac{dx}{\tan(c+dx)} - \frac{B\log(\tan^2(c+dx)+1)}{a} + \frac{B\log(\tan(c+dx))}{d} & \text{for } a=0 \\ \frac{A dx \tan(c+dx)}{2b^2 \tan(c+dx)^2 - 2bd} - \frac{iA dx}{2b \tan(c+dx) - 2bd} - \frac{iA \log(\tan^2(c+dx)+1) \tan(c+dx)}{2b^2 \tan(c+dx)^2 - 2bd} - \frac{A \log(\tan^2(c+dx)+1)}{2b^2 \tan(c+dx)^2 - 2bd} + \frac{2iA \log(\tan(c+dx)) \tan(c+dx)}{2b^2 \tan(c+dx)^2 - 2bd} + \frac{2A \log(\tan(c+dx))}{2b^2 \tan(c+dx)^2 - 2bd} + \frac{A}{2bd \tan(c+dx) - 2bd} + \frac{iB dx \tan(c+dx)}{2bd \tan(c+dx)^2 - 2bd} + \frac{B dx}{2bd \tan(c+dx) - 2bd} + \frac{iB}{2bd \tan(c+dx) - 2bd} & \text{for } a = -ib \\ \frac{A dx \tan(c+dx)}{2b^2 \tan(c+dx)^2 + 2bd} + \frac{iA dx}{2b \tan(c+dx) + 2bd} + \frac{iA \log(\tan^2(c+dx)+1) \tan(c+dx)}{2b^2 \tan(c+dx)^2 + 2bd} - \frac{A \log(\tan^2(c+dx)+1)}{2b^2 \tan(c+dx)^2 + 2bd} - \frac{2iA \log(\tan(c+dx)) \tan(c+dx)}{2b^2 \tan(c+dx)^2 + 2bd} + \frac{2A \log(\tan(c+dx))}{2b^2 \tan(c+dx)^2 + 2bd} + \frac{A}{2bd \tan(c+dx) + 2bd} - \frac{iB dx \tan(c+dx)}{2bd \tan(c+dx)^2 + 2bd} - \frac{B dx}{2bd \tan(c+dx) + 2bd} - \frac{iB}{2bd \tan(c+dx) + 2bd} & \text{for } a = ib \\ \frac{x(A+B\tan(c))\cot(c)}{a+b\tan(c)} & \text{for } d=0 \\ \frac{Aa^2 \log(\tan^2(c+dx)+1)}{2a^3d+2ab^2d} + \frac{2Aa^2 \log(\tan(c+dx))}{2a^3d+2ab^2d} - \frac{2Aab dx}{2a^3d+2ab^2d} - \frac{2Ab^2 \log(\frac{1}{2}+\tan(c+dx))}{2a^3d+2ab^2d} + \frac{2Ab^2 \log(\tan(c+dx))}{2a^3d+2ab^2d} + \frac{2Bb^2 dx}{2a^3d+2ab^2d} + \frac{2Bab \log(\frac{1}{2}+\tan(c+dx))}{2a^3d+2ab^2d} - \frac{Bab \log(\tan^2(c+dx)+1)}{2a^3d+2ab^2d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((zoo\*x\*(A + B\*tan(c))\*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*log(tan(c + d\*x))/d + B\*x)/a, Eq(b, 0)), ((-A\*x - A/(d\*tan(c + d\*x)) - B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*log(tan(c + d\*x))/d)/b, Eq(a, 0)), (A\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*A\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*A\*log(tan(c + d\*x))

```

x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - A*log(tan(c + d*x)
**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*A*log(tan(c + d*x))*tan(c + d
*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*A*log(tan(c + d*x))/(2*b*d*tan(c + d
*x) - 2*I*b*d) + A/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B*d*x*tan(c + d*x)/(2
*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B/(
2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (A*d*x*tan(c + d*x)/(2*b*d*tan
(c + d*x) + 2*I*b*d) + I*A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*A*log(tan
(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - A*log(tan(c
+ d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*A*log(tan(c + d*x))*ta
n(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*A*log(tan(c + d*x))/(2*b*d*ta
n(c + d*x) + 2*I*b*d) + A/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*d*x*tan(c +
d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d)
- I*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(A + B*tan(c))*cot(c)
/(a + b*tan(c)), Eq(d, 0)), (-A*a**2*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2
*a*b**2*d) + 2*A*a**2*log(tan(c + d*x))/(2*a**3*d + 2*a*b**2*d) - 2*A*a*b*d
*x/(2*a**3*d + 2*a*b**2*d) - 2*A*b**2*log(a/b + tan(c + d*x))/(2*a**3*d + 2
*a*b**2*d) + 2*A*b**2*log(tan(c + d*x))/(2*a**3*d + 2*a*b**2*d) + 2*B*a**2*
d*x/(2*a**3*d + 2*a*b**2*d) + 2*B*a*b*log(a/b + tan(c + d*x))/(2*a**3*d + 2
*a*b**2*d) - B*a*b*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d), True))

```

**Giac [A]**

time = 0.55, size = 113, normalized size = 1.41

$$\frac{\frac{2(Ba-Ab)(dx+c)}{a^2+b^2} - \frac{(Aa+Bb)\log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(Bab^2-Ab^3)\log(|b\tan(dx+c)+a|)}{a^3b+ab^3} + \frac{2A\log(|\tan(dx+c)|)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*(B\*a - A\*b)\*(d\*x + c)/(a^2 + b^2) - (A\*a + B\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + 2\*(B\*a\*b^2 - A\*b^3)\*log(abs(b\*tan(d\*x + c) + a))/(a^3\*b + a\*b^3) + 2\*A\*log(abs(tan(d\*x + c)))/a)/d

**Mupad [B]**

time = 7.02, size = 115, normalized size = 1.44

$$\frac{A \ln(\tan(c + dx))}{a d} - \frac{\ln(\tan(c + dx) - i) (-B + A i)}{2 d (-b + a i)} - \frac{\ln(\tan(c + dx) + i) (A - B i)}{2 d (a - b i)} - \frac{b \ln(a + b \tan(c + dx)) (A b - B a)}{a d (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] (A\*log(tan(c + d\*x)))/(a\*d) - (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(a\*1i - b)) - (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(2\*d\*(a - b\*1i)) - (b\*log(a + b\*tan(c + d\*x))\*(A\*b - B\*a))/(a\*d\*(a^2 + b^2))

$$3.272 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=103

$$\frac{(aA + bB)x}{a^2 + b^2} - \frac{A \cot(c + dx)}{ad} - \frac{(Ab - aB) \log(\sin(c + dx))}{a^2 d} + \frac{b^2 (Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{a^2 (a^2 + b^2) d}$$

[Out]  $-(A*a+B*b)*x/(a^2+b^2)-A*\cot(d*x+c)/a/d-(A*b-B*a)*\ln(\sin(d*x+c))/a^2/d+b^2*(A*b-B*a)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/a^2/(a^2+b^2)/d$

**Rubi [A]**

time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {3690, 3732, 3611, 3556}

$$\frac{b^2 (Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{a^2 d (a^2 + b^2)} - \frac{x(aA + bB)}{a^2 + b^2} - \frac{(Ab - aB) \log(\sin(c + dx))}{a^2 d} - \frac{A \cot(c + dx)}{ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x]^2*(A + B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x]), x]$

[Out]  $-(((a*A + b*B)*x)/(a^2 + b^2)) - (A*\text{Cot}[c + d*x])/(a*d) - ((A*b - a*B)*\text{Log}[\text{Sin}[c + d*x]])/(a^2*d) + (b^2*(A*b - a*B)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(a^2*(a^2 + b^2)*d)$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3611

$\text{Int}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]/(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

Rule 3690

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{(m + 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*\text{Tan}[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)], x], x]$

2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&  
 NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &  
 & (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]  
 || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3732

Int[((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2)/(((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])), x\_Symbol] :> Simp[(a\*(A\*c - c\*C + B\*d) + b\*(B\*c - A\*d + C\*d))\*(x/((a^2 + b^2)\*(c^2 + d^2))), x] + (Dist[(A\*b^2 - a\*b\*B + a^2\*C)/((b\*c - a\*d)\*(a^2 + b^2)), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] - Dist[(c^2\*C - B\*c\*d + A\*d^2)/((b\*c - a\*d)\*(c^2 + d^2)), Int[(d - c\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rubi steps

$$\int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{a + b \tan(c + dx)} dx = -\frac{A \cot(c + dx)}{ad} - \frac{\int \frac{\cot(c+dx)(Ab - aB + aA \tan(c+dx) + Ab \tan^2(c+dx))}{a + b \tan(c+dx)} dx}{a}$$

$$= -\frac{(aA + bB)x}{a^2 + b^2} - \frac{A \cot(c + dx)}{ad} - \frac{(Ab - aB) \int \cot(c + dx) dx}{a^2} + \dots$$

$$= -\frac{(aA + bB)x}{a^2 + b^2} - \frac{A \cot(c + dx)}{ad} - \frac{(Ab - aB) \log(\sin(c + dx))}{a^2 d} + \dots$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.60, size = 138, normalized size = 1.34

$$\frac{-\frac{2A \cot(c+dx)}{a} + \frac{i(A+iB) \log(i - \tan(c+dx))}{a+ib} + \frac{2(-Ab+aB) \log(\tan(c+dx))}{a^2} - \frac{(iA+B) \log(i + \tan(c+dx))}{a-ib} + \frac{2b^2(Ab-aB) \log(a+b \tan(c+dx))}{a^2(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((-2\*A\*Cot[c + d\*x])/a + (I\*(A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*b) + (2\*(-(A\*b) + a\*B)\*Log[Tan[c + d\*x]])/a^2 - ((I\*A + B)\*Log[I + Tan[c + d\*x]])/(a - I\*b) + (2\*b^2\*(A\*b - a\*B)\*Log[a + b\*Tan[c + d\*x]])/(a^2\*(a^2 + b^2)))/(2\*d)

**Maple [A]**

time = 0.23, size = 123, normalized size = 1.19

method	result
derivativedivides	$-\frac{A}{a \tan(dx+c)} + \frac{(-Ab+aB) \ln(\tan(dx+c))}{a^2} + \frac{(Ab-aB) \ln(1+\tan^2(dx+c))}{2} + \frac{(-aA-Bb) \arctan(\tan(dx+c))}{a^2+b^2} + \frac{(Ab-aB)b^2 \ln(a+b \tan(dx+c))}{a^2(a^2+b^2)}$
default	$-\frac{A}{a \tan(dx+c)} + \frac{(-Ab+aB) \ln(\tan(dx+c))}{a^2} + \frac{(Ab-aB) \ln(1+\tan^2(dx+c))}{2} + \frac{(-aA-Bb) \arctan(\tan(dx+c))}{a^2+b^2} + \frac{(Ab-aB)b^2 \ln(a+b \tan(dx+c))}{a^2(a^2+b^2)}$
norman	$-\frac{A}{ad} \frac{(aA+Bb)x \tan(dx+c)}{a^2+b^2} + \frac{b^2(Ab-aB) \ln(a+b \tan(dx+c))}{a^2 d(a^2+b^2)} - \frac{(Ab-aB) \ln(\tan(dx+c))}{a^2 d} + \frac{(Ab-aB) \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$
risch	$-\frac{i x B}{i b - a} + \frac{x A}{i b - a} + \frac{2 i A b x}{a^2} + \frac{2 i A b c}{a^2 d} - \frac{2 i x B}{a} - \frac{2 i B c}{a d} - \frac{2 i b^3 A x}{a^2(a^2+b^2)} - \frac{2 i b^3 A c}{a^2 d(a^2+b^2)} + \frac{2 i b^2 B x}{a(a^2+b^2)} + \frac{2 i b^2 B c}{a d(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*(-1/a*A/\tan(dx+c)+1/a^2*(-A*b+B*a)*\ln(\tan(dx+c))+1/(a^2+b^2)*(1/2*(A*b-B*a)*\ln(1+\tan(dx+c)^2)+(-A*a-B*b)*\arctan(\tan(dx+c)))+(A*b-B*a)*b^2/a^2/(a^2+b^2)*\ln(a+b*\tan(dx+c)))$

**Maxima** [A]

time = 0.59, size = 131, normalized size = 1.27

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{2(Bab^2-Ab^3) \log(b \tan(dx+c)+a)}{a^4+a^2b^2} + \frac{(Ba-Ab) \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2(Ba-Ab) \log(\tan(dx+c))}{a^2} + \frac{2A}{a \tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a*b^2 - A*b^3)*\log(b*\tan(d*x + c) + a)/(a^4 + a^2*b^2) + (B*a - A*b)*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*(B*a - A*b)*\log(\tan(d*x + c))/a^2 + 2*A/(a*\tan(d*x + c)))/d$

**Fricas** [A]

time = 1.97, size = 177, normalized size = 1.72

$$\frac{2Aa^3 + 2Aab^2 + 2(Aa^3 + Ba^2b)dx \tan(dx+c) - (Ba^3 - Aa^2b + Bab^2 - Ab^3) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c) + (Bab^2 - Ab^3) \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)}{2(a^4 + a^2b^2)d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/2*(2*A*a^3 + 2*A*a*b^2 + 2*(A*a^3 + B*a^2*b)*d*x*\tan(d*x + c) - (B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*$

$x + c) + (B*a*b^2 - A*b^3)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1))*\tan(d*x + c))/((a^4 + a^2*b^2)*d*\tan(d*x + c))$

**Sympy [C]** Result contains complex when optimal does not.

time = 2.14, size = 2067, normalized size = 20.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((-A*x - A/(d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c + d*x))/d)/a, Eq(b, 0)), ((A*log(tan(c + d*x)**2 + 1)/(2*d) - A*log(tan(c + d*x))/d - A/(2*d*tan(c + d*x)**2) - B*x - B/(d*tan(c + d*x)))/b, Eq(a, 0)), (-3*I*A*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 3*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + 2*A*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 2*I*A*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 3*I*A*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 2*A/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + B*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + 2*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + 2*B*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + B*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)), Eq(a, -I*b)), (3*I*A*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - 3*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 2*A*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 2*I*A*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 3*I*A*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - 2*A/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + B*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - 2*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 2*B*log(tan(c + d*x))*t`



```

an(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + B*tan(c + d*x)
/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)), Eq(a, I*b)), (zoo*A*x/a, E
q(c, -d*x)), (x*(A + B*tan(c))*cot(c)**2/(a + b*tan(c)), Eq(d, 0)), (-2*A*a
**3*d*x*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) -
2*A*a**3/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) + A*a**2*b*I
og(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d
*tan(c + d*x)) - 2*A*a**2*b*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c
+ d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*A*a*b**2/(2*a**4*d*tan(c + d*x) +
2*a**2*b**2*d*tan(c + d*x)) + 2*A*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)
/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*A*b**3*log(tan(c
+ d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) -
B*a**3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a
**2*b**2*d*tan(c + d*x)) + 2*B*a**3*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d
*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*B*a**2*b*d*x*tan(c + d*x)/(
2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*B*a*b**2*log(a/b +
tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d
*x)) + 2*B*a*b**2*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2
*a**2*b**2*d*tan(c + d*x)), True))

```

**Giac** [A]

time = 0.66, size = 157, normalized size = 1.52

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{(Ba-Ab)\log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(Bab^3-Ab^4)\log(|b\tan(dx+c)+a|)}{a^4b+a^2b^3} - \frac{2(Ba-Ab)\log(|\tan(dx+c)|)}{a^2} + \frac{2(Ba\tan(dx+c)-Ab\tan(dx+c)+Aa)}{a^2\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] -1/2\*(2\*(A\*a + B\*b)\*(d\*x + c)/(a^2 + b^2) + (B\*a - A\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + 2\*(B\*a\*b^3 - A\*b^4)\*log(abs(b\*tan(d\*x + c) + a))/(a^4\*b + a^2\*b^3) - 2\*(B\*a - A\*b)\*log(abs(tan(d\*x + c)))/a^2 + 2\*(B\*a\*tan(d\*x + c) - A\*b\*tan(d\*x + c) + A\*a)/(a^2\*tan(d\*x + c)))/d

**Mupad** [B]

time = 7.70, size = 140, normalized size = 1.36

$$\frac{\ln(a + b\tan(c + dx))}{d(a^4 + a^2b^2)} - \frac{(Ab^3 - Ba^2b^2)}{a^2d} - \frac{\ln(\tan(c + dx))}{a^2d} + \frac{(Ab - Ba)}{2d(b + aIi)} + \frac{\ln(\tan(c + dx) + Ii)(A - BIi)}{2d(b + aIi)} - \frac{A \cot(c + dx)}{ad} + \frac{\ln(\tan(c + dx) - Ii)(-B + AIi)}{2d(a + bIi)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] (log(a + b\*tan(c + d\*x))\*(A\*b^3 - B\*a\*b^2))/(d\*(a^4 + a^2\*b^2)) - (log(tan(c + d\*x))\*(A\*b - B\*a))/(a^2\*d) + (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(2\*d\*(a\*1i + b)) - (A\*cot(c + d\*x))/(a\*d) + (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(a + b\*1i))

$$3.273 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=137

$$\frac{(Ab - aB)x}{a^2 + b^2} + \frac{(Ab - aB) \cot(c + dx)}{a^2 d} - \frac{A \cot^2(c + dx)}{2ad} - \frac{(a^2 A - Ab^2 + abB) \log(\sin(c + dx))}{a^3 d} - \frac{b^3 (Ab - aB) \log(\sin(c + dx))}{a^3 d}$$

[Out] (A\*b-B\*a)\*x/(a^2+b^2)+(A\*b-B\*a)\*cot(d\*x+c)/a^2/d-1/2\*A\*cot(d\*x+c)^2/a/d-(A\*a^2-A\*b^2+B\*a\*b)\*ln(sin(d\*x+c))/a^3/d-b^3\*(A\*b-B\*a)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/a^3/(a^2+b^2)/d

**Rubi [A]**

time = 0.37, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{x(Ab - aB)}{a^2 + b^2} + \frac{(Ab - aB) \cot(c + dx)}{a^2 d} - \frac{(a^2 A + abB - Ab^2) \log(\sin(c + dx))}{a^3 d} - \frac{b^3 (Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{a^3 d (a^2 + b^2)} - \frac{A \cot^2(c + dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((A\*b - a\*B)\*x)/(a^2 + b^2) + ((A\*b - a\*B)\*Cot[c + d\*x])/(a^2\*d) - (A\*Cot[c + d\*x]^2)/(2\*a\*d) - ((a^2\*A - A\*b^2 + a\*b\*B)\*Log[Sin[c + d\*x]])/(a^3\*d) - (b^3\*(A\*b - a\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/(a^3\*(a^2 + b^2)\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)], x], x]

2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&  
 NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  
 & (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]  
 || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3730

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) +  
 (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.)  
 + (f\_.)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e +  
 f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 +  
 b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f  
 \*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(  
 m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)  
 \*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan  
 [e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[  
 b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !  
 (ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3732

Int[((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2)/  
 (((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)  
 \*(x\_)])), x\_Symbol] := Simp[(a\*(A\*c - c\*C + B\*d) + b\*(B\*c - A\*d + C\*d))\*(x/  
 ((a^2 + b^2)\*(c^2 + d^2))), x] + (Dist[(A\*b^2 - a\*b\*B + a^2\*C)/((b\*c - a\*d)  
 \*(a^2 + b^2)), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] - Dist  
 [(c^2\*C - B\*c\*d + A\*d^2)/((b\*c - a\*d)\*(c^2 + d^2)), Int[(d - c\*Tan[e + f\*x]  
 )/(c + d\*Tan[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&  
 NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx &= -\frac{A \cot^2(c+dx)}{2ad} - \frac{\int \frac{\cot^2(c+dx)(2(Ab-aB)+2aA \tan(c+dx)+2Ab \tan^2(c+dx))}{a+b \tan(c+dx)} dx}{2a} \\ &= \frac{(Ab-aB) \cot(c+dx)}{a^2d} - \frac{A \cot^2(c+dx)}{2ad} + \frac{\int \frac{\cot(c+dx)(-2(a^2A-Ab^2))}{a+b \tan(c+dx)} dx}{2a} \\ &= \frac{(Ab-aB)x}{a^2+b^2} + \frac{(Ab-aB) \cot(c+dx)}{a^2d} - \frac{A \cot^2(c+dx)}{2ad} - \frac{(b^3(A-Ab^2))}{a^2+b^2} \\ &= \frac{(Ab-aB)x}{a^2+b^2} + \frac{(Ab-aB) \cot(c+dx)}{a^2d} - \frac{A \cot^2(c+dx)}{2ad} - \frac{(a^2A-Ab^2)}{a^2+b^2} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.94, size = 163, normalized size = 1.19

$$\frac{\frac{2(Ab-aB)\cot(c+dx)}{a^2} - \frac{A\cot^2(c+dx)}{a} + \frac{(A+iB)\log(i-\tan(c+dx))}{a+ib} - \frac{2(a^2A-Ab^2+abB)\log(\tan(c+dx))}{a^3} + \frac{(A-iB)\log(i+\tan(c+dx))}{a-ib} + \frac{2b^3(-Ab+aB)\log(a+b\tan(c+dx))}{a^3(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((2\*(A\*b - a\*B)\*Cot[c + d\*x])/a^2 - (A\*Cot[c + d\*x]^2)/a + ((A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*B) - (2\*(a^2\*A - A\*b^2 + a\*b\*B)\*Log[Tan[c + d\*x]])/a^3 + ((A - I\*B)\*Log[I + Tan[c + d\*x]])/(a - I\*B) + (2\*b^3\*(-A\*b) + a\*B)\*Log[a + b\*Tan[c + d\*x]]/(a^3\*(a^2 + b^2)))/(2\*d)

**Maple [A]**

time = 0.28, size = 152, normalized size = 1.11

method	result
derivativedivides	$-\frac{A}{2a \tan(dx+c)^2} - \frac{-Ab+aB}{a^2 \tan(dx+c)} + \frac{(-a^2A+Ab^2- Bab) \ln(\tan(dx+c))}{a^3} + \frac{(aA+Bb) \ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (Ab-aB) \arctan(\tan(dx+c))}{a^2+b^2}$
default	$-\frac{A}{2a \tan(dx+c)^2} - \frac{-Ab+aB}{a^2 \tan(dx+c)} + \frac{(-a^2A+Ab^2- Bab) \ln(\tan(dx+c))}{a^3} + \frac{(aA+Bb) \ln\left(\frac{1+\tan^2(dx+c)}{2}\right) + (Ab-aB) \arctan(\tan(dx+c))}{a^2+b^2}$
norman	$\frac{(Ab-aB) \tan(dx+c) + \frac{(Ab-aB)x(\tan^2(dx+c))}{a^2+b^2} - \frac{A}{2ad}}{\tan(dx+c)^2} + \frac{(aA+Bb) \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{(a^2A-Ab^2+ Bab) \ln(\tan(dx+c))}{a^3d}$
risch	$\frac{xB}{ib-a} + \frac{2iBbc}{a^2d} - \frac{2iAb^2c}{a^3d} + \frac{2iAc}{ad} - \frac{2ib^3Bc}{a^2d(a^2+b^2)} + \frac{ixA}{ib-a} - \frac{2i(iAa e^{2i(dx+c)} - b e^{2i(dx+c)} A + aB e^{2i(dx+c)} + Ab-a)}{a^2d(e^{2i(dx+c)}-1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(-1/2/a\*A/tan(d\*x+c)^2-(-A\*b+B\*a)/a^2/tan(d\*x+c)+1/a^3\*(-A\*a^2+A\*b^2-B\*a\*b)\*ln(tan(d\*x+c))+1/(a^2+b^2)\*(1/2\*(A\*a+B\*b)\*ln(1+tan(d\*x+c)^2)+(A\*b-B\*a)\*arctan(tan(d\*x+c)))-(A\*b-B\*a)\*b^3/a^3/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.54, size = 158, normalized size = 1.15

$$\frac{\frac{2(Ba-Ab)(dx+c)}{a^2+b^2} - \frac{2(Bab^3-Ab^4)\log(b\tan(dx+c)+a)}{a^5+a^3b^2} - \frac{(Aa+Bb)\log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(Aa^2+Bab-Ab^2)\log(\tan(dx+c))}{a^3} + \frac{Aa+2(Ba-Ab)\tan(dx+c)}{a^2\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) - 2*(B*a*b^3 - A*b^4)*\log(b*\tan(d*x + c) + a)/(a^5 + a^3*b^2) - (A*a + B*b)*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2) + 2*(A*a^2 + B*a*b - A*b^2)*\log(\tan(d*x + c))/a^3 + (A*a + 2*(B*a - A*b)*\tan(d*x + c))/(a^2*\tan(d*x + c)^2))/d$$

**Fricas** [A]

time = 1.57, size = 234, normalized size = 1.71

$$\frac{Aa^4 + Aa^2b^2 + (Aa^4 + Ba^3b + Bab^3 - Ab^4) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^2 - (Bab^3 - Ab^4) \log\left(\frac{b^2 \tan(dx+c)^2 - 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2+1}\right) \tan(dx+c)^2 + (Aa^4 + Aa^2b^2 + 2(Ba^4 - Aa^3b)dx) \tan(dx+c)^2 + 2(Ba^4 - Aa^3b + Ba^2b^2 - Aab^3) \tan(dx+c)}{2(a^5 + a^3b^2)d \tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] 
$$-1/2*(A*a^4 + A*a^2*b^2 + (A*a^4 + B*a^3*b + B*a*b^3 - A*b^4)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^2 - (B*a*b^3 - A*b^4)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1))*\tan(d*x + c)^2 + (A*a^4 + A*a^2*b^2 + 2*(B*a^4 - A*a^3*b)*d*x)*\tan(d*x + c)^2 + 2*(B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*\tan(d*x + c))/((a^5 + a^3*b^2)*d*\tan(d*x + c)^2)$$

**Sympy** [C] Result contains complex when optimal does not.

time = 3.26, size = 2596, normalized size = 18.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)`

[Out] 
$$\text{Piecewise}((\text{zoo}*A*x, \text{Eq}(a, 0) \ \& \ \text{Eq}(b, 0) \ \& \ \text{Eq}(c, 0) \ \& \ \text{Eq}(d, 0)), ((A*\log(\tan(c + d*x)**2 + 1)/(2*d) - A*\log(\tan(c + d*x))/d - A/(2*d*\tan(c + d*x)**2) - B*x - B/(d*\tan(c + d*x)))/a, \text{Eq}(b, 0)), ((A*x + A/(d*\tan(c + d*x)) - A/(3*d*\tan(c + d*x)**3) + B*\log(\tan(c + d*x)**2 + 1)/(2*d) - B*\log(\tan(c + d*x))/d - B/(2*d*\tan(c + d*x)**2))/b, \text{Eq}(a, 0)), (-3*A*d*x*\tan(c + d*x)**3/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) + 3*I*A*d*x*\tan(c + d*x)**2/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) + 2*I*A*\log(\tan(c + d*x)**2 + 1)*\tan(c + d*x)**3/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) + 2*A*\log(\tan(c + d*x)**2 + 1)*\tan(c + d*x)**2/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 4*I*A*\log(\tan(c + d*x))*\tan(c + d*x)**3/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 4*A*\log(\tan(c + d*x))*\tan(c + d*x)**2/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 3*A*\tan(c + d*x)**2/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) + I*A*\tan(c + d*x)/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - A/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 3*I*B*d*x*\tan(c + d*x)**3/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 3*B*d*x*\tan(c + d*x)**2/(2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - B*\log(\tan(c + d*x)**2 + 1)*\tan(c + d*x)**3/(2*b*$$

$$\begin{aligned}
& d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) + I*B*\log(\tan(c + d*x)**2 + 1) \\
& * \tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) + 2*B*\log(\tan(c + d*x)) * \tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 2*I*B*\log(\tan(c + d*x)) * \tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 3*I*B*\tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2) - 2*B*\tan(c + d*x) / (2*b*d*\tan(c + d*x)**3 - 2*I*b*d*\tan(c + d*x)**2), Eq(a, -I*b)), (-3*A*d*x*\tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - 3*I*A*d*x*\tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - 2*I*A*\log(\tan(c + d*x)**2 + 1) * \tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) + 2*A*\log(\tan(c + d*x)**2 + 1) * \tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) + 4*I*A*\log(\tan(c + d*x)) * \tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - 4*A*\log(\tan(c + d*x)) * \tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - 3*A*\tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - I*A*\tan(c + d*x) / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - A / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) + 3*I*B*d*x*\tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - 3*B*d*x*\tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - B*\log(\tan(c + d*x)**2 + 1) * \tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - I*B*\log(\tan(c + d*x)**2 + 1) * \tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) + 2*B*\log(\tan(c + d*x)) * \tan(c + d*x)**3 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) + 2*I*B*\log(\tan(c + d*x)) * \tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) + 3*I*B*\tan(c + d*x)**2 / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2) - 2*B*\tan(c + d*x) / (2*b*d*\tan(c + d*x)**3 + 2*I*b*d*\tan(c + d*x)**2), Eq(a, I*b)), (zoo*A*x/a, Eq(c, -d*x)), (x*(A + B*tan(c))*cot(c)**3 / (a + b*tan(c)), Eq(d, 0)), (A*a**4*log(\tan(c + d*x)**2 + 1) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - 2*A*a**4*log(\tan(c + d*x)) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - A*a**4 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) + 2*A*a**3*b*d*x*\tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) + 2*A*a**3*b*\tan(c + d*x) / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - A*a**2*b**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) + 2*A*a*b**3*\tan(c + d*x) / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - 2*A*b**4*log(a/b + \tan(c + d*x)) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) + 2*A*b**4*log(\tan(c + d*x)) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - 2*B*a**4*d*x*\tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - 2*B*a**4*\tan(c + d*x) / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) + B*a**3*b*log(\tan(c + d*x)**2 + 1) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - 2*B*a**3*b*log(\tan(c + d*x)) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) - 2*B*a**2*b**2*\tan(c + d*x) / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2) + 2*B*a*b**3*log(a/b + \tan(c + d*x)) * \tan(c + d*x)**2 / (2*a**5*d*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2)
\end{aligned}$$

$d*\tan(c + d*x)**2) - 2*B*a*b**3*\log(\tan(c + d*x))*\tan(c + d*x)**2/(2*a**5*d$   
 $*\tan(c + d*x)**2 + 2*a**3*b**2*d*\tan(c + d*x)**2), True))$

**Giac [A]**

time = 0.81, size = 214, normalized size = 1.56

$$\frac{\frac{2(Ba-Ab)(dx+c)}{a^2+b^2} - \frac{(Aa+Bb)\log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2(Bab^4-Ab^5)\log(b\tan(dx+c)+a)}{a^5b+a^4b^4} + \frac{2(Aa^2+Bab-Ab^2)\log(\tan(dx+c))}{a^3} - \frac{3Aa^2\tan(dx+c)^2+3Bab\tan(dx+c)^2-3Ab^2\tan(dx+c)^2-2Ba^2\tan(dx+c)+2Aab\tan(dx+c)-Aa^2}{a^3\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) - (A*a + B*b)*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*(B*a*b^4 - A*b^5)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^5*b + a^3*b^3) + 2*(A*a^2 + B*a*b - A*b^2)*\log(\text{abs}(\tan(d*x + c)))/a^3 - (3*A*a^2*\tan(d*x + c)^2 + 3*B*a*b*\tan(d*x + c)^2 - 3*A*b^2*\tan(d*x + c)^2 - 2*B*a^2*\tan(d*x + c) + 2*A*a*b*\tan(d*x + c) - A*a^2)/(a^3*\tan(d*x + c)^2))/d$

**Mupad [B]**

time = 8.15, size = 175, normalized size = 1.28

$$\frac{\cot(c+dx)^2\left(\frac{A}{2a} - \frac{\tan(c+dx)(A-Ba)}{a^2}\right)}{d} + \frac{\ln(\tan(c+dx)-i)(-B+Ai)}{2d(-b+ai)} - \frac{\ln(\tan(c+dx))(Aa^2+Bab-Ab^2)}{a^3d} - \frac{\ln(a+b\tan(c+dx))(Ab^4-Bab^3)}{d(a^5+a^3b^2)} + \frac{\ln(\tan(c+dx)+i)(A-Bi)}{2d(a-bi)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out]  $(\log(\tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*1i - b)) - (\cot(c + d*x)^2*(A/(2*a) - (\tan(c + d*x)*(A*b - B*a))/a^2))/d - (\log(\tan(c + d*x))*(A*a^2 - A*b^2 + B*a*b))/(a^3*d) - (\log(a + b*\tan(c + d*x))*(A*b^4 - B*a*b^3))/(d*(a^5 + a^3*b^2)) + (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a - b*1i))$

$$3.274 \quad \int \frac{\cot^4(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=169

$$\frac{(aA + bB)x}{a^2 + b^2} + \frac{(a^2A - Ab^2 + abB) \cot(c + dx)}{a^3d} + \frac{(Ab - aB) \cot^2(c + dx)}{2a^2d} - \frac{A \cot^3(c + dx)}{3ad} + \frac{(a^2 - b^2)(Ab - aB) \cot^4(c + dx)}{a^4d}$$

[Out] (A\*a+B\*b)\*x/(a^2+b^2)+(A\*a^2-A\*b^2+B\*a\*b)\*cot(d\*x+c)/a^3/d+1/2\*(A\*b-B\*a)\*cot(d\*x+c)^2/a^2/d-1/3\*A\*cot(d\*x+c)^3/a/d+(a^2-b^2)\*(A\*b-B\*a)\*ln(sin(d\*x+c))/a^4/d+b^4\*(A\*b-B\*a)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/a^4/(a^2+b^2)/d

**Rubi [A]**

time = 0.56, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{x(aA + bB)}{a^2 + b^2} + \frac{(Ab - aB) \cot^2(c + dx)}{2a^2d} + \frac{(a^2 - b^2)(Ab - aB) \log(\sin(c + dx))}{a^4d} + \frac{b^4(Ab - aB) \log(a \cos(c + dx) + b \sin(c + dx))}{a^4d(a^2 + b^2)} + \frac{(a^2A + abB - Ab^2) \cot(c + dx)}{a^3d} - \frac{A \cot^3(c + dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^4\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((a\*A + b\*B)\*x)/(a^2 + b^2) + ((a^2\*A - A\*b^2 + a\*b\*B)\*Cot[c + d\*x])/(a^3\*d) + ((A\*b - a\*B)\*Cot[c + d\*x]^2)/(2\*a^2\*d) - (A\*Cot[c + d\*x]^3)/(3\*a\*d) + ((a^2 - b^2)\*(A\*b - a\*B)\*Log[Sin[c + d\*x]])/(a^4\*d) + (b^4\*(A\*b - a\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/(a^4\*(a^2 + b^2)\*d)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2))



```
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rubi steps

$$\int \frac{\cot^4(c + dx)(A + B \tan(c + dx))}{a + b \tan(c + dx)} dx = -\frac{A \cot^3(c + dx)}{3ad} - \frac{\int \frac{\cot^3(c+dx)(3(Ab-aB)+3aA \tan(c+dx)+3Ab \tan^2(c+dx))}{a+b \tan(c+dx)}}{3a}$$

$$= \frac{(Ab - aB) \cot^2(c + dx)}{2a^2d} - \frac{A \cot^3(c + dx)}{3ad} + \frac{\int \frac{\cot^2(c+dx)(-6(a^2A - Ab^2 + abB))}{a^2 + b^2}}{2a^2d}$$

$$= \frac{(a^2A - Ab^2 + abB) \cot(c + dx)}{a^3d} + \frac{(Ab - aB) \cot^2(c + dx)}{2a^2d} - \frac{A \cot^3(c + dx)}{3ad}$$

$$= \frac{(aA + bB)x}{a^2 + b^2} + \frac{(a^2A - Ab^2 + abB) \cot(c + dx)}{a^3d} + \frac{(Ab - aB) \cot^2(c + dx)}{2a^2d}$$

$$= \frac{(aA + bB)x}{a^2 + b^2} + \frac{(a^2A - Ab^2 + abB) \cot(c + dx)}{a^3d} + \frac{(Ab - aB) \cot^2(c + dx)}{2a^2d}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.64, size = 194, normalized size = 1.15

$$\frac{\frac{6(a^2A - Ab^2 + abB) \cot(c+dx)}{a^3} + \frac{3(Ab - aB) \cot^2(c+dx)}{a^2} - \frac{2A \cot^3(c+dx)}{a} + \frac{3(-IA + B) \log(i - \tan(c+dx))}{a+ib} + \frac{6(a-b)(a+b)(Ab - aB) \log(\tan(c+dx))}{a^4} + \frac{3(iA + B) \log(i + \tan(c+dx))}{a-ib} + \frac{6b^4(Ab - aB) \log(a+b \tan(c+dx))}{a^4(a^2 + b^2)}}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^4\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((6\*(a^2\*A - A\*b^2 + a\*b\*B)\*Cot[c + d\*x])/a^3 + (3\*(A\*b - a\*B)\*Cot[c + d\*x]^2)/a^2 - (2\*A\*Cot[c + d\*x]^3)/a + (3\*((-I)\*A + B)\*Log[I - Tan[c + d\*x]])/(a + I\*b) + (6\*(a - b)\*(a + b)\*(A\*b - a\*B)\*Log[Tan[c + d\*x]])/a^4 + (3\*(I\*A + B)\*Log[I + Tan[c + d\*x]])/(a - I\*b) + (6\*b^4\*(A\*b - a\*B)\*Log[a + b\*Tan[c + d\*x]])/(a^4\*(a^2 + b^2)))/(6\*d)

**Maple [A]**

time = 0.28, size = 189, normalized size = 1.12

method	result
derivativedivides	$\frac{(A a^2 b - A b^3 - B a^3 + B a b^2) \ln(\tan(dx+c))}{a^4} - \frac{-Ab+aB}{2a^2 \tan(dx+c)^2} - \frac{A}{3a \tan(dx+c)^3} - \frac{-a^2A+Ab^2-Bab}{a^3 \tan(dx+c)} + \frac{(-Ab+aB) \ln(1+\tan^2(dx+c))}{2} + \dots$
default	$\frac{(A a^2 b - A b^3 - B a^3 + B a b^2) \ln(\tan(dx+c))}{a^4} - \frac{-Ab+aB}{2a^2 \tan(dx+c)^2} - \frac{A}{3a \tan(dx+c)^3} - \frac{-a^2A+Ab^2-Bab}{a^3 \tan(dx+c)} + \frac{(-Ab+aB) \ln(1+\tan^2(dx+c))}{2} + \dots$
norman	$\frac{(aA+Bb)x(\tan^3(dx+c))}{a^2+b^2} + \frac{(a^2A - Ab^2 + Bab)(\tan^2(dx+c))}{a^3d} - \frac{A}{3ad} + \frac{(Ab - aB) \tan(dx+c)}{2a^2d} + \frac{(Ab - aB)(a^2 - b^2) \ln(\tan(dx+c))}{da^4} - \dots$
risch	$\frac{2ib^4Bx}{(a^2+b^2)a^3} - \frac{xA}{ib-a} - \frac{2ib^5Ac}{(a^2+b^2)da^4} - \frac{2iAbc}{a^2d} - \frac{2i(-3iAab e^{4i(dx+c)} + 3iB a^2 e^{4i(dx+c)} - 6A a^2 e^{4i(dx+c)} + 3A b^2 e^{4i(dx+c)})}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/a^4*(A*a^2*b-A*b^3-B*a^3+B*a*b^2)*ln(tan(d*x+c))-1/2*(-A*b+B*a)/a^2/
tan(d*x+c)^2-1/3/a*A/tan(d*x+c)^3-(-A*a^2+A*b^2-B*a*b)/a^3/tan(d*x+c)+1/(a^
2+b^2)*(1/2*(-A*b+B*a)*ln(1+tan(d*x+c)^2)+(A*a+B*b)*arctan(tan(d*x+c)))+(A*
b-B*a)*b^4/a^4/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

**Maxima** [A]

time = 0.50, size = 200, normalized size = 1.18

$$\frac{6(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{6(Bab^4-Ab^5)\log(b\tan(dx+c)+a)}{a^6+a^4b^2} + \frac{3(Ba-Ab)\log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{6(Ba^3-Aa^2b-Bab^2+Ab^3)\log(\tan(dx+c))}{a^4} - \frac{2Aa^2-6(Aa^2+Bab-Ab^2)\tan(dx+c)^2+3(Ba^2-Ab)\tan(dx+c)}{a^3\tan(dx+c)^3} \\ 6d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(6*(A*a + B*b)*(d*x + c)/(a^2 + b^2) - 6*(B*a*b^4 - A*b^5)*log(b*tan(d*
x + c) + a)/(a^6 + a^4*b^2) + 3*(B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
b^2) - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*log(tan(d*x + c))/a^4 - (2*A*a
^2 - 6*(A*a^2 + B*a*b - A*b^2)*tan(d*x + c)^2 + 3*(B*a^2 - A*a*b)*tan(d*x +
c))/(a^3*tan(d*x + c)^3))/d
```

**Fricas** [A]

time = 2.15, size = 293, normalized size = 1.73

$$\frac{2Aa^2+2Aa^2b^2+3(Ba^2-Aa^2b-Bab^2+Ab^3)\log\left(\frac{\tan(dx+c)}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3+3(Bab^4-Ab^5)\log\left(\frac{b^2\tan(dx+c)^2+2ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3+3(Ba^3-Aa^2b+Ba^2b^2-Aa^2b^2-2(Aa^2+Ba^2b)dx)\tan(dx+c)^3-6(Aa^2+Ba^2b+Ba^2b^2-Ab^3)\tan(dx+c)^3+3(Ba^2-Aa^2b+Ba^2b^2-Aa^2b^2)\tan(dx+c)}{6(a^4+a^2b^2)d\tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(2*A*a^5 + 2*A*a^3*b^2 + 3*(B*a^5 - A*a^4*b - B*a*b^4 + A*b^5)*log(tan
(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3 + 3*(B*a*b^4 - A*b^5)*log(
(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d
*x + c)^3 + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*(A*a^5 + B*a^4*b
)*d*x)*tan(d*x + c)^3 - 6*(A*a^5 + B*a^4*b + B*a^2*b^3 - A*a*b^4)*tan(d*x +
c)^2 + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3)*tan(d*x + c))/((a^6 + a
^4*b^2)*d*tan(d*x + c)^3)
```

**Sympy** [C] Result contains complex when optimal does not.

time = 5.51, size = 3012, normalized size = 17.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*A*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((A*x + A/(d*tan(c + d*x)) - A/(3*d*tan(c + d*x)**3) + B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2))/a, Eq(b, 0)), ((-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*log(tan(c + d*x))/d + A/(2*d*tan(c + d*x)**2) - A/(4*d*tan(c + d*x)**4) + B*x + B/(d*tan(c + d*x)) - B/(3*d*tan(c + d*x)**3))/b, Eq(a, 0)), (15*I*A*d*x*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 15*A*d*x*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 6*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 6*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 12*A*log(tan(c + d*x))*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 12*I*A*log(tan(c + d*x))*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 15*I*A*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 9*A*tan(c + d*x)**2/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + I*A*tan(c + d*x)/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 2*A/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 9*B*d*x*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 9*I*B*d*x*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 6*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 6*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 12*I*B*log(tan(c + d*x))*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 9*B*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) + 3*I*B*tan(c + d*x)**2/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3) - 3*B*tan(c + d*x)/(6*b*d*tan(c + d*x)**4 - 6*I*b*d*tan(c + d*x)**3), Eq(a, -I*b)), (-15*I*A*d*x*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) + 15*A*d*x*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) + 6*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) + 6*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 12*A*log(tan(c + d*x))*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 12*I*A*log(tan(c + d*x))*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 15*I*A*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) + 9*A*tan(c + d*x)**2/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - I*A*tan(c + d*x)/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 2*A/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 9*B*d*x*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 9*I*B*d*x*tan(c + d*x)**3/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) - 6*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(6*b*d*tan(c + d*x)**4 + 6*I*b*d*tan(c + d*x)**3) + 6*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3`

$$\begin{aligned} & / (6*b*d*\tan(c + d*x)**4 + 6*I*b*d*\tan(c + d*x)**3) + 12*I*B*\log(\tan(c + d*x)) * \tan(c + d*x)**4 / (6*b*d*\tan(c + d*x)**4 + 6*I*b*d*\tan(c + d*x)**3) - 12*B * \log(\tan(c + d*x)) * \tan(c + d*x)**3 / (6*b*d*\tan(c + d*x)**4 + 6*I*b*d*\tan(c + d*x)**3) \\ & - 9*B*\tan(c + d*x)**3 / (6*b*d*\tan(c + d*x)**4 + 6*I*b*d*\tan(c + d*x)**3) - 3*I*B*\tan(c + d*x)**2 / (6*b*d*\tan(c + d*x)**4 + 6*I*b*d*\tan(c + d*x)**3) \\ & - 3*B*\tan(c + d*x) / (6*b*d*\tan(c + d*x)**4 + 6*I*b*d*\tan(c + d*x)**3), \\ & \text{Eq}(a, I*b), (\text{zoo}*A*x/a, \text{Eq}(c, -d*x)), (x*(A + B*\tan(c))*\cot(c)**4 / (a + b*\tan(c)), \text{Eq}(d, 0)), (6*A*a**5*d*x*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 6*A*a**5*\tan(c + d*x)**2 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 2*A*a**5 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 3*A*a**4*b*\log(\tan(c + d*x)**2 + 1)*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 6*A*a**4*b*\log(\tan(c + d*x))*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 3*A*a**4*b*\tan(c + d*x) / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 2*A*a**3*b**2 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 3*A*a**2*b**3*\tan(c + d*x) / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 6*A*a*b**4*\tan(c + d*x)**2 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 6*A*b**5*\log(a/b + \tan(c + d*x))*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 6*A*b**5*\log(\tan(c + d*x))*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 3*B*a**5*\log(\tan(c + d*x)**2 + 1)*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 6*B*a**5*\log(\tan(c + d*x))*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) - 3*B*a**5*\tan(c + d*x) / (6*a**6*d*\tan(c + d*x)**3 + 6*a**4*b**2*d*\tan(c + d*x)**3) + 6*B*a**4*b*d*x*\tan(c + d*x)**3 / (6*a**6*d*\tan(c + d*x)**3 + 6*a... \end{aligned}$$

**Giac** [A]

time = 0.93, size = 285, normalized size = 1.69

$$\frac{\frac{6(Aa+Ab)(dx+c)}{a^2+b^2} + \frac{3(Ba-Ab)\log(\tan(dx+c)+1)}{a^2+b^2} - \frac{6(Ba^2-Ab^2)\log(b\tan(dx+c)+a)}{a^2+b^2} - \frac{6(Ba^2-Aa^2-2Ab^2+Ab^2)\log(\tan(dx+c))}{a^2} + \frac{11Ba^3\tan(dx+c)^2-11Aa^2b\tan(dx+c)^2-11Bab^2\tan(dx+c)^2+11Aa^3\tan(dx+c)^2+6Aa^2b\tan(dx+c)^2+6Ba^2b\tan(dx+c)^2-6Aab^2\tan(dx+c)^2-3Ba^3\tan(dx+c)+3Aa^2b\tan(dx+c)-2Aa^3}{a^4\tan(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/6*(6*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 3*(B*a - A*b)*\log(\tan(d*x + c))^2 + 1)/(a^2 + b^2) - 6*(B*a*b^5 - A*b^6)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^6*b + a^4*b^3) - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*\log(\text{abs}(\tan(d*x + c)))/a^4 \\ & + (11*B*a^3*\tan(d*x + c)^3 - 11*A*a^2*b*\tan(d*x + c)^3 - 11*B*a*b^2*\tan(d*x + c)^3 + 11*A*b^3*\tan(d*x + c)^3 + 6*A*a^3*\tan(d*x + c)^2 + 6*B*a^2*b*\tan(d*x + c)^2 - 6*A*a*b^2*\tan(d*x + c)^2 - 3*B*a^3*\tan(d*x + c) + 3*A*a^2*b*\tan(d*x + c) - 2*A*a^3)/(a^4*\tan(d*x + c)^3)/d \end{aligned}$$

**Mupad** [B]

time = 8.34, size = 208, normalized size = 1.23

$$\frac{\cot(c+dx)^3 \left( \frac{\tan(c+dx)^2 (Aa^2+Bab-Ab^2)}{a^3} - \frac{A}{3a} + \frac{\tan(c+dx)(Ab-Ba)}{2a^2} \right)}{d} + \frac{\ln(a+b\tan(c+dx))(Ab^2-Bab^2)}{d(a^6+a^2b^2)} - \frac{\ln(\tan(c+dx))(Ba^3-Aa^2b-Bab^2+Ab^3)}{a^4d} - \frac{\ln(\tan(c+dx)+1)(A-B1i)}{2d(b+a1i)} - \frac{\ln(\tan(c+dx)-1)(-B+A1i)}{2d(a+b1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^4\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] (cot(c + d\*x)^3\*((tan(c + d\*x)^2\*(A\*a^2 - A\*b^2 + B\*a\*b))/a^3 - A/(3\*a) + (tan(c + d\*x)\*(A\*b - B\*a))/(2\*a^2)))/d + (log(a + b\*tan(c + d\*x))\*(A\*b^5 - B\*a\*b^4))/(d\*(a^6 + a^4\*b^2)) - (log(tan(c + d\*x))\*(A\*b^3 + B\*a^3 - A\*a^2\*b - B\*a\*b^2))/(a^4\*d) - (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(2\*d\*(a\*1i + b)) - (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(a + b\*1i))

$$3.275 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=208

$$-\frac{(2aAb - a^2B + b^2B)x}{(a^2 + b^2)^2} + \frac{(a^2A - Ab^2 + 2abB) \log(\cos(c + dx))}{(a^2 + b^2)^2 d} + \frac{a^2(a^2Ab + 3Ab^3 - 2a^3B - 4ab^2B) \log(a + b \tan(c + dx))}{b^3(a^2 + b^2)^2 d}$$

[Out]  $-(2Aa^2b - B^2a^2 + B^2b^2)x/(a^2 + b^2)^2 + (Aa^2 - Ab^2 + 2B^2ab) \ln(\cos(dx + c))/(a^2 + b^2)^2/d + a^2(Aa^2b + 3Ab^3 - 2a^3B - 4ab^2B) \ln(a + b \tan(dx + c))/b^3/(a^2 + b^2)^2/d - (Aa^2b - 2B^2a^2 - B^2b^2) \tan(dx + c)/b^2/(a^2 + b^2)/d + a(Ab - B^2a) \tan(dx + c)^2/b/(a^2 + b^2)/d/(a + b \tan(dx + c))$

**Rubi [A]**

time = 0.30, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3686, 3728, 3707, 3698, 31, 3556}

$$\frac{a(Ab - aB) \tan^2(c + dx)}{bd(a^2 + b^2)(a + b \tan(c + dx))} - \frac{(-2a^2B + aAb - b^2B) \tan(c + dx)}{b^2d(a^2 + b^2)} + \frac{(a^2A + 2abB - Ab^2) \log(\cos(c + dx))}{d(a^2 + b^2)^2} - \frac{x(a^2(-B) + 2aAb + b^2B)}{(a^2 + b^2)^2} + \frac{a^2(-2a^3B + a^2Ab - 4ab^2B + 3Ab^3) \log(a + b \tan(c + dx))}{b^3d(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out]  $-(((2a^2Ab - a^2B + b^2B)x)/(a^2 + b^2)^2) + ((a^2A - Ab^2 + 2a^2bB) \text{Log}[\text{Cos}[c + d*x]])/((a^2 + b^2)^2*d) + (a^2(a^2Ab + 3Ab^3 - 2a^3B - 4a^2b^2B) \text{Log}[a + b \text{Tan}[c + d*x]])/(b^3(a^2 + b^2)^2*d) - ((a^2Ab - 2a^2B - b^2B) \text{Tan}[c + d*x])/(b^2(a^2 + b^2)*d) + (a(Ab - aB) \text{Tan}[c + d*x]^2)/(b(a^2 + b^2)*d*(a + b \text{Tan}[c + d*x]))$

**Rule 31**

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 3556**

Int[tan[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3686**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n

```

+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

```

### Rule 3698

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
(f_.)*(x_)])^2), x_Symbol] := Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

```

### Rule 3707

```

Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)
]^2)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*A + b*B -
a*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
+ Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]

```

### Rule 3728

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.
) + (f_.)*(x_)])^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rubi steps





default	$\frac{B \tan(dx+c)}{b^2} + \frac{(-a^2 A + A b^2 - 2 B a b) \ln(1 + \tan^2(dx+c))}{2(a^2 + b^2)^2} + \frac{(-2 A a b + a^2 B - b^2 B) \arctan(\tan(dx+c))}{(a^2 + b^2)^2} + \frac{a^2 (A a^2 b + 3 A b^3 - 2 B a^3 - 4 B a b^2)}{b^3 (a^2 + b^2)^2}$
norman	$\frac{B(\tan^2(dx+c))}{bd} + \frac{(A a^2 b - 2 B a^3 - B a b^2) a}{d b^3 (a^2 + b^2)} - \frac{a(2 A a b - a^2 B + b^2 B) x}{a^4 + 2 a^2 b^2 + b^4} - \frac{b(2 A a b - a^2 B + b^2 B) x \tan(dx+c)}{a^4 + 2 a^2 b^2 + b^4} + \frac{a^2 (A a^2 b + 3 A b^3 - 2 B a^3 - 4 B a b^2)}{(a^4 + 2 a^2 b^2 + b^4) d}$
risch	$-\frac{x B}{2 i b a - a^2 + b^2} + \frac{4 i a^5 B x}{b^3 (a^4 + 2 a^2 b^2 + b^4)} - \frac{6 i a^2 A x}{a^4 + 2 a^2 b^2 + b^4} - \frac{4 i a B x}{b^3} - \frac{i x A}{2 i b a - a^2 + b^2} + \frac{2 i A c}{b^2 d} + \frac{2 i (-2 i B a^3 b e^{2 i (d x + c)})}{(a^4 + 2 a^2 b^2 + b^4) d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] 1/d\*(B/b^2\*tan(d\*x+c)+1/(a^2+b^2)^2\*(1/2\*(-A\*a^2+A\*b^2-2\*B\*a\*b)\*ln(1+tan(d\*x+c)^2)+(-2\*A\*a\*b+B\*a^2-B\*b^2)\*arctan(tan(d\*x+c)))+1/b^3\*a^2\*(A\*a^2\*b+3\*A\*b^3-2\*B\*a^3-4\*B\*a\*b^2)/(a^2+b^2)^2\*ln(a+b\*tan(d\*x+c))+1/b^3\*a^3\*(A\*b-B\*a)/(a^2+b^2)/(a+b\*tan(d\*x+c))

**Maxima [A]**

time = 0.51, size = 220, normalized size = 1.06

$$\frac{2(Ba^2 - 2Aab - Bb^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{2(2Ba^5 - Aa^4b + 4Ba^3b^2 - 3Aa^2b^3) \log(b \tan(dx+c) + a)}{a^4b^3 + 2a^2b^5 + b^7} - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(Ba^4 - Aa^3b)}{a^3b^3 + ab^5 + (a^2b^4 + b^6) \tan(dx+c)} + \frac{2B \tan(dx+c)}{b^2}$$

2 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/2\*(2\*(B\*a^2 - 2\*A\*a\*b - B\*b^2)\*(d\*x + c)/(a^4 + 2\*a^2\*b^2 + b^4) - 2\*(2\*B\*a^5 - A\*a^4\*b + 4\*B\*a^3\*b^2 - 3\*A\*a^2\*b^3)\*log(b\*tan(d\*x + c) + a)/(a^4\*b^3 + 2\*a^2\*b^5 + b^7) - (A\*a^2 + 2\*B\*a\*b - A\*b^2)\*log(tan(d\*x + c)^2 + 1)/(a^4 + 2\*a^2\*b^2 + b^4) - 2\*(B\*a^4 - A\*a^3\*b)/(a^3\*b^3 + a\*b^5 + (a^2\*b^4 + b^6)\*tan(d\*x + c)) + 2\*B\*tan(d\*x + c)/b^2)/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(209) = 418.

time = 1.54, size = 434, normalized size = 2.09

$$\frac{2(Ba^2 - 2Aab - Bb^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{2(2Ba^5 - Aa^4b + 4Ba^3b^2 - 3Aa^2b^3) \log(b \tan(dx+c) + a)}{a^4b^3 + 2a^2b^5 + b^7} - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(Ba^4 - Aa^3b)}{a^3b^3 + ab^5 + (a^2b^4 + b^6) \tan(dx+c)} + \frac{2B \tan(dx+c)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] -1/2\*(2\*B\*a^4\*b^2 - 2\*A\*a^3\*b^3 - 2\*(B\*a^3\*b^3 - 2\*A\*a^2\*b^4 - B\*a\*b^5)\*d\*x - 2\*(B\*a^4\*b^2 + 2\*B\*a^2\*b^4 + B\*b^6)\*tan(d\*x + c)^2 + (2\*B\*a^6 - A\*a^5\*b

$$+ 4*B*a^4*b^2 - 3*A*a^3*b^3 + (2*B*a^5*b - A*a^4*b^2 + 4*B*a^3*b^3 - 3*A*a^2*b^4)*\tan(d*x + c))*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - (2*B*a^6 - A*a^5*b + 4*B*a^4*b^2 - 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5 + (2*B*a^5*b - A*a^4*b^2 + 4*B*a^3*b^3 - 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*\tan(d*x + c))*\log(1/(\tan(d*x + c)^2 + 1)) - 2*(2*B*a^5*b - A*a^4*b^2 + 2*B*a^3*b^3 + B*a*b^5 + (B*a^2*b^4 - 2*A*a*b^5 - B*b^6)*d*x)*\tan(d*x + c)/((a^4*b^4 + 2*a^2*b^6 + b^8)*d*\tan(d*x + c) + (a^5*b^3 + 2*a^3*b^5 + a*b^7)*d)$$

**Sympy** [C] Result contains complex when optimal does not.

time = 1.10, size = 4534, normalized size = 21.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*x\*(A + B\*tan(c))\*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*tan(c + d\*x)\*\*2/(2\*d) + B\*x + B\*tan(c + d\*x)\*\*3/(3\*d) - B\*tan(c + d\*x)/d)/a\*\*2, Eq(b, 0)), (3\*I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 6\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 3\*I\*A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*I\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 2\*A\*log(tan(c + d\*x)\*\*2 + 1)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 5\*I\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*A/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 9\*B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 18\*I\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 9\*B\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 4\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 8\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 4\*B\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 19\*B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 14\*I\*B/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d), Eq(a, -I\*b)), (-3\*I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 6\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 3\*I\*A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d

$\tan(c + dx) - 4b^{**2}d) + 4IA \cdot \log(\tan(c + dx)**2 + 1) \cdot \tan(c + dx) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) - 2A \cdot \log(\tan(c + dx)**2 + 1) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 5IA \cdot \tan(c + dx) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) - 4A / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) - 9B \cdot dx \cdot \tan(c + dx)**2 / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) - 18IB \cdot dx \cdot \tan(c + dx) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 9B \cdot dx / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) - 4IB \cdot \log(\tan(c + dx)**2 + 1) \cdot \tan(c + dx)**2 / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 8B \cdot \log(\tan(c + dx)**2 + 1) \cdot \tan(c + dx) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 4IB \cdot \log(\tan(c + dx)**2 + 1) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 4B \cdot \tan(c + dx)**3 / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 19B \cdot \tan(c + dx) / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d) + 14IB / (4b^{**2}d \cdot \tan(c + dx)**2 + 8Ib^{**2}d \cdot \tan(c + dx) - 4b^{**2}d),$   
 Eq(a, I\*b)), (x\*(A + B\*tan(c))\*tan(c)\*\*3/(a + b\*tan(c))\*\*2, Eq(d, 0)), (2\*A\*a\*\*5\*b\*log(a/b + tan(c + dx))/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) + 2\*A\*a\*\*5\*b/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) + 2\*A\*a\*\*4\*b\*\*2\*log(a/b + tan(c + dx))\*tan(c + dx)/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) + 6\*A\*a\*\*3\*b\*\*3\*log(a/b + tan(c + dx))/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) - A\*a\*\*3\*b\*\*3\*log(tan(c + dx)\*\*2 + 1)/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) + 2\*A\*a\*\*3\*b\*\*3/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) - 4\*A\*a\*\*2\*b\*\*4\*d\*x/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) + 6\*A\*a\*\*2\*b\*\*4\*log(a/b + tan(c + dx))\*tan(c + dx)/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) - A\*a\*\*2\*b\*\*4\*log(tan(c + dx)\*\*2 + 1)\*tan(c + dx)/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) - 4\*A\*a\*b\*\*5\*d\*x\*tan(c + dx)/(2\*a\*\*5\*b\*\*3\*d + 2\*a\*\*4\*b\*\*4\*d\*tan(c + dx) + 4\*a\*\*3\*b\*\*5\*d + 4\*a\*\*2\*b\*\*6\*d\*tan(c + dx) + 2\*a\*b\*\*7\*d + 2\*b\*\*8\*d\*tan(c + dx)) + A...

**Giac [A]**

time = 0.82, size = 290, normalized size = 1.39

$$\frac{2(Ba^2 - 2Aab - Bb^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(2Ba^3 - Aa^2b + 4Ba^2b^2 - 3Aa^2b^2) \log(|b \tan(dx+c) + a|)}{a^4b^2 + 2a^2b^2 + b^4} + \frac{2B \tan(dx+c)}{b^2} + \frac{2(2Ba^3b \tan(dx+c) - Aa^4b^2 \tan(dx+c) + 4Ba^3b^2 \tan(dx+c) - 3Aa^3b^2 \tan(dx+c) + Ba^6 + 3Ba^4b^2 - 2Aa^2b^2)}{(a^4b^2 + 2a^2b^2 + b^4)(b \tan(dx+c) + a)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot (2 \cdot (B \cdot a^2 - 2 \cdot A \cdot a \cdot b - B \cdot b^2) \cdot (d \cdot x + c) / (a^4 + 2 \cdot a^2 \cdot b^2 + b^4) - (A \cdot a^2 + 2 \cdot B \cdot a \cdot b - A \cdot b^2) \cdot \log(\tan(d \cdot x + c)^2 + 1) / (a^4 + 2 \cdot a^2 \cdot b^2 + b^4) - 2 \cdot (2 \cdot B \cdot a^5 - A \cdot a^4 \cdot b + 4 \cdot B \cdot a^3 \cdot b^2 - 3 \cdot A \cdot a^2 \cdot b^3) \cdot \log(\text{abs}(b \cdot \tan(d \cdot x + c) + a)) / (a^4 \cdot b^3 + 2 \cdot a^2 \cdot b^5 + b^7) + 2 \cdot B \cdot \tan(d \cdot x + c) / b^2 + 2 \cdot (2 \cdot B \cdot a^5 \cdot b \cdot \tan(d \cdot x + c) - A \cdot a^4 \cdot b^2 \cdot \tan(d \cdot x + c) + 4 \cdot B \cdot a^3 \cdot b^3 \cdot \tan(d \cdot x + c) - 3 \cdot A \cdot a^2 \cdot b^4 \cdot \tan(d \cdot x + c) + B \cdot a^6 + 3 \cdot B \cdot a^4 \cdot b^2 - 2 \cdot A \cdot a^3 \cdot b^3) / ((a^4 \cdot b^3 + 2 \cdot a^2 \cdot b^5 + b^7) \cdot (b \cdot \tan(d \cdot x + c) + a))) / d$

**Mupad [B]**

time = 7.19, size = 210, normalized size = 1.01

$$\frac{B \tan(c + dx)}{b^2 d} - \frac{\ln(a + b \tan(c + dx)) (2 B a^5 - A a^4 b + 4 B a^3 b^2 - 3 A a^2 b^3)}{d (a^4 b^3 + 2 a^2 b^5 + b^7)} - \frac{\ln(\tan(c + dx) - i) (A + B i)}{2 d (a^2 + a b 2i - b^2)} - \frac{\ln(\tan(c + dx) + i) (B + A i)}{2 d (a^2 1i + 2 a b - b^2 1i)} - \frac{a^2 (B a^2 - A a b)}{b d (\tan(c + dx) b^3 + a b^2) (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out]  $(B \cdot \tan(c + d \cdot x)) / (b^2 \cdot d) - (\log(a + b \cdot \tan(c + d \cdot x)) \cdot (2 \cdot B \cdot a^5 - 3 \cdot A \cdot a^2 \cdot b^3 + 4 \cdot B \cdot a^3 \cdot b^2 - A \cdot a^4 \cdot b)) / (d \cdot (b^7 + 2 \cdot a^2 \cdot b^5 + a^4 \cdot b^3)) - (\log(\tan(c + d \cdot x) - 1i) \cdot (A + B \cdot 1i)) / (2 \cdot d \cdot (a \cdot b \cdot 2i + a^2 - b^2)) - (\log(\tan(c + d \cdot x) + 1i) \cdot (A \cdot 1i + B)) / (2 \cdot d \cdot (2 \cdot a \cdot b + a^2 \cdot 1i - b^2 \cdot 1i)) - (a^2 \cdot (B \cdot a^2 - A \cdot a \cdot b)) / (b \cdot d \cdot (a \cdot b^2 + b^3 \cdot \tan(c + d \cdot x)) \cdot (a^2 + b^2))$

$$3.276 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=157

$$\frac{(a^2 A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} - \frac{(2aAb - a^2 B + b^2 B) \log(\cos(c + dx))}{(a^2 + b^2)^2 d} - \frac{a(2Ab^3 - a(a^2 + 3b^2)B) \log(a + b \tan(c + dx))}{b^2 (a^2 + b^2)^2 d}$$

[Out]  $-(A*a^2 - A*b^2 + 2*B*a*b)*x/(a^2 + b^2)^2 - (2*A*a*b - B*a^2 + B*b^2)*\ln(\cos(d*x + c))/(a^2 + b^2)^2/d - a*(2*A*b^3 - a*(a^2 + 3*b^2)*B)*\ln(a + b*\tan(d*x + c))/b^2/(a^2 + b^2)^2/d - a^2*(A*b - B*a)/b^2/(a^2 + b^2)/d/(a + b*\tan(d*x + c))$

**Rubi [A]**

time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3685, 3707, 3698, 31, 3556}

$$\frac{a^2(Ab - aB)}{b^2 d (a^2 + b^2) (a + b \tan(c + dx))} - \frac{(a^2(-B) + 2aAb + b^2 B) \log(\cos(c + dx))}{d (a^2 + b^2)^2} - \frac{x(a^2 A + 2abB - Ab^2)}{(a^2 + b^2)^2} - \frac{a(2Ab^3 - aB(a^2 + 3b^2)) \log(a + b \tan(c + dx))}{b^2 d (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c + d*x]^2*(A + B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x])^2, x]$

[Out]  $-(((a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2)^2) - ((2*a*A*b - a^2*B + b^2*B)*\text{Log}[\text{Cos}[c + d*x]])/((a^2 + b^2)^2*d) - (a*(2*A*b^3 - a*(a^2 + 3*b^2)*B)*\text{Log}[a + b*\text{Tan}[c + d*x]])/(b^2*(a^2 + b^2)^2*d) - (a^2*(A*b - a*B))/(b^2*(a^2 + b^2)*d*(a + b*\text{Tan}[c + d*x]))$

**Rule 31**

$\text{Int}[(a + (b*x)^{-1}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

**Rule 3556**

$\text{Int}[\tan[(c + (d*x))], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3685**

$\text{Int}[(a + (b*x)*\tan[(e + (f*x))])^2*((A + (B*x)*\tan[(e + (f*x))*x])*((c + (d*x)*\tan[(e + (f*x))])^n), x\_Symbol] \rightarrow \text{Simp}[-(B*c - A*d)*(b*c - a*d)^2*((c + d*\text{Tan}[e + f*x])^{n+1}/(f*d^2*(n+1)*(c^2 + d^2))), x] + \text{Dist}[1/(d*(c^2 + d^2)), \text{Int}[(c + d*\text{Tan}[e + f*x])^{n+1}]*\text{Simp}[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*\text{Tan}[e + f*x] + b^2*B*(c^2 + d^2)*\text{Tan}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*$

$c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[n, -1]$

### Rule 3698

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \rightarrow \text{Dist}[A/(b*f), \text{Subst}[\text{Int}[(a + x)^m, x], x, b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, A, C, m\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3707

$\text{Int}[(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)] + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2)/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(a*A + b*B - a*C)*(x/(a^2 + b^2)), x] + (\text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x]), x], x] - \text{Dist}[(A*b - a*B - b*C)/(a^2 + b^2), \text{Int}[\text{Tan}[e + f*x], x], x]) /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[A*b - a*B - b*C, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^2} dx &= -\frac{a^2(Ab - aB)}{b^2(a^2 + b^2)d(a + b \tan(c + dx))} + \frac{\int \frac{-a(Ab - aB) + b(Ab - aB) \tan(c + dx)}{a + b \tan(c + dx)} dx}{b(a^2 + b^2)} \\ &= -\frac{(a^2A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} - \frac{a^2(Ab - aB)}{b^2(a^2 + b^2)d(a + b \tan(c + dx))} + \frac{(2aAb - a^2B + b^2B) \log(\cos(c + dx))}{(a^2 + b^2)^2 d} \\ &= -\frac{(a^2A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} - \frac{(2aAb - a^2B + b^2B) \log(\cos(c + dx))}{(a^2 + b^2)^2 d} \\ &= -\frac{(a^2A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} - \frac{(2aAb - a^2B + b^2B) \log(\cos(c + dx))}{(a^2 + b^2)^2 d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.48, size = 323, normalized size = 2.06

$\frac{a(2a + b)^2(-Ab^2 + a(a + 2bB)(c + dx) - 2a^2 + b^2)B \log(\cos(c + dx)) + a(-2aB^2 + a(a^2 + 2b^2)B) \log(\cos(c + dx)) + b \sin(c + dx)^2}{2b^2(a^2 + b^2)^2(d + b \tan(c + dx))} + \frac{a(2a + b)(-1 - 4b^2(c + dx) + a^2B(c + dx) - a^2(-2bB(c + dx) + A) + (c + dx)) + a^2(A + B)(c + dx)}{2b^2(a^2 + b^2)^2(d + b \tan(c + dx))} - \frac{2a^2 + b^2}{b^2} B \log(\cos(c + dx)) + a(-2aB^2 + a(a^2 + 2b^2)B) \log(\cos(c + dx)) + b \sin(c + dx)^2}{2b^2(a^2 + b^2)^2(d + b \tan(c + dx))} \tan(c + dx) - \frac{2a(-2aB^2 + a(a^2 + 2b^2)B) \text{ArcTan}(\tan(c + dx)) + b \sin(c + dx)}{2b^2(a^2 + b^2)^2(d + b \tan(c + dx))}$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (a\*(2\*(a + I\*b)^2\*(-(A\*b^2) + a\*(I\*a + 2\*b)\*B)\*(c + d\*x) - 2\*(a^2 + b^2)^2\*B\*Log[Cos[c + d\*x]] + a\*(-2\*A\*b^3 + a\*(a^2 + 3\*b^2)\*B)\*Log[(a\*Cos[c + d\*x] + b\*Sin[c + d\*x])^2]) + b\*(2\*(a + I\*b)\*((-I)\*A\*b^3\*(c + d\*x) + I\*a^3\*B\*(I +

$$c + d*x) - a*b^2*((-2*I)*B*(c + d*x) + A*(I + c + d*x)) + a^2*b*(A + B*(I + c + d*x))) - 2*(a^2 + b^2)^2*B*Log[Cos[c + d*x]] + a*(-2*A*b^3 + a*(a^2 + 3*b^2)*B)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2]*Tan[c + d*x] - (2*I)*a*(-2*A*b^3 + a*(a^2 + 3*b^2)*B)*ArcTan[Tan[c + d*x]]*(a + b*Tan[c + d*x])/ (2*b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))$$

Maple [A]

time = 0.18, size = 155, normalized size = 0.99

method	result
derivativdivides	$\frac{(2Aab - a^2B + b^2B) \ln(1 + \tan^2(dx+c)) + (-a^2A + Ab^2 - 2Bab) \arctan(\tan(dx+c))}{(a^2 + b^2)^2} - \frac{a^2(Ab - aB)}{b^2(a^2 + b^2)(a + b \tan(dx+c))} - \frac{a(2Ab^3 - Ba^3 - 3Bab)}{(a^2 + b^2)^2} d$
default	$\frac{(2Aab - a^2B + b^2B) \ln(1 + \tan^2(dx+c)) + (-a^2A + Ab^2 - 2Bab) \arctan(\tan(dx+c))}{(a^2 + b^2)^2} - \frac{a^2(Ab - aB)}{b^2(a^2 + b^2)(a + b \tan(dx+c))} - \frac{a(2Ab^3 - Ba^3 - 3Bab)}{(a^2 + b^2)^2} d$
norman	$-\frac{a(a^2A - Ab^2 + 2Bab)x}{a^4 + 2a^2b^2 + b^4} - \frac{b(a^2A - Ab^2 + 2Bab)x \tan(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{(Ab - a^2B)a}{db^2(a^2 + b^2)} + \frac{(2Aab - a^2B + b^2B) \ln(1 + \tan^2(dx+c))}{2d(a^4 + 2a^2b^2 + b^4)} - \frac{a(2Ab^3 - Ba^3 - 3Bab)}{(a^2 + b^2)^2} d$
risch	$\frac{4iabAx}{a^4 + 2a^2b^2 + b^4} + \frac{xA}{2iba - a^2 + b^2} + \frac{4iabAc}{(a^4 + 2a^2b^2 + b^4)d} - \frac{2ia^4Bc}{(a^4 + 2a^2b^2 + b^4)d b^2} + \frac{2iBx}{b^2} + \frac{2ia^2A}{(ib+a)d(-ib+a)^2(-ibe^{2i(dx+c)})}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERBOS E)

[Out] 1/d\*(1/(a^2+b^2)^2\*(1/2\*(2\*A\*a\*b-B\*a^2+B\*b^2)\*ln(1+tan(d\*x+c)^2)+(-A\*a^2+A\*b^2-2\*B\*a\*b)\*arctan(tan(d\*x+c)))-a^2\*(A\*b-B\*a)/b^2/(a^2+b^2)/(a+b\*tan(d\*x+c))-a\*(2\*A\*b^3-B\*a^3-3\*B\*a\*b^2)/(a^2+b^2)^2/b^2\*ln(a+b\*tan(d\*x+c)))

Maxima [A]

time = 0.54, size = 197, normalized size = 1.25

$$\frac{\frac{2(Aa^2 + 2Bab - Ab^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{2(Ba^4 + 3Ba^2b^2 - 2Aab^3) \log(b \tan(dx+c) + a)}{a^4b^2 + 2a^2b^4 + b^6} + \frac{(Ba^2 - 2Aab - Bb^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(Ba^3 - Aa^2b)}{a^3b^2 + ab^4 + (a^2b^3 + b^5) \tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] -1/2\*(2\*(A\*a^2 + 2\*B\*a\*b - A\*b^2)\*(d\*x + c)/(a^4 + 2\*a^2\*b^2 + b^4) - 2\*(B\*a^4 + 3\*B\*a^2\*b^2 - 2\*A\*a\*b^3)\*log(b\*tan(d\*x + c) + a)/(a^4\*b^2 + 2\*a^2\*b^4 + b^6) + (B\*a^2 - 2\*A\*a\*b - B\*b^2)\*log(tan(d\*x + c)^2 + 1)/(a^4 + 2\*a^2\*b^2 + b^4) - 2\*(B\*a^3 - A\*a^2\*b)/(a^3\*b^2 + a\*b^4 + (a^2\*b^3 + b^5)\*tan(d\*x + c)))/d



**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(155) = 310.

time = 1.71, size = 311, normalized size = 1.98

$$\frac{2Ba^3b^2 - 2Aa^2b^3 - 2(Aa^3b^2 + 2Ba^2b^3 - Ab^5)dx + (Ba^5 + 3Ba^3b^2 - 2Aa^2b^3 + (Ba^4b + 3Ba^2b^3 - 2Aab^5)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2 + 2ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - (Ba^5 + 2Ba^3b^2 + Bab^5 + (Ba^4b + 2Ba^2b^3 + Bb^5)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)}\right) - 2(Ba^4b - Aa^3b^2 + (Aa^2b^3 + 2Bab^5 - Ab^5)dx)\tan(dx+c)}{2((a^3b^2 + 2a^2b^3 + b^5)d\tan(dx+c) + (a^5b^2 + 2a^3b^4 + ab^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 1/2\*(2\*B\*a^3\*b^2 - 2\*A\*a^2\*b^3 - 2\*(A\*a^3\*b^2 + 2\*B\*a^2\*b^3 - A\*a\*b^4)\*d\*x + (B\*a^5 + 3\*B\*a^3\*b^2 - 2\*A\*a^2\*b^3 + (B\*a^4\*b + 3\*B\*a^2\*b^3 - 2\*A\*a\*b^4)\*tan(d\*x + c))\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)) - (B\*a^5 + 2\*B\*a^3\*b^2 + B\*a\*b^4 + (B\*a^4\*b + 2\*B\*a^2\*b^3 + B\*b^5)\*tan(d\*x + c))\*log(1/(tan(d\*x + c)^2 + 1)) - 2\*(B\*a^4\*b - A\*a^3\*b^2 + (A\*a^2\*b^3 + 2\*B\*a\*b^4 - A\*b^5)\*d\*x)\*tan(d\*x + c))/((a^4\*b^3 + 2\*a^2\*b^5 + b^7)\*d\*tan(d\*x + c) + (a^5\*b^2 + 2\*a^3\*b^4 + a\*b^6)\*d)

**Sympy** [C] Result contains complex when optimal does not.

time = 0.95, size = 3485, normalized size = 22.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*x\*(A + B\*tan(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-A\*x + A\*tan(c + d\*x)/d - B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*tan(c + d\*x)\*\*2/(2\*d))/a\*\*2, Eq(b, 0)), (A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 2\*I\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 3\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*I\*A/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 3\*I\*B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 6\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 3\*I\*B\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 2\*B\*log(tan(c + d\*x)\*\*2 + 1)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 5\*I\*B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*B/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d), Eq(a, -I\*b)), (A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*I\*A\*d\*x\*tan(c + d\*x

$$\begin{aligned}
&)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - A*d*x/( \\
&4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*A*tan(c \\
&+ d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2* \\
&I*A/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*B \\
&*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - \\
&4*b**2*d) + 6*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan \\
&(c + d*x) - 4*b**2*d) + 3*I*B*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan \\
&(c + d*x) - 4*b**2*d) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b* \\
&>**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*B*log(tan( \\
&c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c \\
&+ d*x) - 4*b**2*d) - 2*B*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 \\
&+ 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 5*I*B*tan(c + d*x)/(4*b**2*d*tan(c \\
&+ d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*B/(4*b**2*d*tan(c + d* \\
&x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*(A + B*tan(c)) \\
&)*tan(c)**2/(a + b*tan(c))**2, Eq(d, 0)), (-2*A*a**4*b/(2*a**5*b**2*d + 2*a* \\
&>*4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b \\
&>**6*d + 2*b**7*d*tan(c + d*x)) - 2*A*a**3*b**2*d*x/(2*a**5*b**2*d + 2*a**4* \\
&b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6 \\
&>*d + 2*b**7*d*tan(c + d*x)) - 2*A*a**2*b**3*d*x*tan(c + d*x)/(2*a**5*b**2*d \\
&+ 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) \\
&+ 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 4*A*a**2*b**3*log(a/b + tan(c + d*x \\
&))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b** \\
&5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*A*a**2*b**3*log( \\
&tan(c + d*x)**2 + 1)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b \\
&>**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - \\
&2*A*a**2*b**3/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + \\
&4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*A*a*b \\
&>**4*d*x/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a** \\
&2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 4*A*a*b**4*lo \\
&g(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d \\
&>*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*ta \\
&n(c + d*x)) + 2*A*a*b**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b**2 \\
&*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x \\
&+ 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*A*b**5*d*x*tan(c + d*x)/(2*a**5 \\
&*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c \\
&+ d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*a**5*log(a/b + tan(c + d \\
&>*x))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b \\
&>**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*a**5/(2*a**5 \\
&*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c \\
&+ d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*a**4*b*log(a/b + tan(c + \\
&d*x))*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b* \\
&>**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 6 \\
&*B*a**3*b**2*log(a/b + tan(c + d*x))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + \\
&d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d* \\
&tan(c + d*x)) - B*a**3*b**2*log(tan(c + d*x)**2 + 1)/(2*a**5*b**2*d + 2*a**
\end{aligned}$$

4\*b\*\*3\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*4\*d + 4\*a\*\*2\*b\*\*5\*d\*tan(c + d\*x) + 2\*a\*b\*\*6\*d + 2\*b\*\*7\*d\*tan(c + d\*x)) + 2\*B\*a\*\*3\*b\*\*2/(2\*a\*\*5\*b\*\*2\*d + 2\*a\*\*4\*b\*\*3\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*4\*d + 4\*a\*\*2\*b\*\*5\*d\*...)

**Giac** [A]

time = 0.66, size = 244, normalized size = 1.55

$$\frac{\frac{2(Aa^2+2Bab-Ab^2)(dx+c)}{a^4+2a^2b^2+b^4} + \frac{(Ba^2-2Aab-Bb^2)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} - \frac{2(Ba^4+3Ba^2b^2-2Aab^3)\log(|b\tan(dx+c)+a|)}{a^4b^2+2a^2b^4+b^6} + \frac{2(Ba^4\tan(dx+c)+3Ba^2b^2\tan(dx+c)-2Aab^3\tan(dx+c)+Aa^4+2Ba^2b-Aa^2b^2)}{(a^4b+2a^2b^3+b^5)(b\tan(dx+c)+a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 
$$-1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (B*a^2 - 2*A*a*b - B*b^2)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^4 + 3*B*a^2*b^2 - 2*A*a*b^3)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^4*b^2 + 2*a^2*b^4 + b^6) + 2*(B*a^4*\tan(d*x + c) + 3*B*a^2*b^2*\tan(d*x + c) - 2*A*a*b^3*\tan(d*x + c) + A*a^4 + 2*B*a^3*b - A*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*\tan(d*x + c) + a)))/d$$

**Mupad** [B]

time = 6.56, size = 165, normalized size = 1.05

$$\frac{\ln(\tan(c+dx)+1i)(B+A1i)}{2d(-a^2+ab2i+b^2)} + \frac{\ln(\tan(c+dx)-i)(A+B1i)}{2d(-a^21i+2ab+b^21i)} - \frac{a^2(Ab-BA)}{b^2d(a^2+b^2)(a+b\tan(c+dx))} + \frac{a\ln(a+b\tan(c+dx))(Ba^3+3Ba^2b-2Ab^3)}{b^2d(a^2+b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] 
$$(\log(\tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(a*b*2i - a^2 + b^2)) + (\log(\tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (a^2*(A*b - B*a))/(b^2*d*(a^2 + b^2)*(a + b*\tan(c + d*x))) + (a*\log(a + b*\tan(c + d*x))*(B*a^3 - 2*A*b^3 + 3*B*a*b^2))/(b^2*d*(a^2 + b^2)^2)$$

$$3.277 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=115

$$\frac{(2aAb - a^2B + b^2B)x}{(a^2 + b^2)^2} - \frac{(a^2A - Ab^2 + 2abB) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^2 d} + \frac{a(Ab - aB)}{b(a^2 + b^2)d(a + b \tan(c + dx))}$$

[Out] (2\*A\*a\*b-B\*a^2+B\*b^2)\*x/(a^2+b^2)^2-(A\*a^2-A\*b^2+2\*B\*a\*b)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)^2/d+a\*(A\*b-B\*a)/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3672, 3612, 3611}

$$\frac{a(Ab - aB)}{bd(a^2 + b^2)(a + b \tan(c + dx))} - \frac{(a^2A + 2abB - Ab^2) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^2} + \frac{x(a^2(-B) + 2aAb + b^2B)}{(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] ((2\*a\*A\*b - a^2\*B + b^2\*B)\*x)/(a^2 + b^2)^2 - ((a^2\*A - A\*b^2 + 2\*a\*b\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)^2\*d) + (a\*(A\*b - a\*B))/(b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x]))

Rule 3611

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(x\_)), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3672

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c +

$b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= \frac{a(Ab-aB)}{b(a^2+b^2)d(a+b\tan(c+dx))} + \frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{a+b\tan(c+dx)} dx}{a^2+b^2} \\ &= \frac{(2aAb-a^2B+b^2B)x}{(a^2+b^2)^2} + \frac{a(Ab-aB)}{b(a^2+b^2)d(a+b\tan(c+dx))} - \frac{(a^2A}{(a^2+b^2)^2} \\ &= \frac{(2aAb-a^2B+b^2B)x}{(a^2+b^2)^2} - \frac{(a^2A-Ab^2+2abB)\log(a\cos(c+dx))}{(a^2+b^2)^2 d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.46, size = 140, normalized size = 1.22

$$\frac{\frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2} + \frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2\left((-a^2A+Ab^2-2abB)\log(a+b\tan(c+dx)) - \frac{a(a^2+b^2)(-Ab+aB)}{b(a+b\tan(c+dx))}\right)}{(a^2+b^2)^2}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] (((A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*b)^2 + ((A - I\*B)\*Log[I + Tan[c + d\*x]])/(a - I\*b)^2 + (2\*((-a^2\*A) + A\*b^2 - 2\*a\*b\*B)\*Log[a + b\*Tan[c + d\*x]] - (a\*(a^2 + b^2)\*(-A\*b) + a\*B))/(b\*(a + b\*Tan[c + d\*x])))/(a^2 + b^2)^2)/(2\*d)

**Maple [A]**

time = 0.14, size = 145, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{(a^2A-Ab^2+2Bab)\ln(1+\tan^2(dx+c))}{2} + (2Aab-a^2B+b^2B)\arctan(\tan(dx+c)) + \frac{a(Ab-aB)}{(a^2+b^2)b(a+b\tan(dx+c))} - \frac{(a^2A-Ab^2+2Bab)}{(a^2+b^2)^2}}{d}$
default	$\frac{\frac{(a^2A-Ab^2+2Bab)\ln(1+\tan^2(dx+c))}{2} + (2Aab-a^2B+b^2B)\arctan(\tan(dx+c)) + \frac{a(Ab-aB)}{(a^2+b^2)b(a+b\tan(dx+c))} - \frac{(a^2A-Ab^2+2Bab)}{(a^2+b^2)^2}}{d}$
norman	$\frac{\frac{a(2Aab-a^2B+b^2B)x}{a^4+2a^2b^2+b^4} + \frac{b(2Aab-a^2B+b^2B)x\tan(dx+c)}{a^4+2a^2b^2+b^4} + \frac{a(Ab-aB)}{(a^2+b^2)bd}}{a+b\tan(dx+c)} + \frac{(a^2A-Ab^2+2Bab)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)} - \frac{(a^2A-Ab^2+2Bab)}{(a^2+b^2)^2}$

risch	$\frac{x B}{2i b a - a^2 + b^2} + \frac{i x A}{2i b a - a^2 + b^2} + \frac{2i a^2 A x}{a^4 + 2a^2 b^2 + b^4} - \frac{2i A b^2 x}{a^4 + 2a^2 b^2 + b^4} + \frac{4i B a b x}{a^4 + 2a^2 b^2 + b^4} + \frac{2i a^2 A c}{d(a^4 + 2a^2 b^2 + b^4)} - \frac{2i A}{d(a^4 + 2a^2 b^2 + b^4)}$
-------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out]  $1/d*(1/(a^2+b^2)^2*(1/2*(A*a^2-A*b^2+2*B*a*b)*\ln(1+\tan(d*x+c)^2)+(2*A*a*b-B*a^2+B*b^2)*\arctan(\tan(d*x+c)))+a*(A*b-B*a)/(a^2+b^2)/b/(a+b*\tan(d*x+c))-(A*a^2-A*b^2+2*B*a*b)/(a^2+b^2)^2*\ln(a+b*\tan(d*x+c))$

**Maxima** [A]

time = 0.51, size = 185, normalized size = 1.61

$$\frac{\frac{2(Ba^2-2Aab-Bb^2)(dx+c)}{a^4+2a^2b^2+b^4} + \frac{2(Aa^2+2Bab-Ab^2)\log(b\tan(dx+c)+a)}{a^4+2a^2b^2+b^4} - \frac{(Aa^2+2Bab-Ab^2)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{2(Ba^2-Aab)}{a^3b+ab^3+(a^2b^2+b^4)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")`

[Out]  $-1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(A*a^2 + 2*B*a*b - A*b^2)*\log(b*\tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - (A*a^2 + 2*B*a*b - A*b^2)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*a^2 - A*a*b)/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*\tan(d*x + c)))/d$

**Fricas** [A]

time = 1.28, size = 221, normalized size = 1.92

$$\frac{2Ba^2b - 2Aab^2 + 2(Ba^3 - 2Aa^2b - Bab^2)dx + (Aa^3 + 2Ba^2b - Aab^2 + (Aa^2b + 2Bab^2 - Ab^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2 + 2ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - 2(Ba^3 - Aa^2b - (Ba^2b - 2Aab^2 - Bb^3)dx)\tan(dx+c)}{2((a^4b + 2a^2b^3 + b^5)d\tan(dx+c) + (a^3 + 2a^2b^2 + ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*B*a^2*b - 2*A*a*b^2 + 2*(B*a^3 - 2*A*a^2*b - B*a*b^2)*d*x + (A*a^3 + 2*B*a^2*b - A*a*b^2 + (A*a^2*b + 2*B*a*b^2 - A*b^3)*\tan(d*x + c))*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - 2*(B*a^3 - A*a^2*b - (B*a^2*b - 2*A*a*b^2 - B*b^3)*d*x)*\tan(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d*\tan(d*x + c) + (a^3 + 2*a^2*b^2 + a*b^4)*d)$

**Sympy** [C] Result contains complex when optimal does not.

time = 0.82, size = 2987, normalized size = 25.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*x\*(A + B\*tan(c))/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*x + B\*tan(c + d\*x)/d)/a\*\*2, Eq(b, 0)), (I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - I\*A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + I\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 2\*I\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - B\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 3\*B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*I\*B/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d), Eq(a, -I\*b)), (-I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + I\*A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - I\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*I\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - B\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 3\*B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 2\*I\*B/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d), Eq(a, I\*b)), (x\*(A + B\*tan(c))\*tan(c)/(a + b\*tan(c))\*\*2, Eq(d, 0)), (-2\*A\*a\*\*3\*b\*log(a/b + tan(c + d\*x))/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + A\*a\*\*3\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + 2\*A\*a\*\*3\*b/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + 4\*A\*a\*\*2\*b\*\*2\*d\*x/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) - 2\*A\*a\*\*2\*b\*\*2\*log(a/b + tan(c + d\*x))\*tan(c + d\*x)/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + A\*a\*\*2\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + 4\*A\*a\*b\*\*3\*d\*x\*tan(c + d\*x)/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + 2\*A\*a\*b\*\*3\*log(a/b + tan(c + d\*x))/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) - A\*a\*b\*\*3\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d + 2\*b\*\*6\*d\*tan(c + d\*x)) + 2\*A\*a\*b\*\*3/(2\*a\*\*5\*b\*d + 2\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 4\*a\*\*2\*b\*\*4\*d

$$\begin{aligned} & * \tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) + 2*A*b**4*\log(a/b + \tan(c + d*x)) * \tan(c + d*x) / (2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & A*b**4*\log(\tan(c + d*x)**2 + 1) * \tan(c + d*x) / (2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & 2*B*a**4/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & 2*B*a**3*b*d*x/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & 2*B*a**2*b**2*d*x*\tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & 4*B*a**2*b**2*\log(a/b + \tan(c + d*x))/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) + \\ & 2*B*a**2*b**2*\log(\tan(c + d*x)**2 + 1)/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & 2*B*a**2*b**2/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) + \\ & 2*B*a*b**3*d*x/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) - \\ & 4*B*a*b**3*\log(a/b + \tan(c + d*x)) * \tan(c + d*x) / (2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) + \\ & 2*B*a*b**3*\log(\tan(c + d*x)**2 + 1) * \tan(c + d*x) / (2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) + \\ & 2*B*a*b**3/(2*a**5*b*d + 2*a**4*b**2*d*\tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*\tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*\tan(c + d*x)) \dots \end{aligned}$$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(117) = 234.

time = 0.56, size = 241, normalized size = 2.10

$$\frac{\frac{2(Ba^2 - 2Ab - B^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} + \frac{2(Aa^2b + 2Bab^2 - Ab^3) \log(|b \tan(dx+c) + a|)}{a^4b + 2a^2b^3 + b^5} - \frac{2(Aa^2b^2 \tan(dx+c) + 2Bab^3 \tan(dx+c) - Ab^4 \tan(dx+c) - Ba^4 + 2Aa^3b + Ba^2b^2)}{(a^4b + 2a^2b^3 + b^5)(b \tan(dx+c) + a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (A*a^2 + 2*B*a*b - A*b^2)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(A*a^2*b + 2*B*a*b^2 - A*b^3)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - \\ & 2*(A*a^2*b^2*\tan(d*x + c) + 2*B*a*b^3*\tan(d*x + c) - A*b^4*\tan(d*x + c) - B*a^4 + 2*A*a^3*b + B*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*\tan(d*x + c) + a)))/d \end{aligned}$$

**Mupad** [B]

time = 6.60, size = 163, normalized size = 1.42

$$\frac{a(Ab - Ba)}{bd(a^2 + b^2)(a + b \tan(c + dx))} + \frac{\ln(\tan(c + dx) - i)(A + B \operatorname{li})}{2d(a^2 + ab2i - b^2)} + \frac{\ln(\tan(c + dx) + i)(B + A \operatorname{li})}{2d(a^2 \operatorname{li} + 2ab - b^2 \operatorname{li})} - \frac{\ln(a + b \tan(c + dx))}{d} \left( \frac{A}{a^2 + b^2} - \frac{2b(Ab - Ba)}{(a^2 + b^2)^2} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x)*(A + B*\tan(c + d*x)))/(a + b*\tan(c + d*x))^2, x)$

[Out]  $(\log(\tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (\log(a + b*\tan(c + d*x))*(A/(a^2 + b^2) - (2*b*(A*b - B*a))/(a^2 + b^2)^2))/d + (\log(\tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(2*a*b + a^2*1i - b^2*1i)) + (a*(A*b - B*a))/(b*d*(a^2 + b^2)*(a + b*\tan(c + d*x)))$

$$3.278 \quad \int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=111

$$\frac{(a^2 A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} + \frac{(2aAb - a^2 B + b^2 B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^2 d} - \frac{Ab - aB}{(a^2 + b^2) d(a + b \tan(c + dx))}$$

[Out] (A\*a^2-A\*b^2+2\*B\*a\*b)\*x/(a^2+b^2)^2+(2\*A\*a\*b-B\*a^2+B\*b^2)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)^2/d+(-A\*b+B\*a)/(a^2+b^2)/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3610, 3612, 3611}

$$-\frac{Ab - aB}{d(a^2 + b^2)(a + b \tan(c + dx))} + \frac{(a^2(-B) + 2aAb + b^2 B) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^2} + \frac{x(a^2 A + 2abB - Ab^2)}{(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^2,x]

[Out] ((a^2\*A - A\*b^2 + 2\*a\*b\*B)\*x)/(a^2 + b^2)^2 + ((2\*a\*A\*b - a^2\*B + b^2\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)^2\*d) - (A\*b - a\*B)/((a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x]))

Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && Ne

Q[a\*c + b\*d, 0]

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^2} dx = -\frac{Ab - aB}{(a^2 + b^2) d(a + b \tan(c + dx))} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{a + b \tan(c + dx)} dx}{a^2 + b^2}$$

$$= \frac{(a^2 A - Ab^2 + 2abB) x}{(a^2 + b^2)^2} - \frac{Ab - aB}{(a^2 + b^2) d(a + b \tan(c + dx))} + \frac{(2aAb - a^2 B + b^2 B)}{(a^2 + b^2)}$$

$$= \frac{(a^2 A - Ab^2 + 2abB) x}{(a^2 + b^2)^2} + \frac{(2aAb - a^2 B + b^2 B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^2 d}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.48, size = 190, normalized size = 1.71

$$\frac{B \left( \frac{-ia-b}{a^2+b^2} \log(i-\tan(c+dx)) + \frac{i(a+ib)}{a^2+b^2} \log(i+\tan(c+dx)) + 2b \log(a+b \tan(c+dx)) \right) - (Ab - aB) \left( \frac{i \log(i-\tan(c+dx))}{(a+ib)^2} - \frac{i \log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2b(-2a \log(a+b \tan(c+dx)) + \frac{a^2+b^2}{a+b \tan(c+dx)})}{(a^2+b^2)^2} \right)}{2bd}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^2, x]

[Out] ((B\*(((−I)\*a − b)\*Log[I − Tan[c + d\*x]] + I\*(a + I\*b)\*Log[I + Tan[c + d\*x]] + 2\*b\*Log[a + b\*Tan[c + d\*x]]))/(a^2 + b^2) − (A\*b − a\*B)\*((I\*Log[I − Tan[c + d\*x]])/(a + I\*b)^2 − (I\*Log[I + Tan[c + d\*x]])/(a − I\*b)^2 + (2\*b\*(−2\*a\*Log[a + b\*Tan[c + d\*x]] + (a^2 + b^2)/(a + b\*Tan[c + d\*x])))/(a^2 + b^2)^2)))/(2\*b\*d)

**Maple [A]**

time = 0.12, size = 141, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{(-2Aab+a^2B-b^2B) \ln(1+\tan^2(dx+c))}{2} + (a^2A-Ab^2+2Bab) \arctan(\tan(dx+c))}{(a^2+b^2)^2} - \frac{Ab-aB}{(a^2+b^2)(a+b \tan(dx+c))} + \frac{(2Aab-a^2B+b^2B)}{(a^2+b^2)}$
default	$\frac{\frac{(-2Aab+a^2B-b^2B) \ln(1+\tan^2(dx+c))}{2} + (a^2A-Ab^2+2Bab) \arctan(\tan(dx+c))}{(a^2+b^2)^2} - \frac{Ab-aB}{(a^2+b^2)(a+b \tan(dx+c))} + \frac{(2Aab-a^2B+b^2B)}{(a^2+b^2)}$
norman	$\frac{a(a^2A-Ab^2+2Bab)x}{a^4+2a^2b^2+b^4} + \frac{b(a^2A-Ab^2+2Bab)x \tan(dx+c)}{a^4+2a^2b^2+b^4} + \frac{(Ab-aB)b \tan(dx+c)}{ad(a^2+b^2)} + \frac{(2Aab-a^2B+b^2B) \ln(a+b \tan(dx+c))}{d(a^4+2a^2b^2+b^4)}$
risch	$\frac{i x B}{2i b a - a^2 + b^2} - \frac{x A}{2i b a - a^2 + b^2} - \frac{4i a b A x}{a^4 + 2a^2 b^2 + b^4} + \frac{2i a^2 B x}{a^4 + 2a^2 b^2 + b^4} - \frac{2i B b^2 x}{a^4 + 2a^2 b^2 + b^4} - \frac{4i a b A c}{(a^4 + 2a^2 b^2 + b^4) d} + \frac{2i B c}{(a^4 + 2a^2 b^2 + b^4)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} \left( \frac{1}{(a^2+b^2)^2} \left( \frac{1}{2} (-2Aab+B^2-Bb^2) \ln(1+\tan(dx+c)^2) + (Aa^2-A^2b^2+2Bab) \arctan(\tan(dx+c)) \right) - \frac{(Ab-Ba)}{(a^2+b^2)} \frac{1}{(a+b\tan(dx+c))} + \frac{(2Aab-Ba^2+Bb^2)}{(a^2+b^2)^2} \ln(a+b\tan(dx+c)) \right)$

**Maxima** [A]

time = 0.52, size = 177, normalized size = 1.59

$$\frac{\frac{2(Aa^2+2Bab-Ab^2)(dx+c)}{a^4+2a^2b^2+b^4} - \frac{2(Ba^2-2Aab-Bb^2)\log(b\tan(dx+c)+a)}{a^4+2a^2b^2+b^4} + \frac{(Ba^2-2Aab-Bb^2)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{2(Ba-Ab)}{a^3+ab^2+(a^2b+b^3)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2} \left( \frac{2(Aa^2 + 2Bab - Ab^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{2(Ba^2 - 2Aab - Bb^2)\log(b\tan(dx+c)+a)}{a^4 + 2a^2b^2 + b^4} + \frac{(Ba^2 - 2Aab - Bb^2)\log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} + \frac{2(Ba - Ab)}{a^3 + ab^2 + (a^2b + b^3)\tan(dx+c)} \right) / d$

**Fricas** [A]

time = 1.45, size = 222, normalized size = 2.00

$$\frac{2Bab^2 - 2Ab^3 + 2(Aa^3 + 2Ba^2b - Aab^2)dx - (Ba^2 - 2Aa^2b - Bab^2 + (Ba^2b - 2Aab^2 - Bb^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2 + 2ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - 2(Ba^2b - Aab^2 - (Aa^2b + 2Bab^2 - Ab^3)dx)\tan(dx+c)}{2((a^4b + 2a^2b^3 + b^5)d\tan(dx+c) + (a^5 + 2a^3b^2 + ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{2} \left( \frac{2B^2a^2b^2 - 2A^2b^3 + 2(A^3a + 2B^2a^2b - A^2ab^2)dx - (B^3a^3 - 2A^2a^2b - B^2a^2b^2 + (B^2a^2b - 2A^2a^2b^2 - B^2b^3)\tan(dx+c))\log((b^2\tan(dx+c)^2 + 2ab\tan(dx+c) + a^2)/(\tan(dx+c)^2 + 1)) - 2(B^2a^2b^2 - A^2a^2b^2 - (A^2a^2b + 2B^2a^2b^2 - A^2b^3)dx)\tan(dx+c)}{((a^4b + 2a^2b^3 + b^5)d\tan(dx+c) + (a^5 + 2a^3b^2 + ab^4)d)} \right)$

**Sympy** [C] Result contains complex when optimal does not.

time = 0.79, size = 2878, normalized size = 25.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)`

```
[Out] Piecewise((zoo*x*(A + B*tan(c))/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
((A*x + B*log(tan(c + d*x)**2 + 1)/(2*d))/a**2, Eq(b, 0)), (-A*d*x*tan(c +
d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
2*I*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x)
- 4*b**2*d) + A*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4
*b**2*d) - A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*
x) - 4*b**2*d) + 2*I*A/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x)
- 4*b**2*d) + I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*
d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2
- 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*d*x/(4*b**2*d*tan(c + d*x)**2
- 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*tan(c + d*x)/(4*b**2*d*tan(c +
d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-A*d*x*tan(c
+ d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) -
2*I*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
- 4*b**2*d) + A*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) -
4*b**2*d) - A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d
*x) - 4*b**2*d) - 2*I*A/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
- 4*b**2*d) - I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2
*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**
2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*d*x/(4*b**2*d*tan(c + d*x)**2
+ 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*tan(c + d*x)/(4*b**2*d*tan(c +
d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*(A + B*tan(
c))/(a + b*tan(c))**2, Eq(d, 0)), (2*A*a**3*d*x/(2*a**5*d + 2*a**4*b*d*tan(
c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5
*d*tan(c + d*x)) + 2*A*a**2*b*d*x*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c
+ d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*
d*tan(c + d*x)) + 4*A*a**2*b*log(a/b + tan(c + d*x))/(2*a**5*d + 2*a**4*b*d
*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2
*b**5*d*tan(c + d*x)) - 2*A*a**2*b*log(tan(c + d*x)**2 + 1)/(2*a**5*d + 2*a
**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**
4*d + 2*b**5*d*tan(c + d*x)) - 2*A*a**2*b/(2*a**5*d + 2*a**4*b*d*tan(c + d*
x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan
(c + d*x)) - 2*A*a*b**2*d*x/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b*
**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 4
*A*a*b**2*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c
+ d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*
d*tan(c + d*x)) - 2*A*a*b**2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*
d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) +
2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*A*b**3*d*x*tan(c + d*x)/(2*a**5*d +
2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a
*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*A*b**3/(2*a**5*d + 2*a**4*b*d*tan(c +
d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*t
an(c + d*x)) - 2*B*a**3*log(a/b + tan(c + d*x))/(2*a**5*d + 2*a**4*b*d*tan(
c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5
*d*tan(c + d*x)) + B*a**3*log(tan(c + d*x)**2 + 1)/(2*a**5*d + 2*a**4*b*d*t
```

$$\begin{aligned} & \text{an}(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + 2*b \\ & **5*d*\tan(c + d*x)) + 2*B*a**3/(2*a**5*d + 2*a**4*b*d*\tan(c + d*x) + 4*a**3 \\ & *b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*\tan(c + d*x)) \\ & + 4*B*a**2*b*d*x/(2*a**5*d + 2*a**4*b*d*\tan(c + d*x) + 4*a**3*b**2*d + 4*a* \\ & **2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*\tan(c + d*x)) - 2*B*a**2*b*1 \\ & \text{og}(a/b + \tan(c + d*x))*\tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*\tan(c + d*x) + 4 \\ & *a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*\tan(c + d \\ & *x)) + B*a**2*b*\log(\tan(c + d*x)**2 + 1)*\tan(c + d*x)/(2*a**5*d + 2*a**4*b* \\ & d*\tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + \\ & 2*b**5*d*\tan(c + d*x)) + 4*B*a*b**2*d*x*\tan(c + d*x)/(2*a**5*d + 2*a**4*b*d \\ & *\tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + 2 \\ & *b**5*d*\tan(c + d*x)) + 2*B*a*b**2*\log(a/b + \tan(c + d*x))/(2*a**5*d + 2*a* \\ & **4*b*d*\tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4 \\ & *d + 2*b**5*d*\tan(c + d*x)) - B*a*b**2*\log(\tan(c + d*x)**2 + 1)/(2*a**5*d + \\ & 2*a**4*b*d*\tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a \\ & *b**4*d + 2*b**5*d*\tan(c + d*x)) + 2*B*a*b**2/(2*a**5*d + 2*a**4*b*d*\tan(c \\ & + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a*b**4*d + 2*b**5*d \\ & *\tan(c + d*x)) + 2*B*b**3*\log(a/b + \tan(c + d*x))*\tan(c + d*x)/(2*a**5*d + \\ & 2*a**4*b*d*\tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan(c + d*x) + 2*a* \\ & b**4*d + 2*b**5*d*\tan(c + d*x)) - B*b**3*\log(\tan(c + d*x)**2 + 1)*\tan(c + d \\ & *x)/(2*a**5*d + 2*a**4*b*d*\tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*\tan \\ & (c + d*x) + 2*a*b**4*d + 2*b**5*d*\tan(c + d*x))\dots \end{aligned}$$

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(111) = 222.

time = 0.54, size = 234, normalized size = 2.11

$$\frac{\frac{2(Aa^2+2Ab-Bb^2)(dx+c)}{a^4+2a^2b^2+b^4} + \frac{(Ba^2-2Aab-Bb^2)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} - \frac{2(Ba^2b-2Aab^2-Bb^3)\log(|b\tan(dx+c)+a|)}{a^4b+2a^2b^3+b^5} + \frac{2(Ba^2b\tan(dx+c)-2Aab^2\tan(dx+c)-Bb^3\tan(dx+c)+2Ba^3-3Aa^2b-Ab^3)}{(a^4+2a^2b^2+b^4)(b\tan(dx+c)+a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $\frac{1}{2}*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (B*a^2 - 2*A*a*b - B*b^2)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b - 2*A*a*b^2 - B*b^3)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 2*(B*a^2*b*\tan(d*x + c) - 2*A*a*b^2*\tan(d*x + c) - B*b^3*\tan(d*x + c) + 2*B*a^3 - 3*A*a^2*b - A*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*\tan(d*x + c) + a)))/d$

**Mupad [B]**

time = 6.48, size = 153, normalized size = 1.38

$$\frac{\ln(a + b\tan(c + dx))(-Ba^2 + 2Aab + Bb^2)}{d(a^2 + b^2)^2} - \frac{Ab - Ba}{d(a^2 + b^2)(a + b\tan(c + dx))} - \frac{\ln(\tan(c + dx) + 1i)(B + A1i)}{2d(-a^2 + ab2i + b^2)} - \frac{\ln(\tan(c + dx) - i)(A + B1i)}{2d(-a^2 1i + 2ab + b^2 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B*\tan(c + d*x))/(a + b*\tan(c + d*x))^2, x)$

[Out]  $(\log(a + b*\tan(c + d*x))*(B*b^2 - B*a^2 + 2*A*a*b))/(d*(a^2 + b^2)^2) - (A*b - B*a)/(d*(a^2 + b^2)*(a + b*\tan(c + d*x))) - (\log(\tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(a*b*2i - a^2 + b^2)) - (\log(\tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i))$

$$3.279 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=137

$$-\frac{(2aAb - a^2B + b^2B)x}{(a^2 + b^2)^2} + \frac{A \log(\sin(c + dx))}{a^2d} - \frac{b(3a^2Ab + Ab^3 - 2a^3B) \log(a \cos(c + dx) + b \sin(c + dx))}{a^2(a^2 + b^2)^2d} + \frac{a}{a}$$

[Out]  $-(2*A*a*b - B*a^2 + B*b^2)*x/(a^2 + b^2)^2 + A*\ln(\sin(d*x + c))/a^2/d - b*(3*A*a^2*b + A*b^3 - 2*B*a^3)*\ln(a*\cos(d*x + c) + b*\sin(d*x + c))/a^2/(a^2 + b^2)^2/d + b*(A*b - B*a)/a/(a^2 + b^2)/d/(a + b*\tan(d*x + c))$

**Rubi [A]**

time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3690, 3732, 3611, 3556}

$$\frac{b(Ab - aB)}{ad(a^2 + b^2)(a + b \tan(c + dx))} - \frac{x(a^2(-B) + 2aAb + b^2B)}{(a^2 + b^2)^2} + \frac{A \log(\sin(c + dx))}{a^2d} - \frac{b(-2a^3B + 3a^2Ab + Ab^3) \log(a \cos(c + dx) + b \sin(c + dx))}{a^2d(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x] * (A + B*\text{Tan}[c + d*x])) / (a + b*\text{Tan}[c + d*x])^2, x]$

[Out]  $-(((2*a*A*b - a^2*B + b^2*B)*x)/(a^2 + b^2)^2) + (A*\text{Log}[\text{Sin}[c + d*x]])/(a^2*d) - (b*(3*a^2*A*b + A*b^3 - 2*a^3*B)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(a^2*(a^2 + b^2)^2*d) + (b*(A*b - a*B))/(a*(a^2 + b^2)*d*(a + b*\text{Tan}[c + d*x]))$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3611

$\text{Int}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)] / ((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

Rule 3690

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{(m + 1)} * ((c + d*\text{Tan}[e + f*x])^{(n + 1)} / (f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)} * (c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)]$



```
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(LtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)] + (C_.)*tan[(e_.) + (f_.)*(x_.)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_.)])), x_Symbol] := Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx &= \frac{b(Ab-aB)}{a(a^2+b^2)d(a+b \tan(c+dx))} + \frac{\int \frac{\cot(c+dx)(A(a^2+b^2)-a(Ab-aB) \tan(c+dx))}{a+b \tan(c+dx)} dx}{a(a^2+b^2)d} \\ &= -\frac{(2aAb-a^2B+b^2B)x}{(a^2+b^2)^2} + \frac{b(Ab-aB)}{a(a^2+b^2)d(a+b \tan(c+dx))} + \frac{A \int \frac{\cot(c+dx)}{a+b \tan(c+dx)} dx}{a(a^2+b^2)d} \\ &= -\frac{(2aAb-a^2B+b^2B)x}{(a^2+b^2)^2} + \frac{A \log(\sin(c+dx))}{a^2d} - \frac{b(3a^2Ab+Ab^3-a^3B)}{a(a^2+b^2)d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.58, size = 183, normalized size = 1.34

$$\frac{-\frac{a(a-ib)(A+iB) \log(i-\tan(c+dx))}{2(a+ib)} + \frac{A(a^2+b^2) \log(\tan(c+dx))}{a} - \frac{a(a+ib)(A-ib) \log(i+\tan(c+dx))}{2(a-ib)} + \frac{b(-3a^2Ab-Ab^3+2a^3B) \log(a+b \tan(c+dx))}{a(a^2+b^2)} + \frac{b(Ab-aB)}{a+b \tan(c+dx)}}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-1/2*(a*(a - I*b)*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b) + (A*(a^2 + b
^2)*Log[Tan[c + d*x]])/a - (a*(a + I*b)*(A - I*B)*Log[I + Tan[c + d*x]])/(2
*(a - I*b)) + (b*(-3*a^2*A*b - A*b^3 + 2*a^3*B)*Log[a + b*Tan[c + d*x]])/(a
*(a^2 + b^2)) + (b*(A*b - a*B))/(a + b*Tan[c + d*x]))/(a*(a^2 + b^2)*d)
```

### Maple [A]

time = 0.31, size = 163, normalized size = 1.19

method	result
derivativedivides	$\frac{A \ln(\tan(dx+c))}{a^2} + \frac{(-a^2 A + A b^2 - 2 B a b) \ln(1 + \tan^2(dx+c))}{2(a^2 + b^2)^2} + \frac{(-2 A a b + a^2 B - b^2 B) \arctan(\tan(dx+c))}{(a^2 + b^2)^2} - \frac{b(3 A a^2 b + A b^3 - 2 B a^3) \ln(a)}{a^2(a^2 + b^2)^2}$
default	$\frac{A \ln(\tan(dx+c))}{a^2} + \frac{(-a^2 A + A b^2 - 2 B a b) \ln(1 + \tan^2(dx+c))}{2(a^2 + b^2)^2} + \frac{(-2 A a b + a^2 B - b^2 B) \arctan(\tan(dx+c))}{(a^2 + b^2)^2} - \frac{b(3 A a^2 b + A b^3 - 2 B a^3) \ln(a)}{a^2(a^2 + b^2)^2}$
norman	$\frac{a(2 A a b - a^2 B + b^2 B) x}{a^4 + 2 a^2 b^2 + b^4} - \frac{b(2 A a b - a^2 B + b^2 B) x \tan(dx+c)}{a^4 + 2 a^2 b^2 + b^4} - \frac{(A b^2 - B a b) b \tan(dx+c)}{d a^2 (a^2 + b^2)} + \frac{A \ln(\tan(dx+c))}{a^2 d} - \frac{(a^2 A - A b^2 + 2 B b) \ln(a)}{2 d (a^4 + 2 a^2 b^2 + b^4)}$
risch	$-\frac{x B}{2 i b a - a^2 + b^2} - \frac{4 i B a b c}{d(a^4 + 2 a^2 b^2 + b^4)} + \frac{2 i b^3 A}{(-i a + b) d a (i a + b)^2 (b e^{2 i(dx+c)} + i a e^{2 i(dx+c)} - b + i a)} + \frac{6 i A b^2 c}{d(a^4 + 2 a^2 b^2 + b^4)} - \frac{2 b(3 A a^2 b + A b^3 - 2 B a^3) \ln(a)}{a^2(a^2 + b^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] 1/d\*(A/a^2\*ln(tan(d\*x+c))+1/(a^2+b^2)^2\*(1/2\*(-A\*a^2+A\*b^2-2\*B\*a\*b)\*ln(1+tan(d\*x+c)^2)+(-2\*A\*a\*b+B\*a^2-B\*b^2)\*arctan(tan(d\*x+c)))-b\*(3\*A\*a^2\*b+A\*b^3-2\*B\*a^3)/a^2/(a^2+b^2)^2\*ln(a+b\*tan(d\*x+c))+(A\*b-B\*a)\*b/a/(a^2+b^2)/(a+b\*tan(d\*x+c))

Maxima [A]

time = 0.55, size = 208, normalized size = 1.52

$$\frac{2(Ba^2 - 2Aab - Bb^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} + \frac{2(2Ba^3b - 3Aa^2b^2 - Ab^4) \log(b \tan(dx+c) + a)}{a^6 + 2a^4b^2 + a^2b^4} - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(Bab - Ab^2)}{a^4 + a^2b^2 + (a^3b + ab^3) \tan(dx+c)} + \frac{2A \log(\tan(dx+c))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/2\*(2\*(B\*a^2 - 2\*A\*a\*b - B\*b^2)\*(d\*x + c)/(a^4 + 2\*a^2\*b^2 + b^4) + 2\*(2\*B\*a^3\*b - 3\*A\*a^2\*b^2 - A\*b^4)\*log(b\*tan(d\*x + c) + a)/(a^6 + 2\*a^4\*b^2 + a^2\*b^4) - (A\*a^2 + 2\*B\*a\*b - A\*b^2)\*log(tan(d\*x + c)^2 + 1)/(a^4 + 2\*a^2\*b^2 + b^4) - 2\*(B\*a\*b - A\*b^2)/(a^4 + a^2\*b^2 + (a^3\*b + a\*b^3)\*tan(d\*x + c)) + 2\*A\*log(tan(d\*x + c))/a^2)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(137) = 274.

time = 1.72, size = 323, normalized size = 2.36

$$\frac{2Ba^3b^3 - 2Aab^4 - 2(Ba^3 - 2Aa^2b - Ba^2b^2)dx - (Aa^5 + 2Aa^4b + Ab^5) \tan(dx+c) \log\left(\frac{\tan(dx+c)}{\tan(dx+c)+1}\right) - (2Ba^4b - 3Aa^3b^2 - Ab^4) \tan(dx+c) \log\left(\frac{b \tan(dx+c) + a}{\tan(dx+c)+1}\right) - 2(Ba^2b^2 - Aa^2b^3 + (Ba^4b - 2Aa^3b^2 - Ba^2b^3)dx) \tan(dx+c)}{2((a^6 + 2a^4b^2 + a^2b^4)d \tan(dx+c) + (a^7 + 2a^5b^2 + a^3b^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$-1/2*(2*B*a^2*b^3 - 2*A*a*b^4 - 2*(B*a^5 - 2*A*a^4*b - B*a^3*b^2)*d*x - (A*a^5 + 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b + 2*A*a^2*b^3 + A*b^5)*\tan(d*x + c)) * \log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)) - (2*B*a^4*b - 3*A*a^3*b^2 - A*a*b^4 + (2*B*a^3*b^2 - 3*A*a^2*b^3 - A*b^5)*\tan(d*x + c)) * \log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - 2*(B*a^3*b^2 - A*a^2*b^3 + (B*a^4*b - 2*A*a^3*b^2 - B*a^2*b^3)*d*x)*\tan(d*x + c))/((a^6*b + 2*a^4*b^3 + a^2*b^5)*d*\tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)*d)$$

**Sympy** [C] Result contains complex when optimal does not.

time = 1.80, size = 4447, normalized size = 32.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x)

[Out] Piecewise((zoo\*x\*(A + B\*tan(c))\*cot(c)/tan(c)\*\*2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + A\*log(tan(c + d\*x))/d + B\*x)/a\*\*2, Eq(b, 0)), ((A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - A\*log(tan(c + d\*x))/d - A/(2\*d\*tan(c + d\*x)\*\*2) - B\*x - B/(d\*tan(c + d\*x)))/b\*\*2, Eq(a, 0)), (3\*I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 6\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 3\*I\*A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*I\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 2\*A\*log(tan(c + d\*x)\*\*2 + 1)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - 4\*A\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 8\*I\*A\*log(tan(c + d\*x))\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 4\*A\*log(tan(c + d\*x))/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 3\*I\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 4\*A/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*I\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + B\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) - B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*I\*B/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d), Eq(a, -I\*b)), (-3\*I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 6\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 3\*I\*A\*d\*x/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 + 8\*I\*b\*\*2\*d\*tan(c + d\*x) - 4\*b\*\*2\*d) + 2\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c

$$\begin{aligned}
& + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) \\
& + 4*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8 \\
& *I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*A*log(tan(c + d*x)**2 + 1)/(4*b**2*d \\
& *tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*A*log(tan(c + d* \\
& x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4 \\
& *b**2*d) - 8*I*A*log(tan(c + d*x))*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + \\
& 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*A*log(tan(c + d*x))/(4*b**2*d*tan( \\
& c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*A*tan(c + d*x)/(4*b \\
& **2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*A/(4*b**2*d \\
& *tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - B*d*x*tan(c + d*x) \\
& **2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B \\
& *d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b \\
& **2*d) + B*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2 \\
& *d) - B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - \\
& 4*b**2*d) - 2*I*B/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b \\
& **2*d), Eq(a, I*b)), (x*(A + B*tan(c))*cot(c)/(a + b*tan(c))**2, Eq(d, 0)), \\
& (-A*a**5*log(tan(c + d*x)**2 + 1)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4* \\
& a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan \\
& (c + d*x)) + 2*A*a**5*log(tan(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) \\
& ) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5 \\
& *d*tan(c + d*x)) - 4*A*a**4*b*d*x/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4* \\
& a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan \\
& (c + d*x)) - A*a**4*b*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**7*d + 2* \\
& a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3 \\
& *b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 2*A*a**4*b*log(tan(c + d*x))*tan(c \\
& + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d* \\
& tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 4*A*a**3*b**2* \\
& d*x*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a* \\
& **4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 6*A* \\
& a**3*b**2*log(a/b + tan(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a \\
& **5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan \\
& (c + d*x)) + A*a**3*b**2*log(tan(c + d*x)**2 + 1)/(2*a**7*d + 2*a**6*b*d*tan \\
& (c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2 \\
& *a**2*b**5*d*tan(c + d*x)) + 4*A*a**3*b**2*log(tan(c + d*x))/(2*a**7*d + 2* \\
& a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3 \\
& *b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 2*A*a**3*b**2/(2*a**7*d + 2*a**6*b* \\
& d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d \\
& + 2*a**2*b**5*d*tan(c + d*x)) - 6*A*a**2*b**3*log(a/b + tan(c + d*x))*tan( \\
& c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3* \\
& d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*...
\end{aligned}$$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(137) = 274.

time = 0.75, size = 279, normalized size = 2.04

$$\frac{\frac{2(Ba^2 - 2Aab - Bb^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} + \frac{2(2Ba^3b^2 - 3Aa^2b^3 - Ab^5) \log(|b \tan(dx+c) + a|)}{a^6b + 2a^4b^3 + a^2b^5} + \frac{2A \log(|\tan(dx+c)|)}{a^2} - \frac{2(2Ba^3b^2 \tan(dx+c) - 3Aa^2b^3 \tan(dx+c) - Ab^5 \tan(dx+c) + 3Ba^4b - 4Aa^3b^2 + Ba^2b^3 - 2Aab^4)}{(a^6 + 2a^4b^2 + a^2b^4)(b \tan(dx+c) + a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 1/2\*(2\*(B\*a^2 - 2\*A\*a\*b - B\*b^2)\*(d\*x + c)/(a^4 + 2\*a^2\*b^2 + b^4) - (A\*a^2 + 2\*B\*a\*b - A\*b^2)\*log(tan(d\*x + c)^2 + 1)/(a^4 + 2\*a^2\*b^2 + b^4) + 2\*(2\*B\*a^3\*b^2 - 3\*A\*a^2\*b^3 - A\*b^5)\*log(abs(b\*tan(d\*x + c) + a))/(a^6\*b + 2\*a^4\*b^3 + a^2\*b^5) + 2\*A\*log(abs(tan(d\*x + c)))/a^2 - 2\*(2\*B\*a^3\*b^2\*tan(d\*x + c) - 3\*A\*a^2\*b^3\*tan(d\*x + c) - A\*b^5\*tan(d\*x + c) + 3\*B\*a^4\*b - 4\*A\*a^3\*b^2 + B\*a^2\*b^3 - 2\*A\*a\*b^4)/((a^6 + 2\*a^4\*b^2 + a^2\*b^4)\*(b\*tan(d\*x + c) + a)))/d

**Mupad [B]**

time = 8.00, size = 180, normalized size = 1.31

$$\frac{A \ln(\tan(c + dx))}{a^2 d} - \frac{\ln(\tan(c + dx) - i)(A + B i)}{2d(a^2 + a b 2i - b^2)} - \frac{\ln(\tan(c + dx) + i)(B + A i)}{2d(a^2 i i + 2 a b - b^2 i i)} + \frac{A b^2 - B a b}{a d(a^2 + b^2)(a + b \tan(c + dx))} - \frac{b \ln(a + b \tan(c + dx))(-2 B a^3 + 3 A a^2 b + A b^3)}{a^2 d(a^2 + b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] (A\*log(tan(c + d\*x)))/(a^2\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i))/(2\*d\*(a\*b\*2i + a^2 - b^2)) - (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(2\*d\*(2\*a\*b + a^2\*1i - b^2\*1i)) + (A\*b^2 - B\*a\*b)/(a\*d\*(a^2 + b^2)\*(a + b\*tan(c + d\*x))) - (b\*log(a + b\*tan(c + d\*x))\*(A\*b^3 - 2\*B\*a^3 + 3\*A\*a^2\*b))/(a^2\*d\*(a^2 + b^2)^2)

$$3.280 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=192

$$\frac{(a^2 A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} - \frac{(2Ab - aB) \log(\sin(c + dx))}{a^3 d} + \frac{b^2(4a^2 Ab + 2Ab^3 - 3a^3 B - ab^2 B) \log(a \cos(c + dx))}{a^3 (a^2 + b^2)^2 d}$$

[Out]  $-(A*a^2 - A*b^2 + 2*B*a*b)*x/(a^2 + b^2)^2 - (2*A*b - B*a)*\ln(\sin(d*x + c))/a^3/d + b^2*(4*a^2*Ab + 2*Ab^3 - 3*a^3*B - ab^2*B)\log(a*\cos(c + dx))/a^3/(a^2 + b^2)^2/d - b*(A*a^2 + 2*A*b^2 - B*a*b)/a^2/(a^2 + b^2)/d/(a + b*\tan(d*x + c)) - A*\cot(d*x + c)/a/d/(a + b*\tan(d*x + c))$

**Rubi [A]**

time = 0.36, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$-\frac{(2Ab - aB) \log(\sin(c + dx))}{a^3 d} - \frac{b(a^2 A - abB + 2Ab^2)}{a^2 d (a^2 + b^2) (a + b \tan(c + dx))} - \frac{x(a^2 A + 2abB - Ab^2)}{(a^2 + b^2)^2} + \frac{b^2(-3a^3 B + 4a^2 Ab - ab^2 B + 2Ab^3) \log(a \cos(c + dx) + b \sin(c + dx))}{a^3 d (a^2 + b^2)^2} - \frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out]  $-(((a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2)^2) - ((2*A*b - a*B)*\text{Log}[\text{Sin}[c + d*x]])/(a^3*d) + (b^2*(4*a^2*A*b + 2*A*b^3 - 3*a^3*B - a*b^2*B)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(a^3*(a^2 + b^2)^2*d) - (b*(a^2*A + 2*A*b^2 - a*b*B))/(a^2*(a^2 + b^2)*d*(a + b*\text{Tan}[c + d*x])) - (A*\text{Cot}[c + d*x])/(a*d*(a + b*\text{Tan}[c + d*x]))$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x]

```

2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps







**Sympy [C]** Result contains complex when optimal does not.

time = 2.79, size = 8102, normalized size = 42.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*A\*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((A\*x + A/(d\*tan(c + d\*x)) - A/(3\*d\*tan(c + d\*x)\*\*3) + B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - B\*log(tan(c + d\*x))/d - B/(2\*d\*tan(c + d\*x)\*\*2))/b\*\*2, Eq(a, 0)), (9\*A\*d\*x\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 18\*I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 9\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*I\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 8\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 4\*I\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 8\*I\*A\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 16\*A\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 9\*A\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 14\*I\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*A/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 3\*I\*B\*d\*x\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 6\*B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 3\*I\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 8\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 4\*B\*log(tan(c + d\*x))\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 3\*I\*B\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 4\*B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 4\*B\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x))

\*3 - 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)), Eq(a, -I\*b)), (9\*A\*d\*x\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 18\*I\*A\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 9\*A\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 4\*I\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 8\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*I\*A\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 8\*I\*A\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 16\*A\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 8\*I\*A\*log(tan(c + d\*x))\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 9\*A\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 14\*I\*A\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*A/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 3\*I\*B\*d\*x\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 6\*B\*d\*x\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 3\*I\*B\*d\*x\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) + 4\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b\*\*2\*d\*tan(c + d\*x)) - 4\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*3/(4\*b\*\*2\*d\*tan(c + d\*x)\*\*3 + 8\*I\*b\*\*2\*d\*tan(c + d\*x)\*\*2 - 4\*b...

**Giac** [A]

time = 0.92, size = 362, normalized size = 1.89

$$\frac{2(Aa^2+2Bab-Ab^2)(dx+c) + (Ba^2-2Ab-B^2)\log(\tan(dx+c)+1)}{a^2+2a^2b^2} + \frac{2(3Ba^2b^2-4Aa^2b^2+Bab^2-2Ab^2)\log(B\tan(dx+c)+a)}{a^2+2a^2b^2+ab^2} + \frac{Ba^2\tan(dx+c)^2-2Aa^2b^2\tan(dx+c)^2-Ba^2b^2\tan(dx+c)^2+Bb^2\tan(dx+c)^2-3Ba^2b^2\tan(dx+c)+4Aa^2b^2\tan(dx+c)-2Bab^2\tan(dx+c)+4Ab^2\tan(dx+c)+2Aa^2b^2+2Aab^2}{a^6+2a^2b^4+ab^6}(\tan(dx+c)^2+\tan(dx+c)) - \frac{2(Ba-2Ab)\log(\tan(dx+c))}{a^2}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] -1/2\*(2\*(A\*a^2 + 2\*B\*a\*b - A\*b^2)\*(d\*x + c)/(a^4 + 2\*a^2\*b^2 + b^4) + (B\*a^2 - 2\*A\*a\*b - B\*b^2)\*log(tan(d\*x + c)^2 + 1)/(a^4 + 2\*a^2\*b^2 + b^4) + 2\*(3\*B\*a^3\*b^3 - 4\*A\*a^2\*b^4 + B\*a\*b^5 - 2\*A\*b^6)\*log(abs(b\*tan(d\*x + c) + a))/(a^7\*b + 2\*a^5\*b^3 + a^3\*b^5) + (B\*a^4\*b\*tan(d\*x + c)^2 - 2\*A\*a^3\*b^2\*tan(d\*x + c)^2 - B\*a^2\*b^3\*tan(d\*x + c)^2 + B\*a^5\*tan(d\*x + c) - 3\*B\*a^3\*b^2\*tan

$$\frac{(d*x + c) + 6*A*a^2*b^3*\tan(d*x + c) - 2*B*a*b^4*\tan(d*x + c) + 4*A*b^5*\tan(d*x + c) + 2*A*a^5 + 4*A*a^3*b^2 + 2*A*a*b^4}{(a^6 + 2*a^4*b^2 + a^2*b^4)*(b*\tan(d*x + c)^2 + a*\tan(d*x + c))} - 2*(B*a - 2*A*b)*\log(\text{abs}(\tan(d*x + c)))/a^3)/d$$

**Mupad [B]**

time = 9.27, size = 230, normalized size = 1.20

$$\frac{b^2 \ln(a + b \tan(c + dx)) (-3Ba^3 + 4Aa^2b - Ba^2b^2 + 2Ab^3)}{a^3 d (a^2 + b^2)^2} - \frac{\ln(\tan(c + dx)) (2Ab - Ba)}{a^3 d} + \frac{\ln(\tan(c + dx) + 1i) (B + A1i)}{2d (-a^2 + ab2i + b^2)} + \frac{\ln(\tan(c + dx) - 1i) (A + B1i)}{2d (-a^2 1i + 2ab + b^2 1i)} - \frac{\frac{A}{a} + \frac{\tan(c+dx)(Aa^2b - Ba^2b^2 + 2Ab^3)}{a^2(a^2+b^2)}}{d (b \tan(c + dx))^2 + a \tan(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(2\*d\*(a\*b\*2i - a^2 + b^2)) - (log(tan(c + d\*x))\*(2\*A\*b - B\*a))/(a^3\*d) - (A/a + (tan(c + d\*x)\*(2\*A\*b^3 + A\*a^2\*b - B\*a\*b^2)))/(a^2\*(a^2 + b^2)))/(d\*(a\*tan(c + d\*x) + b\*tan(c + d\*x)^2)) + (log(tan(c + d\*x) - 1i)\*(A + B\*1i))/(2\*d\*(2\*a\*b - a^2\*1i + b^2\*1i)) + (b^2\*log(a + b\*tan(c + d\*x))\*(2\*A\*b^3 - 3\*B\*a^3 + 4\*A\*a^2\*b - B\*a\*b^2))/(a^3\*d\*(a^2 + b^2)^2)

$$3.281 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=250

$$\frac{(2aAb - a^2B + b^2B)x}{(a^2 + b^2)^2} - \frac{(a^2A - 3Ab^2 + 2abB) \log(\sin(c + dx))}{a^4d} - \frac{b^3(5a^2Ab + 3Ab^3 - 4a^3B - 2ab^2B) \log(a + b \tan(c + dx))}{a^4(a^2 + b^2)^2 d}$$

[Out]  $(2Aa^2b - B^2a^2 + B^2b^2)x / (a^2 + b^2)^2 - (Aa^2 - 3Ab^2 + 2Bab) \ln(\sin(dx+c)) / a^4/d - b^3(5Aa^2b + 3Ab^3 - 4a^3B - 2ab^2B) \ln(a \cos(dx+c) + b \sin(dx+c)) / a^4 / (a^2 + b^2)^2 / d + b(2Aa^2b + 3Ab^3 - B^2a^3 - 2B^2ab^2) / a^3 / (a^2 + b^2) / d / (a + b \tan(dx+c)) + 1/2(3Ab - 2B^2a) \cot(dx+c) / a^2 / d / (a + b \tan(dx+c)) - 1/2A \cot(dx+c)^2 / a / d / (a + b \tan(dx+c))$

**Rubi** [A]

time = 0.56, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{x(a^2(-B) + 2aAb + b^2B)}{(a^2 + b^2)^2} + \frac{(3Ab - 2aB) \cot(c + dx)}{2a^2d(a + b \tan(c + dx))} - \frac{(a^2A + 2abB - 3Ab^2) \log(\sin(c + dx))}{a^4d} + \frac{b(a^3(-B) + 2a^2Ab - 2ab^2B + 3Ab^3)}{a^3d(a^2 + b^2)(a + b \tan(c + dx))} - \frac{b^3(-4a^3B + 5a^2Ab - 2ab^2B + 3Ab^3) \log(a \cos(c + dx) + b \sin(c + dx))}{a^4d(a^2 + b^2)^2} - \frac{A \cot^2(c + dx)}{2ad(a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out]  $((2a^2Ab - a^2B + b^2B)x) / (a^2 + b^2)^2 - ((a^2A - 3Ab^2 + 2a^2bB) \text{Log}[\text{Sin}[c + d*x]]) / (a^4d) - (b^3(5a^2Ab + 3Ab^3 - 4a^3B - 2a^2b^2B) \text{Log}[a \text{Cos}[c + d*x] + b \text{Sin}[c + d*x]]) / (a^4(a^2 + b^2)^2d) + (b(2a^2Ab + 3Ab^3 - a^3B - 2a^2b^2B)) / (a^3(a^2 + b^2)d(a + b \tan[c + d*x])) + ((3Ab - 2a^2B) \text{Cot}[c + d*x]) / (2a^2d(a + b \tan[c + d*x])) - (A \text{Cot}[c + d*x]^2) / (2a^2d(a + b \tan[c + d*x]))$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]) / ((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^m \* ((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^n, x\_Symbol] := Si

```

mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] := Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= -\frac{A \cot^2(c+dx)}{2ad(a+b\tan(c+dx))} - \frac{\int \frac{\cot^2(c+dx)(3Ab-2aB+2aA\tan(c+dx)+3Ab\tan(c+dx))}{(a+b\tan(c+dx))^2} dx}{2a} \\
&= \frac{(3Ab-2aB)\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{A \cot^2(c+dx)}{2ad(a+b\tan(c+dx))} + \frac{\int \frac{\cot(c+dx)(3Ab-2aB+2aA\tan(c+dx)+3Ab\tan(c+dx))}{(a+b\tan(c+dx))^2} dx}{2a} \\
&= \frac{b(2a^2Ab+3Ab^3-a^3B-2ab^2B)}{a^3(a^2+b^2)d(a+b\tan(c+dx))} + \frac{(3Ab-2aB)\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{A \cot^2(c+dx)}{2ad(a+b\tan(c+dx))} \\
&= \frac{(2aAb-a^2B+b^2B)x}{(a^2+b^2)^2} + \frac{b(2a^2Ab+3Ab^3-a^3B-2ab^2B)}{a^3(a^2+b^2)d(a+b\tan(c+dx))} + \frac{(3Ab-2aB)\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{A \cot^2(c+dx)}{2ad(a+b\tan(c+dx))} \\
&= \frac{(2aAb-a^2B+b^2B)x}{(a^2+b^2)^2} - \frac{(a^2A-3Ab^2+2abB)\log(\sin(c+dx))}{a^4d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 2.94, size = 220, normalized size = 0.88

$$\frac{-\frac{2(-2Ab+aB)\cot(c+dx)}{a^3} - \frac{A \cot^2(c+dx)}{a^2} + \frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2} - \frac{2(a^2A-3Ab^2+2abB)\log(\tan(c+dx))}{a^4} + \frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2b^3(-5a^2Ab-3Ab^3+4a^3B+2ab^2B)\log(a+b\tan(c+dx))}{a^4(a^2+b^2)^2} + \frac{2b^3(Ab-aB)}{a^3(a^2+b^2)(a+b\tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
[Out] ((-2*(-2*A*b + a*B)*Cot[c + d*x])/a^3 - (A*Cot[c + d*x]^2)/a^2 + ((A + I*B)
 *Log[I - Tan[c + d*x]])/(a + I*b)^2 - (2*(a^2*A - 3*A*b^2 + 2*a*b*B)*Log[Tan
 [c + d*x]])/a^4 + ((A - I*B)*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b^3*(
 -5*a^2*A*b - 3*A*b^3 + 4*a^3*B + 2*a*b^2*B)*Log[a + b*Tan[c + d*x]])/(a^4*(
 a^2 + b^2)^2) + (2*b^3*(A*b - a*B))/(a^3*(a^2 + b^2)*(a + b*Tan[c + d*x]))
 / (2*d)

```

**Maple [A]**

time = 0.39, size = 227, normalized size = 0.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOS
E)

```

```

[Out] 1/d*(-1/2*A/a^2/tan(d*x+c)^2-(-2*A*b+B*a)/a^3/tan(d*x+c)+(-A*a^2+3*A*b^2-2*
 B*a*b)/a^4*ln(tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(A*a^2-A*b^2+2*B*a*b)*ln(1+tan
 (d*x+c)^2)+(2*A*a*b-B*a^2+B*b^2)*arctan(tan(d*x+c)))-b^3*(5*A*a^2*b+3*A*b^3
 -4*B*a^3-2*B*a*b^2)/a^4/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+(A*b-B*a)*b^3/a^3/(a
 ^2+b^2)/(a+b*tan(d*x+c)))

```

**Maxima [A]**

time = 0.53, size = 325, normalized size = 1.30

$$\frac{2(Ba^2 - 2Ab - Bb^2)(dx+c) - 2(4Ba^3b^3 - 5Aa^2b^4 + 2Bab^5 - 3Ab^6) \log(b \tan(dx+c)+a) - \frac{(Aa^2 + 2Bab - Ab^2) \log(\tan(dx+c)^2 + 1)}{a^2 + 2a^2b^2 + b^4} + \frac{Aa^4 + Aa^2b^2 + 2(Ba^3b - 2Aa^2b^2 + 2Bab^3 - 3Ab^4) \tan(dx+c) + (2Ba^4 - 3Aa^3b + 2Ba^2b^2 - 3Aab^3) \tan(dx+c)}{(a^2 + 2a^2b^2 + b^4) \tan(dx+c)^2} + \frac{2(Aa^2 + 2Bab - 3Ab^2) \log(\tan(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 
$$-1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(4*B*a^3*b^3 - 5*A*a^2*b^4 + 2*B*a*b^5 - 3*A*b^6)*\log(b*\tan(d*x + c) + a)/(a^8 + 2*a^6*b^2 + a^4*b^4) - (A*a^2 + 2*B*a*b - A*b^2)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (A*a^4 + A*a^2*b^2 + 2*(B*a^3*b - 2*A*a^2*b^2 + 2*B*a*b^3 - 3*A*b^4)*\tan(d*x + c)^2 + (2*B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2 - 3*A*a*b^3)*\tan(d*x + c))/((a^5*b + a^3*b^3)*\tan(d*x + c)^3 + (a^6 + a^4*b^2)*\tan(d*x + c)^2) + 2*(A*a^2 + 2*B*a*b - 3*A*b^2)*\log(\tan(d*x + c))/a^4)/d$$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 590 vs.  $2(246) = 492$ .

time = 1.47, size = 590, normalized size = 2.36

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$-1/2*(A*a^7 + 2*A*a^5*b^2 + A*a^3*b^4 + (A*a^6*b + 2*A*a^4*b^3 - 2*B*a^3*b^4 + 3*A*a^2*b^5 + 2*(B*a^6*b - 2*A*a^5*b^2 - B*a^4*b^3)*d*x)*\tan(d*x + c)^3 + (A*a^7 + 2*B*a^6*b - 2*A*a^5*b^2 + 4*B*a^4*b^3 - 7*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 + 2*(B*a^7 - 2*A*a^6*b - B*a^5*b^2)*d*x)*\tan(d*x + c)^2 + ((A*a^6*b + 2*B*a^5*b^2 - A*a^4*b^3 + 4*B*a^3*b^4 - 5*A*a^2*b^5 + 2*B*a*b^6 - 3*A*b^7)*\tan(d*x + c)^3 + (A*a^7 + 2*B*a^6*b - A*a^5*b^2 + 4*B*a^4*b^3 - 5*A*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*\tan(d*x + c)^2)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)) - ((4*B*a^3*b^4 - 5*A*a^2*b^5 + 2*B*a*b^6 - 3*A*b^7)*\tan(d*x + c)^3 + (4*B*a^4*b^3 - 5*A*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*\tan(d*x + c)^2)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) + (2*B*a^7 - 3*A*a^6*b + 4*B*a^5*b^2 - 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*\tan(d*x + c))/((a^8*b + 2*a^6*b^3 + a^4*b^5)*d*\tan(d*x + c)^3 + (a^9 + 2*a^7*b^2 + a^5*b^4)*d*\tan(d*x + c)^2)$$

**Sympy [C]** Result contains complex when optimal does not.

time = 4.35, size = 9840, normalized size = 39.36

Too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)`

[Out] `Piecewise((zoo*A*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((A*log(tan(c + d*x)**2 + 1)/(2*d) - A*log(tan(c + d*x))/d - A/(2*d*tan(c + d*x)**2) - B*x - B/(d*tan(c + d*x)))/a**2, Eq(b, 0)), ((-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*log(tan(c + d*x))/d + A/(2*d*tan(c + d*x)**2) - A/(4*d*tan(c + d*x)**4) + B*x + B/(d*tan(c + d*x)) - B/(3*d*tan(c + d*x)**3))/b**2, Eq(a, 0)), (-15*I*A*d*x*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 30*A*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 15*I*A*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 8*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 16*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 8*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 16*A*log(tan(c + d*x))**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 32*I*A*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 16*A*log(tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 15*I*A*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 22*A*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 4*I*A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 2*A/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 9*B*d*x*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 18*I*B*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 9*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 8*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 8*I*B*log(tan(c + d*x))*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 16*B*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 8*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 9*B*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2)`

$c + d*x)**2) - 14*I*B*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 4*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**4 - 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2), E$   
 $q(a, -I*b)), (15*I*A*d*x*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 30*A*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 15*I*A*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 8*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 16*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 8*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 16*A*log(tan(c + d*x))*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 32*I*A*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 16*A*log(tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 15*I*A*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 22*A*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 4*I*A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) - 2*A/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 9*B*d*x*tan(c + d*x)**4/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d*tan(c + d*x)**3 - 4*b**2*d*tan(c + d*x)**2) + 18*I*B*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**4 + 8*I*b**2*d...$

**Giac [A]**

time = 0.99, size = 402, normalized size = 1.61

$$\frac{2 (A^2 - 2 A b - B^2) \log(\tan(d x + c)) - (A^2 + 2 B a - A^2) \log(\tan(d x + c)^2 + 1) + 2 (A^2 B^2 - 5 A^2 B^2 + 2 B^2) \log(\tan(d x + c)) + 2 (A^2 B^2 \tan(d x + c) - 5 A^2 B^2 \tan(d x + c) - 1 A^2 \tan(d x + c) + 5 B^2 \tan(d x + c) - 1 A^2) + 2 (A^2 + 2 B a - 3 A^2) \log(\tan(d x + c)) - 3 A^2 \tan(d x + c) + 6 B a \tan(d x + c) - 9 A^2 \tan(d x + c)^2 - 2 B^2 \tan(d x + c) + 4 A b \tan(d x + c) - 4 A^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $-1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (A*a^2 + 2*B*a*b - A*b^2)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(4*B*a^3*b^4 - 5*A*a^2*b^5 + 2*B*a*b^6 - 3*A*b^7)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^8*b + 2*a^6*b^3 + a^4*b^5) + 2*(4*B*a^3*b^4*\tan(d*x + c) - 5*A*a^2*b^5*\tan(d*x + c) + 2*B*a*b^6*\tan(d*x + c) - 3*A*b^7*\tan(d*x + c) + 5*B*a^4*b^3 - 6*A*a^3*b^4 + 3*B*a^2*b^5 - 4*A*a*b^6)/((a^8 + 2*a^6*b^2 + a^4*b^4)*(b*\tan(d*x + c) + a)) + 2*(A*a^2 + 2*B*a*b - 3*A*b^2)*\log(\text{abs}(\tan(d*x + c)))/a^4 - (3*A*a^2*\tan(d*x + c)^2 + 6*B*a*b*\tan(d*x + c)^2 - 9*A*b^2*\tan(d*x + c)^2 - 2*B*a^2*\tan(d*x + c) + 4*A*a*b*\tan(d*x + c) - A*a^2)/(a^4*\tan(d*x + c)^2))/d$

**Mupad [B]**

time = 10.66, size = 284, normalized size = 1.14

$$\frac{\frac{\tan(c+dx)(3Ab-2Ba) - \frac{A}{2a} + \frac{\tan(c+dx)^2(-Bx^2b^2+2Aa^2b^2-2Ba^2b^2+3A^2b^2)}{a^2(c^2+b^2)}}{d(b\tan(c+dx)^3+a\tan(c+dx)^2)} - \frac{\ln(\tan(c+dx))(Aa^2+2Bab-3Ab^2)}{a^4d} + \frac{\ln(\tan(c+dx)-1)(A+B1i)}{2d(a^2+ab2i-b^2)} - \frac{\ln(a+b\tan(c+dx))(-4Ba^2b^2+5Aa^2b^4-2Ba^2b^5+3A^2b^6)}{d(a^8+2a^6b^2+a^4b^4)} + \frac{\ln(\tan(c+dx)+1i)(B+A1i)}{2d(a^21i+2ab-b^21i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

```
[Out] ((tan(c + d*x)*(3*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (tan(c + d*x)^2*(3*A*b^4 + 2*A*a^2*b^2 - 2*B*a*b^3 - B*a^3*b))/(a^3*(a^2 + b^2)))/(d*(a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) - (log(tan(c + d*x))*(A*a^2 - 3*A*b^2 + 2*B*a*b))/(a^4*d) + (log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(a + b*tan(c + d*x))*(3*A*b^6 + 5*A*a^2*b^4 - 4*B*a^3*b^3 - 2*B*a*b^5))/(d*(a^8 + a^4*b^4 + 2*a^6*b^2)) + (log(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(2*a*b + a^2*1i - b^2*1i))
```



```

mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

```

### Rule 3698

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

```

### Rule 3707

```

Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C, 0]

```

### Rule 3726

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3728

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*

```

```
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} dx &= \frac{a(Ab-aB)\tan^3(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^2(c+dx)(-3a(Ab-aB)+2b(Ab-aB))}{(a+b\tan(c+dx))^3} dx}{2b(a^2+b^2)d} \\
&= \frac{a(Ab-aB)\tan^3(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{a(a^2Ab+5Ab^3-3a^3B-7ab^2)}{2b^2(a^2+b^2)^2d(a+b\tan(c+dx))} \\
&= -\frac{(a^3Ab+3aAb^3-3a^4B-6a^2b^2B-b^4B)\tan(c+dx)}{b^3(a^2+b^2)^2d} + \frac{a(Ab-aB)}{2b(a^2+b^2)} \\
&= \frac{(a^3A-3aAb^2+3a^2bB-b^3B)x}{(a^2+b^2)^3} - \frac{(a^3Ab+3aAb^3-3a^4B-6a^2b^2B)}{b^3(a^2+b^2)} \\
&= \frac{(a^3A-3aAb^2+3a^2bB-b^3B)x}{(a^2+b^2)^3} + \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)}{(a^2+b^2)^3d} \\
&= \frac{(a^3A-3aAb^2+3a^2bB-b^3B)x}{(a^2+b^2)^3} + \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)}{(a^2+b^2)^3d}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.  
time = 6.59, size = 1146, normalized size = 3.46

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
[Out] (a^4*(-(A*b) + a*B)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*(A + B
*Tan[c + d*x]))/(2*(a - I*b)^2*(a + I*b)^2*b^2*d*(A*Cos[c + d*x] + B*Sin[c
+ d*x]))*(a + b*Tan[c + d*x])^3 + ((a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*
(c + d*x)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(A + B*Tan[c +
d*x]))/((a - I*b)^3*(a + I*b)^3*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + b
*Tan[c + d*x])^3) + ((I*a^11*A*b^4 + a^10*A*b^5 + (5*I)*a^9*A*b^6 + 5*a^8*A
*b^7 + (13*I)*a^7*A*b^8 + 13*a^6*A*b^9 + (15*I)*a^5*A*b^10 + 15*a^4*A*b^11
+ (6*I)*a^3*A*b^12 + 6*a^2*A*b^13 - (3*I)*a^12*b^3*B - 3*a^11*b^4*B - (15*I
)*a^10*b^5*B - 15*a^9*b^6*B - (31*I)*a^8*b^7*B - 31*a^7*b^8*B - (29*I)*a^6*
```

$$\begin{aligned}
& b^9 B - 29 a^5 b^{10} B - (10 I) a^4 b^{11} B - 10 a^3 b^{12} B) (c + dx) \operatorname{Sec}[c \\
& + dx]^2 (a \cos[c + dx] + b \sin[c + dx])^3 (A + B \tan[c + dx]) / ((a - I b)^6 (a + I b)^5 b^7 d (A \cos[c + dx] + B \sin[c + dx]) (a + b \tan[c + dx])^3) \\
& - (I (a^6 A b + 3 a^4 A b^3 + 6 a^2 A b^5 - 3 a^7 B - 9 a^5 b^2 B - 10 a^3 b^4 B) \operatorname{ArcTan}[\tan[c + dx]] \operatorname{Sec}[c + dx]^2 (a \cos[c + dx] + b \sin[c + dx])^3 (A + B \tan[c + dx]) / (b^4 (a^2 + b^2)^3 d (A \cos[c + dx] + B \sin[c + dx]) (a + b \tan[c + dx])^3) + ((- (A b) + 3 a B) \operatorname{Log}[\cos[c + dx]] \operatorname{Sec}[c + dx]^2 (a \cos[c + dx] + b \sin[c + dx])^3 (A + B \tan[c + dx]) / (b^4 d (A \cos[c + dx] + B \sin[c + dx]) (a + b \tan[c + dx])^3) + ((a^6 A b + 3 a^4 A b^3 + 6 a^2 A b^5 - 3 a^7 B - 9 a^5 b^2 B - 10 a^3 b^4 B) \operatorname{Log}[(a \cos[c + dx] + b \sin[c + dx])^2] \operatorname{Sec}[c + dx]^2 (a \cos[c + dx] + b \sin[c + dx])^3 (A + B \tan[c + dx]) / (2 b^4 (a^2 + b^2)^3 d (A \cos[c + dx] + B \sin[c + dx]) (a + b \tan[c + dx])^3) + (\operatorname{Sec}[c + dx]^2 (a \cos[c + dx] + b \sin[c + dx])^2 (- (a^4 A b \sin[c + dx]) - 4 a^2 A b^3 \sin[c + dx] + 2 a^5 B \sin[c + dx] + 5 a^3 b^2 B \sin[c + dx]) (A + B \tan[c + dx])) / ((a - I b)^2 (a + I b)^2 b^3 d (A \cos[c + dx] + B \sin[c + dx]) (a + b \tan[c + dx])^3) + (B \operatorname{Sec}[c + dx]^2 (a \cos[c + dx] + b \sin[c + dx])^3 \tan[c + dx] (A + B \tan[c + dx]) / (b^3 d (A \cos[c + dx] + B \sin[c + dx]) (a + b \tan[c + dx])^3)
\end{aligned}$$

**Maple [A]**

time = 0.35, size = 263, normalized size = 0.79

method	result
derivativedivides	$\frac{B \tan(dx+c)}{b^3} + \frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \ln(1 + \tan^2(dx+c))}{2} + \frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^3} + \frac{a^2 (A a^4 b + \dots)}{(a^2 + b^2)^3}$
default	$\frac{B \tan(dx+c)}{b^3} + \frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \ln(1 + \tan^2(dx+c))}{2} + \frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^3} + \frac{a^2 (A a^4 b + \dots)}{(a^2 + b^2)^3}$
norman	$\frac{B (\tan^3(dx+c))}{bd} + \frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) a^2 x}{(a^4 + 2a^2 b^2 + b^4) (a^2 + b^2)} + \frac{b^2 (A a^3 - 3A a b^2 + 3B a^2 b - B b^3) x (\tan^2(dx+c))}{(a^4 + 2a^2 b^2 + b^4) (a^2 + b^2)} + \frac{a (2A a^4 b + 4A a^2 b^3 - 6B a^3 b^2)}{d b^3 (a^2 + b^2)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(dx+c)^4*(A+B*tan(dx+c))/(a+b*tan(dx+c))^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned}
& 1/d*(B/b^3*\tan(dx+c)+1/(a^2+b^2)^3*(1/2*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2) \\
& * \ln(1+\tan(dx+c)^2)+(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*\arctan(\tan(dx+c)))+1 \\
& /b^4*a^2*(A*a^4*b+3*A*a^2*b^3+6*A*b^5-3*B*a^5-9*B*a^3*b^2-10*B*a*b^4)/(a^2+ \\
& b^2)^3*\ln(a+b*\tan(dx+c))-1/2/b^4*a^4*(A*b-B*a)/(a^2+b^2)/(a+b*\tan(dx+c))^ \\
& 2+1/b^4*a^3*(2*A*a^2*b+4*A*b^3-3*B*a^3-5*B*a*b^2)/(a^2+b^2)^2/(a+b*\tan(dx+c))
\end{aligned}$$

**Maxima [A]**

time = 0.60, size = 389, normalized size = 1.18

$$\frac{2(Aa^3+3Ba^2b-3Aab^2-Bb^3)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(3Ba^7-Aa^6b+9Ba^5b^2-3Aa^4b^3+10Ba^3b^4-6Aa^2b^5)\log(b\tan(dx+c)+a)}{a^6b^4+3a^4b^6+3a^2b^8+b^{10}} + \frac{(Ba^3-3Aa^2b-3Ab^2+Ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{5Ba^7-3Aa^6b+9Ba^5b^2-7Aa^4b^3+2(3Ba^6b-2Aa^5b^2+5Ba^4b^3-4Aa^3b^4)\tan(dx+c)}{a^6b^4+2a^4b^6+a^2b^8+(a^4b^6+2a^2b^8+b^{10})\tan(dx+c)^2+2(a^6b^4+2a^4b^6+ab^8)\tan(dx+c)} + \frac{2B\tan(dx+c)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/2\*(2\*(A\*a^3 + 3\*B\*a^2\*b - 3\*A\*a\*b^2 - B\*b^3)\*(d\*x + c)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) - 2\*(3\*B\*a^7 - A\*a^6\*b + 9\*B\*a^5\*b^2 - 3\*A\*a^4\*b^3 + 10\*B\*a^3\*b^4 - 6\*A\*a^2\*b^5)\*log(b\*tan(d\*x + c) + a)/(a^6\*b^4 + 3\*a^4\*b^6 + 3\*a^2\*b^8 + b^10) + (B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*log(tan(d\*x + c)^2 + 1)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) - (5\*B\*a^7 - 3\*A\*a^6\*b + 9\*B\*a^5\*b^2 - 7\*A\*a^4\*b^3 + 2\*(3\*B\*a^6\*b - 2\*A\*a^5\*b^2 + 5\*B\*a^4\*b^3 - 4\*A\*a^3\*b^4)\*tan(d\*x + c))/(a^6\*b^4 + 2\*a^4\*b^6 + a^2\*b^8 + (a^4\*b^6 + 2\*a^2\*b^8 + b^10)\*tan(d\*x + c)^2 + 2\*(a^5\*b^5 + 2\*a^3\*b^7 + a\*b^9)\*tan(d\*x + c)) + 2\*B\*tan(d\*x + c)/b^3)/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(328) = 656.

time = 1.95, size = 890, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] -1/2\*(3\*B\*a^7\*b^2 - A\*a^6\*b^3 + 9\*B\*a^5\*b^4 - 7\*A\*a^4\*b^5 - 2\*(B\*a^6\*b^3 + 3\*B\*a^4\*b^5 + 3\*B\*a^2\*b^7 + B\*b^9)\*tan(d\*x + c)^3 - 2\*(A\*a^5\*b^4 + 3\*B\*a^4\*b^5 - 3\*A\*a^3\*b^6 - B\*a^2\*b^7)\*d\*x - (9\*B\*a^7\*b^2 - 3\*A\*a^6\*b^3 + 23\*B\*a^5\*b^4 - 9\*A\*a^4\*b^5 + 12\*B\*a^3\*b^6 + 4\*B\*a\*b^8 + 2\*(A\*a^3\*b^6 + 3\*B\*a^2\*b^7 - 3\*A\*a\*b^8 - B\*b^9)\*d\*x)\*tan(d\*x + c)^2 + (3\*B\*a^9 - A\*a^8\*b + 9\*B\*a^7\*b^2 - 3\*A\*a^6\*b^3 + 10\*B\*a^5\*b^4 - 6\*A\*a^4\*b^5 + (3\*B\*a^7\*b^2 - A\*a^6\*b^3 + 9\*B\*a^5\*b^4 - 3\*A\*a^4\*b^5 + 10\*B\*a^3\*b^6 - 6\*A\*a^2\*b^7)\*tan(d\*x + c)^2 + 2\*(3\*B\*a^8\*b - A\*a^7\*b^2 + 9\*B\*a^6\*b^3 - 3\*A\*a^5\*b^4 + 10\*B\*a^4\*b^5 - 6\*A\*a^3\*b^6)\*tan(d\*x + c))\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)) - (3\*B\*a^9 - A\*a^8\*b + 9\*B\*a^7\*b^2 - 3\*A\*a^6\*b^3 + 9\*B\*a^5\*b^4 - 3\*A\*a^4\*b^5 + 3\*B\*a^3\*b^6 - A\*a^2\*b^7 + (3\*B\*a^7\*b^2 - A\*a^6\*b^3 + 9\*B\*a^5\*b^4 - 3\*A\*a^4\*b^5 + 9\*B\*a^3\*b^6 - 3\*A\*a^2\*b^7 + 3\*B\*a\*b^8 - A\*b^9)\*tan(d\*x + c)^2 + 2\*(3\*B\*a^8\*b - A\*a^7\*b^2 + 9\*B\*a^6\*b^3 - 3\*A\*a^5\*b^4 + 9\*B\*a^4\*b^5 - 3\*A\*a^3\*b^6 + 3\*B\*a^2\*b^7 - A\*a\*b^8)\*tan(d\*x + c))\*log(1/(tan(d\*x + c)^2 + 1)) - 2\*(3\*B\*a^8\*b - A\*a^7\*b^2 + 6\*B\*a^6\*b^3 - 3\*A\*a^5\*b^4 - 2\*B\*a^4\*b^5 + 4\*A\*a^3\*b^6 + B\*a^2\*b^7 + 2\*(A\*a^4\*b^5 + 3\*B\*a^3\*b^6 - 3\*A\*a^2\*b^7



- B\*a\*b^8)\*d\*x)\*tan(d\*x + c))/((a^6\*b^6 + 3\*a^4\*b^8 + 3\*a^2\*b^10 + b^12)\*d  
\*tan(d\*x + c)^2 + 2\*(a^7\*b^5 + 3\*a^5\*b^7 + 3\*a^3\*b^9 + a\*b^11)\*d\*tan(d\*x +  
c) + (a^8\*b^4 + 3\*a^6\*b^6 + 3\*a^4\*b^8 + a^2\*b^10)\*d)

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [A]

time = 1.20, size = 505, normalized size = 1.53

$$\frac{\frac{1}{2} \frac{2A^2a^3 + 3B^2a^2b - 3A^2ab^2 - B^2b^3}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{B^2a^3 - 3A^2ab^2 - 3B^2a^2b + A^2b^3}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} \log(\tan(dx + c)^2 + 1) - \frac{2(3B^2a^7 - A^2a^6b + 9B^2a^5b^2 - 3A^2a^4b^3 + 10B^2a^3b^4 - 6A^2a^2b^5) \log(\tan(dx + c) + a)}{a^6b^4 + 3a^4b^6 + 3a^2b^8 + b^{10}} + \frac{2B^2a^7 \tan(dx + c)^2 - 3A^2a^6b^3 \tan(dx + c)^2 + 27B^2a^5b^4 \tan(dx + c)^2 - 9A^2a^4b^5 \tan(dx + c)^2 + 30B^2a^3b^6 \tan(dx + c)^2 - 18A^2a^2b^7 \tan(dx + c)^2 + 12B^2a^8b \tan(dx + c) - 2A^2a^7b^2 \tan(dx + c) + 38B^2a^6b^3 \tan(dx + c) - 6A^2a^5b^4 \tan(dx + c) + 50B^2a^4b^5 \tan(dx + c) - 28A^2a^3b^6 \tan(dx + c) + 4B^2a^9 + 13B^2a^7b^2 + A^2a^6b^3 + 21B^2a^5b^4 - 11A^2a^4b^5}{(a^6b^4 + 3a^4b^6 + 3a^2b^8 + b^{10})(\tan(dx + c) + a)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] 1/2\*(2\*(A\*a^3 + 3\*B\*a^2\*b - 3\*A\*a\*b^2 - B\*b^3)\*(d\*x + c)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + (B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*log(tan(d\*x + c)^2 + 1)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) - 2\*(3\*B\*a^7 - A\*a^6\*b + 9\*B\*a^5\*b^2 - 3\*A\*a^4\*b^3 + 10\*B\*a^3\*b^4 - 6\*A\*a^2\*b^5)\*log(abs(b\*tan(d\*x + c) + a))/(a^6\*b^4 + 3\*a^4\*b^6 + 3\*a^2\*b^8 + b^10) + 2\*B\*tan(d\*x + c)/b^3 + (9\*B\*a^7\*b^2\*tan(d\*x + c)^2 - 3\*A\*a^6\*b^3\*tan(d\*x + c)^2 + 27\*B\*a^5\*b^4\*tan(d\*x + c)^2 - 9\*A\*a^4\*b^5\*tan(d\*x + c)^2 + 30\*B\*a^3\*b^6\*tan(d\*x + c)^2 - 18\*A\*a^2\*b^7\*tan(d\*x + c)^2 + 12\*B\*a^8\*b\*tan(d\*x + c) - 2\*A\*a^7\*b^2\*tan(d\*x + c) + 38\*B\*a^6\*b^3\*tan(d\*x + c) - 6\*A\*a^5\*b^4\*tan(d\*x + c) + 50\*B\*a^4\*b^5\*tan(d\*x + c) - 28\*A\*a^3\*b^6\*tan(d\*x + c) + 4\*B\*a^9 + 13\*B\*a^7\*b^2 + A\*a^6\*b^3 + 21\*B\*a^5\*b^4 - 11\*A\*a^4\*b^5)/(a^6\*b^4 + 3\*a^4\*b^6 + 3\*a^2\*b^8 + b^10)\*(b\*tan(d\*x + c) + a)^2)/d

**Mupad** [B]

time = 7.87, size = 335, normalized size = 1.01

$$\frac{B \tan(c + dx)}{b^3 d} + \frac{\ln(\tan(c + dx) - 1) (-B + A 11)}{2d (-a^3 - a^2 b 3i + 3 a b^2 + b^3 11)} + \frac{\ln(\tan(c + dx) + 1) (A - B 11)}{2d (-a^3 11 - 3 a^2 b + a b^2 3i + b^3)} - \frac{\frac{5 B a^2 - 3 A a^2 b + 9 B a^2 b^2 - A a^2 b^3}{24 (a^3 + 2 a^2 b + b^3)} + \frac{\tan(c + dx) (9 B a^2 - 2 A a^2 b + 5 B a^2 b^2 - 4 A a^2 b^3)}{a^2 + 2 a b + a^2}}{d (a^2 b^3 + 2 a b^4 \tan(c + dx) + b^5 \tan(c + dx)^2)} + \frac{a^2 \ln(a + b \tan(c + dx)) (-3 B a^5 + A a^4 b - 9 B a^3 b^2 + 3 A a^2 b^3 - 10 B a b^4 + 6 A b^5)}{b^3 d (a^2 + b^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^4\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(3\*a\*b^2 - a^2\*b^3i - a^3 + b^3\*1i)) - ((5\*B\*a^7 - 7\*A\*a^4\*b^3 + 9\*B\*a^5\*b^2 - 3\*A\*a^6\*b)/(2\*b\*(a^4 + b^4 + 2

$$\begin{aligned}
& *a^2*b^2)) + (\tan(c + d*x)*(3*B*a^6 - 4*A*a^3*b^3 + 5*B*a^4*b^2 - 2*A*a^5*b \\
& ))/(a^4 + b^4 + 2*a^2*b^2))/(d*(a^2*b^3 + b^5*\tan(c + d*x)^2 + 2*a*b^4*\tan( \\
& c + d*x))) + (\log(\tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2*3i - 3*a^2*b - \\
& a^3*1i + b^3)) + (B*\tan(c + d*x))/(b^3*d) + (a^2*\log(a + b*\tan(c + d*x))* \\
& 6*A*b^5 - 3*B*a^5 + 3*A*a^2*b^3 - 9*B*a^3*b^2 + A*a^4*b - 10*B*a*b^4))/(b^4 \\
& *d*(a^2 + b^2)^3)
\end{aligned}$$

$$3.283 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=250

$$\frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B)x}{(a^2 + b^2)^3} + \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B) \log(\cos(c + dx))}{(a^2 + b^2)^3 d} + \frac{a(a^2Ab^3 - 3Ab^5 + a^3B^2)}{(a^2 + b^2)^3 d}$$

[Out]  $-(3Aa^2b - Ab^3 - B a^3 + 3B a^2 b)x / (a^2 + b^2)^3 + (Aa^3 - 3Aa^2b + 3B a^2 b - B b^3) \ln(\cos(dx+c)) / (a^2 + b^2)^3 / d + a(Aa^2b^3 - 3Aa^2b^5 + B a^5 + 3B a^3 b^2 + 6B a^2 b^4) \ln(a+b \tan(dx+c)) / b^3 / (a^2 + b^2)^3 / d + 1/2 a(Ab - B a) \tan(dx+c)^2 / b / (a^2 + b^2) / d / (a+b \tan(dx+c))^2 - a^2(2A^2b^3 - a(a^2 + 3b^2)B) / b^3 / (a^2 + b^2)^2 / d / (a+b \tan(dx+c))$

**Rubi [A]**

time = 0.33, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3686, 3716, 3707, 3698, 31, 3556}

$$\frac{a(Ab - aB) \tan^2(c + dx)}{2bd(a^2 + b^2)(a + b \tan(c + dx))^2} - \frac{a^2(2Ab^3 - aB(a^2 + 3b^2))}{b^3d(a^2 + b^2)^2(a + b \tan(c + dx))} + \frac{(a^3A + 3a^2bB - 3aAb^2 - b^3B) \log(\cos(c + dx))}{d(a^2 + b^2)^3} - \frac{x(a^3(-B) + 3a^2Ab + 3a^2bB - Ab^3)}{(a^2 + b^2)^3} + \frac{a(a^2B + 3a^2b^2B + a^2Ab^3 + 6ab^4B - 3Ab^5) \log(a + b \tan(c + dx))}{b^3d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3, x]

[Out]  $-(((3a^2Ab - Ab^3 - a^3B + 3a^2b^2B)x)/(a^2 + b^2)^3) + ((a^3A - 3a^2Ab^2 + 3a^2b^2B - b^3B) \text{Log}[\text{Cos}[c + d*x]]) / ((a^2 + b^2)^3 d) + (a(a^2Ab^3 - 3a^2Ab^5 + a^5B + 3a^3b^2B + 6a^2b^4B) \text{Log}[a + b \text{Tan}[c + d*x]]) / (b^3(a^2 + b^2)^3 d) + (a(Ab - aB) \text{Tan}[c + d*x]^2) / (2b(a^2 + b^2)d(a + b \text{Tan}[c + d*x])^2) - (a^2(2A^2b^3 - a(a^2 + 3b^2)B)) / (b^3(a^2 + b^2)^2 d(a + b \text{Tan}[c + d*x]))$

**Rule 31**

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 3556**

Int[tan[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3686**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m-1)\*((c + d\*Tan[e + f\*x])^(n+1)/(d\*f\*(n+1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n+1)\*(c^2 + d^2)),

```

Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

```

### Rule 3698

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
(f_.)*(x_)^2], x_Symbol] := Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

```

### Rule 3707

```

Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)^2
]/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
+ Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]

```

### Rule 3716

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)^2], x_Symbol] := Simp[(-b*c - a*d)*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} dx &= \frac{a(Ab-aB)\tan^2(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan(c+dx)(-2a(Ab-aB)+2b(Ab-aB))}{(a+b\tan(c+dx))^2} dx}{2b(a^2+b^2)d} \\
&= \frac{a(Ab-aB)\tan^2(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{a^2(2Ab^3-a(a^2+3b^2))E}{b^3(a^2+b^2)^2d(a+b\tan(c+dx))} \\
&= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} + \frac{a(Ab-aB)\tan^2(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))} \\
&= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} + \frac{(a^3A-3aAb^2+3a^2bB-b^3)E}{(a^2+b^2)^2} \\
&= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} + \frac{(a^3A-3aAb^2+3a^2bB-b^3)E}{(a^2+b^2)^2}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 3.15, size = 462, normalized size = 1.85

Mathematica [C] Result contains complex when optimal does not. time = 3.15, size = 462, normalized size = 1.85

Antiderivative was successfully verified.

```

[In] Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
[Out] (Sec[c + d*x]^2*(a*cos[c + d*x] + b*sin[c + d*x])*(a^3*b^2*(a^2 + b^2)*(A*b
- a*B) - 2*a*b*(a^2 + b^2)*(-3*A*b^3 + a*(a^2 + 4*b^2)*B)*Sin[c + d*x]*(a*
Cos[c + d*x] + b*sin[c + d*x]) + 2*b^3*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^
2*B)*(c + d*x)*(a*cos[c + d*x] + b*sin[c + d*x])^2 + (2*I)*a*(a^2*A*b^3 - 3
*A*b^5 + a^5*B + 3*a^3*b^2*B + 6*a*b^4*B)*(c + d*x)*(a*cos[c + d*x] + b*sin
[c + d*x])^2 - (2*I)*a*(a^2*A*b^3 - 3*A*b^5 + a^5*B + 3*a^3*b^2*B + 6*a*b^4
*B)*ArcTan[Tan[c + d*x]]*(a*cos[c + d*x] + b*sin[c + d*x])^2 - 2*(a^2 + b^2
)^3*B*Log[Cos[c + d*x]]*(a*cos[c + d*x] + b*sin[c + d*x])^2 + a*(a^2*A*b^3
- 3*A*b^5 + a^5*B + 3*a^3*b^2*B + 6*a*b^4*B)*Log[(a*cos[c + d*x] + b*sin[c
+ d*x])^2]*(a*cos[c + d*x] + b*sin[c + d*x])^2*(A + B*Tan[c + d*x]))/(2*b^
3*(a^2 + b^2)^3*d*(A*cos[c + d*x] + B*sin[c + d*x])*(a + b*Tan[c + d*x])^3)

```

**Maple [A]**

time = 0.32, size = 242, normalized size = 0.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOS
E)

```

```

[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*ln(1+tan(d*x+c))^
2)+(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*arctan(tan(d*x+c)))+a*(A*a^2*b^3-3*A*

```

$$b^5 + B*a^5 + 3*B*a^3*b^2 + 6*B*a*b^4) / (a^2 + b^2)^3 / b^3 * \ln(a + b*\tan(d*x + c)) - a^2 * (A*a^2*b + 3*A*b^3 - 2*B*a^3 - 4*B*a*b^2) / b^3 / (a^2 + b^2)^2 / (a + b*\tan(d*x + c)) + 1/2*a^3 * (A*b - B*a) / b^3 / (a^2 + b^2) / (a + b*\tan(d*x + c))^2$$

**Maxima [A]**

time = 0.51, size = 366, normalized size = 1.46

$$\frac{2(Ba^3 - 3Aa^2b - 3Bab^2 + Ab^3)(dx+c)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{2(Ba^6 + 3Ba^4b^2 + Aa^3b^4 + 6Ba^2b^4 - 3Aab^5) \log(b \tan(dx+c) + a)}{a^6b^3 + 3a^4b^2 + 3a^2b^4 + b^6} - \frac{(Aa^3 + 3Ba^2b - 3Aab^2 - Bb^3) \log(\tan(dx+c)^2 + 1)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{3Ba^6 - Aa^5b + 7Ba^4b^2 - 5Aa^3b^4 + 2(2Ba^2b - Aa^4b^2 + 4Ba^3b^3 - 3Aa^2b^4) \tan(dx+c)}{a^6b^3 + 2a^4b^2 + a^2b^4 + (a^6b^2 + 2a^2b^4 + b^6) \tan(dx+c)^2 + 2(a^2b^4 + 2a^2b^6 + ab^8) \tan(dx+c)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/2\*(2\*(B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*(d\*x + c)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + 2\*(B\*a^6 + 3\*B\*a^4\*b^2 + A\*a^3\*b^3 + 6\*B\*a^2\*b^4 - 3\*A\*a\*b^5)\*log(b\*tan(d\*x + c) + a)/(a^6\*b^3 + 3\*a^4\*b^5 + 3\*a^2\*b^7 + b^9) - (A\*a^3 + 3\*B\*a^2\*b - 3\*A\*a\*b^2 - B\*b^3)\*log(tan(d\*x + c)^2 + 1)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + (3\*B\*a^6 - A\*a^5\*b + 7\*B\*a^4\*b^2 - 5\*A\*a^3\*b^3 + 2\*(2\*B\*a^5\*b - A\*a^4\*b^2 + 4\*B\*a^3\*b^3 - 3\*A\*a^2\*b^4)\*tan(d\*x + c))/(a^6\*b^3 + 2\*a^4\*b^5 + a^2\*b^7 + (a^4\*b^5 + 2\*a^2\*b^7 + b^9)\*tan(d\*x + c)^2 + 2\*(a^5\*b^4 + 2\*a^3\*b^6 + a\*b^8)\*tan(d\*x + c)))/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(245) = 490.

time = 1.92, size = 666, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] 1/2\*(B\*a^6\*b^2 + A\*a^5\*b^3 + 7\*B\*a^4\*b^4 - 5\*A\*a^3\*b^5 + 2\*(B\*a^5\*b^3 - 3\*A\*a^4\*b^4 - 3\*B\*a^3\*b^5 + A\*a^2\*b^6)\*d\*x - (3\*B\*a^6\*b^2 - A\*a^5\*b^3 + 9\*B\*a^4\*b^4 - 7\*A\*a^3\*b^5 - 2\*(B\*a^3\*b^5 - 3\*A\*a^2\*b^6 - 3\*B\*a\*b^7 + A\*b^8)\*d\*x)\*tan(d\*x + c)^2 + (B\*a^8 + 3\*B\*a^6\*b^2 + A\*a^5\*b^3 + 6\*B\*a^4\*b^4 - 3\*A\*a^3\*b^5 + (B\*a^6\*b^2 + 3\*B\*a^4\*b^4 + A\*a^3\*b^5 + 6\*B\*a^2\*b^6 - 3\*A\*a\*b^7)\*tan(d\*x + c)^2 + 2\*(B\*a^7\*b + 3\*B\*a^5\*b^3 + A\*a^4\*b^4 + 6\*B\*a^3\*b^5 - 3\*A\*a^2\*b^6)\*tan(d\*x + c))\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)) - (B\*a^8 + 3\*B\*a^6\*b^2 + 3\*B\*a^4\*b^4 + B\*a^2\*b^6 + (B\*a^6\*b^2 + 3\*B\*a^4\*b^4 + 3\*B\*a^2\*b^6 + B\*b^8)\*tan(d\*x + c)^2 + 2\*(B\*a^7\*b + 3\*B\*a^5\*b^3 + 3\*B\*a^3\*b^5 + B\*a\*b^7)\*tan(d\*x + c))\*log(1/(tan(d\*x + c)^2 + 1)) - 2\*(B\*a^7\*b + 3\*B\*a^5\*b^3 - 3\*A\*a^4\*b^4 - 4\*B\*a^3\*b^5 + 3\*A\*a^2\*b^6 - 2\*(B\*a^4\*b^4 - 3\*A\*a^3\*b^5 - 3\*B\*a^2\*b^6 + A\*a\*b^7)\*d\*x)\*tan(d\*x + c))/((a^6\*b^5 + 3\*a^4\*b^7 + 3\*a^2\*b^9 + b^11)\*d\*tan(d\*x + c)^2 + 2\*(a^7\*b^4 + 3\*a^5\*b^6 +



$$3.284 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=189

$$\frac{(a^3 A - 3aAb^2 + 3a^2 bB - b^3 B)x}{(a^2 + b^2)^3} - \frac{(3a^2 Ab - Ab^3 - a^3 B + 3ab^2 B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^3 d} - \frac{2b^2}{2b^2}$$

[Out]  $-(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^3/d-1/2*a^2*(A*b-B*a)/b^2/(a^2+b^2)/d/(a+b*\tan(d*x+c))^2+a*(2*A*b^3-a*(a^2+3*b^2)*B)/b^2/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))$

**Rubi [A]**

time = 0.25, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {3685, 3709, 3612, 3611}

$$-\frac{a^2(Ab-aB)}{2b^2d(a^2+b^2)(a+b\tan(c+dx))^2} + \frac{a(2Ab^3-aB(a^2+3b^2))}{b^2d(a^2+b^2)^2(a+b\tan(c+dx))} - \frac{(a^3(-B)+3a^2Ab+3ab^2B-Ab^3)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)^3} - \frac{x(a^3A+3a^2bB-3aAb^2-b^3B)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out]  $-\left(\frac{(a^3A - 3a^2Ab + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} - \frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B) \text{Log}[a \text{Cos}[c + d*x] + b \text{Sin}[c + d*x]]}{(a^2 + b^2)^3 d} - \frac{a^2(Ab - aB)}{(2b^2(a^2 + b^2)d(a + b \text{Tan}[c + d*x])^2} + \frac{a(2Ab^3 - a(a^2 + 3b^2)B)}{b^2(a^2 + b^2)^2 d(a + b \text{Tan}[c + d*x])}\right)$

Rule 3611

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rule 3685

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^2\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[



```
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

### Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2), x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A - C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx &= -\frac{a^2(Ab - aB)}{2b^2(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{\int \frac{-a(Ab - aB) + b(Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^2} dx}{b(a^2 + b^2)} \\ &= -\frac{a^2(Ab - aB)}{2b^2(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{a(2Ab^3 - a(a^2 + 3b^2))}{b^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\ &= -\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} - \frac{a^2(Ab - aB)}{2b^2(a^2 + b^2)d(a + b \tan(c + dx))} \\ &= -\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} - \frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2)}{2b^2(a^2 + b^2)d(a + b \tan(c + dx))} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 3.52, size = 288, normalized size = 1.52

$$\frac{-\frac{Ab+aB}{b(a+b \tan(c+dx))^2} - \frac{2B \tan(c+dx)}{(a+b \tan(c+dx))^2} + B \left( \frac{i \log(i - \tan(c+dx))}{(a+ib)^2} - \frac{i \log(i + \tan(c+dx))}{(a-ib)^2} + \frac{2b(-2a \log(a+b \tan(c+dx)) + \frac{a^2+b^2}{a+b \tan(c+dx)})}{(a^2+b^2)^2} \right) + (Ab - aB) \left( \frac{i \log(i - \tan(c+dx))}{(a+ib)^2} - \frac{\log(i + \tan(c+dx))}{(a+ib)^2} + \frac{b \left( (-6a^2+2b^2) \log(a+b \tan(c+dx)) + \frac{(a^2+b^2)(a^2+4ab \tan(c+dx))}{(a+b \tan(c+dx))^2} \right)}{(a^2+b^2)^2} \right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3, x]
[Out] (-(A*b + a*B)/(b*(a + b*Tan[c + d*x])^2)) - (2*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^2 + B*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c + d*x])))/(a^2 + b^2)^2) + (A*b - a*B)*((I*Log[I - Tan[c + d*x]])/
```

$$(a + I*b)^3 - \text{Log}[I + \text{Tan}[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*\text{Log}[a + b*\text{Tan}[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x])^2))/(a^2 + b^2)^3)/(2*b*d)$$

Maple [A]

time = 0.19, size = 223, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) \ln(1 + \tan^2(dx+c))}{2} + (-A a^3 + 3A a b^2 - 3B a^2 b + B b^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^3} - \frac{a^2 (Ab - aB)}{2b^2 (a^2 + b^2) (a + b \tan(dx+c)) d}$
default	$\frac{\frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) \ln(1 + \tan^2(dx+c))}{2} + (-A a^3 + 3A a b^2 - 3B a^2 b + B b^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^3} - \frac{a^2 (Ab - aB)}{2b^2 (a^2 + b^2) (a + b \tan(dx+c)) d}$
norman	$-\frac{(2A a b^3 - B a^4 - 3B a^2 b^2) (\tan^2(dx+c))}{2ad (a^4 + 2a^2 b^2 + b^4)} - \frac{a (A a^3 - A a b^2 + 2B a^2 b)}{2db (a^4 + 2a^2 b^2 + b^4)} - \frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) a^2 x}{(a^4 + 2a^2 b^2 + b^4) (a^2 + b^2)} - \frac{b^2 (A a^3 - 3A a b^2 + 3B a^2 b - B b^3)}{(a^4 + 2a^2 b^2 + b^4) (a + b \tan(dx+c))^2}$
risch	$-\frac{i x B}{3i a^2 b - i b^3 - a^3 + 3a b^2} + \frac{x A}{3i a^2 b - i b^3 - a^3 + 3a b^2} + \frac{6i a^2 b A x}{a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6} - \frac{2i A b^3 x}{a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6} - \frac{2i a^3 B a}{a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVERBOSE)

[Out] 1/d\*(1/(a^2+b^2)^3\*(1/2\*(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*ln(1+tan(d\*x+c)^2)+(-A\*a^3+3\*A\*a\*b^2-3\*B\*a^2\*b+B\*b^3)\*arctan(tan(d\*x+c)))-1/2\*a^2\*(A\*b-B\*a)/b^2/(a^2+b^2)/(a+b\*tan(d\*x+c))^2-(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)/(a^2+b^2)^3\*ln(a+b\*tan(d\*x+c))+a\*(2\*A\*b^3-B\*a^3-3\*B\*a\*b^2)/(a^2+b^2)^2/b^2/(a+b\*tan(d\*x+c)))

Maxima [A]

time = 0.58, size = 333, normalized size = 1.76

$$\frac{2(Aa^3+3Ba^2b-3Aab^2-Bb^3)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(Ba^3-3Aa^2b-3Bab^2+Ab^3)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{(Ba^3-3Aa^2b-3Bab^2+Ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{Ba^3+Aa^4b+5Ba^3b^2-3Aa^2b^3+2(Ba^4b+3Ba^2b^3-2Aa^5)\tan(dx+c)}{a^6b^2+2a^4b^4+a^2b^6+(a^4b^4+2a^2b^6+b^8)\tan(dx+c)^2+2(a^5b^3+2a^3b^5+ab^7)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] -1/2\*(2\*(A\*a^3 + 3\*B\*a^2\*b - 3\*A\*a\*b^2 - B\*b^3)\*(d\*x + c)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) - 2\*(B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*log(b\*tan(d\*x + c) + a)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + (B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*log(tan(d\*x + c)^2 + 1)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + (B\*a^5 + A\*a^4\*b + 5\*B\*a^3\*b^2 - 3\*A\*a^2\*b^3 + 2\*(B\*a^4\*b + 3\*B\*a^2\*b^3 - 2\*A\*a\*b^4)\*tan(d\*x + c))/(a^6\*b^2 + 2\*a^4\*b^4 + a^2\*b^6 + (a^4\*b^4 + 2\*a^2\*b^6 + a^2\*b^6 + ab^7)tan(dx+c))

$b^6 + b^8) \tan(dx + c)^2 + 2(a^5 b^3 + 2a^3 b^5 + a b^7) \tan(dx + c)) / d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(184) = 368.

time = 1.65, size = 478, normalized size = 2.53

$$\frac{b^6 - 3Ab^5 - 3A^2b^4 + 3A^3b^3 - 21A^4b^2 - 3A^5b - B^2d^2 \tan(dx+c) + (B^2 + A^2b + 7B^2d^2 - 5A^2b^2 - 21A^3b - 3A^4b^2 - B^2d^2) \tan(dx+c)^2 - (B^2 - 3A^2b - 3B^2d^2 + A^2b^2 - 3A^3b - 3A^4b^2) \tan(dx+c)^3 - 21B^2d^2 - 3A^2b^2 - 3A^3b - 3A^4b^2) \tan(dx+c) + \log\left(\frac{C \tan(dx+c)}{D}\right) + 2(A^2 + 3B^2d^2 - 3A^2b - 21A^3b - 3A^4b^2 - B^2d^2) \tan(dx+c)}{2((A^2 + 3A^2b + 3A^3b^2) \tan(dx+c)^2 + 2(A^2 + 3A^2b + 3A^3b^2 + A^2b^2) \tan(dx+c) + (A^2 + 3A^2b + 3A^3b^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(B*a^5 - 3*A*a^4*b - 5*B*a^3*b^2 + 3*A*a^2*b^3 - 2*(A*a^5 + 3*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3)*dx + (B*a^5 + A*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3 - 2*(A*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*dx)*\tan(dx + c)^2 + (B*a^5 - 3*A*a^4*b - 3*B*a^3*b^2 + A*a^2*b^3 + (B*a^3*b^2 - 3*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*\tan(dx + c)^2 + 2*(B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*\tan(dx + c))*\log((b^2*\tan(dx + c)^2 + 2*a*b*\tan(dx + c) + a^2)/(\tan(dx + c)^2 + 1)) + 2*(A*a^5 + 3*B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4 - 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4)*dx)*\tan(dx + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*\tan(dx + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*\tan(dx + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*2\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))\*\*3,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(184) = 368.

time = 0.79, size = 410, normalized size = 2.17

$$\frac{2(A^2 + 3B^2d^2 - 3A^2b - B^2d^2) \tan(dx+c) + (B^2 - 3A^2b - 3B^2d^2 + A^2b^2) \tan(dx+c)^2 + 2(A^2 + 3A^2b + 3A^3b^2) \tan(dx+c)}{2((A^2 + 3A^2b + 3A^3b^2) \tan(dx+c)^2 + 2(A^2 + 3A^2b + 3A^3b^2 + A^2b^2) \tan(dx+c) + (A^2 + 3A^2b + 3A^3b^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="giac")

```
[Out] -1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b - 3*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + (3*B*a^3*b^4*tan(d*x + c)^2 - 9*A*a^2*b^5*tan(d*x + c)^2 - 9*B*a*b^6*tan(d*x + c)^2 + 3*A*b^7*tan(d*x + c)^2 + 2*B*a^6*b*tan(d*x + c) + 14*B*a^4*b^3*tan(d*x + c) - 22*A*a^3*b^4*tan(d*x + c) - 12*B*a^2*b^5*tan(d*x + c) + 2*A*a*b^6*tan(d*x + c) + B*a^7 + A*a^6*b + 9*B*a^5*b^2 - 11*A*a^4*b^3 - 4*B*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

**Mupad [B]**

time = 6.67, size = 280, normalized size = 1.48

$$\frac{\ln(a + b \tan(c + dx)) (B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3)}{d (a^2 + b^2)^3} - \frac{\ln(\tan(c + dx) - i) (-B + A i)}{2 d (-a^3 - a^2 b 3i + 3 a b^2 + b^3 i i)} - \frac{\ln(\tan(c + dx) + i) (A - B i)}{2 d (-a^3 i i - 3 a^2 b + a b^2 3i + b^3)} - \frac{\frac{a (B a^4 + A a^3 b + 5 B a^2 b^2 - 3 A a b^3) + \tan(c + dx) (B a^4 + 3 B a^2 b^2 - 2 A a b^3)}{2 b^2 (a^4 + 2 a^2 b^2 + b^4)}}{d (a^2 + 2 a b \tan(c + dx) + b^2 \tan(c + dx)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*(A*b^3 + B*a^3 - 3*A*a^2*b - 3*B*a*b^2))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - ((a*(B*a^4 + 5*B*a^2*b^2 - 3*A*a*b^3 + A*a^3*b))/(2*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)*(B*a^4 + 3*B*a^2*b^2 - 2*A*a*b^3)))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x)))
```

$$3.285 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=179

$$\frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B)x}{(a^2 + b^2)^3} - \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^3 d} + \frac{1}{2b(a^2 + b^2)}$$

[Out] (3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*x/(a^2+b^2)^3-(A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)^3/d+1/2\*a\*(A\*b-B\*a)/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^2+(A\*a^2-A\*b^2+2\*B\*a\*b)/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3672, 3610, 3612, 3611}

$$\frac{a(Ab - aB)}{2bd(a^2 + b^2)(a + b \tan(c + dx))^2} + \frac{a^2A + 2abB - Ab^2}{d(a^2 + b^2)^2(a + b \tan(c + dx))} - \frac{(a^3A + 3a^2bB - 3aAb^2 - b^3B) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^3} + \frac{x(a^3(-B) + 3a^2Ab + 3ab^2B - Ab^3)}{(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out] ((3\*a^2\*A\*b - A\*b^3 - a^3\*B + 3\*a\*b^2\*B)\*x)/(a^2 + b^2)^3 - ((a^3\*A - 3\*a\*A\*b^2 + 3\*a^2\*b\*B - b^3\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)^3\*d) + (a\*(A\*b - a\*B))/(2\*b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^2) + (a^2\*A - A\*b^2 + 2\*a\*b\*B)/((a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x]))

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3611**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a

\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

### Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\tan(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx &= \frac{a(Ab - aB)}{2b(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{\int \frac{Ab - aB + (aA + bB) \tan(c + dx)}{(a + b \tan(c + dx))^2} dx}{a^2 + b^2} \\ &= \frac{a(Ab - aB)}{2b(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{a^2A - Ab^2 + 2abB}{(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\ &= \frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B)x}{(a^2 + b^2)^3} + \frac{a(Ab - aB)}{2b(a^2 + b^2)d(a + b \tan(c + dx))} \\ &= \frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B)x}{(a^2 + b^2)^3} - \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)}{(a^2 + b^2)^3} \end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 2.61, size = 188, normalized size = 1.05

$$\frac{\frac{(A+ib) \log(i - \tan(c+dx))}{(a+ib)^3} + \frac{(A-ib) \log(i + \tan(c+dx))}{(a-ib)^3} - \frac{2(a^3A - 3aAb^2 + 3a^2bB - b^3B) \log(a+b \tan(c+dx))}{(a^2+b^2)^3} + \frac{a(Ab-aB)}{b(a^2+b^2)(a+b \tan(c+dx))^2} + \frac{2(a^2A - Ab^2 + 2abB)}{(a^2+b^2)^2(a+b \tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3, x]

[Out] (((A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*b)^3 + ((A - I\*B)\*Log[I + Tan[c + d\*x]])/(a - I\*b)^3 - (2\*(a^3\*A - 3\*a\*A\*b^2 + 3\*a^2\*b\*B - b^3\*B)\*Log[a + b\*Tan[c + d\*x]])/(a^2 + b^2)^3 + (a\*(A\*b - a\*B))/(b\*(a^2 + b^2)\*(a + b\*Tan[c + d\*x])^2) + (2\*(a^2\*A - A\*b^2 + 2\*a\*b\*B))/((a^2 + b^2)^2\*(a + b\*Tan[c + d\*x])))/(2\*d)

**Maple** [A]

time = 0.20, size = 213, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \ln(1 + \tan^2(dx+c))}{2} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) \arctan(\tan(dx+c))}{(a^2+b^2)^3} + \frac{a(Ab-aB)}{2(a^2+b^2)b(a+b \tan(dx+c))}}{d}$
default	$\frac{\frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \ln(1 + \tan^2(dx+c))}{2} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) \arctan(\tan(dx+c))}{(a^2+b^2)^3} + \frac{a(Ab-aB)}{2(a^2+b^2)b(a+b \tan(dx+c))}}{d}$
norman	$\frac{\frac{(A a^2 b^2 - A b^4 + 2B a b^3) \tan(dx+c)}{db(a^4+2a^2b^2+b^4)} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) a^2 x}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{b^2(3A a^2 b - A b^3 - B a^3 + 3B a b^2) x (\tan^2(dx+c))}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{a(3A a^2 b^2 - A b^4 + 2B a b^3)}{(a+b \tan(dx+c))^2}}{d}$
risch	$\frac{x B}{3i a^2 b - i b^3 - a^3 + 3a b^2} + \frac{i x A}{3i a^2 b - i b^3 - a^3 + 3a b^2} + \frac{2i a^3 A x}{a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6} - \frac{6i a b^2 A x}{a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6} + \frac{6i a^2 b B x}{a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} \left( \frac{1}{(a^2+b^2)^3} \left( \frac{1}{2} (A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \ln(1 + \tan^2(dx+c)) + (3A a^2 b - A b^3 - B a^3 + 3B a b^2) \arctan(\tan(dx+c)) \right) + \frac{1}{2} \frac{a(Ab-aB)}{(a^2+b^2)b(a+b \tan(dx+c))} - \frac{(A a^2 b^2 - A b^4 + 2B a b^3) \tan(dx+c)}{db(a^4+2a^2b^2+b^4)} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) a^2 x}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{b^2(3A a^2 b - A b^3 - B a^3 + 3B a b^2) x (\tan^2(dx+c))}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{a(3A a^2 b^2 - A b^4 + 2B a b^3)}{(a+b \tan(dx+c))^2} \right)$$

**Maxima** [A]

time = 0.52, size = 330, normalized size = 1.84

$$\frac{2(Ba^3 - 3Aa^2b - 3Bab^2 + Ab^3)(dx+c)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{2(Aa^3 + 3Ba^2b - 3Aab^2 - Bb^3) \log(b \tan(dx+c) + a)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} - \frac{(Aa^3 + 3Ba^2b - 3Aab^2 - Bb^3) \log(\tan(dx+c)^2 + 1)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{Ba^4 - 3Aa^3b - 3Ba^2b^2 + Aab^3 - 2(Aa^2b^2 + 2Bab^3 - Ab^4) \tan(dx+c)}{a^6b + 2a^4b^3 + a^2b^5 + (a^4b^3 + 2a^2b^5 + b^7) \tan(dx+c)^2 + 2(a^2b^2 + 2a^3b^4 + ab^6) \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] 
$$\frac{-1/2 * (2 * (B * a^3 - 3 * A * a^2 * b - 3 * B * a * b^2 + A * b^3) * (d * x + c) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) + 2 * (A * a^3 + 3 * B * a^2 * b - 3 * A * a * b^2 - B * b^3) * \log(b * \tan(d * x + c) + a) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) - (A * a^3 + 3 * B * a^2 * b - 3 * A * a * b^2 - B * b^3) * \log(\tan(d * x + c)^2 + 1) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) + (B * a^4 - 3 * A * a^3 * b - 3 * B * a^2 * b^2 + A * a * b^3 - 2 * (A * a^2 * b^2 + 2 * B * a * b^3 - A * b^4) * \tan(d * x + c)) / (a^6 * b + 2 * a^4 * b^3 + a^2 * b^5 + (a^4 * b^3 + 2 * a^2 * b^5 + b^7) * \tan(d * x + c)^2 + 2 * (a^5 * b^2 + 2 * a^3 * b^4 + a * b^6) * \tan(d * x + c))}{d}$$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(176) = 352.

time = 1.47, size = 488, normalized size = 2.73

$$\frac{2(Ba^3 - 3Aa^2b - 3Bab^2 + Ab^3)(dx+c)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{2(Aa^3 + 3Ba^2b - 3Aab^2 - Bb^3) \log(b \tan(dx+c) + a)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} - \frac{(Aa^3 + 3Ba^2b - 3Aab^2 - Bb^3) \log(\tan(dx+c)^2 + 1)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{Ba^4 - 3Aa^3b - 3Ba^2b^2 + Aab^3 - 2(Aa^2b^2 + 2Bab^3 - Ab^4) \tan(dx+c)}{a^6b + 2a^4b^3 + a^2b^5 + (a^4b^3 + 2a^2b^5 + b^7) \tan(dx+c)^2 + 2(a^5b^2 + 2a^3b^4 + ab^6) \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$-1/2*(3*B*a^4*b - 5*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 2*(B*a^5 - 3*A*a^4*b - 3*B*a^3*b^2 + A*a^2*b^3)*d*x - (B*a^4*b - 3*A*a^3*b^2 - 5*B*a^2*b^3 + 3*A*a*b^4 - 2*(B*a^3*b^2 - 3*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*d*x)*\tan(d*x + c)^2 + (A*a^5 + 3*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + (A*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*\tan(d*x + c)^2 + 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4)*\tan(d*x + c))*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - 2*(B*a^5 - 2*A*a^4*b - 3*B*a^3*b^2 + 3*A*a^2*b^3 + 2*B*a*b^4 - A*b^5 - 2*(B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*d*x)*\tan(d*x + c)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*\tan(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*\tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)$$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(176) = 352.

time = 0.70, size = 410, normalized size = 2.29

$$\frac{2(Ba^3 - 3Aa^2b - 3Bab^2 + Ab^3)\log(\tan(dx+c))}{a^6 + 3a^4b^2 + b^6} - \frac{(Aa^3 + 3Baa^2b - 3Aaab^2 - Bb^3)\log(\tan(dx+c))}{a^6 + 3a^4b^2 + b^6} + \frac{2(Aa^4b - 3Aa^3b^2 - 3Aa^2b^3 + Ab^4)\log(\tan(dx+c))}{a^6 + 3a^4b^2 + b^6} - \frac{3Aa^5 + 3Baa^4b + 3Aa^3b^2 + 3Aa^2b^3 + Ab^4}{a^6 + 3a^4b^2 + b^6} \tan(dx+c) - \frac{2Aa^4b + 3Aa^3b^2 - 3Aa^2b^3 + Ab^4}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^2 - \frac{18Aa^5 + 11Aa^4b + 7Aa^3b^2 - 7Aa^2b^3 - 7Ab^4}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^3 - \frac{2Aa^6 + 6Aa^5b + 11Aa^4b^2 - 7Aa^3b^3 - 7Aa^2b^4 - 7Ab^5}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^4 - \frac{2Aa^7 + 7Aa^6b + 11Aa^5b^2 + 7Aa^4b^3 + 7Aa^3b^4 + 7Aa^2b^5 + 7Ab^6}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^5 - \frac{2Aa^8 + 8Aa^7b + 14Aa^6b^2 + 14Aa^5b^3 + 14Aa^4b^4 + 14Aa^3b^5 + 14Aa^2b^6 + 14Ab^7}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^6 - \frac{2Aa^9 + 9Aa^8b + 18Aa^7b^2 + 18Aa^6b^3 + 18Aa^5b^4 + 18Aa^4b^5 + 18Aa^3b^6 + 18Aa^2b^7 + 18Ab^8}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^7 - \frac{2Aa^{10} + 10Aa^9b + 20Aa^8b^2 + 20Aa^7b^3 + 20Aa^6b^4 + 20Aa^5b^5 + 20Aa^4b^6 + 20Aa^3b^7 + 20Aa^2b^8 + 20Ab^9}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^8 - \frac{2Aa^{11} + 11Aa^{10}b + 22Aa^9b^2 + 22Aa^8b^3 + 22Aa^7b^4 + 22Aa^6b^5 + 22Aa^5b^6 + 22Aa^4b^7 + 22Aa^3b^8 + 22Aa^2b^9 + 22Ab^{10}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^9 - \frac{2Aa^{12} + 12Aa^{11}b + 24Aa^{10}b^2 + 24Aa^9b^3 + 24Aa^8b^4 + 24Aa^7b^5 + 24Aa^6b^6 + 24Aa^5b^7 + 24Aa^4b^8 + 24Aa^3b^9 + 24Aa^2b^{10} + 24Ab^{11}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{10} - \frac{2Aa^{13} + 13Aa^{12}b + 26Aa^{11}b^2 + 26Aa^{10}b^3 + 26Aa^9b^4 + 26Aa^8b^5 + 26Aa^7b^6 + 26Aa^6b^7 + 26Aa^5b^8 + 26Aa^4b^9 + 26Aa^3b^{10} + 26Aa^2b^{11} + 26Ab^{12}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{11} - \frac{2Aa^{14} + 14Aa^{13}b + 28Aa^{12}b^2 + 28Aa^{11}b^3 + 28Aa^{10}b^4 + 28Aa^9b^5 + 28Aa^8b^6 + 28Aa^7b^7 + 28Aa^6b^8 + 28Aa^5b^9 + 28Aa^4b^{10} + 28Aa^3b^{11} + 28Aa^2b^{12} + 28Ab^{13}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{12} - \frac{2Aa^{15} + 15Aa^{14}b + 30Aa^{13}b^2 + 30Aa^{12}b^3 + 30Aa^{11}b^4 + 30Aa^{10}b^5 + 30Aa^9b^6 + 30Aa^8b^7 + 30Aa^7b^8 + 30Aa^6b^9 + 30Aa^5b^{10} + 30Aa^4b^{11} + 30Aa^3b^{12} + 30Aa^2b^{13} + 30Ab^{14}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{13} - \frac{2Aa^{16} + 16Aa^{15}b + 32Aa^{14}b^2 + 32Aa^{13}b^3 + 32Aa^{12}b^4 + 32Aa^{11}b^5 + 32Aa^{10}b^6 + 32Aa^9b^7 + 32Aa^8b^8 + 32Aa^7b^9 + 32Aa^6b^{10} + 32Aa^5b^{11} + 32Aa^4b^{12} + 32Aa^3b^{13} + 32Aa^2b^{14} + 32Ab^{15}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{14} - \frac{2Aa^{17} + 17Aa^{16}b + 34Aa^{15}b^2 + 34Aa^{14}b^3 + 34Aa^{13}b^4 + 34Aa^{12}b^5 + 34Aa^{11}b^6 + 34Aa^{10}b^7 + 34Aa^9b^8 + 34Aa^8b^9 + 34Aa^7b^{10} + 34Aa^6b^{11} + 34Aa^5b^{12} + 34Aa^4b^{13} + 34Aa^3b^{14} + 34Aa^2b^{15} + 34Ab^{16}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{15} - \frac{2Aa^{18} + 18Aa^{17}b + 36Aa^{16}b^2 + 36Aa^{15}b^3 + 36Aa^{14}b^4 + 36Aa^{13}b^5 + 36Aa^{12}b^6 + 36Aa^{11}b^7 + 36Aa^{10}b^8 + 36Aa^9b^9 + 36Aa^8b^{10} + 36Aa^7b^{11} + 36Aa^6b^{12} + 36Aa^5b^{13} + 36Aa^4b^{14} + 36Aa^3b^{15} + 36Aa^2b^{16} + 36Ab^{17}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{16} - \frac{2Aa^{19} + 19Aa^{18}b + 38Aa^{17}b^2 + 38Aa^{16}b^3 + 38Aa^{15}b^4 + 38Aa^{14}b^5 + 38Aa^{13}b^6 + 38Aa^{12}b^7 + 38Aa^{11}b^8 + 38Aa^{10}b^9 + 38Aa^9b^{10} + 38Aa^8b^{11} + 38Aa^7b^{12} + 38Aa^6b^{13} + 38Aa^5b^{14} + 38Aa^4b^{15} + 38Aa^3b^{16} + 38Aa^2b^{17} + 38Ab^{18}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{17} - \frac{2Aa^{20} + 20Aa^{19}b + 40Aa^{18}b^2 + 40Aa^{17}b^3 + 40Aa^{16}b^4 + 40Aa^{15}b^5 + 40Aa^{14}b^6 + 40Aa^{13}b^7 + 40Aa^{12}b^8 + 40Aa^{11}b^9 + 40Aa^{10}b^{10} + 40Aa^9b^{11} + 40Aa^8b^{12} + 40Aa^7b^{13} + 40Aa^6b^{14} + 40Aa^5b^{15} + 40Aa^4b^{16} + 40Aa^3b^{17} + 40Aa^2b^{18} + 40Ab^{19}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{18} - \frac{2Aa^{21} + 21Aa^{20}b + 42Aa^{19}b^2 + 42Aa^{18}b^3 + 42Aa^{17}b^4 + 42Aa^{16}b^5 + 42Aa^{15}b^6 + 42Aa^{14}b^7 + 42Aa^{13}b^8 + 42Aa^{12}b^9 + 42Aa^{11}b^{10} + 42Aa^{10}b^{11} + 42Aa^9b^{12} + 42Aa^8b^{13} + 42Aa^7b^{14} + 42Aa^6b^{15} + 42Aa^5b^{16} + 42Aa^4b^{17} + 42Aa^3b^{18} + 42Aa^2b^{19} + 42Ab^{20}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{19} - \frac{2Aa^{22} + 22Aa^{21}b + 44Aa^{20}b^2 + 44Aa^{19}b^3 + 44Aa^{18}b^4 + 44Aa^{17}b^5 + 44Aa^{16}b^6 + 44Aa^{15}b^7 + 44Aa^{14}b^8 + 44Aa^{13}b^9 + 44Aa^{12}b^{10} + 44Aa^{11}b^{11} + 44Aa^{10}b^{12} + 44Aa^9b^{13} + 44Aa^8b^{14} + 44Aa^7b^{15} + 44Aa^6b^{16} + 44Aa^5b^{17} + 44Aa^4b^{18} + 44Aa^3b^{19} + 44Aa^2b^{20} + 44Ab^{21}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{20} - \frac{2Aa^{23} + 23Aa^{22}b + 46Aa^{21}b^2 + 46Aa^{20}b^3 + 46Aa^{19}b^4 + 46Aa^{18}b^5 + 46Aa^{17}b^6 + 46Aa^{16}b^7 + 46Aa^{15}b^8 + 46Aa^{14}b^9 + 46Aa^{13}b^{10} + 46Aa^{12}b^{11} + 46Aa^{11}b^{12} + 46Aa^{10}b^{13} + 46Aa^9b^{14} + 46Aa^8b^{15} + 46Aa^7b^{16} + 46Aa^6b^{17} + 46Aa^5b^{18} + 46Aa^4b^{19} + 46Aa^3b^{20} + 46Aa^2b^{21} + 46Ab^{22}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{21} - \frac{2Aa^{24} + 24Aa^{23}b + 48Aa^{22}b^2 + 48Aa^{21}b^3 + 48Aa^{20}b^4 + 48Aa^{19}b^5 + 48Aa^{18}b^6 + 48Aa^{17}b^7 + 48Aa^{16}b^8 + 48Aa^{15}b^9 + 48Aa^{14}b^{10} + 48Aa^{13}b^{11} + 48Aa^{12}b^{12} + 48Aa^{11}b^{13} + 48Aa^{10}b^{14} + 48Aa^9b^{15} + 48Aa^8b^{16} + 48Aa^7b^{17} + 48Aa^6b^{18} + 48Aa^5b^{19} + 48Aa^4b^{20} + 48Aa^3b^{21} + 48Aa^2b^{22} + 48Ab^{23}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{22} - \frac{2Aa^{25} + 25Aa^{24}b + 50Aa^{23}b^2 + 50Aa^{22}b^3 + 50Aa^{21}b^4 + 50Aa^{20}b^5 + 50Aa^{19}b^6 + 50Aa^{18}b^7 + 50Aa^{17}b^8 + 50Aa^{16}b^9 + 50Aa^{15}b^{10} + 50Aa^{14}b^{11} + 50Aa^{13}b^{12} + 50Aa^{12}b^{13} + 50Aa^{11}b^{14} + 50Aa^{10}b^{15} + 50Aa^9b^{16} + 50Aa^8b^{17} + 50Aa^7b^{18} + 50Aa^6b^{19} + 50Aa^5b^{20} + 50Aa^4b^{21} + 50Aa^3b^{22} + 50Aa^2b^{23} + 50Ab^{24}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{23} - \frac{2Aa^{26} + 26Aa^{25}b + 52Aa^{24}b^2 + 52Aa^{23}b^3 + 52Aa^{22}b^4 + 52Aa^{21}b^5 + 52Aa^{20}b^6 + 52Aa^{19}b^7 + 52Aa^{18}b^8 + 52Aa^{17}b^9 + 52Aa^{16}b^{10} + 52Aa^{15}b^{11} + 52Aa^{14}b^{12} + 52Aa^{13}b^{13} + 52Aa^{12}b^{14} + 52Aa^{11}b^{15} + 52Aa^{10}b^{16} + 52Aa^9b^{17} + 52Aa^8b^{18} + 52Aa^7b^{19} + 52Aa^6b^{20} + 52Aa^5b^{21} + 52Aa^4b^{22} + 52Aa^3b^{23} + 52Aa^2b^{24} + 52Ab^{25}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{24} - \frac{2Aa^{27} + 27Aa^{26}b + 54Aa^{25}b^2 + 54Aa^{24}b^3 + 54Aa^{23}b^4 + 54Aa^{22}b^5 + 54Aa^{21}b^6 + 54Aa^{20}b^7 + 54Aa^{19}b^8 + 54Aa^{18}b^9 + 54Aa^{17}b^{10} + 54Aa^{16}b^{11} + 54Aa^{15}b^{12} + 54Aa^{14}b^{13} + 54Aa^{13}b^{14} + 54Aa^{12}b^{15} + 54Aa^{11}b^{16} + 54Aa^{10}b^{17} + 54Aa^9b^{18} + 54Aa^8b^{19} + 54Aa^7b^{20} + 54Aa^6b^{21} + 54Aa^5b^{22} + 54Aa^4b^{23} + 54Aa^3b^{24} + 54Aa^2b^{25} + 54Ab^{26}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{25} - \frac{2Aa^{28} + 28Aa^{27}b + 56Aa^{26}b^2 + 56Aa^{25}b^3 + 56Aa^{24}b^4 + 56Aa^{23}b^5 + 56Aa^{22}b^6 + 56Aa^{21}b^7 + 56Aa^{20}b^8 + 56Aa^{19}b^9 + 56Aa^{18}b^{10} + 56Aa^{17}b^{11} + 56Aa^{16}b^{12} + 56Aa^{15}b^{13} + 56Aa^{14}b^{14} + 56Aa^{13}b^{15} + 56Aa^{12}b^{16} + 56Aa^{11}b^{17} + 56Aa^{10}b^{18} + 56Aa^9b^{19} + 56Aa^8b^{20} + 56Aa^7b^{21} + 56Aa^6b^{22} + 56Aa^5b^{23} + 56Aa^4b^{24} + 56Aa^3b^{25} + 56Aa^2b^{26} + 56Ab^{27}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{26} - \frac{2Aa^{29} + 29Aa^{28}b + 58Aa^{27}b^2 + 58Aa^{26}b^3 + 58Aa^{25}b^4 + 58Aa^{24}b^5 + 58Aa^{23}b^6 + 58Aa^{22}b^7 + 58Aa^{21}b^8 + 58Aa^{20}b^9 + 58Aa^{19}b^{10} + 58Aa^{18}b^{11} + 58Aa^{17}b^{12} + 58Aa^{16}b^{13} + 58Aa^{15}b^{14} + 58Aa^{14}b^{15} + 58Aa^{13}b^{16} + 58Aa^{12}b^{17} + 58Aa^{11}b^{18} + 58Aa^{10}b^{19} + 58Aa^9b^{20} + 58Aa^8b^{21} + 58Aa^7b^{22} + 58Aa^6b^{23} + 58Aa^5b^{24} + 58Aa^4b^{25} + 58Aa^3b^{26} + 58Aa^2b^{27} + 58Ab^{28}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{27} - \frac{2Aa^{30} + 30Aa^{29}b + 60Aa^{28}b^2 + 60Aa^{27}b^3 + 60Aa^{26}b^4 + 60Aa^{25}b^5 + 60Aa^{24}b^6 + 60Aa^{23}b^7 + 60Aa^{22}b^8 + 60Aa^{21}b^9 + 60Aa^{20}b^{10} + 60Aa^{19}b^{11} + 60Aa^{18}b^{12} + 60Aa^{17}b^{13} + 60Aa^{16}b^{14} + 60Aa^{15}b^{15} + 60Aa^{14}b^{16} + 60Aa^{13}b^{17} + 60Aa^{12}b^{18} + 60Aa^{11}b^{19} + 60Aa^{10}b^{20} + 60Aa^9b^{21} + 60Aa^8b^{22} + 60Aa^7b^{23} + 60Aa^6b^{24} + 60Aa^5b^{25} + 60Aa^4b^{26} + 60Aa^3b^{27} + 60Aa^2b^{28} + 60Ab^{29}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{28} - \frac{2Aa^{31} + 31Aa^{30}b + 62Aa^{29}b^2 + 62Aa^{28}b^3 + 62Aa^{27}b^4 + 62Aa^{26}b^5 + 62Aa^{25}b^6 + 62Aa^{24}b^7 + 62Aa^{23}b^8 + 62Aa^{22}b^9 + 62Aa^{21}b^{10} + 62Aa^{20}b^{11} + 62Aa^{19}b^{12} + 62Aa^{18}b^{13} + 62Aa^{17}b^{14} + 62Aa^{16}b^{15} + 62Aa^{15}b^{16} + 62Aa^{14}b^{17} + 62Aa^{13}b^{18} + 62Aa^{12}b^{19} + 62Aa^{11}b^{20} + 62Aa^{10}b^{21} + 62Aa^9b^{22} + 62Aa^8b^{23} + 62Aa^7b^{24} + 62Aa^6b^{25} + 62Aa^5b^{26} + 62Aa^4b^{27} + 62Aa^3b^{28} + 62Aa^2b^{29} + 62Ab^{30}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{29} - \frac{2Aa^{32} + 32Aa^{31}b + 64Aa^{30}b^2 + 64Aa^{29}b^3 + 64Aa^{28}b^4 + 64Aa^{27}b^5 + 64Aa^{26}b^6 + 64Aa^{25}b^7 + 64Aa^{24}b^8 + 64Aa^{23}b^9 + 64Aa^{22}b^{10} + 64Aa^{21}b^{11} + 64Aa^{20}b^{12} + 64Aa^{19}b^{13} + 64Aa^{18}b^{14} + 64Aa^{17}b^{15} + 64Aa^{16}b^{16} + 64Aa^{15}b^{17} + 64Aa^{14}b^{18} + 64Aa^{13}b^{19} + 64Aa^{12}b^{20} + 64Aa^{11}b^{21} + 64Aa^{10}b^{22} + 64Aa^9b^{23} + 64Aa^8b^{24} + 64Aa^7b^{25} + 64Aa^6b^{26} + 64Aa^5b^{27} + 64Aa^4b^{28} + 64Aa^3b^{29} + 64Aa^2b^{30} + 64Ab^{31}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{30} - \frac{2Aa^{33} + 33Aa^{32}b + 66Aa^{31}b^2 + 66Aa^{30}b^3 + 66Aa^{29}b^4 + 66Aa^{28}b^5 + 66Aa^{27}b^6 + 66Aa^{26}b^7 + 66Aa^{25}b^8 + 66Aa^{24}b^9 + 66Aa^{23}b^{10} + 66Aa^{22}b^{11} + 66Aa^{21}b^{12} + 66Aa^{20}b^{13} + 66Aa^{19}b^{14} + 66Aa^{18}b^{15} + 66Aa^{17}b^{16} + 66Aa^{16}b^{17} + 66Aa^{15}b^{18} + 66Aa^{14}b^{19} + 66Aa^{13}b^{20} + 66Aa^{12}b^{21} + 66Aa^{11}b^{22} + 66Aa^{10}b^{23} + 66Aa^9b^{24} + 66Aa^8b^{25} + 66Aa^7b^{26} + 66Aa^6b^{27} + 66Aa^5b^{28} + 66Aa^4b^{29} + 66Aa^3b^{30} + 66Aa^2b^{31} + 66Ab^{32}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{31} - \frac{2Aa^{34} + 34Aa^{33}b + 68Aa^{32}b^2 + 68Aa^{31}b^3 + 68Aa^{30}b^4 + 68Aa^{29}b^5 + 68Aa^{28}b^6 + 68Aa^{27}b^7 + 68Aa^{26}b^8 + 68Aa^{25}b^9 + 68Aa^{24}b^{10} + 68Aa^{23}b^{11} + 68Aa^{22}b^{12} + 68Aa^{21}b^{13} + 68Aa^{20}b^{14} + 68Aa^{19}b^{15} + 68Aa^{18}b^{16} + 68Aa^{17}b^{17} + 68Aa^{16}b^{18} + 68Aa^{15}b^{19} + 68Aa^{14}b^{20} + 68Aa^{13}b^{21} + 68Aa^{12}b^{22} + 68Aa^{11}b^{23} + 68Aa^{10}b^{24} + 68Aa^9b^{25} + 68Aa^8b^{26} + 68Aa^7b^{27} + 68Aa^6b^{28} + 68Aa^5b^{29} + 68Aa^4b^{30} + 68Aa^3b^{31} + 68Aa^2b^{32} + 68Ab^{33}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{32} - \frac{2Aa^{35} + 35Aa^{34}b + 70Aa^{33}b^2 + 70Aa^{32}b^3 + 70Aa^{31}b^4 + 70Aa^{30}b^5 + 70Aa^{29}b^6 + 70Aa^{28}b^7 + 70Aa^{27}b^8 + 70Aa^{26}b^9 + 70Aa^{25}b^{10} + 70Aa^{24}b^{11} + 70Aa^{23}b^{12} + 70Aa^{22}b^{13} + 70Aa^{21}b^{14} + 70Aa^{20}b^{15} + 70Aa^{19}b^{16} + 70Aa^{18}b^{17} + 70Aa^{17}b^{18} + 70Aa^{16}b^{19} + 70Aa^{15}b^{20} + 70Aa^{14}b^{21} + 70Aa^{13}b^{22} + 70Aa^{12}b^{23} + 70Aa^{11}b^{24} + 70Aa^{10}b^{25} + 70Aa^9b^{26} + 70Aa^8b^{27} + 70Aa^7b^{28} + 70Aa^6b^{29} + 70Aa^5b^{30} + 70Aa^4b^{31} + 70Aa^3b^{32} + 70Aa^2b^{33} + 70Ab^{34}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{33} - \frac{2Aa^{36} + 36Aa^{35}b + 72Aa^{34}b^2 + 72Aa^{33}b^3 + 72Aa^{32}b^4 + 72Aa^{31}b^5 + 72Aa^{30}b^6 + 72Aa^{29}b^7 + 72Aa^{28}b^8 + 72Aa^{27}b^9 + 72Aa^{26}b^{10} + 72Aa^{25}b^{11} + 72Aa^{24}b^{12} + 72Aa^{23}b^{13} + 72Aa^{22}b^{14} + 72Aa^{21}b^{15} + 72Aa^{20}b^{16} + 72Aa^{19}b^{17} + 72Aa^{18}b^{18} + 72Aa^{17}b^{19} + 72Aa^{16}b^{20} + 72Aa^{15}b^{21} + 72Aa^{14}b^{22} + 72Aa^{13}b^{23} + 72Aa^{12}b^{24} + 72Aa^{11}b^{25} + 72Aa^{10}b^{26} + 72Aa^9b^{27} + 72Aa^8b^{28} + 72Aa^7b^{29} + 72Aa^6b^{30} + 72Aa^5b^{31} + 72Aa^4b^{32} + 72Aa^3b^{33} + 72Aa^2b^{34} + 72Ab^{35}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{34} - \frac{2Aa^{37} + 37Aa^{36}b + 74Aa^{35}b^2 + 74Aa^{34}b^3 + 74Aa^{33}b^4 + 74Aa^{32}b^5 + 74Aa^{31}b^6 + 74Aa^{30}b^7 + 74Aa^{29}b^8 + 74Aa^{28}b^9 + 74Aa^{27}b^{10} + 74Aa^{26}b^{11} + 74Aa^{25}b^{12} + 74Aa^{24}b^{13} + 74Aa^{23}b^{14} + 74Aa^{22}b^{15} + 74Aa^{21}b^{16} + 74Aa^{20}b^{17} + 74Aa^{19}b^{18} + 74Aa^{18}b^{19} + 74Aa^{17}b^{20} + 74Aa^{16}b^{21} + 74Aa^{15}b^{22} + 74Aa^{14}b^{23} + 74Aa^{13}b^{24} + 74Aa^{12}b^{25} + 74Aa^{11}b^{26} + 74Aa^{10}b^{27} + 74Aa^9b^{28} + 74Aa^8b^{29} + 74Aa^7b^{30} + 74Aa^6b^{31} + 74Aa^5b^{32} + 74Aa^4b^{33} + 74Aa^3b^{34} + 74Aa^2b^{35} + 74Ab^{36}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{35} - \frac{2Aa^{38} + 38Aa^{37}b + 76Aa^{36}b^2 + 76Aa^{35}b^3 + 76Aa^{34}b^4 + 76Aa^{33}b^5 + 76Aa^{32}b^6 + 76Aa^{31}b^7 + 76Aa^{30}b^8 + 76Aa^{29}b^9 + 76Aa^{28}b^{10} + 76Aa^{27}b^{11} + 76Aa^{26}b^{12} + 76Aa^{25}b^{13} + 76Aa^{24}b^{14} + 76Aa^{23}b^{15} + 76Aa^{22}b^{16} + 76Aa^{21}b^{17} + 76Aa^{20}b^{18} + 76Aa^{19}b^{19} + 76Aa^{18}b^{20} + 76Aa^{17}b^{21} + 76Aa^{16}b^{22} + 76Aa^{15}b^{23} + 76Aa^{14}b^{24} + 76Aa^{13}b^{25} + 76Aa^{12}b^{26} + 76Aa^{11}b^{27} + 76Aa^{10}b^{28} + 76Aa^9b^{29} + 76Aa^8b^{30} + 76Aa^7b^{31} + 76Aa^6b^{32} + 76Aa^5b^{33} + 76Aa^4b^{34} + 76Aa^3b^{35} + 76Aa^2b^{36} + 76Ab^{37}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{36} - \frac{2Aa^{39} + 39Aa^{38}b + 78Aa^{37}b^2 + 78Aa^{36}b^3 + 78Aa^{35}b^4 + 78Aa^{34}b^5 + 78Aa^{33}b^6 + 78Aa^{32}b^7 + 78Aa^{31}b^8 + 78Aa^{30}b^9 + 78Aa^{29}b^{10} + 78Aa^{28}b^{11} + 78Aa^{27}b^{12} + 78Aa^{26}b^{13} + 78Aa^{25}b^{14} + 78Aa^{24}b^{15} + 78Aa^{23}b^{16} + 78Aa^{22}b^{17} + 78Aa^{21}b^{18} + 78Aa^{20}b^{19} + 78Aa^{19}b^{20} + 78Aa^{18}b^{21} + 78Aa^{17}b^{22} + 78Aa^{16}b^{23} + 78Aa^{15}b^{24} + 78Aa^{14}b^{25} + 78Aa^{13}b^{26} + 78Aa^{12}b^{27} + 78Aa^{11}b^{28} + 78Aa^{10}b^{29} + 78Aa^9b^{30} + 78Aa^8b^{31} + 78Aa^7b^{32} + 78Aa^6b^{33} + 78Aa^5b^{34} + 78Aa^4b^{35} + 78Aa^3b^{36} + 78Aa^2b^{37} + 78Ab^{38}}{a^6 + 3a^4b^2 + b^6} \tan(dx+c)^{37} - \frac{2Aa^{40} + 40Aa^{39}b + 80Aa^{38$$



Mupad [B]

time = 6.63, size = 282, normalized size = 1.58

$$\frac{\frac{\tan(c+dx)(Aa^2b+2Ba^2b^2-Ab^3)}{a^4+2a^2b^2+b^4} - \frac{Ba^4-3Aa^3b-3Ba^2b^2+Aa^2b^3}{2b(a^4+2a^2b^2+b^4)}}{d(a^2+2ab\tan(c+dx)+b^2\tan(c+dx)^2)} - \frac{\ln(a+b\tan(c+dx))\left(\frac{Aa+3Bb}{(a^2+b^2)^2} - \frac{4b^2(Aa+Bb)}{(a^2+b^2)^3}\right)}{d} - \frac{\ln(\tan(c+dx)-i)(-B+Ai)}{2d(-a^3+3a^2b+ab^2-3b^3)} - \frac{\ln(\tan(c+dx)+i)(A-Bi)}{2d(-a^3+a^2b+3ab^2-b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] ((tan(c + d\*x)\*(A\*a^2\*b - A\*b^3 + 2\*B\*a\*b^2))/(a^4 + b^4 + 2\*a^2\*b^2) - (B\*a^4 - 3\*B\*a^2\*b^2 + A\*a\*b^3 - 3\*A\*a^3\*b)/(2\*b\*(a^4 + b^4 + 2\*a^2\*b^2)))/(d\*(a^2 + b^2\*tan(c + d\*x)^2 + 2\*a\*b\*tan(c + d\*x))) - (log(a + b\*tan(c + d\*x)) \* ((A\*a + 3\*B\*b)/(a^2 + b^2)^2 - (4\*b^2\*(A\*a + B\*b))/(a^2 + b^2)^3))/d - (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(a\*b^2\*3i + 3\*a^2\*b - a^3\*1i - b^3)) - (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(2\*d\*(3\*a\*b^2 + a^2\*b\*3i - a^3 - b^3\*1i))

$$3.286 \quad \int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=175

$$\frac{(a^3 A - 3aAb^2 + 3a^2 bB - b^3 B)x}{(a^2 + b^2)^3} + \frac{(3a^2 Ab - Ab^3 - a^3 B + 3ab^2 B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^3 d} - \frac{1}{2(a^2 + b^2)}$$

[Out] (A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*x/(a^2+b^2)^3+(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)^3/d+1/2\*(-A\*b+B\*a)/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^2+(-2\*A\*a\*b+B\*a^2-B\*b^2)/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3610, 3612, 3611}

$$-\frac{Ab - aB}{2d(a^2 + b^2)(a + b \tan(c + dx))^2} - \frac{a^2(-B) + 2aAb + b^2B}{d(a^2 + b^2)^2(a + b \tan(c + dx))} + \frac{(a^3(-B) + 3a^2Ab + 3ab^2B - Ab^3) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^3} + \frac{x(a^3A + 3a^2bB - 3aAb^2 - b^3B)}{(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^3,x]

[Out] ((a^3\*A - 3\*a\*A\*b^2 + 3\*a^2\*b\*B - b^3\*B)\*x)/(a^2 + b^2)^3 + ((3\*a^2\*A\*b - A\*b^3 - a^3\*B + 3\*a\*b^2\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)^3\*d) - (A\*b - a\*B)/(2\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^2) - (2\*a\*A\*b - a^2\*B + b^2\*B)/((a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x]))

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3611**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

**Rule 3612**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^3} dx = -\frac{Ab - aB}{2(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^2} dx}{a^2 + b^2}$$

$$= -\frac{Ab - aB}{2(a^2 + b^2) d(a + b \tan(c + dx))^2} - \frac{2aAb - a^2B + b^2B}{(a^2 + b^2)^2 d(a + b \tan(c + dx))} + \int \frac{a^2A - aAb + b^2B}{(a + b \tan(c + dx))^2} dx$$

$$= \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} - \frac{Ab - aB}{2(a^2 + b^2) d(a + b \tan(c + dx))^2} - \frac{2aAb - a^2B + b^2B}{(a^2 + b^2)^2 d(a + b \tan(c + dx))} + \int \frac{a^2A - aAb + b^2B}{(a + b \tan(c + dx))^2} dx$$

$$= \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} + \frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B) \log(a \cos(c + dx))}{(a^2 + b^2)^3 d}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 2.47, size = 243, normalized size = 1.39

$$\frac{B \left( \frac{i \log(i - \tan(c + dx))}{(a + ib)^2} - \frac{i \log(i + \tan(c + dx))}{(a - ib)^2} + \frac{2b(-2a \log(a + b \tan(c + dx)) + \frac{a^2 + b^2}{a + b \tan(c + dx)})}{(a^2 + b^2)^2} \right) + (Ab - aB) \left( \frac{i \log(i - \tan(c + dx))}{(a + ib)^3} - \frac{\log(i + \tan(c + dx))}{(a + b)^3} + \frac{b \left( (-6a^2 + 2b^2) \log(a + b \tan(c + dx)) + \frac{(a^2 + b^2)(a^2 + b^2 + 4ab \tan(c + dx))}{(a + b \tan(c + dx))^2} \right)}{(a^2 + b^2)^3} \right)}{2bd}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^3,x]

[Out] -1/2\*(B\*((I\*Log[I - Tan[c + d\*x]])/(a + I\*b)^2 - (I\*Log[I + Tan[c + d\*x]])/(a - I\*b)^2 + (2\*b\*(-2\*a\*Log[a + b\*Tan[c + d\*x]] + (a^2 + b^2)/(a + b\*Tan[c + d\*x])))/(a^2 + b^2)^2) + (A\*b - a\*B)\*((I\*Log[I - Tan[c + d\*x]])/(a + I\*b)^3 - Log[I + Tan[c + d\*x]]/(I\*a + b)^3 + (b\*((-6\*a^2 + 2\*b^2)\*Log[a + b\*Tan[c + d\*x]] + ((a^2 + b^2)\*(5\*a^2 + b^2 + 4\*a\*b\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2))/(a^2 + b^2)^3))/(b\*d)

**Maple [A]**

time = 0.19, size = 208, normalized size = 1.19

method	result
derivativedivides	$\frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \ln(1 + \tan^2(dx + c)) + (A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \arctan(\tan(dx + c))}{(a^2 + b^2)^3} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2)}{(a^2 + b^2)^3} \frac{1}{d}$
default	$\frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \ln(1 + \tan^2(dx + c)) + (A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \arctan(\tan(dx + c))}{(a^2 + b^2)^3} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2)}{(a^2 + b^2)^3} \frac{1}{d}$

norman	$\frac{\left(\frac{A a^3 - 3 A a b^2 + 3 B a^2 b - B b^3}{a^4 + 2 a^2 b^2 + b^4}\right) a^2 x + \frac{b^2 \left(\frac{A a^3 - 3 A a b^2 + 3 B a^2 b - B b^3}{a^4 + 2 a^2 b^2 + b^4}\right) x \left(\tan^2(dx+c)\right) - \frac{3 A a^2 b^2 + A b^4 - 2 B a^3 b}{2 b d \left(a^4 + 2 a^2 b^2 + b^4\right)} + \frac{b \left(2 A a b^2 - B a^2 b + B b^3\right)}{2 d a \left(a^4 + 2 a^2 b^2 + b^4\right)}}{\left(a+b \tan(dx+c)\right)^2}$
risch	$\frac{i x B}{3 i a^2 b - i b^3 - a^3 + 3 a b^2} - \frac{x A}{3 i a^2 b - i b^3 - a^3 + 3 a b^2} - \frac{6 i a^2 b A x}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6} + \frac{2 i A b^3 x}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6} + \frac{2 i a^3 B x}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVERBOSE)

[Out] 1/d\*(1/(a^2+b^2)^3\*(1/2\*(-3\*A\*a^2\*b+A\*b^3+B\*a^3-3\*B\*a\*b^2)\*ln(1+tan(d\*x+c)^2)+(A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*arctan(tan(d\*x+c)))+(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)/(a^2+b^2)^3\*ln(a+b\*tan(d\*x+c))-1/2\*(A\*b-B\*a)/(a^2+b^2)/(a+b\*tan(d\*x+c))^2-(2\*A\*a\*b-B\*a^2+B\*b^2)/(a^2+b^2)^2/(a+b\*tan(d\*x+c)))

Maxima [A]

time = 0.53, size = 321, normalized size = 1.83

$$\frac{2 \left(\frac{A a^3 + 3 B a^2 b - 3 A a b^2 - B b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) (dx+c) - \frac{2 \left(\frac{B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) \log(b \tan(dx+c)+a)}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6} + \frac{\left(\frac{B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) \log\left(\frac{\tan(dx+c)^2+1}{\tan(dx+c)}\right)}{2 d} + \frac{3 B a^3 - 5 A a^2 b - B a b^2 - A b^3 + 2 \left(\frac{B a^2 b - 2 A a b^2 - B b^3}{a^6 + 2 a^4 b^2 + a^2 b^4 + (a^4 b^2 + 2 a^2 b^4 + b^6) \tan(dx+c)^2 + 2(a^6 b + 2 a^3 b^3 + a b^5) \tan(dx+c)}\right) \tan(dx+c)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/2\*(2\*(A\*a^3 + 3\*B\*a^2\*b - 3\*A\*a\*b^2 - B\*b^3)\*(d\*x + c)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) - 2\*(B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*log(b\*tan(d\*x + c) + a)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + (B\*a^3 - 3\*A\*a^2\*b - 3\*B\*a\*b^2 + A\*b^3)\*log(tan(d\*x + c)^2 + 1)/(a^6 + 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6) + (3\*B\*a^3 - 5\*A\*a^2\*b - B\*a\*b^2 - A\*b^3 + 2\*(B\*a^2\*b - 2\*A\*a\*b^2 - B\*b^3)\*tan(d\*x + c))/(a^6 + 2\*a^4\*b^2 + a^2\*b^4 + (a^4\*b^2 + 2\*a^2\*b^4 + b^6)\*tan(d\*x + c)^2 + 2\*(a^5\*b + 2\*a^3\*b^3 + a\*b^5)\*tan(d\*x + c)))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(171) = 342.

time = 1.24, size = 482, normalized size = 2.75

$$\frac{5 B a^3 - 7 A a^2 b - B a b^2 - A b^3 + 2 \left(\frac{A a^3 + 3 B a^2 b - 3 A a b^2 - B b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) \log(b \tan(dx+c)+a) - 2 \left(\frac{B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) \log\left(\frac{\tan(dx+c)^2+1}{\tan(dx+c)}\right) - 2 \left(\frac{B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3}{a^6 + 2 a^4 b^2 + a^2 b^4 + (a^4 b^2 + 2 a^2 b^4 + b^6) \tan(dx+c)^2 + 2(a^6 b + 2 a^3 b^3 + a b^5) \tan(dx+c)}\right) \tan(dx+c)}{2 \left(\frac{A a^3 + 3 B a^2 b - 3 A a b^2 - B b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) (dx+c) - \frac{2 \left(\frac{B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) \log(b \tan(dx+c)+a)}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6} + \frac{\left(\frac{B a^3 - 3 A a^2 b - 3 B a b^2 + A b^3}{a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6}\right) \log\left(\frac{\tan(dx+c)^2+1}{\tan(dx+c)}\right)}{2 d} + \frac{3 B a^3 - 5 A a^2 b - B a b^2 - A b^3 + 2 \left(\frac{B a^2 b - 2 A a b^2 - B b^3}{a^6 + 2 a^4 b^2 + a^2 b^4 + (a^4 b^2 + 2 a^2 b^4 + b^6) \tan(dx+c)^2 + 2(a^6 b + 2 a^3 b^3 + a b^5) \tan(dx+c)}\right) \tan(dx+c)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] 1/2\*(5\*B\*a^3\*b^2 - 7\*A\*a^2\*b^3 - B\*a\*b^4 - A\*b^5 + 2\*(A\*a^5 + 3\*B\*a^4\*b - 3\*A\*a^3\*b^2 - B\*a^2\*b^3)\*d\*x - (3\*B\*a^3\*b^2 - 5\*A\*a^2\*b^3 - 3\*B\*a\*b^4 + A\*b^5 - 2\*(A\*a^3\*b^2 + 3\*B\*a^2\*b^3 - 3\*A\*a\*b^4 - B\*b^5)\*d\*x)\*tan(d\*x + c)^2 - (B\*a^5 - 3\*A\*a^4\*b - 3\*B\*a^3\*b^2 + A\*a^2\*b^3 + (B\*a^3\*b^2 - 3\*A\*a^2\*b^3 - 3\*B\*a\*b^4 + A\*b^5)\*tan(d\*x + c)^2 + 2\*(B\*a^4\*b - 3\*A\*a^3\*b^2 - 3\*B\*a^2\*b^3 + A\*a\*b^4)\*tan(d\*x + c))\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/

$(\tan(dx + c)^2 + 1) - 2*(2*B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + 3*A*a*b^4 + B*b^5 - 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4)*dx)*\tan(dx + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*\tan(dx + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*\tan(dx + c) + (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/(a+b\*tan(dx+c))\*\*3,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(171) = 342.

time = 0.67, size = 409, normalized size = 2.34

$$\frac{2(Aa^3 + 3Ba^2b - 3Aab^2 - Bb^3)\log(\tan(dx+c))}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{(Ba^3 - 3Aa^2b - 3Bab^2 + Ab^3)\log(\tan(dx+c))}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} - \frac{2(Ba^3b - 3Aa^2b^2 - 3Bab^3 + Ab^4)\log(\tan(dx+c))}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{3Ba^3b^2\tan(dx+c)^2 - 9Aa^2b^3\tan(dx+c)^2 - 9Bab^4\tan(dx+c)^2 + 3Aa^3b^5\tan(dx+c)^2 + 3Aa^2b^6\tan(dx+c)^2 - 22Aa^3b^7\tan(dx+c)^2 - 18Bab^8\tan(dx+c)^2 + 2Aa^4b^9\tan(dx+c)^2 - 2Bb^{10}\tan(dx+c)^2}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)(b\tan(dx+c) + a)^2}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="giac")

[Out]  $\frac{1}{2}*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(dx + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*\log(\tan(dx + c))^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b - 3*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*\log(\text{abs}(b*\tan(dx + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + (3*B*a^3*b^2*\tan(dx + c)^2 - 9*A*a^2*b^3*\tan(dx + c)^2 - 9*B*a*b^4*\tan(dx + c)^2 + 3*A*b^5*\tan(dx + c)^2 + 8*B*a^4*b*\tan(dx + c) - 22*A*a^3*b^2*\tan(dx + c) - 18*B*a^2*b^3*\tan(dx + c) + 2*A*a*b^4*\tan(dx + c) - 2*B*b^5*\tan(dx + c) + 6*B*a^5 - 14*A*a^4*b - 7*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 - A*b^5)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*\tan(dx + c) + a)^2)/d$

**Mupad** [B]

time = 6.52, size = 279, normalized size = 1.59

$$\frac{\ln(a + b\tan(c + dx))}{d} \left( \frac{3Ab - Ba}{(a^2 + b^2)^2} - \frac{4b^2(Ab - Ba)}{(a^2 + b^2)^3} \right) - \frac{-3Ba^3 + 5Aa^2b + Ba^2b^2 + Ab^3}{2(a^4 + 2a^2b^2 + b^4)} + \frac{\tan(c + dx)(-Ba^2b + 2Aa^2b^2 + Bb^3)}{a^4 + 2a^2b^2 + b^4} + \frac{\ln(\tan(c + dx) - i)(-B + Ai)}{2d(-a^3 - a^2b^3i + 3ab^2 + b^3i)} + \frac{\ln(\tan(c + dx) + i)(A - Bi)}{2d(-a^3 1i - 3a^2b + ab^2 3i + b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + dx))/(a + b\*tan(c + dx))^3,x)

```
[Out] (log(a + b*tan(c + d*x))*((3*A*b - B*a)/(a^2 + b^2)^2 - (4*b^2*(A*b - B*a))
/(a^2 + b^2)^3))/d - ((A*b^3 - 3*B*a^3 + 5*A*a^2*b + B*a*b^2)/(2*(a^4 + b^4
+ 2*a^2*b^2)) + (tan(c + d*x)*(B*b^3 + 2*A*a*b^2 - B*a^2*b))/(a^4 + b^4 +
2*a^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (log(tan(
c + d*x) - 1i)*(A*1i - B))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log
(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

$$3.287 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=215

$$-\frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B)x}{(a^2 + b^2)^3} + \frac{A \log(\sin(c + dx))}{a^3d} - \frac{b(6a^4Ab + 3a^2Ab^3 + Ab^5 - 3a^5B + a^3b^2B) \log(a \cos(c + dx) + b \sin(c + dx))}{a^3(a^2 + b^2)^3d}$$

[Out]  $-(3Aa^2b - Ab^3 - a^3B + 3Bab^2)x/(a^2 + b^2)^3 + A \ln(\sin(dx + c))/a^3/d - b(6Aa^4b + 3Aa^2b^3 + Ab^5 - 3a^5B + a^3b^2B) \ln(a \cos(dx + c) + b \sin(dx + c))/a^3/(a^2 + b^2)^3/d + 1/2 * b * (Ab - Ba) / a / (a^2 + b^2) / d / (a + b \tan(dx + c))^2 + b * (3Aa^2b + Ab^3 - 2Ba^3) / a^2 / (a^2 + b^2)^2 / d / (a + b \tan(dx + c))$

**Rubi [A]**

time = 0.42, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{A \log(\sin(c + dx))}{a^3d} + \frac{b(Ab - aB)}{2ad(a^2 + b^2)(a + b \tan(c + dx))^2} + \frac{b(-2a^3B + 3a^2Ab + Ab^3)}{a^2d(a^2 + b^2)^2(a + b \tan(c + dx))} - \frac{x(a^3(-B) + 3a^2Ab + 3ab^2B - Ab^3)}{(a^2 + b^2)^3} - \frac{b(-3a^5B + 6a^4Ab + a^3b^2B + 3a^2Ab^3 + Ab^5) \log(a \cos(c + dx) + b \sin(c + dx))}{a^3d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x] * (A + B * \text{Tan}[c + d*x])) / (a + b * \text{Tan}[c + d*x])^3, x]$

[Out]  $-(((3a^2Ab - Ab^3 - a^3B + 3a^2b^2B)x)/(a^2 + b^2)^3) + (A \text{Log}[\text{Sin}[c + d*x]])/(a^3d) - (b(6a^4Ab + 3a^2Ab^3 + Ab^5 - 3a^5B + a^3b^2B) \text{Log}[a \text{Cos}[c + d*x] + b \text{Sin}[c + d*x]])/(a^3(a^2 + b^2)^3d) + (b(Ab - aB))/(2a(a^2 + b^2)d(a + b \text{Tan}[c + d*x])^2) + (b(3a^2Ab + Ab^3 - 2a^3B))/(a^2(a^2 + b^2)^2d(a + b \text{Tan}[c + d*x]))$

**Rule 3556**

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3611**

$\text{Int}[(c_. + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

**Rule 3690**

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)} * ((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*(Ab - a*B)*(a + b*\text{Tan}[e + f*x])^{(m+1)} * ((c + d*\text{Tan}[e + f*x])^{(n+1)}) / (f*(m+1)*(b*c - a*d)*(a^2 + b^2)), x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 + b^2)), x]$

```

2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] := Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx &= \frac{b(Ab-aB)}{2a(a^2+b^2)d(a+b \tan(c+dx))^2} + \frac{\int \frac{\cot(c+dx)(2A(a^2+b^2)-2a(Ab-aB))}{(a+b \tan(c+dx))^2} dx}{2a(a^2+b^2)d} \\
&= \frac{b(Ab-aB)}{2a(a^2+b^2)d(a+b \tan(c+dx))^2} + \frac{b(3a^2Ab+Ab^3-2a^3B)}{a^2(a^2+b^2)^2d(a+b \tan(c+dx))} \\
&= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} + \frac{b(Ab-aB)}{2a(a^2+b^2)d(a+b \tan(c+dx))} \\
&= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} + \frac{A \log(\sin(c+dx))}{a^3d} - \frac{b(6a^4A)}{a^3d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 2.35, size = 254, normalized size = 1.18

$$\frac{-\frac{a(-ib)(A+B) \log(i-\tan(c+dx))}{(a+ib)^2} + \frac{2A(a^2+b^2) \log(\tan(c+dx))}{a^2} - \frac{a(a+ib)(A-B) \log(i+\tan(c+dx))}{(a-ib)^2} - \frac{2b(6a^4Ab+3a^2Ab^3+Ab^5-3a^5B+a^3b^2B) \log(a+b \tan(c+dx))}{a^2(a^2+b^2)^2} + \frac{b(Ab-aB)}{(a+b \tan(c+dx))^2} + \frac{4ab(Ab-aB)}{(a^2+b^2)(a+b \tan(c+dx))} + \frac{2Ab^2}{a^2+ab \tan(c+dx)}}{2a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out] (-(a\*(a - I\*b)\*(A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*b)^2 + (2\*A\*(a^2 + b^2)\*Log[Tan[c + d\*x]])/a^2 - (a\*(a + I\*b)\*(A - I\*B)\*Log[I + Tan[c + d\*x]])/(a - I\*b)^2 - (2\*b\*(6\*a^4\*A\*b + 3\*a^2\*A\*b^3 + A\*b^5 - 3\*a^5\*B + a^3\*b^2\*B)\*Log[a + b\*Tan[c + d\*x]])/(a^2\*(a^2 + b^2)^2 + (b\*(A\*b - a\*B))/(a + b\*Tan[c + d\*x])^2 + (4\*a\*b\*(A\*b - a\*B))/((a^2 + b^2)\*(a + b\*Tan[c + d\*x])) + (2\*A\*b^2)/(a^2 + a\*b\*Tan[c + d\*x]))/(2\*a\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.47, size = 243, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVERBOSE)

[Out] 1/d\*(A/a^3\*ln(tan(d\*x+c))+1/(a^2+b^2)^3\*(1/2\*(-A\*a^3+3\*A\*a\*b^2-3\*B\*a^2\*b+B\*b^3)\*ln(1+tan(d\*x+c)^2)+(-3\*A\*a^2\*b+A\*b^3+B\*a^3-3\*B\*a\*b^2)\*arctan(tan(d\*x+c)))+b\*(3\*A\*a^2\*b+A\*b^3-2\*B\*a^3)/a^2/(a^2+b^2)^2/(a+b\*tan(d\*x+c))-b\*(6\*A\*a^4\*b+3\*A\*a^2\*b^3+A\*b^5-3\*B\*a^5+B\*a^3\*b^2)/a^3/(a^2+b^2)^3\*ln(a+b\*tan(d\*x+c))+1/2\*(A\*b-B\*a)\*b/a/(a^2+b^2)/(a+b\*tan(d\*x+c))^2)

**Maxima [A]**

time = 0.53, size = 372, normalized size = 1.73

$$\frac{\frac{2(Ba^3-3Aa^2b-3Ba^2+Ab^3)(dx+c)}{a^3+3a^2b+3a^2b^2+3b^3} + \frac{2(3Ba^5b-6Aa^4b^2-Ba^3b^3-3Aa^2b^4-Ab^5) \log(b \tan(dx+c)+a)}{a^2+3a^2b^2+3a^2b^4+a^2b^6} - \frac{(Aa^3+3Ba^2b-3Aa^2-Bb^2) \log(\tan(dx+c)^2+1)}{a^3+3a^2b^2+3a^2b^4+b^6} - \frac{5Ba^4b-7Aa^3b^2+Bb^2b^3-3Aa^2b^4+2(3Ba^3b^2-3Aa^2b^3-Ab^5) \tan(dx+c)}{a^3+2a^2b^2+a^2b^4+(a^2b^2+2a^2b^4+a^2b^6) \tan(dx+c)^2+2(a^2b+2a^2b^3+a^2b^5) \tan(dx+c)} + \frac{2A \log(\tan(dx+c))}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*B*a^5*b - 6*A*a^4*b^2 - B*a^3*b^3 - 3*A*a^2*b^4 - A*b^6)*\log(b*\tan(d*x + c) + a)/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6) - (A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*\log(\tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (5*B*a^4*b - 7*A*a^3*b^2 + B*a^2*b^3 - 3*A*a*b^4 + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3 - A*b^5)*\tan(d*x + c))/(a^8 + 2*a^6*b^2 + a^4*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*\tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*\tan(d*x + c)) + 2*A*\log(\tan(d*x + c))/a^3)/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(213) = 426.

time = 1.54, size = 683, normalized size = 3.18

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fricas")`

[Out]  $-\frac{1}{2}*(7*B*a^5*b^3 - 9*A*a^4*b^4 + B*a^3*b^5 - 3*A*a^2*b^6 - 2*(B*a^8 - 3*A*a^7*b - 3*B*a^6*b^2 + A*a^5*b^3)*d*x - (5*B*a^5*b^3 - 7*A*a^4*b^4 - B*a^3*b^5 - A*a^2*b^6 + 2*(B*a^6*b^2 - 3*A*a^5*b^3 - 3*B*a^4*b^4 + A*a^3*b^5)*d*x)*\tan(d*x + c)^2 - (A*a^8 + 3*A*a^6*b^2 + 3*A*a^4*b^4 + A*a^2*b^6 + (A*a^6*b^2 + 3*A*a^4*b^4 + 3*A*a^2*b^6 + A*b^8)*\tan(d*x + c)^2 + 2*(A*a^7*b + 3*A*a^5*b^3 + 3*A*a^3*b^5 + A*a*b^7)*\tan(d*x + c))*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)) - (3*B*a^7*b - 6*A*a^6*b^2 - B*a^5*b^3 - 3*A*a^4*b^4 - A*a^2*b^6 + (3*B*a^5*b^3 - 6*A*a^4*b^4 - B*a^3*b^5 - 3*A*a^2*b^6 - A*b^8)*\tan(d*x + c)^2 + 2*(3*B*a^6*b^2 - 6*A*a^5*b^3 - B*a^4*b^4 - 3*A*a^3*b^5 - A*a*b^7)*\tan(d*x + c))*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - 2*(3*B*a^6*b^2 - 4*A*a^5*b^3 - 3*B*a^4*b^4 + 3*A*a^3*b^5 + A*a*b^7 + 2*(B*a^7*b - 3*A*a^6*b^2 - 3*B*a^5*b^3 + A*a^4*b^4)*d*x)*\tan(d*x + c))/((a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*d*\tan(d*x + c)^2 + 2*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*\tan(d*x + c) + (a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(213) = 426.

time = 1.03, size = 479, normalized size = 2.23

$$\frac{2(Ba^3 - 3Ab^2 - 3Ba^2b + 3Ab^3) \log(\tan(dx+c))}{a^3 + 3a^2b + 3ab^2 + b^3} + \frac{(A^2 + 3Ba^2 - 3Ab^2) \log(\tan(dx+c))}{a^3 + 3a^2b + 3ab^2 + b^3} + \frac{2(3Ba^2b - 3Ab^3 - 3Aa^2b + 3Ab^3) \log(\tan(dx+c))}{a^3 + 3a^2b + 3ab^2 + b^3} + \frac{2A \log(\tan(dx+c))}{a^3} - \frac{9Ba^2 \tan(dx+c)^2 - 18Aa^2 \tan(dx+c)^2 - 9Ab^2 \tan(dx+c)^2 - 9Aa^2 \tan(dx+c)^2 - 9Ab^2 \tan(dx+c)^2 + 22Ba^2 \tan(dx+c) - 22Aa^2 \tan(dx+c) - 22Ab^2 \tan(dx+c) - 22Aa^2 \tan(dx+c) - 22Ab^2 \tan(dx+c) - 8Aa^2 \tan(dx+c) + 14Bb^2 - 25Aa^2 - 19Aa^2 - 19Ab^2 - 8Aa^2}{a^3 + 3a^2b + 3ab^2 + b^3} + \frac{2d}{a^3 + 3a^2b + 3ab^2 + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out]  $\frac{1}{2} * (2 * (B * a^3 - 3 * A * a^2 * b - 3 * B * a * b^2 + A * b^3) * (d * x + c) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) - (A * a^3 + 3 * B * a^2 * b - 3 * A * a * b^2 - B * b^3) * \log(\tan(d * x + c)^2 + 1) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) + 2 * (3 * B * a^5 * b^2 - 6 * A * a^4 * b^3 - B * a^3 * b^4 - 3 * A * a^2 * b^5 - A * b^7) * \log(\text{abs}(b * \tan(d * x + c) + a)) / (a^9 * b + 3 * a^7 * b^3 + 3 * a^5 * b^5 + a^3 * b^7) + 2 * A * \log(\text{abs}(\tan(d * x + c))) / a^3 - (9 * B * a^5 * b^3 * \tan(d * x + c)^2 - 18 * A * a^4 * b^4 * \tan(d * x + c)^2 - 3 * B * a^3 * b^5 * \tan(d * x + c)^2 - 9 * A * a^2 * b^6 * \tan(d * x + c)^2 - 3 * A * b^8 * \tan(d * x + c)^2 + 22 * B * a^6 * b^2 * \tan(d * x + c) - 42 * A * a^5 * b^3 * \tan(d * x + c) - 2 * B * a^4 * b^4 * \tan(d * x + c) - 26 * A * a^3 * b^5 * \tan(d * x + c) - 8 * A * a * b^7 * \tan(d * x + c) + 14 * B * a^7 * b - 25 * A * a^6 * b^2 + 3 * B * a^5 * b^3 - 19 * A * a^4 * b^4 + B * a^3 * b^5 - 6 * A * a^2 * b^6) / ((a^9 + 3 * a^7 * b^2 + 3 * a^5 * b^4 + a^3 * b^6) * (b * \tan(d * x + c) + a)^2)) / d$

**Mupad** [B]

time = 8.38, size = 315, normalized size = 1.47

$$\frac{-5Ba^3b + 7Aa^2b^2 - Ba^2b^2 + 3Ab^3}{2a(a^2 + 2a^2b + b^2)} + \frac{\tan(c+dx) \ln(\tan(c+dx)) - 2Ba^2b + 3Aa^2b^2 + Ab^3}{a^2(a^2 + 2a^2b + b^2)} + \frac{A \ln(\tan(c+dx))}{a^2d} + \frac{\ln(\tan(c+dx) - 1) - (B + A)}{2d(-a^3 + 3a^2b + ab^2 - b^3)} + \frac{\ln(\tan(c+dx) + 1) - (A - B)}{2d(-a^3 + a^2b + 3ab^2 - b^3)} - \frac{b \ln(a + b \tan(c+dx))}{a^3d(a^2 + b^2)} - \frac{(-3Ba^5 + 6Aa^4b + Ba^3b^2 + 3Aa^2b^3 + Ab^4)}{a^3d(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out]  $((3 * A * b^4 + 7 * A * a^2 * b^2 - B * a * b^3 - 5 * B * a^3 * b) / (2 * a * (a^4 + b^4 + 2 * a^2 * b^2)) + (\tan(c + d * x) * (A * b^5 + 3 * A * a^2 * b^3 - 2 * B * a^3 * b^2)) / (a^2 * (a^4 + b^4 + 2 * a^2 * b^2))) / (d * (a^2 + b^2 * \tan(c + d * x)^2 + 2 * a * b * \tan(c + d * x))) + (A * \log(\tan(c + d * x))) / (a^3 * d) + (\log(\tan(c + d * x) - 1) * (A * 1i - B)) / (2 * d * (a * b^2 * 3i + 3 * a^2 * b - a^3 * 1i - b^3)) + (\log(\tan(c + d * x) + 1) * (A - B * 1i)) / (2 * d * (3 * a * b^2 + a^2 * b * 3i - a^3 - b^3 * 1i)) - (b * \log(a + b * \tan(c + d * x)) * (A * b^5 - 3 * B * a^5 + 3 * A * a^2 * b^3 + B * a^3 * b^2 + 6 * A * a^4 * b)) / (a^3 * d * (a^2 + b^2)^3)$

$$3.288 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=287

$$\frac{(a^3 A - 3aAb^2 + 3a^2 bB - b^3 B)x}{(a^2 + b^2)^3} - \frac{(3Ab - aB) \log(\sin(c + dx))}{a^4 d} + \frac{b^2(10a^4 Ab + 9a^2 Ab^3 + 3Ab^5 - 6a^5 B - 3a^4 B^2)}{a^4 d}$$

[Out]  $-(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3-(3*A*b-B*a)*\ln(\sin(d*x+c))$   
 $/a^4/d+b^2*(10*A*a^4*b+9*A*a^2*b^3+3*A*b^5-6*B*a^5-3*B*a^3*b^2-B*a*b^4)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/a^4/(a^2+b^2)^3/d-1/2*b*(2*A*a^2+3*A*b^2-B*a*b)/a^2/(a^2+b^2)/d/(a+b*\tan(d*x+c))^2-A*\cot(d*x+c)/a/d/(a+b*\tan(d*x+c))^2-b*(A*a^4+6*A*a^2*b^2+3*A*b^4-3*B*a^3*b-B*a*b^3)/a^3/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))$

**Rubi [A]**

time = 0.59, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{(3Ab - aB) \log(\sin(c + dx))}{a^4 d} - \frac{b(2a^2 A - abB + 3Ab^2)}{2a^2 d (a^2 + b^2) (a + b \tan(c + dx))^2} - \frac{\pi(a^3 A + 3a^2 bB - 3aAb^2 - b^3 B)}{(a^2 + b^2)^3} - \frac{b(a^4 A - 3a^3 bB + 6a^2 Ab^2 - ab^3 B + 3Ab^4)}{a^3 d (a^2 + b^2)^2 (a + b \tan(c + dx))} + \frac{b^2(-6a^5 B + 10a^4 Ab - 3a^3 b^2 B + 9a^2 Ab^3 - ab^4 B + 3Ab^5) \log(a \cos(c + dx) + b \sin(c + dx))}{a^4 d (a^2 + b^2)^3} - \frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out]  $-(((a^3 A - 3a^2 A b + 3a^2 b B - b^3 B)x)/(a^2 + b^2)^3 - ((3A b - a B) \text{Log}[\text{Sin}[c + d*x]])/(a^4 d) + (b^2(10a^4 A b + 9a^2 A b^3 + 3A b^5 - 6a^5 B - 3a^3 b^2 B - a b^4 B) \text{Log}[a \text{Cos}[c + d*x] + b \text{Sin}[c + d*x]])/(a^4 (a^2 + b^2)^3 d) - (b(2a^2 A + 3A b^2 - a b B))/(2a^2 (a^2 + b^2) d (a + b \text{Tan}[c + d*x])^2) - (A \text{Cot}[c + d*x])/(a d (a + b \text{Tan}[c + d*x])^2) - (b(a^4 A + 6a^2 A b^2 + 3A b^4 - 3a^3 b B - a b^3 B))/(a^3 (a^2 + b^2)^2 d (a + b \text{Tan}[c + d*x]))$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps

$$\int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx = -\frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))^2} - \frac{\int \frac{\cot(c+dx)(3Ab-aB+aA \tan(c+dx)+3Ab \tan^2(c+dx))}{(a+b \tan(c+dx))^3} dx}{a}$$

$$= -\frac{b(2a^2 A + 3Ab^2 - abB)}{2a^2 (a^2 + b^2) d(a + b \tan(c + dx))^2} - \frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))^2} - \frac{b^2(4a^2 Ab + 2Ab^3 - 3a^2 B - ab^2 B)}{2a^2 (a^2 + b^2) d(a + b \tan(c + dx))^2}$$

$$= -\frac{b(2a^2 A + 3Ab^2 - abB)}{2a^2 (a^2 + b^2) d(a + b \tan(c + dx))^2} - \frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))^2} - \frac{b^2(4a^2 Ab + 2Ab^3 - 3a^2 B - ab^2 B)}{2a^2 (a^2 + b^2) d(a + b \tan(c + dx))^2}$$

$$= -\frac{(a^3 A - 3aAb^2 + 3a^2 bB - b^3 B) x}{(a^2 + b^2)^3} - \frac{b(2a^2 A + 3Ab^2 - abB)}{2a^2 (a^2 + b^2) d(a + b \tan(c + dx))^2}$$

$$= -\frac{(a^3 A - 3aAb^2 + 3a^2 bB - b^3 B) x}{(a^2 + b^2)^3} - \frac{(3Ab - aB) \log(\sin(c + dx))}{a^4 d}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 6.29, size = 288, normalized size = 1.00

$$\frac{A \cot(c + dx)}{a^3 d} + \frac{(A + iB) \log(i - \tan(c + dx))}{2(a - ib)^3 d} - \frac{(3Ab - aB) \log(\tan(c + dx))}{a^3 d} - \frac{(i(A + B) \log(i + \tan(c + dx)))}{2(a - ib)^3 d} + \frac{b^2(10a^2 Ab + 9a^2 Ab^3 + 3Ab^5 - 6a^2 B - 3a^2 b^2 B - ab^3 B) \log(a + b \tan(c + dx))}{a^4 (a^2 + b^2)^3 d} - \frac{b^2(Ab - aB)}{2a^2 (a^2 + b^2) d(a + b \tan(c + dx))^2} - \frac{b^2(4a^2 Ab + 2Ab^3 - 3a^2 B - ab^2 B)}{a^4 (a^2 + b^2) d(a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]^3,x]
[Out] -((A*Cot[c + d*x])/(a^3*d)) + ((A + I*B)*Log[I - Tan[c + d*x]])/(2*(I*a - b)^3*d) - ((3*A*b - a*B)*Log[Tan[c + d*x]])/(a^4*d) - ((I*A + B)*Log[I + Tan[c + d*x]])/(2*(a - I*b)^3*d) + (b^2*(10*a^4*A*b + 9*a^2*A*b^3 + 3*A*b^5 - 6*a^5*B - 3*a^3*b^2*B - a*b^4*B)*Log[a + b*Tan[c + d*x]])/(a^4*(a^2 + b^2)^3*d) - (b^2*(A*b - a*B))/(2*a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (b^2*(4*a^2*A*b + 2*A*b^3 - 3*a^3*B - a*b^2*B))/(a^3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

**Maple [A]**  
time = 0.49, size = 289, normalized size = 1.01

method	result
derivativedivides	$-\frac{A}{a^3 \tan(dx+c)} + \frac{(-3Ab+aB) \ln(\tan(dx+c))}{a^4} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) \ln(1 + \tan^2(dx+c))}{2} + \frac{(-A a^3 + 3A a b^2 - 3B a^2 b + B b^3)}{(a^2 + b^2)^3}$
default	$-\frac{A}{a^3 \tan(dx+c)} + \frac{(-3Ab+aB) \ln(\tan(dx+c))}{a^4} + \frac{(3A a^2 b - A b^3 - B a^3 + 3B a b^2) \ln(1 + \tan^2(dx+c))}{2} + \frac{(-A a^3 + 3A a b^2 - 3B a^2 b + B b^3)}{(a^2 + b^2)^3}$

norman	$\frac{b(3Aa^4b+11Aa^2b^3+6Ab^5-4Ba^3b^2-2Ba^4)(\tan^2(dx+c))}{da^3(a^4+2a^2b^2+b^4)} - \frac{A}{ad} - \frac{b^2(Aa^3-3Aab^2+3Ba^2b-Bb^3)x(\tan^3(dx+c))}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{b^2(4Aa^4b+3Aa^3b^2+3Aa^2b^3+3Aab^4+3Ba^3b^2+3Ba^2b^3+3Bab^4+3Bb^5)}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{b^2(4Aa^4b+3Aa^3b^2+3Aa^2b^3+3Aab^4+3Ba^3b^2+3Ba^2b^3+3Bab^4+3Bb^5)}{(a^4+2a^2b^2+b^4)(a^2+b^2)} \tan(dx+c)$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} \left( -\frac{A}{a^3 \tan(dx+c)} + \frac{-3A^*b+B^*a}{a^4 \ln(\tan(dx+c))} + \frac{1}{(a^2+b^2)^3} \left( \frac{1}{2} (3A^*a^2*b-A^*b^3-B^*a^3+3B^*a*b^2) \ln(1+\tan(dx+c)^2) + (-A^*a^3+3A^*a*b^2-3B^*a^2*b+B^*b^3) \arctan(\tan(dx+c)) \right) - \frac{b^2(4A^*a^2*b+2A^*b^3-3B^*a^3-B^*a*b^2)}{a^3(a^2+b^2)^2(a+b*\tan(dx+c))} + \frac{b^2(10A^*a^4*b+9A^*a^2*b^3+3A^*b^5-6B^*a^5-3B^*a^3*b^2-B^*a*b^4)}{a^4(a^2+b^2)^3 \ln(a+b*\tan(dx+c))} - \frac{1}{2} \frac{(A^*b-B^*a)*b^2}{a^2(a^2+b^2)(a+b*\tan(dx+c))^2} \right)$$

**Maxima** [A]

time = 0.61, size = 454, normalized size = 1.58

$$\frac{\frac{2(Aa^3+3Ba^2-3Aa^2B-3B^2)(dx+c)}{a^6+3a^4b+3a^2b^2+b^4} + \frac{2(6Bb^2-10Aa^2b+3Ba^3-9Aa^2b+3Ba^2-3Ab^2)\log(b\tan(dx+c)+a)}{a^6+3a^4b+3a^2b^2+b^4} + \frac{(Bb^3-3Aa^2b-3Bb^2+Ab^2)\log(\tan(dx+c)^2+1)}{a^6+3a^4b+3a^2b^2+b^4} + \frac{2Aa^4+4Aa^2b+2Aa^2b^2+2(Aa^2b^2+6Aa^2b^2-3Aa^2b^2+3Ab^2)\tan(dx+c)^2+(4Aa^2b-7Bb^2+17Aa^2b-3Ba^2b+9Aa^2b^2)\tan(dx+c)}{(a^2+b^2+2a^2b^2+a^2b^2)\tan(dx+c)^2+(a^2+2a^2b^2+a^2b^2)\tan(dx+c)} - \frac{2(Ba-3Ab)\log(\tan(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="maxima")`

[Out] 
$$-\frac{1}{2} \frac{(2(A^*a^3+3B^*a^2*b-3A^*a*b^2-B^*b^3)(dx+c))/(a^6+3a^4*b^2+3a^2*b^4+b^6)+2*(6B^*a^5*b^2-10A^*a^4*b^3+3B^*a^3*b^4-9A^*a^2*b^5+B^*a*b^6-3A^*b^7)*\log(b*\tan(dx+c)+a)/(a^{10}+3a^8*b^2+3a^6*b^4+a^4*b^6)+(B^*a^3-3A^*a^2*b-3B^*a*b^2+A^*b^3)*\log(\tan(dx+c)^2+1)/(a^6+3a^4*b^2+3a^2*b^4+b^6)+(2A^*a^6+4A^*a^4*b^2+2A^*a^2*b^4+2*(A^*a^4*b^2-3B^*a^3*b^3+6A^*a^2*b^4-B^*a*b^5+3A^*b^6)*\tan(dx+c)^2+(4A^*a^5*b-7B^*a^4*b^2+17A^*a^3*b^3-3B^*a^2*b^4+9A^*a*b^5)*\tan(dx+c))/(a^7*b^2+2a^5*b^4+a^3*b^6)*\tan(dx+c)^3+2*(a^8*b+2a^6*b^3+a^4*b^5)*\tan(dx+c)^2+(a^9+2a^7*b^2+a^5*b^4)*\tan(dx+c)-2*(B^*a-3A^*b)*\log(\tan(dx+c))/a^4}{d}$$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 917 vs. 2(283) = 566.

time = 1.77, size = 917, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fricas")`

```
[Out] -1/2*(2*A*a^9 + 6*A*a^7*b^2 + 6*A*a^5*b^4 + 2*A*a^3*b^6 + (7*B*a^5*b^4 - 9*
A*a^4*b^5 + B*a^3*b^6 - 3*A*a^2*b^7 + 2*(A*a^7*b^2 + 3*B*a^6*b^3 - 3*A*a^5*
b^4 - B*a^4*b^5)*d*x)*tan(d*x + c)^3 + 2*(A*a^7*b^2 + 4*B*a^6*b^3 - 2*A*a^5
*b^4 - 3*B*a^4*b^5 + 6*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8 + 2*(A*a^8*b + 3*B
*a^7*b^2 - 3*A*a^6*b^3 - B*a^5*b^4)*d*x)*tan(d*x + c)^2 - ((B*a^7*b^2 - 3*A
*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 + B*a*b^8
- 3*A*b^9)*tan(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 + 3*B*a^6*b^3 - 9*A*a^
5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 + B*a^2*b^7 - 3*A*a*b^8)*tan(d*x + c)^2 +
(B*a^9 - 3*A*a^8*b + 3*B*a^7*b^2 - 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5
+ B*a^3*b^6 - 3*A*a^2*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^
2 + 1)) + ((6*B*a^5*b^4 - 10*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 + B*a*b^
8 - 3*A*b^9)*tan(d*x + c)^3 + 2*(6*B*a^6*b^3 - 10*A*a^5*b^4 + 3*B*a^4*b^5 -
9*A*a^3*b^6 + B*a^2*b^7 - 3*A*a*b^8)*tan(d*x + c)^2 + (6*B*a^7*b^2 - 10*A*
a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 + B*a^3*b^6 - 3*A*a^2*b^7)*tan(d*x + c)
)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))
+ (4*A*a^8*b + 12*A*a^6*b^3 - 9*B*a^5*b^4 + 23*A*a^4*b^5 - 3*B*a^3*b^6 + 9
*A*a^2*b^7 + 2*(A*a^9 + 3*B*a^8*b - 3*A*a^7*b^2 - B*a^6*b^3)*d*x)*tan(d*x +
c))/((a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d*tan(d*x + c)^3 + 2*(a^
11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d*tan(d*x + c)^2 + (a^12 + 3*a^10*b
^2 + 3*a^8*b^4 + a^6*b^6)*d*tan(d*x + c))
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

**Giac** [A]

time = 1.02, size = 560, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(6*B*a^5*b^3 - 10*A*a^4*b
^4 + 3*B*a^3*b^5 - 9*A*a^2*b^6 + B*a*b^7 - 3*A*b^8)*log(abs(b*tan(d*x + c)
+ a))/(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7) - (18*B*a^5*b^4*tan(d*x +
```



$$\begin{aligned} & c)^2 - 30*A*a^4*b^5*\tan(d*x + c)^2 + 9*B*a^3*b^6*\tan(d*x + c)^2 - 27*A*a^2* \\ & b^7*\tan(d*x + c)^2 + 3*B*a*b^8*\tan(d*x + c)^2 - 9*A*b^9*\tan(d*x + c)^2 + 42 \\ & *B*a^6*b^3*\tan(d*x + c) - 68*A*a^5*b^4*\tan(d*x + c) + 26*B*a^4*b^5*\tan(d*x \\ & + c) - 66*A*a^3*b^6*\tan(d*x + c) + 8*B*a^2*b^7*\tan(d*x + c) - 22*A*a*b^8*\tan \\ & n(d*x + c) + 25*B*a^7*b^2 - 39*A*a^6*b^3 + 19*B*a^5*b^4 - 41*A*a^4*b^5 + 6* \\ & B*a^3*b^6 - 14*A*a^2*b^7)/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*(b*\tan( \\ & d*x + c) + a)^2) - 2*(B*a - 3*A*b)*\log(\text{abs}(\tan(d*x + c)))/a^4 + 2*(B*a*\tan( \\ & d*x + c) - 3*A*b*\tan(d*x + c) + A*a)/(a^4*\tan(d*x + c))/d \end{aligned}$$

**Mupad [B]**

time = 11.23, size = 380, normalized size = 1.32

$$\frac{b^2 \ln(a + b \tan(c + dx)) (-6B a^5 + 10A a^4 b - 3B a^3 b^2 + 9A a^2 b^3 - B a b^4 + 3A b^5)}{a^4 d (a^2 + b^2)^3} - \frac{\ln(\tan(c + dx) - 1) (-B + A 11)}{2d (-a^3 - a^2 b 3i + 3a b^2 + b^3 11)} - \frac{\ln(\tan(c + dx)) (3Ab - Ba)}{a^4 d} - \frac{\ln(\tan(c + dx) + 1) (A - B 11)}{2d (-a^3 11 - 3a^2 b + a b^2 3i + b^3)} - \frac{d}{a} + \frac{\text{atan}(dx)^2 (4a^6 b^2 - 3B a^5 b^2 + 6A a^4 b^3 - B a^3 b^4 + 3A b^5)}{d^2 (a^2 \tan(c + dx) + 2a b \tan(c + dx) + b^2 \tan(c + dx)^2)} + \frac{\text{atan}(dx) (4A a^5 b - 7B a^4 b^2 + 17A a^3 b^3 - 3B a^2 b^4 + 9A b^5)}{2d^2 (a^2 \tan(c + dx) + 2a b \tan(c + dx) + b^2 \tan(c + dx)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] (b^2\*log(a + b\*tan(c + d\*x))\*(3\*A\*b^5 - 6\*B\*a^5 + 9\*A\*a^2\*b^3 - 3\*B\*a^3\*b^2 + 10\*A\*a^4\*b - B\*a\*b^4))/(a^4\*d\*(a^2 + b^2)^3) - (log(tan(c + d\*x) - 1i)\*(A\*1i - B))/(2\*d\*(3\*a\*b^2 - a^2\*b\*3i - a^3 + b^3\*1i)) - (log(tan(c + d\*x))\*(3\*A\*b - B\*a))/(a^4\*d) - (log(tan(c + d\*x) + 1i)\*(A - B\*1i))/(2\*d\*(a\*b^2\*3i - 3\*a^2\*b - a^3\*1i + b^3)) - (A/a + (tan(c + d\*x)^2\*(3\*A\*b^6 + 6\*A\*a^2\*b^4 + A\*a^4\*b^2 - 3\*B\*a^3\*b^3 - B\*a\*b^5))/(a^3\*(a^4 + b^4 + 2\*a^2\*b^2)) + (tan(c + d\*x)\*(9\*A\*b^5 + 17\*A\*a^2\*b^3 - 7\*B\*a^3\*b^2 + 4\*A\*a^4\*b - 3\*B\*a\*b^4))/(2\*a^2\*(a^4 + b^4 + 2\*a^2\*b^2)))/(d\*(a^2\*tan(c + d\*x) + b^2\*tan(c + d\*x)^3 + 2\*a\*b\*tan(c + d\*x)^2))

$$3.289 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=352

$$\frac{(3a^2Ab - Ab^3 - a^3B + 3ab^2B)x}{(a^2 + b^2)^3} - \frac{(a^2A - 6Ab^2 + 3abB) \log(\sin(c + dx))}{a^5d} - \frac{b^3(15a^4Ab + 17a^2Ab^3 + 6Ab^5 - 10a^3B + 9a^2b^2B - 3ab^4B)}{(a^2 + b^2)^3}$$

[Out] (3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*x/(a^2+b^2)^3-(A\*a^2-6\*A\*b^2+3\*B\*a\*b)\*ln(sin(d\*x+c))/a^5/d-b^3\*(15\*A\*a^4\*b+17\*A\*a^2\*b^3+6\*A\*b^5-10\*B\*a^3\*b^2-3\*B\*a\*b^4)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/a^5/(a^2+b^2)^3/d+1/2\*b\*(5\*A\*a^2\*b+6\*A\*b^3-2\*B\*a^3-3\*B\*a\*b^2)/a^3/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^2+(2\*A\*b-B\*a)\*cot(d\*x+c)/a^2/d/(a+b\*tan(d\*x+c))^2-1/2\*A\*cot(d\*x+c)^2/a/d/(a+b\*tan(d\*x+c))^2+b\*(3\*A\*a^4\*b+11\*A\*a^2\*b^3+6\*A\*b^5-B\*a^5-6\*B\*a^3\*b^2-3\*B\*a\*b^4)/a^4/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.83, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{(2Ab - aB) \cot(c + dx)}{a^2d(a + b \tan(c + dx))^2} - \frac{(a^2A + 3abB - 6A^2) \log(\sin(c + dx))}{a^5d} + \frac{b(-2a^2B + 5a^2Ab - 3ab^2B + 6A^2)}{2a^2d(a^2 + b^2)(a + b \tan(c + dx))^2} + \frac{a(a^2 - B) + 3a^2Ab + 3ab^2B - AB}{(a^2 + b^2)} + \frac{b(a^2 - B) + 3a^2Ab - 6a^2b^2B + 11a^2Ab^3 - 3ab^4B + 6A^2}{a^4(a^2 + b^2)(a + b \tan(c + dx))} - \frac{b^3(-10a^2B + 15a^4Ab - 9a^2b^2B + 17a^2Ab^3 - 3ab^4B + 6A^2) \log(a \cos(c + dx) + b \sin(c + dx))}{a^5d(a^2 + b^2)} - \frac{A \cot^2(c + dx)}{2a^2d(a + b \tan(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out] ((3\*a^2\*A\*b - A\*b^3 - a^3\*B + 3\*a\*b^2\*B)\*x)/(a^2 + b^2)^3 - ((a^2\*A - 6\*A\*b^2 + 3\*a\*b\*B)\*Log[Sin[c + d\*x]])/(a^5\*d) - (b^3\*(15\*a^4\*A\*b + 17\*a^2\*A\*b^3 + 6\*A\*b^5 - 10\*a^3\*B - 9\*a^3\*b^2\*B - 3\*a\*b^4\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/(a^5\*(a^2 + b^2)^3\*d) + (b\*(5\*a^2\*A\*b + 6\*A\*b^3 - 2\*a^3\*B - 3\*a\*b^2\*B))/(2\*a^3\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^2) + ((2\*A\*b - a\*B)\*Cot[c + d\*x])/(a^2\*d\*(a + b\*Tan[c + d\*x])^2) - (A\*Cot[c + d\*x]^2)/(2\*a\*d\*(a + b\*Tan[c + d\*x])^2) + (b\*(3\*a^4\*A\*b + 11\*a^2\*A\*b^3 + 6\*A\*b^5 - a^5\*B - 6\*a^3\*b^2\*B - 3\*a\*b^4\*B))/(a^4\*(a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x]))

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/(a\_. + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

Rubi steps

$$\begin{aligned}
 \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx &= -\frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^2} - \frac{\int \frac{\cot^2(c+dx)(2(2Ab-aB)+2aA \tan(c+dx)+4Ab)}{(a+b \tan(c+dx))^3} dx}{2a} \\
 &= \frac{(2Ab-aB) \cot(c+dx)}{a^2d(a+b \tan(c+dx))^2} - \frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^2} + \frac{\int \frac{\cot(c+dx)}{(a+b \tan(c+dx))^2} dx}{2a} \\
 &= \frac{b(5a^2Ab+6Ab^3-2a^3B-3ab^2B)}{2a^3(a^2+b^2)d(a+b \tan(c+dx))^2} + \frac{(2Ab-aB) \cot(c+dx)}{a^2d(a+b \tan(c+dx))^2} - \frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^2} \\
 &= \frac{b(5a^2Ab+6Ab^3-2a^3B-3ab^2B)}{2a^3(a^2+b^2)d(a+b \tan(c+dx))^2} + \frac{(2Ab-aB) \cot(c+dx)}{a^2d(a+b \tan(c+dx))^2} - \frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^2} \\
 &= \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} + \frac{b(5a^2Ab+6Ab^3-2a^3B-3ab^2B)}{2a^3(a^2+b^2)d(a+b \tan(c+dx))} \\
 &= \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)x}{(a^2+b^2)^3} - \frac{(a^2A-6Ab^2+3abB) \log(\sin(c+dx))}{a^5d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
 time = 6.33, size = 320, normalized size = 0.91

$$\frac{(3Ab-aB)\cot(c+dx)}{a^2d} - \frac{A \cot^2(c+dx)}{2a^2d} + \frac{(A+iB) \log(i-\tan(c+dx))}{2(a+ib)^2d} - \frac{(a^2A-6Ab^2+3abB) \log(\tan(c+dx))}{a^2d} + \frac{(A-iB) \log(i+\tan(c+dx))}{2(a-ib)^2d} - \frac{b^2(15a^2Ab+17a^2Ab^2+6Ab^3-10a^2B-9a^2b^2B-3ab^2B) \log(a+b \tan(c+dx))}{a^2(a^2+b^2)^2d} + \frac{b^2(Ab-aB)}{2a^2(a^2+b^2)d(a+b \tan(c+dx))^2} + \frac{b^2(5a^2Ab+3Ab^3-4a^2B-2ab^2B)}{a^2(a^2+b^2)^2d(a+b \tan(c+dx))}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
[Out] ((3*A*b - a*B)*Cot[c + d*x])/(a^4*d) - (A*Cot[c + d*x]^2)/(2*a^3*d) + ((A +
I*B)*Log[I - Tan[c + d*x]])/(2*(a + I*b)^3*d) - ((a^2*A - 6*A*b^2 + 3*a*b*B)*
Log[Tan[c + d*x]])/(a^5*d) + ((A - I*B)*Log[I + Tan[c + d*x]])/(2*(a - I
*b)^3*d) - (b^3*(15*a^4*A*b + 17*a^2*A*b^3 + 6*A*b^5 - 10*a^5*B - 9*a^3*b^2
*B - 3*a*b^4*B)*Log[a + b*Tan[c + d*x]])/(a^5*(a^2 + b^2)^3*d) + (b^3*(A*b
- a*B))/(2*a^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (b^3*(5*a^2*A*b + 3*
A*b^3 - 4*a^3*B - 2*a*b^2*B))/(a^4*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
    
```

**Maple [A]**  
 time = 0.58, size = 320, normalized size = 0.91

method	result
derivativedivides	$  -\frac{A}{2a^3 \tan(dx+c)^2} - \frac{-3Ab+aB}{a^4 \tan(dx+c)} + \frac{(-a^2A+6Ab^2-3Bab) \ln(\tan(dx+c))}{a^5} + \frac{(Aa^3-3Aab^2+3Ba^2b-Bb^3) \ln(1+\tan^2(dx+c))}{2} + \frac{(3Aa^2B-3Ab^3)}{(a^2+b^2)^3}  $



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fricas")`

[Out] 
$$-1/2*(A*a^{10} + 3*A*a^8*b^2 + 3*A*a^6*b^4 + A*a^4*b^6 + (A*a^8*b^2 + 3*A*a^6*b^4 - 9*B*a^5*b^5 + 14*A*a^4*b^6 - 3*B*a^3*b^7 + 6*A*a^2*b^8 + 2*(B*a^8*b^2 - 3*A*a^7*b^3 - 3*B*a^6*b^4 + A*a^5*b^5)*d*x)*\tan(d*x + c)^4 + 2*(A*a^9*b + B*a^8*b^2 - 2*B*a^6*b^4 + 6*B*a^4*b^6 - 11*A*a^3*b^7 + 3*B*a^2*b^8 - 6*A*a*b^9 + 2*(B*a^9*b - 3*A*a^8*b^2 - 3*B*a^7*b^3 + A*a^6*b^4)*d*x)*\tan(d*x + c)^3 + (A*a^{10} + 4*B*a^9*b - 8*A*a^8*b^2 + 12*B*a^7*b^3 - 30*A*a^6*b^4 + 2*3*B*a^5*b^5 - 45*A*a^4*b^6 + 9*B*a^3*b^7 - 18*A*a^2*b^8 + 2*(B*a^{10} - 3*A*a^9*b - 3*B*a^8*b^2 + A*a^7*b^3)*d*x)*\tan(d*x + c)^2 + ((A*a^8*b^2 + 3*B*a^7*b^3 - 3*A*a^6*b^4 + 9*B*a^5*b^5 - 15*A*a^4*b^6 + 9*B*a^3*b^7 - 17*A*a^2*b^8 + 3*B*a*b^9 - 6*A*b^{10})*\tan(d*x + c)^4 + 2*(A*a^9*b + 3*B*a^8*b^2 - 3*A*a^7*b^3 + 9*B*a^6*b^4 - 15*A*a^5*b^5 + 9*B*a^4*b^6 - 17*A*a^3*b^7 + 3*B*a^2*b^8 - 6*A*a*b^9)*\tan(d*x + c)^3 + (A*a^{10} + 3*B*a^9*b - 3*A*a^8*b^2 + 9*B*a^7*b^3 - 15*A*a^6*b^4 + 9*B*a^5*b^5 - 17*A*a^4*b^6 + 3*B*a^3*b^7 - 6*A*a^2*b^8)*\tan(d*x + c)^2)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)) - ((10*B*a^5*b^5 - 15*A*a^4*b^6 + 9*B*a^3*b^7 - 17*A*a^2*b^8 + 3*B*a*b^9 - 6*A*b^{10})*\tan(d*x + c)^4 + 2*(10*B*a^6*b^4 - 15*A*a^5*b^5 + 9*B*a^4*b^6 - 17*A*a^3*b^7 + 3*B*a^2*b^8 - 6*A*a*b^9)*\tan(d*x + c)^3 + (10*B*a^7*b^3 - 15*A*a^6*b^4 + 9*B*a^5*b^5 - 17*A*a^4*b^6 + 3*B*a^3*b^7 - 6*A*a^2*b^8)*\tan(d*x + c)^2)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) + 2*(B*a^{10} - 2*A*a^9*b + 3*B*a^8*b^2 - 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 + B*a^4*b^6 - 2*A*a^3*b^7)*\tan(d*x + c))/((a^{11}*b^2 + 3*a^9*b^4 + 3*a^7*b^6 + a^5*b^8)*d*\tan(d*x + c)^4 + 2*(a^{12}*b + 3*a^{10}*b^3 + 3*a^8*b^5 + a^6*b^7)*d*\tan(d*x + c)^3 + (a^{13} + 3*a^{11}*b^2 + 3*a^9*b^4 + a^7*b^6)*d*\tan(d*x + c)^2)$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)`

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 812 vs. 2(346) = 692.

time = 1.22, size = 812, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*(4*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 \\ & + 3*a^2*b^4 + b^6) - 2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*\log(\tan(d*x \\ & + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*(10*B*a^5*b^4 - 15*A*a^4*b^5 \\ & + 9*B*a^3*b^6 - 17*A*a^2*b^7 + 3*B*a*b^8 - 6*A*b^9)*\log(\text{abs}(b*\tan(d*x \\ & + c) + a))/(a^{11}*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7) - (3*A*a^7*b^2*\tan(d \\ & *x + c)^4 + 9*B*a^6*b^3*\tan(d*x + c)^4 - 9*A*a^5*b^4*\tan(d*x + c)^4 - 3*B*a \\ & ^4*b^5*\tan(d*x + c)^4 + 6*A*a^8*b*\tan(d*x + c)^3 + 14*B*a^7*b^2*\tan(d*x + c \\ & )^3 - 6*A*a^6*b^3*\tan(d*x + c)^3 - 34*B*a^5*b^4*\tan(d*x + c)^3 + 56*A*a^4*b \\ & ^5*\tan(d*x + c)^3 - 36*B*a^3*b^6*\tan(d*x + c)^3 + 68*A*a^2*b^7*\tan(d*x + c \\ & )^3 - 12*B*a*b^8*\tan(d*x + c)^3 + 24*A*b^9*\tan(d*x + c)^3 + 3*A*a^9*\tan(d*x \\ & + c)^2 + B*a^8*b*\tan(d*x + c)^2 + 13*A*a^7*b^2*\tan(d*x + c)^2 - 45*B*a^6*b^3 \\ & * \tan(d*x + c)^2 + 88*A*a^5*b^4*\tan(d*x + c)^2 - 52*B*a^4*b^5*\tan(d*x + c)^2 \\ & + 102*A*a^3*b^6*\tan(d*x + c)^2 - 18*B*a^2*b^7*\tan(d*x + c)^2 + 36*A*a*b^8 \\ & * \tan(d*x + c)^2 - 4*B*a^9*\tan(d*x + c) + 8*A*a^8*b*\tan(d*x + c) - 12*B*a^7*b^2 \\ & * \tan(d*x + c) + 24*A*a^6*b^3*\tan(d*x + c) - 12*B*a^5*b^4*\tan(d*x + c) + \\ & 24*A*a^4*b^5*\tan(d*x + c) - 4*B*a^3*b^6*\tan(d*x + c) + 8*A*a^2*b^7*\tan(d*x \\ & + c) - 2*A*a^9 - 6*A*a^7*b^2 - 6*A*a^5*b^4 - 2*A*a^3*b^6)/((a^{10} + 3*a^8*b^2 \\ & + 3*a^6*b^4 + a^4*b^6)*(b*\tan(d*x + c)^2 + a*\tan(d*x + c))^2) + 4*(A*a^2 \\ & + 3*B*a*b - 6*A*b^2)*\log(\text{abs}(\tan(d*x + c)))/a^5/d \end{aligned}$$

**Mupad [B]**

time = 12.87, size = 434, normalized size = 1.23

$$\frac{\frac{\tan(dx+c)(Aa-Ba) - \frac{A}{d} + \frac{\tan(dx+c)^2(-Ba^2+3Aa^2b-4Bb^2+11Aa^2b^2-9Ba^2b^2+4A^2b^2+3Ba^2b^2)}{d^2} + \frac{\tan(dx+c)^3(-4Ba^2+11Aa^2b-17Bb^2+20Aa^2b^2-9Ba^2b^2+3A^2b^2)}{2d^3} + \frac{\ln(a+b\tan(dx+c))\left(\frac{A}{d} - \frac{6AaB}{d^2} + \frac{11A^2}{d^3} + \frac{4B^2(Aa+Ba)}{d^2}\right) - \frac{\ln(\tan(dx+c)-1)(-B+A)}{2d} - \frac{\ln(\tan(dx+c))(A^2+3Ba^2-6A^2)}{a^2d} - \frac{\ln(\tan(dx+c)+1)(A-B)}{2d} - \frac{\ln(-a^2+3a^2b+a^2b^2-B^2)}{2d}}{d^2(a^2\tan(dx+c)+2ab\tan(dx+c)+b^2)\tan^2(dx+c)}}{d^2(a^2\tan(dx+c)+2ab\tan(dx+c)+b^2)\tan^2(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] 
$$\begin{aligned} & ((\tan(c + d*x)*(2*A*b - B*a))/a^2 - A/(2*a) + (\tan(c + d*x)^3*(6*A*b^7 + 11 \\ & *A*a^2*b^5 + 3*A*a^4*b^3 - 6*B*a^3*b^4 - B*a^5*b^2 - 3*B*a*b^6))/(a^4*(a^4 \\ & + b^4 + 2*a^2*b^2)) + (\tan(c + d*x)^2*(18*A*b^6 + 33*A*a^2*b^4 + 11*A*a^4*b \\ & ^2 - 17*B*a^3*b^3 - 9*B*a*b^5 - 4*B*a^5*b))/ (2*a^3*(a^4 + b^4 + 2*a^2*b^2)) \\ & )/(d*(a^2*\tan(c + d*x)^2 + b^2*\tan(c + d*x)^4 + 2*a*b*\tan(c + d*x)^3)) + (1 \\ & \log(a + b*\tan(c + d*x))*(A/a^3 - (A*a + 3*B*b)/(a^2 + b^2)^2 - (6*A*b^2)/a^5 \\ & + (3*B*b)/a^4 + (4*b^2*(A*a + B*b))/(a^2 + b^2)^3))/d - (\log(\tan(c + d*x) \\ & - 1i)*(A*1i - B))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3)) - (\log(\tan(c + \\ & d*x))*(A*a^2 - 6*A*b^2 + 3*B*a*b))/(a^5*d) - (\log(\tan(c + d*x) + 1i)*(A - B \\ & *1i))/(2*d*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) \end{aligned}$$

$$3.290 \quad \int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=351

$$\frac{(a^4 A - 6a^2 A b^2 + A b^4 + 4a^3 b B - 4ab^3 B) x}{(a^2 + b^2)^4} + \frac{(4a^3 A b - 4a A b^3 - a^4 B + 6a^2 b^2 B - b^4 B) \log(\cos(c + dx))}{(a^2 + b^2)^4 d} + \frac{a(4a^3 A b^3 - 4a A b^5 - 4a^2 A b^7 + 4a^2 A b^9 + 4a^2 B b^5 - 4a^2 B b^7 + 4a^2 B b^9 + 10a^3 A b^4 + 10a^3 A b^6 + 10a^3 B b^4 + 10a^3 B b^6 + 10a^3 B b^8 - 4a^4 B^2 - 4a^4 B^4 - 4a^4 B^6 - 4a^4 B^8 - 4a^4 B^{10} - 4a^4 B^{12} - 4a^4 B^{14} - 4a^4 B^{16} - 4a^4 B^{18} - 4a^4 B^{20} - 4a^4 B^{22} - 4a^4 B^{24} - 4a^4 B^{26} - 4a^4 B^{28} - 4a^4 B^{30}) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4 d}$$

[Out] (A\*a^4-6\*A\*a^2\*b^2+A\*b^4+4\*a^3\*b\*B-4\*a\*b^3\*B)\*x/(a^2+b^2)^4+(4\*A\*a^3\*b-4\*A\*a\*b^3-B\*a^4+6\*B\*a^2\*b^2-B\*b^4)\*ln(cos(d\*x+c))/(a^2+b^2)^4/d+a\*(4\*A\*a^2\*b^5-4\*A\*a\*b^7+B\*a^4+4\*B\*a^2\*b^2+5\*B\*a^3\*b^4+10\*B\*a\*b^6)\*ln(a+b\*tan(d\*x+c))/b^4/(a^2+b^2)^4/d+1/3\*a\*(A\*b-B\*a)\*tan(d\*x+c)^3/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^3+1/2\*a\*(2\*A\*b^3-a\*(a^2+3\*b^2)\*B)\*tan(d\*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^2+a^2\*(A\*a^2\*b^3-3\*A\*b^5+B\*a^5+3\*B\*a^3\*b^2+6\*B\*a\*b^4)/b^4/(a^2+b^2)^3/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.55, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3686, 3726, 3716, 3707, 3698, 31, 3556}

$$\frac{a(Ab - aB) \tan^3(c + dx)}{3bd(a^2 + b^2)(a + b \tan(c + dx))^3} + \frac{a(2Ab^2 - aB(a^2 + 3b^2)) \tan^2(c + dx)}{2b^2d(a^2 + b^2)^2(a + b \tan(c + dx))^2} + \frac{a^2(a^3B + 3a^2b^2B + a^2Ab^2 + 6ab^3B - 3Ab^3)}{b^4d(a^2 + b^2)^3(a + b \tan(c + dx))} + \frac{(a^4(-B) + 4a^3Ab + 6a^2b^2B - 4aAb^2 - b^4B) \log(\cos(c + dx))}{d(a^2 + b^2)^4} + \frac{a(a^4A + 4a^3bB - 6a^2b^2B - 4ab^3B + Ab^4)}{(a^2 + b^2)^4} + \frac{a(a^2B + 4a^2b^2B + 5a^3b^2B + 4a^2Ab^2 + 10ab^3B - 4Ab^4) \log(a + b \tan(c + dx))}{b^4d(a^2 + b^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^4\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4,x]

[Out] ((a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 + 4\*a^3\*b\*B - 4\*a\*b^3\*B)\*x)/(a^2 + b^2)^4 + ((4\*a^3\*A\*b - 4\*a\*A\*b^3 - a^4\*B + 6\*a^2\*b^2\*B - b^4\*B)\*Log[Cos[c + d\*x]])/((a^2 + b^2)^4\*d) + (a\*(4\*a^2\*A\*b^5 - 4\*A\*b^7 + a^7\*B + 4\*a^5\*b^2\*B + 5\*a^3\*b^4\*B + 10\*a\*b^6\*B)\*Log[a + b\*Tan[c + d\*x]])/(b^4\*(a^2 + b^2)^4\*d) + (a\*(A\*b - a\*B)\*Tan[c + d\*x]^3)/(3\*b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^3) + (a\*(2\*A\*b^3 - a\*(a^2 + 3\*b^2)\*B)\*Tan[c + d\*x]^2)/(2\*b^2\*(a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x])^2) + (a^2\*(a^2\*A\*b^3 - 3\*A\*b^5 + a^5\*B + 3\*a^3\*b^2\*B + 6\*a\*b^4\*B))/(b^4\*(a^2 + b^2)^3\*d\*(a + b\*Tan[c + d\*x]))

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 3556**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rule 3686**



```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

```

### Rule 3698

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
(f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

```

### Rule 3707

```

Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
+ Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]

```

### Rule 3716

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-b*c - a*d)*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]

```

### Rule 3726

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis

```

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan^4(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^4} dx &= \frac{a(Ab - aB) \tan^3(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{\int \frac{\tan^2(c + dx)(-3a(Ab - aB) + 3b(Ab - aB))}{(a + b \tan(c + dx))^4} dx}{3b(a^2 + b^2)d} \\ &= \frac{a(Ab - aB) \tan^3(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{a(2Ab^3 - a(a^2 + 3b^2)B) \tan^2(c + dx)}{2b^2(a^2 + b^2)^2 d(a + b \tan(c + dx))^2} \\ &= \frac{a(Ab - aB) \tan^3(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{a(2Ab^3 - a(a^2 + 3b^2)B) \tan^2(c + dx)}{2b^2(a^2 + b^2)^2 d(a + b \tan(c + dx))^2} \\ &= \frac{(a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B)x}{(a^2 + b^2)^4} + \frac{a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^2} \\ &= \frac{(a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B)x}{(a^2 + b^2)^4} + \frac{(4a^3Ab - 4aAb^3 - a^2B)}{(a^2 + b^2)^4} \\ &= \frac{(a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B)x}{(a^2 + b^2)^4} + \frac{(4a^3Ab - 4aAb^3 - a^2B)}{(a^2 + b^2)^4} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 6.62, size = 1812, normalized size = 5.16

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
[Out] (((4*I)*a^10*A*b^8 + 4*a^9*A*b^9 + (8*I)*a^8*A*b^10 + 8*a^7*A*b^11 - (8*I)*a^4*A*b^14 - 8*a^3*A*b^15 - (4*I)*a^2*A*b^16 - 4*a*A*b^17 + I*a^15*b^3*B + a^14*b^4*B + (7*I)*a^13*b^5*B + 7*a^12*b^6*B + (20*I)*a^11*b^7*B + 20*a^10*b^8*B + (38*I)*a^9*b^9*B + 38*a^8*b^10*B + (49*I)*a^7*b^11*B + 49*a^6*b^12*B + (35*I)*a^5*b^13*B + 35*a^4*b^14*B + (10*I)*a^3*b^15*B + 10*a^2*b^16*B)*(c + d*x)*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4*(A + B*Tan[c + d*x]))/((a - I*b)^8*(a + I*b)^7*b^7*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + b*Tan[c + d*x])^4) - (I*(4*a^3*A*b^5 - 4*a*A*b^7 + a^8*B + 4*a^6*b^2*B +
```

$$\begin{aligned}
& 5a^4b^4B + 10a^2b^6B) \cdot \text{ArcTan}[\text{Tan}[c + dx]] \cdot \text{Sec}[c + dx]^3 \cdot (a \cdot \text{Cos}[c + dx] + b \cdot \text{Sin}[c + dx])^4 \cdot (A + B \cdot \text{Tan}[c + dx]) / (b^4(a^2 + b^2)^4 \cdot d \cdot (A \cdot \text{Cos}[c + dx] + B \cdot \text{Sin}[c + dx]) \cdot (a + b \cdot \text{Tan}[c + dx])^4) - (B \cdot \text{Log}[\text{Cos}[c + dx]] \cdot \text{Sec}[c + dx]^3 \cdot (a \cdot \text{Cos}[c + dx] + b \cdot \text{Sin}[c + dx])^4 \cdot (A + B \cdot \text{Tan}[c + dx]) / (b^4 \cdot d \cdot (A \cdot \text{Cos}[c + dx] + B \cdot \text{Sin}[c + dx]) \cdot (a + b \cdot \text{Tan}[c + dx])^4) + ((4a^3A \cdot b^5 - 4a \cdot A \cdot b^7 + a^8B + 4a^6b^2B + 5a^4b^4B + 10a^2b^6B) \cdot \text{Log}[(a \cdot \text{Cos}[c + dx] + b \cdot \text{Sin}[c + dx])^2] \cdot \text{Sec}[c + dx]^3 \cdot (a \cdot \text{Cos}[c + dx] + b \cdot \text{Sin}[c + dx])^4 \cdot (A + B \cdot \text{Tan}[c + dx]) / (2b^4(a^2 + b^2)^4 \cdot d \cdot (A \cdot \text{Cos}[c + dx] + B \cdot \text{Sin}[c + dx]) \cdot (a + b \cdot \text{Tan}[c + dx])^4) + (\text{Sec}[c + dx]^3 \cdot (a \cdot \text{Cos}[c + dx] + b \cdot \text{Sin}[c + dx]) \cdot (12a^6A \cdot b^4 \cdot \text{Cos}[c + dx] + 48a^4A \cdot b^6 \cdot \text{Cos}[c + dx] + 36a^2A \cdot b^8 \cdot \text{Cos}[c + dx] - 12a^9b \cdot B \cdot \text{Cos}[c + dx] - 60a^7b^3 \cdot B \cdot \text{Cos}[c + dx] - 108a^5b^5 \cdot B \cdot \text{Cos}[c + dx] - 60a^3b^7 \cdot B \cdot \text{Cos}[c + dx] + 9a^7A \cdot b^3 \cdot (c + dx) \cdot \text{Cos}[c + dx] - 45a^5A \cdot b^5 \cdot (c + dx) \cdot \text{Cos}[c + dx] - 45a^3A \cdot b^7 \cdot (c + dx) \cdot \text{Cos}[c + dx] + 9a \cdot A \cdot b^9 \cdot (c + dx) \cdot \text{Cos}[c + dx] + 36a^6b^4 \cdot B \cdot (c + dx) \cdot \text{Cos}[c + dx] - 36a^2b^8 \cdot B \cdot (c + dx) \cdot \text{Cos}[c + dx] + 8a^6A \cdot b^4 \cdot \text{Cos}[3(c + dx)] - 28a^4A \cdot b^6 \cdot \text{Cos}[3(c + dx)] - 36a^2A \cdot b^8 \cdot \text{Cos}[3(c + dx)] + 6a^9b \cdot B \cdot \text{Cos}[3(c + dx)] + 28a^7b^3 \cdot B \cdot \text{Cos}[3(c + dx)] + 82a^5b^5 \cdot B \cdot \text{Cos}[3(c + dx)] + 60a^3b^7 \cdot B \cdot \text{Cos}[3(c + dx)] + 3a^7A \cdot b^3 \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] - 27a^5A \cdot b^5 \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] + 57a^3A \cdot b^7 \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] - 9a \cdot A \cdot b^9 \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] + 12a^6b^4 \cdot B \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] - 48a^4b^6 \cdot B \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] + 36a^2b^8 \cdot B \cdot (c + dx) \cdot \text{Cos}[3(c + dx)] + 30a^5A \cdot b^5 \cdot \text{Sin}[c + dx] + 84a^3A \cdot b^7 \cdot \text{Sin}[c + dx] + 54a \cdot A \cdot b^9 \cdot \text{Sin}[c + dx] - 3a^{10} \cdot B \cdot \text{Sin}[c + dx] - 33a^8b^2 \cdot B \cdot \text{Sin}[c + dx] - 123a^6b^4 \cdot B \cdot \text{Sin}[c + dx] - 183a^4b^6 \cdot B \cdot \text{Sin}[c + dx] - 90a^2b^8 \cdot B \cdot \text{Sin}[c + dx] + 9a^6A \cdot b^4 \cdot (c + dx) \cdot \text{Sin}[c + dx] - 45a^4A \cdot b^6 \cdot (c + dx) \cdot \text{Sin}[c + dx] - 45a^2A \cdot b^8 \cdot (c + dx) \cdot \text{Sin}[c + dx] + 9A \cdot b^{10} \cdot (c + dx) \cdot \text{Sin}[c + dx] + 36a^5b^5 \cdot B \cdot (c + dx) \cdot \text{Sin}[c + dx] - 36a \cdot b^9 \cdot B \cdot (c + dx) \cdot \text{Sin}[c + dx] - 4a^7A \cdot b^3 \cdot \text{Sin}[3(c + dx)] + 18a^5A \cdot b^5 \cdot \text{Sin}[3(c + dx)] + 4a^3A \cdot b^7 \cdot \text{Sin}[3(c + dx)] - 18a \cdot A \cdot b^9 \cdot \text{Sin}[3(c + dx)] - 3a^{10} \cdot B \cdot \text{Sin}[3(c + dx)] - 11a^8b^2 \cdot B \cdot \text{Sin}[3(c + dx)] - 27a^6b^4 \cdot B \cdot \text{Sin}[3(c + dx)] + 11a^4b^6 \cdot B \cdot \text{Sin}[3(c + dx)] + 30a^2b^8 \cdot B \cdot \text{Sin}[3(c + dx)] + 9a^6A \cdot b^4 \cdot (c + dx) \cdot \text{Sin}[3(c + dx)] - 57a^4A \cdot b^6 \cdot (c + dx) \cdot \text{Sin}[3(c + dx)] + 27a^2A \cdot b^8 \cdot (c + dx) \cdot \text{Sin}[3(c + dx)] - 3A \cdot b^{10} \cdot (c + dx) \cdot \text{Sin}[3(c + dx)] + 36a^5b^5 \cdot B \cdot (c + dx) \cdot \text{Sin}[3(c + dx)] - 48a^3b^7 \cdot B \cdot (c + dx) \cdot \text{Sin}[3(c + dx)] + 12a \cdot b^9 \cdot B \cdot (c + dx) \cdot \text{Sin}[3(c + dx)]) \cdot (A + B \cdot \text{Tan}[c + dx]) / (12(a - I \cdot b)^4 \cdot (a + I \cdot b)^4 \cdot b^3 \cdot d \cdot (A \cdot \text{Cos}[c + dx] + B \cdot \text{Sin}[c + dx]) \cdot (a + b \cdot \text{Tan}[c + dx])^4)
\end{aligned}$$

Maple [A]

time = 0.48, size = 343, normalized size = 0.98

method	result
derivativedivides	$ \frac{(-4Aa^3b + 4Aab^3 + Ba^4 - 6Ba^2b^2 + Bb^4) \ln(1 + \tan^2(dx+c))}{2} + \frac{(Aa^4 - 6Aa^2b^2 + Ab^4 + 4Ba^3b - 4Bab^3) \arctan(\tan(dx+c))}{(a^2+b^2)^4} + \frac{a(4A}{ $

default	$\frac{(-4A a^3 b + 4A a b^3 + B a^4 - 6B a^2 b^2 + B b^4) \ln(1 + \tan^2(dx+c)) + (A a^4 - 6A a^2 b^2 + A b^4 + 4B a^3 b - 4B a b^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^4} + \frac{a(4A a^2}{(a^2 + b^2)^4}$
norman	$\frac{(A a^4 - 6A a^2 b^2 + A b^4 + 4B a^3 b - 4B a b^3) a^3 x}{(a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)(a^2 + b^2)} + \frac{b^3 (A a^4 - 6A a^2 b^2 + A b^4 + 4B a^3 b - 4B a b^3) x (\tan^3(dx+c))}{(a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)(a^2 + b^2)} - \frac{a^3 (2A a^5 b + 4A a^3 b^3 + 26A a^2 b^4 + 4A a b^5 + B a^6)}{6d b^4 (a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} \left( \frac{1}{(a^2+b^2)^4} \left( \frac{1}{2} (-4Aa^3b + 4Aab^3 + Ba^4 - 6Ba^2b^2 + Bb^4) \ln(1 + \tan^2(dx+c)) + (Aa^4 - 6Aa^2b^2 + Ab^4 + 4Ba^3b - 4Bab^3) \arctan(\tan(dx+c)) \right) + \frac{a(4Aa^2 + 4Aa^2b^2 + 4Aa^2b^4 + 4Aa^2b^6 + 4Ab^4 + 4Ab^6 + 4Bab^4 + 4Bab^6)}{(a^2+b^2)^4} \right) - \frac{a^3(2Aa^5b + 4Aa^3b^3 + 26Aa^2b^4 + 4Aab^5 + Ba^6)}{6db^4(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)}$

**Maxima [A]**

time = 0.55, size = 583, normalized size = 1.66

$$\frac{6(Aa^4 + Bb^4 - 4Aa^2b^2 - 4Bb^2a^2) \ln(\tan(dx+c)) + 6(Ba^4 + 4Ba^2b^2 + 4Ab^4 - 4Aa^2b^2) \arctan(\tan(dx+c)) + \frac{3(Ba^4 - 4Aa^3b + 4Aa^2b^2 - 4Aab^3 + Bb^4) \ln(\tan(dx+c))}{a^4 + a^2b^2 + b^4} + \frac{11Ba^4 - 2Aa^3b + 34Ba^2b^2 - 4Aa^2b^3 + 47Ba^2b^4 - 26Aa^2b^5 + 6(3Ba^2b^6 - Aa^2b^7 - 3Aa^2b^8 - 10Ba^2b^9 - 6Aa^2b^{10} - 3Aa^2b^{11} - 3Aa^2b^{12}) \tan(dx+c) + 3(9Ba^2b^{13} - 2Aa^2b^{14} - 20Aa^2b^{15}) \tan(dx+c)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="maxima")`

[Out]  $\frac{1}{6} \left( \frac{6(Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Ba^2b^3 + Ab^4)(dx+c)}{(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) + 6(Ba^8 + 4Ba^6b^2 + 5Ba^4b^4 + 4Aa^3b^5 + 10Ba^2b^6 - 4Aa^2b^7) \log(b \tan(dx+c) + a)} + \frac{3(Ba^4 - 4Aa^3b - 6Ba^2b^2 + 4Aa^2b^3 + Bb^4) \log(\tan(dx+c)^2 + 1)}{(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) + (11Ba^9 - 2Aa^8b + 34Ba^7b^2 - 4Aa^6b^3 + 47Ba^5b^4 - 26Aa^4b^5 + 6(3Ba^3b^7 - Aa^6b^3 + 9Ba^5b^4 - 3Aa^4b^5 + 10Ba^3b^6 - 6Aa^2b^7) \tan(dx+c)^2 + 3(9Ba^8b - 2Aa^7b^2 + 28Ba^6b^3 - 6Aa^5b^4 + 35Ba^4b^5 - 20Aa^3b^6) \tan(dx+c))} + \frac{3(a^7b^6 + 3a^5b^8 + 3a^3b^{10} + a^2b^{12}) \tan(dx+c)^3 + 3(a^7b^6 + 3a^5b^8 + 3a^3b^{10} + a^2b^{12}) \tan(dx+c)^2 + 3(a^8b^5 + 3a^6b^7 + 3a^4b^9 + a^2b^{11}) \tan(dx+c)}{d} \right)$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1113 vs. 2(346) = 692.

time = 1.89, size = 1113, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*B*a^9*b^2 + 6*B*a^7*b^4 + 18*A*a^6*b^5 + 47*B*a^5*b^6 - 26*A*a^4*b^7 - (11*B*a^8*b^3 - 2*A*a^7*b^4 + 42*B*a^6*b^5 - 6*A*a^5*b^6 + 75*B*a^4*b^7 - 48*A*a^3*b^8 - 6*(A*a^4*b^7 + 4*B*a^3*b^8 - 6*A*a^2*b^9 - 4*B*a*b^10 + A*b^11)*d*x)*tan(d*x + c)^3 + 6*(A*a^7*b^4 + 4*B*a^6*b^5 - 6*A*a^5*b^6 - 4*B*a^4*b^7 + A*a^3*b^8)*d*x - 3*(5*B*a^9*b^2 + 18*B*a^7*b^4 + 2*A*a^6*b^5 + 37*B*a^5*b^6 - 30*A*a^4*b^7 - 20*B*a^3*b^8 + 12*A*a^2*b^9 - 6*(A*a^5*b^6 + 4*B*a^4*b^7 - 6*A*a^3*b^8 - 4*B*a^2*b^9 + A*a*b^10)*d*x)*tan(d*x + c)^2 + 3*(B*a^11 + 4*B*a^9*b^2 + 5*B*a^7*b^4 + 4*A*a^6*b^5 + 10*B*a^5*b^6 - 4*A*a^4*b^7 + (B*a^8*b^3 + 4*B*a^6*b^5 + 5*B*a^4*b^7 + 4*A*a^3*b^8 + 10*B*a^2*b^9 - 4*A*a*b^10)*tan(d*x + c)^3 + 3*(B*a^9*b^2 + 4*B*a^7*b^4 + 5*B*a^5*b^6 + 4*A*a^4*b^7 + 10*B*a^3*b^8 - 4*A*a^2*b^9)*tan(d*x + c)^2 + 3*(B*a^10*b + 4*B*a^8*b^3 + 5*B*a^6*b^5 + 4*A*a^5*b^6 + 10*B*a^4*b^7 - 4*A*a^3*b^8)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(B*a^11 + 4*B*a^9*b^2 + 6*B*a^7*b^4 + 4*B*a^5*b^6 + B*a^3*b^8 + (B*a^8*b^3 + 4*B*a^6*b^5 + 6*B*a^4*b^7 + 4*B*a^2*b^9 + B*b^11)*tan(d*x + c)^3 + 3*(B*a^9*b^2 + 4*B*a^7*b^4 + 6*B*a^5*b^6 + 4*B*a^3*b^8 + B*a*b^10)*tan(d*x + c)^2 + 3*(B*a^10*b + 4*B*a^8*b^3 + 6*B*a^6*b^5 + 4*B*a^4*b^7 + B*a^2*b^9)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 3*(2*B*a^10*b + 5*B*a^8*b^3 + 2*A*a^7*b^4 + 12*B*a^6*b^5 - 22*A*a^5*b^6 - 35*B*a^4*b^7 + 20*A*a^3*b^8 - 6*(A*a^6*b^5 + 4*B*a^5*b^6 - 6*A*a^4*b^7 - 4*B*a^3*b^8 + A*a^2*b^9)*d*x)*tan(d*x + c)/((a^8*b^7 + 4*a^6*b^9 + 6*a^4*b^11 + 4*a^2*b^13 + b^15)*d*tan(d*x + c)^3 + 3*(a^9*b^6 + 4*a^7*b^8 + 6*a^5*b^10 + 4*a^3*b^12 + a*b^14)*d*tan(d*x + c)^2 + 3*(a^10*b^5 + 4*a^8*b^7 + 6*a^6*b^9 + 4*a^4*b^11 + a^2*b^13)*d*tan(d*x + c) + (a^11*b^4 + 4*a^9*b^6 + 6*a^7*b^8 + 4*a^5*b^10 + a^3*b^12)*d)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(346) = 692.

time = 1.33, size = 719, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] 
$$\frac{1}{6} \cdot (6 \cdot (A \cdot a^4 + 4 \cdot B \cdot a^3 \cdot b - 6 \cdot A \cdot a^2 \cdot b^2 - 4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot (d \cdot x + c) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) + 3 \cdot (B \cdot a^4 - 4 \cdot A \cdot a^3 \cdot b - 6 \cdot B \cdot a^2 \cdot b^2 + 4 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot \log(\tan(d \cdot x + c)^2 + 1) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) + 6 \cdot (B \cdot a^8 + 4 \cdot B \cdot a^6 \cdot b^2 + 5 \cdot B \cdot a^4 \cdot b^4 + 4 \cdot A \cdot a^3 \cdot b^5 + 10 \cdot B \cdot a^2 \cdot b^6 - 4 \cdot A \cdot a \cdot b^7) \cdot \log(\text{abs}(b \cdot \tan(d \cdot x + c) + a)) / (a^8 \cdot b^4 + 4 \cdot a^6 \cdot b^6 + 6 \cdot a^4 \cdot b^8 + 4 \cdot a^2 \cdot b^{10} + b^{12}) - (11 \cdot B \cdot a^8 \cdot b^2 \cdot \tan(d \cdot x + c)^3 + 44 \cdot B \cdot a^6 \cdot b^4 \cdot \tan(d \cdot x + c)^3 + 55 \cdot B \cdot a^4 \cdot b^6 \cdot \tan(d \cdot x + c)^3 + 44 \cdot A \cdot a^3 \cdot b^7 \cdot \tan(d \cdot x + c)^3 + 110 \cdot B \cdot a^2 \cdot b^8 \cdot \tan(d \cdot x + c)^3 - 44 \cdot A \cdot a \cdot b^9 \cdot \tan(d \cdot x + c)^3 + 15 \cdot B \cdot a^9 \cdot b \cdot \tan(d \cdot x + c)^2 + 6 \cdot A \cdot a^8 \cdot b^2 \cdot \tan(d \cdot x + c)^2 + 60 \cdot B \cdot a^7 \cdot b^3 \cdot \tan(d \cdot x + c)^2 + 24 \cdot A \cdot a^6 \cdot b^4 \cdot \tan(d \cdot x + c)^2 + 51 \cdot B \cdot a^5 \cdot b^5 \cdot \tan(d \cdot x + c)^2 + 186 \cdot A \cdot a^4 \cdot b^6 \cdot \tan(d \cdot x + c)^2 + 270 \cdot B \cdot a^3 \cdot b^7 \cdot \tan(d \cdot x + c)^2 - 96 \cdot A \cdot a^2 \cdot b^8 \cdot \tan(d \cdot x + c)^2 + 6 \cdot B \cdot a^{10} \cdot \tan(d \cdot x + c) + 6 \cdot A \cdot a^9 \cdot b \cdot \tan(d \cdot x + c) + 21 \cdot B \cdot a^8 \cdot b^2 \cdot \tan(d \cdot x + c) + 24 \cdot A \cdot a^7 \cdot b^3 \cdot \tan(d \cdot x + c) - 24 \cdot B \cdot a^6 \cdot b^4 \cdot \tan(d \cdot x + c) + 210 \cdot A \cdot a^5 \cdot b^5 \cdot \tan(d \cdot x + c) + 225 \cdot B \cdot a^4 \cdot b^6 \cdot \tan(d \cdot x + c) - 72 \cdot A \cdot a^3 \cdot b^7 \cdot \tan(d \cdot x + c) + 2 \cdot A \cdot a^{10} - B \cdot a^9 \cdot b + 6 \cdot A \cdot a^8 \cdot b^2 - 26 \cdot B \cdot a^7 \cdot b^3 + 74 \cdot A \cdot a^6 \cdot b^4 + 63 \cdot B \cdot a^5 \cdot b^5 - 18 \cdot A \cdot a^4 \cdot b^6) / ((a^8 \cdot b^3 + 4 \cdot a^6 \cdot b^5 + 6 \cdot a^4 \cdot b^7 + 4 \cdot a^2 \cdot b^9 + b^{11}) \cdot (b \cdot \tan(d \cdot x + c) + a)^3) / d$$

**Mupad [B]**

time = 7.35, size = 486, normalized size = 1.38

$$\frac{\frac{11 B^2 - 24 A^2 b^2 B^2 + 4 A^2 b^2 B^2 + 4 A^2 b^2 B^2 - 20 A^2 b^2}{10^4 (A^2 B^2 - 12 A^2 B^2 + 12 A^2 B^2 - 12 A^2 B^2)} + \frac{\text{atan}(d x + c) (11 B^2 - 24 A^2 b^2 B^2 + 4 A^2 b^2 B^2 + 4 A^2 b^2 B^2 - 20 A^2 b^2)}{2 d (a^8 b^3 + 4 a^6 b^5 + 6 a^4 b^7 + 4 a^2 b^9 + b^{11})} + \frac{\ln(\tan(c + d x) - 1) (A + B)}{2 d (a^8 b^3 + 4 a^6 b^5 + 6 a^4 b^7 + 4 a^2 b^9 + b^{11})} + \frac{\ln(\tan(c + d x) + 1) (B + A)}{2 d (a^8 b^3 + 4 a^6 b^5 + 6 a^4 b^7 + 4 a^2 b^9 + b^{11})} + \frac{a \ln(a + b \tan(c + d x)) (B a^7 + 4 B a^5 b^2 + 5 B a^3 b^4 + 4 A a^2 b^5 + 10 B a b^6 - 4 A b^7)}{b^4 d (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x))^4\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^4,x)

[Out] 
$$\frac{((11 \cdot B \cdot a^9 - 26 \cdot A \cdot a^4 \cdot b^5 - 4 \cdot A \cdot a^6 \cdot b^3 + 47 \cdot B \cdot a^5 \cdot b^4 + 34 \cdot B \cdot a^7 \cdot b^2 - 2 \cdot A \cdot a^8 \cdot b) / (6 \cdot b^4 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2)) + (\tan(c + d \cdot x)^2 \cdot (3 \cdot B \cdot a^7 - 6 \cdot A \cdot a^2 \cdot b^5 - 3 \cdot A \cdot a^4 \cdot b^3 + 10 \cdot B \cdot a^3 \cdot b^4 + 9 \cdot B \cdot a^5 \cdot b^2 - A \cdot a^6 \cdot b)) / (b^2 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2)) + (\tan(c + d \cdot x) \cdot (9 \cdot B \cdot a^8 - 20 \cdot A \cdot a^3 \cdot b^5 - 6 \cdot A \cdot a^5 \cdot b^3 + 35 \cdot B \cdot a^4 \cdot b^4 + 28 \cdot B \cdot a^6 \cdot b^2 - 2 \cdot A \cdot a^7 \cdot b)) / (2 \cdot b^3 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2)) / (d \cdot (a^3 + b^3 \cdot \tan(c + d \cdot x))^3 + 3 \cdot a \cdot b^2 \cdot \tan(c + d \cdot x)^2 + 3 \cdot a^2 \cdot b \cdot \tan(c + d \cdot x)) + (\log(\tan(c + d \cdot x) - 1) \cdot (A + B \cdot 1)) / (2 \cdot d \cdot (4 \cdot a \cdot b^3 - 4 \cdot a^3 \cdot b + a^4 \cdot 1i + b^4 \cdot 1i - a^2 \cdot b^2 \cdot 6i)) + (\log(\tan(c + d \cdot x) + 1) \cdot (A \cdot 1i + B)) / (2 \cdot d \cdot (a \cdot b^3 \cdot 4i - a^3 \cdot b \cdot 4i + a^4 + b^4 - 6 \cdot a^2 \cdot b^2)) + (a \cdot \log(a + b \cdot \tan(c + d \cdot x)) \cdot (B \cdot a^7 - 4 \cdot A \cdot b^7 + 4 \cdot A \cdot a^2 \cdot b^5 + 5 \cdot B \cdot a^3 \cdot b^4 + 4 \cdot B \cdot a^5 \cdot b^2 + 10 \cdot B \cdot a \cdot b^6)) / (b^4 \cdot d \cdot (a^2 + b^2)^4)$$

$$3.291 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=298

$$\frac{(4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x}{(a^2 + b^2)^4} + \frac{(a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B) \log(a \cos(c + dx))}{(a^2 + b^2)^4 d}$$

[Out]  $-(4Aa^3b - 4Aa^2b^3 - B a^4 + 6B a^2b^2 - B b^4) * x / (a^2 + b^2)^4 + (A a^4 - 6A a^2b^2 + A b^4 + 4a^3bB - 4ab^3B) * \ln(a \cos(dx+c) + b \sin(dx+c)) / (a^2 + b^2)^4 / d + 1/3 * a * (A b - B a) * \tan(dx+c)^2 / b / (a^2 + b^2) / d / (a + b \tan(dx+c))^3 + 1/6 * a^2 * (A a^2b - 5A a b^3 + 2B a^3 + 8B a^2b) / b^3 / (a^2 + b^2)^2 / d / (a + b \tan(dx+c))^2 - 1/3 * a * (A a^4b + 5A a^2b^3 - 8A a b^5 + 2B a^5 + 7B a^3b^2 + 17B a^2b^4) / b^3 / (a^2 + b^2)^3 / d / (a + b \tan(dx+c))$

**Rubi** [A]

time = 0.39, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3686, 3716, 3709, 3612, 3611}

$$\frac{a(Ab - aB) \tan^2(c + dx)}{3bd(a^2 + b^2)(a + b \tan(c + dx))^3} + \frac{a^2(2a^3B + a^2Ab + 8ab^2B - 5Ab^3)}{6b^3d(a^2 + b^2)^2(a + b \tan(c + dx))^2} + \frac{(a^4A + 4a^3bB - 6a^2Ab^2 - 4ab^3B + Ab^4) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^4} - \frac{x(a^4(-B) + 4a^3Ab + 6a^2b^2B - 4aAb^3 - b^4B)}{(a^2 + b^2)^4} - \frac{a(2a^3B + a^2Ab + 7a^2b^2B + 5a^2Ab^3 + 17ab^4B - 8Ab^5)}{3b^3d(a^2 + b^2)^3(a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4, x]

[Out]  $-(((4a^3A^3Ab - 4a^2A^2Ab^3 - a^4AB + 6a^2b^2AB - b^4AB)x) / (a^2 + b^2)^4) + ((a^4A - 6a^2A^2Ab^2 + Ab^4 + 4a^3b^3B - 4a^2b^3B) * \text{Log}[a \text{Cos}[c + d*x] + b \text{Sin}[c + d*x]]) / ((a^2 + b^2)^4 * d) + (a * (A^2b - a^2B) * \text{Tan}[c + d*x]^2) / (3 * b * (a^2 + b^2) * d * (a + b * \text{Tan}[c + d*x])^3) + (a^2 * (a^2A^2Ab - 5A^2Ab^3 + 2a^3AB + 8a^2b^2B)) / (6 * b^3 * (a^2 + b^2)^2 * d * (a + b * \text{Tan}[c + d*x])^2) - (a * (a^4A^2b + 5a^2A^2Ab^3 - 8A^2Ab^5 + 2a^5AB + 7a^3b^2B + 17a^2b^4B)) / (3 * b^3 * (a^2 + b^2)^3 * d * (a + b * \text{Tan}[c + d*x]))$

Rule 3611

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && Ne

$Q[a*c + b*d, 0]$

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[(-b*c - a*d)*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]
```

Rubi steps



$$\begin{aligned}
\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4} dx &= \frac{a(Ab-aB)\tan^2(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{\int \frac{\tan(c+dx)(-2a(Ab-aB)+3b(Ab-aB))}{(a+b\tan(c+dx))^4} dx}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} \\
&= \frac{a(Ab-aB)\tan^2(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{a^2(a^2Ab-5Ab^3+2a^3B+8Ab^2)}{6b^3(a^2+b^2)^2d(a+b\tan(c+dx))^3} \\
&= \frac{a(Ab-aB)\tan^2(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{a^2(a^2Ab-5Ab^3+2a^3B+8Ab^2)}{6b^3(a^2+b^2)^2d(a+b\tan(c+dx))^3} \\
&= -\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x}{(a^2+b^2)^4} + \frac{a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} \\
&= -\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x}{(a^2+b^2)^4} + \frac{(a^4A-6a^2Ab^2+5Ab^3)}{(a^2+b^2)^4}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 6.24, size = 465, normalized size = 1.56

$$\frac{B \tan^2(c+dx)}{bd(a+b\tan(c+dx))^2} - \frac{(Ab-2aB)\tan(c+dx)}{bd(a+b\tan(c+dx))^2} - \frac{a^2b^2d-2a^2b}{bd(a+b\tan(c+dx))^2} + \frac{(a^2b^2d-2a^2b)\tan(c+dx)}{bd(a+b\tan(c+dx))^2} - \frac{a^2b^2d-2a^2b}{bd(a+b\tan(c+dx))^2} + \frac{(a^2b^2d-2a^2b)\tan^2(c+dx)}{bd(a+b\tan(c+dx))^2} - \frac{a^2b^2d-2a^2b}{bd(a+b\tan(c+dx))^2} + \frac{(a^2b^2d-2a^2b)\tan^3(c+dx)}{bd(a+b\tan(c+dx))^2} - \frac{a^2b^2d-2a^2b}{bd(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
[Out] -((B*Tan[c + d*x]^2)/(b*d*(a + b*Tan[c + d*x])^3)) - (-1/2*((-A*b) - 2*a*B)
)*Tan[c + d*x]/(b*d*(a + b*Tan[c + d*x])^3) - (-1/3*(a*A*b + 2*a^2*B - 2*b
^2*B)/(b*d*(a + b*Tan[c + d*x])^3) + (((6*a*A*b^3 + 6*b^4*B)*((-1/2*I)*Log
[I - Tan[c + d*x]])/(a + I*b)^4 + ((I/2)*Log[I + Tan[c + d*x]])/(a - I*b)^4
+ (4*a*(a - b)*b*(a + b)*Log[a + b*Tan[c + d*x]]/(a^2 + b^2)^4 - b/(3*(a^
2 + b^2)*(a + b*Tan[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*(a + b*Tan[c + d*x]
)^2) - (b*(3*a^2 - b^2))/((a^2 + b^2)^3*(a + b*Tan[c + d*x])))/b - 6*A*b^2
*(-1/2*Log[I - Tan[c + d*x]]/(I*a - b)^3 + Log[I + Tan[c + d*x]]/(2*(I*a +
b)^3) + (b*(3*a^2 - b^2)*Log[a + b*Tan[c + d*x]]/(a^2 + b^2)^3 - b/(2*(a^2
+ b^2)*(a + b*Tan[c + d*x])^2) - (2*a*b)/((a^2 + b^2)^2*(a + b*Tan[c + d*x]
))))/(3*b*d)/(2*b))/b

```

**Maple [A]**

time = 0.34, size = 318, normalized size = 1.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOS
E)

```

```

[Out] 1/d*(1/(a^2+b^2)^4*(1/2*(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(1
+tan(d*x+c)^2)+(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*arctan(tan(d*

```

$$x+c)))+(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)/(a^2+b^2)^4*\ln(a+b*\tan(d*x+c))-a*(A*a^2*b^3-3*A*b^5+B*a^5+3*B*a^3*b^2+6*B*a*b^4)/(a^2+b^2)^3/b^3/(a+b*\tan(d*x+c))-1/2*a^2*(A*a^2*b+3*A*b^3-2*B*a^3-4*B*a*b^2)/b^3/(a^2+b^2)^2/(a+b*\tan(d*x+c))^2+1/3*a^3*(A*b-B*a)/b^3/(a^2+b^2)/(a+b*\tan(d*x+c))^3$$

**Maxima** [A]

time = 0.55, size = 550, normalized size = 1.85

$$\frac{6 (B a^4 - 4 A a^2 b^2 + 4 A b^4 + 4 B a^3 b - 4 B a b^3) \ln(a + b \tan(dx + c)) + 5 (A^2 + 4 B a^2 b - 6 A a b^2 - 4 B a b^3 + 4 B^2) \ln(\tan(dx + c)) + 5 (A^4 + 4 A^2 b^2 - 6 A^2 b^4 - 4 B a^3 b^2 + 4 B^2) \ln(\tan(dx + c)^2 + 1) - 2 B a^4 + A a^2 b^4 + 4 B a^2 b^2 + 14 A a^2 b^2 + 20 B a^2 b^2 - 11 A a^2 b^4 + (B a^2 + 3 B a^2 + A a^2 + 6 B a^2 - 3 A a^2) \tan(dx + c)^2 + 3 (2 B a^2 + A a^2 + 6 B a^2 + 8 A a^2 + 20 B a^2 - 9 A a^2) \tan(dx + c)}{a^2 + b^2 + 2 a b \tan(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out] 1/6\*(6\*(B\*a^4 - 4\*A\*a^3\*b - 6\*B\*a^2\*b^2 + 4\*A\*a\*b^3 + B\*b^4)\*(d\*x + c)/(a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8) + 6\*(A\*a^4 + 4\*B\*a^3\*b - 6\*A\*a^2\*b^2 - 4\*B\*a\*b^3 + A\*b^4)\*log(b\*tan(d\*x + c) + a)/(a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8) - 3\*(A\*a^4 + 4\*B\*a^3\*b - 6\*A\*a^2\*b^2 - 4\*B\*a\*b^3 + A\*b^4)\*log(tan(d\*x + c)^2 + 1)/(a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8) - (2\*B\*a^8 + A\*a^7\*b + 4\*B\*a^6\*b^2 + 14\*A\*a^5\*b^3 + 26\*B\*a^4\*b^4 - 11\*A\*a^3\*b^5 + 6\*(B\*a^6\*b^2 + 3\*B\*a^4\*b^4 + A\*a^3\*b^5 + 6\*B\*a^2\*b^6 - 3\*A\*a\*b^7)\*tan(d\*x + c)^2 + 3\*(2\*B\*a^7\*b + A\*a^6\*b^2 + 6\*B\*a^5\*b^3 + 8\*A\*a^4\*b^4 + 20\*B\*a^3\*b^5 - 9\*A\*a^2\*b^6)\*tan(d\*x + c))/(a^9\*b^3 + 3\*a^7\*b^5 + 3\*a^5\*b^7 + a^3\*b^9 + (a^6\*b^6 + 3\*a^4\*b^8 + 3\*a^2\*b^10 + b^12)\*tan(d\*x + c)^3 + 3\*(a^7\*b^5 + 3\*a^5\*b^7 + 3\*a^3\*b^9 + a\*b^11)\*tan(d\*x + c)^2 + 3\*(a^8\*b^4 + 3\*a^6\*b^6 + 3\*a^4\*b^8 + a^2\*b^10)\*tan(d\*x + c)))/d

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(289) = 578.

time = 1.49, size = 813, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] 1/6\*(3\*A\*a^7 + 18\*B\*a^6\*b - 30\*A\*a^5\*b^2 - 26\*B\*a^4\*b^3 + 11\*A\*a^3\*b^4 + (2\*B\*a^7 + A\*a^6\*b + 6\*B\*a^5\*b^2 + 18\*A\*a^4\*b^3 + 48\*B\*a^3\*b^4 - 27\*A\*a^2\*b^5 + 6\*(B\*a^4\*b^3 - 4\*A\*a^3\*b^4 - 6\*B\*a^2\*b^5 + 4\*A\*a\*b^6 + B\*b^7)\*d\*x)\*tan(d\*x + c)^3 + 6\*(B\*a^7 - 4\*A\*a^6\*b - 6\*B\*a^5\*b^2 + 4\*A\*a^4\*b^3 + B\*a^3\*b^4)\*d\*x + 3\*(A\*a^7 - 2\*B\*a^6\*b + 16\*A\*a^5\*b^2 + 30\*B\*a^4\*b^3 - 23\*A\*a^3\*b^4 - 12\*B\*a^2\*b^5 + 6\*A\*a\*b^6 + 6\*(B\*a^5\*b^2 - 4\*A\*a^4\*b^3 - 6\*B\*a^3\*b^4 + 4\*A\*a^2\*b^5 + B\*a\*b^6)\*d\*x)\*tan(d\*x + c)^2 + 3\*(A\*a^7 + 4\*B\*a^6\*b - 6\*A\*a^5\*b^2 - 4\*B\*a^4\*b^3 + A\*a^3\*b^4 + (A\*a^4\*b^3 + 4\*B\*a^3\*b^4 - 6\*A\*a^2\*b^5 - 4\*B\*a\*b^6 + A\*b^7)\*tan(d\*x + c)^3 + 3\*(A\*a^5\*b^2 + 4\*B\*a^4\*b^3 - 6\*A\*a^3\*b^4 - 4\*B\*

$$a^2b^5 + Aa^6b^6) \tan(dx + c)^2 + 3(Aa^6b + 4Ba^5b^2 - 6Aa^4b^3 - 4Ba^3b^4 + Aa^2b^5) \tan(dx + c) \log((b^2 \tan(dx + c)^2 + 2a^2 \tan(dx + c) + a^2) / (\tan(dx + c)^2 + 1)) - 3(2Ba^7 - 9Aa^6b - 22Ba^5b^2 + 26Aa^4b^3 + 20Ba^3b^4 - 9Aa^2b^5 - 6(Ba^6b - 4Aa^5b^2 - 6Ba^4b^3 + 4Aa^3b^4 + Ba^2b^5) dx) \tan(dx + c) / ((a^8b^3 + 4a^6b^5 + 6a^4b^7 + 4a^2b^9 + b^{11}) dx \tan(dx + c)^3 + 3(a^9b^2 + 4a^7b^4 + 6a^5b^6 + 4a^3b^8 + ab^{10}) dx \tan(dx + c)^2 + 3(a^{10}b + 4a^8b^3 + 6a^6b^5 + 4a^4b^7 + a^2b^9) dx \tan(dx + c) + (a^{11} + 4a^9b^2 + 6a^7b^4 + 4a^5b^6 + a^3b^8) dx)$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*3\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))\*\*4,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(289) = 578.

time = 1.07, size = 670, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^3\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^4,x, algorithm="giac")

[Out]  $\frac{1}{6} (6(Ba^4 - 4Aa^3b - 6Ba^2b^2 + 4Aa^2b^3 + Bb^4)(dx + c) / (a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) - 3(Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Ba^2b^3 + Ab^4) \log(\tan(dx + c)^2 + 1) / (a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) + 6(Aa^4b + 4Ba^3b^2 - 6Aa^2b^3 - 4Ba^2b^4 + Ab^5) \log(\tan(dx + c) + a) / (a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) - (11Aa^4b^6 \tan(dx + c)^3 + 44Ba^3b^7 \tan(dx + c)^3 - 66Aa^2b^8 \tan(dx + c)^3 - 44Ba^2b^9 \tan(dx + c)^3 + 11Ab^{10} \tan(dx + c)^3 + 6Ba^8b^2 \tan(dx + c)^2 + 24Ba^6b^4 \tan(dx + c)^2 + 39Aa^5b^5 \tan(dx + c)^2 + 186Ba^4b^6 \tan(dx + c)^2 - 210Aa^3b^7 \tan(dx + c)^2 - 96Ba^2b^8 \tan(dx + c)^2 + 15Aa^2b^9 \tan(dx + c)^2 + 6Ba^9b \tan(dx + c) + 3Aa^8b^2 \tan(dx + c) + 24Ba^7b^3 \tan(dx + c) + 60Aa^6b^4 \tan(dx + c) + 210Ba^5b^5 \tan(dx + c) - 201Aa^4b^6 \tan(dx + c) - 72Ba^3b^7 \tan(dx + c) + 6Aa^2b^8 \tan(dx + c) + 2Ba^10 + Aa^9b + 6Ba^8b^2 + 26Aa^7b^3 + 74Ba^6b^4 - 63Aa^5b^5 - 18$

$*B*a^4*b^6)/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^{11})*(b*\tan(dx + c) + a)^3)/d$

**Mupad [B]**

time = 7.19, size = 470, normalized size = 1.58

$$\frac{\ln(a + b \tan(c + dx)) \left( \frac{d}{(a^2 + b^2)^2} - \frac{4b^2(Ab - Ba)}{(a^2 + b^2)^3} + \frac{8B^2(Ab - Ba)}{(a^2 + b^2)^4} \right) - \frac{d^2(2Ba^6 - 4a^5b + 4Ba^4b^2 - 14Aa^3b^3 + 20B^2a^2b^4 - 11Aa^2b^5) + \frac{\tan(c+dx)^2(2a^6 - 3B^2a^4b^2 + 4a^5b^3 + 6Ba^4b^4 - 3Aa^3b^5) + \frac{\tan(c+dx)}{2B^2(a^2 + b^2)^2(2a^6 - 3Aa^4b^2 + 4a^3b^5)}}{d(a^2 + 3a^2b \tan(c + dx) + 3a^2b^2 \tan^2(c + dx) + b^2 \tan^3(c + dx))} - \frac{\ln(\tan(c + dx) + 1)(B + A)}{2d(a^2 + 4a^2b - a^2b^2 - 4a^2b^2 + b^2)} - \frac{\ln(\tan(c + dx) - 1)(A + B)}{2d(a^2 + a^2b - a^2b^2 - a^2b^2 + b^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + dx))^3*(A + B*\tan(c + dx)))/(a + b*\tan(c + dx))^4, x)$

[Out]  $(\log(a + b*\tan(c + dx))*(A/(a^2 + b^2)^2 - (4*b*(2*A*b - B*a))/(a^2 + b^2)^3 + (8*b^3*(A*b - B*a))/(a^2 + b^2)^4))/d - ((a^2*(2*B*a^6 + 14*A*a^3*b^3 + 26*B*a^2*b^4 + 4*B*a^4*b^2 - 11*A*a*b^5 + A*a^5*b))/(6*b^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (\tan(c + dx)^2*(B*a^6 + A*a^3*b^3 + 6*B*a^2*b^4 + 3*B*a^4*b^2 - 3*A*a*b^5))/(b*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*\tan(c + dx)*(2*B*a^6 + 8*A*a^3*b^3 + 20*B*a^2*b^4 + 6*B*a^4*b^2 - 9*A*a*b^5 + A*a^5*b))/(2*b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*\tan(c + dx)^3 + 3*a*b^2*\tan(c + dx)^2 + 3*a^2*b*\tan(c + dx))) - (\log(\tan(c + dx) + 1)*(A*1i + B))/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) - (\log(\tan(c + dx) - 1)*(A + B*1i))/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2))$

$$3.292 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=261

$$\frac{(a^4 A - 6a^2 A b^2 + A b^4 + 4a^3 b B - 4a b^3 B) x}{(a^2 + b^2)^4} - \frac{(4a^3 A b - 4a A b^3 - a^4 B + 6a^2 b^2 B - b^4 B) \log(a \cos(c + dx))}{(a^2 + b^2)^4 d}$$

[Out]  $-(A*a^4-6*A*a^2*b^2+A*b^4+4*a^3*b*B-4*A*b^3*B)*x/(a^2+b^2)^4-(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^4/d-1/3*a^2*(A*b-B*a)/b^2/(a^2+b^2)/d/(a+b*\tan(d*x+c))^3+1/2*a*(2*A*b^3-a*(a^2+3*b^2)*B)/b^2/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^2+(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3/d/(a+b*\tan(d*x+c))$

**Rubi** [A]

time = 0.32, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3685, 3709, 3610, 3612, 3611}

$$-\frac{a^2(Ab - aB)}{3b^2d(a^2 + b^2)(a + b \tan(c + dx))^3} + \frac{a(2Ab^2 - aB(a^2 + 3b^2))}{2b^2d(a^2 + b^2)^2(a + b \tan(c + dx))^2} + \frac{a^2(-B) + 3a^2Ab + 3ab^2B - Ab^3}{d(a^2 + b^2)^3(a + b \tan(c + dx))} - \frac{(a^4(-B) + 4a^3Ab + 6a^2b^2B - 4aAb^3 - b^4B) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^4} - \frac{x(a^4A + 4a^2bB - 6a^2Ab^2 - 4ab^2B + Ab^4)}{(a^2 + b^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4,x]

[Out]  $-(((a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*x)/(a^2 + b^2)^4) - (((4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*(A*b - a*B))/(3*b^2*(a^2 + b^2))*d*(a + b*\text{Tan}[c + d*x])^3) + (a*(2*A*b^3 - a*(a^2 + 3*b^2)*B))/(2*b^2*(a^2 + b^2)^2*d*(a + b*\text{Tan}[c + d*x])^2) + (3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)/((a^2 + b^2)^3*d*(a + b*\text{Tan}[c + d*x]))$

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3611**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

## Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)
)*(x_)]), x_Symbol] := Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

## Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[
(- (B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
+ 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

## Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^4} dx &= -\frac{a^2(Ab - aB)}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{\int \frac{-a(Ab - aB) + b(Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^4} dx}{b(a^2 + b^2)} \\
&= -\frac{a^2(Ab - aB)}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{a(2Ab^3 - a(a^2 + 3b^2))}{2b^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
&= -\frac{a^2(Ab - aB)}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{a(2Ab^3 - a(a^2 + 3b^2))}{2b^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
&= -\frac{(a^4 A - 6a^2 Ab^2 + Ab^4 + 4a^3 bB - 4ab^3 B)x}{(a^2 + b^2)^4} - \frac{a^2(Ab - aB)}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))} \\
&= -\frac{(a^4 A - 6a^2 Ab^2 + Ab^4 + 4a^3 bB - 4ab^3 B)x}{(a^2 + b^2)^4} - \frac{(4a^3 Ab - 4aAb^3 - a^2(Ab - aB))}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 6.02, size = 357, normalized size = 1.37

$$\frac{\frac{2Ab+dB}{(a+b\tan(c+dx))^2} + \frac{2B^2\tan(c+dx)}{(a+b\tan(c+dx))^3} + 3bB \left( \frac{\log\left(\frac{-\tan(c+dx)}{(-a+b)}\right) + \log\left(\frac{\tan(c+dx)}{(a+b)}\right) + \frac{4\left((a^2-2b^2)\log(a+b\tan(c+dx)) - \frac{(c^2+d^2)(a^2+b^2+2ab\tan(c+dx))}{(a^2+b^2)^2}\right)}{(a^2+b^2)^2}\right)}{6b^2d} + b(AB - aB) \left( \frac{-3\log\left(\frac{-\tan(c+dx)}{(a+b)}\right) + 3\log\left(\frac{\tan(c+dx)}{(a-b)}\right) + \frac{2b\left((2a^2-d^2)\log(a+b\tan(c+dx)) - \frac{(c^2+d^2)(2a^4+2a^2b^2+2^4ab\tan(c+dx) + (b^2d^2-3a^4)\tan^2(c+dx))}{(a^2+b^2)^2}\right)}{(a^2+b^2)^2}\right)}{6b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4, x]

[Out] 
$$-1/6*((2*A*b + a*B)/(a + b*\tan[c + d*x])^3 + (3*b*B*\tan[c + d*x])/(a + b*\tan[c + d*x])^3 + 3*b*B*(\log[I - \tan[c + d*x]]/((-I)*a + b)^3 + \log[I + \tan[c + d*x]]/(I*a + b)^3 + (b*((6*a^2 - 2*b^2)*\log[a + b*\tan[c + d*x]] - ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*\tan[c + d*x]))/(a + b*\tan[c + d*x])^2))/(a^2 + b^2)^3 + b*(A*b - a*B)*((-3*I)*\log[I - \tan[c + d*x]]/(a + I*b)^4 + ((3*I)*\log[I + \tan[c + d*x]]/(a - I*b)^4 + (2*b*(12*a*(a^2 - b^2)*\log[a + b*\tan[c + d*x]] - ((a^2 + b^2)*(13*a^4 + 2*a^2*b^2 + b^4 + 3*a*b*(7*a^2 - b^2)*\tan[c + d*x] + (9*a^2*b^2 - 3*b^4)*\tan[c + d*x]^2))/(a + b*\tan[c + d*x])^3))/(a^2 + b^2)^4)/(b^2*d)$$

**Maple [A]**

time = 0.32, size = 301, normalized size = 1.15

method	result
derivativedivides	$\frac{(4Aa^3b - 4Aab^3 - Ba^4 + 6Ba^2b^2 - Bb^4) \ln(1 + \tan^2(dx+c)) + (-Aa^4 + 6Aa^2b^2 - Ab^4 - 4Ba^3b + 4Bab^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^4} - \frac{3b^2(a^2 + b^2)}{3b^2(a^2 + b^2)}$
default	$\frac{(4Aa^3b - 4Aab^3 - Ba^4 + 6Ba^2b^2 - Bb^4) \ln(1 + \tan^2(dx+c)) + (-Aa^4 + 6Aa^2b^2 - Ab^4 - 4Ba^3b + 4Bab^3) \arctan(\tan(dx+c))}{(a^2 + b^2)^4} - \frac{3b^2(a^2 + b^2)}{3b^2(a^2 + b^2)}$
norman	$-\frac{(8Aa^3b^3 - Ba^6 - 6Ba^4b^2 + 3Ba^2b^4) (\tan^2(dx+c))}{2ad(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)} - \frac{a(a^5A - 3Aa^3b^2 + 3Ba^4b - Ba^2b^3)}{3db(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)} - \frac{(Aa^4 - 6Aa^2b^2 + Ab^4 + 4Ba^3b - 4Bab^3)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)} \frac{1}{(a^2 + b^2)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4, x, method=\_RETURNVERBOSE)

[Out] 
$$1/d*(1/(a^2+b^2)^4*(1/2*(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*\ln(1+\tan(d*x+c)^2)+(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*\arctan(\tan(d*x+c)))-1/3*a^2*(A*b-B*a)/b^2/(a^2+b^2)/(a+b*\tan(d*x+c))^3+(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3/(a+b*\tan(d*x+c))-(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)/(a^2+b^2)^4*\ln(a+b*\tan(d*x+c))+1/2*a*(2*A*b^3-B*a^3-3*B*a*b^2)/(a^2+b^2)^2/b^2/(a+b*\tan(d*x+c))^2)$$

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 526 vs.  $2(253) = 506$ .

time = 0.51, size = 526, normalized size = 2.02

$$\frac{\frac{6(A^2+4B^2b-6A^2b^2-4Bb^3+A^3)(dx+c)}{a^2+4a^2b^2+6a^2b^3+4a^2b^4} - \frac{6(Ba^4-4A^2b-6Bb^2b^2+4Aa^3+2b^3)\log(b\tan(dx+c)+a)}{a^2+4a^2b^2+6a^2b^3+4a^2b^4} + \frac{3(Ba^4-4A^2b-6Bb^2b^2+4Aa^3+2b^3)\log(\tan(dx+c)^2+1)}{a^2+4a^2b^2+6a^2b^3+4a^2b^4} + \frac{Ba^7+2Aa^6b+14Ba^5b^2-20Aa^4b^3-11Ba^3b^4+6(Ba^2b^2-3Aa^2b-3Bab^2+A^3)\tan(dx+c)^2+3(Ba^2b+8Ba^2b^2-14Aa^2b^3-9Ba^2b^4+2Aa^3)\tan(dx+c)}{a^2+4a^2b^2+6a^2b^3+4a^2b^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out] 
$$\frac{-1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*\log(b*\tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (B*a^7 + 2*A*a^6*b + 14*B*a^5*b^2 - 20*A*a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5 + 6*(B*a^3*b^4 - 3*A*a^2*b^5 - 3*B*a*b^6 + A*b^7)*\tan(d*x + c)^2 + 3*(B*a^6*b + 8*B*a^4*b^3 - 14*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*\tan(d*x + c))/(a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8 + (a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*\tan(d*x + c)^3 + 3*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*\tan(d*x + c)^2 + 3*(a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*\tan(d*x + c)))/d$$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 836 vs.  $2(253) = 506$ .

time = 2.28, size = 836, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] 
$$\frac{1/6*(3*B*a^7 - 12*A*a^6*b - 30*B*a^5*b^2 + 30*A*a^4*b^3 + 11*B*a^3*b^4 - 2*A*a^2*b^5 + (B*a^6*b + 2*A*a^5*b^2 + 18*B*a^4*b^3 - 30*A*a^3*b^4 - 27*B*a^2*b^5 + 12*A*a*b^6 - 6*(A*a^4*b^3 + 4*B*a^3*b^4 - 6*A*a^2*b^5 - 4*B*a*b^6 + A*b^7)*d*x)*\tan(d*x + c)^3 - 6*(A*a^7 + 4*B*a^6*b - 6*A*a^5*b^2 - 4*B*a^4*b^3 + A*a^3*b^4)*d*x + 3*(B*a^7 + 2*A*a^6*b + 16*B*a^5*b^2 - 24*A*a^4*b^3 - 23*B*a^3*b^4 + 16*A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 - 6*(A*a^5*b^2 + 4*B*a^4*b^3 - 6*A*a^3*b^4 - 4*B*a^2*b^5 + A*a*b^6)*d*x)*\tan(d*x + c)^2 + 3*(B*a^7 - 4*A*a^6*b - 6*B*a^5*b^2 + 4*A*a^4*b^3 + B*a^3*b^4 + (B*a^4*b^3 - 4*A*a^3*b^4 - 6*B*a^2*b^5 + 4*A*a*b^6 + B*b^7)*\tan(d*x + c)^3 + 3*(B*a^5*b^2 - 4*A*a^4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + B*a*b^6)*\tan(d*x + c)^2 + 3*(B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 4*A*a^3*b^4 + B*a^2*b^5)*\tan(d*x + c))*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) + 3*(2*A$$



$$\begin{aligned} & *a^7 + 9*B*a^6*b - 16*A*a^5*b^2 - 26*B*a^4*b^3 + 24*A*a^3*b^4 + 9*B*a^2*b^5 \\ & - 2*A*a*b^6 - 6*(A*a^6*b + 4*B*a^5*b^2 - 6*A*a^4*b^3 - 4*B*a^3*b^4 + A*a^2 \\ & *b^5)*d*x)*\tan(d*x + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^{11}) \\ & *d*\tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^{10}) \\ & *d*\tan(d*x + c)^2 + 3*(a^{10}*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2 \\ & *b^9)*d*\tan(d*x + c) + (a^{11} + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8) \\ & *d) \end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*4,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(253) = 506.

time = 0.89, size = 632, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(B*a^4*b - 4*A*a^3*b^2 - 6*B*a^2*b^3 + 4*A*a*b^4 + B*b^5)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (11*B*a^4*b^5*\tan(d*x + c)^3 - 44*A*a^3*b^6*\tan(d*x + c)^3 - 66*B*a^2*b^7*\tan(d*x + c)^3 + 44*A*a*b^8*\tan(d*x + c)^3 + 11*B*b^9*\tan(d*x + c)^3 + 39*B*a^5*b^4*\tan(d*x + c)^2 - 150*A*a^4*b^5*\tan(d*x + c)^2 - 210*B*a^3*b^6*\tan(d*x + c)^2 + 120*A*a^2*b^7*\tan(d*x + c)^2 + 15*B*a*b^8*\tan(d*x + c)^2 + 6*A*b^9*\tan(d*x + c)^2 + 3*B*a^8*b*\tan(d*x + c) + 60*B*a^6*b^3*\tan(d*x + c) - 174*A*a^5*b^4*\tan(d*x + c) - 201*B*a^4*b^5*\tan(d*x + c) + 96*A*a^3*b^6*\tan(d*x + c) + 6*B*a^2*b^7*\tan(d*x + c) + 6*A*a*b^8*\tan(d*x + c) + B*a^9 + 2*A*a^8*b + 26*B*a^7*b^2 - 62*A*a^6*b^3 - 63*B*a^5*b^4 + 26*A*a^4*b^5 + 2*A*a^2*b^7)/((a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^{10})*(b*\tan(d*x + c) + a)^3))/d \end{aligned}$$

**Mupad [B]**

time = 7.03, size = 446, normalized size = 1.71

$$\frac{\ln(a + b \tan(c + dx)) \left( \frac{B}{(a^2 + b^2)^2} - \frac{4b(Aa + 2Bb)}{(a^2 + b^2)^3} + \frac{8b^2(Aa + 2Bb)}{(a^2 + b^2)^4} \right) - \frac{\tan(c + dx) (D^2 a^2 b^2 - 3Aa^2 b^2 - 3Bb^2 a^2 + A^2 b^2) + (D^2 a^2 - 2Aa^2 b^2 + 14D^2 a^2 b^2 - 20Aa^2 b^2 - 11D^2 a^2 b^2 + 2Aa^2 b^2) + \tan(c + dx) (D^2 a^2 - 2Aa^2 b^2 - 14Aa^2 b^2 - 3Bb^2 a^2 + 2Aa^2 b^2)}{d^2 (a^2 + 3a^2 b \tan(c + dx) + 3a^2 b^2 \tan^2(c + dx) + b^2 \tan^3(c + dx))} - \frac{\ln(\tan(c + dx) - 1) (A + B)}{2d (a^4 - 4a^2 b - a^2 b^2 + 4a^2 b^2 + b^4)} - \frac{\ln(\tan(c + dx) + 1) (B + A)}{2d (a^4 - a^2 b^2 - 6a^2 b^2 + a^2 b^4 + b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^4,x)

[Out] (log(a + b\*tan(c + d\*x))\*(B/(a^2 + b^2)^2 - (4\*b\*(A\*a + 2\*B\*b))/(a^2 + b^2)^3 + (8\*b^3\*(A\*a + B\*b))/(a^2 + b^2)^4))/d - ((tan(c + d\*x)^2\*(A\*b^5 - 3\*A\*a^2\*b^3 + B\*a^3\*b^2 - 3\*B\*a\*b^4))/(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2) + (a\*(B\*a^6 - 20\*A\*a^3\*b^3 - 11\*B\*a^2\*b^4 + 14\*B\*a^4\*b^2 + 2\*A\*a\*b^5 + 2\*A\*a^5\*b))/(6\*b^2\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)) + (tan(c + d\*x)\*(B\*a^6 - 14\*A\*a^3\*b^3 - 9\*B\*a^2\*b^4 + 8\*B\*a^4\*b^2 + 2\*A\*a\*b^5))/(2\*b\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)))/(d\*(a^3 + b^3\*tan(c + d\*x)^3 + 3\*a\*b^2\*tan(c + d\*x)^2 + 3\*a^2\*b\*tan(c + d\*x))) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i))/(2\*d\*(4\*a\*b^3 - 4\*a^3\*b + a^4\*1i + b^4\*1i - a^2\*b^2\*6i)) - (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(2\*d\*(a\*b^3\*4i - a^3\*b\*4i + a^4 + b^4 - 6\*a^2\*b^2))

$$3.293 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=250

$$\frac{(4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x}{(a^2 + b^2)^4} - \frac{(a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^4 d}$$

[Out]  $(4Aa^3b - 4Aab^3 - Ba^4 + 6Bb^2a^2 - Bb^4)x / (a^2 + b^2)^4 - (Aa^4 - 6Aa^2b^2 + Ab^4 + 4a^3bB - 4ab^3B) \log(a \cos(dx+c) + b \sin(dx+c)) / (a^2 + b^2)^4 d + 1/3a(Ab - Ba) / b / (a^2 + b^2) / d / (a + b \tan(dx+c))^3 + 1/2(Aa^2 - Ab^2 + 2Bab) / (a^2 + b^2)^2 / d / (a + b \tan(dx+c))^2 + (Aa^3 - 3Aab^2 + 3Bb^2a - Bb^3) / (a^2 + b^2)^3 / d / (a + b \tan(dx+c))$

**Rubi [A]**

time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3672, 3610, 3612, 3611}

$$\frac{a(Ab - aB)}{3bd(a^2 + b^2)(a + b \tan(c + dx))^3} + \frac{a^2A + 2abB - Ab^2}{2d(a^2 + b^2)^2(a + b \tan(c + dx))^2} + \frac{a^3A + 3a^2bB - 3aAb^2 - b^2B}{d(a^2 + b^2)^3(a + b \tan(c + dx))} - \frac{(a^4A + 4a^3bB - 6a^2Ab^2 - 4ab^3B + Ab^4) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^4} + \frac{x(a^4(-B) + 4a^3Ab + 6a^2b^2B - 4aAb^3 - b^4B)}{(a^2 + b^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4,x]

[Out]  $((4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x) / (a^2 + b^2)^4 - ((a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B) \text{Log}[a \text{Cos}[c + d*x] + b \text{Sin}[c + d*x]]) / ((a^2 + b^2)^4 d) + (a(Ab - aB)) / (3b(a^2 + b^2)d(a + b \text{Tan}[c + d*x])^3) + (a^2A - Ab^2 + 2aAb) / (2(a^2 + b^2)^2 d(a + b \text{Tan}[c + d*x])^2) + (a^3A - 3aAb^2 + 3a^2bB - b^3B) / ((a^2 + b^2)^3 d(a + b \text{Tan}[c + d*x]))$

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1) / (f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3611**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) / ((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

**Rule 3612**

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

### Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4} dx &= \frac{a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^3} dx}{a^2+b^2} \\
&= \frac{a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{a^2A-Ab^2+2abB}{2(a^2+b^2)^2d(a+b\tan(c+dx))} \\
&= \frac{a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{a^2A-Ab^2+2abB}{2(a^2+b^2)^2d(a+b\tan(c+dx))} \\
&= \frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x}{(a^2+b^2)^4} + \frac{a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))} \\
&= \frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x}{(a^2+b^2)^4} - \frac{(a^4A-6a^2Ab^2+Ab^3)}{(a^2+b^2)^4}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.85, size = 248, normalized size = 0.99

$$\frac{\frac{3(A+iB)\log(i-\tan(c+dx))}{(a+ib)^4} + \frac{3(A-iB)\log(i+\tan(c+dx))}{(a-ib)^4} - \frac{6(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^2B)\log(a+b\tan(c+dx))}{(a^2+b^2)^4} + \frac{2a(Ab-aB)}{b(a^2+b^2)(a+b\tan(c+dx))^3} + \frac{3(a^2A-Ab^2+2abB)}{(a^2+b^2)^2(a+b\tan(c+dx))^2} + \frac{6(a^3A-3aAb^2+3a^2bB-b^3B)}{(a^2+b^2)^3(a+b\tan(c+dx))}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4, x]
```

```
[Out] ((3*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^4 + (3*(A - I*B)*Log[I + Tan
[c + d*x]])/(a - I*b)^4 - (6*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a
```

$$*b^3*B)*\text{Log}[a + b*\text{Tan}[c + d*x]]/(a^2 + b^2)^4 + (2*a*(A*b - a*B))/(b*(a^2 + b^2)*(a + b*\text{Tan}[c + d*x])^3) + (3*(a^2*A - A*b^2 + 2*a*b*B))/((a^2 + b^2)^2*(a + b*\text{Tan}[c + d*x])^2) + (6*(a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B))/((a^2 + b^2)^3*(a + b*\text{Tan}[c + d*x]))/(6*d)$$

Maple [A]

time = 0.35, size = 287, normalized size = 1.15

method	result
derivativedivides	$\frac{(A a^4 - 6A a^2 b^2 + A b^4 + 4B a^3 b - 4B a b^3) \ln(1 + \tan^2(dx+c))}{2} + \frac{(4A a^3 b - 4A a b^3 - B a^4 + 6B a^2 b^2 - B b^4) \arctan(\tan(dx+c))}{(a^2+b^2)^4} + \frac{1}{3(a^2+b^2)}$
default	$\frac{(A a^4 - 6A a^2 b^2 + A b^4 + 4B a^3 b - 4B a b^3) \ln(1 + \tan^2(dx+c))}{2} + \frac{(4A a^3 b - 4A a b^3 - B a^4 + 6B a^2 b^2 - B b^4) \arctan(\tan(dx+c))}{(a^2+b^2)^4} + \frac{1}{3(a^2+b^2)}$
norman	$\frac{(4A a^3 b - 4A a b^3 - B a^4 + 6B a^2 b^2 - B b^4) a^3 x}{(a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)(a^2 + b^2)} + \frac{b^3 (4A a^3 b - 4A a b^3 - B a^4 + 6B a^2 b^2 - B b^4) x (\tan^3(dx+c))}{(a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)(a^2 + b^2)} + \frac{a (9A a^4 b^2 - 8a^2 A b^4 - A b^6)}{6b^2 (a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x,method=\_RETURNVERBOSE)

[Out] 1/d\*(1/(a^2+b^2)^4\*(1/2\*(A\*a^4-6\*A\*a^2\*b^2+A\*b^4+4\*B\*a^3\*b-4\*B\*a\*b^3)\*ln(1+tan(d\*x+c)^2)+(4\*A\*a^3\*b-4\*A\*a\*b^3-B\*a^4+6\*B\*a^2\*b^2-B\*b^4)\*arctan(tan(d\*x+c)))+1/3\*a\*(A\*b-B\*a)/(a^2+b^2)/b/(a+b\*tan(d\*x+c))^3+1/2\*(A\*a^2-A\*b^2+2\*B\*a\*b)/(a^2+b^2)^2/(a+b\*tan(d\*x+c))^2+(A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)/(a^2+b^2)^3/(a+b\*tan(d\*x+c))-(A\*a^4-6\*A\*a^2\*b^2+A\*b^4+4\*B\*a^3\*b-4\*B\*a\*b^3)/(a^2+b^2)^4\*ln(a+b\*tan(d\*x+c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(245) = 490.

time = 0.56, size = 523, normalized size = 2.09

$$\frac{\frac{5(Ba^4 - 4Aa^3b - 6Ba^2b^2 + 4Aab^3 + Bb^4)(dx+c)}{a^6 + 4a^4b^2 + 6a^2b^4 + 4a^2b^4 + b^6} + \frac{6(Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Bab^3 + Ab^4) \log(\tan(dx+c)+a)}{a^6 + 4a^4b^2 + 6a^2b^4 + 4a^2b^4 + b^6} - \frac{3(Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Bab^3 + Ab^4) \log(\tan(dx+c)^2+1)}{a^6 + 4a^4b^2 + 6a^2b^4 + 4a^2b^4 + b^6} + \frac{2Ba^6 - 11Aa^5b - 20Ba^4b^2 + 14Aa^3b^3 + 2Ba^2b^4 + Aab^5 - 6(Aa^3b^3 + 3Ba^2b^4 - 3Aab^5 - Bb^6) \tan(dx+c) - 3(5Aa^5b^2 + 14Ba^4b^3 - 12Aa^3b^4 - 2Bab^5 - Ab^6) \tan(dx+c)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^6b^3 + 3a^4b^3 + a^2b^5 + b^7) \tan(dx+c)^2 + 3(a^7b^3 + 3a^5b^3 + 3a^3b^5 + a^5b^7) \tan(dx+c)}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out] -1/6\*(6\*(B\*a^4 - 4\*A\*a^3\*b - 6\*B\*a^2\*b^2 + 4\*A\*a\*b^3 + B\*b^4)\*(d\*x + c)/(a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8) + 6\*(A\*a^4 + 4\*B\*a^3\*b - 6\*A\*a^2\*b^2 - 4\*B\*a\*b^3 + A\*b^4)\*log(b\*tan(d\*x + c) + a)/(a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8) - 3\*(A\*a^4 + 4\*B\*a^3\*b - 6\*A\*a^2\*b^2 - 4\*B\*a\*b^3 + A\*b^4)\*log(tan(d\*x + c)^2 + 1)/(a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8) + (2\*B\*a^6 - 11\*A\*a^5\*b - 20\*B\*a^4\*b^2 + 14\*A\*a^3\*b^3 + 2\*B\*a^2\*b^4 +

$$\frac{Aab^5 - 6(Aa^3b^3 + 3Ba^2b^4 - 3Aab^5 - Bb^6)\tan(dx + c)^2 - 3(5Aa^4b^2 + 14Ba^3b^3 - 12Aa^2b^4 - 2Bab^5 - Ab^6)\tan(dx + c)}{(a^9b + 3a^7b^3 + 3a^5b^5 + a^3b^7 + (a^6b^4 + 3a^4b^6 + 3a^2b^8 + b^{10})\tan(dx + c)^3 + 3(a^7b^3 + 3a^5b^5 + 3a^3b^7 + ab^9)\tan(dx + c)^2 + 3(a^8b^2 + 3a^6b^4 + 3a^4b^6 + a^2b^8)\tan(dx + c))}d$$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 838 vs.  $2(245) = 490$ .

time = 1.46, size = 838, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(dx+c)*(A+B*tan(dx+c))/(a+b*tan(dx+c))^4,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/6*(12Ba^6b - 27Aa^5b^2 - 30Ba^4b^3 + 18Aa^3b^4 + 2Ba^2b^5 \\ & + Aab^6 - (2Ba^5b^2 - 11Aa^4b^3 - 30Ba^3b^4 + 30Aa^2b^5 + 12 \\ & *Bab^6 - 3Ab^7 - 6*(Ba^4b^3 - 4Aa^3b^4 - 6Ba^2b^5 + 4Aab^6 + \\ & Bb^7)*dx)*\tan(dx + c)^3 + 6*(Ba^7 - 4Aa^6b - 6Ba^5b^2 + 4Aa^4b^3 \\ & + Ba^3b^4)*dx - 3*(2Ba^6b - 9Aa^5b^2 - 24Ba^4b^3 + 26Aa^3b^4 \\ & + 16Ba^2b^5 - 9Aab^6 - 2Bb^7 - 6*(Ba^5b^2 - 4Aa^4b^3 - 6Ba^3b^4 \\ & + 4Aa^2b^5 + Bab^6)*dx)*\tan(dx + c)^2 + 3*(Aa^7 + 4Ba^6b \\ & - 6Aa^5b^2 - 4Ba^4b^3 + Aa^3b^4 + (Aa^4b^3 + 4Ba^3b^4 - 6Aa^2b^5 \\ & - 4Bab^6 + Ab^7)*\tan(dx + c)^3 + 3*(Aa^5b^2 + 4Ba^4b^3 - \\ & 6Aa^3b^4 - 4Ba^2b^5 + Aab^6)*\tan(dx + c)^2 + 3*(Aa^6b + 4Ba^5b^2 \\ & - 6Aa^4b^3 - 4Ba^3b^4 + Aa^2b^5)*\tan(dx + c))*\log((b^2*\tan(dx \\ & + c)^2 + 2ab*\tan(dx + c) + a^2)/(\tan(dx + c)^2 + 1)) - 3*(2Ba^7 - 6 \\ & *Aa^6b - 16Ba^5b^2 + 23Aa^4b^3 + 24Ba^3b^4 - 16Aa^2b^5 - 2Bab^6 \\ & - Ab^7 - 6*(Ba^6b - 4Aa^5b^2 - 6Ba^4b^3 + 4Aa^3b^4 + Ba^2b^5)*dx)*\tan(dx + c) \\ & /((a^8b^3 + 4a^6b^5 + 6a^4b^7 + 4a^2b^9 + b^{11})*d*\tan(dx + c)^3 + 3*(a^9b^2 + 4a^7b^4 + 6a^5b^6 + 4a^3b^8 + a \\ & b^{10})*d*\tan(dx + c)^2 + 3*(a^{10}b + 4a^8b^3 + 6a^6b^5 + 4a^4b^7 + a^2b^9)*d*\tan(dx + c) + (a^{11} + 4a^9b^2 + 6a^7b^4 + 4a^5b^6 + a^3b^8) \\ & )*d) \end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(dx+c)*(A+B*tan(dx+c))/(a+b*tan(dx+c))**4,x)`



### 3.294 $\int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^4} dx$

**Optimal.** Leaf size=247

$$\frac{(a^4 A - 6a^2 A b^2 + A b^4 + 4a^3 b B - 4a b^3 B) x}{(a^2 + b^2)^4} + \frac{(4a^3 A b - 4a A b^3 - a^4 B + 6a^2 b^2 B - b^4 B) \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)^4 d}$$

[Out]  $(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*x/(a^2+b^2)^4+(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^4/d+1/3*(-A*b+B*a)/(a^2+b^2)/d/(a+b*\tan(d*x+c))^3+1/2*(-2*A*a*b+B*a^2-B*b^2)/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^2+(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)/(a^2+b^2)^3/d/(a+b*\tan(d*x+c))$

**Rubi [A]**

time = 0.27, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3610, 3612, 3611}

$$\frac{Ab - aB}{3d(a^2 + b^2)(a + b \tan(c + dx))^3} - \frac{a^2(-B) + 2aAb + b^2B}{2d(a^2 + b^2)^2(a + b \tan(c + dx))^2} - \frac{a^3(-B) + 3a^2Ab + 3ab^2B - Ab^3}{d(a^2 + b^2)^3(a + b \tan(c + dx))} + \frac{(a^4(-B) + 4a^3Ab + 6a^2b^2B - 4aAb^3 - b^4B) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^4} + \frac{x(a^4A + 4a^3bB - 6a^2Ab^2 - 4ab^3B + Ab^4)}{(a^2 + b^2)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[c + d*x])/(a + b*\text{Tan}[c + d*x])^4, x]$

[Out]  $((a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*x)/(a^2 + b^2)^4 + ((4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)^4*d) - (A*b - a*B)/(3*(a^2 + b^2)*d*(a + b*\text{Tan}[c + d*x])^3) - (2*a*A*b - a^2*B + b^2*B)/(2*(a^2 + b^2)^2*d*(a + b*\text{Tan}[c + d*x])^2) - (3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)/((a^2 + b^2)^3*d*(a + b*\text{Tan}[c + d*x]))$

**Rule 3610**

$\text{Int}[(c_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] :> \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

**Rule 3611**

$\text{Int}[(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] :> \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

**Rule 3612**



```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^4} dx &= -\frac{Ab - aB}{3(a^2 + b^2) d(a + b \tan(c + dx))^3} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^3} dx}{a^2 + b^2} \\ &= -\frac{Ab - aB}{3(a^2 + b^2) d(a + b \tan(c + dx))^3} - \frac{2aAb - a^2B + b^2B}{2(a^2 + b^2)^2 d(a + b \tan(c + dx))^2} + \frac{\int \frac{a^2}{(a + b \tan(c + dx))^2} dx}{2(a^2 + b^2)^2 d} \\ &= -\frac{Ab - aB}{3(a^2 + b^2) d(a + b \tan(c + dx))^3} - \frac{2aAb - a^2B + b^2B}{2(a^2 + b^2)^2 d(a + b \tan(c + dx))^2} - \frac{3a^2}{(a^2 + b^2)^2 d} \\ &= \frac{(a^4 A - 6a^2 Ab^2 + Ab^4 + 4a^3 bB - 4ab^3 B) x}{(a^2 + b^2)^4} - \frac{Ab - aB}{3(a^2 + b^2) d(a + b \tan(c + dx))^3} \\ &= \frac{(a^4 A - 6a^2 Ab^2 + Ab^4 + 4a^3 bB - 4ab^3 B) x}{(a^2 + b^2)^4} + \frac{(4a^3 Ab - 4aAb^3 - a^4 B + 6a^2 b^2 B)}{(a^2 + b^2)^4} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 6.18, size = 327, normalized size = 1.32

$$\frac{(Ab - aB) \left( \frac{3i \log(-\tan(c+dx))}{(a+b)^2} - \frac{3i \log(i+\tan(c+dx))}{(a-b)^2} - \frac{24i(a-3i)(a+b) \log(a+b \tan(c+dx))}{(a^2+b^2)^2} + \frac{2b}{(a^2+b^2)(a+b \tan(c+dx))^2} + \frac{6ab}{(a^2+b^2)^2(a+b \tan(c+dx))^2} + \frac{6b(3a^2-b^2)}{(a^2+b^2)^2(a+b \tan(c+dx))^2} \right) - B \left( \frac{\log(i-\tan(c+dx))}{(a-b)^2} - \frac{\log(i+\tan(c+dx))}{(a+b)^2} - \frac{24(3a^2-b^2) \log(a+b \tan(c+dx))}{(a^2+b^2)^2} + \frac{b}{(a^2+b^2)(a+b \tan(c+dx))^2} + \frac{6ab}{(a^2+b^2)^2(a+b \tan(c+dx))^2} \right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^4, x]
```

```
[Out] -1/6*((A*b - a*B)*(((3*I)*Log[I - Tan[c + d*x]])/(a + I*b)^4 - ((3*I)*Log[I
+ Tan[c + d*x]])/(a - I*b)^4 - (24*a*(a - b)*b*(a + b)*Log[a + b*Tan[c + d
*x]])/(a^2 + b^2)^4 + (2*b)/((a^2 + b^2)*(a + b*Tan[c + d*x])^3) + (6*a*b)/
((a^2 + b^2)^2*(a + b*Tan[c + d*x])^2) + (6*b*(3*a^2 - b^2))/((a^2 + b^2)^3
*(a + b*Tan[c + d*x])))/(b*d) - (B*(Log[I - Tan[c + d*x]])/(I*a - b)^3 - Lo
g[I + Tan[c + d*x]])/(I*a + b)^3 - (2*b*(3*a^2 - b^2)*Log[a + b*Tan[c + d*x]
])/(a^2 + b^2)^3 + b/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (4*a*b)/((a^2 +
b^2)^2*(a + b*Tan[c + d*x])))/(2*b*d)
```

### Maple [A]

time = 0.32, size = 284, normalized size = 1.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x,method=\_RETURNVERBOSE)

[Out]  $1/d*(1/(a^2+b^2)^4*(1/2*(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*\ln(1+\tan(d*x+c)^2)+(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*\arctan(\tan(d*x+c)))-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3/(a+b*\tan(d*x+c))+(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)/(a^2+b^2)^4*\ln(a+b*\tan(d*x+c))-1/3*(A*b-B*a)/(a^2+b^2)/(a+b*\tan(d*x+c))^3-1/2*(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/(a+b*\tan(d*x+c))^2)$

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs.  $2(239) = 478$ .

time = 0.57, size = 514, normalized size = 2.08

$$\frac{6(Aa^4+4Ba^3b-6Aa^2b^2+4Bab^3+Ab^4)(dx+c) - 6(Ba^4-4Aa^3b+4Aa^2b^2+4Ab^3+Ba^4)\log(\tan(dx+c)+a) + \frac{3(Ba^4-4Aa^3b-6Ba^2b^2+4Aab^3+Ba^4)\log(\tan(dx+c)^2+1)}{a^2+4a^2b^2+6a^2b^4+4a^2b^6+b^8} + \frac{11Ba^5-26Aa^4b-14Ba^3b^2-4Aa^2b^3-Ba^2b^4-2Aab^4-2Aa^2b^5+6(Ba^4b^2-3Aa^3b^2-3Ba^4+Ab^4)\tan(dx+c)^2+3(5Ba^4b-14Aa^3b^2-12Ba^2b^3+2Aab^4+2Aa^2b^5)\tan(dx+c)}{a^2+4a^2b^2+6a^2b^4+4a^2b^6+b^8}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out]  $1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*\log(b*\tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (11*B*a^5 - 26*A*a^4*b - 14*B*a^3*b^2 - 4*A*a^2*b^3 - B*a*b^4 - 2*A*b^5 + 6*(B*a^3*b^2 - 3*A*a^2*b^3 - 3*B*a*b^4 + A*b^5))*\tan(d*x + c)^2 + 3*(5*B*a^4*b - 14*A*a^3*b^2 - 12*B*a^2*b^3 + 2*A*a*b^4 - B*b^5))*\tan(d*x + c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9))*\tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8))*\tan(d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7))*\tan(d*x + c)))/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs.  $2(239) = 478$ .

time = 1.86, size = 815, normalized size = 3.30

$$\frac{1}{6d} \left( (27B^2a^5b^2 - 48A^2a^4b^3 - 18B^2a^3b^4 - 6A^2a^2b^5 - B^2a^2b^6 - 2A^2a^2b^7 - (11B^2a^4b^3 - 26A^2a^3b^4 - 30B^2a^2b^5 + 18A^2a^2b^6 + 3B^2a^2b^7 - 6(A^2a^4b^3 + 4B^2a^3b^4 - 6A^2a^2b^5 - 4B^2a^2b^6 + A^2a^2b^7)*d*x)*\tan(d*x + c)^3 + 6(A^2a^7 + 4B^2a^6b - 6A^2a^5b^2 - 4B^2a^4b^3 + A^2a^3b^4)*d*x - 3(9B^2a^5b^2 - 20A^2a^4b^3 - 26B^2a^3b^4 + 22A^2a^2b^5 + 9B^2a^2b^6 - 2A^2a^2b^7 - 6(A^2a^5b^2 + 4B^2a^4b^3 - 6A^2a^3b^4 - 4B^2a^2b^5 + A^2a^2b^6)*d*x)*\tan(d*x + c)^2 - 3(B^2a^7 - 4A^2a^6b - 6B^2a^5b^2 + 4A^2a^4b^3) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out]  $1/6*(27*B^2*a^5*b^2 - 48*A^2*a^4*b^3 - 18*B^2*a^3*b^4 - 6*A^2*a^2*b^5 - B^2*a^2*b^6 - 2*A^2*a^2*b^7 - (11*B^2*a^4*b^3 - 26*A^2*a^3*b^4 - 30*B^2*a^2*b^5 + 18*A^2*a^2*b^6 + 3*B^2*a^2*b^7 - 6*(A^2*a^4*b^3 + 4*B^2*a^3*b^4 - 6*A^2*a^2*b^5 - 4*B^2*a^2*b^6 + A^2*a^2*b^7)*d*x)*\tan(d*x + c)^3 + 6*(A^2*a^7 + 4*B^2*a^6*b - 6*A^2*a^5*b^2 - 4*B^2*a^4*b^3 + A^2*a^3*b^4)*d*x - 3*(9*B^2*a^5*b^2 - 20*A^2*a^4*b^3 - 26*B^2*a^3*b^4 + 22*A^2*a^2*b^5 + 9*B^2*a^2*b^6 - 2*A^2*a^2*b^7 - 6*(A^2*a^5*b^2 + 4*B^2*a^4*b^3 - 6*A^2*a^3*b^4 - 4*B^2*a^2*b^5 + A^2*a^2*b^6)*d*x)*\tan(d*x + c)^2 - 3*(B^2*a^7 - 4*A^2*a^6*b - 6*B^2*a^5*b^2 + 4*A^2*a^4*b^3$

$$\begin{aligned}
& + B*a^3*b^4 + (B*a^4*b^3 - 4*A*a^3*b^4 - 6*B*a^2*b^5 + 4*A*a*b^6 + B*b^7)* \\
& \tan(d*x + c)^3 + 3*(B*a^5*b^2 - 4*A*a^4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + B \\
& *a*b^6)*\tan(d*x + c)^2 + 3*(B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 4*A*a^3*b \\
& ^4 + B*a^2*b^5)*\tan(d*x + c)*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) \\
& + a^2)/(\tan(d*x + c)^2 + 1)) - 3*(6*B*a^6*b - 12*A*a^5*b^2 - 23*B*a^4*b^3 + \\
& 30*A*a^3*b^4 + 16*B*a^2*b^5 - 2*A*a*b^6 + B*b^7 - 6*(A*a^6*b + 4*B*a^5*b^2 \\
& - 6*A*a^4*b^3 - 4*B*a^3*b^4 + A*a^2*b^5)*d*x)*\tan(d*x + c))/((a^8*b^3 + 4* \\
& a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*\tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a \\
& ^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*\tan(d*x + c)^2 + 3*(a^10*b + 4*a \\
& ^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*d*\tan(d*x + c) + (a^11 + 4*a^9*b^2 \\
& + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*d)
\end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*4,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(239) = 478.

time = 0.75, size = 630, normalized size = 2.55

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="giac")

$$\begin{aligned}
& [Out] \frac{1}{6}*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8 \\
& + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2 \\
& *b^2 + 4*A*a*b^3 + B*b^4)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4 \\
& *b^4 + 4*a^2*b^6 + b^8) - 6*(B*a^4*b - 4*A*a^3*b^2 - 6*B*a^2*b^3 + 4*A*a*b^4 \\
& + B*b^5)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4* \\
& a^2*b^7 + b^9) + (11*B*a^4*b^3*\tan(d*x + c)^3 - 44*A*a^3*b^4*\tan(d*x + c)^3 \\
& - 66*B*a^2*b^5*\tan(d*x + c)^3 + 44*A*a*b^6*\tan(d*x + c)^3 + 11*B*b^7*\tan(d \\
& *x + c)^3 + 39*B*a^5*b^2*\tan(d*x + c)^2 - 150*A*a^4*b^3*\tan(d*x + c)^2 - 21 \\
& 0*B*a^3*b^4*\tan(d*x + c)^2 + 120*A*a^2*b^5*\tan(d*x + c)^2 + 15*B*a*b^6*\tan( \\
& d*x + c)^2 + 6*A*b^7*\tan(d*x + c)^2 + 48*B*a^6*b*\tan(d*x + c) - 174*A*a^5*b \\
& ^2*\tan(d*x + c) - 219*B*a^4*b^3*\tan(d*x + c) + 96*A*a^3*b^4*\tan(d*x + c) - \\
& 6*B*a^2*b^5*\tan(d*x + c) + 6*A*a*b^6*\tan(d*x + c) - 3*B*b^7*\tan(d*x + c) + \\
& 22*B*a^7 - 70*A*a^6*b - 69*B*a^5*b^2 + 14*A*a^4*b^3 - 4*B*a^3*b^4 - 6*A*a^2
\end{aligned}$$

$$\frac{b^5 - B*a*b^6 - 2*A*b^7}{(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) * (b*\tan(dx + c) + a)^3} / d$$

**Mupad [B]**

time = 6.82, size = 442, normalized size = 1.79

$$\frac{-11B^2c^2 + 26A^2b^2 + 14B^2c^2 + 4A^2b^2 + 8A^2b^2 + 2A^2b^2 + \tan(c+dx)(-5B^2a^5 + 14A^2a^2b^2 + 12B^2a^2b^2 - 2A^2b^4 + B^2b^4) - \tan(c+dx)^2(B^2a^2b^2 - 3A^2a^2b^2 - 3B^2a^2b^2 + A^2b^2)}{6(a^2 + 3a^2b^2 + 3a^2b^2 + 3a^2b^2)} \frac{\ln(a + b \tan(c + dx)) \left( \frac{B}{a^2 + b^2} - \frac{4b(A + 2Bb)}{(a^2 + b^2)^2} + \frac{8b^2(A + Bb)}{(a^2 + b^2)^3} \right)}{d(a^3 + 3a^2b \tan(c + dx) + 3ab^2 \tan^2(c + dx) + b^3 \tan^3(c + dx))} + \frac{\ln(\tan(c + dx) - 1)(A + Bb)}{2d(a^2 - 4a^2b - a^2b^2 + 4ab^2 + b^3)} + \frac{\ln(\tan(c + dx) + 1)(B + Ab)}{2d(a^2 - a^2b - 6a^2b^2 + a^2b^3 + b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^4,x)

[Out] (log(tan(c + d\*x) - 1i)\*(A + B\*1i))/(2\*d\*(4\*a\*b^3 - 4\*a^3\*b + a^4\*1i + b^4\*1i - a^2\*b^2\*6i)) - (log(a + b\*tan(c + d\*x))\*(B/(a^2 + b^2)^2 - (4\*b\*(A\*a + 2\*B\*b))/(a^2 + b^2)^3 + (8\*b^3\*(A\*a + B\*b))/(a^2 + b^2)^4))/d - ((2\*A\*b^5 - 11\*B\*a^5 + 4\*A\*a^2\*b^3 + 14\*B\*a^3\*b^2 + 26\*A\*a^4\*b + B\*a\*b^4)/(6\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)) + (tan(c + d\*x)\*(B\*b^5 + 14\*A\*a^3\*b^2 + 12\*B\*a^2\*b^3 - 2\*A\*a\*b^4 - 5\*B\*a^4\*b))/(2\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)) - (tan(c + d\*x)^2\*(A\*b^5 - 3\*A\*a^2\*b^3 + B\*a^3\*b^2 - 3\*B\*a\*b^4))/(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2))/(d\*(a^3 + b^3\*tan(c + d\*x)^3 + 3\*a\*b^2\*tan(c + d\*x)^2 + 3\*a^2\*b\*tan(c + d\*x))) + (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(2\*d\*(a\*b^3\*4i - a^3\*b\*4i + a^4 + b^4 - 6\*a^2\*b^2))

$$3.295 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=302

$$-\frac{(4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x}{(a^2 + b^2)^4} + \frac{A \log(\sin(c + dx))}{a^4d} - \frac{b(10a^6Ab + 5a^4Ab^3 + 4a^2Ab^5 + Ab^7 - 4a^4b^2B - 4a^2b^4B + b^6B)}{a^4d}$$

[Out]  $-(4Aa^3b - 4Aab^3 - B a^4 + 6B a^2 b^2 - B b^4) x / (a^2 + b^2)^4 + A \ln(\sin(dx + c)) / a^4 d - b(10Aa^6b + 5Aa^4b^3 + 4Aa^2b^5 + Ab^7 - 4a^4b^2B - 4a^2b^4B + b^6B) / a^4 d + (A \cos(dx + c) + b \sin(dx + c)) / a^4 (a^2 + b^2)^4 d + 1/3 b (A b - B a) / a (a^2 + b^2) d / (a + b \tan(dx + c))^3 + 1/2 b (3A a^2 b + A b^3 - 2B a^3) / a^2 (a^2 + b^2)^2 d / (a + b \tan(dx + c))^2 + b(6A a^4 b + 3A a^2 b^3 + A b^5 - 3B a^5 + B a^3 b^2) / a^3 (a^2 + b^2)^3 d / (a + b \tan(dx + c))$

**Rubi** [A]

time = 0.61, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{A \log(\sin(c + dx))}{a^4 d} + \frac{b(Ab - aB)}{3ad(a^2 + b^2)(a + b \tan(c + dx))^2} + \frac{b(-2a^2B + 3a^2Ab + Ab^3)}{2a^2d(a^2 + b^2)^2(a + b \tan(c + dx))^2} - \frac{a(a^4(-B) + 4a^3Ab + 6a^2b^2B - 4aAb^3 - b^4B)}{(a^2 + b^2)^4} + \frac{b(-3a^2B + 6a^4Ab + a^2b^2B + 3a^2Ab^3 + Ab^5)}{a^4d(a^2 + b^2)^3(a + b \tan(c + dx))} - \frac{b(-4a^7B + 10a^6Ab + 4a^5b^2B + 5a^4b^3 + 4a^3b^4 + Ab^7) \log(a \cos(c + dx) + b \sin(c + dx))}{a^4d(a^2 + b^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4, x]

[Out]  $-(((4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x) / (a^2 + b^2)^4) + (A \log[\sin[c + d*x]]) / (a^4 d) - (b(10a^6Ab + 5a^4Ab^3 + 4a^2Ab^5 + Ab^7 - 4a^4b^2B + 4a^2b^4B) * \log[a \cos[c + d*x] + b \sin[c + d*x]]) / (a^4 (a^2 + b^2)^4 d) + (b(Ab - aB)) / (3a(a^2 + b^2) * d * (a + b \tan[c + d*x]))^3 + (b(3a^2Ab + Ab^3 - 2a^3B)) / (2a^2(a^2 + b^2)^2 * d * (a + b \tan[c + d*x]))^2 + (b(6a^4Ab + 3a^2Ab^3 + Ab^5 - 3a^5B + a^3b^2B)) / (a^3(a^2 + b^2)^3 * d * (a + b \tan[c + d*x]))$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(c/(b\*f))\*Log[RemoveContent[a\*cos[e + f\*x] + b\*sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3690

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] := Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps

$$\int \frac{\cot(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^4} dx = \frac{b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{\int \frac{\cot(c+dx)(3A(a^2+b^2)-3a(Ab-aB))}{(a+b \tan(c+dx))^4} dx}{3a(a^2+b^2)d(a+b \tan(c+dx))^3}$$

$$= \frac{b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{b(3a^2Ab + Ab^3 - 2a^3B)}{2a^2(a^2 + b^2)^2d(a + b \tan(c + dx))^3}$$

$$= \frac{b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^3} + \frac{b(3a^2Ab + Ab^3 - 2a^3B)}{2a^2(a^2 + b^2)^2d(a + b \tan(c + dx))^3}$$

$$= -\frac{(4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x}{(a^2 + b^2)^4} + \frac{b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^3}$$

$$= -\frac{(4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x}{(a^2 + b^2)^4} + \frac{A \log(\sin(c + dx))}{a^4d}$$

**Mathematica [C]** Result contains complex when optimal does not.  
 time = 2.10, size = 308, normalized size = 1.02

$$\frac{3(-a^4(a-b)^4(A+b) \log(-\tan(c+dx)) + 2A(a^2+b^2)^4 \log(\tan(c+dx)) - a^4(a+b)^4(A-b) \log(1+\tan(c+dx)) - 2b(10a^6Ab + 5a^4Ab^3 + 4a^2Ab^5 + Ab^7 - 4a^7B + 4a^5b^2B) \log(a+b \tan(c+dx)))}{a^2(a^2+b^2)^3} + \frac{2ab(a^2+b^2)(Ab-aB)}{(a+b \tan(c+dx))^3} + \frac{3b(3a^2Ab + Ab^3 - 2a^3B)}{(a+b \tan(c+dx))^2} + \frac{6b(6a^4Ab + 3a^2Ab^3 + Ab^5 - 3a^5B + a^3b^2B)}{a(a^2+b^2)(a+b \tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
[Out] ((3*(-(a^4*(a - I*b)^4*(A + I*B)*Log[I - Tan[c + d*x]]) + 2*A*(a^2 + b^2)^4 *Log[Tan[c + d*x]] - a^4*(a + I*b)^4*(A - I*B)*Log[I + Tan[c + d*x]] - 2*b*(10*a^6*A*b + 5*a^4*A*b^3 + 4*a^2*A*b^5 + A*b^7 - 4*a^7*B + 4*a^5*b^2*B)*Log[a + b*Tan[c + d*x]]))/(a^2*(a^2 + b^2)^2) + (2*a*b*(a^2 + b^2)*(A*b - a*B))/(a + b*Tan[c + d*x])^3 + (3*b*(3*a^2*A*b + A*b^3 - 2*a^3*B))/(a + b*Tan[c + d*x])^2 + (6*b*(6*a^4*A*b + 3*a^2*A*b^3 + A*b^5 - 3*a^5*B + a^3*b^2*B))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x]))/(6*a^2*(a^2 + b^2)^2*d)
```

**Maple [A]**  
 time = 0.63, size = 335, normalized size = 1.11

method	result
derivativedivides	$\frac{A \ln(\tan(dx+c))}{a^4} + \frac{(-A a^4 + 6A a^2 b^2 - A b^4 - 4B a^3 b + 4B a b^3) \ln(1 + \tan^2(dx+c))}{2(a^2 + b^2)^4} + \frac{(-4A a^3 b + 4A a b^3 + B a^4 - 6B a^2 b^2 + B b^4) \arctan\left(\frac{a + b \tan(dx+c)}{a}\right)}{(a^2 + b^2)^4}$
default	$\frac{A \ln(\tan(dx+c))}{a^4} + \frac{(-A a^4 + 6A a^2 b^2 - A b^4 - 4B a^3 b + 4B a b^3) \ln(1 + \tan^2(dx+c))}{2(a^2 + b^2)^4} + \frac{(-4A a^3 b + 4A a b^3 + B a^4 - 6B a^2 b^2 + B b^4) \arctan\left(\frac{a + b \tan(dx+c)}{a}\right)}{(a^2 + b^2)^4}$

norman	$\frac{(4Aa^3b - 4Aa^2b^3 - Ba^4 + 6Ba^2b^2 - Bb^4)a^3x - b^3(4Aa^3b - 4Aa^2b^3 - Ba^4 + 6Ba^2b^2 - Bb^4)x(\tan^3(dx+c)) - b(10Aa^4b^2 + 9a^2Ab^4 + 3a^2b^6)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)(a^2 + b^2)} - \frac{b^3(4Aa^3b - 4Aa^2b^3 - Ba^4 + 6Ba^2b^2 - Bb^4)x(\tan^3(dx+c)) - b(10Aa^4b^2 + 9a^2Ab^4 + 3a^2b^6)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)(a^2 + b^2)} - \frac{b(10Aa^4b^2 + 9a^2Ab^4 + 3a^2b^6)}{da^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(A/a^4*ln(tan(d*x+c))+1/(a^2+b^2)^4*(1/2*(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(1+tan(d*x+c)^2)+(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*arctan(tan(d*x+c)))+1/3*b*(A*b-B*a)/a/(a^2+b^2)/(a+b*tan(d*x+c))^3+1/2*b*(3*A*a^2*b+A*b^3-2*B*a^3)/a^2/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+b*(6*A*a^4*b+3*A*a^2*b^3+A*b^5-3*B*a^5+B*a^3*b^2)/a^3/(a^2+b^2)^3/(a+b*tan(d*x+c))-b*(10*A*a^6*b+5*A*a^4*b^3+4*A*a^2*b^5+A*b^7-4*B*a^7+4*B*a^5*b^2)/a^4/(a^2+b^2)^4*ln(a+b*tan(d*x+c)))
```

**Maxima [A]**

time = 0.57, size = 580, normalized size = 1.92

$$\frac{6 (Ba^4 - 4Aa^3b - 6Ba^2b^2 + 4Aa^2b^3 + Bb^4) \log(\tan(dx+c)) + 6 (4Ba^3b - 10Aa^2b^2 - 4Ba^2b^3 - 5Aa^2b^4 - AB^2) \log(\tan(dx+c)+1) - 3 (Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Ba^2b^3 + AB^2) \log(\tan(dx+c)^2+1) - \frac{20Ba^7b - 47Aa^6b^2 - 4Ba^5b^3 - 34Aa^4b^4 + 2Ba^3b^5 - 11Aa^2b^6 - 6 (3Ba^4b - 6Aa^3b^2 - 3Aa^2b^3 - AB^2) \tan(dx+c) + 3 (14Ba^5b - 27Aa^4b^2 - 10Aa^3b^3 - 5Aa^2b^4) \tan(dx+c) + 6A \log(\tan(dx+c))}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*(Ba^4 - 4Aa^3*b - 6Ba^2*b^2 + 4Aa*b^3 + Bb^4)*(d*x + c)/(a^8 + 4a^6*b^2 + 6a^4*b^4 + 4a^2*b^6 + b^8) + 6*(4Ba^7*b - 10Aa^6*b^2 - 4Ba^5*b^3 - 5Aa^4*b^4 - 4Aa^2*b^6 - Ab^8)*log(b*tan(d*x + c) + a)/(a^12 + 4a^10*b^2 + 6a^8*b^4 + 4a^6*b^6 + a^4*b^8) - 3*(Aa^4 + 4Ba^3*b - 6Aa^2*b^2 - 4Ba*b^3 + Ab^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4a^6*b^2 + 6a^4*b^4 + 4a^2*b^6 + b^8) - (26Ba^7*b - 47Aa^6*b^2 + 4Ba^5*b^3 - 34Aa^4*b^4 + 2Ba^3*b^5 - 11Aa^2*b^6 + 6*(3Ba^5*b^3 - 6Aa^4*b^4 - Ba^3*b^5 - 3Aa^2*b^6 - Ab^8)*tan(d*x + c)^2 + 3*(14Ba^6*b^2 - 27Aa^5*b^3 - 2Ba^4*b^4 - 16Aa^3*b^5 - 5Aa*b^7)*tan(d*x + c))/(a^12 + 3a^10*b^2 + 3a^8*b^4 + a^6*b^6 + (a^9*b^3 + 3a^7*b^5 + 3a^5*b^7 + a^3*b^9)*tan(d*x + c)^3 + 3*(a^10*b^2 + 3a^8*b^4 + 3a^6*b^6 + a^4*b^8)*tan(d*x + c)^2 + 3*(a^11*b + 3a^9*b^3 + 3a^7*b^5 + a^5*b^7)*tan(d*x + c)) + 6A*log(tan(d*x + c))/a^4)/d
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(299) = 598.

time = 2.40, size = 1126, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/6*(48*B*a^8*b^3 - 75*A*a^7*b^4 + 6*B*a^6*b^5 - 42*A*a^5*b^6 + 2*B*a^4*b^7 - 11*A*a^3*b^8 - (26*B*a^7*b^4 - 47*A*a^6*b^5 - 18*B*a^5*b^6 - 6*A*a^4*b^7 - 3*A*a^2*b^9 + 6*(B*a^8*b^3 - 4*A*a^7*b^4 - 6*B*a^6*b^5 + 4*A*a^5*b^6 + B*a^4*b^7)*d*x)*\tan(d*x + c)^3 - 6*(B*a^11 - 4*A*a^10*b - 6*B*a^9*b^2 + 4*A*a^8*b^3 + B*a^7*b^4)*d*x - 3*(20*B*a^8*b^3 - 35*A*a^7*b^4 - 22*B*a^6*b^5 + 12*A*a^5*b^6 + 2*B*a^4*b^7 + 5*A*a^3*b^8 + 2*A*a*b^10 + 6*(B*a^9*b^2 - 4*A*a^8*b^3 - 6*B*a^7*b^4 + 4*A*a^6*b^5 + B*a^5*b^6)*d*x)*\tan(d*x + c)^2 - 3*(A*a^11 + 4*A*a^9*b^2 + 6*A*a^7*b^4 + 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 + 4*A*a^6*b^5 + 6*A*a^4*b^7 + 4*A*a^2*b^9 + A*b^11)*\tan(d*x + c))^3 + 3*(A*a^9*b^2 + 4*A*a^7*b^4 + 6*A*a^5*b^6 + 4*A*a^3*b^8 + A*a*b^10)*\tan(d*x + c)^2 + 3*(A*a^10*b + 4*A*a^8*b^3 + 6*A*a^6*b^5 + 4*A*a^4*b^7 + A*a^2*b^9)*\tan(d*x + c))*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)) - 3*(4*B*a^10*b - 10*A*a^9*b^2 - 4*B*a^8*b^3 - 5*A*a^7*b^4 - 4*A*a^5*b^6 - A*a^3*b^8 + (4*B*a^7*b^4 - 10*A*a^6*b^5 - 4*B*a^5*b^6 - 5*A*a^4*b^7 - 4*A*a^2*b^9 - A*b^11)*\tan(d*x + c))^3 + 3*(4*B*a^8*b^3 - 10*A*a^7*b^4 - 4*B*a^6*b^5 - 5*A*a^5*b^6 - 4*A*a^3*b^8 - A*a*b^10)*\tan(d*x + c)^2 + 3*(4*B*a^9*b^2 - 10*A*a^8*b^3 - 4*B*a^7*b^4 - 5*A*a^6*b^5 - 4*A*a^4*b^7 - A*a^2*b^9)*\tan(d*x + c))*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - 3*(12*B*a^9*b^2 - 20*A*a^8*b^3 - 30*B*a^7*b^4 + 37*A*a^6*b^5 + 2*B*a^5*b^6 + 18*A*a^4*b^7 + 5*A*a^2*b^9 + 6*(B*a^10*b - 4*A*a^9*b^2 - 6*B*a^8*b^3 + 4*A*a^7*b^4 + B*a^6*b^5)*d*x)*\tan(d*x + c))/((a^12*b^3 + 4*a^10*b^5 + 6*a^8*b^7 + 4*a^6*b^9 + a^4*b^11)*d*\tan(d*x + c)^3 + 3*(a^13*b^2 + 4*a^11*b^4 + 6*a^9*b^6 + 4*a^7*b^8 + a^5*b^10)*d*\tan(d*x + c)^2 + 3*(a^14*b + 4*a^12*b^3 + 6*a^10*b^5 + 4*a^8*b^7 + a^6*b^9)*d*\tan(d*x + c) + (a^15 + 4*a^13*b^2 + 6*a^11*b^4 + 4*a^9*b^6 + a^7*b^8)*d) \end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*4,x)

[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(299) = 598.

time = 1.03, size = 722, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="giac")

[Out]  $\frac{1}{6} \cdot (6 \cdot (B \cdot a^4 - 4 \cdot A \cdot a^3 \cdot b - 6 \cdot B \cdot a^2 \cdot b^2 + 4 \cdot A \cdot a \cdot b^3 + B \cdot b^4) \cdot (d \cdot x + c) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) - 3 \cdot (A \cdot a^4 + 4 \cdot B \cdot a^3 \cdot b - 6 \cdot A \cdot a^2 \cdot b^2 - 4 \cdot B \cdot a \cdot b^3 + A \cdot b^4) \cdot \log(\tan(d \cdot x + c)^2 + 1) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) + 6 \cdot (4 \cdot B \cdot a^7 \cdot b^2 - 10 \cdot A \cdot a^6 \cdot b^3 - 4 \cdot B \cdot a^5 \cdot b^4 - 5 \cdot A \cdot a^4 \cdot b^5 - 4 \cdot A \cdot a^2 \cdot b^7 - A \cdot b^9) \cdot \log(\text{abs}(b \cdot \tan(d \cdot x + c) + a)) / (a^{12} \cdot b + 4 \cdot a^{10} \cdot b^3 + 6 \cdot a^8 \cdot b^5 + 4 \cdot a^6 \cdot b^7 + a^4 \cdot b^9) + 6 \cdot A \cdot \log(\text{abs}(\tan(d \cdot x + c)))) / a^4 - (44 \cdot B \cdot a^7 \cdot b^4 \cdot \tan(d \cdot x + c)^3 - 110 \cdot A \cdot a^6 \cdot b^5 \cdot \tan(d \cdot x + c)^3 - 44 \cdot B \cdot a^5 \cdot b^6 \cdot \tan(d \cdot x + c)^3 - 55 \cdot A \cdot a^4 \cdot b^7 \cdot \tan(d \cdot x + c)^3 - 44 \cdot A \cdot a^2 \cdot b^9 \cdot \tan(d \cdot x + c)^3 - 11 \cdot A \cdot b^{11} \cdot \tan(d \cdot x + c)^3 + 150 \cdot B \cdot a^8 \cdot b^3 \cdot \tan(d \cdot x + c)^2 - 366 \cdot A \cdot a^7 \cdot b^4 \cdot \tan(d \cdot x + c)^2 - 120 \cdot B \cdot a^6 \cdot b^5 \cdot \tan(d \cdot x + c)^2 - 219 \cdot A \cdot a^5 \cdot b^6 \cdot \tan(d \cdot x + c)^2 - 6 \cdot B \cdot a^4 \cdot b^7 \cdot \tan(d \cdot x + c)^2 - 156 \cdot A \cdot a^3 \cdot b^8 \cdot \tan(d \cdot x + c)^2 - 39 \cdot A \cdot a \cdot b^{10} \cdot \tan(d \cdot x + c)^2 + 174 \cdot B \cdot a^9 \cdot b^2 \cdot \tan(d \cdot x + c) - 411 \cdot A \cdot a^8 \cdot b^3 \cdot \tan(d \cdot x + c) - 96 \cdot B \cdot a^7 \cdot b^4 \cdot \tan(d \cdot x + c) - 294 \cdot A \cdot a^6 \cdot b^5 \cdot \tan(d \cdot x + c) - 6 \cdot B \cdot a^5 \cdot b^6 \cdot \tan(d \cdot x + c) - 195 \cdot A \cdot a^4 \cdot b^7 \cdot \tan(d \cdot x + c) - 48 \cdot A \cdot a^2 \cdot b^9 \cdot \tan(d \cdot x + c) + 70 \cdot B \cdot a^{10} \cdot b - 157 \cdot A \cdot a^9 \cdot b^2 - 14 \cdot B \cdot a^8 \cdot b^3 - 136 \cdot A \cdot a^7 \cdot b^4 + 6 \cdot B \cdot a^6 \cdot b^5 - 89 \cdot A \cdot a^5 \cdot b^6 + 2 \cdot B \cdot a^4 \cdot b^7 - 22 \cdot A \cdot a^3 \cdot b^8) / ((a^{12} + 4 \cdot a^{10} \cdot b^2 + 6 \cdot a^8 \cdot b^4 + 4 \cdot a^6 \cdot b^6 + a^4 \cdot b^8) \cdot (b \cdot \tan(d \cdot x + c) + a)^3) / d$

Mupad [B]

time = 9.76, size = 484, normalized size = 1.60

$$\frac{\frac{6 \cdot B \cdot a^7 \cdot b^4 \cdot \tan^3(d \cdot x + c) - 110 \cdot A \cdot a^6 \cdot b^5 \cdot \tan^3(d \cdot x + c) - 44 \cdot B \cdot a^5 \cdot b^6 \cdot \tan^3(d \cdot x + c) - 55 \cdot A \cdot a^4 \cdot b^7 \cdot \tan^3(d \cdot x + c) - 44 \cdot A \cdot a^2 \cdot b^9 \cdot \tan^3(d \cdot x + c) - 11 \cdot A \cdot b^{11} \cdot \tan^3(d \cdot x + c) + 150 \cdot B \cdot a^8 \cdot b^3 \cdot \tan^2(d \cdot x + c) - 366 \cdot A \cdot a^7 \cdot b^4 \cdot \tan^2(d \cdot x + c) - 120 \cdot B \cdot a^6 \cdot b^5 \cdot \tan^2(d \cdot x + c) - 219 \cdot A \cdot a^5 \cdot b^6 \cdot \tan^2(d \cdot x + c) - 6 \cdot B \cdot a^4 \cdot b^7 \cdot \tan^2(d \cdot x + c) - 156 \cdot A \cdot a^3 \cdot b^8 \cdot \tan^2(d \cdot x + c) - 39 \cdot A \cdot a \cdot b^{10} \cdot \tan^2(d \cdot x + c) + 174 \cdot B \cdot a^9 \cdot b^2 \cdot \tan(d \cdot x + c) - 411 \cdot A \cdot a^8 \cdot b^3 \cdot \tan(d \cdot x + c) - 96 \cdot B \cdot a^7 \cdot b^4 \cdot \tan(d \cdot x + c) - 294 \cdot A \cdot a^6 \cdot b^5 \cdot \tan(d \cdot x + c) - 6 \cdot B \cdot a^5 \cdot b^6 \cdot \tan(d \cdot x + c) - 195 \cdot A \cdot a^4 \cdot b^7 \cdot \tan(d \cdot x + c) - 48 \cdot A \cdot a^2 \cdot b^9 \cdot \tan(d \cdot x + c) + 70 \cdot B \cdot a^{10} \cdot b - 157 \cdot A \cdot a^9 \cdot b^2 - 14 \cdot B \cdot a^8 \cdot b^3 - 136 \cdot A \cdot a^7 \cdot b^4 + 6 \cdot B \cdot a^6 \cdot b^5 - 89 \cdot A \cdot a^5 \cdot b^6 + 2 \cdot B \cdot a^4 \cdot b^7 - 22 \cdot A \cdot a^3 \cdot b^8}{(a^{12} + 4 \cdot a^{10} \cdot b^2 + 6 \cdot a^8 \cdot b^4 + 4 \cdot a^6 \cdot b^6 + a^4 \cdot b^8) \cdot (b \cdot \tan(d \cdot x + c) + a)^3} \cdot \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^4,x)

[Out]  $((11 \cdot A \cdot b^6 + 34 \cdot A \cdot a^2 \cdot b^4 + 47 \cdot A \cdot a^4 \cdot b^2 - 4 \cdot B \cdot a^3 \cdot b^3 - 2 \cdot B \cdot a \cdot b^5 - 26 \cdot B \cdot a^5 \cdot b) / (6 \cdot a \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2)) + (\tan(c + d \cdot x)^2 \cdot (A \cdot b^8 + 3 \cdot A \cdot a^2 \cdot b^6 + 6 \cdot A \cdot a^4 \cdot b^4 + B \cdot a^3 \cdot b^5 - 3 \cdot B \cdot a^5 \cdot b^3)) / (a^3 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2)) + (\tan(c + d \cdot x) \cdot (5 \cdot A \cdot b^7 + 16 \cdot A \cdot a^2 \cdot b^5 + 27 \cdot A \cdot a^4 \cdot b^3 + 2 \cdot B \cdot a^3 \cdot b^4 - 14 \cdot B \cdot a^5 \cdot b^2)) / (2 \cdot a^2 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2)) / (d \cdot (a^3 + b^3 \cdot \tan(c + d \cdot x)^3 + 3 \cdot a \cdot b^2 \cdot \tan(c + d \cdot x)^2 + 3 \cdot a^2 \cdot b \cdot \tan(c + d \cdot x))) + (A \cdot \log(\tan(c + d \cdot x))) / (a^4 \cdot d) - (\log(\tan(c + d \cdot x) + 1) \cdot (A \cdot 1i + B)) / (2 \cdot d \cdot (4 \cdot a^3 \cdot b - 4 \cdot a \cdot b^3 + a^4 \cdot 1i + b^4 \cdot 1i - a^2 \cdot b^2 \cdot 6i)) - (\log(\tan(c + d \cdot x) - 1) \cdot (A + B \cdot 1i)) / (2 \cdot d \cdot (a^3 \cdot b \cdot 4i - a \cdot b^3 \cdot 4i + a^4 + b^4 - 6 \cdot a^2 \cdot b^2)) - (b \cdot \log(a + b \cdot \tan(c + d \cdot x)) \cdot (A \cdot b^7 - 4 \cdot B \cdot a^7 + 4 \cdot A \cdot a^2 \cdot b^5 + 5 \cdot A \cdot a^4 \cdot b^3 + 4 \cdot B \cdot a^5 \cdot b^2 + 10 \cdot A \cdot a^6 \cdot b)) / (a^4 \cdot d \cdot (a^2 + b^2)^4)$

$$3.296 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=399

$$\frac{(a^4 A - 6a^2 A b^2 + A b^4 + 4a^3 b B - 4a b^3 B) x}{(a^2 + b^2)^4} - \frac{(4A b - a B) \log(\sin(c + dx))}{a^5 d} + \frac{b^2(20a^6 A b + 24a^4 A b^3 + 16a^2 A b^5 + 4A b^7 - 10A b^7 B - 5A b^5 b^2 B - 4A b^3 b^4 B - a b^6 B)}{a^5 d} + \frac{b^2(20a^6 A b + 24a^4 A b^3 + 16a^2 A b^5 + 4A b^7 - 10A b^7 B - 5A b^5 b^2 B - 4A b^3 b^4 B - a b^6 B)}{a^5 d}$$

[Out]  $-(A*a^4-6*A*a^2*b^2+A*b^4+4*a^3*b*B-4*a*b^3*B)*x/(a^2+b^2)^4-(4*A*b-B*a)*\ln(\sin(d*x+c))/a^5/d+b^2*(20*A*a^6*b+24*A*a^4*b^3+16*A*a^2*b^5+4*A*b^7-10*B*a^7-5*B*a^5*b^2-4*B*a^3*b^4-B*a*b^6)*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/a^5/(a^2+b^2)^4/d-1/3*b*(3*A*a^2+4*A*b^2-B*a*b)/a^2/(a^2+b^2)/d/(a+b*\tan(d*x+c))^3-A*\cot(d*x+c)/a/d/(a+b*\tan(d*x+c))^3-1/2*b*(2*A*a^4+8*A*a^2*b^2+4*A*b^4-3*B*a^3*b-B*a*b^3)/a^3/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^2-b*(A*a^6+13*A*a^4*b^2+12*A*a^2*b^4+4*A*b^6-6*B*a^5*b-3*B*a^3*b^3-B*a*b^5)/a^4/(a^2+b^2)^3/d/(a+b*\tan(d*x+c))$

**Rubi** [A]

time = 0.86, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

$$\frac{(4A - aB) \log(\sin(c + dx))}{a^5 d} - \frac{b^2(20a^6 A b + 24a^4 A b^3 + 16a^2 A b^5 + 4A b^7 - 10A b^7 B - 5A b^5 b^2 B - 4A b^3 b^4 B - a b^6 B)}{a^5 d} + \frac{b^2(20a^6 A b + 24a^4 A b^3 + 16a^2 A b^5 + 4A b^7 - 10A b^7 B - 5A b^5 b^2 B - 4A b^3 b^4 B - a b^6 B)}{a^5 d} + \frac{A \cot(c + dx)}{a d (a + b \tan(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4,x]

[Out]  $-(((a^4 A - 6a^2 A b^2 + A b^4 + 4a^3 b B - 4a b^3 B) x)/(a^2 + b^2)^4) - ((4A b - a B) * \text{Log}[\text{Sin}[c + d x]])/(a^5 d) + (b^2 * (20a^6 A b + 24a^4 A b^3 + 16a^2 A b^5 + 4A b^7 - 10a^7 B - 5a^5 b^2 B - 4a^3 b^4 B - a b^6 B) * \text{Log}[a \text{Cos}[c + d x] + b \text{Sin}[c + d x]])/(a^5 * (a^2 + b^2)^4 d) - (b * (3a^2 A + 4A b^2 - a b B))/(3a^2 * (a^2 + b^2) * d * (a + b \text{Tan}[c + d x])^3) - (A \text{Cot}[c + d x])/(a * d * (a + b \text{Tan}[c + d x])^3) - (b * (2a^4 A + 8a^2 A b^2 + 4A b^4 - 3a^3 b B - a b^3 B))/(2a^3 * (a^2 + b^2)^2 * d * (a + b \text{Tan}[c + d x])^2) - (b * (a^6 A + 13a^4 A b^2 + 12a^2 A b^4 + 4A b^6 - 6a^5 b B - 3a^3 b^3 B - a b^5 B))/(a^4 * (a^2 + b^2)^3 * d * (a + b \text{Tan}[c + d x]))$

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] &&

NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

### Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n +
2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] := Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx &= -\frac{A \cot(c+dx)}{ad(a+b \tan(c+dx))^3} - \frac{\int \frac{\cot(c+dx)(4Ab-aB+aA \tan(c+dx)+4Ab \tan^2(c+dx))}{(a+b \tan(c+dx))^4} dx}{a} \\
&= -\frac{b(3a^2A+4Ab^2-abB)}{3a^2(a^2+b^2)d(a+b \tan(c+dx))^3} - \frac{A \cot(c+dx)}{ad(a+b \tan(c+dx))^3} \\
&= -\frac{b(3a^2A+4Ab^2-abB)}{3a^2(a^2+b^2)d(a+b \tan(c+dx))^3} - \frac{A \cot(c+dx)}{ad(a+b \tan(c+dx))^3} \\
&= -\frac{b(3a^2A+4Ab^2-abB)}{3a^2(a^2+b^2)d(a+b \tan(c+dx))^3} - \frac{A \cot(c+dx)}{ad(a+b \tan(c+dx))^3} \\
&= -\frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)x}{(a^2+b^2)^4} - \frac{b(3a^2A+4Ab^2-abB)}{3a^2(a^2+b^2)d(a+b \tan(c+dx))^3} \\
&= -\frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)x}{(a^2+b^2)^4} - \frac{(4Ab-aB) \log(s)}{a^5d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 3.89, size = 357, normalized size = 0.89

$$\frac{-\frac{6A \cot(c+dx)}{a^4} + \frac{3(A+B) \log(-\tan(c+dx))}{(a+b)^4} + \frac{6(-4Ab+aB) \log(\tan(c+dx))}{a^5} - \frac{3(A+B) \log(1+\tan(c+dx))}{(a-b)^4} - \frac{6b^2(-20a^6Ab-24a^4Ab^3-16a^2Ab^5-4Ab^7+10a^7B+5a^5b^2B+4a^3b^4B+aB^6) \log(a+b \tan(c+dx))}{a^7(a^2+b^2)^4} + \frac{2b^2(-Ab+aB)}{a^2(a^2+b^2)(a+b \tan(c+dx))^2} + \frac{3b^2(-4a^2Ab-2Ab^3+3a^2B+aB^2B)}{a^3(a^2+b^2)^2(a+b \tan(c+dx))^2} + \frac{6b^2(-10a^4Ab-9a^2Ab^3-3Ab^5+6a^2B+3a^2b^2B+aB^4B)}{a^4(a^2+b^2)^2(a+b \tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4,x]

[Out] ((-6\*A\*Cot[c + d\*x])/a^4 + ((3\*I)\*(A + I\*B)\*Log[I - Tan[c + d\*x]])/(a + I\*b)^4 + (6\*(-4\*A\*b + a\*B)\*Log[Tan[c + d\*x]])/a^5 - (3\*(I\*A + B)\*Log[I + Tan[c + d\*x]])/(a - I\*b)^4 - (6\*b^2\*(-20\*a^6\*A\*b - 24\*a^4\*A\*b^3 - 16\*a^2\*A\*b^5 - 4\*A\*b^7 + 10\*a^7\*B + 5\*a^5\*b^2\*B + 4\*a^3\*b^4\*B + a\*b^6\*B)\*Log[a + b\*Tan[c + d\*x]])/(a^5\*(a^2 + b^2)^4) + (2\*b^2\*(-(A\*b) + a\*B))/(a^2\*(a^2 + b^2)\*(a + b\*Tan[c + d\*x])^3) + (3\*b^2\*(-4\*a^2\*A\*b - 2\*A\*b^3 + 3\*a^3\*B + a\*b^2\*B))/(a^3\*(a^2 + b^2)^2\*(a + b\*Tan[c + d\*x])^2) + (6\*b^2\*(-10\*a^4\*A\*b - 9\*a^2\*A\*b^3 - 3\*A\*b^5 + 6\*a^5\*B + 3\*a^3\*b^2\*B + a\*b^4\*B))/(a^4\*(a^2 + b^2)^3\*(a + b\*Tan[c + d\*x]))/(6\*d)

**Maple [A]**

time = 0.78, size = 400, normalized size = 1.00

method	result
--------	--------

derivativdivides	$-\frac{A}{a^4 \tan(dx+c)} + \frac{(-4Ab+aB) \ln(\tan(dx+c))}{a^5} + \frac{(4Aa^3b-4Aa^2b^2-Ba^4+6Ba^2b^2-Bb^4) \ln(1+\tan^2(dx+c))}{2} + \frac{(-Aa^4+6Aa^2b^2-Aa^4b^4)}{(a^2+b^2)^4}$
default	$-\frac{A}{a^4 \tan(dx+c)} + \frac{(-4Ab+aB) \ln(\tan(dx+c))}{a^5} + \frac{(4Aa^3b-4Aa^2b^2-Ba^4+6Ba^2b^2-Bb^4) \ln(1+\tan^2(dx+c))}{2} + \frac{(-Aa^4+6Aa^2b^2-Aa^4b^4)}{(a^2+b^2)^4}$
norman	$\frac{b(6Aa^6b+33Aa^4b^3+35Aa^2b^5+12Aab^7-10Ba^5b^2-9Ba^3b^4-3Bab^6)(\tan^2(dx+c))}{d a^3(a^6+3a^4b^2+3a^2b^4+b^6)} - \frac{A}{ad} \frac{b^3(Aa^4-6Aa^2b^2+Ab^4+4Ba^3b-4Bab^3)}{(a^6+3a^4b^2+3a^2b^4+b^6)(a^2+b^2)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} \left( -\frac{A}{a^4 \tan(dx+c)} + \frac{(-4Aa^3b+4Aa^2b^2-Ba^4+6Ba^2b^2-Bb^4) \ln(1+\tan^2(dx+c))}{2} + \frac{(-Aa^4+6Aa^2b^2-Aa^4b^4)}{(a^2+b^2)^4} \right) + \frac{b(6Aa^6b+33Aa^4b^3+35Aa^2b^5+12Aab^7-10Ba^5b^2-9Ba^3b^4-3Bab^6)(\tan^2(dx+c))}{d a^3(a^6+3a^4b^2+3a^2b^4+b^6)} - \frac{A}{ad} \frac{b^3(Aa^4-6Aa^2b^2+Ab^4+4Ba^3b-4Bab^3)}{(a^6+3a^4b^2+3a^2b^4+b^6)(a^2+b^2)}$$

Maxima [A]

time = 0.61, size = 698, normalized size = 1.75

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] 
$$-\frac{1}{6} \left( 6(Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Ba^2b^3 + Ab^4)(dx+c) + (a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) + 6(10Ba^7b^2 - 20Aa^6b^3 + 5Ba^5b^4 - 24Aa^4b^5 + 4Ba^3b^6 - 16Aa^2b^7 + Ba^2b^8 - 4Aa^2b^9) \log(b \tan(dx+c) + a) + 3(Ba^4 - 4Aa^3b - 6Ba^2b^2 + 4Aa^2b^3 + Bb^4) \log(\tan(dx+c)^2 + 1) \right) / (a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) + (6Aa^9 + 18Aa^7b^2 + 18Aa^5b^4 + 6Aa^3b^6 + 6(Aa^6b^3 - 6Ba^5b^4 + 13Aa^4b^5 - 3Ba^3b^6 + 12Aa^2b^7 - Ba^2b^8 + 4Aa^2b^9) \tan(dx+c)^3 + 3(6Aa^7b^2 - 27Ba^6b^3 + 62Aa^5b^4 - 16Ba^4b^5 + 60Aa^3b^6 - 5Ba^2b^7 + 20Aa^2b^8) \tan(dx+c)^2 + (18Aa^8b - 47Ba^7b^2 + 128Aa^6b^3 - 34Ba^5b^4 + 130Aa^4b^5 - 11Ba^3b^6 + 44Aa^2$$

$$\begin{aligned} & *b^7) \cdot \tan(dx + c) / ((a^{10}b^3 + 3a^8b^5 + 3a^6b^7 + a^4b^9) \cdot \tan(dx + \\ & c)^4 + 3(a^{11}b^2 + 3a^9b^4 + 3a^7b^6 + a^5b^8) \cdot \tan(dx + c)^3 + 3(a^{12}b \\ & + 3a^{10}b^3 + 3a^8b^5 + a^6b^7) \cdot \tan(dx + c)^2 + (a^{13} + 3a^{11}b^2 + 3a^9b^4 + a^7b^6) \cdot \tan(dx + c) \\ & - 6(Ba - 4Ab) \cdot \log(\tan(dx + c)) / a^5) / d \end{aligned}$$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. 2(393) = 786.

time = 2.57, size = 1510, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^2\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^4,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/6*(6Aa^{12} + 24Aa^{10}b^2 + 36Aa^8b^4 + 24Aa^6b^6 + 6Aa^4b^8 \\ & + (47Ba^7b^5 - 74Aa^6b^6 + 6Ba^5b^7 - 42Aa^4b^8 + 3Ba^3b^9 - \\ & 12Aa^2b^{10} + 6(Aa^9b^3 + 4Ba^8b^4 - 6Aa^7b^5 - 4Ba^6b^6 + A \\ & a^5b^7) \cdot dx) \cdot \tan(dx + c)^4 + 3(2Aa^9b^3 + 35Ba^8b^4 - 46Aa^7b^5 \\ & - 12Ba^6b^6 + 8Aa^5b^7 - 5Ba^4b^8 + 20Aa^3b^9 - 2Ba^2b^{10} \\ & + 8Aa \cdot b^{11} + 6(Aa^{10}b^2 + 4Ba^9b^3 - 6Aa^8b^4 - 4Ba^7b^5 + A \\ & a^6b^6) \cdot dx) \cdot \tan(dx + c)^3 + 3(6Aa^{10}b^2 + 20Ba^9b^3 - 6Aa^8b^4 \\ & - 37Ba^7b^5 + 80Aa^6b^6 - 18Ba^5b^7 + 68Aa^4b^8 - 5Ba^3b^9 \\ & + 20Aa^2b^{10} + 6(Aa^{11}b + 4Ba^{10}b^2 - 6Aa^9b^3 - 4Ba^8b^4 + \\ & Aa^7b^5) \cdot dx) \cdot \tan(dx + c)^2 - 3((Ba^9b^3 - 4Aa^8b^4 + 4Ba^7b^5 \\ & - 16Aa^6b^6 + 6Ba^5b^7 - 24Aa^4b^8 + 4Ba^3b^9 - 16Aa^2b^{10} + \\ & Ba \cdot b^{11} - 4Ab^{12}) \cdot \tan(dx + c)^4 + 3(Ba^{10}b^2 - 4Aa^9b^3 + 4Ba^8 \\ & b^4 - 16Aa^7b^5 + 6Ba^6b^6 - 24Aa^5b^7 + 4Ba^4b^8 - 16Aa^3b^9 \\ & + Ba^2b^{10} - 4Aa \cdot b^{11}) \cdot \tan(dx + c)^3 + 3(Ba^{11}b - 4Aa^{10}b^2 \\ & + 4Ba^9b^3 - 16Aa^8b^4 + 6Ba^7b^5 - 24Aa^6b^6 + 4Ba^5b^7 - 1 \\ & 6Aa^4b^8 + Ba^3b^9 - 4Aa^2b^{10}) \cdot \tan(dx + c)^2 + (Ba^{12} - 4Aa^{11} \\ & \cdot b + 4Ba^{10}b^2 - 16Aa^9b^3 + 6Ba^8b^4 - 24Aa^7b^5 + 4Ba^6b^6 \\ & - 16Aa^5b^7 + Ba^4b^8 - 4Aa^3b^9) \cdot \tan(dx + c) \cdot \log(\tan(dx + c))^2 \\ & / (\tan(dx + c)^2 + 1) + 3((10Ba^7b^5 - 20Aa^6b^6 + 5Ba^5b^7 - 24 \\ & Aa^4b^8 + 4Ba^3b^9 - 16Aa^2b^{10} + Ba \cdot b^{11} - 4Ab^{12}) \cdot \tan(dx + c) \\ & )^4 + 3(10Ba^8b^4 - 20Aa^7b^5 + 5Ba^6b^6 - 24Aa^5b^7 + 4Ba^4b^8 \\ & \cdot b^8 - 16Aa^3b^9 + Ba^2b^{10} - 4Aa \cdot b^{11}) \cdot \tan(dx + c)^3 + 3(10Ba^9 \\ & \cdot b^3 - 20Aa^8b^4 + 5Ba^7b^5 - 24Aa^6b^6 + 4Ba^5b^7 - 16Aa^4b^8 \\ & + Ba^3b^9 - 4Aa^2b^{10}) \cdot \tan(dx + c)^2 + (10Ba^{10}b^2 - 20Aa^9b^3 \\ & + 5Ba^8b^4 - 24Aa^7b^5 + 4Ba^6b^6 - 16Aa^5b^7 + Ba^4b^8 - \\ & 4Aa^3b^9) \cdot \tan(dx + c) \cdot \log((b^2 \cdot \tan(dx + c)^2 + 2a \cdot b \cdot \tan(dx + c) + a \\ & ^2) / (\tan(dx + c)^2 + 1)) + (18Aa^{11}b + 72Aa^9b^3 - 75Ba^8b^4 + 21 \\ & 6Aa^7b^5 - 42Ba^6b^6 + 162Aa^5b^7 - 11Ba^4b^8 + 44Aa^3b^9 + \\ & 6(Aa^{12} + 4Ba^{11}b - 6Aa^{10}b^2 - 4Ba^9b^3 + Aa^8b^4) \cdot dx) \cdot \tan(dx + c) \end{aligned}$$

```
*x + c))/((a^13*b^3 + 4*a^11*b^5 + 6*a^9*b^7 + 4*a^7*b^9 + a^5*b^11)*d*tan(
d*x + c)^4 + 3*(a^14*b^2 + 4*a^12*b^4 + 6*a^10*b^6 + 4*a^8*b^8 + a^6*b^10)*
d*tan(d*x + c)^3 + 3*(a^15*b + 4*a^13*b^3 + 6*a^11*b^5 + 4*a^9*b^7 + a^7*b^
9)*d*tan(d*x + c)^2 + (a^16 + 4*a^14*b^2 + 6*a^12*b^4 + 4*a^10*b^6 + a^8*b^
8)*d*tan(d*x + c))
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(393) = 786.

time = 1.33, size = 846, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a
^2*b^2 + 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8) + 6*(10*B*a^7*b^3 - 20*A*a^6*b^4 + 5*B*a^5*b^5 - 2
4*A*a^4*b^6 + 4*B*a^3*b^7 - 16*A*a^2*b^8 + B*a*b^9 - 4*A*b^10)*log(abs(b*ta
n(d*x + c) + a))/(a^13*b + 4*a^11*b^3 + 6*a^9*b^5 + 4*a^7*b^7 + a^5*b^9) -
(110*B*a^7*b^5*tan(d*x + c)^3 - 220*A*a^6*b^6*tan(d*x + c)^3 + 55*B*a^5*b^7
*tan(d*x + c)^3 - 264*A*a^4*b^8*tan(d*x + c)^3 + 44*B*a^3*b^9*tan(d*x + c)^
3 - 176*A*a^2*b^10*tan(d*x + c)^3 + 11*B*a*b^11*tan(d*x + c)^3 - 44*A*b^12*
tan(d*x + c)^3 + 366*B*a^8*b^4*tan(d*x + c)^2 - 720*A*a^7*b^5*tan(d*x + c)^
2 + 219*B*a^6*b^6*tan(d*x + c)^2 - 906*A*a^5*b^7*tan(d*x + c)^2 + 156*B*a^4
*b^8*tan(d*x + c)^2 - 600*A*a^3*b^9*tan(d*x + c)^2 + 39*B*a^2*b^10*tan(d*x
+ c)^2 - 150*A*a*b^11*tan(d*x + c)^2 + 411*B*a^9*b^3*tan(d*x + c) - 792*A*a
^8*b^4*tan(d*x + c) + 294*B*a^7*b^5*tan(d*x + c) - 1050*A*a^6*b^6*tan(d*x +
c) + 195*B*a^5*b^7*tan(d*x + c) - 696*A*a^4*b^8*tan(d*x + c) + 48*B*a^3*b^
9*tan(d*x + c) - 174*A*a^2*b^10*tan(d*x + c) + 157*B*a^10*b^2 - 294*A*a^9*b
^3 + 136*B*a^8*b^4 - 414*A*a^7*b^5 + 89*B*a^6*b^6 - 278*A*a^5*b^7 + 22*B*a^
4*b^8 - 70*A*a^3*b^9)/((a^13 + 4*a^11*b^2 + 6*a^9*b^4 + 4*a^7*b^6 + a^5*b^8
```



)\*(b\*tan(d\*x + c) + a)^3) - 6\*(B\*a - 4\*A\*b)\*log(abs(tan(d\*x + c)))/a^5 + 6\*(B\*a\*tan(d\*x + c) - 4\*A\*b\*tan(d\*x + c) + A\*a)/(a^5\*tan(d\*x + c))/d

**Mupad [B]**

time = 11.02, size = 576, normalized size = 1.44

$$\frac{B \ln(a + b \tan(c + dx)) (-10 B^2 a^5 + 5 B^2 b^5 + 24 A^2 d^2 b^5 - 4 B^2 d^2 b^5 + 16 A^2 d^2 b^5 - B^2 d^2 b^5 + 4 A^2 d^2)}{d^2 (d^2 + b^2)} - \frac{\ln(\tan(c + dx)) (4 A^2 - B^2)}{2 d^2 (d^2 - 4 b^2 - 2 B^2 b^2 + 4 A^2 + 4 B^2)} - \frac{\ln(\tan(c + dx)) (A + B)}{2 d^2 (d^2 - 4 b^2 - 2 B^2 b^2 + 4 A^2 + 4 B^2)} - \frac{2}{d^2} + \frac{2 \ln(\tan(c + dx)) (2 A^2 b^2 + 2 B^2 b^2 + 2 A^2 b^2 + 2 B^2 b^2 + 2 A^2 b^2 + 2 B^2 b^2)}{d^2 (d^2 + b^2)} + \frac{2 \ln(\tan(c + dx)) (2 A^2 b^2 + 2 B^2 b^2 + 2 A^2 b^2 + 2 B^2 b^2 + 2 A^2 b^2 + 2 B^2 b^2)}{d^2 (d^2 + b^2)} + \frac{2 \ln(\tan(c + dx)) (2 A^2 b^2 + 2 B^2 b^2 + 2 A^2 b^2 + 2 B^2 b^2 + 2 A^2 b^2 + 2 B^2 b^2)}{d^2 (d^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^4,x)

[Out] (b^2\*log(a + b\*tan(c + d\*x))\*(4\*A\*b^7 - 10\*B\*a^7 + 16\*A\*a^2\*b^5 + 24\*A\*a^4\*b^3 - 4\*B\*a^3\*b^4 - 5\*B\*a^5\*b^2 + 20\*A\*a^6\*b - B\*a\*b^6))/(a^5\*d\*(a^2 + b^2)^4) - (log(tan(c + d\*x))\*(4\*A\*b - B\*a))/(a^5\*d) - (log(tan(c + d\*x) - 1i)\*(A + B\*1i))/(2\*d\*(4\*a\*b^3 - 4\*a^3\*b + a^4\*1i + b^4\*1i - a^2\*b^2\*6i)) - (log(tan(c + d\*x) + 1i)\*(A\*1i + B))/(2\*d\*(a\*b^3\*4i - a^3\*b\*4i + a^4 + b^4 - 6\*a^2\*b^2)) - (A/a + (tan(c + d\*x))^3\*(4\*A\*b^9 + 12\*A\*a^2\*b^7 + 13\*A\*a^4\*b^5 + A\*a^6\*b^3 - 3\*B\*a^3\*b^6 - 6\*B\*a^5\*b^4 - B\*a\*b^8))/(a^4\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)) + (tan(c + d\*x))^2\*(20\*A\*b^8 + 60\*A\*a^2\*b^6 + 62\*A\*a^4\*b^4 + 6\*A\*a^6\*b^2 - 16\*B\*a^3\*b^5 - 27\*B\*a^5\*b^3 - 5\*B\*a\*b^7))/(2\*a^3\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)) + (tan(c + d\*x)\*(44\*A\*b^7 + 130\*A\*a^2\*b^5 + 128\*A\*a^4\*b^3 - 34\*B\*a^3\*b^4 - 47\*B\*a^5\*b^2 + 18\*A\*a^6\*b - 11\*B\*a\*b^6))/(6\*a^2\*(a^6 + b^6 + 3\*a^2\*b^4 + 3\*a^4\*b^2)))/(d\*(a^3\*tan(c + d\*x) + b^3\*tan(c + d\*x))^4 + 3\*a^2\*b\*tan(c + d\*x)^2 + 3\*a\*b^2\*tan(c + d\*x)^3)

$$3.297 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

**Optimal.** Leaf size=477

$$\frac{(4a^3Ab - 4aAb^3 - a^4B + 6a^2b^2B - b^4B)x}{(a^2 + b^2)^4} - \frac{(a^2A - 10Ab^2 + 4abB) \log(\sin(c + dx))}{a^6d} - \frac{b^3(35a^6Ab + 56a^4Ab^3}{a^6d}$$

[Out] (4\*A\*a^3\*b-4\*A\*a\*b^3-B\*a^4+6\*B\*a^2\*b^2-B\*b^4)\*x/(a^2+b^2)^4-(A\*a^2-10\*A\*b^2+4\*B\*a\*b)\*ln(sin(d\*x+c))/a^6/d-b^3\*(35\*A\*a^6\*b+56\*A\*a^4\*b^3+39\*A\*a^2\*b^5+10\*A\*b^7-20\*B\*a^7-24\*B\*a^5\*b^2-16\*B\*a^3\*b^4-4\*B\*a\*b^6)\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/a^6/(a^2+b^2)^4/d+1/3\*b\*(9\*A\*a^2\*b+10\*A\*b^3-3\*B\*a^3-4\*B\*a\*b^2)/a^3/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^3+1/2\*(5\*A\*b-2\*B\*a)\*cot(d\*x+c)/a^2/d/(a+b\*tan(d\*x+c))^3-1/2\*A\*cot(d\*x+c)^2/a/d/(a+b\*tan(d\*x+c))^3+1/2\*b\*(7\*A\*a^4\*b+19\*A\*a^2\*b^3+10\*A\*b^5-2\*B\*a^5-8\*B\*a^3\*b^2-4\*B\*a\*b^4)/a^4/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^2+b\*(4\*A\*a^6\*b+27\*A\*a^4\*b^3+29\*A\*a^2\*b^5+10\*A\*b^7-B\*a^7-13\*B\*a^5\*b^2-12\*B\*a^3\*b^4-4\*B\*a\*b^6)/a^5/(a^2+b^2)^3/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 1.12, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3690, 3730, 3732, 3611, 3556}

(5d-2b)cos(c+dx) / (2a^2b+4ab^2+d^2) \* (A^2+4abB-10B^2)log(sin(c+dx)) / a^2 + (4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x / (a^2+b^2)^4 - (a^2A-10Ab^2+4abB)log(sin(c+dx)) / a^6d - b^3(35a^6Ab+56a^4Ab^3+39a^2Ab^5+10Ab^7-20B\*a^7-24B\*a^5\*b^2-16B\*a^3\*b^4-4B\*a\*b^6)log(a\*cos(c+dx)+b\*sin(c+dx)) / a^6/(a^2+b^2)^4/d + 1/3\*b\*(9A\*a^2\*b+10A\*b^3-3B\*a^3-4B\*a\*b^2)/a^3/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^3 + 1/2\*(5A\*b-2B\*a)\*cot(d\*x+c)/a^2/d/(a+b\*tan(d\*x+c))^3 - 1/2\*A\*cot(d\*x+c)^2/a/d/(a+b\*tan(d\*x+c))^3 + 1/2\*b\*(7A\*a^4\*b+19A\*a^2\*b^3+10A\*b^5-2B\*a^5-8B\*a^3\*b^2-4B\*a\*b^4)/a^4/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^2 + b\*(4A\*a^6\*b+27A\*a^4\*b^3+29A\*a^2\*b^5+10A\*b^7-B\*a^7-13B\*a^5\*b^2-12B\*a^3\*b^4-4B\*a\*b^6)/a^5/(a^2+b^2)^3/d/(a+b\*tan(d\*x+c))

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^4,x]

[Out] ((4\*a^3\*A\*b - 4\*a\*A\*b^3 - a^4\*B + 6\*a^2\*b^2\*B - b^4\*B)\*x)/(a^2 + b^2)^4 - ((a^2\*A - 10\*A\*b^2 + 4\*a\*b\*B)\*Log[Sin[c + d\*x]]/(a^6\*d) - (b^3\*(35\*a^6\*A\*b + 56\*a^4\*A\*b^3 + 39\*a^2\*A\*b^5 + 10\*A\*b^7 - 20\*a^7\*B - 24\*a^5\*b^2\*B - 16\*a^3\*b^4\*B - 4\*a\*b^6\*B)\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]]/(a^6\*(a^2 + b^2)^4\*d) + (b\*(9\*a^2\*A\*b + 10\*A\*b^3 - 3\*a^3\*B - 4\*a\*b^2\*B))/(3\*a^3\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^3) + ((5\*A\*b - 2\*a\*B)\*Cot[c + d\*x])/(2\*a^2\*d\*(a + b\*Tan[c + d\*x])^3) - (A\*Cot[c + d\*x]^2)/(2\*a\*d\*(a + b\*Tan[c + d\*x])^3) + (b\*(7\*a^4\*A\*b + 19\*a^2\*A\*b^3 + 10\*A\*b^5 - 2\*a^5\*B - 8\*a^3\*b^2\*B - 4\*a\*b^4\*B))/(2\*a^4\*(a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x])^2) + (b\*(4\*a^6\*A\*b + 27\*a^4\*A\*b^3 + 29\*a^2\*A\*b^5 + 10\*A\*b^7 - a^7\*B - 13\*a^5\*b^2\*B - 12\*a^3\*b^4\*B - 4\*a\*b^6\*B))/(a^5\*(a^2 + b^2)^3\*d\*(a + b\*Tan[c + d\*x]))

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] := Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3732

```
Int[((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^
2)/(((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)
*(x_)])), x_Symbol] := Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx &= -\frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^3} - \frac{\int \frac{\cot^2(c+dx)(5Ab-2aB+2aA \tan(c+dx)+5Ab \tan(c+dx))}{(a+b \tan(c+dx))^4} dx}{2a} \\
&= \frac{(5Ab-2aB) \cot(c+dx)}{2a^2d(a+b \tan(c+dx))^3} - \frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^3} + \frac{\int \frac{\cot(c+dx)(5Ab-2aB+2aA \tan(c+dx)+5Ab \tan(c+dx))}{(a+b \tan(c+dx))^4} dx}{2a} \\
&= \frac{b(9a^2Ab+10Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d(a+b \tan(c+dx))^3} + \frac{(5Ab-2aB) \cot(c+dx)}{2a^2d(a+b \tan(c+dx))^3} \\
&= \frac{b(9a^2Ab+10Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d(a+b \tan(c+dx))^3} + \frac{(5Ab-2aB) \cot(c+dx)}{2a^2d(a+b \tan(c+dx))^3} \\
&= \frac{b(9a^2Ab+10Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d(a+b \tan(c+dx))^3} + \frac{(5Ab-2aB) \cot(c+dx)}{2a^2d(a+b \tan(c+dx))^3} \\
&= \frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x}{(a^2+b^2)^4} + \frac{b(9a^2Ab+10Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d(a+b \tan(c+dx))^3} \\
&= \frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)x}{(a^2+b^2)^4} - \frac{(a^2A-10Ab^2+4ab^3)}{a^6}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 6.46, size = 417, normalized size = 0.87

$$\frac{(4Ab-aB)\cot(c+dx)}{2a^2} - \frac{A \cot^2(c+dx)}{2ad} - \frac{(A+Ib)\log(1-\tan(c+dx))}{2(a+Ib)^4} - \frac{(a^2d-10Ab^2+4ab^2B)\log(\tan(c+dx))}{a^2d} - \frac{(A-IB)\log(1+\tan(c+dx))}{2(a-IB)^4} - \frac{b^3(35a^6Ab+56a^4Ab^3+39a^2Ab^5+10Ab^7-20a^7B-24a^5b^2B-16a^3b^4B-4ab^6B)\log(a+b \tan(c+dx))}{a^2d(a^2+b^2)^4} + \frac{b^3(4b-aB)}{3a^3(a^2+b^2)^4} - \frac{b^3(10a^2Ab+17a^2Ab^3+6a^2B-10a^2B^2-2a^2B^3)}{3a^3(a^2+b^2)^4(a+b \tan(c+dx))^3} + \frac{b^3(15a^4Ab+17a^2Ab^3+6a^2B^2-10a^2B^3-9a^2B^4-3a^2B^5)}{3a^3(a^2+b^2)^4(a+b \tan(c+dx))^3}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
[Out] ((4*A*b - a*B)*Cot[c + d*x])/(a^5*d) - (A*Cot[c + d*x]^2)/(2*a^4*d) + ((A +
I*B)*Log[I - Tan[c + d*x]])/(2*(a + I*b)^4*d) - ((a^2*A - 10*A*b^2 + 4*a*b
*B)*Log[Tan[c + d*x]])/(a^6*d) + ((A - I*B)*Log[I + Tan[c + d*x]])/(2*(a -
I*b)^4*d) - (b^3*(35*a^6*A*b + 56*a^4*A*b^3 + 39*a^2*A*b^5 + 10*A*b^7 - 20*
a^7*B - 24*a^5*b^2*B - 16*a^3*b^4*B - 4*a*b^6*B)*Log[a + b*Tan[c + d*x]])/(
a^6*(a^2 + b^2)^4*d) + (b^3*(A*b - a*B))/(3*a^3*(a^2 + b^2)*d*(a + b*Tan[c
+ d*x])^3) + (b^3*(5*a^2*A*b + 3*A*b^3 - 4*a^3*B - 2*a*b^2*B))/(2*a^4*(a^2
+ b^2)^2*d*(a + b*Tan[c + d*x])^2) + (b^3*(15*a^4*A*b + 17*a^2*A*b^3 + 6*A*
b^5 - 10*a^5*B - 9*a^3*b^2*B - 3*a*b^4*B))/(a^5*(a^2 + b^2)^3*d*(a + b*Tan[
c + d*x]))

```

**Maple [A]**

time = 0.88, size = 429, normalized size = 0.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x,method=\_RETURNVERBOS E)

[Out]  $1/d*(-1/2*A/a^4/\tan(d*x+c)^2-(-4*A*b+B*a)/a^5/\tan(d*x+c)+(-A*a^2+10*A*b^2-4*B*a*b)/a^6*\ln(\tan(d*x+c))+1/(a^2+b^2)^4*(1/2*(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*\ln(1+\tan(d*x+c)^2)+(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*\arctan(\tan(d*x+c)))+1/2*b^3*(5*A*a^2*b+3*A*b^3-4*B*a^3-2*B*a*b^2)/a^4/(a^2+b^2)^2/(a+b*\tan(d*x+c))^2+b^3*(15*A*a^4*b+17*A*a^2*b^3+6*A*b^5-10*B*a^5-9*B*a^3*b^2-3*B*a*b^4)/a^5/(a^2+b^2)^3/(a+b*\tan(d*x+c))-b^3*(35*A*a^6*b+56*A*a^4*b^3+39*A*a^2*b^5+10*A*b^7-20*B*a^7-24*B*a^5*b^2-16*B*a^3*b^4-4*B*a*b^6)/a^6/(a^2+b^2)^4*\ln(a+b*\tan(d*x+c))+1/3*(A*b-B*a)*b^3/a^3/(a^2+b^2)/(a+b*\tan(d*x+c))^3)$

**Maxima** [A]

time = 0.55, size = 815, normalized size = 1.71

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out]  $-1/6*(6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(20*B*a^7*b^3 - 35*A*a^6*b^4 + 24*B*a^5*b^5 - 56*A*a^4*b^6 + 16*B*a^3*b^7 - 39*A*a^2*b^8 + 4*B*a*b^9 - 10*A*b^10)*\log(b*\tan(d*x + c) + a)/(a^14 + 4*a^12*b^2 + 6*a^10*b^4 + 4*a^8*b^6 + a^6*b^8) - 3*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (3*A*a^10 + 9*A*a^8*b^2 + 9*A*a^6*b^4 + 3*A*a^4*b^6 + 6*(B*a^7*b^3 - 4*A*a^6*b^4 + 13*B*a^5*b^5 - 27*A*a^4*b^6 + 12*B*a^3*b^7 - 29*A*a^2*b^8 + 4*B*a*b^9 - 10*A*b^10)*\tan(d*x + c)^4 + 3*(6*B*a^8*b^2 - 23*A*a^7*b^3 + 62*B*a^6*b^4 - 134*A*a^5*b^5 + 60*B*a^4*b^6 - 145*A*a^3*b^7 + 20*B*a^2*b^8 - 50*A*a*b^9)*\tan(d*x + c)^3 + (18*B*a^9*b - 63*A*a^8*b^2 + 128*B*a^7*b^3 - 296*A*a^6*b^4 + 130*B*a^5*b^5 - 319*A*a^4*b^6 + 44*B*a^3*b^7 - 110*A*a^2*b^8)*\tan(d*x + c)^2 + 3*(2*B*a^10 - 5*A*a^9*b + 6*B*a^8*b^2 - 15*A*a^7*b^3 + 6*B*a^6*b^4 - 15*A*a^5*b^5 + 2*B*a^4*b^6 - 5*A*a^3*b^7)*\tan(d*x + c))/((a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*\tan(d*x + c)^5 + 3*(a^12*b^2 + 3*a^10*b^4 + 3*a^8*b^6 + a^6*b^8)*\tan(d*x + c)^4 + 3*(a^13*b + 3*a^11*b^3 + 3*a^9*b^5 + a^7*b^7)*\tan(d*x + c)^3 + (a^14 + 3*a^12*b^2 + 3*a^10*b^4 + a^8*b^6)*\tan(d*x + c)^2) + 6*(A*a^2 + 4*B*a*b - 10*A*b^2)*\log(\tan(d*x + c))/a^6)/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1732 vs.  $2(467) = 934$ .

time = 2.29, size = 1732, normalized size = 3.63

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/6*(3*A*a^{13} + 12*A*a^{11}*b^2 + 18*A*a^9*b^4 + 12*A*a^7*b^6 + 3*A*a^5*b^8 \\ & + (3*A*a^{10}*b^3 + 12*A*a^8*b^5 - 74*B*a^7*b^6 + 125*A*a^6*b^7 - 42*B*a^5*b^8 \\ & + 102*A*a^4*b^9 - 12*B*a^3*b^{10} + 30*A*a^2*b^{11} + 6*(B*a^{10}*b^3 - 4*A*a^9 \\ & *b^4 - 6*B*a^8*b^5 + 4*A*a^7*b^6 + B*a^6*b^7)*d*x)*\tan(d*x + c)^5 + 3*(3*A \\ & a^{11}*b^2 + 2*B*a^{10}*b^3 + 4*A*a^9*b^4 - 46*B*a^8*b^5 + 63*A*a^7*b^6 + 8*B \\ & a^6*b^7 - 10*A*a^5*b^8 + 20*B*a^4*b^9 - 48*A*a^3*b^{10} + 8*B*a^2*b^{11} - 20*A \\ & a*b^{12} + 6*(B*a^{11}*b^2 - 4*A*a^{10}*b^3 - 6*B*a^9*b^4 + 4*A*a^8*b^5 + B \\ & a^7*b^6)*d*x)*\tan(d*x + c)^4 + 3*(3*A*a^{12}*b + 6*B*a^{11}*b^2 - 11*A \\ & a^{10}*b^3 - 6*B*a^9*b^4 - 32*A*a^8*b^5 + 80*B*a^7*b^6 - 177*A*a^6*b^7 + 68 \\ & B*a^5*b^8 - 165*A*a^4*b^9 + 20*B*a^3*b^{10} - 50*A*a^2*b^{11} + 6*(B*a^{12}*b - 4 \\ & A*a^{11}*b^2 - 6*B*a^{10}*b^3 + 4*A*a^9*b^4 + B*a^8*b^5)*d*x)*\tan(d*x + c)^3 + (3 \\ & A*a^{13} + 18*B*a^{12}*b - 51*A*a^{11}*b^2 + 72*B*a^{10}*b^3 - 234*A*a^9*b^4 + 216 \\ & B*a^8*b^5 - 513*A*a^7*b^6 + 162*B*a^6*b^7 - 399*A*a^5*b^8 + 44*B*a^4*b^9 - 110 \\ & A*a^3*b^{10} + 6*(B*a^{13} - 4*A*a^{12}*b - 6*B*a^{11}*b^2 + 4*A*a^{10}*b^3 + B \\ & a^9*b^4)*d*x)*\tan(d*x + c)^2 + 3*((A*a^{10}*b^3 + 4*B*a^9*b^4 - 6*A*a^8*b^5 + 16 \\ & B*a^7*b^6 - 34*A*a^6*b^7 + 24*B*a^5*b^8 - 56*A*a^4*b^9 + 16*B*a^3*b^{10} - 39 \\ & A*a^2*b^{11} + 4*B*a*b^{12} - 10*A*b^{13})*\tan(d*x + c)^5 + 3*(A*a^{11}*b^2 + 4 \\ & B*a^{10}*b^3 - 6*A*a^9*b^4 + 16*B*a^8*b^5 - 34*A*a^7*b^6 + 24*B*a^6*b^7 - 56 \\ & A*a^5*b^8 + 16*B*a^4*b^9 - 39*A*a^3*b^{10} + 4*B*a^2*b^{11} - 10*A*a*b^{12})*\tan \\ & (d*x + c)^4 + 3*(A*a^{12}*b + 4*B*a^{11}*b^2 - 6*A*a^{10}*b^3 + 16*B*a^9*b^4 - 34 \\ & A*a^8*b^5 + 24*B*a^7*b^6 - 56*A*a^6*b^7 + 16*B*a^5*b^8 - 39*A*a^4*b^9 + 4 \\ & B*a^3*b^{10} - 10*A*a^2*b^{11})*\tan(d*x + c)^3 + (A*a^{13} + 4*B*a^{12}*b - 6 \\ & A*a^{11}*b^2 + 16*B*a^{10}*b^3 - 34*A*a^9*b^4 + 24*B*a^8*b^5 - 56*A*a^7*b^6 + 16 \\ & B*a^6*b^7 - 39*A*a^5*b^8 + 4*B*a^4*b^9 - 10*A*a^3*b^{10})*\tan(d*x + c)^2)*\log \\ & (\tan(d*x + c)^2/( \tan(d*x + c)^2 + 1)) - 3*((20*B*a^7*b^6 - 35*A*a^6*b^7 + 24 \\ & B*a^5*b^8 - 56*A*a^4*b^9 + 16*B*a^3*b^{10} - 39*A*a^2*b^{11} + 4*B*a*b^{12} - 10 \\ & A*b^{13})*\tan(d*x + c)^5 + 3*(20*B*a^8*b^5 - 35*A*a^7*b^6 + 24*B*a^6*b^7 - 56 \\ & A*a^5*b^8 + 16*B*a^4*b^9 - 39*A*a^3*b^{10} + 4*B*a^2*b^{11} - 10*A*a*b^{12})*\tan \\ & (d*x + c)^4 + 3*(20*B*a^9*b^4 - 35*A*a^8*b^5 + 24*B*a^7*b^6 - 56*A*a^6*b^7 + 16 \\ & B*a^5*b^8 - 39*A*a^4*b^9 + 4*B*a^3*b^{10} - 10*A*a^2*b^{11})*\tan(d*x + c)^3 + (20 \\ & B*a^{10}*b^3 - 35*A*a^9*b^4 + 24*B*a^8*b^5 - 56*A*a^7*b^6 + 16*B*a^6*b^7 - 39 \\ & A*a^5*b^8 + 4*B*a^4*b^9 - 10*A*a^3*b^{10})*\tan(d*x + c)^2)*\log((b^2*\tan(d*x + c) \\ & )^2 + 2*a*b*\tan(d*x + c) + a^2)/( \tan(d*x + c)^2 + 1)) + 3*(2*B*a^{13} - 5 \\ & A*a^{12}*b + 8*B*a^{11}*b^2 - 20*A*a^{10}*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 + 8 \\ & B*a^7*b^6 - 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*\tan(d*x + c))/((a^{14}*b^3 \\ & + 4*a^{12}*b^5 + 6*a^{10}*b^7 + 4*a^8*b^9 + a^6*b^{11})*d*\tan(d*x + c)^5 + 3*(a^{15} \\ & b^2 + 4*a^{13}*b^4 + 6*a^{11}*b^6 + 4*a^9*b^8 + a^7*b^{10})*d*\tan(d*x + c)^4 + 3 \\ & *(a^{16}*b + 4*a^{14}*b^3 + 6*a^{12}*b^5 + 4*a^{10}*b^7 + a^8*b^9)*d*\tan(d*x + c)^3 \\ & + (a^{17} + 4*a^{15}*b^2 + 6*a^{13}*b^4 + 4*a^{11}*b^6 + a^9*b^8)*d*\tan(d*x + c)^2) \end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*4,x)**[Out]** Exception raised: AttributeError >> 'NoneType' object has no attribute 'primitive'**Giac** [A]

time = 1.20, size = 903, normalized size = 1.89

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="giac")

**[Out]** 
$$-1/6*(6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 3*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(20*B*a^7*b^4 - 35*A*a^6*b^5 + 24*B*a^5*b^6 - 56*A*a^4*b^7 + 16*B*a^3*b^8 - 39*A*a^2*b^9 + 4*B*a*b^{10} - 10*A*b^{11})*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^{14}*b + 4*a^{12}*b^3 + 6*a^{10}*b^5 + 4*a^8*b^7 + a^6*b^9) + (220*B*a^7*b^6*\tan(d*x + c)^3 - 385*A*a^6*b^7*\tan(d*x + c)^3 + 264*B*a^5*b^8*\tan(d*x + c)^3 - 616*A*a^4*b^9*\tan(d*x + c)^3 + 176*B*a^3*b^{10}*\tan(d*x + c)^3 - 429*A*a^2*b^{11}*\tan(d*x + c)^3 + 44*B*a*b^{12}*\tan(d*x + c)^3 - 110*A*b^{13}*\tan(d*x + c)^3 + 720*B*a^8*b^5*\tan(d*x + c)^2 - 1245*A*a^7*b^6*\tan(d*x + c)^2 + 906*B*a^6*b^7*\tan(d*x + c)^2 - 2040*A*a^5*b^8*\tan(d*x + c)^2 + 600*B*a^4*b^9*\tan(d*x + c)^2 - 1425*A*a^3*b^{10}*\tan(d*x + c)^2 + 150*B*a^2*b^{11}*\tan(d*x + c)^2 - 366*A*a*b^{12}*\tan(d*x + c)^2 + 792*B*a^9*b^4*\tan(d*x + c) - 1350*A*a^8*b^5*\tan(d*x + c) + 1050*B*a^7*b^6*\tan(d*x + c) - 2271*A*a^6*b^7*\tan(d*x + c) + 696*B*a^5*b^8*\tan(d*x + c) - 1596*A*a^4*b^9*\tan(d*x + c) + 174*B*a^3*b^{10}*\tan(d*x + c) - 411*A*a^2*b^{11}*\tan(d*x + c) + 294*B*a^{10}*b^3 - 492*A*a^9*b^4 + 414*B*a^8*b^5 - 853*A*a^7*b^6 + 278*B*a^6*b^7 - 606*A*a^5*b^8 + 70*B*a^4*b^9 - 157*A*a^3*b^{10})/((a^{14} + 4*a^{12}*b^2 + 6*a^{10}*b^4 + 4*a^8*b^6 + a^6*b^8)*(b*\tan(d*x + c) + a)^3) + 6*(A*a^2 + 4*B*a*b - 10*A*b^2)*\log(\text{abs}(\tan(d*x + c)))/a^6 - 3*(3*A*a^2*\tan(d*x + c)^2 + 12*B*a*b*\tan(d*x + c)^2 - 30*A*b^2*\tan(d*x + c)^2 - 2*B*a^2*\tan(d*x + c) + 8*A*a*b*\tan(d*x + c) - A*a^2)/(a^6*\tan(d*x + c)^2))/d$$

**Mupad** [B]

time = 12.38, size = 664, normalized size = 1.39

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\cot(c + d*x))^3*(A + B*\tan(c + d*x)))/(a + b*\tan(c + d*x))^4, x)$

[Out]  $((\tan(c + d*x)*(5*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (\tan(c + d*x))^4*(10*A*b^{10} + 29*A*a^2*b^8 + 27*A*a^4*b^6 + 4*A*a^6*b^4 - 12*B*a^3*b^7 - 13*B*a^5*b^5 - B*a^7*b^3 - 4*B*a*b^9))/(a^5*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (\tan(c + d*x))^3*(50*A*b^9 + 145*A*a^2*b^7 + 134*A*a^4*b^5 + 23*A*a^6*b^3 - 60*B*a^3*b^6 - 62*B*a^5*b^4 - 6*B*a^7*b^2 - 20*B*a*b^8))/(2*a^4*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (\tan(c + d*x))^2*(110*A*b^8 + 319*A*a^2*b^6 + 296*A*a^4*b^4 + 63*A*a^6*b^2 - 130*B*a^3*b^5 - 128*B*a^5*b^3 - 44*B*a*b^7 - 18*B*a^7*b))/(6*a^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3*\tan(c + d*x)^2 + b^3*\tan(c + d*x)^5 + 3*a^2*b*\tan(c + d*x)^3 + 3*a*b^2*\tan(c + d*x)^4)) - (\log(\tan(c + d*x))*(A*a^2 - 10*A*b^2 + 4*B*a*b))/(a^6*d) + (\log(\tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) + (\log(\tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)) - (\log(a + b*\tan(c + d*x))*(10*A*b^{10} + 39*A*a^2*b^8 + 56*A*a^4*b^6 + 35*A*a^6*b^4 - 16*B*a^3*b^7 - 24*B*a^5*b^5 - 20*B*a^7*b^3 - 4*B*a*b^9))/(d*(a^{14} + a^6*b^8 + 4*a^8*b^6 + 6*a^{10}*b^4 + 4*a^{12}*b^2))$



$$3.298 \quad \int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=29

$$\frac{B \log(\cos(c+dx))}{d} + \frac{B \tan^2(c+dx)}{2d}$$

[Out] B\*ln(cos(d\*x+c))/d+1/2\*B\*tan(d\*x+c)^2/d

**Rubi** [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {21, 3554, 3556}

$$\frac{B \tan^2(c+dx)}{2d} + \frac{B \log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] (B\*Log[Cos[c + d\*x]])/d + (B\*Tan[c + d\*x]^2)/(2\*d)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)(aB + bB \tan(c+dx))}{a + b \tan(c+dx)} dx = B \int \tan^3(c+dx) dx$$

$$= \frac{B \tan^2(c+dx)}{2d} - B \int \tan(c+dx) dx$$

$$= \frac{B \log(\cos(c+dx))}{d} + \frac{B \tan^2(c+dx)}{2d}$$

**Mathematica [A]**

time = 0.02, size = 26, normalized size = 0.90

$$\frac{B(2 \log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] (B\*(2\*Log[Cos[c + d\*x]] + Tan[c + d\*x]^2))/(2\*d)

**Maple [A]**

time = 0.07, size = 30, normalized size = 1.03

method	result	size
derivativedivides	$\frac{B \left( \frac{\tan^2(dx+c)}{2} - \frac{\ln(1+\tan^2(dx+c))}{2} \right)}{d}$	30
default	$\frac{B \left( \frac{\tan^2(dx+c)}{2} - \frac{\ln(1+\tan^2(dx+c))}{2} \right)}{d}$	30
norman	$\frac{B \tan^2(dx+c)}{2d} - \frac{B \ln(1+\tan^2(dx+c))}{2d}$	33
risch	$-iBx - \frac{2iBc}{d} + \frac{2Be^{2i(dx+c)}}{d(e^{2i(dx+c)}+1)^2} + \frac{B \ln(e^{2i(dx+c)}+1)}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*B\*(1/2\*tan(d\*x+c)^2-1/2\*ln(1+tan(d\*x+c)^2))

**Maxima [A]**

time = 0.54, size = 30, normalized size = 1.03

$$\frac{B \tan(dx+c)^2 - B \log(\tan(dx+c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*(B\*tan(d\*x + c)^2 - B\*log(tan(d\*x + c)^2 + 1))/d

**Fricas** [A]

time = 1.90, size = 31, normalized size = 1.07

$$\frac{B \tan(dx + c)^2 + B \log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(B\*tan(d\*x + c)^2 + B\*log(1/(tan(d\*x + c)^2 + 1)))/d

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs.  $2(24) = 48$ .

time = 0.39, size = 53, normalized size = 1.83

$$\begin{cases} -\frac{B \log(\tan^2(c+dx)+1)}{2d} + \frac{B \tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb \tan(c)) \tan^3(c)}{a+b \tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((-B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*tan(c + d\*x)\*\*2/(2\*d), Ne(d, 0)), (x\*(B\*a + B\*b\*tan(c))\*tan(c)\*\*3/(a + b\*tan(c)), True))

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs.  $2(27) = 54$ .

time = 0.72, size = 187, normalized size = 6.45

$$\frac{B \log\left(\left|-\frac{\cos(dx+c)+1}{\cos(dx+c)-1} - \frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 2\right|\right) - B \log\left(\left|-\frac{\cos(dx+c)+1}{\cos(dx+c)-1} - \frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 2\right|\right) + \frac{B\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1} + \frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right) + 6B}{\frac{\cos(dx+c)+1}{\cos(dx+c)-1} + \frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] -1/2\*(B\*log(abs(-(cos(d\*x + c) + 1)/(cos(d\*x + c) - 1) - (cos(d\*x + c) - 1)/(cos(d\*x + c) + 1) + 2)) - B\*log(abs(-(cos(d\*x + c) + 1)/(cos(d\*x + c) - 1

) - (cos(d\*x + c) - 1)/(cos(d\*x + c) + 1) - 2)) + (B\*((cos(d\*x + c) + 1)/(cos(d\*x + c) - 1) + (cos(d\*x + c) - 1)/(cos(d\*x + c) + 1)) + 6\*B)/((cos(d\*x + c) + 1)/(cos(d\*x + c) - 1) + (cos(d\*x + c) - 1)/(cos(d\*x + c) + 1) + 2))/d

**Mupad [B]**

time = 6.21, size = 28, normalized size = 0.97

$$\frac{B (\ln (\tan(c + dx)^2 + 1) - \tan(c + dx)^2)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] -(B\*(log(tan(c + d\*x)^2 + 1) - tan(c + d\*x)^2))/(2\*d)

$$3.299 \quad \int \frac{\tan^2(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=16

$$-Bx + \frac{B \tan(c+dx)}{d}$$

[Out]  $-B*x+B*\tan(d*x+c)/d$

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {21, 3554, 8}

$$\frac{B \tan(c+dx)}{d} - Bx$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c + d*x]^2*(a*B + b*B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x]),x]$

[Out]  $-(B*x) + (B*\text{Tan}[c + d*x])/d$

Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 21

$\text{Int}[(u_)*((a_) + (b_)*(v_))^{(m_)*((c_) + (d_)*(v_))^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

Rule 3554

$\text{Int}[(b_)*\tan[(c_) + (d_)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[b*((b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1]$

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx &= B \int \tan^2(c+dx) dx \\ &= \frac{B \tan(c+dx)}{d} - B \int 1 dx \\ &= -Bx + \frac{B \tan(c+dx)}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.01, size = 25, normalized size = 1.56

$$B \left( -\frac{\text{ArcTan}(\tan(c + dx))}{d} + \frac{\tan(c + dx)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] B\*(-(ArcTan[Tan[c + d\*x]]/d) + Tan[c + d\*x]/d)

**Maple [A]**

time = 0.05, size = 22, normalized size = 1.38

method	result	size
norman	$-Bx + \frac{B \tan(dx+c)}{d}$	17
derivativedivides	$\frac{B(\tan(dx+c) - \arctan(\tan(dx+c)))}{d}$	22
default	$\frac{B(\tan(dx+c) - \arctan(\tan(dx+c)))}{d}$	22
risch	$-Bx + \frac{2iB}{d(e^{2i(dx+c)} + 1)}$	26

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*B\*(tan(d\*x+c)-arctan(tan(d\*x+c)))

**Maxima [A]**

time = 0.56, size = 22, normalized size = 1.38

$$\frac{(dx + c)B - B \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] -((d\*x + c)\*B - B\*tan(d\*x + c))/d

**Fricas [A]**

time = 1.31, size = 19, normalized size = 1.19

$$\frac{Bdx - B \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-(B*d*x - B*\tan(d*x + c))/d$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 36 vs.  $2(12) = 24$ .

time = 0.31, size = 36, normalized size = 2.25

$$\begin{cases} -Bx + \frac{B \tan(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb \tan(c)) \tan^2(c)}{a+b \tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((-B\*x + B\*tan(c + d\*x)/d, Ne(d, 0)), (x\*(B\*a + B\*b\*tan(c))\*tan(c)\*\*2/(a + b\*tan(c)), True))

**Giac [A]**

time = 0.57, size = 22, normalized size = 1.38

$$\frac{(dx + c)B - B \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $-((d*x + c)*B - B*\tan(d*x + c))/d$

**Mupad [B]**

time = 6.19, size = 16, normalized size = 1.00

$$\frac{B \tan(c + dx)}{d} - Bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out]  $(B*\tan(c + d*x))/d - B*x$

$$3.300 \quad \int \frac{\tan(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=13

$$-\frac{B \log(\cos(c+dx))}{d}$$

[Out] -B\*ln(cos(d\*x+c))/d

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {21, 3556}

$$-\frac{B \log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -((B\*Log[Cos[c + d\*x]])/d)

Rule 21

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_))^(m\_.)\*((c\_.) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :>  
Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]  
&& EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d\*x,  
a + b\*x])

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d  
\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx &= B \int \tan(c+dx) dx \\ &= -\frac{B \log(\cos(c+dx))}{d} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 13, normalized size = 1.00

$$-\frac{B \log(\cos(c+dx))}{d}$$



Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -((B\*Log[Cos[c + d\*x]])/d)

**Maple [A]**

time = 0.05, size = 18, normalized size = 1.38

method	result	size
derivativedivides	$\frac{B \ln(1+\tan^2(dx+c))}{2d}$	18
default	$\frac{B \ln(1+\tan^2(dx+c))}{2d}$	18
norman	$\frac{B \ln(1+\tan^2(dx+c))}{2d}$	18
risch	$iBx + \frac{2iBc}{d} - \frac{B \ln(e^{2i(dx+c)}+1)}{d}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/2\*B/d\*ln(1+tan(d\*x+c)^2)

**Maxima [A]**

time = 0.53, size = 17, normalized size = 1.31

$$\frac{B \log(\tan(dx+c)^2 + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*B\*log(tan(d\*x + c)^2 + 1)/d

**Fricas [A]**

time = 1.69, size = 19, normalized size = 1.46

$$-\frac{B \log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] -1/2\*B\*log(1/(tan(d\*x + c)^2 + 1))/d

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 37 vs.  $2(12) = 24$ .

time = 0.28, size = 37, normalized size = 2.85

$$\begin{cases} \frac{B \log(\tan^2(c+dx)+1)}{2d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb \tan(c)) \tan(c)}{a+b \tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d), Ne(d, 0)), (x\*(B\*a + B\*b\*tan(c))\*tan(c)/(a + b\*tan(c)), True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 99 vs.  $2(13) = 26$ .

time = 0.47, size = 99, normalized size = 7.62

$$\frac{B \log\left(\left|-\frac{\cos(dx+c)+1}{\cos(dx+c)-1}-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+2\right|\right)-B \log\left(\left|-\frac{\cos(dx+c)+1}{\cos(dx+c)-1}-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-2\right|\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{2} * (B * \log(\frac{-(\cos(dx+c)+1)}{\cos(dx+c)-1} - (\cos(dx+c)-1)/(\cos(dx+c)+1) + 2)) - B * \log(\frac{-(\cos(dx+c)+1)}{\cos(dx+c)-1} - (\cos(dx+c)-1)/(\cos(dx+c)+1) - 2)) / d$

**Mupad [B]**

time = 6.24, size = 17, normalized size = 1.31

$$\frac{B \ln(\tan(c+dx)^2+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] (B\*log(tan(c + d\*x)^2 + 1))/(2\*d)

$$3.301 \quad \int \frac{aB + bB \tan(c + dx)}{a + b \tan(c + dx)} dx$$

Optimal. Leaf size=3

$$Bx$$

[Out] B\*x

Rubi [A]

time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {21, 8}

$$Bx$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]),x]

[Out] B\*x

Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rubi steps

$$\int \frac{aB + bB \tan(c + dx)}{a + b \tan(c + dx)} dx = B \int 1 dx = Bx$$

Mathematica [A]

time = 0.00, size = 3, normalized size = 1.00

$$Bx$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]),x]

[Out]  $Bx$

**Maple** [A]

time = 0.03, size = 4, normalized size = 1.33

method	result	size
default	$Bx$	4
norman	$Bx$	4
risch	$Bx$	4
derivativedivides	$\frac{B \arctan(\tan(dx+c))}{d}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $Bx$

**Maxima** [C] Result contains higher order function than in optimal. Order 3 vs. order 1.

time = 0.52, size = 10, normalized size = 3.33

$$\frac{(dx + c)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $(d*x + c)*B/d$

**Fricas** [A]

time = 1.25, size = 3, normalized size = 1.00

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $Bx$

**Sympy** [A]

time = 0.06, size = 2, normalized size = 0.67

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)`

[Out]  $Bx$

**Giac [C]** Result contains higher order function than in optimal. Order 3 vs. order 1.  
time = 0.43, size = 10, normalized size = 3.33

$$\frac{(dx + c)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] (d\*x + c)\*B/d

**Mupad [B]**

time = 6.26, size = 3, normalized size = 1.00

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(a + b\*tan(c + d\*x)),x)

[Out] B\*x

$$3.302 \quad \int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=12

$$\frac{B \log(\sin(c+dx))}{d}$$

[Out] B\*ln(sin(d\*x+c))/d

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {21, 3556}

$$\frac{B \log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] (B\*Log[Sin[c + d\*x]])/d

Rule 21

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_))^(m\_.)\*((c\_.) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :>  
Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]  
&& EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d\*x,  
a + b\*x])

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx = B \int \cot(c+dx) dx = \frac{B \log(\sin(c+dx))}{d}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 1.67

$$\frac{B(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] (B\*(Log[Cos[c + d\*x]] + Log[Tan[c + d\*x]]))/d

**Maple [A]**

time = 0.10, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{B \ln(\sin(dx+c))}{d}$	13
default	$\frac{B \ln(\sin(dx+c))}{d}$	13
norman	$\frac{B \ln(\tan(dx+c))}{d} - \frac{B \ln(1+\tan^2(dx+c))}{2d}$	31
risch	$-iBx - \frac{2iBc}{d} + \frac{B \ln(e^{2i(dx+c)}-1)}{d}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] B\*ln(sin(d\*x+c))/d

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

time = 0.51, size = 29, normalized size = 2.42

$$\frac{B \log(\tan(dx+c)^2 + 1) - 2B \log(\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] -1/2\*(B\*log(tan(d\*x + c)^2 + 1) - 2\*B\*log(tan(d\*x + c)))/d

**Fricas [A]**

time = 1.28, size = 20, normalized size = 1.67

$$\frac{B \log\left(-\frac{1}{2} \cos(2dx + 2c) + \frac{1}{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*B\*log(-1/2\*cos(2\*d\*x + 2\*c) + 1/2)/d

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 49 vs.  $2(10) = 20$ .

time = 0.45, size = 49, normalized size = 4.08

$$\begin{cases} -\frac{B \log(\tan^2(c+dx)+1)}{2d} + \frac{B \log(\tan(c+dx))}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb \tan(c)) \cot(c)}{a+b \tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((-B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*log(tan(c + d\*x))/d, Ne(d, 0)), (x\*(B\*a + B\*b\*tan(c))\*cot(c)/(a + b\*tan(c)), True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 59 vs.  $2(12) = 24$ .

time = 0.49, size = 59, normalized size = 4.92

$$\frac{B \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2B \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(B\*log(abs(-cos(d\*x + c) + 1)/abs(cos(d\*x + c) + 1)) - 2\*B\*log(abs(-(cos(d\*x + c) - 1)/(cos(d\*x + c) + 1) + 1)))/d

**Mupad [B]**

time = 6.27, size = 27, normalized size = 2.25

$$-\frac{B (\ln(\tan(c + dx)^2 + 1) - 2 \ln(\tan(c + dx)))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] -(B\*(log(tan(c + d\*x)^2 + 1) - 2\*log(tan(c + d\*x))))/(2\*d)



$$3.303 \quad \int \frac{\cot^2(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=17

$$-Bx - \frac{B \cot(c+dx)}{d}$$

[Out] -B\*x-B\*cot(d\*x+c)/d

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {21, 3554, 8}

$$-\frac{B \cot(c+dx)}{d} - Bx$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -(B\*x) - (B\*Cot[c + d\*x])/d

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 3554

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*((b\*Tan[c + d\*x])^(n - 1)/(d\*(n - 1))), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int \frac{\cot^2(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx &= B \int \cot^2(c+dx) dx \\ &= -\frac{B \cot(c+dx)}{d} - B \int 1 dx \\ &= -Bx - \frac{B \cot(c+dx)}{d} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 30, normalized size = 1.76

$$\frac{B \cot(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(c + dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -((B\*Cot[c + d\*x]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d\*x]^2])/d)

**Maple [A]**

time = 0.09, size = 22, normalized size = 1.29

method	result	size
derivativedivides	$\frac{B(-\cot(dx+c)-dx-c)}{d}$	22
default	$\frac{B(-\cot(dx+c)-dx-c)}{d}$	22
risch	$-Bx - \frac{2iB}{d(e^{2i(dx+c)}-1)}$	26
norman	$\frac{-\frac{B}{d} - Bx \tan(dx+c)}{\tan(dx+c)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*B\*(-cot(d\*x+c)-d\*x-c)

**Maxima [A]**

time = 0.67, size = 23, normalized size = 1.35

$$\frac{(dx + c)B + \frac{B}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] -((d\*x + c)\*B + B/tan(d\*x + c))/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.

time = 2.62, size = 42, normalized size = 2.47

$$\frac{Bdx \sin(2dx + 2c) + B \cos(2dx + 2c) + B}{d \sin(2dx + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-(B*d*x*\sin(2*d*x + 2*c) + B*\cos(2*d*x + 2*c) + B)/(d*\sin(2*d*x + 2*c))$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs.  $2(14) = 28$ .

time = 0.41, size = 37, normalized size = 2.18

$$\begin{cases} -Bx - \frac{B \cot(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb \tan(c)) \cot^2(c)}{a+b \tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)`

[Out] `Piecewise((-B*x - B*cot(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*tan(c))*cot(c)**2/(a + b*tan(c)), True))`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs.  $2(17) = 34$ .  
time = 0.49, size = 39, normalized size = 2.29

$$-\frac{2(dx+c)B - B \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{B}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac")`

[Out]  $-1/2*(2*(d*x + c)*B - B*\tan(1/2*d*x + 1/2*c) + B/\tan(1/2*d*x + 1/2*c))/d$

**Mupad** [B]

time = 6.25, size = 16, normalized size = 0.94

$$-\frac{B(\cot(c+dx) + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cot(c + d*x)^2*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)`

[Out]  $-(B*(\cot(c + d*x) + d*x))/d$

$$3.304 \quad \int \frac{\cot^3(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=30

$$-\frac{B \cot^2(c+dx)}{2d} - \frac{B \log(\sin(c+dx))}{d}$$

[Out]  $-1/2*B*\cot(d*x+c)^2/d-B*\ln(\sin(d*x+c))/d$

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {21, 3554, 3556}

$$-\frac{B \cot^2(c+dx)}{2d} - \frac{B \log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[(Cot[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]`

[Out]  $-1/2*(B*\cot[c + d*x]^2)/d - (B*\log[\sin[c + d*x]])/d$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)} dx &= B \int \cot^3(c+dx) dx \\ &= -\frac{B \cot^2(c+dx)}{2d} - B \int \cot(c+dx) dx \\ &= -\frac{B \cot^2(c+dx)}{2d} - \frac{B \log(\sin(c+dx))}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 35, normalized size = 1.17

$$-\frac{B(\cot^2(c+dx) + 2 \log(\cos(c+dx)) + 2 \log(\tan(c+dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -1/2\*(B\*(Cot[c + d\*x]^2 + 2\*Log[Cos[c + d\*x]] + 2\*Log[Tan[c + d\*x]]))/d

**Maple [A]**

time = 0.12, size = 26, normalized size = 0.87

method	result	size
derivativedivides	$\frac{B\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)}{d}$	26
default	$\frac{B\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c))\right)}{d}$	26
norman	$-\frac{B}{2d \tan(dx+c)^2} - \frac{B \ln(\tan(dx+c))}{d} + \frac{B \ln(1+\tan^2(dx+c))}{2d}$	46
risch	$iBx + \frac{2iBc}{d} + \frac{2Be^{2i(dx+c)}}{d(e^{2i(dx+c)}-1)^2} - \frac{B \ln(e^{2i(dx+c)}-1)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERB OSE)

[Out] 1/d\*B\*(-1/2\*cot(d\*x+c)^2-ln(sin(d\*x+c)))

**Maxima [A]**

time = 0.58, size = 40, normalized size = 1.33

$$\frac{B \log(\tan(dx+c)^2 + 1) - 2B \log(\tan(dx+c)) - \frac{B}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(B*\log(\tan(d*x + c)^2 + 1) - 2*B*\log(\tan(d*x + c)) - B/\tan(d*x + c)^2)/d$

**Fricas** [A]

time = 1.51, size = 53, normalized size = 1.77

$$\frac{(B \cos(2 dx + 2 c) - B) \log\left(-\frac{1}{2} \cos(2 dx + 2 c) + \frac{1}{2}\right) - 2 B}{2(d \cos(2 dx + 2 c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-\frac{1}{2}*((B*\cos(2*d*x + 2*c) - B)*\log(-1/2*\cos(2*d*x + 2*c) + 1/2) - 2*B)/(d*\cos(2*d*x + 2*c) - d)$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

time = 0.85, size = 80, normalized size = 2.67

$$\begin{cases} \infty Bx & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ \frac{x(Ba + Bb \tan(c)) \cot^3(c)}{a + b \tan(c)} & \text{for } d = 0 \\ \frac{B \log(\tan^2(c + dx) + 1)}{2d} - \frac{B \log(\tan(c + dx))}{d} - \frac{B}{2d \tan^2(c + dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)`

[Out] `Piecewise((zoo*B*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(B*a + B*b*tan(c))*cot(c)**3/(a + b*tan(c)), Eq(d, 0)), (B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2), True))`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(28) = 56.

time = 0.52, size = 124, normalized size = 4.13

$$\frac{4 B \log\left(\frac{|\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 8 B \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - \frac{\left(B + \frac{4 B (\cos(dx+c)-1)}{\cos(dx+c)+1}\right) (\cos(dx+c)+1)}{\cos(dx+c)-1} - \frac{B (\cos(dx+c)-1)}{\cos(dx+c)+1}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] 
$$-1/8*(4*B*\log(\text{abs}(-\cos(d*x + c) + 1)/\text{abs}(\cos(d*x + c) + 1)) - 8*B*\log(\text{abs}(-(\cos(d*x + c) - 1)/(\cos(d*x + c) + 1) + 1)) - (B + 4*B*(\cos(d*x + c) - 1)/(\cos(d*x + c) + 1))*(\cos(d*x + c) + 1)/(\cos(d*x + c) - 1) - B*(\cos(d*x + c) - 1)/(\cos(d*x + c) + 1))/d$$

**Mupad [B]**

time = 6.24, size = 37, normalized size = 1.23

$$-\frac{B(\cot(c + dx)^2 - \ln(\tan(c + dx)^2 + 1) + 2 \ln(\tan(c + dx)))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] 
$$-(B*(2*\log(\tan(c + d*x)) - \log(\tan(c + d*x)^2 + 1) + \cot(c + d*x)^2))/(2*d)$$

$$3.305 \quad \int \frac{\cot^4(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=31

$$Bx + \frac{B \cot(c+dx)}{d} - \frac{B \cot^3(c+dx)}{3d}$$

[Out] B\*x+B\*cot(d\*x+c)/d-1/3\*B\*cot(d\*x+c)^3/d

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$ , Rules used = {21, 3554, 8}

$$-\frac{B \cot^3(c+dx)}{3d} + \frac{B \cot(c+dx)}{d} + Bx$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^4\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] B\*x + (B\*Cot[c + d\*x])/d - (B\*Cot[c + d\*x]^3)/(3\*d)

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 3554

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*((b\*Tan[c + d\*x])^(n - 1)/(d\*(n - 1))), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps



$$\begin{aligned}
\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)} dx &= B \int \cot^4(c+dx) dx \\
&= -\frac{B \cot^3(c+dx)}{3d} - B \int \cot^2(c+dx) dx \\
&= \frac{B \cot(c+dx)}{d} - \frac{B \cot^3(c+dx)}{3d} + B \int 1 dx \\
&= Bx + \frac{B \cot(c+dx)}{d} - \frac{B \cot^3(c+dx)}{3d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 34, normalized size = 1.10

$$-\frac{B \cot^3(c+dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^4\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -1/3\*(B\*Cot[c + d\*x]^3\*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d\*x]^2])/d

**Maple [A]**

time = 0.10, size = 27, normalized size = 0.87

method	result	size
derivativedivides	$\frac{B \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right)}{d}$	27
default	$\frac{B \left( -\frac{\cot^3(dx+c)}{3} + \cot(dx+c) + dx+c \right)}{d}$	27
norman	$\frac{Bx(\tan^3(dx+c)) + \frac{B(\tan^2(dx+c))}{d} - \frac{B}{3d}}{\tan(dx+c)^3}$	41
risch	$Bx + \frac{4iB(3e^{4i(dx+c)} - 3e^{2i(dx+c)} + 2)}{3d(e^{2i(dx+c)} - 1)^3}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERB  
OSE)

[Out] 1/d\*B\*(-1/3\*cot(d\*x+c)^3+cot(d\*x+c)+d\*x+c)

**Maxima [A]**

time = 0.53, size = 38, normalized size = 1.23

$$\frac{3(dx+c)B + \frac{3B \tan(dx+c)^2 - B}{\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/3\*(3\*(d\*x + c)\*B + (3\*B\*tan(d\*x + c)^2 - B)/tan(d\*x + c)^3)/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(29) = 58.

time = 2.43, size = 90, normalized size = 2.90

$$\frac{4B \cos(2dx+2c)^2 + 2B \cos(2dx+2c) + 3(Bdx \cos(2dx+2c) - Bdx \sin(2dx+2c) - 2B}{3(d \cos(2dx+2c) - d) \sin(2dx+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/3\*(4\*B\*cos(2\*d\*x + 2\*c)^2 + 2\*B\*cos(2\*d\*x + 2\*c) + 3\*(B\*d\*x\*cos(2\*d\*x + 2\*c) - B\*d\*x)\*sin(2\*d\*x + 2\*c) - 2\*B)/((d\*cos(2\*d\*x + 2\*c) - d)\*sin(2\*d\*x + 2\*c))

**Sympy [A]**

time = 0.96, size = 49, normalized size = 1.58

$$\begin{cases} Bx - \frac{B \cot^3(c+dx)}{3d} + \frac{B \cot(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb \tan(c)) \cot^4(c)}{a+b \tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((B\*x - B\*cot(c + d\*x)\*\*3/(3\*d) + B\*cot(c + d\*x)/d, Ne(d, 0)), (x\*(B\*a + B\*b\*tan(c))\*cot(c)\*\*4/(a + b\*tan(c)), True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.  
time = 0.53, size = 69, normalized size = 2.23

$$\frac{B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 24(dx+c)B - 15B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{15B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - B}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/24*(B*tan(1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*B - 15*B*tan(1/2*d*x + 1/2*c)
+ (15*B*tan(1/2*d*x + 1/2*c)^2 - B)/tan(1/2*d*x + 1/2*c)^3)/d
```

**Mupad [B]**

time = 6.29, size = 32, normalized size = 1.03

$$Bx - \frac{\frac{B}{3} - B \tan(c + dx)^2}{d \tan(c + dx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

```
[Out] B*x - (B/3 - B*tan(c + d*x)^2)/(d*tan(c + d*x)^3)
```

$$3.306 \quad \int \frac{\tan^4(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=102

$$\frac{aBx}{a^2+b^2} + \frac{bB \log(\cos(c+dx))}{(a^2+b^2)d} + \frac{a^4B \log(a+b \tan(c+dx))}{b^3(a^2+b^2)d} - \frac{aB \tan(c+dx)}{b^2d} + \frac{B \tan^2(c+dx)}{2bd}$$

[Out] a\*B\*x/(a^2+b^2)+b\*B\*ln(cos(d\*x+c))/(a^2+b^2)/d+a^4\*B\*ln(a+b\*tan(d\*x+c))/b^3/(a^2+b^2)/d-a\*B\*tan(d\*x+c)/b^2/d+1/2\*B\*tan(d\*x+c)^2/b/d

**Rubi [A]**

time = 0.17, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {21, 3647, 3728, 3708, 3698, 31, 3556}

$$\frac{bB \log(\cos(c+dx))}{d(a^2+b^2)} + \frac{aBx}{a^2+b^2} + \frac{a^4B \log(a+b \tan(c+dx))}{b^3d(a^2+b^2)} - \frac{aB \tan(c+dx)}{b^2d} + \frac{B \tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^4\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (a\*B\*x)/(a^2 + b^2) + (b\*B\*Log[Cos[c + d\*x]])/((a^2 + b^2)\*d) + (a^4\*B\*Log[a + b\*Tan[c + d\*x]])/(b^3\*(a^2 + b^2)\*d) - (a\*B\*Tan[c + d\*x])/(b^2\*d) + (B\*Tan[c + d\*x]^2)/(2\*b\*d)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
```

```

+ d*Tan[e + f*x]^(n + 1)/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
+ f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3698

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
(f_.)*(x_)^2], x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

```

### Rule 3708

```

Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
Dist[b*((A - C)/(a^2 + b^2)), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]

```

### Rule 3728

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.
) + (f_.)*(x_)^2], x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= B \int \frac{\tan^4(c+dx)}{a+b\tan(c+dx)} dx \\
&= \frac{B \tan^2(c+dx)}{2bd} + \frac{B \int \frac{\tan(c+dx)(-2a-2b\tan(c+dx)-2a\tan^2(c+dx))}{a+b\tan(c+dx)} dx}{2b} \\
&= -\frac{aB \tan(c+dx)}{b^2d} + \frac{B \tan^2(c+dx)}{2bd} + \frac{B \int \frac{2a^2+2(a^2-b^2)\tan^2(c+dx)}{a+b\tan(c+dx)} dx}{2b^2} \\
&= \frac{aBx}{a^2+b^2} - \frac{aB \tan(c+dx)}{b^2d} + \frac{B \tan^2(c+dx)}{2bd} + \frac{(a^4B) \int \frac{1+\tan^2(c+dx)}{a+b\tan(c+dx)} dx}{b^2(a^2+b^2)} \\
&= \frac{aBx}{a^2+b^2} + \frac{bB \log(\cos(c+dx))}{(a^2+b^2)d} - \frac{aB \tan(c+dx)}{b^2d} + \frac{B \tan^2(c+dx)}{2bd} \\
&= \frac{aBx}{a^2+b^2} + \frac{bB \log(\cos(c+dx))}{(a^2+b^2)d} + \frac{a^4B \log(a+b\tan(c+dx))}{b^3(a^2+b^2)d} - \frac{aB \tan(c+dx)}{b^2d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.32, size = 108, normalized size = 1.06

$$\frac{B \left( \frac{\log(i-\tan(c+dx))}{ia-b} - \frac{\log(i+\tan(c+dx))}{ia+b} + \frac{2a^4 \log(a+b\tan(c+dx))}{b^3(a^2+b^2)} - \frac{2a \tan(c+dx)}{b^2} + \frac{\tan^2(c+dx)}{b} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^4\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] (B\*(Log[I - Tan[c + d\*x]]/(I\*a - b) - Log[I + Tan[c + d\*x]]/(I\*a + b) + (2\*a^4\*Log[a + b\*Tan[c + d\*x]])/(b^3\*(a^2 + b^2)) - (2\*a\*Tan[c + d\*x])/b^2 + Tan[c + d\*x]^2/b))/(2\*d)

**Maple [A]**

time = 0.14, size = 93, normalized size = 0.91

method	result
derivativedivides	$ \frac{B \left( -\frac{b(\tan^2(dx+c))}{2b^2} + a \tan(dx+c) + \frac{b \ln(1+\tan^2(dx+c))}{2a^2+b^2} + a \arctan(\tan(dx+c)) + \frac{a^4 \ln(a+b \tan(dx+c))}{b^3(a^2+b^2)} \right)}{d} $
default	$ \frac{B \left( -\frac{b(\tan^2(dx+c))}{2b^2} + a \tan(dx+c) + \frac{b \ln(1+\tan^2(dx+c))}{2a^2+b^2} + a \arctan(\tan(dx+c)) + \frac{a^4 \ln(a+b \tan(dx+c))}{b^3(a^2+b^2)} \right)}{d} $

norman	$\frac{\frac{B a^2 x}{a^2+b^2} + \frac{a^3 B}{d b^3} + \frac{b B a x \tan(dx+c)}{a^2+b^2} + \frac{B(\tan^3(dx+c))}{2d} - \frac{a B(\tan^2(dx+c))}{2bd}}{a+b \tan(dx+c)} + \frac{a^4 B \ln(a+b \tan(dx+c))}{b^3(a^2+b^2)d} - \frac{B b \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$
risch	$-\frac{x B}{i b-a} + \frac{2 i a^2 B x}{b^3} + \frac{2 i a^2 B c}{b^3 d} - \frac{2 i B x}{b} - \frac{2 i B c}{b d} - \frac{2 i a^4 B x}{b^3(a^2+b^2)} - \frac{2 i a^4 B c}{b^3 d(a^2+b^2)} + \frac{2 B(-i a e^{2 i(dx+c)}+b e^{2 i(dx+c)})}{b^2 d(e^{2 i(dx+c)}+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*B\*(-1/b^2\*(-1/2\*b\*tan(d\*x+c)^2+a\*tan(d\*x+c))+1/(a^2+b^2)\*(-1/2\*b\*ln(1+tan(d\*x+c)^2)+a\*arctan(tan(d\*x+c)))+1/b^3\*a^4/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

Maxima [A]

time = 0.50, size = 104, normalized size = 1.02

$$\frac{2 B a^4 \log(b \tan(dx+c)+a)}{a^2 b^3+b^5} + \frac{2(dx+c) B a}{a^2+b^2} - \frac{B b \log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{B b \tan(dx+c)^2-2 B a \tan(dx+c)}{b^2}$$

$2 d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm  
="maxima")

[Out] 1/2\*(2\*B\*a^4\*log(b\*tan(d\*x + c) + a)/(a^2\*b^3 + b^5) + 2\*(d\*x + c)\*B\*a/(a^2 + b^2) - B\*b\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + (B\*b\*tan(d\*x + c)^2 - 2\*B\*a\*tan(d\*x + c))/b^2)/d

Fricas [A]

time = 2.02, size = 144, normalized size = 1.41

$$\frac{2 B a b^3 d x + B a^4 \log\left(\frac{b^2 \tan(dx+c)^2+2 a b \tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right) + (B a^2 b^2 + B b^4) \tan(dx+c)^2 - (B a^4 - B b^4) \log\left(\frac{1}{\tan(dx+c)^2+1}\right) - 2(B a^3 b + B a b^3) \tan(dx+c)}{2(a^2 b^3 + b^5) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm  
="fricas")

[Out] 1/2\*(2\*B\*a\*b^3\*d\*x + B\*a^4\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)) + (B\*a^2\*b^2 + B\*b^4)\*tan(d\*x + c)^2 - (B\*a^4 - B\*b^4)\*log(1/(tan(d\*x + c)^2 + 1)) - 2\*(B\*a^3\*b + B\*a\*b^3)\*tan(d\*x + c))/((a^2\*b^3 + b^5)\*d)

Sympy [C] Result contains complex when optimal does not.

time = 0.97, size = 782, normalized size = 7.67

$\left\{ \begin{array}{l} \infty B x \tan^3(c) \\ B \left( x + \frac{\tan^2(c+dx) - \tan(c+dx)}{d} \right) \\ \frac{3iBdx \tan(c+dx)}{2bd \tan(c+dx)-2ibd} - \frac{3Bdx}{2bd \tan(c+dx)-2ibd} - \frac{2B \log(\tan^2(c+dx)+1) \tan(c+dx)}{2bd \tan(c+dx)-2ibd} + \frac{2iB \log(\tan^2(c+dx)+1)}{2bd \tan(c+dx)-2ibd} + \frac{B \tan^3(c+dx)}{2bd \tan(c+dx)-2ibd} + \frac{iB \tan^2(c+dx)}{2bd \tan(c+dx)-2ibd} - \frac{B \tan(c+dx)}{2bd \tan(c+dx)-2ibd} + \frac{4iB}{2bd \tan(c+dx)-2ibd} \\ \frac{3iBdx \tan(c+dx)}{2bd \tan(c+dx)+2ibd} - \frac{3Bdx}{2bd \tan(c+dx)+2ibd} - \frac{2B \log(\tan^2(c+dx)+1) \tan(c+dx)}{2bd \tan(c+dx)+2ibd} - \frac{2iB \log(\tan^2(c+dx)+1)}{2bd \tan(c+dx)+2ibd} + \frac{B \tan^3(c+dx)}{2bd \tan(c+dx)+2ibd} - \frac{iB \tan^2(c+dx)}{2bd \tan(c+dx)+2ibd} - \frac{B \tan(c+dx)}{2bd \tan(c+dx)+2ibd} - \frac{4iB}{2bd \tan(c+dx)+2ibd} \\ \frac{2(Ba+Bb \tan(c)) \tan^4(c)}{(a+b \tan(c))^2} \\ \frac{2Ba^4 \log\left(\frac{b^2 \tan^2(c+dx)}{2a^2 b^3 d+2b^5 d}\right) - 2Ba^3 b \tan(c+dx)}{2a^2 b^3 d+2b^5 d} + \frac{Ba^2 b^2 \tan^2(c+dx)}{2a^2 b^3 d+2b^5 d} + \frac{2Bab^3 dx}{2a^2 b^3 d+2b^5 d} - \frac{2Bab^3 \tan(c+dx)}{2a^2 b^3 d+2b^5 d} - \frac{Bb^4 \log(\tan^2(c+dx)+1)}{2a^2 b^3 d+2b^5 d} + \frac{Bb^4 \tan^2(c+dx)}{2a^2 b^3 d+2b^5 d} \end{array} \right.$	for $a = 0 \wedge b = 0 \wedge d = 0$ for $b = 0$ for $a = -ib$ for $a = ib$ for $d = 0$ otherwise
---	---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x\*tan(c)\*\*3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B\*(x + tan(c + d\*x))\*\*3/(3\*d) - tan(c + d\*x)/d)/a, Eq(b, 0)), (-3\*I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - 3\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 2\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + I\*B\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 4\*I\*B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (3\*I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 3\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 2\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - I\*B\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 4\*I\*B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), (x\*(B\*a + B\*b\*tan(c))\*tan(c)\*\*4/(a + b\*tan(c))\*\*2, Eq(d, 0)), (2\*B\*a\*\*4\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d) - 2\*B\*a\*\*3\*b\*tan(c + d\*x)/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d) + B\*a\*\*2\*b\*\*2\*tan(c + d\*x)\*\*2/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d) + 2\*B\*a\*b\*\*3\*d\*x/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d) - 2\*B\*a\*b\*\*3\*tan(c + d\*x)/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d) - B\*b\*\*4\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d) + B\*b\*\*4\*tan(c + d\*x)\*\*2/(2\*a\*\*2\*b\*\*3\*d + 2\*b\*\*5\*d), True))

**Giac [A]**

time = 1.04, size = 105, normalized size = 1.03

$$\frac{\frac{2Ba^4 \log(|b \tan(dx+c)+a|)}{a^2 b^3 + b^5} + \frac{2(dx+c)Ba}{a^2 + b^2} - \frac{Bb \log(\tan(dx+c)^2 + 1)}{a^2 + b^2} + \frac{Bb \tan(dx+c)^2 - 2Ba \tan(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 1/2\*(2\*B\*a^4\*log(abs(b\*tan(d\*x + c) + a))/(a^2\*b^3 + b^5) + 2\*(d\*x + c)\*B\*a/(a^2 + b^2) - B\*b\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + (B\*b\*tan(d\*x + c)^2 - 2\*B\*a\*tan(d\*x + c))/b^2)/d

**Mupad [B]**

time = 6.34, size = 114, normalized size = 1.12

$$\frac{B \tan(c + dx)^2}{2bd} - \frac{B \ln(\tan(c + dx) + 1i)}{2d(b + a1i)} - \frac{Ba \tan(c + dx)}{b^2 d} + \frac{Ba^4 \ln(a + b \tan(c + dx))}{b^3 d(a^2 + b^2)} - \frac{B \ln(\tan(c + dx) - i) 1i}{2d(a + b1i)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int((tan(c + d*x)^4*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
[Out] (B*tan(c + d*x)^2)/(2*b*d) - (B*log(tan(c + d*x) + 1i))/(2*d*(a*1i + b)) -
(B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) - (B*a*tan(c + d*x))/(b^2*d)
+ (B*a^4*log(a + b*tan(c + d*x)))/(b^3*d*(a^2 + b^2))
```

$$3.307 \quad \int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=83

$$-\frac{bBx}{a^2+b^2} + \frac{aB \log(\cos(c+dx))}{(a^2+b^2)d} - \frac{a^3B \log(a+b \tan(c+dx))}{b^2(a^2+b^2)d} + \frac{B \tan(c+dx)}{bd}$$

[Out]  $-b*B*x/(a^2+b^2)+a*B*\ln(\cos(d*x+c))/(a^2+b^2)/d-a^3*B*\ln(a+b*\tan(d*x+c))/b^2/(a^2+b^2)/d+B*\tan(d*x+c)/b/d$

Rubi [A]

time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {21, 3647, 3707, 3698, 31, 3556}

$$\frac{aB \log(\cos(c+dx))}{d(a^2+b^2)} - \frac{bBx}{a^2+b^2} - \frac{a^3B \log(a+b \tan(c+dx))}{b^2d(a^2+b^2)} + \frac{B \tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c + d*x]^3*(a*B + b*B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x])^2, x]$

[Out]  $-((b*B*x)/(a^2 + b^2)) + (a*B*\text{Log}[\text{Cos}[c + d*x]])/((a^2 + b^2)*d) - (a^3*B*\text{Log}[a + b*\text{Tan}[c + d*x]])/(b^2*(a^2 + b^2)*d) + (B*\text{Tan}[c + d*x])/(b*d)$

Rule 21

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

Rule 31

$\text{Int}[(a_.) + (b_.)*(x_))^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3647

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b^2*(a + b*\text{Tan}[e + f*x])^{(m-2)}*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(d*f*(m+n-1)), x] + \text{Dist}[1/(d*(m+n-1)),$

```
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n - 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

### Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^3(c + dx)(aB + bB \tan(c + dx))}{(a + b \tan(c + dx))^2} dx &= B \int \frac{\tan^3(c + dx)}{a + b \tan(c + dx)} dx \\
 &= \frac{B \tan(c + dx)}{bd} + \frac{B \int \frac{-a - b \tan(c + dx) - a \tan^2(c + dx)}{a + b \tan(c + dx)} dx}{b} \\
 &= -\frac{bBx}{a^2 + b^2} + \frac{B \tan(c + dx)}{bd} - \frac{(aB) \int \tan(c + dx) dx}{a^2 + b^2} - \frac{(a^3B) \operatorname{Su}}{b} \\
 &= -\frac{bBx}{a^2 + b^2} + \frac{aB \log(\cos(c + dx))}{(a^2 + b^2)d} + \frac{B \tan(c + dx)}{bd} - \frac{(a^3B) \operatorname{Su}}{b} \\
 &= -\frac{bBx}{a^2 + b^2} + \frac{aB \log(\cos(c + dx))}{(a^2 + b^2)d} - \frac{a^3B \log(a + b \tan(c + dx))}{b^2(a^2 + b^2)d}
 \end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 0.28, size = 92, normalized size = 1.11

$$\frac{B \left( \frac{\log(i - \tan(c + dx))}{a + ib} + \frac{\log(i + \tan(c + dx))}{a - ib} + \frac{2a^3 \log(a + b \tan(c + dx))}{b^2(a^2 + b^2)} - \frac{2 \tan(c + dx)}{b} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] -1/2\*(B\*(Log[I - Tan[c + d\*x]]/(a + I\*b) + Log[I + Tan[c + d\*x]]/(a - I\*b) + (2\*a^3\*Log[a + b\*Tan[c + d\*x]])/(b^2\*(a^2 + b^2)) - (2\*Tan[c + d\*x])/b))/d

**Maple [A]**

time = 0.14, size = 80, normalized size = 0.96

method	result
derivativedivides	$\frac{B \left( \frac{\tan(dx+c)}{b} + \frac{-\frac{a \ln(1+\tan^2(dx+c))}{2} - b \arctan(\tan(dx+c)) - \frac{a^3 \ln(a+b \tan(dx+c))}{b^2(a^2+b^2)}}{a^2+b^2} \right)}{d}$
default	$\frac{B \left( \frac{\tan(dx+c)}{b} + \frac{-\frac{a \ln(1+\tan^2(dx+c))}{2} - b \arctan(\tan(dx+c)) - \frac{a^3 \ln(a+b \tan(dx+c))}{b^2(a^2+b^2)}}{a^2+b^2} \right)}{d}$
norman	$\frac{\frac{B(\tan^2(dx+c))}{d} - \frac{a^2 B}{d b^2} - \frac{b B a x}{a^2+b^2} - \frac{b^2 B x \tan(dx+c)}{a^2+b^2}}{a+b \tan(dx+c)} - \frac{a B \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{a^3 B \ln(a+b \tan(dx+c))}{b^2(a^2+b^2)d}$
risch	$-\frac{ixB}{ib-a} - \frac{2iaBx}{b^2} - \frac{2iaBc}{b^2d} + \frac{2ia^3Bx}{b^2(a^2+b^2)} + \frac{2ia^3Bc}{d(a^2+b^2)b^2} + \frac{2iB}{db(e^{2i(dx+c)}+1)} + \frac{\ln(e^{2i(dx+c)}+1)aB}{b^2d} - \frac{a^3 \ln(e^{2i(dx+c)}+1)}{d(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*B\*(1/b\*tan(d\*x+c)+1/(a^2+b^2)\*(-1/2\*a\*ln(1+tan(d\*x+c)^2)-b\*arctan(tan(d\*x+c)))-1/b^2\*a^3/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.55, size = 89, normalized size = 1.07

$$\frac{\frac{2Ba^3 \log(b \tan(dx+c)+a)}{a^2 b^2 + b^4} + \frac{2(dx+c)Bb}{a^2+b^2} + \frac{Ba \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2B \tan(dx+c)}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] -1/2\*(2\*B\*a^3\*log(b\*tan(d\*x + c) + a)/(a^2\*b^2 + b^4) + 2\*(d\*x + c)\*B\*b/(a^2 + b^2) + B\*a\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) - 2\*B\*tan(d\*x + c)/b)/d

**Fricas** [A]

time = 1.70, size = 119, normalized size = 1.43

$$\frac{2 B b^3 d x + B a^3 \log\left(\frac{b^2 \tan(dx+c)^2 + 2 a b \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - (B a^3 + B a b^2) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) - 2 (B a^2 b + B b^3) \tan(dx+c)}{2 (a^2 b^2 + b^4) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out]  $-1/2*(2*B*b^3*d*x + B*a^3*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - (B*a^3 + B*a*b^2)*\log(1/(\tan(d*x + c)^2 + 1)) - 2*(B*a^2*b + B*b^3)*\tan(d*x + c))/((a^2*b^2 + b^4)*d)$

**Sympy** [C] Result contains complex when optimal does not.

time = 0.79, size = 660, normalized size = 7.95

$$\left\{ \begin{array}{ll} \infty B x \tan^2(c) & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{3 B d x \tan(c+d x)}{2 b d \tan(c+d x)-2 i b d} + \frac{3 i B d x}{2 b d \tan(c+d x)-2 i b d} + \frac{i B \log(\tan^2(c+d x)+1) \tan(c+d x)}{2 b d \tan(c+d x)-2 i b d} + \frac{B \log(\tan^2(c+d x)+1)}{2 b d \tan(c+d x)-2 i b d} + \frac{2 B \tan^2(c+d x)}{2 b d \tan(c+d x)-2 i b d} + \frac{2 i B \tan(c+d x)}{2 b d \tan(c+d x)-2 i b d} + \frac{5 B}{2 b d \tan(c+d x)-2 i b d} & \text{for } a = -i b \\ -\frac{3 B d x \tan(c+d x)}{2 b d \tan(c+d x)+2 i b d} - \frac{3 i B d x}{2 b d \tan(c+d x)+2 i b d} - \frac{i B \log(\tan^2(c+d x)+1) \tan(c+d x)}{2 b d \tan(c+d x)+2 i b d} + \frac{B \log(\tan^2(c+d x)+1)}{2 b d \tan(c+d x)+2 i b d} + \frac{2 B \tan^2(c+d x)}{2 b d \tan(c+d x)+2 i b d} - \frac{2 i B \tan(c+d x)}{2 b d \tan(c+d x)+2 i b d} + \frac{5 B}{2 b d \tan(c+d x)+2 i b d} & \text{for } a = i b \\ B \left( \frac{\log(\tan^2(c+d x)+1)}{2 d} + \frac{\tan^2(c+d x)}{2 d} \right) & \text{for } b = 0 \\ \frac{\pi(B a+B b \tan(c)) \tan^3(c)}{(a+b \tan(c))^2} & \text{for } d = 0 \\ -\frac{2 B a^3 \log\left(\frac{a}{b} + \tan(c+d x)\right)}{2 a^2 b^2 d+2 b^4 d} + \frac{2 B a^2 b \tan(c+d x)}{2 a^2 b^2 d+2 b^4 d} - \frac{B a b^2 \log(\tan^2(c+d x)+1)}{2 a^2 b^2 d+2 b^4 d} - \frac{2 B b^3 d x}{2 a^2 b^2 d+2 b^4 d} + \frac{2 B b^3 \tan(c+d x)}{2 a^2 b^2 d+2 b^4 d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x\*tan(c)\*\*2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-3\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 3\*I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 2\*B\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 2\*I\*B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 5\*B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (-3\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 3\*I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + 2\*B\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 2\*I\*B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + 5\*B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), (B\*(-log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + tan(c + d\*x)\*\*2/(2\*d))/a, Eq(b, 0)), (x\*(B\*a + B\*b\*tan(c))\*tan(c)\*\*3/(a + b\*tan(c))\*\*2, Eq(d, 0)), (-2\*B\*a\*\*3\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*b\*\*2\*d + 2\*b\*\*4\*d) + 2\*B\*a\*\*2\*b\*tan(c + d\*x)/(2\*a\*\*2\*b\*\*2\*d + 2\*b\*\*4\*d) - B\*a\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*b\*\*2\*d + 2\*b\*\*4\*d) - 2\*B\*b\*\*3\*d\*x/(2\*a\*\*2\*b\*\*2\*d + 2\*b\*\*4\*d) + 2\*B\*b\*\*3\*tan(c + d\*x)/(2\*a\*\*2\*b\*\*2\*d + 2\*b\*\*4\*d), True))

**Giac [A]**

time = 0.78, size = 90, normalized size = 1.08

$$\frac{\frac{2Ba^3 \log(|b \tan(dx+c)+a|)}{a^2b^2+b^4} + \frac{2(dx+c)Bb}{a^2+b^2} + \frac{Ba \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2B \tan(dx+c)}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(2*B*a^3*log(abs(b*tan(d*x + c) + a))/(a^2*b^2 + b^4) + 2*(d*x + c)*B*
b/(a^2 + b^2) + B*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*B*tan(d*x + c)/
b)/d
```

**Mupad [B]**

time = 6.37, size = 98, normalized size = 1.18

$$\frac{B \tan(c + dx)}{bd} - \frac{B \ln(\tan(c + dx) + 1i)}{2d(a - b1i)} - \frac{B a^3 \ln(a + b \tan(c + dx))}{b^2 d (a^2 + b^2)} - \frac{B \ln(\tan(c + dx) - i) 1i}{2d(-b + a1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^3*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (B*tan(c + d*x))/(b*d) - (B*log(tan(c + d*x) + 1i))/(2*d*(a - b*1i)) - (B*1
og(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (B*a^3*log(a + b*tan(c + d*x))
)/(b^2*d*(a^2 + b^2))
```

$$3.308 \quad \int \frac{\tan^2(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=81

$$-\frac{aBx}{b^2} + \frac{a^3Bx}{b^2(a^2+b^2)} - \frac{B \log(\cos(c+dx))}{bd} + \frac{a^2B \log(a \cos(c+dx) + b \sin(c+dx))}{b(a^2+b^2)d}$$

[Out]  $-a*B*x/b^2+a^3*B*x/b^2/(a^2+b^2)-B*\ln(\cos(d*x+c))/b/d+a^2*B*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/b/(a^2+b^2)/d$

**Rubi [A]**

time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {21, 3622, 3556, 3565, 3611}

$$\frac{a^2B \log(a \cos(c+dx) + b \sin(c+dx))}{bd(a^2+b^2)} + \frac{a^3Bx}{b^2(a^2+b^2)} - \frac{aBx}{b^2} - \frac{B \log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Tan}[c + d*x]^2*(a*B + b*B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x])^2, x]$

[Out]  $-((a*B*x)/b^2) + (a^3*B*x)/(b^2*(a^2 + b^2)) - (B*\text{Log}[\text{Cos}[c + d*x]])/(b*d) + (a^2*B*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(b*(a^2 + b^2)*d)$

Rule 21

$\text{Int}[(u_*)*((a_*) + (b_*)*(v_))^{(m_*)}*((c_*) + (d_*)*(v_))^{(n_*)}, x\_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

Rule 3556

$\text{Int}[\tan[(c_*) + (d_*)*(x_)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3565

$\text{Int}[(a_*) + (b_*)*\tan[(c_*) + (d_*)*(x_)]^{-1}, x\_Symbol] \rightarrow \text{Simp}[a*(x/(a^2 + b^2)), x] + \text{Dist}[b/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[c + d*x])/(a + b*\text{Tan}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 + b^2, 0]$

Rule 3611

$\text{Int}[(c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)]/((a_*) + (b_*)*\tan[(e_*) + (f_*)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Si}$

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

### Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\tan^2(c + dx)(aB + bB \tan(c + dx))}{(a + b \tan(c + dx))^2} dx &= B \int \frac{\tan^2(c + dx)}{a + b \tan(c + dx)} dx \\ &= -\frac{aBx}{b^2} + \frac{(a^2B) \int \frac{1}{a + b \tan(c + dx)} dx}{b^2} + \frac{B \int \tan(c + dx) dx}{b} \\ &= -\frac{aBx}{b^2} + \frac{a^3Bx}{b^2(a^2 + b^2)} - \frac{B \log(\cos(c + dx))}{bd} + \frac{(a^2B) \int \frac{b - a \tan(c + dx)}{a + b \tan(c + dx)} dx}{b(a^2 + b^2)} \\ &= -\frac{aBx}{b^2} + \frac{a^3Bx}{b^2(a^2 + b^2)} - \frac{B \log(\cos(c + dx))}{bd} + \frac{a^2B \log(a \cos(c + dx) + b \sin(c + dx))}{b(a^2 + b^2)} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.07, size = 79, normalized size = 0.98

$$\frac{B(b(ia + b) \log(i - \tan(c + dx)) + b(-ia + b) \log(i + \tan(c + dx)) + 2a^2 \log(a + b \tan(c + dx)))}{2b(a^2 + b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,
x]
```

```
[Out] (B*(b*(I*a + b)*Log[I - Tan[c + d*x]] + b*((-I)*a + b)*Log[I + Tan[c + d*x]]
+ 2*a^2*Log[a + b*Tan[c + d*x]])/(2*b*(a^2 + b^2)*d)
```

### Maple [A]

time = 0.12, size = 69, normalized size = 0.85

method	result	size
--------	--------	------



derivativedivides	$\frac{B \left( \frac{b \ln(1+\tan^2(dx+c))}{2} - a \arctan(\tan(dx+c)) + \frac{a^2 \ln(a+b \tan(dx+c))}{(a^2+b^2)b} \right)}{d}$	69
default	$\frac{B \left( \frac{b \ln(1+\tan^2(dx+c))}{2} - a \arctan(\tan(dx+c)) + \frac{a^2 \ln(a+b \tan(dx+c))}{(a^2+b^2)b} \right)}{d}$	69
norman	$\frac{-\frac{B a^2 x}{a^2+b^2} - \frac{b B a x \tan(dx+c)}{a^2+b^2}}{a+b \tan(dx+c)} + \frac{a^2 B \ln(a+b \tan(dx+c))}{b d (a^2+b^2)} + \frac{B b \ln(1+\tan^2(dx+c))}{2 d (a^2+b^2)}$	111
risch	$\frac{x B}{i b - a} + \frac{2 i B x}{b} + \frac{2 i B c}{b d} - \frac{2 i a^2 B x}{b (a^2+b^2)} - \frac{2 i a^2 B c}{b d (a^2+b^2)} - \frac{\ln(e^{2i(dx+c)}+1) B}{b d} + \frac{a^2 \ln\left(e^{2i(dx+c)} - \frac{i b + a}{i b - a}\right) B}{b d (a^2+b^2)}$	147

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*B*(1/(a^2+b^2)*(1/2*b*\ln(1+\tan(d*x+c)^2)-a*\arctan(\tan(d*x+c)))+a^2/(a^2+b^2)/b*\ln(a+b*\tan(d*x+c)))$

**Maxima** [A]

time = 0.52, size = 75, normalized size = 0.93

$$\frac{\frac{2 B a^2 \log(b \tan(dx+c)+a)}{a^2 b + b^3} - \frac{2 (dx+c) B a}{a^2 + b^2} + \frac{B b \log(\tan(dx+c)^2 + 1)}{a^2 + b^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm  
="maxima")`

[Out]  $1/2*(2*B*a^2*\log(b*\tan(d*x + c) + a)/(a^2*b + b^3) - 2*(d*x + c)*B*a/(a^2 + b^2) + B*b*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2))/d$

**Fricas** [A]

time = 1.57, size = 95, normalized size = 1.17

$$\frac{2 B a b d x - B a^2 \log\left(\frac{b^2 \tan(dx+c)^2 + 2 a b \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) + (B a^2 + B b^2) \log\left(\frac{1}{\tan(dx+c)^2 + 1}\right)}{2 (a^2 b + b^3) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm  
="fricas")`

[Out]  $-1/2*(2*B*a*b*d*x - B*a^2*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) + (B*a^2 + B*b^2)*\log(1/(\tan(d*x + c)^2 + 1)))/((a^2*b + b^3)*d)$

**Sympy [C]** Result contains complex when optimal does not.

time = 0.65, size = 442, normalized size = 5.46

$$\left\{ \begin{array}{ll} \infty Bx \tan(c) & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{iBdx \tan(c+dx)}{2bd \tan(c+dx)-2ibd} + \frac{Bdx}{2bd \tan(c+dx)-2ibd} + \frac{B \log(\tan^2(c+dx)+1) \tan(c+dx)}{2bd \tan(c+dx)-2ibd} - \frac{iB \log(\tan^2(c+dx)+1)}{2bd \tan(c+dx)-2ibd} - \frac{iB}{2bd \tan(c+dx)-2ibd} & \text{for } a = -ib \\ -\frac{iBdx \tan(c+dx)}{2bd \tan(c+dx)+2ibd} + \frac{Bdx}{2bd \tan(c+dx)+2ibd} + \frac{B \log(\tan^2(c+dx)+1) \tan(c+dx)}{2bd \tan(c+dx)+2ibd} + \frac{iB \log(\tan^2(c+dx)+1)}{2bd \tan(c+dx)+2ibd} + \frac{iB}{2bd \tan(c+dx)+2ibd} & \text{for } a = ib \\ \frac{B(-x + \frac{\tan(c+dx)}{d})}{a} & \text{for } b = 0 \\ \frac{x(Ba+Bb \tan(c)) \tan^2(c)}{(a+b \tan(c))^2} & \text{for } d = 0 \\ \frac{2Ba^2 \log(\frac{a}{b} + \tan(c+dx))}{2a^2bd+2b^3d} - \frac{2Babdx}{2a^2bd+2b^3d} + \frac{Bb^2 \log(\tan^2(c+dx)+1)}{2a^2bd+2b^3d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x\*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (-I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), (B\*(-x + tan(c + d\*x)/d)/a, Eq(b, 0)), (x\*(B\*a + B\*b\*tan(c))\*tan(c)\*\*2/(a + b\*tan(c))\*\*2, Eq(d, 0)), (2\*B\*a\*\*2\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) - 2\*B\*a\*b\*d\*x/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d) + B\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*b\*d + 2\*b\*\*3\*d), True))

**Giac [A]**

time = 0.60, size = 76, normalized size = 0.94

$$\frac{\frac{2Ba^2 \log(|b \tan(dx+c)+a|)}{a^2b+b^3} - \frac{2(dx+c)Ba}{a^2+b^2} + \frac{Bb \log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 1/2\*(2\*B\*a^2\*log(abs(b\*tan(d\*x + c) + a))/(a^2\*b + b^3) - 2\*(d\*x + c)\*B\*a/(a^2 + b^2) + B\*b\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2))/d

**Mupad [B]**

time = 6.41, size = 81, normalized size = 1.00

$$\frac{B \ln(\tan(c + dx) + 1i)}{2d(b + a 1i)} + \frac{B a^2 \ln(a + b \tan(c + dx))}{bd(a^2 + b^2)} + \frac{B \ln(\tan(c + dx) - i) 1i}{2d(a + b 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) + (B*log(tan(c + d*x) + 1i))  
/(2*d*(a*1i + b)) + (B*a^2*log(a + b*tan(c + d*x)))/(b*d*(a^2 + b^2))
```

$$3.309 \quad \int \frac{\tan(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=48

$$\frac{bBx}{a^2 + b^2} - \frac{aB \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2) d}$$

[Out] b\*B\*x/(a^2+b^2)-a\*B\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)/d

Rubi [A]

time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {21, 3612, 3611}

$$\frac{bBx}{a^2 + b^2} - \frac{aB \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (b\*B\*x)/(a^2 + b^2) - (a\*B\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)\*d)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
  NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= B \int \frac{\tan(c+dx)}{a+b\tan(c+dx)} dx \\ &= \frac{bBx}{a^2+b^2} - \frac{(aB) \int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)} dx}{a^2+b^2} \\ &= \frac{bBx}{a^2+b^2} - \frac{aB \log(a \cos(c+dx) + b \sin(c+dx))}{(a^2+b^2)d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.09, size = 67, normalized size = 1.40

$$\frac{B(2(-ia+b)(c+dx) + 2ia\text{ArcTan}(\tan(c+dx)) - a \log((a \cos(c+dx) + b \sin(c+dx))^2))}{2(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (B\*(2\*((-I)\*a + b)\*(c + d\*x) + (2\*I)\*a\*ArcTan[Tan[c + d\*x]] - a\*Log[(a\*Cos[c + d\*x] + b\*Sin[c + d\*x])^2]))/(2\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.10, size = 64, normalized size = 1.33

method	result	size
derivativedivides	$\frac{B \left( \frac{\frac{a \ln(1+\tan^2(dx+c))}{2} + b \arctan(\tan(dx+c))}{a^2+b^2} - \frac{a \ln(a+b \tan(dx+c))}{a^2+b^2} \right)}{d}$	64
default	$\frac{B \left( \frac{\frac{a \ln(1+\tan^2(dx+c))}{2} + b \arctan(\tan(dx+c))}{a^2+b^2} - \frac{a \ln(a+b \tan(dx+c))}{a^2+b^2} \right)}{d}$	64
risch	$\frac{ixB}{ib-a} + \frac{2iBax}{a^2+b^2} + \frac{2iBac}{d(a^2+b^2)} - \frac{\ln\left(e^{2i(dx+c)} - \frac{ib+a}{ib-a}\right)aB}{d(a^2+b^2)}$	95
norman	$\frac{\frac{bBax}{a^2+b^2} + \frac{b^2 Bx \tan(dx+c)}{a^2+b^2}}{a+b \tan(dx+c)} + \frac{aB \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{aB \ln(a+b \tan(dx+c))}{d(a^2+b^2)}$	105

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERB  
OSE)

[Out] 1/d\*B\*(1/(a^2+b^2)\*(1/2\*a\*ln(1+tan(d\*x+c)^2)+b\*arctan(tan(d\*x+c)))-a/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.50, size = 71, normalized size = 1.48

$$\frac{\frac{2(dx+c)Bb}{a^2+b^2} - \frac{2Ba \log(b \tan(dx+c)+a)}{a^2+b^2} + \frac{Ba \log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/2\*(2\*(d\*x + c)\*B\*b/(a^2 + b^2) - 2\*B\*a\*log(b\*tan(d\*x + c) + a)/(a^2 + b^2) + B\*a\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2))/d

**Fricas [A]**

time = 1.33, size = 65, normalized size = 1.35

$$\frac{2Bbdx - Ba \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right)}{2(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 1/2\*(2\*B\*b\*d\*x - B\*a\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)))/((a^2 + b^2)\*d)

**Sympy [C] Result contains complex when optimal does not.**

time = 0.57, size = 282, normalized size = 5.88

$$\left\{ \begin{array}{ll} \tilde{\infty} Bx & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{Bdx \tan(c+dx)}{2bd \tan(c+dx) - 2ibd} - \frac{iBdx}{2bd \tan(c+dx) - 2ibd} - \frac{B}{2bd \tan(c+dx) - 2ibd} & \text{for } a = -ib \\ \frac{Bdx \tan(c+dx)}{2bd \tan(c+dx) + 2ibd} + \frac{iBdx}{2bd \tan(c+dx) + 2ibd} - \frac{B}{2bd \tan(c+dx) + 2ibd} & \text{for } a = ib \\ \frac{x(Ba + Bb \tan(c)) \tan(c)}{(a+b \tan(c))^2} & \text{for } d = 0 \\ \frac{B \log(\tan^2(c+dx)+1)}{2ad} & \text{for } b = 0 \\ -\frac{2Ba \log\left(\frac{a}{b} + \tan(c+dx)\right)}{2a^2d + 2b^2d} + \frac{Ba \log(\tan^2(c+dx)+1)}{2a^2d + 2b^2d} + \frac{2Bbdx}{2a^2d + 2b^2d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - B/(

$2*b*d*\tan(c + d*x) - 2*I*b*d$ , Eq(a,  $-I*b$ )), ( $B*d*x*\tan(c + d*x)/(2*b*d*\tan(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*\tan(c + d*x) + 2*I*b*d) - B/(2*b*d*\tan(c + d*x) + 2*I*b*d)$ , Eq(a,  $I*b$ )), ( $x*(B*a + B*b*\tan(c))*\tan(c)/(a + b*\tan(c))^2$ , Eq(d, 0)), ( $B*\log(\tan(c + d*x)^2 + 1)/(2*a*d)$ , Eq(b, 0)), ( $-2*B*a*\log(a/b + \tan(c + d*x))/(2*a^2*d + 2*b^2*d) + B*a*\log(\tan(c + d*x)^2 + 1)/(2*a^2*d + 2*b^2*d) + 2*B*b*d*x/(2*a^2*d + 2*b^2*d)$ , True))

**Giac [A]**

time = 0.52, size = 76, normalized size = 1.58

$$-\frac{\frac{2 B a b \log(|b \tan(dx+c)+a|)}{a^2 b+b^3} - \frac{2(dx+c) B b}{a^2+b^2} - \frac{B a \log(\tan(dx+c)^2+1)}{a^2+b^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $-1/2*(2*B*a*b*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^2*b + b^3) - 2*(d*x + c)*B*b/(a^2 + b^2) - B*a*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2))/d$

**Mupad [B]**

time = 6.38, size = 79, normalized size = 1.65

$$\frac{B \ln(\tan(c + dx) + 1i)}{2 d (a - b 1i)} - \frac{B a \ln(a + b \tan(c + dx))}{d (a^2 + b^2)} + \frac{B \ln(\tan(c + dx) - i) 1i}{2 d (-b + a 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out]  $(B*\log(\tan(c + d*x) + 1i))/(2*d*(a - b*1i)) + (B*\log(\tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (B*a*\log(a + b*\tan(c + d*x)))/(d*(a^2 + b^2))$

$$3.310 \quad \int \frac{aB + bB \tan(c + dx)}{(a + b \tan(c + dx))^2} dx$$

Optimal. Leaf size=47

$$\frac{aBx}{a^2 + b^2} + \frac{bB \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2)d}$$

[Out] a\*B\*x/(a^2+b^2)+b\*B\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/(a^2+b^2)/d

Rubi [A]

time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {21, 3565, 3611}

$$\frac{bB \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)} + \frac{aBx}{a^2 + b^2}$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^2,x]

[Out] (a\*B\*x)/(a^2 + b^2) + (b\*B\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/((a^2 + b^2)\*d)

Rule 21

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_))^(m\_.)\*((c\_.) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 3565

Int[((a\_.) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> Simp[a\*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/(a\_. + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rubi steps



$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{(a + b \tan(c + dx))^2} dx &= B \int \frac{1}{a + b \tan(c + dx)} dx \\
&= \frac{aBx}{a^2 + b^2} + \frac{(bB) \int \frac{b-a \tan(c+dx)}{a+b \tan(c+dx)} dx}{a^2 + b^2} \\
&= \frac{aBx}{a^2 + b^2} + \frac{bB \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2) d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.04, size = 77, normalized size = 1.64

$$\frac{B((-ia - b) \log(i - \tan(c + dx)) + i(a + ib) \log(i + \tan(c + dx)) + 2b \log(a + b \tan(c + dx)))}{2(a^2 + b^2) d}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^2,x]

[Out] (B\*(((-I)\*a - b)\*Log[I - Tan[c + d\*x]] + I\*(a + I\*b)\*Log[I + Tan[c + d\*x]] + 2\*b\*Log[a + b\*Tan[c + d\*x]]))/(2\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.08, size = 63, normalized size = 1.34

method	result	size
derivativdivides	$B \frac{\left( -\frac{b \ln(1 + \tan^2(dx+c))}{2} + a \arctan(\tan(dx+c)) + \frac{b \ln(a+b \tan(dx+c))}{a^2+b^2} \right)}{d}$	63
default	$B \frac{\left( -\frac{b \ln(1 + \tan^2(dx+c))}{2} + a \arctan(\tan(dx+c)) + \frac{b \ln(a+b \tan(dx+c))}{a^2+b^2} \right)}{d}$	63
risch	$-\frac{x B}{ib-a} - \frac{2ib B x}{a^2+b^2} - \frac{2ib B c}{d(a^2+b^2)} + \frac{b \ln(e^{2i(dx+c)} - \frac{ib+a}{ib-a}) B}{d(a^2+b^2)}$	93
norman	$\frac{B a^2 x}{a^2+b^2} + \frac{b B a x \tan(dx+c)}{a^2+b^2} + \frac{B b \ln(a+b \tan(dx+c))}{d(a^2+b^2)} - \frac{B b \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$	104

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] 1/d\*B\*(1/(a^2+b^2)\*(-1/2\*b\*ln(1+tan(d\*x+c)^2)+a\*arctan(tan(d\*x+c)))+b/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.60, size = 72, normalized size = 1.53

$$\frac{\frac{2(dx+c)Ba}{a^2+b^2} + \frac{2Bb \log(b \tan(dx+c)+a)}{a^2+b^2} - \frac{Bb \log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/2\*(2\*(d\*x + c)\*B\*a/(a^2 + b^2) + 2\*B\*b\*log(b\*tan(d\*x + c) + a)/(a^2 + b^2) - B\*b\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2))/d

**Fricas [A]**

time = 2.89, size = 64, normalized size = 1.36

$$\frac{2Badx + Bb \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right)}{2(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 1/2\*(2\*B\*a\*d\*x + B\*b\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1)))/((a^2 + b^2)\*d)

**Sympy [C] Result contains complex when optimal does not.**

time = 0.54, size = 272, normalized size = 5.79

$$\left\{ \begin{array}{ll} \frac{\infty Bx}{\tan(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{iBdx \tan(c+dx)}{2bd \tan(c+dx) - 2ibd} + \frac{Bdx}{2bd \tan(c+dx) - 2ibd} + \frac{iB}{2bd \tan(c+dx) - 2ibd} & \text{for } a = -ib \\ -\frac{iBdx \tan(c+dx)}{2bd \tan(c+dx) + 2ibd} + \frac{Bdx}{2bd \tan(c+dx) + 2ibd} - \frac{iB}{2bd \tan(c+dx) + 2ibd} & \text{for } a = ib \\ \frac{x(Ba + Bb \tan(c))}{(a + b \tan(c))^2} & \text{for } d = 0 \\ \frac{Bx}{a} & \text{for } b = 0 \\ \frac{2Badx}{2a^2d + 2b^2d} + \frac{2Bb \log\left(\frac{a}{b} + \tan(c+dx)\right)}{2a^2d + 2b^2d} - \frac{Bb \log(\tan^2(c+dx) + 1)}{2a^2d + 2b^2d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x/tan(c), Eq(a, 0) &amp; Eq(b, 0) &amp; Eq(d, 0)), (I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + I\*B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (-I\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - I

\*B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), (x\*(B\*a + B\*b\*tan(c))/(a + b\*tan(c))\*\*2, Eq(d, 0)), (B\*x/a, Eq(b, 0)), (2\*B\*a\*d\*x/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + 2\*B\*b\*log(a/b + tan(c + d\*x))/(2\*a\*\*2\*d + 2\*b\*\*2\*d) - B\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*d + 2\*b\*\*2\*d), True))

**Giac** [A]

time = 0.47, size = 77, normalized size = 1.64

$$\frac{\frac{2 B b^2 \log(|b \tan(dx+c)+a|)}{a^2 b + b^3} + \frac{2 (dx+c) B a}{a^2 + b^2} - \frac{B b \log(\tan(dx+c)^2 + 1)}{a^2 + b^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 1/2\*(2\*B\*b^2\*log(abs(b\*tan(d\*x + c) + a))/(a^2\*b + b^3) + 2\*(d\*x + c)\*B\*a/(a^2 + b^2) - B\*b\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2))/d

**Mupad** [B]

time = 6.31, size = 76, normalized size = 1.62

$$\frac{B b \ln(a + b \tan(c + d x))}{d (a^2 + b^2)} - \frac{B \ln(\tan(c + d x) + 1i)}{2 d (b + a 1i)} - \frac{B \ln(\tan(c + d x) - i) 1i}{2 d (a + b 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^2,x)

[Out] (B\*b\*log(a + b\*tan(c + d\*x)))/(d\*(a^2 + b^2)) - (B\*log(tan(c + d\*x) + 1i))/(2\*d\*(a\*1i + b)) - (B\*log(tan(c + d\*x) - 1i)\*1i)/(2\*d\*(a + b\*1i))

$$3.311 \quad \int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=69

$$-\frac{bBx}{a^2+b^2} + \frac{B \log(\sin(c+dx))}{ad} - \frac{b^2 B \log(a \cos(c+dx) + b \sin(c+dx))}{a(a^2+b^2)d}$$

[Out]  $-b*B*x/(a^2+b^2)+B*\ln(\sin(d*x+c))/a/d-b^2*B*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/a/(a^2+b^2)/d$

**Rubi [A]**

time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {21, 3652, 3611, 3556}

$$-\frac{b^2 B \log(a \cos(c+dx) + b \sin(c+dx))}{ad(a^2+b^2)} - \frac{bBx}{a^2+b^2} + \frac{B \log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x]*(a*B + b*B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x])^2, x]$

[Out]  $-((b*B*x)/(a^2 + b^2)) + (B*\text{Log}[\text{Sin}[c + d*x]])/(a*d) - (b^2*B*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(a*(a^2 + b^2)*d)$

Rule 21

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3611

$\text{Int}[((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

Rule 3652

$\text{Int}[1/(((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)])), x\_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x]$

] + (Dist[b^2/((b\*c - a\*d)\*(a^2 + b^2)), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] - Dist[d^2/((b\*c - a\*d)\*(c^2 + d^2)), Int[(d - c\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cot(c + dx)(aB + bB \tan(c + dx))}{(a + b \tan(c + dx))^2} dx &= B \int \frac{\cot(c + dx)}{a + b \tan(c + dx)} dx \\ &= -\frac{bBx}{a^2 + b^2} + \frac{B \int \cot(c + dx) dx}{a} - \frac{(b^2 B) \int \frac{b-a \tan(c+dx)}{a+b \tan(c+dx)} dx}{a(a^2 + b^2)} \\ &= -\frac{bBx}{a^2 + b^2} + \frac{B \log(\sin(c + dx))}{ad} - \frac{b^2 B \log(a \cos(c + dx) + b \sin(c + dx))}{a(a^2 + b^2)d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.08, size = 79, normalized size = 1.14

$$\frac{B(a(a + ib) \log(i - \cot(c + dx)) + a(a - ib) \log(i + \cot(c + dx)) + 2b^2 \log(b + a \cot(c + dx)))}{2a(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] -1/2\*(B\*(a\*(a + I\*b)\*Log[I - Cot[c + d\*x]] + a\*(a - I\*b)\*Log[I + Cot[c + d\*x]] + 2\*b^2\*Log[b + a\*Cot[c + d\*x]])/(a\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.20, size = 81, normalized size = 1.17

method	result	size
derivativedivides	$\frac{B \left( \frac{\ln(\tan(dx+c))}{a} + \frac{-\frac{a \ln(1+\tan^2(dx+c))}{2} - b \arctan(\tan(dx+c))}{a^2+b^2} - \frac{b^2 \ln(a+b \tan(dx+c))}{a(a^2+b^2)} \right)}{d}$	81
default	$\frac{B \left( \frac{\ln(\tan(dx+c))}{a} + \frac{-\frac{a \ln(1+\tan^2(dx+c))}{2} - b \arctan(\tan(dx+c))}{a^2+b^2} - \frac{b^2 \ln(a+b \tan(dx+c))}{a(a^2+b^2)} \right)}{d}$	81
norman	$\frac{-\frac{bBax}{a^2+b^2} - \frac{b^2 Bx \tan(dx+c)}{a^2+b^2}}{a+b \tan(dx+c)} + \frac{B \ln(\tan(dx+c))}{ad} - \frac{aB \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{b^2 B \ln(a+b \tan(dx+c))}{ad(a^2+b^2)}$	127
risch	$-\frac{ixB}{ib-a} - \frac{2ixB}{a} - \frac{2iBc}{ad} + \frac{2ib^2 Bx}{a(a^2+b^2)} + \frac{2ib^2 Bc}{ad(a^2+b^2)} + \frac{\ln(e^{2i(dx+c)}-1)B}{ad} - \frac{b^2 \ln\left(e^{2i(dx+c)} - \frac{ib+a}{ib-a}\right)B}{ad(a^2+b^2)}$	149

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out]  $1/d*B*(1/a*\ln(\tan(d*x+c))+1/(a^2+b^2)*(-1/2*a*\ln(1+\tan(d*x+c)^2)-b*\arctan(\tan(d*x+c))))-b^2/a/(a^2+b^2)*\ln(a+b*\tan(d*x+c))$

**Maxima** [A]

time = 0.59, size = 88, normalized size = 1.28

$$\frac{\frac{2 B b^2 \log(b \tan(dx+c)+a)}{a^3+ab^2} + \frac{2(dx+c)Bb}{a^2+b^2} + \frac{Ba \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2 B \log(\tan(dx+c))}{a}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")`

[Out]  $-1/2*(2*B*b^2*\log(b*\tan(d*x + c) + a)/(a^3 + a*b^2) + 2*(d*x + c)*B*b/(a^2 + b^2) + B*a*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*B*\log(\tan(d*x + c))/a)/d$

**Fricas** [A]

time = 1.74, size = 104, normalized size = 1.51

$$\frac{2 B a b d x + B b^2 \log\left(\frac{b^2 \tan(dx+c)^2+2 a b \tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right) - (B a^2 + B b^2) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2(a^3 + ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*B*a*b*d*x + B*b^2*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1)) - (B*a^2 + B*b^2)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1)))/((a^3 + a*b^2)*d)$

**Sympy** [C] Result contains complex when optimal does not.

time = 1.45, size = 672, normalized size = 9.74

$$\left\{ \begin{array}{ll} \frac{\infty B x \cot(c)}{\tan(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ B \left( -\frac{\log\left(\frac{\tan^2(c+dx)+1}{2d}\right) + \log\left(\frac{\tan(c+dx)}{d}\right)}{a} \right) & \text{for } b = 0 \\ B \left( -x - \frac{1}{d \tan(c+dx)} \right) & \text{for } a = 0 \\ \frac{B d x \tan(c+dx)}{2 b d \tan(c+dx)-2 i b d} - \frac{i B d x}{2 b d \tan(c+dx)-2 i b d} - \frac{i B \log(\tan^2(c+dx)+1) \tan(c+dx)}{2 b d \tan(c+dx)-2 i b d} - \frac{B \log(\tan^2(c+dx)+1)}{2 b d \tan(c+dx)-2 i b d} + \frac{2 i B \log(\tan(c+dx)) \tan(c+dx)}{2 b d \tan(c+dx)-2 i b d} + \frac{2 B \log(\tan(c+dx))}{2 b d \tan(c+dx)-2 i b d} + \frac{B}{2 b d \tan(c+dx)-2 i b d} & \text{for } a = -i b \\ \frac{B d x \tan(c+dx)}{2 b d \tan(c+dx)+2 i b d} + \frac{i B d x}{2 b d \tan(c+dx)+2 i b d} + \frac{i B \log(\tan^2(c+dx)+1) \tan(c+dx)}{2 b d \tan(c+dx)+2 i b d} - \frac{B \log(\tan^2(c+dx)+1)}{2 b d \tan(c+dx)+2 i b d} - \frac{2 i B \log(\tan(c+dx)) \tan(c+dx)}{2 b d \tan(c+dx)+2 i b d} + \frac{2 B \log(\tan(c+dx))}{2 b d \tan(c+dx)+2 i b d} + \frac{B}{2 b d \tan(c+dx)+2 i b d} & \text{for } a = i b \\ \frac{x(B a+B b \tan(c)) \cot(c)}{(a+b \tan(c))^2} & \text{for } d = 0 \\ -\frac{B a^2 \log(\tan^2(c+dx)+1)}{2 a^3 d+2 a b^2 d} + \frac{2 B a^2 \log(\tan(c+dx))}{2 a^3 d+2 a b^2 d} - \frac{2 B a b d x}{2 a^3 d+2 a b^2 d} - \frac{2 B b^2 \log\left(\frac{x}{2}+\tan(c+dx)\right)}{2 a^3 d+2 a b^2 d} + \frac{2 B b^2 \log(\tan(c+dx))}{2 a^3 d+2 a b^2 d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x\*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B\*(-log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + log(tan(c + d\*x))/d)/a, Eq(b, 0)), (B\*(-x - 1/(d\*tan(c + d\*x)))/b, Eq(a, 0)), (B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) - B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 2\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + 2\*B\*log(tan(c + d\*x))/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d) + B/(2\*b\*d\*tan(c + d\*x) - 2\*I\*b\*d), Eq(a, -I\*b)), (B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B\*d\*x/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) - 2\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + 2\*B\*log(tan(c + d\*x))/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d) + B/(2\*b\*d\*tan(c + d\*x) + 2\*I\*b\*d), Eq(a, I\*b)), (x\*(B\*a + B\*b\*tan(c))\*cot(c)/(a + b\*tan(c))\*\*2, Eq(d, 0)), (-B\*a\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) + 2\*B\*a\*\*2\*log(tan(c + d\*x))/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) - 2\*B\*a\*b\*d\*x/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) - 2\*B\*b\*\*2\*log(a/b + tan(c + d\*x))/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) + 2\*B\*b\*\*2\*log(tan(c + d\*x))/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d), True))

**Giac** [A]

time = 0.57, size = 92, normalized size = 1.33

$$-\frac{\frac{2 B b^3 \log(|b \tan(dx+c)+a|)}{a^3 b + a b^3} + \frac{2(dx+c) B b}{a^2 + b^2} + \frac{B a \log(\tan(dx+c)^2 + 1)}{a^2 + b^2} - \frac{2 B \log(|\tan(dx+c)|)}{a}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] -1/2\*(2\*B\*b^3\*log(abs(b\*tan(d\*x + c) + a))/(a^3\*b + a\*b^3) + 2\*(d\*x + c)\*B\*b/(a^2 + b^2) + B\*a\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) - 2\*B\*log(abs(tan(d\*x + c)))/a)/d

**Mupad** [B]

time = 6.36, size = 99, normalized size = 1.43

$$\frac{B \ln(\tan(c + dx))}{a d} - \frac{B \ln(\tan(c + dx) + 1i)}{2 d (a - b 1i)} - \frac{B b^2 \ln(a + b \tan(c + dx))}{a d (a^2 + b^2)} - \frac{B \ln(\tan(c + dx) - i) 1i}{2 d (-b + a 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] (B\*log(tan(c + d\*x)))/(a\*d) - (B\*log(tan(c + d\*x) + 1i))/(2\*d\*(a - b\*1i)) - (B\*log(tan(c + d\*x) - 1i)\*1i)/(2\*d\*(a\*1i - b)) - (B\*b^2\*log(a + b\*tan(c + d\*x)))/(a\*d\*(a^2 + b^2))

$$3.312 \quad \int \frac{\cot^2(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=85

$$-\frac{aBx}{a^2+b^2} - \frac{B \cot(c+dx)}{ad} - \frac{bB \log(\sin(c+dx))}{a^2d} + \frac{b^3B \log(a \cos(c+dx) + b \sin(c+dx))}{a^2(a^2+b^2)d}$$

[Out]  $-a*B*x/(a^2+b^2)-B*\cot(d*x+c)/a/d-b*B*\ln(\sin(d*x+c))/a^2/d+b^3*B*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/a^2/(a^2+b^2)/d$

**Rubi [A]**

time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ ,

Rules used = {21, 3650, 3732, 3611, 3556}

$$-\frac{aBx}{a^2+b^2} + \frac{b^3B \log(a \cos(c+dx) + b \sin(c+dx))}{a^2d(a^2+b^2)} - \frac{bB \log(\sin(c+dx))}{a^2d} - \frac{B \cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x]^2*(a*B + b*B*\text{Tan}[c + d*x]))/(a + b*\text{Tan}[c + d*x])^2, x]$

[Out]  $-((a*B*x)/(a^2 + b^2)) - (B*\text{Cot}[c + d*x])/(a*d) - (b*B*\text{Log}[\text{Sin}[c + d*x]])/(a^2*d) + (b^3*B*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(a^2*(a^2 + b^2)*d)$

Rule 21

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x\_Symbol] \rightarrow$   
 $\text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^(m + n), x], x] /;$  FreeQ[{a, b, c, d, n}, x]  
 && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplrQ[c + d\*x, a + b\*x])

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$  FreeQ[{c, d}, x]

Rule 3611

$\text{Int}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]/(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[(c/(b*f))*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3650

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]^(m_.)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]^(n_.), x\_Symbol] \rightarrow \text{Simp}[b^2*(a + b*\text{Tan}[e + f*x])^(m + 1)*((c$



```

+ d*Tan[e + f*x]^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3732

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)] + (C_.)*tan[(e_.) + (f_.)*(x_.)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_.)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c + dx)(aB + bB \tan(c + dx))}{(a + b \tan(c + dx))^2} dx &= B \int \frac{\cot^2(c + dx)}{a + b \tan(c + dx)} dx \\
&= -\frac{B \cot(c + dx)}{ad} - \frac{B \int \frac{\cot(c + dx)(b + a \tan(c + dx) + b \tan^2(c + dx))}{a + b \tan(c + dx)} dx}{a} \\
&= -\frac{aBx}{a^2 + b^2} - \frac{B \cot(c + dx)}{ad} - \frac{(bB) \int \cot(c + dx) dx}{a^2} + \frac{(b^3 B) \int}{a^2} \\
&= -\frac{aBx}{a^2 + b^2} - \frac{B \cot(c + dx)}{ad} - \frac{bB \log(\sin(c + dx))}{a^2 d} + \frac{b^3 B \log(a}{a^2}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.29, size = 97, normalized size = 1.14

$$-\frac{B \left( \frac{\cot(c + dx)}{a} - \frac{\log(i - \cot(c + dx))}{2(ia + b)} + \frac{\log(i + \cot(c + dx))}{2(ia - b)} - \frac{b^3 \log(b + a \cot(c + dx))}{a^2(a^2 + b^2)} \right)}{d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,
x]

```

[Out]  $-\left(\frac{B \cdot (\cot[c + dx] / a - \log[I - \cot[c + dx]])}{2 \cdot (I \cdot a + b)} + \log[I + \cot[c + dx]] / (2 \cdot (I \cdot a - b)) - (b^3 \cdot \log[b + a \cdot \cot[c + dx]]) / (a^2 \cdot (a^2 + b^2))\right) / d$

**Maple [A]**

time = 0.20, size = 95, normalized size = 1.12

method	result
derivativdivides	$\frac{B \left( -\frac{1}{a \tan(dx+c)} - \frac{b \ln(\tan(dx+c))}{a^2} + \frac{b \ln(1+\tan^2(dx+c))}{2} - \frac{a \arctan(\tan(dx+c))}{a^2+b^2} + \frac{b^3 \ln(a+b \tan(dx+c))}{a^2(a^2+b^2)} \right)}{d}$
default	$\frac{B \left( -\frac{1}{a \tan(dx+c)} - \frac{b \ln(\tan(dx+c))}{a^2} + \frac{b \ln(1+\tan^2(dx+c))}{2} - \frac{a \arctan(\tan(dx+c))}{a^2+b^2} + \frac{b^3 \ln(a+b \tan(dx+c))}{a^2(a^2+b^2)} \right)}{d}$
norman	$\frac{\frac{B b^2 (\tan^2(dx+c))}{d a^2} - \frac{B}{d} - \frac{B a^2 x \tan(dx+c)}{a^2+b^2} - \frac{b B a x (\tan^2(dx+c))}{a^2+b^2}}{\tan(dx+c)(a+b \tan(dx+c))} + \frac{B b^3 \ln(a+b \tan(dx+c))}{(a^2+b^2)a^2 d} - \frac{B b \ln(\tan(dx+c))}{a^2 d} + \frac{B b \ln(\tan(dx+c))}{a^2 d}$
risch	$\frac{x B}{i b - a} + \frac{2 i B b x}{a^2} + \frac{2 i B b c}{a^2 d} - \frac{2 i b^3 B x}{a^2(a^2+b^2)} - \frac{2 i b^3 B c}{a^2 d(a^2+b^2)} - \frac{2 i B}{d a (e^{2 i(dx+c)} - 1)} - \frac{\ln(e^{2 i(dx+c)} - 1) B b}{a^2 d} + \frac{b^3 \ln(e^{2 i(dx+c)} - 1)}{a^2 d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*B*(-1/a/\tan(dx+c)-1/a^2*b*\ln(\tan(dx+c))+1/(a^2+b^2)*(1/2*b*\ln(1+\tan(dx+c)^2)-a*\arctan(\tan(dx+c)))+b^3/a^2/(a^2+b^2)*\ln(a+b*\tan(dx+c)))$

**Maxima [A]**

time = 0.54, size = 105, normalized size = 1.24

$$\frac{\frac{2 B b^3 \log(b \tan(dx+c)+a)}{a^4+a^2 b^2} - \frac{2(dx+c) B a}{a^2+b^2} + \frac{B b \log(\tan(dx+c)^2+1)}{a^2+b^2} - \frac{2 B b \log(\tan(dx+c))}{a^2} - \frac{2 B}{a \tan(dx+c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm  
="maxima")`

[Out]  $1/2*(2*B*b^3*\log(b*\tan(dx+c)+a)/(a^4+a^2*b^2)-2*(dx+c)*B*a/(a^2+b^2)+B*b*\log(\tan(dx+c)^2+1)/(a^2+b^2)-2*B*b*\log(\tan(dx+c))/a^2-2*B/(a*\tan(dx+c)))/d$

**Fricas [A]**

time = 1.54, size = 147, normalized size = 1.73

$$\frac{2 B a^3 dx \tan(dx+c) - B b^3 \log\left(\frac{b^2 \tan(dx+c)^2 + 2 a b \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) \tan(dx+c) + 2 B a^3 + 2 B a b^2 + (B a^2 b + B b^3) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2 + 1}\right) \tan(dx+c)}{2(a^4 + a^2 b^2) d \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$-1/2*(2*B*a^3*d*x*\tan(d*x + c) - B*b^3*\log((b^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + a^2)/(\tan(d*x + c)^2 + 1))*\tan(d*x + c) + 2*B*a^3 + 2*B*a*b^2 + (B*a^2*b + B*b^3)*\log(\tan(d*x + c)^2/(\tan(d*x + c)^2 + 1))*\tan(d*x + c))/((a^4 + a^2*b^2)*d*\tan(d*x + c))$$

**Sympy** [C] Result contains complex when optimal does not.  
time = 2.63, size = 1137, normalized size = 13.38

$$\int \frac{B(-a - \frac{\log(\tan(c+dx))}{d})}{a + b \tan(c+dx)} dx$$

for a = 0  $\wedge$  b = 0  $\wedge$  c = 0  $\wedge$  d = 0  
for b = 0  
for a = 0  
for a = -ib  
for a = ib  
for c = -dx  
for d = 0  
otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), (B\*(-x - cot(c + d\*x)/d)/a, Eq(b, 0)), (B\*(log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - log(tan(c + d\*x))/d - 1/(2\*d\*tan(c + d\*x)\*\*2))/b, Eq(a, 0)), (-3\*I\*B\*d\*x\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) - 3\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) - B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) + I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) + 2\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) - 2\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) - 3\*I\*B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)) - 2\*B/(2\*b\*d\*tan(c + d\*x)\*\*2 - 2\*I\*b\*d\*tan(c + d\*x)), Eq(a, -I\*b)), (3\*I\*B\*d\*x\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) - 3\*B\*d\*x\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) - B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) - I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) + 2\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) + 2\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) + 3\*I\*B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)) - 2\*B/(2\*b\*d\*tan(c + d\*x)\*\*2 + 2\*I\*b\*d\*tan(c + d\*x)), Eq(a, I\*b)), (zoo\*B\*x/a, Eq(c, -dx)), (x\*(B\*a + B\*b\*tan(c))\*cot(c)\*\*2/(a + b\*tan(c))\*\*2, Eq(d, 0)), (-2\*B\*a\*\*3\*d\*x\*tan(c + d\*x)/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x)) - 2\*B\*a\*\*3/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x)) + B\*a\*\*2\*b\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x))

+ d\*x)) - 2\*B\*a\*\*2\*b\*log(tan(c + d\*x))\*tan(c + d\*x)/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x)) - 2\*B\*a\*b\*\*2/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x)) + 2\*B\*b\*\*3\*log(a/b + tan(c + d\*x))\*tan(c + d\*x)/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x)) - 2\*B\*b\*\*3\*log(tan(c + d\*x))\*tan(c + d\*x)/(2\*a\*\*4\*d\*tan(c + d\*x) + 2\*a\*\*2\*b\*\*2\*d\*tan(c + d\*x)), True))

**Giac [A]**

time = 0.62, size = 122, normalized size = 1.44

$$\frac{\frac{2 B b^4 \log(|b \tan(dx+c)+a|)}{a^4 b + a^2 b^3} - \frac{2 (dx+c) B a}{a^2 + b^2} + \frac{B b \log(\tan(dx+c)^2 + 1)}{a^2 + b^2} - \frac{2 B b \log(|\tan(dx+c)|)}{a^2} + \frac{2 (B b \tan(dx+c) - B a)}{a^2 \tan(dx+c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] 1/2\*(2\*B\*b^4\*log(abs(b\*tan(d\*x + c) + a))/(a^4\*b + a^2\*b^3) - 2\*(d\*x + c)\*B\*a/(a^2 + b^2) + B\*b\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) - 2\*B\*b\*log(abs(tan(d\*x + c)))/a^2 + 2\*(B\*b\*tan(d\*x + c) - B\*a)/(a^2\*tan(d\*x + c)))/d

**Mupad [B]**

time = 6.38, size = 113, normalized size = 1.33

$$\frac{B \ln(\tan(c + dx) + 1i)}{2 d (b + a 1i)} - \frac{B \cot(c + dx)}{a d} - \frac{B b \ln(\tan(c + dx))}{a^2 d} + \frac{B b^3 \ln(a + b \tan(c + dx))}{a^2 d (a^2 + b^2)} + \frac{B \ln(\tan(c + dx) - i) 1i}{2 d (a + b 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^2\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] (B\*log(tan(c + d\*x) - 1i)\*1i)/(2\*d\*(a + b\*1i)) + (B\*log(tan(c + d\*x) + 1i))/(2\*d\*(a\*1i + b)) - (B\*cot(c + d\*x))/(a\*d) - (B\*b\*log(tan(c + d\*x)))/(a^2\*d) + (B\*b^3\*log(a + b\*tan(c + d\*x)))/(a^2\*d\*(a^2 + b^2))

$$3.313 \quad \int \frac{\cot^3(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=112

$$\frac{bBx}{a^2+b^2} + \frac{bB \cot(c+dx)}{a^2d} - \frac{B \cot^2(c+dx)}{2ad} - \frac{(a^2-b^2)B \log(\sin(c+dx))}{a^3d} - \frac{b^4B \log(a \cos(c+dx) + b \sin(c+dx))}{a^3(a^2+b^2)d}$$

[Out] b\*B\*x/(a^2+b^2)+b\*B\*cot(d\*x+c)/a^2/d-1/2\*B\*cot(d\*x+c)^2/a/d-(a^2-b^2)\*B\*ln(sin(d\*x+c))/a^3/d-b^4\*B\*ln(a\*cos(d\*x+c)+b\*sin(d\*x+c))/a^3/(a^2+b^2)/d

**Rubi [A]**

time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {21, 3650, 3730, 3733, 3611, 3556}

$$\frac{bBx}{a^2+b^2} + \frac{bB \cot(c+dx)}{a^2d} - \frac{B(a^2-b^2) \log(\sin(c+dx))}{a^3d} - \frac{b^4B \log(a \cos(c+dx) + b \sin(c+dx))}{a^3d(a^2+b^2)} - \frac{B \cot^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (b\*B\*x)/(a^2 + b^2) + (b\*B\*Cot[c + d\*x])/(a^2\*d) - (B\*Cot[c + d\*x]^2)/(2\*a\*d) - ((a^2 - b^2)\*B\*Log[Sin[c + d\*x]])/(a^3\*d) - (b^4\*B\*Log[a\*Cos[c + d\*x] + b\*Sin[c + d\*x]])/(a^3\*(a^2 + b^2)\*d)

Rule 21

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_.))^(m\_.)\*((c\_.) + (d\_.)\*(v\_.))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3611

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/(a\_. + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3650

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3733

```

Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] := Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^
2 + a^2*c)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
+ f*x]), x], x] - Dist[(c^2*c + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rubi steps

$$\begin{aligned} \int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= B \int \frac{\cot^3(c+dx)}{a+b\tan(c+dx)} dx \\ &= -\frac{B \cot^2(c+dx)}{2ad} - \frac{B \int \frac{\cot^2(c+dx)(2b+2a\tan(c+dx)+2b\tan^2(c+dx))}{a+b\tan(c+dx)} dx}{2a} \\ &= \frac{bB \cot(c+dx)}{a^2d} - \frac{B \cot^2(c+dx)}{2ad} + \frac{B \int \frac{\cot(c+dx)(-2(a^2-b^2)+2b^2\tan(c+dx))}{a+b\tan(c+dx)} dx}{2a^2} \\ &= \frac{bBx}{a^2+b^2} + \frac{bB \cot(c+dx)}{a^2d} - \frac{B \cot^2(c+dx)}{2ad} - \frac{((a^2-b^2)B) \int \frac{\cot(c+dx)}{a+b\tan(c+dx)} dx}{a^2} \\ &= \frac{bBx}{a^2+b^2} + \frac{bB \cot(c+dx)}{a^2d} - \frac{B \cot^2(c+dx)}{2ad} - \frac{(a^2-b^2)B \log(\frac{a+b\tan(c+dx)}{a^2+b^2})}{a^3d} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.44, size = 107, normalized size = 0.96

$$\frac{B \left( -\frac{2b \cot(c+dx)}{a^2} + \frac{\cot^2(c+dx)}{a} - \frac{\log(i-\cot(c+dx))}{a-ib} - \frac{\log(i+\cot(c+dx))}{a+ib} + \frac{2b^4 \log(b+a \cot(c+dx))}{a^3(a^2+b^2)} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] -1/2\*(B\*((-2\*b\*Cot[c + d\*x])/a^2 + Cot[c + d\*x]^2/a - Log[I - Cot[c + d\*x]]/(a - I\*b) - Log[I + Cot[c + d\*x]]/(a + I\*b) + (2\*b^4\*Log[b + a\*Cot[c + d\*x]])/(a^3\*(a^2 + b^2))))/d

**Maple [A]**

time = 0.24, size = 115, normalized size = 1.03

method	result
derivativedivides	$B \left( -\frac{1}{2a \tan(dx+c)^2} + \frac{(-a^2+b^2) \ln(\tan(dx+c))}{a^3} + \frac{b}{a^2 \tan(dx+c)} + \frac{\frac{a \ln(1+\tan^2(dx+c))}{2} + b \arctan(\tan(dx+c))}{a^2+b^2} - \frac{b^4 \ln(a+b \tan(dx+c))}{a^3(a^2+b^2)} \right) \frac{d}{d}$
default	$B \left( -\frac{1}{2a \tan(dx+c)^2} + \frac{(-a^2+b^2) \ln(\tan(dx+c))}{a^3} + \frac{b}{a^2 \tan(dx+c)} + \frac{\frac{a \ln(1+\tan^2(dx+c))}{2} + b \arctan(\tan(dx+c))}{a^2+b^2} - \frac{b^4 \ln(a+b \tan(dx+c))}{a^3(a^2+b^2)} \right) \frac{d}{d}$
norman	$\frac{b^2 B x (\tan^3(dx+c))}{a^2+b^2} + \frac{B b^2 (\tan^2(dx+c))}{d a^2} + \frac{b B a x (\tan^2(dx+c))}{a^2+b^2} - \frac{B}{2d} + \frac{B b \tan(dx+c)}{2ad} + \frac{a B \ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{(a^2-b^2) B \ln(a+b \tan(dx+c))}{a^3(a^2+b^2)}$

risch	$\frac{ixB}{ib-a} + \frac{2ixB}{a} + \frac{2iBc}{ad} - \frac{2iBb^2x}{a^3} - \frac{2iBb^2c}{da^3} + \frac{2ib^4Bx}{(a^2+b^2)a^3} + \frac{2ib^4Bc}{(a^2+b^2)da^3} + \frac{2iB(-ia e^{2i(dx+c)} + b e^{2i(dx+c)} - b)}{da^2(e^{2i(dx+c)} - 1)^2}$
-------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*B\*(-1/2/a/tan(d\*x+c)^2+1/a^3\*(-a^2+b^2)\*ln(tan(d\*x+c))+1/a^2\*b/tan(d\*x+c)+1/(a^2+b^2)\*(1/2\*a\*ln(1+tan(d\*x+c)^2)+b\*arctan(tan(d\*x+c)))-b^4/a^3/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.55, size = 130, normalized size = 1.16

$$\frac{2Bb^4 \log(b \tan(dx+c)+a)}{a^5+a^3b^2} - \frac{2(dx+c)Bb}{a^2+b^2} - \frac{Ba \log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(Ba^2-Bb^2) \log(\tan(dx+c))}{a^3} - \frac{2Bb \tan(dx+c)-Ba}{a^2 \tan(dx+c)^2} \cdot \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm  
="maxima")

[Out] -1/2\*(2\*B\*b^4\*log(b\*tan(d\*x + c) + a)/(a^5 + a^3\*b^2) - 2\*(d\*x + c)\*B\*b/(a^2 + b^2) - B\*a\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + 2\*(B\*a^2 - B\*b^2)\*log(tan(d\*x + c))/a^3 - (2\*B\*b\*tan(d\*x + c) - B\*a)/(a^2\*tan(d\*x + c)^2))/d

**Fricas [A]**

time = 1.79, size = 192, normalized size = 1.71

$$\frac{Bb^4 \log\left(\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) \tan(dx+c)^2 + Ba^4 + Ba^2b^2 + (Ba^4 - Bb^4) \log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2 + 1}\right) \tan(dx+c)^2 - (2Ba^3bdx - Ba^4 - Ba^2b^2) \tan(dx+c)^2 - 2(Ba^2b + Bab^2) \tan(dx+c)}{2(a^5 + a^3b^2)d \tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm  
="fricas")

[Out] -1/2\*(B\*b^4\*log((b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2)/(tan(d\*x + c)^2 + 1))\*tan(d\*x + c)^2 + B\*a^4 + B\*a^2\*b^2 + (B\*a^4 - B\*b^4)\*log(tan(d\*x + c)^2/(tan(d\*x + c)^2 + 1))\*tan(d\*x + c)^2 - (2\*B\*a^3\*b\*d\*x - B\*a^4 - B\*a^2\*b^2)\*tan(d\*x + c)^2 - 2\*(B\*a^3\*b + B\*a\*b^3)\*tan(d\*x + c))/((a^5 + a^3\*b^2)\*d\*tan(d\*x + c)^2)

**Sympy [C]** Result contains complex when optimal does not.

time = 4.18, size = 1397, normalized size = 12.47

$\frac{\int \frac{Bx}{(a+b \tan(dx+c))^2} \cot^3(dx+c) dx}{\dots}$	<p>for a = 0 ^ b = 0 ^ c = 0 ^ d = 0</p> <p>for b = 0</p> <p>for a = 0</p> <p>for a = -b</p> <p>for a = b</p> <p>for c = -d</p> <p>for d = 0</p> <p>otherwise</p>
--	---



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+B\*b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Piecewise((zoo\*B\*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), (B\*(log(tan(c + d\*x)\*\*2 + 1)/(2\*d) - log(tan(c + d\*x))/d - 1/(2\*d\*tan(c + d\*x)\*\*2))/a, Eq(b, 0)), (B\*(x + 1/(d\*tan(c + d\*x)) - 1/(3\*d\*tan(c + d\*x)\*\*3))/b, Eq(a, 0)), (-3\*B\*d\*x\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) + 3\*I\*B\*d\*x\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) + 2\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) + 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 4\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 4\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 3\*B\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) + I\*B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - B/(2\*b\*d\*tan(c + d\*x)\*\*3 - 2\*I\*b\*d\*tan(c + d\*x)\*\*2), Eq(a, -I\*b)), (-3\*B\*d\*x\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 3\*I\*B\*d\*x\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 2\*I\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) + 2\*B\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) + 4\*I\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*3/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 4\*B\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - 3\*B\*tan(c + d\*x)\*\*2/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - I\*B\*tan(c + d\*x)/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2) - B/(2\*b\*d\*tan(c + d\*x)\*\*3 + 2\*I\*b\*d\*tan(c + d\*x)\*\*2), Eq(a, I\*b)), (zoo\*B\*x/a, Eq(c, -d\*x)), (x\*(B\*a + B\*b\*tan(c))\*cot(c)\*\*3/(a + b\*tan(c))\*\*2, Eq(d, 0)), (B\*a\*\*4\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)\*\*2/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) - 2\*B\*a\*\*4\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) - B\*a\*\*4/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) + 2\*B\*a\*\*3\*b\*d\*x\*tan(c + d\*x)\*\*2/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) + 2\*B\*a\*\*3\*b\*tan(c + d\*x)/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) - B\*a\*\*2\*b\*\*2/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) + 2\*B\*a\*b\*\*3\*tan(c + d\*x)/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) - 2\*B\*b\*\*4\*log(a/b + tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2) + 2\*B\*b\*\*4\*log(tan(c + d\*x))\*tan(c + d\*x)\*\*2/(2\*a\*\*5\*d\*tan(c + d\*x)\*\*2 + 2\*a\*\*3\*b\*\*2\*d\*tan(c + d\*x)\*\*2), True))

**Giac** [A]

time = 0.71, size = 165, normalized size = 1.47

$$\frac{2 B b^5 \log(|b \tan(dx+c)+a|)}{a^5 b+a^3 b^3} - \frac{2(dx+c) B b}{a^2+b^2} - \frac{B a \log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(B a^2-B b^2) \log(|\tan(dx+c)|)}{a^3} - \frac{3 B a^2 \tan(dx+c)^2-3 B b^2 \tan(dx+c)^2+2 B a b \tan(dx+c)-B a^2}{a^3 \tan(dx+c)^2}$$


---

2 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+B\*b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out]  $-1/2*(2*B*b^5*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^5*b + a^3*b^3) - 2*(d*x + c)*B*b/(a^2 + b^2) - B*a*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2) + 2*(B*a^2 - B*b^2)*\log(\text{abs}(\tan(d*x + c)))/a^3 - (3*B*a^2*\tan(d*x + c)^2 - 3*B*b^2*\tan(d*x + c)^2 + 2*B*a*b*\tan(d*x + c) - B*a^2)/(a^3*\tan(d*x + c)^2))/d$

**Mupad [B]**

time = 6.43, size = 143, normalized size = 1.28

$$-\frac{\cot(c+dx)^2 \left( \frac{B}{2a} - \frac{B b \tan(c+dx)}{a^2} \right)}{d} + \frac{B \ln(\tan(c+dx) + 1i)}{2d(a-b1i)} - \frac{B \ln(\tan(c+dx)) (a^2 - b^2)}{a^3 d} - \frac{B b^4 \ln(a + b \tan(c+dx))}{d(a^5 + a^3 b^2)} + \frac{B \ln(\tan(c+dx) - i) 1i}{2d(-b+a1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^3\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out]  $(B*\log(\tan(c + d*x) + 1i))/(2*d*(a - b*1i)) - (\cot(c + d*x)^2*(B/(2*a) - (B*b*tan(c + d*x))/a^2))/d + (B*\log(\tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (B*\log(\tan(c + d*x))*(a^2 - b^2))/(a^3*d) - (B*b^4*\log(a + b*tan(c + d*x)))/(d*(a^5 + a^3*b^2))$

$$3.314 \quad \int \frac{3 + \tan(c + dx)}{2 - \tan(c + dx)} dx$$

Optimal. Leaf size=25

$$x - \frac{\log(2 \cos(c + dx) - \sin(c + dx))}{d}$$

[Out] x-ln(2\*cos(d\*x+c)-sin(d\*x+c))/d

Rubi [A]

time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {3612, 3611}

$$x - \frac{\log(2 \cos(c + dx) - \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(3 + Tan[c + d\*x])/(2 - Tan[c + d\*x]),x]

[Out] x - Log[2\*Cos[c + d\*x] - Sin[c + d\*x]]/d

Rule 3611

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{3 + \tan(c + dx)}{2 - \tan(c + dx)} dx &= x - \int \frac{-1 - 2 \tan(c + dx)}{2 - \tan(c + dx)} dx \\ &= x - \frac{\log(2 \cos(c + dx) - \sin(c + dx))}{d} \end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

time = 0.03, size = 62, normalized size = 2.48

$$\frac{\text{ArcTan}(\tan(c + dx))}{d} + \frac{\log(5 - 4(2 - \tan(c + dx)) + (2 - \tan(c + dx))^2)}{2d} - \frac{\log(2 - \tan(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + Tan[c + d\*x])/(2 - Tan[c + d\*x]), x]

[Out] ArcTan[Tan[c + d\*x]]/d + Log[5 - 4\*(2 - Tan[c + d\*x]) + (2 - Tan[c + d\*x])^2]/(2\*d) - Log[2 - Tan[c + d\*x]]/d

**Maple [A]**

time = 0.06, size = 37, normalized size = 1.48

method	result	size
norman	$x - \frac{\ln(-2 + \tan(dx+c))}{d} + \frac{\ln(1 + \tan^2(dx+c))}{2d}$	33
risch	$ix + x + \frac{2ic}{d} - \frac{\ln(e^{2i(dx+c)} + \frac{3}{5} - \frac{4i}{5})}{d}$	33
derivativdivides	$\frac{-\ln(-2 + \tan(dx+c)) + \frac{\ln(1 + \tan^2(dx+c))}{2} + \arctan(\tan(dx+c))}{d}$	37
default	$\frac{-\ln(-2 + \tan(dx+c)) + \frac{\ln(1 + \tan^2(dx+c))}{2} + \arctan(\tan(dx+c))}{d}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+tan(d\*x+c))/(2-tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(-ln(-2+tan(d\*x+c))+1/2\*ln(1+tan(d\*x+c)^2)+arctan(tan(d\*x+c)))

**Maxima [A]**

time = 0.60, size = 35, normalized size = 1.40

$$\frac{2 dx + 2 c + \log(\tan(dx + c)^2 + 1) - 2 \log(\tan(dx + c) - 2)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+tan(d\*x+c))/(2-tan(d\*x+c)), x, algorithm="maxima")

[Out] 1/2\*(2\*d\*x + 2\*c + log(tan(d\*x + c)^2 + 1) - 2\*log(tan(d\*x + c) - 2))/d

**Fricas [A]**

time = 2.01, size = 44, normalized size = 1.76

$$\frac{2 dx - \log\left(\frac{\tan(dx+c)^2 - 4 \tan(dx+c) + 4}{\tan(dx+c)^2 + 1}\right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+tan(d\*x+c))/(2-tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(2\*d\*x - log((tan(d\*x + c)^2 - 4\*tan(d\*x + c) + 4)/(tan(d\*x + c)^2 + 1)))/d

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

time = 0.17, size = 39, normalized size = 1.56

$$\begin{cases} x - \frac{\log(\tan(c+dx)-2)}{d} + \frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } d \neq 0 \\ \frac{x(\tan(c)+3)}{2-\tan(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+tan(d\*x+c))/(2-tan(d\*x+c)),x)

[Out] Piecewise((x - log(tan(c + d\*x) - 2)/d + log(tan(c + d\*x)\*\*2 + 1)/(2\*d), Ne(d, 0)), (x\*(tan(c) + 3)/(2 - tan(c)), True))

**Giac [A]**

time = 0.48, size = 36, normalized size = 1.44

$$\frac{2dx + 2c + \log(\tan(dx + c)^2 + 1) - 2\log(|\tan(dx + c) - 2|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+tan(d\*x+c))/(2-tan(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*d\*x + 2\*c + log(tan(d\*x + c)^2 + 1) - 2\*log(abs(tan(d\*x + c) - 2)))/d

**Mupad [B]**

time = 6.23, size = 49, normalized size = 1.96

$$-\frac{\ln(\tan(c + dx) - 2)}{d} + \frac{\ln(\tan(c + dx) - i) \left(\frac{1}{2} - \frac{1}{2}i\right)}{d} + \frac{\ln(\tan(c + dx) + 1i) \left(\frac{1}{2} + \frac{1}{2}i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(tan(c + d\*x) + 3)/(tan(c + d\*x) - 2),x)

[Out] (log(tan(c + d\*x) - 1i)\*(1/2 - 1i/2))/d - log(tan(c + d\*x) - 2)/d + (log(tan(c + d\*x) + 1i)\*(1/2 + 1i/2))/d

$$3.315 \quad \int \frac{\frac{bB}{a} + B \tan(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=58

$$\frac{2bBx}{a^2 + b^2} - \frac{\left(a - \frac{b^2}{a}\right) B \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2) d}$$

[Out]  $2*b*B*x/(a^2+b^2)-(a-b^2/a)*B*\ln(a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)/d$

Rubi [A]

time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {3612, 3611}

$$\frac{2bBx}{a^2 + b^2} - \frac{B\left(a - \frac{b^2}{a}\right) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[((b\*B)/a + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]),x]

[Out]  $(2*b*B*x)/(a^2 + b^2) - ((a - b^2/a)*B*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)*d)$

Rule 3611

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c/(b\*f))\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

Rule 3612

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(a\*c + b\*d)\*(x/(a^2 + b^2)), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

Rubi steps

$$\int \frac{\frac{bB}{a} + B \tan(c + dx)}{a + b \tan(c + dx)} dx = \frac{2bBx}{a^2 + b^2} - \frac{\left( \left( a - \frac{b^2}{a} \right) B \right) \int \frac{b-a \tan(c+dx)}{a+b \tan(c+dx)} dx}{a^2 + b^2}$$

$$= \frac{2bBx}{a^2 + b^2} - \frac{\left( a - \frac{b^2}{a} \right) B \log(a \cos(c + dx) + b \sin(c + dx))}{(a^2 + b^2) d}$$

**Mathematica [A]**

time = 0.07, size = 65, normalized size = 1.12

$$\frac{B(4ab \operatorname{ArcTan}(\tan(c + dx)) + (a^2 - b^2) (\log(\sec^2(c + dx)) - 2 \log(a + b \tan(c + dx))))}{2a(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*B)/a + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x]),x]

[Out] (B\*(4\*a\*b\*ArcTan[Tan[c + d\*x]] + (a^2 - b^2)\*(Log[Sec[c + d\*x]^2] - 2\*Log[a + b\*Tan[c + d\*x]])))/(2\*a\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.10, size = 85, normalized size = 1.47

method	result
derivativedivides	$\frac{B \left( \frac{(a^2 - b^2) \ln(1 + \tan^2(dx + c))}{2} + 2ab \arctan(\tan(dx + c)) - \frac{(a^2 - b^2) \ln(a + b \tan(dx + c))}{a^2 + b^2} \right)}{da}$
default	$\frac{B \left( \frac{(a^2 - b^2) \ln(1 + \tan^2(dx + c))}{2} + 2ab \arctan(\tan(dx + c)) - \frac{(a^2 - b^2) \ln(a + b \tan(dx + c))}{a^2 + b^2} \right)}{da}$
norman	$\frac{2bBx}{a^2 + b^2} + \frac{B(a^2 - b^2) \ln(1 + \tan^2(dx + c))}{2ad(a^2 + b^2)} - \frac{B(a^2 - b^2) \ln(a + b \tan(dx + c))}{ad(a^2 + b^2)}$
risch	$-\frac{Bxb}{a(ib-a)} + \frac{ixB}{ib-a} + \frac{2iBax}{a^2+b^2} - \frac{2ib^2Bx}{a(a^2+b^2)} + \frac{2iBac}{d(a^2+b^2)} - \frac{2ib^2Bc}{ad(a^2+b^2)} - \frac{\ln(e^{2i(dx+c)} - \frac{ib+a}{ib-a})aB}{d(a^2+b^2)} + \frac{b^2 \ln(e^{2i(dx+c)} - \frac{ib+a}{ib-a})}{ad(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*B/a+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*B/a\*(1/(a^2+b^2)\*(1/2\*(a^2-b^2)\*ln(1+tan(d\*x+c)^2)+2\*a\*b\*arctan(tan(d\*x+c)))-(a^2-b^2)/(a^2+b^2)\*ln(a+b\*tan(d\*x+c)))

**Maxima [A]**

time = 0.71, size = 95, normalized size = 1.64

$$\frac{\frac{4(dx+c)Bb}{a^2+b^2} - \frac{2(Ba^2 - Bb^2) \log(b \tan(dx+c) + a)}{a^3 + ab^2}}{2d} + \frac{(Ba^2 - Bb^2) \log(\tan(dx+c)^2 + 1)}{a^3 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*B/a+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2} * (4 * (d * x + c) * B * b / (a^2 + b^2) - 2 * (B * a^2 - B * b^2) * \log(b * \tan(d * x + c) + a)) / (a^3 + a * b^2) + (B * a^2 - B * b^2) * \log(\tan(d * x + c)^2 + 1) / (a^3 + a * b^2) / d$

**Fricas** [A]

time = 1.16, size = 78, normalized size = 1.34

$$\frac{4 B a b d x - (B a^2 - B b^2) \log\left(\frac{b^2 \tan(dx+c)^2 + 2 a b \tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right)}{2 (a^3 + a b^2) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*B/a+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} * (4 * B * a * b * d * x - (B * a^2 - B * b^2) * \log((b^2 * \tan(d * x + c)^2 + 2 * a * b * \tan(d * x + c) + a^2) / (\tan(d * x + c)^2 + 1))) / ((a^3 + a * b^2) * d)$

**Sympy** [C] Result contains complex when optimal does not.

time = 0.44, size = 235, normalized size = 4.05

$$\left\{ \begin{array}{ll} \text{NaN} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{B \log(\tan^2(c+dx)+1)}{2ad} & \text{for } b = 0 \\ -\frac{B}{bd \tan(c+dx) - ibd} & \text{for } a = -ib \\ -\frac{B}{bd \tan(c+dx) + ibd} & \text{for } a = ib \\ x \frac{B \tan(c) + \frac{Bb}{a}}{a + b \tan(c)} & \text{for } d = 0 \\ -\frac{2Ba^2 \log\left(\frac{a}{b} + \tan(c+dx)\right)}{2a^3d + 2ab^2d} + \frac{Ba^2 \log(\tan^2(c+dx)+1)}{2a^3d + 2ab^2d} + \frac{4Babd x}{2a^3d + 2ab^2d} + \frac{2Bb^2 \log\left(\frac{a}{b} + \tan(c+dx)\right)}{2a^3d + 2ab^2d} - \frac{Bb^2 \log(\tan^2(c+dx)+1)}{2a^3d + 2ab^2d} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*B/a+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*d), Eq(b, 0)), (-B/(b\*d\*tan(c + d\*x) - I\*b\*d), Eq(a, -I\*b)), (-B/(b\*d\*tan(c + d\*x) + I\*b\*d), Eq(a, I\*b)), (x\*(B\*tan(c) + B\*b/a)/(a + b\*tan(c)), Eq(d, 0)), (-2\*B\*a\*\*2\*log(a/b + tan(c + d\*x))/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) + B\*a\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) + 4\*B\*a\*b\*d\*x/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) + 2\*B\*b\*\*2\*log(a/b + tan(c + d\*x))/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d) - B\*b\*\*2\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*3\*d + 2\*a\*b\*\*2\*d), True))

**Giac** [A]

time = 0.52, size = 99, normalized size = 1.71

$$\frac{\frac{4(dx+c)Bb}{a^2+b^2} + \frac{(Ba^2-Bb^2) \log(\tan(dx+c)^2+1)}{a^3+ab^2}}{2d} - \frac{2(Ba^2b-Bb^3) \log(|b \tan(dx+c)+a|)}{a^3b+ab^3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*B/a+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{2} * (4 * (d * x + c) * B * b / (a^2 + b^2) + (B * a^2 - B * b^2) * \log(\tan(d * x + c)^2 + 1) / (a^3 + a * b^2) - 2 * (B * a^2 * b - B * b^3) * \log(\text{abs}(b * \tan(d * x + c) + a)) / (a^3 * b + a * b^3)) / d$

**Mupad [B]**

time = 6.57, size = 112, normalized size = 1.93

$$-\frac{\ln(\tan(c + dx) - i) (Bb + Ba1i)}{2d (ab - a^2 1i)} - \frac{\ln(\tan(c + dx) + 1i) (Ba + Bb1i)}{2d (-a^2 + ab1i)} - \frac{B \ln(a + b \tan(c + dx)) (a^2 - b^2)}{ad (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*tan(c + d\*x) + (B\*b)/a)/(a + b\*tan(c + d\*x)),x)

[Out]  $-(\log(\tan(c + d * x) - 1i) * (B * a * 1i + B * b)) / (2 * d * (a * b - a^2 * 1i)) - (\log(\tan(c + d * x) + 1i) * (B * a + B * b * 1i)) / (2 * d * (a * b * 1i - a^2)) - (B * \log(a + b * \tan(c + d * x)) * (a^2 - b^2)) / (a * d * (a^2 + b^2))$

### 3.316 $\int \frac{a+b \tan(c+dx)}{(b+a \tan(c+dx))^2} dx$

**Optimal.** Leaf size=101

$$-\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^2} + \frac{b(3a^2 - b^2) \log(b \cos(c + dx) + a \sin(c + dx))}{(a^2 + b^2)^2 d} - \frac{a^2 - b^2}{(a^2 + b^2) d (b + a \tan(c + dx))}$$

[Out]  $-a*(a^2-3*b^2)*x/(a^2+b^2)^2+b*(3*a^2-b^2)*\ln(b*\cos(d*x+c)+a*\sin(d*x+c))/(a^2+b^2)^2/d+(-a^2+b^2)/(a^2+b^2)/d/(b+a*\tan(d*x+c))$

**Rubi [A]**

time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3610, 3612, 3611}

$$-\frac{a^2 - b^2}{d(a^2 + b^2)(a \tan(c + dx) + b)} + \frac{b(3a^2 - b^2) \log(a \sin(c + dx) + b \cos(c + dx))}{d(a^2 + b^2)^2} - \frac{ax(a^2 - 3b^2)}{(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])/(b + a*\text{Tan}[c + d*x])^2, x]$

[Out]  $-((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^2) + (b*(3*a^2 - b^2)*\text{Log}[b*\text{Cos}[c + d*x] + a*\text{Sin}[c + d*x]])/((a^2 + b^2)^2*d) - (a^2 - b^2)/((a^2 + b^2)*d*(b + a*\text{Tan}[c + d*x]))$

Rule 3610

$\text{Int}[(a + b*\text{tan}[(e + f*x)]/(c + d*\text{tan}[(e + f*x)] + (f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3611

$\text{Int}[(c + d*\text{tan}[(e + f*x)]/(a + b*\text{tan}[(e + f*x)] + (f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{EqQ}[a*c + b*d, 0]$

Rule 3612

$\text{Int}[(c + d*\text{tan}[(e + f*x)]/(a + b*\text{tan}[(e + f*x)] + (f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[a*c + b*d, 0]$

Q[a\*c + b\*d, 0]

Rubi steps

$$\int \frac{a + b \tan(c + dx)}{(b + a \tan(c + dx))^2} dx = -\frac{a^2 - b^2}{(a^2 + b^2) d(b + a \tan(c + dx))} + \frac{\int \frac{2ab - (a^2 - b^2) \tan(c + dx)}{b + a \tan(c + dx)} dx}{a^2 + b^2}$$

$$= -\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^2} - \frac{a^2 - b^2}{(a^2 + b^2) d(b + a \tan(c + dx))} + \frac{(b(3a^2 - b^2)) \int \frac{a - b \tan(c + dx)}{b + a \tan(c + dx)} dx}{(a^2 + b^2)^2}$$

$$= -\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^2} + \frac{b(3a^2 - b^2) \log(b \cos(c + dx) + a \sin(c + dx))}{(a^2 + b^2)^2 d} - \frac{1}{(a^2 + b^2) d}$$

**Mathematica [C]** Result contains complex when optimal does not.  
 time = 1.64, size = 187, normalized size = 1.85

$$\frac{b(-((a+ib)\log(i-\tan(c+dx)))-(a-ib)\log(i+\tan(c+dx))+2a\log(b+a\tan(c+dx)))}{a^2+b^2} + (a-b)(a+b) \left( \frac{i\log(i-\tan(c+dx))}{(a-ib)^2} - \frac{i\log(i+\tan(c+dx))}{(a+ib)^2} + \frac{2a(2b\log(b+a\tan(c+dx))-\frac{a^2+b^2}{b+a\tan(c+dx)})}{(a^2+b^2)^2} \right)$$

2ad

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])/(b + a\*Tan[c + d\*x])^2,x]

[Out] ((b\*(-((a + I\*b)\*Log[I - Tan[c + d\*x]]) - (a - I\*b)\*Log[I + Tan[c + d\*x]] + 2\*a\*Log[b + a\*Tan[c + d\*x]]))/(a^2 + b^2) + (a - b)\*(a + b)\*((I\*Log[I - Tan[c + d\*x]])/(a - I\*b)^2 - (I\*Log[I + Tan[c + d\*x]])/(a + I\*b)^2 + (2\*a\*(2\*b\*Log[b + a\*Tan[c + d\*x]] - (a^2 + b^2)/(b + a\*Tan[c + d\*x])))/(a^2 + b^2)^2))/(2\*a\*d)

**Maple [A]**

time = 0.12, size = 125, normalized size = 1.24

method	result
derivativedivides	$-\frac{a^2 - b^2}{(a^2 + b^2)(b + a \tan(dx + c))} + \frac{b(3a^2 - b^2) \ln(b + a \tan(dx + c))}{(a^2 + b^2)^2} + \frac{\frac{(-3a^2b + b^3) \ln(1 + \tan^2(dx + c))}{2} + (-a^3 + 3ab^2) \arctan(\tan(dx + c))}{(a^2 + b^2)^2}$
default	$-\frac{a^2 - b^2}{(a^2 + b^2)(b + a \tan(dx + c))} + \frac{b(3a^2 - b^2) \ln(b + a \tan(dx + c))}{(a^2 + b^2)^2} + \frac{\frac{(-3a^2b + b^3) \ln(1 + \tan^2(dx + c))}{2} + (-a^3 + 3ab^2) \arctan(\tan(dx + c))}{(a^2 + b^2)^2}$
norman	$\frac{\frac{(a^2 - b^2)a \tan(dx + c)}{bd(a^2 + b^2)} - \frac{a^2(a^2 - 3b^2)x \tan(dx + c)}{a^4 + 2a^2b^2 + b^4} - \frac{ba(a^2 - 3b^2)x}{a^4 + 2a^2b^2 + b^4}}{b + a \tan(dx + c)} + \frac{b(3a^2 - b^2) \ln(b + a \tan(dx + c))}{d(a^4 + 2a^2b^2 + b^4)} - \frac{b(3a^2 - b^2) \ln(1 + \tan^2(dx + c))}{2d(a^4 + 2a^2b^2 + b^4)}$
risch	$\frac{ixb}{2iba + a^2 - b^2} - \frac{xa}{2iba + a^2 - b^2} - \frac{6ib a^2 x}{a^4 + 2a^2b^2 + b^4} + \frac{2ib^3 x}{a^4 + 2a^2b^2 + b^4} - \frac{6ib a^2 c}{d(a^4 + 2a^2b^2 + b^4)} + \frac{2ib^3 c}{d(a^4 + 2a^2b^2 + b^4)} - \frac{1}{(a^2 + b^2) d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out]  $1/d*(-(a^2-b^2)/(a^2+b^2)/(b+a*\tan(dx+c))+b*(3*a^2-b^2)/(a^2+b^2)^2*\ln(b+a*\tan(dx+c))+1/(a^2+b^2)^2*(1/2*(-3*a^2*b+b^3)*\ln(1+\tan(dx+c)^2)+(-a^3+3*a*b^2)*\arctan(\tan(dx+c))))$

**Maxima** [A]

time = 0.55, size = 161, normalized size = 1.59

$$\frac{\frac{2(a^3-3ab^2)(dx+c)}{a^4+2a^2b^2+b^4} - \frac{2(3a^2b-b^3)\log(a\tan(dx+c)+b)}{a^4+2a^2b^2+b^4} + \frac{(3a^2b-b^3)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{2(a^2-b^2)}{a^2b+b^3+(a^3+ab^2)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x, algorithm="maxima")`

[Out]  $-1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b^3)*\log(a*\tan(d*x + c) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 + (a^3 + a*b^2)*\tan(d*x + c)))/d$

**Fricas** [A]

time = 1.52, size = 191, normalized size = 1.89

$$\frac{2a^4 - 2a^2b^2 + 2(a^3b - 3ab^3)dx - (3a^2b^2 - b^4 + (3a^3b - ab^3)\tan(dx+c))\log\left(\frac{a^2\tan(dx+c)^2+2ab\tan(dx+c)+b^2}{\tan(dx+c)^2+1}\right) - 2(a^3b - ab^3 - (a^4 - 3a^2b^2)dx)\tan(dx+c)}{2((a^5 + 2a^3b^2 + ab^4)d\tan(dx+c) + (a^4b + 2a^2b^3 + b^5)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*d*x - (3*a^2*b^2 - b^4 + (3*a^3*b - a*b^3)*\tan(d*x + c))*\log((a^2*\tan(d*x + c)^2 + 2*a*b*\tan(d*x + c) + b^2)/(\tan(d*x + c)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*d*x)*\tan(d*x + c)/((a^5 + 2*a^3*b^2 + a*b^4)*d*\tan(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d)$

**Sympy** [C] Result contains complex when optimal does not.

time = 0.70, size = 1346, normalized size = 13.33

---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))**2,x)`

```
[Out] Piecewise((zoo*x*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(2*b*d*tan(c +
d*x)**2 + 4*I*b*d*tan(c + d*x) - 2*b*d), Eq(a, -I*b)), (1/(2*b*d*tan(c + d
*x)**2 - 4*I*b*d*tan(c + d*x) - 2*b*d), Eq(a, I*b)), (x*(a + b*tan(c))/(a*t
an(c) + b)**2, Eq(d, 0)), (log(tan(c + d*x)**2 + 1)/(2*b*d), Eq(a, 0)), (-2
*a**4*d*x*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*
tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 2*a**4
/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*
b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 2*a**3*b*d*x/(2*a**5*d*tan(c
+ d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**
4*d*tan(c + d*x) + 2*b**5*d) + 6*a**3*b*log(tan(c + d*x) + b/a)*tan(c + d*x
)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2
*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 3*a**3*b*log(tan(c + d*x)**
2 + 1)*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan
(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) + 6*a**2*b*
**2*d*x*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan
(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) + 6*a**2*b*
**2*log(tan(c + d*x) + b/a)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**
2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 3*
a**2*b**2*log(tan(c + d*x)**2 + 1)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*
a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5
*d) + 6*a*b**3*d*x/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c
+ d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 2*a*b**3*1
og(tan(c + d*x) + b/a)*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4
*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**
5*d) + a*b**3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*d*tan(c + d*x)
+ 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c
+ d*x) + 2*b**5*d) - 2*b**4*log(tan(c + d*x) + b/a)/(2*a**5*d*tan(c + d*x
) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*ta
n(c + d*x) + 2*b**5*d) + b**4*log(tan(c + d*x)**2 + 1)/(2*a**5*d*tan(c + d*
x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*ta
n(c + d*x) + 2*b**5*d) + 2*b**4/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a*
**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d
), True))
```

**Giac [A]**

time = 0.60, size = 199, normalized size = 1.97

$$\frac{\frac{2(a^3 - 3ab^2)(dx+c)}{a^4 + 2a^2b^2 + b^4} + \frac{(3a^2b - b^3) \log(\tan(dx+c)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(3a^3b - ab^3) \log(|a \tan(dx+c) + b|)}{a^3 + 2a^3b^2 + ab^4} + \frac{2(3a^3b \tan(dx+c) - ab^3 \tan(dx+c) + a^4 + 3a^2b^2 - 2b^4)}{(a^4 + 2a^2b^2 + b^4)(a \tan(dx+c) + b)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)
*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^3*b - a*b^3)*log(
```

```
abs(a*tan(d*x + c) + b))/(a^5 + 2*a^3*b^2 + a*b^4) + 2*(3*a^3*b*tan(d*x + c) - a*b^3*tan(d*x + c) + a^4 + 3*a^2*b^2 - 2*b^4)/((a^4 + 2*a^2*b^2 + b^4)*(a*tan(d*x + c) + b))/d
```

**Mupad [B]**

time = 6.63, size = 152, normalized size = 1.50

$$\frac{b \ln(b + a \tan(c + dx)) (3a^2 - b^2)}{d(a^2 + b^2)^2} - \frac{\ln(\tan(c + dx) + 1i) (a - b1i)}{2d(-a^2 1i + 2ab + b^2 1i)} - \frac{a^2 - b^2}{d(a^2 + b^2)(b + a \tan(c + dx))} - \frac{\ln(\tan(c + dx) - i) (a + b1i)}{2d(a^2 1i + 2ab - b^2 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/(b + a*tan(c + d*x))^2,x)
```

```
[Out] (b*log(b + a*tan(c + d*x))*(3*a^2 - b^2))/(d*(a^2 + b^2)^2) - (log(tan(c + d*x) + 1i)*(a - b*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (a^2 - b^2)/(d*(a^2 + b^2)*(b + a*tan(c + d*x))) - (log(tan(c + d*x) - 1i)*(a + b*1i))/(2*d*(2*a*b + a^2*1i - b^2*1i))
```

$$3.317 \quad \int \tan^3(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal. Leaf size=233

$$\frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{\sqrt{a + ib} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d}$$

[Out] (A-I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))\*(a-I\*b)^(1/2)/d+(A+I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))\*(a+I\*b)^(1/2)/d-2\*A\*(a+b\*tan(d\*x+c))^(1/2)/d-2/105\*(14\*A\*a\*b-8\*B\*a^2+35\*B\*b^2)\*(a+b\*tan(d\*x+c))^(3/2)/b^3/d+2/35\*(7\*A\*b-4\*B\*a)\*tan(d\*x+c)\*(a+b\*tan(d\*x+c))^(3/2)/b^2/d+2/7\*B\*tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(3/2)/b/d

Rubi [A]

time = 0.46, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3688, 3728, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(-8a^2B + 14aAb + 35B^2)(a + b \tan(c + dx))^{3/2}}{105b^3d} + \frac{2(7Ab - 4aB) \tan(c + dx)(a + b \tan(c + dx))^{3/2}}{35b^2d} + \frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{\sqrt{a + ib} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} - \frac{2A\sqrt{a + b \tan(c + dx)}}{d} + \frac{2B \tan^2(c + dx)(a + b \tan(c + dx))^{3/2}}{7bd}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^3\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[a - I\*b]\*(A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/d + (Sqrt[a + I\*b]\*(A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/d - (2\*A\*Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*(14\*a\*A\*b - 8\*a^2\*B + 35\*b^2\*B)\*(a + b\*Tan[c + d\*x])^(3/2))/(105\*b^3\*d) + (2\*(7\*A\*b - 4\*a\*B)\*Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^(3/2))/(35\*b^2\*d) + (2\*B\*Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^(3/2))/(7\*b\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rule 3728



```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2], x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

Rubi steps

$$\begin{aligned}
\int \tan^3(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{2B \tan^2(c + dx)(a + b \tan(c + dx))^{3/2}}{7bd} + \frac{2 \int \tan^2(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx}{7bd} \\
&= \frac{2(7Ab - 4aB) \tan(c + dx)(a + b \tan(c + dx))^{3/2}}{35b^2d} \\
&= -\frac{2(14aAb - 8a^2B + 35b^2B)(a + b \tan(c + dx))^{3/2}}{105b^3d} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d} - \frac{2(14aAb - 8a^2B + 35b^2B)(a + b \tan(c + dx))^{3/2}}{105b^3d} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d} - \frac{2(14aAb - 8a^2B + 35b^2B)(a + b \tan(c + dx))^{3/2}}{105b^3d} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d} - \frac{2(14aAb - 8a^2B + 35b^2B)(a + b \tan(c + dx))^{3/2}}{105b^3d} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d} - \frac{2(14aAb - 8a^2B + 35b^2B)(a + b \tan(c + dx))^{3/2}}{105b^3d} \\
&= \frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) + \sqrt{a + ib} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + 2 \sqrt{a + b \tan(c + dx)} (-14a^2Ab - 105Ab^3 + 8a^2B - 35ab^2B - b(-7aAb + 4a^2B + 35b^2B) \tan(c + dx) + 3b^2(7Ab + aB) \tan^2(c + dx) + 15b^3B \tan^3(c + dx))}{105b^3d}
\end{aligned}$$

**Mathematica [A]**

time = 1.86, size = 212, normalized size = 0.91

$$\frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) + \sqrt{a + ib} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + 2 \sqrt{a + b \tan(c + dx)} (-14a^2Ab - 105Ab^3 + 8a^2B - 35ab^2B - b(-7aAb + 4a^2B + 35b^2B) \tan(c + dx) + 3b^2(7Ab + aB) \tan^2(c + dx) + 15b^3B \tan^3(c + dx))}{105b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^3\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]  
 [Out] (Sqrt[a - I\*b]\*(A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/d  
 + (Sqrt[a + I\*b]\*(A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]  
 )/d + (2\*Sqrt[a + b\*Tan[c + d\*x]]\*(-14\*a^2\*A\*b - 105\*A\*b^3 + 8\*a^3\*B - 35\*a  
 \*b^2\*B - b\*(-7\*a\*A\*b + 4\*a^2\*B + 35\*b^2\*B)\*Tan[c + d\*x] + 3\*b^2\*(7\*A\*b + a\*  
 B)\*Tan[c + d\*x]^2 + 15\*b^3\*B\*Tan[c + d\*x]^3))/(105\*b^3\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 739 vs.  
 $2(201) = 402$ .

time = 0.32, size = 740, normalized size = 3.18

method	result
derivativedivides	$\frac{2B(a+b \tan(dx+c))^{\frac{7}{2}}}{7} + \frac{2Ab(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{4Ba(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2Aab(a+b \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{2B a^2(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - \frac{2B b^2}{3}$
default	$\frac{2B(a+b \tan(dx+c))^{\frac{7}{2}}}{7} + \frac{2Ab(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{4Ba(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2Aab(a+b \tan(dx+c))^{\frac{3}{2}}}{3} + \frac{2B a^2(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - \frac{2B b^2}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVE  
 RBOSE)

[Out] 2/d/b^3\*(1/7\*B\*(a+b\*tan(d\*x+c))^(7/2)+1/5\*A\*b\*(a+b\*tan(d\*x+c))^(5/2)-2/5\*B\*  
 a\*(a+b\*tan(d\*x+c))^(5/2)-1/3\*A\*a\*b\*(a+b\*tan(d\*x+c))^(3/2)+1/3\*B\*a^2\*(a+b\*ta  
 n(d\*x+c))^(3/2)-1/3\*B\*b^2\*(a+b\*tan(d\*x+c))^(3/2)-A\*b^3\*(a+b\*tan(d\*x+c))^(1/  
 2)+b^3\*(1/4/b\*(1/2\*(A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b-B\*(a^2+b^2)^(1/2)\*(2\*  
 (a^2+b^2)^(1/2)+2\*a)^(1/2)+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a)\*ln(b\*tan(d\*x+  
 c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))+  
 2\*(2\*A\*(a^2+b^2)^(1/2)\*b-1/2\*(A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b-B\*(a^2+b^2)  
 ^^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a)\*(2\*  
 (a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((2\*(a+b\*ta

$$\begin{aligned} & n(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)} \\ & ))+1/4/b*(1/2*(-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b+B*(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a)*\ln(b*\tan(d*x+c)+ \\ & a-(a+b*\tan(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)}))+2*( \\ & 2*A*(a^2+b^2)^{(1/2)}*b+1/2*(-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b+B*(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\tan(d*x+c))^{(1/2)}-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})) \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^3, x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 9053 vs. 2(195) = 390.

time = 20.36, size = 9053, normalized size = 38.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/420*(420*\sqrt{2}*b^3*d^5*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4}) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(3/4)}*\arctan(((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^3)*d^4*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4})*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4}) + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 + (A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 + (A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a*b^3)*d^2*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}) + \sqrt{2}*(( \end{aligned}$$

$$\begin{aligned}
& 2*(A^4*B + A^2*B^3)*a + (A^5 - A*B^4)*b*d^7*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b - (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (2*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a^2 + (A^7 - A^5*B^2 - 5*A^3*B^4 - 3*A*B^6)*a*b - (A^6*B + A^4*B^3 - A^2*B^5 - B^7)*b^2)*d^5*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)}*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}*\sqrt{((a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(3/4)} + \sqrt{2}*(A*d^7*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + ((A^3 + A*B^2)*a - (A^2*B + B^3)*b)*d^5*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)}*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}*\sqrt{((4*(A^4*B^2 + A^2*B^4)*a^4 + 4*(A^5*B - A*B^5)*a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 + 4*(A^5*B - A*B^5)*a*b^3 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^4)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4}}*\cos(d*x + c) + \sqrt{2}*((4*A^3*B^2*a^3 + 4*(A^4*B - 2*A^2*B^3)*a^2*b + (A^5 - 6*A^3*B^2 + 5*A*B^4)*a*b^2 - (A^4*B - 2*A^2*B^3 + B^5)*b^3)*d^3*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4}}*\cos(d*x + c) + (4*(A^5*B^2 + A^3*B^4)*a^4 + 4*(A^6*B - A^2*B^5)*a^3*b + (A^7 + 3*A^5*B^2 + 3*A^3*B^4 + A*B^6)*a^2*b^2 + 4*(A^6*B - A^2*B^5)*a*b^3 + (A^7 - A^5*B^2 - A^3*B^4 + A*B^6)*b^4)*d*\cos(d*x + c))*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}*\sqrt{((a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(1/4)} + (4*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^5 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^4*b + (A^8 + 4*A^6*B^2 + 6*A^4*B^4 + 4*A^2*B^6 + B^8)*a^3*b^2 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^2*b^3 + (A^8 - 2*A^4*B^4 + B^8)*a*b^4)*\cos(d*x + c) + (4*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^4*b + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^3*b^2 + (A^8 + 4*A^6*B^2 + 6*A^4*B^4 + 4*A^2*B^6 + B^8)*a^2*b^3 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^4 + (A^8 - 2*A^4*B^4 + B^8)*b^5)*\sin(d*x + c))/((a^2 + b^2)*\cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(3/4)}/(4*(A^10*B^2 + 4*A^8*B^4 + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^10)*a^4*b + 4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a^3*b^2 + (A^12 + 6*A^10*B^2 + 15*A^8*B^4 + 20*A^6*B^6 + 15*A^4*B^8 + 6*A^2*B^10 + B^12)*a^2*b^3 + 4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a*b^4 + (A^12 + 2*A^10*B^2 - A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 + B^12)*b^5))*\cos(d*x + c)^3 + 420*\sqrt{2}*b^3*d^5*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*
\end{aligned}$$

$$A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4) + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2))\sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)*((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4}}\arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b + 2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^2 + (A^8 + \dots$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \tan^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*3, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 56.41, size = 1093, normalized size = 4.69

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] atan((d^3\*((16\*(B^2\*b^4 - B^2\*a^2\*b^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^2 + (16\*a\*b^2\*((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^4)\*(-((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)/(4\*d^4))^(1/2)\*1i)/(8\*(B^3\*b^5 + B^3\*a^2\*b^3)))\*(-((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)/(4\*d^4))^(1/2)\*2i - ((2\*B\*(a^2 + b^2))/(3\*b^3\*d) - (4\*B\*a^2)/(3\*b^3\*d))\*(a + b\*tan(c + d\*x))^(3/2) + a tan((d^3\*((16\*(B^2\*b^4 - B^2\*a^2\*b^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^2 - (16\*a\*b^2\*((-B^4\*b^2\*d^4)^(1/2) - B^2\*a\*d^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^4)\*(((B^4\*b^2\*d^4)^(1/2) - B^2\*a\*d^2)/(4\*d^4))^(1/2)\*1i)/(8\*(B^3\*b^5 + B^3\*a^

$$\begin{aligned}
& 2*b^3)) * (((-B^4*b^2*d^4)^{(1/2)} - B^2*a*d^2)/(4*d^4))^{(1/2)} * 2i - (a + b*\tan \\
& (c + d*x))^{(1/2)} * (2*a*((2*B*(a^2 + b^2))/(b^3*d) - (4*B*a^2)/(b^3*d)) + (8* \\
& B*a^3)/(b^3*d) - (4*B*a*(a^2 + b^2))/(b^3*d)) - \operatorname{atan}((A^2*b^4*((-A^4*b^2*d^4)^{(1/2)})/(4*d^4) + (A^2*a)/(4*d^2))^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)} * 32i) / ( \\
& (16*A*b^4*(-A^4*b^2*d^4)^{(1/2)})/d^3 + (16*A*a^2*b^2*(-A^4*b^2*d^4)^{(1/2)})/d \\
& ^3) + (a*b^2*((-A^4*b^2*d^4)^{(1/2)})/(4*d^4) + (A^2*a)/(4*d^2))^{(1/2)} * (a + b* \\
& \tan(c + d*x))^{(1/2)} * (-A^4*b^2*d^4)^{(1/2)} * 32i) / ((16*A*b^4*(-A^4*b^2*d^4)^{(1/2)})/d + (16*A*a^2*b^2*(-A^4*b^2*d^4)^{(1/2)})/d) * (((-A^4*b^2*d^4)^{(1/2)} + A^ \\
& 2*a*d^2)/(4*d^4))^{(1/2)} * 2i + \operatorname{atan}((A^2*b^4*((A^2*a)/(4*d^2) - (-A^4*b^2*d^4)^{(1/2)})/(4*d^4))^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)} * 32i) / ((16*A*b^4*(-A^4*b^2 \\
& *d^4)^{(1/2)})/d^3 + (16*A*a^2*b^2*(-A^4*b^2*d^4)^{(1/2)})/d^3) - (a*b^2*((A^2* \\
& a)/(4*d^2) - (-A^4*b^2*d^4)^{(1/2)})/(4*d^4))^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)} \\
& * (-A^4*b^2*d^4)^{(1/2)} * 32i) / ((16*A*b^4*(-A^4*b^2*d^4)^{(1/2)})/d + (16*A*a^2*b \\
& ^2*(-A^4*b^2*d^4)^{(1/2)})/d) * (-((-A^4*b^2*d^4)^{(1/2)} - A^2*a*d^2)/(4*d^4))^{(1/2)} * 2i - ((2*A*(a^2 + b^2))/(b^2*d) - (2*A*a^2)/(b^2*d)) * (a + b*\tan(c + d \\
& *x))^{(1/2)} + (2*A*(a + b*\tan(c + d*x))^{(5/2)})/(5*b^2*d) + (2*B*(a + b*\tan(c \\
& + d*x))^{(7/2)})/(7*b^3*d) - (2*A*a*(a + b*\tan(c + d*x))^{(3/2)})/(3*b^2*d) - \\
& (4*B*a*(a + b*\tan(c + d*x))^{(5/2)})/(5*b^3*d)
\end{aligned}$$

$$3.318 \quad \int \tan^2(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=186

$$\frac{\sqrt{a - ib} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d}$$

[Out] (I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))\*(a-I\*b)^(1/2)/d-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))\*(a+I\*b)^(1/2)/d-2\*B\*(a+b\*tan(d\*x+c))^(1/2)/d+2/15\*(5\*A\*b-2\*B\*a)\*(a+b\*tan(d\*x+c))^(3/2)/b^2/d+2/5\*B\*tan(d\*x+c)\*(a+b\*tan(d\*x+c))^(3/2)/b/d

**Rubi** [A]

time = 0.30, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3688, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(5Ab - 2aB)(a + b \tan(c + dx))^{3/2}}{15b^2d} + \frac{\sqrt{a - ib} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} + \frac{2B \tan(c + dx)(a + b \tan(c + dx))^{3/2}}{5bd} - \frac{2B \sqrt{a + b \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[a - I\*b]\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/d - (Sqrt[a + I\*b]\*(I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/d - (2\*B\*Sqrt[a + b\*Tan[c + d\*x]])/d + (2\*(5\*A\*b - 2\*a\*B)\*(a + b\*Tan[c + d\*x])^(3/2))/(15\*b^2\*d) + (2\*B\*Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^(3/2))/(5\*b\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3609**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

#### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

#### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

#### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

#### Rubi steps



$$\begin{aligned}
\int \tan^2(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx &= \frac{2B \tan(c+dx)(a+b \tan(c+dx))^{3/2}}{5bd} + \frac{2 \int}{15b^2d} \\
&= \frac{2(5Ab-2aB)(a+b \tan(c+dx))^{3/2}}{15b^2d} + \frac{2B \tan(c+dx) \sqrt{a+b \tan(c+dx)}}{5bd} \\
&= -\frac{2B \sqrt{a+b \tan(c+dx)}}{d} + \frac{2(5Ab-2aB)(a+b \tan(c+dx))^{3/2}}{15b^2d} \\
&= -\frac{2B \sqrt{a+b \tan(c+dx)}}{d} + \frac{2(5Ab-2aB)(a+b \tan(c+dx))^{3/2}}{15b^2d} \\
&= -\frac{2B \sqrt{a+b \tan(c+dx)}}{d} + \frac{2(5Ab-2aB)(a+b \tan(c+dx))^{3/2}}{15b^2d} \\
&= -\frac{2B \sqrt{a+b \tan(c+dx)}}{d} + \frac{2(5Ab-2aB)(a+b \tan(c+dx))^{3/2}}{15b^2d} \\
&= \frac{\sqrt{a-ib} (iA+B) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 1.29, size = 169, normalized size = 0.91

$$\frac{15\sqrt{a-ib} (iA+B) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right) + 15\sqrt{a+ib} (-iA+B) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right) + \frac{2\sqrt{a+b \tan(c+dx)} (5aAb-2a^2B-15b^2B+b(5Ab+aB) \tan(c+dx)+3b^2B \tan^2(c+dx))}{15d}}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

```
[Out] (15*Sqrt[a - I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]
+ 15*Sqrt[a + I*b]*((-I)*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a +
I*b]] + (2*Sqrt[a + b*Tan[c + d*x]]*(5*a*A*b - 2*a^2*B - 15*b^2*B + b*(5*A*
b + a*B)*Tan[c + d*x] + 3*b^2*B*Tan[c + d*x]^2))/b^2)/(15*d)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(158) = 316.

time = 0.14, size = 692, normalized size = 3.72

method	result
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derivativedivides	$\frac{2B(a+b \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2Ab(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - \frac{2Ba(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - 2Bb^2 \sqrt{a+b \tan(dx+c)} - 2b^2$	$\left( \frac{-A \sqrt{2} \sqrt{a+b \tan(dx+c)}}{\dots} \right)$
default	$\frac{2B(a+b \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2Ab(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - \frac{2Ba(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - 2Bb^2 \sqrt{a+b \tan(dx+c)} - 2b^2$	$\left( \frac{-A \sqrt{2} \sqrt{a+b \tan(dx+c)}}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$\frac{2}{d} \frac{1}{b^2} \left( \frac{1}{5} B (a+b \tan(dx+c))^{\frac{5}{2}} + \frac{1}{3} A b (a+b \tan(dx+c))^{\frac{3}{2}} - \frac{1}{3} B a (a+b \tan(dx+c))^{\frac{3}{2}} - B b^2 (a+b \tan(dx+c))^{\frac{1}{2}} - b^2 \left( \frac{1}{4} \frac{1}{b} \left( \frac{1}{2} (-A (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} (a^2+b^2)^{\frac{1}{2}} + A (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}}) * a - B (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * b) * \ln(b \tan(dx+c) + a + (a+b \tan(dx+c))^{\frac{1}{2}}) * (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} + (a^2+b^2)^{\frac{1}{2}} \right) + 2(-2B(a^2+b^2)^{\frac{1}{2}} * b - \frac{1}{2}(-A(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2+b^2)^{\frac{1}{2}} + A(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a - B(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * b) * (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} \right) / (2(a^2+b^2)^{\frac{1}{2}} - 2a)^{\frac{1}{2}} * \arctan\left(\frac{(2(a+b \tan(dx+c))^{\frac{1}{2}} + (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}})}{(2(a^2+b^2)^{\frac{1}{2}} - 2a)^{\frac{1}{2}}}\right) + \frac{1}{4} \frac{1}{b} \left( \frac{1}{2} (-A(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2+b^2)^{\frac{1}{2}} + A(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}}) * a - B(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * b) * \ln(-b \tan(dx+c) - a + (a+b \tan(dx+c))^{\frac{1}{2}}) * (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} - (a^2+b^2)^{\frac{1}{2}} \right) + 2(2B(a^2+b^2)^{\frac{1}{2}} * b + \frac{1}{2}(-A(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2+b^2)^{\frac{1}{2}} + A(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a - B(2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * b) * (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} \right) / (2(a^2+b^2)^{\frac{1}{2}} - 2a)^{\frac{1}{2}} * \arctan\left(\frac{(-2(a+b \tan(dx+c))^{\frac{1}{2}} + (2(a^2+b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}})}{(2(a^2+b^2)^{\frac{1}{2}} - 2a)^{\frac{1}{2}}}\right) \right)$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^2, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8926 vs. 2(152) = 304.

time = 19.04, size = 8926, normalized size = 47.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^2*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/60*(60*sqrt(2)*b^2*d^5*sqrt(-((2*A*B*b - (A^2 - B^2)*a)*d^2*sqrt(((A^4 +
2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^4 + 2*A^2*B^
2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B
^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((4*A^2*B^2*a^2 + 4*(A^3*B - A*
B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*(((A^4 + 2*A^2*B^2 + B^4)*a^2
+ (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^(3/4)*arctan(((2*(A^7*B + 3*A^5*B^3 + 3
*A^3*B^5 + A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7*
B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B
^8)*b^3)*d^4*sqrt((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2
+ B^4)*b^2)/d^4)*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^
4)*b^2)/d^4) + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 +
(A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a^3*b + 2*(A
^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 + (A^10 + 3*A^8*B
^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a*b^3)*d^2*sqrt((4*A^2*B^2*a
^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + sqrt(2)*((
2*(A^3*B^2 + A*B^4)*a + (A^4*B - B^5)*b)*d^7*sqrt((4*A^2*B^2*a^2 + 4*(A^3*B
- A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*sqrt(((A^4 + 2*A^2*B^2 +
B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) + (2*(A^5*B^2 + 2*A^3*B^4 + A*
B^6)*a^2 + (3*A^6*B + 5*A^4*B^3 + A^2*B^5 - B^7)*a*b + (A^7 + A^5*B^2 - A^3
*B^4 - A*B^6)*b^2)*d^5*sqrt((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 -
2*A^2*B^2 + B^4)*b^2)/d^4))*sqrt(-((2*A*B*b - (A^2 - B^2)*a)*d^2*sqrt(((A^
4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^4 + 2*A^2
*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B -
A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x + c) + b*sin(d*x
+ c))/cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4
)*b^2)/d^4)^(3/4) + sqrt(2)*(B*d^7*sqrt((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*
a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2 +
(A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) + ((A^2*B + B^3)*a + (A^3 + A*B^2)*b)*d^
5*sqrt((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2
```

$$\begin{aligned}
 & )/d^4))\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)* \\
 & a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^ \\
 & 4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2 \\
 & *A^2*B^2 + B^4)*b^2))\sqrt{((4*(A^4*B^2 + A^2*B^4)*a^4 + 4*(A^5*B - A*B^5)* \\
 & a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 + 4*(A^5*B - A*B^5)*a*b \\
 & ^3 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^4)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4) \\
 & *a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)*\cos(d*x + c) + \sqrt{2)*((4*A^2*B^3 \\
 & *a^3 + 4*(2*A^3*B^2 - A*B^4)*a^2*b + (5*A^4*B - 6*A^2*B^3 + B^5)*a*b^2 + (A \\
 & ^5 - 2*A^3*B^2 + A*B^4)*b^3)*d^3*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + \\
 & 2*A^2*B^2 + B^4)*b^2)/d^4)*\cos(d*x + c) + (4*(A^4*B^3 + A^2*B^5)*a^4 + 4*( \\
 & A^5*B^2 - A*B^6)*a^3*b + (A^6*B + 3*A^4*B^3 + 3*A^2*B^5 + B^7)*a^2*b^2 + 4* \\
 & (A^5*B^2 - A*B^6)*a*b^3 + (A^6*B - A^4*B^3 - A^2*B^5 + B^7)*b^4)*d*\cos(d*x \\
 & + c))*\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^ \\
 & 2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 \\
 & + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A \\
 & ^2*B^2 + B^4)*b^2))\sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))*(( \\
 & (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(1/4) + (4* \\
 & (A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^5 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7) \\
 & *a^4*b + (A^8 + 4*A^6*B^2 + 6*A^4*B^4 + 4*A^2*B^6 + B^8)*a^3*b^2 + 4*(A^7*B \\
 & + A^5*B^3 - A^3*B^5 - A*B^7)*a^2*b^3 + (A^8 - 2*A^4*B^4 + B^8)*a*b^4)*\cos( \\
 & d*x + c) + (4*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^4*b + 4*(A^7*B + A^5*B^3 - \\
 & A^3*B^5 - A*B^7)*a^3*b^2 + (A^8 + 4*A^6*B^2 + 6*A^4*B^4 + 4*A^2*B^6 + B^8)* \\
 & a^2*b^3 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^4 + (A^8 - 2*A^4*B^4 + \\
 & B^8)*b^5)*\sin(d*x + c))/((a^2 + b^2)*\cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^ \\
 & 4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(3/4)}/(4*(A^{10}*B^2 + 4*A^8*B^4 \\
 & + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^{10})*a^4*b + 4*(A^{11}*B + 3*A^9*B^3 + 2*A^7*B \\
 & ^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^{11})*a^3*b^2 + (A^{12} + 6*A^{10}*B^2 + 15*A^8* \\
 & B^4 + 20*A^6*B^6 + 15*A^4*B^8 + 6*A^2*B^{10} + B^{12})*a^2*b^3 + 4*(A^{11}*B + 3* \\
 & A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^{11})*a*b^4 + (A^{12} + 2*A^{1 \\
 & 0*B^2 - A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^{10} + B^{12})*b^5))*\cos(d*x + \\
 & c)^2 + 60*\sqrt{2)*b^2*d^5*\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + \\
 & 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^4 + 2*A^2*B^ \\
 & 2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B \\
 & ^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A* \\
 & B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*(((A^4 + 2*A^2*B^2 + B^4)*a^2 \\
 & + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(3/4)*\arctan(-((2*(A^7*B + 3*A^5*B^3 + \\
 & 3*A^3*B^5 + A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7 \\
 & *B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^...
 \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 21.57, size = 938, normalized size = 5.04

-----

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] atanh((d^3\*((16\*(A^2\*b^4 - A^2\*a^2\*b^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^2 + (16\*a\*b^2\*((-A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^4)\*(-((-A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)/d^4)^(1/2))/(16\*(A^3\*b^5 + A^3\*a^2\*b^3)))\*(-((-A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)/d^4)^(1/2) - ((2\*B\*(a^2 + b^2))/(b^2\*d) - (2\*B\*a^2)/(b^2\*d))\*(a + b\*tan(c + d\*x))^(1/2) + atanh((d^3\*((16\*(A^2\*b^4 - A^2\*a^2\*b^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^2 - (16\*a\*b^2\*((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^4)\*((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)/d^4)^(1/2))/(16\*(A^3\*b^5 + A^3\*a^2\*b^3)))\*((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)/d^4)^(1/2) - atan((B^2\*b^4\*((-B^4\*b^2\*d^4)^(1/2))/(4\*d^4) + (B^2\*a)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*32i)/((16\*B\*b^4\*((-B^4\*b^2\*d^4)^(1/2))/d^3 + (16\*B\*a^2\*b^2\*((-B^4\*b^2\*d^4)^(1/2))/d^3) + (a\*b^2\*((-B^4\*b^2\*d^4)^(1/2))/(4\*d^4) + (B^2\*a)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*(-B^4\*b^2\*d^4)^(1/2)\*32i)/((16\*B\*b^4\*((-B^4\*b^2\*d^4)^(1/2))/d + (16\*B\*a^2\*b^2\*((-B^4\*b^2\*d^4)^(1/2))/d))\*((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)/(4\*d^4))^(1/2)\*2i + atan((B^2\*b^4\*((B^2\*a)/(4\*d^2) - (-B^4\*b^2\*d^4)^(1/2))/(4\*d^4))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*32i)/((16\*B\*b^4\*((-B^4\*b^2\*d^4)^(1/2))/d^3 + (16\*B\*a^2\*b^2\*((-B^4\*b^2\*d^4)^(1/2))/d^3) - (a\*b^2\*((B^2\*a)/(4\*d^2) - (-B^4\*b^2\*d^4)^(1/2))/(4\*d^4))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*(-B^4\*b^2\*d^4)^(1/2)\*32i)/((16\*B\*b^4\*((-B^4\*b^2\*d^4)^(1/2))/d + (16\*B\*a^2\*b^2\*((-B^4\*b^2\*d^4)^(1/2))/d))\*((-B^4\*b^2\*d^4)^(1/2) - B^2\*a\*d^2)/(4\*d^4))^(1/2)\*2i + (2\*A\*(a + b\*tan(c + d\*x))^(3/2))/(3\*b\*d) + (2\*B\*(a + b\*tan(c + d\*x))^(5/2))/(5\*b^2\*d) - (2\*B\*a\*(a + b\*tan(c + d\*x))^(3/2))/(3\*b^2\*d)

### 3.319 $\int \tan(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=146

$$\frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} + \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd}$$

[Out]  $-(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})*(a-I*b)^{(1/2)}/d-(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})*(a+I*b)^{(1/2)}/d+2*A*(a+b*\tan(d*x+c))^{(1/2)}/d+2/3*B*(a+b*\tan(d*x+c))^{(3/2)}/b/d$

**Rubi [A]**

time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3673, 3609, 3620, 3618, 65, 214}

$$\frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} + \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

[In] `Int[Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $-\left(\frac{\sqrt{a - I*b}*(A - I*B)*\operatorname{ArcTanh}[\sqrt{a + b*\tan[c + d*x]}/\sqrt{a - I*b}]}{d} - \frac{\sqrt{a + I*b}*(A + I*B)*\operatorname{ArcTanh}[\sqrt{a + b*\tan[c + d*x]}/\sqrt{a + I*b}]}{d} + \frac{2*A*\sqrt{a + b*\tan[c + d*x]}}{d} + \frac{2*B*(a + b*\tan[c + d*x])^{(3/2)}}{(3*b*d)}\right)$

**Rule 65**

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

**Rule 214**

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

**Rule 3609**

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int`

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd} + \int (-B + A \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx \\
&= \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd} \\
&= \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd} \\
&= \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd} \\
&= \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd} \\
&= \frac{2A \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3bd} \\
&= -\frac{\sqrt{a - ib} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) - 3\sqrt{a + ib} b(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + 2\sqrt{a + b \tan(c + dx)} (3Ab + aB + bB \tan(c + dx))}{3bd}
\end{aligned}$$

**Mathematica [A]**

time = 0.34, size = 140, normalized size = 0.96

$$\frac{-3\sqrt{a - ib} b(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) - 3\sqrt{a + ib} b(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + 2\sqrt{a + b \tan(c + dx)} (3Ab + aB + bB \tan(c + dx))}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

```
[Out] (-3*Sqrt[a - I*b]*b*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - 3*Sqrt[a + I*b]*b*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*Sqrt[a + b*Tan[c + d*x]]*(3*A*b + a*B + b*B*Tan[c + d*x]))/(3*b*d)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(122) = 244.

time = 0.12, size = 651, normalized size = 4.46

method	result
--------	--------



derivativedivides	$\frac{2B(a+b \tan(dx+c))^{\frac{3}{2}} + 2Ab \sqrt{a + b \tan(dx+c)}}{3} - 2b$	$\left( \frac{A \sqrt{2\sqrt{a^2 + b^2} + 2a} - b - B \sqrt{a^2 + b^2} \sqrt{2\sqrt{a^2 + b^2} + 2a}}{\dots} \right)$
default	$\frac{2B(a+b \tan(dx+c))^{\frac{3}{2}} + 2Ab \sqrt{a + b \tan(dx+c)}}{3} - 2b$	$\left( \frac{A \sqrt{2\sqrt{a^2 + b^2} + 2a} - b - B \sqrt{a^2 + b^2} \sqrt{2\sqrt{a^2 + b^2} + 2a}}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)*(A+B*tan(d*x+c)),x,method=_RETURNVERB  
OSE)`

[Out] 
$$\begin{aligned} & 2/d/b*(1/3*B*(a+b*\tan(d*x+c))^{3/2}+A*b*(a+b*\tan(d*x+c))^{1/2}-b*(1/4/b*(1/ \\ & 2*(A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b-B*(a^2+b^2)^{1/2}*(2*(a^2+b^2)^{1/2}+2 \\ & *a)^{1/2}+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a)*\ln(b*\tan(d*x+c)+a+(a+b*\tan(d*x \\ & +c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}+(a^2+b^2)^{1/2}))+2*(2*A*(a^2+b^2)^{ \\ & (1/2)*b-1/2*(A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b-B*(a^2+b^2)^{1/2}*(2*(a^2+b^ \\ & 2)^{1/2}+2*a)^{1/2}+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a)*(2*(a^2+b^2)^{1/2}+2 \\ & *a)^{1/2})/(2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((2*(a+b*\tan(d*x+c))^{1/2}+( \\ & 2*(a^2+b^2)^{1/2}+2*a)^{1/2})/(2*(a^2+b^2)^{1/2}-2*a)^{1/2}))+1/4/b*(-1/2*( \\ & A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b-B*(a^2+b^2)^{1/2}*(2*(a^2+b^2)^{1/2}+2*a) \\ & ^{1/2}+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a)*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c \\ & ))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}-(a^2+b^2)^{1/2}))+2*(-2*A*(a^2+b^2)^{ \\ & (1/2)*b+1/2*(A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b-B*(a^2+b^2)^{1/2}*(2*(a^2+b^2 \\ & )^{1/2}+2*a)^{1/2}+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a)*(2*(a^2+b^2)^{1/2}+2* \\ & a)^{1/2})/(2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((-2*(a+b*\tan(d*x+c))^{1/2}+( \\ & 2*(a^2+b^2)^{1/2}+2*a)^{1/2})/(2*(a^2+b^2)^{1/2}-2*a)^{1/2})))) \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8737 vs. 2(116) = 232.

time = 17.61, size = 8737, normalized size = 59.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/12*(12*\sqrt{2}*b*d^5*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\arctan(((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^3)*d^4*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^3)*d^2*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4} + \sqrt{2}*((2*(A^4*B + A^2*B^3)*a + (A^5 - A*B^4)*b)*d^7*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (2*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a^2 + (A^7 - A^5*B^2 - 5*A^3*B^4 - 3*A*B^6)*a*b - (A^6*B + A^4*B^3 - A^2*B^5 - B^7)*b^2)*d^5*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4})*\sqrt{((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c)}*((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{3/4} + \sqrt{2}*(A*d^7*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + ((A^3 + A*B^2)*a - (A^2*B + B^3)*b)*d^5*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}$$

```

4))*sqrt(((2*A*B*b - (A^2 - B^2)*a)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2 +
(A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2
*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*
B^2 + B^4)*b^2))*sqrt(((4*(A^4*B^2 + A^2*B^4)*a^4 + 4*(A^5*B - A*B^5)*a^3*b
+ (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 + 4*(A^5*B - A*B^5)*a*b^3 +
(A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^4)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2
+ (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)*cos(d*x + c) + sqrt(2)*((4*A^3*B^2*a^3
+ 4*(A^4*B - 2*A^2*B^3)*a^2*b + (A^5 - 6*A^3*B^2 + 5*A*B^4)*a*b^2 - (A^4*B
- 2*A^2*B^3 + B^5)*b^3)*d^3*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^
2*B^2 + B^4)*b^2)/d^4)*cos(d*x + c) + (4*(A^5*B^2 + A^3*B^4)*a^4 + 4*(A^6*B
- A^2*B^5)*a^3*b + (A^7 + 3*A^5*B^2 + 3*A^3*B^4 + A*B^6)*a^2*b^2 + 4*(A^6*
B - A^2*B^5)*a*b^3 + (A^7 - A^5*B^2 - A^3*B^4 + A*B^6)*b^4)*d*cos(d*x + c))
*sqrt(((2*A*B*b - (A^2 - B^2)*a)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A
^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^
2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2
+ B^4)*b^2))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(((A^4 +
2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^(1/4) + (4*(A^6*B
^2 + 2*A^4*B^4 + A^2*B^6)*a^5 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^4*b
+ (A^8 + 4*A^6*B^2 + 6*A^4*B^4 + 4*A^2*B^6 + B^8)*a^3*b^2 + 4*(A^7*B + A^5
*B^3 - A^3*B^5 - A*B^7)*a^2*b^3 + (A^8 - 2*A^4*B^4 + B^8)*a*b^4)*cos(d*x +
c) + (4*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^4*b + 4*(A^7*B + A^5*B^3 - A^3*B^
5 - A*B^7)*a^3*b^2 + (A^8 + 4*A^6*B^2 + 6*A^4*B^4 + 4*A^2*B^6 + B^8)*a^2*b^
3 + 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^4 + (A^8 - 2*A^4*B^4 + B^8)*b
^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^2
+ (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^(3/4))/(4*(A^10*B^2 + 4*A^8*B^4 + 6*A^
6*B^6 + 4*A^4*B^8 + A^2*B^10)*a^4*b + 4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2
*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a^3*b^2 + (A^12 + 6*A^10*B^2 + 15*A^8*B^4 +
20*A^6*B^6 + 15*A^4*B^8 + 6*A^2*B^10 + B^12)*a^2*b^3 + 4*(A^11*B + 3*A^9*B^
3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a*b^4 + (A^12 + 2*A^10*B^2
- A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 + B^12)*b^5))*cos(d*x + c) + 1
2*sqrt(2)*b*d^5*sqrt(((2*A*B*b - (A^2 - B^2)*a)*d^2*sqrt(((A^4 + 2*A^2*B^2
+ B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4) + (A^4 + 2*A^2*B^2 + B^4)*a^
2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (
A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt(((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
(A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*
A^2*B^2 + B^4)*b^2)/d^4)^(3/4)*arctan(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 +
A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7*B + 3*A^5*
B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a*b^2 +

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*tan(d\*x+c)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 12.05, size = 864, normalized size = 5.92



Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] atanh((d^3\*((16\*(B^2\*b^4 - B^2\*a^2\*b^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^2 + (16\*a\*b^2\*((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^4)\*(-((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)/d^4)^(1/2))/(16\*(B^3\*b^5 + B^3\*a^2\*b^3)))\*(-((-B^4\*b^2\*d^4)^(1/2) + B^2\*a\*d^2)/d^4)^(1/2) + atanh((d^3\*((16\*(B^2\*b^4 - B^2\*a^2\*b^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^2 - (16\*a\*b^2\*((-B^4\*b^2\*d^4)^(1/2) - B^2\*a\*d^2)\*(a + b\*tan(c + d\*x))^(1/2))/d^4)\*((-B^4\*b^2\*d^4)^(1/2) - B^2\*a\*d^2)/d^4)^(1/2))/(16\*(B^3\*b^5 + B^3\*a^2\*b^3)))\*((( -B^4\*b^2\*d^4)^(1/2) - B^2\*a\*d^2)/d^4)^(1/2) - 2\*atanh((32\*A^2\*b^4\*((-A^4\*b^2\*d^4)^(1/2))/(4\*d^4) + (A^2\*a)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2))/((16\*A\*b^4\*(-A^4\*b^2\*d^4)^(1/2))/d^3 + (16\*A\*a^2\*b^2\*(-A^4\*b^2\*d^4)^(1/2))/d^3) + (32\*a\*b^2\*((-A^4\*b^2\*d^4)^(1/2))/(4\*d^4) + (A^2\*a)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*(-A^4\*b^2\*d^4)^(1/2))/((16\*A\*b^4\*(-A^4\*b^2\*d^4)^(1/2))/d + (16\*A\*a^2\*b^2\*(-A^4\*b^2\*d^4)^(1/2))/d))\*((( -A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)/(4\*d^4))^(1/2) + 2\*atanh((32\*A^2\*b^4\*((A^2\*a)/(4\*d^2) - (-A^4\*b^2\*d^4)^(1/2))/(4\*d^4))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2))/((16\*A\*b^4\*(-A^4\*b^2\*d^4)^(1/2))/d^3 + (16\*A\*a^2\*b^2\*(-A^4\*b^2\*d^4)^(1/2))/d^3) - (32\*a\*b^2\*((A^2\*a)/(4\*d^2) - (-A^4\*b^2\*d^4)^(1/2))/(4\*d^4))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*(-A^4\*b^2\*d^4)^(1/2))/((16\*A\*b^4\*(-A^4\*b^2\*d^4)^(1/2))/d + (16\*A\*a^2\*b^2\*(-A^4\*b^2\*d^4)^(1/2))/d))\*(-((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)/(4\*d^4))^(1/2) + (2\*A\*(a + b\*tan(c + d\*x))^(1/2))/d + (2\*B\*(a + b\*tan(c + d\*x))^(3/2))/(3\*b\*d)

### 3.320 $\int \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{a - ib} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{\sqrt{a + ib} (iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} +$$

[Out]  $-(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}*(a-I*b)^{(1/2)/d+(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}*(a+I*b)^{(1/2)/d+2*B*(a+b*\tan(d*x+c))^{(1/2)/d}$

Rubi [A]

time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3609, 3620, 3618, 65, 214}

$$-\frac{\sqrt{a - ib} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{\sqrt{a + ib} (-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} + \frac{2B\sqrt{a + b \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-\left(\left(\operatorname{Sqrt}[a - I*b]*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]]\right)/d + \left(\operatorname{Sqrt}[a + I*b]*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]]\right)/d + (2*B*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d\right)$

Rule 65

$\operatorname{Int}[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)*\left((c_.) + (d_.)*(x_.)\right)^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c - a*(d/b) + d*(x^p/b))^{n}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[\left((a_.) + (b_.)*(x_.)^2\right)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Rt}[-a/b, 2]/a*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3609

$\operatorname{Int}[\left((a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)*\left((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]\right)}, x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}[\left((a + b*\operatorname{Tan}[e + f*x])^{(m-1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\operatorname{Tan}[e + f*x], x\right), x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 + b^2,$

0] && GtQ[m, 0]

### Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{2B \sqrt{a + b \tan(c + dx)}}{d} + \int \frac{aA - bB + (Ab + aB) \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \\
 &= \frac{2B \sqrt{a + b \tan(c + dx)}}{d} + \frac{1}{2}((a - ib)(A - iB)) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \\
 &= \frac{2B \sqrt{a + b \tan(c + dx)}}{d} + \frac{(i(a - ib)(A - iB)) \text{Subst}\left(\int \frac{1 + i \tan(x)}{\sqrt{a + b \tan(x)}} dx, c + dx\right)}{2} \\
 &= \frac{2B \sqrt{a + b \tan(c + dx)}}{d} - \frac{((a - ib)(A - iB)) \text{Subst}\left(\int \frac{1 - i \tan(x)}{\sqrt{a + b \tan(x)}} dx, c + dx\right)}{2} \\
 &= -\frac{\sqrt{a - ib} (iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) + \sqrt{a + ib} (iA - B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) + 2B \sqrt{a + b \tan(c + dx)}}{d}
 \end{aligned}$$

### Mathematica [A]

time = 0.09, size = 120, normalized size = 0.98

$$\frac{-i\sqrt{a-ib}(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+i\sqrt{a+ib}(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2B\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]), x]

```
[Out] ((-I)*Sqrt[a - I*b]*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + I*Sqrt[a + I*b]*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*B*Sqrt[a + b*Tan[c + d*x]])/d
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 631 vs.  $2(102) = 204$ .

time = 0.11, size = 632, normalized size = 5.18

method	result
derivativedivides	$\frac{2B\sqrt{a + b \tan(dx + c)} + \frac{\left(-A\sqrt{2\sqrt{a^2 + b^2} + 2a}\sqrt{a^2 + b^2} + A\sqrt{2\sqrt{a^2 + b^2} + 2a}\right) a^{-B}\sqrt{\dots}}{\dots}}{\dots}$
default	$\frac{2B\sqrt{a + b \tan(dx + c)} + \frac{\left(-A\sqrt{2\sqrt{a^2 + b^2} + 2a}\sqrt{a^2 + b^2} + A\sqrt{2\sqrt{a^2 + b^2} + 2a}\right) a^{-B}\sqrt{\dots}}{\dots}}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*B*(a+b*tan(d*x+c))^(1/2)+1/2/b*(1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*b)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)+(a^2+b^2)^(1/2))+2*(-2*B*(a^2+b^2)^(1/2)*b-1/2*(-A*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*b)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/2/b*(-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*b)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)-(a^2+b^2)^(1/2))+2*(2*B*(a^2+b^2)^(1/2)*b+1/2*(-A*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-B*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*b)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8608 vs. 2(97) = 194.

time = 14.52, size = 8608, normalized size = 70.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{4} \cdot (4 \sqrt{2}) \cdot d^5 \sqrt{-((2ABb - (A^2 - B^2)a) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} - (A^4 + 2A^2B^2 + B^4)a^2 - (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)} \cdot \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot \left( \frac{(A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2}{d^4} \right)^{3/4} \cdot \arctan\left(\frac{(2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b + 2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^2 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^3)}{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4}\right) \cdot d^4 \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (2(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^4 + (A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^3b + 2(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^2b^2 + (A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})ab^3) \cdot d^2 \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} + \sqrt{2} \cdot \left( (2(A^3B^2 + AB^4)a + (A^4B - B^5)b) \cdot d^7 \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (2(A^5B^2 + 2A^3B^4 + AB^6)a^2 + (3A^6B + 5A^4B^3 + A^2B^5 - B^7)ab + (A^7 + A^5B^2 - A^3B^4 - AB^6)b^2) \cdot d^5 \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot \sqrt{-((2ABb - (A^2 - B^2)a) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} - (A^4 + 2A^2B^2 + B^4)a^2 - (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)} \cdot \sqrt{((a \cos(dx + c) + b \sin(dx + c))/\cos(dx + c)) \cdot \left( \frac{(A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2}{d^4} \right)^{3/4} + \sqrt{2} \cdot (B \cdot d^7 \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + ((A^2B + B^3)a + (A^3 + AB^2)b) \cdot d^5 \sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot \sqrt{-((2ABb - (A^2 - B^2)a) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + ($$



$$\begin{aligned}
& (A^4 + 2A^2B^2 + B^4)b^2/d^4) - (A^4 + 2A^2B^2 + B^4)a^2 - (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2))\sqrt{((4(A^4B^2 + A^2B^4)a^4 + 4(A^5B - AB^5)a^3b + (A^6 + 3A^4B^2 + 3A^2B^4 + B^6)a^2b^2 + 4(A^5B - AB^5)ab^3 + (A^6 - A^4B^2 - A^2B^4 + B^6)b^4)d^2\sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4})\cos(dx + c) + \sqrt{2}((4A^2B^3a^3 + 4(2A^3B^2 - AB^4)a^2b + (5A^4B - 6A^2B^3 + B^5)ab^2 + (A^5 - 2A^3B^2 + AB^4)b^3)d^3\sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4})\cos(dx + c) + (4(A^4B^3 + A^2B^5)a^4 + 4(A^5B^2 - AB^6)a^3b + (A^6B + 3A^4B^3 + 3A^2B^5 + B^7)a^2b^2 + 4(A^5B^2 - AB^6)ab^3 + (A^6B - A^4B^3 - A^2B^5 + B^7)b^4)d\cos(dx + c))\sqrt{-((2ABb - (A^2 - B^2)a)d^2\sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4}) - (A^4 + 2A^2B^2 + B^4)a^2 - (A^4 + 2A^2B^2 + B^4)b^2))\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c))((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{1/4} + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^5 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^4b + (A^8 + 4A^6B^2 + 6A^4B^4 + 4A^2B^6 + B^8)a^3b^2 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^2b^3 + (A^8 - 2A^4B^4 + B^8)ab^4)\cos(dx + c) + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^4b + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^3b^2 + (A^8 + 4A^6B^2 + 6A^4B^4 + 4A^2B^6 + B^8)a^2b^3 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)ab^4 + (A^8 - 2A^4B^4 + B^8)b^5)\sin(dx + c))/((a^2 + b^2)\cos(dx + c))((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4}}/(4(A^{10}B^2 + 4A^8B^4 + 6A^6B^6 + 4A^4B^8 + A^2B^{10})a^4b + 4(A^{11}B + 3A^9B^3 + 2A^7B^5 - 2A^5B^7 - 3A^3B^9 - AB^{11})a^3b^2 + (A^{12} + 6A^{10}B^2 + 15A^8B^4 + 20A^6B^6 + 15A^4B^8 + 6A^2B^{10} + B^{12})a^2b^3 + 4(A^{11}B + 3A^9B^3 + 2A^7B^5 - 2A^5B^7 - 3A^3B^9 - AB^{11})ab^4 + (A^{12} + 2A^{10}B^2 - A^8B^4 - 4A^6B^6 - A^4B^8 + 2A^2B^{10} + B^{12})b^5)) + 4\sqrt{2}d^5\sqrt{-((2ABb - (A^2 - B^2)a)d^2\sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4}) - (A^4 + 2A^2B^2 + B^4)a^2 - (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2))\sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4})((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4}\arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b + 2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^2 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^5))}
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

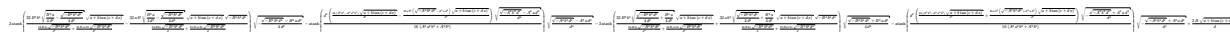
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a), x)

**Mupad [B]**

time = 8.64, size = 845, normalized size = 6.93



Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] 
$$2*\operatorname{atanh}\left(\frac{32*B^2*b^4*(B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^{(1/2)/(4*d^4)}^{(1/2)}}{(16*B*b^4*(-B^4*b^2*d^4)^{(1/2)}/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)}/d^3) - (32*a*b^2*(B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^{(1/2)/(4*d^4)}^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-B^4*b^2*d^4)^{(1/2)})}{(16*B*b^4*(-B^4*b^2*d^4)^{(1/2)}/d + (16*B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)}/d)}\right) * \left( \frac{(-(-B^4*b^2*d^4)^{(1/2)} - B^2*a*d^2)/(4*d^4)^{(1/2)} - \operatorname{atanh}\left(\frac{d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*\tan(c + d*x))^{(1/2)})/d^2 - (16*a*b^2*(-A^4*b^2*d^4)^{(1/2)} - A^2*a*d^2)*(a + b*\tan(c + d*x))^{(1/2)})/d^4}{((-A^4*b^2*d^4)^{(1/2)} - A^2*a*d^2)/d^4}\right)}{(16*(A^3*b^5 + A^3*a^2*b^3))} \right) * \left( \frac{(-A^4*b^2*d^4)^{(1/2)} - A^2*a*d^2}{d^4} \right)^{(1/2)} - 2*\operatorname{atanh}\left(\frac{32*B^2*b^4*((-B^4*b^2*d^4)^{(1/2)}/(4*d^4) + (B^2*a)/(4*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}}{(16*B*b^4*(-B^4*b^2*d^4)^{(1/2)}/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)}/d^3) + (32*a*b^2*((-B^4*b^2*d^4)^{(1/2)}/(4*d^4) + (B^2*a)/(4*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-B^4*b^2*d^4)^{(1/2)})}{(16*B*b^4*(-B^4*b^2*d^4)^{(1/2)}/d + (16*B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)}/d)}\right) * \left( \frac{(-B^4*b^2*d^4)^{(1/2)} + B^2*a*d^2}{(4*d^4)^{(1/2)} - \operatorname{atanh}\left(\frac{d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*\tan(c + d*x))^{(1/2)})/d^2 + (16*a*b^2*(-A^4*b^2*d^4)^{(1/2)} + A^2*a*d^2)*(a + b*\tan(c + d*x))^{(1/2)})/d^4}{(-(-A^4*b^2*d^4)^{(1/2)} + A^2*a*d^2)/d^4}\right)}{(16*(A^3*b^5 + A^3*a^2*b^3))} \right) * \left( \frac{(-A^4*b^2*d^4)^{(1/2)} + A^2*a*d^2}{d^4} \right)^{(1/2)} + (2*B*(a + b*\tan(c + d*x))^{(1/2)})/d$$

$$3.321 \quad \int \cot(c+dx) \sqrt{a + b \tan(c + dx)} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=131

$$\frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{a - ib} (A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{\sqrt{a + ib} (A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d}$$

[Out]  $-2*A*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})*a^{(1/2)}/d+(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})*(a-I*b)^{(1/2)}/d+(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})*(a+I*b)^{(1/2)}/d$

**Rubi** [A]

time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3693, 3620, 3618, 65, 214, 3715}

$$\frac{\sqrt{a - ib} (A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{\sqrt{a + ib} (A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d} - \frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $(-2*\operatorname{Sqrt}[a]*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/d + (\operatorname{Sqrt}[a - I*b]*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d + (\operatorname{Sqrt}[a + I*b]*(A + I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d$

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 214

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 3618

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b`

\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3693

Int[(((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])]/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[Simp[A\*(a\*c + b\*d) + B\*(b\*c - a\*d) - (A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*Tan[e + f\*x], x]/Sqrt[c + d\*Tan[e + f\*x]], x], x] - Dist[(b\*c - a\*d)\*((B\*a - A\*b)/(a^2 + b^2)), Int[(1 + Tan[e + f\*x]^2)/((a + b\*Tan[e + f\*x])\*Sqrt[c + d\*Tan[e + f\*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3715

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rubi steps

$$\begin{aligned}
 \int \cot(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= (aA) \int \frac{\cot(c + dx) (1 + \tan^2(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx + \int \\
 &= \frac{1}{2} (Ab + aB - i(-aA + bB)) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \\
 &= \frac{(2aA) \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{bd} \\
 &= \frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} \\
 &= \frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \dots
 \end{aligned}$$

**Mathematica [A]**

time = 0.38, size = 219, normalized size = 1.67

$$\frac{2\sqrt{a} A \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right) - \frac{(A(b^2+a\sqrt{-b^2})+b(a-\sqrt{-b^2})B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} + \frac{(A(b^2-a\sqrt{-b^2})+b(a+\sqrt{-b^2})B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left(\frac{2\sqrt{a} A \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right]}{\sqrt{a}} - \left(\frac{A(b^2+a\sqrt{-b^2})+b(a-\sqrt{-b^2})B}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right] + \left(\frac{A(b^2-a\sqrt{-b^2})+b(a+\sqrt{-b^2})B}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right]\right) / (\sqrt{-b^2} \sqrt{a-\sqrt{-b^2}}) + \left(\frac{A(b^2-a\sqrt{-b^2})+b(a+\sqrt{-b^2})B}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right] / (\sqrt{-b^2} \sqrt{a+\sqrt{-b^2}}) / d$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.99, size = 29038, normalized size = 221.66

method	result	size
default	Expression too large to display	29038

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*cot(d\*x + c), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 8591 vs. 2(101) = 202.

time = 26.07, size = 17257, normalized size = 131.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& (2A^2B^2 + B^4)b^2/d^4) + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2) / (4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2) \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)} * (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{1/4} + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^5 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^4b + (A^8 + 4A^6B^2 + 6A^4B^4 + 4A^2B^6 + B^8)a^3b^2 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^2b^3 + (A^8 - 2A^4B^4 + B^8)ab^4) \cos(dx + c) + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^4b + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^3b^2 + (A^8 + 4A^6B^2 + 6A^4B^4 + 4A^2B^6 + B^8)a^2b^3 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)ab^4 + (A^8 - 2A^4B^4 + B^8)b^5) \sin(dx + c) / ((a^2 + b^2) \cos(dx + c)) * (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4} / (4(A^{10}B^2 + 4A^8B^4 + 6A^6B^6 + 4A^4B^8 + A^2B^{10})a^4b + 4(A^{11}B + 3A^9B^3 + 2A^7B^5 - 2A^5B^7 - 3A^3B^9 - AB^{11})a^3b^2 + (A^{12} + 6A^{10}B^2 + 15A^8B^4 + 20A^6B^6 + 15A^4B^8 + 6A^2B^{10} + B^{12})a^2b^3 + 4(A^{11}B + 3A^9B^3 + 2A^7B^5 - 2A^5B^7 - 3A^3B^9 - AB^{11})ab^4 + (A^{12} + 2A^{10}B^2 - A^8B^4 - 4A^6B^6 - A^4B^8 + 2A^2B^{10} + B^{12})b^5) + 4\sqrt{2}d^5\sqrt{((2ABb - (A^2 - B^2)a)d^2\sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2) / (4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2) \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4} * (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4} \arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b + 2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^2 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8) \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*(a+b\*tan(dx+c))\*\*(1/2)\*(A+B\*tan(dx+c)),x)

[Out] Integral((A + B\*tan(c + dx))\*sqrt(a + b\*tan(c + dx))\*cot(c + dx), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*(a+b\*tan(dx+c))^(1/2)\*(A+B\*tan(dx+c)),x, algorithm="giac")





$$\begin{aligned}
& a^4 b^8 d^2 - 15 A^2 B a^3 b^9 d^2) / d^5 - (A a^{1/2}) * ((32 * (a + b \tan(c + d * x))^{1/2} * (10 B^2 a^3 b^8 d^2 - 18 A^2 a^3 b^8 d^2 + 16 A B b^{11} d^2 - 6 A^2 a b^{10} d^2 + 6 B^2 a b^{10} d^2 + 24 A B a^2 b^9 d^2)) / d^4 + (A a^{1/2}) * ((32 * (12 A a b^{10} d^4 + 12 A a^3 b^8 d^4)) / d^5 + (32 A a^{1/2} * (16 b^{10} d^4 + 24 a^2 b^8 d^4) * (a + b \tan(c + d * x))^{1/2}) / d^5)) / d)) / d)) / d)) * 2i) / d - \\
& \operatorname{atan}(((((((32 * (12 A a b^{10} d^4 + 12 A a^3 b^8 d^4)) / d^5 - (32 * (16 b^{10} d^4 + 24 a^2 b^8 d^4) * (a + b \tan(c + d * x))^{1/2}) * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2}) / d^4 * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2} - (32 * (a + b \tan(c + d * x))^{1/2} * (10 B^2 a^3 b^8 d^2 - 18 A^2 a^3 b^8 d^2 + 16 A B b^{11} d^2 - 6 A^2 a b^{10} d^2 + 6 B^2 a b^{10} d^2 + 24 A B a^2 b^9 d^2)) / d^4 * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2} - (32 * (3 A^3 a^2 b^{10} d^2 + 3 A^3 a^4 b^8 d^2 + B^3 a^3 b^9 d^2 + B^3 a b^{11} d^2 - 15 A^2 B a b^{11} d^2 - 9 A B^2 a^2 b^{10} d^2 - 9 A B^2 a^4 b^8 d^2 - 15 A^2 B a^3 b^9 d^2)) / d^5 * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2} - (32 * (a + b \tan(c + d * x))^{1/2} * (A^4 b^{12} + B^4 b^{12} + 2 A^2 B^2 b^{12} + 3 A^4 a^4 b^8 + 2 B^4 a^2 b^{10} + B^4 a^4 b^8 + 6 A^2 B^2 a^2 b^{10} - 8 A^3 B a^3 b^9)) / d^4 * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2} * i - (((((32 * (12 A a b^{10} d^4 + 12 A a^3 b^8 d^4)) / d^5 + (32 * (16 b^{10} d^4 + 24 a^2 b^8 d^4) * (a + b \tan(c + d * x))^{1/2}) * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2}) / d^4 * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (2 d^2))^{1/2}) / d^4 * ((2 A^2 B^2 b^2 d^4 - B^4 b^2 d^4 - 4 A^2 B^2 a^2 d^4 - A^4 b^2 d^4 + 4 A B^3 a b d^4 - 4 A^3 B a b d^4)^{1/2}) / (4 d^4) + (A^2 a) / (4 d^2) - (B^2 a) / (4 d^2) - (A B b) / (...
\end{aligned}$$

### 3.322 $\int \cot^2(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=167

$$\frac{(Ab + 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{\sqrt{a} d} + \frac{\sqrt{a - ib} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} \sqrt{a + i}$$

[Out]  $-(A*b+2*B*a)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d/a^{(1/2)}+(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})*(a-I*b)^{(1/2)}/d-(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})*(a+I*b)^{(1/2)}/d-A*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.34, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3689, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(2aB + Ab) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{\sqrt{a} d} + \frac{\sqrt{a - ib} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} - \frac{A \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]),x]$

[Out]  $-\left(\left(\left(A*b + 2*a*B\right)*\operatorname{ArcTanh}\left[\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right]/\operatorname{Sqrt}\left[a\right]\right]\right)/\left(\operatorname{Sqrt}\left[a\right]*d\right)\right) + \left(\operatorname{Sqrt}\left[a - I*b\right]*\left(I*A + B\right)*\operatorname{ArcTanh}\left[\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right]/\operatorname{Sqrt}\left[a - I*b\right]\right)/d - \left(\operatorname{Sqrt}\left[a + I*b\right]*\left(I*A - B\right)*\operatorname{ArcTanh}\left[\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right]/\operatorname{Sqrt}\left[a + I*b\right]\right)/d - \left(A*\operatorname{Cot}\left[c + d*x\right]*\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right)\right)/d$

**Rule 65**

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}, x_{\text{Symbol}}\right] \rightarrow \operatorname{With}\left[\left\{p = \operatorname{Denominator}\left[m_{.}\right]\right\}, \operatorname{Dist}\left[p/b, \operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(p*\left(m_{.} + 1\right) - 1\right)*\left(c_{.} - a_{.}\left(d_{.}/b\right) + d_{.}\left(x^{\left(p/b\right)}\right)^n, x\right], x, \left(a + b*x\right)^{\left(1/p\right)}, x\right]\right] /; \operatorname{FreeQ}\left[\left\{a, b, c, d\right\}, x\right] \&\& \operatorname{NeQ}\left[b*c - a*d, 0\right] \&\& \operatorname{LtQ}\left[-1, m, 0\right] \&\& \operatorname{LeQ}\left[-1, n, 0\right] \&\& \operatorname{LeQ}\left[\operatorname{Denominator}\left[n\right], \operatorname{Denominator}\left[m\right]\right] \&\& \operatorname{IntLinearQ}\left[a, b, c, d, m, n, x\right]$

**Rule 214**

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)^2\right)^{-1}, x_{\text{Symbol}}\right] \rightarrow \operatorname{Simp}\left[\left(\operatorname{Rt}\left[-a/b, 2\right]/a\right)*\operatorname{ArcTanh}\left[x/\operatorname{Rt}\left[-a/b, 2\right]\right], x\right] /; \operatorname{FreeQ}\left[\left\{a, b\right\}, x\right] \&\& \operatorname{NegQ}\left[a/b\right]$

**Rule 3618**

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\operatorname{tan}\left[\left(e_{.}\right) + \left(f_{.}\right)*\left(x_{.}\right)\right]\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\operatorname{tan}\left[\left(e_{.}\right) + \left(f_{.}\right)*\left(x_{.}\right)\right]\right), x_{\text{Symbol}}\right] \rightarrow \operatorname{Dist}\left[c*\left(d/f\right), \operatorname{Subst}\left[\operatorname{Int}\left[\left(a + \left(b/d\right)*x\right)^m/\left(d^2 + c\right)\right], x\right]\right]$

$*x), x], x, d*\text{Tan}[e + f*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)} * \left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m * (1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m * (1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$

### Rule 3689

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)} * \left((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right) * \left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{m+1} * ((c + d*\text{Tan}[e + f*x])^n / (f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(b*(m+1)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1} * (c + d*\text{Tan}[e + f*x])^{n-1} * \text{Simp}[b*B*(b*c*(m+1) + a*d*n) + A*b*(a*c*(m+1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m+1)*\text{Tan}[e + f*x] - b*d*(A*b - a*B)*(m+n+1)*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{LtQ}[0, n, 1] \&\& (\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2*m, 2*n])$

### Rule 3715

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)} * \left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(n_.)} * \left((A_.) + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\right), x\_Symbol] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m * (c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3734

$\text{Int}[\left(\left(\left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(n_.)} * \left((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\right)\right) / \left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n * ((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!GtQ}[n, 0] \&\& \text{!LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
 \int \cot^2(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= -\frac{A \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} - \int \frac{\cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{d} dx \\
 &= -\frac{A \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} - \frac{1}{2}(-A \cot(c + dx) \sqrt{a + b \tan(c + dx)}) \\
 &= -\frac{A \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} - \frac{1}{2}((a - b \tan^2(c + dx)) \sqrt{a + b \tan(c + dx)}) \\
 &= -\frac{A \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} - \frac{(i(a - b \tan^2(c + dx)) \sqrt{a + b \tan(c + dx)})}{d} \\
 &= -\frac{(Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} d} \\
 &= -\frac{(Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.61, size = 235, normalized size = 1.41

$$\frac{(Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\left(\frac{(a^2 + a\sqrt{-b^2})^{1/2} (a - \sqrt{-b^2})^B \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{-b^2}}}\right) + (a^2 - a\sqrt{-b^2})^{1/2} (a + \sqrt{-b^2})^B \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + \sqrt{-b^2}}}\right) - Ab \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d}\right)}{d}$$

Antiderivative was successfully verified.

```

[In] Integrate[Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]), x]
[Out] (-(((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a]) + (((A*(b^2 + a*Sqrt[-b^2]) + b*(a - Sqrt[-b^2]))*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[a - Sqrt[-b^2]] + ((A*(b^2 - a*Sqrt[-b^2]) + b*(a + Sqrt[-b^2]))*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] - A*b*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x])/b)/d
    
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
 time = 1.36, size = 50546, normalized size = 302.67

method	result	size
default	Expression too large to display	50546

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^2, x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 9122 vs. 2(135) = 270.

time = 24.48, size = 18318, normalized size = 109.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] 
$$\begin{aligned} & [-1/4*(4*\sqrt{2}*(a*d^5*\cos(d*x + c))^2 - a*d^5)*\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}*\sqrt{(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{(3/4)}*\arctan(((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^3)*d^4*\sqrt{(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^3)*d^2*\sqrt{(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4} + \sqrt{2}*((2*(A^3*B^2 + A*B^4)*a + (A^4*B - B^5)*b)*d^7*\sqrt{(4* \end{aligned}$$

$$\begin{aligned}
& A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2 / d^4 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} + (2(A^5 B^2 + 2A^3 B^4 + A B^6) a^2 + (3A^6 B + 5A^4 B^3 + A^2 B^5 - B^7) a b \\
& + (A^7 + A^5 B^2 - A^3 B^4 - A B^6) b^2) d^5 \sqrt{(4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) / d^4} \sqrt{-((2A B b - (A^2 - B^2) a) d^2 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2)} \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)} * (((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4)^{3/4} + \sqrt{2} * (B d^7 \sqrt{(4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) / d^4} \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} + ((A^2 B + B^3) a + (A^3 + A B^2) b) d^5 \sqrt{(4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) / d^4} \sqrt{-((2A B b - (A^2 - B^2) a) d^2 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2)} \sqrt{((4(A^4 B^2 + A^2 B^4) a^4 + 4(A^5 B - A B^5) a^3 b + (A^6 + 3A^4 B^2 + 3A^2 B^4 + B^6) a^2 b^2 + 4(A^5 B - A B^5) a b^3 + (A^6 - A^4 B^2 - A^2 B^4 + B^6) b^4) d^2 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} \cos(dx + c) + \sqrt{2} * ((4A^2 B^3 a^3 + 4(2A^3 B^2 - A B^4) a^2 b + (5A^4 B - 6A^2 B^3 + B^5) a b^2 + (A^5 - 2A^3 B^2 + A B^4) b^3) d^3 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} \cos(dx + c) + (4(A^4 B^3 + A^2 B^5) a^4 + 4(A^5 B^2 - A B^6) a^3 b + (A^6 B + 3A^4 B^3 + 3A^2 B^5 + B^7) a^2 b^2 + 4(A^5 B^2 - A B^6) a b^3 + (A^6 B - A^4 B^3 - A^2 B^5 + B^7) b^4) d \cos(dx + c) \sqrt{-((2A B b - (A^2 - B^2) a) d^2 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2)} \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)} * (((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4)^{1/4} + (4(A^6 B^2 + 2A^4 B^4 + A^2 B^6) a^5 + 4(A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a^4 b + (A^8 + 4A^6 B^2 + 6A^4 B^4 + 4A^2 B^6 + B^8) a^3 b^2 + 4(A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a^2 b^3 + (A^8 - 2A^4 B^4 + B^8) a b^4) \cos(dx + c) + (4(A^6 B^2 + 2A^4 B^4 + A^2 B^6) a^4 b + 4(A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a^3 b^2 + (A^8 + 4A^6 B^2 + 6A^4 B^4 + 4A^2 B^6 + B^8) a^2 b^3 + 4(A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a b^4 + (A^8 - 2A^4 B^4 + B^8) b^5) \sin(dx + c) / ((a^2 + b^2) \cos(dx + c)) * (((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4)^{3/4} / (4(A^{10} B^2 + 4A^8 B^4 + 6A^6 B^6 + 4A^4 B^8 + A^2 B^{10}) a^4 b + 4(A^{11} B + 3A^9 B^3 + 2A^7 B^5 - 2A^5 B^7 - 3A^3 B^9 - A B^{11}) a^3 b^2 + (A^{12} + 6A^{10} B^2 + 15A^8 B^4 + 20A^6 B^6 + 15A^4 B^8 + 6A^2 B^{10} + B^{12}) a^2 b^3 + 4(A^{11} B + 3A^9 B^3 + 2A^7 B^5 - 2A^5 B^7 - 3A^3 B^9 - A B^{11}) a b^4 + (A^{12} + 2A^{10} B^2 - A^8 B^4 - 4A^6 B^6 - A^4 B^8 + 2A^2 B^{10} + B^{12}) b^5) + 4 \sqrt{2} * (a d^5 \cos(dx + c)^2 - a d^5) \sqrt{-((2A B b - (A^2 - B^2) a) d^2 \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4} - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 + 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2)} \sqrt{((A^4 + 2A^2 B^2 + B^4) a^2 + (A^4 + 2A^2 B^2 + B^4) b^2) / d^4}
\end{aligned}$$

)<sup>2</sup>)/d<sup>4</sup>) - (A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*a<sup>2</sup> - (A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*b<sup>2</sup>)/(4  
 \*A<sup>2</sup>\*B<sup>2</sup>\*a<sup>2</sup> + 4\*(A<sup>3</sup>\*B - A\*B<sup>3</sup>)\*a\*b + (A<sup>4</sup> - 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*b<sup>2</sup>))\*sqrt((  
 4\*A<sup>2</sup>\*B<sup>2</sup>\*a<sup>2</sup> + 4\*(A<sup>3</sup>\*B - A\*B<sup>3</sup>)\*a\*b + (A<sup>4</sup> - 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*b<sup>2</sup>)/d<sup>4</sup>))\*(  
 ((A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*a<sup>2</sup> + (A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*b<sup>2</sup>)/d<sup>4</sup>)<sup>(3/4)</sup>\*arct  
 an(-((2\*(A<sup>7</sup>\*B + 3\*A<sup>5</sup>\*B<sup>3</sup> + 3\*A<sup>3</sup>\*B<sup>5</sup> + A\*B<sup>7</sup>)\*a<sup>3</sup> + (A<sup>8</sup> + 2\*A<sup>6</sup>\*B<sup>2</sup> - 2\*  
 A<sup>2</sup>\*B<sup>6</sup> - B<sup>8</sup>)\*a<sup>2</sup>\*b + 2\*(A<sup>7</sup>\*B + 3\*A<sup>5</sup>\*B<sup>3</sup> + 3...

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))\*cot(c + d\*x)\*\*2, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm  
 ="giac")

[Out] Timed out

**Mupad [B]**

time = 7.57, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] (atan((((16\*(a + b\*tan(c + d\*x))^(1/2)\*(3\*A<sup>4</sup>\*b<sup>12</sup> + 2\*B<sup>4</sup>\*b<sup>12</sup> + 3\*A<sup>2</sup>\*B<sup>2</sup>\*  
 2\*b<sup>12</sup> + 3\*A<sup>4</sup>\*a<sup>2</sup>\*b<sup>10</sup> + 2\*A<sup>4</sup>\*a<sup>4</sup>\*b<sup>8</sup> + 6\*B<sup>4</sup>\*a<sup>4</sup>\*b<sup>8</sup> + 29\*A<sup>2</sup>\*B<sup>2</sup>\*a<sup>2</sup>\*b<sup>10</sup>  
 - 4\*A\*B<sup>3</sup>\*a\*b<sup>11</sup> + 8\*A<sup>3</sup>\*B\*a\*b<sup>11</sup> + 20\*A\*B<sup>3</sup>\*a<sup>3</sup>\*b<sup>9</sup> - 4\*A<sup>3</sup>\*B\*a<sup>3</sup>\*b<sup>9</sup>))  
 /d<sup>4</sup> + ((A\*b + 2\*B\*a)\*((8\*(12\*B<sup>3</sup>\*a<sup>2</sup>\*b<sup>10</sup>\*d<sup>2</sup> - 20\*A<sup>3</sup>\*a<sup>3</sup>\*b<sup>9</sup>\*d<sup>2</sup> + 12\*B<sup>3</sup>\*  
 3\*a<sup>4</sup>\*b<sup>8</sup>\*d<sup>2</sup> + 28\*A<sup>2</sup>\*B\*b<sup>12</sup>\*d<sup>2</sup> - 20\*A<sup>3</sup>\*a\*b<sup>11</sup>\*d<sup>2</sup> + 60\*A\*B<sup>2</sup>\*a\*b<sup>11</sup>\*d<sup>2</sup>  
 + 60\*A\*B<sup>2</sup>\*a<sup>3</sup>\*b<sup>9</sup>\*d<sup>2</sup> - 8\*A<sup>2</sup>\*B\*a<sup>2</sup>\*b<sup>10</sup>\*d<sup>2</sup> - 36\*A<sup>2</sup>\*B\*a<sup>4</sup>\*b<sup>8</sup>\*d<sup>2</sup>))/d<sup>5</sup>  
 - (((16\*(a + b\*tan(c + d\*x))^(1/2)\*(36\*B<sup>2</sup>\*a<sup>3</sup>\*b<sup>8</sup>\*d<sup>2</sup> - 20\*A<sup>2</sup>\*a<sup>3</sup>\*b<sup>8</sup>\*d<sup>2</sup>  
 2 + 32\*A\*B\*b<sup>11</sup>\*d<sup>2</sup> - 8\*A<sup>2</sup>\*a\*b<sup>10</sup>\*d<sup>2</sup> + 12\*B<sup>2</sup>\*a\*b<sup>10</sup>\*d<sup>2</sup> + 64\*A\*B\*a<sup>2</sup>\*b<sup>9</sup>  
 \*d<sup>2</sup>))/d<sup>4</sup> + ((A\*b + 2\*B\*a)\*((8\*(32\*A\*b<sup>11</sup>\*d<sup>4</sup> + 48\*B\*a\*b<sup>10</sup>\*d<sup>4</sup> + 32\*A\*a<sup>2</sup>

$$\begin{aligned}
& *b^9*d^4 + 48*B*a^3*b^8*d^4))/d^5 - (8*(A*b + 2*B*a)*(32*b^10*d^4 + 48*a^2* \\
& b^8*d^4)*(a + b*\tan(c + d*x))^{(1/2)})/(a^{(1/2)*d^5}))/((2*a^{(1/2)*d})*(A*b + \\
& 2*B*a))/((2*a^{(1/2)*d}))/((2*a^{(1/2)*d})*(A*b + 2*B*a)*1i)/((2*a^{(1/2)*d} + (( \\
& (16*(a + b*\tan(c + d*x))^{(1/2)}*(3*A^4*b^12 + 2*B^4*b^12 + 3*A^2*B^2*b^12 + \\
& 3*A^4*a^2*b^10 + 2*A^4*a^4*b^8 + 6*B^4*a^4*b^8 + 29*A^2*B^2*a^2*b^10 - 4*A* \\
& B^3*a*b^11 + 8*A^3*B*a*b^11 + 20*A*B^3*a^3*b^9 - 4*A^3*B*a^3*b^9))/d^4 - (( \\
& A*b + 2*B*a)*((8*(12*B^3*a^2*b^10*d^2 - 20*A^3*a^3*b^9*d^2 + 12*B^3*a^4*b^8 \\
& *d^2 + 28*A^2*B*b^12*d^2 - 20*A^3*a*b^11*d^2 + 60*A*B^2*a*b^11*d^2 + 60*A*B \\
& ^2*a^3*b^9*d^2 - 8*A^2*B*a^2*b^10*d^2 - 36*A^2*B*a^4*b^8*d^2))/d^5 + (((16* \\
& (a + b*\tan(c + d*x))^{(1/2)}*(36*B^2*a^3*b^8*d^2 - 20*A^2*a^3*b^8*d^2 + 32*A* \\
& B*b^11*d^2 - 8*A^2*a*b^10*d^2 + 12*B^2*a*b^10*d^2 + 64*A*B*a^2*b^9*d^2))/d^ \\
& 4 - ((A*b + 2*B*a)*((8*(32*A*b^11*d^4 + 48*B*a*b^10*d^4 + 32*A*a^2*b^9*d^4 \\
& + 48*B*a^3*b^8*d^4))/d^5 + (8*(A*b + 2*B*a)*(32*b^10*d^4 + 48*a^2*b^8*d^4)* \\
& (a + b*\tan(c + d*x))^{(1/2)})/(a^{(1/2)*d^5}))/((2*a^{(1/2)*d})*(A*b + 2*B*a))/ \\
& (2*a^{(1/2)*d}))/((2*a^{(1/2)*d})*(A*b + 2*B*a)*1i)/((2*a^{(1/2)*d}))/((16*(A^5*b^ \\
& 13 + 2*A*B^4*b^13 + 4*B^5*a*b^12 + 3*A^3*B^2*b^13 + 3*A^5*a^2*b^11 + 2*A^5* \\
& a^4*b^9 + 4*B^5*a^3*b^10 + 7*A^2*B^3*a^3*b^10 + 4*A^2*B^3*a^5*b^8 - A^3*B^2 \\
& *a^2*b^11 - 4*A^3*B^2*a^4*b^9 - A^4*B*a*b^12 - 4*A*B^4*a^2*b^11 - 6*A*B^4*a \\
& ^4*b^9 + 3*A^2*B^3*a*b^12 + 3*A^4*B*a^3*b^10 + 4*A^4*B*a^5*b^8))/d^5 - (((1 \\
& 6*(a + b*\tan(c + d*x))^{(1/2)}*(3*A^4*b^12 + 2*B^4*b^12 + 3*A^2*B^2*b^12 + 3* \\
& A^4*a^2*b^10 + 2*A^4*a^4*b^8 + 6*B^4*a^4*b^8 + 29*A^2*B^2*a^2*b^10 - 4*A*B^ \\
& 3*a*b^11 + 8*A^3*B*a*b^11 + 20*A*B^3*a^3*b^9 - 4*A^3*B*a^3*b^9))/d^4 + ((A* \\
& b + 2*B*a)*((8*(12*B^3*a^2*b^10*d^2 - 20*A^3*a^3*b^9*d^2 + 12*B^3*a^4*b^8*d \\
& ^2 + 28*A^2*B*b^12*d^2 - 20*A^3*a*b^11*d^2 + 60*A*B^2*a*b^11*d^2 + 60*A*B^2 \\
& *a^3*b^9*d^2 - 8*A^2*B*a^2*b^10*d^2 - 36*A^2*B*a^4*b^8*d^2))/d^5 - (((16*(a \\
& + b*\tan(c + d*x))^{(1/2)}*(36*B^2*a^3*b^8*d^2 - 20*A^2*a^3*b^8*d^2 + 32*A*B* \\
& b^11*d^2 - 8*A^2*a*b^10*d^2 + 12*B^2*a*b^10*d^2 + 64*A*B*a^2*b^9*d^2))/d^4 \\
& + ((A*b + 2*B*a)*((8*(32*A*b^11*d^4 + 48*B*a*b^10*d^4 + 32*A*a^2*b^9*d^4 + \\
& 48*B*a^3*b^8*d^4))/d^5 - (8*(A*b + 2*B*a)*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a \\
& + b*\tan(c + d*x))^{(1/2)})/(a^{(1/2)*d^5}))/((2*a^{(1/2)*d})*(A*b + 2*B*a))/((2* \\
& a^{(1/2)*d}))/((2*a^{(1/2)*d})*(A*b + 2*B*a))/((2*a^{(1/2)*d} + (((16*(a + b*\tan \\
& (c + d*x))^{(1/2)}*(3*A^4*b^12 + 2*B^4*b^12 + 3*A^2*B^2*b^12 + 3*A^4*a^2*b^10 \\
& + 2*A^4*a^4*b^8 + 6*B^4*a^4*b^8 + 29*A^2*B^2*a^2*b^10 - 4*A*B^3*a*b^11 + 8 \\
& *A^3*B*a*b^11 + 20*A*B^3*a^3*b^9 - 4*A^3*B*a^3*b^9))/d^4 - ((A*b + 2*B*a)* \\
& (8*(12*B^3*a^2*b^10*d^2 - 20*A^3*a^3*b^9*d^2 + 12*B^3*a^4*b^8*d^2 + 28*A^2* \\
& B*b^12*d^2 - 20*A^3*a*b^11*d^2 + 60*A*B^2*a*b^11*d^2 + 60*A*B^2*a^3*b^9*d^2 \\
& - 8*A^2*B*a^2*b^10*d^2 - 36*A^2*B*a^4*b^8*d^2))/d^5 + (((16*(a + b*\tan(c + \\
& d*x))^{(1/2)}*(36*B^2*a^3*b^8*d^2 - 20*A^2*a^3*b^8*d^2 + 32*A*B*b^11*d^2 - 8 \\
& *A^2*a*b^10*d^2 + 12*B^2*a*b^10*d^2 + 64*A*B*a^2*b^9*d^2))/d^4 - ((A*b + 2* \\
& B*a)*((8*(32*A*b^11*d^4 + 48*B*a*b^10*d^4 + 32*A*a^2*b^9*d^4 + 48*B*a^3*b^8 \\
& *d^4))/d^5 + (8*(A*b + 2*B*a)*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*\tan(c + \\
& d*x))^{(1/2)})/(a^{(1/2)*d^5}))/((2*a^{(1/2)*d})*(A*b + 2*B*a))/((2*a^{(1/2)*d}))/ \\
& ((2*a^{(1/2)*d})*(A*b + 2*B*a))/((2*a^{(1/2)*d}))*1i)/(a^{(1/2)*d} \\
& ) - \operatorname{atan}(((((((8*(32*A*b^11*d^4 + 48*B*a*b^10*d^4 + 32*A*a^2*b^9*d^4 + 48*B \\
& *a^3*b^8*d^4))/d^5 - (16*(32*b^10*d^4 + 48*a^2*b^8*d^4)*(a + b*\tan(c + d*x)
\end{aligned}$$



$$\begin{aligned}
& )^{1/2} * ((2A^2B^2b^2d^4 - B^4b^2d^4 - 4A^2B^2a^2d^4 - A^4b^2d^4 \\
& + 4AB^3ab^2d^4 - 4A^3B^2ab^2d^4)^{1/2} / (4d^4) - (A^2a) / (4d^2) + (B^2a) / (4d^2) + (ABb) / (2d^2))^{1/2} / d^4 * ((2A^2B^2b^2d^4 - B^4b^2d^4 \\
& - 4A^2B^2a^2d^4 - A^4b^2d^4 + 4AB^3ab^2d^4 - 4A^3B^2ab^2d^4)^{1/2} / (4d^4) - (A^2a) / (4d^2) + (B^2a) / (4d^2) + (ABb) / (2d^2))^{1/2} + \\
& (16(a + b \tan(c + dx))^{1/2} * (36B^2a^3b^8d^2 - 20A^2a^3b^8d^2 + 32ABb^{11}d^2 - 8A^2ab^{10}d^2 + 12B^2ab^{10}d^2 + 64ABa^2b^9d^2) \\
& ) / d^4 * ((2A^2B^2b^2d^4 - B^4b^2d^4 - 4A^2B^2a^2d^4 - A^4b^2d^4 \\
& + 4AB^3ab^2d^4 - 4A^3B^2ab^2d^4)^{1/2} / (4d^4) - (A^2a) / (4d^2) + (B^2a) / (4d^2) + (ABb) / (2d^2))^{1/2} - (8(12B^3a^2b^{10}d^2 - 20A^3a^3b^9d^2 + 12B^3a^4b^8d^2 + 28A^2Bb^{12}d^2 - 20A^3ab^{11}d^2 + 60 \\
& * AB^2ab^{11}d^2 + 60AB^2a^3b^9d^2 - 8A^2B^2a^2b^{10}d^2 - 36A^2B^2a^4b^8d^2) / d^5 * ((2A^2B^2b^2d^4 - B^4b^2d^4 - \dots
\end{aligned}$$

### 3.323 $\int \cot^3(c+dx) \sqrt{a + b \tan(c + dx)} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=219

$$\frac{(8a^2A + Ab^2 - 4abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} - \frac{\sqrt{a - ib} (A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d}$$

[Out]  $1/4*(8*A*a^2+A*b^2-4*B*a*b)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/a^{(3/2)}/d-(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})*(a-I*b)^{(1/2)}/d-(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})*(a+I*b)^{(1/2)}/d-1/4*(A*b+4*B*a)*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1/2)}/a/d-1/2*A*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.56, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3689, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2A - 4abB + Ab^2) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} - \frac{\sqrt{a - ib} (A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{\sqrt{a + ib} (A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d} - \frac{(4aB + Ab) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4ad} - \frac{A \cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^3*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]),x]$

[Out]  $((8*a^2*A + A*b^2 - 4*a*b*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(4*a^{(3/2)*d} - (\operatorname{Sqrt}[a - I*b]*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - (\operatorname{Sqrt}[a + I*b]*(A + I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d - ((A*b + 4*a*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(4*a*d) - (A*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3689

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ
[2*m, 2*n])
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\int \cot^3(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx = -\frac{A \cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{2d} - \frac{1}{2} \int \dots$$

$$= -\frac{(Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4ad}$$

$$= -\frac{(Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4ad}$$

$$= -\frac{(Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4ad}$$

$$= -\frac{(Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4ad}$$

$$= \frac{(8a^2A + Ab^2 - 4abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}$$

$$= \frac{(8a^2A + Ab^2 - 4abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}$$

Mathematica [A]

time = 3.17, size = 271, normalized size = 1.24

$$\frac{(8a^2A + Ab^2 - 4abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\frac{(-aAb + Ab\sqrt{-b^2 - a^2B + a\sqrt{-b^2}B}) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{-b^2}}}\right) - (aAb + Ab\sqrt{-b^2 - a^2B + a\sqrt{-b^2}B}) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + \sqrt{-b^2}}}\right)}{\sqrt{a - \sqrt{-b^2}} \sqrt{a + \sqrt{-b^2}}}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
[Out] (((8*a^2*A + A*b^2 - 4*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) + ((4*(-(a*A*b) + A*b*Sqrt[-b^2] + b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[a - Sqrt[-b^2]] - (4*(a*A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] - (b*Cot[c + d*x]*(A*b + 4*a*B + 2*a*A*Cot[c + d*x])*Sqrt[a + b*Tan[c + d*x]])/a)/b)/(4*d)
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 1.82, size = 81276, normalized size = 371.12

method	result	size
default	Expression too large to display	81276

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^3, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 9276 vs. 2(179) = 358.

time = 30.66, size = 18627, normalized size = 85.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [-1/16*(16*sqrt(2)*(a^2*d^5*cos(d*x + c)^2 - a^2*d^5)*sqrt(((2*A*B*b - (A^2 - B^2)*a)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*
```

$$\begin{aligned}
& b^2/d^4) + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2))\sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \cdot (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4} \arctan \\
& (((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b + 2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^2 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^3) \cdot d^4 \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4)} \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (2(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^4 + (A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^3b + 2(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^2b^2 + (A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})ab^3) \cdot d^2 \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4} + \sqrt{2} \cdot ((2(A^4B + A^2B^3)a + (A^5 - AB^4)b) \cdot d^7 \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4}) \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (2(A^6B + 2A^4B^3 + A^2B^5)a^2 + (A^7 - A^5B^2 - 5A^3B^4 - 3AB^6)ab - (A^6B + A^4B^3 - A^2B^5 - B^7)b^2) \cdot d^5 \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4}) \sqrt{((2ABb - (A^2 - B^2)a) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)) \sqrt{(a \cos(dx + c) + b \sin(dx + c))/\cos(dx + c)} \cdot (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{3/4} + \sqrt{2} \cdot (A \cdot d^7 \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4}) \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + ((A^3 + AB^2)a - (A^2B + B^3)b) \cdot d^5 \sqrt{(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4}) \sqrt{((2ABb - (A^2 - B^2)a) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)) \sqrt{((4(A^4B^2 + A^2B^4)a^4 + 4(A^5B - AB^5)a^3b + (A^6 + 3A^4B^2 + 3A^2B^4 + B^6)a^2b^2 + 4(A^5B - AB^5)ab^3 + (A^6 - A^4B^2 - A^2B^4 + B^6)b^4) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4}) \cos(dx + c) + \sqrt{2} \cdot ((4A^3B^2a^3 + 4(A^4B - 2A^2B^3)a^2b + (A^5 - 6A^3B^2 + 5AB^4)ab^2 - (A^4B - 2A^2B^3 + B^5)b^3) \cdot d^3 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4}) \cos(dx + c) + (4(A^5B^2 + A^3B^4)a^4 + 4(A^6B - A^2B^5)a^3b + (A^7 + 3A^5B^2 + 3A^3B^4 + AB^6)a^2b^2 + 4(A^6B - A^2B^5)ab^3 + (A^7 - A^5B^2 - A^3B^4 + AB^6)b^4) \cdot d \cdot \cos(dx + c) \sqrt{((2ABb - (A^2 - B^2)a) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)) \sqrt{(a \cos(dx + c) + b \sin(dx + c))/\cos(dx + c)} \cdot (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{1/4} + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^5 + 4(A^7B + A^
\end{aligned}$$

$$5B^3 - A^3B^5 - AB^7)a^4b + (A^8 + 4A^6B^2 + 6A^4B^4 + 4A^2B^6 + B^8)a^3b^2 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^2b^3 + (A^8 - 2A^4B^4 + B^8)ab^4) \cos(dx + c) + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^4b + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^3b^2 + (A^8 + 4A^6B^2 + 6A^4B^4 + 4A^2B^6 + B^8)a^2b^3 + 4(A^7B + A^5B^3 - A^3B^5 - AB^7)ab^4 + (A^8 - 2A^4B^4 + B^8)b^5) \sin(dx + c) / ((a^2 + b^2) \cos(dx + c))$$

$$*(((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{(3/4)} / (4(A^{10}B^2 + 4A^8B^4 + 6A^6B^6 + 4A^4B^8 + A^2B^{10})a^4b + 4(A^{11}B + 3A^9B^3 + 2A^7B^5 - 2A^5B^7 - 3A^3B^9 - AB^{11})a^3b^2 + (A^{12} + 6A^{10}B^2 + 15A^8B^4 + 20A^6B^6 + 15A^4B^8 + 6A^2B^{10} + B^{12})a^2b^3 + 4(A^{11}B + 3A^9B^3 + 2A^7B^5 - 2A^5B^7 - 3A^3B^9 - AB^{11})ab^4 + (A^{12} + 2A^{10}B^2 - A^8B^4 - 4A^6B^6 - A^4B^8 + 2A^2B^{10} + B^{12})b^5) + 16\sqrt{2}(a^2d^5\cos(dx + c)^2 - a^2d^5)\sqrt{((2ABb - (A^2 - B^2)a)d^2\sqrt{((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)}}$$

$$\sqrt{((4A^2B^2a^2 + 4(A^3B - AB^3)ab + (A^4 - 2A^2B^2 + B^4)b^2)/d^4) * (((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/d^4)^{(3/4)}} \arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b + 2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^2 + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^3)/((A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)))$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*3\*(a+b\*tan(dx+c))\*\*(1/2)\*(A+B\*tan(dx+c)),x)

[Out] Integral((A + B\*tan(c + dx))\*sqrt(a + b\*tan(c + dx))\*cot(c + dx)\*\*3, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^3\*(a+b\*tan(dx+c))^(1/2)\*(A+B\*tan(dx+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 7.90, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + d*x)^3*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x))^{(1/2)}, x)$

[Out]  $(\text{atan}(\frac{(((((2*A^3*b^{14}*d^2 + 2*A^3*a^2*b^{12}*d^2 - 96*A^3*a^4*b^{10}*d^2 - 96*A^3*a^6*b^8*d^2 - 160*B^3*a^3*b^{11}*d^2 - 160*B^3*a^5*b^9*d^2 + 48*A^2*B*a*b^{13}*d^2 - 192*A*B^2*a^2*b^{12}*d^2 + 96*A*B^2*a^4*b^{10}*d^2 + 288*A*B^2*a^6*b^8*d^2 + 528*A^2*B*a^3*b^{11}*d^2 + 480*A^2*B*a^5*b^9*d^2)/(8*a^2*d^5) + (((64*A*a*b^{12}*d^4 + 448*A*a^3*b^{10}*d^4 + 384*A*a^5*b^8*d^4 - 256*B*a^2*b^{11}*d^4 - 256*B*a^4*b^9*d^4)/(8*a^2*d^5) - ((512*a^2*b^{10}*d^4 + 768*a^4*b^8*d^4)*(a + b*\tan(c + d*x))^{(1/2)}*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}))/(64*a^5*d^5))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}))/(8*a^3*d) - ((a + b*\tan(c + d*x))^{(1/2)}*(128*B^2*a^3*b^{10}*d^2 - 576*A^2*a^5*b^8*d^2 - 256*A^2*a^3*b^{10}*d^2 + 320*B^2*a^5*b^8*d^2 - 4*A^2*a*b^{12}*d^2 + 544*A*B*a^2*b^{11}*d^2 + 1024*A*B*a^4*b^9*d^2))/(8*a^2*d^4))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}))/(8*a^3*d) - ((a + b*\tan(c + d*x))^{(1/2)}*(A^2*B^2*b^{14} - A^4*b^{14} + 17*A^4*a^2*b^{12} + 16*A^4*a^4*b^{10} + 96*A^4*a^6*b^8 + 48*B^4*a^2*b^{12} + 48*B^4*a^4*b^{10} + 32*B^4*a^6*b^8 + 95*A^2*B^2*a^2*b^{12} + 448*A^2*B^2*a^4*b^{10} - 8*A*B^3*a*b^{13} + 4*A^3*B*a*b^{13} - 120*A*B^3*a^3*b^{11} + 64*A*B^3*a^5*b^9 - 8*A^3*B*a^3*b^{11} - 320*A^3*B*a^5*b^9))/(8*a^2*d^4))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}*i)/(a^3*d) - (((((2*A^3*b^{14}*d^2 + 2*A^3*a^2*b^{12}*d^2 - 96*A^3*a^4*b^{10}*d^2 - 96*A^3*a^6*b^8*d^2 - 160*B^3*a^3*b^{11}*d^2 - 160*B^3*a^5*b^9*d^2 + 48*A^2*B*a*b^{13}*d^2 - 192*A*B^2*a^2*b^{12}*d^2 + 96*A*B^2*a^4*b^{10}*d^2 + 288*A*B^2*a^6*b^8*d^2 + 528*A^2*B*a^3*b^{11}*d^2 + 480*A^2*B*a^5*b^9*d^2)/(8*a^2*d^5) + (((((64*A*a*b^{12}*d^4 + 448*A*a^3*b^{10}*d^4 + 384*A*a^5*b^8*d^4 - 256*B*a^2*b^{11}*d^4 - 256*B*a^4*b^9*d^4)/(8*a^2*d^5) + ((512*a^2*b^{10}*d^4 + 768*a^4*b^8*d^4)*(a + b*\tan(c + d*x))^{(1/2)}*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}))/(64*a^5*d^5))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}))/(8*a^3*d) + ((a + b*\tan(c + d*x))^{(1/2)}*(128*B^2*a^3*b^{10}*d^2 - 576*A^2*a^5*b^8*d^2 - 256*A^2*a^3*b^{10}*d^2 + 320*B^2*a^5*b^8*d^2 - 4*A^2*a*b^{12}*d^2 + 544*A*B*a^2*b^{11}*d^2 + 1024*A*B*a^4*b^9*d^2))/(8*a^2*d^4))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)}))/(8*a^3*d) + ((a + b*\tan(c + d*x))^{(1/2)}*(A^2*B^2*b^{14} - A^4*b^{14} + 17*A^4*a^2*b^{12} + 16*A^4*a^4*b^{10} + 96*A^4*a^6*b^8 + 48*B^4*a^2*b^{12} + 48*B^4*a^4*b^{10} + 32*B^4*a^6*b^8 + 95*A^2*B^2*a^2*b^{12} + 448*A^2*B^2*a^4*b^{10} - 8*A*B^3*a*b^{13} + 4*A^3*B*a*b^{13} - 120*A*B^3*a^3*b^{11} + 64*A*B^3*a^5*b^9 - 8*A^3*B*a^3*b^{11} - 320*A^3*B*a^5*b^9))/(8*a^2*d^4))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A$



$$\begin{aligned}
& *B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)*i)/(a^3*d))/((A^4*B*b^15 + 7*A^5*a*b^14 + \\
& A^2*B^3*b^15 + 63*A^5*a^3*b^12 + 56*A^5*a^5*b^10 - 16*B^5*a^2*b^13 - 48*B^5 \\
& *a^4*b^11 - 32*B^5*a^6*b^9 - 23*A^2*B^3*a^2*b^13 + 40*A^2*B^3*a^4*b^11 + 64 \\
& *A^2*B^3*a^6*b^9 + 55*A^3*B^2*a^3*b^12 + 112*A^3*B^2*a^5*b^10 + 64*A^3*B^2* \\
& a^7*b^8 - 8*A*B^4*a^3*b^12 + 56*A*B^4*a^5*b^10 + 64*A*B^4*a^7*b^8 + 7*A^3*B \\
& ^2*a*b^14 - 7*A^4*B*a^2*b^13 + 88*A^4*B*a^4*b^11 + 96*A^4*B*a^6*b^9)/(a^2*d \\
& ^5) + (((((2*A^3*b^14*d^2 + 2*A^3*a^2*b^12*d^2 - 96*A^3*a^4*b^10*d^2 - 96*A \\
& ^3*a^6*b^8*d^2 - 160*B^3*a^3*b^11*d^2 - 160*B^3*a^5*b^9*d^2 + 48*A^2*B*a*b^ \\
& 13*d^2 - 192*A*B^2*a^2*b^12*d^2 + 96*A*B^2*a^4*b^10*d^2 + 288*A*B^2*a^6*b^8 \\
& *d^2 + 528*A^2*B*a^3*b^11*d^2 + 480*A^2*B*a^5*b^9*d^2)/(8*a^2*d^5) + (((((6 \\
& 4*A*a*b^12*d^4 + 448*A*a^3*b^10*d^4 + 384*A*a^5*b^8*d^4 - 256*B*a^2*b^11*d^ \\
& 4 - 256*B*a^4*b^9*d^4)/(8*a^2*d^5) - ((512*a^2*b^10*d^4 + 768*a^4*b^8*d^4)* \\
& (a + b*\tan(c + d*x))^{(1/2)}*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16* \\
& B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)})/(64*a^5*d^5))*(64*A^2*a^ \\
& 7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^ \\
& 4*b^3)^{(1/2)})/(8*a^3*d) - ((a + b*\tan(c + d*x))^{(1/2)}*(128*B^2*a^3*b^10*d^2 \\
& - 576*A^2*a^5*b^8*d^2 - 256*A^2*a^3*b^10*d^2 + 320*B^2*a^5*b^8*d^2 - 4*A^2 \\
& *a*b^12*d^2 + 544*A*B*a^2*b^11*d^2 + 1024*A*B*a^4*b^9*d^2))/(8*a^2*d^4))*(6 \\
& 4*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - \\
& 8*A*B*a^4*b^3)^{(1/2)})/(8*a^3*d))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 \\
& + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^{(1/2)})/(8*a^3*d) - ((a + \\
& b*\tan(c + d*x))^{(1/2)}*(A^2*B^2*b^14 - A^4*b^14 + 17*A^4*a^2*b^12 + 16*A^4*a \\
& ^4*b^10 + 96*A^4*a^6*b^8 + 48*B^4*a^2*b^12 + 48*B^4*a^4*b^10 + 32*B^4*a^6*b \\
& ^8 + 95*A^2*B^2*a^2*b^12 + 448*A^2*B^2*a^4*b^10 - 8*A*B^3*a*b^13 + 4*A^3*B* \\
& a*b^13 - 120*A*B^3*a^3*b^11 + 64*A*B^3*a^5*b^9 - 8*A^3*B*a^3*b^11 - 320*A^3 \\
& *B*a^5*b^9))/(8*a^2*d^4))*(64*A^2*a^7 + A^2*a^3...
\end{aligned}$$

### 3.324 $\int \cot^4(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=279

$$\frac{(8a^2Ab - Ab^3 + 16a^3B + 2ab^2B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right) + \sqrt{a - ib} (iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{8a^{5/2}d}$$

[Out]  $\frac{1}{8} \frac{(8A^2a^2b - Ab^3 + 16A^3B + 2Ab^2B) \operatorname{arctanh}\left(\frac{a + b \tan(dx + c)}{a}\right)^{1/2}}{a^{5/2}d} - \frac{(iA + B) \operatorname{arctanh}\left(\frac{a + b \tan(dx + c)}{a - ib}\right)^{1/2}}{(a - ib)^{1/2}} \frac{(a - ib)^{1/2}}{d} + \frac{(iA - B) \operatorname{arctanh}\left(\frac{a + b \tan(dx + c)}{a + ib}\right)^{1/2}}{(a + ib)^{1/2}} \frac{(a + ib)^{1/2}}{d} + \frac{1}{8} \frac{(8A^2a^2 + Ab^2 - 2Ab^2B) \cot(dx + c) (a + b \tan(dx + c))^{1/2}}{a^2d} - \frac{1}{12} \frac{(Ab + 6A^2B) \cot(dx + c)^2 (a + b \tan(dx + c))^{1/2}}{a/d} - \frac{1}{3} \frac{A \cot(dx + c)^3 (a + b \tan(dx + c))^{1/2}}{d}$

**Rubi [A]**

time = 0.74, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3689, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2Ab - Ab^3 + 16a^3B + 2ab^2B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right) + \sqrt{a - ib} (iA + B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) + \sqrt{a + ib} (-iA - B) \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) + (6aB + Ab) \cot(c + dx) \sqrt{a + b \tan(c + dx)} + A \cot^3(c + dx) \sqrt{a + b \tan(c + dx)}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + dx]^4 \sqrt{a + b \operatorname{Tan}[c + dx]} (A + B \operatorname{Tan}[c + dx]), x]$

[Out]  $\frac{(8a^2Ab - Ab^3 + 16a^3B + 2ab^2B) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tan}[c + dx]}}{\sqrt{a}}\right]}{8a^{5/2}d} - \frac{(\sqrt{a - ib} (iA + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tan}[c + dx]}}{\sqrt{a - ib}}\right])}{d} + \frac{(\sqrt{a + ib} (-iA - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Tan}[c + dx]}}{\sqrt{a + ib}}\right])}{d} + \frac{(8a^2A + Ab^2 - 2ab^2B) \operatorname{Cot}[c + dx] \sqrt{a + b \operatorname{Tan}[c + dx]}}{8a^2d} - \frac{(Ab + 6a^2B) \operatorname{Cot}[c + dx]^2 \sqrt{a + b \operatorname{Tan}[c + dx]}}{12ad} - \frac{A \operatorname{Cot}[c + dx]^3 \sqrt{a + b \operatorname{Tan}[c + dx]}}{3d}$

**Rule 65**

$\operatorname{Int}[(a + b \cdot x)^m ((c + d \cdot x)^n), x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p(m+1)-1} (c - a(d/b) + d(x^p/b))^n], x], x, (a + b \cdot x)^{1/p}], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b \cdot c - a \cdot d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a + b \cdot x)^2 (-1), x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a) \operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

Rule 3689

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3715

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

Rule 3730

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[

`b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !  
 (ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))`

### Rule 3734

`Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)  
 + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)  
 + (f_.)*(x_)])], x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n  
 *Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(  
 A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e  
 + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C  
 , n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  
 !GtQ[n, 0] && !LeQ[n, -1]`

### Rubi steps

$$\begin{aligned}
 \int \cot^4(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= -\frac{A \cot^3(c + dx) \sqrt{a + b \tan(c + dx)}}{3d} - \frac{1}{3} \int \frac{A \cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{d} dx \\
 &= -\frac{(Ab + 6aB) \cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{12ad} \\
 &= \frac{(8a^2 A + Ab^2 - 2abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8a^2 d} \\
 &= \frac{(8a^2 A + Ab^2 - 2abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8a^2 d} \\
 &= \frac{(8a^2 A + Ab^2 - 2abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8a^2 d} \\
 &= \frac{(8a^2 A + Ab^2 - 2abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8a^2 d} \\
 &= \frac{(8a^2 Ab - Ab^3 + 16a^3 B + 2ab^2 B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{a + b \tan(c + dx)} \right)}{8a^{5/2} d} \\
 &= \frac{(8a^2 Ab - Ab^3 + 16a^3 B + 2ab^2 B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{a + b \tan(c + dx)} \right)}{8a^{5/2} d}
 \end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 564 vs. 2(279) = 558.

time = 6.29, size = 564, normalized size = 2.02

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
[Out] (2*b^4*((A*b + a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*b^4) - ((a*A - b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(2*a^(3/2)*b^3) + ((a*A*b - A*b*Sqrt[-b^2] - b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(2*b^4*Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a*A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(2*(-b^2)^(5/2)*Sqrt[a + Sqrt[-b^2]]) + ((a*A - b*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(2*a*b^4) - ((A*b + a*B)*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(4*a*b^4) - (A*Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]])/(6*b^4) - (3*(A*b + a*B)*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]/a^(3/2) - (Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]/(a*b))))/(8*a*b^2) + (5*A*((2*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]/(a*b^2) + (3*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]/a^(3/2) - (Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]/(a*b))))/a))/(48*b))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 2.39, size = 118304, normalized size = 424.03

method	result	size
default	Expression too large to display	118304

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^4, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 10076 vs.  $2(235) = 470$ .

time = 34.91, size = 20228, normalized size = 72.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/96*(96*\sqrt{2}*(a^3*d^5*\cos(d*x + c)^4 - 2*a^3*d^5*\cos(d*x + c)^2 + a^3*d^5)*\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}* \sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)*((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4}^{3/4} * \arctan(((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^2 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^3)*d^4*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^3)*d^2*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4} + \sqrt{2}*((2*(A^3*B^2 + A*B^4)*a + (A^4*B - B^5)*b)*d^7*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + (2*(A^5*B^2 + 2*A^3*B^4 + A*B^6)*a^2 + (3*A^6*B + 5*A^4*B^3 + A^2*B^5 - B^7)*a*b + (A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*b^2)*d^5*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4})*\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}*\sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c)}*(((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4)^{3/4} + \sqrt{2}*(B*d^7*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} + ((A^2*B + B^3)*a + (A^3 + A*B^2)*b)*d^5*\sqrt{((4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4})*\sqrt{-((2*A*B*b - (A^2 - B^2)*a)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/d^4} - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)}*\sqrt{((4*(A^4*B^2 + A^2*B^4)*a^4 + 4*(A^5*B - A*B^5)*a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 + 4*(A^5*B - A*B^5)*a*b^3 + (A^6 - A^4*B^2 - \end{aligned}$$

$$\begin{aligned}
& A^2 B^4 + B^6) * b^4) * d^2 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4} * \cos(dx + c) + \sqrt{2} * ((4 * A^2 * B^3 * a^3 + 4 * (2 * A^3 * B^2 - A * B^4) * a^2 * b + (5 * A^4 * B - 6 * A^2 * B^3 + B^5) * a * b^2 + (A^5 - 2 * A^3 * B^2 + A * B^4) * b^3) * d^3 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4} * \cos(dx + c) + (4 * (A^4 * B^3 + A^2 * B^5) * a^4 + 4 * (A^5 * B^2 - A * B^6) * a^3 * b + (A^6 * B + 3 * A^4 * B^3 + 3 * A^2 * B^5 + B^7) * a^2 * b^2 + 4 * (A^5 * B^2 - A * B^6) * a * b^3 + (A^6 * B - A^4 * B^3 - A^2 * B^5 + B^7) * b^4) * d * \cos(dx + c)) * \sqrt{-((2 * A * B * b - (A^2 - B^2) * a) * d^2 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4} - (A^4 + 2 * A^2 * B^2 + B^4) * a^2 - (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / (4 * A^2 * B^2 * a^2 + 4 * (A^3 * B - A * B^3) * a * b + (A^4 - 2 * A^2 * B^2 + B^4) * b^2))} * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)} * (((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4)^{(1/4)} + (4 * (A^6 * B^2 + 2 * A^4 * B^4 + A^2 * B^6) * a^5 + 4 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^4 * b + (A^8 + 4 * A^6 * B^2 + 6 * A^4 * B^4 + 4 * A^2 * B^6 + B^8) * a^3 * b^2 + 4 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^2 * b^3 + (A^8 - 2 * A^4 * B^4 + B^8) * a * b^4) * \cos(dx + c) + (4 * (A^6 * B^2 + 2 * A^4 * B^4 + A^2 * B^6) * a^4 * b + 4 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^3 * b^2 + (A^8 + 4 * A^6 * B^2 + 6 * A^4 * B^4 + 4 * A^2 * B^6 + B^8) * a^2 * b^3 + 4 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a * b^4 + (A^8 - 2 * A^4 * B^4 + B^8) * b^5) * \sin(dx + c)) / ((a^2 + b^2) * \cos(dx + c)) * (((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4)^{(3/4)} / (4 * (A^{10} * B^2 + 4 * A^8 * B^4 + 6 * A^6 * B^6 + 4 * A^4 * B^8 + A^2 * B^{10}) * a^4 * b + 4 * (A^{11} * B + 3 * A^9 * B^3 + 2 * A^7 * B^5 - 2 * A^5 * B^7 - 3 * A^3 * B^9 - A * B^{11}) * a^3 * b^2 + (A^{12} + 6 * A^{10} * B^2 + 15 * A^8 * B^4 + 20 * A^6 * B^6 + 15 * A^4 * B^8 + 6 * A^2 * B^{10} + B^{12}) * a^2 * b^3 + 4 * (A^{11} * B + 3 * A^9 * B^3 + 2 * A^7 * B^5 - 2 * A^5 * B^7 - 3 * A^3 * B^9 - A * B^{11}) * a * b^4 + (A^{12} + 2 * A^{10} * B^2 - A^8 * B^4 - 4 * A^6 * B^6 - A^4 * B^8 + 2 * A^2 * B^{10} + B^{12}) * b^5)) + 96 * \sqrt{2} * (a^3 * d^5 * \cos(dx + c)^4 - 2 * a^3 * d^5 * \cos(dx + c)^2 + a^3 * d^5) * \sqrt{-((2 * A * B * b - (A^2 - B^2) * a) * d^2 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4} - (A^4 + 2 * A^2 * B^2 + B^4) * a^2 - (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / (4 * A^2 * B^2 * a^2 + 4 * (A^3 * B - A * B^3) * a * b + (A^4 - 2 * A^2 * B^2 + B^4) * b^2))} * \sqrt{(4 * A^2 * B^2 * a^2 + 4 * (A^3 * B - A * B^3) * a * b + (A^4 - 2 * A^2 * B^2 + B^4) * b^2) / d^4} * (((A^4 + 2 * A^2 * B^2 + B^4) * a^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^2) / d^4)^{(3/4)} * \arctan(-((2 * (A^7 * B + 3 * A^5 * B^3 + 3 * A^3 * B^5 + A * B^7) * a^3 + (A^8 \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \cot^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*4\*(a+b\*tan(dx+c))\*\*(1/2)\*(A+B\*tan(dx+c)),x)

[Out] Integral((A + B\*tan(c + dx))\*sqrt(a + b\*tan(c + dx))\*cot(c + dx)\*\*4, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 8.46, size = 2500, normalized size = 8.96
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] atan((((((((224*A*a^4*b^11*d^4 - 32*A*a^2*b^13*d^4 + 256*A*a^6*b^9*d^4 + 64*
B*a^3*b^12*d^4 + 448*B*a^5*b^10*d^4 + 384*B*a^7*b^8*d^4)/(a^4*d^5) - ((2048
*a^4*b^10*d^4 + 3072*a^6*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((B^2*a)/(4*d^
2) - (A^2*a)/(4*d^2) - (2*A^2*B^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2*d^4
- A^4*b^2*d^4 + 4*A*B^3*a*b*d^4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*B*b)
/(2*d^2))^(1/2))/(4*a^4*d^4))*((B^2*a)/(4*d^2) - (A^2*a)/(4*d^2) - (2*A^2*B
^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2*d^4 - A^4*b^2*d^4 + 4*A*B^3*a*b*d^
4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*B*b)/(2*d^2))^(1/2) + ((a + b*tan(c
+ d*x))^(1/2)*(16*B^2*a^3*b^12*d^2 - 512*A^2*a^5*b^10*d^2 - 1280*A^2*a^7*b
^8*d^2 - 64*A^2*a^3*b^12*d^2 + 1024*B^2*a^5*b^10*d^2 + 2304*B^2*a^7*b^8*d^2
+ 4*A^2*a*b^14*d^2 - 16*A*B*a^2*b^13*d^2 + 2048*A*B*a^4*b^11*d^2 + 4096*A*
B*a^6*b^9*d^2))/(4*a^4*d^4))*((B^2*a)/(4*d^2) - (A^2*a)/(4*d^2) - (2*A^2*B
^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2*d^4 - A^4*b^2*d^4 + 4*A*B^3*a*b*d^
4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*B*b)/(2*d^2))^(1/2) - (16*A^3*a^3*b^
13*d^2 - 144*A^3*a^5*b^11*d^2 - 160*A^3*a^7*b^9*d^2 - 2*B^3*a^2*b^14*d^2 -
2*B^3*a^4*b^12*d^2 + 96*B^3*a^6*b^10*d^2 + 96*B^3*a^8*b^8*d^2 - (A^2*B*b^16
*d^2)/2 + 2*A*B^2*a*b^15*d^2 + 50*A*B^2*a^3*b^13*d^2 + 528*A*B^2*a^5*b^11*d
^2 + 480*A*B^2*a^7*b^9*d^2 - (49*A^2*B*a^2*b^14*d^2)/2 + 168*A^2*B*a^4*b^12
*d^2 - 96*A^2*B*a^6*b^10*d^2 - 288*A^2*B*a^8*b^8*d^2)/(a^4*d^5))*((B^2*a)/(
4*d^2) - (A^2*a)/(4*d^2) - (2*A^2*B^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2
*d^4 - A^4*b^2*d^4 + 4*A*B^3*a*b*d^4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*
B*b)/(2*d^2))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(A^4*b^16 - A^2*B^2*b^16
- 17*A^4*a^2*b^14 + 208*A^4*a^4*b^12 + 192*A^4*a^6*b^10 + 128*A^4*a^8*b^8 -
4*B^4*a^2*b^14 + 68*B^4*a^4*b^12 + 64*B^4*a^6*b^10 + 384*B^4*a^8*b^8 + 5*A
^2*B^2*a^2*b^14 + 236*A^2*B^2*a^4*b^12 + 1792*A^2*B^2*a^6*b^10 + 4*A*B^3*a*
b^15 + 12*A*B^3*a^3*b^13 + 1280*A*B^3*a^7*b^9 - 60*A^3*B*a^3*b^13 + 512*A^3
*B*a^5*b^11 - 256*A^3*B*a^7*b^9))/(4*a^4*d^4))*((B^2*a)/(4*d^2) - (A^2*a)/(
4*d^2) - (2*A^2*B^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2*d^4 - A^4*b^2*d^4
+ 4*A*B^3*a*b*d^4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*B*b)/(2*d^2))^(1/2
))*1i - (((((((224*A*a^4*b^11*d^4 - 32*A*a^2*b^13*d^4 + 256*A*a^6*b^9*d^4 + 64
*B*a^3*b^12*d^4 + 448*B*a^5*b^10*d^4 + 384*B*a^7*b^8*d^4)/(a^4*d^5) + ((204
```





### 3.325 $\int \tan^2(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=214

$$\frac{(a - ib)^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{3/2}(iA - B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d}$$

[Out] (a-I\*b)^(3/2)\*(I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d-(a+I\*b)^(3/2)\*(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d-2\*(A\*b+B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/3\*B\*(a+b\*tan(d\*x+c))^(3/2)/d+2/35\*(7\*A\*b-2\*B\*a)\*(a+b\*tan(d\*x+c))^(5/2)/b^2/d+2/7\*B\*tan(d\*x+c)\*(a+b\*tan(d\*x+c))^(5/2)/b/d

#### Rubi [A]

time = 0.40, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3688, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(7Ab - 2aB)(a + b \tan(c + dx))^{3/2}}{35b^2d} - \frac{2(aB + Ab)\sqrt{a + b \tan(c + dx)}}{d} + \frac{(a - ib)^{3/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{3/2}(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d} + \frac{2B \tan(c + dx)(a + b \tan(c + dx))^{3/2}}{7bd} - \frac{2B(a + b \tan(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a - I\*b)^(3/2)\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/d - ((a + I\*b)^(3/2)\*(I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/d - (2\*(A\*b + a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*B\*(a + b\*Tan[c + d\*x])^(3/2))/(3\*d) + (2\*(7\*A\*b - 2\*a\*B)\*(a + b\*Tan[c + d\*x])^(5/2))/(35\*b^2\*d) + (2\*B\*Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^(5/2))/(7\*b\*d)

#### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan^2(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= \frac{2B \tan(c + dx)(a + b \tan(c + dx))^{5/2}}{7bd} + \frac{2 \int (a + b \tan(c + dx))^{3/2} dx}{7bd} \\
&= \frac{2(7Ab - 2aB)(a + b \tan(c + dx))^{5/2}}{35b^2d} + \frac{2B \tan(c + dx)(a + b \tan(c + dx))^{3/2}}{7bd} \\
&= -\frac{2B(a + b \tan(c + dx))^{3/2}}{3d} + \frac{2(7Ab - 2aB)(a + b \tan(c + dx))^{5/2}}{35b^2d} \\
&= -\frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{2B(a + b \tan(c + dx))^{3/2}}{7bd} \\
&= -\frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{2B(a + b \tan(c + dx))^{3/2}}{7bd} \\
&= -\frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{2B(a + b \tan(c + dx))^{3/2}}{7bd} \\
&= -\frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{2B(a + b \tan(c + dx))^{3/2}}{7bd} \\
&= \frac{(a - ib)^{3/2}(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 1.68, size = 252, normalized size = 1.18

$$\frac{2(7Ab - 2aB)(a + b \tan(c + dx))^{5/2} + 10B \tan(c + dx)(a + b \tan(c + dx))^{3/2} + \frac{2B(A - iB)}{3} \left( 3\sqrt{a - ib} (ia + b) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) - i\sqrt{a + b \tan(c + dx)} (4a - 3ib + b \tan(c + dx)) \right) + \frac{2B(A + iB)}{3} \left( 3\sqrt{a + ib} (-ia + b) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + i\sqrt{a + b \tan(c + dx)} (4a + 3ib + b \tan(c + dx)) \right)}{35bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

```

[Out] ((2*(7*A*b - 2*a*B)*(a + b*Tan[c + d*x])^(5/2))/b + 10*B*Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2) + (35*b*(A - I*B)*(3*sqrt[a - I*b]*(I*a + b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - I*Sqrt[a + b*Tan[c + d*x]]*(4*a - (3*I)*b + b*Tan[c + d*x])))/3 + (35*b*(A + I*B)*(3*sqrt[a + I*b]*((-I)*a + b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + I*Sqrt[a + b*Tan[c + d*x]]*(4*a + (3*I)*b + b*Tan[c + d*x])))/3)/(35*b*d)

```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(182) = 364.

time = 0.16, size = 984, normalized size = 4.60

method	result
derivativedivides	$\frac{2B(a+b \tan(dx+c))^{\frac{7}{2}}}{7} + \frac{2Ab(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2Ba(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2Bb^2(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - 2Ab^3 \sqrt{a+b \tan(dx+c)}$
default	$\frac{2B(a+b \tan(dx+c))^{\frac{7}{2}}}{7} + \frac{2Ab(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2Ba(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - \frac{2Bb^2(a+b \tan(dx+c))^{\frac{3}{2}}}{3} - 2Ab^3 \sqrt{a+b \tan(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$\begin{aligned} & 2/d/b^2*(1/7*B*(a+b*\tan(d*x+c))^{(7/2)}+1/5*A*b*(a+b*\tan(d*x+c))^{(5/2)}-1/5*B* \\ & a*(a+b*\tan(d*x+c))^{(5/2)}-1/3*B*b^2*(a+b*\tan(d*x+c))^{(3/2)}-A*b^3*(a+b*\tan(d* \\ & x+c))^{(1/2)}-B*a*b^2*(a+b*\tan(d*x+c))^{(1/2)}+b^2*(1/4/b*(1/2*(A*(2*(a^2+b^2)^{ \\ & (1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2+A*(2 \\ & *(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^2-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{( \\ & 1/2)}*b+2*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b)*\ln(b*\tan(d*x+c)+a+(a+b*\tan(d* \\ & x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)}))+2*(2*A*(a^2+b^2) \\ & ^{(1/2)}*b^2+2*B*(a^2+b^2)^{(1/2)}*a*b-1/2*(A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^ \\ & 2+b^2)^{(1/2)}*a-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2+A*(2*(a^2+b^2)^{(1/2)}+2*a \\ & )^{(1/2)}*b^2-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b+2*B*(2*(a^2+b \\ & ^2)^{(1/2)}+2*a)^{(1/2)}*a*b)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)} \\ & -2*a)^{(1/2)}*\arctan((2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}) \\ & / (2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}))+1/4/b*(-1/2*(A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/ \\ & 2)}*(a^2+b^2)^{(1/2)}*a-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2+A*(2*(a^2+b^2)^{(1/ \\ & 2)}+2*a)^{(1/2)}*b^2-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b+2*B*(2* \\ & (a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b)*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{(1/2)}*( \\ & 2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(-2*A*(a^2+b^2)^{(1/2)}*b^2-2 \\ & *B*(a^2+b^2)^{(1/2)}*a*b+1/2*(A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)} \\ & *a-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2+A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^2- \end{aligned}$$

$$B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b+2*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((-2*(a+b*\tan(dx+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}))$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(a+b\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(dx + c) + A)\*(b\*tan(dx + c) + a)^(3/2)\*tan(dx + c)^2, x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^2\*(a+b\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*2\*(a+b\*tan(dx+c))\*\*(3/2)\*(A+B\*tan(dx+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)\*tan(c + d\*x)\*\*2, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 69.45, size = 2993, normalized size = 13.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out] 
$$\log\left(\frac{16A^3ab^3(a^2+b^2)^2}{d^3} - \left(\frac{16b^2((-A^4b^2d^4(3a^2-b^2)^2)^{1/2} - A^2a^3d^2 + 3A^2ab^2d^2)}{d^4}\right)^{1/2}(Ab^3 + Aa^2b + a*d*\left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} - A^2a^3d^2 + 3A^2ab^2d^2}{d^4}\right)^{1/2}(a + b*\tan(c + d*x))^{1/2}\right)/d + \frac{16A^2b^2(a + b*\tan(c + d*x))^{1/2}(a^4 + b^4 - 6a^2b^2)}{d^2} * \left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} - A^2a^3d^2 + 3A^2ab^2d^2}{d^4}\right)^{1/2} / 2 * \left(\frac{6A^4a^2b^4d^4 - A^4b^6d^4 - 9A^4a^4b^2d^4}{4d^4} - \frac{A^2a^3}{4d^2} + \frac{3A^2ab^2}{4d^2}\right)^{1/2} - \log\left(\frac{16A^3ab^3(a^2+b^2)^2}{d^3} - \left(\frac{16b^2((-A^4b^2d^4(3a^2-b^2)^2)^{1/2} + A^2a^3d^2 - 3A^2ab^2d^2)}{d^4}\right)^{1/2}(Ab^3 + Aa^2b - a*d*\left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} + A^2a^3d^2 - 3A^2ab^2d^2}{d^4}\right)^{1/2}(a + b*\tan(c + d*x))^{1/2}\right)/d - \frac{16A^2b^2(a + b*\tan(c + d*x))^{1/2}(a^4 + b^4 - 6a^2b^2)}{d^2} * \left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} + A^2a^3d^2 - 3A^2ab^2d^2}{d^4}\right)^{1/2} / 2 * \left(\frac{6A^4a^2b^4d^4 - A^4b^6d^4 - 9A^4a^4b^2d^4}{4d^4}\right)^{1/2} + \frac{A^2a^3d^2 - 3A^2ab^2d^2}{4d^2} - \log\left(\frac{16A^3ab^3(a^2+b^2)^2}{d^3} - \left(\frac{16b^2((-A^4b^2d^4(3a^2-b^2)^2)^{1/2} - A^2a^3d^2 + 3A^2ab^2d^2)}{d^4}\right)^{1/2}(Ab^3 + Aa^2b - a*d*\left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} - A^2a^3d^2 + 3A^2ab^2d^2}{d^4}\right)^{1/2}(a + b*\tan(c + d*x))^{1/2}\right)/d - \frac{16A^2b^2(a + b*\tan(c + d*x))^{1/2}(a^4 + b^4 - 6a^2b^2)}{d^2} * \left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} - A^2a^3d^2 + 3A^2ab^2d^2}{d^4}\right)^{1/2} / 2 * \left(\frac{6A^4a^2b^4d^4 - A^4b^6d^4 - 9A^4a^4b^2d^4}{4d^4}\right)^{1/2} - \frac{(2B(a^2 + b^2))}{(3b^2d)} - \frac{(2Ba^2)}{(3b^2d)} * (a + b*\tan(c + d*x))^{3/2} + \log\left(\frac{16A^3ab^3(a^2+b^2)^2}{d^3} - \left(\frac{16b^2((-A^4b^2d^4(3a^2-b^2)^2)^{1/2} + A^2a^3d^2 - 3A^2ab^2d^2)}{d^4}\right)^{1/2}(Ab^3 + Aa^2b + a*d*\left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} + A^2a^3d^2 - 3A^2ab^2d^2}{d^4}\right)^{1/2}(a + b*\tan(c + d*x))^{1/2}\right)/d + \frac{16A^2b^2(a + b*\tan(c + d*x))^{1/2}(a^4 + b^4 - 6a^2b^2)}{d^2} * \left(\frac{(-A^4b^2d^4(3a^2-b^2)^2)^{1/2} + A^2a^3d^2 - 3A^2ab^2d^2}{d^4}\right)^{1/2} / 2 * \left(\frac{3A^2ab^2}{4d^2} - \frac{A^2a^3}{4d^2} - \frac{6A^4a^2b^4d^4 - A^4b^6d^4 - 9A^4a^4b^2d^4}{4d^4}\right)^{1/2} - (a + b*\tan(c + d*x))^{1/2} * \frac{2a((2B(a^2 + b^2))}{(b^2d)} - \frac{(2Ba^2)}{(b^2d)} + \frac{(2Ba^3)}{(b^2d)} - \frac{(2Ba(a^2 + b^2))}{(b^2d)} - \log\left(\frac{8B^3b^2(a^2 - b^2)(a^2 + b^2)^2}{d^3} - \left(\frac{(-B^4b^2d^4(3a^2 - b^2))}{d^4}\right)^{1/2}\right)$$

$$\begin{aligned}
& ^2)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(B*a^2 + B*b^2 - d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/d)/2)*(((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/(4*d^4))^{(1/2)} - \log((8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3 - (((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(B*a^2 + B*b^2 - d*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/d)/2)*(-((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/(4*d^4))^{(1/2)} + \log(((((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(B*a^2 + B*b^2 + d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/d)/2 + (8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{(1/2)}/(4*d^4) + (B^2*a^3)/(4*d^2) - (3*B^2*a*b^2)/(4*d^2))^{(1/2)} + \log(((((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(B*a^2 + B*b^2 + d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/d)/2 + (8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((B^2*a^3)/(4*d^2) - (6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{(1/2)}/(4*d^4) - (3*B^2*a*b^2)/(4*d^2))^{(1/2)} - ((2*A*(a^2 + b^2))/(b*d) - (2*A*a^2)/(b*d))* (a + b*\tan(c + d*x))^{(1/2)} + (2*A*(a + b*\tan(c + d*x))^{(5/2)})/(5*b*d) + (2*B*(a + b*\tan(c + d*x))^{(7/2)})/(7*b^2*d) - (2*B*a*(a + b*\tan(c + d*x))^{(5/2)})/(5*b^2*d)
\end{aligned}$$



### 3.326 $\int \tan(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=175

$$\frac{(a-ib)^{3/2}(A-ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(A+ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

[Out]  $-(a-I*b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d-(a+I*b)^{(3/2)}*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+2*(A*a-B*b)*(a+b*\tan(d*x+c))^{(1/2)}/d+2/3*A*(a+b*\tan(d*x+c))^{(3/2)}/d+2/5*B*(a+b*\tan(d*x+c))^{(5/2)}/b/d$

**Rubi** [A]

time = 0.25, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3673, 3609, 3620, 3618, 65, 214}

$$\frac{2(aA-bB)\sqrt{a+b \tan(c+dx)}}{d} - \frac{(a-ib)^{3/2}(A-ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(A+ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2A(a+b \tan(c+dx))^{3/2}}{3d} + \frac{2B(a+b \tan(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c+d*x]*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $-\left(\frac{(a-I*b)^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]]}{d} - \frac{(a+I*b)^{(3/2)}*(A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]]}{d} + \frac{2*(a*A-b*B)*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]}{d} + \frac{2*A*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}}{(3*d)} + \frac{2*B*(a+b*\operatorname{Tan}[c+d*x])^{(5/2)}}{(5*b*d)}\right)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3609

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] := \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}$

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :=> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :=> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :=> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= \frac{2B(a + b \tan(c + dx))^{5/2}}{5bd} + \int (-B + A \tan(c + dx)) dx \\
&= \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} + \frac{2B(a + b \tan(c + dx))^{5/2}}{5bd} \\
&= \frac{2(aA - bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2(aA - bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2(aA - bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2(aA - bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2(aA - bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2(aA - bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{(a - ib)^{3/2}(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{2A(a + b \tan(c + dx))^{3/2}}{3d}
\end{aligned}$$

**Mathematica [A]**

time = 0.92, size = 192, normalized size = 1.10

$$\frac{6B(a+b \tan(c+dx))^{5/2} + 5(A-ib) \left( -3(a-ib)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right) + \sqrt{a+b \tan(c+dx)} (4a-3ib+b \tan(c+dx)) \right) + 5(A+ib) \left( -3(a+ib)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right) + \sqrt{a+b \tan(c+dx)} (4a+3ib+b \tan(c+dx)) \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((6\*B\*(a + b\*Tan[c + d\*x])^(5/2))/b + 5\*(A - I\*B)\*(-3\*(a - I\*b)^(3/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] + Sqrt[a + b\*Tan[c + d\*x]]\*(4\*a - (3\*I)\*b + b\*Tan[c + d\*x])) + 5\*(A + I\*B)\*(-3\*(a + I\*b)^(3/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]] + Sqrt[a + b\*Tan[c + d\*x]]\*(4\*a + (3\*I)\*b + b\*Tan[c + d\*x])))/(15\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(147) = 294.

time = 0.15, size = 950, normalized size = 5.43

method	result
--------	--------

derivativedivides	$\frac{2B(a+b \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2Ab(a+b \tan(dx+c))^{\frac{3}{2}}}{3} + 2Aab \sqrt{a+b \tan(dx+c)} - 2Bb^2 \sqrt{a+b \tan(dx+c)} - 2b$
default	$\frac{2B(a+b \tan(dx+c))^{\frac{5}{2}}}{5} + \frac{2Ab(a+b \tan(dx+c))^{\frac{3}{2}}}{3} + 2Aab \sqrt{a+b \tan(dx+c)} - 2Bb^2 \sqrt{a+b \tan(dx+c)} - 2b$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & 2/d/b*(1/5*B*(a+b*\tan(d*x+c))^{(5/2)}+1/3*A*b*(a+b*\tan(d*x+c))^{(3/2)}+A*a*b*(a \\ & +b*\tan(d*x+c))^{(1/2)}-B*b^2*(a+b*\tan(d*x+c))^{(1/2)}-b*(1/4/b*(1/2*(-A*(2*(a^2 \\ & +b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b+2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}* \\ & a*b-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a+B*(2*(a^2+b^2)^{(1/2)}+ \\ & 2*a)^{(1/2)}*a^2-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^2)*\ln(b*\tan(d*x+c))+a+(a+b* \\ & \tan(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)}+2*(2*A*(a^2 \\ & +b^2)^{(1/2)}*a*b-2*B*(a^2+b^2)^{(1/2)}*b^2-1/2*(-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1 \\ & /2)}*(a^2+b^2)^{(1/2)}*b+2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b-B*(2*(a^2+b^2)^{(1 \\ & /2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a+B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2-B*(2 \\ & *(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^2)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2 \\ & )^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a) \\ & ^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}))+1/4/b*(-1/2*(-A*(2*(a^2+b^2)^{(1/2)}+ \\ & 2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b+2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b-B*(2*(a^2 \\ & +b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a+B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a \\ & ^2-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^2)*\ln(-b*\tan(d*x+c))-a+(a+b*\tan(d*x+c)) \\ & ^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}+2*(-2*A*(a^2+b^2)^{(1/ \\ & 2)}*a*b+2*B*(a^2+b^2)^{(1/2)}*b^2+1/2*(-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b \\ & ^2)^{(1/2)}*b+2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b-B*(2*(a^2+b^2)^{(1/2)}+2*a) \\ & ^{(1/2)}*(a^2+b^2)^{(1/2)}*a+B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2-B*(2*(a^2+b^2) \\ & ^{(1/2)}+2*a)^{(1/2)}*b^2)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2* \end{aligned}$$

$a^{1/2} \arctan\left(\frac{-2(a+b\tan(dx+c))^{1/2} + (2(a^2+b^2)^{1/2} + 2a)^{1/2}}{2(a^2+b^2)^{1/2} - 2a^{1/2}}\right)$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(a+b\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(dx + c) + A)\*(b\*tan(dx + c) + a)^(3/2)\*tan(dx + c), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(a+b\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(a+b\*tan(dx+c))\*\*(3/2)\*(A+B\*tan(dx+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)\*tan(c + d\*x), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(a+b\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="giac")

[Out] Timed out

Mupad [B]

time = 29.60, size = 2868, normalized size = 16.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\tan(c + d*x)*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x))^{3/2}, x)$ 

[Out]  $\log\left(\frac{(16*B^3*a*b^3*(a^2 + b^2)^2)/d^3 - \left(\left(\frac{(16*b^2*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(B*b^3 + B*a^2*b + a*d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(a + b*\tan(c + d*x))^{1/2}}{d} + \frac{(16*B^2*b^2*(a + b*\tan(c + d*x))^{1/2}*(a^4 + b^4 - 6*a^2*b^2))/d^2}{\left(\left(\frac{(-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2}{d^4}\right)^{1/2}\right)^2} * \left(\frac{(6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{1/2}}{(4*d^4)} - \frac{(B^2*a^3)}{(4*d^2)} + \frac{(3*B^2*a*b^2)}{(4*d^2)}\right)^{1/2} - \log\left(\frac{(16*B^3*a*b^3*(a^2 + b^2)^2)/d^3 - \left(\left(\frac{(16*b^2*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(B*b^3 + B*a^2*b - a*d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(a + b*\tan(c + d*x))^{1/2}}{d} - \frac{(16*B^2*b^2*(a + b*\tan(c + d*x))^{1/2}*(a^4 + b^4 - 6*a^2*b^2))/d^2}{\left(\left(\frac{(-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2}{d^4}\right)^{1/2}\right)^2} * \left(\frac{(6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{1/2}}{(4*d^4)} + \frac{(B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)}{(4*d^4)}\right)^{1/2} - \log\left(\frac{(16*B^3*a*b^3*(a^2 + b^2)^2)/d^3 - \left(\left(\frac{(16*b^2*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(B*b^3 + B*a^2*b - a*d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(a + b*\tan(c + d*x))^{1/2}}{d} - \frac{(16*B^2*b^2*(a + b*\tan(c + d*x))^{1/2}*(a^4 + b^4 - 6*a^2*b^2))/d^2}{\left(\left(\frac{(-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2}{d^4}\right)^{1/2}\right)^2} * \left(\frac{(6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{1/2}}{(4*d^4)}\right)^{1/2} - \frac{((2*B*(a^2 + b^2)))/(b*d) - (2*B*a^2)/(b*d)}{(b*d)} * (a + b*\tan(c + d*x))^{1/2} + \log\left(\frac{(16*B^3*a*b^3*(a^2 + b^2)^2)/d^3 - \left(\left(\frac{(16*b^2*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(B*b^3 + B*a^2*b + a*d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4\right)^{1/2}*(a + b*\tan(c + d*x))^{1/2}}{d} + \frac{(16*B^2*b^2*(a + b*\tan(c + d*x))^{1/2}*(a^4 + b^4 - 6*a^2*b^2))/d^2}{\left(\left(\frac{(-B^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2}{d^4}\right)^{1/2}\right)^2} * \left(\frac{(3*B^2*a*b^2)}{(4*d^2)} - \frac{(B^2*a^3)}{(4*d^2)}\right) - \frac{(6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^{1/2}}{(4*d^4)}\right)^{1/2} - \log\left(-\left(\left(\frac{(-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2}{d^4}\right)^{1/2} * \left(\frac{(16*A^2*b^2*(a + b*\tan(c + d*x))^{1/2}*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4\right)^{1/2} * (A*a^2 + A*b^2 + d*((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{1/2} + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4\right)^{1/2} * (a + b*\tan(c + d*x))^{1/2}}{d}\right)/2 - \frac{(8*A^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3}{\left(\left(\frac{(6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^{1/2}}{(4*d^4)}\right)^{1/2} + \frac{(A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)}{(4*d^4)}\right)^{1/2} + \frac{(8*A^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3}{\left(\left(\frac{(6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^{1/2}}{(4*d^4)}\right)^{1/2} + \frac{(A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)}{(4*d^4)}\right)^{1/2}}$

$$\begin{aligned}
& *a*b^2*d^2)/(4*d^4))^{(1/2)} - \log(- ((-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} \\
& - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*A^2*b^2*(a + b*\tan(c + d* \\
& x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(-((-A^4*b^2*d^4*(3*a^2 \\
& - b^2)^2)^{(1/2)} - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^{(1/2)}*(A*a^2 + A*b^2 \\
& + d*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2 \\
& )/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/d))/2 - (8*A^3*b^2*(a^2 - b^2)*(a \\
& ^2 + b^2)^2)/d^3)*(-((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^{(1/2)} \\
& - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/(4*d^4))^{(1/2)} + \log(((((-A^4*b^2*d^4 \\
& *(3*a^2 - b^2)^2)^{(1/2)} + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*A \\
& ^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2* \\
& (((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4 \\
& )^{(1/2)}*(A*a^2 + A*b^2 - d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} + A^2*a^3 \\
& *d^2 - 3*A^2*a*b^2*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/d))/2 - (8* \\
& A^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - \\
& 9*A^4*a^4*b^2*d^4)^{(1/2)}/(4*d^4) + (A^2*a^3)/(4*d^2) - (3*A^2*a*b^2)/(4*d^ \\
& 2))^{(1/2)} + \log(((((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - A^2*a^3*d^2 + 3* \\
& A^2*a*b^2*d^2)/d^4)^{(1/2)}*((16*A^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^4 + b^ \\
& 4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^{(1/2)} - A \\
& ^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^{(1/2)}*(A*a^2 + A*b^2 - d*(((-A^4*b^2*d^ \\
& 4*(3*a^2 - b^2)^2)^{(1/2)} - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^{(1/2)}*(a + b \\
& *tan(c + d*x))^{(1/2)}))/d))/2 - (8*A^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)* \\
& (A^2*a^3)/(4*d^2) - (6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^{(1/2)} \\
& /((4*d^4) - (3*A^2*a*b^2)/(4*d^2))^{(1/2)} + (2*A*(a + b*\tan(c + d*x))^{(3/ \\
& 2)})/(3*d) + (2*A*a*(a + b*\tan(c + d*x))^{(1/2)})/d + (2*B*(a + b*\tan(c + d*x) \\
& )^{(5/2)})/(5*b*d)
\end{aligned}$$

### 3.327 $\int (a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=150

$$\frac{(a - ib)^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{(a + ib)^{3/2}(iA - B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d}$$

[Out]  $-(a - I*b)^{(3/2)}*(I*A + B)*\operatorname{arctanh}((a + b*\tan(d*x + c))^{(1/2)}/(a - I*b)^{(1/2)})/d + (a + I*b)^{(3/2)}*(I*A - B)*\operatorname{arctanh}((a + b*\tan(d*x + c))^{(1/2)}/(a + I*b)^{(1/2)})/d + 2*(A*b + B*a)*(a + b*\tan(d*x + c))^{(1/2)}/d + 2/3*B*(a + b*\tan(d*x + c))^{(3/2)}/d$

**Rubi [A]**

time = 0.21, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2(aB + Ab)\sqrt{a + b \tan(c + dx)}}{d} - \frac{(a - ib)^{3/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{(a + ib)^{3/2}(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-\left(\frac{(a - I*b)^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]]}{d} + \frac{(a + I*b)^{(3/2)}*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]]}{d} + \frac{2*(A*b + a*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}{d} + \frac{2*B*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}}{3*d}\right)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3609**

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m - 1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\operatorname{Tan}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 + b^2,$



0] && GtQ[m, 0]

### Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int (a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx)) dx &= \frac{2B(a + b \tan(c + dx))^{3/2}}{3d} + \int \sqrt{a + b \tan(c + dx)} (aA \\
 &= \frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{2(Ab + aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2B(a + b \tan(c + dx))^{3/2}}{3d} \\
 &= \frac{(a - ib)^{3/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} +
 \end{aligned}$$

### Mathematica [A]

time = 0.36, size = 140, normalized size = 0.93

$$\frac{-3i(a - ib)^{3/2}(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) + 3i(a + ib)^{3/2}(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + 2\sqrt{a + b \tan(c + dx)}(3Ab + 4aB + bB \tan(c + dx))}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((-3\*I)\*(a - I\*b)^(3/2)\*(A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] + (3\*I)\*(a + I\*b)^(3/2)\*(A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]] + 2\*Sqrt[a + b\*Tan[c + d\*x]]\*(3\*A\*b + 4\*a\*B + b\*B\*Tan[c + d\*x]))/(3\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(126) = 252.

time = 0.12, size = 920, normalized size = 6.13

method	result
derivativedivides	$\frac{2B(a+b \tan(dx+c))^{\frac{3}{2}} + 2Ab\sqrt{a+b \tan(dx+c)} + 2Ba\sqrt{a+b \tan(dx+c)} + \frac{(-A\sqrt{2\sqrt{a^2+b^2}} + 2\sqrt{a+b \tan(dx+c)})^{\frac{3}{2}}}{3}}{3}$
default	$\frac{2B(a+b \tan(dx+c))^{\frac{3}{2}} + 2Ab\sqrt{a+b \tan(dx+c)} + 2Ba\sqrt{a+b \tan(dx+c)} + \frac{(-A\sqrt{2\sqrt{a^2+b^2}} + 2\sqrt{a+b \tan(dx+c)})^{\frac{3}{2}}}{3}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(2/3\*B\*(a+b\*tan(d\*x+c))^(3/2)+2\*A\*b\*(a+b\*tan(d\*x+c))^(1/2)+2\*B\*a\*(a+b\*tan(d\*x+c))^(1/2)+1/2/b\*(1/2\*(-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)+2\*a+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^2+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b-2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))+2\*(-2\*A\*(a^2+b^2)^(1/2)\*b^2-2\*B\*(a^2+b^2)^(1/2)\*a\*b-1/2\*(-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^2+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b-2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((2\*(a+b\*tan(d\*x+c))^(1/2)+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)))+1/2/b\*(-1/2\*(-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^2+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b-2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b)\*ln(-b\*tan(d\*x+c)-a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)-(a^2+b^2)^(1/2))+2\*(2\*A\*(a^2+b^2)^(1/2)\*b^2+2\*B\*(a^2+b^2)^(1/2)\*a\*b+1/2\*(-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^2+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b-2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b)\*(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)

$$\frac{(1/2)+2*a)^{(1/2)}}{(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((-2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2))}}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out



$$\begin{aligned}
& 2) - B^2 a^3 d^2 + 3 B^2 a b^2 d^2 / d^4)^{1/2} * ((16 B^2 b^2 (a + b \tan(c + d x))^{1/2} * (a^4 + b^4 - 6 a^2 b^2) / d^2 - (16 a b^2 * (-(-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} - B^2 a^3 d^2 + 3 B^2 a b^2 d^2) / d^4)^{1/2} * (B a^2 + B b^2 + d * (-(-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} - B^2 a^3 d^2 + 3 B^2 a b^2 d^2) / d^4)^{1/2} * (a + b \tan(c + d x))^{1/2})) / d) / 2 - (8 B^3 b^2 (a^2 - b^2) * (a^2 + b^2)^2 / d^3) * (-((6 B^4 a^2 b^4 d^4 - B^4 b^6 d^4 - 9 B^4 a^4 b^2 d^4)^{1/2} - B^2 a^3 d^2 + 3 B^2 a b^2 d^2) / (4 d^4))^{1/2} + \log(((((-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} + B^2 a^3 d^2 - 3 B^2 a b^2 d^2) / d^4)^{1/2} * ((16 B^2 b^2 (a + b \tan(c + d x))^{1/2} * (a^4 + b^4 - 6 a^2 b^2) / d^2 + (16 a b^2 * (((-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} + B^2 a^3 d^2 - 3 B^2 a b^2 d^2) / d^4)^{1/2} * (B a^2 + B b^2 - d * (((-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} + B^2 a^3 d^2 - 3 B^2 a b^2 d^2) / d^4)^{1/2} * (a + b \tan(c + d x))^{1/2}))) / d) / 2 - (8 B^3 b^2 (a^2 - b^2) * (a^2 + b^2)^2 / d^3) * ((6 B^4 a^2 b^4 d^4 - B^4 b^6 d^4 - 9 B^4 a^4 b^2 d^4)^{1/2} / (4 d^4) + (B^2 a^3) / (4 d^2) - (3 B^2 a b^2) / (4 d^2))^{1/2} + \log(((((-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} - B^2 a^3 d^2 + 3 B^2 a b^2 d^2) / d^4)^{1/2} * ((16 B^2 b^2 (a + b \tan(c + d x))^{1/2} * (a^4 + b^4 - 6 a^2 b^2) / d^2 + (16 a b^2 * (-(-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} - B^2 a^3 d^2 + 3 B^2 a b^2 d^2) / d^4)^{1/2} * (B a^2 + B b^2 - d * (-(-B^4 b^2 d^4 (3 a^2 - b^2)^2)^{1/2} - B^2 a^3 d^2 + 3 B^2 a b^2 d^2) / d^4)^{1/2} * (a + b \tan(c + d x))^{1/2}))) / d) / 2 - (8 B^3 b^2 (a^2 - b^2) * (a^2 + b^2)^2 / d^3) * ((B^2 a^3) / (4 d^2) - (6 B^4 a^2 b^4 d^4 - B^4 b^6 d^4 - 9 B^4 a^4 b^2 d^4)^{1/2} / (4 d^4) - (3 B^2 a b^2) / (4 d^2))^{1/2} + (2 B * (a + b \tan(c + d x))^{3/2}) / (3 d) + (2 A b * (a + b \tan(c + d x))^{1/2}) / d + (2 B a * (a + b \tan(c + d x))^{1/2}) / d
\end{aligned}$$

### 3.328 $\int \cot(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=152

$$\frac{2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{3/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{3/2}(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2bB\sqrt{a+b \tan(c+dx)}}{d}$$

[Out]  $-2*a^{(3/2)}*A*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d+(a-I*b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d+(a+I*b)^{(3/2)}*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+2*b*B*(a+b*\tan(d*x+c))^{(1/2)}/d$

Rubi [A]

time = 0.41, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3688, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{3/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{3/2}(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2bB\sqrt{a+b \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-2*a^{(3/2)}*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/d + ((a-I*b)^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/d + ((a+I*b)^{(3/2)}*(A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/d + (2*b*B*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/d$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c$

\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2)/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \frac{2bB \sqrt{a+b \tan(c+dx)}}{d} + 2 \int \frac{\cot(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx \\
&= \frac{2bB \sqrt{a+b \tan(c+dx)}}{d} + 2 \int \frac{\frac{1}{2}(2aAb+a^2I)}{\sqrt{a+b \tan(c+dx)}} dx \\
&= \frac{2bB \sqrt{a+b \tan(c+dx)}}{d} - \frac{1}{2}((a+ib)^2(iA - \\
&= \frac{2bB \sqrt{a+b \tan(c+dx)}}{d} + \frac{(2a^2A) \text{Subst}\left(\int \frac{1}{\sqrt{a+b \tan(c+dx)}} dx\right)}{2a^{3/2}} \\
&= -\frac{2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2bB \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2bB \sqrt{a+b \tan(c+dx)}}{d}
\end{aligned}$$

**Mathematica [A]**

time = 0.23, size = 144, normalized size = 0.95

$$\frac{-2a^{3/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right) + (a-ib)^{3/2}(A-ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right) + (a+ib)^{3/2}(A+ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right) + 2bB \sqrt{a+b \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

```
[Out] (-2*a^(3/2)*A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + (a - I*b)^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (a + I*b)^(3/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b*B*Sqrt[a + b*Tan[c + d*x]])/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.71, size = 41721, normalized size = 274.48

method	result	size
default	Expression too large to display	41721

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```



[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c), x)

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)\*cot(c + d\*x), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out



$$\begin{aligned}
& 3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4* \\
& (A^4*a^6 + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3* \\
& A^4*a^2*b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*a^2 \\
& *b^4 + 6*A^2*B^2*a^4*b^2))^2 - A^2*a^3*d^2 + B^2*a^3*d^2 - 2*A*B*b^3*d^2 \\
& + 3*A^2*a*b^2*d^2 - 3*B^2*a*b^2*d^2 + 6*A*B*a^2*b*d^2)/(4*d^4))^2 * i \\
& - ((((((32*(4*B*a*b^11*d^4 + 12*A*a^2*b^10*d^4 + 12*A*a^4*b^8*d^4 + 4*B*a^3* \\
& b^9*d^4))/d^5 + (32*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x)))^(1/ \\
& 2))*(-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + \\
& 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 + B^4*a \\
& ^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A^4*a^4*b^ \\
& 2 + 3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6*A^2*B^2*a^4*b^2)) \\
& ^2 - A^2*a^3*d^2 + B^2*a^3*d^2 - 2*A*B*b^3*d^2 + 3*A^2*a*b^2*d^2 - 3*B^ \\
& 2*a*b^2*d^2 + 6*A*B*a^2*b*d^2)/(4*d^4))^2)/d^4)*(-(((8*A^2*a^3*d^2 - 8* \\
& B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B \\
& *a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a \\
& ^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*a \\
& a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6*A^2*B^2*a^4*b^2))^2 - A^2*a^3*d^2 + B^2 \\
& *a^3*d^2 - 2*A*B*b^3*d^2 + 3*A^2*a*b^2*d^2 - 3*B^2*a*b^2*d^2 + 6*A*B*a^2*b \\
& d^2)/(4*d^4))^2 + (32*(a + b*tan(c + d*x)))^(1/2)*(28*A^2*a^3*b^10*d^2 - \\
& 18*A^2*a^5*b^8*d^2 - 28*B^2*a^3*b^10*d^2 + 10*B^2*a^5*b^8*d^2 - 16*A*B*b^1 \\
& 3*d^2 + 22*A^2*a*b^12*d^2 - 22*B^2*a*b^12*d^2 + 16*A*B*a^2*b^11*d^2 + 64*A* \\
& B*a^4*b^9*d^2))/d^4)*(-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - \\
& 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 \\
& + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2* \\
& b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6 \\
& *A^2*B^2*a^4*b^2))^2 - A^2*a^3*d^2 + B^2*a^3*d^2 - 2*A*B*b^3*d^2 + 3*A^ \\
& 2*a*b^2*d^2 - 3*B^2*a*b^2*d^2 + 6*A*B*a^2*b*d^2)/(4*d^4))^2 - (32*(3*A^ \\
& 3*a^7*b^8*d^2 - 21*A^3*a^5*b^10*d^2 - 23*A^3*a^3*b^12*d^2 + 2*B^3*a^2*b^13* \\
& d^2 + 4*B^3*a^4*b^11*d^2 + 2*B^3*a^6*b^9*d^2 + A^3*a*b^14*d^2 + A*B^2*a*b^1 \\
& 4*d^2 + 25*A*B^2*a^3*b^12*d^2 + 15*A*B^2*a^5*b^...
\end{aligned}$$

### 3.329 $\int \cot^2(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=169

$$\frac{\sqrt{a} (3Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{(a - ib)^{3/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d}$$

[Out]  $(a-I*b)^{(3/2)}*(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d-(a+I*b)^{(3/2)}*(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d-(3*A*b+2*B*a)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})*a^{(1/2)}/d-a*A*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.40, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3686, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(a-ib)^{3/2}(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a}(2aB+3Ab) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{(a+ib)^{3/2}(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{aA \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^2*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-((\operatorname{Sqrt}[a]*(3*A*b + 2*a*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/d) + ((a - I*b)^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - ((a + I*b)^{(3/2)}*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d - (a*A*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**

$\operatorname{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c$

\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3715

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2)/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \cot^2(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} + \int \frac{c}{d} \\
 &= -\frac{aA \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} + \frac{1}{2}(a \\
 &= -\frac{aA \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} - \frac{1}{2}((c \\
 &= -\frac{aA \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} + \frac{((a - \\
 &= -\frac{\sqrt{a} (3Ab + 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{d} \\
 &= -\frac{\sqrt{a} (3Ab + 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{d}
 \end{aligned}$$

**Mathematica [A]**

time = 0.39, size = 282, normalized size = 1.67

$$\frac{-\sqrt{a} (3Ab + 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right) + (a - b)^{3/2} (A + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - b}} \right) - iaA\sqrt{a + b} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b}} \right) + A\sqrt{a + b} b \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b}} \right) + a\sqrt{a + b} B \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b}} \right) + i\sqrt{a + b} bB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b}} \right) - aA \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(-(\text{Sqrt}[a]*(3*A*b + 2*a*B)*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a]]) + (a - I*b)^{(3/2)}*(I*A + B)*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a - I*b]] - I*a*A*\text{Sqrt}[a + I*b]*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]] + A*\text{Sqrt}[a + I*b]*b*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]] + a*\text{Sqrt}[a + I*b]*B*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]] + I*\text{Sqrt}[a + I*b]*b*B*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]] - a*A*\text{Cot}[c + d*x]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.68, size = 69532, normalized size = 411.43

method	result	size
default	Expression too large to display	69532

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x  
)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**2,  
x)`

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 8.51, size = 2500, normalized size = 14.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out]  $(a^{1/2}) \operatorname{atan}\left(\frac{(a^{1/2}) \left( (16(a + b \tan(c + d x))^{1/2} (2A^4 b^{16} + 2B^4 b^{16} + 4A^2 B^2 b^{16} - A^4 a^2 b^{14} + 66A^4 a^4 b^{12} - A^4 a^6 b^{10} + 2A^4 a^8 b^8 + 8B^4 a^2 b^{14} + 16B^4 a^4 b^{12} - 16B^4 a^6 b^{10} + 6B^4 a^8 b^8 + 25A^2 B^2 a^2 b^{14} - 130A^2 B^2 a^4 b^{12} + 145A^2 B^2 a^6 b^{10} + 12A^2 B^3 a^3 b^{13} - 104A^2 B^3 a^5 b^{11} + 44A^2 B^3 a^7 b^9 - 84A^3 B a^3 b^{13} + 144A^3 B a^5 b^{11} - 12A^3 B a^7 b^9) \right)}{d^4} + (a^{1/2}) (3A^3 b + 2B^3 a) \left( (8(100A^3 a^2 b^{13} d^2 + 44A^3 a^4 b^{11} d^2 - 56A^3 a^6 b^9 d^2 - 92B^3 a^3 b^{12} d^2 - 84B^3 a^5 b^{10} d^2 + 12B^3 a^7 b^8 d^2 + 4B^3 a^9 b^6 d^2 - 92A^2 B a^2 b^{14} d^2 - 216A^2 B^2 a^2 b^{13} d^2 - 48A^2 B^2 a^4 b^{11} d^2 + 168A^2 B^2 a^6 b^9 d^2 + 208A^2 B^2 a^8 b^{12} d^2 + 264A^2 B^2 a^{10} b^{10} d^2 - 36A^2 B^2 a^{12} b^8 d^2) \right)}{d^5} - (a^{1/2}) (3A^3 b + 2B^3 a) \left( (16(a + b \tan(c + d x))^{1/2} (92A^2 a^3 b^{10} d^2 - 20A^2 a^5 b^8 d^2 - 56B^2 a^3 b^{10} d^2 + 36B^2 a^5 b^8 d^2 - 32A^2 B b^{13} d^2 + 44A^2 a^2 b^{12} d^2 - 44B^2 a^2 b^{12} d^2 + 32A^2 B a^2 b^{11} d^2 + 176A^2 B a^4 b^9 d^2) \right)}{d^4} + (a^{1/2}) (3A^3 b + 2B^3 a) \left( (8(80A^2 a^2 b^{11} d^4 + 80A^2 a^3 b^9 d^4 + 48B^2 a^2 b^{10} d^4 + 48B^2 a^4 b^8 d^4) \right)}{d^5} - (8a^{1/2}) (3A^3 b + 2B^3 a) (32b^{10} d^4 + 48a^2 b^8 d^4) (a + b \tan(c + d x))^{1/2} / d^5 \right) / (2d) \right) / (2d) \right) / (2d) (3A^3 b + 2B^3 a) * i) / (2d) + (a^{1/2}) \left( (16(a + b \tan(c + d x))^{1/2} (2A^4 b^{16} + 2B^4 b^{16} + 4A^2 B^2 b^{16} - A^4 a^2 b^{14} + 66A^4 a^4 b^{12} - A^4 a^6 b^{10} + 2A^4 a^8 b^8 + 8B^4 a^2 b^{14} + 16B^4 a^4 b^{12} - 16B^4 a^6 b^{10} + 6B^4 a^8 b^8 + 25A^2 B^2 a^2 b^{14} - 130A^2 B^2 a^4 b^{12} + 145A^2 B^2 a^6 b^{10} + 12A^2 B^3 a^3 b^{13} - 104A^2 B^3 a^5 b^{11} + 44A^2 B^3 a^7 b^9 - 84A^3 B a^3 b^{13} + 144A^3 B a^5 b^{11} - 12A^3 B a^7 b^9) \right)}{d^4} - (a^{1/2}) (3A^3 b + 2B^3 a) \left( (8(100A^3 a^2 b^{13} d^2 + 44A^3 a^4 b^{11} d^2 - 56A^3 a^6 b^9 d^2 - 92B^3 a^3 b^{12} d^2 - 84B^3 a^5 b^{10} d^2 + 12B^3 a^7 b^8 d^2 + 4B^3 a^9 b^6 d^2 - 92A^2 B a^2 b^{14} d^2 - 216A^2 B^2 a^2 b^{13} d^2 - 48A^2 B^2 a^4 b^{11} d^2 + 168A^2 B^2 a^6 b^9 d^2 + 208A^2 B^2 a^8 b^{12} d^2 + 264A^2 B^2 a^{10} b^{10} d^2 - 36A^2 B^2 a^{12} b^8 d^2) \right)}{d^5} + (a^{1/2}) (3A^3 b + 2B^3 a) \left( (16(a + b \tan(c + d x))^{1/2} (92A^2 a^3 b^{10} d^2 - 20A^2 a^5 b^8 d^2 - 56B^2 a^3 b^{10} d^2 + 36B^2 a^5 b^8 d^2 - 32A^2 B b^{13} d^2 + 44A^2 a^2 b^{12} d^2 - 44B^2 a^2 b^{12} d^2 + 32A^2 B a^2 b^{11} d^2 + 176A^2 B a^4 b^9 d^2) \right)}{d^4} - (a^{1/2}) (3A^3 b + 2B^3 a) \left( (8(80A^2 a^2 b^{11} d^4 + 80A^2 a^3 b^9 d^4 + 48B^2 a^2 b^{10} d^4 + 48B^2 a^4 b^8 d^4) \right)}{d^5} + (8a^{1/2}) (3A^3 b + 2B^3 a) (32b^{10} d^4 + 48a^2 b^8 d^4) \right) / d^5 \right)$



$$\begin{aligned}
& * (a + b \tan(c + d*x))^{(1/2)} / d^5) / (2*d)) / (2*d)) / (2*d)) * (3*A*b + 2*B*a) * 1 \\
& i) / (2*d)) / ((16*(6*A^5*a*b^17 + 6*A^5*a^3*b^15 + 6*A^5*a^7*b^11 + 6*A^5*a^9* \\
& b^9 + 4*B^5*a^2*b^16 + 20*B^5*a^4*b^14 + 28*B^5*a^6*b^12 + 12*B^5*a^8*b^10 \\
& + 17*A^2*B^3*a^2*b^16 + 21*A^2*B^3*a^4*b^14 - 5*A^2*B^3*a^6*b^12 - 5*A^2*B^ \\
& 3*a^8*b^10 + 4*A^2*B^3*a^10*b^8 + 42*A^3*B^2*a^3*b^15 + 40*A^3*B^2*a^5*b^13 \\
& + 2*A^3*B^2*a^7*b^11 - 8*A^3*B^2*a^9*b^9 + 6*A*B^4*a*b^17 + 36*A*B^4*a^3*b \\
& ^15 + 40*A*B^4*a^5*b^13 - 4*A*B^4*a^7*b^11 - 14*A*B^4*a^9*b^9 + 12*A^3*B^2* \\
& a*b^17 + 13*A^4*B*a^2*b^16 + A^4*B*a^4*b^14 - 33*A^4*B*a^6*b^12 - 17*A^4*B* \\
& a^8*b^10 + 4*A^4*B*a^10*b^8) / d^5 - (a^{(1/2)} * ((16*(a + b \tan(c + d*x))^{(1/2)} \\
& ) * (2*A^4*b^16 + 2*B^4*b^16 + 4*A^2*B^2*b^16 - A^4*a^2*b^14 + 66*A^4*a^4*b^1 \\
& 2 - A^4*a^6*b^10 + 2*A^4*a^8*b^8 + 8*B^4*a^2*b^14 + 16*B^4*a^4*b^12 - 16*B^ \\
& 4*a^6*b^10 + 6*B^4*a^8*b^8 + 25*A^2*B^2*a^2*b^14 - 130*A^2*B^2*a^4*b^12 + 1 \\
& 45*A^2*B^2*a^6*b^10 + 12*A*B^3*a^3*b^13 - 104*A*B^3*a^5*b^11 + 44*A*B^3*a^7 \\
& *b^9 - 84*A^3*B*a^3*b^13 + 144*A^3*B*a^5*b^11 - 12*A^3*B*a^7*b^9)) / d^4 + (a \\
& ^{(1/2)} * (3*A*b + 2*B*a) * ((8*(100*A^3*a^2*b^13*d^2 + 44*A^3*a^4*b^11*d^2 - 56 \\
& *A^3*a^6*b^9*d^2 - 92*B^3*a^3*b^12*d^2 - 84*B^3*a^5*b^10*d^2 + 12*B^3*a^7*b \\
& ^8*d^2 + 4*B^3*a*b^14*d^2 - 92*A^2*B*a*b^14*d^2 - 216*A*B^2*a^2*b^13*d^2 - \\
& 48*A*B^2*a^4*b^11*d^2 + 168*A*B^2*a^6*b^9*d^2 + 208*A^2*B*a^3*b^12*d^2 + 26 \\
& 4*A^2*B*a^5*b^10*d^2 - 36*A^2*B*a^7*b^8*d^2)) / d^5 - (a^{(1/2)} * (3*A*b + 2*B*a \\
& ) * ((16*(a + b \tan(c + d*x))^{(1/2)} * (92*A^2*a^3*b^10*d^2 - 20*A^2*a^5*b^8*d^2 \\
& - 56*B^2*a^3*b^10*d^2 + 36*B^2*a^5*b^8*d^2 - 32*A*B*b^13*d^2 + 44*A^2*a*b^ \\
& 12*d^2 - 44*B^2*a*b^12*d^2 + 32*A*B*a^2*b^11*d^2 + 176*A*B*a^4*b^9*d^2)) / d^ \\
& 4 + (a^{(1/2)} * (3*A*b + 2*B*a) * ((8*(80*A*a*b^11*d^4 + 80*A*a^3*b^9*d^4 + 48*B \\
& *a^2*b^10*d^4 + 48*B*a^4*b^8*d^4)) / d^5 - (8*a^{(1/2)} * (3*A*b + 2*B*a) * (32*b^1 \\
& 0*d^4 + 48*a^2*b^8*d^4) * (a + b \tan(c + d*x))^{(1/2)} / d^5) / (2*d)) / (2*d)) / ( \\
& 2*d)) * (3*A*b + 2*B*a)) / (2*d) + (a^{(1/2)} * ((16*(a + b \tan(c + d*x))^{(1/2)} * (2* \\
& A^4*b^16 + 2*B^4*b^16 + 4*A^2*B^2*b^16 - A^4*a^2*b^14 + 66*A^4*a^4*b^12 - A \\
& ^4*a^6*b^10 + 2*A^4*a^8*b^8 + 8*B^4*a^2*b^14 + 16*B^4*a^4*b^12 - 16*B^4*a^6 \\
& *b^10 + 6*B^4*a^8*b^8 + 25*A^2*B^2*a^2*b^14 - 130*A^2*B^2*a^4*b^12 + 145*A^ \\
& 2*B^2*a^6*b^10 + 12*A*B^3*a^3*b^13 - 104*A*B^3*a^5*b^11 + 44*A*B^3*a^7*b^9 \\
& - 84*A^3*B*a^3*b^13 + 144*A^3*B*a^5*b^11 - 12*A^3*B*a^7*b^9)) / d^4 - (a^{(1/2)} \\
& ) * (3*A*b + 2*B*a) * ((8*(100*A^3*a^2*b^13*d^2 + 44*A^3*a^4*b^11*d^2 - 56*A^3* \\
& a^6*b^9*d^2 - 92*B^3*a^3*b^12*d^2 - 84*B^3*a^5*b^10*d^2 + 12*B^3*a^7*b^8*d^ \\
& 2 + 4*B^3*a*b^14*d^2 - 92*A^2*B*a*b^14*d^2 - 21...
\end{aligned}$$

$$3.330 \quad \int \cot^3(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=219

$$\frac{(8a^2A - 3Ab^2 - 12abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d} - \frac{(a-ib)^{3/2}(A-ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}$$

[Out]  $-(a-I*b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d-(a+I*b)^{(3/2)}*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+1/4*(8*A*a^2-3*A*b^2-12*B*a*b)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d/a^{(1/2)}-1/4*(5*A*b+4*B*a)*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1/2)}/d-1/2*a*A*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.63, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3686, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2A - 12abB - 3Ab^2) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d} - \frac{(a-ib)^{3/2}(A-ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(A+ib) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b \tan(c+dx)}}{4d} - \frac{aA\cot^2(c+dx)\sqrt{a+b \tan(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^3*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $((8*a^2*A - 3*A*b^2 - 12*a*b*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(4*\operatorname{Sqrt}[a]*d) - ((a-I*b)^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/d - ((a+I*b)^{(3/2)}*(A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/d - ((5*A*b+4*a*B)*\operatorname{Cot}[c+d*x]*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(4*d) - (a*A*\operatorname{Cot}[c+d*x]^2*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^(m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^(m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n \*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \cot^3(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{2d} + \frac{1}{2} \int \cot^2(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx \\
 &= -\frac{(5Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= -\frac{(5Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= -\frac{(5Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= -\frac{(5Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= \frac{(8a^2A - 3Ab^2 - 12abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d} \\
 &= \frac{(8a^2A - 3Ab^2 - 12abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d}
 \end{aligned}$$

### Mathematica [A]

time = 1.67, size = 195, normalized size = 0.89

$$\frac{(8a^2A - 3Ab^2 - 12abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right) - \sqrt{a} \left(4(a - ib)^{3/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) + 4(a + ib)^{3/2}(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) + \cot(c + dx)(5Ab + 4aB + 2aA \cot(c + dx)) \sqrt{a + b \tan(c + dx)}\right)}{4\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((8*a^2*A - 3*A*b^2 - 12*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] -
Sqrt[a]*(4*(a - I*b)^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt
[a - I*b]] + 4*(a + I*b)^(3/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/S
qrt[a + I*b]] + Cot[c + d*x]*(5*A*b + 4*a*B + 2*a*A*Cot[c + d*x])*Sqrt[a +
b*Tan[c + d*x]))/(4*Sqrt[a]*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 2.18, size = 102706, normalized size = 468.98

method	result	size
default	Expression too large to display	102706

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x
)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```



$$\begin{aligned}
& b^6 + 2A^2B^2a^6 + 2A^2B^2b^6 + 3A^4a^2b^4 + 3A^4a^4b^2 + 3B^4a^2b^4 + 3B^4a^4b^2 + 6A^2B^2a^2b^4 + 6A^2B^2a^4b^2)^{(1/2)} + \\
& A^2a^3d^2 - B^2a^3d^2 + 2A*B*b^3d^2 - 3A^2a*b^2d^2 + 3B^2a*b^2d^2 - 6A*B*a^2b*d^2)/(4d^4))^{(1/2)} + ((a + b*\tan(c + d*x))^{(1/2)}*(1088A^2a^3b^{10}d^2 - 576A^2a^5b^8d^2 - 1472B^2a^3b^{10}d^2 + 320B^2a^5b^8d^2 - 512A*B*b^{13}d^2 + 668A^2a*b^{12}d^2 - 704B^2a*b^{12}d^2 + 224A*B*a^2b^{11}d^2 + 2816A*B*a^4b^9d^2))/d^4)*(((8A^2a^3d^2 - 8B^2a^3d^2 + 16A*B*b^3d^2 - 24A^2a*b^2d^2 + 24B^2a*b^2d^2 - 48A*B*a^2b*d^2)^2/64 - d^4*(A^4a^6 + A^4b^6 + B^4a^6 + B^4b^6 + 2A^2B^2a^6 + 2A^2B^2b^6 + 3A^4a^2b^4 + 3A^4a^4b^2 + 3B^4a^2b^4 + 3B^4a^4b^2 + 6A^2B^2a^2b^4 + 6A^2B^2a^4b^2))^{(1/2)} + A^2a^3d^2 - B^2a^3d^2 + 2A*B*b^3d^2 - 3A^2a*b^2d^2 + 3B^2a*b^2d^2 - 6A*B*a^2b*d^2)/(4d^4))^{(1/2)})*(((8A^2a^3d^2 - 8B^2a^3d^2 + 16A*B*b^3d^2 - 24A^2a*b^2d^2 + 24B^2a*b^2d^2 - 48A*B*a^2b*d^2)^2/64 - d^4*(A^4a^6 + A^4b^6 + B^4a^6 + B^4b^6 + 2A^2B^2a^6 + 2A^2B^2b^6 + 3A^4a^2b^4 + 3A^4a^4b^2 + 3B^4a^2b^4 + 3B^4a^4b^2 + 6A^2B^2a^2b^4 + 6A^2B^2a^4b^2))^{(1/2)} + A^2a^3d^2 - B^2a^3d^2 + 2A*B*b^3d^2 - 3A^2a*b^2d^2 + 3B^2a*b^2d^2 - 6A*B*a^2b*d^2)/(4d^4))^{(1/2)} - ((a + b*\tan(c + d*x))^{(1/2)}*(41A^4b^{16} + 32B^4b^{16} + 55A^2B^2b^{16} + 26A^4a^2b^{14} + 553A^4a^4b^{12} - 304A^4a^6b^{10} + 96A^4a^8b^8 - 16B^4a^2b^{14} + 1056B^4a^4b^{12} - 16B^4a^6b^{10} + 32B^4a^8b^8 + 1078A^2B^2a^2b^{14} - 2953A^2B^2a^4b^{12} + 2368A^2B^2a^6b^{10} - 72A*B^3a^3b^{15} + 144A^3B^3a^3b^{15} + 1776A*B^3a^3b^{13} - 2376A*B^3a^5b^{11} + 192A*B^3a^7b^9 - 1080A^3B^3a^3b^{13} + 2120A^3B^3a^5b^{11} - 704A^3B^3a^7b^9))/d^4)*(((8A^2a^3d^2 - 8B^2a^3d^2 + 16A*B*b^3d^2 - 24A^2a*b^2d^2 + 24B^2a*b^2d^2 - 48A*B*a^2b*d^2)^2/64 - d^4*(A^4a^6 + A^4b^6 + B^4a^6 + B^4b^6 + 2A^2B^2a^6 + 2A^2B^2b^6 + 3A^4a^2b^4 + 3A^4a^4b^2 + 3B^4a^2b^4 + 3B^4a^4b^2 + 6A^2B^2a^2b^4 + 6A^2B^2a^4b^2))^{(1/2)} + A^2a^3d^2 - B^2a^3d^2 + 2A*B*b^3d^2 - 3A^2a*b^2d^2 + 3B^2a*b^2d^2 - 6A*B*a^2b*d^2)/(4d^4))^{(1/2)}*1i - (((932A^3a^3b^{12}d^2 + 1344A^3a^5b^{10}d^2 - 192A^3a^7b^8d^2 + 1600B^3a^2b^{13}d^2 + 704B^3a^4b^{11}d^2 - 896B^3a^6b^9d^2 + 348A^2B^3b^{15}d^2 - 604A^3a^3b^{14}d^2 + 1760A*B^2a^3b^{14}d^2 - 3232A*B^2a^5b^{12}d^2 - 4416A*B^2a^7b^{10}d^2 + 576A*B^2a^9b^8d^2 - 3780A^2B^2a^2b^{13}d^2 - 1440A^2B^2a^4b^{11}d^2 + 2688A^2B^2a^6b^9d^2)/(2d^5) - (((384A*b^{12}d^4 + 1280B*a^3b^{11}d^4 - 384A*a^2b^{10}d^4 - 768A*a^4b^8d^4 + 1280B*a^3b^9d^4)/(2d^5) - (512b^{10}d^4 + 768a^2b^8d^4)*(a + b*\tan(c + d*x))^{(1/2)})*(((8A^2a^3d^2 - 8B^2a^3d^2 + 16A*B*b^3d^2 - 24A^2a*b^2d^2 + 24B^2a*b^2d^2 - 48A*B*a^2b*d^2)^2/64 - d^4*(A^4a^6 + A^4b^6 + B^4a^6 + B^4b^6 + 2A^2B^2a^6 + 2A^2B^2b^6 + 3A^4a^2b^4 + 3A^4a^4b^2 + 3B^4a^2b^4 + 3B^4a^4b^2 + 6A^2B^2a^2b^4 + 6A^2B^2a^4b^2))^{(1/2)} + A^2a^3d^2 - B^2a^3d^2 + 2A*B*b^3d^2 - 3A^2a*b^2d^2 + 3B^2a*b^2d^2 - 6A*B*a^2b*d^2)/(4d^4))^{(1/2)}/d^4)*(((8A^2a^3d^2 - 8B^2a^3d^2 + 16A*B*b^3d^2 - 24A^2a*b^2d^2 + 24B^2a*b^2d^2 - 48A*B*a^2b*d^2)^2/64 - d^4*(A^4a^6 + A^4b^6 + B^4a^6 + B^4b^6 + 2A^2B^2a^6 + 2A^2B^2b^6 +
\end{aligned}$$

$$\begin{aligned}
& 3A^4a^2b^4 + 3A^4a^4b^2 + 3B^4a^2b^4 + 3B^4a^4b^2 + 6A^2B^2a^2b^4 + 6A^2B^2a^4b^2))^{(1/2)} + A^2a^3d^2 - B^2a^3d^2 + 2ABb^3d^2 - 3A^2ab^2d^2 + 3B^2ab^2d^2 - 6ABa^2bd^2)/(4d^4))^{(1/2)} \\
& - ((a + b\tan(c + dx))^{(1/2)}(1088A^2a^3b^{10}d^2 - 576A^2a^5b^8d^2 - 1472B^2a^3b^{10}d^2 + 320B^2a^5b^8d^2 - 512ABb^{13}d^2 + 668A^2a^2b^{12}d^2 - 704B^2a^2b^{12}d^2 + 224ABa^2b^{11}d^2 + 2816ABa^4b^9d^2))/d^4)*(((8A^2a^3d^2 - 8B^2a^3d^2 + 1\dots
\end{aligned}$$



### 3.331 $\int \cot^4(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=278

$$\frac{(24a^2Ab + Ab^3 + 16a^3B - 6ab^2B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{(a-ib)^{3/2}(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d}$$

[Out]  $1/8*(24*A*a^2*b+A*b^3+16*B*a^3-6*B*a*b^2)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/a^{1/2})/a^{3/2}/d-(a-I*b)^{3/2}*(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/(a-I*b)^{1/2})/d+(a+I*b)^{3/2}*(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/(a+I*b)^{1/2})/d+1/8*(8*A*a^2-A*b^2-10*B*a*b)*\cot(d*x+c)*(a+b*\tan(d*x+c))^{1/2}/a/d-1/12*(7*A*b+6*B*a)*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{1/2}/d-1/3*a*A*\cot(d*x+c)^3*(a+b*\tan(d*x+c))^{1/2}/d$

**Rubi [A]**

time = 0.85, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3686, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2A - 10abB - Ab^2) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{8ad} + \frac{(16a^2B + 24a^2Ab - 6ab^2B + Ab^3) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{(a-ib)^{3/2}(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{3/2}(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{(6aB+7Ab) \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{12d} - \frac{aA \cot^3(c+dx) \sqrt{a+b \tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^4*(a + b*\operatorname{Tan}[c + d*x])^{3/2}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((24*a^2*A*b + A*b^3 + 16*a^3*B - 6*a*b^2*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(8*a^{3/2}*d) - ((a - I*b)^{3/2}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d + ((a + I*b)^{3/2}*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d + ((8*a^2*A - A*b^2 - 10*a*b*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(8*a*d) - ((7*A*b + 6*a*B)*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(12*d) - (a*A*\operatorname{Cot}[c + d*x]^3*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(3*d)$

**Rule 65**

$\operatorname{Int}[(c_.) + (b_.)*(x_.)^{(m_)}*((c_.) + (d_.)*(x_.)^{(n_)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b})^n), x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(c_.) + (b_.)*(x_.)^{(n_)}]^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3715

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

Rule 3730

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan

$[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& !(\text{ILtQ}[n, -1] \&\& ( !\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3734

$\text{Int}[\frac{((c_.) + (d_.)\tan[(e_.) + (f_.)*(x_)])^{(n_)}*((A_.) + (B_.)\tan[(e_.) + (f_.)*(x_)] + (C_.)\tan[(e_.) + (f_.)*(x_)]^2)}{((a_.) + (b_.)\tan[(e_.) + (f_.)*(x_)])}, x\_Symbol] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x], x] + \text{Dist}[\frac{A*b^2 - a*b*B + a^2*C}{(a^2 + b^2)}, \text{Int}[(c + d*\text{Tan}[e + f*x])^n * ((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0] \&\& !\text{LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned} \int \cot^4(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^3(c + dx) \sqrt{a + b \tan(c + dx)}}{3d} + \frac{1}{3} \\ &= -\frac{(7Ab + 6aB) \cot^2(c + dx) \sqrt{a + b \tan(c + dx)}}{12d} \\ &= \frac{(8a^2A - Ab^2 - 10abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8ad} \\ &= \frac{(8a^2A - Ab^2 - 10abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8ad} \\ &= \frac{(8a^2A - Ab^2 - 10abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8ad} \\ &= \frac{(8a^2A - Ab^2 - 10abB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{8ad} \\ &= \frac{(24a^2Ab + Ab^3 + 16a^3B - 6ab^2B) \tanh^{-1}(\cot(c + dx) \sqrt{a + b \tan(c + dx)})}{8a^{3/2}d} \\ &= \frac{(24a^2Ab + Ab^3 + 16a^3B - 6ab^2B) \tanh^{-1}(\cot(c + dx) \sqrt{a + b \tan(c + dx)})}{8a^{3/2}d} \end{aligned}$$

**Mathematica [A]**

time = 3.71, size = 241, normalized size = 0.87

$$\frac{3(24a^2Ab + Ab^3 + 16a^2B - 6a^2B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{a}\left(-24a(a-b)^{3/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-b}}\right) + 24a(a+ib)^{3/2}(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) - \cot(c+dx)(-24a^2A + 3Ab^2 + 30abB + 2a(7Ab + 6aB)\cot(c+dx) + 8a^2A\cot^2(c+dx))\sqrt{a+b\tan(c+dx)}\right)}{24a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
[Out] (3*(24*a^2*A*b + A*b^3 + 16*a^3*B - 6*a*b^2*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + Sqrt[a]*((-24*I)*a*(a - I*b)^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (24*I)*a*(a + I*b)^(3/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - Cot[c + d*x]*(-24*a^2*A + 3*A*b^2 + 30*a*b*B + 2*a*(7*A*b + 6*a*B)*Cot[c + d*x] + 8*a^2*A*Cot[c + d*x]^2)*Sqrt[a + b*Tan[c + d*x]))/(24*a^(3/2)*d)
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 2.79, size = 145176, normalized size = 522.22

method	result	size
default	Expression too large to display	145176

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

[Out] result too large to display

**Maxima** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

**Fricas** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \cot^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*4\*(a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)\*cot(c + d\*x)\*\*4, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^4\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 9.05, size = 2500, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^4\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out] atan(((((((256\*A\*a\*b^13\*d^4 + 5376\*A\*a^3\*b^11\*d^4 + 5120\*A\*a^5\*b^9\*d^4 - 1536\*B\*a^2\*b^12\*d^4 + 1536\*B\*a^4\*b^10\*d^4 + 3072\*B\*a^6\*b^8\*d^4)/(8\*a^2\*d^5) - ((2048\*a^2\*b^10\*d^4 + 3072\*a^4\*b^8\*d^4)\*(a + b\*tan(c + d\*x))^(1/2))\*(-(8\*A^2\*a^3\*d^2 - 8\*B^2\*a^3\*d^2 + 16\*A\*B\*b^3\*d^2 - 24\*A^2\*a\*b^2\*d^2 + 24\*B^2\*a\*b^2\*d^2 - 48\*A\*B\*a^2\*b\*d^2)^2/64 - d^4\*(A^4\*a^6 + A^4\*b^6 + B^4\*a^6 + B^4\*b^6 + 2\*A^2\*B^2\*a^6 + 2\*A^2\*B^2\*b^6 + 3\*A^4\*a^2\*b^4 + 3\*A^4\*a^4\*b^2 + 3\*B^4\*a^2\*b^4 + 3\*B^4\*a^4\*b^2 + 6\*A^2\*B^2\*a^2\*b^4 + 6\*A^2\*B^2\*a^4\*b^2))))^(1/2) + A^2\*a^3\*d^2 - B^2\*a^3\*d^2 + 2\*A\*B\*b^3\*d^2 - 3\*A^2\*a\*b^2\*d^2 + 3\*B^2\*a\*b^2\*d^2 - 6\*A\*B\*a^2\*b\*d^2)/(4\*d^4))^(1/2))/(4\*a^2\*d^4))\*(-(8\*A^2\*a^3\*d^2 - 8\*B^2\*a^3\*d^2 + 16\*A\*B\*b^3\*d^2 - 24\*A^2\*a\*b^2\*d^2 + 24\*B^2\*a\*b^2\*d^2 - 48\*A\*B\*a^2\*b\*d^2)^2/64 - d^4\*(A^4\*a^6 + A^4\*b^6 + B^4\*a^6 + B^4\*b^6 + 2\*A^2\*B^2\*a^6 + 2\*A^2\*B^2\*b^6 + 3\*A^4\*a^2\*b^4 + 3\*A^4\*a^4\*b^2 + 3\*B^4\*a^2\*b^4 + 3\*B^4\*a^4\*b^2 + 6\*A^2\*B^2\*a^2\*b^4 + 6\*A^2\*B^2\*a^4\*b^2))))^(1/2) + A^2\*a^3\*d^2 - B^2\*a



$$\begin{aligned}
& d^2 - 48* A * B * a^2 * b * d^2)^2 / 64 - d^4 * (A^4 * a^6 + A^4 * b^6 + B^4 * a^6 + B^4 * b^6 + \\
& 2 * A^2 * B^2 * a^6 + 2 * A^2 * B^2 * b^6 + 3 * A^4 * a^2 * b^4 + 3 * A^4 * a^4 * b^2 + 3 * B^4 * a^2 * \\
& b^4 + 3 * B^4 * a^4 * b^2 + 6 * A^2 * B^2 * a^2 * b^4 + 6 * A^2 * B^2 * a^4 * b^2))^{(1/2)} + A^2 * a \\
& ^3 * d^2 - B^2 * a^3 * d^2 + 2 * A * B * b^3 * d^2 - 3 * A^2 * a * b^2 * d^2 + 3 * B^2 * a * b^2 * d^2 - \\
& 6 * A * B * a^2 * b * d^2) / (4 * d^4)^{(1/2)} - ((a + b * \tan(c + d * x))^{(1/2)} * (3008 * A^2 * a^3 \\
& * b^{12} * d^2 + 5888 * A^2 * a^5 * b^{10} * d^2 - 1280 * A^2 * a^7 * b^8 * d^2 - 2672 * B^2 * a^3 * b^{11} \\
& * d^2 - 4352 * B^2 * a^5 * b^{10} * d^2 + 2304 * B^2 * a^7 * b^8 * d^2 + 4 * A^2 * a * b^{14} * d^2 - 2 \\
& 096 * A * B * a^2 * b^{13} * d^2 + 1024 * A * B * a^4 * b^{11} * d^2 + 11264 * A * B * a^6 * b^9 * d^2)) / (4 * a \\
& ^2 * d^4)) * (-((8 * A^2 * a^3 * d^2 - 8 * B^2 * a^3 * d^2 + 1 \dots
\end{aligned}$$

$$3.332 \quad \int \tan^2(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=252

$$\frac{(a-ib)^{5/2}(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(iA-B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

[Out] (a-I\*b)^(5/2)\*(I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d-(a+I\*b)^(5/2)\*(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d-2\*(2\*A\*a\*b+B\*a^2-B\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/3\*(A\*b+B\*a)\*(a+b\*tan(d\*x+c))^(3/2)/d-2/5\*B\*(a+b\*tan(d\*x+c))^(5/2)/d+2/63\*(9\*A\*b-2\*B\*a)\*(a+b\*tan(d\*x+c))^(7/2)/b^2/d+2/9\*B\*tan(d\*x+c)\*(a+b\*tan(d\*x+c))^(7/2)/b/d

Rubi [A]

time = 0.46, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3688, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(a^2B+2aAb-B^2B)\sqrt{a+b \tan(c+dx)}}{d} + \frac{2(0Ab-2aB)(a+b \tan(c+dx))^{7/2}}{63B^2d} - \frac{2(aB+Ab)(a+b \tan(c+dx))^{5/2}}{3d} + \frac{(a-ib)^{5/2}(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2B \tan(c+dx)(a+b \tan(c+dx))^{7/2}}{9bd} - \frac{2B(a+b \tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a - I\*b)^(5/2)\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/d - ((a + I\*b)^(5/2)\*(I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/d - (2\*(2\*a\*A\*b + a^2\*B - b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*(A\*b + a\*B)\*(a + b\*Tan[c + d\*x])^(3/2))/(3\*d) - (2\*B\*(a + b\*Tan[c + d\*x])^(5/2))/(5\*d) + (2\*(9\*A\*b - 2\*a\*B)\*(a + b\*Tan[c + d\*x])^(7/2))/(63\*b^2\*d) + (2\*B\*Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^(7/2))/(9\*b\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]



Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/d/b^2*(1/9*B*(a+b*tan(d*x+c))^(9/2)+1/7*A*b*(a+b*tan(d*x+c))^(7/2)-1/7*B*
a*(a+b*tan(d*x+c))^(7/2)-1/5*B*b^2*(a+b*tan(d*x+c))^(5/2)-1/3*A*b^3*(a+b*ta
n(d*x+c))^(3/2)-1/3*B*a*b^2*(a+b*tan(d*x+c))^(3/2)-2*A*a*b^3*(a+b*tan(d*x+c
))^(1/2)-B*a^2*b^2*(a+b*tan(d*x+c))^(1/2)+B*b^4*(a+b*tan(d*x+c))^(1/2)+b^2*
(1/4/b*(1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-A*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a
^3+3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2-2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*(a^2+b^2)^(1/2)*a*b+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b-B*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*b^3)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(4*A*b^2*(a^2+b^2)^(1/2)*a+2*B*a^2*(
a^2+b^2)^(1/2)*b-2*B*b^3*(a^2+b^2)^(1/2)-1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*(a^2+b^2)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-
A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2
-2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b+3*B*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*a^2*b-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b*(-1/2
*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3*A*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2-2*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)
^(1/2)*a*b+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b-B*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*b^3)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-4*A*b^2*(a^2+b^2)^(1/2)*a-2*B*a^2*(a^2+b^2)^(
1/2)*b+2*B*b^3*(a^2+b^2)^(1/2)+1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a^2-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^2-A*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^3+3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2-2*B*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*a^2*b-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}} \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(5/2)\*tan(c + d\*x)\*\*2, x)

**Giac [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**  
time = 168.24, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] log(((8\*B^3\*a\*b^2\*(a^2 - 3\*b^2)\*(a^2 + b^2)^3)/d^3 - (((((-B^4\*b^2\*d^4\*(5\*a^4 + b^4 - 10\*a^2\*b^2)^2)^(1/2) + B^2\*a^5\*d^2 - 10\*B^2\*a^3\*b^2\*d^2 + 5\*B^2\*a\*b^4\*d^2)/d^4)^(1/2)\*(32\*B\*a^4\*b^2 - 32\*B\*b^6 + 32\*a\*b^2\*d\*((( -B^4\*b^2\*d^4\*(5\*a^4 + b^4 - 10\*a^2\*b^2)^2)^(1/2) + B^2\*a^5\*d^2 - 10\*B^2\*a^3\*b^2\*d^2 + 5

$$\begin{aligned}
& *B^2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)})/(2*d) - (16*B^2*b^2 \\
& *(a + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((( \\
& -B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3 \\
& *b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)})/2)*((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^{(1/2)} \\
& / (4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d^2))^{(1/2)} - (2*a*(2*a*((2*B*(a^2 + b^2))/(b^2*d) - (2*B*a^2)/(b^2*d)) + (2 \\
& *B*a^3)/(b^2*d) - (2*B*a*(a^2 + b^2))/(b^2*d)) - ((2*B*(a^2 + b^2))/(b^2*d) \\
& - (2*B*a^2)/(b^2*d))*(a^2 + b^2))*(a + b*\tan(c + d*x))^{(1/2)} - \log((((((- \\
& B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3* \\
& b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(32*B*b^6 - 32*B*a^4*b^2 + 32*a*b^2*d \\
& *((( -B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2 \\
& *a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/ (2* \\
& d) - (16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4 \\
& *b^2))/d^2)*((( -B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 \\
& - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)})/2 + (8*B^3*a*b^2*(a \\
& ^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110* \\
& B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^{(1/2)} + B^2*a^5 \\
& *d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/(4*d^4))^{(1/2)} - ((2*B*(a^2 + \\
& b^2))/(5*b^2*d) - (2*B*a^2)/(5*b^2*d))*(a + b*\tan(c + d*x))^{(5/2)} - \log(((( \\
& (-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2 \\
& *a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(32*B*b^6 - 32*B*a^4*b^2 + 32*a* \\
& b^2*d*(-((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + \\
& 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)} \\
& )))/(2*d) - (16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15*a^2*b^4 - \\
& 15*a^4*b^2))/d^2)*((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B \\
& ^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)})/2 + (8*B^3*a \\
& *b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*((-((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 \\
& - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^{(1/2)} - \\
& B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/(4*d^4))^{(1/2)} + \log(( \\
& 8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((-((-B^4*b^2*d^4*(5*a^4 + \\
& b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^ \\
& 4*d^2)/d^4)^{(1/2)}*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(-((-B^4*b^2*d^4*(5 \\
& *a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^ \\
& 2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/ (2*d) - (16*B^2*b^2*(a \\
& + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((-B \\
& ^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b \\
& ^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)})/2)*((B^2*a^5)/(4*d^2) - (20*B^4*a^2*b \\
& ^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4* \\
& a^8*b^2*d^4)^{(1/2)}/(4*d^4) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d^2 \\
& ))^{(1/2)} - (a + b*\tan(c + d*x))^{(3/2)}*((2*a*((2*B*(a^2 + b^2))/(b^2*d) - (2 \\
& *B*a^2)/(b^2*d)))/3 + (2*B*a^3)/(3*b^2*d) - (2*B*a*(a^2 + b^2))/(3*b^2*d)) \\
& - \log((8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((-A^4*b^2*d^4*(5*a^ \\
& 4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a \\
& *b^4*d^2)/d^4)^{(1/2)}*(((-((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& + A^2 a^5 d^2 - 10 A^2 a^3 b^2 d^2 + 5 A^2 a b^4 d^2 / d^4)^{(1/2)} * (64 A a^3 \\
& * b^3 + 64 A a b^5 - 32 a b^2 d * (-((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} + A^2 a^5 d^2 - 10 A^2 a^3 b^2 d^2 + 5 A^2 a b^4 d^2) / d^4)^{(1/2)} * ( \\
& a + b \tan(c + d x))^{(1/2)})) / (2 d) - (16 A^2 b^2 (a + b \tan(c + d x))^{(1/2)} * \\
& (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2) / d^2) / 2 * (-((20 A^4 a^2 b^8 d^4 - A^4 \\
& 4 b^{10} d^4 - 110 A^4 a^4 b^6 d^4 + 100 A^4 a^6 b^4 d^4 - 25 A^4 a^8 b^2 d^4)^{(1/2)} + A^2 a^5 d^2 - 10 A^2 a^3 b^2 d^2 + 5 A^2 a b^4 d^2) / (4 d^4))^{(1/2)} \\
& ) - \log((8 A^3 b^3 (3 a^2 - b^2) (a^2 + b^2)^3) / d^3 - ((((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a \\
& a b^4 d^2) / d^4)^{(1/2)} * (((((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} \\
& - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2) / d^4)^{(1/2)} * (64 A a^3 \\
& * b^3 + 64 A a b^5 - 32 a b^2 d * (((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2) / d^4)^{(1/2)} * (a \\
& + b \tan(c + d x))^{(1/2)})) / (2 d) - (16 A^2 b^2 (a + b \tan(c + d x))^{(1/2)} * ( \\
& a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2) / d^2) / 2 * (((20 A^4 a^2 b^8 d^4 - A^4 b^{10} d^4 - 110 A^4 a^4 b^6 d^4 + 100 A^4 a^6 b^4 d^4 - 25 A^4 a^8 b^2 d^4)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2) / (4 d^4))^{(1/2)} \\
& + \log((8 A^3 b^3 (3 a^2 - b^2) (a^2 + b^2)^3) / d \dots
\end{aligned}$$

### 3.333 $\int \tan(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=213

$$\frac{(a - ib)^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{5/2}(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d}$$

[Out]  $-(a-I*b)^{(5/2)*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d-(a+I*b)^{(5/2)*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}/d+2*(A*a^2-A*b^2-2*B*a*b)*(a+b*\tan(d*x+c))^{(1/2)}/d+2/3*(A*a-B*b)*(a+b*\tan(d*x+c))^{(3/2)}/d+2/5*A*(a+b*\tan(d*x+c))^{(5/2)}/d+2/7*B*(a+b*\tan(d*x+c))^{(7/2)}/b/d$

**Rubi [A]**

time = 0.33, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3673, 3609, 3620, 3618, 65, 214}

$$\frac{2(a^2A - 2abB - Ab^2)\sqrt{a + b \tan(c + dx)}}{d} + \frac{2(aA - bB)(a + b \tan(c + dx))^{3/2}}{3d} - \frac{(a - ib)^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{5/2}(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d} + \frac{2A(a + b \tan(c + dx))^{5/2}}{5d} + \frac{2B(a + b \tan(c + dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

[In] `Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]`

[Out]  $-\left(\frac{(a - I*b)^{(5/2)*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]]}{d} - \frac{(a + I*b)^{(5/2)*(A + I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]]}{d} + \frac{2*(a^2*A - A*b^2 - 2*a*b*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}{d} + \frac{2*(a*A - b*B)*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}}{(3*d)} + \frac{2*A*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}}{(5*d)} + \frac{2*B*(a + b*\operatorname{Tan}[c + d*x])^{(7/2)}}{(7*b*d)}\right)$

**Rule 65**

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

**Rule 214**

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

**Rule 3609**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int \tan(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{2B(a + b \tan(c + dx))^{7/2}}{7bd} + \int (-B + A \tan(c + dx))^{5/2} dx \\
&= \frac{2A(a + b \tan(c + dx))^{5/2}}{5d} + \frac{2B(a + b \tan(c + dx))^{7/2}}{7bd} \\
&= \frac{2(aA - bB)(a + b \tan(c + dx))^{3/2}}{3d} + \frac{2A(a + b \tan(c + dx))^{5/2}}{5d} \\
&= \frac{2(a^2A - Ab^2 - 2abB) \sqrt{a + b \tan(c + dx)}}{d} \\
&= \frac{2(a^2A - Ab^2 - 2abB) \sqrt{a + b \tan(c + dx)}}{d} \\
&= \frac{2(a^2A - Ab^2 - 2abB) \sqrt{a + b \tan(c + dx)}}{d} \\
&= \frac{2(a^2A - Ab^2 - 2abB) \sqrt{a + b \tan(c + dx)}}{d} \\
&= \frac{2(a^2A - Ab^2 - 2abB) \sqrt{a + b \tan(c + dx)}}{d} \\
&= -\frac{(a - ib)^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 1.08, size = 258, normalized size = 1.21

$$\frac{2B(a + b \tan(c + dx))^{7/2} - 7(I(A + B) \left( \frac{2}{3}(a + b \tan(c + dx))^{3/2} + \frac{2}{3}(a - ib) \left( -3(a - ib)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) + \sqrt{a + b \tan(c + dx)}(4a - 3b + b \tan(c + dx)) \right) \right) - 7(I(A - B) \left( \frac{2}{3}(a + b \tan(c + dx))^{5/2} + \frac{2}{3}(a + ib) \left( -3(a + ib)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) + \sqrt{a + b \tan(c + dx)}(4a + 3b + b \tan(c + dx)) \right) \right))}{14d}$$

Antiderivative was successfully verified.

```

[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
[Out] ((4*B*(a + b*Tan[c + d*x])^(7/2))/b - (7*I)*(I*A + B)*((2*(a + b*Tan[c + d*x])^(5/2))/5 + (2*(a - I*b)*(-3*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*a - (3*I)*b + b*Tan[c + d*x])))/3) - (7*I)*(I*A - B)*((2*(a + b*Tan[c + d*x])^(5/2))/5 + (2*(a + I*b)*(-3*(a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*a + (3*I)*b + b*Tan[c + d*x])))/3))/(14*d)

```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(181) = 362.

time = 0.16, size = 1282, normalized size = 6.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{2}{d/b} \left( \frac{1}{7} B (a+b \tan(dx+c))^{7/2} + \frac{1}{5} A b (a+b \tan(dx+c))^{5/2} + \frac{1}{3} A a b (a+b \tan(dx+c))^{3/2} - \frac{1}{3} B b^2 (a+b \tan(dx+c))^{3/2} + A a^2 b (a+b \tan(dx+c))^{1/2} - A b^3 (a+b \tan(dx+c))^{1/2} - 2 B a b^2 (a+b \tan(dx+c))^{1/2} - b \left( \frac{1}{4} b \left( \frac{1}{2} (-2 A (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a b + 3 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^2 b - A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} b^3 - B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a^2 + B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^3 - 3 B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} b^2 + B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a b^2 \right) \ln(b \tan(dx+c) + a + (a+b \tan(dx+c))^{1/2} (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} + (a^2+b^2)^{1/2}) + 2 (2 A (a^2+b^2)^{1/2} a^2 b - 2 A b^3 (a^2+b^2)^{1/2} - 4 B (a^2+b^2)^{1/2} a b^2 - \frac{1}{2} (-2 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a b + 3 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^2 b - A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} b^3 - B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a^2 + B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^3 - 3 B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a b^2) (2 (a^2+b^2)^{1/2} - 2 a)^{1/2} \arctan\left(\frac{2 (a+b \tan(dx+c))^{1/2} + (2 (a^2+b^2)^{1/2} + 2 a)^{1/2}}{(2 (a^2+b^2)^{1/2} - 2 a)^{1/2}}\right) + \frac{1}{4} b (-\frac{1}{2} (-2 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a b + 3 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^2 b - A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} b^3 - B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a^2 + B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^3 - 3 B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a b^2) \ln(-b \tan(dx+c) - a + (a+b \tan(dx+c))^{1/2} (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} - (a^2+b^2)^{1/2}) + 2 (-2 A (a^2+b^2)^{1/2} a^2 b + 2 A b^3 (a^2+b^2)^{1/2} + 4 B (a^2+b^2)^{1/2} a b^2 + \frac{1}{2} (-2 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a b + 3 A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^2 b - A (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} b^3 - B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} (a^2+b^2)^{1/2} a^2 + B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a^3 - 3 B (2 (a^2+b^2)^{1/2} + 2 a)^{1/2} a b^2) (2 (a^2+b^2)^{1/2} - 2 a)^{1/2} \arctan\left(\frac{-2 (a+b \tan(dx+c))^{1/2} + (2 (a^2+b^2)^{1/2} + 2 a)^{1/2}}{(2 (a^2+b^2)^{1/2} - 2 a)^{1/2}}\right) \right)$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*tan(d*x + c), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}} \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(5/2)\*tan(c + d\*x), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 79.74, size = 2500, normalized size = 11.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out]  $\log(-(((A^4 b^2 d^4 (5a^4 + b^4 - 10a^2 b^2)^2)^{1/2} + A^2 a^5 d^2 - 10A^2 a^3 b^2 d^2 + 5A^2 a b^4 d^2)/d^4)^{1/2} (32A^6 b^6 - 32A^4 a^4 b^2 + 32a^2 b^2 d^2 * ((-A^4 b^2 d^4 (5a^4 + b^4 - 10a^2 b^2)^2)^{1/2} + A^2 a^5 d^2 - 10A^2 a^3 b^2 d^2 + 5A^2 a b^4 d^2)/d^4)^{1/2} (a + b \tan(c + d x))^{1/2}) / (2d) - (16A^2 b^2 (a + b \tan(c + d x))^{1/2} (a^6 - b^6 + 15a^2 b^4 - 15a^4 b^2) / d^2) * (((-A^4 b^2 d^4 (5a^4 + b^4 - 10a^2 b^2)^2)^{1/2} + A^2 a^5 d^2 - 10A^2 a^3 b^2 d^2 + 5A^2 a b^4 d^2) / d^4)^{1/2} / 2 - (8A^3 a b^2 (a^2 - 3b^2) (a^2 + b^2)^3) / d^3 * ((20A^4 a^2 b^8 d^4 - A^4 b^10 d^4 - 110A^4 a^4 b^6 d^4 + 100A^4 a^6 b^4 d^4 - 25A^4 a^8 b^2 d^4)^{1/2} / (4d^4) + (A^2 a^5) / (4d^2) - (5A^2 a^3 b^2) / (2d^2) + (5A^2 a b^4) / (4$



$$\begin{aligned}
& - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(64*B*a^3*b^3 + 64*B*a*b^5 - 32*a*b^2*d*((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}))/((2*d) - (16*B^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2)*(((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/(4*d^4))^{(1/2)} + \log((8*B^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - (((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*((( (-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(64*B*a^3*b^3 + 64*B*a*b^5 + 32*a*b...
\end{aligned}$$

### 3.334 $\int (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=188

$$\frac{(a - ib)^{5/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{(a + ib)^{5/2} (iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d}$$

[Out]  $-(a-I*b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d+(a+I*b)^{(5/2)}*(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+2*(2*A*a*b+B*a^2-B*b^2)*(a+b*\tan(d*x+c))^{(1/2)}/d+2/3*(A*b+B*a)*(a+b*\tan(d*x+c))^{(3/2)}/d+2/5*B*(a+b*\tan(d*x+c))^{(5/2)}/d$

**Rubi [A]**

time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3609, 3620, 3618, 65, 214}

$$\frac{2(a^2B + 2aAb - b^2B)\sqrt{a + b \tan(c + dx)}}{d} + \frac{2(aB + Ab)(a + b \tan(c + dx))^{3/2}}{3d} - \frac{(a - ib)^{5/2}(B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \frac{(a + ib)^{5/2}(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} + \frac{2B(a + b \tan(c + dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-(((a - I*b)^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d) + ((a + I*b)^{(5/2)}*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d + (2*(2*a*A*b + a^2*B - b^2*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d + (2*(A*b + a*B)*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d) + (2*B*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)})/(5*d)$

Rule 65

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{NegQ}[a/b]$

Rule 3609

$\operatorname{Int}[(a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.))]^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}$

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx &= \frac{2B(a + b \tan(c + dx))^{5/2}}{5d} + \int (a + b \tan(c + dx))^{3/2} (aA \\
&= \frac{2(Ab + aB)(a + b \tan(c + dx))^{3/2}}{3d} + \frac{2B(a + b \tan(c + dx))^{5/2}}{5d} \\
&= \frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2(Ab + aB)(a + b \tan(c + dx))^{3/2}}{5d} \\
&= \frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2(Ab + aB)(a + b \tan(c + dx))^{3/2}}{5d} \\
&= \frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2(Ab + aB)(a + b \tan(c + dx))^{3/2}}{5d} \\
&= \frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2(Ab + aB)(a + b \tan(c + dx))^{3/2}}{5d} \\
&= -\frac{(a - ib)^{5/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} + \dots
\end{aligned}$$

**Mathematica** [A]

time = 0.73, size = 233, normalized size = 1.24

$$\frac{\left( (A - iB) \left( \frac{2}{3}(a + b \tan(c + dx))^{3/2} + \frac{2}{3}(a - ib) \left( -3(a - ib)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) + \sqrt{a + b \tan(c + dx)} (4a - 3ib + b \tan(c + dx)) \right) \right) - (A + iB) \left( \frac{2}{3}(a + b \tan(c + dx))^{3/2} + \frac{2}{3}(a + ib) \left( -3(a + ib)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + \sqrt{a + b \tan(c + dx)} (4a + 3ib + b \tan(c + dx)) \right) \right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((I/2)\*((A - I\*B)\*((2\*(a + b\*Tan[c + d\*x])^(5/2))/5 + (2\*(a - I\*b)\*(-3\*(a - I\*b)^(3/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] + Sqrt[a + b\*Tan[c + d\*x]])\*(4\*a - (3\*I)\*b + b\*Tan[c + d\*x])))/3 - (A + I\*B)\*((2\*(a + b\*Tan[c + d\*x])^(5/2))/5 + (2\*(a + I\*b)\*(-3\*(a + I\*b)^(3/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]] + Sqrt[a + b\*Tan[c + d\*x]])\*(4\*a + (3\*I)\*b + b\*Tan[c + d\*x])))/3))/d

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1248 vs.  $2(160) = 320$ .

time = 0.13, size = 1249, normalized size = 6.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $\frac{1}{d} \left( \frac{2}{5} B (a + b \tan(d*x+c))^{5/2} + \frac{2}{3} A b (a + b \tan(d*x+c))^{3/2} + \frac{2}{3} B a (a + b \tan(d*x+c))^{3/2} + 4 A a b (a + b \tan(d*x+c))^{1/2} + 2 a^2 B (a + b \tan(d*x+c))^{1/2} - 2 B b^2 (a + b \tan(d*x+c))^{1/2} + \frac{1}{2} b ( \frac{1}{2} (-A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * b^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^3 - 3 A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a * b^2 + 2 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a * b - 3 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^2 * b + B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * b^3) * \ln(b * \tan(d*x+c)) + a (a + b \tan(d*x+c))^{1/2} * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} + (a^2 + b^2)^{1/2} \right) + \frac{2 * (-4 A b^2 (a^2 + b^2)^{1/2} * a - 2 B a^2 (a^2 + b^2)^{1/2} * b + 2 B b^3 (a^2 + b^2)^{1/2} - 1/2 * (-A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * b^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^3 - 3 A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a * b^2 + 2 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a * b - 3 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^2 * b + B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * b^3) * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2}}{(2 (a^2 + b^2)^{1/2} - 2 a)^{1/2}} * \arctan \left( \frac{2 (a + b \tan(d*x+c))^{1/2} + (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2}}{(2 (a^2 + b^2)^{1/2} - 2 a)^{1/2}} \right) + \frac{1}{2} b ( - \frac{1}{2} (-A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * b^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^3 - 3 A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a * b^2 + 2 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a * b - 3 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^2 * b + B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * b^3) * \ln(-b * \tan(d*x+c)) - a (a + b \tan(d*x+c))^{1/2} * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} - (a^2 + b^2)^{1/2} \right) + 2 * (4 A b^2 (a^2 + b^2)^{1/2} * a + 2 B a^2 (a^2 + b^2)^{1/2} * b - 2 B b^3 (a^2 + b^2)^{1/2} + 1/2 * (-A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * b^2 + A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^3 - 3 A (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a * b^2 + 2 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a * b - 3 B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^2 * b + B (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * b^3) * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} \right)$



$$2)^{(1/2)} * a * b - 3 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a^2 * b + B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * b^3 * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} / (2 * (a^2 + b^2)^{(1/2)} - 2 * a)^{(1/2)} * \arctan((-2 * (a + b * \tan(d * x + c))^{(1/2)} + (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)}) / (2 * (a^2 + b^2)^{(1/2)} - 2 * a)^{(1/2)})$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")



$$\begin{aligned}
& a^6 - b^6 + 15a^2b^4 - 15a^4b^2)/d^2) * (-((-B^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - B^2*a^5*d^2 + 10*B^2*a^3*b^2*d^2 - 5*B^2*a*b^4*d^2)/d^4)^{(1/2)})/2 - (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*((B^2*a^5)/(4*d^2) - (20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^{(1/2)})/(4*d^4) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d^2))^{(1/2)} + ((4*B*a^2)/d - (2*B*(a^2 + b^2))/d)*(a + b*\tan(c + d*x))^{(1/2)} - \log((((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (64*A*a^3*b^3 + 64*A*a*b^5 + 32*a*b^2*d * (-((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)})))/(2*d) + (16*A^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15a^2*b^4 - 15a^4*b^2))/d^2))/2 - (8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*(-((20*A^4*a^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4*a^8*b^2*d^4)^{(1/2)} + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} - \log((((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (64*A*a^3*b^3 + 64*A*a*b^5 + 32*a*b^2*d * (((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)})))/(2*d) + (16*A^2*b^2*(a + b*\tan(c + d*x))^{(1/2)}*(a^6 - b^6 + 15a^2*b^4 - 15a^4*b^2))/d^2))/2 - (8*A^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*(((20*A^4*a^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4*a^8*b^2*d^4)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} + \log((((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * ((((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (64*A*a^3*b^3 + 64*A*a*b^5 - 32*a*b^2*d * (((-A^4*b^2*d^4*(5a^4 + b^4 - 10a^2*b^2)^2)^{(1/2)} - A^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 5*A^2*a*b^4*d^2)/d^4)^{(1/2)} * (...
\end{aligned}$$

### 3.335 $\int \cot(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=182

$$\frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{5/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}}{d}$$

[Out]  $-2*a^{(5/2)}*A*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d+(a-I*b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d+(a+I*b)^{(5/2)}*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+2*b*(A*b+2*B*a)*(a+b*\tan(d*x+c))^{(1/2)}/d+2/3*b*B*(a+b*\tan(d*x+c))^{(3/2)}/d$

Rubi [A]

time = 0.56, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3688, 3728, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2b(2aB+Ab)\sqrt{a+b \tan(c+dx)}}{d} + \frac{(a-ib)^{5/2}(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2bB(a+b \tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]*(a+b*\operatorname{Tan}[c+d*x])^{(5/2)}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-2*a^{(5/2)}*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/d + ((a-I*b)^{(5/2)}*(A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/d + ((a+I*b)^{(5/2)}*(A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/d + (2*b*(A*b+2*a*B)*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/d + (2*b*B*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)})/(3*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b}))^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^(m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^(m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.
) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^(m*(c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

## Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

## Rubi steps

$$\begin{aligned}
\int \cot(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d} + \frac{2}{3} \int \cot(c + dx) \\
&= \frac{2b(Ab + 2aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2b(Ab + 2aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2b(Ab + 2aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d} \\
&= \frac{2b(Ab + 2aB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d} \\
&= -\frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d} \\
&= -\frac{2a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{2bB(a + b \tan(c + dx))^{3/2}}{3d}
\end{aligned}$$

**Mathematica** [A]

time = 0.78, size = 177, normalized size = 0.97

$$\frac{2\left(-3a^{5/2}A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right) + \frac{3}{2}(a - ib)^{5/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) + \frac{3}{2}(a + ib)^{5/2}(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) + 3b(Ab + 2aB)\sqrt{a + b \tan(c + dx)} + bB(a + b \tan(c + dx))^{3/2}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(2*(-3*a^{(5/2)}*A*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a]] + (3*(a - I*b)^{(5/2)}*(A - I*B)*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a - I*b]])/2 + (3*(a + I*b)^{(5/2)}*(A + I*B)*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]])/2 + 3*b*(A*b + 2*a*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] + b*B*(a + b*\text{Tan}[c + d*x])^{(3/2)})/(3*d)$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 3.06, size = 55566, normalized size = 305.31

method	result	size
default	Expression too large to display	55566

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}} \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(5/2)\*cot(c + d\*x), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 12.29, size = 2500, normalized size = 13.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] 
$$\left(\frac{2A^2b^2 - 2B^2a^2}{d} + \frac{6B^2a^2b}{d}\right)(a + b\tan(c + d*x))^{1/2} - \operatorname{atan}\left(\frac{(32(3A^3a^2b^{16}d^2 + 48A^3a^4b^{14}d^2 - 30A^3a^6b^{12}d^2 - 72A^3a^8b^{10}d^2 + 3A^3a^{10}b^8d^2 + 6B^3a^5b^{13}d^2 + 8B^3a^7b^{11}d^2 + 3B^3a^9b^9d^2 - B^3a^2b^{17}d^2 - A^2B^2a^2b^{16}d^2 - 32A^2B^2a^4b^{14}d^2 + 46A^2B^2a^6b^{12}d^2 + 72A^2B^2a^8b^{10}d^2 - 9A^2B^2a^{10}b^8d^2 - 16A^2B^2a^3b^{15}d^2 + 150A^2B^2a^5b^{13}d^2 + 96A^2B^2a^7b^{11}d^2 - 69A^2B^2a^9b^9d^2)}{d^5} - \frac{(32(4A^2a^2b^{12}d^4 + 16A^2a^3b^{10}d^4 + 12A^2a^5b^8d^4 + 8B^2a^2b^{11}d^4 + 8B^2a^4b^9d^4)}{d^5} - \frac{32(16b^{10}d^4 + 24a^2b^8d^4)(a + b\tan(c + d*x))^{1/2} + ((8B^2a^5d^2 - 8A^2a^5d^2 + 80A^2a^3b^2d^2 - 80B^2a^3b^2d^2 + 16A^2B^2a^5d^2 - 40A^2a^2b^4d^2 + 40B^2a^2b^4d^2 - 160A^2B^2a^2b^3d^2 + 80A^2B^2a^4b^2d^2)^2/64 - d^4(A^4a^{10} + A^4b^{10} + B^4a^{10} + B^4b^{10} + 2A^2B^2a^{10} + 2A^2B^2b^{10} + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5B^4a^2b^8 + 10B^4a^4b^6 + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 + 10A^2B^2a^8b^2)}{d^4} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 + 10B^2a^3b^2d^2 - 2A^2B^2a^5d^2 + 5A^2a^2b^4d^2 - 5B^2a^2b^4d^2 + 20A^2B^2a^2b^3d^2 - 10A^2B^2a^4b^2d^2)/(4d^4)}{d^4}\right)$$



$$\begin{aligned}
& ^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 + \\
& 10A^2B^2a^8b^2))^{(1/2)} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 \\
& + 10B^2a^3b^2d^2 - 2A^2B^2a^5d^2 + 5A^2a^4b^4d^2 - 5B^2a^4b^4d^2 \\
& + 20A^2B^2a^2b^3d^2 - 10A^2B^2a^4b^4d^2)/(4d^4))^{(1/2)} - (32(a + b\tan(c \\
& + dx))^{(1/2)}*(10A^2a^3b^12d^2 + 102A^2a^5b^10d^2 - 18A^2a^7b^8d^2 \\
& - 10B^2a^3b^12d^2 - 102B^2a^5b^10d^2 + 10B^2a^7b^8d^2 + 16A^2B^2a^2b^15d^2 \\
& - 38A^2a^4b^14d^2 + 38B^2a^4b^14d^2 - 120A^2B^2a^2b^13d^2 \\
& - 160A^2B^2a^4b^11d^2 + 104A^2B^2a^6b^9d^2))/d^4)*(((8B^2a^5d^2 - 8A^2a^5d^2 \\
& + 80A^2a^3b^2d^2 - 80B^2a^3b^2d^2 + 16A^2B^2a^5d^2 - 40A^2a^4b^4d^2 \\
& + 40B^2a^4b^4d^2 - 160A^2B^2a^2b^3d^2 + 80A^2B^2a^4b^4d^2) \\
& ^2/64 - d^4(A^4a^10 + A^4b^10 + B^4a^10 + B^4b^10 + 2A^2B^2a^10 + 2 \\
& *A^2B^2b^10 + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5B^4a^2b^8 \\
& + 10B^4a^4b^6 + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 \\
& + 20A^2B^2a^6b^4 + 10A^2B^2a^8b^2))^{(1/2)} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 \\
& + 10B^2a^3b^2d^2 - 2A^2B^2a^5d^2 + 5A^2a^4b^4d^2 - 5B^2a^4b^4d^2 + 20A^2B^2a^2b^3 \\
& d^2 - 10A^2B^2a^4b^4d^2)/(4d^4))^{(1/2)})*(((8B^2a^5d^2 - 8A^2a^5d^2 \\
& + 80A^2a^3b^2d^2 - 80B^2a^3b^2d^2 + 16A^2B^2a^5d^2 - 40A^2a^4b^4d^2 \\
& + 40B^2a^4b^4d^2 - 160A^2B^2a^2b^3d^2 + 80A^2B^2a^4b^4d^2)^2/64 - d^4 \\
& *(A^4a^10 + A^4b^10 + B^4a^10 + B^4b^10 + 2A^2B^2a^10 + 2A^2B^2b^10 \\
& + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5B^4a^2b^8 + 10B^4a^4b^6 \\
& + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 \\
& + 10A^2B^2a^8b^2))^{(1/2)} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 + 10B^2a^3b^2d^2 - 2 \\
& *A^2B^2a^5d^2 + 5A^2a^4b^4d^2 - 5B^2a^4b^4d^2 + 20A^2B^2a^2b^3d^2 - 10A^2B^2a^4b^4d^2 \\
& / (4d^4))^{(1/2)} + (32(a + b\tan(c + dx))^{(1/2)}*(A^4b^20 + B^4b^20 + 2A^2B^2b^20 \\
& + 6A^4a^2b^18 + 15A^4a^4b^16 + 18A^4a^6b^14 + 45A^4a^8b^12 - 24A^4a^10b^10 \\
& + 3A^4a^12b^8 + 6B^4a^2b^18 + 15B^4a^4b^16 + 20B^4a^6b^14 + 15B^4a^8b^12 \\
& + 6B^4a^10b^10 + B^4a^12b^8 + 12A^2B^2a^2b^18 + 30A^2B^2a^4b^16 + 42A^2B^2a^6b^14 \\
& + 42A^2B^2a^10b^10 - 24A^3B^2a^7b^13 + 80A^3B^2a^9b^11 - 24A^3B^2a^11b^9))/d^4) \\
& *(((8B^2a^5d^2 - 8A^2a^5d^2 + 80A^2a^3b^2d^2 - 80B^2a^3b^2d^2 + 16A^2B^2a^5d^2 \\
& - 40A^2a^4b^4d^2 + 40B^2a^4b^4d^2 - 160A^2B^2a^2b^3d^2 + 80A^2B^2a^4b^4d^2) \\
& ^2/64 - d^4(A^4a^10 + A^4b^10 + B^4a^10 + B^4b^10 + 2A^2B^2a^10 + 2A^2B^2b^10 \\
& + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5B^4a^2b^8 + 10B^4a^4b^6 \\
& + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 \\
& + 10A^2B^2a^8b^2))^{(1/2)} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 + 10B^2a^3b^2d^2 \\
& - 2A^2B^2a^5d^2 + 5A^2a^4b^4d^2 - 5B^2a^4b^4d^2 + 20A^2B^2a^2b^3d^2 - 10A^2B^2a^4b^4d^2 \\
& / (4d^4))^{(1/2)} * i - (((32(3A^3a^2b^16d^2 + 48A^3a^4b^14d^2 - 30A^3a^6b^12d^2 \\
& - 72A^3a^8b^10d^2 + 3A^3a^10b^8d^2 + 6B^3a^5b^13d^2 + 8B^3a^7b^11d^2 + 3B^3a^9b^9d^2 \\
& - B^3a^17d^2 - A^2B^2a^17d^2 + 3A^2B^2a^2b^16d^2 - 32A^2B^2a^4b^14d^2 + 4...
\end{aligned}$$

### 3.336 $\int \cot^2(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

Optimal. Leaf size=196

$$\frac{a^{3/2}(5Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{(a - ib)^{5/2}(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d}$$

[Out]  $-a^{3/2}(5Ab + 2aB) \operatorname{arctanh}\left(\frac{(a+b \tan(dx+c))^{1/2}}{a^{1/2}}\right)/d + (a-I*b)^{5/2}(I*A+B) \operatorname{arctanh}\left(\frac{(a+b \tan(dx+c))^{1/2}}{(a-I*b)^{1/2}}\right)/d - (a+I*b)^{5/2}(I*A-B) \operatorname{arctanh}\left(\frac{(a+b \tan(dx+c))^{1/2}}{(a+I*b)^{1/2}}\right)/d + b*(A*a+2*B*b)*(a+b \tan(dx+c))^{1/2}/d - a*A*\cot(dx+c)*(a+b \tan(dx+c))^{3/2}/d$

Rubi [A]

time = 0.57, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3686, 3728, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{a^{3/2}(2aB + 5Ab) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} + \frac{b(aA + 2bB) \sqrt{a + b \tan(c + dx)}}{d} + \frac{(a - ib)^{5/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{5/2}(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d} - \frac{aA \cot(c + dx)(a + b \tan(c + dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]`

[Out]  $-((a^{3/2}(5A*b + 2*a*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/d) + ((a - I*b)^{5/2}(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - ((a + I*b)^{5/2}(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d + (b*(a*A + 2*b*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d - (a*A*\operatorname{Cot}[c + d*x]*(a + b*\operatorname{Tan}[c + d*x])^{3/2})/d$

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 214

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.
) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
```

, 0] && NeQ[a, 0]))

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n *Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\int \cot^2(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx = -\frac{aA \cot(c + dx)(a + b \tan(c + dx))^{3/2}}{d} + \int \dots$$

$$= \frac{b(aA + 2bB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)}{d}$$

$$= \frac{b(aA + 2bB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)}{d}$$

$$= \frac{b(aA + 2bB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)}{d}$$

$$= \frac{b(aA + 2bB) \sqrt{a + b \tan(c + dx)}}{d} - \frac{aA \cot(c + dx)}{d}$$

$$= -\frac{a^{3/2}(5Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d}$$

$$= -\frac{a^{3/2}(5Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 400 vs. 2(196) = 392.  
 time = 0.70, size = 400, normalized size = 2.04

$$\frac{2bB \cot(c + dx)(a + b \tan(c + dx))^{5/2}}{d} - 2 \left( \frac{b(aA + 2bB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{d} - \frac{a^{3/2} b (5Ab + 2aB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{d} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
[Out] (2*b*B*Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2))/d + 2*(-((b*(A*b + 4*a*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d) - 2*(-((-1/4*(a^(5/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (I*Sqrt[a - I*b]*((I/4)*a*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B) - (a*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B))/4)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((-a + I*b)*d) - (I*Sqrt[a + I*b]*((-1/4*I)*a*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B) - (a*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B))/4)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((-a - I*b)*d))/a + ((a^2*A - 2*A*b^2 - 6*a*b*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(4*d))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 2.47, size = 88645, normalized size = 452.27

method	result	size
default	Expression too large to display	88645

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x
)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] Timed out

**Mupad [B]**

time = 9.77, size = 2500, normalized size = 12.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)`

[Out] 
$$\begin{aligned} & (2*B*b^2*(a + b*\tan(c + d*x))^{(1/2)})/d - \operatorname{atan}\left(\frac{(8*(176*A^3*a^5*b^{13}*d^2 - 400*A^3*a^3*b^{15}*d^2 + 488*A^3*a^7*b^{11}*d^2 - 92*A^3*a^9*b^9*d^2 + 12*B^3*a^2*b^{16}*d^2 + 192*B^3*a^4*b^{14}*d^2 - 120*B^3*a^6*b^{12}*d^2 - 288*B^3*a^8*b^{10}*d^2 + 12*B^3*a^{10}*b^8*d^2 + 4*A^3*a*b^{17}*d^2 + 4*A*B^2*a*b^{17}*d^2 + 464*A*B^2*a^3*b^{15}*d^2 - 920*A*B^2*a^5*b^{13}*d^2 - 1104*A*B^2*a^7*b^{11}*d^2 + 276*A*B^2*a^9*b^9*d^2 + 172*A^2*B*a^2*b^{16}*d^2 - 1468*A^2*B*a^4*b^{14}*d^2 - 776*A^2*B*a^6*b^{12}*d^2 + 828*A^2*B*a^8*b^{10}*d^2 - 36*A^2*B*a^{10}*b^8*d^2)}{d^5} - \frac{(8*(16*B*a*b^{12}*d^4 + 128*A*a^2*b^{11}*d^4 + 128*A*a^4*b^9*d^4 + 64*B*a^3*b^{10}*d^4 + 48*B*a^5*b^8*d^4)}{d^5} - (16*(32*b^{10}*d^4 + 48*a^2*b^8*d^4)*(a + b*\tan(c + d*x))^{(1/2)} * (((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^{10} + A^4*b^{10} + B^4*a^{10} + B^4*b^{10} + 2*A^2*B^2*a^{10} + 2*A^2*B^2*b^{10} + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2 \end{aligned}$$

$$\begin{aligned}
& *a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} - A^2*a^5*d^2 + \\
& B^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 10*B^2*a^3*b^2*d^2 + 2*A*B*b^5*d^2 - 5*A \\
& ^2*a*b^4*d^2 + 5*B^2*a*b^4*d^2 - 20*A*B*a^2*b^3*d^2 + 10*A*B*a^4*b*d^2)/(4* \\
& d^4))^{(1/2)}/d^4)*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - \\
& 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - \\
& 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + \\
& B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10* \\
& A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b \\
& ^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b \\
& ^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} - A^2*a^5*d^2 + B^2*a^ \\
& 5*d^2 + 10*A^2*a^3*b^2*d^2 - 10*B^2*a^3*b^2*d^2 + 2*A*B*b^5*d^2 - 5*A^2*a*b \\
& ^4*d^2 + 5*B^2*a*b^4*d^2 - 20*A*B*a^2*b^3*d^2 + 10*A*B*a^4*b*d^2)/(4*d^4))^{ \\
& (1/2)} + (16*(a + b*tan(c + d*x))^{(1/2)}*(20*A^2*a^3*b^12*d^2 + 304*A^2*a^5*b \\
& ^10*d^2 - 20*A^2*a^7*b^8*d^2 - 20*B^2*a^3*b^12*d^2 - 204*B^2*a^5*b^10*d^2 + \\
& 36*B^2*a^7*b^8*d^2 + 32*A*B*b^15*d^2 - 76*A^2*a*b^14*d^2 + 76*B^2*a*b^14*d \\
& ^2 - 240*A*B*a^2*b^13*d^2 - 320*A*B*a^4*b^11*d^2 + 288*A*B*a^6*b^9*d^2))/d^ \\
& 4)*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2* \\
& d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^ \\
& 3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4* \\
& b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 1 \\
& 0*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6 \\
& *b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2 \\
& *a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} - A^2*a^5*d^2 + B^2*a^5*d^2 + 10*A^2* \\
& a^3*b^2*d^2 - 10*B^2*a^3*b^2*d^2 + 2*A*B*b^5*d^2 - 5*A^2*a*b^4*d^2 + 5*B^2* \\
& a*b^4*d^2 - 20*A*B*a^2*b^3*d^2 + 10*A*B*a^4*b*d^2)/(4*d^4))^{(1/2)})*(((8*B^ \\
& 2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A* \\
& B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80* \\
& A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^ \\
& 2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b \\
& ^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^ \\
& 4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + \\
& 10*A^2*B^2*a^8*b^2))^{(1/2)} - A^2*a^5*d^2 + B^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 \\
& - 10*B^2*a^3*b^2*d^2 + 2*A*B*b^5*d^2 - 5*A^2*a*b^4*d^2 + 5*B^2*a*b^4*d^2 - \\
& 20*A*B*a^2*b^3*d^2 + 10*A*B*a^4*b*d^2)/(4*d^4))^{(1/2)} + (16*(a + b*tan(c + \\
& d*x))^{(1/2)}*(2*A^4*b^20 + 2*B^4*b^20 + 4*A^2*B^2*b^20 + 12*A^4*a^2*b^18 + \\
& 55*A^4*a^4*b^16 - 335*A^4*a^6*b^14 + 405*A^4*a^8*b^12 - 13*A^4*a^10*b^10 + \\
& 2*A^4*a^12*b^8 + 12*B^4*a^2*b^18 + 30*B^4*a^4*b^16 + 36*B^4*a^6*b^14 + 90*B \\
& ^4*a^8*b^12 - 48*B^4*a^10*b^10 + 6*B^4*a^12*b^8 + 24*A^2*B^2*a^2*b^18 + 35* \\
& A^2*B^2*a^4*b^16 + 699*A^2*B^2*a^6*b^14 - 1175*A^2*B^2*a^8*b^12 + 349*A^2*B \\
& ^2*a^10*b^10 - 20*A*B^3*a^5*b^15 + 348*A*B^3*a^7*b^13 - 460*A*B^3*a^9*b^11 \\
& + 68*A*B^3*a^11*b^9 + 320*A^3*B*a^5*b^15 - 1300*A^3*B*a^7*b^13 + 600*A^3*B* \\
& a^9*b^11 - 20*A^3*B*a^11*b^9))/d^4)*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80* \\
& A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + \\
& 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4* \\
& a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5
\end{aligned}$$

$$\begin{aligned}
& *A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2* \\
& b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 \\
& + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} - A^ \\
& 2*a^5*d^2 + B^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 10*B^2*a^3*b^2*d^2 + 2*A*B*b \\
& ^5*d^2 - 5*A^2*a*b^4*d^2 + 5*B^2*a*b^4*d^2 - 20*A*B*a^2*b^3*d^2 + 10*A*B*a^ \\
& 4*b*d^2)/(4*d^4))^{(1/2)}*1i - (((8*(176*A^3*a^5*b^13*d^2 - 400*A^3*a^3*b^15* \\
& d^2 + 488*A^3*a^7*b^11*d^2 - 92*A^3*a^9*b^9*d^2\dots
\end{aligned}$$



$$3.337 \quad \int \cot^3(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=220

$$\frac{\sqrt{a} (8a^2 A - 15Ab^2 - 20abB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{4d} - \frac{(a - ib)^{5/2} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d}$$

[Out]  $-(a-I*b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d-(a+I*b)^{(5/2)}*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+1/4*(8*A*a^2-15*A*b^2-20*B*a*b)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})*a^{(1/2)}/d-1/4*a*(7*A*b+4*B*a)*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1/2)}/d-1/2*a*A*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{(3/2)}/d$

**Rubi** [A]

time = 0.60, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3686, 3726, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{\sqrt{a} (8a^2 A - 20abB - 15Ab^2) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{4d} - \frac{(a - ib)^{5/2} (A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{(a + ib)^{5/2} (A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d} - \frac{a(4aB + 7Ab) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} - \frac{aA \cot^2(c + dx) (a + b \tan(c + dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^3*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(\operatorname{Sqrt}[a]*(8*a^2*A - 15*A*b^2 - 20*a*b*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(4*d) - ((a - I*b)^{(5/2)}*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - ((a + I*b)^{(5/2)}*(A + I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d - (a*(7*A*b + 4*a*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(4*d) - (a*A*\operatorname{Cot}[c + d*x]^2*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)})/(2*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
```

$a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[n, -1]$

### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \cot^3(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= -\frac{aA \cot^2(c + dx)(a + b \tan(c + dx))^{3/2}}{2d} + \frac{1}{2} \\
 &= -\frac{a(7Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= -\frac{a(7Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= -\frac{a(7Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= -\frac{a(7Ab + 4aB) \cot(c + dx) \sqrt{a + b \tan(c + dx)}}{4d} \\
 &= \frac{\sqrt{a} (8a^2A - 15Ab^2 - 20abB) \tanh^{-1} \left( \frac{\sqrt{a}}{a + b \tan(c + dx)} \right)}{4d} \\
 &= \frac{\sqrt{a} (8a^2A - 15Ab^2 - 20abB) \tanh^{-1} \left( \frac{\sqrt{a}}{a + b \tan(c + dx)} \right)}{4d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 448 vs. 2(220) = 440.  
time = 1.59, size = 448, normalized size = 2.04

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
[Out] -1/4*(-(Sqrt[a]*(8*a^2*A - 15*A*b^2 - 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]) + 4*(a - I*b)^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + 4*a^2*A*Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + (8*I)*a*A*Sqrt[a + I*b]*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - 4*A*Sqrt[a + I*b]*b^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + (4*I)*a^2*Sqrt[a + I*b]*B*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - 8*a*Sqrt[a + I*b]*b*B*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - (4*I)*Sqrt[a + I*b]*b^2*B*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 9*a*A*b*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + 4*a^2*B*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + 2*a^2*A*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/d
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 2.63, size = 128221, normalized size = 582.82

method	result	size
default	Expression too large to display	128221

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

**Fricas** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

[Out] Timed out

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 9.80, size = 2500, normalized size = 11.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] atan(-(((3708\*A^3\*a^2\*b^16\*d^2 - 6912\*A^3\*a^4\*b^14\*d^2 - 5820\*A^3\*a^6\*b^12\*d^2 + 4608\*A^3\*a^8\*b^10\*d^2 - 192\*A^3\*a^10\*b^8\*d^2 - 6400\*B^3\*a^3\*b^15\*d^2 + 2816\*B^3\*a^5\*b^13\*d^2 + 7808\*B^3\*a^7\*b^11\*d^2 - 1472\*B^3\*a^9\*b^9\*d^2 + 64\*B^3\*a\*b^17\*d^2 - 1856\*A^2\*B\*a\*b^17\*d^2 - 7552\*A\*B^2\*a^2\*b^16\*d^2 + 23488\*A\*B^2\*a^4\*b^14\*d^2 + 16256\*A\*B^2\*a^6\*b^12\*d^2 - 14208\*A\*B^2\*a^8\*b^10\*d^2 + 576\*A\*B^2\*a^10\*b^8\*d^2 + 20504\*A^2\*B\*a^3\*b^15\*d^2 - 5000\*A^2\*B\*a^5\*b^13\*d^2 - 22944\*A^2\*B\*a^7\*b^11\*d^2 + 4416\*A^2\*B\*a^9\*b^9\*d^2)/(2\*d^5) - (((1664\*A\*a\*b^12\*d^4 + 896\*A\*a^3\*b^10\*d^4 - 768\*A\*a^5\*b^8\*d^4 + 2048\*B\*a^2\*b^11\*d^4 + 2048\*B\*a^4\*b^9\*d^4)/(2\*d^5) - ((512\*b^10\*d^4 + 768\*a^2\*b^8\*d^4)\*(a + b\*tan(c + d\*x))^(1/2)\*(((8\*B^2\*a^5\*d^2 - 8\*A^2\*a^5\*d^2 + 80\*A^2\*a^3\*b^2\*d^2 - 80\*B^2\*a^3\*b^2\*d^2 + 16\*A\*B\*b^5\*d^2 - 40\*A^2\*a\*b^4\*d^2 + 40\*B^2\*a\*b^4\*d^2 - 160\*A\*B\*a^2\*b^3\*d^2 + 80\*A\*B\*a^4\*b\*d^2)^2/64 - d^4\*(A^4\*a^10 + A^4\*b^10 + B^4\*a^10 + B^4\*b^10 + 2\*A^2\*B^2\*a^10 + 2\*A^2\*B^2\*b^10 + 5\*A^4\*a^2\*b^8 + 10\*A^4\*a^4\*b^6 + 10\*A^4\*a^6\*b^4 + 5\*A^4\*a^8\*b^2 + 5\*B^4\*a^2\*b^8 + 10\*B^4\*a^4\*b^6 + 10\*B^4\*a^6\*b^4 + 5\*B^4\*a^8\*b^2 + 10\*A^2\*B^2\*a^2\*b^8 + 20\*A^2\*B^2\*a^4\*b^6

$$\begin{aligned}
& + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} + A^2*a^5*d^2 - B^2*a^5*d^2 \\
& - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2 - 2*A*B*b^5*d^2 + 5*A^2*a*b^4*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 - 10*A*B*a^4*b*d^2)/(4*d^4))^{(1/2)}/d^4)*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} + A^2*a^5*d^2 - B^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2 - 2*A*B*b^5*d^2 + 5*A^2*a*b^4*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 - 10*A*B*a^4*b*d^2)/(4*d^4))^{(1/2)} + ((a + b*tan(c + d*x))^{(1/2)}*(580*A^2*a^3*b^12*d^2 - 4224*A^2*a^5*b^10*d^2 + 576*A^2*a^7*b^8*d^2 + 320*B^2*a^3*b^12*d^2 + 4864*B^2*a^5*b^10*d^2 - 320*B^2*a^7*b^8*d^2 - 512*A*B*b^15*d^2 + 1216*A^2*a*b^14*d^2 - 1216*B^2*a*b^14*d^2 + 3840*A*B*a^2*b^13*d^2 + 7520*A*B*a^4*b^11*d^2 - 4608*A*B*a^6*b^9*d^2))/d^4)*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} + A^2*a^5*d^2 - B^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2 - 2*A*B*b^5*d^2 + 5*A^2*a*b^4*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 - 10*A*B*a^4*b*d^2)/(4*d^4))^{(1/2)})*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^{(1/2)} + A^2*a^5*d^2 - B^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2 - 2*A*B*b^5*d^2 + 5*A^2*a*b^4*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 - 10*A*B*a^4*b*d^2)/(4*d^4))^{(1/2)} + ((a + b*tan(c + d*x))^{(1/2)}*(32*A^4*b^20 + 32*B^4*b^20 + 64*A^2*B^2*b^20 - 33*A^4*a^2*b^18 + 4095*A^4*a^4*b^16 - 6399*A^4*a^6*b^14 + 5265*A^4*a^8*b^12 - 1008*A^4*a^10*b^10 + 96*A^4*a^12*b^8 + 192*B^4*a^2*b^18 + 880*B^4*a^4*b^16 - 5360*B^4*a^6*b^14 + 6480*B^4*a^8*b^12 - 208*B^4*a^10*b^10 + 32*B^4*a^12*b^8 + 609*A^2*B^2*a^2*b^18 - 10255*A^2*B^2*a^4*b^16 + 42159*A^2*B^2*a^6*b^14 - 29825*A^2*B^2*a^8*b^12 + 5824*A^2*B^2*a^10*b^10 + 600*A*B^3*a^3*b^17 - 14120*A*B^3*a^5*b^15 + 29800*A*B^3*a^7*b^13 - 10200*A*B^3*a^9*b^11 + 320*A*B^3*a^11*b^9 - 3300*A^3*B*a^3*b^17 + 21200*A^3*B*a^5*b^15 - 26868*A^3*B*a^7*b^13 + 10840*A^3*B*a^9*b^11 - 1088*A^3*B*a^11*b^9))/d^4)*(((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 -
\end{aligned}$$

$$\begin{aligned}
& d^4(A^4a^{10} + A^4b^{10} + B^4a^{10} + B^4b^{10} + 2A^2B^2a^{10} + 2A^2B^2 \\
& *b^{10} + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5 \\
& *B^4a^2b^8 + 10B^4a^4b^6 + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2 \\
& *a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 + 10A^2B^2a^8b^2))^{( \\
& 1/2)} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 + 10B^2a^3b^2d^2 \\
& - 2A*B*b^5d^2 + 5A^2a*b^4d^2 - 5B^2a*b^4d^2 + 20A*B*a^2b^3d^2 - \\
& 10A*B*a^4b*d^2)/(4*d^4))^{(1/2)}*1i - (((3708*A...
\end{aligned}$$

$$3.338 \quad \int \cot^4(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=277

$$\frac{(40a^2Ab - 5Ab^3 + 16a^3B - 30ab^2B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right) + (a-ib)^{5/2}(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{8\sqrt{a}d}$$

[Out]  $-(a-I*b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d+(a+I*b)^{(5/2)}*(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d+1/8*(40*A*a^2*b-5*A*b^3+16*B*a^3-30*B*a*b^2)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d/a^{(1/2)}+1/8*(8*A*a^2-11*A*b^2-18*B*a*b)*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1/2)}/d-1/4*a*(3*A*b+2*B*a)*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{(1/2)}/d-1/3*a*A*\cot(d*x+c)^3*(a+b*\tan(d*x+c))^{(3/2)}/d$

Rubi [A]

time = 0.83, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3686, 3726, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(8a^2A - 18abB - 11A^2B) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{8d} + \frac{(16a^2B + 40a^2Ab - 5A^2B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}d} + \frac{(a-ib)^{5/2}(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{a(2aB+3A^2) \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{4d} - \frac{aA \cot^3(c+dx) (a+b \tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^4*(a+b*\operatorname{Tan}[c+d*x])^{5/2}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $((40*a^2*A*b - 5*A*b^3 + 16*a^3*B - 30*a*b^2*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(8*\operatorname{Sqrt}[a]*d) - ((a-I*b)^{(5/2)}*(I*A+B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/d + ((a+I*b)^{(5/2)}*(I*A-B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/d + ((8*a^2*A - 11*A*b^2 - 18*a*b*B)*\operatorname{Cot}[c+d*x]*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(8*d) - (a*(3*A*b + 2*a*B)*\operatorname{Cot}[c+d*x]^2*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(4*d) - (a*A*\operatorname{Cot}[c+d*x]^3*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)})/(3*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] :> \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$



Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
```

], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

### Rule 3730

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] :> Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx)) dx &= -\frac{aA \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d} + \frac{1}{3} \\
&= -\frac{a(3Ab+2aB) \cot^2(c+dx) \sqrt{a+b\tan(c+dx)}}{4d} \\
&= \frac{(8a^2A-11Ab^2-18abB) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{8d} \\
&= \frac{(8a^2A-11Ab^2-18abB) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{8d} \\
&= \frac{(8a^2A-11Ab^2-18abB) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{8d} \\
&= \frac{(8a^2A-11Ab^2-18abB) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{8d} \\
&= \frac{(40a^2Ab-5Ab^3+16a^3B-30ab^2B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}d} \\
&= \frac{(40a^2Ab-5Ab^3+16a^3B-30ab^2B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}d}
\end{aligned}$$

### Mathematica [A]

time = 4.34, size = 240, normalized size = 0.87

$$\frac{3(40a^2Ab-5Ab^3+16a^3B-30ab^2B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{a}(-24i(a-ib)^{5/2}(A-ib) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) + 24i(a+ib)^{5/2}(A+ib) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) - \cot(c+dx)(-24a^2A+33Ab^2+54abB+2a(13Ab+6aB) \cot(c+dx) + 8a^2A \cot^2(c+dx)) \sqrt{a+b\tan(c+dx)}}{24\sqrt{a}d}$$

Antiderivative was successfully verified.

```

[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
[Out] (3*(40*a^2*A*b - 5*A*b^3 + 16*a^3*B - 30*a*b^2*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + Sqrt[a]*((-24*I)*(a - I*b)^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (24*I)*(a + I*b)^(5/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - Cot[c + d*x]*(-24*a^2*A + 33*A*b^2 + 54*a*b*B + 2*a*(13*A*b + 6*a*B)*Cot[c + d*x] + 8*a^2*A*Cot[c + d*x]^2)*Sqrt[a + b*Tan[c + d*x]])/(24*Sqrt[a]*d)

```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 3.35, size = 171974, normalized size = 620.84

method	result	size
default	Expression too large to display	171974

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

**Fricas** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

**Sympy** [F(-2)]  
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

**Giac** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 10.07, size = 2500, normalized size = 9.03
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (atan((((((a + b*tan(c + d*x))^(1/2)*(153*A^4*b^20 + 128*B^4*b^20 + 231*A^2
*B^2*b^20 - 7*A^4*a^2*b^18 + 9895*A^4*a^4*b^16 - 27465*A^4*a^6*b^14 + 26320
*A^4*a^8*b^12 - 832*A^4*a^10*b^10 + 128*A^4*a^12*b^8 - 132*B^4*a^2*b^18 + 1
6380*B^4*a^4*b^16 - 25596*B^4*a^6*b^14 + 21060*B^4*a^8*b^12 - 4032*B^4*a^10
*b^10 + 384*B^4*a^12*b^8 + 6811*A^2*B^2*a^2*b^18 - 61315*A^2*B^2*a^4*b^16 +
184661*A^2*B^2*a^6*b^14 - 121620*A^2*B^2*a^8*b^12 + 23296*A^2*B^2*a^10*b^1
0 - 300*A*B^3*a*b^19 + 600*A^3*B*a*b^19 + 17860*A*B^3*a^3*b^17 - 91700*A*B^
3*a^5*b^15 + 110172*A*B^3*a^7*b^13 - 43520*A*B^3*a^9*b^11 + 4352*A*B^3*a^11
*b^9 - 12860*A^3*B*a^3*b^17 + 79680*A^3*B*a^5*b^15 - 126700*A^3*B*a^7*b^13
+ 40960*A^3*B*a^9*b^11 - 1280*A^3*B*a^11*b^9))/(64*d^4) + (((3225*A^3*a^3*b
^15*d^2 - 1088*A^3*a^5*b^13*d^2 - 3984*A^3*a^7*b^11*d^2 + 736*A^3*a^9*b^9*d
^2 + 1854*B^3*a^2*b^16*d^2 - 3456*B^3*a^4*b^14*d^2 - 2910*B^3*a^6*b^12*d^2
+ 2304*B^3*a^8*b^10*d^2 - 96*B^3*a^10*b^8*d^2 + (295*A^2*B*b^18*d^2)/2 - 40
7*A^3*a*b^17*d^2 + 1178*A*B^2*a*b^17*d^2 - 10572*A*B^2*a^3*b^15*d^2 + 1930*
A*B^2*a^5*b^13*d^2 + 11472*A*B^2*a^7*b^11*d^2 - 2208*A*B^2*a^9*b^9*d^2 - 47
16*A^2*B*a^2*b^16*d^2 + (22193*A^2*B*a^4*b^14*d^2)/2 + 8568*A^2*B*a^6*b^12*
d^2 - 7104*A^2*B*a^8*b^10*d^2 + 288*A^2*B*a^10*b^8*d^2)/(16*d^5) + (((a +
b*tan(c + d*x))^(1/2)*(320*A^2*a^3*b^12*d^2 - 19456*A^2*a^5*b^10*d^2 + 1280
*A^2*a^7*b^8*d^2 - 2320*B^2*a^3*b^12*d^2 + 16896*B^2*a^5*b^10*d^2 - 2304*B^
2*a^7*b^8*d^2 - 2048*A*B*b^15*d^2 + 4764*A^2*a*b^14*d^2 - 4864*B^2*a*b^14*d
^2 + 14160*A*B*a^2*b^13*d^2 + 30720*A*B*a^4*b^11*d^2 - 18432*A*B*a^6*b^9*d^
2))/(64*d^4) - (((160*A*b^13*d^4 + 832*B*a*b^12*d^4 - 864*A*a^2*b^11*d^4 -
1024*A*a^4*b^9*d^4 + 448*B*a^3*b^10*d^4 - 384*B*a^5*b^8*d^4)/(16*d^5) - ((2
048*b^10*d^4 + 3072*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(256*B^2*a^7 +
25*A^2*a*b^6 - 400*A^2*a^3*b^4 + 1600*A^2*a^5*b^2 + 900*B^2*a^3*b^4 - 960*B
^2*a^5*b^2 + 1280*A*B*a^6*b + 300*A*B*a^2*b^5 - 2560*A*B*a^4*b^3)^(1/2))/(1
024*a*d^5))*(256*B^2*a^7 + 25*A^2*a*b^6 - 400*A^2*a^3*b^4 + 1600*A^2*a^5*b^
2 + 900*B^2*a^3*b^4 - 960*B^2*a^5*b^2 + 1280*A*B*a^6*b + 300*A*B*a^2*b^5 -
2560*A*B*a^4*b^3)^(1/2))/(16*a*d))*(256*B^2*a^7 + 25
```

$$\begin{aligned}
& *A^2*a*b^6 - 400*A^2*a^3*b^4 + 1600*A^2*a^5*b^2 + 900*B^2*a^3*b^4 - 960*B^2 \\
& *a^5*b^2 + 1280*A*B*a^6*b + 300*A*B*a^2*b^5 - 2560*A*B*a^4*b^3)^{(1/2)} / (16* \\
& a*d)) * (256*B^2*a^7 + 25*A^2*a*b^6 - 400*A^2*a^3*b^4 + 1600*A^2*a^5*b^2 + 90 \\
& 0*B^2*a^3*b^4 - 960*B^2*a^5*b^2 + 1280*A*B*a^6*b + 300*A*B*a^2*b^5 - 2560*A \\
& *B*a^4*b^3)^{(1/2)} * i) / (a*d) + (((a + b*\tan(c + d*x))^{(1/2)} * (153*A^4*b^20 + \\
& 128*B^4*b^20 + 231*A^2*B^2*b^20 - 7*A^4*a^2*b^18 + 9895*A^4*a^4*b^16 - 274 \\
& 65*A^4*a^6*b^14 + 26320*A^4*a^8*b^12 - 832*A^4*a^10*b^10 + 128*A^4*a^12*b^8 \\
& - 132*B^4*a^2*b^18 + 16380*B^4*a^4*b^16 - 25596*B^4*a^6*b^14 + 21060*B^4*a \\
& ^8*b^12 - 4032*B^4*a^10*b^10 + 384*B^4*a^12*b^8 + 6811*A^2*B^2*a^2*b^18 - 6 \\
& 1315*A^2*B^2*a^4*b^16 + 184661*A^2*B^2*a^6*b^14 - 121620*A^2*B^2*a^8*b^12 + \\
& 23296*A^2*B^2*a^10*b^10 - 300*A*B^3*a*b^19 + 600*A^3*B*a*b^19 + 17860*A*B^ \\
& 3*a^3*b^17 - 91700*A*B^3*a^5*b^15 + 110172*A*B^3*a^7*b^13 - 43520*A*B^3*a^9 \\
& *b^11 + 4352*A*B^3*a^11*b^9 - 12860*A^3*B*a^3*b^17 + 79680*A^3*B*a^5*b^15 - \\
& 126700*A^3*B*a^7*b^13 + 40960*A^3*B*a^9*b^11 - 1280*A^3*B*a^11*b^9)) / (64*d \\
& ^4) - (((3225*A^3*a^3*b^15*d^2 - 1088*A^3*a^5*b^13*d^2 - 3984*A^3*a^7*b^11* \\
& d^2 + 736*A^3*a^9*b^9*d^2 + 1854*B^3*a^2*b^16*d^2 - 3456*B^3*a^4*b^14*d^2 - \\
& 2910*B^3*a^6*b^12*d^2 + 2304*B^3*a^8*b^10*d^2 - 96*B^3*a^10*b^8*d^2 + (295 \\
& *A^2*B*b^18*d^2) / 2 - 407*A^3*a*b^17*d^2 + 1178*A*B^2*a*b^17*d^2 - 10572*A*B \\
& ^2*a^3*b^15*d^2 + 1930*A*B^2*a^5*b^13*d^2 + 11472*A*B^2*a^7*b^11*d^2 - 2208 \\
& *A*B^2*a^9*b^9*d^2 - 4716*A^2*B*a^2*b^16*d^2 + (22193*A^2*B*a^4*b^14*d^2) / 2 \\
& + 8568*A^2*B*a^6*b^12*d^2 - 7104*A^2*B*a^8*b^10*d^2 + 288*A^2*B*a^10*b^8*d \\
& ^2) / (16*d^5) - (((a + b*\tan(c + d*x))^{(1/2)} * (320*A^2*a^3*b^12*d^2 - 19456* \\
& A^2*a^5*b^10*d^2 + 1280*A^2*a^7*b^8*d^2 - 2320*B^2*a^3*b^12*d^2 + 16896*B^2 \\
& *a^5*b^10*d^2 - 2304*B^2*a^7*b^8*d^2 - 2048*A*B*b^15*d^2 + 4764*A^2*a*b^14* \\
& d^2 - 4864*B^2*a*b^14*d^2 + 14160*A*B*a^2*b^13*d^2 + 30720*A*B*a^4*b^11*d^2 \\
& - 18432*A*B*a^6*b^9*d^2)) / (64*d^4) + (((160*A*b^13*d^4 + 832*B*a*b^12*d^4 \\
& - 864*A*a^2*b^11*d^4 - 1024*A*a^4*b^9*d^4 + 448*B*a^3*b^10*d^4 - 384*B*a^5* \\
& b^8*d^4) / (16*d^5) + ((2048*b^10*d^4 + 3072*a^2*b^8*d^4) * (a + b*\tan(c + d*x) \\
& )^{(1/2)} * (256*B^2*a^7 + 25*A^2*a*b^6 - 400*A^2*a^3*b^4 + 1600*A^2*a^5*b^2 + \\
& 900*B^2*a^3*b^4 - 960*B^2*a^5*b^2 + 1280*A*B*a^6*b + 300*A*B*a^2*b^5 - 2560 \\
& *A*B*a^4*b^3)^{(1/2)} / (1024*a*d^5)) * (256*B^2*a^7 + 25*A^2*a*b^6 - 400*A^2*a^ \\
& 3*b^4 + 1600*A^2*a^5*b^2 + 900*B^2*a^3*b^4 - 960*B^2*a^5*b^2 + 1280*A*B*a^6 \\
& *b + 300*A*B*a^2*b^5 - 2560*A*B*a^4*b^3)^{(1/2)} / (16*a*d)) * (256*B^2*a^7 + 25 \\
& *A^2*a*b^6 - 400*A^2*a^3*b^4 + 1600*A^2*a^5*b^2 + 900*B^2*a^3*b^4 - 960*B^2 \\
& *a^5*b^2 + 1280*A*B*a^6*b + 300*A*B*a^2*b^5 - 2560*A*B*a^4*b^3)^{(1/2)} / (16* \\
& a*d)) * (256*B^2*a^7 + 25*A^2*a*b^6 - 400*A^2*a^3...
\end{aligned}$$

### 3.339 $\int \cot^5(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=342

$$\frac{(128a^4A - 240a^2Ab^2 - 5Ab^4 - 320a^3bB + 40ab^3B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right) + (a-ib)^{5/2}(A-iB)}{64a^{3/2}d}$$

[Out]  $-1/64*(128*A*a^4-240*A*a^2*b^2-5*A*b^4-320*B*a^3*b+40*B*a*b^3)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/a^{1/2})/a^{3/2}/d+(a-I*b)^{5/2}*(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/(a-I*b)^{1/2})/d+(a+I*b)^{5/2}*(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/(a+I*b)^{1/2})/d+1/64*(144*A*a^2*b-5*A*b^3+64*B*a^3-88*B*a*b^2)*\cot(d*x+c)*(a+b*\tan(d*x+c))^{1/2}/a/d+1/96*(48*A*a^2-59*A*b^2-104*B*a*b)*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{1/2}/d-1/24*a*(11*A*b+8*B*a)*\cot(d*x+c)^3*(a+b*\tan(d*x+c))^{1/2}/d-1/4*a*A*\cot(d*x+c)^4*(a+b*\tan(d*x+c))^{3/2}/d$

**Rubi [A]**

time = 1.09, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3686, 3726, 3730, 3734, 3620, 3618, 65, 214, 3715}

$\frac{(48a^4A - 104abB - 59a^2b^2 - d^2)\sqrt{a+b \tan(c+dx)}}{96d}$ ,  $\frac{(64a^2B + 144a^2Ab - 88a^2B^2 - 54b^2)\cot(c+dx)\sqrt{a+b \tan(c+dx)}}{64ad}$ ,  $\frac{(128a^4A - 320a^3bB - 240a^2b^2 + 40ab^3B - 5Ab^4)\tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{64a^{3/2}d}$ ,  $\frac{(a-ib)^{5/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-Ib}}\right)}{d}$ ,  $\frac{(a+ib)^{5/2}(A+ib)\tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$ ,  $\frac{a(8a^2B + 114b)\cot^2(c+dx)\sqrt{a+b \tan(c+dx)}}{24d}$ ,  $\frac{a^2A \cot^2(c+dx)\sqrt{a+b \tan(c+dx)}}{24d}$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^5*(a + b*\operatorname{Tan}[c + d*x])^{5/2}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-1/64*((128*a^4*A - 240*a^2*A*b^2 - 5*A*b^4 - 320*a^3*b*B + 40*a*b^3*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(a^{3/2}*d) + ((a - I*b)^{5/2}*(A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d + ((a + I*b)^{5/2}*(A + I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d + ((144*a^2*A*b - 5*A*b^3 + 64*a^3*B - 88*a*b^2*B)*\operatorname{Cot}[c + d*x]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(64*a*d) + ((48*a^2*A - 59*A*b^2 - 104*a*b*B)*\operatorname{Cot}[c + d*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(96*d) - (a*(11*A*b + 8*a*B)*\operatorname{Cot}[c + d*x]^3*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(24*d) - (a*A*\operatorname{Cot}[c + d*x]^4*(a + b*\operatorname{Tan}[c + d*x])^{3/2})/(4*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_))\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_))\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e



```

+ f*x]]^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx)) dx &= -\frac{aA \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d} + \frac{1}{4} \\
&= -\frac{a(11Ab+8aB) \cot^3(c+dx) \sqrt{a+b\tan(c+dx)}}{24d} \\
&= \frac{(48a^2A-59Ab^2-104abB) \cot^2(c+dx) \sqrt{a+b\tan(c+dx)}}{96d} \\
&= \frac{(144a^2Ab-5Ab^3+64a^3B-88ab^2B) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{64ad} \\
&= \frac{(144a^2Ab-5Ab^3+64a^3B-88ab^2B) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{64ad} \\
&= \frac{(144a^2Ab-5Ab^3+64a^3B-88ab^2B) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{64ad} \\
&= \frac{(144a^2Ab-5Ab^3+64a^3B-88ab^2B) \cot(c+dx) \sqrt{a+b\tan(c+dx)}}{64ad} \\
&= \frac{(128a^4A-240a^2Ab^2-5Ab^4-320a^3bB+64a^3)}{64a^3} \\
&= \frac{(128a^4A-240a^2Ab^2-5Ab^4-320a^3bB+64a^3)}{64a^3}
\end{aligned}$$

**Mathematica [A]**

time = 6.32, size = 622, normalized size = 1.82



Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

```
[Out] (-2*b*B*Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2))/(5*d) - (2*((b*(5*A*b + 2*a*B)*Cot[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]]))/(7*d) - (2*(-1/16*((35*a^2*A - 40*A*b^2 - 72*a*b*B)*Cot[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]])/d - ((7*a*(85*a*A*b + 40*a^2*B - 48*b^2*B)*Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]])/(24*d) - ((35*a^2*(48*a^2*A - 59*A*b^2 - 104*a*b*B)*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(32*d) - (-((-105*a^(5/2)*(128*a^4*A - 240*a^2*A*b^2 - 5
```

$$\begin{aligned} & *A*b^4 - 320*a^3*b*B + 40*a*b^3*B) * \text{ArcTanh}\left[\frac{\text{Sqrt}[a + b*\text{Tan}[c + d*x]]}{\text{Sqrt}[a]} \right] / (32*d) + (I*\text{Sqrt}[a - I*b] * (210*a^4*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B) + (210*I)*a^4*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)) * \text{ArcTanh}\left[\frac{\text{Sqrt}[a + b*\text{Tan}[c + d*x]]}{\text{Sqrt}[a - I*b]} \right] / ((-a + I*b)*d) - (I*\text{Sqrt}[a + I*b] * (210*a^4*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B) - (210*I)*a^4*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)) * \text{ArcTanh}\left[\frac{\text{Sqrt}[a + b*\text{Tan}[c + d*x]]}{\text{Sqrt}[a + I*b]} \right] / ((-a - I*b)*d)) / a - (105*a^2*(144*a^2*A*b - 5*A*b^3 + 64*a^3*B - 88*a*b^2*B) * \text{Cot}[c + d*x] * \text{Sqrt}[a + b*\text{Tan}[c + d*x]] / (32*d)) / (2*a) / (3*a) / (4*a) / 7) / 5 \end{aligned}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 4.31, size = 227162, normalized size = 664.22

method	result	size
default	Expression too large to display	227162

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] Timed out

**Mupad** [B]

time = 10.67, size = 2500, normalized size = 7.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)`

[Out] 
$$\operatorname{atan}\left(\frac{(10240A^4a^4b^4 + 436224A^3a^3b^3 + 229376A^2a^2b^2 + 196608Aa^1b^1 + 81920B^4a^4b^4 + 442368B^3a^3b^3 + 524288B^2a^2b^2 + 131072Ba^1b^1)(a + b \tan(c + d x))^{1/2} \left( (8B^2a^5d^2 - 8A^2a^5d^2 + 80A^2a^3b^2d^2 - 80B^2a^3b^2d^2 + 16ABb^5d^2 - 40A^2a^4b^4d^2 + 40B^2a^4b^4d^2 - 160ABa^2b^3d^2 + 80ABa^4b^4d^2)^2/64 - d^4(A^4a^{10} + A^4b^{10} + B^4a^{10} + B^4b^{10} + 2A^2B^2a^{10} + 2A^2B^2b^{10} + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5B^4a^2b^8 + 10B^4a^4b^6 + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 + 10A^2B^2a^8b^2) \right)^{1/2} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 + 10B^2a^3b^2d^2 - 2ABb^5d^2 + 5A^2a^4b^4d^2 - 5B^2a^4b^4d^2 + 20ABa^2b^3d^2 - 10ABa^4b^4d^2}{(4d^4)^{1/2}}}{(256a^2d^4)} \left( (8B^2a^5d^2 - 8A^2a^5d^2 + 80A^2a^3b^2d^2 - 80B^2a^3b^2d^2 + 16ABb^5d^2 - 40A^2a^4b^4d^2 + 40B^2a^4b^4d^2 - 160ABa^2b^3d^2 + 80ABa^4b^4d^2)^2/64 - d^4(A^4a^{10} + A^4b^{10} + B^4a^{10} + B^4b^{10} + 2A^2B^2a^{10} + 2A^2B^2b^{10} + 5A^4a^2b^8 + 10A^4a^4b^6 + 10A^4a^6b^4 + 5A^4a^8b^2 + 5B^4a^2b^8 + 10B^4a^4b^6 + 10B^4a^6b^4 + 5B^4a^8b^2 + 10A^2B^2a^2b^8 + 20A^2B^2a^4b^6 + 20A^2B^2a^6b^4 + 10A^2B^2a^8b^2) \right)^{1/2} + A^2a^5d^2 - B^2a^5d^2 - 10A^2a^3b^2d^2 + 10B^2a^3b^2d^2 - 2ABb^5d^2 + 5A^2a^4b^4d^2 - 5B^2a^4b^4d^2 + 20ABa^2b^3d^2 - 10ABa^4b^4d^2$$

$$\begin{aligned}
& a^4 * b * d^2 / (4 * d^4)^{(1/2)} + ((a + b * \tan(c + d * x))^{(1/2)} * (320896 * A^2 * a^3 * b^1 \\
& 4 * d^2 + 143360 * A^2 * a^5 * b^12 * d^2 - 1081344 * A^2 * a^7 * b^10 * d^2 + 147456 * A^2 * a^9 \\
& * b^8 * d^2 - 304896 * B^2 * a^3 * b^14 * d^2 - 20480 * B^2 * a^5 * b^12 * d^2 + 1245184 * B^2 * a \\
& ^7 * b^10 * d^2 - 81920 * B^2 * a^9 * b^8 * d^2 + 100 * A^2 * a * b^16 * d^2 - 132672 * A * B * a^2 * b \\
& ^15 * d^2 + 919040 * A * B * a^4 * b^13 * d^2 + 1966080 * A * B * a^6 * b^11 * d^2 - 1179648 * A * B * \\
& a^8 * b^9 * d^2)) / (256 * a^2 * d^4) * (((8 * B^2 * a^5 * d^2 - 8 * A^2 * a^5 * d^2 + 80 * A^2 * a^3 \\
& * b^2 * d^2 - 80 * B^2 * a^3 * b^2 * d^2 + 16 * A * B * b^5 * d^2 - 40 * A^2 * a * b^4 * d^2 + 40 * B^2 * \\
& a * b^4 * d^2 - 160 * A * B * a^2 * b^3 * d^2 + 80 * A * B * a^4 * b * d^2)^2 / 64 - d^4 * (A^4 * a^10 + \\
& A^4 * b^10 + B^4 * a^10 + B^4 * b^10 + 2 * A^2 * B^2 * a^10 + 2 * A^2 * B^2 * b^10 + 5 * A^4 * a^ \\
& 2 * b^8 + 10 * A^4 * a^4 * b^6 + 10 * A^4 * a^6 * b^4 + 5 * A^4 * a^8 * b^2 + 5 * B^4 * a^2 * b^8 + 1 \\
& 0 * B^4 * a^4 * b^6 + 10 * B^4 * a^6 * b^4 + 5 * B^4 * a^8 * b^2 + 10 * A^2 * B^2 * a^2 * b^8 + 20 * A^ \\
& 2 * B^2 * a^4 * b^6 + 20 * A^2 * B^2 * a^6 * b^4 + 10 * A^2 * B^2 * a^8 * b^2))^{(1/2)} + A^2 * a^5 * d \\
& ^2 - B^2 * a^5 * d^2 - 10 * A^2 * a^3 * b^2 * d^2 + 10 * B^2 * a^3 * b^2 * d^2 - 2 * A * B * b^5 * d^2 \\
& + 5 * A^2 * a * b^4 * d^2 - 5 * B^2 * a * b^4 * d^2 + 20 * A * B * a^2 * b^3 * d^2 - 10 * A * B * a^4 * b * d^2 \\
& ) / (4 * d^4)^{(1/2)} + (100 * A^3 * b^20 * d^2 - 16000 * A^3 * a^2 * b^18 * d^2 - 928868 * A^3 * \\
& a^4 * b^16 * d^2 + 1805952 * A^3 * a^6 * b^14 * d^2 + 1489920 * A^3 * a^8 * b^12 * d^2 - 117964 \\
& 8 * A^3 * a^10 * b^10 * d^2 + 49152 * A^3 * a^12 * b^8 * d^2 - 208384 * B^3 * a^3 * b^17 * d^2 + 16 \\
& 51200 * B^3 * a^5 * b^15 * d^2 - 557056 * B^3 * a^7 * b^13 * d^2 - 2039808 * B^3 * a^9 * b^11 * d^2 \\
& + 376832 * B^3 * a^11 * b^9 * d^2 + 8840 * A^2 * B * a * b^19 * d^2 - 53120 * A * B^2 * a^2 * b^18 * d \\
& ^2 + 2411392 * A * B^2 * a^4 * b^16 * d^2 - 5701888 * A * B^2 * a^6 * b^14 * d^2 - 4381696 * A * B^ \\
& 2 * a^8 * b^12 * d^2 + 3637248 * A * B^2 * a^10 * b^10 * d^2 - 147456 * A * B^2 * a^12 * b^8 * d^2 + \\
& 543176 * A^2 * B * a^3 * b^17 * d^2 - 5453504 * A^2 * B * a^5 * b^15 * d^2 + 1016320 * A^2 * B * a^7 * \\
& b^13 * d^2 + 5873664 * A^2 * B * a^9 * b^11 * d^2 - 1130496 * A^2 * B * a^11 * b^9 * d^2) / (512 * a^ \\
& 2 * d^5) * (((8 * B^2 * a^5 * d^2 - 8 * A^2 * a^5 * d^2 + 80 * A^2 * a^3 * b^2 * d^2 - 80 * B^2 * a^3 \\
& * b^2 * d^2 + 16 * A * B * b^5 * d^2 - 40 * A^2 * a * b^4 * d^2 + 40 * B^2 * a * b^4 * d^2 - 160 * A * B * a \\
& ^2 * b^3 * d^2 + 80 * A * B * a^4 * b * d^2)^2 / 64 - d^4 * (A^4 * a^10 + A^4 * b^10 + B^4 * a^10 + \\
& B^4 * b^10 + 2 * A^2 * B^2 * a^10 + 2 * A^2 * B^2 * b^10 + 5 * A^4 * a^2 * b^8 + 10 * A^4 * a^4 * b^ \\
& 6 + 10 * A^4 * a^6 * b^4 + 5 * A^4 * a^8 * b^2 + 5 * B^4 * a^2 * b^8 + 10 * B^4 * a^4 * b^6 + 10 * B^ \\
& 4 * a^6 * b^4 + 5 * B^4 * a^8 * b^2 + 10 * A^2 * B^2 * a^2 * b^8 + 20 * A^2 * B^2 * a^4 * b^6 + 20 * A^ \\
& 2 * B^2 * a^6 * b^4 + 10 * A^2 * B^2 * a^8 * b^2))^{(1/2)} + A^2 * a^5 * d^2 - B^2 * a^5 * d^2 - 10 \\
& * A^2 * a^3 * b^2 * d^2 + 10 * B^2 * a^3 * b^2 * d^2 - 2 * A * B * b^5 * d^2 + 5 * A^2 * a * b^4 * d^2 - 5 \\
& * B^2 * a * b^4 * d^2 + 20 * A * B * a^2 * b^3 * d^2 - 10 * A * B * a^4 * b * d^2) / (4 * d^4)^{(1/2)} - (( \\
& a + b * \tan(c + d * x))^{(1/2)} * (25 * A^2 * B^2 * b^22 - 25 * A^4 * b^22 + 6167 * A^4 * a^2 * b^2 \\
& 0 + 28457 * A^4 * a^4 * b^18 + 993145 * A^4 * a^6 * b^16 - 1616544 * A^4 * a^8 * b^14 + 13465 \\
& 60 * A^4 * a^10 * b^12 - 258048 * A^4 * a^12 * b^10 + 24576 * A^4 * a^14 * b^8 + 9792 * B^4 * a^2 \\
& * b^20 - 448 * B^4 * a^4 * b^18 + 633280 * B^4 * a^6 * b^16 - 1757760 * B^4 * a^8 * b^14 + 168 \\
& 4480 * B^4 * a^10 * b^12 - 53248 * B^4 * a^12 * b^10 + 8192 * B^4 * a^14 * b^8 + 21609 * A^2 * B^ \\
& 2 * a^2 * b^20 + 344599 * A^2 * B^2 * a^4 * b^18 - 3736185 * A^2 * B^2 * a^6 * b^16 + 11758304 * \\
& A^2 * B^2 * a^8 * b^14 - 7782400 * A^2 * B^2 * a^10 * b^12 + 1490944 * A^2 * B^2 * a^12 * b^10 - \\
& 400 * A * B^3 * a * b^21 + 100 * A^3 * B * a * b^21 - 29200 * A * B^3 * a^3 * b^19 + 769040 * A * B^3 * a \\
& ^5 * b^17 - 5051120 * A * B^3 * a^7 * b^15 + 8105600 * A * B^3 * a^9 * b^13 - 2621440 * A * B^3 * a \\
& ^11 * b^11 + 81920 * A * B^3 * a^13 * b^9 - 17800 * A^3 * B * a^3 * b^19 - 977980 * A^3 * B * a^5 * b \\
& ^17 + 5740400 * A^3 * B * a^7 * b^15 - 7032448 * A^3 * B * a^9 * b^13 + 2785280 * A^3 * B * a^11 * \\
& b^11 - 278528 * A^3 * B * a^13 * b^9)) / (256 * a^2 * d^4) * (((8 * B^2 * a^5 * d^2 - 8 * A^2 * a^5 \\
& * d^2 + 80 * A^2 * a^3 * b^2 * d^2 - 80 * B^2 * a^3 * b^2 * d^2 + 16 * A * B * b^5 * d^2 - 40 * A^2 * a *
\end{aligned}$$

$$b^4d^2 + 40B^2ab^4d^2 - 160ABa^2b^3d^2 \dots$$

### 3.340 $\int (-a + b \tan(c + dx))(a + b \tan(c + dx))^{5/2} dx$

**Optimal.** Leaf size=151

$$\frac{(ia - b)(a - ib)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{5/2}(ia + b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d}$$

[Out]  $(I*a-b)*(a-I*b)^{(5/2)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d-(a+I*b)^{(5/2)*(I*a+b)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}/d-2*b*(a^2+b^2)*(a+b*\tan(d*x+c))^{(1/2)}/d+2/5*b*(a+b*\tan(d*x+c))^{(5/2)}/d$

**Rubi [A]**

time = 0.17, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {3609, 12, 3563, 3620, 3618, 65, 214}

$$-\frac{2b(a^2 + b^2) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} + \frac{(-b + ia)(a - ib)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d} - \frac{(a + ib)^{5/2}(b + ia) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Tan}[c + d*x])*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}, x]$

[Out]  $((I*a - b)*(a - I*b)^{(5/2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - ((a + I*b)^{(5/2)*(I*a + b)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d - (2*b*(a^2 + b^2)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/d + (2*b*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)})/(5*d)$

**Rule 12**

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}[\operatorname{Q}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]]$

**Rule 65**

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(m_)*((c_.) + (d_.)*(x_)^{(n_)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c - a*(d/b) + d*(x^p/b))^{(n_)}], x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_) + (b_.)*(x_)^{(2)}]^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

**Rule 3563**

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

### Rule 3609

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3618

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rubi steps



$$\begin{aligned}
\int (-a + b \tan(c + dx))(a + b \tan(c + dx))^{5/2} dx &= \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} + \int (-a^2 - b^2)(a + b \tan(c + dx))^{5/2} dx \\
&= \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} + (-a^2 - b^2) \int (a + b \tan(c + dx))^{5/2} dx \\
&= -\frac{2b(a^2 + b^2) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} \\
&= -\frac{2b(a^2 + b^2) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} \\
&= -\frac{2b(a^2 + b^2) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} \\
&= -\frac{2b(a^2 + b^2) \sqrt{a + b \tan(c + dx)}}{d} + \frac{2b(a + b \tan(c + dx))^{5/2}}{5d} \\
&= \frac{(ia - b)(a - ib)^{5/2} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{2b(a + b \tan(c + dx))^{5/2}}{5d}
\end{aligned}$$

**Mathematica [A]**

time = 0.89, size = 193, normalized size = 1.28

$$\frac{\cos(c + dx)(a - b \tan(c + dx)) \left( 5i(a - ib)^{5/2}(a + ib) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) - 5i(a - ib)(a + ib)^{5/2} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) + 2b\sqrt{a + b \tan(c + dx)}(-4a^2 - 5b^2 + 2ab \tan(c + dx) + b^2 \tan^2(c + dx)) \right)}{5d(a \cos(c + dx) - b \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Tan[c + d\*x])\*(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] (Cos[c + d\*x]\*(a - b\*Tan[c + d\*x])\*((5\*I)\*(a - I\*b)^(5/2)\*(a + I\*b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] - (5\*I)\*(a - I\*b)\*(a + I\*b)^(5/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]] + 2\*b\*Sqrt[a + b\*Tan[c + d\*x]]\*(-4\*a^2 - 5\*b^2 + 2\*a\*b\*Tan[c + d\*x] + b^2\*Tan[c + d\*x]^2)))/(5\*d\*(a\*Cos[c + d\*x] - b\*Sin[c + d\*x]))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(127) = 254.

time = 0.14, size = 682, normalized size = 4.52

method	result
--------	--------

derivativedivides	$2b \left( \frac{(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - a^2 \sqrt{a+b \tan(dx+c)} - b^2 \sqrt{a+b \tan(dx+c)} + (a^2+b^2) \right)$	$\left( \frac{\sqrt{2}\sqrt{a^2+b^2}}{\dots} \right)$
default	$2b \left( \frac{(a+b \tan(dx+c))^{\frac{5}{2}}}{5} - a^2 \sqrt{a+b \tan(dx+c)} - b^2 \sqrt{a+b \tan(dx+c)} + (a^2+b^2) \right)$	$\left( \frac{\sqrt{2}\sqrt{a^2+b^2}}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $2/d*b*(1/5*(a+b*\tan(d*x+c))^{(5/2)}-a^2*(a+b*\tan(d*x+c))^{(1/2)}-b^2*(a+b*\tan(d*x+c))^{(1/2)}+(a^2+b^2)*(1/4/b^2*(1/2*((2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a-(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*a^2+(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*b^2*\ln(b*\tan(d*x+c)+a+(a+b*\tan(d*x+c))^{(1/2)}*(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)}))+2*(2*(a^2+b^2))^{(1/2)}*b^2-1/2*((2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a-(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*a^2+(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*b^2*(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2))^{(1/2)}-2*a)^{(1/2)}*arctan((2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2))^{(1/2)}-2*a)^{(1/2)}))+1/4/b^2*(-1/2*((2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a-(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*a^2+(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*b^2*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{(1/2)}*(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(-2*(a^2+b^2))^{(1/2)}*b^2+1/2*((2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a-(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*a^2+(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)}*b^2*(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2))^{(1/2)}-2*a)^{(1/2)}*arctan((-2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2))^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2))^{(1/2)}-2*a)^{(1/2)))))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError



$$\begin{aligned}
& 2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14} \\
& )/d^4) \cdot \cos(dx + c) + (9a^{16}b^3 + 48a^{14}b^5 + 100a^{12}b^7 + 96a^{10}b^9 \\
& + 30a^8b^{11} - 16a^6b^{13} - 12a^4b^{15} + b^{19}) \cdot d \cdot \cos(dx + c) \cdot \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10} + (a^3 - 3ab^2) \cdot d^2 \cdot \sqrt{(a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4})/(9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10})) \cdot \sqrt{(a \cdot \cos(dx + c) + b \cdot \sin(dx + c))/\cos(dx + c)} \cdot ((a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4)^{1/4} + (9a^{21}b^2 + 66a^{19}b^4 + 205a^{17}b^6 + 344a^{15}b^8 + 322a^{13}b^{10} + 140a^{11}b^{12} - 14a^9b^{14} - 40a^7b^{16} - 11a^5b^{18} + 2a^3b^{20} + ab^{22}) \cdot \cos(dx + c) + (9a^{20}b^3 + 66a^{18}b^5 + 205a^{16}b^7 + 344a^{14}b^9 + 322a^{12}b^{11} + 140a^{10}b^{13} - 14a^8b^{15} - 40a^6b^{17} - 11a^4b^{19} + 2a^2b^{21} + b^{23}) \cdot \sin(dx + c) \cdot ((a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4)^{3/4} + \sqrt{2} \cdot ((3a^{10}b + 11a^8b^3 + 14a^6b^5 + 6a^4b^7 - a^2b^9 - b^{11}) \cdot d^7 \cdot \sqrt{(a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4}) \cdot \sqrt{(9a^{12}b^2 + 30a^{10}b^4 + 31a^8b^6 + 4a^6b^8 - 9a^4b^{10} - 2a^2b^{12} + b^{14})/d^4} + 2 \cdot (3a^{17}b + 20a^{15}b^3 + 56a^{13}b^5 + 84a^{11}b^7 + 70a^9b^9 + 28a^7b^{11} - 4a^5b^{13} - ab^{17}) \cdot d^5 \cdot \sqrt{(9a^{12}b^2 + 30a^{10}b^4 + 31a^8b^6 + 4a^6b^8 - 9a^4b^{10} - 2a^2b^{12} + b^{14})/d^4}) \cdot \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10} + (a^3 - 3ab^2) \cdot d^2 \cdot \sqrt{(a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4})/(9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10})) \cdot \sqrt{(a \cdot \cos(dx + c) + b \cdot \sin(dx + c))/\cos(dx + c)} \cdot ((a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4)^{3/4})/(9a^{34}b^2 + 129a^{32}b^4 + 856a^{30}b^6 + 3480a^{28}b^8 + 9660a^{26}b^{10} + 19292a^{24}b^{12} + 28392a^{22}b^{14} + 30888a^{20}b^{16} + 24310a^{18}b^{18} + 12870a^{16}b^{20} + 3432a^{14}b^{22} - 728a^{12}b^{24} - 1092a^{10}b^{26} - 420a^8b^{28} - 40a^6b^{30} + 24a^4b^{32} + 9a^2b^{34} + b^{36})) \cdot \cos(dx + c)^2 + 20 \cdot \sqrt{2} \cdot d^5 \cdot \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10} + (a^3 - 3ab^2) \cdot d^2 \cdot \sqrt{(a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4})/(9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10})) \cdot ((a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14})/d^4)^{3/4} \cdot \sqrt{(9a^{12}b^2 + 30a^{10}b^4 + 31a^8b^6 + 4a^6b^8 - 9a^4b^{10} - 2a^2b^{12} + b^{14})/d^4} \cdot \arctan(-((3a^{22} + 29a^{20}b^2 + 125a^{18}b^4 + 315a^{16}b^6 + 510a^{14}b^8 + 546a^{12}b^{10} + 378a^{10}b^{12} + 150a^8b^{14} + 15a^6b^{16} - 15a^4b^{18} \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int a^3 \sqrt{a + b \tan(c + dx)} dx - \int (-b^3 \sqrt{a + b \tan(c + dx)} \tan^3(c + dx)) dx - \int (-ab^2 \sqrt{a + b \tan(c + dx)} \tan^2(c + dx)) dx - \int a^2 b \sqrt{a + b \tan(c + dx)} \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] -Integral(a\*\*3\*sqrt(a + b\*tan(c + d\*x)), x) - Integral(-b\*\*3\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*3, x) - Integral(-a\*b\*\*2\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*2, x) - Integral(a\*\*2\*b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 27.93, size = 2500, normalized size = 16.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*tan(c + d\*x))^(5/2)\*(a - b\*tan(c + d\*x)),x)

[Out]  $\log\left(\frac{(8a^3b^3(3a^2 - b^2)(a^2 + b^2)^3)}{d^3} - \frac{((((( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2)/d^4)^{1/2} * (64a^2b^5 + 64a^4b^3 + 32ab^2d * (( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2)/d^4)^{1/2} * (a + b \tan(c + d*x))^{1/2}}{(2*d)} + (16a^2b^2(a + b \tan(c + d*x))^{1/2} * (a^6 - b^6 + 15a^2b^4 - 15a^4b^2))/d^2 * ((( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2)/d^4)^{1/2}}{2} * ((20a^6b^8d^4 - a^4b^{10}d^4 - 110a^8b^6d^4 + 100a^{10}b^4d^4 - 25a^{12}b^2d^4)^{1/2} / (4d^4) - a^7 / (4d^2) - (5a^3b^4) / (4d^2) + (5a^5b^2) / (2d^2))^{1/2} - \log\left(\frac{(8a^3b^3(3a^2 - b^2)(a^2 + b^2)^3)}{d^3} - \frac{((((( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} + a^7d^2 + 5a^3b^4d^2 - 10a^5b^2d^2)/d^4)^{1/2} * (64a^2b^5 + 64a^4b^3 - 32ab^2d * (( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} + a^7d^2 + 5a^3b^4d^2 - 10a^5b^2d^2)/d^4)^{1/2} * (a + b \tan(c + d*x))^{1/2}}{(2*d)} - (16a^2b^2(a + b \tan(c + d*x))^{1/2} * (a^6 - b^6 + 15a^2b^4 - 15a^4b^2))/d^2 * ((( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} + a^7d^2 + 5a^3b^4d^2 - 10a^5b^2d^2)/d^4)^{1/2}}{2} * (-a^7d^2 + (20a^6b^8d^4 - a^4b^{10}d^4 - 110a^8b^6d^4 + 100a^{10}b^4d^4 - 25a^{12}b^2d^4)^{1/2} + 5a^3b^4d^2 - 10a^5b^2d^2) / (4d^4))^{1/2} - \log\left(\frac{(8a^3b^3(3a^2 - b^2)(a^2 + b^2)^3)}{d^3} - \frac{((((( -a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{1/2} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2)/d^4)^{1/2} * (64a^2b^5 +$

$$\begin{aligned}
& 64a^4b^3 - 32a^3b^2d * (((-a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2)/d^4)^{(1/2)} * (a + b \tan(c + dx)) \\
& )^{(1/2)}) / (2d) - (16a^2b^2(a + b \tan(c + dx))^{(1/2)} * (a^6 - b^6 + 15a^2 \\
& 2b^4 - 15a^4b^2))/d^2 * (((-a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2)/d^4)^{(1/2)}) / 2 * (-a^7d^2 - \\
& (20a^6b^8d^4 - a^4b^10d^4 - 110a^8b^6d^4 + 100a^10b^4d^4 - 25a^12b^2d^4)^{(1/2)} \\
& + 5a^3b^4d^2 - 10a^5b^2d^2)/(4d^4))^{(1/2)} + \log((8 \\
& a^3b^3(3a^2 - b^2)(a^2 + b^2)^3)/d^3 - ((((-a^4b^2d^4(5a^4 + b^4 - \\
& 10a^2b^2)^2)^{(1/2)} + a^7d^2 + 5a^3b^4d^2 - 10a^5b^2d^2)/d^4)^{(1/2)} * (64a^2b^5 \\
& + 64a^4b^3 + 32a^3b^2d * (-((-a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& + a^7d^2 + 5a^3b^4d^2 - 10a^5b^2d^2)/d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / (2d) \\
& + (16a^2b^2(a + b \tan(c + dx))^{(1/2)} * (a^6 - b^6 + 15a^2b^4 - 15a^4b^2))/d^2 * (-((-a^4b^2d^4(5a^4 + b^4 - \\
& 10a^2b^2)^2)^{(1/2)} + a^7d^2 + 5a^3b^4d^2 - 10a^5b^2d^2)/d^4)^{(1/2)}) / 2 * ((5a^5b^2) / (2d^2) \\
& - a^7/(4d^2) - (5a^3b^4)/(4d^2) - (20a^6b^8d^4 - a^4b^10d^4 - 110a^8b^6d^4 + 100a^10b^4d^4 - 25a^12b^2d^4)^{(1/2)} / (4d^4))^{(1/2)} \\
& + ((4a^2b)/d - (2b*(a^2 + b^2))/d) * (a + b \tan(c + dx))^{(1/2)} - \log(((((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} + 5 \\
& a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/d^4)^{(1/2)} * (32a^4b^3 - 32b^7 + 32a^3b^2d * (((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& + 5a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / (2d) \\
& + (16(a + b \tan(c + dx))^{(1/2)} * (b^10 - 15a^2b^8 + 15a^4b^6 - a^6b^4))/d^2 * (((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& + 5a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/d^4)^{(1/2)}) / 2 - (8a^5b^5(a^2 - 3b^2)(a^2 + b^2)^3)/d^3 * (((20a^2b^12d^4 - b^14d^4 - 110a^4b^10d^4 + 100a^6b^8 \\
& d^4 - 25a^8b^6d^4)^{(1/2)} + 5a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/(4d^4))^{(1/2)} + \log(- ((((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& + 5a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/d^4)^{(1/2)} * (32b^7 - 32a^4b^3 + 32a^3b^2d * (((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& + 5a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / (2d) \\
& + (16(a + b \tan(c + dx))^{(1/2)} * (b^10 - 15a^2b^8 + 15a^4b^6 - a^6b^4))/d^2 * (((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& + 5a^3b^6d^2 - 10a^5b^4d^2 + a^7b^2d^2)/d^4)^{(1/2)}) / 2 - (8a^5b^5(a^2 - 3b^2)(a^2 + b^2)^3)/d^3 * ((20a^2b^12d^4 - b^14d^4 - 110a^4b^10d^4 + 100a^6b^8 \\
& d^4 - 25a^8b^6d^4)^{(1/2)} / (4d^4) + (5a^3b^6)/(4d^2) - (5a^3b^4)/(2d^2) + (a^5b^2)/(4d^2))^{(1/2)} - \log(((((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& - 5a^3b^6d^2 + 10a^5b^4d^2 - a^7b^2d^2)/d^4)^{(1/2)} * (32a^4b^3 - 32b^7 + 32a^3b^2d * (-((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& - 5a^3b^6d^2 + 10a^5b^4d^2 - a^7b^2d^2)/d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / (2d) \\
& + (16(a + b \tan(c + dx))^{(1/2)} * (b^10 - 15a^2b^8 + 15a^4b^6 - a^6b^4))/d^2 * (-((-b^6d^4(5a^4 + b^4 - 10a^2b^2)^2)^{(1/2)} \\
& - 5a^3b^6d^2 + 10a^5b^4d^2 - a^7b^2d^2)/d^4)^{(1/2)}) / 2 - (8a^5b^5(a^2 - 3b^2)(a^2 + b^2)^3)/d^3 * (-((20a^2b^12d^4 - b^14d^4 - 110a^4b^10d^4 + 100a^6b^8 \\
& d^4 - 25a^8b^6d^4)^{(1/2)} - 5a^3b^6d^2 + 10a^5b^4d^2 - a^7b^2d^2)/(4d^4))^{(1/2)} + \log(- ((((-b^6d^4(5a^4 + b^4 -
\end{aligned}$$

$$10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4...$$

### 3.341 $\int (-a + b \tan(c + dx))(a + b \tan(c + dx))^{3/2} dx$

**Optimal.** Leaf size=408

$$\frac{b(a^2 + b^2) \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} + \frac{b(a^2 + b^2) \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d}$$

[Out]  $-1/2*b*(a^2+b^2)*\operatorname{arctanh}(((a+(a^2+b^2)^{(1/2)})^{(1/2)}-2^{(1/2)}*(a+b*\tan(dx+c))^{(1/2)})/(a-(a^2+b^2)^{(1/2)})^{(1/2)})/d*2^{(1/2)}/(a-(a^2+b^2)^{(1/2)})^{(1/2)}+1/2*b*(a^2+b^2)*\operatorname{arctanh}(((a+(a^2+b^2)^{(1/2)})^{(1/2)}+2^{(1/2)}*(a+b*\tan(dx+c))^{(1/2)})/(a-(a^2+b^2)^{(1/2)})^{(1/2)})/d*2^{(1/2)}/(a-(a^2+b^2)^{(1/2)})^{(1/2)}-1/4*b*(a^2+b^2)*\ln(a+(a^2+b^2)^{(1/2)}-2^{(1/2)}*(a+(a^2+b^2)^{(1/2)})^{(1/2)}*(a+b*\tan(dx+c))^{(1/2)}+b*\tan(dx+c))/d*2^{(1/2)}/(a+(a^2+b^2)^{(1/2)})^{(1/2)}+1/4*b*(a^2+b^2)*\ln(a+(a^2+b^2)^{(1/2)}+2^{(1/2)}*(a+(a^2+b^2)^{(1/2)})^{(1/2)}*(a+b*\tan(dx+c))^{(1/2)}+b*\tan(dx+c))/d*2^{(1/2)}/(a+(a^2+b^2)^{(1/2)})^{(1/2)}+2/3*b*(a+b*\tan(dx+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.36, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3609, 12, 3566, 714, 1143, 648, 632, 212, 642}

$$\frac{b(a^2 + b^2) \log \left( -\sqrt{2} \sqrt{a^2 + b^2} + a \sqrt{a + b \tan(c + dx)} + \sqrt{a^2 + b^2} + a + b \tan(c + dx) \right)}{2\sqrt{2}d\sqrt{a^2 + b^2}} + \frac{b(a^2 + b^2) \log \left( \sqrt{2} \sqrt{a^2 + b^2} + a \sqrt{a + b \tan(c + dx)} + \sqrt{a^2 + b^2} + a + b \tan(c + dx) \right)}{2\sqrt{2}d\sqrt{a^2 + b^2}} - \frac{b(a^2 + b^2) \operatorname{tanh}^{-1} \left( \frac{\sqrt{a^2 + b^2} + a - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2}d\sqrt{a - \sqrt{a^2 + b^2}}} + \frac{b(a^2 + b^2) \operatorname{tanh}^{-1} \left( \frac{\sqrt{a^2 + b^2} + a + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2}d\sqrt{a - \sqrt{a^2 + b^2}}} + \frac{2b(a + b \tan(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Tan}[c + d*x])*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $-((b*(a^2 + b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]] - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]]*d) + (b*(a^2 + b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]] + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]]*d) - (b*(a^2 + b^2)*\operatorname{Log}[a + \operatorname{Sqrt}[a^2 + b^2] + b*\operatorname{Tan}[c + d*x] - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*d) + (b*(a^2 + b^2)*\operatorname{Log}[a + \operatorname{Sqrt}[a^2 + b^2] + b*\operatorname{Tan}[c + d*x] + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*d) + (2*b*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 212



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 632

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 648

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 714

Int[Sqrt[(d\_) + (e\_)\*(x\_)]/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[2\*e, Subst[Int[x^2/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

#### Rule 1143

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*r), Int[x^(m-1)/(q - r\*x + x^2), x], x] - Dist[1/(2\*c\*r), Int[x^(m-1)/(q + r\*x + x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GeQ[m, 1] && LtQ[m, 3] && NegQ[b^2 - 4\*a\*c]

#### Rule 3566

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int

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[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (-a + b \tan(c + dx))(a + b \tan(c + dx))^{3/2} dx &= \frac{2b(a + b \tan(c + dx))^{3/2}}{3d} + \int (-a^2 - b^2) \sqrt{a + b \tan(c + dx)} dx \\
&= \frac{2b(a + b \tan(c + dx))^{3/2}}{3d} + (-a^2 - b^2) \int \sqrt{a + b \tan(c + dx)} dx \\
&= \frac{2b(a + b \tan(c + dx))^{3/2}}{3d} - \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{\sqrt{a + x}}{b^2 + x^2} dx\right)}{d} \\
&= \frac{2b(a + b \tan(c + dx))^{3/2}}{3d} - \frac{(2b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{x^2}{a^2 + b^2 - 2bx} dx\right)}{d} \\
&= \frac{2b(a + b \tan(c + dx))^{3/2}}{3d} - \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2 + b^2 - 2bx}} dx\right)}{d} \\
&= \frac{2b(a + b \tan(c + dx))^{3/2}}{3d} - \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2 + b^2 - 2bx}} dx\right)}{d} \\
&= -\frac{b(a^2 + b^2) \log\left(a + \sqrt{a^2 + b^2} + b \tan(c + dx) - \sqrt{2} \sqrt{\sqrt{a + \sqrt{a^2 + b^2}}}\right)}{2\sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}}} \\
&= -\frac{b(a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}\right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.31, size = 183, normalized size = 0.45

$$\frac{(a - b \tan(c + dx)) \left( 3i\sqrt{a - ib} (a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) \cos(c + dx) - 3i\sqrt{a + ib} (a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) \cos(c + dx) + 2b(a \cos(c + dx) + b \sin(c + dx)) \sqrt{a + b \tan(c + dx)} \right)}{3d(a \cos(c + dx) - b \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Tan[c + d\*x])\*(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((a - b\*Tan[c + d\*x])\*((3\*I)\*Sqrt[a - I\*b]\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]\*Cos[c + d\*x] - (3\*I)\*Sqrt[a + I\*b]\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]\*Cos[c + d\*x] + 2\*b\*(a\*Cos[c + d\*x] + b\*Sin[c + d\*x])\*Sqrt[a + b\*Tan[c + d\*x]]))/(3\*d\*(a\*Cos[c + d\*x] - b\*Sin[c + d\*x]))

Maple [A]

time = 0.14, size = 390, normalized size = 0.96

method	result
derivativedivides	$2b \frac{(a+b \tan(dx+c))^{\frac{3}{2}} + (-a^2-b^2)}{\sqrt{2\sqrt{a^2+b^2}+2a} (\sqrt{a^2+b^2}-a)} \frac{\ln(b \tan(dx+c)+a+\sqrt{a+b \tan(dx+c)})}{\sqrt{a+b \tan(dx+c)}}$
default	$2b \frac{(a+b \tan(dx+c))^{\frac{3}{2}} + (-a^2-b^2)}{\sqrt{2\sqrt{a^2+b^2}+2a} (\sqrt{a^2+b^2}-a)} \frac{\ln(b \tan(dx+c)+a+\sqrt{a+b \tan(dx+c)})}{\sqrt{a+b \tan(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))^(3/2), x, method=\_RETURNVERBOSE)

[Out] 2/d\*b\*(1/3\*(a+b\*tan(d\*x+c))^(3/2)+(-a^2-b^2)\*(-1/4\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*((a^2+b^2)^(1/2)-a)/b^2\*(1/2\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*((2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((2\*(a+b\*tan(d\*x+c))^(1/2)+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)))+1/4\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*((a^2+b^2)^(1/2)-a)/b^2\*(1/2\*ln(-b\*tan(d\*x+c)-a+(a+b\*tan(d\*x+c))^(1/2)\*((2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)-(a^2+b^2)^(1/2))-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))

$$2*a^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((-2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2))}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 4304 vs. 2(335) = 670.

time = 3.04, size = 4304, normalized size = 10.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$\frac{1}{12} * (12 * \sqrt{2} * d^5 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a*d^2*\sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})}) / (a^4b^2 + 2a^2b^4 + b^6) * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} * \arctan(-(\sqrt{2} * (a^6b + 3a^4b^3 + 3a^2b^5 + b^7) * d^5 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a*d^2*\sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})}) / (a^4b^2 + 2a^2b^4 + b^6)) * \sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c)) / \cos(d*x + c)} * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} - \sqrt{2} * d^5 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a*d^2*\sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})}) / (a^4b^2 + 2a^2b^4 + b^6) * \sqrt{(\sqrt{2} * (a^4b^3 + 2a^2b^5 + b^7) * d^3 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a*d^2*\sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})}) / (a^4b^2 + 2a^2b^4 + b^6)) * \sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c)) / \cos(d*x + c)} * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \cos(d*x + c) + (a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10}) * d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4} * \cos(d*x + c) + (a^{13}b^2 + 6a^{11}b^4 + 15a^9b^6 + 20a^7b^8 + 15a^5b^{10} + 6a^3b^{12} + a*b^{14}) * \cos(d*x + c) + (a^{12}b^3 + 6a^{10}b^5 + 15a^8b^7 + 20a^6b^9 + 15a^4b^{11} + 6a^2b^{13} + b^{15}) * \sin(d*x + c) / \cos(d*x + c)$$

$$\begin{aligned}
& + c)) * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} + (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})d^4 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} + (a^{15} + 7a^{13}b^2 + 21a^{11}b^4 + 35a^9b^6 + 35a^7b^8 + 21a^5b^{10} + 7a^3b^{12} + ab^{14}) * d^2 * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4}) / (a^{18}b^2 + 9a^{16}b^4 + 36a^{14}b^6 + 84a^{12}b^8 + 126a^{10}b^{10} + 126a^8b^{12} + 84a^6b^{14} + 36a^4b^{16} + 9a^2b^{18} + b^{20}) * \cos(dx + c) + 12 * \sqrt{2} * d^5 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a * d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})} / (a^4b^2 + 2a^2b^4 + b^6)) * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} * \arctan(-(\sqrt{2} * (a^6b + 3a^4b^3 + 3a^2b^5 + b^7) * d^5 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a * d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})} / (a^4b^2 + 2a^2b^4 + b^6))) * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)} * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} - \sqrt{2} * d^5 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a * d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})} / (a^4b^2 + 2a^2b^4 + b^6)) * \sqrt{-(\sqrt{2} * (a^4b^3 + 2a^2b^5 + b^7) * d^3 * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a * d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})} / (a^4b^2 + 2a^2b^4 + b^6))) * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)} * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \cos(dx + c) - (a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4} * \cos(dx + c) - (a^{13}b^2 + 6a^{11}b^4 + 15a^9b^6 + 20a^7b^8 + 15a^5b^{10} + 6a^3b^{12} + ab^{14}) * \cos(dx + c) - (a^{12}b^3 + 6a^{10}b^5 + 15a^8b^7 + 20a^6b^9 + 15a^4b^{11} + 6a^2b^{13} + b^{15}) * \sin(dx + c)) / \cos(dx + c)) * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(3/4)} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} - (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})d^4 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4} * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4} - (a^{15} + 7a^{13}b^2 + 21a^{11}b^4 + 35a^9b^6 + 35a^7b^8 + 21a^5b^{10} + 7a^3b^{12} + ab^{14}) * d^2 * \sqrt{(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10})/d^4}) / (a^{18}b^2 + 9a^{16}b^4 + 36a^{14}b^6 + 84a^{12}b^8 + 126a^{10}b^{10} + 126a^8b^{12} + 84a^6b^{14} + 36a^4b^{16} + 9a^2b^{18} + b^{20}) * \cos(dx + c) - 3 * \sqrt{2} * ((a^5 + 2a^3b^2 + ab^4) * d^3 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4} * \cos(dx + c) - (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) * d * \cos(dx + c)) * \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + a * d^2 * \sqrt{(a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4})} / (a^4b^2 + 2a^2b^4 + b^6)) * ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})/d^4)^{(1/4)} * \log((\sqrt{2} * (a^4b^3 + 2a^2b^5 +
\end{aligned}$$



$$\begin{aligned}
& 2) * (a^4 + b^4 - 6a^2b^2) / d^2 - (16ab^2 * (((-b^6d^4(3a^2 - b^2)^2)^{(1/2)} - 3a^2b^4d^2 + a^3b^2d^2) / d^4)^{(1/2)} * (a^2b + b^3 + d * (((-b^6d^4(3a^2 - b^2)^2)^{(1/2)} - 3a^2b^4d^2 + a^3b^2d^2) / d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / d * (((-b^6d^4(3a^2 - b^2)^2)^{(1/2)} - 3a^2b^4d^2 + a^3b^2d^2) / d^4)^{(1/2)} / 2 - (8b^5(a^2 - b^2)(a^2 + b^2)^2) / d^3 * (((6a^2b^8d^4 - b^{10}d^4 - 9a^4b^6d^4)^{(1/2)} - 3a^2b^4d^2 + a^3b^2d^2) / (4d^4))^{(1/2)} + \log((((16b^4(a + b \tan(c + dx))^{(1/2)} * (a^4 + b^4 - 6a^2b^2)) / d^2 + (16ab^2 * (-((-b^6d^4(3a^2 - b^2)^2)^{(1/2)} + 3a^2b^4d^2 - a^3b^2d^2) / d^4)^{(1/2)} * (a^2b + b^3 - d * (-((-b^6d^4(3a^2 - b^2)^2)^{(1/2)} + 3a^2b^4d^2 - a^3b^2d^2) / d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / d * (-((-b^6d^4(3a^2 - b^2)^2)^{(1/2)} + 3a^2b^4d^2 - a^3b^2d^2) / d^4)^{(1/2)} / 2 - (8b^5(a^2 - b^2)(a^2 + b^2)^2) / d^3 * ((a^3b^2) / (4d^2) - (3a^2b^4) / (4d^2) - (6a^2b^8d^4 - b^{10}d^4 - 9a^4b^6d^4)^{(1/2)} / (4d^4))^{(1/2)} - \log((((-((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} + a^5d^2 - 3a^3b^2d^2) / d^4)^{(1/2)} * ((16a^2b^2(a + b \tan(c + dx))^{(1/2)} * (a^4 + b^4 - 6a^2b^2)) / d^2 - (16ab^2 * (-((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} + a^5d^2 - 3a^3b^2d^2) / d^4)^{(1/2)} * (a^2b + b^3 - d * (-((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} + a^5d^2 - 3a^3b^2d^2) / d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / d)) / 2 + (16a^4b^3(a^2 + b^2)^2) / d^3 * (-((6a^6b^4d^4 - a^4b^6d^4 - 9a^8b^2d^4)^{(1/2)} + a^5d^2 - 3a^3b^2d^2) / (4d^4))^{(1/2)} - \log(((((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * ((16a^2b^2(a + b \tan(c + dx))^{(1/2)} * (a^4 + b^4 - 6a^2b^2)) / d^2 - (16ab^2 * (-((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * (a^2b + b^3 - d * (((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / d)) / 2 + (16a^4b^3(a^2 + b^2)^2) / d^3 * (((6a^6b^4d^4 - a^4b^6d^4 - 9a^8b^2d^4)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / (4d^4))^{(1/2)} + \log((16a^4b^3(a^2 + b^2)^2) / d^3 - ((((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * ((16a^2b^2(a + b \tan(c + dx))^{(1/2)} * (a^4 + b^4 - 6a^2b^2)) / d^2 + (16ab^2 * (((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * (a^2b + b^3 + d * (((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / d)) / 2 * ((6a^6b^4d^4 - a^4b^6d^4 - 9a^8b^2d^4)^{(1/2)} / (4d^4) - a^5 / (4d^2) + (3a^3b^2) / (4d^2))^{(1/2)} + \log((16a^4b^3(a^2 + b^2)^2) / d^3 - ((((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} + a^5d^2 - 3a^3b^2d^2) / d^4)^{(1/2)} * ((16a^2b^2(a + b \tan(c + dx))^{(1/2)} * (a^4 + b^4 - 6a^2b^2)) / d^2 + (16ab^2 * (((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} - a^5d^2 + 3a^3b^2d^2) / d^4)^{(1/2)} * (a^2b + b^3 + d * (((-a^4b^2d^4(3a^2 - b^2)^2)^{(1/2)} + a^5d^2 - 3a^3b^2d^2) / d^4)^{(1/2)} * (a + b \tan(c + dx))^{(1/2)})) / d)) / 2 * ((3a^3b^2) / (4d^2) - a^5 / (4d^2) - (6a^6b^4d^4 - a^4b^6d^4 - 9a^8b^2d^4)^{(1/2)} / (4d^4))^{(1/2)} + (2b * (a + b \tan(c + dx))^{(3/2)}) / (3d)
\end{aligned}$$

### 3.342 $\int (-a + b \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$

**Optimal.** Leaf size=422

$$\frac{b\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}\right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} + \frac{b\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{\sqrt{a + \sqrt{a^2 + b^2}}}{\sqrt{a - \sqrt{a^2 + b^2}}}\right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d}$$

[Out]  $-1/2*b*\operatorname{arctanh}(((a+(a^2+b^2)^{(1/2)})^{(1/2)}-2^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)})/(a-(a^2+b^2)^{(1/2)})^{(1/2)})*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a-(a^2+b^2)^{(1/2)})^{(1/2)}+1/2*b*\operatorname{arctanh}(((a+(a^2+b^2)^{(1/2)})^{(1/2)}+2^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)})/(a-(a^2+b^2)^{(1/2)})^{(1/2)})*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a-(a^2+b^2)^{(1/2)})^{(1/2)}+1/4*b*\ln(a+(a^2+b^2)^{(1/2)}-2^{(1/2)}*(a+(a^2+b^2)^{(1/2)})^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}+b*\tan(d*x+c))*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a+(a^2+b^2)^{(1/2)})^{(1/2)}-1/4*b*\ln(a+(a^2+b^2)^{(1/2)}+2^{(1/2)}*(a+(a^2+b^2)^{(1/2)})^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}+b*\tan(d*x+c))*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a+(a^2+b^2)^{(1/2)})^{(1/2)}+2*b*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.34, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3609, 12, 3566, 722, 1108, 648, 632, 212, 642}

$$\frac{b\sqrt{a^2+b^2} \log\left(\frac{-\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)}{2\sqrt{2}d\sqrt{a^2+b^2}+a}\right)}{2\sqrt{2}d\sqrt{a^2+b^2}+a} - \frac{b\sqrt{a^2+b^2} \log\left(\frac{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+a+b\tan(c+dx)}{2\sqrt{2}d\sqrt{a^2+b^2}+a}\right)}{2\sqrt{2}d\sqrt{a^2+b^2}+a} - \frac{b\sqrt{a^2+b^2} \operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}+a-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}} + \frac{b\sqrt{a^2+b^2} \operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}+a+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}d\sqrt{a-\sqrt{a^2+b^2}}} + \frac{2b\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Tan}[c + d*x])*Sqrt[a + b*\operatorname{Tan}[c + d*x]], x]$

[Out]  $-((b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]]) - Sqrt[2]*Sqrt[a + b*\operatorname{Tan}[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]]/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) + (b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]]) + Sqrt[2]*Sqrt[a + b*\operatorname{Tan}[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]]]/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]*d) + (b*Sqrt[a^2 + b^2]*Log[a + Sqrt[a^2 + b^2] + b*\operatorname{Tan}[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*\operatorname{Tan}[c + d*x]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) - (b*Sqrt[a^2 + b^2]*Log[a + Sqrt[a^2 + b^2] + b*\operatorname{Tan}[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*\operatorname{Tan}[c + d*x]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (2*b*Sqrt[a + b*\operatorname{Tan}[c + d*x]])/d$

Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$



Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 648

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 722

Int[1/(Sqrt[(d\_) + (e\_)\*(x\_)])\*((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[2\*e, Subst[Int[1/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

Rule 1108

Int[((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(r - x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(r + x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && NegQ[b^2 - 4\*a\*c]

Rule 3566

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (-a + b \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx &= \frac{2b \sqrt{a + b \tan(c + dx)}}{d} + \int \frac{-a^2 - b^2}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{2b \sqrt{a + b \tan(c + dx)}}{d} + (-a^2 - b^2) \int \frac{1}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{2b \sqrt{a + b \tan(c + dx)}}{d} - \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a + x}} dx, x, b \tan(c + dx)\right)}{d} \\
&= \frac{2b \sqrt{a + b \tan(c + dx)}}{d} - \frac{(2b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{a^2 + b^2 - 2ax^2} dx, x, \frac{b \tan(c + dx)}{a}\right)}{d} \\
&= \frac{2b \sqrt{a + b \tan(c + dx)}}{d} - \frac{(b\sqrt{a^2 + b^2}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2 + b^2 - x^2}} dx, x, \frac{b \tan(c + dx)}{a}\right)}{d} \\
&= \frac{2b \sqrt{a + b \tan(c + dx)}}{d} - \frac{(b\sqrt{a^2 + b^2}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2 + b^2 - x^2}} dx, x, \frac{b \tan(c + dx)}{a}\right)}{d} \\
&= \frac{b\sqrt{a^2 + b^2} \log\left(a + \sqrt{a^2 + b^2} + b \tan(c + dx) - \sqrt{2} \sqrt{a + b \tan(c + dx)}\right)}{2\sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}}} \\
&= \frac{b\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}\right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.16, size = 157, normalized size = 0.37

$$\frac{\cos(c + dx)(a - b \tan(c + dx)) \left( i\sqrt{a - ib} (a + ib) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right) - i(a - ib)\sqrt{a + ib} \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right) + 2b\sqrt{a + b \tan(c + dx)} \right)}{d(a \cos(c + dx) - b \sin(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Tan[c + d\*x])\*Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] (Cos[c + d\*x]\*(a - b\*Tan[c + d\*x])\*(I\*Sqrt[a - I\*b]\*(a + I\*b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] - I\*(a - I\*b)\*Sqrt[a + I\*b]\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]] + 2\*b\*Sqrt[a + b\*Tan[c + d\*x]]))/(d\*(a\*Cos[c + d\*x] - b\*Sin[c + d\*x]))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(341) = 682.

time = 0.14, size = 802, normalized size = 1.90

method	result
derivativedivides	$2b \left( \sqrt{a + b \tan(dx + c)} + (-a^2 - b^2) \left( \frac{\left( \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^2 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \right)}{\dots} \right) \right)$
default	$2b \left( \sqrt{a + b \tan(dx + c)} + (-a^2 - b^2) \left( \frac{\left( \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^2 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \right)}{\dots} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVERBOSE)

[Out] 2/d\*b\*((a+b\*tan(d\*x+c))^(1/2)+(-a^2-b^2)\*(1/4/b^2/(a^2+b^2)^(3/2)\*(1/2\*((2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^2+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b^2-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^3-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b^2)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2)))+2\*(2\*a^2\*b^2+2\*b^4-1/2\*((2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^2+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b^2-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^3-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b^2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((2\*(a+b\*tan(d\*x+c))^(1/2)+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2))





$c) + b \sin(dx + c) / \cos(dx + c) * ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) / d^4)^{1/4} \cos(dx + c) - (a^6b^2 + 3a^4b^4 + 3a^2b^6 + b^8) * d^2 \sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6) / d^4} \cos(dx + c) - (a^9b^2 + 4a^7b^4 + 6a^5b^6 + 4a^3b^8 + ab^{10}) \cos(dx + c) - (a^8b^3 + 4a^6b^5 + 6a^4b^7 + 4a^2b^9 + b^{11}) \sin(dx + c) / \cos(dx + c) + 8(a^6b + 3a^4b^3 + 3a^2b^5 + b^7) \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)} / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int a \sqrt{a + b \tan(c + dx)} dx - \int \left( -b \sqrt{a + b \tan(c + dx)} \tan(c + dx) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] -Integral(a\*sqrt(a + b\*tan(c + d\*x)), x) - Integral(-b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))\*(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*tan(d\*x + c) + a)\*(b\*tan(d\*x + c) - a), x)

**Mupad [B]**

time = 9.02, size = 581, normalized size = 1.38

$$\frac{a \left( \frac{a \sqrt{a + b \tan(dx + c)}}{b \sqrt{a + b \tan(dx + c)}} \right) \sqrt{\frac{a + b \tan(dx + c)}{a}} + \frac{a \sqrt{a + b \tan(dx + c)}}{b \sqrt{a + b \tan(dx + c)}} \sqrt{\frac{a + b \tan(dx + c)}{a}} - \frac{a \sqrt{a + b \tan(dx + c)}}{b \sqrt{a + b \tan(dx + c)}} \sqrt{\frac{a + b \tan(dx + c)}{a}} - \frac{a \sqrt{a + b \tan(dx + c)}}{b \sqrt{a + b \tan(dx + c)}} \sqrt{\frac{a + b \tan(dx + c)}{a}}}{\sqrt{\frac{a + b \tan(dx + c)}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*tan(c + d\*x))^(1/2)\*(a - b\*tan(c + d\*x)),x)

[Out] atan((b^6\*((b^3\*1i)/(4\*d^2) + (a\*b^2)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*32i)/((b^8\*16i)/d + (a^2\*b^6\*16i)/d) - (32\*a\*b^5\*((b^3\*1i)/(4\*d^2) + (a\*b^2)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2))/((b^8\*16i)/d + (a^2\*b^6\*16i)/d))\*((a\*b^2 + b^3\*1i)/(4\*d^2))^(1/2)\*2i - atan((b^6\*((a\*b^2)/(4\*d^2) - (b^3\*1i)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*32i)/((b^8\*16i)/d + (a^2\*b^6\*16i)/d) + (32\*a\*b^5\*((a\*b^2)/(4\*d^2) - (b^3\*1i)/(4\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2))/((b^8\*16i)/d + (a^2\*b^6\*16i)/d))\*((a\*b^2 - b^3\*1i)/(4\*d^2))^(1/2)\*2i + atanh((d^3\*((16\*(a^2\*b^4 - a^4\*b^2)\*(a + b\*tan(c + d\*x))^(1/2)

$$\begin{aligned}
& 1/2))/d^2 + (16*a*b^2*(a^2*b*1i + a^3)*(a + b*\tan(c + d*x))^(1/2))/d^2)*(- \\
& a^2*b*1i + a^3)/d^2)^(1/2))/(16*(a^3*b^5 + a^5*b^3))*(-(a^2*b*1i + a^3)/d^ \\
& 2)^(1/2) + \operatorname{atanh}((d^3*((a^2*b*1i - a^3)/d^2)^(1/2)*((16*(a^2*b^4 - a^4*b^2) \\
& *(a + b*\tan(c + d*x))^(1/2))/d^2 - (16*a*b^2*(a^2*b*1i - a^3)*(a + b*\tan(c \\
& + d*x))^(1/2))/d^2))/(16*(a^3*b^5 + a^5*b^3))*((a^2*b*1i - a^3)/d^2)^(1/2) \\
& + (2*b*(a + b*\tan(c + d*x))^(1/2))/d
\end{aligned}$$

$$3.343 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=213

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d} - \frac{2(10aAb - 8a^2B + 15b^2B)}{15b^2d}$$

[Out] (A-I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d/(a-I\*b)^(1/2)+(A+I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d/(a+I\*b)^(1/2)-2/15\*(10\*A\*a\*b-8\*B\*a^2+15\*B\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/b^3/d+2/15\*(5\*A\*b-4\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)/b^2/d+2/5\*B\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^2/b/d

**Rubi [A]**

time = 0.35, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3688, 3728, 3711, 3620, 3618, 65, 214}

$$\frac{-2(-8a^2B + 10aAb + 15b^2B) \sqrt{a+b \tan(c+dx)}}{15b^2d} + \frac{2(5Ab - 4aB) \tan(c+dx) \sqrt{a+b \tan(c+dx)}}{15b^2d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} + \frac{2B \tan^2(c+dx) \sqrt{a+b \tan(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/(Sqrt[a - I\*b]\*d) + ((A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/(Sqrt[a + I\*b]\*d) - (2\*(10\*a\*A\*b - 8\*a^2\*B + 15\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*b^3\*d) + (2\*(5\*A\*b - 4\*a\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*b^2\*d) + (2\*B\*Tan[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*b\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618



```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
```

, 0] && NeQ[a, 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx &= \frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd} + \frac{2\int \frac{\tan(c+dx)(-2aB-\frac{5}{2}bB\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx}{5bd} \\
 &= \frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd} \\
 &= -\frac{2(10aAb-8a^2B+15b^2B)\sqrt{a+b\tan(c+dx)}}{15b^3d} + \frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} \\
 &= -\frac{2(10aAb-8a^2B+15b^2B)\sqrt{a+b\tan(c+dx)}}{15b^3d} + \frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} \\
 &= -\frac{2(10aAb-8a^2B+15b^2B)\sqrt{a+b\tan(c+dx)}}{15b^3d} + \frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} \\
 &= -\frac{2(10aAb-8a^2B+15b^2B)\sqrt{a+b\tan(c+dx)}}{15b^3d} + \frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} \\
 &= \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}
 \end{aligned}$$

**Mathematica [A]**

time = 2.87, size = 170, normalized size = 0.80

$$\frac{15(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{15(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{2\sqrt{a+b\tan(c+dx)}(-10aAb+8a^2B-15b^2B+b(5Ab-4aB)\tan(c+dx)+3b^2B\tan^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((15\*(A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/Sqrt[a - I\*b] + (15\*(A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/Sqrt[a + I\*b] + (2\*Sqrt[a + b\*Tan[c + d\*x]]\*(-10\*a\*A\*b + 8\*a^2\*B - 15\*b^2\*B + b\*(5\*A\*b - 4\*a\*B)\*Tan[c + d\*x] + 3\*b^2\*B\*Tan[c + d\*x]^2))/b^3)/(15\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. 2(183) = 366.

time = 0.15, size = 1503, normalized size = 7.06

method	result	size
derivativedivides	Expression too large to display	1503
default	Expression too large to display	1503

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\tan(dx+c)^3(A+B\tan(dx+c))/(a+b\tan(dx+c))^{1/2}, x, \text{method}=\_RETURNVE$   
RBOSE)

[Out]  $2/d/b^3*(1/5*B*(a+b\tan(dx+c))^{5/2}+1/3*A*b*(a+b\tan(dx+c))^{3/2}-2/3*B*$   
 $a*(a+b\tan(dx+c))^{3/2}-A*a*b*(a+b\tan(dx+c))^{1/2}+a^2*B*(a+b\tan(dx+c))^{1/2}-B*b^2*(a+b\tan(dx+c))^{1/2}+b^3*(1/4/b^2/(a^2+b^2)^{3/2}*(1/2*(-A*$   
 $(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*$   
 $b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}$   
 $*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a*b^3)*\ln(b*\tan(dx+c)+a+(a+b*\tan(dx+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}+(a^2+b^2)^{1/2}))$   
 $+2*(2*A*a^3*b^2+2*A*a*b^4+2*B*a^2*b^3+2*B*b^5-1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*$   
 $(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a*b^3)*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}))/((2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((2*(a+b*\tan(dx+c))^{1/2}+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}))/((2*(a^2+b^2)^{1/2}-2*a)^{1/2}))+1/4/b^2/(a^2+b^2)^{3/2}*(-1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a*b^3)*\ln(-b*\tan(dx+c)-a+(a+b*\tan(dx+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}-(a^2+b^2)^{1/2}))$   
 $+2*(-2*A*a^3*b^2-2*A*a*b^4-2*B*a^2*b^3-2*B*b^5+1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a*b^3)*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}))/((2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((-2*(a+b*\tan(dx+c))^{1/2}+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}))/((2*(a^2+b^2)^{1/2}-2*a)^{1/2}))))$

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^3/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8574 vs. 2(177) = 354.

time = 8.31, size = 8574, normalized size = 40.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/60*(60*\sqrt{2}*(a^2*b^3 + b^5)*d^5*\sqrt{-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/(a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2})/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/(a^2 + b^2)*d^4))^{3/4}*a \\ & \operatorname{rctan}(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^2 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^3 + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^4 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^5)*d^4*\sqrt{((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/(a^2 + b^2)*d^4))} \\ & + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^3)*d^2*\sqrt{((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))} \\ & - \sqrt{2}*((A*a^5 + B*a^4*b + 2*A*a^3*b^2 + 2*B*a^2*b^3 + A*a*b^4 + B*b^5)*d^7*\sqrt{((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/(a^2 + b^2)*d^4))} \\ & + ((A^3 + A*B^2)*a^4 + 2*(A^3 + A*B^2)*a^2*b^2 + (A^3 + A*B^2)*b^4)*d^5*\sqrt{((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)))*\sqrt{-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/(a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2})/(4* \end{aligned}$$

$$\begin{aligned}
& A^2 B^2 a^2 - 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) \sqrt{((4(A^4 B^2 + A^2 B^4) a^4 - 4(A^5 B - A B^5) a^3 b + (A^6 + 3A^4 B^2 + 3A^2 B^4 + B^6) a^2 b^2 - 4(A^5 B - A B^5) a b^3 + (A^6 - A^4 B^2 - A^2 B^4 + B^6) b^4) d^2 \sqrt{(A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4)} \cos(dx + c) + \sqrt{2} * ((4A^3 B^2 a^4 - 4(A^4 B - A^2 B^3) a^3 b + (A^5 + 2A^3 B^2 + A B^4) a^2 b^2 - 4(A^4 B - A^2 B^3) a b^3 + (A^5 - 2A^3 B^2 + A B^4) b^4) d^3 \sqrt{(A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4)} \cos(dx + c) + (4(A^5 B^2 + A^3 B^4) a^3 - 4(A^6 B - A^4 B^3 - 2A^2 B^5) a^2 b + (A^7 - 5A^5 B^2 - A^3 B^4 + 5A B^6) a b^2 + (A^6 B - A^4 B^3 - A^2 B^5 + B^7) b^3) * d \cos(dx + c)) \sqrt{-((2A B a^2 b + 2A B b^3 + (A^2 - B^2) a^3 + (A^2 - B^2) a b^2) d^2 \sqrt{(A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4)}) - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 - 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2)) \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)) * ((A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4))^{1/4} + (4(A^6 B^2 + 2A^4 B^4 + A^2 B^6) a^3 - 4(A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a^2 b + (A^8 - 2A^4 B^4 + B^8) a b^2) \cos(dx + c) + (4(A^6 B^2 + 2A^4 B^4 + A^2 B^6) a^2 b - 4(A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a b^2 + (A^8 - 2A^4 B^4 + B^8) b^3) \sin(dx + c)) / \cos(dx + c)) * ((A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4))^{3/4} + \sqrt{2} * ((2(A^4 B + A^2 B^3) a^6 - (A^5 - 2A^3 B^2 - 3A B^4) a^5 b + (3A^4 B + 4A^2 B^3 + B^5) a^4 b^2 - 2(A^5 - 2A^3 B^2 - 3A B^4) a^3 b^3 + 2(A^2 B^3 + B^5) a^2 b^4 - (A^5 - 2A^3 B^2 - 3A B^4) a b^5 - (A^4 B - B^5) b^6) d^7 \sqrt{(4A^2 B^2 a^2 - 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) / ((a^4 + 2a^2 b^2 + b^4) d^4)} * \sqrt{(A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4)} + (2(A^6 B + 2A^4 B^3 + A^2 B^5) a^5 - (A^7 + A^5 B^2 - A^3 B^4 - A B^6) a^4 b + 4(A^6 B + 2A^4 B^3 + A^2 B^5) a^3 b^2 - 2(A^7 + A^5 B^2 - A^3 B^4 - A B^6) a^2 b^3 + 2(A^6 B + 2A^4 B^3 + A^2 B^5) a b^4 - (A^7 + A^5 B^2 - A^3 B^4 - A B^6) b^5) d^5 \sqrt{(4A^2 B^2 a^2 - 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) / ((a^4 + 2a^2 b^2 + b^4) d^4)} * \sqrt{-((2A B a^2 b + 2A B b^3 + (A^2 - B^2) a^3 + (A^2 - B^2) a b^2) d^2 \sqrt{(A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4)}) - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 - 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2)) \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)) * ((A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4))^{3/4} / (4(A^{10} B^2 + 4A^8 B^4 + 6A^6 B^6 + 4A^4 B^8 + A^2 B^{10}) a^2 b - 4(A^{11} B + 3A^9 B^3 + 2A^7 B^5 - 2A^5 B^7 - 3A^3 B^9 - A B^{11}) a b^2 + (A^{12} + 2A^{10} B^2 - A^8 B^4 - 4A^6 B^6 - A^4 B^8 + 2A^2 B^{10} + B^{12}) b^3) \cos(dx + c)^2 + 60 \sqrt{2} * (a^2 b^3 + b^5) d^5 \sqrt{-((2A B a^2 b + 2A B b^3 + (A^2 - B^2) a^3 + (A^2 - B^2) a b^2) d^2 \sqrt{(A^4 + 2A^2 B^2 + B^4) / ((a^2 + b^2) d^4)}) - (A^4 + 2A^2 B^2 + B^4) a^2 - (A^4 + 2A^2 B^2 + B^4) b^2) / (4A^2 B^2 a^2 - 4(A^3 B - A B^3) a b + (A^4 - 2A^2 B^2 + B^4) b^2) / ((a^4 + 2a^2 b^2 + b^4) d^4) \dots
\end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^3(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3/sqrt(a + b\*tan(c + d\*x)), x)

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [B]

time = 13.97, size = 3054, normalized size = 14.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^3\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] atan((B^2\*b^2\*((-16\*B^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)) - (B^2\*a\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4)))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*32i)/((16\*B^3\*a\*b^3\*d^3)/(a^2\*d^4 + b^2\*d^4) - (4\*B\*b^3\*d^2\*(-16\*B^4\*b^2\*d^4)^(1/2))/(a^2\*d^5 + b^2\*d^5)) + (a\*b^2\*((-16\*B^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)) - (B^2\*a\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4)))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*(-16\*B^4\*b^2\*d^4)^(1/2)\*8i)/((16\*B^3\*a\*b^5\*d^5)/(a^2\*d^4 + b^2\*d^4) - (4\*B\*b^5\*d^4\*(-16\*B^4\*b^2\*d^4)^(1/2))/(a^2\*d^5 + b^2\*d^5) + (16\*B^3\*a^3\*b^3\*d^5)/(a^2\*d^4 + b^2\*d^4) - (4\*B\*a^2\*b^3\*d^4\*(-16\*B^4\*b^2\*d^4)^(1/2))/(a^2\*d^5 + b^2\*d^5)) - (B^2\*a^2\*b^2\*d^2\*((-16\*B^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)) - (B^2\*a\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4)))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*32i)/((16\*B^3\*a\*b^5\*d^5)/(a^2\*d^4 + b^2\*d^4) - (4\*B\*b^5\*d^4\*(-16\*B^4\*b^2\*d^4)^(1/2))/(a^2\*d^5 + b^2\*d^5) + (16\*B^3\*a^3\*b^3\*d^5)/(a^2\*d^4 + b^2\*d^4) - (4\*B\*a^2\*b^3\*d^4\*(-16\*B^4\*b^2\*d^4)^(1/2))/(a^2\*d^5 + b^2\*d^5)))\*((-16\*B^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)) - (B^2\*a\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4)))^(1/2)\*2i - atan((A^2\*b^2\*((A^2\*a\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4)) - (-16\*A^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*

$$\begin{aligned}
& 32i)/((16*A^3*b^2)/d - (16*A^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) + (4*A*a*b^2*d^2*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (a*b^2*((A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*A^4*b^2*d^4)^{(1/2)}*8i)/(16*A^3*b^4*d + 16*A^3*a^2*b^2*d - (16*A^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*A^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*a^3*b^2*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*A*a*b^4*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (A^2*a^2*b^2*d^2*((A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*32i)/(16*A^3*b^4*d + 16*A^3*a^2*b^2*d - (16*A^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*A^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*a^3*b^2*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*A*a*b^4*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*((A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)}*2i - \operatorname{atan}((a*b^2*((-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*A^4*b^2*d^4)^{(1/2)}*8i)/((16*A^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*A^3*a^2*b^2*d - 16*A^3*b^4*d + (16*A^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*a^3*b^2*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*A*a*b^4*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (A^2*b^2*((-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*32i)/((16*A^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) - (16*A^3*b^2)/d + (4*A*a*b^2*d^2*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (A^2*a^2*b^2*d^2*((-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*32i)/((16*A^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*A^3*a^2*b^2*d - 16*A^3*b^4*d + (16*A^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*a^3*b^2*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*A*a*b^4*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*((-16*A^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*2i - ((2*B*(a^2 + b^2))/(b^3*d) - (4*B*a^2)/(b^3*d))*(a + b*\tan(c + d*x))^{(1/2)} - \operatorname{atan}((a*b^2*(-(-16*B^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*B^4*b^2*d^4)^{(1/2)}*8i)/((16*B^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*b^5*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (16*B^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (B^2*b^2*(-(-16*B^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*32i)/((16*B^3*a*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*B*b^3*d^2*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (B^2*a^2*b^2*d^2*(-(-16*B^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*32i)/((16*B^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*b^5*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (16*B^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*(-(-16*B^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*2i + (2*A*(a + b*\tan(c + d*x))^{(3/2)})/(3*b^2*
\end{aligned}$$

$$d) + (2*B*(a + b*\tan(c + d*x))^{5/2})/(5*b^3*d) - (2*A*a*(a + b*\tan(c + d*x))^{1/2})/(b^2*d) - (4*B*a*(a + b*\tan(c + d*x))^{3/2})/(3*b^3*d)$$



$$3.344 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=166

$$\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} - \frac{(iA-B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d} + \frac{2(3Ab-2aB)\sqrt{a-ib}}{3b^2 d}$$

[Out] (I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d/(a-I\*b)^(1/2)-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d/(a+I\*b)^(1/2)+2/3\*(3\*A\*b-2\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/b^2/d+2/3\*B\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)/b/d

Rubi [A]

time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {3688, 3711, 3620, 3618, 65, 214}

$$\frac{2(3Ab-2aB)\sqrt{a+b \tan(c+dx)}}{3b^2 d} + \frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} + \frac{2B \tan(c+dx)\sqrt{a+b \tan(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/(Sqrt[a - I\*b]\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/(Sqrt[a + I\*b]\*d) + (2\*(3\*A\*b - 2\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*b^2\*d) + (2\*B\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*b\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c

\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 - I\*Tan[e + f\*x])), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 + I\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx &= \frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} + \frac{2\int \frac{-aB-\frac{3}{2}bB\tan(c+dx)+\frac{1}{2}(3A^2-3Ab+2aB)}{\sqrt{a+b\tan(c+dx)}} dx}{3b} \\
&= \frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} \\
&= \frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} \\
&= \frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} \\
&= \frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} \\
&= \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}
\end{aligned}$$

**Mathematica [A]**

time = 1.11, size = 139, normalized size = 0.84

$$\frac{3^{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}}{\sqrt{a-ib}} + \frac{3^{(-iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}}{\sqrt{a+ib}} + \frac{2\sqrt{a+b\tan(c+dx)}(3Ab-2aB+bB\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

**[In]** Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

**[Out]** ((3\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/Sqrt[a - I\*b] + (3\*((-I)\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/Sqrt[a + I\*b] + (2\*Sqrt[a + b\*Tan[c + d\*x]]\*(3\*A\*b - 2\*a\*B + b\*B\*Tan[c + d\*x]))/b^2)/(3\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. 2(140) = 280.

time = 0.14, size = 1463, normalized size = 8.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVE RBOSE)

```
[Out] 2/d/b^2*(1/3*B*(a+b*tan(d*x+c))^(3/2)+A*b*(a+b*tan(d*x+c))^(1/2)-B*a*(a+b*tan(d*x+c))^(1/2)-b^2*(1/4/b^2/(a^2+b^2)^(3/2)*(1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*A*a^2*b^3+2*A*b^5-2*B*a^3*b^2-2*B*a*b^4-1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*A*a^2*b^3-2*A*b^5+2*B*a^3*b^2+2*B*a*b^4+1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*tan(d*x + c)^2/sqrt(b*tan(d*x + c) + a), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 8468 vs. 2(134) = 268.

time = 8.12, size = 8468, normalized size = 51.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$-1/12*(12*\sqrt{2}*(a^2*b^2 + b^4)*d^5*\sqrt{((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^{3/4}*\arctan(((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^2 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^3 + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^4 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^5)*d^4*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^3)*d^2*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} - \sqrt{2}*(B*a^5 - A*a^4*b + 2*B*a^3*b^2 - 2*A*a^2*b^3 + B*a*b^4 - A*b^5)*d^7*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + ((A^2*B + B^3)*a^4 + 2*(A^2*B + B^3)*a^2*b^2 + (A^2*B + B^3)*b^4)*d^5*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\sqrt{((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{((4*(A^4*B^2 + A^2*B^4)*a^4 - 4*(A^5*B - A*B^5)*a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 - 4*(A^5*B - A*B^5)*a*b^3 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^4)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)}*\cos(d*x + c) + \sqrt{2}*(4*A^2*B^3*a^4 - 4*(A^3*B^2 - A*B^4)*a^3*b + (A^4*B + 2*A^2*B^3 + B^5)*a^2*b^2 - 4*(A^3*B^2 - A*B^4)*a*b^3 + (A^4*B - 2*A^2*B^3 + B^5)*b^4)*d^3*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)}*\cos(d*x + c) + (4*(A^4*B^3 + A^2*B^5)*a^3 - 4*(2*A^5*B^2 + A^3*B^4 - A*B^6)*a^2*b + (5*A^6*B - A^4*B^3 - 5*A^2*B^5 + B^7)*a*b^2 - (A^7 - A^5*B^2 - A^3*B^4 + A*B^6)*b^3)*d*\cos(d*x + c))*\sqrt{((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A$$

```

*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x + c) + b*sin(d*x
+ c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(1/4) + (4*
(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^3 - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)
*a^2*b + (A^8 - 2*A^4*B^4 + B^8)*a*b^2))*cos(d*x + c) + (4*(A^6*B^2 + 2*A^4*
B^4 + A^2*B^6)*a^2*b - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^2 + (A^8 -
2*A^4*B^4 + B^8)*b^3))*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)
/((a^2 + b^2)*d^4))^(3/4) + sqrt(2))*((2*(A^3*B^2 + A*B^4)*a^6 - (3*A^4*B +
2*A^2*B^3 - B^5)*a^5*b + (A^5 + 4*A^3*B^2 + 3*A*B^4)*a^4*b^2 - 2*(3*A^4*B +
2*A^2*B^3 - B^5)*a^3*b^3 + 2*(A^5 + A^3*B^2)*a^2*b^4 - (3*A^4*B + 2*A^2*B^
3 - B^5)*a*b^5 + (A^5 - A*B^4)*b^6)*d^7*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A
B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt
((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) + (2*(A^5*B^2 + 2*A^3*B^4 + A*B
^6)*a^5 - (A^6*B + A^4*B^3 - A^2*B^5 - B^7)*a^4*b + 4*(A^5*B^2 + 2*A^3*B^4
+ A*B^6)*a^3*b^2 - 2*(A^6*B + A^4*B^3 - A^2*B^5 - B^7)*a^2*b^3 + 2*(A^5*B^2
+ 2*A^3*B^4 + A*B^6)*a*b^4 - (A^6*B + A^4*B^3 - A^2*B^5 - B^7)*b^5)*d^5*sq
rt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*
a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)
) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a
^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x
+ c) + b*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*
d^4))^(3/4))/(4*(A^10*B^2 + 4*A^8*B^4 + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^10)*a
^2*b - 4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*
a*b^2 + (A^12 + 2*A^10*B^2 - A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 + B
^12)*b^3))*cos(d*x + c) + 12*sqrt(2)*(a^2*b^2 + b^4)*d^5*sqrt(((2*A*B*a^2*b
+ 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B
^2 + B^4)/((a^2 + b^2)*d^4)) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B
^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 +
B^4)*b^2))*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 +
B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*((A^4...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^2(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2/sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 10.55, size = 2981, normalized size = 17.96
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] 2*atanh((32*A^2*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*A^3*a*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-16*A^4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*A^2*a^2*b^2*d^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) - atan((a*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2)*8i)/((16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (B^2*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*2i - atan((B^2*b^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)
```

$$\begin{aligned}
& \left(\frac{1}{2}\right) * (a + b * \tan(c + d * x))^{\frac{1}{2}} * 32i / \left( \frac{(16 * B^3 * b^2)}{d} - (16 * B^3 * a^2 * b^2 * d^3) \right. \\
& \left. / (a^2 * d^4 + b^2 * d^4) + (4 * B * a * b^2 * d^2 * (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) \right) \\
& + (a * b^2 * ((B^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4))) - (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}} / (16 * (a^2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} * (a + b * \tan(c + d * x))^{\frac{1}{2}} * (-16 * B^4 * \\
& b^2 * d^4)^{\frac{1}{2}} * 8i / (16 * B^3 * b^4 * d + 16 * B^3 * a^2 * b^2 * d - (16 * B^3 * a^2 * b^4 * d^5) / \\
& (a^2 * d^4 + b^2 * d^4) - (16 * B^3 * a^4 * b^2 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * B * a^3 * b \\
& ^2 * d^4 * (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) + (4 * B * a * b^4 * d^4 * (-16 * B \\
& ^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) - (B^2 * a^2 * b^2 * d^2 * ((B^2 * a * d^2) / (4 * \\
& (a^2 * d^4 + b^2 * d^4)) - (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}} / (16 * (a^2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} * (a + b * \tan(c + d * x))^{\frac{1}{2}} * 32i / (16 * B^3 * b^4 * d + 16 * B^3 * a^2 * b^2 * d - (16 * \\
& B^3 * a^2 * b^4 * d^5) / (a^2 * d^4 + b^2 * d^4) - (16 * B^3 * a^4 * b^2 * d^5) / (a^2 * d^4 + b^2 * \\
& d^4) + (4 * B * a^3 * b^2 * d^4 * (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) + (4 * B \\
& * a * b^4 * d^4 * (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5)) * ((B^2 * a * d^2) / (4 * \\
& a^2 * d^4 + b^2 * d^4)) - (-16 * B^4 * b^2 * d^4)^{\frac{1}{2}} / (16 * (a^2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} * 2i - 2 * \operatorname{atanh}((8 * a * b^2 * (- \\
& (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (16 * (a^2 * d^4 + b^2 * d^4)) - (A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} * (a + b * \tan(c + d * x))^{\frac{1}{2}} * ( \\
& -16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / ((16 * A^3 * a * b^5 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * A * b^5 * \\
& d^4 * (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) + (16 * A^3 * a^3 * b^3 * d^5) / (a^ \\
& 2 * d^4 + b^2 * d^4) + (4 * A * a^2 * b^3 * d^4 * (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 \\
& * d^5)) - (32 * A^2 * b^2 * (- (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (16 * (a^2 * d^4 + b^2 * d^4)) - \\
& (A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} * (a + b * \tan(c + d * x))^{\frac{1}{2}}) / ((16 \\
& * A^3 * a * b^3 * d^3) / (a^2 * d^4 + b^2 * d^4) + (4 * A * b^3 * d^2 * (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (16 * \\
& (a^2 * d^4 + b^2 * d^4) - (A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} * (a + b * \tan \\
& (c + d * x))^{\frac{1}{2}}) / ((16 * A^3 * a * b^5 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * A * b^5 * d^4 * ( \\
& -16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) + (16 * A^3 * a^3 * b^3 * d^5) / (a^2 * d^4 \\
& + b^2 * d^4) + (4 * A * a^2 * b^3 * d^4 * (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}}) / (a^2 * d^5 + b^2 * d^5) \\
& )) * (- (-16 * A^4 * b^2 * d^4)^{\frac{1}{2}} / (16 * (a^2 * d^4 + b^2 * d^4)) - (A^2 * a * d^2) / (4 * (a^ \\
& 2 * d^4 + b^2 * d^4)))^{\frac{1}{2}} + (2 * A * (a + b * \tan(c + d * x))^{\frac{1}{2}}) / (b * d) + (2 * B * (a \\
& + b * \tan(c + d * x))^{\frac{3}{2}}) / (3 * b^2 * d) - (2 * B * a * (a + b * \tan(c + d * x))^{\frac{1}{2}}) / (b \\
& ^2 * d)
\end{aligned}$$



$$3.345 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=124

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} - \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d} + \frac{2B \sqrt{a+b \tan(c+dx)}}{bd}$$

[Out]  $-(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d/(a-I*b)^{(1/2)}-(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}/d/(a+I*b)^{(1/2)}+2*B*(a+b*\tan(d*x+c))^{(1/2)}/b/d$

Rubi [A]

time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3673, 3620, 3618, 65, 214}

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} + \frac{2B \sqrt{a+b \tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In] `Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]`

[Out]  $-\left(\frac{(A-I*B)*\operatorname{ArcTanh}\left[\frac{\sqrt{a+b*\tan[c+d*x]}}{\sqrt{a-I*b}}\right]}{\sqrt{a-I*b}}\right)/\left(\sqrt{a-I*b}*d\right)-\left(\frac{(A+I*B)*\operatorname{ArcTanh}\left[\frac{\sqrt{a+b*\tan[c+d*x]}}{\sqrt{a+I*b}}\right]}{\sqrt{a+I*b}}\right)/\left(\sqrt{a+I*b}*d\right)+\frac{2*B*\sqrt{a+b*\tan[c+d*x]}}{b*d}$

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 214

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 3618

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b`

\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[b\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx &= \frac{2B\sqrt{a+b\tan(c+dx)}}{bd} + \int \frac{-B+A\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
 &= \frac{2B\sqrt{a+b\tan(c+dx)}}{bd} + \frac{1}{2}(-iA-B) \int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
 &= \frac{2B\sqrt{a+b\tan(c+dx)}}{bd} + \frac{(A-iB)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}} dx, x, \sqrt{a+b\tan(c+dx)}\right)}{2d} \\
 &= \frac{2B\sqrt{a+b\tan(c+dx)}}{bd} - \frac{(iA-B)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}} dx, x, \sqrt{a+b\tan(c+dx)}\right)}{bd} \\
 &= -\frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}
 \end{aligned}$$

### Mathematica [A]

time = 0.37, size = 118, normalized size = 0.95

$$\frac{\frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} - \frac{2B\sqrt{a+b\tan(c+dx)}}{b}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] -((((A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/Sqrt[a - I\*b]) + ((A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/Sqrt[a + I\*b] - (2\*B\*Sqrt[a + b\*Tan[c + d\*x]]/b)/d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1416 vs.  $2(104) = 208$ .

time = 0.12, size = 1417, normalized size = 11.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVERB  
OSE)

[Out]  $2/d/b*(B*(a+b*\tan(d*x+c))^{1/2}-b*(1/4/b^2/(a^2+b^2)^{3/2}*(1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^3)*\ln(b*\tan(d*x+c)+a+(a+b*\tan(d*x+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}+(a^2+b^2)^{1/2}))+2*(2*A*a^3*b^2+2*A*a*b^4+2*B*a^2*b^3+2*B*b^5-1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^3)*(2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((2*(a+b*\tan(d*x+c))^{1/2}+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}))/((2*(a^2+b^2)^{1/2}-2*a)^{1/2}))+1/4/b^2/(a^2+b^2)^{3/2}*(-1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^3)*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}-(a^2+b^2)^{1/2}))+2*(-2*A*a^3*b^2-2*A*a*b^4-2*B*a^2*b^3-2*B*b^5+1/2*(-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^2+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^4+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*b^3-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3*b-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^3)*(2*(a^2+b^2)^{1/2}-2*a)^{1/2}$

$$\frac{1}{2} \arctan\left(\frac{-2(a+b\tan(dx+c))^{1/2} + (2(a^2+b^2)^{1/2} + 2a)^{1/2}}{2(a^2+b^2)^{1/2} - 2a}\right)$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(dx + c) + A)\*tan(dx + c)/sqrt(b\*tan(dx + c) + a), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8365 vs. 2(98) = 196.

time = 6.54, size = 8365, normalized size = 67.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{2} (a^2 b + b^3) d^5 \sqrt{-((2 A B a^2 b + 2 A B b^3 + (A^2 - B^2) a^3 + (A^2 - B^2) a b^2) d^2 \sqrt{(A^4 + 2 A^2 B^2 + B^4) / ((a^2 + b^2) d^4)} - (A^4 + 2 A^2 B^2 + B^4) a^2 - (A^4 + 2 A^2 B^2 + B^4) b^2) / (4 A^2 B^2 a^2 - 4 (A^3 B - A B^3) a b + (A^4 - 2 A^2 B^2 + B^4) b^2)} \sqrt{(4 A^2 B^2 a^2 - 4 (A^3 B - A B^3) a b + (A^4 - 2 A^2 B^2 + B^4) b^2) / ((a^4 + 2 a^2 b^2 + b^4) d^4)} \arctan\left(\frac{-((2 (A^7 B + 3 A^5 B^3 + 3 A^3 B^5 + A B^7) a^5 - (A^8 + 2 A^6 B^2 - 2 A^2 B^6 - B^8) a^4 b + 4 (A^7 B + 3 A^5 B^3 + 3 A^3 B^5 + A B^7) a^3 b^2 - 2 (A^8 + 2 A^6 B^2 - 2 A^2 B^6 - B^8) a^2 b^3 + 2 (A^7 B + 3 A^5 B^3 + 3 A^3 B^5 + A B^7) a b^4 - (A^8 + 2 A^6 B^2 - 2 A^2 B^6 - B^8) b^5) d^4 \sqrt{(4 A^2 B^2 a^2 - 4 (A^3 B - A B^3) a b + (A^4 - 2 A^2 B^2 + B^4) b^2) / ((a^4 + 2 a^2 b^2 + b^4) d^4)} \sqrt{(A^4 + 2 A^2 B^2 + B^4) / ((a^2 + b^2) d^4)} + (2 (A^9 B + 4 A^7 B^3 + 6 A^5 B^5 + 4 A^3 B^7 + A B^9) a^4 - (A^{10} + 3 A^8 B^2 + 2 A^6 B^4 - 2 A^4 B^6 - 3 A^2 B^8 - B^{10}) a^3 b + 2 (A^9 B + 4 A^7 B^3 + 6 A^5 B^5 + 4 A^3 B^7 + A B^9) a^2 b^2 - (A^{10} + 3 A^8 B^2 + 2 A^6 B^4 - 2 A^4 B^6 - 3 A^2 B^8 - B^{10}) a b^3) d^2 \sqrt{(4 A^2 B^2 a^2 - 4 (A^3 B - A B^3) a b + (A^4 - 2 A^2 B^2 + B^4) b^2) / ((a^4 + 2 a^2 b^2 + b^4) d^4)} \sqrt{(A^4 + 2 A^2 B^2 + B^4) / ((a^2 + b^2) d^4)} + ((A^3 + A B^2) a^4 + 2 (A^3 + A B^2) a^2 b^2 + (A^3 + A B^2) b^4) d^5 \sqrt{(4 A^2 B^2 a^2 - 4 (A^3 B - A B^3) a b + (A^4 - 2 A^2 B^2 + B^4) b^2)}\right)$

$$\begin{aligned}
& /((a^4 + 2*a^2*b^2 + b^4)*d^4)) * \text{sqrt}(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2 * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)) * \text{sqrt}(((4*(A^4*B^2 + A^2*B^4)*a^4 - 4*(A^5*B - A*B^5)*a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 - 4*(A^5*B - A*B^5)*a*b^3 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^4)*d^2 * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) * \cos(dx + c) + \text{sqrt}(2)*((4*A^3*B^2*a^4 - 4*(A^4*B - A^2*B^3)*a^3*b + (A^5 + 2*A^3*B^2 + A*B^4)*a^2*b^2 - 4*(A^4*B - A^2*B^3)*a*b^3 + (A^5 - 2*A^3*B^2 + A*B^4)*b^4)*d^3 * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) * \cos(dx + c) + (4*(A^5*B^2 + A^3*B^4)*a^3 - 4*(A^6*B - A^4*B^3 - 2*A^2*B^5)*a^2*b + (A^7 - 5*A^5*B^2 - A^3*B^4 + 5*A*B^6)*a*b^2 + (A^6*B - A^4*B^3 - A^2*B^5 + B^7)*b^3)*d * \cos(dx + c)) * \text{sqrt}(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2 * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)) * \text{sqrt}((a * \cos(dx + c) + b * \sin(dx + c))/\cos(dx + c)) * ((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(1/4) + (4*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^3 - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^2*b + (A^8 - 2*A^4*B^4 + B^8)*a*b^2) * \cos(dx + c) + (4*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^2*b - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^2 + (A^8 - 2*A^4*B^4 + B^8)*b^3) * \sin(dx + c))/\cos(dx + c)) * ((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(3/4) + \text{sqrt}(2)*((2*(A^4*B + A^2*B^3)*a^6 - (A^5 - 2*A^3*B^2 - 3*A*B^4)*a^5*b + (3*A^4*B + 4*A^2*B^3 + B^5)*a^4*b^2 - 2*(A^5 - 2*A^3*B^2 - 3*A*B^4)*a^3*b^3 + 2*(A^2*B^3 + B^5)*a^2*b^4 - (A^5 - 2*A^3*B^2 - 3*A*B^4)*a*b^5 - (A^4*B - B^5)*b^6)*d^7 * \text{sqrt}((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) + (2*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a^5 - (A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*a^4*b + 4*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a^3*b^2 - 2*(A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*a^2*b^3 + 2*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a*b^4 - (A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*b^5)*d^5 * \text{sqrt}((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) * \text{sqrt}(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2 * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)) * \text{sqrt}((a * \cos(dx + c) + b * \sin(dx + c))/\cos(dx + c)) * ((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(3/4))/((4*(A^10*B^2 + 4*A^8*B^4 + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^10)*a^2*b - 4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a*b^2 + (A^12 + 2*A^10*B^2 - A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 + B^12)*b^3)) + 4 * \text{sqrt}(2)*(a^2*b + b^3)*d^5 * \text{sqrt}(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2 * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)) * \text{sqrt}(((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) * ((A^4 + 2*A^2*B^2 + ...
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)/sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 9.35, size = 2930, normalized size = 23.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out]  $2*\operatorname{atanh}\left(\frac{32*B^2*b^2*(-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))}{(a + b*\tan(c + d*x))^{1/2}}\right) / \left(\frac{16*B^3*a*b^3*d^3}{a^2*d^4 + b^2*d^4} - \frac{4*B*b^3*d^2*(-16*B^4*b^2*d^4)^{1/2}}{a^2*d^5 + b^2*d^5}\right) + \frac{8*a*b^2*(-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))}{(a + b*\tan(c + d*x))^{1/2}} * \frac{(-16*B^4*b^2*d^4)^{1/2}}{\left(\frac{16*B^3*a*b^5*d^5}{a^2*d^4 + b^2*d^4} - \frac{4*B*b^5*d^4*(-16*B^4*b^2*d^4)^{1/2}}{a^2*d^5 + b^2*d^5}\right) + \frac{16*B^3*a^3*b^3*d^5}{a^2*d^4 + b^2*d^4} - \frac{4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^{1/2}}{a^2*d^5 + b^2*d^5}} - \frac{32*B^2*a^2*b^2*d^2*(-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))}{(a + b*\tan(c + d*x))^{1/2}} / \left(\frac{16*B^3*a*b^5*d^5}{a^2*d^4 + b^2*d^4} - \frac{4*B*b^5*d^4*(-16*B^4*b^2*d^4)^{1/2}}{a^2*d^5 + b^2*d^5}\right) + \frac{16*B^3*a^3*b^3*d^5}{a^2*d^4 + b^2*d^4} - \frac{4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^{1/2}}{a^2*d^5 + b^2*d^5} \Big) * \left(\frac{-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))}{(a + b*\tan(c + d*x))^{1/2}} - 2*\operatorname{atanh}\left(\frac{8*a*b^2*(-16*A^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4))}{(a + b*\tan(c + d*x))^{1/2}}\right)\right)$

$$\begin{aligned}
&)) + (A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} * \\
&(-16 * A^4 * b^2 * d^4)^{(1/2)} / ((16 * A^3 * a^2 * b^4 * d^5) / (a^2 * d^4 + b^2 * d^4) - 16 * A^3 \\
&* a^2 * b^2 * d - 16 * A^3 * b^4 * d + (16 * A^3 * a^4 * b^2 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * A \\
&* a^3 * b^2 * d^4 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5) + (4 * A * a * b^4 * d^4 * \\
&(-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5)) - (32 * A^2 * b^2 * ((-16 * A^4 * b^2 * d \\
&^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)) + (A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} / ((16 * A^3 * a^2 * b^2 * d^3) / (a^2 * d^4 + b^2 * d^4) \\
&- (16 * A^3 * b^2) / d + (4 * A * a * b^2 * d^2 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d \\
&^5)) + (32 * A^2 * a^2 * b^2 * d^2 * ((-16 * A^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4) \\
&)) + (A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} / \\
&((16 * A^3 * a^2 * b^4 * d^5) / (a^2 * d^4 + b^2 * d^4) - 16 * A^3 * a^2 * b^2 * d - 16 * A^3 * b^4 * d \\
&+ (16 * A^3 * a^4 * b^2 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * A * a^3 * b^2 * d^4 * (-16 * A^4 * b^2 \\
&* d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5) + (4 * A * a * b^4 * d^4 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / \\
&(a^2 * d^5 + b^2 * d^5)) * ((-16 * A^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)) + ( \\
&A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} - 2 * \operatorname{atanh}((32 * A^2 * b^2 * ((A^2 * a * d^2 \\
&)) / (4 * (a^2 * d^4 + b^2 * d^4)) - (-16 * A^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4) \\
&))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} / ((16 * A^3 * b^2) / d - (16 * A^3 * a^2 * b^2 * d^3) \\
&/ (a^2 * d^4 + b^2 * d^4) + (4 * A * a * b^2 * d^2 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b \\
&^2 * d^5)) + (8 * a * b^2 * ((A^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)) - (-16 * A^4 * b^2 * d^4 \\
&)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} * (-16 * A^4 \\
&* b^2 * d^4)^{(1/2)} / (16 * A^3 * b^4 * d + 16 * A^3 * a^2 * b^2 * d - (16 * A^3 * a^2 * b^4 * d^5) / (a \\
&^2 * d^4 + b^2 * d^4) - (16 * A^3 * a^4 * b^2 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * A * a^3 * b^2 \\
&* d^4 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5) + (4 * A * a * b^4 * d^4 * (-16 * A^4 \\
&* b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5)) - (32 * A^2 * a^2 * b^2 * d^2 * ((A^2 * a * d^2) / (4 \\
&* (a^2 * d^4 + b^2 * d^4)) - (-16 * A^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} / (16 * A^3 * b^4 * d + 16 * A^3 * a^2 * b^2 * d - (16 * A^3 \\
&* a^2 * b^4 * d^5) / (a^2 * d^4 + b^2 * d^4) - (16 * A^3 * a^4 * b^2 * d^5) / (a^2 * d^4 + b^2 * d^4 \\
&)) + (4 * A * a^3 * b^2 * d^4 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5) + (4 * A * a * \\
&b^4 * d^4 * (-16 * A^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5)) * ((A^2 * a * d^2) / (4 * (a^2 \\
&* d^4 + b^2 * d^4)) - (-16 * A^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} \\
&- 2 * \operatorname{atanh}((8 * a * b^2 * (- (-16 * B^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)) - (B \\
&^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} * (-16 * B^ \\
&4 * b^2 * d^4)^{(1/2)} / ((16 * B^3 * a * b^5 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * B * b^5 * d^4 * (- \\
&16 * B^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5) + (16 * B^3 * a^3 * b^3 * d^5) / (a^2 * d^4 \\
&+ b^2 * d^4) + (4 * B * a^2 * b^3 * d^4 * (-16 * B^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5)) \\
&- (32 * B^2 * b^2 * (- (-16 * B^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)) - (B^2 * a \\
&* d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + d * x))^{(1/2)} / ((16 * B^3 * a \\
&* b^3 * d^3) / (a^2 * d^4 + b^2 * d^4) + (4 * B * b^3 * d^2 * (-16 * B^4 * b^2 * d^4)^{(1/2)}) / (a^2 * \\
&d^5 + b^2 * d^5)) + (32 * B^2 * a^2 * b^2 * d^2 * (- (-16 * B^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d \\
&^4 + b^2 * d^4)) - (B^2 * a * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{(1/2)} * (a + b * \tan(c + \\
&d * x))^{(1/2)} / ((16 * B^3 * a * b^5 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * B * b^5 * d^4 * (-16 * B^ \\
&4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5) + (16 * B^3 * a^3 * b^3 * d^5) / (a^2 * d^4 + b^2 \\
&* d^4) + (4 * B * a^2 * b^3 * d^4 * (-16 * B^4 * b^2 * d^4)^{(1/2)}) / (a^2 * d^5 + b^2 * d^5)) * (- \\
&(-16 * B^4 * b^2 * d^4)^{(1/2)} / (16 * (a^2 * d^4 + b^2 * d^4)) - (B^2 * a * d^2) / (4 * (a^2 * d^4 \\
&+ b^2 * d^4)))^{(1/2)} + (2 * B * (a + b * \tan(c + d * x))^{(1/2)}) / (b * d)
\end{aligned}$$

$$3.346 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=102

$$-\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} + \frac{(iA-B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d}$$

[Out]  $-(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d/(a-I*b)^{(1/2)}+(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d/(a+I*b)^{(1/2)}$

Rubi [A]

time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {3620, 3618, 65, 214}

$$\frac{(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/Sqrt[a+b*\operatorname{Tan}[c+d*x]],x]$

[Out]  $-\left(\left(\left(I*A+B\right)*\operatorname{ArcTanh}\left[\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]/\operatorname{Sqrt}\left[a-I*b\right]\right)\right)/\left(\operatorname{Sqrt}\left[a-I*b\right]*d\right)\right)+\left(\left(\left(I*A-B\right)*\operatorname{ArcTanh}\left[\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]/\operatorname{Sqrt}\left[a+I*b\right]\right)\right)/\left(\operatorname{Sqrt}\left[a+I*b\right]*d\right)\right)$

Rule 65

$\operatorname{Int}[(a_.)+(b_.)*(x_)^{(m_)}*((c_.)+(d_.)*(x_)^{(n_)}), x\_Symbol] :> \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)}, x], x, (a+b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x\} \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_.)+(b_.)*(x_)^2)^{-1}, x\_Symbol] :> \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{NegQ}[a/b]$

Rule 3618

$\operatorname{Int}[(a_.)+(b_.)*\operatorname{tan}[(e_.)+(f_.)*(x_)])^{(m_)}*((c_.)+(d_.)*\operatorname{tan}[(e_.)+(f_.)*(x_)])], x\_Symbol] :> \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a+(b/d)*x)^m/(d^2+c*x), x], x, d*\operatorname{Tan}[e+f*x]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \operatorname{NeQ}[b$



\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx &= \frac{1}{2}(A - iB) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx + \frac{1}{2}(A + iB) \int \frac{1 - i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \\ &= -\frac{(iA - B) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, -i \tan(c + dx)\right)}{2d} + \frac{(iA + B) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, -i \tan(c + dx)\right)}{2d} \\ &= -\frac{(A - iB) \text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{bd} - \frac{(A + iB) \text{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{bd} \\ &= -\frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{\sqrt{a - ib} d} + \frac{(iA - B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{\sqrt{a + ib} d} \end{aligned}$$

### Mathematica [A]

time = 0.07, size = 101, normalized size = 0.99

$$i \left( \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{\sqrt{a - ib}} + \frac{(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{\sqrt{a + ib}} \right) / d$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] (I\*(-(((A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/Sqrt[a - I\*b]) + ((A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/Sqrt[a + I\*b]))/d

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1406 vs.  $2(84) = 168$ .

time = 0.36, size = 1407, normalized size = 13.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b^2/(a^2+b^2)^(3/2)*(1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(2*A*a^2*b^3+2*A*b^5-2*B*a^3*b^2-2*B*a*b^4-1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/2/b^2/(a^2+b^2)^(3/2)*(-1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-(a^2+b^2)^(1/2))+2*(-2*A*a^2*b^3-2*A*b^5+2*B*a^3*b^2+2*B*a*b^4+1/2*(A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2*b+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*b^3-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3*b-A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^3+B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*a-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^2-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8282 vs. 2(79) = 158.

time = 7.61, size = 8282, normalized size = 81.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/4*(4*sqrt(2)*(a^2 + b^2)*d^4*sqrt(((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)
*a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4
)) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*
a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((4*A^2*B^2
*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b
^2 + b^4)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(3/4)*arctan(((
2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5 - (A^8 + 2*A^6*B^2 - 2*A^2*B^
6 - B^8)*a^4*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^2 - 2*(A^8
+ 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^3 + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5
+ A*B^7)*a*b^4 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^5)*d^4*sqrt((4*A^2*B
^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) + (2*(A^9
*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 - (A^10 + 3*A^8*B^2 + 2
*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A
^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 - (A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4
*B^6 - 3*A^2*B^8 - B^10)*a*b^3)*d^2*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)
*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - sqrt(2
)*(B*a^5 - A*a^4*b + 2*B*a^3*b^2 - 2*A*a^2*b^3 + B*a*b^4 - A*b^5)*d^7*sqrt
((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))
+ ((A^2*B + B^3)*a^4 + 2*(A^2*B + B^3)*a^2*b^2 + (A^2*B + B^3)*b^4)*d^5*sq
rt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a
^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a
^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))
+ (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^
2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt(((4*(A^4*B^2
+ A^2*B^4)*a^4 - 4*(A^5*B - A*B^5)*a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 +
B^6)*a^2*b^2 - 4*(A^5*B - A*B^5)*a*b^3 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^
4)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))*cos(d*x + c) + sqrt(
2))*((4*A^2*B^3*a^4 - 4*(A^3*B^2 - A*B^4)*a^3*b + (A^4*B + 2*A^2*B^3 + B^5)*
a^2*b^2 - 4*(A^3*B^2 - A*B^4)*a*b^3 + (A^4*B - 2*A^2*B^3 + B^5)*b^4)*d^3*sq
rt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))*cos(d*x + c) + (4*(A^4*B^3 +
A^2*B^5)*a^3 - 4*(2*A^5*B^2 + A^3*B^4 - A*B^6)*a^2*b + (5*A^6*B - A^4*B^3 -
```

```

5*A^2*B^5 + B^7)*a*b^2 - (A^7 - A^5*B^2 - A^3*B^4 + A*B^6)*b^3)*d*cos(d*x
+ c))*sqrt(((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)
*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))) + (A^4 + 2*A^2*B^2 + B
^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a
*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(1/4) + (4*(A^6*B^
2 + 2*A^4*B^4 + A^2*B^6)*a^3 - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^2*b
+ (A^8 - 2*A^4*B^4 + B^8)*a*b^2)*cos(d*x + c) + (4*(A^6*B^2 + 2*A^4*B^4 + A
^2*B^6)*a^2*b - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^2 + (A^8 - 2*A^4*
B^4 + B^8)*b^3)*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2
+ b^2)*d^4))^(3/4) + sqrt(2)*((2*(A^3*B^2 + A*B^4)*a^6 - (3*A^4*B + 2*A^2*B
^3 - B^5)*a^5*b + (A^5 + 4*A^3*B^2 + 3*A*B^4)*a^4*b^2 - 2*(3*A^4*B + 2*A^2*
B^3 - B^5)*a^3*b^3 + 2*(A^5 + A^3*B^2)*a^2*b^4 - (3*A^4*B + 2*A^2*B^3 - B^5
)*a*b^5 + (A^5 - A*B^4)*b^6)*d^7*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*
b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt((A^4 +
2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) + (2*(A^5*B^2 + 2*A^3*B^4 + A*B^6)*a^5
- (A^6*B + A^4*B^3 - A^2*B^5 - B^7)*a^4*b + 4*(A^5*B^2 + 2*A^3*B^4 + A*B^6
)*a^3*b^2 - 2*(A^6*B + A^4*B^3 - A^2*B^5 - B^7)*a^2*b^3 + 2*(A^5*B^2 + 2*A^
3*B^4 + A*B^6)*a*b^4 - (A^6*B + A^4*B^3 - A^2*B^5 - B^7)*b^5)*d^5*sqrt((4*A
^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^4))*sqrt(((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (
A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) + (A^
4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*
(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x + c) +
b*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(
3/4))/(4*(A^10*B^2 + 4*A^8*B^4 + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^10)*a^2*b -
4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a*b^2 +
(A^12 + 2*A^10*B^2 - A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 + B^12)*b^
3)) + 4*sqrt(2)*(a^2 + b^2)*d^4*sqrt(((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)
)*a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^
4)) + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2
*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((4*A^2*B^
2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*
b^2 + b^4)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^2...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/sqrt(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 8.77, size = 2909, normalized size = 28.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] 
$$2*\operatorname{atanh}\left(\frac{8*a*b^2*(-(-16*A^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))^{1/2}}{(16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5)} - \frac{(32*A^2*b^2*(-(-16*A^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))^{1/2})}{(16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5)}\right) - 2*\operatorname{atanh}\left(\frac{8*a*b^2*((-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))^{1/2}}{(16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5) - (32*B^2*b^2*((-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))^{1/2})}{(16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^{1/2})/(a^2*d^5 + b^2*d^5) - (32*B^2*b^2*((-16*B^4*b^2*d^4)^{1/2}/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4))^{1/2})}{(16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16$$

$$\begin{aligned}
& *B^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4) \\
& ^{(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*B^4*b^2*d^4)^{(1/2)/(16*(a^2*d^4 + b^2*d \\
& ^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2) - 2*atanh((32*A^2*b^2*(- \\
& 16*A^4*b^2*d^4)^{(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + \\
& b^2*d^4)))^{(1/2)*(a + b*tan(c + d*x))^{(1/2)/((16*A^3*a*b^3*d^3)/(a^2*d^4 + \\
& b^2*d^4) - (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5)) + (8 \\
& *a*b^2*(-16*A^4*b^2*d^4)^{(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*( \\
& a^2*d^4 + b^2*d^4)))^{(1/2)*(a + b*tan(c + d*x))^{(1/2)*(-16*A^4*b^2*d^4)^{(1/ \\
& 2)/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4 \\
& )^{(1/2)))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - ( \\
& 4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5)) - (32*A^2*a^2 \\
& *b^2*d^2*(-16*A^4*b^2*d^4)^{(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4 \\
& *(a^2*d^4 + b^2*d^4)))^{(1/2)*(a + b*tan(c + d*x))^{(1/2)/((16*A^3*a*b^5*d^5 \\
& )/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^ \\
& 2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A \\
& ^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*A^4*b^2*d^4)^{(1/2)/(16*(a^2* \\
& d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2) - 2*atanh((32* \\
& B^2*b^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^{(1/2)/(16* \\
& (a^2*d^4 + b^2*d^4)))^{(1/2)*(a + b*tan(c + d*x))^{(1/2)/((16*B^3*b^2)/d - ( \\
& 16*B^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) + (4*B*a*b^2*d^2*(-16*B^4*b^2*d^4)^ \\
& (1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) \\
& - (-16*B^4*b^2*d^4)^{(1/2)/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)*(a + b*tan(c + d \\
& *x))^{(1/2)*(-16*B^4*b^2*d^4)^{(1/2)/(16*B^3*b^4*d + 16*B^3*a^2*b^2*d - (16* \\
& B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2* \\
& d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5) + (4*B \\
& *a*b^4*d^4*(-16*B^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5)) - (32*B^2*a^2*b^2* \\
& d^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^{(1/2)/(16*(a^2 \\
& *d^4 + b^2*d^4)))^{(1/2)*(a + b*tan(c + d*x))^{(1/2)/((16*B^3*b^4*d + 16*B^3* \\
& a^2*b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a^4*b^2*d^5) \\
& / (a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{(1/2))/(a^2*d^5 + \\
& b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^{(1/2))/(a^2*d^5 + b^2*d^5)))* \\
& ((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^{(1/2)/(16*(a^2*d^4 \\
& + b^2*d^4)))^{(1/2)}
\end{aligned}$$

$$3.347 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=131

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d}$$

[Out]  $-2*A*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d/a^{(1/2)}+(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d/(a-I*b)^{(1/2)}+(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d/(a+I*b)^{(1/2)}$

**Rubi [A]**

time = 0.21, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3694, 3620, 3618, 65, 214, 3715}

$$\frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]],x]$

[Out]  $(-2*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a]*d) + ((A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/(\operatorname{Sqrt}[a-I*b]*d) + ((A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/(\operatorname{Sqrt}[a+I*b]*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\operatorname{Tan}[e + f*x]], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{NeQ}[b$

$*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[\left((a_{.}) + (b_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(m_{.})}\left((c_{.}) + (d_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right), x_{\text{Symbol}}] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{IntegerQ}[m]$

### Rule 3694

$\text{Int}[\left(\left((A_{.}) + (B_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)\left((c_{.}) + (d_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(n_{.})}\right) / \left((a_{.}) + (b_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right), x_{\text{Symbol}}] \rightarrow \text{Dist}\left[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A + b*B - (A*b - a*B)*\text{Tan}[e + f*x], x], x], x\right] + \text{Dist}[b*((A*b - a*B)/(a^2 + b^2)), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3715

$\text{Int}[\left((a_{.}) + (b_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(m_{.})}\left((c_{.}) + (d_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^{(n_{.})}\left((A_{.}) + (C_{.})\tan[(e_{.}) + (f_{.})(x_{.})]\right)^2, x_{\text{Symbol}}] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rubi steps



$$\begin{aligned}
\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx &= A \int \frac{\cot(c+dx)(1+\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx + \int \frac{B-A\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
&= \frac{1}{2}(-iA+B) \int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx + \frac{1}{2}(iA+B) \int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
&= \frac{(2A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\tan(c+dx)}\right)}{bd} - \frac{(A-iB)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}} dx, x, \sqrt{a+b\tan(c+dx)}\right)}{bd} \\
&= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(iA-B)\text{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}} dx, x, \sqrt{a+b\tan(c+dx)}\right)}{bd} \\
&= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}
\end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 29.41, size = 11296, normalized size = 86.23

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] Result too large to show

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.18, size = 33052, normalized size = 252.31

method	result	size
default	Expression too large to display	33052

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVERB  
OSE)

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 8380 vs. 2(101) = 202.

time = 16.03, size = 16835, normalized size = 128.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*(4*\sqrt{2})*(a^3 + a*b^2)*d^5*\sqrt{-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))} \\ & - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} \\ & *((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^{3/4}*\arctan(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^2 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^3 + 2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^4 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^5)*d^4*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} \\ & *\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^2 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^3)*d^2*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} \\ & - \sqrt{2})*((A*a^5 + B*a^4*b + 2*A*a^3*b^2 + 2*B*a^2*b^3 + A*a*b^4 + B*b^5)*d^7*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} \\ & *\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + ((A^3 + A*B^2)*a^4 + 2*(A^3 + A*B^2)*a^2*b^2 + (A^3 + A*B^2)*b^4)*d^5*\sqrt{(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} \\ & *\sqrt{-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))} \\ & - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{((4*(A^4*B^2 + A^2*B^4)*a^4 - 4*(A^5*B - A*B^5)*a^3*b + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2*b^2 - 4*(A^5*B - A*B^5)*a*b^3 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^4)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)}}*\cos(d*x + c) \end{aligned}$$

```

+ sqrt(2)*((4*A^3*B^2*a^4 - 4*(A^4*B - A^2*B^3)*a^3*b + (A^5 + 2*A^3*B^2 +
A*B^4)*a^2*b^2 - 4*(A^4*B - A^2*B^3)*a*b^3 + (A^5 - 2*A^3*B^2 + A*B^4)*b^4)
*d^3*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))*cos(d*x + c) + (4*(A^5
*B^2 + A^3*B^4)*a^3 - 4*(A^6*B - A^4*B^3 - 2*A^2*B^5)*a^2*b + (A^7 - 5*A^5*
B^2 - A^3*B^4 + 5*A*B^6)*a*b^2 + (A^6*B - A^4*B^3 - A^2*B^5 + B^7)*b^3)*d*c
os(d*x + c))*sqrt(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2
)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2
*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B -
A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x + c) + b*sin(d*x
+ c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(1/4) + (4
*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^3 - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7
)*a^2*b + (A^8 - 2*A^4*B^4 + B^8)*a*b^2)*cos(d*x + c) + (4*(A^6*B^2 + 2*A^4
*B^4 + A^2*B^6)*a^2*b - 4*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^2 + (A^8
- 2*A^4*B^4 + B^8)*b^3)*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4
)/((a^2 + b^2)*d^4))^(3/4) + sqrt(2)*((2*(A^4*B + A^2*B^3)*a^6 - (A^5 - 2*A
^3*B^2 - 3*A*B^4)*a^5*b + (3*A^4*B + 4*A^2*B^3 + B^5)*a^4*b^2 - 2*(A^5 - 2*
A^3*B^2 - 3*A*B^4)*a^3*b^3 + 2*(A^2*B^3 + B^5)*a^2*b^4 - (A^5 - 2*A^3*B^2 -
3*A*B^4)*a*b^5 - (A^4*B - B^5)*b^6)*d^7*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A
*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqr
t((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) + (2*(A^6*B + 2*A^4*B^3 + A^2*
B^5)*a^5 - (A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*a^4*b + 4*(A^6*B + 2*A^4*B^3 +
A^2*B^5)*a^3*b^2 - 2*(A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*a^2*b^3 + 2*(A^6*B
+ 2*A^4*B^3 + A^2*B^5)*a*b^4 - (A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*b^5)*d^5*s
qrt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2
)*a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^
4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2
*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2
)*d^4))^(3/4))/(4*(A^10*B^2 + 4*A^8*B^4 + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^10)
*a^2*b - 4*(A^11*B + 3*A^9*B^3 + 2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11
)*a*b^2 + (A^12 + 2*A^10*B^2 - A^8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 +
B^12)*b^3)) + 4*sqrt(2)*(a^3 + a*b^2)*d^5*sqrt(-((2*A*B*a^2*b + 2*A*B*b^3
+ (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a
^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 + B^4)*b^2
)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*sq
rt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*d^4))*((A^4 + 2*A^2*B^2 ...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& d^2)^2/4 - (16a^2d^4 + 16b^2d^4)(A^4 + 2A^2B^2 + B^4))^{(1/2)} - 4A^2 \\
& *a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2d^4 + b^2d^4))^{(1/2)} - (32*( \\
& a + b*\tan(c + d*x))^{(1/2)}*(16A*B*b^9d^2 + 18A^2*a*b^8d^2 - 10B^2*a*b^8 \\
& *d^2))/d^4)*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 \\
& + 16b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 \\
& - 8A*B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{(1/2)})*(-(((8A^2*a^2d^2 - 8B^2*a^2 \\
& d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)*(A^4 + 2A^2*B^2 + B^4) \\
& )^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4)) \\
& )^{(1/2)} - (32*(3A^4*b^8 + B^4*b^8)*(a + b*\tan(c + d*x))^{(1/2)})/d^4)*(-(((8 \\
& *A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)*(A \\
& ^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16 \\
& *(a^2*d^4 + b^2*d^4))^{(1/2)}*i)/(((32*(12A^2*B*b^9d^2 + 3A^3*a*b^8d^2 \\
& - 9A*B^2*a*b^8d^2))/d^5 - (((32*(16A*b^10d^4 - 4B*a*b^9d^4 + 12A*a^2 \\
& *b^8d^4))/d^5 - (32*(16b^10d^4 + 24a^2*b^8d^4)*(a + b*\tan(c + d*x))^{( \\
& 1/2)}*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16* \\
& b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B \\
& *b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{(1/2)})/d^4)*(-(((8A^2*a^2d^2 - 8B^2*a^2d^ \\
& 2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{ \\
& (1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{ \\
& (1/2)} + (32*(a + b*\tan(c + d*x))^{(1/2)}*(16A*B*b^9d^2 + 18A^2*a*b^8d^2 - \\
& 10B^2*a*b^8d^2))/d^4)*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 \\
& - (16a^2*d^4 + 16b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + \\
& 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{(1/2)})*(-(((8A^2*a^2d \\
& ^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)*(A^4 + 2A \\
& ^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2*d^ \\
& 4 + b^2*d^4))^{(1/2)} + (32*(3A^4*b^8 + B^4*b^8)*(a + b*\tan(c + d*x))^{(1/2)} \\
& )/d^4)*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 1 \\
& 6b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A \\
& *B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{(1/2)} + (((32*(12A^2*B*b^9d^2 + 3A^3 \\
& *a*b^8d^2 - 9A*B^2*a*b^8d^2))/d^5 - (((32*(16A*b^10d^4 - 4B*a*b^9d^4 \\
& + 12A*a^2*b^8d^4))/d^5 + (32*(16b^10d^4 + 24a^2*b^8d^4)*(a + b*\tan(c \\
& + d*x))^{(1/2)}*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2 \\
& *d^4 + 16b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d \\
& ^2 - 8A*B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{(1/2)})/d^4)*(-(((8A^2*a^2d^2 - \\
& 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)*(A^4 + 2A^2*B^ \\
& 2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2*d^4 + b \\
& ^2*d^4))^{(1/2)} - (32*(a + b*\tan(c + d*x))^{(1/2)}*(16A*B*b^9d^2 + 18A^2*a \\
& *b^8d^2 - 10B^2*a*b^8d^2))/d^4)*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B* \\
& b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)*(A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A \\
& ^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/(16*(a^2*d^4 + b^2*d^4))^{(1/2)})*(-(( \\
& (8A^2*a^2d^2 - 8B^2*a^2d^2 + 16A*B*b^2d^2)^2/4 - (16a^2*d^4 + 16b^2*d^4)* \\
& (A^4 + 2A^2*B^2 + B^4))^{(1/2)} - 4A^2*a^2d^2 + 4B^2*a^2d^2 - 8A*B*b^2d^2)/( \\
& 16*(a^2*d^4 + b^2*d^4))^{(1/2)} - (32*(3A^4*b^8 + B^4*b^8)*(a + b*\tan(c + d \\
& *x))^{(1/2)})/d^4)*(-(((8A^2*a^2d^2 - 8B^2*a^2d^2...
\end{aligned}$$

$$3.348 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=169

$$\frac{(Ab - 2aB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{a^{3/2}d} + \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right)}{\sqrt{a-ib}d} - \frac{(iA - B) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right)}{\sqrt{a+ib}d} - \frac{A \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{ad}$$

[Out] (A\*b-2\*B\*a)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+(I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d/(a-I\*b)^(1/2)-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d/(a+I\*b)^(1/2)-A\*cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(1/2)/a/d

**Rubi [A]**

time = 0.35, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3690, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(Ab - 2aB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{a^{3/2}d} + \frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right)}{d\sqrt{a-ib}} - \frac{(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right)}{d\sqrt{a+ib}} - \frac{A \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((A\*b - 2\*a\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]])/(a^(3/2)\*d) + ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/(Sqrt[a - I\*b]\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/(Sqrt[a + I\*b]\*d) - (A\*Cot[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]])/(a\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3618**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c

$*x), x], x, d*\text{Tan}[e + f*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[\left((a_{.}) + (b_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right), x\_Symbol] \text{:>} \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$

### Rule 3690

$\text{Int}[\left((a_{.}) + (b_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(m_{.})}*\left((A_{.}) + (B_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)*\left((c_{.}) + (d_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(n_{.})}, x\_Symbol] \text{:>} \text{Simp}[b*(A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n+1}/(f*(m+1)*(b*c - a*d)*(a^2 + b^2)), x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[b*B*(b*c*(m+1) + a*d*(n+1)) + A*(a*(b*c - a*d)*(m+1) - b^2*d*(m+n+2)) - (A*b - a*B)*(b*c - a*d)*(m+1)*\text{Tan}[e + f*x] - b*d*(A*b - a*B)*(m+n+2)*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegerQ}[m] || \text{IntegersQ}[2*m, 2*n]) \&\& \text{!(ILtQ}[n, -1] \&\& (\text{!IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3715

$\text{Int}[\left((a_{.}) + (b_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(n_{.})}*\left((A_{.}) + (C_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]^2\right), x\_Symbol] \text{:>} \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3734

$\text{Int}[\left(\left((c_{.}) + (d_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(n_{.})}*\left((A_{.}) + (B_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right) + (C_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]^2\right)/\left((a_{.}) + (b_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right), x\_Symbol] \text{:>} \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!GtQ}[n, 0] \&\& \text{!LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx &= -\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\int \frac{\cot(c+dx)(\frac{1}{2}(Ab-2aB)+aA\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx}{a} \\
&= -\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{\int \frac{aA+aB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx}{a} \\
&= -\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} - \frac{1}{2}(A-iB) \int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
&= -\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} + \frac{(iA-B)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}} dx\right)}{a} \\
&= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad} \\
&= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

**Mathematica [A]**

time = 1.87, size = 201, normalized size = 1.19

$$\frac{b(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(A\sqrt{-b^2}+bB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}} - \frac{(A\sqrt{-b^2}-bB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}} - \frac{Ab\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{a}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
[Out] ((b*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) + ((A*
Sqrt[-b^2] + b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/S
qrt[a - Sqrt[-b^2]] - ((A*Sqrt[-b^2] - b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]
]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] - (A*b*Cot[c + d*x]*Sqrt[a +
b*Tan[c + d*x]]/a)/(b*d)

```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.62, size = 69579, normalized size = 411.71

method	result	size
default	Expression too large to display	69579



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm  
="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*cot(d*x + c)^2/sqrt(b*tan(d*x + c) + a), x)`

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 8726 vs. 2(139) = 278.

time = 14.88, size = 17527, normalized size = 103.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm  
="fricas")`

[Out] 
$$\begin{aligned} & [1/4*(4*(A^5 + 2*A^3*B^2 + A*B^4)*a*\sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c))}/ \\ & \cos(d*x + c))*\cos(d*x + c)*\sin(d*x + c) - 4*\sqrt{2}*((a^4 + a^2*b^2)*d^5*\cos \\ & (d*x + c)^2 - (a^4 + a^2*b^2)*d^5)*\sqrt{((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - \\ & B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2) \\ & )*d^4)} + (A^4 + 2*A^2*B^2 + B^4)*a^2 + (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2 \\ & *B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2))*\sqrt{(4*A^ \\ & 2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2* \\ & a^2*b^2 + b^4)*d^4)}*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^{3/4}*\arct \\ & \text{an}(((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5 - (A^8 + 2*A^6*B^2 - 2*A \\ & ^2*B^6 - B^8)*a^4*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^2 - 2 \\ & *(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^3 + 2*(A^7*B + 3*A^5*B^3 + 3*A^3 \\ & *B^5 + A*B^7)*a*b^4 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^5)*d^4*\sqrt{(4* \\ & A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + \\ & 2*a^2*b^2 + b^4)*d^4)}*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)} + (2 \\ & *(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4 - (A^{10} + 3*A^8*B^ \\ & 2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^3*b + 2*(A^9*B + 4*A^7*B^3 \end{aligned}$$

$$\begin{aligned}
& + 6A^5B^5 + 4A^3B^7 + AB^9)a^2b^2 - (A^{10} + 3A^8B^2 + 2A^6B^4 - \\
& 2A^4B^6 - 3A^2B^8 - B^{10})a^3b^3)d^2\sqrt{(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2)/((a^4 + 2a^2b^2 + b^4)d^4)} - \\
& \sqrt{2}((B^5a^5 - A^4a^4b + 2B^3a^3b^2 - 2A^2a^2b^3 + B^2a^2b^4 - Ab^5)d^7 \\
& \sqrt{(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2) \\
& /((a^4 + 2a^2b^2 + b^4)d^4)}\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)} \\
& + ((A^2B + B^3)a^4 + 2(A^2B + B^3)a^2b^2 + (A^2B + B^3)b^4)d^5 \\
& \sqrt{(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2) \\
& /((a^4 + 2a^2b^2 + b^4)d^4)}\sqrt{((2ABa^2b + 2AB^2b^3 + (A^2 - B^2)a^3 + (A^2 - B^2)a^2b^2)d^2\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)} \\
& + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2)}\sqrt{((4(A^4B^2 + A^2B^4)a^4 - 4(A^5B - AB^5)a^3b + (A^6 + 3A^4B^2 + 3A^2B^4 + B^6)a^2b^2 - 4(A^5B - AB^5)a^2b^3 + (A^6 - A^4B^2 - A^2B^4 + B^6)b^4)d^2\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)}\cos(dx + c) + \sqrt{2}((4A^2B^3a^4 - 4(A^3B^2 - AB^4)a^3b + (A^4B + 2A^2B^3 + B^5)a^2b^2 - 4(A^3B^2 - AB^4)a^2b^3 + (A^4B - 2A^2B^3 + B^5)b^4)d^3\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)}\cos(dx + c) + (4(A^4B^3 + A^2B^5)a^3 - 4(2A^5B^2 + A^3B^4 - AB^6)a^2b + (5A^6B - A^4B^3 - 5A^2B^5 + B^7)a^2b^2 - (A^7 - A^5B^2 - A^3B^4 + AB^6)b^3)d\cos(dx + c))\sqrt{((2ABa^2b + 2AB^2b^3 + (A^2 - B^2)a^3 + (A^2 - B^2)a^2b^2)d^2\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2)}\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c))*((A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4))^{1/4} + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^3 - 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^2b + (A^8 - 2A^4B^4 + B^8)a^2b^2)\cos(dx + c) + (4(A^6B^2 + 2A^4B^4 + A^2B^6)a^2b - 4(A^7B + A^5B^3 - A^3B^5 - AB^7)a^2b^2 + (A^8 - 2A^4B^4 + B^8)b^3)\sin(dx + c))/\cos(dx + c))*((A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4))^{3/4} + \sqrt{2}(((2(A^3B^2 + AB^4)a^6 - (3A^4B + 2A^2B^3 - B^5)a^5b + (A^5 + 4A^3B^2 + 3AB^4)a^4b^2 - 2(3A^4B + 2A^2B^3 - B^5)a^3b^3 + 2(A^5 + A^3B^2)a^2b^4 - (3A^4B + 2A^2B^3 - B^5)a^2b^5 + (A^5 - AB^4)b^6)d^7\sqrt{(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2)/((a^4 + 2a^2b^2 + b^4)d^4)}\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)} + (2(A^5B^2 + 2A^3B^4 + AB^6)a^5 - (A^6B + A^4B^3 - A^2B^5 - B^7)a^4b + 4(A^5B^2 + 2A^3B^4 + AB^6)a^3b^2 - 2(A^6B + A^4B^3 - A^2B^5 - B^7)a^2b^3 + 2(A^5B^2 + 2A^3B^4 + AB^6)a^2b^4 - (A^6B + A^4B^3 - A^2B^5 - B^7)b^5)d^5\sqrt{(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2)/((a^4 + 2a^2b^2 + b^4)d^4)}\sqrt{((2ABa^2b + 2AB^2b^3 + (A^2 - B^2)a^3 + (A^2 - B^2)a^2b^2)d^2\sqrt{(A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4)} + (A^4 + 2A^2B^2 + B^4)a^2 + (A^4 + 2A^2B^2 + B^4)b^2)/(4A^2B^2a^2 - 4(A^3B - AB^3)a^2b + (A^4 - 2A^2B^2 + B^4)b^2)}\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c))*((A^4 + 2A^2B^2 + B^4)/((a^2 + b^2)d^4))^{3/4})/(4(A^{10}B^2 + 4A^8B^4 + 6A^6B^6 + 4A^4B^8 + A^2B^{10})a^2
\end{aligned}$$

\*b - 4\*(A<sup>11</sup>\*B + 3\*A<sup>9</sup>\*B<sup>3</sup> + 2\*A<sup>7</sup>\*B<sup>5</sup> - 2\*A<sup>5</sup>\*B<sup>7</sup> - 3\*A<sup>3</sup>\*B<sup>9</sup> - A\*B<sup>11</sup>)\*a\*b<sup>2</sup> + (A<sup>12</sup> + 2\*A<sup>10</sup>\*B<sup>2</sup> - A<sup>8</sup>\*B<sup>4</sup> - 4\*A<sup>6</sup>\*B<sup>6</sup> - A<sup>4</sup>\*B<sup>8</sup> + 2\*A<sup>2</sup>\*B<sup>10</sup> + B<sup>12</sup>)\*b<sup>3</sup>) - 4\*sqrt(2)\*((a<sup>4</sup> + a<sup>2</sup>\*b<sup>2</sup>)\*d<sup>5</sup>\*cos(dx + c)<sup>2</sup> - (a<sup>4</sup> + a<sup>2</sup>\*b<sup>2</sup>)\*d<sup>5</sup>)\*sqrt(((2\*A\*B\*a<sup>2</sup>\*b + 2\*A\*B\*b<sup>3</sup> + (A<sup>2</sup> - B<sup>2</sup>)\*a<sup>3</sup> + (A<sup>2</sup> - B<sup>2</sup>)\*a\*b<sup>2</sup>)\*d<sup>2</sup>\*sqrt((A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)/((a<sup>2</sup> + b<sup>2</sup>)\*d<sup>4</sup>)) + (A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*a<sup>2</sup> + (A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>)\*b<sup>2</sup>)/(4\*A<sup>2</sup>\*B<sup>2</sup>...

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^2(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*2\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + dx))\*cot(c + dx)\*\*2/sqrt(a + b\*tan(c + dx)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^2\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 8.68, size = 2500, normalized size = 14.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + dx)^2\*(A + B\*tan(c + dx)))/(a + b\*tan(c + dx))^(1/2),x)

[Out] atan(((((((8\*(32\*A\*a\*b<sup>11</sup>\*d<sup>4</sup> + 16\*A\*a<sup>3</sup>\*b<sup>9</sup>\*d<sup>4</sup> - 64\*B\*a<sup>2</sup>\*b<sup>10</sup>\*d<sup>4</sup> - 48\*B\*a<sup>4</sup>\*b<sup>8</sup>\*d<sup>4</sup>))/(a<sup>2</sup>\*d<sup>5</sup>) - (16\*(32\*a<sup>2</sup>\*b<sup>10</sup>\*d<sup>4</sup> + 48\*a<sup>4</sup>\*b<sup>8</sup>\*d<sup>4</sup>)\*(a + b\*tan(c + dx))^(1/2)\*(((8\*A<sup>2</sup>\*a\*d<sup>2</sup> - 8\*B<sup>2</sup>\*a\*d<sup>2</sup> + 16\*A\*B\*b\*d<sup>2</sup>)<sup>2</sup>/4 - (16\*a<sup>2</sup>\*d<sup>4</sup> + 16\*b<sup>2</sup>\*d<sup>4</sup>)\*(A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>))^(1/2) - 4\*A<sup>2</sup>\*a\*d<sup>2</sup> + 4\*B<sup>2</sup>\*a\*d<sup>2</sup> - 8\*A\*B\*b\*d<sup>2</sup>)/(16\*(a<sup>2</sup>\*d<sup>4</sup> + b<sup>2</sup>\*d<sup>4</sup>))^(1/2))/(a<sup>2</sup>\*d<sup>4</sup>))\*(((8\*A<sup>2</sup>\*a\*d<sup>2</sup> - 8\*B<sup>2</sup>\*a\*d<sup>2</sup> + 16\*A\*B\*b\*d<sup>2</sup>)<sup>2</sup>/4 - (16\*a<sup>2</sup>\*d<sup>4</sup> + 16\*b<sup>2</sup>\*d<sup>4</sup>)\*(A<sup>4</sup> + 2\*A<sup>2</sup>\*B<sup>2</sup> + B<sup>4</sup>))^(1/2) - 4\*A<sup>2</sup>\*a\*d<sup>2</sup> + 4\*B<sup>2</sup>\*a\*d<sup>2</sup> - 8\*A\*B\*b\*d<sup>2</sup>)/(16\*(a<sup>2</sup>\*d<sup>4</sup> + b<sup>2</sup>\*d<sup>4</sup>))^(1/2) - (16\*(a + b\*tan(c + dx))^(1/2)\*(20\*A<sup>2</sup>\*a<sup>3</sup>\*b<sup>8</sup>\*d<sup>2</sup> - 36\*B<sup>2</sup>\*a<sup>3</sup>\*b<sup>8</sup>\*d<sup>2</sup> - 4\*A<sup>2</sup>\*a\*b<sup>10</sup>\*d<sup>2</sup> + 48\*A\*B\*a<sup>2</sup>\*b<sup>9</sup>\*d<sup>2</sup>))/(a<sup>2</sup>\*d<sup>4</sup>))\*



$$\begin{aligned}
& B*b*d^2)^{2/4} - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^{1/2} - 4 \\
& *A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4))^{1/2} - ( \\
& 16*(a + b*\tan(c + d*x))^{1/2}*(A^2*B^2*b^{10} - A^4*b^{10} + 2*A^4*a^2*b^8 + 6* \\
& B^4*a^2*b^8 - 4*A*B^3*a*b^9 + 4*A^3*B*a*b^9))/(a^2*d^4))*(((8*A^2*a*d^2 - \\
& 8*B^2*a*d^2 + 16*A*B*b*d^2)^{2/4} - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^ \\
& 2 + B^4))^{1/2} - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b \\
& ^2*d^4))^{1/2} + (((((8*(32*A*a*b^{11}*d^4 + 16*A*a^3*b^9*d^4 - 64*B*a^2*b^{1 \\
& 0*d^4 - 48*B*a^4*b^8*d^4))/(a^2*d^5) + (16*(32*a^2*b^{10}*d^4 + 48*a^4*b^8*d^ \\
& 4)*(a + b*\tan(c + d*x))^{1/2})*(((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2) \\
& ^{2/4} - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^{1/2} - 4*A^2*a*d \\
& ^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4))^{1/2}))/a^2*d^4)) \\
& *(((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^{2/4} - (16*a^2*d^4 + 16*b^2*d \\
& ^4)*(A^4 + 2*A^2*B^2 + B^4))^{1/2} - 4*A^2*a*d^...
\end{aligned}$$

$$3.349 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=224

$$\frac{(8a^2A - 3Ab^2 + 4abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(A + iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2ad}$$

[Out] 1/4\*(8\*A\*a^2-3\*A\*b^2+4\*B\*a\*b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-(A-I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d/(a-I\*b)^(1/2)-(A+I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d/(a+I\*b)^(1/2)+1/4\*(3\*A\*b-4\*B\*a)\*cot(d\*x+c)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d-1/2\*A\*cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^(1/2)/a/d

**Rubi [A]**

time = 0.53, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3690, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(3Ab - 4aB) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{4a^2d} + \frac{(8a^2A + 4abB - 3Ab^2) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{(A + iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((8\*a^2\*A - 3\*A\*b^2 + 4\*a\*b\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]])/(4\*a^(5/2)\*d) - ((A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/(Sqrt[a - I\*b]\*d) - ((A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/(Sqrt[a + I\*b]\*d) + ((3\*A\*b - 4\*a\*B)\*Cot[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]])/(4\*a^2\*d) - (A\*Cot[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]])/(2\*a\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2)/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n \*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx &= -\frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2ad} - \frac{\int \frac{\cot^2(c+dx)(\frac{1}{2}(3Ab-4aB)+2aA)}{\sqrt{a+b \tan(c+dx)}} dx}{2} \\
 &= \frac{(3Ab-4aB) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{4a^2d} - \frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2} \\
 &= \frac{(3Ab-4aB) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{4a^2d} - \frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2} \\
 &= \frac{(3Ab-4aB) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{4a^2d} - \frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2} \\
 &= \frac{(3Ab-4aB) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{4a^2d} - \frac{A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{2} \\
 &= \frac{(8a^2A-3Ab^2+4abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} + \frac{(3Ab-4aB) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{2} \\
 &= \frac{(8a^2A-3Ab^2+4abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{(A-ib) \cot(c+dx) \sqrt{a+b \tan(c+dx)}}{2}
 \end{aligned}$$

**Mathematica** [A]



time = 5.92, size = 241, normalized size = 1.08

$$\frac{(8a^2A - 3Ab^2 + 4abB) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{a} \left( \frac{a^2 \left( \frac{-Ab + \sqrt{-b^2} B}{\sqrt{a - \sqrt{-b^2}}} \right) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a - \sqrt{-b^2}}}\right) - a^2 \left( \frac{Ab + \sqrt{-b^2} B}{\sqrt{a + \sqrt{-b^2}}} \right) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a + \sqrt{-b^2}}}\right) + b \cos(c+dx)(3Ab - 4aB - 2aA \cot(c+dx)) \sqrt{a+b\tan(c+dx)}}{4a^{5/2}d}}{b}}{4a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((8\*a^2\*A - 3\*A\*b^2 + 4\*a\*b\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]] + (Sqrt[a]\*((4\*a^2\*(-A\*b) + Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[a - Sqrt[-b^2]] - (4\*a^2\*(A\*b + Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] + b\*Cot[c + d\*x]\*(3\*A\*b - 4\*a\*B - 2\*a\*A\*Cot[c + d\*x])\*Sqrt[a + b\*Tan[c + d\*x]))/b)/(4\*a^(5/2)\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 2.29, size = 111103, normalized size = 496.00

method	result	size
default	Expression too large to display	111103

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVE RBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^3/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 8846 vs. 2(184) = 368.

time = 27.35, size = 17769, normalized size = 79.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16\*(16\*sqrt(2)\*((a^5 + a^3\*b^2)\*d^5\*cos(d\*x + c)^2 - (a^5 + a^3\*b^2)\*d^5)\*sqrt(-((2\*A\*B\*a^2\*b + 2\*A\*B\*b^3 + (A^2 - B^2)\*a^3 + (A^2 - B^2)\*a\*b^2)\*d^2\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4)) - (A^4 + 2\*A^2\*B^2 + B^4)\*a^2 - (A^4 + 2\*A^2\*B^2 + B^4)\*b^2)/(4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2))\*sqrt((4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4))\*((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4))^(3/4)\*arctan(-((2\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^5 - (A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^4\*b + 4\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^3\*b^2 - 2\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^2\*b^3 + 2\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a\*b^4 - (A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*b^5)\*d^4\*sqrt((4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4))\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4)) + (2\*(A^9\*B + 4\*A^7\*B^3 + 6\*A^5\*B^5 + 4\*A^3\*B^7 + A\*B^9)\*a^4 - (A^10 + 3\*A^8\*B^2 + 2\*A^6\*B^4 - 2\*A^4\*B^6 - 3\*A^2\*B^8 - B^10)\*a^3\*b + 2\*(A^9\*B + 4\*A^7\*B^3 + 6\*A^5\*B^5 + 4\*A^3\*B^7 + A\*B^9)\*a^2\*b^2 - (A^10 + 3\*A^8\*B^2 + 2\*A^6\*B^4 - 2\*A^4\*B^6 - 3\*A^2\*B^8 - B^10)\*a\*b^3)\*d^2\*sqrt((4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4)) - sqrt(2)\*((A\*a^5 + B\*a^4\*b + 2\*A\*a^3\*b^2 + 2\*B\*a^2\*b^3 + A\*a\*b^4 + B\*b^5)\*d^7\*sqrt((4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4))\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4)) + ((A^3 + A\*B^2)\*a^4 + 2\*(A^3 + A\*B^2)\*a^2\*b^2 + (A^3 + A\*B^2)\*b^4)\*d^5\*sqrt((4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4)))\*sqrt(-((2\*A\*B\*a^2\*b + 2\*A\*B\*b^3 + (A^2 - B^2)\*a^3 + (A^2 - B^2)\*a\*b^2)\*d^2\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4)) - (A^4 + 2\*A^2\*B^2 + B^4)\*a^2 - (A^4 + 2\*A^2\*B^2 + B^4)\*b^2)/(4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2))\*sqrt(((4\*(A^4\*B^2 + A^2\*B^4)\*a^4 - 4\*(A^5\*B - A\*B^5)\*a^3\*b + (A^6 + 3\*A^4\*B^2 + 3\*A^2\*B^4 + B^6)\*a^2\*b^2 - 4\*(A^5\*B - A\*B^5)\*a\*b^3 + (A^6 - A^4\*B^2 - A^2\*B^4 + B^6)\*b^4)\*d^2\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4))\*cos(d\*x + c) + sqrt(2)\*((4\*A^3\*B^2\*a^4 - 4\*(A^4\*B - A^2\*B^3)\*a^3\*b + (A^5 + 2\*A^3\*B^2 + A\*B^4)\*a^2\*b^2 - 4\*(A^4\*B - A^2\*B^3)\*a\*b^3 + (A^5 - 2\*A^3\*B^2 + A\*B^4)\*b^4)\*d^3\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4))\*cos(d\*x + c) + (4\*(A^5\*B^2 + A^3\*B^4)\*a^3 - 4\*(A^6\*B - A^4\*B^3 - 2\*A^2\*B^5)\*a^2\*b + (A^7 - 5\*A^5\*B^2 - A^3\*B^4 + 5\*A\*B^6)\*a\*b^2 + (A^6\*B - A^4\*B^3 - A^2\*B^5 + B^7)\*b^3)\*d\*cos(d\*x + c))\*sqrt(-((2\*A\*B\*a^2\*b + 2\*A\*B\*b^3 + (A^2 - B^2)\*a^3 + (A^2 - B^2)\*a\*b^2)\*d^2\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4)) - (A^4 + 2\*A^2\*B^2 + B^4)\*a^2 - (A^4 + 2\*A^2\*B^2 + B^4)\*b^2)/(4\*A^2\*B^2\*a^2 - 4\*(A^3\*B - A\*B^3)\*a\*b + (A^4 - 2\*A^2\*B^2 + B^4)\*b^2))\*sqrt((a\*cos(d\*x + c) + b\*sin(d\*x + c))/cos(d\*x + c))\*((A^4 + 2\*A^2\*B^2 + B^4)/((a^2 + b^2)\*d^4))^(1/4) + (4\*(A^6\*B^2 + 2\*A^4\*B^4 + A^2\*B^6)\*a^3 - 4\*(A^7\*B + A^5\*B^3 - A^3\*B^5 - A\*B^7)\*a^2\*b + (A^8 - 2\*A^4\*B^4 + B^8)\*a\*b^2)\*cos(d\*x + c) + (4\*(A^6\*B^2 + 2\*A^4\*B^4 + A^2\*B^6)\*a^2\*b - 4\*(A^7\*B

```

+ A^5*B^3 - A^3*B^5 - A*B^7)*a*b^2 + (A^8 - 2*A^4*B^4 + B^8)*b^3)*sin(d*x +
c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(3/4) + sqrt
(2)*((2*(A^4*B + A^2*B^3)*a^6 - (A^5 - 2*A^3*B^2 - 3*A*B^4)*a^5*b + (3*A^4*
B + 4*A^2*B^3 + B^5)*a^4*b^2 - 2*(A^5 - 2*A^3*B^2 - 3*A*B^4)*a^3*b^3 + 2*(A
^2*B^3 + B^5)*a^2*b^4 - (A^5 - 2*A^3*B^2 - 3*A*B^4)*a*b^5 - (A^4*B - B^5)*b
^6)*d^7*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B
^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^2 +
b^2)*d^4)) + (2*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a^5 - (A^7 + A^5*B^2 - A^3*B
^4 - A*B^6)*a^4*b + 4*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a^3*b^2 - 2*(A^7 + A^5*
B^2 - A^3*B^4 - A*B^6)*a^2*b^3 + 2*(A^6*B + 2*A^4*B^3 + A^2*B^5)*a*b^4 - (A
^7 + A^5*B^2 - A^3*B^4 - A*B^6)*b^5)*d^5*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A
*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sq
rt(-((2*A*B*a^2*b + 2*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*sq
rt((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2
- (A^4 + 2*A^2*B^2 + B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A
^4 - 2*A^2*B^2 + B^4)*b^2))*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*((A^4 + 2*A^2*B^2 + B^4)/((a^2 + b^2)*d^4))^(3/4))/(4*(A^10*B^2 + 4*A
^8*B^4 + 6*A^6*B^6 + 4*A^4*B^8 + A^2*B^10)*a^2*b - 4*(A^11*B + 3*A^9*B^3 +
2*A^7*B^5 - 2*A^5*B^7 - 3*A^3*B^9 - A*B^11)*a*b^2 + (A^12 + 2*A^10*B^2 - A^
8*B^4 - 4*A^6*B^6 - A^4*B^8 + 2*A^2*B^10 + B^12)*b^3)) + 16*sqrt(2)*((a^5 +
a^3*b^2)*d^5*cos(d*x + c)^2 - (a^5 + a^3*b^2)*d^5)*sqrt(-((2*A*B*a^2*b + 2
*A*B*b^3 + (A^2 - B^2)*a^3 + (A^2 - B^2)*a*b^2)*d^2*sqrt((A^4 + 2*A^2*B^2 +
B^4)/((a^2 + b^2)*d^4)) - (A^4 + 2*A^2*B^2 + B^4)*a^2 - (A^4 + 2*A^2*B^2 +
B^4)*b^2)/(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)
*b^2))*sqrt((4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^3(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*3/sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")



$$\begin{aligned}
& ^{10}d^2 + 64B^2a^3b^{10}d^2 - 320B^2a^5b^8d^2 + 36A^2a^*b^{12}d^2 - 9 \\
& 6A^*B^*a^2b^{11}d^2 + 768A^*B^*a^4b^9d^2)/(a^4d^4))*(-(((8A^2a^*d^2 - 8B^2a^*d^2 + 16A^*B^*b^*d^2)^2/4 - (16a^2d^4 + 16b^2d^4)*(A^4 + 2A^2B^2 \\
& + B^4))^{(1/2)} - 4A^2a^*d^2 + 4B^2a^*d^2 - 8A^*B^*b^*d^2)/(16*(a^2d^4 + b^2d^4)))^{(1/2)} + (64B^3a^2b^{11}d^2 - 192A^3a^5b^8d^2 + 256B^3a^4b^9d^2 + 36A^2B^*b^{13}d^2 + 36A^3a^*b^{12}d^2 - 96A^*B^2a^*b^{12}d^2 - 384A^* \\
& *B^2a^3b^{10}d^2 + 576A^*B^2a^5b^8d^2 + 96A^2B^*a^2b^{11}d^2 - 768A^2 \\
& *B^*a^4b^9d^2)/(2a^4d^5))*(-(((8A^2a^*d^2 - 8B^2a^*d^2 + 16A^*B^*b^*d^2)^2/4 - (16a^2d^4 + 16b^2d^4)*(A^4 + 2A^2B^2 + B^4))^{(1/2)} - 4A^2a^*d^2 + 4B^2a^*d^2 - 8A^*B^*b^*d^2)/(16*(a^2d^4 + b^2d^4)))^{(1/2)} - ((a + b*\tan(c + d*x))^{(1/2)}*(9A^4b^{12} - 9A^2B^2b^{12} - 48A^4a^2b^{10} + 96A^4a^4b^8 - 16B^4a^2b^{10} + 32B^4a^4b^8 + 64A^2B^2a^2b^{10} + 24A^*B^3 \\
& *a^*b^{11} - 24A^3B^*a^*b^{11} - 64A^*B^3a^3b^9 + 64A^3B^*a^3b^9))/(a^4d^4) \\
& )*(-(((8A^2a^*d^2 - 8B^2a^*d^2 + 16A^*B^*b^*d^2)^2/4 - (16a^2d^4 + 16b^2d^4)*(A^4 + 2A^2B^2 + B^4))^{(1/2)} - 4A^2a^*d^2 + 4B^2a^*d^2 - 8A^*B^*b^*d^2)/(16*(a^2d^4 + b^2d^4)))^{(1/2)}*i)/((((((640A^*a^4b^{10}d^4 - 384A^*a^2b^{12}d^4 + 768A^*a^6b^8d^4 + 512B^*a^3b^{11}d^4 + 256B^*a^5b^9d^4)/(2a^4d^5) + ((512a^4b^{10}d^4 + 768a^6b^8d^4)*(a + b*\tan(c + d*x))^{(1/2)}*(-(((8A^2a^*d^2 - 8B^2a^*d^2 + 16A^*B^*b^*d^2)^2/4 - (16a^2d^4 + 16b^2d^4)*(A^4 + 2A^2B^2 + B^4))^{(1/2)} - 4A^2a^*d^2 + 4B^2a^*d^2 - 8A^*B^*b^*d^2)/(16*(a^2d^4 + b^2d^4)))^{(1/2)}))/(a^4d^4))*(-(((8A^2a^*d^2 - 8B^2a^*d^2 + 16A^*B^*b^*d^2)^2/4 - (16a^2d^4 + 16b^2d^4)*(A^4 + 2A^2B^2 + B^4))^{(1/2)} - 4A^2a^*d^2 + 4B^2a^*d^2 - 8A^*B^*b^*d^2)/(16*(a^2d^4 + b^2d^4))))^{(1/2)} - ((a + b*\tan(c + d*x))^{(1/2)}*(576A^2a^5b^8d^2 - 192A^2a^3b^{10}d^2 + 64B^2a^3b^{10}d^2 - 320B^2a^5b^8d^2 + 36A^2a^*b^{12}d^2 - 96A^*B^*a^2b^{11}d^2 + 768A^*B^*a^4b^9d^2))/(a^4d^4))*(-(((8A^2a^*d^2 - 8B^2a^*d^2 + 16A^*B^*b^*d^2)^2/4 - (16a^2d^4 + 16b^2d^4)*(A^4 + 2A^2B^2 + B^4))^{(1/2)} - 4A^2a^*d^2 + 4B^2a^*d^2 - 8A^*B^*b^*d^2)/(16*(a^2d^4 + b^2d^4)))^{(1/2)} + (64B^3a^2b^{11}d^2 - 192A^3a^5b^8d^2 + 256B^3a^4b^9d^2 + 36A^2B^*b^{13}d^2 + 36A^3a^*b^{12}d^2 - 96A^*B^2a^*b^{12}d^2 - 384A^* \\
& A^*B^2a^3b^{10}d^2 + 576A^*B^2a^5b^8d^2 + 96A^2B^*a^2b^{11}d^2 - 768A^2 \\
& *B^*a^4b^9d^2)/(2a^4d^5))*(-(((8A^2a^*d^2 \dots
\end{aligned}$$

$$3.350 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=264

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{3/2} d} + \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{3/2} d} + \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

[Out] (A-I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(3/2)/d+(A+I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(3/2)/d+2/3\*(6\*A\*a^2\*b+3\*A\*b^3-8\*B\*a^3-5\*B\*a\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/b^3/(a^2+b^2)/d-2/3\*(3\*A\*a\*b-4\*B\*a^2-B\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)/b^2/(a^2+b^2)/d+2\*a\*(A\*b-B\*a)\*tan(d\*x+c)^2/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(1/2)

**Rubi [A]**

time = 0.46, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3686, 3728, 3711, 3620, 3618, 65, 214}

$$\frac{2a(Ab - aB) \tan^2(c + dx)}{bd(a^2 + b^2) \sqrt{a + b \tan(c + dx)}} - \frac{2(-4a^2B + 3aAb - b^2B) \tan(c + dx) \sqrt{a + b \tan(c + dx)}}{3b^2d(a^2 + b^2)} + \frac{2(-8a^3B + 6a^2Ab - 5aB^2 + 3Ab^2) \sqrt{a + b \tan(c + dx)}}{3b^2d(a^2 + b^2)} + \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{3/2}} + \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(3/2)\*d) + ((A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(3/2)\*d) + (2\*a\*(A\*b - a\*B)\*Tan[c + d\*x]^2)/(b\*(a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]]) + (2\*(6\*a^2\*A\*b + 3\*A\*b^3 - 8\*a^3\*B - 5\*a\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*b^3\*(a^2 + b^2)\*d) - (2\*(3\*a\*A\*b - 4\*a^2\*B - b^2\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*b^2\*(a^2 + b^2)\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3618**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^(m*(1
- I*Tan[e + f*x])), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^(m*(
1 + I*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.
) + (f_.)*(x_)^2], x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^(m*(c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
```

, 0] && NeQ[a, 0]))

Rubi steps

$$\int \frac{\tan^3(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx = \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{2 \int \frac{\tan(c+dx)(-2a(Ab-aB)+\frac{1}{2}b(Ab-c))}{\sqrt{a+...}}}{\sqrt{a+...}}$$

$$= \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{2(3aAb - 4a^2B - b^2B) \tan(c + dx)}{3b^2(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{2(6a^2Ab + 3Ab^3 - 8a^3B - 5a^2bB)}{3b^3(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{2(6a^2Ab + 3Ab^3 - 8a^3B - 5a^2bB)}{3b^3(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{2(6a^2Ab + 3Ab^3 - 8a^3B - 5a^2bB)}{3b^3(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2a(Ab - aB) \tan^2(c + dx)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{2(6a^2Ab + 3Ab^3 - 8a^3B - 5a^2bB)}{3b^3(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{3/2} d} + \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{3/2} d}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 2.22, size = 300, normalized size = 1.14

$$3iA \left( \frac{\tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{\sqrt{a - ib}} - \frac{\tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{\sqrt{a + ib}} \right) + \frac{2(6aAb - 8a^2B + 3b^2B)}{b^2 \sqrt{a + b \tan(c + dx)}} + \frac{3i(aA + bB) \left( (a + ib) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{3 + b \tan(c + dx)}{2 - ib} \right) - (a - ib) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{3 + b \tan(c + dx)}{2 + ib} \right) \right)}{(a^2 + b^2) \sqrt{a + b \tan(c + dx)}} + \frac{2(3Ab - 4aB) \tan(c + dx)}{b \sqrt{a + b \tan(c + dx)}} + \frac{2B \tan^2(c + dx)}{\sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((3\*I)\*A\*(ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/Sqrt[a - I\*b] - ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/Sqrt[a + I\*b]) + (2\*(6\*a\*A\*b



$$\begin{aligned}
& - 8a^2B + 3b^2B) / (b^2 \sqrt{a + b \tan[c + dx]}) + ((3I)(aA + bB) * \\
& ((a + Ib) \operatorname{Hypergeometric2F1}[-1/2, 1, 1/2, (a + b \tan[c + dx]) / (a - Ib)] \\
& - (a - Ib) \operatorname{Hypergeometric2F1}[-1/2, 1, 1/2, (a + b \tan[c + dx]) / (a + Ib)] \\
& )) / ((a^2 + b^2) \sqrt{a + b \tan[c + dx]}) + (2(3Ab - 4aB) \tan[c + dx] \\
& ) / (b \sqrt{a + b \tan[c + dx]}) + (2B \tan[c + dx]^2) / \sqrt{a + b \tan[c + dx]} \\
& ) / (3bd)
\end{aligned}$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2323 vs.  $2(236) = 472$ .

time = 0.18, size = 2324, normalized size = 8.80

method	result	size
derivativedivides	Expression too large to display	2324
default	Expression too large to display	2324

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(dx+c)^3*(A+B*tan(dx+c))/(a+b*tan(dx+c))^(3/2),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$\begin{aligned}
& 2/d/b^3*(1/3*B*(a+b*\tan(dx+c))^(3/2)+A*b*(a+b*\tan(dx+c))^(1/2)-2*B*a*(a+b \\
& * \tan(dx+c))^(1/2)+a^3*(A*b-B*a)/(a^2+b^2)/(a+b*\tan(dx+c))^(1/2)+b^3/(a^2+ \\
& b^2)*(1/4/b^2/(3*a^2-b^2)/(a^2+b^2)^(3/2)*(1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^( \\
& (1/2)*(a^2+b^2)^(3/2)*a^4+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*b \\
& ^4+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^6-2*A*(2*(a^2+b^2)^(1/ \\
& 2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^2-3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^ \\
& 2+b^2)^(1/2)*a^2*b^4+6*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^2+4*A*(2*(a^2+ \\
& b^2)^(1/2)+2*a)^(1/2)*a^3*b^4-2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^6+3*B*( \\
& 2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5*b+2*B*(2*(a^2+b^2)^(1/2)+2 \\
& *a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2) \\
& ^{(1/2)*a*b^5-3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6*b+B*(2*(a^2+b^2)^(1/2)+2 \\
& *a)^(1/2)*a^4*b^3+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^5-B*(2*(a^2+b^2)^( \\
& (1/2)+2*a)^(1/2)*b^7)*\ln(b*\tan(dx+c)+a+(a+b*\tan(dx+c))^(1/2)*(2*(a^2+b^2) \\
& ^{(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(6*A*a^6*b^2-2*A*a^4*b^4-6*A*a^2*b^6+2 \\
& *A*b^8+12*B*a^5*b^3+8*B*a^3*b^5-4*B*a*b^7-1/2*(-A*(2*(a^2+b^2)^(1/2)+2*a)^( \\
& (1/2)*(a^2+b^2)^(3/2)*a^4+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(3/2)*b^ \\
& 4+A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^6-2*A*(2*(a^2+b^2)^(1/2 \\
& )+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^4*b^2-3*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2 \\
& +b^2)^(1/2)*a^2*b^4+6*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5*b^2+4*A*(2*(a^2+b \\
& ^2)^(1/2)+2*a)^(1/2)*a^3*b^4-2*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*b^6+3*B*(2 \\
& *(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^5*b+2*B*(2*(a^2+b^2)^(1/2)+2* \\
& a)^(1/2)*(a^2+b^2)^(1/2)*a^3*b^3-B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^( \\
& (1/2)*a*b^5-3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6*b+B*(2*(a^2+b^2)^(1/2)+2* \\
& a)^(1/2)*a^4*b^3+3*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b^5-B*(2*(a^2+b^2)^( \\
& (1/2)+2*a)^(1/2)*b^7)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a) \\
& ^{(1/2)*\arctan((2*(a+b*\tan(dx+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(
\end{aligned}$$

$$\begin{aligned} & a^2+b^2)^{(1/2)}-2*a)^{(1/2)})) + 1/4/b^2/(3*a^2-b^2)/(a^2+b^2)^{(3/2)} * (-1/2*(-A*( \\ & 2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4+A*(2*(a^2+b^2)^{(1/2)}+2*a)^{( \\ & (1/2)}*(a^2+b^2)^{(3/2)}*b^4+A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a \\ & ^6-2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^2-3*A*(2*(a^2+b^ \\ & 2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^4+6*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/ \\ & 2)}*a^5*b^2+4*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3*b^4-2*A*(2*(a^2+b^2)^{(1/2)} \\ & +2*a)^{(1/2)}*a*b^6+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b+ \\ & *B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^3*b^3-B*(2*(a^2+b^2)^{(1/ \\ & 2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^5-3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^6*b \\ & +B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4*b^3+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}* \\ & a^2*b^5-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^7)*\ln(-b*\tan(d*x+c))-a+(a+b*\tan(d* \\ & x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}+2*(-6*A*a^6*b^2+ \\ & 2*A*a^4*b^4+6*A*a^2*b^6-2*A*b^8-12*B*a^5*b^3-8*B*a^3*b^5+4*B*a*b^7+1/2*(-A* \\ & (2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4+A*(2*(a^2+b^2)^{(1/2)}+2*a) \\ & ^{(1/2)}*(a^2+b^2)^{(3/2)}*b^4+A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}* \\ & a^6-2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^2-3*A*(2*(a^2+b \\ & ^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^4+6*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1 \\ & /2)}*a^5*b^2+4*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3*b^4-2*A*(2*(a^2+b^2)^{(1/2)} \\ & )+2*a)^{(1/2)}*a*b^6+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b+ \\ & 2*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^3*b^3-B*(2*(a^2+b^2)^{(1 \\ & /2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^5-3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^6* \\ & b+B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4*b^3+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)} \\ & *a^2*b^5-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^7)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)} \\ & )/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((-2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b \\ & ^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}))))) \end{aligned}$$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 21128 vs. 2(229) = 458.

time = 44.66, size = 21128, normalized size = 80.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

```

[Out] -1/12*(12*sqrt(2)*((a^10*b^3 + 3*a^8*b^5 + 2*a^6*b^7 - 2*a^4*b^9 - 3*a^2*b^
11 - b^13)*d^5*cos(d*x + c)^3 + 2*(a^9*b^4 + 4*a^7*b^6 + 6*a^5*b^8 + 4*a^3*
b^10 + a*b^12)*d^5*cos(d*x + c)^2*sin(d*x + c) + (a^8*b^5 + 4*a^6*b^7 + 6*a
^4*b^9 + 4*a^2*b^11 + b^13)*d^5*cos(d*x + c))*sqrt(((A^4 + 2*A^2*B^2 + B^4)
*a^6 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b
^4 + (A^4 + 2*A^2*B^2 + B^4)*b^6 - (6*A*B*a^8*b + 16*A*B*a^6*b^3 + 12*A*B*a^
4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)
*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b
+ 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*
(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B
^2 + B^4)*b^6))*sqrt((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 -
14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*
B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6
)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
d^4))^(3/4)*arctan(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^13 - 3*(A
^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^12*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5
+ A*B^7)*a^11*b^2 - 14*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^10*b^3 - 10*(
A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^4 - 25*(A^8 + 2*A^6*B^2 - 2*A^
2*B^6 - B^8)*a^8*b^5 - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^6 -
20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^7 - 50*(A^7*B + 3*A^5*B^3 + 3
*A^3*B^5 + A*B^7)*a^5*b^8 - 5*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^9 -
28*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^10 + 2*(A^8 + 2*A^6*B^2 -
2*A^2*B^6 - B^8)*a^2*b^11 - 6*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^
12 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^13)*d^4*sqrt((4*A^2*B^2*a^6 - 12
*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B
- A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*
a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*
a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4
)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (2*(A^9*B + 4*A^7*B^3 + 6*A^
5*B^5 + 4*A^3*B^7 + A*B^9)*a^10 - 3*(A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B
^6 - 3*A^2*B^8 - B^10)*a^9*b - 8*(A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6
- 3*A^2*B^8 - B^10)*a^7*b^3 - 12*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7
+ A*B^9)*a^6*b^4 - 6*(A^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8
- B^10)*a^5*b^5 - 16*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a
^4*b^6 - 6*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^8 + (A
^10 + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a*b^9)*d^2*sqrt
((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)
*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 -
12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^
2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(
2)*((A*a^14 + 2*B*a^13*b + 5*A*a^12*b^2 + 12*B*a^11*b^3 + 9*A*a^10*b^4 + 30
*B*a^9*b^5 + 5*A*a^8*b^6 + 40*B*a^7*b^7 - 5*A*a^6*b^8 + 30*B*a^5*b^9 - 9*A*
a^4*b^10 + 12*B*a^3*b^11 - 5*A*a^2*b^12 + 2*B*a*b^13 - A*b^14)*d^7*sqrt((4*

```

$$A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)ab^5 + (A^4 - 2A^2B^2 + B^4)b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4) \sqrt{(A^4 + 2A^2B^2 + B^4) / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} + ((A^3 + AB^2)a^{11} + (A^2B + B^3)a^{10}b + 5(A^3 + AB^2)a^9b^2 + 5(A^2B + B^3)a^8b^3 + 10(A^3 + AB^2)a^7b^4 + 10(A^2B + B^3)a^6b^5 + 10(A^3 + AB^2)a^5b^6 + 10(A^2B + B^3)a^4b^7 + 5(A^3 + AB^2)a^3b^8 + 5(A^2B + B^3)a^2b^9 + (A^3 + AB^2)ab^{10} + (A^2B + B^3)b^{11})d^5 \sqrt{(4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)ab^5 + (A^4 - 2A^2B^2 + B^4)b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)} \sqrt{((A^4 + 2A^2B^2 + B^4)a^6 + 3(A^4 + 2A^2B^2 + B^4)a^4b^2 + 3(A^4 + 2A^2B^2 + B^4)a^2b^4 + (A^4 + 2A^2B^2 + B^4)b^6 - (6ABa^8b + 16ABa^6b^3 + 12ABa^4b^5 - 2ABb^9 + (A^2 - B^2)a^9 - 6(A^2 - B^2)a^5b^4 - 8(A^2 - B^2)a^3b^6 - 3(A^2 - B^2)ab^8)d^2 \sqrt{(A^4 + 2A^2B^2 + B^4) / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)}} / (4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)ab^5 + (A^4 - 2A^2B^2 + B^4)b^6) \sqrt{((4(A^4B^2 + A^2B^4)a^{10} - 12(A^5B - AB^5)a^9b + (9A^6 - 25A^4B^2 - 25\dots$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^3(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*3/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

Mupad [B]

time = 20.58, size = 2500, normalized size = 9.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + dx))^3(A + B\tan(c + dx)))/(a + b\tan(c + dx))^{(3/2)}, x)$

[Out]  $(\log(24A^3a^3b^6d^2 - (((((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (((((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (a + b\tan(c + dx)))^{(1/2)} * (64ab^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5))/4 - 32Ab^{12}d^4 - 96Aa^2b^{10}d^4 - 64Aa^4b^8d^4 + 64Aa^6b^6d^4 + 96Aa^8b^4d^4 + 32Aa^{10}b^2d^4))/4 + (a + b\tan(c + dx))^{(1/2)} * (16A^2b^{10}d^3 + 32A^2a^2b^8d^3 - 32A^2a^6b^4d^3 - 16A^2a^8b^2d^3)) * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)})/4 + 24A^3a^5b^4d^2 + 8A^3a^7b^2d^2 + 8A^3a^9b^0d^2) * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)})/4 + (\log(24A^3a^3b^6d^2 - ((((-((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * ((((-((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (a + b\tan(c + dx)))^{(1/2)} * (64ab^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5))/4 - 32Ab^{12}d^4 - 96Aa^2b^{10}d^4 - 64Aa^4b^8d^4 + 64Aa^6b^6d^4 + 96Aa^8b^4d^4 + 32Aa^{10}b^2d^4))/4 + (a + b\tan(c + dx))^{(1/2)} * (16A^2b^{10}d^3 + 32A^2a^2b^8d^3 - 32A^2a^6b^4d^3 - 16A^2a^8b^2d^3)) * (-((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)})/4 + 24A^3a^5b^4d^2 + 8A^3a^7b^2d^2 + 8A^3a^9b^0d^2) * (-((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)})/4 - \log((((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} * (32Ab^{12}d^4 + (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} * (a + b\tan(c + dx)))^{(1/2)} * (64ab^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5) + 96Aa^2b^{10}d^4 + 64Aa^4b^8d^4 -$

$$\begin{aligned}
& 64*A*a^6*b^6*d^4 - 96*A*a^8*b^4*d^4 - 32*A*a^{10}*b^2*d^4) + (a + b*\tan(c + d \\
& *x))^{(1/2)}*(16*A^2*b^{10}*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16* \\
& A^2*a^8*b^2*d^3))*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2* \\
& d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 4 \\
& 8*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + 24*A^3*a^3*b^6*d^2 + 24*A^3*a^5*b^ \\
& 4*d^2 + 8*A^3*a^7*b^2*d^2 + 8*A^3*a*b^8*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4 \\
& *b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/( \\
& 16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} - \log((( - \\
& ((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2* \\
& a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48* \\
& a^4*b^2*d^4))^{(1/2)}*(32*A*b^{12}*d^4 + (-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^ \\
& 4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6* \\
& d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*\tan(c + d \\
& *x))^{(1/2)}*(64*a*b^{12}*d^5 + 320*a^3*b^{10}*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^ \\
& 6*d^5 + 320*a^9*b^4*d^5 + 64*a^{11}*b^2*d^5) + 96*A*a^2*b^{10}*d^4 + 64*A*a^4*b^ \\
& ^8*d^4 - 64*A*a^6*b^6*d^4 - 96*A*a^8*b^4*d^4 - 32*A*a^{10}*b^2*d^4) + (a + b* \\
& \tan(c + d*x))^{(1/2)}*(16*A^2*b^{10}*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4* \\
& d^3 - 16*A^2*a^8*b^2*d^3))*(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^ \\
& 4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b \\
& ^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + 24*A^3*a^3*b^6*d^2 + 24* \\
& A^3*a^5*b^4*d^2 + 8*A^3*a^7*b^2*d^2 + 8*A^3*a*b^8*d^2)*(-((96*A^4*a^2*b^4*d \\
& ^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2*a \\
& *b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} \\
& ) + (\log((((a + b*\tan(c + d*x))^{(1/2)}*(16*B^2*b^{10}*d^3 + 32*B^2*a^2*b^8*d^3 \\
& - 32*B^2*a^6*b^4*d^3 - 16*B^2*a^8*b^2*d^3) - (((96*B^4*a^2*b^4*d^4 - 16*B \\
& ^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2) \\
& / (a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}*(((96*B^4*a^2 \\
& *b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} - 4*B^2*a^3*d^2 + 12 \\
& *B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}* \\
& (a + b*\tan(c + d*x))^{(1/2)}*(64*a*b^{12}*d^5 + 320...
\end{aligned}$$

$$3.351 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=167

$$\frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{3/2} d} - \frac{(iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{3/2} d} - \frac{2a^2(Ab - B^2)}{b^2(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

[Out] (I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(3/2)/d-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(3/2)/d-2\*a^2\*(A\*b-B\*a)/b^2/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(1/2)+2\*B\*(a+b\*tan(d\*x+c))^(1/2)/b^2/d

**Rubi** [A]

time = 0.28, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {3685, 3711, 3620, 3618, 65, 214}

$$\frac{2a^2(Ab - aB)}{b^2 d (a^2 + b^2) \sqrt{a + b \tan(c + dx)}} + \frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{3/2}} - \frac{(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{3/2}} + \frac{2B \sqrt{a + b \tan(c + dx)}}{b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2),x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(3/2)\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(3/2)\*d) - (2\*a^2\*(A\*b - a\*B))/(b^2\*(a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]]) + (2\*B\*Sqrt[a + b\*Tan[c + d\*x]])/(b^2\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c

\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3685

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(- (B\*c - A\*d)) \* (b\*c - a\*d)^2 \* ((c + d\*Tan[e + f\*x])^(n + 1) / (f\*d^2\*(n + 1) \* (c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1) \* Simp[B\*(b\*c - a\*d)^2 + A\*d\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + d\*(B\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + A\*(2\*a\*b\*c - a^2\*d + b^2\*d)) \* Tan[e + f\*x] + b^2\*B\*(c^2 + d^2) \* Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

### Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1) / (b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m \* Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps



$$\begin{aligned}
\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx &= -\frac{2a^2(Ab-aB)}{b^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{-a(Ab-aB)+b(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}}{b(a^2+b^2)} \\
&= -\frac{2a^2(Ab-aB)}{b^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{2B\sqrt{a+b\tan(c+dx)}}{b^2d} + \frac{2B\sqrt{a+b\tan(c+dx)}}{b^2d} \\
&= -\frac{2a^2(Ab-aB)}{b^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{2B\sqrt{a+b\tan(c+dx)}}{b^2d} \\
&= -\frac{2a^2(Ab-aB)}{b^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{2B\sqrt{a+b\tan(c+dx)}}{b^2d} \\
&= -\frac{2a^2(Ab-aB)}{b^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{2B\sqrt{a+b\tan(c+dx)}}{b^2d} \\
&= \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.84, size = 248, normalized size = 1.49

$$\frac{iB \left( \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} \right) + \frac{-2Ab+4aB}{b\sqrt{a+b\tan(c+dx)}} + \frac{(Ab-aB)\left((-ia+b) {}_2F_1\left(-\frac{1}{2}, 1; \frac{3}{2}; \frac{a+b\tan(c+dx)}{a-ib}\right) + (ia+b) {}_2F_1\left(-\frac{1}{2}, 1; \frac{3}{2}; \frac{a+b\tan(c+dx)}{a+ib}\right)\right)}{(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{2B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (I\*B\*(ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/Sqrt[a - I\*b] - ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/Sqrt[a + I\*b]) + (-2\*A\*b + 4\*a\*B)/(b\*Sqrt[a + b\*Tan[c + d\*x]]) + ((A\*b - a\*B)\*((( -I)\*a + b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] + (I\*a + b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)]))/((a^2 + b^2)\*Sqrt[a + b\*Tan[c + d\*x]]) + (2\*B\*Tan[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]]/(b\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. 2(145) = 290.

time = 0.16, size = 2298, normalized size = 13.76



$$\begin{aligned} &)^{(1/2)} * a^3 * b^4 + 2 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a * b^6) * \ln(-b * \tan(d * x + c) - a \\ &+ (a + b * \tan(d * x + c))^{(1/2)} * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} - (a^2 + b^2)^{(1/2)}) + 2 * (- \\ &12 * A * a^5 * b^3 - 8 * A * a^3 * b^5 + 4 * A * a * b^7 + 6 * B * a^6 * b^2 - 2 * B * a^4 * b^4 - 6 * B * a^2 * b^6 + 2 * B * \\ &b^8 + 1/2 * (3 * A * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * (a^2 + b^2)^{(1/2)} * a^5 * b + 2 * A * (2 * (a^ \\ &2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * (a^2 + b^2)^{(1/2)} * a^3 * b^3 - A * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * \\ &(a^2 + b^2)^{(1/2)} * a * b^5 - 3 * A * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a^6 * b + A * (2 * (a^ \\ &2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a^4 * b^3 + 3 * A * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a^2 * b^5 - A \\ &* (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * b^7 + B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * (a^2 + b^2 \\ &)^{(3/2)} * a^4 - B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * (a^2 + b^2)^{(3/2)} * b^4 - B * (2 * (a^2 + b \\ &^2)^{(1/2)} + 2 * a)^{(1/2)} * (a^2 + b^2)^{(1/2)} * a^6 + 2 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * \\ &(a^2 + b^2)^{(1/2)} * a^4 * b^2 + 3 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * (a^2 + b^2)^{(1/2)} * a \\ &^2 * b^4 - 6 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a^5 * b^2 - 4 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a \\ &)^{(1/2)} * a^3 * b^4 + 2 * B * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a * b^6) * (2 * (a^2 + b^2)^{(1/2)} \\ &+ 2 * a)^{(1/2)} / (2 * (a^2 + b^2)^{(1/2)} - 2 * a)^{(1/2)} * \arctan((-2 * (a + b * \tan(d * x + c))^{(1/2)} \\ &+ (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)}) / (2 * (a^2 + b^2)^{(1/2)} - 2 * a)^{(1/2)})) \end{aligned}$$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 20869 vs. 2(139) = 278.

time = 46.86, size = 20869, normalized size = 124.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (4 * \sqrt{2}) * ((a^{10} * b^2 + 3 * a^8 * b^4 + 2 * a^6 * b^6 - 2 * a^4 * b^8 - 3 * a^2 * b^{10} - b^{12}) * d^5 * \cos(d * x + c)^2 + 2 * (a^9 * b^3 + 4 * a^7 * b^5 + 6 * a^5 * b^7 + 4 * a^3 * b^9 + a * b^{11}) * d^5 * \cos(d * x + c) * \sin(d * x + c) + (a^8 * b^4 + 4 * a^6 * b^6 + 6 * a^4 * b^8 + 4 * a^2 * b^{10} + b^{12}) * d^5) * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^6 + 3 * (A^4 + 2 * A^2 * B^2 + B^4) * a^4 * b^2 + 3 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^4 + (A^4 + 2 * A^2 * B^2 + B^4) * b^6 + (6 * A * B * a^8 * b + 16 * A * B * a^6 * b^3 + 12 * A * B * a^4 * b^5 - 2 * A * B * b^9 + (A^2 - B^2) * a^9 - 6 * (A^2 - B^2) * a^5 * b^4 - 8 * (A^2 - B^2) * a^3 * b^6 - 3 * (A^2 - B^2) * a * b^8) * d^2 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) / ((a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * d^4))} / (4 * A^2 * B^2 * a^6 - 12 * (A^3 * B - A * B^3) * a^5 * b + 3 * (3 * A^4 - 14 * A^2 * B^2 + B^4) * a^4 * b^2 - 12 * A * B * a^3 * b^4 + 3 * (3 * A^4 - 14 * A^2 * B^2 + B^4) * a^2 * b^6 - 12 * A * B * a * b^8 + 3 * (3 * A^4 - 14 * A^2 * B^2 + B^4) * b^8) * \arctan(\frac{a * \tan(d * x + c) + \sqrt{a^2 + b^2} * \tan(d * x + c)}{a + b * \tan(d * x + c)})$

$$\begin{aligned}
& 2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)) * \text{sqrt} \\
& \text{t}((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4) \\
& ) * a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 \\
& - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6) / ((a^{12} + 6*a^{10}*b^2 \\
& ^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) * d^4)) * ((A^4 \\
& + 2*A^2*B^2 + B^4) / ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) * d^4))^{3/4} * \arctan( \\
& - ((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13} - 3*(A^8 + 2*A^6*B^2 - 2* \\
& A^2*B^6 - B^8)*a^{12}*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^2 \\
& - 14*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^3 - 10*(A^7*B + 3*A^5*B^3 + \\
& 3*A^3*B^5 + A*B^7)*a^9*b^4 - 25*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^5 \\
& - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^6 - 20*(A^8 + 2*A^6*B^2 \\
& - 2*A^2*B^6 - B^8)*a^6*b^7 - 50*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a \\
& ^5*b^8 - 5*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^9 - 28*(A^7*B + 3*A^5* \\
& B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{10} + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a \\
& ^2*b^{11} - 6*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{12} + (A^8 + 2*A^6*B \\
& ^2 - 2*A^2*B^6 - B^8)*b^{13}) * d^4 * \text{sqrt}((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5 \\
& *b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - \\
& 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2 \\
& *B^2 + B^4)*b^6) / ((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 \\
& + 6*a^2*b^{10} + b^{12}) * d^4)) * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4) / ((a^6 + 3*a^4*b^2 \\
& + 3*a^2*b^4 + b^6) * d^4)) + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + \\
& A*B^9)*a^{10} - 3*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10}) \\
& *a^9*b - 8*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10}) \\
& *a^7*b^3 - 12*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^6*b^4 - \\
& 6*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10}) *a^5*b^5 - \\
& 16*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4*b^6 - 6*(A^9*B + \\
& 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^8 + (A^{10} + 3*A^8*B^2 + 2 \\
& *A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10}) *a*b^9) * d^2 * \text{sqrt}((4*A^2*B^2*a^6 - 1 \\
& 2*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3* \\
& B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3) \\
& *a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6) / ((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20 \\
& *a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) * d^4)) + \text{sqrt}(2) * ((B*a^{14} - 2*A*a \\
& ^{13}*b + 5*B*a^{12}*b^2 - 12*A*a^{11}*b^3 + 9*B*a^{10}*b^4 - 30*A*a^9*b^5 + 5*B*a^8 \\
& *b^6 - 40*A*a^7*b^7 - 5*B*a^6*b^8 - 30*A*a^5*b^9 - 9*B*a^4*b^{10} - 12*A*a^3 \\
& *b^{11} - 5*B*a^2*b^{12} - 2*A*a*b^{13} - B*b^{14}) * d^7 * \text{sqrt}((4*A^2*B^2*a^6 - 12*(A \\
& ^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - \\
& A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b \\
& ^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6) / ((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6 \\
& *b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) * d^4)) * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4) / ( \\
& (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) * d^4)) + ((A^2*B + B^3)*a^{11} - (A^3 + A \\
& B^2)*a^{10}*b + 5*(A^2*B + B^3)*a^9*b^2 - 5*(A^3 + A*B^2)*a^8*b^3 + 10*(A^2*B \\
& + B^3)*a^7*b^4 - 10*(A^3 + A*B^2)*a^6*b^5 + 10*(A^2*B + B^3)*a^5*b^6 - 10* \\
& (A^3 + A*B^2)*a^4*b^7 + 5*(A^2*B + B^3)*a^3*b^8 - 5*(A^3 + A*B^2)*a^2*b^9 + \\
& (A^2*B + B^3)*a*b^{10} - (A^3 + A*B^2)*b^{11}) * d^5 * \text{sqrt}((4*A^2*B^2*a^6 - 12*(A
\end{aligned}$$

$$\begin{aligned} &^3B - AB^3)a^5b + 3*(3A^4 - 14A^2B^2 + 3B^4)*a^4b^2 + 40*(A^3B - \\ &AB^3)*a^3b^3 - 6*(A^4 - 8A^2B^2 + B^4)*a^2b^4 - 12*(A^3B - AB^3)*a*b \\ &^5 + (A^4 - 2A^2B^2 + B^4)*b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6 \\ &*b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})*d^4))*sqrt(((A^4 + 2A^2B^2 + B^4) \\ &*a^6 + 3*(A^4 + 2A^2B^2 + B^4)*a^4b^2 + 3*(A^4 + 2A^2B^2 + B^4)*a^2b^ \\ &4 + (A^4 + 2A^2B^2 + B^4)*b^6 + (6A*B*a^8*b + 16A*B*a^6*b^3 + 12A*B*a^ \\ &4*b^5 - 2A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2) \\ &*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*sqrt((A^4 + 2A^2B^2 + B^4)/((a^6 + 3* \\ &a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4A^2B^2*a^6 - 12*(A^3B - AB^3)*a^5*b \\ &+ 3*(3A^4 - 14A^2B^2 + 3B^4)*a^4b^2 + 40*(A^3B - AB^3)*a^3b^3 - 6* \\ &(A^4 - 8A^2B^2 + B^4)*a^2b^4 - 12*(A^3B - AB^3)*a*b^5 + (A^4 - 2A^2B \\ &^2 + B^4)*b^6))*sqrt(((4*(A^4B^2 + A^2B^4)*a^{10} - 12*(A^5B - AB^5)*a^9* \\ &b + (9A^6 - 25A^4B^2 - 25A^2B^4 + 9B^6)*a... \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^2(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*2/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 13.28, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^2\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] (log((((a + b\*tan(c + d\*x))^(1/2)\*(16A^2\*b^10\*d^3 + 32A^2\*a^2\*b^8\*d^3 - 32A^2\*a^6\*b^4\*d^3 - 16A^2\*a^8\*b^2\*d^3) + (((96A^4\*a^2\*b^4\*d^4 - 16A^4\*b



$$\begin{aligned}
& 2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} * ((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)} * (64*a*b^{12}*d^5 + 320*a^3*b^{10}*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^{11}*b^2*d^5) + 64*A*a*b^{11}*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4)) * (-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2) * (-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + (\log(24*B^3*a^3*b^6*d^2 - ((((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)} * ((((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)} * (64*a*b^{12}*d^5 + 320*a^3*b^{10}*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^{11}*b^2*d^5))/4 - 32*B*b^{12}*d^4 - 96*B*a^2*b^{10}*d^4 - 64*B*a^4*b^8*d^4 + 64*B*a^6*b...
\end{aligned}$$

$$3.352 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=141

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{3/2} d} - \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{3/2} d} + \frac{2a(Ab - B^2)}{b(a^2 + b^2)d\sqrt{a + b \tan(c + dx)}}$$

[Out]  $-(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(3/2)}/d-(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(3/2)}/d+2*a*(A*b-B*a)/b/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 0.19, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3672, 3620, 3618, 65, 214}

$$\frac{2a(Ab - aB)}{bd(a^2 + b^2)\sqrt{a + b \tan(c + dx)}} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{3/2}} - \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x]*(A + B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $-\left(\left(\left(A - I*B\right)*\operatorname{ArcTanh}\left[\frac{\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{\sqrt{a - I*b}}\right]\right)/\left(\left(a - I*b\right)^{(3/2)*d}\right)\right) - \left(\left(A + I*B\right)*\operatorname{ArcTanh}\left[\frac{\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{\sqrt{a + I*b}}\right]\right)/\left(\left(a + I*b\right)^{(3/2)*d}\right) + \left(2*a*(A*b - a*B)\right)/\left(b*(a^2 + b^2)*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]\right)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c$



\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 - I\*Tan[e + f\*x])), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 + I\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3672

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx &= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{\int \frac{Ab - aB + (aA + bB) \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{a^2 + b^2} \\
 &= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}}}{2(ia + b)} \\
 &= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a - bx}}\right)}{2(a - ib)} \\
 &= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{(i(A + iB)) \text{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - ix}\right)}{(a - ib)} \\
 &= -\frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{3/2} d} - \frac{(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{3/2} d}
 \end{aligned}$$

**Mathematica [A]**

time = 0.90, size = 229, normalized size = 1.62

$$\frac{b(A(b^2 - a\sqrt{-b^2}) - b(a + \sqrt{-b^2})B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{-b^2}}}\right) - b(A(b^2 + a\sqrt{-b^2}) + b(-a + \sqrt{-b^2})B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + \sqrt{-b^2}}}\right) + \frac{2a(Ab - aB)}{\sqrt{a + b \tan(c + dx)}}}{b(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((b\*(A\*(b^2 - a\*Sqrt[-b^2]) - b\*(a + Sqrt[-b^2])\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a - Sqrt[-b^2]]) - (b\*(A\*(b^2 + a\*Sqrt[-b^2]) + b\*(-a + Sqrt[-b^2])\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a + Sqrt[-b^2]]) + (2\*a\*(A\*b - a\*B))/Sqrt[a + b\*Tan[c + d\*x]])/(b\*(a^2 + b^2)\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. 2(121) = 242.

time = 0.13, size = 2275, normalized size = 16.13

method	result	size
derivativedivides	Expression too large to display	2275
default	Expression too large to display	2275

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, method=\_RETURNVERBOSE)

[Out] 2/d/b\*(a\*(A\*b-B\*a)/(a^2+b^2)/(a+b\*tan(d\*x+c))^(1/2)-b/(a^2+b^2)\*(1/4/b^2/(3\*a^2-b^2)/(a^2+b^2)^(3/2)\*(1/2\*(-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^4+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*b^4+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^6-2\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^4\*b^2-3\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^2\*b^4+6\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^5\*b^2+4\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^3\*b^4-2\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b^6+3\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5\*b+2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^3-B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^5-3\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6\*b+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^3+3\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^5-B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^7)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))+2\*(6\*A\*a^6\*b^2-2\*A\*a^4\*b^4-6\*A\*a^2\*b^6+2\*A\*b^8+12\*B\*a^5\*b^3+8\*B\*a^3\*b^5-4\*B\*a\*b^7-1/2\*(-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^4+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*b^4+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^6-2\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^4\*b^2-3\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^2\*

$$\begin{aligned}
& b^4 + 6A(2(a^2+b^2)^{1/2} + 2a)^{1/2} a^5 b^2 + 4A(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& (1/2) a^3 b^4 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} a^2 b^6 + 3B(2(a^2+b^2)^{1/2} \\
& + 2a)^{1/2} (a^2+b^2)^{1/2} a^5 b + 2B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} \\
& (a^2+b^2)^{1/2} a^3 b^3 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^2 b^5 - 3B \\
& (2(a^2+b^2)^{1/2} + 2a)^{1/2} a^6 b + B(2(a^2+b^2)^{1/2} + 2a)^{1/2} a^4 b^3 \\
& + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} a^2 b^5 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& b^7) * (2(a^2+b^2)^{1/2} + 2a)^{1/2} / (2(a^2+b^2)^{1/2} - 2a)^{1/2} * \arctan((2 \\
& * (a+b*\tan(dx+c))^{1/2} + (2(a^2+b^2)^{1/2} + 2a)^{1/2}) / (2(a^2+b^2)^{1/2} - 2 \\
& * a)^{1/2})) + 1/4/b^2 / (3a^2 - b^2) / (a^2+b^2)^{3/2} * (-1/2 * (-A(2(a^2+b^2)^{1/2} \\
& ) + 2a)^{1/2} * (a^2+b^2)^{3/2} * a^4 + A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{3/2} \\
& * b^4 + A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^6 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& * (a^2+b^2)^{1/2} * a^4 b^2 - 3A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^2 b^4 + 6A(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& * a^5 b^2 + 4A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^3 b^4 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^2 b^6 \\
& + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^5 b + 2B(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& * (a^2+b^2)^{1/2} * a^3 b^3 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^2 b^5 - 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& * a^6 b + B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^4 b^3 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^2 b^5 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} \\
& * b^7) * \ln(-b*\tan(dx+c) - a + (a+b*\tan(dx+c))^{1/2} * (2(a^2+b^2)^{1/2} + 2a)^{1/2} - (a^2+b^2)^{1/2}) + 2 * (-6Aa^6 b^2 + 2Aa^4 b^4 + 6Aa^2 b^6 - 2Ab^8 - 12B a^5 b^3 - 8B a^3 b^5 + 4B a b^7 + 1/2 * (-A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{3/2} * a^4 + A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{3/2} * b^4 + A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^6 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^4 b^2 - 3A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^2 b^4 + 6A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^5 b^2 + 4A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^3 b^4 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^2 b^6 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^5 b + 2B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^3 b^3 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^2 b^5 - 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^6 b + B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^4 b^3 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^2 b^5 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} * b^7) * (2(a^2+b^2)^{1/2} + 2a)^{1/2} / (2(a^2+b^2)^{1/2} - 2a)^{1/2} * \arctan((-2(a+b*\tan(dx+c))^{1/2} + (2(a^2+b^2)^{1/2} + 2a)^{1/2}) / (2(a^2+b^2)^{1/2} - 2a)^{1/2}))
\end{aligned}$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B\*tan(dx + c) + A)\*tan(dx + c)/(b\*tan(dx + c) + a)^(3/2), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 20761 vs. 2(115) = 230.

time = 65.12, size = 20761, normalized size = 147.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \cdot (4 \cdot \sqrt{2}) \cdot ((a^{10}b + 3a^8b^3 + 2a^6b^5 - 2a^4b^7 - 3a^2b^9 - b^{11}) \cdot d^5 \cdot \cos(dx + c)^2 + 2(a^9b^2 + 4a^7b^4 + 6a^5b^6 + 4a^3b^8 + ab^{10}) \cdot d^5 \cdot \cos(dx + c) \cdot \sin(dx + c) + (a^8b^3 + 4a^6b^5 + 6a^4b^7 + 4a^2b^9 + b^{11}) \cdot d^5) \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^6 + 3(A^4 + 2A^2B^2 + B^4)a^4b^2 + 3(A^4 + 2A^2B^2 + B^4)a^2b^4 + (A^4 + 2A^2B^2 + B^4)b^6 - (6ABa^8b + 16AB^2a^6b^3 + 12A^2B^3a^4b^5 - 2AB^2b^9 + (A^2 - B^2)a^9 - 6(A^2 - B^2)a^5b^4 - 8(A^2 - B^2)a^3b^6 - 3(A^2 - B^2)a^b^8) \cdot d^2 \cdot \sqrt{(A^4 + 2A^2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4))} / (4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^b^5 + (A^4 - 2A^2B^2 + B^4)b^6) \cdot \sqrt{(4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^b^5 + (A^4 - 2A^2B^2 + B^4)b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot d^4)} \cdot ((A^4 + 2A^2B^2 + B^4) / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4))^{3/4} \cdot \arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13} - 3(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b + 4(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^2 - 14(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^3 - 10(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^4 - 25(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^5 - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^6 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^7 - 50(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^8 - 5(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^9 - 28(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{10} + 2(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b^{11} - 6(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^b^{12} + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{13}) \cdot d^4 \cdot \sqrt{(4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^b^5 + (A^4 - 2A^2B^2 + B^4)b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot d^4)} \cdot \sqrt{(A^4 + 2A^2B^2 + B^4) / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4)} + (2(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^{10} - 3(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^9b - 8(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^7b^3 - 12(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^6b^4 - 6(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^5b^5 - 16(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^4b^6 - 6(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^2b^8 + (A^{10} + 3A^8B^2 + 2A^6$

```

*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^10)*a*b^9)*d^2*sqrt((4*A^2*B^2*a^6 - 12*(A
^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B -
A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b
^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6
*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + sqrt(2)*((A*a^14 + 2*B*a^13*
b + 5*A*a^12*b^2 + 12*B*a^11*b^3 + 9*A*a^10*b^4 + 30*B*a^9*b^5 + 5*A*a^8*b
^6 + 40*B*a^7*b^7 - 5*A*a^6*b^8 + 30*B*a^5*b^9 - 9*A*a^4*b^10 + 12*B*a^3*b^1
1 - 5*A*a^2*b^12 + 2*B*a*b^13 - A*b^14)*d^7*sqrt((4*A^2*B^2*a^6 - 12*(A^3*B
- A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^
3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 +
(A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + ((A^3 + A*B^2)*a^11 + (A^2*B + B^3)
*a^10*b + 5*(A^3 + A*B^2)*a^9*b^2 + 5*(A^2*B + B^3)*a^8*b^3 + 10*(A^3 + A*B
^2)*a^7*b^4 + 10*(A^2*B + B^3)*a^6*b^5 + 10*(A^3 + A*B^2)*a^5*b^6 + 10*(A^2
*B + B^3)*a^4*b^7 + 5*(A^3 + A*B^2)*a^3*b^8 + 5*(A^2*B + B^3)*a^2*b^9 + (A^
3 + A*B^2)*a*b^10 + (A^2*B + B^3)*b^11)*d^5*sqrt((4*A^2*B^2*a^6 - 12*(A^3*B
- A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^
3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 +
(A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6
+ 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^6
+ 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^4 +
(A^4 + 2*A^2*B^2 + B^4)*b^6 - (6*A*B*a^8*b + 16*A*B*a^6*b^3 + 12*A*B*a^4*b^
5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3
*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3
*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4
- 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 +
B^4)*b^6))*sqrt(((4*(A^4*B^2 + A^2*B^4)*a^10 - 12*(A^5*B - A*B^5)*a^9*b +
(9*A^6 - 25*A^4*B^2 - 25*A^2*B^4 + 9*B^6)*a^8*b...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2), x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 12.18, size = 2500, normalized size = 17.73
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (log(- (((((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2)
+ 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 +
3*a^4*b^2*d^4))^(1/2)*(32*A*b^12*d^4 + (((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6
*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*
d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(
1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5
+ 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 96*A*a^2*b^10*d^4 + 64*A*a^4*b^8
*d^4 - 64*A*a^6*b^6*d^4 - 96*A*a^8*b^4*d^4 - 32*A*a^10*b^2*d^4))/4 + (a + b
*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4
*d^3 - 16*A^2*a^8*b^2*d^3))*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^
4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4
+ 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - 24*A^3*a^3*b^6*d^2 - 24*A^3*a
^5*b^4*d^2 - 8*A^3*a^7*b^2*d^2 - 8*A^3*a*b^8*d^2))*(((96*A^4*a^2*b^4*d^4 - 1
6*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d
^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log(-
(((((-(96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4
*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4
*b^2*d^4))^(1/2)*(32*A*b^12*d^4 + ((((-(96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4
- 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 +
b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)
*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 32
0*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 96*A*a^2*b^10*d^4 + 64*A*a^4*b^8*d^4
- 64*A*a^6*b^6*d^4 - 96*A*a^8*b^4*d^4 - 32*A*a^10*b^2*d^4))/4 + (a + b*tan(
c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3
- 16*A^2*a^8*b^2*d^3))*(-(96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^
4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3
*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - 24*A^3*a^3*b^6*d^2 - 24*A^3*a^5*
b^4*d^2 - 8*A^3*a^7*b^2*d^2 - 8*A^3*a*b^8*d^2))*(-(96*A^4*a^2*b^4*d^4 - 16*A
^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)
/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - log((((96
*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*
```

$$\begin{aligned}
& d^2 - 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} * (a + b \tan(c + dx))^{(1/2)} * (64a^12d^5 + 320a^3b^10d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^11b^2d^5) - 32A^2b^12d^4 - 96A^2a^2b^10d^4 - 64A^2a^4b^8d^4 + 64A^2a^6b^6d^4 + 96A^2a^8b^4d^4 + 32A^2a^10b^2d^4) + (a + b \tan(c + dx))^{(1/2)} * (16A^2b^10d^3 + 32A^2a^2b^8d^3 - 32A^2a^6b^4d^3 - 16A^2a^8b^2d^3) * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} - 24A^3a^3b^6d^2 - 24A^3a^5b^4d^2 - 8A^3a^7b^2d^2 - 8A^3a^9b^2d^2) * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} + 4A^2a^3d^2 - 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} - \log(((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} * (a + b \tan(c + dx))^{(1/2)} * (64a^12d^5 + 320a^3b^10d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^11b^2d^5) - 32A^2b^12d^4 - 96A^2a^2b^10d^4 - 64A^2a^4b^8d^4 + 64A^2a^6b^6d^4 + 96A^2a^8b^4d^4 + 32A^2a^10b^2d^4) + (a + b \tan(c + dx))^{(1/2)} * (16A^2b^10d^3 + 32A^2a^2b^8d^3 - 32A^2a^6b^4d^3 - 16A^2a^8b^2d^3) * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} - 24A^3a^3b^6d^2 - 24A^3a^5b^4d^2 - 8A^3a^7b^2d^2 - 8A^3a^9b^2d^2) * (((96A^4a^2b^4d^4 - 16A^4b^6d^4 - 144A^4a^4b^2d^4)^{(1/2)} - 4A^2a^3d^2 + 12A^2ab^2d^2)/(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{(1/2)} + (\log(((a + b \tan(c + dx))^{(1/2)} * (16B^2b^10d^3 + 32B^2a^2b^8d^3 - 32B^2a^6b^4d^3 - 16B^2a^8b^2d^3) + (((96B^4a^2b^4d^4 - 16B^4b^6d^4 - 144B^4a^4b^2d^4)^{(1/2)} - 4B^2a^3d^2 + 12B^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (64B^2a^11d^4 - (((96B^4a^2b^4d^4 - 16B^4b^6d^4 - 144B^4a^4b^2d^4)^{(1/2)} - 4B^2a^3d^2 + 12B^2ab^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (a + b \tan(c + dx))^{(1/2)} \dots
\end{aligned}$$

$$3.353 \quad \int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=138

$$\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{(iA-B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{2(Ab-a^2)}{(a^2+b^2)d\sqrt{a+b \tan(c+dx)}}$$

[Out]  $-(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(3/2)}/d+(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(3/2)}/d-2*(A*b-B*a)/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{a+b \tan(c+dx)}} - \frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} + \frac{(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}, x]$

[Out]  $-(((I*A+B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/((a-I*b)^{(3/2)*d}) + ((I*A-B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/((a+I*b)^{(3/2)*d}) - (2*(A*b-a*B))/((a^2+b^2)*d*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^{n, x}], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3610**

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*c-a*d)*((a+b*\operatorname{Tan}[e+f*x])^{(m+1)/(f*(m+1)*(a^2+b^2)}), x] + \operatorname{Dist}[1/(a^2+b^2), \operatorname{Int}[(a+b*\operatorname{Tan}[e+f*x])^{(m+1)/(f*(m+1)*(a^2+b^2)}], x]$



$^{(m+1)}\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

### Rule 3618

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{IntegerQ}[m]$

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx &= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{a^2 + b^2} \\ &= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{2(a - ib)} + \dots \\ &= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{(i(A + iB)) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a + ibx}} dx\right)}{2(a + ib)d} \\ &= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{(A - iB) \text{Subst}\left(\int \frac{1}{-1 - \frac{ia}{b} + \frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{(a - ib)bd} \\ &= -\frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{3/2}d} + \frac{(iA - B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{(a + ib)^{3/2}d} \end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.13, size = 113, normalized size = 0.82

$$i \frac{\left( \frac{(A-iB) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{a+b \tan(c+dx)}{a-ib}\right)}{a-ib} - \frac{(A+iB) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{a+b \tan(c+dx)}{a+ib}\right)}{a+ib} \right)}{d \sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (I\*(((A - I\*B)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)])/(a - I\*b) - ((A + I\*B)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)])/(a + I\*b)))/(d\*sqrt[a + b\*Tan[c + d\*x]])

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2273 vs.  $2(118) = 236$ .

time = 0.12, size = 2274, normalized size = 16.48

method	result	size
derivativedivides	Expression too large to display	2274
default	Expression too large to display	2274

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, method=\_RETURNVERBOSE)

[Out]  $\frac{1}{d} \frac{(-2(A*b-B*a)/(a^2+b^2)/(a+b*\tan(d*x+c))^{1/2} + 2/(a^2+b^2)*(1/4/b^2/(3*a^2-b^2)/(a^2+b^2)^{3/2}*(1/2*(3*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*(a^2+b^2)^{1/2}*a^3*b^3-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^5-3*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^6*b+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^4*b^3+3*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^2*b^5-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^7+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a^4-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^6+2*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^4*b^2+3*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^4-6*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^5*b^2-4*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^3*b^4+2*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a*b^6)*\ln(b*\tan(d*x+c)+a+(a+b*\tan(d*x+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}+(a^2+b^2)^{1/2})+2*(12*A*a^5*b^3+8*A*a^3*b^5-4*A*a*b^7-6*B*a^6*b^2+2*B*a^4*b^4+6*B*a^2*b^6-2*B*b^8-1/2*(3*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^5*b+2*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3*b^3-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^5-3*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^6*b+A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^4*b^3+3*A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^5-A*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^7+B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{3/2}*a^4-B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^6+2*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^4*b^2+3*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^2*b^4-6*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^5*b^2-4*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^3*b^4+2*B*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a*b^6)}{d \sqrt{a + b \tan(c + d*x)}}$

$$\begin{aligned}
& 2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^2+3*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)} \\
& 2)*(a^2+b^2)^{(1/2)}*a^2*b^4-6*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^5*b^2-4*B*(2 \\
& *(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^3*b^4+2*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a*b^6 \\
& )*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)-2*a)^{(1/2)}*\arctan((2*(a \\
& +b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)-2*a) \\
& ^{(1/2)})))+1/4/b^2/(3*a^2-b^2)/(a^2+b^2)^{(3/2)}*(-1/2*(3*A*(2*(a^2+b^2)^{(1/2)+ \\
& 2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b+2*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2) \\
& )^{(1/2)}*a^3*b^3-A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^5-3*A*( \\
& 2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^6*b+A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^4*b^3+ \\
& 3*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^2*b^5-A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*b \\
& ^7+B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4-B*(2*(a^2+b^2)^{(1/2) \\
& +2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^4-B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{( \\
& 1/2)}*a^6+2*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^2+3*B*(2*( \\
& a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^4-6*B*(2*(a^2+b^2)^{(1/2)+2* \\
& a)^{(1/2)}*a^5*b^2-4*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^3*b^4+2*B*(2*(a^2+b^2) \\
& ^{(1/2)+2*a)^{(1/2)}*a*b^6)*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{(1/2)}*(2*(a^2+ \\
& b^2)^{(1/2)+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(-12*A*a^5*b^3-8*A*a^3*b^5+4*A*a*b \\
& ^7+6*B*a^6*b^2-2*B*a^4*b^4-6*B*a^2*b^6+2*B*b^8+1/2*(3*A*(2*(a^2+b^2)^{(1/2)+ \\
& 2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b+2*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2) \\
& )^{(1/2)}*a^3*b^3-A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^5-3*A*( \\
& 2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^6*b+A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^4*b^3+ \\
& 3*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^2*b^5-A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*b \\
& ^7+B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4-B*(2*(a^2+b^2)^{(1/2) \\
& +2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^4-B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{( \\
& 1/2)}*a^6+2*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^2+3*B*(2*( \\
& a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^4-6*B*(2*(a^2+b^2)^{(1/2)+2* \\
& a)^{(1/2)}*a^5*b^2-4*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^3*b^4+2*B*(2*(a^2+b^2) \\
& ^{(1/2)+2*a)^{(1/2)}*a*b^6)*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)- \\
& 2*a)^{(1/2)}*\arctan((-2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}) \\
& /((2*(a^2+b^2)^{(1/2)-2*a)^{(1/2)})))))
\end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 20729 vs. 2(113) = 226.

time = 52.64, size = 20729, normalized size = 150.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$-1/4*(4*\sqrt{2})*((a^{10} + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^{10})*d^5*\cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d^5*\cos(d*x + c)*\sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^{10})*d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^6 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^4 + (A^4 + 2*A^2*B^2 + B^4)*b^6 + (6*A*B*a^8*b + 16*A*B*a^6*b^3 + 12*A*B*a^4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6))*\sqrt{((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^{3/4}*\arctan(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13} - 3*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12}*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^2 - 14*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^3 - 10*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^4 - 25*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^5 - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^6 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^7 - 50*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^8 - 5*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^9 - 28*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{10} + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^{11} - 6*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{12} + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{13})*d^4*\sqrt{((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4))*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)} + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{10} - 3*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^9*b - 8*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^7*b^3 - 12*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^6*b^4 - 6*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^5*b^5 - 16*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4*b^6 - 6*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^8 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^9)*d^2*\sqrt{((4*A^2*B^2*a^6 - 12*(A^3*B$$

$$\begin{aligned}
& - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + \\
& (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + \text{sqrt}(2)*((B*a^14 - 2*A*a^13*b + \\
& 5*B*a^12*b^2 - 12*A*a^11*b^3 + 9*B*a^10*b^4 - 30*A*a^9*b^5 + 5*B*a^8*b^6 - 40*A*a^7*b^7 - 5*B*a^6*b^8 - 30*A*a^5*b^9 - 9*B*a^4*b^10 - 12*A*a^3*b^11 - \\
& 5*B*a^2*b^12 - 2*A*a*b^13 - B*b^14)*d^7*\text{sqrt}((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*\text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + ((A^2*B + B^3)*a^11 - (A^3 + A*B^2)*a^10*b + 5*(A^2*B + B^3)*a^9*b^2 - 5*(A^3 + A*B^2)*a^8*b^3 + 10*(A^2*B + B^3)*a^7*b^4 - 10*(A^3 + A*B^2)*a^6*b^5 + 10*(A^2*B + B^3)*a^5*b^6 - 10*(A^3 + A*B^2)*a^4*b^7 + 5*(A^2*B + B^3)*a^3*b^8 - 5*(A^3 + A*B^2)*a^2*b^9 + (A^2*B + B^3)*a*b^10 - (A^3 + A*B^2)*b^11)*d^5*\text{sqrt}((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))*\text{sqrt}(((A^4 + 2*A^2*B^2 + B^4)*a^6 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^4 + (A^4 + 2*A^2*B^2 + B^4)*b^6 + (6*A*B*a^8*b + 16*A*B*a^6*b^3 + 12*A*B*a^4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*\text{sqrt}((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6))*\text{sqrt}(((4*(A^4*B^2 + A^2*B^4)*a^10 - 12*(A^5*B - A*B^5)*a^9*b + (9*A^6 - 25*A^4*B^2 - 25*A^2*B^4 + 9*B^6)*a^8*b^2 + \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2), x)

[Out] Integral((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 12.07, size = 2500, normalized size = 18.12
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] (log((((a + b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 3
2*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - (((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^
6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*((((96*A^4*a^2*b^4
*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2
*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a +
b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5
+ 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 64*A*a*b^11*d^4
+ 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3
*d^4))/4)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/
2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 +
3*a^4*b^2*d^4))^(1/2))/4 + 8*A^3*b^9*d^2 + 24*A^3*a^2*b^7*d^2 + 24*A^3*a^4
*b^5*d^2 + 8*A^3*a^6*b^3*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*
A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d
^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a + b*tan(c + d*x))
^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*
a^8*b^2*d^3) - (((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d
^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^
4*d^4 + 3*a^4*b^2*d^4))^(1/2)*((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 1
44*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^
6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(6
4*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a
^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 38
4*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))/4)*(-((96*A^4*a^2*
b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*
A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/
4 + 8*A^3*b^9*d^2 + 24*A^3*a^2*b^7*d^2 + 24*A^3*a^4*b^5*d^2 + 8*A^3*a^6*b^3
*d^2)*(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2)
+ 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*
a^4*b^2*d^4))^(1/2))/4 - log(8*A^3*b^9*d^2 - ((a + b*tan(c + d*x))^(1/2)*(1
6*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d
^3) + (((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) -
4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^
4 + 48*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 - ((96*A^4*a^2*b^4*d^4 - 16*A^
```

$$\begin{aligned}
& 4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/ \\
& (16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 64 \\
& 0*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*A*a^3*b^9*d^4 + 38 \\
& 4*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(((96*A^4*a^2*b^4* \\
& d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2* \\
& a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/ \\
& 2)} + 24*A^3*a^2*b^7*d^2 + 24*A^3*a^4*b^5*d^2 + 8*A^3*a^6*b^3*d^2))*(((96*A^4 \\
& *a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 \\
& + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2* \\
& d^4))^{(1/2)} - \log(8*A^3*b^9*d^2 - ((a + b*\tan(c + d*x))^{(1/2)}*(16*A^2*b^10* \\
& d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) + (-((9 \\
& 6*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3 \\
& *d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4 \\
& *b^2*d^4))^{(1/2)}*(64*A*a*b^11*d^4 - (-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 \\
& - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d \\
& ^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*\tan(c + d* \\
& x))^{(1/2)}*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6 \\
& *d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 256*A*a^3*b^9*d^4 + 384*A*a^5*b \\
& ^7*d^4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(-((96*A^4*a^2*b^4*d^4 - 16 \\
& *A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^ \\
& 2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + 24* \\
& A^3*a^2*b^7*d^2 + 24*A^3*a^4*b^5*d^2 + 8*A^3*a^6*b^3*d^2))*(-((96*A^4*a^2*b^ \\
& 4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^ \\
& 2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{( \\
& 1/2)} + (\log(- ((((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d \\
& ^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^ \\
& 4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}*(32*B*b^12*d^4 + (((((96*B^4*a^2*b^4*d^4 - 16* \\
& B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2 \\
& ))/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}*(a + b*\tan(c + \\
& d*x))^{(1/2)}*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7* \\
& b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 96*B*a^2*b^10*d^4 + 64*B* \\
& a^4*b^8*d^4 - 64*B*a^6*b^6*d^4 - 96*B*a^8*b^4*d^4\dots
\end{aligned}$$

$$3.354 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=171

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}$$

[Out]  $-2*A*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/a^{(3/2)}/d+(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(3/2)}/d+(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(3/2)}/d+2*b*(A*b-B*a)/a/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 0.41, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3690, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b(Ab-aB)}{ad(a^2+b^2)\sqrt{a+b \tan(c+dx)}} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/(a+b*\operatorname{Tan}[c+d*x])^{(3/2)},x]$

[Out]  $(-2*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]]/(a^{(3/2)*d}) + ((A-I*B)*\operatorname{rcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/((a-I*b)^{(3/2)*d}) + ((A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/((a+I*b)^{(3/2)*d}) + (2*b*(A*b-a*B))/(a*(a^2+b^2)*d*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**

$\operatorname{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c$



$*x), x], x, d*\text{Tan}[e + f*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)} * \left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m * (1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m * (1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$

### Rule 3690

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)} * \left((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right) * \left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*(a + b*\text{Tan}[e + f*x])^{m+1} * (c + d*\text{Tan}[e + f*x])^{n+1} / (f*(m+1)*(b*c - a*d)*(a^2 + b^2)), x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1} * (c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B*(b*c*(m+1) + a*d*(n+1)) + A*(a*(b*c - a*d)*(m+1) - b^2*d*(m+n+2)) - (A*b - a*B)*(b*c - a*d)*(m+1)*\text{Tan}[e + f*x] - b*d*(A*b - a*B)*(m+n+2)*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegerQ}[m] || \text{IntegersQ}[2*m, 2*n]) \&\& \text{!(ILtQ}[n, -1] \&\& (\text{!IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3715

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(m_.)} * \left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(n_.)} * \left((A_.) + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\right), x\_Symbol] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m * (c + d*x)^n, x], x, \text{Tan}[e + f*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3734

$\text{Int}[\left(\left((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right)^{(n_.)} * \left((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\right)\right) / \left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\right), x\_Symbol] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n * ((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!GtQ}[n, 0] \&\& \text{!LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx &= \frac{2b(Ab-aB)}{a(a^2+b^2)d\sqrt{a+b \tan(c+dx)}} + \frac{2 \int \frac{\cot(c+dx)(\frac{1}{2}A(a^2+b^2)-\frac{1}{2}a(Ab-aB))}{\sqrt{a+b \tan(c+dx)}} dx}{a(a^2+b^2)} \\
&= \frac{2b(Ab-aB)}{a(a^2+b^2)d\sqrt{a+b \tan(c+dx)}} + \frac{A \int \frac{\cot(c+dx)(1+\tan^2(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx}{a} + \dots \\
&= \frac{2b(Ab-aB)}{a(a^2+b^2)d\sqrt{a+b \tan(c+dx)}} - \frac{((ia+b)(A+iB)) \int \frac{1-i \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx}{2(a^2+b^2)} \\
&= \frac{2b(Ab-aB)}{a(a^2+b^2)d\sqrt{a+b \tan(c+dx)}} + \frac{(2A) \text{Subst} \left( \int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b \tan(c+dx)} \right)}{abd} \\
&= -\frac{2A \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{a^{3/2}d} + \frac{2b(Ab-aB)}{a(a^2+b^2)d\sqrt{a+b \tan(c+dx)}} \\
&= -\frac{2A \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{a^{3/2}d} + \frac{(A-iB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{(a-ib)^{3/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 0.79, size = 186, normalized size = 1.09

$$\frac{-\frac{2A(a^2+b^2) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{\sqrt{a}} + \frac{a(a+ib)(A-iB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right)}{\sqrt{a-ib}} + \frac{a(a-ib)(A+iB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right)}{\sqrt{a+ib}} + \frac{2b(Ab-aB)}{\sqrt{a+b \tan(c+dx)}}}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
[Out] ((-2*A*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]/Sqrt[a] + (a*(a + I*b)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + (a*(a - I*b)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b] + (2*b*(A*b - a*B))/Sqrt[a + b*Tan[c + d*x]])/(a*(a^2 + b^2)*d)

```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 1.67, size = 63939, normalized size = 373.91

method	result	size
--------	--------	------

default	Expression too large to display	63939
---------	---------------------------------	-------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERB  
OSE)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*cot(d*x + c)/(b*tan(d*x + c) + a)^(3/2), x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 21008 vs. 2(139) = 278.

time = 103.71, size = 42091, normalized size = 246.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & [-1/4*(4*\sqrt{2})*((a^{12} + 3*a^{10}*b^2 + 2*a^8*b^4 - 2*a^6*b^6 - 3*a^4*b^8 - \\ & a^2*b^{10})*d^5*\cos(d*x + c)^2 + 2*(a^{11}*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*d^5*\cos(d*x + c)*\sin(d*x + c) + (a^{10}*b^2 + 4*a^8*b^4 + 6*a^6* \\ & b^6 + 4*a^4*b^8 + a^2*b^{10})*d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^6 + 3*(A^4 \\ & + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^4 + (A^4 + 2* \\ & A^2*B^2 + B^4)*b^6 - (6*A*B*a^8*b + 16*A*B*a^6*b^3 + 12*A*B*a^4*b^5 - 2*A*B \\ & *b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3* \\ & (A^2 - B^2)*a*b^8)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a \\ & ^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - \\ & 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2* \\ & B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6 \\ & ))*\sqrt{((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + \\ & 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^ \\ & 2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^{12} + 6* \\ & a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4)} \end{aligned}$$

$$\begin{aligned}
& ((A^4 + 2A^2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4))^{3/4} \operatorname{arctan}(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13} - 3(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b + 4(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^2 - 14(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^3 - 10(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^4 - 25(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^5 - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^6 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^7 - 50(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^8 - 5(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^9 - 28(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{10} + 2(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b^{11} - 6(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^1b^{12} + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{13})d^4 \sqrt{((4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^1b^5 + (A^4 - 2A^2B^2 + B^4)b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}) \sqrt{((A^4 + 2A^2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4))} + (2(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^{10} - 3(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^9b - 8(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^7b^3 - 12(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^6b^4 - 6(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^5b^5 - 16(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^4b^6 - 6(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^2b^8 + (A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})a^1b^9)d^2 \sqrt{((4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^1b^5 + (A^4 - 2A^2B^2 + B^4)b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}) + \sqrt{2} * ((A^14 + 2B^13a + 5A^12b^2 + 12B^11a^3 + 9A^10b^4 + 30B^9a^5 + 5A^8b^6 + 40B^7a^7 - 5A^6b^8 + 30B^5a^9 - 9A^4b^{10} + 12B^3a^{11} - 5A^2b^{12} + 2B^13a^3 - Ab^{14})d^7 \sqrt{((4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^1b^5 + (A^4 - 2A^2B^2 + B^4)b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}) \sqrt{((A^4 + 2A^2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4))} + ((A^3 + AB^2)a^{11} + (A^2B + B^3)a^{10}b + 5(A^3 + AB^2)a^9b^2 + 5(A^2B + B^3)a^8b^3 + 10(A^3 + AB^2)a^7b^4 + 10(A^2B + B^3)a^6b^5 + 10(A^3 + AB^2)a^5b^6 + 10(A^2B + B^3)a^4b^7 + 5(A^3 + AB^2)a^3b^8 + 5(A^2B + B^3)a^2b^9 + (A^3 + AB^2)a^1b^{10} + (A^2B + B^3)b^{11})d^5 \sqrt{((4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)a^1b^5 + (A^4 - 2A^2B^2 + B^4)b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}) \sqrt{((A^4 + 2A^2B^2 + B^4)a^6 + 3(A^4 + 2A^2B^2 + B^4)a^4b^2 + 3(A^4 + 2A^2B^2 + B^4)a^2b^4 + (A^4 + 2A^2B^2 + B^4)b^6 - (6A^8B^2 + 16A^6B^3 + 12A^4B^4 + 12A^2B^5 + 6B^6))d^4}
\end{aligned}$$

$A*B*a^4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)})/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6))*\sqrt{((4*(A^4*B^2 + A^2*B^4)*a^{10} - 12*(A^5*B - A*B^5)*a^9*b + (9*A^6 - 25*A^4*B^2 - 25*A^2*B^4 + 9*...$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 12.91, size = 2500, normalized size = 14.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] atan(-(((8\*A^2\*a^3\*d^2 - 8\*B^2\*a^3\*d^2 - 16\*A\*B\*b^3\*d^2 - 24\*A^2\*a\*b^2\*d^2 + 24\*B^2\*a\*b^2\*d^2 + 48\*A\*B\*a^2\*b\*d^2)^2/4 - (A^4 + 2\*A^2\*B^2 + B^4)\*(16\*a^6\*d^4 + 16\*b^6\*d^4 + 48\*a^2\*b^4\*d^4 + 48\*a^4\*b^2\*d^4))^(1/2) - 4\*A^2\*a^3\*d^2 + 4\*B^2\*a^3\*d^2 + 8\*A\*B\*b^3\*d^2 + 12\*A^2\*a\*b^2\*d^2 - 12\*B^2\*a\*b^2\*d^2 - 24\*A\*B\*a^2\*b\*d^2)/(16\*(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4)))^(1/2)\*((a + b\*tan(c + d\*x))^(1/2)\*(256\*A^2\*a^8\*b^26\*d^7 + 1472\*A^2\*a^10\*b^24\*d^7 + 3712\*A^2\*a^12\*b^22\*d^7 + 6272\*A^2\*a^14\*b^20\*d^7 + 9856\*A^2\*a^16\*b^18\*d^7 + 14336\*A^2\*a^18\*b^16\*d^7 + 15232\*A^2\*a^20\*b^14\*d^7 + 10112\*A^2\*

$$\begin{aligned}
& a^{22}b^{12}d^7 + 3712A^2a^{24}b^{10}d^7 + 576A^2a^{26}b^8d^7 + 832B^2a^{10}b^{24}d^7 + 5504B^2a^{12}b^{22}d^7 + 15232B^2a^{14}b^{20}d^7 + 22400B^2a^{16}b^{18}d^7 + 17920B^2a^{18}b^{16}d^7 + 6272B^2a^{20}b^{14}d^7 - 896B^2a^{22}b^{12}d^7 - 1408B^2a^{24}b^{10}d^7 - 320B^2a^{26}b^8d^7 - 512A^2B^2a^9b^{25}d^7 - 1792A^2B^2a^{11}b^{23}d^7 + 1792A^2B^2a^{13}b^{21}d^7 + 19712A^2B^2a^{15}b^{19}d^7 + 44800A^2B^2a^{17}b^{17}d^7 + 51968A^2B^2a^{19}b^{15}d^7 + 34048A^2B^2a^{21}b^{13}d^7 + 12032A^2B^2a^{23}b^{11}d^7 + 1792A^2B^2a^{25}b^9d^7) + (-(((8A^2a^3d^2 - 8B^2a^3d^2 - 16A^2B^2a^3d^2 - 24A^2a^2b^2d^2 + 24B^2a^2b^2d^2 + 48A^2B^2a^2b^2d^2)^2/4 - (A^4 + 2A^2B^2 + B^4)(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4)))^{1/2} - 4A^2a^3d^2 + 4B^2a^3d^2 + 8A^2B^2a^3d^2 + 12A^2a^2b^2d^2 - 12B^2a^2b^2d^2 - 24A^2B^2a^2b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (512A^2a^8b^{28}d^8 - (a + b*\tan(c + d*x))^{1/2} * (-(((8A^2a^3d^2 - 8B^2a^3d^2 - 16A^2B^2a^3d^2 - 24A^2a^2b^2d^2 + 24B^2a^2b^2d^2 + 48A^2B^2a^2b^2d^2)^2/4 - (A^4 + 2A^2B^2 + B^4)(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4)))^{1/2} - 4A^2a^3d^2 + 4B^2a^3d^2 + 8A^2B^2a^3d^2 + 12A^2a^2b^2d^2 - 12B^2a^2b^2d^2 - 24A^2B^2a^2b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (512a^9b^{28}d^9 + 5376a^{11}b^{26}d^9 + 25344a^{13}b^{24}d^9 + 70656a^{15}b^{22}d^9 + 129024a^{17}b^{20}d^9 + 161280a^{19}b^{18}d^9 + 139776a^{21}b^{16}d^9 + 82944a^{23}b^{14}d^9 + 32256a^{25}b^{12}d^9 + 7424a^{27}b^{10}d^9 + 768a^{29}b^8d^9) + 5248A^2a^{10}b^{26}d^8 + 23936A^2a^{12}b^{24}d^8 + 64000A^2a^{14}b^{22}d^8 + 111104A^2a^{16}b^{20}d^8 + 130816A^2a^{18}b^{18}d^8 + 105728A^2a^{20}b^{16}d^8 + 57856A^2a^{22}b^{14}d^8 + 20480A^2a^{24}b^{12}d^8 + 4224A^2a^{26}b^{10}d^8 + 384A^2a^{28}b^8d^8 - 256A^2B^2a^{11}b^{25}d^8 - 2048B^2a^{13}b^{23}d^8 - 7168B^2a^{15}b^{21}d^8 - 14336B^2a^{17}b^{19}d^8 - 17920B^2a^{19}b^{17}d^8 - 14336B^2a^{21}b^{15}d^8 - 7168B^2a^{23}b^{13}d^8 - 2048B^2a^{25}b^{11}d^8 - 256B^2a^{27}b^9d^8)) * (-(((8A^2a^3d^2 - 8B^2a^3d^2 - 16A^2B^2a^3d^2 - 24A^2a^2b^2d^2 + 24B^2a^2b^2d^2 + 48A^2B^2a^2b^2d^2)^2/4 - (A^4 + 2A^2B^2 + B^4)(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4)))^{1/2} - 4A^2a^3d^2 + 4B^2a^3d^2 + 8A^2B^2a^3d^2 + 12A^2a^2b^2d^2 - 12B^2a^2b^2d^2 - 24A^2B^2a^2b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} - 128A^3a^7b^{26}d^6 - 128A^3a^9b^{24}d^6 + 2592A^3a^{11}b^{22}d^6 + 10976A^3a^{13}b^{20}d^6 + 20384A^3a^{15}b^{18}d^6 + 20832A^3a^{17}b^{16}d^6 + 11872A^3a^{19}b^{14}d^6 + 3232A^3a^{21}b^{12}d^6 + 96A^3a^{23}b^{10}d^6 - 96A^3a^{25}b^8d^6 + 32B^3a^{10}b^{23}d^6 + 224B^3a^{12}b^{21}d^6 + 672B^3a^{14}b^{19}d^6 + 1120B^3a^{16}b^{17}d^6 + 1120B^3a^{18}b^{15}d^6 + 672B^3a^{20}b^{13}d^6 + 224B^3a^{22}b^{11}d^6 + 32B^3a^{24}b^9d^6 - 768A^2B^2a^9b^{24}d^6 - 5088A^2B^2a^{11}b^{22}d^6 - 14112A^2B^2a^{13}b^{20}d^6 - 20832A^2B^2a^{15}b^{18}d^6 - 16800A^2B^2a^{17}b^{16}d^6 - 6048A^2B^2a^{19}b^{14}d^6 + 672A^2B^2a^{21}b^{12}d^6 + 1248A^2B^2a^{23}b^{10}d^6 + 288A^2B^2a^{25}b^8d^6 + 768A^2B^2a^8b^{25}d^6 + 4128A^2B^2a^{10}b^{23}d^6 + 7392A^2B^2a^{12}b^{21}d^6 + 672A^2B^2a^{14}b^{19}d^6 - 16800A^2B^2a^{16}b^{17}d^6 - 27552A^2B^2a^{18}b^{15}d^6 - 20832A^2B^2a^{20}b^{13}d^6 - 7968A^2B^2a^{22}b^{11}d^6 - 1248A^2B^2a^{24}b^9d^6) - (a + b*\tan(c + d*x))^{1/2} * (1120A^4a^{15}b^{16}d^5 - 352A^4a^9b^{22}d^5
\end{aligned}$$

$$\begin{aligned}
& - 672A^4a^{11}b^{20}d^5 - 224A^4a^{13}b^{18}d^5 - 64A^4a^7b^{24}d^5 + 20 \\
& 16A^4a^{17}b^{14}d^5 + 1568A^4a^{19}b^{12}d^5 + 608A^4a^{21}b^{10}d^5 + 96A^4a^{23}b^8d^5 \\
& + 32B^4a^9b^{22}d^5 + 224B^4a^{11}b^{20}d^5 + 672B^4a^{13}b^{18}d^5 + 1120B^4a^{15}b^{16}d^5 \\
& + 1120B^4a^{17}b^{14}d^5 + 672B^4a^{19}b^{12}d^5 + 224B^4a^{21}b^{10}d^5 + 32B^4a^{23}b^8d^5 \\
& + 256A^3B^3a^8b^{23}d^5 + 1792A^3B^3a^{10}b^{21}d^5 + 5376A^3B^3a^{12}b^{19}d^5 + 8960A^3B^3a^{14}b^{17}d^5 \\
& + 8960A^3B^3a^{16}b^{15}d^5 + 5376A^3B^3a^{18}b^{13}d^5 + 1792A^3B^3a^{20}b^{11}d^5 + 256A^3B^3a^{22}b^9d^5 \\
& + 64A^2B^2a^7b^{24}d^5 + 448A^2B^2a^9b^{22}d^5 + 1344A^2B^2a^{11}b^{20}d^5 + 2240A^2B^2a^{13}b^{18}d^5 \\
& + 2240A^2B^2a^{15}b^{16}d^5 + 1344A^2B^2a^{17}b^{14}d^5 + 448A^2B^2a^{19}b^{12}d^5 + 64A^2B^2a^{21}b^{10}d^5) \\
& * (-(((8A^2a^3d^2 - 8B^2a^3d^2 - 16AB^3d^2 - 24A^2ab^2d^2 + 24B^2ab^2d^2 + 48AB^2b^2d^2)^2/4 \\
& - (A^4 + 2A^2B^2 + B^4)*(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 4A^2a^3d^2 + 4\dots
\end{aligned}$$

$$3.355 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=219

$$\frac{(3Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(iA - B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}$$

[Out] (3\*A\*b-2\*B\*a)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+(I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(3/2)/d-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(3/2)/d-b\*(A\*a^2+3\*A\*b^2-2\*B\*a\*b)/a^2/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(1/2)-A\*cot(d\*x+c)/a/d/(a+b\*tan(d\*x+c))^(1/2)

**Rubi [A]**

time = 0.59, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3690, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(3Ab - 2aB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{b(a^2A - 2abB + 3Ab^2)}{a^2d(a^2 + b^2)\sqrt{a+b \tan(c+dx)}} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} - \frac{(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}} - \frac{A \cot(c+dx)}{ad\sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2),x]

[Out] ((3\*A\*b - 2\*a\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]]/(a^(5/2)\*d) + (I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(3/2)\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(3/2)\*d) - (b\*(a^2\*A + 3\*A\*b^2 - 2\*a\*b\*B))/(a^2\*(a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]]) - (A\*Cot[c + d\*x])/(a\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3618**



```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n \*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^2(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx &= -\frac{A \cot(c + dx)}{ad \sqrt{a + b \tan(c + dx)}} - \int \frac{\cot(c + dx) \left( \frac{1}{2}(3Ab - 2aB) + aA \tan(c + dx) + \frac{3}{2}Ab \tan^2(c + dx) \right)}{(a + b \tan(c + dx))^{3/2}} dx \\
 &= -\frac{b(a^2 A + 3Ab^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{A \cot(c + dx)}{ad \sqrt{a + b \tan(c + dx)}} - \int \frac{\cot(c + dx) \left( \frac{1}{2}(3Ab - 2aB) + aA \tan(c + dx) + \frac{3}{2}Ab \tan^2(c + dx) \right)}{(a + b \tan(c + dx))^{3/2}} dx \\
 &= -\frac{b(a^2 A + 3Ab^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{A \cot(c + dx)}{ad \sqrt{a + b \tan(c + dx)}} - \int \frac{\cot(c + dx) \left( \frac{1}{2}(3Ab - 2aB) + aA \tan(c + dx) + \frac{3}{2}Ab \tan^2(c + dx) \right)}{(a + b \tan(c + dx))^{3/2}} dx \\
 &= -\frac{b(a^2 A + 3Ab^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{A \cot(c + dx)}{ad \sqrt{a + b \tan(c + dx)}} - \int \frac{\cot(c + dx) \left( \frac{1}{2}(3Ab - 2aB) + aA \tan(c + dx) + \frac{3}{2}Ab \tan^2(c + dx) \right)}{(a + b \tan(c + dx))^{3/2}} dx \\
 &= -\frac{b(a^2 A + 3Ab^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{A \cot(c + dx)}{ad \sqrt{a + b \tan(c + dx)}} + \frac{(3Ab - 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{a^{5/2} d} - \frac{b(a^2 A + 3Ab^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{(3Ab - 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{a^{5/2} d} + \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{a^{5/2} d} + \frac{b(a^2 A + 3Ab^2 - 2abB)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{A \cot(c + dx)}{ad \sqrt{a + b \tan(c + dx)}}
 \end{aligned}$$

Mathematica [A]

time = 2.51, size = 208, normalized size = 0.95

$$\frac{(3Ab-2aB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right) + a^2 \left( \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}} + \frac{(-iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}} \right) - \frac{b(a^2A+3Ab^2-2abB)}{(a^2+b^2)\sqrt{a+b \tan(c+dx)}} - \frac{aA \cot(c+dx)}{\sqrt{a+b \tan(c+dx)}}}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (((3\*A\*b - 2\*a\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]]/Sqrt[a] + a^2\*((I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/(a - I\*b)^(3/2) + (((-I)\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/(a + I\*b)^(3/2)) - (b\*(a^2\*A + 3\*A\*b^2 - 2\*a\*b\*B))/((a^2 + b^2)\*Sqrt[a + b\*Tan[c + d\*x]]) - (a\*A\*Cot[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]])/(a^2\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 2.97, size = 119757, normalized size = 546.84

method	result	size
default	Expression too large to display	119757

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, method=\_RETURNVE RBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^2/(b\*tan(d\*x + c) + a)^(3/2), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 22675 vs. 2(186) = 372.

time = 119.33, size = 45424, normalized size = 207.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/4*(4*\sqrt{2})*((a^{13} + 3*a^{11}*b^2 + 2*a^9*b^4 - 2*a^7*b^6 - 3*a^5*b^8 - a^{3*b^{10}})*d^5*\cos(d*x + c)^4 - (a^{13} + 2*a^{11}*b^2 - 2*a^9*b^4 - 8*a^7*b^6 - \\ & 7*a^5*b^8 - 2*a^3*b^{10})*d^5*\cos(d*x + c)^2 - (a^{11}*b^2 + 4*a^9*b^4 + 6*a^7*b^6 + 4*a^5*b^8 + a^3*b^{10})*d^5 + 2*((a^{12}*b + 4*a^{10}*b^3 + 6*a^8*b^5 + 4*a^6*b^7 + a^4*b^9)*d^5*\cos(d*x + c)^3 - (a^{12}*b + 4*a^{10}*b^3 + 6*a^8*b^5 + 4*a^6*b^7 + a^4*b^9)*d^5*\cos(d*x + c))*\sin(d*x + c))*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^6 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^4 + (A^4 + 2*A^2*B^2 + B^4)*b^6 + (6*A*B*a^8*b + 16*A*B*a^6*b^3 + 12*A*B*a^4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))}/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6))*\sqrt{(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)}/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^(3/4)*\arctan(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13} - 3*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12}*b + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^2 - 14*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^3 - 10*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^4 - 25*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^5 - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^6 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^7 - 50*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^8 - 5*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^9 - 28*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{10} + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^{11} - 6*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{12} + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{13})*d^4*\sqrt{(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)}/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4))*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{10} - 3*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^9*b - 8*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^7*b^3 - 12*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^6*b^4 - 6*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^5*b^5 - 16*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4*b^6 - 6*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^2*b^8 + (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a*b^9)*d^2*\sqrt{(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2} \end{aligned}$$

```

*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a
^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) +
sqrt(2)*((B*a^14 - 2*A*a^13*b + 5*B*a^12*b^2 - 12*A*a^11*b^3 + 9*B*a^10*b^
4 - 30*A*a^9*b^5 + 5*B*a^8*b^6 - 40*A*a^7*b^7 - 5*B*a^6*b^8 - 30*A*a^5*b^9
- 9*B*a^4*b^10 - 12*A*a^3*b^11 - 5*B*a^2*b^12 - 2*A*a*b^13 - B*b^14)*d^7*sq
rt((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^
4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4
- 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt(
(A^4 + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)) + ((A^2*
B + B^3)*a^11 - (A^3 + A*B^2)*a^10*b + 5*(A^2*B + B^3)*a^9*b^2 - 5*(A^3 + A
*B^2)*a^8*b^3 + 10*(A^2*B + B^3)*a^7*b^4 - 10*(A^3 + A*B^2)*a^6*b^5 + 10*(A
^2*B + B^3)*a^5*b^6 - 10*(A^3 + A*B^2)*a^4*b^7 + 5*(A^2*B + B^3)*a^3*b^8 -
5*(A^3 + A*B^2)*a^2*b^9 + (A^2*B + B^3)*a*b^10 - (A^3 + A*B^2)*b^11)*d^5*sq
rt((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^
4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4
- 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*
b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4))*sqrt
(((A^4 + 2*A^2*B^2 + B^4)*a^6 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4
+ 2*A^2*B^2 + B^4)*a^2*b^4 + (A^4 + 2*A^2*B^2 + B^4)*b^6 + (6*A*B*a^8*b + 1
6*A*B*a^6*b^3 + 12*A*B*a^4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2
)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*sqrt((A^4 + 2*
A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 -
12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^
3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^2(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*2/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 10.33, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(c + dx)^2(A + B\tan(c + dx))/(a + b\tan(c + dx))^{3/2}, x)$

[Out] 
$$\left(\frac{2(Ab^3 - B^2a^2)}{ab^2 + a^3} - \frac{(a + b\tan(c + dx))(3Ab^3 + A^2b - 2B^2a^2)}{a(ab^2 + a^3)}\right) / \left(d(a + b\tan(c + dx))^{3/2} - a d(a + b\tan(c + dx))^{1/2}\right) + \text{atan}\left(\frac{((8A^2a^3d^2 - 8B^2a^3d^2 - 16ABb^3d^2 - 24A^2ab^2d^2 + 24B^2a^2b^2d^2 + 48ABa^2b^2d^2)^{2/4} - (A^4 + 2A^2B^2 + B^4)(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 4A^2a^3d^2 + 4B^2a^3d^2 + 8ABb^3d^2 + 12A^2ab^2d^2 - 12B^2a^2b^2d^2 - 24ABa^2b^2d^2}{(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{1/2}}\right) \cdot \left(\frac{(a + b\tan(c + dx))^{1/2} (576A^2a^{15}b^{28}d^7 + 5184A^2a^{17}b^{26}d^7 + 21568A^2a^{19}b^{24}d^7 + 53888A^2a^{21}b^{22}d^7 + 87808A^2a^{23}b^{20}d^7 + 94976A^2a^{25}b^{18}d^7 + 66304A^2a^{27}b^{16}d^7 + 27008A^2a^{29}b^{14}d^7 + 4288A^2a^{31}b^{12}d^7 - 832A^2a^{33}b^{10}d^7 - 320A^2a^{35}b^8d^7 + 256B^2a^{17}b^{26}d^7 + 1472B^2a^{19}b^{24}d^7 + 3712B^2a^{21}b^{22}d^7 + 6272B^2a^{23}b^{20}d^7 + 9856B^2a^{25}b^{18}d^7 + 14336B^2a^{27}b^{16}d^7 + 15232B^2a^{29}b^{14}d^7 + 10112B^2a^{31}b^{12}d^7 + 3712B^2a^{33}b^{10}d^7 + 576B^2a^{35}b^8d^7 - 768ABa^{16}b^{27}d^7 - 6400ABa^{18}b^{25}d^7 - 25856ABa^{20}b^{23}d^7 - 66304ABa^{22}b^{21}d^7 - 116480ABa^{24}b^{19}d^7 - 141568ABa^{26}b^{17}d^7 - 116480ABa^{28}b^{15}d^7 - 61696ABa^{30}b^{13}d^7 - 18944ABa^{32}b^{11}d^7 - 2560ABa^{34}b^9d^7) - (((8A^2a^3d^2 - 8B^2a^3d^2 - 16ABb^3d^2 - 24A^2ab^2d^2 + 24B^2a^2b^2d^2 + 48ABa^2b^2d^2)^{2/4} - (A^4 + 2A^2B^2 + B^4)(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 4A^2a^3d^2 + 4B^2a^3d^2 + 8ABb^3d^2 + 12A^2ab^2d^2 - 12B^2a^2b^2d^2 - 24ABa^2b^2d^2)}{(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{1/2}}\right) \cdot \left(\frac{(a + b\tan(c + dx))^{1/2} ((8A^2a^3d^2 - 8B^2a^3d^2 - 16ABb^3d^2 - 24A^2ab^2d^2 + 24B^2a^2b^2d^2 + 48ABa^2b^2d^2)^{2/4} - (A^4 + 2A^2B^2 + B^4)(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 4A^2a^3d^2 + 4B^2a^3d^2 + 8ABb^3d^2 + 12A^2ab^2d^2 - 12B^2a^2b^2d^2 - 24ABa^2b^2d^2)}{(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{1/2}}\right) \cdot (512a^{18}b^{28}d^9 + 5376a^{20}b^{26}d^9 + 25344a^{22}b^{24}d^9 + 70656a^{24}b^{22}d^9 + 129024a^{26}b^{20}d^9 + 161280a^{28}b^{18}d^9 + 139776a^{30}b^{16}d^9 + 82944a^{32}b^{14}d^9 + 32256a^{34}b^{12}d^9 + 7424a^{36}b^{10}d^9 + 768a^{38}b^8d^9) - 768Aa^{16}b^{29}d^8 - 7680Aa^{18}b^{27}d^8 - 34304Aa^{20}b^{25}d^8 - 90112Aa^{22}b^{23}d^8 - 154112Aa^{24}b^{21}d^8 - 179200Aa^{26}b^{19}d^8 - 143360Aa^{28}b^{17}d^8 - 77824Aa^{30}b^{15}d^8 - 27392Aa^{32}b^{13}d^8 - 5632Aa^{34}b^{11}d^8)$$

$$\begin{aligned}
& 4*b^{11}*d^8 - 512*A*a^{36}*b^9*d^8 + 512*B*a^{17}*b^{28}*d^8 + 5248*B*a^{19}*b^{26}*d^8 \\
& + 23936*B*a^{21}*b^{24}*d^8 + 64000*B*a^{23}*b^{22}*d^8 + 111104*B*a^{25}*b^{20}*d^8 \\
& + 130816*B*a^{27}*b^{18}*d^8 + 105728*B*a^{29}*b^{16}*d^8 + 57856*B*a^{31}*b^{14}*d^8 + \\
& 20480*B*a^{33}*b^{12}*d^8 + 4224*B*a^{35}*b^{10}*d^8 + 384*B*a^{37}*b^8*d^8) * (((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 - 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 4*A^2*a^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 12*A^2*a*b^2*d^2 - 12*B^2*a*b^2*d^2 - 24*A*B*a^2*b*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2) + 576 \\
& *A^3*a^{15}*b^{27}*d^6 + 3456*A^3*a^{17}*b^{25}*d^6 + 8480*A^3*a^{19}*b^{23}*d^6 + 10976*A^3*a^{21}*b^{21}*d^6 + 8736*A^3*a^{23}*b^{19}*d^6 + 6496*A^3*a^{25}*b^{17}*d^6 + 6496*A^3*a^{27}*b^{15}*d^6 + 5280*A^3*a^{29}*b^{13}*d^6 + 2336*A^3*a^{31}*b^{11}*d^6 + 416 \\
& *A^3*a^{33}*b^9*d^6 + 128*B^3*a^{16}*b^{26}*d^6 + 128*B^3*a^{18}*b^{24}*d^6 - 2592*B^3*a^{20}*b^{22}*d^6 - 10976*B^3*a^{22}*b^{20}*d^6 - 20384*B^3*a^{24}*b^{18}*d^6 - 20832 \\
& *B^3*a^{26}*b^{16}*d^6 - 11872*B^3*a^{28}*b^{14}*d^6 - 3232*B^3*a^{30}*b^{12}*d^6 - 96*B^3*a^{32}*b^{10}*d^6 + 96*B^3*a^{34}*b^8*d^6 - 384*A*B^2*a^{15}*b^{27}*d^6 - 768*A*B^2*a^{17}*b^{25}*d^6 + 4128*A*B^2*a^{19}*b^{23}*d^6 + 18144*A*B^2*a^{21}*b^{21}*d^6 + 2 \\
& 7552*A*B^2*a^{23}*b^{19}*d^6 + 15456*A*B^2*a^{25}*b^{17}*d^6 - 6048*A*B^2*a^{27}*b^{15}*d^6 - 13152*A*B^2*a^{29}*b^{13}*d^6 - 6816*A*B^2*a^{31}*b^{11}*d^6 - 1248*A*B^2*a^{33}*b^9*d^6 + 288*A^2*B*a^{14}*b^{28}*d^6 + 480*A^2*B*a^{16}*b^{26}*d^6 - 2688*A^2*B*a^{18}*b^{24}*d^6 - 8352*A^2*B*a^{20}*b^{22}*d^6 - 3360*A^2*B*a^{22}*b^{20}*d^6 + 1680 \\
& 0*A^2*B*a^{24}*b^{18}*d^6 + 30240*A^2*B*a^{26}*b^{16}*d^6 + 21792*A^2*B*a^{28}*b^{14}*d^6 + 6528*A^2*B*a^{30}*b^{12}*d^6 - 288*A^2*B*a^{34}*b^8*d^6) - (a + b*tan(c + d*x))^(1/2)*(144*A^4*a^{14}*b^{26}*d^5 + 864*A^4*a^{16}*b^{24}*d^5 + 2048*A^4*a^{18}*b^{22}*d^5 + 2240*A^4*a^{20}*b^{20}*d^5 + 672*A^4*a^{22}*b^{18}*d^5 - 896*A^4*a^{24}*b^{16}*d^5 - 896*A^4*a^{26}*b^{14}*d^5 - 192*A^4*a^{28}*b^{12}*d^5 + 80*A^4*a^{30}*b^{10}*d^5 + 32*A^4*a^{32}*b^8*d^5 - 64*B^4*a^{16}*b^{24}*d^5 - 352*B^4*a^{18}*b^{22}*d^5 - 672 \\
& *B^4*a^{20}*b^{20}*d^5 - 224*B^4*a^{22}*b^{18}*d^5 + 1120*B^4*a^{24}*b^{16}*d^5 + 2016*B^4*a^{26}*b^{14}*d^5 + 1568*B^4*a^{28}*b^{12}*d^5 + 608*B^4*a^{30}*b^{10}*d^5 + 96*B^4*a^{32}*b^8*d^5 + 192*A*B^3*a^{15}*b^{25}*d^5 + 896*A*B^3*a^{17}*b^{23}*d^5 + 896*A*B^3*a^{19}*b^{21}*d^5 - 2688*A*B^3*a^{21}*b^{19}*d^5 - 8960*A*B^3*a^{23}*b^{17}*d^5 - 11 \\
& 648*A*B^3*a^{25}*b^{15}*d^5 - 8064*A*B^3*a^{27}*b^{13}...
\end{aligned}$$

$$3.356 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=285

$$\frac{(8a^2A - 15Ab^2 + 12abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(A + iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}$$

[Out]  $1/4*(8*A*a^2-15*A*b^2+12*B*a*b)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/a^{1/2})/a^{7/2}/d - (A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/(a-I*b)^{1/2})/(a-I*b)^{3/2}/d - (A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{1/2}/(a+I*b)^{1/2})/(a+I*b)^{3/2}/d + 1/4*b*(7*A*a^2*b+15*A*b^3-4*B*a^3-12*B*a*b^2)/a^3/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{1/2} + 1/4*(5*A*b-4*B*a)*\cot(d*x+c)/a^2/d/(a+b*\tan(d*x+c))^{1/2} - 1/2*A*\cot(d*x+c)^2/a/d/(a+b*\tan(d*x+c))^{1/2}$

**Rubi [A]**

time = 0.81, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3690, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(5Ab - 4aB) \cot(c+dx)}{4a^2d\sqrt{a+b \tan(c+dx)}} + \frac{(8a^2A + 12abB - 15Ab^2) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} + \frac{b(-4a^3B + 7a^2Ab - 12ab^2B + 15Ab^3)}{4a^3d(a^2+b^2)\sqrt{a+b \tan(c+dx)}} - \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} - \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}} - \frac{A \cot^2(c+dx)}{2ad\sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x])]/(a + b*\operatorname{Tan}[c + d*x])^{3/2}, x]$

[Out]  $((8*a^2*A - 15*A*b^2 + 12*a*b*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(4*a^{7/2}*d) - ((A - I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/((a - I*b)^{3/2}*d) - ((A + I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/((a + I*b)^{3/2}*d) + (b*(7*a^2*A*b + 15*A*b^3 - 4*a^3*B - 12*a*b^2*B))/(4*a^3*(a^2 + b^2)*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]) + ((5*A*b - 4*a*B)*\operatorname{Cot}[c + d*x])/(4*a^2*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]) - (A*\operatorname{Cot}[c + d*x]^2)/(2*a*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{n, x}], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$



Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n +
2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx &= -\frac{A \cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} - \frac{\int \frac{\cot^2(c+dx)(\frac{1}{2}(5Ab-4aB)+2aA\tan(c+dx)+}{(a+b\tan(c+dx))^{3/2}}}{2a}}{2a} \\
&= \frac{(5Ab-4aB)\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{A \cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} + \frac{\int \frac{\cot(c+dx)}{a+b\tan(c+dx)}}{2a} \\
&= \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{(5Ab-4aB)\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{(5Ab-4aB)\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{(5Ab-4aB)\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{(5Ab-4aB)\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{(8a^2A-15Ab^2+12abB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} + \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)\cot(c+dx)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{(8a^2A-15Ab^2+12abB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} + \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)\cot(c+dx)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}}
\end{aligned}$$

### Mathematica [A]

time = 6.15, size = 409, normalized size = 1.44

$$\frac{A \cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} - \frac{(5Ab-4aB)\cot(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} - \frac{\left( \frac{(a^2+a^2)(a^2A-15Ab^2+12abB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d} \right) \cdot \sqrt{a-b} \left( \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-b}} \right) \cdot \sqrt{a+b} \left( \frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+b}} \right)}{(a^2+b^2)^2} \cdot \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)\cot(c+dx)}{4a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] -1/2\*(A\*Cot[c + d\*x]^2)/(a\*d\*Sqrt[a + b\*Tan[c + d\*x]]) - (-1/2\*((5\*A\*b - 4\*a\*B)\*Cot[c + d\*x])/(a\*d\*Sqrt[a + b\*Tan[c + d\*x]]) - ((2\*((a^2 + b^2)\*(8\*a^2\*A - 15\*A\*b^2 + 12\*a\*b\*B))\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]])/(4\*Sqrt[a]\*d) + (I\*Sqrt[a - I\*b]\*(a^3\*(A\*b - a\*B) - I\*a^3\*(a\*A + b\*B))\*ArcTanh[S

$$\frac{\sqrt{a + b \tan[c + dx]} / \sqrt{a - I b}}{((-a + I b) * d) - (I \sqrt{a + I b} * (a^3 * (A * b - a * B) + I * a^3 * (a * A + b * B)) * \operatorname{ArcTanh}[\sqrt{a + b \tan[c + dx]} / \sqrt{a + I b}]) / ((-a - I b) * d)) / (a * (a^2 + b^2)) + (2 * ((b^2 * (-8 * a^2 * A + 15 * A * b^2 - 12 * a * b * B)) / 4 - a * (-2 * a^2 * b * B - (3 * a * b * (5 * A * b - 4 * a * B)) / 4)) / (a * (a^2 + b^2) * d * \sqrt{a + b \tan[c + dx]})) / a / (2 * a)}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 3.49, size = 174418, normalized size = 611.99

method	result	size
default	Expression too large to display	174418

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm  
="maxima")`

[Out] Timed out

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 22970 vs.  
 $2(241) = 482$ .  
time = 173.18, size = 46017, normalized size = 161.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm  
="fricas")`

[Out]  $[1/16 * (16 * \sqrt{2}) * ((a^{14} + 3 * a^{12} * b^2 + 2 * a^{10} * b^4 - 2 * a^8 * b^6 - 3 * a^6 * b^8 - a^4 * b^{10}) * d^5 * \cos(d * x + c)^4 - (a^{14} + 2 * a^{12} * b^2 - 2 * a^{10} * b^4 - 8 * a^8 * b^6 - 7 * a^6 * b^8 - 2 * a^4 * b^{10}) * d^5 * \cos(d * x + c)^2 - (a^{12} * b^2 + 4 * a^{10} * b^4 + 6 * a^8 * b^6 + 4 * a^6 * b^8 + a^4 * b^{10}) * d^5 + 2 * ((a^{13} * b + 4 * a^{11} * b^3 + 6 * a^9 * b^5 + 4 * a^7 * b^7 + a^5 * b^9) * d^5 * \cos(d * x + c)^3 - (a^{13} * b + 4 * a^{11} * b^3 + 6 * a^9 * b^5 + 4 * a^7 * b^7 + a^5 * b^9) * d^5 * \cos(d * x + c)) * \sin(d * x + c)) * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^6 + 3 * (A^4 + 2 * A^2 * B^2 +$

$$\begin{aligned}
& B^4)a^2b^4 + (A^4 + 2A^2B^2 + B^4)b^6 - (6ABa^8b + 16ABa^6b^3 \\
& + 12ABa^4b^5 - 2ABb^9 + (A^2 - B^2)a^9 - 6(A^2 - B^2)a^5b^4 - 8 \\
& (A^2 - B^2)a^3b^6 - 3(A^2 - B^2)ab^8)d^2\sqrt{(A^4 + 2A^2B^2 + B^4)} \\
& /((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4))/((4A^2B^2a^6 - 12(A^3B - A \\
& *B^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a \\
& ^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)ab^5 + (A^ \\
& 4 - 2A^2B^2 + B^4)b^6))\sqrt{(4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + \\
& 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A \\
& ^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B - AB^3)ab^5 + (A^4 - 2A^2B^2 \\
& + B^4)b^6)}((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6 \\
& a^2b^{10} + b^{12})d^4))*((A^4 + 2A^2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b \\
& ^4 + b^6)d^4))^{3/4}\arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)* \\
& a^{13} - 3(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b + 4(A^7B + 3A^5B^3 \\
& + 3A^3B^5 + AB^7)a^{11}b^2 - 14(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10} \\
& *b^3 - 10(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^4 - 25(A^8 + 2A^6 \\
& *B^2 - 2A^2B^6 - B^8)a^8b^5 - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7 \\
& )a^7b^6 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^7 - 50(A^7B + 3 \\
& A^5B^3 + 3A^3B^5 + AB^7)a^5b^8 - 5(A^8 + 2A^6B^2 - 2A^2B^6 - B^8 \\
& )a^4b^9 - 28(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{10} + 2(A^8 + \\
& 2A^6B^2 - 2A^2B^6 - B^8)a^2b^{11} - 6(A^7B + 3A^5B^3 + 3A^3B^5 + \\
& AB^7)ab^{12} + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{13})d^4\sqrt{(4A^2B \\
& ^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + 3B^4)a^4b^2 \\
& + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^2b^4 - 12(A^3B \\
& - AB^3)ab^5 + (A^4 - 2A^2B^2 + B^4)b^6)}((a^{12} + 6a^{10}b^2 + 15a^8 \\
& b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4))*\sqrt{(A^4 + 2A^ \\
& 2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} + (2(A^9B + 4A^7 \\
& *B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a^{10} - 3(A^{10} + 3A^8B^2 + 2A^6B^4 - \\
& 2A^4B^6 - 3A^2B^8 - B^{10})a^9b - 8(A^{10} + 3A^8B^2 + 2A^6B^4 - \\
& 2A^4B^6 - 3A^2B^8 - B^{10})a^7b^3 - 12(A^9B + 4A^7B^3 + 6A^5B^5 \\
& + 4A^3B^7 + AB^9)a^6b^4 - 6(A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 \\
& - 3A^2B^8 - B^{10})a^5b^5 - 16(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 \\
& + AB^9)a^4b^6 - 6(A^9B + 4A^7B^3 + 6A^5B^5 + 4A^3B^7 + AB^9)a \\
& ^2b^8 + (A^{10} + 3A^8B^2 + 2A^6B^4 - 2A^4B^6 - 3A^2B^8 - B^{10})ab^9 \\
& )d^2\sqrt{(4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B \\
& ^2 + 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4) \\
& )a^2b^4 - 12(A^3B - AB^3)ab^5 + (A^4 - 2A^2B^2 + B^4)b^6)}((a^{12} \\
& + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^ \\
& 4) + \sqrt{2}*((Aa^{14} + 2Ba^{13}b + 5Aa^{12}b^2 + 12Ba^{11}b^3 + 9Aa^{10} \\
& b^4 + 30Ba^9b^5 + 5Aa^8b^6 + 40Ba^7b^7 - 5Aa^6b^8 + 30Ba^5 \\
& *b^9 - 9Aa^4b^{10} + 12Ba^3b^{11} - 5Aa^2b^{12} + 2Bab^{13} - Ab^{14})d \\
& ^7\sqrt{(4A^2B^2a^6 - 12(A^3B - AB^3)a^5b + 3(3A^4 - 14A^2B^2 + \\
& 3B^4)a^4b^2 + 40(A^3B - AB^3)a^3b^3 - 6(A^4 - 8A^2B^2 + B^4)a^ \\
& 2b^4 - 12(A^3B - AB^3)ab^5 + (A^4 - 2A^2B^2 + B^4)b^6)}((a^{12} + 6 \\
& a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4))* \\
& \sqrt{(A^4 + 2A^2B^2 + B^4)/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} + (
\end{aligned}$$

$$\begin{aligned}
& (A^3 + A*B^2)*a^{11} + (A^2*B + B^3)*a^{10}*b + 5*(A^3 + A*B^2)*a^9*b^2 + 5*(A^2*B + B^3)*a^8*b^3 + 10*(A^3 + A*B^2)*a^7*b^4 + 10*(A^2*B + B^3)*a^6*b^5 + \\
& 10*(A^3 + A*B^2)*a^5*b^6 + 10*(A^2*B + B^3)*a^4*b^7 + 5*(A^3 + A*B^2)*a^3*b^8 + 5*(A^2*B + B^3)*a^2*b^9 + (A^3 + A*B^2)*a*b^{10} + (A^2*B + B^3)*b^{11})*d \\
& ^5*\sqrt{((4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - \\
& 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4))} \\
& *\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^6 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^2 + 3*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^4 + (A^4 + 2*A^2*B^2 + B^4)*b^6 - (6*A*B*a^8*b \\
& + 16*A*B*a^6*b^3 + 12*A*B*a^4*b^5 - 2*A*B*b^9 + (A^2 - B^2)*a^9 - 6*(A^2 - B^2)*a^5*b^4 - 8*(A^2 - B^2)*a^3*b^6 - 3*(A^2 - B^2)*a*b^8)*d^2*\sqrt{(A^4 \\
& + 2*A^2*B^2 + B^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 4 \\
& 0*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^3(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*3/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 10.28, size = 2500, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x))^3\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

```
[Out] ((2*(A*b^4 - B*a*b^3))/(a*(a^2 + b^2)) + ((a + b*tan(c + d*x))^2*(15*A*b^4
+ 7*A*a^2*b^2 - 12*B*a*b^3 - 4*B*a^3*b))/(4*a^3*(a^2 + b^2)) - ((a + b*tan(
c + d*x))*(25*A*b^4 + 9*A*a^2*b^2 - 20*B*a*b^3 - 4*B*a^3*b))/(4*a^2*(a^2 +
b^2)))/(d*(a + b*tan(c + d*x))^(5/2) - 2*a*d*(a + b*tan(c + d*x))^(3/2) + a
^2*d*(a + b*tan(c + d*x))^(1/2)) + atan((((a + b*tan(c + d*x))^(1/2)*(70464
3072*A^4*a^29*b^20*d^5 - 290979840*A^4*a^23*b^26*d^5 - 465043456*A^4*a^25*b
^24*d^5 - 37224448*A^4*a^27*b^22*d^5 - 58982400*A^4*a^21*b^28*d^5 + 7670333
44*A^4*a^31*b^18*d^5 + 238551040*A^4*a^33*b^16*d^5 + 1572864*A^4*a^35*b^14*
d^5 + 92536832*A^4*a^37*b^12*d^5 + 96468992*A^4*a^39*b^10*d^5 + 25165824*A^
4*a^41*b^8*d^5 + 37748736*B^4*a^23*b^26*d^5 + 226492416*B^4*a^25*b^24*d^5 +
536870912*B^4*a^27*b^22*d^5 + 587202560*B^4*a^29*b^20*d^5 + 176160768*B^4*
a^31*b^18*d^5 - 234881024*B^4*a^33*b^16*d^5 - 234881024*B^4*a^35*b^14*d^5 -
50331648*B^4*a^37*b^12*d^5 + 20971520*B^4*a^39*b^10*d^5 + 8388608*B^4*a^41
*b^8*d^5 - 94371840*A*B^3*a^22*b^27*d^5 - 364904448*A*B^3*a^24*b^25*d^5 + 3
7748736*A*B^3*a^26*b^23*d^5 + 2554331136*A*B^3*a^28*b^21*d^5 + 5989466112*A
*B^3*a^30*b^19*d^5 + 6606028800*A*B^3*a^32*b^17*d^5 + 3787456512*A*B^3*a^34
*b^15*d^5 + 918552576*A*B^3*a^36*b^13*d^5 - 56623104*A*B^3*a^38*b^11*d^5 -
50331648*A*B^3*a^40*b^9*d^5 + 330301440*A^3*B*a^22*b^27*d^5 + 1915748352*A^
3*B*a^24*b^25*d^5 + 4279238656*A^3*B*a^26*b^23*d^5 + 4059037696*A^3*B*a^28*
b^21*d^5 + 154140672*A^3*B*a^30*b^19*d^5 - 2825912320*A^3*B*a^32*b^17*d^5 -
1901068288*A^3*B*a^34*b^15*d^5 + 22020096*A^3*B*a^36*b^13*d^5 + 425721856*
A^3*B*a^38*b^11*d^5 + 117440512*A^3*B*a^40*b^9*d^5 + 58982400*A^2*B^2*a^21*
b^28*d^5 - 124256256*A^2*B^2*a^23*b^26*d^5 - 2202533888*A^2*B^2*a^25*b^24*d
^5 - 6984040448*A^2*B^2*a^27*b^22*d^5 - 10041163776*A^2*B^2*a^29*b^20*d^5 -
6404177920*A^2*B^2*a^31*b^18*d^5 + 289931264*A^2*B^2*a^33*b^16*d^5 + 29931
60192*A^2*B^2*a^35*b^14*d^5 + 1694236672*A^2*B^2*a^37*b^12*d^5 + 318767104*
A^2*B^2*a^39*b^10*d^5) + (-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 - 16*A*B*b^3*d^
2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2)^2/4 - (A^4 + 2*
A^2*B^2 + B^4)*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))
^(1/2) - 4*A^2*a^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 12*A^2*a*b^2*d^2 -
12*B^2*a*b^2*d^2 - 24*A*B*a^2*b*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^
4 + 3*a^4*b^2*d^4)))^(1/2)*(773849088*A^3*a^35*b^16*d^6 - 117964800*A^3*a^2
1*b^30*d^6 - 699924480*A^3*a^23*b^28*d^6 - 1889533952*A^3*a^25*b^26*d^6 - 3
336568832*A^3*a^27*b^24*d^6 - 4495245312*A^3*a^29*b^22*d^6 - 4279238656*A^3
*a^31*b^20*d^6 - 1923088384*A^3*a^33*b^18*d^6 - ((a + b*tan(c + d*x))^(1/2)
*(235929600*A^2*a^22*b^30*d^7 + 1871708160*A^2*a^24*b^28*d^7 + 6295650304*A
^2*a^26*b^26*d^7 + 11144265728*A^2*a^28*b^24*d^7 + 9560915968*A^2*a^30*b^22
*d^7 - 337641472*A^2*a^32*b^20*d^7 - 9307160576*A^2*a^34*b^18*d^7 - 8887730
176*A^2*a^36*b^16*d^7 - 2943352832*A^2*a^38*b^14*d^7 + 621805568*A^2*a^40*b
^12*d^7 + 721420288*A^2*a^42*b^10*d^7 + 150994944*A^2*a^44*b^8*d^7 + 150994
944*B^2*a^24*b^28*d^7 + 1358954496*B^2*a^26*b^26*d^7 + 5653921792*B^2*a^28*
b^24*d^7 + 14126415872*B^2*a^30*b^22*d^7 + 23018340352*B^2*a^32*b^20*d^7 +
24897388544*B^2*a^34*b^18*d^7 + 17381195776*B^2*a^36*b^16*d^7 + 7079985152*
B^2*a^38*b^14*d^7 + 1124073472*B^2*a^40*b^12*d^7 - 218103808*B^2*a^42*b^10*
d^7 - 83886080*B^2*a^44*b^8*d^7 - 377487360*A*B*a^23*b^29*d^7 - 3196059648*
```

$$\begin{aligned}
& A*B*a^{25}*b^{27}*d^7 - 11911823360*A*B*a^{27}*b^{25}*d^7 - 24930942976*A*B*a^{29}*b^{23}*d^7 - 30182211584*A*B*a^{31}*b^{21}*d^7 - 17028874240*A*B*a^{33}*b^{19}*d^7 + 5402263552*A*B*a^{35}*b^{17}*d^7 + 16944988160*A*B*a^{37}*b^{15}*d^7 + 12775849984*A*B*a^{39}*b^{13}*d^7 + 4588568576*A*B*a^{41}*b^{11}*d^7 + 671088640*A*B*a^{43}*b^9*d^7 \\
& ) - ( - ( ( ( ( 8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 - 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2 )^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4) )^{1/2} - 4*A^2*a^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 12*A^2*a*b^2*d^2 - 12*B^2*a*b^2*d^2 - 24*A*B*a^2*b*d^2 ) / ( 16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4) ) )^{1/2} * ( (a + b*\tan(c + d*x))^{1/2} * ( - ( ( ( 8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 - 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2 )^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4) )^{1/2} - 4*A^2*a^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 12*A^2*a*b^2*d^2 - 12*B^2*a*b^2*d^2 - 24*A*B*a^2*b*d^2 ) / ( 16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4) ) )^{1/2} * ( 134217728*a^{27}*b^{28}*d^9 + 1409286144*a^{29}*b^{26}*d^9 + 6643777536*a^{31}*b^{24}*d^9 + 18522046464*a^{33}*b^{22}*d^9 + 33822867456*a^{35}*b^{20}*d^9 + 42278584320*a^{37}*b^{18}*d^9 + 36641439744*a^{39}*b^{16}*d^9 + 21743271936*a^{41}*b^{14}*d^9 + 8455716864*a^{43}*b^{12}*d^9 + 1946157056*a^{45}*b^{10}*d^9 + 201326592*a^{47}*b^8*d^9 ) - 251658240*A*a^{24}*b^{30}*d^8 - 2382364672*A*a^{26}*b^{28}*d^8 - 9948889088*A*a^{28}*b^{26}*d^8 - 23924310016*A*a^{30}*b^{24}*d^8 - 36071014400*A*a^{32}*b^{22}*d^8 - 34292629504*A*a^{34}*b^{20}*d^8 - 18555600896*A*a^{36}*b^{18}*d^8 - 2483027968*A*a^{38}*b^{16}*d^8 + \dots
\end{aligned}$$



$$3.357 \quad \int \frac{\tan^4(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=371

$$-\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(iA-B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2a(Ab-aB)}{3b(a^2+b^2)d(a+ib)}$$

[Out]  $-(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(5/2)}/d+(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(5/2)}/d+2/3*(8*A*a^4*b+17*A*a^2*b^3+3*A*b^5-16*B*a^5-30*B*a^3*b^2-8*B*a*b^4)*(a+b*\tan(d*x+c))^{(1/2)}/b^4/(a^2+b^2)^2/d-2/3*(4*A*a^3*b+10*A*a*b^3-8*B*a^4-15*B*a^2*b^2-B*b^4)*(a+b*\tan(d*x+c))^{(1/2)}*\tan(d*x+c)/b^3/(a^2+b^2)^2/d+2*a*(A*a^2*b+3*A*b^3-2*B*a^3-4*B*a*b^2)*\tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)}+2/3*a*(A*b-B*a)*\tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}$

**Rubi** [A]

time = 0.71, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3686, 3726, 3728, 3711, 3620, 3618, 65, 214}

$$\frac{2a(Ab-aB)\operatorname{atanh}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{3bd(a^2+b^2)(a+b \tan(c+dx))^{5/2}} + \frac{2a(-2a^2B+a^2Ab-4ab^2B+3AB)\operatorname{atanh}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{b^2d(a^2+b^2)\sqrt{a+b \tan(c+dx)}} - \frac{2(-8a^2B+4a^2Ab-15a^2b^2B+10aAb^3-B^2)\operatorname{atanh}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{3b^2d(a^2+b^2)} + \frac{(B+iA)\operatorname{atanh}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}} + \frac{(-B+iA)\operatorname{atanh}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c+d*x])^4*(A+B*\operatorname{Tan}[c+d*x])]/(a+b*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $-(((I*A+B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/((a-I*b)^{(5/2)*d}) + ((I*A-B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/((a+I*b)^{(5/2)*d}) + (2*a*(A*b-a*B)*\operatorname{Tan}[c+d*x]^3)/(3*b*(a^2+b^2)*d*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}) + (2*a*(a^2*A*b+3*A*b^3-2*a^3*B-4*a*b^2*B)*\operatorname{Tan}[c+d*x]^2)/(b^2*(a^2+b^2)^2*d*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]) + (2*(8*a^4*A*b+17*a^2*A*b^3+3*A*b^5-16*a^5*B-30*a^3*b^2*B-8*a*b^4*B)*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(3*b^4*(a^2+b^2)^2*d) - (2*(4*a^3*A*b+10*a*A*b^3-8*a^4*B-15*a^2*b^2*B-b^4*B)*\operatorname{Tan}[c+d*x]*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(3*b^3*(a^2+b^2)^2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3711

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dis

```
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx &= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2\int \frac{\tan^2(c+dx)(-3a(Ab-aB)+\frac{3}{2}b(A^2+B^2))}{(a+b\tan(c+dx))^{5/2}} dx}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-a^2B^2)}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-a^2B^2)}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-a^2B^2)}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-a^2B^2)}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-a^2B^2)}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-a^2B^2)}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}
\end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.





$$\begin{aligned} & \frac{1}{2} + 2a)^{\frac{1}{2}} * a^9 + 20A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^7 * b^2 - 6A * (2 * (a^2 + \\ & b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^5 * b^4 - 28A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^3 * b^6 + 3A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a * b^8 + 10B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a \\ & ^2 + b^2)^{\frac{1}{2}} * a^7 * b - 10B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^5 * \\ & b^3 - 18B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^3 * b^5 + 2B * (2 * (a^2 + \\ & b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a * b^7 - 15B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * ( \\ & \frac{1}{2}) * a^8 * b + 20B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^6 * b^3 + 22B * (2 * (a^2 + b^2)^{\frac{1}{2}} \\ & ) + 2a)^{\frac{1}{2}} * a^4 * b^5 - 12B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^2 * b^7 + B * (2 * (a^2 + b \\ & ^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * b^9) * \ln(-b * \tan(dx+c) - a + (a + b * \tan(dx+c))^{\frac{1}{2}}) * (2 * (a^2 \\ & + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} - (a^2 + b^2)^{\frac{1}{2}}) + 2 * (-30A * a^8 * b^2 + 40A * a^6 * b^4 + 44A * \\ & a^4 * b^6 - 24A * a^2 * b^8 + 2A * b^{10} + 10B * a^9 * b - 40B * a^7 * b^3 + 12B * a^5 * b^5 + 56B * a^3 \\ & * b^7 - 6B * a * b^9 + \frac{1}{2} * (3A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{3}{2}} * a^6 + 5 \\ & * A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{3}{2}} * a^4 * b^2 + A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{3}{2}} * a^2 * b^4 - A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + \\ & b^2)^{\frac{3}{2}} * b^6 + 2A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^8 - 18A * ( \\ & 2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^6 * b^2 - 10A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + \\ & b^2)^{\frac{1}{2}} * a^4 * b^4 + 10A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^2 * b^6 - 5A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^9 + 20A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^7 * b^2 - 6A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^5 * b^4 - 28A * ( \\ & 2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a^3 * b^6 + 3A * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * a * b^8 + 10B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^7 * b - 10B * (2 * (a^2 + b^2)^{\frac{1}{2}} + 2a)^{\frac{1}{2}} * (a^2 + b^2)^{\frac{1}{2}} * a^5 * b^3 - 18B * \dots \end{aligned}$$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^4\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 36379 vs. 2(334) = 668.

time = 169.43, size = 36379, normalized size = 98.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^4\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^(5/2),x, algorithm="fricas")

[Out] 
$$-1/12 * (12 * \sqrt{2}) * ((a^{18} * b^4 + a^{16} * b^6 - 20 * a^{14} * b^8 - 84 * a^{12} * b^{10} - 154 * a^{10} * b^{12} - 154 * a^8 * b^{14} - 84 * a^6 * b^{16} - 20 * a^4 * b^{18} + a^2 * b^{20} + b^{22}) * d^5$$

$$\begin{aligned}
& * \cos(dx + c)^5 + 2*(3a^{16}b^6 + 20a^{14}b^8 + 56a^{12}b^{10} + 84a^{10}b^{12} \\
& + 70a^8b^{14} + 28a^6b^{16} - 4a^2b^{20} - b^{22})d^5 \cos(dx + c)^3 + (a^{14}b^8 + 7a^{12}b^{10} + 21a^{10}b^{12} + 35a^8b^{14} + 35a^6b^{16} + 21a^4b^{18} \\
& + 7a^2b^{20} + b^{22})d^5 \cos(dx + c) + 4*((a^{17}b^5 + 6a^{15}b^7 + 14a^{13}b^9 + 14a^{11}b^{11} - 14a^7b^{15} - 14a^5b^{17} - 6a^3b^{19} - ab^{21})d^5 \\
& * \cos(dx + c)^4 + (a^{15}b^7 + 7a^{13}b^9 + 21a^{11}b^{11} + 35a^9b^{13} + 35a^7b^{15} + 21a^5b^{17} + 7a^3b^{19} + ab^{21})d^5 \cos(dx + c)^2 * \sin(dx \\
& + c)) * \sqrt{((A^4 + 2A^2B^2 + B^4)a^{10} + 5(A^4 + 2A^2B^2 + B^4)a^8b^2 + 10(A^4 + 2A^2B^2 + B^4)a^6b^4 + 10(A^4 + 2A^2B^2 + B^4)a^4b^6 \\
& + 5(A^4 + 2A^2B^2 + B^4)a^2b^8 + (A^4 + 2A^2B^2 + B^4)b^{10} + (10A^3B + 30A^2B^2 + 2A^2B^3)a^{14}b + 30A^2B^2a^{12}b^3 + 2A^2B^3a^{10}b^5 - 90A^2B^2a^8b^7 - 130A^2B^2a^6b^9 \\
& - 70A^2B^2a^4b^{11} - 10A^2B^2a^2b^{13} + 2A^2B^2b^{15} + (A^2 - B^2)a^{15} - 5(A^2 - B^2)a^{13}b^2 - 35(A^2 - B^2)a^{11}b^4 - 65(A^2 - B^2)a^9b^6 - 45(A^2 - B^2)a^7b^8 \\
& + (A^2 - B^2)a^5b^{10} + 15(A^2 - B^2)a^3b^{12} + 5(A^2 - B^2)ab^{14})d^2 * \sqrt{(A^4 + 2A^2B^2 + B^4)/((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})d^4))} / (4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b \\
& + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 \\
& + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)ab^9 + (A^4 - 2A^2B^2 + B^4)b^{10}) * \sqrt{(4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b \\
& + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 \\
& + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)ab^9 + (A^4 - 2A^2B^2 + B^4)b^{10})} / ((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 \\
& + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20})d^4) * ((A^4 + 2A^2B^2 + B^4)/((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})d^4))^{3/4} * \arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{21} - 5(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{20}b - 4(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{19}b^2 - 30(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{18}b^3 - 94(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{17}b^4 - 61(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16}b^5 - 368(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b^6 - 8(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{14}b^7 - 700(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^8 + 182(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^9 - 728(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^{10} + 364(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^{11} - 364(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^{12} + 350(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^{13} + 16(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^{14} + 184(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^{15} + 122(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^{16} + 47(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^{17} + 60(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{18} + 2(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b^{19} + 10(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^{20} - (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{21})d^4 * \sqrt{(4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)
\end{aligned}$$



$$\begin{aligned} &^4)*a^8*b^2 + 240*(A^3*B - A*B^3)*a^7*b^3 - 20*(5*A^4 - 32*A^2*B^2 + 5*B^4) \\ &*a^6*b^4 - 504*(A^3*B - A*B^3)*a^5*b^5 + 10*(11*A^4 - 62*A^2*B^2 + 11*B^4)* \\ &a^4*b^6 + 240*(A^3*B - A*B^3)*a^3*b^7 - 20*(A^4 - 7*A^2*B^2 + B^4)*a^2*b^8 \\ &- 20*(A^3*B - A*B^3)*a*b^9 + (A^4 - 2*A^2*B^2 + B^4)*b^{10}/((a^{20} + 10*a^{18} \\ &*b^2 + 45*a^{16}*b^4 + 120*a^{14}*b^6 + 210*a^{12}*b^8 + 252*a^{10}*b^{10} + 210*a^8* \\ &b^{12} + 120*a^6*b^{14} + 45*a^4*b^{16} + 10*a^2*b^{18} + b^{20})*d^4))*sqrt((A^4 + 2 \\ &*A^2*B^2 + B^4)/((a^{10} + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + \\ &b^{10})*d^4)) + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{16} - \\ &5*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{15}*b - 1 \\ &0*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{14}*b^2 - 15*(A^{10} + \\ &3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{13}*b^3 - 70*(A^9*B \\ &+ 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{12}*b^4 - (A^{10} + 3*A^8*B^2 \\ &+ 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{11}*b^5 - 130*(A^9*B + 4*A^7*B \\ &^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{10}*b^6 + 45*(A^{10} + 3*A^8*B^2 + 2*A^6 \\ &*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^9*b^7 - 90*(A^9*B + 4*A^7*B^3 + 6*A^ \\ &5*B^5 + 4*A^3*B^7 + A*B^9)*a^8*b^8 + 65*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A \\ &^4*B^6 - 3*A^2*B^8 - B^{10})*a^7*b^9 + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A \\ &^3*B^7 + A*B^9)*a^6*b^{10} + 35*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3 \\ &*A^2*B^8 - B^{10})*a^5*b^{11} + 30*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + \\ &A*B^9)*a^4*b^{12} + 5*(A^{10} + 3*A^8*B^2 + 2*A^6*... \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^4(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*4/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 40.79, size = 2500, normalized size = 6.74

Too large to display



$$\begin{aligned}
& b^3 d^4) / 4) * (-((320 A^4 a^2 b^8 d^4 - 16 A^4 b^{10} d^4 - 1760 A^4 a^4 b^6 d^4 \\
& ^4 + 1600 A^4 a^6 b^4 d^4 - 400 A^4 a^8 b^2 d^4)^{(1/2)} + 4 A^2 a^5 d^2 - 40 \\
& * A^2 a^3 b^2 d^2 + 20 A^2 a b^4 d^2) / (a^{10} d^4 + b^{10} d^4 + 5 a^2 b^8 d^4 + \\
& 10 a^4 b^6 d^4 + 10 a^6 b^4 d^4 + 5 a^8 b^2 d^4))^{(1/2)} / 4 + 96 A^3 a^3 b^ \\
& 13 d^2 + 240 A^3 a^5 b^{11} d^2 + 320 A^3 a^7 b^9 d^2 + 240 A^3 a^9 b^7 d^2 + \\
& 96 A^3 a^{11} b^5 d^2 + 16 A^3 a^{13} b^3 d^2 + 16 A^3 a b^{15} d^2) * (-((320 A^4 \\
& a^2 b^8 d^4 - 16 A^4 b^{10} d^4 - 1760 A^4 a^4 b^6 d^4 + 1600 A^4 a^6 b^4 d^4 \\
& 4 - 400 A^4 a^8 b^2 d^4)^{(1/2)} + 4 A^2 a^5 d^2 - 40 A^2 a^3 b^2 d^2 + 20 A^ \\
& 2 a b^4 d^2) / (a^{10} d^4 + b^{10} d^4 + 5 a^2 b^8 d^4 + 10 a^4 b^6 d^4 + 10 a^6 \\
& * b^4 d^4 + 5 a^8 b^2 d^4))^{(1/2)} / 4 - \log(96 A^3 a^3 b^{13} d^2 - ((a + b \tan \\
& (c + d x))^{(1/2)} * (320 A^2 a^4 b^{14} d^3 - 16 A^2 b^{18} d^3 + 1024 A^2 a^6 b^1 \\
& 2 d^3 + 1440 A^2 a^8 b^{10} d^3 + 1024 A^2 a^{10} b^8 d^3 + 320 A^2 a^{12} b^6 d^ \\
& 3 - 16 A^2 a^{16} b^2 d^3) + (((320 A^4 a^2 b^8 d^4 - 16 A^4 b^{10} d^4 - 1760 * \\
& A^4 a^4 b^6 d^4 + 1600 A^4 a^6 b^4 d^4 - 400 A^4 a^8 b^2 d^4)^{(1/2)} - 4 A^2 \\
& * a^5 d^2 + 40 A^2 a^3 b^2 d^2 - 20 A^2 a b^4 d^2) / (16 a^{10} d^4 + 16 b^{10} d^ \\
& 4 + 80 a^2 b^8 d^4 + 160 a^4 b^6 d^4 + 160 a^6 b^4 d^4 + 80 a^8 b^2 d^4))^{( \\
& 1/2)} * (896 A a^6 b^{15} d^4 - 32 A b^{21} d^4 - 160 A a^2 b^{19} d^4 - 128 A a^4 b \\
& ^{17} d^4 - (((320 A^4 a^2 b^8 d^4 - 16 A^4 b^{10} d^4 - 1760 A^4 a^4 b^6 d^4 + \\
& 1600 A^4 a^6 b^4 d^4 - 400 A^4 a^8 b^2 d^4)^{(1/2)} - 4 A^2 a^5 d^2 + 40 A^2 \\
& * a^3 b^2 d^2 - 20 A^2 a b^4 d^2) / (16 a^{10} d^4 + 16 b^{10} d^4 + 80 a^2 b^8 d^ \\
& 4 + 160 a^4 b^6 d^4 + 160 a^6 b^4 d^4 + 80 a^8 b^2 d^4))^{(1/2)} * (a + b \tan(c \\
& + d x))^{(1/2)} * (64 a b^{22} d^5 + 640 a^3 b^{20} d^5 + 2880 a^5 b^{18} d^5 + 7680 \\
& * a^7 b^{16} d^5 + 13440 a^9 b^{14} d^5 + 16128 a^{11} b^{12} d^5 + 13440 a^{13} b^{10} \\
& d^5 + 7680 a^{15} b^8 d^5 + 2880 a^{17} b^6 d^5 + 640 a^{19} b^4 d^5 + 64 a^{21} b^ \\
& 2 d^5) + 3136 A a^8 b^{13} d^4 + 4928 A a^{10} b^{11} d^4 + 4480 A a^{12} b^9 d^4 + \\
& 2432 A a^{14} b^7 d^4 + 736 A a^{16} b^5 d^4 + 96 \dots
\end{aligned}$$

$$3.358 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=261

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2} d} + \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2} d} + \frac{2a(Ab - aB) \tan(c + dx)}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{5/2}}$$

[Out] (A-I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(5/2)/d+(A+I\*B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(5/2)/d-2/3\*a^2\*(A\*a^2\*b+7\*A\*b^3-4\*B\*a^3-10\*B\*a\*b^2)/b^3/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)-2/3\*(A\*a\*b-4\*B\*a^2-3\*B\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/b^3/(a^2+b^2)/d+2/3\*a\*(A\*b-B\*a)\*tan(d\*x+c)^2/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.48, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {3686, 3716, 3711, 3620, 3618, 65, 214}

$$\frac{2a(Ab - aB) \tan^2(c + dx)}{3bd(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} - \frac{2(-4a^2B + aAb - 3b^2B) \sqrt{a + b \tan(c + dx)}}{3b^2d(a^2 + b^2)} - \frac{2a^2(-4a^3B + a^2Ab - 10ab^2B + 7Ab^3)}{3b^2d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{5/2}} + \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(5/2)\*d) + ((A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(5/2)\*d) + (2\*a\*(A\*b - a\*B)\*Tan[c + d\*x]^2)/(3\*b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) - (2\*a^2\*(a^2\*A\*b + 7\*A\*b^3 - 4\*a^3\*B - 10\*a\*b^2\*B))/(3\*b^3\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]]) - (2\*(a\*A\*b - 4\*a^2\*B - 3\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*b^3\*(a^2 + b^2)\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3618**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rule 3716

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(-b\*c - a\*d)\*(c^2\*C - B\*c\*d + A\*d^2)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d^2\*f\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*d\*(A\*c - c\*C + B\*d) + b\*(c^2\*C - B\*c\*d + A\*d^2) + d\*(A\*b\*c + a\*B\*c - b\*c\*C - a\*A\*d + b\*B\*d + a\*C\*d)\*Tan[e + f\*x] + b\*C\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,

-1]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^3(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx &= \frac{2a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2 \int \frac{\tan(c+dx)(-2a(Ab-aB)+\frac{3}{2}b(A+B \tan(c+dx)))}{(a + b \tan(c + dx))^{5/2}} dx}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} \\
 &= \frac{2a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2a^2(a^2 Ab + 7Ab^3 - 4a^3 B - 3b^3)}{3b^3(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2a^2(a^2 Ab + 7Ab^3 - 4a^3 B - 3b^3)}{3b^3(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2a^2(a^2 Ab + 7Ab^3 - 4a^3 B - 3b^3)}{3b^3(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2a^2(a^2 Ab + 7Ab^3 - 4a^3 B - 3b^3)}{3b^3(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \tan^2(c + dx)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2a^2(a^2 Ab + 7Ab^3 - 4a^3 B - 3b^3)}{3b^3(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2}d} + \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2}d}
 \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 2.31, size = 309, normalized size = 1.18

$$\frac{-2(a - ib)(a + ib)(-2aAb + 8a^2B + b^2B) - b^2(a + bB) \left( (a + ib) {}_2F_1 \left( -\frac{3}{2}, 1, -\frac{1}{2}; \frac{a + b \tan(c + dx)}{a - ib} \right) - (a + b) {}_2F_1 \left( -\frac{3}{2}, 1, -\frac{1}{2}; \frac{a + b \tan(c + dx)}{a + ib} \right) \right) - 6(a - ib)(a + ib)(-Ab + 4aB) \tan(c + dx) - 6(a - ib)(a + ib)b^2B \tan^2(c + dx) + 3AB \left( (a + ib) {}_2F_1 \left( -\frac{3}{2}, 1, -\frac{1}{2}; \frac{a + b \tan(c + dx)}{a - ib} \right) - (a + b) {}_2F_1 \left( -\frac{3}{2}, 1, -\frac{1}{2}; \frac{a + b \tan(c + dx)}{a + ib} \right) \right)}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2), x]
```

```
[Out] -1/3*(-2*(a - I*b)*(a + I*b)*(-2*a*A*b + 8*a^2*B + b^2*B) - b^2*(a*A + b*B)
*(I*(a + I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a - I*b)] - (I*a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a + I*b)]) - 6*(a - I*b)*(a + I*b)*b*(-(A*b) + 4*a*B)*Tan[c + d*x] - 6*(a - I*b
```

)\*(a + I\*b)\*b^2\*B\*Tan[c + d\*x]^2 + 3\*A\*b^2\*(I\*(a + I\*b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] - (I\*a + b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)])\*(a + b\*Tan[c + d\*x]))/(b^3\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 3274 vs. 2(231) = 462.

time = 0.20, size = 3275, normalized size = 12.55

method	result	size
derivativedivides	Expression too large to display	3275
default	Expression too large to display	3275

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x,method=\_RETURNVE  
RBOSE)

[Out] 2/d/b^3\*(B\*(a+b\*tan(d\*x+c))^(1/2)-a^2\*(A\*a^2\*b+3\*A\*b^3-2\*B\*a^3-4\*B\*a\*b^2))/(a^2+b^2)^2/(a+b\*tan(d\*x+c))^(1/2)+1/3\*a^3\*(A\*b-B\*a)/(a^2+b^2)/(a+b\*tan(d\*x+c))^(3/2)+b^3/(a^2+b^2)^2\*(1/4/b/(5\*a^4-10\*a^2\*b^2+b^4)/(a^2+b^2)^(3/2)\*(1/2\*(-10\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^7\*b+10\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5\*b^3+18\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^5-2\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^7+15\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^8\*b-20\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6\*b^3-22\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^5+12\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^7-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^9+3\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^6+5\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^4\*b^2+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^2\*b^4-B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*b^6+2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^8-18\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^6\*b^2-10\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^4\*b^4+10\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^2\*b^6-5\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^9+20\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^7\*b^2-6\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^5\*b^4-28\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^3\*b^6+3\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b^8)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))+2\*(10\*A\*a^9\*b-40\*A\*a^7\*b^3+12\*A\*a^5\*b^5+56\*A\*a^3\*b^7-6\*A\*a\*b^9+30\*B\*a^8\*b^2-40\*B\*a^6\*b^4-44\*B\*a^4\*b^6+24\*B\*a^2\*b^8-2\*B\*b^10-1/2\*(-10\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^7\*b+10\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5\*b^3+18\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^5-2\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^7+15\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^8\*b-20\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6\*b^3-22\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^5+12\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^7-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^9+3\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^6+5\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^4\*b^2+B\*(2\*(a^2

$$\begin{aligned}
& +b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^2*b^4-B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^6+2*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}* \\
& ^8-18*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^6*b^2-10*B*(2*(a^2+ \\
& b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^4+10*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^6-5*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^9+20*B*(2 \\
& *(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^7*b^2-6*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^5*b \\
& ^4-28*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^3*b^6+3*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a*b^8)*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)-2*a)^{(1/2)}*ar \\
& ctan((2*(a+b*tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)-2*a)^{(1/2)}))+1/4/b/(5*a^4-10*a^2*b^2+b^4)/(a^2+b^2)^{(3/2)}*(-1/2*(-10* \\
& A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^7*b+10*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b^3+18*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a \\
& ^2+b^2)^{(1/2)}*a^3*b^5-2*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b \\
& ^7+15*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^8*b-20*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^6*b^3-22*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^4*b^5+12*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^2*b^7-A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*b^9+3*B*(2*(a^2+b^2) \\
& ^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^6+5*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^2*b^ \\
& 4-B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^6+2*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^8-18*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^ \\
& 2)^{(1/2)}*a^6*b^2-10*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^4 \\
& +10*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^6-5*B*(2*(a^2+b^2) \\
& )^{(1/2)+2*a)^{(1/2)}*a^9+20*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^7*b^2-6*B*(2*(a \\
& ^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^5*b^4-28*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^3*b^6 \\
& +3*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a*b^8)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c) \\
& ))^{(1/2)}*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(-10*A*a^9*b+40*A \\
& *a^7*b^3-12*A*a^5*b^5-56*A*a^3*b^7+6*A*a*b^9-30*B*a^8*b^2+40*B*a^6*b^4+44*B \\
& *a^4*b^6-24*B*a^2*b^8+2*B*b^10+1/2*(-10*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^ \\
& 2+b^2)^{(1/2)}*a^7*b+10*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b \\
& ^3+18*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^3*b^5-2*A*(2*(a^2+b \\
& ^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^7+15*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^8*b-20*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^6*b^3-22*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^4*b^5+12*A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*a^2*b^7-A*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*b^9+3*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^6+5*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4*b^2+B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^2*b^4-B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^6+2*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^8-18 \\
& *B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^6*b^2-10*B*(2*(a^2+b^2)^{(1/2)+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^4+10*B*(...
\end{aligned}$$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 35969 vs. 2(227) = 454.

time = 192.05, size = 35969, normalized size = 137.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$\frac{1}{12} \cdot (12 \cdot \sqrt{2}) \cdot ((a^{18}b^3 + a^{16}b^5 - 20a^{14}b^7 - 84a^{12}b^9 - 154a^{10}b^{11} - 154a^8b^{13} - 84a^6b^{15} - 20a^4b^{17} + a^2b^{19} + b^{21}) \cdot d^5 \cdot \cos(d \cdot x + c)^4 + 2 \cdot (3a^{16}b^5 + 20a^{14}b^7 + 56a^{12}b^9 + 84a^{10}b^{11} + 70a^8b^{13} + 28a^6b^{15} - 4a^2b^{19} - b^{21}) \cdot d^5 \cdot \cos(d \cdot x + c)^2 + (a^{14}b^7 + 7a^{12}b^9 + 21a^{10}b^{11} + 35a^8b^{13} + 35a^6b^{15} + 21a^4b^{17} + 7a^2b^{19} + b^{21}) \cdot d^5 + 4 \cdot ((a^{17}b^4 + 6a^{15}b^6 + 14a^{13}b^8 + 14a^{11}b^{10} - 14a^7b^{14} - 14a^5b^{16} - 6a^3b^{18} - ab^{20}) \cdot d^5 \cdot \cos(d \cdot x + c)^3 + (a^{15}b^6 + 7a^{13}b^8 + 21a^{11}b^{10} + 35a^9b^{12} + 35a^7b^{14} + 21a^5b^{16} + 7a^3b^{18} + ab^{20}) \cdot d^5 \cdot \cos(d \cdot x + c)) \cdot \sin(d \cdot x + c) \cdot \sqrt{((A^4 + 2A^2B^2 + B^4) \cdot a^{10} + 5(A^4 + 2A^2B^2 + B^4) \cdot a^8b^2 + 10(A^4 + 2A^2B^2 + B^4) \cdot a^6b^4 + 10(A^4 + 2A^2B^2 + B^4) \cdot a^4b^6 + 5(A^4 + 2A^2B^2 + B^4) \cdot a^2b^8 + (A^4 + 2A^2B^2 + B^4) \cdot b^{10} - (10A \cdot B \cdot a^{14}b + 30A \cdot B \cdot a^{12}b^3 + 2A \cdot B \cdot a^{10}b^5 - 90A \cdot B \cdot a^8b^7 - 130A \cdot B \cdot a^6b^9 - 70A \cdot B \cdot a^4b^{11} - 10A \cdot B \cdot a^2b^{13} + 2A \cdot B \cdot b^{15} + (A^2 - B^2) \cdot a^{15} - 5(A^2 - B^2) \cdot a^{13}b^2 - 35(A^2 - B^2) \cdot a^{11}b^4 - 65(A^2 - B^2) \cdot a^9b^6 - 45(A^2 - B^2) \cdot a^7b^8 + (A^2 - B^2) \cdot a^5b^{10} + 15(A^2 - B^2) \cdot a^3b^{12} + 5(A^2 - B^2) \cdot a \cdot b^{14}) \cdot d^2 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4) / ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) \cdot d^4))} / (4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)a \cdot b^9 + (A^4 - 2A^2B^2 + B^4) \cdot b^{10})) \cdot \sqrt{((4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)a \cdot b^9 + (A^4 - 2A^2B^2 + B^4) \cdot b^{10}))} / ((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20}) \cdot d^4) \cdot ((A^4 + 2A^2B^2 + B^4) / ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) \cdot d^4))^{3/4} \cdot \arctan(((2(A^7B + 3A^5B^3 + 3A^3B^5 +$$

$$\begin{aligned}
 & A*B^7)*a^{21} - 5*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{20}*b - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{19}*b^2 - 30*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{18}*b^3 - 94*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{17}*b^4 - 61*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{16}*b^5 - 368*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{15}*b^6 - 8*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{14}*b^7 - 700*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13}*b^8 + 182*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12}*b^9 - 728*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^{10} + 364*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^{11} - 364*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^{12} + 350*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^{13} + 16*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^{14} + 184*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^{15} + 122*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^{16} + 47*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^{17} + 60*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{18} + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^{19} + 10*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{20} - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{21})*d^4*sqrt((4*A^2*B^2*a^{10} - 20*(A^3*B - A*B^3)*a^9*b + 5*(5*A^4 - 26*A^2*B^2 + 5*B^4)*a^8*b^2 + 240*(A^3*B - A*B^3)*a^7*b^3 - 20*(5*A^4 - 32*A^2*B^2 + 5*B^4)*a^6*b^4 - 504*(A^3*B - A*B^3)*a^5*b^5 + 10*(11*A^4 - 62*A^2*B^2 + 11*B^4)*a^4*b^6 + 240*(A^3*B - A*B^3)*a^3*b^7 - 20*(A^4 - 7*A^2*B^2 + B^4)*a^2*b^8 - 20*(A^3*B - A*B^3)*a*b^9 + (A^4 - 2*A^2*B^2 + B^4)*b^{10})/((a^{20} + 10*a^{18}*b^2 + 45*a^{16}*b^4 + 120*a^{14}*b^6 + 210*a^{12}*b^8 + 252*a^{10}*b^{10} + 210*a^8*b^{12} + 120*a^6*b^{14} + 45*a^4*b^{16} + 10*a^2*b^{18} + b^{20})*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^{10} + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^{10})*d^4)) + (2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{16} - 5*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{15}*b - 10*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{14}*b^2 - 15*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{13}*b^3 - 70*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{12}*b^4 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{11}*b^5 - 130*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{10}*b^6 + 45*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^9*b^7 - 90*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^8*b^8 + 65*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^7*b^9 + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^6*b^{10} + 35*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^5*b^{11} + 30*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4*b^{12} + 5*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*...
 \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^3(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)



$$\begin{aligned}
& 4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4 \\
& d^4))^{(1/2)}/4 + (\log(((320B^4a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4 \\
& a^4b^6d^4 + 1600B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} + 4B^2a^5 \\
& 5d^2 - 40B^2a^3b^2d^2 + 20B^2a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2 \\
& b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)}*((a + b \\
& \tan(c + dx))^{(1/2)}*(320B^2a^4b^{14}d^3 - 16B^2b^{18}d^3 + 1024B^2a^6 \\
& b^{12}d^3 + 1440B^2a^8b^{10}d^3 + 1024B^2a^{10}b^8d^3 + 320B^2a^{12}b^6 \\
& d^3 - 16B^2a^{16}b^2d^3) - (((320B^4a^2b^8d^4 - 16B^4b^{10}d^4 - \\
& 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} + \\
& 4B^2a^5d^2 - 40B^2a^3b^2d^2 + 20B^2a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 \\
& + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)}* \\
& (((320B^4a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B \\
& ^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} + 4B^2a^5d^2 - 40B^2a^3b^2 \\
& d^2 + 20B^2a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6 \\
& d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)}*(a + b*\tan(c + dx))^{(1/2)}*(6 \\
& 4a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7b^{16}d^5 + 1 \\
& 3440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 13440a^{13}b^{10}d^5 + 7680a^{15}b \\
& ^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 + 64a^{21}b^2d^5))/4 - 32B \\
& b^{21}d^4 - 160B^2a^2b^{19}d^4 - 128B^2a^4b^{17}d^4 + 896B^2a^6b^{15}d^4 + 3 \\
& 136B^2a^8b^{13}d^4 + 4928B^2a^{10}b^{11}d^4 + 4480B^2a^{12}b^9d^4 + 2432B^2a^{14} \\
& b^7d^4 + 736B^2a^{16}b^5d^4 + 96B^2a^{18}b^3d^4))/4 + 96B^3a^3b^{13} \\
& d^2 + 240B^3a^5b^{11}d^2 + 320B^3a^7b^9d^2 + 240B^3a^9b^7d^2 + \\
& 96B^3a^{11}b^5d^2 + 16B^3a^{13}b^3d^2 + 16B^3a^2b^{15}d^2)*(-((320B^4 \\
& a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 \\
& - 400B^4a^8b^2d^4)^{(1/2)} + 4B^2a^5d^2 - 40B^2a^3b^2d^2 + 20B^2 \\
& a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6 \\
& b^4d^4 + 5a^8b^2d^4))^{(1/2)}/4 - \log(96B^3a^3b^{13}d^2 - (((320B^4 \\
& a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 \\
& - 400B^4a^8b^2d^4)^{(1/2)} - 4B^2a^5d^2 + 40B^2a^3b^2d^2 - 20B^2 \\
& a^2b^4d^2)/(16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + \\
& 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)}*((a + b*\tan(c + dx))^{(1/2)}*(320 \\
& B^2a^4b^{14}d^3 - 16B^2b^{18}d^3 + 1024B^2a^6b^{12}d^3 + 1440B^2a^8b^{10} \\
& d^3 + 1024B^2a^{10}b^8d^3 + 320B^2a^{12}b^6d^3 - 16B^2a^{16}b^2d^3 \\
& 3) + (((320B^4a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600 \\
& B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} - 4B^2a^5d^2 + 40B^2a^3b^2 \\
& d^2 - 20B^2a^2b^4d^2)/(16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 1 \\
& 60a^4b^6d^4 + 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)}*(896B^2a^6b^{15}d \\
& ^4 - 32B^2b^{21}d^4 - 160B^2a^2b^{19}d^4 - 128B^2a^4b^{17}d^4 - (((320B^4 \\
& a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 \\
& - 400B^4a^8b^2d^4)^{(1/2)} - 4B^2a^5d^2 + 40B^2a^3b^2d^2 - 20B^2 \\
& a^2b^4d^2)/(16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + \\
& 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)}*(a + b*\tan(c + dx))^{(1/2)}*(64a^2 \\
& b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 \dots
\end{aligned}$$

$$3.359 \quad \int \frac{\tan^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=198

$$\frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2} d} - \frac{(iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2} d} - \frac{2a^2(Ab - B^2)}{3b^2(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}}$$

[Out] (I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(5/2)/d-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(5/2)/d+2\*a\*(2\*A\*b^3-a\*(a^2+3\*b^2)\*B)/b^2/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)-2/3\*a^2\*(A\*b-B\*a)/b^2/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.35, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {3685, 3709, 3620, 3618, 65, 214}

$$-\frac{2a^2(Ab - aB)}{3b^2d(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{2a(2Ab^3 - aB(a^2 + 3b^2))}{b^2d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} + \frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{5/2}} - \frac{(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2),x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(5/2)\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(5/2)\*d) - (2\*a^2\*(A\*b - a\*B))/(3\*b^2\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*a\*(2\*A\*b^3 - a\*(a^2 + 3\*b^2)\*B))/(b^2\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3618**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c

\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3685

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(- (B\*c - A\*d)) \* (b\*c - a\*d)^2 \* ((c + d\*Tan[e + f\*x])^(n + 1) / (f\*d^2\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1) \* Simp[B\*(b\*c - a\*d)^2 + A\*d\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + d\*(B\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + A\*(2\*a\*b\*c - a^2\*d + b^2\*d))\*Tan[e + f\*x] + b^2\*B\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

### Rule 3709

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*b\*B + a^2\*C)\*((a + b\*Tan[e + f\*x])^(m + 1) / (b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx &= -\frac{2a^2(Ab-aB)}{3b^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{\int \frac{-a(Ab-aB)+b(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}} dx}{b(a^2+b^2)} \\
&= -\frac{2a^2(Ab-aB)}{3b^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(2Ab^3-a(a^2+b^2))}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{2a^2(Ab-aB)}{3b^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(2Ab^3-a(a^2+b^2))}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{2a^2(Ab-aB)}{3b^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(2Ab^3-a(a^2+b^2))}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{2a^2(Ab-aB)}{3b^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2a(2Ab^3-a(a^2+b^2))}{b^2(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{(iA+B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(iA-B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.69, size = 260, normalized size = 1.31

$$\frac{2(a-ib)(a+ib)(Ab+2aB)+b(Ab-aB)\left(i(a+ib) {}_2F_1\left(-\frac{3}{2}, 1, -\frac{1}{2}; \frac{a+b\tan(c+dx)}{a-ib}\right) - (ia+b) {}_2F_1\left(-\frac{3}{2}, 1, -\frac{1}{2}; \frac{a+b\tan(c+dx)}{a+ib}\right)\right) + 6(a-ib)(a+ib)bB\tan(c+dx) + 3bB\left(i(a+ib) {}_2F_1\left(-\frac{1}{2}, 1, \frac{1}{2}; \frac{a+b\tan(c+dx)}{a-ib}\right) - (ia+b) {}_2F_1\left(-\frac{1}{2}, 1, \frac{1}{2}; \frac{a+b\tan(c+dx)}{a+ib}\right)\right)}{3b^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}}(a+b\tan(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] -1/3\*(2\*(a - I\*b)\*(a + I\*b)\*(A\*b + 2\*a\*B) + b\*(A\*b - a\*B)\*(I\*(a + I\*b)\*Hypergeometric2F1[-3/2, 1, -1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] - (I\*a + b)\*Hypergeometric2F1[-3/2, 1, -1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)]) + 6\*(a - I\*b)\*(a + I\*b)\*b\*B\*Tan[c + d\*x] + 3\*b\*B\*(I\*(a + I\*b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] - (I\*a + b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)])\*(a + b\*Tan[c + d\*x])/(b^2\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 3248 vs. 2(174) = 348.

time = 0.14, size = 3249, normalized size = 16.41

method	result	size
derivativedivides	Expression too large to display	3249
default	Expression too large to display	3249

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\tan(dx+c)^2(A+B\tan(dx+c))/(a+b\tan(dx+c))^{5/2}, x, \text{method}=\_RETURNVE$   
RBOSE)

[Out]  $\frac{2}{d/b^2} \left( -\frac{1}{3} a^2 (A b - B a) / (a^2 + b^2) / (a + b \tan(dx+c))^{3/2} + a (2 A a b^3 - B a^3 - 3 B a a b^2) / (a^2 + b^2)^2 / (a + b \tan(dx+c))^{1/2} - b^2 / (a^2 + b^2)^2 (1/4 b / (5 a^4 - 10 a^2 b^2 + b^4) / (a^2 + b^2)^{3/2} * (1/2 * (3 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * a^6 + 5 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * a^4 * b^2 + A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * a^2 * b^4 - A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * b^6 + 2 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^6 * b^2 - 10 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^4 * b^4 + 10 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^2 * b^6 - 5 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^9 + 20 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^7 * b^2 - 6 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^5 * b^4 - 28 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^3 * b^6 + 3 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a * b^8 + 10 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^7 * b - 10 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^5 * b^3 - 18 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^3 * b^5 + 2 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a * b^7 - 15 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^8 * b + 20 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^6 * b^3 + 22 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^4 * b^5 - 12 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^2 * b^7 + B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * b^9) * \ln(b \tan(dx+c)) + (a + b \tan(dx+c))^{1/2} * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} + (a^2 + b^2)^{1/2} + 2 * (30 A a^8 b^2 - 40 A a^6 b^4 - 44 A a^4 b^6 + 24 A a^2 b^8 - 2 A b^{10} - 10 B a^9 b + 40 B a^7 b^3 - 12 B a^5 b^5 - 56 B a^3 b^7 + 6 B a b^9 - 1/2 * (3 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * a^6 + 5 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * a^4 * b^2 + A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * a^2 * b^4 - A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{3/2} * b^6 + 2 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^8 - 18 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^6 * b^2 - 10 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^4 * b^4 + 10 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^2 * b^6 - 5 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^9 + 20 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^7 * b^2 - 6 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^5 * b^4 - 28 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^3 * b^6 + 3 A * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a * b^8 + 10 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^7 * b - 10 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^5 * b^3 - 18 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a^3 * b^5 + 2 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * (a^2 + b^2)^{1/2} * a * b^7 - 15 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^8 * b + 20 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^6 * b^3 + 22 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^4 * b^5 - 12 B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * a^2 * b^7 + B * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} * b^9) * (2 (a^2 + b^2)^{1/2} + 2 a)^{1/2} \right)$





time = 199.07, size = 35709, normalized size = 180.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} \cdot (12 \cdot \sqrt{2}) \cdot ((a^{18}b^2 + a^{16}b^4 - 20a^{14}b^6 - 84a^{12}b^8 - 154a^{10}b^{10} - 154a^8b^{12} - 84a^6b^{14} - 20a^4b^{16} + a^2b^{18} + b^{20}) \cdot d^5 \cdot \cos(d \cdot x + c)^4 + 2 \cdot (3a^{16}b^4 + 20a^{14}b^6 + 56a^{12}b^8 + 84a^{10}b^{10} + 70a^8b^{12} + 28a^6b^{14} - 4a^2b^{18} - b^{20}) \cdot d^5 \cdot \cos(d \cdot x + c)^2 + (a^{14}b^6 + 7a^{12}b^8 + 21a^{10}b^{10} + 35a^8b^{12} + 35a^6b^{14} + 21a^4b^{16} + 7a^2b^{18} + b^{20}) \cdot d^5 + 4 \cdot ((a^{17}b^3 + 6a^{15}b^5 + 14a^{13}b^7 + 14a^{11}b^9 - 14a^7b^{13} - 14a^5b^{15} - 6a^3b^{17} - ab^{19}) \cdot d^5 \cdot \cos(d \cdot x + c)^3 + (a^{15}b^5 + 7a^{13}b^7 + 21a^{11}b^9 + 35a^9b^{11} + 35a^7b^{13} + 21a^5b^{15} + 7a^3b^{17} + ab^{19}) \cdot d^5 \cdot \cos(d \cdot x + c)) \cdot \sin(d \cdot x + c)) \cdot \sqrt{((A^4 + 2A^2B^2 + B^4) \cdot a^{10} + 5(A^4 + 2A^2B^2 + B^4) \cdot a^8b^2 + 10(A^4 + 2A^2B^2 + B^4) \cdot a^6b^4 + 10(A^4 + 2A^2B^2 + B^4) \cdot a^4b^6 + 5(A^4 + 2A^2B^2 + B^4) \cdot a^2b^8 + (A^4 + 2A^2B^2 + B^4) \cdot b^{10} + (10A \cdot B \cdot a^{14}b + 30A \cdot B \cdot a^{12}b^3 + 2A \cdot B \cdot a^{10}b^5 - 90A \cdot B \cdot a^8b^7 - 130A \cdot B \cdot a^6b^9 - 70A \cdot B \cdot a^4b^{11} - 10A \cdot B \cdot a^2b^{13} + 2A \cdot B \cdot b^{15} + (A^2 - B^2) \cdot a^{15} - 5(A^2 - B^2) \cdot a^{13}b^2 - 35(A^2 - B^2) \cdot a^{11}b^4 - 65(A^2 - B^2) \cdot a^9b^6 - 45(A^2 - B^2) \cdot a^7b^8 + (A^2 - B^2) \cdot a^5b^{10} + 15(A^2 - B^2) \cdot a^3b^{12} + 5(A^2 - B^2) \cdot ab^{14}) \cdot d^2 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4) / ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) \cdot d^4))} / (4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)ab^9 + (A^4 - 2A^2B^2 + B^4)b^{10})) \cdot \sqrt{((4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)ab^9 + (A^4 - 2A^2B^2 + B^4)b^{10}) / ((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20}) \cdot d^4)} \cdot ((A^4 + 2A^2B^2 + B^4) / ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) \cdot d^4))^{3/4} \cdot \arctan(-((2(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{21} - 5(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{20}b - 4(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{19}b^2 - 30(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{18}b^3 - 94(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{17}b^4 - 61(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16}b^5 - 368(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b^6 - 8(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{14}b^7 - 700(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^8 + 182(A^8 + 2A^6B^2 - 2$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 23.12, size = 2500, normalized size = 12.63
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (log((((a + b*tan(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 +
1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 32
0*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + (((320*A^4*a^2*b^8*d^4 - 16*A^
4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*
d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d
^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2
*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^4 - 128
*A*a^4*b^17*d^4 - (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*
b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2
+ 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*
d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*tan(c
+ d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*
a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d
^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2
*d^5))/4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480*A*a^12*b^9*d^4
+ 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3*d^4))/4)*(((320
*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^
4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 2
0*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10
*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3*b^13*d^2 - 240*A^3*a^5
*b^11*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^11*b^5*d^2
- 16*A^3*a^13*b^3*d^2 - 16*A^3*a*b^15*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4
*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d
^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^
4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*
d^4))^(1/2))/4 + (log((((a + b*tan(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 -
16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*
a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((-((320*A^4*a
^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4
- 400*A^4*a^8*b^2*d^4)^(1/2) + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*
a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b
```

$$\begin{aligned}
& ^4d^4 + 5a^8b^2d^4))^{(1/2)} * (896A^6b^{15}d^4 - 32Ab^{21}d^4 - 160A^2b^{19}d^4 - 128A^4b^{17}d^4 - ((-(320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4))^{(1/2)} + 4A^2a^5d^2 - 40A^2a^3b^2d^2 + 20A^2ab^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)} * (a + b \tan(c + dx))^{(1/2)} * (64a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7b^{16}d^5 + 13440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 13440a^{13}b^{10}d^5 + 7680a^{15}b^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 + 64a^{21}b^2d^5))/4 + 3136A^8b^{13}d^4 + 4928A^10b^{11}d^4 + 4480A^12b^9d^4 + 2432A^14b^7d^4 + 736A^16b^5d^4 + 96A^18b^3d^4))/4 * (-((320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4))^{(1/2)} + 4A^2a^5d^2 - 40A^2a^3b^2d^2 + 20A^2ab^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)})/4 - 96A^3a^3b^{13}d^2 - 240A^3a^5b^{11}d^2 - 320A^3a^7b^9d^2 - 240A^3a^9b^7d^2 - 96A^3a^{11}b^5d^2 - 16A^3a^{13}b^3d^2 - 16A^3ab^{15}d^2) * (-((320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4))^{(1/2)} + 4A^2a^5d^2 - 40A^2a^3b^2d^2 + 20A^2ab^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)})/4 - \log(-((a + b \tan(c + dx))^{(1/2)} * (320A^2a^4b^{14}d^3 - 16A^2b^{18}d^3 + 1024A^2a^6b^{12}d^3 + 1440A^2a^8b^{10}d^3 + 1024A^2a^{10}b^8d^3 + 320A^2a^{12}b^6d^3 - 16A^2a^{16}b^2d^3) - (((320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4))^{(1/2)} - 4A^2a^5d^2 + 40A^2a^3b^2d^2 - 20A^2ab^4d^2)/(16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)} * (((320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4))^{(1/2)} - 4A^2a^5d^2 + 40A^2a^3b^2d^2 - 20A^2ab^4d^2)/(16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)} * (a + b \tan(c + dx))^{(1/2)} * (64a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7b^{16}d^5 + 13440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 13440a^{13}b^{10}d^5 + 7680a^{15}b^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 + 64a^{21}b^2d^5) - 32Ab^{21}d^4 - 160A^2b^{19}d^4 - 128A^4b^{17}d^4 + 896A^6b^{15}d^4 + 3136A^8b^{13}d^4 + 4928A^10b^{11}d^4 + 4480A^12b^9d^4 + 2432A^14b^7d^4 + 736A^16b^5d^4 + 96A^18b^3d^4)) * ((...
\end{aligned}$$

$$3.360 \quad \int \frac{\tan(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=188

$$\frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2} d} - \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2} d} + \frac{2a(Ab - a^2 - b^2)}{3b(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}}$$

[Out]  $-(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(5/2)}/d-(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(5/2)}/d+2*(A*a^2-A*b^2+2*B*a*b)/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)}+2/3*a*(A*b-B*a)/b/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3672, 3610, 3620, 3618, 65, 214}

$$\frac{2a(Ab - a^2)}{3bd(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{2(a^2A + 2abB - Ab^2)}{d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{5/2}} - \frac{(A + iB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]`

[Out]  $-\left(\frac{(A - I*B)*\operatorname{ArcTanh}\left[\frac{\sqrt{a + b*\tan[c + d*x]}}{\sqrt{a - I*b}}\right]}{(a - I*b)^{(5/2)*d}\right) - \left(\frac{(A + I*B)*\operatorname{ArcTanh}\left[\frac{\sqrt{a + b*\tan[c + d*x]}}{\sqrt{a + I*b}}\right]}{(a + I*b)^{(5/2)*d}\right) + \frac{2*a*(A*b - a^2 - b^2)}{(3*b*(a^2 + b^2)*d*(a + b*\tan[c + d*x])^{(3/2)}} + \frac{2*(a^2*A - A*b^2 + 2*a*b*B)}{(a^2 + b^2)^2*d*\sqrt{a + b*\tan[c + d*x]}}$

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 214

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 3610

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/`

$(f*(m + 1)*(a^2 + b^2))$ ,  $x$ ] + Dist[ $1/(a^2 + b^2)$ , Int[( $a + b*\text{Tan}[e + f*x]$ ) <sup>$m + 1$</sup> \*Simp[ $a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x]$ ,  $x$ ],  $x$ ],  $x$ ] /; FreeQ[{ $a, b, c, d, e, f$ },  $x$ ] && NeQ[ $b*c - a*d, 0$ ] && NeQ[ $a^2 + b^2, 0$ ] && LtQ[ $m, -1$ ]

### Rule 3618

Int[(( $a_.$ ) + ( $b_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )]] <sup>$m_.$</sup> \*(( $c_.$ ) + ( $d_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )]),  $x\_Symbol$ ] :> Dist[ $c*(d/f)$ , Subst[Int[( $a + (b/d)*x$ ) <sup>$m$</sup> / $(d^2 + c*x)$ ],  $x$ ],  $x$ ,  $d*\text{Tan}[e + f*x]$ ],  $x$ ] /; FreeQ[{ $a, b, c, d, e, f, m$ },  $x$ ] && NeQ[ $b*c - a*d, 0$ ] && NeQ[ $a^2 + b^2, 0$ ] && EqQ[ $c^2 + d^2, 0$ ]

### Rule 3620

Int[(( $a_.$ ) + ( $b_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )]] <sup>$m_.$</sup> \*(( $c_.$ ) + ( $d_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )]),  $x\_Symbol$ ] :> Dist[( $c + I*d$ )/2, Int[( $a + b*\text{Tan}[e + f*x]$ ) <sup>$m$</sup> \*( $1 - I*\text{Tan}[e + f*x]$ )],  $x$ ],  $x$ ] + Dist[( $c - I*d$ )/2, Int[( $a + b*\text{Tan}[e + f*x]$ ) <sup>$m$</sup> \*( $1 + I*\text{Tan}[e + f*x]$ )],  $x$ ],  $x$ ] /; FreeQ[{ $a, b, c, d, e, f, m$ },  $x$ ] && NeQ[ $b*c - a*d, 0$ ] && NeQ[ $a^2 + b^2, 0$ ] && NeQ[ $c^2 + d^2, 0$ ] && !IntegerQ[ $m$ ]

### Rule 3672

Int[(( $a_.$ ) + ( $b_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )]] <sup>$m_.$</sup> \*(( $A_.$ ) + ( $B_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )])\*(( $c_.$ ) + ( $d_.$ )\*tan[( $e_.$ ) + ( $f_.$ )\*( $x_.$ )]),  $x\_Symbol$ ] :> Simp[( $b*c - a*d$ )\*( $A*b - a*B$ )\*(( $a + b*\text{Tan}[e + f*x]$ ) <sup>$m + 1$</sup> /( $b*f*(m + 1)*(a^2 + b^2)$ )),  $x$ ] + Dist[ $1/(a^2 + b^2)$ , Int[( $a + b*\text{Tan}[e + f*x]$ ) <sup>$m + 1$</sup> \*Simp[ $a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x]$ ,  $x$ ],  $x$ ],  $x$ ] /; FreeQ[{ $a, b, c, d, e, f, A, B$ },  $x$ ] && NeQ[ $b*c - a*d, 0$ ] && LtQ[ $m, -1$ ] && NeQ[ $a^2 + b^2, 0$ ]

### Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx &= \frac{2a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}} dx}{a^2+b^2} \\
&= \frac{2a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2A-Ab^2+2abB)}{(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2A-Ab^2+2abB)}{(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2A-Ab^2+2abB)}{(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2A-Ab^2+2abB)}{(a^2+b^2)^2 d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(A+iB)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 2.31, size = 325, normalized size = 1.73

$$\frac{3b(-a^2(A\sqrt{-b^2}+bB)+b^2(A\sqrt{-b^2}+bB)+2ab(Ab-\sqrt{-b^2}B))\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right) - 3b(2aAb^2+a^2A\sqrt{-b^2}+A(-b^2)^{3/2}-a^2bB+b^3B+2ab\sqrt{-b^2}B)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right) + \frac{2a(a^2+b^2)(Ab-aB)}{(a+b\tan(c+dx))^{3/2}} + \frac{6b(a^2A-Ab^2+2abB)}{\sqrt{a+b\tan(c+dx)}}}{3b(a^2+b^2)^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((3\*b\*(-(a^2\*(A\*Sqrt[-b^2] + b\*B)) + b^2\*(A\*Sqrt[-b^2] + b\*B) + 2\*a\*b\*(A\*b - Sqrt[-b^2]\*B))\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a - Sqrt[-b^2]]) - (3\*b\*(2\*a\*A\*b^2 + a^2\*A\*Sqrt[-b^2] + A\*(-b^2)^(3/2) - a^2\*b\*B + b^3\*B + 2\*a\*b\*Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a + Sqrt[-b^2]]) + (2\*a\*(a^2 + b^2)\*(A\*b - a\*B))/(a + b\*Tan[c + d\*x])^(3/2) + (6\*b\*(a^2\*A - A\*b^2 + 2\*a\*b\*B))/Sqrt[a + b\*Tan[c + d\*x]])/(3\*b\*(a^2 + b^2)^2\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 3245 vs. 2(164) = 328.

time = 0.13, size = 3246, normalized size = 17.27



method	result	size
derivativedivides	Expression too large to display	3246
default	Expression too large to display	3246

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{2}{d} \frac{1}{b} \left( \frac{1}{3} \frac{A(b-Ba)}{(a^2+b^2)} \frac{1}{(a+b \tan(dx+c))^{3/2}} + b \frac{(Aa^2 - Ab^2 + 2Bab)}{(a^2+b^2)^2} \frac{1}{(a+b \tan(dx+c))^{1/2}} - \frac{b}{(a^2+b^2)^2} \frac{1}{4} \frac{1}{b} (5a^4 - 10a^2b^2 + b^4) \frac{1}{(a^2+b^2)^{3/2}} \right) \frac{1}{2} (-10A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^7 b + 10A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^5 b^3 + 18A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^3 b^5 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a b^7 + 15A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^8 b - 20A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^6 b^3 - 22A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^4 b^5 + 12A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^2 b^7 - A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} b^9 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} a^6 + 5B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} a^4 b^2 + B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} a^2 b^4 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} b^6 + 2B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^8 - 18B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^6 b^2 - 10B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^4 b^4 + 10B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^2 b^6 - 5B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^9 + 20B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^7 b^2 - 6B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^5 b^4 - 28B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^3 b^6 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a b^8) \ln(b \tan(dx+c)) + a + (a+b \tan(dx+c))^{1/2} (2(a^2+b^2)^{1/2} + 2a)^{1/2} + (a^2+b^2)^{1/2} + 2(10Aa^9 b - 40Aa^7 b^3 + 12Aa^5 b^5 + 56Aa^3 b^7 - 6Aa b^9 + 30Ba^8 b^2 - 40Ba^6 b^4 - 44Ba^4 b^6 + 24Ba^2 b^8 - 2Bb^{10} - \frac{1}{2}(-10A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^7 b + 10A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^5 b^3 + 18A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^3 b^5 - 2A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a b^7 + 15A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^8 b - 20A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^6 b^3 - 22A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^4 b^5 + 12A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^2 b^7 - A(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} b^9 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} a^6 + 5B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} a^4 b^2 + B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} a^2 b^4 - B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{3/2} b^6 + 2B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^8 - 18B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^6 b^2 - 10B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^4 b^4 + 10B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^2 b^6 - 5B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^9 + 20B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^7 b^2 - 6B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^5 b^4 - 28B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a^3 b^6 + 3B(2(a^2+b^2)^{1/2} + 2a)^{1/2} (a^2+b^2)^{1/2} a b^8) \frac{1}{(2(a^2+b^2)^{1/2} + 2a)^{1/2}}$$

$$\begin{aligned} & a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}))+1/4/b/(5*a^4-10*a^2*b^2+b^4) \\ & /((a^2+b^2)^{(3/2)}*(-1/2*(-10*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)} \\ & *a^7*b+10*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^5*b^3+18*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)} \\ & *(a^2+b^2)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^3*b^5-2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^7+15*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^8*b-20* \\ & A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^6*b^3-22*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4*b^5+12*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2*b^7-A*(2*(a^2+b^2)^{(1/2)}+2*a) \\ & )^{(1/2)}*b^9+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^6+5*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4*b^2+B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)} \\ & *(a^2+b^2)^{(3/2)}*a^2*b^4-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^6+2*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^8-18*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)} \\ & *(a^2+b^2)^{(1/2)}*a^6*b^2-10*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^4+10*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2*b^6-5*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^9+20*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^7*b^2-6*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5*b^4-28*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3*b^6+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^8)*\ln(-b \\ & *\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(-10*A*a^9*b+40*A*a^7*b^3-12*A*a^5*b^5-56*A*a^3*b^7+6*A*a*b^9-30 \\ & *B*a^8*b^2+40*B*a^6*b^4+44*B*a^4*b^6-24*B*a^2*b^8+2*B*b^10+1/2*(-10*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^7*b+10*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)} \\ & *(a^2+b^2)^{(1/2)}*a^5*b^3+18*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^3*b^5-2*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a*b^7+15*A \\ & *(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^8*b-20*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^6 \\ & *b^3-22*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4*b^5+12*A*(2*(a^2+b^2)^{(1/2)}+2*a) \\ & )^{(1/2)}*a^2*b^7-A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*b^9+3*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^6+5*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^4*b^2+B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*a^2*b^4-B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(3/2)}*b^6+2*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^8-18*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^6*b^2-10*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4*b^4+10*B*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^... \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)/(b\*tan(d\*x + c) + a)^(5/2), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 35726 vs. 2(158) = 316.

time = 140.90, size = 35726, normalized size = 190.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$-1/12*(12*\sqrt{2})*((a^{18}b + a^{16}b^3 - 20a^{14}b^5 - 84a^{12}b^7 - 154a^{10}b^9 - 154a^8b^{11} - 84a^6b^{13} - 20a^4b^{15} + a^2b^{17} + b^{19})d^5*\cos(d*x + c)^4 + 2*(3a^{16}b^3 + 20a^{14}b^5 + 56a^{12}b^7 + 84a^{10}b^9 + 70a^8b^{11} + 28a^6b^{13} - 4a^2b^{17} - b^{19})d^5*\cos(d*x + c)^2 + (a^{14}b^5 + 7a^{12}b^7 + 21a^{10}b^9 + 35a^8b^{11} + 35a^6b^{13} + 21a^4b^{15} + 7a^2b^{17} + b^{19})d^5 + 4*((a^{17}b^2 + 6a^{15}b^4 + 14a^{13}b^6 + 14a^{11}b^8 - 14a^7b^{12} - 14a^5b^{14} - 6a^3b^{16} - ab^{18})d^5*\cos(d*x + c)^3 + (a^{15}b^4 + 7a^{13}b^6 + 21a^{11}b^8 + 35a^9b^{10} + 35a^7b^{12} + 21a^5b^{14} + 7a^3b^{16} + ab^{18})d^5*\cos(d*x + c))*\sin(d*x + c))*\sqrt{((A^4 + 2A^2B^2 + B^4)*a^{10} + 5*(A^4 + 2A^2B^2 + B^4)*a^8b^2 + 10*(A^4 + 2A^2B^2 + B^4)*a^6b^4 + 10*(A^4 + 2A^2B^2 + B^4)*a^4b^6 + 5*(A^4 + 2A^2B^2 + B^4)*a^2b^8 + (A^4 + 2A^2B^2 + B^4)*b^{10} - (10ABa^{14}b + 30ABa^{12}b^3 + 2ABa^{10}b^5 - 90ABa^8b^7 - 130ABa^6b^9 - 70ABa^4b^{11} - 10ABa^2b^{13} + 2ABb^{15} + (A^2 - B^2)*a^{15} - 5*(A^2 - B^2)*a^{13}b^2 - 35*(A^2 - B^2)*a^{11}b^4 - 65*(A^2 - B^2)*a^9b^6 - 45*(A^2 - B^2)*a^7b^8 + (A^2 - B^2)*a^5b^{10} + 15*(A^2 - B^2)*a^3b^{12} + 5*(A^2 - B^2)*ab^{14})d^2*\sqrt{((A^4 + 2A^2B^2 + B^4)/((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})d^4)))/(4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)*ab^9 + (A^4 - 2A^2B^2 + B^4)*b^{10})*\sqrt{((4A^2B^2a^{10} - 20(A^3B - AB^3)a^9b + 5(5A^4 - 26A^2B^2 + 5B^4)a^8b^2 + 240(A^3B - AB^3)a^7b^3 - 20(5A^4 - 32A^2B^2 + 5B^4)a^6b^4 - 504(A^3B - AB^3)a^5b^5 + 10(11A^4 - 62A^2B^2 + 11B^4)a^4b^6 + 240(A^3B - AB^3)a^3b^7 - 20(A^4 - 7A^2B^2 + B^4)a^2b^8 - 20(A^3B - AB^3)*ab^9 + (A^4 - 2A^2B^2 + B^4)*b^{10})/((a^2 + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20})d^4))*((A^4 + 2A^2B^2 + B^4)/((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})d^4))^(3/4)*\arctan(((2*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{21} - 5*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{20}b - 4*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{19}b^2 - 30*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{18}b^3 - 94*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{17}b^4 - 61*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16}b^5 - 368*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b^6 - 8*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{14}b^7 - 700*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^8 + 182*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^9 - 14*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{11}b^{10} - 140*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{10}b^{11} - 70*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^9b^{12} - 700*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^8b^{13} - 350*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^7b^{14} - 350*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^6b^{15} - 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^5b^{16} - 175*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^4b^{17} - 875*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^3b^{18} - 875*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^2b^{19} - 875*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)ab^{20} - 875*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)b^{21})/((A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{20} + 8*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{19}b + 35*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{18}b^2 + 280*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{17}b^3 + 105*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16}b^4 + 700*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b^5 + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{14}b^6 + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^7 + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^8 + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^9 + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^{10} + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^{11} + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^{12} + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^{13} + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^{14} + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^{15} + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^{16} + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{17} + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^2b^{18} + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)ab^{19} + 175*(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{20} + 1050*(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7))$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 22.34, size = 2500, normalized size = 13.30
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] (log((((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 160
0*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3
*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4
*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*tan(c + d*x))^(1/2
)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 1440*B^
2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^2*a^16
*b^2*d^3) + (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^
4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*
B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(896*B*a^6*b^15*d^4
- 32*B*b^21*d^4 - 160*B*a^2*b^19*d^4 - 128*B*a^4*b^17*d^4 - (((320*B^4*a^
2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 -
400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a
*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^
4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 6
40*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^
5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^
17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 3136*B*a^8*b^13*d^4 +
4928*B*a^10*b^11*d^4 + 4480*B*a^12*b^9*d^4 + 2432*B*a^14*b^7*d^4 + 736*B*a^
16*b^5*d^4 + 96*B*a^18*b^3*d^4))/4 - 96*B^3*a^3*b^13*d^2 - 240*B^3*a^5
*b^11*d^2 - 320*B^3*a^7*b^9*d^2 - 240*B^3*a^9*b^7*d^2 - 96*B^3*a^11*b^5*d^2
- 16*B^3*a^13*b^3*d^2 - 16*B^3*a*b^15*d^2)*(((320*B^4*a^2*b^8*d^4 - 16*B^4
*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d
^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^
4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2
*d^4))^(1/2))/4 + (log((((-(320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4
*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^
5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2
*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*
tan(c + d*x))^(1/2)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*
b^12*d^3 + 1440*B^2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6
```

$$\begin{aligned}
& *d^3 - 16B^2a^{16}b^2d^3) + ((-((320B^4a^2b^8d^4 - 16B^4b^{10}d^4 - \\
& 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} + \\
& 4B^2a^5d^2 - 40B^2a^3b^2d^2 + 20B^2a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 \\
& + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)} * \\
& (896B^2a^6b^{15}d^4 - 32B^2b^{21}d^4 - 160B^2a^2b^{19}d^4 - 128B^2a^4b^{17}d^4 \\
& - ((-((320B^4a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 16 \\
& 00B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} + 4B^2a^5d^2 - 40B^2a^3 \\
& b^2d^2 + 20B^2a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4 \\
& b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)} * (a + b \tan(c + d*x))^{(1/2)} \\
& ) * (64a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7b^{16}d^5 \\
& + 13440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 13440a^{13}b^{10}d^5 + 7680a^{15} \\
& b^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 + 64a^{21}b^2d^5))/4 + 3 \\
& 136B^2a^8b^{13}d^4 + 4928B^2a^{10}b^{11}d^4 + 4480B^2a^{12}b^9d^4 + 2432B^2a^{14} \\
& b^7d^4 + 736B^2a^{16}b^5d^4 + 96B^2a^{18}b^3d^4))/4)/4 - 96B^3a^3b^{13} \\
& d^2 - 240B^3a^5b^{11}d^2 - 320B^3a^7b^9d^2 - 240B^3a^9b^7d^2 - \\
& 96B^3a^{11}b^5d^2 - 16B^3a^{13}b^3d^2 - 16B^3a^2b^{15}d^2) * (-((320B^4 \\
& a^2b^8d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 - \\
& 400B^4a^8b^2d^4)^{(1/2)} + 4B^2a^5d^2 - 40B^2a^3b^2d^2 + 20B^2 \\
& a^2b^4d^2)/(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6 \\
& b^4d^4 + 5a^8b^2d^4))^{(1/2)})/4 - \log(- ((320B^4a^2b^8d^4 - 16B^4 \\
& b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} - \\
& 4B^2a^5d^2 + 40B^2a^3b^2d^2 - 20B^2a^2b^4d^2)/(16a^{10} \\
& d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160a^6b^4d^4 + 8 \\
& 0a^8b^2d^4))^{(1/2)} * ((a + b \tan(c + d*x))^{(1/2)} * (320B^2a^4b^{14}d^3 - 1 \\
& 6B^2b^{18}d^3 + 1024B^2a^6b^{12}d^3 + 1440B^2a^8b^{10}d^3 + 1024B^2a^{10} \\
& b^8d^3 + 320B^2a^{12}b^6d^3 - 16B^2a^{16}b^2d^3) - (((320B^4a^2b^8 \\
& d^4 - 16B^4b^{10}d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 - 4 \\
& 00B^4a^8b^2d^4)^{(1/2)} - 4B^2a^5d^2 + 40B^2a^3b^2d^2 - 20B^2a^2b^4 \\
& d^2)/(16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160 \\
& a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)} * (((320B^4a^2b^8d^4 - 16B^4b^{10} \\
& d^4 - 1760B^4a^4b^6d^4 + 1600B^4a^6b^4d^4 - 400B^4a^8b^2d^4)^{(1/2)} - \\
& 4B^2a^5d^2 + 40B^2a^3b^2d^2 - 20B^2a^2b^4d^2)/(16a^{10}d^4 \\
& + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160a^6b^4d^4 + 80a^8 \\
& b^2d^4))^{(1/2)} * (a + b \tan(c + d*x))^{(1/2)} * (64a^2b^{22}d^5 + 640a^3 \\
& b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7b^{16}d^5 + 13440a^9b^{14}d^5 + 16128a^{11} \\
& b^{12}d^5 + 13440a^{13}b^{10}d^5 + 7680a^{15}b^8d^5 + \dots
\end{aligned}$$

$$3.361 \quad \int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=185

$$\frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(iA-B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{2(Ab-B^2)}{3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}}$$

[Out]  $-(I*A+B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})/(a-I*b)^{(5/2)/d+(I*A-B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})/(a+I*b)^{(5/2)/d-2*(2*A*a-b-B*a^2+B*b^2)/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)-2/3*(A*b-B*a)/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}$

**Rubi** [A]

time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2(Ab-aB)}{3d(a^2+b^2)(a+b \tan(c+dx))^{3/2}} - \frac{2(a^2(-B)+2aAb+b^2B)}{d(a^2+b^2)^2 \sqrt{a+b \tan(c+dx)}} - \frac{(B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}} + \frac{(-B+iA) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(a+b*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $-\left(\left(\left(I*A+B\right)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]}{\operatorname{Sqrt}\left[a-I*b\right]}\right]\right)/\left(\left(a-I*b\right)^{\left(5/2\right)*d}\right)\right)+\left(\left(\left(I*A-B\right)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]}{\operatorname{Sqrt}\left[a+I*b\right]}\right]\right)/\left(\left(a+I*b\right)^{\left(5/2\right)*d}\right)\right)-\left(\frac{2*\left(A*b-a*B\right)}{3*\left(a^2+b^2\right)*d*\left(a+b*\operatorname{Tan}\left[c+d*x\right]\right)^{\left(3/2\right)}\right)-\left(\frac{2*\left(2*a*A*b-a^2*B+b^2*B\right)}{\left(a^2+b^2\right)^2*d*\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]}\right)$

Rule 65

$\operatorname{Int}\left[\left(\left(a_{.}\right)+\left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right)+\left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)},x_{\text{Symbol}}\right]:>\operatorname{With}\left[\left\{p=\operatorname{Denominator}\left[m\right]\right\},\operatorname{Dist}\left[p/b,\operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(p*\left(m+1\right)-1\right)}*\left(c-a*\left(d/b\right)+d*\left(x^{p/b}\right)^n,x\right],x,\left(a+b*x\right)^{\left(1/p\right)},x\right]\right]/;\operatorname{FreeQ}\left[\left\{a,b,c,d\right\},x\right]\ \&\&\ \operatorname{NeQ}\left[b*c-a*d,0\right]\ \&\&\ \operatorname{LtQ}\left[-1,m,0\right]\ \&\&\ \operatorname{LeQ}\left[-1,n,0\right]\ \&\&\ \operatorname{LeQ}\left[\operatorname{Denominator}\left[n\right],\operatorname{Denominator}\left[m\right]\right]\ \&\&\ \operatorname{IntLinearQ}\left[a,b,c,d,m,n,x\right]$

Rule 214

$\operatorname{Int}\left[\left(\left(a_{.}\right)+\left(b_{.}\right)*\left(x_{.}\right)^2\right)^{-1},x_{\text{Symbol}}\right]:>\operatorname{Simp}\left[\operatorname{Rt}\left[-a/b,2\right]/a*\operatorname{ArcTanh}\left[x/\operatorname{Rt}\left[-a/b,2\right]\right],x\right]/;\operatorname{FreeQ}\left[\left\{a,b\right\},x\right]\ \&\&\ \operatorname{NegQ}\left[a/b\right]$

Rule 3610

$\operatorname{Int}\left[\left(\left(a_{.}\right)+\left(b_{.}\right)*\operatorname{tan}\left[\left(e_{.}\right)+\left(f_{.}\right)*\left(x_{.}\right)\right]\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right)+\left(d_{.}\right)*\operatorname{tan}\left[\left(e_{.}\right)+\left(f_{.}\right)*\left(x_{.}\right)\right]\right),x_{\text{Symbol}}\right]:>\operatorname{Simp}\left[\left(b*c-a*d\right)*\left(a+b*\operatorname{Tan}\left[e+f*x\right]\right)^{\left(m+1\right)}/\right]$

$(f*(m + 1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m + 1}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

### Rule 3618

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(c + d*\text{tan}[e + f*x]), x\_Symbol] :> \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(c + d*\text{tan}[e + f*x]), x\_Symbol] :> \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{IntegerQ}[m]$

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx &= -\frac{2(Ab - aB)}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx}{a^2 + b^2} \\ &= -\frac{2(Ab - aB)}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx}{a^2 + b^2} \\ &= -\frac{2(Ab - aB)}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx}{a^2 + b^2} \\ &= -\frac{2(Ab - aB)}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} - \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx}{a^2 + b^2} \\ &= -\frac{2(Ab - aB)}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} - \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} + \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx}{a^2 + b^2} \\ &= -\frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2}d} + \frac{(iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2}d} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.



time = 0.14, size = 115, normalized size = 0.62

$$\frac{i \left( -\frac{(A-iB) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{a+b \tan(c+dx)}{a-ib}\right)}{a-ib} + \frac{(A+iB) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{a+b \tan(c+dx)}{a+ib}\right)}{a+ib} \right)}{3d(a+b \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((-1/3\*I)\*(-((A - I\*B)\*Hypergeometric2F1[-3/2, 1, -1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)])/(a - I\*b)) + ((A + I\*B)\*Hypergeometric2F1[-3/2, 1, -1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)])/(a + I\*b))/(d\*(a + b\*Tan[c + d\*x])^(3/2))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 3235 vs. 2(161) = 322.

time = 0.13, size = 3236, normalized size = 17.49

method	result	size
derivativedivides	Expression too large to display	3236
default	Expression too large to display	3236

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, method=\_RETURNVERBOSE)

[Out] 1/d\*(-2/3\*(A\*b-B\*a)/(a^2+b^2)/(a+b\*tan(d\*x+c))^(3/2)-2\*(2\*A\*a\*b-B\*a^2+B\*b^2)/(a^2+b^2)^2/(a+b\*tan(d\*x+c))^(1/2)+2/(a^2+b^2)^2\*(1/4/b/(5\*a^4-10\*a^2\*b^2+b^4)/(a^2+b^2)^(3/2)\*(1/2\*(3\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^6+5\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^4\*b^2+A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^2\*b^4-A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*b^6+2\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^8-18\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^6\*b^2-10\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^4\*b^4+10\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^2\*b^6-5\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^9+20\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^7\*b^2-6\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^5\*b^4-28\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^3\*b^6+3\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b^8+10\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^7\*b-10\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5\*b^3-18\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^5+2\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^7-15\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^8\*b+20\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6\*b^3+22\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^5-12\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^7+B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^9)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))+2\*(30\*A\*a^8\*b^2-40\*A\*a^6\*b^4-44\*A\*a^4\*b^6+24\*A\*a^2\*b^8-2\*A\*b^10-10\*B\*a^9\*b+40\*B\*a^7\*b^3-12\*B\*a^5\*b^5-56\*B\*a^3\*b^7+6\*B\*a\*b^9-1/2\*(3\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(3/2)\*a^6+5\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)



$2+b^2)^{(1/2)+2*a)^{(1/2)*(a^2+b^2)^{(1/2)*a*b^7-1\dots}$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 35653 vs.  $2(156) = 312$ .

time = 151.70, size = 35653, normalized size = 192.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/12*(12*\sqrt{2})*((a^{18} + a^{16}b^2 - 20a^{14}b^4 - 84a^{12}b^6 - 154a^{10}b^8 - 154a^8b^{10} - 84a^6b^{12} - 20a^4b^{14} + a^2b^{16} + b^{18})*d^5*\cos(d \\ & *x + c)^4 + 2*(3a^{16}b^2 + 20a^{14}b^4 + 56a^{12}b^6 + 84a^{10}b^8 + 70a^8b^{10} + 28a^6b^{12} - 4a^2b^{16} - b^{18})*d^5*\cos(d*x + c)^2 + (a^{14}b^4 + \\ & 7a^{12}b^6 + 21a^{10}b^8 + 35a^8b^{10} + 35a^6b^{12} + 21a^4b^{14} + 7a^2b^{16} + b^{18})*d^5 + 4*((a^{17}b + 6a^{15}b^3 + 14a^{13}b^5 + 14a^{11}b^7 - 14 \\ & *a^7b^{11} - 14a^5b^{13} - 6a^3b^{15} - a*b^{17})*d^5*\cos(d*x + c)^3 + (a^{15}b^3 + 7a^{13}b^5 + 21a^{11}b^7 + 35a^9b^9 + 35a^7b^{11} + 21a^5b^{13} + 7 \\ & a^3b^{15} + a*b^{17})*d^5*\cos(d*x + c))*\sin(d*x + c))*\sqrt{((A^4 + 2A^2B^2 + B^4)*a^{10} + 5*(A^4 + 2A^2B^2 + B^4)*a^8b^2 + 10*(A^4 + 2A^2B^2 + B^4) \\ & *a^6b^4 + 10*(A^4 + 2A^2B^2 + B^4)*a^4b^6 + 5*(A^4 + 2A^2B^2 + B^4)*a^2b^8 + (A^4 + 2A^2B^2 + B^4)*b^{10} + (10A*B*a^{14}b + 30A*B*a^{12}b^3 + \\ & 2A*B*a^{10}b^5 - 90A*B*a^8b^7 - 130A*B*a^6b^9 - 70A*B*a^4b^{11} - 10A*B*a^2b^{13} + 2A*B*b^{15} + (A^2 - B^2)*a^{15} - 5*(A^2 - B^2)*a^{13}b^2 - 35*(A \\ & ^2 - B^2)*a^{11}b^4 - 65*(A^2 - B^2)*a^9b^6 - 45*(A^2 - B^2)*a^7b^8 + (A^2 - B^2)*a^5b^{10} + 15*(A^2 - B^2)*a^3b^{12} + 5*(A^2 - B^2)*a*b^{14})*d^2*\sqrt{ \\ & ((A^4 + 2A^2B^2 + B^4)/((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10})*d^4)))/(4A^2B^2a^{10} - 20*(A^3B - AB^3)*a^9b + 5*(5A^4 \\ & - 26A^2B^2 + 5B^4)*a^8b^2 + 240*(A^3B - AB^3)*a^7b^3 - 20*(5A^4 - 32A^2B^2 + 5B^4)*a^6b^4 - 504*(A^3B - AB^3)*a^5b^5 + 10*(11A^4 - 62 \\ & *A^2B^2 + 11B^4)*a^4b^6 + 240*(A^3B - AB^3)*a^3b^7 - 20*(A^4 - 7A^2B^2 + B^4)*a^2b^8 - 20*(A^3B - AB^3)*a*b^9 + (A^4 - 2A^2B^2 + B^4)*b^{11} \end{aligned}$$

$$\begin{aligned}
& 0)) * \text{sqrt}((4*A^2*B^2*a^{10} - 20*(A^3*B - A*B^3)*a^9*b + 5*(5*A^4 - 26*A^2*B^2 \\
& + 5*B^4)*a^8*b^2 + 240*(A^3*B - A*B^3)*a^7*b^3 - 20*(5*A^4 - 32*A^2*B^2 + \\
& 5*B^4)*a^6*b^4 - 504*(A^3*B - A*B^3)*a^5*b^5 + 10*(11*A^4 - 62*A^2*B^2 + 11 \\
& *B^4)*a^4*b^6 + 240*(A^3*B - A*B^3)*a^3*b^7 - 20*(A^4 - 7*A^2*B^2 + B^4)*a^2 \\
& *b^8 - 20*(A^3*B - A*B^3)*a*b^9 + (A^4 - 2*A^2*B^2 + B^4)*b^{10}) / ((a^{20} + 1 \\
& 0*a^{18}*b^2 + 45*a^{16}*b^4 + 120*a^{14}*b^6 + 210*a^{12}*b^8 + 252*a^{10}*b^{10} + 21 \\
& 0*a^8*b^{12} + 120*a^6*b^{14} + 45*a^4*b^{16} + 10*a^2*b^{18} + b^{20}) * d^4)) * ((A^4 + \\
& 2*A^2*B^2 + B^4) / ((a^{10} + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 \\
& + b^{10}) * d^4))^{(3/4)} * \arctan(-((2*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{2 \\
& 1 - 5*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{20}*b - 4*(A^7*B + 3*A^5*B^3 + 3 \\
& *A^3*B^5 + A*B^7)*a^{19}*b^2 - 30*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{18}*b^3 \\
& - 94*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{17}*b^4 - 61*(A^8 + 2*A^6*B^2 \\
& - 2*A^2*B^6 - B^8)*a^{16}*b^5 - 368*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7) \\
& ) * a^{15}*b^6 - 8*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{14}*b^7 - 700*(A^7*B + \\
& 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13}*b^8 + 182*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 \\
& - B^8)*a^{12}*b^9 - 728*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^{10} + 3 \\
& 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^{11} - 364*(A^7*B + 3*A^5*B^3 + \\
& 3*A^3*B^5 + A*B^7)*a^9*b^{12} + 350*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8* \\
& b^{13} + 16*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^{14} + 184*(A^8 + 2*A \\
& ^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^{15} + 122*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A \\
& *B^7)*a^5*b^{16} + 47*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^{17} + 60*(A^7* \\
& B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{18} + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 \\
& - B^8)*a^2*b^{19} + 10*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{20} - (A^ \\
& 8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{21}) * d^4 * \text{sqrt}((4*A^2*B^2*a^{10} - 20*(A^3*B \\
& - A*B^3)*a^9*b + 5*(5*A^4 - 26*A^2*B^2 + 5*B^4)*a^8*b^2 + 240*(A^3*B - A*B \\
& ^3)*a^7*b^3 - 20*(5*A^4 - 32*A^2*B^2 + 5*B^4)*a^6*b^4 - 504*(A^3*B - A*B^3) \\
& *a^5*b^5 + 10*(11*A^4 - 62*A^2*B^2 + 11*B^4)*a^4*b^6 + 240*(A^3*B - A*B^3)* \\
& a^3*b^7 - 20*(A^4 - 7*A^2*B^2 + B^4)*a^2*b^8 - 20*(A^3*B - A*B^3)*a*b^9 + ( \\
& A^4 - 2*A^2*B^2 + B^4)*b^{10}) / ((a^{20} + 10*a^{18}*b^2 + 45*a^{16}*b^4 + 120*a^{14}* \\
& b^6 + 210*a^{12}*b^8 + 252*a^{10}*b^{10} + 210*a^8*b^{12} + 120*a^6*b^{14} + 45*a^4*b \\
& ^{16} + 10*a^2*b^{18} + b^{20}) * d^4)) * \text{sqrt}((A^4 + 2*A^2*B^2 + B^4) / ((a^{10} + 5*a^8 \\
& *b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^{10}) * d^4)) + (2*(A^9*B + 4*A^ \\
& 7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^{16} - 5*(A^{10} + 3*A^8*B^2 + 2*A^6*B \\
& ^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^{15}*b - 10*(A^9*B + 4*A^7*B^3 + 6*A^5*B \\
& ^5 + 4*A^3*B^7 + A*B^9)*a^{14}*b^2 - 15*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4 \\
& *B^6 - 3*A^2*B^8 - B^{10})*a^{13}*b^3 - 70*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A \\
& ^3*B^7 + A*B^9)*a^{12}*b^4 - (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^ \\
& 2*B^8 - B^{10})*a^{11}*b^5 - 130*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A \\
& *B^9)*a^{10}*b^6 + 45*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - \\
& B^{10})*a^9*b^7 - 90*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^8 \\
& *b^8 + 65*(A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^7 \\
& *b^9 + 2*(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^6*b^{10} + 35* \\
& (A^{10} + 3*A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - B^{10})*a^5*b^{11} + 30 \\
& *(A^9*B + 4*A^7*B^3 + 6*A^5*B^5 + 4*A^3*B^7 + A*B^9)*a^4*b^{12} + 5*(A^{10} + 3 \\
& *A^8*B^2 + 2*A^6*B^4 - 2*A^4*B^6 - 3*A^2*B^8 - \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac [F(-1)] Timed out**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 22.95, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out]  $(\log(\frac{((a + b \tan(c + dx))^{1/2} (320A^2a^4b^{14}d^3 - 16A^2b^{18}d^3 + 1024A^2a^6b^{12}d^3 + 1440A^2a^8b^{10}d^3 + 1024A^2a^{10}b^8d^3 + 320A^2a^{12}b^6d^3 - 16A^2a^{16}b^2d^3) - (((320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4)^{1/2} - 4A^2a^5d^2 + 40A^2a^3b^2d^2 - 20A^2ab^4d^2)}{(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4)})^{1/2} \cdot (((320A^4a^2b^8d^4 - 16A^4b^{10}d^4 - 1760A^4a^4b^6d^4 + 1600A^4a^6b^4d^4 - 400A^4a^8b^2d^4)^{1/2} - 4A^2a^5d^2 + 40A^2a^3b^2d^2 - 20A^2ab^4d^2)}{(a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4)})^{1/2} \cdot (a + b \tan(c + dx))^{1/2} \cdot (64a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7b^{16}d^5 + 13440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 13440a^{13}b^{10}d^5 + 7680a^{15}b^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 + 64a^{21}b^2d^5))}{4} - 32A^2b^{21}d^4 - 160A^2a^2b^{19}d^4 - 128A^2a^4b^{17}d^4 + 896A^2a^6b^{15}d^4 + 3136A^2a^8b^{13}d^4 + 4928A^2a^{10}b^{11}d^4 + 4480A^2a^{12}b^9d^4 + 2432A^2a^{14}b^7d^4 + 736A^2a^{16}b^5d^4 + 96A^2a^{18}b^3d^4))}{4} \cdot (((320$

$$\begin{aligned}
& *A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)^{(1/2))/4 + 96*A^3*a^3*b^13*d^2 + 240*A^3*a^5*b^11*d^2 + 320*A^3*a^7*b^9*d^2 + 240*A^3*a^9*b^7*d^2 + 96*A^3*a^11*b^5*d^2 + 16*A^3*a^13*b^3*d^2 + 16*A^3*a*b^15*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)^{(1/2))/4 + (\log((((a + b*\tan(c + d*x))^{(1/2)}*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) - (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)^{(1/2)}*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)^{(1/2)}*((a + b*\tan(c + d*x))^{(1/2)}*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^4 + 896*A*a^6*b^15*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3*d^4))/4)*(-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)^{(1/2))/4 + 96*A^3*a^3*b^13*d^2 + 240*A^3*a^5*b^11*d^2 + 320*A^3*a^7*b^9*d^2 + 240*A^3*a^9*b^7*d^2 + 96*A^3*a^11*b^5*d^2 + 16*A^3*a^13*b^3*d^2 + 16*A^3*a*b^15*d^2)*(-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)^{(1/2))/4 - \log(96*A^3*a^3*b^13*d^2 - ((a + b*\tan(c + d*x))^{(1/2)}*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3 + 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 + 320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4)^{(1/2)}*(896*A*a^6*b^15*d^4 - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^4 - (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4
\end{aligned}$$

$$\begin{aligned} & (4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4)^{(1/2)} * (a + b*\tan(c \\ & + d*x))^{(1/2)} * (64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680 \\ & *a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}* \\ & d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2 \\ & *d^5) + 3136*A*a^8*b^{13}*d^4 + 4928*A*a^{10}*b^{11}*d^4 + 4480*A*a^{12}*b^9*d^4 + \\ & 2432*A*a^{14}*b^7*d^4 + 736*A*a^{16}*b^5*d^4 + 96*... \end{aligned}$$

$$3.362 \quad \int \frac{\cot(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=224

$$\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d}$$

[Out]  $-2*A*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/a^{(5/2)}/d+(A-I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(5/2)}/d+(A+I*B)*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(5/2)}/d+2*b*(3*A*a^2*b+A*b^3-2*B*a^3)/a^2/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)}+2/3*b*(A*b-B*a)/a/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.64, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3690, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{2A \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2b(Ab-aB)}{3ad(a^2+b^2)(a+b \tan(c+dx))^{3/2}} + \frac{2b(-2a^3B+3a^2Ab+Ab^3)}{a^2d(a^2+b^2)^2\sqrt{a+b \tan(c+dx)}} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}} + \frac{(A+iB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x]*(A+B*\operatorname{Tan}[c+d*x]))/(a+b*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $(-2*A*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(a^{(5/2)*d}) + ((A-I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/((a-I*b)^{(5/2)*d}) + ((A+I*B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/((a+I*b)^{(5/2)*d}) + (2*b*(A*b-a*B))/(3*a*(a^2+b^2)*d*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}) + (2*b*(3*a^2*A*b+A*b^3-2*a^3*B))/(a^2*(a^2+b^2)^2*d*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)}, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] :> \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

**Rule 3618**



```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cot(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx = \frac{2b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2 \int \frac{\cot(c + dx)(\frac{3}{2}A(a^2 + b^2) - \frac{3}{2}a(Ab - aB))}{(a + b \tan(c + dx))^{3/2}} dx}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}}$$

$$= \frac{2b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2Ab + Ab^3 - 2a^3B)}{a^2(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2Ab + Ab^3 - 2a^3B)}{a^2(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2Ab + Ab^3 - 2a^3B)}{a^2(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{2b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2Ab + Ab^3 - 2a^3B)}{a^2(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}}$$

$$= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2b(Ab - aB)}{3a(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}}$$

$$= -\frac{2A \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{(a - ib)^{5/2}d}$$

Mathematica [A]

time = 3.27, size = 242, normalized size = 1.08

$$2 \left( \frac{3A(a^2+b^2) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}} \right)}{a^{3/2}} + \frac{3a(a+ib)(A-iB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}} \right)}{2(a-ib)^{3/2}} + \frac{3a(a-ib)(A+iB) \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right)}{2(a+ib)^{3/2}} + \frac{B(Ab-aB)}{(a+b \tan(c+dx))^{3/2}} + \frac{3b(3a^2Ab+Ab^3-2a^2B)}{a(a^2+b^2) \sqrt{a+b \tan(c+dx)}} \right) \frac{1}{3a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] (2\*((-3\*A\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]])/a^(3/2) + (3\*a\*(a + I\*b)\*(A - I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/(2\*(a - I\*b)^(3/2)) + (3\*a\*(a - I\*b)\*(A + I\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/(2\*(a + I\*b)^(3/2)) + (b\*(A\*b - a\*B))/(a + b\*Tan[c + d\*x])^(3/2) + (3\*b\*(3\*a^2\*A\*b + A\*b^3 - 2\*a^3\*B))/(a\*(a^2 + b^2)\*Sqrt[a + b\*Tan[c + d\*x]])))/(3\*a\*(a^2 + b^2)\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 5.10, size = 185603, normalized size = 828.58

method	result	size
default	Expression too large to display	185603

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)/(b\*tan(d\*x + c) + a)^(5/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 15.74, size = 2500, normalized size = 11.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] atan(-((( -(((8\*A^2\*a^5\*d^2 - 8\*B^2\*a^5\*d^2 - 80\*A^2\*a^3\*b^2\*d^2 + 80\*B^2\*a^3\*b^2\*d^2 + 16\*A\*B\*b^5\*d^2 + 40\*A^2\*a\*b^4\*d^2 - 40\*B^2\*a\*b^4\*d^2 - 160\*A\*B\*a^2\*b^3\*d^2 + 80\*A\*B\*a^4\*b\*d^2)^2/4 - (A^4 + 2\*A^2\*B^2 + B^4)\*(16\*a^10\*d^4 + 16\*b^10\*d^4 + 80\*a^2\*b^8\*d^4 + 160\*a^4\*b^6\*d^4 + 160\*a^6\*b^4\*d^4 + 80\*a^8\*b^2\*d^4)))^(1/2) - 4\*A^2\*a^5\*d^2 + 4\*B^2\*a^5\*d^2 + 40\*A^2\*a^3\*b^2\*d^2 - 40\*B^2\*a^3\*b^2\*d^2 - 8\*A\*B\*b^5\*d^2 - 20\*A^2\*a\*b^4\*d^2 + 20\*B^2\*a\*b^4\*d^2 + 80\*A\*B\*a^2\*b^3\*d^2 - 40\*A\*B\*a^4\*b\*d^2)/(16\*(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4)))^(1/2)\*((a + b\*tan(c + d\*x))^(1/2)\*(256\*A^2\*a^15\*b^44\*d^7 + 4608\*A^2\*a^17\*b^42\*d^7 + 40512\*A^2\*a^19\*b^40\*d^7 + 224768\*A^2\*a^21\*b^38\*d^7 + 864768\*A^2\*a^23\*b^36\*d^7 + 2419200\*A^2\*a^25\*b^34\*d^7 + 5055232\*A^2\*a^27\*b^32\*d^7 + 8007168\*A^2\*a^29\*b^30\*d^7

$$\begin{aligned}
& 7 + 9664512*A^2*a^{31}*b^{28}*d^7 + 8859136*A^2*a^{33}*b^{26}*d^7 + 6095232*A^2*a^{35}*b^{24}*d^7 + 3095040*A^2*a^{37}*b^{22}*d^7 + 1164800*A^2*a^{39}*b^{20}*d^7 + 376320 \\
& *A^2*a^{41}*b^{18}*d^7 + 154368*A^2*a^{43}*b^{16}*d^7 + 76288*A^2*a^{45}*b^{14}*d^7 + 2 \\
& 8416*A^2*a^{47}*b^{12}*d^7 + 6144*A^2*a^{49}*b^{10}*d^7 + 576*A^2*a^{51}*b^8*d^7 - 13 \\
& 44*B^2*a^{19}*b^{40}*d^7 - 15872*B^2*a^{21}*b^{38}*d^7 - 81408*B^2*a^{23}*b^{36}*d^7 - \\
& 225792*B^2*a^{25}*b^{34}*d^7 - 302848*B^2*a^{27}*b^{32}*d^7 + 139776*B^2*a^{29}*b^{30}* \\
& d^7 + 1537536*B^2*a^{31}*b^{28}*d^7 + 3587584*B^2*a^{33}*b^{26}*d^7 + 5106816*B^2*a \\
& ^{35}*b^{24}*d^7 + 5051904*B^2*a^{37}*b^{22}*d^7 + 3587584*B^2*a^{39}*b^{20}*d^7 + 1817 \\
& 088*B^2*a^{41}*b^{18}*d^7 + 628992*B^2*a^{43}*b^{16}*d^7 + 132608*B^2*a^{45}*b^{14}*d^7 \\
& + 10752*B^2*a^{47}*b^{12}*d^7 - 1536*B^2*a^{49}*b^{10}*d^7 - 320*B^2*a^{51}*b^8*d^7 \\
& + 512*A*B*a^{18}*b^{41}*d^7 + 1536*A*B*a^{20}*b^{39}*d^7 - 29184*A*B*a^{22}*b^{37}*d^7 \\
& - 283136*A*B*a^{24}*b^{35}*d^7 - 1257984*A*B*a^{26}*b^{33}*d^7 - 3494400*A*B*a^{28}*b \\
& ^{31}*d^7 - 6662656*A*B*a^{30}*b^{29}*d^7 - 9005568*A*B*a^{32}*b^{27}*d^7 - 8566272*A \\
& *B*a^{34}*b^{25}*d^7 - 5344768*A*B*a^{36}*b^{23}*d^7 - 1537536*A*B*a^{38}*b^{21}*d^7 + \\
& 698880*A*B*a^{40}*b^{19}*d^7 + 1071616*A*B*a^{42}*b^{17}*d^7 + 612864*A*B*a^{44}*b^{15} \\
& *d^7 + 201216*A*B*a^{46}*b^{13}*d^7 + 37376*A*B*a^{48}*b^{11}*d^7 + 3072*A*B*a^{50}*b \\
& ^9*d^7) + (-(((8*A^2*a^5*d^2 - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^2*d^2 + 80*B^2* \\
& a^3*b^2*d^2 + 16*A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 - 40*B^2*a*b^4*d^2 - 160*A* \\
& B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^10*d^4 \\
& + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a \\
& ^8*b^2*d^4))^(1/2) - 4*A^2*a^5*d^2 + 4*B^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 4 \\
& 0*B^2*a^3*b^2*d^2 - 8*A*B*b^5*d^2 - 20*A^2*a*b^4*d^2 + 20*B^2*a*b^4*d^2 + 8 \\
& 0*A*B*a^2*b^3*d^2 - 40*A*B*a^4*b*d^2)/(16*(a^10*d^4 + b^10*d^4 + 5*a^2*b^8* \\
& d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)))^(1/2)*(512*A*a^16* \\
& b^46*d^8 - (a + b*tan(c + d*x))^(1/2)*(-(((8*A^2*a^5*d^2 - 8*B^2*a^5*d^2 - \\
& 80*A^2*a^3*b^2*d^2 + 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 \\
& - 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/4 - (A^4 + \\
& 2*A^2*B^2 + B^4)*(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6* \\
& d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - 4*A^2*a^5*d^2 + 4*B^2*a^5* \\
& d^2 + 40*A^2*a^3*b^2*d^2 - 40*B^2*a^3*b^2*d^2 - 8*A*B*b^5*d^2 - 20*A^2*a*b^ \\
& 4*d^2 + 20*B^2*a*b^4*d^2 + 80*A*B*a^2*b^3*d^2 - 40*A*B*a^4*b*d^2)/(16*(a^10 \\
& *d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b \\
& ^2*d^4)))^(1/2)*(512*a^18*b^46*d^9 + 9984*a^20*b^44*d^9 + 92160*a^22*b^42*d \\
& ^9 + 535296*a^24*b^40*d^9 + 2193408*a^26*b^38*d^9 + 6736896*a^28*b^36*d^9 + \\
& 16084992*a^30*b^34*d^9 + 30551040*a^32*b^32*d^9 + 46844928*a^34*b^30*d^9 + \\
& 58499584*a^36*b^28*d^9 + 59744256*a^38*b^26*d^9 + 49900032*a^40*b^24*d^9 + \\
& 33945600*a^42*b^22*d^9 + 18643968*a^44*b^20*d^9 + 8146944*a^46*b^18*d^9 + \\
& 2767872*a^48*b^16*d^9 + 705024*a^50*b^14*d^9 + 126720*a^52*b^12*d^9 + 14336 \\
& *a^54*b^10*d^9 + 768*a^56*b^8*d^9) + 9728*A*a^18*b^44*d^8 + 87936*A*a^20*b^ \\
& 42*d^8 + 502144*A*a^22*b^40*d^8 + 2028544*A*a^24*b^38*d^8 + 6153216*A*a^26* \\
& b^36*d^8 + 14518784*A*a^28*b^34*d^8 + 27243008*A*a^30*b^32*d^8 + 41213952*A \\
& *a^32*b^30*d^8 + 50665472*A*a^34*b^28*d^8 + 50775296*A*a^36*b^26*d^8 + 4144 \\
& 3584*A*a^38*b^24*d^8 + 27409408*A*a^40*b^22*d^8 + 14543872*A*a^42*b^20*d^8 \\
& + 6093312*A*a^44*b^18*d^8 + 1966592*A*a^46*b^16*d^8 + 470528*A*a^48*b^14*d^ \\
& 8 + 78336*A*a^50*b^12*d^8 + 8064*A*a^52*b^10*d^8 + 384*A*a^54*b^8*d^8 + 128
\end{aligned}$$

$$\begin{aligned}
& *B*a^{19}*b^{43}*d^8 + 1664*B*a^{21}*b^{41}*d^8 + 9216*B*a^{23}*b^{39}*d^8 + 25600*B*a^{25}*b^{37}*d^8 + 17920*B*a^{27}*b^{35}*d^8 - 139776*B*a^{29}*b^{33}*d^8 - 652288*B*a^{31}*b^{31}*d^8 - 1610752*B*a^{33}*b^{29}*d^8 - 2745600*B*a^{35}*b^{27}*d^8 - 3477760*B*a^{37}*b^{25}*d^8 - 3367936*B*a^{39}*b^{23}*d^8 - 2515968*B*a^{41}*b^{21}*d^8 - 1444352*B*a^{43}*b^{19}*d^8 - 627200*B*a^{45}*b^{17}*d^8 - 199680*B*a^{47}*b^{15}*d^8 - 44032*B*a^{49}*b^{13}*d^8 - 6016*B*a^{51}*b^{11}*d^8 - 384*B*a^{53}*b^9*d^8) * (-(((8*A^2*a^5*d^2 - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^2*d^2 + 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 - 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - 4*A^2*a^5*d^2 + 4*B^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 \dots
\end{aligned}$$

$$3.363 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=289

$$\frac{(5Ab - 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{a^{7/2}d} + \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2}d} - \frac{(iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2}d} - \frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))^{3/2}}$$

[Out] (5\*A\*b-2\*B\*a)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/a^(1/2))/a^(7/2)/d+(I\*A+B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(5/2)/d-(I\*A-B)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(5/2)/d-b\*(A\*a^4+10\*A\*a^2\*b^2+5\*A\*b^4-6\*B\*a^3\*b-2\*B\*a\*b^3)/a^3/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)-1/3\*b\*(3\*A\*a^2+5\*A\*b^2-2\*B\*a\*b)/a^2/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)-A\*cot(d\*x+c)/a/d/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.84, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3690, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(5Ab - 2aB) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}} \right)}{a^{7/2}d} - \frac{b(3a^2A - 2abB + 5Ab^2)}{3a^2d(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} - \frac{b(a^4A - 6a^2bB + 10a^2Ab^2 - 2ab^2B + 5Ab^4)}{a^3d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} + \frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{5/2}} - \frac{(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{5/2}} - \frac{A \cot(c + dx)}{ad(a + b \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((5\*A\*b - 2\*a\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]]/(a^(7/2)\*d) + (I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(5/2)\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(5/2)\*d) - (b\*(3\*a^2\*A + 5\*A\*b^2 - 2\*a\*b\*B))/(3\*a^2\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) - (A\*Cot[c + d\*x])/(a\*d\*(a + b\*Tan[c + d\*x])^(3/2)) - (b\*(a^4\*A + 10\*a^2\*A\*b^2 + 5\*A\*b^4 - 6\*a^3\*b\*B - 2\*a\*b^3\*B))/(a^3\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n +
2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```



```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx &= -\frac{A \cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{\int \frac{\cot(c+dx)(\frac{1}{2}(5Ab-2aB)+aA\tan(c+dx)+\frac{5}{2}Ab)}{(a+b\tan(c+dx))^{5/2}}}{a} \\
&= -\frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} - \frac{A \cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} \\
&= -\frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} - \frac{A \cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} \\
&= -\frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} - \frac{A \cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} \\
&= -\frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} - \frac{A \cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} \\
&= -\frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} - \frac{A \cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} \\
&= \frac{(5Ab-2aB) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d} - \frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)d(a+b\tan(c+dx))^{3/2}} \\
&= \frac{(5Ab-2aB) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d} + \frac{(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 3.27, size = 306, normalized size = 1.06

$$\frac{b(-3a^2A-5Ab^2+2abB)}{(a^2+b^2)(a+b\tan(c+dx))^{3/2}} - \frac{3aA \cot(c+dx)}{(a+b\tan(c+dx))^{3/2}} + \frac{3 \left( \frac{((a^2+b^2)^2(5Ab-2aB) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right))}{\sqrt{a}} + \frac{a^3(a+b)^2(iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{a^3(a-b)^2(-iA+B) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} - \frac{i(a^4A+10a^2A^2+5Ab^4-6a^2bB-2ab^3B)}{\sqrt{a+b\tan(c+dx)}} \right)}{3a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^2\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((b\*(-3\*a^2\*A - 5\*A\*b^2 + 2\*a\*b\*B))/((a^2 + b^2)\*(a + b\*Tan[c + d\*x])^(3/2)) - (3\*a\*A\*Cot[c + d\*x])/(a + b\*Tan[c + d\*x])^(3/2) + (3\*(((a^2 + b^2)^2\*(5\*A\*b - 2\*a\*B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a]])/Sqrt[a] + (a^3\*(a + I\*b)^2\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/Sqrt[a

- I\*b] + (a^3\*(a - I\*b)^2\*((-I)\*A + B)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/Sqrt[a + I\*b] - (b\*(a^4\*A + 10\*a^2\*A\*b^2 + 5\*A\*b^4 - 6\*a^3\*b\*B - 2\*a\*b^3\*B))/Sqrt[a + b\*Tan[c + d\*x]]/(a\*(a^2 + b^2)^2)/(3\*a^2\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 8.96, size = 339366, normalized size = 1174.28

method	result	size
default	Expression too large to display	339366

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x,method=\_RETURNVE  
RBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm  
="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^2/(b\*tan(d\*x + c) + a)^(5/2), x  
)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm  
="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^2(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& b^3 d^2 + 80 A^2 B^2 a^4 b^2 d^2)^{1/2} - (A^4 + 2 A^2 B^2 + B^4) (16 a^{10} d^4 + 16 \\
& * b^{10} d^4 + 80 a^2 b^8 d^4 + 160 a^4 b^6 d^4 + 160 a^6 b^4 d^4 + 80 a^8 b^2 \\
& * d^4)^{1/2} - 4 A^2 a^5 d^2 + 4 B^2 a^5 d^2 + 40 A^2 a^3 b^2 d^2 - 40 B^2 a^3 b^2 d^2 \\
& - 8 A^2 B^2 a^5 d^2 - 20 A^2 a^2 b^4 d^2 + 20 B^2 a^2 b^4 d^2 + 80 A^2 B^2 \\
& a^2 b^3 d^2 - 40 A^2 B^2 a^4 b^2 d^2) / (16 (a^{10} d^4 + b^{10} d^4 + 5 a^2 b^8 d^4 + \\
& 10 a^4 b^6 d^4 + 10 a^6 b^4 d^4 + 5 a^8 b^2 d^4))^{1/2} * (512 a^{27} b^{46} d^9 \\
& + 9984 a^{29} b^{44} d^9 + 92160 a^{31} b^{42} d^9 + 535296 a^{33} b^{40} d^9 + 219340 \\
& 8 a^{35} b^{38} d^9 + 6736896 a^{37} b^{36} d^9 + 16084992 a^{39} b^{34} d^9 + 30551040 \\
& * a^{41} b^{32} d^9 + 46844928 a^{43} b^{30} d^9 + 58499584 a^{45} b^{28} d^9 + 59744256 \\
& * a^{47} b^{26} d^9 + 49900032 a^{49} b^{24} d^9 + 33945600 a^{51} b^{22} d^9 + 18643968 \\
& * a^{53} b^{20} d^9 + 8146944 a^{55} b^{18} d^9 + 2767872 a^{57} b^{16} d^9 + 705024 a^5 \\
& 9 b^{14} d^9 + 126720 a^{61} b^{12} d^9 + 14336 a^{63} b^{10} d^9 + 768 a^{65} b^8 d^9) \\
& + 1280 A^2 a^{24} b^{47} d^8 + 24320 A^2 a^{26} b^{45} d^8 + 219008 A^2 a^{28} b^{43} d^8 + \\
& 1241984 A^2 a^{30} b^{41} d^8 + 4970496 A^2 a^{32} b^{39} d^8 + 14909440 A^2 a^{34} b^{37} d^8 \\
& + 34746880 A^2 a^{36} b^{35} d^8 + 64356864 A^2 a^{38} b^{33} d^8 + 96092672 A^2 a^{40} b^{31} d^8 \\
& + 116633088 A^2 a^{42} b^{29} d^8 + 115498240 A^2 a^{44} b^{27} d^8 + 93267200 A^2 a^{46} b^{25} d^8 \\
& + 61128704 A^2 a^{48} b^{23} d^8 + 32212992 A^2 a^{50} b^{21} d^8 + 13439488 A^2 a^{52} b^{19} d^8 \\
& + 4334080 A^2 a^{54} b^{17} d^8 + 1040640 A^2 a^{56} b^{15} d^8 + 174848 A^2 a^{58} b^{13} d^8 \\
& + 18304 A^2 a^{60} b^{11} d^8 + 896 A^2 a^{62} b^9 d^8 - 512 B^2 a^{25} b^{46} d^8 - 9728 B^2 a^{27} b^{44} d^8 \\
& - 87936 B^2 a^{29} b^{42} d^8 - 502144 B^2 a^{31} b^{40} d^8 - 2028544 B^2 a^{33} b^{38} d^8 \\
& - 6153216 B^2 a^{35} b^{36} d^8 - 14518784 B^2 a^{37} b^{34} d^8 - 27243008 B^2 a^{39} b^{32} d^8 \\
& - 41213952 B^2 a^{41} b^{30} d^8 - 50665472 B^2 a^{43} b^{28} d^8 - 50775296 B^2 a^{45} b^{26} d^8 \\
& - 41443584 B^2 a^{47} b^{24} d^8 - 27409408 B^2 a^{49} b^{22} d^8 - 14543872 B^2 a^{51} b^{20} d^8 \\
& - 6093312 B^2 a^{53} b^{18} d^8 - 1966592 B^2 a^{55} b^{16} d^8 - 470528 B^2 a^{57} b^{14} d^8 - 78336 B^2 a^{59} b^{12} d^8 \\
& - 8064 B^2 a^{61} b^{10} d^8 - 384 B^2 a^{63} b^8 d^8) - (a + b \tan(c + d x))^{1/2} * (1600 A^2 a^{22} b^{46} d^7 \\
& + 28800 A^2 a^{24} b^{44} d^7 + 244800 A^2 a^{26} b^{42} d^7 + 1304256 A^2 a^{28} b^{40} d^7 + 4880128 A^2 a^{30} b^{38} d^7 \\
& + 13627392 A^2 a^{32} b^{36} d^7 + 29476608 A^2 a^{34} b^{34} d^7 + 50615552 A^2 a^{36} b^{32} d^7 \\
& + 70152576 A^2 a^{38} b^{30} d^7 + 79329536 A^2 a^{40} b^{28} d^7 + 73600384 A^2 a^{42} b^{26} d^7 \\
& + 56025216 A^2 a^{44} b^{24} d^7 + 34754304 A^2 a^{46} b^{22} d^7 + 17296384 A^2 a^{48} b^{20} d^7 \\
& + 6713088 A^2 a^{50} b^{18} d^7 + 1934592 A^2 a^{52} b^{16} d^7 + 377408 A^2 a^{54} b^{14} d^7 \\
& + 39552 A^2 a^{56} b^{12} d^7 + 64 A^2 a^{58} b^{10} d^7 - 320 A^2 a^{60} b^8 d^7 + 256 B^2 a^{24} b^{44} d^7 \\
& + 4608 B^2 a^{26} b^{42} d^7 + 40512 B^2 a^{28} b^{40} d^7 + 224768 B^2 a^{30} b^{38} d^7 + 864768 B^2 a^{32} b^{36} d^7 \\
& + 2419200 B^2 a^{34} b^{34} d^7 + 5055232 B^2 a^{36} b^{32} d^7 + 8007168 B^2 a^{38} b^{30} d^7 \\
& + 9664512 B^2 a^{40} b^{28} d^7 + 8859136 B^2 a^{42} b^{26} d^7 + 6095232 B^2 a^{44} b^{24} d^7 \\
& + 3095040 B^2 a^{46} b^{22} d^7 + 1164800 B^2 a^{48} b^{20} d^7 + 376320 B^2 a^{50} b^{18} d^7 \\
& + 154368 B^2 a^{52} b^{16} d^7 + 76288 B^2 a^{54} b^{14} d^7 + 28416 B^2 a^{56} b^{12} d^7 \\
& + 6144 B^2 a^{58} b^{10} d^7 + 576 B^2 a^{60} b^8 d^7 - 1280 A^2 B^2 a^{23} b^{45} d^7 - 23040 A^2 B^2 a^{25} b^{43} d^7 \\
& - 196352 A^2 B^2 a^{27} b^{41} d^7 - 1046016 A^2 B^2 a^{29} b^{39} d^7 - 3887616 A^2 B^2 a^{31} b^{37} d^7 - 10683904 A^2 B^2 a^{33} b^{35} d^7 \\
& - 22503936 A^2 B^2 a^{35} b^{33} d^7 - 37240320 A^2 B^2 a^{37} b^{31} d^7 - 49347584 A^2 B^2 a^{39} b^{29} d^7 \\
& - 53228032 A^2 B^2 a^{41} b^{27} d^7 - 47443968 A^2 B^2 a^{43} b^{25} d^7 - 35389952 A^2 B^2 a^{45} b^{23} d^7 \dots
\end{aligned}$$

**3.364** 
$$\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=364

$$\frac{(8a^2A - 35Ab^2 + 20abB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{9/2}d} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{5/2}d} - \dots (A +$$

[Out]  $1/4*(8*A*a^2-35*A*b^2+20*B*a*b)*\arctanh((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/a^{(9/2)}/d-(A-I*B)*\arctanh((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(5/2)}/d-(A+I*B)*\arctanh((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(5/2)}/d+1/4*b*(11*A*a^4*b+62*A*a^2*b^3+35*A*b^5-4*B*a^5-40*B*a^3*b^2-20*B*a*b^4)/a^4/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)}+1/12*b*(27*A*a^2*b+35*A*b^3-12*B*a^3-20*B*a*b^2)/a^3/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}+1/4*(7*A*b-4*B*a)*\cot(d*x+c)/a^2/d/(a+b*\tan(d*x+c))^{(3/2)}-1/2*A*\cot(d*x+c)^2/a/d/(a+b*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 1.10, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {3690, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$\frac{(7Ab - 4aB) \cot(c + dx)}{4a^2d(a + b \tan(c + dx))^{5/2}} + \frac{(8a^2A + 20abB - 35Ab^2) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a}}\right)}{4a^{9/2}d} + \frac{b(-12a^2B + 27a^2Ab - 20ab^2B + 35Ab^3)}{12a^2d(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{b(-4a^2B + 11a^2Ab - 40a^2B^2 + 62a^2Ab^2 - 20ab^3B + 35Ab^3)}{4a^2d(a^2 + b^2)\sqrt{a + b \tan(c + dx)}} - \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{d(a - ib)^{5/2}} - \frac{(A + iB) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{d(a + ib)^{5/2}} - \frac{A \cot^2(c + dx)}{2ab(a + b \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2), x]`

[Out] `((8*a^2*A - 35*A*b^2 + 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4*a^(9/2)*d) - ((A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((a - I*b)^(5/2)*d) - ((A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/((a + I*b)^(5/2)*d) + (b*(27*a^2*A*b + 35*A*b^3 - 12*a^3*B - 20*a*b^2*B))/(12*a^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((7*A*b - 4*a*B)*Cot[c + d*x])/(4*a^2*d*(a + b*Tan[c + d*x])^(3/2)) - (A*Cot[c + d*x]^2)/(2*a*d*(a + b*Tan[c + d*x])^(3/2)) + (b*(11*a^4*A*b + 62*a^2*A*b^3 + 35*A*b^5 - 4*a^5*B - 40*a^3*b^2*B - 20*a*b^4*B))/(4*a^4*(a^2 + b^2)^2*d*Sqrt[a + b*Tan[c + d*x]])`

**Rule 65**

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

Rule 3620

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e +

```

f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)])], x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps



$$\begin{aligned}
\int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx &= -\frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^{3/2}} - \frac{\int \frac{\cot^2(c+dx)(\frac{1}{2}(7Ab-4aB)+2aA \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}}}{2a} \\
&= \frac{(7Ab-4aB) \cot(c+dx)}{4a^2d(a+b \tan(c+dx))^{3/2}} - \frac{A \cot^2(c+dx)}{2ad(a+b \tan(c+dx))^{3/2}} + \frac{\int \frac{\cot(c+dx)}{(a+b \tan(c+dx))^{3/2}}}{2a} \\
&= \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}} + \frac{(7Ab-4aB) \cot(c+dx)}{4a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}} + \frac{(7Ab-4aB) \cot(c+dx)}{4a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}} + \frac{(7Ab-4aB) \cot(c+dx)}{4a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}} + \frac{(7Ab-4aB) \cot(c+dx)}{4a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}} + \frac{(7Ab-4aB) \cot(c+dx)}{4a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{(8a^2A-35Ab^2+20abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}d} + \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{(8a^2A-35Ab^2+20abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}d} + \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)d(a+b \tan(c+dx))^{3/2}}
\end{aligned}$$

### Mathematica [A]

time = 4.49, size = 373, normalized size = 1.02

$$\frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{a^2(a^2+b^2)(a+b \tan(c+dx))^{3/2}} + \frac{3(7Ab-4aB) \cot(c+dx)}{a(a+b \tan(c+dx))^{3/2}} - \frac{6A \cot^2(c+dx)}{(a+b \tan(c+dx))^{3/2}} + \frac{(8a^2A-35Ab^2+20abB) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^3\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((b\*(27\*a^2\*A\*b + 35\*A\*b^3 - 12\*a^3\*B - 20\*a\*b^2\*B))/(a^2\*(a^2 + b^2)\*(a + b\*Tan[c + d\*x])^(3/2)) + (3\*(7\*A\*b - 4\*a\*B)\*Cot[c + d\*x])/(a\*(a + b\*Tan[c +

$$d*x])^{(3/2)} - (6*A*Cot[c + d*x]^2)/(a + b*Tan[c + d*x])^{(3/2)} + (3*(((a^2 + b^2)^2*(8*a^2*A - 35*A*b^2 + 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] - (4*a^4*(a + I*b)^2*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] - (4*a^4*(a - I*b)^2*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] + (b*(11*a^4*A*b + 62*a^2*A*b^3 + 35*A*b^5 - 4*a^5*B - 40*a^3*b^2*B - 20*a*b^4*B))/Sqrt[a + b*Tan[c + d*x]])))/(a^3*(a^2 + b^2)^2)/(12*a*d)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 9.67, size = 467697, normalized size = 1284.88

method	result	size
default	Expression too large to display	467697

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm  
="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^3(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/(a + b*tan(c + d*x))**(5/2), x)`

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")`

[Out] Timed out

**Mupad** [B]

time = 12.40, size = 2500, normalized size = 6.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)`

[Out] `((a + b*tan(c + d*x))^3*(35*A*b^6 + 62*A*a^2*b^4 + 11*A*a^4*b^2 - 40*B*a^3*b^3 - 20*B*a*b^5 - 4*B*a^5*b))/(4*(a^8 + a^4*b^4 + 2*a^6*b^2)) - ((a + b*tan(c + d*x))^2*(175*A*b^6 + 310*A*a^2*b^4 + 39*A*a^4*b^2 - 208*B*a^3*b^3 - 100*B*a*b^5 - 12*B*a^5*b))/(12*(a^7 + a^3*b^4 + 2*a^5*b^2)) + (2*(A*b^4 - B*a*b^3))/(3*a*(a^2 + b^2)) + (2*(a + b*tan(c + d*x))*(7*A*b^6 + 13*A*a^2*b^4 - 10*B*a^3*b^3 - 4*B*a*b^5))/(3*a^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + b*tan(c + d*x))^(7/2) - 2*a*d*(a + b*tan(c + d*x))^(5/2) + a^2*d*(a + b*tan(c + d*x))^(3/2)) + atan((((a + b*tan(c + d*x))^(1/2))*(321126400*A^4*a^28*b^44*d^5 + 2422210560*A^4*a^30*b^42*d^5 + 1411383296*A^4*a^32*b^40*d^5 - 54989422592*A^4*a^34*b^38*d^5 - 325864914944*A^4*a^36*b^36*d^5 - 1011294928896*A^4*a^38*b^34*d^5 - 2054783238144*A^4*a^40*b^32*d^5 - 2923490705408*A^4*a^42*b^30*d^5 - 2962565365760*A^4*a^44*b^28*d^5 - 2094150975488*A^4*a^46*b^26*d^5 - 943762440192*A^4*a^48*b^24*d^5 - 175655354368*A^4*a^50*b^22*d^5 + 74523344896*A^4*a^52*b^20*d^5 + 62081990656*A^4*a^54*b^18*d^5 + 17307795456*A^4*a^56*b^16*d^5 + 1629487104*A^4*a^58*b^14*d^5 + 44302336*A^4*a^60*b^12*d^5 + 104857600*A^4*a^62*b^10*d^5 + 25165824*A^4*a^64*b^8*d^5 - 104857600*B^4*a^30*b^42*d^5 - 838860800*B^4*a^32*b^40*d^5 - 838860800*B^4*a^34*b^38*d^5 + 17624465408*B^4*a^36*b^36*d^5 + 114621939712*B^4*a^38*b^34*d^5 + 382445027328*B^4*a^40*b^32*d^5 + 842753114112*B^4*a^42*b^30*d^5 + 1327925035008*B^4*a^44*b^28*d^5 + 1546246946816*B^4*a^46*b^26*d^5 + 1344719028224*B^4*a^48*b^24`

$$\begin{aligned}
& *d^5 + 868489363456*B^4*a^50*b^22*d^5 + 406872653824*B^4*a^52*b^20*d^5 + 13 \\
& 1298492416*B^4*a^54*b^18*d^5 + 26013073408*B^4*a^56*b^16*d^5 + 2214592512*B \\
& ^4*a^58*b^14*d^5 - 75497472*B^4*a^60*b^12*d^5 + 12582912*B^4*a^62*b^10*d^5 \\
& + 8388608*B^4*a^64*b^8*d^5 + 367001600*A*B^3*a^29*b^43*d^5 + 2013265920*A*B \\
& ^3*a^31*b^41*d^5 - 8640266240*A*B^3*a^33*b^39*d^5 - 126919639040*A*B^3*a^35 \\
& *b^37*d^5 - 615681884160*A*B^3*a^37*b^35*d^5 - 1778636554240*A*B^3*a^39*b^3 \\
& 3*d^5 - 3473303142400*A*B^3*a^41*b^31*d^5 - 4786288066560*A*B^3*a^43*b^29*d \\
& ^5 - 4657334190080*A*B^3*a^45*b^27*d^5 - 3034914488320*A*B^3*a^47*b^25*d^5 \\
& - 1043626721280*A*B^3*a^49*b^23*d^5 + 175573565440*A*B^3*a^51*b^21*d^5 + 43 \\
& 8933913600*A*B^3*a^53*b^19*d^5 + 258956328960*A*B^3*a^55*b^17*d^5 + 8162115 \\
& 5840*A*B^3*a^57*b^15*d^5 + 13170114560*A*B^3*a^59*b^13*d^5 + 534773760*A*B^ \\
& 3*a^61*b^11*d^5 - 83886080*A*B^3*a^63*b^9*d^5 - 2936012800*A^3*B*a^29*b^43* \\
& d^5 - 35074867200*A^3*B*a^31*b^41*d^5 - 184945737728*A^3*B*a^33*b^39*d^5 - \\
& 550328336384*A^3*B*a^35*b^37*d^5 - 939509415936*A^3*B*a^37*b^35*d^5 - 61584 \\
& 3364864*A^3*B*a^39*b^33*d^5 + 1152106102784*A^3*B*a^41*b^31*d^5 + 388571018 \\
& 0352*A^3*B*a^43*b^29*d^5 + 5769036562432*A^3*B*a^45*b^27*d^5 + 540496678092 \\
& 8*A^3*B*a^47*b^25*d^5 + 3336906473472*A^3*B*a^49*b^23*d^5 + 1259358650368*A \\
& ^3*B*a^51*b^21*d^5 + 183016357888*A^3*B*a^53*b^19*d^5 - 67249373184*A^3*B*a \\
& ^55*b^17*d^5 - 37419483136*A^3*B*a^57*b^15*d^5 - 4628414464*A^3*B*a^59*b^13 \\
& *d^5 + 874512384*A^3*B*a^61*b^11*d^5 + 218103808*A^3*B*a^63*b^9*d^5 - 32112 \\
& 6400*A^2*B^2*a^28*b^44*d^5 + 618659840*A^2*B^2*a^30*b^42*d^5 + 36924555264* \\
& A^2*B^2*a^32*b^40*d^5 + 273177116672*A^2*B^2*a^34*b^38*d^5 + 1058131673088* \\
& A^2*B^2*a^36*b^36*d^5 + 2584175706112*A^2*B^2*a^38*b^34*d^5 + 4186857013248 \\
& *A^2*B^2*a^40*b^32*d^5 + 4360140488704*A^2*B^2*a^42*b^30*d^5 + 225601807974 \\
& 4*A^2*B^2*a^44*b^28*d^5 - 932366516224*A^2*B^2*a^46*b^26*d^5 - 295124441497 \\
& 6*A^2*B^2*a^48*b^24*d^5 - 2844864282624*A^2*B^2*a^50*b^22*d^5 - 15791124643 \\
& 84*A^2*B^2*a^52*b^20*d^5 - 500252540928*A^2*B^2*a^54*b^18*d^5 - 52598669312 \\
& *A^2*B^2*a^56*b^16*d^5 + 21875392512*A^2*B^2*a^58*b^14*d^5 + 8872787968*A^2 \\
& *B^2*a^60*b^12*d^5 + 1023410176*A^2*B^2*a^62*b^10*d^5) + (-(((8*A^2*a^5*d^2 \\
& - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^2*d^2 + 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 \\
& + 40*A^2*a*b^4*d^2 - 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b \\
& *d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8 \\
& *d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - 4*A^2*a \\
& ^5*d^2 + 4*B^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 40*B^2*a^3*b^2*d^2 - 8*A*B*b^ \\
& 5*d^2 - 20*A^2*a*b^4*d^2 + 20*B^2*a*b^4*d^2 + 80*A*B*a^2*b^3*d^2 - 40*A*B*a \\
& ^4*b*d^2)/(16*(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^ \\
& 6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(1926758400*A^3*a^29*b^46*d^6 - ((-(((8* \\
& A^2*a^5*d^2 - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^2*d^2 + 80*B^2*a^3*b^2*d^2 + 16* \\
& A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 - 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 8 \\
& 0*A*B*a^4*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^10*d^4 + 16*b^10*d^4 + \\
& 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) \\
& ) - 4*A^2*a^5*d^2 + 4*B^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 40*B^2*a^3*b^2*d^2 \\
& - 8*A*B*b^5*d^2 - 20*A^2*a*b^4*d^2 + 20*B^2*a*b^4*d^2 + 80*A*B*a^2*b^3*d^2 \\
& - 40*A*B*a^4*b*d^2)/(16*(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6* \\
& d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(22934454272*A*a^66*b^14*d^8
\end{aligned}$$

$$- 587202560*A*a^{32}*b^{48}*d^8 - 11022630912*A*a^{34}*b^{46}*d^8 - 97861500928*A*a^{36}*b^{44}*d^8 - 545947385856*A*a^{38}*b^{42}*d^8 - 2\dots$$

$$3.365 \quad \int \frac{aB + bB \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Optimal. Leaf size=362

$$\frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d}$$

[Out]  $\frac{1}{2} b B \operatorname{arctanh} \left( \frac{(a + (a^2 + b^2)^{1/2})^{1/2} - 2^{1/2} (a + b \tan(dx + c))^{1/2}}{(a - (a^2 + b^2)^{1/2})^{1/2}} \right) / d \cdot 2^{1/2} / (a - (a^2 + b^2)^{1/2})^{1/2} - \frac{1}{2} b B \operatorname{arctanh} \left( \frac{(a + (a^2 + b^2)^{1/2})^{1/2} + 2^{1/2} (a + b \tan(dx + c))^{1/2}}{(a - (a^2 + b^2)^{1/2})^{1/2}} \right) / d \cdot 2^{1/2} / (a - (a^2 + b^2)^{1/2})^{1/2} + \frac{1}{4} b B \ln(a + (a^2 + b^2)^{1/2} - 2^{1/2} (a + (a^2 + b^2)^{1/2})^{1/2} (a + b \tan(dx + c))^{1/2} + b \tan(dx + c)) / d \cdot 2^{1/2} / (a + (a^2 + b^2)^{1/2})^{1/2} - \frac{1}{4} b B \ln(a + (a^2 + b^2)^{1/2} + 2^{1/2} (a + (a^2 + b^2)^{1/2})^{1/2} (a + b \tan(dx + c))^{1/2} + b \tan(dx + c)) / d \cdot 2^{1/2} / (a + (a^2 + b^2)^{1/2})^{1/2}$

Rubi [A]

time = 0.30, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {21, 3566, 714, 1143, 648, 632, 212, 642}

$$\frac{bB \log \left( \frac{-\sqrt{2} \sqrt{a^2 + b^2} + a \sqrt{a + b \tan(c + dx)} + \sqrt{a^2 + b^2} + a + b \tan(c + dx)}{2\sqrt{2} d \sqrt{a^2 + b^2} + a} \right)}{2\sqrt{2} d \sqrt{a^2 + b^2} + a} - \frac{bB \log \left( \frac{\sqrt{2} \sqrt{a^2 + b^2} + a \sqrt{a + b \tan(c + dx)} + \sqrt{a^2 + b^2} + a + b \tan(c + dx)}{2\sqrt{2} d \sqrt{a^2 + b^2} + a} \right)}{2\sqrt{2} d \sqrt{a^2 + b^2} + a} + \frac{bB \tanh^{-1} \left( \frac{\sqrt{a^2 + b^2} + a - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} d \sqrt{a - \sqrt{a^2 + b^2}}} - \frac{bB \tanh^{-1} \left( \frac{\sqrt{a^2 + b^2} + a + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} d \sqrt{a - \sqrt{a^2 + b^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out]  $(b*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]] - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]]*d) - (b*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]] + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a - \operatorname{Sqrt}[a^2 + b^2]]*d) + (b*B*\operatorname{Log}[a + \operatorname{Sqrt}[a^2 + b^2] + b*\operatorname{Tan}[c + d*x] - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*d) - (b*B*\operatorname{Log}[a + \operatorname{Sqrt}[a^2 + b^2] + b*\operatorname{Tan}[c + d*x] + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + \operatorname{Sqrt}[a^2 + b^2]]*d)$

Rule 21

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_.))^(m\_.)\*((c\_.) + (d\_.)\*(v\_.))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 632

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 648

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 714

Int[Sqrt[(d\_) + (e\_)\*(x\_)]/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[2\*e, Subst[Int[x^2/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

Rule 1143

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*r), Int[x^(m - 1)/(q - r\*x + x^2), x], x] - Dist[1/(2\*c\*r), Int[x^(m - 1)/(q + r\*x + x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GeQ[m, 1] && LtQ[m, 3] && NegQ[b^2 - 4\*a\*c]

Rule 3566

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx &= B \int \sqrt{a + b \tan(c + dx)} dx \\
&= \frac{(bB) \text{Subst} \left( \int \frac{\sqrt{a+x}}{b^2+x^2} dx, x, b \tan(c + dx) \right)}{d} \\
&= \frac{(2bB) \text{Subst} \left( \int \frac{x^2}{a^2+b^2-2ax^2+x^4} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{d} \\
&= \frac{(bB) \text{Subst} \left( \int \frac{x}{\sqrt{a^2+b^2}-\sqrt{2} \sqrt{a+\sqrt{a^2+b^2}} x+x^2} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{\sqrt{2} \sqrt{a + \sqrt{a^2+b^2}} d} \\
&= \frac{(bB) \text{Subst} \left( \int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2} \sqrt{a+\sqrt{a^2+b^2}} x+x^2} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{2d} \\
&= \frac{bB \log \left( a + \sqrt{a^2+b^2} + b \tan(c + dx) - \sqrt{2} \sqrt{a + \sqrt{a^2+b^2}} \sqrt{a + b \tan(c + dx)} \right)}{2\sqrt{2} \sqrt{a + \sqrt{a^2+b^2}} d} \\
&= \frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2+b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2+b^2}}} \right)}{\sqrt{2} \sqrt{a - \sqrt{a^2+b^2}} d} - \frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2+b^2}} + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2+b^2}}} \right)}{\sqrt{2} \sqrt{a - \sqrt{a^2+b^2}} d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.08, size = 88, normalized size = 0.24

$$\frac{iB \left( \sqrt{a - ib} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) \right) - \sqrt{a + ib} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((-I)\*B\*(Sqrt[a - I\*b]\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] - Sqrt[a + I\*b]\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]))/d



**Maple [A]**

time = 0.17, size = 364, normalized size = 1.01

method	result
derivativedivides	$\frac{2Bb}{\left( \sqrt{2\sqrt{a^2 + b^2} + 2a} (\sqrt{a^2 + b^2} - a) \right)} \left( \frac{\ln \left( b \tan(dx+c) + a + \sqrt{a + b \tan(dx+c)} \sqrt{2\sqrt{a^2 + b^2}} \right)}{2} \right)$
default	$\frac{2Bb}{\left( \sqrt{2\sqrt{a^2 + b^2} + 2a} (\sqrt{a^2 + b^2} - a) \right)} \left( \frac{\ln \left( b \tan(dx+c) + a + \sqrt{a + b \tan(dx+c)} \sqrt{2\sqrt{a^2 + b^2}} \right)}{2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*B*b*(-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*(1/2*ln
(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b
^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arct
an((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*((a^2+b^2)^(1/2)-a)/b^2*
(1/2*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)-(a^2+b^2)^(1/2))-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan((-2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2))))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 2049 vs.  $2(293) = 586$ .

time = 2.59, size = 2049, normalized size = 5.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*sqrt(B^4*b^2/d^4)*d^4*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*sqrt(B^4*b^2/d^4)*B^3*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4) - sqrt(2)*sqrt(B^4*b^2/d^4)*d^5*sqrt((sqrt(2)*B^3*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4)*cos(d*x + c) + (B^4*a^2*b^2 + B^4*b^4)*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4)*cos(d*x + c) + (B^6*a^3*b^2 + B^6*a*b^4)*cos(d*x + c) + (B^6*a^2*b^3 + B^6*b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4) + (B^4*a^2 + B^4*b^2)*sqrt(B^4*b^2/d^4)*d^4*sqrt((B^4*a^2 + B^4*b^2)/d^4) + (B^6*a^3 + B^6*a*b^2)*sqrt(B^4*b^2/d^4)*d^2)/(B^8*a^2*b^2 + B^8*b^4) + 4*sqrt(2)*sqrt(B^4*b^2/d^4)*d^4*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4)*arctan(-(sqrt(2)*sqrt(B^4*b^2/d^4)*B^3*b*d^5*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4) - sqrt(2)*sqrt(B^4*b^2/d^4)*d^5*sqrt(-(sqrt(2)*B^3*b^3*d^3*sqrt((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4)*cos(d*x + c) - (B^4*a^2*b^2 + B^4*b^4)*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4)*cos(d*x + c) - (B^6*a^3*b^2 + B^6*a*b^4)*cos(d*x + c) - (B^6*a^2*b^3 + B^6*b^5)*sin(d*x + c))/((a^2 + b^2)*cos(d*x + c)))*sqrt((B^2*a^2 + B^2*b^2 + a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^(3/4) - (B^4*a^2 + B^4*b^2)*sqrt(B^4*b^2/d^4)*d^4*sqrt((B^4*a^2 + B^4*b^2)/d^4) - (B^6*a^3 + B^6*a*b^2)*sqrt(B^4*b^2/d^4)*d^2)/(B^8*a^2*b^2 + B^8*b^4) + sqrt(2)*(B^4*a^2 + B^4*b^2 - B^2*a*d^2*sqrt((B^4*a^2 + B^4*b^2)/d^4))*sqrt((B
```

$$\begin{aligned} &^2*a^2 + B^2*b^2 + a*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4})/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^{(1/4)}*\log((\sqrt{2}*B^3*b^3*d^3*\sqrt{(a*\cos(dx + c) + b*\sin(dx + c))/\cos(dx + c)})*\sqrt{(B^2*a^2 + B^2*b^2 + a*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4})/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^{(3/4)}*\cos(dx + c) + (B^4*a^2*b^2 + B^4*b^4)*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4)*\cos(dx + c) + (B^6*a^3*b^2 + B^6*a*b^4)*\cos(dx + c) + (B^6*a^2*b^3 + B^6*b^5)*\sin(dx + c))/((a^2 + b^2)*\cos(dx + c))) - \sqrt{2}*(B^4*a^2 + B^4*b^2 - B^2*a*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4})*\sqrt{(B^2*a^2 + B^2*b^2 + a*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4})/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^{(1/4)}*\log(-(\sqrt{2}*B^3*b^3*d^3*\sqrt{(a*\cos(dx + c) + b*\sin(dx + c))/\cos(dx + c)})*\sqrt{(B^2*a^2 + B^2*b^2 + a*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4})/(B^2*b^2))*((B^4*a^2 + B^4*b^2)/d^4)^{(3/4)}*\cos(dx + c) - (B^4*a^2*b^2 + B^4*b^4)*d^2*\sqrt{(B^4*a^2 + B^4*b^2)/d^4)*\cos(dx + c) - (B^6*a^3*b^2 + B^6*a*b^4)*\cos(dx + c) - (B^6*a^2*b^3 + B^6*b^5)*\sin(dx + c))/((a^2 + b^2)*\cos(dx + c))))/(B^4*a^2 + B^4*b^2) \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] B\*Integral(sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 8.27, size = 3033, normalized size = 8.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] 2\*atanh((8\*a\*b^2\*(a + b\*tan(c + d\*x))^(1/2)\*(- (-16\*B^4\*a^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)) - (B^2\*a^3\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4))))^(1/2)\*(-1



$$\begin{aligned}
& \left( a^2 d^4 + b^2 d^4 \right)^{1/2} (a + b \tan(c + dx))^{1/2} / \left( (16 B^3 b^5) / d - (16 B^3 a^2 b^5 d^3) / (a^2 d^4 + b^2 d^4) + (4 B a b^3 d^2 (-16 B^4 b^6 d^4)^{1/2}) / (a^2 d^5 + b^2 d^5) \right) \\
& + (8 a b^2 (B^2 a b^2 d^2) / (4 (a^2 d^4 + b^2 d^4))) - (-16 B^4 b^6 d^4)^{1/2} / (16 (a^2 d^4 + b^2 d^4))^{1/2} (a + b \tan(c + dx))^{1/2} \\
& * (-16 B^4 b^6 d^4)^{1/2} / (16 B^3 b^7 d + 16 B^3 a^2 b^5 d - (16 B^3 a^2 b^7 d^5) / (a^2 d^4 + b^2 d^4) - (16 B^3 a^4 b^5 d^5) / (a^2 d^4 + b^2 d^4) \\
& + (4 B a^3 b^3 d^4 (-16 B^4 b^6 d^4)^{1/2}) / (a^2 d^5 + b^2 d^5) + (4 B a b^5 d^4 (-16 B^4 b^6 d^4)^{1/2}) / (a^2 d^5 + b^2 d^5) \\
& - (32 B^2 a^2 b^4 d^2 (B^2 a b^2 d^2) / (4 (a^2 d^4 + b^2 d^4))) - (-16 B^4 b^6 d^4)^{1/2} / (16 (a^2 d^4 + b^2 d^4))^{1/2} (a + b \tan(c + dx))^{1/2} \\
& / (16 B^3 b^7 d + 16 B^3 a^2 b^5 d - (16 B^3 a^2 b^7 d^5) / (a^2 d^4 + b^2 d^4) - (16 B^3 a^4 b^5 d^5) / (a^2 d^4 + b^2 d^4) \\
& + (4 B a^3 b^3 d^4 (-16 B^4 b^6 d^4)^{1/2}) / (a^2 d^5 + b^2 d^5) + (4 B a b^5 d^4 (-16 B^4 b^6 d^4)^{1/2}) / (a^2 d^5 + b^2 d^5) \\
& )) * (B^2 a b^2 d^2) / (4 (a^2 d^4 + b^2 d^4)) - (-16 B^4 b^6 d^4)^{1/2} / (16 (a^2 d^4 + b^2 d^4))^{1/2}
\end{aligned}$$

$$3.366 \quad \int \frac{aB + bB \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx$$

**Optimal.** Leaf size=406

$$\frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}} d}$$

[Out]  $\frac{1}{2} b B \operatorname{arctanh} \left( \frac{(a + (a^2 + b^2)^{1/2})^{1/2} - 2^{1/2} (a + b \tan(dx + c))^{1/2}}{(a - (a^2 + b^2)^{1/2})^{1/2}} \right) / d \sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}$   
 $- \frac{1}{2} b B \operatorname{arctanh} \left( \frac{(a + (a^2 + b^2)^{1/2})^{1/2} + 2^{1/2} (a + b \tan(dx + c))^{1/2}}{(a - (a^2 + b^2)^{1/2})^{1/2}} \right) / d \sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}$   
 $- \frac{1}{4} b B \ln \left( \frac{a + (a^2 + b^2)^{1/2} - 2^{1/2} (a + b \tan(dx + c))^{1/2}}{a + (a^2 + b^2)^{1/2}} \right) / d \sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}$   
 $+ \frac{1}{4} b B \ln \left( \frac{a + (a^2 + b^2)^{1/2} + 2^{1/2} (a + b \tan(dx + c))^{1/2}}{a + (a^2 + b^2)^{1/2}} \right) / d \sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}$

**Rubi [A]**

time = 0.30, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {21, 3566, 722, 1108, 648, 632, 212, 642}

$$\frac{bB \log \left( \frac{-\sqrt{2} \sqrt{a^2 + b^2} + a \sqrt{a + b \tan(c + dx)} + \sqrt{a^2 + b^2} + a + b \tan(c + dx)}{2\sqrt{2} d \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a} \right)}{2\sqrt{2} d \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a} + \frac{bB \log \left( \frac{\sqrt{2} \sqrt{a^2 + b^2} + a \sqrt{a + b \tan(c + dx)} + \sqrt{a^2 + b^2} + a + b \tan(c + dx)}{2\sqrt{2} d \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a} \right)}{2\sqrt{2} d \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a} + \frac{bB \tanh^{-1} \left( \frac{\sqrt{a^2 + b^2} + a - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} d \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}} - \frac{bB \tanh^{-1} \left( \frac{\sqrt{a^2 + b^2} + a + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} d \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out]  $\frac{(b*B*\operatorname{ArcTanh}[\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}] - \sqrt{2} \sqrt{a + b \tan(c + dx)})}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}} - \frac{(b*B*\operatorname{ArcTanh}[\frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}] + \sqrt{2} \sqrt{a + b \tan(c + dx)})}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}}}$   
 $- \frac{(b*B*\operatorname{Log}[\frac{a + \sqrt{a^2 + b^2} + b \tan(c + dx) - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{2\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a}] - \sqrt{2} \sqrt{a + b \tan(c + dx)})}{2\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a}$   
 $+ \frac{(b*B*\operatorname{Log}[\frac{a + \sqrt{a^2 + b^2} + b \tan(c + dx) + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{2\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a}] + \sqrt{2} \sqrt{a + b \tan(c + dx)})}{2\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a^2 + b^2} + a}$

**Rule 21**

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_))^(m\_.)\*((c\_.) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :>  
 Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]  
 && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x,

$a + b*x]$ )

#### Rule 212

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /;$   $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 632

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$   $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x\_Symbol] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /;$   $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 648

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x\_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

#### Rule 722

$\text{Int}[1/(\text{Sqrt}[d + (e \cdot x)] \cdot (a + (c \cdot x)^2)), x\_Symbol] \rightarrow \text{Dist}[2*e, \text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, \text{Sqrt}[d + e*x]], x] /;$   $\text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

#### Rule 1108

$\text{Int}[(a + (b \cdot x)^2 + (c \cdot x)^4)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2*q - b/c, 2]\}, \text{Dist}[1/(2*c*q*r), \text{Int}[(r - x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2*c*q*r), \text{Int}[(r + x)/(q + r*x + x^2), x], x]] /;$   $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NegQ}[b^2 - 4*a*c]$

#### Rule 3566

$\text{Int}[(a + (b \cdot x) \cdot \tan[(c \cdot x) + (d \cdot x)])^{(n)}, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[(a + x)^n/(b^2 + x^2), x], x, b \cdot \text{Tan}[c + d*x]], x] /;$   $\text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

## Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx &= B \int \frac{1}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{(bB) \text{Subst} \left( \int \frac{1}{\sqrt{a + x} (b^2 + x^2)} dx, x, b \tan(c + dx) \right)}{d} \\
&= \frac{(2bB) \text{Subst} \left( \int \frac{1}{a^2 + b^2 - 2ax^2 + x^4} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{d} \\
&= \frac{(bB) \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} - x}{\sqrt{a^2 + b^2} - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} x + x^2} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a + \sqrt{a^2 + b^2}} d} \\
&= \frac{(bB) \text{Subst} \left( \int \frac{1}{\sqrt{a^2 + b^2} - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} x + x^2} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{2\sqrt{a^2 + b^2} d} \\
&= -\frac{bB \log \left( a + \sqrt{a^2 + b^2} + b \tan(c + dx) - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \tan(c + dx)} \right)}{2\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a + \sqrt{a^2 + b^2}} d} \\
&= \frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{bB \tanh^{-1} \left( \frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \tan(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right)}{\sqrt{2} \sqrt{a^2 + b^2} \sqrt{a - \sqrt{a^2 + b^2}} d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.04, size = 88, normalized size = 0.22

$$\frac{iB \left( \frac{\tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{\sqrt{a - ib}} - \frac{\tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{\sqrt{a + ib}} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(3/2), x]



[Out]  $((-I)*B*(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a - I*b]]/\text{Sqrt}[a - I*b] - \text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]]/\text{Sqrt}[a + I*b]))/d$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 777 vs.  $2(327) = 654$ .

time = 0.15, size = 778, normalized size = 1.92

method	result
derivativedivides	$\frac{\left( \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^2 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} b^2 - \sqrt{2\sqrt{a^2 + b^2}} \right)}{2Bb}$
default	$\left( \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^2 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} b^2 - \sqrt{2\sqrt{a^2 + b^2}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $2/d*B*b*(1/4/b^2/(a^2+b^2)^{(3/2)}*(1/2*((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^2-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^2)*\ln(b*\text{tan}(d*x+c)+a+(a+b*\text{tan}(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)}))+2*(2*a^2*b^2+2*b^4-1/2*((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^2-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^2)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\text{tan}(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}))+1/4/b^2/(a^2+b^2)^{(3/2)}*(-1/2*((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^2-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^2)*\ln(-b*\text{tan}(d*x+c)-a+(a+b*\text{tan}(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(-2*a^2*b^2-2*b^4+1/2*((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^2+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^2-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}$

$$\frac{1}{2} * a * b^2 * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan\left(\frac{-2 * (a + b * \tan(dx + c))^{1/2} + (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2}}{2 * (a^2 + b^2)^{1/2} - 2 * a}\right)$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 2127 vs. 2(329) = 658.

time = 1.19, size = 2127, normalized size = 5.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^2 + b^2)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^4*(B^4/((a^2 + b^2)*d^4))^(3/4)*sqrt((B^2*a^2 + B^2*b^2 + (a^3 + a*b^2)*d^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2))*arctan((sqrt(2)*(a^4 + 2*a^2*b^2 + b^4)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^7*sqrt((sqrt(2)*B^5*b^3*d*sqrt((a*cos(dx + c) + b*sin(dx + c))/cos(dx + c)))*(B^4/((a^2 + b^2)*d^4))^(1/4)*sqrt((B^2*a^2 + B^2*b^2 + (a^3 + a*b^2)*d^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2))*cos(dx + c) + B^6*a*b^2*cos(dx + c) + B^6*b^3*sin(dx + c) + (B^4*a^2*b^2 + B^4*b^4)*d^2*sqrt(B^4/((a^2 + b^2)*d^4))*cos(dx + c)/cos(dx + c))*(B^4/((a^2 + b^2)*d^4))^(5/4)*sqrt((B^2*a^2 + B^2*b^2 + (a^3 + a*b^2)*d^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2)) - sqrt(2)*(B^3*a^4*b + 2*B^3*a^2*b^3 + B^3*b^5)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^7*sqrt((a*cos(dx + c) + b*sin(dx + c))/cos(dx + c))*(B^4/((a^2 + b^2)*d^4))^(5/4)*sqrt((B^2*a^2 + B^2*b^2 + (a^3 + a*b^2)*d^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2)) - (B^6*a^4 + 2*B^6*a^2*b^2 + B^6*b^4)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^4*sqrt(B^4/((a^2 + b^2)*d^4)) - (B^8*a^3 + B^8*a*b^2)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2/(B^10*b^2)) + 4*sqrt(2)*(a^2 + b^2)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^4*(B^4/((a^2 + b^2)*d^4))^(3/4)*sqrt((B^2*a^2 + B^2*b^2 + (a^3 + a*b^2)*d^2*sqrt(B^4/((a
```

$$\begin{aligned} & \sqrt{2 + b^2} * d^4)) / (B^2 * b^2)) * \arctan((\sqrt{2} * (a^4 + 2 * a^2 * b^2 + b^4) * \sqrt{B^4 * b^2 / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4)}) * d^7 * \sqrt{-(\sqrt{2} * B^5 * b^3 * d * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)}) * (B^4 / ((a^2 + b^2) * d^4))^{1/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) * \cos(dx + c) - B^6 * a * b^2 * \cos(dx + c) - B^6 * b^3 * \sin(dx + c) - (B^4 * a^2 * b^2 + B^4 * b^4) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)} * \cos(dx + c)) / \cos(dx + c)) * (B^4 / ((a^2 + b^2) * d^4))^{5/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) - \sqrt{2} * (B^3 * a^4 * b + 2 * B^3 * a^2 * b^3 + B^3 * b^5) * \sqrt{B^4 * b^2 / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4)} * d^7 * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)}) * (B^4 / ((a^2 + b^2) * d^4))^{5/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) + (B^6 * a^4 + 2 * B^6 * a^2 * b^2 + B^6 * b^4) * \sqrt{B^4 * b^2 / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4)} * d^4 * \sqrt{B^4 / ((a^2 + b^2) * d^4)} + (B^8 * a^3 + B^8 * a * b^2) * \sqrt{B^4 * b^2 / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4)} * d^2 / (B^{10} * b^2)) + \sqrt{2} * (B^2 * a * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)} - B^4) * (B^4 / ((a^2 + b^2) * d^4))^{1/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) * \log((\sqrt{2} * B^5 * b^3 * d * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)}) * (B^4 / ((a^2 + b^2) * d^4))^{1/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) * \cos(dx + c) + B^6 * a * b^2 * \cos(dx + c) + B^6 * b^3 * \sin(dx + c) + (B^4 * a^2 * b^2 + B^4 * b^4) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)} * \cos(dx + c)) / \cos(dx + c)) - \sqrt{2} * (B^2 * a * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)} - B^4) * (B^4 / ((a^2 + b^2) * d^4))^{1/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) * \log(-(\sqrt{2} * B^5 * b^3 * d * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)}) * (B^4 / ((a^2 + b^2) * d^4))^{1/4} * \sqrt{(B^2 * a^2 + B^2 * b^2 + (a^3 + a * b^2) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)})}) / (B^2 * b^2)) * \cos(dx + c) - B^6 * a * b^2 * \cos(dx + c) - B^6 * b^3 * \sin(dx + c) - (B^4 * a^2 * b^2 + B^4 * b^4) * d^2 * \sqrt{B^4 / ((a^2 + b^2) * d^4)} * \cos(dx + c)) / \cos(dx + c)) / B^4 \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2), x)

[Out] B\*Integral(1/sqrt(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& *d^4 - 144*B^4*a^8*b^2*d^4)^{(1/2)} - 4*B^2*a^5*d^2 + 12*B^2*a^3*b^2*d^2)/(16 \\
& *a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*((a + b*\tan \\
& (c + d*x))^{(1/2)}*(16*B^2*a^2*b^10*d^3 + 32*B^2*a^4*b^8*d^3 - 32*B^2*a^8*b^4 \\
& *d^3 - 16*B^2*a^10*b^2*d^3) + (((96*B^4*a^6*b^4*d^4 - 16*B^4*a^4*b^6*d^4 - \\
& 144*B^4*a^8*b^2*d^4)^{(1/2)} - 4*B^2*a^5*d^2 + 12*B^2*a^3*b^2*d^2)/(16*a^6*d^4 \\
& + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(64*B*a^2*b^11*d^4 \\
& - (((96*B^4*a^6*b^4*d^4 - 16*B^4*a^4*b^6*d^4 - 144*B^4*a^8*b^2*d^4)^{(1/2)} \\
& - 4*B^2*a^5*d^2 + 12*B^2*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4 \\
& *d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(64*a*b^12*d^5 + 3 \\
& 20*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64* \\
& a^11*b^2*d^5) + 256*B*a^4*b^9*d^4 + 384*B*a^6*b^7*d^4 + 256*B*a^8*b^5*d^4 + \\
& 64*B*a^10*b^3*d^4)) + 24*B^3*a^5*b^7*d^2 + 24*B^3*a^7*b^5*d^2 + 8*B^3*a^9* \\
& b^3*d^2)*(((96*B^4*a^6*b^4*d^4 - 16*B^4*a^4*b^6*d^4 - 144*B^4*a^8*b^2*d^4)^{(1/2)} \\
& - 4*B^2*a^5*d^2 + 12*B^2*a^3*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2 \\
& *b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + \log(((a + b*\tan(c + d*x))^{(1/2)}*(16*B \\
& ^2*a^2*b^10*d^3 + 32*B^2*a^4*b^8*d^3 - 32*B^2*a^8*b^4*d^3 - 16*B^2*a^10*b^2 \\
& *d^3) - (-(((8*B^2*a^5*d^2 - 24*B^2*a^3*b^2*d^2)^{2/4} - B^4*a^4*(16*a^6*d^4 \\
& + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + 4*B^2*a^5*d^2 - 12 \\
& *B^2*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))) \\
& ^{(1/2)}*((a + b*\tan(c + d*x))^{(1/2)}*(-(((8*B^2*a^5*d^2 - 24*B^2*a^3*b^2*d^2) \\
& ^{2/4} - B^4*a^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4)) \\
& ^{(1/2)} + 4*B^2*a^5*d^2 - 12*B^2*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2 \\
& *b^4*d^4 + 3*a^4*b^2*d^4)))^{(1/2)}*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a \\
& ^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 64*B*a^ \\
& 2*b^11*d^4 + 256*B*a^4*b^9*d^4 + 384*B*a^6*b^7*d^4 + 256*B*a^8*b^5*d^4 + 64 \\
& *B*a^10*b^3*d^4))*(-(((8*B^2*a^5*d^2 - 24*B^2*a^3*b^2*d^2)^{2/4} - B^4*a^4*(1 \\
& 6*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + 4*B^2*a^ \\
& 5*d^2 - 12*B^2*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4* \\
& b^2*d^4)))^{(1/2)} + 8*B^3*a^3*b^9*d^2 + 24*B^3*a^5*b^7*d^2 + 24*B^3*a^7*b^5* \\
& d^2 + 8*B^3*a^9*b^3*d^2)*(-(((8*B^2*a^5*d^2 - 24*B^2*a^3*b^2*d^2)^{2/4} - B^4 \\
& *a^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} + 4 \\
& *B^2*a^5*d^2 - 12*B^2*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + \\
& 3*a^4*b^2*d^4)))^{(1/2)} + \log(- ((a + b*\tan(c + ...
\end{aligned}$$

$$3.367 \quad \int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=119

$$-\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d}$$

[Out]  $-2*B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/d/a^{(1/2)}+B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/d/(a-I*b)^{(1/2)}+B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d/(a+I*b)^{(1/2)}$

**Rubi [A]**

time = 0.20, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {21, 3655, 3620, 3618, 65, 214, 3715}

$$-\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c + d*x]*(a*B + b*B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $(-2*B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a]*d) + (B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/(\operatorname{Sqrt}[a - I*b]*d) + (B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/(\operatorname{Sqrt}[a + I*b]*d)$

**Rule 21**

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x$   
 $\&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 65**

$\operatorname{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 214**

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b]$

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3655

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*
(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x
])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx &= B \int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
&= -\left( B \int \frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \right) + B \int \frac{\cot(c+dx)(1+\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx \\
&= -\left( \frac{1}{2}(iB) \int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \right) + \frac{1}{2}(iB) \int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
&= -\frac{B \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}} dx, x, i\tan(c+dx)\right)}{2d} - \frac{B \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, i\tan(c+dx)\right)}{2d} \\
&= -\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}} dx, x, i\tan(c+dx)\right)}{2d} \\
&= -\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}
\end{aligned}$$

**Mathematica [A]**

time = 0.10, size = 112, normalized size = 0.94

$$\frac{B \left( -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

```
[Out] (B*((-2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]/Sqrt[a] + ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b]))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.82, size = 20195, normalized size = 169.71

method	result	size
default	Expression too large to display	20195



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F(-1)] Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 2656 vs. 2(93) = 186.

```
time = 2.26, size = 5387, normalized size = 45.27
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a^3 + a*b^2)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d
^5*(B^4/((a^2 + b^2)*d^4))^(3/4)*sqrt((B^2*a^2 + B^2*b^2 - (a^3 + a*b^2)*d
^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2))*arctan(-((B^6*a^4 + 2*B^6*a^2*b^2
+ B^6*b^4)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^4*sqrt(B^4/((a^2
+ b^2)*d^4)) + (B^8*a^3 + B^8*a*b^2)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*
d^4))*d^2 - sqrt(2)*((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(B^4*b^2/((a^4 + 2*a^2*b
^2 + b^4)*d^4))*d^7*sqrt(B^4/((a^2 + b^2)*d^4)) + (B^2*a^4 + 2*B^2*a^2*b^2
+ B^2*b^4)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^5)*sqrt((B^6*a*cos
(d*x + c) + B^6*b*sin(d*x + c) + (B^4*a^2 + B^4*b^2)*d^2*sqrt(B^4/((a^2 + b
^2)*d^4))*cos(d*x + c) + sqrt(2)*(B^5*a*d*cos(d*x + c) + (B^3*a^2 + B^3*b^2
)*d^3*sqrt(B^4/((a^2 + b^2)*d^4))*cos(d*x + c))*sqrt((a*cos(d*x + c) + b*si
n(d*x + c))/cos(d*x + c))*(B^4/((a^2 + b^2)*d^4))^(1/4)*sqrt((B^2*a^2 + B^2
*b^2 - (a^3 + a*b^2)*d^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2)))/cos(d*x +
c))*(B^4/((a^2 + b^2)*d^4))^(3/4)*sqrt((B^2*a^2 + B^2*b^2 - (a^3 + a*b^2)*
d^2*sqrt(B^4/((a^2 + b^2)*d^4)))/(B^2*b^2)) + sqrt(2)*((B^3*a^5 + 2*B^3*a^3
*b^2 + B^3*a*b^4)*sqrt(B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^7*sqrt(B^4/
```

$$\begin{aligned}
& ((a^2 + b^2)d^4) + (B^5a^4 + 2B^5a^2b^2 + B^5b^4)\sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^5\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c)} \\
& \times (B^4/((a^2 + b^2)d^4))^{3/4}\sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)))/(B^{10}b^2) + 4\sqrt{2}(a^3 + ab^2) \\
& \sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^5(B^4/((a^2 + b^2)d^4))^{3/4}\sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2))} \\
& \arctan(((B^6a^4 + 2B^6a^2b^2 + B^6b^4)\sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^4\sqrt{B^4/((a^2 + b^2)d^4)} + (B^8a^3 + B^8ab^2) \\
& \sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^2 + \sqrt{2}) * ((a^5 + 2a^3b^2 + ab^4)\sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^7\sqrt{B^4/((a^2 + b^2)d^4)} + (B^2a^4 + 2B^2a^2b^2 + B^2b^4) \\
& \sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^5)\sqrt{(B^6a\cos(dx + c) + B^6b\sin(dx + c) + (B^4a^2 + B^4b^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)}\cos(dx + c) - \sqrt{2} \\
& (B^5ad\cos(dx + c) + (B^3a^2 + B^3b^2)d^3\sqrt{B^4/((a^2 + b^2)d^4)}\cos(dx + c))\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c)} \\
& \times (B^4/((a^2 + b^2)d^4))^{1/4}\sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)))/\cos(dx + c)} * (B^4/((a^2 + b^2)d^4))^{3/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)} - \sqrt{2} * ((B^3a^5 + 2B^3a^3b^2 + B^3ab^4)\sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^7\sqrt{B^4/((a^2 + b^2)d^4)} \\
& + (B^5a^4 + 2B^5a^2b^2 + B^5b^4)\sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^5)\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c)} * (B^4/((a^2 + b^2)d^4))^{3/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)))/(B^{10}b^2) + 2B^5\sqrt{a}\log(-8ab\cos(dx + c)\sin(dx + c) + (8a^2 - b^2)\cos(dx + c)^2 + b^2 - 4(2a\cos(dx + c)^2 + b\cos(dx + c)\sin(dx + c))\sqrt{a}\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c)))/(\cos(dx + c)^2 - 1)) + \sqrt{2} * (B^2a^2d^3\sqrt{B^4/((a^2 + b^2)d^4)} + B^4ad) * (B^4/((a^2 + b^2)d^4))^{1/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)} * \log((B^6a\cos(dx + c) + B^6b\sin(dx + c) + (B^4a^2 + B^4b^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)}\cos(dx + c) + \sqrt{2} * (B^5ad\cos(dx + c) + (B^3a^2 + B^3b^2)d^3\sqrt{B^4/((a^2 + b^2)d^4)}\cos(dx + c))\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c)} * (B^4/((a^2 + b^2)d^4))^{1/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)))/\cos(dx + c)) - \sqrt{2} * (B^2a^2d^3\sqrt{B^4/((a^2 + b^2)d^4)} + B^4ad) * (B^4/((a^2 + b^2)d^4))^{1/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)} * \log((B^6a\cos(dx + c) + B^6b\sin(dx + c) + (B^4a^2 + B^4b^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)}\cos(dx + c) - \sqrt{2} * (B^5ad\cos(dx + c) + (B^3a^2 + B^3b^2)d^3\sqrt{B^4/((a^2 + b^2)d^4)}\cos(dx + c))\sqrt{(a\cos(dx + c) + b\sin(dx + c))/\cos(dx + c)} * (B^4/((a^2 + b^2)d^4))^{1/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)))/\cos(dx + c)))/(B^4ad), 1/4 * (4\sqrt{2})(a^3 + ab^2)\sqrt{B^4b^2/((a^4 + 2a^2b^2 + b^4)d^4)}d^5 * (B^4/((a^2 + b^2)d^4))^{3/4} \\
& \sqrt{(B^2a^2 + B^2b^2 - (a^3 + ab^2)d^2\sqrt{B^4/((a^2 + b^2)d^4)})/(B^2b^2)} * \arctan(-((B^6a^4 + 2B^6a^2b^2 +
\end{aligned}$$

$B^6 b^4 \sqrt{B^4 b^2 / ((a^4 + 2a^2 b^2 + b^4) d^4)} d^4 \sqrt{B^4 / ((a^2 + b^2) d^4)} + (B^8 a^3 + B^8 a b^2) \sqrt{B^4 b^2 / ((a^4 + 2a^2 b^2 + b^4) d^4)} d^2 - \sqrt{2} ((a^5 + 2a^3 b^2 + a b^4) \sqrt{B^4 b^2 / ((a^4 + 2a^2 b^2 + b^4) d^4)} d^7 \sqrt{B^4 / ((a^2 + b^2) d^4)} + (B^2 a^4 + 2B^2 a^2 b^2 + B^2 b^4) \sqrt{B^4 b^2 / ((a^4 + 2a^2 b^2 + b^4) d^4)} d^5) \sqrt{(B^6 a \cos(dx + c) + B^6 b \sin(dx + c) + (B^4 a^2 + B^4 b^2))}$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\cot(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] B\*Integral(cot(c + d\*x)/sqrt(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 8.77, size = 2142, normalized size = 18.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out]  $- \operatorname{atan}\left(\frac{\left(\frac{32(16B^2 b^{10} d^2 + 12B^2 a^2 b^8 d^2)}{d^3} - (32(16b^{10} d^4 + 24a^2 b^8 d^4)(a + b \tan(c + d*x))^{1/2} (B^2 / (4(a d^2 - b d^2 1i)))^{1/2}\right)}{d^4} (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} + (576 B^2 a b^8 (a + b \tan(c + d*x))^{1/2}) / d^2 (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} - (96 B^3 a b^8) / d^3 (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} - (96 B^4 b^8 (a + b \tan(c + d*x))^{1/2}) / d^4 (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} 1i - \left(\frac{32(16B^2 b^{10} d^2 + 12B^2 a^2 b^8 d^2)}{d^3} + (32(16b^{10} d^4 + 24a^2 b^8 d^4)(a + b \tan(c + d*x))^{1/2} (B^2 / (4(a d^2 - b d^2 1i)))^{1/2}\right)}{d^4} (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} - (576 B^2 a b^8 (a + b \tan(c + d*x))^{1/2}) / d^2 (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} - (96 B^3 a b^8) / d^3 (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} - (96 B^4 b^8 (a + b \tan(c + d*x))^{1/2}) / d^4 (B^2 / (4(a d^2 - b d^2 1i)))^{1/2} 1i\right)$



$$3.368 \quad \int \frac{aB + bB \tan(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx$$

**Optimal.** Leaf size=123

$$\frac{iB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{3/2} d} + \frac{iB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{3/2} d} - \frac{2bB}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

[Out]  $-I*B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})/(a-I*b)^{(3/2)}/d+I*B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})/(a+I*b)^{(3/2)}/d-2*b*B/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {21, 3564, 3620, 3618, 65, 214}

$$\frac{2bB}{d(a^2 + b^2) \sqrt{a + b \tan(c + dx)}} - \frac{iB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{3/2}} + \frac{iB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x])/(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}, x]$

[Out]  $((-I)*B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/((a - I*b)^{(3/2)*d}) + (I*B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/((a + I*b)^{(3/2)*d}) - (2*b*B)/((a^2 + b^2)*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])$

Rule 21

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_))^{(m_*)}*((c_*) + (d_*)*(v_))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 65

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 3564

Int[((a\_) + (b\_.)\*tan[(c\_) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*((a + b\*Tan[c + d\*x])^(n + 1)/(d\*(n + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a - b\*Tan[c + d\*x])\*(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]

#### Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

#### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx &= B \int \frac{1}{(a + b \tan(c + dx))^{3/2}} dx \\
&= -\frac{2bB}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{B \int \frac{a - b \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{a^2 + b^2} \\
&= -\frac{2bB}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{B \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{2(a - ib)} + \frac{B \int \frac{1 - i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{2(a + ib)} \\
&= -\frac{2bB}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{B \text{Subst} \left( \int \frac{1}{(-1+x) \sqrt{a + ibx}} dx, x, -i \tan(c + dx) \right)}{2(ia - b)d} \\
&= -\frac{2bB}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{B \text{Subst} \left( \int \frac{1}{-1 - \frac{ia}{b} + \frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{(a - ib)bd} \\
&= -\frac{iB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{3/2} d} + \frac{iB \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{3/2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.10, size = 106, normalized size = 0.86

$$\frac{B \left( i(a + ib) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{a + b \tan(c + dx)}{a - ib} \right) + (-ia - b) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{a + b \tan(c + dx)}{a + ib} \right) \right)}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(5/2),x]

[Out] (B\*(I\*(a + I\*b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] + ((-I)\*a - b)\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)]))/((a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(103) = 206.

time = 0.14, size = 810, normalized size = 6.59

method	result
--------	--------

derivativedivides	$2Bb \frac{1}{(a^2+b^2) \sqrt{a+b \tan(dx+c)}} + \frac{(\sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3 + \sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3 + \sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3 + \sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3)}{\dots}$
default	$2Bb \frac{1}{(a^2+b^2) \sqrt{a+b \tan(dx+c)}} + \frac{(\sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3 + \sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3 + \sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3 + \sqrt{2\sqrt{a^2+b^2}+2a} \sqrt{a^2+b^2} a^3)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*B*b*(-1/(a^2+b^2)/(a+b*tan(d*x+c))^(1/2)+1/(a^2+b^2)*(1/4/b^2/(a^2+b^2)
^(3/2)*(1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(4*a^3*b^2+4*a*b^4-1/2*((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*(a^2+b^2)^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)*b^4)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)*(-1/2*((2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/
2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^
4)*ln(-b*tan(d*x+c)-a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
(a^2+b^2)^(1/2))+2*(-4*a^3*b^2-4*a*b^4+1/2*((2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(
a^2+b^2)^(1/2)*a^3+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b^2-(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^4)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*tan(d*x+
```



c))^(1/2)+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2))))

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 6756 vs. 2(97) = 194.

time = 1.68, size = 6756, normalized size = 54.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$\frac{1}{4} \cdot (4 \cdot \sqrt{2}) \cdot ((a^{10} + 3a^8b^2 + 2a^6b^4 - 2a^4b^6 - 3a^2b^8 - b^{10}) \cdot d^5 \cdot \cos(dx + c)^2 + 2(a^9b + 4a^7b^3 + 6a^5b^5 + 4a^3b^7 + ab^9) \cdot d^5 \cdot \cos(dx + c) \cdot \sin(dx + c) + (a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10}) \cdot d^5) \cdot \sqrt{(B^2a^6 + 3B^2a^4b^2 + 3B^2a^2b^4 + B^2b^6 + (a^9 - 6a^5b^4 - 8a^3b^6 - 3ab^8) \cdot d^2 \cdot \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4)})} / (9B^2a^4b^2 - 6B^2a^2b^4 + B^2b^6) \cdot (B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4))^{3/4} \cdot \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot d^4)} \cdot \arctan(((3B^6a^{12} + 14B^6a^{10}b^2 + 25B^6a^8b^4 + 20B^6a^6b^6 + 5B^6a^4b^8 - 2B^6a^2b^{10} - B^6b^{12}) \cdot d^4 \cdot \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4)}) \cdot \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot d^4)}) + (3B^8a^9 + 8B^8a^7b^2 + 6B^8a^5b^4 - B^8ab^8) \cdot d^2 \cdot \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot d^4)} + \sqrt{2} \cdot (2(a^{13} + 6a^{11}b^2 + 15a^9b^4 + 20a^7b^6 + 15a^5b^8 + 6a^3b^{10} + ab^{12}) \cdot d^7 \cdot \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4)}) \cdot \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot d^4)}) + (B^2a^{10} + 5B^2a^8b^2 + 10B^2a^6b^4 + 10B^2a^4b^6 + 5B^2a^2b^8 + B^2b^{10}$$

$$\begin{aligned}
& ) * d^5 * \sqrt{((9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) * d^4))} * \sqrt{(B^2a^6 + 3B^2a^4b^2 + 3B^2a^2b^4 + B^2b^6 + (a^9 - 6a^5b^4 - 8a^3b^6 - 3ab^8) * d^2 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)})} / (9B^2a^4b^2 - 6B^2a^2b^4 + B^2b^6) * \sqrt{((9B^4a^8b^2 + 12B^4a^6b^4 - 2B^4a^4b^6 - 4B^4a^2b^8 + B^4b^{10}) * d^2 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)}) * \cos(dx + c) + \sqrt{2} * ((9B^3a^8b^3 + 12B^3a^6b^5 - 2B^3a^4b^7 - 4B^3a^2b^9 + B^3b^{11}) * d^3 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)}) * \cos(dx + c) + 2 * (9B^5a^5b^3 - 6B^5a^3b^5 + B^5ab^7) * d * \cos(dx + c)) * \sqrt{(B^2a^6 + 3B^2a^4b^2 + 3B^2a^2b^4 + B^2b^6 + (a^9 - 6a^5b^4 - 8a^3b^6 - 3ab^8) * d^2 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)})} / (9B^2a^4b^2 - 6B^2a^2b^4 + B^2b^6) * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)) * (B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4))^{1/4} + (9B^6a^5b^2 - 6B^6a^3b^4 + B^6ab^6) * \cos(dx + c) + (9B^6a^4b^3 - 6B^6a^2b^5 + B^6b^7) * \sin(dx + c) / \cos(dx + c)) * (B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4))^{3/4} + \sqrt{2} * (2 * (3B^3a^{15}b + 17B^3a^{13}b^3 + 39B^3a^{11}b^5 + 45B^3a^9b^7 + 25B^3a^7b^9 + 3B^3a^5b^{11} - 3B^3a^3b^{13} - B^3ab^{15}) * d^7 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)}) * \sqrt{((9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) * d^4))} + (3B^5a^{12}b + 14B^5a^{10}b^3 + 25B^5a^8b^5 + 20B^5a^6b^7 + 5B^5a^4b^9 - 2B^5a^2b^{11} - B^5b^{13}) * d^5 * \sqrt{((9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) * d^4))} * \sqrt{(B^2a^6 + 3B^2a^4b^2 + 3B^2a^2b^4 + B^2b^6 + (a^9 - 6a^5b^4 - 8a^3b^6 - 3ab^8) * d^2 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)})} / (9B^2a^4b^2 - 6B^2a^2b^4 + B^2b^6) * \sqrt{(a * \cos(dx + c) + b * \sin(dx + c)) / \cos(dx + c)) * (B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4))^{3/4} / (9B^{10}a^4b^2 - 6B^{10}a^2b^4 + B^{10}b^6) + 4 * \sqrt{2} * ((a^{10} + 3a^8b^2 + 2a^6b^4 - 2a^4b^6 - 3a^2b^8 - b^{10}) * d^5 * \cos(dx + c)^2 + 2 * (a^9b + 4a^7b^3 + 6a^5b^5 + 4a^3b^7 + ab^9) * d^5 * \cos(dx + c) * \sin(dx + c) + (a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10}) * d^5) * \sqrt{(B^2a^6 + 3B^2a^4b^2 + 3B^2a^2b^4 + B^2b^6 + (a^9 - 6a^5b^4 - 8a^3b^6 - 3ab^8) * d^2 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)})} / (9B^2a^4b^2 - 6B^2a^2b^4 + B^2b^6) * (B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4))^{3/4} * \sqrt{((9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) * d^4))} * \arctan(-((3B^6a^{12} + 14B^6a^{10}b^2 + 25B^6a^8b^4 + 20B^6a^6b^6 + 5B^6a^4b^8 - 2B^6a^2b^{10} - B^6b^{12}) * d^4 * \sqrt{B^4 / ((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) * d^4)}) * \sqrt{((9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) * d^4))} + (3B^8a^9 + 8B^8a^7b^2 + 6B^8a^5b^4 - B^8ab^8) * d^2 * \sqrt{((9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) * d^4))} - \sqrt{2} * (2 * (a^{13} + 6a^{11}b^2 + 15a^9b^4 + 20a^7b^6 + 15a^5b^8 + 6a^3b^{10} + ab^{12}) * d^7 * \sqrt{B^4 / ((a^6
\end{aligned}$$

+ 3\*a^4\*b^2 + 3\*a^2\*b^4 + b^6)\*d^4))\*sqrt((9\*B^4\*a^4\*b^2 - 6\*B^4\*a^2\*b^4 + B^4\*b^6)/((a^12 + 6\*a^10\*b^2 + 15\*a^8\*b^4 + 20...

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{a\sqrt{a + b \tan(c + dx)} + b\sqrt{a + b \tan(c + dx)} \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] B\*Integral(1/(a\*sqrt(a + b\*tan(c + d\*x)) + b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 18.41, size = 2500, normalized size = 20.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] (log(16\*B^3\*a^4\*b^15\*d^2 - (((320\*B^4\*a^6\*b^8\*d^4 - 16\*B^4\*a^4\*b^10\*d^4 - 1760\*B^4\*a^8\*b^6\*d^4 + 1600\*B^4\*a^10\*b^4\*d^4 - 400\*B^4\*a^12\*b^2\*d^4)^(1/2) - 4\*B^2\*a^7\*d^2 - 20\*B^2\*a^3\*b^4\*d^2 + 40\*B^2\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*((((320\*B^4\*a^6\*b^8\*d^4 - 16\*B^4\*a^4\*b^10\*d^4 - 1760\*B^4\*a^8\*b^6\*d^4 + 1600\*B^4\*a^10\*b^4\*d^4 - 400\*B^4\*a^12\*b^2\*d^4)^(1/2) - 4\*B^2\*a^7\*d^2 - 20\*B^2\*a^3\*b^4\*d^2 + 40\*B^2\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*((((320\*B^4\*a^6\*b^8\*d^4 - 16\*B^4\*a^4\*b^10\*d^4 - 1760\*B^4\*a^8\*b^6\*d^4 + 1600\*B^4\*a^10\*b^4\*d^4 - 400\*B^4\*a^12\*b^2\*d^4)^(1/2) - 4\*B^2\*a^7\*d^2 - 20\*B^2\*a^3\*b^4\*d^2 + 40\*B^2\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*(64\*a\*b^22\*d^5 + 640\*a^3\*b^20\*d^5 + 2880\*a^5\*b^18\*d^5 + 7680\*a^7\*b^16\*d^5 + 13440\*a^9\*b^14\*d^5 + 28800\*a^11\*b^14\*d^5 + 38400\*a^13\*b^12\*d^5 + 38400\*a^15\*b^10\*d^5 + 28800\*a^17\*b^8\*d^5 + 17280\*a^19\*b^6\*d^5 + 8640\*a^21\*b^4\*d^5 + 3456\*a^23\*b^2\*d^5 + 1152\*a^25\*b^2\*d^5 + 288\*a^27\*d^5))

$$\begin{aligned}
& 14*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 28 \\
& 80*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5)/4 - 32*B*a*b^{21}*d^4 \\
& - 160*B*a^3*b^{19}*d^4 - 128*B*a^5*b^{17}*d^4 + 896*B*a^7*b^{15}*d^4 + 3136*B*a^9 \\
& *b^{13}*d^4 + 4928*B*a^{11}*b^{11}*d^4 + 4480*B*a^{13}*b^9*d^4 + 2432*B*a^{15}*b^7*d^4 \\
& + 736*B*a^{17}*b^5*d^4 + 96*B*a^{19}*b^3*d^4)/4 - (a + b*\tan(c + d*x))^{(1/2)} \\
& *(320*B^2*a^6*b^{14}*d^3 - 16*B^2*a^2*b^{18}*d^3 + 1024*B^2*a^8*b^{12}*d^3 + 1440 \\
& *B^2*a^{10}*b^{10}*d^3 + 1024*B^2*a^{12}*b^8*d^3 + 320*B^2*a^{14}*b^6*d^3 - 16*B^2* \\
& a^{18}*b^2*d^3))/4 + 96*B^3*a^6*b^{13}*d^2 + 240*B^3*a^8*b^{11}*d^2 + 320*B^3*a^ \\
& 10*b^9*d^2 + 240*B^3*a^{12}*b^7*d^2 + 96*B^3*a^{14}*b^5*d^2 + 16*B^3*a^{16}*b^3*d \\
& ^2)*(((320*B^4*a^6*b^8*d^4 - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6*d^4 + 1 \\
& 600*B^4*a^{10}*b^4*d^4 - 400*B^4*a^{12}*b^2*d^4)^{(1/2)} - 4*B^2*a^7*d^2 - 20*B^2 \\
& *a^3*b^4*d^2 + 40*B^2*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 1 \\
& 0*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2))/4 + (\log(16*B^3*a^4 \\
& *b^{15}*d^2 - ((-(320*B^4*a^6*b^8*d^4 - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6 \\
& *d^4 + 1600*B^4*a^{10}*b^4*d^4 - 400*B^4*a^{12}*b^2*d^4)^{(1/2)} + 4*B^2*a^7*d^2 \\
& + 20*B^2*a^3*b^4*d^2 - 40*B^2*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 \\
& + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}*((-(320*B^4*a^6*b^8*d^4 \\
& - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^{10}*b^4*d^4 - 400*B^4*a \\
& ^{12}*b^2*d^4)^{(1/2)} + 4*B^2*a^7*d^2 + 20*B^2*a^3*b^4*d^2 - 40*B^2*a^5*b^2*d^2) \\
& /((a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + \\
& 5*a^8*b^2*d^4))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(64*a*b^{22}*d^5 + 640*a^3* \\
& b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 161 \\
& 28*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6* \\
& d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5))/4 - 32*B*a*b^{21}*d^4 - 160*B*a^3* \\
& b^{19}*d^4 - 128*B*a^5*b^{17}*d^4 + 896*B*a^7*b^{15}*d^4 + 3136*B*a^9*b^{13}*d^4 + \\
& 4928*B*a^{11}*b^{11}*d^4 + 4480*B*a^{13}*b^9*d^4 + 2432*B*a^{15}*b^7*d^4 + 736*B*a^ \\
& 17*b^5*d^4 + 96*B*a^{19}*b^3*d^4)/4 - (a + b*\tan(c + d*x))^{(1/2)}*(320*B^2*a^ \\
& 6*b^{14}*d^3 - 16*B^2*a^2*b^{18}*d^3 + 1024*B^2*a^8*b^{12}*d^3 + 1440*B^2*a^{10}*b^ \\
& 10*d^3 + 1024*B^2*a^{12}*b^8*d^3 + 320*B^2*a^{14}*b^6*d^3 - 16*B^2*a^{18}*b^2*d^3 \\
& ))/4 + 96*B^3*a^6*b^{13}*d^2 + 240*B^3*a^8*b^{11}*d^2 + 320*B^3*a^{10}*b^9*d^2 + \\
& 240*B^3*a^{12}*b^7*d^2 + 96*B^3*a^{14}*b^5*d^2 + 16*B^3*a^{16}*b^3*d^2)*(((320*B^4*a^ \\
& 6*b^8*d^4 - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^{10}*b^4*d^4 \\
& - 400*B^4*a^{12}*b^2*d^4)^{(1/2)} + 4*B^2*a^7*d^2 + 20*B^2*a^3*b^4*d^2 - 40*B^2*a^5*b^2*d^2) \\
& /((a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + \\
& 5*a^8*b^2*d^4))^{(1/2))/4 - \log(16*B^3*a^4*b^{15}*d^2 - \\
& (((320*B^4*a^6*b^8*d^4 - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^{10}*b^4*d^4 \\
& - 400*B^4*a^{12}*b^2*d^4)^{(1/2)} - 4*B^2*a^7*d^2 - 20*B^2*a^3*b^4*d^2 + 40*B^2*a^5*b^2*d^2) \\
& /((a^{10}*d^4 + b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(((320*B^4*a^ \\
& 6*b^8*d^4 - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^{10}*b^4*d^4 \\
& - 400*B^4*a^{12}*b^2*d^4)^{(1/2)} - 4*B^2*a^7*d^2 - 20*B^2*a^3*b^4*d^2 + 40
\end{aligned}$$

$$\begin{aligned}
& *B^2*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6 \\
& *d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(896*B*a^7*b^{15}*d^4 - 32*B* \\
& a*b^{21}*d^4 - 160*B*a^3*b^{19}*d^4 - 128*B*a^5*b^{17}*d^4 - (((320*B^4*a^6*b^8*d \\
& ^4 - 16*B^4*a^4*b^{10}*d^4 - 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^{10}*b^4*d^4 - 4 \\
& 00*B^4*a^{12}*b^2*d^4))^{(1/2)} - 4*B^2*a^7*d^2 - 20*B^2*a^3*b^4*d^2 + 40*B^2*a^ \\
& 5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + \\
& 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(64*a*b \\
& ^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440* \\
& a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}...
\end{aligned}$$

$$3.369 \quad \int \frac{\cot(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=154

$$-\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}$$

[Out]  $-2*B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/a^{(1/2)})/a^{(3/2)}/d+B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(3/2)}/d+B*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(3/2)}/d+2*b^2*B/a/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 0.35, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {21, 3650, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b^2B}{ad(a^2+b^2)\sqrt{a+b \tan(c+dx)}} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x]*(a*B+b*B*\operatorname{Tan}[c+d*x]))/(a+b*\operatorname{Tan}[c+d*x])^{(5/2)},x]$

[Out]  $(-2*B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a]])/(a^{(3/2)*d})+(B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a-I*b]])/((a-I*b)^{(3/2)*d})+(B*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/\operatorname{Sqrt}[a+I*b]])/((a+I*b)^{(3/2)*d})+(2*b^2*B)/(a*(a^2+b^2)*d*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])$

Rule 21

$\operatorname{Int}[(u_.)*((a_.)+(b_.)*(v_.))^{(m_.)*((c_.)+(d_.)*(v_.))^{(n_.)},x\_Symbol] :>$   
 $\operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c+d*v)^{(m+n)},x],x] /;$   $\operatorname{FreeQ}[\{a,b,c,d,n\},x]$   
 $\&\& \operatorname{EqQ}[b*c-a*d,0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \|\operatorname{SimplerQ}[c+d*x, a+b*x])$

Rule 65

$\operatorname{Int}[(a_.)+(b_.)*(x_.))^{(m_.)*((c_.)+(d_.)*(x_.))^{(n_.)},x\_Symbol] :>$   $\operatorname{With}[\{p=\operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)},x],x,(a+b*x)^{(1/p)}],x] /;$   $\operatorname{FreeQ}[\{a,b,c,d\},x] \&\& \operatorname{NeQ}[b*c-a*d,0] \&\& \operatorname{LtQ}[-1,m,0] \&\& \operatorname{LeQ}[-1,n,0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a,b,c,d,m,n,x]$

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3650

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^n, x\_Symbol] := Simp[b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d))), x] + Dist[1/((m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^n)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

Int((((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2))/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&

!GtQ[n, 0] &amp;&amp; !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}} dx &= B \int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}} dx \\
&= \frac{2b^2 B}{a(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{(2B) \int \frac{\cot(c+dx)(\frac{1}{2}(a^2+b^2)-\frac{1}{2}ab\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx}{a(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2b^2 B}{a(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{B \int \frac{\cot(c+dx)(1+\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx}{a} \\
&= \frac{2b^2 B}{a(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{B \int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx}{2(ia-b)} \\
&= \frac{2b^2 B}{a(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{B \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}}\right)}{2(a-ib)} \\
&= -\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b^2 B}{a(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{2B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 0.78, size = 166, normalized size = 1.08

$$\frac{B \left( -\frac{2(a^2+b^2) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{a(a+ib) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{a(a-ib) \tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{2b^2}{\sqrt{a+b\tan(c+dx)}} \right)}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2), x]
```

```
[Out] (B*((-2*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]/Sqrt[a] + (a*(a + I*b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] +
```



$$\frac{(a*(a - I*b)*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/\text{Sqrt}[a + I*b]])/\text{Sqrt}[a + I*b] + (2*b^2)/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])}{(a*(a^2 + b^2)*d)}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 1.16, size = 39351, normalized size = 255.53

method	result	size
default	Expression too large to display	39351

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,algorithm="maxima")`

[Out] Timed out

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 6958 vs. 2(126) = 252.

time = 2.89, size = 13991, normalized size = 90.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,algorithm="fricas")`

[Out] 
$$\begin{aligned} & [1/4*(4*\text{sqrt}(2)*((a^{12} + 3*a^{10}*b^2 + 2*a^8*b^4 - 2*a^6*b^6 - 3*a^4*b^8 - a^2*b^{10})*d^5*\cos(d*x + c)^2 + 2*(a^{11}*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 \\ & + a^3*b^9)*d^5*\cos(d*x + c)*\sin(d*x + c) + (a^{10}*b^2 + 4*a^8*b^4 + 6*a^6*b^6 + 4*a^4*b^8 + a^2*b^{10})*d^5)*\text{sqrt}((B^2*a^6 + 3*B^2*a^4*b^2 + 3*B^2*a^2*b^4 + B^2*b^6 - (a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^2*\text{sqrt}(B^4/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4)))/(9*B^2*a^4*b^2 - 6*B^2*a^2*b^4 + B^2*b^6))*B^4/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^4))^{3/4}*\text{sqrt}((9*B^4*a^4*b^2 - 6*B^4*a^2*b^4 + B^4*b^6)/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*d^4))*\arctan(-((3*B^6*a^{12} + 14*B^6*a^{10}*b^2 + 25*B^6*a^8*b^4 + 20*B^6*a^6*b^6 + 5*B^6*a^4*b^8 - 2*B^6*a^2*b^{10} - \end{aligned}$$

$$\begin{aligned}
& B^6 b^{12} d^4 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)} \sqrt{(9B^4 a^4 b^2 - 6B^4 a^2 b^4 + B^4 b^6) / ((a^{12} + 6a^{10} b^2 + 15a^8 b^4 + 20a^6 b^6 + 15a^4 b^8 + 6a^2 b^{10} + b^{12}) d^4)} + (3B^8 a^9 + 8B^8 a^7 b^2 + 6B^8 a^5 b^4 - B^8 a^3 b^8) d^2 \sqrt{(9B^4 a^4 b^2 - 6B^4 a^2 b^4 + B^4 b^6) / ((a^{12} + 6a^{10} b^2 + 15a^8 b^4 + 20a^6 b^6 + 15a^4 b^8 + 6a^2 b^{10} + b^{12}) d^4)} + \sqrt{2} ((a^{14} + 5a^{12} b^2 + 9a^{10} b^4 + 5a^8 b^6 - 5a^6 b^8 - 9a^4 b^{10} - 5a^2 b^{12} - b^{14}) d^7 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) \sqrt{(9B^4 a^4 b^2 - 6B^4 a^2 b^4 + B^4 b^6) / ((a^{12} + 6a^{10} b^2 + 15a^8 b^4 + 20a^6 b^6 + 15a^4 b^8 + 6a^2 b^{10} + b^{12}) d^4)} + (B^2 a^{11} + 5B^2 a^9 b^2 + 10B^2 a^7 b^4 + 10B^2 a^5 b^6 + 5B^2 a^3 b^8 + B^2 a b^{10}) d^5 \sqrt{(9B^4 a^4 b^2 - 6B^4 a^2 b^4 + B^4 b^6) / ((a^{12} + 6a^{10} b^2 + 15a^8 b^4 + 20a^6 b^6 + 15a^4 b^8 + 6a^2 b^{10} + b^{12}) d^4)}) \sqrt{(B^2 a^6 + 3B^2 a^4 b^2 + 3B^2 a^2 b^4 + B^2 b^6 - (a^9 - 6a^5 b^4 - 8a^3 b^6 - 3a b^8) d^2 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) / (9B^2 a^4 b^2 - 6B^2 a^2 b^4 + B^2 b^6)) \sqrt{((9B^4 a^8 + 12B^4 a^6 b^2 - 2B^4 a^4 b^4 - 4B^4 a^2 b^6 + B^4 b^8) d^2 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) \cos(dx + c) + \sqrt{2} ((9B^3 a^9 + 12B^3 a^7 b^2 - 2B^3 a^5 b^4 - 4B^3 a^3 b^6 + B^3 a b^8) d^3 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) \cos(dx + c) + (9B^5 a^6 - 15B^5 a^4 b^2 + 7B^5 a^2 b^4 - B^5 b^6) d \cos(dx + c) \sqrt{(B^2 a^6 + 3B^2 a^4 b^2 + 3B^2 a^2 b^4 + B^2 b^6 - (a^9 - 6a^5 b^4 - 8a^3 b^6 - 3a b^8) d^2 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) / (9B^2 a^4 b^2 - 6B^2 a^2 b^4 + B^2 b^6)) \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)} * (B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4))^{1/4} + (9B^6 a^5 - 6B^6 a^3 b^2 + B^6 a b^4) \cos(dx + c) + (9B^6 a^4 b - 6B^6 a^2 b^3 + B^6 b^5) \sin(dx + c) / \cos(dx + c) * (B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4))^{3/4} + \sqrt{2} ((3B^3 a^{16} + 14B^3 a^{14} b^2 + 22B^3 a^{12} b^4 + 6B^3 a^{10} b^6 - 20B^3 a^8 b^8 - 22B^3 a^6 b^{10} - 6B^3 a^4 b^{12} + 2B^3 a^2 b^{14} + B^3 b^{16}) d^7 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) \sqrt{(9B^4 a^4 b^2 - 6B^4 a^2 b^4 + B^4 b^6) / ((a^{12} + 6a^{10} b^2 + 15a^8 b^4 + 20a^6 b^6 + 15a^4 b^8 + 6a^2 b^{10} + b^{12}) d^4)} + (3B^5 a^{13} + 14B^5 a^{11} b^2 + 25B^5 a^9 b^4 + 20B^5 a^7 b^6 + 5B^5 a^5 b^8 - 2B^5 a^3 b^{10} - B^5 a b^{12}) d^5 \sqrt{(9B^4 a^4 b^2 - 6B^4 a^2 b^4 + B^4 b^6) / ((a^{12} + 6a^{10} b^2 + 15a^8 b^4 + 20a^6 b^6 + 15a^4 b^8 + 6a^2 b^{10} + b^{12}) d^4)}) \sqrt{(B^2 a^6 + 3B^2 a^4 b^2 + 3B^2 a^2 b^4 + B^2 b^6 - (a^9 - 6a^5 b^4 - 8a^3 b^6 - 3a b^8) d^2 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) / (9B^2 a^4 b^2 - 6B^2 a^2 b^4 + B^2 b^6)) \sqrt{(a \cos(dx + c) + b \sin(dx + c)) / \cos(dx + c)} * (B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4))^{3/4} / (9B^{10} a^4 b^2 - 6B^{10} a^2 b^4 + B^{10} b^6)) + 4 \sqrt{2} ((a^{12} + 3a^{10} b^2 + 2a^8 b^4 - 2a^6 b^6 - 3a^4 b^8 - a^2 b^{10}) d^5 \cos(dx + c)^2 + 2(a^{11} b + 4a^9 b^3 + 6a^7 b^5 + 4a^5 b^7 + a^3 b^9) d^5 \cos(dx + c) \sin(dx + c) + (a^{10} b^2 + 4a^8 b^4 + 6a^6 b^6 + 4a^4 b^8 + a^2 b^{10}) d^5) \sqrt{(B^2 a^6 + 3B^2 a^4 b^2 + 3B^2 a^2 b^4 + B^2 b^6 - (a^9 - 6a^5 b^4 - 8a^3 b^6 - 3a b^8) d^2 \sqrt{B^4 / ((a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) d^4)}) / (9B^2 a^4 b^2 - 6B^2 a^2 b^4 + B^2 b^6)) * (B^4 / ((
\end{aligned}$$

$$a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)^{(3/4)} \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)} \arctan(((3B^6a^{12} + 14B^6a^{10}b^2 + 25B^6a^8b^4 + 20B^6a^6b^6 + 5B^6a^4b^8 - 2B^6a^2b^{10} - B^6b^{12})d^4 \sqrt{B^4/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)} + (3B^8a^9 + 8B^8a^7b^2 + 6B^8a^5b^4 - B^8a^3b^8)d^2 \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)} - \sqrt{2} * ((a^{14} + 5a^{12}b^2 + 9a^{10}b^4 + 5a^8b^6 - 5a^6b^8 - 9a^4b^{10} - 5a^2b^{12} - b^{14})d^7 \sqrt{B^4/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} \sqrt{(9B^4a^4b^2 - 6B^4a^2b^4 + B^4b^6)/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}))$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\cot(c + dx)}{a \sqrt{a + b \tan(c + dx)} + b \sqrt{a + b \tan(c + dx)} \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] B\*Integral(cot(c + d\*x)/(a\*sqrt(a + b\*tan(c + d\*x)) + b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.76, size = 2500, normalized size = 16.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] (log((((96B^4a^2b^4d^4 - 16B^4b^6d^4 - 144B^4a^4b^2d^4)^(1/2) + 4B^2a^3d^2 - 12B^2a\*b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a

$$\begin{aligned}
& ^4b^2d^4)^{(1/2)} * (((((((96B^4a^2b^4d^4 - 16B^4b^6d^4 - 144B^4a^4 \\
& *b^2d^4)^{(1/2)} + 4B^2a^3d^2 - 12B^2a*b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2 \\
& b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (512B^8a^8b^28d^8 - (((96B^4a^2b^4 \\
& 4d^4 - 16B^4b^6d^4 - 144B^4a^4b^2d^4)^{(1/2)} + 4B^2a^3d^2 - 12B^2 \\
& a*b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (a \\
& + b*\tan(c + d*x))^{(1/2)} * (512a^9b^28d^9 + 5376a^11b^26d^9 + 25344a^13 \\
& *b^24d^9 + 70656a^15b^22d^9 + 129024a^17b^20d^9 + 161280a^19b^18d^9 \\
& ^9 + 139776a^21b^16d^9 + 82944a^23b^14d^9 + 32256a^25b^12d^9 + 742 \\
& 4a^27b^10d^9 + 768a^29b^8d^9))/4 + 5248B^10a^10b^26d^8 + 23936B^12a^12 \\
& b^24d^8 + 64000B^14a^14b^22d^8 + 111104B^16a^16b^20d^8 + 130816B^18a^18 \\
& *b^18d^8 + 105728B^20a^20b^16d^8 + 57856B^22a^22b^14d^8 + 20480B^24a^24b \\
& ^12d^8 + 4224B^26a^26b^10d^8 + 384B^28a^28b^8d^8))/4 + (a + b*\tan(c + d* \\
& x))^{(1/2)} * (256B^2a^8b^26d^7 + 1472B^2a^10b^24d^7 + 3712B^2a^12b^22 \\
& d^7 + 6272B^2a^14b^20d^7 + 9856B^2a^16b^18d^7 + 14336B^2a^18b^16 \\
& d^7 + 15232B^2a^20b^14d^7 + 10112B^2a^22b^12d^7 + 3712B^2a^24 \\
& *b^10d^7 + 576B^2a^26b^8d^7)) * (((96B^4a^2b^4d^4 - 16B^4b^6d^4 - \\
& 144B^4a^4b^2d^4)^{(1/2)} + 4B^2a^3d^2 - 12B^2a*b^2d^2)/(a^6d^4 + \\
& b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)}/4 - 128B^3a^7b^26d^6 - \\
& 128B^3a^9b^24d^6 + 2592B^3a^11b^22d^6 + 10976B^3a^13b^20d^6 + \\
& 20384B^3a^15b^18d^6 + 20832B^3a^17b^16d^6 + 11872B^3a^19b^14d^6 \\
& + 3232B^3a^21b^12d^6 + 96B^3a^23b^10d^6 - 96B^3a^25b^8d^6))/4 \\
& - (a + b*\tan(c + d*x))^{(1/2)} * (1120B^4a^15b^16d^5 - 352B^4a^9b^22d^5 \\
& - 672B^4a^11b^20d^5 - 224B^4a^13b^18d^5 - 64B^4a^7b^24d^5 + 20 \\
& 16B^4a^17b^14d^5 + 1568B^4a^19b^12d^5 + 608B^4a^21b^10d^5 + 96B^4 \\
& a^23b^8d^5)) * (((96B^4a^2b^4d^4 - 16B^4b^6d^4 - 144B^4a^4b^2 \\
& d^4)^{(1/2)} + 4B^2a^3d^2 - 12B^2a*b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4 \\
& d^4 + 3a^4b^2d^4))^{(1/2)}/4 + (\log(((96B^4a^2b^4d^4 - 16B^4b^6d^4 - \\
& 144B^4a^4b^2d^4)^{(1/2)} - 4B^2a^3d^2 + 12B^2a*b^2d^2)/(a^6d^4 + b^6d^4 + \\
& 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (((96B^4a^2b^4d^4 - 16B^4b^6d^4 - \\
& 144B^4a^4b^2d^4)^{(1/2)} - 4B^2a^3d^2 + 12B^2a*b^2d^2)/(a^6d^4 + b^6d^4 + \\
& 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (512B^8a^8b^28d^8 - (((96B^4a^2b^4d^4 - \\
& 16B^4b^6d^4 - 144B^4a^4b^2d^4)^{(1/2)} - 4B^2a^3d^2 + 12B^2a*b^2d^2)/(a^6d^4 + \\
& b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{(1/2)} * (a + b*\tan(c + d*x))^{(1/2)} * (512a^9b^ \\
& 28d^9 + 5376a^11b^26d^9 + 25344a^13b^24d^9 + 70656a^15b^22d^9 + 1 \\
& 29024a^17b^20d^9 + 161280a^19b^18d^9 + 139776a^21b^16d^9 + 82944a^ \\
& ^23b^14d^9 + 32256a^25b^12d^9 + 7424a^27b^10d^9 + 768a^29b^8d^9) \\
& )/4 + 5248B^10a^10b^26d^8 + 23936B^12a^12b^24d^8 + 64000B^14a^14b^22d^8 \\
& + 111104B^16a^16b^20d^8 + 130816B^18a^18b^18d^8 + 105728B^20a^20b^16d^8 \\
& + 57856B^22a^22b^14d^8 + 20480B^24a^24b^12d^8 + 4224B^26a^26b^10d^8 + 38 \\
& 4B^28a^28b^8d^8))/4 + (a + b*\tan(c + d*x))^{(1/2)} * (256B^2a^8b^26d^7 + 1 \\
& 472B^2a^10b^24d^7 + 3712B^2a^12b^22d^7 + 6272B^2a^14b^20d^7 + 9 \\
& 856B^2a^16b^18d^7 + 14336B^2a^18b^16d^7 + 15232B^2a^20b^14d^7 + \\
& 10112B^2a^22b^12d^7 + 3712B^2a^24b^10d^7 + 576B^2a^26b^8d^7)) * \\
& (-(96B^4a^2b^4d^4 - 16B^4b^6d^4 - 144B^4a^4b^2d^4)^{(1/2)} - 4B^
\end{aligned}$$

$$\begin{aligned}
& 2*a^3*d^2 + 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}/4 - 128*B^3*a^7*b^26*d^6 - 128*B^3*a^9*b^24*d^6 + 2592*B^3*a^{11}*b^{22}*d^6 + 10976*B^3*a^{13}*b^{20}*d^6 + 20384*B^3*a^{15}*b^{18}*d^6 + 20832*B^3*a^{17}*b^{16}*d^6 + 11872*B^3*a^{19}*b^{14}*d^6 + 3232*B^3*a^{21}*b^{12}*d^6 + 96*B^3*a^{23}*b^{10}*d^6 - 96*B^3*a^{25}*b^8*d^6))/4 - (a + b*tan(c + d*x))^{(1/2)}*(1120*B^4*a^{15}*b^{16}*d^5 - 352*B^4*a^9*b^{22}*d^5 - 672*B^4*a^{11}*b^{20}*d^5 - 224*B^4*a^{13}*b^{18}*d^5 - 64*B^4*a^7*b^{24}*d^5 + 2016*B^4*a^{17}*b^{14}*d^5 + 1568*B^4*a^{19}*b^{12}*d^5 + 608*B^4*a^{21}*b^{10}*d^5 + 96*B^4*a^{23}*b^8*d^5))*(-((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}/4 - \log((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*tan(c + d*x))^{(1/2)}*(512*a^9*b^{28}*d^9 + 5376*a^{11}*b^{26}*d^9 + 25344*a^{13}*b^{24}*d^9 + 70656*a^{15}*b^{22}*d^9 + 129024*a^{17}*b^{20}*d^9 + 161280*a^{19}*b^{18}*d^9 + 139776*a^{21}*b^{16}*d^9 + 82944*a^{23}*b^{14}*d^9 + \dots
\end{aligned}$$

$$3.370 \quad \int \frac{-a+b \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=102

$$\frac{(ia-b) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} - \frac{(ia+b) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d}$$

[Out] (I\*a-b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d/(a-I\*b)^(1/2)-(I\*a+b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d/(a+I\*b)^(1/2)

Rubi [A]

time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {3620, 3618, 65, 214}

$$\frac{(-b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{(b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In] Int[(-a + b\*Tan[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((I\*a - b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/(Sqrt[a - I\*b]\*d) - ((I\*a + b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/(Sqrt[a + I\*b]\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^(m)/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b

\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned} \int \frac{-a + b \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx &= \frac{1}{2}(-a - ib) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx + \frac{1}{2}(-a + ib) \int \frac{1 - i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \\ &= \frac{(ia - b) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a - ibx}} dx, x, i \tan(c + dx)\right)}{2d} + \frac{(ia + b) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a + ibx}} dx, x, i \tan(c + dx)\right)}{2d} \\ &= \frac{(a - ib) \text{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{bd} + \frac{(a + ib) \text{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{bd} \\ &= \frac{(ia - b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{\sqrt{a - ib} d} - \frac{(ia + b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{\sqrt{a + ib} d} \end{aligned}$$

### Mathematica [A]

time = 0.11, size = 109, normalized size = 1.07

$$\frac{i \left( (a + ib)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right) - (a - ib)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right) \right)}{\sqrt{a - ib} \sqrt{a + ib} d}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Tan[c + d\*x])/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] (I\*((a + I\*b)^(3/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]] - (a - I\*b)^(3/2)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/(Sqrt[a - I\*b]\*Sqrt[a + I\*b]\*d)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(84) = 168.

time = 0.15, size = 783, normalized size = 7.68

method	result
derivativedivides	$\frac{\left( -\sqrt{2\sqrt{a^2+b^2}+2a}\sqrt{a^2+b^2}a^3-\sqrt{2\sqrt{a^2+b^2}+2a}\sqrt{a^2+b^2}ab^2+\sqrt{2\sqrt{a^2+b^2}} \right)}{2b}$
default	$\frac{\left( -\sqrt{2\sqrt{a^2+b^2}+2a}\sqrt{a^2+b^2}a^3-\sqrt{2\sqrt{a^2+b^2}+2a}\sqrt{a^2+b^2}ab^2+\sqrt{2\sqrt{a^2+b^2}} \right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{2/d*b*(1/4/b^2/(a^2+b^2)^{3/2}*(1/2*(-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^4-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^4)*\ln(b*\tan(d*x+c)+a+(a+b*\tan(d*x+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}+(a^2+b^2)^{1/2}))+2*(-4*a^3*b^2-4*a*b^4-1/2*(-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^4-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^4)*(2*(a^2+b^2)^{1/2}+2*a)^{1/2})}{(2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((2*(a+b*\tan(d*x+c))^{1/2}+(2*(a^2+b^2)^{1/2}+2*a)^{1/2})/(2*(a^2+b^2)^{1/2}-2*a)^{1/2}))+1/4/b^2/(a^2+b^2)^{3/2}*(-1/2*(-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^4-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^4)*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{1/2}*(2*(a^2+b^2)^{1/2}+2*a)^{1/2}-(a^2+b^2)^{1/2}))+2*(4*a^3*b^2+4*a*b^4+1/2*(-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a^3-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*(a^2+b^2)^{1/2}*a*b^2+(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*a^4-(2*(a^2+b^2)^{1/2}+2*a)^{1/2}*b^4)*(2*(a^2+b^2)^{1/2}+2*a)^{1/2})}{(2*(a^2+b^2)^{1/2}-2*a)^{1/2}*\arctan((-2*(a+b*\tan(d*x+c))^{1/2}+(2*(a^2+b^2)^{1/2}+2*a)^{1/2})/(2*(a^2+b^2)^{1/2}-2*a)^{1/2})))}$$

**Maxima** [F(-2)]



time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 3540 vs. 2(78) = 156.

time = 1.47, size = 3540, normalized size = 34.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$-1/4*(4*\sqrt{2}*(a^2 + b^2)*d^4*\sqrt{(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^5 - 2*a^3*b^2 - 3*a*b^4)*d^2*\sqrt{(a^2 + b^2)/d^4}})/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*((a^2 + b^2)/d^4)^{3/4}*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\arctan(((3*a^8 + 8*a^6*b^2 + 6*a^4*b^4 - b^8)*d^4*\sqrt{(a^2 + b^2)/d^4}*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} + (3*a^9 + 8*a^7*b^2 + 6*a^5*b^4 - a*b^8)*d^2*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} + \sqrt{2}*(2*a*d^7*\sqrt{(a^2 + b^2)/d^4}*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} + (a^2 + b^2)*d^5*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}))*\sqrt{(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^5 - 2*a^3*b^2 - 3*a*b^4)*d^2*\sqrt{(a^2 + b^2)/d^4}})/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*\sqrt{((9*a^8*b^2 + 12*a^6*b^4 - 2*a^4*b^6 - 4*a^2*b^8 + b^{10})*d^2*\sqrt{(a^2 + b^2)/d^4}*\cos(d*x + c) + \sqrt{2}*((9*a^6*b^3 + 3*a^4*b^5 - 5*a^2*b^7 + b^9)*d^3*\sqrt{(a^2 + b^2)/d^4}*\cos(d*x + c) + 2*(9*a^7*b^3 + 3*a^5*b^5 - 5*a^3*b^7 + a*b^9)*d*\cos(d*x + c))*\sqrt{(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^5 - 2*a^3*b^2 - 3*a*b^4)*d^2*\sqrt{(a^2 + b^2)/d^4}})/(9*a^4*b^2 - 6*a^2*b^4 + b^6))*\sqrt{(a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))*((a^2 + b^2)/d^4)^{1/4} + (9*a^9*b^2 + 12*a^7*b^4 - 2*a^5*b^6 - 4*a^3*b^8 + a*b^{10})*\cos(d*x + c) + (9*a^8*b^3 + 12*a^6*b^5 - 2*a^4*b^7 - 4*a^2*b^9 + b^{11})*\sin(d*x + c))/\cos(d*x + c))*((a^2 + b^2)/d^4)^{3/4} + \sqrt{2}*(2*(3*a^5*b + 2*a^3*b^3 - a*b^5)*d^7*\sqrt{(a^2 + b^2)/d^4}*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)} + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d^5*\sqrt{(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}))*\sqrt{(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + (a^5 - 2*a^3*b^2 - 3*a*b^4)*d^2*\sqrt{(a^2 + b^2)/d^4}})$$

$$\begin{aligned}
& )/d^4))/((9a^4b^2 - 6a^2b^4 + b^6))\sqrt{(a\cos(dx + c) + b\sin(dx + c))} \\
& )/\cos(dx + c))*((a^2 + b^2)/d^4)^{(3/4)}/(9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10}) + 4\sqrt{2}*(a^2 + b^2)*d^4\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6))*((a^2 + b^2)/d^4)^{(3/4)}\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)}*\arctan(-((3a^8 + 8a^6b^2 + 6a^4b^4 - b^8)*d^4\sqrt{(a^2 + b^2)/d^4})\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)} + (3a^9 + 8a^7b^2 + 6a^5b^4 - ab^8)*d^2\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)} - \sqrt{2}*(2a*d^7\sqrt{(a^2 + b^2)/d^4})\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)} + (a^2 + b^2)*d^5\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)}))\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6))\sqrt{((9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10})*d^2\sqrt{(a^2 + b^2)/d^4})\cos(dx + c) - \sqrt{2}*((9a^6b^3 + 3a^4b^5 - 5a^2b^7 + b^9)*d^3\sqrt{(a^2 + b^2)/d^4})\cos(dx + c) + 2*(9a^7b^3 + 3a^5b^5 - 5a^3b^7 + ab^9)*d*\cos(dx + c))\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6))\sqrt{(a*\cos(dx + c) + b*\sin(dx + c))/\cos(dx + c))*((a^2 + b^2)/d^4)^{(1/4)} + (9a^9b^2 + 12a^7b^4 - 2a^5b^6 - 4a^3b^8 + ab^{10})*\cos(dx + c) + (9a^8b^3 + 12a^6b^5 - 2a^4b^7 - 4a^2b^9 + b^{11})*\sin(dx + c))/\cos(dx + c))*((a^2 + b^2)/d^4)^{(3/4)} - \sqrt{2}*(2*(3a^5b + 2a^3b^3 - ab^5)*d^7\sqrt{(a^2 + b^2)/d^4})\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)} + (3a^6b + 5a^4b^3 + a^2b^5 - b^7)*d^5\sqrt{(9a^4b^2 - 6a^2b^4 + b^6)/((a^4 + 2a^2b^2 + b^4)*d^4)}))\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6))\sqrt{(a*\cos(dx + c) + b*\sin(dx + c))/\cos(dx + c))*((a^2 + b^2)/d^4)^{(3/4)}/(9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10}) + \sqrt{2}*(a^4 + 2a^2b^2 + b^4 - (a^3 - 3ab^2)*d^2\sqrt{(a^2 + b^2)/d^4})\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6))*((a^2 + b^2)/d^4)^{(1/4)}*\log(((9a^8b^2 + 12a^6b^4 - 2a^4b^6 - 4a^2b^8 + b^{10})*d^2\sqrt{(a^2 + b^2)/d^4})\cos(dx + c) + \sqrt{2}*((9a^6b^3 + 3a^4b^5 - 5a^2b^7 + b^9)*d^3\sqrt{(a^2 + b^2)/d^4})\cos(dx + c) + 2*(9a^7b^3 + 3a^5b^5 - 5a^3b^7 + ab^9)*d*\cos(dx + c))\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6))\sqrt{(a*\cos(dx + c) + b*\sin(dx + c))/\cos(dx + c))*((a^2 + b^2)/d^4)^{(1/4)} + (9a^9b^2 + 12a^7b^4 - 2a^5b^6 - 4a^3b^8 + ab^{10})*\cos(dx + c) + (9a^8b^3 + 12a^6b^5 - 2a^4b^7 - 4a^2b^9 + b^{11})*\sin(dx + c))/\cos(dx + c)) - \sqrt{2}*(a^4 + 2a^2b^2 + b^4 - (a^3 - 3ab^2)*d^2\sqrt{(a^2 + b^2)/d^4})\sqrt{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6 + (a^5 - 2a^3b^2 - 3ab^4)*d^2\sqrt{(a^2 + b^2)/d^4)}/(9a^4b^2 - 6a^2b^4 + b^6)...
\end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{\sqrt{a + b \tan(c + dx)}} dx - \int \left( -\frac{b \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] -Integral(a/sqrt(a + b\*tan(c + d\*x)), x) - Integral(-b\*tan(c + d\*x)/sqrt(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 8.51, size = 2731, normalized size = 26.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a - b\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out]  $2*\operatorname{atanh}\left(\frac{(32*a^2*b^2*((-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4))}^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}\right)/((16*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*a*b^3*d^2*((-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4))}^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*a^4*b^2*d^4)^{(1/2)})/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*((-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*((-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (32*a^4*b^2*d^2*((-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4))}^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)})/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*((-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*((-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*((-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4))}^{(1/2)} - 2*\operatorname{atanh}\left(\frac{(32*a^2*b^4*d^2*((-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4))}^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}\right)/((16*a^2*b^7*d^5)/(a^2*d^4$

$$\begin{aligned}
& + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) \\
& ) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4* \\
& (-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (32*b^4*((-16*b^6*d^4)^{(1/2)})/(1 \\
& 6*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b* \\
& \tan(c + d*x))^{(1/2)}/((16*a^2*b^5*d^3)/(a^2*d^4 + b^2*d^4) - (16*b^5)/d + ( \\
& 4*a*b^3*d^2*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*b^6* \\
& d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{( \\
& 1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*b^6*d^4)^{(1/2)}/((16*a^2*b^7*d^5)/(a^ \\
& 2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^ \\
& 2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3 \\
& *d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*(-16*b^6*d^4)^{(1/2)}/(16*(a \\
& ^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} - 2*\operatorname{atanh}(( \\
& 32*b^4*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2* \\
& d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}/((16*b^5)/d - (16*a^2*b^ \\
& 5*d^3)/(a^2*d^4 + b^2*d^4) + (4*a*b^3*d^2*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b \\
& ^2*d^5)) - (32*a^2*b^4*d^2*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6* \\
& d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}/(16* \\
& b^7*d + 16*a^2*b^5*d - (16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d \\
& ^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2* \\
& d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (8*a*b^2* \\
& ((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b \\
& ^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*b^6*d^4)^{(1/2)}/(16*b^7*d + \\
& 16*a^2*b^5*d - (16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^ \\
& 2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + \\
& (4*a^3*b^3*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*((a*b^2*d^2)/(4*( \\
& a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)} - \\
& 2*\operatorname{atanh}((8*a*b^2*(- (-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^ \\
& 3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*(-16*a^4*b \\
& ^2*d^4)^{(1/2)}/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^ \\
& 2*d^4 + b^2*d^4) + (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d \\
& ^5) + (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (32*a^2* \\
& b^2*(- (-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2 \\
& *d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}/((16*a^4*b^3*d^3)/(a^2* \\
& d^4 + b^2*d^4) + (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) \\
& + (32*a^4*b^2*d^2*(- (-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a \\
& ^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}/((16*a^4 \\
& *b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*a \\
& ^3*b^3*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a*b^5*d^4*(-16 \\
& *a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*(- (-16*a^4*b^2*d^4)^{(1/2)})/(16*( \\
& a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}
\end{aligned}$$

$$3.371 \quad \int \frac{-a+b \tan(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=132

$$\frac{(ia-b) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(ia+b) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{4ab}{(a^2+b^2)d\sqrt{a+b \tan(c+dx)}}$$

[Out] (I\*a-b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(3/2)/d-(I\*a+b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(3/2)/d+4\*a\*b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(1/2)

**Rubi [A]**

time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b \tan(c+dx)}} + \frac{(-b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{3/2}} - \frac{(b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}}\right)}{d(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(-a + b\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((I\*a - b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(3/2)\*d) - ((I\*a + b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(3/2)\*d) + (4\*a\*b)/((a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])

$(m + 1) \text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

### Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned} \int \frac{-a + b \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx &= \frac{4ab}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{\int \frac{-a^2 + b^2 + 2ab \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{a^2 + b^2} \\ &= \frac{4ab}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} - \frac{(a - ib) \int \frac{1 - i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{2(a + ib)} - \frac{(a + ib) \int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx}{2(a - ib)} \\ &= \frac{4ab}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(a + ib) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a - ibx}} dx, x, i \tan(c + dx)\right)}{2(ia + b)d} \\ &= \frac{4ab}{(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(a - ib) \text{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{(a + ib)bd} \\ &= \frac{(ia - b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{3/2}d} - \frac{(ia + b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{(a + ib)^{3/2}d} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.25, size = 154, normalized size = 1.17

$$\frac{i \cos(c + dx) \left( (a + ib)^2 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{a + b \tan(c + dx)}{a - ib}\right) - (a - ib)^2 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{a + b \tan(c + dx)}{a + ib}\right) \right) (a - b \tan(c + dx))}{(a - ib)(a + ib)d(a \cos(c + dx) - b \sin(c + dx)) \sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((-I)\*Cos[c + d\*x]\*((a + I\*b)^2\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] - (a - I\*b)^2\*Hypergeometric2F1[-1/2, 1, 1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)])\*(a - b\*Tan[c + d\*x]))/((a - I\*b)\*(a + I\*b)\*d\*(a\*Cos[c + d\*x] - b\*Sin[c + d\*x])\*Sqrt[a + b\*Tan[c + d\*x]])

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 927 vs. 2(112) = 224.

time = 0.15, size = 928, normalized size = 7.03

method	result
derivativedivides	$2b \frac{\frac{2a}{(a^2+b^2) \sqrt{a + b \tan(dx + c)}}}{\left( \frac{-\sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 \right)}$
default	$2b \frac{\frac{2a}{(a^2+b^2) \sqrt{a + b \tan(dx + c)}}}{\left( \frac{-\sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 + \sqrt{2\sqrt{a^2 + b^2} + 2a} \sqrt{a^2 + b^2} a^4 \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, method=\_RETURNVERBOSE)

[Out] 2/d\*b\*(2\*a/(a^2+b^2)/(a+b\*tan(d\*x+c))^(1/2)+1/(a^2+b^2)\*(1/4/b^2/(a^2+b^2)^(3/2)\*(1/2\*(-(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^5-2\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^3\*b^2-3\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a\*b^4)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^

$$2+b^2)^{(1/2)}+2*(-6*a^4*b^2-4*a^2*b^4+2*b^6-1/2*(-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^4+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5-2*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3*b^2-3*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^4)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)))+1/4/b^2/(a^2+b^2)^{(3/2)}*(-1/2*(-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^4+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5-2*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3*b^2-3*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^4)*\ln(-b*\tan(d*x+c)-a+(a+b*\tan(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-(a^2+b^2)^{(1/2)}))+2*(6*a^4*b^2+4*a^2*b^4-2*b^6+1/2*(-(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*a^4+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}*b^4+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5-2*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3*b^2-3*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a*b^4)*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((-2*(a+b*\tan(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)))/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)))))$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 6318 vs. 2(106) = 212.

time = 1.66, size = 6318, normalized size = 47.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$-1/4*(4*\sqrt{2}*((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^5*\cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^5*\cos(d*x + c)*\sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^5)*\sqrt{(a^{10} + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^{10} + (a^{11} - 7*a^9*b^2 - 22*a^7*b^4 - 14*a^5*b^6 + 5*a^3*b^8 + 5*a*b^{10})*d^2*\sqrt{1/((a^2 + b^2)*d^4)}})/(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^{10})*\sqrt{(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^{10})}/((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 +$$





$$- 14a^5b^6 + 5a^3b^8 + 5ab^{10})d^2\sqrt{1/((a^2 + b^2)d^4)})/(25a^8b^2 - 100a^6b^4 + 110a^4b^6 - 20a^2b^8 + b^{10})\sqrt{(25a^8b^2 - 100a^6b^4 + 110a^4b^6 - 20a^2b^8 + b^{10})/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}*(1/((a^2 + b^2)d^4))^{3/4}*\arctan(-((5a^{12} + 10a^{10}b^2 - 9a^8b^4 - 36a^6b^6 - 29a^4b^8 - 6a^2b^{10} + b^{12})d^4*\sqrt{(25a^8b^2 - 100a^6b^4 + 110a^4b^6 - 20a^2b^8 + b^{10})/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}*\sqrt{1/((a^2 + b^2)d^4)} + (5a^{11} + 5a^9b^2 - 14a^7b^4 - 22a^5b^6 - 7a^3b^8 + ab^{10})d^2*\sqrt{(25a^8b^2 - 100a^6b^4 + 110a^4b^6 - 20a^2b^8 + b^{10})/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)} + \sqrt{2})*((3a^6 + 5a^4b^2 + a^2b^4 - b^6)d^7*\sqrt{(25a^8b^2 - 100a^6b^4 + 110a^4b^6 - 20a^2b^8 + b^{10})/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}*\sqrt{1/((a^2 + b^2)d^4)} + 2*(a^5 + 2a^3b^2 + ab^4)d^5*\sqrt{(25a^8b^2 - 100a^6b^4 + 110a^4b^6 - 20a^2b^8 + b^{10})/((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12})d^4)}))$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{a\sqrt{a+b\tan(c+dx)} + b\sqrt{a+b\tan(c+dx)} \tan(c+dx)} dx - \int \left( -\frac{b\tan(c+dx)}{a\sqrt{a+b\tan(c+dx)} + b\sqrt{a+b\tan(c+dx)} \tan(c+dx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] -Integral(a/(a\*sqrt(a + b\*tan(c + d\*x)) + b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)), x) - Integral(-b\*tan(c + d\*x)/(a\*sqrt(a + b\*tan(c + d\*x)) + b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.92, size = 2500, normalized size = 18.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a - b\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out]  $\log(-(((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 12ab^4d^2 + 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (32b^{13}d^4 + 96a^2b^{11}d^4 + 64a^4b^9d^4 - 64a^6b^7d^4 - 96a^8b^5d^4 - 32a^{10}b^3d^4 + (a + b \tan(c + dx))^{1/2} * (((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 12ab^4d^2 + 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (64ab^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5)) + (a + b \tan(c + dx))^{1/2} * (16b^{12}d^3 + 32a^2b^{10}d^3 - 32a^6b^6d^3 - 16a^8b^4d^3)) * (((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 12ab^4d^2 + 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} - 8ab^{11}d^2 - 24a^3b^9d^2 - 24a^5b^7d^2 - 8a^7b^5d^2) * (((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} - 12ab^4d^2 + 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} + \log(-(-(24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} + 12ab^4d^2 - 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (32b^{13}d^4 + 96a^2b^{11}d^4 + 64a^4b^9d^4 - 64a^6b^7d^4 - 96a^8b^5d^4 - 32a^{10}b^3d^4 + (a + b \tan(c + dx))^{1/2} * (-(((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} + 12ab^4d^2 - 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (64ab^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5)) + (a + b \tan(c + dx))^{1/2} * (16b^{12}d^3 + 32a^2b^{10}d^3 - 32a^6b^6d^3 - 16a^8b^4d^3)) * (-(((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} + 12ab^4d^2 - 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} - 8ab^{11}d^2 - 24a^3b^9d^2 - 24a^5b^7d^2 - 8a^7b^5d^2) * (-(((24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2} + 12ab^4d^2 - 4a^3b^2d^2)/(16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} + (\log((((96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4)^{1/2} - 4a^5d^2 + 12a^3b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4))^{1/2} * ((a + b \tan(c + dx))^{1/2} * (16a^2b^{10}d^3 + 32a^4b^8d^3 - 32a^8b^4d^3 - 16a^{10}b^2d^3) + (((96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4)^{1/2} - 4a^5d^2 + 12a^3b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (64a^2b^{11}d^4 + 256a^4b^9d^4 + 384a^6b^7d^4 + 256a^8b^5d^4 + 64a^{10}b^3d^4 - (((96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4)^{1/2} - 4a^5d^2 + 12a^3b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (a + b \tan(c + dx))^{1/2} * (64ab^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5))/4)/4)/4 - 8a^3b^9d^2 - 24a^5b^7d^2 - 24a^7b^5d^2 - 8a^9b^3d^2) * (((96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4)^{1/2} - 4a^5d^2 + 12a^3b^2d^2)/(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)))^{1/2} * (a + b \tan(c + dx))^{1/2} * (16b^{12}d^3 + 32a^2b^{10}d^3 - 32a^6b^6d^3 - 16a^8b^4d^3))$

$$\begin{aligned}
& a^8 b^2 d^4)^{(1/2)} - 4 a^5 d^2 + 12 a^3 b^2 d^2) / (a^6 d^4 + b^6 d^4 + 3 a^2 \\
& * b^4 d^4 + 3 a^4 b^2 d^4))^{(1/2)} / 4 + (\log((( - ((96 a^6 b^4 d^4 - 16 a^4 b^6 \\
& * d^4 - 144 a^8 b^2 d^4)^{(1/2)} + 4 a^5 d^2 - 12 a^3 b^2 d^2) / (a^6 d^4 + b^6 d^4 + 3 a^2 b^4 d^4 + 3 a^4 b^2 d^4))^{(1/2)} * ((a + b \tan(c + d x))^{(1/2)} * (16 \\
& * a^2 b^{10} d^3 + 32 a^4 b^8 d^3 - 32 a^8 b^4 d^3 - 16 a^{10} b^2 d^3) + (( - ((9 \\
& 6 a^6 b^4 d^4 - 16 a^4 b^6 d^4 - 144 a^8 b^2 d^4)^{(1/2)} + 4 a^5 d^2 - 12 a^3 \\
& b^2 d^2) / (a^6 d^4 + b^6 d^4 + 3 a^2 b^4 d^4 + 3 a^4 b^2 d^4))^{(1/2)} * (64 a^2 b^{11} d^4 + 256 a^4 b^9 d^4 + 384 a^6 b^7 d^4 + 256 a^8 b^5 d^4 + 64 a^{10} \\
& * b^3 d^4 - (( - ((96 a^6 b^4 d^4 - 16 a^4 b^6 d^4 - 144 a^8 b^2 d^4)^{(1/2)} + \\
& 4 a^5 d^2 - 12 a^3 b^2 d^2) / (a^6 d^4 + b^6 d^4 + 3 a^2 b^4 d^4 + 3 a^4 b^2 d^4))^{(1/2)} * (a + b \tan(c + d x))^{(1/2)} * (64 a^2 b^{12} d^5 + 320 a^3 b^{10} d^5 + \\
& 640 a^5 b^8 d^5 + 640 a^7 b^6 d^5 + 320 a^9 b^4 d^5 + 64 a^{11} b^2 d^5)) / 4)) / 4 - 8 a^3 b^9 d^2 - 24 a^5 b^7 d^2 - 24 a^7 b^5 d^2 - 8 a^9 b^3 d^2) * ( \\
& - ((96 a^6 b^4 d^4 - 16 a^4 b^6 d^4 - 144 a^8 b^2 d^4)^{(1/2)} + 4 a^5 d^2 - 1 \\
& 2 a^3 b^2 d^2) / (a^6 d^4 + b^6 d^4 + 3 a^2 b^4 d^4 + 3 a^4 b^2 d^4))^{(1/2)} / \\
& 4 - \log(- ((96 a^6 b^4 d^4 - 16 a^4 b^6 d^4 - 144 a^8 b^2 d^4)^{(1/2)} - 4 a^5 \\
& d^2 + 12 a^3 b^2 d^2) / (16 a^6 d^4 + 16 b^6 d^4 + 48 a^2 b^4 d^4 + 48 a^4 \\
& * b^2 d^4))^{(1/2)} * ((a + b \tan(c + d x))^{(1/2)} * (16 a^2 b^{10} d^3 + 32 a^4 b^8 \\
& d^3 - 32 a^8 b^4 d^3 - 16 a^{10} b^2 d^3) - (((96 a^6 b^4 d^4 - 16 a^4 b^6 d^4 \\
& - 144 a^8 b^2 d^4)^{(1/2)} - 4 a^5 d^2 + 12 a^3 b^2 d^2) / (16 a^6 d^4 + 16 b^6 \\
& d^4 + 48 a^2 b^4 d^4 + 48 a^4 b^2 d^4))^{(1/2)} * (64 a^2 b^{11} d^4 + 256 a^4 \\
& * b^9 d^4 + 384 a^6 b^7 d^4 + 256 a^8 b^5 d^4 + \dots
\end{aligned}$$

$$3.372 \quad \int \frac{-a + b \tan(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx$$

**Optimal.** Leaf size=174

$$\frac{(ia - b) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2} d} - \frac{(ia + b) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2} d} + \frac{4ab}{3(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}}$$

[Out] (I\*a-b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(5/2)/d-(I\*a+b)\*arctanh((a+b\*tan(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(5/2)/d+2\*b\*(3\*a^2-b^2)/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)+4/3\*a\*b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {3610, 3620, 3618, 65, 214}

$$\frac{2b(3a^2 - b^2)}{d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} + \frac{4ab}{3d(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{(-b + ia) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{d(a - ib)^{5/2}} - \frac{(b + ia) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}} \right)}{d(a + ib)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(-a + b\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((I\*a - b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a - I\*b]])/((a - I\*b)^(5/2)\*d) - ((I\*a + b)\*ArcTanh[Sqrt[a + b\*Tan[c + d\*x]]/Sqrt[a + I\*b]])/((a + I\*b)^(5/2)\*d) + (4\*a\*b)/(3\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*b\*(3\*a^2 - b^2))/((a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 214**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3610**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/

```
(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_.)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_.)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int \frac{-a + b \tan(c + dx)}{(a + b \tan(c + dx))^{5/2}} dx &= \frac{4ab}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{\int \frac{-a^2 + b^2 + 2ab \tan(c + dx)}{(a + b \tan(c + dx))^{3/2}} dx}{a^2 + b^2} \\
&= \frac{4ab}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} + \frac{\int -a}{(a - b) \sqrt{a + b \tan(c + dx)}} \\
&= \frac{4ab}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} - \frac{(a - b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{5/2}d} \\
&= \frac{4ab}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} - \frac{(ia - b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{5/2}d} \\
&= \frac{4ab}{3(a^2 + b^2)d(a + b \tan(c + dx))^{3/2}} + \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} - \frac{(ia + b) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{(a + ib)^{5/2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.21, size = 156, normalized size = 0.90

$$\frac{i \cos(c + dx) \left( (a + ib)^2 {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}, \frac{a+b \tan(c+dx)}{a-ib}\right) - (a - ib)^2 {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}, \frac{a+b \tan(c+dx)}{a+ib}\right) \right) (a - b \tan(c + dx))}{3(a - ib)(a + ib)d(a \cos(c + dx) - b \sin(c + dx))(a + b \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((-1/3\*I)\*Cos[c + d\*x]\*((a + I\*b)^2\*Hypergeometric2F1[-3/2, 1, -1/2, (a + b\*Tan[c + d\*x])/(a - I\*b)] - (a - I\*b)^2\*Hypergeometric2F1[-3/2, 1, -1/2, (a + b\*Tan[c + d\*x])/(a + I\*b)])\*(a - b\*Tan[c + d\*x]))/((a - I\*b)\*(a + I\*b)\*d\*(a\*Cos[c + d\*x] - b\*Sin[c + d\*x])\*(a + b\*Tan[c + d\*x])^(3/2))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. 2(150) = 300.

time = 0.16, size = 1182, normalized size = 6.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, method=\_RETURNVERBOSE)

[Out] 2/d\*b\*(-1/(a^2+b^2)^2\*(-3\*a^2+b^2)/(a+b\*tan(d\*x+c))^(1/2)+2/3\*a/(a^2+b^2)/(a+b\*tan(d\*x+c))^(3/2)+1/(a^2+b^2)^2\*(1/4/b^2/(a^2+b^2)^(3/2)\*(1/2\*(-2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5+2\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^2+3\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6-5\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^2-5\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^6)\*ln(b\*tan(d\*x+c)+a+(a+b\*tan(d\*x+c))^(1/2)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))+2\*(-8\*a^5\*b^2+8\*a\*b^6-1/2\*(-2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5+2\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^2+3\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6-5\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^2-5\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^6)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((2\*(a+b\*tan(d\*x+c))^(1/2)+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)))+1/4/b^2/(a^2+b^2)^(3/2)\*(-1/2\*(-2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^5+2\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a^3\*b^2+3\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)\*a\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^6-5\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^4\*b^2-5\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a^2\*b^4+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*b^6)\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((-2\*(a+

$$b \cdot \tan(dx+c)^{(1/2)} + (2 \cdot (a^2+b^2)^{(1/2)} + 2 \cdot a)^{(1/2)} / (2 \cdot (a^2+b^2)^{(1/2)} - 2 \cdot a)^{(1/2)}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 10036 vs. 2(144) = 288.

time = 3.16, size = 10036, normalized size = 57.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$\frac{1}{12} \cdot (12 \cdot \sqrt{2}) \cdot ((a^{14} - a^{12}b^2 - 19a^{10}b^4 - 45a^8b^6 - 45a^6b^8 - 19a^4b^{10} - a^2b^{12} + b^{14}) \cdot d^5 \cos(dx+c)^4 + 2 \cdot (3a^{12}b^2 + 14a^{10}b^4 + 25a^8b^6 + 20a^6b^8 + 5a^4b^{10} - 2a^2b^{12} - b^{14}) \cdot d^5 \cos(dx+c)^2 + (a^{10}b^4 + 5a^8b^6 + 10a^6b^8 + 10a^4b^{10} + 5a^2b^{12} + b^{14}) \cdot d^5 + 4 \cdot ((a^{13}b + 4a^{11}b^3 + 5a^9b^5 - 5a^5b^9 - 4a^3b^{11} - ab^{13}) \cdot d^5 \cos(dx+c)^3 + (a^{11}b^3 + 5a^9b^5 + 10a^7b^7 + 10a^5b^9 + 5a^3b^{11} + ab^{13}) \cdot d^5 \cos(dx+c)) \cdot \sin(dx+c) \cdot \sqrt{(a^{14} + 7a^{12}b^2 + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14} + (a^{17} - 16a^{15}b^2 - 60a^{13}b^4 - 32a^{11}b^6 + 110a^9b^8 + 176a^7b^{10} + 84a^5b^{12} - 7ab^{16}) \cdot d^2 \sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4)}}) / (49a^{12}b^2 - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} - 42a^2b^{12} + b^{14})) \cdot \sqrt{(49a^{12}b^2 - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} - 42a^2b^{12} + b^{14}) / ((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20}) \cdot d^4)} \cdot (1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6) \cdot d^4))^{3/4} \cdot \arctan(-((7a^{20} + 14a^{18}b^2 - 77a^{16}b^4 - 344a^{14}b^6 - 546a^{12}b^8 - 364a^{10}b^{10} + 14a^8b^{12} + 168a^6b^{14} + 91a^4b^{16} + 14a^2b^{18} - b^{20}) \cdot d^4 \cdot \sqrt{(49a^{12}b^2 - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} - 42a^2b^{12} + b^{14}) / ((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20}) \cdot d^4)})$$



$$\begin{aligned}
& d^4) \sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} + (7a^{17} - 84a^{13} \\
& *b^4 - 176a^{11}b^6 - 110a^9b^8 + 32a^7b^{10} + 60a^5b^{12} + 16a^3b^{14} \\
& - ab^{16})d^2\sqrt{(49a^{12}b^2 - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 \\
& + 511a^4b^{10} - 42a^2b^{12} + b^{14})/((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 \\
& + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} \\
& + 45a^4b^{16} + 10a^2b^{18} + b^{20})d^4)} + \sqrt{2}*(4*(a^{15} + 5a^{13}b^2 \\
& + 9a^{11}b^4 + 5a^9b^6 - 5a^7b^8 - 9a^5b^{10} - 5a^3b^{12} - ab^{14})d^7 \\
& * \sqrt{(49a^{12}b^2 - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} \\
& - 42a^2b^{12} + b^{14})/((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 \\
& + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} + 45a^4b^{16} \\
& + 10a^2b^{18} + b^{20})d^4)} * \sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} \\
& + (3a^{12} + 14a^{10}b^2 + 25a^8b^4 + 20a^6b^6 + 5a^4b^8 - 2a^2b^{10} \\
& - b^{12})d^5\sqrt{(49a^{12}b^2 - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 \\
& + 511a^4b^{10} - 42a^2b^{12} + b^{14})/((a^{20} + 10a^{18}b^2 + 45a^{16}b^4 \\
& + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} + 120a^6b^{14} \\
& + 45a^4b^{16} + 10a^2b^{18} + b^{20})d^4)} * \sqrt{(a^{14} + 7a^{12}b^2 + 21a^{10}b^4 \\
& + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14} + (a^{17} - 16a^{15}b^2 \\
& - 60a^{13}b^4 - 32a^{11}b^6 + 110a^9b^8 + 176a^7b^{10} + 84a^5b^{12} - 7a^3b^{16}) \\
& * d^2\sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)})) / (49a^{12}b^2 - 490a^{10}b^4 \\
& + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} - 42a^2b^{12} + b^{14}) * \sqrt{((49a^{20}b^2 - 294a^{18}b^4 \\
& - 147a^{16}b^6 + 1848a^{14}b^8 + 1778a^{12}b^{10} - 1316a^{10}b^{12} - 1518a^8b^{14} + 312a^6b^{16} \\
& + 349a^4b^{18} - 38a^2b^{20} + b^{22})d^2\sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} \\
& * \cos(dx + c) + \sqrt{2}*((147a^{20}b^3 - 1078a^{18}b^5 + 931a^{16}b^7 + 4760a^{14}b^9 \\
& - 1274a^{12}b^{11} - 4452a^{10}b^{13} + 1214a^8b^{15} + 1240a^6b^{17} - 505a^4b^{19} \\
& + 42a^2b^{21} - b^{23})d^3\sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)} * \cos(dx + c) \\
& + 4*(49a^{17}b^3 - 490a^{15}b^5 + 1470a^{13}b^7 - 994a^{11}b^9 - 1008a^9b^{11} + 1442a^7b^{13} \\
& - 510a^5b^{15} + 42a^3b^{17} - ab^{19})d * \cos(dx + c)) * \sqrt{(a^{14} + 7a^{12}b^2 \\
& + 21a^{10}b^4 + 35a^8b^6 + 35a^6b^8 + 21a^4b^{10} + 7a^2b^{12} + b^{14} + (a^{17} - 16a^{15}b^2 \\
& - 60a^{13}b^4 - 32a^{11}b^6 + 110a^9b^8 + 176a^7b^{10} + 84a^5b^{12} - 7a^3b^{16}) \\
& * d^2\sqrt{1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4)})) / (49a^{12}b^2 - 490a^{10}b^4 \\
& + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} - 42a^2b^{12} + b^{14}) * \sqrt{(a * \cos(dx + c) \\
& + b * \sin(dx + c)) / \cos(dx + c)} * (1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4))^{1/4} \\
& + (49a^{17}b^2 - 392a^{15}b^4 + 588a^{13}b^6 + 1064a^{11}b^8 - 938a^9b^{10} - 504a^7b^{12} \\
& + 428a^5b^{14} - 40a^3b^{16} + ab^{18}) * \cos(dx + c) + (49a^{16}b^3 - 392a^{14}b^5 \\
& + 588a^{12}b^7 + 1064a^{10}b^9 - 938a^8b^{11} - 504a^6b^{13} + 428a^4b^{15} - 40a^2b^{17} \\
& + b^{19}) * \sin(dx + c)) / \cos(dx + c)) * (1/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)d^4))^{3/4} \\
& + \sqrt{2}*(4*(7a^{23}b + 7a^{21}b^3 - 91a^{19}b^5 - 267a^{17}b^7 - 202a^{15}b^9 + 182a^{13}b^{11} \\
& + 378a^{11}b^{13} + 154a^9b^{15} - 77a^7b^{17} - 77a^5b^{19} - 15a^3b^{21} + ab^{23})d^7\sqrt{(49a^{12}b^2 \\
& - 490a^{10}b^4 + 1519a^8b^6 - 1484a^6b^8 + 511a^4b^{10} - 42a^2b^{12} + b^{14}) / ((a^{20} \\
& + 10a^{18}b^2 + 45a^{16}b^4 + 120a^{14}b^6 + 210a^{12}b^8 + 252a^{10}b^{10} + 210a^8b^{12} \\
& + 120a^6b^{14} + 45a^4b^{16} + 10a^2b^{18} + b^{20})d^4)}
\end{aligned}$$

$\sqrt{2b^{18} + b^{20}d^4}) \sqrt{1/((a^6 + 3a^4b^2 \dots$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{a^2\sqrt{a+b\tan(c+dx)} + 2ab\sqrt{a+b\tan(c+dx)}\tan(c+dx) + b^2\sqrt{a+b\tan(c+dx)}\tan^2(c+dx)} dx - \int \left( \frac{b\tan(c+dx)}{a^2\sqrt{a+b\tan(c+dx)} + 2ab\sqrt{a+b\tan(c+dx)}\tan(c+dx) + b^2\sqrt{a+b\tan(c+dx)}\tan^2(c+dx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2), x)

[Out] -Integral(a/(a\*\*2\*sqrt(a + b\*tan(c + d\*x)) + 2\*a\*b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x) + b\*\*2\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*2), x) - Integral(-b\*tan(c + d\*x)/(a\*\*2\*sqrt(a + b\*tan(c + d\*x)) + 2\*a\*b\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x) + b\*\*2\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 21.26, size = 2500, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a - b\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(5/2), x)

[Out] (log((((-(4\*a^7\*d^2 + (320\*a^6\*b^8\*d^4 - 16\*a^4\*b^10\*d^4 - 1760\*a^8\*b^6\*d^4 + 1600\*a^10\*b^4\*d^4 - 400\*a^12\*b^2\*d^4)^(1/2) + 20\*a^3\*b^4\*d^2 - 40\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*(896\*a^7\*b^15\*d^4 - 32\*a\*b^21\*d^4 - 160\*a^3\*b^19\*d^4 - 128\*a^5\*b^17\*d^4 - ((a + b\*tan(c + d\*x))^(1/2)\*(-(4\*a^7\*d^2 + (320\*a^6\*b^8\*d^4 - 16\*a^4\*b^10\*d^4 - 1760\*a^8\*b^6\*d^4 + 1600\*a^10\*b^4\*d^4 - 400\*a^12\*b^2\*d^4)^(1/2) + 20\*a^3\*b^4\*d^2 - 40\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*(64\*a\*b^22\*d^5 + 640\*a^3\*b^20\*d^5 + 2880\*a^5\*b^18\*d^5 + 7680\*a^7\*b^16\*d^5 + 13440\*a^9\*b^14\*d^5 + 16128\*a^11\*b^12\*d^5 + 13440\*a^13\*b^10\*d^5 + 7680\*a^15\*b^8\*d^5 + 2880\*a^17\*b^6\*d^5 + 640\*a^19\*b^4\*d^5 + 64\*a^21\*b^2\*d^5))/4 + 3136\*a^9\*b^13\*d^4 + 4928\*a^11\*b^11\*d^4 + 4480\*a^13\*b^9\*d^4 + 2432\*a^15\*b^7\*d^4 + 736\*a^17\*b^5\*d^4 + 96\*a^19\*b^3\*d^4))/4 + (a + b\*tan(c + d\*x))^(1/2)\*(320

$$\begin{aligned}
& *a^6b^{14}d^3 - 16a^2b^{18}d^3 + 1024a^8b^{12}d^3 + 1440a^{10}b^{10}d^3 + \\
& 1024a^{12}b^8d^3 + 320a^{14}b^6d^3 - 16a^{18}b^2d^3) * (- (4a^7d^2 + (32 \\
& 0a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^6d^4 + 1600a^{10}b^4d^4 - 40 \\
& 0a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a^5b^2d^2) / (a^{10}d^4 + b^{10}d \\
& ^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2 \\
& )) / 4 - 16a^4b^{15}d^2 - 96a^6b^{13}d^2 - 240a^8b^{11}d^2 - 320a^{10}b^9 \\
& d^2 - 240a^{12}b^7d^2 - 96a^{14}b^5d^2 - 16a^{16}b^3d^2) * (- (4a^7d^2 + \\
& (320a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^6d^4 + 1600a^{10}b^4d^4 - \\
& 400a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a^5b^2d^2) / (a^{10}d^4 + b^{1 \\
& 0}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{( \\
& 1/2)) / 4 - \log((( - (4a^7d^2 + (320a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8 \\
& b^6d^4 + 1600a^{10}b^4d^4 - 400a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 4 \\
& 0a^5b^2d^2) / (16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^ \\
& 4 + 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)} * ((a + b \tan(c + d*x))^{(1/2)} * ( - \\
& (4a^7d^2 + (320a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^6d^4 + 1600a \\
& ^{10}b^4d^4 - 400a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a^5b^2d^2) / (1 \\
& 6a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160a^6b^4d \\
& ^4 + 80a^8b^2d^4))^{(1/2)} * (64a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^ \\
& 18d^5 + 7680a^7b^{16}d^5 + 13440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 134 \\
& 40a^{13}b^{10}d^5 + 7680a^{15}b^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 \\
& + 64a^{21}b^2d^5) - 32a^2b^{21}d^4 - 160a^3b^{19}d^4 - 128a^5b^{17}d^4 + \\
& 896a^7b^{15}d^4 + 3136a^9b^{13}d^4 + 4928a^{11}b^{11}d^4 + 4480a^{13}b^9 \\
& d^4 + 2432a^{15}b^7d^4 + 736a^{17}b^5d^4 + 96a^{19}b^3d^4) - (a + b \tan( \\
& c + d*x))^{(1/2)} * (320a^6b^{14}d^3 - 16a^2b^{18}d^3 + 1024a^8b^{12}d^3 + 1 \\
& 440a^{10}b^{10}d^3 + 1024a^{12}b^8d^3 + 320a^{14}b^6d^3 - 16a^{18}b^2d^3) \\
& ) * (- (4a^7d^2 + (320a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^6d^4 + 16 \\
& 00a^{10}b^4d^4 - 400a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a^5b^2d^2 \\
& ) / (16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + 160a^6b \\
& ^4d^4 + 80a^8b^2d^4))^{(1/2)} - 16a^4b^{15}d^2 - 96a^6b^{13}d^2 - 240a^ \\
& ^8b^{11}d^2 - 320a^{10}b^9d^2 - 240a^{12}b^7d^2 - 96a^{14}b^5d^2 - 16a^ \\
& ^{16}b^3d^2) * (- (4a^7d^2 + (320a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^ \\
& 6d^4 + 1600a^{10}b^4d^4 - 400a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a \\
& ^5b^2d^2) / (16a^{10}d^4 + 16b^{10}d^4 + 80a^2b^8d^4 + 160a^4b^6d^4 + \\
& 160a^6b^4d^4 + 80a^8b^2d^4))^{(1/2)} + (\log((( - (4a^7d^2 - (320a^6b \\
& ^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^6d^4 + 1600a^{10}b^4d^4 - 400a^{12} \\
& b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a^5b^2d^2) / (a^{10}d^4 + b^{10}d^4 + 5 \\
& a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)} * ((( - ( \\
& 4a^7d^2 - (320a^6b^8d^4 - 16a^4b^{10}d^4 - 1760a^8b^6d^4 + 1600a^ \\
& ^{10}b^4d^4 - 400a^{12}b^2d^4)^{(1/2)} + 20a^3b^4d^2 - 40a^5b^2d^2) / (a^ \\
& ^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8 \\
& b^2d^4))^{(1/2)} * (896a^7b^{15}d^4 - (( - (4a^7d^2 - (320a^6b^8d^4 - 16 \\
& a^4b^{10}d^4 - 1760a^8b^6d^4 + 1600a^{10}b^4d^4 - 400a^{12}b^2d^4)^{(1/ \\
& 2)} + 20a^3b^4d^2 - 40a^5b^2d^2) / (a^{10}d^4 + b^{10}d^4 + 5a^2b^8d^4 \\
& + 10a^4b^6d^4 + 10a^6b^4d^4 + 5a^8b^2d^4))^{(1/2)} * (a + b \tan(c + d* \\
& x))^{(1/2)} * (64a^2b^{22}d^5 + 640a^3b^{20}d^5 + 2880a^5b^{18}d^5 + 7680a^7*
\end{aligned}$$

$$\begin{aligned}
& b^{16}d^5 + 13440a^9b^{14}d^5 + 16128a^{11}b^{12}d^5 + 13440a^{13}b^{10}d^5 + \\
& 7680a^{15}b^8d^5 + 2880a^{17}b^6d^5 + 640a^{19}b^4d^5 + 64a^{21}b^2d^5 \\
& ))/4 - 160a^3b^{19}d^4 - 128a^5b^{17}d^4 - 32a^7b^{15}d^4 + 3136a^9b^{13}d^4 \\
& + 4928a^{11}b^{11}d^4 + 4480a^{13}b^9d^4 + 2432a^{15}b^7d^4 + 736a^{17}b^5d^4 \\
& + 96a^{19}b^3d^4))/4 + (a + b\tan(c + d*x))^{(1/2)}*(320a^6b^{14}d^3 \\
& - 16a^2b^{18}d^3 + 1024a^8b^{12}d^3 + 1440a^{10}b^{10}d^3 + 1024a^{12}b^8d^3 \\
& + 320a^{14}b^6d^3 - 16a^{18}b^2d^3))/4 - 16a^4b^{15}d^2 - 96a^6b^{13}d^2 \\
& - 240a^8b^{11}d^2 - 320a^{10}b^9d^2 - 240a^{12}b^7d^2 - 96a^{14}b^5d^2 \\
& - 16a^{16}b^3d^2)*(-(4a^7d^2 - (32\dots
\end{aligned}$$

$$3.373 \quad \int \frac{1+i \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=45

$$-\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d}$$

[Out]  $-2*I*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d/(a-I*b)^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3618, 65, 214}

$$-\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(1 + I*\operatorname{Tan}[c + d*x])/Sqrt[a + b*\operatorname{Tan}[c + d*x]], x]$

[Out]  $((-2*I)*\operatorname{ArcTanh}[Sqrt[a + b*\operatorname{Tan}[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^{n}, x], x, (a+b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\operatorname{Tan}[e + f*x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 + b^2, 0] \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx &= \frac{i \text{Subst} \left( \int \frac{1}{(-1+x)\sqrt{a - ibx}} dx, x, i \tan(c + dx) \right)}{d} \\
&= -\frac{2 \text{Subst} \left( \int \frac{1}{-1 - \frac{ia}{b} + \frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)} \right)}{bd} \\
&= -\frac{2i \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib}} \right)}{\sqrt{a - ib} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.85, size = 70, normalized size = 1.56

$$-\frac{2i \tanh^{-1} \left( \frac{\sqrt{a - \frac{ib(-1 + e^{2i(c+dx)})}}{1 + e^{2i(c+dx)}}}{\sqrt{a - ib}} \right)}{\sqrt{a - ib} d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]], x]
```

```
[Out] ((-2*I)*ArcTanh[Sqrt[a - (I*b*(-1 + E^((2*I)*(c + d*x))))]/(1 + E^((2*I)*(c + d*x)))]/Sqrt[a - I*b])/(Sqrt[a - I*b]*d)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(36) = 72.

time = 0.13, size = 729, normalized size = 16.20

method	result
derivativedivides	$ \frac{(-i\sqrt{a^2 + b^2} - ia + b) \ln \left( \frac{b \tan(dx+c) + a + \sqrt{a + b \tan(dx+c)}}{\sqrt{2\sqrt{a^2 + b^2} + 2a + \sqrt{a^2 + b^2}}} \right)}{2} + \dots $







$2*(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*\cos(2*d*x + 2*c)^2 + 2*(a^4 + a^2*b^2)*\cos(2*d*x + 2*c))*\sin(2*d*x + 2*c)^2 + 4*(a^4 + a^2*b^2)*\cos(2*d*x + 2*c) + 4*(a^3*b + a*b^3 + (a^3*b + a*b^3)*\cos(2*d*x + 2*c)^2 + 2*(a^3*b + a*b^3)*\cos(2*d*x + 2*c))*\sin(2*d*x + 2*c)^{(1/4)}*(a*b*\cos(2*d*x + 2*c) + b^2*\sin(2*d*x + 2*c) + a*b)*\cos(1/2*\arctan2(-2*(a*b*\cos(2*d*x + 2*c))^2 - a*b*\sin(2*d*x + 2*c)^2 + a*b*\cos(2*d*x + 2*c) - (a^2 + (a^2 - b^2)*\cos(2*d*x + 2*c))*\sin(2*d*x + 2*c))/b^2, (2*a^2*\cos(2*d*x + 2*c) + (a^2 - b^2)*\cos(2*d*x + 2*c)^2 - (a^2 - b^2)*\sin(2*d*x + 2*c)^2 + a^2 + b^2 + 2*(2*a*b*\cos(2*d*x + 2*c) + a*b)*\sin(2*d*x + 2*c))/b^2))/b + 2*((a^4 + 2*a^2*b^2 + b^4)*\cos(2*d*x + 2*c)^4 + (a^4 + 2*a^2*b^2 + b^4)*\sin(2*d*x + 2*c)^4 + a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + a^2*b^2)*\cos(2*d*x + 2*c)^3 + 4*(a^3*b + a*b^3)*\sin(2*d*x + 2*c)^3 + 2*(3*a^4 + 2*a^2*b^2 - b^4)*\cos(2*d*x + 2*c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*\cos(2*d*x + 2*c)^2 + 2*(a^4 + a^2*b^2)*\cos(2*d*x + 2*c))*\sin(2*d*x + 2*c)^2 + 4*(a^4 + a^2*b^2)*\cos(2*d*x + 2*c) + 4*(a^3*b + a*b^3 + (a^3*b + a*b^3))...$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs.  $2(33) = 66$ .

time = 1.18, size = 249, normalized size = 5.53

$$\frac{1}{4} \sqrt{\frac{4i}{(a+b)d^2}} \log\left(\left(\frac{((a+b)d^{2d+2c}) + (a+b)d \sqrt{\frac{(a-b)d^{2d+2c} + a+ib}{e^{2d+2c} + 1}} \sqrt{\frac{4i}{(a+b)d^2}} + 2(a-ib)e^{2d+2c} + 2a\right) e^{-2d-2c}\right) - \frac{1}{4} \sqrt{\frac{4i}{(a+b)d^2}} \log\left(\left(\frac{((-i-a-b)d^{2d+2c}) + (-i-a-b)d \sqrt{\frac{(a-b)d^{2d+2c} + a+ib}{e^{2d+2c} + 1}} \sqrt{\frac{4i}{(a+b)d^2}} + 2(a-ib)e^{2d+2c} + 2a\right) e^{-2d-2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{-4I/((Ia + b)d^2)} \log(\frac{((Ia + b)d e^{(2I*d*x + 2I*c)} + (Ia + b)d) \sqrt{((a - I*b) e^{(2I*d*x + 2I*c)} + a + I*b)/(e^{(2I*d*x + 2I*c)} + 1)}}{(-4I/((Ia + b)d^2))} + 2*(a - I*b) e^{(2I*d*x + 2I*c)} + 2*a) e^{(-2I*d*x - 2I*c)}) - \frac{1}{4} \sqrt{-4I/((Ia + b)d^2)} \log(\frac{((-Ia - b)d e^{(2I*d*x + 2I*c)} + (-Ia - b)d) \sqrt{((a - I*b) e^{(2I*d*x + 2I*c)} + a + I*b)/(e^{(2I*d*x + 2I*c)} + 1)}}{(-4I/((Ia + b)d^2))} + 2*(a - I*b) e^{(2I*d*x + 2I*c)} + 2*a) e^{(-2I*d*x - 2I*c)})$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \left( -\frac{i}{\sqrt{a + b \tan(c + dx)}} \right) dx + \int \frac{\tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out]  $I*(\text{Integral}(-I/\sqrt{a + b*\tan(c + d*x)}, x) + \text{Integral}(\tan(c + d*x)/\sqrt{a + b*\tan(c + d*x)}, x))$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(33) = 66.  
time = 0.51, size = 153, normalized size = 3.40

$$4 \arctan \left( \frac{2 \left( i \sqrt{b \tan(dx+c) + a} \sqrt{a^2 + b^2} \sqrt{b \tan(dx+c) + a} \right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} \sqrt{2a + 2\sqrt{a^2 + b^2}} \sqrt{2a + 2\sqrt{a^2 + b^2}}} \right) \\ \frac{d \left( -\frac{ib}{a + \sqrt{a^2 + b^2}} + 1 \right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] 4\*arctan(-2\*(I\*sqrt(b\*tan(d\*x + c) + a)\*a + I\*sqrt(a^2 + b^2)\*sqrt(b\*tan(d\*x + c) + a))/(sqrt(2\*a + 2\*sqrt(a^2 + b^2))\*a - I\*sqrt(2\*a + 2\*sqrt(a^2 + b^2))\*b + sqrt(a^2 + b^2)\*sqrt(2\*a + 2\*sqrt(a^2 + b^2)))/(sqrt(2\*a + 2\*sqrt(a^2 + b^2))\*d\*(-I\*b/(a + sqrt(a^2 + b^2)) + 1))

**Mupad [B]**

time = 8.70, size = 1410, normalized size = 31.33

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)\*1i + 1)/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] 2\*atanh((32\*b^2\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2))/((a^2\*b^2\*d^2\*64i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (b^2\*16i)/d + (64\*a\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) - (128\*a^2\*b^2\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2))/((a^2\*b^4\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (a^2\*b^2\*64i)/d - (b^4\*64i)/d + (256\*a^3\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (a^4\*b^2\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (256\*a\*b^5\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) + (a\*b^3\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*128i)/((a^2\*b^4\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (a^2\*b^2\*64i)/d - (b^4\*64i)/d + (256\*a^3\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (a^4\*b^2\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (256\*a\*b^5\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)))\*(-a - b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2)^(1/2) + (log(d\*(-1/(d^2\*(a - b\*1i))))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)\*1i + 1)\*(-1/(a\*d^2 - b\*d^2\*1i))^(1/2))/2 - log(d\*(-1/(d^2\*(a - b\*1i))))^(1/2)\*(a + b\*tan(c + d\*x))^(1/2) + 1i)\*(-1/(4\*(a\*d^2 - b\*d^2\*1i)))^(1/2) + (log(16\*b^2\*(a + b\*tan(c + d\*x))^(1/2) + 16\*b^3\*d\*(-1/(d^2\*(a - b\*1i))))^(1/2) - (16\*a\*b^2\*(a + b\*tan(c + d\*x))^(1/2))/(a - b\*1i))\*(-1/(a\*d^2 - b\*d^2\*1i))^(1/2))/2 - log(16\*b^3\*d\*(-1/(d^2\*(a - b\*1i))))^(1/2) - 16\*b^2\*(a + b\*tan(c + d\*x))^(1/2) + (16\*a\*b^2\*(a + b\*tan(c + d\*x))^(1/2))/(a - b\*1i))\*(-1/(4\*(a\*d^2 - b\*d^2\*1i)))^(1/2)

$$\begin{aligned}
& 2) + 2*\operatorname{atanh}\left(\frac{32*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{1/2}*(a + b*\tan(c + d*x))^{1/2}}{(b^4*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)} + \frac{a*b^3*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{1/2}*(a + b*\tan(c + d*x))^{1/2}*128i}{(b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3)} - \frac{256*a^3*b^3*d^2}{(4*a^2*d^3 + 4*b^2*d^3)} - \frac{256*a*b^5*d^2}{(4*a^2*d^3 + 4*b^2*d^3)} - \frac{128*a^2*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{1/2}*(a + b*\tan(c + d*x))^{1/2}}{(b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3)} - \frac{256*a^3*b^3*d^2}{(4*a^2*d^3 + 4*b^2*d^3)} - \frac{256*a*b^5*d^2}{(4*a^2*d^3 + 4*b^2*d^3)}\right)*(-a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2)^{1/2}
\end{aligned}$$

$$3.374 \quad \int \frac{1-i \tan(c+dx)}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=45

$$\frac{2i \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right)}{\sqrt{a+ib} d}$$

[Out]  $2*I*\operatorname{arctanh}((a+b*\tan(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/d/(a+I*b)^{(1/2)}$

Rubi [A]

time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3618, 65, 214}

$$\frac{2i \tanh^{-1} \left( \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+ib}} \right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(1 - I*\operatorname{Tan}[c + d*x])/Sqrt[a + b*\operatorname{Tan}[c + d*x]], x]$

[Out]  $((2*I)*\operatorname{ArcTanh}[Sqrt[a + b*\operatorname{Tan}[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{n, x}], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 214

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

Rule 3618

$\operatorname{Int}[(a_. + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \operatorname{Dist}[c*(d/f), \operatorname{Subst}[\operatorname{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\operatorname{Tan}[e + f*x]], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 + b^2, 0] \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1 - i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx &= -\frac{i \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, -i \tan(c + dx)\right)}{d} \\
&= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{-1+\frac{ia}{b}-\frac{ix^2}{b}} dx, x, \sqrt{a + b \tan(c + dx)}\right)}{bd} \\
&= \frac{2i \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{\sqrt{a + ib} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 45, normalized size = 1.00

$$\frac{2i \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib}}\right)}{\sqrt{a + ib} d}$$

Antiderivative was successfully verified.

`[In] Integrate[(1 - I*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]], x]``[Out] ((2*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)`**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(36) = 72.

time = 0.11, size = 741, normalized size = 16.47

method	result
derivativedivides	$ \frac{(-i\sqrt{a^2 + b^2} - ia - b) \ln\left(\frac{b \tan(dx+c)+a + \sqrt{a + b \tan(dx+c)} \sqrt{2\sqrt{a^2 + b^2} + 2a + \sqrt{a^2 + b^2}}}{2}\right)}{2} $
default	$ \frac{(-i\sqrt{a^2 + b^2} - ia - b) \ln\left(\frac{b \tan(dx+c)+a + \sqrt{a + b \tan(dx+c)} \sqrt{2\sqrt{a^2 + b^2} + 2a + \sqrt{a^2 + b^2}}}{2}\right)}{2} $

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{d} \left( \frac{-1}{(2(a^2+b^2)^{1/2}+2a)^{1/2}} \frac{1}{(a^2+b^2)^{1/2}} \left( \frac{1}{2} (-I(a^2+b^2)^{1/2} - I(a-b)) \ln(b \tan(dx+c) + a + (a+b \tan(dx+c))^{1/2}) \right) \right. \\ \left. + \frac{2(a^2+b^2)^{1/2} + 2a}{(a^2+b^2)^{1/2}} + 2(-I(2(a^2+b^2)^{1/2} + 2a)^{1/2} a - (2(a^2+b^2)^{1/2} + 2a)^{1/2} * b - \frac{1}{2} (-I(a^2+b^2)^{1/2} - I(a-b)) \right) \frac{1}{(2(a^2+b^2)^{1/2} - 2a)^{1/2}} \arctan \left( \frac{2(a+b \tan(dx+c))^{1/2} + (2(a^2+b^2)^{1/2} + 2a)^{1/2}}{(2(a^2+b^2)^{1/2} - 2a)^{1/2}} \right) - \frac{1}{(2(a^2+b^2)^{1/2} + 2a)^{1/2}} \frac{1}{(a^2+b^2)^{1/2}} \left( (a^2+b^2)^{1/2} * a + a^2 + b^2 \right) \left( -\frac{1}{2} (-2I(a^2+b^2)^{1/2} * a^2 - I(a^2+b^2)^{1/2} * b^2 - 2Ia^3 - 2Ia*b^2 - (a^2+b^2)^{1/2} * a * b - a^2 * b - b^3) \right) \ln(-b \tan(dx+c) - a + (a+b \tan(dx+c))^{1/2}) \right. \\ \left. + \frac{2(a^2+b^2)^{1/2} + 2a}{(a^2+b^2)^{1/2}} + 2(I(2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a^2 + I(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^3 + I(2(a^2+b^2)^{1/2} + 2a)^{1/2} * a * b^2 + (2(a^2+b^2)^{1/2} + 2a)^{1/2} * (a^2+b^2)^{1/2} * a * b + (2(a^2+b^2)^{1/2} + 2a)^{1/2} * a^2 * b + (2(a^2+b^2)^{1/2} + 2a)^{1/2} * b^3 + \frac{1}{2} (-2I(a^2+b^2)^{1/2} * a^2 - I(a^2+b^2)^{1/2} * b^2 - 2Ia^3 - 2Ia*b^2 - (a^2+b^2)^{1/2} * a * b - a^2 * b - b^3) \right) \frac{1}{(2(a^2+b^2)^{1/2} - 2a)^{1/2}} \arctan \left( \frac{-2(a+b \tan(dx+c))^{1/2} + (2(a^2+b^2)^{1/2} + 2a)^{1/2}}{(2(a^2+b^2)^{1/2} - 2a)^{1/2}} \right) \right)$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more details)Is

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs.  $2(33) = 66$ .

time = 1.17, size = 267, normalized size = 5.93

$$\frac{1}{4} \sqrt{\frac{4i}{(-ia+b)d^2}} \log \left( \frac{\left( ((a-b)de^{2i dx+2i c} + (i a - b)d) \sqrt{\frac{(a-b)e^{2i dx+2i c} + a + ib}{e^{2i dx+2i c} + 1}} \sqrt{\frac{4i}{(-ia+b)d^2}} + 2ae^{2i dx+2i c} + 2a + 2ib \right) e^{-2i dx-2i c}}{(-ia+b)d} \right) + \frac{1}{4} \sqrt{\frac{4i}{(-ia+b)d^2}} \log \left( \frac{\left( (-ia+b)de^{2i dx+2i c} + (-ia+b)d) \sqrt{\frac{(a-b)e^{2i dx+2i c} + a + ib}{e^{2i dx+2i c} + 1}} \sqrt{\frac{4i}{(-ia+b)d^2}} + 2ae^{2i dx+2i c} + 2a + 2ib \right) e^{-2i dx-2i c}}{(-ia+b)d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")`

```
[Out] -1/4*sqrt(4*I/((-I*a + b)*d^2))*log((((I*a - b)*d*e^(2*I*d*x + 2*I*c) + (I*a - b)*d)*sqrt(((a - I*b)*e^(2*I*d*x + 2*I*c) + a + I*b)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(4*I/((-I*a + b)*d^2)) + 2*a*e^(2*I*d*x + 2*I*c) + 2*a + 2*I*b)*e^(-2*I*d*x - 2*I*c)/((-I*a + b)*d)) + 1/4*sqrt(4*I/((-I*a + b)*d^2))*log(((((-I*a + b)*d*e^(2*I*d*x + 2*I*c) + (-I*a + b)*d)*sqrt(((a - I*b)*e^(2*I*d*x + 2*I*c) + a + I*b)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(4*I/((-I*a + b)*d^2)) + 2*a*e^(2*I*d*x + 2*I*c) + 2*a + 2*I*b)*e^(-2*I*d*x - 2*I*c)/((-I*a + b)*d))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-i \left( \int \frac{i}{\sqrt{a + b \tan(c + dx)}} dx + \int \frac{\tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] -I*(Integral(I/sqrt(a + b*tan(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(a + b*tan(c + d*x)), x))
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(33) = 66.

time = 0.51, size = 153, normalized size = 3.40

$$\frac{4 \arctan \left( \frac{2 \left( -i \sqrt{b \tan(dx + c) + a} \sqrt{a - i \sqrt{a^2 + b^2}} \sqrt{b \tan(dx + c) + a} \right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} \sqrt{a + i \sqrt{2a + 2\sqrt{a^2 + b^2}}} \sqrt{b + \sqrt{a^2 + b^2}} \sqrt{2a + 2\sqrt{a^2 + b^2}}} \right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} d \left( \frac{ib}{a + \sqrt{a^2 + b^2}} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4*arctan(-2*(-I*sqrt(b*tan(d*x + c) + a)*a - I*sqrt(a^2 + b^2)*sqrt(b*tan(d*x + c) + a))/(sqrt(2*a + 2*sqrt(a^2 + b^2))*a + I*sqrt(2*a + 2*sqrt(a^2 + b^2))*b + sqrt(a^2 + b^2)*sqrt(2*a + 2*sqrt(a^2 + b^2)))/(sqrt(2*a + 2*sqrt(a^2 + b^2))*d*(I*b/(a + sqrt(a^2 + b^2)) + 1))
```

**Mupad [B]**

time = 7.62, size = 1410, normalized size = 31.33

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(-(\tan(c + d*x)*i - 1)/(a + b*\tan(c + d*x))^{(1/2)}, x)$

[Out]  $(\log(d*(-1/(d^2*(a - b*i)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)} + i)*(-1/(a*d^2 - b*d^2*i))^{(1/2)})/2 - 2*\text{atanh}((32*b^2*((b*i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)})/((a^2*b^2*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (b^2*16i)/d + (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*((b*i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)})/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*((b*i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*128i)/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-a - b*i)/(4*a^2*d^2 + 4*b^2*d^2)^{(1/2)} - \log(d*(-1/(d^2*(a - b*i)))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*i + 1)*(-1/(4*(a*d^2 - b*d^2*i)))^{(1/2)} + (\log(16*b^2*(a + b*\tan(c + d*x))^{(1/2)} + 16*b^3*d*(-1/(d^2*(a - b*i)))^{(1/2)} - (16*a*b^2*(a + b*\tan(c + d*x))^{(1/2)})/(a - b*i))*(-1/(a*d^2 - b*d^2*i))^{(1/2)})/2 - \log(16*b^3*d*(-1/(d^2*(a - b*i)))^{(1/2)} - 16*b^2*(a + b*\tan(c + d*x))^{(1/2)} + (16*a*b^2*(a + b*\tan(c + d*x))^{(1/2)})/(a - b*i))*(-1/(4*(a*d^2 - b*d^2*i)))^{(1/2)} + 2*\text{atanh}((32*b^2*((b*i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)})/((b^4*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*((b*i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)}*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*((b*i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^{(1/2)}*(a + b*\tan(c + d*x))^{(1/2)})/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-a - b*i)/(4*a^2*d^2 + 4*b^2*d^2)^{(1/2)}$



$$3.375 \quad \int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}} dx$$

Optimal. Leaf size=30

$$-\sqrt{2} \operatorname{ArcTan}\left(\frac{1-3\tan(x)}{\sqrt{2}\sqrt{4+3\tan(x)}}\right)$$

[Out]  $-\arctan(1/2*(1-3*\tan(x))*2^{(1/2)}/(4+3*\tan(x))^{(1/2)})*2^{(1/2)}$

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3616, 209}

$$-\sqrt{2} \operatorname{ArcTan}\left(\frac{1-3\tan(x)}{\sqrt{2}\sqrt{3\tan(x)+4}}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(3 + \text{Tan}[x])/\text{Sqrt}[4 + 3*\text{Tan}[x]], x]$

[Out]  $-(\text{Sqrt}[2]*\text{ArcTan}[(1 - 3*\text{Tan}[x])]/(\text{Sqrt}[2]*\text{Sqrt}[4 + 3*\text{Tan}[x]]))$

Rule 209

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))*\text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3616

$\text{Int}[(c_ + (d_)*\tan[(e_ + (f_)*(x_))]/\text{Sqrt}[(a_ + (b_)*\tan[(e_ + (f_)*(x_))]]], x\_Symbol] \rightarrow \text{Dist}[-2*(d^2/f), \text{Subst}[\text{Int}[1/(2*b*c*d - 4*a*d^2 + x^2), x], x, (b*c - 2*a*d - b*d*\tan[e + f*x])/\text{Sqrt}[a + b*\tan[e + f*x]]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{EqQ}[2*a*c*d - b*(c^2 - d^2), 0]$

Rubi steps

$$\begin{aligned} \int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}} dx &= -\left(2\text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\right)\right) \\ &= -\sqrt{2} \tan^{-1}\left(\frac{1-3\tan(x)}{\sqrt{2}\sqrt{4+3\tan(x)}}\right) \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.12, size = 69, normalized size = 2.30

$$\left(\frac{1}{5} - \frac{3i}{5}\right) \sqrt{4-3i} \tanh^{-1}\left(\frac{\sqrt{4+3\tan(x)}}{\sqrt{4-3i}}\right) + \left(\frac{1}{5} + \frac{3i}{5}\right) \sqrt{4+3i} \tanh^{-1}\left(\frac{\sqrt{4+3\tan(x)}}{\sqrt{4+3i}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + Tan[x])/Sqrt[4 + 3\*Tan[x]], x]

[Out] (1/5 - (3\*I)/5)\*Sqrt[4 - 3\*I]\*ArcTanh[Sqrt[4 + 3\*Tan[x]]/Sqrt[4 - 3\*I]] + (1/5 + (3\*I)/5)\*Sqrt[4 + 3\*I]\*ArcTanh[Sqrt[4 + 3\*Tan[x]]/Sqrt[4 + 3\*I]]

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(25) = 50.

time = 0.19, size = 54, normalized size = 1.80

method	result
derivativedivides	$\sqrt{2} \arctan\left(\frac{(2\sqrt{4+3\tan(x)} - 3\sqrt{2})\sqrt{2}}{2}\right) + \sqrt{2} \arctan\left(\frac{(2\sqrt{4+3\tan(x)} + 3\sqrt{2})\sqrt{2}}{2}\right)$
default	$\sqrt{2} \arctan\left(\frac{(2\sqrt{4+3\tan(x)} - 3\sqrt{2})\sqrt{2}}{2}\right) + \sqrt{2} \arctan\left(\frac{(2\sqrt{4+3\tan(x)} + 3\sqrt{2})\sqrt{2}}{2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+tan(x))/(4+3\*tan(x))^(1/2), x, method=\_RETURNVERBOSE)

[Out] 2^(1/2)\*arctan(1/2\*(2\*(4+3\*tan(x))^(1/2)-3\*2^(1/2))\*2^(1/2))+2^(1/2)\*arctan(1/2\*(2\*(4+3\*tan(x))^(1/2)+3\*2^(1/2))\*2^(1/2))

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+tan(x))/(4+3\*tan(x))^(1/2), x, algorithm="maxima")

[Out] integrate((tan(x) + 3)/sqrt(3\*tan(x) + 4), x)

**Fricas [A]**

time = 1.32, size = 28, normalized size = 0.93

$$\sqrt{2} \arctan\left(\frac{3\sqrt{2}\tan(x) - \sqrt{2}}{2\sqrt{3}\tan(x) + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+tan(x))/(4+3*tan(x))^(1/2),x, algorithm="fricas")`

[Out] `sqrt(2)*arctan(1/2*(3*sqrt(2)*tan(x) - sqrt(2))/sqrt(3*tan(x) + 4))`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x) + 3}{\sqrt{3 \tan(x) + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+tan(x))/(4+3*tan(x))**(1/2),x)`

[Out] `Integral((tan(x) + 3)/sqrt(3*tan(x) + 4), x)`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs.  $2(25) = 50$ .  
time = 0.45, size = 65, normalized size = 2.17

$$\sqrt{2} \arctan\left(\frac{1}{250} \cdot 25^{\frac{3}{4}} \sqrt{10} \left(3 \cdot 25^{\frac{1}{4}} \sqrt{10} + 10 \sqrt{3 \tan(x) + 4}\right)\right) + \sqrt{2} \arctan\left(-\frac{1}{250} \cdot 25^{\frac{3}{4}} \sqrt{10} \left(3 \cdot 25^{\frac{1}{4}} \sqrt{10} - 10 \sqrt{3 \tan(x) + 4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+tan(x))/(4+3*tan(x))^(1/2),x, algorithm="giac")`

[Out] `sqrt(2)*arctan(1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) + 10*sqrt(3*tan(x) + 4))) + sqrt(2)*arctan(-1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) - 10*sqrt(3*tan(x) + 4)))`

**Mupad** [B]

time = 0.68, size = 31, normalized size = 1.03

$$\sqrt{2} \left( \operatorname{atan}\left(\sqrt{6 \tan(x) + 8} \left(\frac{1}{10} - \frac{3}{10}i\right)\right) + \operatorname{atan}\left(\sqrt{6 \tan(x) + 8} \left(\frac{1}{10} + \frac{3}{10}i\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(x) + 3)/(3*tan(x) + 4)^(1/2),x)`

[Out] `2^(1/2)*(atan((6*tan(x) + 8)^(1/2)*(1/10 - 3i/10)) + atan((6*tan(x) + 8)^(1/2)*(1/10 + 3i/10)))`

$$3.376 \quad \int \frac{1-3 \tan(x)}{\sqrt{4+3 \tan(x)}} dx$$

Optimal. Leaf size=27

$$\sqrt{2} \tanh^{-1} \left( \frac{3 + \tan(x)}{\sqrt{2} \sqrt{4 + 3 \tan(x)}} \right)$$

[Out] arctanh(1/2\*(3+tan(x))\*2^(1/2)/(4+3\*tan(x))^(1/2))\*2^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3616, 213}

$$\sqrt{2} \tanh^{-1} \left( \frac{\tan(x) + 3}{\sqrt{2} \sqrt{3 \tan(x) + 4}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3\*Tan[x])/Sqrt[4 + 3\*Tan[x]],x]

[Out] Sqrt[2]\*ArcTanh[(3 + Tan[x])/(Sqrt[2]\*Sqrt[4 + 3\*Tan[x]])]

Rule 213

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[b, 2])^(-1))\*ArcTanh[Rt[b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3616

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*(d^2/f), Subst[Int[1/(2\*b\*c\*d - 4\*a\*d^2 + x^2), x], x, (b\*c - 2\*a\*d - b\*d\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[2\*a\*c\*d - b\*(c^2 - d^2), 0]

Rubi steps

$$\begin{aligned} \int \frac{1-3 \tan(x)}{\sqrt{4+3 \tan(x)}} dx &= - \left( 18 \text{Subst} \left( \int \frac{1}{-162+x^2} dx, x, \frac{27+9 \tan(x)}{\sqrt{4+3 \tan(x)}} \right) \right) \\ &= \sqrt{2} \tanh^{-1} \left( \frac{3 + \tan(x)}{\sqrt{2} \sqrt{4 + 3 \tan(x)}} \right) \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.08, size = 65, normalized size = 2.41

$$\frac{1}{5} \left( (3+i)\sqrt{4-3i} \tanh^{-1} \left( \frac{\sqrt{4+3\tan(x)}}{\sqrt{4-3i}} \right) + (3-i)\sqrt{4+3i} \tanh^{-1} \left( \frac{\sqrt{4+3\tan(x)}}{\sqrt{4+3i}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3\*Tan[x])/Sqrt[4 + 3\*Tan[x]], x]

[Out] ((3 + I)\*Sqrt[4 - 3\*I]\*ArcTanh[Sqrt[4 + 3\*Tan[x]]/Sqrt[4 - 3\*I]] + (3 - I)\*Sqrt[4 + 3\*I]\*ArcTanh[Sqrt[4 + 3\*Tan[x]]/Sqrt[4 + 3\*I]])/5

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

time = 0.06, size = 52, normalized size = 1.93

method	result
derivativedivides	$-\frac{\sqrt{2} \ln\left(9+3\tan(x)-3\sqrt{4+3\tan(x)}\sqrt{2}\right)}{2} + \frac{\sqrt{2} \ln\left(9+3\tan(x)+3\sqrt{4+3\tan(x)}\sqrt{2}\right)}{2}$
default	$-\frac{\sqrt{2} \ln\left(9+3\tan(x)-3\sqrt{4+3\tan(x)}\sqrt{2}\right)}{2} + \frac{\sqrt{2} \ln\left(9+3\tan(x)+3\sqrt{4+3\tan(x)}\sqrt{2}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-3\*tan(x))/(4+3\*tan(x))^(1/2), x, method=\_RETURNVERBOSE)

[Out] -1/2\*2^(1/2)\*ln(9+3\*tan(x)-3\*(4+3\*tan(x))^(1/2)\*2^(1/2))+1/2\*2^(1/2)\*ln(9+3\*tan(x)+3\*(4+3\*tan(x))^(1/2)\*2^(1/2))

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-3\*tan(x))/(4+3\*tan(x))^(1/2), x, algorithm="maxima")

[Out] -integrate((3\*tan(x) - 1)/sqrt(3\*tan(x) + 4), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

time = 1.20, size = 47, normalized size = 1.74

$$\frac{1}{2} \sqrt{2} \log \left( \frac{\tan(x)^2 + 2 \left( \sqrt{2} \tan(x) + 3\sqrt{2} \right) \sqrt{3 \tan(x) + 4} + 12 \tan(x) + 17}{\tan(x)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-3\*tan(x))/(4+3\*tan(x))^(1/2),x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*log((tan(x)^2 + 2\*(sqrt(2)\*tan(x) + 3\*sqrt(2))\*sqrt(3\*tan(x) + 4) + 12\*tan(x) + 17)/(tan(x)^2 + 1))

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3 \tan(x)}{\sqrt{3 \tan(x) + 4}} dx - \int \left( -\frac{1}{\sqrt{3 \tan(x) + 4}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-3\*tan(x))/(4+3\*tan(x))^(1/2),x)

[Out] -Integral(3\*tan(x)/sqrt(3\*tan(x) + 4), x) - Integral(-1/sqrt(3\*tan(x) + 4), x)

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44.

time = 0.43, size = 57, normalized size = 2.11

$$\frac{1}{2} \sqrt{2} \log \left( \frac{3}{5} \cdot 25^{\frac{1}{4}} \sqrt{10} \sqrt{3 \tan(x) + 4} + 3 \tan(x) + 9 \right) - \frac{1}{2} \sqrt{2} \log \left( -\frac{3}{5} \cdot 25^{\frac{1}{4}} \sqrt{10} \sqrt{3 \tan(x) + 4} + 3 \tan(x) + 9 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-3\*tan(x))/(4+3\*tan(x))^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*log(3/5\*25^(1/4)\*sqrt(10)\*sqrt(3\*tan(x) + 4) + 3\*tan(x) + 9) - 1/2\*sqrt(2)\*log(-3/5\*25^(1/4)\*sqrt(10)\*sqrt(3\*tan(x) + 4) + 3\*tan(x) + 9)

**Mupad [B]**

time = 7.11, size = 35, normalized size = 1.30

$$\sqrt{2} \left( \operatorname{atan} \left( \sqrt{6 \tan(x) + 8} \left( \frac{1}{10} - \frac{3}{10}i \right) \right) - \operatorname{atan} \left( \sqrt{6 \tan(x) + 8} \left( \frac{1}{10} + \frac{3}{10}i \right) \right) \right) \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3\*tan(x) - 1)/(3\*tan(x) + 4)^(1/2),x)

[Out] 2^(1/2)\*(atan((6\*tan(x) + 8)^(1/2)\*(1/10 - 3i/10)) - atan((6\*tan(x) + 8)^(1/2)\*(1/10 + 3i/10)))\*i

$$3.377 \quad \int \frac{4-3 \tan(a+bx)}{\sqrt{4+3 \tan(a+bx)}} dx$$

**Optimal.** Leaf size=85

$$-\frac{9 \operatorname{ArcTan}\left(\frac{1-3 \tan(a+bx)}{\sqrt{2} \sqrt{4+3 \tan(a+bx)}}\right)}{5\sqrt{2} b} + \frac{13 \tanh^{-1}\left(\frac{3+\tan(a+bx)}{\sqrt{2} \sqrt{4+3 \tan(a+bx)}}\right)}{5\sqrt{2} b}$$

[Out]  $-9/10 \cdot \arctan(1/2 \cdot (1-3 \cdot \tan(b \cdot x+a)) \cdot 2^{(1/2)} / (4+3 \cdot \tan(b \cdot x+a))^{(1/2)}) / b \cdot 2^{(1/2)} + 13/10 \cdot \operatorname{arctanh}(1/2 \cdot (3+\tan(b \cdot x+a)) \cdot 2^{(1/2)} / (4+3 \cdot \tan(b \cdot x+a))^{(1/2)}) / b \cdot 2^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {3617, 3616, 209, 213}

$$\frac{13 \tanh^{-1}\left(\frac{\tan(a+bx)+3}{\sqrt{2} \sqrt{3 \tan(a+bx)+4}}\right)}{5\sqrt{2} b} - \frac{9 \operatorname{ArcTan}\left(\frac{1-3 \tan(a+bx)}{\sqrt{2} \sqrt{3 \tan(a+bx)+4}}\right)}{5\sqrt{2} b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(4 - 3 \cdot \operatorname{Tan}[a + b \cdot x]) / \operatorname{Sqrt}[4 + 3 \cdot \operatorname{Tan}[a + b \cdot x]], x]$

[Out]  $(-9 \cdot \operatorname{ArcTan}[(1 - 3 \cdot \operatorname{Tan}[a + b \cdot x]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[4 + 3 \cdot \operatorname{Tan}[a + b \cdot x]])]) / (5 \cdot \operatorname{Sqrt}[2] \cdot b) + (13 \cdot \operatorname{ArcTanh}[(3 + \operatorname{Tan}[a + b \cdot x]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[4 + 3 \cdot \operatorname{Tan}[a + b \cdot x]])]) / (5 \cdot \operatorname{Sqrt}[2] \cdot b)$

Rule 209

$\operatorname{Int}[(a + (b \cdot x)^{-1}), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1 / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[b, 2])) \cdot \operatorname{ArcTan}[\operatorname{Rt}[b, 2] \cdot (x / \operatorname{Rt}[a, 2])], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 213

$\operatorname{Int}[(a + (b \cdot x)^{-1}), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2] \cdot \operatorname{Rt}[b, 2])^{-1}) \cdot \operatorname{ArcTanh}[\operatorname{Rt}[b, 2] \cdot (x / \operatorname{Rt}[-a, 2])], x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3616

$\operatorname{Int}[(c + (d \cdot \tan[e + (f \cdot x)]) / \operatorname{Sqrt}[(a + (b \cdot \tan[e + (f \cdot x)])]), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[-2 \cdot (d^2 / f), \operatorname{Subst}[\operatorname{Int}[1 / (2 \cdot b \cdot c \cdot d - 4 \cdot a \cdot d^2 + x^2), x], x, (b \cdot c - 2 \cdot a \cdot d - b \cdot d \cdot \operatorname{Tan}[e + f \cdot x]) / \operatorname{Sqrt}[a + b \cdot \operatorname{Tan}[e + f \cdot x]]], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 + b^2, 0]

&& NeQ[c^2 + d^2, 0] && EqQ[2\*a\*c\*d - b\*(c^2 - d^2), 0]

### Rule 3617

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2\*q), Int[(a\*c + b\*d + c\*q + (b\*c - a\*d + d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] - Dist[1/(2\*q), Int[(a\*c + b\*d - c\*q + (b\*c - a\*d - d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2\*a\*c\*d - b\*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])

### Rubi steps

$$\begin{aligned} \int \frac{4 - 3 \tan(a + bx)}{\sqrt{4 + 3 \tan(a + bx)}} dx &= \frac{1}{10} \int \frac{27 + 9 \tan(a + bx)}{\sqrt{4 + 3 \tan(a + bx)}} dx - \frac{1}{10} \int \frac{-13 + 39 \tan(a + bx)}{\sqrt{4 + 3 \tan(a + bx)}} dx \\ &= -\frac{81 \operatorname{Subst}\left(\int \frac{1}{162 + x^2} dx, x, \frac{9 - 27 \tan(a + bx)}{\sqrt{4 + 3 \tan(a + bx)}}\right)}{5b} + \frac{1521 \operatorname{Subst}\left(\int \frac{1}{-27378 + x^2} dx, x, \frac{3 + \tan(a + bx)}{\sqrt{4 + 3 \tan(a + bx)}}\right)}{5b} \\ &= -\frac{9 \tan^{-1}\left(\frac{1 - 3 \tan(a + bx)}{\sqrt{2} \sqrt{4 + 3 \tan(a + bx)}}\right)}{5\sqrt{2} b} + \frac{13 \tanh^{-1}\left(\frac{3 + \tan(a + bx)}{\sqrt{2} \sqrt{4 + 3 \tan(a + bx)}}\right)}{5\sqrt{2} b} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.06, size = 75, normalized size = 0.88

$$\frac{(3 - 4i) \tanh^{-1}\left(\frac{\sqrt{4 + 3 \tan(a + bx)}}{\sqrt{4 - 3i}}\right)}{\sqrt{4 - 3i} b} + \frac{(3 + 4i) \tanh^{-1}\left(\frac{\sqrt{4 + 3 \tan(a + bx)}}{\sqrt{4 + 3i}}\right)}{\sqrt{4 + 3i} b}$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 3\*Tan[a + b\*x])/Sqrt[4 + 3\*Tan[a + b\*x]],x]

[Out] ((3 - 4\*I)\*ArcTanh[Sqrt[4 + 3\*Tan[a + b\*x]]/Sqrt[4 - 3\*I]])/(Sqrt[4 - 3\*I]\*b) + ((3 + 4\*I)\*ArcTanh[Sqrt[4 + 3\*Tan[a + b\*x]]/Sqrt[4 + 3\*I]])/(Sqrt[4 + 3\*I]\*b)

### Maple [A]

time = 0.22, size = 134, normalized size = 1.58

method	result
--------	--------



derivativedivides	$\frac{13\sqrt{2} \ln\left(9+3\tan(bx+a)-3\sqrt{4+3\tan(bx+a)}\sqrt{2}\right)}{20} + \frac{9\sqrt{2} \arctan\left(\frac{(2\sqrt{4+3\tan(bx+a)}-3\sqrt{2})}{2}\right)}{10}$
default	$\frac{13\sqrt{2} \ln\left(9+3\tan(bx+a)-3\sqrt{4+3\tan(bx+a)}\sqrt{2}\right)}{20} + \frac{9\sqrt{2} \arctan\left(\frac{(2\sqrt{4+3\tan(bx+a)}-3\sqrt{2})}{2}\right)}{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{b} * (-13/20 * 2^{(1/2)} * \ln(9+3*\tan(b*x+a)-3*(4+3*\tan(b*x+a))^{(1/2)}*2^{(1/2)}) + 9/10 * 2^{(1/2)} * \arctan(1/2*(2*(4+3*\tan(b*x+a))^{(1/2)}-3*2^{(1/2)}) * 2^{(1/2)}) + 13/20 * 2^{(1/2)} * \ln(9+3*\tan(b*x+a)+3*(4+3*\tan(b*x+a))^{(1/2)}*2^{(1/2)}) + 9/10 * 2^{(1/2)} * \arctan(1/2*(2*(4+3*\tan(b*x+a))^{(1/2)}+3*2^{(1/2)}) * 2^{(1/2)})$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `-integrate((3*tan(b*x + a) - 4)/sqrt(3*tan(b*x + a) + 4), x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(71) = 142.

time = 1.11, size = 855, normalized size = 10.06

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x, algorithm="fricas")`

[Out]  $\frac{1}{58500} * 25^{(1/4)} * (44 * b^2 * \sqrt{b^{(-4)}} + 125) * \sqrt{-11000 * b^2 * \sqrt{b^{(-4)}}} + 31250 * (b^{(-4)})^{(1/4)} * \log(25/39 * (4875 * b^2 * \sqrt{b^{(-4)}}) * \cos(b*x + a) + 25^{(1/4)} * (5 * b^3 * \sqrt{b^{(-4)}}) * \cos(b*x + a) + 8 * b * \cos(b*x + a)) * \sqrt{-11000 * b^2 * \sqrt{b^{(-4)}}} + 31250) * \sqrt{((4 * \cos(b*x + a) + 3 * \sin(b*x + a)) / \cos(b*x + a)) * (b^{(-4)})^{(1/4)} + 3900 * \cos(b*x + a) + 2925 * \sin(b*x + a)) / \cos(b*x + a)} - \frac{1}{58500} * 25^{(1/4)} * (44 * b^2 * \sqrt{b^{(-4)}} + 125) * \sqrt{-11000 * b^2 * \sqrt{b^{(-4)}}} + 31250 * (b^{(-4)})^{(1/4)} * \log(25/39 * (4875 * b^2 * \sqrt{b^{(-4)}}) * \cos(b*x + a) - 25^{(1/4)} * (5 * b^3 * \sqrt{b^{(-4)}}) * \cos(b*x + a) + 8 * b * \cos(b*x + a)) * \sqrt{-11000 * b^2 * \sqrt{b^{(-4)}}}$

$$b^{-4}) + 31250) \sqrt{(4 \cos(bx + a) + 3 \sin(bx + a)) / \cos(bx + a)} (b^{-4})^{1/4} + 3900 \cos(bx + a) + 2925 \sin(bx + a) / \cos(bx + a) - 1/125 \cdot 25^{1/4} \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250} (b^{-4})^{1/4} \arctan(1/73125 \cdot 25^{3/4} \sqrt{1/39} (5 b^5 \sqrt{b^{-4}} + 8 b^3) \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250}) + 31250) \sqrt{(4875 b^2 \sqrt{b^{-4}} \cos(bx + a) + 25^{1/4} (5 b^3 \sqrt{b^{-4}} \cos(bx + a) + 8 b \cos(bx + a)) \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250}) \sqrt{(4 \cos(bx + a) + 3 \sin(bx + a)) / \cos(bx + a)} (b^{-4})^{1/4} + 3900 \cos(bx + a) + 2925 \sin(bx + a) / \cos(bx + a) (b^{-4})^{3/4} - 1/14625 \cdot 25^{3/4} (5 b^5 \sqrt{b^{-4}} + 8 b^3) \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250}) \sqrt{(4 \cos(bx + a) + 3 \sin(bx + a)) / \cos(bx + a)} (b^{-4})^{3/4} - 4/3 b^2 \sqrt{b^{-4}} - 5/3 - 1/125 \cdot 25^{1/4} \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250} (b^{-4})^{1/4} \arctan(1/73125 \cdot 25^{3/4} \sqrt{1/39} (5 b^5 \sqrt{b^{-4}} + 8 b^3) \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250}) \sqrt{(4875 b^2 \sqrt{b^{-4}} \cos(bx + a) - 25^{1/4} (5 b^3 \sqrt{b^{-4}} \cos(bx + a) + 8 b \cos(bx + a)) \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250}) \sqrt{(4 \cos(bx + a) + 3 \sin(bx + a)) / \cos(bx + a)} (b^{-4})^{1/4} + 3900 \cos(bx + a) + 2925 \sin(bx + a) / \cos(bx + a) (b^{-4})^{3/4} - 1/14625 \cdot 25^{3/4} (5 b^5 \sqrt{b^{-4}} + 8 b^3) \sqrt{-11000 b^2 \sqrt{b^{-4}} + 31250}) \sqrt{(4 \cos(bx + a) + 3 \sin(bx + a)) / \cos(bx + a)} (b^{-4})^{3/4} + 4/3 b^2 \sqrt{b^{-4}} + 5/3$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3 \tan(a + bx)}{\sqrt{3 \tan(a + bx) + 4}} dx - \int \left( -\frac{4}{\sqrt{3 \tan(a + bx) + 4}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-3\*tan(b\*x+a))/(4+3\*tan(b\*x+a))^(1/2), x)

[Out] -Integral(3\*tan(a + b\*x)/sqrt(3\*tan(a + b\*x) + 4), x) - Integral(-4/sqrt(3\*tan(a + b\*x) + 4), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4-3\*tan(b\*x+a))/(4+3\*tan(b\*x+a))^(1/2), x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 7.23, size = 147, normalized size = 1.73

$$\operatorname{atan}\left(b \sqrt{\frac{-\frac{16}{b^2} - \frac{121}{161^2}}{2}} \sqrt{3 \tan(a + bx) + 4}\right) \sqrt{\frac{-\frac{16}{b^2} - \frac{121}{161^2}}{2}} - \operatorname{atan}\left(b \sqrt{\frac{-\frac{16}{b^2} + \frac{121}{161^2}}{2}} \sqrt{3 \tan(a + bx) + 4}\right) \sqrt{\frac{-\frac{16}{b^2} + \frac{121}{161^2}}{2}} + 2 \operatorname{atanh}\left(\frac{2b \sqrt{\frac{\frac{9}{25} - \frac{27}{161^2}}{3}} \sqrt{3 \tan(a + bx) + 4}}{\frac{9}{25} - \frac{27}{161^2}}\right) + 2 \operatorname{atanh}\left(\frac{2b \sqrt{\frac{\frac{9}{25} + \frac{27}{161^2}}{3}} \sqrt{3 \tan(a + bx) + 4}}{\frac{9}{25} + \frac{27}{161^2}}\right) \sqrt{\frac{\frac{9}{25} - \frac{27}{161^2}}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(-(3*\tan(a + b*x) - 4)/(3*\tan(a + b*x) + 4)^{(1/2)}, x)$

[Out]  $\text{atan}\left(\frac{b\sqrt{-16/25 - 12i/25} \sqrt{3\tan(a + b*x) + 4}}{2}\right) \sqrt{-16/25 - 12i/25} + 2i - \text{atan}\left(\frac{b\sqrt{-16/25 + 12i/25} \sqrt{3\tan(a + b*x) + 4}}{2}\right) \sqrt{-16/25 + 12i/25} + 2*\text{atanh}\left(\frac{2*b\sqrt{(9/25 - 27i/100)} \sqrt{3\tan(a + b*x) + 4}}{3}\right) \sqrt{9/25 - 27i/100} + 2*\text{atanh}\left(\frac{2*b\sqrt{(9/25 + 27i/100)} \sqrt{3\tan(a + b*x) + 4}}{3}\right) \sqrt{9/25 + 27i/100}$

$$3.378 \quad \int \tan^2(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=278

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-1/2*(a*(A-B)-b*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a*(A-B)-b*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(b*(A-B)+a*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*(b*(A-B)+a*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*(A*b+B*a)*\tan(d*x+c)^{(1/2)}/d+2/3*(A*a-B*b)*\tan(d*x+c)^{(3/2)}/d+2/5*(A*b+B*a)*\tan(d*x+c)^{(5/2)}/d+2/7*b*B*\tan(d*x+c)^{(7/2)}/d$

**Rubi [A]**

time = 0.22, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3673, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2(aB + Ab)\tan^2(c+dx)}{5d} + \frac{2(aA - bB)\tan^2(c+dx)}{3d} - \frac{2(aB + Ab)\sqrt{\tan(c+dx)}}{d} - \frac{(a(A+B) + b(A-B))\log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2}d}\right)}{2\sqrt{2}d} + \frac{(a(A+B) + b(A-B))\log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2}d}\right)}{2\sqrt{2}d} + \frac{2aB\tan^2(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((a*(A - B) - b*(A + B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(\text{Sqrt}[2]*d) - ((a*(A - B) - b*(A + B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(\text{Sqrt}[2]*d) - ((b*(A - B) + a*(A + B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/(2*\text{Sqrt}[2]*d) + ((b*(A - B) + a*(A + B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/(2*\text{Sqrt}[2]*d) - (2*(A*b + a*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/d + (2*(a*A - b*B)*\text{Tan}[c + d*x]^{(3/2)})/(3*d) + (2*(A*b + a*B)*\text{Tan}[c + d*x]^{(5/2)})/(5*d) + (2*b*B*\text{Tan}[c + d*x]^{(7/2)})/(7*d)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3609

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \ :> \ \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{2bB \tan^{\frac{7}{2}}(c + dx)}{7d} + \int \tan^{\frac{5}{2}}(c + dx)(aA - bB) \\
&= \frac{2(Ab + aB) \tan^{\frac{5}{2}}(c + dx)}{5d} + \frac{2bB \tan^{\frac{7}{2}}(c + dx)}{7d} \\
&= \frac{2(aA - bB) \tan^{\frac{3}{2}}(c + dx)}{3d} + \frac{2(Ab + aB) \tan^{\frac{5}{2}}(c + dx)}{5d} \\
&= -\frac{2(Ab + aB) \sqrt{\tan(c + dx)}}{d} + \frac{2(aA - bB) \tan^{\frac{5}{2}}(c + dx)}{3d} \\
&= -\frac{2(Ab + aB) \sqrt{\tan(c + dx)}}{d} + \frac{2(aA - bB) \tan^{\frac{5}{2}}(c + dx)}{3d} \\
&= -\frac{2(Ab + aB) \sqrt{\tan(c + dx)}}{d} + \frac{2(aA - bB) \tan^{\frac{5}{2}}(c + dx)}{3d} \\
&= -\frac{2(Ab + aB) \sqrt{\tan(c + dx)}}{d} + \frac{2(aA - bB) \tan^{\frac{5}{2}}(c + dx)}{3d} \\
&= -\frac{2(Ab + aB) \sqrt{\tan(c + dx)}}{d} + \frac{2(aA - bB) \tan^{\frac{5}{2}}(c + dx)}{3d} \\
&= -\frac{(b(A - B) + a(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
&= \frac{(a(A - B) - b(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica** [C] Result contains complex when optimal does not.

time = 1.13, size = 151, normalized size = 0.54

$$\frac{-105\sqrt{-1}(ia + b)(A - iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{\tan(c + dx)}}{105d}\right) + 105(-1)^{3/4}(a + ib)(A + iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{\tan(c + dx)}}{105d}\right) + 2\sqrt{\tan(c + dx)}(-105(Ab + aB) + 35(aA - bB)\tan(c + dx) + 21(Ab + aB)\tan^2(c + dx) + 15bB\tan^3(c + dx))}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]), x]
```

```
[Out] (-105*(-1)^(1/4)*(I*a + b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
+ 105*(-1)^(3/4)*(a + I*b)*(A + I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
```

$$+ 2*\text{Sqrt}[\text{Tan}[c + d*x]]*(-105*(A*b + a*B) + 35*(a*A - b*B)*\text{Tan}[c + d*x] + 2*1*(A*b + a*B)*\text{Tan}[c + d*x]^2 + 15*b*B*\text{Tan}[c + d*x]^3)/(105*d)$$

**Maple [A]**

time = 0.06, size = 275, normalized size = 0.99

method	result
derivativedivides	$\frac{2Bb \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{2Ab \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ba \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2aA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - \frac{2Bb \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2Ab \left( \sqrt{\tan(dx+c)} \right)$
default	$\frac{2Bb \left( \tan^{\frac{7}{2}}(dx+c) \right)}{7} + \frac{2Ab \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ba \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2aA \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - \frac{2Bb \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - 2Ab \left( \sqrt{\tan(dx+c)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $1/d*(2/7*B*b*\tan(d*x+c)^{(7/2)}+2/5*A*b*\tan(d*x+c)^{(5/2)}+2/5*B*a*\tan(d*x+c)^{(5/2)}+2/3*a*A*\tan(d*x+c)^{(3/2)}-2/3*B*b*\tan(d*x+c)^{(3/2)}-2*A*b*\tan(d*x+c)^{(1/2)}-2*B*a*\tan(d*x+c)^{(1/2)}+1/4*(A*b+B*a)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})))+1/4*(-A*a+B*b)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima [A]**

time = 0.51, size = 227, normalized size = 0.82

120\*B\*tan(dx+c)^3+168\*(B\*a+A\*b)\*tan(dx+c)^2-210\*sqrt(2)\*(A-B)\*a-(A+B)\*b\*arctan(1/2\*sqrt(2)\*(sqrt(2)+2\*sqrt(tan(dx+c))))-210\*sqrt(2)\*(A-B)\*a-(A+B)\*b\*arctan(-1/2\*sqrt(2)\*(sqrt(2)-2\*sqrt(tan(dx+c))))+105\*sqrt(2)\*((A+B)\*a+(A-B)\*b)\*log(sqrt(2)\*sqrt(tan(dx+c))+tan(dx+c)+1)-105\*sqrt(2)\*((A+B)\*a+(A-B)\*b)\*log(-sqrt(2)\*sqrt(tan(dx+c))+tan(dx+c)+1)+280\*(A\*a-B\*b)\*tan(dx+c)^3-840\*(B\*a+A\*b)\*sqrt(tan(dx+c))/d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out]  $1/420*(120*B*b*\tan(d*x+c)^{(7/2)}+168*(B*a+A*b)*\tan(d*x+c)^{(5/2)}-210*\text{sqrt}(2)*((A-B)*a-(A+B)*b)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2)+2*\text{sqrt}(\tan(d*x+c))))-210*\text{sqrt}(2)*((A-B)*a-(A+B)*b)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2)-2*\text{sqrt}(\tan(d*x+c))))+105*\text{sqrt}(2)*((A+B)*a+(A-B)*b)*\log(\text{sqrt}(2)*\text{sqrt}(\tan(d*x+c))+\tan(d*x+c)+1)-105*\text{sqrt}(2)*((A+B)*a+(A-B)*b)*\log(-\text{sqrt}(2)*\text{sqrt}(\tan(d*x+c))+\tan(d*x+c)+1)+280*(A*a-B*b)*\tan(d*x+c)^3-840*(B*a+A*b)*\text{sqrt}(\tan(d*x+c)))/d$







Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 15.35, size = 1522, normalized size = 5.47
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

```
[Out] atan((A^2*a^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^
4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A*b*(2*A^4*a^2*b^
2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^3)/d + (16*A^3*a*
b^2)/d) - (A^2*b^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A
^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A*b*(2*A^4*a
^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^3)/d + (16*A
^3*a*b^2)/d))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4
) + (A^2*a*b)/(2*d^2))^(1/2)*2i - atan((A^2*a^2*tan(c + d*x)^(1/2)*((A^2*a*
b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))
^(1/2)*32i)/((16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^
4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a*b^2)/d) - (A^2*b^2*tan(c + d*x)^(1/2)*((A
^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*
d^4))^(1/2)*32i)/((16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4
- A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a*b^2)/d))*((A^2*a*b)/(2*d^2) - (2*A^4
*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2)*2i + atan((B
^2*a^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4
)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*32i)/((16*B*a*(2*B^4*a^2*b^2*d^4
- B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*b^3)/d + (16*B^3*a^2*b)/
d) - (B^2*b^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*
a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*32i)/((16*B*a*(2*B^4*a^2*
b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*b^3)/d + (16*B^3*
a^2*b)/d))*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4)
- (B^2*a*b)/(2*d^2))^(1/2)*2i - atan((B^2*a^2*tan(c + d*x)^(1/2)*((2*B^4*a
^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(
1/2)*32i)/((16*B^3*b^3)/d + (16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4
*a^4*d^4)^(1/2))/d^3 - (16*B^3*a^2*b)/d) - (B^2*b^2*tan(c + d*x)^(1/2)*((2*
B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d
^2))^(1/2)*32i)/((16*B^3*b^3)/d + (16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4
- B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*a^2*b)/d))*((2*B^4*a^2*b^2*d^4 - B^4*b^
4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*2i + (2*A*a*t
an(c + d*x)^(3/2))/(3*d) - (2*A*b*tan(c + d*x)^(1/2))/d - (2*B*a*tan(c + d*
```

$$\begin{aligned} & x)^{(1/2)})/d + (2*A*b*\tan(c + d*x)^{(5/2)})/(5*d) + (2*B*a*\tan(c + d*x)^{(5/2)}) \\ & / (5*d) - (2*B*b*\tan(c + d*x)^{(3/2)})/(3*d) + (2*B*b*\tan(c + d*x)^{(7/2)})/(7*d) \\ & ) \end{aligned}$$

$$3.379 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=254

$$\frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-1/2*(b*(A-B)+a*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(b*(A-B)+a*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a*(A-B)-b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*(a*(A-B)-b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2*(A*a-B*b)*\tan(d*x+c)^{(1/2)}/d+2/3*(A*b+B*a)*\tan(d*x+c)^{(3/2)}/d+2/5*b*B*\tan(d*x+c)^{(5/2)}/d$

**Rubi [A]**

time = 0.18, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3673, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A+B)+b(A-B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A+B)+b(A-B))\text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d} + \frac{(a(A-B)-b(A+B))\log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2}d}\right)}{2\sqrt{2}d} - \frac{(a(A-B)-b(A+B))\log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2}d}\right)}{2\sqrt{2}d} + \frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((b*(A - B) + a*(A + B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(\text{Sqrt}[2]*d) - ((b*(A - B) + a*(A + B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(\text{Sqrt}[2]*d) + ((a*(A - B) - b*(A + B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/(2*\text{Sqrt}[2]*d) - ((a*(A - B) - b*(A + B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/(2*\text{Sqrt}[2]*d) + (2*(a*A - b*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/d + (2*(A*b + a*B)*\text{Tan}[c + d*x]^{(3/2)})/(3*d) + (2*b*B*\text{Tan}[c + d*x]^{(5/2)})/(5*d)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^m\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[B

\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= \frac{2bB \tan^{\frac{5}{2}}(c + dx)}{5d} + \int \tan^{\frac{3}{2}}(c + dx)(aA - bB) dx \\
 &= \frac{2(Ab + aB) \tan^{\frac{3}{2}}(c + dx)}{3d} + \frac{2bB \tan^{\frac{5}{2}}(c + dx)}{5d} \\
 &= \frac{2(aA - bB) \sqrt{\tan(c + dx)}}{d} + \frac{2(Ab + aB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= \frac{2(aA - bB) \sqrt{\tan(c + dx)}}{d} + \frac{2(Ab + aB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= \frac{2(aA - bB) \sqrt{\tan(c + dx)}}{d} + \frac{2(Ab + aB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= \frac{2(aA - bB) \sqrt{\tan(c + dx)}}{d} + \frac{2(Ab + aB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= \frac{2(aA - bB) \sqrt{\tan(c + dx)}}{d} + \frac{2(Ab + aB) \tan^{\frac{3}{2}}(c + dx)}{3d} \\
 &= \frac{(a(A - B) - b(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
 &= \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.68, size = 134, normalized size = 0.53

$$\frac{15\sqrt{-1}(a - ib)(A - iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c + dx)}\right) + 15\sqrt{-1}(a + ib)(A + iB)\text{tanh}^{-1}\left((-1)^{3/4}\sqrt{\tan(c + dx)}\right) + 2\sqrt{\tan(c + dx)}(15(aA - bB) + 5(Ab + aB)\tan(c + dx) + 3bB\tan^2(c + dx))}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]), x]

[Out] (15\*(-1)^(1/4)\*(a - I\*b)\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 15\*(-1)^(1/4)\*(a + I\*b)\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*Sqrt[Tan[c + d\*x]]\*(15\*(a\*A - b\*B) + 5\*(A\*b + a\*B)\*Tan[c + d\*x] + 3\*b\*B\*Tan[c + d\*x]^2))/(15\*d)

**Maple [A]**

time = 0.05, size = 253, normalized size = 1.00

method	result
derivativedivides	$\frac{2Bb \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ab \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{2Ba \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2aA \left( \sqrt{\tan}(dx+c) \right) - 2Bb \left( \sqrt{\tan}(dx+c) \right) + \frac{(-aA+L)}{d}$
default	$\frac{2Bb \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ab \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{2Ba \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2aA \left( \sqrt{\tan}(dx+c) \right) - 2Bb \left( \sqrt{\tan}(dx+c) \right) + \frac{(-aA+L)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} \left( \frac{2}{5} B b \tan^{\frac{5}{2}}(dx+c) + \frac{2}{3} A b \tan^{\frac{3}{2}}(dx+c) + \frac{2}{3} B a \tan^{\frac{3}{2}}(dx+c) + 2 a A \tan^{\frac{1}{2}}(dx+c) - 2 B b \tan^{\frac{1}{2}}(dx+c) + \frac{1}{4} (-A a + B b) 2^{\frac{1}{2}} \left( \ln \left( \frac{(1+2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}} + \tan(dx+c))}{(1-2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}} + \tan(dx+c))} \right) + 2 \arctan(1+2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}}) + 2 \arctan(-1+2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}}) \right) + \frac{1}{4} (-A b - B a) 2^{\frac{1}{2}} \left( \ln \left( \frac{(1-2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}} + \tan(dx+c))}{(1+2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}} + \tan(dx+c))} \right) + 2 \arctan(1+2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}}) + 2 \arctan(-1+2^{\frac{1}{2}} \tan(dx+c)^{\frac{1}{2}}) \right) \right)$

**Maxima [A]**

time = 0.52, size = 210, normalized size = 0.83

$\frac{24 B b \tan(dx+c)^5 - 30 \sqrt{2} (A+B)a + (A-B)b \arctan\left(\frac{1}{2} \sqrt{2} + 2 \sqrt{\tan(dx+c)}\right) - 30 \sqrt{2} (A+B)a + (A-B)b \arctan\left(-\frac{1}{2} \sqrt{2} - 2 \sqrt{\tan(dx+c)}\right) - 15 \sqrt{2} (A-B)a - (A+B)b \log\left(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + 15 \sqrt{2} (A-B)a - (A+B)b \log\left(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + 40 (B a + A b) \tan(dx+c)^{\frac{3}{2}} + 120 (A a - B b) \sqrt{\tan(dx+c)}}{60 d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{60} \left( 24 B b \tan(dx+c)^{\frac{5}{2}} - 30 \sqrt{2} \left( (A+B)a + (A-B)b \right) \arctan\left(\frac{1}{2} \sqrt{2} \left( \sqrt{2} + 2 \sqrt{\tan(dx+c)} \right)\right) - 30 \sqrt{2} \left( (A+B)a + (A-B)b \right) \arctan\left(-\frac{1}{2} \sqrt{2} \left( \sqrt{2} - 2 \sqrt{\tan(dx+c)} \right)\right) - 15 \sqrt{2} \left( (A-B)a - (A+B)b \right) \log\left(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + 15 \sqrt{2} \left( (A-B)a - (A+B)b \right) \log\left(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + 40 (B a + A b) \tan(dx+c)^{\frac{3}{2}} + 120 (A a - B b) \sqrt{\tan(dx+c)} \right) / d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 13541 vs. 2(216) = 432.

time = 26.64, size = 13541, normalized size = 53.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/60*(60*\sqrt{2}*d^5*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 + 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^{(3/4)}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4}*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} + \sqrt{2}*((A*a - B*b)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} - ((A^2*B + B^3)*a^3 + (A^3 + A*B^2)*a^2*b + (A^2*B + B^3)*a*b^2 + (A^3 + A*B^2)*b^3)*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4})*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 + 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{(((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^6 - 8*(A^5*B - A*B^5)*a^5*b - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^4*b^2 - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^2*b^4 + 8*(A^5*B - A*B^5)*a*b^5 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^6)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\cos(d*x + c) + \sqrt{2}*((A^4*B - 2*A^2*B^3 + B^5)*a^5 + (A^5 - 10*A^3*B^2 + 9*A*B^4)*a^4*b - 2*(5*A^4*B - 14*A^2*B^3 + B^5)*a^3*b^2 - 2*(A^5 - 14*A^3*B^2 + 5*A*B^4)*a^2*b^3 + (9*A^4*B - 10*A^2*B^3 + B^5)*a*b^4 + (A^5 - 2*A^3*B^2 + A*B^4)*b^5)*d^3*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\cos(d$$



```

*x + c) - ((A^7 - A^5*B^2 - A^3*B^4 + A*B^6)*a^7 - (9*A^6*B - A^4*B^3 - 9*A
^2*B^5 + B^7)*a^6*b - (A^7 - 25*A^5*B^2 - 17*A^3*B^4 + 9*A*B^6)*a^5*b^2 + (
A^6*B - 17*A^4*B^3 - 17*A^2*B^5 + B^7)*a^4*b^3 - (A^7 - 17*A^5*B^2 - 17*A^3
*B^4 + A*B^6)*a^3*b^4 + (9*A^6*B - 17*A^4*B^3 - 25*A^2*B^5 + B^7)*a^2*b^5 +
(A^7 - 9*A^5*B^2 - A^3*B^4 + 9*A*B^6)*a*b^6 - (A^6*B - A^4*B^3 - A^2*B^5 +
B^7)*b^7)*d*cos(d*x + c))*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A
^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 + 2*(A*B*a^2 - A*B*b^2
+ (A^2 - B^2)*a*b)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B
^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4))/((A^4 - 2*A^2*B^2 +
B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8
*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*sqrt(sin(d*x + c)/co
s(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b
^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^(1/4) + ((A^8 - 2*A^4*B^4 + B^8)*a^8
- 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^7*b + 16*(A^6*B^2 + 2*A^4*B^4 +
A^2*B^6)*a^6*b^2 - 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^5*b^3 - 2*(A^8 -
16*A^6*B^2 - 34*A^4*B^4 - 16*A^2*B^6 + B^8)*a^4*b^4 + 8*(A^7*B + A^5*B^3 -
A^3*B^5 - A*B^7)*a^3*b^5 + 16*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^2*b^6 + 8*
(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^7 + (A^8 - 2*A^4*B^4 + B^8)*b^8)*si
n(d*x + c))/cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^
2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^(3/4) - sqrt(2)*(((A^5
- A*B^4)*a^5 - (5*A^4*B + 4*A^2*B^3 - B^5)*a^4*b + 4*(A^3*B^2 + A*B^4)*a^3
*b^2 - 4*(A^4*B + A^2*B^3)*a^2*b^3 - (A^5 - 4*A^3*B^2 - 5*A*B^4)*a*b^4 + (A
^4*B - B^5)*b^5)*d^7*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2
+ B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)*sqrt(((A^4 - 2*A^2*B^2
+ B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 +
8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4) - ((A^6*B + A^
4*B^3 - A^2*B^5 - B^7)*a^7 + (A^7 - 3*A^5*B^2 - ...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 11.62, size = 1492, normalized size = 5.87
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

```
[Out] atan((A^2*a^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^
4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A^3*b^3)/d + (16*
A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^
2*b)/d) - (A^2*b^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A
^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A^3*b^3)/d +
(16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A
^3*a^2*b)/d))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4
) - (A^2*a*b)/(2*d^2))^(1/2)*2i - atan((A^2*a^2*tan(c + d*x)^(1/2)*(- (2*A^
4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2
))^(1/2)*32i)/((16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2
))/d^3 - (16*A^3*b^3)/d + (16*A^3*a^2*b)/d) - (A^2*b^2*tan(c + d*x)^(1/2)*(-
(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)
/(2*d^2))^(1/2)*32i)/((16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^
4)^(1/2))/d^3 - (16*A^3*b^3)/d + (16*A^3*a^2*b)/d))*(- (2*A^4*a^2*b^2*d^4 -
A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)*2i - a
tan((B^2*a^2*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2) - (2*B^4*a^2*b^2*d^4 - B
^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*B^3*a^3)/d + (16*B
*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*a*b
^2)/d) - (B^2*b^2*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2) - (2*B^4*a^2*b^2*d^
4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*B^3*a^3)/d +
(16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^
3*a*b^2)/d))*((B^2*a*b)/(2*d^2) - (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^
4*d^4)^(1/2)/(4*d^4))^(1/2)*2i + atan((B^2*a^2*tan(c + d*x)^(1/2)*((2*B^4*a
^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) + (B^2*a*b)/(2*d^2))^(
1/2)*32i)/((16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/
d^3 - (16*B^3*a^3)/d + (16*B^3*a*b^2)/d) - (B^2*b^2*tan(c + d*x)^(1/2)*((2*
B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) + (B^2*a*b)/(2*d
^2))^(1/2)*32i)/((16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1
/2))/d^3 - (16*B^3*a^3)/d + (16*B^3*a*b^2)/d))*((2*B^4*a^2*b^2*d^4 - B^4*b^
4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) + (B^2*a*b)/(2*d^2))^(1/2)*2i + (2*A*a*t
an(c + d*x)^(1/2))/d + (2*A*b*tan(c + d*x)^(3/2))/(3*d) + (2*B*a*tan(c + d*
x)^(3/2))/(3*d) - (2*B*b*tan(c + d*x)^(1/2))/d + (2*B*b*tan(c + d*x)^(5/2)
)/(5*d)
```

$$3.380 \quad \int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=229

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out] 1/2\*(a\*(A-B)-b\*(A+B))\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/2\*(a\*(A-B)-b\*(A+B))\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/4\*(b\*(A-B)+a\*(A+B))\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)-1/4\*(b\*(A-B)+a\*(A+B))\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)+2\*(A\*b+B\*a)\*tan(d\*x+c)^(1/2)/d+2/3\*b\*B\*tan(d\*x+c)^(3/2)/d

**Rubi [A]**

time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3673, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2(aB + Ab)\sqrt{\tan(c+dx)}}{d} + \frac{(a(A+B) + b(A-B))\log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d}\right)}{2\sqrt{2} d} - \frac{(a(A+B) + b(A-B))\log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d}\right)}{2\sqrt{2} d} + \frac{2bB \tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] -(((a\*(A - B) - b\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + ((a\*(A - B) - b\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + ((b\*(A - B) + a\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) - ((b\*(A - B) + a\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) + (2\*(A\*b + a\*B)\*Sqrt[Tan[c + d\*x]])/d + (2\*b\*B\*Tan[c + d\*x]^(3/2))/(3\*d)

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 642**

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)), x]

$x])^m \text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
 \int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))(A+B \tan(c+dx)) dx &= \frac{2bB \tan^{\frac{3}{2}}(c+dx)}{3d} + \int \sqrt{\tan(c+dx)} (aA \\
 &= \frac{2(Ab+aB) \sqrt{\tan(c+dx)}}{d} + \frac{2bB \tan^{\frac{3}{2}}(c+dx)}{3d} \\
 &= \frac{2(Ab+aB) \sqrt{\tan(c+dx)}}{d} + \frac{2bB \tan^{\frac{3}{2}}(c+dx)}{3d} \\
 &= \frac{2(Ab+aB) \sqrt{\tan(c+dx)}}{d} + \frac{2bB \tan^{\frac{3}{2}}(c+dx)}{3d} \\
 &= \frac{2(Ab+aB) \sqrt{\tan(c+dx)}}{d} + \frac{2bB \tan^{\frac{3}{2}}(c+dx)}{3d} \\
 &= \frac{(b(A-B) + a(A+B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d} \\
 &= -\frac{(a(A-B) - b(A+B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.28, size = 114, normalized size = 0.50

$$\frac{3\sqrt{-1}(ia+b)(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - 3(-1)^{3/4}(a+ib)(A+iB)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 2\sqrt{\tan(c+dx)}(3Ab+3aB+bB\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(3*(-1)^{1/4}*(I*a + b)*(A - I*B)*\text{ArcTan}[(-1)^{3/4}*\text{Sqrt}[\text{Tan}[c + d*x]]] - 3*(-1)^{3/4}*(a + I*b)*(A + I*B)*\text{ArcTanh}[(-1)^{3/4}*\text{Sqrt}[\text{Tan}[c + d*x]]] + 2*\text{Sqrt}[\text{Tan}[c + d*x]]*(3*A*b + 3*a*B + b*B*\text{Tan}[c + d*x]))/(3*d)$

**Maple [A]**

time = 0.05, size = 229, normalized size = 1.00

method	result
--------	--------

derivativedivides	$\frac{2Bb \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2Ab \left( \sqrt{\tan}(dx+c) \right) + 2Ba \left( \sqrt{\tan}(dx+c) \right) + \frac{(-Ab-aB)\sqrt{2}}{\ln \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan}(dx+c)}{\sqrt{\tan}(dx+c)} \right) \right)}$
default	$\frac{2Bb \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2Ab \left( \sqrt{\tan}(dx+c) \right) + 2Ba \left( \sqrt{\tan}(dx+c) \right) + \frac{(-Ab-aB)\sqrt{2}}{\ln \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan}(dx+c)}{\sqrt{\tan}(dx+c)} \right) \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} \left( \frac{2}{3} B b \tan^{\frac{3}{2}}(d x+c)+2 A b \tan^{\frac{1}{2}}(d x+c)+2 B a \tan^{\frac{1}{2}}(d x+c)+\frac{1}{4}(-A b-B a) 2^{\frac{1}{2}} \ln \left( \frac{1+2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)}{1-2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)} \right)+2 \arctan \left( \frac{1+2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)}{1-2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)} \right)+2 \arctan \left( -\frac{1+2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)}{1-2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)} \right)+\frac{1}{4}(A a-B b) 2^{\frac{1}{2}} \ln \left( \frac{1-2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)}{1+2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)} \right)+2 \arctan \left( \frac{1+2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)}{1-2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)} \right)+2 \arctan \left( -\frac{1+2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)}{1-2^{\frac{1}{2}} \tan^{\frac{1}{2}}(d x+c)} \right) \right)$

**Maxima** [A]

time = 0.54, size = 192, normalized size = 0.84

$\frac{8 B b \tan (d x+c)^{\frac{3}{2}}+6 \sqrt{2}((A-B) a-(A+B) b) \arctan \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)+6 \sqrt{2}((A-B) a-(A+B) b) \arctan \left( -\frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)-3 \sqrt{2}((A+B) a+(A-B) b) \log \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)+3 \sqrt{2}((A+B) a+(A-B) b) \log \left( -\frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)+24(B a+A b) \sqrt{\tan (d x+c)}}{12 d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{12} \left( 8 B b \tan^{\frac{3}{2}}(d x+c)+6 \sqrt{2}((A-B) a-(A+B) b) \arctan \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)+6 \sqrt{2}((A-B) a-(A+B) b) \arctan \left( -\frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)-3 \sqrt{2}((A+B) a+(A-B) b) \log \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)+3 \sqrt{2}((A+B) a+(A-B) b) \log \left( -\frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan (d x+c)}}{\sqrt{\tan (d x+c)}} \right) \right)+24(B a+A b) \sqrt{\tan (d x+c)} \right) / d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 13378 vs. 2(195) = 390.

time = 24.19, size = 13378, normalized size = 58.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/12*(12*\sqrt{2}*d^5*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)}*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^{(3/4)}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4)*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4})*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} - \sqrt{2}*((B*a + A*b)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4})*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} + ((A^3 + A*B^2)*a^3 - (A^2*B + B^3)*a^2*b + (A^3 + A*B^2)*a*b^2 - (A^2*B + B^3)*b^3)*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4})*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)}*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{(((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^6 - 8*(A^5*B - A*B^5)*a^5*b - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^4*b^2 - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^2*b^4 + 8*(A^5*B - A*B^5)*a*b^5 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^6)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4})*\cos(d*x + c) + \sqrt{2}*((A^5 - 2*A^3*B^2 + A*B^4)*a^5 - (9*A^4*B - 10*A^2*B^3 + B^5)*a^4*b - 2*(A^5 - 14*A^3*B^2 + 5*A*B^4)*a^3*b^2 + 2*(5*A^4*B - 14*A^2*B^3 + B^5)*a^2*b^3 + (A^5 - 10*A^3*B^2 + 9*A*B^4)*a*b^4 - (A^4*B - 2*A^2*B^3 + B^5)*b^5)*d^3*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4})*\cos(d*x + c) + ((A^6*B - A^4*B^3 - A^2*B^5 + B^7)*a^7 + (A^7 - 9*A^5*B^2 - A^3*B^4 + 9*A*B^6)*a^6*b - (9*A^6*B - 17*A^4*B^3 - 25*A^2*B^5 + B^7)*a^5*b^2 - ($$





[Out] Timed out

**Mupad [B]**

time = 9.52, size = 1456, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\tan(c + d*x)^{(1/2)}*(A + B*\tan(c + d*x))*(a + b*\tan(c + d*x)),x)$

[Out] 
$$2*\operatorname{atanh}\left(\frac{(32*A^2*a^2*\tan(c + d*x)^{(1/2)}*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/(4*d^4) + (A^2*a*b)/(2*d^2))^{(1/2)}}{(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/d^3 - (16*A^3*a^3)/d + (16*A^3*a*b^2)/d} - \frac{(32*A^2*b^2*\tan(c + d*x)^{(1/2)}*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/(4*d^4) + (A^2*a*b)/(2*d^2))^{(1/2)}}{(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/d^3 - (16*A^3*a^3)/d + (16*A^3*a*b^2)/d}\right)*\left(\frac{(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)}}{(4*d^4) + (A^2*a*b)/(2*d^2)}\right)^{(1/2)} - 2*\operatorname{atanh}\left(\frac{(32*A^2*a^2*\tan(c + d*x)^{(1/2)}*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/(4*d^4))^{(1/2)}}{(16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/d^3 - (16*A^3*a*b^2)/d} - \frac{(32*A^2*b^2*\tan(c + d*x)^{(1/2)}*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/(4*d^4))^{(1/2)}}{(16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)})/d^3 - (16*A^3*a*b^2)/d}\right)*\left(\frac{(A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{(1/2)}}{(4*d^4)}\right)^{(1/2)} - \operatorname{atan}\left(\frac{(B^2*a^2*\tan(c + d*x)^{(1/2)}*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/(4*d^4) - (B^2*a*b)/(2*d^2))^{(1/2)}*32i}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/d^3 - (16*B^3*b^3)/d + (16*B^3*a^2*b)/d} - \frac{(B^2*b^2*\tan(c + d*x)^{(1/2)}*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/(4*d^4) - (B^2*a*b)/(2*d^2))^{(1/2)}*32i}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/d^3 - (16*B^3*b^3)/d + (16*B^3*a^2*b)/d}\right)*\left(- \frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)}}{(4*d^4)} - \frac{(B^2*a*b)/(2*d^2)}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/d^3 - (16*B^3*b^3)/d + (16*B^3*a^2*b)/d}\right)*2i + \operatorname{atan}\left(\frac{(B^2*a^2*\tan(c + d*x)^{(1/2)}*((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/(4*d^4) - (B^2*a*b)/(2*d^2))^{(1/2)}*32i}{(16*B^3*b^3)/d + (16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/d^3 - (16*B^3*a^2*b)/d} - \frac{(B^2*b^2*\tan(c + d*x)^{(1/2)}*((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/(4*d^4) - (B^2*a*b)/(2*d^2))^{(1/2)}*32i}{(16*B^3*b^3)/d + (16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/d^3 - (16*B^3*a^2*b)/d}\right)*\left(\frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)}}{(4*d^4)} - \frac{(B^2*a*b)/(2*d^2)}{(16*B^3*b^3)/d + (16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)})/d^3 - (16*B^3*a^2*b)/d}\right)*2i + \frac{(2*A*b*\tan(c + d*x)^{(1/2)})/d + (2*B*a*\tan(c + d*x)^{(1/2)})/d + (2*B*b*\tan(c + d*x)^{(3/2)})/(3*d)}{3*d}$$

$$3.381 \quad \int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=205

$$\frac{(b(A-B) + a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(b(A-B) + a(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $1/2*(b*(A-B)+a*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*(b*(A-B)+a*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(a*(A-B)-b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*(a*(A-B)-b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2*b*B*\tan(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3673, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A+B)+b(A-B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a(A+B)+b(A-B))\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d} - \frac{(a(A-B)-b(A+B))\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} + \frac{(a(A-B)-b(A+B))\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} + \frac{2bB\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

[Out]  $-(((b*(A-B) + a*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/(\operatorname{Sqrt}[2]*d) + ((b*(A-B) + a*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/(\operatorname{Sqrt}[2]*d) - ((a*(A-B) - b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + ((a*(A-B) - b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + (2*b*B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d$

Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] :> Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3673

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan(c + dx))(A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{2bB \sqrt{\tan(c + dx)}}{d} + \int \frac{aA - bB + (Ab + aB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx \\
 &= \frac{2bB \sqrt{\tan(c + dx)}}{d} + \frac{2 \text{Subst}\left(\int \frac{aA - bB + (Ab + aB)x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
 &= \frac{2bB \sqrt{\tan(c + dx)}}{d} + \frac{(b(A - B) + a(A + B)) \text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
 &= \frac{2bB \sqrt{\tan(c + dx)}}{d} + \frac{(b(A - B) + a(A + B)) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\tan(c + dx)}\right)}{2d} \\
 &= -\frac{(a(A - B) - b(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{2\sqrt{2} d} \\
 &= -\frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
 time = 0.11, size = 94, normalized size = 0.46

$$\frac{\sqrt[4]{-1} (a - ib)(A - iB) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right) + \sqrt[4]{-1} (a + ib)(A + iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right) - 2bB \sqrt{\tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

```

[In] Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
[Out] -(((-1)^(1/4)*(a - I*b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (-1)^(1/4)*(a + I*b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 2*b*B*Sqrt[Tan[c + d*x]])/d)
    
```

**Maple [A]**  
 time = 0.05, size = 203, normalized size = 0.99

method	result
derivativedivides	$  \frac{2Bb \left( \sqrt{\tan(dx+c)} \right) + \frac{(aA - Bb) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4}}{4}  $

default	$\frac{2Bb\left(\sqrt{\tan(dx+c)}\right) + \frac{(aA-Bb)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}\right)\right) + 2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)}{4}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} \cdot \left( \frac{2Bb \tan(dx+c)^{1/2} + \frac{1}{4} (Aa - Bb) 2^{1/2} \left( \ln \left( \frac{1 + 2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1 - 2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan(1 + 2^{1/2} \tan(dx+c)^{1/2}) \right) + \frac{1}{4} (Ab + Ba) 2^{1/2} \left( \ln \left( \frac{1 - 2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1 + 2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan(1 + 2^{1/2} \tan(dx+c)^{1/2}) \right) + 2 \arctan(-1 + 2^{1/2} \tan(dx+c)^{1/2}) \right)$

**Maxima** [A]

time = 0.54, size = 174, normalized size = 0.85

$$\frac{2\sqrt{2}(A+B)a + (A-B)b \arctan\left(\frac{1}{\sqrt{2}}(\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}(A+B)a + (A-B)b \arctan\left(-\frac{1}{\sqrt{2}}(\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) + \sqrt{2}((A-B)a - (A+B)b) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - \sqrt{2}((A-B)a - (A+B)b) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + 8Bb\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{1}{4} \cdot \left( 2\sqrt{2} \left( (A+B)a + (A-B)b \right) \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{\tan(dx+c)}\right)\right) + 2\sqrt{2} \left( (A+B)a + (A-B)b \right) \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{\tan(dx+c)}\right)\right) + \sqrt{2} \left( (A-B)a - (A+B)b \right) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - \sqrt{2} \left( (A-B)a - (A+B)b \right) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + 8Bb\sqrt{\tan(dx+c)} \right) / d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 13180 vs. 2(175) = 350.

time = 24.92, size = 13180, normalized size = 64.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]  $\frac{1}{4} \cdot \left( 4\sqrt{2} d^5 \sqrt{\left( (A^4 + 2A^2B^2 + B^4)a^4 + 2(A^4 + 2A^2B^2 + B^4)a^2b^2 + (A^4 + 2A^2B^2 + B^4)b^4 + 2(A^2B^2 - A^2B^2 + A^2 - B^2)ab \right)} + d^2 \sqrt{\left( (A^4 + 2A^2B^2 + B^4)a^4 + 2(A^4 + 2A^2B^2 + B^4)a^2b^2 + (A^4 + 2A^2B^2 + B^4)b^4 + 2(A^2B^2 - A^2B^2 + A^2 - B^2)ab \right)}$

$$\begin{aligned}
& ) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4) / ((A^4 - 2 * A^2 * B^2 + B^4) * a^4 \\
& - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 + 8 * (A^3 * B \\
& - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) * (((A^4 + 2 * A^2 * B^2 + B^4) * a^4 \\
& + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4)^{(3/4)} * \text{sqrt}(((A^4 - 2 * A^2 * B^2 + B^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - \\
& 10 * A^2 * B^2 + B^4) * a^2 * b^2 + 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B \\
& ^4) * b^4) / d^4) * \arctan(-(((A^8 + 2 * A^6 * B^2 - 2 * A^2 * B^6 - B^8) * a^8 - 4 * (A^7 * B \\
& + 3 * A^5 * B^3 + 3 * A^3 * B^5 + A * B^7) * a^7 * b + 2 * (A^8 + 2 * A^6 * B^2 - 2 * A^2 * B^6 - B \\
& ^8) * a^6 * b^2 - 12 * (A^7 * B + 3 * A^5 * B^3 + 3 * A^3 * B^5 + A * B^7) * a^5 * b^3 - 12 * (A^7 * \\
& B + 3 * A^5 * B^3 + 3 * A^3 * B^5 + A * B^7) * a^3 * b^5 - 2 * (A^8 + 2 * A^6 * B^2 - 2 * A^2 * B^6 \\
& - B^8) * a^2 * b^6 - 4 * (A^7 * B + 3 * A^5 * B^3 + 3 * A^3 * B^5 + A * B^7) * a * b^7 - (A^8 + \\
& 2 * A^6 * B^2 - 2 * A^2 * B^6 - B^8) * b^8) * d^4 * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 \\
& * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4) * \text{sqrt}(( \\
& (A^4 - 2 * A^2 * B^2 + B^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 \\
& + B^4) * a^2 * b^2 + 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) / d^4 \\
& + \text{sqrt}(2) * ((A * a - B * b) * d^7 * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + \\
& 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4) * \text{sqrt}(((A^4 - 2 \\
& * A^2 * B^2 + B^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * \\
& a^2 * b^2 + 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) / d^4) - ((A \\
& ^2 * B + B^3) * a^3 + (A^3 + A * B^2) * a^2 * b + (A^2 * B + B^3) * a * b^2 + (A^3 + A * B^2) \\
& * b^3) * d^5 * \text{sqrt}(((A^4 - 2 * A^2 * B^2 + B^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * ( \\
& A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 + 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 \\
& + B^4) * b^4) / d^4) * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 \\
& + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4 + 2 * (A * B * a^2 - A * B * b^2 + (A^2 \\
& - B^2) * a * b) * d^2 * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) \\
& * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4)) / ((A^4 - 2 * A^2 * B^2 + B^4) * a^4 \\
& - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 + 8 * (A^3 * B \\
& - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) * \text{sqrt}((((A^6 - A^4 * B^2 - A^2 \\
& * B^4 + B^6) * a^6 - 8 * (A^5 * B - A * B^5) * a^5 * b - (A^6 - 17 * A^4 * B^2 - 17 * A^2 * B^4 \\
& + B^6) * a^4 * b^2 - (A^6 - 17 * A^4 * B^2 - 17 * A^2 * B^4 + B^6) * a^2 * b^4 + 8 * (A^5 * B - \\
& A * B^5) * a * b^5 + (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * b^6) * d^2 * \text{sqrt}(((A^4 + 2 * A^2 \\
& * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) \\
& * b^4) / d^4) * \cos(d * x + c) + \text{sqrt}(2) * (((A^4 * B - 2 * A^2 * B^3 + B^5) * a^5 + (A^5 \\
& - 10 * A^3 * B^2 + 9 * A * B^4) * a^4 * b - 2 * (5 * A^4 * B - 14 * A^2 * B^3 + B^5) * a^3 * b^2 - 2 * \\
& (A^5 - 14 * A^3 * B^2 + 5 * A * B^4) * a^2 * b^3 + (9 * A^4 * B - 10 * A^2 * B^3 + B^5) * a * b^4 + \\
& (A^5 - 2 * A^3 * B^2 + A * B^4) * b^5) * d^3 * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * ( \\
& A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4) * \cos(d * x \\
& + c) - ((A^7 - A^5 * B^2 - A^3 * B^4 + A * B^6) * a^7 - (9 * A^6 * B - A^4 * B^3 - 9 * A^2 * \\
& B^5 + B^7) * a^6 * b - (A^7 - 25 * A^5 * B^2 - 17 * A^3 * B^4 + 9 * A * B^6) * a^5 * b^2 + (A^6 \\
& * B - 17 * A^4 * B^3 - 17 * A^2 * B^5 + B^7) * a^4 * b^3 - (A^7 - 17 * A^5 * B^2 - 17 * A^3 * B^4 \\
& + A * B^6) * a^3 * b^4 + (9 * A^6 * B - 17 * A^4 * B^3 - 25 * A^2 * B^5 + B^7) * a^2 * b^5 + (A \\
& ^7 - 9 * A^5 * B^2 - A^3 * B^4 + 9 * A * B^6) * a * b^6 - (A^6 * B - A^4 * B^3 - A^2 * B^5 + B^7) \\
& * b^7) * d * \cos(d * x + c) * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * \\
& B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4 + 2 * (A * B * a^2 - A * B * b^2 + ( \\
& A^2 - B^2) * a * b) * d^2 * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2
\end{aligned}$$

$$\begin{aligned}
& + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4) / ((A^4 - 2 * A^2 * B^2 + B^4) \\
& ) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 + 8 * (A \\
& ^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) * \sqrt{\sin(dx + c) / \cos(d \\
& * x + c)} * (((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 \\
& + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4)^{(1/4)} + ((A^8 - 2 * A^4 * B^4 + B^8) * a^8 - \\
& 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^7 * b + 16 * (A^6 * B^2 + 2 * A^4 * B^4 + A^2 \\
& * B^6) * a^6 * b^2 - 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^5 * b^3 - 2 * (A^8 - 16 \\
& * A^6 * B^2 - 34 * A^4 * B^4 - 16 * A^2 * B^6 + B^8) * a^4 * b^4 + 8 * (A^7 * B + A^5 * B^3 - A^ \\
& 3 * B^5 - A * B^7) * a^3 * b^5 + 16 * (A^6 * B^2 + 2 * A^4 * B^4 + A^2 * B^6) * a^2 * b^6 + 8 * (A^ \\
& 7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a * b^7 + (A^8 - 2 * A^4 * B^4 + B^8) * b^8) * \sin(d \\
& * x + c) / \cos(dx + c) * (((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + \\
& B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4)^{(3/4)} - \sqrt{2} * (((A^5 - \\
& A * B^4) * a^5 - (5 * A^4 * B + 4 * A^2 * B^3 - B^5) * a^4 * b + 4 * (A^3 * B^2 + A * B^4) * a^3 * b^ \\
& 2 - 4 * (A^4 * B + A^2 * B^3) * a^2 * b^3 - (A^5 - 4 * A^3 * B^2 - 5 * A * B^4) * a * b^4 + (A^4 * \\
& B - B^5) * b^5) * d^7 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + \\
& B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4} * \sqrt{((A^4 - 2 * A^2 * B^2 + B \\
& ^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 + 8 * \\
& (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) / d^4} - ((A^6 * B + A^4 * B \\
& ^3 - A^2 * B^5 - B^7) * a^7 + (A^7 - 3 * A^5 * B^2 - 9 * \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))/sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 8.89, size = 1420, normalized size = 6.93







$$3.382 \quad \int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\tan^3(c+dx)} dx$$

**Optimal.** Leaf size=205

$$\frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-1/2*(a*(A-B)-b*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a*(A-B)-b*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(b*(A-B)+a*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*(b*(A-B)+a*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*a*A/d/\tan(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3672, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{(a(A+B) + b(A-B)) \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{(a(A+B) + b(A-B)) \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{2aA}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Tan}[c + d*x]^{(3/2)}, x]$

[Out]  $((a*(A - B) - b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) - ((a*(A - B) - b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) - ((b*(A - B) + a*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) + ((b*(A - B) + a*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) - (2*a*A)/(d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]))$

Rule 210

$\operatorname{Int}[(a + (b_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 631

$\operatorname{Int}[(a + (b_*)*(x_*) + (c_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{q = 1 - 4*S \operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \operatorname{RationalQ}[q] \&\& (\operatorname{EqQ}[q^2, 1] \parallel \operatorname{!RationalQ}[b^2 - 4*a*c]) /; \operatorname{FreeQ}\{a, b, c\}, x \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\operatorname{Int}[(d + (e_*)*(x_*)]/((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2), x\_Symbol] \rightarrow \operatorname{Simp}[d*(\operatorname{Log}[\operatorname{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \operatorname{FreeQ}\{a, b, c, d,$

e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3672

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(b\*c - a\*d)\*(A\*b - a\*B)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2aA}{d\sqrt{\tan(c + dx)}} + \int \frac{Ab + aB - (aA - bB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2aA}{d\sqrt{\tan(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{Ab + aB + (-aA + bB)x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
&= -\frac{2aA}{d\sqrt{\tan(c + dx)}} + \frac{(b(A - B) + a(A + B)) \text{Subst}\left(\int \frac{1-x}{1+x} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
&= -\frac{2aA}{d\sqrt{\tan(c + dx)}} - \frac{(b(A - B) + a(A + B)) \text{Subst}\left(\int \frac{1}{-1-x} dx, x, \sqrt{\tan(c + dx)}\right)}{2\sqrt{2}d} \\
&= -\frac{(b(A - B) + a(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) + (b(A - B) + a(A + B)) \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2}d} \\
&= \frac{(a(A - B) - b(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [A]**

time = 0.41, size = 158, normalized size = 0.77

$$\frac{-2\sqrt{2}(a(A - B) - b(A + B))(\text{ArcTan}(1 - \sqrt{2} \sqrt{\tan(c + dx)}) - \text{ArcTan}(1 + \sqrt{2} \sqrt{\tan(c + dx)})) + \sqrt{2}(b(A - B) + a(A + B))(\log(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx))) + \frac{2aA}{\sqrt{\tan(c + dx)}}}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2),x]

[Out] -1/4\*(-2\*Sqrt[2]\*(a\*(A - B) - b\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]) + Sqrt[2]\*(b\*(A - B) + a\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + (8\*a\*A)/Sqrt[Tan[c + d\*x]]/d

**Maple [A]**

time = 0.05, size = 203, normalized size = 0.99

method	result
derivativedivides	$ -\frac{2aA}{\sqrt{\tan(dx + c)}} + \frac{(Ab + aB)\sqrt{2} \left( \ln\left(\frac{1 + \sqrt{2} \left(\sqrt{\tan(dx + c)} + \tan(dx + c)\right)}{1 - \sqrt{2} \left(\sqrt{\tan(dx + c)} + \tan(dx + c)\right)}\right) + 2 \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)} + \tan(dx + c)\right)\right) \right)}{4} $

default	$-\frac{2aA}{\sqrt{\tan(dx+c)}} + \frac{(Ab+aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{d} * (-2*a*A/\tan(d*x+c)^{(1/2)} + 1/4*(A*b+B*a)*2^{(1/2)} * (\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)} + \tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)} + \tan(d*x+c))) + 2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}) + 2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})) + 1/4*(-A*a+B*b)*2^{(1/2)} * (\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)} + \tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)} + \tan(d*x+c))) + 2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}) + 2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [A]

time = 0.53, size = 174, normalized size = 0.85

$$\frac{2\sqrt{2}((A-B)a - (A+B)b)\arctan\left(\frac{1}{\sqrt{2}}(\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}((A-B)a - (A+B)b)\arctan\left(-\frac{1}{\sqrt{2}}(\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) - \sqrt{2}((A+B)a + (A-B)b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \sqrt{2}((A+B)a + (A-B)b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \frac{2aA}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="maxima")`

[Out]  $-\frac{1}{4} * (2*\sqrt{2} * ((A - B)*a - (A + B)*b) * \arctan(1/2*\sqrt{2} * (\sqrt{2} + 2*\sqrt{\tan(dx+c)})) + 2*\sqrt{2} * ((A - B)*a - (A + B)*b) * \arctan(-1/2*\sqrt{2} * (\sqrt{2} - 2*\sqrt{\tan(dx+c)})) - \sqrt{2} * ((A + B)*a + (A - B)*b) * \log(\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2} * ((A + B)*a + (A - B)*b) * \log(-\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + 8*A*a/\sqrt{\tan(dx+c)})/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 13523 vs. 2(175) = 350.

time = 31.72, size = 13523, normalized size = 65.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="fricas")`

[Out]  $\frac{1}{4} * (4*\sqrt{2} * (d^5*\cos(dx+c)^2 - d^5) * \sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B$

$$\begin{aligned}
& a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2 \\
& *(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)} / ((A^4 \\
& - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B \\
& ^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4) * (((A^ \\
& 4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2 \\
& *B^2 + B^4)*b^4)/d^4)^{3/4} * \sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - \\
& A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 \\
& + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4)} * \arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 \\
& - B^8)*a^8 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2* \\
& A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A* \\
& B^7)*a^5*b^3 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 \\
& + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + \\
& A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*\sqrt{((A^4 + 2 \\
& *A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 \\
& + B^4)*b^4)/d^4)} * \sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3* \\
& b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2 \\
& *A^2*B^2 + B^4)*b^4)/d^4)} - \sqrt{2} * ((B*a + A*b)*d^7*\sqrt{((A^4 + 2*A^2*B^2 \\
& + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b \\
& ^4)/d^4)} * \sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A \\
& ^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 \\
& + B^4)*b^4)/d^4)} + ((A^3 + A*B^2)*a^3 - (A^2*B + B^3)*a^2*b + (A^3 + A*B^2 \\
& )*a*b^2 - (A^2*B + B^3)*b^3)*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3 \\
& *B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)* \\
& a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4)} * \sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^ \\
& 4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A* \\
& B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + \\
& 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)} / ((A^ \\
& 4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + \\
& B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4) * \sqrt{ \\
& (((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^6 - 8*(A^5*B - A*B^5)*a^5*b - (A^6 - 1 \\
& 7*A^4*B^2 - 17*A^2*B^4 + B^6)*a^4*b^2 - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^ \\
& 6)*a^2*b^4 + 8*(A^5*B - A*B^5)*a*b^5 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^6) \\
& *d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 \\
& + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)} * \cos(dx + c) + \sqrt{2} * (((A^5 - 2*A^3*B \\
& ^2 + A*B^4)*a^5 - (9*A^4*B - 10*A^2*B^3 + B^5)*a^4*b - 2*(A^5 - 14*A^3*B^2 \\
& + 5*A*B^4)*a^3*b^2 + 2*(5*A^4*B - 14*A^2*B^3 + B^5)*a^2*b^3 + (A^5 - 10*A^3 \\
& *B^2 + 9*A*B^4)*a*b^4 - (A^4*B - 2*A^2*B^3 + B^5)*b^5)*d^3*\sqrt{((A^4 + 2*A \\
& ^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + \\
& B^4)*b^4)/d^4)} * \cos(dx + c) + ((A^6*B - A^4*B^3 - A^2*B^5 + B^7)*a^7 + (A^7 \\
& - 9*A^5*B^2 - A^3*B^4 + 9*A*B^6)*a^6*b - (9*A^6*B - 17*A^4*B^3 - 25*A^2*B^ \\
& 5 + B^7)*a^5*b^2 - (A^7 - 17*A^5*B^2 - 17*A^3*B^4 + A*B^6)*a^4*b^3 - (A^6*B \\
& - 17*A^4*B^3 - 17*A^2*B^5 + B^7)*a^3*b^4 - (A^7 - 25*A^5*B^2 - 17*A^3*B^4 \\
& + 9*A*B^6)*a^2*b^5 + (9*A^6*B - A^4*B^3 - 9*A^2*B^5 + B^7)*a*b^6 + (A^7 - A \\
& ^5*B^2 - A^3*B^4 + A*B^6)*b^7)*d*\cos(dx + c) * \sqrt{((A^4 + 2*A^2*B^2 + B^4 \\
& )*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2
\end{aligned}$$

```

*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4))/
((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*
sqrt(sin(d*x + c)/cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^(1/4) + ((A^8 - 2*A^4*B^4 + B^8)*a^8 - 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^7*b + 16*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^6*b^2 - 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^5*b^3 - 2*(A^8 - 16*A^6*B^2 - 34*A^4*B^4 - 16*A^2*B^6 + B^8)*a^4*b^4 + 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^3*b^5 + 16*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^2*b^6 + 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^7 + (A^8 - 2*A^4*B^4 + B^8)*b^8)*sin(d*x + c))/cos(d*x + c))*(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^(3/4) + sqrt(2)*(((A^4*B - B^5)*a^5 + (A^5 - 4*A^3*B^2 - 5*A*B^4)*a^4*b - 4*(A^4*B + A^2*B^3)*a^3*b^2 - 4*(A^3*B^2 + A*B^4)*a^2*b^3 - (5*A^4*B + 4*A^2*B^3 - B^5)*a*b^4 - (A^5 - A*B^4)*b^5)*d^7*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4) + ((A^7 + A^5*B^2 - A^3*B^4 - A*B^6)*a^7 ...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2), x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))/tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 8.81, size = 1420, normalized size = 6.93



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B \cdot \tan(c + d \cdot x)) \cdot (a + b \cdot \tan(c + d \cdot x))) / \tan(c + d \cdot x)^{(3/2)}, x)$

[Out]  $2 \cdot \text{atanh}\left(\frac{32 A^2 a^2 d^3 \tan(c + d x)^{1/2} \left(\frac{A^2 a b}{2 d^2} - (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} / (4 d^4)\right)^{1/2}}{16 A b (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} + 16 A^3 a^3 d^2 - 16 A^3 a b^2 d^2} - \frac{32 A^2 b^2 d^3 \tan(c + d x)^{1/2} \left(\frac{A^2 a b}{2 d^2} - (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} / (4 d^4)\right)^{1/2}}{16 A b (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} + 16 A^3 a^3 d^2 - 16 A^3 a b^2 d^2}\right) \cdot \left(\frac{A^2 a b}{2 d^2} - (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} / (4 d^4)\right)^{1/2} - 2 \cdot \text{atanh}\left(\frac{32 A^2 a^2 d^3 \tan(c + d x)^{1/2} \left(\frac{A^2 a b}{2 d^2} - (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} / (4 d^4)\right)^{1/2}}{16 A b (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} - 16 A^3 a^3 d^2 + 16 A^3 a b^2 d^2} - \frac{32 A^2 b^2 d^3 \tan(c + d x)^{1/2} \left(\frac{A^2 a b}{2 d^2} - (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} / (4 d^4)\right)^{1/2}}{16 A b (2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4)^{1/2} - 16 A^3 a^3 d^2 + 16 A^3 a b^2 d^2}\right) \cdot \left(\frac{2 A^4 a^2 b^2 d^4 - A^4 b^4 d^4 - A^4 a^4 d^4}{4 d^4} + \frac{A^2 a b}{2 d^2}\right)^{1/2} - 2 \cdot \text{atanh}\left(\frac{32 B^2 a^2 \tan(c + d x)^{1/2} \left(- (2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4)^{1/2} / (4 d^4) - \frac{B^2 a b}{2 d^2}\right)^{1/2}}{(16 B a (2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4)^{1/2}) / d^3 - (16 B^3 b^3) / d + (16 B^3 a^2 b) / d} - \frac{32 B^2 b^2 \tan(c + d x)^{1/2} \left(- (2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4)^{1/2} / (4 d^4) - \frac{B^2 a b}{2 d^2}\right)^{1/2}}{(16 B a (2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4)^{1/2}) / d^3 - (16 B^3 b^3) / d + (16 B^3 a^2 b) / d}\right) \cdot \left(\frac{2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4}{4 d^4} - \frac{B^2 a b}{2 d^2}\right)^{1/2} + 2 \cdot \text{atanh}\left(\frac{32 B^2 a^2 \tan(c + d x)^{1/2} \left(\frac{2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4}{4 d^4} - \frac{B^2 a b}{2 d^2}\right)^{1/2}}{(16 B^3 b^3) / d + (16 B a (2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4)^{1/2}) / d^3 - (16 B^3 a^2 b) / d} - \frac{32 B^2 b^2 \tan(c + d x)^{1/2} \left(\frac{2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4}{4 d^4} - \frac{B^2 a b}{2 d^2}\right)^{1/2}}{(16 B^3 b^3) / d + (16 B a (2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4)^{1/2}) / d^3 - (16 B^3 a^2 b) / d}\right) \cdot \left(\frac{2 B^4 a^2 b^2 d^4 - B^4 b^4 d^4 - B^4 a^4 d^4}{4 d^4} - \frac{B^2 a b}{2 d^2}\right)^{1/2} - (2 A a) / (d \cdot \tan(c + d x)^{1/2})$

$$3.383 \quad \int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=229

$$\frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-1/2*(b*(A-B)+a*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(b*(A-B)+a*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a*(A-B)-b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*(a*(A-B)-b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*(A*b+B*a)/d/\tan(d*x+c)^{(1/2)}-2/3*a*A/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3672, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A+B)+b(A-B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a(A+B)+b(A-B))\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d} - \frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}} + \frac{(a(A-B)-b(A+B))\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{(a(A-B)-b(A+B))\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out]  $((b*(A-B) + a*(A+B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(\text{Sqrt}[2]*d) - ((b*(A-B) + a*(A+B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]])/(\text{Sqrt}[2]*d) + ((a*(A-B) - b*(A+B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/(2*\text{Sqrt}[2]*d) - ((a*(A-B) - b*(A+B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/(2*\text{Sqrt}[2]*d) - (2*a*A)/(3*d*\text{Tan}[c + d*x]^{(3/2)}) - (2*(A*b + a*B))/(d*\text{Sqrt}[\text{Tan}[c + d*x]])$

Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642



```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3672

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
```

)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} + \int \frac{Ab + aB - (aA - bB) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} dx \\
 &= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + aB)}{d \sqrt{\tan(c + dx)}} + \int \frac{-aA + bB - (A - B) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx \\
 &= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + aB)}{d \sqrt{\tan(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{-aA + bB + Bx}{1 + x^2} dx\right)}{d} \\
 &= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + aB)}{d \sqrt{\tan(c + dx)}} - \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(\sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2}d} \\
 &= -\frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + aB)}{d \sqrt{\tan(c + dx)}} - \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) + \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2}d} \\
 &= \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}d} - \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}d} - \frac{2(Ab + aB)}{d \sqrt{\tan(c + dx)}} - \frac{2aA}{3d \tan^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

**Mathematica [A]**

time = 0.49, size = 178, normalized size = 0.78

$$\frac{6\sqrt{2}(b(A-B) + a(A+B))\left(\text{ArcTan}\left(1 - \sqrt{2}\sqrt{\tan(c+dx)}\right) - \text{ArcTan}\left(1 + \sqrt{2}\sqrt{\tan(c+dx)}\right)\right) + 3\sqrt{2}(a(A-B) - b(A+B))\left(\log\left(1 - \sqrt{2}\sqrt{\tan(c+dx)} + \tan(c+dx)\right) - \log\left(1 + \sqrt{2}\sqrt{\tan(c+dx)} + \tan(c+dx)\right)\right) - \frac{8aA}{\tan^{\frac{3}{2}}(c+dx)} - \frac{24(Ab+aB)}{\sqrt{\tan(c+dx)}}}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (6\*Sqrt[2]\*(b\*(A - B) + a\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]) + 3\*Sqrt[2]\*(a\*(A - B) - b\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) - (8\*a\*A)/Tan[c + d\*x]^(3/2) - (24\*(A\*b + a\*B))/Sqrt[Tan[c + d\*x]]/(12\*d)

**Maple [A]**

time = 0.05, size = 222, normalized size = 0.97

method	result
derivativedivides	$-\frac{2(Ab+aB)}{\sqrt{\tan(dx+c)}} - \frac{2aA}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{(-aA+Bb)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2 \arctan \left( 1+\sqrt{2} \right) \right)}{4}$
default	$-\frac{2(Ab+aB)}{\sqrt{\tan(dx+c)}} - \frac{2aA}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{(-aA+Bb)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2 \arctan \left( 1+\sqrt{2} \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x,method=\_RETURNVERB  
OSE)

[Out] 1/d\*(-2\*(A\*b+B\*a)/tan(d\*x+c)^(1/2)-2/3\*a\*A/tan(d\*x+c)^(3/2)+1/4\*(-A\*a+B\*b)\*  
2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(  
1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*  
tan(d\*x+c)^(1/2)))+1/4\*(-A\*b-B\*a)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+t  
an(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(  
d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))))

**Maxima [A]**

time = 0.50, size = 192, normalized size = 0.84

$$\frac{6\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{\sqrt{2}}(\sqrt{2}+2\sqrt{\tan(dx+c)})\right)+6\sqrt{2}((A+B)a+(A-B)b)\arctan\left(-\frac{1}{\sqrt{2}}(\sqrt{2}-2\sqrt{\tan(dx+c)})\right)+3\sqrt{2}((A-B)a-(A+B)b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-3\sqrt{2}((A-B)a-(A+B)b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\frac{5(Aa+3Bb)\sqrt{2}\tan(dx+c)}{\tan(dx+c)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] -1/12\*(6\*sqrt(2)\*((A + B)\*a + (A - B)\*b)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sq  
rt(tan(d\*x + c)))) + 6\*sqrt(2)\*((A + B)\*a + (A - B)\*b)\*arctan(-1/2\*sqrt(2)\*  
(sqrt(2) - 2\*sqrt(tan(d\*x + c)))) + 3\*sqrt(2)\*((A - B)\*a - (A + B)\*b)\*log(s  
qrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) - 3\*sqrt(2)\*((A - B)\*a - (A +  
B)\*b)\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) + 8\*(A\*a + 3\*(B\*  
a + A\*b)\*tan(d\*x + c))/tan(d\*x + c)^(3/2))/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 13671 vs.  
2(195) = 390.

time = 26.06, size = 13671, normalized size = 59.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] 
$$-1/12*(12*\sqrt{2}*(d^5*\cos(dx + c)^2 - d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 + 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^{3/4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4}*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} + \sqrt{2}*((A*a - B*b)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} - ((A^2*B + B^3)*a^3 + (A^3 + A*B^2)*a^2*b + (A^2*B + B^3)*a*b^2 + (A^3 + A*B^2)*b^3)*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4}))*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 + 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^6 - 8*(A^5*B - A*B^5)*a^5*b - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^4*b^2 - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^2*b^4 + 8*(A^5*B - A*B^5)*a*b^5 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^6)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\cos(dx + c) + \sqrt{2}*((A^4*B - 2*A^2*B^3 + B^5)*a^5 + (A^5 - 10*A^3*B^2 + 9*A*B^4)*a^4*b - 2*(5*A^4*B - 14*A^2*B^3 + B^5)*a^3*b^2 - 2*(A^5 - 14*A^3*B^2 + 5*A*B^4)*a^2*b^3 + (9*A^4*B - 10*A^2*B^3 + B^5)*a*b^4 + (A^5 - 2*A^3*B^2 + A*B^4)*b^5)*d^3*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)}}$$

$$\begin{aligned}
& + B^4) * b^4) / d^4) * \cos(dx + c) - ((A^7 - A^5 * B^2 - A^3 * B^4 + A * B^6) * a^7 - ( \\
& 9 * A^6 * B - A^4 * B^3 - 9 * A^2 * B^5 + B^7) * a^6 * b - (A^7 - 25 * A^5 * B^2 - 17 * A^3 * B^4 \\
& + 9 * A * B^6) * a^5 * b^2 + (A^6 * B - 17 * A^4 * B^3 - 17 * A^2 * B^5 + B^7) * a^4 * b^3 - (A^ \\
& 7 - 17 * A^5 * B^2 - 17 * A^3 * B^4 + A * B^6) * a^3 * b^4 + (9 * A^6 * B - 17 * A^4 * B^3 - 25 * A \\
& ^2 * B^5 + B^7) * a^2 * b^5 + (A^7 - 9 * A^5 * B^2 - A^3 * B^4 + 9 * A * B^6) * a * b^6 - (A^6 * \\
& B - A^4 * B^3 - A^2 * B^5 + B^7) * b^7) * d * \cos(dx + c)) * \sqrt{((A^4 + 2 * A^2 * B^2 + \\
& B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4 \\
& + 2 * (A * B * a^2 - A * B * b^2 + (A^2 - B^2) * a * b) * d^2 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) \\
& * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4 \\
& )) / ((A^4 - 2 * A^2 * B^2 + B^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 \\
& * B^2 + B^4) * a^2 * b^2 + 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4 \\
& )) * \sqrt{\sin(dx + c) / \cos(dx + c)} * (((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + 2 * (A^4 + \\
& 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4)^{(1/4)} + ((A^8 \\
& - 2 * A^4 * B^4 + B^8) * a^8 - 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^7 * b + 16 * \\
& (A^6 * B^2 + 2 * A^4 * B^4 + A^2 * B^6) * a^6 * b^2 - 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * \\
& B^7) * a^5 * b^3 - 2 * (A^8 - 16 * A^6 * B^2 - 34 * A^4 * B^4 - 16 * A^2 * B^6 + B^8) * a^4 * b^4 \\
& + 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^3 * b^5 + 16 * (A^6 * B^2 + 2 * A^4 * B^4 \\
& + A^2 * B^6) * a^2 * b^6 + 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a * b^7 + (A^8 - 2 \\
& * A^4 * B^4 + B^8) * b^8) * \sin(dx + c) / \cos(dx + c)) * (((A^4 + 2 * A^2 * B^2 + B^4) * \\
& a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4) \\
& ^{(3/4)} - \sqrt{2} * (((A^5 - A * B^4) * a^5 - (5 * A^4 * B + 4 * A^2 * B^3 - B^5) * a^4 * b + \\
& 4 * (A^3 * B^2 + A * B^4) * a^3 * b^2 - 4 * (A^4 * B + A^2 * B^3) * a^2 * b^3 - (A^5 - 4 * A^3 * B^ \\
& 2 - 5 * A * B^4) * a * b^4 + (A^4 * B - B^5) * b^5) * d^7 * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) * a \\
& ^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4) / d^4} * \\
& \sqrt{((A^4 - 2 * A^2 * B^2 + B^4) * a^4 - 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^ \\
& ^2 * B^2 + B^4) * a^2 * b^2 + 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b \\
& ^4) / d^4} - ((A^6 * B + A^4 * B^3 - A^2 * B^5 - B^7) * a \dots
\end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))/tan(c + d\*x)\*\*(5/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 10.27, size = 1448, normalized size = 6.32

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)))/tan(c + d\*x)^(5/2),x)

[Out] 
$$2*\operatorname{atanh}\left(\frac{(32*A^2*a^2*d^3*\tan(c+d*x)^{1/2}*(-(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) - (A^2*a*b)/(2*d^2))^{1/2}}{(16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2} - 16*A^3*b^3*d^2 + 16*A^3*a^2*b*d^2) - (32*A^2*b^2*d^3*\tan(c+d*x)^{1/2}*(-(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) - (A^2*a*b)/(2*d^2))^{1/2}}\right) / (16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2} - 16*A^3*b^3*d^2 + 16*A^3*a^2*b*d^2) - (32*A^2*b^2*d^3*\tan(c+d*x)^{1/2}*(-(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) - (A^2*a*b)/(2*d^2))^{1/2} - 2*\operatorname{atanh}\left(\frac{(32*A^2*a^2*d^3*\tan(c+d*x)^{1/2})*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) - (A^2*a*b)/(2*d^2))^{1/2}}{(16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2} + 16*A^3*b^3*d^2 - 16*A^3*a^2*b*d^2) - (32*A^2*b^2*d^3*\tan(c+d*x)^{1/2})*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) - (A^2*a*b)/(2*d^2))^{1/2}}\right) / (16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2} + 16*A^3*b^3*d^2 - 16*A^3*a^2*b*d^2) - (32*A^2*b^2*d^3*\tan(c+d*x)^{1/2})*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) - (A^2*a*b)/(2*d^2))^{1/2} + 2*\operatorname{atanh}\left(\frac{(32*B^2*a^2*d^3*\tan(c+d*x)^{1/2})*((B^2*a*b)/(2*d^2) - (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2})/(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2} + 16*B^3*a^3*d^2 - 16*B^3*a*b^2*d^2) - (32*B^2*b^2*d^3*\tan(c+d*x)^{1/2})*((B^2*a*b)/(2*d^2) - (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2})/(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}\right) / (16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2} - 16*B^3*a^3*d^2 + 16*B^3*a*b^2*d^2) - (32*B^2*b^2*d^3*\tan(c+d*x)^{1/2})*((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2})/(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2} + 2*\operatorname{atanh}\left(\frac{(32*B^2*a^2*d^3*\tan(c+d*x)^{1/2})*((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2})/(4*d^4) + (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2} - 16*B^3*a^3*d^2 + 16*B^3*a*b^2*d^2) - (32*B^2*b^2*d^3*\tan(c+d*x)^{1/2})*((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2})/(4*d^4) + (B^2*a*b)/(2*d^2))^{1/2}}\right) / (16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2} - 16*B^3*a^3*d^2 + 16*B^3*a*b^2*d^2) - (32*B^2*b^2*d^3*\tan(c+d*x)^{1/2})*((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2})/(4*d^4) + (B^2*a*b)/(2*d^2))^{1/2} - ((2*A*a)/3 + 2*A*b*\tan(c+d*x))/(d*\tan(c+d*x)^{3/2}) - (2*B*a)/(d*\tan(c+d*x)^{1/2})$$

$$3.384 \quad \int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

**Optimal.** Leaf size=254

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out] 1/2\*(a\*(A-B)-b\*(A+B))\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/2\*(a\*(A-B)-b\*(A+B))\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/4\*(b\*(A-B)+a\*(A+B))\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)-1/4\*(b\*(A-B)+a\*(A+B))\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)+2\*(A\*a-B\*b)/d/tan(d\*x+c)^(1/2)-2/5\*a\*A/d/tan(d\*x+c)^(5/2)-2/3\*(A\*b+B\*a)/d/tan(d\*x+c)^(3/2)

**Rubi** [A]

time = 0.18, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3672, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{2(aB + Ab)}{3d \tan^2(c+dx)} + \frac{2(aA - bB)}{d \sqrt{\tan(c+dx)}} + \frac{(a(A+B) + b(A-B)) \log(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1)}{2\sqrt{2} d} - \frac{(a(A+B) + b(A-B)) \log(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1)}{2\sqrt{2} d} - \frac{2aA}{5d \tan^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2),x]

[Out] -(((a\*(A - B) - b\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + ((a\*(A - B) - b\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + ((b\*(A - B) + a\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) - ((b\*(A - B) + a\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) - (2\*a\*A)/(5\*d\*Tan[c + d\*x]^(5/2)) - (2\*(A\*b + a\*B))/(3\*d\*Tan[c + d\*x]^(3/2)) + (2\*(a\*A - b\*B))/(d\*Sqrt[Tan[c + d\*x]])

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c]
```

Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3672

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b
```



```
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} + \int \frac{Ab + aB - (aA - bB) \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)} \\
&= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \int \frac{-aA + bB - (aA - bB) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(aA - bB)}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(aA - bB)}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(aA - bB)}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{2aA}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(aA - bB)}{d \sqrt{\tan(c + dx)}} \\
&= \frac{(b(A - B) + a(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{2\sqrt{2} d} \\
&= -\frac{(a(A - B) - b(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.77, size = 198, normalized size = 0.78

$$\frac{30\sqrt{2}(a(A - B) - b(A + B)) \left( \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) - \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right) \right) - 15\sqrt{2}(b(A - B) + a(A + B)) \left( \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) - \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) \right) + \frac{2aA}{\tan^{\frac{5}{2}}(c + dx)} + \frac{4b(A + B)}{\tan^{\frac{3}{2}}(c + dx)} - \frac{2(aA - bB)}{\sqrt{\tan(c + dx)}}}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2),x]

[Out] -1/60\*(30\*sqrt[2]\*(a\*(A - B) - b\*(A + B))\*(ArcTan[1 - sqrt[2]\*sqrt[Tan[c + d\*x]]] - ArcTan[1 + sqrt[2]\*sqrt[Tan[c + d\*x]]]) - 15\*sqrt[2]\*(b\*(A - B) +

a\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + (24\*a\*A)/Tan[c + d\*x]^(5/2) + (40\*(A\*b + a\*B))/Tan[c + d\*x]^(3/2) - (120\*(a\*A - b\*B))/Sqrt[Tan[c + d\*x]]/d

**Maple [A]**

time = 0.05, size = 240, normalized size = 0.94

method	result
derivativedivides	$\frac{(-Ab-aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2 \arctan \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) \right)}{3 \tan(dx+c)^{\frac{3}{2}} - \sqrt{\tan(dx+c)} - 5 \tan(dx+c)^{\frac{5}{2}}} + \dots$
default	$\frac{(-Ab-aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2 \arctan \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) \right)}{3 \tan(dx+c)^{\frac{3}{2}} - \sqrt{\tan(dx+c)} - 5 \tan(dx+c)^{\frac{5}{2}}} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x,method=\_RETURNVERBOSE)

[Out] 1/d\*(-2/3\*(A\*b+B\*a)/tan(d\*x+c)^(3/2)-2\*(-A\*a+B\*b)/tan(d\*x+c)^(1/2)-2/5\*a\*A/tan(d\*x+c)^(5/2)+1/4\*(-A\*b-B\*a)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(A\*a-B\*b)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))))

**Maxima [A]**

time = 0.51, size = 211, normalized size = 0.83

$\frac{30\sqrt{2}(A-B)a-(A+B)b}{60d} \operatorname{arctan}\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})\right) + 30\sqrt{2}(A-B)a-(A+B)b \operatorname{arctan}\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})\right) - 15\sqrt{2}(A+B)a+(A-B)b \log(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1) + 15\sqrt{2}(A+B)a+(A-B)b \log(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1) + \frac{2(15(A-B)\tan(dx+c)^2-3Aa-5(Ba-B)\tan(dx+c))}{\tan(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/60\*(30\*sqrt(2)\*((A - B)\*a - (A + B)\*b)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(tan(d\*x + c)))) + 30\*sqrt(2)\*((A - B)\*a - (A + B)\*b)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(tan(d\*x + c)))) - 15\*sqrt(2)\*((A + B)\*a + (A - B)\*b)\*log(sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) + 15\*sqrt(2)\*((A + B)\*a + (A - B)\*b)\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) + 8\*(15\*(A\*a

- B\*b)\*tan(d\*x + c)^2 - 3\*A\*a - 5\*(B\*a + A\*b)\*tan(d\*x + c))/tan(d\*x + c)^(5/2))/d

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 14358 vs. 2(216) = 432.

time = 28.89, size = 14358, normalized size = 56.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 
$$-1/60*(60*\sqrt{2}*(d^5*\cos(d*x + c)^4 - 2*d^5*\cos(d*x + c)^2 + d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)^{(3/4)}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4}*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 - 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 - 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} - \sqrt{2}*(B*a + A*b)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4} + ((A^3 + A*B^2)*a^3 - (A^2*B + B^3)*a^2*b + (A^3 + A*B^2)*a*b^2 - (A^2*B + B^3)*b^3)*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/d^4})*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4}})/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{(((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^6 - 8*(A^5*B$$

```

- A*B^5)*a^5*b - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^4*b^2 - (A^6 - 17
*A^4*B^2 - 17*A^2*B^4 + B^6)*a^2*b^4 + 8*(A^5*B - A*B^5)*a*b^5 + (A^6 - A^4
*B^2 - A^2*B^4 + B^6)*b^6)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 +
2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)*cos(d*x + c)
+ sqrt(2)*(((A^5 - 2*A^3*B^2 + A*B^4)*a^5 - (9*A^4*B - 10*A^2*B^3 + B^5)*a^
4*b - 2*(A^5 - 14*A^3*B^2 + 5*A*B^4)*a^3*b^2 + 2*(5*A^4*B - 14*A^2*B^3 + B^
5)*a^2*b^3 + (A^5 - 10*A^3*B^2 + 9*A*B^4)*a*b^4 - (A^4*B - 2*A^2*B^3 + B^5)
*b^5)*d^3*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2
*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4)/d^4)*cos(d*x + c) + ((A^6*B - A^4*B^3 -
A^2*B^5 + B^7)*a^7 + (A^7 - 9*A^5*B^2 - A^3*B^4 + 9*A*B^6)*a^6*b - (9*A^6*B
B - 17*A^4*B^3 - 25*A^2*B^5 + B^7)*a^5*b^2 - (A^7 - 17*A^5*B^2 - 17*A^3*B^4
+ A*B^6)*a^4*b^3 - (A^6*B - 17*A^4*B^3 - 17*A^2*B^5 + B^7)*a^3*b^4 - (A^7
- 25*A^5*B^2 - 17*A^3*B^4 + 9*A*B^6)*a^2*b^5 + (9*A^6*B - A^4*B^3 - 9*A^2*B
^5 + B^7)*a*b^6 + (A^7 - A^5*B^2 - A^3*B^4 + A*B^6)*b^7)*d*cos(d*x + c))*sq
rt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4
+ 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^2 - A*B*b^2 + (A^2 - B^2)*a*b)*d^2*sqrt((
(A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*
A^2*B^2 + B^4)*b^4)/d^4))/((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3)*
a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A^4
- 2*A^2*B^2 + B^4)*b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(((A^4 + 2*A^2*B^
2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*
b^4)/d^4)^(1/4) + ((A^8 - 2*A^4*B^4 + B^8)*a^8 - 8*(A^7*B + A^5*B^3 - A^3*B
^5 - A*B^7)*a^7*b + 16*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^6*b^2 - 8*(A^7*B +
A^5*B^3 - A^3*B^5 - A*B^7)*a^5*b^3 - 2*(A^8 - 16*A^6*B^2 - 34*A^4*B^4 - 16
*A^2*B^6 + B^8)*a^4*b^4 + 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^3*b^5 + 1
6*(A^6*B^2 + 2*A^4*B^4 + A^2*B^6)*a^2*b^6 + 8*(A^7*B + A^5*B^3 - A^3*B^5 -
A*B^7)*a*b^7 + (A^8 - 2*A^4*B^4 + B^8)*b^8)*sin(d*x + c))/cos(d*x + c))*(((
A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A
^2*B^2 + B^4)*b^4)/d^4)^(3/4) + sqrt(2)*(((A^4*B - B^5)*a^5 + (A^5 - 4*A^3*
B^2 - 5*A*B^4)*a^4*b - 4*(A^4*B + A^2*B^3)*a^3*b^2 - 4*(A^3*B^2 + A*B^4)*a^
2*b^3 - (5*A^4*B + 4*A^2*B^3 - B^5)*a*b^4 - (A^5 - A*B^4)*b^5)*d^7*sqrt(((A
^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^
2*B^2 + B^4)*b^4)/d^4)*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 - 8*(A^3*B - A*B^3
)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 + 8*(A^3*B - A*B^3)*a*b^3 + (A
^4 - 2*A^2*B^2 + B^4)*b^4)/d^4) + ((A^7 + A^5*B...

```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))/tan(c + d\*x)\*\*(7/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 12.50, size = 1473, normalized size = 5.80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)))/tan(c + d\*x)^(7/2),x)

[Out] 
$$2*\operatorname{atanh}\left(\frac{(32*A^2*a^2*d^3*\tan(c+d*x))^{1/2}*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) + (A^2*a*b)/(2*d^2))^{1/2}}{(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2}) - 16*A^3*a^3*d^2 + 16*A^3*a*b^2*d^2} - \frac{(32*A^2*b^2*d^3*\tan(c+d*x))^{1/2}*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2})/(4*d^4) + (A^2*a*b)/(2*d^2))^{1/2}}{(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2}) - 16*A^3*a^3*d^2 + 16*A^3*a*b^2*d^2}\right) * \left(\frac{(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2}}{(4*d^4) + (A^2*a*b)/(2*d^2))^{1/2}}{(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2}) + 16*A^3*a^3*d^2 - 16*A^3*a*b^2*d^2} - \frac{(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2}}{(4*d^4) + (A^2*a*b)/(2*d^2))^{1/2}}{(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^{1/2}) + 16*A^3*a^3*d^2 - 16*A^3*a*b^2*d^2}\right) * \left(\frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}}{(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}) - 16*B^3*b^3*d^2 + 16*B^3*a^2*b*d^2} - \frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}}{(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}) - 16*B^3*b^3*d^2 + 16*B^3*a^2*b*d^2}\right) * \left(\frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}}{(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}) + 16*B^3*b^3*d^2 - 16*B^3*a^2*b*d^2} - \frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}}{(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}) + 16*B^3*b^3*d^2 - 16*B^3*a^2*b*d^2}\right) * \left(\frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}}{(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}) + 16*B^3*b^3*d^2 - 16*B^3*a^2*b*d^2} - \frac{(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}}{(4*d^4) - (B^2*a*b)/(2*d^2))^{1/2}}{(16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{1/2}) + 16*B^3*b^3*d^2 - 16*B^3*a^2*b*d^2}\right)$$

$$\begin{aligned}
& *d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)}/(4*d^4) - (B^2*a*b)/(2*d^2))^{(1/2)} \\
& / (16*B*a*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)} + 16*B^3*b^3 \\
& *d^2 - 16*B^3*a^2*b*d^2)) * ((2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^{(1/2)} \\
& / (4*d^4) - (B^2*a*b)/(2*d^2))^{(1/2)} - ((2*A*a)/5 + (2*A*b*\tan(c + d*x) \\
& )/3 - 2*A*a*\tan(c + d*x)^2)/(d*\tan(c + d*x)^{(5/2)}) - ((2*B*a)/3 + 2*B*b*\tan \\
& (c + d*x))/(d*\tan(c + d*x)^{(3/2)})
\end{aligned}$$

$$3.385 \quad \int \tan^5(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=394

$$\frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \dots$$

[Out]  $-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*(2*A*a*b+B*a^2-B*b^2)*\tan(d*x+c)^{(1/2)}/d+2/3*(A*a^2-A*b^2-2*B*a*b)*\tan(d*x+c)^{(3/2)}/d+2/5*(2*A*a*b+B*a^2-B*b^2)*\tan(d*x+c)^{(5/2)}/d+2/63*b*(9*A*b+11*B*a)*\tan(d*x+c)^{(7/2)}/d+2/9*b*B*\tan(d*x+c)^{(7/2)}*(a+b*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.43, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3688, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \dots$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c + d*x]^{(5/2)}*(a + b*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) - ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) - ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) + ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) - (2*(2*a*A*b + a^2*B - b^2*B))*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d + (2*(a^2*A - A*b^2 - 2*a*b*B))*\operatorname{Tan}[c + d*x]^{(3/2)}/(3*d) + (2*(2*a*A*b + a^2*B - b^2*B))*\operatorname{Tan}[c + d*x]^{(5/2)}/(5*d) + (2*b*(9*A*b + 11*a*B))*\operatorname{Tan}[c + d*x]^{(7/2)}/(63*d) + (2*b*B*\operatorname{Tan}[c + d*x]^{(7/2)}*(a + b*\operatorname{Tan}[c + d*x]))/(9*d)$

**Rule 210**

$\operatorname{Int}[(a + b*x)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

Rule 3609

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```



```
t[b*Tan[e + f*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx &= \frac{2bB \tan^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))}{9d} + \frac{2}{9} \int \tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx \\
&= \frac{2b(9Ab+11aB) \tan^{\frac{7}{2}}(c+dx)}{63d} + \frac{2bB \tan^{\frac{7}{2}}(c+dx)}{9d} \\
&= \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{5}{2}}(c+dx)}{5d} + \frac{2b(9Ab+11aB) \tan^{\frac{7}{2}}(c+dx)}{63d} \\
&= \frac{2(a^2A-Ab^2-2abB) \tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{5}{2}}(c+dx)}{5d} \\
&= -\frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2(a^2A-Ab^2-2abB) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= -\frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2(a^2A-Ab^2-2abB) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= -\frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2(a^2A-Ab^2-2abB) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= -\frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2(a^2A-Ab^2-2abB) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= -\frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2(a^2A-Ab^2-2abB) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= -\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B)) \log(\sqrt{\tan(c+dx)})}{2\sqrt{2}d} \\
&= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B)) \tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 3.90, size = 205, normalized size = 0.52

$$\frac{-315\sqrt{-1}(a-ib)^2(A+B)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+315(-1)^{3/4}(a+ib)^2(A+iB)\text{tanh}^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}(-315(2aAb+a^2B-b^2B)+105(a^2A-Ab^2-2abB)\tan(c+dx)+63(2aAb+a^2B-b^2B)\tan^2(c+dx)+45(Ab+2aB)\tan^3(c+dx)+35^2B\tan^4(c+dx))}{315d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] (-315\*(-1)^(1/4)\*(a - I\*b)^2\*(I\*A + B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 315\*(-1)^(3/4)\*(a + I\*b)^2\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*Sqrt[Tan[c + d\*x]]\*(-315\*(2\*a\*A\*b + a^2\*B - b^2\*B) + 105\*(a^2\*A - A\*b^2 - 2\*a\*b\*B)\*Tan[c + d\*x] + 63\*(2\*a\*A\*b + a^2\*B - b^2\*B)\*Tan[c + d\*x]^2 + 45\*b\*(A\*b + 2\*a\*B)\*Tan[c + d\*x]^3 + 35\*b^2\*B\*Tan[c + d\*x]^4)/(315\*d)

**Maple [A]**

time = 0.05, size = 374, normalized size = 0.95

method	result
derivativedivides	$\frac{2b^2B \left(\tan^{\frac{9}{2}}(dx+c)\right)}{9} + \frac{2Ab^2 \left(\tan^{\frac{7}{2}}(dx+c)\right)}{7} + \frac{4Bab \left(\tan^{\frac{7}{2}}(dx+c)\right)}{7} + \frac{4Aab \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} + \frac{2Ba^2 \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} - \frac{2Bb^2 \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}$
default	$\frac{2b^2B \left(\tan^{\frac{9}{2}}(dx+c)\right)}{9} + \frac{2Ab^2 \left(\tan^{\frac{7}{2}}(dx+c)\right)}{7} + \frac{4Bab \left(\tan^{\frac{7}{2}}(dx+c)\right)}{7} + \frac{4Aab \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} + \frac{2Ba^2 \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} - \frac{2Bb^2 \left(\tan^{\frac{5}{2}}(dx+c)\right)}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/d*(2/9*b^2*B*tan(d*x+c)^(9/2)+2/7*A*b^2*tan(d*x+c)^(7/2)+4/7*B*a*b*tan(d*
x+c)^(7/2)+4/5*A*a*b*tan(d*x+c)^(5/2)+2/5*B*a^2*tan(d*x+c)^(5/2)-2/5*B*b^2*
tan(d*x+c)^(5/2)+2/3*a^2*A*tan(d*x+c)^(3/2)-2/3*A*b^2*tan(d*x+c)^(3/2)-4/3*
B*a*b*tan(d*x+c)^(3/2)-4*A*a*b*tan(d*x+c)^(1/2)-2*B*a^2*tan(d*x+c)^(1/2)+2*
b^2*B*tan(d*x+c)^(1/2)+1/4*(2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A
*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+
2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
```

**Maxima [A]**

time = 0.51, size = 329, normalized size = 0.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/1260*(280*B*b^2*tan(d*x + c)^(9/2) + 360*(2*B*a*b + A*b^2)*tan(d*x + c)^(
7/2) + 504*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^(5/2) - 630*sqrt(2)*((A -
B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt
(tan(d*x + c)))) - 630*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 315*sqrt(2)*((A + B
)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d
*x + c) + 1) - 315*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(
-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 840*(A*a^2 - 2*B*a*b - A*
b^2)*tan(d*x + c)^(3/2) - 2520*(B*a^2 + 2*A*a*b - B*b^2)*sqrt(tan(d*x + c))
)/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 26023 vs.  $2(348) = 696$ .

time = 197.88, size = 26023, normalized size = 66.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{1260} \cdot (1260 \cdot \sqrt{2} \cdot d^5 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4}) / ((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^7b + (A^4 - 2A^2B^2 + B^4)b^8) / d^4) \cdot \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^7b + (A^4 - 2A^2B^2 + B^4)b^8) / d^4} \cdot \arctan(-((A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16} - 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^3 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^4 - 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^5 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^6 - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^7 - 90(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^8 + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^9 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^{10} + 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^{11} - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^{12} + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{13} + 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^2b^{15} + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{16}) / d^4} \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4} \cdot \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^7b + (A^4 - 2A^2B^2 + B^4)b^8) / d^4} + \sqrt{2} \cdot ((B^2a^2 + 2A^2ab - B^2b^2) \cdot d^7 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4})$

$$\begin{aligned}
& + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + \\
& (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)*\text{sqrt}(((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A \\
& ^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - \\
& A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - \\
& A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3 \\
& )*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4) + ((A^3 + A*B^2)*a^6 - 2*(A^2*B \\
& + B^3)*a^5*b + (A^3 + A*B^2)*a^4*b^2 - 4*(A^2*B + B^3)*a^3*b^3 - (A^3 + A* \\
& B^2)*a^2*b^4 - 2*(A^2*B + B^3)*a*b^5 - (A^3 + A*B^2)*b^6)*d^5*\text{sqrt}(((A^4 - \\
& 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3 \\
& *B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19* \\
& B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4) \\
& *a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4))*\text{sq} \\
& \text{rt}(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^ \\
& 4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2 \\
& *A^2*B^2 + B^4)*b^8 - 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)* \\
& a^3*b - 2*(A^2 - B^2)*a*b^3)*d^2*\text{sqrt}(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 \\
& + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + \\
& 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4))/((A^4 - 2*A^2 \\
& *B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4) \\
& *a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)* \\
& a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2* \\
& b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*\text{sqrt}((((A^6 \\
& - A^4*B^2 - A^2*B^4 + B^6)*a^12 - 16*(A^5*B - A*B^5)*a^11*b - 2*(5*A^6 - 37 \\
& *A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^10*b^2 + 80*(A^5*B - A*B^5)*a^9*b^3 + 15*( \\
& A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^8*b^4 + 96*(A^5*B - A*B^5)*a^7*b^5 + 4*(13 \\
& *A^6 - 45*A^4*B^2 - 45*A^2*B^4 + 13*B^6)*a^6*b^6 - 96*(A^5*B - A*B^5)*a^5*b \\
& ^7 + 15*(A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^4*b^8 - 80*(A^5*B - A*B^5)*a^3*b^ \\
& 9 - 2*(5*A^6 - 37*A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^2*b^10 + 16*(A^5*B - A*B^ \\
& 5)*a*b^11 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^12)*d^2*\text{sqrt}(((A^4 + 2*A^2*B^ \\
& 2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4 \\
& )*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 \\
& )/d^4)*\text{cos}(d*x + c) + \text{sqrt}(2)*(((A^5 - 2*A^3*B^2...
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 \tan^{\frac{5}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)), x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*2\*tan(c + d\*x)\*\*(5/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

[Out] Timed out

**Mupad [B]**

time = 25.93, size = 2500, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

```
[Out] atan((B^2*a^4*tan(c + d*x)^(1/2)*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B
^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/(
4*d^4) - (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b
^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3
- (192*B^3*a^3*b^3)/d - (16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a
^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 + (32*B^3*a*b^
5)/d + (32*B^3*a^5*b)/d) + (B^2*b^4*tan(c + d*x)^(1/2)*((B^2*a*b^3)/d^2 - (
12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^
4*a^6*b^2*d^4)^(1/2))/(4*d^4) - (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B*a^2*(12*B
^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^
6*b^2*d^4)^(1/2))/d^3 - (192*B^3*a^3*b^3)/d - (16*B*b^2*(12*B^4*a^2*b^6*d^4
- B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/
2))/d^3 + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d) - (B^2*a^2*b^2*tan(c + d*x)^(
1/2)*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 -
38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/(4*d^4) - (B^2*a^3*b)/d^2)^(
1/2)*192i)/((16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*
B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 - (192*B^3*a^3*b^3)/d - (1
6*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^
4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d))*
((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4
*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/(4*d^4) - (B^2*a^3*b)/d^2)^(1/2)*2
i + atan((B^2*a^4*tan(c + d*x)^(1/2)*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B
^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/(4*d^4) + (B^2*
a*b^3)/d^2 - (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B*b^2*(12*B^4*a^2*b^6*d^4 - B
^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/
d^3 - (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^
4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 - (192*B^3*a^3*b^3)/d + (32*B^3*
```



$$3.386 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=360

$$\frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{(2(a^2A - Ab^2 - 2a^2bB) \tan(c+dx) + (2b^2B - 7a^2bB + 9a^2B) \tan^2(c+dx) + (2b^2B - 7a^2bB + 9a^2B) \tan^3(c+dx))}{35d} + \frac{(2b^2B - 7a^2bB + 9a^2B) \tan^5(c+dx)}{35d}$$

[Out]  $-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2*(A*a^2-A*b^2-2*B*a*b)*\tan(d*x+c)^{(1/2)}/d+2/3*(2*A*a*b+B*a^2-B*b^2)*\tan(d*x+c)^{(3/2)}/d+2/35*b*(7*A*b+9*B*a)*\tan(d*x+c)^{(5/2)}/d+2/7*b*B*\tan(d*x+c)^{(5/2)}*(a+b*\tan(d*x+c))/d$

**Rubi [A]**

time = 0.38, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3688, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{(a^2(A-B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A-B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{(2(a^2A - Ab^2 - 2a^2bB) \tan(c+dx) + (2b^2B - 7a^2bB + 9a^2B) \tan^2(c+dx) + (2b^2B - 7a^2bB + 9a^2B) \tan^3(c+dx))}{35d} + \frac{(2b^2B - 7a^2bB + 9a^2B) \tan^5(c+dx)}{35d}$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) - ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) + ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) - ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) + (2*(a^2*A - A*b^2 - 2*a*b*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d + (2*(2*a*A*b + a^2*B - b^2*B)*\operatorname{Tan}[c + d*x]^(3/2))/(3*d) + (2*b*(7*A*b + 9*a*B)*\operatorname{Tan}[c + d*x]^(5/2))/(35*d) + (2*b*B*\operatorname{Tan}[c + d*x]^(5/2)*(a + b*\operatorname{Tan}[c + d*x]))/(7*d)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)



], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3688

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3711

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx &= \frac{2bB \tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))}{7d} + \frac{2}{7} \int \\
&= \frac{2b(7Ab+9aB) \tan^{\frac{5}{2}}(c+dx)}{35d} + \frac{2bB \tan^{\frac{5}{2}}(c+dx)}{7d} \\
&= \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2b(7Ab+9aB) \tan^{\frac{5}{2}}(c+dx)}{35d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\tan(c+dx)}}{d} + \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\tan(c+dx)}}{d} + \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\tan(c+dx)}}{d} + \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\tan(c+dx)}}{d} + \frac{2(2aAb+a^2B-b^2B) \tan^{\frac{3}{2}}(c+dx)}{3d} \\
&= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B)) \log \sqrt{2}}{2\sqrt{2}} \\
&= \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B)) \tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.42, size = 178, normalized size = 0.49

$$\frac{105\sqrt{-1}(a-ib)^2(A-iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+105\sqrt{-1}(a+ib)^2(A+iB)\text{tanh}^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}(105(a^2A-Ab^2-2abB)+35(2aAb+a^2B-b^2B)\tan(c+dx)+21b(Ab+2aB)\tan^2(c+dx)+15b^2B\tan^3(c+dx))}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (105\*(-1)^(1/4)\*(a - I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 105\*(-1)^(1/4)\*(a + I\*b)^2\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*Sqrt[Tan[c + d\*x]]\*(105\*(a^2\*A - A\*b^2 - 2\*a\*b\*B) + 35\*(2\*a\*A\*b + a^2\*B - b^2\*B)\*Tan[c + d\*x] + 21\*b\*(A\*b + 2\*a\*B)\*Tan[c + d\*x]^2 + 15\*b^2\*B\*Tan[c + d\*x]^3)/(105\*d)

**Maple [A]**

time = 0.05, size = 333, normalized size = 0.92



**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 25761 vs. 2(318) = 636.

time = 152.95, size = 25761, normalized size = 71.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{420} \cdot (420 \cdot \sqrt{2}) \cdot d^5 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8 + 2(ABa^4 - 6ABa^2b^2 + ABb^4 + 2(A^2 - B^2)a^3b - 2(A^2 - B^2)a^2b^3) \cdot d^2 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4}}{(A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8)} \cdot \left( \frac{(A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8}{d^4} \right)^{3/4} \cdot \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8) / d^4} \cdot \arctan\left(\frac{(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16} - 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^3 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^4 - 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^5 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^6 - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^7 - 90(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^8 + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^9 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^{10} + 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^{11} - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^{12} + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{13} + 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^2b^{15} + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{16}}{d^4} \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4} \cdot \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8) / d^4} - \sqrt{2} \cdot ((Aa^2 - 2Bab - Ab^2) \cdot d^7 \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4}}{(A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8)} \right)$

```

6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^
4 + 2*A^2*B^2 + B^4)*b^8)/d^4)*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B
- A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B
^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B
^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a
*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4) - ((A^2*B + B^3)*a^6 + 2*(A^3 + A*
B^2)*a^5*b + (A^2*B + B^3)*a^4*b^2 + 4*(A^3 + A*B^2)*a^3*b^3 - (A^2*B + B^3
)*a^2*b^4 + 2*(A^3 + A*B^2)*a*b^5 - (A^2*B + B^3)*b^6)*d^5*sqrt(((A^4 - 2*A
^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^
4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4
)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^
2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4))*sqrt(
((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 +
2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^
2*B^2 + B^4)*b^8 + 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)*a^3
*b - 2*(A^2 - B^2)*a*b^3)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 +
2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A
^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)))/((A^4 - 2*A^2*B^
2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^
6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4
*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6
+ 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*sqrt((((A^6 - A
^4*B^2 - A^2*B^4 + B^6)*a^12 - 16*(A^5*B - A*B^5)*a^11*b - 2*(5*A^6 - 37*A^
4*B^2 - 37*A^2*B^4 + 5*B^6)*a^10*b^2 + 80*(A^5*B - A*B^5)*a^9*b^3 + 15*(A^6
- A^4*B^2 - A^2*B^4 + B^6)*a^8*b^4 + 96*(A^5*B - A*B^5)*a^7*b^5 + 4*(13*A^
6 - 45*A^4*B^2 - 45*A^2*B^4 + 13*B^6)*a^6*b^6 - 96*(A^5*B - A*B^5)*a^5*b^7
+ 15*(A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^4*b^8 - 80*(A^5*B - A*B^5)*a^3*b^9 -
2*(5*A^6 - 37*A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^2*b^10 + 16*(A^5*B - A*B^5)*
a*b^11 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^12)*d^2*sqrt(((A^4 + 2*A^2*B^2 +
B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a
^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d
^4)*cos(d*x + c) + sqrt(2)*(((A^4*B - 2*A^2*B^3...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 \tan^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*2\*tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 18.48, size = 2500, normalized size = 6.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] atan((B^2\*a^4\*tan(c + d\*x)^(1/2)\*((12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4) - (B^2\*a\*b^3)/d^2 + (B^2\*a^3\*b)/d^2)^(1/2)\*32i)/((16\*B^3\*b^6)/d - (16\*B^3\*a^6)/d - (112\*B^3\*a^2\*b^4)/d + (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) + (B^2\*b^4\*tan(c + d\*x)^(1/2)\*((12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4) - (B^2\*a\*b^3)/d^2 + (B^2\*a^3\*b)/d^2)^(1/2)\*32i)/((16\*B^3\*b^6)/d - (16\*B^3\*a^6)/d - (112\*B^3\*a^2\*b^4)/d + (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) - (B^2\*a^2\*b^2\*tan(c + d\*x)^(1/2)\*((12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4) - (B^2\*a\*b^3)/d^2 + (B^2\*a^3\*b)/d^2)^(1/2)\*192i)/((16\*B^3\*b^6)/d - (16\*B^3\*a^6)/d - (112\*B^3\*a^2\*b^4)/d + (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3))\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4) - (B^2\*a\*b^3)/d^2 + (B^2\*a^3\*b)/d^2)^(1/2)\*2i - atan((B^2\*a^4\*tan(c + d\*x)^(1/2)\*((B^2\*a^3\*b)/d^2 - (B^2\*a\*b^3)/d^2 - (12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4))^(1/2)\*32i)/((16\*B^3\*a^6)/d - (16\*B^3\*b^6)/d + (112\*B^3\*a^2\*b^4)/d - (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) + (B^2\*b^4\*tan(c + d\*x)^(1/2)\*((B^2\*a^3\*b)/d^2 - (B^2\*a\*b^3)/d^2 - (12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4))^(1/2)\*32i)/((16\*B^3\*a^6)/d - (16\*B^3\*b^6)/d + (112\*B^3\*a^2\*b^4)/d - (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3)





### 3.387 $\int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=326

$$\frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}$$

```
[Out] 1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d
*2^(1/2)+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d
*2^(1/2)+1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d
*2^(1/2)-1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d
*2^(1/2)+2*(2*A*a*b+B*a^2-B*b^2)*tan(d*x+c)^(1/2)/d+2/15*b*(5*A*b+7*B*a)*tan(d*x+c)^(3/2)/d+2/5*b*B*tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))/d
```

**Rubi** [A]

time = 0.34, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3688, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{(a^2(A-B)-2ab(A+B)-b^2(A-B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^2(A-B)-2ab(A+B)-b^2(A-B))\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2a^2b+2ab^2-2b^3}{d}\ln\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\right) + \frac{(a^2(A+B)+2ab(A-B)-b^2(A+B))\ln\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\right)}{2\sqrt{2}d} + \frac{2a^2b+5ab^2+5b^3}{15d}\tan(c+dx) + \frac{2B\tan^2(c+dx)(a+b\tan(c+dx))}{5d}$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

```
[Out] -(((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Tan[c + d*x]])/d + (2*b*(5*A*b + 7*a*B)*Tan[c + d*x]^(3/2))/(15*d) + (2*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]))/(5*d)
```

Rule 210

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3688

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3711

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^2 (A+B \tan(c+dx)) dx &= \frac{2bB \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))}{5d} + \frac{2}{5} \int \\
&= \frac{2b(5Ab+7aB) \tan^{\frac{3}{2}}(c+dx)}{15d} + \frac{2bB \tan^{\frac{3}{2}}(c}{ \\
&= \frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2b}{ \\
&= \frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2b}{ \\
&= \frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2b}{ \\
&= \frac{2(2aAb+a^2B-b^2B) \sqrt{\tan(c+dx)}}{d} + \frac{2b}{ \\
&= \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B)) \log}{2\sqrt{2}} \\
&= -\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B)) \tan}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.81, size = 151, normalized size = 0.46

$$\frac{15\sqrt{-1}(a-ib)^2(iA+B)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-15(-1)^{3/4}(a+ib)^2(A+iB)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+2\sqrt{\tan(c+dx)}(15(2aAb+a^2B-b^2B)+5b(Ab+2aB)\tan(c+dx)+3b^2B\tan^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (15\*(-1)^(1/4)\*(a - I\*b)^2\*(I\*A + B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - 15\*(-1)^(3/4)\*(a + I\*b)^2\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*Sqrt[Tan[c + d\*x]]\*(15\*(2\*a\*A\*b + a^2\*B - b^2\*B) + 5\*b\*(A\*b + 2\*a\*B)\*Tan[c + d\*x] + 3\*b^2\*B\*Tan[c + d\*x]^2))/(15\*d)

**Maple [A]**

time = 0.05, size = 292, normalized size = 0.90

method	result
--------	--------

derivativedivides	$\frac{2Bb^2 \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ab^2 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{4Bab \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 4Aab \left( \sqrt{\tan(dx+c)} \right) + 2Ba^2 \left( \sqrt{\tan(dx+c)} \right) - 2$
default	$\frac{2Bb^2 \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ab^2 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + \frac{4Bab \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 4Aab \left( \sqrt{\tan(dx+c)} \right) + 2Ba^2 \left( \sqrt{\tan(dx+c)} \right) - 2$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*(2/5*B*b^2*\tan(d*x+c)^{(5/2)}+2/3*A*b^2*\tan(d*x+c)^{(3/2)}+4/3*B*a*b*\tan(d*x+c)^{(3/2)}+4*A*a*b*\tan(d*x+c)^{(1/2)}+2*B*a^2*\tan(d*x+c)^{(1/2)}-2*b^2*B*\tan(d*x+c)^{(1/2)}+1/4*(-2*A*a*b-B*a^2+B*b^2)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})))+1/4*(A*a^2-A*b^2-2*B*a*b)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [A]

time = 0.51, size = 275, normalized size = 0.84

$\frac{24B^2 \tan(dx+c)^5 + 30\sqrt{2}(A-B)^2 \tan(dx+c)^4 + 30\sqrt{2}(A-B)^2 \tan(dx+c)^3 + 30\sqrt{2}(A-B)^2 \tan(dx+c)^2 + 30\sqrt{2}(A-B)^2 \tan(dx+c) + 30\sqrt{2}(A-B)^2}{120(B^2 + 2Aa - Bb)\sqrt{\tan(dx+c)}} - 15\sqrt{2}(A+B)^2 \tan(dx+c)^2 + 15\sqrt{2}(A+B)^2 \tan(dx+c) + 15\sqrt{2}(A+B)^2 + 15\sqrt{2}(A+B)^2 \ln\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1}{\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c) + 1}\right) + 40(2B^2 + 2Aa - Bb)\sqrt{\tan(dx+c)} + 120(B^2 + 2Aa - Bb)\sqrt{\tan(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out]  $1/60*(24*B*b^2*\tan(d*x+c)^{(5/2)}+30*\sqrt{2}*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(d*x+c)}))+30*\sqrt{2}*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(d*x+c)}))-15*\sqrt{2}*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*\log(\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1)+15*\sqrt{2}*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*\log(-\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1)+40*(2*B*a*b+A*b^2)*\tan(d*x+c)^{(3/2)}+120*(B*a^2+2*A*a*b-B*b^2)*\sqrt{\tan(d*x+c)}/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 25469 vs. 2(288) = 576.

time = 196.12, size = 25469, normalized size = 78.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/60*(60*\sqrt{2}*d^5*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 - 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)*a^3*b - 2*(A^2 - B^2)*a*b^3)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4}))/((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)^{3/4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4}*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{16} - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{15*b} - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13*b^3} - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12*b^4} - 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11*b^5} - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10*b^6} - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^7 - 90*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^8 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^9 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^{10} + 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^{11} - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^{12} + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{13} + 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{15} + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{16})*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4} + \sqrt{2})*((B*a^2 + 2*A*a*b - B*b^2)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a$$

$$\begin{aligned}
& *b^7 + (A^4 - 2A^2B^2 + B^4)*b^8)/d^4) + ((A^3 + A*B^2)*a^6 - 2*(A^2*B + \\
& B^3)*a^5*b + (A^3 + A*B^2)*a^4*b^2 - 4*(A^2*B + B^3)*a^3*b^3 - (A^3 + A*B^2 \\
& )*a^2*b^4 - 2*(A^2*B + B^3)*a*b^5 - (A^3 + A*B^2)*b^6)*d^5*\sqrt{((A^4 - 2A \\
& ^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4 \\
& 4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4 \\
& )*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^ \\
& 2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4))*\sqrt{ \\
& ((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + \\
& 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^ \\
& 2*B^2 + B^4)*b^8 - 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)*a^3 \\
& *b - 2*(A^2 - B^2)*a*b^3)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + \\
& 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A \\
& ^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)}/((A^4 - 2*A^2*B^ \\
& 2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^ \\
& 6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4 \\
& *b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 \\
& + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*\sqrt{(((A^6 - A \\
& ^4*B^2 - A^2*B^4 + B^6)*a^12 - 16*(A^5*B - A*B^5)*a^11*b - 2*(5*A^6 - 37*A^ \\
& 4*B^2 - 37*A^2*B^4 + 5*B^6)*a^10*b^2 + 80*(A^5*B - A*B^5)*a^9*b^3 + 15*(A^6 \\
& - A^4*B^2 - A^2*B^4 + B^6)*a^8*b^4 + 96*(A^5*B - A*B^5)*a^7*b^5 + 4*(13*A^ \\
& 6 - 45*A^4*B^2 - 45*A^2*B^4 + 13*B^6)*a^6*b^6 - 96*(A^5*B - A*B^5)*a^5*b^7 \\
& + 15*(A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^4*b^8 - 80*(A^5*B - A*B^5)*a^3*b^9 - \\
& 2*(5*A^6 - 37*A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^2*b^10 + 16*(A^5*B - A*B^5)* \\
& a*b^11 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^12)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + \\
& B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a \\
& ^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d \\
& ^4)*\cos(d*x + c) + \sqrt{2}*(((A^5 - 2*A^3*B^2 + \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*2\*sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 12.96, size = 2500, normalized size = 7.67
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

```
[Out] atan((A^2*a^4*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A
^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^
6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*A^3*a^6)/d - (16*A^3*b^6)/d + (11
2*A^3*a^2*b^4)/d - (112*A^3*a^4*b^2)/d + (32*A*a*b*(12*A^4*a^2*b^6*d^4 - A^
4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^(1/2))/d
^3) + (A^2*b^4*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*
A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a
^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*A^3*a^6)/d - (16*A^3*b^6)/d + (1
12*A^3*a^2*b^4)/d - (112*A^3*a^4*b^2)/d + (32*A*a*b*(12*A^4*a^2*b^6*d^4 - A
^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^(1/2))/
d^3) - (A^2*a^2*b^2*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 -
(12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*
A^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*192i)/((16*A^3*a^6)/d - (16*A^3*b^6)/
d + (112*A^3*a^2*b^4)/d - (112*A^3*a^4*b^2)/d + (32*A*a*b*(12*A^4*a^2*b^6*d
^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^(
1/2))/d^3)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*
b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^(1/2)/(4*d
^4))^(1/2)*2i - atan((B^2*a^4*tan(c + d*x)^(1/2)*((12*B^4*a^2*b^6*d^4 - B^4
*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*
d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B*b^2*(12*B^4*a^2
*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*
d^4)^(1/2))/d^3 - (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4
- 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 - (192*B^3*a^3*b^3)/
d + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d) + (B^2*b^4*tan(c + d*x)^(1/2)*((12
*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*
a^6*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^(1/2)*32i)/
((16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4
*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 - (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4
*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^
3 - (192*B^3*a^3*b^3)/d + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d) - (B^2*a^2*b
^2*tan(c + d*x)^(1/2)*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38
*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a*b^3)/d^2 - (B
^2*a^3*b)/d^2)^(1/2)*192i)/((16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B
```



$$\begin{aligned}
&^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 - (16B^2a^2(12B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12 \\
&*B^4a^6b^2d^4)^{(1/2)}/d^3 - (192B^3a^3b^3)/d + (32B^3a^5b)/d + (32 \\
&*B^3a^5b)/d)) * ((12B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/(4d^4) + (B^2a^3b^3)/d^2 - (B^2a^3 \\
&*b)/d^2)^{(1/2)} * 2i - \operatorname{atan}((B^2a^4 \tan(c + dx))^{(1/2)} * ((B^2a^3b^3)/d^2 - (12 \\
&*B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/(4d^4) - (B^2a^3b^3)/d^2)^{(1/2)} * 32i) / ((16B^2a^2(12B^4 \\
&*a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 - (192B^3a^3b^3)/d - (16B^2a^2(12B^4a^2b^6d^4 - \\
&B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 + (32B^3a^5b^5)/d + (32B^3a^5b)/d) + (B^2b^4 \tan(c + dx))^{(1/2)} * \\
&((B^2a^3b^3)/d^2 - (12B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/(4d^4) - (B^2a^3b^3)/d^2)^{(1/2)} * 3 \\
&2i) / ((16B^2a^2(12B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 - (192B^3a^3b^3)/d - (16B^2a^2 \\
&*b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 + (32B^3a^5b^5)/d + (32B^3a^5b)/d) - (B^2a^2 \tan(c + dx))^{(1/2)} * ((B^2a^3b^3)/d^2 - (12B^4a^2b^6d^4 - B^4b^8d^4 - \\
&B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/(4d^4) - (B^2a^3b^3)/d^2)^{(1/2)} * 192i) / ((16B^2a^2(12B^4a^2b^6d^4 - B^4b^8d^4 - \\
&B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 - (192 \\
&*B^3a^3b^3)/d - (16B^2a^2(12B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/d^3 + (32B^3a^5b^5)/d + \\
&(32B^3a^5b)/d)) * ((B^2a^3b^3)/d^2 - (12B^4a^2b^6d^4 - B^4b^8d^4 - B^4a^8d^4 - 38B^4a^4b^4d^4 + 12B^4a^6b^2d^4)^{(1/2)}/(4d^4) - (B^2 \\
&a^3b^3)/d^2)^{(1/2)} * 2i - \operatorname{atan}((A^2a^4 \tan(c + dx))^{(1/2)} * ((12A^4a^2b^6d^4 - A^4b^8d^4 - A^4a^8d^4 - 38A^4a^4b^4d^4 + 12A^4a^6b^2d^4)^{(1/2)}/(4d^4) - (A^2a^3b^3)/d^2 + (A^2a^3b^3)/d^2)^{(1/2)} * 32i) / ((16A^3b^6)/ \\
&d - (16A^3a^6)/d - (112A^3a^2b^4)/d + (112A^3a^4b^2)/d + (32A^2a^3b^3) \\
&(12A^4a^2b^6d^4 - A^4b^8d^4 - A^4a^8d^4 - 38A^4a^4b^4d^4 + 12A^4a^6b^2d^4)^{(1/2)}/d^3) + (A^2b^4 \tan(c + dx))^{(1/2)} * ((12A^4a^2b^6d^4 - A^4b^8d^4 - A^4a^8d^4 - 38A^4a^4b^4d^4 + 12A^4a^6b^2d^4)^{(1/2)}/(4d^4) - (A^2a^3b^3)/d^2 + (A^2a^3b^3)/d \dots
\end{aligned}$$

$$3.388 \quad \int \frac{(a+b \tan(c+dx))^2 (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=294

$$\frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d$   
 $+2^{(1/2)+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d$   
 $-1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)+\tan(d*x+c)})/d$   
 $+1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)+\tan(d*x+c)})/d$   
 $+2/3*b*(3*A*b+5*B*a)*\tan(d*x+c)^{(1/2)+2/3*b*B*\tan(d*x+c)^{(1/2)*(a+b*\tan(d*x+c))}/d$

**Rubi [A]**

time = 0.30, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3688, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\tan(c+dx)}\right)}{2\sqrt{2} d} + \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{\tan(c+dx)}\right)}{2\sqrt{2} d} + \frac{2b(3aB + 5a^2) \sqrt{\tan(c+dx)}}{3d} + \frac{2b^2 \sqrt{\tan(c+dx)} (a + b \tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x])/ \operatorname{Sqrt}[\operatorname{Tan}[c + d*x]], x]$

[Out]  $-(((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) + (2*b*(3*A*b + 5*a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (3*d) + (2*b*B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])) / (3*d)$

**Rule 210**

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&\& \operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0]$

**Rule 631**

$\operatorname{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{q = 1 - 4*S\operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /;$   $\operatorname{RationalQ}[q] \&\& (\operatorname{EqQ}[q^2, 1] \parallel \operatorname{!RationalQ}[b^2 - 4*a*c]) /;$   $\operatorname{FreeQ}[a, b, c], x$

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

#### Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

#### Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\tan[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3688

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \ :> \ \text{Simp}[b*B*(a + b*\tan[e + f*x])^{(m-1)}*((c + d*\tan[e + f*x])^{(n+1)})/(d*f*(m+n)), x] + \text{Dist}[1/(d*(m+n)), \text{Int}[(a + b*\tan[e + f*x])^{(m-2)}*(c + d*\tan[e + f*x])^n*\text{Simp}[a^2*A*d*(m+n) - b*B*(b*c*(m-1) + a*d*(n+1)) + d*(m+n)*(2*a*A*b + B*(a^2 - b^2))*\tan[e + f*x] - (b*B*(b*c - a*d)*(m-1) - b*(A*b + a*B)*d*(m+n))*\tan[e + f*x]^2, x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e,$

f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] :> Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}a(3aA - b)}{\sqrt{\tan(c + dx)}} dx \\
 &= \frac{2b(3Ab + 5aB) \sqrt{\tan(c + dx)}}{3d} + \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))}{3d} \\
 &= \frac{2b(3Ab + 5aB) \sqrt{\tan(c + dx)}}{3d} + \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))}{3d} \\
 &= \frac{2b(3Ab + 5aB) \sqrt{\tan(c + dx)}}{3d} + \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))}{3d} \\
 &= \frac{2b(3Ab + 5aB) \sqrt{\tan(c + dx)}}{3d} + \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))}{3d} \\
 &= -\frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
 &= -\frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.35, size = 119, normalized size = 0.40

$$\frac{-3\sqrt{-1}(a - ib)^2(A - iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c + dx)}\right) - 3\sqrt{-1}(a + ib)^2(A + iB)\text{tanh}^{-1}\left((-1)^{3/4}\sqrt{\tan(c + dx)}\right) + 2b\sqrt{\tan(c + dx)}(3Ab + 6aB + bB\tan(c + dx))}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

```
[Out] (-3*(-1)^(1/4)*(a - I*b)^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
- 3*(-1)^(1/4)*(a + I*b)^2*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
+ 2*b*Sqrt[Tan[c + d*x]]*(3*A*b + 6*a*B + b*B*Tan[c + d*x]))/(3*d)
```

**Maple [A]**

time = 0.05, size = 251, normalized size = 0.85

method	result
derivativedivides	$\frac{2b^2B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2Ab^2 \left( \sqrt{\tan(dx+c)} \right) + 4Bab \left( \sqrt{\tan(dx+c)} \right) + \frac{(a^2A - Ab^2 - 2Bab) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}} \right) \right) \right)}{3}$
default	$\frac{2b^2B \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2Ab^2 \left( \sqrt{\tan(dx+c)} \right) + 4Bab \left( \sqrt{\tan(dx+c)} \right) + \frac{(a^2A - Ab^2 - 2Bab) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}}{1-\sqrt{2}} \left( \frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}} \right) \right) \right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/d*(2/3*b^2*B*tan(d*x+c)^(3/2)+2*A*b^2*tan(d*x+c)^(1/2)+4*B*a*b*tan(d*x+c)
^(1/2)+1/4*(A*a^2-A*b^2-2*B*a*b)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(2*A*a*b+B*a^2-B*b^
2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c
)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2))))
```

**Maxima [A]**

time = 0.52, size = 248, normalized size = 0.84

$8B^2 \tan(dx+c)^3 + 6\sqrt{2}(A+B)^2 + 2(A-B)ab - (A+B)^2 \arctan\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\sqrt{\tan(dx+c)}\right) + 6\sqrt{2}(A+B)^2 + 2(A-B)ab - (A+B)^2 \arctan\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\sqrt{\tan(dx+c)}\right) + 3\sqrt{2}(A-B)^2 - 2(A+B)ab - (A-B)^2 \log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1}{\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c)+1}\right) - 3\sqrt{2}(A-B)^2 - 2(A+B)ab - (A-B)^2 \log\left(\frac{-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)} - \tan(dx+c)+1}\right) + 24(2Bab + Ab^2)\sqrt{\tan(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/12*(8*B*b^2*tan(d*x + c)^(3/2) + 6*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b -
(A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sqrt
(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A -
B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*((A
- B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + t
an(d*x + c) + 1) + 24*(2*B*a*b + A*b^2)*sqrt(tan(d*x + c))/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 25195 vs.  $2(258) = 516$ .

time = 163.17, size = 25195, normalized size = 85.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 
$$-1/12*(12*\sqrt{2}*d^5*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 + 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)*a^3*b - 2*(A^2 - B^2)*a*b^3)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4})/((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)^{(3/4)}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4)}*\arctan((((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^16 - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^15*b - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^13*b^3 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^12*b^4 - 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^11*b^5 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^10*b^6 - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^7 - 90*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^8 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^9 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^10 + 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^11 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^12 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^13 + 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^15 + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^16)*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4)} - \sqrt{2}*(((A*a^2 - 2*B*a*b - A*b^2)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4})$$

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*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4
+ 2*A^2*B^2 + B^4)*b^8)/d^4)*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B
- A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B
^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^
3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*
b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4) - ((A^2*B + B^3)*a^6 + 2*(A^3 + A*B
^2)*a^5*b + (A^2*B + B^3)*a^4*b^2 + 4*(A^3 + A*B^2)*a^3*b^3 - (A^2*B + B^3)
*a^2*b^4 + 2*(A^3 + A*B^2)*a*b^5 - (A^2*B + B^3)*b^6)*d^5*sqrt(((A^4 - 2*A^
2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4
)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)
*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2
*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4))*sqrt((
(A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 +
2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2
*B^2 + B^4)*b^8 + 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)*a^3*
b - 2*(A^2 - B^2)*a*b^3)*d^2*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2
*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^
2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4))/((A^4 - 2*A^2*B^2
+ B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6
*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*
b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6
+ 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*sqrt((((A^6 - A^
4*B^2 - A^2*B^4 + B^6)*a^12 - 16*(A^5*B - A*B^5)*a^11*b - 2*(5*A^6 - 37*A^4
*B^2 - 37*A^2*B^4 + 5*B^6)*a^10*b^2 + 80*(A^5*B - A*B^5)*a^9*b^3 + 15*(A^6
- A^4*B^2 - A^2*B^4 + B^6)*a^8*b^4 + 96*(A^5*B - A*B^5)*a^7*b^5 + 4*(13*A^6
- 45*A^4*B^2 - 45*A^2*B^4 + 13*B^6)*a^6*b^6 - 96*(A^5*B - A*B^5)*a^5*b^7 +
15*(A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^4*b^8 - 80*(A^5*B - A*B^5)*a^3*b^9 -
2*(5*A^6 - 37*A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^2*b^10 + 16*(A^5*B - A*B^5)*a
*b^11 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^12)*d^2*sqrt(((A^4 + 2*A^2*B^2 +
B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^
4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^
4)*cos(d*x + c) + sqrt(2)*(((A^4*B - 2*A^2*B^3 ...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^2}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2), x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*2/sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 9.78, size = 2500, normalized size = 8.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2)/tan(c + d\*x)^(1/2),x)

[Out] atan((B^2\*a^4\*tan(c + d\*x)^(1/2)\*((B^2\*a^3\*b)/d^2 - (B^2\*a\*b^3)/d^2 - (12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4))^(1/2)\*32i)/((16\*B^3\*a^6)/d - (16\*B^3\*b^6)/d + (112\*B^3\*a^2\*b^4)/d - (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) + (B^2\*b^4\*tan(c + d\*x)^(1/2)\*((B^2\*a^3\*b)/d^2 - (B^2\*a\*b^3)/d^2 - (12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4))^(1/2)\*32i)/((16\*B^3\*a^6)/d - (16\*B^3\*b^6)/d + (112\*B^3\*a^2\*b^4)/d - (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) - (B^2\*a^2\*b^2\*tan(c + d\*x)^(1/2)\*((B^2\*a^3\*b)/d^2 - (B^2\*a\*b^3)/d^2 - (12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4))^(1/2)\*192i)/((16\*B^3\*a^6)/d - (16\*B^3\*b^6)/d + (112\*B^3\*a^2\*b^4)/d - (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) - atan((B^2\*a^4\*tan(c + d\*x)^(1/2)\*((12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4) - (B^2\*a\*b^3)/d^2 + (B^2\*a^3\*b)/d^2)^(1/2)\*32i)/((16\*B^3\*b^6)/d - (16\*B^3\*a^6)/d - (112\*B^3\*a^2\*b^4)/d + (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2))/d^3) + (B^2\*b^4\*tan(c + d\*x)^(1/2)\*((12\*B^4\*a^2\*b^6\*d^4 - B^4\*b^8\*d^4 - B^4\*a^8\*d^4 - 38\*B^4\*a^4\*b^4\*d^4 + 12\*B^4\*a^6\*b^2\*d^4)^(1/2)/(4\*d^4) - (B^2\*a\*b^3)/d^2 + (B^2\*a^3\*b)/d^2)^(1/2)\*32i)/((16\*B^3\*b^6)/d - (16\*B^3\*a^6)/d - (112\*B^3\*a^2\*b^4)/d + (112\*B^3\*a^4\*b^2)/d + (32\*B\*a\*b\*(12\*B^4





$$3.389 \quad \int \frac{(a+b \tan(c+dx))^2 (A+B \tan(c+dx))}{\tan^3(c+dx)} dx$$

**Optimal.** Leaf size=276

$$\frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} (a^2(A-B) - b^2(A-B) - 2ab$$

[Out]  $-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*a^2*A/d/\tan(d*x+c)^{(1/2)}+2*b^2*B*\tan(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.22, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3685, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d}\right)}{2\sqrt{2} d} - \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d}\right)}{2\sqrt{2} d} - \frac{2a^2A}{d\sqrt{\tan(c+dx)}} - \frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out]  $((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/(\operatorname{Sqrt}[2]*d) - ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/(\operatorname{Sqrt}[2]*d) - ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (2*a^2*A)/(d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) + (2*b^2*B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3685

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^2\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(- (B\*c - A\*d)\*(b\*c - a\*d)^2\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*d^2\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[B\*(b\*c - a\*d)^2 + A\*d\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + d\*(B\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + A\*(2\*a\*b\*c - a^2\*d + b^2\*d))\*Tan[e + f\*x] + b^2\*B\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

## Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_.)] + (C_.)*tan[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2a^2 A}{d \sqrt{\tan(c + dx)}} + \int \frac{a(2Ab + aB) - (a^2 A - Ab^2 - 2abB)}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2a^2 A}{d \sqrt{\tan(c + dx)}} + \frac{2b^2 B \sqrt{\tan(c + dx)}}{d} + \int \frac{2aAb + a^2 B}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2a^2 A}{d \sqrt{\tan(c + dx)}} + \frac{2b^2 B \sqrt{\tan(c + dx)}}{d} + \frac{2 \text{Subst}\left(\int \frac{2aA}{\sqrt{u}} du\right)}{d} \\
&= -\frac{2a^2 A}{d \sqrt{\tan(c + dx)}} + \frac{2b^2 B \sqrt{\tan(c + dx)}}{d} - \frac{(a^2(A - B) - 2abB)}{d} \\
&= -\frac{2a^2 A}{d \sqrt{\tan(c + dx)}} + \frac{2b^2 B \sqrt{\tan(c + dx)}}{d} - \frac{(a^2(A - B) - 2abB)}{d} \\
&= -\frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
&= \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.60, size = 211, normalized size = 0.76

$$\frac{-8(Ab + 3aB) - 8(a^2A - Ab^2 - 2abB) {}_2F_1\left(-\frac{1}{2}, 1; \frac{3}{2}; -\tan^2(c + dx)\right) - \sqrt{2}(2aAb + a^2B - b^2B) \left(2\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) - 2\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right) + \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) - \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)\right) \sqrt{\tan(c + dx)} + 8bB(a + b \tan(c + dx))}{4d \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2), x]
```

```
[Out] (-8*b*(A*b + 3*a*B) - 8*(a^2*A - A*b^2 - 2*a*b*B)*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2] - Sqrt[2]*(2*a*A*b + a^2*B - b^2*B)*(2*ArcTan[1 - S
```

```

qrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log
[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]] + 8*b*B*(a + b*Tan[c + d*x]))
/(4*d*Sqrt[Tan[c + d*x]])

```

**Maple [A]**

time = 0.05, size = 238, normalized size = 0.86

method	result
derivativedivides	$2b^2B\left(\sqrt{\tan(dx+c)}\right) - \frac{2a^2A}{\sqrt{\tan(dx+c)}} + \frac{(2Aab+a^2B-b^2B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)}$
default	$2b^2B\left(\sqrt{\tan(dx+c)}\right) - \frac{2a^2A}{\sqrt{\tan(dx+c)}} + \frac{(2Aab+a^2B-b^2B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)

```

```

[Out] 1/d*(2*b^2*B*tan(d*x+c)^(1/2)-2*a^2*A/tan(d*x+c)^(1/2)+1/4*(2*A*a*b+B*a^2-B
*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*tan(d*x+c)^(1/2)))+1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((1-2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))

```

**Maxima [A]**

time = 0.52, size = 240, normalized size = 0.87

$\frac{8Bb^2\sqrt{\tan(dx+c)}-2\sqrt{2}(A-B)a^2-2(A+B)ab-(A-B)^2\arctan\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)-2\sqrt{2}(A-B)a^2-2(A+B)ab-(A-B)^2\arctan\left(-\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+\sqrt{2}(A+B)a^2+2(A-B)ab-(A+B)^2\log\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)-\sqrt{2}(A+B)a^2+2(A-B)ab-(A+B)^2\log\left(-\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm
="maxima")

```

```

[Out] 1/4*(8*B*b^2*sqrt(tan(d*x + c)) - 2*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b -
(A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 2*sqrt(
2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*
b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A + B)*
a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*
x + c) + 1) - 8*A*a^2/sqrt(tan(d*x + c))/d

```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 25667 vs.  $2(244) = 488$ .

time = 193.33, size = 25667, normalized size = 93.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \cdot (4 \cdot \sqrt{2}) \cdot (d^5 \cos(d \cdot x + c)^2 - d^5) \cdot \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8 - 2(ABa^4 - 6AB^2a^2b^2 + AB^3b^4 + 2(A^2 - B^2)a^3b - 2(A^2 - B^2)a^2b^3) \cdot d^2 \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4}) / ((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8) \cdot (((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4)^{3/4} \cdot \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8) / d^4} \cdot \arctan(-(((A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16} - 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{13}b^3 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^4 - 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^5 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{10}b^6 - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b^7 - 90(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^8 + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^7b^9 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b^{10} + 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^{11} - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^4b^{12} + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^3b^{13} + 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^2b^{15} + (A^8 + 2A^6B^2 - 2A^2B^6 - B^8)b^{16}) \cdot d^4 \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4} \cdot \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 - 16(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 + 112(A^3B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 - 112(A^3B - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 + 16(A^3B - AB^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8) / d^4} + \sqrt{2} \cdot ((B^2a^2 + 2A^2a^2b - B^2b^2) \cdot d^7 \sqrt{((A^4 + 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8) / d^4})$

$$\begin{aligned} &^2 + B^4) * a^6 * b^2 + 6 * (A^4 + 2 * A^2 * B^2 + B^4) * a^4 * b^4 + 4 * (A^4 + 2 * A^2 * B^2 \\ &+ B^4) * a^2 * b^6 + (A^4 + 2 * A^2 * B^2 + B^4) * b^8) / d^4) * \text{sqrt}(((A^4 - 2 * A^2 * B^2 + \\ &B^4) * a^8 - 16 * (A^3 * B - A * B^3) * a^7 * b - 4 * (3 * A^4 - 22 * A^2 * B^2 + 3 * B^4) * a^6 * b \\ &^2 + 112 * (A^3 * B - A * B^3) * a^5 * b^3 + 2 * (19 * A^4 - 102 * A^2 * B^2 + 19 * B^4) * a^4 * b^4 \\ &- 112 * (A^3 * B - A * B^3) * a^3 * b^5 - 4 * (3 * A^4 - 22 * A^2 * B^2 + 3 * B^4) * a^2 * b^6 + \\ &16 * (A^3 * B - A * B^3) * a * b^7 + (A^4 - 2 * A^2 * B^2 + B^4) * b^8) / d^4) + ((A^3 + A * B^2) * a^6 \\ &- 2 * (A^2 * B + B^3) * a^5 * b + (A^3 + A * B^2) * a^4 * b^2 - 4 * (A^2 * B + B^3) * a^3 * b^3 - \\ &(A^3 + A * B^2) * a^2 * b^4 - 2 * (A^2 * B + B^3) * a * b^5 - (A^3 + A * B^2) * b^6) * \\ &d^5 * \text{sqrt}(((A^4 - 2 * A^2 * B^2 + B^4) * a^8 - 16 * (A^3 * B - A * B^3) * a^7 * b - 4 * (3 * A^4 \\ &- 22 * A^2 * B^2 + 3 * B^4) * a^6 * b^2 + 112 * (A^3 * B - A * B^3) * a^5 * b^3 + 2 * (19 * A^4 - \\ &102 * A^2 * B^2 + 19 * B^4) * a^4 * b^4 - 112 * (A^3 * B - A * B^3) * a^3 * b^5 - 4 * (3 * A^4 - 22 \\ &* A^2 * B^2 + 3 * B^4) * a^2 * b^6 + 16 * (A^3 * B - A * B^3) * a * b^7 + (A^4 - 2 * A^2 * B^2 + B \\ &^4) * b^8) / d^4) * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^8 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) \\ &) * a^6 * b^2 + 6 * (A^4 + 2 * A^2 * B^2 + B^4) * a^4 * b^4 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a \\ &^2 * b^6 + (A^4 + 2 * A^2 * B^2 + B^4) * b^8 - 2 * (A * B * a^4 - 6 * A * B * a^2 * b^2 + A * B * b^4 \\ &+ 2 * (A^2 - B^2) * a^3 * b - 2 * (A^2 - B^2) * a * b^3) * d^2 * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + \\ &B^4) * a^8 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^6 * b^2 + 6 * (A^4 + 2 * A^2 * B^2 + B^4) * a^ \\ &4 * b^4 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^6 + (A^4 + 2 * A^2 * B^2 + B^4) * b^8) / d^ \\ &4)) / (((A^4 - 2 * A^2 * B^2 + B^4) * a^8 - 16 * (A^3 * B - A * B^3) * a^7 * b - 4 * (3 * A^4 - 22 \\ &* A^2 * B^2 + 3 * B^4) * a^6 * b^2 + 112 * (A^3 * B - A * B^3) * a^5 * b^3 + 2 * (19 * A^4 - 102 * A \\ &^2 * B^2 + 19 * B^4) * a^4 * b^4 - 112 * (A^3 * B - A * B^3) * a^3 * b^5 - 4 * (3 * A^4 - 22 * A^2 * \\ &B^2 + 3 * B^4) * a^2 * b^6 + 16 * (A^3 * B - A * B^3) * a * b^7 + (A^4 - 2 * A^2 * B^2 + B^4) * b \\ &^8)) * \text{sqrt}((((A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * a^12 - 16 * (A^5 * B - A * B^5) * a^11 * \\ &b - 2 * (5 * A^6 - 37 * A^4 * B^2 - 37 * A^2 * B^4 + 5 * B^6) * a^10 * b^2 + 80 * (A^5 * B - A * B^5) \\ &) * a^9 * b^3 + 15 * (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * a^8 * b^4 + 96 * (A^5 * B - A * B^5) \\ &) * a^7 * b^5 + 4 * (13 * A^6 - 45 * A^4 * B^2 - 45 * A^2 * B^4 + 13 * B^6) * a^6 * b^6 - 96 * (A^5 \\ &* B - A * B^5) * a^5 * b^7 + 15 * (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * a^4 * b^8 - 80 * (A^5 * \\ &B - A * B^5) * a^3 * b^9 - 2 * (5 * A^6 - 37 * A^4 * B^2 - 37 * A^2 * B^4 + 5 * B^6) * a^2 * b^10 + \\ &16 * (A^5 * B - A * B^5) * a * b^11 + (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * b^12) * d^2 * \text{sqrt} \\ &(((A^4 + 2 * A^2 * B^2 + B^4) * a^8 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^6 * b^2 + 6 * (A^4 \\ &+ 2 * A^2 * B^2 + B^4) * a^4 * b^4 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^6 + (A^4 + 2 * A \\ &^2 * B^2 + B^4) * b^8) / d^4) * \text{cos}(d * x + c) + \text{sqrt}(2) * \dots \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^2}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2), x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*2/tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 8.89, size = 2500, normalized size = 9.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2)/tan(c + d\*x)^(3/2),x)

[Out] 
$$2*\operatorname{atanh}\left(\frac{32*A^2*a^4*d^3*\tan(c+d*x)^{1/2}*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}/(4*d^4))^{1/2}}{(16*A^3*a^6*d^2 - 16*A^3*b^6*d^2 + 112*A^3*a^2*b^4*d^2 - 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}}\right) + (32*A^2*b^4*d^3*\tan(c+d*x)^{1/2}*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}/(4*d^4))^{1/2}}{(16*A^3*a^6*d^2 - 16*A^3*b^6*d^2 + 112*A^3*a^2*b^4*d^2 - 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}}) - (192*A^2*a^2*b^2*d^3*\tan(c+d*x)^{1/2}*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}/(4*d^4))^{1/2}}{(16*A^3*a^6*d^2 - 16*A^3*b^6*d^2 + 112*A^3*a^2*b^4*d^2 - 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}}) - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{1/2}/(4*d^4))^{1/2} - 2*\operatorname{atanh}\left(\frac{32*B^2*a^4*\tan(c+d*x)^{1/2}*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}/(4*d^4))^{1/2}}{(16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}\right)/d^3 - (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2})/d^3 - (192*B^3*a^3*b^3)/d + (32*B^3*a*b^5)/d + (32*B^2*b^4*\tan(c+d*x)^{1/2}*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}/(4*d^4))^{1/2}}{(16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}) - (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^{1/2}}{(16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}) - (B^2*a^3*b)/d^2)^{1/2}}{(16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}) - (B^2*a^3*b)/d^2)^{1/2}}$$



$$\begin{aligned}
& ^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 1 \\
& 2*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 - (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 \\
& - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 - (192 \\
& *B^3*a^3*b^3)/d + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d - (192*B^2*a^2*b^2*t \\
& an(c + d*x)^{(1/2)}*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4 \\
& *a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)/(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a \\
& ^3*b)/d^2)^{(1/2)}/((16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^ \\
& 4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 - (16*B*a^2*(12*B^4 \\
& *a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6* \\
& b^2*d^4)^{(1/2)}/d^3 - (192*B^3*a^3*b^3)/d + (32*B^3*a*b^5)/d + (32*B^3*a^5* \\
& b)/d))*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^ \\
& 4 + 12*B^4*a^6*b^2*d^4)^{(1/2)/(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^ \\
& (1/2) - 2*atanh((32*B^2*a^4*tan(c + d*x)^{(1/2)}*((B^2*a*b^3)/d^2 - (12*B^4*a \\
& ^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^ \\
& 2*d^4)^{(1/2)/(4*d^4) - (B^2*a^3*b)/d^2)^{(1/2)}/((16*B*a^2*(12*B^4*a^2*b^6*d \\
& ^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{( \\
& 1/2)}/d^3 - (192*B^3*a^3*b^3)/d - (16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d \\
& ^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 + (3 \\
& 2*B^3*a*b^5)/d + (32*B^3*a^5*b)/d) + (32*B^2*b^4*tan(c + d*x)^{(1/2)}*((B^2*a \\
& *b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^ \\
& 4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)/(4*d^4) - (B^2*a^3*b)/d^2)^{(1/2)}/((16*B* \\
& a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + \\
& 12*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 - (192*B^3*a^3*b^3)/d - (16*B*b^2*(12*B^4*a^ \\
& 2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2 \\
& *d^4)^{(1/2)}/d^3 + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d - (192*B^2*a^2*b^2* \\
& tan(c + d*x)^{(1/2)}*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B \\
& ^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)/(4*d^4) - (B^2* \\
& a^3*b)/d^2)^{(1/2)}/((16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d \\
& ^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 - (192*B^3*a^3*b^3 \\
& )/d - (16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^ \\
& 4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)}/d^3 + (32*B^3*a*b^5)/d + (32*B^3*a^5 \\
& *b)/d))*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 \\
& - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{(1/2)/(4*d^4) - (B^2*a^3*b)/d^2) \\
& ^{(1/2) - 2*atanh((32*A^2*a^4*d^3*tan(c + d*x)^{(1/2)}*((12*A^4*a^2*b^6*d^4 - \\
& A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{(1/2)/ \\
& (4*d^4) - (A^2*a*b^3)/d^2 + (A^2*a^3*b)/d^2)^{(1/2)}/(16*A^3*b^6*d^2 - 16*A^ \\
& 3*a^6*d^2 - 112*A^3*a^2*b^4*d^2 + 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*A^4*a^ \\
& 2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2 \\
& *d^4)^{(1/2)) + (32*A^2*b^4*d^3*tan(c + d*x)^{(1/2)}*((12*A^4*a^2*b^6*d^4 - A^ \\
& 4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^{(1/2)/(4 \\
& *d^4) - (A^2*a*b^3)/d^2 + (A^2*a^3*b)/d^2)^{(1/2)...}
\end{aligned}$$

$$3.390 \quad \int \frac{(a+b \tan(c+dx))^2 (A+B \tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=283

$$\frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} \quad (2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)$$

[Out]  $-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*a*(2*A*b+B*a)/d/\tan(d*x+c)^{(1/2)}-2/3*a^2*A/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.25, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3685, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{2a^2 A}{3d \tan^3(c+dx)} - \frac{2a(aB+2Ab)}{4d \sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x])]/\operatorname{Tan}[c + d*x]^{(5/2)}, x]$

[Out]  $((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) - ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*d) + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) - ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]/(2*\operatorname{Sqrt}[2]*d) - (2*a^2*A)/(3*d*\operatorname{Tan}[c + d*x]^{(3/2)}) - (2*a*(2*A*b + a*B))/(d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

**Rule 210**

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

$\operatorname{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{With}[\{q = 1 - 4*Simplify[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /;$  RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3685

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^2\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(- (B\*c - A\*d))\*(b\*c - a\*d)^2\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*d^2\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[B\*(b\*c - a\*d)^2 + A\*d\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + d\*(B\*(a^2\*c - b^2\*c + 2\*a\*b\*d) + A\*(2\*a\*b\*c - a^2\*d + b^2\*d))\*Tan[e + f\*x] + b^2\*B\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

## Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2a^2 A}{3d \tan^{\frac{3}{2}}(c + dx)} + \int \frac{a(2Ab + aB) - (a^2 A - Ab^2 - 2abA)}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2a^2 A}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{d \sqrt{\tan(c + dx)}} + \int \frac{-a^2 A + Ab^2 + 2abA}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2a^2 A}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{d \sqrt{\tan(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{-a^2 A + Ab^2 + 2abA}{\tan^{\frac{3}{2}}(c + dx)} dx\right)}{d} \\
&= -\frac{2a^2 A}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{d \sqrt{\tan(c + dx)}} - \frac{(a^2(A - B) - b^2(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
&= -\frac{2a^2 A}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{d \sqrt{\tan(c + dx)}} + \frac{(a^2(A - B) - b^2(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
&= \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
&= \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.47, size = 119, normalized size = 0.42

$$\frac{2(-a^2 A + Ab^2 + 2abB) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)\right) - 6(2aAb + a^2 B - b^2 B) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\tan^2(c + dx)\right) \tan(c + dx) - 2b(Ab + 2aB + 3bB \tan(c + dx))}{3d \tan^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),
x]
```

```
[Out] (2*(-(a^2*A) + A*b^2 + 2*a*b*B)*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] - 6*(2*a*A*b + a^2*B - b^2*B)*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]*Tan[c + d*x] - 2*b*(A*b + 2*a*B + 3*b*B*Tan[c + d*x]))/(3*d*Tan[c + d*x]^(3/2))
```

**Maple [A]**

time = 0.05, size = 243, normalized size = 0.86

method	result
derivativedivides	$-\frac{2a(2Ab+aB)}{\sqrt{\tan(dx+c)}} - \frac{2a^2A}{3\tan(dx+c)^{\frac{3}{2}}} + \frac{(-a^2A+Ab^2+2Bab)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \right)}{4}$
default	$-\frac{2a(2Ab+aB)}{\sqrt{\tan(dx+c)}} - \frac{2a^2A}{3\tan(dx+c)^{\frac{3}{2}}} + \frac{(-a^2A+Ab^2+2Bab)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVE RBOSE)
```

```
[Out] 1/d*(-2*a*(2*A*b+B*a)/tan(d*x+c)^(1/2)-2/3*a^2*A/tan(d*x+c)^(3/2)+1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-2*A*a*b-B*a^2+2*B*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

**Maxima [A]**

time = 0.53, size = 248, normalized size = 0.88

$\frac{6\sqrt{2}(A+B)^2+2(A-B)ab-(A+B)^2\arctan\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)+6\sqrt{2}(A+B)^2+2(A-B)ab-(A+B)^2\arctan\left(-\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)+3\sqrt{2}(A-B)^2+2(A+B)ab-(A-B)^2\log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}\right)+\frac{6\sqrt{2}(A+B)^2+2(A-B)ab-(A+B)^2\arctan\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)+6\sqrt{2}(A+B)^2+2(A-B)ab-(A+B)^2\arctan\left(-\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)+3\sqrt{2}(A-B)^2+2(A+B)ab-(A-B)^2\log\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}{-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1}\right)}{12d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2)+2*sqrt(tan(d*x+c))))+6*sqrt(2)*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)-2*sqrt(tan(d*x+c))))+3*sqrt(2)*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*log(sqrt(2)*sqrt(tan(d*x+c))+tan(d*x+c)+1)-3*sqrt(2)*((A-B)*a^2-2*(A+B)*a*b-(
```

$(A - B) \cdot b^2 \cdot \log(-\sqrt{2} \cdot \sqrt{\tan(dx + c)} + \tan(dx + c) + 1) + 8 \cdot (A \cdot a^2 + 3 \cdot (B \cdot a^2 + 2 \cdot A \cdot a \cdot b) \cdot \tan(dx + c)) / \tan(dx + c)^{(3/2)} / d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 25680 vs.  $2(249) = 498$ .

time = 222.96, size = 25680, normalized size = 90.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(dx+c))^2\*(A+B\*tan(dx+c))/tan(dx+c)^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} \cdot (12 \cdot \sqrt{2} \cdot (d^5 \cdot \cos(dx + c))^2 - d^5) \cdot \sqrt{((A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^6 \cdot b^2 + 6 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^4 \cdot b^4 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^2 \cdot b^6 + (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot b^8 + 2 \cdot (A \cdot B \cdot a^4 - 6 \cdot A \cdot B \cdot a^2 \cdot b^2 + A \cdot B \cdot b^4 + 2 \cdot (A^2 - B^2) \cdot a^3 \cdot b - 2 \cdot (A^2 - B^2) \cdot a \cdot b^3) \cdot d^2 \cdot \sqrt{((A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^6 \cdot b^2 + 6 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^4 \cdot b^4 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^2 \cdot b^6 + (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot b^8) / d^4)} / ((A^4 - 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 - 16 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^7 \cdot b - 4 \cdot (3 \cdot A^4 - 22 \cdot A^2 \cdot B^2 + 3 \cdot B^4) \cdot a^6 \cdot b^2 + 112 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^5 \cdot b^3 + 2 \cdot (19 \cdot A^4 - 102 \cdot A^2 \cdot B^2 + 19 \cdot B^4) \cdot a^4 \cdot b^4 - 112 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^3 \cdot b^5 - 4 \cdot (3 \cdot A^4 - 22 \cdot A^2 \cdot B^2 + 3 \cdot B^4) \cdot a^2 \cdot b^6 + 16 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a \cdot b^7 + (A^4 - 2 \cdot A^2 \cdot B^2 + B^4) \cdot b^8) \cdot (((A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^6 \cdot b^2 + 6 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^4 \cdot b^4 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^2 \cdot b^6 + (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot b^8) / d^4)^{(3/4)} \cdot \sqrt{((A^4 - 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 - 16 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^7 \cdot b - 4 \cdot (3 \cdot A^4 - 22 \cdot A^2 \cdot B^2 + 3 \cdot B^4) \cdot a^6 \cdot b^2 + 112 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^5 \cdot b^3 + 2 \cdot (19 \cdot A^4 - 102 \cdot A^2 \cdot B^2 + 19 \cdot B^4) \cdot a^4 \cdot b^4 - 112 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^3 \cdot b^5 - 4 \cdot (3 \cdot A^4 - 22 \cdot A^2 \cdot B^2 + 3 \cdot B^4) \cdot a^2 \cdot b^6 + 16 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a \cdot b^7 + (A^4 - 2 \cdot A^2 \cdot B^2 + B^4) \cdot b^8) / d^4} \cdot \operatorname{arctan}(\frac{((A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot a^{16} - 8 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^{15} \cdot b - 40 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^{13} \cdot b^3 - 20 \cdot (A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot a^{12} \cdot b^4 - 72 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^{11} \cdot b^5 - 64 \cdot (A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot a^{10} \cdot b^6 - 40 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^9 \cdot b^7 - 90 \cdot (A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot a^8 \cdot b^8 + 40 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^7 \cdot b^9 - 64 \cdot (A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot a^6 \cdot b^{10} + 72 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^5 \cdot b^{11} - 20 \cdot (A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot a^4 \cdot b^{12} + 40 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a^3 \cdot b^{13} + 8 \cdot (A^7 \cdot B + 3 \cdot A^5 \cdot B^3 + 3 \cdot A^3 \cdot B^5 + A \cdot B^7) \cdot a \cdot b^{15} + (A^8 + 2 \cdot A^6 \cdot B^2 - 2 \cdot A^2 \cdot B^6 - B^8) \cdot b^{16}) \cdot d^4 \cdot \sqrt{((A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^6 \cdot b^2 + 6 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^4 \cdot b^4 + 4 \cdot (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^2 \cdot b^6 + (A^4 + 2 \cdot A^2 \cdot B^2 + B^4) \cdot b^8) / d^4} \cdot \sqrt{((A^4 - 2 \cdot A^2 \cdot B^2 + B^4) \cdot a^8 - 16 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^7 \cdot b - 4 \cdot (3 \cdot A^4 - 22 \cdot A^2 \cdot B^2 + 3 \cdot B^4) \cdot a^6 \cdot b^2 + 112 \cdot (A^3 \cdot B - A \cdot B^3) \cdot a^5 \cdot b^3 + 2 \cdot (19 \cdot A^4 - 102 \cdot A^2 \cdot B^2 + 19 \cdot B^4) \cdot a^4 \cdot b^4 -$

$$\begin{aligned}
& 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16* \\
& (A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4) - \text{sqrt}(2)*((A*a^2 \\
& - 2*B*a*b - A*b^2)*d^7*\text{sqrt}(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 \\
& + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 \\
& + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4)*\text{sqrt}(((A^4 - 2*A^2*B^2 \\
& + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6* \\
& b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b \\
& ^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + \\
& 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4) - ((A^2*B + B \\
& ^3)*a^6 + 2*(A^3 + A*B^2)*a^5*b + (A^2*B + B^3)*a^4*b^2 + 4*(A^3 + A*B^2)*a \\
& ^3*b^3 - (A^2*B + B^3)*a^2*b^4 + 2*(A^3 + A*B^2)*a*b^5 - (A^2*B + B^3)*b^6) \\
& *d^5*\text{sqrt}(((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^ \\
& 4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - \\
& 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 2 \\
& 2*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + \\
& B^4)*b^8)/d^4))*\text{sqrt}(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^ \\
& 4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)* \\
& a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 + 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^ \\
& 4 + 2*(A^2 - B^2)*a^3*b - 2*(A^2 - B^2)*a*b^3)*d^2*\text{sqrt}(((A^4 + 2*A^2*B^2 + \\
& B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a \\
& ^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d \\
& ^4))/((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 2 \\
& 2*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102* \\
& A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2 \\
& *B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)* \\
& b^8))*\text{sqrt}(((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^12 - 16*(A^5*B - A*B^5)*a^11 \\
& *b - 2*(5*A^6 - 37*A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^10*b^2 + 80*(A^5*B - A*B \\
& ^5)*a^9*b^3 + 15*(A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^8*b^4 + 96*(A^5*B - A*B^ \\
& 5)*a^7*b^5 + 4*(13*A^6 - 45*A^4*B^2 - 45*A^2*B^4 + 13*B^6)*a^6*b^6 - 96*(A^ \\
& 5*B - A*B^5)*a^5*b^7 + 15*(A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^4*b^8 - 80*(A^5 \\
& *B - A*B^5)*a^3*b^9 - 2*(5*A^6 - 37*A^4*B^2 - 37*A^2*B^4 + 5*B^6)*a^2*b^10 \\
& + 16*(A^5*B - A*B^5)*a*b^11 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^12)*d^2*\text{sqr} \\
& t(((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 \\
& + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2* \\
& A^2*B^2 + B^4)*b^8)/d^4)*\cos(d*x + c) + \text{sqrt}(2) \dots
\end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^2}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2),x)







$$3.391 \quad \int \frac{(a+b \tan(c+dx))^2 (A+B \tan(c+dx))}{\tan^7(c+dx)} dx$$

**Optimal.** Leaf size=317

$$\frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{2a^2(A+B) - b^2(A+B)}{2\sqrt{2}d} \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right)$$

[Out] 1/2\*(a^2\*(A-B)-b^2\*(A-B)-2\*a\*b\*(A+B))\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/d \*2^(1/2)+1/2\*(a^2\*(A-B)-b^2\*(A-B)-2\*a\*b\*(A+B))\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/4\*(2\*a\*b\*(A-B)+a^2\*(A+B)-b^2\*(A+B))\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)-1/4\*(2\*a\*b\*(A-B)+a^2\*(A+B)-b^2\*(A+B))\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)+2\*(A\*a^2-A\*b^2-2\*B\*a\*b)/d/tan(d\*x+c)^(1/2)-2/5\*a^2\*A/d/tan(d\*x+c)^(5/2)-2/3\*a\*(2\*A\*b+B\*a)/d/tan(d\*x+c)^(3/2)

**Rubi [A]**

time = 0.26, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3685, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2(a^2(A+B) - 2ab(A+B) - b^2(A+B)) \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{(a^2(A+B) + 2ab(A+B) - b^2(A+B)) \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{(a^2(A+B) + 2ab(A+B) - b^2(A+B)) \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{1 - \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{2(a^2(A+B) - 2ab(A+B) - b^2(A+B)) \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d} + \frac{2(a^2(A+B) - 2ab(A+B) - b^2(A+B)) \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{1 + \sqrt{2} \sqrt{\tan(c+dx)}}\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] -(((a^2\*(A - B) - b^2\*(A - B) - 2\*a\*b\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + ((a^2\*(A - B) - b^2\*(A - B) - 2\*a\*b\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + ((2\*a\*b\*(A - B) + a^2\*(A + B) - b^2\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) - ((2\*a\*b\*(A - B) + a^2\*(A + B) - b^2\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) - (2\*a^2\*A)/(5\*d\*Tan[c + d\*x]^(5/2)) - (2\*a\*(2\*A\*b + a\*B))/(3\*d\*Tan[c + d\*x]^(3/2)) + (2\*(a^2\*A - A\*b^2 - 2\*a\*b\*B))/(d\*Sqrt[Tan[c + d\*x]])

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3610

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A - C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2a^2 A}{5d \tan^{\frac{5}{2}}(c + dx)} + \int \frac{a(2Ab + aB) - (a^2 A - Ab^2 - 2a)}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2 A}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \int \frac{-a^2 A + Ab^2 - 2a}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2 A}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(a^2 A - Ab^2 - 2a)}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2 A}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(a^2 A - Ab^2 - 2a)}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2 A}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(a^2 A - Ab^2 - 2a)}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2 A}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2a(2Ab + aB)}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2(a^2 A - Ab^2 - 2a)}{d \sqrt{\tan(c + dx)}} \\
&= \frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} d} \\
&= -\frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.41, size = 120, normalized size = 0.38

$$\frac{2((-3a^2A + 3Ab^2 + 6abB) {}_2F_1\left(-\frac{5}{4}, 1; -\frac{1}{4}; -\tan^2(c + dx)\right) - 5(2aAb + a^2B - b^2B) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)\right) \tan(c + dx) - b(3Ab + 6aB + 5bB \tan(c + dx)))}{15d \tan^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] (2\*((-3\*a^2\*A + 3\*A\*b^2 + 6\*a\*b\*B)\*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d\*x]^2] - 5\*(2\*a\*A\*b + a^2\*B - b^2\*B)\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x] - b\*(3\*A\*b + 6\*a\*B + 5\*b\*B\*Tan[c + d\*x])))/(15\*d\*Tan[c + d\*x]^(5/2))

**Maple [A]**

time = 0.05, size = 270, normalized size = 0.85

method	result
derivativedivides	$\frac{2(-a^2A+Ab^2+2Bab)}{\sqrt{\tan(dx+c)}} - \frac{2a(2Ab+aB)}{3\tan(dx+c)^{\frac{3}{2}}} - \frac{2a^2A}{5\tan(dx+c)^{\frac{5}{2}}} + \frac{(-2Aab-a^2B+b^2B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}$
default	$\frac{2(-a^2A+Ab^2+2Bab)}{\sqrt{\tan(dx+c)}} - \frac{2a(2Ab+aB)}{3\tan(dx+c)^{\frac{3}{2}}} - \frac{2a^2A}{5\tan(dx+c)^{\frac{5}{2}}} + \frac{(-2Aab-a^2B+b^2B)\sqrt{2}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*(-2*(-A*a^2+A*b^2+2*B*a*b)/\tan(dx+c)^{(1/2)}-2/3*a*(2*A*b+B*a)/\tan(dx+c)^{(3/2)}-2/5*a^2*A/\tan(dx+c)^{(5/2)}+1/4*(-2*A*a*b-B*a^2+B*b^2)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c))/(1-2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c))))+2*\arctan(1+2^{(1/2)}*\tan(dx+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(dx+c)^{(1/2)})))+1/4*(A*a^2-A*b^2-2*B*a*b)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c))/(1+2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c))))+2*\arctan(1+2^{(1/2)}*\tan(dx+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(dx+c)^{(1/2)}))$

**Maxima** [A]

time = 0.52, size = 276, normalized size = 0.87

$\frac{30\sqrt{2}(A-B)a^2-2(A+B)ab-(A-B)B\arctan\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\sqrt{\tan(dx+c)}\right)+30\sqrt{2}(A-B)a^2-2(A+B)ab-(A-B)B\arctan\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\sqrt{\tan(dx+c)}\right)-15\sqrt{2}(A+B)a^2+2(A-B)ab-(A+B)B\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+15\sqrt{2}(A+B)a^2+2(A-B)ab-(A+B)B\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\frac{\sqrt{1+2^{(1/2)}*\tan(dx+c)^{(1/2)}+\tan(dx+c)}}{\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm  
="maxima")`

[Out]  $1/60*(30*\sqrt{2}*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)}))+30*\sqrt{2}*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)}))-15*\sqrt{2}*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*\log(\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+15*\sqrt{2}*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*\log(-\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)-8*(3*A*a^2-15*(A*a^2-2*B*a*b-A*b^2)*\tan(dx+c)^2+5*(B*a^2+2*A*a*b)*\tan(dx+c))/\tan(dx+c)^{(5/2)}/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 26849 vs. 2(279) = 558.

time = 151.03, size = 26849, normalized size = 84.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 
$$-1/60*(60*\sqrt{2}*(d^5*\cos(d*x + c)^4 - 2*d^5*\cos(d*x + c)^2 + d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 - 2*(A*B*a^4 - 6*A*B*a^2*b^2 + A*B*b^4 + 2*(A^2 - B^2)*a^3*b - 2*(A^2 - B^2)*a*b^3)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4})/((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))/d^4)^{3/4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4}*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{16} - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{15}*b - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13}*b^3 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12}*b^4 - 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^5 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^6 - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^7 - 90*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^8 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^9 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^{10} + 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^{11} - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^{12} + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^{13} + 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{15} + (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{16})*d^4*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4}*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 + 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/d^4} + \sqrt{2}*((B*a^2 + 2*A*a*b - B*b^2)*d^7*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8)/d^4})*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 - 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 + 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 - 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A$$

$$\begin{aligned} & \wedge 2 * B^2 + 3 * B^4) * a^2 * b^6 + 16 * (A^3 * B - A * B^3) * a * b^7 + (A^4 - 2 * A^2 * B^2 + B^4) \\ & * b^8) / d^4) + ((A^3 + A * B^2) * a^6 - 2 * (A^2 * B + B^3) * a^5 * b + (A^3 + A * B^2) * a^4 \\ & * b^2 - 4 * (A^2 * B + B^3) * a^3 * b^3 - (A^3 + A * B^2) * a^2 * b^4 - 2 * (A^2 * B + B^3) * a \\ & * b^5 - (A^3 + A * B^2) * b^6) * d^5 * \text{sqrt}(((A^4 - 2 * A^2 * B^2 + B^4) * a^8 - 16 * (A^3 * B \\ & - A * B^3) * a^7 * b - 4 * (3 * A^4 - 22 * A^2 * B^2 + 3 * B^4) * a^6 * b^2 + 112 * (A^3 * B - A * B^3) \\ & * a^5 * b^3 + 2 * (19 * A^4 - 102 * A^2 * B^2 + 19 * B^4) * a^4 * b^4 - 112 * (A^3 * B - A * B^3) \\ & * a^3 * b^5 - 4 * (3 * A^4 - 22 * A^2 * B^2 + 3 * B^4) * a^2 * b^6 + 16 * (A^3 * B - A * B^3) * a \\ & * b^7 + (A^4 - 2 * A^2 * B^2 + B^4) * b^8) / d^4)) * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^8 \\ & + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^6 * b^2 + 6 * (A^4 + 2 * A^2 * B^2 + B^4) * a^4 * b^4 + 4 \\ & * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^6 + (A^4 + 2 * A^2 * B^2 + B^4) * b^8 - 2 * (A * B * a^4 \\ & - 6 * A * B * a^2 * b^2 + A * B * b^4 + 2 * (A^2 - B^2) * a^3 * b - 2 * (A^2 - B^2) * a * b^3) * d^2 \\ & * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^8 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^6 * b^2 + 6 * \\ & (A^4 + 2 * A^2 * B^2 + B^4) * a^4 * b^4 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^6 + (A^4 \\ & + 2 * A^2 * B^2 + B^4) * b^8) / d^4)) / ((A^4 - 2 * A^2 * B^2 + B^4) * a^8 - 16 * (A^3 * B - A * \\ & B^3) * a^7 * b - 4 * (3 * A^4 - 22 * A^2 * B^2 + 3 * B^4) * a^6 * b^2 + 112 * (A^3 * B - A * B^3) * a^5 \\ & * b^3 + 2 * (19 * A^4 - 102 * A^2 * B^2 + 19 * B^4) * a^4 * b^4 - 112 * (A^3 * B - A * B^3) * a^3 \\ & * b^5 - 4 * (3 * A^4 - 22 * A^2 * B^2 + 3 * B^4) * a^2 * b^6 + 16 * (A^3 * B - A * B^3) * a * b^7 + \\ & (A^4 - 2 * A^2 * B^2 + B^4) * b^8) * \text{sqrt}((((A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * a^12 \\ & - 16 * (A^5 * B - A * B^5) * a^11 * b - 2 * (5 * A^6 - 37 * A^4 * B^2 - 37 * A^2 * B^4 + 5 * B^6) * a \\ & ^10 * b^2 + 80 * (A^5 * B - A * B^5) * a^9 * b^3 + 15 * (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * a \\ & ^8 * b^4 + 96 * (A^5 * B - A * B^5) * a^7 * b^5 + 4 * (13 * A^6 - 45 * A^4 * B^2 - 45 * A^2 * B^4 + \\ & 13 * B^6) * a^6 * b^6 - 96 * (A^5 * B - A * B^5) * a^5 * b^7 + 15 * (A^6 - A^4 * B^2 - A^2 * B^4 \\ & + B^6) * a^4 * b^8 - 80 * (A^5 * B - A * B^5) * a^3 * b^9 - 2 * (5 * A^6 - 37 * A^4 * B^2 - 37 * A^2 * B^4 \\ & + 5 * B^6) * a^2 * b^10 + 16 * (A^5 * B - A * B^5) * a * b^11 + (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * b^12) * d^2 * \\ & \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^8 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^6 * b^2 + 6 * (A^4 + 2 * A^2 * B^2 + \\ & B^4) * a^4 * b^4 + 4 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^6 + (A^4 + 2 * A^2 * B^2 + B^4) * b^8) / d^4) \dots \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^2}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*2/tan(c + d\*x)\*\*(7/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 12.73, size = 2500, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2)/tan(c + d\*x)^(7/2),x)

[Out] 
$$2*\operatorname{atanh}\left(\frac{32*B^2*a^4*d^3*\tan(c+d*x)^{1/2}*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}{(4*d^4) - (B^2*a^3*b)/d^2)^{1/2}}{(16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2)\right) + (32*B^2*b^4*d^3*\tan(c+d*x)^{1/2}*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2})/(4*d^4) - (B^2*a^3*b)/d^2)^{1/2} / (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2} - 16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2) - (192*B^2*a^2*b^2*d^3*\tan(c+d*x)^{1/2}*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2})/(4*d^4) - (B^2*a^3*b)/d^2)^{1/2} / (16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2} - 16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2) * ((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2})/(4*d^4) - (B^2*a^3*b)/d^2)^{1/2} + 2*\operatorname{atanh}\left(\frac{32*B^2*a^4*d^3*\tan(c+d*x)^{1/2}*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}{(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^{1/2}}{(16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2)\right) + (32*B^2*b^4*d^3*\tan(c+d*x)^{1/2}*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}}{(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^{1/2}} / (16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^{1/2}} - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2)$$



$$3.392 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=463

$$\frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} (3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B))$$

```
[Out] -1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)-1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*tan(d*x+c)^(1/2)/d+2/3*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*tan(d*x+c)^(3/2)/d+2/45*b*(27*A*a*b+22*B*a^2-9*B*b^2)*tan(d*x+c)^(5/2)/d+2/63*b^2*(9*A*b+13*B*a)*tan(d*x+c)^(7/2)/d+2/9*b*B*tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2/d
```

**Rubi** [A]

time = 0.56, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3688, 3718, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*d) - ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*d) + ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*d) + (2*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Sqrt[Tan[c + d*x]])/d + (2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Tan[c + d*x]^(3/2))/(3*d) + (2*b*(27*a*A*b + 22*a^2*B - 9*b^2*B)*Tan[c + d*x]^(5/2))/(45*d) + (2*b^2*(9*A*b + 13*a*B)*Tan[c + d*x]^(7/2))/(63*d) + (2*b*B*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2)/(9*d)
```

**Rule 210**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)]*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3688

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3711

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rule 3718

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)
*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f
_)*(x_)]^2), x_Symbol] := Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]
```

### Rubi steps



**Maple [A]**

time = 0.05, size = 432, normalized size = 0.93

method	result
derivativedivides	$\frac{2B b^3 \left(\tan \frac{9}{2}(dx+c)\right)}{9} + \frac{2A b^3 \left(\tan \frac{7}{2}(dx+c)\right)}{7} + \frac{6B a b^2 \left(\tan \frac{7}{2}(dx+c)\right)}{7} + \frac{6A a b^2 \left(\tan \frac{5}{2}(dx+c)\right)}{5} + \frac{6B a^2 b \left(\tan \frac{5}{2}(dx+c)\right)}{5} - \frac{2B b^3 \left(\tan \frac{3}{2}(dx+c)\right)}{3}$
default	$\frac{2B b^3 \left(\tan \frac{9}{2}(dx+c)\right)}{9} + \frac{2A b^3 \left(\tan \frac{7}{2}(dx+c)\right)}{7} + \frac{6B a b^2 \left(\tan \frac{7}{2}(dx+c)\right)}{7} + \frac{6A a b^2 \left(\tan \frac{5}{2}(dx+c)\right)}{5} + \frac{6B a^2 b \left(\tan \frac{5}{2}(dx+c)\right)}{5} - \frac{2B b^3 \left(\tan \frac{3}{2}(dx+c)\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*(2/9*B*b^3*\tan(d*x+c)^{(9/2)}+2/7*A*b^3*\tan(d*x+c)^{(7/2)}+6/7*B*a*b^2*\tan(d*x+c)^{(7/2)}+6/5*A*a*b^2*\tan(d*x+c)^{(5/2)}+6/5*B*a^2*b*\tan(d*x+c)^{(5/2)}-2/5*B*b^3*\tan(d*x+c)^{(5/2)}+2*A*a^2*b*\tan(d*x+c)^{(3/2)}-2/3*A*b^3*\tan(d*x+c)^{(3/2)}+2/3*B*a^3*\tan(d*x+c)^{(3/2)}-2*B*a*b^2*\tan(d*x+c)^{(3/2)}+2*A*a^3*\tan(d*x+c)^{(1/2)}-6*A*a*b^2*\tan(d*x+c)^{(1/2)}-6*B*a^2*b*\tan(d*x+c)^{(1/2)}+2*B*b^3*\tan(d*x+c)^{(1/2)}+1/4*(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima [A]**

time = 0.51, size = 398, normalized size = 0.86

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out]  $1/1260*(280*B*b^3*\tan(d*x+c)^{(9/2)}+360*(3*B*a*b^2+A*b^3)*\tan(d*x+c)^{(7/2)}+504*(3*B*a^2*b+3*A*a*b^2-B*b^3)*\tan(d*x+c)^{(5/2)}-630*\sqrt{2}*((A+B)*a^3+3*(A-B)*a^2*b-3*(A+B)*a*b^2-(A-B)*b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(d*x+c)})))-630*\sqrt{2}*((A+B)*a^3+3*(A-B)*a^2*b-3*(A+B)*a*b^2-(A-B)*b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(d*x+c)}))-315*\sqrt{2}*((A-B)*a^3-3*(A+B)*a^2*b-3$

```

*(A - B)*a*b^2 + (A + B)*b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c)
+ 1) + 315*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A +
B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 840*(B*a^3 +
3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c)^(3/2) + 2520*(A*a^3 - 3*B*a^2*
b - 3*A*a*b^2 + B*b^3)*sqrt(tan(d*x + c))/d

```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm
="fricas")

```

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 \tan^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)

```

```

[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*tan(c + d*x)**(3/2),
x)

```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm
="giac")

```

[Out] Timed out

**Mupad** [B]

time = 32.65, size = 2500, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.





$$\begin{aligned}
& d^3 - (16 \tan(c + dx)^{1/2} (A^2 a^6 - A^2 b^6 + 15 A^2 a^2 b^4 - 15 A^2 a^4 b^2)) / d^2 * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{1/2} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{1/2} * i - ((8 (4 A a^3 d^2 - 12 A a b^2 d^2) * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{1/2} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{1/2}) / d^3 + \\
& (16 \tan(c + dx)^{1/2} (A^2 a^6 - A^2 b^6 + 15 A^2 a^2 b^4 - 15 A^2 a^4 b^2)) / d^2 * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{1/2} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{1/2} * i) / ((16 (3 A^3 a^8 b - A^3 b^9 + 6 A^3 a^4 b^5 + 8 A^3 a^6 b^3)) / d^3 + ((8 (4 A a^3 d^2 - 12 A a b^2 d^2) * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{1/2} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{1/2}) / d^3 - (16 \tan(c + dx)^{1/2} (A^2 a^6 - A^2 b^6 + 15 A^2 a^2 b^4 - 15 A^2 a^4 b^2)) / d^2 * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{1/2} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{1/2} + \\
& ((8 (4 A a^3 d^2 - 12 A a b^2 d^2) * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{1/2} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{1/2}) / d^3 + (5 \dots
\end{aligned}$$

### 3.393 $\int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^3 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=421

$$\frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d} + \dots$$

```
[Out] 1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))/d*2^(1/2)+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+
B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)+1/4*(3*a^2*b*(A-B)-b^3*(A-
B)+a^3*(A+B)-3*a*b^2*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(
1/2)-1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*tan
(d*x+c)^(1/2)/d+2/21*b*(21*A*a*b+18*B*a^2-7*B*b^2)*tan(d*x+c)^(3/2)/d+2/35*
b^2*(7*A*b+11*B*a)*tan(d*x+c)^(5/2)/d+2/7*b*B*tan(d*x+c)^(3/2)*(a+b*tan(d*x
+c))^2/d
```

**Rubi [A]**

time = 0.48, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3688, 3718, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

```
[Out] -(((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*ArcTan[1
- Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*d)) + ((a^3*(A - B) - 3*a*b^2*(A -
B) - 3*a^2*b*(A + B) + b^3*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d) + ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A
+ B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*d) - (
(3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*Log[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*d) + (2*(3*a^2*A*b - A*
b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Tan[c + d*x]]/d + (2*b*(21*a*A*b + 18*a^2*B
- 7*b^2*B)*Tan[c + d*x]^(3/2))/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Tan[c + d*x
]^(5/2))/(35*d) + (2*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2)/(7*d)
```

**Rule 210**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

$\text{t}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3688

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \text{:> Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{(m - 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)}/(d*f*(m + n))), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 2)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*\text{Tan}[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegersQ}[2*m, 2*n]) \ \&\& \ !(\text{IGtQ}[n, 1] \ \& \ ( \ !\text{IntegerQ}[m] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{NeQ}[a, 0])))$

### Rule 3711

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\}, x\_Symbol] \text{:> Simp}[C*((a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1))), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A - C + B*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \ \&\& \ \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \ \&\& \ !\text{LeQ}[m, -1]$

### Rule 3718

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\}, x\_Symbol] \text{:> Simp}[b*C*\text{Tan}[e + f*x]*((c + d*\text{Tan}[e + f*x])^{(n + 1)}/(d*f*(n + 2))), x] - \text{Dist}[1/(d*(n + 2)), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*\text{Tan}[e + f*x] - (a*C*d*(n + 2) - b*(c*C - B*d*(n + 2)))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ !\text{LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^3 (A+B \tan(c+dx)) dx &= \frac{2bB \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2}{7d} + \frac{2}{7} \int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^3 (A+B \tan(c+dx)) dx \\
&= \frac{2b^2(7Ab+11aB) \tan^{\frac{5}{2}}(c+dx)}{35d} + \frac{2bB \tan^{\frac{3}{2}}(c+dx)}{7d} \\
&= \frac{2b(21aAb+18a^2B-7b^2B) \tan^{\frac{3}{2}}(c+dx)}{21d} + \frac{2}{7} \int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^3 (A+B \tan(c+dx)) dx \\
&= \frac{2(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \sqrt{\tan(c+dx)}}{d} \\
&= \frac{2(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \sqrt{\tan(c+dx)}}{d} \\
&= \frac{2(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \sqrt{\tan(c+dx)}}{d} \\
&= \frac{2(3a^2Ab - Ab^3 + a^3B - 3ab^2B) \sqrt{\tan(c+dx)}}{d} \\
&= \frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \sqrt{\tan(c+dx)}}{d} \\
&= \frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B)) \sqrt{\tan(c+dx)}}{d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.31, size = 197, normalized size = 0.47

$$\frac{2\left(\frac{105}{2}(a-ib)^4(A+B)\left(\sqrt{-1}\operatorname{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}\right)+\frac{105}{2}(a+ib)^4(-1A+B)\left(\sqrt{-1}\operatorname{tanh}^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}\right)+5b(21aAb+18a^2B-7b^2B)\tan^3(c+dx)+3b^2(7Ab+11aB)\tan^5(c+dx)+15bB\tan^3(c+dx)(a+b\tan(c+dx))^2\right)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] (2\*((105\*(a - I\*b)^3\*(I\*A + B)\*((-1)^(1/4)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + Sqrt[Tan[c + d\*x]]))/2 + (105\*(a + I\*b)^3\*((-I)\*A + B)\*((-1)^(1/4)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + Sqrt[Tan[c + d\*x]]))/2 + 5\*b\*(21\*a\*A\*b + 18\*a^2\*B - 7\*b^2\*B)\*Tan[c + d\*x]^(3/2) + 3\*b^2\*(7\*A\*b + 11\*a\*B)\*Tan[c + d\*x]^(5/2) + 15\*b\*B\*Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2)/(105\*d)

**Maple [A]**

time = 0.05, size = 372, normalized size = 0.88

method	result
derivativedivides	$\frac{2Bb^3\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7} + \frac{2Ab^3\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} + \frac{6Bab^2\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} + 2Aab^2\left(\tan^{\frac{3}{2}}(dx+c)\right) + 2Ba^2b\left(\tan^{\frac{3}{2}}(dx+c)\right) - \frac{2Bb^3}{5}$
default	$\frac{2Bb^3\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7} + \frac{2Ab^3\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} + \frac{6Bab^2\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5} + 2Aab^2\left(\tan^{\frac{3}{2}}(dx+c)\right) + 2Ba^2b\left(\tan^{\frac{3}{2}}(dx+c)\right) - \frac{2Bb^3}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/d*(2/7*B*b^3*tan(d*x+c)^(7/2)+2/5*A*b^3*tan(d*x+c)^(5/2)+6/5*B*a*b^2*tan(
d*x+c)^(5/2)+2*A*a*b^2*tan(d*x+c)^(3/2)+2*B*a^2*b*tan(d*x+c)^(3/2)-2/3*B*b^
3*tan(d*x+c)^(3/2)+6*A*a^2*b*tan(d*x+c)^(1/2)-2*A*b^3*tan(d*x+c)^(1/2)+2*B*
a^3*tan(d*x+c)^(1/2)-6*B*a*b^2*tan(d*x+c)^(1/2)+1/4*(-3*A*a^2*b+A*b^3-B*a^3
+3*B*a*b^2)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*2^(1/2
)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2)))
```

**Maxima [A]**

time = 0.53, size = 363, normalized size = 0.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/420*(120*B*b^3*tan(d*x + c)^(7/2) + 168*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^(
5/2) + 210*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A +
B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 210*sqrt(2)
*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 105*sqrt(2)*((A + B)*a^3 + 3*
(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(sqrt(2)*sqrt(tan(d*x + c
)) + tan(d*x + c) + 1) + 105*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A
+ B)*a*b^2 - (A - B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
1) + 280*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*tan(d*x + c)^(3/2) + 840*(B*a^3 +
3*A*a^2*b - 3*B*a*b^2 - A*b^3)*sqrt(tan(d*x + c))/d
```

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*3\*sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**  
time = 20.83, size = 2500, normalized size = 5.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out] tan(c + d\*x)^(1/2)\*((2\*B\*a^3)/d - (6\*B\*a\*b^2)/d) - tan(c + d\*x)^(1/2)\*((2\*A\*b^3)/d - (6\*A\*a^2\*b)/d) - tan(c + d\*x)^(3/2)\*((2\*B\*b^3)/(3\*d) - (2\*B\*a^2\*b)/d) - atan((((8\*(4\*A\*b^3\*d^2 - 12\*A\*a^2\*b\*d^2))\*((3\*A^2\*a\*b^5)/(2\*d^2) - (5\*A^2\*a^3\*b^3)/d^2 - (30\*A^4\*a^2\*b^10\*d^4 - A^4\*b^12\*d^4 - A^4\*a^12\*d^4 - 25





$$\begin{aligned}
& \left( \frac{3A^2a^5b}{(2d^2)^{1/2}} \right) / d^3 + (16 \tan(c + dx)^{1/2} (A^2a^6 - A^2b^6 + 15A^2a^2b^4 - 15A^2a^4b^2)) / d^2 * ((30A^4a^2b^{10}d^4 - A^4b^{12}d^4 - A^4a^{12}d^4 - 255A^4a^4b^8d^4 + 452A^4a^6b^6d^4 - 255A^4a^8b^4d^4 + 30A^4a^{10}b^2d^4)^{1/2} / (4d^4) - (5A^2a^3b^3) / d^2 + (3A^2ab^5) / (2d^2) + (3A^2a^5b) / (2d^2)^{1/2} * i) / (((8(4Ab^3d^2 - 12Aa^2bd^2) * ((30A^4a^2b^{10}d^4 - A^4b^{12}d^4 - A^4a^{12}d^4 - 255A^4a^4b^8d^4 + 452A^4a^6b^6d^4 - 255A^4a^8b^4d^4 + 30A^4a^{10}b^2d^4)^{1/2} / (4d^4) - (5A^2a^3b^3) / d^2 + (3A^2ab^5) / (2d^2) + (3A^2a^5b) / (2d^2)^{1/2}) / d^3 - (16 \tan(c + dx)^{1/2} (A^2a^6 - A^2b^6 + 15A^2a^2b^4 - 15A^2a^4b^2)) / d^2 * ((30A^4a^2b^{10}d^4 - A^4b^{12}d^4 - A^4a^{12}d^4 - 255A^4a^4b^8d^4 + 452A^4a^6b^6d^4 - 255A^4a^8b^4d^4 + 30A^4a^{10}b^2d^4)^{1/2} / (4d^4) - (5A^2a^3b^3) / d^2 + (3A^2ab^5) / (2d^2) + (3A^2a^5b) / (2d^2)^{1/2}) / d^3 - (16 * (3A^3ab^8 - A^3a^9 + 8A^3a^3b^6 + 6A^3a^5b^4)) / d^3 + ((8(4Ab^3d^2 - 12Aa^2bd^2) * ((30A^4a^2b^{10}d^4 - A^4b^{12}d^4 - A^4a^{12}d^4 - 255A^4a^4b^8d^4 + 452A^4a^6b^6d^4 - 255A^4a^8b^4d^4 + 30A^4a^{10}b^2d^4)^{1/2} / (4d^4) - (5A^2a^3b^3) / d^2 + (3A^2ab^5) / (2d^2) + (3A^2a^5b) / (2d^2)^{1/2}) / d^3 + (16 \tan(c + dx)^{1/2} (A^2a^6 - \dots
\end{aligned}$$

$$3.394 \quad \int \frac{(a+b \tan(c+dx))^3 (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=380

$$\frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) + (3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out] 1/2\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)+a^3\*(A+B)-3\*a\*b^2\*(A+B))\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/2\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)+a^3\*(A+B)-3\*a\*b^2\*(A+B))\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)-1/4\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)-3\*a^2\*b\*(A+B)+b^3\*(A+B))\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)+1/4\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)-3\*a^2\*b\*(A+B)+b^3\*(A+B))\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)+2/5\*b\*(15\*A\*a\*b+14\*B\*a^2-5\*B\*b^2)\*tan(d\*x+c)^(1/2)/d+2/15\*b^2\*(5\*A\*b+9\*B\*a)\*tan(d\*x+c)^(3/2)/d+2/5\*b\*B\*tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^2/d

**Rubi** [A]

time = 0.44, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3688, 3718, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$ ,  $\frac{\operatorname{ArcTan}\left[\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]],x]

[Out] -(((3\*a^2\*b\*(A - B) - b^3\*(A - B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*d)) + ((3\*a^2\*b\*(A - B) - b^3\*(A - B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]/(Sqrt[2]\*d) - ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(2\*Sqrt[2]\*d) + ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]/(2\*Sqrt[2]\*d) + (2\*b\*(15\*a\*A\*b + 14\*a^2\*B - 5\*b^2\*B)\*Sqrt[Tan[c + d\*x]]/(5\*d) + (2\*b^2\*(5\*A\*b + 9\*a\*B)\*Tan[c + d\*x]^(3/2))/(15\*d) + (2\*b\*B\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^2)/(5\*d)

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3688

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
```

```
[e + f*x]^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rule 3718

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Simp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && !LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^3 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2}{5d} + \frac{2}{5} \int \frac{(a + b \tan(c + dx))^3 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx \\
&= \frac{2b^2(5Ab + 9aB) \tan^{\frac{3}{2}}(c + dx)}{15d} + \frac{2bB \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2}{5d} \\
&= \frac{2b(15aAb + 14a^2B - 5b^2B) \sqrt{\tan(c + dx)}}{5d} + \frac{2b^2(5Ab + 9aB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{2b(15aAb + 14a^2B - 5b^2B) \sqrt{\tan(c + dx)}}{5d} + \frac{2b^2(5Ab + 9aB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{2b(15aAb + 14a^2B - 5b^2B) \sqrt{\tan(c + dx)}}{5d} + \frac{2b^2(5Ab + 9aB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{2b(15aAb + 14a^2B - 5b^2B) \sqrt{\tan(c + dx)}}{5d} + \frac{2b^2(5Ab + 9aB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{2b(15aAb + 14a^2B - 5b^2B) \sqrt{\tan(c + dx)}}{5d} + \frac{2b^2(5Ab + 9aB) \tan^{\frac{3}{2}}(c + dx)}{15d} \\
&= \frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B))}{2\sqrt{2}d} \\
&= \frac{(3a^2b(A - B) - b^3(A - B) + a^3(A + B) - 3ab^2(A + B))}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.95, size = 153, normalized size = 0.40

$$\frac{-15\sqrt{-1}(a - ib)^3(A - iB)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c + dx)}\right) - 15\sqrt{-1}(a + ib)^3(A + iB)\text{tanh}^{-1}\left((-1)^{3/4}\sqrt{\tan(c + dx)}\right) + 2b\sqrt{\tan(c + dx)}(15(3aAb + 3a^2B - b^2B) + 5b(Ab + 3aB)\tan(c + dx) + 3b^2B\tan^2(c + dx))}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

[Out] (-15\*(-1)^(1/4)\*(a - I\*b)^3\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - 15\*(-1)^(1/4)\*(a + I\*b)^3\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*b\*Sqrt[Tan[c + d\*x]]\*(15\*(3\*a\*A\*b + 3\*a^2\*B - b^2\*B) + 5\*b\*(A\*b + 3\*a\*B)\*Tan[c + d\*x] + 3\*b^2\*B\*Tan[c + d\*x]^2))/(15\*d)

**Maple [A]**

time = 0.05, size = 314, normalized size = 0.83

method	result
--------	--------

derivativedivides	$\frac{2Bb^3 \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ab^3 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2Bab^2 \left( \tan^{\frac{3}{2}}(dx+c) \right) + 6Aab^2 \left( \sqrt{\tan}(dx+c) \right) + 6Ba^2b \left( \sqrt{\tan}(dx+c) \right)$
default	$\frac{2Bb^3 \left( \tan^{\frac{5}{2}}(dx+c) \right)}{5} + \frac{2Ab^3 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2Bab^2 \left( \tan^{\frac{3}{2}}(dx+c) \right) + 6Aab^2 \left( \sqrt{\tan}(dx+c) \right) + 6Ba^2b \left( \sqrt{\tan}(dx+c) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*(2/5*B*b^3*\tan(d*x+c)^{(5/2)}+2/3*A*b^3*\tan(d*x+c)^{(3/2)}+2*B*a*b^2*\tan(d*x+c)^{(3/2)}+6*A*a*b^2*\tan(d*x+c)^{(1/2)}+6*B*a^2*b*\tan(d*x+c)^{(1/2)}-2*B*b^3*\tan(d*x+c)^{(1/2)}+1/4*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*2^{(1/2)}*(\ln((1+2^{(1/2)})*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [A]

time = 0.51, size = 327, normalized size = 0.86

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $1/60*(24*B*b^3*\tan(d*x+c)^{(5/2)}+30*\sqrt{2}*((A+B)*a^3+3*(A-B)*a^2*b-3*(A+B)*a*b^2-(A-B)*b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(d*x+c)}))+30*\sqrt{2}*((A+B)*a^3+3*(A-B)*a^2*b-3*(A+B)*a*b^2-(A-B)*b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(d*x+c)}))+15*\sqrt{2}*((A-B)*a^3-3*(A+B)*a^2*b-3*(A-B)*a*b^2+(A+B)*b^3)*\log(\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1)-15*\sqrt{2}*((A-B)*a^3-3*(A+B)*a^2*b-3*(A-B)*a*b^2+(A+B)*b^3)*\log(-\sqrt{2}*\sqrt{\tan(d*x+c)}+\tan(d*x+c)+1)+40*(3*B*a*b^2+A*b^3)*\tan(d*x+c)^{(3/2)}+120*(3*B*a^2*b+3*A*a*b^2-B*b^3)*\sqrt{\tan(d*x+c)}/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^3}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*3/sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.98, size = 2500, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3)/tan(c + d\*x)^(1/2),x)

[Out] atan((((8\*(4\*A\*a^3\*d^2 - 12\*A\*a\*b^2\*d^2)\*((5\*A^2\*a^3\*b^3)/d^2 - (30\*A^4\*a^2\*b^10\*d^4 - A^4\*b^12\*d^4 - A^4\*a^12\*d^4 - 255\*A^4\*a^4\*b^8\*d^4 + 452\*A^4\*a^6\*b^6\*d^4 - 255\*A^4\*a^8\*b^4\*d^4 + 30\*A^4\*a^10\*b^2\*d^4)^(1/2)/(4\*d^4) - (3\*A^2\*a\*b^5)/(2\*d^2) - (3\*A^2\*a^5\*b)/(2\*d^2))^(1/2))/d^3 - (16\*tan(c + d\*x)^(1/2)\*(A^2\*a^6 - A^2\*b^6 + 15\*A^2\*a^2\*b^4 - 15\*A^2\*a^4\*b^2))/d^2)\*((5\*A^2\*a^3\*b^3)/d^2 - (30\*A^4\*a^2\*b^10\*d^4 - A^4\*b^12\*d^4 - A^4\*a^12\*d^4 - 255\*A^4\*a^4\*b^8\*d^4 + 452\*A^4\*a^6\*b^6\*d^4 - 255\*A^4\*a^8\*b^4\*d^4 + 30\*A^4\*a^10\*b^2\*d^4)^(1/2)/(4\*d^4) - (3\*A^2\*a\*b^5)/(2\*d^2) - (3\*A^2\*a^5\*b)/(2\*d^2))^(1/2)\*1i - ((8\*(4\*A\*a^3\*d^2 - 12\*A\*a\*b^2\*d^2)\*((5\*A^2\*a^3\*b^3)/d^2 - (30\*A^4\*a^2\*b^10\*





$$\begin{aligned}
& A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - \\
& 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} / (4 d^4) + (5 A^2 a^3 b^3) / \\
& d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{(1/2)} / d^3 - (16 \tan(c \\
& + d x)^{(1/2)} (A^2 a^6 - A^2 b^6 + 15 A^2 a^2 b^4 - 15 A^2 a^4 b^2)) / d^2 * ( \\
& (30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + \\
& 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} / (4 d \\
& ^4) + (5 A^2 a^3 b^3) / d^2 - (3 A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{(1/2)} \\
& + ((8 (4 A a^3 d^2 - 12 A a b^2 d^2) * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} \\
& * d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a \\
& ^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} / (4 d^4) + (5 A^2 a^3 b^3) / d^2 - (3 \\
& A^2 a b^5) / (2 d^2) - (3 A^2 a^5 b) / (2 d^2))^{(1/2)} / d^3 + (16 \tan(c + d x)^{(1/2)} * (A^2 a^6 - A^2 b^6 + 15 A^2 a^2 b^4 - 15 A^2 a^4 b^2)) / d^2) * ((30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 \dots
\end{aligned}$$

$$3.395 \quad \int \frac{(a+b \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^3(c+dx)} dx$$

**Optimal.** Leaf size=374

$$\frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) (a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B))}{\sqrt{2} d}$$

[Out]  $-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2*b*(2*A*a^2+A*b^2+3*B*a*b)*\tan(d*x+c)^{(1/2)}/d+2/3*b^2*(3*A*a+B*b)*\tan(d*x+c)^{(3/2)}/d-2*a*A*(a+b*\tan(d*x+c))^2/d/\tan(d*x+c)^{(1/2)}$

**Rubi** [A]

time = 0.43, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3686, 3718, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{3ab^2(a+b)\sqrt{2}\sqrt{\tan(c+dx)}}{d\sqrt{2}}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}, \frac{a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)}{d}$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2),x]

[Out]  $((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - ((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B)) * \operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]] / (2*\operatorname{Sqrt}[2]*d) + ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B)) * \operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]] / (2*\operatorname{Sqrt}[2]*d) + (2*b*(2*a^2*A + A*b^2 + 3*a*b*B) * \operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / d + (2*b^2*(3*a*A + b*B) * \operatorname{Tan}[c + d*x]^(3/2)) / (3*d) - (2*a*A*(a + b*\operatorname{Tan}[c + d*x])^2) / (d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

Rule 210

$\operatorname{Int}[(a + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1} * \operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
```

```

Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

```

### Rule 3711

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

```

### Rule 3718

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^2}{d \sqrt{\tan(c + dx)}} + 2 \int \frac{(a + b \tan(c + dx)) (\frac{1}{2}a(3 \\
&= \frac{2b^2(3aA + bB) \tan^{\frac{3}{2}}(c + dx)}{3d} - \frac{2aA(a + b \tan(c + dx))^2}{d \sqrt{\tan(c + dx)}} \\
&= \frac{2b(2a^2A + Ab^2 + 3abB) \sqrt{\tan(c + dx)}}{d} + \frac{2b^2(3aA + bB)}{3d} \\
&= \frac{2b(2a^2A + Ab^2 + 3abB) \sqrt{\tan(c + dx)}}{d} + \frac{2b^2(3aA + bB)}{3d} \\
&= \frac{2b(2a^2A + Ab^2 + 3abB) \sqrt{\tan(c + dx)}}{d} + \frac{2b^2(3aA + bB)}{3d} \\
&= \frac{2b(2a^2A + Ab^2 + 3abB) \sqrt{\tan(c + dx)}}{d} + \frac{2b^2(3aA + bB)}{3d} \\
&= \frac{(3a^2b(A - B) - b^3(A - B) + a^3(A + B) - 3ab^2(A + B))}{2\sqrt{2}d} \\
&= \frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B))}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.71, size = 264, normalized size = 0.71

$$\frac{8(-12a^2b - 17a^2B + 3b^2B) - 3(8(a^3A - 3a^2Ab^2 - 3a^2b^2B) + b^3B) \operatorname{Hypergeometric2F1}[-1/4, 1, 3/4, -\tan(c + dx)] + \sqrt{2}(3a^2A - Ab^2 + a^2B - 3abB) \left( 2 \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) - 2 \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right) + \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) - \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) \right) \sqrt{\tan(c + dx)} + 8(3aA + 7aB)(a + b \tan(c + dx)) + 8B(a + b \tan(c + dx))^2}{12\sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out] (8\*b\*(-12\*a\*A\*b - 17\*a^2\*B + 3\*b^2\*B) - 3\*(8\*(a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2] + Sqrt[2]\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]] + 8\*b\*(3\*A\*b + 7\*a\*B)\*(a + b\*Tan[c + d\*x]) + 8\*b\*B\*(a + b\*Tan[c + d\*x])^2)/(12\*d\*Sqrt[Tan[c + d\*x]])

**Maple [A]**

time = 0.05, size = 286, normalized size = 0.76

method	result
derivativedivides	$\frac{2B b^3 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2A b^3 \left( \sqrt{\tan(dx+c)} \right) + 6B a b^2 \left( \sqrt{\tan(dx+c)} \right) - \frac{2A a^3}{\sqrt{\tan(dx+c)}} + \frac{(3A a^2 b - A b^3 + B a^3)}{\sqrt{\tan(dx+c)}}$
default	$\frac{2B b^3 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} + 2A b^3 \left( \sqrt{\tan(dx+c)} \right) + 6B a b^2 \left( \sqrt{\tan(dx+c)} \right) - \frac{2A a^3}{\sqrt{\tan(dx+c)}} + \frac{(3A a^2 b - A b^3 + B a^3)}{\sqrt{\tan(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*(2/3\*B\*b^3\*tan(d\*x+c)^(3/2)+2\*A\*b^3\*tan(d\*x+c)^(1/2)+6\*B\*a\*b^2\*tan(d\*x+c)^(1/2)-2\*A\*a^3/tan(d\*x+c)^(1/2)+1/4\*(3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/4\*(-A\*a^3+3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))

**Maxima [A]**

time = 0.50, size = 310, normalized size = 0.83

1/12\*(8\*B\*b^3\*tan(d\*x+c)^(3/2)-24\*A\*a^3/sqrt(tan(d\*x+c))-6\*sqrt(2)\*((A-B)\*a^3-3\*(A+B)\*a^2\*b-3\*(A-B)\*a\*b^2+(A+B)\*b^3)\*arctan(1/2\*sqrt(2)\*(sqrt(2)+2\*sqrt(tan(d\*x+c))))-6\*sqrt(2)\*((A-B)\*a^3-3\*(A+B)\*a^2\*b-3\*(A-B)\*a\*b^2+(A+B)\*b^3)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)-2\*sqrt(tan(d\*x+c))))+3\*sqrt(2)\*((A+B)\*a^3+3\*(A-B)\*a^2\*b-3\*(A+B)\*a\*b^2-(A-B)\*b^3)\*log(sqrt(2)\*sqrt(tan(d\*x+c))+tan(d\*x+c)+1)-3\*sqrt(2)\*((A+B)\*a^3+3\*(A-B)\*a^2\*b-3\*(A+B)\*a\*b^2-(A-B)\*b^3)\*log(-sqrt(2)\*sqrt(tan(d\*x+c))+tan(d\*x+c)+1)+24\*(3\*B\*a\*b^2+A\*b^3)\*sqrt(tan(d\*x+c)))/d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm  
="maxima")

[Out] 1/12\*(8\*B\*b^3\*tan(d\*x+c)^(3/2)-24\*A\*a^3/sqrt(tan(d\*x+c))-6\*sqrt(2)\*((A-B)\*a^3-3\*(A+B)\*a^2\*b-3\*(A-B)\*a\*b^2+(A+B)\*b^3)\*arctan(1/2\*sqrt(2)\*(sqrt(2)+2\*sqrt(tan(d\*x+c))))-6\*sqrt(2)\*((A-B)\*a^3-3\*(A+B)\*a^2\*b-3\*(A-B)\*a\*b^2+(A+B)\*b^3)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)-2\*sqrt(tan(d\*x+c))))+3\*sqrt(2)\*((A+B)\*a^3+3\*(A-B)\*a^2\*b-3\*(A+B)\*a\*b^2-(A-B)\*b^3)\*log(sqrt(2)\*sqrt(tan(d\*x+c))+tan(d\*x+c)+1)-3\*sqrt(2)\*((A+B)\*a^3+3\*(A-B)\*a^2\*b-3\*(A+B)\*a\*b^2-(A-B)\*b^3)\*log(-sqrt(2)\*sqrt(tan(d\*x+c))+tan(d\*x+c)+1)+24\*(3\*B\*a\*b^2+A\*b^3)\*sqrt(tan(d\*x+c))/d

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^3}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*3/tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**  
time = 9.93, size = 2500, normalized size = 6.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3)/tan(c + d\*x)^(3/2),x)

[Out] atan((((8\*(4\*B\*a^3\*d^2 - 12\*B\*a\*b^2\*d^2)\*((5\*B^2\*a^3\*b^3)/d^2 - (30\*B^4\*a^2\*b^10\*d^4 - B^4\*b^12\*d^4 - B^4\*a^12\*d^4 - 255\*B^4\*a^4\*b^8\*d^4 + 452\*B^4\*a^6\*b^6\*d^4 - 255\*B^4\*a^8\*b^4\*d^4 + 30\*B^4\*a^10\*b^2\*d^4)^(1/2)/(4\*d^4) - (3\*B^2\*a\*b^5)/(2\*d^2) - (3\*B^2\*a^5\*b)/(2\*d^2))^(1/2))/d^3 - (16\*tan(c + d\*x)^(1/2)





$$\begin{aligned}
& A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - \\
& 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} + 16 A^3 a^9 d^2 - 736 A^3 \\
& a^3 b^6 d^2 + 960 A^3 a^5 b^4 d^2 - 288 A^3 a^7 b^2 d^2 - 48 A a^2 b (30 A \\
& ^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A \\
& ^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} + 48 A^3 a \\
& a b^8 d^2) + (A^2 a^2 b^4 d^3 \tan(c + d x))^{(1/2)} * ((30 A^4 a^2 b^{10} d^4 - A^ \\
& 4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 \\
& * A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} / (4 d^4) - (5 A^2 a^3 b^3) / d^2 \\
& + (3 A^2 a b^5) / (2 d^2) + (3 A^2 a^5 b) / (2 d^2))^{(1/2)} * 480 i) / (16 A b^3 (30 \\
& * A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 \\
& * A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} + 16 A^ \\
& 3 a^9 d^2 - 736 A^3 a^3 b^6 d^2 + 960 A^3 a^5 b^4 d^2 - 288 A^3 a^7 b^2 d^2 \\
& - 48 A a^2 b (30 A^4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^ \\
& a^4 b^8 d^4 + 452 A^4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^ \\
& ^4)^{(1/2)} + 48 A^3 a b^8 d^2) - (A^2 a^4 b^2 d^3 \tan(c + d x))^{(1/2)} * ((30 A^ \\
& 4 a^2 b^{10} d^4 - A^4 b^{12} d^4 - A^4 a^{12} d^4 - 255 A^4 a^4 b^8 d^4 + 452 A^ \\
& 4 a^6 b^6 d^4 - 255 A^4 a^8 b^4 d^4 + 30 A^4 a^{10} b^2 d^4)^{(1/2)} / (4 d^4) - \\
& (5 A^2 a^3 b^3) / d^2 + (3 A^2 a b^5) / (2 d^2) + (\dots
\end{aligned}$$



```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
```

```

Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

```

### Rule 3711

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

```

### Rule 3716

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^2}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{(a + b \tan(c + dx)) (\frac{1}{2}a)}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(7Ab + 3aB)}{3d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^2}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{-\frac{1}{2}a}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(7Ab + 3aB)}{3d \sqrt{\tan(c + dx)}} + \frac{2b^2(aA + 3bB) \sqrt{\tan(c + dx)}}{3d} - \frac{2a}{3} \int \frac{1}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(7Ab + 3aB)}{3d \sqrt{\tan(c + dx)}} + \frac{2b^2(aA + 3bB) \sqrt{\tan(c + dx)}}{3d} - \frac{2a}{3} \int \frac{1}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(7Ab + 3aB)}{3d \sqrt{\tan(c + dx)}} + \frac{2b^2(aA + 3bB) \sqrt{\tan(c + dx)}}{3d} - \frac{2a}{3} \int \frac{1}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(7Ab + 3aB)}{3d \sqrt{\tan(c + dx)}} + \frac{2b^2(aA + 3bB) \sqrt{\tan(c + dx)}}{3d} - \frac{2a}{3} \int \frac{1}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B)) \sqrt{\tan(c + dx)}}{2\sqrt{2} d} \\
&= \frac{(3a^2b(A - B) - b^3(A - B) + a^3(A + B) - 3ab^2(A + B)) \sqrt{\tan(c + dx)}}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.89, size = 165, normalized size = 0.44

$$\frac{2((a^3A - 3aAb^2 - 3a^2bB + b^3B) {}_2F_1(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)) + 3(3a^2Ab - Ab^3 + a^3B - 3ab^2B) {}_2F_1(-\frac{1}{4}, 1; \frac{3}{4}; -\tan^2(c + dx)) \tan(c + dx) + b(3aAb + 3a^2B - b^2B + 3b(Ab + 3aB) \tan(c + dx) - 3b^2B \tan^2(c + dx)))}{3d \tan^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (-2\*((a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d\*x]^2] + 3\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x] + b\*(3\*a\*A\*b + 3\*a^2\*B - b^2\*B + 3\*b\*(A\*b + 3\*a\*B)\*Tan[c + d\*x] - 3\*b^2\*B\*Tan[c + d\*x]^2))/(3\*d\*Tan[c + d\*x]^(3/2))

**Maple [A]**

time = 0.06, size = 278, normalized size = 0.75

method	result
derivativedivides	$\frac{-\frac{2Aa^3}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{2a^2(3Ab+aB)}{\sqrt{\tan(dx+c)}}+2Bb^3\left(\sqrt{\tan(dx+c)}\right)+\frac{(-Aa^3+3Aab^2+3Ba^2b-Bb^3)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{1+\sqrt{\tan(dx+c)}}{1-\sqrt{\tan(dx+c)}}\right)\right)\right)}{\sqrt{\tan(dx+c)}}}{\sqrt{\tan(dx+c)}}$
default	$\frac{-\frac{2Aa^3}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{2a^2(3Ab+aB)}{\sqrt{\tan(dx+c)}}+2Bb^3\left(\sqrt{\tan(dx+c)}\right)+\frac{(-Aa^3+3Aab^2+3Ba^2b-Bb^3)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\left(\frac{1+\sqrt{\tan(dx+c)}}{1-\sqrt{\tan(dx+c)}}\right)\right)\right)}{\sqrt{\tan(dx+c)}}}{\sqrt{\tan(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/d*(-2/3*A*a^3/tan(d*x+c)^(3/2)-2*a^2*(3*A*b+B*a)/tan(d*x+c)^(1/2)+2*B*b^3
*tan(d*x+c)^(1/2)+1/4*(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*2^(1/2)*(ln((1+2^(
1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
+1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 310, normalized size = 0.83

$\frac{24B^2\sqrt{2}\sqrt{\tan(dx+c)}-6\sqrt{2}(A+B)^2(a^3+3Aa^2b-3(A-B)a^2b-3(A+B)a^2b-3(A-B)b^3)\arctan(1/2\sqrt{2}(\sqrt{2}+\sqrt{\tan(dx+c)})-6\sqrt{2}(A+B)a^3+3(A-B)a^2b-3(A+B)a^2b-3(A-B)b^3)\arctan(-1/2\sqrt{2}(\sqrt{2}-\sqrt{\tan(dx+c)})-3\sqrt{2}(A-B)a^3-3(A+B)a^2b-3(A-B)a^2b+3(A+B)b^3)\log(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+3\sqrt{2}(A-B)a^3-3(A+B)a^2b-3(A-B)a^2b+3(A+B)b^3)\log(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1)-8(Aa^3+3(Ba^3+3Aa^2b)\tan(dx+c))/\tan(dx+c)^{3/2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] 1/12*(24*B*b^3*sqrt(tan(d*x + c)) - 6*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*
b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan
(d*x + c)))) - 6*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 -
(A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 3*sqr
t(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(sq
rt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 3*sqrt(2)*((A - B)*a^3 - 3*(
A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c
)) + tan(d*x + c) + 1) - 8*(A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/tan
(d*x + c)^(3/2))/d
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^3}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*3*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*3/tan(c + d*x)**(5/2), x)
```

**Giac [F(-1)]** Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 10.15, size = 2500, normalized size = 6.72
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/tan(c + d*x)^(5/2),x)
```

```
[Out] 2*atanh(((32*A^2*a^6*d^3*tan(c + d*x)^(1/2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2))/(4*d^4) - (3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2))/(16*A*a^3*(30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2) + 16*A^3*b^9*d^2 - 288*A^3*a^2*b^7*d^2 + 960*A^3*a^4*b^5*d^2 - 736*A^3*a^6*b^3*d^2 - 48*A*a*b^2*(30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2) +
```







$$3.397 \quad \int \frac{(a+b \tan(c+dx))^3 (A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

**Optimal.** Leaf size=380

$$\frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2/5*a*(5*A*a^2-14*A*b^2-15*B*a*b)/d/\tan(d*x+c)^{(1/2)}-2/15*a^2*(9*A*b+5*B*a)/d/\tan(d*x+c)^{(3/2)}-2/5*a*A*(a+b*\tan(d*x+c))^2/d/\tan(d*x+c)^{(5/2)}$

**Rubi [A]**

time = 0.40, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3686, 3716, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right]}{2\sqrt{2} d} - \frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right]}{2\sqrt{2} d} - \frac{(2a^2(9Ab + 5aB))}{15d \tan(c+dx)^{(3/2)}} + \frac{(2a(5a^2A - 14Ab^2 - 15a*b*B))}{5d \sqrt{\tan(c+dx)}} - \frac{(2aA(a + b \tan(c+dx))^2)}{5d \tan(c+dx)^{(5/2)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Tan}[c + d*x]^{(7/2)}), x]$

[Out]  $-(((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/( \operatorname{Sqrt}[2]*d) + ((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/( \operatorname{Sqrt}[2]*d) + ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (2*a^2*(9*A*b + 5*a*B))/(15*d*\operatorname{Tan}[c + d*x]^{(3/2)}) + (2*a*(5*a^2*A - 14*A*b^2 - 15*a*b*B))/(5*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) - (2*a*A*(a + b*\operatorname{Tan}[c + d*x])^2)/(5*d*\operatorname{Tan}[c + d*x]^{(5/2)})$

**Rule 210**

$\operatorname{Int}[(a + b*x)^2*(-1), x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
)*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
```

```

Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

```

### Rule 3709

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

```

### Rule 3716

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(a + b \tan(c + dx)) (\frac{1}{2}a)}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(9Ab + 5aB)}{15d \tan^{\frac{3}{2}}(c + dx)} - \frac{2aA(a + b \tan(c + dx))^2}{5d \tan^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{-\frac{1}{2}a}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2a^2(9Ab + 5aB)}{15d \tan^{\frac{3}{2}}(c + dx)} + \frac{2a(5a^2A - 14Ab^2 - 15abB)}{5d \sqrt{\tan(c + dx)}} - \frac{2aA}{5d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2(9Ab + 5aB)}{15d \tan^{\frac{3}{2}}(c + dx)} + \frac{2a(5a^2A - 14Ab^2 - 15abB)}{5d \sqrt{\tan(c + dx)}} - \frac{2aA}{5d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2(9Ab + 5aB)}{15d \tan^{\frac{3}{2}}(c + dx)} + \frac{2a(5a^2A - 14Ab^2 - 15abB)}{5d \sqrt{\tan(c + dx)}} - \frac{2aA}{5d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2(9Ab + 5aB)}{15d \tan^{\frac{3}{2}}(c + dx)} + \frac{2a(5a^2A - 14Ab^2 - 15abB)}{5d \sqrt{\tan(c + dx)}} - \frac{2aA}{5d \sqrt{\tan(c + dx)}} \\
&= \frac{(3a^2b(A - B) - b^3(A - B) + a^3(A + B) - 3ab^2(A + B))}{2\sqrt{2}d} \\
&= -\frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B))}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.88, size = 166, normalized size = 0.44

$$\frac{2(3(a^3A - 3aAb^2 - 3a^2bB + b^3B) {}_2F_1(-\frac{5}{4}, 1; -\frac{1}{4}; -\tan^2(c + dx)) + 5(3a^2Ab - Ab^3 + a^2B - 3ab^2B) {}_2F_1(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)) \tan(c + dx) + b(9aAb + 9a^2B - 3b^2B + 5b(Ab + 3aB) \tan(c + dx) + 15b^2B \tan^2(c + dx)))}{15d \tan^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] (-2\*(3\*(a^3\*A - 3\*a\*A\*b^2 - 3\*a^2\*b\*B + b^3\*B)\*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d\*x]^2] + 5\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x] + b\*(9\*a\*A\*b + 9\*a^2\*B - 3\*b^2\*B + 5\*b\*(A\*b + 3\*a\*B)\*Tan[c + d\*x] + 15\*b^2\*B\*Tan[c + d\*x]^2))/(15\*d\*Tan[c + d\*x]^(5/2))

**Maple [A]**

time = 0.06, size = 290, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{2Aa^3}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2a^2(3Ab+aB)}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{2a(a^2A-3Ab^2-3Bab)}{\sqrt{\tan(dx+c)}}}{(-3Aa^2b+Ab^3-Ba^3+3Bab^2)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)} \right) \right)}$
default	$\frac{-\frac{2Aa^3}{5 \tan(dx+c)^{\frac{5}{2}}} - \frac{2a^2(3Ab+aB)}{3 \tan(dx+c)^{\frac{3}{2}}} + \frac{2a(a^2A-3Ab^2-3Bab)}{\sqrt{\tan(dx+c)}}}{(-3Aa^2b+Ab^3-Ba^3+3Bab^2)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right)} \right) \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNVE  
RBOSE)`

[Out]  $1/d*(-2/5*A*a^3/\tan(d*x+c)^{(5/2)}-2/3*a^2*(3*A*b+B*a)/\tan(d*x+c)^{(3/2)}+2*a*(A*a^2-3*A*b^2-3*B*a*b)/\tan(d*x+c)^{(1/2)}+1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/4*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima [A]**

time = 0.51, size = 327, normalized size = 0.86

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm  
="maxima")`

[Out]  $1/60*(30*\sqrt{2}*((A-B)*a^3-3*(A+B)*a^2*b-3*(A-B)*a*b^2+(A+B)*b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)}))+30*\sqrt{2}*((A-B)*a^3-3*(A+B)*a^2*b-3*(A-B)*a*b^2+(A+B)*b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)}))-15*\sqrt{2}*((A+B)*a^3+3*(A-B)*a^2*b-3*(A+B)*a*b^2-(A-B)*b^3)*\log(\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+15*\sqrt{2}*((A+B)*a^3+3*(A-B)*a^2*b-3*(A+B)*a*b^2-(A-B)*b^3)*\log(-\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)-8*(3*A*a^3-15*(A*a^3-3*B*a^2*b-3*A*a*b^2)*\tan(dx+c)^2+5*(B*a^3+3*A*a^2*b)*\tan(dx+c))/\tan(dx+c)^{(5/2)}/d$

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^3}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*3/tan(c + d\*x)\*\*(7/2), x)

**Giac [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**  
time = 12.20, size = 2500, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3)/tan(c + d\*x)^(7/2),x)

[Out] 2\*atanh(((32\*B^2\*a^6\*d^3\*tan(c + d\*x)^(1/2))\*((5\*B^2\*a^3\*b^3)/d^2 - (30\*B^4\*a^2\*b^10\*d^4 - B^4\*b^12\*d^4 - B^4\*a^12\*d^4 - 255\*B^4\*a^4\*b^8\*d^4 + 452\*B^4\*a^6\*b^6\*d^4 - 255\*B^4\*a^8\*b^4\*d^4 + 30\*B^4\*a^10\*b^2\*d^4)^(1/2))/(4\*d^4) - (3\*B^2\*a\*b^5)/(2\*d^2) - (3\*B^2\*a^5\*b)/(2\*d^2))^(1/2))/(16\*B^3\*b^9\*d^2 + 16\*B\*a



$$\begin{aligned}
& ^3(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 \\
& + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)} - \\
& 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - 736*B^3*a^6*b^3*d^2 + 48*B^3*a^8*b*d^2 - 48*B*a*b^2*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - \\
& 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2))} - (32*B^2*b^6*d^3*\tan(c + d*x)^{(1/2)}*((5*B^2*a^3*b^3)/d^2 - \\
& (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)})/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^{(1/2)})/(16*B^3*b^9 \\
& *d^2 + 16*B*a^3*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2 \\
& *d^4)^{(1/2)} - 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - 736*B^3*a^6*b^3*d^2 + 48*B^3*a^8*b*d^2 - 48*B*a*b^2*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4 \\
& a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2))} + (480*B^2*a^2*b^4*d^3*\tan(c + d*x)^{(1/2)}*((5*B^2*a^3*b^3)/d^2 - \\
& (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)})/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^{(1/2)})/(16*B^3*b^9*d^2 + 16*B*a^3*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4 \\
& a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)} - 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - \\
& 736*B^3*a^6*b^3*d^2 + 48*B^3*a^8*b*d^2 - 48*B*a*b^2*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - \\
& 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2))} - (480*B^2*a^2*b^4*d^3*\tan(c + d*x)^{(1/2)}*((5*B^2*a^3*b^3)/d^2 - (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 \\
& - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)})/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2 \\
& a^5*b)/(2*d^2))^{(1/2)})/(16*B^3*b^9*d^2 + 16*B*a^3*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 2 \\
& 55*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)} - 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - 736*B^3*a^6*b^3*d^2 + 48*B^3*a^8*b*d^2 - 48*B*a*b^2*(30* \\
& B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2))}*((5*B^2 \\
& a^3*b^3)/d^2 - (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2 \\
& *d^4)^{(1/2)})/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^{(1/2)} + 2*\operatorname{atanh}((32*B^2*a^6*d^3*\tan(c + d*x)^{(1/2)}*((30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4 \\
& a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)})/(4*d^4) + (5*B^2*a^3*b^3)/d^2 - \\
& (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^{(1/2)})/(16*B^3*b^9*d^2 - 16* \\
& B*a^3*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^{(1/2)} \\
& ) - 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - 736*B^3*a^6*b^3*d^2 + 48*B^3 \\
& a^8*b*d^2 + 48*B*a*b^2*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 \\
& - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4
\end{aligned}$$



$$3.398 \quad \int \frac{\tan^5(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=325

$$\frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $-2*a^{5/2}*(A*b-B*a)*\arctan(b^{1/2}*\tan(d*x+c)^{1/2}/a^{1/2})/b^{5/2}/(a^2+b^2)/d-1/2*(a*(A-B)+b*(A+B))*\arctan(-1+2^{1/2}*\tan(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}-1/2*(a*(A-B)+b*(A+B))*\arctan(1+2^{1/2}*\tan(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+1/4*(b*(A-B)-a*(A+B))*\ln(1-2^{1/2}*\tan(d*x+c)^{1/2}+\tan(d*x+c))/(a^2+b^2)/d*2^{1/2}-1/4*(b*(A-B)-a*(A+B))*\ln(1+2^{1/2}*\tan(d*x+c)^{1/2}+\tan(d*x+c))/(a^2+b^2)/d*2^{1/2}+2*(A*b-B*a)*\tan(d*x+c)^{1/2}/b^2/d+2/3*B*\tan(d*x+c)^{3/2}/b/d$

**Rubi [A]**

time = 0.64, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3688, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(b(A-B) - a(A+B)) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d (a^2 + b^2)}\right)}{2\sqrt{2} d (a^2 + b^2)} - \frac{(b(A-B) - a(A+B)) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d (a^2 + b^2)}\right)}{2\sqrt{2} d (a^2 + b^2)} - \frac{2a^{5/2}(Ab - aB) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2}}\right)}{b^{5/2} d (a^2 + b^2)} + \frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{b d} + \frac{2B \tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out]  $((a*(A - B) + b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - (2*a^{5/2}*(A*b - a*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a]] / (b^{5/2}*(a^2 + b^2)*d) + ((b*(A - B) - a*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((b*(A - B) - a*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*(A*b - a*B))*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] / (b^2*d) + (2*B*\operatorname{Tan}[c + d*x]^{3/2}) / (3*b*d)$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 210**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3688

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3728

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)])^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3734

```
Int[(((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  
!GtQ[n, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx &= \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{2 \int \frac{\sqrt{\tan(c+dx)} \left(-\frac{3aB}{2} - \frac{3}{2}bB \tan(c+dx) + \frac{3}{2}(Ab-aB)\right)}{a+b \tan(c+dx)}}{3b} \\
 &= \frac{2(Ab-aB) \sqrt{\tan(c+dx)}}{b^2d} + \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{4 \int \frac{-\frac{3}{4}a(Ab-aB) - \dots}{\dots}}{\dots} \\
 &= \frac{2(Ab-aB) \sqrt{\tan(c+dx)}}{b^2d} + \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{4 \int \frac{-\frac{3}{4}b^2(Ab-aB) \dots}{\sqrt{\tan(c+dx)}}}{3b^2} \\
 &= \frac{2(Ab-aB) \sqrt{\tan(c+dx)}}{b^2d} + \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3bd} + \frac{8 \text{Subst}\left(\int \frac{-\frac{3}{4}b^2 \dots}{\dots}\right)}{\dots} \\
 &= \frac{2(Ab-aB) \sqrt{\tan(c+dx)}}{b^2d} + \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3bd} - \frac{(2a^3(Ab-aB))}{\dots} \\
 &= -\frac{2a^{5/2}(Ab-aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)d} + \frac{2(Ab-aB) \sqrt{\tan(c+dx)}}{b^2d} \\
 &= -\frac{2a^{5/2}(Ab-aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)d} + \frac{(b(A-B) - a)}{\dots} \\
 &= \frac{(a(A-B) + b(A+B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(a(A-B) + b(A+B))}{\dots}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.75, size = 187, normalized size = 0.58

$$\frac{-3\sqrt{-1}(a+ib)b^{5/2}(iA+B)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 6a^{5/2}(-Ab+aB)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + 3\sqrt{-1}b^{5/2}(ia+b)(A+iB)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 2\sqrt{b}(a^2+b^2)\sqrt{\tan(c+dx)}(3Ab-3aB+bB \tan(c+dx))}{3b^{5/2}(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] (-3\*(-1)^(1/4)\*(a + I\*b)\*b^(5/2)\*(I\*A + B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 6\*a^(5/2)\*(-A\*b) + a\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a

```
]] + 3*(-1)^(1/4)*b^(5/2)*(I*a + b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c
+ d*x]]] + 2*Sqrt[b]*(a^2 + b^2)*Sqrt[Tan[c + d*x]]*(3*A*b - 3*a*B + b*B*T
an[c + d*x]))/(3*b^(5/2)*(a^2 + b^2)*d)
```

**Maple [A]**

time = 0.10, size = 292, normalized size = 0.90

method	result
derivativedivides	$\frac{2Bb \left( \tan^{\frac{3}{2}}(dx+c) \right) + 2Ab \left( \sqrt{\tan(dx+c)} \right) - 2Ba \left( \sqrt{\tan(dx+c)} \right)}{b^2} + \frac{(-Ab+aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) \right)}{b^2}$
default	$\frac{2Bb \left( \tan^{\frac{3}{2}}(dx+c) \right) + 2Ab \left( \sqrt{\tan(dx+c)} \right) - 2Ba \left( \sqrt{\tan(dx+c)} \right)}{b^2} + \frac{(-Ab+aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) \right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/d*(2/b^2*(1/3*B*b*tan(d*x+c)^(3/2)+A*b*tan(d*x+c)^(1/2)-B*a*tan(d*x+c)^(1
/2))+2/(a^2+b^2)*(1/8*(-A*b+B*a)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*a-B*b)*2^(1/2)*
(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+
c)^(1/2))))-2/b^2*a^3*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(
1/2)/(a*b)^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 258, normalized size = 0.79

$$\frac{24(Ba^4 - Aa^3b) \arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) - 3(2\sqrt{2}(A-B)a + (A+B)b) \arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{\tan(dx+c)}}\right) + 2\sqrt{2}(A-B)a + (A+B)b \arctan\left(-\frac{1}{2}\sqrt{2-2\sqrt{\tan(dx+c)}}\right) - \sqrt{2}(A+B)a - (A-B)b \arctan\left(\sqrt{2}\sqrt{\tan(dx+c)+\tan(dx+c)+1}\right) + \sqrt{2}(A+B)a - (A-B)b \arctan\left(-\sqrt{2}\sqrt{\tan(dx+c)+\tan(dx+c)+1}\right) + \frac{1}{2}(B\tan(dx+c)^2 - (B-a)\sqrt{\tan(dx+c)})}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/12*(24*(B*a^4 - A*a^3*b)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b^2
+ b^4)*sqrt(a*b)) - 3*(2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2
)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a - (
```

$$(A - B)*b)*\log(\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1) + \sqrt{2}*((A + B)*a - (A - B)*b)*\log(-\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1))/(a^2 + b^2) + 8*(B*b*\tan(dx + c)^{(3/2)} - 3*(B*a - A*b)*\sqrt{\tan(dx + c)})/b^2)/d$$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 12568 vs. 2(281) = 562.

time = 62.24, size = 25248, normalized size = 77.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(5/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c)),x, algorithm="fricas")

[Out] [-1/12\*(12\*sqrt(2)\*(a^6\*b^2 + 3\*a^4\*b^4 + 3\*a^2\*b^6 + b^8)\*d^5\*sqrt(((A^4 + 2\*A^2\*B^2 + B^4)\*a^4 + 2\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^2\*b^2 + (A^4 + 2\*A^2\*B^2 + B^4)\*b^4 - 2\*(A\*B\*a^6 + A\*B\*a^4\*b^2 - A\*B\*a^2\*b^4 - A\*B\*b^6 - (A^2 - B^2)\*a^5\*b - 2\*(A^2 - B^2)\*a^3\*b^3 - (A^2 - B^2)\*a\*b^5)\*d^2\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4)))/((A^4 - 2\*A^2\*B^2 + B^4)\*a^4 + 8\*(A^3\*B - A\*B^3)\*a^3\*b - 2\*(A^4 - 10\*A^2\*B^2 + B^4)\*a^2\*b^2 - 8\*(A^3\*B - A\*B^3)\*a\*b^3 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^4))\*sqrt(((A^4 - 2\*A^2\*B^2 + B^4)\*a^4 + 8\*(A^3\*B - A\*B^3)\*a^3\*b - 2\*(A^4 - 10\*A^2\*B^2 + B^4)\*a^2\*b^2 - 8\*(A^3\*B - A\*B^3)\*a\*b^3 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^4)/((a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8)\*d^4))\*((A^4 + 2\*A^2\*B^2 + B^4)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4))^(3/4)\*arctan((((A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^8 + 4\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^7\*b + 2\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^6\*b^2 + 12\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^5\*b^3 + 12\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^3\*b^5 - 2\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^2\*b^6 + 4\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a\*b^7 - (A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*b^8)\*d^4\*sqrt(((A^4 - 2\*A^2\*B^2 + B^4)\*a^4 + 8\*(A^3\*B - A\*B^3)\*a^3\*b - 2\*(A^4 - 10\*A^2\*B^2 + B^4)\*a^2\*b^2 - 8\*(A^3\*B - A\*B^3)\*a\*b^3 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^4)/((a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8)\*d^4))\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4)) - sqrt(2)\*((B\*a^9 - A\*a^8\*b + 4\*B\*a^7\*b^2 - 4\*A\*a^6\*b^3 + 6\*B\*a^5\*b^4 - 6\*A\*a^4\*b^5 + 4\*B\*a^3\*b^6 - 4\*A\*a^2\*b^7 + B\*a\*b^8 - A\*b^9)\*d^7\*sqrt(((A^4 - 2\*A^2\*B^2 + B^4)\*a^4 + 8\*(A^3\*B - A\*B^3)\*a^3\*b - 2\*(A^4 - 10\*A^2\*B^2 + B^4)\*a^2\*b^2 - 8\*(A^3\*B - A\*B^3)\*a\*b^3 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^4)/((a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8)\*d^4))\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^4 + 2\*a^2\*b^2 + b^4)\*d^4)) + ((A^3 + A\*B^2)\*a^7 + (A^2\*B + B^3)\*a^6\*b + 3\*(A^3 + A\*B^2)\*a^5\*b^2 + 3\*(A^2\*B + B^3)\*a^4\*b^3 + 3\*(A^3 + A\*B^2)\*a^3\*b^4 + 3\*(A^2\*B + B^3)\*a^2\*b^5 + (A^3 + A\*B^2)\*a\*b^6 + (A^2\*B + B^3)\*b^7)\*d^5\*sqrt(((A^4 - 2\*A^2\*B^2 + B^4)\*a^4 + 8\*(A^3\*B - A\*B^3)\*a^3\*b - 2\*(A^4 - 10\*A^2\*B^2 + B^4)\*a^2\*b^2 - 8\*(A^3\*B - A\*B^3)\*a\*b^3 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^4)/((a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8)\*d^4))\*sqrt(((A^4



$$\begin{aligned}
& + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2* \\
& B^2 + B^4)*b^4 - 2*(A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - \\
& B^2)*a^5*b - 2*(A^2 - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*\sqrt{(A^4 + 2*A \\
& ^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)}))/((A^4 - 2*A^2*B^2 + B^4)*a^4 \\
& + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - \\
& A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{(((A^6 - A^4*B^2 - A^2*B \\
& ^4 + B^6)*a^6 + 8*(A^5*B - A*B^5)*a^5*b - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + \\
& B^6)*a^4*b^2 - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^2*b^4 - 8*(A^5*B - A \\
& *B^5)*a*b^5 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^6)*d^2*\sqrt{(A^4 + 2*A^2*B^ \\
& 2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*\cos(dx + c) + \sqrt{2}*(((A^5 - 2*A \\
& ^3*B^2 + A*B^4)*a^7 + (9*A^4*B - 10*A^2*B^3 + B^5)*a^6*b - (A^5 - 26*A^3*B^ \\
& 2 + 9*A*B^4)*a^5*b^2 - (A^4*B - 18*A^2*B^3 + B^5)*a^4*b^3 - (A^5 - 18*A^3*B \\
& ^2 + A*B^4)*a^3*b^4 - (9*A^4*B - 26*A^2*B^3 + B^5)*a^2*b^5 + (A^5 - 10*A^3* \\
& B^2 + 9*A*B^4)*a*b^6 + (A^4*B - 2*A^2*B^3 + B^5)*b^7)*d^3*\sqrt{(A^4 + 2*A^2 \\
& *B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))*\cos(dx + c) + ((A^6*B - A^4*B^3 \\
& - A^2*B^5 + B^7)*a^5 - (A^7 - 9*A^5*B^2 - A^3*B^4 + 9*A*B^6)*a^4*b - 2*(5* \\
& A^6*B - 9*A^4*B^3 - 13*A^2*B^5 + B^7)*a^3*b^2 + 2*(A^7 - 13*A^5*B^2 - 9*A^3 \\
& *B^4 + 5*A*B^6)*a^2*b^3 + (9*A^6*B - A^4*B^3 - 9*A^2*B^5 + B^7)*a*b^4 - (A^ \\
& 7 - A^5*B^2 - A^3*B^4 + A*B^6)*b^5)*d*\cos(dx + c))*\sqrt{((A^4 + 2*A^2*B^2 \\
& + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^ \\
& 4 - 2*(A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*b - \\
& 2*(A^2 - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4) \\
& /((a^4 + 2*a^2*b^2 + b^4)*d^4)}))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - \\
& A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 \\
& + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{\sin(dx + c)/\cos(dx + c))*((A^4 + 2* \\
& A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{1/4} + ((A^8 - 2*A^4*B^4 + B \\
& ^8)*a^4 + 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a^3*b - 2*(A^8 - 8*A^6*B^2 \\
& - 18*A^4*B^4 - 8*A^2*B^6 + B^8)*a^2*b^2 - 8*(A^7*B + A^5*B^3 - A^3*B^5 - A* \\
& B^7)*a*b^3 + (A^8 - 2*A^4*B^4 + B^8)*b^4)*\sin(dx + c))/\cos(dx + c))*((A^4 \\
& + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4} + \sqrt{2}*(((A^4*B \\
& - B^5)*a^{11} - (A^5 - 4*A^3*B^2 - 5*A*B^4)*a^{10}*b - (A^4*B + 4*A^2*B^3 + 3* \\
& B^5)*a^9*b^2 - (3*A^5 - 16*A^3*B^2 - 19*A*B^4)*a^8*b^3 - 2*(7*A^4*B + 8*A^2 \\
& *B^3 + B^5)*a^7*b^4 - 2*(A^5 - 12*A^3*B^2 - 13*A*B^4)*a^6*b^5 - 2*(13*A^4*B \\
& + 12*A^2*B^3 - B^5)*a^5*b^6 + 2*(A^5 + 8*A^3*B^2 + 7*A*B^4)*a^4*b^7 - (19* \\
& A^4*B + 16*A^2*B^3 - 3*B^5)*a^3*b^8 + (3*A^5 + 4*A^3*B^2 + A*B^4)*a^2*b^9 - \\
& (5*A^4*B + 4*A^2*B^3 - B^5)*a*b^{10} + (A^5 - A*B^4)*b^{11})*d^7*\sqrt{((A^4 - \\
& 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b \dots}
\end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{5}{2}}(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& d^4 - A^4(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2 \\
& )/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)} - (32*(A^3a^2b^5d^2 - \\
& 15A^3a^4b^3d^2 + 12A^3a^6b^*d^2))/(b^*d^5))*(((64A^4a^2b^2d^4 - A^4 \\
& 4*(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a \\
& ^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)} - (32*\tan(c + d*x))^{(1/2)}*(2A^4a \\
& ^6 - A^4b^6))/(b^*d^4))*(((64A^4a^2b^2d^4 - A^4*(16a^4d^4 + 16b^4d^ \\
& 4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2 \\
& b^2d^4))^{(1/2)}*i)/((((((32*(4A^*a^*b^8d^4 + 8A^*a^3b^6d^4 + 4A^*a^5b^ \\
& 4d^4))/(b^*d^5) - (32*\tan(c + d*x))^{(1/2)}*(((64A^4a^2b^2d^4 - A^4*(16a^ \\
& 4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4d^4 + \\
& b^4d^4 + 2a^2b^2d^4))^{(1/2)}*(16b^10d^4 + 16a^2b^8d^4 - 16a^4b^ \\
& 6d^4 - 16a^6b^4d^4))/(b^*d^4))*(((64A^4a^2b^2d^4 - A^4*(16a^4d^4 + \\
& 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4d^4 + b^4d^ \\
& 4 + 2a^2b^2d^4))^{(1/2)} + (32*\tan(c + d*x))^{(1/2)}*(4A^2a^3b^5d^2 + 2 \\
& A^2a^5b^3d^2 - 14A^2a^*b^7d^2 + 16A^2a^7b^*d^2))/(b^*d^4))*(((64A^4a \\
& ^2b^2d^4 - A^4*(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2 \\
& *a^*b^*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)} - (32*(A^3a^2b^ \\
& 5d^2 - 15A^3a^4b^3d^2 + 12A^3a^6b^*d^2))/(b^*d^5))*(((64A^4a^2b^2 \\
& d^4 - A^4*(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2 \\
& )/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)} + (32*\tan(c + d*x))^{(1/2)}* \\
& (2A^4a^6 - A^4b^6))/(b^*d^4))*(((64A^4a^2b^2d^4 - A^4*(16a^4d^4 + 1 \\
& 6b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4d^4 + b^4d^4 \\
& + 2a^2b^2d^4))^{(1/2)} + (((((32*(4A^*a^*b^8d^4 + 8A^*a^3b^6d^4 + 4A^*a \\
& ^5b^4d^4))/(b^*d^5) + (32*\tan(c + d*x))^{(1/2)}*(((64A^4a^2b^2d^4 - A^4*( \\
& 16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4 \\
& d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)}*(16b^10d^4 + 16a^2b^8d^4 - 16a^ \\
& ^4b^6d^4 - 16a^6b^4d^4))/(b^*d^4))*(((64A^4a^2b^2d^4 - A^4*(16a^4 \\
& d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4d^4 + b \\
& ^4d^4 + 2a^2b^2d^4))^{(1/2)} - (32*\tan(c + d*x))^{(1/2)}*(4A^2a^3b^5d^2 \\
& + 2A^2a^5b^3d^2 - 14A^2a^*b^7d^2 + 16A^2a^7b^*d^2))/(b^*d^4))*(((64 \\
& *A^4a^2b^2d^4 - A^4*(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - \\
& 8A^2a^*b^*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)} - (32*(A^3a \\
& ^2b^5d^2 - 15A^3a^4b^3d^2 + 12A^3a^6b^*d^2))/(b^*d^5))*(((64A^4a^2 \\
& *b^2d^4 - A^4*(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^* \\
& b^*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)} - (32*\tan(c + d*x))^{( \\
& 1/2)}*(2A^4a^6 - A^4b^6))/(b^*d^4))*(((64A^4a^2b^2d^4 - A^4*(16a^4d^ \\
& 4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - 8A^2a^*b^*d^2)/(16*(a^4d^4 + b^4 \\
& *d^4 + 2a^2b^2d^4))^{(1/2)} - (64*(A^5a^5 - A^5a^3b^2))/(b^*d^5))*(((6 \\
& 4A^4a^2b^2d^4 - A^4*(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - \\
& 8A^2a^*b^*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))^{(1/2)}*2i - \operatorname{atan}(( \\
& (((((32*(12B^*a^2b^9d^4 + 24B^*a^4b^7d^4 + 12B^*a^6b^5d^4))/(b^3d^5) \\
& - (32*\tan(c + d*x))^{(1/2)}*(((64B^4a^2b^2d^4 - B^4*(16a^4d^4 + 16b^4 \\
& d^4 + 32a^2b^2d^4))^{(1/2)} + 8B^2a^*b^*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^ \\
& 2b^2d^4))^{(1/2)}*(16b^12d^4 + 16a^2b^10d^4 \dots
\end{aligned}$$

$$3.399 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=297

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $2*a^{(3/2)}*(A*b-B*a)*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/b^{(3/2)}/(a^2+b^2)/d+1/2*(b*(A-B)-a*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(b*(A-B)-a*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a*(A-B)+b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}-1/4*(a*(A-B)+b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+2*B*\tan(d*x+c)^{(1/2)}/b/d$

**Rubi [A]**

time = 0.41, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3688, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(a(A-B) + b(A+B)) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d (a^2 + b^2)}\right)}{2\sqrt{2} d (a^2 + b^2)} - \frac{(a(A-B) + b(A+B)) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d (a^2 + b^2)}\right)}{2\sqrt{2} d (a^2 + b^2)} + \frac{2a^{3/2}(Ab - aB) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2} d (a^2 + b^2)} + \frac{2B \sqrt{\tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out]  $-(((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d)) + ((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*a^{(3/2)}*(A*b - a*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a]])/(b^{(3/2)}*(a^2 + b^2)*d) + ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(b*d)$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

```
t[b*Tan[e + f*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :>
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx &= \frac{2B \sqrt{\tan(c+dx)}}{bd} + \frac{2 \int \frac{-\frac{aB}{2} - \frac{1}{2}bB \tan(c+dx) + \frac{1}{2}(Ab-aB) \tan^2(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))} dx}{b} \\
&= \frac{2B \sqrt{\tan(c+dx)}}{bd} + \frac{2 \int \frac{-\frac{1}{2}b(aA+bB) + \frac{1}{2}b(Ab-aB) \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{b(a^2+b^2)} + \frac{(a^2)}{b} \\
&= \frac{2B \sqrt{\tan(c+dx)}}{bd} + \frac{4 \text{Subst}\left(\int \frac{-\frac{1}{2}b(aA+bB) + \frac{1}{2}b(Ab-aB)x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{b(a^2+b^2)d} \\
&= \frac{2B \sqrt{\tan(c+dx)}}{bd} + \frac{(2a^2(Ab-aB)) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{\tan(c+dx)}\right)}{b(a^2+b^2)d} \\
&= \frac{2a^{3/2}(Ab-aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)d} + \frac{2B \sqrt{\tan(c+dx)}}{bd} \\
&= \frac{2a^{3/2}(Ab-aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)d} + \frac{(a(A-B) + b)}{b} \\
&= -\frac{(b(A-B) - a(A+B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(b)}{b}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.23, size = 165, normalized size = 0.56

$$\frac{\sqrt{-1}(a+ib)b^{3/2}(A-ib)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - 2a^{3/2}(-Ab+aB)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{-1}(a-ib)b^{3/2}(A+ib)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 2\sqrt{b}(a^2+b^2)B\sqrt{\tan(c+dx)}}{b^{3/2}(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] ((-1)^(1/4)\*(a + I\*b)\*b^(3/2)\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - 2\*a^(3/2)\*(-A\*b) + a\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] + (-1)^(1/4)\*(a - I\*b)\*b^(3/2)\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*Sqrt[b]\*(a^2 + b^2)\*B\*Sqrt[Tan[c + d\*x]]/(b^(3/2)\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.10, size = 265, normalized size = 0.89

method	result
--------	--------

derivativedivides	$\frac{2B\left(\sqrt{\tan(dx+c)}\right)}{b} + \frac{(-aA-Bb)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)}{4}$
default	$\frac{2B\left(\sqrt{\tan(dx+c)}\right)}{b} + \frac{(-aA-Bb)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERB  
OSE)`

[Out]  $1/d*(2*B/b*\tan(d*x+c)^{(1/2)}+2/(a^2+b^2)*(1/8*(-A*a-B*b)*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+1/8*(A*b-B*a)*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+2/b*a^2*(A*b-B*a)/(a^2+b^2)/(a*b)^{(1/2)*\arctan(b*\tan(d*x+c)^{(1/2)}/(a*b)^{(1/2))}$

**Maxima** [A]

time = 0.55, size = 234, normalized size = 0.79

$$\frac{8\sqrt{2}\sqrt{a^2-A^2}\arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)-8B\sqrt{\tan(dx+c)}+2\sqrt{2}\sqrt{(A+B)-(A-B)\arctan\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{2}\sqrt{\tan(dx+c)}}\right)}+2\sqrt{2}\sqrt{(A+B)-(A-B)\arctan\left(\frac{-1+\sqrt{2}\sqrt{\tan(dx+c)}}{1+\sqrt{2}\sqrt{\tan(dx+c)}}\right)}+\sqrt{2}\sqrt{(A-B)a+(A+B)b}\log\left(\sqrt{2}\sqrt{\tan(dx+c)+\tan(dx+c)+1}\right)-\sqrt{2}\sqrt{(A-B)a+(A+B)b}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)+\tan(dx+c)+1}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/4*(8*(B*a^3 - A*a^2*b)*\arctan(b*\sqrt{\tan(d*x + c)})/\sqrt{a*b})/((a^2*b + b^3)*\sqrt{a*b}) - 8*B*\sqrt{\tan(d*x + c)}/b + (2*\sqrt{2})*((A + B)*a - (A - B)*b)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(d*x + c)})) + 2*\sqrt{2}*((A + B)*a - (A - B)*b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(d*x + c)})) + \sqrt{2}*((A - B)*a + (A + B)*b)*\log(\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1) - \sqrt{2}*((A - B)*a + (A + B)*b)*\log(-\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1)/(a^2 + b^2)/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 12392 vs. 2(257) = 514.

time = 30.10, size = 24896, normalized size = 83.82

Too large to display





$$\begin{aligned}
& B^3 + B^5) * a^7 - (A^5 - 10 * A^3 * B^2 + 9 * A * B^4) * a^6 * b - (9 * A^4 * B - 26 * A^2 * B^3 \\
& + B^5) * a^5 * b^2 + (A^5 - 18 * A^3 * B^2 + A * B^4) * a^4 * b^3 - (A^4 * B - 18 * A^2 * B^3 \\
& + B^5) * a^3 * b^4 + (A^5 - 26 * A^3 * B^2 + 9 * A * B^4) * a^2 * b^5 + (9 * A^4 * B - 10 * A^2 * B^3 \\
& + B^5) * a * b^6 - (A^5 - 2 * A^3 * B^2 + A * B^4) * b^7) * d^3 * \sqrt{(A^4 + 2 * A^2 * B^2 \\
& + B^4) / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4)} * \cos(d * x + c) - ((A^7 - A^5 * B^2 - A^3 * \\
& B^4 + A * B^6) * a^5 + (9 * A^6 * B - A^4 * B^3 - 9 * A^2 * B^5 + B^7) * a^4 * b - 2 * (A^7 - 1 \\
& 3 * A^5 * B^2 - 9 * A^3 * B^4 + 5 * A * B^6) * a^3 * b^2 - 2 * (5 * A^6 * B - 9 * A^4 * B^3 - 13 * A^2 * \\
& B^5 + B^7) * a^2 * b^3 + (A^7 - 9 * A^5 * B^2 - A^3 * B^4 + 9 * A * B^6) * a * b^4 + (A^6 * B - \\
& A^4 * B^3 - A^2 * B^5 + B^7) * b^5) * d * \cos(d * x + c)) * \sqrt{((A^4 + 2 * A^2 * B^2 + B^4) \\
& ) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4 + 2 \\
& * (A * B * a^6 + A * B * a^4 * b^2 - A * B * a^2 * b^4 - A * B * b^6 - (A^2 - B^2) * a^5 * b - 2 * (A^2 \\
& - B^2) * a^3 * b^3 - (A^2 - B^2) * a * b^5) * d^2 * \sqrt{(A^4 + 2 * A^2 * B^2 + B^4) / ((a^4 \\
& + 2 * a^2 * b^2 + b^4) * d^4)}} / ((A^4 - 2 * A^2 * B^2 + B^4) * a^4 + 8 * (A^3 * B - A * B^3) \\
& ) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 - 8 * (A^3 * B - A * B^3) * a * b^3 + (A \\
& ^4 - 2 * A^2 * B^2 + B^4) * b^4) * \sqrt{\sin(d * x + c) / \cos(d * x + c)} * ((A^4 + 2 * A^2 * B^2 \\
& + B^4) / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4))^{1/4} + ((A^8 - 2 * A^4 * B^4 + B^8) * a^4 \\
& + 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^3 * b - 2 * (A^8 - 8 * A^6 * B^2 - 18 * \\
& A^4 * B^4 - 8 * A^2 * B^6 + B^8) * a^2 * b^2 - 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * \\
& a * b^3 + (A^8 - 2 * A^4 * B^4 + B^8) * b^4) * \sin(d * x + c) / \cos(d * x + c) * ((A^4 + 2 * \\
& A^2 * B^2 + B^4) / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4))^{3/4} - \sqrt{2} * (((A^5 - A * B^4) * a^{11} \\
& + (5 * A^4 * B + 4 * A^2 * B^3 - B^5) * a^{10} * b + (3 * A^5 + 4 * A^3 * B^2 + A * B^4) * \\
& a^9 * b^2 + (19 * A^4 * B + 16 * A^2 * B^3 - 3 * B^5) * a^8 * b^3 + 2 * (A^5 + 8 * A^3 * B^2 + 7 * \\
& A * B^4) * a^7 * b^4 + 2 * (13 * A^4 * B + 12 * A^2 * B^3 - B^5) * a^6 * b^5 - 2 * (A^5 - 12 * A^3 * \\
& B^2 - 13 * A * B^4) * a^5 * b^6 + 2 * (7 * A^4 * B + 8 * A^2 * B^3 + B^5) * a^4 * b^7 - (3 * A^5 - \\
& 16 * A^3 * B^2 - 19 * A * B^4) * a^3 * b^8 + (A^4 * B + 4 * A^2 * B^3 + 3 * B^5) * a^2 * b^9 - (A^5 \\
& - 4 * A^3 * B^2 - 5 * A * B^4) * a * b^{10} - (A^4 * B - B^5) * b^{11}) * d^7 * \sqrt{((A^4 - 2 * A^2 \\
& * B^2 + B^4) * a^4 + 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (...}
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 11.82, size = 2500, normalized size = 8.42
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

```
[Out] atan(((((((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b^4*d^4))/(b*d^5)
- (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d
^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2
*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b
^4*d^4))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32
*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d
^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(4*B^2*a^3*b^5*d^2 + 2*B^2*a^5*b^3*d^2
- 14*B^2*a*b^7*d^2 + 16*B^2*a^7*b*d^2))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B
^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a
^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32*(B^3*a^2*b^5*d^2 - 15*B^3*a
^4*b^3*d^2 + 12*B^3*a^6*b*d^2))/(b*d^5))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a
^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4
+ b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*B^4*a^6 - B^4
*b^6))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a
^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4
)))^(1/2)*1i - ((((((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b^4*d^4))/
(b*d^5) + (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 +
16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4
+ 2*a^2*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 -
16*a^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*
d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a
^2*b^2*d^4)))^(1/2) - (32*tan(c + d*x)^(1/2)*(4*B^2*a^3*b^5*d^2 + 2*B^2*a^5*
b^3*d^2 - 14*B^2*a*b^7*d^2 + 16*B^2*a^7*b*d^2))/(b*d^4))*(((64*B^4*a^2*b^2
*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d
^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32*(B^3*a^2*b^5*d^2
- 15*B^3*a^4*b^3*d^2 + 12*B^3*a^6*b*d^2))/(b*d^5))*(((64*B^4*a^2*b^2*d^4 - B
^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a
^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32*tan(c + d*x)^(1/2)*(2*B^4*a
^6 - B^4*b^6))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d
^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a
^2*b^2*d^4)))^(1/2)*1i)/(((((((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b
^4*d^4))/(b*d^5) - (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a
^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4
+ 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 +
```

$$\begin{aligned}
& (b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4)/(b*d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} + (32*\tan(c + d*x))^{(1/2)}*(4*B^2*a^3*b^5*d^2 + 2*B^2*a^5*b^3*d^2 - 14*B^2*a*b^7*d^2 + 16*B^2*a^7*b*d^2))/(b*d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*(B^3*a^2*b^5*d^2 - 15*B^3*a^4*b^3*d^2 + 12*B^3*a^6*b*d^2))/(b*d^5))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} + (32*\tan(c + d*x))^{(1/2)}*(2*B^4*a^6 - B^4*b^6))/(b*d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} + (((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b^4*d^4))/(b*d^5) + (32*\tan(c + d*x))^{(1/2)}*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*\tan(c + d*x))^{(1/2)}*(4*B^2*a^3*b^5*d^2 + 2*B^2*a^5*b^3*d^2 - 14*B^2*a*b^7*d^2 + 16*B^2*a^7*b*d^2))/(b*d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*(B^3*a^2*b^5*d^2 - 15*B^3*a^4*b^3*d^2 + 12*B^3*a^6*b*d^2))/(b*d^5))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*\tan(c + d*x))^{(1/2)}*(2*B^4*a^6 - B^4*b^6))/(b*d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (64*(B^5*a^5 - B^5*a^3*b^2))/(b*d^5))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*2i + \operatorname{atan}(\frac{(((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b^4*d^4))/(b*d^5) - (32*\tan(c + d*x))^{(1/2)}*(-((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} + 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16...
\end{aligned}$$

$$3.400 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=278

$$\frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $1/2*(a*(A-B)+b*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(a*(A-B)+b*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(b*(A-B)-a*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2+b^2/d*2^{(1/2)}+1/4*(b*(A-B)-a*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2+b^2/d*2^{(1/2)}-2*(A*b-B*a)*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})*a^{(1/2)}/(a^2+b^2)/d/b^{(1/2)}$

Rubi [A]

time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3693, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a(A-B)+b(A+B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)} + \frac{(a(A-B)+b(A+B))\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)} - \frac{2\sqrt{a}(Ab-aB)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{2}d(a^2+b^2)} - \frac{(b(A-B)-a(A+B))\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{(b(A-B)-a(A+B))\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c+dx]]*(A+B*\operatorname{Tan}[c+dx]))/(a+b*\operatorname{Tan}[c+dx]),x]$

[Out]  $-(((a*(A-B)+b*(A+B))*\operatorname{ArcTan}[1-\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]]])/(\operatorname{Sqrt}[2]*(a^2+b^2)*d)) + ((a*(A-B)+b*(A+B))*\operatorname{ArcTan}[1+\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]])/(\operatorname{Sqrt}[2]*(a^2+b^2)*d) - (2*\operatorname{Sqrt}[a]*(A*b-a*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]])/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[b]*(a^2+b^2)*d) - ((b*(A-B)-a*(A+B))*\operatorname{Log}[1-\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]]+\operatorname{Tan}[c+dx]])/(2*\operatorname{Sqrt}[2]*(a^2+b^2)*d) + ((b*(A-B)-a*(A+B))*\operatorname{Log}[1+\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]]+\operatorname{Tan}[c+dx]])/(2*\operatorname{Sqrt}[2]*(a^2+b^2)*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 210

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

$\text{t}[b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3693

$\text{Int}[(((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]])/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \text{:> Dist}[1/(a^2 + b^2), \text{Int}[\text{Simp}[A*(a*c + b*d) + B*(b*c - a*d) - (A*(b*c - a*d) - B*(a*c + b*d))*\text{Tan}[e + f*x], x]/\text{Sqrt}[c + d*\text{Tan}[e + f*x]], x], x] - \text{Dist}[(b*c - a*d)*((B*a - A*b)/(a^2 + b^2)), \text{Int}[(1 + \text{Tan}[e + f*x]^2)/((a + b*\text{Tan}[e + f*x])*\text{Sqrt}[c + d*\text{Tan}[e + f*x]]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3715

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_.) + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \text{:> Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \ \&\& \ \text{EqQ}[A, C]$

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{a+b \tan(c+dx)} dx &= \frac{\int \frac{Ab-aB+(aA+bB) \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{a^2+b^2} - \frac{(a(Ab-aB)) \int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}} dx}{a^2+b^2} \\ &= \frac{2 \text{Subst}\left(\int \frac{Ab-aB+(aA+bB)x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} - \frac{(a(Ab-aB)) \int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}} dx}{(a^2+b^2)d} \\ &= -\frac{(2a(Ab-aB)) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} + \frac{(b(A-B)) \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} \\ &= -\frac{2\sqrt{a} (Ab-aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} (a^2+b^2)d} - \frac{(b(A-B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} (a^2+b^2)d} \\ &= -\frac{2\sqrt{a} (Ab-aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} (a^2+b^2)d} - \frac{(b(A-B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} (a^2+b^2)d} \\ &= -\frac{(a(A-B) + b(A+B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2+b^2)d} + \end{aligned}$$

**Mathematica [A]**

time = 0.28, size = 195, normalized size = 0.70

$$\frac{2\sqrt{2}(a(A-B)+b(A+B))(\text{ArcTan}(1-\sqrt{2}\sqrt{\tan(c+dx)})-\text{ArcTan}(1+\sqrt{2}\sqrt{\tan(c+dx)})) + \frac{s\sqrt{a}(ab-a^2)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}} - \sqrt{2}(b(-A+B)+a(A+B))(\log(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx))-\log(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)))}{4(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/4*(2*Sqrt[2]*(a*(A - B) + b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (8*Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[b] - Sqrt[2]*(b*(-A + B) + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/((a^2 + b^2)*d)
```

**Maple [A]**

time = 0.10, size = 244, normalized size = 0.88

method	result
derivativedivides	$\frac{(Ab-aB)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$
default	$\frac{(Ab-aB)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)*(1/8*(A*b-B*a)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(A*a+B*b)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-2*a*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))
```

**Maxima [A]**

time = 0.53, size = 216, normalized size = 0.78

$$\frac{8(Ba^2-Ab^2)\arctan\left(\frac{1+\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) + 2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) + 2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right) - \sqrt{2}((A+B)a-(A-B)b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right) + \sqrt{2}((A+B)a-(A-B)b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{4d}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/4*(8*(B*a^2 - A*a*b)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*
sqrt(a*b)) + (2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(-1/2*s
qrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a - (A - B)*b)*
log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a - (
A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2))
/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 12316 vs. 2(240) = 480.

time = 22.66, size = 24744, normalized size = 89.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt(((A^4 + 2*A^2*
B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4
)*b^4 - 2*(A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*
b - 2*(A^2 - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*sqrt((A^4 + 2*A^2*B^2 +
B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*
B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a
*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*
(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B
^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)
)^(3/4)*arctan((((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 + 4*(A^7*B + 3*A^5
*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6
*b^2 + 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 + 12*(A^7*B + 3*A
^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)
*a^2*b^6 + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B
^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B
- A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*
b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))
- sqrt(2)*((B*a^9 - A*a^8*b + 4*B*a^7*b^2 - 4*A*a^6*b^3 + 6*B*a^5*b^4 - 6*A
*a^4*b^5 + 4*B*a^3*b^6 - 4*A*a^2*b^7 + B*a*b^8 - A*b^9)*d^7*sqrt(((A^4 - 2*
A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a
^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a
```



**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\tan(c + dx)}}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x))/(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.12, size = 2500, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] atan((((32\*(13\*A^3\*a^2\*b^4\*d^2 + A^3\*a^4\*b^2\*d^2))/d^5 + ((32\*(12\*A\*a\*b^7\*d^4 + 24\*A\*a^3\*b^5\*d^4 + 12\*A\*a^5\*b^3\*d^4))/d^5 - (32\*tan(c + d\*x)^(1/2)\*((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(16\*b^9\*d^4 + 16\*a^2\*b^7\*d^4 - 16\*a^4\*b^5\*d^4 - 16\*a^6\*b^3\*d^4))/d^4)\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) + (32\*tan(c + d\*x)^(1/2)\*(20\*A^2\*a^3\*b^4\*d^2 + 2\*A^2\*a^5\*b^2\*d^2 - 14\*A^2\*a\*b^6\*d^2))/d^4)\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) - (32\*tan(c + d\*x)^(1/2)\*(A^4\*b^5 - 2\*A^4\*a^2\*b^3))/d^4)\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*i - (((32\*(13\*A^3\*a^2\*b^4\*d^2 + A^3\*a^4\*b

$$\begin{aligned}
& \text{^2*d^2))/d^5 + (((32*(12*A*a*b^7*d^4 + 24*A*a^3*b^5*d^4 + 12*A*a^5*b^3*d^4) \\
& )/d^5 + (32*\tan(c + d*x)^{(1/2)}*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16 \\
& *b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + \\
& 2*a^2*b^2*d^4)))^{(1/2)}*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16* \\
& a^6*b^3*d^4))/d^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 3 \\
& 2*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d \\
& ^4)))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(20*A^2*a^3*b^4*d^2 + 2*A^2*a^5*b^2*d^ \\
& 2 - 14*A^2*a*b^6*d^2))/d^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^ \\
& 4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2* \\
& a^2*b^2*d^4)))^{(1/2)}*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 \\
& + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^ \\
& 2*d^4)))^{(1/2)} + (32*\tan(c + d*x)^{(1/2)}*(A^4*b^5 - 2*A^4*a^2*b^3))/d^4)*((( \\
& 64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} \\
& - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*1i)/(((32 \\
& *(13*A^3*a^2*b^4*d^2 + A^3*a^4*b^2*d^2))/d^5 + (((32*(12*A*a*b^7*d^4 + 24*A \\
& *a^3*b^5*d^4 + 12*A*a^5*b^3*d^4))/d^5 - (32*\tan(c + d*x)^{(1/2)}*(((64*A^4*a^ \\
& 2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a \\
& *b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(16*b^9*d^4 + 16*a^ \\
& 2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*A^4*a^2*b^2*d^4 - \\
& A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16* \\
& (a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} + (32*\tan(c + d*x)^{(1/2)}*(20*A^ \\
& 2*a^3*b^4*d^2 + 2*A^2*a^5*b^2*d^2 - 14*A^2*a*b^6*d^2))/d^4)*(((64*A^4*a^2*b \\
& ^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b* \\
& d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(((64*A^4*a^2*b^2*d^4 \\
& - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/( \\
& 16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(A^ \\
& 4*b^5 - 2*A^4*a^2*b^3))/d^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b \\
& ^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2 \\
& *a^2*b^2*d^4)))^{(1/2)} + (((32*(13*A^3*a^2*b^4*d^2 + A^3*a^4*b^2*d^2))/d^5 + \\
& ((32*(12*A*a*b^7*d^4 + 24*A*a^3*b^5*d^4 + 12*A*a^5*b^3*d^4))/d^5 + (32*ta \\
& n(c + d*x)^{(1/2)}*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32* \\
& a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4 \\
& )))^{(1/2)}*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/ \\
& d^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4) \\
& )^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} - \\
& (32*\tan(c + d*x)^{(1/2)}*(20*A^2*a^3*b^4*d^2 + 2*A^2*a^5*b^2*d^2 - 14*A^2*a*b \\
& ^6*d^2))/d^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2 \\
& *b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))) \\
& ^{(1/2)}*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d \\
& ^4))^{(1/2)} - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} \\
& + (32*\tan(c + d*x)^{(1/2)}*(A^4*b^5 - 2*A^4*a^2*b^3))/d^4)*(((64*A^4*a^2*b^2 \\
& *d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*A^2*a*b*d^ \\
& 2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} + (64*A^5*a*b^3)/d^5)* ( \\
& ((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} \\
& ) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*2i - ata
\end{aligned}$$

$$\begin{aligned}
& n(((((((32*(4*B*a^2*b^6*d^4 + 8*B*a^4*b^4*d^4 + 4*B*a^6*b^2*d^4))/d^5 - (32 \\
& * \tan(c + d*x)^{(1/2)} * (((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + \\
& 32*a^2*b^2*d^4))^{(1/2)} + 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2* \\
& d^4)))^{(1/2)} * (16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4 \\
& ))/d^4 * (((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d \\
& ^4))^{(1/2)} + 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} \\
& + (32*\tan(c + d*x)^{(1/2)} * (14*B^2*a^5*b^2*d^2 - \dots
\end{aligned}$$

$$3.401 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))} dx$$

Optimal. Leaf size=278

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $-1/2*(b*(A-B)-a*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(b*(A-B)-a*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(a*(A-B)+b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+1/4*(a*(A-B)+b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+2*(A*b-B*a)*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})*b^{(1/2)}/(a^2+b^2)/d/a^{(1/2)}$

Rubi [A]

time = 0.24, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3694, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{2\sqrt{b} (Ab - aB) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d (a^2 + b^2)} - \frac{(a(A-B) + b(A+B)) \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)} + \frac{(a(A-B) + b(A+B)) \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x]) / (\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] * (a + b*\operatorname{Tan}[c + d*x])), x]$

[Out]  $((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*\operatorname{Sqrt}[b]*(A*b - a*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / \operatorname{Sqrt}[a]] / (\operatorname{Sqrt}[a]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 210

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] :> \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

$\text{t}[b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3694

$\text{Int}[(((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{\text{n}_.})/((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \text{:> Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A + b*B - (A*b - a*B)*\text{Tan}[e + f*x], x], x] + \text{Dist}[b*(A*b - a*B)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3715

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{\text{m}_.})*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{\text{n}_.})*((A_.) + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \text{:> Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \ \&\& \ \text{EqQ}[A, C]$

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx &= \frac{\int \frac{aA + bB - (Ab - aB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{a^2 + b^2} + \frac{(b(Ab - aB)) \int \frac{1 + \tan^2(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx}{a^2 + b^2} \\ &= \frac{2 \text{Subst}\left(\int \frac{aA + bB + (-Ab + aB)x^2}{1 + x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2) d} + \frac{(b(Ab - aB)) \int \frac{1 + \tan^2(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx}{a^2 + b^2} \\ &= \frac{(2b(Ab - aB)) \text{Subst}\left(\int \frac{1}{a + bx^2} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2) d} - \frac{(b(A - B)) \int \frac{1}{a + bx^2} dx, x, \sqrt{\tan(c + dx)}}{(a^2 + b^2) d} \\ &= \frac{2\sqrt{b} (Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2) d} - \frac{(b(A - B) - a(A + B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2) d} \\ &= \frac{2\sqrt{b} (Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2) d} - \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2) d} \\ &= \frac{(b(A - B) - a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(b(A + B) + a(A - B)) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} \end{aligned}$$



**Mathematica [A]**

time = 0.26, size = 194, normalized size = 0.70

$$\frac{2\sqrt{2}(b(-A+B)+a(A+B))(\text{ArcTan}(1-\sqrt{2}\sqrt{\tan(c+dx)})-\text{ArcTan}(1+\sqrt{2}\sqrt{\tan(c+dx)})) + \frac{8\sqrt{b(-Ab+B)}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \sqrt{2}(a(A-B)+b(A+B))(\log(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx))-\log(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)))}{4(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])),x]

[Out]  $-1/4*(2*\text{Sqrt}[2]*(b*(-A + B) + a*(A + B))*(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]] - \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]) + (8*\text{Sqrt}[b]*(-A*b) + a*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/\text{Sqrt}[a] + \text{Sqrt}[2]*(a*(A - B) + b*(A + B))*(\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]])/((a^2 + b^2)*d)$

**Maple [A]**

time = 0.09, size = 244, normalized size = 0.88

method	result
derivativedivides	$\frac{(aA+Bb)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$
default	$\frac{(aA+Bb)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c)),x,method=\_RETURNVERB OSE)

[Out]  $1/d*(2/(a^2+b^2)*(1/8*(A*a+B*b)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*b+B*a)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+2*b*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))$

**Maxima [A]**

time = 0.52, size = 217, normalized size = 0.78

$$\frac{8(Bab-Ab^2)\text{arctan}\left(\frac{1+\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) - 2\sqrt{2}((A+B)a-(A-B)b)\text{arctan}\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) + 2\sqrt{2}((A+B)a-(A-B)b)\text{arctan}\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right) + \sqrt{2}((A-B)a+(A+B)b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right) - \sqrt{2}((A-B)a+(A+B)b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{(a^2+b^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(8*(B*a*b - A*b^2)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B)*a + (A + B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a + (A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 + b^2)/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 12422 vs.  $2(240) = 480$ .

time = 26.61, size = 24848, normalized size = 89.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(2)*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 + 2*(A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*b - 2*(A^2 - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4)*arctan((((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 + 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 + 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8)*d^4*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt(2)*((A*a^9 + B*a^8*b + 4*A*a^7*b^2 + 4*B*a^6*b^3 + 6*A*a^5*b^4 + 6*B*a^4*b^5 + 4*A*a^3*b^6 + 4*B*a^2*b^7 + A*a*b^8 + B*b^9)*d^7*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*
```

$$\begin{aligned}
& a^6 b^2 + 6 a^4 b^4 + 4 a^2 b^6 + b^8) d^4) \sqrt{(A^4 + 2 A^2 B^2 + B^4) /} \\
& (a^4 + 2 a^2 b^2 + b^4) d^4) - ((A^2 B + B^3) a^7 - (A^3 + A B^2) a^6 b + \\
& 3 (A^2 B + B^3) a^5 b^2 - 3 (A^3 + A B^2) a^4 b^3 + 3 (A^2 B + B^3) a^3 b^4 \\
& - 3 (A^3 + A B^2) a^2 b^5 + (A^2 B + B^3) a b^6 - (A^3 + A B^2) b^7) d^5 s \\
& \text{qrt}(((A^4 - 2 A^2 B^2 + B^4) a^4 + 8 (A^3 B - A B^3) a^3 b - 2 (A^4 - 10 A^ \\
& 2 B^2 + B^4) a^2 b^2 - 8 (A^3 B - A B^3) a b^3 + (A^4 - 2 A^2 B^2 + B^4) b^ \\
& 4) / ((a^8 + 4 a^6 b^2 + 6 a^4 b^4 + 4 a^2 b^6 + b^8) d^4))) \sqrt{((A^4 + 2 A \\
& ^2 B^2 + B^4) a^4 + 2 (A^4 + 2 A^2 B^2 + B^4) a^2 b^2 + (A^4 + 2 A^2 B^2 + \\
& B^4) b^4 + 2 (A B a^6 + A B a^4 b^2 - A B a^2 b^4 - A B b^6 - (A^2 - B^2) a \\
& ^5 b - 2 (A^2 - B^2) a^3 b^3 - (A^2 - B^2) a b^5) d^2 \sqrt{(A^4 + 2 A^2 B^2 \\
& + B^4) / ((a^4 + 2 a^2 b^2 + b^4) d^4))} / ((A^4 - 2 A^2 B^2 + B^4) a^4 + 8 (A \\
& ^3 B - A B^3) a^3 b - 2 (A^4 - 10 A^2 B^2 + B^4) a^2 b^2 - 8 (A^3 B - A B^3 \\
& ) a b^3 + (A^4 - 2 A^2 B^2 + B^4) b^4) \sqrt{(((A^6 - A^4 B^2 - A^2 B^4 + B \\
& ^6) a^6 + 8 (A^5 B - A B^5) a^5 b - (A^6 - 17 A^4 B^2 - 17 A^2 B^4 + B^6) a \\
& ^4 b^2 - (A^6 - 17 A^4 B^2 - 17 A^2 B^4 + B^6) a^2 b^4 - 8 (A^5 B - A B^5) a \\
& b^5 + (A^6 - A^4 B^2 - A^2 B^4 + B^6) b^6) d^2 \sqrt{(A^4 + 2 A^2 B^2 + B^ \\
& 4) / ((a^4 + 2 a^2 b^2 + b^4) d^4)) \cos(dx + c) + \sqrt{2} * (((A^4 B - 2 A^2 B \\
& ^3 + B^5) a^7 - (A^5 - 10 A^3 B^2 + 9 A B^4) a^6 b - (9 A^4 B - 26 A^2 B^3 \\
& + B^5) a^5 b^2 + (A^5 - 18 A^3 B^2 + A B^4) a^4 b^3 - (A^4 B - 18 A^2 B^3 + \\
& B^5) a^3 b^4 + (A^5 - 26 A^3 B^2 + 9 A B^4) a^2 b^5 + (9 A^4 B - 10 A^2 B^ \\
& 3 + B^5) a b^6 - (A^5 - 2 A^3 B^2 + A B^4) b^7) d^3 \sqrt{(A^4 + 2 A^2 B^2 + \\
& B^4) / ((a^4 + 2 a^2 b^2 + b^4) d^4)) \cos(dx + c) - ((A^7 - A^5 B^2 - A^3 B \\
& ^4 + A B^6) a^5 + (9 A^6 B - A^4 B^3 - 9 A^2 B^5 + B^7) a^4 b - 2 (A^7 - 13 \\
& * A^5 B^2 - 9 A^3 B^4 + 5 A B^6) a^3 b^2 - 2 (5 A^6 B - 9 A^4 B^3 - 13 A^2 B \\
& ^5 + B^7) a^2 b^3 + (A^7 - 9 A^5 B^2 - A^3 B^4 + 9 A B^6) a b^4 + (A^6 B - \\
& A^4 B^3 - A^2 B^5 + B^7) b^5) d \cos(dx + c) \sqrt{((A^4 + 2 A^2 B^2 + B^4) \\
& * a^4 + 2 (A^4 + 2 A^2 B^2 + B^4) a^2 b^2 + (A^4 + 2 A^2 B^2 + B^4) b^4 + 2 \\
& (A B a^6 + A B a^4 b^2 - A B a^2 b^4 - A B b^6 - (A^2 - B^2) a^5 b - 2 (A^2 \\
& - B^2) a^3 b^3 - (A^2 - B^2) a b^5) d^2 \sqrt{(A^4 + 2 A^2 B^2 + B^4) / ((a^4 \\
& + 2 a^2 b^2 + b^4) d^4))} / ((A^4 - 2 A^2 B^2 + B^4) a^4 + 8 (A^3 B - A B^3) \\
& * a^3 b - 2 (A^4 - 10 A^2 B^2 + B^4) a^2 b^2 - 8 (A^3 B - A B^3) a b^3 + (A^ \\
& 4 - 2 A^2 B^2 + B^4) b^4) \sqrt{\sin(dx + c) / \cos(dx + c)} * ((A^4 + 2 A^2 B^ \\
& 2 + B^4) / ((a^4 + 2 a^2 b^2 + b^4) d^4))^{1/4} + ((A^8 - 2 A^4 B^4 + B^8) a^ \\
& 4 + 8 (A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a^3 b - 2 (A^8 - 8 A^6 B^2 - 18 A \\
& ^4 B^4 - 8 A^2 B^6 + B^8) a^2 b^2 - 8 (A^7 B + A^5 B^3 - A^3 B^5 - A B^7) a \\
& * b^3 + (A^8 - 2 A^4 B^4 + B^8) b^4) \sin(dx + c) / \cos(dx + c) * ((A^4 + 2 A \\
& ^2 B^2 + B^4) / ((a^4 + 2 a^2 b^2 + b^4) d^4))^{3/4} - \sqrt{2} * (((A^5 - A B^4 \\
& ) a^{11} + (5 A^4 B + 4 A^2 B^3 - B^5) a^{10} b + (3 A^5 + 4 A^3 B^2 + A B^4) a \\
& ^9 b^2 + (19 A^4 B + 16 A^2 B^3 - 3 B^5) a^8 b^3 + 2 (A^5 + 8 A^3 B^2 + 7 A \\
& * B^4) a^7 b^4 + 2 (13 A^4 B + 12 A^2 B^3 - B^5) a^6 b^5 - 2 (A^5 - 12 A^3 B \\
& ^2 - 13 A B^4) a^5 b^6 + 2 (7 A^4 B + 8 A^2 B^3 + B^5) a^4 b^7 - (3 A^5 - 1 \\
& 6 A^3 B^2 - 19 A B^4) a^3 b^8 + (A^4 B + 4 A^2 B^3 + 3 B^5) a^2 b^9 - (A^5 \\
& - 4 A^3 B^2 - 5 A B^4) a b^{10} - (A^4 B - B^5) b^{11}) d^7 \sqrt{((A^4 - 2 A^2 * \\
& B^2 + B^4) a^4 + 8 (A^3 B - A B^3) a^3 b - 2 (A \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx)) \sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)``[Out] Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*sqrt(tan(c + d*x))), x)`**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="giac")``[Out] Timed out`**Mupad [B]**

time = 11.84, size = 2500, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)`

```
[Out] atan((((32*(13*B^3*a^2*b^4*d^2 + B^3*a^4*b^2*d^2))/d^5 + ((32*(12*B*a*b^7*d^4 + 24*B*a^3*b^5*d^4 + 12*B*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)*((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(20*B^2*a^3*b^4*d^2 + 2*B^2*a^5*b^2*d^2 - 14*B^2*a*b^6*d^2))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32*tan(c + d*x)^(1/2)*(B^4*b^5 - 2*B^4*a^2*b^3))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i - (((32*(13*B^3*a^2*b^4*d^2 + B^3*a^4*b
```

$$\begin{aligned}
&^2*d^2))/d^5 + (((32*(12*B*a*b^7*d^4 + 24*B*a^3*b^5*d^4 + 12*B*a^5*b^3*d^4) \\
&)/d^5 + (32*\tan(c + d*x)^{(1/2)}*((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16 \\
&*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + \\
&2*a^2*b^2*d^4)))^{(1/2)}*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16* \\
&a^6*b^3*d^4))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 3 \\
&2*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d \\
&^4)))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(20*B^2*a^3*b^4*d^2 + 2*B^2*a^5*b^2*d^ \\
&2 - 14*B^2*a*b^6*d^2))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^ \\
&4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2* \\
&a^2*b^2*d^4)))^{(1/2)}*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 \\
&+ 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^ \\
&2*d^4)))^{(1/2)} + (32*\tan(c + d*x)^{(1/2)}*(B^4*b^5 - 2*B^4*a^2*b^3))/d^4)*((( \\
&64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} \\
&- 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*1i)/(((32 \\
&*(13*B^3*a^2*b^4*d^2 + B^3*a^4*b^2*d^2))/d^5 + (((32*(12*B*a*b^7*d^4 + 24*B \\
&*a^3*b^5*d^4 + 12*B*a^5*b^3*d^4))/d^5 - (32*\tan(c + d*x)^{(1/2)}*((64*B^4*a^ \\
&2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a \\
&*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(16*b^9*d^4 + 16*a^ \\
&2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*B^4*a^2*b^2*d^4 - \\
&B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16* \\
&(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} + (32*\tan(c + d*x)^{(1/2)}*(20*B^ \\
&2*a^3*b^4*d^2 + 2*B^2*a^5*b^2*d^2 - 14*B^2*a*b^6*d^2))/d^4)*(((64*B^4*a^2*b \\
&^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b \\
&d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(((64*B^4*a^2*b^2*d^4 \\
&- B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/( \\
&16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(B^ \\
&4*b^5 - 2*B^4*a^2*b^3))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b \\
&^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2 \\
&*a^2*b^2*d^4)))^{(1/2)} + (((32*(13*B^3*a^2*b^4*d^2 + B^3*a^4*b^2*d^2))/d^5 + \\
&(((32*(12*B*a*b^7*d^4 + 24*B*a^3*b^5*d^4 + 12*B*a^5*b^3*d^4))/d^5 + (32*ta \\
&n(c + d*x)^{(1/2)}*((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32* \\
&a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4 \\
&))))^{(1/2)}*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/ \\
&d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4) \\
&))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} - \\
&(32*\tan(c + d*x)^{(1/2)}*(20*B^2*a^3*b^4*d^2 + 2*B^2*a^5*b^2*d^2 - 14*B^2*a*b \\
&^6*d^2))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2 \\
&*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))) \\
&^{(1/2)}*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d \\
&^4))^{(1/2)} - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} \\
&+ (32*\tan(c + d*x)^{(1/2)}*(B^4*b^5 - 2*B^4*a^2*b^3))/d^4)*(((64*B^4*a^2*b^2 \\
&*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b*d^ \\
&2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} + (64*B^5*a*b^3)/d^5))* \\
&(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} \\
&- 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*2i + ata
\end{aligned}$$

$$\begin{aligned}
& n\left(\left(\left(\left(32*(13*B^3*a^2*b^4*d^2 + B^3*a^4*b^2*d^2)\right)/d^5 + \left(\left(32*(12*B*a*b^7*d^4 + 24*B*a^3*b^5*d^4 + 12*B*a^5*b^3*d^4)\right)/d^5 - (32*\tan(c + d*x))^{(1/2)}*-\left(\left(64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4)\right)^{(1/2)} + 8*B^2*a*b*d^2\right)/\left(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)\right)\right)^{(1/2)}*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4)\right)/d^4\right)*-\left(\left(64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4)\right)^{(1/2)} + 8*B^2*a*b*d^2\right)/\left(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)\right)\right)\dots
\end{aligned}$$

$$3.402 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=297

$$\frac{(a(A-B)+b(A+B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(a(A-B)+b(A+B))\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}$$

[Out]  $-2*b^{3/2}*(A*b-B*a)*\arctan(b^{1/2}*\tan(d*x+c)^{1/2}/a^{1/2})/a^{3/2}/(a^2+b^2)/d-1/2*(a*(A-B)+b*(A+B))*\arctan(-1+2^{1/2}*\tan(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}-1/2*(a*(A-B)+b*(A+B))*\arctan(1+2^{1/2}*\tan(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+1/4*(b*(A-B)-a*(A+B))*\ln(1-2^{1/2}*\tan(d*x+c)^{1/2}+\tan(d*x+c))/(a^2+b^2)/d*2^{1/2}-1/4*(b*(A-B)-a*(A+B))*\ln(1+2^{1/2}*\tan(d*x+c)^{1/2}+\tan(d*x+c))/(a^2+b^2)/d*2^{1/2}-2*A/a/d/\tan(d*x+c)^{1/2}$

**Rubi** [A]

time = 0.42, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3690, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a-B)+k(A+B)\text{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)}{\sqrt{2}d(a^2+b^2)} - \frac{(a-B)+k(A+B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}}+1\right)}{\sqrt{2}d(a^2+b^2)} + \frac{(A-B)-a(A+B)\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}}+1\right)}{2\sqrt{2}d(a^2+b^2)} - \frac{(A-B)-a(A+B)\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}}+1\right)}{2\sqrt{2}d(a^2+b^2)} - \frac{2b^{3/2}(A-b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2+b^2)} - \frac{2A}{a\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])),x]

[Out]  $((a*(A-B)+b*(A+B))*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])/(\text{Sqrt}[2]*(a^2+b^2)*d) - ((a*(A-B)+b*(A+B))*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]])/(\text{Sqrt}[2]*(a^2+b^2)*d) - (2*b^{3/2}*(A*b-a*B))*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a]]/(a^{3/2}*(a^2+b^2)*d) + ((b*(A-B)-a*(A+B))*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(2*\text{Sqrt}[2]*(a^2+b^2)*d) - ((b*(A-B)-a*(A+B))*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(2*\text{Sqrt}[2]*(a^2+b^2)*d) - (2*A)/(a*d*\text{Sqrt}[\text{Tan}[c+d*x]])$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^n, x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr



$\text{Int}[b \cdot \tan[e + f \cdot x]], x] /;$  FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3690

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (c + d \cdot \tan[e + f \cdot x])^n, x] \text{ :> } \text{Simp}[b \cdot (A \cdot b - a \cdot B) \cdot (a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^{n+1} / (f \cdot (m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B \cdot (b \cdot c \cdot (m+1) + a \cdot d \cdot (n+1)) + A \cdot (a \cdot (b \cdot c - a \cdot d) \cdot (m+1) - b^2 \cdot d \cdot (m+n+2)) - (A \cdot b - a \cdot B) \cdot (b \cdot c - a \cdot d) \cdot (m+1) \cdot \tan[e + f \cdot x] - b \cdot d \cdot (A \cdot b - a \cdot B) \cdot (m+n+2) \cdot \tan[e + f \cdot x]^2, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2 \cdot m, 2 \cdot n]) && !(LtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^m \cdot (c + d \cdot \tan[e + f \cdot x])^n \cdot (A + C \cdot \tan[e + f \cdot x])^2, x] \text{ :> } \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x, \tan[e + f \cdot x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

$\text{Int}[(c + d \cdot \tan[e + f \cdot x])^n \cdot (A + B \cdot \tan[e + f \cdot x] + C \cdot \tan[e + f \cdot x]^2) / (a + b \cdot \tan[e + f \cdot x]), x] \text{ :> } \text{Dist}[1 / (a^2 + b^2), \text{Int}[(c + d \cdot \tan[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B + a \cdot (A - C) + (a \cdot B - b \cdot (A - C)) \cdot \tan[e + f \cdot x], x], x], x] + \text{Dist}[(A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) / (a^2 + b^2), \text{Int}[(c + d \cdot \tan[e + f \cdot x])^n \cdot ((1 + \tan[e + f \cdot x]^2) / (a + b \cdot \tan[e + f \cdot x])), x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx &= -\frac{2A}{ad\sqrt{\tan(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(Ab - aB) + \frac{1}{2}aA \tan(c + dx) + \frac{1}{2}Ab \tan^2(c + dx)}{\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} dx}{a} \\
&= -\frac{2A}{ad\sqrt{\tan(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}a(Ab - aB) + \frac{1}{2}a(aA + bB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{a(a^2 + b^2)} - \frac{(b^2(Ab - aB)) \int \frac{1}{a + b \tan^2(c + dx)} dx}{a(a^2 + b^2)} \\
&= -\frac{2A}{ad\sqrt{\tan(c + dx)}} - \frac{4 \text{Subst}\left(\int \frac{\frac{1}{2}a(Ab - aB) + \frac{1}{2}a(aA + bB)x^2}{1 + x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{a(a^2 + b^2)d} \\
&= -\frac{2A}{ad\sqrt{\tan(c + dx)}} - \frac{(2b^2(Ab - aB)) \text{Subst}\left(\int \frac{1}{a + bx^2} dx, x, \sqrt{\tan(c + dx)}\right)}{a(a^2 + b^2)d} \\
&= -\frac{2b^{3/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2 + b^2)d} - \frac{2A}{ad\sqrt{\tan(c + dx)}} \\
&= -\frac{2b^{3/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2 + b^2)d} + \frac{(b(A - B) - a(A - B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d} - \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.39, size = 153, normalized size = 0.52

$$\frac{2b^{3/2}(-Ab + aB) \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2 + b^2)} + \frac{\sqrt[4]{-1} a^{(-ia + b)(A - iB)} \text{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{a^2 + b^2}\right) + (ia + b)(A + iB) \tanh^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{a^2 + b^2}\right)}{ad} - \frac{2A}{\sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
[Out] ((2*b^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)) + ((-1)^(1/4)*a*(((I)*a + b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (I*a + b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) - (2*A)/Sqrt[Tan[c + d*x]]/(a*d)

```

**Maple [A]**

time = 0.10, size = 265, normalized size = 0.89

method	result
derivativedivides	$-\frac{2A}{a\sqrt{\tan(dx+c)}} + \frac{(-Ab+aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \left( 1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)) \right) \right)}{4}$
default	$-\frac{2A}{a\sqrt{\tan(dx+c)}} + \frac{(-Ab+aB)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \left( 1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERB OSE)`

[Out]  $\frac{1}{d} \left( -\frac{2A}{a\sqrt{\tan(dx+c)}} + \frac{2}{(a^2+b^2)^{3/2}} \left( \frac{1}{8} (-A^2+B^2) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \left( 1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)) \right) \right) \right. \right.$   
 $\left. \left. + \frac{1}{8} (-A^2-B^2) \sqrt{2} \left( \ln \left( \frac{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \left( -1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)) \right) \right) \right) \right.$   
 $\left. - \frac{2A}{a\sqrt{\tan(dx+c)}} + \frac{2}{(a^2+b^2)^{3/2}} \left( \frac{1}{8} (-A^2+B^2) \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \left( 1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)) \right) \right) \right. \right.$   
 $\left. \left. + \frac{1}{8} (-A^2-B^2) \sqrt{2} \left( \ln \left( \frac{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))} \right) + 2\arctan \left( -1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c)) \right) \right) \right) \right)$

**Maxima** [A]

time = 0.53, size = 235, normalized size = 0.79

$$\frac{8(Ba^2 - Ab^2) \arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) - 2\sqrt{2}((A-B)a + (A+B)b) \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}((A-B)a + (A+B)b) \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) - \sqrt{2}((A+B)a - (A-B)b) \log(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1) - \sqrt{2}((A+B)a - (A-B)b) \log(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1)}{a^2 \sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{4} \left( 8(B^2 a^2 - A^2 b^2) \arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{a^2 b}}\right) / \sqrt{a^2 b} \right) / ((a^2 + a^2 b^2) \sqrt{a^2 b}) - (2\sqrt{2}((A-B)a + (A+B)b) \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}((A-B)a + (A+B)b) \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) - \sqrt{2}((A+B)a - (A-B)b) \log(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2}((A+B)a - (A-B)b) \log(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1)) / (a^2 + b^2) - 8A / (a\sqrt{\tan(dx+c)}) / d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 12924 vs. 2(257) = 514.

time = 72.60, size = 25852, normalized size = 87.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*(4*\sqrt{2})*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*\cos(d*x + c)^2 \\ & - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)* \\ & a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*( \\ & A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*b - 2*(A^2 \\ & - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^4 \\ & + 2*a^2*b^2 + b^4)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)* \\ & a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 \\ & - 2*A^2*B^2 + B^4)*b^4))*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A* \\ & B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + \\ & (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} \\ & *((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4}*\arctan \\ & \left(\frac{(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 + 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 + 12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b^6 + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^8}{(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4}\right) \\ & - \sqrt{2}*(B*a^9 - A*a^8*b + 4*B*a^7*b^2 - 4*A*a^6*b^3 + 6*B*a^5*b^4 - 6*A*a^4*b^5 + 4*B*a^3*b^6 - 4*A*a^2*b^7 + B*a*b^8 - A*b^9)*d^7*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} \\ & + \sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - \sqrt{2}*(B*a^9 - A*a^8*b + 4*B*a^7*b^2 - 4*A*a^6*b^3 + 6*B*a^5*b^4 - 6*A*a^4*b^5 + 4*B*a^3*b^6 - 4*A*a^2*b^7 + B*a*b^8 - A*b^9)*d^7*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}} \\ & + ((A^3 + A*B^2)*a^7 + (A^2*B + B^3)*a^6*b + 3*(A^3 + A*B^2)*a^5*b^2 + 3*(A^2*B + B^3)*a^4*b^3 + 3*(A^3 + A*B^2)*a^3*b^4 + 3*(A^2*B + B^3)*a^2*b^5 + (A^3 + A*B^2)*a*b^6 + (A^2*B + B^3)*b^7)*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}} \\ & + \sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*b - 2*(A^2 - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4))*\sqrt{((A^6 - A^4*B^2 - A^2*B^4 + B^6)*a^6 + 8*(A^5*B - A*B^5)*a^5*b - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^4*b^2 - (A^6 - 17*A^4*B^2 - 17*A^2*B^4 + B^6)*a^2*b^4 - 8*(A^5*B - A*B^5)*a*b^5 + (A^6 - A^4*B^2 - A^2*B^4 + B^6)*b^6)*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2 \end{aligned}$$

```

*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + sqrt(2)*(((A^5 - 2*A^3*B^2 + A*B^4)*a^
7 + (9*A^4*B - 10*A^2*B^3 + B^5)*a^6*b - (A^5 - 26*A^3*B^2 + 9*A*B^4)*a^5*b
^2 - (A^4*B - 18*A^2*B^3 + B^5)*a^4*b^3 - (A^5 - 18*A^3*B^2 + A*B^4)*a^3*b^
4 - (9*A^4*B - 26*A^2*B^3 + B^5)*a^2*b^5 + (A^5 - 10*A^3*B^2 + 9*A*B^4)*a*b
^6 + (A^4*B - 2*A^2*B^3 + B^5)*b^7)*d^3*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4
+ 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) + ((A^6*B - A^4*B^3 - A^2*B^5 + B^7)*
a^5 - (A^7 - 9*A^5*B^2 - A^3*B^4 + 9*A*B^6)*a^4*b - 2*(5*A^6*B - 9*A^4*B^3
- 13*A^2*B^5 + B^7)*a^3*b^2 + 2*(A^7 - 13*A^5*B^2 - 9*A^3*B^4 + 5*A*B^6)*a^
2*b^3 + (9*A^6*B - A^4*B^3 - 9*A^2*B^5 + B^7)*a*b^4 - (A^7 - A^5*B^2 - A^3*
B^4 + A*B^6)*b^5)*d*cos(d*x + c))*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^4 + 2*(A^
4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4 - 2*(A*B*a^6 + A
*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*b - 2*(A^2 - B^2)*a^3*
b^3 - (A^2 - B^2)*a*b^5)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2
+ b^4)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(
A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^
2 + B^4)*b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4)/((a
^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + ((A^8 - 2*A^4*B^4 + B^8)*a^4 + 8*(A^7*B
+ A^5*B^3 - A^3*B^5 - A*B^7)*a^3*b - 2*(A^8 - 8*A^6*B^2 - 18*A^4*B^4 - 8*A
^2*B^6 + B^8)*a^2*b^2 - 8*(A^7*B + A^5*B^3 - A^3*B^5 - A*B^7)*a*b^3 + (A^8
- 2*A^4*B^4 + B^8)*b^4)*sin(d*x + c))/cos(d*x + c))*((A^4 + 2*A^2*B^2 + B^4
)/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*(((A^4*B - B^5)*a^11 - (A^
5 - 4*A^3*B^2 - 5*A*B^4)*a^10*b - (A^4*B + 4*A^2*B^3 + 3*B^5)*a^9*b^2 - (3*
A^5 - 16*A^3*B^2 - 19*A*B^4)*a^8*b^3 - 2*(7*A^4*B + 8*A^2*B^3 + B^5)*a^7*b^
4 - 2*(A^5 - 12*A^3*B^2 - 13*A*B^4)*a^6*b^5 - 2*(13*A^4*B + 12*A^2*B^3 - B^
5)*a^5*b^6 + 2*(A^5 + 8*A^3*B^2 + 7*A*B^4)*a^4*b^7 - (19*A^4*B + 16*A^2*B^3
- 3*B^5)*a^3*b^8 + (3*A^5 + 4*A^3*B^2 + A*B^4)*a^2*b^9 - (5*A^4*B + 4*A^2*
B^3 - B^5)*a*b^10 + (A^5 - A*B^4)*b^11)*d^7*sqr...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.60, size = 2500, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))),x)

[Out] atan(((tan(c + d\*x)^(1/2)\*(64\*A^4\*a^7\*b^7\*d^5 - 32\*A^4\*a^9\*b^5\*d^5) + (((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(tan(c + d\*x)^(1/2)\*(512\*A^2\*a^8\*b^8\*d^7 - 448\*A^2\*a^10\*b^6\*d^7 + 128\*A^2\*a^12\*b^4\*d^7 + 64\*A^2\*a^14\*b^2\*d^7) - (((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(tan(c + d\*x)^(1/2)\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(512\*a^9\*b^9\*d^9 + 512\*a^11\*b^7\*d^9 - 512\*a^13\*b^5\*d^9 - 512\*a^15\*b^3\*d^9) - 512\*A\*a^8\*b^9\*d^8 - 640\*A\*a^10\*b^7\*d^8 + 256\*A\*a^12\*b^5\*d^8 + 384\*A\*a^14\*b^3\*d^8))\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) + 128\*A^3\*a^7\*b^8\*d^6 - 32\*A^3\*a^11\*b^4\*d^6 - 32\*A^3\*a^13\*b^2\*d^6))\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*i + (tan(c + d\*x)^(1/2)\*(64\*A^4\*a^7\*b^7\*d^5 - 32\*A^4\*a^9\*b^5\*d^5) + (((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(tan(c + d\*x)^(1/2)\*(512\*A^2\*a^8\*b^8\*d^7 - 448\*A^2\*a^10\*b^6\*d^7 + 128\*A^2\*a^12\*b^4\*d^7 + 64\*A^2\*a^14\*b^2\*d^7) - (((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(tan(c + d\*x)^(1/2)\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(512\*a^9\*b^9\*d^9 + 512\*a^11\*b^7\*d^9 - 512\*a^13\*b^5\*d^9 - 512\*a^15\*b^3\*d^9) + 512\*A\*a^8\*b^9\*d^8 + 640\*A\*a^10\*b^7\*d^8 - 256\*A\*a^12\*b^5\*d^8 - 384\*A\*a^14\*b^3\*d^8))\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) - 128\*A^3\*a^7\*b^8\*d^6 + 32\*A^3\*a^11\*b^4\*d^6 + 32\*A^3\*a^13\*b^2\*d^6))\*(((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*i)/((tan(c + d\*x)^(1/2)\*(64\*A^4\*a^7\*b^7\*d^5 - 32\*A^4\*a^9\*b^5\*d^5) + (((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)



$$3.403 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx$$

Optimal. Leaf size=325

$$-\frac{(b(A-B)-a(A+B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(b(A-B)-a(A+B))\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}$$

[Out]  $2*b^{5/2}*(A*b-B*a)*\arctan(b^{1/2}*\tan(d*x+c)^{1/2}/a^{1/2})/a^{5/2}/(a^2+b^2)/d+1/2*(b*(A-B)-a*(A+B))*\arctan(-1+2^{1/2}*\tan(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+1/2*(b*(A-B)-a*(A+B))*\arctan(1+2^{1/2}*\tan(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+1/4*(a*(A-B)+b*(A+B))*\ln(1-2^{1/2}*\tan(d*x+c)^{1/2}+\tan(d*x+c))/(a^2+b^2)/d*2^{1/2}-1/4*(a*(A-B)+b*(A+B))*\ln(1+2^{1/2}*\tan(d*x+c)^{1/2}+\tan(d*x+c))/(a^2+b^2)/d*2^{1/2}+2*(A*b-B*a)/a^2/d/\tan(d*x+c)^{1/2}-2/3*A/a/d/\tan(d*x+c)^{3/2}$

Rubi [A]

time = 0.64, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(b(A-B)-a(A+B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)} + \frac{(b(A-B)-a(A+B))\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)} + \frac{(a(A-B)+b(A+B))\log\left(\frac{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}{2\sqrt{2}d(a^2+b^2)}\right)}{2\sqrt{2}d(a^2+b^2)} - \frac{(a(A-B)+b(A+B))\log\left(\frac{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}{2\sqrt{2}d(a^2+b^2)}\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{2(Ab-aB)}{a^2d\sqrt{\tan(c+dx)}} - \frac{2b^{5/2}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d(a^2+b^2)} - \frac{2A}{3a^2d\tan(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])),x]

[Out]  $-(((b*(A-B)-a*(A+B))*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]])/( \text{Sqrt}[2]*(a^2+b^2)*d)) + ((b*(A-B)-a*(A+B))*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]])/( \text{Sqrt}[2]*(a^2+b^2)*d) + (2*b^{5/2}*(A*b-a*B))*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a]]/(a^{5/2}*(a^2+b^2)*d) + ((a*(A-B)+b*(A+B))*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(2*\text{Sqrt}[2]*(a^2+b^2)*d) - ((a*(A-B)+b*(A+B))*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c+d*x]]+\text{Tan}[c+d*x]])/(2*\text{Sqrt}[2]*(a^2+b^2)*d) - (2*A)/(3*a*d*\text{Tan}[c+d*x]^{3/2}) + (2*(A*b-a*B))/(a^2*d*\text{Sqrt}[\text{Tan}[c+d*x]])$

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^n, x], x, (a+b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
(IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3734

```
Int((((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_)
+ (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
```

$A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)$ , Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))} dx &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)} - \frac{2 \int \frac{\frac{3}{2}(Ab - aB) + \frac{3}{2}aA \tan(c + dx) + \frac{3}{2}Ab \tan^2(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx}{3a} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{a^2 d \sqrt{\tan(c + dx)}} + \frac{4 \int \frac{-\frac{3}{4}(a^2 A - Ab^2 + abB) - \frac{3}{4}a^2 A \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{\sqrt{\tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{a^2 d \sqrt{\tan(c + dx)}} + \frac{4 \int \frac{-\frac{3}{4}a^2(aA + bB) + \frac{3}{4}a^2(A - bB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{3a^2(a^2 + b^2)} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{a^2 d \sqrt{\tan(c + dx)}} + \frac{8 \text{Subst}\left(\int \frac{-\frac{3}{4}a^2(aA + bB) + \frac{3}{4}a^2(A - bB) \tan(u)}{\sqrt{\tan(u)}} du\right)}{3a^2(a^2 + b^2)} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{a^2 d \sqrt{\tan(c + dx)}} + \frac{(2b^3(Ab - aB)) \text{Subst}\left(\int \frac{-\frac{3}{4}a^2(aA + bB) + \frac{3}{4}a^2(A - bB) \tan(u)}{\sqrt{\tan(u)}} du\right)}{3a^2(a^2 + b^2)} \\
 &= \frac{2b^{5/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2 + b^2)d} - \frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)} \\
 &= \frac{2b^{5/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2 + b^2)d} + \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2 + b^2)d} \\
 &= -\frac{(b(A - B) - a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d} + \frac{(b(A + B) + a(A - B)) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 2.24, size = 174, normalized size = 0.54

$$\frac{{}_3\sqrt{-1}^{(A-iB)} \text{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{a - ib}\right) + \frac{6b^{5/2}(Ab - aB) \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2 + b^2)} + \frac{{}_3\sqrt{-1}^{(A+iB)} \tanh^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{a + ib}\right) - \frac{2(aA + (-3Ab + 3aB) \tan(c + dx))}{a^2 \tan^{\frac{3}{2}}(c + dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
[Out] ((3*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]/(a - I*b) +
(6*b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]/(a^(5/2)*(a^2 + b^2)) +
(3*(-1)^(1/4)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]/(a + I*b) -
(2*(a*A + (-3*A*b + 3*a*B)*Tan[c + d*x]))/(a^2*Tan[c + d*x]^(3/2)))/(3*d)
```

**Maple [A]**

time = 0.10, size = 286, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{2A}{3a \tan(dx+c)^{\frac{3}{2}}} - \frac{2(-Ab+aB)}{a^2 \sqrt{\tan(dx+c)}} + \frac{(-aA-Bb)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{\frac{2}{\tan(dx+c)}} \right)}{4}}$
default	$\frac{-\frac{2A}{3a \tan(dx+c)^{\frac{3}{2}}} - \frac{2(-Ab+aB)}{a^2 \sqrt{\tan(dx+c)}} + \frac{(-aA-Bb)\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{\frac{2}{\tan(dx+c)}} \right)}{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*A/a/tan(d*x+c)^(3/2)-2/a^2*(-A*b+B*a)/tan(d*x+c)^(1/2)+2/(a^2+b^2)
)*(1/8*(-A*a-B*b)*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)
*(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))) + 2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)) + 2*
arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))) + 1/8*(A*b-B*a)*2^(1/2)*(ln((1-2^(1/2)*t
an(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))) + 2*arct
an(1+2^(1/2)*tan(d*x+c)^(1/2)) + 2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))) + 2/a^
2*(A*b-B*a)*b^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)
))
```

**Maxima [A]**

time = 0.53, size = 256, normalized size = 0.79

$$\frac{24 (Bb^3 - Ab^3) \operatorname{atan}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) + 3 \left( 2\sqrt{2} ((A+B)a - (A-B)b) \operatorname{atan}\left(\frac{1}{\sqrt{2}} \sqrt{2 + \sqrt{\tan(dx+c)}}\right) + 2\sqrt{2} ((A+B)a - (B-A)b) \operatorname{atan}\left(-\frac{1}{\sqrt{2}} \sqrt{2 - \sqrt{\tan(dx+c)}}\right) + \sqrt{2} ((A-B)a + (A+B)b) \log\left(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c)\right) - \sqrt{2} ((A-B)a + (A+B)b) \log\left(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c)\right) \right) + \frac{6(Aa+3(Ba-Ab)\tan(dx+c))}{a^2 \tan(dx+c)^3}}{(a^2+b^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(24*(B*a*b^3 - A*b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4 +
a^2*b^2)*sqrt(a*b)) + 3*(2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sqrt(
2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B)*a +
(A + B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A
- B)*a + (A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/((
a^2 + b^2) + 8*(A*a + 3*(B*a - A*b)*tan(d*x + c))/(a^2*tan(d*x + c)^(3/2)))
/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 13055 vs. 2(281) = 562.

time = 63.76, size = 26114, normalized size = 80.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/12*(12*sqrt(2)*((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^5*cos(d*x + c)
^2 - (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d^5)*sqrt(((A^4 + 2*A^2*B^2 +
B^4)*a^4 + 2*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^2 + (A^4 + 2*A^2*B^2 + B^4)*b^4
+ 2*(A*B*a^6 + A*B*a^4*b^2 - A*B*a^2*b^4 - A*B*b^6 - (A^2 - B^2)*a^5*b - 2*
(A^2 - B^2)*a^3*b^3 - (A^2 - B^2)*a*b^5)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((
a^4 + 2*a^2*b^2 + b^4)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*
B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 +
(A^4 - 2*A^2*B^2 + B^4)*b^4)*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B
- A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*
b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4
)*arctan((((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8 + 4*(A^7*B + 3*A^5*B^3 +
3*A^3*B^5 + A*B^7)*a^7*b + 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^2 +
12*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^3 + 12*(A^7*B + 3*A^5*B^3
+ 3*A^3*B^5 + A*B^7)*a^3*b^5 - 2*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^2*b
^6 + 4*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^7 - (A^8 + 2*A^6*B^2 - 2
*A^2*B^6 - B^8)*b^8)*d^4*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^4 + 8*(A^3*B - A*B
^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2 - 8*(A^3*B - A*B^3)*a*b^3 +
(A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + sqrt
(2)*((A*a^9 + B*a^8*b + 4*A*a^7*b^2 + 4*B*a^6*b^3 + 6*A*a^5*b^4 + 6*B*a^4*b
^5 + 4*A*a^3*b^6 + 4*B*a^2*b^7 + A*a*b^8 + B*b^9)*d^7*sqrt(((A^4 - 2*A^2*B^
2 + B^4)*a^4 + 8*(A^3*B - A*B^3)*a^3*b - 2*(A^4 - 10*A^2*B^2 + B^4)*a^2*b^2
- 8*(A^3*B - A*B^3)*a*b^3 + (A^4 - 2*A^2*B^2 + B^4)*b^4)/((a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^4 +
2*a^2*b^2 + b^4)*d^4)) - ((A^2*B + B^3)*a^7 - (A^3 + A*B^2)*a^6*b + 3*(A^2*
```

$$\begin{aligned}
& B + B^3) * a^5 * b^2 - 3 * (A^3 + A * B^2) * a^4 * b^3 + 3 * (A^2 * B + B^3) * a^3 * b^4 - 3 * (A \\
& ^3 + A * B^2) * a^2 * b^5 + (A^2 * B + B^3) * a * b^6 - (A^3 + A * B^2) * b^7) * d^5 * \text{sqrt}(((A \\
& ^4 - 2 * A^2 * B^2 + B^4) * a^4 + 8 * (A^3 * B - A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + \\
& B^4) * a^2 * b^2 - 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) / ((a^ \\
& 8 + 4 * a^6 * b^2 + 6 * a^4 * b^4 + 4 * a^2 * b^6 + b^8) * d^4))) * \text{sqrt}(((A^4 + 2 * A^2 * B^2 \\
& + B^4) * a^4 + 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^ \\
& 4 + 2 * (A * B * a^6 + A * B * a^4 * b^2 - A * B * a^2 * b^4 - A * B * b^6 - (A^2 - B^2) * a^5 * b - \\
& 2 * (A^2 - B^2) * a^3 * b^3 - (A^2 - B^2) * a * b^5) * d^2 * \text{sqrt}((A^4 + 2 * A^2 * B^2 + B^4) \\
& / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4))) / ((A^4 - 2 * A^2 * B^2 + B^4) * a^4 + 8 * (A^3 * B - \\
& A * B^3) * a^3 * b - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 - 8 * (A^3 * B - A * B^3) * a * b^3 \\
& + (A^4 - 2 * A^2 * B^2 + B^4) * b^4) * \text{sqrt}((((A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * a^6 \\
& + 8 * (A^5 * B - A * B^5) * a^5 * b - (A^6 - 17 * A^4 * B^2 - 17 * A^2 * B^4 + B^6) * a^4 * b^2 \\
& - (A^6 - 17 * A^4 * B^2 - 17 * A^2 * B^4 + B^6) * a^2 * b^4 - 8 * (A^5 * B - A * B^5) * a * b^5 + \\
& (A^6 - A^4 * B^2 - A^2 * B^4 + B^6) * b^6) * d^2 * \text{sqrt}((A^4 + 2 * A^2 * B^2 + B^4) / ((a^ \\
& 4 + 2 * a^2 * b^2 + b^4) * d^4)) * \cos(dx + c) + \text{sqrt}(2) * (((A^4 * B - 2 * A^2 * B^3 + B^ \\
& 5) * a^7 - (A^5 - 10 * A^3 * B^2 + 9 * A * B^4) * a^6 * b - (9 * A^4 * B - 26 * A^2 * B^3 + B^5) * \\
& a^5 * b^2 + (A^5 - 18 * A^3 * B^2 + A * B^4) * a^4 * b^3 - (A^4 * B - 18 * A^2 * B^3 + B^5) * a \\
& ^3 * b^4 + (A^5 - 26 * A^3 * B^2 + 9 * A * B^4) * a^2 * b^5 + (9 * A^4 * B - 10 * A^2 * B^3 + B^5 \\
& ) * a * b^6 - (A^5 - 2 * A^3 * B^2 + A * B^4) * b^7) * d^3 * \text{sqrt}((A^4 + 2 * A^2 * B^2 + B^4) / ( \\
& (a^4 + 2 * a^2 * b^2 + b^4) * d^4)) * \cos(dx + c) - ((A^7 - A^5 * B^2 - A^3 * B^4 + A * \\
& B^6) * a^5 + (9 * A^6 * B - A^4 * B^3 - 9 * A^2 * B^5 + B^7) * a^4 * b - 2 * (A^7 - 13 * A^5 * B^ \\
& 2 - 9 * A^3 * B^4 + 5 * A * B^6) * a^3 * b^2 - 2 * (5 * A^6 * B - 9 * A^4 * B^3 - 13 * A^2 * B^5 + B^ \\
& 7) * a^2 * b^3 + (A^7 - 9 * A^5 * B^2 - A^3 * B^4 + 9 * A * B^6) * a * b^4 + (A^6 * B - A^4 * B^3 \\
& - A^2 * B^5 + B^7) * b^5) * d * \cos(dx + c)) * \text{sqrt}(((A^4 + 2 * A^2 * B^2 + B^4) * a^4 + \\
& 2 * (A^4 + 2 * A^2 * B^2 + B^4) * a^2 * b^2 + (A^4 + 2 * A^2 * B^2 + B^4) * b^4 + 2 * (A * B * a^ \\
& 6 + A * B * a^4 * b^2 - A * B * a^2 * b^4 - A * B * b^6 - (A^2 - B^2) * a^5 * b - 2 * (A^2 - B^2) \\
& * a^3 * b^3 - (A^2 - B^2) * a * b^5) * d^2 * \text{sqrt}((A^4 + 2 * A^2 * B^2 + B^4) / ((a^4 + 2 * a^ \\
& 2 * b^2 + b^4) * d^4))) / ((A^4 - 2 * A^2 * B^2 + B^4) * a^4 + 8 * (A^3 * B - A * B^3) * a^3 * b \\
& - 2 * (A^4 - 10 * A^2 * B^2 + B^4) * a^2 * b^2 - 8 * (A^3 * B - A * B^3) * a * b^3 + (A^4 - 2 * A \\
& ^2 * B^2 + B^4) * b^4) * \text{sqrt}(\sin(dx + c) / \cos(dx + c)) * ((A^4 + 2 * A^2 * B^2 + B^4 \\
& ) / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4))^{1/4} + ((A^8 - 2 * A^4 * B^4 + B^8) * a^4 + 8 * ( \\
& A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a^3 * b - 2 * (A^8 - 8 * A^6 * B^2 - 18 * A^4 * B^4 \\
& - 8 * A^2 * B^6 + B^8) * a^2 * b^2 - 8 * (A^7 * B + A^5 * B^3 - A^3 * B^5 - A * B^7) * a * b^3 + \\
& (A^8 - 2 * A^4 * B^4 + B^8) * b^4) * \sin(dx + c) / \cos(dx + c)) * ((A^4 + 2 * A^2 * B^2 \\
& + B^4) / ((a^4 + 2 * a^2 * b^2 + b^4) * d^4))^{3/4} - \text{sqrt}(2) * (((A^5 - A * B^4) * a^{11} \\
& + (5 * A^4 * B + 4 * A^2 * B^3 - B^5) * a^{10} * b + (3 * A^5 + 4 * A^3 * B^2 + A * B^4) * a^9 * b^2 \\
& + (19 * A^4 * B + 16 * A^2 * B^3 - 3 * B^5) * a^8 * b^3 + 2 * (A^5 + 8 * A^3 * B^2 + 7 * A * B^4) * a \\
& ^7 * b^4 + 2 * (13 * A^4 * B + 12 * A^2 * B^3 - B^5) * a^6 * b^5 - 2 * (A^5 - 12 * A^3 * B^2 - 13 \\
& * A * B^4) * a^5 * b^6 + 2 * (7 * A^4 * B + 8 * A^2 * B^3 + B^5) * a^4 * b^7 - (3 * A^5 - 16 * A^3 * B \\
& ^2 - 19 * A * B^4) * a^3 * b^8 + (A^4 * B + 4 * A^2 * B^3 + 3 * B^5) * a^2 * b^9 - (A^5 - 4 * A^3 \\
& * B^2 - 5 * A * B^4) * a * b^{10} - (A^4 * B - B^5) * b^{11}) * d^{\dots}
\end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx)) \tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2)/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*(5/2)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 12.97, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))),x)

[Out] atan(((tan(c + d\*x)^(1/2)\*(64\*A^4\*a^14\*b^9\*d^5 + 32\*A^4\*a^18\*b^5\*d^5) + (- (64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*((( -((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(tan(c + d\*x)^(1/2)\*(-((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\* (512\*a^18\*b^9\*d^9 + 512\*a^20\*b^7\*d^9 - 512\*a^22\*b^5\*d^9 - 512\*a^24\*b^3\*d^9) - 512\*A\*a^16\*b^10\*d^8 - 512\*A\*a^18\*b^8\*d^8 + 384\*A\*a^20\*b^6\*d^8 + 256\*A\*a^22\*b^4\*d^8 - 128\*A\*a^24\*b^2\*d^8) - tan(c + d\*x)^(1/2)\*(512\*A^2\*a^15\*b^10\*d^7 + 448\*A^2\*a^19\*b^6\*d^7 - 128\*A^2\*a^21\*b^4\*d^7 - 64\*A^2\*a^23\*b^2\*d^7))\*(-((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) + 384\*A^3\*a^15\*b^9\*d^6 - 32\*A^3\*a^19\*b^5\*d^6 - 32\*A^3\*a^21\*b^3\*d^6))\*(-((64\*A^4\*a^2\*b^2\*d^4 - A^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*A^2\*a\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) - 8\*A^2\*a\*

$$\begin{aligned}
& b^4 d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * i + (\tan(c + d*x))^{1/2} * \\
& (64A^4 a^{14} b^9 d^5 + 32A^4 a^{18} b^5 d^5) + (-((64A^4 a^2 b^2 d^4 - \\
& A^4(16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16 \\
& (a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * (((-(64A^4 a^2 b^2 d^4 - A^4 \\
& (16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 \\
& d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * (\tan(c + d*x))^{1/2} * (-((64A^4 a^2 \\
& b^2 d^4 - A^4(16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b \\
& d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * (512a^{18} b^9 d^9 + \\
& 512a^{20} b^7 d^9 - 512a^{22} b^5 d^9 - 512a^{24} b^3 d^9) + 512A a^{16} b^{10} d^8 \\
& + 512A a^{18} b^8 d^8 - 384A a^{20} b^6 d^8 - 256A a^{22} b^4 d^8 + 128A a^{24} \\
& b^2 d^8) - \tan(c + d*x)^{1/2} * (512A^2 a^{15} b^{10} d^7 + 448A^2 a^{19} b^6 \\
& d^7 - 128A^2 a^{21} b^4 d^7 - 64A^2 a^{23} b^2 d^7)) * (-((64A^4 a^2 b^2 d^4 - \\
& A^4(16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (1 \\
& 6(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} - 384A^3 a^{15} b^9 d^6 + 32A^3 \\
& a^{19} b^5 d^6 + 32A^3 a^{21} b^3 d^6)) * (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 \\
& d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + \\
& b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * i) / ((\tan(c + d*x))^{1/2} * (64A^4 a^{14} b^9 \\
& d^5 + 32A^4 a^{18} b^5 d^5) + (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16 \\
& b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + \\
& 2a^2 b^2 d^4))^{1/2} * (((-(64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16b^4 \\
& d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + 2a \\
& ^2 b^2 d^4))^{1/2} * (\tan(c + d*x))^{1/2} * (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 \\
& d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + \\
& b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * (512a^{18} b^9 d^9 + 512a^{20} b^7 d^9 - 51 \\
& 2a^{22} b^5 d^9 - 512a^{24} b^3 d^9) + 512A a^{16} b^{10} d^8 + 512A a^{18} b^8 d^8 \\
& - 384A a^{20} b^6 d^8 - 256A a^{22} b^4 d^8 + 128A a^{24} b^2 d^8) - \tan(c \\
& + d*x)^{1/2} * (512A^2 a^{15} b^{10} d^7 + 448A^2 a^{19} b^6 d^7 - 128A^2 a^{21} b^4 \\
& d^7 - 64A^2 a^{23} b^2 d^7)) * (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 1 \\
& 6b^4 d^4 + 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 \\
& + 2a^2 b^2 d^4))^{1/2} - 384A^3 a^{15} b^9 d^6 + 32A^3 a^{19} b^5 d^6 + 32A^3 \\
& a^{21} b^3 d^6)) * (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16b^4 d^4 + \\
& 32a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 \\
& d^4))^{1/2} - (\tan(c + d*x))^{1/2} * (64A^4 a^{14} b^9 d^5 + 32A^4 a^{18} b^5 d^5 \\
& ^5) + (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4 \\
& ^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * \\
& (((-(64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4))^{1/2} \\
& ^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2} * (\tan \\
& (c + d*x))^{1/2} * (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16b^4 d^4 + 32 \\
& a^2 b^2 d^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4 \\
& ^4))^{1/2} * (512a^{18} b^9 d^9 + 512a^{20} b^7 d^9 - 512a^{22} b^5 d^9 - 512a^{2 \\
& 4} b^3 d^9) - 512A a^{16} b^{10} d^8 - 512A a^{18} b^8 d^8 + 384A a^{20} b^6 d^8 \\
& + 256A a^{22} b^4 d^8 - 128A a^{24} b^2 d^8) - \tan(c + d*x)^{1/2} * (512A^2 a^{15} \\
& b^{10} d^7 + 448A^2 a^{19} b^6 d^7 - 128A^2 a^{21} b^4 d^7 - 64A^2 a^{23} b^2 \\
& d^7)) * (-((64A^4 a^2 b^2 d^4 - A^4(16a^4 d^4 + 16b^4 d^4 + 32a^2 b^2 d^4 \\
& ^4))^{1/2} - 8A^2 a b d^2) / (16(a^4 d^4 + b^4 d^4 + 2a^2 b^2 d^4))^{1/2}
\end{aligned}$$



$$\begin{aligned}
& + 384A^3a^{15}b^9d^6 - 32A^3a^{19}b^5d^6 - 32A^3a^{21}b^3d^6) * (-((6 \\
& 4A^4a^2b^2d^4 - A^4(16a^4d^4 + 16b^4d^4 + 32a^2b^2d^4))^{(1/2)} - \\
& 8A^2a*b*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4)))^{(1/2)} + 64A^5a^{14} \\
& b^8d^4) * (-((64A^4a^2b^2d^4 - A^4(16a^4d^4 + 16b^4d^4 + 32a^2 \\
& *b^2d^4))^{(1/2)} - 8A^2a*b*d^2)/(16*(a^4d^4 + b^4d^4 + 2a^2b^2d^4))) \\
& ^{(1/2)} * 2i + \operatorname{atan}(((\tan(c + d*x))^{(1/2)} * (64A^4a^{14}b^9d^5 + 32A^4a^{18}b^ \\
& 5d^5) + (((64A^4a^2b^2d^4 - A^4(16a^4d^4 \dots
\end{aligned}$$



ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

Rule 3728

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*(b\*c\*m + a\*d\*(n + 1)) + d\*(A\*b + a\*B - b\*C)\*(m + n + 1)\*Tan[e + f\*x] - (C\*m\*(b\*c - a\*d) - b\*B\*d\*(m + n + 1))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && (!IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3734

Int[(((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])

```

+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx &= \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{b(a^2+b^2)d(a+b \tan(c+dx))} + \frac{\int \sqrt{\tan(c+dx)}^{(-\frac{3}{2}a(Ab-aB)+)}{}}{b(a^2+b^2)d(a+b \tan(c+dx))} \\
&= -\frac{(aAb-3a^2B-2b^2B) \sqrt{\tan(c+dx)}}{b^2(a^2+b^2)d} + \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{b(a^2+b^2)d(a+b \tan(c+dx))} \\
&= -\frac{(aAb-3a^2B-2b^2B) \sqrt{\tan(c+dx)}}{b^2(a^2+b^2)d} + \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{b(a^2+b^2)d(a+b \tan(c+dx))} \\
&= -\frac{(aAb-3a^2B-2b^2B) \sqrt{\tan(c+dx)}}{b^2(a^2+b^2)d} + \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{b(a^2+b^2)d(a+b \tan(c+dx))} \\
&= -\frac{(aAb-3a^2B-2b^2B) \sqrt{\tan(c+dx)}}{b^2(a^2+b^2)d} + \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{b(a^2+b^2)d(a+b \tan(c+dx))} \\
&= \frac{a^{3/2}(a^2Ab+5Ab^3-3a^3B-7ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)^2 d} \\
&= \frac{a^{3/2}(a^2Ab+5Ab^3-3a^3B-7ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)^2 d} \\
&= \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B)) \tan^{-1}\left(1-\sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.55, size = 275, normalized size = 0.63

$$\frac{2 \left( (-Ab+3aB) \sqrt{\tan(c+dx)} + \frac{(a^2Ab+2Ab^3-3a^3B-7ab^2B) \sqrt{\tan(c+dx)}}{2(a^2+b^2)} + bB \tan^3(c+dx) - \frac{\left( \sqrt{-1}^{(a+b) \operatorname{Im} \sqrt{2} (A+B)} \operatorname{ArcTan} \left( (-1)^{1/4} \sqrt{\tan(c+dx)} \right) \right)^{a+2i} (-a^2Ab-5Ab^3+3a^3B+7ab^2B) \operatorname{ArcTan} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}} \right) + (-1)^{3/4} \sqrt{(a+b) \operatorname{Im} \sqrt{2} (A+B)} \tan^{-1} \left( (-1)^{1/4} \sqrt{\tan(c+dx)} \right) \right)^{(a+b) \operatorname{Im} (c+dx)}}{b^2 d (a+b \tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] (2\*((-(A\*b) + 3\*a\*B)\*Sqrt[Tan[c + d\*x]] + ((a^2\*A\*b + 2\*A\*b^3 - 3\*a^3\*B - 4\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(2\*(a^2 + b^2)) + b\*B\*Tan[c + d\*x]^(3/2) - ((-1)^(1/4)\*(a + I\*b)^2\*b^(5/2)\*(I\*A + B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + a^(3/2)\*(-(a^2\*A\*b) - 5\*A\*b^3 + 3\*a^3\*B + 7\*a\*b^2\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] + (-1)^(3/4)\*b^(5/2)\*(I\*a + b)^2\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]])\*(a + b\*Tan[c + d\*x]))/(2\*Sqrt[b]\*(a^2 + b^2)^2))/(b^2\*d\*(a + b\*Tan[c + d\*x]))

Maple [A]

time = 0.11, size = 351, normalized size = 0.81

method	result
derivativedivides	$\frac{2B\left(\sqrt{\tan(dx+c)}\right)}{b^2} + \frac{(-2Aab+a^2B-b^2B)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$
default	$\frac{2B\left(\sqrt{\tan(dx+c)}\right)}{b^2} + \frac{(-2Aab+a^2B-b^2B)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVE RBOSE)

[Out] 1/d\*(2\*B/b^2\*tan(d\*x+c)^(1/2)+2/(a^2+b^2)^2\*(1/8\*(-2\*A\*a\*b+B\*a^2-B\*b^2)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/8\*(-A\*a^2+A\*b^2-2\*B\*a\*b)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+2\*a^2/b^2/(a^2+b^2)^2\*((-1/2\*A\*a^2\*b-1/2\*A\*b^3+1/2\*B\*a^3+1/2\*B\*a\*b^2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))+1/2\*(A\*a^2\*b+5\*A\*b^3-3\*B\*a^3-7\*B\*a\*b^2)/(a\*b)^(1/2)\*arctan(b\*tan(d\*x+c)^(1/2)/(a\*b)^(1/2))))

Maxima [A]

time = 0.52, size = 377, normalized size = 0.86

$$\frac{1}{d} \left( \frac{2B\sqrt{\tan(dx+c)}}{b^2} + \frac{(-2Aab+a^2B-b^2B)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4} + \frac{2a^2}{(a^2+b^2)^2} \left( \frac{(-1/2Aa^2b-1/2Ab^3+1/2Ba^3+1/2Bab^2)\tan(dx+c)^{1/2}}{a+b\tan(dx+c)} + \frac{1}{2} \frac{(Aa^2b+5Ab^3-3Ba^3-7Bab^2)}{(ab)^{1/2}} \arctan\left(\frac{b\tan(dx+c)^{1/2}}{(ab)^{1/2}}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(4*(3*B*a^5 - A*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3)*arctan(b*sqrt(tan(d
*x + c))/sqrt(a*b))/((a^4*b^2 + 2*a^2*b^4 + b^6)*sqrt(a*b)) - 4*(B*a^3 - A*
a^2*b)*sqrt(tan(d*x + c))/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d*x + c))
+ (2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)
*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b
- (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqr
t(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sqrt(tan(d*x +
c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b
^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)/(a^4 + 2*a^2*b^2 +
b^4) - 8*B*sqrt(tan(d*x + c))/b^2)/d
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 25059 vs. 2(397) = 794.

time = 123.05, size = 50230, normalized size = 115.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*((a^14*b^2 + 5*a^12*b^4 + 9*a^10*b^6 + 5*a^8*b^8 - 5*a^6*b^
10 - 9*a^4*b^12 - 5*a^2*b^14 - b^16)*d^5*cos(d*x + c)^2 + 2*(a^13*b^3 + 6*a
^11*b^5 + 15*a^9*b^7 + 20*a^7*b^9 + 15*a^5*b^11 + 6*a^3*b^13 + a*b^15)*d^5*
cos(d*x + c)*sin(d*x + c) + (a^12*b^4 + 6*a^10*b^6 + 15*a^8*b^8 + 20*a^6*b^
10 + 15*a^4*b^12 + 6*a^2*b^14 + b^16)*d^5)*sqrt(((A^4 + 2*A^2*B^2 + B^4)*a^
8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 +
4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 - 2*(A*B*a
^12 - 2*A*B*a^10*b^2 - 17*A*B*a^8*b^4 - 28*A*B*a^6*b^6 - 17*A*B*a^4*b^8 - 2
*A*B*a^2*b^10 + A*B*b^12 - 2*(A^2 - B^2)*a^11*b - 6*(A^2 - B^2)*a^9*b^3 - 4
*(A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A^2 - B^2)*a^3*b^9 + 2*(A
^2 - B^2)*a*b^11)*d^2*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B -
A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)
*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*
a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7
+ (A^4 - 2*A^2*B^2 + B^4)*b^8))*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^
3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B -
A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A
*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)
*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))/(a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 5
```

$$\begin{aligned}
& 6a^{10}b^6 + 70a^8b^8 + 56a^6b^{10} + 28a^4b^{12} + 8a^2b^{14} + b^{16})d^4 \\
& 4)) * ((A^4 + 2A^2B^2 + B^4) / ((a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) \\
& 8)d^4))^{(3/4)} * \arctan(-(((A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{16} + 8(A^7B \\
& B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{15}b + 40(A^7B + 3A^5B^3 + 3A^3B^5 \\
& ^5 + AB^7)a^{13}b^3 - 20(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^{12}b^4 + 72 \\
& *(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^{11}b^5 - 64(A^8 + 2A^6B^2 - 2 \\
& *A^2B^6 - B^8)a^{10}b^6 + 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^9b \\
& ^7 - 90(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^8b^8 - 40(A^7B + 3A^5B^3 \\
& + 3A^3B^5 + AB^7)a^7b^9 - 64(A^8 + 2A^6B^2 - 2A^2B^6 - B^8)a^6b \\
& ^{10} - 72(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^5b^{11} - 20(A^8 + 2A^6 \\
& B^2 - 2A^2B^6 - B^8)a^4b^{12} - 40(A^7B + 3A^5B^3 + 3A^3B^5 + AB \\
& ^7)a^3b^{13} - 8(A^7B + 3A^5B^3 + 3A^3B^5 + AB^7)a^2b^{15} + (A^8 + 2 \\
& A^6B^2 - 2A^2B^6 - B^8)b^{16})d^4 * \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 + 16 \\
& *(A^3B - AB^3)a^7b - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 - 112(A^3B \\
& B - AB^3)a^5b^3 + 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 + 112(A^3B \\
& - AB^3)a^3b^5 - 4(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 - 16(A^3B - A \\
& B^3)a^2b^7 + (A^4 - 2A^2B^2 + B^4)b^8) / ((a^{16} + 8a^{14}b^2 + 28a^{12}b^4 \\
& + 56a^{10}b^6 + 70a^8b^8 + 56a^6b^{10} + 28a^4b^{12} + 8a^2b^{14} + b^{16} \\
& )d^4)} * \sqrt{((A^4 + 2A^2B^2 + B^4) / ((a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) \\
& d^4)) + \sqrt{2} * ((B*a^{18} - 2A*a^{17}b + 7B*a^{16}b^2 - 16A*a^{15} \\
& *b^3 + 20B*a^{14}b^4 - 56A*a^{13}b^5 + 28B*a^{12}b^6 - 112A*a^{11}b^7 + 14 \\
& B*a^{10}b^8 - 140A*a^9b^9 - 14B*a^8b^{10} - 112A*a^7b^{11} - 28B*a^6b^{12} \\
& - 56A*a^5b^{13} - 20B*a^4b^{14} - 16A*a^3b^{15} - 7B*a^2b^{16} - 2A*a^2b^{17} \\
& - B*b^{18})d^7 * \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 + 16(A^3B - AB^3)a^7b \\
& - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 - 112(A^3B - AB^3)a^5b^3 + \\
& 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 + 112(A^3B - AB^3)a^3b^5 - 4 \\
& *(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 - 16(A^3B - AB^3)a^2b^7 + (A^4 - 2 \\
& *A^2B^2 + B^4)b^8) / ((a^{16} + 8a^{14}b^2 + 28a^{12}b^4 + 56a^{10}b^6 + 70a^8 \\
& b^8 + 56a^6b^{10} + 28a^4b^{12} + 8a^2b^{14} + b^{16})d^4)} * \sqrt{((A^4 + 2 \\
& *A^2B^2 + B^4) / ((a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8)d^4)) + (( \\
& A^3 + AB^2)a^{14} + 2(A^2B + B^3)a^{13}b + 5(A^3 + AB^2)a^{12}b^2 + 12 \\
& (A^2B + B^3)a^{11}b^3 + 9(A^3 + AB^2)a^{10}b^4 + 30(A^2B + B^3)a^9b^5 \\
& + 5(A^3 + AB^2)a^8b^6 + 40(A^2B + B^3)a^7b^7 - 5(A^3 + AB^2)a^6 \\
& b^8 + 30(A^2B + B^3)a^5b^9 - 9(A^3 + AB^2)a^4b^{10} + 12(A^2B + B \\
& ^3)a^3b^{11} - 5(A^3 + AB^2)a^2b^{12} + 2(A^2B + B^3)a^2b^{13} - (A^3 + A \\
& *B^2)b^{14})d^5 * \sqrt{((A^4 - 2A^2B^2 + B^4)a^8 + 16(A^3B - AB^3)a^7b \\
& - 4(3A^4 - 22A^2B^2 + 3B^4)a^6b^2 - 112(A^3B - AB^3)a^5b^3 + \\
& 2(19A^4 - 102A^2B^2 + 19B^4)a^4b^4 + 112(A^3B - AB^3)a^3b^5 - 4 \\
& *(3A^4 - 22A^2B^2 + 3B^4)a^2b^6 - 16(A^3B - AB^3)a^2b^7 + (A^4 - 2 \\
& *A^2B^2 + B^4)b^8) / ((a^{16} + 8a^{14}b^2 + 28a^{12}b^4 + 56a^{10}b^6 + 70a^8 \\
& b^8 + 56a^6b^{10} + 28a^4b^{12} + 8a^2b^{14} + b^{16})d^4)} * \sqrt{((A^4 + \\
& 2A^2B^2 + B^4)a^8 + 4(A^4 + 2A^2B^2 + B^4)a^6b^2 + 6(A^4 + 2A^2B^2 + \\
& B^4)a^4b^4 + 4(A^4 + 2A^2B^2 + B^4)a^2b^6 + (A^4 + 2A^2B^2 + B^4)b^8 - \\
& 2(AB*a^{12} - 2AB*a^{10}b^2 - 17AB*a^8b^4 - 28AB*a^6b^6 - 17AB*a^4b^8 - \\
& 2AB*a^2b^{10} + AB*b^{12} - 2(A^2 - B^2)a^{11}b - 6(A^
\end{aligned}$$



$2 - B^2) * a^9 * b^3 - 4 * (A^2 - B^2) * a^7 * b^5 + 4 * (A^2 - B^2) * a^5 * b^7 + 6 * (A^2 - B^2) * a^3 * b^9 + 2 * (A^2 - B^2) * a * b^{11}) * d^2 * \text{sqrt}(\dots$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{5}{2}}(c + dx)}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(5/2)/(a + b\*tan(c + d\*x))\*\*2, x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 36.19, size = 2500, normalized size = 5.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] (log((((((((((128\*b^3\*tan(c + d\*x)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2) - (128\*B\*a\*b^2\*(7\*a^4 + b^4 + 8\*a^2\*b^2))/d)\*(4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (64\*B^2\*a\*tan(c + d\*x)^(1/2)\*(18\*a^10 - 15\*b^10 + 17\*a^2\*b^8 - a^4\*b^6 + 97\*a^6\*b^4 + 84\*a^8\*b^2))/(b^2\*d^2\*(a^2 + b^2)^2))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (32\*B^3\*a^4\*(127\*a^2\*b^6 - 112\*b^8 - 9\*a^8 + 173\*a^4\*b^4 + 21\*a^6\*b^2))/(b^3\*d^3\*(a^2 + b^2)^3))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 - (16\*B^4\*tan(c + d\*x)^(1/2)\*(9\*a^12 + 2\*b^12 + 4\*a^2\*b^10 + 2\*a^4\*b^8 - 49\*a^6\*b^6 + 7\*a^8\*b^4 + 33\*a^



$$\begin{aligned}
& )^2)^{(1/2)} + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4)^{(1/2)} \\
& ))/4 + (64*B^2*a*\tan(c + d*x)^{(1/2)}*(18*a^{10} - 15*b^{10} + 17*a^2*b^8 - a^4*b \\
& ^6 + 97*a^6*b^4 + 84*a^8*b^2))/(b^2*d^2*(a^2 + b^2)^2))*(-(4*(-B^4*d^4*(a^4 \\
& + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(d^4*(a \\
& ^2 + b^2)^4))^{(1/2)})/4 - (32*B^3*a^4*(127*a^2*b^6 - 112*b^8 - 9*a^8 + 173*a \\
& ^4*b^4 + 21*a^6*b^2))/(b^3*d^3*(a^2 + b^2)^3))*(-(4*(-B^4*d^4*(a^4 + b^4 - \\
& 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2) \\
& ^4))^{(1/2)})/4 - (16*B^4*\tan(c + d*x)^{(1/2)}*(9*a^{12} + 2*b^{12} + 4*a^2*b^{10} + \\
& 2*a^4*b^8 - 49*a^6*b^6 + 7*a^8*b^4 + 33*a^{10}*b^2))/(b^3*d^4*(a^2 + b^2)^4)) \\
& *(-(4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)}...
\end{aligned}$$



ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int((((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx &= \frac{a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d(a+b \tan(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+b(Ab-aB) \tan(c+dx)}{\sqrt{\tan(c+dx)}}}{b(a^2+b^2)} \\
&= \frac{a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d(a+b \tan(c+dx))} + \frac{\int \frac{-b(a^2A-Ab^2+2abB)+b(2aAb-a^2B)}{\sqrt{\tan(c+dx)}}}{b(a^2+b^2)^2} \\
&= \frac{a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d(a+b \tan(c+dx))} + \frac{2 \text{Subst}\left(\int \frac{-b(a^2A-Ab^2+2abB)+b(2aAb-a^2B)}{1+x^4}\right)}{b(a^2+b^2)} \\
&= \frac{a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d(a+b \tan(c+dx))} + \frac{(a^2Ab-3Ab^3+a^3B+5ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)^2 d} \\
&= \frac{\sqrt{a}(a^2Ab-3Ab^3+a^3B+5ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)^2 d} \\
&= \frac{\sqrt{a}(a^2Ab-3Ab^3+a^3B+5ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)^2 d} \\
&= -\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B)) \tan^{-1}\left(1-\sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.21, size = 230, normalized size = 0.59

$$\frac{\sqrt{a}(a^2Ab-3Ab^3+a^3B+5ab^2B) \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{-1} b^{3/2} ((a+b)^2(A-B) \text{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + (a-b)^2(A+B) \text{tanh}^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right))}{\sqrt{b}(a^2+b^2)^2} - \frac{2B \sqrt{\tan(c+dx)}}{a+b \tan(c+dx)} + \frac{(aAb+a^2B+2b^2B) \sqrt{\tan(c+dx)}}{(a^2+b^2)(a+b \tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] ((Sqrt[a]\*(a^2\*A\*b - 3\*A\*b^3 + a^3\*B + 5\*a\*b^2\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])]/Sqrt[a]] + (-1)^(1/4)\*b^(3/2)\*((a + I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + (a - I\*b)^2\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]))/(Sqrt[b]\*(a^2 + b^2)^2 - (2\*B\*Sqrt[Tan[c + d\*x]])/(a + b\*Tan[c + d\*x]) + ((a\*A\*b + a^2\*B + 2\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/((a^2 + b^2)\*(a + b\*Tan[c + d\*x])))/(b\*d)

**Maple [A]**

time = 0.11, size = 336, normalized size = 0.86

method	result
derivativedivides	$\frac{(-a^2A+Ab^2-2Bab)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$
default	$\frac{(-a^2A+Ab^2-2Bab)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)+2\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*(2/(a^2+b^2)^2\*(1/8\*(-A\*a^2+A\*b^2-2\*B\*a\*b)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/8\*(2\*A\*a\*b-B\*a^2+B\*b^2)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+2\*a/(a^2+b^2)^2\*(1/2\*(A\*a^2\*b+A\*b^3-B\*a^3-B\*a\*b^2)/b\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))+1/2\*(A\*a^2\*b-3\*A\*b^3+B\*a^3+5\*B\*a\*b^2)/b/(a\*b)^(1/2)\*arctan(b\*tan(d\*x+c)^(1/2)/(a\*b)^(1/2))))

Maxima [A]

time = 0.51, size = 354, normalized size = 0.91

$$\frac{1}{4} \frac{(A^2 a^4 + A^2 a^3 b + 5 A^2 a^2 b^2 - 3 A^2 a b^3) \arctan\left(\frac{b \sqrt{\tan(dx+c)}}{\sqrt{a b}}\right) + (A^2 a^2 b - A^2 a b^2) \sqrt{\tan(dx+c)} + (A^2 a^3 b + a^2 b^3 + (A^2 b^2 + b^4) \tan(dx+c)) \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + \sqrt{\tan(dx+c)})\right) - (2 \sqrt{2} (A+B) a^2 - 2(A-B) a b - (A+B) b^2) \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} - \sqrt{\tan(dx+c)})\right) + 2 \sqrt{2} (A+B) a^2 - 2(A-B) a b - (A+B) b^2}{(a^2 + b^2)^2 \sqrt{a b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*(4\*(B\*a^4 + A\*a^3\*b + 5\*B\*a^2\*b^2 - 3\*A\*a\*b^3)\*arctan(b\*sqrt(tan(d\*x + c))/sqrt(a\*b)))/((a^4\*b + 2\*a^2\*b^3 + b^5)\*sqrt(a\*b)) - 4\*(B\*a^2 - A\*a\*b)\*sqrt(tan(d\*x + c))/(a^3\*b + a\*b^3 + (a^2\*b^2 + b^4)\*tan(d\*x + c)) - (2\*sqrt(2))\*((A + B)\*a^2 - 2\*(A - B)\*a\*b - (A + B)\*b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*((A + B)\*a^2 - 2\*(A - B)\*a\*b - (A + B)\*b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(tan(d\*x + c)))) + sqrt(2)\*((A -



$B)a^2 + 2*(A + B)*a*b - (A - B)*b^2)*\log(\sqrt{2}*\sqrt{\tan(dx + c)}) + \tan(dx + c) + 1) - \sqrt{2}*((A - B)a^2 + 2*(A + B)a*b - (A - B)b^2)*\log(-\sqrt{2}*\sqrt{\tan(dx + c)}) + \tan(dx + c) + 1))/(a^4 + 2*a^2*b^2 + b^4))/d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 24919 vs. 2(352) = 704.

time = 111.30, size = 49950, normalized size = 127.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(3/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{4}*(4*\sqrt{2}*((a^{14}*b + 5*a^{12}*b^3 + 9*a^{10}*b^5 + 5*a^8*b^7 - 5*a^6*b^9 - 9*a^4*b^{11} - 5*a^2*b^{13} - b^{15})*d^5*\cos(dx + c)^2 + 2*(a^{13}*b^2 + 6*a^{11}*b^4 + 15*a^9*b^6 + 20*a^7*b^8 + 15*a^5*b^{10} + 6*a^3*b^{12} + a*b^{14})*d^5*\cos(dx + c)*\sin(dx + c) + (a^{12}*b^3 + 6*a^{10}*b^5 + 15*a^8*b^7 + 20*a^6*b^9 + 15*a^4*b^{11} + 6*a^2*b^{13} + b^{15})*d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 + 2*(A*B*a^{12} - 2*A*B*a^{10}*b^2 - 17*A*B*a^8*b^4 - 28*A*B*a^6*b^6 - 17*A*B*a^4*b^8 - 2*A*B*a^2*b^{10} + A*B*b^{12} - 2*(A^2 - B^2)*a^{11}*b - 6*(A^2 - B^2)*a^9*b^3 - 4*(A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A^2 - B^2)*a^3*b^9 + 2*(A^2 - B^2)*a*b^{11})*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))/((a^{16} + 8*a^{14}*b^2 + 28*a^{12}*b^4 + 56*a^{10}*b^6 + 70*a^8*b^8 + 56*a^6*b^{10} + 28*a^4*b^{12} + 8*a^2*b^{14} + b^{16})*d^4))*((A^4 + 2*A^2*B^2 + B^4))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))^(3/4)*\arctan((((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{16} + 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{15}*b + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{13}*b^3 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12}*b^4 + 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^5 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^6 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^7 - 90*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^8 - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^9 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^{10} - 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^{11} - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^{12} - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a$

```

^3*b^13 - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^15 + (A^8 + 2*A^6*B
^2 - 2*A^2*B^6 - B^8)*b^16)*d^4*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3
*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A
*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*
B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*
a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56
*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4
))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*d^4)) - sqrt(2)*((A*a^18 + 2*B*a^17*b + 7*A*a^16*b^2 + 16*B*a^15*b^3
+ 20*A*a^14*b^4 + 56*B*a^13*b^5 + 28*A*a^12*b^6 + 112*B*a^11*b^7 + 14*A*a^1
0*b^8 + 140*B*a^9*b^9 - 14*A*a^8*b^10 + 112*B*a^7*b^11 - 28*A*a^6*b^12 + 56
*B*a^5*b^13 - 20*A*a^4*b^14 + 16*B*a^3*b^15 - 7*A*a^2*b^16 + 2*B*a*b^17 - A
*b^18)*d^7*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b - 4
*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19
*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A
^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*
B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^
8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt((A^4 + 2*A^2*
B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - ((A^2*B
+ B^3)*a^14 - 2*(A^3 + A*B^2)*a^13*b + 5*(A^2*B + B^3)*a^12*b^2 - 12*(A^3
+ A*B^2)*a^11*b^3 + 9*(A^2*B + B^3)*a^10*b^4 - 30*(A^3 + A*B^2)*a^9*b^5 + 5
*(A^2*B + B^3)*a^8*b^6 - 40*(A^3 + A*B^2)*a^7*b^7 - 5*(A^2*B + B^3)*a^6*b^8
- 30*(A^3 + A*B^2)*a^5*b^9 - 9*(A^2*B + B^3)*a^4*b^10 - 12*(A^3 + A*B^2)*a
^3*b^11 - 5*(A^2*B + B^3)*a^2*b^12 - 2*(A^3 + A*B^2)*a*b^13 - (A^2*B + B^3)
*b^14)*d^5*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b - 4
*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19
*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A
^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*
B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8*b^
8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(((A^4 + 2*A^
2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 +
B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)
*b^8 + 2*(A*B*a^12 - 2*A*B*a^10*b^2 - 17*A*B*a^8*b^4 - 28*A*B*a^6*b^6 - 17*
A*B*a^4*b^8 - 2*A*B*a^2*b^10 + A*B*b^12 - 2*(A^2 - B^2)*a^11*b - 6*(A^2 - B
^2)*a^9*b^3 - 4*(A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A^2 - B^2)
*a^3*b^9 + 2*(A^2 - B^2)*a*b^11)*d^2*sqrt((A^4 ...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x))\*\*2, x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 33.22, size = 2500, normalized size = 6.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] (log((((((((((128\*b^3\*tan(c + d\*x)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*((4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*A^2\*a\*b^3\*d^2 + 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2) - (128\*A\*a\*b^2\*(5\*b^4 - a^4 + 4\*a^2\*b^2))/d)\*(4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*A^2\*a\*b^3\*d^2 + 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (64\*A^2\*a\*b^2\*tan(c + d\*x)^(1/2)\*(a^6 - 15\*b^6 + 35\*a^2\*b^4 - 13\*a^4\*b^2))/(d^2\*(a^2 + b^2)^2))\*((4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*A^2\*a\*b^3\*d^2 + 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (32\*A^3\*a^2\*b\*(a^6 - 39\*b^6 + 43\*a^2\*b^4 - 13\*a^4\*b^2))/(d^3\*(a^2 + b^2)^3))\*((4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*A^2\*a\*b^3\*d^2 + 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 - (16\*A^4\*b\*tan(c + d\*x)^(1/2)\*(a^8 + 2\*b^8 - 5\*a^2\*b^6 + 17\*a^4\*b^4 - 7\*a^6\*b^2))/(d^4\*(a^2 + b^2)^4))\*((4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*A^2\*a\*b^3\*d^2 + 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (16\*A^5\*a\*b^4\*(a^2 - 3\*b^2))/(d^5\*(a^2 + b^2)^4))\*((((192\*A^4\*a^2\*b^6\*d^4 - 16\*A^4\*b^8\*d^4 - 16\*A^4\*a^8\*d^4 - 608\*A^4\*a^4\*b^4\*d^4 + 192\*A^4\*a^6\*b^2\*d^4)^(1/2) - 16\*A^2\*a\*b^3\*d^2 + 16\*A^2\*a^3\*b\*d^2)/(a^8\*d^4 + b^8\*d^4 + 4\*a^2\*b^6\*d^4 + 6\*a^4\*b^4\*d^4 + 4\*a^6\*b^2\*d^4))^(1/2))/4 + (log((((((((((128\*b^3\*tan(c + d\*x)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*(-(4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) + 16\*A^2\*a\*b^3\*d^2 - 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2) - (128\*A\*a\*b^2\*(5\*b^4 - a^4 + 4\*a^2\*b^2))/d)\*(-(4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) + 16\*A^2\*a\*b^3\*d^2 - 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (64\*A^2\*a\*b^2\*tan(c + d\*x)^(1/2)\*(a^6 - 15\*b^6 + 35\*a^2\*b^4 - 13\*a^4\*b^2))/(d^2\*(a^2 + b^2)^2))\*(-(4\*(-A^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) + 16\*A^2\*a\*b^3\*d^2 - 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2) + 16\*A^2\*a\*b^3\*d^2 - 16\*A^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2)



$$3.406 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=391

$$\frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

```
[Out] 1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^2/d*2^(1/2)+1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^2/d*2^(1/2)-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/(a^2+b^2)^2/d/a^(1/2)/b^(1/2)-(A*b-B*a)*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]

time = 0.52, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3689, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{a^2 + b^2}}\right)}{\sqrt{2} d (a^2 + b^2)^2} + \frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{a^2 + b^2}}\right)}{\sqrt{2} d (a^2 + b^2)^2} - \frac{(b^2 - ab) \ln(\tan(c+dx))}{2d^2 (a^2 + b^2)} - \frac{(-a^2(A+B) + 2ab(A-B) + b^2(A+B)) \ln(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)})}{2\sqrt{2} d (a^2 + b^2)} + \frac{(-a^2(A+B) + 2ab(A-B) + b^2(A+B)) \ln(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)})}{2\sqrt{2} d (a^2 + b^2)} - \frac{(a^2 - b^2 + 2ab(A+B) - ab) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{a^2 + b^2}}\right)}{\sqrt{2} d (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2*(A - B) - b^2*(A - B) + 2*a*b*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^2*d)) + ((a^2*(A - B) - b^2*(A - B) + 2*a*b*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*(a^2 + b^2)^2*d) - (((3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B))*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a^2 + b^2)^2*d) - ((2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) + ((2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)^2*d) - ((A*b - a*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_) ]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

Rule 3734

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\tan(c+dx)}(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx &= -\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d(a+b \tan(c+dx))} - \frac{\int \frac{-\frac{1}{2}b(Ab-aB)-b(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}(a+b \tan(c+dx))}{b(a^2+b^2)} \\
 &= -\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d(a+b \tan(c+dx))} - \frac{\int \frac{-b(2aAb-a^2B+b^2B)-b(a^2A-Ab^2)}{\sqrt{\tan(c+dx)}}(a+b \tan(c+dx))}{b(a^2+b^2)^2} \\
 &= -\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d(a+b \tan(c+dx))} - \frac{2\text{Subst}\left(\int \frac{-b(2aAb-a^2B+b^2B)-b(a^2A-Ab^2)}{1+x} dx\right)}{b} \\
 &= -\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d(a+b \tan(c+dx))} - \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)}{b} \\
 &= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a^2+b^2)^2d} \\
 &= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a^2+b^2)^2d} \\
 &= -\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^2d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.92, size = 220, normalized size = 0.56

$$\frac{\sqrt{a}(-3a^2Ab+Ab^3+a^3B-3a^2b^2B)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{-1}a^{(a+b)^2(iA+B)}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{\tan(c+dx)}}{a^2+b^2}\right) + (a-b)^2(-iA+B)\text{tanh}^{-1}\left(\frac{(-1)^{3/4}\sqrt{\tan(c+dx)}}{a^2+b^2}\right)}{\sqrt{b}(a^2+b^2)} + \frac{(-Ab+aB)\sqrt{\tan(c+dx)} + \frac{b(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{a+b \tan(c+dx)}}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] ((Sqrt[a]\*(-3\*a^2\*A\*b + A\*b^3 + a^3\*B - 3\*a\*b^2\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/(Sqrt[b]\*(a^2 + b^2)) + ((-1)^(1/4)\*a\*((a + I\*b)^2\*(I\*A + B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + (a - I\*b)^2\*((-I)\*A + B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]])/(a^2 + b^2) + (- (A\*b) + a\*B)\*Sqrt[Tan[c + d\*x]] + (b\*(A\*b - a\*B)\*Tan[c + d\*x]^(3/2))/(a + b\*Tan[c + d\*x]))/(a\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.11, size = 332, normalized size = 0.85



method	result
derivativedivides	$\frac{(2Aab - a^2B + b^2B)\sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4} + 2 \arctan \left( - \right)$
default	$\frac{(2Aab - a^2B + b^2B)\sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4} + 2 \arctan \left( - \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVE  
RBOSE)`

[Out]  $\frac{1}{d} \cdot \frac{2}{(a^2+b^2)^2} \cdot \frac{1}{8} \cdot (2Aab - B^2a^2 + B^2b^2) \cdot 2^{1/2} \cdot \left( \ln \left( \frac{(1+2^{1/2}) \tan(dx+c)^{1/2} + \tan(dx+c)}{(1-2^{1/2}) \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{-1+2^{1/2} \tan(dx+c)^{1/2}} \right) + 1/8 \cdot (Aa^2 - Ab^2 + 2Bab) \cdot 2^{1/2} \cdot \left( \ln \left( \frac{(1-2^{1/2}) \tan(dx+c)^{1/2} + \tan(dx+c)}{(1+2^{1/2}) \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{-1+2^{1/2} \tan(dx+c)^{1/2}} \right) \right) - \frac{2}{(a^2+b^2)^2} \cdot \left( \frac{1}{2} Aa^2b + \frac{1}{2} Ab^3 - \frac{1}{2} B^2a^3 - \frac{1}{2} B^2ab^2 \right) \cdot \frac{\tan(dx+c)^{1/2}}{(a+b \tan(dx+c))} + \frac{1}{2} \cdot \frac{(3Aa^2b - Ab^3 - B^2a^3 + 3B^2ab^2)}{(ab)^{1/2}} \cdot \arctan \left( \frac{b \tan(dx+c)^{1/2}}{(ab)^{1/2}} \right) \right)$

**Maxima [A]**

time = 0.52, size = 341, normalized size = 0.87

$$\frac{(Aa^2 - 3Ab^2 + 2B^2a^2) \arctan \left( \frac{\sqrt{\tan(dx+c)}}{ab} \right) + \frac{4(Ba - Ab) \sqrt{\tan(dx+c)}}{a^2b^2} + \frac{2\sqrt{2} \left( (A-B)a^2 + 2(A+B)ab - (A-B)b^2 \right) \arctan \left( \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right) \right) + 2\sqrt{2} \left( (A-B)a^2 + 2(A+B)ab - (A-B)b^2 \right) \arctan \left( \sqrt{2} \left( \sqrt{\tan(dx+c)} - \tan(dx+c) \right) \right) - \sqrt{2} \left( (A-B)a^2 - 2(A+B)ab + (A-B)b^2 \right) \ln \left( \sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) + \sqrt{2} \left( (A-B)a^2 - 2(A+B)ab + (A-B)b^2 \right) \ln \left( \sqrt{2} \sqrt{\tan(dx+c)} - \tan(dx+c) + 1 \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm  
="maxima")`

[Out]  $\frac{1}{4} \cdot (4 \cdot (B^2a^3 - 3Aa^2b - 3B^2ab^2 + Ab^3) \cdot \arctan \left( \frac{b \sqrt{\tan(dx+c)}}{\sqrt{ab}} \right) / \sqrt{ab} + ((a^4 + 2a^2b^2 + b^4) \cdot \sqrt{ab}) + 4 \cdot (Ba - Ab) \cdot \sqrt{\tan(dx+c)}) / (a^3 + ab^2 + (a^2b + b^3) \cdot \tan(dx+c)) + (2 \cdot \sqrt{2} \cdot ((A - B) \cdot a^2 + 2 \cdot (A + B) \cdot ab - (A - B) \cdot b^2) \cdot \arctan \left( \frac{1}{2} \cdot \sqrt{2} \cdot (\sqrt{2} + 2 \cdot \sqrt{\tan(dx+c)}) \right) + 2 \cdot \sqrt{2} \cdot ((A - B) \cdot a^2 + 2 \cdot (A + B) \cdot ab - (A - B) \cdot b^2) \cdot \arctan \left( -\frac{1}{2} \cdot \sqrt{2} \cdot (\sqrt{2} - 2 \cdot \sqrt{\tan(dx+c)}) \right) - \sqrt{2} \cdot ((A + B) \cdot a^2 - 2 \cdot (A$

- B)\*a\*b - (A + B)\*b^2)\*log(sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) + sqrt(2)\*((A + B)\*a^2 - 2\*(A - B)\*a\*b - (A + B)\*b^2)\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1))/(a^4 + 2\*a^2\*b^2 + b^4))/d

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 25027 vs. 2(347) = 694.

time = 123.47, size = 50058, normalized size = 128.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] [-1/4\*(4\*sqrt(2)\*((a^15\*b + 5\*a^13\*b^3 + 9\*a^11\*b^5 + 5\*a^9\*b^7 - 5\*a^7\*b^9 - 9\*a^5\*b^11 - 5\*a^3\*b^13 - a\*b^15)\*d^5\*cos(d\*x + c)^2 + 2\*(a^14\*b^2 + 6\*a^12\*b^4 + 15\*a^10\*b^6 + 20\*a^8\*b^8 + 15\*a^6\*b^10 + 6\*a^4\*b^12 + a^2\*b^14)\*d^5\*cos(d\*x + c)\*sin(d\*x + c) + (a^13\*b^3 + 6\*a^11\*b^5 + 15\*a^9\*b^7 + 20\*a^7\*b^9 + 15\*a^5\*b^11 + 6\*a^3\*b^13 + a\*b^15)\*d^5)\*sqrt(((A^4 + 2\*A^2\*B^2 + B^4)\*a^8 + 4\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^6\*b^2 + 6\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^4\*b^4 + 4\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^2\*b^6 + (A^4 + 2\*A^2\*B^2 + B^4)\*b^8 - 2\*(A\*B\*a^12 - 2\*A\*B\*a^10\*b^2 - 17\*A\*B\*a^8\*b^4 - 28\*A\*B\*a^6\*b^6 - 17\*A\*B\*a^4\*b^8 - 2\*A\*B\*a^2\*b^10 + A\*B\*b^12 - 2\*(A^2 - B^2)\*a^11\*b - 6\*(A^2 - B^2)\*a^9\*b^3 - 4\*(A^2 - B^2)\*a^7\*b^5 + 4\*(A^2 - B^2)\*a^5\*b^7 + 6\*(A^2 - B^2)\*a^3\*b^9 + 2\*(A^2 - B^2)\*a\*b^11)\*d^2\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)/((a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8)\*d^4)))/((A^4 - 2\*A^2\*B^2 + B^4)\*a^8 + 16\*(A^3\*B - A\*B^3)\*a^7\*b - 4\*(3\*A^4 - 22\*A^2\*B^2 + 3\*B^4)\*a^6\*b^2 - 112\*(A^3\*B - A\*B^3)\*a^5\*b^3 + 2\*(19\*A^4 - 102\*A^2\*B^2 + 19\*B^4)\*a^4\*b^4 + 112\*(A^3\*B - A\*B^3)\*a^3\*b^5 - 4\*(3\*A^4 - 22\*A^2\*B^2 + 3\*B^4)\*a^2\*b^6 - 16\*(A^3\*B - A\*B^3)\*a\*b^7 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^8)\*sqrt(((A^4 - 2\*A^2\*B^2 + B^4)\*a^8 + 16\*(A^3\*B - A\*B^3)\*a^7\*b - 4\*(3\*A^4 - 22\*A^2\*B^2 + 3\*B^4)\*a^6\*b^2 - 112\*(A^3\*B - A\*B^3)\*a^5\*b^3 + 2\*(19\*A^4 - 102\*A^2\*B^2 + 19\*B^4)\*a^4\*b^4 + 112\*(A^3\*B - A\*B^3)\*a^3\*b^5 - 4\*(3\*A^4 - 22\*A^2\*B^2 + 3\*B^4)\*a^2\*b^6 - 16\*(A^3\*B - A\*B^3)\*a\*b^7 + (A^4 - 2\*A^2\*B^2 + B^4)\*b^8)/((a^16 + 8\*a^14\*b^2 + 28\*a^12\*b^4 + 56\*a^10\*b^6 + 70\*a^8\*b^8 + 56\*a^6\*b^10 + 28\*a^4\*b^12 + 8\*a^2\*b^14 + b^16)\*d^4))\*((A^4 + 2\*A^2\*B^2 + B^4)/((a^8 + 4\*a^6\*b^2 + 6\*a^4\*b^4 + 4\*a^2\*b^6 + b^8)\*d^4))^(3/4)\*arctan(-(((A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^16 + 8\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^15\*b + 40\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^13\*b^3 - 20\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^12\*b^4 + 72\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^11\*b^5 - 64\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^10\*b^6 + 40\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^9\*b^7 - 90\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^8\*b^8 - 40\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^7\*b^9 - 64\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^6\*b^10 - 72\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 + A\*B^7)\*a^5\*b^11 - 20\*(A^8 + 2\*A^6\*B^2 - 2\*A^2\*B^6 - B^8)\*a^4\*b^12 - 40\*(A^7\*B + 3\*A^5\*B^3 + 3\*A^3\*B^5 +

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A*B^7)*a^3*b^13 - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^15 + (A^8
+ 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^16)*d^4*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8
+ 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(
A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A
^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B
- A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12
*b^4 + 56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 +
b^16)*d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*d^4)) + sqrt(2)*((B*a^18 - 2*A*a^17*b + 7*B*a^16*b^2 - 16*A*
a^15*b^3 + 20*B*a^14*b^4 - 56*A*a^13*b^5 + 28*B*a^12*b^6 - 112*A*a^11*b^7 +
14*B*a^10*b^8 - 140*A*a^9*b^9 - 14*B*a^8*b^10 - 112*A*a^7*b^11 - 28*B*a^6*
b^12 - 56*A*a^5*b^13 - 20*B*a^4*b^14 - 16*A*a^3*b^15 - 7*B*a^2*b^16 - 2*A*a
*b^17 - B*b^18)*d^7*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*
a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^
3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5
- 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4
- 2*A^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 +
70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt((A^4
+ 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))
+ ((A^3 + A*B^2)*a^14 + 2*(A^2*B + B^3)*a^13*b + 5*(A^3 + A*B^2)*a^12*b^2 +
12*(A^2*B + B^3)*a^11*b^3 + 9*(A^3 + A*B^2)*a^10*b^4 + 30*(A^2*B + B^3)*a^
9*b^5 + 5*(A^3 + A*B^2)*a^8*b^6 + 40*(A^2*B + B^3)*a^7*b^7 - 5*(A^3 + A*B^2
)*a^6*b^8 + 30*(A^2*B + B^3)*a^5*b^9 - 9*(A^3 + A*B^2)*a^4*b^10 + 12*(A^2*B
+ B^3)*a^3*b^11 - 5*(A^3 + A*B^2)*a^2*b^12 + 2*(A^2*B + B^3)*a*b^13 - (A^3
+ A*B^2)*b^14)*d^5*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*
a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^
3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5
- 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4
- 2*A^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 +
70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(((A
^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*
A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B
^2 + B^4)*b^8 - 2*(A*B*a^12 - 2*A*B*a^10*b^2 - 17*A*B*a^8*b^4 - 28*A*B*a^6*
b^6 - 17*A*B*a^4*b^8 - 2*A*B*a^2*b^10 + A*B*b^12 - 2*(A^2 - B^2)*a^11*b - 6
*(A^2 - B^2)*a^9*b^3 - 4*(A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A
^2 - B^2)*a^3*b^9 + 2*(A^2 - B^2)*a*b^11)*d^2*s...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\tan(c + dx)}}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)





$$3.407 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=391

$$\frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} \quad (2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)$$

[Out]  $-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)^2/d*2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)^2/d*2^{(1/2)}+(5*A*a^2*b+A*b^3-3*B*a^3+B*a*b^2)*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})*b^{(1/2)}/a^{(3/2)}/(a^2+b^2)^2/d+b*(A*b-B*a)*\tan(d*x+c)^{(1/2)}/a/(a^2+b^2)/d/(a+b*\tan(d*x+c))$

Rubi [A]

time = 0.53, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3690, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(-a^2(A+B)+2ab(A-B)+b^2(A+B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)^2} - \frac{(-a^2(A+B)+2ab(A-B)+b^2(A+B))\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)^2} + \frac{M(a-b)\sqrt{\tan(c+dx)}}{d(a^2+b^2)(a+b\tan(c+dx))} - \frac{(a^2(A-B)+2ab(A+B)-b^2(A-B))\log(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1)}{2\sqrt{2}d(a^2+b^2)^2} + \frac{(a^2(A-B)+2ab(A+B)-b^2(A-B))\log(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1)}{2\sqrt{2}d(a^2+b^2)^2} + \frac{\sqrt{2}(-3a^2b+5a^2b^3+a^2b^3+ab^3)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{a}\right)}{a^{3/2}d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(a+b*\operatorname{Tan}[c+d*x])^2),x]$

[Out]  $((2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\operatorname{ArcTan}[1-\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]/(\operatorname{Sqrt}[2]*(a^2+b^2)^2*d)-((2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\operatorname{ArcTan}[1+\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]/(\operatorname{Sqrt}[2]*(a^2+b^2)^2*d)+(\operatorname{Sqrt}[b]*(5*a^2*A*b+A*b^3-3*a^3*B+a*b^2*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a]]/(a^{(3/2)}*(a^2+b^2)^2*d)-((a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\operatorname{Log}[1-\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]+\operatorname{Tan}[c+d*x]]/(2*\operatorname{Sqrt}[2]*(a^2+b^2)^2*d)+((a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\operatorname{Log}[1+\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]+\operatorname{Tan}[c+d*x]]/(2*\operatorname{Sqrt}[2]*(a^2+b^2)^2*d)+b*(A*b-a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]/(a*(a^2+b^2)*d*(a+b*\operatorname{Tan}[c+d*x]))$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3734

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2} dx &= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d(a + b \tan(c + dx))} + \frac{\int \frac{\frac{1}{2}(2a^2 A + Ab^2 + abB) - a(Ab - aB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{a(a^2 + b^2)} \\
&= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d(a + b \tan(c + dx))} + \frac{\int \frac{a(a^2 A - Ab^2 + 2abB) - a(2aAb - a^2 B)}{\sqrt{\tan(c + dx)}} dx}{a(a^2 + b^2)^2} \\
&= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d(a + b \tan(c + dx))} + \frac{2 \text{Subst}\left(\int \frac{a(a^2 A - Ab^2 + 2abB) - a}{1 + a \tan^2(x)} dx\right)}{a(a^2 + b^2)^2} \\
&= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d(a + b \tan(c + dx))} + \frac{(b(5a^2 Ab + Ab^3 - 3a^3 B + a^2 B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2} (a^2 + b^2)^2 d} \\
&= \frac{\sqrt{b} (5a^2 Ab + Ab^3 - 3a^3 B + ab^2 B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2} (a^2 + b^2)^2 d} \\
&= \frac{\sqrt{b} (5a^2 Ab + Ab^3 - 3a^3 B + ab^2 B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2} (a^2 + b^2)^2 d} \\
&= \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.74, size = 204, normalized size = 0.52

$$\frac{\sqrt{b} (5a^2 Ab + Ab^3 - 3a^3 B + ab^2 B) \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2)} + \frac{\sqrt{-1} \left(-a(a + ib)^2 (A - iB) \text{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{a^2 + b^2}\right) - a(a - ib)^2 (A + iB) \text{tanh}^{-1}\left(\frac{(-1)^{3/4} \sqrt{\tan(c + dx)}}{a^2 + b^2}\right)\right)}{a(a^2 + b^2) d} + \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{a + b \tan(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^2), x]

[Out] ((Sqrt[b]\*(5\*a^2\*A\*b + A\*b^3 - 3\*a^3\*B + a\*b^2\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/(Sqrt[a]\*(a^2 + b^2)) + ((-1)^(1/4)\*(-(a\*(a + I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]])] - a\*(a - I\*b)^2\*(A + I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]])]/(a^2 + b^2) + (b\*(A\*b - a\*B)\*Sqrt[Tan[c + d\*x]])/(a + b\*Tan[c + d\*x]))/(a\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.11, size = 336, normalized size = 0.86

method	result
derivativedivides	$\frac{(a^2A - Ab^2 + 2Bab)\sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan(-1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right)) \right)}{4}$
default	$\frac{(a^2A - Ab^2 + 2Bab)\sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan(-1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right)) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)^2*(1/8*(A*a^2-A*b^2+2*B*a*b)*2^(1/2)*(ln((1+2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))) + 2*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2)) + 2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))) + 1/8*(-2*A*
a*b+B*a^2-B*b^2)*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(
1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))) + 2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)) + 2*a
rctan(-1+2^(1/2)*tan(d*x+c)^(1/2))) + 2*b/(a^2+b^2)^2*(1/2*(A*a^2*b+A*b^3-B*
a^3-B*a*b^2)/a*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)) + 1/2*(5*A*a^2*b+A*b^3-3*B*a
^3+B*a*b^2)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))))
```

**Maxima [A]**

time = 0.53, size = 356, normalized size = 0.91

$$\frac{(A+B \tan(dx+c)) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan(-1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right)) \right)}{4(a^2+b^2)^2} + \frac{2b}{(a^2+b^2)^2} \left( \frac{1}{2} \frac{Aa^2b + Ab^3 - Ba^3 - Babb^2}{a \tan(dx+c)^{1/2} (a+b \tan(dx+c))} + \frac{1}{2} \frac{5Aa^2b + Ab^3 - 3Ba^3 + Babb^2}{a (ab)^{1/2} \arctan \left( \frac{b \tan(dx+c)^{1/2}}{(ab)^{1/2}} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(4*(3*B*a^3*b - 5*A*a^2*b^2 - B*a*b^3 - A*b^4)*arctan(b*sqrt(tan(d*x +
c))/sqrt(a*b))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a*b)) + 4*(B*a*b - A*b^2)*s
qrt(tan(d*x + c))/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c)) - (2*sqrt(
2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)
*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A -
```

$B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*\log(\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1) - \sqrt{2}*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*\log(-\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1))/(a^4 + 2*a^2*b^2 + b^4))/d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 25042 vs. 2(355) = 710.

time = 117.14, size = 50087, normalized size = 128.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/tan(dx+c)^(1/2)/(a+b\*tan(dx+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*(4*\sqrt{2}*((a^{15} + 5*a^{13}*b^2 + 9*a^{11}*b^4 + 5*a^9*b^6 - 5*a^7*b^8 - \\ & 9*a^5*b^{10} - 5*a^3*b^{12} - a*b^{14})*d^5*\cos(dx + c)^2 + 2*(a^{14}*b + 6*a^{12}* \\ & b^3 + 15*a^{10}*b^5 + 20*a^8*b^7 + 15*a^6*b^9 + 6*a^4*b^{11} + a^2*b^{13})*d^5*\cos(dx + c)*\sin(dx + c) + (a^{13}*b^2 + 6*a^{11}*b^4 + 15*a^9*b^6 + 20*a^7*b^8 \\ & + 15*a^5*b^{10} + 6*a^3*b^{12} + a*b^{14})*d^5)*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 \\ & + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + \\ & 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 + 2*(A*B*a^{12} \\ & - 2*A*B*a^{10}*b^2 - 17*A*B*a^8*b^4 - 28*A*B*a^6*b^6 - 17*A*B*a^4*b^8 - 2* \\ & A*B*a^2*b^{10} + A*B*b^{12} - 2*(A^2 - B^2)*a^{11}*b - 6*(A^2 - B^2)*a^9*b^3 - 4* \\ & (A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A^2 - B^2)*a^3*b^9 + 2*(A^2 \\ & - B^2)*a*b^{11})*d^2*\sqrt{(A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4* \\ & *b^4 + 4*a^2*b^6 + b^8)*d^4)}]/((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A \\ & *B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)* \\ & a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a \\ & ^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 \\ & + (A^4 - 2*A^2*B^2 + B^4)*b^8))*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3* \\ & *B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A \\ & *B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A* \\ & B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)* \\ & a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^{16} + 8*a^{14}*b^2 + 28*a^{12}*b^4 + 56 \\ & *a^{10}*b^6 + 70*a^8*b^8 + 56*a^6*b^{10} + 28*a^4*b^{12} + 8*a^2*b^{14} + b^{16})*d^4 \\ & )*((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) \\ & )*d^4))^{3/4}*\arctan(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{16} + 8*(A^7*B \\ & + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{15}*b + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 \\ & + A*B^7)*a^{13}*b^3 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{12}*b^4 + 72*( \\ & A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^5 - 64*(A^8 + 2*A^6*B^2 - 2*A \\ & ^2*B^6 - B^8)*a^{10}*b^6 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^9*b^7 \\ & - 90*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^8 - 40*(A^7*B + 3*A^5*B^3 + \\ & 3*A^3*B^5 + A*B^7)*a^7*b^9 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^6*b^{10} \\ & - 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^{11} - 20*(A^8 + 2*A^6* \\ & B^2 - 2*A^2*B^6 - B^8)*a^4*b^{12} - 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7 \end{aligned}$$

```

)*a^3*b^13 - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^15 + (A^8 + 2*A^
6*B^2 - 2*A^2*B^6 - B^8)*b^16)*d^4*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(
A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B
- A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B -
A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^
3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 +
56*a^10*b^6 + 70*a^8*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*
d^4))*sqrt((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4)) - sqrt(2)*((A*a^18 + 2*B*a^17*b + 7*A*a^16*b^2 + 16*B*a^15*b
^3 + 20*A*a^14*b^4 + 56*B*a^13*b^5 + 28*A*a^12*b^6 + 112*B*a^11*b^7 + 14*A*
a^10*b^8 + 140*B*a^9*b^9 - 14*A*a^8*b^10 + 112*B*a^7*b^11 - 28*A*a^6*b^12 +
56*B*a^5*b^13 - 20*A*a^4*b^14 + 16*B*a^3*b^15 - 7*A*a^2*b^16 + 2*B*a*b^17
- A*b^18)*d^7*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b
- 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*
(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(
3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A
^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8
*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt((A^4 + 2*A
^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - ((A^
2*B + B^3)*a^14 - 2*(A^3 + A*B^2)*a^13*b + 5*(A^2*B + B^3)*a^12*b^2 - 12*(A
^3 + A*B^2)*a^11*b^3 + 9*(A^2*B + B^3)*a^10*b^4 - 30*(A^3 + A*B^2)*a^9*b^5
+ 5*(A^2*B + B^3)*a^8*b^6 - 40*(A^3 + A*B^2)*a^7*b^7 - 5*(A^2*B + B^3)*a^6*
b^8 - 30*(A^3 + A*B^2)*a^5*b^9 - 9*(A^2*B + B^3)*a^4*b^10 - 12*(A^3 + A*B^2
)*a^3*b^11 - 5*(A^2*B + B^3)*a^2*b^12 - 2*(A^3 + A*B^2)*a*b^13 - (A^2*B + B
^3)*b^14)*d^5*sqrt(((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b
- 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*
(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(
3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A
^2*B^2 + B^4)*b^8)/((a^16 + 8*a^14*b^2 + 28*a^12*b^4 + 56*a^10*b^6 + 70*a^8
*b^8 + 56*a^6*b^10 + 28*a^4*b^12 + 8*a^2*b^14 + b^16)*d^4))*sqrt(((A^4 + 2
*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^
2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B
^4)*b^8 + 2*(A*B*a^12 - 2*A*B*a^10*b^2 - 17*A*B*a^8*b^4 - 28*A*B*a^6*b^6 -
17*A*B*a^4*b^8 - 2*A*B*a^2*b^10 + A*B*b^12 - 2*(A^2 - B^2)*a^11*b - 6*(A^2
- B^2)*a^9*b^3 - 4*(A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A^2 - B
^2)*a^3*b^9 + 2*(A^2 - B^2)*a*b^11)*d^2*sqrt((A...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^2 \sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*2\*sqrt(tan(c + d\*x))), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 33.53, size = 2500, normalized size = 6.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^2),x)

[Out] (log(- (((((((((256\*B\*b^3\*(2\*a^4 - b^4 + a^2\*b^2))/d - 128\*b^3\*tan(c + d\*x)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) + 16\*B^2\*a\*b^3\*d^2 - 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) + 16\*B^2\*a\*b^3\*d^2 - 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2)/4 - (64\*B^2\*a\*b^2\*tan(c + d\*x)^(1/2)\*(a^6 + 17\*b^6 - 29\*a^2\*b^4 + 19\*a^4\*b^2))/(d^2\*(a^2 + b^2)^2))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) + 16\*B^2\*a\*b^3\*d^2 - 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2)/4 + (32\*B^3\*a\*b^2\*(a^6 + 13\*b^6 - 45\*a^2\*b^4 + 39\*a^4\*b^2))/(d^3\*(a^2 + b^2)^3))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) + 16\*B^2\*a\*b^3\*d^2 - 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2)/4 - (16\*B^4\*b^3\*tan(c + d\*x)^(1/2)\*(9\*a^6 - 3\*b^6 + 3\*a^2\*b^4 - 17\*a^4\*b^2))/(d^4\*(a^2 + b^2)^4))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) + 16\*B^2\*a\*b^3\*d^2 - 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2)/4 - (8\*B^5\*b^3\*(9\*a^4 - b^4))/(d^5\*(a^2 + b^2)^4))\*(((192\*B^4\*a^2\*b^6\*d^4 - 16\*B^4\*b^8\*d^4 - 16\*B^4\*a^8\*d^4 - 608\*B^4\*a^4\*b^4\*d^4 + 192\*B^4\*a^6\*b^2\*d^4)^(1/2) + 16\*B^2\*a\*b^3\*d^2 - 16\*B^2\*a^3\*b\*d^2)/(a^8\*d^4 + b^8\*d^4 + 4\*a^2\*b^6\*d^4 + 6\*a^4\*b^4\*d^4 + 4\*a^6\*b^2\*d^4))^1/2)/4 + (log(- (((((((((256\*B\*b^3\*(2\*a^4 - b^4 + a^2\*b^2))/d - 128\*b^3\*tan(c + d\*x)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*(-(4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2))\*(-4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2)/4 - (64\*B^2\*a\*b^2\*tan(c + d\*x)^(1/2)\*(a^6 + 17\*b^6 - 29\*a^2\*b^4 + 19\*a^4\*b^2))/(d^2\*(a^2 + b^2)^2))\*(-4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2))^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^1/2)/4 + (32\*B^3\*



$$3.408 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=439

$$\frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} - \frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

```
[Out] -b^(3/2)*(7*A*a^2*b+3*A*b^3-5*B*a^3-B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)
)/a^(1/2))/a^(5/2)/(a^2+b^2)^2/d-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arct
an(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2*(A-B)-b^2*(A
-B)+2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^2/d*2^(1/2)+
1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+
c))/(a^2+b^2)^2/d*2^(1/2)-1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*ln(1+2^(1/2
)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^2/d*2^(1/2)+(-2*A*a^2-3*A*b^2+B*a*
b)/a^2/(a^2+b^2)/d/tan(d*x+c)^(1/2)+b*(A*b-B*a)/a/(a^2+b^2)/d/tan(d*x+c)^(1
/2)/(a+b*tan(d*x+c))
```

**Rubi** [A]

time = 0.74, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{\tan(c+dx)}}\right)}{\sqrt{2} (a^2 + b^2)^2 d} - \frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{\tan(c+dx)}}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{b^2(A-B) - a^2(A+B) + 2ab(A+B)}{4(a^2 + b^2)^2 d} \ln\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{b^2(A-B) - a^2(A+B) + 2ab(A+B)}{4(a^2 + b^2)^2 d} \ln\left(\frac{1 + \sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{\tan(c+dx)}}\right) + \frac{(-2Aa^2 - 3Ab^2 + Bba)}{a^2(a^2 + b^2)d} \frac{1}{\tan(c+dx)^{1/2}} + \frac{b(Ab - Ba)}{a(a^2 + b^2)d} \frac{1}{\tan(c+dx)^{1/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

```
[Out] ((a^2*(A - B) - b^2*(A - B) + 2*a*b*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]]]/(Sqrt[2]*(a^2 + b^2)^2*d) - ((a^2*(A - B) - b^2*(A - B) + 2*a*b*(
A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^2*d) -
(b^(3/2)*(7*a^2*A*b + 3*A*b^3 - 5*a^3*B - a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x])/Sqrt[a]]]/(a^(5/2)*(a^2 + b^2)^2*d) + ((2*a*b*(A - B) - a^2*(A
+ B) + b^2*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2
*Sqrt[2]*(a^2 + b^2)^2*d) - ((2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*Lo
g[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^2*
d) - (2*a^2*A + 3*A*b^2 - a*b*B)/(a^2*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]]) + (
b*(A*b - a*B))/(a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]))
```

**Rule 65**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a



\*c]

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2} dx &= \frac{b(Ab - aB)}{a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} + \int \frac{\frac{1}{2}(2a^2A + 3Ab^2 - abB)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2} dx \\
&= -\frac{2a^2A + 3Ab^2 - abB}{a^2(a^2 + b^2)d\sqrt{\tan(c + dx)}} + \frac{b(Ab - aB)}{a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} \\
&= -\frac{2a^2A + 3Ab^2 - abB}{a^2(a^2 + b^2)d\sqrt{\tan(c + dx)}} + \frac{b(Ab - aB)}{a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} \\
&= -\frac{2a^2A + 3Ab^2 - abB}{a^2(a^2 + b^2)d\sqrt{\tan(c + dx)}} + \frac{b(Ab - aB)}{a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} \\
&= -\frac{2a^2A + 3Ab^2 - abB}{a^2(a^2 + b^2)d\sqrt{\tan(c + dx)}} + \frac{b(Ab - aB)}{a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} \\
&= -\frac{b^{3/2}(7a^2Ab + 3Ab^3 - 5a^3B - ab^2B) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2 + b^2)^2d} \\
&= -\frac{b^{3/2}(7a^2Ab + 3Ab^3 - 5a^3B - ab^2B) \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2 + b^2)^2d} \\
&= \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)^2d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.53, size = 239, normalized size = 0.54

$$\frac{b^{3/2}(-7a^2Ab-3Ab^3+5a^3B+ab^2B)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt[4]{-1} a^{-(a+b)^2(A-iB)}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{\tan(c+dx)}}{a^2+b^2}\right) + (a-b)^2(A+iB)\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{\tan(c+dx)}}{a^2+b^2}\right) + \frac{-2a^2A-3Ab^2+abB}{a\sqrt{\tan(c+dx)}} + \frac{b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2), x]

[Out] ((b^(3/2)\*(-7\*a^2\*A\*b - 3\*A\*b^3 + 5\*a^3\*B + a\*b^2\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/(a^(3/2)\*(a^2 + b^2)) + ((-1)^(1/4)\*a\*((-I)\*(a + I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + I\*(a - I\*b)^2\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]))/(a^2 + b^2) + (-2\*a^2\*A - 3\*A\*b^2 + a\*b\*B)/(a\*Sqrt[Tan[c + d\*x]]) + (b\*(A\*b - a\*B))/(Sqrt[Tan[c + d\*x]])\*(a + b\*Tan[c + d\*x]))/(a\*(a^2 + b^2)\*d)

Maple [A]

time = 0.11, size = 352, normalized size = 0.80

method	result
derivativedivides	$-\frac{2A}{a^2\sqrt{\tan(dx+c)}} + \frac{(-2Aab+a^2B-b^2B)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)\right)}{4}$
default	$-\frac{2A}{a^2\sqrt{\tan(dx+c)}} + \frac{(-2Aab+a^2B-b^2B)\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}{1-\sqrt{2}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)}\right)+2\arctan\left(1+\sqrt{2}\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}+\tan(dx+c)\right)\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVE RBOSE)

[Out] 1/d\*(-2\*A/a^2/tan(d\*x+c)^(1/2)+2/(a^2+b^2)^2\*(1/8\*(-2\*A\*a\*b+B\*a^2-B\*b^2)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/8\*(-A\*a^2+A\*b^2-2\*B\*a\*b)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))))-2\*b^2/a^2/(

$$a^2+b^2)^2 * ((1/2 * A * a^2 * b + 1/2 * A * b^3 - 1/2 * B * a^3 - 1/2 * B * a * b^2) * \tan(dx+c)^{(1/2)} / (a+b * \tan(dx+c)) + 1/2 * (7 * A * a^2 * b + 3 * A * b^3 - 5 * B * a^3 - B * a * b^2) / (a * b)^{(1/2)} * \arctan(b * \tan(dx+c)^{(1/2)} / (a * b)^{(1/2)}))$$

**Maxima** [A]

time = 0.52, size = 396, normalized size = 0.90

$$\frac{1/4 * (4 * (5 * B * a^3 * b^2 - 7 * A * a^2 * b^3 + B * a * b^4 - 3 * A * b^5) * \arctan(b * \sqrt{\tan(dx+c)}) / \sqrt{a * b} - ((a^6 + 2 * a^4 * b^2 + a^2 * b^4) * \sqrt{a * b}) - (2 * \sqrt{2} * ((A - B) * a^2 + 2 * (A + B) * a * b - (A - B) * b^2) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 * \sqrt{\tan(dx+c)})) + 2 * \sqrt{2} * ((A - B) * a^2 + 2 * (A + B) * a * b - (A - B) * b^2) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 * \sqrt{\tan(dx+c)})) - \sqrt{2} * ((A + B) * a^2 - 2 * (A - B) * a * b - (A + B) * b^2) * \log(\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2} * ((A + B) * a^2 - 2 * (A - B) * a * b - (A + B) * b^2) * \log(-\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1)) / (a^4 + 2 * a^2 * b^2 + b^4) - 4 * (2 * A * a^3 + 2 * A * a * b^2 + (2 * A * a^2 * b - B * a * b^2 + 3 * A * b^3) * \tan(dx+c)) / ((a^4 * b + a^2 * b^3) * \tan(dx+c)^{(3/2)} + (a^5 + a^3 * b^2) * \sqrt{\tan(dx+c)})) / d}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*(4\*(5\*B\*a^3\*b^2 - 7\*A\*a^2\*b^3 + B\*a\*b^4 - 3\*A\*b^5)\*arctan(b\*sqrt(tan(dx + c)))/sqrt(a\*b))/((a^6 + 2\*a^4\*b^2 + a^2\*b^4)\*sqrt(a\*b)) - (2\*sqrt(2)\*((A - B)\*a^2 + 2\*(A + B)\*a\*b - (A - B)\*b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(tan(dx + c)))) + 2\*sqrt(2)\*((A - B)\*a^2 + 2\*(A + B)\*a\*b - (A - B)\*b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(tan(dx + c)))) - sqrt(2)\*((A + B)\*a^2 - 2\*(A - B)\*a\*b - (A + B)\*b^2)\*log(sqrt(2)\*sqrt(tan(dx + c)) + tan(dx + c) + 1) + sqrt(2)\*((A + B)\*a^2 - 2\*(A - B)\*a\*b - (A + B)\*b^2)\*log(-sqrt(2)\*sqrt(tan(dx + c)) + tan(dx + c) + 1))/(a^4 + 2\*a^2\*b^2 + b^4) - 4\*(2\*A\*a^3 + 2\*A\*a\*b^2 + (2\*A\*a^2\*b - B\*a\*b^2 + 3\*A\*b^3)\*tan(dx + c))/((a^4\*b + a^2\*b^3)\*tan(dx + c)^(3/2) + (a^5 + a^3\*b^2)\*sqrt(tan(dx + c)))/d

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 26965 vs. 2(396) = 792.

time = 134.74, size = 53933, normalized size = 122.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] [1/4\*(4\*sqrt(2)\*((a^16 + 5\*a^14\*b^2 + 9\*a^12\*b^4 + 5\*a^10\*b^6 - 5\*a^8\*b^8 - 9\*a^6\*b^10 - 5\*a^4\*b^12 - a^2\*b^14)\*d^5\*cos(dx + c)^4 - (a^16 + 4\*a^14\*b^2 + 3\*a^12\*b^4 - 10\*a^10\*b^6 - 25\*a^8\*b^8 - 24\*a^6\*b^10 - 11\*a^4\*b^12 - 2\*a^2\*b^14)\*d^5\*cos(dx + c)^2 - (a^14\*b^2 + 6\*a^12\*b^4 + 15\*a^10\*b^6 + 20\*a^8\*b^8 + 15\*a^6\*b^10 + 6\*a^4\*b^12 + a^2\*b^14)\*d^5 + 2\*((a^15\*b + 6\*a^13\*b^3 + 15\*a^11\*b^5 + 20\*a^9\*b^7 + 15\*a^7\*b^9 + 6\*a^5\*b^11 + a^3\*b^13)\*d^5\*cos(dx + c)^3 - (a^15\*b + 6\*a^13\*b^3 + 15\*a^11\*b^5 + 20\*a^9\*b^7 + 15\*a^7\*b^9 + 6\*a^5\*b^11 + a^3\*b^13)\*d^5\*cos(dx + c))\*sin(dx + c))\*sqrt((A^4 + 2\*A^2\*B^2 + B^4)\*a^8 + 4\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^6\*b^2 + 6\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^4\*b^4 + 4\*(A^4 + 2\*A^2\*B^2 + B^4)\*a^2\*b^6 + (A^4 + 2\*A^2\*B^2 + B^4)\*b^8 - 2\*(A\*B\*a^12 - 2\*A\*B\*a^10\*b^2 - 17\*A\*B\*a^8\*b^4 - 28\*A\*B\*a^6\*b^6 - 17\*A\*B\*a^4\*b^8 - 2\*A\*B\*a^2\*b^10 + A\*B\*b^12 - 2\*(A^2 - B^2)\*a^11\*b - 6\*(A^2 - B^2)\*a

$$\begin{aligned}
& ^9b^3 - 4*(A^2 - B^2)*a^7*b^5 + 4*(A^2 - B^2)*a^5*b^7 + 6*(A^2 - B^2)*a^3* \\
& b^9 + 2*(A^2 - B^2)*a*b^{11})*d^2*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6* \\
& b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))/((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16 \\
& *(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B \\
& B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B \\
& - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A* \\
& B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8))*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^ \\
& 8 + 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112 \\
& *(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112* \\
& (A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B \\
& B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^{16} + 8*a^{14}*b^2 + 28*a^{ \\
& 12*b^4 + 56*a^{10}*b^6 + 70*a^8*b^8 + 56*a^6*b^{10} + 28*a^4*b^{12} + 8*a^2*b^{14} \\
& + b^{16})*d^4))*((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^ \\
& 2*b^6 + b^8)*d^4))^{3/4}*\arctan(-(((A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{16} \\
& + 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{15}*b + 40*(A^7*B + 3*A^5*B^3 \\
& + 3*A^3*B^5 + A*B^7)*a^{13}*b^3 - 20*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^{1 \\
& 2*b^4 + 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^{11}*b^5 - 64*(A^8 + 2*A \\
& ^6*B^2 - 2*A^2*B^6 - B^8)*a^{10}*b^6 + 40*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A* \\
& B^7)*a^9*b^7 - 90*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^8*b^8 - 40*(A^7*B + \\
& 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^7*b^9 - 64*(A^8 + 2*A^6*B^2 - 2*A^2*B^6 - \\
& B^8)*a^6*b^{10} - 72*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a^5*b^{11} - 20*( \\
& A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*a^4*b^{12} - 40*(A^7*B + 3*A^5*B^3 + 3*A^3 \\
& *B^5 + A*B^7)*a^3*b^{13} - 8*(A^7*B + 3*A^5*B^3 + 3*A^3*B^5 + A*B^7)*a*b^{15} + \\
& (A^8 + 2*A^6*B^2 - 2*A^2*B^6 - B^8)*b^{16})*d^4*\sqrt{((A^4 - 2*A^2*B^2 + B^4 \\
& )*a^8 + 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - \\
& 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + \\
& 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*( \\
& A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^{16} + 8*a^{14}*b^2 + 2 \\
& 8*a^{12}*b^4 + 56*a^{10}*b^6 + 70*a^8*b^8 + 56*a^6*b^{10} + 28*a^4*b^{12} + 8*a^2*b \\
& ^{14} + b^{16})*d^4))*\sqrt{((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^ \\
& 4 + 4*a^2*b^6 + b^8)*d^4))} + \sqrt{2})*((B*a^{18} - 2*A*a^{17}*b + 7*B*a^{16}*b^2 - \\
& 16*A*a^{15}*b^3 + 20*B*a^{14}*b^4 - 56*A*a^{13}*b^5 + 28*B*a^{12}*b^6 - 112*A*a^{11} \\
& *b^7 + 14*B*a^{10}*b^8 - 140*A*a^9*b^9 - 14*B*a^8*b^{10} - 112*A*a^7*b^{11} - 28* \\
& B*a^6*b^{12} - 56*A*a^5*b^{13} - 20*B*a^4*b^{14} - 16*A*a^3*b^{15} - 7*B*a^2*b^{16} - \\
& 2*A*a*b^{17} - B*b^{18})*d^7*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A \\
& *B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)* \\
& a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a \\
& ^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 \\
& + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^{16} + 8*a^{14}*b^2 + 28*a^{12}*b^4 + 56*a^{10} \\
& b^6 + 70*a^8*b^8 + 56*a^6*b^{10} + 28*a^4*b^{12} + 8*a^2*b^{14} + b^{16})*d^4))*\sqrt{ \\
& t((A^4 + 2*A^2*B^2 + B^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)* \\
& d^4))} + ((A^3 + A*B^2)*a^{14} + 2*(A^2*B + B^3)*a^{13}*b + 5*(A^3 + A*B^2)*a^{12} \\
& *b^2 + 12*(A^2*B + B^3)*a^{11}*b^3 + 9*(A^3 + A*B^2)*a^{10}*b^4 + 30*(A^2*B + B \\
& ^3)*a^9*b^5 + 5*(A^3 + A*B^2)*a^8*b^6 + 40*(A^2*B + B^3)*a^7*b^7 - 5*(A^3 + \\
& A*B^2)*a^6*b^8 + 30*(A^2*B + B^3)*a^5*b^9 - 9*(A^3 + A*B^2)*a^4*b^{10} + 12*
\end{aligned}$$

$$(A^2*B + B^3)*a^3*b^{11} - 5*(A^3 + A*B^2)*a^2*b^{12} + 2*(A^2*B + B^3)*a*b^{13} - (A^3 + A*B^2)*b^{14}*d^5*\sqrt{((A^4 - 2*A^2*B^2 + B^4)*a^8 + 16*(A^3*B - A*B^3)*a^7*b - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^6*b^2 - 112*(A^3*B - A*B^3)*a^5*b^3 + 2*(19*A^4 - 102*A^2*B^2 + 19*B^4)*a^4*b^4 + 112*(A^3*B - A*B^3)*a^3*b^5 - 4*(3*A^4 - 22*A^2*B^2 + 3*B^4)*a^2*b^6 - 16*(A^3*B - A*B^3)*a*b^7 + (A^4 - 2*A^2*B^2 + B^4)*b^8)/((a^{16} + 8*a^{14}*b^2 + 28*a^{12}*b^4 + 56*a^{10}*b^6 + 70*a^8*b^8 + 56*a^6*b^{10} + 28*a^4*b^{12} + 8*a^2*b^{14} + b^{16})*d^4)}*\sqrt{((A^4 + 2*A^2*B^2 + B^4)*a^8 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^6*b^2 + 6*(A^4 + 2*A^2*B^2 + B^4)*a^4*b^4 + 4*(A^4 + 2*A^2*B^2 + B^4)*a^2*b^6 + (A^4 + 2*A^2*B^2 + B^4)*b^8 - 2*(A*B*a^{12} - 2*A*B*a^{10}*...$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 25.69, size = 2500, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^2),x)

[Out] (log((((((((((128\*b^3\*tan(c + d\*x)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2) + (128\*B\*b^2\*(2\*b^6 - a^6 + 9\*a^2\*b^4 + 6\*a^4\*b^2))/(a\*d))\*((4\*(-B^4\*d^4\*(a^4 + b^4 - 6\*a^2\*b^2)^2)^(1/2) - 16\*B^2\*a\*b^3\*d^2 + 16\*B^2\*a^3\*b\*d^2)/(d^4\*(a^2 + b^2)^4))^(1/2))/4 + (64\*B^2\*b^2\*tan(c + d\*x)^(1/2)\*(2\*b^8 - a^8 + 5\*a^2\*b^6 + 67\*a^4\*b^4 - a^6\*b^2))/(a\*d^2\*(a^2 +

$$\begin{aligned}
& b^2)^2) * ((4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} - 16 * B^2 * a * b^3 * d^2 \\
& + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4))^{1/2} / 4 - (32 * B^3 * b^5 * (25 * a^6 + \\
& b^6 - 13 * a^2 * b^4 - 85 * a^4 * b^2)) / (a^2 * d^3 * (a^2 + b^2)^3) * ((4 * (-B^4 * d^4 * (a^4 \\
& + b^4 - 6 * a^2 * b^2)^2)^{1/2} - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a \\
& ^2 + b^2)^4))^{1/2} / 4 + (16 * B^4 * b^5 * \tan(c + d * x)^{1/2} * (b^6 - 27 * a^6 + 7 * a \\
& ^2 * b^4 + 11 * a^4 * b^2)) / (a^2 * d^4 * (a^2 + b^2)^4) * ((4 * (-B^4 * d^4 * (a^4 + b^4 - 6 \\
& * a^2 * b^2)^2)^{1/2} - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^ \\
& 4))^{1/2} / 4 + (16 * B^5 * b^6 * (5 * a^2 + b^2)) / (a * d^5 * (a^2 + b^2)^4) * (((192 * B^4 \\
& * a^2 * b^6 * d^4 - 16 * B^4 * b^8 * d^4 - 16 * B^4 * a^8 * d^4 - 608 * B^4 * a^4 * b^4 * d^4 + 192 * \\
& B^4 * a^6 * b^2 * d^4)^{1/2} - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (a^8 * d^4 + b^ \\
& 8 * d^4 + 4 * a^2 * b^6 * d^4 + 6 * a^4 * b^4 * d^4 + 4 * a^6 * b^2 * d^4))^{1/2} / 4 + (\log(((( \\
& (((((128 * b^3 * \tan(c + d * x)^{1/2} * (a^2 - b^2) * (a^2 + b^2)^2 * (-4 * (-B^4 * d^4 * (a \\
& ^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} + 16 * B^2 * a * b^3 * d^2 - 16 * B^2 * a^3 * b * d^2) / (d^4 * \\
& (a^2 + b^2)^4))^{1/2} + (128 * B * b^2 * (2 * b^6 - a^6 + 9 * a^2 * b^4 + 6 * a^4 * b^2)) / ( \\
& a * d)) * (-4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} + 16 * B^2 * a * b^3 * d^2 - \\
& 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4))^{1/2} / 4 + (64 * B^2 * b^2 * \tan(c + d * x)^ \\
& (1/2) * (2 * b^8 - a^8 + 5 * a^2 * b^6 + 67 * a^4 * b^4 - a^6 * b^2)) / (a * d^2 * (a^2 + b^2)^ \\
& 2)) * (-4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} + 16 * B^2 * a * b^3 * d^2 - 16 \\
& * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4))^{1/2} / 4 - (32 * B^3 * b^5 * (25 * a^6 + b^6 - \\
& 13 * a^2 * b^4 - 85 * a^4 * b^2)) / (a^2 * d^3 * (a^2 + b^2)^3) * (-4 * (-B^4 * d^4 * (a^4 + b \\
& ^4 - 6 * a^2 * b^2)^2)^{1/2} + 16 * B^2 * a * b^3 * d^2 - 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + \\
& b^2)^4))^{1/2} / 4 + (16 * B^4 * b^5 * \tan(c + d * x)^{1/2} * (b^6 - 27 * a^6 + 7 * a^2 * b \\
& ^4 + 11 * a^4 * b^2)) / (a^2 * d^4 * (a^2 + b^2)^4) * (-4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^ \\
& 2 * b^2)^2)^{1/2} + 16 * B^2 * a * b^3 * d^2 - 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4)) \\
& ^{1/2} / 4 + (16 * B^5 * b^6 * (5 * a^2 + b^2)) / (a * d^5 * (a^2 + b^2)^4) * (-((192 * B^4 * a \\
& ^2 * b^6 * d^4 - 16 * B^4 * b^8 * d^4 - 16 * B^4 * a^8 * d^4 - 608 * B^4 * a^4 * b^4 * d^4 + 192 * B^ \\
& 4 * a^6 * b^2 * d^4)^{1/2} + 16 * B^2 * a * b^3 * d^2 - 16 * B^2 * a^3 * b * d^2) / (a^8 * d^4 + b^8 * \\
& d^4 + 4 * a^2 * b^6 * d^4 + 6 * a^4 * b^4 * d^4 + 4 * a^6 * b^2 * d^4))^{1/2} / 4 - \log((16 * B^ \\
& 5 * b^6 * (5 * a^2 + b^2)) / (a * d^5 * (a^2 + b^2)^4) - (((((((((128 * b^3 * \tan(c + d * x)^ ( \\
& 1/2) * (a^2 - b^2) * (a^2 + b^2)^2 * ((4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} \\
& - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4))^{1/2} - (128 \\
& * B * b^2 * (2 * b^6 - a^6 + 9 * a^2 * b^4 + 6 * a^4 * b^2)) / (a * d)) * ((4 * (-B^4 * d^4 * (a^4 + b \\
& ^4 - 6 * a^2 * b^2)^2)^{1/2} - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + \\
& b^2)^4))^{1/2} / 4 + (64 * B^2 * b^2 * \tan(c + d * x)^{1/2} * (2 * b^8 - a^8 + 5 * a^2 * b^ \\
& 6 + 67 * a^4 * b^4 - a^6 * b^2)) / (a * d^2 * (a^2 + b^2)^2)) * ((4 * (-B^4 * d^4 * (a^4 + b^4 \\
& - 6 * a^2 * b^2)^2)^{1/2} - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^ \\
& 2)^4))^{1/2} / 4 + (32 * B^3 * b^5 * (25 * a^6 + b^6 - 13 * a^2 * b^4 - 85 * a^4 * b^2)) / (a^ \\
& 2 * d^3 * (a^2 + b^2)^3) * ((4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} - 16 * B \\
& ^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4))^{1/2} / 4 + (16 * B^4 * b^ \\
& 5 * \tan(c + d * x)^{1/2} * (b^6 - 27 * a^6 + 7 * a^2 * b^4 + 11 * a^4 * b^2)) / (a^2 * d^4 * (a^2 \\
& + b^2)^4) * ((4 * (-B^4 * d^4 * (a^4 + b^4 - 6 * a^2 * b^2)^2)^{1/2} - 16 * B^2 * a * b^3 * d \\
& ^2 + 16 * B^2 * a^3 * b * d^2) / (d^4 * (a^2 + b^2)^4))^{1/2} / 4 * (((192 * B^4 * a^2 * b^6 * d^ \\
& 4 - 16 * B^4 * b^8 * d^4 - 16 * B^4 * a^8 * d^4 - 608 * B^4 * a^4 * b^4 * d^4 + 192 * B^4 * a^6 * b^2 \\
& * d^4)^{1/2} - 16 * B^2 * a * b^3 * d^2 + 16 * B^2 * a^3 * b * d^2) / (16 * a^8 * d^4 + 16 * b^8 * d^4 \\
& + 64 * a^2 * b^6 * d^4 + 96 * a^4 * b^4 * d^4 + 64 * a^6 * b^2 * d^4))^{1/2} - \log((16 * B^5 * b
\end{aligned}$$

$$\begin{aligned}
& ^6(5a^2 + b^2))/(a^5d^5(a^2 + b^2)^4) - (((((((((128b^3 \tan(c + dx)^{1/2} \\
& )*(a^2 - b^2)*(a^2 + b^2)^2*(-4*(-B^4d^4(a^4 + b^4 - 6a^2b^2)^2)^{1/2} \\
& + 16B^2a^3b^3d^2 - 16B^2a^3b^3d^2)/(d^4(a^2 + b^2)^4))^{1/2} - (128B \\
& *b^2*(2b^6 - a^6 + 9a^2b^4 + 6a^4b^2))/(a*d))*(-4*(-B^4d^4(a^4 + b^4 \\
& - 6a^2b^2)^2)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^3b^3d^2)/(d^4(a^2 + \\
& b^2)^4))^{1/2})/4 + (64B^2b^2 \tan(c + dx)^{1/2}*(2b^8 - a^8 + 5a^2b^6 \\
& + 67a^4b^4 - a^6b^2))/(a^2d^2(a^2 + b^2)^2))*(-4*(-B^4d^4(a^4 + b^4 \\
& - 6a^2b^2)^2)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^3b^3d^2)/(d^4(a^2 + b^2)^4))^{1/2})/4 + (32B^3b^5(25a^6 + b^6 - 13a^2b^4 - 85a^4b^2))/(a^2 \\
& d^3(a^2 + b^2)^3))*(-4*(-B^4d^4(a^4 + b^4 - 6a^2b^2)^2)^{1/2} + 16B \\
& B^2a^3b^3d^2 - 16B^2a^3b^3d^2)/(d^4(a^2 + b^2)^4))^{1/2})/4 + (16B^4b \\
& ^5 \tan(c + dx)^{1/2}*(b^6 - 27a^6 + 7a^2b^4 + 11a^4b^2))/(a^2d^4(a^2 \\
& + b^2)^4))*(-4*(-B^4d^4(a^4 + b^4 - 6a^2b^2)^2)^{1/2} + 16B^2a^3b^3 \\
& *d^2 - 16B^2a^3b^3d^2)/(d^4(a^2 + b^2)^4))^{1/2})/4)*(-((192B^4a^2b^6 \\
& *d^4 - 16B^4b^8d^4 - 16B^4a^8d^4 - 608B^4a^4b^4d^4 + 192B^4a^6 \\
& b^2d^4)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^3*...
\end{aligned}$$



$$3.409 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=493

$$\frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

[Out]  $b^{5/2}*(9*A*a^2*b+5*A*b^3-7*B*a^3-3*B*a*b^2)*\arctan(b^{1/2}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/a^{7/2}/(a^2+b^2)^2/d+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)^2/d+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)^2/d+1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2+b^2)/d+1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/a^2+b^2)/d+(4*A*a^2*b+5*A*b^3-2*B*a^3-3*B*a*b^2)/a^3/(a^2+b^2)/d/\tan(d*x+c)^{(1/2)}+1/3*(-2*A*a^2-5*A*b^2+3*B*a*b)/a^2/(a^2+b^2)/d/\tan(d*x+c)^{(3/2)}+b*(A*b-B*a)/a/(a^2+b^2)/d/\tan(d*x+c)^{(3/2)}/(a+b*\tan(d*x+c))$

**Rubi [A]**

time = 0.98, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

(-a^2(A+B)+b^2(A+B)-2\*a\*b\*(A+B))\*ArcTan[1-Sqrt[2]\*Sqrt[Tan[c+d\*x]]]/(Sqrt[2]\*(a^2+b^2)^2\*d)+((2\*a\*b\*(A-B)-a^2\*(A+B)+b^2\*(A+B))\*ArcTan[1+Sqrt[2]\*Sqrt[Tan[c+d\*x]]])/(Sqrt[2]\*(a^2+b^2)^2\*d)+((2\*a\*b\*(A-B)-a^2\*(A+B)+b^2\*(A+B))\*Log[1-Sqrt[2]\*Sqrt[Tan[c+d\*x]]+Tan[c+d\*x]])/(2\*Sqrt[2]\*(a^2+b^2)^2\*d)-((a^2\*(A-B)-b^2\*(A-B)+2\*a\*b\*(A+B))\*Log[1+Sqrt[2]\*Sqrt[Tan[c+d\*x]]+Tan[c+d\*x]])/(2\*Sqrt[2]\*(a^2+b^2)^2\*d)-((2\*a^2\*A+5\*A\*b^2-3\*a\*b\*B)/(3\*a^2\*(a^2+b^2)\*d\*Tan[c+d\*x])^(3/2))+((4\*a^2\*A\*b+5\*A\*b^3-2\*a^3\*B-3\*a\*b^2\*B)/(a^3\*(a^2+b^2)\*d\*Sqrt[Tan[c+d\*x]])+((b\*(A\*b-a\*B))/(a\*(a^2+b^2)\*d\*Tan[c+d\*x])^(3/2)\*(a+b\*Tan[c+d\*x]))

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^2), x]

[Out]  $-(((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)) + ((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) + (b^{5/2}*(9*a^2*A*b + 5*A*b^3 - 7*a^3*B - 3*a*b^2*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a]]) / (a^{7/2}*(a^2 + b^2)^2*d) + ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - (2*a^2*A + 5*A*b^2 - 3*a*b*B) / (3*a^2*(a^2 + b^2)*d*\operatorname{Tan}[c + d*x]^{(3/2)}) + (4*a^2*A*b + 5*A*b^3 - 2*a^3*B - 3*a*b^2*B) / (a^3*(a^2 + b^2)*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) + (b*(A*b - a*B)) / (a*(a^2 + b^2)*d*\operatorname{Tan}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x]))$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) +

$d*(x^p/b)^n, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 210

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

#### Rule 211

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

#### Rule 631

$\text{Int}[(a_ + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[(d_ + (e_.)*(x_))/((a_ + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[(d_ + (e_.)*(x_)^2)/((a_ + (c_.)*(x_)^4), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[(d_ + (e_.)*(x_)^2)/((a_ + (c_.)*(x_)^4), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

#### Rule 1182

$\text{Int}[(d_ + (e_.)*(x_)^2)/((a_ + (c_.)*(x_)^4), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + D$

ist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]]], x\_Symbol] :> Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] :> Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] :> Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

## Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

## Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^2} dx &= \frac{b(Ab - aB)}{a(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} + \int \frac{\frac{1}{2}(2a^2A + 5Ab^2 - 3abB)}{\dots} \\
&= -\frac{2a^2A + 5Ab^2 - 3abB}{3a^2(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)} + \frac{b(Ab - aB)}{a(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} \\
&= -\frac{2a^2A + 5Ab^2 - 3abB}{3a^2(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2Ab + 5Ab^3 - 2a^3B - 3ab^2B}{a^3(a^2 + b^2) d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2A + 5Ab^2 - 3abB}{3a^2(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2Ab + 5Ab^3 - 2a^3B - 3ab^2B}{a^3(a^2 + b^2) d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2A + 5Ab^2 - 3abB}{3a^2(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2Ab + 5Ab^3 - 2a^3B - 3ab^2B}{a^3(a^2 + b^2) d \sqrt{\tan(c + dx)}} \\
&= -\frac{2a^2A + 5Ab^2 - 3abB}{3a^2(a^2 + b^2) d \tan^{\frac{3}{2}}(c + dx)} + \frac{4a^2Ab + 5Ab^3 - 2a^3B - 3ab^2B}{a^3(a^2 + b^2) d \sqrt{\tan(c + dx)}} \\
&= \frac{b^{5/2}(9a^2Ab + 5Ab^3 - 7a^3B - 3ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{7/2}(a^2 + b^2)^2 d} \\
&= \frac{b^{5/2}(9a^2Ab + 5Ab^3 - 7a^3B - 3ab^2B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{7/2}(a^2 + b^2)^2 d} \\
&= -\frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)^2 d}
\end{aligned}$$



$$*b^2)^{1/2} * (\ln((1-2^{1/2}) \tan(dx+c)^{1/2} + \tan(dx+c)) / (1+2^{1/2}) \tan(dx+c)^{1/2} + \tan(dx+c)) + 2 \arctan(1+2^{1/2}) \tan(dx+c)^{1/2} + 2 \arctan(-1+2^{1/2}) \tan(dx+c)^{1/2})) + 2b^3/a^3 / (a^2+b^2)^{1/2} * ((1/2)Aa^2b + (1/2)Ab^3 - 1/2 * Ba^3 - 1/2 * B * a * b^2) \tan(dx+c)^{1/2} / (a+b \tan(dx+c)) + 1/2 * (9Aa^2b + 5Ab^3 - 7Ba^3 - 3B * a * b^2) / (a*b)^{1/2} * \arctan(b \tan(dx+c)^{1/2} / (a*b)^{1/2}))$$

**Maxima** [A]

time = 0.53, size = 449, normalized size = 0.91

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/tan(dx+c)^(5/2)/(a+b\*tan(dx+c))^2,x, algorithm="maxima")

[Out] 
$$-1/12 * (12 * (7 * B * a^3 * b^3 - 9 * A * a^2 * b^4 + 3 * B * a * b^5 - 5 * A * b^6) * \arctan(b * \sqrt{\tan(dx+c)} / \sqrt{a*b}) / ((a^7 + 2 * a^5 * b^2 + a^3 * b^4) * \sqrt{a*b}) + 4 * (2 * A * a^4 + 2 * A * a^2 * b^2 + 3 * (2 * B * a^3 * b - 4 * A * a^2 * b^2 + 3 * B * a * b^3 - 5 * A * b^4) * \tan(dx+c)^2 + 2 * (3 * B * a^4 - 5 * A * a^3 * b + 3 * B * a^2 * b^2 - 5 * A * a * b^3) * \tan(dx+c)) / ((a^5 * b + a^3 * b^3) * \tan(dx+c)^{5/2} + (a^6 + a^4 * b^2) * \tan(dx+c)^{3/2}) + 3 * (2 * \sqrt{2}) * ((A+B) * a^2 - 2 * (A-B) * a * b - (A+B) * b^2) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 * \sqrt{\tan(dx+c)})) + 2 * \sqrt{2} * ((A+B) * a^2 - 2 * (A-B) * a * b - (A+B) * b^2) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 * \sqrt{\tan(dx+c)})) + \sqrt{2} * ((A-B) * a^2 + 2 * (A+B) * a * b - (A-B) * b^2) * \log(\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) - \sqrt{2} * ((A-B) * a^2 + 2 * (A+B) * a * b - (A-B) * b^2) * \log(-\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1)) / (a^4 + 2 * a^2 * b^2 + b^4)) / d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/tan(dx+c)^(5/2)/(a+b\*tan(dx+c))^2,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/tan(dx+c)\*\*(5/2)/(a+b\*tan(dx+c))\*\*2,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 22.90, size = 2500, normalized size = 5.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^2),x)

[Out]  $(\log(80A^5a^{24}b^{20}d^4 - (((192A^4a^2b^6d^4 - 16A^4b^8d^4 - 16A^4a^8d^4 - 608A^4a^4b^4d^4 + 192A^4a^6b^2d^4)^{(1/2)} - 16A^2a^3b^3d^2 + 16A^2a^3b^3d^2)/(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4))^{(1/2)} * (\tan(c + d*x)^{(1/2)} * (9472A^4a^{31}b^{15}d^5 - 3040A^4a^{23}b^{23}d^5 - 9056A^4a^{25}b^{21}d^5 - 12352A^4a^{27}b^{19}d^5 - 4256A^4a^{29}b^{17}d^5 - 400A^4a^{21}b^{25}d^5 + 13760A^4a^{33}b^{13}d^5 + 7744A^4a^{35}b^{11}d^5 + 1968A^4a^{37}b^9d^5 + 224A^4a^{39}b^7d^5 + 32A^4a^{41}b^5d^5) + (((192A^4a^2b^6d^4 - 16A^4b^8d^4 - 16A^4a^8d^4 - 608A^4a^4b^4d^4 + 192A^4a^6b^2d^4)^{(1/2)} - 16A^2a^3b^3d^2 + 16A^2a^3b^3d^2)/(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4))^{(1/2)} * (12928A^3a^{25}b^{23}d^6 - 800A^3a^{21}b^{27}d^6 - 2080A^3a^{23}b^{25}d^6 - ((\tan(c + d*x)^{(1/2)} * (3200A^2a^{22}b^{28}d^7 + 33920A^2a^{24}b^{26}d^7 + 158208A^2a^{26}b^{24}d^7 + 425536A^2a^{28}b^{22}d^7 + 727296A^2a^{30}b^{20}d^7 + 820672A^2a^{32}b^{18}d^7 + 615936A^2a^{34}b^{16}d^7 + 304256A^2a^{36}b^{14}d^7 + 98432A^2a^{38}b^{12}d^7 + 22016A^2a^{40}b^{10}d^7 + 3072A^2a^{42}b^8d^7 - 704A^2a^{44}b^6d^7 - 512A^2a^{46}b^4d^7 - 64A^2a^{48}b^2d^7) + (((192A^4a^2b^6d^4 - 16A^4b^8d^4 - 16A^4a^8d^4 - 608A^4a^4b^4d^4 + 192A^4a^6b^2d^4)^{(1/2)} - 16A^2a^3b^3d^2 + 16A^2a^3b^3d^2)/(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4))^{(1/2)} * (1280A^3a^{24}b^{28}d^8 - (\tan(c + d*x)^{(1/2)} * (((192A^4a^2b^6d^4 - 16A^4b^8d^4 - 16A^4a^8d^4 - 608A^4a^4b^4d^4 + 192A^4a^6b^2d^4)^{(1/2)} - 16A^2a^3b^3d^2 + 16A^2a^3b^3d^2)/(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4))^{(1/2)} * (512a^{27}b^{27}d^9 + 5120a^{29}b^{25}d^9 + 22528a^{31}b^{23}d^9 + 56320a^{33}b^{21}d^9 + 84480a^{35}b^{19}d^9 + 67584a^{37}b^{17}d^9 - 67584a^{41}b^{13}d^9 - 84480a^{43}b$

$$\begin{aligned}
& \cdot 11 \cdot d^9 - 56320 \cdot a^{45} \cdot b^9 \cdot d^9 - 22528 \cdot a^{47} \cdot b^7 \cdot d^9 - 5120 \cdot a^{49} \cdot b^5 \cdot d^9 - 512 \\
& \cdot a^{51} \cdot b^3 \cdot d^9) / 4 + 13824 \cdot A \cdot a^{26} \cdot b^{26} \cdot d^8 + 66944 \cdot A \cdot a^{28} \cdot b^{24} \cdot d^8 + 190848 \cdot \\
& A \cdot a^{30} \cdot b^{22} \cdot d^8 + 352640 \cdot A \cdot a^{32} \cdot b^{20} \cdot d^8 + 435840 \cdot A \cdot a^{34} \cdot b^{18} \cdot d^8 + 354048 \cdot \\
& A \cdot a^{36} \cdot b^{16} \cdot d^8 + 169728 \cdot A \cdot a^{38} \cdot b^{14} \cdot d^8 + 24576 \cdot A \cdot a^{40} \cdot b^{12} \cdot d^8 - 21760 \cdot A \cdot \\
& a^{42} \cdot b^{10} \cdot d^8 - 13440 \cdot A \cdot a^{44} \cdot b^8 \cdot d^8 - 2176 \cdot A \cdot a^{46} \cdot b^6 \cdot d^8 + 384 \cdot A \cdot a^{48} \cdot b^4 \\
& \cdot d^8 + 128 \cdot A \cdot a^{50} \cdot b^2 \cdot d^8) / 4) \cdot (((192 \cdot A^4 \cdot a^2 \cdot b^6 \cdot d^4 - 16 \cdot A^4 \cdot b^8 \cdot d^4 - 16 \\
& \cdot A^4 \cdot a^8 \cdot d^4 - 608 \cdot A^4 \cdot a^4 \cdot b^4 \cdot d^4 + 192 \cdot A^4 \cdot a^6 \cdot b^2 \cdot d^4)^{(1/2)} - 16 \cdot A^2 \cdot a \\
& b^3 \cdot d^2 + 16 \cdot A^2 \cdot a^3 \cdot b \cdot d^2) / (a^8 \cdot d^4 + b^8 \cdot d^4 + 4 \cdot a^2 \cdot b^6 \cdot d^4 + 6 \cdot a^4 \cdot b^4 \cdot \\
& d^4 + 4 \cdot a^6 \cdot b^2 \cdot d^4))^{(1/2)} / 4 + 78464 \cdot A^3 \cdot a^{27} \cdot b^{21} \cdot d^6 + 183616 \cdot A^3 \cdot a^{29} \cdot \\
& b^{19} \cdot d^6 + 238400 \cdot A^3 \cdot a^{31} \cdot b^{17} \cdot d^6 + 184960 \cdot A^3 \cdot a^{33} \cdot b^{15} \cdot d^6 + 84608 \cdot A^3 \cdot \\
& a^{35} \cdot b^{13} \cdot d^6 + 20704 \cdot A^3 \cdot a^{37} \cdot b^{11} \cdot d^6 + 2016 \cdot A^3 \cdot a^{39} \cdot b^9 \cdot d^6) / 4) / 4 + 5 \\
& 44 \cdot A^5 \cdot a^{26} \cdot b^{18} \cdot d^4 + 1520 \cdot A^5 \cdot a^{28} \cdot b^{16} \cdot d^4 + 2240 \cdot A^5 \cdot a^{30} \cdot b^{14} \cdot d^4 + 18 \\
& 40 \cdot A^5 \cdot a^{32} \cdot b^{12} \cdot d^4 + 800 \cdot A^5 \cdot a^{34} \cdot b^{10} \cdot d^4 + 144 \cdot A^5 \cdot a^{36} \cdot b^8 \cdot d^4) \cdot (((192 \\
& \cdot A^4 \cdot a^2 \cdot b^6 \cdot d^4 - 16 \cdot A^4 \cdot b^8 \cdot d^4 - 16 \cdot A^4 \cdot a^8 \cdot d^4 - 608 \cdot A^4 \cdot a^4 \cdot b^4 \cdot d^4 + \\
& 192 \cdot A^4 \cdot a^6 \cdot b^2 \cdot d^4)^{(1/2)} - 16 \cdot A^2 \cdot a \cdot b^3 \cdot d^2 + 16 \cdot A^2 \cdot a^3 \cdot b \cdot d^2) / (a^8 \cdot d^4 \\
& + b^8 \cdot d^4 + 4 \cdot a^2 \cdot b^6 \cdot d^4 + 6 \cdot a^4 \cdot b^4 \cdot d^4 + 4 \cdot a^6 \cdot b^2 \cdot d^4))^{(1/2)} / 4 + (\log \\
& (80 \cdot A^5 \cdot a^{24} \cdot b^{20} \cdot d^4 - ((-((192 \cdot A^4 \cdot a^2 \cdot b^6 \cdot d^4 - 16 \cdot A^4 \cdot b^8 \cdot d^4 - 16 \cdot A^4 \cdot \\
& a^8 \cdot d^4 - 608 \cdot A^4 \cdot a^4 \cdot b^4 \cdot d^4 + 192 \cdot A^4 \cdot a^6 \cdot b^2 \cdot d^4)^{(1/2)} + 16 \cdot A^2 \cdot a \cdot b^3 \cdot d \\
& ^2 - 16 \cdot A^2 \cdot a^3 \cdot b \cdot d^2) / (a^8 \cdot d^4 + b^8 \cdot d^4 + 4 \cdot a^2 \cdot b^6 \cdot d^4 + 6 \cdot a^4 \cdot b^4 \cdot d^4 + \\
& 4 \cdot a^6 \cdot b^2 \cdot d^4))^{(1/2)} \cdot (\tan(c + d \cdot x))^{(1/2)} \cdot (9472 \cdot A^4 \cdot a^{31} \cdot b^{15} \cdot d^5 - 3040 \cdot A \\
& ^4 \cdot a^{23} \cdot b^{23} \cdot d^5 - 9056 \cdot A^4 \cdot a^{25} \cdot b^{21} \cdot d^5 - 12352 \cdot A^4 \cdot a^{27} \cdot b^{19} \cdot d^5 - 4256 \cdot \\
& A^4 \cdot a^{29} \cdot b^{17} \cdot d^5 - 400 \cdot A^4 \cdot a^{21} \cdot b^{25} \cdot d^5 + 13760 \cdot A^4 \cdot a^{33} \cdot b^{13} \cdot d^5 + 7744 \cdot \\
& A^4 \cdot a^{35} \cdot b^{11} \cdot d^5 + 1968 \cdot A^4 \cdot a^{37} \cdot b^9 \cdot d^5 + 224 \cdot A^4 \cdot a^{39} \cdot b^7 \cdot d^5 + 32 \cdot A^4 \cdot a \\
& ^{41} \cdot b^5 \cdot d^5) + ((-((192 \cdot A^4 \cdot a^2 \cdot b^6 \cdot d^4 - 16 \cdot A^4 \cdot b^8 \cdot d^4 - 16 \cdot A^4 \cdot a^8 \cdot d^4 - \\
& 608 \cdot A^4 \cdot a^4 \cdot b^4 \cdot d^4 + 192 \cdot A^4 \cdot a^6 \cdot b^2 \cdot d^4)^{(1/2)} + 16 \cdot A^2 \cdot a \cdot b^3 \cdot d^2 - 16 \cdot A \\
& ^2 \cdot a^3 \cdot b \cdot d^2) / (a^8 \cdot d^4 + b^8 \cdot d^4 + 4 \cdot a^2 \cdot b^6 \cdot d^4 + 6 \cdot a^4 \cdot b^4 \cdot d^4 + 4 \cdot a^6 \cdot b^2 \\
& \cdot d^4))^{(1/2)} \cdot (12928 \cdot A^3 \cdot a^{25} \cdot b^{23} \cdot d^6 - 800 \cdot A^3 \cdot a^{21} \cdot b^{27} \cdot d^6 - 2080 \cdot A^3 \cdot a \\
& ^{23} \cdot b^{25} \cdot d^6 - ((\tan(c + d \cdot x))^{(1/2)} \cdot (3200 \cdot A^2 \cdot a^{22} \cdot b^{28} \cdot d^7 + 33920 \cdot A^2 \cdot a^2 \\
& 4 \cdot b^{26} \cdot d^7 + 158208 \cdot A^2 \cdot a^{26} \cdot b^{24} \cdot d^7 + 425536 \cdot A^2 \cdot a^{28} \cdot b^{22} \cdot d^7 + 727296 \cdot A \\
& ^2 \cdot a^{30} \cdot b^{20} \cdot d^7 + 820672 \cdot A^2 \cdot a^{32} \cdot b^{18} \cdot d^7 + 615936 \cdot A^2 \cdot a^{34} \cdot b^{16} \cdot d^7 + 30 \\
& 4256 \cdot A^2 \cdot a^{36} \cdot b^{14} \cdot d^7 + 98432 \cdot A^2 \cdot a^{38} \cdot b^{12} \cdot d^7 + 22016 \cdot A^2 \cdot a^{40} \cdot b^{10} \cdot d^7 \\
& + 3072 \cdot A^2 \cdot a^{42} \cdot b^8 \cdot d^7 - 704 \cdot A^2 \cdot a^{44} \cdot b^6 \cdot d^7 - 512 \cdot A^2 \cdot a^{46} \cdot b^4 \cdot d^7 - 64 \cdot \\
& A^2 \cdot a^{48} \cdot b^2 \cdot d^7) + ((-((192 \cdot A^4 \cdot a^2 \cdot b^6 \cdot d^4 - 16 \cdot A^4 \cdot b^8 \cdot d^4 - 16 \cdot A^4 \cdot a^8 \cdot \\
& d^4 - 608 \cdot A^4 \cdot a^4 \cdot b^4 \cdot d^4 + 192 \cdot A^4 \cdot a^6 \cdot b^2 \cdot d^4)^{(1/2)} + 16 \cdot A^2 \cdot a \cdot b^3 \cdot d^2 - \\
& 16 \cdot A^2 \cdot a^3 \cdot b \cdot d^2) / (a^8 \cdot d^4 + b^8 \cdot d^4 + 4 \cdot a^2 \cdot b^6 \cdot d^4 + 6 \cdot a^4 \cdot b^4 \cdot d^4 + 4 \cdot a \\
& ^6 \cdot b^2 \cdot d^4))^{(1/2)} \cdot (1280 \cdot A \cdot a^{24} \cdot b^{28} \cdot d^8 - (\tan(c + d \cdot x))^{(1/2)} \cdot ((192 \cdot A^4 \cdot \\
& a^2 \cdot b^6 \cdot d^4 - 16 \cdot A^4 \cdot b^8 \cdot d^4 - 16 \cdot A^4 \cdot a^8 \cdot d^4 - 608 \cdot A^4 \cdot a^4 \cdot b^4 \cdot d^4 + 192 \cdot A \\
& ^4 \cdot a^6 \cdot b^2 \cdot d^4)^{(1/2)} + 16 \cdot A^2 \cdot a \cdot b^3 \cdot d^2 - 16 \cdot A^2 \cdot a^3 \cdot b \cdot d^2) / (a^8 \cdot d^4 + b^8 \\
& \cdot d^4 + 4 \cdot a^2 \cdot b^6 \cdot d^4 + 6 \cdot a^4 \cdot b^4 \cdot d^4 + 4 \cdot a^6 \cdot b^2 \cdot d^4))^{(1/2)} \cdot (512 \cdot a^{27} \cdot b^{27} \\
& \cdot d^9 + 5120 \cdot a^{29} \cdot b^{25} \cdot d^9 + 22528 \cdot a^{31} \cdot b^{23} \cdot d^9 + 56320 \cdot a^{33} \cdot b^{21} \cdot d^9 + 844 \\
& 80 \cdot a^{35} \cdot b^{19} \cdot d^9 + 67584 \cdot a^{37} \cdot b^{17} \cdot d^9 - 67584 \cdot a^{41} \cdot b^{13} \cdot d^9 - 84480 \cdot a^{43} \cdot b \\
& ^{11} \cdot d^9 - 56320 \cdot a^{45} \cdot b^9 \cdot d^9 - 22528 \cdot a^{47} \cdot b^7 \cdot d^9 \dots
\end{aligned}$$



$$3.410 \quad \int \frac{\tan^7(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=600

$$\frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} (3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)$$

```
[Out] 1/4*a^(3/2)*(3*A*a^4*b+6*A*a^2*b^3+35*A*b^5-15*B*a^5-46*B*a^3*b^2-63*B*a*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(7/2)/(a^2+b^2)^3/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a^3*(A-B)-3*a*b^2*(A+B)-b^3*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a^3*(A-B)-3*a*b^2*(A+B)-b^3*(A+B))*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*A*a^3*b+11*A*a*b^3-15*B*a^4-31*B*a^2*b^2-8*B*b^4)*tan(d*x+c)^(1/2)/b^3/(a^2+b^2)^2/d+1/2*a*(A*b-B*a)*tan(d*x+c)^(5/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*a*(A*a^2*b+9*A*b^3-5*B*a^3-13*B*a*b^2)*tan(d*x+c)^(3/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

**Rubi** [A]

time = 1.10, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3686, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^(3/2)*(3*a^4*A*b + 6*a^2*A*b^3 + 35*A*b^5 - 15*a^5*B - 46*a^3*b^2*B - 63*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]]/Sqrt[a])]/(4*b^(7/2)*(a^2 + b^2)^3*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^3*d) + ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((3*a^3*A*b + 11*a*A*b^3 - 15*a^4*B - 31*a^2*b^2*B - 8*b^4*B)*Sqrt[Tan[c + d*x]]/(4*b^3*(a^2 + b^2)^2*d) + (a*(A*b - a*B)*Tan[c + d*x]^(5/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

$x])^2) + (a*(a^2*A*b + 9*A*b^3 - 5*a^3*B - 13*a*b^2*B)*\text{Tan}[c + d*x]^{(3/2)})/(4*b^2*(a^2 + b^2)^2*d*(a + b*\text{Tan}[c + d*x]))$

#### Rule 65

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{n, x}], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 210

$\text{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{(-1)}*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

#### Rule 211

$\text{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

#### Rule 631

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{(-1)}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[(d_.) + (e_.)*(x_.)^2]/((a_.) + (c_.)*(x_.)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[(d_.) + (e_.)*(x_.)^2]/((a_.) + (c_.)*(x_.)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

$eQ[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[\frac{d*q + a*e}{2*a*c}, \text{Int}[\frac{q + c*x^2}{a + c*x^4}, x], x] + \text{Dist}[\frac{d*q - a*e}{2*a*c}, \text{Int}[\frac{q - c*x^2}{a + c*x^4}, x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

### Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)])}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[\frac{b*c + d*x^2}{b^2 + x^4}, x], x, \text{Sqrt}[b*\tan[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3686

$\text{Int}[\frac{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}]^{(m_.)} * \frac{(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[\frac{(b*c - a*d)*(B*c - A*d)*(a + b*\tan[e + f*x])^{(m-1)}*(c + d*\tan[e + f*x])^{(n+1)}}{(d*f*(n+1)*(c^2 + d^2))}, x] - \text{Dist}[1/(d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\tan[e + f*x])^{(m-2)}*(c + d*\tan[e + f*x])^{(n+1)}*\text{Simp}[a*A*d*(b*d*(m-1) - a*c*(n+1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m-1) + a*d*(n+1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n+1)*\tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m+n) - b*B*(c^2*(m-1) - d^2*(n+1)))*\tan[e + f*x]^2, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[m] \parallel \text{IntegersQ}[2*m, 2*n])$

### Rule 3715

$\text{Int}[\frac{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}]^{(m_.)} * \frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{(A_.) + (C_.)*\tan[(e_.) + (f_.)*(x_.)]}^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, \tan[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3726

$\text{Int}[\frac{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}]^{(m_.)} * \frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]} + \frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{(C_.) + (D_.)*\tan[(e_.) + (f_.)*(x_.)]}^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[\frac{A*d^2 + c*(c*C - B*d)*(a + b*\tan[e + f*x])^m*(c + d*\tan[e + f*x])^{(n+1)}}{(d*f*(n+1)*(c^2 + d^2))}, x] - \text{Dist}[1/(d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\tan[e + f*x])^{(m-1)}*(c + d*\tan[e + f*x])^{(n+1)}*\text{Simp}[A*d*(b*d*m - a*c*(n+1)) + (c*C - B*d)*(b*c*m + a*d*($

```

n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3728

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_.)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)] + (C_.)*tan[(e_.
) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_.)] + (C_.)*tan[(e_.) + (f_.)*(x_.)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_.)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} dx &= \frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} + \int \frac{\tan^{\frac{3}{2}}(c+dx)(-\frac{5}{2}a(Ab-aB)+2b(A}}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} dx \\
&= \frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2b(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{a(a^2Ab+9Ab^3-5a^3B-1}}{4b^2(a^2+b^2)^2d(a+b\tan(c+dx))} \\
&= -\frac{(3a^3Ab+11aAb^3-15a^4B-31a^2b^2B-8b^4B)\sqrt{\tan(c+dx)}}{4b^3(a^2+b^2)^2d} \\
&= -\frac{(3a^3Ab+11aAb^3-15a^4B-31a^2b^2B-8b^4B)\sqrt{\tan(c+dx)}}{4b^3(a^2+b^2)^2d} \\
&= -\frac{(3a^3Ab+11aAb^3-15a^4B-31a^2b^2B-8b^4B)\sqrt{\tan(c+dx)}}{4b^3(a^2+b^2)^2d} \\
&= -\frac{(3a^3Ab+11aAb^3-15a^4B-31a^2b^2B-8b^4B)\sqrt{\tan(c+dx)}}{4b^3(a^2+b^2)^2d} \\
&= \frac{a^{3/2}(3a^4Ab+6a^2Ab^3+35Ab^5-15a^5B-46a^3b^2B-63ab^4B)\tan^{\frac{3}{2}}(c+dx)}{4b^{7/2}(a^2+b^2)^3d} \\
&= \frac{a^{3/2}(3a^4Ab+6a^2Ab^3+35Ab^5-15a^5B-46a^3b^2B-63ab^4B)\tan^{\frac{3}{2}}(c+dx)}{4b^{7/2}(a^2+b^2)^3d} \\
&= \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\tan^{-1}(c+dx)}{\sqrt{2}(a^2+b^2)^3d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 5.06, size = 1194, normalized size = 1.99

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(7/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3, x]

[Out] (3\*a^(15/2)\*A\*b\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] + 6\*a^(11/2)\*A\*b^3\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] + 35\*a^(7/2)\*A\*b^5\*ArcTan

$$\begin{aligned}
& \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} - 15a^{17/2} B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 46a^{13/2} b^2 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] - 63a^{9/2} b^4 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 3a^7 A b^{3/2} \sqrt{\tan[c + dx]} - 14a^5 A b^{7/2} \sqrt{\tan[c + dx]} - 11a^3 A b^{11/2} \sqrt{\tan[c + dx]} \right. \\
& \left. + 15a^8 \sqrt{b} B \sqrt{\tan[c + dx]} + 46a^6 b^{5/2} B \sqrt{\tan[c + dx]} + 39a^4 b^{9/2} B \sqrt{\tan[c + dx]} \right. \\
& \left. + 8a^2 b^{13/2} B \sqrt{\tan[c + dx]} + 6a^{13/2} A b^2 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. + 12a^{9/2} A b^4 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. + 70a^{5/2} A b^6 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 30a^{15/2} b B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 92a^{11/2} b^3 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 126a^{7/2} b^5 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 5a^6 A b^{5/2} \tan[c + dx]^{3/2} - 18a^4 A b^{9/2} \tan[c + dx]^{3/2} - 13a^2 A b^{13/2} \tan[c + dx]^{3/2} \right. \\
& \left. + 25a^7 b^{3/2} B \tan[c + dx]^{3/2} + 74a^5 b^{7/2} B \tan[c + dx]^{3/2} + 65a^3 b^{11/2} B \tan[c + dx]^{3/2} \right. \\
& \left. + 16a b^{15/2} B \tan[c + dx]^{3/2} + 3a^{11/2} A b^3 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. + 6a^{7/2} A b^5 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. + 35a^{3/2} A b^7 \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 15a^{13/2} b^2 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 46a^{9/2} b^4 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. - 63a^{5/2} b^6 B \operatorname{ArcTan} \left[ \frac{\sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{a}} \right] \right. \\
& \left. + 8a^6 b^{5/2} B \tan[c + dx]^{5/2} + 24a^4 b^{9/2} B \tan[c + dx]^{5/2} + 24a^2 b^{13/2} B \tan[c + dx]^{5/2} \right. \\
& \left. + 8b^{17/2} B \tan[c + dx]^{5/2} - 4(-1)^{1/4} (a + I b)^3 b^{7/2} (A - I B) \operatorname{ArcTan} \left[ (-1)^{3/4} \sqrt{\tan[c + dx]} \right] \right. \\
& \left. + 4(-1)^{1/4} (-a + I b)^3 b^{7/2} (A + I B) \operatorname{ArcTan} \left[ (-1)^{3/4} \sqrt{\tan[c + dx]} \right] \right. \\
& \left. \right) / (4b^{7/2} (a^2 + b^2)^3 d (a + b \tan[c + dx])^2)
\end{aligned}$$

**Maple [A]**

time = 0.12, size = 466, normalized size = 0.78

method	result
derivativedivides	$ \frac{2B \left( \sqrt{\tan(dx+c)} \right)}{b^3} + \frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \right) \right)}{4} $

default	$\frac{2B\left(\sqrt{\tan(dx+c)}\right)}{b^3} + \frac{(Aa^3 - 3Aab^2 + 3Ba^2b - Bb^3)\sqrt{2} \left( \ln\left(\frac{1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)}{1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)}\right) + 2\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \right)}{4}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(7/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVE  
RBOSE)

[Out]  $\frac{1}{d} \cdot \left( \frac{2B}{b^3} \tan(dx+c)^{1/2} + \frac{2}{(a^2+b^2)^{3/2}} \left( \frac{1}{8} (Aa^3 - 3Aab^2 + 3Ba^2b - Bb^3) 2^{1/2} \left( \ln\left(\frac{1+2^{1/2}\tan(dx+c)^{1/2} + \tan(dx+c)}{1-2^{1/2}\tan(dx+c)^{1/2} + \tan(dx+c)}\right) + 2\arctan\left(\frac{1+2^{1/2}\tan(dx+c)^{1/2}}{1-2^{1/2}\tan(dx+c)^{1/2}}\right) \right) \right) \right) + \frac{2a^2}{b^3} \frac{1}{(a^2+b^2)^{3/2}} \left( \left( -\frac{5}{8}Aa^4b^2 - \frac{9}{4}a^2Ab^4 - \frac{13}{8}Ab^6 + \frac{9}{8}Ba^5b + \frac{13}{4}Ba^3b^3 + \frac{17}{8}Bab^5 \right) \tan(dx+c)^{3/2} - \frac{1}{8}a(3Aa^4b + 14Aa^2b^3 + 11Ab^5 - 7Ba^5 - 22Ba^3b^2 - 15Bab^4) \tan(dx+c)^{1/2} \right) / (a+b\tan(dx+c))^2 + \frac{1}{8} \frac{(3Aa^4b + 6Aa^2b^3 + 35Ab^5 - 15Ba^5 - 46Ba^3b^2 - 63Bab^4)}{(ab)^{1/2}} \arctan\left(\frac{b\tan(dx+c)^{1/2}}{(ab)^{1/2}}\right) \right)$

**Maxima** [A]

time = 0.53, size = 571, normalized size = 0.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(7/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm  
="maxima")

[Out] 
$$-\frac{1}{4} \left( (15Ba^7 - 3Aa^6b + 46Ba^5b^2 - 6Aa^4b^3 + 63Ba^3b^4 - 35Aa^2b^5) \arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) / \left( (a^6b^3 + 3a^4b^5 + 3a^2b^7 + b^9) \sqrt{ab} \right) - (2\sqrt{2}) \left( (A+B)a^3 - 3(A-B)a^2b - 3(A+B)ab^2 + (A-B)b^3 \right) \arctan\left(\frac{1/2\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})}{\sqrt{2} - 2\sqrt{\tan(dx+c)}}\right) + \sqrt{2} \left( (A-B)a^3 + 3(A+B)a^2b - 3(A-B)ab^2 - (A+B)b^3 \right) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - \sqrt{2} \left( (A-B)a^3 + 3(A+B)a^2b - 3(A-B)ab^2 - (A+B)b^3 \right) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) \right) / (a^6 + 3a^4b^2 + 3a^2b^4 + b^6) - \left( (9Ba^5b - 5Aa^4b^2 + 17Ba^3b^3 - 13Aa^2b^4) \tan(dx+c)^{3/2} + (7Ba^6 - 3Aa^5b + 15Ba^4b^2 - 11Aa^3b^3) \sqrt{\tan(dx+c)} \right) / (a^6b^3 + 2a^4b^5$$

$+ a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*\tan(dx + c)^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*\tan(dx + c) - 8*B*\sqrt{\tan(dx + c)}/b^3)/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(7/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*(7/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))\*\*3,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(7/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 60.84, size = 2500, normalized size = 4.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + dx)^(7/2)\*(A + B\*tan(c + dx)))/(a + b\*tan(c + dx))^3,x)

[Out] (log((((((((128\*b^3\*tan(c + dx)^(1/2)\*(a^2 - b^2)\*(a^2 + b^2)^2\*((4\*(-B^4\*d^4\*(a^6 - b^6 + 15\*a^2\*b^4 - 15\*a^4\*b^2)^2)^(1/2) + 80\*B^2\*a^3\*b^3\*d^2 - 24\*B^2\*a\*b^5\*d^2 - 24\*B^2\*a^5\*b\*d^2)/(d^4\*(a^2 + b^2)^6))^(1/2) - (64\*B\*a\*b





$$\begin{aligned}
& + 24*B^2*a^5*b*d^2)/(a^{12}*d^4 + b^{12}*d^4 + 6*a^2*b^{10}*d^4 + 15*a^4*b^8*d^4 \\
& + 20*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^{10}*b^2*d^4))^{(1/2)}/4 - \log(- ((( \\
& (((128*b^3*\tan(c + d*x))^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*d^4*(a^6 \\
& - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a* \\
& b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)} + (64*B*a*b*(15*a^4 \\
& + 2*b^4 + 41*a^2*b^2))/d*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^ \\
& 2)^2)^{(1/2)} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^ \\
& 4*(a^2 + b^2)^6))^{(1/2)}/4 + (8*B^2*a*\tan(c + d*x))^{(1/2)}*(225*a^{14} - 184*b^ \\
& 14 + 608*a^2*b^{12} - 272*a^4*b^{10} + 3937*a^6*b^8 + 5804*a^8*b^6 + 4006*a^{10}* \\
& b^4 + 1380*a^{12}*b^2))/(b^4*d^2*(a^2 + b^2)^4))*((4*(-B^4*d^4*(a^6 - b^6 + 1 \\
& 5*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - \\
& 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 + (2*B^3*a^2*(1125*a^{14} + 1 \\
& 6*b^{14} + 6112*a^2*b^{12} - 17727*a^4*b^{10} - 23239*a^6*b^8 - 11174*a^8*b^6 + 2 \\
& 930*a^{10}*b^4 + 3525*a^{12}*b^2))/(b^4*d^3*(a^2 + b^2)^6))*((4*(-B^4*d^4*(a^6 \\
& - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b \\
& ^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (B^4*\tan(c + d*x) \\
& )^{(1/2)}*(32*b^{18} - 225*a^{18} + 128*a^2*b^{16} + 192*a^4*b^{14} - 3841*a^6*b^{12} + \\
& 18050*a^8*b^{10} + 26801*a^{10}*b^8 + 16860*a^{12}*b^6 + 4049*a^{14}*b^4 - 30*a^{16} \\
& *b^2))/(b^5*d^4*(a^2 + b^2)^8))*((4*(-B^4*d^4*(...
\end{aligned}$$

$$3.411 \quad \int \frac{\tan^5(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=534

$$\frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) (a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B))}{\sqrt{2} (a^2 + b^2)^3 d}$$

```
[Out] -1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(A*a^4*b+18*A*a^2*b^3-15*A*b^5+3*B*a^5+6*B*a^3*b^2+35*B*a*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)/(a^2+b^2)^3/d+1/2*a*(A*b-B*a)*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*a*(A*a^2*b-7*A*b^3+3*B*a^3+11*B*a*b^2)*tan(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

**Rubi [A]**

time = 0.79, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3686, 3726, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) + (Sqrt[a]*(a^4*A*b + 18*a^2*A*b^3 - 15*A*b^5 + 3*a^5*B + 6*a^3*b^2*B + 35*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]/(4*b^(5/2)*(a^2 + b^2)^3*d) + ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^3*d) + (a*(A*b - a*B)*Tan[c + d*x]^(3/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (a*(a^2*A*b - 7*A*b^3 + 3*a^3*B + 11*a*b^2*B)*Sqrt[Tan[c + d*x]]/(4*b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)^2]), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) +
(f_)*(x_)^2]), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
```

], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2)/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])], x\_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n \*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx &= \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{2b(a^2+b^2)d(a+b \tan(c+dx))^2} + \frac{\int \sqrt{\tan(c+dx)}^{(-\frac{3}{2}a(Ab-aB))}}{dx} \\
 &= \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{2b(a^2+b^2)d(a+b \tan(c+dx))^2} - \frac{a(a^2Ab-7Ab^3+3a^3B+11ab^2B)}{4b^2(a^2+b^2)^2d(a+b \tan(c+dx))} \\
 &= \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{2b(a^2+b^2)d(a+b \tan(c+dx))^2} - \frac{a(a^2Ab-7Ab^3+3a^3B+11ab^2B)}{4b^2(a^2+b^2)^2d(a+b \tan(c+dx))} \\
 &= \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{2b(a^2+b^2)d(a+b \tan(c+dx))^2} - \frac{a(a^2Ab-7Ab^3+3a^3B+11ab^2B)}{4b^2(a^2+b^2)^2d(a+b \tan(c+dx))} \\
 &= \frac{a(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{2b(a^2+b^2)d(a+b \tan(c+dx))^2} - \frac{a(a^2Ab-7Ab^3+3a^3B+11ab^2B)}{4b^2(a^2+b^2)^2d(a+b \tan(c+dx))} \\
 &= \frac{\sqrt{a}(a^4Ab+18a^2Ab^3-15Ab^5+3a^5B+6a^3b^2B+35ab^4B) \tan^{-1}\left(\frac{a+b \tan(c+dx)}{\sqrt{a}}\right)}{4b^{5/2}(a^2+b^2)^3d} \\
 &= \frac{\sqrt{a}(a^4Ab+18a^2Ab^3-15Ab^5+3a^5B+6a^3b^2B+35ab^4B) \tan^{-1}\left(\frac{a+b \tan(c+dx)}{\sqrt{a}}\right)}{4b^{5/2}(a^2+b^2)^3d} \\
 &= \frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B)) \tan^{-1}\left(\frac{a+b \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{2}(a^2+b^2)^3d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 4.47, size = 372, normalized size = 0.70

$$\frac{2 \left( (AB + 3aB) \sqrt{\tan(c+dx)} + \frac{a^2 b^2 + 4a^2 b^2 \sqrt{\tan(c+dx)}}{2 \sqrt{\tan(c+dx)}} - 3B \tan^2(c+dx) + \frac{(a+b \sqrt{\tan(c+dx)})^2 (a^2 b^2 + 4a^2 b^2 \sqrt{\tan(c+dx)}) \sqrt{\tan(c+dx)}}{2 \sqrt{\tan(c+dx)}} + \left( \frac{a^2 b^2 + 4a^2 b^2 \sqrt{\tan(c+dx)}}{2 \sqrt{\tan(c+dx)}} \right) \operatorname{ArcTan} \left( \frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{\tan(c+dx)}} \right) + \sqrt{-1} a^{5/2} b^{5/2} (a-b) \operatorname{ArcTan} \left( \frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{\tan(c+dx)}} \right) - (a+b) \sqrt{\tan(c+dx)} \operatorname{ArcTan} \left( \frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{\tan(c+dx)}} \right) \right)}{3B d (a + b \tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3, x]

[Out] (2\*(-((A\*b + 3\*a\*B)\*Sqrt[Tan[c + d\*x]]) + ((a^2\*A\*b + 4\*A\*b^3 + 3\*a^3\*B)\*Sqrt[Tan[c + d\*x]])/(4\*(a^2 + b^2)) - 3\*b\*B\*Tan[c + d\*x]^(3/2) + ((a + b\*Tan[c + d\*x])\*((3\*a^(5/2)\*Sqrt[b]\*(a^2 + b^2)\*(a^3\*A\*b + 9\*a\*A\*b^3 + 3\*a^4\*B + 3\*a^2\*b^2\*B + 8\*b^4\*B)\*Sqrt[Tan[c + d\*x]])/8 + ((3\*a^3\*(a^4\*A\*b + 18\*a^2\*A\*b^3 - 15\*A\*b^5 + 3\*a^5\*B + 6\*a^3\*b^2\*B + 35\*a\*b^4\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/8 + (3\*(-1)^(1/4)\*a^(5/2)\*b^(5/2)\*((I\*a - b)^3\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - (I\*a + b)^3\*(A + I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]))/(2\*(a + b\*Tan[c + d\*x])))/(a^(5/2)\*Sqrt[b]\*(a^2 + b^2)^3))/(3\*b^2\*d\*(a + b\*Tan[c + d\*x])^2)

**Maple [A]**

time = 0.12, size = 451, normalized size = 0.84

method	result
derivativedivides	$\frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$\frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3, x, method=\_RETURNVE RBOSE)

[Out] 1/d\*(2/(a^2+b^2)^3\*(1/8\*(-3\*A\*a^2\*b+A\*b^3+B\*a^3-3\*B\*a\*b^2)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))

$$\begin{aligned} &)) + 1/8 * (-A * a^3 + 3 * A * a * b^2 - 3 * B * a^2 * b + B * b^3) * 2^{(1/2)} * (\ln((1 - 2^{(1/2)} * \tan(d * x + c))^{(1/2)} + \tan(d * x + c)) / (1 + 2^{(1/2)} * \tan(d * x + c)^{(1/2)} + \tan(d * x + c))) + 2 * \arctan(1 + 2^{(1/2)} * \tan(d * x + c)^{(1/2)}) + 2 * \arctan(-1 + 2^{(1/2)} * \tan(d * x + c)^{(1/2)})) + 2 * a / (a^2 + b^2) \\ &^3 * ((1/8 * (A * a^4 * b + 10 * A * a^2 * b^3 + 9 * A * b^5 - 5 * B * a^5 - 18 * B * a^3 * b^2 - 13 * B * a * b^4) / b * \tan(d * x + c)^{(3/2)} - 1/8 * a * (A * a^4 * b - 6 * A * a^2 * b^3 - 7 * A * b^5 + 3 * B * a^5 + 14 * B * a^3 * b^2 + 11 * B * a * b^4) / b^2 * \tan(d * x + c)^{(1/2)}) / (a + b * \tan(d * x + c))^2 + 1/8 * (A * a^4 * b + 18 * A * a^2 * b^3 - 15 * A * b^5 + 3 * B * a^5 + 6 * B * a^3 * b^2 + 35 * B * a * b^4) / b^2 / (a * b)^{(1/2)} * \arctan(b * \tan(d * x + c)^{(1/2)} / (a * b)^{(1/2)})) \end{aligned}$$

**Maxima [A]**

time = 0.55, size = 551, normalized size = 1.03

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{4} * ((3 * B * a^6 + A * a^5 * b + 6 * B * a^4 * b^2 + 18 * A * a^3 * b^3 + 35 * B * a^2 * b^4 - 15 * A * a * b^5) * \arctan(b * \sqrt{\tan(d * x + c)} / \sqrt{a * b}) / (\sqrt{a^6 * b^2 + 3 * a^4 * b^4 + 3 * a^2 * b^6 + b^8} * \sqrt{a * b}) - (2 * \sqrt{2} * ((A - B) * a^3 + 3 * (A + B) * a^2 * b - 3 * (A - B) * a * b^2 - (A + B) * b^3) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 * \sqrt{\tan(d * x + c)}) + 2 * \sqrt{2} * ((A - B) * a^3 + 3 * (A + B) * a^2 * b - 3 * (A - B) * a * b^2 - (A + B) * b^3) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 * \sqrt{\tan(d * x + c)}))) - \sqrt{2} * ((A + B) * a^3 - 3 * (A - B) * a^2 * b - 3 * (A + B) * a * b^2 + (A - B) * b^3) * \log(\sqrt{2} * \sqrt{\tan(d * x + c)} + \tan(d * x + c) + 1) + \sqrt{2} * ((A + B) * a^3 - 3 * (A - B) * a^2 * b - 3 * (A + B) * a * b^2 + (A - B) * b^3) * \log(-\sqrt{2} * \sqrt{\tan(d * x + c)} + \tan(d * x + c) + 1)) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) - ((5 * B * a^4 * b - A * a^3 * b^2 + 1 * 3 * B * a^2 * b^3 - 9 * A * a * b^4) * \tan(d * x + c)^{(3/2)} + (3 * B * a^5 + A * a^4 * b + 11 * B * a^3 * b^2 - 7 * A * a^2 * b^3) * \sqrt{\tan(d * x + c)}) / (a^6 * b^2 + 2 * a^4 * b^4 + a^2 * b^6 + (a^4 * b^4 + 2 * a^2 * b^6 + b^8) * \tan(d * x + c)^2 + 2 * (a^5 * b^3 + 2 * a^3 * b^5 + a * b^7) * \tan(d * x + c)) / d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 52.61, size = 2500, normalized size = 4.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] 
$$\begin{aligned} & (\log(\frac{(64A^2ab^3(11a^2 - 13b^2))/d + 128b^3 \tan(c + dx)^{1/2} (a^2 - b^2)(a^2 + b^2)^2 ((4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2))^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2a^5bd^2 - 24A^2a^5bd^2)/(d^4(a^2 + b^2)^6))^{1/2} ((4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2))^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2a^5bd^2 - 24A^2a^5bd^2)/(d^4(a^2 + b^2)^6))^{1/2})/4 + (8A^2a \tan(c + dx)^{1/2} (a^{10} - 184b^{10} + 833a^2b^8 - 812a^4b^6 + 262a^6b^4 + 44a^8b^2))/(d^2(a^2 + b^2)^4) \\ & * ((4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2))^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2a^5bd^2 - 24A^2a^5bd^2)/(d^4(a^2 + b^2)^6))^{1/2})/4 + (2A^3a^2(5a^{10} - 1199b^{10} + 5017a^2b^8 - 5142a^4b^6 + 1106a^6b^4 + 181a^8b^2))/(d^3(a^2 + b^2)^6) * ((4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2))^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2a^5bd^2 - 24A^2a^5bd^2)/(d^4(a^2 + b^2)^6))^{1/2})/4 + (A^4 \tan(c + dx)^{1/2} (a^{14} - 32b^{14} + 97a^2b^{12} - 2082a^4b^{10} + 3631a^6b^8 - 2300a^8b^6 + 79a^{10}b^4 + 30a^{12}b^2))/(b^4d^4(a^2 + b^2)^8) * ((4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2))^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2a^5bd^2 - 24A^2a^5bd^2)/(d^4(a^2 + b^2)^6))^{1/2})/4 + (A^5a(a^{10} - 120b^{10} + 249a^2b^8 - 388a^4b^6 + 302a^6b^4 + 36a^8b^2))/(2b^5d^5(a^2 + b^2)^8) * ((480A^4a^2b^{10}d^4 - 16A^4b^{12}d^4 - 16A^4a^{12}d^4 - 4080A^4a^4b^8d^4 + 7232A^4a^6b^6d^4 - 4080A^4a^8b^4d^4 + 480A^4a^{10}b^2d^4)^{1/2} + 80A^2a^3b^3d^2 - 24A^2a^5bd^2 - 24A^2a^5bd^2) \end{aligned}$$



$$- 24A^2a^5bd^2 / (16a^{12}d^4 + 16b^{12}d^4 \dots)$$

$$3.412 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=533

$$\frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}$$

[Out] 1/2\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)-a^3\*(A+B)+3\*a\*b^2\*(A+B))\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/(a^2+b^2)^3/d\*2^(1/2)+1/2\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)-a^3\*(A+B)+3\*a\*b^2\*(A+B))\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/(a^2+b^2)^3/d\*2^(1/2)+1/4\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)+3\*a^2\*b\*(A+B)-b^3\*(A+B))\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(a^2+b^2)^3/d\*2^(1/2)-1/4\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)+3\*a^2\*b\*(A+B)-b^3\*(A+B))\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(a^2+b^2)^3/d\*2^(1/2)+1/4\*(3\*A\*a^4\*b-26\*A\*a^2\*b^3+3\*A\*b^5+B\*a^5+18\*B\*a^3\*b^2-15\*B\*a\*b^4)\*arctan(b^(1/2)\*tan(d\*x+c)^(1/2)/a^(1/2))/b^(3/2)/(a^2+b^2)^3/d/a^(1/2)+1/2\*a\*(A\*b-B\*a)\*tan(d\*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^2+1/4\*(3\*A\*a^2\*b-5\*A\*b^3+B\*a^3+9\*B\*a\*b^2)\*tan(d\*x+c)^(1/2)/b/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.80, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3686, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$\frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out] -(((3\*a^2\*b\*(A - B) - b^3\*(A - B) - a^3\*(A + B) + 3\*a\*b^2\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*(a^2 + b^2)^3\*d) + ((3\*a^2\*b\*(A - B) - b^3\*(A - B) - a^3\*(A + B) + 3\*a\*b^2\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*(a^2 + b^2)^3\*d) + ((3\*a^4\*A\*b - 26\*a^2\*A\*b^3 + 3\*A\*b^5 + a^5\*B + 18\*a^3\*b^2\*B - 15\*a\*b^4\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/(4\*Sqrt[a]\*b^(3/2)\*(a^2 + b^2)^3\*d) + ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) + 3\*a^2\*b\*(A + B) - b^3\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*(a^2 + b^2)^3\*d) - ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) + 3\*a^2\*b\*(A + B) - b^3\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*(a^2 + b^2)^3\*d) + (a\*(A\*b - a\*B)\*Sqrt[Tan[c + d\*x]])/(2\*b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^2) + ((3\*a^2\*A\*b - 5\*A\*b^3 + a^3\*B + 9\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(4\*b\*(a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x]))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
+ (f_)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx &= \frac{a(Ab - aB) \sqrt{\tan(c + dx)}}{2b(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{\int \frac{-\frac{1}{2}a(Ab - aB) + 2b(Ab - aB) \tan(c + dx)}{\sqrt{\tan(c + dx)}}}{2b(a^2 + b^2) d(a + b \tan(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\tan(c + dx)}}{2b(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{(3a^2 Ab - 5Ab^3 + a^3 B + 9ab^2)}{4b(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\tan(c + dx)}}{2b(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{(3a^2 Ab - 5Ab^3 + a^3 B + 9ab^2)}{4b(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\tan(c + dx)}}{2b(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{(3a^2 Ab - 5Ab^3 + a^3 B + 9ab^2)}{4b(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\tan(c + dx)}}{2b(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{(3a^2 Ab - 5Ab^3 + a^3 B + 9ab^2)}{4b(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{(3a^4 Ab - 26a^2 Ab^3 + 3Ab^5 + a^5 B + 18a^3 b^2 B - 15ab^4 B) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}} \right)}{4\sqrt{a} b^{3/2} (a^2 + b^2)^3 d} \\
 &= \frac{(3a^4 Ab - 26a^2 Ab^3 + 3Ab^5 + a^5 B + 18a^3 b^2 B - 15ab^4 B) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}} \right)}{4\sqrt{a} b^{3/2} (a^2 + b^2)^3 d} \\
 &= \frac{(3a^2 b(A - B) - b^3(A - B) - a^3(A + B) + 3ab^2(A + B)) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}} \right)}{\sqrt{2} (a^2 + b^2)^3 d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 3.76, size = 333, normalized size = 0.62

$$\frac{-4B\sqrt{\tan(c+dx)} + \frac{(3a^2b^2 + 3ab^2)\sqrt{\tan(c+dx)}}{2\sqrt{a}} - \frac{2(a+b\tan(c+dx))\left(-\frac{1}{2}a^2\sqrt{b}\sqrt{\tan(c+dx)}(3a^2b-5Ab^3+a^3B+9ab^2)\sqrt{\tan(c+dx)} + \left(-\frac{1}{2}a^2(3a^2b-26a^2Ab^3+3Ab^5+a^5B+18a^3b^2B-15ab^4B)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) - \sqrt{-1}a^{3/2}b^{3/2}\left((a+b)\text{ArcTan}\left(-\frac{1}{\sqrt{a}}\sqrt{\tan(c+dx)}\right) + (a-b)\text{ArcTan}\left(-\frac{1}{\sqrt{a}}\sqrt{\tan(c+dx)}\right)\right)\right)}{6bd(a+b\tan(c+dx))^2}}{6bd(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3, x]

[Out] (-4\*B\*Sqrt[Tan[c + d\*x]] + ((3\*a\*A\*b + a^2\*B + 4\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(a^2 + b^2) - (2\*(a + b\*Tan[c + d\*x])\*((-3\*a^(5/2)\*Sqrt[b]\*(a^2 + b^2)\*(3\*a^2\*A\*b - 5\*A\*b^3 + a^3\*B + 9\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/4 + ((-3\*a^2\*(3\*a^4\*A\*b - 26\*a^2\*A\*b^3 + 3\*A\*b^5 + a^5\*B + 18\*a^3\*b^2\*B - 15\*a\*b^4\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/4 - 3\*(-1)^(1/4)\*a^(5/2)\*b^(3/2)\*



$$(a + I*b)^3*(A - I*B)*\text{ArcTan}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]] + (a - I*b)^3*(A + I*B)*\text{ArcTanh}[(-1)^{(3/4)}*\text{Sqrt}[\text{Tan}[c + d*x]]])*(a + b*\text{Tan}[c + d*x])/(a^{5/2}*\text{Sqrt}[b]*(a^2 + b^2)^3)/(6*b*d*(a + b*\text{Tan}[c + d*x])^2)$$

**Maple [A]**

time = 0.12, size = 450, normalized size = 0.84

method	result
derivativedivides	$\frac{(-A a^3 + 3A a b^2 - 3B a^2 b + B b^3) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$\frac{(-A a^3 + 3A a b^2 - 3B a^2 b + B b^3) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVE  
RBOSE)`

[Out]  $\frac{1}{d} \frac{2}{(a^2+b^2)^3} \frac{1}{8} (-A a^3 + 3A a b^2 - 3B a^2 b + B b^3) 2^{1/2} (\ln((1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c))/(1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c))) + 2 \arctan(1+2^{1/2} \tan(dx+c)^{1/2}) + 2 \arctan(-1+2^{1/2} \tan(dx+c)^{1/2})) + \frac{1}{8} (3A a^2 b - A b^3 - B a^3 + 3B a b^2) 2^{1/2} (\ln((1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c))/(1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c))) + 2 \arctan(1+2^{1/2} \tan(dx+c)^{1/2}) + 2 \arctan(-1+2^{1/2} \tan(dx+c)^{1/2})) + \frac{2}{(a^2+b^2)^3} ((\frac{3}{8} A a^4 b - \frac{1}{4} A a^2 b^3 - \frac{5}{8} A b^5 + \frac{1}{8} B a^5 + \frac{5}{4} B a^3 b^2 + \frac{9}{8} B a b^4) \tan(dx+c)^{3/2} + \frac{1}{8} a (5A a^4 b + 2A a^2 b^3 - 3A b^5 - B a^5 + 6B a^3 b^2 + 7B a b^4) / b \tan(dx+c)^{1/2}) / (a+b \tan(dx+c))^2 + \frac{1}{8} (3A a^4 b - 26A a^2 b^3 + 3A b^5 + B a^5 + 18B a^3 b^2 - 15B a b^4) / b (a*b)^{1/2} \arctan(b \tan(dx+c)^{1/2} / (a*b)^{1/2}))$

**Maxima [A]**

time = 0.53, size = 538, normalized size = 1.01

([In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="maxima")`

```
[Out] 1/4*((B*a^5 + 3*A*a^4*b + 18*B*a^3*b^2 - 26*A*a^2*b^3 - 15*B*a*b^4 + 3*A*b^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((B*a^3*b + 3*A*a^2*b^2 + 9*B*a*b^3 - 5*A*b^4)*tan(d*x + c)^(3/2) - (B*a^4 - 5*A*a^3*b - 7*B*a^2*b^2 + 3*A*a*b^3)*sqrt(tan(d*x + c)))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b^5 + b^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c))/d
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

[Out] Timed out

Mupad [B]

time = 53.11, size = 2500, normalized size = 4.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x))^{3/2}*(A + B*\tan(c + d*x)))/(a + b*\tan(c + d*x))^3, x)$ 

[Out]  $(\log(\frac{(64*B*a*b^3*(11*a^2 - 13*b^2))/d + 128*b^3*\tan(c + d*x)^{1/2}*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2}}{(4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2}}/4 + (8*B^2*a*\tan(c + d*x)^{1/2}*(a^{10} - 184*b^{10} + 833*a^2*b^8 - 812*a^4*b^6 + 262*a^6*b^4 + 44*a^8*b^2))/(d^2*(a^2 + b^2)^4)) * ((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2})/4 + (2*B^3*a^2*(5*a^{10} - 1199*b^{10} + 5017*a^2*b^8 - 5142*a^4*b^6 + 1106*a^6*b^4 + 181*a^8*b^2))/(d^3*(a^2 + b^2)^6)) * ((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2})/4 + (B^4*\tan(c + d*x)^{1/2}*(a^{14} - 32*b^{14} + 97*a^2*b^{12} - 2082*a^4*b^{10} + 3631*a^6*b^8 - 2300*a^8*b^6 + 79*a^{10}*b^4 + 30*a^{12}*b^2))/(b*d^4*(a^2 + b^2)^8)) * ((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2})/4 + (B^5*a*(a^{10} - 120*b^{10} + 249*a^2*b^8 - 388*a^4*b^6 + 302*a^6*b^4 + 36*a^8*b^2))/(2*b*d^5*(a^2 + b^2)^8)) * (((480*B^4*a^2*b^{10}*d^4 - 16*B^4*b^{12}*d^4 - 16*B^4*a^{12}*d^4 - 4080*B^4*a^4*b^8*d^4 + 7232*B^4*a^6*b^6*d^4 - 4080*B^4*a^8*b^4*d^4 + 480*B^4*a^{10}*b^2*d^4)^{1/2} + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(a^{12}*d^4 + b^{12}*d^4 + 6*a^2*b^{10}*d^4 + 15*a^4*b^8*d^4 + 20*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^{10}*b^2*d^4)^{1/2})/4 + (\log(\frac{(64*B*a*b^3*(11*a^2 - 13*b^2))/d + 128*b^3*\tan(c + d*x)^{1/2}*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} - 80*B^2*a^3*b^3*d^2 + 24*B^2*a*b^5*d^2 + 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2}}{(4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} - 80*B^2*a^3*b^3*d^2 + 24*B^2*a*b^5*d^2 + 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2}}/4 + (8*B^2*a*\tan(c + d*x)^{1/2}*(a^{10} - 184*b^{10} + 833*a^2*b^8 - 812*a^4*b^6 + 262*a^6*b^4 + 44*a^8*b^2))/(d^2*(a^2 + b^2)^4)) * (-(4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} - 80*B^2*a^3*b^3*d^2 + 24*B^2*a*b^5*d^2 + 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2})/4 + (2*B^3*a^2*(5*a^{10} - 1199*b^{10} + 5017*a^2*b^8 - 5142*a^4*b^6 + 1106*a^6*b^4 + 181*a^8*b^2))/(d^3*(a^2 + b^2)^6)) * (-(4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))^2)^{1/2} - 80*B^2*a^3*b^3*d^2 + 24*B^2*a*b^5*d^2 + 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{1/2})/4 + (B^4*\tan(c + d*x)^{1/2}*(a^{14} - 32*b^{14} + 97*a^2*b^{12} - 2082*a^4*b^{10} + 3631*a^6*b^8 - 2300*a^8*b^6 + 79*a^{10}*b^4 + 30*a^{12}*b^2)))/($

$$\begin{aligned}
& b^2 d^4 (a^2 + b^2)^8) * (- (4 * (-B^4 d^4 (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)^2)^{1/2} - 80 B^2 a^3 b^3 d^2 + 24 B^2 a^5 b^5 d^2 + 24 B^2 a^5 b^5 d^2) / (d^4 (a^2 + b^2)^6))^{1/2} / 4 + (B^5 a (a^{10} - 120 b^{10} + 249 a^2 b^8 - 388 a^4 b^6 + 302 a^6 b^4 + 36 a^8 b^2)) / (2 b^5 d^5 (a^2 + b^2)^8) * (- ((480 B^4 a^2 b^{10} d^4 - 16 B^4 b^{12} d^4 - 16 B^4 a^{12} d^4 - 4080 B^4 a^4 b^8 d^4 + 7232 B^4 a^6 b^6 d^4 - 4080 B^4 a^8 b^4 d^4 + 480 B^4 a^{10} b^2 d^4)^{1/2} - 80 B^2 a^3 b^3 d^2 + 24 B^2 a^5 b^5 d^2 + 24 B^2 a^5 b^5 d^2) / (a^{12} d^4 + b^{12} d^4 + 6 a^2 b^{10} d^4 + 15 a^4 b^8 d^4 + 20 a^6 b^6 d^4 + 15 a^8 b^4 d^4 + 6 a^{10} b^2 d^4))^{1/2} / 4 - \log((((((((((64 B^3 a^3 b^3 (11 a^2 - 13 b^2)) / d - 128 b^3 * \tan(c + d x)^{1/2} (a^2 - b^2) (a^2 + b^2)^2 ((4 * (-B^4 d^4 (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)^2)^{1/2} + 80 B^2 a^3 b^3 d^2 - 24 B^2 a^5 b^5 d^2 - 24 B^2 a^5 b^5 d^2) / (d^4 (a^2 + b^2)^6))^{1/2}) * ((4 * (-B^4 d^4 (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)^2)^{1/2} + 80 B^2 a^3 b^3 d^2 - 24 B^2 a^5 b^5 d^2 - 24 B^2 a^5 b^5 d^2) / (d^4 (a^2 + b^2)^6))^{1/2} / 4 - (8 B^2 a * \tan(c + d x)^{1/2} * (a^{10} - 184 b^{10} + 833 a^2 b^8 - 812 a^4 b^6 + 262 a^6 b^4 + 44 a^8 b^2)) / (d^2 (a^2 + b^2)^4) * ((4 * (-B^4 d^4 (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)^2)^{1/2} + 80 B^2 a^3 b^3 d^2 - 24 B^2 a^5 b^5 d^2 - 24 B^2 a^5 b^5 d^2) / (d^4 (a^2 + b^2)^6))^{1/2} / 4 + (2 B^3 a^2 (5 a^{10} - 1199 b^{10} + 5017 a^2 b^8 - 5142 a^4 b^6 + 1106 a^6 b^4 + 181 a^8 b^2)) / (d^3 (a^2 + b^2)^6) * ((4 * (-B^4 d^4 (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)^2)^{1/2} + 80 B^2 a^3 b^3 d^2 - 24 B^2 a^5 b^5 d^2 - 24 B^2 a^5 b^5 d^2) / (d^4 (a^2 + b^2)^6))^{1/2} / 4 - (B^4 * \tan(c + d x)^{1/2} * (a^{14} - 32 b^{14} + 97 a^2 b^{12} - 2082 a^4 b^{10} + 3631 a^6 b^8 - 2300 a^8 b^6 + 79 a^{10} b^4 + 30 a^{12} b^2)) / (b^5 d^4 (a^2 + b^2)^8) * ((4 * (-B^4 d^4 (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)^2)^{1/2} + 80 B^2 a^3 b^3 d^2 - 24 B^2 a^5 b^5 d^2 - 24 B^2 a^5 b^5 d^2) / (d^4 (a^2 + b^2)^6))^{1/2} / 4 + (B^5 a (a^{10} - 120 b^{10} + 249 a^2 b^8 - 388 a^4 b^6 + 302 a^6 b^4 + 36 a^8 b^2)) / (2 b^5 d^5 (a^2 + b^2)^8) * (((480 B^4 a^2 b^{10} d^4 - 16 B^4 b^{12} d^4 - 16 B^4 a^{12} d^4 - 4080 B^4 a^4 b^8 d^4 + 7232 B^4 a^6 b^6 d^4 - 4080 B^4 a^8 b^4 d^4 + 480 B^4 a^{10} b^2 d^4)^{1/2} + 80 B^2 a^3 b^3 d^2 - 24 B^2 a^5 b^5 d^2 - 24 B^2 a^5 b^5 d^2) / (16 a^{12} d^4 + 16 b^{12} d^4 ...
\end{aligned}$$

$$3.413 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=531

$$\frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} + \dots$$

[Out]  $1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(15*A*a^4*b-18*A*a^2*b^3-A*b^5-3*B*a^5+26*B*a^3*b^2-3*B*a*b^4)*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/a^{(3/2)}/(a^2+b^2)^3/d/b^{(1/2)}-1/2*(A*b-B*a)*\tan(d*x+c)^{(1/2)}/(a^2+b^2)/d/(a+b*\tan(d*x+c))^2-1/4*(7*A*a^2*b-A*b^3-3*B*a^3+5*B*a*b^2)*\tan(d*x+c)^{(1/2)}/a/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))$

**Rubi** [A]

time = 0.84, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3689, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$\frac{(A-b)\sqrt{2}}{2(a^2+b^2)^3 d} \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right) + \frac{(A+b)\sqrt{2}}{2(a^2+b^2)^3 d} \operatorname{ArcTan}\left(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right) - \frac{1}{4} \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B)) \ln\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{(a^2+b^2)^3 d} + \frac{1}{4} \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B)) \ln\left(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{(a^2+b^2)^3 d} - \frac{1}{4} \frac{(15Aa^4b-18Aa^2b^3-Ab^5-3Ba^5+26Ba^3b^2-3Bab^4) \operatorname{ArcTan}\left(\frac{b\sqrt{\tan(c+dx)}}{a}\right)}{(a^2+b^2)^3 d} - \frac{1}{2} \frac{(Ab-Ba) \tan(c+dx)}{(a^2+b^2)d} - \frac{1}{4} \frac{(7Aa^2b-Ab^3-3Ba^3+5Bab^2) \tan(c+dx)}{(a^2+b^2)^2 d}$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out]  $-(((a^3*(A-B) - 3*a*b^2*(A-B) + 3*a^2*b*(A+B) - b^3*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) + ((a^3*(A-B) - 3*a*b^2*(A-B) + 3*a^2*b*(A+B) - b^3*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - ((15*a^4*A*b - 18*a^2*A*b^3 - A*b^5 - 3*a^5*B + 26*a^3*b^2*B - 3*a*b^4*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a]]) / (4*a^{(3/2)}*\operatorname{Sqrt}[b]*(a^2 + b^2)^3*d) - ((3*a^2*b*(A-B) - b^3*(A-B) - a^3*(A+B) + 3*a*b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) + ((3*a^2*b*(A-B) - b^3*(A-B) - a^3*(A+B) + 3*a*b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - ((A*b - a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (2*(a^2 + b^2)*d*(a + b*\operatorname{Tan}[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B + 5*a*b^2*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (4*a*(a^2 + b^2)^2*d*(a + b*\operatorname{Tan}[c + d*x]))$

Rule 65

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ
[2*m, 2*n])
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)^2]), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) +
(f_)*(x_)^2]), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3734

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2]))/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n \*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx &= -\frac{(Ab-aB) \sqrt{\tan(c+dx)}}{2(a^2+b^2) d(a+b \tan(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}b(Ab-aB)-2b(aA+bB) \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{2b(a^2+b^2) d(a+b \tan(c+dx))^2} \\
 &= -\frac{(Ab-aB) \sqrt{\tan(c+dx)}}{2(a^2+b^2) d(a+b \tan(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B+5a^4) \tan^{-1}(\sqrt{\tan(c+dx)})}{4a(a^2+b^2)^2 d(a+b \tan(c+dx))} \\
 &= -\frac{(Ab-aB) \sqrt{\tan(c+dx)}}{2(a^2+b^2) d(a+b \tan(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B+5a^4) \tan^{-1}(\sqrt{\tan(c+dx)})}{4a(a^2+b^2)^2 d(a+b \tan(c+dx))} \\
 &= -\frac{(Ab-aB) \sqrt{\tan(c+dx)}}{2(a^2+b^2) d(a+b \tan(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B+5a^4) \tan^{-1}(\sqrt{\tan(c+dx)})}{4a(a^2+b^2)^2 d(a+b \tan(c+dx))} \\
 &= -\frac{(Ab-aB) \sqrt{\tan(c+dx)}}{2(a^2+b^2) d(a+b \tan(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B+5a^4) \tan^{-1}(\sqrt{\tan(c+dx)})}{4a(a^2+b^2)^2 d(a+b \tan(c+dx))} \\
 &= -\frac{(15a^4Ab-18a^2Ab^3-Ab^5-3a^5B+26a^3b^2B-3ab^4B) \tan^{-1}(\sqrt{\tan(c+dx)})}{4a^{3/2}\sqrt{b} (a^2+b^2)^3 d} \\
 &= -\frac{(15a^4Ab-18a^2Ab^3-Ab^5-3a^5B+26a^3b^2B-3ab^4B) \tan^{-1}(\sqrt{\tan(c+dx)})}{4a^{3/2}\sqrt{b} (a^2+b^2)^3 d} \\
 &= -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B)) \tan^{-1}(\sqrt{\tan(c+dx)})}{\sqrt{2} (a^2+b^2)^3 d}
 \end{aligned}$$



**Mathematica [C]** Result contains complex when optimal does not.  
 time = 4.01, size = 344, normalized size = 0.65

$$\frac{b(Ab - aB)\tan^2(c + dx) - (Ab - aB)\sqrt{\tan(c + dx)}(a + b\tan(c + dx)) + \frac{2a + b\tan(c + dx)}{2a^2 + b^2} \left( \frac{1}{4} a^{3/2} b^{3/2} (a^2 + b^2) - \frac{1}{2} a^{5/2} b^{3/2} + \frac{1}{4} a^{3/2} b^{5/2} - \frac{1}{2} a^{1/2} b^{5/2} + \frac{1}{4} a^{1/2} b^{3/2} \right) \sqrt{\tan(c + dx)} - \left( \frac{1}{4} a^{3/2} b^{3/2} (a^2 + b^2) - \frac{1}{2} a^{5/2} b^{3/2} + \frac{1}{4} a^{3/2} b^{5/2} - \frac{1}{2} a^{1/2} b^{5/2} + \frac{1}{4} a^{1/2} b^{3/2} \right) \sqrt{\tan(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c + dx)}}{\sqrt{a}}\right) - \sqrt{-1} a^{3/2} b^{3/2} (a - b) \operatorname{ArcTan}\left(-\frac{1}{\sqrt{a}} \sqrt{\tan(c + dx)}\right) - (a + b) \operatorname{tanh}^{-1}\left(-\frac{1}{\sqrt{a}} \sqrt{\tan(c + dx)}\right) \right)}{2a(a^2 + b^2)d(a + b\tan(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3, x]

[Out] (b\*(A\*b - a\*B)\*Tan[c + d\*x]^(3/2) - (A\*b - a\*B)\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x]) + (2\*(a + b\*Tan[c + d\*x]))\*((a^(3/2)\*b^(3/2)\*(a^2 + b^2)\*(-7\*a^2\*A\*b + A\*b^3 + 3\*a^3\*B - 5\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/4 - (-1/4\*(a\*b\*(-15\*a^4\*A\*b + 18\*a^2\*A\*b^3 + A\*b^5 + 3\*a^5\*B - 26\*a^3\*b^2\*B + 3\*a\*b^4\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]]) + (-1)^(1/4)\*a^(5/2)\*b^(3/2)\*((I\*a - b)^3\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - (I\*a + b)^3\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]))\*(a + b\*Tan[c + d\*x]))/(a^(3/2)\*b^(3/2)\*(a^2 + b^2)^2)/(2\*a\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^2)

**Maple [A]**  
 time = 0.12, size = 451, normalized size = 0.85

method	result
derivativedivides	$\frac{(3Aa^2b - Ab^3 - Ba^3 + 3Bab^2)\sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$\frac{(3Aa^2b - Ab^3 - Ba^3 + 3Bab^2)\sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVE RBOSE)

[Out] 1/d\*(2/(a^2+b^2))^3\*(1/8\*(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))) + 2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))) + 1/8\*(A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)^(1/2))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)^(1/2))))

$$\frac{1}{2} + \tan(dx+c) / (1 + 2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)) + 2 \arctan(1 + 2^{1/2} \tan(dx+c)^{1/2}) - 2 / (a^2 + b^2)^{3/2} \left( \frac{1}{8} b (7Aa^4b + 6Aa^2b^3 - Ab^5 - 3Ba^5 + 2Ba^3b^2 + 5Ba^2b^4) / a \tan(dx+c)^{3/2} + (9/8Aa^4b + 5/4Aa^2b^3 - 5/8Ba^5 + 3/8Ba^2b^4 + 1/8Ab^5 - 1/4Ba^3b^2) \tan(dx+c)^{1/2} \right) / (a + b \tan(dx+c))^2 + 1/8 (15Aa^4b - 18Aa^2b^3 - Ab^5 - 3Ba^5 + 26Ba^3b^2 - 3Ba^2b^4) / a (ab)^{1/2} \arctan(b \tan(dx+c)^{1/2}) / (ab)^{1/2}$$

**Maxima [A]**

time = 0.53, size = 537, normalized size = 1.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(1/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{4} \left( (3Ba^5 - 15Aa^4b - 26Ba^3b^2 + 18Aa^2b^3 + 3Ba^2b^4 + Ab^5) \arctan(b \sqrt{\tan(dx+c)}) / \sqrt{ab} \right) / (a^7 + 3a^5b^2 + 3a^3b^4 + ab^6) \sqrt{ab} + (2\sqrt{2} \left( (A-B)a^3 + 3(A+B)a^2b - 3(A-B)ab^2 - (A+B)b^3 \right) \arctan(1/2\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})) + 2\sqrt{2} \left( (A-B)a^3 + 3(A+B)a^2b - 3(A-B)ab^2 - (A+B)b^3 \right) \arctan(-1/2\sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})) - \sqrt{2} \left( (A+B)a^3 - 3(A-B)a^2b - 3(A+B)ab^2 + (A-B)b^3 \right) \log(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2} \left( (A+B)a^3 - 3(A-B)a^2b - 3(A+B)ab^2 + (A-B)b^3 \right) \log(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1)) / (a^6 + 3a^4b^2 + 3a^2b^4 + b^6) + ((3Ba^3b - 7Aa^2b^2 - 5Ba^2b^3 + Ab^4) \tan(dx+c)^{3/2} + (5Ba^4 - 9Aa^3b - 3Ba^2b^2 - Aa^2b^3) \sqrt{\tan(dx+c)}) / (a^7 + 2a^5b^2 + a^3b^4 + (a^5b^2 + 2a^3b^4 + ab^6) \tan(dx+c)^2 + 2(a^6b + 2a^4b^3 + a^2b^5) \tan(dx+c)) / d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(1/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)`

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="giac")`

[Out] Timed out

**Mupad** [B]

time = 52.40, size = 2500, normalized size = 4.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)`

[Out] 
$$\begin{aligned} & \left( \log\left(\frac{(64Ab^3(b^4 - 10a^4 + 15a^2b^2))}{(ad)} + 128b^3 \tan(c + d x)^{1/2} (a^2 - b^2)(a^2 + b^2)^2 \left( (4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2)^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2ab^5d^2 - 24A^2a^5b^3d^2 \right) \right)}{(d^4(a^2 + b^2)^6)^{1/2}} \right) \left( (4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2)^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2ab^5d^2 - 24A^2a^5b^3d^2) \right) / (d^4(a^2 + b^2)^6)^{1/2} \\ & + \frac{(8A^2b^2 \tan(c + d x)^{1/2} (8a^{10} + b^{10} - 148a^2b^8 + 902a^4b^6 - 812a^6b^4 + 193a^8b^2))}{(ad^2(a^2 + b^2)^4)} \left( (4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2)^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2ab^5d^2 - 24A^2a^5b^3d^2) \right) / (d^4(a^2 + b^2)^6)^{1/2} \\ & - \frac{(2A^3b^2(16a^{12} + b^{12} - 71a^2b^{10} - 1382a^4b^8 + 5266a^6b^6 - 4539a^8b^4 + 1189a^{10}b^2))}{(a^2d^3(a^2 + b^2)^6)} \left( (4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2)^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2ab^5d^2 - 24A^2a^5b^3d^2) \right) / (d^4(a^2 + b^2)^6)^{1/2} \\ & - \frac{(A^4b^3 \tan(c + d x)^{1/2} (2a^2b^{10} - b^{12} - 225a^{12} + 49a^4b^8 + 2460a^6b^6 - 3631a^8b^4 + 1922a^{10}b^2))}{(a^2d^4(a^2 + b^2)^8)} \left( (4(-A^4d^4(a^6 - b^6 + 15a^2b^4 - 15a^4b^2)^2)^{1/2} + 80A^2a^3b^3d^2 - 24A^2ab^5d^2 - 24A^2a^5b^3d^2) \right) / (d^4(a^2 + b^2)^6)^{1/2} \\ & + \frac{(A^5b^3(7b^8 - 225a^8 + 116a^2b^6 - 270a^4b^4 + 420a^6b^2))}{(2a^2d^5(a^2 + b^2)^8)} \left( (480A^4a^2b^{10}d^4 - 16A^4b^{12}d^4 - 16A^4a^{12}d^4 - 4080A^4a^4b^8d^4 + 7232A^4a^6b^6d^4 - 4080A^4a^8b^4d^4 + 480A^4a^{10}b^2d^4) \right)^{1/2} + 80A^2a^3b^3d^2 - 24A^2ab^5d^2 - 24A^2a^5b^3d^2 \end{aligned}$$

$$\begin{aligned}
& *a^5*b*d^2)/(a^{12}*d^4 + b^{12}*d^4 + 6*a^2*b^{10}*d^4 + 15*a^4*b^8*d^4 + 20*a^6* \\
& *b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^{10}*b^2*d^4)^{(1/2)}/4 - ((\tan(c + d*x)^{(1/2)} \\
& )*(A*b^3 + 9*A*a^2*b))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (\tan(c + d*x)^{(3/2)}*(A \\
& *b^4 - 7*A*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*d + b^2*d*\tan(c + \\
& d*x)^2 + 2*a*b*d*\tan(c + d*x)) - ((\tan(c + d*x)^{(3/2)}*(5*B*b^3 - 3*B*a^2*b) \\
& )/(4*(a^4 + b^4 + 2*a^2*b^2)) - (a*\tan(c + d*x)^{(1/2)}*(5*B*a^2 - 3*B*b^2)))/ \\
& (4*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*d + b^2*d*\tan(c + d*x)^2 + 2*a*b*d*\tan(c \\
& + d*x)) + (\log((((((((((64*A*b^3*(b^4 - 10*a^4 + 15*a^2*b^2))/(a*d) + 128*b \\
& ^3*\tan(c + d*x)^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-A^4*d^4*(a^6 - b^6 + \\
& 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 \\
& + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}*(-(4*(-A^4*d^4*(a^6 - b^6 + \\
& 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 \\
& + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}))/4 + (8*A^2*b^2*\tan(c + d*x) \\
& ^{(1/2)}*(8*a^{10} + b^{10} - 148*a^2*b^8 + 902*a^4*b^6 - 812*a^6*b^4 + 193*a^8*b^2) \\
& ^2))/(a*d^2*(a^2 + b^2)^4))*(-(4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4 \\
& *b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/ \\
& (d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (2*A^3*b^2*(16*a^{12} + b^{12} - 71*a^2*b^{10} - \\
& 1382*a^4*b^8 + 5266*a^6*b^6 - 4539*a^8*b^4 + 1189*a^{10}*b^2))/(a^2*d^3*(a^2 \\
& + b^2)^6))*(-(4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - \\
& 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6) \\
& )^{(1/2)}/4 - (A^4*b^3*\tan(c + d*x)^{(1/2)}*(2*a^2*b^{10} - b^{12} - 225*a^{12} + \\
& 49*a^4*b^8 + 2460*a^6*b^6 - 3631*a^8*b^4 + 1922*a^{10}*b^2))/(a^2*d^4*(a^2 + \\
& b^2)^8))*(-(4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80 \\
& *A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6) \\
& )^{(1/2)}/4 + (A^5*b^3*(7*b^8 - 225*a^8 + 116*a^2*b^6 - 270*a^4*b^4 + 420*a^6 \\
& *b^2))/(2*a*d^5*(a^2 + b^2)^8))*(-((480*A^4*a^2*b^{10}*d^4 - 16*A^4*b^{12}*d^4 \\
& - 16*A^4*a^{12}*d^4 - 4080*A^4*a^4*b^8*d^4 + 7232*A^4*a^6*b^6*d^4 - 4080*A^4 \\
& *a^8*b^4*d^4 + 480*A^4*a^{10}*b^2*d^4)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 24*A^2*a* \\
& b^5*d^2 + 24*A^2*a^5*b*d^2)/(a^{12}*d^4 + b^{12}*d^4 + 6*a^2*b^{10}*d^4 + 15*a^4* \\
& b^8*d^4 + 20*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^{10}*b^2*d^4))^{(1/2)}/4 - \log \\
& (((((((((((64*A*b^3*(b^4 - 10*a^4 + 15*a^2*b^2))/(a*d) - 128*b^3*\tan(c + d*x) \\
& )^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 1 \\
& 5*a^4*b^2)^2)^{(1/2)} + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b* \\
& d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15 \\
& *a^4*b^2)^2)^{(1/2)} + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d \\
& ^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (8*A^2*b^2*\tan(c + d*x)^{(1/2)}*(8*a^{10} + \\
& b^{10} - 148*a^2*b^8 + 902*a^4*b^6 - 812*a^6*b^4 + 193*a^8*b^2))/(a*d^2*(a^2 \\
& + b^2)^4))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} + \\
& 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6) \\
& )^{(1/2)}/4 - (2*A^3*b^2*(16*a^{12} + b^{12} - 71*a^2*b^{10} - 1382*a^4*b^8 + 52 \\
& 66*a^6*b^6 - 4539*a^8*b^4 + 1189*a^{10}*b^2))/(a^2*d^3*(a^2 + b^2)^6))*((4*(- \\
& A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} + 80*A^2*a^3*b^3*d^2 \\
& - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 + (A^ \\
& 4*b^3*\tan(c + d*x)^{(1/2)}*(2*a^2*b^{10} - b^{12} - 225*a^{12} + 49*a^4*b^8 + 2460* \\
& a^6*b^6 - 3631*a^8*b^4 + 1922*a^{10}*b^2))/(a^2*d^4*(a^2 + b^2)^8))*((4*(-A^4
\end{aligned}$$

$$*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(\dots}$$



Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n +
2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
(IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
+ (f_)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```



```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

#### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^3} dx &= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{2a(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{\int \frac{\frac{1}{2}(4a^2A + 3Ab^2 + abB) - 2a(Ab - aB)}{\sqrt{\tan(c + dx)}} dx}{2a(a^2 + b^2)} \\
 &= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{2a(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{b(11a^2Ab + 3Ab^3 - 7a^3B + ab^2B)}{4a^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{2a(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{b(11a^2Ab + 3Ab^3 - 7a^3B + ab^2B)}{4a^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{2a(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{b(11a^2Ab + 3Ab^3 - 7a^3B + ab^2B)}{4a^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{b(Ab - aB) \sqrt{\tan(c + dx)}}{2a(a^2 + b^2) d(a + b \tan(c + dx))^2} + \frac{b(11a^2Ab + 3Ab^3 - 7a^3B + ab^2B)}{4a^2(a^2 + b^2)^2 d(a + b \tan(c + dx))} \\
 &= \frac{\sqrt{b} (35a^4Ab + 6a^2Ab^3 + 3Ab^5 - 15a^5B + 18a^3b^2B + ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{4a^{5/2} (a^2 + b^2)^3 d} \\
 &= \frac{\sqrt{b} (35a^4Ab + 6a^2Ab^3 + 3Ab^5 - 15a^5B + 18a^3b^2B + ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{4a^{5/2} (a^2 + b^2)^3 d} \\
 &= \frac{(3a^2b(A - B) - b^3(A - B) - a^3(A + B) + 3ab^2(A + B)) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{2} (a^2 + b^2)^3 d}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 2.94, size = 288, normalized size = 0.54

$$\frac{2 \left( \frac{1}{2} \sqrt{b} (35a^4Ab + 6a^2Ab^3 + 3Ab^5 - 15a^5B + 18a^3b^2B + ab^4B) \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right) - 2 \sqrt{-1} a^{5/2} (a + b)^2 (A - iB) \operatorname{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right) + (a - b)^2 (A + iB) \operatorname{tanh}^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right) \right)}{4a^{5/2} (a^2 + b^2)^3 d} + \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{(a + b \tan(c + dx))^2} + \frac{b(11a^2Ab + 3Ab^3 - 7a^3B + ab^2B) \sqrt{\tan(c + dx)}}{a(a^2 + b^2)(a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^3), x]

[Out] ((2\*((Sqrt[b]\*(35\*a^4\*A\*b + 6\*a^2\*A\*b^3 + 3\*A\*b^5 - 15\*a^5\*B + 18\*a^3\*b^2\*B + a\*b^4\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/2 - 2\*(-1)^(1/4)\*a^(5/2)\*((a + I\*b)^3\*(A - I\*B)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + (a - I\*b)^3\*(A + I\*B)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]))/(a^(3/2)\*(a^2 + b^2))

$$b^2)^2 + (2*b*(A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a + b*\text{Tan}[c + d*x])^2 + (b*(11*a^2*A*b + 3*A*b^3 - 7*a^3*B + a*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a*(a^2 + b^2)*(a + b*\text{Tan}[c + d*x]))/(4*a*(a^2 + b^2)*d)$$

**Maple [A]**

time = 0.12, size = 452, normalized size = 0.85

method	result
derivativedivides	$\frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$\frac{(A a^3 - 3A a b^2 + 3B a^2 b - B b^3) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*(2/(a^2+b^2)^3\*(1/8\*(A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))) + 2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))) + 1/8\*(-3\*A\*a^2\*b+A\*b^3+B\*a^3-3\*B\*a\*b^2)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))) + 2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))) + 2\*b/(a^2+b^2)^3\*((1/8\*b\*(11\*A\*a^4\*b+14\*A\*a^2\*b^3+3\*A\*b^5-7\*B\*a^5-6\*B\*a^3\*b^2+B\*a\*b^4)/a^2\*tan(d\*x+c)^(3/2)+1/8\*(13\*A\*a^4\*b+18\*A\*a^2\*b^3+5\*A\*b^5-9\*B\*a^5-10\*B\*a^3\*b^2-B\*a\*b^4)/a\*tan(d\*x+c)^(1/2))/(a+b\*tan(d\*x+c))^2+1/8\*(35\*A\*a^4\*b+6\*A\*a^2\*b^3+3\*A\*b^5-15\*B\*a^5+18\*B\*a^3\*b^2+B\*a\*b^4)/a^2/(a\*b)^(1/2)\*arctan(b\*tan(d\*x+c)^(1/2)/(a\*b)^(1/2))))

**Maxima [A]**

time = 0.52, size = 551, normalized size = 1.03

\*\*\*\*\*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-1/4*((15*B*a^5*b - 35*A*a^4*b^2 - 18*B*a^3*b^3 - 6*A*a^2*b^4 - B*a*b^5 - 3*A*b^6)*\arctan(b*\sqrt{\tan(d*x + c)})/\sqrt{a*b})/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*\sqrt{a*b}) - (2*\sqrt{2}*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(d*x + c)}))) + 2*\sqrt{2}*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(d*x + c)}))) + \sqrt{2}*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*\log(\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1) - \sqrt{2}*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*\log(-\sqrt{2}*\sqrt{\tan(d*x + c)} + \tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((7*B*a^3*b^2 - 11*A*a^2*b^3 - B*a*b^4 - 3*A*b^5)*\tan(d*x + c)^(3/2) + (9*B*a^4*b - 13*A*a^3*b^2 + B*a^2*b^3 - 5*A*a*b^4)*\sqrt{\tan(d*x + c)})/(a^8 + 2*a^6*b^2 + a^4*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*\tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*\tan(d*x + c))/d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")



$$\begin{aligned}
& *b^{12} - 8*a^{12} + 36*a^2*b^{10} + 430*a^4*b^8 - 188*a^6*b^6 + 1497*a^8*b^4 + 3 \\
& 2*a^{10}*b^2)/(a^3*d^2*(a^2 + b^2)^4))*(-4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 \\
& 4 - 15*a^4*b^2)^2)^{(1/2)} + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a \\
& ^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (2*A^3*b^3*(45*b^{12} - 16*a^{12} + 3 \\
& 33*a^2*b^{10} + 146*a^4*b^8 + 1178*a^6*b^6 - 9791*a^8*b^4 + 1161*a^{10}*b^2))/( \\
& a^3*d^3*(a^2 + b^2)^6))*(-4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2 \\
& )^2)^{(1/2)} + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4 \\
& *(a^2 + b^2)^6))^{(1/2)}/4 + (A^4*b^5*tan(c + d*x))^{(1/2)}*(18*a^2*b^{10} - 9*b^{12} \\
& - 1257*a^{12} - 71*a^4*b^8 + 892*a^6*b^6 + 857*a^8*b^4 + 6802*a^{10}*b^2))/( \\
& a^4*d^4*(a^2 + b^2)^8))*(-4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2 \\
& )^2)^{(1/2)} + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4 \\
& *(a^2 + b^2)^6))^{(1/2)}/4 + (A^5*b^6*(1505*a^8 + 9*b^8 + 60*a^2*b^6 + 318*a \\
& ^4*b^4 + 748*a^6*b^2))/(2*a^4*d^5*(a^2 + b^2)^8))*(-((480*A^4*a^2*b^{10}*d^4 \\
& - 16*A^4*b^{12}*d^4 - 16*A^4*a^{12}*d^4 - 4080*A^4*a^4*b^8*d^4 + 7232*A^4*a^6*b \\
& ^6*d^4 - 4080*A^4*a^8*b^4*d^4 + 480*A^4*a^{10}*b^2*d^4)^{(1/2)} + 80*A^2*a^3*b^ \\
& 3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(a^{12}*d^4 + b^{12}*d^4 + 6*a^2*b \\
& ^{10}*d^4 + 15*a^4*b^8*d^4 + 20*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^{10}*b^2*d^4 \\
& ))^{(1/2)}/4 - \log((((((((((64*A*b^2*(3*b^6 - 2*a^6 + 3*a^2*b^4 + 22*a^4*b^2 \\
& ))/(a^2*d) - 128*b^3*tan(c + d*x))^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-A^4 \\
& *d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + \\
& 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}*((4*(-A^4* \\
& d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 2 \\
& 4*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (8*A^2* \\
& b^2*tan(c + d*x))^{(1/2)}*(9*b^{12} - 8*a^{12} + 36*a^2*b^{10} + 430*a^4*b^8 - 188*a \\
& ^6*b^6 + 1497*a^8*b^4 + 32*a^{10}*b^2))/(a^3*d^2*(a^2 + b^2)^4))*((4*(-A^4*d^ \\
& 4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 24* \\
& A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (2*A^3*b^ \\
& 3*(45*b^{12} - 16*a^{12} + 333*a^2*b^{10} + 146*a^4*b^8 + 1178*a^6*b^6 - 9791*a^8 \\
& *b^4 + 1161*a^{10}*b^2))/(a^3*d^3*(a^2 + b^2)^6))*((4*(-A^4*d^4*(a^6 - b^6 + \\
& 15*a^2*b^4 - 15*a^4*b^2)^2)^{(1/2)} - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + \\
& 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^{(1/2)}/4 - (A^4*b^5*tan(c + d*x))^{(1 \\
& /2)}*(18*a^2*b^{10} - 9*b^{12} - 1257*a^{12} - 71*a^4*...
\end{aligned}$$

$$3.415 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=601

$$\frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} (a^3(A-B) -$$

```
[Out] -1/4*b^(3/2)*(63*A*a^4*b+46*A*a^2*b^3+15*A*b^5-35*B*a^5-6*B*a^3*b^2-3*B*a*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(7/2)/(a^2+b^2)^3/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(a^2+b^2)^3/d*2^(1/2)+1/4*(-8*A*a^4-31*A*a^2*b^2-15*A*b^4+11*B*a^3*b+3*B*a*b^3)/a^3/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)+1/2*b*(A*b-B*a)/a/(a^2+b^2)/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2+1/4*b*(13*A*a^2*b+5*A*b^3-9*B*a^3-B*a*b^2)/a^2/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))
```

**Rubi** [A]

time = 1.08, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

```
[Out] ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) - (b^(3/2)*(63*a^4*A*b + 46*a^2*A*b^3 + 15*A*b^5 - 35*a^5*B - 6*a^3*b^2*B - 3*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]/(4*a^(7/2)*(a^2 + b^2)^3*d) + ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*(a^2 + b^2)^3*d) - (8*a^4*A + 31*a^2*A*b^2 + 15*A*b^4 - 11*a^3*b*B - 3*a*b^3*B)/(4*a^3*(a^2 + b^2)^2*d*Sqrt[Tan[c + d*x]]) + (b*(A*b - a*B))/(2*a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])
```

)^2) + (b\*(13\*a^2\*A\*b + 5\*A\*b^3 - 9\*a^3\*B - a\*b^2\*B))/(4\*a^2\*(a^2 + b^2)^2\*d\*sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x]))

#### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x])/b], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; Fre



$eQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 1182

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] :> With[\{q = Rt[a*c, 2]\}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[\{a, c, d, e\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& NeQ[c*d^2 - a*e^2, 0] \&\& NegQ[(-a)*c]$

### Rule 3615

$Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]]], x\_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[\{b, c, d, e, f\}, x] \&\& NeQ[c^2 - d^2, 0] \&\& NeQ[c^2 + d^2, 0]$

### Rule 3690

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] :> Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, n\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 + b^2, 0] \&\& NeQ[c^2 + d^2, 0] \&\& LtQ[m, -1] \&\& (IntegerQ[m] || IntegersQ[2*m, 2*n]) \&\& !(ILtQ[n, -1] \&\& (!IntegerQ[m] || (EqQ[c, 0] \&\& NeQ[a, 0])))$

### Rule 3715

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x\_Symbol] :> Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& EqQ[A, C]$

### Rule 3730

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2), x\_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*($

```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_))*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^3} dx &= \frac{b(Ab - aB)}{2a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))^2} + \int \frac{\frac{1}{2}(4a^2A + 5Ab^2)}{\dots} \\
&= \frac{b(Ab - aB)}{2a(a^2 + b^2)d\sqrt{\tan(c + dx)}(a + b \tan(c + dx))^2} + \frac{b(13a^2A + 13Ab^2)}{4a^2(a^2 + b^2)} \\
&= -\frac{8a^4A + 31a^2Ab^2 + 15Ab^4 - 11a^3bB - 3ab^3B}{4a^3(a^2 + b^2)^2d\sqrt{\tan(c + dx)}} + \frac{b(13a^2A + 13Ab^2)}{2a(a^2 + b^2)d\sqrt{\tan(c + dx)}} \\
&= -\frac{8a^4A + 31a^2Ab^2 + 15Ab^4 - 11a^3bB - 3ab^3B}{4a^3(a^2 + b^2)^2d\sqrt{\tan(c + dx)}} + \frac{b(13a^2A + 13Ab^2)}{2a(a^2 + b^2)d\sqrt{\tan(c + dx)}} \\
&= -\frac{8a^4A + 31a^2Ab^2 + 15Ab^4 - 11a^3bB - 3ab^3B}{4a^3(a^2 + b^2)^2d\sqrt{\tan(c + dx)}} + \frac{b(13a^2A + 13Ab^2)}{2a(a^2 + b^2)d\sqrt{\tan(c + dx)}} \\
&= -\frac{8a^4A + 31a^2Ab^2 + 15Ab^4 - 11a^3bB - 3ab^3B}{4a^3(a^2 + b^2)^2d\sqrt{\tan(c + dx)}} + \frac{b(13a^2A + 13Ab^2)}{2a(a^2 + b^2)d\sqrt{\tan(c + dx)}} \\
&= -\frac{b^{3/2}(63a^4Ab + 46a^2Ab^3 + 15Ab^5 - 35a^5B - 6a^3b^2B - 3ab^4B)}{4a^{7/2}(a^2 + b^2)^3d} \\
&= -\frac{b^{3/2}(63a^4Ab + 46a^2Ab^3 + 15Ab^5 - 35a^5B - 6a^3b^2B - 3ab^4B)}{4a^{7/2}(a^2 + b^2)^3d} \\
&= \frac{(a^3(A - B) - 3ab^2(A - B) + 3a^2b(A + B) - b^3(A + B)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{\tan(c + dx)}}{a + b \tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)^3d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 5.72, size = 341, normalized size = 0.57

$$\frac{-\sqrt{a^2 + b^2}(8a^4A + 31a^2Ab^2 + 15Ab^4 - 11a^3bB - 3ab^3B) + \left(b^{3/2}(-63a^4Ab - 46a^2Ab^3 - 15Ab^5 + 35a^5B + 6a^3b^2B + 3ab^4B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c + dx)}}{\sqrt{a^2 + b^2}}\right) + \sqrt{-1}a^{7/2}((a - b)^2(A - B)\text{ArcTan}((-1)^{1/4}\sqrt{\tan(c + dx)}) - (a + b)^2(A + B)\text{tanh}^{-1}((-1)^{1/4}\sqrt{\tan(c + dx)}))\right)\sqrt{\tan(c + dx)}}{4a(a^2 + b^2)d\sqrt{\tan(c + dx)}} + \frac{2b(13a^2A + 13Ab^2)}{(a^2 + b^2)^2} + \frac{b(13a^2A + 13Ab^2)}{a(a^2 + b^2)(a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^3), x]

[Out] ((- (Sqrt[a]\*(a^2 + b^2)\*(8\*a^4\*A + 31\*a^2\*A\*b^2 + 15\*A\*b^4 - 11\*a^3\*b\*B - 3\*a\*b^3\*B)) + (b^(3/2)\*(-63\*a^4\*A\*b - 46\*a^2\*A\*b^3 - 15\*A\*b^5 + 35\*a^5\*B + 6

$a^3 b^2 B + 3 a b^4 B) \operatorname{ArcTan}[\operatorname{Sqrt}[b] \operatorname{Sqrt}[\operatorname{Tan}[c + d x]]] / \operatorname{Sqrt}[a] + 4 (-1)^{1/4} a^{7/2} ((I a - b)^3 (A - I B) \operatorname{ArcTan}[(-1)^{3/4} \operatorname{Sqrt}[\operatorname{Tan}[c + d x]]] - (I a + b)^3 (A + I B) \operatorname{ArcTanh}[(-1)^{3/4} \operatorname{Sqrt}[\operatorname{Tan}[c + d x]]]) \operatorname{Sqrt}[\operatorname{Tan}[c + d x]] / (a^{5/2} (a^2 + b^2)^2) + (2 b (A b - a B)) / (a + b \operatorname{Tan}[c + d x])^2 + (b (13 a^2 A b + 5 A b^3 - 9 a^3 B - a b^2 B)) / (a (a^2 + b^2) (a + b \operatorname{Tan}[c + d x])) / (4 a (a^2 + b^2) d \operatorname{Sqrt}[\operatorname{Tan}[c + d x]])$

**Maple [A]**

time = 0.12, size = 466, normalized size = 0.78

method	result
derivativedivides	$-\frac{a^3 \sqrt{\tan(dx+c)}^{2A}}{a^3 \sqrt{\tan(dx+c)}} + \frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan(1 + \dots) \right)}{4}$
default	$-\frac{a^3 \sqrt{\tan(dx+c)}^{2A}}{a^3 \sqrt{\tan(dx+c)}} + \frac{(-3A a^2 b + A b^3 + B a^3 - 3B a b^2) \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan(1 + \dots) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVE  
RBOSE)

[Out] 1/d\*(-2\*A/a^3/tan(d\*x+c)^(1/2)+2/(a^2+b^2)^3\*(1/8\*(-3\*A\*a^2\*b+A\*b^3+B\*a^3-3\*B\*a\*b^2)\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/8\*(-A\*a^3+3\*A\*a\*b^2-3\*B\*a^2\*b+B\*b^3)\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))-2\*b^2/a^3/(a^2+b^2)^3\*((15/8\*A\*a^4\*b^2+11/4\*a^2\*A\*b^4+7/8\*A\*b^6-11/8\*B\*a^5\*b-7/4\*B\*a^3\*b^3-3/8\*B\*a\*b^5)\*tan(d\*x+c)^(3/2)+1/8\*a\*(17\*A\*a^4\*b+26\*A\*a^2\*b^3+9\*A\*b^5-13\*B\*a^5-18\*B\*a^3\*b^2-5\*B\*a\*b^4)\*tan(d\*x+c)^(1/2))/(a+b\*tan(d\*x+c))^2+1/8\*(63\*A\*a^4\*b+46\*A\*a^2\*b^3+15\*A\*b^5-35\*B\*a^5-6\*B\*a^3\*b^2-3\*B\*a\*b^4)/(a\*b)^(1/2)\*arctan(b\*tan(d\*x+c)^(1/2)/(a\*b)^(1/2)))

**Maxima [A]**

time = 0.53, size = 607, normalized size = 1.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$\frac{1}{4} \left( (35B a^5 b^2 - 63A a^4 b^3 + 6B a^3 b^4 - 46A a^2 b^5 + 3B a b^6 - 15A b^7) \arctan\left(\frac{b \sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) / \left( (a^9 + 3a^7 b^2 + 3a^5 b^4 + a^3 b^6) \sqrt{ab} \right) - (8A a^6 + 16A a^4 b^2 + 8A a^2 b^4 + (8A a^4 b^2 - 11B a^3 b^3 + 31A a^2 b^4 - 3B a b^5 + 15A b^6) \tan(dx+c)^2 + (16A a^5 b - 13B a^4 b^2 + 49A a^3 b^3 - 5B a^2 b^4 + 25A a b^5) \tan(dx+c) \right) / \left( (a^7 b^2 + 2a^5 b^4 + a^3 b^6) \tan(dx+c)^{5/2} + 2(a^8 b + 2a^6 b^3 + a^4 b^5) \tan(dx+c)^{3/2} + (a^9 + 2a^7 b^2 + a^5 b^4) \sqrt{\tan(dx+c)} \right) - (2\sqrt{2}) \left( (A-B) a^3 + 3(A+B) a^2 b - 3(A-B) a b^2 - (A+B) b^3 \right) \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2} \left( (A-B) a^3 + 3(A+B) a^2 b - 3(A-B) a b^2 - (A+B) b^3 \right) \arctan\left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) - \sqrt{2} \left( (A+B) a^3 - 3(A-B) a^2 b - 3(A+B) a b^2 + (A-B) b^3 \right) \log\left(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \sqrt{2} \left( (A+B) a^3 - 3(A-B) a^2 b - 3(A+B) a b^2 + (A-B) b^3 \right) \log\left(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) \right) / (a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) / d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 48.48, size = 2500, normalized size = 4.16
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((B*tan(c + d*x)^(1/2)*(5*b^4 + 13*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2))
+ (B*b*tan(c + d*x)^(3/2)*(3*b^4 + 11*a^2*b^2))/(4*a^2*(a^4 + b^4 + 2*a^2*b^2)))
/(a^2*d + b^2*d*tan(c + d*x)^2 + 2*a*b*d*tan(c + d*x)) - ((2*A)/a + (A
*tan(c + d*x)^2*(15*b^6 + 31*a^2*b^4 + 8*a^4*b^2))/(4*a^3*(a^4 + b^4 + 2*a^2*b^2))
+ (A*tan(c + d*x)*(16*a^4*b + 25*b^5 + 49*a^2*b^3))/(4*a^2*(a^4 + b^4 + 2*a^2*b^2)))
/(a^2*d*tan(c + d*x)^(1/2) + b^2*d*tan(c + d*x)^(5/2) + 2*
a*b*d*tan(c + d*x)^(3/2)) + (log(29491200*A^5*a^22*b^35*d^4 - ((tan(c + d*x)
)^(1/2)*(7610564608*A^4*a^27*b^33*d^5 - 597688320*A^4*a^23*b^37*d^5 - 16714
30144*A^4*a^25*b^35*d^5 - 58982400*A^4*a^21*b^39*d^5 + 85774565376*A^4*a^29
*b^31*d^5 + 385487994880*A^4*a^31*b^29*d^5 + 1104303620096*A^4*a^33*b^27*d^5
+ 2240523796480*A^4*a^35*b^25*d^5 + 3345249468416*A^4*a^37*b^23*d^5 + 371
7287903232*A^4*a^39*b^21*d^5 + 3053967114240*A^4*a^41*b^19*d^5 + 1807474491
392*A^4*a^43*b^17*d^5 + 726513221632*A^4*a^45*b^15*d^5 + 170768990208*A^4*a^47*b^13*d^5
+ 10492051456*A^4*a^49*b^11*d^5 - 4917821440*A^4*a^51*b^9*d^5
- 923009024*A^4*a^53*b^7*d^5 + 8388608*A^4*a^55*b^5*d^5) + (((((480*A^4*a^2*
b^10*d^4 - 16*A^4*b^12*d^4 - 16*A^4*a^12*d^4 - 4080*A^4*a^4*b^8*d^4 + 7232*
A^4*a^6*b^6*d^4 - 4080*A^4*a^8*b^4*d^4 + 480*A^4*a^10*b^2*d^4)^(1/2) + 80*A^2*a^3*b^3*d^2
- 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(a^12*d^4 + b^12*d^4
+ 6*a^2*b^10*d^4 + 15*a^4*b^8*d^4 + 20*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^10*b^2*d^4))
^(1/2)*((((((480*A^4*a^2*b^10*d^4 - 16*A^4*b^12*d^4 - 16*A^4*a^12*d^4 - 4080*A^4*a^4*b^8*d^4
+ 7232*A^4*a^6*b^6*d^4 - 4080*A^4*a^8*b^4*d^4 + 480*A^4*a^10*b^2*d^4)^(1/2) + 80*A^2*a^3*b^3*d^2
- 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(a^12*d^4 + b^12*d^4 + 6*a^2*b^10*d^4 + 15*a^4*b^8*d^4 + 20
*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^10*b^2*d^4))^(1/2)*((tan(c + d*x)^(1/2)
)*(((480*A^4*a^2*b^10*d^4 - 16*A^4*b^12*d^4 - 16*A^4*a^12*d^4 - 4080*A^4*a^4*b^8*d^4
+ 7232*A^4*a^6*b^6*d^4 - 4080*A^4*a^8*b^4*d^4 + 480*A^4*a^10*b^2*d^4)^(1/2) + 80*A^2*a^3*b^3*d^2
- 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(a^12*d^4 + b^12*d^4 + 6*a^2*b^10*d^4 + 15*a^4*b^8*d^4 + 20
*a^6*b^6*d^4 + 15*a^8*b^4*d^4 + 6*a^10*b^2*d^4))^(1/2)*(134217728*a^27*b^45*d^9 + 2550136832*a^29
*b^43*d^9 + 22817013760*a^31*b^41*d^9 + 127506841600*a^33*b^39*d^9 + 49727
6682240*a^35*b^37*d^9 + 1430626762752*a^37*b^35*d^9 + 3121367482368*a^39*b^
```

$$\begin{aligned}
& 33*d^9 + 5202279137280*a^41*b^31*d^9 + 6502848921600*a^43*b^29*d^9 + 563580 \\
& 2398720*a^45*b^27*d^9 + 2254320959488*a^47*b^25*d^9 - 2254320959488*a^49*b^ \\
& 23*d^9 - 5635802398720*a^51*b^21*d^9 - 6502848921600*a^53*b^19*d^9 - 520227 \\
& 9137280*a^55*b^17*d^9 - 3121367482368*a^57*b^15*d^9 - 1430626762752*a^59*b^ \\
& 13*d^9 - 497276682240*a^61*b^11*d^9 - 127506841600*a^63*b^9*d^9 - 228170137 \\
& 60*a^65*b^7*d^9 - 2550136832*a^67*b^5*d^9 - 134217728*a^69*b^3*d^9)/4 + 25 \\
& 1658240*A*a^24*b^45*d^8 + 5049942016*A*a^26*b^43*d^8 + 48368713728*A*a^28*b \\
& ^41*d^8 + 293819383808*A*a^30*b^39*d^8 + 1268458192896*A*a^32*b^37*d^8 + 41 \\
& 32731617280*A*a^34*b^35*d^8 + 10531192700928*A*a^36*b^33*d^8 + 214628239933 \\
& 44*A*a^38*b^31*d^8 + 35469618315264*A*a^40*b^29*d^8 + 47896904859648*A*a^42 \\
& *b^27*d^8 + 52983958077440*A*a^44*b^25*d^8 + 47896904859648*A*a^46*b^23*d^8 \\
& + 35090285461504*A*a^48*b^21*d^8 + 20487396655104*A*a^50*b^19*d^8 + 923062 \\
& 2916608*A*a^52*b^17*d^8 + 2994733056000*A*a^54*b^15*d^8 + 565576728576*A*a^ \\
& 56*b^13*d^8 - 18572378112*A*a^58*b^11*d^8 - 50281316352*A*a^60*b^9*d^8 - 16 \\
& 089350144*A*a^62*b^7*d^8 - 2516582400*A*a^64*b^5*d^8 - 167772160*A*a^66*b^3 \\
& *d^8)/4 - \tan(c + d*x)^{(1/2)}*(471859200*A^2*a^22*b^44*d^7 + 9500098560*A^2 \\
& *a^24*b^42*d^7 + 91857354752*A^2*a^26*b^40*d^7 + 564502986752*A^2*a^28*b^38 \\
& *d^7 + 2464648527872*A^2*a^30*b^36*d^7 + 8104469069824*A^2*a^32*b^34*d^7 + \\
& 20769933361152*A^2*a^34*b^32*d^7 + 42351565209600*A^2*a^36*b^30*d^7 + 69534 \\
& 945902592*A^2*a^38*b^28*d^7 + 92434029608960*A^2*a^40*b^26*d^7 + 9950871735 \\
& 5008*A^2*a^42*b^24*d^7 + 86342935511040*A^2*a^44*b^22*d^7 + 59767095558144* \\
& A^2*a^46*b^20*d^7 + 32432589897728*A^2*a^48*b^18*d^7 + 13411815522304*A^2*a \\
& ^50*b^16*d^7 + 4030457708544*A^2*a^52*b^14*d^7 + 805425905664*A^2*a^54*b^12 \\
& *d^7 + 86608183296*A^2*a^56*b^10*d^7 + 1612709888*A^2*a^58*b^8*d^7 + 167772 \\
& 16*A^2*a^60*b^6*d^7 + 167772160*A^2*a^62*b^4*d^7 + 16777216*A^2*a^64*b^2*d^ \\
& 7))*(((480*A^4*a^2*b^10*d^4 - 16*A^4*b^12*d^4 - 16*A^4*a^12*d^4 - 4080*A^4* \\
& a^4*b^8*d^4 + 7232*A^4*a^6*b^6*d^4 - 4080*A^4*a^8*b^4*d^4 + 480*A^4*a^10*b^ \\
& 2*d^4))^{(1/2)} + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(a \\
& ^12*d^4 + b^12*d^4 + 6*a^2*b^10*d^4 + 15*a^4*b^8*d^4 + 20*a^6*b^6*d^4 + 15* \\
& a^8*b^4*d^4 + 6*a^10*b^2*d^4))^{(1/2)}/4 - 117964800*A^3*a^21*b^42*d^6 - 841 \\
& 482240*A^3*a^23*b^40*d^6 + 3829399552*A^3*a^25*b^38*d^6 + 78068580352*A^3*a \\
& ^27*b^36*d^6 + 497438162944*A^3*a^29*b^34*d^6 + 1899895980032*A^3*a^31*b^32 \\
& *d^6 + 4972695519232*A^3*a^33*b^30*d^6 + 9371195015168*A^3*a^35*b^28*d^6 + \\
& 12890720436224*A^3*a^37*b^26*d^6 + 12726089809920*A^3*a^39*b^24*d^6 + 83669 \\
& 61197056*A^3*a^41*b^22*d^6 + 2597662490624*A^3*a^43*b^20*d^6 - 117183610880 \\
& 0*A^3*a^45*b^18*d^6 - 1986881650688*A^3*a^47*b^...
\end{aligned}$$

$$3.416 \quad \int \frac{\tan^5(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=156

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-1/2*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2/3*B*\tan(d*x+c)^{(3/2)}/d$

**Rubi [A]**

time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2B \tan^3(c+dx)}{3d} - \frac{B \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{B \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x]^{(5/2)}*(a*B + b*B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + (2*B*\operatorname{Tan}[c + d*x]^{(3/2)})/(3*d)$

Rule 21

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*)^{(m_*)})*((c_*) + (d_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 303

$\operatorname{Int}[(x_*)^2/((a_*) + (b_*)*(x_*)^4), x\_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}\{a,$



b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &  
& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k =  
Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n  
)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F  
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b  
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := S  
imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e  
/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; Fre  
eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3554

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*((b\*Tan[c + d  
\*x])^(n - 1)/(d\*(n - 1))), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x],  
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[  
x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)} dx &= B \int \tan^{\frac{5}{2}}(c+dx) dx \\
&= \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3d} - B \int \sqrt{\tan(c+dx)} dx \\
&= \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3d} - \frac{B \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3d} - \frac{(2B) \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{B \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} - \frac{B \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= \frac{2B \tan^{\frac{3}{2}}(c+dx)}{3d} - \frac{B \text{Subst}\left(\int \frac{1}{1-\sqrt{2}xx^2} dx, x, \sqrt{\tan(c+dx)}\right)}{2d} \\
&= -\frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{2\sqrt{2}d} + \frac{B \log\left(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{2\sqrt{2}d} \\
&= \frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 38, normalized size = 0.24

$$-\frac{2B(-1 + {}_2F_1\left(\frac{3}{4}, 1, \frac{7}{4}, -\tan^2(c+dx)\right)) \tan^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(5/2)\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] (-2\*B\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2])\*Tan[c + d\*x]^(3/2))/(3\*d)

**Maple [A]**

time = 0.05, size = 102, normalized size = 0.65

method	result
derivativedivides	$B \left( \frac{2 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - \frac{\sqrt{2} \left( \ln \left( \frac{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( 1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right)}{d}$
default	$B \left( \frac{2 \left( \tan^{\frac{3}{2}}(dx+c) \right)}{3} - \frac{\sqrt{2} \left( \ln \left( \frac{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( 1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(5/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out]  $1/d*B*(2/3*\tan(d*x+c)^{(3/2)}-1/4*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [A]

time = 0.51, size = 123, normalized size = 0.79

$$\frac{8B \tan(dx+c)^{\frac{5}{2}} - 3 \left( 2\sqrt{2} \arctan \left( \frac{1}{2}\sqrt{2} \left( \sqrt{2} + 2\sqrt{\tan(dx+c)} \right) \right) + 2\sqrt{2} \arctan \left( -\frac{1}{2}\sqrt{2} \left( \sqrt{2} - 2\sqrt{\tan(dx+c)} \right) \right) - \sqrt{2} \log \left( \sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) + \sqrt{2} \log \left( -\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) \right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(5/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/12*(8*B*\tan(d*x+c)^{(3/2)} - 3*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\tan(d*x+c)}))) + 2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\tan(d*x+c)})) - \sqrt{2}*\log(\sqrt{2}*\sqrt{\tan(d*x+c)} + \tan(d*x+c) + 1) + \sqrt{2}*\log(-\sqrt{2}*\sqrt{\tan(d*x+c)} + \tan(d*x+c) + 1))*B/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs.  $2(124) = 248$ .

time = 1.73, size = 592, normalized size = 3.79

$$\frac{8B \tan(dx+c)^{\frac{5}{2}} - 3 \left( 2\sqrt{2} \arctan \left( \frac{1}{2}\sqrt{2} \left( \sqrt{2} + 2\sqrt{\tan(dx+c)} \right) \right) + 2\sqrt{2} \arctan \left( -\frac{1}{2}\sqrt{2} \left( \sqrt{2} - 2\sqrt{\tan(dx+c)} \right) \right) - \sqrt{2} \log \left( \sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) + \sqrt{2} \log \left( -\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) \right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{12} * (12 * \sqrt{2} * d * (B^4/d^4)^{1/4} * \arctan(-(\sqrt{2} * B^3 * d * (B^4/d^4)^{1/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} + B^4 - \sqrt{2} * d * (B^4/d^4)^{1/4} * \sqrt{(\sqrt{2} * B^3 * d^3 * (B^4/d^4)^{3/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} * \cos(d*x + c) + B^4 * d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) + B^6 * \sin(d*x + c))/\cos(d*x + c)})) / B^4 * \cos(d*x + c) + 12 * \sqrt{2} * d * (B^4/d^4)^{1/4} * \arctan(-(\sqrt{2} * B^3 * d * (B^4/d^4)^{1/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} - B^4 - \sqrt{2} * d * (B^4/d^4)^{1/4} * \sqrt{-(\sqrt{2} * B^3 * d^3 * (B^4/d^4)^{3/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} * \cos(d*x + c) - B^4 * d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) - B^6 * \sin(d*x + c))/\cos(d*x + c)})) / B^4 * \cos(d*x + c) + 3 * \sqrt{2} * d * (B^4/d^4)^{1/4} * \cos(d*x + c) * \log((\sqrt{2} * B^3 * d^3 * (B^4/d^4)^{3/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} * \cos(d*x + c) + B^4 * d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) + B^6 * \sin(d*x + c))/\cos(d*x + c)) - 3 * \sqrt{2} * d * (B^4/d^4)^{1/4} * \cos(d*x + c) * \log(-(\sqrt{2} * B^3 * d^3 * (B^4/d^4)^{3/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} * \cos(d*x + c) - B^4 * d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) - B^6 * \sin(d*x + c))/\cos(d*x + c)) + 8 * B * \sqrt{\sin(d*x + c)/\cos(d*x + c)} * \sin(d*x + c) / (d * \cos(d*x + c))$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \tan^{\frac{5}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(tan(c + d\*x)\*\*(5/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.61, size = 2500, normalized size = 16.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((\tan(c + d*x))^{5/2}*(B*a + B*b*\tan(c + d*x)))/(a + b*\tan(c + d*x)),x)$

[Out]  $\text{atan}(\frac{(((((32*(4*B*a^2*b^8*d^4 + 8*B*a^4*b^6*d^4 + 4*B*a^6*b^4*d^4))/(b*d^5) - (32*\tan(c + d*x))^{1/2}*((((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} + (32*\tan(c + d*x))^{1/2}*(4*B^2*a^5*b^5*d^2 - 14*B^2*a^3*b^7*d^2 + 2*B^2*a^7*b^3*d^2 + 16*B^2*a^9*b*d^2))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - (32*(B^3*a^5*b^5*d^2 - 15*B^3*a^7*b^3*d^2 + 12*B^3*a^9*b*d^2))/(b*d^5))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - (32*(B^3*a^5*b^5*d^2 - 15*B^3*a^7*b^3*d^2 + 12*B^3*a^9*b*d^2))/(b*d^5))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} + (32*\tan(c + d*x))^{1/2}*(2*B^4*a^{10} - B^4*a^4*b^6))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} * 1i - ((((((32*(4*B*a^2*b^8*d^4 + 8*B*a^4*b^6*d^4 + 4*B*a^6*b^4*d^4))/(b*d^5) + (32*\tan(c + d*x))^{1/2}*((((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - (32*\tan(c + d*x))^{1/2}*(4*B^2*a^5*b^5*d^2 - 14*B^2*a^3*b^7*d^2 + 2*B^2*a^7*b^3*d^2 + 16*B^2*a^9*b*d^2))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - (32*(B^3*a^5*b^5*d^2 - 15*B^3*a^7*b^3*d^2 + 12*B^3*a^9*b*d^2))/(b*d^5))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - (32*\tan(c + d*x))^{1/2}*(2*B^4*a^{10} - B^4*a^4*b^6))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} * 1i)/(((((((32*(4*B*a^2*b^8*d^4 + 8*B*a^4*b^6*d^4 + 4*B*a^6*b^4*d^4))/(b*d^5) - (32*\tan(c + d*x))^{1/2}*((((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} + (32*\tan(c + d*x))^{1/2}*(4*B^2*a^5*b^5*d^2 - 14*B^2*a^3*b^7*d^2 + 2*B^2*a^7*b^3*d^2 + 16*B^2*a^9*b*d^2))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2} - (32*(B^3*a^5*b^5*d^2 - 15*B^3*a^7*b^3*d^2 + 12*B^3*a^9*b*d^2))/(b*d^5))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{1/2} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{1/2}$

$$\begin{aligned}
& 2*d^4)^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} + (32*\tan(c + d*x)^{(1/2)}*(2*B^4*a^{10} - B^4*a^4*b^6))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} + (((((32*(4*B*a^2*b^8*d^4 + 8*B*a^4*b^6*d^4 + 4*B*a^6*b^4*d^4))/(b*d^5) + (32*\tan(c + d*x)^{(1/2)}*((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*(16*b^{10}*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4)))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(4*B^2*a^5*b^5*d^2 - 14*B^2*a^3*b^7*d^2 + 2*B^2*a^7*b^3*d^2 + 16*B^2*a^9*b*d^2))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*(B^3*a^5*b^5*d^2 - 15*B^3*a^7*b^3*d^2 + 12*B^3*a^9*b*d^2))/(b*d^5))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(2*B^4*a^{10} - B^4*a^4*b^6))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (64*(B^5*a^{10} - B^5*a^8*b^2))/(b*d^5))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*2i - \operatorname{atan}((((((32*(12*B*a^2*b^7*d^4 + 24*B*a^4*b^5*d^4 + 12*B*a^6*b^3*d^4))/d^5 - (32*\tan(c + d*x)^{(1/2)}*((64*B^4*a^2*...
\end{aligned}$$

$$3.417 \quad \int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=154

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-1/2*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+2*B*\tan(d*x+c)^{(1/2)}/d$

**Rubi** [A]

time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2B \sqrt{\tan(c+dx)}}{d} + \frac{B \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{B \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x])^{(3/2)}*(a*B + b*B*\operatorname{Tan}[c + d*x])]/(a + b*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x])/(2*\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x])/(2*\operatorname{Sqrt}[2]*d) + (2*B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d$

Rule 21

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*)^{(m_*)}*((c_*) + (d_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \& \& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 217

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^4)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}\{a, b$

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 335

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

### Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```



IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)} dx &= B \int \tan^{\frac{3}{2}}(c+dx) dx \\
&= \frac{2B\sqrt{\tan(c+dx)}}{d} - B \int \frac{1}{\sqrt{\tan(c+dx)}} dx \\
&= \frac{2B\sqrt{\tan(c+dx)}}{d} - \frac{B \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{2B\sqrt{\tan(c+dx)}}{d} - \frac{(2B) \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= \frac{2B\sqrt{\tan(c+dx)}}{d} - \frac{B \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= \frac{2B\sqrt{\tan(c+dx)}}{d} - \frac{B \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(c+dx)}\right)}{2d} \\
&= \frac{B \log\left(1 - \sqrt{2}\sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{2\sqrt{2}d} - \frac{B \log\left(1 + \sqrt{2}\sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{2\sqrt{2}d} \\
&= \frac{B \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B \tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 138, normalized size = 0.90

$$\frac{B(2\sqrt{2} \operatorname{ArcTan}(1 - \sqrt{2}\sqrt{\tan(c+dx)}) - 2\sqrt{2} \operatorname{ArcTan}(1 + \sqrt{2}\sqrt{\tan(c+dx)}) + \sqrt{2} \log(1 - \sqrt{2}\sqrt{\tan(c+dx)} + \tan(c+dx)) - \sqrt{2} \log(1 + \sqrt{2}\sqrt{\tan(c+dx)} + \tan(c+dx)) + 8\sqrt{\tan(c+dx)})}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] (B\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + 8\*Sqrt[Tan[c + d\*x]]))/(4\*d)

Maple [A]

time = 0.05, size = 102, normalized size = 0.66

method	result
derivativedivides	$B \left[ \frac{2 \left( \sqrt{\tan(dx+c)} \right) - \frac{\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4}}{d} \right]$
default	$B \left[ \frac{2 \left( \sqrt{\tan(dx+c)} \right) - \frac{\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4}}{d} \right]$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x,method=\_RETURN VERBOSE)

[Out] 1/d\*B\*(2\*tan(d\*x+c)^(1/2)-1/4\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))

**Maxima [A]**

time = 0.51, size = 123, normalized size = 0.80

$$\frac{2\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}B\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}B\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-8B\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] -1/4\*(2\*sqrt(2)\*B\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*B\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(tan(d\*x + c)))) + sqrt(2)\*B\*log(sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) - sqrt(2)\*B\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) - 8\*B\*sqrt(tan(d\*x + c)))/d

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(124) = 248.

time = 1.49, size = 529, normalized size = 3.44

$$\frac{\sqrt{2}B\sqrt{\tan(dx+c)}\arctan\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}}{1+\sqrt{\tan(dx+c)}}\right)+\sqrt{2}B\sqrt{\tan(dx+c)}\arctan\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}}{1-\sqrt{\tan(dx+c)}}\right)+\sqrt{2}B\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}B\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-8B\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (4 * \sqrt{2} * d * (B^4/d^4)^{1/4} * \arctan(-(\sqrt{2} * B * d^3 * (B^4/d^4)^{3/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} - \sqrt{2} * d^3 * (B^4/d^4)^{3/4} * \sqrt{(\sqrt{2} * B * d * (B^4/d^4)^{1/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)}) * \cos(d*x + c) + d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) + B^2 * \sin(d*x + c))/\cos(d*x + c)}) + B^4)/B^4 + 4 * \sqrt{2} * d * (B^4/d^4)^{1/4} * \arctan(-(\sqrt{2} * B * d^3 * (B^4/d^4)^{3/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)} - \sqrt{2} * d^3 * (B^4/d^4)^{3/4} * \sqrt{-(\sqrt{2} * B * d * (B^4/d^4)^{1/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)}) * \cos(d*x + c) - d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) - B^2 * \sin(d*x + c))/\cos(d*x + c)}) - B^4)/B^4 - \sqrt{2} * d * (B^4/d^4)^{1/4} * \log((\sqrt{2} * B * d * (B^4/d^4)^{1/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)}) * \cos(d*x + c) + d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) + B^2 * \sin(d*x + c))/\cos(d*x + c)) + \sqrt{2} * d * (B^4/d^4)^{1/4} * \log(-(\sqrt{2} * B * d * (B^4/d^4)^{1/4} * \sqrt{\sin(d*x + c)/\cos(d*x + c)}) * \cos(d*x + c) - d^2 * \sqrt{B^4/d^4} * \cos(d*x + c) - B^2 * \sin(d*x + c))/\cos(d*x + c)) + 8 * B * \sqrt{\sin(d*x + c)/\cos(d*x + c)})/d$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \tan^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(tan(c + d\*x)\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 11.29, size = 2500, normalized size = 16.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)



$$\begin{aligned}
& ))^{(1/2)} - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} \\
& - (64*(B^5*a^3*b^6 - B^5*a^5*b^4))/d^5 + (((32*(B^3*a^2*b^7*d^2 - 15*B^3*a \\
& ^4*b^5*d^2 + 12*B^3*a^6*b^3*d^2))/d^5 - (((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6* \\
& d^4 + 4*B*a^5*b^4*d^4))/d^5 + (32*\tan(c + d*x)^{(1/2)}*((64*B^4*a^2*b^6*d^4 \\
& - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b^3*d \\
& ^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*(16*b^9*d^4 + 16*a^2*b^ \\
& 7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*B^4*a^2*b^6*d^4 - B^4* \\
& b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b^3*d^2)/(1 \\
& 6*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} - (32*\tan(c + d*x)^{(1/2)}*(4*B \\
& ^2*a^3*b^6*d^2 + 2*B^2*a^5*b^4*d^2 + 16*B^2*a^7*b^2*d^2 - 14*B^2*a*b^8*d^2) \\
& )/d^4)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^ \\
& 2*d^4))^{(1/2)} - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{( \\
& 1/2)})*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^ \\
& 2*d^4))^{(1/2)} - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{( \\
& 1/2)} - (32*\tan(c + d*x)^{(1/2)}*(B^4*b^9 - 2*B^4*a^6*b^3))/d^4)*(((64*B^4*a^ \\
& 2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B \\
& ^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}))*(((64*B^4*a \\
& ^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8* \\
& B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)})*2i - \operatorname{atan}((( \\
& (((32*(4*B*a^3*b^6*d^4 + 8*B*a^5*b^4*d^4 + 4*B*a^7*b^2*d^4))/d^5 - (32*\tan \\
& (c + d*x)^{(1/2)}*((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + \\
& 32*a^2*b^2*d^4))^{(1/2)} + 8*B^2*a^3*b*d^2)/(16*(...
\end{aligned}$$

$$3.418 \quad \int \frac{\sqrt{\tan(c+dx)} (aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=138

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d}$$

[Out] 1/2\*B\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/2\*B\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/4\*B\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)-1/4\*B\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)

**Rubi [A]**

time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {21, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{B \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{B \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Tan[c + d\*x]]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] -((B\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d)) + (B\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) + (B\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) - (B\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d)

Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 303

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a,

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &  
& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k =  
Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n  
)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F  
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b  
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := S  
imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e  
(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; Fre  
eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[  
x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !  
IntegerQ[n]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)} (aB + bB \tan(c+dx))}{a + b \tan(c+dx)} dx &= B \int \sqrt{\tan(c+dx)} dx \\
&= \frac{B \operatorname{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{(2B) \operatorname{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= -\frac{B \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{B \operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\
&= \frac{B \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(c+dx)}\right)}{2d} + \frac{B \operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(c+dx)}\right)}{2d} \\
&= \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{2\sqrt{2}d} - \frac{B \log\left(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right)}{2\sqrt{2}d} \\
&= -\frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 36, normalized size = 0.26

$$\frac{2B {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(c+dx)\right) \tan^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Tan[c + d\*x]]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] (2\*B\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(3/2))/(3\*d)

**Maple [A]**

time = 0.08, size = 90, normalized size = 0.65

method	result
derivativedivides	$ \frac{B\sqrt{2} \left( \ln\left(\frac{1-\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}{1+\sqrt{2}(\sqrt{\tan(dx+c)}+\tan(dx+c))}\right) + 2\arctan\left(1+\sqrt{2}(\sqrt{\tan(dx+c)})\right) + 2\arctan\left(-1+\sqrt{2}(\sqrt{\tan(dx+c)})\right) \right)}{4d} $



default	$\frac{B\sqrt{2} \left( \ln \left( \frac{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \right) \right)}{4d}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out]  $\frac{1}{4}d*B*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima** [A]

time = 0.51, size = 109, normalized size = 0.79

$$\frac{(2\sqrt{2} \arctan(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})))+2\sqrt{2} \arctan(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)}))-\sqrt{2} \log(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+\sqrt{2} \log(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1))B}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{4}*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)})))+2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)}))-\sqrt{2}*\log(\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+\sqrt{2}*\log(-\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1))*B/d$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(110) = 220.

time = 1.22, size = 525, normalized size = 3.80

$$\frac{-\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1}{\sqrt{2} \sqrt{\tan(dx+c)} - \tan(dx+c) - 1}\right) + \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)} - \tan(dx+c) - 1}{\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1}\right) - \sqrt{2} \log\left(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \sqrt{2} \log\left(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]  $-\sqrt{2}*(B^4/d^4)^{(1/4)}*\arctan(-(\sqrt{2})*B^3*d*(B^4/d^4)^{(1/4)}*\sqrt{\sin(dx+c)/\cos(dx+c)})+B^4-\sqrt{2}*d*(B^4/d^4)^{(1/4)}*\sqrt{((\sqrt{2})*B^3*d^3*(B^4/d^4)^{(3/4)}*\sqrt{\sin(dx+c)/\cos(dx+c)}*\cos(dx+c)+B^4*d^2*\sqrt{B^4/d^4}*\cos(dx+c)+B^6*\sin(dx+c))/\cos(dx+c)}}/B^4)-\sqrt{2}*(B^4/d^4)^{(1/4)}*\arctan(-(\sqrt{2})*B^3*d*(B^4/d^4)^{(1/4)}*\sqrt{\sin(dx+c)/\cos(dx+c)})-\sqrt{2}*(B^4/d^4)^{(1/4)}*\sqrt{((\sqrt{2})*B^3*d^3*(B^4/d^4)^{(3/4)}*\sqrt{\sin(dx+c)/\cos(dx+c)}*\cos(dx+c)-B^4*d^2*\sqrt{B^4/d^4}*\cos(dx+c)-B^6*\sin(dx+c))/\cos(dx+c)}}/B^4)-1/4*\sqrt{2}*(B^4$

$$\begin{aligned} & /d^4)^{(1/4)} * \log((\sqrt{2} * B^3 * d^3 * (B^4/d^4)^{(3/4)} * \sqrt{\sin(dx + c)/\cos(dx + c)} * \cos(dx + c) + B^4 * d^2 * \sqrt{B^4/d^4} * \cos(dx + c) + B^6 * \sin(dx + c) / \cos(dx + c)) + 1/4 * \sqrt{2} * (B^4/d^4)^{(1/4)} * \log(-(\sqrt{2} * B^3 * d^3 * (B^4/d^4)^{(3/4)} * \sqrt{\sin(dx + c)/\cos(dx + c)} * \cos(dx + c) - B^4 * d^2 * \sqrt{B^4/d^4} * \cos(dx + c) - B^6 * \sin(dx + c) / \cos(dx + c))) \end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)\*\*(1/2)\*(a\*B+b\*B\*tan(dx+c))/(a+b\*tan(dx+c)),x)

[Out] B\*Integral(sqrt(tan(c + dx)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(dx+c)^(1/2)\*(a\*B+b\*B\*tan(dx+c))/(a+b\*tan(dx+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 11.08, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + dx)^(1/2)\*(B\*a + B\*b\*tan(c + dx)))/(a + b\*tan(c + dx)),x)

[Out] atan((((32\*(13\*B^3\*a^5\*b^4\*d^2 + B^3\*a^7\*b^2\*d^2))/d^5 + (((32\*(12\*B\*a^2\*b^7\*d^4 + 24\*B\*a^4\*b^5\*d^4 + 12\*B\*a^6\*b^3\*d^4))/d^5 - (32\*tan(c + dx)^(1/2) \* (((64\*B^4\*a^6\*b^2\*d^4 - B^4\*a^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*B^2\*a^3\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) \* (16\*b^9\*d^4 + 16\*a^2\*b^7\*d^4 - 16\*a^4\*b^5\*d^4 - 16\*a^6\*b^3\*d^4))/d^4) \* (((64\*B^4\*a^6\*b^2\*d^4 - B^4\*a^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*B^2\*a^3\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) + (32\*tan(c + dx)^(1/2) \* (20\*B^2\*a^5\*b^4\*d^2 - 14\*B^2\*a^3\*b^6\*d^2 + 2\*B^2\*a^7\*b^2\*d^2))/d^4) \* (((64\*B^4\*a^6\*b^2\*d^4 - B^4\*a^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*B^2\*a^3\*b\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4))



$$\begin{aligned}
& B^4 a^6 b^2 d^4 - B^4 a^4 (16 a^4 d^4 + 16 b^4 d^4 + 32 a^2 b^2 d^4)^{1/2} \\
& - 8 B^2 a^3 b d^2 / (16 (a^4 d^4 + b^4 d^4 + 2 a^2 b^2 d^4))^{1/2} + (32 \tan(c + d x)^{1/2} (B^4 a^4 b^5 - 2 B^4 a^6 b^3) / d^4) * (((64 B^4 a^6 b^2 d^4 \\
& - B^4 a^4 (16 a^4 d^4 + 16 b^4 d^4 + 32 a^2 b^2 d^4))^{1/2} - 8 B^2 a^3 b d^2) / (16 (a^4 d^4 + b^4 d^4 + 2 a^2 b^2 d^4))^{1/2} + (64 B^5 a^6 b^3) / d^5 \\
& ) * (((64 B^4 a^6 b^2 d^4 - B^4 a^4 (16 a^4 d^4 + 16 b^4 d^4 + 32 a^2 b^2 d^4))^{1/2} - 8 B^2 a^3 b d^2) / (16 (a^4 d^4 + b^4 d^4 + 2 a^2 b^2 d^4))^{1/2} \\
& ) * 2i + \operatorname{atan}((((32 (13 B^3 a^5 b^4 d^2 + B^3 a^7 b^2 d^2)) / d^5 + ((32 (12 B a^2 b^7 d^4 + 24 B a^4 b^5 d^4 + 12 B a^6 b^3 d^4)) / d^5 - (32 \tan(c + d x) \\
& )^{1/2} * (-((64 B^4 a^6 b^2 d^4 - B^4 a^4 (16 a^4 d^4 + 16 b^4 d^4 + 32 a^2 b^2 d^4))^{1/2} + 8 B^2 a^3 b d^2) / (16 (a^4 d^4 + b^4 d^4 + 2 a^2 b^2 d^4)) \\
& )^{1/2} * (16 b^9 d^4 + 16 a^2 b^7 d^4 - 16 a^4 b \dots
\end{aligned}$$

$$3.419 \quad \int \frac{aB + bB \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=138

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d}$$

[Out] 1/2\*B\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)+1/2\*B\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))/d\*2^(1/2)-1/4\*B\*ln(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)+1/4\*B\*ln(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/d\*2^(1/2)

**Rubi** [A]

time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {21, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{B \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{B \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])),x]

[Out] -((B\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d)) + (B\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/(Sqrt[2]\*d) - (B\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d) + (B\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])/(2\*Sqrt[2]\*d)

Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}

}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n))^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx &= B \int \frac{1}{\sqrt{\tan(c + dx)}} dx \\
&= \frac{B \text{Subst} \left( \int \frac{1}{\sqrt{x} (1+x^2)} dx, x, \tan(c + dx) \right)}{d} \\
&= \frac{(2B) \text{Subst} \left( \int \frac{1}{1+x^4} dx, x, \sqrt{\tan(c + dx)} \right)}{d} \\
&= \frac{B \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)} \right)}{d} + \frac{B \text{Subst} \left( \int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)} \right)}{d} \\
&= \frac{B \text{Subst} \left( \int \frac{1}{1-\sqrt{2} x+x^2} dx, x, \sqrt{\tan(c + dx)} \right)}{2d} + \frac{B \text{Subst} \left( \int \frac{1}{1+\sqrt{2} x+x^2} dx, x, \sqrt{\tan(c + dx)} \right)}{2d} \\
&= -\frac{B \log \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx) \right)}{2\sqrt{2} d} + \frac{B \log \left( 1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx) \right)}{2\sqrt{2} d} \\
&= -\frac{B \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} \right)}{\sqrt{2} d} + \frac{B \tan^{-1} \left( 1 + \sqrt{2} \sqrt{\tan(c + dx)} \right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 110, normalized size = 0.80

$$\frac{B(-2\text{ArcTan}(1 - \sqrt{2} \sqrt{\tan(c + dx)}) + 2\text{ArcTan}(1 + \sqrt{2} \sqrt{\tan(c + dx)}) - \log(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)) + \log(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)))}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])), x]
```

```
[Out] (B*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(2*Sqrt[2]*d)
```

**Maple [A]**

time = 0.07, size = 90, normalized size = 0.65

method	result
derivativedivides	$ \frac{B\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2\arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2\arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4d} $





$\log(\sqrt{2} * B * d * (B^4/d^4)^{1/4} * \sqrt{\sin(dx + c)/\cos(dx + c)} * \cos(dx + c) + d^2 * \sqrt{B^4/d^4} * \cos(dx + c) + B^2 * \sin(dx + c))/\cos(dx + c) - 1/4 * \sqrt{2} * (B^4/d^4)^{1/4} * \log(-\sqrt{2} * B * d * (B^4/d^4)^{1/4} * \sqrt{\sin(dx + c)/\cos(dx + c)} * \cos(dx + c) - d^2 * \sqrt{B^4/d^4} * \cos(dx + c) - B^2 * \sin(dx + c))/\cos(dx + c)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(dx+c))/tan(dx+c)\*\*(1/2)/(a+b\*tan(dx+c)),x)

[Out] B\*Integral(1/sqrt(tan(c + dx)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(dx+c))/tan(dx+c)^(1/2)/(a+b\*tan(dx+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 11.12, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + dx))/(tan(c + dx)^(1/2)\*(a + b\*tan(c + dx))),x)

[Out] atan((((32\*(13\*B^3\*a^2\*b^7\*d^2 + B^3\*a^4\*b^5\*d^2))/d^5 + ((32\*(12\*B\*a\*b^8\*d^4 + 24\*B\*a^3\*b^6\*d^4 + 12\*B\*a^5\*b^4\*d^4))/d^5 - (32\*tan(c + dx)^(1/2)\*((64\*B^4\*a^2\*b^6\*d^4 - B^4\*b^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*B^2\*a\*b^3\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2)\*(16\*b^9\*d^4 + 16\*a^2\*b^7\*d^4 - 16\*a^4\*b^5\*d^4 - 16\*a^6\*b^3\*d^4))/d^4)\*((64\*B^4\*a^2\*b^6\*d^4 - B^4\*b^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*B^2\*a\*b^3\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))^(1/2) + (32\*tan(c + dx)^(1/2)\*(20\*B^2\*a^3\*b^6\*d^2 + 2\*B^2\*a^5\*b^4\*d^2 - 14\*B^2\*a\*b^8\*d^2))/d^4)\*(((64\*B^4\*a^2\*b^6\*d^4 - B^4\*b^4\*(16\*a^4\*d^4 + 16\*b^4\*d^4 + 32\*a^2\*b^2\*d^4))^(1/2) - 8\*B^2\*a\*b^3\*d^2)/(16\*(a^4\*d^4 + b^4\*d^4 + 2\*a^2\*b^2\*d^4)))



$$\begin{aligned}
& (6*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4)^{(1/2)} - 8*B^2*a*b^3*d^2)/(16*(a^4 \\
& *d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)} + (32*\tan(c + d*x)^{(1/2)}*(B^4*b^9 - \\
& 2*B^4*a^2*b^7))/d^4)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4* \\
& d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2* \\
& a^2*b^2*d^4))^{(1/2)} + (64*B^5*a*b^8)/d^5))*(((64*B^4*a^2*b^6*d^4 - B^4*b^4 \\
& *(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a*b^3*d^2)/(16*( \\
& a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*2i - \operatorname{atan}((((32*(5*B^3*a^4*b^5 \\
& + B^3*a^6*b^3))/d^3 - (((32*(16*B*a*b^8*d^2 + 28*B*a^3*b^6*d^2 + 8*B*a^5*b^ \\
& 4*d^2 - 4*B*a^7*b^2*d^2))/d^3 - (32*\tan(c + d*x)^{(1/2)}*((64*B^4*a^6*b^2*d^ \\
& 4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} + 8*B^2*a^3*b \\
& *d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))^{(1/2)}*(16*b^9*d^4 + 16*a^2* \\
& b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4\dots
\end{aligned}$$

$$3.420 \quad \int \frac{aB + bB \tan(c+dx)}{\tan^3(c+dx)(a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=154

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-1/2*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}+1/4*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2*B/d/\tan(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{2B}{d\sqrt{\tan(c+dx)}} - \frac{B \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{B \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x])]/(\operatorname{Tan}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x])), x]$

[Out]  $(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (2*B)/(d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

Rule 21

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_.))^{(m_.)*((c_.) + (d_.)*(v_.))^{(n_.)}, x\_Symbol] := \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2]^{(-1)}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] || \operatorname{LtQ}[b, 0])$

Rule 303

$\operatorname{Int}[(x_)^2/((a_.) + (b_.)*(x_)^4), x\_Symbol] := \operatorname{With}[\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}[\{a,$

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &  
& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k =  
Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n  
)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F  
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b  
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := S  
imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e  
/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; Fre  
eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3555

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*Tan[c + d\*x]  
)^(n + 1)/(b\*d\*(n + 1)), x] - Dist[1/b^2, Int[(b\*Tan[c + d\*x])^(n + 2), x],  
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[  
x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx &= B \int \frac{1}{\tan^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2B}{d\sqrt{\tan(c + dx)}} - B \int \sqrt{\tan(c + dx)} dx \\
&= -\frac{2B}{d\sqrt{\tan(c + dx)}} - \frac{B \operatorname{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(c + dx)\right)}{d} \\
&= -\frac{2B}{d\sqrt{\tan(c + dx)}} - \frac{(2B) \operatorname{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
&= -\frac{2B}{d\sqrt{\tan(c + dx)}} + \frac{B \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} - \frac{BS}{d} \\
&= -\frac{2B}{d\sqrt{\tan(c + dx)}} - \frac{B \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(c + dx)}\right)}{2d} \\
&= -\frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{2\sqrt{2}d} + \frac{B \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{2\sqrt{2}d} \\
&= \frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}d} - \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 34, normalized size = 0.22

$$-\frac{2B {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\tan^2(c + dx)\right)}{d\sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])),x]

[Out] (-2\*B\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2])/(d\*Sqrt[Tan[c + d\*x]])

**Maple [A]**

time = 0.05, size = 102, normalized size = 0.66

method	result
derivativedivides	$B \left( \frac{\sqrt{2} \left( \ln \left( \frac{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{\sqrt{\tan(dx+c)}^2} \right) \frac{1}{4}$
default	$B \left( \frac{\sqrt{2} \left( \ln \left( \frac{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{\sqrt{\tan(dx+c)}^2} \right) \frac{1}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out]  $1/d*B*(-2/\tan(d*x+c)^{(1/2)}-1/4*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))$

**Maxima [A]**

time = 0.52, size = 122, normalized size = 0.79

$$\frac{(2\sqrt{2}\arctan(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)}))+2\sqrt{2}\arctan(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)}))-\sqrt{2}\log(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+\sqrt{2}\log(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1))B+\frac{8B}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/4*((2*\sqrt{2})*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)})))+2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)}))- \sqrt{2}*\log(\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+\sqrt{2}*\log(-\sqrt{2}*\sqrt{\tan(dx+c)}+\tan(dx+c)+1))*B+8*B/\sqrt{\tan(dx+c)}/d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(124) = 248.

time = 1.56, size = 642, normalized size = 4.17

$$\frac{(2\sqrt{2}\arctan(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)}))+2\sqrt{2}\arctan(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)}))-\sqrt{2}\log(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1)+\sqrt{2}\log(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1))B+\frac{8B}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (8 * B * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) * \cos(d*x + c) * \sin(d*x + c) + 4 * (\sqrt{2} * d * \cos(d*x + c)^2 - \sqrt{2} * d * (B^4 / d^4)^{1/4} * \arctan(-(\sqrt{2} * B^3 * d * (B^4 / d^4)^{1/4} * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) + B^4 - \sqrt{2} * d * (B^4 / d^4)^{1/4} * \sqrt{((\sqrt{2} * B^3 * d^3 * (B^4 / d^4)^{3/4} * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) * \cos(d*x + c) + B^4 * d^2 * \sqrt{B^4 / d^4} * \cos(d*x + c) + B^6 * \sin(d*x + c)) / \cos(d*x + c)}) / B^4) + 4 * (\sqrt{2} * d * \cos(d*x + c)^2 - \sqrt{2} * d * (B^4 / d^4)^{1/4} * \arctan(-(\sqrt{2} * B^3 * d * (B^4 / d^4)^{1/4} * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) - B^4 - \sqrt{2} * d * (B^4 / d^4)^{1/4} * \sqrt{-(\sqrt{2} * B^3 * d^3 * (B^4 / d^4)^{3/4} * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) * \cos(d*x + c) - B^4 * d^2 * \sqrt{B^4 / d^4} * \cos(d*x + c) - B^6 * \sin(d*x + c)) / \cos(d*x + c)}) / B^4) + (\sqrt{2} * d * \cos(d*x + c)^2 - \sqrt{2} * d * (B^4 / d^4)^{1/4} * \log((\sqrt{2} * B^3 * d^3 * (B^4 / d^4)^{3/4} * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) * \cos(d*x + c) + B^4 * d^2 * \sqrt{B^4 / d^4} * \cos(d*x + c) + B^6 * \sin(d*x + c)) / \cos(d*x + c)) - (\sqrt{2} * d * \cos(d*x + c)^2 - \sqrt{2} * d * (B^4 / d^4)^{1/4} * \log(-(\sqrt{2} * B^3 * d^3 * (B^4 / d^4)^{3/4} * \sqrt{\sin(d*x + c) / \cos(d*x + c)}) * \cos(d*x + c) - B^4 * d^2 * \sqrt{B^4 / d^4} * \cos(d*x + c) - B^6 * \sin(d*x + c)) / \cos(d*x + c))) / (d * \cos(d*x + c)^2 - d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(tan(c + d\*x)\*\*(-3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [B]**

time = 10.89, size = 2500, normalized size = 16.23

Too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((B*a + B*b*\tan(c + d*x))/(\tan(c + d*x)^{(3/2)}*(a + b*\tan(c + d*x))),x)$

[Out]  $\text{atan}(((\tan(c + d*x))^{(1/2)}*(64*B^4*a^2*b^7*d^5 - 32*B^4*a^4*b^5*d^5) + ((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(\tan(c + d*x))^{(1/2)}*(128*B^2*a^5*b^4*d^7 - 448*B^2*a^3*b^6*d^7 + 64*B^2*a^7*b^2*d^7 + 512*B^2*a*b^8*d^7) - (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(\tan(c + d*x))^{(1/2)}*((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(512*b^9*d^9 + 512*a^2*b^7*d^9 - 512*a^4*b^5*d^9 - 512*a^6*b^3*d^9) - 512*B*b^9*d^8 - 640*B*a^2*b^7*d^8 + 256*B*a^4*b^5*d^8 + 384*B*a^6*b^3*d^8))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} - 32*B^3*a^5*b^4*d^6 - 32*B^3*a^7*b^2*d^6 + 128*B^3*a*b^8*d^6))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*i + (\tan(c + d*x))^{(1/2)}*(64*B^4*a^2*b^7*d^5 - 32*B^4*a^4*b^5*d^5) + (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*((\tan(c + d*x))^{(1/2)}*(128*B^2*a^5*b^4*d^7 - 448*B^2*a^3*b^6*d^7 + 64*B^2*a^7*b^2*d^7 + 512*B^2*a*b^8*d^7) - (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(\tan(c + d*x))^{(1/2)}*((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(512*b^9*d^9 + 512*a^2*b^7*d^9 - 512*a^4*b^5*d^9 - 512*a^6*b^3*d^9) + 512*B*b^9*d^8 + 640*B*a^2*b^7*d^8 - 256*B*a^4*b^5*d^8 - 384*B*a^6*b^3*d^8))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)} + 32*B^3*a^5*b^4*d^6 + 32*B^3*a^7*b^2*d^6 - 128*B^3*a*b^8*d^6))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*i)/((\tan(c + d*x))^{(1/2)}*(64*B^4*a^2*b^7*d^5 - 32*B^4*a^4*b^5*d^5) + (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*((\tan(c + d*x))^{(1/2)}*(128*B^2*a^5*b^4*d^7 - 448*B^2*a^3*b^6*d^7 + 64*B^2*a^7*b^2*d^7 + 512*B^2*a*b^8*d^7) - (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(\tan(c + d*x))^{(1/2)}*((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(512*b^9*d^9 + 512*a^2*b^7*d^9 - 512*a^4*b^5*d^9 - 512*a^6*b^3*d^9) - 512*B*b^9*d^8 - 640*B*a^2*b^7*d^8 + 256*B*a^4*b^5*d^8 +$



$$3.421 \quad \int \frac{aB + bB \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=156

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-1/2*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-1/4*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/d*2^{(1/2)}-2/3*B/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{2B}{3d \tan^3(c+dx)} + \frac{B \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{B \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x])/( \operatorname{Tan}[c + d*x]^{(5/2)}*(a + b*\operatorname{Tan}[c + d*x])) , x]$

[Out]  $(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/( \operatorname{Sqrt}[2]*d) - (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/( \operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (2*B)/(3*d*\operatorname{Tan}[c + d*x]^{(3/2)})$

Rule 21

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*)^{(m_*)}*((c_*) + (d_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 217

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^4)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}\{a, b$

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 335

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

### Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))} dx &= B \int \frac{1}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2B}{3d \tan^{\frac{3}{2}}(c + dx)} - B \int \frac{1}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2B}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{B \text{Subst}\left(\int \frac{1}{\sqrt{x} (1+x^2)} dx, x, \tan(c + dx)\right)}{d} \\
&= -\frac{2B}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{(2B) \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
&= -\frac{2B}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{B \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{d} - \frac{B}{d} \\
&= -\frac{2B}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{B \text{Subst}\left(\int \frac{1}{1-\sqrt{2} x+x^2} dx, x, \sqrt{\tan(c + dx)}\right)}{2d} \\
&= \frac{B \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{2\sqrt{2} d} - \frac{B \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right)}{2\sqrt{2} d} \\
&= \frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d} - \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 36, normalized size = 0.23

$$-\frac{2B {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)\right)}{3d \tan^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])), x]

[Out] (-2\*B\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d\*x]^2])/(3\*d\*Tan[c + d\*x]^(3/2))

**Maple [A]**

time = 0.05, size = 102, normalized size = 0.65

method	result
derivativedivides	$B \frac{\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$B \frac{\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d*B*(-1/4*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2)))-2/3/tan(d*x+c)^(3/2))
```

**Maxima [A]**

time = 0.50, size = 124, normalized size = 0.79

$$\frac{6\sqrt{2} B \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) + 6\sqrt{2} B \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) + 3\sqrt{2} B \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - 3\sqrt{2} B \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \frac{8B}{\tan(dx+c)^{3/2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6
*sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2
)*B*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*B*log(-s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*B/tan(d*x + c)^(3/2))/d
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(124) = 248.

time = 1.32, size = 615, normalized size = 3.94

$$\frac{6\sqrt{2} B \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{\tan(dx+c)})\right) + 6\sqrt{2} B \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{\tan(dx+c)})\right) + 3\sqrt{2} B \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - 3\sqrt{2} B \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \frac{8B}{\tan(dx+c)^{3/2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{12} \left( 8B \sqrt{\sin(dx+c)/\cos(dx+c)} \cos(dx+c)^2 + 12(\sqrt{2}d \cos(dx+c)^2 - \sqrt{2}d)(B^4/d^4)^{1/4} \arctan(-(\sqrt{2}Bd^3(B^4/d^4)^{3/4} \sqrt{\sin(dx+c)/\cos(dx+c)} - \sqrt{2}d^3(B^4/d^4)^{3/4} \sqrt{(\sqrt{2}Bd(B^4/d^4)^{1/4} \sqrt{\sin(dx+c)/\cos(dx+c)} \cos(dx+c) + d^2 \sqrt{B^4/d^4} \cos(dx+c) + B^2 \sin(dx+c))/\cos(dx+c)}) + B^4/B^4) + 12(\sqrt{2}d \cos(dx+c)^2 - \sqrt{2}d)(B^4/d^4)^{1/4} \arctan(-(\sqrt{2}Bd^3(B^4/d^4)^{3/4} \sqrt{\sin(dx+c)/\cos(dx+c)} - \sqrt{2}d^3(B^4/d^4)^{3/4} \sqrt{-(\sqrt{2}Bd(B^4/d^4)^{1/4} \sqrt{\sin(dx+c)/\cos(dx+c)} \cos(dx+c) - d^2 \sqrt{B^4/d^4} \cos(dx+c) - B^2 \sin(dx+c))/\cos(dx+c)}) - B^4/B^4) - 3(\sqrt{2}d \cos(dx+c)^2 - \sqrt{2}d)(B^4/d^4)^{1/4} \log((\sqrt{2}Bd(B^4/d^4)^{1/4} \sqrt{\sin(dx+c)/\cos(dx+c)} \cos(dx+c) + d^2 \sqrt{B^4/d^4} \cos(dx+c) + B^2 \sin(dx+c))/\cos(dx+c)) + 3(\sqrt{2}d \cos(dx+c)^2 - \sqrt{2}d)(B^4/d^4)^{1/4} \log(-(\sqrt{2}Bd(B^4/d^4)^{1/4} \sqrt{\sin(dx+c)/\cos(dx+c)} \cos(dx+c) - d^2 \sqrt{B^4/d^4} \cos(dx+c) - B^2 \sin(dx+c))/\cos(dx+c)) \right) / (d \cos(dx+c)^2 - d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\tan^{\frac{5}{2}}(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2)/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(tan(c + d\*x)\*\*(-5/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 11.59, size = 2500, normalized size = 16.03

Too large to display





$$\begin{aligned}
& *d^9 + 512*a^{11}*b^7*d^9 - 512*a^{13}*b^5*d^9 - 512*a^{15}*b^3*d^9) + 512*B*a^8* \\
& b^{10}*d^8 + 512*B*a^{10}*b^8*d^8 - 384*B*a^{12}*b^6*d^8 - 256*B*a^{14}*b^4*d^8 + 1 \\
& 28*B*a^{16}*b^2*d^8))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 \\
& ^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a \\
& ^2*b^2*d^4)))^{(1/2)} + 384*B^3*a^9*b^9*d^6 - 32*B^3*a^{13}*b^5*d^6 - 32*B^3*a^ \\
& 15*b^3*d^6))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32 \\
& *a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2* \\
& d^4)))^{(1/2)} - (\tan(c + d*x)^{(1/2)}*(64*B^4*a^9*b^9*d^5 + 32*B^4*a^{13}*b^5*d^ \\
& 5) - (-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2 \\
& *d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{( \\
& 1/2)}*((\tan(c + d*x)^{(1/2)}*(512*B^2*a^8*b^{10}*d^7 + 448*B^2*a^{12}*b^6*d^7 - 12 \\
& 8*B^2*a^{14}*b^4*d^7 - 64*B^2*a^{16}*b^2*d^7) - (-((64*B^4*a^6*b^2*d^4 - B^4*a^ \\
& 4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16* \\
& (a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(\tan(c + d*x)^{(1/2)}*(-((64*B^4* \\
& a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8 \\
& *B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*(512*a^9*b^ \\
& 9*d^9 + 512*a^{11}*b^7*d^9 - 512*a^{13}*b^5*d^9 - 512*a^{15}*b^3*d^9) - 512*B*a^8 \\
& *b^{10}*d^8 - 512*B*a^{10}*b^8*d^8 + 384*B*a^{12}*b^6*d^8 + 256*B*a^{14}*b^4*d^8 - \\
& 128*B*a^{16}*b^2*d^8))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4* \\
& d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2* \\
& a^2*b^2*d^4)))^{(1/2)} - 384*B^3*a^9*b^9*d^6 + 32*B^3*a^{13}*b^5*d^6 + 32*B^3*a^ \\
& ^15*b^3*d^6))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 3 \\
& 2*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2 \\
& *d^4)))^{(1/2)} + 64*B^5*a^{10}*b^8*d^4))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16* \\
& a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^{(1/2)} - 8*B^2*a^3*b*d^2)/(16*(a^4*d \\
& ^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^{(1/2)}*2i + \operatorname{atan}...
\end{aligned}$$

$$3.422 \quad \int \frac{\tan^5(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=256

$$\frac{(a+b)B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2+b^2) d} - \frac{(a+b)B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2+b^2) d} - \frac{2a^{5/2} B \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2} (a^2+b^2) d}$$

[Out]  $-2*a^{(5/2)}*B*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/b^{(3/2)}/(a^2+b^2)/d-1/2*(a+b)*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(a+b)*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(a-b)*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+1/4*(a-b)*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+2*B*\tan(d*x+c)^{(1/2)}/b/d$

**Rubi [A]**

time = 0.30, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$ , Rules used = {21, 3647, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{B(a+b) \operatorname{ArcTan}\left(\frac{1 - \sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{2} \sqrt{a^2+b^2}}\right)}{\sqrt{2} d(a^2+b^2)} - \frac{B(a+b) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)} + 1}{\sqrt{2} \sqrt{a^2+b^2}}\right)}{\sqrt{2} d(a^2+b^2)} - \frac{B(a-b) \log\left(\frac{\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d(a^2+b^2)}\right)}{2\sqrt{2} d(a^2+b^2)} + \frac{B(a-b) \log\left(\frac{\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1}{2\sqrt{2} d(a^2+b^2)}\right)}{2\sqrt{2} d(a^2+b^2)} - \frac{2a^{5/2} B \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2} d(a^2+b^2)} + \frac{2B \sqrt{\tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x]^{(5/2)}*(a*B + b*B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x])^2, x]$

[Out]  $((a + b)*B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a + b)*B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - (2*a^{(5/2)}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a]])/(b^{(3/2)}*(a^2 + b^2)*d) - ((a - b)*B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + ((a - b)*B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(b*d)$

**Rule 21**

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_.))^{(m_.)*((c_.) + (d_.)*(v_.))^{(n_.)}, x\_Symbol] :> \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 65**

$\operatorname{Int}[(a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}$

$[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 210

$\text{Int}[\{(a_) + (b_)*(x_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1} * \text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 211

$\text{Int}[\{(a_) + (b_)*(x_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

### Rule 631

$\text{Int}[\{(a_) + (b_)*(x_) + (c_)*(x_)^2\}^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 642

$\text{Int}[\{(d_) + (e_)*(x_)\}/\{(a_) + (b_)*(x_) + (c_)*(x_)^2\}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1176

$\text{Int}[\{(d_) + (e_)*(x_)^2\}/\{(a_) + (c_)*(x_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 1179

$\text{Int}[\{(d_) + (e_)*(x_)^2\}/\{(a_) + (c_)*(x_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 1182

$\text{Int}[\{(d_) + (e_)*(x_)^2\}/\{(a_) + (c_)*(x_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}\{a,$

$c, d, e, x$  &&  $\text{NeQ}[c*d^2 + a*e^2, 0]$  &&  $\text{NeQ}[c*d^2 - a*e^2, 0]$  &&  $\text{NegQ}[(-a)*c]$

### Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\sqrt{(b_.)\tan[(e_.) + (f_.)x]}}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \sqrt{b*\tan[e + f*x]}], x] /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3647

$\text{Int}[\frac{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}]^m * ((c_.) + (d_.)\tan[(e_.) + (f_.)x])^n, x\_Symbol] \rightarrow \text{Simp}[b^2*(a + b*\tan[e + f*x])^{m-2} * (c + d*\tan[e + f*x])^{n+1} / (d*f*(m+n-1)), x] + \text{Dist}[1/(d*(m+n-1)), \text{Int}[(a + b*\tan[e + f*x])^{m-3} * (c + d*\tan[e + f*x])^n * \text{Simp}[a^3*d*(m+n-1) - b^2*(b*c*(m-2) + a*d*(1+n)) + b*d*(m+n-1)*(3*a^2 - b^2)*\tan[e + f*x] - b^2*(b*c*(m-2) - a*d*(3*m+2*n-4))*\tan[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{IntegerQ}[2*m] \&\& \text{GtQ}[m, 2] \&\& (\text{GeQ}[n, -1] || \text{IntegerQ}[m]) \&\& !( \text{IGtQ}[n, 2] \&\& ( !\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])) )$

### Rule 3715

$\text{Int}[\frac{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}]^m * ((A_.) + (C_.)\tan[(e_.) + (f_.)x])^2, x\_Symbol] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m * (c + d*x)^n, x], x, \tan[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3734

$\text{Int}[\frac{((c_.) + (d_.)\tan[(e_.) + (f_.)x])^n * ((A_.) + (B_.)\tan[(e_.) + (f_.)x]) + (C_.)\tan[(e_.) + (f_.)x]^2)}{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}], x\_Symbol] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\tan[e + f*x])^n * \text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\tan[e + f*x], x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\tan[e + f*x])^n * (1 + \tan[e + f*x]^2)/(a + b*\tan[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0] \&\& !\text{LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= B \int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)} dx \\
&= \frac{2B\sqrt{\tan(c+dx)}}{bd} + \frac{(2B) \int \frac{-\frac{a}{2}-\frac{1}{2}b\tan(c+dx)-\frac{1}{2}a\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))} dx}{b} \\
&= \frac{2B\sqrt{\tan(c+dx)}}{bd} + \frac{(2B) \int \frac{-\frac{b^2}{2}-\frac{1}{2}ab\tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{b(a^2+b^2)} - \frac{(a^3B) \int \frac{1}{\sqrt{\tan(c+dx)}} dx}{b(a^2+b^2)} \\
&= \frac{2B\sqrt{\tan(c+dx)}}{bd} + \frac{(4B)\text{Subst}\left(\int \frac{-\frac{b^2}{2}-\frac{1}{2}abx^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{b(a^2+b^2)d} \\
&= \frac{2B\sqrt{\tan(c+dx)}}{bd} + \frac{((a-b)B)\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} \\
&= -\frac{2a^{5/2}B \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)d} + \frac{2B\sqrt{\tan(c+dx)}}{bd} \\
&= -\frac{2a^{5/2}B \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)d} - \frac{(a-b)B \log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{2(a^2+b^2)d} \\
&= \frac{(a+b)B \tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(a+b)B \tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.14, size = 156, normalized size = 0.61

$$\frac{B\left(\sqrt{-1}b^{3/2}(-ia+b)\text{ArcTan}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - 2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) + \sqrt{-1}b^{3/2}(ia+b)\tanh^{-1}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) + 2a^2\sqrt{b}\sqrt{\tan(c+dx)} + 2b^{5/2}\sqrt{\tan(c+dx)}\right)}{b^{3/2}(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(5/2)\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] (B\*((-1)^(1/4)\*b^(3/2)\*((-I)\*a + b)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - 2\*a^(5/2)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] + (-1)^(1/4)\*b^(3/2)\*(I\*a + b)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] + 2\*a^2\*Sqrt[b]\*Sqrt[Tan[c + d\*x]] + 2\*b^(5/2)\*Sqrt[Tan[c + d\*x]])/(b^(3/2)\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.09, size = 242, normalized size = 0.95

method	result
derivativedivides	$B \left[ \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{b} + \frac{b \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right]$
default	$B \left[ \frac{2 \left( \sqrt{\tan(dx+c)} \right)}{b} + \frac{b \sqrt{2} \left( \ln \left( \frac{1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right]$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(5/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*B*(2/b*tan(d*x+c)^(1/2)+2/(a^2+b^2)*(-1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*a*2^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-2/b*a^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 188, normalized size = 0.73

$$\frac{8Ba^2 \arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) + \left(2\sqrt{2}(a+b) \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) + 2\sqrt{2}(a+b) \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right) - \sqrt{2}(a-b) \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) + \sqrt{2}(a-b) \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right)\right)B}{(a^2+b^2)\sqrt{ab}} - \frac{8B\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*B*a^3*arctan(b*sqrt(tan(d*x+c))/sqrt(a*b))/((a^2*b+b^3)*sqrt(a*b)) + (2*sqrt(2)*(a+b)*arctan(1/2*sqrt(2)*(sqrt(2)+2*sqrt(tan(d*x+c)))) + 2*sqrt(2)*(a+b)*arctan(-1/2*sqrt(2)*(sqrt(2)-2*sqrt(tan(d*x+c)))) - sqrt(2)*(a-b)*log(sqrt(2)*sqrt(tan(d*x+c))+tan(d*x+c)+1) + sqrt(2)*(a-b)*log(-sqrt(2)*sqrt(tan(d*x+c))+tan(d*x+c)+1))*B/(a^2+b^2) - 8*B*sqrt(tan(d*x+c))/b)/d
```



$$\begin{aligned}
& b^2 + B^4 b^4) / ((a^8 + 4a^6 b^2 + 6a^4 b^4 + 4a^2 b^6 + b^8) d^4) \arctan \\
& n(((B^6 a^8 + 2B^6 a^6 b^2 - 2B^6 a^2 b^6 - B^6 b^8) d^4 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)}) \\
& \sqrt{(B^4 a^4 - 2B^4 a^2 b^2 + B^4 b^4) / ((a^8 + 4a^6 b^2 + 6a^4 b^4 + 4a^2 b^6 + b^8) d^4)}) \\
& + \sqrt{2} * ((a^8 b + 4a^6 b^3 + 6a^4 b^5 + 4a^2 b^7 + b^9) d^7 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)}) \\
& \sqrt{(B^4 a^4 - 2B^4 a^2 b^2 + B^4 b^4) / ((a^8 + 4a^6 b^2 + 6a^4 b^4 + 4a^2 b^6 + b^8) d^4)}) \\
& - (B^2 a^7 + 3B^2 a^5 b^2 + 3B^2 a^3 b^4 + B^2 a b^6) d^5 \sqrt{(B^4 a^4 - 2B^4 a^2 b^2 + B^4 b^4) / ((a^8 + 4a^6 b^2 + 6a^4 b^4 + 4a^2 b^6 + b^8) d^4)}) \\
& \sqrt{(B^2 a^4 + 2B^2 a^2 b^2 + B^2 b^4 + 2(a^5 b + 2a^3 b^3 + a b^5) d^2 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)})} / (B^2 a^4 - 2B^2 a^2 b^2 + B^2 b^4) \\
& \sqrt{((B^4 a^6 - B^4 a^4 b^2 - B^4 a^2 b^4 + B^4 b^6) d^2 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)}) \cos(dx + c) - \sqrt{2} * ((B^3 a^7 - B^3 a^5 b^2 - B^3 a^3 b^4 + B^3 a b^6) d^3 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)}) \cos(dx + c) - (B^5 a^4 b - 2B^5 a^2 b^3 + B^5 b^5) d \cos(dx + c)} \\
& \sqrt{(B^2 a^4 + 2B^2 a^2 b^2 + B^2 b^4 + 2(a^5 b + 2a^3 b^3 + a b^5) d^2 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)})} / (B^2 a^4 - 2B^2 a^2 b^2 + B^2 b^4) \\
& \sqrt{\sin(dx + c) / \cos(dx + c)} * (B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4))^{1/4} + (B^6 a^4 - 2B^6 a^2 b^2 + B^6 b^4) \sin(dx + c) / \cos(dx + c) * (B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4))^{3/4} + \sqrt{2} * ((B^3 a^{10} b + 3B^3 a^8 b^3 + 2B^3 a^6 b^5 - 2B^3 a^4 b^7 - 3B^3 a^2 b^9 - B^3 b^{11}) d^7 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)}) \sqrt{(B^4 a^4 - 2B^4 a^2 b^2 + B^4 b^4) / ((a^8 + 4a^6 b^2 + 6a^4 b^4 + 4a^2 b^6 + b^8) d^4)}) \\
& - (B^5 a^9 + 2B^5 a^7 b^2 - 2B^5 a^5 b^4 - B^5 a^3 b^6 - B^5 a b^8) d^5 \sqrt{(B^4 a^4 - 2B^4 a^2 b^2 + B^4 b^4) / ((a^8 + 4a^6 b^2 + 6a^4 b^4 + 4a^2 b^6 + b^8) d^4)}) \\
& \sqrt{(B^2 a^4 + 2B^2 a^2 b^2 + B^2 b^4 + 2(a^5 b + 2a^3 b^3 + a b^5) d^2 \sqrt{B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4)})} / (B^2 a^4 - 2B^2 a^2 b^2 + B^2 b^4) \\
& \sqrt{\sin(dx + c) / \cos(dx + c)} * (B^4 / ((a^4 + 2a^2 b^2 + b^4) d^4))^{3/4} / (B^{10} a^4 - 2B^{10} a^2 b^2 + B^{10} b^4) + 2B^5 a^2 \sqrt{-a/b} \log(-(6a b \cos(dx + c) \sin(dx + c) - (a^2 - b^2) \cos(dx + c)^2 - b^2 - 4(a b \cos(dx + c)^2 - b^2 \cos(dx + c) \sin(dx + c)) \sqrt{-a/b} \sqrt{\sin(dx + c) / \cos(dx + c)}) / (2a b \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2)) - \sqrt{2} * (2(B^2 a^3 b^2 + B^2 a b^4) \dots
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\tan^{\frac{5}{2}}(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] B\*Integral(tan(c + d\*x)\*\*(5/2)/(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out





$$\begin{aligned}
& * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} - 16B^2a^3b^3d^2 + 16B^2a^5b^2d^2) / \\
& (d^4(a^2 + b^2)^4))^{(1/2)} / 4 + (16B^4a^4 \tan(c + dx)^{(1/2)} * (a^{10} - 2b^{10} \\
& - 4a^2b^8 - 27a^4b^6 + 15a^6b^4 + 9a^8b^2)) / (b^4d^4(a^2 + b^2)^4) \\
& )) * (-4 * (-B^4a^4d^4(a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} - 16B^2a^3b^3d^2 \\
& + 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 + (8B^5a^7(a^6 + 10b^6 \\
& + 27a^2b^4 + 10a^4b^2)) / (b^5d^5(a^2 + b^2)^4)) * (-((192B^4a^6b^6d^4 \\
& - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4)^{(1/2)} - 16B^2a^3b^3d^2 + 16B^2a^5b^2d^2) / (a^8d^4 + b^8d^4 \\
& + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4))^{(1/2)} / 4 - \log((8B^5a^7(a^6 + 10b^6 + 27a^2b^4 + 10a^4b^2)) / (b^5d^5(a^2 + b^2)^4) - ((((((128b^3 \tan(c + dx)^{(1/2)} * (a^2 - b^2) * (a^2 + b^2)^2 * ((4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} - (768B^4a^3b^3(a^2 + b^2)) / d) * ((4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 + (64B^2a^3 \tan(c + dx)^{(1/2)} * (2a^8 + 15b^8 - 17a^2b^6 + 51a^4b^4 + 21a^6b^2)) / (d^2(a^2 + b^2)^2)) * ((4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 - (32B^3a^4(4a^8 + b^8 - 77a^2b^6 + 47a^4b^4 + 33a^6b^2)) / (d^3(a^2 + b^2)^3)) * ((4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 + (16B^4a^4 \tan(c + dx)^{(1/2)} * (a^{10} - 2b^{10} - 4a^2b^8 - 27a^4b^6 + 15a^6b^4 + 9a^8b^2)) / (b^4d^4(a^2 + b^2)^4)) * ((4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 * (((192B^4a^6b^6d^4 - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4)^{(1/2)} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2) / (16a^8d^4 + 16b^8d^4 + 64a^2b^6d^4 + 96a^4b^4d^4 + 64a^6b^2d^4))^{(1/2)} - \log((8B^5a^7(a^6 + 10b^6 + 27a^2b^4 + 10a^4b^2)) / (b^5d^5(a^2 + b^2)^4) - (((((((128b^3 \tan(c + dx)^{(1/2)} * (a^2 - b^2) * (a^2 + b^2)^2 * (-4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} - 16B^2a^3b^3d^2 + 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} - (768B^4a^3b^3(a^2 + b^2)) / d) * (-4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} - 16B^2a^3b^3d^2 + 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 + (64B^2a^3 \tan(c + dx)^{(1/2)} * (2a^8 + 15b^8 - 17a^2b^6 + 51a^4b^4 + 21a^6b^2)) / (d^2(a^2 + b^2)^2)) * (-4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} - 16B^2a^3b^3d^2 + 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 - (32B^3a^4(4a^8 + b^8 - 77a^2b^6 + 47a^4b^4 + 33a^6b^2)) / (d^3(a^2 + b^2)^3)) * (-4 * (-B^4a^4d^4 * (a^4 + b^4 - 6a^2b^2)^2)^{(1/2)} - 16B^2a^3b^3d^2 + 16B^2a^5b^2d^2) / (d^4(a^2 + b^2)^4))^{(1/2)} / 4 + (16B^4a^4 \tan(c + dx)^{(1/2)} * (a^{10} - 2b^{10} - 4a^2b^8 - 27a^4b^6 + 15a^6b^4 + 9a^8b^2) \dots
\end{aligned}$$

$$3.423 \quad \int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=237

$$\frac{(a-b)B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(a-b)B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{2a^{3/2} B \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}}$$

[Out]  $-1/2*(a-b)*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(a-b)*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a+b)*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}-1/4*(a+b)*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+2*a^{(3/2)}*B*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/(a^2+b^2)/d/b^{(1/2)}$

**Rubi [A]**

time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {21, 3654, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{B(a-b) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{B(a-b) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{B(a+b) \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)} - \frac{B(a+b) \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)} + \frac{2a^{3/2} B \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} d (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x])^{(3/2)}*(a*B + b*B*\operatorname{Tan}[c + d*x])]/(a + b*\operatorname{Tan}[c + d*x])^2, x]$

[Out]  $((a-b)*B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a-b)*B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*a^{(3/2)}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[b]*(a^2 + b^2)*d) + ((a+b)*B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a+b)*B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

**Rule 21**

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*))^{(m_*)}*((c_*) + (d_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n, x\} \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 65**

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Den}$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a

\*c]

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3654

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(3/2)/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]^(n_))*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= B \int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)} dx \\
&= \frac{B \int \frac{-a+b\tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{a^2+b^2} + \frac{(a^2B) \int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))} dx}{a^2+b^2} \\
&= \frac{(2B)\text{Subst}\left(\int \frac{-a+bx^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} + \frac{(a^2B)\text{Subst}\left(\int \frac{1+x^2}{\sqrt{x}(a+bx)} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} \\
&= \frac{(2a^2B)\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{\tan(c+dx)}\right)}{(a^2+b^2)d} - \frac{((a-b)B)\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\tan(c+dx)}\right)}{2(a^2+b^2)d} \\
&= \frac{2a^{3/2}B \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}(a^2+b^2)d} - \frac{((a-b)B)\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\tan(c+dx)}\right)}{2(a^2+b^2)d} \\
&= \frac{2a^{3/2}B \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}(a^2+b^2)d} + \frac{(a+b)B \log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)d} \\
&= \frac{(a-b)B \tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(a-b)B \tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.17, size = 228, normalized size = 0.96

$$\frac{B\left(3a\left(2\sqrt{2}\sqrt{b}\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\sqrt{b}\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+8\sqrt{a}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{2}\sqrt{b}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\sqrt{b}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)+8b^{3/2}{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\tan^4(c+dx)\right)}{12\sqrt{b}(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (B\*(3\*a\*(2\*Sqrt[2]\*Sqrt[b]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 2\*Sqrt[2]\*Sqrt[b]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 8\*Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] + Sqrt[2]\*Sqrt[b]\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Sqrt[2]\*Sqrt[b]\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + 8\*b^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2\*Tan[c + d\*x]^(3/2)))/(12\*Sqrt[b]\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.09, size = 226, normalized size = 0.95

method	result
derivativedivides	$B \frac{\left( a\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$B \frac{\left( a\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] 1/d\*B\*(2/(a^2+b^2)\*(-1/8\*a\*2^(1/2)\*(ln((1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2)))+1/8\*b\*2^(1/2)\*(ln((1-2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c))/(1+2^(1/2)\*tan(d\*x+c)^(1/2)+tan(d\*x+c)))+2\*arctan(1+2^(1/2)\*tan(d\*x+c)^(1/2))+2\*arctan(-1+2^(1/2)\*tan(d\*x+c)^(1/2))))+2\*a^2/(a^2+b^2)/(a\*b)^(1/2)\*arctan(b\*tan(d\*x+c)^(1/2)/(a\*b)^(1/2))

**Maxima** [A]

time = 0.50, size = 173, normalized size = 0.73

$$\frac{8 B a^2 \arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) - \left(2\sqrt{2}^{(a-b)} \arctan\left(\frac{1}{2}\sqrt{2}^{(2+\sqrt{\tan(dx+c)})}\right) + 2\sqrt{2}^{(a-b)} \arctan\left(-\frac{1}{2}\sqrt{2}^{(2-\sqrt{\tan(dx+c)})}\right) + \sqrt{2}^{(a+b)} \log\left(\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right) - \sqrt{2}^{(a+b)} \log\left(-\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1\right)\right) B}{(a^2+b^2)\sqrt{ab}} \quad 4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*(8\*B\*a^2\*arctan(b\*sqrt(tan(d\*x + c))/sqrt(a\*b))/((a^2 + b^2)\*sqrt(a\*b)) - (2\*sqrt(2)\*(a - b)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*(a - b)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(tan(d\*x + c)))) + sqrt(2)\*(a + b)\*log(sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1) - sqrt(2)\*(a + b)\*log(-sqrt(2)\*sqrt(tan(d\*x + c)) + tan(d\*x + c) + 1))\*B/(a^2 + b^2))/d

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 4374 vs. 2(199) = 398.





```

a^2*b^2 + b^4)*d^4))*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - sqrt(2)*((a^9 + 4*a^7*b^2 + 6*a
^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqr
t((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*d^4)) + (B^2*a^6*b + 3*B^2*a^4*b^3 + 3*B^2*a^2*b^5 + B^2*b^7)*d
^5*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4)))*sqrt((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4
- 2*B^2*a^2*b^2 + B^2*b^4))*sqrt(((B^4*a^6 - B^4*a^4*b^2 - B^4*a^2*b^4 + B
^4*b^6)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*
((B^3*a^6*b - B^3*a^4*b^3 - B^3*a^2*b^5 + B^3*b^7)*d^3*sqrt(B^4/((a^4 + 2*a
^2*b^2 + b^4)*d^4))*cos(d*x + c) + (B^5*a^5 - 2*B^5*a^3*b^2 + B^5*a*b^4)*d*
cos(d*x + c))*sqrt((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b
^3 + a*b^5)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a
^2*b^2 + B^2*b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 +
b^4)*d^4))^(1/4) + (B^6*a^4 - 2*B^6*a^2*b^2 + B^6*b^4)*sin(d*x + c))/cos(d*
x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) - sqrt(2)*((B^3*a^11 + 3*
B^3*a^9*b^2 + 2*B^3*a^7*b^4 - 2*B^3*a^5*b^6 - 3*B^3*a^3*b^8 - B^3*a*b^10)*d
^7*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 +
B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + (B^5*a^8*
b + 2*B^5*a^6*b^3 - 2*B^5*a^4*b^7 - B^5*b^9)*d^5*sqrt((B^4*a^4 - 2*B^4*a^2*
b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt
((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqr
t(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*
sqrt(sin(d*x + c)/cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/
(B^10*a^4 - 2*B^10*a^2*b^2 + B^10*b^4)) - 2*B^5*a*sqrt(-a/b)*log(-(6*a*b*co
s(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 - b^2 + 4*(a*b*cos(d*x
+ c))^2 - b^2*cos(d*x + c)*sin(d*x + c))*sqrt(-a/b)*sqrt(sin(d*x + c)/cos(d
*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)) + sqrt(2)*(2*(B^2*a^3*b + B^2*a*b^3)*d^3*s...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\tan^{\frac{3}{2}}(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+B\*b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] B\*Integral(tan(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out



$$\begin{aligned}
& ^3*d^2 - 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4)^{(1/2)}/4 + (8*B^5*a^2*b^4*(a^6 + 10*b^6 + 27*a^2*b^4 + 10*a^4*b^2))/(d^5*(a^2 + b^2)^4))*(-((192*B^4*a^2*b^10*d^4 - 16*B^4*b^12*d^4 - 608*B^4*a^4*b^8*d^4 + 192*B^4*a^6*b^6*d^4 - 16*B^4*a^8*b^4*d^4)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a*b^5*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^{(1/2)}/4 - \log((8*B^5*a^2*b^4*(a^6 + 10*b^6 + 27*a^2*b^4 + 10*a^4*b^2))/(d^5*(a^2 + b^2)^4) - (((((((((128*b^3*\tan(c + d*x)^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)} - (768*B*a^2*b^4*(a^2 + b^2))/d)*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4 + (64*B^2*a*b^2*\tan(c + d*x)^{(1/2)}*(2*a^8 + 15*b^8 - 17*a^2*b^6 + 51*a^4*b^4 + 21*a^6*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4 - (32*B^3*a*b^3*(4*a^8 + b^8 - 77*a^2*b^6 + 47*a^4*b^4 + 33*a^6*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4 + (16*B^4*b^3*\tan(c + d*x)^{(1/2)}*(a^10 - 2*b^10 - 4*a^2*b^8 - 27*a^4*b^6 + 15*a^6*b^4 + 9*a^8*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4)*(((192*B^4*a^2*b^10*d^4 - 16*B^4*b^12*d^4 - 608*B^4*a^4*b^8*d^4 + 192*B^4*a^6*b^6*d^4 - 16*B^4*a^8*b^4*d^4)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(16*a^8*d^4 + 16*b^8*d^4 + 64*a^2*b^6*d^4 + 96*a^4*b^4*d^4 + 64*a^6*b^2*d^4))^{(1/2)} - \log((8*B^5*a^2*b^4*(a^6 + 10*b^6 + 27*a^2*b^4 + 10*a^4*b^2))/(d^5*(a^2 + b^2)^4) - (((((((((128*b^3*\tan(c + d*x)^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)} - (768*B*a^2*b^4*(a^2 + b^2))/d)*(-(4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4 + (64*B^2*a*b^2*\tan(c + d*x)^{(1/2)}*(2*a^8 + 15*b^8 - 17*a^2*b^6 + 51*a^4*b^4 + 21*a^6*b^2))/(d^2*(a^2 + b^2)^2))*(-(4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4 - (32*B^3*a*b^3*(4*a^8 + b^8 - 77*a^2*b^6 + 47*a^4*b^4 + 33*a^6*b^2))/(d^3*(a^2 + b^2)^3))*(-(4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}/4 + (16*B^4*b^3*\tan(c + d*x)^{(1/2)}*(a^10 - 2*b^10 - 4*a^2*b^8 - 27*a^4*b^6 + 15*...
\end{aligned}$$

$$3.424 \quad \int \frac{\sqrt{\tan(c+dx)} (aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=237

$$\frac{(a+b)B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2+b^2) d} + \frac{(a+b)B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} (a^2+b^2) d} - \frac{2\sqrt{a} \sqrt{b} B \operatorname{ArcTan}\left(\frac{\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{d(a^2+b^2)}$$

[Out]  $1/2*(a+b)*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(a+b)*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a-b)*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}-1/4*(a-b)*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}-2*B*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})*a^{(1/2)}*b^{(1/2)}/(a^2+b^2)/d$

**Rubi [A]**

time = 0.18, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {21, 3653, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{B(a+b) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right)}{\sqrt{2} d(a^2+b^2)} + \frac{B(a+b) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{\sqrt{2} d(a^2+b^2)} - \frac{2\sqrt{a} \sqrt{b} B \operatorname{ArcTan}\left(\frac{\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{d(a^2+b^2)} + \frac{B(a-b) \log\left(\tan(c+dx) - \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d(a^2+b^2)} - \frac{B(a-b) \log\left(\tan(c+dx) + \sqrt{2} \sqrt{\tan(c+dx)} + 1\right)}{2\sqrt{2} d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(a*B+b*B*\operatorname{Tan}[c+d*x]))/(a+b*\operatorname{Tan}[c+d*x])^2,x]$

[Out]  $-(((a+b)*B*\operatorname{ArcTan}[1-\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]])/(\operatorname{Sqrt}[2]*(a^2+b^2)*d)) + ((a+b)*B*\operatorname{ArcTan}[1+\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]])/(\operatorname{Sqrt}[2]*(a^2+b^2)*d) - (2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a]])/(a^2+b^2)*d + ((a-b)*B*\operatorname{Log}[1-\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]+\operatorname{Tan}[c+d*x]])/(2*\operatorname{Sqrt}[2]*(a^2+b^2)*d) - ((a-b)*B*\operatorname{Log}[1+\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]+\operatorname{Tan}[c+d*x]])/(2*\operatorname{Sqrt}[2]*(a^2+b^2)*d)$

**Rule 21**

$\operatorname{Int}[(u_*)*((a_*)+(b_*)(v_*))^{(m_*)}*((c_*)+(d_*)(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c+d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c-a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c+d*x, a+b*x])$

**Rule 65**

$\operatorname{Int}[(a_*)(x_*)^{(m_*)}*((c_*)+(d_*)(x_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)}, x], x, (a+b*x)^{(1/p)}], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3653

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c - a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[d*((b*c - a*d)/(c^2 + d^2)), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :> Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)} (aB + bB \tan(c+dx))}{(a + b \tan(c+dx))^2} dx &= B \int \frac{\sqrt{\tan(c+dx)}}{a + b \tan(c+dx)} dx \\
&= \frac{B \int \frac{b+ax^2}{\sqrt{\tan(c+dx)}} dx}{a^2 + b^2} - \frac{(abB) \int \frac{1+\tan^2(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))} dx}{a^2 + b^2} \\
&= \frac{(2B) \text{Subst} \left( \int \frac{b+ax^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)} \right)}{(a^2 + b^2) d} - \frac{(abB) \text{Subst} \left( \int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)} \right)}{(a^2 + b^2) d} \\
&= -\frac{((a-b)B) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)} \right)}{(a^2 + b^2) d} - \frac{(2abB) \text{Subst} \left( \int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)} \right)}{(a^2 + b^2) d} \\
&= -\frac{2\sqrt{a} \sqrt{b} B \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}} \right)}{(a^2 + b^2) d} + \frac{((a-b)B) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(c+dx)} \right)}{(a^2 + b^2) d} \\
&= -\frac{2\sqrt{a} \sqrt{b} B \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}} \right)}{(a^2 + b^2) d} + \frac{(a-b)B \log \left( \frac{1-\sqrt{\tan(c+dx)}}{1+\sqrt{\tan(c+dx)}} \right)}{(a^2 + b^2) d} \\
&= -\frac{(a+b)B \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c+dx)} \right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a+b)B \tan^{-1} \left( 1 + \sqrt{2} \sqrt{\tan(c+dx)} \right)}{\sqrt{2} (a^2 + b^2) d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.11, size = 205, normalized size = 0.86

$$\frac{B \left( -6\sqrt{2} b \text{ArcTan} \left( 1 - \sqrt{2} \sqrt{\tan(c+dx)} \right) + 6\sqrt{2} b \text{ArcTan} \left( 1 + \sqrt{2} \sqrt{\tan(c+dx)} \right) - 24\sqrt{a} \sqrt{b} \text{ArcTan} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}} \right) - 3\sqrt{2} b \log \left( 1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx) \right) + 3\sqrt{2} b \log \left( 1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx) \right) + 8a {}_2F_1 \left( \frac{1}{4}, 1; \frac{5}{4}; -\tan^2(c+dx) \tan^3(c+dx) \right) \right)}{12(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Tan[c + d\*x]]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (B\*(-6\*Sqrt[2]\*b\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 6\*Sqrt[2]\*b\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 24\*Sqrt[a]\*Sqrt[b]\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] - 3\*Sqrt[2]\*b\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + 3\*Sqrt[2]\*b\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + 8\*a\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2\*Tan[c + d\*x]^(3/2)])/(12\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.09, size = 225, normalized size = 0.95

method	result
derivativedivides	$B \frac{\left( b\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$B \frac{\left( b\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETU  
RNVERBOSE)`

[Out]  $\frac{1}{d} B \left( \frac{2}{(a^2+b^2)} * \left( \frac{1}{8} b^2 \sqrt{2} \left( \ln \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{1-2^{1/2} \tan(dx+c)^{1/2}} \right) + 2 \arctan \left( \frac{-1+2^{1/2} \tan(dx+c)^{1/2}}{1+2^{1/2} \tan(dx+c)^{1/2}} \right) \right) + \frac{1}{8} a^2 \sqrt{2} \left( \ln \left( \frac{1-2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)}{1+2^{1/2} \tan(dx+c)^{1/2} + \tan(dx+c)} \right) + 2 \arctan \left( \frac{1+2^{1/2} \tan(dx+c)^{1/2}}{1-2^{1/2} \tan(dx+c)^{1/2}} \right) + 2 \arctan \left( \frac{-1+2^{1/2} \tan(dx+c)^{1/2}}{1+2^{1/2} \tan(dx+c)^{1/2}} \right) \right) - 2 a b \sqrt{2} \arctan \left( \frac{b \tan(dx+c)^{1/2}}{a \sqrt{2} \sqrt{a^2+b^2}} \right) \right) / (a^2+b^2) / (a^2+b^2) \sqrt{ab}$

**Maxima [A]**

time = 0.51, size = 172, normalized size = 0.73

$$\frac{8 B a b \arctan \left( \frac{b \sqrt{\tan(dx+c)}}{\sqrt{ab}} \right) - \left( 2 \sqrt{2} (a+b) \arctan \left( \frac{1}{2} \sqrt{2} (\sqrt{2} + 2 \sqrt{\tan(dx+c)}) \right) + 2 \sqrt{2} (a-b) \arctan \left( -\frac{1}{2} \sqrt{2} (\sqrt{2} - 2 \sqrt{\tan(dx+c)}) \right) - \sqrt{2} (a-b) \log \left( \sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) + \sqrt{2} (a-b) \log \left( -\sqrt{2} \sqrt{\tan(dx+c)} + \tan(dx+c) + 1 \right) \right) B}{(a^2+b^2) \sqrt{ab}} \frac{1}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo  
rithm="maxima")`

[Out] 
$$-1/4 * (8 * B * a * b * \arctan(b * \sqrt{\tan(dx+c)}) / \sqrt{ab}) / ((a^2 + b^2) * \sqrt{ab}) - (2 * \sqrt{2} * (a + b) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 * \sqrt{\tan(dx+c)}))) + 2 * \sqrt{2} * (a + b) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 * \sqrt{\tan(dx+c)}))) - \sqrt{2} * (a - b) * \log(\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) + \sqrt{2} * (a - b) * \log(-\sqrt{2} * \sqrt{\tan(dx+c)} + \tan(dx+c) + 1) * B / (a^2 + b^2) / d$$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 4483 vs. 2(199) = 398.



time = 12.52, size = 8971, normalized size = 37.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*(4*\sqrt{2}*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}]^{3/4}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}*\arctan(-((B^6*a^8 + 2*B^6*a^6*b^2 - 2*B^6*a^4*b^4 - B^6*b^8)*d^4*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} - \sqrt{2}*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}) \\ & - (B^2*a^7 + 3*B^2*a^5*b^2 + 3*B^2*a^3*b^4 + B^2*a*b^6)*d^5*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)})*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{((B^4*a^6 - B^4*a^4*b^2 - B^4*a^2*b^4 + B^4*b^6)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\cos(d*x + c) + \sqrt{2}*((B^3*a^7 - B^3*a^5*b^2 - B^3*a^3*b^4 + B^3*a*b^6)*d^3*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\cos(d*x + c) - (B^5*a^4*b - 2*B^5*a^2*b^3 + B^5*b^5)*d*\cos(d*x + c))*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4)} \\ & *\sqrt{(\sin(d*x + c)/\cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{1/4} + (B^6*a^4 - 2*B^6*a^2*b^2 + B^6*b^4)*\sin(d*x + c)/\cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4} - \sqrt{2}*((B^3*a^{10}*b + 3*B^3*a^8*b^3 + 2*B^3*a^6*b^5 - 2*B^3*a^4*b^7 - 3*B^3*a^2*b^9 - B^3*b^{11})*d^7*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} - (B^5*a^9 + 2*B^5*a^7*b^2 - 2*B^5*a^3*b^6 - B^5*a*b^8)*d^5*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}})*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4)}*\sqrt{(\sin(d*x + c)/\cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4}}/(B^{10}*a^4 - 2*B^{10}*a^2*b^2 + B^{10}*b^4)} + 4*\sqrt{2}*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4)} \\ & *\sqrt{(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}*\arctan((B^6*a^8 + 2*B^6*a^6*b^2 - 2*B^6*a^4*b^4 - B^6*b^8)*d^4*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}}) \end{aligned}$$

```

a^2*b^2 + b^4)*d^4))*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b^3 + 6
*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqr
t((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*d^4)) - (B^2*a^7 + 3*B^2*a^5*b^2 + 3*B^2*a^3*b^4 + B^2*a*b^6)*d
^5*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*d^4)))*sqrt((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4
- 2*B^2*a^2*b^2 + B^2*b^4))*sqrt(((B^4*a^6 - B^4*a^4*b^2 - B^4*a^2*b^4 + B
^4*b^6)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*
((B^3*a^7 - B^3*a^5*b^2 - B^3*a^3*b^4 + B^3*a*b^6)*d^3*sqrt(B^4/((a^4 + 2*a
^2*b^2 + b^4)*d^4))*cos(d*x + c) - (B^5*a^4*b - 2*B^5*a^2*b^3 + B^5*b^5)*d*
cos(d*x + c))*sqrt((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^
3 + a*b^5)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a
^2*b^2 + B^2*b^4))*sqrt(sin(d*x + c)/cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 +
b^4)*d^4))^(1/4) + (B^6*a^4 - 2*B^6*a^2*b^2 + B^6*b^4)*sin(d*x + c))/cos(d*x
+ c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((B^3*a^10*b +
3*B^3*a^8*b^3 + 2*B^3*a^6*b^5 - 2*B^3*a^4*b^7 - 3*B^3*a^2*b^9 - B^3*b^11)*d
^7*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 +
B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (B^5*a^9
+ 2*B^5*a^7*b^2 - 2*B^5*a^3*b^6 - B^5*a*b^8)*d^5*sqrt((B^4*a^4 - 2*B^4*a^2*
b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt
((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqr
t(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*
sqrt(sin(d*x + c)/cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/
(B^10*a^4 - 2*B^10*a^2*b^2 + B^10*b^4)) - 2*sqrt(-a*b)*B^5*log(-(6*a*b*cos(
d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 - b^2 - 4*(a*cos(d*x + c
)^2 - b*cos(d*x + c)*sin(d*x + c))*sqrt(-a*b)*sqrt(sin(d*x + c)/cos(d*x + c
)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) -
sqrt(2)*(2*(B^2*a^3*b + B^2*a*b^3)*d^3*sqrt(B^...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\sqrt{\tan(c + dx)}}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] B\*Integral(sqrt(tan(c + d\*x))/(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
ithm="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 31.52, size = 2500, normalized size = 10.55
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(- (((((((((256*B*a*b^3*(2*a^4 - b^4 + a^2*b^2))/d - 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (64*B^2*a^3*b^2*tan(c + d*x)^(1/2)*(a^6 + 17*b^6 - 29*a^2*b^4 + 19*a^4*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*a^4*b^2*(a^6 + 13*b^6 - 45*a^2*b^4 + 39*a^4*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (16*B^4*a^4*b^3*tan(c + d*x)^(1/2)*(9*a^6 - 3*b^6 + 3*a^2*b^4 - 17*a^4*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (8*B^5*a^5*b^3*(9*a^4 - b^4))/(d^5*(a^2 + b^2)^4))*(((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 + (log(- (((((((((256*B*a*b^3*(2*a^4 - b^4 + a^2*b^2))/d - 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (64*B^2*a^3*b^2*tan(c + d*x)^(1/2)*(a^6 + 17*b^6 - 29*a^2*b^4 + 19*a^4*b^2))/(d^2*(a^2 + b^2)^2))*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*a^4*b^2*(a^6 + 13*b^6 - 45*a^2*b^4 + 39*a^4*b^2))/(d^3*(a^2 + b^2)^3))*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (16*B^4*a^4*b^3*tan(c + d*x)^(1/2)*(9*a^6 - 3*b^6 + 3*a^2*b^4 - 17*a^4*b^2))/(d^4*(a^2 + b^2)^4))*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (8*B^5*a^5*b^3*(9*a^4 - b^4))/(d
```

$$\begin{aligned}
& ^5*(a^2 + b^2)^4)) * (-((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^{(1/2)})/4 - \log(- (((((((((256*B*a*b^3*(2*a^4 - b^4 + a^2*b^2))/d + 128*b^3*\tan(c + d*x)^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}))/4 + (64*B^2*a^3*b^2*\tan(c + d*x)^{(1/2)}*(a^6 + 17*b^6 - 29*a^2*b^4 + 19*a^4*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 + (32*B^3*a^4*b^2*(a^6 + 13*b^6 - 45*a^2*b^4 + 39*a^4*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 + (16*B^4*a^4*b^3*\tan(c + d*x)^{(1/2)}*(9*a^6 - 3*b^6 + 3*a^2*b^4 - 17*a^4*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 - (8*B^5*a^5*b^3*(9*a^4 - b^4))/(d^5*(a^2 + b^2)^4))*(((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(16*a^8*d^4 + 16*b^8*d^4 + 64*a^2*b^6*d^4 + 96*a^4*b^4*d^4 + 64*a^6*b^2*d^4))^{(1/2)} - \log(- (((((((((256*B*a*b^3*(2*a^4 - b^4 + a^2*b^2))/d + 128*b^3*\tan(c + d*x)^{(1/2)}*(a^2 - b^2)*(a^2 + b^2)^2*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)}))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 + (64*B^2*a^3*b^2*\tan(c + d*x)^{(1/2)}*(a^6 + 17*b^6 - 29*a^2*b^4 + 19*a^4*b^2))/(d^2*(a^2 + b^2)^2))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 + (32*B^3*a^4*b^2*(a^6 + 13*b^6 - 45*a^2*b^4 + 39*a^4*b^2))/(d^3*(a^2 + b^2)^3))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 + (16*B^4*a^4*b^3*\tan(c + d*x)^{(1/2)}*(9*a^6 - 3*b^6 + 3*a^2*b^4 - 17*a^4*b^2))/(d^4*(a^2 + b^2)^4))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^{(1/2)})/4 - (8*B^5*a^5*b^3*(9*a^4 - b^4))/(d^5*(a^2 + b^2)^4))*(-((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*...
\end{aligned}$$

$$3.425 \quad \int \frac{aB + bB \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2} dx$$

Optimal. Leaf size=237

$$\frac{(a - b)B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a - b)B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{2b^{3/2} B \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{a}\right)}{\sqrt{a}}$$

[Out]  $1/2*(a-b)*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(a-b)*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(a+b)*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+1/4*(a+b)*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+2*b^{(3/2)}*B*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/(a^2+b^2)/d/a^{(1/2)}$

**Rubi** [A]

time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {21, 3655, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{B(a-b)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)} + \frac{B(a-b)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)} + \frac{2b^{3/2}B\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{a}\right)}{\sqrt{a}d(a^2+b^2)} - \frac{B(a+b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{B(a+b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x]) / (\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^2), x]$

[Out]  $-(((a - b)*B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d)) + ((a - b)*B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*b^{(3/2)}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / \operatorname{Sqrt}[a]]) / (\operatorname{Sqrt}[a]*(a^2 + b^2)*d) - ((a + b)*B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + ((a + b)*B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

Rule 21

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*))^{(m_*)}*((c_*) + (d_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 65

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(1/p)}, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a

\*c]

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3655

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] :> Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^2} dx &= B \int \frac{1}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx \\
&= \frac{B \int \frac{a - b \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{a^2 + b^2} + \frac{(b^2 B) \int \frac{1 + \tan^2(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))} dx}{a^2 + b^2} \\
&= \frac{(2B) \text{Subst}\left(\int \frac{a - bx^2}{1 + x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2) d} + \frac{(b^2 B) \text{Subst}\left(\int \frac{1}{\sqrt{\tan(c + dx)}} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2) d} \\
&= \frac{((a - b)B) \text{Subst}\left(\int \frac{1 + x^2}{1 + x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2) d} + \frac{(2b^2 B) \text{Subst}\left(\int \frac{1}{\sqrt{\tan(c + dx)}} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2) d} \\
&= \frac{2b^{3/2} B \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2) d} + \frac{((a - b)B) \text{Subst}\left(\int \frac{1}{1 - v^2} dv, v, \sqrt{\tan(c + dx)}\right)}{2(a^2 + b^2) d} \\
&= \frac{2b^{3/2} B \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{a} (a^2 + b^2) d} - \frac{(a + b)B \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2} (a^2 + b^2) d} \\
&= -\frac{(a - b)B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a - b)B \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{\sqrt{2} (a^2 + b^2) d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.13, size = 226, normalized size = 0.95

$$\frac{B \left( -6\sqrt{2} a^{3/2} \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) + 6\sqrt{2} a^{3/2} \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right) + 24b^{3/2} \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right) - 3\sqrt{2} a^{3/2} \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) + 3\sqrt{2} a^{3/2} \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) - 8\sqrt{a} b {}_2F_1\left(\frac{3}{4}, 1, \frac{7}{4}, -\tan^2(c + dx)\right) \tan^2(c + dx) \right)}{12\sqrt{a} (a^2 + b^2) d}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^2), x]

[Out] (B\*(-6\*Sqrt[2]\*a^(3/2)\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 6\*Sqrt[2]\*a^(3/2)\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 24\*b^(3/2)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]] - 3\*Sqrt[2]\*a^(3/2)\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + 3\*Sqrt[2]\*a^(3/2)\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - 8\*Sqrt[a]\*b\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(3/2)))/(12\*Sqrt[a]\*(a^2 + b^2)\*d)

**Maple [A]**

time = 0.09, size = 226, normalized size = 0.95



method	result
derivativedivides	$B \frac{\left( a\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$
default	$B \frac{\left( a\sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) + 2 \arctan \left( -1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)`

[Out]  $1/d*B*(2/(a^2+b^2)*(1/8*a*2^{(1/2)}*(\ln((1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}))-1/8*b*2^{(1/2)}*(\ln((1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))/(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c)))+2*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})+2*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})))+2*b^2/(a^2+b^2)/(a*b)^{(1/2)}*\arctan(b*\tan(d*x+c)^{(1/2)}/(a*b)^{(1/2))}$

**Maxima [A]**

time = 0.52, size = 172, normalized size = 0.73

$$\frac{8B^2 \arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) + \left(2\sqrt{2}^{(a-b)} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}^{(a-b)} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{\tan(dx+c)})\right) + \sqrt{2}^{(a+b)} \log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1\right) - \sqrt{2}^{(a+b)} \log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1\right)\right)^B}{(a^2+b^2)\sqrt{ab}} \quad 4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,algorithm="maxima")`

[Out]  $1/4*(8*B*b^2*\arctan(b*\sqrt{\tan(dx+c)}/\sqrt{a*b})/((a^2+b^2)*\sqrt{a*b}) + (2*\sqrt{2}*(a-b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{\tan(dx+c)}))) + 2*\sqrt{2}*(a-b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{\tan(dx+c)}))) + \sqrt{2}*(a+b)*\log(\sqrt{2}*\sqrt{\tan(dx+c)} + \tan(dx+c) + 1) - \sqrt{2}*(a+b)*\log(-\sqrt{2}*\sqrt{\tan(dx+c)} + \tan(dx+c) + 1))*B/(a^2+b^2))/d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 4482 vs. 2(199) = 398.

time = 12.13, size = 8968, normalized size = 37.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/4*(4*\sqrt{2}*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}^{3/4}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}*\arctan(((B^6*a^8 + 2*B^6*a^6*b^2 - 2*B^6*a^4*b^4 - B^6*b^8)*d^4*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}) + \sqrt{2}*((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d^7*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}) + (B^2*a^6*b + 3*B^2*a^4*b^3 + 3*B^2*a^2*b^5 + B^2*b^7)*d^5*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)})*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{((B^4*a^6 - B^4*a^4*b^2 - B^4*a^2*b^4 + B^4*b^6)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\cos(d*x + c) + \sqrt{2}*((B^3*a^6*b - B^3*a^4*b^3 - B^3*a^2*b^5 + B^3*b^7)*d^3*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\cos(d*x + c) + (B^5*a^5 - 2*B^5*a^3*b^2 + B^5*a*b^4)*d*\cos(d*x + c))*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{\sin(d*x + c)/\cos(d*x + c)}*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{1/4} + (B^6*a^4 - 2*B^6*a^2*b^2 + B^6*b^4)*\sin(d*x + c))/\cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4} + \sqrt{2}*((B^3*a^11 + 3*B^3*a^9*b^2 + 2*B^3*a^7*b^4 - 2*B^3*a^5*b^6 - 3*B^3*a^3*b^8 - B^3*a*b^10)*d^7*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)})*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}) + (B^5*a^8*b + 2*B^5*a^6*b^3 - 2*B^5*a^4*b^5 - B^5*b^9)*d^5*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)})*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{\sin(d*x + c)/\cos(d*x + c)}*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4})/(B^10*a^4 - 2*B^10*a^2*b^2 + B^10*b^4) + 4*\sqrt{2}*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^5*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}})/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}*\arctan(-((B^6*a^8 + 2*B^6*a^6*b^2 - 2*B^6*a^4*b^4 - B^6*b^8)*d^4*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}))} \end{aligned}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith="giac")
```

```
[Out] Timed out
```

**Mupad [B]**

```
time = 31.62, size = 2500, normalized size = 10.55
```

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (log((((((((((((128*B*b^2*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/d + 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^2*a*b^2*tan(c + d*x)^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (32*B^3*a*b^5*(25*a^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*a^2*b^5*tan(c + d*x)^(1/2)*(b^6 - 27*a^6 + 7*a^2*b^4 + 11*a^4*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^5*a^4*b^6*(5*a^2 + b^2))/(d^5*(a^2 + b^2)^4))*((((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 + (log((((((((((((128*B*b^2*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/d + 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^2*a*b^2*tan(c + d*x)^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(d^2*(a^2 + b^2)^2))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (32*B^3*a*b^5*(25*a^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(d^3*(a^2 + b^2)^3))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*a^2*b^5*tan(c + d*x)^(1/2)*(b^6 - 27*a^6 + 7*a^2*b^4 + 11*a^4*b^2))/(d^4*(a^2 + b^2)^4))*(-4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B
```

$$\begin{aligned}
& \sqrt[5]{a^4 b^6 (5a^2 + b^2)} / (d^5 (a^2 + b^2)^4) * (-((192B^4 a^6 b^6 d^4 - 16 \\
& B^4 a^4 b^8 d^4 - 16B^4 a^{12} d^4 - 608B^4 a^8 b^4 d^4 + 192B^4 a^{10} b^2 \\
& d^4)^{(1/2)} + 16B^2 a^3 b^3 d^2 - 16B^2 a^5 b d^2) / (a^8 d^4 + b^8 d^4 + 4 \\
& a^2 b^6 d^4 + 6a^4 b^4 d^4 + 4a^6 b^2 d^4))^{(1/2)} / 4 - \log((((((((128B \\
& b^2 (2b^6 - a^6 + 9a^2 b^4 + 6a^4 b^2)) / d - 128b^3 \tan(c + dx)^{(1/2)} \\
& (a^2 - b^2) (a^2 + b^2)^2 ((4(-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2)^2)^{(1/2)} \\
& - 16B^2 a^3 b^3 d^2 + 16B^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/2)})) * ((4 \\
& (-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2)^2)^{(1/2)} - 16B^2 a^3 b^3 d^2 + 16B \\
& ^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/2)} / 4 - (64B^2 a b^2 \tan(c + dx)^{(1/2)} \\
& (2b^8 - a^8 + 5a^2 b^6 + 67a^4 b^4 - a^6 b^2)) / (d^2 (a^2 + b^2)^2) * \\
& ((4(-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2)^2)^{(1/2)} - 16B^2 a^3 b^3 d^2 + 1 \\
& 6B^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/2)} / 4 - (32B^3 a b^5 (25a^6 + b^ \\
& 6 - 13a^2 b^4 - 85a^4 b^2)) / (d^3 (a^2 + b^2)^3) * ((4(-B^4 a^4 d^4 (a^4 + \\
& b^4 - 6a^2 b^2)^2)^{(1/2)} - 16B^2 a^3 b^3 d^2 + 16B^2 a^5 b d^2) / (d^4 (a \\
& ^2 + b^2)^4))^{(1/2)} / 4 - (16B^4 a^2 b^5 \tan(c + dx)^{(1/2)} (b^6 - 27a^6 + \\
& 7a^2 b^4 + 11a^4 b^2)) / (d^4 (a^2 + b^2)^4) * ((4(-B^4 a^4 d^4 (a^4 + b^4 \\
& - 6a^2 b^2)^2)^{(1/2)} - 16B^2 a^3 b^3 d^2 + 16B^2 a^5 b d^2) / (d^4 (a^2 + \\
& b^2)^4))^{(1/2)} / 4 + (16B^5 a^4 b^6 (5a^2 + b^2)) / (d^5 (a^2 + b^2)^4) * (( \\
& (192B^4 a^6 b^6 d^4 - 16B^4 a^4 b^8 d^4 - 16B^4 a^{12} d^4 - 608B^4 a^8 b^ \\
& ^4 d^4 + 192B^4 a^{10} b^2 d^4)^{(1/2)} - 16B^2 a^3 b^3 d^2 + 16B^2 a^5 b d^ \\
& 2) / (16a^8 d^4 + 16b^8 d^4 + 64a^2 b^6 d^4 + 96a^4 b^4 d^4 + 64a^6 b^2 * \\
& d^4))^{(1/2)} - \log((((((((128B b^2 (2b^6 - a^6 + 9a^2 b^4 + 6a^4 b^2)) \\
& / d - 128b^3 \tan(c + dx)^{(1/2)} (a^2 - b^2) (a^2 + b^2)^2 (-4(-B^4 a^4 d^ \\
& 4 (a^4 + b^4 - 6a^2 b^2)^2)^{(1/2)} + 16B^2 a^3 b^3 d^2 - 16B^2 a^5 b d^2) \\
& / (d^4 (a^2 + b^2)^4))^{(1/2)} * (-4(-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2)^2)^{ \\
& (1/2)} + 16B^2 a^3 b^3 d^2 - 16B^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/2)} / \\
& 4 - (64B^2 a b^2 \tan(c + dx)^{(1/2)} (2b^8 - a^8 + 5a^2 b^6 + 67a^4 b^4 \\
& - a^6 b^2)) / (d^2 (a^2 + b^2)^2) * (-4(-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2) \\
& ^2)^{(1/2)} + 16B^2 a^3 b^3 d^2 - 16B^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/ \\
& 2)} / 4 - (32B^3 a b^5 (25a^6 + b^6 - 13a^2 b^4 - 85a^4 b^2)) / (d^3 (a^2 + \\
& b^2)^3) * (-4(-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2)^2)^{(1/2)} + 16B^2 a^3 b \\
& ^3 d^2 - 16B^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/2)} / 4 - (16B^4 a^2 b^5 \\
& \tan(c + dx)^{(1/2)} (b^6 - 27a^6 + 7a^2 b^4 + 11a^4 b^2)) / (d^4 (a^2 + b^ \\
& 2)^4) * (-4(-B^4 a^4 d^4 (a^4 + b^4 - 6a^2 b^2)^2)^{(1/2)} + 16B^2 a^3 b^3 \\
& d^2 - 16B^2 a^5 b d^2) / (d^4 (a^2 + b^2)^4))^{(1/2)} / 4 + (16B^5 a^4 b^6 (5 \\
& a^2 + b^2)) / (d^5 (a^2 + b^2)^4) * (-((192B^4 a...
\end{aligned}$$

$$3.426 \quad \int \frac{aB + bB \tan(c + dx)}{\tan^2(c + dx)(a + b \tan(c + dx))^2} dx$$

**Optimal.** Leaf size=256

$$\frac{(a + b)B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(a + b)B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{2b^{5/2} B \operatorname{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2} (a^2 + b^2) d}$$

[Out]  $-2*b^{(5/2)}*B*\arctan(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(1/2)})/a^{(3/2)}/(a^2+b^2)/d-1/2*(a+b)*B*\arctan(-1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(a+b)*B*\arctan(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(a-b)*B*\ln(1-2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}+1/4*(a-b)*B*\ln(1+2^{(1/2)}*\tan(d*x+c)^{(1/2)}+\tan(d*x+c))/(a^2+b^2)/d*2^{(1/2)}-2*B/a/d/\tan(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.30, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$ , Rules used = {21, 3650, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{B(a+b)\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)} - \frac{B(a+b)\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)} - \frac{B(a-b)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{B(a-b)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} - \frac{2b^{5/2}B\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d(a^2+b^2)} - \frac{2B}{ad\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x]) / (\operatorname{Tan}[c + d*x]^{(3/2)} * (a + b*\operatorname{Tan}[c + d*x])^2), x]$

[Out]  $((a + b)*B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a + b)*B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - (2*b^{(5/2)}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / \operatorname{Sqrt}[a]]) / (a^{(3/2)}*(a^2 + b^2)*d) - ((a - b)*B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + ((a - b)*B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] + \operatorname{Tan}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - (2*B) / (a*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

**Rule 21**

$\operatorname{Int}[(u_.) * ((a_.) + (b_.) * (v_.)^{(m_.)})^{(n_.)} * ((c_.) + (d_.) * (v_.)^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x$  &&  $\operatorname{EqQ}[b*c - a*d, 0]$  &&  $\operatorname{IntegerQ}[m]$  &&  $(\neg \operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 65**

$\operatorname{Int}[(a_.) + (b_.) * (x_.)^{(m_.)} * ((c_.) + (d_.) * (x_.)^{(n_.)}), x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)} * (c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x$  &&  $\operatorname{NeQ}$

$[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 210

$\text{Int}[\{(a_) + (b_)*(x_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1} * \text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 211

$\text{Int}[\{(a_) + (b_)*(x_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

### Rule 631

$\text{Int}[\{(a_) + (b_)*(x_) + (c_)*(x_)^2\}^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 642

$\text{Int}[\{(d_) + (e_)*(x_)\}/\{(a_) + (b_)*(x_) + (c_)*(x_)^2\}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1176

$\text{Int}[\{(d_) + (e_)*(x_)^2\}/\{(a_) + (c_)*(x_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 1179

$\text{Int}[\{(d_) + (e_)*(x_)^2\}/\{(a_) + (c_)*(x_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 1182

$\text{Int}[\{(d_) + (e_)*(x_)^2\}/\{(a_) + (c_)*(x_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}\{a,$

$c, d, e, x$  &&  $\text{NeQ}[c*d^2 + a*e^2, 0]$  &&  $\text{NeQ}[c*d^2 - a*e^2, 0]$  &&  $\text{NegQ}[(-a)*c]$

### Rule 3615

$\text{Int}[\frac{(c + (d \cdot \tan(e) + f \cdot x))}{\sqrt{(b \cdot \tan(e) + f \cdot x)}}, x_{\text{Symbol}}] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b \cdot c + d \cdot x^2)/(b^2 + x^4), x], x, \sqrt{b \cdot \tan(e + f \cdot x)}], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3650

$\text{Int}[\frac{(a + (b \cdot \tan(e) + f \cdot x))^m \cdot (c + (d \cdot \tan(e) + f \cdot x))^n}{x_{\text{Symbol}}}] \rightarrow \text{Simp}[b^2 \cdot (a + b \cdot \tan(e + f \cdot x))^{m+1} \cdot (c + d \cdot \tan(e + f \cdot x))^{n+1} / (f \cdot (m+1) \cdot (a^2 + b^2) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[1 / ((m+1) \cdot (a^2 + b^2) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot \tan(e + f \cdot x))^{m+1} \cdot (c + d \cdot \tan(e + f \cdot x))^n \cdot \text{Simp}[a \cdot (b \cdot c - a \cdot d) \cdot (m+1) - b^2 \cdot d \cdot (m+n+2) - b \cdot (b \cdot c - a \cdot d) \cdot (m+1) \cdot \tan(e + f \cdot x) - b^2 \cdot d \cdot (m+n+2) \cdot \tan(e + f \cdot x)^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{IntegerQ}[2 \cdot m] \&\& \text{LtQ}[m, -1] \&\& (\text{LtQ}[n, 0] \mid \mid \text{IntegerQ}[m]) \&\& (!\text{LtQ}[n, -1] \&\& (!\text{IntegerQ}[m] \mid \mid (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3715

$\text{Int}[\frac{(a + (b \cdot \tan(e) + f \cdot x))^m \cdot (c + (d \cdot \tan(e) + f \cdot x))^n \cdot (A + (C \cdot \tan(e) + f \cdot x))^2}{x_{\text{Symbol}}}] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x, \tan(e + f \cdot x)], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n, x\} \&\& \text{EqQ}[A, C]$

### Rule 3734

$\text{Int}[\frac{(c + (d \cdot \tan(e) + f \cdot x))^n \cdot (A + (B \cdot \tan(e) + f \cdot x) + (C \cdot \tan(e) + f \cdot x))^2)}{(a + (b \cdot \tan(e) + f \cdot x))}, x_{\text{Symbol}}] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d \cdot \tan(e + f \cdot x))^n \cdot \text{Simp}[b \cdot B + a \cdot (A - C) + (a \cdot B - b \cdot (A - C)) \cdot \tan(e + f \cdot x), x], x], x] + \text{Dist}[(A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C)/(a^2 + b^2), \text{Int}[(c + d \cdot \tan(e + f \cdot x))^n \cdot (1 + \tan(e + f \cdot x)^2)/(a + b \cdot \tan(e + f \cdot x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0] \&\& !\text{LeQ}[n, -1]$

### Rubi steps



$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2} dx &= B \int \frac{1}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx \\
&= -\frac{2B}{ad\sqrt{\tan(c + dx)}} - \frac{(2B) \int \frac{\frac{b}{2} + \frac{1}{2}a \tan(c + dx) + \frac{1}{2}b \tan^2(c + dx)}{\sqrt{\tan(c + dx)}(a + b \tan(c + dx))} dx}{a} \\
&= -\frac{2B}{ad\sqrt{\tan(c + dx)}} - \frac{(2B) \int \frac{\frac{ab}{2} + \frac{1}{2}a^2 \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{a(a^2 + b^2)} - \frac{(b^3 B) \int \frac{1}{\sqrt{\tan(c + dx)}} dx}{a^2 + b^2} \\
&= -\frac{2B}{ad\sqrt{\tan(c + dx)}} - \frac{(4B) \text{Subst}\left(\int \frac{\frac{ab}{2} + \frac{a^2 x^2}{2}}{1 + x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{a(a^2 + b^2)d} \\
&= -\frac{2B}{ad\sqrt{\tan(c + dx)}} + \frac{((a - b)B) \text{Subst}\left(\int \frac{1 - x^2}{1 + x^4} dx, x, \sqrt{\tan(c + dx)}\right)}{(a^2 + b^2)d} \\
&= -\frac{2b^{5/2} B \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2 + b^2)d} - \frac{2B}{ad\sqrt{\tan(c + dx)}} - \frac{(a - b)B \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{2\sqrt{2}(a^2 + b^2)d} \\
&= \frac{(a + b)B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d} - \frac{(a + b)B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.35, size = 132, normalized size = 0.52

$$\frac{B \left( (-1)^{3/4} (a + ib) \text{ArcTan}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right) - \frac{2b^{5/2} \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \sqrt{-1} (ia + b) \tanh^{-1}\left((-1)^{3/4} \sqrt{\tan(c + dx)}\right) - \frac{2(a^2 + b^2)}{a \sqrt{\tan(c + dx)}} \right)}{(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2), x]

[Out] (B\*(-((-1)^(3/4)\*(a + I\*b)\*ArcTan[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]]) - (2\*b^(5/2)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/a^(3/2) + (-1)^(1/4)\*(I\*a + b)\*ArcTanh[(-1)^(3/4)\*Sqrt[Tan[c + d\*x]]] - (2\*(a^2 + b^2))/(a\*Sqrt[Tan[c + d\*x]])))/((a^2 + b^2)\*d)

**Maple [A]**

time = 0.09, size = 242, normalized size = 0.95

method	result
derivativedivides	$B \left[ -\frac{2}{a \sqrt{\tan(dx+c)}} + \frac{b \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right]$
default	$B \left[ -\frac{2}{a \sqrt{\tan(dx+c)}} + \frac{b \sqrt{2} \left( \ln \left( \frac{1+\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)}{1-\sqrt{2} \left( \sqrt{\tan(dx+c)} + \tan(dx+c) \right)} \right) + 2 \arctan \left( 1+\sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right)}{4} \right]$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*B*(-2/a/tan(d*x+c)^(1/2)+2/(a^2+b^2)*(-1/8*b*2^(1/2)*(ln((1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))/(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))) + 2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2)) + 2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))) - 1/8*a*2
^(1/2)*(ln((1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/(1+2^(1/2)*tan(d*x+c)^(1
/2)+tan(d*x+c))) + 2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)) + 2*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))) - 2/a*b^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/
(a*b)^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 188, normalized size = 0.73

$$\frac{8B^3 \arctan\left(\frac{\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+ab^2)\sqrt{ab}} + \frac{(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) + 2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right) - \sqrt{2}(a-b)\log\left(\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1\right) + \sqrt{2}(a-b)\log\left(-\sqrt{2}\sqrt{\tan(dx+c)} + \tan(dx+c)+1\right))B}{a^2+ab^2} + \frac{8B}{a\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] -1/4*(8*B*b^3*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b)))/((a^3 + a*b^2)*sqrt(a*
b)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))
)) - sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sq
```

$\text{rt}(2)*(a - b)*\log(-\sqrt{2}*\sqrt{\tan(dx + c)} + \tan(dx + c) + 1))*B/(a^2 + b^2) + 8*B/(a*\sqrt{\tan(dx + c)})/d$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 4796 vs. 2(216) = 432.

time = 12.92, size = 9596, normalized size = 37.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(dx+c))/tan(dx+c)^(3/2)/(a+b\*tan(dx+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{4}*(4*\sqrt{2}*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*\cos(dx + c)^2 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5)*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}}/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}^{3/4}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}*\arctan(-((B^6*a^8 + 2*B^6*a^6*b^2 - 2*B^6*a^2*b^6 - B^6*b^8)*d^4*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} - \sqrt{2}*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d^7*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} - (B^2*a^7 + 3*B^2*a^5*b^2 + 3*B^2*a^3*b^4 + B^2*a*b^6)*d^5*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}))*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}}/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{((B^4*a^6 - B^4*a^4*b^2 - B^4*a^2*b^4 + B^4*b^6)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\cos(dx + c) + \sqrt{2}*((B^3*a^7 - B^3*a^5*b^2 - B^3*a^3*b^4 + B^3*a*b^6)*d^3*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\cos(dx + c) - (B^5*a^4*b - 2*B^5*a^2*b^3 + B^5*b^5)*d*\cos(dx + c))*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}}/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{\sin(dx + c)/\cos(dx + c)}*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{1/4} + (B^6*a^4 - 2*B^6*a^2*b^2 + B^6*b^4)*\sin(dx + c)/\cos(dx + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4} - \sqrt{2}*((B^3*a^{10}*b + 3*B^3*a^8*b^3 + 2*B^3*a^6*b^5 - 2*B^3*a^4*b^7 - 3*B^3*a^2*b^9 - B^3*b^{11})*d^7*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)} - (B^5*a^9 + 2*B^5*a^7*b^2 - 2*B^5*a^3*b^6 - B^5*a*b^8)*d^5*\sqrt{(B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)}))*\sqrt{(B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*\sqrt{B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)}}/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*\sqrt{\sin(dx + c)/\cos(dx + c)}*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^{3/4}}/(B^{10}*a^4 - 2*B^{10}*a^2*b^2 + B$

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^10*b^4)) + 4*sqrt(2)*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5*cos(d*x +
c)^2 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d^5)*sqrt((B^2*a^4 + 2*B^2*a^2
*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(B^4/((a^4 + 2*a^2*b
^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*(B^4/((a^4 + 2*a^2*b
^2 + b^4)*d^4))^(3/4)*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*arctan(((B^6*a^8 + 2*B^6*a^6*b^2
- 2*B^6*a^4*b^4 - B^6*b^8)*d^4*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt
((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*d^4)) + sqrt(2)*((a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b
^9)*d^7*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*sqrt((B^4*a^4 - 2*B^4*a^2*b
^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (B^2*
a^7 + 3*B^2*a^5*b^2 + 3*B^2*a^3*b^4 + B^2*a*b^6)*d^5*sqrt((B^4*a^4 - 2*B^4*
a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*
sqrt((B^2*a^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2
*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b
^4))*sqrt(((B^4*a^6 - B^4*a^4*b^2 - B^4*a^2*b^4 + B^4*b^6)*d^2*sqrt(B^4/((a
^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d*x + c) - sqrt(2)*((B^3*a^7 - B^3*a^5*b^2 -
B^3*a^3*b^4 + B^3*a*b^6)*d^3*sqrt(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))*cos(d
*x + c) - (B^5*a^4*b - 2*B^5*a^2*b^3 + B^5*b^5)*d*cos(d*x + c))*sqrt((B^2*a
^4 + 2*B^2*a^2*b^2 + B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(B^4/(
(a^4 + 2*a^2*b^2 + b^4)*d^4)))/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*sqrt(si
n(d*x + c)/cos(d*x + c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(1/4) + (B^6*a
^4 - 2*B^6*a^2*b^2 + B^6*b^4)*sin(d*x + c)/cos(d*x + c))*(B^4/((a^4 + 2*a^
2*b^2 + b^4)*d^4))^(3/4) + sqrt(2)*((B^3*a^10*b + 3*B^3*a^8*b^3 + 2*B^3*a^6
*b^5 - 2*B^3*a^4*b^7 - 3*B^3*a^2*b^9 - B^3*b^11)*d^7*sqrt(B^4/((a^4 + 2*a^2
*b^2 + b^4)*d^4))*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (B^5*a^9 + 2*B^5*a^7*b^2 - 2*B^5*a
^3*b^6 - B^5*a*b^8)*d^5*sqrt((B^4*a^4 - 2*B^4*a^2*b^2 + B^4*b^4)/((a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)))*sqrt((B^2*a^4 + 2*B^2*a^2*b^2
+ B^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d^2*sqrt(B^4/((a^4 + 2*a^2*b^2 +
b^4)*d^4)))/(B^2*a^4 - 2*B^2*a^2*b^2 + B^2*b^4))*sqrt(sin(d*x + c)/cos(d*x
+ c))*(B^4/((a^4 + 2*a^2*b^2 + b^4)*d^4))^(3/4))/(B^10*a^4 - 2*B^10*a^2*b
^2 + B^10*b^4)) + 8*(B^5*a^2 + B^5*b^2)*sqrt(sin(d*x + c)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + sqrt(2)*((B^4*a^3 + B^4*a*b^2)*d*cos(d*x + c)^2 - (
B^4*a^3 + B^4*a*b^2)*d - 2*((B^2*a^4*b + B^2*a^...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{a \tan^{\frac{3}{2}}(c + dx) + b \tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] B\*Integral(1/(a\*tan(c + d\*x)\*\*(3/2) + b\*tan(c + d\*x)\*\*(5/2)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] Timed out

**Mupad** [B]

time = 24.71, size = 2500, normalized size = 9.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^2),x)

[Out] 
$$\left(\log\left(\frac{\left(\left(\left(192B^4a^6b^6d^4 - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4\right)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2\right)}{\left(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4\right)^{1/2}}\right)\right)^{1/2} \cdot \left(\frac{\left(\left(\left(192B^4a^6b^6d^4 - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4\right)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2\right)}{\left(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4\right)^{1/2}}\right)\right)^{1/2} \cdot \left(\left(\tan(c + d*x)\right)^{1/2} \cdot \left(\left(1152B^2a^8b^{26}d^7 + 13440B^2a^{10}b^{24}d^7 + 69056B^2a^{12}b^{22}d^7 + 202752B^2a^{14}b^{20}d^7 + 372800B^2a^{16}b^{18}d^7 + 443136B^2a^{18}b^{16}d^7 + 337792B^2a^{20}b^{14}d^7 + 156160B^2a^{22}b^{12}d^7 + 37632B^2a^{24}b^{10}d^7 + 3200B^2a^{26}b^8d^7 + 704B^2a^{28}b^6d^7 + 512B^2a^{30}b^4d^7 + 64B^2a^{32}b^2d^7\right) - \left(\left(\left(192B^4a^6b^6d^4 - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4\right)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2\right)}{\left(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4\right)^{1/2}}\right) \cdot \left(\left(\tan(c + d*x)\right)^{1/2} \cdot \left(\left(\left(192B^4a^6b^6d^4 - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4\right)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2\right)}{\left(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4\right)^{1/2}}\right) \cdot \left(512a^9b^{27}d^9 + 5120a^{11}b^{25}d^9 + 22528a^{13}b^{23}d^9 + 56320a^{15}b^{21}d^9 + 84480a^{17}b^{19}d^9 + 67584a^{19}b^{17}d^9 - 67584a^{23}b^{13}d^9 - 84480a^{25}b^{11}d^9 - 56320a^{27}b^9d^9 - 22528a^{29}b^7d^9 - 5120a^{31}b^5d^9 - 512a^{33}b^3d^9\right)/4 + 768B^2a^8b^{27}d^8 + 8704B^2a^{10}b^{25}d^8 + 44288B^2a^{12}b^{23}d^8 + 133120B^2a^{14}b^{21}d^8 + 261120B^2a^{16}b^{19}d^8 + 347136B^2a^{18}b^{17}d^8 + 311808B^2a^{20}b^{15}d^8 + 178176B^2a^{22}b^{13}d^8 + 49920B^2a^{24}b^{11}d^8 - 7680B^2a^{26}b^9d^8 - 12032B^2a^{28}b^7d^8 - 4096B^2a^{30}b^5d^8 - 512B^2a^{32}b^3d^8\right)/4\right) \cdot \left(\left(\left(192B^4a^6b^6d^4 - 16B^4a^4b^8d^4 - 16B^4a^{12}d^4 - 608B^4a^8b^4d^4 + 192B^4a^{10}b^2d^4\right)^{1/2} + 16B^2a^3b^3d^2 - 16B^2a^5b^2d^2\right)}{\left(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4\right)^{1/2}}\right)$$

$$\begin{aligned}
&^4 + 192*B^4*a^{10}*b^2*d^4)^{(1/2)} + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/( \\
&a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^{(1/2)})/ \\
&4 - 1152*B^3*a^9*b^{24}*d^6 - 8448*B^3*a^{11}*b^{22}*d^6 - 23776*B^3*a^{13}*b^{20}*d^ \\
&6 - 29664*B^3*a^{15}*b^{18}*d^6 - 6528*B^3*a^{17}*b^{16}*d^6 + 26496*B^3*a^{19}*b^{14}* \\
&d^6 + 33984*B^3*a^{21}*b^{12}*d^6 + 18624*B^3*a^{23}*b^{10}*d^6 + 5376*B^3*a^{25}*b^8 \\
&*d^6 + 1152*B^3*a^{27}*b^6*d^6 + 288*B^3*a^{29}*b^4*d^6 + 32*B^3*a^{31}*b^2*d^6)) \\
&/4 - \tan(c + d*x)^{(1/2)}*(144*B^4*a^9*b^{23}*d^5 + 1248*B^4*a^{11}*b^{21}*d^5 + 42 \\
&24*B^4*a^{13}*b^{19}*d^5 + 6720*B^4*a^{15}*b^{17}*d^5 + 3872*B^4*a^{17}*b^{15}*d^5 - 28 \\
&16*B^4*a^{19}*b^{13}*d^5 - 5632*B^4*a^{21}*b^{11}*d^5 - 3136*B^4*a^{23}*b^9*d^5 - 560 \\
&*B^4*a^{25}*b^7*d^5 + 32*B^4*a^{27}*b^5*d^5)))/4 + 72*B^5*a^{10}*b^{21}*d^4 + 648*B \\
&^5*a^{12}*b^{19}*d^4 + 2440*B^5*a^{14}*b^{17}*d^4 + 5000*B^5*a^{16}*b^{15}*d^4 + 6040*B \\
&^5*a^{18}*b^{13}*d^4 + 4312*B^5*a^{20}*b^{11}*d^4 + 1688*B^5*a^{22}*b^9*d^4 + 280*B^5 \\
&*a^{24}*b^7*d^4)*(((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^{12}*d^4 \\
&- 608*B^4*a^8*b^4*d^4 + 192*B^4*a^{10}*b^2*d^4)^{(1/2)} + 16*B^2*a^3*b^3*d^2 \\
&- 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4* \\
&a^6*b^2*d^4))^{(1/2)})/4 - (2*B + (B*\tan(c + d*x)*(2*a^2*b + 3*b^3))/(a*(a^2 \\
&+ b^2)))/((a*d*\tan(c + d*x)^{(1/2)} + b*d*\tan(c + d*x)^{(3/2)})) + (\log((( -(192* \\
&B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^{12}*d^4 - 608*B^4*a^8*b^4*d^ \\
&4 + 192*B^4*a^{10}*b^2*d^4)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(a \\
&^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^{(1/2)}*(( \\
&-((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^{12}*d^4 - 608*B^4*a^ \\
&8*b^4*d^4 + 192*B^4*a^{10}*b^2*d^4)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b \\
&*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^{( \\
&1/2)}*(((\tan(c + d*x)^{(1/2)}*(1152*B^2*a^8*b^{26}*d^7 + 13440*B^2*a^{10}*b^{24}*d^ \\
&7 + 69056*B^2*a^{12}*b^{22}*d^7 + 202752*B^2*a^{14}*b^{20}*d^7 + 372800*B^2*a^{16}*b^{ \\
&18}*d^7 + 443136*B^2*a^{18}*b^{16}*d^7 + 337792*B^2*a^{20}*b^{14}*d^7 + 156160*B^2*a \\
&^{22}*b^{12}*d^7 + 37632*B^2*a^{24}*b^{10}*d^7 + 3200*B^2*a^{26}*b^8*d^7 + 704*B^2*a^ \\
&^{28}*b^6*d^7 + 512*B^2*a^{30}*b^4*d^7 + 64*B^2*a^{32}*b^2*d^7) - (( -(192*B^4*a^6 \\
&*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^{12}*d^4 - 608*B^4*a^8*b^4*d^4 + 192 \\
&*B^4*a^{10}*b^2*d^4)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(a^8*d^4 \\
&+ b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^{(1/2)}*((\tan(c + \\
&d*x)^{(1/2)}*(-((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^{12}*d^4 \\
&- 608*B^4*a^8*b^4*d^4 + 192*B^4*a^{10}*b^2*d^4)^{(1/2)} - 16*B^2*a^3*b^3*d^2 + \\
&16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^ \\
&6*b^2*d^4))^{(1/2)}*(512*a^9*b^{27}*d^9 + 5120*a^{11}*b^{25}*d^9 + 22528*a^{13}*b^{23}* \\
&d^9 + 56320*a^{15}*b^{21}*d^9 + 84480*a^{17}*b^{19}*d^9 + 67584*a^{19}*b^{17}*d^9 - 675 \\
&84*a^{23}*b^{13}*d^9 - 84480*a^{25}*b^{11}*d^9 - 56320*a^{27}*b^9*d^9 - 22528*a^{29}*b^ \\
&7*d^9 - 5120*a^{31}*b^5*d^9 - 512*a^{33}*b^3*d^9))/4 + 768*B*a^8*b^{27}*d^8 + 870 \\
&4*B*a^{10}*b^{25}*d^8 + 44288*B*a^{12}*b^{23}*d^8 + 133120*B*a^{14}*b^{21}*d^8 + 261120 \\
&*B*a^{16}*b^{19}*d^8 + 347136*B*a^{18}*b^{17}*d^8 + 311808*B*a^{20}*b^{15}*d^8 + 178176 \\
&*B*a^{22}*b^{13}*d^8 + 49920*B*a^{24}*b^{11}*d^8 - 7680...
\end{aligned}$$

$$3.427 \quad \int \tan^{\frac{3}{2}}(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=264

$$\frac{\sqrt{ia - b} (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) (4aAb - a^2B - 8b^2B) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{4b^{3/2}d}{d}$$

[Out]  $1/4*(4*A*a*b-B*a^2-8*B*b^2)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/b^{(3/2)}/d+(I*A-B)*\operatorname{arctan}((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}/d+(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}/d+1/4*(4*A*b-B*a)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/b/d+1/2*B*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(3/2)}/b/d$

**Rubi [A]**

time = 1.36, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a^2(-B) + 4aAb - 8b^2B) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{4b^{3/2}d} + \frac{\sqrt{-b + ia} (-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(4Ab - aB) \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{4bd} + \frac{\sqrt{b + ia} (B + iA) \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{B \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}}{2bd}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Tan}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]),x]$

[Out]  $(\operatorname{Sqrt}[I*a - b]*(I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d + ((4*a*A*b - a^2*B - 8*b^2*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(4*b^{(3/2)}*d) + (\operatorname{Sqrt}[I*a + b]*(I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d + ((4*A*b - a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(4*b*d) + (B*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)})/(2*b*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] := \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}*(c + d*x)^{(n)}, x], x, (e + f*x)^{(1/q)}, x]]]$

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x,  $(a + b*x)^{(1/q)/(c + d*x)^{(1/q)}$   
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]  
&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3688

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3728

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*(b\*c\*m + a\*d\*(n + 1)) + d\*(A\*b + a\*B - b\*C)\*(m + n + 1)\*Tan[e + f\*x] - (C\*m



```

*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps



```
t[a + b*Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 4*(-1)^(3/4)*Sqrt[a + I*b
]*b*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[Tan[c + d*x]]*(a + b*Tan[
c + d*x])*(4*A*b + a*B + 2*b*B*Tan[c + d*x])))/(4*b^(3/2)*d*Sqrt[a + b*Tan[
c + d*x]])
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 1.58, size = 2182154, normalized size = 8265.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)*(A+B*tan(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2),
x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^(3/2)*(A+B*tan(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \tan^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c)),x, algorith="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2), x)

$$3.428 \quad \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=201

$$\frac{\sqrt{ia - b} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(2Ab + aB) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{b} d} - \frac{\sqrt{ia}}{d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a-b)^(1/2)/d+(2\*A\*b+B\*a)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/b^(1/2)-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a+b)^(1/2)/d+B\*tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 1.02, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3691, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{\sqrt{-b+ia}(A+iB)\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}d} - \frac{\sqrt{b+ia}(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[I\*a - b]\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d + ((2\*A\*b + a\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[b]\*d) - (Sqrt[I\*a + b]\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d + (B\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/d

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3691

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(m + n), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (A\*b\*c + a\*B\*c + a\*A\*d - b\*B\*d)\*(m + n)\*Tan[e + f\*x] + (A\*b\*d\*(m + n) + B\*(a\*d\*m + b\*c\*n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, m, 1] && LtQ[0, n, 1]

### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2, x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xprand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} + \dots \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} + \dots \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} + \dots \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} - \dots \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} - \dots \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} + \dots \\
 &= \frac{(2Ab+aB) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{b} d} \\
 &= \frac{\sqrt{ia-b} (A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.85, size = 239, normalized size = 1.19

$$\frac{\sqrt{-1} \sqrt{-a+ib} (A-iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) + \sqrt{-1} \sqrt{a+ib} (A+iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) + B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)} + \frac{(2Ab+aB) \sinh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a} \sqrt{b} \sqrt{1+\frac{b \tan(c+dx)}{a}}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),
x]
```

```
[Out] ((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (-1)^(1/4)*Sqrt[a + I*b]*(A + I
*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]])] + B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + ((2*A*b + a*B)*Ar
cSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[a + b*Tan[c + d*x]])/(Sqrt
[a]*Sqrt[b]*Sqrt[1 + (b*Tan[c + d*x])/a])/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 296.35, size = 2178538, normalized size = 10838.50

method	result	size
derivativedivides	Expression too large to display	2178538
default	Expression too large to display	2178538

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c)),
x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \sqrt{\tan(c + dx)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x)), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c)), x)`

**Mupad [B]**

time = 118.04, size = 2500, normalized size = 12.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)`

[Out] `((2*B*a*tan(c + d*x)^(1/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (2*B*a*b*tan(c + d*x)^(3/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^3)/(d + (b^2*d*tan(c + d*x)^2)/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^4 - (2*b*d*tan(c + d*x))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2) - atan((((A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2)*(((A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2)*(((A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2)*(((274877906944*(1600*a^12*b^34*d^8 - 16640*a^14*b^32*d^8 + 22784*a^16*b^30*d^8 + 106496*a^18*b^28*d^8 + 65536*a^20*b^26*d^8))/d^8 - (274877906944*tan(c + d*x)*(1600*a^12*b^35*d^8 - 48000*a^14*b^33*d^8 + 155136*a^16*b^31*d^8 + 466944*a^18*b^29*d^8 + 262144*a^20*b^27*d^8))/(d^8*((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2))*((A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2)))^(1/2) - (219902325552*tan(c + d*x)^(1/2)*(2048*A*a^20*b^27*d^6 - 12536*A*a^16*b^31*d^6 - 3328*A*a^18*b^29*d^6 - 7160*A*a^14*b^33*d^6 + 240*B*a^13*b^34*d^6 - 720*B*a^15*b^32*d^6 + 5696*B*a^17*b^30*d^6 + 14848*B*a^19*b^28*d^6 + 8192*B*a^21*b^26*d^6))/(d^7*((a + b*tan(c + d*x))^(1/2) - a^(1/2))))*(A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2) - (274877906944*(1200*A^2*a^12*b^35*d^6 - 1600*A^2*a^14*b^33*d^6 + 272464*A^2*a^16*b^31*d^6 + 573952*A^2*a^18*b^29*d^6 + 299008*A^2*a^20*b^27*d^6 - 144`

$$\begin{aligned}
& 0*B^2*a^{12}*b^{35}*d^6 + 8352*B^2*a^{14}*b^{33}*d^6 - 320*B^2*a^{16}*b^{31}*d^6 + 2688 \\
& 0*B^2*a^{18}*b^{29}*d^6 + 102400*B^2*a^{20}*b^{27}*d^6 + 65536*B^2*a^{22}*b^{25}*d^6 - \\
& 11200*A*B*a^{13}*b^{34}*d^6 - 25856*A*B*a^{15}*b^{32}*d^6 + 303168*A*B*a^{17}*b^{30}*d^6 \\
& + 661504*A*B*a^{19}*b^{28}*d^6 + 344064*A*B*a^{21}*b^{26}*d^6)/d^8 + (2748779069 \\
& 44*\tan(c + d*x)*(1200*A^2*a^{12}*b^{36}*d^6 - 32160*A^2*a^{14}*b^{34}*d^6 + 1125488 \\
& *A^2*a^{16}*b^{32}*d^6 + 2445312*A^2*a^{18}*b^{30}*d^6 + 1351680*A^2*a^{20}*b^{28}*d^6 \\
& + 65536*A^2*a^{22}*b^{26}*d^6 - 1440*B^2*a^{12}*b^{36}*d^6 + 32256*B^2*a^{14}*b^{34}*d^6 \\
& - 41440*B^2*a^{16}*b^{32}*d^6 + 68096*B^2*a^{18}*b^{30}*d^6 + 405504*B^2*a^{20}*b^{28} \\
& *d^6 + 262144*B^2*a^{22}*b^{26}*d^6 - 16320*A*B*a^{13}*b^{35}*d^6 - 34112*A*B*a^{15} \\
& *b^{33}*d^6 + 1294592*A*B*a^{17}*b^{31}*d^6 + 2656256*A*B*a^{19}*b^{29}*d^6 + 1343488 \\
& *A*B*a^{21}*b^{27}*d^6)/(d^8*((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2)) + (219 \\
& 902325552*\tan(c + d*x)^{(1/2)}*(7600*A^3*a^{18}*b^{30}*d^4 - 3902*A^3*a^{16}*b^{32} \\
& *d^4 - 4590*A^3*a^{14}*b^{34}*d^4 + 6912*A^3*a^{20}*b^{28}*d^4 - 168*B^3*a^{13}*b^{35}*d^4 \\
& + 982*B^3*a^{15}*b^{33}*d^4 + 2494*B^3*a^{17}*b^{31}*d^4 + 9024*B^3*a^{19}*b^{29}*d^4 \\
& + 15872*B^3*a^{21}*b^{27}*d^4 + 8192*B^3*a^{23}*b^{25}*d^4 + 4922*A*B^2*a^{14}*b^{34} \\
& *d^4 + 21906*A*B^2*a^{16}*b^{32}*d^4 + 69720*A*B^2*a^{18}*b^{30}*d^4 + 95744*A*B^2* \\
& a^{20}*b^{28}*d^4 + 43008*A*B^2*a^{22}*b^{26}*d^4 - 180*A^2*B*a^{13}*b^{35}*d^4 + 4486* \\
& A^2*B*a^{15}*b^{33}*d^4 + 52154*A^2*B*a^{17}*b^{31}*d^4 + 93568*A^2*B*a^{19}*b^{29}*d^4 \\
& + 46080*A^2*B*a^{21}*b^{27}*d^4)/(d^7*((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})) \\
& )*((A^2*b - A^2*a*i + B^2*a*i - B^2*b + 2*A*B*a + A*B*b*i)/(4*d^2))^{(1/2)} \\
& ) + (274877906944*(300*A^4*a^{12}*b^{36}*d^4 + 2360*A^4*a^{14}*b^{34}*d^4 + 149228* \\
& A^4*a^{16}*b^{32}*d^4 + 307152*A^4*a^{18}*b^{30}*d^4 + 164352*A^4*a^{20}*b^{28}*d^4 + 4 \\
& 096*A^4*a^{22}*b^{26}*d^4 + 484*B^4*a^{12}*b^{36}*d^4 + 328*B^4*a^{14}*b^{34}*d^4 + 230 \\
& 8*B^4*a^{16}*b^{32}*d^4 - 16048*B^4*a^{18}*b^{30}*d^4 - 43008*B^4*a^{20}*b^{28}*d^4 - 2 \\
& 4576*B^4*a^{22}*b^{26}*d^4 + 7880*A*B^3*a^{13}*b^{35}*d^4 + 28592*A*B^3*a^{15}*b^{33}*d^4 \\
& - 55832*A*B^3*a^{17}*b^{31}*d^4 - 100672*A*B^3*a^{19}*b^{29}*d^4 + 57344*A*B^3*a^{21} \\
& *b^{27}*d^4 + 81920*A*B^3*a^{23}*b^{25}*d^4 - 5880*A^3*B*a^{13}*b^{35}*d^4 + 47408 \\
& *A^3*B*a^{15}*b^{33}*d^4 + 569576*A^3*B*a^{17}*b^{31}*d^4 + 1004928*A^3*B*a^{19}*b^{29} \\
& *d^4 + 489472*A^3*B*a^{21}*b^{27}*d^4 - 320*A^2*B^2*a^{12}*b^{36}*d^4 + 1264*A^2*B^2 \\
& *a^{14}*b^{34}*d^4 - 27264*A^2*B^2*a^{16}*b^{32}*d^4 + 294128*A^2*B^2*a^{18}*b^{30}*d^4 \\
& + 711168*A^2*B^2*a^{20}*b^{28}*d^4 + 389120*A^2*B^2*a^{22}*b^{26}*d^4)/d^8 - (27 \\
& 4877906944*\tan(c + d*x)*(300*A^4*a^{12}*b^{37}*d^4 - 6920*A^4*a^{14}*b^{35}*d^4 + 7 \\
& 38524*A^4*a^{16}*b^{33}*d^4 + 1819680*A^4*a^{18}*b^{31}*d^4 + 1384960*A^4*a^{20}*b^{29} \\
& *d^4 + 311296*A^4*a^{22}*b^{27}*d^4 + 484*B^4*a^{12}*b^{37}*d^4 - 5368*B^4*a^{14}*b^{35} \\
& *d^4 + 1236*B^4*a^{16}*b^{33}*d^4 - 88064*B^4*a^{18}*b^{31}*d^4 - 205824*B^4*a^{20}* \\
& b^{29}*d^4 - 110592*B^4*a^{22}*b^{27}*d^4 + 11192*A*B^3*a^{13}*b^{36}*d^4 + 45712*A*B^3 \\
& *a^{15}*b^{34}*d^4 - 394344*A*B^3*a^{17}*b^{32}*d^4 - 599296*A*B^3*a^{19}*b^{30}*d^4 \\
& + 124928*A*B^3*a^{21}*b^{28}*d^4 + 294912*A*B^3*a^{23}*b^{26}*d^4 - 8680*A^3*B*a^{13} \\
& *b^{36}*d^4 + 163152*A^3*B*a^{15}*b^{34}*d^4 + 2287096*A^3*B*a^{17}*b^{32}*d^4 + 4405 \\
& 760*A^3*B*a^{19}*b^{30}*d^4 + 2617344*A^3*B*a^{21}*b^{28}*d^4 + 327680*A^3*B*a^{23}*b^{26} \\
& *d^4 - 320*A^2*B^2*a^{12}*b^{37}*d^4 + 23984*A^2*B^2*a^{14}*b^{35}*d^4 - 212608* \\
& A^2*B^2*a^{16}*b^{33}*d^4 + 865008*A^2*B^2*a^{18}*b^{31}*d^4 + 2561024*A^2*B^2*a^{20} \\
& *b^{29}*d^4 + 1523712*A^2*B^2*a^{22}*b^{27}*d^4 + 65536*A^2*B^2*a^{24}*b^{25}*d^4)/(( \\
& d^8*((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2)) - (219902325552*\tan(c + d*x) \\
& )^{(1/2)}*(3641*A^5*a^{18}*b^{31}*d^2 - 55*A^5*a^{16}*b^{33}*d^2 - 960*A^5*a^{14}*b^{35}
\end{aligned}$$

$$\begin{aligned} & d^2 + 2864*A^5*a^{20}*b^{29}*d^2 + 128*A^5*a^{22}*b^{27}*d^2 + 39*B^5*a^{13}*b^{36}*d^2 \\ & - 341*B^5*a^{15}*b^{34}*d^2 - 2362*B^5*a^{17}*b^{32}*d^2 - 6942*B^5*a^{19}*b^{30}*d^2 \\ & - 8544*B^5*a^{21}*b^{28}*d^2 - 3584*B^5*a^{23}*b^{26}*d^2 \dots \end{aligned}$$

$$3.429 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

**Optimal.** Leaf size=169

$$\frac{\sqrt{ia - b} (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{2\sqrt{b} B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{\sqrt{ia + b} (iA + B) \operatorname{ArcTan}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}/d+2*B*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*b^{(1/2)}/d-(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}/d$

**Rubi [A]**

time = 0.42, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3695, 3697, 3696, 95, 209, 212, 3715, 65, 223}

$$\frac{\sqrt{-b+ia}(-B+iA)\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{\sqrt{b+ia}(B+iA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]], x]$

[Out]  $-\left(\frac{\operatorname{Sqrt}[I*a - b]*(I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]]}{d} + \frac{(2*\operatorname{Sqrt}[b]*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{d} - \frac{\operatorname{Sqrt}[I*a + b]*(I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]]}{d}\right)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}]/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[m + n + 1, 0] \&\& \operatorname{RationalQ}[n] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{SimplerQ}[a + b*x, c + d*x]$

Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3695

Int[(Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Int[Simp[a\*A - b\*B + (A\*b + a\*B)\*Tan[e + f\*x], x]/(Sqrt[a + b\*Tan[e + f\*x]]\*Sqrt[c + d\*Tan[e + f\*x]]), x] + Dist[b\*B, Int[(1 + Tan[e + f\*x]^2)/(Sqrt[a + b\*Tan[e + f\*x]]\*Sqrt[c + d\*Tan[e + f\*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :>
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= (bB) \int \frac{1 + \tan^2(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx + \int \frac{aA - bB}{\sqrt{\tan(c + dx)}} dx \\ &= \frac{1}{2}((a - ib)(A - iB)) \int \frac{1 + i \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\ &= \frac{((a - ib)(A - iB)) \text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c + dx)\right)}{2d} \\ &= \frac{((a - ib)(A - iB)) \text{Subst}\left(\int \frac{1}{1-(ia+bx)^2} dx, x, \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} \\ &= -\frac{\sqrt{ia - b} (iA - B) \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \end{aligned}$$

**Mathematica [A]**

time = 0.56, size = 204, normalized size = 1.21

$$\frac{\sqrt{-1} \left( \sqrt{-a + ib} (iA + B) \text{ArcTan}\left(\frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) + \sqrt{a + ib} (-iA + B) \text{ArcTan}\left(\frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \right) + \frac{2\sqrt{a} \sqrt{b} B \sinh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right) \sqrt{1 + \frac{b \tan(c + dx)}{a}}}{\sqrt{a + b \tan(c + dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]], x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-a + I*b]*(I*A + B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]) + Sqrt[a + I*b]*((-I)*A + B)*ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]] + (2*Sqrt[a]*Sqrt[b]*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]]/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.78, size = 2175963, normalized size = 12875.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/sqrt(tan(d*x + c)), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x)`

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorith="giac")

[Out] Timed out

**Mupad [B]**

time = 17.99, size = 1141, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/tan(c + d\*x)^(1/2),x)

[Out] 
$$\begin{aligned} & \operatorname{atanh}\left(\frac{a^{3/2}d\tan(c+d*x)^{1/2}\left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(-A^4a^2d^4\right)^{1/2}-a d \tan(c+d*x)^{1/2}\left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(a+b \tan(c+d*x)\right)^{1/2}\left(-A^4a^2d^4\right)^{1/2}+A^2a^{3/2}bd^3 \tan(c+d*x)^{1/2} \\ & \left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2}-A^2a^3bd^3 \tan(c+d*x)^{1/2} \\ & \left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2}\left(a+b \tan(c+d*x)\right)^{1/2} \\ & \left(A^3a^3d^2-A^2a^2bd^2\right)^{1/2}-A^2a^3bd^3 \tan(c+d*x)^{1/2} \\ & \left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2}-A^2a^3bd^3 \tan(c+d*x)^{1/2} \\ & \left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(a+b \tan(c+d*x)\right)^{1/2}+A^3a^2bd^2 \tan(c+d*x)+A^2a^{1/2}b\left(a+b \tan(c+d*x)\right)^{1/2} \\ & \left(-A^4a^2d^4\right)^{1/2}\right)\left(\left(-A^4a^2d^4\right)^{1/2}-A^2bd^2\right)}{d^4}\right)^{1/2}-\operatorname{atanh}\left(\frac{a^{3/2}d\tan(c+d*x)^{1/2}\left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(-A^4a^2d^4\right)^{1/2}-a d \tan(c+d*x)^{1/2}\left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(a+b \tan(c+d*x)\right)^{1/2}\left(-A^4a^2d^4\right)^{1/2}-A^2a^{3/2}bd^3 \tan(c+d*x)^{1/2} \\ & \left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2}+A^2a^3bd^3 \tan(c+d*x)^{1/2} \\ & \left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2}\left(a+b \tan(c+d*x)\right)^{1/2} \\ & \left(A^3a^3d^2+A^2a^2bd^2\right)^{1/2}+A^2a^3bd^3 \tan(c+d*x)^{1/2} \\ & \left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2}+A^2a^3bd^3 \tan(c+d*x)^{1/2} \\ & \left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(a+b \tan(c+d*x)\right)^{1/2}+A^3a^2bd^2 \tan(c+d*x)-A^2a^{1/2}b\left(a+b \tan(c+d*x)\right)^{1/2} \\ & \left(-A^4a^2d^4\right)^{1/2}\right)\left(-\left(-A^4a^2d^4\right)^{1/2}+A^2bd^2\right)}{d^4}\right)^{1/2} \\ & +\operatorname{atanh}\left(\frac{2\left(a^{1/2}d\tan(c+d*x)^{1/2}\left(-B^4a^2d^4\right)^{1/2}+B^2bd^2\right)}{d^4}\right)^{1/2}}{2}-\left(d \tan(c+d*x)^{1/2}\left(\left(-B^4a^2d^4\right)^{1/2}+B^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(a+b \tan(c+d*x)\right)^{1/2}\right)}{2}\right) \\ & \left(B\left(a+b \tan(c+d*x)\right)-a^{1/2}\left(a+b \tan(c+d*x)\right)^{1/2}\right)\right)\left(\left(-B^4a^2d^4\right)^{1/2}+B^2bd^2\right)}{d^4}\right)^{1/2} \\ & +\operatorname{atanh}\left(\frac{2\left(a^{1/2}d\tan(c+d*x)^{1/2}\left(-\left(-B^4a^2d^4\right)^{1/2}-B^2bd^2\right)}{d^4}\right)^{1/2}}{2}-\left(d \tan(c+d*x)^{1/2}\left(-\left(-B^4a^2d^4\right)^{1/2}-B^2bd^2\right)}{d^4}\right)^{1/2} \\ & \left(a+b \tan(c+d*x)\right)^{1/2}\right)}{2}\right) \\ & \left(B\left(a+b \tan(c+d*x)\right)-a^{1/2}\left(a+b \tan(c+d*x)\right)^{1/2}\right)\right)\left(-\left(-B^4a^2d^4\right)^{1/2}-B^2bd^2\right)}{d^4}\right)^{1/2} \\ & +\left(4B^2b^{1/2}\right)\operatorname{atanh}\left(\frac{b^{1/2} \tan(c+d*x)^{1/2}}{\left(a+b \tan(c+d*x)\right)^{1/2}-a^{1/2}}\right)}{d} \end{aligned}$$



$$3.430 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Optimal. Leaf size=154

$$-\frac{\sqrt{ia - b} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{\sqrt{ia + b} (A - iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}/d+(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}/d-2*A*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}$

Rubi [A]

time = 0.34, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {3689, 3697, 3696, 95, 209, 212}

$$-\frac{\sqrt{-b + ia} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{\sqrt{b + ia} (A - iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{2A\sqrt{a + b \tan(c + dx)}}{d\sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/\operatorname{Tan}[c + d*x]^{(3/2)}, x]$

[Out]  $-\left(\frac{\operatorname{Sqrt}[I*a - b]*(A + I*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}{d} + \frac{\operatorname{Sqrt}[I*a + b]*(A - I*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}{d} - \frac{2*A*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}{d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]}\right)$

Rule 95

$\operatorname{Int}[(((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)})/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[m + n + 1, 0] \&\& \operatorname{RationalQ}[n] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{SimplerQ}[a + b*x, c + d*x]$

Rule 209

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ
[2*m, 2*n])
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - 2 \int \frac{\frac{1}{2}(-Ab - aB) + \frac{1}{2}(aA + bB)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{1}{2}((ia - b)(A + iB)) \int \frac{1}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{((ia + b)(A - iB)) \text{Subst}\left(\int \frac{1}{\sqrt{t}} dt, \sqrt{\tan(c + dx)}\right)}{d} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{((ia + b)(A - iB)) \text{Subst}\left(\int \frac{1}{\sqrt{t}} dt, \sqrt{\tan(c + dx)}\right)}{d} \\
&= -\frac{\sqrt{ia - b} (A + iB) \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \dots
\end{aligned}$$

**Mathematica [A]**

time = 0.55, size = 169, normalized size = 1.10

$$\frac{\sqrt[4]{-1} \sqrt{-a + ib} (A - iB) \text{ArcTan}\left(\frac{\sqrt[4]{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) + \sqrt[4]{-1} \sqrt{a + ib} (A + iB) \text{ArcTan}\left(\frac{\sqrt[4]{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) + \frac{2A \sqrt{a + b \tan(c + dx)}}{\sqrt{\tan(c + dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((( -1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((( -1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (-1)^(1/4)*Sqrt[a + I*b]*(A + I*B)*ArcTan[((( -1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (2*A*Sqrt[a + b*Tan[c + d*x]])/Sqrt[Tan[c + d*x]])/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.92, size = 2178365, normalized size = 14145.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))/tan(c + d\*x)\*\*(3/2), x)

**Giac [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(3/2), x  
)
```

$$3.431 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Optimal. Leaf size=199

$$\frac{\sqrt{ia - b} (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{\sqrt{ia + b} (iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a-b)^(1/2)/d+(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a+b)^(1/2)/d-2/3\*(A\*b+3\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(1/2)-2/3\*A\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.50, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{\sqrt{-b+ia}(-B+iA)\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}} + \frac{\sqrt{b+ia}(B+iA)\operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (Sqrt[I\*a - b]\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d + (Sqrt[I\*a + b]\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*A\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*d\*Tan[c + d\*x]^(3/2)) - (2\*(A\*b + 3\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*a\*d\*Sqrt[Tan[c + d\*x]])

Rule 95

Int[(((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)))/((e\_) + (f\_)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan

$[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& !(\text{ILtQ}[n, -1] \&\& (!\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2}{3} \int \frac{\frac{1}{2}(-Ab - 3aB) + \frac{3}{2}(aA + bB) \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} dx \\ &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3ad \sqrt{\tan(c + dx)}} \\ &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3ad \sqrt{\tan(c + dx)}} \\ &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3ad \sqrt{\tan(c + dx)}} \\ &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3ad \sqrt{\tan(c + dx)}} \\ &= \frac{\sqrt{ia - b} (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} + \dots \end{aligned}$$

**Mathematica [A]**

time = 0.98, size = 194, normalized size = 0.97

$$\frac{-3\sqrt{-1}\sqrt{-a+ib}(iA+B)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(-1)^{3/4}\sqrt{-a+ib}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\frac{2\sqrt{a+b\tan(c+dx)}(aA+(Ab+3aB)\tan(c+dx))}{a\tan^2(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out]  $(-3*(-1)^{1/4}*\text{Sqrt}[-a + I*b]*(I*A + B)*\text{ArcTan}[((-1)^{1/4}*\text{Sqrt}[-a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]] + 3*(-1)^{3/4}*\text{Sqrt}[a + I*b]*(A + I*B)*\text{ArcTan}[((-1)^{1/4}*\text{Sqrt}[a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]] - (2*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]*(a*A + (A*b + 3*a*B)*\text{Tan}[c + d*x]))/(a*\text{Tan}[c + d*x]^{3/2}))/ (3*d)$



**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.48, size = 2181119, normalized size = 10960.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/tan(d*x + c)^(5/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(5/2), x)`

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/tan(c + d\*x)^(5/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/tan(c + d\*x)^(5/2), x)

$$3.432 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Optimal. Leaf size=250

$$\frac{\sqrt{ia - b} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{\sqrt{ia + b} (A - iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a-b)^(1/2)/d-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a+b)^(1/2)/d+2/15\*(15\*A\*a^2+2\*A\*b^2-5\*B\*a\*b)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(1/2)-2/5\*A\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)-2/15\*(A\*b+5\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.69, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2A - 5abB + 2Ab^2) \sqrt{a + b \tan(c + dx)}}{15a^2d \sqrt{\tan(c + dx)}} + \frac{\sqrt{-b + ia} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{2(5aB + Ab) \sqrt{a + b \tan(c + dx)}}{15ad \tan^2(c + dx)} - \frac{\sqrt{b + ia} (A - iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] (Sqrt[I\*a - b]\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d - (Sqrt[I\*a + b]\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*A\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*d\*Tan[c + d\*x]^(5/2)) - (2\*(A\*b + 5\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a\*d\*Tan[c + d\*x]^(3/2)) + (2\*(15\*a^2\*A + 2\*A\*b^2 - 5\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a^2\*d\*Sqrt[Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[(((a\_) + (b\_.)\*(x\_)^2)^(n\_))^(m\_)\*((c\_) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(

```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2}{5} \int \frac{\frac{1}{2}(-Ab - 5aB) + \frac{5}{2}(aA + bB) \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)} dx \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15ad \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15ad \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15ad \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15ad \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15ad \tan^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5d \tan^{\frac{5}{2}}(c + dx)} - \frac{2(Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15ad \tan^{\frac{3}{2}}(c + dx)} \\
&= \frac{\sqrt{ia - b} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 1.65, size = 226, normalized size = 0.90

$$\frac{15\sqrt{-1}\sqrt{-a+ib}(A-iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+15\sqrt{-1}\sqrt{-a+ib}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{2\sqrt{a+b\tan(c+dx)}(-3a^2A-a(Ab+5aB)\tan(c+dx)+(15a^2A+2Aa^2-5abB)\tan^2(c+dx))}{a^2\tan^2(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2), x]

```

```
[Out] (15*(-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 15*(-1)^(1/4)*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2*A - a*(A*b + 5*a*B)*Tan[c + d*x] + (15*a^2*A + 2*A*b^2 - 5*a*b*B)*Tan[c + d*x]^2))/(a^2*Tan[c + d*x]^(5/2)))/(15*d)
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 1.02, size = 2183172, normalized size = 8732.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))/tan(c + d\*x)\*\*(7/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/tan(c + d\*x)^(7/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/tan(c + d\*x)^(7/2), x)

$$3.433 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx$$

Optimal. Leaf size=314

$$\frac{\sqrt{ia - b} (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{\sqrt{ia + b} (iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}/d-(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}/d+2/105*(35*A*a^2*b-8*A*b^3+105*B*a^3+14*B*a*b^2)*(a+b*\tan(d*x+c))^{(1/2)}/a^3/d/\tan(d*x+c)^{(1/2)}-2/7*A*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(7/2)}-2/35*(A*b+7*B*a)*(a+b*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(5/2)}+2/105*(35*A*a^2+4*A*b^2-7*B*a*b)*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d/\tan(d*x+c)^{(3/2)}$

Rubi [A]

time = 0.89, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^2A - 7Ab + 4Ab^2)\sqrt{a + b\tan(c + dx)}}{105a^2d\tan^3(c + dx)} + \frac{2(105a^2B + 35a^2Ab + 14a^2B^2 - 8Ab^3)\sqrt{a + b\tan(c + dx)}}{105a^2d\sqrt{\tan(c + dx)}} - \frac{\sqrt{-b + ia}(-B + iA)\operatorname{ArcTan}\left(\frac{\sqrt{-b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d} - \frac{2(7aB + Ab)\sqrt{a + b\tan(c + dx)}}{35ad\tan^3(c + dx)} - \frac{\sqrt{b + ia}(B + iA)\operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d} - \frac{2A\sqrt{a + b\tan(c + dx)}}{7d\tan^3(c + dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*(A + B*\operatorname{Tan}[c + d*x])]/\operatorname{Tan}[c + d*x]^{(9/2)}, x]$

[Out]  $-\left(\frac{(\operatorname{Sqrt}[I*a - b]*(I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{d} - \frac{(\operatorname{Sqrt}[I*a + b]*(I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{d} - \frac{(2*A*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{(7*d*\operatorname{Tan}[c + d*x]^{(7/2)})} - \frac{(2*(A*b + 7*a*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{(35*a*d*\operatorname{Tan}[c + d*x]^{(5/2)})} + \frac{(2*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{(105*a^2*d*\operatorname{Tan}[c + d*x]^{(3/2)})} + \frac{(2*(35*a^2*A*b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{(105*a^3*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])}\right)$

Rule 95

$\operatorname{Int}[\frac{((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}}{((e_.) + (f_.)*(x_.))}, x\_Symbol] :> \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /;$  FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 +

b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rubi steps

$$\int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx = -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2}{7} \int \frac{\frac{1}{2}(-Ab - 7aB) + \frac{7}{2}(aA + bB) \tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)} dx$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{7d \tan^{\frac{7}{2}}(c + dx)} - \frac{2(Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35ad \tan^{\frac{5}{2}}(c + dx)}$$

$$= -\frac{\sqrt{ia - b} (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}$$

Mathematica [A]

time = 2.57, size = 265, normalized size = 0.84

$$\frac{105 \sqrt{-1} \sqrt{-a + ib} (iA + B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) - 105(-1)^{3/4} \sqrt{a + ib} (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + 2 \sqrt{a + b \tan(c + dx)} (-15a^3 A - 3a^2 (Ab + 7aB) \tan(c + dx)) + (25a^2 A + 4AaB - 7aB) \tan^2(c + dx) + (35a^2 Ab - 8Aa^3 + 105a^2 B + 14aB^2) \tan^3(c + dx)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2),x]

[Out]  $(105*(-1)^{1/4}*\text{Sqrt}[-a + I*b]*(I*A + B)*\text{ArcTan}[((-1)^{1/4}*\text{Sqrt}[-a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]] - 105*(-1)^{3/4}*\text{Sqrt}[a + I*b]*(A + I*B)*\text{ArcTan}[((-1)^{1/4}*\text{Sqrt}[a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]] + (2*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]*(-15*a^3*A - 3*a^2*(A*b + 7*a*B)*\text{Tan}[c + d*x] + a*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*\text{Tan}[c + d*x]^2 + (35*a^2*A*b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*\text{Tan}[c + d*x]^3))/(a^3*\text{Tan}[c + d*x]^{7/2}))/ (105*d)$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.51, size = 2184224, normalized size = 6956.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)/tan(d\*x + c)^(9/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\tan(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(9/2), x)
```

### 3.434 $\int \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{\frac{3}{2}}(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=323

$$\frac{(ia - b)^{3/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(6a^2 Ab - 16Ab^3 - a^3 B - 24ab^2 B) \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a - b \tan(c + dx)}}\right)}{8b^{3/2}d}$$

[Out]  $(I*a-b)^{(3/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+1/8*(6*A*a^2*b-16*A*b^3-B*a^3-24*B*a*b^2)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/b^{(3/2)}/d+(I*a+b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+1/8*(6*A*a*b-B*a^2-8*B*b^2)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/b/d+1/12*(6*A*b-B*a)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(3/2)}/b/d+1/3*B*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(5/2)}/b/d$

**Rubi [A]**

time = 1.82, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a^2(-d) + 6aAb - 8b^2B) \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{8bd} + \frac{(a^2(-d) + 6a^2Ab - 24ab^2B - 16Ab^3) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{8b^{3/2}d} + \frac{(-b + ia)^{3/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(6Ab - aB) \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}}{12bd} + \frac{(b + ia)^{3/2}(A - iB) \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{B \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{5/2}}{3bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^{(3/2)}*(a + b*\text{Tan}[c + d*x])^{(3/2)}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $((I*a - b)^{(3/2)}*(A + I*B)*\text{ArcTan}[(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d + ((6*a^2*A*b - 16*A*b^3 - a^3*B - 24*a*b^2*B)*\text{ArcTan}[\text{h}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])]/(8*b^{(3/2)}*d) + ((I*a + b)^{(3/2)}*(A - I*B)*\text{ArcTan}[\text{h}[(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])]/d + ((6*a*A*b - a^2*B - 8*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(8*b*d) + ((6*A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(3/2)})/(12*b*d) + (B*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(5/2)})/(3*b*d)$

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\amp; \text{NeQ}[b*c - a*d, 0] \&\amp; \text{LtQ}[-1, m, 0] \&\amp; \text{LeQ}[-1, n, 0] \&\amp; \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\amp; \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(GtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```

b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \frac{B \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{5/2}}{3bd} + \int \\
&= \frac{(6Ab-aB) \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2}}{12bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6aAb-a^2B-8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{8bd} \\
&= \frac{(6a^2Ab-16Ab^3-a^3B-24ab^2B) \tanh^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{8b^{3/2}d} \\
&= \frac{(ia-b)^{3/2}(A+iB) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.80, size = 347, normalized size = 1.07

$$\frac{24\sqrt{c} \sqrt{c-a+b}^{3/2} (A+B) \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + 24(-1)^{3/2} (a+b)^{3/2} (A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) - 3(-6aAb+a^2B+8b^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)} + 2(6Ab-aB) \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2} + 8B \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{5/2}}{8bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]



```
[Out] (24*(-1)^(1/4)*(-a + I*b)^(3/2)*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(3/4)*(a + I*b)^(
(3/2)*b*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]] - 3*(-6*a*A*b + a^2*B + 8*b^2*B)*Sqrt[Tan[c + d*x]]*S
qrt[a + b*Tan[c + d*x]] + 2*(6*A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c +
d*x])^(3/2) + 8*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2) - (3*Sqrt[
a]*(-6*a^2*A*b + 16*A*b^3 + a^3*B + 24*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c
+ d*x]])))/(24*b*d)
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.53, size = 2401996, normalized size = 7436.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(3/2
), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algor
ithm="fricas")
```

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)`

[Out] `int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)`

### 3.435 $\int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=268

$$\frac{(a + ib)^2 (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia - b} d} + \frac{(12aAb + 3a^2B - 8b^2B) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{4\sqrt{b} d}$$

[Out]  $(I*a+b)^{(3/2)}*(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+(a+I*b)^2*(I*A-B)*\operatorname{arctan}((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a-b)^{(1/2)}+1/4*(12*A*a*b+3*B*a^2-8*B*b^2)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/b^{(1/2)}+1/4*(4*A*b+5*B*a)*\operatorname{atan}(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d+1/2*b*B*(a+b*\tan(d*x+c))^{(1/2)}*\tan(d*x+c)^{(3/2)}/d$

**Rubi** [A]

time = 1.68, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(3a^2B + 12aAb - 8b^2B) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{4\sqrt{b} d} + \frac{(a + ib)^2 (-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d\sqrt{-b + ia}} + \frac{(5aB + 4Ab) \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{4d} + \frac{(b + ia)^{3/2} (B + iA) \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{bB \tan^2(c + dx) \sqrt{a + b \tan(c + dx)}}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((a + I*b)^2*(I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[I*a - b]*d) + ((12*a*A*b + 3*a^2*B - 8*b^2*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(4*\operatorname{Sqrt}[b]*d) + ((I*a + b)^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/d + ((4*A*b + 5*a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(4*d) + (b*B*\operatorname{Tan}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(2*d)$

Rule 65

$\operatorname{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p*(m+1)-1} * (c - a*(d/b) + d*(x^p/b))^{n/p}, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 95

$\operatorname{Int}[(a + b*x)^m * (c + d*x)^n / (e + f*x), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{q*(m+1)} * (c + d*x)^n / (e + f*x), x], x, (a + b*x)^{(1/q)}, x]]$

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x,  $(a + b*x)^{(1/q)/(c + d*x)^{(1/q)}$   
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]  
&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3688

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

#### Rule 3728

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*(b\*c\*m + a\*d\*(n + 1)) + d\*(A\*b + a\*B - b\*C)\*(m + n + 1)\*Tan[e + f\*x] - (C\*m

```

*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx &= \frac{bB \tan^{3/2}(c+dx) \sqrt{a+b \tan(c+dx)}}{2d} + \frac{1}{2} \\
&= \frac{(4Ab+5aB) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{4d} \\
&= \frac{(4Ab+5aB) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{4d} \\
&= \frac{(4Ab+5aB) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{4d} \\
&= \frac{(4Ab+5aB) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{4d} \\
&= \frac{(4Ab+5aB) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{4d} \\
&= \frac{(4Ab+5aB) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{4d} \\
&= \frac{(12aAb+3a^2B-8b^2B) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{4\sqrt{b}d} \\
&= \frac{(a+ib)^2(iA-B) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b}d}
\end{aligned}$$

**Mathematica [A]**

time = 1.63, size = 290, normalized size = 1.08

$$\frac{-4\sqrt{-1}(-a+ib)^{3/2}(A-iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)+4\sqrt{-1}(a+ib)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)+(4Ab+5aB)\sqrt{\tan(c+dx)}\sqrt{a+b \tan(c+dx)}+2bB \tan^2(c+dx)\sqrt{a+b \tan(c+dx)}+\frac{\sqrt{a}\sqrt{(12aAb+3a^2B-8b^2B)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b}\sqrt{a+b \tan(c+dx)}}}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (-4\*(-1)^(1/4)\*(-a + I\*b)^(3/2)\*(A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]] + 4\*(-1)^(1/4)\*(a + I\*b)^(3/2)\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a +

$$b \cdot \tan[c + d \cdot x]] + (4 \cdot A \cdot b + 5 \cdot a \cdot B) \cdot \sqrt{\tan[c + d \cdot x]} \cdot \sqrt{a + b \cdot \tan[c + d \cdot x]} + 2 \cdot b \cdot B \cdot \tan[c + d \cdot x]^{3/2} \cdot \sqrt{a + b \cdot \tan[c + d \cdot x]} + (\sqrt{a} \cdot (12 \cdot a \cdot A \cdot b + 3 \cdot a^2 \cdot B - 8 \cdot b^2 \cdot B) \cdot \operatorname{ArcSinh}[(\sqrt{b} \cdot \sqrt{\tan[c + d \cdot x]}) / \sqrt{a}] \cdot \sqrt{1 + (b \cdot \tan[c + d \cdot x]) / a}) / (\sqrt{b} \cdot \sqrt{a + b \cdot \tan[c + d \cdot x]})) / (4 \cdot d)$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.51, size = 2400957, normalized size = 8958.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c)), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)\*sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\tan(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2), x)



$$3.436 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=204

$$\frac{(ia-b)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{b}(2Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $-(I*a-b)^{(3/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d - (I*a+b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d + (2*A*b+3*B*a)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*b^{(1/2)}/d + b*B*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 1.24, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3688, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(-b+ia)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(b+ia)^{3/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{bB\sqrt{\tan(c+dx)}}{d\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])^{(3/2)}*(A + B*\text{Tan}[c + d*x])/ \text{Sqrt}[\text{Tan}[c + d*x]], x]$

[Out]  $-(((I*a - b)^{(3/2)}*(A + I*B)*\text{ArcTan}[(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d) + (\text{Sqrt}[b]*(2*A*b + 3*a*B)*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d - ((I*a + b)^{(3/2)}*(A - I*B)*\text{ArcTanh}[(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d + (b*B*\text{Sqrt}[\text{Tan}[c + d*x]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d$

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)} / ((e_.) + (f_.)*(x_.))^{(q_.)}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)} / (b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)} / (c + d*x)^{(1/q)}, x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n] \&\& \text{LtQ}[-1, m, 0] \&\& \text{SimplerQ}[a + b*x, c + d*x]$

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & (& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3736

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} + \int \frac{\frac{1}{2}a(2aA -)}{\sqrt{\tan(c + dx)}} dx \\
&= \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(2aA -)}{\sqrt{\tan(c + dx)}} dx, \tan(c + dx), u\right)}{d} \\
&= \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} + \frac{\text{Subst}\left(\int \left(\frac{1}{2}a(2aA -)\right) dx, \tan(c + dx), u\right)}{d} \\
&= \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} + \frac{\text{Subst}\left(\int \frac{a^2 A}{\sqrt{\tan(c + dx)}} dx, \tan(c + dx), u\right)}{d} \\
&= \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} + \frac{\text{Subst}\left(\int \left(-\frac{1}{2}a(2aA -)\right) dx, \tan(c + dx), u\right)}{d} \\
&= \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} + \frac{((a + ib)^2 (iA + B)) \sqrt{a + b \tan(c + dx)}}{d} \\
&= \frac{\sqrt{b} (2Ab + 3aB) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{bB \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}{d} \\
&= -\frac{(ia - b)^{3/2} (A + iB) \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 0.66, size = 243, normalized size = 1.19

$$\frac{-\sqrt{-1}(-a + ib)^{3/2}(iA + B)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a + ib}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) - (-1)^{3/4}(a + ib)^{3/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a + ib}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) + bB\sqrt{\tan(c + dx)}\sqrt{a + b \tan(c + dx)} + \frac{\sqrt{a}\sqrt{b}(2Ab + 3aB)\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c + dx)}}{\sqrt{a}}\right)\sqrt{1 + \frac{b \tan(c + dx)}{a}}}{\sqrt{a + b \tan(c + dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*
x]], x]
```

[Out] 
$$\begin{aligned} & -((-1)^{1/4}(-a + I*b)^{3/2}*(I*A + B)*\text{ArcTan}[\frac{(-1)^{1/4}\sqrt{-a + I*b}*\sqrt{\tan[c + d*x]}}{\sqrt{a + b*\tan[c + d*x]}}]) - (-1)^{3/4}*(a + I*b)^{3/2} \\ & *(A + I*B)*\text{ArcTan}[\frac{(-1)^{1/4}\sqrt{a + I*b}*\sqrt{\tan[c + d*x]}}{\sqrt{a + b*\tan[c + d*x]}}] + b*B*\sqrt{\tan[c + d*x]}\sqrt{a + b*\tan[c + d*x]} + (\sqrt{a} \\ & *\sqrt{b}*(2*A*b + 3*a*B)*\text{ArcSinh}[\frac{\sqrt{b}*\sqrt{\tan[c + d*x]}}{\sqrt{a}}]*\sqrt{1 + (b*\tan[c + d*x])/a})/\sqrt{a + b*\tan[c + d*x]})/d \end{aligned}$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.54, size = 2398415, normalized size = 11756.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\tan(d*x+c))^{3/2}*(A+B*\tan(d*x+c))/\tan(d*x+c)^{1/2}, x)$

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\tan(d*x+c))^{3/2}*(A+B*\tan(d*x+c))/\tan(d*x+c)^{1/2}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((B*\tan(d*x + c) + A)*(b*\tan(d*x + c) + a)^{3/2}/\sqrt{\tan(d*x + c)}, x)$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\tan(d*x+c))^{3/2}*(A+B*\tan(d*x+c))/\tan(d*x+c)^{1/2}, x, \text{algorithm}="fricas")$

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)/sqrt(tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(1/2), x)

$$3.437 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

Optimal. Leaf size=209

$$\frac{(a+ib)^2(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}d} + \frac{2b^{3/2}B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} (ia+b)^3$$

[Out]  $2*b^{(3/2)}*B*\text{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d-(I*a+b)^{(3/2)}*(I*A+B)*\text{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d-(a+I*b)^2*(I*A-B)*\text{arctan}((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a-b)^{(1/2)}-2*a*A*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}$

Rubi [A]

time = 1.21, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3686, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a+ib)^2(-B+iA)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{(b+ia)^{3/2}(B+iA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{2b^{3/2}B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*\text{Tan}[c+d*x])^{(3/2)}*(A+B*\text{Tan}[c+d*x])]/\text{Tan}[c+d*x]^{(3/2)},x]$

[Out]  $-(((a+I*b)^2*(I*A-B)*\text{ArcTan}[(\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(\text{Sqrt}[I*a-b]*d) + (2*b^{(3/2)}*B*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - ((I*a+b)^{(3/2)}*(I*A+B)*\text{ArcTanh}[(\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - (2*a*A*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(d*\text{Sqrt}[\text{Tan}[c+d*x]])$

Rule 65

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x\_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 95

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)})/((e_.) + (f_.)*(x_)), x\_Symbol] :> \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}]/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}, x]] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n]$

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)^2], x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

## Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{3/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + 2 \int \frac{\frac{1}{2}a(2Ab + aB) - \frac{1}{2}(a^2)}{\sqrt{\tan(c + dx)}} dx \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{\frac{1}{2}a(2Ab + aB) + \frac{1}{2}(-a^2)}{\sqrt{x} \sqrt{a + bx}} dx\right)}{\sqrt{\tan(c + dx)}} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{2 \text{Subst}\left(\int \left(\frac{b^2 B}{2\sqrt{x} \sqrt{a + bx}}\right) dx\right)}{\sqrt{\tan(c + dx)}} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{\text{Subst}\left(\int \frac{2aAb + a^2 B - b^2 B - (a^2)}{\sqrt{x} \sqrt{a + bx}} dx\right)}{\sqrt{\tan(c + dx)}} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} + \frac{\text{Subst}\left(\int \left(\frac{a^2 A - Ab^2 - 2abB + i(a^2)}{2(i-x)\sqrt{x} \sqrt{a + bx}}\right) dx\right)}{\sqrt{\tan(c + dx)}} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{((a - ib)^2 (A - iB)) \text{Subst}\left(\int \frac{1}{\sqrt{x} \sqrt{a + bx}} dx\right)}{\sqrt{\tan(c + dx)}} \\
&= \frac{2b^{3/2} B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{2aA \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{(a + ib)^2 (iA - B) \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia - b} d} + \dots
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 38.24, size = 121803, normalized size = 582.79

Result too large to show

Warning: Unable to verify antiderivative.



[In] Integrate[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2),x]

[Out] Result too large to show

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.54, size = 2397265, normalized size = 11470.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x)

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^{\frac{3}{2}}}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2)/tan(c + d\*x)^(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(3/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(3/2), x)

$$3.438 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^{5/2}(c+dx)} dx$$

**Optimal.** Leaf size=196

$$\frac{(ia-b)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{3/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $(I*a-b)^{(3/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+(I*a+b)^{(3/2)}*(A-I*B)*\text{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d-2/3*(4*A*b+3*B*a)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/3*a*A*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.58, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{(-b+ia)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}} + \frac{(b+ia)^{3/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{3/2}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out]  $((I*a-b)^{(3/2)}*(A+I*B)*\text{ArcTan}[\frac{\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}])/d + ((I*a+b)^{(3/2)}*(A-I*B)*\text{ArcTanh}[\frac{\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}])/d - (2*a*A*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(3*d*\text{Tan}[c+d*x]^{(3/2)}) - (2*(4*A*b+3*a*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(3*d*\text{Sqrt}[\text{Tan}[c+d*x]])$

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m+1)-1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 212**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[

$b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& !$   
 $(\text{ILtQ}[n, -1] \&\& ( !\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{\frac{1}{2}a(4Ab + 3aB) - \frac{3}{2}}{\tan^{\frac{5}{2}}(c + dx)} dx \\ &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)} - \frac{2(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{3d \sqrt{\tan(c + dx)}} \\ &= \frac{(ia - b)^{3/2} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.66, size = 238, normalized size = 1.21

$$\frac{3\sqrt{-1} \left( (-a + ib)^{3/2} (A + B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + i(a + ib)^{3/2} (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \right) \tan^{\frac{3}{2}}(c + dx) - 3bB \sqrt{a + b \tan(c + dx)} + (-2aA + 3bB) \sqrt{a + b \tan(c + dx)} - 2(4Ab + 3aB) \tan(c + dx) \sqrt{a + b \tan(c + dx)}}{3d \tan^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

[Out] (3\*(-1)^(1/4)\*((-a + I\*b)^(3/2)\*(I\*A + B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]]) + I\*(a + I\*b)^(3/2)\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]])\*Tan[c + d\*x]^(3/2) - 3\*b\*B\*Sqrt[a + b\*Tan[c + d\*x]] + (-2\*a\*A + 3\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 2\*(4\*A\*b + 3\*a\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]]/(3\*d\*Tan[c + d\*x]^(3/2))

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.54, size = 2398858, normalized size = 12239.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(5/2), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}}}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/tan(c + d*x)**(5/2), x)`

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(5/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(5/2), x)

$$3.439 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

Optimal. Leaf size=259

$$\frac{(a+ib)^2(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}d} + \frac{(ia+b)^{3/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out] (I\*a+b)^(3/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d+(a+I\*b)^2\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a-b)^(1/2)+2/15\*(15\*A\*a^2-3\*A\*b^2-20\*B\*a\*b)\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(1/2)-2/5\*a\*A\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)-2/15\*(6\*A\*b+5\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.77, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{15ad\sqrt{\tan(c+dx)}} + \frac{(a+ib)^2(-B+iA)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{15d\tan^2(c+dx)} + \frac{(b+ia)^{3/2}(B+iA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] ((a + I\*b)^2\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]/(Sqrt[I\*a - b]\*d) + ((I\*a + b)^(3/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/d - (2\*a\*A\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*d\*Tan[c + d\*x]^(5/2)) - (2\*(6\*A\*b + 5\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*d\*Tan[c + d\*x]^(3/2)) + (2\*(15\*a^2\*A - 3\*A\*b^2 - 20\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a\*d\*Sqrt[Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])



Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
    
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{7/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} + \frac{2}{5} \int \frac{\frac{1}{2}a(6Ab + 5aB) - \frac{5}{2}a}{\tan^{5/2}(c + dx)} dx \\
 &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} - \frac{2(6Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
 &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} - \frac{2(6Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
 &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} - \frac{2(6Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
 &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} - \frac{2(6Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
 &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} - \frac{2(6Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
 &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{5d \tan^{5/2}(c + dx)} - \frac{2(6Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} \\
 &= \frac{(a + ib)^2 (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d} + \dots
 \end{aligned}$$

**Mathematica [A]**

time = 2.00, size = 286, normalized size = 1.10

$$\frac{-30\sqrt{-1} a \left( (-a + ib)^{3/2} (A - iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) - (a + ib)^{3/2} (A + iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \right) \tan^3(c + dx) - 15abB \sqrt{a + b \tan(c + dx)} - 3a(4a - 5bB) \sqrt{a + b \tan(c + dx)} - 4a(6Ab + 5aB) \tan(c + dx) \sqrt{a + b \tan(c + dx)} + 4(15a^2A - 3Ab^2 - 20abB) \tan^2(c + dx) \sqrt{a + b \tan(c + dx)}}{30ab \tan^3(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2),x]

[Out] (-30\*(-1)^(1/4)\*a\*((-a + I\*b)^(3/2)\*(A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])]/Sqrt[a + b\*Tan[c + d\*x]]) - (a + I\*b)^(3/2)\*(A + I

\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Tan[c + d\*x]^(5/2) - 15\*a\*b\*B\*Sqrt[a + b\*Tan[c + d\*x]] - 3\*a\*(4\*a\*A - 5\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 4\*a\*(6\*A\*b + 5\*a\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]] + 4\*(15\*a^2\*A - 3\*A\*b^2 - 20\*a\*b\*B)\*Tan[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]]/(30\*a\*d\*Tan[c + d\*x]^(5/2))

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.54, size = 2400946, normalized size = 9270.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x)

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^{\frac{3}{2}}}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2)/tan(c + d\*x)^(7/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\tan(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(7/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(7/2), x)

$$3.440 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

**Optimal.** Leaf size=311

$$\frac{(ia-b)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{3/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $-(I*a-b)^{(3/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d - (I*a+b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d + 2/105*(140*A*a^2*b+6*A*b^3+105*B*a^3-21*B*a*b^2)*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d/\tan(d*x+c)^{(1/2)} - 2/7*a*A*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(7/2)} - 2/35*(8*A*b+7*B*a)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(5/2)} + 2/105*(35*A*a^2-3*A*b^2-42*B*a*b)*(a+b*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(3/2)}$

**Rubi** [A]

time = 1.00, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^2A - 42abB - 3Ab^2)\sqrt{a+b\tan(c+dx)}}{105ad\tan^3(c+dx)} + \frac{2(105a^2B + 140a^2Ab - 21ab^2B + 6Ab^3)\sqrt{a+b\tan(c+dx)}}{105a^2d\sqrt{\tan(c+dx)}} - \frac{(-b+ia)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{35d\tan^3(c+dx)} - \frac{(b+ia)^{3/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2a\sqrt{a+b\tan(c+dx)}}{7d\tan^3(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])^{(3/2)}*(A + B*\text{Tan}[c + d*x])/ \text{Tan}[c + d*x]^{(9/2)}, x]$

[Out]  $-(((I*a-b)^{(3/2)}*(A+I*B)*\text{ArcTan}[(\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d) - ((I*a+b)^{(3/2)}*(A-I*B)*\text{ArcTanh}[(\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - (2*a*A*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(7*d*\text{Tan}[c+d*x]^{(7/2)}) - (2*(8*A*b+7*a*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(35*d*\text{Tan}[c+d*x]^{(5/2)}) + (2*(35*a^2*A-3*A*b^2-42*a*b*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(105*a*d*\text{Tan}[c+d*x]^{(3/2)}) + (2*(140*a^2*A*b+6*A*b^3+105*a^3*B-21*a*b^2*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/(105*a^2*d*\text{Sqrt}[\text{Tan}[c+d*x]])$

**Rule 95**

$\text{Int}[(a_.* + (b_.*)*(x_.)^{(m_*)}*((c_.* + (d_.*)*(x_.)^{(n_*)}))/((e_.* + (f_.*)*(x_.*)), x\_Symbol] :> \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n] \&\& \text{LtQ}[-1, m, 0] \&\& \text{SimplerQ}[a + b*x, c + d*x]$

**Rule 209**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
```

```

f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n* Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\tan^{9/2}(c + dx)} dx &= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} + \frac{2}{7} \int \frac{\frac{1}{2}a(8Ab + 7aB) - \frac{7}{2}}{\tan^{5/2}(c + dx)} dx \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2aA \sqrt{a + b \tan(c + dx)}}{7d \tan^{7/2}(c + dx)} - \frac{2(8Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{(ia - b)^{3/2} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}
\end{aligned}$$

### Mathematica [A]

time = 3.53, size = 346, normalized size = 1.11

$$-\frac{3a^2 b \sqrt{a + b \tan(c + dx)} - 5a^2 (bA - 7bB) \sqrt{a + b \tan(c + dx)} - 6a^2 (bA + 7bB) \tan(c + dx) \sqrt{a + b \tan(c + dx)} + a \tan^2(c + dx) \left( -10d(-1)^{3/2} a^2 \left( (-a + b)^{3/2} (A - iB) \operatorname{ArcTan} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + (a + b)^{3/2} (A + iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1 - \frac{a + b \tan(c + dx)}{a + b \tan(c + dx)}}}{\sqrt{a + b \tan(c + dx)}} \right) \right) \tan^3(c + dx) + 2a(35a^2 A - 3aB^2 - 43abB) \sqrt{a + b \tan(c + dx)}^2 + 214ab^2 Ab + 64b^3 + 105a^2 B - 21ab^2 B) \tan(c + dx) \sqrt{a + b \tan(c + dx)}}{105d^2 \tan^7(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2), x]
```

```
[Out] (-35*a^3*b*B*Sqrt[a + b*Tan[c + d*x]] - 5*a^3*(6*a*A - 7*b*B)*Sqrt[a + b*Tan[c + d*x]] - 6*a^3*(8*A*b + 7*a*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + a*Tan[c + d*x]^2*(-105*(-1)^(3/4)*a^2*((-a + I*b)^(3/2)*(A - I*B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (a + I*b)^(3/2)*(A + I*B)*ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]])*Tan[c + d*x]^(3/2) + 2*a*(35*a^2*A - 3*A*b^2 - 42*a*b*B)*Sqrt[a + b*Tan[c + d*x]] + 2*(140*a^2*A*b + 6*A*b^3 + 105*a^3*B - 21*a*b^2*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(105*a^3*d*Tan[c + d*x]^(7/2))
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.55, size = 2403086, normalized size = 7726.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2), x)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2), x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(9/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2), x, algorithm="fricas")
```

```
[Out] Timed out
```



**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)``[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep`**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="giac")``[Out] Timed out`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\tan(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(9/2),x)``[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(9/2), x)`

$$3.441 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=382

$$\frac{(ia-b)^{3/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{3/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out] (I\*a-b)^(3/2)\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d-(I\*a+b)^(3/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d-2/315\*(315\*A\*a^4-63\*A\*a^2\*b^2+8\*A\*b^4-420\*B\*a^3\*b-18\*B\*a\*b^3)\*(a+b\*tan(d\*x+c))^(1/2)/a^3/d/tan(d\*x+c)^(1/2)-2/9\*a\*A\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(9/2)-2/63\*(10\*A\*b+9\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2)+2/105\*(21\*A\*a^2-A\*b^2-24\*B\*a\*b)\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(5/2)+2/315\*(126\*A\*a^2\*b+4\*A\*b^3+105\*B\*a^3-9\*B\*a\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 1.22, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(21a^4 - 24ab^2 - 4B^2)\sqrt{a+b\tan(c+dx)}}{105d\tan(c+dx)} + \frac{2(105a^2B + 126a^2b - 9ab^2B + 4A^2)\sqrt{a+b\tan(c+dx)}}{315a^2d\tan(c+dx)} - \frac{2(315a^4 - 420a^2b - 63a^2B^2 - 24a^2B + 8A^2)\sqrt{a+b\tan(c+dx)}}{315a^2d\sqrt{\tan(c+dx)}} + \frac{(-b+ia)^{3/2}(-B+IA)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(9aB + 10Ab)\sqrt{a+b\tan(c+dx)}}{63d\tan(c+dx)} - \frac{(b+ia)^{3/2}(B+IA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] ((I\*a - b)^(3/2)\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/d - ((I\*a + b)^(3/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/d - (2\*a\*A\*Sqrt[a + b\*Tan[c + d\*x]])/(9\*d\*Tan[c + d\*x]^(9/2)) - (2\*(10\*A\*b + 9\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(63\*d\*Tan[c + d\*x]^(7/2)) + (2\*(21\*a^2\*A - A\*b^2 - 24\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(105\*a\*d\*Tan[c + d\*x]^(5/2)) + (2\*(126\*a^2\*A\*b + 4\*A\*b^3 + 105\*a^3\*B - 9\*a\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(315\*a^2\*d\*Tan[c + d\*x]^(3/2)) - (2\*(315\*a^4\*A - 63\*a^2\*A\*b^2 + 8\*A\*b^4 - 420\*a^3\*b\*B - 18\*a\*b^3\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(315\*a^3\*d\*Sqrt[Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rubi steps



$$\begin{aligned} & ] + 4*\tan[c + d*x]^2*(6*a^3*(21*a^2*A - A*b^2 - 24*a*b*B)*\sqrt{a + b*\tan[c + d*x]} \\ & + a*\tan[c + d*x]*(315*(-1)^{(1/4)}*a^3*((-a + I*b)^{(3/2)}*(A - I*B)*\text{ArcTan} \\ & \text{rcTan}[((-1)^{(1/4)}*\sqrt{-a + I*b}*\sqrt{\tan[c + d*x]})/\sqrt{a + b*\tan[c + d*x]} \\ & ]]) - (a + I*b)^{(3/2)}*(A + I*B)*\text{ArcTan}[((-1)^{(1/4)}*\sqrt{a + I*b}*\sqrt{\tan[c + d*x]} \\ & )/\sqrt{a + b*\tan[c + d*x]})]*\tan[c + d*x]^{(3/2)} + 2*a*(126*a^2*A*b \\ & + 4*A*b^3 + 105*a^3*B - 9*a*b^2*B)*\sqrt{a + b*\tan[c + d*x]} - 2*(315*a^4*A \\ & - 63*a^2*A*b^2 + 8*A*b^4 - 420*a^3*b*B - 18*a*b^3*B)*\tan[c + d*x]*\sqrt{a + b*\tan[c + d*x]} \\ & ))/(1260*a^4*d*\tan[c + d*x]^{(9/2)}) \end{aligned}$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.52, size = 2405433, normalized size = 6296.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(11/2),x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(11/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\tan(c + dx)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(11/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/tan(c + d\*x)^(11/2),x)

$$3.442 \quad \int \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

Optimal. Leaf size=397

$$\frac{(ia - b)^{5/2}(iA - B)\text{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(40a^3Ab - 320aAb^3 - 5a^4B - 240a^2b^2B + 128b^4)}{64b^{3/2}d}$$

[Out]  $-(I*a-b)^{(5/2)}*(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d+1/64*(40*A*a^3*b-320*A*a*b^3-5*B*a^4-240*B*a^2*b^2+128*B*b^4)*\arctanh(b^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/b^{(3/2)}/d-(I*a+b)^{(5/2)}*(I*A+B)*\arctanh((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d+1/64*(40*A*a^2*b-64*A*b^3-5*B*a^3-112*B*a*b^2)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/b/d+1/96*(40*A*a*b-5*B*a^2-48*B*b^2)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(3/2)}/b/d+1/24*(8*A*b-B*a)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(5/2)}/b/d+1/4*B*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(7/2)}/b/d$

Rubi [A]

time = 2.21, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$(-5d^2B + 40aAb - 48d^2B) \sqrt{a + b \tan(c + dx)}^{(5/2)}$ ,  $(-5d^2B + 40a^2Ab - 112d^2B - 64d^2B) \sqrt{a + b \tan(c + dx)}^{(3/2)}$ ,  $(-5d^2B + 40a^3Ab - 240d^2B + 128d^2B) \tan^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)$ ,  $(-b + a)^{(5/2)} \sqrt{a + b \tan(c + dx)}$ ,  $(Bb - aB) \sqrt{a + b \tan(c + dx)}$ ,  $(Bb - aB) \sqrt{a + b \tan(c + dx)}^{(3/2)}$ ,  $(b + a)^{(5/2)} \tan^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)$ ,  $B \sqrt{a + b \tan(c + dx)}^{(7/2)}$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^{(3/2)}*(a + b*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(((I*a - b)^{(5/2)}*(I*A - B)*\text{ArcTan}[(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d) + ((40*a^3*A*b - 320*a*A*b^3 - 5*a^4*B - 240*a^2*b^2*B + 128*b^4*B)*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(64*b^{(3/2)}*d) - ((I*a + b)^{(5/2)}*(I*A + B)*\text{ArcTanh}[(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d + ((40*a^2*A*b - 64*A*b^3 - 5*a^3*B - 112*a*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(64*b*d) + ((40*a*A*b - 5*a^2*B - 48*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(3/2)})/(96*b*d) + ((8*A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(5/2)})/(24*b*d) + (B*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(7/2)})/(4*b*d)$

Rule 65

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Den}$



ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 95

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3728

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps



[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(-192*(-1)^{1/4}*(-a + I*b)^{5/2}*b*(I*A + B)*\text{ArcTan}[\frac{(-1)^{1/4}*\sqrt{-a + I*b}*\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + b*\text{Tan}[c + d*x]}}] + 192*(-1)^{3/4}*(a + I*b)^{5/2}*b*(A + I*B)*\text{ArcTan}[\frac{(-1)^{1/4}*\sqrt{a + I*b}*\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + b*\text{Tan}[c + d*x]}}] - 3*(-40*a^2*A*b + 64*A*b^3 + 5*a^3*B + 112*a*b^2*B)*\sqrt{\text{Tan}[c + d*x]}*\sqrt{a + b*\text{Tan}[c + d*x]} - 2*(-40*a*A*b + 5*a^2*B + 4*8*b^2*B)*\sqrt{\text{Tan}[c + d*x]}*(a + b*\text{Tan}[c + d*x])^{3/2} + 8*(8*A*b - a*B)*\sqrt{\text{Tan}[c + d*x]}*(a + b*\text{Tan}[c + d*x])^{5/2} + 48*B*\sqrt{\text{Tan}[c + d*x]}*(a + b*\text{Tan}[c + d*x])^{7/2} - (3*\sqrt{a}*(-40*a^3*A*b + 320*a*A*b^3 + 5*a^4*B + 2*40*a^2*b^2*B - 128*b^4*B)*\text{ArcSinh}[\frac{\sqrt{b}*\sqrt{\text{Tan}[c + d*x]}}{\sqrt{a}}]*\sqrt{1 + (b*\text{Tan}[c + d*x])/a}]/(\sqrt{b}*\sqrt{a + b*\text{Tan}[c + d*x]})))/(192*b*d)$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.60, size = 2659561, normalized size = 6699.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(5/2)\*tan(d\*x + c)^(3/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algo  
ithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)

### 3.443 $\int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=316

$$\frac{(ia - b)^{5/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(30a^2Ab - 16Ab^3 + 5a^3B - 40ab^2B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{b}}\right)}{8\sqrt{b}d}$$

[Out]  $-(I*a-b)^{(5/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d+(I*a+b)^{(5/2)}*(A-I*B)*\text{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d+1/8*(30*A*a^2*b-16*A*b^3+5*B*a^3-40*B*a*b^2)*\text{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d/b^{(1/2)}+1/8*(14*A*a*b+5*B*a^2-8*B*b^2)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d+1/4*(2*A*b+3*B*a)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(3/2)}/d+1/3*b*B*\tan(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 2.17, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(5a^2B + 14aAb - 8B^2)\sqrt{\tan(c + dx)}\sqrt{a + b \tan(c + dx)}}{8d} + \frac{(5a^2B + 30a^2Ab - 40aB^2)\tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{8\sqrt{b}d} - \frac{(b + ia)^{5/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(30aB + 2Ab)\sqrt{\tan(c + dx)}(a + b \tan(c + dx))^{3/2}}{8d} + \frac{(b + ia)^{5/2}(A - iB)\tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{8B \tan^2(c + dx)(a + b \tan(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{5/2}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\frac{((I*a - b)^{(5/2)}*(A + I*B)*\text{ArcTan}[(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])}{d} + \frac{((30*a^2*A*b - 16*A*b^3 + 5*a^3*B - 40*a*b^2*B)*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])}{(8*\text{Sqrt}[b]*d)} + \frac{((I*a + b)^{(5/2)}*(A - I*B)*\text{ArcTanh}[(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])}{d} + \frac{((14*a*A*b + 5*a^2*B - 8*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])}{(8*d)} + \frac{((2*A*b + 3*a*B)*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(3/2)})}{(4*d)} + \frac{(b*B*\text{Tan}[c + d*x]^{(3/2)}*(a + b*\text{Tan}[c + d*x])^{(3/2)})}{(3*d)}$

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```

b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps





```
[Out] (24*(-1)^(1/4)*(-a + I*b)^(5/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(1/4)*(a + I*b)^(5
/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]] + 3*(14*a*A*b + 5*a^2*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqr
t[a + b*Tan[c + d*x]] + 6*(2*A*b + 3*a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c +
d*x])^(3/2) + 8*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2) + (3*Sqr
t[a]*(30*a^2*A*b - 16*A*b^3 + 5*a^3*B - 40*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[T
an[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Ta
n[c + d*x]]))/(24*d)
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.58, size = 2657119, normalized size = 8408.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c)
), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algor
ithm="fricas")
```

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\tan(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)`

[Out] `int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)`

$$3.444 \quad \int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=260

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{b}(20aAb+15a^2B-8b^2B)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d}$$

[Out] (I\*a-b)^(5/2)\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d+(I\*a+b)^(5/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d+1/4\*(20\*A\*a\*b+15\*B\*a^2-8\*B\*b^2)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*b^(1/2)/d+1/4\*b\*(4\*A\*b+7\*B\*a)\*tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/d+1/2\*b\*B\*tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(3/2)/d

**Rubi [A]**

time = 1.65, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ ,

Rules used = {3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{\sqrt{b}(15a^2B+20aAb-8b^2B)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d} + \frac{(-b+ia)^{5/2}(-B+iA)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b(7aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4d} + \frac{(b+ia)^{5/2}(B+iA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

[Out] ((I\*a - b)^(5/2)\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d + (Sqrt[b]\*(20\*a\*A\*b + 15\*a^2\*B - 8\*b^2\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(4\*d) + ((I\*a + b)^(5/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d + (b\*(4\*A\*b + 7\*a\*B)\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(4\*d) + (b\*B\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2))/(2\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)]

], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3688

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3728

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)^2], x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*(b\*c\*m + a\*d\*(n + 1)) + d\*(A\*b + a\*B - b\*C)\*(m + n + 1)\*Tan[e + f\*x] - (C\*m\*(b\*c - a\*d) - b\*B\*d\*(m + n + 1))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps



$*x]] + 2*b*B*\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{(3/2)} + (\text{Sqrt}[a]*\text{Sqrt}[b]*(20*a*A*b + 15*a^2*B - 8*b^2*B)*\text{ArcSinh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]]*\text{Sqrt}[1 + (b*\text{Tan}[c + d*x])/a])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/(4*d)$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.57, size = 2654895, normalized size = 10211.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/sqrt(tan(d*x + c)), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)`



[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2)/sqrt(tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(1/2), x)

$$3.445 \quad \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^3(c+dx)} dx$$

Optimal. Leaf size=241

$$\frac{(ia-b)^{5/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b^{3/2}(2Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $(I*a-b)^{(5/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+b^{(3/2)}*(2*A*b+5*B*a)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d-(I*a+b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+b*(2*A*a+B*b)*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d-2*a*A*(a+b*\tan(d*x+c))^{(3/2)}/d/\tan(d*x+c)^{(1/2)}$

Rubi [A]

time = 1.67, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3686, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(-b+ia)^{5/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} - \frac{(b+ia)^{5/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out]  $((I*a-b)^{(5/2)}*(A+I*B)*\text{ArcTan}[(\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d + (b^{(3/2)}*(2*A*b+5*a*B)*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - ((I*a+b)^{(5/2)}*(A-I*B)*\text{ArcTanh}[(\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d + (b*(2*a*A+b*B)*\text{Sqrt}[\text{Tan}[c+d*x]]*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - (2*a*A*(a+b*\text{Tan}[c+d*x])^{(3/2)})/(d*\text{Sqrt}[\text{Tan}[c+d*x]])$

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^n, x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 95

Int((((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m+1)-1)/(b\*e-a\*f-(d\*e-c\*f)\*x^q), x], x, (a+b\*x)^(1/q)/(c+d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m+n+1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3728

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*(b\*c\*m + a\*d\*(n + 1)) + d\*(A\*b + a\*B - b\*C)\*(m + n + 1)\*Tan[e + f\*x] - (C\*m\*(b\*c - a\*d) - b\*B\*d\*(m + n + 1))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps



**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.59, size = 2653774, normalized size = 11011.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2), x)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2), x, algorithm="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}}}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2), x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)/tan(c + d*x)**(3/2), x)`

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
ithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(3/2), x
)
```

$$3.446 \quad \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

Optimal. Leaf size=240

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2b^{5/2}B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} (ia+b)$$

[Out]  $-(I*a-b)^{(5/2)}*(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d+2*b^{(5/2)}*B*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d-(I*a+b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})/d-2*a*(2*A*b+B*a)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/3*a*A*(a+b*\tan(d*x+c))^{(3/2)}/d/\tan(d*x+c)^{(3/2)}$

Rubi [A]

time = 1.45, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3686, 3726, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(-b+ia)^{5/2}(-B+IA)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{(b+ia)^{5/2}(B+IA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^2(c+dx)} + \frac{2b^{5/2}B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*\text{Tan}[c+d*x])^{5/2}*(A+B*\text{Tan}[c+d*x])/(\text{Tan}[c+d*x])^{5/2},x]$

[Out]  $-\left(\frac{(I*a-b)^{(5/2)}*(I*A-B)*\text{ArcTan}[\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}\right)/d + \left(\frac{2*b^{(5/2)}*B*\text{ArcTanh}[\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c+d*x]]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}\right)/d - \left(\frac{(I*a+b)^{(5/2)}*(I*A+B)*\text{ArcTanh}[\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}\right)/d - \left(\frac{2*a*(2*A*b+a*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}{(d*\text{Sqrt}[\text{Tan}[c+d*x]])}\right) - \left(\frac{2*a*A*(a+b*\text{Tan}[c+d*x])^{(3/2)}}{(3*d*\text{Tan}[c+d*x])^{(3/2)}}\right)$

Rule 65

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 95

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)} / ((e_.) + (f_.)*(x_.)), x\_Symbol] :> \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)} / (b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)} / (c + d*x)^{(1/q)}]$



], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*d\*(b\*d\*m - a\*c\*(n + 1)) + (c\*C - B\*d)\*(b\*c\*m + a\*d\*(

```

n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{5/2}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} + \frac{2}{3} \int \frac{\sqrt{a + b \tan(c + dx)}}{\tan^{3/2}(c + dx)} dx \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{3d \tan^{3/2}(c + dx)} \\
&= -\frac{2b^{5/2} B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} - \frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} \\
&= -\frac{(ia - b)^{5/2} (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} - \frac{2a(2Ab + aB) \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 38.64, size = 139636, normalized size = 581.82

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2),x]

[Out] Result too large to show

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.60, size = 2654078, normalized size = 11058.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(5/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{5}{2}}}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)/tan(c + d*x)**(5/2), x)`

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(5/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(5/2), x)

$$3.447 \quad \int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

Optimal. Leaf size=247

$$\frac{(ia-b)^{5/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{5/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $-(I*a-b)^{(5/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+(I*a+b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d+2/15*(15*A*a^2-23*A*b^2-35*B*a*b)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(1/2)}-2/15*a*(8*A*b+5*B*a)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(3/2)}-2/5*a*A*(a+b*\tan(d*x+c))^{(3/2)}/d/\tan(d*x+c)^{(5/2)}$

Rubi [A]

time = 0.79, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {3686, 3726, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2A - 35abB - 23A^2b^2)\sqrt{a+b\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}} - \frac{(-b+ia)^{5/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{15d\tan^2(c+dx)} + \frac{(b+ia)^{5/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^2(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x])]/\text{Tan}[c + d*x]^{(7/2)}, x]$

[Out]  $-\left(\frac{(I*a-b)^{(5/2)}*(A+I*B)*\text{ArcTan}[\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}\right)/d + \left(\frac{(I*a+b)^{(5/2)}*(A-I*B)*\text{ArcTanh}[\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]]]}{\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}\right)/d - \frac{(2*a*(8*A*b+5*a*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}{(15*d*\text{Tan}[c+d*x]^{(3/2)})} + \frac{(2*(15*a^2*A-23*A*b^2-35*a*b*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}{(15*d*\text{Sqrt}[\text{Tan}[c+d*x]])} - \frac{(2*a*A*(a+b*\text{Tan}[c+d*x])^{(3/2)})}{(5*d*\text{Tan}[c+d*x]^{(5/2)})}$

Rule 95

$\text{Int}[\frac{((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}}{((e_.) + (f_.)*(x_.))}, x\_Symbol] :> \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n] \&\& \text{LtQ}[-1, m, 0] \&\& \text{SimplerQ}[a + b*x, c + d*x]$

Rule 209

$\text{Int}[\frac{((a_.) + (b_.)*(x_.)^2)^{(-1)}}{x\_Symbol}] :> \text{Simp}[\frac{1}{(\text{Rt}[a, 2]*\text{Rt}[b, 2])}] * \text{ArcTan}[\frac{\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])}{\text{Rt}[a, 2]}], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{GtQ}[b, 0])$

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*d\*(b\*d\*m - a\*c\*(n + 1)) + (c\*C - B\*d)\*(b\*c\*m + a\*d\*(n + 1)) - d\*(n + 1)\*((A - C)\*(b\*c - a\*d) + B\*(a\*c + b\*d))\*Tan[e + f\*x] - b\*

$(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*\text{Tan}[e + f*x]^2, x], x$   
 $], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{LtQ}[n, -1]$

### Rule 3730

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] :> \text{Simp}[(A*b^2 - a*(b*B - a*C))*(a + b*\text{Tan}[e + f*x])^{(m + 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))], x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*\text{Tan}[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& (!\text{IntegerQ}[n, -1] \&\& (!\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{7/2}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^{3/2}}{5d \tan^{5/2}(c + dx)} + \frac{2}{5} \int \frac{\sqrt{a + b \tan(c + dx)}}{\tan^{5/2}(c + dx)} dx \\ &= -\frac{2a(8Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{5d \tan^{5/2}(c + dx)} \\ &= -\frac{2a(8Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2(15a^2A - 23aAb)}{15d \tan^{3/2}(c + dx)} \\ &= -\frac{2a(8Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2(15a^2A - 23aAb)}{15d \tan^{3/2}(c + dx)} \\ &= -\frac{2a(8Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2(15a^2A - 23aAb)}{15d \tan^{3/2}(c + dx)} \\ &= -\frac{2a(8Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2(15a^2A - 23aAb)}{15d \tan^{3/2}(c + dx)} \\ &= -\frac{2a(8Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{15d \tan^{3/2}(c + dx)} + \frac{2(15a^2A - 23aAb)}{15d \tan^{3/2}(c + dx)} \\ &= -\frac{(ia - b)^{5/2} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} \end{aligned}$$



**Mathematica [A]**

time = 1.37, size = 321, normalized size = 1.30

$$\frac{60\sqrt{T}\left((-a+b)^{5/2}(A-iB)\operatorname{ArcTan}\left(\frac{\sqrt{-T}\sqrt{-a+B}\sqrt{\tan(c+dx)}}{\sqrt{-1+\tan(c+dx)}}\right)+(a+b)^{5/2}(A+iB)\operatorname{ArcTan}\left(\frac{\sqrt{-T}\sqrt{-a+B}\sqrt{\tan(c+dx)}}{\sqrt{-1+\tan(c+dx)}}\right)\right)\tan^2(c+dx)+15(-2A+aB)\sqrt{a+B\tan(c+dx)}-3(8a^2A-10aAb^2-15a^2B)\sqrt{a+B\tan(c+dx)}-4(22aAb+10a^2B-15a^2B)\tan(c+dx)\sqrt{a+B\tan(c+dx)}+8(15a^2A-21aAb^2-35a^2B)\tan^2(c+dx)\sqrt{a+B\tan(c+dx)}-60B(a+B\tan(c+dx))^{5/2}}{60d\tan^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(7/2), x]

[Out] (60\*(-1)^(1/4)\*((-a + I\*b)^(5/2)\*(A - I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]]) + (a + I\*b)^(5/2)\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]/Sqrt[a + b\*Tan[c + d\*x]])\*Tan[c + d\*x]^(5/2) + 15\*b\*(-2\*A\*b + a\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 3\*(8\*a^2\*A - 10\*A\*b^2 - 15\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 4\*(22\*a\*A\*b + 10\*a^2\*B - 15\*b^2\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]] + 8\*(15\*a^2\*A - 23\*A\*b^2 - 35\*a\*b\*B)\*Tan[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]] - 60\*b\*B\*(a + b\*Tan[c + d\*x])^(3/2))/(60\*d\*Tan[c + d\*x]^(5/2))

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.62, size = 2654930, normalized size = 10748.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2), x)

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2), x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(7/2),x)

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\tan(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(7/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(7/2), x)

$$3.448 \quad \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\tan^2(c+dx)} dx$$

**Optimal.** Leaf size=309

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{5/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out] (I\*a-b)^(5/2)\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d+(I\*a+b)^(5/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d+2/105\*(245\*A\*a^2\*b-15\*A\*b^3+105\*B\*a^3-161\*B\*a\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(1/2)-2/35\*a\*(10\*A\*b+7\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2)+2/105\*(35\*A\*a^2-45\*A\*b^2-77\*B\*a\*b)\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(3/2)-2/7\*a\*A\*(a+b\*tan(d\*x+c))^(3/2)/d/tan(d\*x+c)^(7/2)

**Rubi [A]**

time = 1.03, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {3686, 3726, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^2A - 77abB - 45A^2B)\sqrt{a+b\tan(c+dx)}}{105d\tan^3(c+dx)} + \frac{2(105a^2B + 245a^2Ab - 161a^2B^2 - 15A^2B)\sqrt{a+b\tan(c+dx)}}{105d\sqrt{\tan(c+dx)}} + \frac{(-b+ia)^{5/2}(-B+iA)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{35d\tan^3(c+dx)} + \frac{(b+ia)^{5/2}(B+iA)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^3(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] ((I\*a - b)^(5/2)\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d + ((I\*a + b)^(5/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*a\*(10\*A\*b + 7\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(35\*d\*Tan[c + d\*x]^(5/2)) + (2\*(35\*a^2\*A - 45\*A\*b^2 - 77\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(3/2)) + (2\*(245\*a^2\*A\*b - 15\*A\*b^3 + 105\*a^3\*B - 161\*a\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(105\*a\*d\*Sqrt[Tan[c + d\*x]]) - (2\*a\*A\*(a + b\*Tan[c + d\*x])^(3/2))/(7\*d\*Tan[c + d\*x]^(7/2))

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 3726

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
```

```

f*x]]^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{9/2}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^{3/2}}{7d \tan^{7/2}(c + dx)} + \frac{2}{7} \int \frac{\sqrt{a + b \tan(c + dx)}}{\tan^{7/2}(c + dx)} dx \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{7d \tan^{7/2}(c + dx)} \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2(35a^2A - 45aAb + 7a^2B) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2(35a^2A - 45aAb + 7a^2B) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2(35a^2A - 45aAb + 7a^2B) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2(35a^2A - 45aAb + 7a^2B) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2(35a^2A - 45aAb + 7a^2B) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= -\frac{2a(10Ab + 7aB) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} + \frac{2(35a^2A - 45aAb + 7a^2B) \sqrt{a + b \tan(c + dx)}}{35d \tan^{5/2}(c + dx)} \\
&= \frac{(ia - b)^{5/2} (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} +
\end{aligned}$$

**Mathematica [A]**

time = 4.13, size = 381, normalized size = 1.23

439777 ((-1 + 9i)^(1/4) + 9i) ArcTan[...]

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(9/2), x]

[Out] (420\*(-1)^(1/4)\*a\*((-a + I\*b)^(5/2)\*(I\*A + B)\*ArcTan[...]) + (a + I\*b)^(5/2)\*((-I)\*A + B)\*ArcTan[...]\*Tan[c + d\*x]^(7/2) - 35\*a\*b\*(4\*A\*b + a\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 5\*a\*(24\*a^2\*A - 28\*A\*b^2 - 49\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 6\*a\*(60\*a\*A\*b + 28\*a^2\*B - 35\*b^2\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]] + 8\*a\*(35\*a^2\*A - 45\*A\*b^2 - 77\*a\*b\*B)\*Tan[c + d\*x]^2\*Sqrt[a + b\*Tan[c + d\*x]] +

$$8*(245*a^2*A*b - 15*A*b^3 + 105*a^3*B - 161*a*b^2*B)*\text{Tan}[c + d*x]^3*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] - 210*a*b*B*(a + b*\text{Tan}[c + d*x])^{(3/2)}/(420*a*d*\text{Tan}[c + d*x]^{(7/2)})$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.66, size = 2657093, normalized size = 8599.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(9/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(9/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\tan(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(9/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/tan(c + d\*x)^(9/2), x)



$$3.449 \quad \int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{11/2}(c+dx)} dx$$

Optimal. Leaf size=378

$$\frac{(ia-b)^{5/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{5/2}(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out] (I\*a-b)^(5/2)\*(A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d - (I\*a+b)^(5/2)\*(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d - 2/315\*(315\*A\*a^4-483\*A\*a^2\*b^2-10\*A\*b^4-735\*B\*a^3\*b+45\*B\*a\*b^3)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(1/2) - 2/21\*a\*(4\*A\*b+3\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(7/2) + 2/105\*(21\*A\*a^2-25\*A\*b^2-45\*B\*a\*b)\*(a+b\*tan(d\*x+c))^(1/2)/d/tan(d\*x+c)^(5/2) + 2/315\*(231\*A\*a^2\*b-5\*A\*b^3+105\*B\*a^3-135\*B\*a\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(3/2) - 2/9\*a\*A\*(a+b\*tan(d\*x+c))^(3/2)/d/tan(d\*x+c)^(9/2)

Rubi [A]

time = 1.27, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {3686, 3726, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(1a^4 - 45abB - 25a^2B^2)\sqrt{a + b\tan(c + dx)}}{315d\tan^3(c + dx)} - \frac{2(10a^2B + 231a^2Ab - 135a^2B^2 - 5aB^3)\sqrt{a + b\tan(c + dx)}}{315d\tan^3(c + dx)} - \frac{2(315a^4 - 735a^2b^2 - 483a^2a^2B - 10aB^3)\sqrt{a + b\tan(c + dx)}}{315a^2d\sqrt{\tan(c + dx)}} + \frac{(-b + ia)^{5/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d} - \frac{2a\tanh(B + 4Ab)\sqrt{a + b\tan(c + dx)}}{21d\tan^3(c + dx)} - \frac{(b + ia)^{5/2}(A - iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d} - \frac{2a(A + 3ab(c + dx))^{3/2}}{9d\tan^3(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] ((I\*a - b)^(5/2)\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d - ((I\*a + b)^(5/2)\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/d - (2\*a\*(4\*A\*b + 3\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(21\*d\*Tan[c + d\*x]^(7/2)) + (2\*(21\*a^2\*A - 25\*A\*b^2 - 45\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(105\*d\*Tan[c + d\*x]^(5/2)) + (2\*(231\*a^2\*A\*b - 5\*A\*b^3 + 105\*a^3\*B - 135\*a\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(315\*a\*d\*Tan[c + d\*x]^(3/2)) - (2\*(315\*a^4\*A - 483\*a^2\*A\*b^2 - 10\*A\*b^4 - 735\*a^3\*b\*B + 45\*a\*b^3\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(315\*a^2\*d\*Sqrt[Tan[c + d\*x]]) - (2\*a\*A\*(a + b\*Tan[c + d\*x])^(3/2))/(9\*d\*Tan[c + d\*x]^(9/2))

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3726

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^{3/2}}{9d \tan^{\frac{9}{2}}(c + dx)} + \frac{2}{9} \int \frac{\sqrt{a + b \tan(c + dx)}}{\tan^{\frac{9}{2}}(c + dx)} dx \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(4Ab + 3aB) \sqrt{a + b \tan(c + dx)}}{21d \tan^{\frac{7}{2}}(c + dx)} + \frac{2(21a^2A - 25A^2) \sqrt{a + b \tan(c + dx)}}{9d \tan^{\frac{9}{2}}(c + dx)} \\
&= \frac{(ia - b)^{5/2} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 6.51, size = 458, normalized size = 1.21

Integrate[(a + b Tan[c + d x])^(5/2) (A + B Tan[c + d x]) / Tan[c + d x]^(11/2), x]

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(11/2), x]

[Out] (-315\*a^4\*b\*(2\*A\*b + a\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 35\*a^4\*(16\*a^2\*A - 18\*A\*b^2 - 33\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 40\*a^4\*(38\*a\*A\*b + 18\*a^2\*B - 21\*b^2\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]] - 840\*a^4\*b\*B\*(a + b\*Tan[c

$$+ d*x]]^{(3/2)} + 8*\text{Tan}[c + d*x]^2*(6*a^4*(21*a^2*A - 25*A*b^2 - 45*a*b*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] + a^2*\text{Tan}[c + d*x]*(-315*(-1)^{(1/4)}*a^2*((-a + I*b)^{(5/2)}*(A - I*B)*\text{ArcTan}[((-1)^{(1/4)}*\text{Sqrt}[-a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])] + (a + I*b)^{(5/2)}*(A + I*B)*\text{ArcTan}[((-1)^{(1/4)}*\text{Sqrt}[a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])]*\text{Tan}[c + d*x]^{(3/2)} + 2*a*(231*a^2*A*b - 5*A*b^3 + 105*a^3*B - 135*a*b^2*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] - 2*(315*a^4*A - 483*a^2*A*b^2 - 10*A*b^4 - 735*a^3*b*B + 45*a*b^3*B)*\text{Tan}[c + d*x]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])))/(2520*a^4*d*\text{Tan}[c + d*x]^{(9/2)})$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.62, size = 2659448, normalized size = 7035.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)`

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, algorithm="giac")`

[Out] Timed out

**Mupad** [F(-1)]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(11/2),x)`

[Out] `\text{Hanged}`

$$3.450 \quad \int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\tan^{13/2}(c+dx)} dx$$

**Optimal.** Leaf size=460

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{5/2}(iA+B)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $-(I*a-b)^{(5/2)}*(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d - (I*a+b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d - 2/3465*(8085*A*a^4*b - 495*A*a^2*b^3 + 40*A*b^5 + 3465*B*a^5 - 5313*B*a^3*b^2 - 110*B*a*b^4)*(a+b*\tan(d*x+c))^{(1/2)}/a^3/d/\tan(d*x+c)^{(1/2)} - 2/99*a*(14*A*b + 11*B*A)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(9/2)} + 2/693*(99*A*a^2 - 113*A*b^2 - 209*B*a*b)*(a+b*\tan(d*x+c))^{(1/2)}/d/\tan(d*x+c)^{(7/2)} + 2/1155*(495*A*a^2*b - 5*A*b^3 + 231*B*a^3 - 275*B*a*b^2)*(a+b*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(5/2)} - 2/3465*(1155*A*a^4 - 1485*A*a^2*b^2 - 20*A*b^4 - 2541*B*a^3*b + 55*B*a*b^3)*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d/\tan(d*x+c)^{(3/2)} - 2/11*a*A*(a+b*\tan(d*x+c))^{(3/2)}/d/\tan(d*x+c)^{(11/2)}$

**Rubi [A]**

time = 1.55, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {3686, 3726, 3730, 3697, 3696, 95, 209, 212}

$\frac{3084*A - 2964*B - 1114*B^2}{693d^2 \tan^2(c+dx)}$   $\frac{2103*B + 454*A - 2754*B^2 - 549*B^3}{1155d^2 \tan^2(c+dx)}$   $\frac{21125*A^4 - 25414*B^2 - 14852*AP + 5548*B^3 - 2614*B^4}{3465d^2 \tan^2(c+dx)}$   $\frac{214852*B + 88882*A - 53334*B^2 - 4952*AP - 10548*B^3 + 4618*B^4}{3465d^2 \tan^2(c+dx)}$   $\frac{(-b + a \tan^2(c+dx) + c) \operatorname{ArcTan}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}$   $\frac{343148 + 11480*B}{99d \tan^2(c+dx)}$   $\frac{(b + a \tan^2(c+dx) + c) \operatorname{Arctanh}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}$   $\frac{2541*A + 55*B}{11d \tan^2(c+dx)}$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(13/2), x]

[Out]  $-(((I*a-b)^{(5/2)}*(I*A-B)*\text{ArcTan}[\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]]]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - ((I*a+b)^{(5/2)}*(I*A+B)*\text{ArcTanh}[\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]]]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])/d - (2*a*(14*A*b + 11*a*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]/(99*d*\text{Tan}[c+d*x]^{(9/2)}) + (2*(99*a^2*A - 113*A*b^2 - 209*a*b*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]/(693*d*\text{Tan}[c+d*x]^{(7/2)}) + (2*(495*a^2*A*b - 5*A*b^3 + 231*a^3*B - 275*a*b^2*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]/(1155*a*d*\text{Tan}[c+d*x]^{(5/2)}) - (2*(1155*a^4*A - 1485*a^2*A*b^2 - 20*A*b^4 - 2541*a^3*b*B + 55*a*b^3*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]/(3465*a^2*d*\text{Tan}[c+d*x]^{(3/2)}) - (2*(8085*a^4*A*b - 495*a^2*A*b^3 + 40*A*b^5 + 3465*a^5*B - 5313*a^3*b^2*B - 110*a*b^4*B)*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]/(3465*a^3*d*\text{Sqrt}[\text{Tan}[c+d*x]]) - (2*a*A*(a+b*\text{Tan}[c+d*x])^{(3/2)})/(11*d*\text{Tan}[c+d*x]^{(11/2)})$

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

#### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])
```

#### Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

#### Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x]
```



$\text{an}[e + f*x]^n*(1 + I*\text{Tan}[e + f*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3726

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \rightarrow \text{Simp}[(A*d^2 + c*(c*C - B*d))*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(d*f*(n + 1)*(c^2 + d^2)), x] - \text{Dist}[1/(d*(n + 1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*\text{Tan}[e + f*x] - b*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*\text{Tan}[e + f*x]^2, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

### Rule 3730

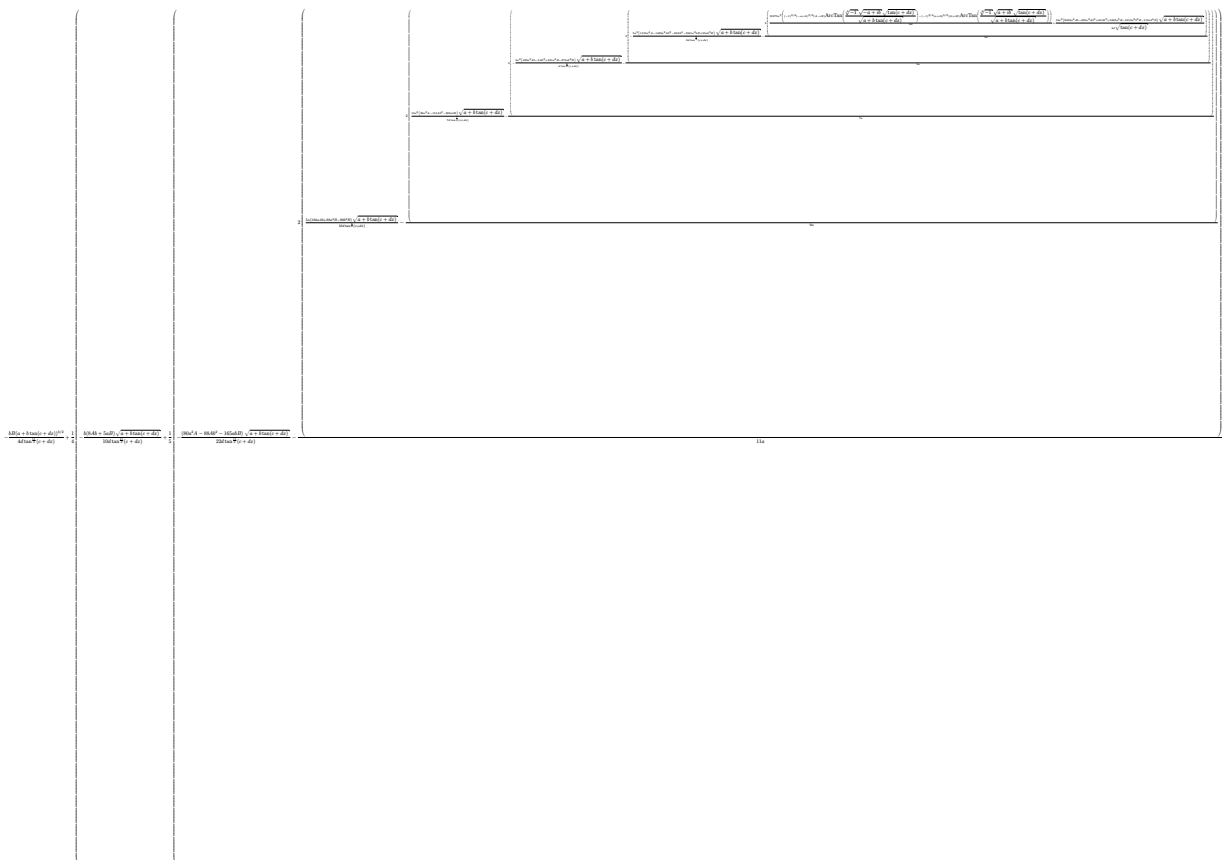
$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \rightarrow \text{Simp}[(A*b^2 - a*(b*B - a*C))*(a + b*\text{Tan}[e + f*x])^{(m + 1)}*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*\text{Tan}[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*\text{Tan}[e + f*x]^2, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{\tan^{\frac{13}{2}}(c + dx)} dx &= -\frac{2aA(a + b \tan(c + dx))^{3/2}}{11d \tan^{\frac{11}{2}}(c + dx)} + \frac{2}{11} \int \frac{\sqrt{a + b \tan(c + dx)}}{\tan^{\frac{11}{2}}(c + dx)} dx \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} - \frac{2aA(a + b \tan(c + dx))^{3/2}}{11d \tan^{\frac{11}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{2a(14Ab + 11aB) \sqrt{a + b \tan(c + dx)}}{99d \tan^{\frac{9}{2}}(c + dx)} + \frac{2(99a^2A - 11aB)}{99d \tan^{\frac{9}{2}}(c + dx)} \\
&= -\frac{(ia - b)^{5/2} (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 6.71, size = 632, normalized size = 1.37



Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(13/2),x]

[Out] 
$$-1/4*(b*B*(a + b*\text{Tan}[c + d*x])^{3/2})/(d*\text{Tan}[c + d*x]^{11/2}) + (-1/10*(b*(8*A*b + 5*a*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(d*\text{Tan}[c + d*x]^{11/2}) + (-1/22*((80*a^2*A - 88*A*b^2 - 165*a*b*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(d*\text{Tan}[c + d*x]^{11/2}) - (2*((5*a*(184*a*A*b + 88*a^2*B - 99*b^2*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(18*d*\text{Tan}[c + d*x]^{9/2}) - (2*((10*a^2*(99*a^2*A - 113*A*b^2 - 209*a*b*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(7*d*\text{Tan}[c + d*x]^{7/2}) - (2*((-3*a^2*(495*a^2*A*b - 5*A*b^3 + 231*a^3*B - 275*a*b^2*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(d*\text{Tan}[c + d*x]^{5/2}) - (2*((-5*a^2*(1155*a^4*A - 1485*a^2*A*b^2 - 20*A*b^4 - 2541*a^3*b*B + 55*a*b^3*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(2*d*\text{Tan}[c + d*x]^{3/2})) - (2*((51975*a^5*((-1)^{3/4})*(-a + I*b)^{5/2}*(A - I*B)*\text{ArcTan}[((-1)^{1/4})*\text{Sqrt}[-a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]) - (-1)^{3/4}*(a + I*b)^{5/2}*(A + I*B)*\text{ArcTan}[((-1)^{1/4})*\text{Sqrt}[a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])))/(8*d) + (15*a^2*(8085*a^4*A*b - 495*a^2*A*b^3 + 40*A*b^5 + 3465*a^5*B - 5313*a^3*b^2*B - 110*a*b^4*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(4*d*\text{Sqrt}[\text{Tan}[c + d*x]])))/(3*a))/(5*a))/(7*a))/(9*a))/(11*a))/5)/4$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.67, size = 2660696, normalized size = 5784.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\tan(dx+c))^{5/2}*(A+B*\tan(dx+c))/\tan(dx+c)^{13/2}, x)$

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\tan(dx+c))^{5/2}*(A+B*\tan(dx+c))/\tan(dx+c)^{13/2}, x, \text{algorithm}="maxima")$

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\tan(dx+c))^{5/2}*(A+B*\tan(dx+c))/\tan(dx+c)^{13/2}, x, \text{algorithm}="fricas")$

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\tan(dx+c))^{5/2}*(A+B*\tan(dx+c))/\tan(dx+c)^{13/2}, x)$

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\tan(c + dx)^{13/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(13/2),x
)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(13/2),
x)
```

$$3.451 \quad \int \frac{(a+b \tan(c+dx))^{5/2} \left( \frac{3bB}{2a} + B \tan(c+dx) \right)}{\tan^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=253

$$\frac{(ia-b)^{5/2}(2a-3ib)B \operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{2ad} + \frac{2b^{5/2}B \operatorname{tanh}^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d} - \frac{(2a+b \tan(c+dx))^{5/2}}{d \tan(c+dx)}$$

[Out]  $1/2*(I*a-b)^{(5/2)*(2*a-3*I*b)*B*\arctan((I*a-b)^{(1/2)*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c)^{(1/2)})/a/d+2*b^{(5/2)*B*\operatorname{arctanh}(b^{(1/2)*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c)^{(1/2)})/d-1/2*(2*a+3*I*b)*(I*a+b)^{(5/2)*B*\operatorname{arctanh}((I*a+b)^{(1/2)*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c)^{(1/2)})/a/d-2*(a^2+3*b^2)*B*(a+b*\tan(d*x+c)^{(1/2)/d/\tan(d*x+c)^{(1/2)-b*B*(a+b*\tan(d*x+c))^{(3/2)/d/\tan(d*x+c)^{(3/2)}$

Rubi [A]

time = 1.70, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$ , Rules used = {3686, 3726, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{2B(a^2+3b^2)\sqrt{a+b \tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{B(2a-3ib)(-b+ia)^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{2ad} + \frac{2b^{5/2}B \operatorname{tanh}^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d} - \frac{bB(a+b \tan(c+dx))^{3/2}}{d \tan^2(c+dx)} - \frac{B(2a+3ib)(b+ia)^{5/2} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{2ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+b*\operatorname{Tan}[c+d*x])^{5/2}*((3*b*B)/(2*a)+B*\operatorname{Tan}[c+d*x])/ \operatorname{Tan}[c+d*x]^{5/2}, x]$

[Out]  $((I*a-b)^{(5/2)*(2*a-(3*I)*b)*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a-b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]])/(2*a*d)+(2*b^{(5/2)*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]])/d-((2*a+(3*I)*b)*(I*a+b)^{(5/2)*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a+b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]])/(2*a*d)-(2*(a^2+3*b^2)*B*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]/(d*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])-(b*B*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)})/(d*\operatorname{Tan}[c+d*x]^{(3/2)})$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^{n,x}], x, (a+b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 95

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}/((e_.) + (f_.)*(x_.)), x\_Symbol] :> \operatorname{With}\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)*(c-a*(d/b)+d*(x^q/b))^{n,x}], x, (e+f*x)^{(1/q)}, x]]$

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e

```

+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{5/2} \left(\frac{3bB}{2a} + B \tan(c + dx)\right)}{\tan^{5/2}(c + dx)} dx &= -\frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} + \frac{2}{3} \int \frac{\sqrt{a + b \tan(c + dx)}}{\tan^{3/2}(c + dx)} dx \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= -\frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\tan(c + dx)}} - \frac{bB(a + b \tan(c + dx))^{3/2}}{d \tan^{3/2}(c + dx)} \\
&= \frac{2b^{5/2} B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} - \frac{2(a^2 + 3b^2) B \sqrt{a + b \tan(c + dx)}}{d} \\
&= \frac{(ia - b)^{5/2} (2a - 3ib) B \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{2ad}
\end{aligned}$$

**Mathematica [A]**

time = 2.97, size = 356, normalized size = 1.41

$$\frac{B \cos(c + dx) (3b + 2a \tan(c + dx)) \left(4\sqrt{a} b^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \tan^2(c + dx) \sqrt{a + b \tan(c + dx)} - \sqrt{1 + \frac{b \tan(c + dx)}{a}} \left(\sqrt{-1} (-a + ib)^{5/2} (2a + 3ib) \operatorname{ArcTan}\left(\frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \tan^2(c + dx) + \sqrt{-1} (a + ib)^{5/2} (2a - 3ib) \operatorname{ArcTan}\left(\frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \tan^2(c + dx) + 2a \sqrt{a + b \tan(c + dx)} (ab + (2a^2 + 3b^2) \tan(c + dx))\right)}{2ad(3b \cos(c + dx) + 2a \sin(c + dx)) \tan^2(c + dx) \sqrt{1 + \frac{b \tan(c + dx)}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(5/2)\*((3\*b\*B)/(2\*a) + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(5/2), x]

```
[Out] (B*Cos[c + d*x]*(3*b + 2*a*Tan[c + d*x])*(4*Sqrt[a]*b^(5/2)*ArcSinh[(Sqrt[b]
]*Sqrt[Tan[c + d*x]])/Sqrt[a])*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]
- Sqrt[1 + (b*Tan[c + d*x])/a]*((-1)^(1/4)*(-a + I*b)^(5/2)*(2*a + (3*I)*b)
*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]])*Tan[c + d*x]^(3/2) + (-1)^(1/4)*(a + I*b)^(5/2)*(2*a - (3*I)*b)*ArcTa
n[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])*T
an[c + d*x]^(3/2) + 2*a*Sqrt[a + b*Tan[c + d*x]]*(a*b + (2*a^2 + 7*b^2)*Tan
[c + d*x])))/(2*a*d*(3*b*Cos[c + d*x] + 2*a*Sin[c + d*x])*Tan[c + d*x]^(3/
2)*Sqrt[1 + (b*Tan[c + d*x])/a])
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.45, size = 1491744, normalized size = 5896.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2), x)
```

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),
x, algorithm="maxima")
```

```
[Out] 1/2*integrate((2*B*tan(d*x + c) + 3*B*b/a)*(b*tan(d*x + c) + a)^(5/2)/tan(d
*x + c)^(5/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),
x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \left( \int \frac{2a^3 \sqrt{a + b \tan(c + dx)}}{\tan^2(c + dx)} dx + \int \frac{3b^3 \sqrt{a + b \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx + \int \frac{6ab^2 \sqrt{a + b \tan(c + dx)}}{\tan^2(c + dx)} dx + \int 2ab^2 \sqrt{a + b \tan(c + dx)} \sqrt{\tan(c + dx)} dx + \int \frac{3a^2 b \sqrt{a + b \tan(c + dx)}}{\tan^2(c + dx)} dx + \int \frac{4a^2 b \sqrt{a + b \tan(c + dx)}}{\sqrt{\tan(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

```
[Out] B*(Integral(2*a**3*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x) + Integral(3*b**3*sqrt(a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x) + Integral(6*a*b**2*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x) + Integral(2*a*b**2*sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x)), x) + Integral(3*a**2*b*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(5/2), x) + Integral(4*a**2*b*sqrt(a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x))/(2*a)
```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B \tan(c + dx) + \frac{3Bb}{2a}\right) (a + b \tan(c + dx))^{5/2}}{\tan(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(5/2),x)
```

```
[Out] int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(5/2), x)
```

$$3.452 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=206

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b}d} + \frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{b^{3/2}d} - \frac{(A-iB)\text{tan}^{-1}\left(\frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{ia-b}}\right)}{b^{3/2}d}$$

[Out]  $(2A*b-B*a)*\text{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/b^{(3/2)}/d - (A+I*B)*\text{arctan}((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d / (I*a-b)^{(1/2)} - (A-I*B)*\text{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d / (I*a+b)^{(1/2)} + B*\tan(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/b/d$

Rubi [A]

time = 0.97, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3688, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{b^{3/2}d} - \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}} + \frac{B\sqrt{\tan(c+dx)}\sqrt{a+b \tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In] `Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]], x]`

[Out] `-(((A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])]/(Sqrt[I*a - b]*d)) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])]/(b^(3/2)*d) - ((A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])]/(Sqrt[I*a + b]*d) + (B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(b*d)`

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 95

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]`

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

#### Rule 3736

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

## Rule 6857

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionE  
xpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ  
[n, 0]

## Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} + \frac{\int \frac{-\frac{aB}{2}-bB \tan(c+dx)+\frac{1}{2}(2Ab)}{\sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} dx}{b} \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} + \frac{\text{Subst}\left(\int \frac{-\frac{aB}{2}-bBx+\frac{1}{2}(2Ab)}{\sqrt{x} \sqrt{a+bx}} dx\right)}{b} \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} + \frac{\text{Subst}\left(\int \left(\frac{2Ab-aB}{2\sqrt{x} \sqrt{a+bx}}\right) dx\right)}{b} \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} - \frac{\text{Subst}\left(\int \frac{Ab+bBx}{\sqrt{x} \sqrt{a+bx}} dx\right)}{b} \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} - \frac{\text{Subst}\left(\int \left(\frac{iAb-bB}{2(i-x)\sqrt{x} \sqrt{a+bx}}\right) dx\right)}{b} \\
 &= \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} - \frac{(iA-B)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}} dx\right)}{b} \\
 &= \frac{(2Ab-aB) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{b^{3/2}d} + \frac{B \sqrt{\tan(c+dx)}}{b} \\
 &= -\frac{(A+iB) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b}d} + \frac{(2Ab-aB) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{b^{3/2}d}
 \end{aligned}$$

## Mathematica [A]

time = 1.24, size = 245, normalized size = 1.19

$$\frac{\sqrt{-1} b^{(A+B)} \text{ArcTan}\left(\frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{(-1)^{3/4} b^{(A+B)} \text{ArcTan}\left(\frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{a+ib}} + \frac{B \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}{bd} + \frac{\sqrt{a} (2Ab-aB) \sinh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{1+\frac{b \tan(c+dx)}{a}}}{\sqrt{b} \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] (((-1)^(1/4)\*b\*(I\*A + B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[-a + I\*b] + ((-1)^(3/4)\*b\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[a + I\*b] + B\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]] + (Sqrt[a]\*(2\*A\*b - a\*B)\*ArcSinh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]]\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(Sqrt[b]\*Sqrt[a + b\*Tan[c + d\*x]])/(b\*d)

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 1.01, size = 1890398, normalized size = 9176.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^(3/2)/sqrt(b\*tan(d\*x + c) + a),x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/sqrt(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{3/2} (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2), x)



$$3.453 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=168

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b} d} - \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a-b)^(1/2)+2\*B\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/b^(1/2)-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a+b)^(1/2)

**Rubi [A]**

time = 0.38, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3695, 3697, 3696, 95, 209, 212, 3715, 65, 223}

$$\frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) + (2\*B\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[b]\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d)

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3695

```
Int[(Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)])*((A_) + (B_)*tan[(e_) + (f_)*(x_)])]/Sqrt[(c_) + (d_)*tan[(e_) + (f_)*(x_)], x_Symbol] := Int[Simp[a*A - b*B + (A*b + a*B)*Tan[e + f*x], x]/(Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]), x] + Dist[b*B, Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\tan(c+dx)} (A + B \tan(c+dx))}{\sqrt{a + b \tan(c+dx)}} dx &= B \int \frac{1 + \tan^2(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a + b \tan(c+dx)}} dx + \int \frac{-B}{\sqrt{\tan(c+dx)}} dx \\ &= \frac{1}{2}(-iA - B) \int \frac{1 + i \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a + b \tan(c+dx)}} dx + \frac{1}{2}(iA - B) \int \frac{1 - i \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a + b \tan(c+dx)}} dx \\ &= \frac{(iA - B) \operatorname{Subst}\left(\int \frac{1}{(1+ix)\sqrt{x} \sqrt{a+bx}} dx, x, \tan(c+dx)\right)}{2d} + \frac{(iA - B) \operatorname{Subst}\left(\int \frac{1}{1-(-ia+bx)x^2} dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{2d} \\ &= \frac{(iA - B) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.85, size = 205, normalized size = 1.22

$$\frac{\sqrt{-1} \left( -\frac{{}_{(A-iB)}\operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{{}_{(A+iB)}\operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{a+ib}} \right) + \frac{2\sqrt{a} B \operatorname{sinh}^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right) \sqrt{1+\frac{b \tan(c+dx)}{a}}}{\sqrt{b}\sqrt{a+b \tan(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]]
, x]
```

```
[Out] ((-1)^(1/4)*(-(((A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + ((A + I*B)*ArcTan[((-1)^(
1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a +
I*b]) + (2*Sqrt[a]*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 +
(b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]))/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.59, size = 1886894, normalized size = 11231.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/sqrt(b*tan(d*x + c) + a), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/sqrt(a + b*tan(c + d*x)), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorith="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(tan(d\*x + c))/sqrt(b\*tan(d\*x + c) + a), x)

Mupad [B]

time = 91.16, size = 2500, normalized size = 14.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] atan((((B^2 - A^2 + A\*B\*2i)/(4\*(a\*d^2\*i + b\*d^2)))^(1/2)\*(((B^2 - A^2 + A\*B\*2i)/(4\*(a\*d^2\*i + b\*d^2)))^(1/2)\*(((B^2 - A^2 + A\*B\*2i)/(4\*(a\*d^2\*i + b\*d^2)))^(1/2)\*(((274877906944\*(1600\*a^12\*b^34\*d^8 - 16640\*a^14\*b^32\*d^8 + 22784\*a^16\*b^30\*d^8 + 106496\*a^18\*b^28\*d^8 + 65536\*a^20\*b^26\*d^8))/d^8 - (274877906944\*tan(c + d\*x)\*(1600\*a^12\*b^35\*d^8 - 48000\*a^14\*b^33\*d^8 + 155136\*a^16\*b^31\*d^8 + 466944\*a^18\*b^29\*d^8 + 262144\*a^20\*b^27\*d^8))/(d^8\*((a + b\*tan(c + d\*x))^(1/2) - a^(1/2))^2))\*((B^2 - A^2 + A\*B\*2i)/(4\*(a\*d^2\*i + b\*d^2)))^(1/2) - (219902325552\*tan(c + d\*x)^(1/2)\*(240\*A\*a^13\*b^33\*d^6 + 3064\*A\*a^15\*b^31\*d^6 + 8960\*A\*a^17\*b^29\*d^6 + 6144\*A\*a^19\*b^27\*d^6 + 6920\*B\*a^14\*b^32\*d^6 + 9472\*B\*a^16\*b^30\*d^6 - 5632\*B\*a^18\*b^28\*d^6 - 8192\*B\*a^20\*b^26\*d^6))/(d^7\*((a + b\*tan(c + d\*x))^(1/2) - a^(1/2))))\*((B^2 - A^2 + A\*B\*2i)/(4\*(a\*d^2\*i + b\*d^2)))^(1/2) - (274877906944\*(19216\*A^2\*a^14\*b^31\*d^6 - 1440\*A^2\*a^12\*b^33\*d^6 - 22016\*A^2\*a^16\*b^29\*d^6 - 45056\*A^2\*a^18\*b^27\*d^6 + 1200\*B^2\*a^12\*b^33\*d^6 - 16640\*B^2\*a^14\*b^31\*d^6 + 279040\*B^2\*a^16\*b^29\*d^6 + 561152\*B^2\*a^18\*b^27\*d^6 + 262144\*B^2\*a^20\*b^25\*d^6 + 16480\*A\*B\*a^13\*b^32\*d^6 - 25792\*A\*B\*a^15\*b^30\*d^6 + 34816\*A\*B\*a^17\*b^28\*d^6 + 81920\*A\*B\*a^19\*b^26\*d^6))/d^8 + (274877906944\*tan(c + d\*x)\*(46704\*A^2\*a^14\*b^32\*d^6 - 1440\*A^2\*a^12\*b^34\*d^6 - 137216\*A^2\*a^16\*b^30\*d^6 - 122880\*A^2\*a^18\*b^28\*d^6 + 65536\*A^2\*a^20\*b^26\*d^6 + 1200\*B^2\*a^12\*b^34\*d^6 - 52320\*B^2\*a^14\*b^32\*d^6 + 1200640\*B^2\*a^16\*b^30\*d^6 + 2306048\*B^2\*a^18\*b^28\*d^6 + 1048576\*B^2\*a^20\*b^26\*d^6 + 21600\*A\*B\*a^13\*b^33\*d^6 - 121856\*A\*B\*a^15\*b^31\*d^6 + 276480\*A\*B\*a^17\*b^29\*d^6 + 425984\*A\*B\*a^19\*b^27\*d^6))/d^8\*((a + b\*tan(c + d\*x))^(1/2) - a^(1/2))^2) - (219902325552\*tan(c + d\*x)^(1/2)\*(168\*A^3\*a^13\*b^32\*d^4 + 1200\*A^3\*a^15\*b^30\*d^4 + 2816\*A^3\*a^17\*b^28\*d^4 - 4770\*B^3\*a^14\*b^31\*d^4 + 17920\*B^3\*a^16\*b^29\*d^4 + 57344\*B^3\*a^18\*b^27\*d^4 + 32768\*B^3\*a^20\*b^25\*d^4 + 180\*A\*B^2\*a^13\*b^32\*d^4 - 26736\*A\*B^2\*a^15\*b^30\*d^4 - 48896\*A\*B^2\*a^17\*b^28\*d^4 - 16384\*A\*B^2\*a^19\*b^26\*d^4 + 4746\*A^2\*B\*a^14\*b^31\*d^4 - 2048\*A^2\*B\*a^16\*b^29\*d^4 - 12288\*A^2\*B\*a^18\*b^27\*d^4))/d^7\*((a + b\*tan(c + d\*x))^(1/2) - a^(1/2))))\*((B^2 - A^2 + A\*B\*2i)/(4\*(a\*d^2\*i + b\*d^2)))^(1/2) + (274877906944\*(484\*A^4\*a^12\*b^32\*d^4 - 6448\*A^4\*a^14\*b^30\*d^4 + 8704\*A^4\*a^16\*b^28\*d^4 + 4096\*A^4\*a^18\*b^26\*d^4 + 300\*B^4\*a^12\*b^32\*d^4 - 5040\*B^4\*a^14\*b^30\*d^4 + 217600\*B^4\*a^16\*b^28\*d^4 + 217088\*B^4\*a^18\*b^26\*d^4 + 7720\*A\*



$$3.454 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=123

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} + \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a-b)^(1/2)+(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a+b)^(1/2)

**Rubi [A]**

time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3697, 3696, 95, 209, 212}

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]),x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]/(Sqrt[I\*a - b]\*d) + ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]/(Sqrt[I\*a + b]\*d))

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Q[a, 0] || LtQ[b, 0])

### Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx &= \frac{1}{2}(A - iB) \int \frac{1 + i \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx + \frac{1}{2}(A + iB) \int \frac{1 - i \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\ &= \frac{(A - iB) \text{Subst}\left(\int \frac{1}{(1-ix)\sqrt{x}\sqrt{a+bx}} dx, x, \tan(c + dx)\right)}{2d} + \frac{(A + iB) \text{Subst}\left(\int \frac{1}{(1+ix)\sqrt{x}\sqrt{a+bx}} dx, x, \tan(c + dx)\right)}{2d} \\ &= \frac{(A - iB) \text{Subst}\left(\int \frac{1}{1-(ia+b)x^2} dx, x, \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(A + iB) \text{Subst}\left(\int \frac{1}{1+(ia+b)x^2} dx, x, \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} \\ &= \frac{(A + iB) \tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia-b}d} + \frac{(A - iB) \tanh^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia-b}d} \end{aligned}$$

### Mathematica [A]

time = 0.15, size = 137, normalized size = 1.11

$$\frac{\sqrt[4]{-1} \left( -\frac{{}^{(iA+B)}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{{}^{(-iA+B)}\text{ArcTan}\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} \right)}{d}$$

Antiderivative was successfully verified.



[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]),x]

[Out]  $((-1)^{1/4} * (-(((I*A + B) * \text{ArcTan}[\frac{(-1)^{1/4} * \sqrt{-a + I*b} * \sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + b*\text{Tan}[c + d*x]}}]) / \sqrt{-a + I*b})) + (((-I)*A + B) * \text{ArcTan}[\frac{(-1)^{1/4} * \sqrt{a + I*b} * \sqrt{\text{Tan}[c + d*x]}}{\sqrt{a + b*\text{Tan}[c + d*x]}}]) / \sqrt{a + I*b})) / d$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.62, size = 1879756, normalized size = 15282.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)/(sqrt(b\*tan(d\*x + c) + a)\*sqrt(tan(d\*x + c))), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)} \sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& 1/2) + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(1/2)}*40704i)/((a + b*\tan(c \\
& + d*x))^{(1/2)} - a^{(1/2)}) + (B^5*a^6*b^3*d^7*\tan(c + d*x)^{(1/2)}*(-((-16*B^4* \\
& a^2*d^4)^{(1/2)} + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(1/2)}*38144i)/((a \\
& + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B^3*a^2*b^8*d^9*\tan(c + d*x)^{(1/2)}*(- \\
& ((-16*B^4*a^2*d^4)^{(1/2)} + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(3/2)}*12 \\
& 9024i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B^3*a^4*b^6*d^9*\tan(c + d* \\
& x)^{(1/2)}*(-((-16*B^4*a^2*d^4)^{(1/2)} + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4 \\
& ))^{(3/2)}*325632i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B^3*a^6*b^4*d^9 \\
& *\tan(c + d*x)^{(1/2)}*(-((-16*B^4*a^2*d^4)^{(1/2)} + 4*B^2*b*d^2)/(16*a^2*d^4 + \\
& 16*b^2*d^4))^{(3/2)}*305152i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B^3* \\
& a^8*b^2*d^9*\tan(c + d*x)^{(1/2)}*(-((-16*B^4*a^2*d^4)^{(1/2)} + 4*B^2*b*d^2)/(1 \\
& 6*a^2*d^4 + 16*b^2*d^4))^{(3/2)}*98304i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2 \\
& )))/(64*a^5*(-16*B^4*a^2*d^4)^{(3/2)} + 49*a*b^4*(-16*B^4*a^2*d^4)^{(3/2)} + 11 \\
& 2*a^3*b^2*(-16*B^4*a^2*d^4)^{(3/2)} - 192*B^6*a^3*b^5*d^6 + 64*B^6*a^5*b^3*d^ \\
& 6 + 1024*B^4*a^7*d^4*(-16*B^4*a^2*d^4)^{(1/2)} - (119*a*b^5*\tan(c + d*x)*(-16 \\
& *B^4*a^2*d^4)^{(3/2)})/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2 - (128*a^5*b* \\
& \tan(c + d*x)*(-16*B^4*a^2*d^4)^{(3/2)})/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2 \\
& )})^2 + 736*B^4*a^3*b^4*d^4*(-16*B^4*a^2*d^4)^{(1/2)} + 1792*B^4*a^5*b^2*d^4*(- \\
& 16*B^4*a^2*d^4)^{(1/2)} - (248*a^3*b^3*\tan(c + d*x)*(-16*B^4*a^2*d^4)^{(3/2)})/ \\
& ((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2 + 16*B^4*a*b^6*d^4*(-16*B^4*a^2*d^ \\
& 4)^{(1/2)} - (192*B^6*a^3*b^6*d^6*\tan(c + d*x))/((a + b*\tan(c + d*x))^{(1/2)} - \\
& a^{(1/2)})^2 + (64*B^6*a^5*b^4*d^6*\tan(c + d*x))/((a + b*\tan(c + d*x))^{(1/2)} \\
& - a^{(1/2)})^2 + (16*B^4*a*b^7*d^4*\tan(c + d*x)*(-16*B^4*a^2*d^4)^{(1/2)})/((a \\
& + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2 - (2048*B^4*a^7*b*d^4*\tan(c + d*x)*(- \\
& 16*B^4*a^2*d^4)^{(1/2)})/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2 - (1952*B^4 \\
& *a^3*b^5*d^4*\tan(c + d*x)*(-16*B^4*a^2*d^4)^{(1/2)})/((a + b*\tan(c + d*x))^{(1 \\
& /2)} - a^{(1/2)})^2 - (3968*B^4*a^5*b^3*d^4*\tan(c + d*x)*(-16*B^4*a^2*d^4)^{(1/ \\
& 2)})/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)})^2))*(-((-16*B^4*a^2*d^4)^{(1/2)} + \\
& 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(1/2)}*2i - \operatorname{atan}(((B^5*b^9*d^7*\tan( \\
& c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4)^{(1/2)} - 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^ \\
& 2*d^4))^{(1/2)}*1280i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B^3*b^10*d^9 \\
& *\tan(c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4)^{(1/2)} - 4*B^2*b*d^2)/(16*a^2*d^4 + \\
& 16*b^2*d^4))^{(3/2)}*10240i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B*b^11 \\
& *d^11*\tan(c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4)^{(1/2)} - 4*B^2*b*d^2)/(16*a^2*d \\
& ^4 + 16*b^2*d^4))^{(5/2)}*20480i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/2)}) + (B \\
& *a^10*b*d^11*\tan(c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4)^{(1/2)} - 4*B^2*b*d^2)/(1 \\
& 6*a^2*d^4 + 16*b^2*d^4))^{(5/2)}*196608i)/((a + b*\tan(c + d*x))^{(1/2)} - a^{(1/ \\
& 2)}) + (B^5*a^8*b*d^7*\tan(c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4)^{(1/2)} - 4*B^2*b \\
& *d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(1/2)}*12288i)/((a + b*\tan(c + d*x))^{(1/2)} \\
& - a^{(1/2)}) + (B*a^2*b^9*d^11*\tan(c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4)^{(1/2)} - \\
& 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(5/2)}*274432i)/((a + b*\tan(c + d*x \\
& ))^{(1/2)} - a^{(1/2)}) + (B*a^4*b^7*d^11*\tan(c + d*x)^{(1/2)}*(((-16*B^4*a^2*d^4 \\
& )^{(1/2)} - 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^{(5/2)}*897024i)/((a + b*ta \\
& n(c + d*x))^{(1/2)} - a^{(1/2)}) + (B*a^6*b^5*d^11*\tan(c + d*x)^{(1/2)}*(((-16*B^ \\
& 4*a^2*d^4)^{(1/2)} - 4*B^2*b*d^2)/(16*a^2*d^4 + 1...
\end{aligned}$$

$$3.455 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=159

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia - b} d} + \frac{(iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia + b} d} - \frac{2A \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a-b)^{(1/2)}+(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a+b)^{(1/2)}-2*A*(a+b*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(1/2)}$

Rubi [A]

time = 0.33, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {3690, 3697, 3696, 95, 209, 212}

$$\frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d\sqrt{-b + ia}} + \frac{(B + iA) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d\sqrt{b + ia}} - \frac{2A \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])]/(\operatorname{Tan}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]), x]$

[Out]  $-\left(\frac{(I*A - B)*\operatorname{ArcTan}[\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]}{\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}\right)/(\operatorname{Sqrt}[I*a - b]*d) + \left(\frac{(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]]}{\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]}\right)/(\operatorname{Sqrt}[I*a + b]*d) - \frac{(2*A*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])}{(a*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])}$

Rule 95

$\operatorname{Int}[((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{With}[q = \operatorname{Denominator}[m], \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \operatorname{FreeQ}[a, b, c, d, e, f, x] \&\& \operatorname{EqQ}[m + n + 1, 0] \&\& \operatorname{RationalQ}[n] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{SimplerQ}[a + b*x, c + d*x]$

Rule 209

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[a, b, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{GtQ}[b, 0])]$

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{2 \int \frac{-\frac{aB}{2} + \frac{1}{2} a A \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}}}{a} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{1}{2}(-iA - B) \int \frac{1 + i \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{(iA - B) \text{Subst} \left( \int \frac{1}{(1+ix) \sqrt{x} \sqrt{a + bx}} \right)}{2d} \\
&= -\frac{2A \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{(iA - B) \text{Subst} \left( \int \frac{1}{1 - (-ia+b)x^2} dx, x \right)}{d} \\
&= -\frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d} + \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{ia - b}} \right)}{\sqrt{ia - b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.32, size = 172, normalized size = 1.08

$$\frac{\sqrt[4]{-1} (A - iB) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{-a + ib}} - \frac{\sqrt[4]{-1} (A + iB) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{a + ib}} - \frac{2A \sqrt{a + b \tan(c + dx)}}{a \sqrt{\tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

```
[Out] (((-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[-a + I*b] - ((-1)^(1/4)*(A + I*B)*ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[a + I*b] - (2*A*Sqrt[a + b*Tan[c + d*x]]/(a*Sqrt[Tan[c + d*x]]))/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.63, size = 1887172, normalized size = 11869.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2)), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)} \tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2)), x)
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{3/2} \sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)), x  
)
```



$$3.456 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=203

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} - \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d} - \frac{2A \sqrt{a+b \tan(c+dx)}}{3ad \tan(c+dx)}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a-b)^{(1/2)}-(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a+b)^{(1/2)}+2/3*(2*A*b-3*B*a)*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d/\tan(d*x+c)^{(1/2)}-2/3*A*(a+b*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.48, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(2Ab-3aB)\sqrt{a+b \tan(c+dx)}}{3a^2 d \sqrt{\tan(c+dx)}} - \frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{-b+ia}} - \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{b+ia}} - \frac{2A \sqrt{a+b \tan(c+dx)}}{3ad \tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(\operatorname{Tan}[c+d*x]^{(5/2)}*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]),x]$

[Out]  $-(((A+I*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a-b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(\operatorname{Sqrt}[I*a-b]*d))-((A-I*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a+b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(\operatorname{Sqrt}[I*a+b]*d)-(2*A*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(3*a*d*\operatorname{Tan}[c+d*x]^{(3/2)})+(2*(2*A*b-3*a*B)*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/(3*a^2*d*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])$

**Rule 95**

$\operatorname{Int}[(((a_.)+(b_.)*(x_.))^{(m_.)*((c_.)+(d_.)*(x_.))^{(n_.)})/((e_.)+(f_.)*(x_.)), x\_Symbol] := \operatorname{With}\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}/(b*e-a*f-(d*e-c*f)*x^q), x], x, (a+b*x)^{(1/q)}/(c+d*x)^{(1/q)}], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[m+n+1, 0] \ \&\& \operatorname{RationalQ}[n] \ \&\& \operatorname{LtQ}[-1, m, 0] \ \&\& \operatorname{SimplerQ}[a+b*x, c+d*x]$

**Rule 209**

$\operatorname{Int}[((a_.)+(b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

**Rule 212**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[

$b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& !$   
 $(\text{ILtQ}[n, -1] \&\& ( !\text{IntegerQ}[m] \mid\mid (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx = -\frac{2A \sqrt{a + b \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} - \frac{2 \int \frac{\frac{1}{2}(2Ab - 3aB) + \frac{3}{2}aA \tan(c + dx) + Ab \tan^2(c + dx)}{\tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx}{3a}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(2Ab - 3aB) \sqrt{a + b \tan(c + dx)}}{3a^2 d \sqrt{\tan(c + dx)}}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(2Ab - 3aB) \sqrt{a + b \tan(c + dx)}}{3a^2 d \sqrt{\tan(c + dx)}}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(2Ab - 3aB) \sqrt{a + b \tan(c + dx)}}{3a^2 d \sqrt{\tan(c + dx)}}$$

$$= -\frac{2A \sqrt{a + b \tan(c + dx)}}{3ad \tan^{\frac{3}{2}}(c + dx)} + \frac{2(2Ab - 3aB) \sqrt{a + b \tan(c + dx)}}{3a^2 d \sqrt{\tan(c + dx)}}$$

$$= -\frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d}$$

**Mathematica [A]**

time = 1.24, size = 195, normalized size = 0.96

$$\frac{{}_3\sqrt{-1}^{(iA+B)} \text{ArcTan} \left( \frac{{}^4\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{-a + ib}} + \frac{{}_3(-1)^{3/4(A+iB)} \text{ArcTan} \left( \frac{{}^4\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{a + ib}} - \frac{2 \sqrt{a + b \tan(c + dx)} (aA + (-2Ab + 3aB) \tan(c + dx))}{a^2 \tan^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]]), x]

[Out] ((3\*(-1)^(1/4)\*(I\*A + B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[-a + I\*b] + (3\*(-1)^(3/4)\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[a + I\*b] - (2\*Sqrt[a + b\*Tan[c + d\*x]]\*(a\*A + (-2\*A\*b + 3\*a\*B)\*Tan[c + d\*x]))/(a^2\*Tan[c + d\*x]^(3/2))/(3\*d)

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.94, size = 1890767, normalized size = 9314.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(5/2)), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)} \tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2)/tan(d*x+c)**(5/2),x)`

[Out] `Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(5/2)), x)`

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2)/tan(d\*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{5/2} \sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^(1/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^(1/2)), x)

$$3.457 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=256

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia - b} d} - \frac{(iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia + b} d} - \frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a-b)^(1/2)-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a+b)^(1/2)+2/15\*(15\*A\*a^2-8\*A\*b^2+10\*B\*a\*b)\*(a+b\*tan(d\*x+c))^(1/2)/a^3/d/tan(d\*x+c)^(1/2)-2/5\*A\*(a+b\*tan(d\*x+c))^(1/2)/a/d/tan(d\*x+c)^(5/2)+2/15\*(4\*A\*b-5\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d/tan(d\*x+c)^(3/2)

Rubi [A]

time = 0.67, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{5}{2}}(c + dx)} + \frac{2(15a^2 A + 10abB - 8Ab^2) \sqrt{a + b \tan(c + dx)}}{15a^3 d \sqrt{\tan(c + dx)}} + \frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d \sqrt{-b + ia}} - \frac{(B + iA) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d \sqrt{b + ia}} - \frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(7/2)\*Sqrt[a + b\*Tan[c + d\*x]]), x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d) - (2\*A\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*a\*d\*Tan[c + d\*x]^(5/2)) + (2\*(4\*A\*b - 5\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a^2\*d\*Tan[c + d\*x]^(3/2)) + (2\*(15\*a^2\*A - 8\*A\*b^2 + 10\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a^3\*d\*Sqrt[Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(LtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)

\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan  
 [e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[  
 b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !  
 (ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} - \frac{2 \int \frac{\frac{1}{2}(4Ab - 5aB) + \frac{5}{2}aA \tan(c + dx) + 2Ab \tan^2(c + dx)}{\tan^{\frac{5}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx}{5a} \\
 &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} + \frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} + \frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} + \frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} + \frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} + \frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{2A \sqrt{a + b \tan(c + dx)}}{5ad \tan^{\frac{5}{2}}(c + dx)} + \frac{2(4Ab - 5aB) \sqrt{a + b \tan(c + dx)}}{15a^2 d \tan^{\frac{3}{2}}(c + dx)} \\
 &= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d} - \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{ia - b}} \right)}{5ad}
 \end{aligned}$$

**Mathematica [A]**

time = 3.57, size = 227, normalized size = 0.89

$$\frac{-\frac{{}^{15}\sqrt{-1} (A-B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{{}^{15}\sqrt{-1} (A+B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}}}{15d} + \frac{2 \sqrt{a+b \tan(c+dx)} (-3a^2 A - a(-4Ab+5aB) \tan(c+dx) + (15a^2 A - 8Ab^2 + 10abB) \tan^2(c+dx))}{a^3 \tan^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(7/2)\*Sqrt[a + b\*Tan[c + d\*x]), x]



```
[Out] ((-15*(-1)^(1/4)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[-a + I*b] + (15*(-1)^(1/4)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[a + I*b] + (2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2*A - a*(-4*A*b + 5*a*B)*Tan[c + d*x] + (15*a^2*A - 8*A*b^2 + 10*a*b*B)*Tan[c + d*x]^2))/(a^3*Tan[c + d*x]^(5/2))/(15*d)
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.61, size = 1891860, normalized size = 7390.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(7/2)), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)} \tan^{\frac{7}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2)/tan(d\*x+c)\*\*(7/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/(sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*(7/2)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2)/tan(d\*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{7/2} \sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(7/2)\*(a + b\*tan(c + d\*x))^(1/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(7/2)\*(a + b\*tan(c + d\*x))^(1/2)), x)

$$3.458 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=219

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{3/2}d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{b^{3/2}d} - \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{ia - b}}\right)}{(ia - b)^{3/2}d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a-b)^{(3/2)}/d+2*B*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/b^{(3/2)}/d-(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a+b)^{(3/2)}/d+2*a*(A*b-B*a)*\tan(d*x+c)^{(1/2)}/b/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 1.29, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {3686, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{bd(a^2 + b^2) \sqrt{a + b \tan(c + dx)}} - \frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} - \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{3/2}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{b^{3/2}d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x])]/(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $-(((I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/((I*a - b)^{(3/2)}*d) + (2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(b^{(3/2)}*d) - ((I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/((I*a + b)^{(3/2)}*d) + (2*a*(A*b - a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(b*(a^2 + b^2)*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] := \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}]/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[m + n + 1, 0] \&\& \operatorname{RationalQ}[n]$

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)^2], x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

## Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx &= \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} + \frac{2 \int \frac{-\frac{1}{2}a(Ab-aB)+\frac{1}{2}b(Ab-aB) \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} dx}{b(a^2+b^2)} \\
&= \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} + \frac{2 \text{Subst} \left( \int \frac{-\frac{1}{2}a(Ab-aB)+\frac{1}{2}b(Ab-aB) \tan(c+dx)}{\sqrt{x} \sqrt{a+b \tan(c+dx)}} dx \right)}{b(a^2+b^2)} \\
&= \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} + \frac{2 \text{Subst} \left( \int \left( \frac{(a^2+b^2)B}{2\sqrt{x} \sqrt{a+b \tan(c+dx)}} \right) dx \right)}{b(a^2+b^2)} \\
&= \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} - \frac{\text{Subst} \left( \int \frac{b(aA+bB)-b(Ab-aB)x}{\sqrt{x} \sqrt{a+b \tan(c+dx)} (1+x^2)} dx \right)}{b(a^2+b^2)} \\
&= \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} - \frac{\text{Subst} \left( \int \left( \frac{b(Ab-aB)+ib(aA+bB)}{2(i-x)\sqrt{x} \sqrt{a+b \tan(c+dx)}} \right) dx \right)}{b(a^2+b^2)} \\
&= \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} - \frac{((ia+b)(A+iB)) \text{Subst} \left( \int \frac{1}{\sqrt{x} \sqrt{a+b \tan(c+dx)}} dx \right)}{b(a^2+b^2)} \\
&= \frac{2B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{b^{3/2} d} + \frac{2a(Ab-aB) \sqrt{\tan(c+dx)}}{b(a^2+b^2) d \sqrt{a+b \tan(c+dx)}} \\
&= -\frac{(iA-B) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{(ia-b)^{3/2} d} + \frac{2B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{b^{3/2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 39.01, size = 177751, normalized size = 811.65

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] Result too large to show

**Maple** [B] result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 4.06, size = 1561438, normalized size = 7129.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x)

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^(3/2)/(b\*tan(d\*x + c) + a)^(3/2), x)

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x)

$$3.459 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=170

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{(ia+b)^{3/2}d} - \frac{2(Ab-aB)}{(a^2+b^2)d \sqrt{a+b \tan(c+dx)}}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a-b)^{(3/2)}/d+(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a+b)^{(3/2)}/d-2*(A*b-B*a)*\tan(d*x+c)^{(1/2)}/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)}$

Rubi [A]

time = 0.41, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {3689, 3697, 3696, 95, 209, 212}

$$\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b \tan(c+dx)}} - \frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(-b+ia)^{3/2}} + \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(A+B*\operatorname{Tan}[c+d*x]))/(a+b*\operatorname{Tan}[c+d*x])^{(3/2)},x]$

[Out]  $-\left(\left(\left(A+I*B\right)*\operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}\left[I*a-b\right]*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c+d*x\right]\right]}{\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]}\right)\right)/\left(\left(I*a-b\right)^{(3/2)*d}\right)+\left(\left(A-I*B\right)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}\left[I*a+b\right]*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c+d*x\right]\right]}{\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]}\right)\right)/\left(\left(I*a+b\right)^{(3/2)*d}\right)-\left(2*\left(A*b-a*B\right)*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c+d*x\right]\right]\right)/\left(\left(a^2+b^2\right)*d*\operatorname{Sqrt}\left[a+b*\operatorname{Tan}\left[c+d*x\right]\right]\right)$

Rule 95

$\operatorname{Int}\left[\left(\left(a_{.}\right)+\left(b_{.}\right)*\left(x_{.}\right)^{\left(m_{.}\right)}\right)*\left(\left(c_{.}\right)+\left(d_{.}\right)*\left(x_{.}\right)^{\left(n_{.}\right)}\right)/\left(\left(e_{.}\right)+\left(f_{.}\right)*\left(x_{.}\right)^{\left(q_{.}\right)}\right),x_{\text{Symbol}}\right]:>\operatorname{With}\left[\left\{q=\operatorname{Denominator}\left[m\right]\right\},\operatorname{Dist}\left[q,\operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(q*\left(m+1\right)-1\right)}/\left(b*e-a*f-\left(d*e-c*f\right)*x^q\right),x\right],x,\left(a+b*x\right)^{\left(1/q\right)}/\left(c+d*x\right)^{\left(1/q\right)}\right],x\right] /; \operatorname{FreeQ}\left[\left\{a,b,c,d,e,f\right\},x\right] \&\& \operatorname{EqQ}\left[m+n+1,0\right] \&\& \operatorname{RationalQ}\left[n\right] \&\& \operatorname{LtQ}\left[-1,m,0\right] \&\& \operatorname{SimplerQ}\left[a+b*x,c+d*x\right]$

Rule 209

$\operatorname{Int}\left[\left(\left(a_{.}\right)+\left(b_{.}\right)*\left(x_{.}\right)^2\right)^{-1},x_{\text{Symbol}}\right]:>\operatorname{Simp}\left[\left(1/\left(\operatorname{Rt}\left[a,2\right]*\operatorname{Rt}\left[b,2\right]\right)\right)*\operatorname{ArcTan}\left[\operatorname{Rt}\left[b,2\right]*\left(x/\operatorname{Rt}\left[a,2\right]\right)\right],x\right] /; \operatorname{FreeQ}\left[\left\{a,b\right\},x\right] \&\& \operatorname{PosQ}\left[a/b\right] \&\& \left(\operatorname{GtQ}\left[a,0\right] \mid \mid \operatorname{GtQ}\left[b,0\right]\right)$

Rule 212



```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 3689

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ
[2*m, 2*n])
```

### Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx &= -\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}b(Ab-aB)-\frac{1}{2}b(aA+bB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b(a^2+b^2)} \\
&= -\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{((ia+b)(A+iB))\int \frac{1}{\sqrt{\tan(c+dx)}}}{2(a^2+b^2)} \\
&= -\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{((ia+b)(A+iB))\text{Subst}\left(\frac{1}{\sqrt{\tan(c+dx)}}\right)}{2(a^2+b^2)} \\
&= -\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{((ia+b)(A+iB))\text{Subst}\left(\frac{1}{\sqrt{\tan(c+dx)}}\right)}{2(a^2+b^2)} \\
&= -\frac{(A+iB)\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 1.04, size = 239, normalized size = 1.41

$$-\frac{\sqrt{-1} a(a+ib)(A-iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\sqrt{-1} a(a-ib)(A+iB)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} + \frac{2b(Ab-aB)\tan^2(c+dx)}{\sqrt{a+b\tan(c+dx)}} + \frac{2(-Ab+aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

```
[Out] (-((( -1)^(1/4)*a*(a + I*b)*(A - I*B)*ArcTan[((( -1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + (( -1)^(1/4)*a*(a - I*b)*(A + I*B)*ArcTan[((( -1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (2*b*(A*b - a*B)*Tan[c + d*x]^(3/2))/Sqrt[a + b*Tan[c + d*x]] + 2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(a*(a^2 + b^2)*d)
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 1.11, size = 1561041, normalized size = 9182.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2), x)
```

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(tan(d\*x + c))/(b\*tan(d\*x + c) + a)^(3/2), x)

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\tan(c + dx)}}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x))/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x  
)

$$3.460 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=175

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{3/2}d} + \frac{(iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia + b)^{3/2}d} + \frac{2b(Ab - a)}{a(a^2 + b^2)d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a-b)^(3/2)/d+(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a+b)^(3/2)/d+2\*b\*(A\*b-B\*a)\*tan(d\*x+c)^(1/2)/a/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(1/2)

**Rubi** [A]

time = 0.38, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {3690, 3697, 3696, 95, 209, 212}

$$\frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{ad(a^2 + b^2) \sqrt{a + b \tan(c + dx)}} + \frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} + \frac{(B + iA) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)),x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a - b)^(3/2)\*d) + ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a + b)^(3/2)\*d) + (2\*b\*(A\*b - a\*B)\*Sqrt[Tan[c + d\*x]])/(a\*(a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 212**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} dx &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{2 \int \frac{\frac{1}{2}a(aA + bB) - \frac{1}{2}a(Ab - aB)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx}{a(a^2 + b^2)} \\
&= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \int \frac{1}{\sqrt{\tan(c + dx)}} dx}{2(a^2 + b^2)} \\
&= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \text{Subst} \left( \int \frac{1}{(1 - ix)} dx \right)}{2(a^2 + b^2)} \\
&= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{a(a^2 + b^2) d \sqrt{a + b \tan(c + dx)}} + \frac{(A - iB) \text{Subst} \left( \int \frac{1}{1 - (ia + b)x} dx \right)}{2(a^2 + b^2)} \\
&= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{3/2} d} + \frac{(iA + B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{3/2} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.53, size = 202, normalized size = 1.15

$$\frac{\sqrt[4]{-1}^{(-ia+b)(A-iB)} \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{\sqrt[4]{-1}^{(a-ib)(-iA+B)} \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} + \frac{2b(Ab-aB) \sqrt{\tan(c+dx)}}{a \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)), x]

[Out] (((-1)^(1/4)\*((-I)\*a + b)\*(A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[-a + I\*b] + ((-1)^(1/4)\*(a - I\*b)\*((-I)\*A + B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[a + I\*b] + (2\*b\*(A\*b - a\*B)\*Sqrt[Tan[c + d\*x]])/(a\*Sqrt[a + b\*Tan[c + d\*x]])/(a^2 + b^2)\*d

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 1.06, size = 1561075, normalized size = 8920.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2), x)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^(3/2)\*sqrt(tan(d\*x + c))), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}} \sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*(3/2)\*sqrt(tan(c + d\*x))), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")



[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(3/2)), x  
)

$$3.461 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=216

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d} - \frac{ad\sqrt{\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}} - \frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a-b)^(3/2)/d-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a+b)^(3/2)/d-2\*A/a/d/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)-2\*b\*(A\*a^2+2\*A\*b^2-B\*a\*b)\*tan(d\*x+c)^(1/2)/a^2/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(1/2)

Rubi [A]

time = 0.56, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$-\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{a^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}} - \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}} - \frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)),x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a - b)^(3/2)\*d) - ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a + b)^(3/2)\*d) - (2\*A)/(a\*d\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]) - (2\*b\*(a^2\*A + 2\*A\*b^2 - a\*b\*B)\*Sqrt[Tan[c + d\*x]])/(a^2\*(a^2 + b^2)\*d\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(LtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)

$(A*b - a*B - b*C)*\text{Tan}[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& (!\text{LtQ}[n, -1] \&\& (!\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} dx = -\frac{2A}{ad \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(2Ab - aB) + \frac{1}{2}aA \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{a \sqrt{a + b \tan(c + dx)}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{2b(a^2 A + 2Ab^2 - ab^2)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{2b(a^2 A + 2Ab^2 - ab^2)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{2b(a^2 A + 2Ab^2 - ab^2)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{2b(a^2 A + 2Ab^2 - ab^2)}{a^2 (a^2 + b^2) d \sqrt{a + b \tan(c + dx)}}$$

$$= \frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{3/2} d} - \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{3/2} d}$$

**Mathematica [A]**

time = 1.39, size = 248, normalized size = 1.15

$$\frac{\sqrt[4]{-1} \left( \frac{(a+ib)(A-iB) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} - \frac{(a-ib)(A+iB) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right)}{a^2 + b^2} - \frac{2A}{a \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} - \frac{2b(a^2 A + 2Ab^2 - ab^2) \sqrt{\tan(c+dx)}}{a^2 (a^2 + b^2) \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)),x]

[Out] (((-1)^(1/4)\*(((a + I\*b)\*(A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[-a + I\*b] - ((a - I\*b)\*(A + I

$*B) * \text{ArcTan} \left( \frac{((-1)^{1/4} \sqrt{a + I*b} \sqrt{\text{Tan}[c + d*x]}) / \sqrt{a + b*\text{Tan}[c + d*x]}}{\sqrt{a + I*b}} \right) / (a^2 + b^2) - (2*A) / (a*\sqrt{\text{Tan}[c + d*x]} * \sqrt{a + b*\text{Tan}[c + d*x]}) - (2*b*(a^2*A + 2*A*b^2 - a*b*B) * \sqrt{\text{Tan}[c + d*x]}) / (a^2 * (a^2 + b^2) * \sqrt{a + b*\text{Tan}[c + d*x]}) / d$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 1.02, size = 1561981, normalized size = 7231.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}} \tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)`

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*(3/2)\*tan(c + d\*x)\*\*(3/2)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F(-1)]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] \text{Hanged}

$$3.462 \quad \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=276

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{3/2}d} - \frac{(iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia + b)^{3/2}d} - \frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a-b)^{(3/2)}/d - (I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a+b)^{(3/2)}/d + 2/3*(4*A*b-3*B*a)/a^2/d/\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)} + 2/3*b*(5*A*a^2*b+8*A*b^3-3*B*a^3-6*B*a*b^2)*\tan(d*x+c)^{(1/2)}/a^3/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(1/2)} - 2/3*A/a/d/(a+b*\tan(d*x+c))^{(1/2)}/\tan(d*x+c)^{(3/2)}$

**Rubi** [A]

time = 0.77, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(4Ab - 3aB)}{3a^2d\sqrt{\tan(c + dx)}\sqrt{a + b\tan(c + dx)}} + \frac{2b(-3a^2B + 5a^2Ab - 6ab^2B + 8Ab^3)\sqrt{\tan(c + dx)}}{3a^3d(a^2 + b^2)\sqrt{a + b\tan(c + dx)}} - \frac{(-B + iA)\operatorname{ArcTan}\left(\frac{\sqrt{-b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} - \frac{(B + iA)\operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d(b + ia)^{3/2}} - \frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Tan}[c + d*x]^{(5/2)}*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}), x]$

[Out]  $-(((I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/((I*a - b)^{(3/2)}*d) - ((I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/((I*a + b)^{(3/2)}*d) - (2*A)/(3*a*d*\operatorname{Tan}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]) + (2*(4*A*b - 3*a*B))/(3*a^2*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]) + (2*b*(5*a^2*A*b + 8*A*b^3 - 3*a^3*B - 6*a*b^2*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(3*a^3*(a^2 + b^2)*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])$

**Rule 95**

$\operatorname{Int}[(a_.* + (b_.*)(x_*)^m)((c_.* + (d_.*)(x_*)^n))/((e_.* + (f_.*)(x_*)^q), x\_Symbol] := \operatorname{With}\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[m + n + 1, 0] \ \&\& \operatorname{RationalQ}[n] \ \&\& \operatorname{LtQ}[-1, m, 0] \ \&\& \operatorname{SimplerQ}[a + b*x, c + d*x]$

**Rule 209**

$\operatorname{Int}[(a_.* + (b_.*)(x_*)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{GtQ}[a$

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f



```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} dx &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} - \frac{2 \int \frac{\frac{1}{2}(4Ab - 3aB) + \frac{3}{2}aA \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)} dx}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} + \frac{2(4Ab)}{3a^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} + \frac{2(4Ab)}{3a^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} + \frac{2(4Ab)}{3a^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} + \frac{2(4Ab)}{3a^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} + \frac{2(4Ab)}{3a^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} + \frac{2(4Ab)}{3a^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{3/2} d} - \frac{(iA + B) \tan^{-1} \left( \frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia + b)^{3/2} d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.93, size = 299, normalized size = 1.08

$$\frac{\frac{3\sqrt{-1} a^{(a+b)(A+B)} \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{3\sqrt{-1} a^{(a+b)(A+B)} \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}}}{a^{2+2b}} - \frac{2A}{3ad \tan^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} + \frac{8Ab-6aB}{a \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} + \frac{2b(5a^2Ab+8Aa^2b^2-3a^2B-6ab^2B) \sqrt{\tan(c+dx)}}{a^2(a^2+b^2) \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(3/2)),x]

[Out] 
$$\left( \frac{(3(-1)^{1/4} a ((a + I b) (I A + B) \operatorname{ArcTan}[\frac{(-1)^{1/4} \sqrt{-a + I b} \sqrt{\tan[c + d x]}}{\sqrt{a + b \tan[c + d x]}}]) / \sqrt{-a + I b} + ((I a + b) (A + I B) \operatorname{ArcTan}[\frac{(-1)^{1/4} \sqrt{a + I b} \sqrt{\tan[c + d x]}}{\sqrt{a + b \tan[c + d x]}}]) / \sqrt{a + I b}}{(a^2 + b^2)} - \frac{2 A}{\tan[c + d x]^{3/2} \sqrt{a + b \tan[c + d x]}} + \frac{8 A b - 6 a B}{a \sqrt{\tan[c + d x]} \sqrt{a + b \tan[c + d x]}} + \frac{2 b (5 a^2 A b + 8 A b^3 - 3 a^3 B - 6 a b^2 B) \sqrt{\tan[c + d x]}}{a^2 (a^2 + b^2) \sqrt{a + b \tan[c + d x]}} \right) / (3 a d)$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 1.12, size = 1564046, normalized size = 5666.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(3/2),x)

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}} \tan^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**(5/2)), x)
```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{5/2} (a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

$$3.463 \quad \int \frac{\tan^5(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=282

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{5/2}d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{b^{5/2}d} - \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{ia}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia + b)^{5/2}d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a-b)^(5/2)/d+2\*B\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/b^(5/2)/d-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a+b)^(5/2)/d+2\*a\*(2\*A\*b^3-a\*(a^2+3\*b^2)\*B)\*tan(d\*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)+2/3\*a\*(A\*b-B\*a)\*tan(d\*x+c)^(3/2)/b/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(3/2)

Rubi [A]

time = 1.76, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ ,

Rules used = {3686, 3726, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{2a(Ab - aB) \tan^3(c + dx)}{3bd(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{2a(2Ab^3 - aB(a^2 + 3b^2)) \sqrt{\tan(c + dx)}}{b^2d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} + \frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{5/2}} - \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{5/2}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a - b)^(5/2)\*d) + (2\*B\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a + b)^(5/2)\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a + b)^(5/2)\*d) + (2\*a\*(A\*b - a\*B)\*Tan[c + d\*x]^(3/2))/(3\*b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*a\*(2\*A\*b^3 - a\*(a^2 + 3\*b^2)\*B)\*Sqrt[Tan[c + d\*x]])/(b^2\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)\*(c + d\*x)^(n\_)/(e + f\*x), x], x, (a + b\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e + f\*x, 0] && IntLinearQ[a, b, c, d, m, n, x]

$- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)/(c + d*x)^{(1/q)}$   
 $], x]] /; FreeQ[{a, b, c, d, e, f}, x] \&\& EqQ[m + n + 1, 0] \&\& RationalQ[n]$   
 $\&\& LtQ[-1, m, 0] \&\& SimplerQ[a + b*x, c + d*x]$

### Rule 211

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] \&\& PosQ[a/b]$

### Rule 212

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] \&\& NegQ[a/b] \&\& (GtQ[a, 0] || LtQ[b, 0])$

### Rule 214

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] \&\& NegQ[a/b]$

### Rule 223

$Int[1/Sqrt[(a_) + (b_)*(x_)^2], x\_Symbol] \rightarrow Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] \&\& !GtQ[a, 0]$

### Rule 3686

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^{(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^{(n_)}}, x\_Symbol] \rightarrow Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^{(m - 1)*((c + d*Tan[e + f*x])^{(n + 1)/(d*f*(n + 1)*(c^2 + d^2))})}, x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^{(m - 2)*((c + d*Tan[e + f*x])^{(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 + b^2, 0] \&\& NeQ[c^2 + d^2, 0] \&\& GtQ[m, 1] \&\& LtQ[n, -1] \&\& (IntegerQ[m] || IntegersQ[2*m, 2*n])$

### Rule 3726

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^{(n_)}}, x\_Symbol] \rightarrow Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^{(n + 1)/(d*f*(n + 1)*(c^2 + d^2))}), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^{(m - 1)*((c + d*Tan[e$

```

+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{\frac{5}{2}}} dx &= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2\int\sqrt{\tan(c+dx)}\left(-\frac{3}{2}a(Ab-aB)\right)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} + \frac{2a(2Ab^3-a(a^2+3b^2)B)}{b^2(a^2+b^2)^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d} + \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)d(a+b\tan(c+dx))^{\frac{3}{2}}} \\
&= -\frac{(iA-B)\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}(a+ib)^2d} + \frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 40.11, size = 265550, normalized size = 941.67

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Tan[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] Result too large to show

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 1.40, size = 2979638, normalized size = 10566.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\tan(dx+c)^{(5/2)}*(A+B*\tan(dx+c))/(a+b*\tan(dx+c))^{(5/2)}, x)$

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\tan(dx+c)^{(5/2)}*(A+B*\tan(dx+c))/(a+b*\tan(dx+c))^{(5/2)}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((B*\tan(dx + c) + A)*\tan(dx + c)^{(5/2)}/(b*\tan(dx + c) + a)^{(5/2)}, x)$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\tan(dx+c)^{(5/2)}*(A+B*\tan(dx+c))/(a+b*\tan(dx+c))^{(5/2)}, x, \text{algorithm}="fricas")$

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\tan(dx+c)**(5/2)*(A+B*\tan(dx+c))/(a+b*\tan(dx+c))**(5/2), x)$

[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
ithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{5/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2), x
)
```

$$3.464 \quad \int \frac{\tan^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=244

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} + \frac{2a(Ab-a^2)}{3b(a^2+b^2)d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a-b)^(5/2)/d+(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a+b)^(5/2)/d+2/3\*(2\*A\*a^2\*b-4\*A\*b^3+B\*a^3+7\*B\*a\*b^2)\*tan(d\*x+c)^(1/2)/b/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)+2/3\*a\*(A\*b-B\*a)\*tan(d\*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)

Rubi [A]

time = 0.65, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2a(Ab-a^2)\sqrt{\tan(c+dx)}}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}} + \frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{3bd(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}} + \frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}} + \frac{(A-iB)\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/((I\*a - b)^(5/2)\*d) + ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/((I\*a + b)^(5/2)\*d) + (2\*a\*(A\*b - a\*B)\*Sqrt[Tan[c + d\*x]])/(3\*b\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*(2\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B + 7\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(3\*b\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
    
```

Rubi steps

$$\begin{aligned}
 \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx &= \frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2 \int \frac{-\frac{1}{2}a(Ab - aB) + \frac{3}{2}b(Ab - aB) \tan(c + dx)}{\sqrt{\tan(c + dx)}} dx}{3b(a^2 + b^2) d} \\
 &= \frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2(2a^2Ab - 4Ab^3 + a^3B + 7a^2b^2)}{3b(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2(2a^2Ab - 4Ab^3 + a^3B + 7a^2b^2)}{3b(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2(2a^2Ab - 4Ab^3 + a^3B + 7a^2b^2)}{3b(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2(2a^2Ab - 4Ab^3 + a^3B + 7a^2b^2)}{3b(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{2a(Ab - aB) \sqrt{\tan(c + dx)}}{3b(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2(2a^2Ab - 4Ab^3 + a^3B + 7a^2b^2)}{3b(a^2 + b^2)^2 d \sqrt{a + b \tan(c + dx)}} \\
 &= \frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{5/2} d} + \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{5/2} d}
 \end{aligned}$$

Mathematica [A]

time = 2.04, size = 308, normalized size = 1.26

$$\frac{\frac{3B \sqrt{\tan(c + dx)}}{(a + b \tan(c + dx))^{5/2}} + \frac{(2aAb + a^2B + 3b^2B) \sqrt{\tan(c + dx)}}{(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{3\sqrt{-1} b \left( \frac{{}_{(a+ib)^2(A+iB)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{{}_{(a-ib)^2(A+iB)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right)}{(a^2 + b^2)^2}}{3bd}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2), x]
    
```

```

[Out] ((-3*B*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x])^(3/2) + ((2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*(a + b*Tan[c + d*x])^(3/2)) + (3*(-1)^(1/4)*b*((a + I*b)^2*(I*A + B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqr
    
```

$$\frac{\tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} \bigg/ \sqrt{-a + I b} + (I(a - I b)^2 * (A + I B) * \text{ArcTan}[\frac{(-1)^{1/4} \sqrt{a + I b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}] \bigg/ \sqrt{a + I b}) + (2*(2*a^2*A*b - 4*A*b^3 + a^3*B + 7*a*b^2*B) * \sqrt{\tan(c + dx)} \bigg/ \sqrt{a + b \tan(c + dx)}) \bigg/ (a^2 + b^2)^2 \bigg/ (3*b*d)$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 1.48, size = 2976654, normalized size = 12199.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)`

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int((tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2), x)

$$3.465 \quad \int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=244

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{(ia - b)^{5/2} d} + \frac{(iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{(ia + b)^{5/2} d} - \frac{2(Ab - a^2)}{3(a^2 + b^2)}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a-b)^{(5/2)}/d+(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a+b)^{(5/2)}/d-2/3*(5*A*a^2*b-A*b^3-2*B*a^3+4*B*a*b^2)*\tan(d*x+c)^{(1/2)}/a/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)}-2/3*(A*b-B*a)*\tan(d*x+c)^{(1/2)}/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.65, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{3d(a^2 + b^2)(a + b \tan(c+dx))^{3/2}} - \frac{2(-2a^3B + 5a^2Ab + 4ab^2B - Ab^3) \sqrt{\tan(c+dx)}}{3ad(a^2 + b^2)^2 \sqrt{a + b \tan(c+dx)}} - \frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{d(-b + ia)^{5/2}} + \frac{(B + iA) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{d(b + ia)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}, x]$

[Out]  $-(((I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]))/((I*a - b)^{(5/2)}*d) + ((I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]))/((I*a + b)^{(5/2)}*d) - (2*(A*b - a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(3*(a^2 + b^2)*d*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B + 4*a*b^2*B)*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(3*a*(a^2 + b^2)^2*d*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])$

Rule 95

$\operatorname{Int}[( (a_.) + (b_.)*(x_.)^{(m_.)} ) / ( (e_.) + (f_.)*(x_.) ), x\_Symbol] := \operatorname{With}\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)} / (b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)} / (c + d*x)^{(1/q)}], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[m + n + 1, 0] \ \&\& \operatorname{RationalQ}[n] \ \&\& \operatorname{LtQ}[-1, m, 0] \ \&\& \operatorname{SimplerQ}[a + b*x, c + d*x]$

Rule 209

$\operatorname{Int}[( (a_.) + (b_.)*(x_.)^2 )^{(-1)}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{GtQ}[b, 0])$

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ
[2*m, 2*n])
```

Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
+ (f_)*(x_)^2], x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```



$[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{ILtQ}[n, -1] \ \&\& \ ( !\text{IntegerQ}[m] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{NeQ}[a, 0])))$

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\tan(c+dx)} (A + B \tan(c+dx))}{(a + b \tan(c+dx))^{5/2}} dx &= -\frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{3(a^2 + b^2) d (a + b \tan(c+dx))^{3/2}} - \frac{2 \int \frac{-\frac{1}{2}b(Ab-aB) - \frac{3}{2}b(aA+bB)}{\sqrt{\tan(c+dx)}} dx}{3b} \\
 &= -\frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{3(a^2 + b^2) d (a + b \tan(c+dx))^{3/2}} - \frac{2(5a^2 Ab - Ab^3 - 2a^3 B)}{3a(a^2 + b^2)^2 d \sqrt{\tan(c+dx)}} \\
 &= -\frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{3(a^2 + b^2) d (a + b \tan(c+dx))^{3/2}} - \frac{2(5a^2 Ab - Ab^3 - 2a^3 B)}{3a(a^2 + b^2)^2 d \sqrt{\tan(c+dx)}} \\
 &= -\frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{3(a^2 + b^2) d (a + b \tan(c+dx))^{3/2}} - \frac{2(5a^2 Ab - Ab^3 - 2a^3 B)}{3a(a^2 + b^2)^2 d \sqrt{\tan(c+dx)}} \\
 &= -\frac{2(Ab - aB) \sqrt{\tan(c+dx)}}{3(a^2 + b^2) d (a + b \tan(c+dx))^{3/2}} - \frac{2(5a^2 Ab - Ab^3 - 2a^3 B)}{3a(a^2 + b^2)^2 d \sqrt{\tan(c+dx)}} \\
 &= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}} \right)}{\sqrt{ia - b} (a + ib)^2 d} + \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}} \right)}{\sqrt{ia - b} (a + ib)^2 d}
 \end{aligned}$$

**Mathematica [A]**

time = 2.24, size = 320, normalized size = 1.31

$$\frac{\frac{2b(Ab-aB) \tan^{\frac{3}{2}}(c+dx)}{(a+b \tan(c+dx))^{3/2}} + \frac{6b(2aAb-a^2B+b^2B) \tan^2(c+dx)}{(a^2+b^2) \sqrt{a+b \tan(c+dx)}} + \frac{\left( \frac{\sqrt{-1} \sqrt{-(a+ib)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} \right) + \frac{\sqrt{-1} \sqrt{(a-ib)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}}}{a^2+b^2} \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}}}{3a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((2\*b\*(A\*b - a\*B)\*Tan[c + d\*x]^(3/2))/(a + b\*Tan[c + d\*x])^(3/2) + (6\*b\*(2\*a\*A\*b - a^2\*B + b^2\*B)\*Tan[c + d\*x]^(3/2))/((a^2 + b^2)\*Sqrt[a + b\*Tan[c + d\*x]]) + (3\*(-((-1)^(1/4))\*a\*(a + I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[-a + I\*b]) +

$$\frac{((-1)^{1/4} * a * (a - I * b)^2 * (A + I * B) * \text{ArcTan}[\frac{((-1)^{1/4} * \text{Sqrt}[a + I * b] * \text{Sqrt}[\text{Tan}[c + d * x]])}{\text{Sqrt}[a + b * \text{Tan}[c + d * x]]}]}{\text{Sqrt}[a + I * b] + 2 * (-2 * a * A * b + a^2 * B - b^2 * B) * \text{Sqrt}[\text{Tan}[c + d * x]] * \text{Sqrt}[a + b * \text{Tan}[c + d * x]]} / (a^2 + b^2) / (3 * a * (a^2 + b^2) * d)$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 1.37, size = 2978176, normalized size = 12205.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(5/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\tan(c + dx)}}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)`

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(tan(c + d\*x))/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int((tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2), x)

$$3.466 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=247

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{(ia+b)^{5/2}d} + \frac{2b(Ab-)}{3a(a^2+b^2)}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a-b)^{(5/2)}/d-(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/(I*a+b)^{(5/2)}/d+2/3*b*(8*A*a^2*b+2*A*b^3-5*B*a^3+B*a*b^2)*\tan(d*x+c)^{(1/2)}/a^2/(a^2+b^2)^2/d/(a+b*\tan(d*x+c))^{(1/2)}+2/3*b*(A*b-B*a)*\tan(d*x+c)^{(1/2)}/a/(a^2+b^2)/d/(a+b*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.61, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ ,

Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b \tan(c+dx))^{3/2}} + \frac{2b(-5a^3B+8a^2Ab+ab^2B+2Ab^3)\sqrt{\tan(c+dx)}}{3a^2d(a^2+b^2)^2\sqrt{a+b \tan(c+dx)}} - \frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(-b+ia)^{5/2}} - \frac{(A-iB) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]*(a+b*\operatorname{Tan}[c+d*x])^{(5/2)})],x]$

[Out]  $-(((A+I*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a-b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/((I*a-b)^{(5/2)}*d))-((A-I*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a+b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])/((I*a+b)^{(5/2)}*d)+(2*b*(A*b-a*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(3*a*(a^2+b^2)*d*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)})+(2*b*(8*a^2*A*b+2*A*b^3-5*a^3*B+a*b^2*B)*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(3*a^2*(a^2+b^2)^2*d*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]])]$

Rule 95

$\operatorname{Int}[(((a_.)+(b_.)*(x_.))^{(m_.)}*((c_.)+(d_.)*(x_.))^{(n_.)})/((e_.)+(f_.)*(x_.)),x\_Symbol] :> \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}/(b*e-a*f-(d*e-c*f)*x^q),x],x,(a+b*x)^{(1/q)}/(c+d*x)^{(1/q)}],x]] /; \operatorname{FreeQ}[\{a,b,c,d,e,f\},x] \&\& \operatorname{EqQ}[m+n+1,0] \&\& \operatorname{RationalQ}[n] \&\& \operatorname{LtQ}[-1,m,0] \&\& \operatorname{SimplerQ}[a+b*x,c+d*x]$

Rule 209

$\operatorname{Int}[((a_.)+(b_.)*(x_.)^2)^{(-1)},x\_Symbol] :> \operatorname{Simp}[(1/(\operatorname{Rt}[a,2]*\operatorname{Rt}[b,2]))*A \operatorname{rcTan}[\operatorname{Rt}[b,2]*(x/\operatorname{Rt}[a,2])],x] /; \operatorname{FreeQ}[\{a,b\},x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a,0] \mid \mid \operatorname{GtQ}[b,0])]$

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(LtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
    
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{5/2}} dx &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{3a(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2 \int \frac{\frac{1}{2}(3a^2A + 2Ab^2 + abB) - \frac{3}{2}a^3}{\sqrt{\tan(c + dx)}} dx}{\sqrt{\tan(c + dx)}} \\
 &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{3a(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2b(8a^2Ab + 2Ab^3 - 5a^3)}{3a^2(a^2 + b^2)^2 d \sqrt{\tan(c + dx)}} \\
 &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{3a(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2b(8a^2Ab + 2Ab^3 - 5a^3)}{3a^2(a^2 + b^2)^2 d \sqrt{\tan(c + dx)}} \\
 &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{3a(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2b(8a^2Ab + 2Ab^3 - 5a^3)}{3a^2(a^2 + b^2)^2 d \sqrt{\tan(c + dx)}} \\
 &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{3a(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2b(8a^2Ab + 2Ab^3 - 5a^3)}{3a^2(a^2 + b^2)^2 d \sqrt{\tan(c + dx)}} \\
 &= \frac{2b(Ab - aB) \sqrt{\tan(c + dx)}}{3a(a^2 + b^2) d(a + b \tan(c + dx))^{3/2}} + \frac{2b(8a^2Ab + 2Ab^3 - 5a^3)}{3a^2(a^2 + b^2)^2 d \sqrt{\tan(c + dx)}} \\
 &= -\frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{5/2} d} - \frac{(A - iB) \tan^{-1} \left( \frac{\sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(a - ib)^{5/2} d}
 \end{aligned}$$

Mathematica [A]

time = 1.66, size = 273, normalized size = 1.11

$$\frac{-3 \sqrt{-1} \left( \frac{(a+ib)^2(iA+B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{i(a-ib)^2(A+iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right) + \frac{2b(a^2+b^2)(Ab-aB) \sqrt{\tan(c+dx)}}{a(a+b \tan(c+dx))^{3/2}} + \frac{2b(8a^2Ab+2Ab^3-5a^3B+ab^2B) \sqrt{\tan(c+dx)}}{a^2 \sqrt{a+b \tan(c+dx)}}}{3(a^2+b^2)^2 d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x]
    
```

```

[Out] (-3*(-1)^(1/4)*(((a + I*b)^2*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (I*(a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b
    
```

$$\frac{\text{Tan}[c + d*x]]}{\text{Sqrt}[a + I*b]} + (2*b*(a^2 + b^2)*(A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a*(a + b*\text{Tan}[c + d*x])^{(3/2)}) + (2*b*(8*a^2*A*b + 2*A*b^3 - 5*a^3*B + a*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a^2*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(3*(a^2 + b^2)^{2*d})$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 4.35, size = 2978185, normalized size = 12057.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c))), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(5/2)), x)



$$3.467 \quad \int \frac{A+B \tan(c+dx)}{\tan^3(c+dx)(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=301

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{5/2}d} - \frac{(iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia + b)^{5/2}d} - \frac{ad \sqrt{\tan(c + dx)}}{ad \sqrt{\tan(c + dx)}}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a-b)^(5/2)/d-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a+b)^(5/2)/d-2/3\*b\*(3\*A\*a^4+17\*A\*a^2\*b^2+8\*A\*b^4-8\*B\*a^3\*b-2\*B\*a\*b^3)\*tan(d\*x+c)^(1/2)/a^3/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)-2\*A/a/d/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2)-2/3\*b\*(3\*A\*a^2+4\*A\*b^2-B\*a\*b)\*tan(d\*x+c)^(1/2)/a^2/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)

**Rubi** [A]

time = 0.79, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{-2b(3a^2A - abB + 4Ab^2) \sqrt{\tan(c + dx)}}{3a^2d(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^4A - 8a^2bB + 17a^2Ab^2 - 2ab^2B + 8Ab^4) \sqrt{\tan(c + dx)}}{3a^3d(a^2 + b^2)^2 \sqrt{a + b \tan(c + dx)}} + \frac{(-B + iA) \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{5/2}} - \frac{(B + iA) \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{5/2}} - \frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(5/2)),x]

[Out] (((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a - b)^(5/2)\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a + b)^(5/2)\*d) - (2\*A)/(a\*d\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)) - (2\*b\*(3\*a^2\*A + 4\*A\*b^2 - a\*b\*B)\*Sqrt[Tan[c + d\*x]])/(3\*a^2\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) - (2\*b\*(3\*a^4\*A + 17\*a^2\*A\*b^2 + 8\*A\*b^4 - 8\*a^3\*b\*B - 2\*a\*b^3\*B)\*Sqrt[Tan[c + d\*x]])/(3\*a^3\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{5/2}} dx = -\frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2 \int \frac{\frac{1}{2}(4Ab - aB) + \frac{1}{2}aA}{\sqrt{\tan(c + dx)}} dx}{\dots}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - \dots)}{3a^2 (a^2 + b^2) d(a + b \tan(c + dx))^{3/2}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - \dots)}{3a^2 (a^2 + b^2) d(a + b \tan(c + dx))^{3/2}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - \dots)}{3a^2 (a^2 + b^2) d(a + b \tan(c + dx))^{3/2}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - \dots)}{3a^2 (a^2 + b^2) d(a + b \tan(c + dx))^{3/2}}$$

$$= -\frac{2A}{ad \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - \dots)}{3a^2 (a^2 + b^2) d(a + b \tan(c + dx))^{3/2}}$$

$$= -\frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} (a + ib)^2 d} - \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} (a + ib)^2 d}$$

**Mathematica [A]**

time = 2.43, size = 326, normalized size = 1.08

$$\frac{6aA}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - abB) \sqrt{\tan(c + dx)}}{(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{3\sqrt{-1} a^3 \left( \frac{(a + ib)^2 (a - ib) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{-a + ib}} \right) - (a - ib)^2 (a + ib) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{a(a^2 + b^2)^2} - \frac{2b(3a^2 A + 4Ab^2 - abB) \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(5/2)),x]

[Out] 
$$\begin{aligned} &((-6*a*A)/(\text{Sqrt}[\text{Tan}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^{3/2}) - (2*b*(3*a^2*A + 4*A*b^2 - a*b*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/((a^2 + b^2)*(a + b*\text{Tan}[c + d*x])^{3/2}) \\ &+ (3*(-1)^{1/4}*a^3*((a + I*b)^2*(A - I*B)*\text{ArcTan}[((-1)^{1/4}*\text{Sqrt}[-a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/\text{Sqrt}[-a + I*b] - ((a - I*b)^2*(A + I*B)*\text{ArcTan}[((-1)^{1/4}*\text{Sqrt}[a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/\text{Sqrt}[a + I*b]) \\ &- (2*b*(3*a^4*A + 17*a^2*A*b^2 + 8*A*b^4 - 8*a^3*b*B - 2*a*b^3*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/(a*(a^2 + b^2)^2)/(3*a^2*d) \end{aligned}$$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 4.23, size = 2979708, normalized size = 9899.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(5/2),x)

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{3/2} (a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(5/2)), x)

$$3.468 \quad \int \frac{A+B \tan(c+dx)}{\tan^2(c+dx)(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=359

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{(A-iB)\text{tanh}^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} - \frac{2A}{3ad \tan^{\frac{3}{2}}(c+dx)}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a-b)^(5/2)/d+(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/(I\*a+b)^(5/2)/d+2/3\*b\*(8\*A\*a^4\*b+30\*A\*a^2\*b^3+16\*A\*b^5-3\*B\*a^5-17\*B\*a^3\*b^2-8\*B\*a\*b^4)\*tan(d\*x+c)^(1/2)/a^4/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))^(1/2)+2\*(2\*A\*b-B\*a)/a^2/d/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2)+2/3\*b\*(7\*A\*a^2\*b+8\*A\*b^3-3\*B\*a^3-4\*B\*a\*b^2)\*tan(d\*x+c)^(1/2)/a^3/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^(3/2)-2/3\*A/a/d/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 1.06, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(2Ab - aB)}{a^2d\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}} + \frac{2b(-3a^2B + 7a^2Ab - 4a^2B + 8Ab^2)\sqrt{\tan(c+dx)}}{3a^2d(a^2+b^2)(a+b\tan(c+dx))^{5/2}} + \frac{2b(-3a^2B + 8a^2Ab - 17a^2b^2B + 30a^2Ab^2 - 8a^2B + 16Ab^2)\sqrt{\tan(c+dx)}}{3a^2d(a^2+b^2)^2\sqrt{a+b\tan(c+dx)}} + \frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}} + \frac{(A-iB)\text{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}} - \frac{2A}{3ad \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(5/2)), x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a - b)^(5/2)\*d) + ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/((I\*a + b)^(5/2)\*d) - (2\*A)/(3\*a\*d\*Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*(2\*A\*b - a\*B))/(a^2\*d\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*b\*(7\*a^2\*A\*b + 8\*A\*b^3 - 3\*a^3\*B - 4\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(3\*a^3\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*b\*(8\*a^4\*A\*b + 30\*a^2\*A\*b^3 + 16\*A\*b^5 - 3\*a^5\*B - 17\*a^3\*b^2\*B - 8\*a\*b^4\*B)\*Sqrt[Tan[c + d\*x]])/(3\*a^4\*(a^2 + b^2)^2\*d\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
```

```
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^{5/2}} dx &= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} - \frac{2 \int \frac{\frac{3}{2}(2Ab - aB) + \frac{3}{2}aA \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} dx}{3} \\
&= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2(2Ab - aB)}{a^2 d \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2(2Ab - aB)}{a^2 d \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2(2Ab - aB)}{a^2 d \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2(2Ab - aB)}{a^2 d \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2(2Ab - aB)}{a^2 d \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2A}{3ad \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2(2Ab - aB)}{a^2 d \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} \\
&= \frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{5/2} d} + \frac{(A - iB) \tanh^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{5/2} d}
\end{aligned}$$

Mathematica [A]



time = 2.29, size = 383, normalized size = 1.07

$$\frac{\frac{6A}{\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}} - \frac{6(6Ab-3aB)}{a\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}} + \frac{6b(7a^2Ab+8Ab^3-3a^2B-4a^2B)\sqrt{\tan(c+dx)}}{a^2(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{9(-1)^{3/4}} \left( \frac{(-1)^{3/4} \operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{(-1)^{3/4} \operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} \right) \frac{6b(a^4+4a^2b^2+4b^4-3a^2B-3a^2B-3a^2B-3a^2B)\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(5/2)), x]

[Out] ((-6\*A)/(Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)) + (6\*(6\*A\*b - 3\*a\*B))/(a\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)) + (6\*b\*(7\*a^2\*A\*b + 8\*A\*b^3 - 3\*a^3\*B - 4\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(a^2\*(a^2 + b^2)\*(a + b\*Tan[c + d\*x])^(3/2)) + (9\*(-1)^(3/4)\*a^4\*((a + I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[-a + I\*b] + ((a - I\*b)^2\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[a + I\*b]) + (6\*b\*(8\*a^4\*A\*b + 30\*a^2\*A\*b^3 + 16\*A\*b^5 - 3\*a^5\*B - 17\*a^3\*b^2\*B - 8\*a\*b^4\*B)\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]/(a^3\*(a^2 + b^2)^2)/(9\*a\*d)

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 1.43, size = 2982515, normalized size = 8307.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2), x)

[Out] result too large to display

**Maxima [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] Timed out

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(5/2)/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/tan(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\tan(c + dx)^{5/2} (a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(tan(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^(5/2)), x)

$$3.469 \quad \int \frac{\tan^3(c+dx)(aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{B \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b} d} - \frac{B \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d}$$

[Out]  $-B \arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a-b)^{(1/2)}+2*B \operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/b^{(1/2)}-B \operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a+b)^{(1/2)}$

**Rubi [A]**

time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {21, 3656, 924, 65, 223, 212, 926, 95, 211, 214}

$$\frac{B \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b} d} - \frac{B \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Tan}[c + d*x])^{(3/2)}*(a*B + b*B*\operatorname{Tan}[c + d*x])]/(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}, x]$

[Out]  $-((B*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[I*a - b]*d)) + (2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[b]*d) - (B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[I*a + b]*d)$

**Rule 21**

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 65**

$\operatorname{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 95

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 924

```
Int[(((d_.) + (e_.)*(x_)^(m_))/(Sqrt[(f_.) + (g_.)*(x_)^(n_)]*((a_.) + (c_.)*(x_)^2)), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d + e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_))*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB + bB \tan(c+dx))}{(a + b \tan(c+dx))^{3/2}} dx &= B \int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a + b \tan(c+dx)}} dx \\
&= \frac{B \operatorname{Subst}\left(\int \frac{x^{3/2}}{\sqrt{a + bx}} dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{B \operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{x} \sqrt{a + bx}} - \frac{1}{\sqrt{x} \sqrt{a + bx} (1+x^2)}\right) dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{B \operatorname{Subst}\left(\int \frac{1}{\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c+dx)\right)}{d} - \frac{B \operatorname{Subst}\left(\int \frac{1}{\sqrt{x} \sqrt{a + bx} (1+x^2)} dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{B \operatorname{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x} \sqrt{a + bx}} + \frac{i}{2\sqrt{x} (i+x)\sqrt{a + bx}}\right) dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{(iB) \operatorname{Subst}\left(\int \frac{1}{(i-x)\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c+dx)\right)}{2d} - \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{i - (-a+ib)x} dx, x, \tan(c+dx)\right)}{2d} \\
&= \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{\sqrt{b} d} - \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{i - (-a+ib)x} dx, x, \tan(c+dx)\right)}{2d} \\
&= -\frac{B \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{\sqrt{ia - b} d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{\sqrt{b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.60, size = 190, normalized size = 1.23

$$\frac{B \left( (-1)^{3/4} \left( \frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}}\right)}{\sqrt{a+ib}} \right) + \frac{2\sqrt{a} \sinh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right) \sqrt{1 + \frac{b \tan(c+dx)}{a}}}{\sqrt{b} \sqrt{a + b \tan(c+dx)}} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]
)^(3/2), x]
```

```
[Out] (B*((-1)^(3/4)*(ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[-a + I*b] + ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[a + I*b]) + (2*Sqrt[a]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]))/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.39, size = 944366, normalized size = 6092.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^(3/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*b*tan(d*x + c) + B*a)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^(3/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\tan^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] B\*Integral(tan(c + d\*x)\*\*(3/2)/sqrt(a + b\*tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^{3/2} (Ba + Bb \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(3/2)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int((tan(c + d\*x)^(3/2)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x)

$$3.470 \quad \int \frac{\sqrt{\tan(c+dx)} (aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=117

$$\frac{iB \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} - \frac{iB \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d}$$

[Out] I\*B\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a-b)^(1/2)-I\*B\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a+b)^(1/2)

Rubi [A]

time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {21, 3656, 924, 95, 211, 214}

$$\frac{iB \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{iB \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Tan[c + d\*x]]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (I\*B\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) - (I\*B\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d)

Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 95

Int[(((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_))/((e\_) + (f\_)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]



Rule 211

$\text{Int}[\{(a\_.) + (b\_.) \cdot (x\_.)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a) \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]], x] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 214

$\text{Int}[\{(a\_.) + (b\_.) \cdot (x\_.)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) \cdot \text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 924

$\text{Int}[\{(d\_.) + (e\_.) \cdot (x\_.)\}^{(m\_)} / (\text{Sqrt}[(f\_.) + (g\_.) \cdot (x\_.)] \cdot \{(a\_.) + (c\_.) \cdot (x\_.)^2\}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[1/(\text{Sqrt}[d + e \cdot x] \cdot \text{Sqrt}[f + g \cdot x]), (d + e \cdot x)^{(m + 1/2)} / (a + c \cdot x^2), x], x] \text{ /; FreeQ}\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \ \text{IGtQ}[m + 1/2, 0]$

Rule 3656

$\text{Int}[\{(a\_.) + (b\_.) \cdot \tan[(e\_.) + (f\_.) \cdot (x\_.)]\}^{(m\_)} \cdot \{(c\_.) + (d\_.) \cdot \tan[(e\_.) + (f\_.) \cdot (x\_.)]\}^{(n\_)}, x\_Symbol] \rightarrow \text{With}\{\{ff = \text{FreeFactors}[\text{Tan}[e + f \cdot x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(a + b \cdot ff \cdot x)^m \cdot (c + d \cdot ff \cdot x)^n / (1 + ff^2 \cdot x^2)], x], x, \text{Tan}[e + f \cdot x]/ff], x] \text{ /; FreeQ}\{a, b, c, d, e, f, m, n\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)} (aB + bB \tan(c+dx))}{(a + b \tan(c+dx))^{3/2}} dx &= B \int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}} dx \\
&= \frac{B \operatorname{Subst} \left( \int \frac{\sqrt{x}}{\sqrt{a+bx} (1+x^2)} dx, x, \tan(c+dx) \right)}{d} \\
&= \frac{B \operatorname{Subst} \left( \int \left( -\frac{1}{2(i-x)\sqrt{x}\sqrt{a+bx}} + \frac{1}{2\sqrt{x}(i+x)\sqrt{a+bx}} \right) dx, x, \tan(c+dx) \right)}{d} \\
&= -\frac{B \operatorname{Subst} \left( \int \frac{1}{(i-x)\sqrt{x}\sqrt{a+bx}} dx, x, \tan(c+dx) \right)}{2d} + \frac{B \operatorname{Subst} \left( \int \frac{1}{(i+x)\sqrt{x}\sqrt{a+bx}} dx, x, \tan(c+dx) \right)}{2d} \\
&= \frac{B \operatorname{Subst} \left( \int \frac{1}{i-(-a+ib)x^2} dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d} - \frac{B \operatorname{Subst} \left( \int \frac{1}{i+(-a+ib)x^2} dx, x, \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d} \\
&= \frac{iB \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{ia-b} d} - \frac{iB \tanh^{-1} \left( \frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{ia+b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.07, size = 124, normalized size = 1.06

$$\frac{\sqrt[4]{-1} B \left( -\frac{\operatorname{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{\operatorname{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

```
[Out] ((-1)^(1/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b])/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.43, size = 940499, normalized size = 8038.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*b*tan(d*x + c) + B*a)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(3/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)`

[Out] `B*Integral(sqrt(tan(c + d*x))/sqrt(a + b*tan(c + d*x)), x)`

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")`

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\tan(c + dx)} (Ba + Bb \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^(1/2)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x)

[Out] int((tan(c + d\*x)^(1/2)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x)

$$3.471 \quad \int \frac{aB + bB \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{B \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d}$$

[Out] B\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a-b)^(1/2)+B\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))/d/(I\*a+b)^(1/2)

Rubi [A]

time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {21, 3656, 926, 95, 211, 214}

$$\frac{B \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{B \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/(Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)), x]

[Out] (B\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) + (B\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d)

Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

Rule 95

Int[(((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_))/((e\_) + (f\_)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 926

Int[(((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))^(n\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)^n, 1/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[m] && !IntegerQ[n]

#### Rule 3656

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*((c + d\*ff\*x)^n/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

#### Rubi steps

$$\begin{aligned}
 \int \frac{aB + bB \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} dx &= B \int \frac{1}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\
 &= \frac{B \operatorname{Subst}\left(\int \frac{1}{\sqrt{x} \sqrt{a + bx} (1+x^2)} dx, x, \tan(c + dx)\right)}{d} \\
 &= \frac{B \operatorname{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{x} \sqrt{a + bx}} + \frac{i}{2\sqrt{x} (i+x)\sqrt{a + bx}}\right) dx, x\right)}{d} \\
 &= \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{(i-x)\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c + dx)\right)}{2d} + \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c + dx)\right)}{2d} \\
 &= \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{i - (-a+ib)x^2} dx, x, \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(iB) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c + dx)\right)}{2d} \\
 &= \frac{B \tan^{-1}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia - b} d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia + b} d}
 \end{aligned}$$

**Mathematica [A]**

time = 0.07, size = 125, normalized size = 1.13

$$\frac{(-1)^{3/4} B \left( -\frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{a+ib}} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])
^(3/2)), x]
```

```
[Out] ((-1)^(3/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]])/Sqrt[-a + I*b]) - ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/Sqrt[a + I*b])/d
```

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.

time = 0.37, size = 940264, normalized size = 8470.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2), x)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2), x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x
+ c))), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{a + b \tan(c + dx)} \sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] B\*Integral(1/(sqrt(a + b\*tan(c + d\*x))\*sqrt(tan(c + d\*x))), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B a + B b \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] int((B\*a + B\*b\*tan(c + d\*x))/(tan(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(3/2)), x)



$$3.472 \quad \int \frac{aB + bB \tan(c + dx)}{\tan^2(c + dx)(a + b \tan(c + dx))^{3/2}} dx$$

**Optimal.** Leaf size=150

$$-\frac{iB \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d} + \frac{iB \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d} - \frac{2B \sqrt{a+b \tan(c+dx)}}{ad \sqrt{\tan(c+dx)}}$$

[Out]  $-I*B*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a-b)^{(1/2)}+I*B*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})/d/(I*a+b)^{(1/2)}-2*B*(a+b*\tan(d*x+c))^{(1/2)}/a/d/\tan(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {21, 3650, 12, 3656, 924, 95, 211, 214}

$$-\frac{iB \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{2B \sqrt{a+b \tan(c+dx)}}{ad \sqrt{\tan(c+dx)}} + \frac{iB \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x])]/(\operatorname{Tan}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}), x]$

[Out]  $((-I)*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[I*a - b]*d) + (I*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(\operatorname{Sqrt}[I*a + b]*d) - (2*B*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(a*d*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

**Rule 21**

$\operatorname{Int}[(u_*)((a_) + (b_*)(v_))^{(m_)}*((c_) + (d_*)(v_))^{(n_)}], x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (\operatorname{!IntegerQ}[n] \operatorname{||} \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 95**

$\operatorname{Int}[(((a_) + (b_*)(x_))^{(m_)}*((c_) + (d_*)(x_))^{(n_)}))/((e_) + (f_*)(x_)), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1))}], x, (e + f*x)^q], x]$

```

- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

```

#### Rule 211

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

#### Rule 214

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

#### Rule 924

```

Int[((d_) + (e_)*(x_)^m)/(Sqrt[(f_) + (g_)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d + e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]

```

#### Rule 3650

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^m*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^n, x_Symbol] := Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 3656

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^m*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^n, x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)], x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} dx &= B \int \frac{1}{\tan^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx \\
&= -\frac{2B \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{(2B) \int \frac{a \sqrt{\tan(c + dx)}}{2 \sqrt{a + b \tan(c + dx)}} dx}{a} \\
&= -\frac{2B \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - B \int \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} dx \\
&= -\frac{2B \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{B \text{Subst} \left( \int \frac{\sqrt{x}}{\sqrt{a + bx} (1+x^2)} dx, x, \tan(c + dx) \right)}{d} \\
&= -\frac{2B \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{B \text{Subst} \left( \int \left( -\frac{1}{2(i-x)\sqrt{x} \sqrt{a + bx}} \right) dx, x, \tan(c + dx) \right)}{d} \\
&= -\frac{2B \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} + \frac{B \text{Subst} \left( \int \frac{1}{(i-x)\sqrt{x} \sqrt{a + bx}} dx, x, \tan(c + dx) \right)}{2d} \\
&= -\frac{2B \sqrt{a + b \tan(c + dx)}}{ad \sqrt{\tan(c + dx)}} - \frac{B \text{Subst} \left( \int \frac{1}{i - (-a+ib)x^2} dx, x, \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} \\
&= -\frac{iB \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d} + \frac{iB \tanh^{-1} \left( \frac{\sqrt{ia + b}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia + b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.28, size = 158, normalized size = 1.05

$$\frac{B \left( \frac{\sqrt[4]{-1} \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{-a + ib}} - \frac{\sqrt[4]{-1} \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{a + ib}} - \frac{2 \sqrt{a + b \tan(c + dx)}}{a \sqrt{\tan(c + dx)}} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)), x]
```

```
[Out] (B*(((−1)^(1/4)*ArcTan[((−1)^(1/4)*Sqrt[−a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[−a + I*b] − ((−1)^(1/4)*ArcTan[((−1)^(1/4)*Sqrt[
```

$a + I*b] * \text{Sqrt}[\text{Tan}[c + d*x]] / \text{Sqrt}[a + b*\text{Tan}[c + d*x]] / \text{Sqrt}[a + I*b] - (2 * \text{Sqrt}[a + b*\text{Tan}[c + d*x]] / (a * \text{Sqrt}[\text{Tan}[c + d*x]])) / d$

**Maple [B]** result has leaf size over 500,000. Avoiding possible recursion issues.  
time = 0.42, size = 944401, normalized size = 6296.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)`

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{a + b \tan(c + dx)} \tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)`

[Out] `B*Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2)), x)`

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/tan(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B a + B b \tan(c + d x)}{\tan(c + d x)^{3/2} (a + b \tan(c + d x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] int((B\*a + B\*b\*tan(c + d\*x))/(tan(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)), x)

### 3.473 $\int (a + b \tan(c + dx))^{2/3} (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=379

$$-\frac{1}{4}(a-ib)^{2/3}(A-iB)x - \frac{1}{4}(a+ib)^{2/3}(A+iB)x + \frac{\sqrt{3}(a-ib)^{2/3}(iA+B)\text{ArcTan}\left(\frac{1 + \sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2d}$$

[Out]  $-1/4*(a-I*b)^{(2/3)*(A-I*B)*x - 1/4*(a+I*b)^{(2/3)*(A+I*B)*x - 1/4*(a+I*b)^{(2/3)*(I*A-B)*\ln(\cos(d*x+c))/d + 1/4*(a-I*b)^{(2/3)*(I*A+B)*\ln(\cos(d*x+c))/d + 3/4*(a-I*b)^{(2/3)*(I*A+B)*\ln((a-I*b)^{(1/3)-(a+b*\tan(d*x+c))^{(1/3)})/d - 3/4*(a+I*b)^{(2/3)*(I*A-B)*\ln((a+I*b)^{(1/3)-(a+b*\tan(d*x+c))^{(1/3)})/d + 1/2*(a-I*b)^{(2/3)*(I*A+B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)/(a-I*b)^{(1/3)})}*3^{(1/2)})}*3^{(1/2)}/d - 1/2*(a+I*b)^{(2/3)*(I*A-B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)/(a+I*b)^{(1/3)})}*3^{(1/2)})}*3^{(1/2)}/d + 3/2*B*(a+b*\tan(d*x+c))^{(2/3)}/d$

**Rubi [A]**

time = 0.30, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {3609, 3620, 3618, 57, 631, 210, 31}

$$\frac{\sqrt{3}(a-ib)^{2/3}(iA+B)\text{ArcTan}\left(\frac{1 + \sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2d} - \frac{\sqrt{3}(a+ib)^{2/3}(-iA-B)\text{ArcTan}\left(\frac{1 + \sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2d} + \frac{3(a-ib)^{2/3}(B+iA)\ln\left(\frac{-\sqrt{a+b\tan(c+dx)} + \sqrt{a-ib}}{d}\right)}{4d} - \frac{3(a+ib)^{2/3}(-B-iA)\ln\left(\frac{-\sqrt{a+b\tan(c+dx)} + \sqrt{a-ib}}{d}\right)}{4d} + \frac{(a-ib)^{2/3}(B+iA)\ln(\cos(c+dx))}{4d} - \frac{(a+ib)^{2/3}(-B-iA)\ln(\cos(c+dx))}{4d} + \frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Tan[c + d\*x])^(2/3)\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-1/4*((a - I*b)^{(2/3)*(A - I*B)*x} - ((a + I*b)^{(2/3)*(A + I*B)*x})/4 + (\text{Sqrt}[3]*(a - I*b)^{(2/3)*(I*A + B)*\text{ArcTan}[(1 + (2*(a + b*\text{Tan}[c + d*x])^{(1/3)})/(a - I*b)^{(1/3)})/\text{Sqrt}[3]]]/(2*d) - (\text{Sqrt}[3]*(a + I*b)^{(2/3)*(I*A - B)*\text{ArcTan}[(1 + (2*(a + b*\text{Tan}[c + d*x])^{(1/3)})/(a + I*b)^{(1/3)})/\text{Sqrt}[3]]]/(2*d) - ((a + I*b)^{(2/3)*(I*A - B)*\text{Log}[\text{Cos}[c + d*x]]}/(4*d) + ((a - I*b)^{(2/3)*(I*A + B)*\text{Log}[\text{Cos}[c + d*x]]}/(4*d) + (3*(a - I*b)^{(2/3)*(I*A + B)*\text{Log}[(a - I*b)^{(1/3) - (a + b*\text{Tan}[c + d*x])^{(1/3)}]}/(4*d) - (3*(a + I*b)^{(2/3)*(I*A - B)*\text{Log}[(a + I*b)^{(1/3) - (a + b*\text{Tan}[c + d*x])^{(1/3)}]}/(4*d) + (3*B*(a + b*\text{Tan}[c + d*x])^{(2/3)})/(2*d)$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 57**

Int[1/(((a\_) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(1/3)), x\_Symbol] := With[{q = Rt[(b\*c - a\*d)/b, 3]}, Simp[-Log[RemoveContent[a + b\*x, x]]/(2\*b\*q), x]

] + (Dist[3/(2\*b), Subst[Int[1/(q^2 + q\*x + x^2), x], x, (c + d\*x)^(1/3)], x] - Dist[3/(2\*b\*q), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x])) /;  
FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &  
& (LtQ[a, 0] || LtQ[b, 0])

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

### Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
\int (a + b \tan(c + dx))^{2/3} (A + B \tan(c + dx)) dx &= \frac{3B(a + b \tan(c + dx))^{2/3}}{2d} + \int \frac{aA - bB + (Ab + aB) \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx \\
&= \frac{3B(a + b \tan(c + dx))^{2/3}}{2d} + \frac{1}{2}((a - ib)(A - iB)) \int \frac{1 + i \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx \\
&= \frac{3B(a + b \tan(c + dx))^{2/3}}{2d} + \frac{(i(a - ib)(A - iB)) \text{Subst}\left(\int \frac{1 + i \tan(u)}{\sqrt[3]{a + b \tan(u)}} du\right)}{2d} \\
&= -\frac{1}{4}(a - ib)^{2/3}(A - iB)x - \frac{1}{4}(a + ib)^{2/3}(A + iB)x - \frac{(a + ib)^{2/3}(A + iB)}{\sqrt{3}} \text{ArcTan}\left(\frac{\sqrt{3}(a + b \tan(c + dx))^{1/3} + (a + ib)}{(a + ib)^{1/3} + \sqrt{3}(a + b \tan(c + dx))^{1/3}}\right) \\
&= -\frac{1}{4}(a - ib)^{2/3}(A - iB)x - \frac{1}{4}(a + ib)^{2/3}(A + iB)x - \frac{(a + ib)^{2/3}(A + iB)}{\sqrt{3}} \text{ArcTan}\left(\frac{\sqrt{3}(a + b \tan(c + dx))^{1/3} + (a + ib)}{(a + ib)^{1/3} + \sqrt{3}(a + b \tan(c + dx))^{1/3}}\right) \\
&= -\frac{1}{4}(a - ib)^{2/3}(A - iB)x - \frac{1}{4}(a + ib)^{2/3}(A + iB)x + \frac{(a + ib)^{2/3}(A + iB)}{\sqrt{3}} \text{ArcTan}\left(\frac{\sqrt{3}(a + b \tan(c + dx))^{1/3} + (a + ib)}{(a + ib)^{1/3} + \sqrt{3}(a + b \tan(c + dx))^{1/3}}\right)
\end{aligned}$$

**Mathematica [A]**

time = 0.64, size = 263, normalized size = 0.69

$$\frac{(A - iB) \left( (a - ib)^{2/3} \left( 2\sqrt{3} \text{ArcTan}\left(\frac{1 + \frac{2\sqrt{3}(a + b \tan(c + dx))^{1/3}}{\sqrt{3}(a + ib)}}{1 + \tan(c + dx)}\right) - \log(i + \tan(c + dx)) + 3 \log(\sqrt{a - ib} - \sqrt{a + b \tan(c + dx)}) + 3(a + b \tan(c + dx))^{2/3} \right) - (A + iB) \left( (a + ib)^{2/3} \left( 2\sqrt{3} \text{ArcTan}\left(\frac{1 + \frac{2\sqrt{3}(a + b \tan(c + dx))^{1/3}}{\sqrt{3}(a + ib)}}{1 - \tan(c + dx)}\right) + 3 \log(\sqrt{a + ib} - \sqrt{a + b \tan(c + dx)}) + 3(a + b \tan(c + dx))^{2/3} \right) \right)}{4d}$$

Antiderivative was successfully verified.

**[In]** Integrate[(a + b\*Tan[c + d\*x])^(2/3)\*(A + B\*Tan[c + d\*x]),x]

**[Out]** ((I/4)\*((A - I\*B)\*((a - I\*b)^(2/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(a + b\*Tan[c + d\*x])^(1/3)))/(a - I\*b)^(1/3)]/Sqrt[3]] - Log[I + Tan[c + d\*x]] + 3\*Log[(a - I\*b)^(1/3) - (a + b\*Tan[c + d\*x])^(1/3])) + 3\*(a + b\*Tan[c + d\*x])^(2/3)) - (A + I\*B)\*((a + I\*b)^(2/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(a + b\*Tan[c + d\*x])^(1/3)))/(a + I\*b)^(1/3)]/Sqrt[3]] - Log[I - Tan[c + d\*x]] + 3\*Log[(a + I\*b)^(1/3) - (a + b\*Tan[c + d\*x])^(1/3])) + 3\*(a + b\*Tan[c + d\*x])^(2/3))))/d

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 2.11, size = 99, normalized size = 0.26

method	result
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derivativedivides	$\frac{\frac{3(a+b \tan(dx+c))^{\frac{2}{3}} B}{2} + \left( \frac{\sum_{-R=\text{RootOf}(-Z^6-2a-Z^3+a^2+b^2)} \left( \frac{(Ab+aB)R^4+B(-a^2-b^2)R}{-R^5-R^2 a} \right) \ln \left( (a+b \tan(dx+c))^{\frac{1}{3}} - R \right)}{d}}{2}}{d}$
default	$\frac{\frac{3(a+b \tan(dx+c))^{\frac{2}{3}} B}{2} + \left( \frac{\sum_{-R=\text{RootOf}(-Z^6-2a-Z^3+a^2+b^2)} \left( \frac{(Ab+aB)R^4+B(-a^2-b^2)R}{-R^5-R^2 a} \right) \ln \left( (a+b \tan(dx+c))^{\frac{1}{3}} - R \right)}{d}}{2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] `1/d*(3/2*(a+b*tan(d*x+c))^(2/3)*B+1/2*sum(((A*b+B*a)*_R^4+B*(-a^2-b^2)*_R)/(-R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2)))`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(2/3), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)`

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(2/3), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(2/3)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] undef

**Mupad [B]**

time = 21.87, size = 2500, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(2/3),x)

[Out]  $\log\left(\frac{((2*(-B^6*a^2*b^2*d^6)^{1/2} + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^{2/3} * ((2*(-B^6*a^2*b^2*d^6)^{1/2} + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^{1/3} * (1944*a*b^4 * ((2*(-B^6*a^2*b^2*d^6)^{1/2} + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^{2/3} * (a^2 + b^2) - (1944*B^2*b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^2)}{2} + \frac{(972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3}{4} + \frac{(243*B^5*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^5}{4} + \frac{(((-4*B^6*a^2*b^2*d^6)^{1/2} + B^3*a^2*d^3 - B^3*b^2*d^3)/(8*d^6))^{1/3} + \log\left(\frac{(-2*(-B^6*a^2*b^2*d^6)^{1/2} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6}{(((-2*(-B^6*a^2*b^2*d^6)^{1/2} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^{1/3} * (1944*a*b^4 * ((-2*(-B^6*a^2*b^2*d^6)^{1/2} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^{2/3} * (a^2 + b^2) - (1944*B^2*b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^2)}{2} + \frac{(972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3}{4} + \frac{(243*B^5*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^5}{4} - \frac{((-4*B^6*a^2*b^2*d^6)^{1/2} - B^3*a^2*d^3 + B^3*b^2*d^3)/(8*d^6)}{(8*d^6)^{1/3} + \log\left(\frac{((-A^6*d^6*(a^2 - b^2)^2)^{1/2} - 2*A^3*a*b*d^3)/d^6}{((1944*a*b^4 * ((-A^6*d^6*(a^2 - b^2)^2)^{1/2} - 2*A^3*a*b*d^3)/d^6)^{2/3} * (a^2 + b^2) + (1944*A^2*b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^2} * \frac{((-A^6*d^6*(a^2 - b^2)^2)^{1/2} - 2*A^3*a*b*d^3)/d^6}{(((-A^6*d^6*(a^2 - b^2)^2)^{1/2} - 2*A^3*a*b*d^3)/d^6)^{1/3}}}{2} + \frac{(972*A^3*b^5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3}{4} + \frac{(486*A^5*a*b^5*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^5}{4} + \frac{(2*A^6*a^2*b^2*d^6 - A^6*b^4*d^6 - A^6*a^4*d^6)^{1/2}}{(8*d^6) - (A^3*a*b)/(4*d^3)}^{1/3} + \log\left(\frac{((-A^6*d^6*(a^2 - b^2)^2)^{1/2} + 2*A^3*a*b*d^3)/d^6}{((1944*a*b^4 * ((-A^6*d^6*(a^2 - b^2)^2)^{1/2} + 2*A^3*a*b*d^3)/d^6)^{2/3} * (a^2 + b^2) + (1944*A^2*b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^2} * \frac{((-A^6*d^6*(a^2 - b^2)^2)^{1/2} + 2*A^3*a*b*d^3)/d^6}{(((-A^6*d^6*(a^2 - b^2)^2)^{1/2} + 2*A^3*a*b*d^3)/d^6)^{1/3}}}{2} + \frac{(972*A^3*b^5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3}{4} + \frac{(486*A^5*a*b^5*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{1/3})/d^5}{4} - \frac{(2*A^6*a^2*b^2*d^6 - A^6*b^4*d^6 - A^6*a^4*d^6)^{1/2}}{(8*d^6) - (A^3*a*b)/(4*d^3)}^{1/3}$

$$\begin{aligned}
& /2)/(8*d^6) - (A^3*a*b)/(4*d^3))^{(1/3)} + (\log((243*B^5*b^4*(a^2 - b^2)*(a^2 \\
& + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^5 - ((3^{(1/2)*1i} - 1)^2*((3^{(1/2)* \\
& 1i} - 1)*((1944*B^2*b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^2 - 486* \\
& a*b^4*(3^{(1/2)*1i} - 1)^2*((2*(-B^6*a^2*b^2*d^6)^{(1/2)} + B^3*a^2*d^3 - B^3*b \\
& ^2*d^3)/d^6)^{(2/3)}*(a^2 + b^2))*((2*(-B^6*a^2*b^2*d^6)^{(1/2)} + B^3*a^2*d^3 \\
& - B^3*b^2*d^3)/d^6)^{(1/3)})/4 - (972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^ \\
& 3)*((2*(-B^6*a^2*b^2*d^6)^{(1/2)} + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^{(2/3)}/16 \\
& )*(3^{(1/2)*1i} - 1)*((-4*B^6*a^2*b^2*d^6)^{(1/2)}/(8*d^6) + (B^3*a^2)/(8*d^3) \\
& - (B^3*b^2)/(8*d^3))^{(1/3)}/2 - (\log(((3^{(1/2)*1i} + 1)^2*((3^{(1/2)*1i} + 1) \\
& *((1944*B^2*b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^2 - 486*a*b^4*( \\
& 3^{(1/2)*1i} + 1)^2*((2*(-B^6*a^2*b^2*d^6)^{(1/2)} + B^3*a^2*d^3 - B^3*b^2*d^3) \\
& /d^6)^{(2/3)}*(a^2 + b^2))*((2*(-B^6*a^2*b^2*d^6)^{(1/2)} + B^3*a^2*d^3 - B^3*b \\
& ^2*d^3)/d^6)^{(1/3)})/4 + (972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3)*((2* \\
& (-B^6*a^2*b^2*d^6)^{(1/2)} + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^{(2/3)}/16 + (243 \\
& *B^5*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^5*(3^{(1/2) \\
& }*1i + 1)*((-4*B^6*a^2*b^2*d^6)^{(1/2)}/(8*d^6) + (B^3*a^2)/(8*d^3) - (B^3*b^ \\
& 2)/(8*d^3))^{(1/3)}/2 + (\log((243*B^5*b^4*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*t \\
& an(c + d*x))^{(1/3)})/d^5 - ((3^{(1/2)*1i} - 1)^2*((3^{(1/2)*1i} - 1)*((1944*B^2 \\
& *b^4*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^2 - 486*a*b^4*(3^{(1/2)*1i \\
& - 1)^2*(-(2*(-B^6*a^2*b^2*d^6)^{(1/2)} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^{(2/3) \\
& }*(a^2 + b^2))*(-(2*(-B^6*a^2*b^2*d^6)^{(1/2)} - B^3*a^2*d^3 + B^3*b^2*d^3)/d \\
& ^6)^{(1/3)}/4 - (972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3)*(-(2*(-B^6*a^ \\
& 2*b^2*d^6)^{(1/2)} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^{(2/3)}/16*(3^{(1/2)*1i} - \\
& 1)*((B^3*a^2)/(8*d^3) - (-4*B^6*a^2*b^2*d^6)^{(1/2)}/(8*d^6) - (B^3*b^2)/(8* \\
& d^3))^{(1/3)}/2 - (\log(((3^{(1/2)*1i} + 1)^2*((3^{(1/2)*1i} + 1)*((1944*B^2*b^4 \\
& *(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^2 - 486*a*b^4*(3^{(1/2)*1i} + 1) \\
& ^2*(-(2*(-B^6*a^2*b^2*d^6)^{(1/2)} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^{(2/3)}*(a \\
& ^2 + b^2))*(-(2*(-B^6*a^2*b^2*d^6)^{(1/2)} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^ \\
& (1/3)}/4 + (972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3)*(-(2*(-B^6*a^2*b^ \\
& 2*d^6)^{(1/2)} - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^{(2/3)}/16 + (243*B^5*b^4*(a^ \\
& 2 - b^2)*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^5*(3^{(1/2)*1i} + 1)*(( \\
& B^3*a^2)/(8*d^3) - (-4*B^6*a^2*b^2*d^6)^{(1/2)}/(8*d^6) - (B^3*b^2)/(8*d^3))^{ \\
& (1/3)}/2 - \log((486*A^5*a*b^5*(a^2 + b^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^5 \\
& - (((3^{(1/2)*1i})/2 - 1/2)*(((3^{(1/2)*1i})/2 + 1/2)*((1944*A^2*b^4*(a^2 + b \\
& ^2)^2*(a + b*\tan(c + d*x))^{(1/3)})/d^2 + 1944*a*b^4*((3^{(1/2)*1i})/2 - 1/2)* \\
& ((-A^6*d^6*(a^2 - b^2)^2)^{(1/2)} - 2*A^3*a*b*d^3)/d^6)^{(2/3)}*(a^2 + b^2))*(( \\
& (-A^6*d^6*(a^2 - b^2)^2)^{(1/2)} - 2*A^3*a*b*d^3)/d^6)^{(1/3)}/2 - (972*A^3*b^ \\
& 5*(3*a^4 - b^4 + 2*a^2*b^2))/d^3)*(((A^6*d^6*(a^2 - b^2)^2)^{(1/2)} - 2*A^3* \\
& a*b*d^3)/d^6)^{(2/3)}/4)*((3^{(1/2)*1i})/2 + 1/2)*(((4*A^6*a^2*b^2*d^6 - d^6*( \\
& A^6*a^4 + A^6*b^4 + 2*A^6*a^2*b^2))^{(1/2)} - 2*A^3*a*b*d^3)/(8*d^6))^{(1/3)} + \\
& \log((486*A^5*a*b^5*(a^2 + b^2)^2*(a + b*\tan(c \dots
\end{aligned}$$

### 3.474 $\int \sqrt[3]{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$

Optimal. Leaf size=377

$$-\frac{1}{4}\sqrt[3]{a - ib} (A - iB)x - \frac{1}{4}\sqrt[3]{a + ib} (A + iB)x - \frac{\sqrt{3} \sqrt[3]{a - ib} (iA + B) \operatorname{ArcTan}\left(\frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}}\right)}{2d} + \dots$$

[Out]  $-1/4*(a-I*b)^{(1/3)}*(A-I*B)*x - 1/4*(a+I*b)^{(1/3)}*(A+I*B)*x - 1/4*(a+I*b)^{(1/3)}*(I*A-B)*\ln(\cos(d*x+c))/d + 1/4*(a-I*b)^{(1/3)}*(I*A+B)*\ln(\cos(d*x+c))/d + 3/4*(a-I*b)^{(1/3)}*(I*A+B)*\ln((a-I*b)^{(1/3)} - (a+b*\tan(d*x+c))^{(1/3)})/d - 3/4*(a+I*b)^{(1/3)}*(I*A-B)*\ln((a+I*b)^{(1/3)} - (a+b*\tan(d*x+c))^{(1/3)})/d - 1/2*(a-I*b)^{(1/3)}*(I*A+B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)})/(a-I*b)^{(1/3)})*3^{(1/2)}*3^{(1/2)}/d + 1/2*(a+I*b)^{(1/3)}*(I*A-B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)})/(a+I*b)^{(1/3)})*3^{(1/2)}*3^{(1/2)}/d + 3*B*(a+b*\tan(d*x+c))^{(1/3)}/d$

**Rubi [A]**

time = 0.27, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {3609, 3620, 3618, 59, 631, 210, 31}

$$\frac{\sqrt{3}\sqrt{-B}(B+iA)\operatorname{ArcTan}\left(\frac{1+\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2d} + \frac{\sqrt{3}\sqrt{a+B}(-B+iA)\operatorname{ArcTan}\left(\frac{1+\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2d} + \frac{3\sqrt{a-B}(B+iA)\log\left(\frac{-\sqrt{a+b\tan(c+dx)}+\sqrt{a-ib}}{4d}\right)}{4d} + \frac{3\sqrt{a+B}(-B+iA)\log\left(\frac{-\sqrt{a+b\tan(c+dx)}+\sqrt{a-ib}}{4d}\right)}{4d} + \frac{\sqrt{a+B}(-B+iA)\log(\cos(c+dx))}{4d} + \frac{\sqrt{a-B}(B+iA)\log(\cos(c+dx))}{4d} + \frac{1}{2}\sqrt{a-B}(A-iB) - \frac{1}{2}\sqrt{a+B}(A+iB) + \frac{3B\sqrt{a+b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Tan[c + d*x])^(1/3)*(A + B*Tan[c + d*x]),x]`

[Out]  $-1/4*((a - I*b)^{(1/3)}*(A - I*B)*x) - ((a + I*b)^{(1/3)}*(A + I*B)*x)/4 - (\operatorname{Sqrt}[3]*(a - I*b)^{(1/3)}*(I*A + B)*\operatorname{ArcTan}[(1 + (2*(a + b*\tan[c + d*x])^{(1/3)})/(a - I*b)^{(1/3)})/\operatorname{Sqrt}[3]])/(2*d) + (\operatorname{Sqrt}[3]*(a + I*b)^{(1/3)}*(I*A - B)*\operatorname{ArcTan}[(1 + (2*(a + b*\tan[c + d*x])^{(1/3)})/(a + I*b)^{(1/3)})/\operatorname{Sqrt}[3]])/(2*d) - ((a + I*b)^{(1/3)}*(I*A - B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(4*d) + ((a - I*b)^{(1/3)}*(I*A + B)*\operatorname{Log}[\operatorname{Cos}[c + d*x]])/(4*d) + (3*(a - I*b)^{(1/3)}*(I*A + B)*\operatorname{Log}[(a - I*b)^{(1/3)} - (a + b*\tan[c + d*x])^{(1/3)})]/(4*d) - (3*(a + I*b)^{(1/3)}*(I*A - B)*\operatorname{Log}[(a + I*b)^{(1/3)} - (a + b*\tan[c + d*x])^{(1/3)})]/(4*d) + (3*B*(a + b*\tan[c + d*x])^{(1/3)})/d$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 59

`Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),`

$x] + (-\text{Dist}[3/(2*b*q), \text{Subst}[\text{Int}[1/(q^2 + q*x + x^2)], x], x, (c + d*x)^{(1/3)}], x] - \text{Dist}[3/(2*b*q^2), \text{Subst}[\text{Int}[1/(q - x)], x], x, (c + d*x)^{(1/3)}], x]) /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{PosQ}[(b*c - a*d)/b]$

### Rule 210

$\text{Int}[(a + (b*x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 631

$\text{Int}[(a + (b*x) + (c*x)^2)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 3609

$\text{Int}[(a + (b*\tan[e + f*x]) + (c + (d*\tan[e + f*x]) + (f*x)))^m, x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\tan[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\tan[e + f*x])^{m-1}*\text{Simp}[a*c - b*d + (b*c + a*d)*\tan[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$

### Rule 3618

$\text{Int}[(a + (b*\tan[e + f*x]) + (c + (d*\tan[e + f*x]) + (f*x)))^m, x\_Symbol] \rightarrow \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\tan[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[(a + (b*\tan[e + f*x]) + (c + (d*\tan[e + f*x]) + (f*x)))^m, x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\tan[e + f*x])^m*(1 - I*\tan[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\tan[e + f*x])^m*(1 + I*\tan[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$

### Rubi steps

$$\begin{aligned}
 \int \sqrt[3]{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{3B \sqrt[3]{a + b \tan(c + dx)}}{d} + \int \frac{aA - bB + (Ab + aB) \tan(c + dx)}{(a + b \tan(c + dx))^{2/3}} dx \\
 &= \frac{3B \sqrt[3]{a + b \tan(c + dx)}}{d} + \frac{1}{2}((a - ib)(A - iB)) \int \frac{1 + i \tan(c + dx)}{(a + b \tan(c + dx))^{2/3}} dx \\
 &= \frac{3B \sqrt[3]{a + b \tan(c + dx)}}{d} + \frac{(i(a - ib)(A - iB)) \text{Subst}\left(\int \frac{1 + i \tan(u)}{(a + b \tan(u))^{2/3}} du\right)}{2} \\
 &= -\frac{1}{4} \sqrt[3]{a - ib} (A - iB)x - \frac{1}{4} \sqrt[3]{a + ib} (A + iB)x - \frac{\sqrt[3]{a + ib}}{\sqrt[3]{a - ib}} \\
 &= -\frac{1}{4} \sqrt[3]{a - ib} (A - iB)x - \frac{1}{4} \sqrt[3]{a + ib} (A + iB)x - \frac{\sqrt[3]{a + ib}}{\sqrt[3]{a - ib}} \\
 &= -\frac{1}{4} \sqrt[3]{a - ib} (A - iB)x - \frac{1}{4} \sqrt[3]{a + ib} (A + iB)x - \frac{\sqrt[3]{a + ib}}{\sqrt[3]{a - ib}}
 \end{aligned}$$

**Mathematica [A]**

time = 0.59, size = 347, normalized size = 0.92

$$\left( (A - iB) \left( -\frac{1}{4} \sqrt[3]{a - ib} \left( 2\sqrt{3} \operatorname{ArcTan} \left( \frac{-\sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}} \right) - 2 \log \left( \sqrt[3]{a - ib} - \sqrt[3]{a + b \tan(c + dx)} \right) + \log \left( (a - ib)^{2/3} + \sqrt[3]{a - ib} \sqrt[3]{a + b \tan(c + dx)} + (a + b \tan(c + dx))^{2/3} \right) \right) + 3\sqrt[3]{a + b \tan(c + dx)} \right) - (A + iB) \left( -\frac{1}{4} \sqrt[3]{a + ib} \left( 2\sqrt{3} \operatorname{ArcTan} \left( \frac{-\sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a + ib}} \right) - 2 \log \left( \sqrt[3]{a + ib} - \sqrt[3]{a + b \tan(c + dx)} \right) + \log \left( (a + ib)^{2/3} + \sqrt[3]{a + ib} \sqrt[3]{a + b \tan(c + dx)} + (a + b \tan(c + dx))^{2/3} \right) \right) + 3\sqrt[3]{a + b \tan(c + dx)} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^(1/3)*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((I/2)*((A - I*B)*(-1/2*((a - I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3)))/(a - I*b)^(1/3)]/Sqrt[3]] - 2*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)])) + 3*(a + b*Tan[c + d*x])^(1/3)) - (A + I*B)*(-1/2*((a + I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3)))/(a + I*b)^(1/3)]/Sqrt[3]] - 2*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)] + Log[(a + I*b)^(2/3) + (a + I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)])) + 3*(a + b*Tan[c + d*x])^(1/3)))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.15, size = 97, normalized size = 0.26

method	result
--------	--------

derivativedivides	$\frac{3B(a+b \tan(dx+c))^{\frac{1}{3}} + \left( \frac{\left( \frac{(Ab+aB)R^3 - a^2B - b^2B}{R^5 - R^2 a} \right) \ln \left( \frac{(a+b \tan(dx+c))^{\frac{1}{3}} - R}{R} \right)}{\sum_{-R=\text{RootOf}(-Z^6 - 2aZ^3 + a^2 + b^2)} -R^5 - R^2 a} \right)}{d}$
default	$\frac{3B(a+b \tan(dx+c))^{\frac{1}{3}} + \left( \frac{\left( \frac{(Ab+aB)R^3 - a^2B - b^2B}{R^5 - R^2 a} \right) \ln \left( \frac{(a+b \tan(dx+c))^{\frac{1}{3}} - R}{R} \right)}{\sum_{-R=\text{RootOf}(-Z^6 - 2aZ^3 + a^2 + b^2)} -R^5 - R^2 a} \right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] `1/d*(3*B*(a+b*tan(d*x+c))^(1/3)+1/2*sum(((A*b+B*a)*_R^3-a^2*B-b^2*B)/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2)))`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(1/3), x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 34380 vs. 2(281) = 562.

time = 152.03, size = 34380, normalized size = 91.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `1/4*(2*d*((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(1/6)*cos(2/3*arctan((d^6*sqrt(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6) + ((3*A^2*B - B^3)*a + (A^3 - 3*A*B^2)*b)*d^3)*sqrt(((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6)/((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2))*log(2*((A^6 - 2*A^4*B^2 - 3*A^2*B^4)*a - (3*A^5*B + 2*A^3*B^3 - A*B^5)*b)*d^4*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^(1/3))*sqrt(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(1/6)*sqrt(((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6)`

$$\begin{aligned}
& 4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6)*\sin(2/3*\arctan((d^6*\sqrt{((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6}) + ((3*A^2*B - B^3)*a + (A^3 - 3*A*B^2)*b)*d^3)*\sqrt{((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6})/((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2))) - 2*((A^8*B - 5*A^6*B^3 + 3*A^4*B^5 + 9*A^2*B^7)*a^2 - 2*(3*A^7*B^2 - 7*A^5*B^4 - 7*A^3*B^6 + 3*A*B^8)*a*b + (9*A^6*B^3 + 3*A^4*B^5 - 5*A^2*B^7 + B^9)*b^2)*d*((a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))^(1/3)*(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(1/6)*\cos(2/3*\arctan((d^6*\sqrt{((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6}) + ((3*A^2*B - B^3)*a + (A^3 - 3*A*B^2)*b)*d^3)*\sqrt{((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6})/((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2))) + ((A^8 - 5*A^6*B^2 + 3*A^4*B^4 + 9*A^2*B^6)*a^2 - 2*(3*A^7*B - 7*A^5*B^3 - 7*A^3*B^5 + 3*A*B^7)*a*b + (9*A^6*B^2 + 3*A^4*B^4 - 5*A^2*B^6 + B^8)*b^2)*d^2*(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(1/3) + ((A^10 - 4*A^8*B^2 - 2*A^6*B^4 + 12*A^4*B^6 + 9*A^2*B^8)*a^2 - 2*(3*A^9*B - 4*A^7*B^3 - 14*A^5*B^5 - 4*A^3*B^7 + 3*A*B^9)*a*b + (9*A^8*B^2 + 12*A^6*B^4 - 2*A^4*B^6 - 4*A^2*B^8 + B^10)*b^2)*((a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))^(2/3)) - 8*d*(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(1/6)*\arctan(-(((A^6 - 2*A^4*B^2 - 3*A^2*B^4)*a - (3*A^5*B + 2*A^3*B^3 - A*B^5)*b)*d^8*((a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))^(1/3)*(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(5/6)*\sqrt{((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6}*\cos(2/3*\arctan((d^6*\sqrt{((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6}) + ((3*A^2*B - B^3)*a + (A^3 - 3*A*B^2)*b)*d^3)*\sqrt{((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6})/((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2))) - ((A^10*B - 6*A^6*B^5 - 8*A^4*B^7 - 3*A^2*B^9)*a^3 - (3*A^9*B^2 + 8*A^7*B^4 + 6*A^5*B^6 - A*B^10)*a^2*b + (A^10*B - 6*A^6*B^5 - 8*A^4*B^7 - 3*A^2*B^9)*a*b^2 - (3*A^9*B^2 + 8*A^7*B^4 + 6*A^5*B^6 - A*B^10)*b^3)*d^3*\sqrt{((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6} - (((A^8*B - 5*A^6*B^3 + 3*A^4*B^5 + 9*A^2*B^7)*a^2 - 2*(3*A^7*B^2 - 7*A^5*B^4 - 7*A^3*B^6 + 3*A*B^8)*a*b + (9*A^6*B^3 + 3*A^4*B^5 - 5*A^2*B^7 + B^9)*b^2)*d^5*((a*\cos(d*x + c) + b*\sin(d*x + c))/\cos(d*x + c))^(1/3)*(((A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*a^2 + (A^6 + 3*A^4*B^2 + 3*A^2*B^4 + B^6)*b^2)/d^6)^(5/6) - ((A^14 - 2*A^12*B^2 - 9*A^10*B^4 + 4*A^8*B^6 + 31*A^6*B^8 + 30*A^4*B^10 + 9*A^2*B^12)*a^4 - 2*(3*A^13*B + 2*A^11*B^3 - 19*A^9*B^5 - 36*A^7*B^7 - 19*A^5*B^9 + 2*
\end{aligned}$$



$$A^3B^{11} + 3A^2B^{13})a^3b + (A^{14} + 7A^{12}B^2 + 21A^{10}B^4 + 35A^8B^6 + 35A^6B^8 + 21A^4B^{10} + 7A^2B^{12} + B^{14})a^2b^2 - 2(3A^{13}B + 2A^{11}B^3 - 19A^9B^5 - 36A^7B^7 - 19A^5B^9 + 2A^3B^{11} + 3A^2B^{13})a^2b^3 + (9A^{12}B^2 + 30A^{10}B^4 + 31A^8B^6 + 4A^6B^8 - 9A^4B^{10} - 2A^2B^{12} + B^{14})b^4) \cos(2/3 \arctan((d^6 \sqrt{((A^6 + 3A^4B^2 + 3A^2B^4 + B^6)a^2 + (A^6 + 3A^4B^2 + 3A^2B^4 + B^6)b^2)/d^6}) + ((3A^2B - B^3)a + (A^3 - 3A^2B^2)b)d^3) \sqrt{((A^6 - 6A^4B^2 + 9A^2B^4)a^2 - 2(3A^5B - 10A^3B^3 + 3A^2B^5)a^2b + (9A^4B^2 - 6A^2B^4 + B^6)b^2)/d^6}) / ((A^6 - 6A^4B^2 + 9A^2B^4)a^2 - 2(3A^5B - 10A^3B^3 + 3A^2B^5)a^2b + (9A^4B^2 - 6A^2B^4 + B^6)b^2)) \sin(2/3 \arctan((d^6 \sqrt{((A^6 + 3A^4B^2 + 3A^2B^4 + B^6)a^2 + (A^6 + 3A^4B^2 + 3A^2B^4 + B^6)b^2)/d^6}) + ((3A^2B - B^3)a + (A^3 - 3A^2B^2)b \dots$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt[3]{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/3)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(1/3), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/3)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] undef

**Mupad** [B]

time = 17.68, size = 2537, normalized size = 6.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/3),x)

[Out] log(a\*d^7\*(((A^6\*a^2\*d^6)^(1/2) - A^3\*b\*d^3)/d^6)^(4/3) + A\*b\*(a + b\*tan(c + d\*x))^(1/3)\*(-A^6\*a^2\*d^6)^(1/2) - A^4\*a^2\*d^3\*(a + b\*tan(c + d\*x))^(1/3) + 2\*A^3\*a\*b\*d^4\*(((A^6\*a^2\*d^6)^(1/2) - A^3\*b\*d^3)/d^6)^(1/3))\*(((A^6\*a^2\*d^6)^(1/2) - A^3\*b\*d^3)/(8\*d^6))^(1/3) + log(A\*b\*(a + b\*tan(c + d\*x))^(1

$$\begin{aligned}
& /3)*(-A^6*a^2*d^6)^{(1/2)} + A^4*a^2*d^3*(a + b*\tan(c + d*x))^{(1/3)} + a*d*(- \\
& (-A^6*a^2*d^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(1/3)}*(-A^6*a^2*d^6)^{(1/2)} - A^3*a*b \\
& *d^4*(-((-A^6*a^2*d^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(1/3)}*(-((-A^6*a^2*d^6)^{(1/2)} \\
& /2) + A^3*b*d^3)/(8*d^6))^{(1/3)} + \log(d*((-B^6*b^2*d^6)^{(1/2)} + B^3*a*d^3)/ \\
& d^6)^{(1/3)} - B*(a + b*\tan(c + d*x))^{(1/3)}*(((-B^6*b^2*d^6)^{(1/2)} + B^3*a*d \\
& ^3)/(8*d^6))^{(1/3)} + \log(d*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3)/d^6)^{(1/3)} \\
& - B*(a + b*\tan(c + d*x))^{(1/3)})*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3)/(8*d^6 \\
& ))^{(1/3)} + (\log(- ((3^{(1/2)}*1i - 1)*((3^{(1/2)}*1i - 1)^2*(1944*a*b^4*(3^{(1/2)} \\
& /2)*1i - 1)*((-B^6*b^2*d^6)^{(1/2)} + B^3*a*d^3)/d^6)^{(1/3)}*(a^2 + b^2) - (38 \\
& 88*B*a*b^4*(a^2 + b^2)*(a + b*\tan(c + d*x))^{(1/3)})/d)*((-B^6*b^2*d^6)^{(1/2)} \\
& ) + B^3*a*d^3)/d^6)^{(2/3)}/16 - (972*B^3*b^4*(a^4 - b^4))/d^3)*((-B^6*b^2* \\
& d^6)^{(1/2)} + B^3*a*d^3)/d^6)^{(1/3)}/4 - (486*B^4*b^4*(a^4 - b^4)*(a + b*\tan \\
& (c + d*x))^{(1/3)})/d^4*(3^{(1/2)}*1i - 1)*((-B^6*b^2*d^6)^{(1/2)}/(8*d^6) + (B^ \\
& 3*a)/(8*d^3))^{(1/3)}/2 - (\log(- ((3^{(1/2)}*1i + 1)*((3^{(1/2)}*1i + 1)^2*(194 \\
& 4*a*b^4*(3^{(1/2)}*1i + 1)*((-B^6*b^2*d^6)^{(1/2)} + B^3*a*d^3)/d^6)^{(1/3)}*(a^ \\
& 2 + b^2) + (3888*B*a*b^4*(a^2 + b^2)*(a + b*\tan(c + d*x))^{(1/3)})/d)*((-B^6 \\
& *b^2*d^6)^{(1/2)} + B^3*a*d^3)/d^6)^{(2/3)}/16 + (972*B^3*b^4*(a^4 - b^4))/d^3 \\
& )*(((-B^6*b^2*d^6)^{(1/2)} + B^3*a*d^3)/d^6)^{(1/3)}/4 - (486*B^4*b^4*(a^4 - b \\
& ^4)*(a + b*\tan(c + d*x))^{(1/3)})/d^4*(3^{(1/2)}*1i + 1)*((-B^6*b^2*d^6)^{(1/2)} \\
& / (8*d^6) + (B^3*a)/(8*d^3))^{(1/3)}/2 + (\log(- ((3^{(1/2)}*1i - 1)*((3^{(1/2)}* \\
& 1i - 1)^2*(1944*a*b^4*(3^{(1/2)}*1i - 1)*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3) \\
& /d^6)^{(1/3)}*(a^2 + b^2) - (3888*B*a*b^4*(a^2 + b^2)*(a + b*\tan(c + d*x))^{(1 \\
& /3)}/d)*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3)/d^6)^{(2/3)}/16 - (972*B^3*b^4* \\
& (a^4 - b^4))/d^3)*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3)/d^6)^{(1/3)}/4 - (486 \\
& *B^4*b^4*(a^4 - b^4)*(a + b*\tan(c + d*x))^{(1/3)})/d^4*(3^{(1/2)}*1i - 1)*((B^ \\
& 3*a)/(8*d^3) - (-B^6*b^2*d^6)^{(1/2)}/(8*d^6))^{(1/3)}/2 - (\log(- ((3^{(1/2)}*1i \\
& + 1)*((3^{(1/2)}*1i + 1)^2*(1944*a*b^4*(3^{(1/2)}*1i + 1)*(-((-B^6*b^2*d^6)^{( \\
& 1/2)} - B^3*a*d^3)/d^6)^{(1/3)}*(a^2 + b^2) + (3888*B*a*b^4*(a^2 + b^2)*(a + b \\
& *tan(c + d*x))^{(1/3)})/d)*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3)/d^6)^{(2/3)}/1 \\
& 6 + (972*B^3*b^4*(a^4 - b^4))/d^3)*(-((-B^6*b^2*d^6)^{(1/2)} - B^3*a*d^3)/d^6 \\
& )^{(1/3)}/4 - (486*B^4*b^4*(a^4 - b^4)*(a + b*\tan(c + d*x))^{(1/3)})/d^4*(3^{( \\
& 1/2)}*1i + 1)*((B^3*a)/(8*d^3) - (-B^6*b^2*d^6)^{(1/2)}/(8*d^6))^{(1/3)}/2 + (3 \\
& *B*(a + b*\tan(c + d*x))^{(1/3)})/d + (\log((((3^{(1/2)}*1i - 1)^2*((3888*A*b^5* \\
& (a^2 + b^2)*(a + b*\tan(c + d*x))^{(1/3)})/d + 1944*a*b^4*(3^{(1/2)}*1i - 1)*((( \\
& -A^6*a^2*d^6)^{(1/2)} - A^3*b*d^3)/d^6)^{(1/3)}*(a^2 + b^2))*((-A^6*a^2*d^6)^{( \\
& 1/2)} - A^3*b*d^3)/d^6)^{(2/3)}/16 + (1944*A^3*a*b^5*(a^2 + b^2))/d^3*(3^{(1/ \\
& 2)}*1i - 1)*((-A^6*a^2*d^6)^{(1/2)} - A^3*b*d^3)/d^6)^{(1/3)}/4 - (486*A^4*b^4 \\
& *(a^4 - b^4)*(a + b*\tan(c + d*x))^{(1/3)})/d^4*(3^{(1/2)}*1i - 1)*((-A^6*a^2*d \\
& ^6)^{(1/2)}/(8*d^6) - (A^3*b)/(8*d^3))^{(1/3)}/2 + (\log((((3^{(1/2)}*1i - 1)^2* \\
& ((3888*A*b^5*(a^2 + b^2)*(a + b*\tan(c + d*x))^{(1/3)})/d + 1944*a*b^4*(3^{(1/2)} \\
& )*1i - 1)*(-((-A^6*a^2*d^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(1/3)}*(a^2 + b^2))*(- \\
& (-A^6*a^2*d^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(2/3)}/16 + (1944*A^3*a*b^5*(a^2 + b^ \\
& 2))/d^3*(3^{(1/2)}*1i - 1)*(-((-A^6*a^2*d^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(1/3)}/ \\
& 4 - (486*A^4*b^4*(a^4 - b^4)*(a + b*\tan(c + d*x))^{(1/3)})/d^4*(3^{(1/2)}*1i - \\
& 1)*(- (-A^6*a^2*d^6)^{(1/2)}/(8*d^6) - (A^3*b)/(8*d^3))^{(1/3)}/2 - (\log(- ((
\end{aligned}$$

$$\begin{aligned}
& ((3^{(1/2)}*1i + 1)^2*((3888*A*b^5*(a^2 + b^2)*(a + b*\tan(c + d*x))^{(1/3)})/d \\
& - 1944*a*b^4*(3^{(1/2)}*1i + 1)*((( -A^6*a^2*d^6)^{(1/2)} - A^3*b*d^3)/d^6)^{(1/3)} \\
& )*(a^2 + b^2))*((( -A^6*a^2*d^6)^{(1/2)} - A^3*b*d^3)/d^6)^{(2/3)}/16 + (1944*A \\
& ^3*a*b^5*(a^2 + b^2))/d^3*(3^{(1/2)}*1i + 1)*((( -A^6*a^2*d^6)^{(1/2)} - A^3*b* \\
& d^3)/d^6)^{(1/3)}/4 - (486*A^4*b^4*(a^4 - b^4)*(a + b*\tan(c + d*x))^{(1/3)})/d \\
& ^4*(3^{(1/2)}*1i + 1)*(( -A^6*a^2*d^6)^{(1/2)}/(8*d^6) - (A^3*b)/(8*d^3))^{(1/3)} \\
& )/2 - (\log(- (((3^{(1/2)}*1i + 1)^2*((3888*A*b^5*(a^2 + b^2)*(a + b*\tan(c + \\
& d*x))^{(1/3)})/d - 1944*a*b^4*(3^{(1/2)}*1i + 1)*(( -A^6*a^2*d^6)^{(1/2)} + A^3* \\
& b*d^3)/d^6)^{(1/3)}*(a^2 + b^2))*(( -A^6*a^2*d^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(2 \\
& /3)}/16 + (1944*A^3*a*b^5*(a^2 + b^2))/d^3*(3^{(1/2)}*1i + 1)*(( -A^6*a^2*d \\
& ^6)^{(1/2)} + A^3*b*d^3)/d^6)^{(1/3)}/4 - (486*A^4*b^4*(a^4 - b^4)*(a + b*\tan( \\
& c + d*x))^{(1/3)})/d^4*(3^{(1/2)}*1i + 1)*(( -A^6*a^2*d^6)^{(1/2)}/(8*d^6) - (A \\
& ^3*b)/(8*d^3))^{(1/3)}/2
\end{aligned}$$

$$3.475 \quad \int \frac{A+B \tan(c+dx)}{\sqrt[3]{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=357

$$\frac{(A-iB)x}{4\sqrt[3]{a-ib}} - \frac{(A+iB)x}{4\sqrt[3]{a+ib}} + \frac{\sqrt{3}(iA+B)\text{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2\sqrt[3]{a-ib}d} - \frac{\sqrt{3}(iA-B)\text{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a+ib}}\right)}{2\sqrt[3]{a+ib}d}$$

[Out]  $-1/4*(A-I*B)*x/(a-I*b)^{(1/3)}-1/4*(A+I*B)*x/(a+I*b)^{(1/3)}-1/4*(I*A-B)*\ln(\cos(d*x+c))/(a+I*b)^{(1/3)}/d+1/4*(I*A+B)*\ln(\cos(d*x+c))/(a-I*b)^{(1/3)}/d+3/4*(I*A+B)*\ln((a-I*b)^{(1/3)}-(a+b*\tan(d*x+c))^{(1/3)})/(a-I*b)^{(1/3)}/d-3/4*(I*A-B)*\ln((a+I*b)^{(1/3)}-(a+b*\tan(d*x+c))^{(1/3)})/(a+I*b)^{(1/3)}/d+1/2*(I*A+B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)})/(a-I*b)^{(1/3}))*3^{(1/2)}*3^{(1/2)}/(a-I*b)^{(1/3)}/d-1/2*(I*A-B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)})/(a+I*b)^{(1/3}))*3^{(1/2)}*3^{(1/2)}/(a+I*b)^{(1/3)}/d$

**Rubi [A]**

time = 0.19, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {3620, 3618, 57, 631, 210, 31}

$$\frac{\sqrt{3}(B+iA)\text{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{24\sqrt[3]{a-ib}} - \frac{\sqrt{3}(-B+iA)\text{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a+ib}}\right)}{24\sqrt[3]{a+ib}} + \frac{3(B+iA)\log(-\sqrt[3]{a+b \tan(c+dx)}+\sqrt[3]{a-ib})}{4d\sqrt[3]{a-ib}} - \frac{3(-B+iA)\log(-\sqrt[3]{a+b \tan(c+dx)}+\sqrt[3]{a+ib})}{4d\sqrt[3]{a+ib}} - \frac{(-B+iA)\log(\cos(c+dx))}{4d\sqrt[3]{a+ib}} + \frac{(B+iA)\log(\cos(c+dx))}{4d\sqrt[3]{a-ib}} - \frac{\pi(A-iB)}{4\sqrt[3]{a-ib}} - \frac{\pi(A+iB)}{4\sqrt[3]{a+ib}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(1/3), x]

[Out]  $-1/4*((A-I*B)*x)/(a-I*b)^{(1/3)} - ((A+I*B)*x)/(4*(a+I*b)^{(1/3)}) + (Sqrt[3]*(I*A+B)*ArcTan[(1+(2*(a+b*Tan[c+d*x])^{(1/3)})/(a-I*b)^{(1/3)})]/Sqrt[3])/(2*(a-I*b)^{(1/3)*d)} - (Sqrt[3]*(I*A-B)*ArcTan[(1+(2*(a+b*Tan[c+d*x])^{(1/3)})/(a+I*b)^{(1/3)})]/Sqrt[3])/(2*(a+I*b)^{(1/3)*d)} - ((I*A-B)*Log[Cos[c+d*x]])/(4*(a+I*b)^{(1/3)*d)} + ((I*A+B)*Log[Cos[c+d*x]])/(4*(a-I*b)^{(1/3)*d)} + (3*(I*A+B)*Log[(a-I*b)^{(1/3)}-(a+b*Tan[c+d*x])^{(1/3)}])/4*(a-I*b)^{(1/3)*d} - (3*(I*A-B)*Log[(a+I*b)^{(1/3)}-(a+b*Tan[c+d*x])^{(1/3)}])/4*(a+I*b)^{(1/3)*d}$

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(1/3)), x_Symbol] := With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

### Rule 210

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 631

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx &= \frac{1}{2}(A - iB) \int \frac{1 + i \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx + \frac{1}{2}(A + iB) \int \frac{1 - i \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx \\
&= -\frac{(iA - B) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{a + ibx}} dx, x, -i \tan(c + dx)\right)}{2d} + \frac{(iA + B) \text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{a + ibx}} dx, x, i \tan(c + dx)\right)}{2d} \\
&= -\frac{(A - iB)x}{4\sqrt[3]{a - ib}} - \frac{(A + iB)x}{4\sqrt[3]{a + ib}} - \frac{(iA - B) \log(\cos(c + dx))}{4\sqrt[3]{a + ib} d} + \frac{(iA + B) \log(\cos(c + dx))}{4\sqrt[3]{a - ib} d} \\
&= -\frac{(A - iB)x}{4\sqrt[3]{a - ib}} - \frac{(A + iB)x}{4\sqrt[3]{a + ib}} - \frac{(iA - B) \log(\cos(c + dx))}{4\sqrt[3]{a + ib} d} + \frac{(iA + B) \log(\cos(c + dx))}{4\sqrt[3]{a - ib} d} \\
&\quad + \frac{\sqrt{3} (iA + B) \tan^{-1}\left(\frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}}\right)}{2\sqrt[3]{a - ib} d} \\
&= -\frac{(A - iB)x}{4\sqrt[3]{a - ib}} - \frac{(A + iB)x}{4\sqrt[3]{a + ib}} + \frac{\sqrt{3} (iA + B) \tan^{-1}\left(\frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}}\right)}{2\sqrt[3]{a - ib} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.30, size = 227, normalized size = 0.64

$$\frac{i \left( \frac{(A-iB) \left( 2\sqrt{3} \operatorname{ArcTan} \left( \frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}} \right) - \log(i + \tan(c + dx)) + 3 \log(\sqrt{a - ib} - \sqrt[3]{a + b \tan(c + dx)}) \right)}{\sqrt[3]{a - ib}} - \frac{(A+iB) \left( 2\sqrt{3} \operatorname{ArcTan} \left( \frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a + ib}} \right) - \log(i - \tan(c + dx)) + 3 \log(\sqrt{a + ib} - \sqrt[3]{a + b \tan(c + dx)}) \right)}{\sqrt[3]{a + ib}} \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(1/3), x]

[Out] ((I/4)\*(((A - I\*B)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(a + b\*Tan[c + d\*x])^(1/3)))/(a - I\*b)^(1/3)]/Sqrt[3]] - Log[I + Tan[c + d\*x]] + 3\*Log[(a - I\*b)^(1/3) - (a + b\*Tan[c + d\*x])^(1/3)]))/(a - I\*b)^(1/3) - ((A + I\*B)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(a + b\*Tan[c + d\*x])^(1/3)))/(a + I\*b)^(1/3)]/Sqrt[3]] - Log[I - Tan[c + d\*x]] + 3\*Log[(a + I\*b)^(1/3) - (a + b\*Tan[c + d\*x])^(1/3)]))/(a + I\*b)^(1/3)))/d

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.26, size = 72, normalized size = 0.20

method	result	size
--------	--------	------

derivativedivides	$\frac{\sum_{R=\text{RootOf}(\_Z^6-2a\_Z^3+a^2+b^2)} \frac{(B\_R^4+(Ab-aB)\_R) \ln((a+b \tan(dx+c))^{\frac{1}{3}}-\_R)}{\_R^5-\_R^2 a}}{2d}$	72
default	$\frac{\sum_{R=\text{RootOf}(\_Z^6-2a\_Z^3+a^2+b^2)} \frac{(B\_R^4+(Ab-aB)\_R) \ln((a+b \tan(dx+c))^{\frac{1}{3}}-\_R)}{\_R^5-\_R^2 a}}{2d}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)`

[Out] `1/2/d*sum((B*_R^4+(A*b-B*a)*_R)/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2))`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)/(b*tan(d*x + c) + a)^(1/3), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/3),x)`

[Out] `Integral((A + B*tan(c + d*x))/(a + b*tan(c + d*x))**(1/3), x)`

**Giac [A]**

time = 3.17, size = 147, normalized size = 0.41

$$\frac{\left(\frac{A^3-3iA^2B-3AB^2+iB^3}{8ia+8b}\right)^{\frac{1}{3}} \log\left(-a+ib - (-a^2+2iab+b^2)^{\frac{1}{3}}(b\tan(dx+c)+a)^{\frac{1}{3}}\right) + \left(-\frac{A^3+3iA^2B-3AB^2-iB^3}{8ia-8b}\right)^{\frac{1}{3}} \log\left(-a-ib + (a^2+2iab-b^2)^{\frac{1}{3}}(b\tan(dx+c)+a)^{\frac{1}{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/3),x, algorithm="giac")

**[Out]** (((A^3 - 3\*I\*A^2\*B - 3\*A\*B^2 + I\*B^3)/(8\*I\*a + 8\*b))^(1/3)\*log(-a + I\*b - (-a^2 + 2\*I\*a\*b + b^2)^(1/3)\*(b\*tan(d\*x + c) + a)^(1/3)) + (- (A^3 + 3\*I\*A^2\*B - 3\*A\*B^2 - I\*B^3)/(8\*I\*a - 8\*b))^(1/3)\*log(-a - I\*b + (a^2 + 2\*I\*a\*b - b^2)^(1/3)\*(b\*tan(d\*x + c) + a)^(1/3)))/d

**Mupad [B]**

time = 18.25, size = 2500, normalized size = 7.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((A + B\*tan(c + d\*x))/(a + b\*tan(c + d\*x))^(1/3),x)

**[Out]** log(((((-B^6\*b^2\*d^6)^(1/2) + B^3\*a\*d^3)/(d^6\*(a^2 + b^2)))^(2/3)\*(972\*a\*b^4\*(-B^6\*b^2\*d^6)^(1/2) - 972\*B^3\*b^6\*d^3 + 972\*B^2\*b^6\*d^4\*(a + b\*tan(c + d\*x))^(1/3)\*((( -B^6\*b^2\*d^6)^(1/2) + B^3\*a\*d^3)/(d^6\*(a^2 + b^2)))^(1/3) - 972\*B^2\*a^2\*b^4\*d^4\*(a + b\*tan(c + d\*x))^(1/3)\*((( -B^6\*b^2\*d^6)^(1/2) + B^3\*a\*d^3)/(d^6\*(a^2 + b^2)))^(1/3)))/(4\*d^6) + (243\*B^5\*a\*b^4\*(a + b\*tan(c + d\*x))^(1/3))/d^5)\*((-64\*B^6\*b^2\*d^6)^(1/2)/(64\*(a^2\*d^6 + b^2\*d^6)) + (B^3\*a\*d^3)/(8\*(a^2\*d^6 + b^2\*d^6)))^(1/3) + log(((1944\*a\*b^4\*(a^2 + b^2)\*((-A^6\*a^2\*d^6)^(1/2) + A^3\*b\*d^3)/(d^6\*(a^2 + b^2)))^(2/3) + (1944\*A^2\*b^4\*(a^2 - b^2)\*(a + b\*tan(c + d\*x))^(1/3))/d^2)\*((-A^6\*a^2\*d^6)^(1/2) + A^3\*b\*d^3))/(8\*d^6\*(a^2 + b^2)) + (243\*A^5\*b^5\*(a + b\*tan(c + d\*x))^(1/3))/d^5)\*((-64\*A^6\*a^2\*d^6)^(1/2)/(64\*(a^2\*d^6 + b^2\*d^6)) + (A^3\*b\*d^3)/(8\*(a^2\*d^6 + b^2\*d^6)))^(1/3) + log((243\*B^5\*a\*b^4\*(a + b\*tan(c + d\*x))^(1/3))/d^5 - ((-8\*(-B^6\*b^2\*d^6)^(1/2) - 8\*B^3\*a\*d^3)/(d^6\*(a^2 + b^2)))^(2/3)\*(972\*a\*b^4\*(-B^6\*b^2\*d^6)^(1/2) + 972\*B^3\*b^6\*d^3 - 486\*B^2\*b^6\*d^4\*(a + b\*tan(c + d\*x))^(1/3)\*((-8\*(-B^6\*b^2\*d^6)^(1/2) - 8\*B^3\*a\*d^3)/(d^6\*(a^2 + b^2)))^(1/3) + 486\*B^2\*a^2\*b^4\*d^4\*(a + b\*tan(c + d\*x))^(1/3)\*((-8\*(-B^6\*b^2\*d^6)^(1/2) - 8\*B^3\*a\*d^3)/(d^6\*(a^2 + b^2)))^(1/3)))/(16\*d^6))\*((B^3\*a\*d^3)/(8\*(a^2\*d^6 + b^2\*d^6)) - (-64\*B^6\*b^2\*d^6)^(1/2)/(64\*(a^2\*d^6 + b^2\*d^6)))^(1/3) + log((243\*A^5\*b^5\*(a + b\*tan(c + d\*x))^(1/3))/d^5 - ((486\*a\*b^4\*(a^2 + b^2)\*((-A^6\*a^2\*d^6)^(1/2) - 8\*A^3\*b\*d^3)/(d^6\*(a^2 + b^2)))^(2/3) + (1944\*A^2\*b^4\*(a^2 - b^2)\*(a + b\*tan(c + d\*x))^(1/3))/d^2)\*((-A^6\*a^2\*d^6)^(1/2) - A^3\*b\*d^3))/(8\*d^6\*(a^2 + b^2))\*((A^3\*b\*d^3)/(8\*(a^2\*d^6 + b^2\*d^6)) - (-64\*A^6\*a^2\*d^6)^(1/2)/(64\*(a^2\*d^6 + b^2\*d^6)))^(1/3) - log((((486\*a\*b^4\*((3^(1/2)\*1i)/2 - 1/2)\*(a^2 + b^2)\*((-A^6\*a^2\*d^6)^(1/2) + 8\*A^3\*b\*d^3)/(d^6\*(a





$$3.476 \quad \int \frac{A+B \tan(c+dx)}{(a+b \tan(c+dx))^{2/3}} dx$$

Optimal. Leaf size=357

$$\frac{(A-ib)x}{4(a-ib)^{2/3}} - \frac{(A+ib)x}{4(a+ib)^{2/3}} - \frac{\sqrt{3}(iA+B) \operatorname{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2(a-ib)^{2/3}d} + \frac{\sqrt{3}(iA-B) \operatorname{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a+ib}}\right)}{2(a+ib)^{2/3}d}$$

[Out]  $-1/4*(A-I*B)*x/(a-I*b)^{(2/3)}-1/4*(A+I*B)*x/(a+I*b)^{(2/3)}-1/4*(I*A-B)*\ln(\cos(d*x+c))/(a+I*b)^{(2/3)}/d+1/4*(I*A+B)*\ln(\cos(d*x+c))/(a-I*b)^{(2/3)}/d+3/4*(I*A+B)*\ln((a-I*b)^{(1/3)}-(a+b*\tan(d*x+c))^{(1/3)})/(a-I*b)^{(2/3)}/d-3/4*(I*A-B)*\ln((a+I*b)^{(1/3)}-(a+b*\tan(d*x+c))^{(1/3)})/(a+I*b)^{(2/3)}/d-1/2*(I*A+B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)})/(a-I*b)^{(1/3}))*3^{(1/2)})/(a-I*b)^{(2/3)}/d+1/2*(I*A-B)*\arctan(1/3*(1+2*(a+b*\tan(d*x+c))^{(1/3)})/(a+I*b)^{(1/3}))*3^{(1/2)})/(a+I*b)^{(2/3)}/d$

Rubi [A]

time = 0.20, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {3620, 3618, 59, 631, 210, 31}

$$\frac{\sqrt{3}(B+iA) \operatorname{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a-ib}}\right)}{2d(a-ib)^{2/3}} + \frac{\sqrt{3}(-B+iA) \operatorname{ArcTan}\left(\frac{1+\sqrt[3]{a+b \tan(c+dx)}}{\sqrt[3]{a+ib}}\right)}{2d(a+ib)^{2/3}} + \frac{3(B+iA) \log\left(\frac{-\sqrt[3]{a+b \tan(c+dx)} + \sqrt[3]{a-ib}}{4d(a-ib)^{2/3}}\right)}{4d(a-ib)^{2/3}} - \frac{3(-B+iA) \log\left(\frac{-\sqrt[3]{a+b \tan(c+dx)} + \sqrt[3]{a+ib}}{4d(a+ib)^{2/3}}\right)}{4d(a+ib)^{2/3}} - \frac{(-B+iA) \log(\cos(c+dx))}{4d(a-ib)^{2/3}} + \frac{(B+iA) \log(\cos(c+dx))}{4d(a+ib)^{2/3}} - \frac{\pi(A-IB)}{4(a-ib)^{2/3}} - \frac{\pi(A+IB)}{4(a+ib)^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(a+b*\operatorname{Tan}[c+d*x])^{(2/3)},x]$

[Out]  $-1/4*((A-I*B)*x)/(a-I*b)^{(2/3)} - ((A+I*B)*x)/(4*(a+I*b)^{(2/3)}) - (\operatorname{Sqrt}[3]*(I*A+B)*\operatorname{ArcTan}[(1+(2*(a+b*\operatorname{Tan}[c+d*x])^{(1/3)})/(a-I*b)^{(1/3)})]/\operatorname{Sqrt}[3])/(2*(a-I*b)^{(2/3)*d}) + (\operatorname{Sqrt}[3]*(I*A-B)*\operatorname{ArcTan}[(1+(2*(a+b*\operatorname{Tan}[c+d*x])^{(1/3)})/(a+I*b)^{(1/3)})]/\operatorname{Sqrt}[3])/(2*(a+I*b)^{(2/3)*d}) - ((I*A-B)*\operatorname{Log}[\operatorname{Cos}[c+d*x]])/(4*(a+I*b)^{(2/3)*d}) + ((I*A+B)*\operatorname{Log}[\operatorname{Cos}[c+d*x]])/(4*(a-I*b)^{(2/3)*d}) + (3*(I*A+B)*\operatorname{Log}[(a-I*b)^{(1/3)} - (a+b*\operatorname{Tan}[c+d*x])^{(1/3)}])/(4*(a-I*b)^{(2/3)*d}) - (3*(I*A-B)*\operatorname{Log}[(a+I*b)^{(1/3)} - (a+b*\operatorname{Tan}[c+d*x])^{(1/3)}])/(4*(a+I*b)^{(2/3)*d})$

Rule 31

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 59

$\operatorname{Int}[1/(((a_.) + (b_.)*(x_.)^{(c_.)}) + (d_.)*(x_.)^{(2/3)}), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[(b*c - a*d)/b, 3]\}, \operatorname{Simp}[-\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/(2*b*q^2),$

$x] + (-\text{Dist}[3/(2*b*q), \text{Subst}[\text{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{(1/3)}], x] - \text{Dist}[3/(2*b*q^2), \text{Subst}[\text{Int}[1/(q - x), x], x, (c + d*x)^{(1/3)}], x]) /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{PosQ}[(b*c - a*d)/b]$

### Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 631

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 3618

$\text{Int}[(a_ + (b_)*\tan[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\tan[(e_ + (f_)*(x_))]), x\_Symbol] \rightarrow \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[(a_ + (b_)*\tan[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\tan[(e_ + (f_)*(x_))]), x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m*(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{2/3}} dx &= \frac{1}{2}(A - iB) \int \frac{1 + i \tan(c + dx)}{(a + b \tan(c + dx))^{2/3}} dx + \frac{1}{2}(A + iB) \int \frac{1 - i \tan(c + dx)}{(a + b \tan(c + dx))^{2/3}} dx \\
&= -\frac{(iA - B) \operatorname{Subst}\left(\int \frac{1}{(-1+x)(a+ibx)^{2/3}} dx, x, -i \tan(c + dx)\right)}{2d} + \frac{(iA + B) \operatorname{Subst}\left(\int \frac{1}{(-1+x)(a+ibx)^{2/3}} dx, x, i \tan(c + dx)\right)}{2d} \\
&= -\frac{(A - iB)x}{4(a - ib)^{2/3}} - \frac{(A + iB)x}{4(a + ib)^{2/3}} - \frac{(iA - B) \log(\cos(c + dx))}{4(a + ib)^{2/3}d} + \frac{(iA + B) \log(\cos(c + dx))}{4(a - ib)^{2/3}d} \\
&= -\frac{(A - iB)x}{4(a - ib)^{2/3}} - \frac{(A + iB)x}{4(a + ib)^{2/3}} - \frac{(iA - B) \log(\cos(c + dx))}{4(a + ib)^{2/3}d} + \frac{(iA + B) \log(\cos(c + dx))}{4(a - ib)^{2/3}d} \\
&= -\frac{(A - iB)x}{4(a - ib)^{2/3}} - \frac{(A + iB)x}{4(a + ib)^{2/3}} - \frac{\sqrt{3} (iA + B) \tan^{-1}\left(\frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}}\right)}{2(a - ib)^{2/3}d}
\end{aligned}$$

**Mathematica [A]**

time = 0.16, size = 305, normalized size = 0.85

$$\left( \frac{(A-iB) \left( 2\sqrt{3} \operatorname{ArcTan}\left(\frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}}\right) - 2 \log\left(\frac{\sqrt{a - ib} - \sqrt{a + b \tan(c + dx)}}{\sqrt{a - ib} + \sqrt{a + b \tan(c + dx)}}\right) + \log\left(\frac{(a - ib)^{2/3} + \sqrt{a - ib} \sqrt{a + b \tan(c + dx)}}{(a + ib)^{2/3} + \sqrt{a + ib} \sqrt{a + b \tan(c + dx)}}\right) \right)}{(a - ib)^{2/3}} + \frac{(A+iB) \left( 2\sqrt{3} \operatorname{ArcTan}\left(\frac{1 + \sqrt[3]{a + b \tan(c + dx)}}{\sqrt[3]{a - ib}}\right) - 2 \log\left(\frac{\sqrt{a + ib} - \sqrt{a + b \tan(c + dx)}}{\sqrt{a + ib} + \sqrt{a + b \tan(c + dx)}}\right) + \log\left(\frac{(a + ib)^{2/3} + \sqrt{a + ib} \sqrt{a + b \tan(c + dx)}}{(a - ib)^{2/3} + \sqrt{a - ib} \sqrt{a + b \tan(c + dx)}}\right) \right)}{(a + ib)^{2/3}} \right) / d$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(a + b\*Tan[c + d\*x])^(2/3), x]

```
[Out] ((I/4)*(-(((A - I*B)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3)))/(a - I*b)^(1/3))/Sqrt[3]] - 2*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)]))/(a - I*b)^(2/3)) + ((A + I*B)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3)))/(a + I*b)^(1/3))/Sqrt[3]] - 2*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)] + Log[(a + I*b)^(2/3) + (a + I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)]))/(a + I*b)^(2/3)))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.30, size = 69, normalized size = 0.19

method	result	size
--------	--------	------

derivativedivides	$\frac{\sum_{R=\text{RootOf}(\_Z^6-2a\_Z^3+a^2+b^2)} \frac{(B\_R^3+Ab-aB) \ln((a+b \tan(dx+c))^{\frac{1}{3}}-\_R)}{\_R^5-\_R^2 a}}{2d}$	69
default	$\frac{\sum_{R=\text{RootOf}(\_Z^6-2a\_Z^3+a^2+b^2)} \frac{(B\_R^3+Ab-aB) \ln((a+b \tan(dx+c))^{\frac{1}{3}}-\_R)}{\_R^5-\_R^2 a}}{2d}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)`

[Out] `1/2/d*sum((B*_R^3+A*b-B*a)/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2))`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)/(b*tan(d*x + c) + a)^(2/3), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(2/3),x)`

[Out] `Integral((A + B*tan(c + d*x))/(a + b*tan(c + d*x))**(2/3), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] undef
```

**Mupad [B]**

time = 21.70, size = 2500, normalized size = 7.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(2/3),x)
```

```
[Out] log((((16*(-B^6*a^2*b^2*d^6)^(1/2) + 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(1/3)*(1944*a*b^4*(-B^6*a^2*b^2*d^6)^(1/2) - 1944*B^3*a*b^6*d^3 + 243*B*b^8*d^5*(a + b*tan(c + d*x))^(1/3)*((16*(-B^6*a^2*b^2*d^6)^(1/2) + 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3) - 243*B*a^4*b^4*d^5*(a + b*tan(c + d*x))^(1/3)*((16*(-B^6*a^2*b^2*d^6)^(1/2) + 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3)))/(4*d^6*(a^2 + b^2)) + (486*B^4*b^4*(a + b*tan(c + d*x))^(1/3))/d^4)*((((16*B^3*a^2*d^3 - 16*B^3*b^2*d^3)^2/4 - B^6*(64*a^4*d^6 + 64*b^4*d^6 + 128*a^2*b^2*d^6))^(1/2) + 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(64*(a^4*d^6 + b^4*d^6 + 2*a^2*b^2*d^6)))^(1/3) + log((486*B^4*b^4*(a + b*tan(c + d*x))^(1/3))/d^4 - (((-16*(-B^6*a^2*b^2*d^6)^(1/2) - 8*B^3*a^2*d^3 + 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(1/3)*(1944*a*b^4*(-B^6*a^2*b^2*d^6)^(1/2) + 1944*B^3*a*b^6*d^3 - 243*B*b^8*d^5*(a + b*tan(c + d*x))^(1/3)*(-16*(-B^6*a^2*b^2*d^6)^(1/2) - 8*B^3*a^2*d^3 + 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3) + 243*B*a^4*b^4*d^5*(a + b*tan(c + d*x))^(1/3)*(-16*(-B^6*a^2*b^2*d^6)^(1/2) - 8*B^3*a^2*d^3 + 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3)))/(4*d^6*(a^2 + b^2)))*(-((((16*B^3*a^2*d^3 - 16*B^3*b^2*d^3)^2/4 - B^6*(64*a^4*d^6 + 64*b^4*d^6 + 128*a^2*b^2*d^6))^(1/2) - 8*B^3*a^2*d^3 + 8*B^3*b^2*d^3)/(64*(a^4*d^6 + b^4*d^6 + 2*a^2*b^2*d^6)))^(1/3) + log((((1944*a*b^4*(a^2 + b^2)*((8*(-A^6*d^6*(a^2 - b^2)^2)^(1/2) + 16*A^3*a*b*d^3)/(d^6*(a^2 + b^2)^2))^(1/3) + (7776*A*a*b^5*(a + b*tan(c + d*x))^(1/3))/d)*((8*(-A^6*d^6*(a^2 - b^2)^2)^(1/2) + 16*A^3*a*b*d^3)/(d^6*(a^2 + b^2)^2))^(2/3))/16 - (972*A^3*b^5)/d^3)*((8*(-A^6*d^6*(a^2 - b^2)^2)^(1/2) + 16*A^3*a*b*d^3)/(d^6*(a^2 + b^2)^2))^(1/3))/4 - (486*A^4*b^4*(a + b*tan(c + d*x))^(1/3))/d^4)*(((256*A^6*a^2*b^2*d^6 - A^6*(64*a^4*d^6 + 64*b^4*d^6 + 128*a^2*b^2*d^6))^(1/2) + 16*A^3*a*b*d^3)/(64*(a^4*d^6 + b^4*d^6 + 2*a^2*b^2*d^6)))^(1/3) + log((((1944*a*b^4*(a^2 + b^2)*(-8*(-A^6*d^6*(a^2 - b^2)^2)^(1/2) - 16*A^3*a*b*d^3)/(d^6*(a^2 + b^2)^2))^(1/3) + (7776*A*a*b^5*(a
```

$$\begin{aligned}
& + b \cdot \tan(c + d \cdot x)^{(1/3)} / d \cdot (- (8 \cdot (-A^6 \cdot d^6 \cdot (a^2 - b^2)^2)^{(1/2)} - 16 \cdot A^3 \cdot a \cdot b \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(2/3)} / 16 - (972 \cdot A^3 \cdot b^5) / d^3 \cdot (- (8 \cdot (-A^6 \cdot d^6 \cdot (a^2 - b^2)^2)^{(1/2)} - 16 \cdot A^3 \cdot a \cdot b \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} / 4 - (486 \cdot A^4 \cdot b^4 \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d^4 \cdot (- ((256 \cdot A^6 \cdot a^2 \cdot b^2 \cdot d^6 - A^6 \cdot (64 \cdot a^4 \cdot d^6 + 64 \cdot b^4 \cdot d^6 + 128 \cdot a^2 \cdot b^2 \cdot d^6))^{(1/2)} - 16 \cdot A^3 \cdot a \cdot b \cdot d^3) / (64 \cdot (a^4 \cdot d^6 + b^4 \cdot d^6 + 2 \cdot a^2 \cdot b^2 \cdot d^6)))^{(1/3)} + \log((486 \cdot B^4 \cdot b^4 \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d^4 - ((3^{(1/2)} \cdot 1i - 1) \cdot (((3^{(1/2)} \cdot 1i + 1) \cdot (972 \cdot a \cdot b^4 \cdot (3^{(1/2)} \cdot 1i - 1) \cdot (a^2 + b^2) \cdot ((16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} - (3888 \cdot B \cdot b^4 \cdot (a^2 - b^2) \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d) \cdot ((16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(2/3)} / 32 + (972 \cdot B^3 \cdot a \cdot b^4) / d^3 \cdot ((16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} / 8) \cdot (((3^{(1/2)} \cdot ((-256 \cdot B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (64 \cdot a^4 \cdot d^6 + 64 \cdot b^4 \cdot d^6 + 128 \cdot a^2 \cdot b^2 \cdot d^6))^{(1/3)} \cdot 1i) / 2 - (((-256 \cdot B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (64 \cdot a^4 \cdot d^6 + 64 \cdot b^4 \cdot d^6 + 128 \cdot a^2 \cdot b^2 \cdot d^6))^{(1/3)} / 2) + \log((486 \cdot B^4 \cdot b^4 \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d^4 - ((3^{(1/2)} \cdot 1i - 1) \cdot (((3^{(1/2)} \cdot 1i + 1) \cdot (972 \cdot a \cdot b^4 \cdot (3^{(1/2)} \cdot 1i - 1) \cdot (a^2 + b^2) \cdot (- (16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} - (3888 \cdot B \cdot b^4 \cdot (a^2 - b^2) \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d) \cdot (- (16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(2/3)} / 32 + (972 \cdot B^3 \cdot a \cdot b^4) / d^3 \cdot (- (16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} / 8) \cdot ((3^{(1/2)} \cdot (- ((-256 \cdot B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (64 \cdot a^4 \cdot d^6 + 64 \cdot b^4 \cdot d^6 + 128 \cdot a^2 \cdot b^2 \cdot d^6))^{(1/3)} \cdot 1i) / 2 - (- ((-256 \cdot B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (64 \cdot a^4 \cdot d^6 + 64 \cdot b^4 \cdot d^6 + 128 \cdot a^2 \cdot b^2 \cdot d^6))^{(1/3)} / 2) - \log(- (((3^{(1/2)} \cdot 1i) / 2 + 1/2) \cdot (((3^{(1/2)} \cdot 1i) / 2 - 1/2) \cdot (1944 \cdot a \cdot b^4 \cdot ((3^{(1/2)} \cdot 1i) / 2 + 1/2) \cdot (a^2 + b^2) \cdot ((16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} + (3888 \cdot B \cdot b^4 \cdot (a^2 - b^2) \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d) \cdot ((16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(2/3)} / 16 + (972 \cdot B^3 \cdot a \cdot b^4) / d^3 \cdot ((16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} / 4 - (486 \cdot B^4 \cdot b^4 \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d^4) \cdot ((3^{(1/2)} \cdot 1i) / 2 + 1/2) \cdot (((16 \cdot B^3 \cdot a^2 \cdot d^3 - 16 \cdot B^3 \cdot b^2 \cdot d^3)^2 / 4 - B^6 \cdot (64 \cdot a^4 \cdot d^6 + 64 \cdot b^4 \cdot d^6 + 128 \cdot a^2 \cdot b^2 \cdot d^6))^{(1/2)} + 8 \cdot B^3 \cdot a^2 \cdot d^3 - 8 \cdot B^3 \cdot b^2 \cdot d^3) / (64 \cdot (a^4 \cdot d^6 + b^4 \cdot d^6 + 2 \cdot a^2 \cdot b^2 \cdot d^6)))^{(1/3)} - \log(- (((3^{(1/2)} \cdot 1i) / 2 + 1/2) \cdot (((3^{(1/2)} \cdot 1i) / 2 - 1/2) \cdot (1944 \cdot a \cdot b^4 \cdot ((3^{(1/2)} \cdot 1i) / 2 + 1/2) \cdot (a^2 + b^2) \cdot (- (16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(1/3)} + (3888 \cdot B \cdot b^4 \cdot (a^2 - b^2) \cdot (a + b \cdot \tan(c + d \cdot x))^{(1/3)} / d) \cdot (- (16 \cdot (-B^6 \cdot a^2 \cdot b^2 \cdot d^6)^{(1/2)} - 8 \cdot B^3 \cdot a^2 \cdot d^3 + 8 \cdot B^3 \cdot b^2 \cdot d^3) / (d^6 \cdot (a^2 + b^2)^2))^{(2/3)} / 16 + (972 \cdot B^3 \cdot a \cdot b^4) / \dots
\end{aligned}$$

$$3.477 \quad \int \frac{i - \tan(e + fx)}{\sqrt[3]{c + d \tan(e + fx)}} dx$$

Optimal. Leaf size=148

$$\frac{\frac{ix}{2\sqrt[3]{c-id}} - \frac{\sqrt{3} \operatorname{ArcTan}\left(\frac{1 + \frac{\sqrt[3]{c+d \tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{\sqrt[3]{c-id} f} - \frac{\log(\cos(e+fx))}{2\sqrt[3]{c-id} f} - \frac{3 \log\left(\sqrt[3]{c-id} - \sqrt[3]{c+d \tan(e+fx)}\right)}{2\sqrt[3]{c-id} f}}$$

[Out]  $-1/2*I*x/(c-I*d)^{(1/3)} - 1/2*\ln(\cos(f*x+e))/(c-I*d)^{(1/3)}/f - 3/2*\ln((c-I*d)^{(1/3)} - (c+d*\tan(f*x+e))^{(1/3)})/(c-I*d)^{(1/3)}/f - \arctan(1/3*(1+2*(c+d*\tan(f*x+e))^{(1/3)})/(c-I*d)^{(1/3)})*3^{(1/2)}*3^{(1/2)}/(c-I*d)^{(1/3)}/f$

Rubi [A]

time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {3618, 57, 631, 210, 31}

$$\frac{\sqrt{3} \operatorname{ArcTan}\left(\frac{1 + \frac{\sqrt[3]{c+d \tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{f\sqrt[3]{c-id}} - \frac{3 \log\left(-\sqrt[3]{c+d \tan(e+fx)} + \sqrt[3]{c-id}\right)}{2f\sqrt[3]{c-id}} - \frac{\log(\cos(e+fx))}{2f\sqrt[3]{c-id}} - \frac{ix}{2\sqrt[3]{c-id}}$$

Antiderivative was successfully verified.

[In] `Int[(I - Tan[e + f*x])/(c + d*Tan[e + f*x])^(1/3), x]`

[Out]  $((-1/2*I)*x)/(c - I*d)^{(1/3)} - (\operatorname{Sqrt}[3]*\operatorname{ArcTan}[(1 + (2*(c + d*\operatorname{Tan}[e + f*x])^{(1/3)})/(c - I*d)^{(1/3)})/\operatorname{Sqrt}[3]])/(c - I*d)^{(1/3)*f} - \operatorname{Log}[\operatorname{Cos}[e + f*x]]/(2*(c - I*d)^{(1/3)*f} - (3*\operatorname{Log}[(c - I*d)^{(1/3)} - (c + d*\operatorname{Tan}[e + f*x])^{(1/3)}])/(2*(c - I*d)^{(1/3)*f})$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 57

`Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] := With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]`



Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{i - \tan(e + fx)}{\sqrt[3]{c + d \tan(e + fx)}} dx &= -\frac{i \operatorname{Subst}\left(\int \frac{1}{(1+ix)\sqrt[3]{c-dx}} dx, x, -\tan(e+fx)\right)}{f} \\ &= -\frac{ix}{2\sqrt[3]{c-id}} - \frac{\log(\cos(e+fx))}{2\sqrt[3]{c-id}f} - \frac{3 \operatorname{Subst}\left(\int \frac{1}{(c-id)^{2/3} + \sqrt[3]{c-id} x + x^2} dx, x, \sqrt[3]{c+ d \tan(e+fx)}\right)}{2f} \\ &= -\frac{ix}{2\sqrt[3]{c-id}} - \frac{\log(\cos(e+fx))}{2\sqrt[3]{c-id}f} - \frac{3 \log\left(\sqrt[3]{c-id} - \sqrt[3]{c+ d \tan(e+fx)}\right)}{2\sqrt[3]{c-id}f} \\ &= -\frac{ix}{2\sqrt[3]{c-id}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{c+ d \tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{\sqrt[3]{c-id}f} - \frac{\log(\cos(e+fx))}{2\sqrt[3]{c-id}f} \end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.98, size = 109, normalized size = 0.74

$$\frac{3 \left( c - \frac{id(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}} \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, 1; \frac{5}{3}; \frac{ic + \frac{d(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}}{ic+d} \right)}{2(c-id)f}$$

Antiderivative was successfully verified.

[In] Integrate[(I - Tan[e + f\*x])/(c + d\*Tan[e + f\*x])^(1/3), x]

[Out] (3\*(c - (I\*d\*(-1 + E^((2\*I)\*(e + f\*x))))/(1 + E^((2\*I)\*(e + f\*x))))^(2/3)\*Hypergeometric2F1[2/3, 1, 5/3, (I\*c + (d\*(-1 + E^((2\*I)\*(e + f\*x))))/(1 + E^((2\*I)\*(e + f\*x))))/(I\*c + d)]/(2\*(c - I\*d)\*f)

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 0.34, size = 42, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sum_{-R=\text{RootOf}(\_Z^3+id-c)} \frac{\ln\left((c+d \tan(fx+e))^{\frac{1}{3}} - \_R\right)}{-R}}{f}$	42
default	$-\frac{\sum_{-R=\text{RootOf}(\_Z^3+id-c)} \frac{\ln\left((c+d \tan(fx+e))^{\frac{1}{3}} - \_R\right)}{-R}}{f}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int((I-tan(f\*x+e))/(c+d\*tan(f\*x+e))^(1/3), x, method=\_RETURNVERBOSE)

[Out] -1/f\*sum(1/\_R\*ln((c+d\*tan(f\*x+e))^(1/3)-\_R), \_R=RootOf(\_Z^3+I\*d-c))

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((I-tan(f\*x+e))/(c+d\*tan(f\*x+e))^(1/3), x, algorithm="maxima")

[Out] -integrate((tan(f\*x + e) - I)/(d\*tan(f\*x + e) + c)^(1/3), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(112) = 224.

time = 1.93, size = 284, normalized size = 1.92

$$\frac{1}{2}(\sqrt{3}-1)\left(\frac{i}{ic+d\sqrt{f}}\right)^{\frac{1}{3}} \log\left(\frac{1}{2}(\sqrt{3}(ic+d\sqrt{f})+(c-id\sqrt{f})\left(-\frac{i}{ic+d\sqrt{f}}\right)^{\frac{1}{3}}+\left(\frac{(c-id\sqrt{f})^{\frac{2}{3}}(ic+d\sqrt{f})}{2^{\frac{2}{3}}(ic+d\sqrt{f})}\right)^{\frac{1}{3}})\right)+\frac{1}{2}(-\sqrt{3}-1)\left(\frac{i}{ic+d\sqrt{f}}\right)^{\frac{1}{3}} \log\left(\frac{1}{2}(\sqrt{3}(-ic-d\sqrt{f})+(c-id\sqrt{f})\left(-\frac{i}{ic+d\sqrt{f}}\right)^{\frac{1}{3}}+\left(\frac{(c-id\sqrt{f})^{\frac{2}{3}}(ic+d\sqrt{f})}{2^{\frac{2}{3}}(ic+d\sqrt{f})}\right)^{\frac{1}{3}})\right)+\left(\frac{i}{ic+d\sqrt{f}}\right)^{\frac{1}{3}} \log\left(-\frac{i}{ic+d\sqrt{f}}\right)^{\frac{1}{3}}+\left(\frac{(c-id\sqrt{f})^{\frac{2}{3}}(ic+d\sqrt{f})}{2^{\frac{2}{3}}(ic+d\sqrt{f})}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((I-tan(f\*x+e))/(c+d\*tan(f\*x+e))^(1/3),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(I*\sqrt{3} - 1)*(-I/((I*c + d)*f^3))^{1/3}*\log(1/2*(\sqrt{3}*(I*c + d)*f^2 + (c - I*d)*f^2)*(-I/((I*c + d)*f^3))^{2/3} + (((c - I*d)*e^{2*I*f*x + 2*I*e}) + c + I*d)/(e^{2*I*f*x + 2*I*e} + 1))^{1/3}) + 1/2*(-I*\sqrt{3} - 1)*(-I/((I*c + d)*f^3))^{1/3}*\log(1/2*(\sqrt{3}*(-I*c - d)*f^2 + (c - I*d)*f^2)*(-I/((I*c + d)*f^3))^{2/3} + (((c - I*d)*e^{2*I*f*x + 2*I*e}) + c + I*d)/(e^{2*I*f*x + 2*I*e} + 1))^{1/3}) + (-I/((I*c + d)*f^3))^{1/3}*\log(-(c - I*d)*f^2*(-I/((I*c + d)*f^3))^{2/3} + (((c - I*d)*e^{2*I*f*x + 2*I*e}) + c + I*d)/(e^{2*I*f*x + 2*I*e} + 1))^{1/3})$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \left( -\frac{i}{\sqrt[3]{c + d \tan(e + fx)}} \right) dx - \int \frac{\tan(e + fx)}{\sqrt[3]{c + d \tan(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((I-tan(f\*x+e))/(c+d\*tan(f\*x+e))^(1/3),x)

[Out] -Integral(-I/(c + d\*tan(e + f\*x))^(1/3), x) - Integral(tan(e + f\*x)/(c + d\*tan(e + f\*x))^(1/3), x)

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(112) = 224.

time = 0.62, size = 910, normalized size = 6.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((I-tan(f\*x+e))/(c+d\*tan(f\*x+e))^(1/3),x, algorithm="giac")

[Out]  $-(c - I*d)^{2/3}*\log((d*\tan(f*x + e) + c)^{1/3} - (c - I*d)^{1/3})/(c*f - I*d*f) - (\sqrt{3}*(c^2 + d^2)^{1/3}*c*\cos(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 - \sqrt{3}*(c^2 + d^2)^{1/3}*c*\sin(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + 2*(c^2 + d^2)^{1/3}*c*\cos(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))*\sin(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))*\arctan(1/3*\sqrt{3}*(2*(d*\tan(f*x + e) + c)^{1/3} + (c - I*d)^{1/3}))/((c^2 + d^2)*f) - I*(\sqrt{3}*(c^2 + d^2)^{1/3}*d*\cos(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 - \sqrt{3}*(c^2 + d^2)^{1/3}*d*\sin(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + 2*(c^2 + d^2)^{1/3}*d*\cos(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))*\sin(1/6*\pi*\text{sgn}(c))*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))*\arctan(1/3*\sqrt{3}*(2*(d*\tan(f*x + e) + c)^{1/3} + (c -$

$$\begin{aligned} & I*d)^{(1/3)} / ((c - I*d)^{(1/3)}) / ((c^2 + d^2)*f) - 1/2*(2*\sqrt{3}*(c^2 + d^2)^{(1/3)}*c*\cos(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))*\sin(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c)) - (c^2 + d^2)^{(1/3)}*c*\cos(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + (c^2 + d^2)^{(1/3)}*c*\sin(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2)*\log((c^2 + d^2)^{(1/3)}*\cos(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + (c^2 + d^2)^{(1/3)}*\sin(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + (d*\tan(f*x + e) + c)^{(2/3)} + (d*\tan(f*x + e) + c)^{(1/3)}*(c - I*d)^{(1/3)}) / ((c^2 + d^2)*f) - 1/2*I*(2*\sqrt{3}*(c^2 + d^2)^{(1/3)}*d*\cos(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))*\sin(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c)) - (c^2 + d^2)^{(1/3)}*d*\cos(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + (c^2 + d^2)^{(1/3)}*d*\sin(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2)*\log((c^2 + d^2)^{(1/3)}*\cos(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + (c^2 + d^2)^{(1/3)}*\sin(1/6*\pi*\text{sgn}(c)*\text{sgn}(d) - 1/6*\pi*\text{sgn}(d) - 1/3*\arctan(d/c))^2 + (d*\tan(f*x + e) + c)^{(2/3)} + (d*\tan(f*x + e) + c)^{(1/3)}*(c - I*d)^{(1/3)}) / ((c^2 + d^2)*f) \end{aligned}$$

Mupad [B]

time = 20.67, size = 2500, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(-(\tan(e + f*x) - 1i)/(c + d*\tan(e + f*x))^{(1/3)}, x)$

[Out]  $\log(d^5*f*(c + d*\tan(e + f*x))^{(1/3)}*243i + ((1944*d^4*f^4*(c^2 - d^2)*(c + d*\tan(e + f*x))^{(1/3)} - 3*2^{(1/3)}*c*d^4*f^6*(c^2 + d^2)*(-(11664*f^3*((c^2*d^12*(c^2 + d^2)^2)/f^6)^{(1/2)} + d^9*11664i + c^2*d^7*11664i)/(d^6*f^3*(c^2 + d^2)^2))^{(2/3)}*(11664*f^3*((c^2*d^12*(c^2 + d^2)^2)/f^6)^{(1/2)} + d^9*11664i + c^2*d^7*11664i)/(93312*d^6*f^3*(c^2 + d^2)^2))*(-(f^3*((4*(729*d^8 + 729*c^2*d^6)*(46656*d^10 + 93312*c^2*d^8 + 46656*c^4*d^6))/f^6 - (11664*d^9 + 11664*c^2*d^7)^2/f^6)^{(1/2)} + d^9*11664i + c^2*d^7*11664i)/(93312*f^3*(d^10 + 2*c^2*d^8 + c^4*d^6)))^{(1/3)} + \log(d^5*f*(c + d*\tan(e + f*x))^{(1/3)}*243i + ((1944*d^4*f^4*(c^2 - d^2)*(c + d*\tan(e + f*x))^{(1/3)} - 3*2^{(1/3)}*c*d^4*f^6*(c^2 + d^2)*(-(d^9*11664i - 11664*f^3*((c^2*d^12*(c^2 + d^2)^2)/f^6)^{(1/2)} + c^2*d^7*11664i)/(d^6*f^3*(c^2 + d^2)^2))^{(2/3)}*(d^9*11664i - 11664*f^3*((c^2*d^12*(c^2 + d^2)^2)/f^6)^{(1/2)} + c^2*d^7*11664i)/(93312*d^6*f^3*(c^2 + d^2)^2))*(-(d^9*11664i - f^3*((4*(729*d^8 + 729*c^2*d^6)*(46656*d^10 + 93312*c^2*d^8 + 46656*c^4*d^6))/f^6 - (11664*d^9 + 11664*c^2*d^7)^2/f^6)^{(1/2)} + c^2*d^7*11664i)/(93312*f^3*(d^10 + 2*c^2*d^8 + c^4*d^6)))^{(1/3)} + (\log(-((-1/(f^3*(c - d*1i)))^{(2/3)}*((-1/(f^3*(c - d*1i)))^{(1/3)}*((1944*d^4*(c^2 - d^2)*(c + d*\tan(e + f*x))^{(1/3)})/f^2 - 1944*c*d^4*(-1/(f^3*(c - d*1i)))^{(2/3)}*(c^2 + d^2)))/2 - (972*d^4*(c^2 + d^2))/f^3)/4 - (243*c*d^4*(c + d*\tan(e + f*x))^{(1/3)})/f^5*(-1/(c*f^3 - d*f^3*1i))^{(1/3)})/2 + \log$



$$\begin{aligned} & /f^6)^{(1/2)} + c^2*d^7*11664i)/(93312*f^3*(d^{10} + 2*c^2*d^8 + c^4*d^6))^{(1/} \\ & 3) + (\log(((972*d^4*(c^2 + d^2))/f^3 - ((3^{(1/2)}*1i - 1)*((1944*d^4*(c^2 - \\ & d^2)*(c + d*\tan(e + f*x))^{(1/3)}))/f^2 - 1944*c*... \end{aligned}$$

$$3.478 \quad \int \frac{d - c \tan(e + fx)}{(c + d \tan(e + fx))^{2/3}} dx$$

**Optimal.** Leaf size=299

$$-\frac{1}{4}i\sqrt[3]{c-id}x + \frac{1}{4}i\sqrt[3]{c+id}x + \frac{\sqrt{3}\sqrt[3]{c-id}\operatorname{ArcTan}\left(\frac{1+\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}\right)}{2f} + \frac{\sqrt{3}\sqrt[3]{c+id}\operatorname{ArcTan}\left(\frac{1+\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}\right)}{2f}$$

[Out]  $-1/4*I*(c-I*d)^{(1/3)*x} + 1/4*I*(c+I*d)^{(1/3)*x} - 1/4*(c-I*d)^{(1/3)}*\ln(\cos(f*x+e))/f - 1/4*(c+I*d)^{(1/3)}*\ln(\cos(f*x+e))/f - 3/4*(c-I*d)^{(1/3)}*\ln((c-I*d)^{(1/3)} - (c+d*\tan(f*x+e))^{(1/3)})/f - 3/4*(c+I*d)^{(1/3)}*\ln((c+I*d)^{(1/3)} - (c+d*\tan(f*x+e))^{(1/3)})/f + 1/2*(c-I*d)^{(1/3)}*\arctan(1/3*(1+2*(c+d*\tan(f*x+e))^{(1/3)})/(c-I*d)^{(1/3)})*3^{(1/2)}*3^{(1/2)}/f + 1/2*(c+I*d)^{(1/3)}*\arctan(1/3*(1+2*(c+d*\tan(f*x+e))^{(1/3)})/(c+I*d)^{(1/3)})*3^{(1/2)}*3^{(1/2)}/f$

**Rubi [A]**

time = 0.24, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {3620, 3618, 59, 631, 210, 31}

$$\frac{\sqrt{3}\sqrt{c-id}\operatorname{ArcTan}\left(\frac{1+\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}\right)}{2f} + \frac{\sqrt{3}\sqrt{c+id}\operatorname{ArcTan}\left(\frac{1+\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}\right)}{2f} - \frac{3\sqrt{c-id}\log(-\sqrt{c+d\tan(e+fx)}+\sqrt{c-id})}{4f} - \frac{3\sqrt{c+id}\log(-\sqrt{c+d\tan(e+fx)}+\sqrt{c+id})}{4f} - \frac{\sqrt{c-id}\log(\cos(e+fx))}{4f} - \frac{\sqrt{c+id}\log(\cos(e+fx))}{4f} - \frac{1}{4}i\sqrt[3]{c-id}x + \frac{1}{4}i\sqrt[3]{c+id}x$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d - c*\operatorname{Tan}[e + f*x])/(c + d*\operatorname{Tan}[e + f*x])^{(2/3)}, x]$

[Out]  $(-1/4*I)*(c - I*d)^{(1/3)*x} + (I/4)*(c + I*d)^{(1/3)*x} + (\operatorname{Sqrt}[3]*(c - I*d)^{(1/3)}*\operatorname{ArcTan}[(1 + (2*(c + d*\operatorname{Tan}[e + f*x])^{(1/3)})/(c - I*d)^{(1/3)})/\operatorname{Sqrt}[3]])/(2*f) + (\operatorname{Sqrt}[3]*(c + I*d)^{(1/3)}*\operatorname{ArcTan}[(1 + (2*(c + d*\operatorname{Tan}[e + f*x])^{(1/3)})/(c + I*d)^{(1/3)})/\operatorname{Sqrt}[3]])/(2*f) - ((c - I*d)^{(1/3)}*\operatorname{Log}[\operatorname{Cos}[e + f*x]])/(4*f) - ((c + I*d)^{(1/3)}*\operatorname{Log}[\operatorname{Cos}[e + f*x]])/(4*f) - (3*(c - I*d)^{(1/3)}*\operatorname{Log}[(c - I*d)^{(1/3)} - (c + d*\operatorname{Tan}[e + f*x])^{(1/3)}])/(4*f) - (3*(c + I*d)^{(1/3)}*\operatorname{Log}[(c + I*d)^{(1/3)} - (c + d*\operatorname{Tan}[e + f*x])^{(1/3)}])/(4*f)$

**Rule 31**

$\operatorname{Int}[(a + (b_*)*(x_*)^{(-1)}, x\_Symbol] := \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

**Rule 59**

$\operatorname{Int}[1/(((a_*) + (b_*)*(x_*)^{(2/3)}), x\_Symbol] := \operatorname{With}[q = \operatorname{Rt}[(b*c - a*d)/b, 3], \operatorname{Simp}[-\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/(2*b*q^2), x] + (-\operatorname{Dist}[3/(2*b*q), \operatorname{Subst}[\operatorname{Int}[1/(q^2 + q*x + x^2), x], x, (c + d*x)^{(1/3)}], x, (c + d*x)^{(1/3)}], x, (c + d*x)^{(1/3)}], x, (c + d*x)^{(1/3)}]$

3)], x] - Dist[3/(2\*b\*q^2), Subst[Int[1/(q - x), x], x, (c + d\*x)^(1/3)], x  
 )] /; FreeQ[{a, b, c, d}, x] && PosQ[(b\*c - a\*d)/b]

#### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 3618

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

#### Rule 3620

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

#### Rubi steps



$$\begin{aligned}
\int \frac{d - c \tan(e + fx)}{(c + d \tan(e + fx))^{2/3}} dx &= \frac{1}{2}(-ic + d) \int \frac{1 - i \tan(e + fx)}{(c + d \tan(e + fx))^{2/3}} dx + \frac{1}{2}(ic + d) \int \frac{1 + i \tan(e + fx)}{(c + d \tan(e + fx))^{2/3}} dx \\
&= -\frac{(c - id) \text{Subst}\left(\int \frac{1}{(-1+x)(c-idx)^{2/3}} dx, x, i \tan(e + fx)\right)}{2f} - \frac{(c + id) \text{Subst}\left(\int \frac{1}{(-1+x)(c+idx)^{2/3}} dx, x, i \tan(e + fx)\right)}{2f} \\
&= -\frac{1}{4}i\sqrt[3]{c - id} x + \frac{1}{4}i\sqrt[3]{c + id} x - \frac{\sqrt[3]{c - id} \log(\cos(e + fx))}{4f} - \frac{\sqrt[3]{c + id} \log(\cos(e + fx))}{4f} \\
&= -\frac{1}{4}i\sqrt[3]{c - id} x + \frac{1}{4}i\sqrt[3]{c + id} x - \frac{\sqrt[3]{c - id} \log(\cos(e + fx))}{4f} - \frac{\sqrt[3]{c + id} \log(\cos(e + fx))}{4f} \\
&= -\frac{1}{4}i\sqrt[3]{c - id} x + \frac{1}{4}i\sqrt[3]{c + id} x + \frac{\sqrt{3} \sqrt[3]{c - id} \tan^{-1}\left(\frac{1 + \sqrt[3]{c + d \tan(e + fx)}}{\sqrt[3]{c - id}}\right)}{2f}
\end{aligned}$$

**Mathematica [A]**

time = 0.31, size = 330, normalized size = 1.10

$$\frac{2\sqrt{3}\sqrt{c-id}\text{ArcTan}\left(\frac{\sqrt[3]{c+d\tan(e+fx)}}{\sqrt{3}}\right) + 2\sqrt{3}\sqrt{c+id}\text{ArcTan}\left(\frac{\sqrt[3]{c+d\tan(e+fx)}}{\sqrt{3}}\right) - 2\sqrt{c-id}\log(\sqrt{c-id} - \sqrt{c+d\tan(e+fx)}) - 2\sqrt{c+id}\log(\sqrt{c+id} - \sqrt{c+d\tan(e+fx)}) + \sqrt{c-id}\log((c-id)^{1/3} + \sqrt{c-id}\sqrt{c+d\tan(e+fx)}) + (c+d\tan(e+fx))^{1/3} + \sqrt{c+id}\log((c+id)^{1/3} + \sqrt{c+id}\sqrt{c+d\tan(e+fx)}) + (c+d\tan(e+fx))^{1/3}}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[(d - c\*Tan[e + f\*x])/(c + d\*Tan[e + f\*x])^(2/3), x]

[Out] (2\*sqrt(3)\*(c - I\*d)^(1/3)\*ArcTan[(1 + (2\*(c + d\*Tan[e + f\*x])^(1/3))/(c - I\*d)^(1/3))/sqrt(3)] + 2\*sqrt(3)\*(c + I\*d)^(1/3)\*ArcTan[(1 + (2\*(c + d\*Tan[e + f\*x])^(1/3))/(c + I\*d)^(1/3))/sqrt(3)] - 2\*(c - I\*d)^(1/3)\*Log[(c - I\*d)^(1/3) - (c + d\*Tan[e + f\*x])^(1/3)] - 2\*(c + I\*d)^(1/3)\*Log[(c + I\*d)^(1/3) - (c + d\*Tan[e + f\*x])^(1/3)] + (c - I\*d)^(1/3)\*Log[(c - I\*d)^(2/3) + (c - I\*d)^(1/3)\*(c + d\*Tan[e + f\*x])^(1/3) + (c + d\*Tan[e + f\*x])^(2/3)] + (c + I\*d)^(1/3)\*Log[(c + I\*d)^(2/3) + (c + I\*d)^(1/3)\*(c + d\*Tan[e + f\*x])^(1/3) + (c + d\*Tan[e + f\*x])^(2/3)]/(4\*f)

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.

time = 0.30, size = 72, normalized size = 0.24

method	result	size
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derivativedivides	$\frac{\sum_{R=\text{RootOf}(\_Z^6-2c\_Z^3+c^2+d^2)} \frac{(-R^3 c-c^2-d^2) \ln\left(\frac{(c+d \tan(fx+e))^{\frac{1}{3}}-R}{-R^5-R^2 c}\right)}{2f}}{-R^5-R^2 c}$	72
default	$\frac{\sum_{R=\text{RootOf}(\_Z^6-2c\_Z^3+c^2+d^2)} \frac{(-R^3 c-c^2-d^2) \ln\left(\frac{(c+d \tan(fx+e))^{\frac{1}{3}}-R}{-R^5-R^2 c}\right)}{2f}}{-R^5-R^2 c}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x,method=_RETURNVERBOSE)`

[Out] `-1/2/f*sum((\_R^3*c-c^2-d^2)/(\_R^5-\_R^2*c)*ln((c+d*tan(f*x+e))^(1/3)-\_R),\_R=RootOf(\_Z^6-2*\_Z^3*c+c^2+d^2))`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x, algorithm="maxima")`

[Out] `-integrate((c*tan(f*x + e) - d)/(d*tan(f*x + e) + c)^(2/3), x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 2784 vs. 2(223) = 446.

time = 1.88, size = 2784, normalized size = 9.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x, algorithm="fricas")`

[Out] `1/2*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2))*log(2*f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) - 2*((c^2 + d^2)/f^6)^(1/6)*arctan((sqrt(2*f*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3))*f^5*((c^2 + d^2)/f^6)^(5/6) - f^5*((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3)*((c^2 + d^2)/f^6)^(5/6) - (c^2 + d^2)*cos(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2))`



```

sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^2 + d^2)/f^6)^(
1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3)) - 1/4*(sqrt(
3)*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^
3)*sqrt(d^2/f^6)/d^2)) + ((c^2 + d^2)/f^6)^(1/6)*cos(2/3*arctan((f^6*sqrt(
c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)))*log(-sqrt(3)*f*((c*cos(f*x +
e) + d*sin(f*x + e))/cos(f*x + e))^(1/3))*((c^2 + d^2)/f^6)^(1/6)*sin(2/3*ar
ctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) - f*((c*cos(f*
x + e) + d*sin(f*x + e))/cos(f*x + e))^(1/3))*((c^2 + d^2)/f^6)^(1/6)*cos(2/
3*arctan((f^6*sqrt((c^2 + d^2)/f^6) + c*f^3)*sqrt(d^2/f^6)/d^2)) + f^2*((c^
2 + d^2)/f^6)^(1/3) + ((c*cos(f*x + e) + d*sin(f*x + e))/cos(f*x + e))^(2/3
))

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-\int \left( -\frac{d}{(c + d \tan(e + fx))^{\frac{2}{3}}} \right) dx - \int \frac{c \tan(e + fx)}{(c + d \tan(e + fx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d-c\*tan(f\*x+e))/(c+d\*tan(f\*x+e))\*\*(2/3),x)

[Out] -Integral(-d/(c + d\*tan(e + f\*x))\*\*(2/3), x) - Integral(c\*tan(e + f\*x)/(c + d\*tan(e + f\*x))\*\*(2/3), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d-c\*tan(f\*x+e))/(c+d\*tan(f\*x+e))^(2/3),x, algorithm="giac")

[Out] undef

**Mupad [B]**

time = 22.17, size = 2500, normalized size = 8.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d - c\*tan(e + f\*x))/(c + d\*tan(e + f\*x))^(2/3),x)

[Out] log((((1944\*c\*d^4\*(c^2 + d^2)\*((8\*(-d^6\*f^6\*(c^2 - d^2)^2)^(1/2) + 16\*c\*d^4\*f^3)/(f^6\*(c^2 + d^2)^2))^(1/3) + (7776\*c\*d^6\*(c + d\*tan(e + f\*x))^(1/3))/f)\*((8\*(-d^6\*f^6\*(c^2 - d^2)^2)^(1/2) + 16\*c\*d^4\*f^3)/(f^6\*(c^2 + d^2)^2))^(2/3))/16 - (972\*d^8)/f^3)\*((8\*(-d^6\*f^6\*(c^2 - d^2)^2)^(1/2) + 16\*c\*d^4\*f^3)/(f^6\*(c^2 + d^2)^2))^(1/3) + (7776\*c\*d^6\*(c + d\*tan(e + f\*x))^(1/3))/f)

$$\begin{aligned}
&^3)/(f^6*(c^2 + d^2)^2))^{(1/3)}/4 - (486*d^8*(c + d*\tan(e + f*x))^{(1/3)})/f^4 \\
&4)*(((256*c^2*d^8*f^6 - d^6*(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/2)} + 16*c*d^4*f^3)/(64*(c^4*f^6 + d^4*f^6 + 2*c^2*d^2*f^6)))^{(1/3)} + \log(((1944*c*d^4*(c^2 + d^2)*(-8*(-d^6*f^6*(c^2 - d^2)^2)^{(1/2)} - 16*c*d^4*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)} + (7776*c*d^6*(c + d*\tan(e + f*x))^{(1/3)})/f)*(-8*(-d^6*f^6*(c^2 - d^2)^2)^{(1/2)} - 16*c*d^4*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)}/16 - (972*d^8)/f^3)*(-8*(-d^6*f^6*(c^2 - d^2)^2)^{(1/2)} - 16*c*d^4*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)}/4 - (486*d^8*(c + d*\tan(e + f*x))^{(1/3)})/f^4 \\
&*(-((256*c^2*d^8*f^6 - d^6*(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/2)} - 16*c*d^4*f^3)/(64*(c^4*f^6 + d^4*f^6 + 2*c^2*d^2*f^6)))^{(1/3)} + \log(-(486*c^4*d^4*(c + d*\tan(e + f*x))^{(1/3)})/f^4 - (((16*(-c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)}*(1944*c*d^4*(-c^8*d^2*f^6)^{(1/2)} + 1944*c^4*d^6*f^3 + 243*c^5*d^4*f^5*(c + d*\tan(e + f*x))^{(1/3)})*((16*(-c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)} - 243*c*d^8*f^5*(c + d*\tan(e + f*x))^{(1/3)}*((16*(-c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)}))/((16*c^5*f^3 - 16*c^3*d^2*f^3)^2/4 - c^6*(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(64*(c^4*f^6 + d^4*f^6 + 2*c^2*d^2*f^6)))^{(1/3)} + \log(((16*(-c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)}*(1944*c*d^4*(-c^8*d^2*f^6)^{(1/2)} - 1944*c^4*d^6*f^3 - 243*c^5*d^4*f^5*(c + d*\tan(e + f*x))^{(1/3)}*(-(16*(-c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)} + 243*c*d^8*f^5*(c + d*\tan(e + f*x))^{(1/3)}*(-(16*(-c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)}))/((16*c^5*f^3 - 16*c^3*d^2*f^3)^2/4 - c^6*(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(64*(c^4*f^6 + d^4*f^6 + 2*c^2*d^2*f^6)))^{(1/3)} + \log(((3^{(1/2)}*1i - 1)*((3^{(1/2)}*1i + 1)*(972*c*d^4*(3^{(1/2)}*1i - 1)*(c^2 + d^2)*(16*(-c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)} + (3888*c*d^4*(c^2 - d^2)*(c + d*\tan(e + f*x))^{(1/3)})/f)*((16*(-c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)}/32 - (972*c^4*d^4)/f^3)*((16*(-c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)}/8 - (486*c^4*d^4*(c + d*\tan(e + f*x))^{(1/3)})/f^4)*((3^{(1/2)}*((-256*c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/3)}*1i)/2 - (((-256*c^8*d^2*f^6)^{(1/2)} - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/3)}/2 + \log(((3^{(1/2)}*1i - 1)*((3^{(1/2)}*1i + 1)*(972*c*d^4*(3^{(1/2)}*1i - 1)*(c^2 + d^2)*(-16*(-c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)} + (3888*c*d^4*(c^2 - d^2)*(c + d*\tan(e + f*x))^{(1/3)})/f)*(-16*(-c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(2/3)}/32 - (972*c^4*d^4)/f^3)*(-16*(-c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^{(1/3)}/8 - (486*c^4*d^4*(c + d*\tan(e + f*x))^{(1/3)})/f^4)*((3^{(1/2)}*((-256*c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^6))^{(1/3)}*1i)/2 - (((-256*c^8*d^2*f^6)^{(1/2)} + 8*c^5*f^3 - 8*c^3*d^2*f^3)/(64*c^4*f^6 +
\end{aligned}$$



### 3.479 $\int \tan^m(c+dx)(a+b \tan(c+dx))^4(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=403

$$\frac{b(Ab^3(12 + 7m + m^2) + 4ab^2B(12 + 7m + m^2) - 2a^3B(19 + 8m + m^2) - a^2Ab(68 + 37m + 5m^2)) \tan^{1+m}}{d(1+m)(3+m)(4+m)}$$

```
[Out] -b*(A*b^3*(m^2+7*m+12)+4*a*b^2*B*(m^2+7*m+12)-2*a^3*B*(m^2+8*m+19)-a^2*A*b*(5*m^2+37*m+68))*tan(d*x+c)^(1+m)/d/(4+m)/(m^2+4*m+3)+(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*hypergeom([1, 1/2+1/2*m], [3/2+1/2*m], -tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1+m)+b^2*(2*a*A*b*(4+m)^2-b^2*B*(m^2+7*m+12)+a^2*B*(m^2+9*m+26))*tan(d*x+c)^(2+m)/d/(2+m)/(3+m)/(4+m)+(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*hypergeom([1, 1+1/2*m], [2+1/2*m], -tan(d*x+c)^2)*tan(d*x+c)^(2+m)/d/(2+m)+b*(A*b*(4+m)+a*B*(7+m))*tan(d*x+c)^(1+m)*(a+b*tan(d*x+c))^2/d/(3+m)/(4+m)+b*B*tan(d*x+c)^(1+m)*(a+b*tan(d*x+c))^3/d/(4+m)
```

**Rubi [A]**

time = 0.89, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3688, 3728, 3718, 3711, 3619, 3557, 371}

$\frac{b^2 B (a^2 b^2 (m^2 + 7m + 12) + 4ab^2 B (m^2 + 7m + 12) - 2a^3 B (m^2 + 8m + 19) - a^2 A b (m^2 + 37m + 68)) \tan^{1+m}(c+dx)}{d(1+m)(3+m)(4+m)} + \frac{(Aa^4 - 6Aa^2b^2 + Ab^4 - 4a^3bB + 4ab^3B) \text{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan^2(c+dx)\right] \tan^{1+m}(c+dx)}{d(1+m)} + \frac{b^2 (2aAb(4+m)^2 - b^2 B (m^2 + 7m + 12) + a^2 B (m^2 + 9m + 26)) \tan^{2+m}(c+dx)}{d(2+m)(3+m)(4+m)} + \frac{(4a^3ab - 4aAb^3 + a^4B - 6a^2b^2B + b^4B) \text{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, -\tan^2(c+dx)\right] \tan^{2+m}(c+dx)}{d(2+m)} + \frac{b(Ab(4+m) + aB(7+m)) \tan^{1+m}(c+dx) (a + b \tan(c+dx))^2}{d(3+m)(4+m)} + \frac{bB \tan^{1+m}(c+dx) (a + b \tan(c+dx))^3}{d(4+m)}$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

```
[Out] -((b*(A*b^3*(12 + 7*m + m^2) + 4*a*b^2*B*(12 + 7*m + m^2) - 2*a^3*B*(19 + 8*m + m^2) - a^2*A*b*(68 + 37*m + 5*m^2))*Tan[c + d*x]^(1 + m))/(d*(1 + m)*(3 + m)*(4 + m)) + ((a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) + (b^2*(2*a*A*b*(4 + m)^2 - b^2*B*(12 + 7*m + m^2) + a^2*B*(26 + 9*m + m^2))*Tan[c + d*x]^(2 + m))/(d*(2 + m)*(3 + m)*(4 + m)) + ((4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m)) + (b*(A*b*(4 + m) + a*B*(7 + m))*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^2)/(d*(3 + m)*(4 + m)) + (b*B*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^3)/(d*(4 + m))
```

**Rule 371**

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p * ((c*x)^(m + 1)/(c*(m + 1)) * Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 3619

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2 + d^2, 0] && !IntegerQ[2\*m]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

Rule 3718

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] := Simp[b\*C\*Tan[e + f\*x]\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 2))), x] - Dist[1/(d\*(n + 2)), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*c\*C - a\*A\*d\*(n + 2) - (A\*b + a\*B - b\*C)\*d\*(n + 2)\*Tan[e + f\*x] - (a\*C\*d\*(n + 2) - b\*(c\*C - B\*d\*(n + 2)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && !LtQ[n, -1]

Rule 3728



```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rubi steps

$$\begin{aligned}
\int \tan^m(c + dx)(a + b \tan(c + dx))^4(A + B \tan(c + dx)) dx &= \frac{bB \tan^{1+m}(c + dx)(a + b \tan(c + dx))^3}{d(4 + m)} + \int \\
&= \frac{b(Ab(4 + m) + aB(7 + m)) \tan^{1+m}(c + dx)}{d(3 + m)(4 + m)} \\
&= \frac{b^2(2aAb(4 + m)^2 - b^2B(12 + 7m + m^2) + a}{d(2 + m)(12 + 7m + m^2)} \\
&= \frac{b(Ab^3(12 + 7m + m^2) + 4ab^2B(12 + 7m + m^2) + a^2B)}{d(2 + m)(12 + 7m + m^2)} \\
&= \frac{b(Ab^3(12 + 7m + m^2) + 4ab^2B(12 + 7m + m^2) + a^2B)}{d(2 + m)(12 + 7m + m^2)} \\
&= \frac{b(Ab^3(12 + 7m + m^2) + 4ab^2B(12 + 7m + m^2) + a^2B)}{d(2 + m)(12 + 7m + m^2)} \\
&= \frac{b(Ab^3(12 + 7m + m^2) + 4ab^2B(12 + 7m + m^2) + a^2B)}{d(2 + m)(12 + 7m + m^2)}
\end{aligned}$$

### Mathematica [A]

time = 3.68, size = 355, normalized size = 0.88

Integrate[(a + b Tan[c + d x])^m (A + B Tan[c + d x])^4 (Tan[c + d x])^(1 + m), x]

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^4\*(A + B\*Tan[c + d\*x]),x]

[Out] (Tan[c + d\*x]^(1 + m)\*(-(b\*(2 + m)\*(A\*b^3\*(12 + 7\*m + m^2) + 4\*a\*b^2\*B\*(12 + 7\*m + m^2) - 2\*a^3\*B\*(19 + 8\*m + m^2) - a^2\*A\*b\*(68 + 37\*m + 5\*m^2)))) + (a^4\*A - 6\*a^2\*A\*b^2 + A\*b^4 - 4\*a^3\*b\*B + 4\*a\*b^3\*B)\*(2 + m)\*(3 + m)\*(4 + m)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2] + b^2\*(1 + m)

$$\begin{aligned} &*(2*a*A*b*(4+m)^2 - b^2*B*(12+7*m+m^2) + a^2*B*(26+9*m+m^2))*\text{Tan}[ \\ &c+d*x] + (4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*(1+m)*(3 \\ &+m)*(4+m)*\text{Hypergeometric2F1}[1, (2+m)/2, (4+m)/2, -\text{Tan}[c+d*x]^2]*\text{T} \\ &\text{an}[c+d*x] + b*(1+m)*(2+m)*(A*b*(4+m) + a*B*(7+m))*(a+b*\text{Tan}[c+ \\ &d*x])^2 + b*B*(1+m)*(2+m)*(3+m)*(a+b*\text{Tan}[c+d*x])^3)/(d*(1+m)*( \\ &2+m)*(3+m)*(4+m)) \end{aligned}$$

**Maple [F]**

time = 0.48, size = 0, normalized size = 0.00

$$\int (\tan^m(dx+c)) (a+b \tan(dx+c))^4 (A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)`

[Out] `int(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)`

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x+c)+A)*(b*tan(d*x+c)+a)^4*tan(d*x+c)^m,x)`

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral((B*b^4*tan(d*x+c)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*tan(d*x+c)^4 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*tan(d*x+c)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)*tan(d*x+c)^2 + (B*a^4 + 4*A*a^3*b)*tan(d*x+c))*tan(d*x+c)^m,x)`

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B \tan(c+dx)) (a+b \tan(c+dx))^4 \tan^m(c+dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*4\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*4\*tan(c + d\*x)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^4\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^4\*tan(d\*x + c)^m, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^4,x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^4, x)

$$3.480 \quad \int \tan^m(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=267

$$\frac{b(3aAb(3+m) - b^2B(3+m) + 2a^2B(4+m)) \tan^{1+m}(c+dx)}{d(1+m)(3+m)} + \frac{(a^3A - 3aAb^2 - 3a^2bB + b^3B) {}_2F_1\left(1, \frac{1+m}{2}; d(1+m)\right)}{d(1+m)}$$

[Out]  $b*(3*a*A*b*(3+m) - b^2*B*(3+m) + 2*a^2*B*(4+m))*\tan(d*x+c)^{(1+m)}/d/(1+m)/(3+m) + (A*a^3 - 3*A*a*b^2 - 3*B*a^2*b + B*b^3)*\text{hypergeom}([1, 1/2+1/2*m], [3/2+1/2*m], -\tan(d*x+c)^2)*\tan(d*x+c)^{(1+m)}/d/(1+m) + b^2*(A*b*(3+m) + a*B*(5+m))*\tan(d*x+c)^{(2+m)}/d/(2+m)/(3+m) + (3*A*a^2*b - A*b^3 + B*a^3 - 3*B*a*b^2)*\text{hypergeom}([1, 1+1/2*m], [2+1/2*m], -\tan(d*x+c)^2)*\tan(d*x+c)^{(2+m)}/d/(2+m) + b*B*\tan(d*x+c)^{(1+m)}*(a+b*\tan(d*x+c))^2/d/(3+m)$

**Rubi [A]**

time = 0.47, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3688, 3718, 3711, 3619, 3557, 371}

$$\frac{b(2a^2B(m+4) + 3aAb(m+3) - b^2B(m+3)) \tan^{m+1}(c+dx)}{d(m+1)(m+3)} + \frac{(a^3A - 3aAb^2 - 3a^2bB + b^3B) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; -\tan^2(c+dx)\right)}{d(m+1)} + \frac{(a^3B + 3a^2Ab - 3ab^2B - Ab^3) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; -\tan^2(c+dx)\right)}{d(m+2)} + \frac{b^2(aB(m+5) + Ab(m+3)) \tan^{m+1}(c+dx)}{d(m+2)(m+3)} + \frac{bB \tan^{m+1}(c+dx)(a+b \tan(c+dx))^2}{d(m+3)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(b*(3*a*A*b*(3+m) - b^2*B*(3+m) + 2*a^2*B*(4+m))*\text{Tan}[c + d*x]^{(1+m)})/(d*(1+m)*(3+m)) + ((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*\text{Hypergeometric2F1}[1, (1+m)/2, (3+m)/2, -\text{Tan}[c + d*x]^2]*\text{Tan}[c + d*x]^{(1+m)})/(d*(1+m)) + (b^2*(A*b*(3+m) + a*B*(5+m))*\text{Tan}[c + d*x]^{(2+m)})/(d*(2+m)*(3+m)) + (((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*\text{Hypergeometric2F1}[1, (2+m)/2, (4+m)/2, -\text{Tan}[c + d*x]^2]*\text{Tan}[c + d*x]^{(2+m)})/(d*(2+m)) + (b*B*\text{Tan}[c + d*x]^{(1+m)}*(a + b*\text{Tan}[c + d*x])^2)/(d*(3+m))$

**Rule 371**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*((c\*x)^(m+1)/(c\*(m+1)))\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

**Rule 3557**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3619

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2 + d^2, 0] && !IntegerQ[2\*m]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3711

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[C\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

Rule 3718

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[b\*C\*Tan[e + f\*x]\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 2))), x] - Dist[1/(d\*(n + 2)), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*c\*C - a\*A\*d\*(n + 2) - (A\*b + a\*B - b\*C)\*d\*(n + 2)\*Tan[e + f\*x] - (a\*C\*d\*(n + 2) - b\*(c\*C - B\*d\*(n + 2)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \tan^m(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \frac{bB \tan^{1+m}(c+dx)(a+b \tan(c+dx))^2}{d(3+m)} + \frac{\int \tan^m(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx}{d(3+m)} \\
&= \frac{b^2(Ab(3+m)+aB(5+m)) \tan^{2+m}(c+dx)}{d(2+m)(3+m)} \\
&= \frac{b(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m))}{d(1+m)(3+m)} \\
&= \frac{b(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m))}{d(1+m)(3+m)} \\
&= \frac{b(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m))}{d(1+m)(3+m)} \\
&= \frac{b(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m))}{d(1+m)(3+m)}
\end{aligned}$$

**Mathematica [A]**

time = 1.75, size = 232, normalized size = 0.87

$$\frac{\tan^{1+m}(c+dx)(b(2+m)(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m))+c^2A-3aAb^2-3a^2B(2+m)(3+m)_2F_1(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx))+b^2(1+m)(Ab(3+m)+aB(5+m))\tan(c+dx)+(3a^2Ab-Ab^2+a^2B-3ab^2B)(1+m)(3+m)_2F_1(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx))\tan(c+dx)+5B(1+m)(2+m)(a+b \tan(c+dx))^2}{d(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]), x]
```

```
[Out] (Tan[c + d*x]^(1 + m)*(b*(2 + m)*(3*a*A*b*(3 + m) - b^2*B*(3 + m) + 2*a^2*B*(4 + m)) + (a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*(2 + m)*(3 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2] + b^2*(1 + m)*(A*b*(3 + m) + a*B*(5 + m))*Tan[c + d*x] + (3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*(1 + m)*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x] + b*B*(1 + m)*(2 + m)*(a + b*Tan[c + d*x])^2))/(d*(1 + m)*(2 + m)*(3 + m))
```

**Maple [F]**

time = 0.33, size = 0, normalized size = 0.00

$$\int (\tan^m(dx+c))(a+b \tan(dx+c))^3(A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)), x)
```

```
[Out] int(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)), x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*tan(d*x + c)^m, x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^3*tan(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*tan(d*x + c)^3 + 3*(B*a^2*b + A*a*b^2)*tan(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*tan(d*x + c))*tan(d*x + c)^m, x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*tan(c + d*x)**m, x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*tan(d*x + c)^m, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3, x)



### 3.481 $\int \tan^m(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=194

$$\frac{b(Ab(2+m) + aB(3+m)) \tan^{1+m}(c+dx)}{d(1+m)(2+m)} + \frac{(a^2A - Ab^2 - 2abB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{d(1+m)}$$

[Out] b\*(A\*b\*(2+m)+a\*B\*(3+m))\*tan(d\*x+c)^(1+m)/d/(1+m)/(2+m)+(A\*a^2-A\*b^2-2\*B\*a\*b)\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/d/(1+m)+(2\*A\*a\*b+B\*a^2-B\*b^2)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/d/(2+m)+b\*B\*tan(d\*x+c)^(1+m)\*(a+b\*tan(d\*x+c))/d/(2+m)

**Rubi [A]**

time = 0.23, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3688, 3711, 3619, 3557, 371}

$$\frac{(a^2A - 2abB - Ab^2) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\tan^2(c+dx)\right)}{d(m+1)} + \frac{(a^2B + 2aAb - b^2B) \tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\tan^2(c+dx)\right)}{d(m+2)} + \frac{b(aB(m+3) + Ab(m+2)) \tan^{m+1}(c+dx)}{d(m+1)(m+2)} + \frac{bB \tan^{m+1}(c+dx)(a+b \tan(c+dx))}{d(m+2)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (b\*(A\*b\*(2 + m) + a\*B\*(3 + m))\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)\*(2 + m)) + ((a^2\*A - A\*b^2 - 2\*a\*b\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)) + ((2\*a\*A\*b + a^2\*B - b^2\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/(d\*(2 + m)) + (b\*B\*Tan[c + d\*x]^(1 + m)\*(a + b\*Tan[c + d\*x]))/(d\*(2 + m))

**Rule 371**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*((c\*x)^(m+1)/(c\*(m+1)))\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

**Rule 3557**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

**Rule 3619**

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[

$b \cdot \tan[e + f \cdot x]^{(m + 1)}, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x\} \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{IntegerQ}[2 \cdot m]$

### Rule 3688

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^{(m)} \cdot ((A + B \cdot \tan[e + f \cdot x]) + (f \cdot x)) \cdot ((c + d \cdot \tan[e + f \cdot x])^{(n)}), x\_Symbol] :> \text{Simp}[b \cdot B \cdot (a + b \cdot \tan[e + f \cdot x])^{(m - 1)} \cdot ((c + d \cdot \tan[e + f \cdot x])^{(n + 1)} / (d \cdot f \cdot (m + n))), x] + \text{Dist}[1 / (d \cdot (m + n)), \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{(m - 2)} \cdot (c + d \cdot \tan[e + f \cdot x])^{(n)} \cdot \text{Simp}[a^2 \cdot A \cdot d \cdot (m + n) - b \cdot B \cdot (b \cdot c \cdot (m - 1) + a \cdot d \cdot (n + 1)) + d \cdot (m + n) \cdot (2 \cdot a \cdot A \cdot b + B \cdot (a^2 - b^2)) \cdot \tan[e + f \cdot x] - (b \cdot B \cdot (b \cdot c - a \cdot d) \cdot (m - 1) - b \cdot (A \cdot b + a \cdot B) \cdot d \cdot (m + n)) \cdot \tan[e + f \cdot x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\& (\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2 \cdot m, 2 \cdot n]) \&\& !( \text{IGtQ}[n, 1] \& \& ( !\text{IntegerQ}[m] \mid \mid (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0]) ) ) )$

### Rule 3711

$\text{Int}[(a + b \cdot \tan[e + f \cdot x])^{(m)} \cdot ((A + B \cdot \tan[e + f \cdot x]) + (f \cdot x)) + (C \cdot \tan[e + f \cdot x])^2), x\_Symbol] :> \text{Simp}[C \cdot ((a + b \cdot \tan[e + f \cdot x])^{(m + 1)} / (b \cdot f \cdot (m + 1))), x] + \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{(m)} \cdot \text{Simp}[A - C + B \cdot \tan[e + f \cdot x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x\} \&\& \text{NeQ}[A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C, 0] \&\& !\text{LeQ}[m, -1]$

### Rubi steps

$$\begin{aligned} \int \tan^m(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \frac{bB \tan^{1+m}(c + dx)(a + b \tan(c + dx))}{d(2 + m)} + \int \tan^m(c + dx)(a + b \tan(c + dx)) dx \\ &= \frac{b(Ab(2 + m) + aB(3 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} + \int \tan^m(c + dx)(a + b \tan(c + dx)) dx \\ &= \frac{b(Ab(2 + m) + aB(3 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} + \int \tan^m(c + dx)(a + b \tan(c + dx)) dx \\ &= \frac{b(Ab(2 + m) + aB(3 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} + \int \tan^m(c + dx)(a + b \tan(c + dx)) dx \\ &= \frac{b(Ab(2 + m) + aB(3 + m)) \tan^{1+m}(c + dx)}{d(1 + m)(2 + m)} + \int \tan^m(c + dx)(a + b \tan(c + dx)) dx \end{aligned}$$

### Mathematica [A]

time = 0.51, size = 155, normalized size = 0.80

$$\frac{\tan^{1+m}(c + dx) \left( \frac{b(Ab(2+m) + aB(3+m))}{1+m} + \frac{(a^2A - Ab^2 - 2abB)(2+m) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c + dx)\right)}{1+m} + (2aAb + a^2B - b^2B) {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; -\tan^2(c + dx)\right) \tan(c + dx) + bB(a + b \tan(c + dx)) \right)}{d(2 + m)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (Tan[c + d\*x]^(1 + m)\*((b\*(A\*b\*(2 + m) + a\*B\*(3 + m)))/(1 + m) + ((a^2\*A - A\*b^2 - 2\*a\*b\*B)\*(2 + m)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2])/(1 + m) + (2\*a\*A\*b + a^2\*B - b^2\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x] + b\*B\*(a + b\*Tan[c + d\*x]))/(d\*(2 + m))

Maple [F]

time = 0.27, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + b \tan(dx + c))^2 (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^2\*tan(d\*x + c)^m, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*b^2\*tan(d\*x + c)^3 + A\*a^2 + (2\*B\*a\*b + A\*b^2)\*tan(d\*x + c)^2 + (B\*a^2 + 2\*A\*a\*b)\*tan(d\*x + c))\*tan(d\*x + c)^m, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*2\*tan(c + d\*x)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^2\*tan(d\*x + c)^m, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2, x)

$$3.482 \quad \int \tan^m(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=127

$$\frac{bB \tan^{1+m}(c+dx)}{d(1+m)} + \frac{(aA - bB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{d(1+m)} + \frac{(Ab + aB) {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; -\tan^2(c+dx)\right) \tan^{2+m}(c+dx)}{d(2+m)}$$

[Out] b\*B\*tan(d\*x+c)^(1+m)/d/(1+m)+(A\*a-B\*b)\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/d/(1+m)+(A\*b+B\*a)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/d/(2+m)

Rubi [A]

time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3673, 3619, 3557, 371}

$$\frac{(aA - bB) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\tan^2(c+dx)\right)}{d(m+1)} + \frac{(aB + Ab) \tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\tan^2(c+dx)\right)}{d(m+2)} + \frac{bB \tan^{m+1}(c+dx)}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] (b\*B\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)) + ((a\*A - b\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/(d\*(1 + m)) + ((A\*b + a\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/(d\*(2 + m))

Rule 371

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[a^p \* ((c\*x)^(m + 1)/(c\*(m + 1)))\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt Q[p, 0] || GtQ[a, 0])

Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 3619

Int[((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c, Int[(b\*Tan[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2

+ d^2, 0] && !IntegerQ[2\*m]

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \tan^m(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx &= \frac{bB \tan^{1+m}(c+dx)}{d(1+m)} + \int \tan^m(c+dx)(aA-bB) dx \\ &= \frac{bB \tan^{1+m}(c+dx)}{d(1+m)} + (Ab+aB) \int \tan^{1+m}(c+dx) dx \\ &= \frac{bB \tan^{1+m}(c+dx)}{d(1+m)} + \frac{(Ab+aB) \operatorname{Subst}\left(\int \frac{x^{1+m}}{1+x^2} dx\right)}{d} \\ &= \frac{bB \tan^{1+m}(c+dx)}{d(1+m)} + \frac{(aA-bB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)}{d} \end{aligned}$$

### Mathematica [A]

time = 0.32, size = 108, normalized size = 0.85

$$\frac{\tan^{1+m}(c+dx) \left( \frac{bB}{1+m} + \frac{(aA-bB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)}{1+m} + \frac{(Ab+aB) {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; -\tan^2(c+dx)\right) \tan(c+dx)}{2+m} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] (Tan[c + d*x]^(1 + m)*((b*B)/(1 + m) + ((a*A - b*B)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -Tan[c + d*x]^2])/(1 + m) + ((A*b + a*B)*Hypergeometric
2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(2 + m))/d
```

### Maple [F]

time = 0.22, size = 0, normalized size = 0.00

$$\int (\tan^m(dx+c))(a+b \tan(dx+c))(A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

[Out] `int(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral((B*b*tan(d*x + c)^2 + A*a + (B*a + A*b)*tan(d*x + c))*tan(d*x + c)^m, x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx)) \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*tan(c + d*x)**m, x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)), x)



$$3.483 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=185

$$\frac{(aA + bB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{(a^2 + b^2) d(1+m)} + \frac{b(Ab - aB) {}_2F_1\left(1, 1+m; 2+m; -\frac{b \tan(c+dx)}{a}\right)}{a(a^2 + b^2) d(1+m)}$$

[Out] (A\*a+B\*b)\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/(a^2+b^2)/d/(1+m)+b\*(A\*b-B\*a)\*hypergeom([1, 1+m], [2+m], -b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1+m)/a/(a^2+b^2)/d/(1+m)-(A\*b-B\*a)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/(a^2+b^2)/d/(2+m)

Rubi [A]

time = 0.21, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3694, 3619, 3557, 371, 3715, 66}

$$\frac{b(Ab - aB) \tan^{m+1}(c+dx) {}_2F_1\left(1, m+1; m+2; -\frac{b \tan(c+dx)}{a}\right)}{ad(m+1)(a^2+b^2)} + \frac{(aA + bB) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\tan^2(c+dx)\right)}{d(m+1)(a^2+b^2)} - \frac{(Ab - aB) \tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\tan^2(c+dx)\right)}{d(m+2)(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] ((a\*A + b\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/((a^2 + b^2)\*d\*(1 + m)) + (b\*(A\*b - a\*B)\*Hypergeometric2F1[1, 1 + m, 2 + m, -(b\*Tan[c + d\*x])/a])\*Tan[c + d\*x]^(1 + m))/(a\*(a^2 + b^2)\*d\*(1 + m)) - ((A\*b - a\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/((a^2 + b^2)\*d\*(2 + m))

Rule 66

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[c^n\*((b\*x)^(m + 1)/(b\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

Rule 371

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*((c\*x)^(m + 1)/(c\*(m + 1)))\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !

IntegerQ[n]

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2 + d^2, 0] && !IntegerQ[2*m]
```

Rule 3694

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan[e + f*x], x], x], x] + Dist[b*((A*b - a*B)/(a^2 + b^2)), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx &= \frac{\int \tan^m(c+dx)(aA+bB-(Ab-aB) \tan(c+dx)) dx}{a^2+b^2} + \frac{(bAb-a^2)}{a^2+b^2} \\
&= -\frac{(Ab-aB) \int \tan^{1+m}(c+dx) dx}{a^2+b^2} + \frac{(aA+bB) \int \tan^m(c+dx) dx}{a^2+b^2} \\
&= \frac{b(Ab-aB) {}_2F_1\left(1, 1+m; 2+m; -\frac{b \tan(c+dx)}{a}\right) \tan^{1+m}(c+dx)}{a(a^2+b^2) d(1+m)} - \frac{b(Ab-aB) \int \tan^{1+m}(c+dx) dx}{a(a^2+b^2) d(1+m)} \\
&= \frac{(aA+bB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{(a^2+b^2) d(1+m)} + \frac{b(Ab-aB) \int \tan^{1+m}(c+dx) dx}{a(a^2+b^2) d(1+m)}
\end{aligned}$$

**Mathematica [A]**

time = 0.64, size = 144, normalized size = 0.78

$$\frac{\tan^{1+m}(c+dx) \left( (aA+bB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) + \frac{(Ab-aB) (b(2+m) {}_2F_1\left(1, 1+m; 2+m; -\frac{b \tan(c+dx)}{a}\right) - a(1+m) {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; -\tan^2(c+dx)\right) \tan(c+dx))}{a(2+m)} \right)}{(a^2+b^2) d(1+m)}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

[Out] (Tan[c + d\*x]^(1 + m)\*((a\*A + b\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2] + ((A\*b - a\*B)\*(b\*(2 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -(b\*Tan[c + d\*x])/a]) - a\*(1 + m)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]))/(a\*(2 + m)))/((a^2 + b^2)\*d\*(1 + m))

**Maple [F]**

time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{a + b \tan(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/(a + b\*tan(c + d\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)), x)

$$3.484 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=282

$$\frac{(a^2A - Ab^2 + 2abB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{(a^2 + b^2)^2 d(1+m)} + \frac{b(a^2Ab(2-m) - Ab^3m + ab^2B)}{(a^2 + b^2)^2 d(1+m)}$$

[Out] (A\*a^2-A\*b^2+2\*B\*a\*b)\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/(a^2+b^2)^2/d/(1+m)+b\*(a^2\*A\*b\*(2-m)-A\*b^3\*m+a\*b^2\*B\*(1+m)-a^3\*(-B\*m+B))\*hypergeom([1, 1+m], [2+m], -b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1+m)/a^2/(a^2+b^2)^2/d/(1+m)-(2\*A\*a\*b-B\*a^2+B\*b^2)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/(a^2+b^2)^2/d/(2+m)+b\*(A\*b-B\*a)\*tan(d\*x+c)^(1+m)/a/(a^2+b^2)/d/(a+b\*tan(d\*x+c))

Rubi [A]

time = 0.47, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3690, 3734, 3619, 3557, 371, 3715, 66}

$$\frac{(a^2A + 2abB - Ab^2) \tan^{m+1}(c+dx) {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\tan^2(c+dx)\right)}{d(m+1)(a^2+b^2)^2} - \frac{(a^2(-B) + 2aAb + b^2B) \tan^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\tan^2(c+dx)\right)}{d(m+2)(a^2+b^2)^2} + \frac{b(Ab - aB) \tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b \tan(c+dx))} + \frac{b(-a^2(B-Bm) + a^2Ab(2-m) + ab^2B(m+1) - Ab^3m) \tan^{m+1}(c+dx) {}_2F_1\left(1, m+1; m+2; -\frac{\tan^2(c+dx)}{a^2+b^2}\right)}{a^2d(m+1)(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] ((a^2\*A - A\*b^2 + 2\*a\*b\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/((a^2 + b^2)^2\*d\*(1 + m)) + (b\*(a^2\*A\*b\*(2 - m) - A\*b^3\*m + a\*b^2\*B\*(1 + m) - a^3\*(B - B\*m))\*Hypergeometric2F1[1, 1 + m, 2 + m, -(b\*Tan[c + d\*x])/a])\*Tan[c + d\*x]^(1 + m))/(a^2\*(a^2 + b^2)^2\*d\*(1 + m)) - ((2\*a\*A\*b - a^2\*B + b^2\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/((a^2 + b^2)^2\*d\*(2 + m)) + (b\*(A\*b - a\*B)\*Tan[c + d\*x]^(1 + m))/(a\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x]))

Rule 66

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[c^n\*((b\*x)^(m+1)/(b\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

Rule 371

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_))^(n\_)]^(p\_), x\_Symbol] :> Simp[a^p\*((c\*x)^(m+1)/(c\*(m+1)))\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, (-b)\*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt

$Q[p, 0] \parallel GtQ[a, 0]$

### Rule 3557

$\text{Int}[(b \cdot \tan[c] + d \cdot x)^n, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b \cdot \tan[c + d \cdot x]], x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& ! \text{IntegerQ}[n]$

### Rule 3619

$\text{Int}[(b \cdot \tan[e] + f \cdot x)^m \cdot (c + d \cdot \tan[e] + f \cdot x), x\_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b \cdot \tan[e + f \cdot x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \cdot \tan[e + f \cdot x])^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& ! \text{IntegerQ}[2 \cdot m]$

### Rule 3690

$\text{Int}[(a + b \cdot \tan[e] + f \cdot x)^m \cdot (A + B \cdot \tan[e] + f \cdot x) \cdot (c + d \cdot \tan[e] + f \cdot x)^n, x\_Symbol] \rightarrow \text{Simp}[b \cdot (A \cdot b - a \cdot B) \cdot (a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^{n+1} / (f \cdot (m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B \cdot (b \cdot c \cdot (m+1) + a \cdot d \cdot (n+1)) + A \cdot (a \cdot (b \cdot c - a \cdot d) \cdot (m+1) - b^2 \cdot d \cdot (m+n+2)) - (A \cdot b - a \cdot B) \cdot (b \cdot c - a \cdot d) \cdot (m+1) \cdot \tan[e + f \cdot x] - b \cdot d \cdot (A \cdot b - a \cdot B) \cdot (m+n+2) \cdot \tan[e + f \cdot x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegerQ}[m] \parallel \text{IntegersQ}[2 \cdot m, 2 \cdot n]) \&\& !( \text{ILtQ}[n, -1] \&\& ( ! \text{IntegerQ}[m] \parallel (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])) )$

### Rule 3715

$\text{Int}[(a + b \cdot \tan[e] + f \cdot x)^m \cdot (c + d \cdot \tan[e] + f \cdot x) \cdot (A + C \cdot \tan[e] + f \cdot x)^2, x\_Symbol] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x, \tan[e + f \cdot x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3734

$\text{Int}[(c + d \cdot \tan[e] + f \cdot x)^n \cdot (A + B \cdot \tan[e] + f \cdot x) \cdot (C + d \cdot \tan[e] + f \cdot x)^2 / ((a + b \cdot \tan[e] + f \cdot x)), x\_Symbol] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d \cdot \tan[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B + a \cdot (A - C) + (a \cdot B - b \cdot (A - C)) \cdot \tan[e + f \cdot x], x], x], x] + \text{Dist}[(A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) / (a^2 + b^2), \text{Int}[(c + d \cdot \tan[e + f \cdot x])^n \cdot ((1 + \tan[e + f \cdot x]^2) / (a + b \cdot \tan[e + f \cdot x])), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\&$

!GtQ[n, 0] &amp;&amp; !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx &= \frac{b(Ab-aB) \tan^{1+m}(c+dx)}{a(a^2+b^2) d(a+b \tan(c+dx))} + \frac{\int \frac{\tan^m(c+dx)(a^2A-Ab^2m+abB(1+m))}{(a+b \tan(c+dx))^2} dx}{a(a^2+b^2) d(a+b \tan(c+dx))} \\
&= \frac{b(Ab-aB) \tan^{1+m}(c+dx)}{a(a^2+b^2) d(a+b \tan(c+dx))} + \frac{\int \tan^m(c+dx) (a^2A-Ab^2m+abB(1+m))}{(a^2+b^2)^2} \\
&= \frac{b(Ab-aB) \tan^{1+m}(c+dx)}{a(a^2+b^2) d(a+b \tan(c+dx))} + \frac{(a^2A-Ab^2+2abB) \int \tan^m(c+dx)}{(a^2+b^2)^2} \\
&= -\frac{b(a^3B(1-m)-a^2Ab(2-m)+Ab^3m-ab^2B(1+m)) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{a^2(a^2+b^2)^2 d(1+m)} \\
&= \frac{(a^2A-Ab^2+2abB) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{(a^2+b^2)^2 d(1+m)}
\end{aligned}$$

Mathematica [A]

time = 2.01, size = 239, normalized size = 0.85

$$\frac{\tan^{1+m}(c+dx) \left( \frac{b(-a^2Ab(-2+m)+a^2B(-1+m)-Ab^2m+ab^2B(1+m)) {}_2F_1\left(1, 1+m, 2+m; -\frac{b \tan(c+dx)}{a}\right)}{a(a^2+b^2)(1+m)} + \frac{b(Ab-aB)}{a+b \tan(c+dx)} + \frac{a \left( \frac{(a^2A-Ab^2+2abB) {}_2F_1\left(1, \frac{1+m}{2}, \frac{3+m}{2}; -\tan^2(c+dx)\right)}{1+m} + \frac{(-2aAb+a^2B-b^2B) {}_2F_1\left(1, \frac{2+m}{2}, \frac{4+m}{2}; -\tan^2(c+dx)\right) \tan(c+dx)}{2+m} \right)}{a^2+b^2} \right)}{a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2,x]

[Out] (Tan[c + d\*x]^(1 + m)\*((b\*(-(a^2\*A\*b\*(-2 + m)) + a^3\*B\*(-1 + m) - A\*b^3\*m + a\*b^2\*B\*(1 + m))\*Hypergeometric2F1[1, 1 + m, 2 + m, -(b\*Tan[c + d\*x])/a])/(a\*(a^2 + b^2)\*(1 + m)) + (b\*(A\*b - a\*B))/(a + b\*Tan[c + d\*x]) + (a\*((a^2\*A - A\*b^2 + 2\*a\*b\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]))/(1 + m) + ((-2\*a\*A\*b + a^2\*B - b^2\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x])/(2 + m))/(a^2 + b^2))/(a\*(a^2 + b^2)\*d)

Maple [F]

time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx+c))(A+B \tan(dx+c))}{(a+b \tan(dx+c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)`

[Out] `int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^2, x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")`

[Out] `integral((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2), x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)`

[Out] `Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x))**2, x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="giac")`



[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2, x)

$$3.485 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=438

$$\frac{(a^3 A - 3aAb^2 + 3a^2 bB - b^3 B) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{(a^2 + b^2)^3 d(1+m)} \quad b(Ab^5(1-m)m + ab^4 Bm)$$

[Out] (A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*hypergeom([1, 1/2+1/2\*m], [3/2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(1+m)/(a^2+b^2)^3/d/(1+m)-1/2\*b\*(A\*b^5\*(1-m)\*m+a\*b^4\*B\*m\*(1+m)-2\*a^3\*b^2\*B\*(-m^2+m+3)+2\*a^2\*A\*b^3\*(-m^2+3\*m+1)-a^4\*A\*b\*(m^2-5\*m+6)+a^5\*B\*(m^2-3\*m+2))\*hypergeom([1, 1+m], [2+m], -b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1+m)/a^3/(a^2+b^2)^3/d/(1+m)-(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*hypergeom([1, 1+1/2\*m], [2+1/2\*m], -tan(d\*x+c)^2)\*tan(d\*x+c)^(2+m)/(a^2+b^2)^3/d/(2+m)+1/2\*b\*(A\*b-B\*a)\*tan(d\*x+c)^(1+m)/a/(a^2+b^2)/d/(a+b\*tan(d\*x+c))^2+1/2\*b\*(A\*b^3\*(1-m)-a^3\*B\*(3-m)+a^2\*A\*b\*(5-m)+a\*b^2\*B\*(1+m))\*tan(d\*x+c)^(1+m)/a^2/(a^2+b^2)^2/d/(a+b\*tan(d\*x+c))

**Rubi [A]**

time = 0.88, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3690, 3730, 3734, 3619, 3557, 371, 3715, 66}

$\frac{b(Ab - a^2)\text{atan}^2(c+dx)}{2d(a^2+b^2)(a+b\tan(c+dx))}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$ ,  $\frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)\text{atan}^2(c+dx)}{d(a^2+b^2)^3}$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out] ((a^3\*A - 3\*a\*A\*b^2 + 3\*a^2\*b\*B - b^3\*B)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(1 + m))/((a^2 + b^2)^3\*d\*(1 + m)) - (b\*(A\*b^5\*(1 - m)\*m + a\*b^4\*B\*m\*(1 + m) - 2\*a^3\*b^2\*B\*(3 + m - m^2) + 2\*a^2\*A\*b^3\*(1 + 3\*m - m^2) - a^4\*A\*b\*(6 - 5\*m + m^2) + a^5\*B\*(2 - 3\*m + m^2))\*Hypergeometric2F1[1, 1 + m, 2 + m, -(b\*Tan[c + d\*x])/a])\*Tan[c + d\*x]^(1 + m))/(2\*a^3\*(a^2 + b^2)^3\*d\*(1 + m)) - ((3\*a^2\*A\*b - A\*b^3 - a^3\*B + 3\*a\*b^2\*B)\*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(2 + m))/((a^2 + b^2)^3\*d\*(2 + m)) + (b\*(A\*b - a\*B)\*Tan[c + d\*x]^(1 + m))/(2\*a\*(a^2 + b^2)\*d\*(a + b\*Tan[c + d\*x])^2) + (b\*(A\*b^3\*(1 - m) - a^3\*B\*(3 - m) + a^2\*A\*b\*(5 - m) + a\*b^2\*B\*(1 + m))\*Tan[c + d\*x]^(1 + m))/(2\*a^2\*(a^2 + b^2)^2\*d\*(a + b\*Tan[c + d\*x]))

**Rule 66**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[c^n\*((b\*x)^(m + 1)/(b\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0]))

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1))]*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
```

```
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx = \frac{b(Ab - aB) \tan^{1+m}(c + dx)}{2a(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{\int \frac{\tan^m(c + dx)(2a^2A + Ab^2(1 - m) + ab^2m)}{(a + b \tan(c + dx))^3} dx}{2a(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{b(Ab^3(1 - m) - a^3B(3 - m))}{2a^2(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{b(Ab^3(1 - m) - a^3B(3 - m))}{2a^2(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{b(Ab^3(1 - m) - a^3B(3 - m))}{2a^2(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{b(Ab^3(1 - m) - a^3B(3 - m))}{2a^2(a^2 + b^2)d(a + b \tan(c + dx))^2} + \frac{b(Ab^5(1 - m)m + ab^4Bm(1 + m) - 2a^3b^2B(3 + m - m^2) + 2a^2b^2B(1 - m))}{2a^2(a^2 + b^2)d(1 + m)} = \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c + dx)\right)}{(a^2 + b^2)^3 d(1 + m)}$$

Mathematica [A]

time = 5.53, size = 370, normalized size = 0.84

$$\frac{\tan^{1+m}(c + dx) \left( \frac{b(Ab - aB)}{(a + b \tan(c + dx))^3} + \frac{b(-a^2Ab - 5am)a^2B(-3m) - Ab^3(-1+m)a^2B(1+m)}{4a^2B^2(c + \tan(c + dx))} + \frac{B^2(m)(Ab^3(-1+m) - a^2Bm(1+m) + 2a^2B^2(3+m - m^2) + a^2Ab(-5m + m^2) + 2a^2Ab^2(-1 - 3m + m^2) - a^2B(2 - 3m + m^2))}{a^2(c + b^2)^2(1 + m)} \right)}{2a(a^2 + b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
[Out] (Tan[c + d*x]^(1 + m)*((b*(A*b - a*B))/(a + b*Tan[c + d*x])^2 + (b*(-a^2*A
*b*(-5 + m)) + a^3*B*(-3 + m) - A*b^3*(-1 + m) + a*b^2*B*(1 + m)))/(a*(a^2
+ b^2)*(a + b*Tan[c + d*x])) + (b*(2 + m)*(A*b^5*(-1 + m)*m - a*b^4*B*m*(1
+ m) + 2*a^3*b^2*B*(3 + m - m^2) + a^4*A*b*(6 - 5*m + m^2) + 2*a^2*A*b^3*(-
1 - 3*m + m^2) - a^5*B*(2 - 3*m + m^2))*Hypergeometric2F1[1, 1 + m, 2 + m,
-((b*Tan[c + d*x])/a)] + 2*a^3*((a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*(2
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2] + (-3*a^2*
A*b + A*b^3 + a^3*B - 3*a*b^2*B)*(1 + m)*Hypergeometric2F1[1, (2 + m)/2, (4
+ m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]))/(a^2*(a^2 + b^2)^2*(1 + m)*(2 + m
)))/(2*a*(a^2 + b^2)*d)
```

**Maple [F]**

time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + b \tan(dx + c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^3, x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fr
icas")
```

[Out] integral((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b^3\*tan(d\*x + c)^3 + 3\*a\*b^2\*tan(d\*x + c)^2 + 3\*a^2\*b\*tan(d\*x + c) + a^3), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/(a + b\*tan(c + d\*x))\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3, x)

$$3.486 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^4} dx$$

Optimal. Leaf size=659

$$\frac{(a^4A - 6a^2Ab^2 + Ab^4 + 4a^3bB - 4ab^3B) {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right) \tan^{1+m}(c+dx)}{(a^2 + b^2)^4 d(1+m)} - \frac{b(ab^6Bm(1 -$$

```
[Out] (A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*hypergeom([1, 1/2+1/2*m], [3/2+1/2*m], -tan(d*x+c)^2)*tan(d*x+c)^(1+m)/(a^2+b^2)^4/d/(1+m)-1/6*b*(a*b^6*B*m*(-m^2+1)+3*a^2*A*b^5*m*(m^2-5*m+2)+A*b^7*m*(m^2-3*m+2)+3*a^3*b^4*B*(-m^3+2*m^2+5*m+2)+a^7*B*(-m^3+6*m^2-11*m+6)-a^6*A*b*(-m^3+9*m^2-26*m+24)+3*a^4*A*b^3*(m^3-7*m^2+10*m+8)-3*a^5*b^2*B*(m^3-4*m^2-m+12))*hypergeom([1, 1+m], [2+m], -b*tan(d*x+c)/a)*tan(d*x+c)^(1+m)/a^4/(a^2+b^2)^4/d/(1+m)-(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*hypergeom([1, 1+1/2*m], [2+1/2*m], -tan(d*x+c)^2)*tan(d*x+c)^(2+m)/(a^2+b^2)^4/d/(2+m)+1/3*b*(A*b-B*a)*tan(d*x+c)^(1+m)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/6*b*(A*b^3*(2-m)-a^3*B*(5-m)+a^2*A*b*(8-m)+a*b^2*B*(1+m))*tan(d*x+c)^(1+m)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+1/6*b*(a*b^4*B*(-m^2+1)+2*a^3*b^2*B*(-m^2+3*m+7)+a^4*A*b*(m^2-9*m+26)+2*a^2*A*b^3*(m^2-6*m+2)-a^5*B*(m^2-6*m+11)+A*b^5*(m^2-3*m+2))*tan(d*x+c)^(1+m)/a^3/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]

time = 1.65, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3690, 3730, 3734, 3619, 3557, 371, 3715, 66}

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/((a^2 + b^2)^4*d*(1 + m)) - (b*(a*b^6*B*m*(1 - m^2) + 3*a^2*A*b^5*m*(2 - 5*m + m^2) + A*b^7*m*(2 - 3*m + m^2) + 3*a^3*b^4*B*(2 + 5*m + 2*m^2 - m^3) + a^7*B*(6 - 11*m + 6*m^2 - m^3) - a^6*A*b*(24 - 26*m + 9*m^2 - m^3) + 3*a^4*A*b^3*(8 + 10*m - 7*m^2 + m^3) - 3*a^5*b^2*B*(12 - m - 4*m^2 + m^3))*Hypergeometric2F1[1, 1 + m, 2 + m, -(b*Tan[c + d*x])/a]*Tan[c + d*x]^(1 + m))/(6*a^4*(a^2 + b^2)^4*d*(1 + m)) - ((4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/((a^2 + b^2)^4*d*(2 + m)) + (b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + (b*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1 + m))*Tan[c + d*x]^(1 + m))/(6*a^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (b*(a*b^4*B*(1 - m^2) + 2*a^3*b^2*B*(7 +
```

$3*m - m^2) + a^4*A*b*(26 - 9*m + m^2) + 2*a^2*A*b^3*(2 - 6*m + m^2) - a^5*B*(11 - 6*m + m^2) + A*b^5*(2 - 3*m + m^2))*Tan[c + d*x]^(1 + m)/(6*a^3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))$

#### Rule 66

$Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x\_Symbol] \rightarrow Simp[c^n*((b*x)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x] /; FreeQ[\{b, c, d, m, n\}, x] \&\& !IntegerQ[m] \&\& (IntegerQ[n] || (GtQ[c, 0] \&\& !(EqQ[n, -2^(-1)] \&\& EqQ[c^2 - d^2, 0] \&\& GtQ[-d/(b*c), 0])))$

#### Rule 371

$Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_))^(n_)]^(p_), x\_Symbol] \rightarrow Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[\{a, b, c, m, n, p\}, x] \&\& !IGtQ[p, 0] \&\& (ILtQ[p, 0] || GtQ[a, 0])$

#### Rule 3557

$Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x\_Symbol] \rightarrow Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[\{b, c, d, n\}, x] \&\& !IntegerQ[n]$

#### Rule 3619

$Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])], x\_Symbol] \rightarrow Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[\{b, c, d, e, f, m\}, x] \&\& NeQ[c^2 + d^2, 0] \&\& !IntegerQ[2*m]$

#### Rule 3690

$Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x\_Symbol] \rightarrow Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, n\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 + b^2, 0] \&\& NeQ[c^2 + d^2, 0] \&\& LtQ[m, -1] \&\& (IntegerQ[m] || IntegersQ[2*m, 2*n]) \&\& !(ILtQ[n, -1] \&\& (!IntegerQ[m] || (EqQ[c, 0] \&\& NeQ[a, 0])))$

#### Rule 3715



```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)^2], x_Symbol] :>
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

```

#### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2], x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4} dx &= \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{3a(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{\int \frac{\tan^m(c+dx)(3a^2A+Ab^2(2-m)+ab)}{d(a+b\tan(c+dx))^4} dx}{(a^2+b^2)^4} \\
&= \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{3a(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{b(Ab^3(2-m)-a^3B(5-m))}{6a^2(a^2+b^2)} \\
&= \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{3a(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{b(Ab^3(2-m)-a^3B(5-m))}{6a^2(a^2+b^2)} \\
&= \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{3a(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{b(Ab^3(2-m)-a^3B(5-m))}{6a^2(a^2+b^2)} \\
&= \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{3a(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{b(Ab^3(2-m)-a^3B(5-m))}{6a^2(a^2+b^2)} \\
&= \frac{b(ab^6Bm(1-m^2)+3a^2Ab^5m(2-5m+m^2)+Ab^7m(2-3m))}{(a^2+b^2)^4d(1+m)} \\
&= \frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)}{(a^2+b^2)^4} {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\tan^2(c+dx)\right)
\end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 1901 vs. 2(659) = 1318.  
time = 6.19, size = 1901, normalized size = 2.88

Antiderivative was successfully verified.

```

[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4, x]
[Out] (b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + (((-a*(-3*a*b*(A*b - a*B) - a*b*(A*b - a*B)*(2 - m))) + b^2*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m)))*Tan[c + d*x]^(1 + m))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (((b^2*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m))) - a*(-6*a^2*b*(2*a*A*b - a^2*B + b^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1 + m))))*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) + (((a^2*b*(6*a^3*(2*a*A*b - a^2*B + b^2*B) - b^2*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1 + m)) + b*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m)))) - a^2*m*(b^2*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m))))

```

$$\begin{aligned} & )) - a*(-6*a^2*b*(2*a*A*b - a^2*B + b^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - \\ & a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1 + m))) + b^2*((a^2 - b^2*m)*( \\ & -(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b \\ & ^2*(2 - m) + a*b*B*(1 + m))) + a*(1 + m)*(-6*a^2*b*(2*a*A*b - a^2*B + b^2*B \\ & ) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B* \\ & (1 + m))))*Hypergeometric2F1[1, 1 + m, 2 + m, -(b*Tan[c + d*x])/a]*Tan[c \\ & + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(1 + m)) + (((-a*b*(6*a^3*(2*a*A*b - a^2 \\ & *B + b^2*B) - b^2*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) \\ & + a*b^2*B*(1 + m)) + b*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2 \\ & *(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m)))) + a*((a^2 - b^2*m)*( \\ & -(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b \\ & ^2*(2 - m) + a*b*B*(1 + m))) + a*(1 + m)*(-6*a^2*b*(2*a*A*b - a^2*B + b^2*B \\ & ) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B* \\ & (1 + m))) + m*(b^2*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 \\ & - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m))) - a*(-6*a^2*b*(2*a*A*b - a \\ & ^2*B + b^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m \\ & ) + a*b^2*B*(1 + m)))))*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -Ta \\ & n[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) + (((-a^2*(6*a^3*(2*a*A*b - \\ & a^2*B + b^2*B) - b^2*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - \\ & m) + a*b^2*B*(1 + m)) + b*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + \\ & b^2*(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m)))) - b*((a^2 - b^2*m) \\ & *(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + \\ & A*b^2*(2 - m) + a*b*B*(1 + m))) + a*(1 + m)*(-6*a^2*b*(2*a*A*b - a^2*B + b \\ & ^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^ \\ & 2*B*(1 + m))) + m*(b^2*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2 \\ & *(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m))) - a*(-6*a^2*b*(2*a*A*b \\ & - a^2*B + b^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 \\ & - m) + a*b^2*B*(1 + m)))))*Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, \\ & -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m))/(a^2 + b^2))/(a*(a^2 + \\ & b^2))/(2*a*(a^2 + b^2))/(3*a*(a^2 + b^2)) \end{aligned}$$

**Maple [F]**

time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + b \tan(dx + c))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a)^4, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b^4\*tan(d\*x + c)^4 + 4\*a\*b^3\*tan(d\*x + c)^3 + 6\*a^2\*b^2\*tan(d\*x + c)^2 + 4\*a^3\*b\*tan(d\*x + c) + a^4), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(a + b \tan(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*4,x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/(a + b\*tan(c + d\*x))\*\*4, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^4,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a)^4, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + b \tan(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^4,x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^4, x)

$$3.487 \quad \int \tan^m(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=193

$$\frac{a^2(A + iB)F_1\left(1 + m; -\frac{5}{2}, 1; 2 + m; -\frac{b \tan(c+dx)}{a}, -i \tan(c + dx)\right) \tan^{1+m}(c + dx) \sqrt{a + b \tan(c + dx)} + a^2(A - iB)F_1\left(1 + m; -\frac{5}{2}, 1; 2 + m; \frac{b \tan(c+dx)}{a}, i \tan(c + dx)\right) \tan^{1+m}(c + dx) \sqrt{a + b \tan(c + dx)}}{2d(1 + m) \sqrt{1 + \frac{b \tan(c + dx)}{a}}}$$

[Out] 1/2\*a^2\*(A+I\*B)\*AppellF1(1+m,1,-5/2,2+m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(1+b\*tan(d\*x+c)/a)^(1/2)+1/2\*a^2\*(A-I\*B)\*AppellF1(1+m,1,-5/2,2+m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(1+b\*tan(d\*x+c)/a)^(1/2)

**Rubi** [A]

time = 0.28, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3684, 3683, 140, 138}

$$\frac{a^2(A + iB) \tan^{m+1}(c + dx) \sqrt{a + b \tan(c + dx)} F_1\left(m + 1; -\frac{5}{2}, 1; m + 2; -\frac{b \tan(c+dx)}{a}, -i \tan(c + dx)\right)}{2d(m+1) \sqrt{\frac{b \tan(c + dx)}{a} + 1}} + \frac{a^2(A - iB) \tan^{m+1}(c + dx) \sqrt{a + b \tan(c + dx)} F_1\left(m + 1; -\frac{5}{2}, 1; m + 2; -\frac{b \tan(c+dx)}{a}, i \tan(c + dx)\right)}{2d(m+1) \sqrt{\frac{b \tan(c + dx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (a^2\*(A + I\*B)\*AppellF1[1 + m, -5/2, 1, 2 + m, -(b\*Tan[c + d\*x])/a], (-I)\*Tan[c + d\*x])\*Tan[c + d\*x]^(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]/(2\*d\*(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a]) + (a^2\*(A - I\*B)\*AppellF1[1 + m, -5/2, 1, 2 + m, -(b\*Tan[c + d\*x])/a], I\*Tan[c + d\*x])\*Tan[c + d\*x]^(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]/(2\*d\*(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])

Rule 138

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m + 1)/(b\*(m + 1)))\*AppellF1[m + 1, -n, -p, m + 2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] & & !IntegerQ[m] & & !IntegerQ[n] & & GtQ[c, 0] & & (IntegerQ[p] || GtQ[e, 0])

Rule 140

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] & & !IntegerQ[m] & & !IntegerQ[n] & & !GtQ[c, 0]

## Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

## Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^m(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \frac{1}{2}(A - iB) \int (1 + i \tan(c + dx)) \tan^m(c + dx) dx \\
&= \frac{(A - iB) \text{Subst}\left(\int \frac{x^m(a+bx)^{5/2}}{1-ix} dx, x, \tan(c + dx)\right)}{2d} \\
&= \frac{\left(a^2(A - iB) \sqrt{a + b \tan(c + dx)}\right) \text{Subst}\left(\int x^m dx, x, \tan(c + dx)\right)}{2d \sqrt{1 + \frac{b \tan(c + dx)}{a}}} \\
&= \frac{a^2(A + iB) F_1\left(1 + m; -\frac{5}{2}, 1; 2 + m; -\frac{b \tan(c + dx)}{a}\right)}{2d(1 + m)}
\end{aligned}$$

**Mathematica** [F]

time = 30.83, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x  
]

**Maple** [F]

time = 0.37, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + b \tan(dx + c))^{\frac{5}{2}} (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(5/2)\*tan(d\*x + c)^m, x  
)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*b^2\*tan(d\*x + c)^3 + A\*a^2 + (2\*B\*a\*b + A\*b^2)\*tan(d\*x + c)^2 + (B\*a^2 + 2\*A\*a\*b)\*tan(d\*x + c))\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x  
)

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)



### 3.488 $\int \tan^m(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$

Optimal. Leaf size=189

$$\frac{a(A + iB)F_1\left(1 + m; -\frac{3}{2}, 1; 2 + m; -\frac{b \tan(c + dx)}{a}, -i \tan(c + dx)\right) \tan^{1+m}(c + dx) \sqrt{a + b \tan(c + dx)} + a(A - iB)F_1\left(1 + m; -\frac{3}{2}, 1; 2 + m; \frac{b \tan(c + dx)}{a}, i \tan(c + dx)\right) \tan^{1+m}(c + dx) \sqrt{a + b \tan(c + dx)}}{2d(1 + m) \sqrt{1 + \frac{b \tan(c + dx)}{a}}}$$

[Out]  $1/2*a*(A+I*B)*\text{AppellF1}(1+m, 1, -3/2, 2+m, -I*\tan(d*x+c), -b*\tan(d*x+c)/a)*(a+b*\tan(d*x+c))^{1/2}*\tan(d*x+c)^{(1+m)}/d/(1+m)/(1+b*\tan(d*x+c)/a)^{(1/2)}+1/2*a*(A-I*B)*\text{AppellF1}(1+m, 1, -3/2, 2+m, I*\tan(d*x+c), -b*\tan(d*x+c)/a)*(a+b*\tan(d*x+c))^{1/2}*\tan(d*x+c)^{(1+m)}/d/(1+m)/(1+b*\tan(d*x+c)/a)^{(1/2)}$

**Rubi** [A]

time = 0.29, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3684, 3683, 140, 138}

$$\frac{a(A + iB) \tan^{m+1}(c + dx) \sqrt{a + b \tan(c + dx)} F_1\left(m + 1; -\frac{3}{2}, 1; m + 2; -\frac{b \tan(c + dx)}{a}, -i \tan(c + dx)\right)}{2d(m + 1) \sqrt{\frac{b \tan(c + dx)}{a} + 1}} + \frac{a(A - iB) \tan^{m+1}(c + dx) \sqrt{a + b \tan(c + dx)} F_1\left(m + 1; -\frac{3}{2}, 1; m + 2; -\frac{b \tan(c + dx)}{a}, i \tan(c + dx)\right)}{2d(m + 1) \sqrt{\frac{b \tan(c + dx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^m*(a + b*\text{Tan}[c + d*x])^{3/2}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(a*(A + I*B)*\text{AppellF1}[1 + m, -3/2, 1, 2 + m, -((b*\text{Tan}[c + d*x])/a), (-I)*\text{Tan}[c + d*x]]*\text{Tan}[c + d*x]^{(1 + m)}*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/(2*d*(1 + m)*\text{Sqrt}[1 + (b*\text{Tan}[c + d*x])/a]) + (a*(A - I*B)*\text{AppellF1}[1 + m, -3/2, 1, 2 + m, -((b*\text{Tan}[c + d*x])/a), I*\text{Tan}[c + d*x]]*\text{Tan}[c + d*x]^{(1 + m)}*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]/(2*d*(1 + m)*\text{Sqrt}[1 + (b*\text{Tan}[c + d*x])/a])$

Rule 138

$\text{Int}[(b_.*(x_))^{(m_)}*((c_)+(d_.*(x_))^{(n_)}*((e_)+(f_.*(x_))^{(p_)}), x_]$   
 Symbol] :>  $\text{Simp}[c^n*e^p*((b*x)^{(m+1)}/(b*(m+1)))*\text{AppellF1}[m+1, -n, -p, m+2, (-d)*(x/c), (-f)*(x/e)], x] /;$  FreeQ[{b, c, d, e, f, m, n, p}, x] & & !IntegerQ[m] & & !IntegerQ[n] & & GtQ[c, 0] & & (IntegerQ[p] || GtQ[e, 0])

Rule 140

$\text{Int}[(b_.*(x_))^{(m_)}*((c_)+(d_.*(x_))^{(n_)}*((e_)+(f_.*(x_))^{(p_)}), x_]$   
 Symbol] :>  $\text{Dist}[c^n*\text{IntPart}[n]*((c + d*x)^{\text{FracPart}[n]}/(1 + d*(x/c))^{\text{FracPart}[n]}], \text{Int}[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /;$  FreeQ[{b, c, d, e, f, m, n, p}, x] & & !IntegerQ[m] & & !IntegerQ[n] & & !GtQ[c, 0]

Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^m(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx &= \frac{1}{2}(A - iB) \int (1 + i \tan(c + dx)) \tan^m(c + dx) dx \\
&= \frac{(A - iB) \text{Subst}\left(\int \frac{x^m(a+bx)^{3/2}}{1-ix} dx, x, \tan(c + dx)\right)}{2d} \\
&= \frac{\left(a(A - iB) \sqrt{a + b \tan(c + dx)}\right) \text{Subst}\left(\int x^m dx, x, \tan(c + dx)\right)}{2d \sqrt{1 + \frac{b \tan(c + dx)}{a}}} \\
&= \frac{a(A + iB) F_1\left(1 + m; -\frac{3}{2}, 1; 2 + m; -\frac{b \tan(c + dx)}{a}\right)}{2d(1 + m)}
\end{aligned}$$

**Mathematica** [F]

time = 9.76, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x  
]

**Maple** [F]

time = 0.31, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + b \tan(dx + c))^{\frac{3}{2}} (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(3/2)\*tan(d\*x + c)^m, x  
)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*b\*tan(d\*x + c)^2 + A\*a + (B\*a + A\*b)\*tan(d\*x + c))\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)\*tan(c + d\*x)\*\*m, x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2), x)

$$3.489 \quad \int \tan^m(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=187

$$\frac{(A + iB)F_1\left(1 + m; -\frac{1}{2}, 1; 2 + m; -\frac{b \tan(c+dx)}{a}, -i \tan(c + dx)\right) \tan^{1+m}(c + dx) \sqrt{a + b \tan(c + dx)} (A - iB)F_1\left(1 + m; -\frac{1}{2}, 1; 2 + m; \frac{b \tan(c+dx)}{a}, i \tan(c + dx)\right)}{2d(1 + m) \sqrt{1 + \frac{b \tan(c + dx)}{a}}}$$

[Out] 1/2\*(A+I\*B)\*AppellF1(1+m,1,-1/2,2+m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(1+b\*tan(d\*x+c)/a)^(1/2)+1/2\*(A-I\*B)\*AppellF1(1+m,1,-1/2,2+m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(1+b\*tan(d\*x+c)/a)^(1/2)

**Rubi [A]**

time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3684, 3683, 140, 138}

$$\frac{(A + iB) \tan^{m+1}(c + dx) \sqrt{a + b \tan(c + dx)} F_1\left(m + 1; -\frac{1}{2}, 1; m + 2; -\frac{b \tan(c+dx)}{a}, -i \tan(c + dx)\right)}{2d(m + 1) \sqrt{\frac{b \tan(c + dx)}{a} + 1}} + \frac{(A - iB) \tan^{m+1}(c + dx) \sqrt{a + b \tan(c + dx)} F_1\left(m + 1; -\frac{1}{2}, 1; m + 2; -\frac{b \tan(c+dx)}{a}, i \tan(c + dx)\right)}{2d(m + 1) \sqrt{\frac{b \tan(c + dx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A + I\*B)\*AppellF1[1 + m, -1/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), (-I)\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]/(2\*d\*(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a]) + ((A - I\*B)\*AppellF1[1 + m, -1/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]/(2\*d\*(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])

**Rule 138**

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m + 1)/(b\*(m + 1)))\*AppellF1[m + 1, -n, -p, m + 2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] & & !IntegerQ[m] & & !IntegerQ[n] & & GtQ[c, 0] & & (IntegerQ[p] || GtQ[e, 0])

**Rule 140**

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] & & !IntegerQ[m] & & !IntegerQ[n] & & !GtQ[c, 0]

Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^m(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= \frac{1}{2} (A - iB) \int (1 + i \tan(c + dx)) \tan^m(c + dx) dx \\
&= \frac{(A - iB) \text{Subst} \left( \int \frac{x^m \sqrt{a + bx}}{1 - ix} dx, x, \tan(c + dx) \right)}{2d} \\
&= \frac{\left( (A - iB) \sqrt{a + b \tan(c + dx)} \right) \text{Subst} \left( \int \frac{x^m}{1 - ix} dx, x, \tan(c + dx) \right)}{2d \sqrt{1 + \frac{b \tan(c + dx)}{a}}} \\
&= \frac{(A + iB) F_1 \left( 1 + m; -\frac{1}{2}, 1; 2 + m; -\frac{b \tan(c + dx)}{a} \right)}{2d(1 + m)}
\end{aligned}$$

Mathematica [F]

time = 3.50, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^m\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^m\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.31, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) \sqrt{a + b \tan(dx + c)} (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))\*tan(c + d\*x)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2), x)



$$3.490 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=187

$$\frac{(A+iB)F_1\left(1+m; \frac{1}{2}, 1; 2+m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \tan^{1+m}(c+dx) \sqrt{1+\frac{b \tan(c+dx)}{a}}}{2d(1+m)\sqrt{a+b \tan(c+dx)}} + \frac{(A-iB)F_1\left(1+m; \frac{1}{2}, 1; 2+m; \frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right) \tan^{1+m}(c+dx) \sqrt{1+\frac{b \tan(c+dx)}{a}}}{2d(1+m)\sqrt{a+b \tan(c+dx)}}$$

[Out] 1/2\*(A+I\*B)\*AppellF1(1+m,1,1/2,2+m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(1+b\*tan(d\*x+c)/a)^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(a+b\*tan(d\*x+c))^(1/2)+1/2\*(A-I\*B)\*AppellF1(1+m,1,1/2,2+m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(1+b\*tan(d\*x+c)/a)^(1/2)\*tan(d\*x+c)^(1+m)/d/(1+m)/(a+b\*tan(d\*x+c))^(1/2)

Rubi [A]

time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3684, 3683, 140, 138}

$$\frac{(A+iB) \tan^{m+1}(c+dx) \sqrt{\frac{b \tan(c+dx)}{a} + 1} F_1\left(m+1; \frac{1}{2}, 1; m+2; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right)}{2d(m+1)\sqrt{a+b \tan(c+dx)}} + \frac{(A-iB) \tan^{m+1}(c+dx) \sqrt{\frac{b \tan(c+dx)}{a} + 1} F_1\left(m+1; \frac{1}{2}, 1; m+2; -\frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right)}{2d(m+1)\sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((A + I\*B)\*AppellF1[1 + m, 1/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), (-I)\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(2\*d\*(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]) + ((A - I\*B)\*AppellF1[1 + m, 1/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(2\*d\*(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 138

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m+1)/(b\*(m+1)))\*AppellF1[m+1, -n, -p, m+2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 140

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 3683

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]

```

#### Rule 3684

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx &= \frac{1}{2}(A-iB) \int \frac{(1+i\tan(c+dx))\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx + \frac{1}{2}(A+iB) \int \frac{(1-i\tan(c+dx))\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}} dx \\
&= \frac{(A-iB)\text{Subst}\left(\int \frac{x^m}{(1-ix)\sqrt{a+bx}} dx, x, \tan(c+dx)\right)}{2d} + \frac{(A+iB)\text{Subst}\left(\int \frac{x^m}{(1+ix)\sqrt{a+bx}} dx, x, \tan(c+dx)\right)}{2d} \\
&= \frac{\left((A-iB)\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^m}{(1-ix)\sqrt{1+\frac{bx}{a}}} dx, x, \tan(c+dx)\right)}{2d\sqrt{a+b\tan(c+dx)}} + \frac{\left((A+iB)\sqrt{1+\frac{b\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{x^m}{(1+ix)\sqrt{1+\frac{bx}{a}}} dx, x, \tan(c+dx)\right)}{2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{(A+iB)F_1\left(1+m; \frac{1}{2}, 1; 2+m; -\frac{b\tan(c+dx)}{a}, -i\tan(c+dx)\right)\tan^m(c+dx)}{2d(1+m)\sqrt{a+b\tan(c+dx)}}
\end{aligned}$$

#### Mathematica [F]

time = 5.69, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]], x]
```

[Out] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]], x]

**Maple** [F]

time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{\sqrt{a + b \tan(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/sqrt(b\*tan(d\*x + c) + a), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2), x)

$$3.491 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=193

$$\frac{(A+iB)F_1\left(1+m; \frac{3}{2}, 1; 2+m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \tan^{1+m}(c+dx) \sqrt{1 + \frac{b \tan(c+dx)}{a}}}{2ad(1+m) \sqrt{a+b \tan(c+dx)}} + \frac{(A-iB)F_1\left(1+m; \frac{3}{2}, 1; 2+m; \frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right) \tan^{1+m}(c+dx) \sqrt{1 + \frac{b \tan(c+dx)}{a}}}{2ad(1+m) \sqrt{a+b \tan(c+dx)}}$$

[Out] 1/2\*(A+I\*B)\*AppellF1(1+m,1,3/2,2+m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(1+b\*tan(d\*x+c)/a)^(1/2)\*tan(d\*x+c)^(1+m)/a/d/(1+m)/(a+b\*tan(d\*x+c))^(1/2)+1/2\*(A-I\*B)\*AppellF1(1+m,1,3/2,2+m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(1+b\*tan(d\*x+c)/a)^(1/2)\*tan(d\*x+c)^(1+m)/a/d/(1+m)/(a+b\*tan(d\*x+c))^(1/2)

**Rubi [A]**

time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3684, 3683, 140, 138}

$$\frac{(A+iB) \tan^{m+1}(c+dx) \sqrt{\frac{b \tan(c+dx)}{a}} + 1 F_1\left(m+1; \frac{3}{2}, 1; m+2; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right)}{2ad(m+1) \sqrt{a+b \tan(c+dx)}} + \frac{(A-iB) \tan^{m+1}(c+dx) \sqrt{\frac{b \tan(c+dx)}{a}} + 1 F_1\left(m+1; \frac{3}{2}, 1; m+2; -\frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right)}{2ad(m+1) \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2),x]

[Out] ((A + I\*B)\*AppellF1[1 + m, 3/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), (-I)\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(2\*a\*d\*(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]) + ((A - I\*B)\*AppellF1[1 + m, 3/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(2\*a\*d\*(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 138

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m+1)/(b\*(m+1)))\*AppellF1[m+1, -n, -p, m+2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 140

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

#### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx &= \frac{1}{2}(A-iB) \int \frac{(1+i \tan(c+dx)) \tan^m(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx + \frac{1}{2}(A+iB) \int \frac{(1-i \tan(c+dx)) \tan^m(c+dx)}{(a+b \tan(c+dx))^{3/2}} dx \\ &= \frac{(A-iB) \text{Subst}\left(\int \frac{x^m}{(1-ix)(a+bx)^{3/2}} dx, x, \tan(c+dx)\right)}{2d} + \frac{(A+iB) \text{Subst}\left(\int \frac{x^m}{(1+ix)(a+bx)^{3/2}} dx, x, \tan(c+dx)\right)}{2d} \\ &= \frac{\left((A-iB) \sqrt{1+\frac{b \tan(c+dx)}{a}}\right) \text{Subst}\left(\int \frac{x^m}{(1-ix)\left(1+\frac{bx}{a}\right)^{3/2}} dx, x, \tan(c+dx)\right)}{2ad \sqrt{a+b \tan(c+dx)}} + \frac{\left((A+iB) \sqrt{1-\frac{b \tan(c+dx)}{a}}\right) \text{Subst}\left(\int \frac{x^m}{(1+ix)\left(1+\frac{bx}{a}\right)^{3/2}} dx, x, \tan(c+dx)\right)}{2ad \sqrt{a+b \tan(c+dx)}} \\ &= \frac{(A+iB) F_1\left(1+m; \frac{3}{2}, 1; 2+m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \tan^{m+1}(c+dx)}{2ad(1+m) \sqrt{a+b \tan(c+dx)}} \end{aligned}$$

#### Mathematica [F]

time = 13.94, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

[Out] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

**Maple** [F]

time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + b \tan(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a)^(3/2), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m/(b^2\*tan(d\*x + c)^2 + 2\*a\*b\*tan(d\*x + c) + a^2), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2), x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/(a + b\*tan(c + d\*x))\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x)



$$3.492 \quad \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=193

$$\frac{(A+iB)F_1\left(1+m; \frac{5}{2}, 1; 2+m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \tan^{1+m}(c+dx) \sqrt{1 + \frac{b \tan(c+dx)}{a}}}{2a^2 d(1+m) \sqrt{a+b \tan(c+dx)}} + \frac{(A-iB)F_1\left(1+m; \frac{5}{2}, 1; 2+m; \frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right) \tan^{1+m}(c+dx) \sqrt{1 + \frac{b \tan(c+dx)}{a}}}{2a^2 d(1+m) \sqrt{a+b \tan(c+dx)}}$$

[Out] 1/2\*(A+I\*B)\*AppellF1(1+m,1,5/2,2+m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(1+b\*tan(d\*x+c)/a)^(1/2)\*tan(d\*x+c)^(1+m)/a^2/d/(1+m)/(a+b\*tan(d\*x+c))^(1/2)+1/2\*(A-I\*B)\*AppellF1(1+m,1,5/2,2+m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(1+b\*tan(d\*x+c)/a)^(1/2)\*tan(d\*x+c)^(1+m)/a^2/d/(1+m)/(a+b\*tan(d\*x+c))^(1/2)

**Rubi [A]**

time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3684, 3683, 140, 138}

$$\frac{(A+iB) \tan^{m+1}(c+dx) \sqrt{\frac{b \tan(c+dx)}{a} + 1} F_1\left(m+1; \frac{5}{2}, 1; m+2; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right)}{2a^2 d(m+1) \sqrt{a+b \tan(c+dx)}} + \frac{(A-iB) \tan^{m+1}(c+dx) \sqrt{\frac{b \tan(c+dx)}{a} + 1} F_1\left(m+1; \frac{5}{2}, 1; m+2; -\frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right)}{2a^2 d(m+1) \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2),x]

[Out] ((A + I\*B)\*AppellF1[1 + m, 5/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), (-I)\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(2\*a^2\*d\*(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]]) + ((A - I\*B)\*AppellF1[1 + m, 5/2, 1, 2 + m, -((b\*Tan[c + d\*x])/a), I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(2\*a^2\*d\*(1 + m)\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 138

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m+1)/(b\*(m+1)))\*AppellF1[m+1, -n, -p, m+2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 140

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

#### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx &= \frac{1}{2}(A-iB) \int \frac{(1+i \tan(c+dx)) \tan^m(c+dx)}{(a+b \tan(c+dx))^{5/2}} dx + \frac{1}{2}(A+iB) \int \frac{(1-i \tan(c+dx)) \tan^m(c+dx)}{(a+b \tan(c+dx))^{5/2}} dx \\ &= \frac{(A-iB) \text{Subst}\left(\int \frac{x^m}{(1-ix)(a+bx)^{5/2}} dx, x, \tan(c+dx)\right)}{2d} + \frac{(A+iB) \text{Subst}\left(\int \frac{x^m}{(1+ix)(a+bx)^{5/2}} dx, x, \tan(c+dx)\right)}{2d} \\ &= \frac{\left((A-iB) \sqrt{1+\frac{b \tan(c+dx)}{a}}\right) \text{Subst}\left(\int \frac{x^m}{(1-ix)\left(1+\frac{bx}{a}\right)^{5/2}} dx, x, \tan(c+dx)\right)}{2a^2 d \sqrt{a+b \tan(c+dx)}} + \frac{\left((A+iB) \sqrt{1-\frac{b \tan(c+dx)}{a}}\right) \text{Subst}\left(\int \frac{x^m}{(1+ix)\left(1+\frac{bx}{a}\right)^{5/2}} dx, x, \tan(c+dx)\right)}{2a^2 d \sqrt{a+b \tan(c+dx)}} \\ &= \frac{(A+iB) F_1\left(1+m; \frac{5}{2}, 1; 2+m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \tan^{m+1}(c+dx)}{2a^2 d (1+m) \sqrt{a+b \tan(c+dx)}} \end{aligned}$$

#### Mathematica [F]

time = 60.48, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),
x]
```

[Out] Integrate[(Tan[c + d\*x]^m\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

**Maple** [F]

time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m(dx + c))(A + B \tan(dx + c))}{(a + b \tan(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x)

[Out] int(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*tan(d\*x + c)^m/(b\*tan(d\*x + c) + a)^(5/2), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)\*tan(d\*x + c)^m/(b^3\*tan(d\*x + c)^3 + 3\*a\*b^2\*tan(d\*x + c)^2 + 3\*a^2\*b\*tan(d\*x + c) + a^3), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \tan^m(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2), x)

[Out] Integral((A + B\*tan(c + d\*x))\*tan(c + d\*x)\*\*m/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c + dx)^m (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int((tan(c + d\*x)^m\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2), x)

### 3.493 $\int \tan^m(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=183

$$\frac{(A+iB)F_1\left(1+m; -n, 1; 2+m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \tan^{1+m}(c+dx)(a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)}{2d(1+m)}$$

[Out] 1/2\*(A+I\*B)\*AppellF1(1+m,1,-n,2+m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1+m)\*(a+b\*tan(d\*x+c))^n/d/(1+m)/((1+b\*tan(d\*x+c)/a)^n)+1/2\*(A-I\*B)\*AppellF1(1+m,1,-n,2+m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1+m)\*(a+b\*tan(d\*x+c))^n/d/(1+m)/((1+b\*tan(d\*x+c)/a)^n)

**Rubi [A]**

time = 0.20, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {3684, 3683, 140, 138}

$$\frac{(A+iB) \tan^{m+1}(c+dx)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) + (A-iB) \tan^{m+1}(c+dx)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right)}{2d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A + I\*B)\*AppellF1[1 + m, -n, 1, 2 + m, -((b\*Tan[c + d\*x])/a), (-I)\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*(a + b\*Tan[c + d\*x])^n)/(2\*d\*(1 + m)\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[1 + m, -n, 1, 2 + m, -((b\*Tan[c + d\*x])/a), I\*Tan[c + d\*x]]\*Tan[c + d\*x]^(1 + m)\*(a + b\*Tan[c + d\*x])^n)/(2\*d\*(1 + m)\*(1 + (b\*Tan[c + d\*x])/a)^n)

Rule 138

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m + 1)/(b\*(m + 1)))\*AppellF1[m + 1, -n, -p, m + 2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 140

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

#### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

#### Rubi steps

$$\begin{aligned} \int \tan^m(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx &= \frac{1}{2}(A - iB) \int (1 + i \tan(c + dx)) \tan^m(c + dx) \\ &= \frac{(A - iB) \text{Subst}\left(\int \frac{x^m(a+bx)^n}{1-ix} dx, x, \tan(c + dx)\right)}{2d} \\ &= \frac{\left((A - iB)(a + b \tan(c + dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)\right)}{2} \\ &= \frac{(A + iB)F_1\left(1 + m; -n, 1; 2 + m; -\frac{b \tan(c+dx)}{a}\right)}{2} \end{aligned}$$

#### Mathematica [F]

time = 1.69, size = 0, normalized size = 0.00

$$\int \tan^m(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

```
[Out] Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

**Maple [F]**

time = 0.31, size = 0, normalized size = 0.00

$$\int (\tan^m(dx + c)) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^m, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^m, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \tan^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*tan(c + d\*x)\*\*m, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^m, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int(tan(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)



$$3.494 \quad \int \tan^4(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=387

$$\frac{(Ab^3(2+n)(3+n)(4+n) - a(b^2B(3+n)(4+n) - 2a(3aB - Ab(4+n))))(a+b \tan(c+dx))^{1+n}}{b^4d(1+n)(2+n)(3+n)(4+n)} + \frac{(A$$

[Out]  $-(A*b^3*(2+n)*(3+n)*(4+n)-a*(b^2*B*(3+n)*(4+n)-2*a*(3*a*B-A*b*(4+n)))*(a+b*\tan(d*x+c))^{(1+n)}/b^4/d/(1+n)/(2+n)/(3+n)/(4+n)+1/2*(A-I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a-I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(I*a+b)/d/(1+n)-1/2*(A+I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a+I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(I*a-b)/d/(1+n)-(b^2*B*(3+n)*(4+n)-2*a*(3*a*B-A*b*(4+n))*\tan(d*x+c)*(a+b*\tan(d*x+c))^{(1+n)}/b^3/d/(2+n)/(3+n)/(4+n)-(3*a*B-A*b*(4+n))*\tan(d*x+c)^2*(a+b*\tan(d*x+c))^{(1+n)}/b^2/d/(3+n)/(4+n)+B*\tan(d*x+c)^3*(a+b*\tan(d*x+c))^{(1+n)}/b/d/(4+n)$

Rubi [A]

time = 0.73, antiderivative size = 385, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3688, 3728, 3711, 3620, 3618, 70}

$\frac{\tan^2(c+dx)(6b^2B-2aAbn-4)-B^2Bn^2(3n+4)(a+b\tan(c+dx))^{2n}}{B^2(n+2)(n+3)(n+4)}$ ,  $\frac{(6b^2B-2a^2Abn-4)-aB^2Bn^2(3n+4)+4B^2n^2(2n+3)(n+4)(a+b\tan(c+dx))^{2n}}{B^2(n+1)(n+2)(n+3)(n+4)}$ ,  $\frac{\tan^2(c+dx)(3aB-Abn+4)(a+b\tan(c+dx))^{2n}}{B^2(n+3)(n+4)}$ ,  $\frac{(A-I^2B)(a+b\tan(c+dx))^{2n}F_1(1, n+1, n+2, \frac{a+b\tan(c+dx)}{a-I*b})}{2d(n+1)(n+2)}$ ,  $\frac{(A+I^2B)(a+b\tan(c+dx))^{2n}F_1(1, n+1, n+2, \frac{a+b\tan(c+dx)}{a+I*b})}{2d(n+1)(n+2)}$ ,  $\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{2n}}{Bd(n+4)}$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out]  $-(((6*a^3*B - 2*a^2*A*b*(4+n) - a*b^2*B*(3+n)*(4+n) + A*b^3*(2+n)*(3+n)*(4+n))*(a+b*\tan[c+d*x])^{(1+n)})/(b^4*d*(1+n)*(2+n)*(3+n)*(4+n))) + ((A-I*B)*\text{Hypergeometric2F1}[1, 1+n, 2+n, (a+b*\tan[c+d*x])/(a-I*b)]*(a+b*\tan[c+d*x])^{(1+n)})/(2*(I*a+b)*d*(1+n)) - ((A+I*B)*\text{Hypergeometric2F1}[1, 1+n, 2+n, (a+b*\tan[c+d*x])/(a+I*b)]*(a+b*\tan[c+d*x])^{(1+n)})/(2*(I*a-b)*d*(1+n)) + ((6*a^2*B - 2*a*A*b*(4+n) - b^2*B*(3+n)*(4+n))*\tan[c+d*x]*(a+b*\tan[c+d*x])^{(1+n)})/(b^3*d*(2+n)*(3+n)*(4+n)) - ((3*a*B - A*b*(4+n))*\tan[c+d*x]^2*(a+b*\tan[c+d*x])^{(1+n)})/(b^2*d*(3+n)*(4+n)) + (B*\tan[c+d*x]^3*(a+b*\tan[c+d*x])^{(1+n)})/(b*d*(4+n))$

Rule 70

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m+1)/(b^(n+1)\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(a+b\*x)/(b\*c-a\*d)], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rubi steps

$$\begin{aligned}
 \int \tan^4(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx &= \frac{B \tan^3(c + dx)(a + b \tan(c + dx))^{1+n}}{bd(4 + n)} + \int \tan^3(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx \\
 &= -\frac{(3aB - Ab(4 + n)) \tan^2(c + dx)(a + b \tan(c + dx))^{1+n}}{b^2d(3 + n)(4 + n)} + \int \tan^2(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx \\
 &= \frac{(6a^2B - 2aAb(4 + n) - b^2B(3 + n)(4 + n)) \tan(c + dx)(a + b \tan(c + dx))^{1+n}}{b^3d(2 + n)(3 + n)} + \int \tan(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx \\
 &= -\frac{(6a^3B - 2a^2Ab(4 + n) - ab^2B(3 + n)(4 + n)) (a + b \tan(c + dx))^{1+n}}{b^4d(1 + n)} + \int (a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx \\
 &= -\frac{(6a^3B - 2a^2Ab(4 + n) - ab^2B(3 + n)(4 + n)) (a + b \tan(c + dx))^{1+n}}{b^4d(1 + n)} + \int (a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx \\
 &= -\frac{(6a^3B - 2a^2Ab(4 + n) - ab^2B(3 + n)(4 + n)) (a + b \tan(c + dx))^{1+n}}{b^4d(1 + n)} + \int (a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx \\
 &= -\frac{(6a^3B - 2a^2Ab(4 + n) - ab^2B(3 + n)(4 + n)) (a + b \tan(c + dx))^{1+n}}{b^4d(1 + n)} + \int (a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx
 \end{aligned}$$

**Mathematica [A]**

time = 3.93, size = 384, normalized size = 0.99

(a + b tan(c + dx))^(1+n) ((3a^2B - 2aAb(4+n) - b^2B(3+n)(4+n)) tan^2(c + dx)(a + b tan(c + dx))^(1+n) + (6a^2B - 2aAb(4+n) - b^2B(3+n)(4+n)) tan(c + dx)(a + b tan(c + dx))^(1+n) - (6a^3B - 2a^2Ab(4+n) - ab^2B(3+n)(4+n)) (a + b tan(c + dx))^(1+n)) / (b^2d(3+n)(4+n)) + (6a^3B - 2a^2Ab(4+n) - ab^2B(3+n)(4+n)) (a + b tan(c + dx))^(1+n) / (b^4d(1+n)) + Integrate[(a + b tan(c + dx))^n (A + B tan(c + dx)), x]

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^4\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a + b\*Tan[c + d\*x])^(1 + n)\*(I\*((2\*I)\*(a - I\*b)\*(a + I\*b)\*(6\*a^3\*B - 2\*a^2\*A\*b\*(4 + n) - a\*b^2\*B\*(3 + n)\*(4 + n) + A\*b^3\*(2 + n)\*(3 + n)\*(4 + n)) - (a + I\*b)\*b^4\*(A - I\*B)\*(24 + 26\*n + 9\*n^2 + n^3)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)] + (a - I\*b)\*b^4\*(A + I\*B)\*(24 + 26\*n + 9\*n^2 + n^3)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)]) + 2\*(a - I\*b)\*(a + I\*b)\*b\*(1 + n)\*(6\*a^2\*B - 2\*a\*A\*b\*(4 + n) - b^2\*B\*(3 + n)\*(4 + n))\*Tan[c + d\*x] - 2\*(a - I\*b)\*(a + I\*b)\*b^2\*(1 + n)\*(2 + n)\*(3\*a\*B - A\*b\*(4 + n))\*Tan[c + d\*x]^2 + 2\*(a - I\*b)\*(a + I\*b)\*b^3\*B\*(1 + n)\*(2 + n)\*(3 + n)\*Tan[c + d\*x]^3)/(2\*(a - I\*b)\*(a + I\*b)\*b^4\*d\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n))

**Maple [F]**

time = 0.28, size = 0, normalized size = 0.00

$$\int (\tan^4(dx + c)) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^4\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^4\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^4, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^4\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c)^5 + A\*tan(d\*x + c)^4)\*(b\*tan(d\*x + c) + a)^n, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \tan^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*4\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*tan(c + d\*x)\*\*4, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^4 (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

### 3.495 $\int \tan^3(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$

Optimal. Leaf size=291

$$\frac{(2a^2B - aAb(3+n) - b^2B(6+5n+n^2))(a+b \tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} + \frac{(iA+B) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right)}{2(ia+b)d(1+n)}$$

[Out]  $(2*a^2*B - a*A*b*(3+n) - b^2*B*(n^2+5*n+6))*(a+b*\tan(d*x+c))^{(1+n)}/b^3/d/(1+n)/(2+n)/(3+n)+1/2*(I*A+B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a-I*b))* (a+b*\tan(d*x+c))^{(1+n)}/(I*a+b)/d/(1+n)+1/2*(A+I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a+I*b))* (a+b*\tan(d*x+c))^{(1+n)}/(a+I*b)/d/(1+n)-(2*a*B - A*b*(3+n))*\tan(d*x+c)*(a+b*\tan(d*x+c))^{(1+n)}/b^2/d/(2+n)/(3+n)+B*\tan(d*x+c)^2*(a+b*\tan(d*x+c))^{(1+n)}/b/d/(3+n)$

Rubi [A]

time = 0.41, antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {3688, 3728, 3711, 3620, 3618, 70}

$$\frac{(2a^2B - aAb(n+3) - b^2B(n+2)(n+3))(a+b \tan(c+dx))^{n+1}}{b^3d(n+1)(n+2)(n+3)} - \frac{\tan(c+dx)(2aB - Ab(n+3))(a+b \tan(c+dx))^{n+1}}{b^2d(n+2)(n+3)} + \frac{(B+iA)(a+b \tan(c+dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-ib}\right)}{2d(n+1)(b+ia)} + \frac{(A+iB)(a+b \tan(c+dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)} + \frac{B \tan^2(c+dx)(a+b \tan(c+dx))^{n+1}}{bd(n+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^3*(a + b*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $((2*a^2*B - a*A*b*(3+n) - b^2*B*(2+n)*(3+n))*(a+b*\text{Tan}[c+d*x])^{(1+n)})/(b^3*d*(1+n)*(2+n)*(3+n)) + ((I*A+B)*\text{Hypergeometric2F1}[1, 1+n, 2+n, (a+b*\text{Tan}[c+d*x])/(a-I*b)]*(a+b*\text{Tan}[c+d*x])^{(1+n)})/(2*(I*a+b)*d*(1+n)) + ((A+I*B)*\text{Hypergeometric2F1}[1, 1+n, 2+n, (a+b*\text{Tan}[c+d*x])/(a+I*b)]*(a+b*\text{Tan}[c+d*x])^{(1+n)})/(2*(a+I*b)*d*(1+n)) - ((2*a*B - A*b*(3+n))*\text{Tan}[c+d*x]*(a+b*\text{Tan}[c+d*x])^{(1+n)})/(b^2*d*(2+n)*(3+n)) + (B*\text{Tan}[c+d*x]^2*(a+b*\text{Tan}[c+d*x])^{(1+n)})/(b*d*(3+n))$

Rule 70

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^{n+1}*(a+b*x)^{(m+1)}/(b^{(n+1)}*(m+1))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*((a+b*x)/(b*c - a*d))], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 3618

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c$

$*x), x], x, d*\text{Tan}[e + f*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3620

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}, x\_Symbol] :> \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^{m*(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^{m*(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{IntegerQ}[m]$

### Rule 3688

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}*\{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] :> \text{Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{(m - 1)}*\{(c + d*\text{Tan}[e + f*x])^{(n + 1)}/(d*f*(m + n))\}, x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 2)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*\text{Tan}[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\& (\text{IntegerQ}[m] \|\| \text{IntegersQ}[2*m, 2*n]) \&\& !( \text{IGtQ}[n, 1] \& \& ( !\text{IntegerQ}[m] \|\| (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3711

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\}, x\_Symbol] :> \text{Simp}[C*(a + b*\text{Tan}[e + f*x])^{(m + 1)}/(b*f*(m + 1)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A - C + B*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \&\& \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \&\& !\text{LeQ}[m, -1]$

### Rule 3728

$\text{Int}[\{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(m_.)}*\{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]\}^{(n_.)}*\{(A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^2\}, x\_Symbol] :> \text{Simp}[C*(a + b*\text{Tan}[e + f*x])^{m*(c + d*\text{Tan}[e + f*x])^{(n + 1)}/(d*f*(m + n + 1))\}, x] + \text{Dist}[1/(d*(m + n + 1)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*\text{Tan}[e + f*x] - (C*m*(b*c - a*d) - b*B*d*(m + n + 1))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 0] \&\& !( \text{IGtQ}[n, 0] \&\& ( !\text{IntegerQ}[m] \|\| (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rubi steps

$$\begin{aligned}
\int \tan^3(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx &= \frac{B \tan^2(c+dx)(a+b \tan(c+dx))^{1+n}}{bd(3+n)} + \frac{\int \tan(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx}{b^2d(2+n)(3+n)} \\
&= -\frac{(2aB - Ab(3+n)) \tan(c+dx)(a+b \tan(c+dx))^n}{b^2d(2+n)(3+n)} \\
&= \frac{(2a^2B - aAb(3+n) - b^2B(2+n)(3+n)) (a+b \tan(c+dx))^n}{b^3d(1+n)(2+n)(3+n)} \\
&= \frac{(2a^2B - aAb(3+n) - b^2B(2+n)(3+n)) (a+b \tan(c+dx))^n}{b^3d(1+n)(2+n)(3+n)} \\
&= \frac{(2a^2B - aAb(3+n) - b^2B(2+n)(3+n)) (a+b \tan(c+dx))^n}{b^3d(1+n)(2+n)(3+n)} \\
&= \frac{(2a^2B - aAb(3+n) - b^2B(2+n)(3+n)) (a+b \tan(c+dx))^n}{b^3d(1+n)(2+n)(3+n)}
\end{aligned}$$

**Mathematica [A]**

time = 1.61, size = 281, normalized size = 0.97

$$\frac{(a+b \tan(c+dx))^{1+n} (2(a-b)(a+b)(2a^2B - aAb(3+n) - b^2B(2+n)(3+n)) + (a+b)^2(A - bD)(6+5n+n^2) {}_2F_1(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+b}) + (a-b)^2(A + bD)(6+5n+n^2) {}_2F_1(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+b}) - 2(a-b)(a+b)(1+n)(2aB - Ab(3+n)) \tan(c+dx) + 2(a-b)(a+b)^2B(1+n)(2+n) \tan^2(c+dx)}{2(a-b)(a+b)^2d(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(2*(a - I*b)*(a + I*b)*(2*a^2*B - a*A*b*(3 + n) - b^2*B*(2 + n)*(3 + n)) + (a + I*b)*b^3*(A - I*B)*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)] + (a - I*b)*b^3*(A + I*B)*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)] - 2*(a - I*b)*(a + I*b)*b*(1 + n)*(2*a*B - A*b*(3 + n))*Tan[c + d*x] + 2*(a - I*b)*(a + I*b)*b^2*B*(1 + n)*(2 + n)*Tan[c + d*x]^2)/(2*(a - I*b)*(a + I*b)*b^3*d*(1 + n)*(2 + n)*(3 + n))
```

**Maple [F]**

time = 0.25, size = 0, normalized size = 0.00

$$\int (\tan^3(dx+c))(a+b \tan(dx+c))^n(A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)), x)
```

```
[Out] int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)), x)
```



**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^3, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c)^4 + A\*tan(d\*x + c)^3)\*(b\*tan(d\*x + c) + a)^n, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \tan^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*3\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*tan(c + d\*x)\*\*3, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^3 (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

### 3.496 $\int \tan^2(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$

Optimal. Leaf size=219

$$\frac{(aB - Ab(2+n))(a + b \tan(c + dx))^{1+n}}{b^2 d(1+n)(2+n)} + \frac{(iA + B) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right) (a + b \tan(c + dx))}{2(a-ib)d(1+n)}$$

[Out]  $-(a*B-A*b*(2+n))*(a+b*\tan(d*x+c))^{(1+n)}/b^2/d/(1+n)/(2+n)+1/2*(I*A+B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a-I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(a-I*b)/d/(1+n)+1/2*(A+I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a+I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(I*a-b)/d/(1+n)+B*\tan(d*x+c)*(a+b*\tan(d*x+c))^{(1+n)}/b/d/(2+n)$

Rubi [A]

time = 0.24, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {3688, 3711, 3620, 3618, 70}

$$\frac{(aB - Ab(n+2))(a + b \tan(c + dx))^{n+1}}{b^2 d(n+1)(n+2)} + \frac{(B + iA)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)} + \frac{(A + iB)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a+ib}\right)}{2d(n+1)(-b+ia)} + \frac{B \tan(c + dx)(a + b \tan(c + dx))^{n+1}}{bd(n+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left(\left(\left(a*B - A*b*(2+n)\right)*(a + b*\text{Tan}[c + d*x])^{(1+n)}\right)/\left(b^2*d*(1+n)*(2+n)\right)\right) + \left(\left(I*A + B\right)*\text{Hypergeometric2F1}\left[1, 1+n, 2+n, (a + b*\text{Tan}[c + d*x])/(a - I*b)\right]*(a + b*\text{Tan}[c + d*x])^{(1+n)}\right)/\left(2*(a - I*b)*d*(1+n)\right) + \left(\left(A + I*B\right)*\text{Hypergeometric2F1}\left[1, 1+n, 2+n, (a + b*\text{Tan}[c + d*x])/(a + I*b)\right]*(a + b*\text{Tan}[c + d*x])^{(1+n)}\right)/\left(2*(I*a - b)*d*(1+n)\right) + \left(B*\text{Tan}[c + d*x]*(a + b*\text{Tan}[c + d*x])^{(1+n)}\right)/\left(b*d*(2+n)\right)$

Rule 70

$\text{Int}[\left((a_) + (b_)*(x_)\right)^{(m_)*\left((c_) + (d_)*(x_)\right)^{(n_)}, x\_Symbol] := \text{Simp}[(b*c - a*d)^n*(a + b*x)^{(m+1)}/(b^{(n+1)}*(m+1))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(a + b*x)/(b*c - a*d)], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3618

$\text{Int}[\left((a_) + (b_)*\tan[(e_) + (f_)*(x_)]\right)^{(m_)*\left((c_) + (d_)*\tan[(e_) + (f_)*(x_)]\right)}, x\_Symbol] := \text{Dist}[c*(d/f), \text{Subst}[\text{Int}[(a + (b/d)*x)^m/(d^2 + c*x), x], x, d*\text{Tan}[e + f*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$

## Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

## Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

## Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

## Rubi steps

$$\begin{aligned}
\int \tan^2(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx &= \frac{B \tan(c + dx)(a + b \tan(c + dx))^{1+n}}{bd(2 + n)} + \frac{\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{b^2 d(1 + n)(2 + n)} \\
&= -\frac{(aB - Ab(2 + n))(a + b \tan(c + dx))^{1+n}}{b^2 d(1 + n)(2 + n)} + \frac{\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{b^2 d(1 + n)(2 + n)} \\
&= -\frac{(aB - Ab(2 + n))(a + b \tan(c + dx))^{1+n}}{b^2 d(1 + n)(2 + n)} + \frac{\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{b^2 d(1 + n)(2 + n)} \\
&= -\frac{(aB - Ab(2 + n))(a + b \tan(c + dx))^{1+n}}{b^2 d(1 + n)(2 + n)} + \frac{\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{b^2 d(1 + n)(2 + n)} \\
&= -\frac{(aB - Ab(2 + n))(a + b \tan(c + dx))^{1+n}}{b^2 d(1 + n)(2 + n)} + \frac{\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{b^2 d(1 + n)(2 + n)}
\end{aligned}$$

**Mathematica [A]**

time = 0.95, size = 169, normalized size = 0.77

$$\frac{(a + b \tan(c + dx))^{1+n} \left( \frac{4Ab - 2aB + 2Abn}{b + bn} + \frac{b(iA+B)(2+n) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right)}{(a-ib)(1+n)} + \frac{b(-iA+B)(2+n) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+ib}\right)}{(a+ib)(1+n)} + 2B \tan(c + dx) \right)}{2bd(2+n)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d\*x]^2\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a + b\*Tan[c + d\*x])^(1 + n)\*((4\*A\*b - 2\*a\*B + 2\*A\*b\*n)/(b + b\*n) + (b\*(I\*A + B)\*(2 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)])/((a - I\*b)\*(1 + n)) + (b\*((-I)\*A + B)\*(2 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)])/((a + I\*b)\*(1 + n)) + 2\*B\*Tan[c + d\*x]))/(2\*b\*d\*(2 + n))

**Maple [F]**

time = 0.26, size = 0, normalized size = 0.00

$$\int (\tan^2(dx + c)) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^2, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c)^3 + A\*tan(d\*x + c)^2)\*(b\*tan(d\*x + c) + a)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \tan^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*2\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*tan(c + d\*x)\*\*2, x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^2, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c + dx)^2 (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int(tan(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)

### 3.497 $\int \tan(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=168

$$\frac{B(a+b \tan(c+dx))^{1+n}}{bd(1+n)} - \frac{(A-iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right) (a+b \tan(c+dx))^{1+n}}{2(a-ib)d(1+n)} - \frac{(A+iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+ib}\right) (a+b \tan(c+dx))^{1+n}}{2(a+ib)d(1+n)}$$

[Out] B\*(a+b\*tan(d\*x+c))^(1+n)/b/d/(1+n)-1/2\*(A-I\*B)\*hypergeom([1, 1+n], [2+n], (a+b\*tan(d\*x+c))/(a-I\*b))\*(a+b\*tan(d\*x+c))^(1+n)/(a-I\*b)/d/(1+n)-1/2\*(A+I\*B)\*hypergeom([1, 1+n], [2+n], (a+b\*tan(d\*x+c))/(a+I\*b))\*(a+b\*tan(d\*x+c))^(1+n)/(a+I\*b)/d/(1+n)

**Rubi [A]**

time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {3673, 3620, 3618, 70}

$$-\frac{(A-iB)(a+b \tan(c+dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)} - \frac{(A+iB)(a+b \tan(c+dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)} + \frac{B(a+b \tan(c+dx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] (B\*(a + b\*Tan[c + d\*x])^(1 + n))/(b\*d\*(1 + n)) - ((A - I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)]\*(a + b\*Tan[c + d\*x])^(1 + n))/(2\*(a - I\*b)\*d\*(1 + n)) - ((A + I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)]\*(a + b\*Tan[c + d\*x])^(1 + n))/(2\*(a + I\*b)\*d\*(1 + n))

**Rule 70**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

**Rule 3618**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

**Rule 3620**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \tan(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx &= \frac{B(a + b \tan(c + dx))^{1+n}}{bd(1+n)} + \int (-B + A \tan(c + dx)) (a + b \tan(c + dx))^n dx \\ &= \frac{B(a + b \tan(c + dx))^{1+n}}{bd(1+n)} + \frac{1}{2}(-iA - B) \int (1 + i \tan(c + dx)) (a + b \tan(c + dx))^n dx \\ &= \frac{B(a + b \tan(c + dx))^{1+n}}{bd(1+n)} + \frac{(A - iB) \text{Subst}\left(\int (1 + i \tan(x)) (a + b \tan(x))^n dx\right)}{2} \\ &= \frac{B(a + b \tan(c + dx))^{1+n}}{bd(1+n)} - \frac{(A - iB) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a + b \tan(c + dx)}{a + ib}\right)}{2d(1+n)} \end{aligned}$$

### Mathematica [A]

time = 0.19, size = 125, normalized size = 0.74

$$\frac{\left(\frac{2B}{b} - \frac{(A-iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right)}{a-ib} - \frac{(A+iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+ib}\right)}{a+ib}\right) (a + b \tan(c + dx))^{1+n}}{2d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

```
[Out] (((2*B)/b - ((A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b)])/(a - I*b) - ((A + I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[c + d*x])^(1 + n))/(2
*d*(1 + n))
```



**Maple [F]**

time = 0.23, size = 0, normalized size = 0.00

$$\int \tan(dx + c) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c)^2 + A\*tan(d\*x + c))\*(b\*tan(d\*x + c) + a)^n, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \tan(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*tan(c + d\*x), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx) (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

### 3.498 $\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=143

$$\frac{(A - iB) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a + b \tan(c + dx)}{a - ib}\right) (a + b \tan(c + dx))^{1+n}}{2(ia + b)d(1 + n)} + \frac{(iA - B) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a + b \tan(c + dx)}{a + ib}\right)}{2(a + ib)d(1 + n)}$$

[Out] 1/2\*(A-I\*B)\*hypergeom([1, 1+n], [2+n], (a+b\*tan(d\*x+c))/(a-I\*b))\*(a+b\*tan(d\*x+c))^(1+n)/(I\*a+b)/d/(1+n)+1/2\*(I\*A-B)\*hypergeom([1, 1+n], [2+n], (a+b\*tan(d\*x+c))/(a+I\*b))\*(a+b\*tan(d\*x+c))^(1+n)/(a+I\*b)/d/(1+n)

**Rubi [A]**

time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3620, 3618, 70}

$$\frac{(A - iB)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a + b \tan(c + dx)}{a - ib}\right)}{2d(n + 1)(b + ia)} + \frac{(-B + iA)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a + b \tan(c + dx)}{a + ib}\right)}{2d(n + 1)(a + ib)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A - I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)]\*(a + b\*Tan[c + d\*x])^(1 + n))/(2\*(I\*a + b)\*d\*(1 + n)) + ((I\*A - B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)]\*(a + b\*Tan[c + d\*x])^(1 + n))/(2\*(a + I\*b)\*d\*(1 + n))

**Rule 70**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

**Rule 3618**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

**Rule 3620**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x]

$1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ !\text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned} \int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx &= \frac{1}{2} (A - iB) \int (1 + i \tan(c + dx)) (a + b \tan(c + dx))^n dx + \frac{1}{2} (A + iB) \int (1 - i \tan(c + dx)) (a + b \tan(c + dx))^n dx \\ &= -\frac{(iA - B) \text{Subst}\left(\int \frac{(a+ibx)^n}{-1+x} dx, x, -i \tan(c + dx)\right)}{2d} + \frac{(iA + B) \text{Subst}\left(\int \frac{(a-ibx)^n}{-1+x} dx, x, i \tan(c + dx)\right)}{2d} \\ &= -\frac{(iA + B) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a+b \tan(c+dx)}{a-ib}\right) (a + b \tan(c + dx))^{1+n}}{2(a - ib)d(1 + n)} \end{aligned}$$

**Mathematica** [A]

time = 0.13, size = 120, normalized size = 0.84

$$\frac{i \left( -\frac{(A-iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right)}{a-ib} + \frac{(A+iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+ib}\right)}{a+ib} \right) (a + b \tan(c + dx))^{1+n}}{2d(1 + n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] ((I/2)\*(-(((A - I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)])/(a - I\*b)) + ((A + I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)])/(a + I\*b))\*(a + b\*Tan[c + d\*x])^(1 + n))/(d\*(1 + n))

**Maple** [F]

time = 0.22, size = 0, normalized size = 0.00

$$\int (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)

### 3.499 $\int \cot(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=190

$$\frac{(iA + B) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a+b \tan(c+dx)}{a-ib}\right) (a + b \tan(c + dx))^{1+n}}{2(ia + b)d(1 + n)} + \frac{(A + iB) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a+b \tan(c+dx)}{a+ib}\right)}{2(a + ib)d(1 + n)}$$

[Out] 1/2\*(I\*A+B)\*hypergeom([1, 1+n], [2+n], (a+b\*tan(d\*x+c))/(a-I\*b))\*(a+b\*tan(d\*x+c))^(1+n)/(I\*a+b)/d/(1+n)+1/2\*(A+I\*B)\*hypergeom([1, 1+n], [2+n], (a+b\*tan(d\*x+c))/(a+I\*b))\*(a+b\*tan(d\*x+c))^(1+n)/(a+I\*b)/d/(1+n)-A\*hypergeom([1, 1+n], [2+n], 1+b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^(1+n)/a/d/(1+n)

**Rubi [A]**

time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {3694, 3620, 3618, 70, 3715, 67}

$$\frac{(B + iA)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \tan(c+dx)}{a-ib}\right)}{2d(n+1)(b+ia)} + \frac{(A + iB)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)} - \frac{A(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{b \tan(c+dx)}{a} + 1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((I\*A + B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)])\*(a + b\*Tan[c + d\*x])^(1 + n)/(2\*(I\*a + b)\*d\*(1 + n)) + ((A + I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)])\*(a + b\*Tan[c + d\*x])^(1 + n)/(2\*(a + I\*b)\*d\*(1 + n)) - (A\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b\*Tan[c + d\*x])/a])\*(a + b\*Tan[c + d\*x])^(1 + n)/(a\*d\*(1 + n))

Rule 67

Int[((b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((c + d\*x)^(n + 1)/(d\*(n + 1)\*(-d/(b\*c))^m)\*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b\*c), 0])

Rule 70

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rule 3694

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan[
e + f*x], x], x] + Dist[b*((A*b - a*B)/(a^2 + b^2)), Int[(c + d*Tan[e +
f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ
[c^2 + d^2, 0]
```

### Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rubi steps

$$\begin{aligned} \int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx &= A \int \cot(c + dx)(a + b \tan(c + dx))^n (1 + \tan^2(c + dx)) dx \\ &= \frac{1}{2}(-iA + B) \int (1 - i \tan(c + dx))(a + b \tan(c + dx))^n dx \\ &= -\frac{A {}_2F_1\left(1, 1 + n; 2 + n; 1 + \frac{b \tan(c + dx)}{a}\right) (a + b \tan(c + dx))}{ad(1 + n)} \\ &= \frac{(A - iB) {}_2F_1\left(1, 1 + n; 2 + n; \frac{a + b \tan(c + dx)}{a - ib}\right) (a + b \tan(c + dx))}{2(a - ib)d(1 + n)} \end{aligned}$$

**Mathematica [A]**

time = 0.27, size = 169, normalized size = 0.89

$$\frac{(a(a+ib)(A-ib) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b\tan(c+dx)}{a-ib}\right) + (a-ib)\left(a(A+ib) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b\tan(c+dx)}{a+ib}\right) - 2A(a+ib) {}_2F_1\left(1, 1+n; 2+n; 1 + \frac{b\tan(c+dx)}{a}\right)\right)) (a+b\tan(c+dx))^{1+n}}{2a(a-ib)(a+ib)d(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a\*(a + I\*b)\*(A - I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a - I\*b)] + (a - I\*b)\*(a\*(A + I\*B)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Tan[c + d\*x])/(a + I\*b)] - 2\*A\*(a + I\*b)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b\*Tan[c + d\*x])/a]))\*(a + b\*Tan[c + d\*x])^(1 + n))/(2\*a\*(a - I\*b)\*(a + I\*b)\*d\*(1 + n))

**Maple [F]**

time = 0.24, size = 0, normalized size = 0.00

$$\int \cot(dx + c) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")



[Out] integral((B\*cot(d\*x + c)\*tan(d\*x + c) + A\*cot(d\*x + c))\*(b\*tan(d\*x + c) + a)<sup>n</sup>, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \cot(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*cot(c + d\*x), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*(a+b\*tan(d\*x+c))<sup>n</sup>\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)<sup>n</sup>\*cot(d\*x + c), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx) (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))<sup>n</sup>,x)

[Out] int(cot(c + d\*x)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))<sup>n</sup>, x)

$$3.500 \quad \int \cot^2(c+dx)(a+b \tan(c+dx))^n (A+B \tan(c+dx)) dx$$

Optimal. Leaf size=228

$$\frac{A \cot(c+dx)(a+b \tan(c+dx))^{1+n}}{ad} - \frac{(A-iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right) (a+b \tan(c+dx))^{1+n}}{2(ia+b)d(1+n)}$$

[Out]  $-A*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1+n)}/a/d-1/2*(A-I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a-I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(I*a+b)/d/(1+n)+1/2*(A+I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a+I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(I*a-b)/d/(1+n)-(A*b*n+B*a)*\text{hypergeom}([1, 1+n], [2+n], 1+b*\tan(d*x+c)/a)*(a+b*\tan(d*x+c))^{(1+n)}/a^2/d/(1+n)$

Rubi [A]

time = 0.31, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {3690, 3734, 3620, 3618, 70, 3715, 67}

$$\frac{(aB + Abn)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{b \tan(c + dx)}{a}\right)}{a^2 d(n+1)} - \frac{(A - iB)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a + b \tan(c + dx)}{a - ib}\right)}{2d(n+1)(b + ia)} + \frac{(A + iB)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{b \tan(c + dx)}{a + ib}\right)}{2d(n+1)(-b + ia)} - \frac{A \cot(c + dx)(a + b \tan(c + dx))^{n+1}}{ad}$$

Antiderivative was successfully verified.

[In] `Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]`

[Out]  $-((A*\text{Cot}[c + d*x]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(a*d)) - ((A - I*B)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a - I*b)]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(2*(I*a + b)*d*(1 + n)) + ((A + I*B)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a + I*b)]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(2*(I*a - b)*d*(1 + n)) - ((a*B + A*b*n)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + (b*\text{Tan}[c + d*x])/a]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(a^2*d*(1 + n))$

Rule 67

`Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])`

Rule 70

`Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(b*c - a*d)^(n)*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]`

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n +
2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx &= -\frac{A \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{ad} - \frac{\int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{ad} \\
&= -\frac{A \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{ad} - \frac{\int (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx}{ad} \\
&= -\frac{A \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{ad} - \frac{1}{2}(A + B) \frac{\int (a + b \tan(c + dx))^n dx}{ad} \\
&= -\frac{A \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{ad} - \frac{(aB + B^2) \int (a + b \tan(c + dx))^n dx}{2ad} \\
&= -\frac{A \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{ad} + \frac{(iA + B) \int (a + b \tan(c + dx))^n dx}{ad}
\end{aligned}$$

**Mathematica [A]**

time = 0.29, size = 202, normalized size = 0.89

$$\frac{(a^2(a+ib)(A-iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a-ib}\right) - (a-ib) \left(a^2(A+iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+b \tan(c+dx)}{a+ib}\right) + 2(-ia+b) \left(aB {}_2F_1\left(1, 1+n; 2+n; 1 + \frac{b \tan(c+dx)}{a}\right) - Ab {}_2F_1\left(2, 1+n; 2+n; 1 + \frac{b \tan(c+dx)}{a}\right)\right)\right)}{2a^2(a-ib)(-ia+b)d(1+n)} (a + b \tan(c + dx))^{1+n}$$

Antiderivative was successfully verified.

`[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]`

```
[Out] ((a^2*(a + I*b)*(A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)] - (a - I*b)*(a^2*(A + I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)] + 2*((-I)*a + b)*(a*B*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] - A*B*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]))*(a + b*Tan[c + d*x])^(1 + n))/(2*a^2*(a - I*b)*((-I)*a + b)*d*(1 + n))
```

**Maple [F]**

time = 0.28, size = 0, normalized size = 0.00

$$\int (\cot^2(dx + c)) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)), x)``[Out] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)), x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^2, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*cot(d\*x + c)^2\*tan(d\*x + c) + A\*cot(d\*x + c)^2)\*(b\*tan(d\*x + c) + a)^n, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \cot^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*2\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*cot(c + d\*x)\*\*2, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^2\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^2 (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int(cot(c + d\*x)^2\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)

### 3.501 $\int \cot^3(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$

Optimal. Leaf size=292

$$\frac{(2aB - Ab(1 - n)) \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{2a^2d} - \frac{A \cot^2(c + dx)(a + b \tan(c + dx))^{1+n}}{2ad} - \frac{(iA + B) {}_2F_1}{2ad}$$

[Out]  $-1/2*(2*a*B-A*b*(1-n))*\cot(d*x+c)*(a+b*\tan(d*x+c))^{(1+n)}/a^2/d-1/2*A*\cot(d*x+c)^2*(a+b*\tan(d*x+c))^{(1+n)}/a/d-1/2*(I*A+B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a-I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(I*a+b)/d/(1+n)-1/2*(A+I*B)*\text{hypergeom}([1, 1+n], [2+n], (a+b*\tan(d*x+c))/(a+I*b))*(a+b*\tan(d*x+c))^{(1+n)}/(a+I*b)/d/(1+n)+1/2*(2*a^2*A-2*a*b*B*n+A*b^2*(1-n)*n)*\text{hypergeom}([1, 1+n], [2+n], 1+b*\tan(d*x+c)/a)*(a+b*\tan(d*x+c))^{(1+n)}/a^3/d/(1+n)$

Rubi [A]

time = 0.56, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {3690, 3730, 3734, 3620, 3618, 70, 3715, 67}

$$\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{2a^2d} + \frac{(2a^2A-2abBn+AP(1-n)n)(a+b\tan(c+dx))^{n+1}}{2a^2d(n+1)} {}_2F_1(1, n+1; n+2; \frac{b\tan(c+dx)}{a+Ib}) - \frac{(B+iA)(a+b\tan(c+dx))^{n+1}}{2d(n+1)(b+in)} {}_2F_1(1, n+1; n+2; \frac{a+b\tan(c+dx)}{a+Ib}) - \frac{(A+iB)(a+b\tan(c+dx))^{n+1}}{2d(n+1)(a+ib)} {}_2F_1(1, n+1; n+2; \frac{a+b\tan(c+dx)}{a+Ib}) - \frac{A \cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^3*(a + b*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-1/2*((2*a*B - A*b*(1 - n))*\text{Cot}[c + d*x]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(a^2*d) - (A*\text{Cot}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(2*a*d) - ((I*A + B)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a - I*b)]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(2*(I*a + b)*d*(1 + n)) - ((A + I*B)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a + I*b)]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(2*(a + I*b)*d*(1 + n)) + ((2*a^2*A - 2*a*b*B*n + A*b^2*(1 - n)*n)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + (b*\text{Tan}[c + d*x])/a]*(a + b*\text{Tan}[c + d*x])^{(1 + n)})/(2*a^3*d*(1 + n))$

Rule 67

$\text{Int}[(b_*)*(x_*)^{(m_*)}((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}/(d*(n+1)*(-d/(b*c))^{(m)})*\text{Hypergeometric2F1}[-m, n+1, n+2, 1+d*(x/c)], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-d/(b*c), 0])$

Rule 70

$\text{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^n*((a + b*x)^{(m+1)}/(b^{(n+1)}*(m+1)))*\text{Hypergeometric2F1}[-n, m$

+ 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

### Rule 3618

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[c\*(d/f), Subst[Int[(a + (b/d)\*x)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3620

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 - I\*Tan[e + f\*x])), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 + I\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3690

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3730

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f

```

*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
 \int \cot^3(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx &= -\frac{A \cot^2(c + dx)(a + b \tan(c + dx))^{1+n}}{2ad} - \int \cot^2(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx \\
 &= -\frac{(2aB - Ab(1 - n)) \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{2a^2d} - \int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx \\
 &= -\frac{(2aB - Ab(1 - n)) \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{2a^2d} - \int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx \\
 &= -\frac{(2aB - Ab(1 - n)) \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{2a^2d} - \int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx \\
 &= -\frac{(2aB - Ab(1 - n)) \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{2a^2d} - \int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx \\
 &= -\frac{(2aB - Ab(1 - n)) \cot(c + dx)(a + b \tan(c + dx))^{1+n}}{2a^2d} - \int \cot(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx
 \end{aligned}$$

### Mathematica [A]

time = 0.37, size = 230, normalized size = 0.79

$$\frac{\left( a^2(a+ib)(A-iB) {}_2F_1\left(1, 1+n; 2+n; \frac{a+ib \tan(c+dx)}{a+ib}\right) + (a-ib) \left( a^2(A+ib) {}_2F_1\left(1, 1+n; 2+n; \frac{a+ib \tan(c+dx)}{a+ib}\right) - 2(a+ib) \left( a^2 A {}_2F_1\left(1, 1+n; 2+n; 1 + \frac{b \tan(c+dx)}{a}\right) + b \left( a B {}_2F_1\left(2, 1+n; 2+n; 1 + \frac{b \tan(c+dx)}{a}\right) - A B {}_2F_1\left(3, 1+n; 2+n; 1 + \frac{b \tan(c+dx)}{a}\right) \right) \right) \right) (a + b \tan(c + dx))^{1+n}}{2a^2(a-ib)(a+ib)d(1+n)}$$

Antiderivative was successfully verified.



[In] Integrate[Cot[c + d\*x]^3\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] 
$$-1/2*((a^3*(a + I*b)*(A - I*B)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a - I*b)] + (a - I*b)*(a^3*(A + I*B)*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a + I*b)] - 2*(a + I*b)*(a^2*A*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + (b*\text{Tan}[c + d*x])/a] + b*(a*B*\text{Hypergeometric2F1}[2, 1 + n, 2 + n, 1 + (b*\text{Tan}[c + d*x])/a] - A*b*\text{Hypergeometric2F1}[3, 1 + n, 2 + n, 1 + (b*\text{Tan}[c + d*x])/a]))*(a + b*\text{Tan}[c + d*x])^(1 + n))/(a^3*(a - I*b)*(a + I*b)*d*(1 + n))$$

Maple [F]

time = 0.28, size = 0, normalized size = 0.00

$$\int (\cot^3(dx + c)) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^3, x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*cot(d\*x + c)^3\*tan(d\*x + c) + A\*cot(d\*x + c)^3)\*(b\*tan(d\*x + c) + a)^n, x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \cot^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*3\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*cot(c + d\*x)\*\*3, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^3\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^3, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^3 (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int(cot(c + d\*x)^3\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)

$$3.502 \quad \int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt[4]{-1} a(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2a(A-iB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(iA+B) \cot^{\frac{3}{2}}(c+dx)}{3d}$$

[Out]  $2*(-1)^{(1/4)}*a*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-2/3*a*(I*A+B)*\cot(d*x+c)^{(3/2)}/d-2/5*a*A*\cot(d*x+c)^{(5/2)}/d+2*a*(A-I*B)*\cot(d*x+c)^{(1/2)}/d$

**Rubi** [A]

time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3662, 3673, 3609, 3614, 214}

$$-\frac{2a(B+iA) \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{2a(A-iB) \sqrt{\cot(c+dx)}}{d} + \frac{2\sqrt[4]{-1} a(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2aA \cot^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^{(7/2)}*(a+I*a*\operatorname{Tan}[c+d*x])*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(2*(-1)^{(1/4)}*a*(A-I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]])/d+(2*a*(A-I*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/d-(2*a*(I*A+B)*\operatorname{Cot}[c+d*x]^{(3/2)})/(3*d)-(2*a*A*\operatorname{Cot}[c+d*x]^{(5/2)})/(5*d)$

**Rule 214**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$   $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b]$

**Rule 3609**

$\operatorname{Int}[(a_+ + (b_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)])^{(m_+)}*((c_+ + (d_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)])^{-1}), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m-1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\operatorname{Tan}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \ \operatorname{GtQ}[m, 0]$

**Rule 3614**

$\operatorname{Int}[(c_+ + (d_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)])/ \operatorname{Sqrt}[(b_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /;$   $\operatorname{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \operatorname{EqQ}[c^2 + d^2, 0]$

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist [g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ [p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \int \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))(B + A \cot(c + dx)) dx \\
&= -\frac{2aA \cot^{\frac{5}{2}}(c + dx)}{5d} + \int \cot^{\frac{3}{2}}(c + dx)(-a(A - B \cot(c + dx))) dx \\
&= -\frac{2a(iA + B) \cot^{\frac{3}{2}}(c + dx)}{3d} - \frac{2aA \cot^{\frac{5}{2}}(c + dx)}{5d} \\
&= \frac{2a(A - iB) \sqrt{\cot(c + dx)}}{d} - \frac{2a(iA + B) \cot^{\frac{3}{2}}(c + dx)}{3d} \\
&= \frac{2a(A - iB) \sqrt{\cot(c + dx)}}{d} - \frac{2a(iA + B) \cot^{\frac{3}{2}}(c + dx)}{3d} \\
&= \frac{2\sqrt[4]{-1} a(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 263 vs.  $2(103) = 206$ .

time = 1.57, size = 263, normalized size = 2.55

$$\frac{a(i + \cot(c + dx))(B + A \cot(c + dx))(\cos(dx) - i \sin(dx)) \sin^2(c + dx) \left( -\frac{2i(A - iB)e^{-ic} \tanh^{-1}\left(\frac{\sqrt{-1 + e^{2i(c+dx)}}}{1 + e^{2i(c+dx)}}\right)}{\sqrt{-1 + e^{2i(c+dx)}}} \frac{i(1 + e^{2i(c+dx)})}{\sqrt{-1 + e^{2i(c+dx)}}} - \frac{1}{15} \sqrt{\cot(c + dx)} \csc^2(c + dx) (\cos(c) - i \sin(c)) (-12A + 15iB + 3(6A - 5iB) \cos(2(c + dx)) + 5(iA + B) \sin(2(c + dx))) \right)}{d(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
[Out] (a*(I + Cot[c + d*x])*(B + A*Cot[c + d*x])*(Cos[d*x] - I*Sin[d*x])*Sin[c +
d*x]^2*(((2*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x))]/(1 + E^((
2*I)*(c + d*x)))])/(E^(I*c)*Sqrt[(-1 + E^((2*I)*(c + d*x))]/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]
) - (Sqrt[Cot[c + d*x])*Csc[c + d*x]^2*(Cos[c] - I*Sin[c])*(-12*A + (15*I)*
B + 3*(6*A - (5*I)*B)*Cos[2*(c + d*x)] + 5*(I*A + B)*Sin[2*(c + d*x)]))/15)
)/(d*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 24.11, size = 2945, normalized size = 28.59

method	result	size
default	Expression too large to display	2945

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
[Out] -1/15*a/d*2^(1/2)*(-15*I*A*cos(d*x+c)^2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c)^(1/2)+15*I*B*cos(d*x+c)^2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c)^(1/2)-15*I*A*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+
c)^(1/2)+15*I*A*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)-15*I*B*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
)^(1/2)+15*I*A*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)-15
*I*A*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)+1
5*I*B*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)+
15*I*A*cos(d*x+c)^2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
```

$$\begin{aligned}
& (1/2), 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 5*B*\cos(dx+c) \\
& ^2*\sin(dx+c)*2^{(1/2)} - 15*I*B*\cos(dx+c)^3*2^{(1/2)} + 15*I*B*\cos(dx+c)*2^{(1/2)} + 18*A*\cos(dx+c)^3*2^{(1/2)} \\
& - 15*A*\cos(dx+c)*2^{(1/2)} + 15*A*((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} \\
& - 15*B*((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticF}((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 5*I*A*\cos(dx+c)^2*\sin(dx+c)*2^{(1/2)} + 15*B*((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 15*B*\cos(dx+c) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 15*I*A*((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} - 15*A*\cos(dx+c)^3 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 15*B*\cos(dx+c)^3 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticF} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} - 15*B*\cos(dx+c)^3 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 15*A*\cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 15*B*\cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticF} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} - 15*B*\cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} + 15*A*\cos(dx+c) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticPi} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2 + 1/2*I, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) ^{(1/2)} - 15*B*\cos(dx+c) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c)) ^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c)) ^{(1/2)} * \text{EllipticF} \\
& ((-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c)) ^{(1/2)}, 1/2*2^{(1/2)}) * (-\cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \dots
\end{aligned}$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs.  $2(81) = 162$ .

time = 0.52, size = 190, normalized size = 1.84

$$\frac{15 \left( 2\sqrt{2} (i-1) A + (i+1) B \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}} \right)}{\sqrt{\tan(dx+c)}} \right) + 2\sqrt{2} (i-1) A + (i+1) B \arctan \left( -\frac{1}{\sqrt{2} \left( \sqrt{2} - \frac{2}{\sqrt{\tan(dx+c)}} \right)} \right) + \sqrt{2} (i+1) A + (i-1) B \log \left( \frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\operatorname{arctan}(1)} + 1 \right) - \sqrt{2} (i-1) A + (i+1) B \log \left( -\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\operatorname{arctan}(1)} + 1 \right) + \frac{15(i+1)A}{\sqrt{\tan(dx+c)}} + \frac{15(-1+i)B}{\operatorname{arctan}(1)^2} - \frac{31A}{\operatorname{arctan}(1)^2}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/60\*(15\*(2\*sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(d\*x + c)))) + sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*log(sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) - sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1))\*a + 120\*(A - I\*B)\*a/sqrt(tan(d\*x + c)) + 40\*(-I\*A - B)\*a/tan(d\*x + c)^(3/2) - 24\*A\*a/tan(d\*x + c)^(5/2))/d

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(81) = 162.

time = 1.34, size = 434, normalized size = 4.21

$$\frac{15 \left( (d^{2i+1} - 2d^{2i} + d) \sqrt{\frac{(-1)^i d^2 - 2AB + B^2}{d^2}} \log \left( \frac{\left( (d^{2i} - 2d^{2i-1} + d) \sqrt{\frac{(-1)^i d^2 - 2AB + B^2}{d^2}} \sqrt{\frac{d^{2i+1} + 1}{d^{2i+1} - 1}} \right) + \dots}{\dots} \right) - 15 \left( (d^{2i+1} - 2d^{2i} + d) \sqrt{\frac{(-1)^i d^2 - 2AB + B^2}{d^2}} \log \left( \frac{\left( (d^{2i} - 2d^{2i-1} + d) \sqrt{\frac{(-1)^i d^2 - 2AB + B^2}{d^2}} \sqrt{\frac{d^{2i+1} + 1}{d^{2i+1} - 1}} \right) + \dots}{\dots} \right) - 4 \left( (23A - 20B) d^{2i+1} - 4 \left( (4A - 5B) d^{2i} + (13A - 10B) d \right) \sqrt{\frac{d^{2i+1} + 1}{d^{2i+1} - 1}} \right)}{30 \left( (d^{2i+1} - 2d^{2i} + d) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] -1/30\*(15\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*log(-2\*((A - I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) - (I\*d\*e^(2\*I\*d\*x + 2\*I\*c) - I\*d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))\*e^(-2\*I\*d\*x - 2\*I\*c)/((I\*A + B)\*a) - 15\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*log(-2\*((A - I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) - (-I\*d\*e^(2\*I\*d\*x + 2\*I\*c) + I\*d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^2/d^2)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))\*e^(-2\*I\*d\*x - 2\*I\*c)/((I\*A + B)\*a) - 4\*((23\*A - 20\*I\*B)\*a\*e^(4\*I\*d\*x + 4\*I\*c) - 6\*(4\*A - 5\*I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) + (13\*A - 10\*I\*B)\*a)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))/(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(7/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)\*cot(d\*x + c)^(7/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i), x)



$$3.503 \quad \int \cot^2(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=78

$$\frac{2\sqrt[4]{-1} a(iA + B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d} - \frac{2a(iA + B) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{3/2}(c + dx)}{3d}$$

[Out]  $-2*(-1)^{(1/4)}*a*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-2/3*a*A*\cot(d*x+c)^{(3/2)}/d-2*a*(I*A+B)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3662, 3673, 3609, 3614, 214}

$$\frac{2a(B + iA) \sqrt{\cot(c + dx)}}{d} - \frac{2\sqrt[4]{-1} a(B + iA) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d} - \frac{2aA \cot^{3/2}(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(5/2)}*(a + I*a*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(-2*(-1)^{(1/4)}*a*(I*A + B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d - (2*a*(I*A + B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/d - (2*a*A*\operatorname{Cot}[c + d*x]^{(3/2)})/(3*d)$

Rule 214

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 3609

$\operatorname{Int}[(a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])^{(m_)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a + b*\operatorname{Tan}[e + f*x])^m/(f*m)), x] + \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m-1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\operatorname{Tan}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{GtQ}[m, 0]$

Rule 3614

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]]/\operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \int \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))(B + A \cot(c + dx)) dx \\
 &= -\frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} + \int \sqrt{\cot(c + dx)} (-a(A + B \cot(c + dx))) dx \\
 &= -\frac{2a(iA + B) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2a(iA + B) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2\sqrt[4]{-1} a(iA + B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d}
 \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 161 vs.  $2(78) = 156$ .  
time = 1.91, size = 161, normalized size = 2.06

$$\frac{2ae^{-ic} \sqrt{\cot(c + dx)} (i + \cot(c + dx))(B + A \cot(c + dx))(\cos(dx) - i \sin(dx)) \sin^2(c + dx) \left(3iA + 3B + A \cot(c + dx) - 3i(A - iB) \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right) \sqrt{i \tan(c + dx)}\right)}{3d(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]), x]
[Out] (-2*a*Sqrt[Cot[c + d*x]]*(I + Cot[c + d*x])*(B + A*Cot[c + d*x])*(Cos[d*x] - I*Sin[d*x])*Sin[c + d*x]^2*((3*I)*A + 3*B + A*Cot[c + d*x] - (3*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Sqrt[I*Tan[c + d*x]])/(3*d*E^(I*c)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 22.71, size = 1538, normalized size = 19.72

method	result	size
default	Expression too large to display	1538

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$-1/3*a/d*2^{(1/2)}*(-3*I*A*\cos(d*x+c)*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c)))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+3*I*B*\cos(d*x+c)*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})-3*I*B*\cos(d*x+c)*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-3*I*A*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-3*A*\cos(d*x+c)*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})+3*A*\cos(d*x+c)*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+3*I*B*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})-3*I*B*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-3*B*\cos(d*x+c)*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-3*A*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})+3*A*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-3*B*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos$$

$$(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}))+3*I*A*2^{(1/2)*\cos(d*x+c)*\sin(d*x+c)+A*2^{(1/2)*\cos(d*x+c)^2+3*B*2^{(1/2)*\cos(d*x+c)*\sin(d*x+c)}*(\cos(d*x+c)/\sin(d*x+c))^{(5/2)*\sin(d*x+c)/\cos(d*x+c)}^3$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs.  $2(62) = 124$ .  
 time = 0.51, size = 174, normalized size = 2.23

$$\frac{3\left(2\sqrt{2}(-i+1)A+(i-1)B\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-i+1)A+(i-1)B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}(i-1)A+(i+1)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\cos(dx+c)}+1\right)+\sqrt{2}(i-1)A+(i+1)B\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\cos(dx+c)}+1\right)\right)^2-\frac{24(-i+1)A-3A}{\sqrt{\tan(dx+c)}}+\frac{-3A}{\cos(dx+c)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] -1/12\*(3\*(2\*sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(d\*x + c)))) - sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*log(sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) + sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1))\*a - 24\*(-I\*A - B)\*a/sqrt(tan(d\*x + c)) + 8\*A\*a/tan(d\*x + c)^(3/2))/d

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 382 vs.  $2(62) = 124$ .  
 time = 1.23, size = 382, normalized size = 4.90

$$\frac{3\left(d\sqrt{d^2+1}-d\right)\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\log\left(\frac{3\left(d\sqrt{d^2+1}-d\right)\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\log\left(\frac{\sqrt{d^2+1}}{d}\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\right)+3\left(d\sqrt{d^2+1}-d\right)\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\log\left(\frac{\sqrt{d^2+1}}{d}\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\right)}{6\left(d\sqrt{d^2+1}-d\right)\sqrt{\frac{d^2+2AB-1B^2}{d^2}}}\right)+4\left(4A+3B\right)a\sqrt{d^2+1}+(-2A-3B)a\sqrt{\frac{d^2+1}{d^2}}}{6\left(d\sqrt{d^2+1}-d\right)\sqrt{\frac{d^2+2AB-1B^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] -1/6\*(3\*(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a^2/d^2)\*log(-2\*((A - I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) + (d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a^2/d^2)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))\*e^(-2\*I\*d\*x - 2\*I\*c)/((I\*A + B)\*a) - 3\*(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a^2/d^2)\*log(-2\*((A - I\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) - (d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a^2/d^2)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))\*e^(-2\*I\*d\*x - 2\*I\*c)/((I\*A + B)\*a) + 4\*((4\*I\*A + 3\*B)\*a\*e^(2\*I\*d\*x + 2\*I\*c) + (-2\*I\*A - 3\*B)\*a)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))/(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)

**Sympy [F(-1)]** Timed out  
 time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)\*cot(d\*x + c)^(5/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i), x)

$$3.504 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=53

$$\frac{2\sqrt[4]{-1} a(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2aA \sqrt{\cot(c+dx)}}{d}$$

[Out]  $-2*(-1)^{(1/4)}*a*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-2*a*A*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3662, 3673, 3614, 214}

$$\frac{2aA \sqrt{\cot(c+dx)}}{d} - \frac{2\sqrt[4]{-1} a(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x]),x]$

[Out]  $(-2*(-1)^{(1/4)}*a*(A - I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/d - (2*a*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/d$

**Rule 214**

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b]$

**Rule 3614**

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])/\operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f, x\} \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

**Rule 3662**

$\operatorname{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)})*((a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b + a*\operatorname{Cot}[e + f*x])^m*(d + c*\operatorname{Cot}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, p, x\} \ \&\& \operatorname{!IntegerQ}[p] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IntegerQ}[n]$

**Rule 3673**

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))(A + B \tan(c + dx)) dx &= \int \frac{(ia + a \cot(c + dx))(B + A \cot(c + dx))}{\sqrt{\cot(c + dx)}} dx \\ &= -\frac{2aA \sqrt{\cot(c + dx)}}{d} + \int \frac{-a(A - iB) + a \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx \\ &= -\frac{2aA \sqrt{\cot(c + dx)}}{d} + \frac{(2a^2(A - iB)^2) \operatorname{Subst}(\int \frac{1}{\sqrt{u}} du, u = \cot(c + dx))}{d} \\ &= -\frac{2\sqrt[4]{-1} a(A - iB) \tanh^{-1}\left(\frac{(-1)^{3/4} \sqrt{\cot(c + dx)}}{1 + \sqrt{\cot(c + dx)}}\right)}{d} \end{aligned}$$

**Mathematica [A]**

time = 0.98, size = 92, normalized size = 1.74

$$\frac{2ae^{-ic} \sqrt{\cot(c + dx)} (\cos(c) + i \sin(c)) \left( -A + (A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) \sqrt{i \tan(c + dx)} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] (2\*a\*Sqrt[Cot[c + d\*x]]\*(Cos[c] + I\*Sin[c])\*(-A + (A - I\*B)\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x))]])\*Sqrt[I\*Tan[c + d\*x]])/(d\*E^(I\*c))

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 23.96, size = 1425, normalized size = 26.89

method	result	size
default	Expression too large to display	1425

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVE  
RBOSE)

```
[Out] -a/d*2^(1/2)*(I*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-I
*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi
((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*B*c
os(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-A*cos(d*x
+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+I*A*(-(cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c
))/sin(d*x+c))^(1/2),1/2*2^(1/2))-I*A*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))+B*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)-B*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+I*B*
(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-A*((cos(d*x+c)-1+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2)-B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/
sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+A*cos(
d*x+c)*2^(1/2))*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(3/2)/cos(d*x+c)^2
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs.  $2(43) = 86$ .

time = 0.52, size = 155, normalized size = 2.92

$$\frac{(2\sqrt{2}^{(i-1)A+(i+1)B})\arctan\left(\frac{1}{\sqrt{2}}\left(\sqrt{2}+\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}^{(i-1)A+(i+1)B}\arctan\left(-\frac{1}{\sqrt{2}}\left(\sqrt{2}-\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}^{-(i+1)A+(i-1)B}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\sqrt{2}^{-(i+1)A+(i-1)B}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)}{4d}+ \frac{A}{\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```



[Out]  $-1/4*((2*\sqrt{2})*((I - 1)*A + (I + 1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(dx + c)}))) + 2*\sqrt{2}*((I - 1)*A + (I + 1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(dx + c)}))) + \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1))*a + 8*A*a/\sqrt{\tan(dx + c)}/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs.  $2(43) = 86$ .  
time = 1.94, size = 316, normalized size = 5.96

$$\frac{4Aa\sqrt{\frac{1+e^{2(dx+2c)}+1}{e^{2(dx+2c)}-1}} - \sqrt{\frac{(-1A^2-2AB+1B^2)^2}{d^2}} d \log\left(\frac{2\left(\frac{(A+B)\sqrt{e^{2(dx+2c)}-1}}{(d)^{2(dx+2c)+1}}\sqrt{\frac{(-1A^2-2AB+1B^2)^2}{d^2}}\sqrt{\frac{1+e^{2(dx+2c)}+1}{e^{2(dx+2c)}-1}}\right)^{d^2-2(dx+2c)}}{(A+B)^2}\right) + \sqrt{\frac{(-1A^2-2AB+1B^2)^2}{d^2}} d \log\left(\frac{2\left(\frac{(A-B)\sqrt{e^{2(dx+2c)}-1}}{(d)^{2(dx+2c)+1}}\sqrt{\frac{(-1A^2-2AB+1B^2)^2}{d^2}}\sqrt{\frac{1+e^{2(dx+2c)}+1}{e^{2(dx+2c)}-1}}\right)^{d^2-2(dx+2c)}}{(A+B)^2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^(3/2)*(a+I*a*tan(dx+c))*(A+B*tan(dx+c)),x, algorithm="fricas")`

[Out]  $-1/2*(4*A*a*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}) - \sqrt{(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*d*\log(-2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} - (I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))})*e^{(-2*I*d*x - 2*I*c)}/((I*A + B)*a)) + \sqrt{(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*d*\log(-2*((A - I*B)*a*e^{(2*I*d*x + 2*I*c)} - (-I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))})*e^{(-2*I*d*x - 2*I*c)}/((I*A + B)*a)))/d$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int(-iA\cot^{\frac{3}{2}}(c+dx))dx + \int A\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)dx + \int B\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)dx + \int(-iB\tan(c+dx)\cot^{\frac{3}{2}}(c+dx))dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)**(3/2)*(a+I*a*tan(dx+c))*(A+B*tan(dx+c)),x)`

[Out]  $I*a*(\text{Integral}(-I*A*\cot(c + d*x)**(3/2), x) + \text{Integral}(A*\tan(c + d*x)*\cot(c + d*x)**(3/2), x) + \text{Integral}(B*\tan(c + d*x)**2*\cot(c + d*x)**(3/2), x) + \text{Integral}(-I*B*\tan(c + d*x)*\cot(c + d*x)**(3/2), x))$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)\*cot(d\*x + c)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) \text{ li}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i),x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i), x)

### 3.505 $\int \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))(A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=55

$$\frac{2\sqrt[4]{-1} a(iA + B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d} + \frac{2iaB}{d\sqrt{\cot(c + dx)}}$$

[Out]  $2*(-1)^{(1/4)}*a*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2*I*a*B/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3662, 3672, 3614, 214}

$$\frac{2\sqrt[4]{-1} a(B + iA) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d} + \frac{2iaB}{d\sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(2*(-1)^{(1/4)}*a*(I*A + B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d + ((2*I)*a*B)/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])$

**Rule 214**

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b]$

**Rule 3614**

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])/ \operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /;$   $\operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \ \operatorname{EqQ}[c^2 + d^2, 0]$

**Rule 3662**

$\operatorname{Int}[(\cot[(e_.) + (f_.)*(x_)])*(g_.)^{(p_.)}*((a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b + a*\operatorname{Cot}[e + f*x])^m*(d + c*\operatorname{Cot}[e + f*x])^n, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, g, p\}, x \ \&\& \ !\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))(A+B \tan(c+dx)) dx &= \int \frac{(ia+a \cot(c+dx))(B+A \cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \\ &= \frac{2iaB}{d\sqrt{\cot(c+dx)}} + \int \frac{a(iA+B)+a(A-ia \cot(c+dx))}{\sqrt{\cot(c+dx)}} dx \\ &= \frac{2iaB}{d\sqrt{\cot(c+dx)}} + \frac{(2a^2(iA+B)^2) \text{Subst}\left(\int \frac{1}{\sqrt{1-u}} du\right)}{d} \\ &= \frac{2a^2(iA+B)^2 \tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt{\cot(c+dx)}}{1+i \tan(c+dx)}\right)}{d} \end{aligned}$$

Mathematica [A]

time = 1.94, size = 108, normalized size = 1.96

$$\frac{2ae^{-ic}(\cos(c) + i \sin(c)) \left( (A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) + iB \sqrt{i \tan(c+dx)} \right)}{d \sqrt{\cot(c+dx)} \sqrt{i \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

```
[Out] (2*a*(Cos[c] + I*Sin[c])*((A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))]/(1 + E^((2*I)*(c + d*x)))] + I*B*Sqrt[I*Tan[c + d*x]]))/(d*E^(I*c)*Sqrt[Cot[c + d*x]]*Sqrt[I*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 63.47, size = 784, normalized size = 14.25

method	result
--------	--------

default	$a\sqrt{2} \sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}} (\cos(dx+c)+1)^2 (-1+\cos(dx+c)) \left( iA \sin(dx+c) \sqrt{\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}} \sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}} \right)$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $a/d*2^{(1/2)}*(\cos(d*x+c)/\sin(d*x+c))^{(1/2)}*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*$   
 $(I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\text{Elliptic}$   
 $\text{Pi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+I*B$   
 $*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\text{EllipticPi}(($   
 $-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}$   
 $*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\text{EllipticF}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})-A*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\text{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+A*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\text{EllipticF}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})+B*\sin(d*x+c)*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\text{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+I*B*2^{(1/2)}*\cos(d*x+c)-I*B*2^{(1/2)}/\sin(d*x+c)^3/\cos(d*x+c)$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs.  $2(43) = 86$ .  
time = 0.51, size = 155, normalized size = 2.82

$$\frac{8iB\sqrt{\tan(dx+c)} + (2\sqrt{2}(-i+1)A + (i-1)B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}(-i+1)A + (i-1)B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) - \sqrt{2}(i-1)A + (i+1)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\cos(dx+c)+1}\right) + \sqrt{2}(i-1)A + (i+1)B\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\cos(dx+c)+1}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/4*(8*I*B*a*\sqrt{\tan(d*x+c)} + (2*\sqrt{2})*(-I+1)*A + (I-1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(d*x+c)})) + 2*\sqrt{2}*(-I+1)*A + (I-1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(d*x+c)})) - \sqrt{2}*(I-1)*A + (I+1)*B)*\log(\sqrt{2}/\sqrt{\tan(d*x+c)} + 1/\tan(d*x+c) + 1) + \sqrt{2}*((I-1)*A + (I+1)*B)*\log(-\sqrt{2}/\sqrt{\tan(d*x+c)} + 1/\tan(d*x+c) + 1))*a/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs.  $2(43) = 86$ .  
time = 1.20, size = 364, normalized size = 6.62

$$\frac{(d e^{2 d x+2 c}+d) \sqrt{\frac{A^2+2 A B-i B^2}{d^2}} \log \left( \frac{2 \left( (A-i B) e^{2 d x+2 c}-(d e^{2 d x+2 c}-d) \sqrt{\frac{A^2+2 A B-i B^2}{d^2}} \sqrt{\frac{e^{2 d x+2 c}+1}{e^{2 d x+2 c}-1}} \right)}{(A+i B) a} \right) - (d e^{2 d x+2 c}+d) \sqrt{\frac{A^2+2 A B-i B^2}{d^2}} \log \left( \frac{2 \left( (A-i B) e^{2 d x+2 c}-(d e^{2 d x+2 c}-d) \sqrt{\frac{A^2+2 A B-i B^2}{d^2}} \sqrt{\frac{e^{2 d x+2 c}+1}{e^{2 d x+2 c}-1}} \right)}{(A+i B) a} \right)}{2(d e^{2 d x+2 c}+d)} + 4(B a e^{2 d x+2 c}-B a) \sqrt{\frac{e^{2 d x+2 c}+1}{e^{2 d x+2 c}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} * ((d * e^{(2 * I * d * x + 2 * I * c)} + d) * \text{sqrt}(- (I * A^2 + 2 * A * B - I * B^2) * a^2 / d^2) * \log(-2 * ((A - I * B) * a * e^{(2 * I * d * x + 2 * I * c)} + (d * e^{(2 * I * d * x + 2 * I * c)} - d) * \text{sqrt}(- (I * A^2 + 2 * A * B - I * B^2) * a^2 / d^2) * \text{sqrt}((I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1))) * e^{(-2 * I * d * x - 2 * I * c)} / ((I * A + B) * a)) - (d * e^{(2 * I * d * x + 2 * I * c)} + d) * \text{sqrt}(- (I * A^2 + 2 * A * B - I * B^2) * a^2 / d^2) * \log(-2 * ((A - I * B) * a * e^{(2 * I * d * x + 2 * I * c)} - (d * e^{(2 * I * d * x + 2 * I * c)} - d) * \text{sqrt}(- (I * A^2 + 2 * A * B - I * B^2) * a^2 / d^2) * \text{sqrt}((I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1))) * e^{(-2 * I * d * x - 2 * I * c)} / ((I * A + B) * a)) + 4 * (B * a * e^{(2 * I * d * x + 2 * I * c)} - B * a) * \text{sqrt}((I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1))) / (d * e^{(2 * I * d * x + 2 * I * c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$i a \left( \int (-i A \sqrt{\cot(c + dx)}) dx + \int A \tan(c + dx) \sqrt{\cot(c + dx)} dx + \int B \tan^2(c + dx) \sqrt{\cot(c + dx)} dx + \int (-i B \tan(c + dx) \sqrt{\cot(c + dx)}) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out]  $I * a * (\text{Integral}(-I * A * \text{sqrt}(\cot(c + d * x)), x) + \text{Integral}(A * \tan(c + d * x) * \text{sqrt}(\cot(c + d * x)), x) + \text{Integral}(B * \tan(c + d * x) ** 2 * \text{sqrt}(\cot(c + d * x)), x) + \text{Integral}(-I * B * \tan(c + d * x) * \text{sqrt}(\cot(c + d * x)), x))$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li),x)`

[Out] `int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li), x)`

$$3.506 \quad \int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=80

$$\frac{2\sqrt[4]{-1} a(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2iaB}{3d \cot^{3/2}(c+dx)} + \frac{2a(iA+B)}{d\sqrt{\cot(c+dx)}}$$

[Out]  $2*(-1)^{(1/4)}*a*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2/3*I*a*B/d/c$   
 $\operatorname{ot}(d*x+c)^{(3/2)}+2*a*(I*A+B)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3662, 3672, 3610, 3614, 214}

$$\frac{2a(B+ia)}{d\sqrt{\cot(c+dx)}} + \frac{2\sqrt[4]{-1} a(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2iaB}{3d \cot^{3/2}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(a + I*a*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x])}{\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]}, x]$

[Out]  $(2*(-1)^{(1/4)}*a*(A - I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d + (((2*I)/3)*a*B)/(d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (2*a*(I*A + B))/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])$

**Rule 214**

$\operatorname{Int}[\frac{(a_) + (b_)*(x_)^2}{(x_)^2}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{\operatorname{Rt}[-a/b, 2]}{a}*\operatorname{ArcTanh}[\frac{x}{\operatorname{Rt}[-a/b, 2]}], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

**Rule 3610**

$\operatorname{Int}[\frac{(a_ + (b_)*\operatorname{tan}[e_ + (f_)*(x_)])^{(m_)}*((c_ + (d_)*\operatorname{tan}[e_ + (f_)*(x_)])^{(m_)} + (f_)*(x_))}{(f_)*(x_)}], x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^{(m+1)})}{(f*(m+1)*(a^2 + b^2))}, x] + \operatorname{Dist}[1/(a^2 + b^2), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m+1)}*\operatorname{Simp}[a*c + b*d - (b*c - a*d)*\operatorname{Tan}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{LtQ}[m, -1]$

**Rule 3614**

$\operatorname{Int}[\frac{(c_ + (d_)*\operatorname{tan}[e_ + (f_)*(x_)])}{\operatorname{Sqrt}[(b_)*\operatorname{tan}[e_ + (f_)*(x_) + (c_ + (d_)*\operatorname{tan}[e_ + (f_)*(x_)])]}], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

**Rule 3662**



```
Int[(cot[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.))*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist [g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ [p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(b*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \int \frac{(ia + a \cot(c + dx))(B + A \cot(c + dx))}{\cot^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2iaB}{3d \cot^{\frac{3}{2}}(c + dx)} + \int \frac{a(iA + B) + a(A - iB) \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2iaB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2a(iA + B)}{d \sqrt{\cot(c + dx)}} + \int \frac{a(A - iB) - a \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx \\
 &= \frac{2iaB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2a(iA + B)}{d \sqrt{\cot(c + dx)}} + \frac{(2a^2(A - iB)^2) \operatorname{Su}}{d} \\
 &= \frac{2\sqrt[4]{-1} a(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d} + \frac{2a^2(A - iB)^2}{3d}
 \end{aligned}$$

### Mathematica [A]

time = 1.35, size = 96, normalized size = 1.20

$$\frac{2a \left( iB + 3(iA + B) \cot(c + dx) + \frac{3^{A-iB} \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right)}{(i \tan(c+dx))^{3/2}} \right)}{3d \cot^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],
x]
```

```
[Out] (2*a*(I*B + 3*(I*A + B)*Cot[c + d*x] + (3*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((
2*I)*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))])))/(I*Tan[c + d*x]^(3/2))/(3*d
*Cot[c + d*x]^(3/2))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 46.21, size = 889, normalized size = 11.11

method	result	size
default	Expression too large to display	889

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3*a/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(3*I*A*EllipticPi((-cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*A*EllipticF(
(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*B*EllipticPi((
-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*cos(d*x+c)*sin(d*x+c)+3*A*Ellip
ticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*cos(d*x+c)*sin(d*x+c)+3*
B*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*cos(d*x+c)*sin(d
*x+c)-3*B*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*cos(d*x+c)*sin(d*x+c
)+3*I*A*2^(1/2)*cos(d*x+c)^2+I*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*A*2^(1/2
)*cos(d*x+c)-I*B*2^(1/2)*sin(d*x+c)+3*B*2^(1/2)*cos(d*x+c)^2-3*B*2^(1/2)*cos
(d*x+c))/cos(d*x+c)/sin(d*x+c)^4/(cos(d*x+c)/sin(d*x+c))^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs.  $2(62) = 124$ .  
time = 0.52, size = 177, normalized size = 2.21

$$\frac{8 \left( i B a - \frac{3 i A d \sin(c)}{\cos(d c)} \tan(d x + c) \right)^2 + 3 \left( 2 \sqrt{2} (i-1) A + (i+1) B \right) \arctan \left( \frac{1}{\sqrt{2}} \left( \sqrt{2} + \frac{2}{\sqrt{\tan(d x + c)}} \right) \right) + 2 \sqrt{2} (i-1) A + (i+1) B \arctan \left( -\frac{1}{\sqrt{2}} \left( \sqrt{2} - \frac{2}{\sqrt{\tan(d x + c)}} \right) \right) + \sqrt{2} (i+1) A + (i-1) B \log \left( \frac{\sqrt{2}}{\sqrt{\tan(d x + c)}} + \frac{1}{\cos(d x + c)} + 1 \right) - \sqrt{2} (i+1) A + (i-1) B \log \left( -\frac{\sqrt{2}}{\sqrt{\tan(d x + c)}} + \frac{1}{\cos(d x + c)} + 1 \right) \right) e^{i A x + B x + C}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{12} * (8 * (I * B * a - 3 * (-I * A - B) * a / \tan(d * x + c)) * \tan(d * x + c)^{(3/2)} + 3 * (2 * \sqrt{2} * ((I - 1) * A + (I + 1) * B) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 / \sqrt{\tan(d * x + c)}))) + 2 * \sqrt{2} * ((I - 1) * A + (I + 1) * B) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 / \sqrt{\tan(d * x + c)}))) + \sqrt{2} * (- (I + 1) * A + (I - 1) * B) * \log(\sqrt{2} / \sqrt{\tan(d * x + c)}) + 1 / \tan(d * x + c) + 1 - \sqrt{2} * (- (I + 1) * A + (I - 1) * B) * \log(-\sqrt{2} / \sqrt{\tan(d * x + c)} + 1 / \tan(d * x + c) + 1)) * a) / d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs.  $2(62) = 124$ .

time = 2.35, size = 429, normalized size = 5.36

$$\frac{3 \left( (d^{2I+1} + 2d^{2I} + d) \sqrt{\frac{(A^2 - 2AB + B^2) \tan^2(dx+c)}{d^2}} \log\left(\frac{1 + (A - I*B) \tan(dx+c) \sqrt{\frac{(A^2 - 2AB + B^2) \tan^2(dx+c)}{d^2}}}{2 \sqrt{\tan^2(dx+c) - 1}}\right) - 3 \left( (d^{2I+1} + 2d^{2I} + d) \sqrt{\frac{(A^2 - 2AB + B^2) \tan^2(dx+c)}{d^2}} \log\left(\frac{1 + (A - I*B) \tan(dx+c) \sqrt{\frac{(A^2 - 2AB + B^2) \tan^2(dx+c)}{d^2}}}{2 \sqrt{\tan^2(dx+c) - 1}}\right) - 4 \left( (I * A - 4 * B) d^{2I+1} + 2 * B d^{2I} - (I * A - 2 * B) \right) \sqrt{\frac{(A^2 - 2AB + B^2) \tan^2(dx+c)}{d^2}} \right)}{d \left( (d^{2I+1} + 2d^{2I} + d) \sqrt{\frac{(A^2 - 2AB + B^2) \tan^2(dx+c)}{d^2}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $-1/6 * (3 * (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \sqrt{-(-I * A^2 - 2 * A * B + I * B^2) * a^2 / d^2} * \log(-2 * ((A - I * B) * a * e^{(2 * I * d * x + 2 * I * c)} - (I * d * e^{(2 * I * d * x + 2 * I * c)} - I * d) * \sqrt{-(-I * A^2 - 2 * A * B + I * B^2) * a^2 / d^2} * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * e^{(-2 * I * d * x - 2 * I * c)} / ((I * A + B) * a)) - 3 * (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d) * \sqrt{-(-I * A^2 - 2 * A * B + I * B^2) * a^2 / d^2} * \log(-2 * ((A - I * B) * a * e^{(2 * I * d * x + 2 * I * c)} - (I * d * e^{(2 * I * d * x + 2 * I * c)} + I * d) * \sqrt{-(-I * A^2 - 2 * A * B + I * B^2) * a^2 / d^2} * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * e^{(-2 * I * d * x - 2 * I * c)} / ((I * A + B) * a)) - 4 * ((3 * A - 4 * I * B) * a * e^{(4 * I * d * x + 4 * I * c)} + 2 * I * B * a * e^{(2 * I * d * x + 2 * I * c)} - (3 * A - 2 * I * B) * a) * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) / (d * e^{(4 * I * d * x + 4 * I * c)} + 2 * d * e^{(2 * I * d * x + 2 * I * c)} + d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int \left( -\frac{iA}{\sqrt{\cot(c+dx)}} \right) dx + \int \frac{A \tan(c+dx)}{\sqrt{\cot(c+dx)}} dx + \int \frac{B \tan^2(c+dx)}{\sqrt{\cot(c+dx)}} dx + \int \left( -\frac{iB \tan(c+dx)}{\sqrt{\cot(c+dx)}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out]  $I * a * (\text{Integral}(-I * A / \sqrt{\cot(c + d * x)}, x) + \text{Integral}(A * \tan(c + d * x) / \sqrt{\cot(c + d * x)}, x) + \text{Integral}(B * \tan(c + d * x) ** 2 / \sqrt{\cot(c + d * x)}, x) + \text{Integral}(-I * B * \tan(c + d * x) / \sqrt{\cot(c + d * x)}, x))$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)/sqrt(cot(d\*x + c)), x )

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i))/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i))/cot(c + d\*x)^(1/2), x)

$$3.507 \quad \int \frac{(a+ia \tan(c+dx))(A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=105

$$-\frac{2\sqrt[4]{-1} a(iA+B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2iaB}{5d \cot^{\frac{5}{2}}(c+dx)} + \frac{2a(iA+B)}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2a(A-iB)}{d \sqrt{\cot(c+dx)}}$$

[Out]  $-2*(-1)^{(1/4)}*a*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2/5*I*a*B/d/\cot(d*x+c)^{(5/2)}+2/3*a*(I*A+B)/d/\cot(d*x+c)^{(3/2)}+2*a*(A-I*B)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3662, 3672, 3610, 3614, 214}

$$\frac{2a(B+iA)}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2a(A-iB)}{d \sqrt{\cot(c+dx)}} - \frac{2\sqrt[4]{-1} a(B+iA) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2iaB}{5d \cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])*(A + B*\operatorname{Tan}[c + d*x])/Cot[c + d*x]^{(3/2)}, x]$

[Out]  $(-2*(-1)^{(1/4)}*a*(I*A + B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[Cot[c + d*x]])/d + ((2*I)/5)*a*B/(d*Cot[c + d*x]^{(5/2)}) + (2*a*(I*A + B))/(3*d*Cot[c + d*x]^{(3/2)}) + (2*a*(A - I*B))/(d*\operatorname{Sqrt}[Cot[c + d*x]])$

**Rule 214**

$\operatorname{Int}[(a_) + (b_)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b]$

**Rule 3610**

$\operatorname{Int}[(a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]]^{(m_)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^{(m+1)})/(f*(m+1)*(a^2 + b^2)), x] + \operatorname{Dist}[1/(a^2 + b^2), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m+1)}*\operatorname{Simp}[a*c + b*d - (b*c - a*d)*\operatorname{Tan}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \ \operatorname{LtQ}[m, -1]$

**Rule 3614**

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]]/\operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /;$   $\operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \ \operatorname{EqQ}[c^2 + d^2, 0]$

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.))*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.))*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(b*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx &= \int \frac{(ia + a \cot(c + dx))(B + A \cot(c + dx))}{\cot^{\frac{7}{2}}(c + dx)} dx \\ &= \frac{2iaB}{5d \cot^{\frac{5}{2}}(c + dx)} + \int \frac{a(iA + B) + a(A - iB) \cot(c + dx)}{\cot^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{2iaB}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2a(iA + B)}{3d \cot^{\frac{3}{2}}(c + dx)} + \int \frac{a(A - iB) - a(iA + B) \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2iaB}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2a(iA + B)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2a(A - iB)}{d \sqrt{\cot(c + dx)}} + \int \frac{a(iA + B) \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2iaB}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2a(iA + B)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2a(A - iB)}{d \sqrt{\cot(c + dx)}} + \frac{2\sqrt[4]{-1} a(iA + B) \tanh^{-1}\left(\frac{(-1)^{3/4} \sqrt{\cot(c + dx)}}{1}\right)}{d} + \frac{a(iA + B)}{5d} \end{aligned}$$

Mathematica [A]

time = 1.71, size = 133, normalized size = 1.27

$$a \left( \frac{\sec^2(c + dx)(3(5A - 4iB) + 3(5A - 6iB) \cos(2(c + dx)) + 5(iA + B) \sin(2(c + dx))) - \frac{30(A - iB) \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}\right)}{\sqrt{i \tan(c + dx)}}}{15d \sqrt{\cot(c + dx)}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),
x]
```

```
[Out] (a*(Sec[c + d*x]^2*(3*(5*A - (4*I)*B) + 3*(5*A - (6*I)*B)*Cos[2*(c + d*x)]
+ 5*(I*A + B)*Sin[2*(c + d*x)]) - (30*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)
*(c + d*x)))/(1 + E^((2*I)*(c + d*x))]])/Sqrt[I*Tan[c + d*x]])/(15*d*Sqrt
[Cot[c + d*x]])
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 49.46, size = 971, normalized size = 9.25

method	result	size
default	Expression too large to display	971

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/15*a/d*2^(1/2)*(-1+cos(d*x+c))*(-15*I*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+15*I*B*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-
sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)+3*I*B*cos(d*x+c)*2^(1/2)+15
*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*Elliptic
Pi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin
(d*x+c)-15*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^
2*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(
d*x+c)-15*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2
*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))*sin(d*x+c)+18*I*B*2^(1/2)*cos(d*x+c)^2-15*I*A*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+15*A*cos(d*x+c)^3*
2^(1/2)-18*I*B*2^(1/2)*cos(d*x+c)^3-3*I*B*2^(1/2)+5*B*cos(d*x+c)^2*sin(d*x+
c)*2^(1/2)-15*A*2^(1/2)*cos(d*x+c)^2-5*I*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)-5*
B*2^(1/2)*cos(d*x+c)*sin(d*x+c)+5*I*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(cos
(d*x+c)+1)^2/cos(d*x+c)/(cos(d*x+c)/sin(d*x+c))^(3/2)/sin(d*x+c)^5
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs.  $2(81) = 162$ .

time = 0.52, size = 191, normalized size = 1.82

$$\frac{8(-3iBa - \frac{15A^2Bd}{\tan(d*x+c)} - \frac{15A^2Bd}{\tan(d*x+c)}) \tan(dx+c)^2 + 15(2\sqrt{2}(-i+1)A+(i-1)B) \arctan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\tan(dx+c)}{\tan(dx+c)}}\right) + 2\sqrt{2}(-i+1)A+(i-1)B \arctan\left(-\frac{1}{\sqrt{2}}\sqrt{\frac{\tan(dx+c)}{\tan(dx+c)}}\right) - \sqrt{2}((i-1)A+(i+1)B) \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) + \sqrt{2}((i-1)A+(i+1)B) \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*(8*(-3*I*B*a - 5*(I*A + B)*a/tan(d*x + c) - 15*(A - I*B)*a/tan(d*x + c)^2)*tan(d*x + c)^(5/2) + 15*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(81) = 162.  
time = 1.49, size = 482, normalized size = 4.59

$$\frac{15(16a^2d^2 + 3d^2a^2 + 3d^2a^2 + d^2) \sqrt{\frac{15A^2Bd}{\tan(d*x+c)}} \log\left(\frac{\sqrt{\frac{15A^2Bd}{\tan(d*x+c)}}}{\sqrt{\frac{15A^2Bd}{\tan(d*x+c)}}}\right) - 15(16a^2d^2 + 3d^2a^2 + 3d^2a^2 + d^2) \sqrt{\frac{15A^2Bd}{\tan(d*x+c)}} \log\left(\frac{\sqrt{\frac{15A^2Bd}{\tan(d*x+c)}}}{\sqrt{\frac{15A^2Bd}{\tan(d*x+c)}}}\right) + 4(20A + 23B)a^2e^{(6I*d*x + 6I*c)} + (10I*A + B)a^2e^{(4I*d*x + 4I*c)} + (-20I*A - 11B)a^2e^{(2I*d*x + 2I*c)} + (-10I*A - 13B)a^2 \sqrt{\frac{15A^2Bd}{\tan(d*x+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))) * e^(-2*I*d*x - 2*I*c)/((I*A + B)*a) - 15*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))) * e^(-2*I*d*x - 2*I*c)/((I*A + B)*a) + 4*((20*I*A + 23*B)*a^2*e^(6*I*d*x + 6*I*c) + (10*I*A + B)*a^2*e^(4*I*d*x + 4*I*c) + (-20*I*A - 11*B)*a^2*e^(2*I*d*x + 2*I*c) + (-10*I*A - 13*B)*a^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int \left( -\frac{iA}{\cot^{\frac{3}{2}}(c+dx)} \right) dx + \int \frac{A \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \tan^2(c+dx)}{\cot^{\frac{3}{2}}(c+dx)} dx + \int \left( \frac{iB \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)} \right) dx \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2),x)

[Out] I\*a\*(Integral(-I\*A/cot(c + d\*x)\*\*(3/2), x) + Integral(A\*tan(c + d\*x)/cot(c + d\*x)\*\*(3/2), x) + Integral(B\*tan(c + d\*x)\*\*2/cot(c + d\*x)\*\*(3/2), x) + Integral(-I\*B\*tan(c + d\*x)/cot(c + d\*x)\*\*(3/2), x))

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)/cot(d\*x + c)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})}{\cot(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i))/cot(c + d\*x)^(3/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i))/cot(c + d\*x)^(3/2), x)

$$3.508 \quad \int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=128

$$\frac{4\sqrt[4]{-1} a^2(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{4a^2(A-iB) \sqrt{\cot(c+dx)}}{d} - \frac{2a^2(7iA+5B) \cot^{\frac{3}{2}}(c+dx)}{15d}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-2/15*a^2*(7*I*A+5*B)*\cot(d*x+c)^{(3/2)}/d-2/5*A*\cot(d*x+c)^{(3/2)}*(I*a^2+a^2*\cot(d*x+c))/d+4*a^2*(A-I*B)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.24, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3662, 3675, 3673, 3609, 3614, 214}

$$-\frac{2a^2(5B+7iA) \cot^{\frac{3}{2}}(c+dx)}{15d} + \frac{4a^2(A-iB) \sqrt{\cot(c+dx)}}{d} + \frac{4\sqrt[4]{-1} a^2(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2A \cot^{\frac{3}{2}}(c+dx) (a^2 \cot(c+dx) + ia^2)}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^{(7/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^2*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(4*(-1)^{(1/4)}*a^2*(A-I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]])/d + (4*a^2*(A-I*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/d - (2*a^2*((7*I)*A+5*B)*\operatorname{Cot}[c+d*x]^{(3/2)})/(15*d) - (2*A*\operatorname{Cot}[c+d*x]^{(3/2)}*(I*a^2+a^2*\operatorname{Cot}[c+d*x]))/(5*d)$

**Rule 214**

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b]$

**Rule 3609**

$\operatorname{Int}[(a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Simp}[d*((a+b*\operatorname{Tan}[e+f*x])^m/(f*m)), x] + \operatorname{Int}[(a+b*\operatorname{Tan}[e+f*x])^{(m-1)}*\operatorname{Simp}[a*c-b*d+(b*c+a*d)*\operatorname{Tan}[e+f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \operatorname{NeQ}[a^2+b^2, 0] \ \&\& \operatorname{GtQ}[m, 0]$

**Rule 3614**

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])/\operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c-d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e+f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f, x\} \ \&\& \operatorname{EqQ}[c^2+d^2, 0]$

## Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist [g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ [p] && IntegerQ[m] && IntegerQ[n]
```

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \int \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^2(B + \\
&= -\frac{2A \cot^{\frac{3}{2}}(c + dx) (ia^2 + a^2 \cot(c + dx))}{5d} - \\
&= -\frac{2a^2(7iA + 5B) \cot^{\frac{3}{2}}(c + dx)}{15d} - \frac{2A \cot^{\frac{3}{2}}(c + dx)}{d} - \\
&= \frac{4a^2(A - iB) \sqrt{\cot(c + dx)}}{d} - \frac{2a^2(7iA + 5B)}{d} - \\
&= \frac{4a^2(A - iB) \sqrt{\cot(c + dx)}}{d} - \frac{2a^2(7iA + 5B)}{d} - \\
&= \frac{4\sqrt{-1} a^2(A - iB) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c + dx)} \right)}{d}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 272 vs. 2(128) = 256.  
 time = 2.43, size = 272, normalized size = 2.12

$$\frac{a^2(i + \cot(c + dx))^2(B + A \cot(c + dx)) \sin^3(c + dx) \left( -\frac{4i(A - iB)e^{-2ic} \operatorname{tanh}^{-1}\left(\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}\right)}{\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{\frac{i(1 + e^{2i(c+dx)}}{-1 + e^{2i(c+dx)}}}} - \frac{1}{15} \sqrt{\cot(c + dx)} \operatorname{csc}^2(c + dx) (\cos(2c) - i \sin(2c)) (-27A + 30iB + (33A - 30iB) \cos(2(c + dx)) + 5(2iA + B) \sin(2(c + dx))) \right)}{d(\cos(dx) + i \sin(dx))^2(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]), x]
```

```
[Out] (a^2*(I + Cot[c + d*x])^2*(B + A*Cot[c + d*x])*Sin[c + d*x]^3*(((4*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))])/(E^((2*I)*c)*Sqrt[(-1 + E^((2*I)*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))] - (Sqrt[Cot[c + d*x])*Csc[c + d*x]^2*(Cos[2*c] - I*Sin[2*c])*(-27*A + (30*I)*B + (33*A - (30*I)*B)*Cos[2*(c + d*x)] + 5*((2*I)*A + B)*Sin[2*(c + d*x)]))/15))/(d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
 time = 25.40, size = 2947, normalized size = 23.02

method	result	size
default	Expression too large to display	2947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)), x, method=_RETURN VERBOSE)
```

```
[Out] -1/15*a^2/d^2^(1/2)*(5*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-30*I*A*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2), 1/2*2^(1/2))*((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+30*I*A*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))*((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-30*I*B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))*((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+33*A*cos(d*x+c)^3*2^(1/2)-30*A*cos(d*x+c)*2^(1/2)+30*I*A*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2), 1/2*2^(1/2))*((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-30*I*A*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))
```





$(I \cdot e^{(2I \cdot d \cdot x + 2I \cdot c)} + I) / (e^{(2I \cdot d \cdot x + 2I \cdot c)} - 1) / (d \cdot e^{(4I \cdot d \cdot x + 4I \cdot c)} - 2 \cdot d \cdot e^{(2I \cdot d \cdot x + 2I \cdot c)} + d)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(7/2)\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^2\*cot(d\*x + c)^(7/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2, x)

$$3.509 \quad \int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=103

$$\frac{4\sqrt[4]{-1} a^2(iA+B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(5iA+3B) \sqrt{\cot(c+dx)}}{3d} - \frac{2A \sqrt{\cot(c+dx)} (ia^2 \cot(c+dx) + ia^2)}{3d}$$

[Out]  $-4*(-1)^{(1/4)}*a^2*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-2/3*a^2*(5*I*A+3*B)*\cot(d*x+c)^{(1/2)}/d-2/3*A*(I*a^2+a^2*\cot(d*x+c))*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3662, 3675, 3673, 3614, 214}

$$\frac{2a^2(3B+5iA) \sqrt{\cot(c+dx)}}{3d} - \frac{4\sqrt[4]{-1} a^2(B+iA) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2A \sqrt{\cot(c+dx)} (a^2 \cot(c+dx) + ia^2)}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^{(5/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^2*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-4*(-1)^{(1/4)}*a^2*(I*A+B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]])/d - (2*a^2*((5*I)*A+3*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/(3*d) - (2*A*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*(I*a^2+a^2*\operatorname{Cot}[c+d*x]))/(3*d)$

Rule 214

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 3614

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]/\operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

Rule 3662

$\operatorname{Int}[(\operatorname{cot}[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}*((a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b+a*\operatorname{Cot}[e + f*x])^m*(d+c*\operatorname{Cot}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, p\}, x \ \&\& \operatorname{IntegerQ}[p] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IntegerQ}[n]$



Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rule 3675

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \int \frac{(ia + a \cot(c + dx))^2(B + A \cot(c + dx))}{\sqrt{\cot(c + dx)}} dx \\ &= -\frac{2A\sqrt{\cot(c + dx)}(ia^2 + a^2 \cot(c + dx))}{3d} \\ &= -\frac{2a^2(5iA + 3B)\sqrt{\cot(c + dx)}}{3d} - \frac{2A\sqrt{\cot(c + dx)}}{3d} \\ &= -\frac{2a^2(5iA + 3B)\sqrt{\cot(c + dx)}}{3d} - \frac{2A\sqrt{\cot(c + dx)}}{3d} \\ &= -\frac{4\sqrt[4]{-1} a^2(iA + B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d} \end{aligned}$$

Mathematica [A]

time = 2.12, size = 174, normalized size = 1.69

$$\frac{2a^2 e^{-2ic} \sqrt{\cot(c + dx)} (B + A \cot(c + dx)) \sin(c + dx) (\cos(2(c + dx)) + i \sin(2(c + dx))) \left(6iA + 3B + A \cot(c + dx) - 6i(A - iB) \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}\right) \sqrt{i \tan(c + dx)}\right)}{3d(\cos(dx) + i \sin(dx))^2(A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]), x]
```

```
[Out] (-2*a^2*Sqrt[Cot[c + d*x]]*(B + A*Cot[c + d*x])*Sin[c + d*x]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*((6*I)*A + 3*B + A*Cot[c + d*x] - (6*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sqrt[I*Tan[c + d*x]]))/(3*d*E^((2*I)*c)*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 25.38, size = 1541, normalized size = 14.96

method	result	size
default	Expression too large to display	1541

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURN VERBOSE)
```

```
[Out] 1/3*a^2/d^2^(1/2)*(6*I*A*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+6*I*B*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)-6*I*B*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)-6*A*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)+6*I*A*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+6*A*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+6*B*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)+6*I*B*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)-6*I*B*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)-6*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
```

$$\begin{aligned} & / \sin(dx+c)^{1/2} * \text{EllipticPi}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2} \\ & , 1/2+1/2*I, 1/2*2^{1/2}) + 6*A*\sin(dx+c)*(-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c) \\ & )^{1/2} * ((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2} * ((-1+\cos(dx+c))/\sin \\ & (dx+c))^{1/2} * \text{EllipticF}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2* \\ & 2^{1/2}) + 6*B*\sin(dx+c)*(-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c) \\ & )^{1/2} * ((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2} * ((-1+\cos(dx+c))/\sin(dx+c))^{1/2} * \\ & \text{EllipticPi}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) \\ & - 6*I*A*\cos(dx+c)*\sin(dx+c)*2^{1/2} - A*2^{1/2}*\cos(dx+c)^2 - 3*B*2^{1/2} \\ & * \cos(dx+c)*\sin(dx+c) * (\cos(dx+c)/\sin(dx+c))^{5/2} * \sin(dx+c)/\cos(dx+c) \\ & \sim 3 \end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs.  $2(83) = 166$ .  
time = 0.52, size = 180, normalized size = 1.75

$$\frac{3\left(2\sqrt{2}(-i+1)A+(i-1)B\right)\arctan\left(\frac{1\sqrt{2}\left(\sqrt{2}+\frac{1}{\sqrt{\tan(dx+c)}}\right)}{\sqrt{\tan(dx+c)}}\right)+2\sqrt{2}(-i+1)A+(i-1)B\arctan\left(-\frac{1\sqrt{2}\left(\sqrt{2}-\frac{1}{\sqrt{\tan(dx+c)}}\right)}{\sqrt{\tan(dx+c)}}\right)-\sqrt{2}((i-1)A+(i+1)B)\log\left(\frac{-\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)+1}\right)+\sqrt{2}((i-1)A+(i+1)B)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)+1}\right)}{6d} - \frac{3(2A-3B)\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{3A\sqrt{2}}{\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(5/2)\*(a+I\*a\*tan(dx+c))^2\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out]  $-1/6*(3*(2*\sqrt{2})*(-(I+1)*A+(I-1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2/\sqrt{\tan(dx+c)})) + 2*\sqrt{2}*(-(I+1)*A+(I-1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2/\sqrt{\tan(dx+c)})) - \sqrt{2}*((I-1)*A+(I+1)*B)*\log(\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1) + \sqrt{2}*((I-1)*A+(I+1)*B)*\log(-\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1)*a^2 - 12*(-2*I*A-B)*a^2/\sqrt{\tan(dx+c)} + 4*A*a^2/\tan(dx+c)^{3/2})/d$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs.  $2(83) = 166$ .  
time = 1.23, size = 394, normalized size = 3.83

$$\frac{3\sqrt{-1(A^2+2AB-1B^2)a^2}}{d}\log\left(\frac{2\left(\frac{(A-1B)a^{2d+2c-1}\sqrt{-1(A^2+2AB-1B^2)a^2}}{d}\sqrt{\frac{e^{2(dx+c)+1}}{2d+2c-1}}\right)^{2d+2c-1}}{(-1-A)B^2}\right)-3\sqrt{-1(A^2+2AB-1B^2)a^2}}{d}\log\left(\frac{2\left(\frac{(A+1B)a^{2d+2c-1}\sqrt{-1(A^2+2AB-1B^2)a^2}}{d}\sqrt{\frac{e^{2(dx+c)+1}}{2d+2c-1}}\right)^{2d+2c-1}}{(-1+A)B^2}\right)+2(7A+3B)a^{2d+2c}+(-5A-3B)a^2\sqrt{\frac{e^{2(dx+c)+1}}{2d+2c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(5/2)\*(a+I\*a\*tan(dx+c))^2\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out]  $-1/3*(3*\sqrt{-1(A^2+2AB-1B^2)a^4/d^2}*(d*e^{2I*dx+2I*c}-d)*\log(2*((A-I*B)*a^2*e^{2I*dx+2I*c}+\sqrt{-1(A^2+2AB-1B^2)a^4/d^2}*(d*e^{2I*dx+2I*c}-d)*\sqrt{(I*e^{2I*dx+2I*c}+I)/(e^{2I*dx+2I*c}-1)}))e^{-2I*dx-2I*c}/((-1A-B)*a^2) - 3*\sqrt{-1(A^2+2AB-1B^2)a^4/d^2}*(d*e^{2I*dx+2I*c}-d)*\log(2*((A-I*B)*a^2*e^{2I*dx+2I*c}-\sqrt{-1(A^2+2AB-1B^2)a^4/d^2}*(d*e^{2I*dx+2I*c}-d)*\sqrt{(I*e^{2I*dx+2I*c}+I)/(e^{2I*dx+2I*c}-1)}))$

```
) * e^(-2*I*d*x - 2*I*c) / ((-I*A - B) * a^2) + 2 * ((7*I*A + 3*B) * a^2 * e^(2*I*d*x + 2*I*c) + (-5*I*A - 3*B) * a^2) * sqrt((I * e^(2*I*d*x + 2*I*c) + I) / (e^(2*I*d*x + 2*I*c) - 1)) / (d * e^(2*I*d*x + 2*I*c) - d)
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2, x)
```

$$3.510 \quad \int \cot^2(c+dx)(a+ia \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=99

$$\frac{4\sqrt{-1} a^2(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(A+iB) \sqrt{\cot(c+dx)}}{d} + \frac{2iB(ia^2 + a^2 \cot(c+dx))}{d \sqrt{\cot(c+dx)}}$$

[Out]  $-4*(-1)^{(1/4)}*a^2*(A-I*B)*\operatorname{arctanh}\left((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)}\right)/d+2*I*B*(I*a^2+a^2*\cot(d*x+c))/d/\cot(d*x+c)^{(1/2)}-2*a^2*(A+I*B)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.21, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3662, 3674, 3673, 3614, 214}

$$\frac{2a^2(A+iB) \sqrt{\cot(c+dx)}}{d} - \frac{4\sqrt{-1} a^2(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2iB(a^2 \cot(c+dx) + ia^2)}{d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^{(3/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^2*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-4*(-1)^{(1/4)}*a^2*(A-I*B)*\operatorname{ArcTanh}\left[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]\right])/d - (2*a^2*(A+I*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/d + ((2*I)*B*(I*a^2+a^2*\operatorname{Cot}[c+d*x]))/(d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])$

**Rule 214**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3614**

$\operatorname{Int}[(c_+ + (d_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)])/ \operatorname{Sqrt}[(b_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)]]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /;$  FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

**Rule 3662**

$\operatorname{Int}[(\cot[e_+ + (f_+)*(x_+)]*(g_+))^{(p_+)}*((a_+ + (b_+)*\operatorname{tan}[e_+ + (f_+)*(x_+)])^{(m_+)})^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b+a*\operatorname{Cot}[e + f*x])^m*(d+c*\operatorname{Cot}[e + f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

## Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

## Rule 3674

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \int \frac{(ia + a \cot(c + dx))^2(B + A \cot(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2iB(ia^2 + a^2 \cot(c + dx))}{d\sqrt{\cot(c + dx)}} + 2 \int \frac{(ia + a \cot(c + dx))^2}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2a^2(A + iB)\sqrt{\cot(c + dx)}}{d} + \frac{2iB(ia^2 + a^2 \cot(c + dx))}{d\sqrt{\cot(c + dx)}} \\
&= -\frac{2a^2(A + iB)\sqrt{\cot(c + dx)}}{d} + \frac{2iB(ia^2 + a^2 \cot(c + dx))}{d\sqrt{\cot(c + dx)}} \\
&= -\frac{4\sqrt[4]{-1} a^2(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.13, size = 163, normalized size = 1.65

$$\frac{2a^2 e^{-2ic} \cos(c + dx) \sqrt{\cot(c + dx)} (\cos(2(c + dx)) + i \sin(2(c + dx))) (A + B \tan(c + dx)) \left( A - 2(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \right) \sqrt{i \tan(c + dx)} + B \tan(c + dx) \right)}{d(\cos(dx) + i \sin(dx))^2 (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),
x]
```

```
[Out] (-2*a^2*Cos[c + d*x]*Sqrt[Cot[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*
x)])*(A + B*Tan[c + d*x])*(A - 2*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c +
d*x))]/(1 + E^((2*I)*(c + d*x)))])*Sqrt[I*Tan[c + d*x]] + B*Tan[c + d*x])
/(d*E^((2*I)*c)*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x])
)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 67.12, size = 1416, normalized size = 14.30

method	result	size
default	Expression too large to display	1416

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -a^2/d*2^(1/2)*(2*I*A*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)
-2*I*A*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+2*I
*B*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x
+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)+2*I*A*(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(
d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*I*A*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-2*A*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*cos(d*x+c)+2*B*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x
+c)+2*I*B*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*B*((1-cos
(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)-2*A*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
```

$$\begin{aligned} & ) * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \text{EllipticPi}(((1 - \cos(dx+c) + \sin(dx+c)) / \sin(dx+c))^{(1/2)}, 1/2 + 1/2 * I, 1/2 * 2^{(1/2)}) + 2 * B * ((1 - \cos(dx+c) + \sin(dx+c)) / \sin(dx+c))^{(1/2)} * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \text{EllipticF}(((1 - \cos(dx+c) + \sin(dx+c)) / \sin(dx+c))^{(1/2)}, 1/2 * 2^{(1/2)}) - 2 * B * ((1 - \cos(dx+c) + \sin(dx+c)) / \sin(dx+c))^{(1/2)} * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{(1/2)} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \text{EllipticPi}(((1 - \cos(dx+c) + \sin(dx+c)) / \sin(dx+c))^{(1/2)}, 1/2 + 1/2 * I, 1/2 * 2^{(1/2)}) + A * \cos(dx+c) * 2^{(1/2)} + B * \sin(dx+c) * 2^{(1/2)} * \sin(dx+c) * (\cos(dx+c) / \sin(dx+c))^{(3/2)} / \cos(dx+c)^2 \end{aligned}$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs.  $2(81) = 162$ .  
time = 0.53, size = 174, normalized size = 1.76

$$\frac{4 B^2 \sqrt{\tan(dx+c)} - (2 \sqrt{2}(-i-1) A - (i+1) B) \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 2 \sqrt{2}(-i-1) A - (i+1) B \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) - \sqrt{2}(-i+1) A + (i-1) B \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)}}\right) + \sqrt{2}(-i+1) A + (i-1) B \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} - \frac{1}{\sqrt{\tan(dx+c)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(a+I\*a\*tan(dx+c))^2\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/2 * (4 * B * a^2 * \sqrt{\tan(dx+c)} - (2 * \sqrt{2}) * (-I - 1) * A - (I + 1) * B) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 / \sqrt{\tan(dx+c)})) + 2 * \sqrt{2} * (-I - 1) * A - (I + 1) * B * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 / \sqrt{\tan(dx+c)})) - \sqrt{2} * (-I + 1) * A + (I - 1) * B * \log(\sqrt{2} / \sqrt{\tan(dx+c)} + 1 / \tan(dx+c) + 1) + \sqrt{2} * (-I + 1) * A + (I - 1) * B * \log(-\sqrt{2} / \sqrt{\tan(dx+c)} + 1 / \tan(dx+c) + 1) * a^2 + 4 * A * a^2 / \sqrt{\tan(dx+c)} \end{aligned} / d$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs.  $2(81) = 162$ .  
time = 2.29, size = 385, normalized size = 3.89

$$\frac{\sqrt{\frac{-1 * A^2 - 2 * A * B + 1 * B^2}{d^2}} \left( \frac{1}{d} \log\left(\frac{\sqrt{\frac{-1 * A^2 - 2 * A * B + 1 * B^2}{d^2}} \sqrt{\frac{1 * e^{2 * I * dx} + 1}{2 * e^{2 * I * dx} - 1}}}{\sqrt{\frac{-1 * A^2 - 2 * A * B + 1 * B^2}{d^2}}}\right) - \sqrt{\frac{-1 * A^2 - 2 * A * B + 1 * B^2}{d^2}} \left( \frac{1}{d} \log\left(\frac{\sqrt{\frac{-1 * A^2 - 2 * A * B + 1 * B^2}{d^2}} \sqrt{\frac{1 * e^{2 * I * dx} + 1}{2 * e^{2 * I * dx} - 1}}}{\sqrt{\frac{-1 * A^2 - 2 * A * B + 1 * B^2}{d^2}}}\right) - 2 * ((A - I * B) * e^{2 * I * dx} + (A + I * B) * e^{-2 * I * dx}) \sqrt{\frac{1 * e^{2 * I * dx} + 1}{2 * e^{2 * I * dx} - 1}} \right)}{d * (2 * e^{2 * I * dx} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(a+I\*a\*tan(dx+c))^2\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & (\sqrt{(-1 * A^2 - 2 * A * B + 1 * B^2)} * a^4 / d^2) * (d * e^{(2 * I * dx + 2 * I * c)} + d) * \log(2 * ((A - I * B) * a^2 * e^{(2 * I * dx + 2 * I * c)} - \sqrt{(-1 * A^2 - 2 * A * B + 1 * B^2)} * a^4 / d^2) * (I * d * e^{(2 * I * dx + 2 * I * c)} - I * d) * \sqrt{(I * e^{(2 * I * dx + 2 * I * c)} + I)} / (e^{(2 * I * dx + 2 * I * c)} - 1)) * e^{(-2 * I * dx - 2 * I * c)} / ((-I * A - B) * a^2) - \sqrt{(-1 * A^2 - 2 * A * B + 1 * B^2)} * a^4 / d^2 * (d * e^{(2 * I * dx + 2 * I * c)} + d) * \log(2 * ((A - I * B) * a^2 * e^{(2 * I * dx + 2 * I * c)} - \sqrt{(-1 * A^2 - 2 * A * B + 1 * B^2)} * a^4 / d^2) * (-I * d * e^{(2 * I * dx + 2 * I * c)} + I * d) * \sqrt{(I * e^{(2 * I * dx + 2 * I * c)} + I)} / (e^{(2 * I * dx + 2 * I * c)} - 1)) * e^{(-2 * I * dx - 2 * I * c)} / ((-I * A - B) * a^2) - 2 * ((A - I * B) * a^2 * e^{(2 * I * dx} \end{aligned}$$



+ 2\*I\*c) + (A + I\*B)\*a^2)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))/(d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int (-A \cot^3(c+dx)) dx + \int A \tan^2(c+dx) \cot^3(c+dx) dx + \int (-B \tan(c+dx) \cot^3(c+dx)) dx + \int B \tan^3(c+dx) \cot^3(c+dx) dx + \int (-2iA \tan(c+dx) \cot^3(c+dx)) dx + \int (-2iB \tan^2(c+dx) \cot^3(c+dx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] -a\*\*2\*(Integral(-A\*cot(c + d\*x)\*\*(3/2), x) + Integral(A\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*(3/2), x) + Integral(-B\*tan(c + d\*x)\*cot(c + d\*x)\*\*(3/2), x) + Integral(B\*tan(c + d\*x)\*\*3\*cot(c + d\*x)\*\*(3/2), x) + Integral(-2\*I\*A\*tan(c + d\*x)\*cot(c + d\*x)\*\*(3/2), x) + Integral(-2\*I\*B\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*(3/2), x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^2\*cot(d\*x + c)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+dx)^{3/2} (A+B \tan(c+dx)) (a+a \tan(c+dx) 1i)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2, x)

### 3.511 $\int \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^2 (A + B \tan(c + dx)) dx$

**Optimal.** Leaf size=105

$$\frac{4\sqrt[4]{-1} a^2 (iA + B) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c + dx)} \right)}{d} - \frac{2a^2 (3A - 5iB)}{3d \sqrt{\cot(c + dx)}} + \frac{2iB (ia^2 + a^2 \cot(c + dx))}{3d \cot^{3/2}(c + dx)}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2/3*I*B*(I*a^2+a^2*\cot(d*x+c))/d/\cot(d*x+c)^{(3/2)}-2/3*a^2*(3*A-5*I*B)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.21, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3662, 3674, 3672, 3614, 214}

$$-\frac{2a^2(3A - 5iB)}{3d\sqrt{\cot(c + dx)}} + \frac{4\sqrt[4]{-1} a^2 (B + iA) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c + dx)} \right)}{d} + \frac{2iB (a^2 \cot(c + dx) + ia^2)}{3d \cot^{3/2}(c + dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(4*(-1)^{(1/4)}*a^2*(I*A + B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d - (2*a^2*(3*A - (5*I)*B))/(3*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (((2*I)/3)*B*(I*a^2 + a^2*\operatorname{Cot}[c + d*x]))/(d*\operatorname{Cot}[c + d*x]^{(3/2)})$

**Rule 214**

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b]$

**Rule 3614**

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])/ \operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)]]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f, x\} \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

**Rule 3662**

$\operatorname{Int}[(\cot[(e_.) + (f_.)*(x_)])*(g_.)^{(p_.)}*((a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b + a*\operatorname{Cot}[e + f*x])^m*(d + c*\operatorname{Cot}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, p\}, x\} \&\& \operatorname{IntegerQ}$

[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

### Rule 3674

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^2 (A+B \tan(c+dx)) dx &= \int \frac{(ia+a \cot(c+dx))^2 (B+A \cot(c+dx))}{\cot^{\frac{5}{2}}(c+dx)} dx \\ &= \frac{2iB(ia^2+a^2 \cot(c+dx))}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2}{3} \int \frac{(ia+a \cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{2a^2(3A-5iB)}{3d \sqrt{\cot(c+dx)}} + \frac{2iB(ia^2+a^2 \cot(c+dx))}{3d \cot^{\frac{3}{2}}(c+dx)} \\ &= -\frac{2a^2(3A-5iB)}{3d \sqrt{\cot(c+dx)}} + \frac{2iB(ia^2+a^2 \cot(c+dx))}{3d \cot^{\frac{3}{2}}(c+dx)} \\ &= \frac{4\sqrt{-1} a^2 (iA+B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} \end{aligned}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 254 vs. 2(105) = 210.





**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int (-A\sqrt{\cot(c+dx)}) dx + \int A \tan^2(c+dx)\sqrt{\cot(c+dx)} dx + \int (-B \tan(c+dx)\sqrt{\cot(c+dx)}) dx + \int B \tan^3(c+dx)\sqrt{\cot(c+dx)} dx + \int (-2iA \tan(c+dx)\sqrt{\cot(c+dx)}) dx + \int (-2iB \tan^2(c+dx)\sqrt{\cot(c+dx)}) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] -a\*\*2\*(Integral(-A\*sqrt(cot(c + d\*x)), x) + Integral(A\*tan(c + d\*x)\*\*2\*sqrt(cot(c + d\*x)), x) + Integral(-B\*tan(c + d\*x)\*sqrt(cot(c + d\*x)), x) + Integral(B\*tan(c + d\*x)\*\*3\*sqrt(cot(c + d\*x)), x) + Integral(-2\*I\*A\*tan(c + d\*x)\*sqrt(cot(c + d\*x)), x) + Integral(-2\*I\*B\*tan(c + d\*x)\*\*2\*sqrt(cot(c + d\*x)), x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^2\*sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c+dx)} (A + B \tan(c+dx)) (a + a \tan(c+dx) i)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2, x)

$$3.512 \quad \int \frac{(a+ia \tan(c+dx))^2(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=130

$$\frac{4\sqrt[4]{-1} a^2(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(5A-7iB)}{15d \cot^{3/2}(c+dx)} + \frac{4a^2(iA+B)}{d\sqrt{\cot(c+dx)}} + \frac{2iB(ia^2+a^2 \cot(c+dx))}{5d \cot^{5/2}(c+dx)}$$

[Out]  $4*(-1)^{(1/4)}*a^2*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-2/15*a^2*(5*A-7*I*B)/d/\cot(d*x+c)^{(3/2)}+2/5*I*B*(I*a^2+a^2*\cot(d*x+c))/d/\cot(d*x+c)^{(5/2)}+4*a^2*(I*A+B)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.24, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3662, 3674, 3672, 3610, 3614, 214}

$$-\frac{2a^2(5A-7iB)}{15d \cot^{3/2}(c+dx)} + \frac{4a^2(B+iA)}{d\sqrt{\cot(c+dx)}} + \frac{4\sqrt[4]{-1} a^2(A-iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{2iB(a^2 \cot(c+dx) + ia^2)}{5d \cot^{5/2}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x])/ \operatorname{Sqrt}[\operatorname{Cot}[c + d*x]], x]$

[Out]  $(4*(-1)^{(1/4)}*a^2*(A - I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/d - (2*a^2*(5*A - (7*I)*B))/(15*d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (4*a^2*(I*A + B))/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (((2*I)/5)*B*(I*a^2 + a^2*\operatorname{Cot}[c + d*x]))/(d*\operatorname{Cot}[c + d*x]^{(5/2)})$

**Rule 214**

$\operatorname{Int}[(a_) + (b_)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$   $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b]$

**Rule 3610**

$\operatorname{Int}[(a_) + (b_)*\operatorname{tan}[(e_) + (f_)*(x_)]]^{(m_)*((c_) + (d_)*\operatorname{tan}[(e_) + (f_)*(x_)])}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^{(m+1)}/(f*(m+1)*(a^2 + b^2))), x] + \operatorname{Dist}[1/(a^2 + b^2), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m+1)}*\operatorname{Simp}[a*c + b*d - (b*c - a*d)*\operatorname{Tan}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \ \operatorname{LtQ}[m, -1]$

**Rule 3614**

$\operatorname{Int}[(c_) + (d_)*\operatorname{tan}[(e_) + (f_)*(x_)])/ \operatorname{Sqrt}[(b_)*\operatorname{tan}[(e_) + (f_)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*$

$\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.))^{\text{p}_.}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}, x\_Symbol] \text{:> Dist}[g^{\text{m} + \text{n}}, \text{Int}[(g*\text{Cot}[e + f*x])^{\text{p} - \text{m} - \text{n}}*(b + a*\text{Cot}[e + f*x])^{\text{m}}*(d + c*\text{Cot}[e + f*x])^{\text{n}}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x\} \&\& \text{!IntegerQ}[\text{p}] \&\& \text{IntegerQ}[\text{m}] \&\& \text{IntegerQ}[\text{n}]$

### Rule 3672

$\text{Int}[(\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}*((\text{A}_. + \text{B}_.*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \text{:> Simp}[(b*c - a*d)*(A*b - a*B)*(\text{a} + \text{b}*\text{Tan}[e + f*x])^{\text{m} + 1}/(\text{b}*f^{\text{m} + 1}*(\text{a}^2 + \text{b}^2))], x] + \text{Dist}[1/(\text{a}^2 + \text{b}^2), \text{Int}[(\text{a} + \text{b}*\text{Tan}[e + f*x])^{\text{m} + 1}*\text{Simp}[\text{a}*A*c + \text{b}*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - \text{b}*B*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[\text{m}, -1] \&\& \text{NeQ}[\text{a}^2 + \text{b}^2, 0]$

### Rule 3674

$\text{Int}[(\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}*((\text{A}_. + \text{B}_.*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}, x\_Symbol] \text{:> Simp}[(-\text{a}^2)*(B*c - A*d)*(\text{a} + \text{b}*\text{Tan}[e + f*x])^{\text{m} - 1}*((c + d*\text{Tan}[e + f*x])^{\text{n} + 1}/(\text{d}*f*(\text{b}*c + \text{a}*d)*(\text{n} + 1))), x] - \text{Dist}[\text{a}/(\text{d}*(\text{b}*c + \text{a}*d)*(\text{n} + 1)), \text{Int}[(\text{a} + \text{b}*\text{Tan}[e + f*x])^{\text{m} - 1}*(c + d*\text{Tan}[e + f*x])^{\text{n} + 1}*\text{Simp}[\text{A}*b*d*(\text{m} - \text{n} - 2) - B*(\text{b}*c*(\text{m} - 1) + \text{a}*d*(\text{n} + 1)) + (\text{a}*A*d*(\text{m} + \text{n}) - B*(\text{a}*c*(\text{m} - 1) + \text{b}*d*(\text{n} + 1)))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[\text{a}^2 + \text{b}^2, 0] \&\& \text{GtQ}[\text{m}, 1] \&\& \text{LtQ}[\text{n}, -1]$

### Rubi steps



$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \int \frac{(ia + a \cot(c + dx))^2 (B + A \cot(c + dx))}{\cot^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2iB(ia^2 + a^2 \cot(c + dx))}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(ia + a \cot(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= -\frac{2a^2(5A - 7iB)}{15d \cot^{\frac{3}{2}}(c + dx)} + \frac{2iB(ia^2 + a^2 \cot(c + dx))}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{(ia + a \cot(c + dx))}{\cot^{\frac{1}{2}}(c + dx)} dx \\
&= -\frac{2a^2(5A - 7iB)}{15d \cot^{\frac{3}{2}}(c + dx)} + \frac{4a^2(iA + B)}{d \sqrt{\cot(c + dx)}} + \frac{2iB(ia^2 + a^2 \cot(c + dx))}{5d \cot^{\frac{5}{2}}(c + dx)} \\
&= -\frac{2a^2(5A - 7iB)}{15d \cot^{\frac{3}{2}}(c + dx)} + \frac{4a^2(iA + B)}{d \sqrt{\cot(c + dx)}} + \frac{2iB(ia^2 + a^2 \cot(c + dx))}{5d \cot^{\frac{5}{2}}(c + dx)} \\
&= \frac{4\sqrt{-1} a^2 (A - iB) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c + dx)} \right)}{d} - \frac{1}{1}
\end{aligned}$$

**Mathematica [A]**

time = 2.63, size = 133, normalized size = 1.02

$$\frac{a^2 \left( \sec^2(c + dx)(30iA + 27B + (30iA + 33B) \cos(2(c + dx)) - 5(A - 2iB) \sin(2(c + dx))) - \frac{60i(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}} \right)}{\sqrt{i \tan(c + dx)}} \right)}{15d \sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

```
[Out] (a^2*(Sec[c + d*x]^2*((30*I)*A + 27*B + ((30*I)*A + 33*B)*Cos[2*(c + d*x)] - 5*(A - (2*I)*B)*Sin[2*(c + d*x)]) - ((60*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])/Sqrt[I*Tan[c + d*x]])/(15*d*Sqrt[Cot[c + d*x]])
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 47.80, size = 971, normalized size = 7.47

method	result	size
default	Expression too large to display	971

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x,method=\_RETURN  
VERBOSE)

[Out]  $\frac{1}{15}a^2/d^{2^{1/2}}*(-1+\cos(d*x+c))*(-30*I*B*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2},1/2+1/2*I,1/2*2^{1/2})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{1/2}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2}*\cos(d*x+c)^2-30*I*A*2^{1/2}*\cos(d*x+c)^2-30*I*A*EllipticF((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2},1/2*2^{1/2})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{1/2}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2}*\cos(d*x+c)^2+30*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{1/2}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2}*\cos(d*x+c)^2*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2},1/2+1/2*I,1/2*2^{1/2})*\sin(d*x+c)+30*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{1/2}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2}*\cos(d*x+c)^2*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2},1/2+1/2*I,1/2*2^{1/2})*\sin(d*x+c)-30*B*EllipticF((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2},1/2*2^{1/2})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{1/2}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2}*\cos(d*x+c)^2+10*I*B*\sin(d*x+c)*2^{1/2}*\cos(d*x+c)^2+30*I*A*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2},1/2+1/2*I,1/2*2^{1/2})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{1/2}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{1/2}*\cos(d*x+c)^2+30*I*A*2^{1/2}*\cos(d*x+c)^3-5*A*\sin(d*x+c)*2^{1/2}*\cos(d*x+c)^2-10*I*B*\cos(d*x+c)*\sin(d*x+c)*2^{1/2}+33*B*2^{1/2}*\cos(d*x+c)^3+5*A*\cos(d*x+c)*\sin(d*x+c)*2^{1/2}-33*B*2^{1/2}*\cos(d*x+c)^2-3*B*2^{1/2}*\cos(d*x+c)+3*B*2^{1/2})*(\cos(d*x+c)+1)^2/(\cos(d*x+c)/\sin(d*x+c))^{1/2}/\cos(d*x+c)^2/\sin(d*x+c)^4$

**Maxima** [A]

time = 0.53, size = 199, normalized size = 1.53

$$\frac{4(3Ba^2 + \frac{15AaBd^2}{\sqrt{\tan(dx+c)}} - \frac{15AaBd^2}{\sqrt{\tan(dx+c)}})\tan(dx+c)^3 + 15(2\sqrt{2}^{-(i-1)A-(i+1)B})\arctan\left(\frac{1}{\sqrt{2}}\left(\sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}^{-(i-1)A-(i+1)B}\arctan\left(-\frac{1}{\sqrt{2}}\left(\sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) - \sqrt{2}^{-(i+1)A+(i-1)B}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)+1}}\right) + \sqrt{2}^{-(i+1)A+(i-1)B}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)+1}}\right)}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $-1/30*(4*(3*B*a^2 + 5*(A - 2*I*B)*a^2/\tan(d*x + c) - 30*(I*A + B)*a^2/\tan(d*x + c)^2)*\tan(d*x + c)^{(5/2)} + 15*(2*\sqrt{2})*(-(I - 1)*A - (I + 1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(d*x + c)})) + 2*\sqrt{2}*(-(I - 1)*A - (I + 1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(d*x + c)})) - \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) + \sqrt{2}*(-(I + 1)*A + (I - 1)*B)*\log(-\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1)*a^2/d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(104) = 208.



[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^2/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^2}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2)/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^2)/cot(c + d\*x)^(1/2), x)

$$3.513 \quad \int \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=171

$$\frac{8\sqrt{-1} a^3(iA + B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{8a^3(iA + B) \sqrt{\cot(c+dx)}}{d} + \frac{8a^3(23A - 21iB) \cot^{\frac{3}{2}}(c+dx)}{105d}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+8/105*a^3*(23*A-21*I*B)*\cot(d*x+c)^{(3/2)}/d-2/7*a*A*\cot(d*x+c)^{(3/2)}*(I*a+a*\cot(d*x+c))^2/d-2/35*(11*I*A+7*B)*\cot(d*x+c)^{(3/2)}*(I*a^3+a^3*\cot(d*x+c))/d+8*a^3*(I*A+B)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.35, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3662, 3675, 3673, 3609, 3614, 214}

$$\frac{8a^3(23A - 21iB) \cot^{\frac{3}{2}}(c+dx)}{105d} - \frac{2(7B + 11iA) \cot^{\frac{3}{2}}(c+dx)(a^3 \cot(c+dx) + ia^3)}{35d} + \frac{8a^3(B + iA) \sqrt{\cot(c+dx)}}{d} + \frac{8\sqrt{-1} a^3(B + iA) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2aA \cot^{\frac{3}{2}}(c+dx)(a \cot(c+dx) + ia)^2}{7d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out]  $(8*(-1)^{(1/4)}*a^3*(I*A + B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\sqrt{\cot[c + d*x]}])/d + (8*a^3*(I*A + B)*\sqrt{\cot[c + d*x]})/d + (8*a^3*(23*A - (21*I)*B)*\cot[c + d*x]^{(3/2)})/(105*d) - (2*a*A*\cot[c + d*x]^{(3/2)}*(I*a + a*\cot[c + d*x])^2)/(7*d) - (2*((11*I)*A + 7*B)*\cot[c + d*x]^{(3/2)}*(I*a^3 + a^3*\cot[c + d*x]))/(35*d)$

**Rule 214**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 3609**

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

**Rule 3614**

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2\*(c^2/f), Subst[Int[1/(b\*c - d\*x^2), x], x, Sqrt[b\*

$\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{EqQ}[c^2 + d^2, 0]$

### Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]))^{(m_)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)])^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[g^{(m+n)}, \text{Int}[(g*\cot[e + f*x])^{(p-m-n)}*(b + a*\cot[e + f*x])^m*(d + c*\cot[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

### Rule 3673

$\text{Int}(((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]))^{(m_)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[B*d*((a + b*\tan[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Int}[(a + b*\tan[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\tan[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!LeQ}[m, -1]$

### Rule 3675

$\text{Int}(((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]))^{(m_)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)])^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[b*B*(a + b*\tan[e + f*x])^{(m-1)}*((c + d*\tan[e + f*x])^{(n+1)})/(d*f*(m+n)), x] + \text{Dist}[1/(d*(m+n)), \text{Int}[(a + b*\tan[e + f*x])^{(m-1)}*(c + d*\tan[e + f*x])^n*\text{Simp}[a*A*d*(m+n) + B*(a*c*(m-1) - b*d*(n+1)) - (B*(b*c - a*d)*(m-1) - d*(A*b + a*B)*(m+n))*\tan[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{!LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \int \sqrt{\cot(c+dx)} (ia+a \cot(c+dx))^3(B+ \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c+dx)(ia+a \cot(c+dx))^2}{7d} - \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c+dx)(ia+a \cot(c+dx))^2}{7d} - \\
&= \frac{8a^3(23A-21iB) \cot^{\frac{3}{2}}(c+dx)}{105d} - \frac{2aA \cot^{\frac{3}{2}}(c+dx)}{7d} \\
&= \frac{8a^3(iA+B) \sqrt{\cot(c+dx)}}{d} + \frac{8a^3(23A-21iB)}{105d} \\
&= \frac{8a^3(iA+B) \sqrt{\cot(c+dx)}}{d} + \frac{8a^3(23A-21iB)}{105d} \\
&= \frac{8\sqrt{-1} a^3(iA+B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 3.99, size = 161, normalized size = 0.94

$$\frac{a^3 \sqrt{\cot(c+dx)} \left( -\csc^3(c+dx)((-95A+105iB) \cos(c+dx) + 5(31A-21iB) \cos(3(c+dx)) + 42(-17iA-19B + (23iA+21B) \cos(2(c+dx))) \sin(c+dx)) - 1680i(A-iB) \tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right) \sqrt{i \tan(c+dx)} \right)}{210d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (a^3\*Sqrt[Cot[c + d\*x]]\*(-(Csc[c + d\*x]^3\*((-95\*A + (105\*I)\*B)\*Cos[c + d\*x] + 5\*(31\*A - (21\*I)\*B)\*Cos[3\*(c + d\*x)] + 42\*((-17\*I)\*A - 19\*B + ((23\*I)\*A + 21\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x])) - (1680\*I)\*(A - I\*B)\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]])\*Sqrt[I\*Tan[c + d\*x]])/(210\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 24.18, size = 3132, normalized size = 18.32

method	result	size
default	Expression too large to display	3132

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

```
[Out] -1/105*a^3/d^2^(1/2)*(-420*I*B*cos(d*x+c)^3*sin(d*x+c)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+420*I*B*cos(d*x+c)^3*sin(d*x+c)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-420*I*A*cos(d*x+c)^2*sin(d*x+c)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1
/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-420*I*B*cos(d*x+c)^2*sin
(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+420*I*B*cos
(d*x+c)^2*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+420*I*A*c
os(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)+420*I*B*cos(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)-420*I*B*cos(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)+420*I*A*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)+420*I*B*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)-420*I*B*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)-420*A*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*
sin(d*x+c)+420*B*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin
(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*co
s(d*x+c)*sin(d*x+c)+420*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1
+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)*cos(d*x+c)^2*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*sin(d*x+c)-420*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d
```



```

*x+c))^(1/2)*cos(d*x+c)^2*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))
^(1/2),1/2*2^(1/2))*sin(d*x+c)-420*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d
*x+c))^(1/2)*cos(d*x+c)^2*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+420*A*cos(d*x+c)^3*sin(d*x+c)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-420*A*cos(d*x+c)^3*sin
(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-420*B*cos(d*x+c)^3*sin
(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-420*I*A*cos
(d*x+c)^3*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2
)+483*I*A*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)-420*I*A*cos(d*x+c)*sin(d*x+c)*2^(
1/2)+420*A*sin(d*x+c)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d
*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*El
lipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-420*A*si
n(d*x+c)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)...

```

**Maxima [A]**

time = 0.53, size = 218, normalized size = 1.27

$$\frac{105 \left( 2\sqrt{2}(-i+1)A+(i-1)B \right) \arctan \left( \frac{1}{2}\sqrt{2} \left( \sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}} \right) \right) + 2\sqrt{2}(-i+1)A+(i-1)B \arctan \left( -\frac{1}{2}\sqrt{2} \left( \sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}} \right) \right) - \sqrt{2}(i-1)A+(i+1)B \log \left( \frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)+1}} \right) + \sqrt{2}(i-1)A+(i+1)B \log \left( \frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} - \frac{1}{\sqrt{\tan(dx+c)+1}} \right)}{105d} + \frac{214A+214iB}{105d \sqrt{\tan(dx+c)}} + \frac{214A-214iB}{105d \sqrt{\tan(dx+c)}} - \frac{84A}{105d \sqrt{\tan(dx+c)}} - \frac{84B}{105d \sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/105\*(105\*(2\*sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(d\*x + c)))) - sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*log(sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) + sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1))\*a^3 + 840\*(I\*A + B)\*a^3/sqrt(tan(d\*x + c)) + 70\*(4\*A - 3\*I\*B)\*a^3/tan(d\*x + c)^(3/2) + 42\*(-3\*I\*A - B)\*a^3/tan(d\*x + c)^(5/2) - 30\*A\*a^3/tan(d\*x + c)^(7/2))/d

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs.  $2(139) = 278$ .

time = 2.37, size = 508, normalized size = 2.97

$$\frac{105 \sqrt{2} \sqrt{\tan(dx+c)} \arctan \left( \frac{1}{2}\sqrt{2} \left( \sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}} \right) \right) + 2\sqrt{2} \sqrt{\tan(dx+c)} \arctan \left( -\frac{1}{2}\sqrt{2} \left( \sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}} \right) \right) - \sqrt{2} \sqrt{\tan(dx+c)} \log \left( \frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)+1}} \right) + \sqrt{2} \sqrt{\tan(dx+c)} \log \left( \frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} - \frac{1}{\sqrt{\tan(dx+c)+1}} \right)}{105d} + \frac{214A+214iB}{105d \sqrt{\tan(dx+c)}} + \frac{214A-214iB}{105d \sqrt{\tan(dx+c)}} - \frac{84A}{105d \sqrt{\tan(dx+c)}} - \frac{84B}{105d \sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 2/105*(105*\sqrt{-(I*A^2 + 2*A*B - I*B^2)}*a^6/d^2)*(d*e^{(6*I*d*x + 6*I*c)} - \\ & 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} - d)*\log(2*((A - I*B)*a^3 \\ & *e^{(2*I*d*x + 2*I*c)} + \sqrt{-(I*A^2 + 2*A*B - I*B^2)}*a^6/d^2)*(d*e^{(2*I*d*x \\ & + 2*I*c)} - d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))) \\ & *e^{(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)} - 105*\sqrt{-(I*A^2 + 2*A*B - I*B^2)} \\ & *a^6/d^2)*(d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x \\ & + 2*I*c)} - d)*\log(2*((A - I*B)*a^3*e^{(2*I*d*x + 2*I*c)} - \sqrt{-(I*A^2 + 2* \\ & A*B - I*B^2)}*a^6/d^2)*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{(I*e^{(2*I*d*x + 2*I* \\ & c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))) *e^{(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)} \\ & - 2*((-319*I*A - 273*B)*a^3*e^{(6*I*d*x + 6*I*c)} + 2*(323*I*A + 336*B)*a^3*e \\ & ^{(4*I*d*x + 4*I*c)} + (-551*I*A - 567*B)*a^3*e^{(2*I*d*x + 2*I*c)} + 4*(41*I*A \\ & + 42*B)*a^3)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))/ \\ & (d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} \\ & - d) \end{aligned}$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(9/2)\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^3\*cot(d\*x + c)^(9/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) li)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

$$3.514 \quad \int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=146

$$\frac{8\sqrt[4]{-1} a^3(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} + \frac{16a^3(6A - 5iB) \sqrt{\cot(c+dx)}}{15d} - \frac{2aA \sqrt{\cot(c+dx)} (ia)}{5d}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+16/15*a^3*(6*A-5*I*B)*\cot(d*x+c)^{(1/2)}/d-2/5*a*A*(I*a+a*\cot(d*x+c))^2*\cot(d*x+c)^{(1/2)}/d-2/15*(9*I*A+5*B)*(I*a^3+a^3*\cot(d*x+c))*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.31, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3662, 3675, 3673, 3614, 214}

$$\frac{16a^3(6A - 5iB) \sqrt{\cot(c+dx)}}{15d} - \frac{2(5B + 9iA) \sqrt{\cot(c+dx)} (a^3 \cot(c+dx) + ia^3)}{15d} + \frac{8\sqrt[4]{-1} a^3(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{2aA \sqrt{\cot(c+dx)} (a \cot(c+dx) + ia)^2}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(7/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(8*(-1)^{(1/4)}*a^3*(A - I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d + (16*a^3*(6*A - (5*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]/(15*d) - (2*a*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(I*a + a*\operatorname{Cot}[c + d*x])^2)/(5*d) - (2*((9*I)*A + 5*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(I*a^3 + a^3*\operatorname{Cot}[c + d*x]))/(15*d)$

Rule 214

$\operatorname{Int}[(a_) + (b_)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 3614

$\operatorname{Int}[(c_) + (d_)*\operatorname{tan}[e_] + (f_)*(x_)]/\operatorname{Sqrt}[(b_)*\operatorname{tan}[e_] + (f_)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

Rule 3662

$\operatorname{Int}[(\operatorname{cot}[e_] + (f_)*(x_)]*(g_))^{(p)}*((a_) + (b_)*\operatorname{tan}[e_] + (f_)*(x_))^{(m)}*((c_) + (d_)*\operatorname{tan}[e_] + (f_)*(x_))^{(n)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b + a*\operatorname{Cot}[e + f*x])^m*(d + c*\operatorname{Cot}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, p\}, x \ \&\& \operatorname{IntegerQ}[p] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IntegerQ}[n]$

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Rule 3675

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[
e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && G
tQ[m, 1] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \int \frac{(ia + a \cot(c + dx))^3(B + A \cot(c + dx))}{\sqrt{\cot(c + dx)}} \\
&= -\frac{2aA\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))^2}{5d} \\
&= -\frac{2aA\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))^2}{5d} \\
&= \frac{16a^3(6A - 5iB)\sqrt{\cot(c + dx)}}{15d} - \frac{2aA\sqrt{\cot(c + dx)}}{15d} \\
&= \frac{16a^3(6A - 5iB)\sqrt{\cot(c + dx)}}{15d} - \frac{2aA\sqrt{\cot(c + dx)}}{15d} \\
&= \frac{8\sqrt{-1} a^3(A - iB) \tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c + dx)}\right)}{d}
\end{aligned}$$

Mathematica [A]

time = 3.22, size = 132, normalized size = 0.90

$$\frac{a^3\sqrt{\cot(c + dx)}\left(\csc^2(c + dx)(-57A + 45iB + 9(7A - 5iB)\cos(2(c + dx)) + 5(3iA + B)\sin(2(c + dx))) + 120(A - iB)\tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c + dx)}}{1 + e^{2i(c + dx)}}}\right)\sqrt{i\tan(c + dx)}\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),
x]
```

```
[Out] -1/15*(a^3*sqrt[Cot[c + d*x]]*(Csc[c + d*x]^2*(-57*A + (45*I)*B + 9*(7*A -
(5*I)*B)*Cos[2*(c + d*x)] + 5*((3*I)*A + B)*Sin[2*(c + d*x)]) + 120*(A - I*
B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]]*sqrt
[I*Tan[c + d*x]]))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 21.36, size = 2947, normalized size = 20.18

method	result	size
default	Expression too large to display	2947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/15*a^3/d^2^(1/2)*(5*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-45*I*B*cos(d*x+c)^
3*2^(1/2)+45*I*B*cos(d*x+c)*2^(1/2)+63*A*cos(d*x+c)^3*2^(1/2)-60*A*cos(d*x+
c)*2^(1/2)+60*A*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)
))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-60*
B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+60*B*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2)-60*I*A*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c)
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c)
))/sin(d*x+c))^(1/2)-60*I*B*cos(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(
d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c)
))/sin(d*x+c))^(1/2)-60*I*A*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c)
))/sin(d*x+c))^(1/2)+60*I*A*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x
+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2)+60*I*B*cos(d*x+c)^3*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)
)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2)-60*I*A*cos(d*x+c)^2*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2
```

$$\begin{aligned}
& ) * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * I * A * \cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} \\
& ) * \text{EllipticF}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 * 2^{1/2}) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * I * B * \cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} \\
& ) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * I * A * \cos(dx+c) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} \\
& ) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 15 * I * A * \cos(dx+c)^2 * \sin(dx+c) * 2^{1/2} + 60 * I * A * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} \\
& ) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} - 60 * I * A * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticF}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} - 60 * I * B * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * B * \cos(dx+c) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} - 60 * A * \cos(dx+c)^3 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * B * \cos(dx+c)^3 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticF}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} - 60 * B * \cos(dx+c)^3 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} - 60 * A * \cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * B * \cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticF}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} - 60 * B * \cos(dx+c)^2 * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c))^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) \\
& ) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \\
& )^{1/2} + 60 * A * \cos \dots
\end{aligned}$$

**Maxima [A]**

time = 0.56, size = 200, normalized size = 1.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 + 30*(4*A - 3*I*B)*a^3/sqrt(tan(d*x + c)) + 10*(-3*I*A - B)*a^3/tan(d*x + c)^(3/2) - 6*A*a^3/tan(d*x + c)^(5/2))/d
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 449 vs.  $2(118) = 236$ .

time = 1.54, size = 449, normalized size = 3.08

$$\frac{2 \left( 15 \sqrt{\frac{15A^2 - 2AB + B^2}{d}} \left( \frac{15 \sqrt{\frac{15A^2 - 2AB + B^2}{d}} \log\left(\frac{\sqrt{\frac{15A^2 - 2AB + B^2}{d}} \sqrt{\frac{15A^2 - 2AB + B^2}{d}} + 1\right)}{\sqrt{\frac{15A^2 - 2AB + B^2}{d}}} \right) - 15 \sqrt{\frac{15A^2 - 2AB + B^2}{d}} \log\left(\frac{\sqrt{\frac{15A^2 - 2AB + B^2}{d}} \sqrt{\frac{15A^2 - 2AB + B^2}{d}} - 1\right)}{\sqrt{\frac{15A^2 - 2AB + B^2}{d}}} \right) - 15 \sqrt{\frac{15A^2 - 2AB + B^2}{d}} \log\left(\frac{\sqrt{\frac{15A^2 - 2AB + B^2}{d}} \sqrt{\frac{15A^2 - 2AB + B^2}{d}} + 1\right)}{\sqrt{\frac{15A^2 - 2AB + B^2}{d}}} \right) - 2 \left( (39A - 25B) \sqrt{\frac{15A^2 - 2AB + B^2}{d}} - 3(19A - 15B) \sqrt{\frac{15A^2 - 2AB + B^2}{d}} + 4(6A - 5B) \sqrt{\frac{15A^2 - 2AB + B^2}{d}} \right) \sqrt{\frac{15A^2 - 2AB + B^2}{d}}}{15(d \sqrt{\frac{15A^2 - 2AB + B^2}{d}} + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2))*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2))*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3) - 15*sqrt(-(-I*A^2 - 2*A*B + I*B^2))*a^6/d^2*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2))*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3) - 2*((39*A - 25*I*B)*a^3*e^(4*I*d*x + 4*I*c) - 3*(19*A - 15*I*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*(6*A - 5*I*B)*a^3)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

$$3.515 \quad \int \cot^2(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=138

$$\frac{8\sqrt[4]{-1} a^3(iA+B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{16ia^3 A \sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(ia+a \cot(c+dx))^2}{d \sqrt{\cot(c+dx)}}$$

[Out]  $-8*(-1)^{(1/4)}*a^3*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2*I*a*B*(I*a+a*\cot(d*x+c))^{2/d}/\cot(d*x+c)^{(1/2)}-16/3*I*a^3*A*\cot(d*x+c)^{(1/2)}/d-2/3*(A+3*I*B)*(I*a^3+a^3*\cot(d*x+c))*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.30, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3662, 3674, 3675, 3673, 3614, 214}

$$\frac{2(A+3iB)\sqrt{\cot(c+dx)}(a^3 \cot(c+dx)+ia^3)}{3d} - \frac{8\sqrt[4]{-1} a^3(B+iA) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{16ia^3 A \sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(a \cot(c+dx)+ia)^2}{d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^{(5/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^3*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $(-8*(-1)^{(1/4)}*a^3*(I*A+B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]])/d - ((16*I)/3)*a^3*A*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]/d + ((2*I)*a*B*(I*a+a*\operatorname{Cot}[c+d*x])^2)/(d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]) - (2*(A+(3*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*(I*a^3+a^3*\operatorname{Cot}[c+d*x]))/(3*d)$

**Rule 214**

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

**Rule 3614**

$\operatorname{Int}[(c_+ + (d_+)*\operatorname{tan}[(e_+ + (f_+)*(x_+)])/ \operatorname{Sqrt}[(b_+)*\operatorname{tan}[(e_+ + (f_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

**Rule 3662**

$\operatorname{Int}[(\cot[(e_+ + (f_+)*(x_+)])*(g_+))^{(p_+)}*((a_+ + (b_+)*\operatorname{tan}[(e_+ + (f_+)*(x_+)])^{(m_+)}*((c_+ + (d_+)*\operatorname{tan}[(e_+ + (f_+)*(x_+)])^{(n_+)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b+a*\operatorname{Cot}[e + f*x])^m*(d+c*\operatorname{Cot}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \ \&\& \operatorname{!IntegerQ}$

[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3673

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[B\*d\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1))), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A\*c - B\*d + (B\*c + A\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1]

### Rule 3674

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rule 3675

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \int \frac{(ia+a \cot(c+dx))^3(B+A \cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2iaB(ia+a \cot(c+dx))^2}{d\sqrt{\cot(c+dx)}} + 2 \int \frac{(ia+a \cot(c+dx))^3}{\cot^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2iaB(ia+a \cot(c+dx))^2}{d\sqrt{\cot(c+dx)}} - \frac{2(A+3iB)\sqrt{\cot(c+dx)}}{d} \\
&= -\frac{16ia^3A\sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(ia+a \cot(c+dx))^2}{d\sqrt{\cot(c+dx)}} \\
&= -\frac{16ia^3A\sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(ia+a \cot(c+dx))^2}{d\sqrt{\cot(c+dx)}} \\
&= -\frac{8\sqrt[4]{-1}a^3(iA+B)\tanh^{-1}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.69, size = 146, normalized size = 1.06

$$\frac{a^3\sqrt{\cot(c+dx)}\csc(c+dx)\sec(c+dx)\left(A+3iB+(A-3iB)\cos(2(c+dx))+9iA\sin(2(c+dx))+3B\sin(2(c+dx))-12i(A-iB)\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sin(2(c+dx))\sqrt{i\tan(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]), x]
```

```
[Out] -1/3*(a^3*Sqrt[Cot[c + d*x]]*Csc[c + d*x]*Sec[c + d*x]*(A + (3*I)*B + (A - (3*I)*B)*Cos[2*(c + d*x)] + (9*I)*A*Sin[2*(c + d*x)] + 3*B*Sin[2*(c + d*x)] - (12*I)*(A - I*B)*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*Sin[2*(c + d*x)]*Sqrt[I*Tan[c + d*x]]))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 68.58, size = 1562, normalized size = 11.32

method	result	size
default	Expression too large to display	1562

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^3/d*2^(1/2)*(3*I*B*2^(1/2)-12*I*B*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)-12*I*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+12*A*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)-12*A*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*sin(d*x+c)+12*I*B*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)-12*B*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*sin(d*x+c)-12*I*A*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)+9*I*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)+12*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-12*B*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*I*B*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)-3*I*B*cos(d*x+c)^2*2^(1/2)+A*2^(1/2)*cos(d*x+c)^2+3*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)+12*I*B*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c))*((cos(d*x+c)/sin(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^3
```

**Maxima [A]**

time = 0.55, size = 194, normalized size = 1.41

$$\frac{-6iBd^2\sqrt{\tan(dx+c)} - 3(2\sqrt{-(i+1)A+(i-1)B})\operatorname{arctan}\left(\frac{1}{\sqrt{2}}\left(\sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{-(i+1)A+(i-1)B}\operatorname{arctan}\left(-\frac{1}{\sqrt{2}}\left(\sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) - \sqrt{2}(i-1)A+(i+1)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) + \sqrt{2}(i-1)A+(i+1)B\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right)}{3d^2 + \frac{4i-3A-2Bd}{\sqrt{\tan(dx+c)}} - \frac{3d^2}{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{3}(-6IBa^3\sqrt{\tan(dx+c)} - 3(2\sqrt{2})(-I+1)A + (I-1)B)\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}})\right) + 2\sqrt{2}(-I+1)A + (I-1)B\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - \frac{2}{\sqrt{\tan(dx+c)}})\right) - \sqrt{2}((I-1)A + (I+1)B)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) + \sqrt{2}((I-1)A + (I+1)B)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right)a^3 + 6(-3IA - B)a^3/\sqrt{\tan(dx+c)} - 2Aa^3/\tan(dx+c)^{3/2}/d$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs.  $2(110) = 220$ .  
time = 1.41, size = 409, normalized size = 2.96

$$2 \left( 3 \sqrt{\frac{(A^2+2AB-1)B^2}{d^2}} (d^{(4I+1)x+c-d}) \log\left(\frac{(1+(-1)^{2I+1} \sqrt{\frac{(A^2+2AB-1)B^2}{d^2}} (d^{(4I+1)x+c-d}) \sqrt{\frac{(d^{(4I+1)x+c-d}}{d^{2I+1}} - 1)}}{(-1)^{2I+1}}}\right) - 3 \sqrt{\frac{(A^2+2AB-1)B^2}{d^2}} (d^{(4I+1)x+c-d}) \log\left(\frac{(1+(-1)^{2I+1} \sqrt{\frac{(A^2+2AB-1)B^2}{d^2}} (d^{(4I+1)x+c-d}) \sqrt{\frac{(d^{(4I+1)x+c-d}}{d^{2I+1}} - 1)}}{(-1)^{2I+1}}}\right) + 2 \left( (5A+3B)a^{3I+3}e^{(4I+1)(x+c-d)} + (A-3B)a^{3I+3}e^{-(4I+1)(x+c-d)} \sqrt{\frac{(d^{(4I+1)x+c-d}}{d^{2I+1}} - 1)}{d^{2I+1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $-2/3(3\sqrt{-(IA^2+2AB-IB^2)}a^6/d^2)(d^e^{(4I*d*x+4I*c)} - d) \log(2((A-IB)a^3e^{(2I*d*x+2I*c)} + \sqrt{-(IA^2+2AB-IB^2)}a^6/d^2)(d^e^{(2I*d*x+2I*c)} - d)\sqrt{(Ie^{(2I*d*x+2I*c)} + I)/(e^{(2I*d*x+2I*c)} - 1)})e^{(-2I*d*x-2I*c)}/((-IA-B)a^3) - 3\sqrt{-(IA^2+2AB-IB^2)}a^6/d^2)(d^e^{(4I*d*x+4I*c)} - d)\log(2((A-IB)a^3e^{(2I*d*x+2I*c)} - \sqrt{-(IA^2+2AB-IB^2)}a^6/d^2)(d^e^{(2I*d*x+2I*c)} - d)\sqrt{(Ie^{(2I*d*x+2I*c)} + I)/(e^{(2I*d*x+2I*c)} - 1)})e^{(-2I*d*x-2I*c)}/((-IA-B)a^3) + 2((5IA+3B)a^3e^{(4I*d*x+4I*c)} + (IA-3B)a^3e^{(2I*d*x+2I*c)} - 4IAa^3)\sqrt{(Ie^{(2I*d*x+2I*c)} + I)/(e^{(2I*d*x+2I*c)} - 1)))/(d^e^{(4I*d*x+4I*c)} - d)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

$$3.516 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=142

$$\frac{8\sqrt{-1} a^3(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{16ia^3B \sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(ia + a \cot(c+dx))^2}{3d \cot^{\frac{3}{2}}(c+dx)}$$

[Out]  $-8*(-1)^{(1/4)}*a^3*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2/3*I*a*B*(I*a+a*\cot(d*x+c))^2/d/\cot(d*x+c)^{(3/2)}-2/3*(3*A-7*I*B)*(I*a^3+a^3*\cot(d*x+c))/d/\cot(d*x+c)^{(1/2)}-16/3*I*a^3*B*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.31, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3662, 3674, 3673, 3614, 214}

$$\frac{2(3A - 7iB)(a^3 \cot(c+dx) + ia^3)}{3d \sqrt{\cot(c+dx)}} - \frac{8\sqrt{-1} a^3(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d} - \frac{16ia^3B \sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(a \cot(c+dx) + ia)^2}{3d \cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(-8*(-1)^{(1/4)}*a^3*(A - I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d - ((16*I)/3)*a^3*B*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]/d + (((2*I)/3)*a*B*(I*a + a*\operatorname{Cot}[c + d*x])^2)/(d*\operatorname{Cot}[c + d*x]^{(3/2)}) - (2*(3*A - (7*I)*B)*(I*a^3 + a^3*\operatorname{Cot}[c + d*x]))/(3*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])$

**Rule 214**

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

**Rule 3614**

$\operatorname{Int}[(c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])/\operatorname{Sqrt}[(b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\operatorname{Tan}[e + f*x]]], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

**Rule 3662**

$\operatorname{Int}[(\operatorname{cot}[(e_.) + (f_.)*(x_.)])*(g_.)^{(p_.)}*((a_.) + (b_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\operatorname{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[g^{(m+n)}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{(p-m-n)}*(b + a*\operatorname{Cot}[e + f*x])^m*(d + c*\operatorname{Cot}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, p\}, x \ \&\& \operatorname{!IntegerQ}$



[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

### Rule 3674

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3(A + B \tan(c + dx)) dx &= \int \frac{(ia + a \cot(c + dx))^3(B + A \cot(c + dx))}{\cot^{\frac{5}{2}}(c + dx)} \\
 &= \frac{2iaB(ia + a \cot(c + dx))^2}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2}{3} \int \frac{(ia + a \cot(c + dx))^3}{\cot^{\frac{3}{2}}(c + dx)} \\
 &= \frac{2iaB(ia + a \cot(c + dx))^2}{3d \cot^{\frac{3}{2}}(c + dx)} - \frac{2(3A - 7iB)(ia + a \cot(c + dx))^3}{3d \sqrt{c + dx}} \\
 &= -\frac{16ia^3B \sqrt{\cot(c + dx)}}{3d} + \frac{2iaB(ia + a \cot(c + dx))^2}{3d \cot^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{16ia^3B \sqrt{\cot(c + dx)}}{3d} + \frac{2iaB(ia + a \cot(c + dx))^2}{3d \cot^{\frac{3}{2}}(c + dx)} \\
 &= -\frac{8\sqrt{-1} a^3(A - iB) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c + dx)}\right)}{d}
 \end{aligned}$$

**Mathematica** [A]

time = 2.78, size = 132, normalized size = 0.93

$$\frac{a^3 \sqrt{\cot(c+dx)} \left( \sec^2(c+dx)(3A+iB+(3A-iB)\cos(2(c+dx))+(3iA+9B)\sin(2(c+dx))) - 24(A-iB)\tanh^{-1}\left(\sqrt{\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\sqrt{i\tan(c+dx)} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] -1/3\*(a^3\*sqrt[Cot[c + d\*x]]\*(Sec[c + d\*x]^2\*(3\*A + I\*B + (3\*A - I\*B)\*Cos[2\*(c + d\*x)] + ((3\*I)\*A + 9\*B)\*Sin[2\*(c + d\*x)]) - 24\*(A - I\*B)\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]]\*sqrt[I\*Tan[c + d\*x]]))/d

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 85.03, size = 1539, normalized size = 10.84

method	result	size
default	Expression too large to display	1539

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] -1/3\*a^3/d\*2^(1/2)\*(12\*I\*A\*cos(d\*x+c)^2\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^1/2)\*((-1+cos(d\*x+c))/sin(d\*x+c))^1/2\*EllipticF((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2,1/2\*2^(1/2))\*((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2+I\*B\*2^(1/2)-12\*I\*A\*cos(d\*x+c)^2\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^1/2)\*((-1+cos(d\*x+c))/sin(d\*x+c))^1/2\*EllipticPi((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2,1/2+1/2\*I,1/2\*2^(1/2))\*((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2-12\*A\*cos(d\*x+c)^2\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^1/2)\*((-1+cos(d\*x+c))/sin(d\*x+c))^1/2\*EllipticPi((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2,1/2+1/2\*I,1/2\*2^(1/2))\*((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2+12\*I\*B\*cos(d\*x+c)^2\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^1/2)\*((-1+cos(d\*x+c))/sin(d\*x+c))^1/2\*EllipticPi((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2,1/2+1/2\*I,1/2\*2^(1/2))\*((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2+12\*B\*cos(d\*x+c)^2\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^1/2)\*((-1+cos(d\*x+c))/sin(d\*x+c))^1/2\*EllipticF((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2,1/2\*2^(1/2))\*((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2-I\*B\*cos(d\*x+c)^2\*2^(1/2)-12\*A\*cos(d\*x+c)\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^1/2)\*((-1+cos(d\*x+c))/sin(d\*x+c))^1/2\*EllipticPi((-cos(d\*x+c)-1-sin(d\*x+c))/sin(d\*x+c))^1/2,1

$$\begin{aligned} & /2+1/2*I, 1/2*2^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-12*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+12*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2*2^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-12*I*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+12*I*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*A*2^{(1/2)}*\cos(d*x+c)^2+9*B*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)+12*I*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2*2^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)})*\sin(d*x+c)*(\cos(d*x+c)/\sin(d*x+c))^{(3/2)}/\cos(d*x+c)^3 \end{aligned}$$

**Maxima [A]**

time = 0.51, size = 197, normalized size = 1.39

$$\frac{3\left(2\sqrt{2}(i-1)A+(i+1)B\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(i-1)A+(i+1)B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}(i+1)A+(i-1)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}+1}+1\right)-\sqrt{2}(i-1)A+(i-1)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}-1}+1\right)}{3d}+\frac{4A^2}{\sqrt{\tan(dx+c)}}-2\left(-12A^2-\frac{36A^2B^2}{\tan(dx+c)}\right)\tan(dx+c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/3*(3*(2*\sqrt{2})*((I-1)*A+(I+1)*B)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2/\sqrt{\tan(dx+c)})) + 2*\sqrt{2}*((I-1)*A+(I+1)*B)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2/\sqrt{\tan(dx+c)})) + \sqrt{2}*(-(I+1)*A+(I-1)*B)*\log(\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1) - \sqrt{2}*(-(I+1)*A+(I-1)*B)*\log(-\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1)*a^3 + 6*A*a^3/\sqrt{\tan(dx+c)} - 2*(-I*B*a^3 - 3*(I*A+3*B)*a^3/\tan(dx+c))*\tan(dx+c)^{(3/2)}/d$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 442 vs.  $2(112) = 224$ .

time = 1.67, size = 442, normalized size = 3.11

$$\frac{2\left(3\sqrt{\frac{-1+A^2-2AB+B^2}{d}}\left(\frac{1}{\sqrt{\tan(dx+c)}}+\frac{1}{\sqrt{\tan(dx+c)}}\right)\log\left(\frac{1}{\sqrt{\tan(dx+c)}}+\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{\frac{-1+A^2-2AB+B^2}{d}}\left(\frac{1}{\sqrt{\tan(dx+c)}}-\frac{1}{\sqrt{\tan(dx+c)}}\right)\log\left(\frac{1}{\sqrt{\tan(dx+c)}}-\frac{1}{\sqrt{\tan(dx+c)}}\right)}{3(dB^2\sqrt{\tan(dx+c)}+2dA^2\sqrt{\tan(dx+c)})}+\frac{2\left(3A^2-6B^2\sqrt{\tan(dx+c)}+(3A+6B)\sqrt{\tan(dx+c)}+6B\right)\sqrt{\tan(dx+c)}}{3dB^2\sqrt{\tan(dx+c)}+2dA^2\sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

```
[Out] 2/3*(3*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d
*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-2*I*d*x - 2*I*c)/((
-I*A - B)*a^3)) - 3*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*d*x +
4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*
I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2))*(-I*d*e^(2*I*d*x + 2*I*c) +
I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-2*I*d
*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((3*A - 5*I*B)*a^3*e^(4*I*d*x + 4*I*c) +
(3*A + I*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*I*B*a^3)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
+ 2*I*c) + d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-ia^2 \int A \cot^3(c+dx) dx + \int (-3A \tan(c+dx) \cot^3(c+dx)) dx + \int A \tan^3(c+dx) \cot^3(c+dx) dx + \int (-3B \tan^2(c+dx) \cot^3(c+dx)) dx + \int B \tan^3(c+dx) \cot^3(c+dx) dx + \int (-3iA \tan^2(c+dx) \cot^3(c+dx)) dx + \int iB \tan(c+dx) \cot^3(c+dx) dx + \int (-3iB \tan^3(c+dx) \cot^3(c+dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

```
[Out] -I*a**3*(Integral(I*A*cot(c + d*x)**(3/2), x) + Integral(-3*A*tan(c + d*x)*
cot(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)**3*cot(c + d*x)**(3/2), x
) + Integral(-3*B*tan(c + d*x)**2*cot(c + d*x)**(3/2), x) + Integral(B*tan(
c + d*x)**4*cot(c + d*x)**(3/2), x) + Integral(-3*I*A*tan(c + d*x)**2*cot(c
+ d*x)**(3/2), x) + Integral(I*B*tan(c + d*x)*cot(c + d*x)**(3/2), x) + In
tegral(-3*I*B*tan(c + d*x)**3*cot(c + d*x)**(3/2), x))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2),
x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+dx)^{3/2} (A+B \tan(c+dx)) (a+a \tan(c+dx) li)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^3,x)
```

```
[Out] int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^3, x)
```

$$3.517 \quad \int \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^3 (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=148

$$\frac{8\sqrt{-1} a^3 (iA + B) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c + dx)} \right)}{d} - \frac{16a^3 (5A - 6iB)}{15d \sqrt{\cot(c + dx)}} + \frac{2iaB (ia + a \cot(c + dx))^2}{5d \cot^{5/2}(c + dx)} - \frac{2(5A - 9iB) (a^3 \cot(c + dx) + ia^3)}{15d \cot^{3/2}(c + dx)}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(I*A+B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d+2/5*I*a*B*(I*a+a*\cot(d*x+c))^2/d/\cot(d*x+c)^{(5/2)}-2/15*(5*A-9*I*B)*(I*a^3+a^3*\cot(d*x+c))/d/\cot(d*x+c)^{(3/2)}-16/15*a^3*(5*A-6*I*B)/d/\cot(d*x+c)^{(1/2)}$

**Rubi** [A]

time = 0.33, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {3662, 3674, 3672, 3614, 214}

$$-\frac{2(5A - 9iB)(a^3 \cot(c + dx) + ia^3)}{15d \cot^{3/2}(c + dx)} - \frac{16a^3(5A - 6iB)}{15d \sqrt{\cot(c + dx)}} + \frac{8\sqrt{-1} a^3 (B + iA) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c + dx)} \right)}{d} + \frac{2iaB(a \cot(c + dx) + ia)^2}{5d \cot^{5/2}(c + dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(8*(-1)^{(1/4)}*a^3*(I*A + B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d - (16*a^3*(5*A - (6*I)*B))/(15*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (((2*I)/5)*a*B*(I*a + a*\operatorname{Cot}[c + d*x])^2)/(d*\operatorname{Cot}[c + d*x]^{(5/2)}) - (2*(5*A - (9*I)*B)*(I*a^3 + a^3*\operatorname{Cot}[c + d*x]))/(15*d*\operatorname{Cot}[c + d*x]^{(3/2)})$

Rule 214

$\operatorname{Int}[(a + b*x^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3614

$\operatorname{Int}[(c + d*\tan[e + f*x])/(\operatorname{Sqrt}[b*\tan[e + f*x] + (f*x + e)]), x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x, \operatorname{Sqrt}[b*\tan[e + f*x]]], x] /;$  FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]

Rule 3662

$\operatorname{Int}[(\cot[e + f*x] + (f*x + e)^p)*(a + b*\tan[e + f*x] + (f*x + e)^m)*(c + d*\tan[e + f*x] + (f*x + e)^n), x\_Symbol] \rightarrow \operatorname{Dist}[g^{m+n}, \operatorname{Int}[(g*\operatorname{Cot}[e + f*x])^{p-m-n}*(b + a*\operatorname{Cot}[e + f*x])^m*(d + c*\operatorname{Cot}[e + f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ

[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
-1] && NeQ[a^2 + b^2, 0]
```

### Rule 3674

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^3 (A+B \tan(c+dx)) dx &= \int \frac{(ia+a \cot(c+dx))^3 (B+A \cot(c+dx))}{\cot^{\frac{7}{2}}(c+dx)} dx \\
&= \frac{2iaB(ia+a \cot(c+dx))^2}{5d \cot^{\frac{5}{2}}(c+dx)} + \frac{2}{5} \int \frac{(ia+a \cot(c+dx))^3 (B+A \cot(c+dx))}{\cot^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2iaB(ia+a \cot(c+dx))^2}{5d \cot^{\frac{5}{2}}(c+dx)} - \frac{2(5A-9iB)}{15d \sqrt{\cot(c+dx)}} \\
&= -\frac{16a^3(5A-6iB)}{15d \sqrt{\cot(c+dx)}} + \frac{2iaB(ia+a \cot(c+dx))}{5d \cot^{\frac{5}{2}}(c+dx)} \\
&= -\frac{16a^3(5A-6iB)}{15d \sqrt{\cot(c+dx)}} + \frac{2iaB(ia+a \cot(c+dx))}{5d \cot^{\frac{5}{2}}(c+dx)} \\
&= \frac{8\sqrt[4]{-1} a^3 (iA+B) \tanh^{-1}\left((-1)^{3/4} \sqrt{\cot(c+dx)}\right)}{d}
\end{aligned}$$

**Mathematica** [A]

time = 3.07, size = 140, normalized size = 0.95

$$\frac{a^3 \sec^2(c + dx) \left( -45A + 57iB - 9(5A - 7iB) \cos(2(c + dx)) - 5(iA + 3B) \sin(2(c + dx)) + \frac{120(A - iB) \tanh^{-1} \left( \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \right) \cos^2(c + dx)}{\sqrt{i \tan(c + dx)}} \right)}{15d \sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (a^3\*Sec[c + d\*x]^2\*(-45\*A + (57\*I)\*B - 9\*(5\*A - (7\*I)\*B)\*Cos[2\*(c + d\*x)] - 5\*(I\*A + 3\*B)\*Sin[2\*(c + d\*x)] + (120\*(A - I\*B)\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1 + E^((2\*I)\*(c + d\*x)))]])\*Cos[c + d\*x]^2)/Sqrt[I\*Tan[c + d\*x]])/(15\*d\*Sqrt[Cot[c + d\*x]])

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 69.45, size = 961, normalized size = 6.49

method	result	size
default	Expression too large to display	961

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)), x, method=\_RETURN VERBOSE)

[Out] -1/15\*a^3/d\*2^(1/2)\*(-1+cos(d\*x+c))\*(3\*I\*B\*2^(1/2)\*cos(d\*x+c)-60\*I\*A\*sin(d\*x+c)\*EllipticPi(((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2), 1/2+1/2\*I, 1/2\*2^(1/2))\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2+63\*I\*B\*2^(1/2)\*cos(d\*x+c)^2+60\*A\*sin(d\*x+c)\*EllipticPi(((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2), 1/2+1/2\*I, 1/2\*2^(1/2))\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2-60\*A\*sin(d\*x+c)\*EllipticF(((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2), 1/2\*2^(1/2))\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2-60\*B\*sin(d\*x+c)\*EllipticPi(((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2), 1/2+1/2\*I, 1/2\*2^(1/2))\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2-3\*I\*B\*2^(1/2)-60\*I\*B\*sin(d\*x+c)\*EllipticPi(((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2), 1/2+1/2\*I, 1/2\*2^(1/2))\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*((cos(d\*x+c)-1+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*((1-cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2-63\*I\*B\*2^(1/2)\*cos(d\*x+c)^3+45\*A\*cos(d\*x+c)^3\*2^(1/2)+15\*B\*cos(d\*x+c)^2\*sin(d\*x+c)\*2^(1/2)





) $\sqrt{(Ie^{(2I*d*x + 2I*c)} + I)/(e^{(2I*d*x + 2I*c)} - 1))}/(d*e^{(6I*d*x + 6I*c)} + 3*d*e^{(4I*d*x + 4I*c)} + 3*d*e^{(2I*d*x + 2I*c)} + d)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-i^2 \left( \int iA\sqrt{\cot(c+dx)} dx + \int (-3A \tan(c+dx)\sqrt{\cot(c+dx)}) dx + \int A \tan^2(c+dx)\sqrt{\cot(c+dx)} dx + \int (-3B \tan^2(c+dx)\sqrt{\cot(c+dx)}) dx + \int B \tan^4(c+dx)\sqrt{\cot(c+dx)} dx + \int (-3iA \tan^2(c+dx)\sqrt{\cot(c+dx)}) dx + \int iB \tan(c+dx)\sqrt{\cot(c+dx)} dx + \int (-3iB \tan^2(c+dx)\sqrt{\cot(c+dx)}) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out]  $-Ia**3*(\text{Integral}(I*A*\sqrt{\cot(c + dx)}, x) + \text{Integral}(-3*A*\tan(c + dx)*\sqrt{\cot(c + dx)}, x) + \text{Integral}(A*\tan(c + dx)**3*\sqrt{\cot(c + dx)}, x) + \text{Integral}(-3*B*\tan(c + dx)**2*\sqrt{\cot(c + dx)}, x) + \text{Integral}(B*\tan(c + dx)**4*\sqrt{\cot(c + dx)}, x) + \text{Integral}(-3*I*A*\tan(c + dx)**2*\sqrt{\cot(c + dx)}, x) + \text{Integral}(I*B*\tan(c + dx)*\sqrt{\cot(c + dx)}, x) + \text{Integral}(-3*I*B*\tan(c + dx)**3*\sqrt{\cot(c + dx)}, x))$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^3\*sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3, x)

$$3.518 \quad \int \frac{(a+ia \tan(c+dx))^3 (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=173

$$\frac{8\sqrt{-1} a^3 (A - iB) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c+dx)} \right)}{d} - \frac{8a^3 (21A - 23iB)}{105d \cot^{3/2}(c+dx)} + \frac{8a^3 (iA + B)}{d \sqrt{\cot(c+dx)}} + \frac{2iaB (ia + a \cot(c+dx))}{7d \cot^{7/2}(c+dx)}$$

[Out]  $8*(-1)^{(1/4)}*a^3*(A-I*B)*\operatorname{arctanh}((-1)^{(3/4)}*\cot(d*x+c)^{(1/2)})/d-8/105*a^3*(21*A-23*I*B)/d/\cot(d*x+c)^{(3/2)}+2/7*I*a*B*(I*a+a*\cot(d*x+c))^2/d/\cot(d*x+c)^{(7/2)}-2/35*(7*A-11*I*B)*(I*a^3+a^3*\cot(d*x+c))/d/\cot(d*x+c)^{(5/2)}+8*a^3*(I*A+B)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.36, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3662, 3674, 3672, 3610, 3614, 214}

$$-\frac{8a^3(21A-23iB)}{105d \cot^{3/2}(c+dx)} - \frac{2(7A-11iB)(a^3 \cot(c+dx) + ia^3)}{35d \cot^{5/2}(c+dx)} + \frac{8a^3(B+iA)}{d \sqrt{\cot(c+dx)}} + \frac{8\sqrt{-1} a^3 (A - iB) \tanh^{-1} \left( (-1)^{3/4} \sqrt{\cot(c+dx)} \right)}{d} + \frac{2iaB(a \cot(c+dx) + ia)^2}{7d \cot^{7/2}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x])/ \operatorname{Sqrt}[\operatorname{Cot}[c + d*x]], x]$

[Out]  $(8*(-1)^{(1/4)}*a^3*(A - I*B)*\operatorname{ArcTanh}[(-1)^{(3/4)}*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/d - (8*a^3*(21*A - (23*I)*B))/(105*d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (8*a^3*(I*A + B))/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (((2*I)/7)*a*B*(I*a + a*\operatorname{Cot}[c + d*x])^2)/(d*\operatorname{Cot}[c + d*x]^{(7/2)}) - (2*(7*A - (11*I)*B)*(I*a^3 + a^3*\operatorname{Cot}[c + d*x]))/(35*d*\operatorname{Cot}[c + d*x]^{(5/2)})$

**Rule 214**

$\operatorname{Int}[(a + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2])/a]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

**Rule 3610**

$\operatorname{Int}[(a + (b_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)])^{(m_*)}*((c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)])], x\_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^{(m+1)})/(f*(m+1)*(a^2 + b^2)), x] + \operatorname{Dist}[1/(a^2 + b^2), \operatorname{Int}[(a + b*\operatorname{Tan}[e + f*x])^{(m+1)}*\operatorname{Simp}[a*c + b*d - (b*c - a*d)*\operatorname{Tan}[e + f*x], x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{LtQ}[m, -1]$

**Rule 3614**

$\operatorname{Int}[(c + (d_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)])/ \operatorname{Sqrt}[(b_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]]], x\_Symbol] \rightarrow \operatorname{Dist}[2*(c^2/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c - d*x^2), x], x], \operatorname{Sqrt}[b*$

$\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2 + d^2, 0]$

### Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.))^{\text{p}_.}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}, x\_Symbol] \text{:>} \text{Dist}[g^{\text{m} + \text{n}}, \text{Int}[(g*\text{Cot}[e + f*x])^{\text{p} - \text{m} - \text{n}}*(b + a*\text{Cot}[e + f*x])^{\text{m}}*(d + c*\text{Cot}[e + f*x])^{\text{n}}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ !\text{IntegerQ}[\text{p}] \ \&\& \ \text{IntegerQ}[\text{m}] \ \&\& \ \text{IntegerQ}[\text{n}]$

### Rule 3672

$\text{Int}[(\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}((\text{A}_. + (\text{B}_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \text{:>} \text{Simp}[(b*c - a*d)*(A*b - a*B)*(\text{a} + b*\text{Tan}[e + f*x])^{\text{m} + 1}/(b*f*(\text{m} + 1)*(a^2 + b^2))], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(\text{a} + b*\text{Tan}[e + f*x])^{\text{m} + 1}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[\text{m}, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

### Rule 3674

$\text{Int}[(\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}((\text{A}_. + (\text{B}_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}, x\_Symbol] \text{:>} \text{Simp}[(-a^2)*(B*c - A*d)*(\text{a} + b*\text{Tan}[e + f*x])^{\text{m} - 1}*((c + d*\text{Tan}[e + f*x])^{\text{n} + 1}/(d*f*(b*c + a*d)*(n + 1))), x] - \text{Dist}[a/(d*(b*c + a*d)*(n + 1)), \text{Int}[(\text{a} + b*\text{Tan}[e + f*x])^{\text{m} - 1}*(c + d*\text{Tan}[e + f*x])^{\text{n} + 1}*\text{Simp}[A*b*d*(\text{m} - \text{n} - 2) - B*(b*c*(\text{m} - 1) + a*d*(\text{n} + 1)) + (a*A*d*(\text{m} + \text{n}) - B*(a*c*(\text{m} - 1) + b*d*(\text{n} + 1)))*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[\text{m}, 1] \ \&\& \ \text{LtQ}[\text{n}, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \int \frac{(ia + a \cot(c + dx))^3 (B + A \cot(c + dx))}{\cot^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{2iaB(ia + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{(ia + a \cot(c + dx))^2}{\cot^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2iaB(ia + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} - \frac{2(7A - 11iB)(ia^3 + a^3 \cot(c + dx))}{35d \cot^{\frac{5}{2}}(c + dx)} \\
&= -\frac{8a^3(21A - 23iB)}{105d \cot^{\frac{3}{2}}(c + dx)} + \frac{2iaB(ia + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} - \frac{2(7A - 11iB)(ia^3 + a^3 \cot(c + dx))}{35d \cot^{\frac{5}{2}}(c + dx)} \\
&= -\frac{8a^3(21A - 23iB)}{105d \cot^{\frac{3}{2}}(c + dx)} + \frac{8a^3(iA + B)}{d \sqrt{\cot(c + dx)}} + \frac{2iaB(ia + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} \\
&= -\frac{8a^3(21A - 23iB)}{105d \cot^{\frac{3}{2}}(c + dx)} + \frac{8a^3(iA + B)}{d \sqrt{\cot(c + dx)}} + \frac{2iaB(ia + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} \\
&= \frac{8\sqrt[4]{-1} a^3 (A - iB) \tanh^{-1}\left(\frac{(-1)^{3/4} \sqrt{\cot(c + dx)}}{1}\right)}{d} - \frac{8a^3(21A - 23iB)}{105d \cot^{\frac{3}{2}}(c + dx)} + \frac{8a^3(iA + B)}{d \sqrt{\cot(c + dx)}} + \frac{2iaB(ia + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)}
\end{aligned}$$

**Mathematica [A]**

time = 5.25, size = 298, normalized size = 1.72

$$\frac{a^3(i + \cot(c + dx))^3(B + A \cot(c + dx)) \left( -8(A - iB)e^{-3ic} \sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{\frac{i(1 + e^{2i(c+dx)})}{-1 + e^{2i(c+dx)}}} \tanh^{-1}\left(\sqrt{\frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}}\right) + \frac{(10(21A + 25B) + 21(A + 31B) \cos(2(c + dx))) + 21(59A - 57iB) \cot(c + dx) + 21(21A - 23iB) \cos(3(c + dx)) \cos(c + dx) \sec^2(c + dx) (i \cos(3c) + \sin(3c))}{210 \cot^2(c + dx)} \right) \sin^4(c + dx)}{d(\cos(dx) + i \sin(dx))^3 (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (a^3*(I + Cot[c + d*x])^3*(B + A*Cot[c + d*x])*((-8*(A - I*B)*Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((2*I)*(c + d*x)))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])/E^((3*I)*c) + ((10*((21*I)*A + 25*B) + ((21*I)*A + 31*B)*Cos[2*(c + d*x)]) + 21*(59*A - (57*I)*B)*Cot[c + d*x] + 21*(21*A - (23*I)*B)*Cos[3*(c + d*x)]*Csc[c + d*x])*Sec[c + d*x]^2*(I*Cos[3*c] + Sin[3*c]))/(210*Cot[c + d*x]^(3/2))*Sin[c + d*x]^4/(d*(Cos[d*x] + I*Sin[d*x])^3*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 62.24, size = 1043, normalized size = 6.03

method	result	size
--------	--------	------

default	Expression too large to display	1043
---------	---------------------------------	------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{105}a^3/d^2^{(1/2)}*(-1+\cos(d*x+c))*(-21*I*A^2^{(1/2)}*\cos(d*x+c)^2+15*I*B*\sin(d*x+c)*2^{(1/2)}+21*I*A*\cos(d*x+c)*2^{(1/2)}+420*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+420*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-420*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3*\text{EllipticF}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})-441*I*A^2^{(1/2)}*\cos(d*x+c)^3+441*I*A^2^{(1/2)}*\cos(d*x+c)^4-105*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^3+420*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+483*B^2^{(1/2)}*\cos(d*x+c)^4-155*I*B*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^2+105*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^2-420*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3*\text{EllipticF}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})-483*B^2^{(1/2)}*\cos(d*x+c)^3-420*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+155*I*B*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^3-63*B^2^{(1/2)}*\cos(d*x+c)^2-15*I*B*\cos(d*x+c)*\sin(d*x+c)*2^{(1/2)}+63*B^2^{(1/2)}*\cos(d*x+c)*(\cos(d*x+c)+1)^2/\cos(d*x+c)^3/\sin(d*x+c)^4/(\cos(d*x+c)/\sin(d*x+c))^{(1/2)}$$

**Maxima [A]**

time = 0.52, size = 221, normalized size = 1.28

$$\frac{2(15Ba^3 - \frac{15A^2d^2a^3}{\cos^2(c)} + \frac{15A^2d^2a^3}{\sin^2(c)} - \frac{15A^2d^2a^3}{\tan^2(c)}) \tan(dcx+c)^2 - 105(2\sqrt{2}((1-1)A+(1+1)B) \arctan(\frac{1}{\sqrt{2}}(\sqrt{2} + \frac{d}{\sqrt{\tan(dx+c)}})) + 2\sqrt{2}((1-1)A+(1+1)B) \arctan(\frac{1}{\sqrt{2}}(\sqrt{2} - \frac{d}{\sqrt{\tan(dx+c)}})) + \sqrt{2}(-(1+1)A+(1-1)B) \log(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)}) - \sqrt{2}(-(1+1)A+(1-1)B) \log(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)}))}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] 
$$-1/105*(2*(15*I*B*a^3 - 21*(-I*A - 3*B)*a^3/\tan(d*x + c) + 35*(3*A - 4*I*B)*a^3/\tan(d*x + c)^2 - 420*(I*A + B)*a^3/\tan(d*x + c)^3)*\tan(d*x + c)^{(7/2)}$$

- 105\*(2\*sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c)))) + 2\*sqrt(2)\*((I - 1)\*A + (I + 1)\*B)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(d\*x + c)))) + sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*log(sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) - sqrt(2)\*(-(I + 1)\*A + (I - 1)\*B)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1))\*a^3/d

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(139) = 278.  
time = 1.85, size = 563, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] -2/105\*(105\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2))\*a^6/d^2)\*(d\*e^(8\*I\*d\*x + 8\*I\*c) + 4\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 4\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(2\*((A - I\*B)\*a^3\*e^(2\*I\*d\*x + 2\*I\*c) - sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2))\*a^6/d^2)\*(I\*d\*e^(2\*I\*d\*x + 2\*I\*c) - I\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-2\*I\*d\*x - 2\*I\*c)/((-I\*A - B)\*a^3)) - 105\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2))\*a^6/d^2)\*(d\*e^(8\*I\*d\*x + 8\*I\*c) + 4\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 4\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(2\*((A - I\*B)\*a^3\*e^(2\*I\*d\*x + 2\*I\*c) - sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2))\*a^6/d^2)\*(-I\*d\*e^(2\*I\*d\*x + 2\*I\*c) + I\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-2\*I\*d\*x - 2\*I\*c)/((-I\*A - B)\*a^3)) - 2\*((273\*A - 319\*I\*B)\*a^3\*e^(8\*I\*d\*x + 8\*I\*c) + 3\*(133\*A - 109\*I\*B)\*a^3\*e^(6\*I\*d\*x + 6\*I\*c) - 5\*(21\*A - 19\*I\*B)\*a^3\*e^(4\*I\*d\*x + 4\*I\*c) - 3\*(133\*A - 129\*I\*B)\*a^3\*e^(2\*I\*d\*x + 2\*I\*c) - 4\*(42\*A - 41\*I\*B)\*a^3)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))/(d\*e^(8\*I\*d\*x + 8\*I\*c) + 4\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 6\*d\*e^(4\*I\*d\*x + 4\*I\*c) + 4\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-ia^3 \left( \int \frac{iA}{\sqrt{\cot(c+dx)}} dx + \int \left( \frac{3A \tan(c+dx)}{\sqrt{\cot(c+dx)}} \right) dx + \int \frac{A \tan^3(c+dx)}{\sqrt{\cot(c+dx)}} dx + \int \left( \frac{-3B \tan^2(c+dx)}{\sqrt{\cot(c+dx)}} \right) dx + \int \frac{B \tan^4(c+dx)}{\sqrt{\cot(c+dx)}} dx + \int \left( \frac{-3iA \tan^2(c+dx)}{\sqrt{\cot(c+dx)}} \right) dx + \int \frac{iB \tan(c+dx)}{\sqrt{\cot(c+dx)}} dx + \int \left( \frac{-3iB \tan^3(c+dx)}{\sqrt{\cot(c+dx)}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] -I\*a\*\*3\*(Integral(I\*A/sqrt(cot(c + d\*x)), x) + Integral(-3\*A\*tan(c + d\*x)/sqrt(cot(c + d\*x)), x) + Integral(A\*tan(c + d\*x)\*\*3/sqrt(cot(c + d\*x)), x) + Integral(-3\*B\*tan(c + d\*x)\*\*2/sqrt(cot(c + d\*x)), x) + Integral(B\*tan(c + d\*x)\*\*4/sqrt(cot(c + d\*x)), x) + Integral(-3\*I\*A\*tan(c + d\*x)\*\*2/sqrt(cot(c + d\*x)), x) + Integral(I\*B\*tan(c + d\*x)/sqrt(cot(c + d\*x)), x) + Integral(-3\*I\*B\*tan(c + d\*x)\*\*3/sqrt(cot(c + d\*x)), x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^3/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^3}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^3)/cot(c + d\*x)^(1/2), x)

$$3.519 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=297

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) \left((6+i)A + (1+4i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left((7-5i)A + (5+3i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{4\sqrt{2} ad}$$

[Out]  $-1/6*(7*A+3*I*B)*\cot(d*x+c)^{(3/2)}/a/d+1/2*(A+I*B)*\cot(d*x+c)^{(5/2)}/d/(I*a+a*\cot(d*x+c))+1/8-1/8*I*((6+I)*A+(1+4*I)*B)*\arctan(-1+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a/d*2^{(1/2)}+1/8*((7-5*I)*A+(5+3*I)*B)*\arctan(1+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a/d*2^{(1/2)}+1/16*((7+5*I)*A+(-5+3*I)*B)*\ln(1+\cot(d*x+c)-2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a/d*2^{(1/2)}+1/16*((-7-5*I)*A+(5-3*I)*B)*\ln(1+\cot(d*x+c)+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a/d*2^{(1/2)}+5/2*(I*A-B)*\cot(d*x+c)^{(1/2)}/a/d$

**Rubi [A]**

time = 0.35, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) \left((6+i)A + (1+4i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} + \frac{\left((7-5i)A + (5+3i)B\right) \text{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{4\sqrt{2} ad} + \frac{(A+iB)\cot^3(c+dx)}{24a\cot(c+dx)+16i} - \frac{(7A+3iB)\cot^3(c+dx)}{6ad} + \frac{5(-B+iA)\sqrt{\cot(c+dx)}}{2ad} + \frac{((7+5i)A - (5-3i)B)\log(\cot(c+dx) - \sqrt{2}\sqrt{\cot(c+dx)} + 1)}{8\sqrt{2} ad} + \frac{((5-3i)B - (7+5i)A)\log(\cot(c+dx) + \sqrt{2}\sqrt{\cot(c+dx)} + 1)}{8\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x]^{(5/2)}*(A + B*\text{Tan}[c + d*x]))/(a + I*a*\text{Tan}[c + d*x]),x]$

[Out]  $((-1/4 + I/4)*((6 + I)*A + (1 + 4*I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]) / (\text{Sqrt}[2]*a*d) + (((7 - 5*I)*A + (5 + 3*I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]) / (4*\text{Sqrt}[2]*a*d) + (5*(I*A - B)*\text{Sqrt}[\text{Cot}[c + d*x]]) / (2*a*d) - ((7*A + (3*I)*B)*\text{Cot}[c + d*x]^{(3/2)}) / (6*a*d) + ((A + I*B)*\text{Cot}[c + d*x]^{(5/2)}) / (2*d*(I*a + a*\text{Cot}[c + d*x])) + (((7 + 5*I)*A - (5 - 3*I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]) / (8*\text{Sqrt}[2]*a*d) + (((-7 - 5*I)*A + (5 - 3*I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]) / (8*\text{Sqrt}[2]*a*d)$

Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 631

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] := \text{With}\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{Free}$



$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

Rule 3609

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(m_.))*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Simp[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^{\frac{5}{2}}(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx &= \int \frac{\cot^{\frac{5}{2}}(c + dx)(B + A \cot(c + dx))}{ia + a \cot(c + dx)} dx \\
 &= \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} + \frac{\int \cot^{\frac{3}{2}}(c + dx) \left(-\frac{5}{2}a(iA - B) + \frac{1}{2}a(7A + 3iB)\right) dx}{2a^2} \\
 &= -\frac{(7A + 3iB) \cot^{\frac{3}{2}}(c + dx)}{6ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} + \frac{\int \sqrt{\cot(c + dx)} dx}{2d(ia + a \cot(c + dx))} \\
 &= \frac{5(iA - B) \sqrt{\cot(c + dx)}}{2ad} - \frac{(7A + 3iB) \cot^{\frac{3}{2}}(c + dx)}{6ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} \\
 &= \frac{5(iA - B) \sqrt{\cot(c + dx)}}{2ad} - \frac{(7A + 3iB) \cot^{\frac{3}{2}}(c + dx)}{6ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} \\
 &= \frac{5(iA - B) \sqrt{\cot(c + dx)}}{2ad} - \frac{(7A + 3iB) \cot^{\frac{3}{2}}(c + dx)}{6ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} \\
 &= \frac{5(iA - B) \sqrt{\cot(c + dx)}}{2ad} - \frac{(7A + 3iB) \cot^{\frac{3}{2}}(c + dx)}{6ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} \\
 &= \frac{5(iA - B) \sqrt{\cot(c + dx)}}{2ad} - \frac{(7A + 3iB) \cot^{\frac{3}{2}}(c + dx)}{6ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))} \\
 &= -\frac{((7 - 5i)A + (5 + 3i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{4\sqrt{2} ad} + \frac{(A + iB) \cot^{\frac{5}{2}}(c + dx)}{2d(ia + a \cot(c + dx))}
 \end{aligned}$$

**Mathematica [A]**

time = 1.49, size = 247, normalized size = 0.83

$$\frac{(\cos(dx) + i \sin(dx)) \left( (1-i) \sec(c+dx) \left( (6+i)A + (1+4i)B \right) \operatorname{ArcSin}(\cos(c+dx) - \sin(c+dx)) + ((-1-6i)A + (4+i)B) \log \left( \frac{\cos(c+dx) + \sin(c+dx) + \sqrt{\sin(2(c+dx))}}{\cos(c) + i \sin(c)} \right) + \frac{1}{2} \cot(c+dx) \sec(c+dx) (\cos(dx) - i \sin(dx)) (-19A - 15iB + (11A + 15iB) \cos(2(c+dx)) + (8A - 12iB) \sin(2(c+dx))) \right) (A + B \tan(c+dx))}{8i \sqrt{\cos(c+dx)} (A \cos(c+dx) + B \sin(c+dx)) (a + i \tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),
x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*((1 - I)*Csc[c + d*x]*(((6 + I)*A + (1 + 4*I)*B)*A
rcSin[Cos[c + d*x] - Sin[c + d*x]] + ((-1 - 6*I)*A + (4 + I)*B)*Log[Cos[c +
d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*(Cos[c] + I*Sin[c])*Sqrt[Si
n[2*(c + d*x)]) + (2*Cot[c + d*x]*Csc[c + d*x]*(Cos[d*x] - I*Sin[d*x])*(-19
*A - (15*I)*B + (11*A + (15*I)*B)*Cos[2*(c + d*x)] + ((8*I)*A - 12*B)*Sin[2
*(c + d*x)]))/3*(A + B*Tan[c + d*x]))/(8*d*Sqrt[Cot[c + d*x]]*(A*Cos[c + d
*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 25.80, size = 2598, normalized size = 8.75

method	result	size
default	Expression too large to display	2598

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/12/a/d*2^(1/2)*(3*I*A*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c
)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)*sin(d*x+c)-18*I*A*cos(d*x+c)*sin(d*x+c)*((cos(d*x+c)-1+sin(
d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cos(d*
x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+3*I*B*cos(d*x+c)*sin(d*x+c)*((cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-12*I*B*cos(d*x+c)*sin(d*x+c)*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+9*I*B*cos(d*x+c)*si
n(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2
^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-3*A*EllipticPi((-cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(
```

$$\begin{aligned}
& d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*\cos(d*x+c)*\sin(d*x+c)+3*B*EllipticPi \\
& ((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2+1/2*I,1/2*2^{\wedge}(1/2))*((-1+ \\
& \cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)* \\
& (-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*\cos(d*x+c)*\sin(d*x+c)+3*I*B*c \\
& \cos(d*x+c)^{\wedge}4*2^{\wedge}(1/2)+3*B*\cos(d*x+c)^{\wedge}3*\sin(d*x+c)*2^{\wedge}(1/2)-3*I*B*\cos(d*x+c)^{\wedge}2* \\
& 2^{\wedge}(1/2)+3*A*\cos(d*x+c)^{\wedge}4*2^{\wedge}(1/2)+21*A*\sin(d*x+c)*(-\cos(d*x+c)-1-\sin(d*x+c) \\
& )/\sin(d*x+c))^{\wedge}(1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d \\
& *x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge} \\
& (1/2),1/2*2^{\wedge}(1/2))-3*A*\sin(d*x+c)*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge} \\
& (1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+ \\
& c))^{\wedge}(1/2)*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2+1/2* \\
& I,1/2*2^{\wedge}(1/2))+3*B*\sin(d*x+c)*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2) \\
& *((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge} \\
& (1/2)*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2+1/2*I,1/ \\
& 2*2^{\wedge}(1/2))+12*B*\sin(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((- \\
& 1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin( \\
& d*x+c))^{\wedge}(1/2),1/2-1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c) \\
& )^{\wedge}(1/2)-18*A*\sin(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+c \\
& \cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x \\
& +c))^{\wedge}(1/2),1/2-1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge} \\
& (1/2)-15*B*2^{\wedge}(1/2)*\cos(d*x+c)*\sin(d*x+c)+15*I*A*\cos(d*x+c)*\sin(d*x+c)*2^{\wedge}(1/2) \\
& )-3*I*A*\cos(d*x+c)^{\wedge}3*\sin(d*x+c)*2^{\wedge}(1/2)+3*I*A*\sin(d*x+c)*((\cos(d*x+c)-1+\sin \\
& (d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi(( \\
& -\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2+1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d \\
& *x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)-18*I*A*\sin(d*x+c)*((\cos(d*x+c)-1+\sin( \\
& d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi(( \\
& -\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2-1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d \\
& x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)+3*I*B*\sin(d*x+c)*((\cos(d*x+c)-1+\sin(d \\
& x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi(( \\
& -\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2+1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d*x+ \\
& c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)-12*I*B*\sin(d*x+c)*((\cos(d*x+c)-1+\sin(d*x \\
& +c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi(( \\
& -\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2-1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d*x+c) \\
& )-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)+9*I*B*\sin(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c) \\
& ))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticF((-\cos(d \\
& x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2*2^{\wedge}(1/2))*(-\cos(d*x+c)-1-\sin(d*x+c) \\
& ))/\sin(d*x+c))^{\wedge}(1/2)-18*A*\cos(d*x+c)*\sin(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/ \\
& \sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi((-\cos(d*x+ \\
& c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2-1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d*x+c)-1-s \\
& in(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)+12*B*\cos(d*x+c)*\sin(d*x+c)*((\cos(d*x+c)-1+\sin( \\
& d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticPi(( \\
& -\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2),1/2-1/2*I,1/2*2^{\wedge}(1/2))*(-\cos(d \\
& x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)-7*A*2^{\wedge}(1/2)*\cos(d*x+c)^{\wedge}2+21*A*(-\cos(d \\
& *x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c) \\
& )^{\wedge}(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c))^{\wedge}(1/2)*EllipticF((-\cos(d*x+c)-1-\sin(d*
\end{aligned}$$

$x+c)/\sin(d*x+c))^{(1/2)}, 1/2*2^{(1/2)})*\cos(d*x+c)*\sin(d*x+c))*\sin(d*x+c)*(\cos(d*x+c)/\sin(d*x+c))^{(5/2)}/\cos(d*x+c)^3$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(222) = 444.

time = 1.67, size = 716, normalized size = 2.41



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/24*(3*(a*d*e^{(4*I*d*x + 4*I*c)} - a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)}*\log(-2*((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 3*(a*d*e^{(4*I*d*x + 4*I*c)} - a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)}*\log(2*((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)} - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 6*(a*d*e^{(4*I*d*x + 4*I*c)} - a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2)}*\log(((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2)} + 3*A + 2*I*B)*e^{(-2*I*d*x - 2*I*c)/(a*d)} + 6*(a*d*e^{(4*I*d*x + 4*I*c)} - a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2)}*\log(-((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2)} - 3*A - 2*I*B)*e^{(-2*I*d*x - 2*I*c)/(a*d)} - 2*((19*I*A - 27*B)*e^{(4*I*d*x + 4*I*c)} - 2*(19*I*A - 15*B)*e^{(2*I*d*x + 2*I*c)} + 3*I*A - 3*B)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))/(a*d*e^{(4*I*d*x + 4*I*c)} - a*d*e^{(2*I*d*x + 2*I*c)}) \end{aligned}$$

**Sympy [F(-1)]** Timed out  
 time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**  
 time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(5/2)/(I\*a\*tan(d\*x + c) + a), x)

**Mupad [F]**  
 time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{5/2} (A + B \tan(c + dx))}{a + a \tan(c + dx) \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] int((cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i), x)

$$3.520 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=268

$$\frac{((-5-3i)A+(3-i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad} + \frac{((5+3i)A-(3-i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad}$$

[Out] 1/2\*(A+I\*B)\*cot(d\*x+c)^(3/2)/d/(I\*a+a\*cot(d\*x+c))-1/8\*((-5-3\*I)\*A+(3-I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)+1/8\*((5+3\*I)\*A+(-3+I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)+(-1/16+1/16\*I)\*((4+I)\*A+(1+2\*I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)+1/16\*((5-3\*I)\*A+(3+I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)-1/2\*(5\*A+I\*B)\*cot(d\*x+c)^(1/2)/a/d

**Rubi** [A]

time = 0.30, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(3-i)B-(5+3i)A\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad} + \frac{(5+3i)A-(3-i)B\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{4\sqrt{2}ad} + \frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2i(a\cot(c+dx)+ia)} - \frac{(5A+iB)\sqrt{\cot(c+dx)}}{2ad} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left((4+i)A+(1+2i)B\right)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}ad} + \frac{(5-3i)A+(3+i)B\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{8\sqrt{2}ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x]),x]

[Out] (((-5-3\*I)\*A+(3-I)\*B)\*ArcTan[1-Sqrt[2]\*Sqrt[Cot[c+d\*x]]]/(4\*Sqrt[2]\*a\*d) + (((5+3\*I)\*A-(3-I)\*B)\*ArcTan[1+Sqrt[2]\*Sqrt[Cot[c+d\*x]]]/(4\*Sqrt[2]\*a\*d) - ((5\*A+I\*B)\*Sqrt[Cot[c+d\*x]]/(2\*a\*d) + ((A+I\*B)\*Cot[c+d\*x]^(3/2))/(2\*d\*(I\*a+a\*Cot[c+d\*x])) - ((1/8-I/8)\*((4+I)\*A+(1+2\*I)\*B)\*Log[1-Sqrt[2]\*Sqrt[Cot[c+d\*x]]+Cot[c+d\*x]]/(Sqrt[2]\*a\*d) + (((5-3\*I)\*A+(3+I)\*B)\*Log[1+Sqrt[2]\*Sqrt[Cot[c+d\*x]]+Cot[c+d\*x]]/(8\*Sqrt[2]\*a\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist
```



```
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx &= \int \frac{\cot^{\frac{3}{2}}(c+dx)(B+A \cot(c+dx))}{ia+a \cot(c+dx)} dx \\
&= \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} + \frac{\int \sqrt{\cot(c+dx)} \left(-\frac{3}{2}a(iA-B) + \frac{1}{2}a(5A+3iB)\right)}{2a^2} dx \\
&= -\frac{(5A+iB) \sqrt{\cot(c+dx)}}{2ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} + \frac{\int -\frac{1}{2}a(5A+3iB) \sqrt{\cot(c+dx)} dx}{2a^2} \\
&= -\frac{(5A+iB) \sqrt{\cot(c+dx)}}{2ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} + \frac{\text{Subst}\left(\int -\frac{1}{2}a(5A+3iB) \sqrt{u} du\right)}{2a^2} \\
&= -\frac{(5A+iB) \sqrt{\cot(c+dx)}}{2ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} + \frac{((5+3i)A - (3-i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{4\sqrt{2} ad} \\
&= -\frac{(5A+iB) \sqrt{\cot(c+dx)}}{2ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} + \frac{((5+3i)A - (3-i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{4\sqrt{2} ad} \\
&= -\frac{(5A+iB) \sqrt{\cot(c+dx)}}{2ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} - \frac{((5-3i)A - (3+i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{4\sqrt{2} ad} \\
&= -\frac{(5A+iB) \sqrt{\cot(c+dx)}}{2ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{2d(ia+a \cot(c+dx))} + \frac{((5+3i)A - (3-i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{4\sqrt{2} ad} + \frac{((5-3i)A - (3+i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{4\sqrt{2} ad}
\end{aligned}$$

### Mathematica [A]

time = 1.11, size = 223, normalized size = 0.83

$$\frac{(\cos(dx) + i \sin(dx)) \left( \cot(c+dx) (-4 \cos(dx) + 4i \sin(dx)) (4A \cos(c+dx) + (5iA - B) \sin(c+dx)) + \csc(c+dx) \left( (3-5i)A + (1+3i)B \right) \text{ArcSin}(\cos(c+dx) - \sin(c+dx)) - (1+i) \left( (4+i)A + (1+2i)B \right) \log(\cos(c+dx) + \sin(c+dx) + \sqrt{\sin(2(c+dx))}) \right) (i \cos(c) - \sin(c)) \sqrt{\sin(2(c+dx))}}{8d \sqrt{\cot(c+dx)} (A \cos(c+dx) + B \sin(c+dx)) (a + ia \tan(c+dx))} (A + B \tan(c+dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),
x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*(Cot[c + d*x]*(-4*Cos[d*x] + (4*I)*Sin[d*x])*(4*A*
Cos[c + d*x] + ((5*I)*A - B)*Sin[c + d*x]) + Csc[c + d*x]*(((3 - 5*I)*A + (
1 + 3*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] - (1 + I)*((4 + I)*A + (1 +
2*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*(I*Cos[
c] - Sin[c])*Sqrt[Sin[2*(c + d*x)]]*(A + B*Tan[c + d*x]))/(8*d*Sqrt[Cot[c
+ d*x]]*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 24.35, size = 2431, normalized size = 9.07

method	result	size
default	Expression too large to display	2431

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4/a/d*2^(1/2)*(4*A*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2
)+2*B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)-I*A*cos(d*x+c
)^2*sin(d*x+c)*2^(1/2)+B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-4*I*A*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)+3*I*A*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c)^(1/2)-I*B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(
1/2)+A*cos(d*x+c)^3*2^(1/2)-5*A*cos(d*x+c)*2^(1/2)+A*((cos(d*x+c)-1+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c)^(1/2)-3*B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*
x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2)+B*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)+I*B*cos(d
```

$$\begin{aligned}
& *x+c)^3*2^{(1/2)}-I*B*\cos(d*x+c)*2^{(1/2)}+B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-3*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*B*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+4*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+I*A*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+2*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+I*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-4*I*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-I*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(\cos(d*x+c)/\sin(d*x+c))^{(3/2)}*\sin(d*x+c)/\cos(d*x+c)^2
\end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a), x
)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{a + a \tan(c + dx) \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i), x)
```

$$3.521 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{a+ia \tan(c+dx)} dx$$

**Optimal.** Leaf size=235

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2+i)A+B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2+i)A+B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad}$$

[Out]  $(-1/8+1/8*I)*((2+I)*A+B)*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(-1/8+1/8*I)*((2+I)*A+B)*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}-1/16*((3+I)*A-(1+I)*B)*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+1/16*((3+I)*A-(1+I)*B)*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+1/2*(A+I*B)*\cot(d*x+c)^{(1/2)}/d/(I*a+a*\cot(d*x+c))$

**Rubi [A]**

time = 0.24, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3662, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (B + (2+i)A) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (B + (2+i)A) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} ad} + \frac{(A+iB) \sqrt{\cot(c+dx)}}{2d(a \cot(c+dx) + ia)} - \frac{(3+i)A - (1+i)B \log\left(\frac{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}{8\sqrt{2} ad}\right)}{8\sqrt{2} ad} + \frac{(3+i)A - (1+i)B \log\left(\frac{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}{8\sqrt{2} ad}\right)}{8\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/(a + I*a*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((1/4 - I/4)*((2 + I)*A + B)*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]/(\operatorname{Sqrt}[2]*a*d) - ((1/4 - I/4)*((2 + I)*A + B)*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]/(\operatorname{Sqrt}[2]*a*d) + ((A + I*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(2*d*(I*a + a*\operatorname{Cot}[c + d*x])) - (((3 + I)*A - (1 + I)*B)*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(8*\operatorname{Sqrt}[2]*a*d) + (((3 + I)*A - (1 + I)*B)*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(8*\operatorname{Sqrt}[2]*a*d)$

Rule 210

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 631

$\operatorname{Int}[(a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)^{-1}, x\_Symbol] := \operatorname{With}\{q = 1 - 4*S\operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \operatorname{RationalQ}[q] \&\& (\operatorname{EqQ}[q^2, 1] \parallel \operatorname{!RationalQ}[b^2 - 4*a*c]) /; \operatorname{FreeQ}\{a, b, c\}, x \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 3676

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
```

```
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] & & NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx = \int \frac{\sqrt{\cot(c + dx)} (B + A \cot(c + dx))}{ia + a \cot(c + dx)} dx$$

$$= \frac{(A + iB) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\int \frac{-\frac{1}{2}a(iA-B) + \frac{1}{2}a(3A-iB) \cot(c+dx)}{\sqrt{\cot(c + dx)}} dx}{2a^2}$$

$$= \frac{(A + iB) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(iA-B) - \frac{1}{2}a(3A-iB)x^2}{1+x^4} dx, x, a^2d\right)}{a^2d}$$

$$= \frac{(A + iB) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{((3 + i)A - (1 + i)B) \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, 4ad\right)}{4ad}$$

$$= \frac{(A + iB) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} - \frac{((3 + i)A - (1 + i)B) \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{-1-x^2} dx, x, 8\sqrt{2} ad\right)}{8\sqrt{2} ad}$$

$$= \frac{(A + iB) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} - \frac{((3 + i)A - (1 + i)B) \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{8\sqrt{2} ad}$$

$$= \frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2 + i)A + B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) ((2 + i)A + B) \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{8\sqrt{2} ad}$$

**Mathematica [A]**

time = 0.78, size = 199, normalized size = 0.85

$$\frac{(\cos(dx) + i \sin(dx)) \left(4(A + iB) \cos(c + dx) (\cos(dx) - i \sin(dx)) + (1 + i) \csc(c + dx) \left( ((2 + i)A + B) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) + (-1 - 2i)A + iB \right) \log\left(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}\right)\right)}{8d \sqrt{\cot(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) (a + ia \tan(c + dx))} (A + B \tan(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]), x]
```

```
[Out] ((Cos[d*x] + I*Sin[d*x])*4*(A + I*B)*Cos[c + d*x]*(Cos[d*x] - I*Sin[d*x]) + (1 + I)*Csc[c + d*x]*(((2 + I)*A + B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + ((-1 - 2*I)*A + I*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*(I*Cos[c] - Sin[c])*Sqrt[Sin[2*(c + d*x)]]*(A + B*Tan[c + d*x]))/(
```



$8*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 21.01, size = 1139, normalized size = 4.85

method	result	size
default	Expression too large to display	1139

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out]  $\frac{1}{4} \frac{a}{d} 2^{\frac{1}{2}} * (-I * B * \cos(d*x+c)^2 * 2^{\frac{1}{2}} + I * B * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \sin(d*x+c) * \text{EllipticPi}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 + 1/2 * I, 1/2 * 2^{\frac{1}{2}}) + I * B * \cos(d*x+c)^3 * 2^{\frac{1}{2}} + I * A * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \sin(d*x+c) * \text{EllipticPi}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 + 1/2 * I, 1/2 * 2^{\frac{1}{2}}) - I * A * \cos(d*x+c)^2 * \sin(d*x+c) * 2^{\frac{1}{2}} + 3 * A * \sin(d*x+c) * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \text{EllipticF}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 * 2^{\frac{1}{2}}) - 2 * A * \sin(d*x+c) * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \text{EllipticPi}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 - 1/2 * I, 1/2 * 2^{\frac{1}{2}}) * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} - A * \sin(d*x+c) * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \text{EllipticPi}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 + 1/2 * I, 1/2 * 2^{\frac{1}{2}}) - 2 * I * A * \sin(d*x+c) * \text{EllipticPi}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 - 1/2 * I, 1/2 * 2^{\frac{1}{2}}) * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} + B * \sin(d*x+c) * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \text{EllipticPi}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 + 1/2 * I, 1/2 * 2^{\frac{1}{2}}) + A * \cos(d*x+c)^3 * 2^{\frac{1}{2}} - I * B * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * ((\cos(d*x+c) - 1 + \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * (-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}} * \sin(d*x+c) * \text{EllipticF}((-\cos(d*x+c) - 1 - \sin(d*x+c)) / \sin(d*x+c))^{\frac{1}{2}}, 1/2 * 2^{\frac{1}{2}}) + B * \cos(d*x+c)^2 * \sin(d*x+c) * 2^{\frac{1}{2}} + I * A * \cos(d*x+c) * \sin(d*x+c) * 2^{\frac{1}{2}} - A * 2^{\frac{1}{2}} * \cos(d*x+c)^2 - B * 2^{\frac{1}{2}} * \cos(d*x+c) * \sin(d*x+c)) * (\cos(d*x+c) + 1)^2 * (-1 + \cos(d*x+c)) * (\cos(d*x+c) / \sin(d*x+c))^{\frac{1}{2}} / \cos(d*x+c) / \sin(d*x+c)^3$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs.  $2(178) = 356$ .  
time = 2.37, size = 570, normalized size = 2.43

$$\frac{\left( \frac{A \sqrt{\cot(c+dx)}}{\tan(c+dx)-i} dx + \int \frac{B \tan(c+dx) \sqrt{\cot(c+dx)}}{\tan(c+dx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/8*(a*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-
2*((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) + (A - I*B)*e^
(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((-I*A^2 - 2*A
*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(2*((a*d*e^(2*I*d*x + 2*I*c)
- a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I
*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d
*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*A^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*lo
g(-((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) - 1))*sqrt(I*A^2/(a^2*d^2)) + A)*e^(-2*I*d*x - 2*I*c)/(a*d))
+ 2*a*d*sqrt(I*A^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2
*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sq
rt(I*A^2/(a^2*d^2)) - A)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2*((-I*A + B)*e^(2*I
*d*x + 2*I*c) + I*A - B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1)))e^(-2*I*d*x - 2*I*c)/(a*d)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \sqrt{\cot(c+dx)}}{\tan(c+dx)-i} dx + \int \frac{B \tan(c+dx) \sqrt{\cot(c+dx)}}{\tan(c+dx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*(Integral(A*sqrt(cot(c + d*x))/(tan(c + d*x) - I), x) + Integral(B*tan(c
+ d*x)*sqrt(cot(c + d*x))/(tan(c + d*x) - I), x))/a
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:  
INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{a+a \tan(c+dx) \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i),x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i), x)

$$3.522 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))} dx$$

**Optimal.** Leaf size=237

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad}$$

[Out]  $(-1/8+1/8*I)*(A+(2-I)*B)*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(-1/8+1/8*I)*(A+(2-I)*B)*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(1/16+1/16*I)*(A-(2+I)*B)*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}-(1/16+1/16*I)*(A-(2+I)*B)*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+1/2*(I*A-B)*\cot(d*x+c)^{(1/2)}/d/(I*a+a*\cot(d*x+c))$

**Rubi [A]**

time = 0.25, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3662, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \text{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} ad} + \frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - (2 + i)B) \log\left(\frac{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}{\sqrt{2} ad}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - (2 + i)B) \log\left(\frac{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}{\sqrt{2} ad}\right)}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])),x]

[Out]  $((1/4 - I/4)*(A + (2 - I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a*d) - ((1/4 - I/4)*(A + (2 - I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a*d) + ((I*A - B)*\text{Sqrt}[\text{Cot}[c + d*x]])/(2*d*(I*a + a*\text{Cot}[c + d*x])) + ((1/8 + I/8)*(A - (2 + I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a*d) - ((1/8 + I/8)*(A - (2 + I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a*d)$

Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1))/(2*f*m*(
```

$b*c - a*d$ )),  $x$ ] + Dist[ $1/(2*a*m*(b*c - a*d)$ ), Int[( $a + b*\text{Tan}[e + f*x]$ ) <sup>$m$</sup>  + 1)\*( $c + d*\text{Tan}[e + f*x]$ ) <sup>$n$</sup> \*Simp[ $A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x]$ ,  $x$ ],  $x$ ] /; FreeQ[{ $a, b, c, d, e, f, A, B, n$ },  $x$ ] && NeQ[ $b*c - a*d, 0$ ] && EqQ[ $a^2 + b^2, 0$ ] && LtQ[ $m, 0$ ] && !GtQ[ $n, 0$ ]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + ia \tan(c + dx))} dx &= \int \frac{B + A \cot(c + dx)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} dx \\ &= \frac{(iA - B) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\int \frac{\frac{1}{2}a(A-3iB) - \frac{1}{2}a(iA-B) \cot(c+dx)}{\sqrt{\cot(c + dx)}} dx}{2a^2} \\ &= \frac{(iA - B) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(A-3iB) + \frac{1}{2}a(iA-B)x^2}{1+x^4} dx, x\right)}{a^2 d} \\ &= \frac{(iA - B) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\left(\left(\frac{1}{4} + \frac{i}{4}\right) (A - (2 + i)B)\right) \text{Subst}\left(\int \frac{1}{1+x^4} dx, x\right)}{ad} \\ &= \frac{(iA - B) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\left(\left(\frac{1}{8} + \frac{i}{8}\right) (A - (2 + i)B)\right) \text{Subst}\left(\int \frac{1}{1+x^4} dx, x\right)}{\sqrt{2} ad} \\ &= \frac{(iA - B) \sqrt{\cot(c + dx)}}{2d(ia + a \cot(c + dx))} + \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - (2 + i)B) \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} ad} \\ &= \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A + (2 - i)B) \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} ad} \end{aligned}$$

**Mathematica** [A]

time = 0.91, size = 198, normalized size = 0.84

$$\frac{(\cos(dx) + i \sin(dx)) \left( 4(A + iB) \cos(c + dx) (i \cos(dx) + \sin(dx)) + (1 + i) \csc(c + dx) \left( (A + (2 - i)B) \text{ArcSin}(\cos(c + dx) - \sin(c + dx)) + i(A - (2 + i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}) \right) \right) (i \cos(c) - \sin(c)) \sqrt{\sin(2(c + dx))}}{8d \sqrt{\cot(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) (a + ia \tan(c + dx))} (A + B \tan(c + dx))$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])), x]

[Out] ((Cos[d\*x] + I\*Sin[d\*x])\*(4\*(A + I\*B)\*Cos[c + d\*x]\*(I\*Cos[d\*x] + Sin[d\*x]) + (1 + I)\*Csc[c + d\*x]\*(A + (2 - I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] + I\*(A - (2 + I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*(I\*Cos[c] - Sin[c])\*Sqrt[Sin[2\*(c + d\*x)]]\*(A + B\*Tan[c + d\*x])/(8\*d

```
*Sqrt[Cot[c + d*x]]*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x]
))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 23.52, size = 6977, normalized size = 29.44

method	result	size
default	Expression too large to display	6977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs.  $2(178) = 356$ .

time = 1.23, size = 571, normalized size = 2.41

$$\frac{1}{8} \left( \frac{a^2 d^2 \sqrt{A^2 + 2AB - B^2}}{(a^2 d^2)^{3/2}} \log\left(\frac{e^{2Ix + 2Ic} - 1}{e^{2Ix + 2Ic} + 1}\right) \sqrt{A^2 + 2AB - B^2} + (A - IB) e^{2Ix + 2Ic} \sqrt{A^2 + 2AB - B^2} \right) \frac{1}{(a^2 d^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/8*(a*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2
*((I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((I*A^2 + 2*
A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((-I*a*d*e^(2*I*d*x + 2*
I*c) + I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*s
qrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(
-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d*x + 2*I
```

\*c)\*log(-((a\*d\*e^(2\*I\*d\*x + 2\*I\*c) - a\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))\*sqrt(I\*B^2/(a^2\*d^2)) + B)\*e^(-2\*I\*d\*x - 2\*I\*c)/(a\*d)) + 2\*a\*d\*sqrt(I\*B^2/(a^2\*d^2))\*e^(2\*I\*d\*x + 2\*I\*c)\*log(((a\*d\*e^(2\*I\*d\*x + 2\*I\*c) - a\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))\*sqrt(I\*B^2/(a^2\*d^2)) - B)\*e^(-2\*I\*d\*x - 2\*I\*c)/(a\*d)) + 2\*((A + I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) - A - I\*B)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))\*e^(-2\*I\*d\*x - 2\*I\*c)/(a\*d)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A}{\tan(c+dx) \sqrt{\cot(c+dx)} - i \sqrt{\cot(c+dx)}} dx + \int \frac{B \tan(c+dx)}{\tan(c+dx) \sqrt{\cot(c+dx)} - i \sqrt{\cot(c+dx)}} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c)),x)

[Out] -I\*(Integral(A/(tan(c + d\*x)\*sqrt(cot(c + d\*x)) - I\*sqrt(cot(c + d\*x))), x) + Integral(B\*tan(c + d\*x)/(tan(c + d\*x)\*sqrt(cot(c + d\*x)) - I\*sqrt(cot(c + d\*x))), x))/a

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + a \tan(c + dx) i)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)), x)



$$3.523 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))} dx$$

**Optimal.** Leaf size=276

$$\frac{((1-3i)A+(3+5i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((1+2i)A-(4+i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad}$$

[Out]  $-1/8*((1-3*I)*A+(3+5*I)*B)*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(1/8+1/8*I)*((1+2*I)*A-(4+I)*B)*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}-(1/16+1/16*I)*((2+I)*A+(1+4*I)*B)*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+(1/16+1/16*I)*((2+I)*A+(1+4*I)*B)*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a/d*2^{(1/2)}+1/2*(-A-5*I*B)/a/d/\cot(d*x+c)^{(1/2)}+1/2*(I*A-B)/d/(I*a+a*\cot(d*x+c))/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.30, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((1-3i)A+(3+5i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((1+2i)A-(4+i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad} + \frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)} - \frac{A+5iB}{2ad\sqrt{\cot(c+dx)}} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((2+i)A+(1+4i)B)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}ad} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((2+i)A+(1+4i)B)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])),x]

[Out]  $((1-3*I)*A+(3+5*I)*B)*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]]/(4*\text{Sqrt}[2]*a*d) + ((1/4+I/4)*((1+2*I)*A-(4+I)*B)*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]]/(\text{Sqrt}[2]*a*d) - (A+(5*I)*B)/(2*a*d*\text{Sqrt}[\text{Cot}[c+d*x]]) + (I*A-B)/(2*d*\text{Sqrt}[\text{Cot}[c+d*x]]*(I*a+a*\text{Cot}[c+d*x])) - ((1/8+I/8)*((2+I)*A+(1+4*I)*B)*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]+\text{Cot}[c+d*x]]/(\text{Sqrt}[2]*a*d) + ((1/8+I/8)*((2+I)*A+(1+4*I)*B)*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]+\text{Cot}[c+d*x]]/(\text{Sqrt}[2]*a*d)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c]
```

Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
```

$[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e+f*x])^{(p-m-n)}*(b+a*\text{Cot}[e+f*x])^m*(d+c*\text{Cot}[e+f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3677

$\text{Int}[(a_+ + (b_-)*\tan[(e_-) + (f_-)*(x_-)])^{(m_+)}*((A_-) + (B_-)*\tan[(e_-) + (f_-)*(x_-)])^{(n_-)}, x\_Symbol] := \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} dx \\ &= \frac{iA - B}{2d\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))} + \frac{\int \frac{-\frac{1}{2}a(A+5iB) - \frac{3}{2}a(iA-B)\cot}{\cot^{\frac{3}{2}}(c+dx)} dx}{2a^2} \\ &= -\frac{A + 5iB}{2ad\sqrt{\cot(c + dx)}} + \frac{iA - B}{2d\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))} + \dots \\ &= -\frac{A + 5iB}{2ad\sqrt{\cot(c + dx)}} + \frac{iA - B}{2d\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))} + \dots \\ &= -\frac{A + 5iB}{2ad\sqrt{\cot(c + dx)}} + \frac{iA - B}{2d\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))} + \dots \\ &= -\frac{A + 5iB}{2ad\sqrt{\cot(c + dx)}} + \frac{iA - B}{2d\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))} - \dots \\ &= -\frac{A + 5iB}{2ad\sqrt{\cot(c + dx)}} + \frac{iA - B}{2d\sqrt{\cot(c + dx)}(ia + a \cot(c + dx))} - \dots \\ &= \frac{((1 - 3i)A + (3 + 5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{4\sqrt{2} ad} - \dots \end{aligned}$$

**Mathematica [A]**

time = 1.10, size = 214, normalized size = 0.78

$$\frac{(\cos(dx) + i \sin(dx)) \left( (-4 \cos(dx) + 4i \sin(dx)) (A + 5iB) \cos(c + dx) - 4B \sin(c + dx) - \cos(c + dx) \left( (1 - 3i)A + (3 + 5i)B \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i)(2 + i)A + (1 + 4i)B \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx)))} \right) \right) (\cos(c) + i \sin(c)) \sqrt{\sin(2(c + dx))} (A + B \tan(c + dx))}{8d \sqrt{\cot(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) (a + i a \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])), x]

[Out] ((Cos[d\*x] + I\*Sin[d\*x])\*((-4\*Cos[d\*x] + (4\*I)\*Sin[d\*x])\*((A + (5\*I)\*B)\*Cos[c + d\*x] - 4\*B\*Sin[c + d\*x]) - Csc[c + d\*x]\*(((1 - 3\*I)\*A + (3 + 5\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] - (1 + I)\*((2 + I)\*A + (1 + 4\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*(Cos[c] + I\*Sin[c])\*Sqrt[Sin[2\*(c + d\*x)]]\*(A + B\*Tan[c + d\*x]))/(8\*d\*Sqrt[Cot[c + d\*x]]\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x]))

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 72.58, size = 8548, normalized size = 30.97

method	result	size
default	Expression too large to display	8548

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(199) = 398.

time = 1.80, size = 700, normalized size = 2.54

$$\frac{(\cos(dx) + i \sin(dx)) \left( (-4 \cos(dx) + 4i \sin(dx)) (A + 5iB) \cos(c + dx) - 4B \sin(c + dx) - \cos(c + dx) \left( (1 - 3i)A + (3 + 5i)B \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i)(2 + i)A + (1 + 4i)B \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx)))} \right) \right) (\cos(c) + i \sin(c)) \sqrt{\sin(2(c + dx))} (A + B \tan(c + dx))}{8d \sqrt{\cot(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) (a + i a \tan(c + dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/8*((a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)}*\log(-2*((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - (a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)}*\log(2*((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)} - (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} + 2*(a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2)}*\log(-((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*\sqrt{(I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2)} + A + 2*I*B)*e^{(-2*I*d*x - 2*I*c)/(a*d)} - 2*(a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)})*\sqrt{(I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2)}*\log(((a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*\sqrt{(I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2)} - A - 2*I*B)*e^{(-2*I*d*x - 2*I*c)/(a*d)} - 2*((I*A - 9*B)*e^{(4*I*d*x + 4*I*c)} + 8*B*e^{(2*I*d*x + 2*I*c)} - I*A + B)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))/(a*d*e^{(4*I*d*x + 4*I*c)} + a*d*e^{(2*I*d*x + 2*I*c)}) \end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A}{\tan(c+dx) \cot^{\frac{3}{2}}(c+dx) - i \cot^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \tan(c+dx)}{\tan(c+dx) \cot^{\frac{3}{2}}(c+dx) - i \cot^{\frac{3}{2}}(c+dx)} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+I\*a\*tan(d\*x+c)),x)

[Out] 
$$-I*(\text{Integral}(A/(\tan(c + d*x)*\cot(c + d*x)**(3/2) - I*\cot(c + d*x)**(3/2)), x) + \text{Integral}(B*\tan(c + d*x)/(\tan(c + d*x)*\cot(c + d*x)**(3/2) - I*\cot(c + d*x)**(3/2)), x))/a$$

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)\*cot(d\*x + c)^(3/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + d x)}{\cot(c + d x)^{3/2} (a + a \tan(c + d x) i)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)), x)

$$3.524 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))} dx$$

**Optimal.** Leaf size=307

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad}$$

[Out] 1/6\*(-3\*A-7\*I\*B)/a/d/cot(d\*x+c)^(3/2)+1/2\*(I\*A-B)/d/cot(d\*x+c)^(3/2)/(I\*a+a\*cot(d\*x+c)-(1/8+1/8\*I)\*((4+I)\*A+(1+6\*I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)-(1/8+1/8\*I)\*((4+I)\*A+(1+6\*I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)+1/16\*((3-5\*I)\*A+(5+7\*I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)+(1/16+1/16\*I)\*((1+4\*I)\*A-(6+I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a/d\*2^(1/2)-5/2\*(I\*A-B)/a/d/cot(d\*x+c)^(1/2)

**Rubi [A]**

time = 0.33, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} ad} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((4+i)A + (1+6i)B\right) \text{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} ad} + \frac{-B + iA}{2a \cot(c+dx) \left(\cot(c+dx) + ia\right)} - \frac{3A + 7B}{6a \cot(c+dx)} - \frac{5(-B + iA)}{2ad \sqrt{\cot(c+dx)}} + \frac{((3-5i)A + (5+7i)B) \log\left(\frac{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}\right)}{8\sqrt{2} ad} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) \left((1+4i)A - (6+i)B\right) \log\left(\frac{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}\right)}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/((Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])),x]

[Out] ((1/4 + I/4)\*((4 + I)\*A + (1 + 6\*I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*a\*d) - ((1/4 + I/4)\*((4 + I)\*A + (1 + 6\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*a\*d) - (3\*A + (7\*I)\*B)/(6\*a\*d\*Cot[c + d\*x]^(3/2)) - (5\*(I\*A - B))/(2\*a\*d\*Sqrt[Cot[c + d\*x]]) + (I\*A - B)/(2\*d\*Cot[c + d\*x]^(3/2)\*(I\*a + a\*Cot[c + d\*x])) + (((3 - 5\*I)\*A + (5 + 7\*I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(8\*Sqrt[2]\*a\*d) + ((1/8 + I/8)\*((1 + 4\*I)\*A - (6 + I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(Sqrt[2]\*a\*d)

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]/b), x] \ /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[2(d/e), 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2(d/e), 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NeQ}[c^2d^2 + a^2e^2, 0] \ \&\& \ \text{NeQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[(-a)c]$

Rule 3610

$\text{Int}[\frac{((a_.) + (b_.)\tan[(e_.) + (f_.)x])^{(m_.)}((c_.) + (d_.)\tan[(e_.) + (f_.)x])}{(f_.)x^2 + b^2)}, x\_Symbol] \rightarrow \text{Simp}[(b^2c - a^2d)((a + b\tan[e + fx])^{(m+1)}/(f(m+1)(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b\tan[e + fx])^{(m+1)}\text{Simp}[ac + bd - (b^2c - a^2d)\tan[e + fx], x], x], x] \ /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[b^2c - a^2d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\sqrt{(b_.)\tan[(e_.) + (f_.)x]}}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b^2c + dx^2)/(b^2 + x^4), x], x, \text{Sqrt}[b^2\tan[e + fx]]], x] \ /; \text{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3662



```

Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]

```

### Rule 3677

```

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(ia + a \cot(c + dx))} dx \\
&= \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} + \frac{\int \frac{-\frac{1}{2}a(3A+7iB) - \frac{5}{2}a(iA-B) \cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)} dx}{2a^2} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} + \frac{\int \frac{-\frac{5}{2}a(iA-B) \cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)} dx}{2a^2} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} - \frac{5(iA - B)}{2ad \sqrt{\cot(c + dx)}} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} - \frac{5(iA - B)}{2ad \sqrt{\cot(c + dx)}} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} - \frac{5(iA - B)}{2ad \sqrt{\cot(c + dx)}} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} - \frac{5(iA - B)}{2ad \sqrt{\cot(c + dx)}} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} - \frac{5(iA - B)}{2ad \sqrt{\cot(c + dx)}} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} \\
&= -\frac{3A + 7iB}{6ad \cot^{\frac{3}{2}}(c + dx)} - \frac{5(iA - B)}{2ad \sqrt{\cot(c + dx)}} + \frac{iA - B}{2d \cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))} \\
&= \frac{((3 + 5i)A - (5 - 7i)B) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{4\sqrt{2} ad} - \frac{((3 + 5i)A - (5 - 7i)B)}{4\sqrt{2} ad}
\end{aligned}$$

**Mathematica [A]**

time = 1.37, size = 242, normalized size = 0.79

$$\frac{(\cos(dx) + i \sin(dx)) \left( (-1 - i) \cos(c + dx) \left( (4 + i)A + (1 + 6i)B \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) + (-1 - 4i)A + (6 + i)B \log \left( \frac{\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx))}}{\cos(c) + i \sin(c)} \right) \right) + \frac{1}{2} \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) \left( -15A + 19B + (-15A + 11B) \cos(2(c + dx)) + 4(3A + 2iB) \sin(2(c + dx)) \right) \right) (A + B \tan(c + dx))}{8d \sqrt{\cot(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) (a + ia \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])), x]

[Out] ((Cos[d\*x] + I\*Sin[d\*x])\*((-1 - I)\*Csc[c + d\*x]\*(((4 + I)\*A + (1 + 6\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] + ((-1 - 4\*I)\*A + (6 + I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)])])\*(Cos[c] + I\*Sin[c])\*Sqrt[Sin[2\*(c + d\*x)]] + (2\*Sec[c + d\*x]\*(Cos[d\*x] - I\*Sin[d\*x])\*((-15\*I)\*A + 19\*B))

$$B + ((-15*I)*A + 11*B)*\text{Cos}[2*(c + d*x)] + 4*(3*A + (2*I)*B)*\text{Sin}[2*(c + d*x)])))/3*(A + B*\text{Tan}[c + d*x]))/(8*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))*(a + I*a*\text{Tan}[c + d*x]))$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 83.83, size = 8761, normalized size = 28.54

method	result	size
default	Expression too large to display	8761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 794 vs.  $2(222) = 444$ .

time = 1.53, size = 794, normalized size = 2.59

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/24*(3*(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*
d*x + 2*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*log(-2*((I*a*d*e^(2*I
*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*
I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*
e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)
/(a^2*d^2))*log(-2*((-I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt((I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2
```

\*d^2)) + (A - I\*B)\*e^(2\*I\*d\*x + 2\*I\*c))\*e^(-2\*I\*d\*x - 2\*I\*c)/(I\*A + B)) + 6\*(a\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 2\*a\*d\*e^(4\*I\*d\*x + 4\*I\*c) + a\*d\*e^(2\*I\*d\*x + 2\*I\*c))\*sqrt((-4\*I\*A^2 + 12\*A\*B + 9\*I\*B^2)/(a^2\*d^2))\*log(-((a\*d\*e^(2\*I\*d\*x + 2\*I\*c) - a\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))\*sqrt((-4\*I\*A^2 + 12\*A\*B + 9\*I\*B^2)/(a^2\*d^2)) + 2\*I\*A - 3\*B)\*e^(-2\*I\*d\*x - 2\*I\*c)/(a\*d)) - 6\*(a\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 2\*a\*d\*e^(4\*I\*d\*x + 4\*I\*c) + a\*d\*e^(2\*I\*d\*x + 2\*I\*c))\*sqrt((-4\*I\*A^2 + 12\*A\*B + 9\*I\*B^2)/(a^2\*d^2))\*log(((a\*d\*e^(2\*I\*d\*x + 2\*I\*c) - a\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))\*sqrt((-4\*I\*A^2 + 12\*A\*B + 9\*I\*B^2)/(a^2\*d^2)) - 2\*I\*A + 3\*B)\*e^(-2\*I\*d\*x - 2\*I\*c)/(a\*d)) + 2\*((27\*A + 19\*I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) + (3\*A + 19\*I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) - (27\*A + 35\*I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) - 3\*A - 3\*I\*B)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))/(a\*d\*e^(6\*I\*d\*x + 6\*I\*c) + 2\*a\*d\*e^(4\*I\*d\*x + 4\*I\*c) + a\*d\*e^(2\*I\*d\*x + 2\*I\*c))

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A}{\tan(c+dx) \cot^{\frac{5}{2}}(c+dx) - i \cot^{\frac{5}{2}}(c+dx)} dx + \int \frac{B \tan(c+dx)}{\tan(c+dx) \cot^{\frac{5}{2}}(c+dx) - i \cot^{\frac{5}{2}}(c+dx)} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(5/2)/(a+I\*a\*tan(d\*x+c)),x)

[Out] -I\*(Integral(A/(tan(c + d\*x)\*cot(c + d\*x)\*\*(5/2) - I\*cot(c + d\*x)\*\*(5/2)), x) + Integral(B\*tan(c + d\*x)/(tan(c + d\*x)\*cot(c + d\*x)\*\*(5/2) - I\*cot(c + d\*x)\*\*(5/2)), x))/a

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)\*cot(d\*x + c)^(5/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + a \tan(c + dx) i)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

$$3.525 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=317

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((2+23i)A - (7+2i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{\left((25+21i)A - (9-5i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^2 d}$$

[Out]  $1/8*(7*A+3*I*B)*\cot(d*x+c)^{(3/2)}/a^2/d/(I+\cot(d*x+c))+1/4*(A+I*B)*\cot(d*x+c)^{(5/2)}/d/(I*a+a*\cot(d*x+c))^2+(1/32-1/32*I)*((2+23*I)*A-(7+2*I)*B)*\arctan(-1+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a^2/d*2^{(1/2)}+1/32*((25+21*I)*A+(-9+5*I)*B)*\arctan(1+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a^2/d*2^{(1/2)}+(-1/64+1/64*I)*((23+2*I)*A+(2+7*I)*B)*\ln(1+\cot(d*x+c)-2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a^2/d*2^{(1/2)}+(1/64-1/64*I)*((23+2*I)*A+(2+7*I)*B)*\ln(1+\cot(d*x+c)+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/a^2/d*2^{(1/2)}-5/8*(5*A+I*B)*\cot(d*x+c)^{(1/2)}/a^2/d$

**Rubi [A]**

time = 0.45, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(h-h)(2+23i)A-(7+2i)B \text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d} + \frac{(25+21i)A-(9-5i)B \text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}a^2d} + \frac{(7A+3iB)\cot^3(c+dx)}{8a^2d(\cot(c+dx)+i)} - \frac{5(A+iB)\sqrt{\cot(c+dx)}}{8a^2d} - \frac{(h-h)((23+2i)A+(2+7i)B)\log(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1)}{\sqrt{2}a^2d} + \frac{(h-h)((23+2i)A+(2+7i)B)\log(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1)}{\sqrt{2}a^2d} + \frac{(A+iB)\cot^5(c+dx)}{4a^2d(\cot(c+dx)+i)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cot}[c + d*x]^{(3/2)}*(A + B*\text{Tan}[c + d*x]))/(a + I*a*\text{Tan}[c + d*x])^2, x]$

[Out]  $((-1/16 + I/16)*((2 + 23*I)*A - (7 + 2*I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]/(\text{Sqrt}[2]*a^2*d) + (((25 + 21*I)*A - (9 - 5*I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]/(16*\text{Sqrt}[2]*a^2*d) - (5*(5*A + I*B)*\text{Sqrt}[\text{Cot}[c + d*x]]/(8*a^2*d) + ((7*A + (3*I)*B)*\text{Cot}[c + d*x]^{(3/2)})/(8*a^2*d*(I + \text{Cot}[c + d*x])) + ((A + I*B)*\text{Cot}[c + d*x]^{(5/2)})/(4*d*(I*a + a*\text{Cot}[c + d*x])^2) - ((1/32 - I/32)*((23 + 2*I)*A + (2 + 7*I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a^2*d) + ((1/32 - I/32)*((23 + 2*I)*A + (2 + 7*I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a^2*d)$

**Rule 210**

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

**Rule 631**

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] := \text{With}\{q = 1 - 4*S\text{implify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{Free}$

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3609

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \ :> \ \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)}*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] & & NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx &= \int \frac{\cot^{\frac{5}{2}}(c+dx)(B+A \cot(c+dx))}{(ia+a \cot(c+dx))^2} dx \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} + \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)(-\frac{5}{2}a(ia-B)+\frac{1}{2}a(9A+iB) \cot(c+dx))}{ia+a \cot(c+dx)}}{4a^2} \\
&= \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} + \frac{\int \sqrt{\cot(c+dx)}}{4d(i+\cot(c+dx))} \\
&= -\frac{5(5A+iB) \sqrt{\cot(c+dx)}}{8a^2d} + \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} \\
&= -\frac{5(5A+iB) \sqrt{\cot(c+dx)}}{8a^2d} + \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} \\
&= -\frac{5(5A+iB) \sqrt{\cot(c+dx)}}{8a^2d} + \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} \\
&= -\frac{5(5A+iB) \sqrt{\cot(c+dx)}}{8a^2d} + \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} \\
&= -\frac{5(5A+iB) \sqrt{\cot(c+dx)}}{8a^2d} + \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} \\
&= -\frac{5(5A+iB) \sqrt{\cot(c+dx)}}{8a^2d} + \frac{(7A+3iB) \cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))} + \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{4d(ia+a \cot(c+dx))^2} \\
&= -\frac{((25+21i)A - (9-5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^2d} + \dots
\end{aligned}$$

### Mathematica [A]

time = 1.34, size = 256, normalized size = 0.81

$\frac{\sec(c+dx)(\cos(dx)+\sin(dx))^2(\cos(c+dx)((21-25i)A+(5+9i)B)\text{ArcSin}(\cos(c+dx)-\sin(c+dx))-(1+i)((23+2i)A+(2+7i)B)\log(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))})+(\cos(2c)-\sin(2c))\sqrt{\sin(2(c+dx))}+\cot(c+dx)(-2\cos(2dx)+2i\sin(2dx))(-9A-5iB+(41A+5iB)\cos(2(c+dx))+(43A-7iB)\sin(2(c+dx)))(A+B \tan(c+dx))}{32d\sqrt{\cot(c+dx)}(A\cos(c+dx)+B\sin(c+dx))(a+ia \tan(c+dx))^2}$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2, x]

[Out] (Sec[c + d\*x]\*(Cos[d\*x] + I\*Sin[d\*x])^2\*(Csc[c + d\*x]\*(((21 - 25\*I)\*A + (5 + 9\*I)\*B)\*ArcSin[Cos[c + d\*x] - Sin[c + d\*x]] - (1 + I)\*((23 + 2\*I)\*A + (2 + 7\*I)\*B)\*Log[Cos[c + d\*x] + Sin[c + d\*x] + Sqrt[Sin[2\*(c + d\*x)]]])\*(I\*Cos[2\*c] - Sin[2\*c])\*Sqrt[Sin[2\*(c + d\*x)]] + Cot[c + d\*x]\*(-2\*Cos[2\*d\*x] + (2 + I)\*Sin[2\*d\*x])\*(-9\*A - (5\*I)\*B + (41\*A + (5\*I)\*B)\*Cos[2\*(c + d\*x)] + ((43\*I)\*A - 7\*B)\*Sin[2\*(c + d\*x)])\*(A + B\*Tan[c + d\*x]))/(32\*d\*Sqrt[Cot[c + d\*x]])\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^2)



$$\begin{aligned}
& x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-2*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+9*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+4*I*A*\cos(d*x+c)^4*\sin(d*x+c)*2^{(1/2)}+7*I*A*\cos(d*x+c)^2*\sin(d*x+c)*2^{(1/2)}-23*A*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-7*B*\cos(d*x+c)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-4*A*\cos(d*x+c)^5*2^{(1/2)}-7*I*B*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*B*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}))+23*I*A*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-2*I*A*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticPi((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-21*I*A*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*EllipticF((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)}))*\sin(d*x+c)*(\cos(d*x+c)/\sin(d*x+c))^{(3/2)}/\cos(d*x+c)^2
\end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs.  $2(234) = 468$ .

time = 1.66, size = 669, normalized size = 2.11



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$-1/32*(2*a^2*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(-2*((I*a^2*d*e^{(2*I*d*x + 2*I*c)} - I*a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)}) - 2*a^2*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(-2*((-I*a^2*d*e^{(2*I*d*x + 2*I*c)} + I*a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)}) - a^2*d*\sqrt{(-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} - a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)} + 23*I*A - 7*B)*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} + a^2*d*\sqrt{(-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(-1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} - a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)} - 23*I*A + 7*B)*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} + 2*(6*(7*A + I*B)*e^{(4*I*d*x + 4*I*c)} - (9*A + 5*I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*e^{(-4*I*d*x - 4*I*c)/(a^2*d)}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cot^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx + \int \frac{B \tan(c+dx) \cot^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] 
$$-(\text{Integral}(A*\cot(c + d*x)**(3/2)/(\tan(c + d*x)**2 - 2*I*\tan(c + d*x) - 1), x) + \text{Integral}(B*\tan(c + d*x)*\cot(c + d*x)**(3/2)/(\tan(c + d*x)**2 - 2*I*\tan(c + d*x) - 1), x))/a**2$$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(I\*a\*tan(d\*x + c) + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2, x)

$$3.526 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=284

$$\frac{((9-5i)A+(1-3i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} + \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((-2+7i)A+(1+2i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d}$$

[Out] 1/4\*(A+I\*B)\*cot(d\*x+c)^(3/2)/d/(I\*a+a\*cot(d\*x+c))^2-1/32\*((9-5\*I)\*A+(1-3\*I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+(1/32+1/32\*I)\*((-2+7\*I)\*A+(1+2\*I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+(1/64+1/64\*I)\*((-7+2\*I)\*A+(2+I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+1/64\*((9+5\*I)\*A-(1+3\*I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+1/8\*(5\*A+I\*B)\*cot(d\*x+c)^(1/2)/a^2/d/(I+cot(d\*x+c))

**Rubi [A]**

time = 0.40, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3662, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(9-5i)A+(1-3i)B\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} + \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((-2+7i)A+(1+2i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d} + \frac{(5A+I B)\sqrt{\cot(c+dx)}}{32\sqrt{2}\cot(c+dx)+1} + \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((-2+7i)B-(1+2i)A)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}a^2d} + \frac{(9+5i)A-(1+3i)B\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{32\sqrt{2}a^2d} + \frac{(A+I B)\cot\frac{1}{2}(c+dx)}{4\cot(c+dx)+20}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^2,x]

[Out] (((9 - 5\*I)\*A + (1 - 3\*I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(16\*Sqrt[2]\*a^2\*d) + ((1/16 + I/16)\*((-2 + 7\*I)\*A + (1 + 2\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(Sqrt[2]\*a^2\*d) + ((5\*A + I\*B)\*Sqrt[Cot[c + d\*x]])/(8\*a^2\*d\*(I + Cot[c + d\*x])) + ((A + I\*B)\*Cot[c + d\*x]^(3/2))/(4\*d\*(I\*a + a\*Cot[c + d\*x])^2) + ((1/32 + I/32)\*((-7 + 2\*I)\*A + (2 + I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(Sqrt[2]\*a^2\*d) + (((9 + 5\*I)\*A - (1 + 3\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^2\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \ :> \ \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3676

```

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (A + B \tan(c+dx))}{(a + ia \tan(c+dx))^2} dx &= \int \frac{\cot^{\frac{3}{2}}(c+dx)(B + A \cot(c+dx))}{(ia + a \cot(c+dx))^2} dx \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c+dx)}{4d(ia + a \cot(c+dx))^2} + \frac{\int \frac{\sqrt{\cot(c+dx)} (-\frac{3}{2}a(iA-B) + \frac{1}{2}a(7A-iB))}{ia+a \cot(c+dx)} dx}{4a^2} \\
&= \frac{(5A + iB) \sqrt{\cot(c+dx)}}{8a^2d(i + \cot(c+dx))} + \frac{(A + iB) \cot^{\frac{3}{2}}(c+dx)}{4d(ia + a \cot(c+dx))^2} + \frac{\int -\frac{1}{2}a^2(5A - iB)}{4a^2} dx \\
&= \frac{(5A + iB) \sqrt{\cot(c+dx)}}{8a^2d(i + \cot(c+dx))} + \frac{(A + iB) \cot^{\frac{3}{2}}(c+dx)}{4d(ia + a \cot(c+dx))^2} + \frac{\text{Subst}\left(\int -\frac{1}{2}a^2(5A - iB)}{4a^2} dx\right)}{4a^2} \\
&= \frac{(5A + iB) \sqrt{\cot(c+dx)}}{8a^2d(i + \cot(c+dx))} + \frac{(A + iB) \cot^{\frac{3}{2}}(c+dx)}{4d(ia + a \cot(c+dx))^2} + \frac{((9 + 5i)A - (9 + 5i)B)}{16\sqrt{2}a^2d} \\
&= \frac{(5A + iB) \sqrt{\cot(c+dx)}}{8a^2d(i + \cot(c+dx))} + \frac{(A + iB) \cot^{\frac{3}{2}}(c+dx)}{4d(ia + a \cot(c+dx))^2} + \frac{((9 + 5i)A - (9 + 5i)B)}{16\sqrt{2}a^2d} \\
&= \frac{(5A + iB) \sqrt{\cot(c+dx)}}{8a^2d(i + \cot(c+dx))} + \frac{(A + iB) \cot^{\frac{3}{2}}(c+dx)}{4d(ia + a \cot(c+dx))^2} + \frac{((9 + 5i)A - (9 + 5i)B)}{16\sqrt{2}a^2d} \\
&= \frac{((9 - 5i)A + (1 - 3i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} + \frac{((9 + 5i)A - (9 + 5i)B)}{16\sqrt{2}a^2d}
\end{aligned}$$

**Mathematica [A]**

time = 1.15, size = 243, normalized size = 0.86

$\frac{\cos(c+dx)(\cos(dx)+i\sin(dx))^2(4\cos(c+dx)\cos(2dx)-i\sin(2dx))((7A+3iB)\cos(c+dx)+(5(A-B)\sin(c+dx))+\cos(c+dx))\left(\frac{(5+9i)A+(3+i)B}{32}\text{ArcSin}(\cos(c+dx)-\sin(c+dx))-1\right)+((2+7i)A+(1-2i)B)\log(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2(c+dx))})}{32\sqrt{\cot(c+dx)}(A\cos(c+dx)+B\sin(c+dx))(a+ia\tan(c+dx))^2}\left(\cos(2c)-\sin(2c)\right)\sqrt{\sin(2(c+dx))}+(A+B\tan(c+dx))\right)}{16\sqrt{2}a^2d}$

Antiderivative was successfully verified.

```

[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^
2, x]

```



```
[Out] (Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(4*Cos[c + d*x]*(Cos[2*d*x] - I*Sin[2*d*x])*((7*A + (3*I)*B)*Cos[c + d*x] + ((5*I)*A - B)*Sin[c + d*x]) + Csc[c + d*x]*(((5 + 9*I)*A + (3 + I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] - (1 + I)*((2 + 7*I)*A + (1 - 2*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*(I*Cos[2*c] - Sin[2*c])*Sqrt[Sin[2*(c + d*x)]]*(A + B*Tan[c + d*x]))/(32*d*Sqrt[Cot[c + d*x]]*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^2)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 25.35, size = 1517, normalized size = 5.34

method	result	size
default	Expression too large to display	1517

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a^2/d*2^(1/2)*(-B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-4*B*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)-4*I*B*cos(d*x+c)^5*2^(1/2)+4*B*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)-3*A*cos(d*x+c)^3*2^(1/2)+4*A*cos(d*x+c)^4*2^(1/2)-9*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*B*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*B*cos(d*x+c)^3*2^(1/2)+B*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)+4*I*2^(1/2)*B*cos(d*x+c)^4-4*I*2^(1/2)*A*cos(d*x+c)^3*sin(d*x+c)+5*I*2^(1/2)*A*cos(d*x+c)^2*sin(d*x+c)-5*I*2^(1/2)*A*cos(d*x+c)*sin(d*x+c)-I*B*cos(d*x+c)^2*2^(1/2)+B*2^(1/2)*cos(d*x+c)*sin(d*x+c)+3*A*2^(1/2)*cos(d*x+c)^2-2*I*A*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+7*I*A*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*I*B*sin(d*x+c)*((-1+cos(d*x+c))/si
```

$$\begin{aligned} & n(d*x+c))^{(1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)*(-(\cos(d*x+c)- \\ & 1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)*\text{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin( \\ & d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}-I*B*\sin(d*x+c)*(((-1+\cos(d*x+c))/\sin(d* \\ & x+c))^{(1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)*(-(\cos(d*x+c)-1-\sin \\ & n(d*x+c))/\sin(d*x+c))^{(1/2)*\text{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+ \\ & c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)}+3*I*B*\sin(d*x+c)*(((-1+\cos(d*x+c))/\sin(d*x+ \\ & c))^{(1/2)*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)*(-(\cos(d*x+c)-1-\sin( \\ & d*x+c))/\sin(d*x+c))^{(1/2)*\text{EllipticF}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c)) \\ & ^{(1/2)},1/2*2^{(1/2)}+4*I*A*\cos(d*x+c)^4*\sin(d*x+c)*2^{(1/2)}-4*A*\cos(d*x+c)^5* \\ & 2^{(1/2)})*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*(\cos(d*x+c)/\sin(d*x+c))^{(1/2)}/\cos \\ & (d*x+c)/\sin(d*x+c)^3 \end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(211) = 422.

time = 1.04, size = 666, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/32*(2*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}* \\ & \log(-2*((a^2*d*e^{(2*I*d*x + 2*I*c)} - a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I \\ & )/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)} + (A \\ & - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - 2*a^2*d*\sqrt{ \\ & (-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(2*((a^2*d*e^{(2* \\ & I*d*x + 2*I*c)} - a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I* \\ & c)} - 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)} - (A - I*B)*e^{(2*I*d*x + \\ & 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - a^2*d*\sqrt{(49*I*A^2 + 14*A*B - I \\ & *B^2)/(a^4*d^2)}*e^{(4*I*d*x + 4*I*c)}*\log(-1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} - \\ & a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(4 \\ & 9*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2)} + 7*A - I*B)*e^{(-2*I*d*x - 2*I*c)}/(a^2 \end{aligned}$$

\*d)) + a<sup>2</sup>\*d\*sqrt((49\*I\*A<sup>2</sup> + 14\*A\*B - I\*B<sup>2</sup>)/(a<sup>4</sup>\*d<sup>2</sup>))\*e<sup>(4\*I\*d\*x + 4\*I\*c)</sup>)\*log(1/8\*((a<sup>2</sup>\*d\*e<sup>(2\*I\*d\*x + 2\*I\*c)</sup> - a<sup>2</sup>\*d)\*sqrt((I\*e<sup>(2\*I\*d\*x + 2\*I\*c)</sup> + I)/(e<sup>(2\*I\*d\*x + 2\*I\*c)</sup> - 1))\*sqrt((49\*I\*A<sup>2</sup> + 14\*A\*B - I\*B<sup>2</sup>)/(a<sup>4</sup>\*d<sup>2</sup>)) - 7\*A + I\*B)\*e<sup>(-2\*I\*d\*x - 2\*I\*c)</sup>/(a<sup>2</sup>\*d)) - 2\*(2\*(3\*I\*A - B)\*e<sup>(4\*I\*d\*x + 4\*I\*c)</sup> - (5\*I\*A - B)\*e<sup>(2\*I\*d\*x + 2\*I\*c)</sup> - I\*A + B)\*sqrt((I\*e<sup>(2\*I\*d\*x + 2\*I\*c)</sup> + I)/(e<sup>(2\*I\*d\*x + 2\*I\*c)</sup> - 1)))\*e<sup>(-4\*I\*d\*x - 4\*I\*c)</sup>/(a<sup>2</sup>\*d)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sqrt{\cot(c+dx)}}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx + \int \frac{B \tan(c+dx) \sqrt{\cot(c+dx)}}{\tan^2(c+dx)-2i \tan(c+dx)-1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] -(Integral(A\*sqrt(cot(c + d\*x))/(tan(c + d\*x)\*\*2 - 2\*I\*tan(c + d\*x) - 1), x) + Integral(B\*tan(c + d\*x)\*sqrt(cot(c + d\*x))/(tan(c + d\*x)\*\*2 - 2\*I\*tan(c + d\*x) - 1), x))/a\*\*2

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/(I\*a\*tan(d\*x + c) + a)^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c+dx)} (A + B \tan(c+dx))}{(a + a \tan(c+dx) 1i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2,x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^2, x)

$$3.527 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=274

$$\frac{((-1+3i)A+(1+3i)B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} + \frac{((-1+3i)A+(1+3i)B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d}$$

[Out] 1/32\*((-1+3\*I)\*A+(1+3\*I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+1/32\*((-1+3\*I)\*A+(1+3\*I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+1/64\*((1+3\*I)\*A+(1-3\*I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)-1/64\*((1+3\*I)\*A+(1-3\*I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+1/8\*(3\*I\*A+B)\*cot(d\*x+c)^(1/2)/a^2/d/(I+cot(d\*x+c))+1/4\*(A+I\*B)\*cot(d\*x+c)^(1/2)/d/(I\*a+a\*cot(d\*x+c))^2

**Rubi [A]**

time = 0.37, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{((1+3i)B-(1-3i)A)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} + \frac{((1+3i)B-(1-3i)A)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}a^2d} + \frac{(B+3iA)\sqrt{\cot(c+dx)}}{8a^2(\cot(c+dx)+1)} + \frac{((1+3i)A+(1-3i)B)\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{32\sqrt{2}a^2d} - \frac{((1+3i)A+(1-3i)B)\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{32\sqrt{2}a^2d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{4i(a\cot(c+dx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^2), x]

[Out] -1/16\*(((1+3\*I)\*A+(1+3\*I)\*B)\*ArcTan[1-Sqrt[2]\*Sqrt[Cot[c+d\*x]]])/(Sqrt[2]\*a^2\*d)+(((1+3\*I)\*A+(1+3\*I)\*B)\*ArcTan[1+Sqrt[2]\*Sqrt[Cot[c+d\*x]]])/(16\*Sqrt[2]\*a^2\*d)+(((3\*I)\*A+B)\*Sqrt[Cot[c+d\*x]])/(8\*a^2\*d\*(I+Cot[c+d\*x]))+((A+I\*B)\*Sqrt[Cot[c+d\*x]])/(4\*d\*(I\*a+a\*Cot[c+d\*x])^2)+(((1+3\*I)\*A+(1-3\*I)\*B)\*Log[1-Sqrt[2]\*Sqrt[Cot[c+d\*x]]+Cot[c+d\*x]])/(32\*Sqrt[2]\*a^2\*d)-(((1+3\*I)\*A+(1-3\*I)\*B)\*Log[1+Sqrt[2]\*Sqrt[Cot[c+d\*x]]+Cot[c+d\*x]])/(32\*Sqrt[2]\*a^2\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)])\*(g\_)^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Sim

$p[(-(A*b - a*B))*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^n/(2*a*f*m)),$   
 $x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*\text{Tan}[e + f*x], x], x], x] /;$ 
 $\text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&$   
 $\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]$

### Rule 3677

$\text{Int}[(a + b*\text{tan}[(e + f*x)])^m*((A + B)*\text{tan}[(e + f*x)] + (f*(x)))]^n, x\_Symbol] \rightarrow \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n + 1)})/(2*f*m*(b*c - a*d)), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$ 
 $\text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, 0] \&\& \text{!GtQ}[n, 0]$

### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^2} dx &= \int \frac{\sqrt{\cot(c + dx)} (B + A \cot(c + dx))}{(ia + a \cot(c + dx))^2} dx \\
 &= \frac{(A + iB) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{\int \frac{-\frac{1}{2}a(iA - B) + \frac{1}{2}a(5A - 3iB) \cot(c + dx)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} dx}{4a^2} \\
 &= \frac{(3iA + B) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(A + iB) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a^2(A + B \cot(c + dx))}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} dx}{4a^2} \\
 &= \frac{(3iA + B) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(A + iB) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{u}} du\right)}{4a^2} \\
 &= \frac{(3iA + B) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(A + iB) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} - \frac{((1 + 3i) \sqrt{\cot(c + dx)})}{4a^2} \\
 &= \frac{(3iA + B) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(A + iB) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{((1 + 3i) \sqrt{\cot(c + dx)})}{4a^2} \\
 &= \frac{(3iA + B) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(A + iB) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{((1 + 3i) \sqrt{\cot(c + dx)})}{4a^2} \\
 &= -\frac{((-1 + 3i)A + (1 + 3i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{16\sqrt{2} a^2d} + \dots
 \end{aligned}$$

**Mathematica [A]**

time = 1.18, size = 243, normalized size = 0.89

$$\frac{(B + A \cot(c + dx)) \cos(c + dx) \cos(dx) + \sin(dx)^2 (1 \cos(c + dx) (\cos(2dx) + \sin(2dx)) ((3A - 1B) \cos(c + dx) + (A + 3B) \sin(c + dx)) + (1 - I) \cos(c + dx) \left( ((1 + 2i)A + (2 + i)B) \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) + ((-2 - i)A + (1 + 2i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx)))} \right) + (\cos(2c) - \sin(2c)) \sqrt{\sin(2(c + dx))}}{32a^2 \sqrt{\cot(c + dx)} (i + \cot(c + dx))^2 (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((B + A*Cot[c + d*x])*Csc[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(4*Cos[c + d*x]
*(I*Cos[2*d*x] + Sin[2*d*x])*((3*A - I*B)*Cos[c + d*x] + (I*A + 3*B)*Sin[c
+ d*x]) + (1 - I)*Csc[c + d*x]*(((1 + 2*I)*A + (2 + I)*B)*ArcSin[Cos[c + d
*x] - Sin[c + d*x]] + ((-2 - I)*A + (1 + 2*I)*B)*Log[Cos[c + d*x] + Sin[c +
d*x] + Sqrt[Sin[2*(c + d*x)]])*(I*Cos[2*c] - Sin[2*c])*Sqrt[Sin[2*(c + d*
x)]])/(32*a^2*d*Sqrt[Cot[c + d*x]]*(I + Cot[c + d*x])^2*(A*Cos[c + d*x] +
B*Sin[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 23.59, size = 13774, normalized size = 50.27

method	result	size
default	Expression too large to display	13774

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 663 vs.  $2(211) = 422$ .

time = 1.95, size = 663, normalized size = 2.42

$$\frac{(B + A \cot(c + dx)) \cos(c + dx) \cos(dx) + \sin(dx)^2 (1 \cos(c + dx) (\cos(2dx) + \sin(2dx)) ((3A - 1B) \cos(c + dx) + (A + 3B) \sin(c + dx)) + (1 - I) \cos(c + dx) \left( ((1 + 2i)A + (2 + i)B) \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) + ((-2 - i)A + (1 + 2i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2(c + dx)))} \right) + (\cos(2c) - \sin(2c)) \sqrt{\sin(2(c + dx))}}{32a^2 \sqrt{\cot(c + dx)} (i + \cot(c + dx))^2 (A \cos(c + dx) + B \sin(c + dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\frac{1}{32} * (2 * a^2 * d * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^4 * d^2)}) * e^{(4 * I * d * x + 4 * I * c)} * \log(-2 * ((I * a^2 * d * e^{(2 * I * d * x + 2 * I * c)} - I * a^2 * d) * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^4 * d^2)}) + (A - I * B) * e^{(2 * I * d * x + 2 * I * c)} * e^{(-2 * I * d * x - 2 * I * c) / (I * A + B)} - 2 * a^2 * d * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^4 * d^2)} * e^{(4 * I * d * x + 4 * I * c)} * \log(-2 * ((-I * a^2 * d * e^{(2 * I * d * x + 2 * I * c)} + I * a^2 * d) * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * \sqrt{(I * A^2 + 2 * A * B - I * B^2) / (a^4 * d^2)}) + (A - I * B) * e^{(2 * I * d * x + 2 * I * c)} * e^{(-2 * I * d * x - 2 * I * c) / (I * A + B)} + a^2 * d * \sqrt{((-I * A^2 + 2 * A * B + I * B^2) / (a^4 * d^2))} * e^{(4 * I * d * x + 4 * I * c)} * \log(1/8 * ((a^2 * d * e^{(2 * I * d * x + 2 * I * c)} - a^2 * d) * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * \sqrt{((-I * A^2 + 2 * A * B + I * B^2) / (a^4 * d^2))} + I * A - B) * e^{(-2 * I * d * x - 2 * I * c) / (a^2 * d)} - a^2 * d * \sqrt{((-I * A^2 + 2 * A * B + I * B^2) / (a^4 * d^2))} * e^{(4 * I * d * x + 4 * I * c)} * \log(-1/8 * ((a^2 * d * e^{(2 * I * d * x + 2 * I * c)} - a^2 * d) * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * \sqrt{((-I * A^2 + 2 * A * B + I * B^2) / (a^4 * d^2))} - I * A + B) * e^{(-2 * I * d * x - 2 * I * c) / (a^2 * d)} + 2 * (2 * (A - I * B) * e^{(4 * I * d * x + 4 * I * c)} - (A - 3 * I * B) * e^{(2 * I * d * x + 2 * I * c)} - A - I * B) * \sqrt{(I * e^{(2 * I * d * x + 2 * I * c)} + I) / (e^{(2 * I * d * x + 2 * I * c)} - 1)}) * e^{(-4 * I * d * x - 4 * I * c) / (a^2 * d)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\tan^2(c+dx)\sqrt{\cot(c+dx)} - 2i \tan(c+dx)\sqrt{\cot(c+dx)} - \sqrt{\cot(c+dx)}} dx + \int \frac{B \tan(c+dx)}{\tan^2(c+dx)\sqrt{\cot(c+dx)} - 2i \tan(c+dx)\sqrt{\cot(c+dx)} - \sqrt{\cot(c+dx)}} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out]  $-(\text{Integral}(A/(\tan(c + d*x)**2*\sqrt{\cot(c + d*x)} - 2*I*\tan(c + d*x)*\sqrt{\cot(c + d*x)} - \sqrt{\cot(c + d*x)})), x) + \text{Integral}(B*\tan(c + d*x)/(\tan(c + d*x)**2*\sqrt{\cot(c + d*x)} - 2*I*\tan(c + d*x)*\sqrt{\cot(c + d*x)} - \sqrt{\cot(c + d*x)})), x)/a**2$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^2\*sqrt(cot(d\*x + c))), x)



**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + a \tan(c + dx) i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^2),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^2), x)

$$3.528 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=284

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((2+i)A + (7-2i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{((1+3i)A + (9+5i)B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^2 d}$$

[Out] (1/32+1/32\*I)\*((2+I)\*A+(7-2\*I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d \*2^(1/2)+1/32\*((1+3\*I)\*A+(9+5\*I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d \*d\*2^(1/2)+1/64\*((1-3\*I)\*A+(-9+5\*I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+(1/64+1/64\*I)\*((1+2\*I)\*A+(2-7\*I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^2/d\*2^(1/2)+1/8\*(A+5\*I\*B)\*cot(d\*x+c)^(1/2)/a^2/d/(I+cot(d\*x+c))+1/4\*(I\*A-B)\*cot(d\*x+c)^(1/2)/d/(I\*a+a\*cot(d\*x+c))^2

**Rubi [A]**

time = 0.39, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3662, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((2+i)A + (7-2i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{((1+3i)A + (9+5i)B) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{16\sqrt{2} a^2 d} + \frac{(A+5iB) \sqrt{\cot(c+dx)}}{8a^2 \cot(c+dx)+1} + \frac{((1-3i)A - (9-5i)B) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{32\sqrt{2} a^2 d} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((1+2i)A + (2-7i)B) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} a^2 d} + \frac{(-B+iA) \sqrt{\cot(c+dx)}}{4d(a \cot(c+dx) + ia)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^2), x]

[Out] ((-1/16 - I/16)\*((2 + I)\*A + (7 - 2\*I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*a^2\*d) + (((1 + 3\*I)\*A + (9 + 5\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(16\*Sqrt[2]\*a^2\*d) + ((A + (5\*I)\*B)\*Sqrt[Cot[c + d\*x]])/(8\*a^2\*d\*(I + Cot[c + d\*x])) + ((I\*A - B)\*Sqrt[Cot[c + d\*x]])/(4\*d\*(I\*a + a\*Cot[c + d\*x])^2) + (((1 - 3\*I)\*A - (9 - 5\*I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(32\*Sqrt[2]\*a^2\*d) + ((1/32 + I/32)\*((1 + 2\*I)\*A + (2 - 7\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(Sqrt[2]\*a^2\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \ :> \ \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx &= \int \frac{B + A \cot(c + dx)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^2} dx \\
&= \frac{(iA - B) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{\int \frac{\frac{1}{2}a(A - 7iB) - \frac{3}{2}a(iA - B) \cot(c + dx)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} dx}{4a^2} \\
&= \frac{(A + 5iB) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(iA - B) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{\int \frac{-\frac{3}{2}a^2(iA - B)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} dx}{4a^2} \\
&= \frac{(A + 5iB) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(iA - B) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \text{Subst}\left(\int \frac{-\frac{3}{2}a^2(iA - B)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} dx, \sqrt{\cot(c + dx)}, u\right) \\
&= \frac{(A + 5iB) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(iA - B) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) \sqrt{\cot(c + dx)}}{4a^2} \\
&= \frac{(A + 5iB) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(iA - B) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{((1 - 3i) \sqrt{\cot(c + dx)})}{16a^2} \\
&= \frac{(A + 5iB) \sqrt{\cot(c + dx)}}{8a^2d(i + \cot(c + dx))} + \frac{(iA - B) \sqrt{\cot(c + dx)}}{4d(ia + a \cot(c + dx))^2} + \frac{((1 - 3i) \sqrt{\cot(c + dx)})}{16a^2} \\
&= -\frac{((1 + 3i)A + (9 + 5i)B) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{16\sqrt{2} a^2 d} + \frac{((1 - 3i) \sqrt{\cot(c + dx)})}{16a^2}
\end{aligned}$$

**Mathematica [A]**

time = 1.38, size = 241, normalized size = 0.85

$\frac{\sec(c + dx)(\cos(dx) + i \sin(dx))^2 \left( 4 \cos(c + dx) \cos(2dx) - i \sin(2dx) \left( (A + 5iB) \cos(c + dx) + (3iA - 7B) \sin(c + dx) \right) - (1 + i) \cos(c + dx) \left( \frac{((1 + 2i)A + (2 + 7i)B) \text{ArcSin}(\cos(c + dx) - \sin(c + dx))}{\sqrt{\cot(c + dx)}} + ((-2 + i)A + (7 + 2i)B) \log(\cos(c + dx) + \sin(c + dx) + \sqrt{\cot(c + dx)}) \right) \right)}{32i \sqrt{\cot(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) (a + ia \tan(c + dx))^2}$

Antiderivative was successfully verified.



$$\frac{I}{(e^{(2I*d*x + 2I*c)} - 1)} * \sqrt{\frac{(-I*A^2 - 2*A*B + I*B^2)}{(a^4*d^2)}} + (A - I*B) * e^{(2I*d*x + 2I*c)} * e^{(-2I*d*x - 2I*c)} / (I*A + B) - 2*a^2*d * \sqrt{\frac{(-I*A^2 - 2*A*B + I*B^2)}{(a^4*d^2)}} * e^{(4I*d*x + 4I*c)} * \log(2 * ((a^2*d * e^{(2I*d*x + 2I*c)} - a^2*d) * \sqrt{\frac{(I * e^{(2I*d*x + 2I*c)} + I)}{(e^{(2I*d*x + 2I*c)} - 1)}} * \sqrt{\frac{(-I*A^2 - 2*A*B + I*B^2)}{(a^4*d^2)}} - (A - I*B) * e^{(2I*d*x + 2I*c)} * e^{(-2I*d*x - 2I*c)} / (I*A + B) + a^2*d * \sqrt{\frac{(I*A^2 + 14*A*B - 49*I*B^2)}{(a^4*d^2)}} * e^{(4I*d*x + 4I*c)} * \log(-1/8 * ((a^2*d * e^{(2I*d*x + 2I*c)} - a^2*d) * \sqrt{\frac{(I * e^{(2I*d*x + 2I*c)} + I)}{(e^{(2I*d*x + 2I*c)} - 1)}} * \sqrt{\frac{(I*A^2 + 14*A*B - 49*I*B^2)}{(a^4*d^2)}} + A - 7*I*B) * e^{(-2I*d*x - 2I*c)} / (a^2*d)) - a^2*d * \sqrt{\frac{(I*A^2 + 14*A*B - 49*I*B^2)}{(a^4*d^2)}} * e^{(4I*d*x + 4I*c)} * \log(1/8 * ((a^2*d * e^{(2I*d*x + 2I*c)} - a^2*d) * \sqrt{\frac{(I * e^{(2I*d*x + 2I*c)} + I)}{(e^{(2I*d*x + 2I*c)} - 1)}} * \sqrt{\frac{(I*A^2 + 14*A*B - 49*I*B^2)}{(a^4*d^2)}}) - A + 7*I*B) * e^{(-2I*d*x - 2I*c)} / (a^2*d)) + 2 * (2 * (I*A - 3*B) * e^{(4I*d*x + 4I*c)} - (3*I*A - 7*B) * e^{(2I*d*x + 2I*c)} + I*A - B) * \sqrt{\frac{(I * e^{(2I*d*x + 2I*c)} + I)}{(e^{(2I*d*x + 2I*c)} - 1)}} * e^{(-4I*d*x - 4I*c)} / (a^2*d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\tan^2(c+dx) \cot^{\frac{3}{2}}(c+dx) - 2i \tan(c+dx) \cot^{\frac{3}{2}}(c+dx) - \cot^{\frac{3}{2}}(c+dx)} dx + \int \frac{B \tan(c+dx)}{\tan^2(c+dx) \cot^{\frac{3}{2}}(c+dx) - 2i \tan(c+dx) \cot^{\frac{3}{2}}(c+dx) - \cot^{\frac{3}{2}}(c+dx)} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] -(Integral(A/(tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*(3/2) - 2\*I\*tan(c + d\*x)\*cot(c + d\*x)\*\*(3/2) - cot(c + d\*x)\*\*(3/2)), x) + Integral(B\*tan(c + d\*x)/(tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*(3/2) - 2\*I\*tan(c + d\*x)\*cot(c + d\*x)\*\*(3/2) - cot(c + d\*x)\*\*(3/2)), x))/a\*\*2

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^2\*cot(d\*x + c)^(3/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + a \tan(c + dx) li)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

$$3.529 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=319

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((2+7i)A - (23+2i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{\left((9+5i)A - (25-21i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^2 d}$$

[Out]  $(1/32-1/32*I)*((2+7*I)*A-(23+2*I)*B)*\arctan(-1+2^{1/2}*\cot(d*x+c)^{(1/2)})/a^2/d*2^{1/2}+1/32*((9+5*I)*A+(-25+21*I)*B)*\arctan(1+2^{1/2}*\cot(d*x+c)^{(1/2)})/a^2/d*2^{1/2}+(-1/64+1/64*I)*((7+2*I)*A+(2+23*I)*B)*\ln(1+\cot(d*x+c)-2^{1/2}*\cot(d*x+c)^{(1/2)})/a^2/d*2^{1/2}+(1/64-1/64*I)*((7+2*I)*A+(2+23*I)*B)*\ln(1+\cot(d*x+c)+2^{1/2}*\cot(d*x+c)^{(1/2)})/a^2/d*2^{1/2}+5/8*(I*A-5*B)/a^2/d/\cot(d*x+c)^{(1/2)}+1/8*(3*A+7*I*B)/a^2/d/(I+\cot(d*x+c))/\cot(d*x+c)^{(1/2)}+1/4*(I*A-B)/d/(I*a+a*\cot(d*x+c))^2/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.45, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(\frac{1}{16} - \frac{i}{16}) \left((2+7i)A - (23+2i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^2 d} + \frac{\left((9+5i)A - (25-21i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^2 d} + \frac{5(-5B + IA)}{8a^2 d \sqrt{\cot(c+dx)}} + \frac{3A + 7IB}{8a^2 d \sqrt{\cot(c+dx)} \sqrt{\cot(c+dx) + 1}} + \frac{(\frac{1}{16} - \frac{i}{16}) \left((7+2i)A + (2+23i)B\right) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} a^2 d} + \frac{(\frac{1}{16} - \frac{i}{16}) \left((7+2i)A + (2+23i)B\right) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} a^2 d} + \frac{-B + IA}{4d \sqrt{\cot(c+dx)} \sqrt{\cot(c+dx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^2), x]

[Out]  $((-1/16 + I/16)*((2+7*I)*A - (23+2*I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]/(\text{Sqrt}[2]*a^2*d) + (((9+5*I)*A - (25-21*I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]/(16*\text{Sqrt}[2]*a^2*d) + (5*(I*A - 5*B))/(8*a^2*d*\text{Sqrt}[\text{Cot}[c + d*x]]) + (3*A + (7*I)*B)/(8*a^2*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(I + \text{Cot}[c + d*x])) + (I*A - B)/(4*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(I*a + a*\text{Cot}[c + d*x])^2) - ((1/32 - I/32)*((7+2*I)*A + (2+23*I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a^2*d) + ((1/32 - I/32)*((7+2*I)*A + (2+23*I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(\text{Sqrt}[2]*a^2*d)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free



$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

Rule 3610

$\text{Int}[\frac{((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}{(f_.)*(x_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)])}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

```

Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]

```

### Rule 3677

```

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))^2} dx \\
&= \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A+9iB) - \frac{5}{2}a(iA-B)}{\cot^{\frac{3}{2}}(c+dx)(ia+a \cot(c+dx))} dx}{4a^2} \\
&= \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{5(iA - 5B)}{8a^2 d \sqrt{\cot(c + dx)}} + \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{5(iA - 5B)}{8a^2 d \sqrt{\cot(c + dx)}} + \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{5(iA - 5B)}{8a^2 d \sqrt{\cot(c + dx)}} + \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{5(iA - 5B)}{8a^2 d \sqrt{\cot(c + dx)}} + \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{5(iA - 5B)}{8a^2 d \sqrt{\cot(c + dx)}} + \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{5(iA - 5B)}{8a^2 d \sqrt{\cot(c + dx)}} + \frac{3A + 7iB}{8a^2 d \sqrt{\cot(c + dx)} (i + \cot(c + dx))} + \frac{iA - B}{4d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))} \\
&= \frac{((9 + 5i)A - (25 - 21i)B) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{16\sqrt{2} a^2 d} + \dots
\end{aligned}$$

**Mathematica [A]**

time = 1.56, size = 249, normalized size = 0.78

$$\frac{\sec(c + dx) \cos(dx) + i \sin(dx) \left( \cos(c + dx) \left( (5 - 9i)A + (21 + 25i)B \right) \operatorname{ArcSin}(\cos(c + dx) - \sin(c + dx)) - (1 + i) \left( (7 + 2i)A + (2 + 23i)B \right) \log \left( \cos(c + dx) + \sin(c + dx) + \sqrt{\sin(2c + 2dx)} \right) \right) \left( i \cos(2c) - \sin(2c) \right) \sqrt{\sin(2c + 2dx)} + 2 \left( i \cos(2dx) + \sin(2dx) \right) (5A + 9iB + (5A + 4iB) \cos(2c + 2dx)) + (7iA - 43B) \sin(2c + 2dx)}{32d \sqrt{\cot(c + dx)} \left( A \cos(c + dx) + B \sin(c + dx) \right) (a + ia \tan(c + dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/((Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2), x]
```

```
[Out] (Sec[c + d*x]*(Cos[d*x] + I*Sin[d*x])^2*(Csc[c + d*x]*(((5 - 9*I)*A + (21 + 25*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] - (1 + I)*((7 + 2*I)*A + (2 + 23*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)])])*(I*Cos[2*c] - Sin[2*c])*Sqrt[Sin[2*(c + d*x)]] + 2*(I*Cos[2*d*x] + Sin[2*d*x])*(5
```

$$*A + (9*I)*B + (5*A + (41*I)*B)*\text{Cos}[2*(c + d*x)] + ((7*I)*A - 43*B)*\text{Sin}[2*(c + d*x)]))*(A + B*\text{Tan}[c + d*x])/(32*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^2)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 73.33, size = 13802, normalized size = 43.27

method	result	size
default	Expression too large to display	13802

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 761 vs.  $2(234) = 468$ .  
time = 2.15, size = 761, normalized size = 2.39

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] -1/32*(2*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((I*A^
2 + 2*A*B - I*B^2)/(a^4*d^2))*log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*
d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2
+ 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
2*I*c)/(I*A + B)) - 2*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c
))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*log(-2*((-I*a^2*d*e^(2*I*d*x + 2
*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*
```

$$e^{(-2*I*d*x - 2*I*c)/(I*A + B)} - (a^2*d*e^{(6*I*d*x + 6*I*c)} + a^2*d*e^{(4*I*d*x + 4*I*c)})*\sqrt{(-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2)}*\log(1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} - a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2)} + 7*I*A - 23*B)*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} + (a^2*d*e^{(6*I*d*x + 6*I*c)} + a^2*d*e^{(4*I*d*x + 4*I*c)})*\sqrt{(-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2)}*\log(-1/8*((a^2*d*e^{(2*I*d*x + 2*I*c)} - a^2*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2)} - 7*I*A + 23*B)*e^{(-2*I*d*x - 2*I*c)/(a^2*d)} - 2*(6*(A + 7*I*B)*e^{(6*I*d*x + 6*I*c)} - (A + 33*I*B)*e^{(4*I*d*x + 4*I*c)} - 2*(3*A + 5*I*B)*e^{(2*I*d*x + 2*I*c)} + A + I*B)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))/(a^2*d*e^{(6*I*d*x + 6*I*c)} + a^2*d*e^{(4*I*d*x + 4*I*c)})$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(5/2)/(a+I\*a\*tan(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^2\*cot(d\*x + c)^(5/2)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + a \tan(c + dx) \operatorname{li})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*li)^2),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*li)^2), x)

$$3.530 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=367

$$\frac{\left(\frac{1}{16} - \frac{i}{16}\right) \left((1+29i)A - (6+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{\left((30+28i)A - (7-5i)B\right) \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^3 d}$$

[Out] 1/6\*(A+I\*B)\*cot(d\*x+c)^(7/2)/d/(I\*a+a\*cot(d\*x+c))^3+1/12\*(5\*A+2\*I\*B)\*cot(d\*x+c)^(5/2)/a/d/(I\*a+a\*cot(d\*x+c))^2+7/24\*(4\*A+I\*B)\*cot(d\*x+c)^(3/2)/d/(I\*a^3+a^3\*cot(d\*x+c))+(1/32-1/32\*I)\*((1+29\*I)\*A-(6+I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/32\*((30+28\*I)\*A+(-7+5\*I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+(-1/64+1/64\*I)\*((29+I)\*A+(1+6\*I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+(1/64-1/64\*I)\*((29+I)\*A+(1+6\*I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-5/8\*(6\*A+I\*B)\*cot(d\*x+c)^(1/2)/a^3/d

**Rubi [A]**

time = 0.60, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(b-a)(1+29i)A-(6+i)B \text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^3d} + \frac{(30+28i)A-(7-5i)B \text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}a^3d} + \frac{7(4A+iB)\cot(c+dx)}{24(a^3\cot(c+dx)+a^3)} + \frac{(b-a)(29+iA+(1+6i)B)\log(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1)}{\sqrt{2}a^3d} + \frac{(b-a)(29+iA+(1+6i)B)\log(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1)}{\sqrt{2}a^3d} + \frac{(A+iB)\cot(c+dx)}{6(a^3\cot(c+dx)+a^3)} + \frac{(5A+2iB)\cot(c+dx)}{12a^3\cot(c+dx)+10a^3}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] ((-1/16 + I/16)\*((1 + 29\*I)\*A - (6 + I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*a^3\*d) + (((30 + 28\*I)\*A - (7 - 5\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(16\*Sqrt[2]\*a^3\*d) - (5\*(6\*A + I\*B)\*Sqrt[Cot[c + d\*x]])/(8\*a^3\*d) + ((A + I\*B)\*Cot[c + d\*x]^(7/2))/(6\*d\*(I\*a + a\*Cot[c + d\*x])^3) + ((5\*A + (2\*I)\*B)\*Cot[c + d\*x]^(5/2))/(12\*a\*d\*(I\*a + a\*Cot[c + d\*x])^2) + (7\*(4\*A + I\*B)\*Cot[c + d\*x]^(3/2))/(24\*d\*(I\*a^3 + a^3\*Cot[c + d\*x])) - ((1/32 - I/32)\*((29 + I)\*A + (1 + 6\*I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(Sqrt[2]\*a^3\*d) + ((1/32 - I/32)\*((29 + I)\*A + (1 + 6\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(Sqrt[2]\*a^3\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(m_.))*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3676

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Simp[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \int \frac{\cot^{\frac{7}{2}}(c+dx)(B+A \cot(c+dx))}{(ia+a \cot(c+dx))^3} dx \\
&= \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{\int \frac{\cot^{\frac{5}{2}}(c+dx)(-\frac{7}{2}a(ia-B)+\frac{1}{2}a(13A+iB) \cot(c+dx))}{(ia+a \cot(c+dx))^2} dx}{6a^2} \\
&= \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)(-\frac{5}{2}a(ia-B)+\frac{1}{2}a(13A+iB) \cot(c+dx))}{(ia+a \cot(c+dx))^2} dx}{6a^2} \\
&= \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{7(4A+iB)}{24d(ia+a \cot(c+dx))} \\
&= -\frac{5(6A+iB) \sqrt{\cot(c+dx)}}{8a^3d} + \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} \\
&= -\frac{5(6A+iB) \sqrt{\cot(c+dx)}}{8a^3d} + \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} \\
&= -\frac{5(6A+iB) \sqrt{\cot(c+dx)}}{8a^3d} + \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} \\
&= -\frac{5(6A+iB) \sqrt{\cot(c+dx)}}{8a^3d} + \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} \\
&= -\frac{5(6A+iB) \sqrt{\cot(c+dx)}}{8a^3d} + \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} \\
&= -\frac{((30+28i)A-(7-5i)B) \tan^{-1}\left(1-\sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^3d} + \frac{(A+iB) \cot^{\frac{7}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(5A+2iB) \cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2}
\end{aligned}$$

**Mathematica [A]**

time = 2.03, size = 284, normalized size = 0.77

$$\frac{\sec^2(c+dx) \cos(dx) + i \sin(dx)^2 \left( \frac{1}{2} \cot^2(c+dx) \cos(3dx) - i \sin(3dx) \cos(c+dx) - (145A+19B) \cos(c+dx) + 6(-18A+2B+7(-7A+B) \cos(2(c+dx))) \sin(c+dx) + \sec(c+dx) \left( (28-30)A + (5+75)B \sin(\cos(c+dx)) - \sin(c+dx) - (1+(29+i)A+(1+6)B) \log(\cos(c+dx) + \sin(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)}) \right) \right) \left( \cos(3c) - \sin(3c) \right) \sqrt{\cot(c+dx)}}{32d^2 \sqrt{\cot(c+dx)} (A \cos(c+dx) + B \sin(c+dx)) (a + i a \tan(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3, x]

[Out] (Sec[c + d\*x]^2\*(Cos[d\*x] + I\*Sin[d\*x])^3\*((2\*Cot[c + d\*x]\*(Cos[3\*d\*x] - I\*Sin[3\*d\*x])\*((49\*A + (19\*I)\*B)\*Cos[c + d\*x] - (145\*A + (19\*I)\*B)\*Cos[3\*(c + d\*x)] + 6\*((-19\*I)\*A + 2\*B + 7\*((-7\*I)\*A + B)\*Cos[2\*(c + d\*x)]))\*Sin[c + d\*

$x)))/3 + \text{Csc}[c + d*x]*(((28 - 30*I)*A + (5 + 7*I)*B)*\text{ArcSin}[\text{Cos}[c + d*x] - \text{Sin}[c + d*x]] - (1 + I)*((29 + I)*A + (1 + 6*I)*B)*\text{Log}[\text{Cos}[c + d*x] + \text{Sin}[c + d*x] + \text{Sqrt}[\text{Sin}[2*(c + d*x)]]])*(I*\text{Cos}[3*c] - \text{Sin}[3*c])* \text{Sqrt}[\text{Sin}[2*(c + d*x)]]*(A + B*\text{Tan}[c + d*x]))/(32*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x])*(a + I*a*\text{Tan}[c + d*x])^3)$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 28.93, size = 2577, normalized size = 7.02

method	result	size
default	Expression too large to display	2577

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\text{cot}(d*x+c)^{(3/2)}*(A+B*\text{tan}(d*x+c))/(a+I*a*\text{tan}(d*x+c))^3, x, \text{method}=\_RETURN \text{VERBOSE})$

[Out]  $1/48/a^3/d^{2^{(1/2)}}*(16*2^{(1/2)}*A*\text{cos}(d*x+c)^7-16*I*2^{(1/2)}*\text{sin}(d*x+c)*A*\text{cos}(d*x+c)^6-16*I*2^{(1/2)}*\text{sin}(d*x+c)*A*\text{cos}(d*x+c)^4-28*I*2^{(1/2)}*\text{sin}(d*x+c)*A*\text{cos}(d*x+c)^2+87*A*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticPi}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)})*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}+18*B*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticPi}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)})*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}+3*I*2^{(1/2)}*B*\text{cos}(d*x+c)^3-15*I*2^{(1/2)}*B*\text{cos}(d*x+c)+16*2^{(1/2)}*\text{sin}(d*x+c)*B*\text{cos}(d*x+c)^6+16*I*2^{(1/2)}*B*\text{cos}(d*x+c)^7+7*B*\text{cos}(d*x+c)^2*\text{sin}(d*x+c)*2^{(1/2)}+4*B*\text{cos}(d*x+c)^4*\text{sin}(d*x+c)*2^{(1/2)}-4*I*B*\text{cos}(d*x+c)^5*2^{(1/2)}+3*I*A*\text{cos}(d*x+c)*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}*\text{EllipticPi}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)})+18*A*\text{cos}(d*x+c)^3*2^{(1/2)}-90*A*\text{cos}(d*x+c)*2^{(1/2)}+3*A*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticPi}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)})*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}-21*B*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticF}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2*2^{(1/2)})*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}+3*B*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticPi}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)})*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}+84*I*A*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticF}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2*2^{(1/2)})+3*I*A*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}*((\text{cos}(d*x+c)-1+\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*((-1+\text{cos}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}*\text{EllipticPi}((-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)})-87*I*A*(-\text{cos}(d*x+c)-1-\text{sin}(d*x+c))/\text{sin}(d*x+c)^{(1/2)}*((\text{cos}(d*x+c)-$

$$\begin{aligned}
& 1 + \sin(dx+c) / \sin(dx+c)^{1/2} * ((-1 + \cos(dx+c)) / \sin(dx+c))^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2} \Big) - 3 * I \\
& * B * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& * \text{EllipticPi} \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2} \Big) + 18 * I * B * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2} \Big) \\
& + 3 * B * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2} \Big) * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& + 3 * A * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2} \Big) * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& - 21 * B * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticF} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 * 2^{1/2} \Big) * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& + 87 * A * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2} \Big) * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& + 18 * B * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2} \Big) * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& + 8 * A * \cos(dx+c)^{5/2} + 84 * I * A * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticF} \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 * 2^{1/2} \Big) - 87 * I * A * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2} \Big) - 3 * I * B * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2} \Big) + 18 * I * B * \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} \\
& * \cos(dx+c) * \left( \frac{\cos(dx+c) - 1 + \sin(dx+c)}{\sin(dx+c)} \right)^{1/2} * \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right)^{1/2} * \text{EllipticPi} \\
& \left( \frac{-\cos(dx+c) - 1 - \sin(dx+c)}{\sin(dx+c)} \right)^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2} \Big) * \left( \frac{\cos(dx+c)}{\sin(dx+c)} \right)^{3/2} * \sin(dx+c) / \cos(dx+c)^2
\end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
 expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(276) = 552.  
 time = 1.15, size = 688, normalized size = 1.87



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$-1/96*(3*a^3*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(-2*((I*a^3*d*e^{(2*I*d*x + 2*I*c)} - I*a^3*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)}) - 3*a^3*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(-2*((-I*a^3*d*e^{(2*I*d*x + 2*I*c)} + I*a^3*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)} + (A - I*B)*e^{(2*I*d*x + 2*I*c)}*e^{(-2*I*d*x - 2*I*c)/(I*A + B)}) - 3*a^3*d*\sqrt{(-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(1/8*((a^3*d*e^{(2*I*d*x + 2*I*c)} - a^3*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)} + 29*I*A - 6*B)*e^{(-2*I*d*x - 2*I*c)/(a^3*d)} + 3*a^3*d*\sqrt{(-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)}*e^{(6*I*d*x + 6*I*c)}*\log(-1/8*((a^3*d*e^{(2*I*d*x + 2*I*c)} - a^3*d)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)} - 29*I*A + 6*B)*e^{(-2*I*d*x - 2*I*c)/(a^3*d)} + 2*(2*(73*A + 10*I*B)*e^{(6*I*d*x + 6*I*c)} - (41*A + 14*I*B)*e^{(4*I*d*x + 4*I*c)} - (8*A + 5*I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*e^{(-6*I*d*x - 6*I*c)/(a^3*d)})$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \frac{A \cot^{\frac{3}{2}}(c+dx)}{\tan^3(c+dx) - 3i \tan^2(c+dx) - 3 \tan(c+dx) + i} dx + \int \frac{B \tan(c+dx) \cot^{\frac{3}{2}}(c+dx)}{\tan^3(c+dx) - 3i \tan^2(c+dx) - 3 \tan(c+dx) + i} dx \right) / a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] 
$$I*(\text{Integral}(A*\cot(c + d*x)**(3/2)/(\tan(c + d*x)**3 - 3*I*\tan(c + d*x)**2 - 3*\tan(c + d*x) + I), x) + \text{Integral}(B*\tan(c + d*x)*\cot(c + d*x)**(3/2)/(\tan(c + d*x)**3 - 3*I*\tan(c + d*x)**2 - 3*\tan(c + d*x) + I), x))/a**3$$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(I\*a\*tan(d\*x + c) + a)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) 1i)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3, x)

$$3.531 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=318

$$\frac{((-7+5i)A+2iB)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((-7+5i)A+2iB)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d}$$

[Out] 1/6\*(A+I\*B)\*cot(d\*x+c)^(5/2)/d/(I\*a+a\*cot(d\*x+c))^3+1/12\*(4\*A+I\*B)\*cot(d\*x+c)^(3/2)/a/d/(I\*a+a\*cot(d\*x+c))^2+1/32\*((-7+5\*I)\*A+2\*I\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/32\*((-7+5\*I)\*A+2\*I\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-1/64\*((7+5\*I)\*A-2\*I\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/64\*((7+5\*I)\*A-2\*I\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+5/8\*A\*cot(d\*x+c)^(1/2)/d/(I\*a^3+a^3\*cot(d\*x+c))

**Rubi [A]**

time = 0.51, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3662, 3676, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(2iB - (7-5i)A)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{(2iB - (7-5i)A)\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{16\sqrt{2}a^3d} - \frac{(7+5i)A-2iB}{32\sqrt{2}a^3d} \log\left(\frac{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}\right) + \frac{(7+5i)A-2iB}{32\sqrt{2}a^3d} \log\left(\frac{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}\right) + \frac{5A\sqrt{\cot(c+dx)}}{8d(a^2\cot(c+dx)+a^2)} + \frac{(A+iB)\cot^3(c+dx)}{6d(a\cot(c+dx)+ia)^2} + \frac{(4+iB)\cot^3(c+dx)}{12ad(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^3,x]

[Out] -1/16\*(((7 + 5\*I)\*A + (2\*I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(Sqrt[2]\*a^3\*d) + (((7 + 5\*I)\*A + (2\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(16\*Sqrt[2]\*a^3\*d) + ((A + I\*B)\*Cot[c + d\*x]^(5/2))/(6\*d\*(I\*a + a\*Cot[c + d\*x])^3) + ((4\*A + I\*B)\*Cot[c + d\*x]^(3/2))/(12\*a\*d\*(I\*a + a\*Cot[c + d\*x])^2) + (5\*A\*Sqrt[Cot[c + d\*x]])/(8\*d\*(I\*a^3 + a^3\*Cot[c + d\*x])) - (((7 + 5\*I)\*A - (2\*I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d) + (((7 + 5\*I)\*A - (2\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)])\*(g\_)^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

## Rule 3676

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

## Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx &= \int \frac{\cot^{\frac{5}{2}}(c+dx)(B+A \cot(c+dx))}{(ia+a \cot(c+dx))^3} dx \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)(-\frac{5}{2}a(ia-B)+\frac{1}{2}a(11A-iB) \cot(c+dx))}{(ia+a \cot(c+dx))^2} dx}{6a^2} \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(4A+iB) \cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{\int \sqrt{\cot(c+dx)}}{8d(ia^3-3a^2d)} dx \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(4A+iB) \cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{5A\sqrt{\cot(c+dx)}}{8d(ia^3-3a^2d)} \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(4A+iB) \cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{5A\sqrt{\cot(c+dx)}}{8d(ia^3-3a^2d)} \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(4A+iB) \cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{5A\sqrt{\cot(c+dx)}}{8d(ia^3-3a^2d)} \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(4A+iB) \cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{5A\sqrt{\cot(c+dx)}}{8d(ia^3-3a^2d)} \\
&= \frac{(A+iB) \cot^{\frac{5}{2}}(c+dx)}{6d(ia+a \cot(c+dx))^3} + \frac{(4A+iB) \cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a \cot(c+dx))^2} + \frac{5A\sqrt{\cot(c+dx)}}{8d(ia^3-3a^2d)} \\
&= -\frac{((-7+5i)A+2iB) \tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((-7+5i)A+2iB) \tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d}
\end{aligned}$$

## Mathematica [A]

time = 1.41, size = 258, normalized size = 0.81

$\frac{\cot^2(c+dx)\cos(dx)+i\sin(dx)^2(\cos(c+dx)((5+7i)A+2B)\operatorname{ArcSin}(\cos(c+dx)-\sin(c+dx))-(1+0(1+6i)A+(1-i)B)\log(\cos(c+dx)+\sin(c+dx)+\sqrt{\sin(2c+4d)})}{32a\sqrt{\cot(c+dx)}(A\cos(c+dx)+B\sin(c+dx))(a+ia\tan(c+dx))}$



Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^3*(Csc[c + d*x]*((5 + 7*I)*A + 2*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] - (1 + I)*((1 + 6*I)*A + (1 - I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*(I*Cos[3*c] - Sin[3*c])*Sqrt[Sin[2*(c + d*x)]] + (4*Cos[c + d*x]*(Cos[3*d*x] - I*Sin[3*d*x]))*(6*A + (3*I)*B + 3*(7*A + I*B)*Cos[2*(c + d*x)] + ((19*I)*A - B)*Sin[2*(c + d*x)]))/3*(A + B*Tan[c + d*x]))/(32*d*Sqrt[Cot[c + d*x]]*(A*Cos[c + d*x] + B*Sin[c + d*x]))*(a + I*a*Tan[c + d*x])^3)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 25.10, size = 1581, normalized size = 4.97

method	result	size
default	Expression too large to display	1581

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/a^3/d^2^(1/2)*(16*2^(1/2)*A*cos(d*x+c)^7-16*I*2^(1/2)*sin(d*x+c)*A*cos(d*x+c)^6+16*2^(1/2)*sin(d*x+c)*B*cos(d*x+c)^6+16*I*2^(1/2)*B*cos(d*x+c)^7-16*2^(1/2)*B*cos(d*x+c)^5*sin(d*x+c)-16*I*2^(1/2)*B*cos(d*x+c)^6-8*I*2^(1/2)*B*cos(d*x+c)^5+8*I*2^(1/2)*B*cos(d*x+c)^4-2*I*2^(1/2)*B*cos(d*x+c)^3+2*I*2^(1/2)*B*cos(d*x+c)^2+7*A*cos(d*x+c)^3*2^(1/2)+16*I*2^(1/2)*A*cos(d*x+c)^5*sin(d*x+c)-12*I*2^(1/2)*A*cos(d*x+c)^4*sin(d*x+c)+12*I*2^(1/2)*A*cos(d*x+c)^3*sin(d*x+c)-15*I*2^(1/2)*A*cos(d*x+c)^2*sin(d*x+c)+15*I*2^(1/2)*A*cos(d*x+c)*sin(d*x+c)-4*A*cos(d*x+c)^4*2^(1/2)+21*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*B*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*B*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-18*A*sin(d*x+c)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)-7*A*2^(1/2)*cos(d*x+c)^2-16*2^(1/2)*A*cos(d*x+c)^6+4*A*cos(d*x+c)^5
```

$$c)^5 \cdot 2^{1/2} + 3 \cdot I \cdot A \cdot \sin(d \cdot x + c) \cdot \text{EllipticPi}\left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right) / \sin(d \cdot x + c)^{1/2}, 1/2 + 1/2 \cdot I, 1/2 \cdot 2^{1/2}) \cdot \left(\frac{-1 + \cos(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{\cos(d \cdot x + c) - 1 + \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} - 18 \cdot I \cdot A \cdot \sin(d \cdot x + c) \cdot \text{EllipticPi}\left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right) / \sin(d \cdot x + c)^{1/2}, 1/2 - 1/2 \cdot I, 1/2 \cdot 2^{1/2}) \cdot \left(\frac{-1 + \cos(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{\cos(d \cdot x + c) - 1 + \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} + 3 \cdot I \cdot B \cdot \sin(d \cdot x + c) \cdot \text{EllipticPi}\left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right) / \sin(d \cdot x + c)^{1/2}, 1/2 + 1/2 \cdot I, 1/2 \cdot 2^{1/2}) \cdot \left(\frac{-1 + \cos(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{\cos(d \cdot x + c) - 1 + \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} + 3 \cdot I \cdot B \cdot \sin(d \cdot x + c) \cdot \text{EllipticPi}\left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right) / \sin(d \cdot x + c)^{1/2}, 1/2 - 1/2 \cdot I, 1/2 \cdot 2^{1/2}) \cdot \left(\frac{-1 + \cos(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{\cos(d \cdot x + c) - 1 + \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} - 6 \cdot I \cdot B \cdot \sin(d \cdot x + c) \cdot \text{EllipticF}\left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right) / \sin(d \cdot x + c)^{1/2}, 1/2 \cdot 2^{1/2}) \cdot \left(\frac{-1 + \cos(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{\cos(d \cdot x + c) - 1 + \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{-\cos(d \cdot x + c) - 1 - \sin(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} \cdot \left(\frac{\cos(d \cdot x + c) + 1}{\sin(d \cdot x + c)}\right)^2 \cdot (-1 + \cos(d \cdot x + c)) \cdot \left(\frac{\cos(d \cdot x + c)}{\sin(d \cdot x + c)}\right)^{1/2} / \sin(d \cdot x + c)^3 / \cos(d \cdot x + c)$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 683 vs.  $2(247) = 494$ .

time = 1.66, size = 683, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{96} \cdot (3 \cdot a^3 \cdot d \cdot \sqrt{(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) / (a^6 \cdot d^2)}) \cdot e^{(6 \cdot I \cdot d \cdot x + 6 \cdot I \cdot c)} \cdot \log(-2 \cdot ((a^3 \cdot d \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} - a^3 \cdot d) \cdot \sqrt{(I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I)}) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} - 1)) \cdot \sqrt{(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) / (a^6 \cdot d^2)}) + (A - I \cdot B) \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} \cdot e^{(-2 \cdot I \cdot d \cdot x - 2 \cdot I \cdot c)} / (I \cdot A + B) - 3 \cdot a^3 \cdot d \cdot \sqrt{(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) / (a^6 \cdot d^2)} \cdot e^{(6 \cdot I \cdot d \cdot x + 6 \cdot I \cdot c)} \cdot \log(2 \cdot ((a^3 \cdot d \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} - a^3 \cdot d) \cdot \sqrt{(I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I)}) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} - 1)))$

c) - 1)) \* sqrt((-I\*A^2 - 2\*A\*B + I\*B^2)/(a^6\*d^2)) - (A - I\*B)\*e^(2\*I\*d\*x + 2\*I\*c)) \* e^(-2\*I\*d\*x - 2\*I\*c)/(I\*A + B) - 3\*a^3\*d\*sqrt((36\*I\*A^2 + 12\*A\*B - I\*B^2)/(a^6\*d^2)) \* e^(6\*I\*d\*x + 6\*I\*c) \* log(-1/8\*((a^3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - a^3\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)) \* sqrt((36\*I\*A^2 + 12\*A\*B - I\*B^2)/(a^6\*d^2)) + 6\*A - I\*B)\*e^(-2\*I\*d\*x - 2\*I\*c)/(a^3\*d)) + 3\*a^3\*d\*sqrt((36\*I\*A^2 + 12\*A\*B - I\*B^2)/(a^6\*d^2)) \* e^(6\*I\*d\*x + 6\*I\*c) \* log(1/8\*((a^3\*d\*e^(2\*I\*d\*x + 2\*I\*c) - a^3\*d)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)) \* sqrt((36\*I\*A^2 + 12\*A\*B - I\*B^2)/(a^6\*d^2)) - 6\*A + I\*B)\*e^(-2\*I\*d\*x - 2\*I\*c)/(a^3\*d)) - 2\*(2\*(10\*I\*A - B)\*e^(6\*I\*d\*x + 6\*I\*c) - (14\*I\*A + B)\*e^(4\*I\*d\*x + 4\*I\*c) - (5\*I\*A - 2\*B)\*e^(2\*I\*d\*x + 2\*I\*c) - I\*A + B)\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))) \* e^(-6\*I\*d\*x - 6\*I\*c)/(a^3\*d)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \sqrt{\cot(c+dx)}}{\tan^3(c+dx) - 3i \tan^2(c+dx) - 3 \tan(c+dx) + i} dx + \int \frac{B \tan(c+dx) \sqrt{\cot(c+dx)}}{\tan^3(c+dx) - 3i \tan^2(c+dx) - 3 \tan(c+dx) + i} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] I\*(Integral(A\*sqrt(cot(c + d\*x))/(tan(c + d\*x)\*\*3 - 3\*I\*tan(c + d\*x)\*\*2 - 3\*tan(c + d\*x) + I), x) + Integral(B\*tan(c + d\*x)\*sqrt(cot(c + d\*x))/(tan(c + d\*x)\*\*3 - 3\*I\*tan(c + d\*x)\*\*2 - 3\*tan(c + d\*x) + I), x))/a\*\*3

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/(I\*a\*tan(d\*x + c) + a)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c+dx)} (A + B \tan(c+dx))}{(a + a \tan(c+dx) i)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3,x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^3, x)

$$3.532 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=316

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((1+i)A+B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) ((1+i)A+B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d}$$

[Out] 1/6\*(A+I\*B)\*cot(d\*x+c)^(3/2)/d/(I\*a+a\*cot(d\*x+c))^3+(1/32+1/32\*I)\*((1+I)\*A+B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+(1/32+1/32\*I)\*((1+I)\*A+B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/64\*(2\*I\*A+(1-I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-1/64\*(2\*I\*A+(1-I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/4\*A\*cot(d\*x+c)^(1/2)/a/d/(I\*a+a\*cot(d\*x+c))^2+1/8\*(2\*I\*A+B)\*cot(d\*x+c)^(1/2)/d/(I\*a^3+a^3\*cot(d\*x+c))

**Rubi [A]**

time = 0.49, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) (B + (1+i)A) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) (B + (1+i)A) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} a^3 d} + \frac{(B+2iA)\sqrt{\cot(c+dx)}}{8d(a^3 \cot(c+dx) + ia^3)} - \frac{(2iA+(1-i)B) \log(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1)}{32\sqrt{2} a^3 d} - \frac{(2iA+(1-i)B) \log(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1)}{32\sqrt{2} a^3 d} + \frac{(A+iB) \cot^2(c+dx)}{64d(a \cot(c+dx) + ia)} + \frac{A\sqrt{\cot(c+dx)}}{32d(a \cot(c+dx) + ia)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out] ((-1/16 - I/16)\*((1 + I)\*A + B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]])/(Sqrt[2]\*a^3\*d) + ((1/16 + I/16)\*((1 + I)\*A + B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])/(Sqrt[2]\*a^3\*d) + ((A + I\*B)\*Cot[c + d\*x]^(3/2))/(6\*d\*(I\*a + a\*Cot[c + d\*x])^3) + (A\*Sqrt[Cot[c + d\*x]])/(4\*a\*d\*(I\*a + a\*Cot[c + d\*x])^2) + (((2\*I)\*A + B)\*Sqrt[Cot[c + d\*x]])/(8\*d\*(I\*a^3 + a^3\*Cot[c + d\*x])) + (((2\*I)\*A + (1 - I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d) - (((2\*I)\*A + (1 - I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \rightarrow \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3676

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^3} dx &= \int \frac{\cot^{\frac{3}{2}}(c + dx)(B + A \cot(c + dx))}{(ia + a \cot(c + dx))^3} dx \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{\int \frac{\sqrt{\cot(c + dx)} (-\frac{3}{2}a(ia-B) + \frac{3}{2}a(3A-ia))}{(ia+a \cot(c+dx))^2} dx}{6a^2} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{\int \frac{-3ia}{\sqrt{\cot(c + dx)}} dx}{8d(ia^3)} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{(2iA + ia^3)}{8d(ia^3)} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{(2iA + ia^3)}{8d(ia^3)} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{(2iA + ia^3)}{8d(ia^3)} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{(2iA + ia^3)}{8d(ia^3)} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{(2iA + ia^3)}{8d(ia^3)} \\
&= \frac{(A + iB) \cot^{\frac{3}{2}}(c + dx)}{6d(ia + a \cot(c + dx))^3} + \frac{A \sqrt{\cot(c + dx)}}{4ad(ia + a \cot(c + dx))^2} + \frac{(2iA + ia^3)}{8d(ia^3)} \\
&= \frac{(\frac{1}{16} + \frac{i}{16}) ((1 + i)A + B) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{\sqrt{2} a^3 d} + \dots
\end{aligned}$$

### Mathematica [A]

time = 2.20, size = 272, normalized size = 0.86

$$\frac{e^{-4i(c+dx)} \left( (A + iB + Ae^{2i(c+dx)} - 2iBe^{2i(c+dx)}) (-1 - 2e^{2i(c+dx)} + e^{4i(c+dx)} + 2e^{6i(c+dx)}) - 3Ae^{4i(c+dx)} \sqrt{-1 + e^{4i(c+dx)}} \operatorname{ArcTan} \left( \sqrt{-1 + e^{4i(c+dx)}} \right) - 6(A - iB) e^{6i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \sqrt{1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}} \right) \right) \sqrt{\cot(c+dx)} \sec(c+dx) (\cos(3(c+dx)) - i \sin(3(c+dx)))}{96a^3 d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out] (((A + I\*B + A\*E^((2\*I)\*(c + d\*x)) - (2\*I)\*B\*E^((2\*I)\*(c + d\*x)))\*(-1 - 2\*E^((2\*I)\*(c + d\*x)) + E^((4\*I)\*(c + d\*x)) + 2\*E^((6\*I)\*(c + d\*x)))) - 3\*A\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((4\*I)\*(c + d\*x))]\*ArcTan[Sqrt[-1 + E^((4\*I)\*(c + d\*x))]]) - 6\*(A - I\*B)\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))]/(1

$$+ E^{((2*I)*(c + d*x))}] * \text{Sqrt}[\text{Cot}[c + d*x]] * \text{Sec}[c + d*x] * (\text{Cos}[3*(c + d*x)] - I * \text{Sin}[3*(c + d*x)]) / (96*a^3*d * E^{((4*I)*(c + d*x))})$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 24.88, size = 15195, normalized size = 48.09

method	result	size
default	Expression too large to display	15195

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs.  $2(247) = 494$ .

time = 1.46, size = 635, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] 1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*l
og(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (
A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sq
r t((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((-I*a^3*d
e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I
d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 24*a^3*d*sqrt(-1/64*I*A^2/(
```



$$a^6 d^2) e^{(6 I d x + 6 I c)} \log\left(\frac{1}{8} (8 a^3 d e^{(2 I d x + 2 I c)} - a^3 d) \sqrt{\frac{I e^{(2 I d x + 2 I c)} + I}{e^{(2 I d x + 2 I c)} - 1}} \sqrt{-\frac{1}{64} I A^2 / (a^6 d^2)} + I A\right) e^{(-2 I d x - 2 I c)} / (a^3 d) - 24 a^3 d \sqrt{-\frac{1}{64} I A^2 / (a^6 d^2)} e^{(6 I d x + 6 I c)} \log\left(-\frac{1}{8} (8 a^3 d e^{(2 I d x + 2 I c)} - a^3 d) \sqrt{\frac{I e^{(2 I d x + 2 I c)} + I}{e^{(2 I d x + 2 I c)} - 1}} \sqrt{-\frac{1}{64} I A^2 / (a^6 d^2)} - I A\right) e^{(-2 I d x - 2 I c)} / (a^3 d) + 2 (2 (A - 2 I B) e^{(6 I d x + 6 I c)} + (A + 4 I B) e^{(4 I d x + 4 I c)} - (2 A - I B) e^{(2 I d x + 2 I c)} - A - I B) \sqrt{\frac{I e^{(2 I d x + 2 I c)} + I}{e^{(2 I d x + 2 I c)} - 1}} e^{(-6 I d x - 6 I c)} / (a^3 d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \frac{A}{\tan^3(c+dx) \sqrt{\cot(c+dx)} - 3 \tan^2(c+dx) \sqrt{\cot(c+dx)} - 3 \tan(c+dx) \sqrt{\cot(c+dx)} + 1} dx + \int \frac{B \tan(c+dx)}{\tan^3(c+dx) \sqrt{\cot(c+dx)} - 3 \tan^2(c+dx) \sqrt{\cot(c+dx)} - 3 \tan(c+dx) \sqrt{\cot(c+dx)} + 1} dx \right) / a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] I\*(Integral(A/(tan(c + d\*x)\*\*3\*sqrt(cot(c + d\*x)) - 3\*I\*tan(c + d\*x)\*\*2\*sqrt(cot(c + d\*x)) - 3\*tan(c + d\*x)\*sqrt(cot(c + d\*x)) + I\*sqrt(cot(c + d\*x))), x) + Integral(B\*tan(c + d\*x)/(tan(c + d\*x)\*\*3\*sqrt(cot(c + d\*x)) - 3\*I\*tan(c + d\*x)\*\*2\*sqrt(cot(c + d\*x)) - 3\*tan(c + d\*x)\*sqrt(cot(c + d\*x)) + I\*sqrt(cot(c + d\*x))), x))/a\*\*3

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^3\*sqrt(cot(d\*x + c))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + d x)}{\sqrt{\cot(c + d x)} (a + a \tan(c + d x) i)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^3),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*1i)^3), x)

$$3.533 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=308

$$\frac{((1+i)A+2B)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((1+i)A+2B)\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{(A+2B)\text{ArcTan}\left(\frac{1+\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{16\sqrt{2}a^3d}$$

[Out] 1/32\*((1+I)\*A+2\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/32\*((1+I)\*A+2\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-1/64\*((-1+I)\*A+2\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/64\*((-1+I)\*A+2\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/6\*(A+I\*B)\*cot(d\*x+c)^(1/2)/d/(I\*a+a\*cot(d\*x+c))^3+1/12\*(2\*I\*A+B)\*cot(d\*x+c)^(1/2)/a/d/(I\*a+a\*cot(d\*x+c))^2+1/8\*A\*cot(d\*x+c)^(1/2)/d/(I\*a^3+a^3\*cot(d\*x+c))

**Rubi [A]**

time = 0.48, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3676, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(2B+(1+i)A)\text{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{16\sqrt{2}a^3d} + \frac{(2B+(1+i)A)\text{ArcTan}\left(\frac{1+\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{16\sqrt{2}a^3d} - \frac{(2B-(1-i)A)\log\left(\frac{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{32\sqrt{2}a^3d} + \frac{(2B-(1-i)A)\log\left(\frac{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{32\sqrt{2}a^3d} + \frac{A\sqrt{\cot(c+dx)}}{8d(a^2\cot(c+dx)+ia^2)} + \frac{(B+2A)\sqrt{\cot(c+dx)}}{12ad(a\cot(c+dx)+ia)^2} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3),x]

[Out] -1/16\*(((1 + I)\*A + 2\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(Sqrt[2]\*a^3\*d) + (((1 + I)\*A + 2\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(16\*Sqrt[2]\*a^3\*d) + ((A + I\*B)\*Sqrt[Cot[c + d\*x]])/(6\*d\*(I\*a + a\*Cot[c + d\*x])^3) + (((2\*I)\*A + B)\*Sqrt[Cot[c + d\*x]])/(12\*a\*d\*(I\*a + a\*Cot[c + d\*x])^2) + (A\*Sqrt[Cot[c + d\*x]])/(8\*d\*(I\*a^3 + a^3\*Cot[c + d\*x])) - (((-1 + I)\*A + 2\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d) + (((-1 + I)\*A + 2\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \rightarrow \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3676

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x
])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*
A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} dx &= \int \frac{\sqrt{\cot(c + dx)} (B + A \cot(c + dx))}{(ia + a \cot(c + dx))^3} dx \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{\int \frac{-\frac{1}{2}a(iA - B) + \frac{1}{2}a(7A - 5iB) \cot(c + dx)}{\sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^2} dx}{6a^2} \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{(2iA + B) \sqrt{\cot(c + dx)}}{12ad(ia + a \cot(c + dx))^2} + \frac{\int \frac{-3ia}{\sqrt{\cot(c + dx)}} dx}{8d(ia^3)} \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{(2iA + B) \sqrt{\cot(c + dx)}}{12ad(ia + a \cot(c + dx))^2} + \frac{A \sqrt{\cot(c + dx)}}{8d(ia^3)} \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{(2iA + B) \sqrt{\cot(c + dx)}}{12ad(ia + a \cot(c + dx))^2} + \frac{A \sqrt{\cot(c + dx)}}{8d(ia^3)} \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{(2iA + B) \sqrt{\cot(c + dx)}}{12ad(ia + a \cot(c + dx))^2} + \frac{A \sqrt{\cot(c + dx)}}{8d(ia^3)} \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{(2iA + B) \sqrt{\cot(c + dx)}}{12ad(ia + a \cot(c + dx))^2} + \frac{A \sqrt{\cot(c + dx)}}{8d(ia^3)} \\
&= \frac{(A + iB) \sqrt{\cot(c + dx)}}{6d(ia + a \cot(c + dx))^3} + \frac{(2iA + B) \sqrt{\cot(c + dx)}}{12ad(ia + a \cot(c + dx))^2} + \frac{A \sqrt{\cot(c + dx)}}{8d(ia^3)} \\
&= \frac{((1 + i)A + 2B) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{16\sqrt{2} a^3 d} + \frac{((1 + i)A \sqrt{\cot(c + dx)})}{8d(ia^3)}
\end{aligned}$$

### Mathematica [A]

time = 2.07, size = 274, normalized size = 0.89

$$\frac{e^{-4i(c+dx)} \left( (1 - 2e^{2i(c+dx)} - e^{4i(c+dx)} + 2e^{6i(c+dx)}) (B - B e^{2i(c+dx)} - iA(1 + 2e^{2i(c+dx)})) - 3B e^{6i(c+dx)} \sqrt{-1 + e^{4i(c+dx)}} \operatorname{ArcTan} \left( \sqrt{-1 + e^{4i(c+dx)}} \right) + 6(iA + B) e^{6i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \sqrt{1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{-1 + e^{2i(c+dx)}}{1 + e^{2i(c+dx)}} \right) \right) \sqrt{\cot(c + dx)} \operatorname{sech}(c + dx) (\cos(3(c + dx)) - i \sin(3(c + dx)))}{96a^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/((Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out] (((1 - 2\*E^((2\*I)\*(c + d\*x)) - E^((4\*I)\*(c + d\*x)) + 2\*E^((6\*I)\*(c + d\*x))) \* (B - B\*E^((2\*I)\*(c + d\*x)) - I\*A\*(1 + 2\*E^((2\*I)\*(c + d\*x)))) - 3\*B\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((4\*I)\*(c + d\*x))]\*ArcTan[Sqrt[-1 + E^((4\*I)\*(c + d\*x))]]) + 6\*(I\*A + B)\*E^((6\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Sqrt[1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[Sqrt[(-1 + E^((2\*I)\*(c + d\*x)))/(1 + E

$$\frac{\sqrt{\cot(c+dx)} \sec(c+dx) (\cos(3(c+dx)) - \sin(3(c+dx)))}{96 a^3 d e^{4(c+dx)}}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 22.40, size = 13448, normalized size = 43.66

method	result	size
default	Expression too large to display	13448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURN
VERBOSE)
```

[Out] result too large to display

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorit
hm="maxima")
```

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 637 vs.  $2(247) = 494$ .

time = 1.69, size = 637, normalized size = 2.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorit
hm="fricas")
```

[Out] 
$$\begin{aligned} & -\frac{1}{96} (3a^3 d \sqrt{(-IA^2 - 2AB + IB^2)/(a^6 d^2)}) e^{6Ix + 6Ic} \\ & * \log(-2((a^3 d e^{2Ix + 2Ic}) - a^3 d) \sqrt{(Ie^{2Ix + 2Ic} + I)/(e^{2Ix + 2Ic} - 1)}) \sqrt{(-IA^2 - 2AB + IB^2)/(a^6 d^2)} + (A \\ & - IB) e^{2Ix + 2Ic}) e^{-2Ix - 2Ic} / (IA + B) - 3a^3 d \sqrt{ \\ & ((-IA^2 - 2AB + IB^2)/(a^6 d^2))} e^{6Ix + 6Ic} * \log(2((a^3 d e^{2 \\ & Ix + 2Ic}) - a^3 d) \sqrt{(Ie^{2Ix + 2Ic} + I)/(e^{2Ix + 2I \\ & c} - 1)}) \sqrt{(-IA^2 - 2AB + IB^2)/(a^6 d^2)} - (A - IB) e^{2Ix + \\ & 2Ic}) e^{-2Ix - 2Ic} / (IA + B) - 24a^3 d \sqrt{-1/64 IB^2 / (a^6 d} \end{aligned}$$

$$\begin{aligned} &^2))e^{(6I*d*x + 6I*c)}\log(1/8*(8*(a^3*d*e^{(2I*d*x + 2I*c)} - a^3*d)*\sqrt{ \\ &t((I*e^{(2I*d*x + 2I*c)} + I)/(e^{(2I*d*x + 2I*c)} - 1))*\sqrt{-1/64*I*B^2/( \\ &a^6*d^2)} + I*B)*e^{(-2I*d*x - 2I*c)/(a^3*d)} + 24*a^3*d*\sqrt{-1/64*I*B^2/ \\ &(a^6*d^2)})*e^{(6I*d*x + 6I*c)}\log(-1/8*(8*(a^3*d*e^{(2I*d*x + 2I*c)} - a^3 \\ &*d)*\sqrt{((I*e^{(2I*d*x + 2I*c)} + I)/(e^{(2I*d*x + 2I*c)} - 1))*\sqrt{-1/64* \\ &I*B^2/(a^6*d^2)} - I*B)*e^{(-2I*d*x - 2I*c)/(a^3*d)} + 2*(2*(2I*A + B)*e^{ \\ &(6I*d*x + 6I*c)} - (4I*A + 5*B)*e^{(4I*d*x + 4I*c)} - (I*A - 4*B)*e^{(2I* \\ &d*x + 2I*c)} + I*A - B)*\sqrt{((I*e^{(2I*d*x + 2I*c)} + I)/(e^{(2I*d*x + 2I* \\ &c)} - 1)))*e^{(-6I*d*x - 6I*c)/(a^3*d)} \end{aligned}$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+I\*a\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^3\*cot(d\*x + c)^(3/2)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + a \tan(c + dx) li)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*li)^3),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*li)^3), x)

$$3.534 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=310

$$\frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{16\sqrt{2} a^3 d} + \frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{16\sqrt{2} a^3 d}$$

[Out] 1/32\*(2\*A+(5-7\*I)\*B)\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/32\*(2\*A+(5-7\*I)\*B)\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)-1/64\*(2\*A-(5+7\*I)\*B)\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/64\*(2\*A-(5+7\*I)\*B)\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/a^3/d\*2^(1/2)+1/6\*(I\*A-B)\*cot(d\*x+c)^(1/2)/d/(I\*a+a\*cot(d\*x+c))^3+1/12\*(A+4\*I\*B)\*cot(d\*x+c)^(1/2)/a/d/(I\*a+a\*cot(d\*x+c))^2+5/8\*B\*cot(d\*x+c)^(1/2)/d/(I\*a^3+a^3\*cot(d\*x+c))

**Rubi [A]**

time = 0.50, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3662, 3677, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{16\sqrt{2} a^3 d} + \frac{(2A + (5 - 7i)B) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c + dx)} + 1\right)}{16\sqrt{2} a^3 d} - \frac{(2A - (5 + 7i)B) \log\left(\cot(c + dx) - \sqrt{2} \sqrt{\cot(c + dx)} + 1\right)}{32\sqrt{2} a^3 d} + \frac{(2A - (5 + 7i)B) \log\left(\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)} + 1\right)}{32\sqrt{2} a^3 d} + \frac{5B \sqrt{\cot(c + dx)}}{8d(a^2 \cot(c + dx) + ia^2)} + \frac{(-B + ia) \sqrt{\cot(c + dx)}}{6d(a \cot(c + dx) + ia)^2} + \frac{(A + 4iB) \sqrt{\cot(c + dx)}}{12ad(a \cot(c + dx) + ia)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^3),x]

[Out] -1/16\*((2\*A + (5 - 7\*I)\*B)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(Sqrt[2]\*a^3\*d) + ((2\*A + (5 - 7\*I)\*B)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(16\*Sqrt[2]\*a^3\*d) + ((I\*A - B)\*Sqrt[Cot[c + d\*x]])/(6\*d\*(I\*a + a\*Cot[c + d\*x])^3) + ((A + (4\*I)\*B)\*Sqrt[Cot[c + d\*x]])/(12\*a\*d\*(I\*a + a\*Cot[c + d\*x])^2) + (5\*B\*Sqrt[Cot[c + d\*x]])/(8\*d\*(I\*a^3 + a^3\*Cot[c + d\*x])) - ((2\*A - (5 + 7\*I)\*B)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d) + ((2\*A - (5 + 7\*I)\*B)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(32\*Sqrt[2]\*a^3\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free



$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \rightarrow \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3677



Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/((Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Csc[c + d*x]^2*Sec[c + d*x]^3*(A*Cos[c + d*x] + B*Sin[c + d*x])*(A + (19*I)*B - (A + (19*I)*B)*Cos[4*(c + d*x)] + 6*A*Cos[3*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] - (15 + 21*I)*B*Cos[3*(c + d*x)]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]] + (6*I)*A*Sin[2*(c + d*x)] - 12*B*Sin[2*(c + d*x)] + (6*I)*A*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Sin[3*(c + d*x)] + (21 - 15*I)*B*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Sin[3*(c + d*x)] + (3 + 3*I)*((1 + I)*A + (6 - I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]) - (3*I)*A*Sin[4*(c + d*x)] + 21*B*Sin[4*(c + d*x)])/(96*a^3*d*(I + Cot[c + d*x])^3*(A + B*Tan[c + d*x]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 25.59, size = 17109, normalized size = 55.19

method	result	size
default	Expression too large to display	17109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs.  $2(245) = 490$ .

time = 1.57, size = 685, normalized size = 2.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*
log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) +
(A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sq
rt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((-I*a^3*d
*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I
*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sqrt((-I*A^2 - 12*
A*B + 36*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*d*e^(2*I*d*x +
2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*sqrt((-I*A^2 - 12*A*B + 36*I*B^2)/(a^6*d^2)) + I*A + 6*B)*e^(-2*I*d*x - 2
*I*c)/(a^3*d)) + 3*a^3*d*sqrt((-I*A^2 - 12*A*B + 36*I*B^2)/(a^6*d^2))*e^(6*
I*d*x + 6*I*c)*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 12*A*B + 36*I*
B^2)/(a^6*d^2)) - I*A - 6*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 2*(2*(A + 10*I
*B)*e^(6*I*d*x + 6*I*c) - (5*A + 26*I*B)*e^(4*I*d*x + 4*I*c) + (4*A + 7*I*B
)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)/((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2)
), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + a \tan(c + dx) \operatorname{li})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*li)^3),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*li)^3), x)

$$3.535 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=367

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((1+6i)A - (29+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{\left((5-7i)A + (28+30i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^3 d}$$

[Out]  $(-1/32-1/32*I)*((1+6*I)*A-(29+I)*B)*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+1/32*((5-7*I)*A+(28+30*I)*B)*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+(1/64+1/64*I)*((6+I)*A+(1+29*I)*B)*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}-(1/64+1/64*I)*((6+I)*A+(1+29*I)*B)*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/a^3/d*2^{(1/2)}+5/8*(A+6*I*B)/a^3/d/\cot(d*x+c)^{(1/2)}+1/6*(I*A-B)/d/(I*a+a*\cot(d*x+c))^3/\cot(d*x+c)^{(1/2)}+1/12*(2*A+5*I*B)/a/d/(I*a+a*\cot(d*x+c))^2/\cot(d*x+c)^{(1/2)}-7/24*(I*A-4*B)/d/(I*a^3+a^3*\cot(d*x+c))/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.60, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3662, 3677, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((1+6i)A - (29+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{\left((5-7i)A + (28+30i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{16\sqrt{2} a^3 d} + \frac{7iAB + iA^2}{24d\sqrt{\cot(c+dx)}(a^3 \cot(c+dx) + a^3)} + \frac{5A + 6B}{16d\sqrt{\cot(c+dx)}} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((1+6i)A - (29+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) \left((1+6i)A - (29+i)B\right) \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} a^3 d} + \frac{2A + 5B}{16d\sqrt{\cot(c+dx)}(a^3 \cot(c+dx) + a^3)} + \frac{-B + iA}{16d\sqrt{\cot(c+dx)}(a^3 \cot(c+dx) + a^3)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(7/2)\*(a + I\*a\*Tan[c + d\*x])^3), x]

[Out]  $((1/16 + I/16)*((1+6*I)*A - (29+I)*B)*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]/(\text{Sqrt}[2]*a^3*d) + (((5-7*I)*A + (28+30*I)*B)*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]/(16*\text{Sqrt}[2]*a^3*d) + (5*(A + (6*I)*B))/(8*a^3*d*\text{Sqrt}[\text{Cot}[c + d*x]]) + (I*A - B)/(6*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(I*a + a*\text{Cot}[c + d*x])^3) + (2*A + (5*I)*B)/(12*a*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(I*a + a*\text{Cot}[c + d*x])^2) - (7*(I*A - 4*B))/(24*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(I*a^3 + a^3*\text{Cot}[c + d*x])) + ((1/32 + I/32)*((6+I)*A + (1+29*I)*B)*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]/(\text{Sqrt}[2]*a^3*d) - ((1/32 + I/32)*((6+I)*A + (1+29*I)*B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]/(\text{Sqrt}[2]*a^3*d)$

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

NeQ[c^2 + d^2, 0]

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3677

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{7}{2}}(c + dx)(a + ia \tan(c + dx))^3} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(ia + a \cot(c + dx))^3} dx \\
&= \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \int \frac{-\frac{1}{2}a(A+13iB) - \frac{7}{2}a(ia-B)}{\cot^{\frac{3}{2}}(c+dx)(ia+a \cot(c+dx))^3} dx \\
&= \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{5(A + 6iB)}{8a^3 d \sqrt{\cot(c + dx)}} + \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{5(A + 6iB)}{8a^3 d \sqrt{\cot(c + dx)}} + \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{5(A + 6iB)}{8a^3 d \sqrt{\cot(c + dx)}} + \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{5(A + 6iB)}{8a^3 d \sqrt{\cot(c + dx)}} + \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{5(A + 6iB)}{8a^3 d \sqrt{\cot(c + dx)}} + \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{5(A + 6iB)}{8a^3 d \sqrt{\cot(c + dx)}} + \frac{iA - B}{6d \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3} \\
&= \frac{((5 - 7i)A + (28 + 30i)B) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{16\sqrt{2} a^3 d} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3}
\end{aligned}$$

**Mathematica [A]**

time = 2.39, size = 280, normalized size = 0.76

$$\frac{((5 - 7i)A + (28 + 30i)B) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{16\sqrt{2} a^3 d} + \frac{2A + 5B}{12ad \sqrt{\cot(c + dx)} (ia + a \cot(c + dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3), x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^3*((2*(Cos[3*d*x] - I*Sin[3*d*x]))*(9*A + (33*I)*B)*Cos[c + d*x] + 21*(A + (7*I)*B)*Cos[3*(c + d*x)] + (2*I)*(
```

$$19A + (97I)B + (19A + (145I)B)\cos[2(c + dx)]\sin[c + dx]/3 - I \cdot \csc[c + dx] \cdot ((7 + 5I)A - (30 - 28I)B) \cdot \arcsin[\cos[c + dx] - \sin[c + dx]] + (1 - I) \cdot ((6 + I)A + (1 + 29I)B) \cdot \log[\cos[c + dx] + \sin[c + dx] + \sqrt{\sin[2(c + dx)]}] \cdot (\cos[3c] + I\sin[3c]) \cdot \sqrt{\sin[2(c + dx)]} \cdot (A + B \tan[c + dx]) / (32d \sqrt{\cot[c + dx]} \cdot (A \cos[c + dx] + B \sin[c + dx])) \cdot (a + I a \tan[c + dx])^3$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 77.66, size = 18612, normalized size = 50.71

method	result	size
default	Expression too large to display	18612

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 783 vs.  $2(274) = 548$ .

time = 1.60, size = 783, normalized size = 2.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*log(-2*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*
```

```

sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*log(2*((a^3*d*e^(2*I*d*x + 2*I*c)
- a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((
-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I
*d*x - 2*I*c)/(I*A + B)) + 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x
+ 6*I*c))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2))*log(1/8*((a^3*d*
e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2)) + 6*A + 29*I
*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^
(6*I*d*x + 6*I*c))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2))*log(-1/
8*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2)) -
6*A - 29*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2*(10*I*A - 73*B)*e^(8*I*d
*x + 8*I*c) + 3*(-2*I*A + 35*B)*e^(6*I*d*x + 6*I*c) - (19*I*A - 49*B)*e^(4*
I*d*x + 4*I*c) + 3*(2*I*A - 3*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(a^3*d*e^(8*I*d*x + 8*I*c
) + a^3*d*e^(6*I*d*x + 6*I*c))

```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)/((I*a*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2)
), x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{7/2} (a + a \tan(c + dx) i)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

$$3.536 \quad \int \cot^2(c+dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$$

Optimal. Leaf size=198

$$\frac{(1+i)\sqrt{a} (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d} + \frac{2(13A - 5iB)\sqrt{\cot(c+dx)}}{d}$$

[Out]  $(-1-I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-2/15*(I*A+5*B)*\cot(d*x+c)^{(3/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d-2/5*A*\cot(d*x+c)^{(5/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d+2/15*(13*A-5*I*B)*\cot(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

Rubi [A]

time = 0.42, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {4326, 3679, 12, 3625, 211}

$$\frac{2(5B + iA)\cot^{\frac{3}{2}}(c + dx)\sqrt{a + ia \tan(c + dx)}}{15d} + \frac{2(13A - 5iB)\sqrt{\cot(c + dx)}\sqrt{a + ia \tan(c + dx)}}{15d} - \frac{(1+i)\sqrt{a} (A - iB)\sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a + ia \tan(c+dx)}} \right)}{d} - \frac{2A \cot^{\frac{3}{2}}(c + dx)\sqrt{a + ia \tan(c + dx)}}{5d}$$

Antiderivative was successfully verified.

[In] `Int[Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $((-1 - I)*\operatorname{Sqrt}[a]*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d + (2*(13*A - (5*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d) - (2*(I*A + 5*B)*\operatorname{Cot}[c + d*x]^{(3/2)}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d) - (2*A*\operatorname{Cot}[c + d*x]^{(5/2)}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(5*d)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 211

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 3625

`Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; F`

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3679

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(A\*d - B\*c)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(a\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*(b\*d\*m - a\*c\*(n + 1)) - B\*(b\*c\*m + a\*d\*(n + 1)) - a\*(B\*c - A\*d)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

### Rule 4326

Int[(cot[(a\_) + (b\_)\*(x\_)])\*(c\_)^(m\_)\*(u\_), x\_Symbol] :> Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{\sqrt{a+ia \tan(c+dx)}}{\cot^{\frac{7}{2}}(c+dx)} dx \\
&= -\frac{2A \cot^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{5d} + \frac{2B \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= -\frac{2(iA+5B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2(13A-5iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2(13A-5iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2(13A-5iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2(13A-5iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{(1-i)\sqrt{a} (iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{15d(-1+e^{2i(c+dx)})^2}
\end{aligned}$$

**Mathematica [A]**

time = 1.61, size = 188, normalized size = 0.95

$$\frac{e^{-i(c+dx)} \left( -20iB e^{3i(c+dx)} (-1 + e^{2i(c+dx)}) + 2A e^{i(c+dx)} (15 - 20e^{2i(c+dx)} + 17e^{4i(c+dx)}) - 15(A - iB) (-1 + e^{2i(c+dx)})^{5/2} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d(-1+e^{2i(c+dx)})^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]), x]
```

```
[Out] (((-20*I)*B*E^((3*I)*(c + d*x))*(-1 + E^((2*I)*(c + d*x))) + 2*A*E^(I*(c + d*x))*(15 - 20*E^((2*I)*(c + d*x)) + 17*E^((4*I)*(c + d*x))) - 15*(A - I*B)*(-1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*E^(I*(c + d*x))*(-1 + E^((2*I)*(c + d*x)))^2)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2242 vs. 2(161) = 322.

time = 60.81, size = 2243, normalized size = 11.33

method	result	size
default	Expression too large to display	2243

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/30/d*2^{(1/2)}*(15*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))+30*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+30*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+30*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+15*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+26*A*2^{(1/2)}+20*B*\cos(d*x+c)^2*\sin(d*x+c)*2^{(1/2)}+34*A*\cos(d*x+c)^3*2^{(1/2)}-28*A*\cos(d*x+c)*2^{(1/2)}+34*I*A*2^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)^2-2*I*A*2^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)-30*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-30*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-15*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))-30*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-30*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-15*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))-30*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-30*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-15*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+30*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+30*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+15*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)$$

$$\begin{aligned} & -\cos(dx+c)+1)/((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1))-10*I*B*2^{1/2}+30*I*A*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\cos(dx+c)^2*\arctan((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}+1)-10*B*2^{1/2}*\cos(dx+c)*\sin(dx+c)-10*2^{1/2}*B*\sin(dx+c)-32*A*2^{1/2}*\cos(dx+c)^2+30*A*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\arctan((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}+1)+30*A*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\arctan((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}-1)+15*A*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\ln(-((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1)/((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}*\sin(dx+c)-\sin(dx+c)-\cos(dx+c)+1)))-30*B*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\arctan((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}+1)+10*I*B*2^{1/2}*\cos(dx+c)^2-26*I*A*2^{1/2}*\sin(dx+c)+20*I*B*2^{1/2}*\cos(dx+c)-30*B*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\arctan((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}-1)-15*B*\sin(dx+c)*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\ln(-((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}*\sin(dx+c)-\sin(dx+c)-\cos(dx+c)+1)/((( -1+\cos(dx+c))/\sin(dx+c))^{1/2}*2^{1/2}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1)))-20*I*B*2^{1/2}*\cos(dx+c)^3*\sin(dx+c)*(\cos(dx+c)/\sin(dx+c))^{7/2}*((I*\sin(dx+c)+\cos(dx+c))*a/\cos(dx+c))^{1/2}/(I*\sin(dx+c)+\cos(dx+c)-1)/\cos(dx+c)^3 \end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1409 vs.  $2(151) = 302$ .  
time = 1.05, size = 1409, normalized size = 7.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(7/2)\*(a+I\*a\*tan(dx+c))^(1/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out]  $\frac{1}{30}*(\sqrt{\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1})*((30*((I+1)*A-(I-1)*B)*\cos(3dx+3c)+(-39I+39)*A+(25I-25)*B)*\cos(dx+c)+30*((I-1)*A+(I+1)*B)*\sin(3dx+3c)+(-39I-39)*A-(25I+25)*B)*\sin(dx+c))*\cos(3/2*\arctan^2(\sin(2dx+2c),\cos(2dx+2c)-1))+30*(-(I-1)*A-(I+1)*B)*\cos(3dx+3c)+((39I-39)*A+(25I+25)*B)*\cos(dx+c)+30*((I+1)*A-(I-1)*B)*\sin(3dx+3c)+(-39I+39)*A+(25I-25)*B)*\sin(dx+c))*\sin(3/2*\arctan^2(\sin(2dx+2c),\cos(2dx+2c)-1)))*\sqrt{a}+15*(2*(-(I-1)*A-(I+1)*B)*\cos(2dx+2c)^2+(-(I-1)*A-(I+1)*B)*\sin(2dx+2c)^2+2*((I-1)*A+(I+1)*B)*\cos(2dx+2c)-(I-1)*A-(I+1)*B)*\arctan^2(2*(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1))^{1/4}*\sin(1/2*\arctan^2(\sin(2dx+2c),\cos(2dx+2c)-1))+2*\sin(dx+c),2*(\cos(2dx+2c)^2+\sin(2dx+2c)^2-2\cos(2dx+2c)+1))^{1/4}*\cos(1/2*\arctan^2(\sin(2dx+2c),\cos(2dx+2c)-1))+2*\cos(dx+c))+(-(I+1)*A+(I-1)*B)*\cos(2dx+2c)^2+(-(I+1)*A+(I-1)$





```
*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1)))e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(2)*(d*e^(4*I*d*x + 4*I*c
) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*log(
-4*((A - I*B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-(-
I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-I*d*x - I*c)/(I*A + B
)) - 4*sqrt(2)*((17*A - 10*I*B)*e^(5*I*d*x + 5*I*c) - 10*(2*A - I*B)*e^(3*I
*d*x + 3*I*c) + 15*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4
*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(7/2
), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x
)
```

```
[Out] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

$$3.537 \quad \int \cot^2(c+dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=155

$$\frac{(1+i)\sqrt{a} (iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d} - \frac{2(iA+3B)\sqrt{\cot(c+dx)}}{d}$$

[Out] (1+I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*a^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-2/3\*A\*cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/3\*(I\*A+3\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 0.30, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {4326, 3679, 12, 3625, 211}

$$-\frac{2(3B+iA)\sqrt{\cot(c+dx)}\sqrt{a+ia \tan(c+dx)}}{3d} + \frac{(1+i)\sqrt{a}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia \tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] ((1 + I)\*Sqrt[a]\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d - (2\*(I\*A + 3\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d) - (2\*A\*Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && Ne

$Q[c^2 + d^2, 0]$

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{5}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx = \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \cot^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d} + \frac{2B \cot^{\frac{1}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d}$$

$$= -\frac{2(iA + 3B) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{3d}$$

$$= -\frac{2(iA + 3B) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{3d}$$

$$= -\frac{2(iA + 3B) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{3d}$$

$$= \frac{(1 + i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}$$

Mathematica [A]

time = 1.17, size = 162, normalized size = 1.05

$$\frac{e^{-i(c+dx)} \left( 4iAe^{3i(c+dx)} + 6Be^{i(c+dx)}(-1 + e^{2i(c+dx)}) - 3i(A - iB)(-1 + e^{2i(c+dx)})^{3/2} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c+dx)} \sqrt{a + ia \tan(c+dx)}}{3d(-1 + e^{2i(c+dx)})}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] 
$$-1/3 * (((4*I)*A * E^{((3*I)*(c + d*x))} + 6*B * E^{(I*(c + d*x))} * (-1 + E^{((2*I)*(c + d*x))}) - (3*I)*(A - I*B) * (-1 + E^{((2*I)*(c + d*x))})^{3/2} * \text{ArcTanh}[E^{(I*(c + d*x))} / \text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] * \text{Sqrt}[\text{Cot}[c + d*x]] * \text{Sqrt}[a + I*a * \text{Tan}[c + d*x]]) / (d * E^{(I*(c + d*x))} * (-1 + E^{((2*I)*(c + d*x))}))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1983 vs.  $2(126) = 252$ .

time = 60.72, size = 1984, normalized size = 12.80

method	result	size
default	Expression too large to display	1984

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] 
$$-1/6/d * 2^{(1/2)} * (-2*A * 2^{(1/2)} - 2*A * \cos(d*x+c) * 2^{(1/2)} + 6*I*A * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \arctan((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)+1} + 6*I*A * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \arctan((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} - 1) + 3*I*A * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \ln((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / (-(( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \cos(d*x+c) + \sin(d*x+c) - 1)) - 6*I*B * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \arctan((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)+1} - 6*I*B * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \arctan((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} - 1) - 3*I*B * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \ln((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / ((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1)) + 4*I*A * \cos(d*x+c) * 2^{(1/2)} * \sin(d*x+c) - 3*B * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \ln((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / (-(( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \cos(d*x+c) + \sin(d*x+c) - 1)) - 6*I*A * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \arctan((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} - 1) - 3*I*A * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * \ln((( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / (-(( -1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \cos(d*x+c) + \sin(d*x+c) - 1)) + 6*I*B * ((-1 + \cos(d*x+c))/\sin(d*x+c))^{(1/2)} * a$$

```

rctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+6*I*B*((−1+cos(d*x+c))/
sin(d*x+c))^(1/2)*arctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)−1)+3*I*
B*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln((−((−1+cos(d*x+c))/sin(d*x+c))^(1/2)
)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)−1)/(((−1+cos(d*x+c))/sin(d*x+c))
^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)−1))+6*A*cos(d*x+c)^2*((−1+c
os(d*x+c))/sin(d*x+c))^(1/2)*arctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)+1)+6*I*B*2^(1/2)−6*A*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((−1+cos
(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)−6*A*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)
)*arctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)−1)−3*A*((−1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*ln((−((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+
c)+cos(d*x+c)+sin(d*x+c)−1)/(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin
(d*x+c)+sin(d*x+c)+cos(d*x+c)−1))−6*B*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*ar
ctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)−6*B*((−1+cos(d*x+c))/sin
(d*x+c))^(1/2)*arctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)−1)+6*B*2^(
1/2)*cos(d*x+c)*sin(d*x+c)−6*2^(1/2)*B*sin(d*x+c)+4*A*2^(1/2)*cos(d*x+c)^2+
6*A*cos(d*x+c)^2*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((−1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*2^(1/2)−1)+3*A*cos(d*x+c)^2*((−1+cos(d*x+c))/sin(d*x+c)
^(1/2)*ln((−((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)
)+sin(d*x+c)−1)/(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(
d*x+c)+cos(d*x+c)−1))+6*B*cos(d*x+c)^2*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*a
rctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+6*B*cos(d*x+c)^2*((−1+c
os(d*x+c))/sin(d*x+c))^(1/2)*arctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)−1)+3*B*cos(d*x+c)^2*((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(((−1+cos(d*x
+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)−1)/((−(−1+c
os(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)−1))−6
*I*B*cos(d*x+c)^2*2^(1/2)−2*I*A*2^(1/2)*sin(d*x+c)−6*I*A*((−1+cos(d*x+c))/s
in(d*x+c))^(1/2)*arctan(((−1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1))*sin(
d*x+c)*(cos(d*x+c)/sin(d*x+c))^(5/2)*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c)
))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)−1)/cos(d*x+c)^2

```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs.  $2(118) = 236$ .  
time = 0.74, size = 1157, normalized size = 7.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, alg
orithm="maxima")
```

```
[Out] 1/6*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((3*(-(I - 1)*A - (I + 1)*B)*cos(3*d*x + 3*c) + (-(I - 1)*A + (3*I + 3)*
B)*cos(d*x + c) + 3*((I + 1)*A - (I - 1)*B)*sin(3*d*x + 3*c) + ((I + 1)*A +
(3*I - 3)*B)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) - 1)) + (3*(-(I + 1)*A + (I - 1)*B)*cos(3*d*x + 3*c) + (-(I + 1)*A - (3
```

```

*I - 3)*B)*cos(d*x + c) + 3*(-(I - 1)*A - (I + 1)*B)*sin(3*d*x + 3*c) + (-(
I - 1)*A + (3*I + 3)*B)*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) - 1)))*sqrt(a) + 3*(2*((-(I + 1)*A + (I - 1)*B)*cos(2*d*x + 2
*c)^2 + (-(I + 1)*A + (I - 1)*B)*sin(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1
)*B)*cos(2*d*x + 2*c) - (I + 1)*A + (I - 1)*B)*arctan2(2*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + (((I - 1)*A + (I +
1)*B)*cos(2*d*x + 2*c)^2 + ((I - 1)*A + (I + 1)*B)*sin(2*d*x + 2*c)^2 + 2*
(-(I - 1)*A - (I + 1)*B)*cos(2*d*x + 2*c) + (I - 1)*A + (I + 1)*B)*log(4*co
s(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))
^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)
) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))))
)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sqrt(a) + 2*((( -(I - 1)*A - (3*I + 3)*B)*cos(d*x + c) + ((I + 1)*A - (3*I
- 3)*B)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + ((-(I - 1)*A - (3*I + 3)*B)*cos(
d*x + c) + ((I + 1)*A - (3*I - 3)*B)*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 2*(
((I - 1)*A + (3*I + 3)*B)*cos(d*x + c) + (-(I + 1)*A + (3*I - 3)*B)*sin(d*x
+ c))*cos(2*d*x + 2*c) + (-(I - 1)*A - (3*I + 3)*B)*cos(d*x + c) + ((I + 1
)*A - (3*I - 3)*B)*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) - 1)) + (((-(I + 1)*A + (3*I - 3)*B)*cos(d*x + c) + (-(I - 1)*A -
(3*I + 3)*B)*sin(d*x + c))*cos(2*d*x + 2*c)^2 + ((-(I + 1)*A + (3*I - 3)*B)
*cos(d*x + c) + (-(I - 1)*A - (3*I + 3)*B)*sin(d*x + c))*sin(2*d*x + 2*c)^2
+ 2*(((I + 1)*A - (3*I - 3)*B)*cos(d*x + c) + ((I - 1)*A + (3*I + 3)*B)*si
n(d*x + c))*cos(2*d*x + 2*c) + (-(I + 1)*A + (3*I - 3)*B)*cos(d*x + c) + (-(
I - 1)*A - (3*I + 3)*B)*sin(d*x + c))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(5/4)*d)

```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 430 vs.  $2(118) = 236$ .

time = 1.84, size = 430, normalized size = 2.77

$$\frac{3\sqrt{2}(a^{2d^2+2d})\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\log\left(\frac{(-1+2a^{2d^2+2d})\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\sqrt{\frac{d^2+2d+1}{2d^2+1}}\sqrt{\frac{d^2+2d+1}{2d^2+1}}}{a^{2d^2+2d}}\right)-3\sqrt{2}(a^{2d^2+2d})\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\log\left(\frac{(-1+2a^{2d^2+2d})\sqrt{\frac{d^2+2AB-1B^2}{d^2}}\sqrt{\frac{d^2+2d+1}{2d^2+1}}\sqrt{\frac{d^2+2d+1}{2d^2+1}}}{a^{2d^2+2d}}\right)+4\sqrt{2}(2d+3B)\sqrt{2d^2+1}\sqrt{\frac{d^2+2d+1}{2d^2+1}}}{6(d^{2d^2+2d})}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")

```

```

[Out] -1/6*(3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a
/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*sqr

```

```
t(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/(I*A
+ B)) - 3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2
)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c) - d)*
sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/(
I*A + B)) + 4*sqrt(2)*((2*I*A + 3*B)*e^(3*I*d*x + 3*I*c) - 3*B*e^(I*d*x + I
*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(5/2
), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x
)
```

```
[Out] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```



$$3.538 \quad \int \cot^{\frac{3}{2}}(c+dx) \sqrt{a + ia \tan(c + dx)} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=110

$$\frac{(1+i)\sqrt{a}(A-iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d} - \frac{2A\sqrt{\cot(c+dx)}}{d}$$

[Out] (1+I)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*a^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-2\*A\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d

**Rubi** [A]

time = 0.19, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {4326, 3679, 12, 3625, 211}

$$\frac{(1+i)\sqrt{a}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] ((1 + I)\*Sqrt[a]\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d - (2\*A\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/d

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx)) dx = \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{a + ia \tan(c + dx)}}{\sqrt{\cot(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} + \frac{2B \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d}$$

$$= -\frac{2A \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} - \frac{(1 - i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}$$

Mathematica [A]

time = 1.17, size = 112, normalized size = 1.02

$$\frac{e^{-i(c+dx)} \left( -2Ae^{i(c+dx)} + (A - iB) \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((-2*A*E^{I*(c + d*x)} + (A - I*B)*\sqrt{-1 + E^{(2*I)*(c + d*x)}})*\text{ArcTanh}[E^{I*(c + d*x)}/\sqrt{-1 + E^{(2*I)*(c + d*x)}}])*\sqrt{\text{Cot}[c + d*x]}*\sqrt{a + I*a*\text{Tan}[c + d*x]}/(d*E^{I*(c + d*x)})$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1031 vs.  $2(91) = 182$ .  
time = 61.80, size = 1032, normalized size = 9.38

method	result	size
default	Expression too large to display	1032

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out]  $-1/2/d*2^{(1/2)}*(2*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+I*A*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+I*B*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1))*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*I*A*2^{(1/2)}*\sin(d*x+c)-2*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}-2*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-A*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1))*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+2*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+B*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+2*A*\cos(d*x+c)*2^{(1/2)}-2*A*2^{(1/2)}*(\cos(d*x+c)/\sin(d*x+c))^{(3/2)}*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}*\sin(d*x+c)/(I*\sin(d*x+c)+\cos(d*x+c)-1)/\cos(d*x+c)$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 558 vs.  $2(85) = 170$ .

time = 0.62, size = 558, normalized size = 5.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*((2\*((I - 1)\*A + (I + 1)\*B)\*arctan2(2\*(cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 - 2\*cos(2\*d\*x + 2\*c) + 1)^(1/4)\*sin(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1)) + 2\*sin(d\*x + c), 2\*(cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 - 2\*cos(2\*d\*x + 2\*c) + 1)^(1/4)\*cos(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1)) + 2\*cos(d\*x + c)) - ((I + 1)\*A + (I - 1)\*B)\*log(4\*cos(d\*x + c)^2 + 4\*sin(d\*x + c)^2 + 4\*sqrt(cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 - 2\*cos(2\*d\*x + 2\*c) + 1)\*(cos(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1))^2 + sin(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1))^2) + 8\*(cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 - 2\*cos(2\*d\*x + 2\*c) + 1)^(1/4)\*(cos(d\*x + c)\*cos(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1)) + sin(d\*x + c)\*sin(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1))))\*(cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 - 2\*cos(2\*d\*x + 2\*c) + 1)^(1/4)\*sqrt(a) + 4\*((-I + 1)\*A\*cos(d\*x + c) - (I - 1)\*A\*sin(d\*x + c))\*cos(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1)) + ((I - 1)\*A\*cos(d\*x + c) - (I + 1)\*A\*sin(d\*x + c))\*sin(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c) - 1)))\*sqrt(a))/((cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 - 2\*cos(2\*d\*x + 2\*c) + 1)^(1/4)\*d)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(85) = 170.  
time = 1.57, size = 370, normalized size = 3.36

$$\frac{4\sqrt{2}A\sqrt{\frac{e^{2dix+2ic}-1}{2e^{dix+ic}-1}}\sqrt{\frac{e^{2dix+2ic}+1}{2e^{dix+ic}+1}}e^{dix+ic}-\sqrt{2}d\sqrt{-1A^2-2AB+1B^2}\log\left(\frac{4\left(\frac{(A-B)\sqrt{e^{2dix+2ic}-1}}{2e^{dix+ic}-1}\sqrt{\frac{-1A^2-2AB+1B^2}{d^2}}\sqrt{\frac{e^{2dix+2ic}+1}{2e^{dix+ic}+1}}\sqrt{\frac{e^{2dix+2ic}+1}{2e^{dix+ic}+1}}\right)}{1+2B}}\right)+\sqrt{2}d\sqrt{-1A^2-2AB+1B^2}\log\left(\frac{4\left(\frac{(A-B)\sqrt{e^{2dix+2ic}-1}}{2e^{dix+ic}-1}\sqrt{\frac{-1A^2-2AB+1B^2}{d^2}}\sqrt{\frac{e^{2dix+2ic}+1}{2e^{dix+ic}+1}}\sqrt{\frac{e^{2dix+2ic}+1}{2e^{dix+ic}+1}}\right)}{1+2B}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] -1/2\*(4\*sqrt(2)\*A\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))\*e^(I\*d\*x + I\*c) - sqrt(2)\*d\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*log(-4\*((A - I\*B)\*a\*e^(I\*d\*x + I\*c) - (I\*d\*e^(2\*I\*d\*x + 2\*I\*c) - I\*d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))\*e^(-I\*d\*x - I\*c)/(I\*A + B)) + sqrt(2)\*d\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*log(-4\*((A - I\*B)\*a\*e^(I\*d\*x + I\*c) - (-I\*d\*e^(2\*I\*d\*x + 2\*I\*c) + I\*d)\*sqrt(-(-I\*A^2 - 2\*A\*B + I\*B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))

) $\sqrt{(Ie^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1))e^{(-I dx - I c)/(IA + B)}/d$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)`

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)`

[Out] `int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)`

$$3.539 \quad \int \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=152

$$\frac{2(-1)^{3/4} \sqrt{a} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} (1-i) \sqrt{a} (A-iB) \tan(c+dx)}{d} + \dots$$

[Out]  $-2*(-1)^{(3/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(1-I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.28, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {4326, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(1-i)\sqrt{a}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2(-1)^{3/4}\sqrt{a}B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $(-2*(-1)^{(3/4)}*\operatorname{Sqrt}[a]*B*\operatorname{ArcTan}(((1+i)\sqrt{a}\sqrt{\tan(c+dx)})/\operatorname{Sqrt}[a+I*a*\tan(c+dx)]))/\operatorname{Sqrt}[a+I*a*\tan(c+dx)]*\operatorname{Sqrt}[\cot(c+dx)]*\operatorname{Sqrt}[\tan(c+dx)]/d + ((1-I)*\operatorname{Sqrt}[a]*(A-I*B)*\operatorname{ArcTanh}(((1+I)*\operatorname{Sqrt}[a]\sqrt{\tan(c+dx)})/\operatorname{Sqrt}[a+I*a*\tan(c+dx)]))/\operatorname{Sqrt}[\cot(c+dx)]*\operatorname{Sqrt}[\tan(c+dx)]/d$

**Rule 65**

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^(n), x], x, (a+b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c-a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

**Rule 209**

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

### Rule 4326

Int[(cot[(a\_) + (b\_)\*(x\_)])\*(c\_)^(m\_)\*(u\_), x\_Symbol] := Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

### Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)} (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{\cot(c+dx)}} dx \\
&= - \left( \left( (-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \right) \int \frac{\sqrt{a+ia \tan(c+dx)}}{\sqrt{\cot(c+dx)}} dx \\
&= \frac{\left( 2ia^2(-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\left( (-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)} \\
&= \frac{(1-i)\sqrt{a} (A-iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{(1-i)\sqrt{a} (A-iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)} \\
&= \frac{2(-1)^{3/4} \sqrt{a} B \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\left( (-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}
\end{aligned}$$

**Mathematica [A]**

time = 1.93, size = 241, normalized size = 1.59

$$\frac{e^{-i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}} \left( (-4iA-4B) \log \left( e^{i(c+dx)} + \sqrt{-1+e^{2i(c+dx)}} \right) + \sqrt{2} B \left( \log \left( 1-3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \right) - \log \left( 1-3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \right) \right) \right) \sqrt{a+ia \tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

```
[Out] (Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))])/(-1 + E^((2*I)*(c + d*x)))*((( -4*I)*A - 4*B)*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt[2]*B*(Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] - Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]]))*Sqrt[a + I*a*Tan[c + d*x]]/(4*d*E^(I*(c + d*x)))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 878 vs.  $2(121) = 242$ .

time = 63.57, size = 879, normalized size = 5.78

method	result
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default	$\frac{\sqrt{2} \sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}} \sqrt{\frac{(i \sin(dx+c) + \cos(dx+c))a}{\cos(dx+c)}} (-1 + \cos(dx+c)) \left( 2iB \sqrt{2} \arctan\left(\sqrt{\frac{-1 + \cos(dx+c)}{\sin(dx+c)}}\right) + 2iA \arctan\left(\sqrt{\frac{-1 + \cos(dx+c)}{\sin(dx+c)}}\right) \right)}{}$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/2/d*2^{(1/2)}*(\cos(d*x+c)/\sin(d*x+c))^{(1/2)}*((I*\sin(d*x+c)+\cos(d*x+c))*a/c \\ & \cos(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))*(2*I*B*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)})+2*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+ \\ & I*B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)-2*I*B*\arctan((( -1+\cos( \\ & d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+I*A*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{( \\ & 1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x \\ & +c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1))+2*I*A*\arctan((( -1+c \\ & os(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-I*B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}+1)-I*B*\ln((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d \\ & *x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}* \\ & \sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))-2*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+ \\ & c))^{(1/2)}*2^{(1/2)}-1)-B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)+B*2 \\ & ^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)-2*B*2^{(1/2)}*\arctan((( -1+\cos \\ & (d*x+c))/\sin(d*x+c))^{(1/2)})+A*\ln((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/ \\ & 2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}* \\ & 2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))+2*A*\arctan((( -1+\cos(d*x+c))/\si \\ & n(d*x+c))^{(1/2)}*2^{(1/2)}+1)+2*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{ \\ & (1/2)}-1)+2*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+B*\ln((( - \\ & 1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1) \\ & /((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x \\ & +c)-1))+2*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1))/(I*\sin(d* \\ & x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,alg  
orithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c)  
) , x)`

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(115) = 230.  
time = 1.24, size = 576, normalized size = 3.79

$$\frac{1}{2} \sqrt{2} \sqrt{-\frac{(A^2 - 2AB - B^2)a}{d^2}} \log\left(\frac{-4((A - IB)a e^{(Id*x + Ic)} + (d e^{(2Id*x + 2Ic)} - d) \sqrt{-\frac{(A^2 - 2AB - B^2)a}{d^2}}) \sqrt{a/(e^{(2Id*x + 2Ic)} + 1)} \sqrt{(I e^{(2Id*x + 2Ic)} + I)/(e^{(2Id*x + 2Ic)} - 1)) e^{-(Id*x - Ic)}}{(IA + B)}}{-4((A - IB)a e^{(Id*x + Ic)} - (d e^{(2Id*x + 2Ic)} - d) \sqrt{-\frac{(A^2 - 2AB - B^2)a}{d^2}}) \sqrt{a/(e^{(2Id*x + 2Ic)} + 1)} \sqrt{(I e^{(2Id*x + 2Ic)} + I)/(e^{(2Id*x + 2Ic)} - 1))} e^{-(Id*x - Ic)}}{(IA + B)}} - \frac{1}{4} \sqrt{4IB^2 a/d^2} \log\left(\frac{-16(3B a^2 e^{(2Id*x + 2Ic)} - B a^2 + \sqrt{2}(I a d e^{(3Id*x + 3Ic)} - I a d e^{(Id*x + Ic)}) \sqrt{4IB^2 a/d^2} \sqrt{a/(e^{(2Id*x + 2Ic)} + 1)} \sqrt{(I e^{(2Id*x + 2Ic)} + I)/(e^{(2Id*x + 2Ic)} - 1))} e^{-(2Id*x - 2Ic)}}{B}}{16(3B a^2 e^{(2Id*x + 2Ic)} - B a^2 + \sqrt{2}(-I a d e^{(3Id*x + 3Ic)} + I a d e^{(Id*x + Ic)}) \sqrt{4IB^2 a/d^2} \sqrt{a/(e^{(2Id*x + 2Ic)} + 1)} \sqrt{(I e^{(2Id*x + 2Ic)} + I)/(e^{(2Id*x + 2Ic)} - 1))} e^{-(2Id*x - 2Ic)}}{B}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*log(-4\*((A - I\*B)\*a\*e^(I\*d\*x + I\*c) + (d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-I\*d\*x - I\*c)/(I\*A + B)) - 1/2\*sqrt(2)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*log(-4\*((A - I\*B)\*a\*e^(I\*d\*x + I\*c) - (d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-I\*d\*x - I\*c)/(I\*A + B)) - 1/4\*sqrt(4\*I\*B^2\*a/d^2)\*log(-16\*(3\*B\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) - B\*a^2 + sqrt(2)\*(I\*a\*d\*e^(3\*I\*d\*x + 3\*I\*c) - I\*a\*d\*e^(I\*d\*x + I\*c))\*sqrt(4\*I\*B^2\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-2\*I\*d\*x - 2\*I\*c)/B) + 1/4\*sqrt(4\*I\*B^2\*a/d^2)\*log(-16\*(3\*B\*a^2\*e^(2\*I\*d\*x + 2\*I\*c) - B\*a^2 + sqrt(2)\*(-I\*a\*d\*e^(3\*I\*d\*x + 3\*I\*c) + I\*a\*d\*e^(I\*d\*x + I\*c))\*sqrt(4\*I\*B^2\*a/d^2)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-2\*I\*d\*x - 2\*I\*c)/B)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(c + dx) - i)} (A + B \tan(c + dx)) \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(I\*a\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx) i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*i)^(1/2), x)

[Out] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*i)^(1/2), x)

$$3.540 \quad \int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Optimal. Leaf size=192

$$\frac{(-1)^{3/4} \sqrt{a} (2A - iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (1 + i) \sqrt{a} (A - B) \operatorname{ArcTan}\left(\frac{(1 + i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)}}{d}$$

[Out]  $-(-1)^{(3/4)}*(2*A-I*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(1+I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*a^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+B*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(1/2)}$

Rubi [A]

time = 0.39, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} \sqrt{a} (2A - iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) (1 + i) \sqrt{a} (A - iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{tanh}^{-1}\left(\frac{(1 + i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) + \frac{B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x]))/\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]], x]$

[Out]  $-((( -1)^{(3/4)}*\operatorname{Sqrt}[a]*(2*A - I*B)*\operatorname{ArcTan}((( -1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d - (((1 + I)*\operatorname{Sqrt}[a]*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d + (B*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_)^{(m_)})*((c_. + (d_.)*(x_)^{(n_)})^{(p_)})^{-1}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^{(p/b)})^{(n)})^{-1}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)} dx \\
&= \frac{B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)} dx}{d \sqrt{\cot(c + dx)}} \\
&= \frac{B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{\left( (iA + B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)} dx}{d \sqrt{\cot(c + dx)}} \\
&= \frac{B \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} + \frac{\left( 2ia^2(iA + B) \sqrt{\cot(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)} dx}{d \sqrt{\cot(c + dx)}} \\
&= - \frac{(1 - i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= - \frac{(1 - i) \sqrt{a} (iA + B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= - \frac{\sqrt[4]{-1} \sqrt{a} (2iA + B) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica** [F]

time = 4.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + ia \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

[Out] Integrate[(Sqrt[a + I\*a\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]], x]

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3840 vs.  $2(155) = 310$ .

time = 65.86, size = 3841, normalized size = 20.01

method	result	size
default	Expression too large to display	3841

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x,method=\_RE  
TURNVERBOSE)

[Out] 
$$\begin{aligned} & -1/4/d*2^{(1/2)}*(I*B*\cos(d*x+c)*2^{(1/2)}*\sin(d*x+c)*\ln((( -1+\cos(d*x+c))/\sin(d*x+c) \\ & *x+c))^{(1/2)}-1)+2*B*\cos(d*x+c)^2*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c) \\ & )^{(1/2)})-4*I*A*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1) \\ & -4*I*A*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1) \\ & -2*I*A*\cos(d*x+c)^2*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c) \\ & +\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}* \\ & \sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))-2*I*B*\cos(d*x+c)^2*\ln((( -1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos( \\ & d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1))-4*I* \\ & B*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-4*I*B*\cos \\ & (d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+4*I*A*\cos( \\ & d*x+c)*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+4*I*A*\cos(d*x+c) \\ & )*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+2*I*A*\cos(d*x+c)*\ln( \\ & (( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c) \\ & -1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos \\ & (d*x+c)-1))+2*I*B*\cos(d*x+c)*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\ & *\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2 \\ & ^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1))+4*I*B*\cos(d*x+c)*\arctan((( -1+co \\ & s(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+4*I*B*\cos(d*x+c)*\arctan((( -1+\cos(d*x \\ & +c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-2*I*B*2^{(1/2)}*(( -1+\cos(d*x+c))/\sin(d*x+c) \\ & )^{(1/2)}-2*A*\cos(d*x+c)*\sin(d*x+c)*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\ & *\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2 \\ & ^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1))-4*A*\cos(d*x+c)*\sin(d*x+c)* \\ & \arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-4*A*\cos(d*x+c)*\sin(d*x \\ & +c)*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-B*\cos(d*x+c)*2^{(1/2)} \\ & *\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)+B*\cos(d*x+c)*2^{(1/2)}*\ln((( -1+co \\ & s(d*x+c))/\sin(d*x+c))^{(1/2)}+1)-2*B*\cos(d*x+c)*2^{(1/2)}*\arctan((( -1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}+4*B*\cos(d*x+c)*\sin(d*x+c)*\arctan((( -1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*2^{(1/2)}+1)+4*B*\cos(d*x+c)*\sin(d*x+c)*\arctan((( -1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+2*B*\cos(d*x+c)*\sin(d*x+c)*\ln((( -1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos( \end{aligned}$$

$$\begin{aligned}
& d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1)) + 2*B* \\
& 2^{(1/2)} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * \sin(d*x+c) - 2*B*\cos(d*x+c)^2 * 2^{(1/2)} * \\
& ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} - 2*A*\cos(d*x+c) * \ln((( -1 + \cos(d*x+c)) / \\
& \sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / ( -((-1 + \cos(d* \\
& x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \cos(d*x+c) + \sin(d*x+c) - 1)) - 4*A*co \\
& s(d*x+c) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} + 1) - 4*A*\cos(d*x+c) \\
& ) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} - 1) + 2*I*A*\cos(d*x+c)^2 * 2^{(1/2)} * \\
& \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} - 1) - 2*I*A*\cos(d*x+c)^2 * 2^{(1/2)} * \\
& \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} + 1) + 4*I*A*\cos(d*x+c)^2 * 2^{(1/2)} * \arctan( \\
& ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)}) + 2*I*B*\cos(d*x+c)^2 * 2^{(1/2)} * ((-1 + \cos(d*x \\
& +c)) / \sin(d*x+c))^{(1/2)} - I*B*\cos(d*x+c)^2 * 2^{(1/2)} * \ln((( -1 + \cos(d*x+c)) / \sin(d*x \\
& +c))^{(1/2)} - 1) + 2*I*B*\cos(d*x+c)^2 * 2^{(1/2)} * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c) \\
& )^{(1/2)}) - 2*I*A*\cos(d*x+c)^2 * 2^{(1/2)} * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} - 1) + \\
& 2*I*A*\cos(d*x+c)^2 * 2^{(1/2)} * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} + 1) - 4*I*A*\cos \\
& (d*x+c)^2 * 2^{(1/2)} * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)}) + 4*I*A*\cos(d*x+c) \\
& * \sin(d*x+c) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} + 1) + 4*I*A*\cos(d \\
& *x+c) * \sin(d*x+c) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} - 1) + 2*I* \\
& A*\cos(d*x+c) * \sin(d*x+c) * \ln((-((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin \\
& (d*x+c) + \cos(d*x+c) + \sin(d*x+c) - 1) / (((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} \\
& ) * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) - I*B*\cos(d*x+c)^2 * 2^{(1/2)} * \ln((( -1 + \cos(d \\
& *x+c)) / \sin(d*x+c))^{(1/2)} + 1) - 2*I*B*\cos(d*x+c)^2 * 2^{(1/2)} * \arctan((( -1 + \cos(d*x+c) \\
& ) / \sin(d*x+c))^{(1/2)}) + 2*I*B*\cos(d*x+c) * \sin(d*x+c) * \ln((( (-1 + \cos(d*x+c)) / \sin(d \\
& *x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / ( -((-1 + \cos(d*x+c)) \\
& / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} * \sin(d*x+c) + \cos(d*x+c) + \sin(d*x+c) - 1)) + 4*I*B*\cos(d \\
& *x+c) * \sin(d*x+c) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} + 1) + 4*I*B \\
& * \cos(d*x+c) * \sin(d*x+c) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * 2^{(1/2)} - 1) \\
& - 2*I*B*2^{(1/2)} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * \sin(d*x+c) - 2*A*\cos(d*x+c) \\
& * 2^{(1/2)} * \sin(d*x+c) * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} - 1) + 2*A*\cos(d*x+c) \\
& * 2^{(1/2)} * \sin(d*x+c) * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} + 1) + 4*A*\cos(d*x+c) \\
& * 2^{(1/2)} * \sin(d*x+c) * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)}) + I*B*\cos(d*x+ \\
& c)^2 * 2^{(1/2)} * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} - 1) - 2*B*\cos(d*x+c)^2 * 2^{(1/2)} * \\
& ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} * \sin(d*x+c) - B*\cos(d*x+c)^2 * 2^{(1/2)} * \sin(d*x+ \\
& c) * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} - 1) + B*\cos(d*x+c)^2 * 2^{(1/2)} * \sin(d*x+c) \\
& * \ln((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} + 1) - 2*B*\cos(d*x+c)^2 * 2^{(1/2)} * \sin(d*x+c) \\
& * \arctan((( -1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)}) + I*B*\cos(d*x+c)^2 * 2^{(1/2)} * \ln((( - \\
& 1 + \cos(d*x+c)) / \sin(d*x+c))^{(1/2)} + 1) + 4*B*\cos(d*x+...
\end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")



[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(I\*a\*tan(d\*x + c) + a)/sqrt(cot(d\*x + c)), x)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(148) = 296.

time = 1.54, size = 793, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*(2*\sqrt{2}*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*} \\ & a/d^2)*\log(-4*((A - I*B)*a*e^{(I*d*x + I*c)} - (I*d*e^{(2*I*d*x + 2*I*c)} - I*d \\ & )*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{ \\ & ((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))*e^{(-I*d*x - I*c)} \\ & )/(I*A + B)) - 2*\sqrt{2}*(d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{-(-I*A^2 - 2*A*B \\ & + I*B^2)*a/d^2}*\log(-4*((A - I*B)*a*e^{(I*d*x + I*c)} - (-I*d*e^{(2*I*d*x + 2*I*c)} \\ & + I*d)*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{ \\ & ((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))*e^{(-I \\ & *d*x - I*c)}/(I*A + B)) + 4*\sqrt{2}*(I*B*e^{(3*I*d*x + 3*I*c)} - I*B*e^{(I*d*x \\ & + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/ \\ & (e^{(2*I*d*x + 2*I*c)} - 1)) - (d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(4*I*A^2 + 4* \\ & A*B - I*B^2)*a/d^2}*\log(-16*(3*(2*I*A + B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-2*I* \\ & A - B)*a^2 + 2*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{ \\ & ((4*I*A^2 + 4*A*B - I*B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((I \\ & *e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))*e^{(-2*I*d*x - 2*I*c)}/ \\ & (2*I*A + B)) + (d*e^{(2*I*d*x + 2*I*c)} + d)*\sqrt{(4*I*A^2 + 4*A*B - I*B^2)*a \\ & /d^2}*\log(-16*(3*(2*I*A + B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-2*I*A - B)*a^2 - 2 \\ & *\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{(4*I*A^2 + 4* \\ & A*B - I*B^2)*a/d^2}*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((I*e^{(2*I*d*x + \\ & 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))*e^{(-2*I*d*x - 2*I*c)/(2*I*A + B)))/ \\ & (d*e^{(2*I*d*x + 2*I*c)} + d) \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia(\tan(c + dx) - i)}(A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] Integral(sqrt(I\*a\*(tan(c + d\*x) - I))\*(A + B\*tan(c + d\*x))/sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(I\*a\*tan(d\*x + c) + a)/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + a \tan(c + dx)} \operatorname{li}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(1/2))/cot(c + d\*x)^(1/2), x)

$$3.541 \quad \int \cot^2(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=245

$$\frac{(2 - 2i)a^{3/2}(A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d} + \frac{4a(67iA + 63B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d}$$

[Out] (2-2\*I)\*a^(3/2)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+4/105\*a\*(19\*A-21\*I\*B)\*cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/35\*a\*(8\*I\*A+7\*B)\*cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/7\*a\*A\*cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d+4/105\*a\*(67\*I\*A+63\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 0.58, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3674, 3679, 12, 3625, 211}

$$\frac{(2-2i)a^{3/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a(7B+8iA)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{4a(19A-21iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{4a(63B+67iA)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{105d} - \frac{2aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (((2 - 2\*I)\*a^(3/2)\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + (4\*a\*((67\*I)\*A + 63\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d) + (4\*a\*(19\*A - (21\*I)\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d) - (2\*a\*((8\*I)\*A + 7\*B)\*Cot[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(35\*d) - (2\*a\*A\*Cot[c + d\*x]^(7/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(7\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

$^2*x^2)$ ,  $x$ ,  $\text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]]$ ,  $x$  /;  $\text{FreeQ}\{a, b, c, d, e, f, x\}$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{NeQ}[c^2 + d^2, 0]$

#### Rule 3674

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(n_)}$ ,  $x_{\text{Symbol}}$   $\rightarrow$   $\text{Simp}[(-a^2)*(B*c - A*d)*(a + b*\text{Tan}[e + f*x])^{(m-1)}*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(d*f*(b*c + a*d)*(n+1))$ ,  $x$  -  $\text{Dist}[a/(d*(b*c + a*d)*(n+1))$ ,  $\text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)}*(c + d*\text{Tan}[e + f*x])^{(n+1)}*\text{Simp}[A*b*d*(m-n-2) - B*(b*c*(m-1) + a*d*(n+1)) + (a*A*d*(m+n) - B*(a*c*(m-1) + b*d*(n+1)))*\text{Tan}[e + f*x]$ ,  $x$ ],  $x$ ],  $x$  /;  $\text{FreeQ}\{a, b, c, d, e, f, A, B\}$ ,  $x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{GtQ}[m, 1]$  &&  $\text{LtQ}[n, -1]$

#### Rule 3679

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(m_)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)]])^{(n_)}$ ,  $x_{\text{Symbol}}$   $\rightarrow$   $\text{Simp}[(A*d - B*c)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{(n+1)})/(f*(n+1)*(c^2 + d^2))$ ,  $x$  -  $\text{Dist}[1/(a*(n+1)*(c^2 + d^2))$ ,  $\text{Int}[(a + b*\text{Tan}[e + f*x])^m*(c + d*\text{Tan}[e + f*x])^{(n+1)}*\text{Simp}[A*(b*d*m - a*c*(n+1)) - B*(b*c*m + a*d*(n+1)) - a*(B*c - A*d)*(m+n+1)*\text{Tan}[e + f*x]$ ,  $x$ ],  $x$ ],  $x$  /;  $\text{FreeQ}\{a, b, c, d, e, f, A, B, m\}$ ,  $x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$  &&  $\text{LtQ}[n, -1]$

#### Rule 4326

$\text{Int}[(\text{cot}[(a_.) + (b_.)*(x_)]*(c_.)^{(m_)}*(u_)$ ,  $x_{\text{Symbol}}$   $\rightarrow$   $\text{Dist}[(c*\text{Cot}[a + b*x])^m*(c*\text{Tan}[a + b*x])^m$ ,  $\text{Int}[\text{ActivateTrig}[u]/(c*\text{Tan}[a + b*x])^m$ ,  $x$ ],  $x$  /;  $\text{FreeQ}\{a, b, c, m\}$ ,  $x$  &&  $! \text{IntegerQ}[m]$  &&  $\text{KnownTangentIntegrandQ}[u, x]$

#### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{7}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{7d} \\
&= -\frac{2a(8iA+7B) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{35d} \\
&= \frac{4a(19A-21iB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a(67iA+63B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a(67iA+63B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a(67iA+63B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a(67iA+63B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{(2+2i)a^{3/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d \operatorname{sech}^5(c+dx)(A \cos(c+dx)+B \sin(c+dx))}
\end{aligned}$$

### Mathematica [A]

time = 4.16, size = 320, normalized size = 1.31

$$\frac{\left( -2i\sqrt{2}(A-iB)e^{-2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{1(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\operatorname{tanh}^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) - \frac{1}{21}\sqrt{\cot(c+dx)}\operatorname{sech}^5(c+dx)\sqrt{\sec(c+dx)}(\cos(c+dx)-i\sin(c+dx))(7(A+6iB)\cos(c+dx)+(53A-42iB)\cos(3(c+dx))+2(-110iA-105B+(158iA+147B)\cos(2(c+dx)))\sin(c+dx))\right)}{d \operatorname{sech}^5(c+dx)(A \cos(c+dx)+B \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((((-2\*I)\*Sqrt[2]\*(A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*Sqrt[E^(I\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x))))\*Sqrt[(I\*(1 + E^((2\*I)\*(c + d\*x))))]/(-1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]/E^((2\*I)\*(c + d\*x)) - (Sqrt[Cot[c + d\*x]]\*Csc[c + d\*x]^3\*Sqrt[Sec[c + d\*x]]\*(Cos[c + d\*x] - I\*Sin[c + d\*x])\*(7\*(A + (6\*I)\*B)\*Cos[c + d\*x] + (53\*A - (42\*I)\*B)\*Cos[3\*(c + d\*x)] + 2\*((-110\*I)\*A - 105\*B + ((158\*I)\*A + 147\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x]))/210)\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/(d\*Sec[c + d\*x]^(5/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3123 vs.  $2(200) = 400$ .  
time = 69.31, size = 3124, normalized size = 12.75

method	result	size
default	Expression too large to display	3124

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/105/d*a*2^{(1/2)}*(134*I*A*2^{(1/2)}*\sin(d*x+c)+210*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+210*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}+105*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))-210*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}-210*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}-105*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))-189*I*B*2^{(1/2)}*\cos(d*x+c)^4+42*I*B*2^{(1/2)}*\cos(d*x+c)^3+315*I*B*2^{(1/2)}*\cos(d*x+c)^2-42*I*B*2^{(1/2)}*\cos(d*x+c)+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}+105*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+134*A*2^{(1/2)}-147*B*\cos(d*x+c)^2*\sin(d*x+c)*2^{(1/2)}+189*B*\cos(d*x+c)^3*\sin(d*x+c)*2^{(1/2)}-126*I*B*2^{(1/2)}-53*A*\cos(d*x+c)^3*2^{(1/2)}+38*A*\cos(d*x+c)*2^{(1/2)}-420*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+211*A*\cos(d*x+c)^4*2^{(1/2)}+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}+210*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+210*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}+105*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+134*A*2^{(1/2)}-147*B*\cos(d*x+c)^2*\sin(d*x+c)*2^{(1/2)}+189*B*\cos(d*x+c)^3*\sin(d*x+c)*2^{(1/2)}-126*I*B*2^{(1/2)}-53*A*\cos(d*x+c)^3*2^{(1/2)}+38*A*\cos(d*x+c)*2^{(1/2)}-420*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+211*A*\cos(d*x+c)^4*2^{(1/2)}+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}+210*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)+1}+210*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)-1}+105*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)))$$

$$\begin{aligned}
& \frac{1}{2} * 2^{(1/2)-1} - 210 * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \ln(- \\
& ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) \\
& + 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1) \\
& - 210 * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \ln(-((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1)) \\
& - 168 * B * 2^{(1/2)} * \cos(dx+c) * \sin(dx+c) + 105 * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \ln(-((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1)) \\
& + 105 * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \ln(-((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1)) \\
& + 126 * 2^{(1/2)} * B * \sin(dx+c) - 330 * A * 2^{(1/2)} * \cos(dx+c)^2 + 210 * I * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^4 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} + 1) + 210 * I * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^4 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} - 1) \\
& + 105 * I * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^4 * \ln(-((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1)) \\
& - 210 * I * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^4 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} + 1) - 210 * I * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^4 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} - 1) \\
& - 105 * I * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^4 * \ln(-((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1)) \\
& + 211 * I * A * 2^{(1/2)} * \cos(dx+c)^3 * \sin(dx+c) - 158 * I * A * 2^{(1/2)} * \cos(dx+c)^2 * \sin(dx+c) - 420 * I * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} + 1) - 420 * I * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} - 1) \\
& - 210 * I * A * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \ln(-((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1) / (((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} * \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1)) \\
& + 420 * I * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \arctan((( -1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * 2^{(1/2)} + 1) + 420 * I * B * ((-1 + \cos(dx+c)) / \sin(dx+c))^{(1/2)} * \cos(dx+c)^2 * \arctan...
\end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3787 vs.  $2(187) = 374$ .  
time = 3.03, size = 3787, normalized size = 15.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(9/2)\*(a+I\*a\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

```
[Out] -1/420*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*(3*(280*(-(I - 1)*A - (I + 1)*B)*a*cos(7*d*x + 7*c) + 140*((I - 1)*A +
(3*I + 3)*B)*a*cos(5*d*x + 5*c) + 7*(-(19*I - 19)*A - (29*I + 29)*B)*a*cos(
3*d*x + 3*c) + (-47*I - 47)*A + (63*I + 63)*B)*a*cos(d*x + c) + 280*((I +
1)*A - (I - 1)*B)*a*sin(7*d*x + 7*c) + 140*(-(I + 1)*A + (3*I - 3)*B)*a*sin
(5*d*x + 5*c) + 7*((19*I + 19)*A - (29*I - 29)*B)*a*sin(3*d*x + 3*c) + ((47
*I + 47)*A + (63*I - 63)*B)*a*sin(d*x + c))*cos(7/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) - 1)) + 4*(((141*I - 141)*A + (119*I + 119)*B)*a*cos(d
*x + c) + (-141*I + 141)*A + (119*I - 119)*B)*a*sin(d*x + c))*cos(2*d*x +
2*c)^2 + ((141*I - 141)*A + (119*I + 119)*B)*a*cos(d*x + c) + (((141*I - 14
1)*A + (119*I + 119)*B)*a*cos(d*x + c) + (-141*I + 141)*A + (119*I - 119)*
B)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (-141*I + 141)*A + (119*I - 119)*B
)*a*sin(d*x + c) + 210*((-(I - 1)*A - (I + 1)*B)*a*cos(2*d*x + 2*c)^2 + -(
I - 1)*A - (I + 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I + 1)*B)*a*co
s(2*d*x + 2*c) + (-141*I + 141)*A + (119*I - 119)*B)*a*cos(3*d*x + 3*c) + 2*((-141*I
- 141)*A - (119*I + 119)*B)*a*cos(d*x + c) + ((141*I + 141)*A - (119*I - 1
19)*B)*a*sin(d*x + c))*cos(2*d*x + 2*c) + 210*(((I + 1)*A - (I - 1)*B)*a*co
s(2*d*x + 2*c)^2 + ((I + 1)*A - (I - 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*(-(I +
1)*A + (I - 1)*B)*a*cos(2*d*x + 2*c) + ((I + 1)*A - (I - 1)*B)*a*sin(3*d*x
+ 3*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 3*(280*
(-(I + 1)*A + (I - 1)*B)*a*cos(7*d*x + 7*c) + 140*((I + 1)*A - (3*I - 3)*B)
*a*cos(5*d*x + 5*c) + 7*(-(19*I + 19)*A + (29*I - 29)*B)*a*cos(3*d*x + 3*c)
+ (-47*I + 47)*A - (63*I - 63)*B)*a*cos(d*x + c) + 280*(-(I - 1)*A - (I +
1)*B)*a*sin(7*d*x + 7*c) + 140*((I - 1)*A + (3*I + 3)*B)*a*sin(5*d*x + 5*c
) + 7*(-(19*I - 19)*A - (29*I + 29)*B)*a*sin(3*d*x + 3*c) + (-47*I - 47)*A
+ (63*I + 63)*B)*a*sin(d*x + c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) - 1)) + 4*(((141*I + 141)*A - (119*I - 119)*B)*a*cos(d*x + c) +
((141*I - 141)*A + (119*I + 119)*B)*a*sin(d*x + c))*cos(2*d*x + 2*c)^2 + ((
141*I + 141)*A - (119*I - 119)*B)*a*cos(d*x + c) + (((141*I + 141)*A - (119
*I - 119)*B)*a*cos(d*x + c) + ((141*I - 141)*A + (119*I + 119)*B)*a*sin(d*x
+ c))*sin(2*d*x + 2*c)^2 + ((141*I - 141)*A + (119*I + 119)*B)*a*sin(d*x +
c) + 210*((-(I + 1)*A + (I - 1)*B)*a*cos(2*d*x + 2*c)^2 + -(I + 1)*A + (I
- 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*a*cos(2*d*x + 2*c
) + (-141*I + 141)*A + (119*I - 119)*B)*a*cos(3*d*x + 3*c) + 2*((-141*I + 141)*A + (
119*I - 119)*B)*a*cos(d*x + c) + (-141*I - 141)*A - (119*I + 119)*B)*a*sin
(d*x + c))*cos(2*d*x + 2*c) + 210*((-(I - 1)*A - (I + 1)*B)*a*cos(2*d*x + 2
*c)^2 + -(I - 1)*A - (I + 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I +
1)*B)*a*cos(2*d*x + 2*c) + (-141*I - 141)*A - (119*I + 119)*B)*a*sin(3*d*x + 3*c))*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 420*(2*(
(-(I + 1)*A + (I - 1)*B)*a*cos(2*d*x + 2*c)^4 + -(I + 1)*A + (I - 1)*B)*a*
sin(2*d*x + 2*c)^4 + 4*((I + 1)*A - (I - 1)*B)*a*cos(2*d*x + 2*c)^3 + 6*(-(
I + 1)*A + (I - 1)*B)*a*cos(2*d*x + 2*c)^2 + 4*((I + 1)*A - (I - 1)*B)*a*co
s(2*d*x + 2*c) + 2*((-(I + 1)*A + (I - 1)*B)*a*cos(2*d*x + 2*c)^2 + 2*((I +
1)*A - (I - 1)*B)*a*cos(2*d*x + 2*c) + (-141*I + 141)*A + (119*I - 119)*B)*a)*arctan2(2*(cos(2*d*x + 2*c)^2 + s
```



```

in(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + (((I - 1)*A + (I + 1)*B
)*a*cos(2*d*x + 2*c)^4 + ((I - 1)*A + (I + 1)*B)*a*sin(2*d*x + 2*c)^4 + 4*(
-(I - 1)*A - (I + 1)*B)*a*cos(2*d*x + 2*c)^3 + 6*((I - 1)*A + (I + 1)*B)*a*
cos(2*d*x + 2*c)^2 + 4*(-(I - 1)*A - (I + 1)*B)*a*cos(2*d*x + 2*c) + 2*((I
- 1)*A + (I + 1)*B)*a*cos(2*d*x + 2*c)^2 + 2*(-(I - 1)*A - (I + 1)*B)*a*co
s(2*d*x + 2*c) + ((I - 1)*A + (I + 1)*B)*a*sin(2*d*x + 2*c)^2 + ((I - 1)*A
+ (I + 1)*B)*a*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) - 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) - 1))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + ((((-1249*I - 1249)*A - (1183*I + 1183)
*B)*a*cos(d*x + c) + ((1249*I + 1249)*A - (1183*I - 1183)*B)*a*sin(d*x + c)
)*cos(2*d*x + 2*c)^2 + (-1249*I - 1249)*A - (1183*I + 1183)*B)*a*cos(d*x +
c) + ((-1249*I - 1249)*A - (1183*I + 1183)*B)*a*cos(d*x + c) + ((1249*I +
1249)*A - (1183*I - 1183)*B)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + ((1249*I
+ 1249)*A - (1183*I - 1183)*B)*a*sin(d*x + c) ...

```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs.  $2(187) = 374$ .

time = 1.41, size = 568, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")

```

```

[Out] 1/105*(105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6
*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A -
I*B)*a^2*e^(I*d*x + I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I
*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-I*d*x - I*c)/((-I*A - B)*a) - 1
05*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6*I*c) -
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A - I*B)*a^2
*e^(I*d*x + I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2
*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1)))e^(-I*d*x - I*c)/((-I*A - B)*a) - 2*sqrt(2)
*((-211*I*A - 189*B)*a*e^(7*I*d*x + 7*I*c) + 7*(53*I*A + 57*B)*a*e^(5*I*d*x
+ 5*I*c) + 35*(-11*I*A - 9*B)*a*e^(3*I*d*x + 3*I*c) + 105*(I*A + B)*a*e^(I

```

$*d*x + I*c))\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))}/(d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} - d)$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(9/2)\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(9/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

$$3.542 \quad \int \cot^2(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=201

$$\frac{(2 + 2i)a^{3/2}(A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d} + \frac{4a(9A - 10iB)}{d}$$

[Out]  $(-2-2*I)*a^{3/2}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{1/2}*\tan(d*x+c)^{1/2}/(a+I*a*\tan(d*x+c))^{1/2})*\cot(d*x+c)^{1/2}*\tan(d*x+c)^{1/2}/d-2/15*a*(6*I*A+5*B)*\cot(d*x+c)^{3/2}*(a+I*a*\tan(d*x+c))^{1/2}/d-2/5*a*A*\cot(d*x+c)^{5/2}*(a+I*a*\tan(d*x+c))^{1/2}/d+4/15*a*(9*A-10*I*B)*\cot(d*x+c)^{1/2}*(a+I*a*\tan(d*x+c))^{1/2}/d$

**Rubi** [A]

time = 0.46, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3674, 3679, 12, 3625, 211}

$$\frac{(2 + 2i)a^{3/2}(A - iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} - \frac{2a(5B + 6iA) \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{15d} + \frac{4a(9A - 10iB) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{15d} - \frac{2aA \cot^3(c + dx) \sqrt{a + ia \tan(c + dx)}}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{7/2}*(a + I*a*\operatorname{Tan}[c + d*x])^{3/2}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((-2 - 2*I)*a^{3/2}*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d + (4*a*(9*A - (10*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d) - (2*a*((6*I)*A + 5*B)*\operatorname{Cot}[c + d*x]^{3/2}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d) - (2*a*A*\operatorname{Cot}[c + d*x]^{5/2}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(5*d)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

$\operatorname{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_) + (b_.)*\tan[(e_.) + (f_.)*(x_)]]/\operatorname{Sqrt}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

#### Rule 3674

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

```

#### Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

```

#### Rule 4326

```

Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

#### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{5d} \\
&= -\frac{2a(6iA+5B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{4a(9A-10iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{4a(9A-10iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{4a(9A-10iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{4a(9A-10iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= -\frac{(2-2i)a^{3/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d \operatorname{sech}^3(c+dx)(A \cos(c+dx)+B \sin(c+dx))}
\end{aligned}$$

**Mathematica [A]**

time = 2.97, size = 289, normalized size = 1.44

$$\frac{\left( -2\sqrt{2}(A-iB)e^{-2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\frac{\sqrt{\cot(c+dx)}\csc^2(c+dx)(-15A+20iB+(21A-20iB)\cos(2(c+dx))+(6A+5iB)\sin(2(c+dx)))(-1+i\tan(c+dx))}{15\sqrt{\sec(c+dx)}}\right)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{d \operatorname{sech}^3(c+dx)(A \cos(c+dx)+B \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(7/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((-2\*sqrt(2)\*(A - I\*B)\*sqrt(-1 + E^((2\*I)\*(c + d\*x))))\*sqrt[E^(I\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x))))\*sqrt[(I\*(1 + E^((2\*I)\*(c + d\*x))))]/(-1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[E^(I\*(c + d\*x))/sqrt(-1 + E^((2\*I)\*(c + d\*x)))]/E^((2\*I)\*(c + d\*x)) + (sqrt[Cot[c + d\*x]]\*Csc[c + d\*x]^2\*(-15\*A + (20\*I)\*B + (21\*A - (20\*I)\*B)\*Cos[2\*(c + d\*x)] + ((6\*I)\*A + 5\*B)\*Sin[2\*(c + d\*x)])\*(-1 + I\*Tan[c + d\*x]))/(15\*sqrt[Sec[c + d\*x]])\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/(d\*Sec[c + d\*x]^(5/2)\*(A\*cos[c + d\*x] + B\*sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2243 vs. 2(164) = 328.

time = 57.48, size = 2244, normalized size = 11.16

method	result	size
default	Expression too large to display	2244

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/15/d*a*2^(1/2)*(15*I*A*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^2*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*
x+c)-cos(d*x+c)+1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+s
in(d*x+c)+cos(d*x+c)-1))+30*I*B*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+30*I*B
*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan(((1+cos
(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+30*I*A*sin(d*x+c)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1
/2)-1)+15*I*B*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*ln
(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x
+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-co
s(d*x+c)+1))+18*A*2^(1/2)+25*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-18*I*A*sin(d
*x+c)*2^(1/2)+25*I*B*2^(1/2)*cos(d*x+c)+27*A*cos(d*x+c)^3*2^(1/2)-24*A*cos(
d*x+c)*2^(1/2)-30*I*A*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)-30*I*A*sin(d*x+c)*((-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1
)-15*I*A*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(-(((1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((1+cos(d
*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))-30*I*
B*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2)+1)-30*I*B*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)-15*I*B*sin(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*
2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))-30*A*sin(d*x+c)*((-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)+1)-30*A*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)-15*A*sin(d*x+c)*
((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*ln(-(((1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((1+cos(d*x+c))/
sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))+30*B*sin(d*x
+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))
/sin(d*x+c))^(1/2)*2^(1/2)+1)+30*B*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+15*
B*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*ln(-(((1+cos(
d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((1
```

$$\begin{aligned}
& +\cos(d*x+c)/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)) \\
& -25*I*B*2^{(1/2)}*\cos(d*x+c)^3-20*I*B*2^{(1/2)}+30*I*A*\sin(d*x+c)*((-1+\cos(d*x+ \\
& c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\
& )*2^{(1/2)}+1)-5*B*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)-20*2^{(1/2)}*B*\sin(d*x+c)-21*A \\
& *2^{(1/2)}*\cos(d*x+c)^2+30*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*ar \\
& ctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+30*A*\sin(d*x+c)*((-1+\cos \\
& (d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\
& )-1)+15*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-((( -1+\cos(d*x+c) \\
& ))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos( \\
& d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))-30*B \\
& *\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin( \\
& d*x+c))^{(1/2)}*2^{(1/2)}+1)-30*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\
& *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-15*B*\sin(d*x+c)*((-1+ \\
& \cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\
& )*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}* \\
& 2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))+27*I*A*\sin(d*x+c)*2^{(1/2)}*\cos( \\
& d*x+c)^2-6*I*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)+20*I*2^{(1/2)}*B*\cos(d*x+c)^2)*s \\
& \sin(d*x+c)*(\cos(d*x+c)/\sin(d*x+c))^{(7/2)}*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d* \\
& x+c))^{(1/2)}/(I*\sin(d*x+c)+\cos(d*x+c)-1)/\cos(d*x+c)^3
\end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs.  $2(153) = 306$ .  
time = 0.91, size = 1457, normalized size = 7.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $1/15*(2*\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2} - 2*\cos(2*d*x + 2*c) + 1)*((15*((I + 1)*A - (I - 1)*B)*a*\cos(3*d*x + 3*c) + (-16*I + 16)*A + (15*I - 15)*B)*a*\cos(d*x + c) + 15*((I - 1)*A + (I + 1)*B)*a*\sin(3*d*x + 3*c) + (-16*I - 16)*A - (15*I + 15)*B)*a*\sin(d*x + c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + (15*(-(I - 1)*A - (I + 1)*B)*a*\cos(3*d*x + 3*c) + ((16*I - 16)*A + (15*I + 15)*B)*a*\cos(d*x + c) + 15*((I + 1)*A - (I - 1)*B)*a*\sin(3*d*x + 3*c) + (-16*I + 16)*A + (15*I - 15)*B)*a*\sin(d*x + c))*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)))*\sqrt{a} + 15*(2*((-I - 1)*A - (I + 1)*B)*a*\cos(2*d*x + 2*c)^2 + (-I - 1)*A - (I + 1)*B)*a*\sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I + 1)*B)*a*\cos(2*d*x + 2*c) + (-I - 1)*A - (I + 1)*B)*a*\arctan2(2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + 2*\sin(d*x + c), 2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + 2*\cos(d*x + c)) + ((-I + 1)*A + (I - 1)*B)*a*\cos(2*d*x$

$$\begin{aligned}
& + 2*c)^2 + (-I + 1)*A + (I - 1)*B)*a*\sin(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*a*\cos(2*d*x + 2*c) + (-I + 1)*A + (I - 1)*B)*a*\log(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 4*\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))^2) + 8*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))))*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*\sqrt{a} \\
& + 2*((15*((I + 1)*A - (I - 1)*B)*a*\cos(5*d*x + 5*c) + 5*(-(I + 1)*A + (4*I - 4)*B)*a*\cos(3*d*x + 3*c) + ((2*I + 2)*A - (5*I - 5)*B)*a*\cos(d*x + c) + 15*((I - 1)*A + (I + 1)*B)*a*\sin(5*d*x + 5*c) + 5*(-(I - 1)*A - (4*I + 4)*B)*a*\sin(3*d*x + 3*c) + ((2*I - 2)*A + (5*I + 5)*B)*a*\sin(d*x + c))*\cos(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + (((-3*I + 3)*A + (5*I - 5)*B)*a*\cos(d*x + c) + (-3*I - 3)*A - (5*I + 5)*B)*a*\sin(d*x + c))*\cos(2*d*x + 2*c)^2 + (-3*I + 3)*A + (5*I - 5)*B)*a*\cos(d*x + c) + ((-3*I + 3)*A + (5*I - 5)*B)*a*\cos(d*x + c) + (-3*I - 3)*A - (5*I + 5)*B)*a*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 + (-3*I - 3)*A - (5*I + 5)*B)*a*\sin(d*x + c) + 2*((3*I + 3)*A - (5*I - 5)*B)*a*\cos(d*x + c) + ((3*I - 3)*A + (5*I + 5)*B)*a*\sin(d*x + c))*\cos(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + (15*(-(I - 1)*A - (I + 1)*B)*a*\cos(5*d*x + 5*c) + 5*((I - 1)*A + (4*I + 4)*B)*a*\cos(3*d*x + 3*c) + (-2*I - 2)*A - (5*I + 5)*B)*a*\cos(d*x + c) + 15*((I + 1)*A - (I - 1)*B)*a*\sin(5*d*x + 5*c) + 5*(-(I + 1)*A + (4*I - 4)*B)*a*\sin(3*d*x + 3*c) + ((2*I + 2)*A - (5*I - 5)*B)*a*\sin(d*x + c))*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + (((3*I - 3)*A + (5*I + 5)*B)*a*\cos(d*x + c) + (-3*I + 3)*A + (5*I - 5)*B)*a*\sin(d*x + c))*\cos(2*d*x + 2*c)^2 + ((3*I - 3)*A + (5*I + 5)*B)*a*\cos(d*x + c) + ((3*I - 3)*A + (5*I + 5)*B)*a*\cos(d*x + c) + (-3*I + 3)*A + (5*I - 5)*B)*a*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 + (-3*I + 3)*A + (5*I - 5)*B)*a*\sin(d*x + c) + 2*((-3*I - 3)*A - (5*I + 5)*B)*a*\cos(d*x + c) + ((3*I + 3)*A - (5*I - 5)*B)*a*\sin(d*x + c))*\cos(2*d*x + 2*c))*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))*\sqrt{a})/((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(5/4)}*d)
\end{aligned}$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs.  $2(153) = 306$ .  
time = 1.23, size = 510, normalized size = 2.54

$$\frac{15\sqrt{2}\sqrt{-2A^2-2AB+I^2B^2}\sqrt{a^3/d^2}\sqrt{d^4e^{4I*d*x+4I*c}-2d^2e^{2I*d*x+2I*c}+d}\log(4*((A-I*B)*a^2e^{I*d*x+I*c}-2d^2e^{2I*d*x+2I*c}+d))}{(15*((I+1)*A-(I-1)*B)*a*\cos(5*d*x+5*c)+5*((I-1)*A+(4*I+4)*B)*a*\cos(3*d*x+3*c)+(-2*I-2)*A-(5*I+5)*B)*a*\cos(d*x+c)+15*((I+1)*A-(I-1)*B)*a*\sin(5*d*x+5*c)+5*((I-1)*A+(4*I-4)*B)*a*\sin(3*d*x+3*c)+((2*I+2)*A-(5*I-5)*B)*a*\sin(d*x+c))*\cos(5/2*\arctan2(\sin(2*d*x+2*c),\cos(2*d*x+2*c)-1))+(((3*I-3)*A+(5*I+5)*B)*a*\cos(d*x+c)+(-3*I+3)*A+(5*I-5)*B)*a*\sin(d*x+c))*\cos(2*d*x+2*c)^2+((3*I-3)*A+(5*I+5)*B)*a*\cos(d*x+c)+((3*I-3)*A+(5*I+5)*B)*a*\cos(d*x+c)+(-3*I+3)*A+(5*I-5)*B)*a*\sin(d*x+c))*\sin(2*d*x+2*c)^2+(-3*I+3)*A+(5*I-5)*B)*a*\sin(d*x+c)+2*((-3*I-3)*A-(5*I+5)*B)*a*\cos(d*x+c)+((3*I+3)*A-(5*I-5)*B)*a*\sin(d*x+c))*\cos(2*d*x+2*c))*\sin(1/2*\arctan2(\sin(2*d*x+2*c),\cos(2*d*x+2*c)-1))*\sqrt{a})/((\cos(2*d*x+2*c)^2+\sin(2*d*x+2*c)^2-2*\cos(2*d*x+2*c)+1)^{(5/4)}*d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] -1/15\*(15\*sqrt(2)\*sqrt(-I\*A^2 - 2\*A\*B + I\*B^2)\*a^3/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) - 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(4\*((A - I\*B)\*a^2\*e^(I\*d\*x + I\*c) -



```

sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a) - 15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a) - 2*sqrt(2)*((27*A - 25*I*B)*a*e^(5*I*d*x + 5*I*c) - 10*(3*A - 4*I*B)*a*e^(3*I*d*x + 3*I*c) + 15*(A - I*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)

```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/2), x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^(3/2),x)
```

$$3.543 \quad \int \cot^2(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=157

$$\frac{(2 + 2i)a^{3/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d} - \frac{2a(4iA + 3B) \sqrt{\cot(c + dx)}}{d}$$

[Out] (2+2\*I)\*a^(3/2)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-2/3\*a\*A\*cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/3\*a\*(4\*I\*A+3\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3674, 3679, 12, 3625, 211}

$$\frac{(2 + 2i)a^{3/2}(B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} - \frac{2a(3B + 4iA) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{3d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx) \sqrt{a + ia \tan(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((2 + 2\*I)\*a^(3/2)\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d - (2\*a\*((4\*I)\*A + 3\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d) - (2\*a\*A\*Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && Ne

$Q[c^2 + d^2, 0]$

Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia)}{\dots} \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{3d} + \\
&= -\frac{2a(4iA+3B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3d} \\
&= -\frac{2a(4iA+3B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3d} \\
&= -\frac{2a(4iA+3B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3d} \\
&= \frac{(2+2i)a^{3/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d \sec^{\frac{5}{2}}(c+dx)(A \cos(c+dx)+B \sin(c+dx))}
\end{aligned}$$

**Mathematica [A]**

time = 2.71, size = 259, normalized size = 1.65

$$\frac{\left(2\sqrt{2}(iA+B)e^{-2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-\frac{2(-i+\cot(c+dx))(A\csc(c+dx)+(4iA+3B)\sec(c+dx))}{3\sqrt{\cot(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\right)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{d \sec^{\frac{5}{2}}(c+dx)(A \cos(c+dx)+B \sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] (((2*Sqrt[2]*(I*A + B)*Sqrt[-1 + E^((2*I)*(c + d*x))])*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]/E^((2*I)*(c + d*x)) - (2*(-I + Cot[c + d*x]))*(A*Csc[c + d*x] + ((4*I)*A + 3*B)*Sec[c + d*x]))/(3*Sqrt[Cot[c + d*x]]*Sec[c + d*x]^(3/2))*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/(d*Sec[c + d*x]^(5/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1984 vs. 2(128) = 256.

time = 66.98, size = 1985, normalized size = 12.64

method	result	size
--------	--------	------

default	Expression too large to display	1985
---------	---------------------------------	------

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/3/d*a*2^{(1/2)}*(3*I*B*2^{(1/2)}-4*A*2^{(1/2)}+5*I*A*\cos(d*x+c)*2^{(1/2)}*\sin(d*x+c) \\ & -A*\cos(d*x+c)*2^{(1/2)}+6*I*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+6*I*A*\cos(d*x+c)^2 \\ & *((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}-1)+3*I*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)-6*I*B*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-6*I*B*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-3*I*B*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\ln((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)-3*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)-6*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-3*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)+6*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+6*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+3*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\ln((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)+6*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-4*I*A*2^{(1/2)}*\sin(d*x+c)-6*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-6*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-3*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\ln((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\cos(d*x+c)+\sin(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)-6*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-6*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+3*B*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)-3*I*B*\cos(d*x+c)^2*2^{(1/2)} \\ & -3*2^{(1/2)}*B*\sin(d*x+c)+5*A*2^{(1/2)}*\cos(d*x+c)^2+6*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1) \end{aligned}$$





**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)



$$3.544 \quad \int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=186

$$\frac{2\sqrt{-1} a^{3/2} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (2 + 2i) a^{3/2} (A - iB) \tan(c + dx)}{d} + \dots$$

[Out]  $2*(-1)^{(1/4)}*a^{(3/2)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(2+2*I)*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-2*a*A*\cot(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.40, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3674, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(2+2i)a^{3/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{2\sqrt{-1}a^{3/2}B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(2*(-1)^{(1/4)}*a^{(3/2)}*B*\operatorname{ArcTan}[\frac{((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d + ((2 + 2*I)*a^{(3/2)}*(A - I*B)*\operatorname{ArcTanh}[\frac{((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d - (2*a*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*A \operatorname{rcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \|\| \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

## Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\tan(c+dx)} dx \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{d} \\
&= \frac{(2-2i)a^{3/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} \\
&= \frac{(2-2i)a^{3/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} \\
&= \frac{2\sqrt[4]{-1} a^{3/2} B \tan^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.53, size = 286, normalized size = 1.54

$$\frac{a \cos(c+dx) \sqrt{\cot(c+dx)} \left( 4\sqrt{2} A e^{i(c+dx)} - 4\sqrt{2} (A-iB) \sqrt{-1+e^{2i(c+dx)}} \log\left(\frac{e^{i(c+dx)} + \sqrt{-1+e^{2i(c+dx)}}}{-iB \sqrt{-1+e^{2i(c+dx)}}}\right) \log\left(\frac{1-3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}}{1-3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}}\right) + iB \sqrt{-1+e^{2i(c+dx)}} \log\left(\frac{1-3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}}{1-3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}}}\right) \right) \sqrt{a+ia \tan(c+dx)}}{\sqrt{2} d (1+e^{2i(c+dx)})}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*
x]), x]
```

```
[Out] -((a*Cos[c + d*x]*Sqrt[Cot[c + d*x]]*(4*Sqrt[2]*A*E^(I*(c + d*x)) - 4*Sqrt[
2]*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))])*Log[E^(I*(c + d*x)) + Sqrt[-1 +
E^((2*I)*(c + d*x))]] - I*B*Sqrt[-1 + E^((2*I)*(c + d*x))])*Log[1 - 3*E^((2
```

$$*I)*(c + d*x)) - 2*\text{Sqrt}[2]*E^{(I*(c + d*x))*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]] + I*B*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]*\text{Log}[1 - 3*E^{((2*I)*(c + d*x))} + 2*\text{Sqrt}[2]*E^{(I*(c + d*x))*\text{Sqrt}[-1 + E^{((2*I)*(c + d*x))}]]]*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]]/(\text{Sqrt}[2]*d*(1 + E^{((2*I)*(c + d*x))}))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1365 vs.  $2(150) = 300$ .  
time = 68.71, size = 1366, normalized size = 7.34

method	result	size
default	Expression too large to display	1366

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/2/d*a*2^{(1/2)}*(-I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\ & )*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)+4*I*B*\sin(d*x+c)*((-1+\cos(d*x+c)) \\ & )/\sin(d*x+c)^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+2* \\ & I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-(((1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c)) \\ & )/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+4*I*A*\sin \\ & (d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c) \\ & ))^{(1/2)}*2^{(1/2)}+1)+2*I*A*\sin(d*x+c)*2^{(1/2)}-2*I*B*\sin(d*x+c)*((-1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})-B* \\ & \sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\ln(((1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}-1)+B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\ & *\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)-2*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*2^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})+I*B*\sin \\ & (d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\ln(((1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}-1)+2*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(- \\ & (((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+ \\ & 1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c) \\ & -1))+4*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos \\ & (d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-4*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-2*A*\sin \\ & (d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c) \\ & ))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c) \\ & ))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))-4*A*\sin(d*x+c)*((- \\ & 1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2 \\ & ^{(1/2)}+1)+4*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos \\ & (d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+4*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+4*B*\sin \\ & (d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c) \\ & ))^{(1/2)}*2^{(1/2)}+1)+2*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(- \end{aligned}$$

$$\frac{((-1+\cos(dx+c))/\sin(dx+c))^{1/2} \cdot 2^{1/2} \cdot \sin(dx+c) - \sin(dx+c) - \cos(dx+c) + 1}{((-1+\cos(dx+c))/\sin(dx+c))^{1/2} \cdot 2^{1/2} \cdot \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1} + 2A \cos(dx+c) \cdot 2^{1/2} - 2A \cdot 2^{1/2} \cdot \sin(dx+c) \cdot \frac{\cos(dx+c)}{\sin(dx+c)} \cdot \frac{(I \sin(dx+c) + \cos(dx+c)) \cdot a / \cos(dx+c)^{1/2}}{(I \sin(dx+c) + \cos(dx+c) - 1) / \cos(dx+c)}$$

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(a+I\*a\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 674 vs. 2(142) = 284.

time = 1.49, size = 674, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(a+I\*a\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4 \cdot (8 \cdot \sqrt{2}) \cdot A \cdot a \cdot \sqrt{a / (e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I) / (e^{(2I \cdot dx + 2I \cdot c)} - 1)} \cdot e^{(I \cdot dx + I \cdot c)} - 4 \cdot \sqrt{2} \cdot \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^3 / d^2} \cdot d \cdot \log(4 \cdot ((A - I \cdot B) \cdot a^2 \cdot e^{(I \cdot dx + I \cdot c)} - \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^3 / d^2}) \cdot (I \cdot d \cdot e^{(2I \cdot dx + 2I \cdot c)} - I \cdot d) \cdot \sqrt{a / (e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I) / (e^{(2I \cdot dx + 2I \cdot c)} - 1)}) \cdot e^{(-I \cdot dx - I \cdot c)} / ((-I \cdot A - B) \cdot a)) + 4 \cdot \sqrt{2} \cdot \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^3 / d^2} \cdot d \cdot \log(4 \cdot ((A - I \cdot B) \cdot a^2 \cdot e^{(I \cdot dx + I \cdot c)} - \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^3 / d^2}) \cdot (-I \cdot d \cdot e^{(2I \cdot dx + 2I \cdot c)} + I \cdot d) \cdot \sqrt{a / (e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I) / (e^{(2I \cdot dx + 2I \cdot c)} - 1)}) \cdot e^{(-I \cdot dx - I \cdot c)} / ((-I \cdot A - B) \cdot a)) + \sqrt{-4 \cdot I \cdot B^2 \cdot a^3 / d^2} \cdot d \cdot \log(-16 \cdot (3 \cdot B \cdot a^2 \cdot e^{(2I \cdot dx + 2I \cdot c)} - B \cdot a^2 + \sqrt{2}) \cdot \sqrt{-4 \cdot I \cdot B^2 \cdot a^3 / d^2}) \cdot (d \cdot e^{(3I \cdot dx + 3I \cdot c)} - d \cdot e^{(I \cdot dx + I \cdot c)}) \cdot \sqrt{a / (e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I) / (e^{(2I \cdot dx + 2I \cdot c)} - 1)}) \cdot e^{(-2I \cdot dx - 2I \cdot c)} / B - \sqrt{-4 \cdot I \cdot B^2 \cdot a^3 / d^2} \cdot d \cdot \log(-16 \cdot (3 \cdot B \cdot a^2 \cdot e^{(2I \cdot dx + 2I \cdot c)} - B \cdot a^2 - \sqrt{2}) \cdot \sqrt{-4 \cdot I \cdot B^2 \cdot a^3 / d^2}) \cdot (d \cdot e^{(3I \cdot dx + 3I \cdot c)} - d \cdot e^{(I \cdot dx + I \cdot c)}) \cdot \sqrt{a / (e^{(2I \cdot dx + 2I \cdot c)} + 1)} \cdot \sqrt{(I \cdot e^{(2I \cdot dx + 2I \cdot c)} + I) / (e^{(2I \cdot dx + 2I \cdot c)} - 1)}) \cdot e^{(-2I \cdot dx - 2I \cdot c)} / B) / d \end{aligned}$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

$$3.545 \quad \int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=196

$$\frac{(-1)^{3/4} a^{3/2} (2iA+3B) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} (2-2i) a^{3/2}}{d} +$$

[Out]  $-(-1)^{3/4} a^{3/2} (2iA+3B) \operatorname{arctan}\left(\frac{(-1)^{3/4} a^{1/2} \tan(dx+c)^{1/2}}{(a+I a \tan(dx+c))^{1/2}}\right) \cot(dx+c)^{1/2} \tan(dx+c)^{1/2} / d + (2-2i) a^{3/2} (A-I B) \operatorname{arctanh}\left(\frac{(1+I) a^{1/2} \tan(dx+c)^{1/2}}{(a+I a \tan(dx+c))^{1/2}}\right) \cot(dx+c)^{1/2} \tan(dx+c)^{1/2} / d + I a B (a+I a \tan(dx+c))^{1/2} / d \cot(dx+c)^{1/2}$

**Rubi** [A]

time = 0.41, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3675, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} a^{3/2} (3B+2iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{(2-2i) a^{3/2} (A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d} + \frac{iaB \sqrt{a+ia \tan(c+dx)}}{d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*(a+I*a*\operatorname{Tan}[c+d*x])^{3/2}*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $-((( -1)^{3/4} a^{3/2} ((2I)A+3B) \operatorname{ArcTan}[\frac{(-1)^{3/4} \operatorname{Sqrt}[a] \operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]}{\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]}] \operatorname{Sqrt}[\operatorname{Cot}[c+d*x]] \operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]]) / d + ((2-2I) a^{3/2} (A-I B) \operatorname{ArcTanh}[\frac{(1+I) \operatorname{Sqrt}[a] \operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]}{\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]}] \operatorname{Sqrt}[\operatorname{Cot}[c+d*x]] \operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]) / d + (I*a*B \operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]) / (d \operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{n, x}], x, (a + b*x)^{1/p}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \|\| \operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)]]/Sqrt[(c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3675

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]



## Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

## Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int (a+ \\
&= \frac{iaB \sqrt{a+ia \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \left( \sqrt{\cot(c+dx)} \right) \int (a+ \\
&= \frac{iaB \sqrt{a+ia \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \left( 2a(A-ia) \sqrt{\cot(c+dx)} \right) \\
&= \frac{iaB \sqrt{a+ia \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{(4ia^3(A-ia) \sqrt{\cot(c+dx)})}{d \sqrt{\cot(c+dx)}} \\
&= - \frac{(2+2i)a^{3/2}(iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a+ia \tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d \sqrt{\cot(c+dx)}} \\
&= - \frac{(2+2i)a^{3/2}(iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a+ia \tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d \sqrt{\cot(c+dx)}} \\
&= \frac{\sqrt[4]{-1} a^{3/2} (2A-3iB) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a+ia \tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{d \sqrt{\cot(c+dx)}}
\end{aligned}$$

**Mathematica [A]**

time = 4.57, size = 360, normalized size = 1.84

$$\frac{(a+ia \tan(c+dx))^{3/2} (A+B \tan(c+dx)) \left( \sqrt{2} e^{-2i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \sqrt{\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}} (-16i(A-ia) \log(e^{i(c+dx)} + \sqrt{-1+e^{2i(c+dx)}}) + \sqrt{2}(2A+3B) (\log(1-3e^{2i(c+dx)} - 2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}) - \log(1-3e^{2i(c+dx)} + 2\sqrt{2}e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}})) \right) + \frac{4B \sqrt{\tan(c+dx)}}{\sqrt{\cot(c+dx)} \sqrt{\sec(c+dx)}} \right)}{8d \sec^3(c+dx) (A \cos(c+dx) + B \sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*
x]), x]
```

```
[Out] ((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])*((Sqrt[2]*Sqrt[-1 + E^((
2*I)*(c + d*x))]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1
```

$$+ E^{\left((2I)(c + dx)\right)} / \left(-1 + E^{\left((2I)(c + dx)\right)}\right) \left[ (-16I)(A - I B) \operatorname{Log} \left[ E^{I(c + dx)} + \sqrt{-1 + E^{\left((2I)(c + dx)\right)}} \right] + \sqrt{2} \left( (2I)A + 3B \right) \left( \operatorname{Log} \left[ 1 - 3E^{\left((2I)(c + dx)\right)} - 2\sqrt{2} E^{I(c + dx)} \sqrt{-1 + E^{\left((2I)(c + dx)\right)}} \right] - \operatorname{Log} \left[ 1 - 3E^{\left((2I)(c + dx)\right)} + 2\sqrt{2} E^{I(c + dx)} \sqrt{-1 + E^{\left((2I)(c + dx)\right)}} \right] \right) \right] / \left( E^{\left((2I)(c + dx)\right)} + (8B(I + \tan[c + dx])) / \left( \sqrt{\cot[c + dx]} \sqrt{\sec[c + dx]} \right) \right) / \left( 8d \sec[c + dx]^{5/2} \right) \left( A \cos[c + dx] + B \sin[c + dx] \right)$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs.  $2(158) = 316$ .

time = 60.92, size = 1290, normalized size = 6.58

method	result	size
default	Expression too large to display	1290

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(dx+c)^(1/2)*(a+I*a*tan(dx+c))^(3/2)*(A+B*tan(dx+c)),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/4/d*2^{(1/2)}*a*(-1+\cos(dx+c))*(4*A*\cos(dx+c)*\ln\left(\frac{-((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\cos(dx+c)+\sin(dx+c)-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1}\right)+4*B*\cos(dx+c)*\ln\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\cos(dx+c)+\sin(dx+c)-1}\right)+3*B*\cos(dx+c)*2^{(1/2)}*\ln\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}+1}\right)-6*B*\cos(dx+c)*2^{(1/2)}*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}+1}\right)+8*A*\cos(dx+c)*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}+1}\right)+8*I*A*\cos(dx+c)*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}+1}\right)+4*I*A*\cos(dx+c)*\ln\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1}\right)+8*I*A*\cos(dx+c)*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}+1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}-1}\right)-8*I*B*\cos(dx+c)*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}+1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}-1}\right)-4*I*B*\cos(dx+c)*\ln\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\cos(dx+c)+\sin(dx+c)-1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}*\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1}\right)-2*B*\cos(dx+c)*2^{(1/2)}*\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}-1\right)-4*I*A*\cos(dx+c)*2^{(1/2)}*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}-1\right)-2*I*B*2^{(1/2)}*\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}-1\right)+8*B*\cos(dx+c)*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}+1}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}*2^{(1/2)}-1}\right)-2*B*2^{(1/2)}*\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}-1\right)+2*A*\cos(dx+c)*2^{(1/2)}*\ln\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}+1}\right)-4*A*\cos(dx+c)*2^{(1/2)}*\arctan\left(\frac{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}}{((-1+\cos(dx+c))}{\sin(dx+c)})^{(1/2)}-1}\right)-2*I*A*\cos(dx+c)*\ln \end{aligned}$$

$$\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+1\right)^{1/2}+2IA\cos(dx+c)\ln\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-1\right)^{1/2}-3IB\cos(dx+c)\ln\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+1\right)^{1/2}+6IB\cos(dx+c)\arctan\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+3IB\cos(dx+c)\ln\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-1\right)^{1/2}\left(\frac{I\sin(dx+c)+\cos(dx+c)}{a/\cos(dx+c)}\right)^{1/2}\left(\frac{\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}/\left(\frac{I\sin(dx+c)+\cos(dx+c)-1}{\cos(dx+c)}\right)/\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(a+I\*a\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(dx + c) + A)\*(I\*a\*tan(dx + c) + a)^(3/2)\*sqrt(cot(dx + c)), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(148) = 296.

time = 1.37, size = 827, normalized size = 4.22



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(a+I\*a\*tan(dx+c))^(3/2)\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} \cdot (4 \cdot \sqrt{2}) \cdot \sqrt{-(IA^2 + 2AB - IB^2)} \cdot a^3/d^2 \cdot (d \cdot e^{(2I dx + 2Ic)} + d) \cdot \log(4 \cdot ((A - IB) \cdot a^2 \cdot e^{(I dx + Ic)} + \sqrt{-(IA^2 + 2AB - IB^2)}) \cdot a^3/d^2 \cdot (d \cdot e^{(2I dx + 2Ic)} - d) \cdot \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \cdot \sqrt{((I \cdot e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1))) \cdot e^{(-I dx - Ic)}/((-IA - B) \cdot a)} - 4 \cdot \sqrt{2} \cdot \sqrt{-(IA^2 + 2AB - IB^2)} \cdot a^3/d^2 \cdot (d \cdot e^{(2I dx + 2Ic)} + d) \cdot \log(4 \cdot ((A - IB) \cdot a^2 \cdot e^{(I dx + Ic)} - \sqrt{-(IA^2 + 2AB - IB^2)}) \cdot a^3/d^2 \cdot (d \cdot e^{(2I dx + 2Ic)} - d) \cdot \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \cdot \sqrt{((I \cdot e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1))) \cdot e^{(-I dx - Ic)}/((-IA - B) \cdot a)} + 4 \cdot \sqrt{2} \cdot (B \cdot a \cdot e^{(3I dx + 3Ic)} - B \cdot a \cdot e^{(I dx + Ic)}) \cdot \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \cdot \sqrt{((I \cdot e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1)) - \sqrt{(-4IA^2 - 12AB + 9IB^2)} \cdot a^3/d^2 \cdot (d \cdot e^{(2I dx + 2Ic)} + d) \cdot \log(-16 \cdot (3 \cdot (2IA + 3B) \cdot a^2 \cdot e^{(2I dx + 2Ic)} + (-2IA - 3B) \cdot a^2 + 2 \cdot \sqrt{2}) \cdot \sqrt{(-4IA^2 - 12AB + 9IB^2)}) \cdot a^3/d^2 \cdot (I \cdot d \cdot e^{(3I dx + 3Ic)} - I \cdot d \cdot e^{(I dx + Ic)}) \cdot \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \cdot \sqrt{((I \cdot e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1))) \cdot e^{(-2I dx - 2Ic)}/(2IA + 3B)} + \sqrt{(-4IA^2 - 12AB + 9IB^2)} \cdot a$

$$\frac{1}{d^2} \left( d e^{2Ix+2Ic} + d \right) \log \left( -16(3(2IA+3B)a^2 e^{2Ix+2Ic} + (-2IA-3B)a^2 + 2\sqrt{2}\sqrt{(-4IA^2-12AB+9IB^2)a^3/d^2}(-I d e^{3Ix+3Ic} + I d e^{Ix+Ic})) \sqrt{a/(e^{2Ix+2Ic}+1)} \sqrt{(I e^{2Ix+2Ic}+1)/(e^{2Ix+2Ic}-1)}) e^{-(2Ix-2Ic)/(2IA+3B)} \right) / (d e^{2Ix+2Ic} + d)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx) - i))^{\frac{3}{2}} (A + B \tan(c+dx)) \sqrt{\cot(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*(A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)\*sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c+dx)} (A + B \tan(c+dx)) (a + a \tan(c+dx) 1i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

$$3.546 \quad \int \frac{(a+ia \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=244

$$\frac{(-1)^{3/4}a^{3/2}(12A - 11iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{4d} (2+2i)a^3$$

[Out]  $-1/4*(-1)^{(3/4)}*a^{(3/2)}*(12*A-11*I*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(2+2*I)*a^{(3/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+1/2*I*a*B*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(3/2)}+1/4*a*(4*I*A+5*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(1/2)}$

**Rubi** [A]

time = 0.56, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {4326, 3675, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4}a^{3/2}(12A - 11iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) - (2+2i)a^{3/2}(A - iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + \frac{a(5B+4iA)\sqrt{a+ia \tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{iaB\sqrt{a+ia \tan(c+dx)}}{2d\cot^3(c+dx)}}{4d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[c + d*x])^{(3/2)}*(A + B*\text{Tan}[c + d*x])}{\text{Sqrt}[\text{Cot}[c + d*x]]}, x]$

[Out]  $-1/4*((-1)^{(3/4)}*a^{(3/2)}*(12*A - (11*I)*B)*\text{ArcTan}[\frac{((-1)^{(3/4)}*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])}{\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]}]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])/d - ((2 + 2*I)*a^{(3/2)}*(A - I*B)*\text{ArcTanh}[\frac{((1 + I)*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])}{\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]}]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])/d + ((I/2)*a*B*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(d*\text{Cot}[c + d*x]^{(3/2)}) + (a*((4*I)*A + 5*B)*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(4*d*\text{Sqrt}[\text{Cot}[c + d*x]])$

**Rule 65**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\text{Int}[\frac{(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] := \text{Simp}[\frac{1}{\text{Rt}[a, 2]*\text{Rt}[b, 2]]]*\text{ArcTan}[\frac{\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])}{\text{Rt}[a, 2]}], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a$

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3678

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x] - Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(c + dx))^{3/2}(A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + \\
 &= \frac{iaB \sqrt{a + ia \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{1}{2} \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \\
 &= \frac{iaB \sqrt{a + ia \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{a(4iA + 5B) \sqrt{a + ia \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
 &= \frac{iaB \sqrt{a + ia \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{a(4iA + 5B) \sqrt{a + ia \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
 &= \frac{iaB \sqrt{a + ia \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{a(4iA + 5B) \sqrt{a + ia \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
 &= \frac{(2 - 2i)a^{3/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
 &= \frac{(2 - 2i)a^{3/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
 &= \frac{\sqrt[4]{-1} a^{3/2}(12iA + 11B) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{4d}
 \end{aligned}$$

**Mathematica [A]**

time = 4.02, size = 441, normalized size = 1.81

$$\frac{ia^2(c + dx)\sqrt{\cot(c + dx)} \operatorname{atanh}\left(\frac{a + ia \tan(c + dx)}{\sqrt{a + ia \tan(c + dx)}}\right) + 4a^2 \sqrt{\cot(c + dx)} \operatorname{atanh}\left(\frac{a + ia \tan(c + dx)}{\sqrt{a + ia \tan(c + dx)}}\right) + 4a^2 \sqrt{\cot(c + dx)} \operatorname{atanh}\left(\frac{a + ia \tan(c + dx)}{\sqrt{a + ia \tan(c + dx)}}\right) + \dots}{16(A \sin(c + dx) + B \cos(c + dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[Cot[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])*(-(Sqrt[2]*(Sqrt[2]*(12*A - (11*I)*B)*Log[(2*E^(((5*I)/2)*c)*(Sqrt[2] - I*Sqrt[2]*E^(I*(c + d*x)) + (2*I)*Sqrt[-1 + E^((2*I)*(c + d*x)]))]/((12*A - (11*I)*B)*(-I + E^(I*(c + d*x))))] + Sqrt[2]*(-12*A + (11*I)*B)*Log[(2*E^(((5*I)/2)*c)*((-I)*Sqrt[2] + Sqrt[2]*E^(I*(c + d*x)) + 2*Sqrt[-1 + E^((2*I)*(c + d*x)]))]/(((12*I)*A + 11*B)*(I + E^(I*
```



$$(c + d*x)))) + 32*(A - I*B)*\text{Log}[(\text{Cos}[c] - I*\text{Sin}[c])*(\text{Cos}[c + d*x] + I*\text{Sin}[c + d*x] + \text{Sqrt}[-1 + \text{Cos}[2*(c + d*x)] + I*\text{Sin}[2*(c + d*x)]])] * \text{Sqrt}[I*(I + \text{Cot}[c + d*x])* \text{Sin}[c + d*x]^2*(\text{Cos}[2*c + d*x] - I*\text{Sin}[2*c + d*x])] + 4*(I*\text{Cos}[c] + \text{Sin}[c])* \text{Tan}[c + d*x]*(4*A - (5*I)*B + 2*B*\text{Tan}[c + d*x])]/(16*d*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4489 vs.  $2(194) = 388$ .

time = 63.26, size = 4490, normalized size = 18.40

method	result	size
default	Expression too large to display	4490

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/16/d*2^{(1/2)}*a*(-24*I*A*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})) \\ & *\cos(d*x+c)^2*\sin(d*x+c)-14*B*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & +11*B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & +8*A*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & +8*A*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & -24*I*A*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})*\cos(d*x+c)^2 \\ & +32*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & +32*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & +16*I*B*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))*\cos(d*x+c)^2*\sin(d*x+c) \\ & +8*I*A*2^{(1/2)}*\cos(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+4*I*B*2^{(1/2)}*\sin(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & +24*A*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})*\cos(d*x+c)^2*\sin(d*x+c) \\ & +12*A*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & -12*A*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & -11*B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & -22*B*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})*\cos(d*x+c)^2*\sin(d*x+c) \\ & +12*I*A*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)*\cos(d*x+c)^2-12*I*A*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)*\cos(d*x+c)^2+32*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & +32*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)*\cos(d*x+c)^2*\sin(d*x+c) \\ & +16*I*A*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))*\cos(d*x+c)^2*\sin(d*x+c) \\ & -22*I*B*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})*\cos(d*x+c)^2-11*I*B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)*\cos(d*x+c)^2+11*I*B*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)*\cos(d*x+c)^2+16*A*\ln(-((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))*\cos(d*x+c)^2*\sin(d*x+c) \end{aligned}$$

$$\begin{aligned}
& 1/2) * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) / (((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} \\
& ) * 2^{1/2} * \sin(d*x+c) - \sin(d*x+c) - \cos(d*x+c) + 1) * \cos(d*x+c)^3 - 32*B * \arctan((( - \\
& 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} - 1) * \cos(d*x+c)^3 - 32*B * \arctan((( - 1 + \cos \\
& s(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} + 1) * \cos(d*x+c)^3 - 16*B * \ln(-((( - 1 + \cos(d*x+ \\
& c)) / \sin(d*x+c))^{1/2} * 2^{1/2} * \sin(d*x+c) - \sin(d*x+c) - \cos(d*x+c) + 1) / (((-1 + \cos \\
& (d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} * \sin(d*x+c) + \sin(d*x+c) + \cos(d*x+c) - 1) * \cos \\
& (d*x+c)^3 + 12*I*A*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} + 1) * \cos(d*x+c \\
& )^2 * \sin(d*x+c) - 12*I*A*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 1) * \cos( \\
& d*x+c)^2 * \sin(d*x+c) - 22*I*B*2^{1/2} * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} \\
& )) * \cos(d*x+c)^2 * \sin(d*x+c) - 11*I*B*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{( \\
& 1/2) + 1) * \cos(d*x+c)^2 * \sin(d*x+c) + 11*I*B*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+ \\
& c))^{1/2} - 1) * \cos(d*x+c)^2 * \sin(d*x+c) + 8*I*A*2^{1/2} * \cos(d*x+c) * \sin(d*x+c) * (( \\
& - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 8*I*A*2^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) * ((-1 \\
& + \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 14*I*B*2^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) * ((-1 + \\
& \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 10*I*B*2^{1/2} * \cos(d*x+c) * \sin(d*x+c) * ((-1 + \cos \\
& (d*x+c)) / \sin(d*x+c))^{1/2} - 11*I*B*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{( \\
& 1/2) - 1) * \cos(d*x+c)^3 + 14*I*B*2^{1/2} * \cos(d*x+c)^3 * ((-1 + \cos(d*x+c)) / \sin(d*x+c \\
& ))^{1/2} - 4*I*B*2^{1/2} * \cos(d*x+c)^2 * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 14*I \\
& * B*2^{1/2} * \cos(d*x+c) * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} + 24*I*A*2^{1/2} * \ar \\
& \tan((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2}) * \cos(d*x+c)^3 - 12*I*A*2^{1/2} * \ln((( - 1 \\
& + \cos(d*x+c)) / \sin(d*x+c))^{1/2} + 1) * \cos(d*x+c)^3 + 12*I*A*2^{1/2} * \ln((( - 1 + \cos(d \\
& *x+c)) / \sin(d*x+c))^{1/2} - 1) * \cos(d*x+c)^3 - 8*I*A*2^{1/2} * \cos(d*x+c)^3 * ((-1 + \cos \\
& s(d*x+c)) / \sin(d*x+c))^{1/2} + 22*I*B*2^{1/2} * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x+ \\
& c))^{1/2}) * \cos(d*x+c)^3 + 11*I*B*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} \\
& ) + 1) * \cos(d*x+c)^3 - 22*B * \cos(d*x+c)^2 * 2^{1/2} * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x \\
& +c))^{1/2}) + 4*B*2^{1/2} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * \sin(d*x+c) - 4*B * \cos \\
& os(d*x+c)^2 * 2^{1/2} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} + 14*B * \cos(d*x+c) * 2^{1 \\
& /2} * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} + 12*A*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin \\
& (d*x+c))^{1/2} - 1) * \cos(d*x+c)^3 + 22*B*2^{1/2} * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x \\
& +c))^{1/2}) * \cos(d*x+c)^3 - 11*B*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} \\
& + 1) * \cos(d*x+c)^3 + 11*B*2^{1/2} * \ln((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 1) * \cos( \\
& d*x+c)^3 - 14*B*2^{1/2} * \cos(d*x+c)^3 * ((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} - 32*A * \\
& \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} - 1) * \cos(d*x+c)^2 * \sin(d*x+c \\
& ) - 32*A * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} + 1) * \cos(d*x+c)^2 * \sin \\
& n(d*x+c) - 32*I*A * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} - 1) * \cos(d* \\
& x+c)^3 - 32*I*A * \arctan((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} + 1) * \cos(d*x+ \\
& c)^3 - 16*I*A * \ln(-((( - 1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} * \sin(d*x+c) - \sin( \\
& d*x+c) - \cos(d*x+c) + 1) / (((-1 + \cos(d*x+c)) / \sin(d*x+c))^{1/2} * 2^{1/2} * \sin(d*x+c) \\
& + \sin(d*x+c) + \cos(d*x+c) - 1) * \cos(d*x+c)^3 - 32*I*B * \dots
\end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)/sqrt(cot(d\*x + c)), x)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 913 vs.  $2(182) = 364$ .  
time = 1.53, size = 913, normalized size = 3.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/16*(16*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((A - I*B)*a^2*e^{(I*d*x + I*c)} - \sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})) * e^{(-I*d*x - I*c)} / ((-I*A - B)*a)) - 16*\sqrt{2}*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((A - I*B)*a^2*e^{(I*d*x + I*c)} - \sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2}*(-I*d*e^{(2*I*d*x + 2*I*c)} + I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})) * e^{(-I*d*x - I*c)} / ((-I*A - B)*a)) - 4*\sqrt{2}*((4*A - 7*I*B)*a*e^{(5*I*d*x + 5*I*c)} + 4*I*B*a*e^{(3*I*d*x + 3*I*c)} - (4*A - 3*I*B)*a*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)} - \sqrt{(144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-16*(3*(12*I*A + 11*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-12*I*A - 11*B)*a^2 + 2*\sqrt{2}*\sqrt{(144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2}*(d*e^{(3*I*d*x + 3*I*c)} - d*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})) * e^{(-2*I*d*x - 2*I*c)} / (12*I*A + 11*B)) + \sqrt{(144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2}*(d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(-16*(3*(12*I*A + 11*B)*a^2*e^{(2*I*d*x + 2*I*c)} + (-12*I*A - 11*B)*a^2 - 2*\sqrt{2}*\sqrt{(144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2}*(d*e^{(3*I*d*x + 3*I*c)} - d*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})) * e^{(-2*I*d*x - 2*I*c)} / (12*I*A + 11*B)) / (d*e^{(4*I*d*x + 4*I*c)} + 2*d*e^{(2*I*d*x + 2*I*c)} + d) \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx) - i))^{\frac{3}{2}}(A + B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*(3/2)\*(A + B\*tan(c + d\*x))/sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(3/2)/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{3/2}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(3/2))/cot(c + d\*x)^(1/2), x)

$$3.547 \quad \int \cot^{\frac{11}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=297

$$\frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d} - \frac{8a^2(197A - 195iB)}{d}$$

[Out] (4+4\*I)\*a^(5/2)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+8/315\*a^2\*(59\*I\*A+60\*B)\*cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d+2/105\*a^2\*(46\*A-45\*I\*B)\*cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/21\*a^2\*(4\*I\*A+3\*B)\*cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-8/315\*a^2\*(197\*A-195\*I\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/9\*a\*A\*cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)/d

**Rubi [A]**

time = 0.70, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3674, 3679, 12, 3625, 211}

$$\frac{(4+4i)a^{5/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{8a^2(197A-195iB)\cot^2(c+dx)\sqrt{\tan(c+dx)}}{21d} + \frac{2a^2(46A-45iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{2a^2(59iA+60B)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{315d} - \frac{8a^2(197A-195iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{315d} - \frac{2a^2\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(11/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((4 + 4\*I)\*a^(5/2)\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d - (8\*a^2\*(197\*A - (195\*I)\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(315\*d) + (8\*a^2\*((59\*I)\*A + 60\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(315\*d) + (2\*a^2\*(46\*A - (45\*I)\*B)\*Cot[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d) - (2\*a^2\*((4\*I)\*A + 3\*B)\*Cot[c + d\*x]^(7/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(21\*d) - (2\*a\*A\*Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(9\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_)
+ (f_)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{11}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}}{9d} \\
&= -\frac{2a^2(4iA+3B) \cot^{\frac{7}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{21d} \\
&= \frac{2a^2(46A-45iB) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{8a^2(59iA+60B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{315d} \\
&= -\frac{8a^2(197A-195iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{315d} \\
&= -\frac{8a^2(197A-195iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{315d} \\
&= -\frac{8a^2(197A-195iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{315d} \\
&= \frac{(4-4i)a^{5/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d \sec^3(c+dx)(A \cos(c+dx)+B \sin(c+dx))}
\end{aligned}$$

**Mathematica [A]**

time = 6.17, size = 354, normalized size = 1.19

$$\frac{\left(4\sqrt{2}(A-IB)e^{-3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)+\frac{\sqrt{\cot(c+dx)}\operatorname{arctanh}\left(\frac{\sqrt{\cot(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\cos(2c)-\sin(2c)(-2331A+2205iB+12(251A-260i)B)\cos(2c)+4e^{i(c+dx)}+9(1+9i)B)\cos(4c+4d)+2(251A-260i)B\cos(2c)+4e^{i(c+dx)}+9(1+9i)B)\cos(4c+4d)+2(251A-260i)B\cos(2c)+4e^{i(c+dx)}+9(1+9i)B)\cos(4c+4d)}{d \sec^3(c+dx)(A \cos(c+dx)+B \sin(c+dx))}\right)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(11/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((4\*sqrt(2)\*(A - I\*B)\*sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*sqrt[(I\*(1 + E^((2\*I)\*(c + d\*x))))]/(-1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[E^(I\*(c + d\*x))/sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/E^((3\*I)\*(c + d\*x)) + (sqrt[Cot[c + d\*x]]\*Csc[c + d\*x]^4\*sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c]))\*(-2331\*A + (2205\*I)\*B + 12\*(251\*A - (260\*I)\*B)\*Cos[2\*

$$(c + d*x)] + (-961*A + (915*I)*B)*\text{Cos}[4*(c + d*x)] + (282*I)*A*\text{Sin}[2*(c + d*x)] + 390*B*\text{Sin}[2*(c + d*x)] - (331*I)*A*\text{Sin}[4*(c + d*x)] - 285*B*\text{Sin}[4*(c + d*x)])) / (1260*(\text{Cos}[d*x] + I*\text{Sin}[d*x])^2)*(a + I*a*\text{Tan}[c + d*x])^(5/2)*(A + B*\text{Tan}[c + d*x])) / (d*\text{Sec}[c + d*x]^(7/2)*(A*\text{Cos}[c + d*x] + B*\text{Sin}[c + d*x]))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3411 vs.  $2(244) = 488$ .  
time = 61.64, size = 3412, normalized size = 11.49

method	result	size
default	Expression too large to display	3412

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(11/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R ETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/315/d*a^2*2^{(1/2)}*(-788*A^2^{(1/2)}-1935*B*\cos(d*x+c)^2*\sin(d*x+c)*2^{(1/2)} \\ & +1200*B*\cos(d*x+c)^4*\sin(d*x+c)*2^{(1/2)}-285*B*\cos(d*x+c)^3*\sin(d*x+c)*2^{(1/2)} \\ & -2281*A*\cos(d*x+c)^3*2^{(1/2)}+1024*A*\cos(d*x+c)*2^{(1/2)}+630*I*A*((-1+\cos(d \\ & *x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\ln(-(((1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c)) \\ & / \sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))+2520*A*\sin \\ & (d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+ \\ & c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+2520*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x \\ & +c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1 \\ & )+1260*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-((( \\ & -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1 \\ & )/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x \\ & +c)+1))-2520*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*a \\ & rctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-2520*B*\sin(d*x+c)*((-1+ \\ & \cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\arctan(((1+\cos(d*x+c))/\sin(d*x+ \\ & c))^{(1/2)}*2^{(1/2)}-1)-1260*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*c \\ & \cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin \\ & (d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c) \\ & +\sin(d*x+c)+\cos(d*x+c)-1))-961*A*\cos(d*x+c)^4*2^{(1/2)}+1260*I*A*((-1+\cos(d*x \\ & +c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*2^{(1/2)}+1)+1260*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d* \\ & x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+1260 \\ & *I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+ \\ & \cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+1260*I*B*((-1+\cos(d*x+c))/\sin(d*x \\ & +c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & )*2^{(1/2)}-1)+630*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d* \\ & x+c)*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+ \\ & \cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d* \end{aligned}$$



$$\begin{aligned}
& x+c)-\cos(d*x+c)+1))-1260*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))-2520*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-2520*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-2520*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-2520*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-1260*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)))+780*I*B*2^{(1/2)}-1260*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-1260*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-630*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)))+630*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)))+1260*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+1260*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+1292*I*A*2^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)-331*I*A*2^{(1/2)}*\cos(d*x+c)^3*\sin(d*x+c)-1950*I*A*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+630*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)))+1260*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+240*B*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)+780*2^{(1/2)}*B*\sin(d*x+c)-1200*I*B*2^{(1/2)}*\cos(d*x+c)^5+915*I*B*2^{(1/2)}*\cos(d*x+c)^4+2220*I*B*2^{(1/2)}*\cos(d*x+c)^3-1695*I*B*2^{(1/2)}*\cos(d*x+c)^2+788*I*A*2^{(1/2)}*\sin(d*x+c)-1020*I*B*2^{(1/2)}*\cos(d*x+c)+1714*A*2^{(1/2)}*\cos(d*x+c)^2-1260*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-1260*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-630*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}\dots
\end{aligned}$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4543 vs.  $2(229) = 458$ .

time = 5.35, size = 4543, normalized size = 15.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(11/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/1260*(\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2} - 2*\cos(2*d*x + 2*c) \\ & + 1)*((5040*((I + 1)*A - (I - 1)*B)*a^2*\cos(7*d*x + 7*c) + 16800*(-(I + 1)* \\ & A + (I - 1)*B)*a^2*\cos(5*d*x + 5*c) + 20496*((I + 1)*A - (I - 1)*B)*a^2*\cos \\ & (3*d*x + 3*c) + (-(9071*I + 9071)*A + (8841*I - 8841)*B)*a^2*\cos(d*x + c) + \\ & 5040*((I - 1)*A + (I + 1)*B)*a^2*\sin(7*d*x + 7*c) + 16800*(-(I - 1)*A - (I \\ & + 1)*B)*a^2*\sin(5*d*x + 5*c) + 20496*((I - 1)*A + (I + 1)*B)*a^2*\sin(3*d*x \\ & + 3*c) + (-(9071*I - 9071)*A - (8841*I + 8841)*B)*a^2*\sin(d*x + c))*\cos(7/ \\ & 2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + 8*((-(121*I + 121)*A + \\ & (75*I - 75)*B)*a^2*\cos(d*x + c) + (-(121*I - 121)*A - (75*I + 75)*B)*a^2*s \\ & \sin(d*x + c) + ((-(121*I + 121)*A + (75*I - 75)*B)*a^2*\cos(d*x + c) + (-(121 \\ & *I - 121)*A - (75*I + 75)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c)^2 + ((-(121 \\ & *I + 121)*A + (75*I - 75)*B)*a^2*\cos(d*x + c) + (-(121*I - 121)*A - (75*I + \\ & 75)*B)*a^2*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 + 630*((I + 1)*A - (I - 1)*B) \\ & *a^2*\cos(2*d*x + 2*c)^2 + ((I + 1)*A - (I - 1)*B)*a^2*\sin(2*d*x + 2*c)^2 + \\ & 2*(-(I + 1)*A + (I - 1)*B)*a^2*\cos(2*d*x + 2*c) + ((I + 1)*A - (I - 1)*B)*a \\ & ^2*\cos(3*d*x + 3*c) + 2*((121*I + 121)*A - (75*I - 75)*B)*a^2*\cos(d*x + c) \\ & + ((121*I - 121)*A + (75*I + 75)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c) + \\ & 630*((I - 1)*A + (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + ((I - 1)*A + (I + 1)* \\ & B)*a^2*\sin(2*d*x + 2*c)^2 + 2*(-(I - 1)*A - (I + 1)*B)*a^2*\cos(2*d*x + 2*c) \\ & + ((I - 1)*A + (I + 1)*B)*a^2*\sin(3*d*x + 3*c))*\cos(3/2*\arctan2(\sin(2*d*x \\ & + 2*c), \cos(2*d*x + 2*c) - 1)) + (5040*(-(I - 1)*A - (I + 1)*B)*a^2*\cos(7* \\ & d*x + 7*c) + 16800*((I - 1)*A + (I + 1)*B)*a^2*\cos(5*d*x + 5*c) + 20496*(-( \\ & I - 1)*A - (I + 1)*B)*a^2*\cos(3*d*x + 3*c) + ((9071*I - 9071)*A + (8841*I + \\ & 8841)*B)*a^2*\cos(d*x + c) + 5040*((I + 1)*A - (I - 1)*B)*a^2*\sin(7*d*x + 7 \\ & *c) + 16800*(-(I + 1)*A + (I - 1)*B)*a^2*\sin(5*d*x + 5*c) + 20496*((I + 1)* \\ & A - (I - 1)*B)*a^2*\sin(3*d*x + 3*c) + (-(9071*I + 9071)*A + (8841*I - 8841) \\ & *B)*a^2*\sin(d*x + c))*\sin(7/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - \\ & 1)) + 8*((121*I - 121)*A + (75*I + 75)*B)*a^2*\cos(d*x + c) + (-(121*I + 12 \\ & 1)*A + (75*I - 75)*B)*a^2*\sin(d*x + c) + (((121*I - 121)*A + (75*I + 75)*B) \\ & *a^2*\cos(d*x + c) + (-(121*I + 121)*A + (75*I - 75)*B)*a^2*\sin(d*x + c))*\co \\ & s(2*d*x + 2*c)^2 + (((121*I - 121)*A + (75*I + 75)*B)*a^2*\cos(d*x + c) + ( \\ & -(121*I + 121)*A + (75*I - 75)*B)*a^2*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 + 630 \\ & *(((I - 1)*A - (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + (-(I - 1)*A - (I + 1)*B) \\ & )*a^2*\sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I + 1)*B)*a^2*\cos(2*d*x + 2*c) + \\ & (-(I - 1)*A - (I + 1)*B)*a^2*\cos(3*d*x + 3*c) + 2*((-(121*I - 121)*A - (7 \\ & 5*I + 75)*B)*a^2*\cos(d*x + c) + ((121*I + 121)*A - (75*I - 75)*B)*a^2*\sin(d \\ & *x + c))*\cos(2*d*x + 2*c) + 630*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2* \\ & c)^2 + ((I + 1)*A - (I - 1)*B)*a^2*\sin(2*d*x + 2*c)^2 + 2*(-(I + 1)*A + (I \\ & - 1)*B)*a^2*\cos(2*d*x + 2*c) + ((I + 1)*A - (I - 1)*B)*a^2*\sin(3*d*x + 3*c) \\ & ))*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))*\sqrt{a} + 2520 \end{aligned}$$

```

*(2*((-I - 1)*A - (I + 1)*B)*a^2*cos(2*d*x + 2*c)^4 + (-I - 1)*A - (I + 1)
)*B)*a^2*sin(2*d*x + 2*c)^4 + 4*((I - 1)*A + (I + 1)*B)*a^2*cos(2*d*x + 2*c
)^3 + 6*(-I - 1)*A - (I + 1)*B)*a^2*cos(2*d*x + 2*c)^2 + 4*((I - 1)*A + (I
+ 1)*B)*a^2*cos(2*d*x + 2*c) + (-I - 1)*A - (I + 1)*B)*a^2 + 2*((-I - 1)
*A - (I + 1)*B)*a^2*cos(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I + 1)*B)*a^2*cos(
2*d*x + 2*c) + (-I - 1)*A - (I + 1)*B)*a^2)*sin(2*d*x + 2*c)^2)*arctan2(2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) +
((-I + 1)*A + (I - 1)*B)*a^2*cos(2*d*x + 2*c)^4 + (-I + 1)*A + (I - 1)*B)
)*a^2*sin(2*d*x + 2*c)^4 + 4*((I + 1)*A - (I - 1)*B)*a^2*cos(2*d*x + 2*c)^3
+ 6*(-I + 1)*A + (I - 1)*B)*a^2*cos(2*d*x + 2*c)^2 + 4*((I + 1)*A - (I - 1)
)*B)*a^2*cos(2*d*x + 2*c) + (-I + 1)*A + (I - 1)*B)*a^2 + 2*((-I + 1)*A +
(I - 1)*B)*a^2*cos(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*a^2*cos(2*d*
x + 2*c) + (-I + 1)*A + (I - 1)*B)*a^2)*sin(2*d*x + 2*c)^2)*log(4*cos(d*x
+ c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
- 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) +
8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + si
n(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))))*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a
) + ((5040*((I + 1)*A - (I - 1)*B)*a^2*cos(9*d*x + 9*c) + 840*(-7*I + 7)*A
+ (13*I - 13)*B)*a^2*cos(7*d*x + 7*c) + 126*((53*I + 53)*A - (83*I - 83)*B
)*a^2*cos(5*d*x + 5*c) + 3*(-167*I + 167)*A + (1777*I - 1777)*B)*a^2*cos(3
*d*x + 3*c) + (-857*I + 857)*A - (753*I - 753)...

```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs.  $2(229) = 458$ .  
time = 1.54, size = 629, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(11/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $2/315*(315*\sqrt{2})*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)*\log(4*((A - I*B)*a^3*e^{(I*d*x + I*c)} - \sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*(I*d*e^{(2*I*d*x + 2*I*c)} - I*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*e^{(-I*d*x - I*c)/((-I*A - B)*a^2)} - 315*\sqrt{2})*\sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x$

$$\begin{aligned}
& + 4*I*c) - 4*d*e^{(2*I*d*x + 2*I*c) + d}*\log(4*((A - I*B)*a^3*e^{(I*d*x + I*c)} \\
& - \sqrt{-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2}*(-I*d*e^{(2*I*d*x + 2*I*c)} + I*d) \\
& *\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)} \\
& *e^{(-I*d*x - I*c)/((-I*A - B)*a^2)} - 2*\sqrt{2}*(2*(3*23*A - 300*I*B)*a^2*e^{(9*I*d*x + 9*I*c)} - 27*(61*A - 65*I*B)*a^2*e^{(7*I*d*x + 7*I*c)} \\
& + 63*(37*A - 35*I*B)*a^2*e^{(5*I*d*x + 5*I*c)} - 1365*(A - I*B)*a^2*e^{(3*I*d*x + 3*I*c)} \\
& + 315*(A - I*B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)} \\
& )/(d*e^{(8*I*d*x + 8*I*c)} - 4*d*e^{(6*I*d*x + 6*I*c)} + 6*d*e^{(4*I*d*x + 4*I*c)} - 4*d*e^{(2*I*d*x + 2*I*c)} + d)
\end{aligned}$$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(11/2)\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(11/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(11/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{11/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(11/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

[Out] int(cot(c + d\*x)^(11/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

$$3.548 \quad \int \cot^{\frac{9}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=251

$$\frac{(4 - 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d} + \frac{4a^2(130iA + 133B)}{d}$$

[Out] (4-4\*I)\*a^(5/2)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+2/105\*a^2\*(80\*A-77\*I\*B)\*cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/35\*a^2\*(10\*I\*A+7\*B)\*cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d+4/105\*a^2\*(130\*I\*A+133\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/d-2/7\*a\*A\*cot(d\*x+c)^(7/2)\*(a+I\*a\*tan(d\*x+c))^(3/2)/d

**Rubi** [A]

time = 0.60, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3674, 3679, 12, 3625, 211}

$$\frac{(4-4i)a^{5/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)-\frac{2a^2(7B+10iA)\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{2a^2(80A-77iB)\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d}+\frac{4a^2(133B+130iA)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{105d}-\frac{2iA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{7d}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (((4 - 4\*I)\*a^(5/2)\*(A - I\*B)\*ArcTanh[(((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + (4\*a^2\*((130\*I)\*A + 133\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d) + (2\*a^2\*(80\*A - (77\*I)\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(105\*d) - (2\*a^2\*((10\*I)\*A + 7\*B)\*Cot[c + d\*x]^(5/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(35\*d) - (2\*a\*A\*Cot[c + d\*x]^(7/2)\*(a + I\*a\*Tan[c + d\*x])^(3/2))/(7\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_)
+ (f_)*(x_)]], x_Symbol] :> Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

#### Rule 3674

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

#### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

#### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

#### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{9}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}}{7d} \\
&= -\frac{2a^2(10iA+7B) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{35d} \\
&= \frac{2a^2(80A-77iB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a^2(130iA+133B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a^2(130iA+133B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= \frac{4a^2(130iA+133B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{105d} \\
&= -\frac{(4+4i)a^{5/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a+ia \tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{105d}
\end{aligned}$$

### Mathematica [A]

time = 5.37, size = 332, normalized size = 1.32

$$\frac{\left( -4i\sqrt{2}(A-iB)e^{-3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-\frac{\sqrt{\cot(c+dx)}\cos^2(c+dx)\sqrt{\sec(c+dx)}(\cos(2c)-i\sin(2c))((-35A+77iB)\cos(c+dx)+(95A-77iB)\cos(3(c+dx))+2(-215A-245B+(305A+287B)\cos(2(c+dx)))\sin(c+dx))}{110(\cos(d)x+\sin(d)x)}}{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))} \right) / d \sec^2(c+dx)(A \cos(c+dx)+B \sin(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(9/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((((-4\*I)\*Sqrt[2]\*(A - I\*B)\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sqrt[(I\*(1 + E^((2\*I)\*(c + d\*x))))/(-1 + E^((2\*I)\*(c + d\*x)))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])/E^((3\*I)\*(c + d\*x)) - (Sqrt[Cot[c + d\*x]]\*Csc[c + d\*x]^3\*Sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c])\*((-35\*A + (77\*I)\*B)\*Cos[c + d\*x] + (95\*A - (77\*I)\*B)\*Cos[3\*(c + d\*x)] + 2\*((-215\*I)\*A - 245\*B + ((305\*I)\*A + 287\*B)\*Cos[2\*(c + d\*x)])\*Sin[c + d\*x]))/(210\*(Cos[d\*x] + I\*Sin[d\*x])^2))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]))/(d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal.  $3125$  vs.  $2(206) = 412$ .  
time = 68.29, size = 3126, normalized size = 12.45

method	result	size
default	Expression too large to display	3126

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/105/d*a^2*2^{(1/2)}*(420*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4 \\ & * \arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+420*A*((-1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}* \\ & 2^{(1/2)}-1)+210*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\ln(-((( -1+ \\ & \cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/ \\ & (((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c) \\ & -1))+420*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\arctan((( -1+\cos( \\ & d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+420*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/ \\ & 2)}*\cos(d*x+c)^4*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+210*B* \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4*\ln(-((( -1+\cos(d*x+c))/\sin(d \\ & *x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d*x+c))/ \\ & \sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+210*I*A*\ln(- \\ & (((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c) \\ & )-1)/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos( \\ & d*x+c)+1))*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4+420*I*A*\arctan(( \\ & (-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)*((-1+\cos(d*x+c))/\sin(d*x+c))^{( \\ & 1/2)}*\cos(d*x+c)^4+420*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\ & -1)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4-420*I*B*\arctan((( -1+\cos \\ & (d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*co \\ & s(d*x+c)^4-420*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*((- \\ & 1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4-210*I*B*\ln(-((( -1+\cos(d*x+c))/ \\ & \sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/((( -1+\cos(d*x \\ & +c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))*((-1+co \\ & s(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^4+400*I*A*2^{(1/2)}*\cos(d*x+c)^3*\sin(d \\ & *x+c)-305*I*A*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-420*I*A*\ln(-((( -1+\cos(d*x+c) \\ & )/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/((( -1+\cos(d* \\ & x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))*((-1+c \\ & os(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2-840*I*A*\arctan((( -1+\cos(d*x+c))/s \\ & in(d*x+c))^{(1/2)}*2^{(1/2)}+1)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2 \\ & -840*I*A*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*((-1+\cos(d*x+ \\ & c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2+840*I*B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+ \\ & c))^{(1/2)}*2^{(1/2)}+1)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2+840*I* \\ & B*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)*((-1+\cos(d*x+c))/\sin \end{aligned}$$



$$\begin{aligned}
& (d*x+c)^{(1/2)}*\cos(d*x+c)^2+420*I*B*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\
& *2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\
& *2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))*((1+\cos(d*x+c))/\sin(d*x \\
& +c))^{(1/2)}*\cos(d*x+c)^2-340*I*A*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)+260*A*2^{(1/2)} \\
& -287*B*\cos(d*x+c)^2*\sin(d*x+c)*2^{(1/2)}-266*I*B*2^{(1/2)}+364*B*\cos(d*x+c)^3*s \\
& \sin(d*x+c)*2^{(1/2)}-95*A*\cos(d*x+c)^3*2^{(1/2)}+80*A*\cos(d*x+c)*2^{(1/2)}-840*A*c \\
& \cos(d*x+c)^2*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin( \\
& d*x+c))^{(1/2)}*2^{(1/2)}+1)+400*A*\cos(d*x+c)^4*2^{(1/2)}+420*A*((1+\cos(d*x+c))/ \\
& \sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+420* \\
& A*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1 \\
& /2)}*2^{(1/2)}-1)+420*B*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x \\
& +c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)+420*B*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}* \\
& \arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-420*A*((1+\cos(d*x+c)) \\
& / \sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{( \\
& 1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)} \\
& )*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))-420*B*((1+\cos(d*x+c))/\sin(d \\
& *x+c))^{(1/2)}*\cos(d*x+c)^2*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*s \\
& \sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1 \\
& /2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))-343*B*2^{(1/2)}*\cos(d*x+c)*\sin(d*x+c \\
& )+210*A*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c) \\
& )^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1)/(((1+\cos(d*x+c))/\sin(d \\
& *x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1))+210*B*((1+\cos(d* \\
& x+c))/\sin(d*x+c))^{(1/2)}*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin \\
& (d*x+c)+\sin(d*x+c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\
& )*\sin(d*x+c)-\sin(d*x+c)-\cos(d*x+c)+1))+266*2^{(1/2)}*B*\sin(d*x+c)-645*A*2^{(1/ \\
& 2)}*\cos(d*x+c)^2-840*A*\cos(d*x+c)^2*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arcta \\
& n(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-840*B*\cos(d*x+c)^2*((1+\cos \\
& (d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)} \\
& )+1)-840*B*\cos(d*x+c)^2*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\arctan(((1+\cos( \\
& d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-364*I*B*2^{(1/2)}*\cos(d*x+c)^4+77*I*B*2^{( \\
& 1/2)}*\cos(d*x+c)^3+630*I*B*2^{(1/2)}*\cos(d*x+c)^2+260*I*A*2^{(1/2)}*\sin(d*x+c)+ \\
& 210*I*A*\ln(-(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)+\sin(d*x+ \\
& c)+\cos(d*x+c)-1)/(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}*\sin(d*x+c)-\sin \\
& (d*x+c)-\cos(d*x+c)+1))*((1+\cos(d*x+c))/\sin(d*x...
\end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4087 vs.  $2(193) = 386$ .  
time = 2.13, size = 4087, normalized size = 16.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, alg  
orithm="maxima")

[Out] 
$$\begin{aligned}
& -1/105*(\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2} - 2*\cos(2*d*x + 2*c) + \\
& 1)*(3*(140*(-(I - 1)*A - (I + 1)*B)*a^2*\cos(7*d*x + 7*c) + 140*((I - 1)*A \\
& + (2*I + 2)*B)*a^2*\cos(5*d*x + 5*c) + 21*(-(4*I - 4)*A - (9*I + 9)*B)*a^2*\cos(3*d*x + 3*c) \\
& + ((4*I - 4)*A + (49*I + 49)*B)*a^2*\cos(d*x + c) + 140*((I + 1)*A - (I - 1)*B)*a^2*\sin(7*d*x + 7*c) \\
& + 140*(-(I + 1)*A + (2*I - 2)*B)*a^2*\sin(5*d*x + 5*c) + 21*((4*I + 4)*A - (9*I - 9)*B)*a^2*\sin(3*d*x + 3*c) \\
& + (-4*I + 4)*A + (49*I - 49)*B)*a^2*\sin(d*x + c))*\cos(7/2*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c) - 1)) + 4*((65*I - 65)*A + (56*I + 56)*B)*a^2*\cos(d*x + c) + (-65*I + 65)*A + (56*I - 56)*B)*a^2*\sin(d*x + c) \\
& + (((65*I - 65)*A + (56*I + 56)*B)*a^2*\cos(d*x + c) + (-65*I + 65)*A + (56*I - 56)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c)^2 \\
& + (((65*I - 65)*A + (56*I + 56)*B)*a^2*\cos(d*x + c) + (-65*I + 65)*A + (56*I - 56)*B)*a^2*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 \\
& + 105*((-(I - 1)*A - (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + (-(I - 1)*A - (I + 1)*B)*a^2*\sin(2*d*x + 2*c)^2 \\
& + 2*((I - 1)*A + (I + 1)*B)*a^2*\cos(2*d*x + 2*c) + (-(I - 1)*A - (I + 1)*B)*a^2*\cos(3*d*x + 3*c) \\
& + 2*((-(65*I - 65)*A - (56*I + 56)*B)*a^2*\cos(d*x + c) + ((65*I + 65)*A - (56*I - 56)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c) \\
& + 105*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + ((I + 1)*A - (I - 1)*B)*a^2*\sin(2*d*x + 2*c)^2 \\
& + 2*((I + 1)*A + (I - 1)*B)*a^2*\cos(2*d*x + 2*c) + ((I + 1)*A - (I - 1)*B)*a^2*\sin(3*d*x + 3*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c) - 1)) + 3*(140*(-(I + 1)*A + (I - 1)*B)*a^2*\cos(7*d*x + 7*c) + 140*((I + 1)*A - (2*I - 2)*B)*a^2*\cos(5*d*x + 5*c) \\
& + 21*(-(4*I + 4)*A + (9*I - 9)*B)*a^2*\cos(3*d*x + 3*c) + ((4*I + 4)*A - (49*I - 49)*B)*a^2*\cos(d*x + c) + 140*(-(I - 1)*A - (I + 1)*B)*a^2*\sin(7*d*x + 7*c) \\
& + 140*((I - 1)*A + (2*I + 2)*B)*a^2*\sin(5*d*x + 5*c) + 21*(-(4*I - 4)*A - (9*I + 9)*B)*a^2*\sin(3*d*x + 3*c) + ((4*I - 4)*A + (49*I + 49)*B)*a^2*\sin(d*x + c))*\sin(7/2*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c) - 1)) + 4*((65*I + 65)*A - (56*I - 56)*B)*a^2*\cos(d*x + c) + ((65*I - 65)*A + (56*I + 56)*B)*a^2*\sin(d*x + c) \\
& + (((65*I + 65)*A - (56*I - 56)*B)*a^2*\cos(d*x + c) + ((65*I - 65)*A + (56*I + 56)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c)^2 \\
& + (((65*I + 65)*A - (56*I - 56)*B)*a^2*\cos(d*x + c) + ((65*I - 65)*A + (56*I + 56)*B)*a^2*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 \\
& + 105*((-(I + 1)*A + (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + (-(I + 1)*A + (I - 1)*B)*a^2*\sin(2*d*x + 2*c)^2 \\
& + 2*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2*c) + (-(I + 1)*A + (I - 1)*B)*a^2*\cos(3*d*x + 3*c) \\
& + 2*((-(65*I + 65)*A + (56*I - 56)*B)*a^2*\cos(d*x + c) + (-65*I - 65)*A - (56*I + 56)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c) \\
& + 105*((-(I - 1)*A - (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + (-(I - 1)*A - (I + 1)*B)*a^2*\sin(2*d*x + 2*c)^2 \\
& + 2*((I - 1)*A + (I + 1)*B)*a^2*\cos(2*d*x + 2*c) + (-(I - 1)*A - (I + 1)*B)*a^2*\sin(3*d*x + 3*c))*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \\
& \cos(2*d*x + 2*c) - 1)))\sqrt{a} + 210*(2*((-(I + 1)*A + (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^4 + (-(I + 1)*A + (I - 1)*B)*a^2*\sin(2*d*x + 2*c)^4 \\
& + 4*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^3 + 6*((-(I + 1)*A + (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + 4*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2*c) \\
& + (-(I + 1)*A + (I - 1)*B)*a^2 + 2*((-(I + 1)*A + (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2*c) \\
& + (-(I + 1)*A + (I - 1)*B)*a^2)*\sin(2*d*x + 2*c)^2)*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))
\end{aligned}$$

$$\begin{aligned} & n2(2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)} \\ & * \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + 2*\sin(d*x + c), \\ & 2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)} \\ & * \cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + 2*\cos(d*x + c) \\ & ) + (((I - 1)*A + (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^4 + ((I - 1)*A + (I + 1) \\ & *B)*a^2*\sin(2*d*x + 2*c)^4 + 4*(-(I - 1)*A - (I + 1)*B)*a^2*\cos(2*d*x + 2*c) \\ & )^3 + 6*((I - 1)*A + (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + 4*(-(I - 1)*A - (I \\ & + 1)*B)*a^2*\cos(2*d*x + 2*c) + ((I - 1)*A + (I + 1)*B)*a^2 + 2*((I - 1)*A \\ & + (I + 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + 2*(-(I - 1)*A - (I + 1)*B)*a^2*\cos(2 \\ & *d*x + 2*c) + ((I - 1)*A + (I + 1)*B)*a^2)*\sin(2*d*x + 2*c)^2*\log(4*\cos(d* \\ & x + c)^2 + 4*\sin(d*x + c)^2 + 4*\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2} \\ & - 2*\cos(2*d*x + 2*c) + 1)*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\ & 2*c) - 1))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))^2) \\ & + 8*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)} \\ & )*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) + \\ & \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))))*(\cos \\ & (2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*\sqrt{ \\ & (a + (((-(608*I - 608)*A - (581*I + 581)*B)*a^2*\cos(d*x + c) + ((608*I + 6 \\ & 08)*A - (581*I - 581)*B)*a^2*\sin(d*x + c) + (((-(608*I - 608)*A - (581*I + 5 \\ & 81)*B)*a^2*\cos(d*x + c) + ((608*I + 608)*A - (581*I - 581)*B)*a^2*\sin(d*x + \\ & c))*\cos(2*d*x + 2*c)^2 + (((-(608*I - 608)*A - (581*I + 581)*B)*a^2*\cos(d*x \\ & + c) + ((608*I + 608)*A - (581*I - 581)*B)*a^2)...} \end{aligned}$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 577 vs.  $2(193) = 386$ .

time = 1.89, size = 577, normalized size = 2.30

$$\frac{\left( \frac{\sqrt{2} \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{6I dx + 6Ic} - 3d e^{4I dx + 4Ic} + 3d e^{2I dx + 2Ic} - d) \log(4((A - I^2 B) a^3 e^{I dx + Ic} + \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{2I dx + 2Ic} - d) \sqrt{a/(e^{2I dx + 2Ic} + 1)) \sqrt{(I e^{2I dx + 2Ic} + I)/(e^{2I dx + 2Ic} - 1)) e^{-I dx - Ic} / ((-IA - B) a^2)}) - 105 \sqrt{2} \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{6I dx + 6Ic} - 3d e^{4I dx + 4Ic} + 3d e^{2I dx + 2Ic} - d) \log(4((A - I^2 B) a^3 e^{I dx + Ic} - \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{2I dx + 2Ic} - d) \sqrt{a/(e^{2I dx + 2Ic} + 1)) \sqrt{(I e^{2I dx + 2Ic} + I)/(e^{2I dx + 2Ic} - 1)) e^{-I dx - Ic} / ((-IA - B) a^2)}) - 2 \sqrt{2} (2(-100IA - 91B) a^2 e^{7I dx + 7Ic} + 7(55IA + 61B) a^2 e^{5I dx + 5Ic} + 350(-IA - B) a^2 e^{3I dx + 3Ic} + 105(IA + B) \right)}{2 \sqrt{2} \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{6I dx + 6Ic} - 3d e^{4I dx + 4Ic} + 3d e^{2I dx + 2Ic} - d) \log(4((A - I^2 B) a^3 e^{I dx + Ic} + \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{2I dx + 2Ic} - d) \sqrt{a/(e^{2I dx + 2Ic} + 1)) \sqrt{(I e^{2I dx + 2Ic} + I)/(e^{2I dx + 2Ic} - 1)) e^{-I dx - Ic} / ((-IA - B) a^2)}) - 105 \sqrt{2} \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{6I dx + 6Ic} - 3d e^{4I dx + 4Ic} + 3d e^{2I dx + 2Ic} - d) \log(4((A - I^2 B) a^3 e^{I dx + Ic} - \sqrt{-I^2 A^2 + 2AB - I^2 B^2} a^{5/d^2} (d e^{2I dx + 2Ic} - d) \sqrt{a/(e^{2I dx + 2Ic} + 1)) \sqrt{(I e^{2I dx + 2Ic} + I)/(e^{2I dx + 2Ic} - 1)) e^{-I dx - Ic} / ((-IA - B) a^2)}) - 2 \sqrt{2} (2(-100IA - 91B) a^2 e^{7I dx + 7Ic} + 7(55IA + 61B) a^2 e^{5I dx + 5Ic} + 350(-IA - B) a^2 e^{3I dx + 3Ic} + 105(IA + B) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $2/105*(105*\sqrt{2}*\sqrt{-I*A^2 + 2*A*B - I*B^2}*a^{5/d^2}*(d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} - d)*\log(4*((A - I*B)*a^3*e^{(I*d*x + I*c)} + \sqrt{-I*A^2 + 2*A*B - I*B^2}*a^{5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) - 105*\sqrt{2}*\sqrt{-I*A^2 + 2*A*B - I*B^2}*a^{5/d^2}*(d*e^{(6*I*d*x + 6*I*c)} - 3*d*e^{(4*I*d*x + 4*I*c)} + 3*d*e^{(2*I*d*x + 2*I*c)} - d)*\log(4*((A - I*B)*a^3*e^{(I*d*x + I*c)} - \sqrt{-I*A^2 + 2*A*B - I*B^2}*a^{5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) - 2*\sqrt{2}*(2*(-100*I*A - 91*B)*a^2*e^{(7*I*d*x + 7*I*c)} + 7*(55*I*A + 61*B)*a^2*e^{(5*I*d*x + 5*I*c)} + 350*(-I*A - B)*a^2*e^{(3*I*d*x + 3*I*c)} + 105*(I*A + B)$

```
*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4
*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(9/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(9
/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x
)
```

```
[Out] int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

$$3.549 \quad \int \cot^2(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=205

$$\frac{(4 + 4i)a^{5/2}(A - iB) \tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d} + \frac{2a^2(38A - 35iB)}{d}$$

[Out]  $(-4-4*I)*a^{5/2}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{1/2}*\tan(d*x+c)^{1/2}/(a+I*a*\tan(d*x+c))^{1/2})*\cot(d*x+c)^{1/2}*\tan(d*x+c)^{1/2}/d-2/15*a^2*(8*I*A+5*B)*\cot(d*x+c)^{3/2}*(a+I*a*\tan(d*x+c))^{1/2}/d+2/15*a^2*(38*A-35*I*B)*\cot(d*x+c)^{1/2}*(a+I*a*\tan(d*x+c))^{1/2}/d-2/5*a*A*\cot(d*x+c)^{5/2}*(a+I*a*\tan(d*x+c))^{3/2}/d$

**Rubi** [A]

time = 0.47, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3674, 3679, 12, 3625, 211}

$$\frac{(4+4i)a^{5/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2(5B+8iA)\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2a^2(38A-35iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{7/2}*(a + I*a*\operatorname{Tan}[c + d*x])^{5/2}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((-4 - 4*I)*a^{5/2}*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/d + (2*a^2*(38*A - (35*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d) - (2*a^2*((8*I)*A + 5*B)*\operatorname{Cot}[c + d*x]^{3/2}*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/(15*d) - (2*a*A*\operatorname{Cot}[c + d*x]^{5/2}*(a + I*a*\operatorname{Tan}[c + d*x])^{3/2})/(5*d)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

$\operatorname{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_) + (b_.)*\tan[(e_.) + (f_.)*(x_)]]/\operatorname{Sqrt}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]

```

#### Rule 3674

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Dist[a/(d*(b*c + a*d)*(n + 1)), Int[(
a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
- 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b*
d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

```

#### Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]

```

#### Rule 4326

```

Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

#### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c+dx)(a+ia \tan(c+dx))^{\frac{5}{2}}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{\frac{5}{2}}}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{\frac{3}{2}}}{5d} \\
&= -\frac{2a^2(8iA+5B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2a^2(38A-35iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2a^2(38A-35iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{2a^2(38A-35iB) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{15d} \\
&= \frac{(4-4i)a^{5/2}(iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{d \sec^{\frac{1}{2}}(c+dx)(A \cos(c+dx)+B \sin(c+dx))}
\end{aligned}$$

### Mathematica [A]

time = 4.28, size = 306, normalized size = 1.49

$$\frac{\left( -4\sqrt{2}(A-iB)e^{-3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)-\frac{\sqrt{\cot(c+dx)}\csc^2(c+dx)\sqrt{\sec(c+dx)}(\cos(2c-i\sin(2c))(-35(A-iB)+(41A-35iB)\cos(2(c+dx))+(11A+5B)\sin(2(c+dx))))}{15(\cos(dx)+i\sin(dx))^2}}{d \sec^{\frac{1}{2}}(c+dx)(A \cos(c+dx)+B \sin(c+dx))} \right) (a+ia \tan(c+dx))^{\frac{5}{2}}(A+B \tan(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(7/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((-4\*sqrt(2)\*(A - I\*B)\*sqrt(-1 + E^((2\*I)\*(c + d\*x))))\*sqrt[E^(I\*(c + d\*x))]/(1 + E^((2\*I)\*(c + d\*x))))\*sqrt[(I\*(1 + E^((2\*I)\*(c + d\*x))))]/(-1 + E^((2\*I)\*(c + d\*x))))\*ArcTanh[E^(I\*(c + d\*x))/sqrt(-1 + E^((2\*I)\*(c + d\*x)))]/E^((3\*I)\*(c + d\*x)) - (sqrt[Cot[c + d\*x]]\*Csc[c + d\*x]^2\*sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c])\*(-35\*(A - I\*B) + (41\*A - (35\*I)\*B)\*Cos[2\*(c + d\*x)] + ((11\*I)\*A + 5\*B)\*Sin[2\*(c + d\*x)]))/(15\*(Cos[d\*x] + I\*Sin[d\*x])^2))\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x])/(d\*Sec[c + d\*x]^(7/2)\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2213 vs. 2(168) = 336.

time = 62.41, size = 2214, normalized size = 10.80

method	result	size
default	Expression too large to display	2214

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/15/d*a^2*2^(1/2)*(60*I*A*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2)+1)*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)+38*A*2^(1/
2)-35*I*B*2^(1/2)+30*I*B*cos(d*x+c)^2*ln((((1+cos(d*x+c))/sin(d*x+c))^(1/2
)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(-(1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1))*sin(d*x+c)*((1+cos(d*
x+c))/sin(d*x+c))^(1/2)+40*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+52*A*cos(d*x+c
)^3*2^(1/2)-49*A*cos(d*x+c)*2^(1/2)-60*A*sin(d*x+c)*((1+cos(d*x+c))/sin(d*
x+c))^(1/2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+
1)-60*A*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan((
(1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+60*B*sin(d*x+c)*((1+cos(d*x+c
))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)
*2^(1/2)+1)+60*B*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2
*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+60*I*A*cos(d*x+c)^2*a
rctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x+c)*((1+cos(d*x
+c))/sin(d*x+c))^(1/2)+30*I*A*cos(d*x+c)^2*ln((((1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((1+cos(d*x+c))/sin(d
*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*sin(d*x+c)*((1+c
os(d*x+c))/sin(d*x+c))^(1/2)+60*I*B*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/si
n(d*x+c))^(1/2)*2^(1/2)+1)*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)+60
*I*B*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(
d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)+52*I*A*cos(d*x+c)^2*2^(1/2)*sin(d
*x+c)-11*I*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)-60*I*A*arctan(((1+cos(d*x+c))/s
in(d*x+c))^(1/2)*2^(1/2)+1)*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)-3
0*A*cos(d*x+c)^2*ln((((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+
sin(d*x+c)+cos(d*x+c)-1)/(-(1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d
*x+c)+cos(d*x+c)+sin(d*x+c)-1))*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/
2)+30*B*cos(d*x+c)^2*ln((((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*
x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*s
in(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c)
)^(1/2)-60*I*A*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x+
c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)-30*I*A*ln((((1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1)/(((1+cos(d*x+c))/sin
(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))*sin(d*x+c)*((1
+cos(d*x+c))/sin(d*x+c))^(1/2)-60*I*B*arctan(((1+cos(d*x+c))/sin(d*x+c))^(
1/2)*2^(1/2)+1)*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)-60*I*B*arctan
(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)*sin(d*x+c)*((1+cos(d*x+c))/
```



$$\begin{aligned} & \sin(dx+c)^{1/2} - 30*I*B*\ln\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} * 2^{1/2} * \sin(dx+c) \\ & + \sin(dx+c) + \cos(dx+c) - 1 / \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * 2^{1/2} * \sin(dx+c) \\ & + \cos(dx+c) + \sin(dx+c) - 1 \Big) * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} \\ & - 5*B*2^{1/2} * \cos(dx+c) * \sin(dx+c) - 35*2^{1/2} * B * \sin(dx+c) - 41*A*2^{1/2} \\ & * \cos(dx+c)^2 + 60*A * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * \arctan\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} \\ & * 2^{1/2} + 1 \Big) + 60*A * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * \arctan\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} \\ & * 2^{1/2} - 1 \Big) - 60*B * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * \arctan\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} \\ & * 2^{1/2} + 1 \Big) - 60*B * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * \arctan\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} \\ & * 2^{1/2} - 1 \Big) + 30*A * \ln\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} * 2^{1/2} * \sin(dx+c) + \sin(dx+c) + \cos(dx+c) - 1 / \\ & \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * 2^{1/2} * \sin(dx+c) + \cos(dx+c) + \sin(dx+c) - 1 \Big) * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} \\ & - 30*B * \ln\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right)^{1/2} * 2^{1/2} * \sin(dx+c) + \cos(dx+c) + \sin(dx+c) - 1 / \\ & \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} * 2^{1/2} * \sin(dx+c) + \cos(dx+c) + \sin(dx+c) - 1 \Big) * \sin(dx+c) * \left( \frac{(-1+\cos(dx+c))}{\sin(dx+c)} \right)^{1/2} \\ & - 40*I*B * \cos(dx+c)^3 * 2^{1/2} + 35*I * B * \cos(dx+c)^2 * 2^{1/2} - 38*I*A * 2^{1/2} * \sin(dx+c) + 40*I*B * \cos(dx+c) * 2^{1/2} \\ & * \left( \frac{\cos(dx+c)}{\sin(dx+c)} \right)^{7/2} * \left( I * \sin(dx+c) + \cos(dx+c) \right) * a / \cos(dx+c)^{1/2} * \sin(dx+c) / \left( I * \sin(dx+c) + \cos(dx+c) - 1 \right) / \cos(dx+c)^3 \end{aligned}$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs.  $2(157) = 314$ .  
time = 0.96, size = 1545, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^(7/2)*(a+I*a*tan(dx+c))^(5/2)*(A+B*tan(dx+c)),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 2/15 * (\sqrt{\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2} - 2\cos(2dx + 2c) + 1) * \\ & ((30 * ((I + 1) * A - (I - 1) * B) * a^2 * \cos(3dx + 3c) + (-31 * I + 31) * A + (25 * I - 25) * B) * a^2 * \cos(dx + c) + 30 * ((I - 1) * A + (I + 1) * B) * a^2 * \sin(3dx + 3c) \\ & + (-31 * I - 31) * A - (25 * I + 25) * B) * a^2 * \sin(dx + c) * \cos(3/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) - 1)) \\ & + (30 * (-I - 1) * A - (I + 1) * B) * a^2 * \cos(3dx + 3c) + ((31 * I - 31) * A + (25 * I + 25) * B) * a^2 * \cos(dx + c) + 30 * ((I + 1) * A - (I - 1) * B) * a^2 * \sin(3dx + 3c) \\ & + (-31 * I + 31) * A + (25 * I - 25) * B) * a^2 * \sin(dx + c) * \sin(3/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) - 1)) * \sqrt{a} \\ & + 15 * (2 * ((-I - 1) * A - (I + 1) * B) * a^2 * \cos(2dx + 2c)^2 + (-I - 1) * A - (I + 1) * B) * a^2 * \sin(2dx + 2c)^2 \\ & + 2 * ((I - 1) * A + (I + 1) * B) * a^2 * \cos(2dx + 2c) + (-I - 1) * A - (I + 1) * B) * a^2 * \arctan2(2 * (\cos(2dx + 2c))^2 + \sin(2dx + 2c)^2 - 2 * \cos(2dx + 2c) + 1)^{1/4} * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) - 1)) \\ & + 2 * \sin(dx + c), 2 * (\cos(2dx + 2c))^2 + \sin(2dx + 2c)^2 - 2 * \cos(2dx + 2c) + 1)^{1/4} * \cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) - 1)) \\ & + 2 * \cos(dx + c)) + ((-I + 1) * A + ( \end{aligned}$$

$$\begin{aligned} & (I - 1)*B)*a^2*\cos(2*d*x + 2*c)^2 + (- (I + 1)*A + (I - 1)*B)*a^2*\sin(2*d*x + \\ & 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*a^2*\cos(2*d*x + 2*c) + (- (I + 1)*A + (I \\ & - 1)*B)*a^2)*\log(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 4*\sqrt{\cos(2*d*x + \\ & 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1}*(\cos(1/2*\arctan2(\sin( \\ & 2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \\ & \cos(2*d*x + 2*c) - 1))^2) + 8*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2* \\ & \cos(2*d*x + 2*c) + 1)^{(1/4)}*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \\ & \cos(2*d*x + 2*c) - 1)) + \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos \\ & (2*d*x + 2*c) - 1))))*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2* \\ & d*x + 2*c) + 1)^{(1/4)}*\sqrt{a} + ((30*((I + 1)*A - (I - 1)*B)*a^2*\cos(5*d*x \\ & + 5*c) + 5*(- (5*I + 5)*A + (11*I - 11)*B)*a^2*\cos(3*d*x + 3*c) + ((7*I + 7) \\ & *A - (25*I - 25)*B)*a^2*\cos(d*x + c) + 30*((I - 1)*A + (I + 1)*B)*a^2*\sin(5 \\ & *d*x + 5*c) + 5*(- (5*I - 5)*A - (11*I + 11)*B)*a^2*\sin(3*d*x + 3*c) + ((7*I \\ & - 7)*A + (25*I + 25)*B)*a^2*\sin(d*x + c))*\cos(5/2*\arctan2(\sin(2*d*x + 2*c) \\ & , \cos(2*d*x + 2*c) - 1)) + 4*((- (2*I + 2)*A + (5*I - 5)*B)*a^2*\cos(d*x + c) \\ & + (- (2*I - 2)*A - (5*I + 5)*B)*a^2*\sin(d*x + c) + ((- (2*I + 2)*A + (5*I - \\ & 5)*B)*a^2*\cos(d*x + c) + (- (2*I - 2)*A - (5*I + 5)*B)*a^2*\sin(d*x + c))*\cos \\ & (2*d*x + 2*c)^2 + ((- (2*I + 2)*A + (5*I - 5)*B)*a^2*\cos(d*x + c) + (- (2*I - \\ & 2)*A - (5*I + 5)*B)*a^2*\sin(d*x + c))*\sin(2*d*x + 2*c)^2 + 2*(( (2*I + 2)*A \\ & - (5*I - 5)*B)*a^2*\cos(d*x + c) + ((2*I - 2)*A + (5*I + 5)*B)*a^2*\sin(d*x \\ & + c))*\cos(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) \\ & - 1)) + (30*(- (I - 1)*A - (I + 1)*B)*a^2*\cos(5*d*x + 5*c) + 5*((5*I - 5)*A \\ & + (11*I + 11)*B)*a^2*\cos(3*d*x + 3*c) + (- (7*I - 7)*A - (25*I + 25)*B)*a^2* \\ & \cos(d*x + c) + 30*((I + 1)*A - (I - 1)*B)*a^2*\sin(5*d*x + 5*c) + 5*(- (5*I + \\ & 5)*A + (11*I - 11)*B)*a^2*\sin(3*d*x + 3*c) + ((7*I + 7)*A - (25*I - 25)*B) \\ & *a^2*\sin(d*x + c))*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1)) \\ & + 4*(( (2*I - 2)*A + (5*I + 5)*B)*a^2*\cos(d*x + c) + (- (2*I + 2)*A + (5*I - \\ & 5)*B)*a^2*\sin(d*x + c) + (( (2*I - 2)*A + (5*I + 5)*B)*a^2*\cos(d*x + c) + ( \\ & - (2*I + 2)*A + (5*I - 5)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c)^2 + (( (2*I - \\ & 2)*A + (5*I + 5)*B)*a^2*\cos(d*x + c) + (- (2*I + 2)*A + (5*I - 5)*B)*a^2*\sin \\ & (d*x + c))*\sin(2*d*x + 2*c)^2 + 2*((- (2*I - 2)*A - (5*I + 5)*B)*a^2*\cos(d* \\ & x + c) + ((2*I + 2)*A - (5*I - 5)*B)*a^2*\sin(d*x + c))*\cos(2*d*x + 2*c))*\sin \\ & (1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) - 1))*\sqrt{a})/((\cos(2*d* \\ & x + 2*c)^2 + \sin(2*d*x + 2*c)^2 - 2*\cos(2*d*x + 2*c) + 1)^{(5/4)}*d) \end{aligned}$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs.  $2(157) = 314$ .  
 time = 1.21, size = 515, normalized size = 2.51

$$\frac{\left( \frac{1}{10} \sqrt{\frac{100 a^2 - 100 a^2 \cos^2(d x + c) + 100 a^2 \sin^2(d x + c)}{a^2}} \sqrt{\frac{100 a^2 - 100 a^2 \cos^2(d x + c) + 100 a^2 \sin^2(d x + c)}{a^2}} \sqrt{\frac{100 a^2 - 100 a^2 \cos^2(d x + c) + 100 a^2 \sin^2(d x + c)}{a^2}} \right) \dots}{10 \sqrt{\frac{100 a^2 - 100 a^2 \cos^2(d x + c) + 100 a^2 \sin^2(d x + c)}{a^2}} \sqrt{\frac{100 a^2 - 100 a^2 \cos^2(d x + c) + 100 a^2 \sin^2(d x + c)}{a^2}} \sqrt{\frac{100 a^2 - 100 a^2 \cos^2(d x + c) + 100 a^2 \sin^2(d x + c)}{a^2}}} \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")
```

```
[Out] -2/15*(15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4
*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) -
sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 15*sqrt(2)*sqrt(-(-I
*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*
I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B + I*
B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*
x - I*c)/((-I*A - B)*a^2)) - 2*sqrt(2)*(2*(13*A - 10*I*B)*a^2*e^(5*I*d*x +
5*I*c) - 35*(A - I*B)*a^2*e^(3*I*d*x + 3*I*c) + 15*(A - I*B)*a^2*e^(I*d*x +
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c)
+ d)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(7
/2), x)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) \operatorname{li})^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^(5/2),x
)
```

```
[Out] int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*li)^(5/2),
x)
```

$$3.550 \quad \int \cot^2(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=230

$$\frac{2(-1)^{3/4}a^{5/2}B \operatorname{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} (4+4i)a^{5/2}(iA+B) \tan(c+dx)}{d} + \dots$$

[Out]  $2*(-1)^{(3/4)}*a^{(5/2)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(4+4*I)*a^{(5/2)}*(I*A+B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-2*a^2*(2*I*A+B)*\cot(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/d-2/3*a*A*\cot(d*x+c)^{(3/2)}*(a+I*a*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.51, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3674, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(4+4i)a^{5/2}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{2(-1)^{3/4}a^{5/2}B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2(B+2iA)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(5/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $(2*(-1)^{(3/4)}*a^{(5/2)}*B*\operatorname{ArcTan}[\frac{(-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d + ((4 + 4*I)*a^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\frac{(1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]}{\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]]}])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d - (2*a^2*((2*I)*A + B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])/d - (2*a*A*\operatorname{Cot}[c + d*x]^{(3/2)}*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)})/(3*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_)^m)*((c_. + (d_.)*(x_)^n), x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*A \operatorname{rcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \|\operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1)), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx))}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}}{3d} \\
&= -\frac{2a^2(2iA + B) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} \\
&= -\frac{2a^2(2iA + B) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} \\
&= -\frac{2a^2(2iA + B) \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}}{d} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= \frac{(4 + 4i)a^{5/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= \frac{2(-1)^{3/4}a^{5/2}B \tan^{-1} \left( \frac{(-1)^{3/4}\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

Mathematica [A]

time = 7.57, size = 396, normalized size = 1.72

$$\frac{\left( \sqrt{2} e^{-3i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{\frac{i(1 + e^{2i(c+dx)})}{-1 + e^{2i(c+dx)}}} \left( i(A + B) \log \left( e^{i(c+dx)} + \sqrt{-1 + e^{2i(c+dx)}} \right) + \sqrt{2} B \left( -\log \left( 1 - 3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \right) + \log \left( 1 - 3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \right) \right) - \frac{1 \sqrt{\cos(c + dx)} (A \cos(c + dx) + B \sin(c + dx)) \operatorname{arctan} \left( \frac{\cos(2c + 2dx)}{\sin(2c + 2dx)} \right)}{\sqrt{\cos(c + dx)} (1 + \cos(2c + 2dx))} \right) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{4d \sec^2(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] (((Sqrt[2]*Sqrt[-1 + E^((2*I)*(c + d*x))])*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))])*Sqrt[(I*(1 + E^((2*I)*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(16*(I*A + B)*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] + Sqrt[2]*B*(-Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] + Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])))/E^((3*I)*(c + d*x)) - (8*Sqrt[Cot[c + d*x]]*(A*Csc[c + d*x] + ((7*I)*A + 3*B)*Sec[c + d*x])*(Cos[2*c] - I*Sin[2*c]))/(Sqrt[Sec[c + d*x]]*(3*Cos[2*d*x] + (3*I)*Sin[2*d*x]))*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])/(4*d*Sec[c + d*x]^(7/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2596 vs.  $2(186) = 372$ .

time = 71.14, size = 2597, normalized size = 11.29

method	result	size
default	Expression too large to display	2597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*a^2*2^(1/2)*(3*I*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-14*A*2^(1/2)-3*I*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-6*I*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2))+3*I*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+3*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-6*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2))-3*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+24*I*A*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+12*I*A*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(-((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+cos(d*x+c)+sin(d*x+c)-1))+24*I*A*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)-24*I*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)-24*I*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)-12*I*B*cos(d*x+c)^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))+16*I*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)-2*A*cos(d*x+c)*2^(1/2)-12*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2
```





time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(5/2), x)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 780 vs. 2(176) = 352.

time = 1.47, size = 780, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/12*(24*\sqrt{2}*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(4*((A - I*B)*a^3*e^{(I*d*x + I*c)} + \sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}* \\ & \sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) - 24*\sqrt{2}*\sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(4*((A - I*B)*a^3*e^{(I*d*x + I*c)} - \sqrt{-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}* \\ & \sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))e^{(-I*d*x - I*c)}/((-I*A - B)*a^2)) + 8*\sqrt{2}*((8*I*A + 3*B)*a^2*e^{(3*I*d*x + 3*I*c)} + 3*(-2*I*A - B)*a^2*e^{(I*d*x + I*c)})*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))} - 3*\sqrt{4*I*B^2*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-16*(3*B*a^3*e^{(2*I*d*x + 2*I*c)} - B*a^3 + \sqrt{2}*\sqrt{4*I*B^2*a^5/d^2}*(I*d*e^{(3*I*d*x + 3*I*c)} - I*d*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))e^{(-2*I*d*x - 2*I*c)/(B*a)} + 3*\sqrt{4*I*B^2*a^5/d^2}*(d*e^{(2*I*d*x + 2*I*c)} - d)*\log(-16*(3*B*a^3*e^{(2*I*d*x + 2*I*c)} - B*a^3 + \sqrt{2}*\sqrt{4*I*B^2*a^5/d^2}*(-I*d*e^{(3*I*d*x + 3*I*c)} + I*d*e^{(I*d*x + I*c)}))*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))e^{(-2*I*d*x - 2*I*c)/(B*a)})) / (d*e^{(2*I*d*x + 2*I*c)} - d) \end{aligned}$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(5/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

$$3.551 \quad \int \cot^2(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=236

$$\frac{(-1)^{3/4} a^{5/2} (2A - 5iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (4 + 4i) a^{5/2} (A + B \tan(c + dx))}{d} +$$

[Out]  $(-1)^{3/4} a^{5/2} (2A - 5iB) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (4 + 4i) a^{5/2} (A + B \tan(c + dx)) / d + ((4 + 4i) a^{5/2} (A - iB) \operatorname{ArcTan}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}) / d + (a^2 ((2i)A - B) \sqrt{a + ia \tan(c + dx)}) / (d \sqrt{\cot(c + dx)}) - (2aA \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}) / d$

**Rubi** [A]

time = 0.54, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {4326, 3674, 3675, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} a^{5/2} (2A - 5iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) + ((4 + 4i) a^{5/2} (A - iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}}\right) + \frac{a^2 (-B + 2iA) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{2aA \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}}{d}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{3/2} * (a + I*a*\operatorname{Tan}[c + d*x])^{5/2} * (A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((-1)^{3/4} a^{5/2} (2A - (5i)B) \operatorname{ArcTan}\left[\frac{(-1)^{3/4} \sqrt{a} \sqrt{\operatorname{Tan}[c + d*x]}}{\sqrt{a + I*a*\operatorname{Tan}[c + d*x]}}\right] \sqrt{\operatorname{Cot}[c + d*x]} \sqrt{\operatorname{Tan}[c + d*x]}) / d + ((4 + 4i) a^{5/2} (A - iB) \operatorname{ArcTan}\left[\frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Tan}[c + d*x]}}{\sqrt{a + I*a*\operatorname{Tan}[c + d*x]}}\right] \sqrt{\operatorname{Cot}[c + d*x]} \sqrt{\operatorname{Tan}[c + d*x]}) / d + (a^2 ((2i)A - B) \sqrt{a + I*a*\operatorname{Tan}[c + d*x]}) / (d \sqrt{\operatorname{Cot}[c + d*x]}) - (2aA \sqrt{\operatorname{Cot}[c + d*x]} * (a + I*a*\operatorname{Tan}[c + d*x])^{3/2}) / d$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)} * ((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)} * (c - a*(d/b) + d*(x^p/b))^{n/p}, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& \operatorname{GtQ}[a$

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3674

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-a^2)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(b\*c + a\*d)\*(n + 1))), x] - Dist[a/(d\*(b\*c + a\*d)\*(n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*b\*d\*(m - n - 2) - B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + (a\*A\*d\*(m + n) - B\*(a\*c\*(m - 1) + b\*d\*(n + 1)))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dis

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{(a + ia \tan(c + dx))^{5/2}}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx \\
&= \frac{2aA \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}}{d} \\
&= \frac{a^2(2iA - B) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{2aA \sqrt{a + ia \tan(c + dx)}}{d} \\
&= \frac{a^2(2iA - B) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{2aA \sqrt{a + ia \tan(c + dx)}}{d} \\
&= \frac{a^2(2iA - B) \sqrt{a + ia \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{2aA \sqrt{a + ia \tan(c + dx)}}{d} \\
&= \frac{(4 - 4i)a^{5/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= \frac{(4 - 4i)a^{5/2}(iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d} \\
&= \frac{\sqrt[4]{-1} a^{5/2}(2iA + 5B) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 5.72, size = 387, normalized size = 1.64

$$\frac{\left( \sqrt{2} e^{-3i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \sqrt{\frac{e^{2i(c+dx)}}{1 + e^{2i(c+dx)}}} \sqrt{\frac{i(1 + e^{2i(c+dx)})}{-1 + e^{2i(c+dx)}}} (32(A - iB) \log(e^{i(c+dx)} + \sqrt{-1 + e^{2i(c+dx)}}) - \sqrt{2}(2A - 5iB) (\log(1 - 3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) - \log(1 - 3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}})) - \frac{5B + 2A \cos(c+dx) \sqrt{\cos(c+dx) \cot(c+dx)}}{\sqrt{\cos(c+dx) (\cos(4c) + \cos(4dx))}} \right) (a + ia \tan(c + dx))^{5/2} (A + B \tan(c + dx))}{8d \sec^2(c + dx) (A \cos(c + dx) + B \sin(c + dx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] (((Sqrt[2]\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*Sqrt[E^(I\*(c + d\*x))/(1 + E^((2\*I)\*(c + d\*x)))]\*Sqrt[(I\*(1 + E^((2\*I)\*(c + d\*x))))]/(-1 + E^((2\*I)\*(c + d\*x)))]\*(32\*(A - I\*B)\*Log[E^(I\*(c + d\*x)) + Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - Sqrt[2]\*(2\*A - (5\*I)\*B)\*(Log[1 - 3\*E^((2\*I)\*(c + d\*x)) - 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - Log[1 - 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]]]))/E^((3\*I)\*(c + d\*x)) - (8\*(B + 2\*A\*Cot[c + d\*x])\*Sqrt[Sec[c + d\*x]]\*(Cos[2\*c] - I\*Sin[2\*c]))/



```

*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+16*I*A*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+8*I*A*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))+16*I*B*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+16*I*B*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+8*I*B*sin(d*x+c)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))+2*A*sin(d*x+c)*2^(1/2)*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-2*I*B*2^(1/2)*cos(d*x+c)^2+2*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)-2*2^(1/2)*B*sin(d*x+c)+4*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/sin(d*x+c))^(3/2)*((I*sin(d*x+c)+cos(d*x+c))*a/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^2

```

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] Timed out

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(182) = 364.

time = 1.35, size = 849, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} \cdot (8 \cdot \sqrt{2}) \cdot \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^5 / d^2} \cdot (d \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c) + d} \cdot \log(4 \cdot ((A - I \cdot B) \cdot a^3 \cdot e^{(I \cdot d \cdot x + I \cdot c)} - \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^5 / d^2}) \cdot (I \cdot d \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} - I \cdot d) \cdot \sqrt{a / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + 1)}) \cdot \sqrt{((I \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I) / (e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} - 1))} \cdot e^{(-I \cdot d \cdot x - I \cdot c)} / ((-I \cdot A - B) \cdot a^2)) - 8 \cdot \sqrt{2} \cdot \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^5 / d^2} \cdot (d \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + d) \cdot \log(4 \cdot ((A - I \cdot B) \cdot a^3 \cdot e^{(I \cdot d \cdot x + I \cdot c)} - \sqrt{-(-I \cdot A^2 - 2 \cdot A \cdot B + I \cdot B^2) \cdot a^5 / d^2}) \cdot (-I \cdot d \cdot e^{(2 \cdot I \cdot d \cdot x + 2 \cdot I \cdot c)} + I \cdot d) \cdot \sqrt{a / (e$



$$\begin{aligned} & \left( e^{2I dx + 2I c} + 1 \right) \sqrt{\left( I e^{2I dx + 2I c} + I \right) / \left( e^{2I dx + 2I c} - 1 \right)} \cdot e^{-I dx - I c} / \left( (-I A - B) a^2 \right) - 4 \sqrt{2} \cdot \left( (2A - I B) a^2 e^{3I dx + 3I c} + (2A + I B) a^2 e^{I dx + I c} \right) \sqrt{a / \left( e^{2I dx + 2I c} + 1 \right)} \\ & \sqrt{\left( I e^{2I dx + 2I c} + I \right) / \left( e^{2I dx + 2I c} - 1 \right)} - \sqrt{\left( 4I A^2 + 20A B - 25I B^2 \right) a^5 / d^2} \cdot \left( d e^{2I dx + 2I c} + d \right) \log \left( -16 \cdot \left( 3 \cdot (2I A + 5B) a^3 e^{2I dx + 2I c} + (-2I A - 5B) a^3 + 2 \sqrt{2} \sqrt{\left( 4I A^2 + 20A B - 25I B^2 \right) a^5 / d^2} \right) \cdot \left( d e^{3I dx + 3I c} - d e^{I dx + I c} \right) \right) \\ & \sqrt{a / \left( e^{2I dx + 2I c} + 1 \right)} \sqrt{\left( I e^{2I dx + 2I c} + I \right) / \left( e^{2I dx + 2I c} - 1 \right)} \cdot e^{-2I dx - 2I c} / \left( (2I A + 5B) a \right) + \sqrt{\left( 4I A^2 + 20A B - 25I B^2 \right) a^5 / d^2} \cdot \left( d e^{2I dx + 2I c} + d \right) \log \left( -16 \cdot \left( 3 \cdot (2I A + 5B) a^3 e^{2I dx + 2I c} + (-2I A - 5B) a^3 - 2 \sqrt{2} \sqrt{\left( 4I A^2 + 20A B - 25I B^2 \right) a^5 / d^2} \right) \cdot \left( d e^{3I dx + 3I c} - d e^{I dx + I c} \right) \right) \\ & \sqrt{a / \left( e^{2I dx + 2I c} + 1 \right)} \sqrt{\left( I e^{2I dx + 2I c} + I \right) / \left( e^{2I dx + 2I c} - 1 \right)} \cdot e^{-2I dx - 2I c} / \left( (2I A + 5B) a \right) \Big/ \left( d e^{2I dx + 2I c} + d \right) \end{aligned}$$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2),x)

$$3.552 \quad \int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=246

$$\frac{(-1)^{3/4} a^{5/2} (20iA + 23B) \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} (4-4i) a^{5/2}}{4d} +$$

[Out]  $-1/4*(-1)^{(3/4)}*a^{(5/2)}*(20*I*A+23*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(4-4*I)*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-1/4*a^2*(4*A-7*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(1/2)}+1/2*I*a*B*(a+I*a*\tan(d*x+c))^{(3/2)}/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.55, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3675, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4} a^{5/2} (23B + 20iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + (4-4i) a^{5/2} (A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{tanh}^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) - \frac{a^2 (4A-7iB) \sqrt{a+ia \tan(c+dx)}}{4d \sqrt{\cot(c+dx)}} + \frac{iaB(a+ia \tan(c+dx))^{3/2}}{2d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cot}[c+dx]]*(a+I*a*\operatorname{Tan}[c+dx])^{(5/2)}*(A+B*\operatorname{Tan}[c+dx]),x]$

[Out]  $-1/4*((-1)^{(3/4)}*a^{(5/2)}*((20*I)*A+23*B)*\operatorname{ArcTan}[( (-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]]]/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+dx]])*\operatorname{Sqrt}[\operatorname{Cot}[c+dx]]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]])/d+((4-4*I)*a^{(5/2)}*(A-I*B)*\operatorname{ArcTanh}[( (1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]]]/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+dx]])*\operatorname{Sqrt}[\operatorname{Cot}[c+dx]]*\operatorname{Sqrt}[\operatorname{Tan}[c+dx]])/d-(a^2*(4*A-(7*I)*B)*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+dx]])/(4*d*\operatorname{Sqrt}[\operatorname{Cot}[c+dx]])+( (I/2)*a*B*(a+I*a*\operatorname{Tan}[c+dx])^{(3/2)})/(d*\operatorname{Sqrt}[\operatorname{Cot}[c+dx]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a$

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3675

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n) + B\*(a\*c\*(m - 1) - b\*d\*(n + 1)) - (B\*(b\*c - a\*d)\*(m - 1) - d\*(A\*b + a\*B)\*(m + n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*

d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx)) dx = \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= \frac{iaB(a+ia \tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= -\frac{a^2(4A-7iB)\sqrt{a+ia \tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= -\frac{a^2(4A-7iB)\sqrt{a+ia \tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= -\frac{a^2(4A-7iB)\sqrt{a+ia \tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= -\frac{(4+4i)a^{5/2}(iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a}}{\sqrt{a+ia \tan(c+dx)}} \right)}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= -\frac{(4+4i)a^{5/2}(iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a}}{\sqrt{a+ia \tan(c+dx)}} \right)}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

$$= \frac{\sqrt[4]{-1} a^{5/2} (20A-23iB) \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a}}{\sqrt{a+ia \tan(c+dx)}} \right)}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

Mathematica [A]

time = 5.10, size = 447, normalized size = 1.82

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx)) \left( \sqrt[4]{-1} \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a}}{\sqrt{a+ia \tan(c+dx)}} \right) \right) + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{4d\sqrt{\cot(c+dx)}} + \frac{1}{2} \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^3*Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])*(Sqrt[2]*(Sqrt[2]*((-20*I)*A - 23*B)*Log[(-2*E^(((7*I)/2)*c)*(I*Sqrt[2] + Sqrt[2]*E^(I*(c + d*x)) - 2*Sqrt[-1 + E^((2*I)*(c + d*x))])])]/((20*A - (23*I)*B)*(-I + E^(I*(c + d*x)))) + Sqrt[2]*((20*I)*A + 23*B)*Log[(-2*E^(((7*I)/2)*c)*((-I)*Sqrt[2] + Sqrt[2]*E^(I*(c + d*x)) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))])])]/((20*A - (23*I)*B)*(I + E^(I*(c + d*x)))) - (64*I)*(A - I*B)*Log[(Cos[c] - I*Sin[c])*(Cos[c + d*x] + I*Sin[c + d*x] + Sqrt[-1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])])]*Sqrt[I*(I + Cot[c + d*x])*Sin[c + d*x]^2*(Cos[3*c + d*x] - I*Sin[3*c + d*x]) - 4*(Cos[2*c] - I*Sin[2*c])*Tan[c + d*x]*(4*A - (9*I)*B + 2*B*Tan[c + d*x]))]/(16*d*(Cos[d*x] + I*Sin[d*x])^2*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1529 vs.  $2(196) = 392$ .

time = 63.80, size = 1530, normalized size = 6.22

method	result	size
default	Expression too large to display	1530

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*2^(1/2)*a^2*(-1+cos(d*x+c))*(22*I*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-8*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-64*I*A*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)-32*I*A*cos(d*x+c)^2*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))-64*I*A*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+64*I*B*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)-1)+64*I*B*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)+1)+32*I*B*cos(d*x+c)^2*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))+46*B*cos(d*x+c)^2*2^(1/2)*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2))+22*B*cos(d*x+c)^2*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-32*A*cos(d*x+c)^2*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1))-32*B*cos(d*x+c)^2*ln(-(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)+sin(d*x+c)+cos(d*x+c)-1)/(((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2)*sin(d*x+c)-sin(d*x+c)-cos(d*x+c)+1))+18*B*cos(d*x+c)^2*(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-4*B*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-64*A*cos(d*x+c)^2*arctan(((1+cos(d*x+c))/sin(d*x+c))^(1/2))
```

$$\begin{aligned}
& *2^{(1/2)+1}-64*A*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)-64*B*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}+1)-64*B*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)}-1)+ \\
& 20*A*\cos(d*x+c)^2*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)-20*A*\cos(d*x+c)^2*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)+40*A*\cos(d*x+c)^2*2^{(1/2)}*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})-23*B*\cos(d*x+c)^2*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)+23*B*\cos(d*x+c)^2*2^{(1/2)}*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)+8*I*A*2^{(1/2)}*\cos(d*x+c)^2*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-20*I*A*2^{(1/2)}*\cos(d*x+c)^2*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)+40*I*A*2^{(1/2)}*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})+20*I*A*2^{(1/2)}*\cos(d*x+c)^2*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)-23*I*B*2^{(1/2)}*\cos(d*x+c)^2*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-1)-46*I*B*2^{(1/2)}*\cos(d*x+c)^2*\arctan((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})+23*I*B*2^{(1/2)}*\cos(d*x+c)^2*\ln((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+1)+8*I*A*2^{(1/2)}*\cos(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+4*I*B*2^{(1/2)}*\sin(d*x+c)*(( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*(\cos(d*x+c)/\sin(d*x+c))^{(1/2)}*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}/(I*\sin(d*x+c)+\cos(d*x+c)-1)/\cos(d*x+c)^2/((( -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})
\end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, alg orithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*sqrt(cot(d\*x + c)), x)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(184) = 368.

time = 0.84, size = 923, normalized size = 3.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, alg orithm="fricas")

[Out] 1/16\*(32\*sqrt(2)\*sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a^5/d^2)\*(d\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*d\*e^(2\*I\*d\*x + 2\*I\*c) + d)\*log(4\*((A - I\*B)\*a^3\*e^(I\*d\*x + I\*c) + sqrt(-(I\*A^2 + 2\*A\*B - I\*B^2)\*a^5/d^2)\*(d\*e^(2\*I\*d\*x + 2\*I\*c) - d)\*sqrt(a/(e^(2\*I\*d\*x + 2\*I\*c) + 1))\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))e^(-I\*d\*x - I\*c)/((-I\*A - B)\*a^2) - 32\*sqrt(2)\*sqrt(-(I\*A^2 + 2

```

*A*B - I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d
)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/
d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*
A - B)*a^2)) + 4*sqrt(2)*((4*I*A + 11*B)*a^2*e^(5*I*d*x + 5*I*c) - 4*B*a^2*
e^(3*I*d*x + 3*I*c) + (-4*I*A - 7*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
) - sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c)
+ 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(20*I*A + 23*B)*a^3*e^(2*I*d*x +
2*I*c) + (-20*I*A - 23*B)*a^3 + 2*sqrt(2)*sqrt((-400*I*A^2 - 920*A*B + 529
*I*B^2)*a^5/d^2)*(I*d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1)))*e^(-2*I*d*x - 2*I*c)/((20*I*A + 23*B)*a)) + sqrt((-400*I*A^2 - 92
0*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c
) + d)*log(-16*(3*(20*I*A + 23*B)*a^3*e^(2*I*d*x + 2*I*c) + (-20*I*A - 23*B
)*a^3 + 2*sqrt(2)*sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(-I*d*e^
(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x -
2*I*c)/((20*I*A + 23*B)*a)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*
c) + d)

```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, alg  
orithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)\*sqrt(cot(d\*x +  
c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x  
)
```

```
[Out] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),  
x)
```



$$3.553 \quad \int \frac{(a+ia \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=292

$$\frac{(-1)^{3/4}a^{5/2}(46A - 45iB)\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{8d} (4+4i)a^5$$

[Out]  $-1/8*(-1)^{(3/4)}*a^{(5/2)}*(46*A-45*I*B)*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(4+4*I)*a^{(5/2)}*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-1/4*a^2*(2*A-3*I*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(3/2)}+1/8*a^2*(18*I*A+19*B)*(a+I*a*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(1/2)}+1/3*I*a*B*(a+I*a*\tan(d*x+c))^{(3/2)}/d/\cot(d*x+c)^{(3/2)}$

**Rubi [A]**

time = 0.70, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {4326, 3675, 3678, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{(-1)^{3/4}a^{5/2}(46A - 45iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + (4+4i)a^{5/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) - \frac{a^2(2A-3iB)\sqrt{a+ia \tan(c+dx)}}{4d\cot^3(c+dx)} + \frac{a^2(19B+18iA)\sqrt{a+ia \tan(c+dx)}}{8d\sqrt{\cot(c+dx)}} + \frac{iaB(a+ia \tan(c+dx))^{3/2}}{3d\cot^3(c+dx)}}{8d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{((a + I*a*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x]))}{\text{Sqrt}[\text{Cot}[c + d*x]]}, x]$

[Out]  $-1/8*((-1)^{(3/4)}*a^{(5/2)}*(46*A - (45*I)*B)*\text{ArcTan}[\frac{((-1)^{(3/4)}*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])}{\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]}]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])/d - ((4 + 4*I)*a^{(5/2)}*(A - I*B)*\text{ArcTanh}[\frac{((1 + I)*\text{Sqrt}[a]*\text{Sqrt}[\text{Tan}[c + d*x]])}{\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]]}]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]])/d - (a^2*(2*A - (3*I)*B)*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(4*d*\text{Cot}[c + d*x]^{(3/2)}) + (a^2*((18*I)*A + 19*B)*\text{Sqrt}[a + I*a*\text{Tan}[c + d*x]])/(8*d*\text{Sqrt}[\text{Cot}[c + d*x]]) + ((I/3)*a*B*(a + I*a*\text{Tan}[c + d*x])^{(3/2)})/(d*\text{Cot}[c + d*x]^{(3/2)})$

**Rule 65**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))^{(m_.)}*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol]}{Denominator[m]}, x] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{(p/b)})^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3625

```
Int[Sqrt[(a_) + (b_.)*tan[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*tan[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3675

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_) + (f_.)*(x_)])^((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c - a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && !LtQ[n, -1]
```

### Rule 3678

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_) + (f_.)*(x_)])^((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_) + (f_.)*(x_)])^((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
```

```
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps



$$(46A - (45I)B)(-I + E^{(I(c + dx))}) + \text{Sqrt}[2](-46A + (45I)B)\text{Log}[(2E^{((7I)/2)c})(-I)\text{Sqrt}[2] + \text{Sqrt}[2]E^{(I(c + dx))} + 2\text{Sqrt}[-1 + E^{((2I)(c + dx))}])]/((46I)A + 45B)(I + E^{(I(c + dx))}) + 128(A - I)B\text{Log}[(\text{Cos}[c] - I\text{Sin}[c])(\text{Cos}[c + dx] + I\text{Sin}[c + dx] + \text{Sqrt}[-1 + \text{Cos}[2(c + dx)] + I\text{Sin}[2(c + dx)]))] \text{Sqrt}[I(I + \text{Cot}[c + dx])\text{Sin}[c + dx]^2(\text{Cos}[3c + dx] - I\text{Sin}[3c + dx])] + (2\text{Sec}[c + dx]^2(\text{Cos}[2c] - I\text{Sin}[2c]))((54I)A + 49B + ((54I)A + 65B)\text{Cos}[2(c + dx)] + (-12A + (26I)B)\text{Sin}[2(c + dx)])\text{Tan}[c + dx])/3)/(32d(\text{Cos}[dx] + I\text{Sin}[dx])^2(A\text{Cos}[c + dx] + B\text{Sin}[c + dx]))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4850 vs.  $2(234) = 468$ .  
time = 69.05, size = 4851, normalized size = 16.61

method	result	size
default	Expression too large to display	4851

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(dx+c))^(5/2)*(A+B*tan(dx+c))/cot(dx+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/96/d^{1/2}a^2(24IA^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)-138IA^2^{1/2}\ln(((1+\cos(dx+c))/\sin(dx+c))^{1/2}+1)\cos(dx+c)^4+138IA^2^{1/2}\ln(((1+\cos(dx+c))/\sin(dx+c))^{1/2}-1)\cos(dx+c)^4+276IA^2^{1/2}\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})\cos(dx+c)^4-132IA^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)^4+135IB^2^{1/2}\ln(((1+\cos(dx+c))/\sin(dx+c))^{1/2}+1)\cos(dx+c)^4-276IA^2^{1/2}\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})\cos(dx+c)^3-24IA^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)^3+384IB\sin(dx+c)\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})^2(1/2)+1)\cos(dx+c)^3+138IA\sin(dx+c)^2(1/2)\ln(((1+\cos(dx+c))/\sin(dx+c))^{1/2}+1)\cos(dx+c)^3-52IB^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)^3+132IA^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)^2-198IB^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)^2+16IB\sin(dx+c)^2(1/2)((-1+\cos(dx+c))/\sin(dx+c))^{1/2}+52IB^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)-135IB^2^{1/2}\ln(((1+\cos(dx+c))/\sin(dx+c))^{1/2}-1)\cos(dx+c)^4+270IB^2^{1/2}(1/2)\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})\cos(dx+c)^4+182IB^2^{1/2}((-1+\cos(dx+c))/\sin(dx+c))^{1/2}\cos(dx+c)^4+384IA\sin(dx+c)\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})^2(1/2)+1)\cos(dx+c)^3+384IA\sin(dx+c)\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})^2(1/2)-1)\cos(dx+c)^3+192IA\sin(dx+c)\ln(-(((1+\cos(dx+c))/\sin(dx+c))^{1/2})^2(1/2)\sin(dx+c)-\sin(dx+c)-\cos(dx+c)+1)/(((1+\cos(dx+c))/\sin(dx+c))^{1/2})^2(1/2)\sin(dx+c)+\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)^3+138IA^2^{1/2}\ln(((1+\cos(dx+c))/\sin(dx+c))^{1/2}+1)\cos(dx+c)^3-24A^2^{1/2}\cos(dx+c)\sin(dx+c)((1+\cos(dx+c))/\sin(dx+c))^{1/2}-384B\arctan(((1+\cos(dx+c))/\sin(dx+c))^{1/2})$$





$a/(e^{(2I*d*x + 2I*c)} + 1))*sqrt((I*e^{(2I*d*x + 2I*c)} + I)/(e^{(2I*d*x + 2I*c)} - 1)))*e^{(-2I*d*x - 2I*c)/((46I*A + 45B)*a))}/(d*e^{(6I*d*x + 6I*c)} + 3*d*e^{(4I*d*x + 4I*c)} + 3*d*e^{(2I*d*x + 2I*c)} + d)$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^(5/2)/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^{5/2}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^(5/2))/cot(c + d\*x)^(1/2), x)



$$3.554 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=211

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{a} d} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia \tan(c+dx)}}$$

[Out] (1/2+1/2\*I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/a^(1/2)+(A+I\*B)\*cot(d\*x+c)^(3/2)/d/(a+I\*a\*tan(d\*x+c))^(1/2)-1/3\*(5\*A+3\*I\*B)\*cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d+1/3\*(7\*I\*A-9\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d

Rubi [A]

time = 0.44, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3677, 3679, 12, 3625, 211}

$$\frac{(5A+3iB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{3ad} + \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia \tan(c+dx)}} + \frac{(-9B+7iA) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3ad} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (B+iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((1/2 + I/2)\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(Sqrt[a]\*d) + (A + I\*B)\*Cot[c + d\*x]^(3/2)/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) + (((7\*I)\*A - 9\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*a\*d) - ((5\*A + (3\*I)\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(3\*a\*d)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

#### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

```

#### Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

```

#### Rule 4326

```

Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx \\
&= \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx}{d \sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} - \frac{(5A+3iB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}}{3ad} \\
&= \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(7iA-9B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3ad} \\
&= \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(7iA-9B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3ad} \\
&= \frac{(A+iB) \cot^{\frac{3}{2}}(c+dx)}{d \sqrt{a+ia \tan(c+dx)}} + \frac{(7iA-9B) \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{3ad} \\
&= \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)}}{\sqrt{a} d}
\end{aligned}$$

**Mathematica [A]**

time = 2.47, size = 166, normalized size = 0.79

$$\frac{\sqrt{\cot(c+dx)} \csc(c+dx) \sec(c+dx) \left( -9A - 9iB + \frac{3}{2}(A-iB)e^{-i(c+dx)}(-1+e^{2i(c+dx)})^{3/2} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}} \right) + (5A+9iB) \cos(2(c+dx)) + 2iA \sin(2(c+dx)) - 6B \sin(2(c+dx)) \right)}{6d \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]], x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Csc[c + d*x]*Sec[c + d*x]*(-9*A - (9*I)*B + (3*(A - I*B)*(-1 + E^((2*I)*(c + d*x))))^(3/2)*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/(2*E^(I*(c + d*x))) + (5*A + (9*I)*B)*Cos[2*(c + d*x)] + (2*I)*A*Sin[2*(c + d*x)] - 6*B*Sin[2*(c + d*x)])/(6*d*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs.  $2(171) = 342$ .

time = 65.64, size = 683, normalized size = 3.24

method	result
default	$\left(-\frac{1}{6}-\frac{i}{6}\right)\left(-3iB\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{2}(\cos^2(dx+c))\arctan\left(\frac{1}{2}+\frac{i}{2}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{2}\right)-9iB(\cos^2(dx+c))+5iA(\cos^2(dx+c))\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} &(-1/6-1/6*I)/d/a*(-3*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2)*\cos(d*x+c) \\ &+c)^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2))-9*I*B*\cos(d*x+c) \\ &^2+5*I*A*\cos(d*x+c)^2+3*A*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2)*\cos(d*x+c) \\ &^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2))+3*B*((-1+\cos(d*x+c))/\sin(d*x+c)) \\ &^(1/2)*\sin(d*x+c)*2^(1/2)*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c)) \\ &^(1/2)*2^(1/2))+3*B*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*\sin(d*x+c)*2^(1/2)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c)) \\ &^(1/2)*2^(1/2))-6*I*B*\sin(d*x+c)*\cos(d*x+c)-3*A*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c)) \\ &^(1/2)*2^(1/2))-2*I*A*\sin(d*x+c)*\cos(d*x+c)-7*I*A+3*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*\sin(d*x+c)*2^(1/2)*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c)) \\ &^(1/2)*2^(1/2))+9*I*B-2*A*\cos(d*x+c)*\sin(d*x+c)-5*A*\cos(d*x+c)^2+6*B*\cos(d*x+c)*\sin(d*x+c)-9*B*\cos(d*x+c)^2+3*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*\sin(d*x+c)*2^(1/2)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2))+3*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*2^(1/2))+7*A+9*B)*\sin(d*x+c)*(\cos(d*x+c)/\sin(d*x+c))^(5/2)*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^(1/2)/(I*\sin(d*x+c)+\cos(d*x+c))/\cos(d*x+c)^2 \end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(161) = 322.

time = 0.88, size = 487, normalized size = 2.31

$$\frac{2\sqrt{2}\sqrt{a^2+ab^2}\sqrt{\frac{1B+2AB-1D^2}{a^2}}\log\left(\frac{\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{1B+2AB-1D^2}{a^2}}}{2\sqrt{a^2+ab^2}}\sqrt{\frac{1B+2AB-1D^2}{a^2}}\right)^2-1}{\frac{1B+2AB-1D^2}{a^2}}\right)}{2\sqrt{2}\sqrt{a^2+ab^2}\sqrt{\frac{1B+2AB-1D^2}{a^2}}\log\left(\frac{\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{1B+2AB-1D^2}{a^2}}}{2\sqrt{a^2+ab^2}}\sqrt{\frac{1B+2AB-1D^2}{a^2}}\right)^2-1}{\frac{1B+2AB-1D^2}{a^2}}\right)}+2\sqrt{2}\sqrt{(-2A+2B)\sqrt{a^2+ab^2}}+2B\sqrt{a^2+ab^2}+2B(A-B)\sqrt{a^2+ab^2}-2(A+2B)\sqrt{\frac{1B+2AB-1D^2}{a^2}}\sqrt{\frac{1B+2AB-1D^2}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/12*(3*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)})*\log(-4*((A - I*B)*a*e^{(I*d*x + I*c)} + (a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))*\sqrt{-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)}})*e^{(-I*d*x - I*c)/(I*A + B)} - 3*\sqrt{2}*(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})*\sqrt{-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)})*\log(-4*((A - I*B)*a*e^{(I*d*x + I*c)} - (a*d*e^{(2*I*d*x + 2*I*c)} - a*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))*\sqrt{-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)}})*e^{(-I*d*x - I*c)/(I*A + B)} + 2*\sqrt{2}*((-7*I*A + 15*B)*e^{(4*I*d*x + 4*I*c)} + 18*(I*A - B)*e^{(2*I*d*x + 2*I*c)} - 3*I*A + 3*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)))/(a*d*e^{(3*I*d*x + 3*I*c)} - a*d*e^{(I*d*x + I*c)})} \end{aligned}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(5/2)/sqrt(I\*a\*tan(d\*x + c) + a), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{5/2} (A + B \tan(c + dx))}{\sqrt{a + a \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

```
[Out] int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

$$3.555 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=163

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{a} d} + \frac{(A+iB) \sqrt{\cot(c+dx)}}{d \sqrt{a+ia \tan(c+dx)}}$$

[Out] (1/2+1/2\*I)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/a^(1/2)+(A+I\*B)\*cot(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))^(1/2)-(3\*A+I\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a/d

**Rubi [A]**

time = 0.32, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3677, 3679, 12, 3625, 211}

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{d\sqrt{a+ia \tan(c+dx)}} - \frac{(3A+iB)\sqrt{\cot(c+dx)}\sqrt{a+ia \tan(c+dx)}}{ad} + \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((1/2 + I/2)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(Sqrt[a]\*d) + ((A + I\*B)\*Sqrt[Cot[c + d\*x]])/(d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((3\*A + I\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(a\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

### Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
] && LtQ[n, -1]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
]
```

### Rubi steps



$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} dx \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} dx}{\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(3A+iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(3A+iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(3A+iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} \\
&= \frac{\left(\frac{1}{2} - \frac{i}{2}\right)(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{\sqrt{a}d}
\end{aligned}$$

**Mathematica [A]**

time = 1.70, size = 165, normalized size = 1.01

$$\frac{e^{-2i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(A-5Ae^{2i(c+dx)}-iB(-1+e^{2i(c+dx)})+(A-iB)e^{i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\right)\sqrt{\cot(c+dx)}}{\sqrt{2}ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]], x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(A - 5*A*E^((2*I)*(c + d*x)) - I*B*(-1 + E^((2*I)*(c + d*x))) + (A - I*B)*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]])/(Sqrt[2]*a*d*E^((2*I)*(c + d*x)))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs.  $2(133) = 266$ .

time = 65.51, size = 482, normalized size = 2.96

method	result
--------	--------

default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\left(iA\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sin(dx+c)\sqrt{2}\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\sqrt{2}\right)-iB\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right)\right)}{\dots}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} &(-1/2-1/2*I)/d/a*(I*A*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)) \\ &*2^(1/2))*2^(1/2)*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)-I*B*\arctan \\ &((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2))*2^(1/2))*2^(1/2)*\cos(d*x+c)* \\ &((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)+A*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin \\ &(d*x+c))^(1/2))*2^(1/2))*2^(1/2)*\cos(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1 \\ &/2)+B*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2)*\sin(d*x+c)*2^(1/2)*\arctan((1/2+1/2 \\ &*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2))*2^(1/2))-I*B*\arctan((1/2+1/2*I)*((-1 \\ &+\cos(d*x+c))/\sin(d*x+c))^(1/2))*2^(1/2))*2^(1/2)*((-1+\cos(d*x+c))/\sin(d*x+c) \\ &)^(1/2)+A*((-1+\cos(d*x+c))/\sin(d*x+c))^(1/2))*2^(1/2)*\arctan((1/2+1/2*I)*((-1 \\ &+\cos(d*x+c))/\sin(d*x+c))^(1/2))*2^(1/2))+3*I*A*\sin(d*x+c)-2*I*A*\cos(d*x+c)+ \\ &I*B*\sin(d*x+c)+3*A*\sin(d*x+c)+2*A*\cos(d*x+c)-B*\sin(d*x+c))*\sin(d*x+c)*(\cos \\ &(d*x+c)/\sin(d*x+c))^(3/2)*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^(1/2)/(I* \\ &\sin(d*x+c)+\cos(d*x+c))/\cos(d*x+c) \end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs.  $2(125) = 250$ .

time = 0.72, size = 426, normalized size = 2.61

$$\left(\frac{\sqrt{2}a\sqrt{-1+A^2-2AB+B^2}}{a^2}\log\left(\frac{\sqrt{2}a\sqrt{-1+A^2-2AB+B^2}}{a^2}\sqrt{\frac{a^2+1}{2a^2+1}}\sqrt{\frac{a^2-1}{2a^2-1}}\sqrt{\frac{-1+A^2-2AB+B^2}{a^2}}\right)-\sqrt{2}a\sqrt{-1+A^2-2AB+B^2}\log\left(\frac{\sqrt{2}a\sqrt{-1+A^2-2AB+B^2}}{a^2}\sqrt{\frac{a^2+1}{2a^2+1}}\sqrt{\frac{a^2-1}{2a^2-1}}\sqrt{\frac{-1+A^2-2AB+B^2}{a^2}}\right)\right)+2\sqrt{2}(5A+1B)a^{2+2i-1}A-B\sqrt{\frac{a^2+1}{2a^2+1}}\sqrt{\frac{a^2-1}{2a^2-1}}\right)^{i+2i-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")`

```
[Out] -1/4*(sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B) - sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B) + 2*sqrt(2)*((5*A + I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(-I*d*x - I*c)/(a*d)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{\sqrt{ia(\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{\sqrt{a + a \tan(c + dx)} \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

$$3.556 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx$$

Optimal. Leaf size=119

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{a} d} + \frac{A+iB}{d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}$$

[Out] (1/2-1/2\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/a^(1/2)+(A+I\*B)/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)

Rubi [A]

time = 0.20, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {4326, 3677, 12, 3625, 211}

$$\frac{A+iB}{d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{\left(\frac{1}{2} - \frac{i}{2}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]],x]

[Out] ((1/2 - I/2)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[a]\*d) + (A + I\*B)/(d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && Ne

$Q[c^2 + d^2, 0]$

### Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+ia \tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} dx \\ &= \frac{A+iB}{d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} \\ &= \frac{A+iB}{d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \right)}{\sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} \\ &= \frac{A+iB}{d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} - \frac{\left( ia(A-iB) \sqrt{\cot(c+dx)} \right)}{\sqrt{\tan(c+dx)} \sqrt{a+ia \tan(c+dx)}} \\ &= - \frac{\left( \frac{1}{2} + \frac{i}{2} \right) (iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{\sqrt{a} d} \sqrt{\cot(c+dx)} \end{aligned}$$

**Mathematica** [A]

time = 1.38, size = 156, normalized size = 1.31

$$\frac{e^{-2i(c+dx)} \sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \left( (-iA+B)(-1+e^{2i(c+dx)}) - i(A-iB)e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c+dx)}}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + I\*a\*Tan[c + d\*x]], x]

[Out] (Sqrt[(a\*E^((2\*I)\*(c + d\*x)))/(1 + E^((2\*I)\*(c + d\*x)))]\*(((-I)\*A + B)\*(-1 + E^((2\*I)\*(c + d\*x))) - I\*(A - I\*B)\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Sqrt[Cot[c + d\*x]])/(Sqrt[2]\*a\*d\*E^((2\*I)\*(c + d\*x)))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(95) = 190.

time = 60.10, size = 431, normalized size = 3.62

method	result
default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right) \left(iA\sqrt{2} \sin(dx+c) \arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right) \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}} \sqrt{2}\right) - iB\sqrt{2} \cos(dx+c) \arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right) \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right)\right)}{\sqrt{2} ad}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(1/2), x, method=\_RE TURNVERBOSE)

[Out] (-1/2-1/2\*I)/d/a\*(I\*A\*2^(1/2)\*sin(d\*x+c)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-I\*B\*2^(1/2)\*cos(d\*x+c)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+A\*2^(1/2)\*cos(d\*x+c)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+I\*A\*sin(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+B\*2^(1/2)\*sin(d\*x+c)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+I\*B\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-I\*B\*sin(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-A\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-A\*sin(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-B\*sin(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*(cos(d\*x+c)/sin(d\*x+c))^(1/2)\*((I\*sin(d\*x+c)+cos(d\*x+c))\*a/cos(d\*x+c))^(1/2)/(I\*sin(d\*x+c)+cos(d\*x+c))/((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs.  $2(89) = 178$ .

time = 1.18, size = 423, normalized size = 3.55

$$\frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{-1A^2+2AB-IB^2}}{a}\right) e^{i d x} \log\left(\frac{x \left( (A+B) e^{i d x} - (A-B) e^{-i d x} \right) \sqrt{\frac{a}{2 a^2 d^2 + 1}} \sqrt{\frac{a}{2 a^2 d^2 + 1}} \sqrt{\frac{-1A^2+2AB-IB^2}{a}}}{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{-1A^2+2AB-IB^2}}{a}\right) e^{i d x} \log\left(\frac{x \left( (A+B) e^{i d x} - (A-B) e^{-i d x} \right) \sqrt{\frac{a}{2 a^2 d^2 + 1}} \sqrt{\frac{a}{2 a^2 d^2 + 1}} \sqrt{\frac{-1A^2+2AB-IB^2}{a}}}{-2 \sqrt{2} (A-B) e^{i d x} - (A+B) \sqrt{\frac{a}{2 a^2 d^2 + 1}} \sqrt{\frac{a}{2 a^2 d^2 + 1}}}\right)}{1 a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")`

[Out]  $\frac{1}{4} \sqrt{2} a d \sqrt{-1A^2 + 2AB - IB^2} / (a d^2) e^{i d x + I c} \log(-4((A - IB) a e^{i d x + I c} + (a d e^{2 i d x + 2 I c} - a d) \sqrt{a / (e^{2 i d x + 2 I c} + 1)}) \sqrt{(I e^{2 i d x + 2 I c} + I) / (e^{2 i d x + 2 I c} - 1)}) \sqrt{-1A^2 + 2AB - IB^2} / (a d^2)) e^{-i d x - I c} / (IA + B) - \sqrt{2} a d \sqrt{-1A^2 + 2AB - IB^2} / (a d^2) e^{i d x + I c} \log(-4((A - IB) a e^{i d x + I c} - (a d e^{2 i d x + 2 I c} - a d) \sqrt{a / (e^{2 i d x + 2 I c} + 1)}) \sqrt{(I e^{2 i d x + 2 I c} + I) / (e^{2 i d x + 2 I c} - 1)}) \sqrt{-1A^2 + 2AB - IB^2} / (a d^2)) e^{-i d x - I c} / (IA + B) - 2 \sqrt{2} ((IA - B) e^{2 i d x + 2 I c} - IA + B) \sqrt{a / (e^{2 i d x + 2 I c} + 1)}) \sqrt{(I e^{2 i d x + 2 I c} + I) / (e^{2 i d x + 2 I c} - 1)}) e^{-i d x - I c} / (a d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{\sqrt{ia(\tan(c + dx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{\sqrt{a + a \tan(c + dx)} \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2), x)
```



$$3.557 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} dx$$

**Optimal.** Leaf size=196

$$\frac{2\sqrt{-1} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left(\frac{1}{2} + \frac{i}{2}\right) (A-iB) \tanh^{-1}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d}$$

[Out]  $-2*(-1)^{(1/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/a^{(1/2)}-(1/2+1/2*I)*(A-I*B)*a*\operatorname{rctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/a^{(1/2)}+(I*A-B)/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)}$

**Rubi [A]**

time = 0.40, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{-B+iA}{d\sqrt{\cot(c+dx)}\sqrt{a+ia \tan(c+dx)}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) (A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d} - \frac{2\sqrt{-1} B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]),x]$

[Out]  $(-2*(-1)^{(1/4)}*B*\operatorname{ArcTan}((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(\operatorname{Sqrt}[a]*d) - ((1/2 + I/2)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(\operatorname{Sqrt}[a]*d) + (I*A - B)/(d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(1/p)}, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 209**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*A*\operatorname{rcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \|\operatorname{GtQ}[b, 0])$

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} dx \\
&= \frac{iA - B}{d \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) (A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} \\
&= \frac{iA - B}{d \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} + \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) (A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} \\
&= \frac{iA - B}{d \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} + \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) (A + B \tan(c + dx))}{\sqrt{a + ia \tan(c + dx)}} \\
&= -\frac{\left( \frac{1}{2} - \frac{i}{2} \right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{\sqrt{a} d} \\
&= -\frac{\left( \frac{1}{2} - \frac{i}{2} \right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{\sqrt{a} d} \\
&= -\frac{2\sqrt{-1} B \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{\sqrt{a} d}
\end{aligned}$$

**Mathematica [A]**

time = 2.46, size = 227, normalized size = 1.16

$$\frac{e^{-2i(c+dx)} \sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}} \left( (A+iB)(-1+e^{2i(c+dx)}) - (A-iB)e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}} \right) - 2i\sqrt{2} B e^{i(c+dx)} \sqrt{-1+e^{2i(c+dx)}} \tanh^{-1} \left( \frac{\sqrt{2} e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c+dx)}}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]), x]
```

```
[Out] (Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*((A + I*B)*(-1 + E^((2*I)*(c + d*x))) - (A - I*B)*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]] - (2*I)*Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[(Sqrt[2]*E^(I*(c + d*x)))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]/(Sqrt[2]*a*d*E^((2*I)*(c + d*x)))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 804 vs.  $2(155) = 310$ .

time = 68.16, size = 805, normalized size = 4.11

method	result
default	$\left(\frac{1}{2} + \frac{i}{2}\right) \left( iB \ln \left( \sqrt{\frac{-1 + \cos(dx+c)}{\sin(dx+c)}} + 1 \right) - iB \cos(dx+c) \ln \left( \sqrt{\frac{-1 + \cos(dx+c)}{\sin(dx+c)}} + 1 \right) + iB \cos(dx+c) \ln \left( \sqrt{\frac{-1 + \cos(dx+c)}{\sin(dx+c)}} - i \right) + i \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2+1/2*I)/d/a*(I*B*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-I*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+I*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+I*B*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-A*sin(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-I*B*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-I*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+I*B*sin(d*x+c)*2^(1/2)*arctan((1/2+1/2*I)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+I*A*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*B*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+B*2^(1/2)*cos(d*x+c)*arctan((1/2+1/2*I)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+A*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*B*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*A*2^(1/2)*arctan((1/2+1/2*I)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+I*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+I*A*2^(1/2)*cos(d*x+c)*arctan((1/2+1/2*I)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-B*sin(d*x+c)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)+B*sin(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-B*sin(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+B*sin(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-B*sin(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-B*2^(1/2)*arctan((1/2+1/2*I)*((1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*cos(d*x+c)*((I*sin(d*x+c)+cos(d*x+c))/sin(d*x+c)/((1+cos(d*x+c))/sin(d*x+c))^(1/2)/(cos(d*x+c)/sin(d*x+c))^(1/2))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(148) = 296.

time = 1.36, size = 736, normalized size = 3.76



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{4} \sqrt{2} a d \sqrt{-(-I A^2 - 2 A B + I B^2)/(a d^2)} e^{(I d x + I c)} \log(-4((A - I B) a e^{(I d x + I c)} + (I a d e^{(2 I d x + 2 I c)} - I a d) \sqrt{a/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} - 1)} \sqrt{-(-I A^2 - 2 A B + I B^2)/(a d^2)}) e^{(-I d x - I c)} / (I A + B) - \sqrt{2} a d \sqrt{-(-I A^2 - 2 A B + I B^2)/(a d^2)} e^{(I d x + I c)} \log(-4((A - I B) a e^{(I d x + I c)} + (-I a d e^{(2 I d x + 2 I c)} + I a d) \sqrt{a/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} - 1)} \sqrt{-(-I A^2 - 2 A B + I B^2)/(a d^2)}) e^{(-I d x - I c)} / (I A + B) + a d \sqrt{-4 I B^2/(a d^2)} e^{(I d x + I c)} \log(-16(3 B a^2 e^{(2 I d x + 2 I c)} - B a^2 + \sqrt{2} (a^2 d e^{(3 I d x + 3 I c)} - a^2 d e^{(I d x + I c)}) \sqrt{a/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} - 1)} \sqrt{-4 I B^2/(a d^2)}) e^{(-2 I d x - 2 I c)} / B - a d \sqrt{-4 I B^2/(a d^2)} e^{(I d x + I c)} \log(-16(3 B a^2 e^{(2 I d x + 2 I c)} - B a^2 - \sqrt{2} (a^2 d e^{(3 I d x + 3 I c)} - a^2 d e^{(I d x + I c)}) \sqrt{a/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} - 1)} \sqrt{-4 I B^2/(a d^2)}) e^{(-2 I d x - 2 I c)} / B + 2 \sqrt{2} ((A + I B) e^{(2 I d x + 2 I c)} - A - I B) \sqrt{a/(e^{(2 I d x + 2 I c)} + 1)}) \sqrt{(I e^{(2 I d x + 2 I c)} + I)/(e^{(2 I d x + 2 I c)} - 1)}) e^{(-I d x - I c)} / (a d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{ia(\tan(c + dx) - i)} \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/(sqrt(I\*a\*(tan(c + d\*x) - I))\*sqrt(cot(c + d\*x))), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/(sqrt(I\*a\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} \sqrt{a + a \tan(c + dx)} i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*i)^(1/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*i)^(1/2)), x)

$$3.558 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=214

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{3/2}d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))}$$

[Out] (1/4+1/4\*I)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/a^(3/2)/d+1/6\*(11\*A+5\*I\*B)\*cot(d\*x+c)^(1/2)/a/d/(a+I\*a\*tan(d\*x+c))^(1/2)-1/6\*(25\*A+7\*I\*B)\*cot(d\*x+c)^(1/2)\*(a+I\*a\*tan(d\*x+c))^(1/2)/a^2/d+1/3\*(A+I\*B)\*cot(d\*x+c)^(1/2)/d/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.48, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} - \frac{(25A+7iB)\sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{6a^2d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(11A+5iB)\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] ((1/4 + I/4)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(a^(3/2)\*d) + (A + I\*B)\*Sqrt[Cot[c + d\*x]]/(3\*d\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + ((11\*A + (5\*I)\*B)\*Sqrt[Cot[c + d\*x]])/(6\*a\*d\*Sqrt[a + I\*a\*Tan[c + d\*x]]) - ((25\*A + (7\*I)\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])/(6\*a^2\*d)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

#### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

```

#### Rule 3679

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, -1]

```

#### Rule 4326

```

Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

#### Rubi steps



$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))} dx \\
&= \frac{(A+iB) \sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))} dx}{3a} \\
&= \frac{(A+iB) \sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(11A+5iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))} dx}{3a} \\
&= \frac{(A+iB) \sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(11A+5iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} - \frac{(25A+15iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(A+iB) \sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(11A+5iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} - \frac{(25A+15iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} \\
&= \frac{(A+iB) \sqrt{\cot(c+dx)}}{3d(a+ia \tan(c+dx))^{3/2}} + \frac{(11A+5iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} - \frac{(25A+15iB) \sqrt{\cot(c+dx)}}{6ad \sqrt{a+ia \tan(c+dx)}} \\
&= \frac{\left( \frac{1}{4} - \frac{i}{4} \right) (iA+B) \tanh^{-1} \left( \frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)}}{a^{3/2} d}
\end{aligned}$$

### Mathematica [A]

time = 2.64, size = 195, normalized size = 0.91

$$\frac{\left( iB(-1-7e^{2i(c+dx)}+8e^{4i(c+dx)})+A(-1-13e^{2i(c+dx)}+38e^{4i(c+dx)})-3(A-iB)e^{3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}} \tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right) \right) \cot^{\frac{3}{2}}(c+dx)}{3ad(1+e^{2i(c+dx)})^2(i+\cot(c+dx))\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] -1/3\*((I\*B\*(-1 - 7\*E^((2\*I)\*(c + d\*x)) + 8\*E^((4\*I)\*(c + d\*x))) + A\*(-1 - 13\*E^((2\*I)\*(c + d\*x)) + 38\*E^((4\*I)\*(c + d\*x))) - 3\*(A - I\*B)\*E^((3\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Cot[c + d\*x]^(3/2))/(a\*d\*(1 + E^((2\*I)\*(c + d\*x)))^2\*(I + Cot[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(172) = 344.

time = 66.18, size = 648, normalized size = 3.03



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/12*(3*\sqrt{1/2}*a^2*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^3*d^2)})*e^{(3*I*d*x + 3*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(I*a^2*d*e^{(2*I*d*x + 2*I*c)} - I*a^2*d) \\ & * \sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^3*d^2)}) + (A - I*B)*a*e^{(I*d*x + I*c)}*e^{(-I*d*x - I*c)/(I*A + B)} - 3*\sqrt{1/2}*a^2*d*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^3*d^2)}*e^{(3*I*d*x + 3*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(-I*a^2*d*e^{(2*I*d*x + 2*I*c)} + I*a^2*d) \\ & * \sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)})*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(I*A^2 + 2*A*B - I*B^2)/(a^3*d^2)}) + (A - I*B)*a*e^{(I*d*x + I*c)}*e^{(-I*d*x - I*c)/(I*A + B)} + \sqrt{2}*(2*(19*A + 4*I*B)*e^{(4*I*d*x + 4*I*c)} - (13*A + 7*I*B)*e^{(2*I*d*x + 2*I*c)} - A - I*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*e^{(-3*I*d*x - 3*I*c)/(a^2*d)} \end{aligned}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2)/(I\*a\*(tan(c + d\*x) - I))\*\*(3/2), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(I\*a\*tan(d\*x + c) + a)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) li)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

$$3.559 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=168

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{3/2}d} + \frac{A+iB}{3d\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}}$$

[Out] (1/4-1/4\*I)\*(A-I\*B)\*arctanh(((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/a^(3/2)/d+1/6\*(7\*A+I\*B)/a/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)+1/3\*(A+I\*B)/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

Rubi [A]

time = 0.34, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {4326, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{A+iB}{3d\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{7A+iB}{6ad\sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2), x]

[Out] ((1/4 - I/4)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(a^(3/2)\*d) + (A + I\*B)/(3\*d\*Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (7\*A + I\*B)/(6\*a\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 4326

Int[(cot[(a\_) + (b\_)\*(x\_)]\*(c\_))^(m\_)\*(u\_), x\_Symbol] := Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{3/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{3/2}} dx \\
 &= \frac{A+iB}{3d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{6ad \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} \\
 &= \frac{A+iB}{3d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{7A+B}{6ad \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} \\
 &= \frac{A+iB}{3d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{7A+B}{6ad \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} \\
 &= \frac{A+iB}{3d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{7A+B}{6ad \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} \\
 &= -\frac{\left( \frac{1}{4} + \frac{i}{4} \right) (iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)}}{a^{3/2}d}
 \end{aligned}$$

**Mathematica [A]**

time = 2.18, size = 192, normalized size = 1.14

$$\frac{e^{-2i(c+dx)} \left( (-1 + e^{2i(c+dx)}) (B + 2B e^{2i(c+dx)} - iA(1 + 8e^{2i(c+dx)})) - 3i(A - iB) e^{3i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c+dx)} \csc(c+dx) \sec(c+dx)}{12ad(i + \cot(c+dx)) \sqrt{a + ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(3/2),x]

[Out] (((-1 + E^((2\*I)\*(c + d\*x)))\*(B + 2\*B\*E^((2\*I)\*(c + d\*x))) - I\*A\*(1 + 8\*E^((2\*I)\*(c + d\*x)))) - (3\*I)\*(A - I\*B)\*E^((3\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))])\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Sqrt[Cot[c + d\*x]]\*Csc[c + d\*x]\*Sec[c + d\*x]/(12\*a\*d\*E^((2\*I)\*(c + d\*x))\*(I + Cot[c + d\*x])\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 852 vs.  $2(134) = 268$ .

time = 63.08, size = 853, normalized size = 5.08

method	result	size
default	Expression too large to display	853

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x,method=\_RETURNVERBOSE)

[Out] (1/12-1/12\*I)/d/a^2\*(-7\*I\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2+3\*I\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*sin(d\*x+c)\*cos(d\*x+c)+I\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2-6\*I\*B\*sin(d\*x+c)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*cos(d\*x+c)+6\*A\*sin(d\*x+c)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*cos(d\*x+c)+9\*I\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*sin(d\*x+c)\*cos(d\*x+c)-6\*I\*A\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*cos(d\*x+c)^2+3\*I\*B\*sin(d\*x+c)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+7\*I\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-6\*B\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*cos(d\*x+c)^2+9\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*sin(d\*x+c)\*cos(d\*x+c)+7\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2-3\*A\*sin(d\*x+c)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+3\*I\*A\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-3\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*sin(d\*x+c)\*cos(d\*x+c)+B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^2+3\*B\*2^(1/2)\*cos(d\*x+c)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+3\*I\*A\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*cos(d\*x+c)-I\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))

$$\begin{aligned} & \sqrt[1/2]{3B^2 + 3B^2 \sqrt[1/2]{\arctan\left(\frac{1/2 + 1/2I}{(-1 + \cos(dx+c)) / \sin(dx+c)}\right)} \sqrt[1/2]{(-1 + \cos(dx+c)) / \sin(dx+c)} \\ & - 7A \sqrt[1/2]{(-1 + \cos(dx+c)) / \sin(dx+c)} - B \sqrt[1/2]{(-1 + \cos(dx+c)) / \sin(dx+c)} \\ & \sqrt[1/2]{\cos(dx+c) / \sin(dx+c)} \sqrt[1/2]{(I \sin(dx+c) + \cos(dx+c))} \frac{a}{\cos(dx+c)} \\ & \sqrt[1/2]{(2I \sin(dx+c) \cos(dx+c) + 2 \cos(dx+c)^2 - 1)} \sqrt[1/2]{(-1 + \cos(dx+c)) / \sin(dx+c)} \end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(126) = 252.

time = 1.51, size = 463, normalized size = 2.76

$$\frac{\left( \sqrt[3]{2} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \right) \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}}}{\sqrt[3]{2} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}} \sqrt[3]{\frac{12B^2 - 12AB + 12B^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(A+B\*tan(dx+c))/(a+I\*a\*tan(dx+c))^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \frac{1}{12} (3 \sqrt[3]{1/2} a^2 d \sqrt[3]{(-I A^2 - 2 A B + I B^2) / (a^3 d^2)}) e^{(3 I d x + 3 I c)} \log(-4 (\sqrt[3]{2} \sqrt[3]{1/2} (a^2 d e^{(2 I d x + 2 I c)} - a^2 d) \sqrt[3]{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt[3]{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)} \sqrt[3]{(-I A^2 - 2 A B + I B^2) / (a^3 d^2)} + (A - I B) a e^{(I d x + I c)}) e^{(-I d x - I c)} / (I A + B) - 3 \sqrt[3]{1/2} a^2 d \sqrt[3]{(-I A^2 - 2 A B + I B^2) / (a^3 d^2)} e^{(3 I d x + 3 I c)} \log(4 (\sqrt[3]{2} \sqrt[3]{1/2} (a^2 d e^{(2 I d x + 2 I c)} - a^2 d) \sqrt[3]{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt[3]{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)} \sqrt[3]{(-I A^2 - 2 A B + I B^2) / (a^3 d^2)} - (A - I B) a e^{(I d x + I c)}) e^{(-I d x - I c)} / (I A + B) - \sqrt[3]{2} (2 (4 I A - B) e^{(4 I d x + 4 I c)} - (7 I A - B) e^{(2 I d x + 2 I c)} - I A + B) \sqrt[3]{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt[3]{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)}) e^{(-3 I d x - 3 I c)} / (a^2 d) \end{aligned}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{(ia(\tan(c + dx) - i))^{3/2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x))/(I\*a\*(tan(c + d\*x) - I))\*\*(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/(I\*a\*tan(d\*x + c) + a)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2),x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(3/2), x)

$$3.560 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=170

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{3/2}d} + \frac{iA - B}{3d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}}$$

[Out]  $(-1/4-1/4*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(3/2)}/d+1/6*(I*A+5*B)/a/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)}+1/3*(I*A-B)/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.34, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3676, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{3/2}d} + \frac{-B+iA}{3d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{5B+iA}{6ad \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}), x]$

[Out]  $((-1/4 - I/4)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(a^{(3/2)}*d) + (I*A - B)/(3*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}) + (I*A + 5*B)/(6*a*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

$\operatorname{Int}(((a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol) := \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]], x\_Symbol] := \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a^2*x^2), x], x, \operatorname{Sqrt}[c + d*\operatorname{Tan}[e + f*x]]/\operatorname{Sqrt}[a + b*\operatorname{Tan}[e + f*x]]], x] /;$  F

reeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(-(A\*b - a\*B))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3677

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(a\*A + b\*B)\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(2\*f\*m\*(b\*c - a\*d))), x] + Dist[1/(2\*a\*m\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(b\*c\*m - a\*d\*(2\*m + n + 1)) + B\*(a\*c\*m - b\*d\*(n + 1)) + d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

### Rule 4326

Int[(cot[(a\_) + (b\_)\*(x\_)])\*(c\_)^(m\_)\*(u\_), x\_Symbol] :> Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx \\
&= \frac{iA - B}{3d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx}{3d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}} \\
&= \frac{iA - B}{3d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{iA}{6ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{3d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{iA}{6ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{3d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{3/2}} + \frac{iA}{6ad \sqrt{\cot(c + dx)}} \\
&= - \frac{\left( \frac{1}{4} - \frac{i}{4} \right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{3/2} d}
\end{aligned}$$

**Mathematica [A]**

time = 2.19, size = 190, normalized size = 1.12

$$\frac{e^{-2i(c+dx)} \left( (-1 + e^{2i(c+dx)}) (A + 2Ae^{2i(c+dx)} - iB(-1 + 4e^{2i(c+dx)})) - 3(A - iB)e^{3i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1 + e^{2i(c+dx)}}} \right) \right) \sqrt{\cot(c + dx)} \csc(c + dx)}{12ad(i + \cot(c + dx)) \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (((-1 + E^((2*I)*(c + d*x)))*(A + 2*A*E^((2*I)*(c + d*x)) - I*B*(-1 + 4*E^((2*I)*(c + d*x)))) - 3*(A - I*B)*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))])*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sqrt[Cot[c + d*x]]*Csc[c + d*x]*Sec[c + d*x])/(12*a*d*E^((2*I)*(c + d*x))*(I + Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 866 vs.  $2(136) = 272$ .

time = 64.30, size = 867, normalized size = 5.10

method	result	size
default	Expression too large to display	867

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & (-1/12+1/12*I)/d/a^2*(3*I*B*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))) \\ & ^{(1/2)*2^{(1/2)}}*2^{(1/2)}*\cos(d*x+c)-I*A*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d* \\ & x+c))^{(1/2)}+5*I*B*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+6*A*\arcta \\ & n((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}}*2^{(1/2)}*\cos(d*x+c) \\ & )^2-3*I*B*\sin(d*x+c)*\cos(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-6*I*B*\ar \\ & ctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}}*2^{(1/2)}*\cos(d* \\ & x+c)^2+6*B*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}}*s \\ & in(d*x+c)*2^{(1/2)}*\cos(d*x+c)-3*I*A*\sin(d*x+c)*\cos(d*x+c)*((-1+\cos(d*x+c))/s \\ & in(d*x+c))^{(1/2)}+I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+3*I*B*\arctan((1/2+1 \\ & /2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}}*2^{(1/2)}-3*A*2^{(1/2)}*\cos(d \\ & *x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})+3*A*(( \\ & -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)-A*((-1+\cos(d*x+c))/s \\ & in(d*x+c))^{(1/2)}*\cos(d*x+c)^2-3*B*2^{(1/2)}*\sin(d*x+c)*\arctan((1/2+1/2*I)*((- \\ & 1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})+6*I*A*\arctan((1/2+1/2*I)*((-1+\cos( \\ & d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)-3*B*((-1+c \\ & os(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)*\cos(d*x+c)-5*B*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*\cos(d*x+c)^2-3*A*2^{(1/2)}*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c)) \\ & )/\sin(d*x+c))^{(1/2)*2^{(1/2)}})-5*I*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-3*I*A* \\ & arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})*\sin(d*x+c)*2 \\ & ^{(1/2)}+A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+5*B*((-1+\cos(d*x+c))/\sin(d*x+c) \\ & )^{(1/2)}*\cos(d*x+c)*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}/(2*I*\sin \\ & (d*x+c)*\cos(d*x+c)+2*\cos(d*x+c)^2-1)/\sin(d*x+c)/((-1+\cos(d*x+c))/\sin(d*x+c) \\ & )^{(1/2)}/(\cos(d*x+c)/\sin(d*x+c))^{(1/2)} \end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs.  $2(126) = 252$ .



**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*i)^(3/2)), x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + a\*tan(c + d\*x)\*i)^(3/2)), x)

$$3.561 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=243

$$\frac{2(-1)^{3/4} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{3/2} d} + \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (iA+B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{2ad \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{-B+iA}{3d \cot^2(c+dx)(a+ia \tan(c+dx))^{3/2}} + \frac{A+3iB}{2ad \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}$$

[Out]  $2*(-1)^{(3/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(3/2)}/d+(1/4+1/4*I)*(I*A+B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(3/2)}/d+1/2*(A+3*I*B)/a/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)}+1/3*(I*A-B)/d/\cot(d*x+c)^{(3/2)}/(a+I*a*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.53, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) (B+iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{3/2} d} + \frac{2(-1)^{3/4} B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{3/2} d} + \frac{-B+iA}{3d \cot^2(c+dx)(a+ia \tan(c+dx))^{3/2}} + \frac{A+3iB}{2ad \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(\operatorname{Cot}[c+d*x]^{(3/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}), x]$

[Out]  $(2*(-1)^{(3/4)}*B*\operatorname{ArcTan}[\frac{(-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]}{\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]}]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(\operatorname{a}^{(3/2)}*d) + ((1/4 + I/4)*(I*A+B)*\operatorname{ArcTanh}[\frac{(1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]}{\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]}]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(\operatorname{a}^{(3/2)}*d) + (I*A - B)/(3*d*\operatorname{Cot}[c+d*x]^{(3/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}) + (A+(3*I)*B)/(2*a*d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 209

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] :> \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))* \operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a$



, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n/(2\*a\*f\*m), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m - n) - a\*A\*(m + n))\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]

## Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_.), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

## Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{3/2}} dx \\
&= \frac{iA - B}{3d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right)}{2ad \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{iA - B}{3d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}} + \frac{A + 3B \tan(c + dx)}{2ad \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{iA - B}{3d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}} + \frac{A + 3B \tan(c + dx)}{2ad \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{iA - B}{3d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{3/2}} + \frac{A + 3B \tan(c + dx)}{2ad \sqrt{\cot(c + dx)} \sqrt{a + ia \tan(c + dx)}} \\
&= \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{3/2}d} \\
&= \frac{\left(\frac{1}{4} + \frac{i}{4}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{3/2}d} \\
&= \frac{2(-1)^{3/4} B \tan^{-1} \left( \frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{3/2}d}
\end{aligned}$$

## Mathematica [A]

time = 4.51, size = 388, normalized size = 1.60

$$\frac{c^{-3/2} \sqrt{\cot(c + dx)} \left( (-A + B + 5iAe^{2i(c+dx)} - 11Be^{2i(c+dx)} - 4iAe^{4i(c+dx)} + 10Be^{4i(c+dx)} + 3(A + B)e^{6i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \log(e^{i(c+dx)} + \sqrt{-1 + e^{2i(c+dx)}}) - 3\sqrt{2} Be^{3i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \log(1 - 3e^{2i(c+dx)} - 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) + 3\sqrt{2} Be^{5i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}} \log(1 - 3e^{2i(c+dx)} + 2\sqrt{2} e^{i(c+dx)} \sqrt{-1 + e^{2i(c+dx)}}) \right) \operatorname{sech}(c + dx) (A + B \tan(c + dx))}{12i(A \cos(c + dx) + B \sin(c + dx))(a + ia \tan(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*((-I)*A + B + (5*I)*A*E^((2*I)*(c + d*x)) - 11*B*E^((2*I)*(c + d*x)) - (4*I)*A*E^((4*I)*(c + d*x)) + 10*B*E^((4*I)*(c + d*x)) + 3*(I*A + B)*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Log[E^(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]] - 3*Sqrt[2]*B*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Log[1 - 3*E^((2*I)*(c + d*x)) - 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]] + 3*Sqrt[2]*B*E^((3*I)*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*Log[1 - 3*E^((2*I)*(c + d*x)) + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]*(A + B*Tan[c + d*x])/(12*d*E^((2*I)*(c + d*x))*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1515 vs.  $2(192) = 384$ .

time = 66.83, size = 1516, normalized size = 6.24

method	result	size
default	Expression too large to display	1516

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12-1/12*I)/d/a^2*(-6*I*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-5*I*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*A*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*2^(1/2)*cos(d*x+c)^2+3*A*2^(1/2)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))-11*I*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*B*2^(1/2)*sin(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+3*A*2^(1/2)*cos(d*x+c)*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))+5*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-11*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2-6*B*arctan((1/2+1/2*I)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-5*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+11*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-12*B*cos(d*x+c)^2*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)+12*B*cos(d*x+c)^2*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)-12*B*cos(d*x+c)^2*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)+12*B*cos(d*x+c)^2*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)+6*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-1)-6*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+1)+6*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)-I)-6*B*cos(d*x+c)*ln(((1+cos(d*x+c))/sin(d*x+c))^(1/2)+I)-3*A*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)-9*B*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)+6*B*ln(((1+cos(d*x+c))
```

$$\begin{aligned} & )/\sin(d*x+c))^{(1/2)-1}-6*B*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)+1}+6*B*\ln( \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)-1}-6*B*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)+1} \\ & +5*I*A*\cos(d*x+c)^2*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)+11*I*B*\cos(d* \\ & x+c)^2*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)-3*I*B*2^{(1/2)}*\arctan((1/2+1/2*I)* \\ & ((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})+6*I*B*\sin(d*x+c)*\ln(((1+\cos(d* \\ & x+c))/\sin(d*x+c))^{(1/2)-1}-6*I*B*\sin(d*x+c)*\ln(((1+\cos(d*x+c))/\sin(d*x+c)) \\ & ^{(1/2)+1}+6*I*B*\sin(d*x+c)*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)-1}-6*I*B*s \\ & in(d*x+c)*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)+1}+12*I*B*\cos(d*x+c)*\sin(d* \\ & x+c)*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)+1}-12*I*B*\cos(d*x+c)*\sin(d*x+c)* \\ & \ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)-1}+12*I*B*\cos(d*x+c)*\sin(d*x+c)*\ln((( \\ & -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)+1}+6*I*B*2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+ \\ & 1/2*I)*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})+3*I*A*((1+\cos(d*x+c))/s \\ & in(d*x+c))^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)+3*I*A*2^{(1/2)}*\sin(d*x+c)*\arctan((1/2 \\ & +1/2*I)*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})-9*I*B*((1+\cos(d*x+c))/ \\ & \sin(d*x+c))^{(1/2)}*\cos(d*x+c)*\sin(d*x+c)-3*I*B*2^{(1/2)}*\cos(d*x+c)*\arctan((1/ \\ & 2+1/2*I)*((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)*2^{(1/2)}})-12*I*B*\cos(d*x+c)*\sin( \\ & d*x+c)*\ln(((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)-1})*\cos(d*x+c)^2*((I*\sin(d*x+c) \\ & )+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}/(2*I*\sin(d*x+c)*\cos(d*x+c)+2*\cos(d*x+c)^2 \\ & -1)/((1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}/\sin(d*x+c)^2/(\cos(d*x+c)/\sin(d*x+c)) \\ & ^{(3/2)} \end{aligned}$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(181) = 362.

time = 1.42, size = 782, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] 
$$-1/12*(3*\sqrt{1/2}*a^2*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)})*e^{(3*I*d*x + 3*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(a^2*d*e^{(2*I*d*x + 2*I*c)} - a^2*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} + I))$$

$x + 2Ic) - 1))\sqrt{(-IA^2 - 2AB + IB^2)/(a^3d^2)} + (A - IB)a e^{(I dx + Ic)} e^{(-I dx - Ic)/(IA + B)} - 3\sqrt{1/2} a^2 d \sqrt{(-IA^2 - 2AB + IB^2)/(a^3d^2)} e^{(3I dx + 3Ic)} \log(4(\sqrt{2}\sqrt{1/2})(a^2 d e^{(2I dx + 2Ic)} - a^2 d) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1)}) \sqrt{(-IA^2 - 2AB + IB^2)/(a^3d^2)} - (A - IB)a e^{(I dx + Ic)} e^{(-I dx - Ic)/(IA + B)} - 3a^2 d \sqrt{4IB^2/(a^3d^2)} e^{(3I dx + 3Ic)} \log(-16(3B a^2 e^{(2I dx + 2Ic)} - B a^2 + \sqrt{2})(I a^3 d e^{(3I dx + 3Ic)} - I a^3 d e^{(I dx + Ic)}) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1)}) \sqrt{4IB^2/(a^3d^2)}) e^{(-2I dx - 2Ic)/B} + 3a^2 d \sqrt{4IB^2/(a^3d^2)} e^{(3I dx + 3Ic)} \log(-16(3B a^2 e^{(2I dx + 2Ic)} - B a^2 + \sqrt{2})(-I a^3 d e^{(3I dx + 3Ic)} + I a^3 d e^{(I dx + Ic)}) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1)}) \sqrt{4IB^2/(a^3d^2)}) e^{(-2I dx - 2Ic)/B} + \sqrt{2}(2(2IA - 5B) e^{(4I dx + 4Ic)} - (5IA - 11B) e^{(2I dx + 2Ic)} + IA - B) \sqrt{a/(e^{(2I dx + 2Ic)} + 1)}) \sqrt{(I e^{(2I dx + 2Ic)} + I)/(e^{(2I dx + 2Ic)} - 1)}) e^{(-3I dx - 3Ic)/(a^2 d)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{3}{2}} \cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)\*\*(3/2)/(a+I\*a\*tan(dx+c))\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + dx))/((I\*a\*(tan(c + dx) - I))\*\*(3/2)\*cot(c + dx)\*\*(3/2)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)^(3/2)/(a+I\*a\*tan(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(dx + c) + A)/((I\*a\*tan(dx + c) + a)^(3/2)\*cot(dx + c)^(3/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + a \tan(c + dx) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2))  
,x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2))  
, x)
```

$$3.562 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=260

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{5/2}d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia \tan(c+dx))}$$

[Out]  $(1/8+1/8*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(5/2)}/d+1/60*(151*A+41*I*B)*\cot(d*x+c)^{(1/2)}/a^2/d/(a+I*a*\tan(d*x+c))^{(1/2)}-1/60*(317*A+67*I*B)*\cot(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^{(1/2)}/a^3/d+1/5*(A+I*B)*\cot(d*x+c)^{(1/2)}/d/(a+I*a*\tan(d*x+c))^{(5/2)}+1/30*(17*A+7*I*B)*\cot(d*x+c)^{(1/2)}/a/d/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi [A]**

time = 0.64, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3677, 3679, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} - \frac{(317A+67iB)\sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}}{60a^2d} + \frac{(151A+41iB)\sqrt{\cot(c+dx)}}{60a^2d\sqrt{a+ia \tan(c+dx)}} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia \tan(c+dx))^{3/2}} + \frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c+d*x])^{(3/2)}*(A+B*\operatorname{Tan}[c+d*x])]/(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}, x]$

[Out]  $((1/8+I/8)*(A-I*B)*\operatorname{ArcTanh}(((1+I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(a^{(5/2)}*d) + (A+I*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]/(5*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}) + ((17*A+(7*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/(30*a*d*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}) + ((151*A+(41*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/(60*a^2*d*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]) - ((317*A+(67*I)*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])/(60*a^3*d)$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

**Rule 211**

$\operatorname{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 3625

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*tan[(e_)
+ (f_)*(x_)]], x_Symbol] := Dist[-2*a*(b/f), Subst[Int[1/(a*c - b*d - 2*a
^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]
```

Rule 3677

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]
```

Rule 3679

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps



$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{\frac{5}{2}}} dx &= \left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right) \int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{\frac{5}{2}}} dx \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{\frac{5}{2}}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right) \int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{\frac{5}{2}}} dx}{5a^2} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{\frac{5}{2}}} + \frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{\frac{3}{2}}} + \frac{\left(\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\right) \int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{\frac{5}{2}}} dx}{60a} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{\frac{5}{2}}} + \frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{\frac{3}{2}}} + \frac{(151A+107iB)\sqrt{\cot(c+dx)}}{60a} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{\frac{5}{2}}} + \frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{\frac{3}{2}}} + \frac{(151A+107iB)\sqrt{\cot(c+dx)}}{60a} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{\frac{5}{2}}} + \frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{\frac{3}{2}}} + \frac{(151A+107iB)\sqrt{\cot(c+dx)}}{60a} \\
&= \frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{\frac{5}{2}}} + \frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{\frac{3}{2}}} + \frac{(151A+107iB)\sqrt{\cot(c+dx)}}{60a} \\
&= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)(iA+B)\tanh^{-1}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}}{a^{\frac{5}{2}}d}
\end{aligned}$$

### Mathematica [A]

time = 5.14, size = 200, normalized size = 0.77

$$\frac{\cot^3(c+dx)\sec(c+dx)\left(-20(-17A-4iB+(23A+4iB)\cos(2(c+dx)))\csc(c+dx)+15(A-iB)e^{2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)\csc(2(c+dx))+(-149iA+19B+(-466iA+86B)\cos(2(c+dx)))\sec(c+dx)\right)}{60a^2d(i+\cot(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] (Cot[c + d\*x]^(3/2)\*Sec[c + d\*x]\*(-20\*(-17\*A - (4\*I)\*B + (23\*A + (4\*I)\*B)\*Cos[2\*(c + d\*x)])\*Csc[c + d\*x] + 15\*(A - I\*B)\*E^((2\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*ArcTanh[E^(I\*(c + d\*x))/Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Csc[2\*(c + d\*x)] + ((-149\*I)\*A + 19\*B + ((-466\*I)\*A + 86\*B)\*Cos[2\*(c + d\*x)])

x]])\*Sec[c + d\*x]))/(60\*a^2\*d\*(I + Cot[c + d\*x])^2\*sqrt[a + I\*a\*Tan[c + d\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(210) = 420.  
time = 68.52, size = 764, normalized size = 2.94

method	result
default	$\left(\frac{1}{120} + \frac{i}{120}\right) \sin(dx+c) \left(\frac{\cos(dx+c)}{\sin(dx+c)}\right)^{\frac{3}{2}} \sqrt{\frac{(i \sin(dx+c) + \cos(dx+c))a}{\cos(dx+c)}} \left(41B \cos(dx+c) \sin(dx+c) - 151A \cos(dx+c) \sin(dx+c) - 317A - \dots\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x,method=\_RE  
TURNVERBOSE)

[Out] (1/120+1/120\*I)/d/a^3\*sin(d\*x+c)\*(cos(d\*x+c)/sin(d\*x+c))^(3/2)\*((I\*sin(d\*x+c)+cos(d\*x+c))\*a/cos(d\*x+c))^(1/2)\*(41\*B\*cos(d\*x+c)\*sin(d\*x+c)-151\*A\*cos(d\*x+c)\*sin(d\*x+c)-317\*A-67\*B+32\*A\*cos(d\*x+c)^4-8\*B\*cos(d\*x+c)^4+27\*B\*cos(d\*x+c)^2+117\*A\*cos(d\*x+c)^2+48\*B\*cos(d\*x+c)^5\*sin(d\*x+c)-48\*A\*cos(d\*x+c)^5\*sin(d\*x+c)+48\*B\*cos(d\*x+c)^6+48\*A\*cos(d\*x+c)^6-56\*A\*cos(d\*x+c)^3\*sin(d\*x+c)+16\*B\*cos(d\*x+c)^3\*sin(d\*x+c)+15\*I\*A\*sin(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-15\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))+317\*I\*A-67\*I\*B-48\*I\*A\*cos(d\*x+c)^6+48\*I\*B\*cos(d\*x+c)^6-32\*I\*A\*cos(d\*x+c)^4-8\*I\*B\*cos(d\*x+c)^4-117\*I\*A\*cos(d\*x+c)^2+27\*I\*B\*cos(d\*x+c)^2+15\*I\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-15\*A\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*2^(1/2)\*cos(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+15\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*sin(d\*x+c)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))-48\*I\*B\*sin(d\*x+c)\*cos(d\*x+c)^5-56\*I\*A\*sin(d\*x+c)\*cos(d\*x+c)^3-16\*I\*B\*sin(d\*x+c)\*cos(d\*x+c)^3-151\*I\*A\*sin(d\*x+c)\*cos(d\*x+c)-41\*I\*B\*sin(d\*x+c)\*cos(d\*x+c)-48\*I\*A\*sin(d\*x+c)\*cos(d\*x+c)^5+15\*I\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2)\*cos(d\*x+c)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2)))/cos(d\*x+c)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")



[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(I\*a\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + a\*tan(c + d\*x)\*1i)^(5/2), x)

$$3.563 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=214

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{5/2}d} + \frac{A+iB}{5d\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}}$$

[Out] (1/8-1/8\*I)\*(A-I\*B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c)^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/a^(5/2)/d+1/60\*(67\*A-3\*I\*B)/a^2/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)+1/5\*(A+I\*B)/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(13\*A+3\*I\*B)/a/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.49, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {4326, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} + \frac{67A - 3iB}{60a^2d\sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{A+iB}{5d\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13A+3iB}{30ad\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + I\*a\*Tan[c + d\*x])^(5/2), x]

[Out] ((1/8 - I/8)\*(A - I\*B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(a^(5/2)\*d) + (A + I\*B)/(5\*d\*Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + (13\*A + (3\*I)\*B)/(30\*a\*d\*Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (67\*A - (3\*I)\*B)/(60\*a^2\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3625**

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && Ne
Q[c^2 + d^2, 0]

```

#### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m -
b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& LtQ[m, 0] && !GtQ[n, 0]

```

#### Rule 4326

```

Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^{5/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{5/2}} dx \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{\left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13A}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13B}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13A}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13B}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13A}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{A+iB}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} + \frac{13B}{30ad \sqrt{\cot(c+dx)}} \\
&= \frac{\left( \frac{1}{8} + \frac{i}{8} \right) (iA+B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)}}{a^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 3.79, size = 167, normalized size = 0.78

$$\frac{\cot^{\frac{3}{2}}(c+dx) \sec^2(c+dx) \left( \frac{30(A-iB)e^{3i(c+dx)} \tanh^{-1} \left( \frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}} \right)}{\sqrt{-1+e^{2i(c+dx)}}} + 2(19A+9iB+(86A+6iB)\cos(2(c+dx))+80iA\sin(2(c+dx))) \right)}{120a^2d(i+\cot(c+dx))^2\sqrt{a+ia \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2), x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Sec[c + d*x]^2*((30*(A - I*B)*E^((3*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]]/Sqrt[-1 + E^((2*I)*(c + d*x))] + 2*(19*A + (9*I)*B + (86*A + (6*I)*B)*Cos[2*(c + d*x)] + (80*I)*A*Sin[2*(c + d*x)])))/(120*a^2*d*(I + Cot[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal.  $1077$  vs.  $2(172) = 344$ .  
time = 65.53, size = 1078, normalized size = 5.04

method	result	size
default	Expression too large to display	1078

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & (-1/120+1/120*I)/d/a^3*(60*I*B*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) \\ & *2^{1/2}) *2^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) - 160*A * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} \\ & * \cos(d*x+c)^3 + 30*A * \sin(d*x+c) * 2^{1/2} * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) \\ & * \cos(d*x+c) + 3*B * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \sin(d*x+c) + 67*A * \sin(d*x+c) * \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} + 160*A * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \cos(d*x+c) - 60*A * \\ & \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) \\ & + 60*I*A * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c)^3 - \\ & 30*I*A * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c)^2 - \\ & 172*I*A * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) - 12*I*B * \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) - 45*I*A * \arctan((1/2+1/2*I)* \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c) - 15*I*B * \arctan((1/2+1/2*I)* \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \sin(d*x+c) + 160*I*A * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} \\ & * \cos(d*x+c)^3 - 160*I*A * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \cos(d*x+c) + 67*I*A * \\ & ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \sin(d*x+c) - 3*I*B * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} \\ & * \sin(d*x+c) + 15*A * \sin(d*x+c) * 2^{1/2} * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) \\ & * 2^{1/2} - 45*B * 2^{1/2} * \cos(d*x+c) * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * \\ & 2^{1/2} + 60*B * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c)^3 - \\ & 172*A * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} * \cos(d*x+c)^2 * \sin(d*x+c) + 12*B * ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} \\ & * \cos(d*x+c)^2 * \sin(d*x+c) + 15*I*A * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * \\ & 2^{1/2} * 2^{1/2} - 30*I*B * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c) * \\ & \sin(d*x+c) + 15*B * 2^{1/2} * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} - \\ & 30*B * 2^{1/2} * \arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2}) * 2^{1/2} * \cos(d*x+c)^2 * \\ & (\cos(d*x+c)/\sin(d*x+c))^{1/2} * ((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{1/2} / (4*I*\cos(d*x+c)^2 * \\ & \sin(d*x+c) + 4*\cos(d*x+c)^3 - I*\sin(d*x+c) - 3*\cos(d*x+c)) / ((-1+\cos(d*x+c))/\sin(d*x+c))^{1/2} \end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError





Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(I*a*tan(d*x + c) + a)^(5
/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{(a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2)
,x)
```

```
[Out] int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2)
, x)
```

$$3.564 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=216

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{5/2}d} + \frac{iA}{5d \sqrt{\cot(c+dx)}} (a$$

[Out]  $(-1/8-1/8*I)*(A-I*B)*\operatorname{arctanh}((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(5/2)}/d+1/60*(-3*I*A+13*B)/a^{2/d}/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(I*A-B)/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(5/2)}+1/30*(3*I*A+7*B)/a/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi** [A]

time = 0.49, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3676, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}} \right)}{a^{5/2}d} - \frac{-13B + 3iA}{60a^2d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{7B + 3iA}{30ad \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}} + \frac{-B + iA}{5d \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}), x]$

[Out]  $((-1/8 - I/8)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(a^{(5/2)}*d) + (I*A - B)/(5*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^{(5/2)}) + ((3*I)*A + 7*B)/(30*a*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + I*a*\operatorname{Tan}[c + d*x])^{(3/2)}) - ((3*I)*A - 13*B)/(60*a^2*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[c + d*x]])$

Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 211

$\operatorname{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]]/\operatorname{Sqrt}[(c_*) + (d_*)*\operatorname{tan}[(e_*) + (f_*)*(x_)]], x\_Symbol] \rightarrow \operatorname{Dist}[-2*a*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(a*c - b*d - 2*a$

```

^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

#### Rule 3676

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(- (A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)), x] + Dist[1/(2*a^2*m), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

```

#### Rule 3677

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(b*c - a*d))), x] + Dist[1/(2*a*m*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

```

#### Rule 4326

```

Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{5/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{5/2} \left( \sqrt{\cot(c + dx)} \right)} dx \\
&= \frac{iA - B}{5d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{5/2}} \\
&= \frac{iA - B}{5d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{5/2}} + \frac{1}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{5/2}} + \frac{1}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{5/2}} + \frac{1}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^{5/2}} + \frac{1}{30ad \sqrt{\cot(c + dx)}} \\
&= -\frac{\left( \frac{1}{8} - \frac{i}{8} \right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 4.15, size = 168, normalized size = 0.78

$$\frac{\cot^{\frac{3}{2}}(c + dx) \sec^2(c + dx) \left( -\frac{30i(A - iB)e^{3i(c + dx)} \tanh^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{-1 + e^{2i(c + dx)}}} \right)}{\sqrt{-1 + e^{2i(c + dx)}}} + 2(9iA + B + 2(3iA + 7B) \cos(2(c + dx)) + 20iB \sin(2(c + dx))) \right)}{120a^2d(i + \cot(c + dx))^2 \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)), x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Sec[c + d*x]^2*((( -30*I)*(A - I*B)*E^((3*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/Sqrt[-1 + E^((2*I)*(c + d*x))] + 2*((9*I)*A + B + 2*((3*I)*A + 7*B)*Cos[2*(c + d*x)] + (20*I)*B*Sin[2*(c + d*x)])))/(120*a^2*d*(I + Cot[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal.  $1091$  vs.  $2(174) = 348$ .  
time = 64.15, size = 1092, normalized size = 5.06

method	result	size
default	Expression too large to display	1092

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\begin{aligned} & (-1/120+1/120*I)/d/a^3*(60*I*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+ \\ & 1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+13*B*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}*\sin(d*x+c)-30*A*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x \\ & +c))^{(1/2)}*2^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2+15*A*2^{(1/2)}*\arctan((1/2+1/2*I)*(( \\ & -1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+3*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}-60*I*B*2^{(1/2)}*\cos(d*x+c)^3*\arctan((1/2+1/2*I)*((-1+\cos(d* \\ & x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-12*I*A*\sin(d*x+c)*\cos(d*x+c)^2*((-1+\cos(d* \\ & x+c))/\sin(d*x+c))^{(1/2)}-28*I*B*\sin(d*x+c)*\cos(d*x+c)^2*((-1+\cos(d*x+c))/\sin \\ & (d*x+c))^{(1/2)}+30*I*B*2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+ \\ & c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-15*I*A*\sin(d*x+c)*2^{(1/2)}*\arctan((1/2+1/2*I) \\ & *((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+45*I*B*2^{(1/2)}*\cos(d*x+c)*\arctan \\ & ((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+60*B*\sin(d*x+c)* \\ & 2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}* \\ & 2^{(1/2)})-30*I*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d \\ & *x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-15*B*2^{(1/2)}*\sin(d*x+c)*\arctan((1/2+1/2*I) \\ & )*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+40*B*\cos(d*x+c)*((-1+\cos(d*x+ \\ & c))/\sin(d*x+c))^{(1/2)}-40*B*\cos(d*x+c)^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}- \\ & 45*A*2^{(1/2)}*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/ \\ & 2)}*2^{(1/2)})-30*B*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1 \\ & /2)})*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)+12*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}* \\ & \cos(d*x+c)^2*\sin(d*x+c)-28*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^ \\ & 2*\sin(d*x+c)+60*A*2^{(1/2)}*\cos(d*x+c)^3*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/ \\ & \sin(d*x+c))^{(1/2)}*2^{(1/2)})-15*I*B*2^{(1/2)}*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c) \\ & ))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+40*I*B*\cos(d*x+c)^3*((-1+\cos(d*x+c))/\sin(d*x+ \\ & c))^{(1/2)}-3*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+13*I*B*\sin(d* \\ & x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-40*I*B*\cos(d*x+c)*((-1+\cos(d*x+c))/ \\ & \sin(d*x+c))^{(1/2)}*\cos(d*x+c)*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)} \\ & )/(4*I*\cos(d*x+c)^2*\sin(d*x+c)+4*\cos(d*x+c)^3-I*\sin(d*x+c)-3*\cos(d*x+c))/\sin \\ & (d*x+c)/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}/(\cos(d*x+c)/\sin(d*x+c))^{(1/2)} \end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs.  $2(162) = 324$ .

time = 1.18, size = 482, normalized size = 2.23

$$\frac{\left( \frac{1}{120} \sqrt{\frac{1}{2}} \sqrt{\frac{15 \sqrt{2} \sqrt{1/2} a^3 d \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)}}{a^5 d^2}} \log\left( \frac{-4 \sqrt{2} \sqrt{1/2} (I a^3 d e^{(2 I d x + 2 I c)} - I a^3 d) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)}}{1} \right) \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} + (A - I B) a e^{(I d x + I c)} e^{(-I d x - I c)} / (I A + B) \right) - 15 \sqrt{1/2} a^3 d \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} e^{(5 I d x + 5 I c)} \log\left( \frac{-4 \sqrt{2} \sqrt{1/2} (I a^3 d e^{(2 I d x + 2 I c)} - I a^3 d) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)}}{1} \right) \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} + (A - I B) a e^{(I d x + I c)} e^{(-I d x - I c)} / (I A + B) \right) + \sqrt{2} \left( (3 A - 17 I B) e^{(6 I d x + 6 I c)} + 2 (3 A + 8 I B) e^{(4 I d x + 4 I c)} - 2 (3 A - 2 I B) e^{(2 I d x + 2 I c)} - 3 A - 3 I B \right) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)} e^{(-5 I d x - 5 I c)} / (a^3 d) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{120} \sqrt{1/2} a^3 d \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} e^{(5 I d x + 5 I c)} \log\left( \frac{-4 \sqrt{2} \sqrt{1/2} (I a^3 d e^{(2 I d x + 2 I c)} - I a^3 d) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)}}{1} \right) \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} + (A - I B) a e^{(I d x + I c)} e^{(-I d x - I c)} / (I A + B) \right) - 15 \sqrt{1/2} a^3 d \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} e^{(5 I d x + 5 I c)} \log\left( \frac{-4 \sqrt{2} \sqrt{1/2} (I a^3 d e^{(2 I d x + 2 I c)} - I a^3 d) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)}}{1} \right) \sqrt{(I A^2 + 2 A B - I B^2) / (a^5 d^2)} + (A - I B) a e^{(I d x + I c)} e^{(-I d x - I c)} / (I A + B) \right) + \sqrt{2} \left( (3 A - 17 I B) e^{(6 I d x + 6 I c)} + 2 (3 A + 8 I B) e^{(4 I d x + 4 I c)} - 2 (3 A - 2 I B) e^{(2 I d x + 2 I c)} - 3 A - 3 I B \right) \sqrt{a / (e^{(2 I d x + 2 I c)} + 1)} \sqrt{(I e^{(2 I d x + 2 I c)} + I) / (e^{(2 I d x + 2 I c)} - 1)} e^{(-5 I d x - 5 I c)} / (a^3 d)$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)/((I*a*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x +
c))), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2))
,x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2))
, x)
```



$$3.565 \quad \int \frac{A+B \tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=214

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (iA + B) \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{5/2}d} + \frac{iA - B}{5d \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}}$$

[Out] (1/8+1/8\*I)\*(I\*A+B)\*arctanh((1+I)\*a^(1/2)\*tan(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/a^(5/2)/d+1/60\*(13\*A-37\*I\*B)/a^2/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(1/2)+1/5\*(I\*A-B)/d/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(5/2)+1/30\*(A+11\*I\*B)/a/d/cot(d\*x+c)^(1/2)/(a+I\*a\*tan(d\*x+c))^(3/2)

**Rubi** [A]

time = 0.49, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4326, 3676, 3677, 12, 3625, 211}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (B + iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{(1+i)\sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2}d} + \frac{13A - 37iB}{60a^2d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{-B + iA}{5d \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{5/2}} + \frac{A + 11iB}{30ad \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/((Cot[c + d\*x])^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out] ((1/8 + I/8)\*(I\*A + B)\*ArcTanh[((1 + I)\*Sqrt[a]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*a\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(a^(5/2)\*d) + (I\*A - B)/(5\*d\*Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)) + (A + (11\*I)\*B)/(30\*a\*d\*Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^(3/2)) + (13\*A - (37\*I)\*B)/(60\*a^2\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + I\*a\*Tan[c + d\*x]])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3625

Int[Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a

$^2*x^2), x], x, \text{Sqrt}[c + d*\text{Tan}[e + f*x]]/\text{Sqrt}[a + b*\text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3676

$\text{Int}[(a_ + (b_)*\text{tan}[(e_ + (f_)*(x_)]))^m*((A_ + (B_)*\text{tan}[(e_ + (f_)*(x_)]))*(c_ + (d_)*\text{tan}[(e_ + (f_)*(x_)]))^n), x\_Symbol] :> \text{Simp}[(-A*b - a*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^n/(2*a*f*m)), x] + \text{Dist}[1/(2*a^2*m), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^{n-1}*\text{Simp}[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a*A*(m + n))*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

#### Rule 3677

$\text{Int}[(a_ + (b_)*\text{tan}[(e_ + (f_)*(x_)]))^m*((A_ + (B_)*\text{tan}[(e_ + (f_)*(x_)]))*(c_ + (d_)*\text{tan}[(e_ + (f_)*(x_)]))^n), x\_Symbol] :> \text{Simp}[(a*A + b*B)*(a + b*\text{Tan}[e + f*x])^m*((c + d*\text{Tan}[e + f*x])^{n+1}/(2*f*m*(b*c - a*d))), x] + \text{Dist}[1/(2*a*m*(b*c - a*d)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m - b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*\text{Tan}[e + f*x], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && !GtQ[n, 0]

#### Rule 4326

$\text{Int}[(\text{cot}[(a_ + (b_)*(x_)]*(c_))]^m*(u_), x\_Symbol] :> \text{Dist}[(c*\text{Cot}[a + b*x])^m*(c*\text{Tan}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Tan}[a + b*x])^m, x], x] /;$  FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{5/2}} dx \\
&= \frac{iA - B}{5d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right)}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{iA - B}{5d \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{30ad \sqrt{\cot(c + dx)}} \\
&= \frac{\left( \frac{1}{8} + \frac{i}{8} \right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 4.22, size = 169, normalized size = 0.79

$$\frac{\cot^{\frac{3}{2}}(c + dx) \sec^2(c + dx) \left( 2A + 22iB - \frac{30(A - iB)e^{3i(c + dx)} \tanh^{-1} \left( \frac{e^{i(c + dx)}}{\sqrt{-1 + e^{2i(c + dx)}}} \right)}{\sqrt{-1 + e^{2i(c + dx)}}} + 4(7A - 13iB) \cos(2(c + dx)) + 40(iA + B) \sin(2(c + dx)) \right)}{120a^2d(i + \cot(c + dx))^2 \sqrt{a + ia \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)), x]
```

```
[Out] (Cot[c + d*x]^(3/2)*Sec[c + d*x]^2*(2*A + (22*I)*B - (30*(A - I*B)*E^((3*I)*(c + d*x))*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/Sqrt[-1 + E^((2*I)*(c + d*x))] + 4*(7*A - (13*I)*B)*Cos[2*(c + d*x)] + 40*(I*A + B)*Sin[2*(c + d*x)])/(120*a^2*d*(I + Cot[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1211 vs.  $2(172) = 344$ .  
time = 73.49, size = 1212, normalized size = 5.66

method	result	size
default	Expression too large to display	1212

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RE  
TURNVERBOSE)`

[Out]  $(1/120+1/120*I)/d/a^3*(60*I*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-40*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3+37*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\sin(d*x+c)+13*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+40*I*A*\cos(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+37*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-30*A*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^2+15*A*2^{(1/2)}*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-13*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}+40*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)-60*I*B*2^{(1/2)}*\cos(d*x+c)^3*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+30*I*B*2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-15*I*A*\sin(d*x+c)*2^{(1/2)}*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+45*I*B*2^{(1/2)}*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+60*B*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)^2*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-30*I*A*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-28*I*A*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2-52*I*B*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2-15*B*2^{(1/2)}*\sin(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+40*B*\cos(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-40*B*\cos(d*x+c)^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-45*A*2^{(1/2)}*\cos(d*x+c)*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})*2^{(1/2)}*\sin(d*x+c)*2^{(1/2)}*\cos(d*x+c)+28*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-52*B*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+60*A*2^{(1/2)}*\cos(d*x+c)^3*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})-40*I*A*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^3-15*I*B*2^{(1/2)}*\arctan((1/2+1/2*I)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*2^{(1/2)})+40*I*B*\cos(d*x+c)^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}-40*I*B*\cos(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*((I*\sin(d*x+c)+\cos(d*x+c))*a/\cos(d*x+c))^{(1/2)}/(4*I*\cos(d*x+c)^2*\sin(d*x+c)+4*\cos(d*x+c)^3-I*\sin(d*x+c)-3*\cos(d*x+c))/\sin(d*x+c)^2/((\cos(d*x+c)/\sin(d*x+c))^{(3/2)}/((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)})$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, alg  
orithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und  
efined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than  
twice the leaf count of optimal. 482 vs. 2(160) = 320.

time = 1.22, size = 482, normalized size = 2.25

$$\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}\right) \log\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}}{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}}}\right) - 15\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}} \left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}}{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}}}\right) - \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}} \left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}}{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}{a^3d\sqrt{(-IA^2-2AB+IB^2)/(a^5d^2)}}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, alg  
orithm="fricas")

[Out]  $-1/120*(15*\sqrt{1/2}*a^3*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2)})*e^{(5*I*d*x + 5*I*c)}*\log(-4*(\sqrt{2}*\sqrt{1/2}*(a^3*d*e^{(2*I*d*x + 2*I*c)} - a^3*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2)}) + (A - I*B)*a*e^{(I*d*x + I*c)}*e^{(-I*d*x - I*c)/(I*A + B)} - 15*\sqrt{1/2}*a^3*d*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2)})*e^{(5*I*d*x + 5*I*c)}*\log(4*(\sqrt{2}*\sqrt{1/2}*(a^3*d*e^{(2*I*d*x + 2*I*c)} - a^3*d)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)}*\sqrt{(-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2)}) - (A - I*B)*a*e^{(I*d*x + I*c)}*e^{(-I*d*x - I*c)/(I*A + B)} - \sqrt{2}*((-17*I*A - 23*B)*e^{(6*I*d*x + 6*I*c)} - 2*(-8*I*A - 17*B)*e^{(4*I*d*x + 4*I*c)} - 2*(-2*I*A + 7*B)*e^{(2*I*d*x + 2*I*c)} - 3*I*A + 3*B)*\sqrt{a/(e^{(2*I*d*x + 2*I*c)} + 1)}*\sqrt{(I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1)})*e^{(-5*I*d*x - 5*I*c)/(a^3*d)}$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(3/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)), x)

$$3.566 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+ia \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=289

$$\frac{2\sqrt[4]{-1} B \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{a^{5/2} d} + \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + \frac{2\sqrt{-1} B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) - \frac{-7B+iA}{4a^2 d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{A+3iB}{6a d \cot^2(c+dx) (a+ia \tan(c+dx))^{3/2}} + \frac{-B+iA}{5d \cot^2(c+dx) (a+ia \tan(c+dx))^{3/2}}}{a^{5/2} d}$$

[Out]  $2*(-1)^{(1/4)}*B*\arctan((-1)^{(3/4)}*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(5/2)}/d+(1/8+1/8*I)*(A-I*B)*\arctanh((1+I)*a^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/a^{(5/2)}/d+1/4*(-I*A+7*B)/a^2/d/\cot(d*x+c)^{(1/2)}/(a+I*a*\tan(d*x+c))^{(1/2)}+1/5*(I*A-B)/d/\cot(d*x+c)^{(5/2)}/(a+I*a*\tan(d*x+c))^{(5/2)}+1/6*(A+3*I*B)/a/d/\cot(d*x+c)^{(3/2)}/(a+I*a*\tan(d*x+c))^{(3/2)}$

**Rubi** [A]

time = 0.67, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$ , Rules used = {4326, 3676, 3682, 3625, 211, 3680, 65, 223, 209}

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) (A - iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{(1+i) \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right) + \frac{2\sqrt{-1} B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{(-1)^{3/4} \sqrt{a} \sqrt{\tan(c+dx)}}{\sqrt{a+ia \tan(c+dx)}}\right)}{a^{5/2} d} - \frac{-7B+iA}{4a^2 d \sqrt{\cot(c+dx)} \sqrt{a+ia \tan(c+dx)}} + \frac{A+3iB}{6a d \cot^2(c+dx) (a+ia \tan(c+dx))^{3/2}} + \frac{-B+iA}{5d \cot^2(c+dx) (a+ia \tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out]  $(2*(-1)^{(1/4)}*B*\operatorname{ArcTan}((-1)^{(3/4)}*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(a^{(5/2)}*d) + ((1/8 + I/8)*(A - I*B)*\operatorname{ArcTanh}(((1 + I)*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/(a^{(5/2)}*d) + (I*A - B)/(5*d*\operatorname{Cot}[c+d*x]^{(5/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^{(5/2)}) + (A + (3*I)*B)/(6*a*d*\operatorname{Cot}[c+d*x]^{(3/2)}*(a+I*a*\operatorname{Tan}[c+d*x])^{(3/2)}) - (I*A - 7*B)/(4*a^2*d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[c+d*x]])$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^n, x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 209**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3625

Int[Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*a\*(b/f), Subst[Int[1/(a\*c - b\*d - 2\*a^2\*x^2), x], x, Sqrt[c + d\*Tan[e + f\*x]]/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3676

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(- (A\*b - a\*B)) \* (a + b\*Tan[e + f\*x])^m \* ((c + d\*Tan[e + f\*x])^n / (2\*a\*f\*m)), x] + Dist[1/(2\*a^2\*m), Int[(a + b\*Tan[e + f\*x])^(m+1) \* (c + d\*Tan[e + f\*x])^(n-1) \* Simp[A\*(a\*c\*m + b\*d\*n) - B\*(b\*c\*m + a\*d\*n) - d\*(b\*B\*(m-n) - a\*A\*(m+n)) \* Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]

### Rule 3680

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m-1) \* (c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A\*b + a\*B, 0]

### Rule 3682

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A\*b + a\*B)/b, Int[(a + b\*Tan[e + f\*x])^m \* (c + d\*Tan[e + f\*x])^n, x], x] - Dist[B/b, Int[(a + b\*Tan[e + f\*x])^m \* (c + d\*Tan[e + f\*x])^n \* (a - b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A\*b + a\*B, 0]



## Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

## Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{5}{2}}(c + dx)(A + B \tan(c + dx))}{(a + ia \tan(c + dx))^{5/2}} dx \\
&= \frac{iA - B}{5d \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right)}{6ad \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{iA - B}{5d \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{6ad \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{iA - B}{5d \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{6ad \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{iA - B}{5d \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{6ad \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{iA - B}{5d \cot^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} + \frac{A + B \tan(c + dx)}{6ad \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} \\
&= \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{5/2}d} \\
&= \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (iA + B) \tanh^{-1} \left( \frac{(1+i)\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{5/2}d} \\
&= \frac{2\sqrt{-1} B \tan^{-1} \left( \frac{(-1)^{3/4}\sqrt{a} \sqrt{\tan(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{a^{5/2}d}
\end{aligned}$$

**Mathematica** [A]

time = 5.70, size = 426, normalized size = 1.47

$$\frac{e^{-30i\sqrt{a^2+d^2}}(34+35B-144B^{3/2}d-24B^2d^{3/2}+34B^3d^2+144B^4d^{3/2}-234B^5d^2-123B^6d^{3/2}+15(A-ID)^2\sqrt{a^2+d^2}\log\left(\frac{e^{i(c+dx)}+1}{e^{i(c+dx)}-1}\right)+30\sqrt{2}B^2d^{3/2}\sqrt{1+e^{2i(c+dx)}}\log\left(\frac{1-3e^{2i(c+dx)}}{2\sqrt{e^{2i(c+dx)}-1+e^{2i(c+dx)}}}\right)-30\sqrt{2}B^2d^{3/2}\sqrt{1+e^{2i(c+dx)}}\log\left(\frac{1-3e^{2i(c+dx)}}{2\sqrt{e^{2i(c+dx)}-1+e^{2i(c+dx)}}}\right))\sec^{5/2}(c+dx)(A+B\tan(c+dx))}{120(A\cos(c+dx)+B\sin(c+dx))(a+\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^(5/2)), x]

[Out] (Sqrt[Cot[c + d\*x]]\*(3\*A + (3\*I)\*B - 14\*A\*E^((2\*I)\*(c + d\*x)) - (24\*I)\*B\*E^((2\*I)\*(c + d\*x)) + 34\*A\*E^((4\*I)\*(c + d\*x)) + (144\*I)\*B\*E^((4\*I)\*(c + d\*x)) - 23\*A\*E^((6\*I)\*(c + d\*x)) - (123\*I)\*B\*E^((6\*I)\*(c + d\*x)) + 15\*(A - I\*B)\*E^((5\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Log[E^(I\*(c + d\*x)) + Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] + (30\*I)\*Sqrt[2]\*B\*E^((5\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Log[1 - 3\*E^((2\*I)\*(c + d\*x)) - 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]] - (30\*I)\*Sqrt[2]\*B\*E^((5\*I)\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]\*Log[1 - 3\*E^((2\*I)\*(c + d\*x)) + 2\*Sqrt[2]\*E^(I\*(c + d\*x))\*Sqrt[-1 + E^((2\*I)\*(c + d\*x))]])\*Sec[c + d\*x]^2\*(A + B\*Tan[c + d\*x])/(120\*d\*E^((3\*I)\*(c + d\*x))\*(A\*Cos[c + d\*x] + B\*Sin[c + d\*x])\*(a + I\*a\*Tan[c + d\*x])^(5/2))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2157 vs. 2(230) = 460.

time = 69.96, size = 2158, normalized size = 7.47

method	result	size
default	Expression too large to display	2158

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x,method=\_RETURNVERBOSE)

[Out] (-1/120-1/120\*I)/d/a^3\*(60\*I\*B\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*2^(1/2)\*cos(d\*x+c)^2\*sin(d\*x+c)+40\*A\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*cos(d\*x+c)^3-60\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+I)\*sin(d\*x+c)-60\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+1)\*sin(d\*x+c)+60\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-I)\*sin(d\*x+c)+60\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-1)\*sin(d\*x+c)-60\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+I)-60\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+1)+60\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-I)+60\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-1)+30\*A\*sin(d\*x+c)\*2^(1/2)\*arctan((1/2+1/2\*I)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*2^(1/2))\*cos(d\*x+c)+147\*B\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)\*sin(d\*x+c)+40\*I\*A\*cos(d\*x+c)\*((-1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-120\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-1)\*cos(d\*x+c)^2+120\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+I)\*cos(d\*x+c)^2+120\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)+1)\*cos(d\*x+c)^2-120\*I\*B\*ln(((1+cos(d\*x+c))/sin(d\*x+c))^(1/2)-I)\*cos(d\*x+c)^2+

$$\begin{aligned}
& 240*B*\ln\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+I*\cos(dx+c)^2*\sin(dx+c)+240* \\
& B*\ln\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+1*\cos(dx+c)^2*\sin(dx+c)-240*B*\ln \\
& \left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-I*\cos(dx+c)^2*\sin(dx+c)-37*A*\sin(dx \\
& +c)*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-40*A*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1 \\
& /2)*\cos(dx+c)-60*A*\arctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2 \\
& ^{1/2})*\cos(dx+c)^2*\sin(dx+c)+60*I*A*\arctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2 \\
& ^{1/2})*\cos(dx+c)^3-30*I*A*\arctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2 \\
& ^{1/2})*\cos(dx+c)^2-45* \\
& I*A*\arctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})* \\
& \cos(dx+c)-15*I*B*\arctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})* \\
& \sin(dx+c)+252*I*B*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}*\cos(dx \\
& +c)^2*\sin(dx+c)-240*B*\cos(dx+c)*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+240*B* \\
& \cos(dx+c)^3*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-37*I*A*\left(\frac{-1+\cos(dx+c)}{\sin \\
& (dx+c)}\right)^{1/2}*\sin(dx+c)-240*I*B*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}*\cos(dx \\
& x+c)-147*I*B*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}*\sin(dx+c)-180*I*B*\ln\left(\left(\frac{-1+ \\
& \cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-1\right)*\cos(dx+c)+52*I*A*\left(\frac{-1+\cos(dx+c)}{\sin(dx \\
& +c)}\right)^{1/2}*\cos(dx+c)^2*\sin(dx+c)+15*A*\sin(dx+c)*2^{1/2}*\arctan\left(\frac{1/2+1/2* \\
& I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})-45*B*2^{1/2}*\cos(dx+c)*\arct \\
& an\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})+60*B*\arctan\left(\frac{1/2 \\
& +1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})*2^{1/2}*\cos(dx+c)^3+52 \\
& *A*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}*\cos(dx+c)^2*\sin(dx+c)-252*B*\left(\frac{-1+co \\
& s(dx+c)}{\sin(dx+c)}\right)^{1/2}*\cos(dx+c)^2*\sin(dx+c)+15*I*A*\arctan\left(\frac{1/2+1/2* \\
& I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})*2^{1/2}-30*I*B*\arctan\left(\frac{1/2+1 \\
& /2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})*2^{1/2}*\cos(dx+c)*\sin(dx \\
& x+c)-40*I*A*\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}*\cos(dx+c)^3+15*B*2^{1/2}*\ar \\
& ctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})-30*B*2^{1/2})*a \\
& rctan\left(\frac{1/2+1/2*I}{\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}}\right)*2^{1/2})*\cos(dx+c)^2- \\
& 240*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-1\right)*\cos(dx+c)^2*\sin(dx+c)-120* \\
& B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+I\right)*\cos(dx+c)*\sin(dx+c)-120*B*\ln\left(\left(\frac{-1+co \\
& s(dx+c)}{\sin(dx+c)}\right)^{1/2}-I\right)*\cos(dx+c)*\sin(dx+c)+120*B*\ln\left(\left(\frac{-1+\cos(dx+ \\
& c)}{\sin(dx+c)}\right)^{1/2}-1\right)*\cos(dx+c)*\sin(dx+c)+240*I*B*\left(\frac{-1+\cos(dx+c)}{\sin \\
& (dx+c)}\right)^{1/2}*\cos(dx+c)^3+240*I*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-1 \\
& \right)*\cos(dx+c)^3+240*I*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}-I\right)*\cos(dx+c)^ \\
& 3+180*I*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+I\right)*\cos(dx+c)+180*I*B*\ln\left(\left(\frac{- \\
& -1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+1\right)*\cos(dx+c)-180*I*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin \\
& (dx+c)}\right)^{1/2}-I\right)*\cos(dx+c)-240*I*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2} \\
& +I\right)*\cos(dx+c)^3-240*I*B*\ln\left(\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}+1\right)*\cos(dx \\
& +c)^3)*\cos(dx+c)^3*(I*\sin(dx+c)+\cos(dx+c))*a/\cos(dx+c)^{1/2}/(4*I*\cos \\
& (dx+c)^2*\sin(dx+c)+4*\cos(dx+c)^3-I*\sin(dx+c)-3*\cos(dx+c))/(\cos(dx+c)/ \\
& \sin(dx+c))^{5/2}/\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)^{1/2}/\sin(dx+c)^3
\end{aligned}$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs.  $2(217) = 434$ .

time = 1.28, size = 799, normalized size = 2.76



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d
*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*B)*a
*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d*sqrt((I*
A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/
2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2
+ 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
/(I*A + B)) + 30*a^3*d*sqrt(-4*I*B^2/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-16
*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c)
- a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I*B^2/(a^5*d^2)))*e^(-2
*I*d*x - 2*I*c)/B) - 30*a^3*d*sqrt(-4*I*B^2/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*
log(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 - sqrt(2)*(a^4*d*e^(3*I*d*x +
3*I*c) - a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I*B^2/(a^5*d^2)
))*e^(-2*I*d*x - 2*I*c)/B) + sqrt(2)*((23*A + 123*I*B)*e^(6*I*d*x + 6*I*c) -
2*(17*A + 72*I*B)*e^(4*I*d*x + 4*I*c) + 2*(7*A + 12*I*B)*e^(2*I*d*x + 2*I*
c) - 3*A - 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(5/2)/(a+I\*a\*tan(d\*x+c))\*\*(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+I\*a\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((I\*a\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(5/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + a \tan(c + dx) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + a\*tan(c + d\*x)\*1i)^(5/2)), x)

### 3.567 $\int \cot^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=179

$$\frac{(A - iB)F_1(1 - m; 1 - n, 1; 2 - m; -i \tan(c + dx), i \tan(c + dx)) \cot^{-1+m}(c + dx)(1 + i \tan(c + dx))^{-n}(a + i \tan(c + dx))^n}{d(1 - m)}$$

[Out] (A-I\*B)\*AppellF1(1-m,1-n,1,2-m,-I\*tan(d\*x+c),I\*tan(d\*x+c))\*cot(d\*x+c)^(-1+m)\*(a+I\*a\*tan(d\*x+c))^n/d/(1-m)/((1+I\*tan(d\*x+c))^n+I\*B\*cot(d\*x+c)^(-1+m)\*hypergeom([1-m, 1-n],[2-m],-I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/(1-m)/((1+I\*tan(d\*x+c))^n))

**Rubi [A]**

time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {4326, 3682, 3645, 140, 138, 3680, 68, 66}

$$\frac{(A - iB) \cot^{m-1}(c + dx)(1 + i \tan(c + dx))^{-n}(a + i \tan(c + dx))^n F_1(1 - m; 1 - n, 1; 2 - m; -i \tan(c + dx), i \tan(c + dx))}{d(1 - m)} + \frac{iB \cot^{m-1}(c + dx)(1 + i \tan(c + dx))^{-n}(a + i \tan(c + dx))^n {}_2F_1(1 - m, 1 - n; 2 - m; -i \tan(c + dx))}{d(1 - m)}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A - I\*B)\*AppellF1[1 - m, 1 - n, 1, 2 - m, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x])\*Cot[c + d\*x]^(-1 + m)\*(a + I\*a\*Tan[c + d\*x])^n/(d\*(1 - m)\*(1 + I\*Tan[c + d\*x])^n) + (I\*B\*Cot[c + d\*x]^(-1 + m)\*Hypergeometric2F1[1 - m, 1 - n, 2 - m, (-I)\*Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*(1 - m)\*(1 + I\*Tan[c + d\*x])^n)

**Rule 66**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[c^n\*((b\*x)^(m + 1)/(b\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0]))

**Rule 68**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

**Rule 138**

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

#### Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] := Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

#### Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

#### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

#### Rubi steps

$$\begin{aligned}
\int \cot^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx &= (\cot^m(c+dx) \tan^m(c+dx)) \int \tan^{-m}(c+dx) dx \\
&= -\left( ((-A+iB) \cot^m(c+dx) \tan^m(c+dx)) \right) \\
&\quad (ia^2(-A+iB) \cot^m(c+dx) \tan^m(c+dx)) \\
&= -\frac{(ia(-A+iB) \cot^m(c+dx)(1+i \tan(c+dx)))}{(A-iB)F_1(1-m; 1-n, 1; 2-m; -i \tan(c+dx))}
\end{aligned}$$

**Mathematica [F]**

time = 12.05, size = 0, normalized size = 0.00

$$\int \cot^m(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Cot[c + d\*x]^m\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 3.79, size = 0, normalized size = 0.00

$$\int (\cot^m(dx+c))(a+ia \tan(dx+c))^n(A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(cot(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")



[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^m, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(((A - I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c) / (e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))^m/(e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \cot^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*m\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*n\*(A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*m, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^m\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^m, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^m (A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(cot(c + d\*x)^m\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

$$3.568 \quad \int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=247

$$\frac{2(3B + 2iAn)\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^n}{3d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^n}{3d} - \frac{2(A-iB)F_1\left(\frac{1}{2}, 1-n, 1, \frac{3}{2}, -I \tan(c+dx)\right)}{3d}$$

[Out]  $-2/3*A*\cot(d*x+c)^{(3/2)}*(a+I*a*\tan(d*x+c))^n/d-2/3*(3*B+2*I*A*n)*\cot(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d-2*(A-I*B)*\text{AppellF1}(1/2, 1-n, 1, 3/2, -I*\tan(d*x+c), I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/\cot(d*x+c)^{(1/2)}/((1+I*\tan(d*x+c))^n)-2/3*(1-2*n)*(3*I*B-2*A*n)*\text{hypergeom}([1/2, 1-n], [3/2], -I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/\cot(d*x+c)^{(1/2)}/((1+I*\tan(d*x+c))^n)$

**Rubi [A]**

time = 0.57, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {4326, 3679, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(A-iB)(1+i \tan(c+dx))^{-(a+ia \tan(c+dx))} F_1\left(\frac{1}{2}, 1-n, 1, \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}} - \frac{2(1-2n)[-2An+3iB](1+i \tan(c+dx))^{-(a+ia \tan(c+dx))} {}_2F_1\left(\frac{1}{2}, 1-n, \frac{3}{2}; -i \tan(c+dx)\right)}{3d\sqrt{\cot(c+dx)}} - \frac{2(3B+2iAn)\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^n}{3d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^n}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(-2*(3*B + (2*I)*A*n)*\text{Sqrt}[\text{Cot}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(3*d) - (2*A*\text{Cot}[c + d*x]^{(3/2)}*(a + I*a*\text{Tan}[c + d*x])^n)/(3*d) - (2*(A - I*B)*\text{AppellF1}[1/2, 1 - n, 1, 3/2, (-I)*\text{Tan}[c + d*x], I*\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(d*\text{Sqrt}[\text{Cot}[c + d*x]]*(1 + I*\text{Tan}[c + d*x])^n) - (2*(1 - 2*n)*((3*I)*B - 2*A*n)*\text{Hypergeometric2F1}[1/2, 1 - n, 3/2, (-I)*\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(3*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(1 + I*\text{Tan}[c + d*x])^n)$

Rule 66

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[c^n\*((b\*x)^(m+1)/(b\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

Rule 68

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

$a^2 + b^2, 0]$  && EqQ[A\*b + a\*B, 0]

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^n}{\sqrt{\cot(c+dx)}} dx \\
&= -\frac{2A \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^n}{3d} + \frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d} \\
&= -\frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d} \\
&= -\frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d} \\
&= -\frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d} \\
&= -\frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d} \\
&= -\frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d} \\
&= -\frac{2(3B+2iAn) \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n}{3d}
\end{aligned}$$

**Mathematica [F]**

time = 6.67, size = 0, normalized size = 0.00

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Cot[c + d\*x]^(5/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.62, size = 0, normalized size = 0.00

$$\int \left( \cot^{\frac{5}{2}}(dx+c) \right) (a+ia \tan(dx+c))^n (A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(dx+c)^{(5/2)}*(a+I*a*\tan(dx+c))^n*(A+B*\tan(dx+c)),x)$

[Out]  $\text{int}(\cot(dx+c)^{(5/2)}*(a+I*a*\tan(dx+c))^n*(A+B*\tan(dx+c)),x)$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\cot(dx+c)^{(5/2)}*(a+I*a*\tan(dx+c))^n*(A+B*\tan(dx+c)),x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((B*\tan(dx+c) + A)*(I*a*\tan(dx+c) + a)^n*\cot(dx+c)^{(5/2)}, x)$

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\cot(dx+c)^{(5/2)}*(a+I*a*\tan(dx+c))^n*(A+B*\tan(dx+c)),x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(-((A - I*B)*e^{(4*I*d*x + 4*I*c)} + 2*A*e^{(2*I*d*x + 2*I*c)} + A + I*B)*(2*a*e^{(2*I*d*x + 2*I*c)}/(e^{(2*I*d*x + 2*I*c)} + 1))^n*\text{sqrt}((I*e^{(2*I*d*x + 2*I*c)} + I)/(e^{(2*I*d*x + 2*I*c)} - 1))/(e^{(4*I*d*x + 4*I*c)} - 2*e^{(2*I*d*x + 2*I*c)} + 1), x)$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\cot(dx+c)**(5/2)*(a+I*a*\tan(dx+c))**n*(A+B*\tan(dx+c)),x)$

[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

$$3.569 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=194

$$\frac{2A\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^n}{d} + \frac{2(iA+B)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)(1+i \tan(c+dx))}{d\sqrt{\cot(c+dx)}}$$

[Out]  $-2A*\cot(d*x+c)^{(1/2)}*(a+I*a*\tan(d*x+c))^n/d+2*(I*A+B)*\text{AppellF1}(1/2, 1-n, 1, 3/2, -I*\tan(d*x+c), I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/\cot(d*x+c)^{(1/2)/((1+I*\tan(d*x+c))^n)-2*I*A*(1-2*n)*\text{hypergeom}([1/2, 1-n], [3/2], -I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/\cot(d*x+c)^{(1/2)/((1+I*\tan(d*x+c))^n)}$

**Rubi [A]**

time = 0.38, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {4326, 3679, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(B+iA)(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}} - \frac{2iA(1-2n)(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}} - \frac{2A\sqrt{\cot(c+dx)}(a+ia \tan(c+dx))^n}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^{(3/2)}*(a + I*a*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $(-2*A*\text{Sqrt}[\text{Cot}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/d + (2*(I*A + B)*\text{AppellF1}[1/2, 1 - n, 1, 3/2, (-I)*\text{Tan}[c + d*x], I*\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(d*\text{Sqrt}[\text{Cot}[c + d*x]]*(1 + I*\text{Tan}[c + d*x])^n) - ((2*I)*A*(1 - 2*n)*\text{Hypergeometric2F1}[1/2, 1 - n, 3/2, (-I)*\text{Tan}[c + d*x]]*(a + I*a*\text{Tan}[c + d*x])^n)/(d*\text{Sqrt}[\text{Cot}[c + d*x]]*(1 + I*\text{Tan}[c + d*x])^n)$

**Rule 66**

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*))^{(n_*)}, x\_Symbol] \rightarrow \text{Simp}[c^n*((b*x)^{(m+1)}/(b*(m+1)))*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x] /; \text{FreeQ}[\{b, c, d, m, n\}, x] \&\& \text{IntegerQ}[m] \&\& (\text{IntegerQ}[n] \parallel (\text{GtQ}[c, 0] \&\& \text{!}(\text{EqQ}[n, -2^{(-1)}]) \&\& \text{EqQ}[c^2 - d^2, 0] \&\& \text{GtQ}[-d/(b*c), 0]))$

**Rule 68**

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*))^{(n_*)}, x\_Symbol] \rightarrow \text{Dist}[c^{\text{IntPart}[n]}*((c + d*x)^{\text{FracPart}[n]/(1 + d*(x/c))^{\text{FracPart}[n]})}, \text{Int}[(b*x)^m*(1 + d*(x/c))^n, x], x] /; \text{FreeQ}[\{b, c, d, m, n\}, x] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& \text{!GtQ}[c, 0] \&\& \text{!GtQ}[-d/(b*c), 0] \&\& ((\text{RationalQ}[m] \&\& \text{!}(\text{EqQ}[n, -2^{(-1)}]) \&\& \text{EqQ}[c^2 - d^2, 0])) \parallel \text{!RationalQ}[n]$

**Rule 129**



```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

#### Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3679

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Dist[1/(a*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f
*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*m
+ a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

#### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx = \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^n}{d} + \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^n}{d} + \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^n}{d} + \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^n}{d} - \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^n}{d} - \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

$$= -\frac{2A \sqrt{\cot(c + dx)} (a + ia \tan(c + dx))^n}{d} + \int \frac{(a + ia \tan(c + dx))^n}{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} dx$$

Mathematica [F]

time = 12.89, size = 0, normalized size = 0.00

$$\int \cot^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Cot[c + d\*x]^(3/2)\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.60, size = 0, normalized size = 0.00

$$\int \left( \cot^{\frac{3}{2}}(dx + c) \right) (a + ia \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^(3/2), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral(((I\*A + B)\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A - B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c))/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))/(e^(2\*I\*d\*x + 2\*I\*c) - 1), x)

**Sympy [F(-1)]** Timed out  
 time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**  
 time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^(3/2), x)

**Mupad [F]**  
 time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n,x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n, x)

$$3.570 \quad \int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=158

$$\frac{2(A-iB)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right) (1+i \tan(c+dx))^{-n} (a+ia \tan(c+dx))^n}{d\sqrt{\cot(c+dx)}} + \frac{2iB}{d\sqrt{\cot(c+dx)}}$$

[Out] 2\*(A-I\*B)\*AppellF1(1/2,1-n,1,3/2,-I\*tan(d\*x+c),I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/cot(d\*x+c)^(1/2)/((1+I\*tan(d\*x+c))^n)+2\*I\*B\*hypergeom([1/2, 1-n],[3/2],-I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/cot(d\*x+c)^(1/2)/((1+I\*tan(d\*x+c))^n)

**Rubi [A]**

time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4326, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(A-iB)(1+i \tan(c+dx))^{-n} (a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}} + \frac{2iB(1+i \tan(c+dx))^{-n} (a+ia \tan(c+dx))^n {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] (2\*(A - I\*B)\*AppellF1[1/2, 1 - n, 1, 3/2, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*Sqrt[Cot[c + d\*x]]\*(1 + I\*Tan[c + d\*x])^n) + ((2\*I)\*B\*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)\*Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*Sqrt[Cot[c + d\*x]]\*(1 + I\*Tan[c + d\*x])^n)

**Rule 66**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[c^n\*((b\*x)^(m + 1)/(b\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

**Rule 68**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n, x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

**Rule 129**

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

#### Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

#### Rule 3682

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

#### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \\
 &= - \left( \left( (-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \right. \\
 &\quad \left. + \left( ia^2(-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \right) \\
 &= - \frac{\left( 2a^3(-A+iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{d \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} \\
 &= \frac{2iB {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right) (1+ia \tan(c+dx))^n (A+B \tan(c+dx))}{d \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} \\
 &= \frac{2(A-iB) {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right) (1+ia \tan(c+dx))^n (A+B \tan(c+dx))}{d \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}
 \end{aligned}$$

**Mathematica [F]**

time = 11.78, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(c+dx)} (a+ia \tan(c+dx))^n (A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

```
[Out] Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

**Maple [F]**

time = 0.65, size = 0, normalized size = 0.00

$$\int \left( \sqrt{\cot(dx+c)} \right) (a+ia \tan(dx+c))^n (A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
[Out] int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c) / (e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))/(e^(2*I*d*x + 2*I*c) + 1), x)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c + dx) - i))^n (A + B \tan(c + dx)) \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*sqrt(cot(c + d*x)), x)
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

$$3.571 \quad \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=215

$$\frac{2B(a+ia \tan(c+dx))^n}{d(1+2n)\sqrt{\cot(c+dx)}} - \frac{2(iA+B)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right) (1+i \tan(c+dx))^{-n}(a)}{d\sqrt{\cot(c+dx)}}$$

[Out] 2\*B\*(a+I\*a\*tan(d\*x+c))^n/d/(1+2\*n)/cot(d\*x+c)^(1/2)-2\*(I\*A+B)\*AppellF1(1/2, 1-n, 1, 3/2, -I\*tan(d\*x+c), I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/cot(d\*x+c)^(1/2)/((1+I\*tan(d\*x+c))^n)+2\*(2\*B\*n+I\*A\*(1+2\*n))\*hypergeom([1/2, 1-n], [3/2], -I\*tan(d\*x+c))\*(a+I\*a\*tan(d\*x+c))^n/d/(1+2\*n)/cot(d\*x+c)^(1/2)/((1+I\*tan(d\*x+c))^n)

**Rubi [A]**

time = 0.40, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {4326, 3678, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(B+iA)(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d\sqrt{\cot(c+dx)}} + \frac{2(2Bn+iA(2n+1))(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; -i \tan(c+dx)\right)}{d(2n+1)\sqrt{\cot(c+dx)}} + \frac{2B(a+ia \tan(c+dx))^n}{d(2n+1)\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]], x]

[Out] (2\*B\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*(1 + 2\*n)\*Sqrt[Cot[c + d\*x]]) - (2\*(I\*A + B)\*AppellF1[1/2, 1 - n, 1, 3/2, (-I)\*Tan[c + d\*x], I\*Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*Sqrt[Cot[c + d\*x]]\*(1 + I\*Tan[c + d\*x])^n) + (2\*(2\*B\*n + I\*A\*(1 + 2\*n))\*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)\*Tan[c + d\*x]]\*(a + I\*a\*Tan[c + d\*x])^n)/(d\*(1 + 2\*n)\*Sqrt[Cot[c + d\*x]]\*(1 + I\*Tan[c + d\*x])^n)

**Rule 66**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[c^n\*((b\*x)^(m+1)/(b\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

**Rule 68**

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Rule 3680

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

## Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

## Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

## Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n dx \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(1 + 2n) \sqrt{\cot(c + dx)}} + \frac{\left( 2\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n dx}{d(1 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(1 + 2n) \sqrt{\cot(c + dx)}} - \frac{\left( (iA + B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n dx}{d(1 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(1 + 2n) \sqrt{\cot(c + dx)}} - \frac{\left( ia^2(iA + B) \sqrt{\cot(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n dx}{d(1 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(1 + 2n) \sqrt{\cot(c + dx)}} + \frac{\left( 2a^3(iA + B) \sqrt{\cot(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^n dx}{d(1 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(1 + 2n) \sqrt{\cot(c + dx)}} + \frac{2(2Bn + iA(1 + 2n)) {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}; -\frac{a + ia \tan(c + dx)}{\cot(c + dx)}\right)}{d(1 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(1 + 2n) \sqrt{\cot(c + dx)}} - \frac{2(iA + B) F_1\left(\frac{1}{2}; 1 - n, 1; \frac{3}{2}; -\frac{a + ia \tan(c + dx)}{\cot(c + dx)}\right)}{d(1 + 2n) \sqrt{\cot(c + dx)}}
\end{aligned}$$

**Mathematica [F]**

time = 7.69, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification is not applicable to the result.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]],x]

[Out] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]], x]

**Maple** [F]

time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(dx + c))^n (A + B \tan(dx + c))}{\sqrt{\cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x)

[Out] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/sqrt(cot(d\*x + c)), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((( -I\*A - B)\*e^(4\*I\*d\*x + 4\*I\*c) + 2\*B\*e^(2\*I\*d\*x + 2\*I\*c) + I\*A - B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((I\*e^(2\*I\*d\*x

+ 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))/(e^(4\*I\*d\*x + 4\*I\*c) + 2\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^n (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I))\*\*n\*(A + B\*tan(c + d\*x))/sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) 1i)^n}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/cot(c + d\*x)^(1/2), x)

$$3.572 \quad \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=291

$$\frac{2B(a+ia \tan(c+dx))^n}{d(3+2n)\cot^{\frac{3}{2}}(c+dx)} - \frac{2(2iBn-A(3+2n))(a+ia \tan(c+dx))^n}{d(1+2n)(3+2n)\sqrt{\cot(c+dx)}} - \frac{2(A-iB)F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx)\right)}{d(2n+1)(2n+3)\sqrt{\cot(c+dx)}}$$

[Out]  $2*B*(a+I*a*\tan(d*x+c))^n/d/(3+2*n)/\cot(d*x+c)^{(3/2)}-2*(2*I*B*n-A*(3+2*n))*(a+I*a*\tan(d*x+c))^n/d/(4*n^2+8*n+3)/\cot(d*x+c)^{(1/2)}-2*(A-I*B)*\text{AppellF1}(1/2, 1-n, 1, 3/2, -I*\tan(d*x+c), I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/\cot(d*x+c)^{(1/2)}/((1+I*\tan(d*x+c))^n)+2*(2*A*n*(3+2*n)-I*B*(4*n^2+6*n+3))*\text{hypergeom}([1/2, 1-n], [3/2], -I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/(4*n^2+8*n+3)/\cot(d*x+c)^{(1/2)}/((1+I*\tan(d*x+c))^n)$

**Rubi [A]**

time = 0.61, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {4326, 3678, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(A-iB)(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; -i \tan(c+dx)\right)}{d \sqrt{\cot(c+dx)}} + \frac{2(2An(2n+3)-iB(4n^2+6n+3))(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n}{d(2n+1)(2n+3)\sqrt{\cot(c+dx)}} - \frac{2(-A(2n+3)+2iBn)(a+ia \tan(c+dx))^n}{d(2n+1)(2n+3)\sqrt{\cot(c+dx)}} + \frac{2B(a+ia \tan(c+dx))^n}{d(2n+3)\cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a+I*a*\text{Tan}[c+d*x])^n*(A+B*\text{Tan}[c+d*x])}{\text{Cot}[c+d*x]^{(3/2)}}, x]$

[Out]  $(2*B*(a+I*a*\text{Tan}[c+d*x])^n)/(d*(3+2*n)*\text{Cot}[c+d*x]^{(3/2)}) - (2*((2*I)*B*n - A*(3+2*n))*(a+I*a*\text{Tan}[c+d*x])^n)/(d*(1+2*n)*(3+2*n)*\text{Sqrt}[\text{Cot}[c+d*x]]) - (2*(A-I*B)*\text{AppellF1}[1/2, 1-n, 1, 3/2, (-I)*\text{Tan}[c+d*x], I*\text{Tan}[c+d*x]]*(a+I*a*\text{Tan}[c+d*x])^n)/(d*\text{Sqrt}[\text{Cot}[c+d*x]]*(1+I*\text{Tan}[c+d*x])^n) + (2*(2*A*n*(3+2*n) - I*B*(3+6*n+4*n^2))*\text{Hypergeometric2F1}[1/2, 1-n, 3/2, (-I)*\text{Tan}[c+d*x]]*(a+I*a*\text{Tan}[c+d*x])^n)/(d*(1+2*n)*(3+2*n)*\text{Sqrt}[\text{Cot}[c+d*x]]*(1+I*\text{Tan}[c+d*x])^n)$

**Rule 66**

$\text{Int}[\frac{(b*x)^m*((c_1)+(d_1)*(x_1))^{n_1}}{(b*(m+1))^{m+1}}*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x]$   
 /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

**Rule 68**

$\text{Int}[\frac{(b*x)^m*((c_1)+(d_1)*(x_1))^{n_1}}{(b*(m+1))^{m+1}}*\text{Hypergeometric2F1}[-n, m+1, m+2, (-d)*(x/c)], x]$   
 /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b\*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b\*c), 0])))

(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

#### Rule 129

Int[((e\_.)\*(x\_))^(p\_)\*((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

#### Rule 440

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 441

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[a^IntPart[p]\*((a + b\*x^n)^FracPart[p]/(1 + b\*(x^n/a))^FracPart[p]), Int[(1 + b\*(x^n/a))^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 3645

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(b/f), Subst[Int[(a + x)^(m - 1)\*((c + (d/b)\*x)^n/(b^2 + a\*x)], x], x, b\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3678

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(a\*(m + n)), Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (a\*A\*d\*(m + n) - B\*(b\*d\*m - a\*c\*n))\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]

#### Rule 3680

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b\*(B/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x]



```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x] - Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \tan^{\frac{3}{2}}(c + dx) (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} + \frac{\left( 2\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \tan^{\frac{3}{2}}(c + dx) (a + ia \tan(c + dx))^n (A + B \tan(c + dx)) dx}{2\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}} \\
&= \frac{2B(a + ia \tan(c + dx))^n}{d(3 + 2n) \cot^{\frac{3}{2}}(c + dx)} - \frac{2(2iBn - A(3 + 2n))(a + ia \tan(c + dx))^n}{d(1 + 2n)(3 + 2n) \sqrt{\cot(c + dx)}}
\end{aligned}$$

**Mathematica [F]**

time = 10.48, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(c + dx))^n (A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification is not applicable to the result.

[In] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Cot[c + d\*x]^(3/2), x]

[Out] Integrate[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Cot[c + d\*x]^(3/2), x]

**Maple [F]**

time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(dx + c))^n (A + B \tan(dx + c))}{\cot(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x)

[Out] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(3/2), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] integral(-((A - I\*B)\*e^(6\*I\*d\*x + 6\*I\*c) - (A - 3\*I\*B)\*e^(4\*I\*d\*x + 4\*I\*c) - (A + 3\*I\*B)\*e^(2\*I\*d\*x + 2\*I\*c) + A + I\*B)\*(2\*a\*e^(2\*I\*d\*x + 2\*I\*c)/(e^(2\*I\*d\*x + 2\*I\*c) + 1))^n\*sqrt((I\*e^(2\*I\*d\*x + 2\*I\*c) + I)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))/(e^(6\*I\*d\*x + 6\*I\*c) + 3\*e^(4\*I\*d\*x + 4\*I\*c) + 3\*e^(2\*I\*d\*x + 2\*I\*c) + 1), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c + dx) - i))^n (A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2),x)

[Out] Integral((I\*a\*(tan(c + d\*x) - I)\*\*n\*(A + B\*tan(c + d\*x))/cot(c + d\*x)\*\*(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n}{\cot(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/cot(c + d\*x)^(3/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/cot(c + d\*x)^(3/2), x)

$$3.573 \quad \int \frac{(a+ia \tan(c+dx))^n (A+B \tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=383

$$\frac{2B(a+ia \tan(c+dx))^n}{d(5+2n) \cot^{\frac{5}{2}}(c+dx)} - \frac{2(2iBn - A(5+2n))(a+ia \tan(c+dx))^n}{d(3+2n)(5+2n) \cot^{\frac{3}{2}}(c+dx)} - \frac{2(2iAn(5+2n) + B(15+10n+4n^2))}{d(1+2n)(3+2n)(5+2n)}$$

[Out]  $2*B*(a+I*a*\tan(d*x+c))^n/d/(5+2*n)/\cot(d*x+c)^{(5/2)}-2*(2*I*B*n-A*(5+2*n))*((a+I*a*\tan(d*x+c))^n/d/(3+2*n)/(5+2*n)/\cot(d*x+c)^{(3/2)}-2*(2*I*A*n*(5+2*n)+B*(4*n^2+10*n+15))*(a+I*a*\tan(d*x+c))^n/d/(5+2*n)/(4*n^2+8*n+3)/\cot(d*x+c)^{(1/2)}+2*(I*A+B)*\text{AppellF1}(1/2,1-n,1,3/2,-I*\tan(d*x+c),I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/\cot(d*x+c)^{(1/2))/((1+I*\tan(d*x+c))^n)-2*(4*B*n*(2*n^2+8*n+9)+I*A*(8*n^3+32*n^2+36*n+15))*\text{hypergeom}([1/2,1-n],[3/2],-I*\tan(d*x+c))*(a+I*a*\tan(d*x+c))^n/d/(5+2*n)/(4*n^2+8*n+3)/\cot(d*x+c)^{(1/2))/((1+I*\tan(d*x+c))^n)$

**Rubi [A]**

time = 0.87, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {4326, 3678, 3682, 3645, 129, 441, 440, 3680, 68, 66}

$$\frac{2(B+A)(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}, 1-n, 1, \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d \sqrt{\cot(c+dx)}} - \frac{2(4Bn(2n^2+8n+9)+A(8n^3+32n^2+36n+15))(1+i \tan(c+dx))^{-n}(a+ia \tan(c+dx))^n F_1\left(\frac{1}{2}, 1-n, 1, \frac{3}{2}; -i \tan(c+dx), i \tan(c+dx)\right)}{d(2n+1)(2n+3)(2n+5) \sqrt{\cot(c+dx)}} - \frac{2(B(dx^2+10n+15)+2A(2n+5))(a+ia \tan(c+dx))^n}{d(2n+1)(2n+3)(2n+5) \sqrt{\cot(c+dx)}} - \frac{2(-A(2n+5)+2iBn)(a+ia \tan(c+dx))^n}{d(2n+3)(2n+5) \cot^2(c+dx)} + \frac{2B(a+ia \tan(c+dx))^n}{d(2n+5) \cot^4(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Cot[c + d\*x]^(5/2),x]

[Out]  $(2*B*(a+I*a*\tan[c+d*x])^n)/(d*(5+2*n)*\cot[c+d*x]^{(5/2)}) - (2*((2*I)*B*n - A*(5+2*n))*(a+I*a*\tan[c+d*x])^n)/(d*(3+2*n)*(5+2*n)*\cot[c+d*x]^{(3/2)}) - (2*((2*I)*A*n*(5+2*n) + B*(15+10*n+4*n^2))*(a+I*a*\tan[c+d*x])^n)/(d*(1+2*n)*(3+2*n)*(5+2*n)*\text{Sqrt}[\cot[c+d*x]]) + (2*(I*A+B)*\text{AppellF1}[1/2,1-n,1,3/2,(-I)*\tan[c+d*x],I*\tan[c+d*x]]*(a+I*a*\tan[c+d*x])^n)/(d*\text{Sqrt}[\cot[c+d*x]]*(1+I*\tan[c+d*x])^n) - (2*(4*B*n*(9+8*n+2*n^2) + I*A*(15+36*n+32*n^2+8*n^3))*\text{Hypergeometric2F1}[1/2,1-n,3/2,(-I)*\tan[c+d*x]]*(a+I*a*\tan[c+d*x])^n)/(d*(1+2*n)*(3+2*n)*(5+2*n)*\text{Sqrt}[\cot[c+d*x]]*(1+I*\tan[c+d*x])^n)$

Rule 66

Int[((b\_.)\*(x\_)^(m\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[c^n\*((b\*x)^(m+1)/(b\*(m+1)))\*Hypergeometric2F1[-n, m+1, m+2, (-d)\*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0]) && GtQ[-d/(b\*c), 0]))

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

#### Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

#### Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q},
x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rule 3645

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(b/f), Subst[Int[(a + x)^(m - 1)*((c
+ (d/b)*x)^n/(b^2 + a*x)], x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d
, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^
2, 0]
```

#### Rule 3678

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Dist[
1/(a*(m + n)), Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Simp
[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*Ta
n[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

#### Rule 3680

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[b*(B/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

### Rule 3682

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[(A*b + a*B)/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
- Dist[B/b, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[e
+ f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*
d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps





**Maple [F]**

time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \tan(dx + c))^n (A + B \tan(dx + c))}{\cot(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2),x)

[Out] int((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(5/2), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2),x, algorithm="fricas")

```
[Out] integral(((I*A + B)*e^(8*I*d*x + 8*I*c) - 2*(I*A + 2*B)*e^(6*I*d*x + 6*I*c)
+ 6*B*e^(4*I*d*x + 4*I*c) - 2*(-I*A + 2*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*
(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))/(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x
+ 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*I*c) + 1), x)
```

**Sympy [F(-1)] Timed out**

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(5/2),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(I\*a\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(5/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + a \tan(c + dx) i)^n}{\cot(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/cot(c + d\*x)^(5/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + a\*tan(c + d\*x)\*1i)^n)/cot(c + d\*x)^(5/2), x)

$$3.574 \quad \int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=229

$$\frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-2/3*a*A*\cot(d*x+c)^{(3/2)}/d+1/2*(b*(A-B)+a*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*(b*(A-B)+a*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a*(A-B)-b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(a*(A-B)-b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-2*(A*b+B*a)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.21, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$ , Rules used = {3662, 3673, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A+B)+b(A-B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A+B)+b(A-B))\text{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d} + \frac{(a(A-B)-b(A+B))\log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{(a(A-B)-b(A+B))\log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{2aA\cot^{\frac{1}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-(((b*(A-B) + a*(A+B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]])/(\text{Sqrt}[2]*d)) + ((b*(A-B) + a*(A+B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]])/(\text{Sqrt}[2]*d) - (2*(A*b + a*B)*\text{Sqrt}[\text{Cot}[c + d*x]])/d - (2*a*A*\text{Cot}[c + d*x]^{(3/2)})/(3*d) + ((a*(A-B) - b*(A+B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d) - ((a*(A-B) - b*(A+B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d)$

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 642**

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := S  
 imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
 e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
 2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e  
 /(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &  
 & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
 -2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
 x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; Fre  
 eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
 a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + D  
 ist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a,  
 c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)  
 \*c]

#### Rule 3609

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) +  
 (f\_)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Int  
 [(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x  
 , x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2,  
 0] && GtQ[m, 0]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_  
 )]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr  
 t[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&  
 NeQ[c^2 + d^2, 0]

#### Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)])\*(g\_)^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(  
 x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist  
 [g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c

$\text{Cot}[e + f*x]^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

### Rule 3673

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]]^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)]), x\_Symbol] :> \text{Simp}[B*d*((a + b*\text{Tan}[e + f*x])^{(m + 1)/(b*f*(m + 1))}), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{!LeQ}[m, -1]$

### Rubi steps

$$\begin{aligned}
 \int \cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))(A + B \tan(c + dx)) dx &= \int \sqrt{\cot(c + dx)} (b + a \cot(c + dx))(B + A \cot(c + dx)) dx \\
 &= -\frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} + \int \sqrt{\cot(c + dx)} (-aA + bB) dx \\
 &= -\frac{2(Ab + aB) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2(Ab + aB) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2(Ab + aB) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2(Ab + aB) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{2(Ab + aB) \sqrt{\cot(c + dx)}}{d} - \frac{2aA \cot^{\frac{3}{2}}(c + dx)}{3d} \\
 &= -\frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d}
 \end{aligned}$$

### Mathematica [A]

time = 0.61, size = 198, normalized size = 0.86

$$\frac{\sqrt{\cot(c + dx)} \left( 6\sqrt{2} (b(A - B) + a(A + B)) \left( \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) - \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c + dx)}\right) \right) + 3\sqrt{2} (a(A - B) - b(A + B)) \left( \log\left(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) - \log\left(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)\right) \right) - \frac{2aA}{\tan^2(c + dx)} - \frac{2bB}{\tan(c + dx)} \right) \sqrt{\tan(c + dx)}}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

```
[Out] (Sqrt[Cot[c + d*x]]*(6*Sqrt[2]*(b*(A - B) + a*(A + B))*(ArcTan[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + 3*Sqrt[2]*(
a*(A - B) - b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
- Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - (8*a*A)/Tan[c + d*x
]^(3/2) - (24*(A*b + a*B))/Sqrt[Tan[c + d*x]]*Sqrt[Tan[c + d*x]]/(12*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 26.67, size = 4490, normalized size = 19.61

method	result	size
default	Expression too large to display	4490

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/6/d*2^(1/2)*(-6*A*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*a-3*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*b+3*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((
cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*a-3*A*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*b+6*B*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos
(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*b-3*B*
sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin
(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a-3*B*si
n(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-3*B*sin(
d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x
+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-3*B*sin(d*
x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c)
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-6*A*sin(d*x+
c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c)
)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticF((-cos(d*x+
```



[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{12} * (6 * \sqrt{2} * ((A + B) * a + (A - B) * b) * \arctan(\frac{1}{2} * \sqrt{2} * (\sqrt{2} + 2 / \sqrt{\tan(d * x + c)})) + 6 * \sqrt{2} * ((A + B) * a + (A - B) * b) * \arctan(-\frac{1}{2} * \sqrt{2} * (\sqrt{2} - 2 / \sqrt{\tan(d * x + c)})) - 3 * \sqrt{2} * ((A - B) * a - (A + B) * b) * \log(\sqrt{2} / \sqrt{\tan(d * x + c)} + 1 / \tan(d * x + c) + 1) + 3 * \sqrt{2} * ((A - B) * a - (A + B) * b) * \log(-\sqrt{2} / \sqrt{\tan(d * x + c)} + 1 / \tan(d * x + c) + 1) - 8 * A * a / \tan(d * x + c)^{(3/2)} - 24 * (B * a + A * b) / \sqrt{\tan(d * x + c)}) / d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)\*cot(d\*x + c)^(5/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + b \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)), x)



$$3.575 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=205

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $\frac{1}{2}(a(A-B) - b(A+B)) \arctan(-1 + 2^{1/2} \cot(dx+c)^{1/2}) / d + \frac{1}{2}(a(A-B) - b(A+B)) \arctan(1 + 2^{1/2} \cot(dx+c)^{1/2}) / d - \frac{1}{4}(b(A-B) + a(A+B)) \ln(1 + \cot(dx+c) - 2^{1/2} \cot(dx+c)^{1/2}) / d + \frac{1}{4}(b(A-B) + a(A+B)) \ln(1 + \cot(dx+c) + 2^{1/2} \cot(dx+c)^{1/2}) / d - 2aA \cot(dx+c)^{1/2} / d$

**Rubi** [A]

time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3662, 3673, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A-B) - b(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{(a(A-B) - b(A+B))\text{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{(a(A+B) + b(A-B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{(a(A+B) + b(A-B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{2aA \sqrt{\cot(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\frac{((a(A-B) - b(A+B)) \text{ArcTan}[1 - \text{Sqrt}[2] \text{Sqrt}[\text{Cot}[c + d*x]])]}{(\text{Sqrt}[2] * d)} + \frac{((a(A-B) - b(A+B)) \text{ArcTan}[1 + \text{Sqrt}[2] \text{Sqrt}[\text{Cot}[c + d*x]])]}{(\text{Sqrt}[2] * d)} - \frac{(2*a*A \text{Sqrt}[\text{Cot}[c + d*x]])}{d} - \frac{((b(A-B) + a(A+B)) \text{Log}[1 - \text{Sqrt}[2] \text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])}{(2*\text{Sqrt}[2]*d)} + \frac{((b(A-B) + a(A+B)) \text{Log}[1 + \text{Sqrt}[2] \text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])}{(2*\text{Sqrt}[2]*d)}$

Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[
e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 3673

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)])*(c_) + (d_)*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
```

$x])^m \text{Simp}[A*c - B*d + (B*c + A*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
 \int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))dx &= \int \frac{(b+a\cot(c+dx))(B+A\cot(c+dx))}{\sqrt{\cot(c+dx)}} dx \\
 &= -\frac{2aA\sqrt{\cot(c+dx)}}{d} + \int \frac{-aA+bB+(Ab-)}{\sqrt{\cot(c+dx)}} dx \\
 &= -\frac{2aA\sqrt{\cot(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{aA-bB+(-Ab)}{1+x^4} dx\right)}{d} \\
 &= -\frac{2aA\sqrt{\cot(c+dx)}}{d} + \frac{(b(A-B)+a(A+B))\tan^{-1}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{d} \\
 &= -\frac{2aA\sqrt{\cot(c+dx)}}{d} - \frac{(b(A-B)+a(A+B))\tan^{-1}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{d} \\
 &= -\frac{2aA\sqrt{\cot(c+dx)}}{d} - \frac{(b(A-B)+a(A+B))\tan^{-1}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{d} \\
 &= -\frac{(a(A-B)-b(A+B))\tan^{-1}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)}{\sqrt{2}d}
 \end{aligned}$$

**Mathematica [A]**

time = 0.31, size = 179, normalized size = 0.87

$$\frac{\sqrt{\cot(c+dx)}\left(2\sqrt{2}(a(A-B)-b(A+B))\left(\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)-\sqrt{2}(b(A-B)+a(A+B))\left(\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)-\frac{8aA}{\sqrt{\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[Cot[c + d\*x]]\*(2\*Sqrt[2]\*(a\*(A - B) - b\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]) - Sqrt[2]\*(b\*(A - B) + a\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) - (8\*a\*A)/Sqrt[Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]])/(4\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 27.58, size = 4230, normalized size = 20.63

method	result	size
default	Expression too large to display	4230

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*2^(1/2)*(I*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)
)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*b+I*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*
((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))*a-I*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((
cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*a-I*A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((co
s(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*b-I*B*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*E
llipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*a-I*B*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*
x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*b+I*B*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellip
ticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
a+I*B*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)
-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*Ellipti
cPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-
A*cos(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a-A*co
s(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-c
os(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-A*cos(d*
x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-A*cos(d*x+c)
*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)
-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b+2*A*cos(d*x+c)*
```



b)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) - 8\*A\*a/sqrt(tan(d\*x + c)))/d

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)\*cot(d\*x + c)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)), x)

$$3.576 \quad \int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=205

$$\frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(b(A-B) + a(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-1/2*(b*(A-B)+a*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(b*(A-B)+a*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(a*(A-B)-b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a*(A-B)-b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+2*b*B/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {3662, 3672, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A+B)+b(A-B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(a(A+B)+b(A-B))\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}d} - \frac{(a(A-B)-b(A+B))\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d} + \frac{(a(A-B)-b(A+B))\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d} + \frac{2bB}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((b*(A-B) + a*(A+B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d) - ((b*(A-B) + a*(A+B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d) + (2*b*B)/(d*\text{Sqrt}[\text{Cot}[c + d*x]]) - ((a*(A-B) - b*(A+B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d) + ((a*(A-B) - b*(A+B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d)$

Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[
e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 3672

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
```



)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*A\*c + b\*B\*c + A\*b\*d - a\*B\*d - (A\*b\*c - a\*B\*c - a\*A\*d - b\*B\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))(A+B \tan(c+dx)) dx &= \int \frac{(b+a \cot(c+dx))(B+A \cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \\
 &= \frac{2bB}{d\sqrt{\cot(c+dx)}} + \int \frac{Ab+aB+(aA-bB)}{\sqrt{\cot(c+dx)}} dx \\
 &= \frac{2bB}{d\sqrt{\cot(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{-Ab-aB+(-aA+bB)}{1+x^4} dx\right)}{d\sqrt{\cot(c+dx)}} \\
 &= \frac{2bB}{d\sqrt{\cot(c+dx)}} - \frac{(b(A-B)+a(A+B))}{d\sqrt{\cot(c+dx)}} \\
 &= \frac{2bB}{d\sqrt{\cot(c+dx)}} - \frac{(b(A-B)+a(A+B))}{d\sqrt{\cot(c+dx)}} \\
 &= \frac{2bB}{d\sqrt{\cot(c+dx)}} - \frac{(a(A-B)-b(A+B))}{d\sqrt{\cot(c+dx)}} \\
 &= \frac{(b(A-B)+a(A+B)) \tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}
 \end{aligned}$$

**Mathematica [A]**

time = 0.14, size = 178, normalized size = 0.87

$$\frac{\sqrt{\cot(c+dx)} \left( 2\sqrt{2} (b(A-B)+a(A+B)) \left( \text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right) - \text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right) \right) + \sqrt{2} (a(A-B)-b(A+B)) \left( \log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right) - \log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right) \right) - 8bB\sqrt{\tan(c+dx)} \right) \sqrt{\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]),x]

[Out] -1/4\*(Sqrt[Cot[c + d\*x]]\*(2\*Sqrt[2]\*(b\*(A - B) + a\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]])] + Sqrt[2]\*(a\*(A - B) - b\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) - 8\*b\*B\*Sqrt[Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]]/d



$$c) - 1 - \sin(dx+c) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 + 1/2 * I, 1/2 * 2 \wedge (1/2))) * b - 2 * A * \sin(dx+c) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticF}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 * 2 \wedge (1/2)) * a - B * \sin(dx+c) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 - 1/2 * I, 1/2 * 2 \wedge (1/2)) * a - B * \sin(dx+c) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 - 1/2 * I, 1/2 * 2 \wedge (1/2)) * b - B * \sin(dx+c) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 + 1/2 * I, 1/2 * 2 \wedge (1/2)) * a - B * \sin(dx+c) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticPi}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 + 1/2 * I, 1/2 * 2 \wedge (1/2)) * b + 2 * B * \sin(dx+c) * (- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((\cos(dx+c) - 1 + \sin(dx+c)) / \sin(dx+c) \wedge (1/2) * ((-1 + \cos(dx+c)) / \sin(dx+c) \wedge (1/2) * \text{EllipticF}((- \cos(dx+c) - 1 - \sin(dx+c)) / \sin(dx+c) \wedge (1/2), 1/2 * 2 \wedge (1/2))) * b - 2 * B * 2 \wedge (1/2) * \cos(dx+c) * b + 2 * B * 2 \wedge (1/2) * b) / \cos(dx+c) / \sin(dx+c) \wedge 3$$

**Maxima** [A]

time = 0.59, size = 178, normalized size = 0.87

$$\frac{2\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A+B)a+(A-B)b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}((A-B)a-(A+B)b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}((A-B)a-(A+B)b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-8Bb\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(a+b\*tan(dx+c))\*(A+B\*tan(dx+c)),x, algorithm="maxima")

[Out] 
$$-1/4*(2*\text{sqrt}(2)*((A+B)*a+(A-B)*b)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2)+2/\text{sqrt}(\tan(dx+c))))+2*\text{sqrt}(2)*((A+B)*a+(A-B)*b)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2)-2/\text{sqrt}(\tan(dx+c))))-\text{sqrt}(2)*((A-B)*a-(A+B)*b)*\log(\text{sqrt}(2)/\text{sqrt}(\tan(dx+c))+1/\tan(dx+c)+1)+\text{sqrt}(2)*((A-B)*a-(A+B)*b)*\log(-\text{sqrt}(2)/\text{sqrt}(\tan(dx+c))+1/\tan(dx+c)+1)-8*B*b*\text{sqrt}(\tan(dx+c))/d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(a+b\*tan(dx+c))\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx)) \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)),x)

[Out] int(cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x)), x)

$$3.577 \quad \int \frac{(a+b \tan(c+dx))(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=229

$$\frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $2/3*b*B/d/\cot(d*x+c)^{(3/2)} - 1/2*(a*(A-B) - b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)} - 1/2*(a*(A-B) - b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)} + 1/4*(b*(A-B) + a*(A+B))*\ln(1+\cot(d*x+c) - 2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)} - 1/4*(b*(A-B) + a*(A+B))*\ln(1+\cot(d*x+c) + 2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)} + 2*(A*b + B*a)/d/\cot(d*x+c)^{(1/2)}$

**Rubi** [A]

time = 0.19, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$ , Rules used = {3662, 3672, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(a(A-B) - b(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2(aB + Ab)}{d \sqrt{\cot(c+dx)}} + \frac{(a(A+B) + b(A-B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{(a(A+B) + b(A-B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{2bB}{3d \cot^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]],x]

[Out]  $((a*(A-B) - b*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - ((a*(A-B) - b*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(\operatorname{Sqrt}[2]*d) + (2*b*B)/(3*d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (2*(A*b + a*B))/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + ((b*(A-B) + a*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - ((b*(A-B) + a*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3610

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

#### Rule 3615

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3662

Int[(cot[(e\_.) + (f\_.)\*(x\_)])\*(g\_.)^(p\_)\*((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c

$\text{Cot}[e + f*x]^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

### Rule 3672

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] := \text{Simp}[(b*c - a*d)*(A*b - a*B)*((a + b*\text{Tan}[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 + b^2))], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*A*c + b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan(c + dx))(A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \int \frac{(b + a \cot(c + dx))(B + A \cot(c + dx))}{\cot^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{2bB}{3d \cot^{\frac{3}{2}}(c + dx)} + \int \frac{Ab + aB + (aA - bB) \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2bB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB)}{d \sqrt{\cot(c + dx)}} + \int \frac{aA - bB - (Ab - bB)}{\sqrt{\cot(c + dx)}} dx \\ &= \frac{2bB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB)}{d \sqrt{\cot(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{-aA + bB + (Ab - bB)}{1 + u^2} du\right)}{\sqrt{\cot(c + dx)}} \\ &= \frac{2bB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB)}{d \sqrt{\cot(c + dx)}} - \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(\sqrt{\cot(c + dx)}\right)}{\sqrt{\cot(c + dx)}} \\ &= \frac{2bB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB)}{d \sqrt{\cot(c + dx)}} + \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(\sqrt{\cot(c + dx)}\right)}{\sqrt{\cot(c + dx)}} \\ &= \frac{2bB}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab + aB)}{d \sqrt{\cot(c + dx)}} + \frac{(b(A - B) + a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} \end{aligned}$$

### Mathematica [A]

time = 0.33, size = 198, normalized size = 0.86

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
[Out] -1/12*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(6*Sqrt[2]*(a*(A - B) - b*(A +
B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]]]) - 3*Sqrt[2]*(b*(A - B) + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]) - 24*(A*b + a*B)*Sqrt[Tan[c + d*x]] - 8*b*B*Tan[c + d*x]^(3/2))/d
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 86.37, size = 2401, normalized size = 10.48

method	result	size
default	Expression too large to display	2401

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/6/d*2^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(3*I*B*cos(d*x+c)*sin(d*x+c)
*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c
)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+3*I*B*cos(d*x+c)
*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((
-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-3*I*A
*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/s
in(d*x+c))^(1/2)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*E
llipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*a-3*I*A*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(
1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*b-3*I*B*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/si
n(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*a+3*I*A*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-3*I*B*cos(d*x+c)*((-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(cos(d*x+c)-
1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b+3*I*A*cos(d*x+c)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi((-cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+3*A*cos(d*x+c)
```



```

*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+3*A*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-6*A*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*b+3*A*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+3*A*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b+3*B*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-3*B*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-6*B*cos(d*x+c)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*a+3*B*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-3*B*cos(d*x+c)*sin(d*x+c)*(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b+6*A*2^(1/2)*cos(d*x+c)^2*b+6*B*2^(1/2)*cos(d*x+c)^2*a+2*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*b-6*A*cos(d*x+c)*2^(1/2)*b-6*B*cos(d*x+c)*2^(1/2)*a-2*B*2^(1/2)*sin(d*x+c)*b)/sin(d*x+c)^4/cos(d*x+c)/(cos(d*x+c)/sin(d*x+c))^(1/2)

```

**Maxima [A]**

time = 0.52, size = 198, normalized size = 0.86

$$\frac{6\sqrt{2}((A-B)a - (A+B)b)\arctan\left(\frac{1}{\sqrt{2}}\left(\sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 6\sqrt{2}((A-B)a - (A+B)b)\arctan\left(-\frac{1}{\sqrt{2}}\left(\sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 3\sqrt{2}((A+B)a + (A-B)b)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) - 3\sqrt{2}((A+B)a + (A-B)b)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) - 8\left(\frac{Bb + \frac{1}{2}Aa}{\tan(dx+c)}\right)\tan(dx+c)^3}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/12\*(6\*sqrt(2)\*((A - B)\*a - (A + B)\*b)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c)))) + 6\*sqrt(2)\*((A - B)\*a - (A + B)\*b)\*arctan(-1/2\*sqrt(2)\*

$(\sqrt{2} - 2/\sqrt{\tan(dx + c)}) + 3\sqrt{2}*((A + B)a + (A - B)b)*\log(\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - 3\sqrt{2}*((A + B)a + (A - B)b)*\log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - 8*(Bb + 3*(Ba + Ab)/\tan(dx + c))*\tan(dx + c)^{(3/2)}/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(dx+c))\*(A+B\*tan(dx+c))/cot(dx+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(dx+c))\*(A+B\*tan(dx+c))/cot(dx+c)\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + dx))\*(a + b\*tan(c + dx))/sqrt(cot(c + dx)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(dx+c))\*(A+B\*tan(dx+c))/cot(dx+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(dx + c) + A)\*(b\*tan(dx + c) + a)/sqrt(cot(dx + c)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + dx))\*(a + b\*tan(c + dx)))/cot(c + dx)^(1/2),x)

[Out] int(((A + B\*tan(c + dx))\*(a + b\*tan(c + dx)))/cot(c + dx)^(1/2), x)

$$3.578 \quad \int \cot^2(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=326

$$\frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} \quad (a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)$$

[Out]  $-2/15*a*(7*A*b+5*B*a)*\cot(d*x+c)^{(3/2)}/d-2/5*a*A*\cot(d*x+c)^{(3/2)}*(b+a*\cot(d*x+c))/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}-1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+\cot(d*x+c))+2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}+2*(A*a^2-A*b^2-2*B*a*b)*\cot(d*x+c)^{(1/2)}/d$

**Rubi** [A]

time = 0.41, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3662, 3688, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2a^2x - 2abx - b^2x}{d} \sqrt{\cot(c+dx)} + \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{Im}\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{Im}\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{2a^2b + 2ab^2}{15d} \cot(c+dx) - \frac{2a^2 \cot^2(c+dx) + 2ab \cot(c+dx) + b^2}{15d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(7/2)}*(a + b*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) + (2*(a^2*A - A*b^2 - 2*a*b*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / d - (2*a*(7*A*b + 5*a*B)*\operatorname{Cot}[c + d*x]^{(3/2)}) / (15*d) - (2*a*A*\operatorname{Cot}[c + d*x]^{(3/2)}*(b + a*\operatorname{Cot}[c + d*x])) / (5*d) + ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) - ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d)$

Rule 210

$\operatorname{Int}[(a_0 + (b_0)*(x_0)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1} * \operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{PosQ}[a/b] \& \& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 631

$\operatorname{Int}[(a_0 + (b_0)*(x_0) + (c_0)*(x_0)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{q = 1 - 4*S \operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)$

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c)]
```

#### Rule 3609

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist [g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ [p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan [e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx &= \int \sqrt{\cot(c+dx)}(b+a \cot(c+dx))^2(B+A \cot(c+dx)) dx \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c+dx)(b+a \cot(c+dx))}{5d} - \frac{2}{5} \int \sqrt{\cot(c+dx)}(b+a \cot(c+dx))^2 dx \\
&= -\frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} - \frac{2aA \cot^{\frac{3}{2}}(c+dx)}{5d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} \\
&= \frac{2(a^2A-Ab^2-2abB) \sqrt{\cot(c+dx)}}{d} - \frac{2a(7Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} \\
&= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B)) \tan^{-1}(\sqrt{\cot(c+dx)})}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [A]**

time = 1.18, size = 255, normalized size = 0.78

$$\frac{\sqrt{\cot(c+dx)} \left( 30\sqrt{2} (a^2(A-B) + b^2(-A+B) - 2ab(A+B)) \left( \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) - \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right) \right) - 15\sqrt{2} (2ab(A-B) + a^2(A+B) - b^2(A+B)) \left( \log\left(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right) - \log\left(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)\right) \right) + \frac{2a^2A}{\sqrt{2}\sqrt{\cot(c+dx)}} + \frac{2b^2B}{\sqrt{2}\sqrt{\cot(c+dx)}} - \frac{2ab(A+B)}{\sqrt{\tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{66d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(7/2)\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]), x]

[Out] -1/60\*(Sqrt[Cot[c + d\*x]]\*(30\*Sqrt[2]\*(a^2\*(A - B) + b^2\*(-A + B) - 2\*a\*b\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]) - 15\*Sqrt[2]\*(2\*a\*b\*(A - B) + a^2\*(A + B) - b^2\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + (24\*a^2\*A)/Tan[c + d\*x]^(5/2) + (40\*a\*(2\*A\*b + a\*B))/Tan[c + d\*x]^(3/2) - (120\*(a^2\*A - A\*b^2 - 2\*a\*b\*B))/Sqrt[Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]]/d

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 28.24, size = 13170, normalized size = 40.40

method	result	size
default	Expression too large to display	13170

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima** [A]

time = 0.60, size = 279, normalized size = 0.86

$$\frac{30\sqrt{2}((A-B)a^2-2(A+B)ab-(A-B)^2b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{1}{\tan(dx+c)}\right)\right)+30\sqrt{2}((A-B)a^2-2(A+B)ab-(A-B)^2b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{1}{\tan(dx+c)}\right)\right)+15\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)^2b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-15\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)^2b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}-\frac{1}{\tan(dx+c)}+1\right)+\frac{24A^2a^2}{\sqrt{\tan(dx+c)}}+\frac{24A^2ab}{\sqrt{\tan(dx+c)}}+\frac{24A^2b^2}{\sqrt{\tan(dx+c)}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] 
$$\begin{aligned} & -1/60*(30*\sqrt{2}*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(d*x + c)})) + 30*\sqrt{2}*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(d*x + c)})) \\ & + 15*\sqrt{2}*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*\log(\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) - 15*\sqrt{2}*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*\log(-\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) + 24*A*a^2/\tan(d*x + c)^(5/2) - 120*(A*a^2 - 2*B*a*b - A*b^2)/\sqrt{\tan(d*x + c)} + 40*(B*a^2 + 2*A*a*b)/\tan(d*x + c)^(3/2))/d \end{aligned}$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(7/2)\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^2\*cot(d\*x + c)^(7/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2,x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2, x)



$$3.579 \quad \int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=294

$$\frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $*2^{(1/2)}+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $2^{(1/2)}-2/3*a*(5*A*b+3*B*a)*\cot(d*x+c)^{(1/2)}/d$   
 $-2/3*a*A*(b+a*\cot(d*x+c))*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.36, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3662, 3688, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right) + (a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right) + (a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right) + \frac{2ab(A+B) + 5ab}{d} \sqrt{\cot(c+dx)} - \frac{2ab \sqrt{\cot(c+dx)}}{d} \cot(c+dx) + \frac{2ab \sqrt{\cot(c+dx)}}{d} \cot(c+dx) + \frac{2ab \sqrt{\cot(c+dx)}}{d} \cot(c+dx)}{\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-(((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) + ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - (2*a*(5*A*b + 3*a*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / (3*d) - (2*a*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(b + a*\operatorname{Cot}[c + d*x])) / (3*d) + ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) - ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.) )^{(p_.)} * ((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)} * ((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \ :> \ \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)} * (b + a*\text{Cot}[e + f*x])^m * (d + c*\text{Cot}[e + f*x])^n, x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3688

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3711

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \int \frac{(b + a \cot(c + dx))^2(B + A \cot(c + dx))}{\sqrt{\cot(c + dx)}} dx \\
&= -\frac{2aA\sqrt{\cot(c + dx)}(b + a \cot(c + dx))}{3d} - \frac{2aA\sqrt{\cot(c + dx)}}{3d} \\
&= -\frac{2a(5Ab + 3aB)\sqrt{\cot(c + dx)}}{3d} - \frac{2aA\sqrt{\cot(c + dx)}}{3d} \\
&= -\frac{2a(5Ab + 3aB)\sqrt{\cot(c + dx)}}{3d} - \frac{2aA\sqrt{\cot(c + dx)}}{3d} \\
&= -\frac{2a(5Ab + 3aB)\sqrt{\cot(c + dx)}}{3d} - \frac{2aA\sqrt{\cot(c + dx)}}{3d} \\
&= -\frac{2a(5Ab + 3aB)\sqrt{\cot(c + dx)}}{3d} - \frac{2aA\sqrt{\cot(c + dx)}}{3d} \\
&= -\frac{2a(5Ab + 3aB)\sqrt{\cot(c + dx)}}{3d} - \frac{2aA\sqrt{\cot(c + dx)}}{3d} \\
&= -\frac{(2ab(A - B) + a^2(A + B) - b^2(A + B)) \tan(c + dx)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.74, size = 226, normalized size = 0.77

$$\frac{\sqrt{\cot(c+dx)} \left( 6\sqrt{2} (2ab(A-B) + a^2(A+B) - b^2(A+B)) (\text{ArcTan}(1 - \sqrt{2} \sqrt{\tan(c+dx)}) - \text{ArcTan}(1 + \sqrt{2} \sqrt{\tan(c+dx)})) + 3\sqrt{2} (a^2(A-B) + b^2(-A+B) - 2ab(A+B)) (\log(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx))) - \frac{6\sqrt{2}ab}{\tan^2(c+dx)} - \frac{2ab^2(a+b)}{\sqrt{\tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[Cot[c + d\*x]]\*(6\*Sqrt[2]\*(2\*a\*b\*(A - B) + a^2\*(A + B) - b^2\*(A + B))\*  
 (ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d  
 \*x]]]) + 3\*Sqrt[2]\*(a^2\*(A - B) + b^2\*(-A + B) - 2\*a\*b\*(A + B))\*(Log[1 - Sq  
 rt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x  
 ]] + Tan[c + d\*x])) - (8\*a^2\*A)/Tan[c + d\*x]^(3/2) - (24\*a\*(2\*A\*b + a\*B))/S  
 qrt[Tan[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(12\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 28.58, size = 6783, normalized size = 23.07

method	result	size
default	Expression too large to display	6783

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVE  
RBOSE)

[Out] result too large to display

**Maxima [A]**

time = 0.51, size = 252, normalized size = 0.86

$$\frac{6\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{\tan(dx+c)}+\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)b^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{\tan(dx+c)}-\frac{1}{\sqrt{\tan(dx+c)}}\right)\right)-3\sqrt{2}((A-B)a^2-2(A+B)ab-(A-B)b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+3\sqrt{2}((A-B)a^2-2(A+B)ab-(A-B)b^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-\frac{8Aa^2}{\tan^2(dx+c)}-\frac{24Aa^2ab}{\sqrt{\tan(dx+c)}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm  
="maxima")

[Out] 1/12\*(6\*sqrt(2)\*((A + B)\*a^2 + 2\*(A - B)\*a\*b - (A + B)\*b^2)\*arctan(1/2\*sqrt  
 (2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c)))) + 6\*sqrt(2)\*((A + B)\*a^2 + 2\*(A - B)\*  
 a\*b - (A + B)\*b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(d\*x + c)))) -  
 3\*sqrt(2)\*((A - B)\*a^2 - 2\*(A + B)\*a\*b - (A - B)\*b^2)\*log(sqrt(2)/sqrt(tan(  
 d\*x + c)) + 1/tan(d\*x + c) + 1) + 3\*sqrt(2)\*((A - B)\*a^2 - 2\*(A + B)\*a\*b -  
 (A - B)\*b^2)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) - 8\*A\*a^  
 2/tan(d\*x + c)^(3/2) - 24\*(B\*a^2 + 2\*A\*a\*b)/sqrt(tan(d\*x + c))/d

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]**  
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2), x)
```

**Mupad [F]**  
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

$$3.580 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^2(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=276

$$\frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $*2^{(1/2)}+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $*2^{(1/2)}-1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $*2^{(1/2)}+1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d$   
 $+2*b^2*B/d/\cot(d*x+c)^{(1/2)}-2*a^2*A*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.30, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3662, 3685, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{2a^2A\sqrt{\cot(c+dx)}}{d} + \frac{2b^2B}{4\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x])^2*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-(((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d)) + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) + (2*b^2*B) / (d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) - (2*a^2*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / d - ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]] / (2*\operatorname{Sqrt}[2]*d) + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]] / (2*\operatorname{Sqrt}[2]*d))$

**Rule 210**

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b] \ \&\& \ (\operatorname{LtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

**Rule 631**

$\operatorname{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{With}\{q = 1 - 4*S\operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /;$   $\operatorname{RationalQ}[q] \ \&\& \ (\operatorname{EqQ}[q^2, 1] \ || \ !\operatorname{RationalQ}[b^2 - 4*a*c]) /;$   $\operatorname{FreeQ}[a, b, c, x]$

$Q[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

#### Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[(-a)*c]$

#### Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.)^{(p_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}), x\_Symbol] \rightarrow \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

#### Rule 3685

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(- (B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
+ 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

```

### Rule 3711

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> Simp[C*((a
+ b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2(A + B \tan(c + dx)) dx &= \int \frac{(b + a \cot(c + dx))^2(B + A \cot(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2b^2B}{d\sqrt{\cot(c + dx)}} - \int \frac{-b(Ab + 2aB) - (2aAb)}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2b^2B}{d\sqrt{\cot(c + dx)}} - \frac{2a^2A\sqrt{\cot(c + dx)}}{d} - \int \frac{2aAb}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2b^2B}{d\sqrt{\cot(c + dx)}} - \frac{2a^2A\sqrt{\cot(c + dx)}}{d} - \frac{2aAb}{d} \\
&= \frac{2b^2B}{d\sqrt{\cot(c + dx)}} - \frac{2a^2A\sqrt{\cot(c + dx)}}{d} + \frac{(a^2)}{d} \\
&= \frac{2b^2B}{d\sqrt{\cot(c + dx)}} - \frac{2a^2A\sqrt{\cot(c + dx)}}{d} + \frac{(a^2)}{d} \\
&= \frac{2b^2B}{d\sqrt{\cot(c + dx)}} - \frac{2a^2A\sqrt{\cot(c + dx)}}{d} - \frac{(2aAb)}{d} \\
&= -\frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \tan(c + dx)}{\sqrt{2} d}
\end{aligned}$$



**Mathematica [A]**

time = 0.53, size = 221, normalized size = 0.80

$$\frac{\sqrt{\cot(c+dx)} \left( 2\sqrt{2} (a^2(A-B) + b^2(-A+B) - 2ab(A+B)) (\text{ArcTan}(1 - \sqrt{2} \sqrt{\tan(c+dx)}) - \text{ArcTan}(1 + \sqrt{2} \sqrt{\tan(c+dx)})) - \sqrt{2} (2ab(A-B) + a^2(A+B) - b^2(A+B)) (\log(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx))) - \frac{8a^2A}{\sqrt{\tan(c+dx)}} + 8b^2B \sqrt{\tan(c+dx)} \right) \sqrt{\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
[Out] (Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B) - 2*a*b*(A + B))
*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]) - Sqrt[2]*(2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(Log[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]) - (8*a^2*A)/Sqrt[Tan[c + d*x]] + 8*b^2*B*Sqrt[Tan[c + d*x
]])*Sqrt[Tan[c + d*x]])/(4*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 76.87, size = 6315, normalized size = 22.88

method	result	size
default	Expression too large to display	6315

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

[Out] result too large to display

**Maxima [A]**

time = 0.52, size = 244, normalized size = 0.88

$$\frac{8Bb^2\sqrt{\tan(dx+c)} + 2\sqrt{2}(A-B)a^2 - 2(A+B)ab - (A-B)^2\arctan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\tan(dx+c)}{\tan(dx+c)+1}}\right) + 2\sqrt{2}(A-B)a^2 - 2(A+B)ab - (A-B)^2\arctan\left(-\frac{1}{\sqrt{2}}\sqrt{\frac{\tan(dx+c)}{\tan(dx+c)+1}}\right) + \sqrt{2}(A+B)a^2 + 2(A-B)ab - (A+B)^2\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)} + 1} + 1\right) - \sqrt{2}(A+B)a^2 + 2(A-B)ab - (A+B)^2\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)} + 1} + 1\right) - \frac{8a^2A}{\sqrt{\tan(dx+c)}} - \frac{8b^2B}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/4*(8*B*b^2*sqrt(tan(d*x + c)) + 2*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b -
(A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(
2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*
b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A + B
)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/ta
n(d*x + c) + 1) - 8*A*a^2/sqrt(tan(d*x + c))/d
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 \cot^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2*cot(c + d*x)**(3/2), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2), x)`

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)`

[Out] `int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)`

$$3.581 \quad \int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^2 (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=283

$$\frac{(2ab(A-B) + a^2(A+B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} (2ab(A-B) + a^2(A+B) - b^2(A+B))$$

[Out]  $2/3*b^2*B/d/\cot(d*x+c)^{(3/2)}-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(-1+2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\arctan(1+2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+2*b*(A*b+2*B*a)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.28, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {3662, 3685, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A+B) + 2ab(A-B) - b^2(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{2b(2aB + Ab)}{d\sqrt{\cot(c+dx)}} - \frac{2bB}{3d\cot^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^2\*(A + B\*Tan[c + d\*x]),x]

[Out]  $((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - ((2*a*b*(A-B) + a^2*(A+B) - b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) + (2*b^2*B) / (3*d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (2*b*(A*b + 2*a*B)) / (d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) - ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) + ((a^2*(A-B) - b^2*(A-B) - 2*a*b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 631**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 642

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1176

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1179

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1182

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[(-a)*c]$

Rule 3615

$\text{Int}[\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \ :> \ \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.) )^{(p_.)} * ((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)} * ((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \ :> \ \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)} * (b + a*\text{Cot}[e + f*x])^m * (d + c*\text{Cot}[e + f*x])^n, x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3685

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
+ 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

```

### Rule 3709

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 -
a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^2 (A+B \tan(c+dx)) dx &= \int \frac{(b+a \cot(c+dx))^2 (B+A \cot(c+dx))}{\cot^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2b^2 B}{3d \cot^{\frac{3}{2}}(c+dx)} - \int \frac{-b(Ab+2aB) - (2a^2 B)}{d \sqrt{\cot(c+dx)}} dx \\
&= \frac{2b^2 B}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)}{d \sqrt{\cot(c+dx)}} - \int \frac{-2a^2 B}{d \sqrt{\cot(c+dx)}} dx \\
&= \frac{2b^2 B}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)}{d \sqrt{\cot(c+dx)}} + \frac{2a^2 B}{d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2 B}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)}{d \sqrt{\cot(c+dx)}} + \frac{a^2 (2b(Ab+2aB) + 2a^2 B)}{d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2 B}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)}{d \sqrt{\cot(c+dx)}} + \frac{a^2 (2ab(A-B) + a^2(A+B) - b^2(A+B))}{\sqrt{2} d}
\end{aligned}$$





$c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)})-3*B*\sin(d*x+c)*\cos(d*x+c)*a^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*(...$

**Maxima [A]**

time = 0.52, size = 254, normalized size = 0.90

$$\frac{6\sqrt{2}((A+B)^2+2(A-B)ab-(A+B)^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{1}{\tan(dx+c)}\right)\right)+6\sqrt{2}((A+B)^2+2(A-B)ab-(A+B)^2)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{1}{\tan(dx+c)}\right)\right)-3\sqrt{2}((A-B)^2-2(A+B)ab-(A-B)^2)\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}\right)+3\sqrt{2}((A-B)^2-2(A+B)ab-(A-B)^2)\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}\right)-8\left(\frac{B^2+2ABa^2}{\tan(dx+c)}\right)\tan(dx+c)^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/12*(6*\sqrt{2}*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2/\sqrt{\tan(dx+c)}))+6*\sqrt{2}*((A+B)*a^2+2*(A-B)*a*b-(A+B)*b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2/\sqrt{\tan(dx+c)}))-3*\sqrt{2}*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*\log(\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1)+3*\sqrt{2}*((A-B)*a^2-2*(A+B)*a*b-(A-B)*b^2)*\log(-\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1)-8*(B*b^2+3*(2*B*a*b+A*b^2)/\tan(dx+c))*\tan(dx+c)^{(3/2)}/d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a+b\*tan(d\*x+c))\*\*2\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*2\*sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*sqrt(cot(d*x + c)), x
)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

$$3.582 \quad \int \frac{(a+b \tan(c+dx))^2 (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=317

$$\frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - b^2(A-B) - 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{2b^2 B}{5d \cot(c+dx)} + \frac{2b^2 B}{3d \cot(c+dx)^{3/2}} + \frac{2b^2 B}{d \cot(c+dx)^{5/2}} + \frac{2b^2 B}{d \cot(c+dx)^{7/2}} + \frac{2b^2 B}{d \cot(c+dx)^{9/2}} + \frac{2b^2 B}{d \cot(c+dx)^{11/2}} + \frac{2b^2 B}{d \cot(c+dx)^{13/2}} + \frac{2b^2 B}{d \cot(c+dx)^{15/2}} + \frac{2b^2 B}{d \cot(c+dx)^{17/2}} + \frac{2b^2 B}{d \cot(c+dx)^{19/2}} + \frac{2b^2 B}{d \cot(c+dx)^{21/2}} + \frac{2b^2 B}{d \cot(c+dx)^{23/2}} + \frac{2b^2 B}{d \cot(c+dx)^{25/2}} + \frac{2b^2 B}{d \cot(c+dx)^{27/2}} + \frac{2b^2 B}{d \cot(c+dx)^{29/2}} + \frac{2b^2 B}{d \cot(c+dx)^{31/2}}$$

[Out]  $2/5*b^2*B/d/\cot(d*x+c)^{(5/2)}+2/3*b*(A*b+2*B*a)/d/\cot(d*x+c)^{(3/2)}-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+2*(2*A*a*b+B*a^2-B*b^2)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3662, 3685, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$$\frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{(a^2(A-B) - 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2b^2 B}{5d \cot(c+dx)} + \frac{2b^2 B}{3d \cot(c+dx)^{3/2}} + \frac{2b^2 B}{d \cot(c+dx)^{5/2}} + \frac{2b^2 B}{d \cot(c+dx)^{7/2}} + \frac{2b^2 B}{d \cot(c+dx)^{9/2}} + \frac{2b^2 B}{d \cot(c+dx)^{11/2}} + \frac{2b^2 B}{d \cot(c+dx)^{13/2}} + \frac{2b^2 B}{d \cot(c+dx)^{15/2}} + \frac{2b^2 B}{d \cot(c+dx)^{17/2}} + \frac{2b^2 B}{d \cot(c+dx)^{19/2}} + \frac{2b^2 B}{d \cot(c+dx)^{21/2}} + \frac{2b^2 B}{d \cot(c+dx)^{23/2}} + \frac{2b^2 B}{d \cot(c+dx)^{25/2}} + \frac{2b^2 B}{d \cot(c+dx)^{27/2}} + \frac{2b^2 B}{d \cot(c+dx)^{29/2}} + \frac{2b^2 B}{d \cot(c+dx)^{31/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b \operatorname{Tan}[c + d*x])^2 (A + B \operatorname{Tan}[c + d*x]) / \operatorname{Sqrt}[\operatorname{Cot}[c + d*x]], x]$

[Out]  $((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / (\operatorname{Sqrt}[2]*d) - ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / (\operatorname{Sqrt}[2]*d) + (2*b^2*B) / (5*d*\operatorname{Cot}[c + d*x]^{(5/2)}) + (2*b*(A*b + 2*a*B)) / (3*d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (2*(2*a*A*b + a^2*B - b^2*B)) / (d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]] / (2*\operatorname{Sqrt}[2]*d) - ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]] / (2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

$\operatorname{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{With}[\{q = 1 - 4*S \operatorname{implify}[a*(c/b^2)]\}, \operatorname{Dist}[-2/b, \operatorname{Subst}[\operatorname{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)$

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3610

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^(m + 1)/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

#### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3662

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(- (B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A - C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \int \frac{(b + a \cot(c + dx))^2 (B + A \cot(c + dx))}{\cot^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} - \int \frac{-b(Ab + 2aB) - (2aAb + a^2 B - b^2)}{\cot^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} - \int \frac{-2aAb - a^2 B + b^2}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(2aAb + a^2 B - b^2)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(2aAb + a^2 B - b^2)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(2aAb + a^2 B - b^2)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(2aAb + a^2 B - b^2)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(2aAb + a^2 B - b^2)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{2b^2 B}{5d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(Ab + 2aB)}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(2aAb + a^2 B - b^2)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{(a^2(A - B) - b^2(A - B) - 2ab(A + B)) \tan^{-1}\left(1 - \sqrt{2}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.71, size = 255, normalized size = 0.80

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( 30\sqrt{2} (a^2(A-B) + b^2(-A+B) - 2ab(A+B)) (\text{ArcTan}[1 - \sqrt{2} \sqrt{\tan(c+dx)}] - \text{ArcTan}[1 + \sqrt{2} \sqrt{\tan(c+dx)}]) - 15\sqrt{2} (2ab(A-B) + a^2(A+B) - b^2(A+B)) (\log(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx))) - 120(2aAb + a^2B - b^2) \sqrt{\tan(c+dx)} - 40(Ab + 2aB) \tan^2(c+dx) - 24b^2 \tan^4(c+dx) \right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

```
[Out] -1/60*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(30*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B) - 2*a*b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) - 15*Sqrt[2]*(2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - 120*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Tan[c + d*x]] - 40*b*(A*b + 2*a*B)*Tan[c + d*x]^(3/2) - 24*b^2*B*Tan[c + d*x]^(5/2))/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 70.13, size = 3748, normalized size = 11.82

method	result	size
default	Expression too large to display	3748

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$\begin{aligned} & -1/30/d*2^{(1/2)}*(-1+\cos(d*x+c))*(6*b^2*B*2^{(1/2)}-15*B*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*a^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+15*B*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*b^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-15*B*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*a^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+15*B*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*b^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+30*B*EllipticF((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*a^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-30*B*EllipticF((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*b^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-6*B*\cos(d*x+c)*2^{(1/2)}*b^2-15*A*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*a^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+15*A*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*b^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-15*A*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*a^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+15*A*EllipticPi((-cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*b^2*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-60*A*2^{(1/2)}*\cos(d*x+c)^3*a*b-10*A*2^{(1/2)}*b^2*\cos(d*x+c)^2*\sin(d*x+c)+60*A*2^{(1/2)}*\cos(d*x+c)^2*a*b+10*A* \end{aligned}$$

```

cos(d*x+c)*2^(1/2)*b^2*sin(d*x+c)-20*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b+
60*A*EllipticF((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1
/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a
*b+30*B*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^
2*sin(d*x+c)*a*b+30*B*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-
1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/
2)*cos(d*x+c)^2*sin(d*x+c)*a*b+15*I*A*EllipticPi((-cos(d*x+c)-1-sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2*((-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*I*A*EllipticPi((-cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(
d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-15*I*A*Ellip
ticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x
+c)+15*I*A*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*b^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d
*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)^2*sin(d*x+c)-15*I*B*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((co
s(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*
x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*I*B*EllipticPi((-cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*((-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*I*B*EllipticPi((-co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*((-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2)*cos...

```

**Maxima [A]**

time = 0.58, size = 282, normalized size = 0.89

$\frac{8(2Bb^2 + \frac{15Aa^2b}{\sqrt{a^2+c^2}} + \frac{15Bb^2a}{\sqrt{a^2+c^2}})\sin(dx+c)^2 - 30\sqrt{2}(A-B)a^2 - 2(A+B)ab - (A-B)B)\operatorname{atan}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{a^2+c^2}{\tan(dx+c)}}\right) - 30\sqrt{2}(A-B)a^2 - 2(A+B)ab - (A-B)B)\operatorname{atan}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{a^2+c^2}{\tan(dx+c)}}\right) - 15\sqrt{2}(A+B)a^2 + 2(A-B)ab - (A+B)B)\ln\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) + 15\sqrt{2}(A+B)a^2 + 2(A-B)ab - (A+B)B)\ln\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/60\*(8\*(3\*B\*b^2 + 5\*(2\*B\*a\*b + A\*b^2)/tan(d\*x + c) + 15\*(B\*a^2 + 2\*A\*a\*b - B\*b^2)/tan(d\*x + c)^2)\*tan(d\*x + c)^(5/2) - 30\*sqrt(2)\*((A - B)\*a^2 - 2\*(A

+ B)\*a\*b - (A - B)\*b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(d\*x + c))) - 30\*sqrt(2)\*((A - B)\*a^2 - 2\*(A + B)\*a\*b - (A - B)\*b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(d\*x + c)))) - 15\*sqrt(2)\*((A + B)\*a^2 + 2\*(A - B)\*a\*b - (A + B)\*b^2)\*log(sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1) + 15\*sqrt(2)\*((A + B)\*a^2 + 2\*(A - B)\*a\*b - (A + B)\*b^2)\*log(-sqrt(2)/sqrt(tan(d\*x + c)) + 1/tan(d\*x + c) + 1))/d

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^2}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^2/sqrt(cot(c + d\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^2\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^2/sqrt(cot(d\*x + c)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^2}{\sqrt{\cot(c + dx)}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/cot(c + d*x)^(1/2),x)`

[Out] `int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/cot(c + d*x)^(1/2), x)`

$$3.583 \quad \int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=421

$$\frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $\frac{2/21*a*(7*A*a^2-18*A*b^2-21*B*a*b)*\cot(d*x+c)^{(3/2)}/d-2/35*a^2*(11*A*b+7*B*a)*\cot(d*x+c)^{(5/2)}/d-2/7*a*A*\cot(d*x+c)^{(3/2)}*(b+a*\cot(d*x+c))^2/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.57, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3662, 3688, 3718, 3711, 3609, 3615, 1182, 1176, 631, 210, 1179, 642}

3662: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 3688: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 3718: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 3711: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 3609: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 3615: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 1182: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 1176: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 631: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 210: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 1179: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ... 642: Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x] -> ...

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out]  $((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) + (2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / d + (2*a*(7*a^2*A - 18*A*b^2 - 21*a*b*B)*\operatorname{Cot}[c + d*x]^{(3/2)}) / (21*d) - (2*a^2*(11*A*b + 7*a*B)*\operatorname{Cot}[c + d*x]^{(5/2)}) / (35*d) - (2*a*A*\operatorname{Cot}[c + d*x]^{(3/2)}*(b + a*\operatorname{Cot}[c + d*x])^2) / (7*d) - ((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) + ((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

Rule 3609

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

$\int [b \tan[e + f x]] dx /;$  FreeQ[{b, c, d, e, f}, x] && NeQ[c<sup>2</sup> - d<sup>2</sup>, 0] && NeQ[c<sup>2</sup> + d<sup>2</sup>, 0]

### Rule 3662

$\int ((\cot[e + f x] + (f x) g)^{p+1} ((a + b \tan[e + f x])^{m+1} (c + d \tan[e + f x])^n) dx$  := Dist[g<sup>m+n</sup>, Int[(g Cot[e + f x])<sup>p-m-n</sup> (b + a Cot[e + f x])<sup>m</sup> (d + c Cot[e + f x])<sup>n</sup>, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3688

$\int ((a + b \tan[e + f x])^m ((A + B \tan[e + f x])^{n+1} (c + d \tan[e + f x])^{m+n}) dx$  := Simp[b B (a + b Tan[e + f x])<sup>m-1</sup> ((c + d Tan[e + f x])<sup>n+1</sup> / (d f (m+n))), x] + Dist[1/(d(m+n)), Int[(a + b Tan[e + f x])<sup>m-2</sup> (c + d Tan[e + f x])<sup>n</sup> Simp[a<sup>2</sup> A d (m+n) - b B (b c (m-1) + a d (n+1)) + d (m+n) (2 a A b + B (a<sup>2</sup> - b<sup>2</sup>)) Tan[e + f x] - (b B (b c - a d) (m-1) - b (A b + a B) d (m+n)) Tan[e + f x]<sup>2</sup>, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b c - a d, 0] && NeQ[a<sup>2</sup> + b<sup>2</sup>, 0] && NeQ[c<sup>2</sup> + d<sup>2</sup>, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2 m, 2 n]) && !(IGtQ[n, 1] & (& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3711

$\int ((a + b \tan[e + f x])^m ((A + B \tan[e + f x])^{n+1} (c + d \tan[e + f x])^{m+1} (e + f x)^2) dx$  := Simp[C (a + b Tan[e + f x])<sup>m+1</sup> / (b f (m+1)), x] + Int[(a + b Tan[e + f x])<sup>m</sup> Simp[A - C + B Tan[e + f x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A b<sup>2</sup> - a b B + a<sup>2</sup> C, 0] && !LeQ[m, -1]

### Rule 3718

$\int ((a + b \tan[e + f x])^m ((c + d \tan[e + f x])^{n+1} (e + f x)^2) dx$  := Simp[b C Tan[e + f x] ((c + d Tan[e + f x])<sup>n+1</sup> / (d f (n+2))), x] - Dist[1/(d(n+2)), Int[(c + d Tan[e + f x])<sup>n</sup> Simp[b c C - a A d (n+2) - (A b + a B - b C) d (n+2) Tan[e + f x] - (a C d (n+2) - b (c C - B d (n+2))) Tan[e + f x]<sup>2</sup>, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b c - a d, 0] && NeQ[c<sup>2</sup> + d<sup>2</sup>, 0] && !LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \int \sqrt{\cot(c+dx)} (b+a \cot(c+dx))^3(B+A \cot(c+dx)) dx \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c+dx)(b+a \cot(c+dx))^2}{7d} - \frac{2a^2B \cot^{\frac{3}{2}}(c+dx)}{7d} \\
&= -\frac{2a^2(11Ab+7aB) \cot^{\frac{5}{2}}(c+dx)}{35d} - \frac{2aA \cot^{\frac{3}{2}}(c+dx)}{7d} \\
&= \frac{2a(7a^2A-18Ab^2-21abB) \cot^{\frac{3}{2}}(c+dx)}{21d} - \frac{2a^2B \cot^{\frac{3}{2}}(c+dx)}{7d} \\
&= \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B) \sqrt{\cot(c+dx)}}{d} \\
&= \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B) \sqrt{\cot(c+dx)}}{d} \\
&= \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B) \sqrt{\cot(c+dx)}}{d} \\
&= \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B) \sqrt{\cot(c+dx)}}{d} \\
&= \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B) \sqrt{\cot(c+dx)}}{d} \\
&= \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2B) \sqrt{\cot(c+dx)}}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.29, size = 326, normalized size = 0.77

$$\frac{2 \sqrt{\cot(c+dx)} \left( -\frac{(3a^2b(A-B)+b^3(A-B)+a^3(A+B)-3ab^2B) \left( \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right) - \operatorname{ArcTan}\left(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right)\right)}{2\sqrt{2}} - \frac{(a^3(A-B)+3ab^2(A-B)-3a^2b(A+B)+b^3(A+B)) \left( \ln\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}+i\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}+i\sqrt{\tan(c+dx)}}\right) - \ln\left(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}+i\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}+i\sqrt{\tan(c+dx)}}\right)\right)}{4\sqrt{2}} - \frac{a^2A}{7 \tan^2(c+dx)} - \frac{a^2(3a^2+2B)}{5 \tan^2(c+dx)} + \frac{a^2(A-3ab^2-3aB)}{3 \tan^2(c+dx)} + \frac{3a^2Ab-Ab^3+a^3B-3ab^2B}{\sqrt{\tan(c+dx)}} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (2\*sqrt[Cot[c + d\*x]]\*(-1/2\*((3\*a^2\*b\*(A - B) + b^3\*(-A + B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]])/Sqrt[2] - ((a^3\*(A - B) + 3\*a\*b^2\*(-A + B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]))/(4\*Sqrt[2]) - (a^3\*A)/(7\*Tan[c + d\*x]^(7/2)) - (a^2\*(3\*A\*b + a\*B))/(5\*Tan[c + d\*x]^(5/2))

$$+ (a*(a^2*A - 3*A*b^2 - 3*a*b*B))/(3*\text{Tan}[c + d*x]^{(3/2)}) + (3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)/\text{Sqrt}[\text{Tan}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]/d$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 44.04, size = 18343, normalized size = 43.57

method	result	size
default	Expression too large to display	18343

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.51, size = 366, normalized size = 0.87

$$\frac{105\sqrt{2}(A+B)a^3-31A-31a^2B-3(A-B)B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+210\sqrt{2}(A+B)a^3+3(A-B)a^2b-3(A+B)a^2b^2-(A-B)b^3}{2\sqrt{2}} + \frac{105\sqrt{2}(A+B)a^3-31A-31a^2B-3(A-B)B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-105\sqrt{2}(A-B)a^3-31A-31a^2B-3(A+B)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+1\right)+105\sqrt{2}(A-B)a^3-31A-31a^2B-3(A+B)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}-1\right)}{2\sqrt{2}} + \frac{105\sqrt{2}(A+B)a^3-31A-31a^2B-3(A-B)B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+210\sqrt{2}(A+B)a^3+3(A-B)a^2b-3(A+B)a^2b^2-(A-B)b^3}{2\sqrt{2}} - \frac{105\sqrt{2}(A+B)a^3-31A-31a^2B-3(A-B)B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-105\sqrt{2}(A-B)a^3-31A-31a^2B-3(A+B)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+1\right)+105\sqrt{2}(A-B)a^3-31A-31a^2B-3(A+B)B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}-1\right)}{2\sqrt{2}} + \frac{120Aa^3}{\tan(dx+c)^{7/2}} - \frac{840(Ba^3+3Aa^2b-3Ba^2b^2-Ab^3)}{\sqrt{\tan(dx+c)}} - \frac{280(Aa^3-3Ba^2b-3Aa^2b^2)}{\tan(dx+c)^{3/2}} + \frac{168(Ba^3+3Aa^2b)}{\tan(dx+c)^{5/2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] 
$$-1/420*(210*\text{sqrt}(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*\text{arctan}(1/2*\text{sqrt}(2)*( \text{sqrt}(2) + 2/\text{sqrt}(\tan(d*x + c)))) + 210*\text{sqrt}(2) * ((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*\text{arctan}(-1/2*\text{sqrt}(2)*( \text{sqrt}(2) - 2/\text{sqrt}(\tan(d*x + c)))) - 105*\text{sqrt}(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*\log(\text{sqrt}(2)/\text{sqrt}(\tan(d*x + c)) + 1/\tan(d*x + c) + 1) + 105*\text{sqrt}(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*\log(-\text{sqrt}(2)/\text{sqrt}(\tan(d*x + c)) + 1/\tan(d*x + c) + 1) + 120*A*a^3/\tan(d*x + c)^{(7/2)} - 840*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)/\text{sqrt}(\tan(d*x + c)) - 280*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)/\tan(d*x + c)^{(3/2)} + 168*(B*a^3 + 3*A*a^2*b)/\tan(d*x + c)^{(5/2)}/d$$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(9/2)\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**  
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^3\*cot(d\*x + c)^(9/2), x)

**Mupad [F]**  
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3, x)

$$3.584 \quad \int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=380

$$\frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) (a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B))}{\sqrt{2} d}$$

[Out]  $-2/15*a^2*(9*A*b+5*B*a)*\cot(d*x+c)^{(3/2)}/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(-1+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/d*2^{(1/2)}-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(1+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)*\cot(d*x+c)^{(1/2)})}/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)*\cot(d*x+c)^{(1/2)})}/d*2^{(1/2)}+2/5*a*(5*A*a^2-14*A*b^2-15*B*a*b)*\cot(d*x+c)^{(1/2)}/d-2/5*a*A*(b+a*\cot(d*x+c))^2*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.50, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3662, 3688, 3718, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(7/2)}*(a + b*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])]/(\operatorname{Sqrt}[2]*d) - ((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])]/(\operatorname{Sqrt}[2]*d) + (2*a*(5*a^2*A - 14*A*b^2 - 15*a*b*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(5*d) - (2*a^2*(9*A*b + 5*a*B)*\operatorname{Cot}[c + d*x]^{(3/2)})/(15*d) - (2*a*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(b + a*\operatorname{Cot}[c + d*x])^2)/(5*d) + ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

$\operatorname{Int}(((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$



Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
```

$\text{Cot}[e + f*x]^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

### Rule 3688

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] :> \text{Simp}[b*B*(a + b*\tan[e + f*x])^{(m - 1)*((c + d*\tan[e + f*x])^{(n + 1)/(d*f*(m + n))}), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\tan[e + f*x])^{(m - 2)*(c + d*\tan[e + f*x])^n*\text{Simp}[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*\tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*\tan[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 1] \&\& (\text{IntegerQ}[m] \|\ \text{IntegersQ}[2*m, 2*n]) \&\& \text{!(IGtQ}[n, 1] \& \& (!\text{IntegerQ}[m] \|\ (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3711

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)] + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] :> \text{Simp}[C*((a + b*\tan[e + f*x])^{(m + 1)/(b*f*(m + 1))}), x] + \text{Int}[(a + b*\tan[e + f*x])^m*\text{Simp}[A - C + B*\tan[e + f*x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x] \&\& \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \&\& \text{!LeQ}[m, -1]$

### Rule 3718

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)] + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] :> \text{Simp}[b*C*\tan[e + f*x]*((c + d*\tan[e + f*x])^{(n + 1)/(d*f*(n + 2))}), x] - \text{Dist}[1/(d*(n + 2)), \text{Int}[(c + d*\tan[e + f*x])^n*\text{Simp}[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*\tan[e + f*x] - (a*C*d*(n + 2) - b*(c*C - B*d*(n + 2)))*\tan[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!LtQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \int \frac{(b+a \cot(c+dx))^3(B+A \cot(c+dx))}{\sqrt{\cot(c+dx)}} dx \\
&= -\frac{2aA \sqrt{\cot(c+dx)}(b+a \cot(c+dx))^2}{5d} - \frac{2a^2(9Ab+5aB) \cot^{\frac{3}{2}}(c+dx)}{15d} - \frac{2aA \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{2a(5a^2A-14Ab^2-15abB) \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{2a(5a^2A-14Ab^2-15abB) \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{2a(5a^2A-14Ab^2-15abB) \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{2a(5a^2A-14Ab^2-15abB) \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{2a(5a^2A-14Ab^2-15abB) \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{2a(5a^2A-14Ab^2-15abB) \sqrt{\cot(c+dx)}}{5d} \\
&= \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+\dots)}{\sqrt{\cot(c+dx)}}
\end{aligned}$$

**Mathematica [A]**

time = 1.50, size = 286, normalized size = 0.75

$$\frac{2\sqrt{\cot(c+dx)} \left( \frac{(a^3(A-B)+3ab^2(-A+B)-3a^2b(A+B)+b^3(A+B))(\operatorname{ArcTan}(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)})}-\operatorname{ArcTan}(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)})})}{2\sqrt{2}} \right) + \frac{(3a^3(A-B)+b^3(-A+B)+a^3(A+B)-3ab^2(A+B))(\log(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)}{1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)})-\log(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)}{1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)})}{2\sqrt{2}} - \frac{a^3A}{5\sqrt{2}\sqrt{\cot(c+dx)}} - \frac{a^3(3ab+2B)}{3\sqrt{2}\sqrt{\cot(c+dx)}} + \frac{a^3(A-14B^2-3ab)}{\sqrt{\tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(7/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]),x]

[Out] (2\*sqrt[Cot[c + d\*x]]\*(-1/2\*((a^3\*(A - B) + 3\*a\*b^2\*(-A + B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]))/Sqrt[2] + ((3\*a^2\*b\*(A - B) + b^3\*(-A + B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*(Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]))/(4\*Sqrt[2]) - (a^3\*A)/(5\*Tan[c + d\*x]^(5/2)) - (a^2\*(3\*A\*b + a\*B))/(3\*Tan[c + d\*x]^(3/2)) + (a\*(a^2\*A - 3\*A\*b^2 - 3\*a\*b\*B))/Sqrt[Tan[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 41.65, size = 17340, normalized size = 45.63

method	result	size
default	Expression too large to display	17340

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima** [A]

time = 0.51, size = 330, normalized size = 0.87

$$\frac{15\sqrt{2}(A-B)^2-3(A+B)^2-3(A-B)^2+3(A+B)^2\arctan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{2-\tan(d*x+c)}{2+\tan(d*x+c)}}\right)+15\sqrt{2}(A-B)^2-3(A+B)^2-3(A-B)^2+3(A+B)^2\arctan\left(-\frac{1}{\sqrt{2}}\sqrt{\frac{2-\tan(d*x+c)}{2+\tan(d*x+c)}}\right)+15\sqrt{2}(A+B)^2-3(A-B)^2-3(A+B)^2\arctan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{2-\tan(d*x+c)}{2+\tan(d*x+c)}}\right)-15\sqrt{2}(A+B)^2-3(A-B)^2-3(A+B)^2\arctan\left(-\frac{1}{\sqrt{2}}\sqrt{\frac{2-\tan(d*x+c)}{2+\tan(d*x+c)}}\right)+\frac{15\sqrt{2}(A+B)^2-3(A-B)^2-3(A+B)^2}{\sqrt{2+\tan(d*x+c)}}}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] -1/60*(30*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B
)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 30*sqrt(2)*((
A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(-1/2*s
qrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 15*sqrt(2)*((A + B)*a^3 + 3*(A -
B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) +
1/tan(d*x + c) + 1) - 15*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B
)*a*b^2 - (A - B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1
) + 24*A*a^3/tan(d*x + c)^(5/2) - 120*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)/sqrt(
tan(d*x + c)) + 40*(B*a^3 + 3*A*a^2*b)/tan(d*x + c)^(3/2))/d
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(7/2)\*(a+b\*tan(d\*x+c))\*\*3\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^3\*cot(d\*x + c)^(7/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3,x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3, x)

$$3.585 \quad \int \cot^2(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=374

$$\frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $-2/3*a^2*(A*a+3*B*b)*\cot(d*x+c)^{(3/2)}/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}-1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+2*b*B*(b+a*\cot(d*x+c))^2/d/\cot(d*x+c)^{(1/2)}-2*a*(3*A*a*b+B*a^2+2*B*b^2)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.51, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3662, 3686, 3718, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)}{2d} \operatorname{ArcTan}\left(\frac{1 - \sqrt{2} \sqrt{\cot(c+dx)}}{1 + \sqrt{2} \sqrt{\cot(c+dx)}}\right) + \frac{3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)}{2d} \operatorname{ArcTan}\left(\frac{1 + \sqrt{2} \sqrt{\cot(c+dx)}}{1 - \sqrt{2} \sqrt{\cot(c+dx)}}\right) - \frac{2a^2(aA + 3bB) \cot(c+dx)^{3/2}}{3d} + \frac{2bB(b + a \cot(c+dx))^2}{d \sqrt{\cot(c+dx)}} + ((a^3(A-B) - 3a^2b(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{Log}[1 - \sqrt{2} \sqrt{\cot(c+dx)} + \cot(c+dx)] + (a^3(A-B) - 3a^2b(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{Log}[1 + \sqrt{2} \sqrt{\cot(c+dx)} + \cot(c+dx)]) / (2 \sqrt{2} d)$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(5/2)}*(a + b*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-(((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d)) + ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*d) - (2*a*(3*a*A*b + a^2*B + 2*b^2*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / d - (2*a^2*(a*A + 3*b*B)*\operatorname{Cot}[c + d*x]^{(3/2)}) / (3*d) + (2*b*B*(b + a*\operatorname{Cot}[c + d*x])^2) / (d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
```

[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[C*((a +
b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rule 3718

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
!LtQ[n, -1]
```

### Rubi steps



$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \int \frac{(b+a \cot(c+dx))^3(B+A \cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2bB(b+a \cot(c+dx))^2}{d \sqrt{\cot(c+dx)}} - 2 \int \frac{(b+a \cot(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)} dx \\
&= -\frac{2a^2(aA+3bB) \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{2bB(b+a \cot(c+dx))^2}{d \sqrt{\cot(c+dx)}} \\
&= -\frac{2a(3aAb+a^2B+2b^2B) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{2a(3aAb+a^2B+2b^2B) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{2a(3aAb+a^2B+2b^2B) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{2a(3aAb+a^2B+2b^2B) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{2a(3aAb+a^2B+2b^2B) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{2a(3aAb+a^2B+2b^2B) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B))}{d}
\end{aligned}$$

**Mathematica [A]**

time = 1.29, size = 270, normalized size = 0.72

$$\frac{2 \sqrt{\cot(c+dx)} \left( \frac{(3a^2b(A-B)+b^3(A-B)+a^3(A+B)) \left( \frac{\text{ArcTan}\left[1-\sqrt{2}\sqrt{\tan(c+dx)}\right]-\text{ArcTan}\left[1+\sqrt{2}\sqrt{\tan(c+dx)}\right]}{2\sqrt{2}} \right) + \frac{(a^3(A-B)+3ab^2(-A+B)-3a^2(A+B)+b^3(A+B)) \left( \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{4\sqrt{2}} \right) - \frac{2a^2A}{3a^2(c+dx)} - \frac{2^{\frac{3}{2}}b^2aB}{\sqrt{\tan(c+dx)}} + b^2B\sqrt{\tan(c+dx)} \right) \sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```

[In] Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]), x]
[Out] (2*sqrt[Cot[c + d*x]]*(((3*a^2*b*(A - B) + b^3*(-A + B) + a^3*(A + B) - 3*a
*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]))/(2*sqrt[2]) + ((a^3*(A - B) + 3*a*b^2*(-A + B) - 3*a^
2*b*(A + B) + b^3*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(4*sqrt[2]) - (a
^3*A)/(3*Tan[c + d*x]^(3/2)) - (a^2*(3*A*b + a*B))/Sqrt[Tan[c + d*x]] + b^3
*B*Sqrt[Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/d

```



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

**Giac** [F]

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2), x)
```

**Mupad** [F]

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

```
[Out] int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

$$3.586 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=372

$$\frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (a^3(A-B) - 3a^2b^2(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $2/3*b*B*(b+a*\cot(d*x+c))^2/d/\cot(d*x+c)^{(3/2)}+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\ln(1+\cot(d*x+c))+2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}+2/3*b^2*(3*A*b+7*B*a)/d/\cot(d*x+c)^{(1/2)}-2/3*a^2*(3*A*a+B*b)*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.46, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3662, 3686, 3716, 3711, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{2^{(1/2)}*(a+b*\cot(d*x+c))}{\sqrt{2}} - \frac{2^{(1/2)}*(a-b-3*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))}{\sqrt{2}} \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right) - \frac{2^{(1/2)}*(a-b-3*b^2*(A+B)+b^3*(A+B))}{\sqrt{2}} \operatorname{ArcTan}\left(\frac{1+\sqrt{2}\sqrt{\cot(c+dx)}}{1-\sqrt{2}\sqrt{\cot(c+dx)}}\right) - \frac{2^{(1/2)}*(a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))}{\sqrt{2}} \ln(1+\cot(c+dx)) + \frac{2^{(1/2)}*(a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))}{\sqrt{2}} \ln(1-\cot(c+dx)) + \frac{2^{(1/2)}*(a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))}{\sqrt{2}} \ln(1+\cot(c+dx)) + \frac{2^{(1/2)}*(a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))}{\sqrt{2}} \ln(1-\cot(c+dx)) + \frac{2^{(1/2)}*(a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))}{\sqrt{2}} \ln(1+\cot(c+dx)) + \frac{2^{(1/2)}*(a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))}{\sqrt{2}} \ln(1-\cot(c+dx))$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c+d*x]^{(3/2)}*(a+b*\operatorname{Tan}[c+d*x])^3*(A+B*\operatorname{Tan}[c+d*x]),x]$

[Out]  $-(((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]]) / (\operatorname{Sqrt}[2]*d)) + ((a^3*(A-B) - 3*a*b^2*(A-B) - 3*a^2*b*(A+B) + b^3*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]]) / (\operatorname{Sqrt}[2]*d) + (2*b^2*(3*A*b + 7*a*B)) / (3*d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]) - (2*a^2*(3*a*A + b*B)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]) / (3*d) + (2*b*B*(b + a*\operatorname{Cot}[c+d*x])^2) / (3*d*\operatorname{Cot}[c+d*x]^{(3/2)}) - ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]] + \operatorname{Cot}[c+d*x]] / (2*\operatorname{Sqrt}[2]*d) + ((3*a^2*b*(A-B) - b^3*(A-B) + a^3*(A+B) - 3*a*b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]] + \operatorname{Cot}[c+d*x]] / (2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 631**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
```

[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*((a
+ b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f
_.)*(x_)])^2, x_Symbol] := Simp[(-b*c - a*d)*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
+ d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
-1]
```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx &= \int \frac{(b+a \cot(c+dx))^3(B+A \cot(c+dx))}{\cot^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2bB(b+a \cot(c+dx))^2}{3d \cot^{\frac{3}{2}}(c+dx)} - \frac{2}{3} \int \frac{(b+a \cot(c+dx))^2}{\cot^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} + \frac{2bB(b+a \cot(c+dx))^2}{3d \cot^{\frac{3}{2}}(c+dx)} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{2b^2(3Ab+7aB)}{3d \sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB) \sqrt{\cot(c+dx)}}{3d} \\
&= \frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B))}{3d}
\end{aligned}$$

**Mathematica [A]**

time = 1.31, size = 270, normalized size = 0.73

$$\frac{2 \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( \frac{(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B)) (\text{ArcTan}[1 - \sqrt{2} \sqrt{\tan(c+dx)}] - \text{ArcTan}[1 + \sqrt{2} \sqrt{\tan(c+dx)})}{2\sqrt{2}} \dots \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

[Out] (2\*sqrt[Cot[c + d\*x]]\*sqrt[Tan[c + d\*x]]\*(((a^3\*(A - B) + 3\*a\*b^2\*(-A + B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*(ArcTan[1 - Sqrt[2]\*sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*sqrt[Tan[c + d\*x]]]))/(2\*sqrt[2]) - ((3\*a^2\*b\*(A - B) + b^3\*(-A + B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*(Log[1 - Sqrt[2]\*sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]))/(4\*sqrt[2]) - (a^3\*A)/sqrt[Tan[c + d\*x]] + b^2\*(A\*b + 3\*a\*B)\*sqrt[Tan[c + d\*x]] + (b^3\*B\*Tan[c + d\*x]^(3/2))/3)/d

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 69.73, size = 8811, normalized size = 23.69

method	result	size
default	Expression too large to display	8811

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.51, size = 316, normalized size = 0.85

$$\frac{-4\sqrt{2}(A+Ba^2-3A+3Bb^2-3A+3B) \operatorname{arctan}\left(\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right) - 4\sqrt{2}(A+Ba^2-3A+3Bb^2-3A+3B) \operatorname{arctan}\left(-\frac{1+\sqrt{2}\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right) - 3\sqrt{2}(A+Ba^2+3A-3Bb^2-3A+3B) \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right) + 3\sqrt{2}(A+Ba^2+3A-3Bb^2-3A+3B) \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)}}\right) - 3\left(B^2 + \frac{3ABa^2}{\tan(dx+c)}\right) \tan(dx+c)^{3/2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm  
="maxima")`

[Out] 
$$\begin{aligned} & -1/12*(24*A*a^3/\sqrt{\tan(dx+c)} - 6*\sqrt{2}*((A-B)*a^3 - 3*(A+B)*a^2*b - 3*(A-B)*a*b^2 + (A+B)*b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(dx+c)})) \\ & - 6*\sqrt{2}*((A-B)*a^3 - 3*(A+B)*a^2*b - 3*(A-B)*a*b^2 + (A+B)*b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(dx+c)})) - 3*\sqrt{2} \\ & *((A+B)*a^3 + 3*(A-B)*a^2*b - 3*(A+B)*a*b^2 - (A-B)*b^3)*\log(\sqrt{2}/\sqrt{\tan(dx+c)} + 1/\tan(dx+c) + 1) + 3*\sqrt{2}*((A+B)*a^3 + \\ & 3*(A-B)*a^2*b - 3*(A+B)*a*b^2 - (A-B)*b^3)*\log(-\sqrt{2}/\sqrt{\tan(dx+c)} + 1/\tan(dx+c) + 1) - 8*(B*b^3 + 3*(3*B*a*b^2 + A*b^3)/\tan(dx+c) \\ & )*\tan(dx+c)^{(3/2)}/d \end{aligned}$$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 \cot^{\frac{3}{2}}(c + dx) dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*cot(c + d*x)**(3/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)`

[Out] `int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)`

$$3.587 \quad \int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^3 (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=380

$$\frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $2/15*b^2*(5*A*b+9*B*a)/d/\cot(d*x+c)^{(3/2)}+2/5*b*B*(b+a*\cot(d*x+c))^2/d/\cot(d*x+c)^{(5/2)}-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}+1/4*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*\ln(1+\cot(d*x+c))+2^{(1/2)}*\cot(d*x+c)^{(1/2)}/d*2^{(1/2)}+2/5*b*(15*A*a*b+14*B*a^2-5*B*b^2)/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.47, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3662, 3686, 3716, 3709, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{3a^2b^2 + 15ab^2 - 3b^3}{4\sqrt{\cot(c+dx)}}, \frac{a^3(A+B) + 3a^2b(A-B) - 3ab^2(A+B) - b^3(A-B)}{\sqrt{2}d} \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right), \frac{a^3(A+B) + 3a^2b(A-B) - 3ab^2(A+B) - b^3(A-B)}{\sqrt{2}d} \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right), \frac{a^3(A-B) - 3a^2b(A+B) - 3ab^2(A-B) + b^3(A+B)}{2\sqrt{2}d} \ln\left(\frac{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}\right), \frac{a^3(A-B) - 3a^2b(A+B) - 3ab^2(A-B) + b^3(A+B)}{2\sqrt{2}d} \ln\left(\frac{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}\right), \frac{2^2 \ln(b + a \cot(c+dx))}{5d \cot(c+dx)}, \frac{2 \ln(b + a \cot(c+dx))}{5d \cot(c+dx)}$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^3*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(\operatorname{Sqrt}[2]*d) + (2*b^2*(5*A*b + 9*a*B))/(15*d*\operatorname{Cot}[c + d*x]^{(3/2)}) + (2*b*(15*A*a*b + 14*a^2*B - 5*b^2*B))/(5*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (2*b*B*(b + a*\operatorname{Cot}[c + d*x])^2)/(5*d*\operatorname{Cot}[c + d*x]^{(5/2)}) - ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

**Rule 210**

$\operatorname{Int}(((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1}*(-1))*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
```

$\text{Cot}[e + f*x]^n, x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3686

$\text{Int}[(a + b*\tan[e + f*x])^m * ((A + B*\tan[e + f*x]) + (c + d*\tan[e + f*x])^n), x\_Symbol] :> \text{Simp}[(b*c - a*d)*(B*c - A*d)*(a + b*\tan[e + f*x])^{m-1} * ((c + d*\tan[e + f*x])^{n+1} / (d*f*(n+1)*(c^2 + d^2))), x] - \text{Dist}[1/(d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\tan[e + f*x])^{m-2} * (c + d*\tan[e + f*x])^{n+1} * \text{Simp}[a*A*d*(b*d*(m-1) - a*c*(n+1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m-1) + a*d*(n+1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n+1)*\tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m+n) - b*B*(c^2*(m-1) - d^2*(n+1)))*\tan[e + f*x]^2, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3709

$\text{Int}[(a + b*\tan[e + f*x])^m * ((A + B*\tan[e + f*x]) + (c + d*\tan[e + f*x])^2), x\_Symbol] :> \text{Simp}[(A*b^2 - a*b*B + a^2*C) * ((a + b*\tan[e + f*x])^{m+1} / (b*f*(m+1)*(a^2 + b^2))), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\tan[e + f*x])^{m+1} * \text{Simp}[b*B + a*(A - C) - (A*b - a*B - b*C)*\tan[e + f*x], x], x], x] /;$  FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rule 3716

$\text{Int}[(a + b*\tan[e + f*x]) * ((c + d*\tan[e + f*x]) + (c + d*\tan[e + f*x])^n) * ((A + B*\tan[e + f*x]) + (c + d*\tan[e + f*x])^2), x\_Symbol] :> \text{Simp}[(-b*c - a*d) * (c^2*C - B*c*d + A*d^2) * ((c + d*\tan[e + f*x])^{n+1} / (d^2*f*(n+1)*(c^2 + d^2))), x] + \text{Dist}[1/(d*(c^2 + d^2)), \text{Int}[(c + d*\tan[e + f*x])^{n+1} * \text{Simp}[a*d*(A*c - c*C + B*d) + b*(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*\tan[e + f*x] + b*C*(c^2 + d^2)*\tan[e + f*x]^2, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^3 (A+B \tan(c+dx)) dx &= \int \frac{(b+a \cot(c+dx))^3 (B+A \cot(c+dx))}{\cot^{\frac{7}{2}}(c+dx)} \\
&= \frac{2bB(b+a \cot(c+dx))^2}{5d \cot^{\frac{5}{2}}(c+dx)} - \frac{2}{5} \int \frac{(b+a \cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a \cot(c+dx))}{5d \cot^{\frac{5}{2}}(c+dx)} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2)}{5d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2)}{5d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2)}{5d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2)}{5d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2)}{5d \sqrt{\cot(c+dx)}} \\
&= \frac{2b^2(5Ab+9aB)}{15d \cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2)}{5d \sqrt{\cot(c+dx)}} \\
&= \frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - \dots)}{5d \sqrt{\cot(c+dx)}}
\end{aligned}$$

**Mathematica [A]**

time = 0.79, size = 287, normalized size = 0.76

$$\frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{-3a^2b(A-B)+a^3(A+B)-3a^2b(A-B)+a^3(A+B)}{2\sqrt{2}}\left(\frac{\text{ArcTan}\left[\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}\right)-\frac{\text{ArcTan}\left[\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{1-\sqrt{2}\sqrt{\tan(c+dx)}}\right]}{\sqrt{2}}\right)+b^2(3aB+3a^2B-b^2B)\sqrt{\tan(c+dx)}+\frac{1}{2}b^2(A+B)\tan(c+dx)+\frac{1}{2}b^2B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]), x]

```

[Out] (2*sqrt(Cot[c + d*x])*sqrt(Tan[c + d*x])*(-1/2*((3*a^2*b*(A - B) + b^3*(-A + B) + a^3*(A + B) - 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*sqrt(Tan[c + d*x]]) - ArcTan[1 + Sqrt[2]*sqrt(Tan[c + d*x]])/Sqrt[2] - ((a^3*(A - B) + 3*a*b^2*(-A + B) - 3*a^2*b*(A + B) + b^3*(A + B))*(Log[1 - Sqrt[2]*sqrt(Tan[c + d*x]]) + Tan[c + d*x]] - Log[1 + Sqrt[2]*sqrt(Tan[c + d*x]]) + Tan[c + d*x]))/(4*sqrt[2]) + b*(3*a*A*b + 3*a^2*B - b^2*B)*sqrt(Tan[c + d*x]) + (b^2*(A*b + 3*a*B)*Tan[c + d*x]^(3/2))/3 + (b^3*B*Tan[c + d*x]^(5/2))/5)/d

```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 64.80, size = 4875, normalized size = 12.83

method	result	size
default	Expression too large to display	4875

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVE  
RBOSE)`

[Out] 
$$-1/30/d*2^{(1/2)}*(-1+\cos(d*x+c))*(-30*\sin(d*x+c)*B*b^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticF}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)}+15*\sin(d*x+c)*A*\cos(d*x+c)^2*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*a^3+15*\sin(d*x+c)*A*\cos(d*x+c)^2*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*b^3+15*\sin(d*x+c)*A*a^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}))+15*\sin(d*x+c)*A*b^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}))-30*\sin(d*x+c)*A*\cos(d*x+c)^2*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*\text{EllipticF}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*a^3-15*\sin(d*x+c)*B*a^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)}))+15*\sin(d*x+c)*B*b^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)}))-15*\sin(d*x+c)*B*a^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}))+15*\sin(d*x+c)*B*b^3*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*\cos(d*x+c)^2*\text{EllipticPi}(((1-\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)}))+6*B*2^{(1/2)}*b^3-90*A*2^{(1/2)}*\cos(d*x+c)^3*a*b^2-90*B*2^{(1/2)}*\cos(d*x+c)^3*a^2*b+10*2^{(1/2)}*\sin(d*x+c)*A*\cos(d*x+c)*b^3+90*2^{(1/2)}*A*\cos(d*x+c)^2*a*b^2+90*2^{(1/2)}*B*\cos(d*x+c)$$



$$3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(dx + c)})) - 30*\sqrt{2}*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(dx + c)})) + 15*\sqrt{2}*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*\log(\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - 15*\sqrt{2}*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*\log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1))/d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(a+b\*tan(dx+c))^3\*(A+B\*tan(dx+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*(1/2)\*(a+b\*tan(dx+c))\*\*3\*(A+B\*tan(dx+c)),x)

[Out] Integral((A + B\*tan(c + dx))\*(a + b\*tan(c + dx))\*\*3\*sqrt(cot(c + dx)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(a+b\*tan(dx+c))^3\*(A+B\*tan(dx+c)),x, algorithm="giac")

[Out] integrate((B\*tan(dx + c) + A)\*(b\*tan(dx + c) + a)^3\*sqrt(cot(dx + c)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^3 dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

**3.588** 
$$\int \frac{(a+b \tan(c+dx))^3 (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=421

$$\frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} (a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right) + \dots$$

[Out] 2/35\*b^2\*(7\*A\*b+11\*B\*a)/d/cot(d\*x+c)^(5/2)+2/21\*b\*(21\*A\*a\*b+18\*B\*a^2-7\*B\*b^2)/d/cot(d\*x+c)^(3/2)+2/7\*b\*B\*(b+a\*cot(d\*x+c))^2/d/cot(d\*x+c)^(7/2)-1/2\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)-3\*a^2\*b\*(A+B)+b^3\*(A+B))\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/d\*2^(1/2)-1/2\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)-3\*a^2\*b\*(A+B)+b^3\*(A+B))\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/d\*2^(1/2)+1/4\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)+a^3\*(A+B)-3\*a\*b^2\*(A+B))\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/d\*2^(1/2)-1/4\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)+a^3\*(A+B)-3\*a\*b^2\*(A+B))\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/d\*2^(1/2)+2\*(3\*A\*a^2\*b-A\*b^3+B\*a^3-3\*B\*a\*b^2)/d/cot(d\*x+c)^(1/2)

**Rubi [A]**

time = 0.52, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3662, 3686, 3716, 3709, 3610, 3615, 1182, 1176, 631, 210, 1179, 642}

$\frac{2(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{2(a^3(A-B) - 3ab^2(A-B) - 3a^2b(A+B) + b^3(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{2b^2(7Ab + 11Ba)}{35d \cot(c + dx)^{5/2}} + \frac{2b(21Aab + 18A^2a - 7Bb^2)}{21d \cot(c + dx)^{3/2}} + \frac{2bB(b + a \cot(c + dx))^2}{7d \cot(c + dx)^{7/2}} + \frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx)\right]}{2\sqrt{2} d} - \frac{(3a^2b(A-B) - b^3(A-B) + a^3(A+B) - 3ab^2(A+B)) \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx)\right]}{2\sqrt{2} d} + \frac{2(3Aa^2b - Ab^3 + Ba^3 - 3Bab^2)}{d \cot(c + dx)^{1/2}}$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]],x]

[Out] ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*d) - ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*d) + (2\*b^2\*(7\*A\*b + 11\*a\*B))/(35\*d\*Cot[c + d\*x]^(5/2)) + (2\*b\*(21\*a\*A\*b + 18\*a^2\*B - 7\*b^2\*B))/(21\*d\*Cot[c + d\*x]^(3/2)) + (2\*(3\*a^2\*A\*b - A\*b^3 + a^3\*B - 3\*a\*b^2\*B))/(d\*Sqrt[Cot[c + d\*x]]) + (2\*b\*B\*(b + a\*Cot[c + d\*x])^2)/(7\*d\*Cot[c + d\*x]^(7/2)) + ((3\*a^2\*b\*(A - B) - b^3\*(A - B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(2\*Sqrt[2]\*d) - ((3\*a^2\*b\*(A - B) - b^3\*(A - B) + a^3\*(A + B) - 3\*a\*b^2\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(2\*Sqrt[2]\*d)

**Rule 210**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 631

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)*c]
```

Rule 3610

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

$t[b*\tan[e + f*x]]$ ,  $x$  /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

Int[(cot[(e\_.) + (f\_.)\*(x\_.)]\*(g\_.))^ (p\_.)\*((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^ (m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^ (n\_.), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^ (m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^ (n\_.), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3709

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^ (m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*b\*B + a^2\*C)\*((a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rule 3716

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^ (n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] := Simp[(-b\*c - a\*d)\*(c^2\*C - B\*c\*d + A\*d^2)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d^2\*f\*(n + 1)\*(c^2 + d^2))), x] + Dist[1/(d\*(c^2 + d^2)), Int[(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*d\*(A\*c - c\*C + B\*d) + b\*(c^2\*C - B\*c\*d + A\*d^2) + d\*(A\*b\*c + a\*B\*c - b\*c\*C - a\*A\*d + b\*B\*d + a\*C\*d)\*Tan[e + f\*x] + b\*C\*(c^2 + d^2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan(c + dx))^3 (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \int \frac{(b + a \cot(c + dx))^3 (B + A \cot(c + dx))}{\cot^{\frac{9}{2}}(c + dx)} dx \\
 &= \frac{2bB(b + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} - \frac{2}{7} \int \frac{(b + a \cot(c + dx)) (-\frac{1}{2}b)}{\cot^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2bB(b + a \cot(c + dx))^2}{7d \cot^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{\frac{1}{2}b(2)}{\cot^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2bB}{7} \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(3a^2)}{7} \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(3a^2)}{7} \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(3a^2)}{7} \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(3a^2)}{7} \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(3a^2)}{7} \\
 &= \frac{2b^2(7Ab + 11aB)}{35d \cot^{\frac{5}{2}}(c + dx)} + \frac{2b(21aAb + 18a^2B - 7b^2B)}{21d \cot^{\frac{3}{2}}(c + dx)} + \frac{2(3a^2)}{7} \\
 &= \frac{(a^3(A - B) - 3ab^2(A - B) - 3a^2b(A + B) + b^3(A + B))}{\sqrt{2} d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.59, size = 327, normalized size = 0.78

$$\frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))}{\sqrt{2}}\operatorname{ArcTan}\left(\frac{-\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{\tan(c+dx)}}\right)-\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\sqrt{\tan(c+dx)}}\right)\right)+\frac{(2b^2(7Ab+11aB)+2b(21aAb+18a^2B-7b^2B)+2(3a^2))\sqrt{\tan(c+dx)}}{d}}{\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^3\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]], x]

[Out] (2\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-1/2\*((a^3\*(A - B) + 3\*a\*b^2\*(-A + B) - 3\*a^2\*b\*(A + B) + b^3\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]))/Sqrt[2] + ((3\*a^2\*b\*(A - B) +

$$b^3(-A + B) + a^3(A + B) - 3ab^2(A + B))(\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + dx]] + \text{Tan}[c + dx]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + dx]] + \text{Tan}[c + dx]])/(4*\text{Sqrt}[2]) + (3a^2Ab - Ab^3 + a^3B - 3ab^2B)*\text{Sqrt}[\text{Tan}[c + dx]] + (b*(3aAb + 3a^2B - b^2B)*\text{Tan}[c + dx]^{(3/2)})/3 + (b^2*(Ab + 3aB)*\text{Tan}[c + dx]^{(5/2)})/5 + (b^3B*\text{Tan}[c + dx]^{(7/2)})/7)/d$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 75.60, size = 5039, normalized size = 11.97

method	result	size
default	Expression too large to display	5039

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.55, size = 370, normalized size = 0.88

$$\frac{(1120a^3b^3 + 1120a^2b^3 + 1120ab^3 + 1120b^3) \sqrt{2} \sqrt{\tan(d*x+c)} + 1120a^3b^3 + 1120a^2b^3 + 1120ab^3 + 1120b^3}{1120a^3b^3 + 1120a^2b^3 + 1120ab^3 + 1120b^3} \sqrt{2} \sqrt{\tan(d*x+c)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{1}{420} * (8 * (15 * B * b^3 + 21 * (3 * B * a * b^2 + A * b^3) / \tan(d*x + c) + 35 * (3 * B * a^2 * b + 3 * A * a * b^2 - B * b^3) / \tan(d*x + c)^2 + 105 * (B * a^3 + 3 * A * a^2 * b - 3 * B * a * b^2 - A * b^3) / \tan(d*x + c)^3) * \tan(d*x + c)^{(7/2)} - 210 * \text{sqrt}(2) * ((A - B) * a^3 - 3 * (A + B) * a^2 * b - 3 * (A - B) * a * b^2 + (A + B) * b^3) * \arctan(1/2 * \text{sqrt}(2) * (\text{sqrt}(2) + 2 / \text{sqrt}(\tan(d*x + c)))) - 210 * \text{sqrt}(2) * ((A - B) * a^3 - 3 * (A + B) * a^2 * b - 3 * (A - B) * a * b^2 + (A + B) * b^3) * \arctan(-1/2 * \text{sqrt}(2) * (\text{sqrt}(2) - 2 / \text{sqrt}(\tan(d*x + c)))) - 105 * \text{sqrt}(2) * ((A + B) * a^3 + 3 * (A - B) * a^2 * b - 3 * (A + B) * a * b^2 - (A - B) * b^3) * \log(\text{sqrt}(2) / \text{sqrt}(\tan(d*x + c)) + 1 / \tan(d*x + c) + 1) + 105 * \text{sqrt}(2) * ((A + B) * a^3 + 3 * (A - B) * a^2 * b - 3 * (A + B) * a * b^2 - (A - B) * b^3) * \log(-\text{sqrt}(2) / \text{sqrt}(\tan(d*x + c)) + 1 / \tan(d*x + c) + 1)) / d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^3}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*3\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*3/sqrt(cot(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^3\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^3/sqrt(cot(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^3}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3)/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^3)/cot(c + d\*x)^(1/2), x)

$$3.589 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))^5}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=325

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $-2*b^{(5/2)}*(A*b-B*a)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/a^{(5/2)}/(a^2+b^2)/d-2/3*A*\cot(d*x+c)^{(3/2)}/a/d-1/2*(b*(A-B)-a*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(b*(A-B)-a*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a*(A-B)+b*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}-1/4*(a*(A-B)+b*(A+B))*\ln(1+\cot(d*x+c))+2^{(1/2)}*\cot(d*x+c)^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}+2*(A*b-B*a)*\cot(d*x+c)^{(1/2)}/a^2/d$

**Rubi [A]**

time = 0.74, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3688, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(a(A-B) + b(A+B)) \log\left(\frac{\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1}{2\sqrt{2} d (a^2 + b^2)}\right)}{2\sqrt{2} d (a^2 + b^2)} - \frac{(a(A-B) + b(A+B)) \log\left(\frac{\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1}{2\sqrt{2} d (a^2 + b^2)}\right)}{2\sqrt{2} d (a^2 + b^2)} + \frac{2(Ab - aB) \sqrt{\cot(c+dx)}}{a^2 d} - \frac{2b^{5/2}(Ab - aB) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\cot(c+dx)}}{\sqrt{2}}\right)}{a^{5/2} d (a^2 + b^2)} - \frac{2A \cot^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c + d*x]^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - (2*b^{(5/2)}*(A*b - a*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]]/(a^{(5/2)}*(a^2 + b^2)*d) + (2*(A*b - a*B))*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]/(a^2*d) - (2*A*\operatorname{Cot}[c + d*x]^{(3/2)})/(3*a*d) + ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 210**



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3688

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] :>
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3728

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

## Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

## Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx &= \int \frac{\cot^{\frac{5}{2}}(c+dx)(B+A \cot(c+dx))}{b+a \cot(c+dx)} dx \\
&= -\frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad} - \frac{2 \int \frac{\sqrt{\cot(c+dx)} \left(\frac{3Ab}{2} + \frac{3}{2}aA \cot(c+dx) + \frac{3}{2}(Ab-aB)\right)}{b+a \cot(c+dx)} dx}{3a} \\
&= \frac{2(Ab-aB) \sqrt{\cot(c+dx)}}{a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{4 \int \frac{\frac{3}{4}b(Ab-aB) - \frac{3}{4}a^2}{\sqrt{\cot(c+dx)}} dx}{3a^2} \\
&= \frac{2(Ab-aB) \sqrt{\cot(c+dx)}}{a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{4 \int \frac{\frac{3}{4}a^2(Ab-aB) - \frac{3}{4}a^2}{\sqrt{\cot(c+dx)}} dx}{3a^2} \\
&= \frac{2(Ab-aB) \sqrt{\cot(c+dx)}}{a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad} + \frac{8 \text{Subst}\left(\int \frac{-\frac{3}{4}a^2}{\sqrt{\cot(c+dx)}} dx\right)}{3a^2} \\
&= \frac{2(Ab-aB) \sqrt{\cot(c+dx)}}{a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad} - \frac{(2b^3(Ab-aB))}{3a^2} \\
&= -\frac{2b^{5/2}(Ab-aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)d} + \frac{2(Ab-aB) \sqrt{\cot(c+dx)}}{a^2d} \\
&= -\frac{2b^{5/2}(Ab-aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)d} + \frac{2(Ab-aB) \sqrt{\cot(c+dx)}}{a^2d} \\
&= \frac{(b(A-B) - a(A+B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(b(A-B) - a(A+B)) \sqrt{\cot(c+dx)}}{\sqrt{2}(a^2+b^2)d}
\end{aligned}$$

**Mathematica** [A]

time = 1.02, size = 272, normalized size = 0.84

$$\frac{\sqrt{\cot(c+dx)} \left( \frac{a\sqrt{2} (b(-A+B)+(A+B)) (\text{ArcTan}(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{a+2b}) - \text{ArcTan}(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{a+2b}))}{a^2+2b^2} + \frac{2ab^2(-A+B) \text{ArcTan}(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{a+2b})}{a^2+2b^2} - \frac{2\sqrt{2} (b(A-B)+(A+B)) (\ln(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{a+2b}) - \ln(\frac{1+\sqrt{2}\sqrt{\tan(c+dx)}}{a+2b}))}{a^2+2b^2} + \frac{2b}{a\sqrt{2}(a+2b)} + \frac{2b(-ab+2b)}{a^2\sqrt{\tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
[Out] -1/12*(Sqrt[Cot[c + d*x]]*((-6*Sqrt[2]*(b*(-A + B) + a*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (24*b^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(5/2)*(a^2 + b^2)) - (3*Sqrt[2]*(a*(A - B) + b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) + (8*A)/(a*Tan[c + d*x]^(3/2)) + (24*(-(A*b) + a*B))/(a^2*Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]]/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 38.60, size = 22300, normalized size = 68.62

method	result	size
default	Expression too large to display	22300

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [A]**

time = 0.53, size = 262, normalized size = 0.81

$$\frac{24(Ba^2 - B^2) \arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right) + \left(\sqrt{2} \sqrt{(A+B)(-A-B)} \arctan\left(\frac{1}{\sqrt{2}} \sqrt{\frac{a}{\tan(dx+c)}}\right) + \sqrt{2} \sqrt{(A-B)(-A-B)} \arctan\left(\frac{1}{\sqrt{2}} \sqrt{\frac{a}{\tan(dx+c)}}\right)\right) \sqrt{2} \sqrt{(A-B)(A+B)} \ln\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)}\right) + \sqrt{2} \sqrt{(A-B)(A+B)} \ln\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} - \frac{1}{\tan(dx+c)}\right) - \left(\frac{A}{\tan(dx+c)} + \frac{2(Ba - B^2)}{\sqrt{\tan(dx+c)}}\right)}{(a^2 + b^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(24*(B*a*b^3 - A*b^4)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4 + a^2*b^2)*sqrt(a*b)) + 3*(2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)*a + (A + B)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a + (A + B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^2 + b^2) - 8*(A*a/tan(d*x + c)^(3/2) + 3*(B*a - A*b)/sqrt(tan(d*x + c)))/a^2/d
```

**Fricas** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]  
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(5/2)/(b\*tan(d\*x + c) + a), x)

**Mupad** [F]  
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{5/2} (A + B \tan(c + dx))}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] int((cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)), x)

$$3.590 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=297

$$\frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $2*b^{(3/2)}*(A*b-B*a)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/a^{(3/2)}/(a^2+b^2)/d+1/2*(a*(A-B)+b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(a*(A-B)+b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(b*(A-B)-a*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(b*(A-B)-a*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-2*A*\cot(d*x+c)^{(1/2)}/a/d$

**Rubi [A]**

time = 0.50, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3662, 3688, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(b(A-B) - a(A+B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)} - \frac{(b(A-B) - a(A+B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)} + \frac{2b^{3/2}(Ab - aB) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2} d (a^2 + b^2)} - \frac{2A \sqrt{\cot(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c + d*x]^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-(((a*(A - B) + b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d)) + ((a*(A - B) + b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*b^{(3/2)}*(A*b - a*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/(a^{(3/2)}*(a^2 + b^2)*d) - (2*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(a*d) + ((b*(A - B) - a*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((b*(A - B) - a*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 210**

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

$\text{t}[b*\text{Tan}[e + f*x]]], x] /;$  FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.))^{\text{p}_.}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}, x\_Symbol] := \text{Dist}[g^{\text{m} + \text{n}}, \text{Int}[(g*\text{Cot}[e + f*x])^{\text{p} - \text{m} - \text{n}}*(b + a*\text{Cot}[e + f*x])^{\text{m}}*(d + c*\text{Cot}[e + f*x])^{\text{n}}, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3688

$\text{Int}[(\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}((\text{A}_. + \text{B}_.*\tan[(e_.) + (f_.)*(x_.)])*(\text{c}_. + \text{d}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}), x\_Symbol] := \text{Simp}[b*B*(a + b*\text{Tan}[e + f*x])^{\text{m} - 1}*((c + d*\text{Tan}[e + f*x])^{\text{n} + 1}/(d*f*(m + n))), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{\text{m} - 2}*(c + d*\text{Tan}[e + f*x])^{\text{n}}*\text{Simp}[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*\text{Tan}[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*\text{Tan}[e + f*x]^2, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

$\text{Int}[(\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{m}_.}((\text{c}_. + \text{d}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}*(\text{A}_. + (\text{C}_.*\tan[(e_.) + (f_.)*(x_.)]^2)), x\_Symbol] := \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^{\text{m}}*(c + d*x)^{\text{n}}, x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

$\text{Int}[(\text{c}_. + \text{d}_.*\tan[(e_.) + (f_.)*(x_.)])^{\text{n}_.}((\text{A}_. + \text{B}_.*\tan[(e_.) + (f_.)*(x_.)] + (\text{C}_.*\tan[(e_.) + (f_.)*(x_.)]^2))/((\text{a}_. + \text{b}_.*\tan[(e_.) + (f_.)*(x_.)])), x\_Symbol] := \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^{\text{n}}*\text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^{\text{n}}*((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps



$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)} dx &= \int \frac{\cot^{\frac{3}{2}}(c+dx)(B+A\cot(c+dx))}{b+a\cot(c+dx)} dx \\
&= -\frac{2A\sqrt{\cot(c+dx)}}{ad} - \frac{2\int \frac{\frac{Ab}{2} + \frac{1}{2}aA\cot(c+dx) + \frac{1}{2}(Ab-aB)\cot^2(c+dx)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))} dx}{a} \\
&= -\frac{2A\sqrt{\cot(c+dx)}}{ad} - \frac{2\int \frac{\frac{1}{2}a(aA+bB) + \frac{1}{2}a(Ab-aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}} dx}{a(a^2+b^2)} \quad (b^2) \\
&= -\frac{2A\sqrt{\cot(c+dx)}}{ad} - \frac{4\text{Subst}\left(\int \frac{-\frac{1}{2}a(aA+bB) - \frac{1}{2}a(Ab-aB)x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{a(a^2+b^2)d} \\
&= -\frac{2A\sqrt{\cot(c+dx)}}{ad} + \frac{(2b^2(Ab-aB))\text{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \sqrt{\cot(c+dx)}\right)}{a(a^2+b^2)d} \\
&= \frac{2b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}(a^2+b^2)d} - \frac{2A\sqrt{\cot(c+dx)}}{ad} \\
&= \frac{2b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}(a^2+b^2)d} - \frac{2A\sqrt{\cot(c+dx)}}{ad} \\
&= -\frac{(a(A-B)+b(A+B))\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(a(A-B)+b(A+B))\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}(a^2+b^2)d}
\end{aligned}$$

### Mathematica [A]

time = 0.58, size = 249, normalized size = 0.84

$$\frac{\sqrt{\cot(c+dx)} \left( \frac{2\sqrt{2}^{(A-B)+(A+B)} (\text{ArcTan}[1-\sqrt{2}\sqrt{\tan(c+dx)}] - \text{ArcTan}[1+\sqrt{2}\sqrt{\tan(c+dx)}])}{a^2+b^2} + \frac{8b^{3/2}(-Ab+aB)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2+b^2)} - \frac{\sqrt{2}^{(A-B)+(A+B)} (\log(1-\sqrt{2}\sqrt{\tan(c+dx)+\tan(c+dx)}) - \log(1+\sqrt{2}\sqrt{\tan(c+dx)+\tan(c+dx)}))}{a^2+b^2} - \frac{8A}{\sqrt{\tan(c+dx)}} \right)}{4d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
[Out] (Sqrt[Cot[c + d*x]]*((2*Sqrt[2]*(a*(A - B) + b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]])]/(a^2 + b^2) + (8*b^(3/2)*(-A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*(a^2 + b^2)) - (Sqrt[2]*(b*(-A + B) + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) - (8*A)/(a*Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(4*d)

```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 42.86, size = 20614, normalized size = 69.41

method	result	size
default	Expression too large to display	20614

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.53, size = 239, normalized size = 0.80

$$\frac{8 (B a^2 - A^2) \arctan\left(\frac{\sqrt{a b} \sqrt{\tan(dx+c)}}{\sqrt{a^2 + b^2}}\right) - 2 \sqrt{2} (A - B) a + (A + B) b \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + \frac{b}{\sqrt{\tan(dx+c)}}\right)\right) + 2 \sqrt{2} (A - B) a + (A + B) b \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - \frac{b}{\sqrt{\tan(dx+c)}}\right)\right) + \sqrt{2} (A + B) a - (A - B) b \ln\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)} + \frac{1}{\tan(dx+c)}}\right) - \sqrt{2} (A + B) a - (A - B) b \ln\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)} + \frac{1}{\tan(dx+c)}}\right) + \frac{8 A}{a \sqrt{\tan(dx+c)}}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-1/4*(8*(B*a*b^2 - A*b^3)*\arctan(a/(\sqrt{a*b}*\sqrt{\tan(d*x + c)})))/((a^3 + a*b^2)*\sqrt{a*b}) - (2*\sqrt{2}*((A - B)*a + (A + B)*b)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(d*x + c)}))) + 2*\sqrt{2}*((A - B)*a + (A + B)*b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(d*x + c)}))) + \sqrt{2}*((A + B)*a - (A - B)*b)*\log(\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) - \sqrt{2}*((A + B)*a - (A - B)*b)*\log(-\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1))/((a^2 + b^2) + 8*A/(a*\sqrt{\tan(d*x + c)}))/d$$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(b\*tan(d\*x + c) + a), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)), x)

$$3.591 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=278

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $1/2*(b*(A-B)-a*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(b*(A-B)-a*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(a*(A-B)+b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a*(A-B)+b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-2*(A*b-B*a)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/(a^2+b^2)/d/a^{(1/2)}$

Rubi [A]

time = 0.31, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3662, 3693, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} + \frac{(b(A-B) - a(A+B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{2\sqrt{b} (Ab - aB) \operatorname{ArcTan}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a} d (a^2 + b^2)} - \frac{(a(A-B) + b(A+B)) \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)} + \frac{(a(A-B) + b(A+B)) \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(A + B*\operatorname{Tan}[c + d*x])\right)/(a + b*\operatorname{Tan}[c + d*x]), x\right]$

[Out]  $-(((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)*d)) + ((b*(A - B) - a*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)*d) - (2*\operatorname{Sqrt}[b]*(A*b - a*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/( \operatorname{Sqrt}[a]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + ((a*(A - B) + b*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 210

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

$\int [b \tan[e + f x]]^m, x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3662

$\int [(\cot[e + f x] + (f x) g)^p (a + b \tan[e + f x])^m (c + d \tan[e + f x])^n, x] \text{Symbol} \rightarrow \text{Dist}[g^{m+n}, \int (g \cot[e + f x])^{p-m-n} (b + a \cot[e + f x])^m (d + c \cot[e + f x])^n, x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ \text{!IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

### Rule 3693

$\int [((A + B \tan[e + f x]) \sqrt{c + d \tan[e + f x]}) / (a + b \tan[e + f x]), x] \text{Symbol} \rightarrow \text{Dist}[1 / (a^2 + b^2), \int [\text{Simp}[A(a c + b d) + B(b c - a d) - (A(b c - a d) - B(a c + b d)) \tan[e + f x], x] / \sqrt{c + d \tan[e + f x]}, x], x] - \text{Dist}[(b c - a d) (B a - A b) / (a^2 + b^2), \int [(1 + \tan[e + f x]^2) / (a + b \tan[e + f x]) \sqrt{c + d \tan[e + f x]}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b c - a d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3715

$\int [(a + b \tan[e + f x])^m (c + d \tan[e + f x])^n (A + C \tan[e + f x])^2, x] \text{Symbol} \rightarrow \text{Dist}[A/f, \text{Subst}[\int (a + b x)^m (c + d x)^n, x], \tan[e + f x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \ \&\& \ \text{EqQ}[A, C]$

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (A + B \tan(c+dx))}{a + b \tan(c+dx)} dx &= \int \frac{\sqrt{\cot(c+dx)} (B + A \cot(c+dx))}{b + a \cot(c+dx)} dx \\
&= \frac{\int \frac{-Ab+aB+(aA+bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}} dx}{a^2 + b^2} + \frac{(b(Ab - aB)) \int \frac{1+\cot^2(c+dx)}{\sqrt{\cot(c+dx)}} dx}{a^2 + b^2} \\
&= \frac{2\text{Subst}\left(\int \frac{Ab-aB+(-aA-bB)x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2 + b^2) d} + \frac{(b(Ab - aB)) \text{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2 + b^2) d} \\
&= -\frac{(2b(Ab - aB)) \text{Subst}\left(\int \frac{1}{b+ax^2} dx, x, \sqrt{\cot(c+dx)}\right)}{(a^2 + b^2) d} + \frac{(b(A - B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a} (a^2 + b^2) d} \\
&= -\frac{2\sqrt{b} (Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a} (a^2 + b^2) d} + \frac{(b(A - B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a} (a^2 + b^2) d} \\
&= -\frac{2\sqrt{b} (Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a} (a^2 + b^2) d} - \frac{(a(A - B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a} (a^2 + b^2) d} \\
&= -\frac{(b(A - B) - a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \dots
\end{aligned}$$

**Mathematica [A]**

time = 0.29, size = 215, normalized size = 0.77

$$\frac{\sqrt{\cot(c+dx)} \left( -2\sqrt{2}(b(-A+B) + a(A+B)) (\text{ArcTan}(1 - \sqrt{2} \sqrt{\tan(c+dx)}) - \text{ArcTan}(1 + \sqrt{2} \sqrt{\tan(c+dx)})) + \frac{2\sqrt{b}(ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{2}(a(A-B) + b(A+B)) (\log(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx))) \right) \sqrt{\tan(c+dx)}}{4(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]),x]

```

[Out] (Sqrt[Cot[c + d*x]]*(-2*Sqrt[2]*(b*(-A + B) + a*(A + B))*(ArcTan[1 - Sqrt[2]
]*Sqrt[Tan[c + d*x]]) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (8*Sqrt[b
]*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a] - Sqrt[
2]*(a*(A - B) + b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x
]])/(4*(a^2 + b^2)*d)

```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 38.32, size = 4035, normalized size = 14.51

method	result	size
default	Expression too large to display	4035

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/d*2^(1/2)/a/(a^2+b^2)^(3/2)/(a+b*(a^2+b^2)^(1/2))/(-b+(a^2+b^2)^(1/2)-a
)*(cos(d*x+c)/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*(I*B*(a^2+b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b+I*B*(a^2+b^2)^(1/2)*Ell
ipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*a*b^3+I*A*(a^2+b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b+I*A*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b^2+I*A*(a^2
+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*a*b^3+I*A*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^3*b-I*A*(a^2+b^2)^(3/2)*Ell
ipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*a*b-I*A*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*a^3*b+3*A*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b^2+A*(a^2+b
^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*a*b^3-I*A*(a^2+b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^2-I*A*(a^2+b^2)^(1/2)*Ellipti
cPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^4
-I*B*(a^2+b^2)^(3/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*a^2-I*B*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^4+I*A*(a^2+b^2)^(3/2
)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))*a^2+I*A*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^4+I*B*(a^2+b^2)^(3/2)*EllipticPi(((1-co
s(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2+B*(a^2+b^
2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*a^2*b^2+B*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^3-B*(a^2+b^2)^(1/2)*Elliptic
Pi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^3*
b+B*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*a^2*b^2+B*(a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b^3-B*(a^2+b^2)^(1/
2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*a^3*b-2*a^2*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),
```



$a/(a+b+(a^2+b^2)^{1/2}), 1/2*2^{(1/2)})*B*b^2*(a^2+b^2)^{1/2}+I*B*(a^2+b^2)^{1/2}$   
 $*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2-1/2*I, 1/2*2^{(1/2)})$   
 $*a^4-A*(a^2+b^2)^{3/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2+1/2*I, 1/2*2^{(1/2)})$   
 $*a^2+2*A*(a^2+b^2)^{3/2}*EllipticF(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2*2^{(1/2)})$   
 $*a^2+2*A*(a^2+b^2)^{3/2}*EllipticF(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2*2^{(1/2)})$   
 $*b^2+B*(a^2+b^2)^{3/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2+1/2*I, 1/2*2^{(1/2)})$   
 $*a^2-B*(a^2+b^2)^{1/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2-1/2*I, 1/2*2^{(1/2)})$   
 $*a^4-B*(a^2+b^2)^{1/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2+1/2*I, 1/2*2^{(1/2)})$   
 $*a^4-2*a^4*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, a/(a+b+(a^2+b^2)^{1/2}), 1/2*2^{(1/2)})$   
 $*B*b-2*a^2*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, a/(a+b+(a^2+b^2)^{1/2}), 1/2*2^{(1/2)})$   
 $*B*b^3+A*(a^2+b^2)^{1/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2-1/2*I, 1/2*2^{(1/2)})$   
 $*a^4+A*(a^2+b^2)^{1/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2+1/2*I, 1/2*2^{(1/2)})$   
 $*a^4+2*a*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, a/(a+b+(a^2+b^2)^{1/2}), 1/2*2^{(1/2)})$   
 $*b^4*A-2*a^3*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, -a/(-b+(a^2+b^2)^{1/2})-a, 1/2*2^{(1/2)})$   
 $*A*b^2-2*a*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, -a/(-b+(a^2+b^2)^{1/2})-a, 1/2*2^{(1/2)})$   
 $*b^4*A+B*(a^2+b^2)^{3/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2-1/2*I, 1/2*2^{(1/2)})$   
 $*a^2-2*A*(a^2+b^2)^{1/2}*EllipticF(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2*2^{(1/2)})$   
 $*a^4-2*A*(a^2+b^2)^{1/2}*EllipticF(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2*2^{(1/2)})$   
 $*b^4+2*a^3*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, a/(a+b+(a^2+b^2)^{1/2}), 1/2*2^{(1/2)})$   
 $*A*b^2+2*a^4*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, -a/(-b+(a^2+b^2)^{1/2})-a, 1/2*2^{(1/2)})$   
 $*B*b+2*a^2*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, -a/(-b+(a^2+b^2)^{1/2})-a, 1/2*2^{(1/2)})$   
 $*B*b^3-A*(a^2+b^2)^{3/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2-1/2*I, 1/2*2^{(1/2)})$   
 $*a^2-I*A*(a^2+b^2)^{1/2}*EllipticPi(((1-\cos(dx+c)+\sin(dx+c))/\sin(dx+c))^{1/2}, 1/2-1/2*I, 1/2*2^{(1/2)})$   
 $*a^2*b^2-I*A*(a^2+b^2)^{1/2}...$

**Maxima [A]**

time = 0.52, size = 221, normalized size = 0.79

$$\frac{8(Bab-Ab^2) \operatorname{arctan}\left(\frac{\sqrt{ab} \sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) - 2\sqrt{2}((A+B)a-(A-B)b) \operatorname{arctan}\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{a}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}((A+B)a-(A-B)b) \operatorname{arctan}\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - \frac{a}{\sqrt{\tan(dx+c)}}\right)\right) - \sqrt{2}((A-B)a+(A+B)b) \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)}} + 1\right) + \sqrt{2}((A-B)a+(A+B)b) \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)}} + 1\right)}{(a^2+b^2)\sqrt{ab}}$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(1/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c)),x, algorithm="maxima")

[Out] 1/4\*(8\*(B\*a\*b - A\*b^2)\*arctan(a/(sqrt(a\*b)\*sqrt(tan(dx + c))))/((a^2 + b^2)\*sqrt(a\*b)) - (2\*sqrt(2)\*((A + B)\*a - (A - B)\*b)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2/sqrt(tan(dx + c)))) + 2\*sqrt(2)\*((A + B)\*a - (A - B)\*b)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2/sqrt(tan(dx + c)))) - sqrt(2)\*((A - B)\*a + (A + B)\*b

) $\log(\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) + \sqrt{2}*((A - B)*a + (A + B)*b)*\log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1))/(a^2 + b^2))/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^(1/2)*(A+B*tan(dx+c))/(a+b*tan(dx+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)**(1/2)*(A+B*tan(dx+c))/(a+b*tan(dx+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x)), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^(1/2)*(A+B*tan(dx+c))/(a+b*tan(dx+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(dx + c) + A)*sqrt(cot(dx + c))/(b*tan(dx + c) + a), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)`

[Out] `int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)), x)`

$$3.592 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))} dx$$

Optimal. Leaf size=278

$$\frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(a(A-B) + b(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $-1/2*(a*(A-B)+b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(a*(A-B)+b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(b*(A-B)-a*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(b*(A-B)-a*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+2*(A*b-B*a)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})*a^{(1/2)}/(a^2+b^2)/d/b^{(1/2)}$

Rubi [A]

time = 0.31, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3662, 3694, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a(A-B)+b(A+B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)} - \frac{(a(A-B)+b(A+B))\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)} + \frac{2\sqrt{a}\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d(a^2+b^2)} - \frac{(b(A-B)-a(A+B))\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{(b(A-B)-a(A+B))\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])),x]$

[Out]  $((a*(A - B) + b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + (2*\operatorname{Sqrt}[a]*(A*b - a*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/(\operatorname{Sqrt}[b]*(a^2 + b^2)*d) - ((b*(A - B) - a*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d) + ((b*(A - B) - a*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 210

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] := \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&$

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

$\text{t}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3662

$\text{Int}[(\cot[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \text{ :> } \text{Dist}[g^{(m+n)}, \text{Int}[(g*\text{Cot}[e + f*x])^{(p-m-n)}*(b + a*\text{Cot}[e + f*x])^m*(d + c*\text{Cot}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

#### Rule 3694

$\text{Int}[(((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)})/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \text{ :> } \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*A + b*B - (A*b - a*B)*\text{Tan}[e + f*x], x], x] + \text{Dist}[b*((A*b - a*B)/(a^2 + b^2)), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*((1 + \text{Tan}[e + f*x]^2)/(a + b*\text{Tan}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

#### Rule 3715

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_) + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] \text{ :> } \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, C, m, n\}, x] \ \&\& \ \text{EqQ}[A, C]$

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))} dx &= \int \frac{B + A \cot(c + dx)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))} dx \\
&= \frac{\int \frac{aA + bB + (Ab - aB) \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx}{a^2 + b^2} - \frac{(a(Ab - aB)) \int \frac{1 + \cot^2(c + dx)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))} dx}{a^2 + b^2} \\
&= \frac{2 \text{Subst}\left(\int \frac{-aA - bB + (-Ab + aB)x^2}{1 + x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{(a^2 + b^2) d} - \frac{(a(Ab - aB)) \text{Subst}\left(\int \frac{1}{b + ax^2} dx, x, \sqrt{\cot(c + dx)}\right)}{(a^2 + b^2) d} + \frac{(b(A - B) - a(A - B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{\sqrt{b} (a^2 + b^2) d} \\
&= \frac{2\sqrt{a} (Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{\sqrt{b} (a^2 + b^2) d} - \frac{(b(A - B) - a(A - B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{\sqrt{b} (a^2 + b^2) d} \\
&= \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d}
\end{aligned}$$

**Mathematica [A]**

time = 0.30, size = 215, normalized size = 0.77

$$\frac{\sqrt{\cot(c + dx)} \left( 2\sqrt{2} (a(A - B) + b(A + B)) \left( \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right) - \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right) \right) + \frac{a\sqrt{a} (Ab - aB) \text{ArcTan}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \sqrt{2} (b(A - B) + a(A + B)) \left( \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)} + \tan(c + dx)\right) - \log\left(1 + \sqrt{2} \sqrt{\cot(c + dx)} + \tan(c + dx)\right) \right) \right)}{4(a^2 + b^2)d} \sqrt{\tan(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])),x]

[Out]  $-1/4 * (\text{Sqrt}[\text{Cot}[c + d*x]] * (2 * \text{Sqrt}[2] * (a * (A - B) + b * (A + B)) * (\text{ArcTan}[1 - \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[c + d*x]]] - \text{ArcTan}[1 + \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[c + d*x]]]) + (8 * \text{Sqrt}[a] * (A * b - a * B) * \text{ArcTan}[(\text{Sqrt}[b] * \text{Sqrt}[\text{Tan}[c + d*x]]) / \text{Sqrt}[a]]) / \text{Sqrt}[b] - \text{Sqrt}[2] * (b * (-A + B) + a * (A + B)) * (\text{Log}[1 - \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[c + d*x]]] + \text{Tan}[c + d*x] - \text{Log}[1 + \text{Sqrt}[2] * \text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x])) * \text{Sqrt}[\text{Tan}[c + d*x]]) / ((a^2 + b^2) * d)$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 25.60, size = 3684, normalized size = 13.25

method	result	size
default	Expression too large to display	3684

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{2}d^{1/2}(a^2+b^2)^{3/2}(a+b+(a^2+b^2)^{1/2})/(-b+(a^2+b^2)^{1/2}-a)*$   
 $((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2}$   
 $*(-(\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}*(\cos(dx+c)+1)^2*(-1+\cos$   
 $(dx+c))*(-3IA(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin$   
 $(dx+c))^{1/2},1/2+1/2I,1/2*2^{1/2})a^2b-IB*(a^2+b^2)^{3/2}EllipticPi$   
 $((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2+1/2I,1/2*2^{1/2})a-IB*$   
 $(a^2+b^2)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2$   
 $-1/2I,1/2*2^{1/2})b-IB*(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(d$   
 $x+c))/\sin(dx+c))^{1/2},1/2+1/2I,1/2*2^{1/2})b^3-IB*(a^2+b^2)^{1/2}Ell$   
 $ipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2}$   
 $)a^3+IA(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c)$   
 $)^{1/2},1/2-1/2I,1/2*2^{1/2})a^3+IA(a^2+b^2)^{1/2}EllipticPi((-\cos(dx$   
 $+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})b^3+IB*(a^2+b^$   
 $2)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2+1/2I$   
 $,1/2*2^{1/2})b+IB*(a^2+b^2)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/$   
 $\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})a+2*BEllipticPi((-\cos(dx+c)-1-s$   
 $in(dx+c))/\sin(dx+c))^{1/2},a/(a+b+(a^2+b^2)^{1/2}),1/2*2^{1/2})a^2*b^2-A$   
 $(a^2+b^2)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1$   
 $/2+1/2I,1/2*2^{1/2})b+A*(a^2+b^2)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(d$   
 $x+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})a-A*(a^2+b^2)^{3/2}Elliptic$   
 $Pi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})b-A$   
 $(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1$   
 $/2+1/2I,1/2*2^{1/2})a^3+2*AEllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(d$   
 $x+c))^{1/2},-a/(-b+(a^2+b^2)^{1/2}-a),1/2*2^{1/2})a^3*b+2*AEllipticPi((-$   
 $\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},-a/(-b+(a^2+b^2)^{1/2}-a),1/2*2^$   
 $(1/2))a*b^3-2*B*(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin$   
 $(dx+c))^{1/2},-a/(-b+(a^2+b^2)^{1/2}-a),1/2*2^{1/2})a^3-2*AEllipticPi((-$   
 $\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},a/(a+b+(a^2+b^2)^{1/2}),1/2*2^$   
 $(1/2))a^3*b+B*(a^2+b^2)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(d$   
 $x+c))^{1/2},1/2-1/2I,1/2*2^{1/2})b-2*B*(a^2+b^2)^{1/2}EllipticPi((-\cos($   
 $dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},a/(a+b+(a^2+b^2)^{1/2}),1/2*2^{1/2})$   
 $a^3+IB*(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))$   
 $^{1/2},1/2+1/2I,1/2*2^{1/2})a^3+IB*(a^2+b^2)^{1/2}EllipticPi((-\cos(dx$   
 $+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})b^3-IA*(a^2+b^2$   
 $)^{3/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,$   
 $1/2*2^{1/2})a+2*IA*(a^2+b^2)^{1/2}EllipticPi((-\cos(dx+c)-1-\sin(dx+c))/s$

$\text{in}(d*x+c))^{(1/2)}, a/(a+b+(a^2+b^2)^{(1/2)}), 1/2*2^{(1/2)}) * a^2*b+2*A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, -a/(-b+(a^2+b^2)^{(1/2)}-a), 1/2*2^{(1/2)}) * a^2*b-2*A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, -a/(-b+(a^2+b^2)^{(1/2)}-a), 1/2*2^{(1/2)}) * a*b^2+I*A*(a^2+b^2)^{(3/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * a+I*A*(a^2+b^2)^{(3/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * b-2*B * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, -a/(-b+(a^2+b^2)^{(1/2)}-a), 1/2*2^{(1/2)}) * a^2*b^2-A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * a^2*b+2*B*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, -a/(-b+(a^2+b^2)^{(1/2)}-a), 1/2*2^{(1/2)}) * a^2*b-I*A*(a^2+b^2)^{(3/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)}) * b-I*A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * a^3-I*A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * b^3-3*B*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * a^2*b-3*B*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * a*b^2-3*B*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)}) * a^2*b-A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)}) * a^2*b-3*B*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)}) * a*b^2+A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2-1/2*I, 1/2*2^{(1/2)}) * a*b^2+2*B*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, a/(a+b+(a^2+b^2)^{(1/2)}), 1/2*2^{(1/2)}) * a^2*b-2*A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, a/(a+b+(a^2+b^2)^{(1/2)}), 1/2*2^{(1/2)}) * a*b^2+A*(a^2+b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, 1/2*2^{(1/2)}) * a*b^2-2*B * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, -a/(-b+(a^2+b^2)^{(1/2)}-a), 1/2*2^{(1/2)}) * a^4+2*B * \text{EllipticPi}((- (\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, a/(a...$

**Maxima [A]**

time = 0.51, size = 220, normalized size = 0.79

$$\frac{8(Ba^2 - Ab) \arctan\left(\frac{a}{\sqrt{ab} \sqrt{\tan(dx+c)}}\right) + 2\sqrt{2}((A-B)a + (A+B)b) \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}((A-B)a - (A+B)b) \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - \frac{1}{\sqrt{\tan(dx+c)}}\right)\right) + \sqrt{2}((A+B)a - (A-B)b) \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)+1}}\right) - \sqrt{2}((A+B)a - (A-B)b) \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{\tan(dx+c)+1}}\right)}{(a^2+b^2)\sqrt{ab}}$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/4*(8*(B*a^2 - A*a*b)*\arctan(a/(\text{sqrt}(a*b)*\text{sqrt}(\tan(d*x + c))))/((a^2 + b^2)*\text{sqrt}(a*b)) + (2*\text{sqrt}(2))*((A - B)*a + (A + B)*b)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2/\text{sqrt}(\tan(d*x + c)))) + 2*\text{sqrt}(2)*((A - B)*a + (A + B)*b)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2/\text{sqrt}(\tan(d*x + c)))) + \text{sqrt}(2)*((A + B)*a - (A - B)*$



$b \cdot \log(\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - \sqrt{2} \cdot ((A + B) \cdot a - (A - B) \cdot b) \cdot \log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) / (a^2 + b^2) / d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx)) \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*sqrt(cot(c + d\*x))), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c))), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))), x)

$$3.593 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))} dx$$

Optimal. Leaf size=297

$$\frac{(b(A-B) - a(A+B))\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d} - \frac{(b(A-B) - a(A+B))\text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2) d}$$

[Out]  $-2*a^{(3/2)}*(A*b-B*a)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(a^2+b^2)/d-1/2*(b*(A-B)-a*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/2*(b*(A-B)-a*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a*(A-B)+b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}-1/4*(a*(A-B)+b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+2*B/b/d/\cot(d*x+c)^{(1/2)}$

Rubi [A]

time = 0.51, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3662, 3690, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(A-B) - a(A+B)\text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d(a^2 + b^2)} - \frac{(A-B) - a(A+B)\text{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d(a^2 + b^2)} + \frac{(a(A-B) + b(A+B)) \log(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1)}{2\sqrt{2} d(a^2 + b^2)} - \frac{(a(A-B) + b(A+B)) \log(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1)}{2\sqrt{2} d(a^2 + b^2)} - \frac{2a^{3/2}(A-b)\text{ArcTan}\left(\frac{\sqrt{2} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d(a^2 + b^2)} + \frac{2B}{b d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])),x]

[Out]  $((b*(A - B) - a*(A + B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*(a^2 + b^2)*d) - ((b*(A - B) - a*(A + B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*(a^2 + b^2)*d) - (2*a^{(3/2)}*(A*b - a*B))*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[\text{Cot}[c + d*x]])/\text{Sqrt}[b]])/(b^{(3/2)}*(a^2 + b^2)*d) + (2*B)/(b*d*\text{Sqrt}[\text{Cot}[c + d*x]]) + ((a*(A - B) + b*(A + B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)*d) - ((a*(A - B) + b*(A + B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)*d)$

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &

& (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqr

$\int [b \cdot \tan[e + f \cdot x]] \cdot x \, dx$  /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

$\int ((\cot[e] + (f \cdot x) \cdot g)^{p \cdot (a + b \cdot \tan[e + f \cdot x])})^{m \cdot (c + d \cdot \tan[e + f \cdot x])} \cdot x \, dx$  /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3690

$\int ((a + b \cdot \tan[e + f \cdot x])^m \cdot (A + B \cdot \tan[e + f \cdot x]) \cdot (c + d \cdot \tan[e + f \cdot x])^n) \cdot x \, dx$  /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2 \cdot m, 2 \cdot n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

$\int ((a + b \cdot \tan[e + f \cdot x])^m \cdot (c + d \cdot \tan[e + f \cdot x]) \cdot (A + C \cdot \tan[e + f \cdot x])^2) \cdot x \, dx$  /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

$\int (((c + d \cdot \tan[e + f \cdot x])^n \cdot (A + B \cdot \tan[e + f \cdot x]) \cdot (C + D \cdot \tan[e + f \cdot x])^2) / ((a + b \cdot \tan[e + f \cdot x]) \cdot (c + d \cdot \tan[e + f \cdot x]))) \cdot x \, dx$  /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(b + a \cot(c + dx))} dx \\
&= \frac{2B}{bd\sqrt{\cot(c + dx)}} + \frac{2 \int \frac{\frac{1}{2}(Ab - aB) - \frac{1}{2}bB \cot(c + dx) - \frac{1}{2}aB \cot^2(c + dx)}{\sqrt{\cot(c + dx)}(b + a \cot(c + dx))} dx}{b} \\
&= \frac{2B}{bd\sqrt{\cot(c + dx)}} + \frac{2 \int \frac{\frac{1}{2}b(Ab - aB) - \frac{1}{2}b(aA + bB) \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx}{b(a^2 + b^2)} + \frac{(a^2(Ab - aB)) \text{Subst}\left(\int \frac{1}{b + ax^2} dx, x, \sqrt{\cot(c + dx)}\right)}{b(a^2 + b^2)d} \\
&= \frac{2B}{bd\sqrt{\cot(c + dx)}} + \frac{4 \text{Subst}\left(\int \frac{-\frac{1}{2}b(Ab - aB) + \frac{1}{2}b(aA + bB)x^2}{1 + x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{b(a^2 + b^2)d} \\
&= \frac{2B}{bd\sqrt{\cot(c + dx)}} - \frac{(2a^2(Ab - aB)) \text{Subst}\left(\int \frac{1}{b + ax^2} dx, x, \sqrt{\cot(c + dx)}\right)}{b(a^2 + b^2)d} \\
&= -\frac{2a^{3/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^{3/2}(a^2 + b^2)d} + \frac{2B}{bd\sqrt{\cot(c + dx)}} \\
&= -\frac{2a^{3/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^{3/2}(a^2 + b^2)d} + \frac{2B}{bd\sqrt{\cot(c + dx)}} \\
&= \frac{(b(A - B) - a(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d} - \frac{(b(A - B) - a(A + B)) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)d}
\end{aligned}$$

**Mathematica [A]**

time = 0.38, size = 251, normalized size = 0.85

$$\frac{\sqrt{\cot(c + dx)} \left( 2\sqrt{2}b^{3/2}(b(A - B) - a(A + B)) (\text{ArcTan}[1 - \sqrt{2}\sqrt{\cot(c + dx)}] - \text{ArcTan}[1 + \sqrt{2}\sqrt{\cot(c + dx)}]) + 8a^{3/2}(b(A - B) - a(A + B)) \text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\cot(c + dx)}}{\sqrt{b}}\right) - \sqrt{2}b^{3/2}(a(A - B) + b(A + B)) (\log(1 - \sqrt{2}\sqrt{\cot(c + dx)} + \tan(c + dx)) - \log(1 + \sqrt{2}\sqrt{\cot(c + dx)} + \tan(c + dx))) - 8\sqrt{b}(a^2 + b^2)B\sqrt{\cot(c + dx)} \right) \sqrt{\tan(c + dx)}}{4b^{3/2}(a^2 + b^2)d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
[Out] -1/4*(Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*b^(3/2)*(b*(A - B) - a*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + 8*a^(3/2)*(-A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] - Sqrt[2]*b^(3/2)*(a*(A - B) + b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - 8*Sqrt[b]*(a^2 + b^2)*B*Sqrt[Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(b^(3/2)*(a^2 + b^2)*d)

```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 76.53, size = 9725, normalized size = 32.74

method	result	size
default	Expression too large to display	9725

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.55, size = 238, normalized size = 0.80

$$\frac{8 \sqrt{a^2 - A^2} \arctan\left(\frac{\sqrt{ab} \sqrt{\tan(dx+c)}}{\sqrt{a^2+b^2}}\right) + \frac{8B \sqrt{\tan(dx+c)}}{1} + \frac{2\sqrt{2} \sqrt{(A+B)a - (A-B)b} \arctan\left(\frac{1}{2} \sqrt{\frac{\sqrt{2} \sqrt{a^2+b^2}}{\sqrt{\tan(dx+c)}}}\right) + 2\sqrt{2} \sqrt{(A+B)a - (A-B)b} \arctan\left(-\frac{1}{2} \sqrt{\frac{\sqrt{2} \sqrt{a^2+b^2}}{\sqrt{\tan(dx+c)}}}\right) - \sqrt{2} \sqrt{(A-B)a + (A+B)b} \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{a^2+b^2}}\right) + \sqrt{2} \sqrt{(A-B)a + (A+B)b} \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} - \frac{1}{\sqrt{a^2+b^2}}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{4} * (8 * (B * a^3 - A * a^2 * b) * \arctan(a / (\sqrt{a * b} * \sqrt{\tan(d * x + c)})) / ((a^2 * b + b^3) * \sqrt{a * b}) + 8 * B * \sqrt{\tan(d * x + c)} / b + (2 * \sqrt{2} * ((A + B) * a - (A - B) * b) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 / \sqrt{\tan(d * x + c)})) + 2 * \sqrt{2} * ((A + B) * a - (A - B) * b) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 / \sqrt{\tan(d * x + c)})) - \sqrt{2} * ((A - B) * a + (A + B) * b) * \log(\sqrt{2} / \sqrt{\tan(d * x + c)} + 1 / \sqrt{a^2 + b^2}) + \sqrt{2} * ((A - B) * a + (A + B) * b) * \log(-\sqrt{2} / \sqrt{\tan(d * x + c)} + 1 / \sqrt{a^2 + b^2})) / (a^2 + b^2) / d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2)), x )

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)\*cot(d\*x + c)^(3/2)), x )

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + b \tan(c + dx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))), x)

$$3.594 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=325

$$-\frac{(a(A-B)+b(A+B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d} + \frac{(a(A-B)+b(A+B))\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)d}$$

[Out]  $2*a^{5/2}*(A*b-B*a)*\arctan(a^{1/2}*\cot(d*x+c)^{1/2}/b^{1/2})/b^{5/2}/(a^2+b^2)/d+2/3*B/b/d/\cot(d*x+c)^{3/2}+1/2*(a*(A-B)+b*(A+B))*\arctan(-1+2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+1/2*(a*(A-B)+b*(A+B))*\arctan(1+2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+1/4*(b*(A-B)-a*(A+B))*\ln(1+\cot(d*x+c)-2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}-1/4*(b*(A-B)-a*(A+B))*\ln(1+\cot(d*x+c)+2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)/d*2^{1/2}+2*(A*b-B*a)/b^2/d/\cot(d*x+c)^{1/2}$

**Rubi [A]**

time = 0.72, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a(A-B)+b(A+B))\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d(a^2+b^2)} + \frac{(a(A-B)+b(A+B))\text{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}d(a^2+b^2)} + \frac{(b(A-B)-a(A+B))\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} - \frac{(b(A-B)-a(A+B))\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{2a^{5/2}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d(a^2+b^2)} + \frac{2(Ab-aB)}{b^2d\sqrt{\cot(c+dx)}} + \frac{2B}{3bd\cot(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])),x]

[Out]  $-(((a*(A-B)+b*(A+B))*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]])/\text{Sqrt}[2]*(a^2+b^2)*d) + ((a*(A-B)+b*(A+B))*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]])/\text{Sqrt}[2]*(a^2+b^2)*d) + (2*a^{5/2}*(A*b-a*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[\text{Cot}[c+d*x]])/\text{Sqrt}[b]])/(b^{5/2}*(a^2+b^2)*d) + (2*B)/(3*b*d*\text{Cot}[c+d*x]^{3/2}) + (2*(A*b-a*B))/(b^2*d*\text{Sqrt}[\text{Cot}[c+d*x]]) + ((b*(A-B)-a*(A+B))*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]+\text{Cot}[c+d*x]])/(2*\text{Sqrt}[2]*(a^2+b^2)*d) - ((b*(A-B)-a*(A+B))*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c+d*x]]+\text{Cot}[c+d*x]])/(2*\text{Sqrt}[2]*(a^2+b^2)*d)$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^(n), x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 210**



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :>
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)])^2, x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
```

(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(b + a \cot(c + dx))} dx \\
 &= \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{2 \int \frac{\frac{3}{2}(Ab - aB) - \frac{3}{2}bB \cot(c + dx) - \frac{3}{2}aB \cot^2(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(b + a \cot(c + dx))} dx}{3b} \\
 &= \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{b^2 d \sqrt{\cot(c + dx)}} + \frac{4 \int \frac{-\frac{3}{4}(aAb - a^2 B + b^2 B) - \frac{3}{4}Ab^2 \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx}{\sqrt{\cot(c + dx)}} \\
 &= \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{b^2 d \sqrt{\cot(c + dx)}} + \frac{4 \int \frac{-\frac{3}{4}b^2(aA + bB) - \frac{3}{4}b^2(Ab - aB) \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx}{3b^2 (a^2 + b^2)} \\
 &= \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{b^2 d \sqrt{\cot(c + dx)}} + \frac{8 \text{Subst}\left(\int \frac{\frac{3}{4}b^2(aA + bB) + \frac{3}{4}b^2(Ab - aB) \cot(x)}{1 + x^4} dx\right)}{3b^2 (a^2 + b^2)} \\
 &= \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{2(Ab - aB)}{b^2 d \sqrt{\cot(c + dx)}} + \frac{(2a^3(Ab - aB)) \text{Subst}\left(\int \frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}} dx\right)}{b^2 (a^2 + b^2)} \\
 &= \frac{2a^{5/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^{5/2} (a^2 + b^2) d} + \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{(2a^3(Ab - aB)) \text{Subst}\left(\int \frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}} dx\right)}{b^2 (a^2 + b^2)} \\
 &= \frac{2a^{5/2}(Ab - aB) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^{5/2} (a^2 + b^2) d} + \frac{2B}{3bd \cot^{\frac{3}{2}}(c + dx)} + \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a(A - B) + b(A + B)) \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2) d}
 \end{aligned}$$

**Mathematica [A]**

time = 0.65, size = 272, normalized size = 0.84

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( \frac{e^{\sqrt{2} \cot(c + dx)} (a(A - B) + b(A + B)) (\text{ArcTan}[-\sqrt{2} \sqrt{\tan(c + dx)}] - \text{ArcTan}[\sqrt{2} \sqrt{\tan(c + dx)}])}{2\sqrt{2}} + \frac{24a^{5/2}(-Ab + aB) \text{ArcTan}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2 + b^2)} - \frac{2\sqrt{2} (b(A - B) + a(A + B)) \log(-\sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx))}{2\sqrt{2}} - \frac{2\sqrt{2} (b(A - B) + a(A + B)) \log(\sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx))}{2\sqrt{2}} + \frac{2b(A - B) \sqrt{\tan(c + dx)}}{2\sqrt{2}} + \frac{2B \tan^{\frac{3}{2}}(c + dx)}{3bd} \right)}{12d}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((6*Sqrt[2]*(a*(A - B) + b*(A + B))*
(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]]]))/(a^2 + b^2) + (24*a^(5/2)*(-A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c

```

$$\frac{+ d*x]]/\text{Sqrt}[a]]/(b^{(5/2)}*(a^2 + b^2)) - (3*\text{Sqrt}[2]*(b*(-A + B) + a*(A + B))*(\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]))/(a^2 + b^2) + (24*(A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]]/b^2 + (8*B*\text{Tan}[c + d*x]^{(3/2)})/b)/(12*d)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 74.21, size = 11951, normalized size = 36.77

method	result	size
default	Expression too large to display	11951

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVERB  
OSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.55, size = 262, normalized size = 0.81

$$\frac{24 (b^4 - a^4) \arctan\left(\frac{a}{\sqrt{ab} \sqrt{\tan(dx+c)}}\right) - 8 (ab - \frac{1}{\sqrt{ab}}) \arctan\left(\frac{a}{\sqrt{ab} \sqrt{\tan(dx+c)}}\right) - 3 \left(2 \sqrt{2} ((A-B)a + (A+B)b) \arctan\left(\frac{1}{\sqrt{2}} \left(\sqrt{2} - \frac{a}{\sqrt{\tan(dx+c)}}\right)\right) + 2 \sqrt{2} ((A-B)a + (A+B)b) \arctan\left(-\frac{1}{\sqrt{2}} \left(\sqrt{2} - \frac{a}{\sqrt{\tan(dx+c)}}\right)\right) + \sqrt{2} ((A+B)a - (A-B)b) \arctan\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{ab} \sqrt{\tan(dx+c)}}\right) - \sqrt{2} ((A+B)a - (A-B)b) \arctan\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\sqrt{ab} \sqrt{\tan(dx+c)}}\right)\right)}{(a^2 + b^2) \sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\frac{-1/12*(24*(B*a^4 - A*a^3*b)*\arctan(a/(\text{sqrt}(a*b)*\text{sqrt}(\tan(d*x + c))))/(a^2*b^2 + b^4)*\text{sqrt}(a*b)) - 8*(B*b - 3*(B*a - A*b)/\tan(d*x + c))*\tan(d*x + c)^{(3/2)}/b^2 - 3*(2*\text{sqrt}(2)*((A - B)*a + (A + B)*b)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2/\text{sqrt}(\tan(d*x + c)))) + 2*\text{sqrt}(2)*((A - B)*a + (A + B)*b)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2/\text{sqrt}(\tan(d*x + c)))) + \text{sqrt}(2)*((A + B)*a - (A - B)*b)*\log(\text{sqrt}(2)/\text{sqrt}(\tan(d*x + c)) + 1/\tan(d*x + c) + 1) - \text{sqrt}(2)*((A + B)*a - (A - B)*b)*\log(-\text{sqrt}(2)/\text{sqrt}(\tan(d*x + c)) + 1/\tan(d*x + c) + 1))/(a^2 + b^2))/d$$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx)) \cot^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)``[Out] Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*cot(c + d*x)**(5/2)), x)`**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm="giac")``[Out] Timed out`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + b \tan(c + dx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)``[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))), x)`

$$3.595 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=438

$$\frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

[Out]  $b^{3/2}*(7*A*a^2*b+3*A*b^3-5*B*a^3-B*a*b^2)*\arctan(a^{1/2}*\cot(d*x+c)^{1/2})/b^{1/2})/a^{5/2}/(a^2+b^2)^2/d+b*(A*b-B*a)*\cot(d*x+c)^{3/2}/a/(a^2+b^2)/d/(b+a*\cot(d*x+c))+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\arctan(-1+2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)^2/d*2^{1/2}+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\arctan(1+2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)^2/d*2^{1/2}+1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\ln(1+\cot(d*x+c)-2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)^2/d*2^{1/2}-1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\ln(1+\cot(d*x+c)+2^{1/2}*\cot(d*x+c)^{1/2})/(a^2+b^2)^2/d*2^{1/2}-(2*A*a^2+3*A*b^2-B*a*b)*\cot(d*x+c)^{1/2}/a^2/(a^2+b^2)/d$

**Rubi** [A]

time = 0.87, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3686, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{1 - \sqrt{2} \sqrt{\cot(c+dx)}}{1 + \sqrt{2} \sqrt{\cot(c+dx)}}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{1 + \sqrt{2} \sqrt{\cot(c+dx)}}{1 - \sqrt{2} \sqrt{\cot(c+dx)}}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{b^3(A-B) + 3ab^2(A+B) - 5a^2b(A-B) - a^3(A+B)}{a^2(a^2 + b^2)^2 d} \arctan\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right) + \frac{b^2(A-B) + 2ab(A+B) - a^2(A+B)}{a^2(a^2 + b^2)^2 d} \ln\left(\frac{1 - \sqrt{2} \sqrt{\cot(c+dx)}}{1 + \sqrt{2} \sqrt{\cot(c+dx)}}\right) + \frac{b^2(A-B) + 2ab(A+B) - a^2(A+B)}{a^2(a^2 + b^2)^2 d} \ln\left(\frac{1 + \sqrt{2} \sqrt{\cot(c+dx)}}{1 - \sqrt{2} \sqrt{\cot(c+dx)}}\right) - \frac{(2Aa^2 + 3Ab^2 - Bab) \cot(c+dx)^{1/2}}{a^2(a^2 + b^2)^2 d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c + d*x]^{3/2}*(A + B*\operatorname{Tan}[c + d*x]))/(a + b*\operatorname{Tan}[c + d*x])^2, x]$

[Out]  $-(((a^2*(A - B) - b^2*(A - B) + 2*a*b*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)) + ((a^2*(A - B) - b^2*(A - B) + 2*a*b*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) + (b^{3/2}*(7*a^2*A*b + 3*A*b^3 - 5*a^3*B - a*b^2*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/(a^{5/2}*(a^2 + b^2)^2*d) - ((2*a^2*A + 3*A*b^2 - a*b*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(a^2*(a^2 + b^2)*d) + (b*(A*b - a*B)*\operatorname{Cot}[c + d*x]^{3/2})/(a*(a^2 + b^2)*d*(b + a*\operatorname{Cot}[c + d*x])) + ((2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - ((2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}$

$[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 210

$\text{Int}[\{(a\_)+(b\_)*(x\_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 211

$\text{Int}[\{(a\_)+(b\_)*(x\_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b]$

### Rule 631

$\text{Int}[\{(a\_)+(b\_)*(x\_)+(c\_)*(x\_)^2\}^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 642

$\text{Int}[\{(d\_)+(e\_)*(x\_)\}/\{(a\_)+(b\_)*(x\_)+(c\_)*(x\_)^2\}, x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1176

$\text{Int}[\{(d\_)+(e\_)*(x\_)^2\}/\{(a\_)+(c\_)*(x\_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 1179

$\text{Int}[\{(d\_)+(e\_)*(x\_)^2\}/\{(a\_)+(c\_)*(x\_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 1182

$\text{Int}[\{(d\_)+(e\_)*(x\_)^2\}/\{(a\_)+(c\_)*(x\_)^4\}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] /; \text{FreeQ}\{a,$



c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)]\*(g\_))^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3728

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n + 1))), x] + Dist[1/(d\*(m + n + 1)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*A\*d\*(m + n + 1) - C\*

```
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)])], x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x]))], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx &= \int \frac{\cot^{\frac{5}{2}}(c+dx)(B+A\cot(c+dx))}{(b+a\cot(c+dx))^2} dx \\
&= \frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{a(a^2+b^2)d(b+a\cot(c+dx))} - \int \frac{\sqrt{\cot(c+dx)}^{(-\frac{3}{2}b(Ab-aB)+a)}}{a(a^2+b^2)d(b+a\cot(c+dx))} dx \\
&= -\frac{(2a^2A+3Ab^2-abB)\sqrt{\cot(c+dx)}}{a^2(a^2+b^2)d} + \frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{a(a^2+b^2)d(b+a\cot(c+dx))} \\
&= -\frac{(2a^2A+3Ab^2-abB)\sqrt{\cot(c+dx)}}{a^2(a^2+b^2)d} + \frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{a(a^2+b^2)d(b+a\cot(c+dx))} \\
&= -\frac{(2a^2A+3Ab^2-abB)\sqrt{\cot(c+dx)}}{a^2(a^2+b^2)d} + \frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{a(a^2+b^2)d(b+a\cot(c+dx))} \\
&= -\frac{(2a^2A+3Ab^2-abB)\sqrt{\cot(c+dx)}}{a^2(a^2+b^2)d} + \frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{a(a^2+b^2)d(b+a\cot(c+dx))} \\
&= \frac{b^{3/2}(7a^2Ab+3Ab^3-5a^3B-ab^2B)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)^2d} \\
&= \frac{b^{3/2}(7a^2Ab+3Ab^3-5a^3B-ab^2B)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)^2d} \\
&= -\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^2d}
\end{aligned}$$

**Mathematica [A]**

time = 3.39, size = 383, normalized size = 0.87

$$\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{\sqrt{2}\sqrt{a-b}\sqrt{c+dx}\sqrt{a+b}\left(\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{a-b}}\right)-\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{a+b}}\right)\right)}{a^2(a^2+b^2)}\right)+\frac{b^{3/2}(7a^2Ab+3Ab^3-5a^3B-ab^2B)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}(a^2+b^2)^2d}-\frac{\sqrt{2}(a^2(A-B)-b^2(A-B)+2ab(A+B))\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^2d}}{a^2(a^2+b^2)^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*((2\*Sqrt[2]\*(a^2\*(A - B) + b^2\*(-A + B) + 2\*a\*b\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]])] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]))/(a^2 + b^2)^2 + (4\*b^(3/2)\*(-A\*b) + a\*B)\*ArcT



[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x))\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(b\*tan(d\*x + c) + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2, x)

$$3.596 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=392

$$\frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

[Out]  $1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-(5*A*a^2*b+A*b^3-3*B*a^3+B*a*b^2)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})*b^{(1/2)}/a^{(3/2)}/(a^2+b^2)^2/d+b*(A*b-B*a)*\cot(d*x+c)^{(1/2)}/a/(a^2+b^2)/d/(b+a*\cot(d*x+c))$

**Rubi [A]**

time = 0.62, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3662, 3686, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(-a^2(A+B)+2ab(A-B)+b^2(A+B))\operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{\sqrt{2(a^2+b^2)}}\right)}{\sqrt{2}d(a^2+b^2)^2} + \frac{(-a^2(A+B)+2ab(A-B)+b^2(A+B))\operatorname{ArcTan}\left(\frac{1+\sqrt{2}\sqrt{\cot(c+dx)}}{\sqrt{2(a^2+b^2)}}\right)}{\sqrt{2}d(a^2+b^2)^2} - \frac{b(A-b)}{4(a^2+b^2)}\ln\left(\frac{1+\cot(c+dx)-2^{1/2}\cot(c+dx)^{1/2}}{1+\cot(c+dx)+2^{1/2}\cot(c+dx)^{1/2}}\right)}{d(a^2+b^2)^2} + \frac{b(A-b)}{4(a^2+b^2)}\ln\left(\frac{1+\cot(c+dx)+2^{1/2}\cot(c+dx)^{1/2}}{1+\cot(c+dx)-2^{1/2}\cot(c+dx)^{1/2}}\right)}{d(a^2+b^2)^2} - \frac{(5Aa^2b+Ab^3-3Ba^3+Bab^2)\operatorname{ArcTan}\left(\frac{a^{1/2}\cot(c+dx)^{1/2}}{b^{1/2}}\right)}{d(a^2+b^2)^2} + \frac{b(Ab-Ba)\cot(c+dx)^{1/2}}{da(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*(A+B*\operatorname{Tan}[c+d*x]))/(a+b*\operatorname{Tan}[c+d*x])^2,x]$

[Out]  $-(((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)) + ((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - (\operatorname{Sqrt}[b]*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*b^2*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]])/\operatorname{Sqrt}[b]])/(a^{(3/2)}*(a^2 + b^2)^2*d) + (b*(A*b - a*B))*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]/(a*(a^2 + b^2)*d*(b + a*\operatorname{Cot}[c+d*x])) - ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]] + \operatorname{Cot}[c+d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) + ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]] + \operatorname{Cot}[c+d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)]\*(g\_))^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

Int[(((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])], x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}



, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  
 !GtQ[n, 0] && !LeQ[n, -1]

Rubi steps

$$\int \frac{\sqrt{\cot(c+dx)} (A + B \tan(c+dx))}{(a + b \tan(c+dx))^2} dx = \int \frac{\cot^{\frac{3}{2}}(c+dx)(B + A \cot(c+dx))}{(b + a \cot(c+dx))^2} dx$$

$$= \frac{b(Ab - aB) \sqrt{\cot(c+dx)}}{a(a^2 + b^2) d(b + a \cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}b(Ab - aB) + a(Ab - aB) \cot(c+dx)}{\sqrt{\cot(c+dx)}} dx}{a(a^2 + b^2) d(b + a \cot(c+dx))}$$

$$= \frac{b(Ab - aB) \sqrt{\cot(c+dx)}}{a(a^2 + b^2) d(b + a \cot(c+dx))} - \frac{\int \frac{a(2aAb - a^2B + b^2B) - a(a^2A - Ab^2)}{\sqrt{\cot(c+dx)}} dx}{a(a^2 + b^2) d(b + a \cot(c+dx))}$$

$$= \frac{b(Ab - aB) \sqrt{\cot(c+dx)}}{a(a^2 + b^2) d(b + a \cot(c+dx))} - \frac{2 \text{Subst}\left(\int \frac{-a(2aAb - a^2B + b^2B) + a^2A - Ab^2}{1 + \cot^2(u)} du\right)}{a(a^2 + b^2) d(b + a \cot(c+dx))}$$

$$= \frac{b(Ab - aB) \sqrt{\cot(c+dx)}}{a(a^2 + b^2) d(b + a \cot(c+dx))} - \frac{(b(5a^2Ab + Ab^3 - 3a^3B + ab^2B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2} (a^2 + b^2)^2 d}$$

$$= -\frac{\sqrt{b} (5a^2Ab + Ab^3 - 3a^3B + ab^2B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2} (a^2 + b^2)^2 d}$$

$$= -\frac{\sqrt{b} (5a^2Ab + Ab^3 - 3a^3B + ab^2B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2} (a^2 + b^2)^2 d}$$

$$= -\frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

**Mathematica [A]**

time = 1.64, size = 341, normalized size = 0.87

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( b\sqrt{2} (2ab(A - B) - a^2(A + B) + b^2(A + B)) (\text{ArcTan}(1 - \sqrt{2} \sqrt{\tan(c+dx)}) - \text{ArcTan}(1 + \sqrt{2} \sqrt{\tan(c+dx)})) + \frac{a\sqrt{b} (2a^2Ab + Ab^3 - 3a^3B + ab^2B)}{2a^2} \frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{a}} + \frac{a\sqrt{b} (2a^2Ab + Ab^3 - 3a^3B + ab^2B) \text{ArcTan}\left(\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{2a^2} - \sqrt{2} (a(A - B) + b^2(-A + B) + 2ab(A + B)) (\log(1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx))) + \frac{b(2a^2Ab + Ab^3 - 3a^3B + ab^2B)}{2a^2} \right)}{4(a^2 + b^2)^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^2, x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(2\*Sqrt[2]\*(2\*a\*b\*(A - B) - a^2\*(A + B) + b^2\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqr

$t[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + (4*\text{Sqrt}[b]*(a^2 + b^2)*(A*b - a*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/a^{3/2} + (8*\text{Sqrt}[b]*(2*a*A*b - a^2*B + b^2*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/\text{Sqrt}[a] - \text{Sqrt}[2]*(a^2*(A - B) + b^2*(-A + B) + 2*a*b*(A + B))*(\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]) + (4*b*(a^2 + b^2)*(A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a*(a + b*\text{Tan}[c + d*x])))/(4*(a^2 + b^2)^2*d$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.12, size = 35577, normalized size = 90.76

method	result	size
default	Expression too large to display	35577

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.57, size = 362, normalized size = 0.92

$$\frac{4 \sqrt{2} B a^2 - 5 A a^2 B + 4 B^2 a^2 \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right) + \sqrt{2} \sqrt{(A+B \tan(dx+c))^2 - (A-B)^2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right) + \sqrt{2} \sqrt{(A+B \tan(dx+c))^2 - (A-B)^2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right) - \sqrt{2} \sqrt{(A-B \tan(dx+c))^2 - (A+B)^2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right) + \sqrt{2} \sqrt{(A-B \tan(dx+c))^2 - (A+B)^2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{2} \sqrt{\tan(dx+c)}}\right)}{4 a^4 (a^2 + b^2 \tan(dx+c)) \sqrt{\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm  
="maxima")`

[Out]  $\frac{1}{4} * (4 * (3 * B * a^3 * b - 5 * A * a^2 * b^2 - B * a * b^3 - A * b^4) * \arctan(a / (\text{sqrt}(a * b) * \text{sqrt}(\tan(d * x + c)))) / ((a^5 + 2 * a^3 * b^2 + a * b^4) * \text{sqrt}(a * b)) - (2 * \text{sqrt}(2) * ((A + B) * a^2 - 2 * (A - B) * a * b - (A + B) * b^2) * \arctan(1/2 * \text{sqrt}(2) * (\text{sqrt}(2) + 2 / \text{sqrt}(\tan(d * x + c)))) + 2 * \text{sqrt}(2) * ((A + B) * a^2 - 2 * (A - B) * a * b - (A + B) * b^2) * \arctan(-1/2 * \text{sqrt}(2) * (\text{sqrt}(2) - 2 / \text{sqrt}(\tan(d * x + c)))) - \text{sqrt}(2) * ((A - B) * a^2 + 2 * (A + B) * a * b - (A - B) * b^2) * \log(\text{sqrt}(2) / \text{sqrt}(\tan(d * x + c)) + 1 / \tan(d * x + c) + 1) + \text{sqrt}(2) * ((A - B) * a^2 + 2 * (A + B) * a * b - (A - B) * b^2) * \log(-\text{sqrt}(2) / \text{sqrt}(\tan(d * x + c)) + 1 / \tan(d * x + c) + 1)) / (a^4 + 2 * a^2 * b^2 + b^4) - 4 * (B * a * b - A * b^2) / ((a^3 * b + a * b^3 + (a^4 + a^2 * b^2) / \tan(d * x + c)) * \text{sqrt}(\tan(d * x + c))) / d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x))/(a + b\*tan(c + d\*x))\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/(b\*tan(d\*x + c) + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2,x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^2, x)

$$3.597 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=390

$$\frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} \quad \frac{(a^2(A-B) - b^2(A-B) + 2ab(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

[Out]  $-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+1/4*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/(a^2+b^2)^2/d/a^{(1/2)}/b^{(1/2)}-(A*b-B*a)*\cot(d*x+c)^{(1/2)}/(a^2+b^2)/d/(b+a*\cot(d*x+c))$

Rubi [A]

time = 0.61, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3662, 3689, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d (a^2 + b^2)} - \frac{(ab - a^2 b) \sqrt{\cot(c+dx)}}{d (a^2 + b^2) \sqrt{\cot(c+dx)} + b} - \frac{(-a^2(A+B) + 2ab(A-B) + b^2(A+B)) \ln(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1)}{2 \sqrt{2} d (a^2 + b^2)} - \frac{(-a^2(A+B) + 2ab(A-B) + b^2(A+B)) \ln(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1)}{2 \sqrt{2} d (a^2 + b^2)} - \frac{(a^2(A-B) + 2ab(A+B) - b^2(A-B)) \operatorname{ArcTan}\left(\frac{\sqrt{2} \sqrt{\cot(c+dx)}}{\sqrt{2}}\right)}{\sqrt{2} d (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^2), x]$

[Out]  $((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/(\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) + ((3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*(a^2 + b^2)^2*d) - ((A*b - a*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/((a^2 + b^2)*d*(b + a*\operatorname{Cot}[c + d*x])) - ((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) + ((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)\*c]

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) + A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^2} dx &= \int \frac{\sqrt{\cot(c + dx)} (B + A \cot(c + dx))}{(b + a \cot(c + dx))^2} dx \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{(a^2 + b^2) d(b + a \cot(c + dx))} + \frac{\int \frac{-\frac{1}{2}a(Ab - aB) + a(aA + bB) \cot(c + dx)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} dx}{a(a^2 + b^2)} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{(a^2 + b^2) d(b + a \cot(c + dx))} + \frac{\int \frac{a(a^2 A - Ab^2 + 2abB) + a(2aAb - a^2 B)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} dx}{a(a^2 + b^2)^2} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{(a^2 + b^2) d(b + a \cot(c + dx))} + \frac{2 \text{Subst}\left(\int \frac{-a(a^2 A - Ab^2 + 2abB) - a^2 B}{(1 + u)^2} du\right)}{a(a^2 + b^2)^2} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{(a^2 + b^2) d(b + a \cot(c + dx))} + \frac{(3a^2 Ab - Ab^3 - a^3 B + 3ab^2 B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{\sqrt{a} \sqrt{b} (a^2 + b^2)^2 d} \\
&= \frac{(3a^2 Ab - Ab^3 - a^3 B + 3ab^2 B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{\sqrt{a} \sqrt{b} (a^2 + b^2)^2 d} \\
&= \frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}
\end{aligned}$$

**Mathematica [A]**

time = 1.80, size = 336, normalized size = 0.86

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( 2\sqrt{2} (a^2(A - B) + b^2(-A + B) + 2ab(A + B)) (\text{ArcTan}[1 - \sqrt{2} \sqrt{\tan(c + dx)}] - \text{ArcTan}[1 + \sqrt{2} \sqrt{\tan(c + dx)}]) - \frac{a^2(A - B) + b^2(-A + B) + 2ab(A + B)}{\sqrt{2} \sqrt{a}} \frac{\sqrt{2} \sqrt{\tan(c + dx)}}{\sqrt{a}} + \sqrt{2} (a^2(A - B) - a^2(A + B) + b^2(A + B)) (\log(1 - \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx)) - \log(1 + \sqrt{2} \sqrt{\tan(c + dx)} + \tan(c + dx))) - \frac{a^2(A - B) + b^2(-A + B) + 2ab(A + B)}{2\sqrt{2} \sqrt{a}} \right)}{4(a^2 + b^2)^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^2), x]

[Out] -1/4\*(Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(2\*Sqrt[2]\*(a^2\*(A - B) + b^2\*(-A + B) + 2\*a\*b\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]) - (4\*(a^2 + b^2)\*(-A\*b) + a\*B)\*ArcTan[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]])/(Sqrt[a]\*Sqrt[b]) + (8\*Sqrt[b]\*(a^2\*A -

$$A*b^2 + 2*a*b*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]/Sqrt[a] + Sqrt[2]*(2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - (4*(a^2 + b^2)*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x]))/((a^2 + b^2)^2*d)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 42.06, size = 40165, normalized size = 102.99

method	result	size
default	Expression too large to display	40165

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^2,x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [A]**

time = 0.53, size = 347, normalized size = 0.89

$$\frac{(B^2 - 3AB + 3B^2 + a^2) \arctan\left(\frac{\sqrt{2} \sqrt{a+b \tan(dx+c)}}{\sqrt{a^2 + b^2 \tan^2(dx+c)}}\right) + \sqrt{2} \sqrt{(a+B \tan(dx+c))} \arctan\left(\frac{\sqrt{2} \sqrt{a+b \tan(dx+c)}}{\sqrt{a^2 + b^2 \tan^2(dx+c)}}\right) + \sqrt{2} \sqrt{(a+B \tan(dx+c))} \arctan\left(\frac{\sqrt{2} \sqrt{a+b \tan(dx+c)}}{\sqrt{a^2 + b^2 \tan^2(dx+c)}}\right) - \sqrt{2} \sqrt{(a+B \tan(dx+c))} \arctan\left(\frac{\sqrt{2} \sqrt{a+b \tan(dx+c)}}{\sqrt{a^2 + b^2 \tan^2(dx+c)}}\right) - \sqrt{2} \sqrt{(a+B \tan(dx+c))} \arctan\left(\frac{\sqrt{2} \sqrt{a+b \tan(dx+c)}}{\sqrt{a^2 + b^2 \tan^2(dx+c)}}\right) - \frac{1}{(a^2 + b^2 \tan^2(dx+c)) \sqrt{a^2 + b^2 \tan^2(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="maxima")

[Out] 
$$-1/4*(4*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*\arctan(a/(\sqrt{a*b})*\sqrt{\tan(d*x + c)}))/((a^4 + 2*a^2*b^2 + b^4)*\sqrt{a*b}) + (2*\sqrt{2})*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(d*x + c)})) + 2*\sqrt{2}*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(d*x + c)})) + \sqrt{2}*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*\log(\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) - \sqrt{2}*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*\log(-\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1))/((a^4 + 2*a^2*b^2 + b^4) - 4*(B*a - A*b)/((a^2*b + b^3 + (a^3 + a*b^2)/\tan(d*x + c))*\sqrt{\tan(d*x + c)}))/d$$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")



[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^2 \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*2\*sqrt(cot(c + d\*x))), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^2\*sqrt(cot(d\*x + c))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^2),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^2), x)

$$3.598 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))^2} dx$$

**Optimal.** Leaf size=392

$$\frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d} \quad \frac{(2ab(A-B) - a^2(A+B) + b^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}$$

[Out]  $-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/4*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-(A*a^2*b-3*A*b^3+B*a^3+5*B*a*b^2)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})*a^{(1/2)}/b^{(3/2)}/(a^2+b^2)^2/d+a*(A*b-B*a)*\cot(d*x+c)^{(1/2)}/b/(a^2+b^2)/d/(b+a*\cot(d*x+c))$

**Rubi [A]**

time = 0.61, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {3662, 3690, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$$\frac{(-b^2(A+B)+2ab(A-B)+a^2(A+B))\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^2d} - \frac{(-b^2(A+B)+2ab(A-B)+a^2(A+B))\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}(a^2+b^2)^2d} + \frac{a^2(b-a)^2\ln\left(\frac{\cot(c+dx)}{1+\cot(c+dx)}\right)}{4(a^2+b^2)(a+b)^2} + \frac{(a^2(A-B)+2ab(A+B)-b^2(A-B))\ln\left(\frac{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)} - \frac{(a^2(A-B)+2ab(A+B)-b^2(A-B))\ln\left(\frac{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)} + \frac{\sqrt{2}(a^2b+b^3-3ab^2)\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{4\sqrt{2}(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2), x]

[Out]  $((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - ((2*a*b*(A-B) - a^2*(A+B) + b^2*(A+B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]) / (\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - (\operatorname{Sqrt}[a]*(a^2*A*b - 3*A*b^3 + a^3*B + 5*a*b^2*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]] / (b^{(3/2)}*(a^2 + b^2)^2*d) + (a*(A*b - a*B))*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] / (b*(a^2 + b^2)*d*(b + a*\operatorname{Cot}[c + d*x])) + ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d) - ((a^2*(A-B) - b^2*(A-B) + 2*a*b*(A+B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*(a^2 + b^2)^2*d)$

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^n, x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)]\*(g\_))^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3734

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*((1 + Tan[e + f\*x]^2)/(a + b\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C

, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  
!GtQ[n, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^2} dx &= \int \frac{B + A \cot(c + dx)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} dx \\
 &= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{b(a^2 + b^2) d(b + a \cot(c + dx))} - \frac{\int \frac{\frac{1}{2}(-aAb - a^2B - 2b^2B) - b(Ab - aB) \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx}{b(a^2 + b^2) d(b + a \cot(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{b(a^2 + b^2) d(b + a \cot(c + dx))} - \frac{\int \frac{-b(2aAb - a^2B + b^2B) + b(a^2A - Ab^2 + 2b^2A)}{\sqrt{\cot(c + dx)}} dx}{b(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{b(a^2 + b^2) d(b + a \cot(c + dx))} - \frac{2 \text{Subst}\left(\int \frac{b(2aAb - a^2B + b^2B) - b(a^2A - Ab^2 + 2b^2A)}{1 + x^4} dx\right)}{b(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
 &= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{b(a^2 + b^2) d(b + a \cot(c + dx))} - \frac{(a(a^2Ab - 3Ab^3 + a^3B + 5ab^2B)) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^3/2 (a^2 + b^2)^2 d} \\
 &= -\frac{\sqrt{a} (a^2Ab - 3Ab^3 + a^3B + 5ab^2B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^3/2 (a^2 + b^2)^2 d} \\
 &= -\frac{\sqrt{a} (a^2Ab - 3Ab^3 + a^3B + 5ab^2B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^3/2 (a^2 + b^2)^2 d} \\
 &= \frac{(2ab(A - B) - a^2(A + B) + b^2(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2)^2 d}
 \end{aligned}$$

**Mathematica [A]**

time = 1.60, size = 342, normalized size = 0.87

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( -2\sqrt{2}d(a-B) - a^2(A+B) + b^2(A+B) \right) \left( \text{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) - \text{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right) \right) + \frac{\sqrt{2} \sqrt{a} \sqrt{\cot(c+dx)} \sqrt{b} \left( \frac{2\sqrt{2}ab(A-B) - a^2(A+B) + b^2(A+B)}{2} \right) \text{ArcTan}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{\cot(c+dx)}}{\sqrt{b}}\right) + \sqrt{2} \sqrt{a} \sqrt{\cot(c+dx)} \sqrt{b} \left( \frac{2\sqrt{2}ab(A-B) - a^2(A+B) + b^2(A+B)}{2} \right) \left( \ln\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) - \ln\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right) \right) + \frac{b \sqrt{2} \sqrt{a} \sqrt{\cot(c+dx)} \sqrt{b} \left( \frac{2\sqrt{2}ab(A-B) - a^2(A+B) + b^2(A+B)}{2} \right)}{4(a^2 + b^2)d}}{4(a^2 + b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^2), x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-2\*Sqrt[2]\*(2\*a\*b\*(A - B) - a^2\*(A + B) + b^2\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]) - ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]])/(Sqrt[2]\*(a^2 + b^2)\*d)

```
rt[2]*Sqrt[Tan[c + d*x]]) + (4*Sqrt[a]*(a^2 + b^2)*(A*b - a*B)*ArcTan[(Sqr
t[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(3/2) + (8*Sqrt[a]*(-2*A*b^3 + a*(a^2
+ 3*b^2)*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(3/2) + Sqrt[2]
*(a^2*(A - B) + b^2*(-A + B) + 2*a*b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
) + (4*a*(a^2 + b^2)*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(b*(a + b*Tan[c + d*x]
))))/(4*(a^2 + b^2)^2*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 43.85, size = 40146, normalized size = 102.41

method	result	size
default	Expression too large to display	40146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [A]**

time = 0.54, size = 360, normalized size = 0.92

$$\frac{4(Ba + A^2B^2 - 3AB^2) \arctan\left(\frac{\sqrt{2}\sqrt{\tan(dx+c)}}{\sqrt{a^2+b^2}\sqrt{ab}}\right) + \sqrt{2}((A+B)^2 - (A-B)ab - (A+B)^2) \arctan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\tan(dx+c)}{a^2+b^2}}\right) + \sqrt{2}((A+B)^2 - (A-B)ab - (A+B)^2) \arctan\left(-\frac{1}{\sqrt{2}}\sqrt{\frac{\tan(dx+c)}{a^2+b^2}}\right) - \sqrt{2}((A-B)^2 + (A+B)ab - (A-B)^2) \arctan\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}\sqrt{\frac{a^2+b^2}{ab}}\right) + \sqrt{2}((A-B)^2 + (A+B)ab - (A-B)^2) \arctan\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}\sqrt{\frac{a^2+b^2}{ab}}\right)}{(a^2+b^2)^2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(4*(B*a^4 + A*a^3*b + 5*B*a^2*b^2 - 3*A*a*b^3)*arctan(a/(sqrt(a*b)*sqr
t(tan(d*x + c))))/((a^4*b + 2*a^2*b^3 + b^5)*sqrt(a*b)) - (2*sqrt(2)*((A +
B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(
tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)*a^2 +
2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x +
c) + 1) + sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(-sqrt(2)/
sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) + 4*(B*a^
2 - A*a*b)/((a^2*b^2 + b^4 + (a^3*b + a*b^3)/tan(d*x + c))*sqrt(tan(d*x + c
))))/d
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^2 \cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*2,x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*2\*cot(c + d\*x)\*\*(3/2)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^2\*cot(d\*x + c)^(3/2)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^2),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^2), x)





ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 210

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-Rt[-a, 2]\*Rt[-b, 2])^(-1)\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 631

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1182

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[(-a)]

\*c]

### Rule 3615

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3662

Int[(cot[(e\_) + (f\_)\*(x\_)]\*(g\_))^(p\_)\*((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[g^(m + n), Int[(g\*Cot[e + f\*x])^(p - m - n)\*(b + a\*Cot[e + f\*x])^m\*(d + c\*Cot[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3715

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(

```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^2} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(b + a \cot(c + dx))^2} dx \\
&= \frac{a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))} - \int \frac{\frac{1}{2}(aAb - 3a^2B - 2b^2B)}{\dots} \\
&= -\frac{aAb - 3a^2B - 2b^2B}{b^2(a^2 + b^2) d \sqrt{\cot(c + dx)}} + \frac{a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))} \\
&= -\frac{aAb - 3a^2B - 2b^2B}{b^2(a^2 + b^2) d \sqrt{\cot(c + dx)}} + \frac{a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))} \\
&= -\frac{aAb - 3a^2B - 2b^2B}{b^2(a^2 + b^2) d \sqrt{\cot(c + dx)}} + \frac{a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))} \\
&= -\frac{aAb - 3a^2B - 2b^2B}{b^2(a^2 + b^2) d \sqrt{\cot(c + dx)}} + \frac{a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))} \\
&= -\frac{a^3/2(a^2Ab + 5Ab^3 - 3a^3B - 7ab^2B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^{5/2}(a^2 + b^2)^2 d} \\
&= -\frac{a^3/2(a^2Ab + 5Ab^3 - 3a^3B - 7ab^2B) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{\cot(c + dx)}}{\sqrt{b}}\right)}{b^{5/2}(a^2 + b^2)^2 d} \\
&= -\frac{(a^2(A - B) - b^2(A - B) + 2ab(A + B)) \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2}(a^2 + b^2)^2 d}
\end{aligned}$$

**Mathematica [A]**

time = 2.10, size = 390, normalized size = 0.89

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( \frac{2\sqrt{2}(a^2b^2-3a^2b+3ab^2+3b^3) \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}\sqrt{\tan(c+dx)}}\right) - \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{2(a^2+b^2)} + \frac{a^{3/2}(a^2Ab+5Ab^3-3a^3B-7ab^2B) \operatorname{ArcTan}\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}(a^2+b^2)^2} - \frac{\sqrt{2}(a^2(A-B)-b^2(A-B)+2ab(A+B)) \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{2(a^2+b^2)} - \frac{\sqrt{2}(a^2(A-B)-b^2(A-B)+2ab(A+B)) \operatorname{ArcTan}\left(\frac{1-\sqrt{2}\sqrt{\tan(c+dx)}}{\sqrt{2}\sqrt{\tan(c+dx)}}\right)}{2(a^2+b^2)} + \frac{a(Ab-aB)}{b(a^2+b^2)d\sqrt{\cot(c+dx)}(b+a\cot(c+dx))} \right)}{\sqrt{2}(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2), x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B) + 2*a*b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]) - ArcTan[1 + S
```

$$\frac{\sqrt{2} \sqrt{\tan(c+dx)}}{(a^2+b^2)^2 + (4a^{3/2}(-Ab) + aB) \operatorname{Arctan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right) / (b^{5/2}(a^2+b^2)) + (8a^{3/2})(a^2Ab + 3Ab^3 - 2a^3B - 4ab^2B) \operatorname{Arctan}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right) / (b^{5/2}(a^2+b^2)^2) - (\sqrt{2}(2ab(-A+B) + a^2(A+B) - b^2(A+B)) \operatorname{Log}[1 - \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)] - \operatorname{Log}[1 + \sqrt{2} \sqrt{\tan(c+dx)} + \tan(c+dx)]) / (a^2+b^2)^2 + (8B \sqrt{\tan(c+dx)}) / b^2 + (4a^2(-Ab) + aB) \sqrt{\tan(c+dx)} / (b^2(a^2+b^2)(a+b \tan(c+dx)))} / (4d)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 72.48, size = 42740, normalized size = 97.80

method	result	size
default	Expression too large to display	42740

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.56, size = 402, normalized size = 0.92

$$\frac{(3B^2 - A^2B^2 + 5A^2B) \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{ab}}\right) + \sqrt{2} (A-B)^2 (1+A-B) \operatorname{arctan}\left(\sqrt{2} \sqrt{\frac{\tan(dx+c)}{\tan(dx+c)+1}}\right) + \sqrt{2} (A-B)^2 (1+A-B) \operatorname{arctan}\left(-\sqrt{2} \sqrt{\frac{\tan(dx+c)}{\tan(dx+c)+1}}\right) + \sqrt{2} (A-B)^2 (1+A-B) \operatorname{arctan}\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)+1}}\right) - \sqrt{2} (A-B)^2 (1+A-B) \operatorname{arctan}\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)+1}}\right) + \frac{(A+B)(A^2+B^2+2AB) \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\sqrt{\tan(dx+c)+1}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm  
="maxima")`

[Out]  $\frac{1}{4} (4(3Ba^5 - Aa^4b + 7Ba^3b^2 - 5Aa^2b^3) \operatorname{arctan}(a/(\sqrt{a*b} \sqrt{\tan(dx+c)})) / ((a^4b^2 + 2a^2b^4 + b^6) \sqrt{a*b}) + (2\sqrt{2}) * ((A-B)a^2 + 2(A+B)ab - (A-B)b^2) \operatorname{arctan}(1/2\sqrt{2} * (\sqrt{2} + 2/\sqrt{\tan(dx+c)})) + 2\sqrt{2} * ((A-B)a^2 + 2(A+B)ab - (A-B)b^2) \operatorname{arctan}(-1/2\sqrt{2} * (\sqrt{2} - 2/\sqrt{\tan(dx+c)})) + \sqrt{2} * ((A+B)a^2 - 2(A-B)ab - (A+B)b^2) \log(\sqrt{2}/\sqrt{\tan(dx+c)}) + 1/\tan(dx+c) + 1) - \sqrt{2} * ((A+B)a^2 - 2(A-B)ab - (A+B)b^2) \log(-\sqrt{2}/\sqrt{\tan(dx+c)}) + 1/\tan(dx+c) + 1) / (a^4 + 2a^2b^2 + b^4) + 4(2Ba^2b + 2Bb^3 + (3Ba^3 - Aa^2b + 2Ba^2b^2)/\tan(dx+c)) / ((a^2b^3 + b^5)/\sqrt{\tan(dx+c)} + (a^3b^2 + ab^4)/\tan(dx+c)^{(3/2)}) / d$

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

[Out] Timed out

**Sympy** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
```

[Out] Timed out

**Giac** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

[Out] Timed out

**Mupad** [F]  
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2), x)
```

$$3.600 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=601

$$\frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}$$

[Out]  $1/4*b^{(3/2)}*(63*A*a^4*b+46*A*a^2*b^3+15*A*b^5-35*B*a^5-6*B*a^3*b^2-3*B*a*b^4)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/a^{(7/2)}/(a^2+b^2)^3/d+1/2*b*(A*b-B*a)*\cot(d*x+c)^{(5/2)}/a/(a^2+b^2)/d/(b+a*\cot(d*x+c))^2+1/4*b*(13*A*a^2*b+5*A*b^3-9*B*a^3-B*a*b^2)*\cot(d*x+c)^{(3/2)}/a^2/(a^2+b^2)^2/d/(b+a*\cot(d*x+c))+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(8*A*a^4+31*A*a^2*b^2+15*A*b^4-11*B*a^3*b-3*B*a*b^3)*\cot(d*x+c)^{(1/2)}/a^3/(a^2+b^2)^2/d$

**Rubi** [A]

time = 1.25, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$ , Rules used = {3662, 3686, 3726, 3728, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cot}[c + d*x])^{(3/2)}*(A + B*\operatorname{Tan}[c + d*x])]/(a + b*\operatorname{Tan}[c + d*x])^3, x]$

[Out]  $-(((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^3*d)) + ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^3*d)) + (b^{(3/2)}*(63*a^4*A*b + 46*a^2*A*b^3 + 15*A*b^5 - 35*a^5*B - 6*a^3*b^2*B - 3*a*b^4*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/(4*a^{(7/2)}*(a^2 + b^2)^3*d) - ((8*a^4*A + 31*a^2*A*b^2 + 15*A*b^4 - 11*a^3*b*B - 3*a*b^3*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(4*a^3*(a^2 + b^2)^2*d) + (b*(A*b - a*B)*\operatorname{Cot}[c + d*x]^{(5/2)})/(2*a*(a^2 + b^2)*d*(b + a*\operatorname{Cot}[c + d*x])^2) + (b*(13*a^2*A*b + 5*A*b^3 - 9*a^3*B - a*b^2*B)*\operatorname{Cot}[c + d*x]^{(3/2)})/(4*a^2*(a^2 + b^2)^2*d*(b + a*\operatorname{Cot}[c + d*x])) + ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d)$

$3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B)*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x] + \text{Cot}[c + d*x]]]/(2*\text{Sqrt}[2]*(a^2 + b^2)^{3*d})$

#### Rule 65

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{n, x}], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 210

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \& \& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

#### Rule 211

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

#### Rule 631

$\text{Int}[(a_. + (b_.)*(x_.) + (c_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 642

$\text{Int}[(d_. + (e_.)*(x_.))/((a_. + (b_.)*(x_.) + (c_.)*(x_.)^2), x\_Symbol] \rightarrow \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1176

$\text{Int}[(d_. + (e_.)*(x_.)^2)/((a_. + (c_.)*(x_.)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \& \& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

#### Rule 1179

$\text{Int}[(d_. + (e_.)*(x_.)^2)/((a_. + (c_.)*(x_.)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{Fre}$



$eQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 1182

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] :> With[\{q = Rt[a*c, 2]\}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[\{a, c, d, e\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& NeQ[c*d^2 - a*e^2, 0] \&\& NegQ[(-a)*c]$

### Rule 3615

$Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]]], x\_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[\{b, c, d, e, f\}, x] \&\& NeQ[c^2 - d^2, 0] \&\& NeQ[c^2 + d^2, 0]$

### Rule 3662

$Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] :> Dist[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] /; FreeQ[\{a, b, c, d, e, f, g, p\}, x] \&\& !IntegerQ[p] \&\& IntegerQ[m] \&\& IntegerQ[n]$

### Rule 3686

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] :> Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 + b^2, 0] \&\& NeQ[c^2 + d^2, 0] \&\& GtQ[m, 1] \&\& LtQ[n, -1] \&\& (IntegerQ[m] || IntegersQ[2*m, 2*n])$

### Rule 3715

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x\_Symbol] :> Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& EqQ[A, C]$

Rule 3726

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2], x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3728

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2], x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} dx &= \int \frac{\cot^{\frac{7}{2}}(c+dx)(B+A\cot(c+dx))}{(b+a\cot(c+dx))^3} dx \\
&= \frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2a(a^2+b^2)d(b+a\cot(c+dx))^2} - \int \frac{\cot^{\frac{3}{2}}(c+dx)(-\frac{5}{2}b(Ab-aB)+2a(AB))}{(b+a\cot(c+dx))^3} dx \\
&= \frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2a(a^2+b^2)d(b+a\cot(c+dx))^2} + \frac{b(13a^2Ab+5Ab^3-9a^3B-9a^2bB)}{4a^2(a^2+b^2)^2d(b+a\cot(c+dx))} \\
&= -\frac{(8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B)\sqrt{\cot(c+dx)}}{4a^3(a^2+b^2)^2d} \\
&= -\frac{(8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B)\sqrt{\cot(c+dx)}}{4a^3(a^2+b^2)^2d} \\
&= -\frac{(8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B)\sqrt{\cot(c+dx)}}{4a^3(a^2+b^2)^2d} \\
&= -\frac{(8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B)\sqrt{\cot(c+dx)}}{4a^3(a^2+b^2)^2d} \\
&= \frac{b^{3/2}(63a^4Ab+46a^2Ab^3+15Ab^5-35a^5B-6a^3b^2B-3ab^4B)\tan^{-1}\left(\frac{\sqrt{\cot(c+dx)}}{b}\right)}{4a^{7/2}(a^2+b^2)^3d} \\
&= \frac{b^{3/2}(63a^4Ab+46a^2Ab^3+15Ab^5-35a^5B-6a^3b^2B-3ab^4B)\tan^{-1}\left(\frac{\sqrt{\cot(c+dx)}}{b}\right)}{4a^{7/2}(a^2+b^2)^3d} \\
&= -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\tan^{-1}\left(\frac{\sqrt{\cot(c+dx)}}{b}\right)}{\sqrt{2}(a^2+b^2)^3d}
\end{aligned}$$

**Mathematica [A]**

time = 5.87, size = 578, normalized size = 0.96

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3, x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*Sqrt[2]*(a^3*(A - B) + 3*a*b^2*(
-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]]) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]])/(a^2 + b^2)^3 - (4*b^(3/2)
*(3*a^2*A*b + A*b^3 - 2*a^3*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]
)/(a^(7/2)*(a^2 + b^2)^2) - (8*b^(3/2)*(6*a^4*A*b + 3*a^2*A*b^3 + A*b^5 - 3
*a^5*B + a^3*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(7/2)*
(a^2 + b^2)^3) - (Sqrt[2]*(b^3*(A - B) + 3*a^2*b*(-A + B) + a^3*(A + B) - 3
*a*b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2)^3 - (8*A)/(a^3*
Sqrt[Tan[c + d*x]]) + (2*b^2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/(a^2*(a^2 +
b^2)*(a + b*Tan[c + d*x])^2) - (4*b^2*(3*a^2*A*b + A*b^3 - 2*a^3*B)*Sqrt[T
an[c + d*x]])/(a^3*(a^2 + b^2)^2*(a + b*Tan[c + d*x])) - (3*b^2*(A*b - a*B)
*(ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*Sqrt[b]) + Sqrt[Tan
[c + d*x]]/(a*(a + b*Tan[c + d*x])))/(a^2*(a^2 + b^2)))/(4*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 42.26, size = 156936, normalized size = 261.12

method	result	size
default	Expression too large to display	156936

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [A]**

time = 0.55, size = 577, normalized size = 0.96

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Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] -1/4*((35*B*a^5*b^2 - 63*A*a^4*b^3 + 6*B*a^3*b^4 - 46*A*a^2*b^5 + 3*B*a*b^6
- 15*A*b^7)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^9 + 3*a^7*b^2 + 3
*a^5*b^4 + a^3*b^6)*sqrt(a*b)) - (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b
- 3*(A - B)*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d
*x + c)))) + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (
A + B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)
*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(sqrt(2)
)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a^3 - 3*(A -
B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) +
```

$$\frac{1/\tan(dx + c) + 1)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6) - ((11Ba^3b^3 - 15Aa^2b^4 + 3Bab^5 - 7Ab^6)/\sqrt{\tan(dx + c)} + (13Ba^4b^2 - 17Aa^3b^3 + 5Ba^2b^4 - 9Aab^5)/\tan(dx + c)^{(3/2)})/(a^7b^2 + 2a^5b^4 + a^3b^6 + 2(a^8b + 2a^6b^3 + a^4b^5)/\tan(dx + c) + (a^9 + 2a^7b^2 + a^5b^4)/\tan(dx + c)^2) + 8A/(a^3\sqrt{\tan(dx + c)})}/d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{(a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*(3/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))\*\*3,x)

[Out] Integral((A + B\*tan(c + dx))\*cot(c + dx)\*\*(3/2)/(a + b\*tan(c + dx))\*\*3, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(A+B\*tan(dx+c))/(a+b\*tan(dx+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(dx + c) + A)\*cot(dx + c)^(3/2)/(b\*tan(dx + c) + a)^3, x)

**Mupad** [F(-1)]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + dx)^(3/2)\*(A + B\*tan(c + dx)))/(a + b\*tan(c + dx))^3,x)

[Out] \text{Hanged}

$$3.601 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx$$

**Optimal.** Leaf size=534

$$\frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}$$

[Out] 1/2\*b\*(A\*b-B\*a)\*cot(d\*x+c)^(3/2)/a/(a^2+b^2)/d/(b+a\*cot(d\*x+c))^2+1/2\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)-a^3\*(A+B)+3\*a\*b^2\*(A+B))\*arctan(-1+2^(1/2)\*cot(d\*x+c)^(1/2))/(a^2+b^2)^3/d\*2^(1/2)+1/2\*(3\*a^2\*b\*(A-B)-b^3\*(A-B)-a^3\*(A+B)+3\*a\*b^2\*(A+B))\*arctan(1+2^(1/2)\*cot(d\*x+c)^(1/2))/(a^2+b^2)^3/d\*2^(1/2)-1/4\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)+3\*a^2\*b\*(A+B)-b^3\*(A+B))\*ln(1+cot(d\*x+c)-2^(1/2)\*cot(d\*x+c)^(1/2))/(a^2+b^2)^3/d\*2^(1/2)+1/4\*(a^3\*(A-B)-3\*a\*b^2\*(A-B)+3\*a^2\*b\*(A+B)-b^3\*(A+B))\*ln(1+cot(d\*x+c)+2^(1/2)\*cot(d\*x+c)^(1/2))/(a^2+b^2)^3/d\*2^(1/2)-1/4\*(35\*A\*a^4\*b+6\*A\*a^2\*b^3+3\*A\*b^5-15\*B\*a^5+18\*B\*a^3\*b^2+B\*a\*b^4)\*arctan(a^(1/2)\*cot(d\*x+c)^(1/2)/b^(1/2))\*b^(1/2)/a^(5/2)/(a^2+b^2)^3/d+1/4\*b\*(11\*A\*a^2\*b+3\*A\*b^3-7\*B\*a^3+B\*a\*b^2)\*cot(d\*x+c)^(1/2)/a^2/(a^2+b^2)^2/d/(b+a\*cot(d\*x+c))

**Rubi [A]**

time = 0.90, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3686, 3726, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

$\frac{1}{2} \sqrt{2} \frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)^3 d} + \frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)^3 d} - \frac{1}{4} \frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \ln\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)^3 d} - \frac{1}{4} \frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \ln\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{(a^2+b^2)^3 d} + \frac{1}{4} \frac{(35Aa^4b + 6Aa^2b^3 + 3Ab^5 - 15B a^5 + 18B a^3b^2 + B a b^4) \operatorname{ArcTan}\left(\frac{a^{1/2} \cot(c+dx)^{1/2}}{b^{1/2}}\right)}{(a^2+b^2)^3 d} + \frac{1}{4} \frac{b(11Aa^2b + 3Ab^3 - 7Ba^3 + B a b^2) \cot(c+dx)^{1/2}}{a^2 (a^2+b^2)^2 d} \frac{1}{(b+a \cot(c+dx))}$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^3,x]

[Out] -(((3\*a^2\*b\*(A - B) - b^3\*(A - B) - a^3\*(A + B) + 3\*a\*b^2\*(A + B))\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*(a^2 + b^2)^3\*d)) + ((3\*a^2\*b\*(A - B) - b^3\*(A - B) - a^3\*(A + B) + 3\*a\*b^2\*(A + B))\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/(Sqrt[2]\*(a^2 + b^2)^3\*d) - (Sqrt[b]\*(35\*a^4\*A\*b + 6\*a^2\*A\*b^3 + 3\*A\*b^5 - 15\*a^5\*B + 18\*a^3\*b^2\*B + a\*b^4\*B)\*ArcTan[(Sqrt[a]\*Sqrt[Cot[c + d\*x]])/Sqrt[b]]/(4\*a^(5/2)\*(a^2 + b^2)^3\*d) + (b\*(A\*b - a\*B)\*Cot[c + d\*x]^(3/2))/(2\*a\*(a^2 + b^2)\*d\*(b + a\*Cot[c + d\*x])^2) + (b\*(11\*a^2\*A\*b + 3\*A\*b^3 - 7\*a^3\*B + a\*b^2\*B)\*Sqrt[Cot[c + d\*x]]/(4\*a^2\*(a^2 + b^2)^2\*d\*(b + a\*Cot[c + d\*x])) - ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) + 3\*a^2\*b\*(A + B) - b^3\*(A + B))\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(2\*Sqrt[2]\*(a^2 + b^2)^3\*d) + ((a^3\*(A - B) - 3\*a\*b^2\*(A - B) + 3\*a^2\*b\*(A + B) - b^3\*(A + B))\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(2\*Sqrt[2]\*(a^2 + b^2)^3\*d))

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(-(Rt[-a, 2]\*Rt[-b, 2])^(-1))\*ArcTan[Rt[-b, 2]\*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726



```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

#### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+b \tan(c+dx))^3} dx &= \int \frac{\cot^{\frac{5}{2}}(c+dx)(B+A \cot(c+dx))}{(b+a \cot(c+dx))^3} dx \\
&= \frac{b(Ab-aB) \cot^{\frac{3}{2}}(c+dx)}{2a(a^2+b^2)d(b+a \cot(c+dx))^2} - \frac{\int \frac{\sqrt{\cot(c+dx)}(-\frac{3}{2}b(Ab-aB))}{(b+a \cot(c+dx))^3} dx}{2a(a^2+b^2)d(b+a \cot(c+dx))^2} \\
&= \frac{b(Ab-aB) \cot^{\frac{3}{2}}(c+dx)}{2a(a^2+b^2)d(b+a \cot(c+dx))^2} + \frac{b(11a^2Ab+3Ab^3-7a^3B+7a^2b^2B+ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{4a^2(a^2+b^2)^2 d(b+a \cot(c+dx))} \\
&= \frac{b(Ab-aB) \cot^{\frac{3}{2}}(c+dx)}{2a(a^2+b^2)d(b+a \cot(c+dx))^2} + \frac{b(11a^2Ab+3Ab^3-7a^3B+7a^2b^2B+ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{4a^2(a^2+b^2)^2 d(b+a \cot(c+dx))} \\
&= \frac{b(Ab-aB) \cot^{\frac{3}{2}}(c+dx)}{2a(a^2+b^2)d(b+a \cot(c+dx))^2} + \frac{b(11a^2Ab+3Ab^3-7a^3B+7a^2b^2B+ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{4a^2(a^2+b^2)^2 d(b+a \cot(c+dx))} \\
&= \frac{b(Ab-aB) \cot^{\frac{3}{2}}(c+dx)}{2a(a^2+b^2)d(b+a \cot(c+dx))^2} + \frac{b(11a^2Ab+3Ab^3-7a^3B+7a^2b^2B+ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{4a^2(a^2+b^2)^2 d(b+a \cot(c+dx))} \\
&= \frac{\sqrt{b}(35a^4Ab+6a^2Ab^3+3Ab^5-15a^5B+18a^3b^2B+ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{4a^{5/2}(a^2+b^2)^3 d} \\
&= \frac{\sqrt{b}(35a^4Ab+6a^2Ab^3+3Ab^5-15a^5B+18a^3b^2B+ab^4B) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{4a^{5/2}(a^2+b^2)^3 d} \\
&= -\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B)) \tan^{-1}\left(\frac{\sqrt{b} \cot^{\frac{1}{2}}(c+dx)}{a+b \cot(c+dx)}\right)}{\sqrt{2}(a^2+b^2)^3 d}
\end{aligned}$$

**Mathematica [A]**

time = 4.39, size = 520, normalized size = 0.97

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3, x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[2]*(3*a^2*b*(A - B) + b^3*(-A + B) - a^3*(A + B) + 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
```

$$\begin{aligned} & x]] - \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]) + (4*\text{Sqrt}[b]*(a^2 + b^2)*(2* \\ & a*A*b - a^2*B + b^2*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/a^{(3/2)} \\ & ) - (8*\text{Sqrt}[b]*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqr} \\ & t[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/\text{Sqrt}[a] - \text{Sqrt}[2]*(a^3*(A - B) + 3*a*b^2*(-A + B) \\ & ) + 3*a^2*b*(A + B) - b^3*(A + B))*(\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Ta} \\ & n[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]) + (2*b*(a \\ & ^2 + b^2)^2*(A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a*(a + b*\text{Tan}[c + d*x])^2) + (4 \\ & *b*(a^2 + b^2)*(2*a*A*b - a^2*B + b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(a*(a + b*\text{Tan}[ \\ & c + d*x])) + (3*b*(a^2 + b^2)^2*(A*b - a*B)*(ArcTan[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d \\ & *x]])/\text{Sqrt}[a]])/a^{(3/2)*\text{Sqrt}[b]} + \text{Sqrt}[\text{Tan}[c + d*x]]/(a*(a + b*\text{Tan}[c + d*x \\ & ]))))/a)/(4*(a^2 + b^2)^3*d) \end{aligned}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 27.68, size = 100811, normalized size = 188.78

method	result	size
default	Expression too large to display	100811

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima [A]**

time = 0.54, size = 558, normalized size = 1.04

$$\frac{\frac{1}{4} \left( \frac{15 B a^5 b - 35 A a^4 b^2 - 18 B a^3 b^3 - 6 A a^2 b^4 - B a b^5 - 3 A b^6}{(a^2 + b^2)^2} \arctan\left(\frac{a}{\sqrt{a b} \sqrt{\tan(d x + c)}}\right) - \frac{(a^8 + 3 a^6 b^2 + 3 a^4 b^4 + a^2 b^6) \sqrt{a b}}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)} - \frac{2 \sqrt{2} (A + B) a^3 - 3 (A - B) a^2 b - 3 (A + B) a b^2 + (A - B) b^3}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)} \arctan\left(\frac{1}{\sqrt{2}} \sqrt{\frac{2}{\tan(d x + c)}}\right) + \frac{2 \sqrt{2} (A + B) a^3 - 3 (A - B) a^2 b - 3 (A + B) a b^2 + (A - B) b^3}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)} \arctan\left(-\frac{1}{\sqrt{2}} \sqrt{\frac{2}{\tan(d x + c)}}\right) - \sqrt{2} \left( (A - B) a^3 + 3 (A + B) a^2 b - 3 (A - B) a b^2 - (A + B) b^3 \right) \log\left(\frac{\sqrt{2}}{\sqrt{\tan(d x + c)}}\right) + \frac{1}{\tan(d x + c)} + 1 \right) + \sqrt{2} \left( (A - B) a^3 + 3 (A + B) a^2 b - 3 (A - B) a b^2 - (A + B) b^3 \right) \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(d x + c)}}\right) + \frac{1}{\tan(d x + c)} + 1 \right)}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) - \frac{((7 B a^3 b^2 - 11 A a^2 b^3 - B a b^4 - 3 A b^5) \sqrt{\tan(d x + c)} + (9 B a^4 b - 13 A a^3 b^2 + B a^2 b^3 - 5 A a b^4) \tan(d x + c)^{(3/2)})}{(a^6 b^2 + 2 a^4 b^4 + a^2 b^6)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm  
="maxima")`

[Out] 
$$\frac{1}{4} * \left( (15 B a^5 b - 35 A a^4 b^2 - 18 B a^3 b^3 - 6 A a^2 b^4 - B a b^5 - 3 A b^6) \arctan\left(\frac{a}{\sqrt{a b} \sqrt{\tan(d x + c)}}\right) - \frac{(a^8 + 3 a^6 b^2 + 3 a^4 b^4 + a^2 b^6) \sqrt{a b}}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)} - \frac{2 \sqrt{2} (A + B) a^3 - 3 (A - B) a^2 b - 3 (A + B) a b^2 + (A - B) b^3}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)} \arctan\left(\frac{1}{\sqrt{2}} \sqrt{\frac{2}{\tan(d x + c)}}\right) + \frac{2 \sqrt{2} (A + B) a^3 - 3 (A - B) a^2 b - 3 (A + B) a b^2 + (A - B) b^3}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)} \arctan\left(-\frac{1}{\sqrt{2}} \sqrt{\frac{2}{\tan(d x + c)}}\right) - \sqrt{2} \left( (A - B) a^3 + 3 (A + B) a^2 b - 3 (A - B) a b^2 - (A + B) b^3 \right) \log\left(\frac{\sqrt{2}}{\sqrt{\tan(d x + c)}}\right) + \frac{1}{\tan(d x + c)} + 1 \right) + \sqrt{2} \left( (A - B) a^3 + 3 (A + B) a^2 b - 3 (A - B) a b^2 - (A + B) b^3 \right) \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(d x + c)}}\right) + \frac{1}{\tan(d x + c)} + 1 \right) / (a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) - \frac{((7 B a^3 b^2 - 11 A a^2 b^3 - B a b^4 - 3 A b^5) \sqrt{\tan(d x + c)} + (9 B a^4 b - 13 A a^3 b^2 + B a^2 b^3 - 5 A a b^4) \tan(d x + c)^{(3/2)})}{(a^6 b^2 + 2 a^4 b^4 + a^2 b^6)}$$

$6 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)/\tan(d*x + c) + (a^8 + 2*a^6*b^2 + a^4*b^4)/\tan(d*x + c)^2)/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{(a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x))/(a + b\*tan(c + d\*x))\*\*3, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/(b\*tan(d\*x + c) + a)^3, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3,x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^3, x)

$$3.602 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=534

$$\frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) (a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B))}{\sqrt{2} (a^2 + b^2)^3 d}$$

[Out]  $-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\ln(1+\cot(d*x+c))+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(15*A*a^4*b-18*A*a^2*b^3-A*b^5-3*B*a^5+26*B*a^3*b^2-3*B*a*b^4)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/a^{(3/2)}/(a^2+b^2)^3/d/b^{(1/2)}+1/2*b*(A*b-B*a)*\cot(d*x+c)^{(1/2)}/a/(a^2+b^2)/d/(b+a*\cot(d*x+c))^2-1/4*(9*A*a^2*b+A*b^3-5*B*a^3+3*B*a*b^2)*\cot(d*x+c)^{(1/2)}/a/(a^2+b^2)^2/d/(b+a*\cot(d*x+c))$

**Rubi** [A]

time = 0.92, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3686, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[c + d*x])]/(\text{Sqrt}[\text{Cot}[c + d*x]]*(a + b*\text{Tan}[c + d*x])^3), x]$

[Out]  $((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])]/(\text{Sqrt}[2]*(a^2 + b^2)^3*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]])]/(\text{Sqrt}[2]*(a^2 + b^2)^3*d) + ((15*a^4*A*b - 18*a^2*A*b^3 - A*b^5 - 3*a^5*B + 26*a^3*b^2*B - 3*a*b^4*B)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[\text{Cot}[c + d*x]])/\text{Sqrt}[b]])/(4*a^{(3/2)}*\text{Sqrt}[b]*(a^2 + b^2)^3*d) + (b*(A*b - a*B)*\text{Sqrt}[\text{Cot}[c + d*x]])/(2*a*(a^2 + b^2)*d*(b + a*\text{Cot}[c + d*x])^2) - ((9*a^2*A*b + A*b^3 - 5*a^3*B + 3*a*b^2*B)*\text{Sqrt}[\text{Cot}[c + d*x]])/(4*a*(a^2 + b^2)^2*d*(b + a*\text{Cot}[c + d*x])) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^3*d) + ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^3*d)$

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3686

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
+ f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

#### Rubi steps



$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^3} dx &= \int \frac{\cot^{\frac{3}{2}}(c + dx)(B + A \cot(c + dx))}{(b + a \cot(c + dx))^3} dx \\
&= \frac{b(Ab - aB) \sqrt{\cot(c + dx)}}{2a(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{\int \frac{-\frac{1}{2}b(Ab - aB) + 2a(Ab - aB) \cot(c + dx)}{\sqrt{\cot(c + dx)}} dx}{2a} \\
&= \frac{b(Ab - aB) \sqrt{\cot(c + dx)}}{2a(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{(9a^2Ab + Ab^3 - 5a^3B + 3a^2B)}{4a(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{b(Ab - aB) \sqrt{\cot(c + dx)}}{2a(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{(9a^2Ab + Ab^3 - 5a^3B + 3a^2B)}{4a(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{b(Ab - aB) \sqrt{\cot(c + dx)}}{2a(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{(9a^2Ab + Ab^3 - 5a^3B + 3a^2B)}{4a(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{b(Ab - aB) \sqrt{\cot(c + dx)}}{2a(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{(9a^2Ab + Ab^3 - 5a^3B + 3a^2B)}{4a(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{b(Ab - aB) \sqrt{\cot(c + dx)}}{2a(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{(9a^2Ab + Ab^3 - 5a^3B + 3a^2B)}{4a(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{(15a^4Ab - 18a^2Ab^3 - Ab^5 - 3a^5B + 26a^3b^2B - 3ab^4B) \tan^{-1}}{4a^{3/2}\sqrt{b} (a^2 + b^2)^3 d} \\
&= \frac{(15a^4Ab - 18a^2Ab^3 - Ab^5 - 3a^5B + 26a^3b^2B - 3ab^4B) \tan^{-1}}{4a^{3/2}\sqrt{b} (a^2 + b^2)^3 d} \\
&= \frac{(a^3(A - B) - 3ab^2(A - B) + 3a^2b(A + B) - b^3(A + B)) \tan^{-1}}{\sqrt{2} (a^2 + b^2)^3 d}
\end{aligned}$$

**Mathematica [A]**

time = 6.24, size = 558, normalized size = 1.04

$$\frac{(15a^4Ab - 18a^2Ab^3 - Ab^5 - 3a^5B + 26a^3b^2B - 3ab^4B) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{\cot(c+dx)}}{a^2 + b^2}\right] + (a^3(A - B) - 3ab^2(A - B) + 3a^2b(A + B) - b^3(A + B)) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{\cot(c+dx)}}{a^2 + b^2}\right]}{4a^{3/2}\sqrt{b}(a^2 + b^2)^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^3), x]

[Out] (2\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-1/4\*((a^3\*(A - B) - 3\*a\*b^2\*(A - B) + 3\*a^2\*b\*(A + B) - b^3\*(A + B))\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c

$$\begin{aligned}
& + d*x]]] - \text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]])]/(a^2 + b^2)^3 \\
& - (\text{Sqrt}[b]*(a^2*A - A*b^2 + 2*a*b*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/ \\
& \text{Sqrt}[a]]/(2*a^{(3/2)}*(a^2 + b^2)^2) - (\text{Sqrt}[b]*(a^3*A - 3*a*A*b^2 + 3*a^2*b*B \\
& - b^3*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(a^2 + b^2 \\
& )^3) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(\text{Sqrt}[2]* \\
& \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]] - \text{Sqrt}[2]*\text{Log}[1 + \\
& \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]))/(8*(a^2 + b^2)^3) - ((A*b - a* \\
& B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(4*(a^2 + b^2)*(a + b*\text{Tan}[c + d*x])^2) - (b*(a^2*A - \\
& A*b^2 + 2*a*b*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(2*a*(a^2 + b^2)^2*(a + b*\text{Tan}[c + d*x \\
& ])) - (3*(A*b - a*B)*(ArcTan[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/(a^{(3/2)} \\
& * \text{Sqrt}[b]) + \text{Sqrt}[\text{Tan}[c + d*x]]/(a*(a + b*\text{Tan}[c + d*x]))))/(8*(a^2 + b^2))) \\
& /d
\end{aligned}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 26.71, size = 102181, normalized size = 191.35

method	result	size
default	Expression too large to display	102181

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [A]**

time = 0.55, size = 544, normalized size = 1.02

$$\frac{1}{4} \left( \frac{3B^2 a^5 - 15A^2 a^4 b - 26B^2 a^3 b^2 + 18A^2 a^2 b^3 + 3B^2 a b^4 + A^2 b^5}{(a^2 + b^2)^3} \arctan\left(\frac{a}{\sqrt{ab} \sqrt{\tan(dx+c)}}\right) + \frac{2\sqrt{2}((A-B)a^3 + 3(A+B)a^2 b - 3(A-B)ab^2 - (A+B)b^3) \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2/\sqrt{\tan(dx+c)})\right) + 2\sqrt{2}((A-B)a^3 + 3(A+B)a^2 b - 3(A-B)ab^2 - (A+B)b^3) \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2/\sqrt{\tan(dx+c)})\right) + \sqrt{2}((A+B)a^3 - 3(A-B)a^2 b - 3(A+B)ab^2 + (A-B)b^3) \log(\sqrt{2}/\sqrt{\tan(dx+c)}) + 1/\tan(dx+c) + 1 - \sqrt{2}((A+B)a^3 - 3(A-B)a^2 b - 3(A+B)ab^2 + (A-B)b^3) \log(-\sqrt{2}/\sqrt{\tan(dx+c)}) + 1/\tan(dx+c) + 1}{(a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6)} - \frac{((3B^2 a^3 b - 7A^2 a^2 b^2 - 5B^2 a b^3 + A^2 b^4)/\sqrt{\tan(dx+c)} + (5B^2 a^4 - 9A^2 a^3 b - 3B^2 a^2 b^4) \sqrt{\tan(dx+c)})}{(a^2 + b^2)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/4*((3*B^2*a^5 - 15*A^2*a^4*b - 26*B^2*a^3*b^2 + 18*A^2*a^2*b^3 + 3*B^2*a*b^4 + A^2*b^5)* \\
& \arctan(a/(\text{sqrt}(a*b)*\text{sqrt}(\text{tan}(d*x + c))))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)* \\
& \text{sqrt}(a*b)) + (2*\text{sqrt}(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)* \\
& \arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2/\text{sqrt}(\text{tan}(d*x + c)))) + 2*\text{sqrt}(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3) \\
& * \arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2/\text{sqrt}(\text{tan}(d*x + c)))) + \text{sqrt}(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3) \\
& * \log(\text{sqrt}(2)/\text{sqrt}(\text{tan}(d*x + c))) + 1/\text{tan}(d*x + c) + 1 - \text{sqrt}(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3) \\
& * \log(-\text{sqrt}(2)/\text{sqrt}(\text{tan}(d*x + c))) + 1/\text{tan}(d*x + c) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((3*B^2*a^3*b - 7*A^2*a^2*b^2 - 5*B^2*a*b^3 + A^2*b^4)/ \\
& \text{sqrt}(\text{tan}(d*x + c)) + (5*B^2*a^4 - 9*A^2*a^3*b - 3*B^2*a^2*b^4) \sqrt{\text{tan}(d*x + c)})/(a^2 + b^2)^3
\end{aligned}$$

$$2 - A*a*b^3)/\tan(d*x + c)^{(3/2)})/(a^5*b^2 + 2*a^3*b^4 + a*b^6 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)/\tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)/\tan(d*x + c)^2))/d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^3 \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*3\*sqrt(cot(c + d\*x))), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^3\*sqrt(cot(d\*x + c))), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^3),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^3), x)

$$3.603 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=530

$$\frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) (3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B))}{\sqrt{2} (a^2 + b^2)^3 d}$$

[Out]  $-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\ln(1+\cot(d*x+c))-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(3*A*a^4*b-26*A*a^2*b^3+3*A*b^5+B*a^5+18*B*a^3*b^2-15*B*a*b^4)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/b^{(3/2)}/(a^2+b^2)^3/d/a^{(1/2)}-1/2*(A*b-B*a)*\cot(d*x+c)^{(1/2)}/(a^2+b^2)/d/(b+a*\cot(d*x+c))^2+1/4*(5*A*a^2*b-3*A*b^3-B*a^3+7*B*a*b^2)*\cot(d*x+c)^{(1/2)}/b/(a^2+b^2)^2/d/(b+a*\cot(d*x+c))$

Rubi [A]

time = 0.92, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3689, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

-----

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])]/(\operatorname{Cot}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x])^3), x]$

[Out]  $((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]/(\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]]/(\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - ((3*a^4*A*b - 26*a^2*A*b^3 + 3*A*b^5 + a^5*B + 18*a^3*b^2*B - 15*a*b^4*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]])/(4*\operatorname{Sqrt}[a]*b^{(3/2)}*(a^2 + b^2)^3*d) - ((A*b - a*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(2*(a^2 + b^2)*d*(b + a*\operatorname{Cot}[c + d*x])^2) + ((5*a^2*A*b - 3*A*b^3 - a^3*B + 7*a*b^2*B)*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(4*b*(a^2 + b^2)^2*d*(b + a*\operatorname{Cot}[c + d*x])) + ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d)$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1) - 1)}*(c - a*(d/b) +$

$d*(x^p/b)^n, x], x, (a + b*x)^{1/p}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 210

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(-\text{Rt}[-a, 2]*\text{Rt}[-b, 2])^{-1})*\text{ArcTan}[\text{Rt}[-b, 2]*(x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 211

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

### Rule 631

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] :> \text{With}[\{q = 1 - 4*\text{Simplify}[a*(c/b^2)]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 642

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x\_Symbol] :> \text{Simp}[d*(\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]/b), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1176

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] :> \text{With}[\{q = \text{Rt}[2*(d/e), 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 1179

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] :> \text{With}[\{q = \text{Rt}[-2*(d/e), 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 1182

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] :> \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + D$

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

### Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ
[2*m, 2*n])
```

### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```

*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^3} dx &= \int \frac{\sqrt{\cot(c + dx)} (B + A \cot(c + dx))}{(b + a \cot(c + dx))^3} dx \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{2(a^2 + b^2) d(b + a \cot(c + dx))^2} + \frac{\int \frac{-\frac{1}{2}a(Ab - aB) + 2a(aA + bB) \cot(c + dx)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} dx}{2a(a^2 + b^2) d} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{2(a^2 + b^2) d(b + a \cot(c + dx))^2} + \frac{(5a^2 Ab - 3Ab^3 - a^3 B + 7ab^2)}{4b(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{2(a^2 + b^2) d(b + a \cot(c + dx))^2} + \frac{(5a^2 Ab - 3Ab^3 - a^3 B + 7ab^2)}{4b(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{2(a^2 + b^2) d(b + a \cot(c + dx))^2} + \frac{(5a^2 Ab - 3Ab^3 - a^3 B + 7ab^2)}{4b(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= -\frac{(Ab - aB) \sqrt{\cot(c + dx)}}{2(a^2 + b^2) d(b + a \cot(c + dx))^2} + \frac{(5a^2 Ab - 3Ab^3 - a^3 B + 7ab^2)}{4b(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= -\frac{(3a^4 Ab - 26a^2 Ab^3 + 3Ab^5 + a^5 B + 18a^3 b^2 B - 15ab^4 B) \tan^{-1} \left( \frac{\sqrt{a} \sqrt{\cot(c + dx)}}{b + a \cot(c + dx)} \right)}{4\sqrt{a} b^{3/2} (a^2 + b^2)^3 d} \\
&= -\frac{(3a^4 Ab - 26a^2 Ab^3 + 3Ab^5 + a^5 B + 18a^3 b^2 B - 15ab^4 B) \tan^{-1} \left( \frac{\sqrt{a} \sqrt{\cot(c + dx)}}{b + a \cot(c + dx)} \right)}{4\sqrt{a} b^{3/2} (a^2 + b^2)^3 d} \\
&= \frac{(3a^2 b(A - B) - b^3(A - B) - a^3(A + B) + 3ab^2(A + B)) \tan^{-1} \left( \frac{\sqrt{2} \sqrt{\cot(c + dx)}}{b + a \cot(c + dx)} \right)}{\sqrt{2} (a^2 + b^2)^3 d}
\end{aligned}$$

**Mathematica [A]**

time = 4.31, size = 518, normalized size = 0.98

$$\frac{(3a^2 b(A - B) - b^3(A - B) - a^3(A + B) + 3ab^2(A + B)) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{\cot(c + dx)}}{b + a \cot(c + dx)}\right] - \frac{(Ab - aB) \sqrt{\cot(c + dx)}}{2(a^2 + b^2) d(b + a \cot(c + dx))^2}}{4\sqrt{a} b^{3/2} (a^2 + b^2)^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^3), x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-2\*Sqrt[2]\*(3\*a^2\*b\*(A - B) + b^3\*(A - B) - a^3\*(A + B) + 3\*a\*b^2\*(A + B))\*(ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d



$$\begin{aligned}
& *x]] - \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]) + (8*\text{Sqrt}[b]*(-3*a^2*A*b + \\
& A*b^3 + a^3*B - 3*a*b^2*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/\text{Sqrt}[a] \\
& + (4*(a^2 + b^2)*(-2*A*b^3 + a*(a^2 + 3*b^2)*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/(\text{Sqrt}[a]*b^{(3/2)}) + \text{Sqrt}[2]*(a^3*(A - B) + 3*a*b^2 \\
& *(-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] \\
& + \text{Tan}[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]]) + \\
& (2*a*(a^2 + b^2)^2*(A*b - a*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(b*(a + b*\text{Tan}[c + d*x]) \\
& ^2) + (4*(a^2 + b^2)*(-2*A*b^3 + a*(a^2 + 3*b^2)*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(b*(a + b*\text{Tan}[c + d*x])) \\
& + (3*(a^2 + b^2)^2*(A*b - a*B)*(\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/(\text{Sqrt}[a]*\text{Sqrt}[b]) + \text{Sqrt}[\text{Tan}[c + d*x]]/(a + b*\text{Tan}[c \\
& + d*x]))/b)/(4*(a^2 + b^2)^3*d)
\end{aligned}$$

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 26.02, size = 102237, normalized size = 192.90

method	result	size
default	Expression too large to display	102237

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVE  
RBOSE)`

[Out] result too large to display

**Maxima** [A]

time = 0.54, size = 545, normalized size = 1.03

$$\frac{1}{4} \frac{(B a^5 + 3 A a^4 b + 18 B a^3 b^2 - 26 A a^2 b^3 - 15 B a b^4 + 3 A b^5) \arctan\left(\frac{a}{\sqrt{a b}} \sqrt{\tan(d x + c)}\right) - (2 \sqrt{2} ((A + B) a^3 - 3(A - B) a^2 b - 3(A + B) a b^2 + (A - B) b^3) \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2/\sqrt{\tan(d x + c)})\right) + 2 \sqrt{2} ((A + B) a^3 - 3(A - B) a^2 b - 3(A + B) a b^2 + (A - B) b^3) \arctan\left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2/\sqrt{\tan(d x + c)})\right) - \sqrt{2} ((A - B) a^3 + 3(A + B) a^2 b - 3(A - B) a b^2 - (A + B) b^3) \log(\sqrt{2}/\sqrt{\tan(d x + c)}) + 1/\tan(d x + c) + 1) + \sqrt{2} ((A - B) a^3 + 3(A + B) a^2 b - 3(A - B) a b^2 - (A + B) b^3) \log(-\sqrt{2}/\sqrt{\tan(d x + c)}) + 1/\tan(d x + c) + 1)}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) - ((B a^3 b + 3 A a^2 b^2 + 9 B a b^3 - 5 A a b^4)/\sqrt{\tan(d x + c)} - (B a^4 - 5 A a^3 b - 7 B a^2 b^2 + 3 A a b^3)/\tan(d x + c)^{(3/2)})/(a^4 b^3 + 2 a^2 b^5 + b^7 + 2(a^5 b^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorithm  
="maxima")`

[Out] 
$$\begin{aligned}
& -1/4*((B*a^5 + 3*A*a^4*b + 18*B*a^3*b^2 - 26*A*a^2*b^3 - 15*B*a*b^4 + 3*A*b \\
& ^5)*\arctan(a/(\text{sqrt}(a*b)*\text{sqrt}(\text{tan}(d*x + c))))/((a^6*b + 3*a^4*b^3 + 3*a^2*b^ \\
& 5 + b^7)*\text{sqrt}(a*b)) - (2*\text{sqrt}(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B) \\
& *a*b^2 + (A - B)*b^3)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2/\text{sqrt}(\text{tan}(d*x + c)))) \\
& + 2*\text{sqrt}(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3) \\
& *\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2/\text{sqrt}(\text{tan}(d*x + c)))) - \text{sqrt}(2)*((A - B)*a \\
& ^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*\log(\text{sqrt}(2)/\text{sqrt}(\text{tan}( \\
& d*x + c)) + 1/\text{tan}(d*x + c) + 1) + \text{sqrt}(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - \\
& 3*(A - B)*a*b^2 - (A + B)*b^3)*\log(-\text{sqrt}(2)/\text{sqrt}(\text{tan}(d*x + c)) + 1/\text{tan}(d*x \\
& + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((B*a^3*b + 3*A*a^2*b^2 + \\
& 9*B*a*b^3 - 5*A*a*b^4)/\text{sqrt}(\text{tan}(d*x + c)) - (B*a^4 - 5*A*a^3*b - 7*B*a^2*b^2 \\
& + 3*A*a*b^3)/\text{tan}(d*x + c)^(3/2))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 2*(a^5*b^2 +
\end{aligned}$$

$2*a^3*b^4 + a*b^6)/\tan(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)/\tan(d*x + c)^2)/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^3),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^3), x)

$$3.604 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=534

$$\frac{(a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + (a^3(A-B) - 3ab^2(A-B) + 3a^2b(A+B) - b^3(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}$$

```
[Out] 1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*cot
(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(
A+B)-b^3*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/
4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*ln(1+cot(d*x+c)-2^(1/2)
*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(
A+B)+3*a*b^2*(A+B))*ln(1+cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2))/(a^2+b^2)^3/d
*2^(1/2)-1/4*(A*a^4*b+18*A*a^2*b^3-15*A*b^5+3*B*a^5+6*B*a^3*b^2+35*B*a*b^4)
*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))*a^(1/2)/b^(5/2)/(a^2+b^2)^3/d+1/2
*a*(A*b-B*a)*cot(d*x+c)^(1/2)/b/(a^2+b^2)/d/(b+a*cot(d*x+c))^2-1/4*a*(A*a^2
*b-7*A*b^3+3*B*a^3+11*B*a*b^2)*cot(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(b+a*cot(
d*x+c))
```

**Rubi** [A]

time = 0.93, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[c + d*x])/((Cot[c + d*x])^(5/2)*(a + b*Tan[c + d*x])^3), x]
```

```
[Out] -(((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*ArcTan[1
- Sqrt[2]*Sqrt[Cot[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d)) + ((a^3*(A - B)
- 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]]/(Sqrt[2]*(a^2 + b^2)^3*d) - (Sqrt[a]*(a^4*A*b + 18*a^2*A*b^
3 - 15*A*b^5 + 3*a^5*B + 6*a^3*b^2*B + 35*a*b^4*B))*ArcTan[(Sqrt[a]*Sqrt[Cot
[c + d*x]]/Sqrt[b])]/(4*b^(5/2)*(a^2 + b^2)^3*d) + (a*(A*b - a*B)*Sqrt[Cot
[c + d*x]]/(2*b*(a^2 + b^2)*d*(b + a*Cot[c + d*x])^2) - (a*(a^2*A*b - 7*A*
b^3 + 3*a^3*B + 11*a*b^2*B))*Sqrt[Cot[c + d*x]]/(4*b^2*(a^2 + b^2)^2*d*(b +
a*Cot[c + d*x])) + ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2
*(A + B))*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]*(a
^2 + b^2)^3*d) - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A
+ B))*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]*(a^2
+ b^2)^3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

#### Rule 1182

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a
*c]
```

#### Rule 3615

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

#### Rule 3662

```
Int[(cot[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*tan[(e_) + (f_)*(
x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist
[g^(m + n), Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c*
Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && !IntegerQ
[p] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n +
2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3715

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

#### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
```

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^3} dx &= \int \frac{B + A \cot(c + dx)}{\sqrt{\cot(c + dx)} (b + a \cot(c + dx))^3} dx \\
&= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{2b(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{\int \frac{\frac{1}{2}(-aAb - 3a^2B - 4b^2B) - 2b(Ab - aB)}{\sqrt{\cot(c + dx)}} dx}{2b(a^2 + b^2)} \\
&= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{2b(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{a(a^2Ab - 7Ab^3 + 3a^3B + 11b^3A)}{4b^2(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{2b(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{a(a^2Ab - 7Ab^3 + 3a^3B + 11b^3A)}{4b^2(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{2b(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{a(a^2Ab - 7Ab^3 + 3a^3B + 11b^3A)}{4b^2(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{2b(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{a(a^2Ab - 7Ab^3 + 3a^3B + 11b^3A)}{4b^2(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{a(Ab - aB) \sqrt{\cot(c + dx)}}{2b(a^2 + b^2) d(b + a \cot(c + dx))^2} - \frac{a(a^2Ab - 7Ab^3 + 3a^3B + 11b^3A)}{4b^2(a^2 + b^2)^2 d(b + a \cot(c + dx))} \\
&= \frac{\sqrt{a} (a^4Ab + 18a^2Ab^3 - 15Ab^5 + 3a^5B + 6a^3b^2B + 35ab^4B) \tan^{-1}\left(\frac{\sqrt{a} \cot(c + dx)}{b + a \cot(c + dx)}\right)}{4b^{5/2} (a^2 + b^2)^3 d} \\
&= \frac{\sqrt{a} (a^4Ab + 18a^2Ab^3 - 15Ab^5 + 3a^5B + 6a^3b^2B + 35ab^4B) \tan^{-1}\left(\frac{\sqrt{a} \cot(c + dx)}{b + a \cot(c + dx)}\right)}{4b^{5/2} (a^2 + b^2)^3 d} \\
&= -\frac{(a^3(A - B) - 3ab^2(A - B) + 3a^2b(A + B) - b^3(A + B)) \tan^{-1}\left(\frac{\sqrt{a} \cot(c + dx)}{b + a \cot(c + dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}
\end{aligned}$$

**Mathematica [A]**

time = 4.52, size = 547, normalized size = 1.02

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3), x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[2]*(a^3*(A - B) + 3*a*b^2*(-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*]
```

$$\begin{aligned} & x]] - \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]]]) + (4*\text{Sqrt}[a]*(a^2 + b^2)*(a^2 \\ & *A*b + 3*A*b^3 - 2*a^3*B - 4*a*b^2*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/ \\ & \text{Sqrt}[a]])/b^{(5/2)} + (8*\text{Sqrt}[a]*(a^2*A*b^3 - 3*A*b^5 + a^5*B + 3*a^3*b^2*B + \\ & 6*a*b^4*B)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a]])/b^{(5/2)} + \text{Sqrt}[2] \\ & *(3*a^2*b*(A - B) + b^3*(-A + B) - a^3*(A + B) + 3*a*b^2*(A + B))*(\text{Log}[1 - \\ & \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d*x]] + \text{Tan}[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[c + d \\ & *x]] + \text{Tan}[c + d*x]]) + (2*a^2*(a^2 + b^2)^2*(-(A*b) + a*B)*\text{Sqrt}[\text{Tan}[c + d* \\ & x]])/(b^2*(a + b*\text{Tan}[c + d*x])^2) + (4*a*(a^2 + b^2)*(a^2*A*b + 3*A*b^3 - 2 \\ & *a^3*B - 4*a*b^2*B)*\text{Sqrt}[\text{Tan}[c + d*x]])/(b^2*(a + b*\text{Tan}[c + d*x])) - (3*(a^2 \\ & + b^2)^2*(A*b - a*B)*((\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])/\text{Sqrt}[a \\ & ]])/ \text{Sqrt}[b] + (a*\text{Sqrt}[\text{Tan}[c + d*x]])/(a + b*\text{Tan}[c + d*x]))/b^2)/(4*(a^2 + \\ & b^2)^3*d) \end{aligned}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 23.70, size = 102127, normalized size = 191.25

method	result	size
default	Expression too large to display	102127

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^3,x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [A]**

time = 0.55, size = 556, normalized size = 1.04

$$\frac{\frac{1}{4} \sqrt{2} \left( (3B a^6 + A a^5 b + 6B a^4 b^2 + 18A a^3 b^3 + 35B a^2 b^4 - 15A a b^5) \arctan\left(\frac{a}{\sqrt{a b} \sqrt{\tan(d x + c)}}\right) - (a^6 b^2 + 3a^4 b^4 + 3a^2 b^6 + b^8) \sqrt{a b} \right) - (2\sqrt{2}((A - B)a^3 + 3(A + B)a^2 b - 3(A - B)a b^2 - (A + B)b^3) \arctan\left(\frac{1}{2\sqrt{2}}(\sqrt{2} + 2/\sqrt{\tan(d x + c)})\right) + 2\sqrt{2}((A - B)a^3 + 3(A + B)a^2 b - 3(A - B)a b^2 - (A + B)b^3) \arctan\left(-\frac{1}{2\sqrt{2}}(\sqrt{2} - 2/\sqrt{\tan(d x + c)})\right) + \sqrt{2}((A + B)a^3 - 3(A - B)a^2 b - 3(A + B)a b^2 + (A - B)b^3) \log(\sqrt{2}/\sqrt{\tan(d x + c)} + 1/\tan(d x + c) + 1) - \sqrt{2}((A + B)a^3 - 3(A - B)a^2 b - 3(A + B)a b^2 + (A - B)b^3) \log(-\sqrt{2}/\sqrt{\tan(d x + c)} + 1/\tan(d x + c) + 1))}{(a^6 + 3a^4 b^2 + 3a^2 b^4 + b^6) + ((5B a^4 b - A a^3 b^2 + 13B a^2 b^3 - 9A a b^4)/\sqrt{\tan(d x + c)} + (3B a^5 + A a^4 b + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/4*((3*B*a^6 + A*a^5*b + 6*B*a^4*b^2 + 18*A*a^3*b^3 + 35*B*a^2*b^4 - 15*A \\ & *a*b^5)*\arctan(a/(\text{sqrt}(a*b)*\text{sqrt}(\text{tan}(d*x + c))))/((a^6*b^2 + 3*a^4*b^4 + 3* \\ & a^2*b^6 + b^8)*\text{sqrt}(a*b)) - (2*\text{sqrt}(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*( \\ & A - B)*a*b^2 - (A + B)*b^3)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2/\text{sqrt}(\text{tan}(d*x + \\ & c)))) + 2*\text{sqrt}(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B \\ & )*b^3)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2/\text{sqrt}(\text{tan}(d*x + c)))) + \text{sqrt}(2)*((A \\ & + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*\log(\text{sqrt}(2)/\text{sqrt} \\ & \text{t}(\text{tan}(d*x + c)) + 1/\text{tan}(d*x + c) + 1) - \text{sqrt}(2)*((A + B)*a^3 - 3*(A - B)*a^2 \\ & *b - 3*(A + B)*a*b^2 + (A - B)*b^3)*\log(-\text{sqrt}(2)/\text{sqrt}(\text{tan}(d*x + c)) + 1/\text{ta} \\ & \text{n}(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((5*B*a^4*b - A*a^3* \\ & b^2 + 13*B*a^2*b^3 - 9*A*a*b^4)/\text{sqrt}(\text{tan}(d*x + c)) + (3*B*a^5 + A*a^4*b + 1 \end{aligned}$$



$$1*B*a^3*b^2 - 7*A*a^2*b^3)/\tan(dx + c)^{(3/2)})/(a^4*b^4 + 2*a^2*b^6 + b^8 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)/\tan(dx + c) + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)/\tan(dx + c)^2))/d$$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)^(5/2)/(a+b\*tan(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)\*\*(5/2)/(a+b\*tan(dx+c))\*\*3,x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)^(5/2)/(a+b\*tan(dx+c))^3,x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^3),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^3), x)

$$3.605 \quad \int \frac{A+B \tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=600

$$\frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d} + \frac{(3a^2b(A-B) - b^3(A-B) - a^3(A+B) + 3ab^2(A+B)) \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} (a^2 + b^2)^3 d}$$

[Out]  $-1/4*a^{(3/2)}*(3*A*a^4*b+6*A*a^2*b^3+35*A*b^5-15*B*a^5-46*B*a^3*b^2-63*B*a*b^4)*\arctan(a^{(1/2)}*\cot(d*x+c)^{(1/2)}/b^{(1/2)})/b^{(7/2)}/(a^2+b^2)^3/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(-3*A*a^3*b-11*A*a*b^3+15*B*a^4+31*B*a^2*b^2+8*B*b^4)/b^3/(a^2+b^2)^2/d/\cot(d*x+c)^{(1/2)}+1/2*a*(A*b-B*a)/b/(a^2+b^2)/d/(b+a*\cot(d*x+c))^2/\cot(d*x+c)^{(1/2)}+1/4*a*(A*a^2*b+9*A*b^3-5*B*a^3-13*B*a*b^2)/b^2/(a^2+b^2)^2/d/(b+a*\cot(d*x+c))/\cot(d*x+c)^{(1/2)}$

Rubi [A]

time = 1.22, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$ , Rules used = {3662, 3690, 3730, 3734, 3615, 1182, 1176, 631, 210, 1179, 642, 3715, 65, 211}

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Cot}[c + d*x]^{(7/2)}*(a + b*\operatorname{Tan}[c + d*x])^3), x]$

[Out]  $-(((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^3*d)) + ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) - (a^{(3/2)}*(3*a^4*A*b + 6*a^2*A*b^3 + 35*A*b^5 - 15*a^5*B - 46*a^3*b^2*B - 63*a*b^4*B))*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/\operatorname{Sqrt}[b]]/(4*b^{(7/2)}*(a^2 + b^2)^3*d) - (3*a^3*A*b + 11*a*A*b^3 - 15*a^4*B - 31*a^2*b^2*B - 8*b^4*B)/(4*b^3*(a^2 + b^2)^2*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (a*(A*b - a*B))/(2*b*(a^2 + b^2)*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(b + a*\operatorname{Cot}[c + d*x])^2) + (a*(a^2*A*b + 9*A*b^3 - 5*a^3*B - 13*a*b^2*B))/(4*b^2*(a^2 + b^2)^2*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(b + a*\operatorname{Cot}[c + d*x])) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d) + ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*(a^2 + b^2)^3*d)$

$$(A - B) + 3a^2b(A + B) - b^3(A + B)) \cdot \text{Log}[1 + \text{Sqrt}[2] \cdot \text{Sqrt}[\text{Cot}[c + dx]] + \text{Cot}[c + dx]] / (2 \cdot \text{Sqrt}[2] \cdot (a^2 + b^2)^{3d})$$
Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$eQ\{a, c, d, e, x\} \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 1182

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] \rightarrow With\{q = Rt[a*c, 2]\}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] \;/; FreeQ\{a, c, d, e\}, x\} \&\& NeQ[c*d^2 + a*e^2, 0] \&\& NeQ[c*d^2 - a*e^2, 0] \&\& NegQ[(-a)*c]$

### Rule 3615

$Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]]], x\_Symbol] \rightarrow Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] \;/; FreeQ\{b, c, d, e, f\}, x\} \&\& NeQ[c^2 - d^2, 0] \&\& NeQ[c^2 + d^2, 0]$

### Rule 3662

$Int[(cot[(e_) + (f_)*(x_)])*(g_)^p*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^m*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^n], x\_Symbol] \rightarrow Dist[g^{m+n}, Int[(g*Cot[e + f*x])^{p-m-n}*(b + a*Cot[e + f*x])^m*(d + c*Cot[e + f*x])^n, x], x] \;/; FreeQ\{a, b, c, d, e, f, g, p\}, x\} \&\& !IntegerQ[p] \&\& IntegerQ[m] \&\& IntegerQ[n]$

### Rule 3690

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^m*((A_) + (B_)*tan[(e_) + (f_)*(x_)])^n*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^n], x\_Symbol] \rightarrow Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^{m+1}*((c + d*Tan[e + f*x])^{n+1}/(f*(m+1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m+1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^{m+1}*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m+1) + a*d*(n+1)) + A*(a*(b*c - a*d)*(m+1) - b^2*d*(m+n+2)) - (A*b - a*B)*(b*c - a*d)*(m+1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m+n+2)*Tan[e + f*x]^2, x], x], x] \;/; FreeQ\{a, b, c, d, e, f, A, B, n\}, x\} \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 + b^2, 0] \&\& NeQ[c^2 + d^2, 0] \&\& LtQ[m, -1] \&\& (IntegerQ[m] || IntegersQ[2*m, 2*n]) \&\& !(ILtQ[n, -1] \&\& (!IntegerQ[m] || (EqQ[c, 0] \&\& NeQ[a, 0])))$

### Rule 3715

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^m*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^n*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2], x\_Symbol] \rightarrow Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] \;/; FreeQ\{a, b, c, d, e, f, A, C, m, n\}, x\} \&\& EqQ[A, C]$

Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3734

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{7}{2}}(c + dx)(a + b \tan(c + dx))^3} dx &= \int \frac{B + A \cot(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(b + a \cot(c + dx))^3} dx \\
&= \frac{a(Ab - aB)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} - \int \frac{\frac{1}{2}(aAb - 5a^2B - 4b^3)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} dx \\
&= \frac{a(Ab - aB)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)} (b + a \cot(c + dx))^2} + \frac{a(a^2Ab - 5a^2B - 4b^3)}{4b^2(a^2 + b^2)^2 d \sqrt{\cot(c + dx)}} \\
&= -\frac{3a^3Ab + 11aAb^3 - 15a^4B - 31a^2b^2B - 8b^4B}{4b^3(a^2 + b^2)^2 d \sqrt{\cot(c + dx)}} + \frac{a(a^2Ab - 5a^2B - 4b^3)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)}} \\
&= -\frac{3a^3Ab + 11aAb^3 - 15a^4B - 31a^2b^2B - 8b^4B}{4b^3(a^2 + b^2)^2 d \sqrt{\cot(c + dx)}} + \frac{a(a^2Ab - 5a^2B - 4b^3)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)}} \\
&= -\frac{3a^3Ab + 11aAb^3 - 15a^4B - 31a^2b^2B - 8b^4B}{4b^3(a^2 + b^2)^2 d \sqrt{\cot(c + dx)}} + \frac{a(a^2Ab - 5a^2B - 4b^3)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)}} \\
&= -\frac{3a^3Ab + 11aAb^3 - 15a^4B - 31a^2b^2B - 8b^4B}{4b^3(a^2 + b^2)^2 d \sqrt{\cot(c + dx)}} + \frac{a(a^2Ab - 5a^2B - 4b^3)}{2b(a^2 + b^2) d \sqrt{\cot(c + dx)}} \\
&= -\frac{a^{3/2}(3a^4Ab + 6a^2Ab^3 + 35Ab^5 - 15a^5B - 46a^3b^2B - 63ab^4B) \tan^{-1}(\sqrt{\cot(c + dx)})}{4b^{7/2}(a^2 + b^2)^3 d} \\
&= -\frac{a^{3/2}(3a^4Ab + 6a^2Ab^3 + 35Ab^5 - 15a^5B - 46a^3b^2B - 63ab^4B) \tan^{-1}(\sqrt{\cot(c + dx)})}{4b^{7/2}(a^2 + b^2)^3 d} \\
&= -\frac{(3a^2b(A - B) - b^3(A - B) - a^3(A + B) + 3ab^2(A + B)) \tan^{-1}(\sqrt{\cot(c + dx)})}{\sqrt{2}(a^2 + b^2)^3 d}
\end{aligned}$$

**Mathematica [A]**

time = 5.97, size = 599, normalized size = 1.00

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(7/2)\*(a + b\*Tan[c + d\*x])^3), x]

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*Sqrt[2]*(b^3*(A - B) + 3*a^2*b*(-A + B) + a^3*(A + B) - 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]])] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2)^3 + (4*a^(3/2))*(-2*a^2*A*b - 4*A*b^3 + 3*a^3*B + 5*a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]/(b^(7/2)*(a^2 + b^2)^2) + (8*a^(3/2)*(a^4*A*b + 3*a^2*A*b^3 + 6*A*b^5 - 3*a^5*B - 9*a^3*b^2*B - 10*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]/(b^(7/2)*(a^2 + b^2)^3) - (Sqrt[2]*(a^3*(A - B) + 3*a*b^2*(-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2)^3 + (8*B*Sqrt[Tan[c + d*x]])/b^3 + (2*a^3*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(b^3*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (4*a^2*(-2*a^2*A*b - 4*A*b^3 + 3*a^3*B + 5*a*b^2*B)*Sqrt[Tan[c + d*x]])/(b^3*(a^2 + b^2)^2*(a + b*Tan[c + d*x])) + (3*(A*b - a*B)*((a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[b] + (a^2*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x]))/(b^3*(a^2 + b^2)))/(4*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 51.42, size = 104911, normalized size = 174.85

method	result	size
default	Expression too large to display	104911

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVE  
RBOSE)
```

```
[Out] result too large to display
```

**Maxima [A]**

time = 0.55, size = 612, normalized size = 1.02

```
1/4*((15*B*a^7 - 3*A*a^6*b + 46*B*a^5*b^2 - 6*A*a^4*b^3 + 63*B*a^3*b^4 - 35  
*A*a^2*b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^6*b^3 + 3*a^4*b^5  
+ 3*a^2*b^7 + b^9)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b -  
3*(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*  
x + c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A  
- B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*  
((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(2)  
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a^3 + 3*(A + B
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorithm  
="maxima")
```

```
[Out] 1/4*((15*B*a^7 - 3*A*a^6*b + 46*B*a^5*b^2 - 6*A*a^4*b^3 + 63*B*a^3*b^4 - 35  
*A*a^2*b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^6*b^3 + 3*a^4*b^5  
+ 3*a^2*b^7 + b^9)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b -  
3*(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*  
x + c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A  
- B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*  
((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(2)  
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a^3 + 3*(A + B
```

) $a^2b - 3(A - B)ab^2 - (A + B)b^3) \log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1)/(a^6 + 3a^4b^2 + 3a^2b^4 + b^6) + (8Ba^4b^2 + 16B^2a^2b^4 + 8B^2b^6 + (25Ba^5b - 5A^2a^4b^2 + 49B^2a^3b^3 - 13A^2a^2b^4 + 16B^2ab^5)/\tan(dx + c) + (15Ba^6 - 3A^2a^5b + 31B^2a^4b^2 - 11A^2a^3b^3 + 8B^2a^2b^4)/\tan(dx + c)^2)/((a^4b^5 + 2a^2b^7 + b^9)/\sqrt{\tan(dx + c)} + 2(a^5b^4 + 2a^3b^6 + ab^8)/\tan(dx + c)^{3/2} + (a^6b^3 + 2a^4b^5 + a^2b^7)/\tan(dx + c)^{5/2}))/d$

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)^(7/2)/(a+b\*tan(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)\*\*(7/2)/(a+b\*tan(dx+c))\*\*3,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(dx+c))/cot(dx+c)^(7/2)/(a+b\*tan(dx+c))^3,x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{7/2} (a + b \tan(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + dx))/(cot(c + dx)^(7/2)\*(a + b\*tan(c + dx))^3),x)

[Out] int((A + B\*tan(c + dx))/(cot(c + dx)^(7/2)\*(a + b\*tan(c + dx))^3), x)



$$3.606 \quad \int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=156

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{2\sqrt{2} d} - \frac{B \log\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-2/3*B*\cot(d*x+c)^{(3/2)}/d+1/2*B*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*B*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{B \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{B \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(\cot[c + d*x]^{(5/2)}*(a*B + b*B*\tan[c + d*x])\right)/(a + b*\tan[c + d*x]), x\right]$

[Out]  $-((B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\cot[c + d*x]]])/(\operatorname{Sqrt}[2]*d)) + (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\cot[c + d*x]]])/(\operatorname{Sqrt}[2]*d) - (2*B*\cot[c + d*x]^{(3/2)})/(3*d) + (B*\log[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\cot[c + d*x]] + \cot[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (B*\log[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\cot[c + d*x]] + \cot[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

**Rule 21**

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*)^{(m_*)}*((c_*) + (d_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

**Rule 210**

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{(-1)}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

**Rule 303**

$\operatorname{Int}[(x_*)^2/((a_*) + (b_*)*(x_*)^4), x\_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}\{a,$

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &  
& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k =  
Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n  
)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F  
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)  
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := S  
imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e  
(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; Fre  
eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3554

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*((b\*Tan[c + d  
\*x])^(n - 1)/(d\*(n - 1))), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x],  
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[  
x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)} dx &= B \int \cot^{\frac{5}{2}}(c+dx) dx \\
&= -\frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} - B \int \sqrt{\cot(c+dx)} dx \\
&= -\frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{B \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{(2B) \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= -\frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{B \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} + \\
&= -\frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{B \text{Subst}\left(\int \frac{1}{1-\sqrt{2}xx^2} dx, x, \sqrt{\cot(c+dx)}\right)}{2d} \\
&= -\frac{2B \cot^{\frac{3}{2}}(c+dx)}{3d} + \frac{B \log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}d} \\
&= -\frac{B \tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B \tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.05, size = 38, normalized size = 0.24

$$\frac{2B \cot^{\frac{3}{2}}(c+dx) \left(-1 + {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(5/2)\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] (2\*B\*Cot[c + d\*x]^(3/2)\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(3\*d)

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 28.29, size = 1275, normalized size = 8.17

method	result	size
default	Expression too large to display	1275

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(5/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out] 
$$-1/6*B/d*2^{(1/2)}*(-1+\cos(d*x+c))^{2*}(-3*I*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*I*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-6*\sin(d*x+c)*\cos(d*x+c)*\text{EllipticF}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-3*I*\sin(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*I*\sin(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*\sin(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+3*\sin(d*x+c)*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-6*\sin(d*x+c)*\text{EllipticF}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+2*2^{(1/2)}*\cos(d*x+c)^2*(\cos(d*x+c)+1)^2*(\cos(d*x+c)/\sin(d*x+c))^{(5/2)}/\cos(d*x+c)^3/\sin(d*x+c)^3$$

**Maxima [A]**

time = 0.54, size = 127, normalized size = 0.81

$$3 \left( 2\sqrt{2} \arctan \left( \frac{1}{2}\sqrt{2} \left( \sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}} \right) \right) + 2\sqrt{2} \arctan \left( -\frac{1}{2}\sqrt{2} \left( \sqrt{2} - \frac{2}{\sqrt{\tan(dx+c)}} \right) \right) - \sqrt{2} \log \left( \frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1 \right) + \sqrt{2} \log \left( -\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1 \right) \right) B - \frac{8B}{\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{12} * (3 * (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2/\sqrt{\tan(d*x + c)}))) + 2 * \sqrt{2} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2/\sqrt{\tan(d*x + c)}))) - \sqrt{2} * \log(\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) + \sqrt{2} * \log(-\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1)) * B - 8 * B / \tan(d*x + c)^{(3/2)} / d$

**Fricas** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \cot^{\frac{5}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(cot(c + d\*x)\*\*(5/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*b\*tan(d\*x + c) + B\*a)\*cot(d\*x + c)^(5/2)/(b\*tan(d\*x + c) + a), x)

**Mupad** [B]

time = 9.99, size = 64, normalized size = 0.41

$$\frac{(-1)^{1/4} B \operatorname{atan}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c + dx)}}\right)}{d} - \frac{2 B \left(\frac{1}{\tan(c + dx)}\right)^{3/2}}{3 d} - \frac{(-1)^{1/4} B \operatorname{atanh}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c + dx)}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(5/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

```
[Out] ((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c + d*x))^(1/2)))/d - (2*B*(1/tan(c +  
d*x))^(3/2))/(3*d) - ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/2))  
)/d
```

$$3.607 \quad \int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=154

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{2B \sqrt{\cot(c+dx)}}{d} - \frac{B \log\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d}$$

[Out]  $1/2*B*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*B*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-2*B*\cot(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{2B \sqrt{\cot(c+dx)}}{d} - \frac{B \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{B \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(\operatorname{Cot}[c + d*x]^{(3/2)}*(a*B + b*B*\operatorname{Tan}[c + d*x])\right)/(a + b*\operatorname{Tan}[c + d*x]), x\right]$

[Out]  $-((B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*d)) + (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*d) - (2*B*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/d - (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

Rule 21

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_*)^{(m_*)}*((c_*) + (d_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] || \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1}*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] || \operatorname{LtQ}[b, 0])$

Rule 217

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^4)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}\{a, b$

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 335

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

### Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```



IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)} dx &= B \int \cot^{\frac{3}{2}}(c+dx) dx \\
&= -\frac{2B\sqrt{\cot(c+dx)}}{d} - B \int \frac{1}{\sqrt{\cot(c+dx)}} dx \\
&= -\frac{2B\sqrt{\cot(c+dx)}}{d} + \frac{B \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)} dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{2B\sqrt{\cot(c+dx)}}{d} + \frac{(2B) \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= -\frac{2B\sqrt{\cot(c+dx)}}{d} + \frac{B \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= -\frac{2B\sqrt{\cot(c+dx)}}{d} + \frac{B \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(c+dx)}\right)}{2d} \\
&= -\frac{2B\sqrt{\cot(c+dx)}}{d} - \frac{B \log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}d} \\
&= -\frac{B \tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B \tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 138, normalized size = 0.90

$$\frac{B(2\sqrt{2}\operatorname{ArcTan}(1-\sqrt{2}\sqrt{\cot(c+dx)})-2\sqrt{2}\operatorname{ArcTan}(1+\sqrt{2}\sqrt{\cot(c+dx)})+8\sqrt{\cot(c+dx)}+\sqrt{2}\log(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx))-\sqrt{2}\log(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] -1/4\*(B\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 8\*Sqrt[Cot[c + d\*x]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/d

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 26.60, size = 969, normalized size = 6.29

method	result	size
default	Expression too large to display	969

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/2*B/d*2^(1/2)*(-I*cos(d*x+c)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+I*cos(d*x+c)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*((c
os(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-I*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*((cos(d*x+c)-1+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*(-cos(d
*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d
*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-cos(d*x+c)*EllipticPi((-co
s(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+
c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)-cos(d*x+c)*(-cos(d*x+c)-1-sin(d*x+
c))/sin(d*x+c)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)
)^(1/2)*((cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d
*x+c))^(1/2)-(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*((cos(d*x+c)-1+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*2^(1
/2)*cos(d*x+c)*cos(d*x+c)/sin(d*x+c)^(3/2)*sin(d*x+c)/cos(d*x+c)^2
```

**Maxima [A]**

time = 0.52, size = 127, normalized size = 0.82

$$\frac{2\sqrt{2}B \operatorname{arctan}\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}B \operatorname{arctan}\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - \frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + \sqrt{2}B \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) - \sqrt{2}B \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) - \frac{8B}{\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*s
qrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*B*
```

$\log(\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - \sqrt{2}*B*\log(-\sqrt{2}/\sqrt{\tan(dx + c)} + 1/\tan(dx + c) + 1) - 8*B/\sqrt{\tan(dx + c)}/d$

**Fricas** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(a\*B+b\*B\*tan(dx+c))/(a+b\*tan(dx+c)),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \cot^{\frac{3}{2}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)\*\*(3/2)\*(a\*B+b\*B\*tan(dx+c))/(a+b\*tan(dx+c)),x)

[Out] B\*Integral(cot(c + dx)\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(dx+c)^(3/2)\*(a\*B+b\*B\*tan(dx+c))/(a+b\*tan(dx+c)),x, algorithm="giac")

[Out] integrate((B\*b\*tan(dx + c) + B\*a)\*cot(dx + c)^(3/2)/(b\*tan(dx + c) + a), x)

**Mupad** [B]

time = 9.40, size = 65, normalized size = 0.42

$$\frac{2B \sqrt{\frac{1}{\tan(c+dx)}}}{d} - \frac{(-1)^{1/4} B \operatorname{atan}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right) \operatorname{li}}{d} - \frac{(-1)^{1/4} B \operatorname{atanh}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right) \operatorname{li}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + dx)^(3/2)\*(B\*a + B\*b\*tan(c + dx)))/(a + b\*tan(c + dx)),x)

[Out] - (2\*B\*(1/tan(c + dx))^(1/2))/d - ((-1)^(1/4)\*B\*atan((-1)^(1/4)\*(1/tan(c + dx))^(1/2))\*1i)/d - ((-1)^(1/4)\*B\*atanh((-1)^(1/4)\*(1/tan(c + dx))^(1/2))\*1i)/d

$$3.608 \quad \int \frac{\sqrt{\cot(c+dx)} (aB+bB \tan(c+dx))}{a+b \tan(c+dx)} dx$$

**Optimal.** Leaf size=138

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + B \log\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-1/2*B*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*B*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}$

**Rubi [A]**

time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {21, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} - \frac{B \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} + \frac{B \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a*B + b*B*\operatorname{Tan}[c + d*x])\right)/(a + b*\operatorname{Tan}[c + d*x]), x\right]$

[Out]  $(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]])/(\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

Rule 21

$\operatorname{Int}[(a_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2])^{-1})*\operatorname{ArcTan}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 303

$\operatorname{Int}[(x_.)^2/((a_.) + (b_.)*(x_.)^4), x\_Symbol] \rightarrow \operatorname{With}[\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \operatorname{Dist}[1/(2*s), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /; \operatorname{FreeQ}\{a,$

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &  
& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k =  
Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n  
)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F  
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b  
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := S  
imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e  
(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; Fre  
eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[  
x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !  
IntegerQ[n]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}(aB + bB \tan(c+dx))}{a + b \tan(c+dx)} dx &= B \int \sqrt{\cot(c+dx)} dx \\
&= \frac{B \operatorname{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \cot(c+dx)\right)}{d} \\
&= \frac{(2B) \operatorname{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= \frac{B \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} - \frac{B \operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(c+dx)}\right)}{d} \\
&= \frac{B \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(c+dx)}\right)}{2d} - \frac{B \operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\cot(c+dx)}\right)}{2d} \\
&= -\frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c+dx)} + \cot(c+dx)\right)}{2\sqrt{2}d} + \frac{B \log\left(1 + \sqrt{2} \sqrt{\cot(c+dx)} + \cot(c+dx)\right)}{2\sqrt{2}d} \\
&= \frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 36, normalized size = 0.26

$$-\frac{2B \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cot[c + d\*x]]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x]), x]

[Out] (-2\*B\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2])/(3\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 22.89, size = 324, normalized size = 2.35

method	result
default	$-\frac{B\sqrt{2} \sqrt{\frac{\cos(dx+c)}{\sin(dx+c)}} (-1+\cos(dx+c)) \sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}} \sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}} \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}}{3d} \left( i E \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/2*B/d*2^{(1/2)}*(\cos(d*x+c)/\sin(d*x+c))^{(1/2)}*(-1+\cos(d*x+c))*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*(I*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})-I*\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})-2*\text{EllipticF}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2*2^{(1/2)})+\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})+\text{EllipticPi}((-\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})/\sin(d*x+c)^2/\cos(d*x+c)*(\cos(d*x+c)+1)^2$$

**Maxima [A]**

time = 0.54, size = 113, normalized size = 0.82

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)B}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,algorithm="maxima")`

[Out] 
$$-1/4*(2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2)+2/\text{sqrt}(\tan(dx+c))))+2*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2)-2/\text{sqrt}(\tan(dx+c))))-\text{sqrt}(2)*\log(\text{sqrt}(2)/\text{sqrt}(\tan(dx+c))+1/\tan(dx+c)+1)+\text{sqrt}(2)*\log(-\text{sqrt}(2)/\text{sqrt}(\tan(dx+c))+1/\tan(dx+c)+1))*B/d$$

**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \sqrt{\cot(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(sqrt(cot(c + d\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(a\*B+b\*B\*tan(d\*x+c))/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*b\*tan(d\*x + c) + B\*a)\*sqrt(cot(d\*x + c))/(b\*tan(d\*x + c) + a), x)

**Mupad** [B]

time = 8.89, size = 42, normalized size = 0.30

$$\frac{(-1)^{1/4} B \left( \operatorname{atan} \left( (-1)^{1/4} \sqrt{\frac{1}{\tan(c + dx)}} \right) - \operatorname{atanh} \left( (-1)^{1/4} \sqrt{\frac{1}{\tan(c + dx)}} \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(B\*a + B\*b\*tan(c + d\*x)))/(a + b\*tan(c + d\*x)),x)

[Out] -((-1)^(1/4)\*B\*(atan((-1)^(1/4)\*(1/tan(c + d\*x))^(1/2)) - atanh((-1)^(1/4)\*(1/tan(c + d\*x))^(1/2)))/d



$$3.609 \quad \int \frac{aB + bB \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=138

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{2\sqrt{2} d}$$

[Out]  $-1/2*B*\arctan(-1+2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/2*B*\arctan(1+2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1+\cot(d*x+c)-2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1+\cot(d*x+c)+2^{(1/2)*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)})$

**Rubi** [A]

time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ ,

Rules used = {21, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} - \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{B \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{B \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x]) / (\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] * (a + b*\operatorname{Tan}[c + d*x])), x]$

[Out]  $(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / (\operatorname{Sqrt}[2]*d) - (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) / (\operatorname{Sqrt}[2]*d) + (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]]) / (2*\operatorname{Sqrt}[2]*d)$

Rule 21

$\operatorname{Int}[(u_*) * ((a_*) + (b_*) * (v_*)^{(m_*)}) * ((c_*) + (d_*) * (v_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \mid\mid \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 210

$\operatorname{Int}[(a_*) + (b_*) * (x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(-\operatorname{Rt}[-a, 2] * \operatorname{Rt}[-b, 2])^{-1} * \operatorname{ArcTan}[\operatorname{Rt}[-b, 2] * (x/\operatorname{Rt}[-a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \mid\mid \operatorname{LtQ}[b, 0])$

Rule 217

$\operatorname{Int}[(a_*) + (b_*) * (x_*)^4)^{-1}, x\_Symbol] \rightarrow \operatorname{With}\{r = \operatorname{Numerator}[\operatorname{Rt}[a/b, 2]], s = \operatorname{Denominator}[\operatorname{Rt}[a/b, 2]]\}, \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \operatorname{Dist}[1/(2*r), \operatorname{Int}[(r + s*x^2)/(a + b*x^4), x], x] /; \operatorname{FreeQ}\{a, b$

}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n))^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))} dx &= B \int \frac{1}{\sqrt{\cot(c + dx)}} dx \\
&= \frac{B \text{Subst} \left( \int \frac{1}{\sqrt{x} (1+x^2)} dx, x, \cot(c + dx) \right)}{d} \\
&= \frac{(2B) \text{Subst} \left( \int \frac{1}{1+x^4} dx, x, \sqrt{\cot(c + dx)} \right)}{d} \\
&= \frac{B \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(c + dx)} \right)}{d} - \frac{B \text{Subst} \left( \int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(c + dx)} \right)}{d} \\
&= \frac{B \text{Subst} \left( \int \frac{1}{1-\sqrt{2} x+x^2} dx, x, \sqrt{\cot(c + dx)} \right)}{2d} - \frac{B \text{Subst} \left( \int \frac{1}{1+\sqrt{2} x+x^2} dx, x, \sqrt{\cot(c + dx)} \right)}{2d} \\
&= \frac{B \log \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx) \right)}{2\sqrt{2} d} - \frac{B \log \left( 1 + \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx) \right)}{2\sqrt{2} d} \\
&= \frac{B \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right)}{\sqrt{2} d} - \frac{B \tan^{-1} \left( 1 + \sqrt{2} \sqrt{\cot(c + dx)} \right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 110, normalized size = 0.80

$$\frac{B \left( 2 \text{ArcTan} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right) - 2 \text{ArcTan} \left( 1 + \sqrt{2} \sqrt{\cot(c + dx)} \right) + \log \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx) \right) - \log \left( 1 + \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx) \right) \right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])), x]
```

```
[Out] (B*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]) + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(2*Sqrt[2]*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 24.37, size = 284, normalized size = 2.06

method	result
default	$ -\frac{B\sqrt{2} \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}} \sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}} \sqrt{\frac{-\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}} (\cos(dx+c)+1)^2 (-1+\cos(dx+c))}{d} $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/2*B/d*2^(1/2)*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((cos(d*x+c)-1+sin(d*x+
c))/sin(d*x+c))^(1/2)*(-cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c)^(1/2)*(cos(d*
x+c)+1)^2*(-1+cos(d*x+c))*(I*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/sin(d*x
+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-cos(d*x+c)-1-sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((-cos(d*x+c)-1-sin(d*x
+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-cos(d*x+c)-1-si
n(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+c)^3/(cos(d*x+c
)/sin(d*x+c))^(1/2)
```

**Maxima [A]**

time = 0.52, size = 116, normalized size = 0.84

$$\frac{2\sqrt{2}B \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}B \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - \frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + \sqrt{2}B \log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right) - \sqrt{2}B \log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}} + \frac{1}{\tan(dx+c)} + 1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*
sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*B
*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*B*log(-sqrt
(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catde
f: division by zero
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{\cot(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] B*Integral(1/sqrt(cot(c + d*x)), x)
```

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

**Mupad** [B]

time = 8.96, size = 47, normalized size = 0.34

$$\frac{(-1)^{1/4} B \operatorname{atan}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right) \operatorname{li}}{d} + \frac{(-1)^{1/4} B \operatorname{atanh}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right) \operatorname{li}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)
```

```
[Out] ((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c + d*x))^(1/2))*1i)/d + ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/2))*1i)/d
```

$$3.610 \quad \int \frac{aB + bB \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))} dx$$

**Optimal.** Leaf size=154

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{2B}{d \sqrt{\cot(c+dx)}} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{d \sqrt{\cot(c+dx)}} - \frac{B \log\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right)}{d \sqrt{\cot(c+dx)}}$$

[Out]  $1/2*B*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/2*B*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1+\cot(d*x+c)-2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+2*B/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{\sqrt{2} d} + \frac{2B}{d \sqrt{\cot(c+dx)}} + \frac{B \log\left(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d} - \frac{B \log\left(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] `Int[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]`

[Out]  $-(B*\operatorname{ArcTan}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*d) + (B*\operatorname{ArcTan}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]])/( \operatorname{Sqrt}[2]*d) + (2*B)/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) + (B*\operatorname{Log}[1 - \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d) - (B*\operatorname{Log}[1 + \operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]] + \operatorname{Cot}[c + d*x]])/(2*\operatorname{Sqrt}[2]*d)$

**Rule 21**

`Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])`

**Rule 210**

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

**Rule 303**

`Int[(x_)^2/((a_.) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a,`

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &  
& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k =  
Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n  
)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F  
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
implify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b  
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free  
Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := S  
imp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e  
/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &  
& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],  
x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; Fre  
eQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3555

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*Tan[c + d\*x]  
)^(n + 1)/(b\*d\*(n + 1)), x] - Dist[1/b^2, Int[(b\*Tan[c + d\*x])^(n + 2), x],  
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[  
x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx &= B \int \frac{1}{\cot^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2B}{d \sqrt{\cot(c + dx)}} - B \int \sqrt{\cot(c + dx)} dx \\
&= \frac{2B}{d \sqrt{\cot(c + dx)}} + \frac{B \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \cot(c + dx)\right)}{d} \\
&= \frac{2B}{d \sqrt{\cot(c + dx)}} + \frac{(2B) \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{d} \\
&= \frac{2B}{d \sqrt{\cot(c + dx)}} - \frac{B \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{d} + \frac{B \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{d} \\
&= \frac{2B}{d \sqrt{\cot(c + dx)}} + \frac{B \text{Subst}\left(\int \frac{1}{1-\sqrt{2} x+x^2} dx, x, \sqrt{\cot(c + dx)}\right)}{2d} + \frac{B \text{Subst}\left(\int \frac{1}{1+\sqrt{2} x+x^2} dx, x, \sqrt{\cot(c + dx)}\right)}{2d} \\
&= \frac{2B}{d \sqrt{\cot(c + dx)}} + \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx)\right)}{2\sqrt{2} d} - \frac{B \log\left(1 + \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx)\right)}{2\sqrt{2} d} \\
&= -\frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.03, size = 34, normalized size = 0.22

$$\frac{2B {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right)}{d \sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])),x]

[Out] (2\*B\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2])/(d\*Sqrt[Cot[c + d\*x]])



**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 49.61, size = 658, normalized size = 4.27

method	result
default	$B\sqrt{2} \left( i \sin(dx+c) \operatorname{EllipticPi} \left( \sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}, \frac{1}{2}-\frac{i}{2}, \frac{\sqrt{2}}{2} \right) \sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}} \sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out] 
$$\begin{aligned} & 1/2*B/d*2^{(1/2)}*(I*\operatorname{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, \\ & 1/2-1/2*I, 1/2*2^{(1/2)})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos \\ & (d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x \\ & +c))^{(1/2)}-I*\operatorname{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1 \\ & /2*I, 1/2*2^{(1/2)})*\sin(d*x+c)*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c) \\ & )-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{( \\ & 1/2)}+\sin(d*x+c)*\operatorname{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/ \\ & 2-1/2*I, 1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin( \\ & d*x+c))/\sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+\sin \\ & (d*x+c)*\operatorname{EllipticPi}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2+1/2*I, \\ & 1/2*2^{(1/2)})*((-1+\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/ \\ & \sin(d*x+c))^{(1/2)}*(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}-2*\sin(d*x+c \\ & )*\operatorname{EllipticF}((-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}, 1/2*2^{(1/2)})*((-1 \\ & +\cos(d*x+c))/\sin(d*x+c))^{(1/2)}*((\cos(d*x+c)-1+\sin(d*x+c))/\sin(d*x+c))^{(1/2)} \\ & *(-(\cos(d*x+c)-1-\sin(d*x+c))/\sin(d*x+c))^{(1/2)}+2*2^{(1/2)}*\cos(d*x+c)-2*2^{(1/ \\ & 2)}*(-1+\cos(d*x+c))*\cos(d*x+c)*(\cos(d*x+c)+1)^2/(\cos(d*x+c)/\sin(d*x+c))^{(3/ \\ & 2)}/\sin(d*x+c)^5 \end{aligned}$$

**Maxima [A]**

time = 0.55, size = 126, normalized size = 0.82

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+\sqrt{2}\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)\right)B+8B\sqrt{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorit  
hm="maxima")`

[Out] 
$$\begin{aligned} & 1/4*((2*\sqrt{2})*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2/\sqrt{\tan(d*x + c)}))) + 2*\sqrt{2} \\ & *\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2/\sqrt{\tan(d*x + c)})) - \sqrt{2}*\log(\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1) + \sqrt{2}*\log(-\sqrt{2}/\sqrt{\tan(d*x + c)} + 1/\tan(d*x + c) + 1))*B + 8*B*\sqrt{\tan(d*x + c)}/d \end{aligned}$$

**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

```
[Out] B*Integral(cot(c + d*x)**(-3/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)*cot(d*x + c)^(3/2)), x)
```

**Mupad [B]**

time = 8.98, size = 64, normalized size = 0.42

$$\frac{2B}{d\sqrt{\frac{1}{\tan(c+dx)}}} + \frac{(-1)^{1/4} B \operatorname{atan}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right)}{d} - \frac{(-1)^{1/4} B \operatorname{atanh}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*B)/(d*(1/tan(c + d*x))^(1/2)) + ((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c + d*x))^(1/2)))/d - ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/2)))/d
```

$$3.611 \quad \int \frac{aB + bB \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))} dx$$

**Optimal.** Leaf size=156

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} - \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{B \log\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d}$$

[Out]  $2/3*B/d/\cot(d*x+c)^{(3/2)}+1/2*B*\arctan(-1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}$   
 $+1/2*B*\arctan(1+2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}-1/4*B*\ln(1+\cot(d*x+c)-$   
 $2^{(1/2)}*\cot(d*x+c)^{(1/2)})/d*2^{(1/2)}+1/4*B*\ln(1+\cot(d*x+c)+2^{(1/2)}*\cot(d*x+c)$   
 $)^{(1/2)})/d*2^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {21, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{B \operatorname{ArcTan}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{B \operatorname{ArcTan}\left(\sqrt{2} \sqrt{\cot(c + dx)} + 1\right)}{\sqrt{2} d} + \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} - \frac{B \log\left(\cot(c + dx) - \sqrt{2} \sqrt{\cot(c + dx)} + 1\right)}{2\sqrt{2} d} + \frac{B \log\left(\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)} + 1\right)}{2\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] `Int[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]`

[Out]  $-\left(\frac{B \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\cot[c + d*x]}\right]}{\sqrt{2} d}\right) + \left(\frac{B \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\cot[c + d*x]}\right]}{\sqrt{2} d}\right) + \frac{(2*B)}{(3*d*\cot[c + d*x]^{(3/2)})} - \left(\frac{B*\log\left[1 - \sqrt{2} \sqrt{\cot[c + d*x]} + \cot[c + d*x]\right]}{(2*\sqrt{2}*d)}\right) + \left(\frac{B*\log\left[1 + \sqrt{2} \sqrt{\cot[c + d*x]} + \cot[c + d*x]\right]}{(2*\sqrt{2}*d)}\right)$

Rule 21

`Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])`

Rule 210

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(1 - 1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 217

`Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}`

}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 335

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*(x^(k\*n)/c^n))^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 631

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[a\*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2\*c\*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 642

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[d\*(Log[RemoveContent[a + b\*x + c\*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1176

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[2\*(d/e), 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1179

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-2\*(d/e), 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3555

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*Tan[c + d\*x])^(n + 1)/(b\*d\*(n + 1)), x] - Dist[1/b^2, Int[(b\*Tan[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

### Rule 3557

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))} dx &= B \int \frac{1}{\cot^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} - B \int \frac{1}{\sqrt{\cot(c + dx)}} dx \\
&= \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{B \text{Subst}\left(\int \frac{1}{\sqrt{x} (1+x^2)} dx, x, \cot(c + dx)\right)}{d} \\
&= \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{(2B) \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{d} \\
&= \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{B \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{d} + \frac{B \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(c + dx)}\right)}{d} \\
&= \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} + \frac{B \text{Subst}\left(\int \frac{1}{1-\sqrt{2} x+x^2} dx, x, \sqrt{\cot(c + dx)}\right)}{2d} \\
&= \frac{2B}{3d \cot^{\frac{3}{2}}(c + dx)} - \frac{B \log\left(1 - \sqrt{2} \sqrt{\cot(c + dx)} + \cot(c + dx)\right)}{2\sqrt{2} d} \\
&= -\frac{B \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d} + \frac{B \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c + dx)}\right)}{\sqrt{2} d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.03, size = 36, normalized size = 0.23

$$\frac{2B {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx)\right)}{3d \cot^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*B + b\*B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])),x]

[Out] (2\*B\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2])/(3\*d\*Cot[c + d\*x]^(3/2))

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 47.77, size = 546, normalized size = 3.50

method	result
default	$B\sqrt{2}(-1+\cos(dx+c))\left(3i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\right)$ EllipticP

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURN  
VERBOSE)`

[Out]  $\frac{1}{6}B/d^{2^{1/2}}(-1+\cos(dx+c))*(3I\cos(dx+c)*\text{EllipticPi}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})*((-\cos(dx+c))/\sin(dx+c))^{1/2}*((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2}*(-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}-3I\cos(dx+c)*\text{EllipticPi}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2+1/2I,1/2*2^{1/2})*((-\cos(dx+c))/\sin(dx+c))^{1/2}*((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2}*(-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}-3\cos(dx+c)*(-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}*((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2}*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}*\text{EllipticPi}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2-1/2I,1/2*2^{1/2})-3\cos(dx+c)*\text{EllipticPi}((-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2},1/2+1/2I,1/2*2^{1/2})*(-\cos(dx+c)-1-\sin(dx+c))/\sin(dx+c))^{1/2}*((\cos(dx+c)-1+\sin(dx+c))/\sin(dx+c))^{1/2}*((-1+\cos(dx+c))/\sin(dx+c))^{1/2}+2*2^{1/2}*\cos(dx+c)-2*2^{1/2})*\cos(dx+c)*(\cos(dx+c)+1)^2/\sin(dx+c)^5/(\cos(dx+c)/\sin(dx+c))^{5/2}$

**Maxima [A]**

time = 0.54, size = 128, normalized size = 0.82

$$\frac{6\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+3\sqrt{2}B\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)-3\sqrt{2}B\log\left(-\frac{\sqrt{2}}{\sqrt{\tan(dx+c)}}+\frac{1}{\tan(dx+c)}+1\right)+8B\tan(dx+c)^3}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,algorit  
hm="maxima")`

[Out]  $\frac{1}{12}(6*\sqrt{2}*B*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2/\sqrt{\tan(dx+c)}))+6*\sqrt{2}*B*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2/\sqrt{\tan(dx+c)}))+3*\sqrt{2}*B*\log(\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1)-3*\sqrt{2}*B*\log(-\sqrt{2}/\sqrt{\tan(dx+c)}+1/\tan(dx+c)+1)+8*B*\tan(dx+c)^{(3/2)})/d$

**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\cot^{\frac{5}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)\*\*(5/2)/(a+b\*tan(d\*x+c)),x)

[Out] B\*Integral(cot(c + d\*x)\*\*(-5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*b\*tan(d\*x + c) + B\*a)/((b\*tan(d\*x + c) + a)\*cot(d\*x + c)^(5/2)), x)

**Mupad [B]**

time = 9.49, size = 65, normalized size = 0.42

$$\frac{2B}{3d \left(\frac{1}{\tan(c+dx)}\right)^{3/2}} - \frac{(-1)^{1/4} B \operatorname{atan}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right) \operatorname{li}}{d} - \frac{(-1)^{1/4} B \operatorname{atanh}\left((-1)^{1/4} \sqrt{\frac{1}{\tan(c+dx)}}\right) \operatorname{li}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))),x)

[Out] (2\*B)/(3\*d\*(1/tan(c + d\*x))^(3/2)) - ((-1)^(1/4)\*B\*atan((-1)^(1/4)\*(1/tan(c + d\*x))^(1/2))\*1i)/d - ((-1)^(1/4)\*B\*atanh((-1)^(1/4)\*(1/tan(c + d\*x))^(1/2))\*1i)/d

$$3.612 \quad \int \cot^{\frac{9}{2}}(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=354

$$\frac{\sqrt{ia - b} (iA - B) \text{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} - \sqrt{ia + b} (iA + B) \text{ArcTan}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d - (I*A+B)*\text{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d + 2/105*(35*A*a^2+4*A*b^2-7*B*a*b)*\cot(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d - 2/35*(A*b+7*B*a)*\cot(d*x+c)^{(5/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a/d - 2/7*A*\cot(d*x+c)^{(7/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d + 2/105*(35*A*a^2*b-8*A*b^3+105*B*a^3+14*B*a*b^2)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a^3/d$

**Rubi [A]**

time = 0.97, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^4 - 7ab^2 + 4a^2b^2)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{35a^4} - \frac{2(105a^3b + 35a^2b^2 + 14ab^3 - 8a^2b^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{105a^4} - \frac{\sqrt{-b+ia}\sqrt{(a+b)\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(7a^2 + 4b)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{35a^2} - \frac{\sqrt{b+ia}\sqrt{(a+b)\tan(c+dx)}\sqrt{\cot(c+dx)}\text{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{24\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{7a}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(9/2)\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left(\left(\text{Sqrt}[I*a - b]*(I*A - B)*\text{ArcTan}\left[\left(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/d - \left(\text{Sqrt}[I*a + b]*(I*A + B)*\text{ArcTanh}\left[\left(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/d + \left(2*(35*a^2*A*b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right)/(10*5*a^3*d) + \left(2*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*\text{Cot}[c + d*x]^{(3/2)}*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right)/(105*a^2*d) - \left(2*(A*b + 7*a*B)*\text{Cot}[c + d*x]^{(5/2)}*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right)/(35*a*d) - \left(2*A*\text{Cot}[c + d*x]^{(7/2)}*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right)/(7*d)$

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]



Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] :=

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps



$+ d*x] + a*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*\text{Tan}[c + d*x]^2 + (35*a^2*A*b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*\text{Tan}[c + d*x]^3)))/(105*a^3*d)$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 30.51, size = 44507, normalized size = 125.73

method	result	size
default	Expression too large to display	44507

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(9/2),x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2), x)

$$3.613 \quad \int \cot^2(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=290

$$\frac{\sqrt{ia - b} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + \sqrt{ia + b} (A - iB) \tan(c + dx)}{d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a-b)^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a+b)^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-2/15\*(A\*b+5\*B\*a)\*cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d-2/5\*A\*cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(1/2)/d+2/15\*(15\*A\*a^2+2\*A\*b^2-5\*B\*a\*b)\*cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d

**Rubi [A]**

time = 0.79, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2A - 5abB + 2A^2) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^2d} + \frac{\sqrt{-b+ia} (A+iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d} - \frac{2(5aB + Ab) \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{15ad} - \frac{\sqrt{b+ia} (A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d} - \frac{2A \cot^2(c+dx) \sqrt{a+b \tan(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(7/2)\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[I\*a - b]\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d - (Sqrt[I\*a + b]\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d + (2\*(15\*a^2\*A + 2\*A\*b^2 - 5\*a\*b\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a^2\*d) - (2\*(A\*b + 5\*a\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a\*d) - (2\*A\*Cot[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*d)

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(

```
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\int \cot^{\frac{7}{2}}(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx = \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{a + b \tan(c + dx)}}{\cot^{\frac{5}{2}}(c + dx)} dx$$

$$= -\frac{2A \cot^{\frac{5}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}}{5d} - \frac{1}{5} \left( 2(Ab + 5aB) \cot^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)} \right)$$

$$= -\frac{2(Ab + 5aB) \cot^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}}{15ad}$$

$$= \frac{2(15a^2A + 2Ab^2 - 5abB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{15a^2d}$$

$$= \frac{2(15a^2A + 2Ab^2 - 5abB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{15a^2d}$$

$$= \frac{2(15a^2A + 2Ab^2 - 5abB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{15a^2d}$$

$$= \frac{2(15a^2A + 2Ab^2 - 5abB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{15a^2d}$$

$$= \frac{\sqrt{ia - b} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}$$

**Mathematica [A]**

time = 1.47, size = 252, normalized size = 0.87

$$\frac{\cot^{\frac{7}{2}}(c + dx) \left( 15\sqrt{-1} a^2 \sqrt{-a + ib} (A - iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \tan^{\frac{3}{2}}(c + dx) + 15\sqrt{-1} a^2 \sqrt{a + ib} (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \tan^{\frac{3}{2}}(c + dx) + 2\sqrt{a + b \tan(c + dx)} (-3a^2A - a(Ab + 5aB) \tan(c + dx) + (15a^2A + 2Ab^2 - 5abB) \tan^2(c + dx)) \right)}{15a^2d}$$



Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),
x]
```

```
[Out] (Cot[c + d*x]^(5/2)*(15*(-1)^(1/4)*a^2*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(5/2) + 15*(-1)^(1/4)*a^2*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(5/2) + 2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2*A - a*(A*b + 5*a*B)*Tan[c + d*x] + (15*a^2*A + 2*A*b^2 - 5*a*b*B)*Tan[c + d*x]^2))/(15*a^2*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.06, size = 43145, normalized size = 148.78

method	result	size
default	Expression too large to display	43145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(7/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(7/2)\*(a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F(-2)]**  
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**  
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2), x)

$$3.614 \quad \int \cot^2(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=239

$$\frac{\sqrt{ia-b} (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + \sqrt{ia+b} (iA + B) \operatorname{ArcTan}\left(\frac{\sqrt{ib+a} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a-b)^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a+b)^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-2/3\*A\*cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/3\*(A\*b+3\*B\*a)\*cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d

**Rubi** [A]

time = 0.60, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{\sqrt{-b+ia}(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(3aB+Ab)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} + \frac{\sqrt{b+ia}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[I\*a - b]\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d + (Sqrt[I\*a + b]\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d - (2\*(A\*b + 3\*a\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*a\*d) - (2\*A\*Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*d)

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ
[2*m, 2*n])
```

Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
+ (f_)*(x_)^2], x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

$[e + f*x]^2, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{ILtQ}[n, -1] \ \&\& \ ( !\text{IntegerQ}[m] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{NeQ}[a, 0])))$

### Rule 4326

$\text{Int}[(\cot[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_.), x\_Symbol] :> \text{Dist}[(c*\text{Cot}[a + b*x])^m*(c*\text{Tan}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Tan}[a + b*x])^m, x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ \text{KnownTangentIntegrandQ}[u, x]$

### Rubi steps

$$\begin{aligned} \int \cot^{\frac{5}{2}}(c + dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{\cot(c + dx)}} dx \\ &= -\frac{2A \cot^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}}{3d} - \frac{1}{3} \int \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{\cot(c + dx)}} dx \\ &= -\frac{2(Ab + 3aB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{3ad} \\ &= -\frac{2(Ab + 3aB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{3ad} \\ &= -\frac{2(Ab + 3aB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{3ad} \\ &= -\frac{2(Ab + 3aB) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}}{3ad} \\ &= \frac{\sqrt{ia - b} (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d} \end{aligned}$$

### Mathematica [A]

time = 1.25, size = 216, normalized size = 0.90

$$\frac{\cot^{\frac{3}{2}}(c + dx) \left( -3\sqrt{-1} a \sqrt{-a + ib} (iA + B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \tan^{\frac{3}{2}}(c + dx) + 3(-1)^{\frac{3}{4}} a \sqrt{a + ib} (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \tan^{\frac{3}{2}}(c + dx) - 2\sqrt{a + b \tan(c + dx)} (aA + (Ab + 3aB) \tan(c + dx)) \right)}{3ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]), x]

[Out]  $(\cot[c + d*x]^{(3/2)}*(-3*(-1)^{(1/4)}*a*\sqrt{-a + I*b}*(I*A + B)*\text{ArcTan}[((-1)^{(1/4)}*\sqrt{-a + I*b}*\sqrt{\tan[c + d*x]})/\sqrt{a + b*\tan[c + d*x]})]*\tan[c + d*x]^{(3/2)} + 3*(-1)^{(3/4)}*a*\sqrt{a + I*b}*(A + I*B)*\text{ArcTan}[((-1)^{(1/4)}*\sqrt{a + I*b}*\sqrt{\tan[c + d*x]})/\sqrt{a + b*\tan[c + d*x]})]*\tan[c + d*x]^{(3/2)} - 2*\sqrt{a + b*\tan[c + d*x]}*(a*A + (A*b + 3*a*B)*\tan[c + d*x]))/(3*a*d)$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.12, size = 21822, normalized size = 91.31

method	result	size
default	Expression too large to display	21822

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(5/2),x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(5/2)\*(a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2), x)

$$3.615 \quad \int \cot^{\frac{3}{2}}(c+dx) \sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=194

$$\frac{\sqrt{ia - b} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + \sqrt{ia + b} (A - iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-2*A*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 0.42, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4326, 3689, 3697, 3696, 95, 209, 212}

$$\frac{\sqrt{-b+ia} (A+iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + \sqrt{b+ia} (A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) - 2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $-\left(\frac{\sqrt{I*a - b} (A + I*B) \operatorname{ArcTan}[\sqrt{I*a - b} \sqrt{\tan(c + d*x)}]}{\sqrt{a + b \tan(c + d*x)}}\right) \sqrt{\cot(c + d*x)} \sqrt{\tan(c + d*x)} / d + \left(\frac{\sqrt{I*a + b} (A - I*B) \operatorname{ArcTanh}[\sqrt{I*a + b} \sqrt{\tan(c + d*x)}]}{\sqrt{a + b \tan(c + d*x)}}\right) \sqrt{\cot(c + d*x)} \sqrt{\tan(c + d*x)} / d - (2*A*\sqrt{\cot(c + d*x)}*\sqrt{a + b \tan(c + d*x)}) / d$

**Rule 95**

`Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]`

**Rule 209**

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

**Rule 212**



```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 3689

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
1)*(a^2 + b^2))), x] + Dist[1/(b*(m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n) +
A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan[
e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegersQ
[2*m, 2*n])
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{\cot(c+dx)}} dx \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} - \frac{2B \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} - \frac{2B \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} - \frac{2B \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} - \frac{2B \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{\sqrt{ia-b} (A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 0.32, size = 189, normalized size = 0.97

$$\frac{\sqrt{\cot(c+dx)} \left( \sqrt[4]{-1} \sqrt{-a+ib} (A-iB) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\tan(c+dx)} + \sqrt[4]{-1} \sqrt{a+ib} (A+iB) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\tan(c+dx)} + 2A \sqrt{a+b \tan(c+dx)} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]), x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] + (-1)^(1/4)*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] + 2*A*Sqrt[a + b*Tan[c + d*x]]))/d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 27.97, size = 21149, normalized size = 109.02

method	result	size
default	Expression too large to display	21149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a

ssumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2), x)

$$3.616 \quad \int \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=229

$$\frac{\sqrt{ia-b} (iA-B) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + 2\sqrt{b} B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a-b)^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+2*B*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*b^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*(I*a+b)^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d$

**Rubi [A]**

time = 0.48, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4326, 3695, 3697, 3696, 95, 209, 212, 3715, 65, 223}

$$\frac{\sqrt{-b+ia} (-B+iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) - \sqrt{b+ia} (B+iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + 2\sqrt{b} B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]`

[Out]  $-\left(\left(\operatorname{Sqrt}[I*a-b]\right)\left(I*A-B\right)\operatorname{ArcTan}\left[\left(\operatorname{Sqrt}[I*a-b]\right)\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]\right]/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]\right)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]/d + \left(2*\operatorname{Sqrt}[b]*B*\operatorname{ArcTanh}\left[\left(\operatorname{Sqrt}[b]\right)\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]\right]/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]\right)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]/d - \left(\operatorname{Sqrt}[I*a+b]\right)\left(I*A+B\right)\operatorname{ArcTanh}\left[\left(\operatorname{Sqrt}[I*a+b]\right)\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]\right]/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]\right)*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]/d$

**Rule 65**

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b))^n, x], x, (a+b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c-a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

**Rule 95**

`Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m+1)-1)/(b*e-a*f-(d*e-c*f)*x^q], x], x, (a+b*x)^(1/q)/(c+d*x)^(1/q)]`

], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3695

Int[(Sqrt[(a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Int[Simp[a\*A - b\*B + (A\*b + a\*B)\*Tan[e + f\*x], x]/(Sqrt[a + b\*Tan[e + f\*x]]\*Sqrt[c + d\*Tan[e + f\*x]]), x] + Dist[b\*B, Int[(1 + Tan[e + f\*x]^2)/(Sqrt[a + b\*Tan[e + f\*x]]\*Sqrt[c + d\*Tan[e + f\*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

#### Rule 3696

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

#### Rule 3697

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
  Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{\sqrt{a+b \tan(c+dx)}}{\sqrt{\tan(c+dx)}} dx \\
&= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{aA - bB \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx \\
&= \frac{1}{2} \left( (a-ib)(A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{aA - bB \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx \\
&= \frac{\left( (a-ib)(A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{aA - bB \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{2} \\
&= \frac{\left( (a-ib)(A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{aA - bB \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx}{2} \\
&= \frac{\sqrt{ia-b} (iA-B) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{2}
\end{aligned}$$

Mathematica [A]

time = 0.51, size = 238, normalized size = 1.04

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( \sqrt{-1} \left( \sqrt{-a+ib} (iA+B) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) + \sqrt{a+ib} (-iA+B) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \right) \sqrt{a+b \tan(c+dx)} + 2\sqrt{a} \sqrt{b} B \sinh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}} \right) \sqrt{1 + \frac{b \tan(c+dx)}{a}} \right)}{d \sqrt{a+b \tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),
x]
```

[Out]  $(\sqrt{\cot[c + dx]} \sqrt{\tan[c + dx]} * ((-1)^{1/4} * (\sqrt{-a + I*b} * (I*A + B) * \text{ArcTan}[\frac{((-1)^{1/4} * \sqrt{-a + I*b} * \sqrt{\tan[c + dx]})}{\sqrt{a + b*\tan[c + dx]}}] + \sqrt{a + I*b} * ((-I)*A + B) * \text{ArcTan}[\frac{((-1)^{1/4} * \sqrt{a + I*b} * \sqrt{\tan[c + dx]})}{\sqrt{a + b*\tan[c + dx]}}]) * \sqrt{a + b*\tan[c + dx]} + 2*\sqrt{a} * \sqrt{b} * B * \text{ArcSinh}[\frac{\sqrt{b} * \sqrt{\tan[c + dx]})}{\sqrt{a}}] * \sqrt{1 + (b*\tan[c + dx])/a}))/ (d*\sqrt{a + b*\tan[c + dx]})$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.66, size = 8336, normalized size = 36.40

method	result	size
default	Expression too large to display	8336

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(dx+c)^(1/2)*(a+b*tan(dx+c))^(1/2)*(A+B*tan(dx+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^(1/2)*(a+b*tan(dx+c))^(1/2)*(A+B*tan(dx+c)),x,algorithm="maxima")`

[Out] `integrate((B*tan(dx + c) + A)*sqrt(b*tan(dx + c) + a)*sqrt(cot(dx + c)),x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(dx+c)^(1/2)*(a+b*tan(dx+c))^(1/2)*(A+B*tan(dx+c)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} \sqrt{\cot(c + dx)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x)),
x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

$$3.617 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

**Optimal.** Leaf size=261

$$\frac{\sqrt{ia - b} (A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (2Ab + aB) \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a-b)^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+(2\*A\*b+B\*a)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/b^(1/2)-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*(I\*a+b)^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+B\*(a+b\*tan(d\*x+c))^(1/2)/d/cot(d\*x+c)^(1/2)

**Rubi [A]**

time = 1.10, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4326, 3691, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{\sqrt{-b+ia} (A + iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) + (aB + 2Ab) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) - \sqrt{b+ia} (A - iB) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) + \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]],x]

[Out] (Sqrt[I\*a - b]\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + ((2\*A\*b + a\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[b]\*d) - (Sqrt[I\*a + b]\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + (B\*Sqrt[a + b\*Tan[c + d\*x]])/(d\*Sqrt[Cot[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)\*(c + d\*x)^n, x], x, (a + b\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e + f\*x, 0] && IntLinearQ[a, b, c, d, m, n, x]

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3691

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[B\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^n/(f\*(m + n))), x] + Dist[1/(m + n), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*A\*c\*(m + n) - B\*(b\*c\*m + a\*d\*n) + (A\*b\*c + a\*B\*c + a\*A\*d - b\*B\*d)\*(m + n)\*Tan[e + f\*x] + (A\*b\*d\*(m + n) + B\*(a\*d\*m + b\*c\*n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, m, 1] && LtQ[0, n, 1]

#### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} + \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx}{\sqrt{\cot(c + dx)}} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx}{\sqrt{\cot(c + dx)}} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx}{\sqrt{\cot(c + dx)}} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx}{\sqrt{\cot(c + dx)}} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{d \sqrt{\cot(c + dx)}} + \frac{\left( (a - ib)(A - iB) \sqrt{\cot(c + dx)} \int \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} dx \right)}{d \sqrt{\cot(c + dx)}} \\
&= \frac{(2Ab + aB) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{\sqrt{b} d} \\
&= \frac{\sqrt{ia - b} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{d}
\end{aligned}$$

**Mathematica [A]**

time = 2.28, size = 293, normalized size = 1.12

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( \sqrt{-1} \sqrt{-a + ib} (A - iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{a + b \tan(c + dx)} + \sqrt{-1} \sqrt{a + ib} (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{a + b \tan(c + dx)} + B \sqrt{\tan(c + dx)} (a + b \tan(c + dx)) + \frac{(2Ab + aB) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{\sqrt{a} \sqrt{b} \sqrt{1 + \frac{b \tan(c + dx)}{a}}} \right)}{d \sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])
```

```
*x]])*Sqrt[a + b*Tan[c + d*x]] + (-1)^(1/4)*Sqrt[a + I*b]*(A + I*B)*ArcTan[
((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqr
t[a + b*Tan[c + d*x]] + B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]) + ((2*A*b
+ a*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*(a + b*Tan[c + d*x]))
/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*Tan[c + d*x])/a]))/(d*Sqrt[a + b*Tan[c + d*x
]])
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.36, size = 23915, normalized size = 91.63

method	result	size
default	Expression too large to display	23915

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/sqrt(cot(d*x + c)),
x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/sqrt(cot(c + d*x)),
x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/cot(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/cot(c + d*x)^(1/2), x
)
```

$$3.618 \quad \int \frac{\sqrt{a + b \tan(c + dx)} (A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Optimal. Leaf size=324

$$\frac{\sqrt{ia - b} (iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (4aAb - a^2B - 8b^2B)}{d} +$$

[Out]  $\frac{1}{4} * (4 * A * a * b - B * a^2 - 8 * B * b^2) * \operatorname{arctanh}(b^{(1/2)} * \tan(dx + c)^{(1/2)} / (a + b * \tan(dx + c)))^{(1/2)} * \cot(dx + c)^{(1/2)} * \tan(dx + c)^{(1/2)} / b^{(3/2)} / d + (I * A - B) * \operatorname{arctan}((I * a - b)^{(1/2)} * \tan(dx + c)^{(1/2)} / (a + b * \tan(dx + c)))^{(1/2)} * (I * a - b)^{(1/2)} * \cot(dx + c)^{(1/2)} * \tan(dx + c)^{(1/2)} / d + (I * A + B) * \operatorname{arctanh}((I * a + b)^{(1/2)} * \tan(dx + c)^{(1/2)} / (a + b * \tan(dx + c)))^{(1/2)} * (I * a + b)^{(1/2)} * \cot(dx + c)^{(1/2)} * \tan(dx + c)^{(1/2)} / d + \frac{1}{4} * (4 * A * b - B * a) * (a + b * \tan(dx + c))^{(1/2)} / b / d / \cot(dx + c)^{(1/2)} + \frac{1}{2} * B * (a + b * \tan(dx + c))^{(3/2)} / b / d / \cot(dx + c)^{(1/2)}$

Rubi [A]

time = 1.41, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a^2(-B) + 4aAb - 8b^2B) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{4b^{3/2}d} + \frac{\sqrt{-b + ia} (-B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(4Ab - aB) \sqrt{a + b \tan(c + dx)}}{4bd \sqrt{\cot(c + dx)}} + \frac{\sqrt{b + ia} (B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{B(a + b \tan(c + dx))^{3/2}}{2bd \sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[a + b * \operatorname{Tan}[c + d * x]] * (A + B * \operatorname{Tan}[c + d * x])) / \operatorname{Cot}[c + d * x]^{(3/2)}, x]$

[Out]  $(\operatorname{Sqrt}[I * a - b] * (I * A - B) * \operatorname{ArcTan}[(\operatorname{Sqrt}[I * a - b] * \operatorname{Sqrt}[\operatorname{Tan}[c + d * x]]) / \operatorname{Sqrt}[a + b * \operatorname{Tan}[c + d * x]]) * \operatorname{Sqrt}[\operatorname{Cot}[c + d * x]] * \operatorname{Sqrt}[\operatorname{Tan}[c + d * x]] / d + ((4 * a * A * b - a^2 * B - 8 * b^2 * B) * \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[\operatorname{Tan}[c + d * x]]) / \operatorname{Sqrt}[a + b * \operatorname{Tan}[c + d * x]]) * \operatorname{Sqrt}[\operatorname{Cot}[c + d * x]] * \operatorname{Sqrt}[\operatorname{Tan}[c + d * x]] / (4 * b^{(3/2)} * d) + (\operatorname{Sqrt}[I * a + b] * (I * A + B) * \operatorname{ArcTanh}[(\operatorname{Sqrt}[I * a + b] * \operatorname{Sqrt}[\operatorname{Tan}[c + d * x]]) / \operatorname{Sqrt}[a + b * \operatorname{Tan}[c + d * x]]) * \operatorname{Sqrt}[\operatorname{Cot}[c + d * x]] * \operatorname{Sqrt}[\operatorname{Tan}[c + d * x]] / d + ((4 * A * b - a * B) * \operatorname{Sqrt}[a + b * \operatorname{Tan}[c + d * x]]) / (4 * b * d * \operatorname{Sqrt}[\operatorname{Cot}[c + d * x]]) + (B * (a + b * \operatorname{Tan}[c + d * x])^{(3/2)}) / (2 * b * d * \operatorname{Sqrt}[\operatorname{Cot}[c + d * x]])$

Rule 65

$\operatorname{Int}[(a_. + (b_.) * (x_.))^{(m_.)} * ((c_.) + (d_.) * (x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p * (m + 1) - 1)} * (c - a * (d/b) + d * (x^{(p/b)})^n), x], x, (a + b * x)^{(1/p)}, x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b * c - a * d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 95



```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```

b*Tan[e + f*x]]^(m - 1)*(c + d*Tan[e + f*x]]^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps



[In] Integrate[(Sqrt[a + b\*Tan[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Cot[c + d\*x]^(3/2),x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-((-4\*a\*A\*b + a^2\*B + 8\*b^2\*B)\*ArcSinh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]]\*(a + b\*Tan[c + d\*x])) + Sqrt[a]\*Sqrt[b]\*Sqrt[1 + (b\*Tan[c + d\*x])/a]\*(-4\*(-1)^(1/4)\*Sqrt[-a + I\*b]\*b\*(I\*A + B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[a + b\*Tan[c + d\*x]] + 4\*(-1)^(3/4)\*Sqrt[a + I\*b]\*b\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[a + b\*Tan[c + d\*x]] + Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])\*(4\*A\*b + a\*B + 2\*b\*B\*Tan[c + d\*x])))/(4\*Sqrt[a]\*b^(3/2)\*d\*Sqrt[a + b\*Tan[c + d\*x]]\*Sqrt[1 + (b\*Tan[c + d\*x])/a])

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 30.20, size = 28578, normalized size = 88.20

method	result	size
default	Expression too large to display	28578

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(b\*tan(d\*x + c) + a)/cot(d\*x + c)^(3/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(a + b\*tan(c + d\*x))/cot(c + d\*x)\*\*(3/2), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(1/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{a + b \tan(c + dx)}}{\cot(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/cot(c + d\*x)^(3/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(1/2))/cot(c + d\*x)^(3/2), x)

$$3.619 \quad \int \cot^{\frac{11}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}(A + B \tan(c + dx)) dx$$

Optimal. Leaf size=422

$$(ia - b)^{3/2}(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (ia + b)^{3/2}(iA + B)$$


---


$$d$$

[Out] (I\*a-b)^(3/2)\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-(I\*a+b)^(3/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+2/315\*(126\*A\*a^2\*b+4\*A\*b^3+105\*B\*a^3-9\*B\*a\*b^2)\*cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d+2/105\*(21\*A\*a^2-A\*b^2-24\*B\*a\*b)\*cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d-2/63\*(10\*A\*b+9\*B\*a)\*cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/9\*a\*A\*cot(d\*x+c)^(9/2)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/315\*(315\*A\*a^4-63\*A\*a^2\*b^2+8\*A\*b^4-420\*B\*a^3\*b-18\*B\*a\*b^3)\*cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/a^3/d

Rubi [A]

time = 1.32, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3686, 3730, 3697, 3696, 95, 209, 212}

223\*A - 24\*B - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*B + 126\*A^2\*b - 8\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*A^2 - 40\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), (-b+sqrt(a-b)\*sqrt(tan(c+dx)))/sqrt(a+b\*tan(c+dx)), 210\*B + 126\*A^2\*b - 8\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*A^2 - 40\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*B + 126\*A^2\*b - 8\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*A^2 - 40\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*B + 126\*A^2\*b - 8\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx)), 210\*A^2 - 40\*B^2\*a^3 - 40\*sqrt(a-b)\*sqrt(tan(c+dx))

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(11/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] ((I\*a - b)^(3/2)\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d - ((I\*a + b)^(3/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d - (2\*(315\*a^4\*A - 63\*a^2\*A\*b^2 + 8\*A\*b^4 - 420\*a^3\*b\*B - 18\*a\*b^3\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(315\*a^3\*d) + (2\*(126\*a^2\*A\*b + 4\*A\*b^3 + 105\*a^3\*B - 9\*a\*b^2\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(315\*a^2\*d) + (2\*(21\*a^2\*A - A\*b^2 - 24\*a\*b\*B)\*Cot[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(105\*a\*d) - (2\*(10\*A\*b + 9\*a\*B)\*Cot[c + d\*x]^(7/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(63\*d) - (2\*a\*A\*Cot[c + d\*x]^(9/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(9\*d)

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)

], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

Rubi steps



$$\begin{aligned}
\int \cot^{\frac{11}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{9}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{9d} + \frac{2(10Ab+9aB) \cot^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{63d} \\
&= \frac{2(21a^2A-Ab^2-24abB) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{105ad} \\
&= \frac{2(126a^2Ab+4Ab^3+105a^3B-9ab^2B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315a^2d} \\
&= -\frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315a^2d} \\
&= -\frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} \\
&= -\frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} \\
&= -\frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} \\
&= -\frac{(a+ib)^2(iA-B) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b}}
\end{aligned}$$

**Mathematica [A]**

time = 6.00, size = 411, normalized size = 0.97

$$\frac{-(a+ib)^2 \left( \frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} - \frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} + \frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} - \frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB) \sqrt{a+b \tan(c+dx)}}{315a^2d} \right)}{\sqrt{ia-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(11/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] -1/1260\*(Cot[c + d\*x]^(9/2)\*(315\*a^4\*b\*B\*Sqrt[a + b\*Tan[c + d\*x]] + 35\*a^4\*(8\*a\*A - 9\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] + 40\*a^4\*(10\*A\*b + 9\*a\*B)\*Tan[c +

$$d*x]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] - 4*\text{Tan}[c + d*x]^2*(6*a^3*(21*a^2*A - A*b^2 - 24*a*b*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] + a*\text{Tan}[c + d*x]*(315*(-1)^{(1/4)}*a^3*((-a + I*b)^{(3/2)}*(A - I*B)*\text{ArcTan}[((-1)^{(1/4)}*\text{Sqrt}[-a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])]/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]] - (a + I*b)^{(3/2)}*(A + I*B)*\text{ArcTan}[((-1)^{(1/4)}*\text{Sqrt}[a + I*b]*\text{Sqrt}[\text{Tan}[c + d*x]])]/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])]*\text{Tan}[c + d*x]^{(3/2)} + 2*a*(126*a^2*A*b + 4*A*b^3 + 105*a^3*B - 9*a*b^2*B)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]] - 2*(315*a^4*A - 63*a^2*A*b^2 + 8*A*b^4 - 420*a^3*b*B - 18*a*b^3*B)*\text{Tan}[c + d*x]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])))/ (a^4*d)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.51, size = 74462, normalized size = 176.45

method	result	size
default	Expression too large to display	74462

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(11/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(11/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(11/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{11/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)`

[Out] `int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)`

$$3.620 \quad \int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{\frac{3}{2}}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=351

$$\frac{(ia-b)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d} (ia+b)^{3/2}(A-iB)$$

[Out]  $-(I*a-b)^{(3/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(I*a+b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+2/105*(35*A*a^2-3*A*b^2-42*B*a*b)*\cot(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a/d-2/35*(8*A*b+7*B*a)*\cot(d*x+c)^{(5/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d-2/7*a*A*\cot(d*x+c)^{(7/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d+2/105*(140*A*a^2*b+6*A*b^3+105*B*a^3-21*B*a*b^2)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d$

**Rubi [A]**

time = 1.10, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ ,

Rules used = {4326, 3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35a^4 - 4abB - 34B^2)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{105d} - \frac{2(105a^2B + 140a^2Ab - 21a^2B^2 + 64B^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{105d} - \frac{(-b+ia)^{3/2}(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2(7aB + 8Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{35d} - \frac{(b+ia)^{3/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{105d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left(\left(\left(I*a-b\right)^{(3/2)}*(A+I*B)*\operatorname{ArcTan}\left[\left(\operatorname{Sqrt}\left[I*a-b\right]*\operatorname{Sqrt}\left[\tan\left[c+d*x\right]\right]\right)/\operatorname{Sqrt}\left[a+b*\tan\left[c+d*x\right]\right]\right)*\operatorname{Sqrt}\left[\cot\left[c+d*x\right]\right]*\operatorname{Sqrt}\left[\tan\left[c+d*x\right]\right]/d\right)-\left(\left(I*a+b\right)^{(3/2)}*(A-I*B)*\operatorname{ArcTanh}\left[\left(\operatorname{Sqrt}\left[I*a+b\right]*\operatorname{Sqrt}\left[\tan\left[c+d*x\right]\right]\right)/\operatorname{Sqrt}\left[a+b*\tan\left[c+d*x\right]\right]\right)*\operatorname{Sqrt}\left[\cot\left[c+d*x\right]\right]*\operatorname{Sqrt}\left[\tan\left[c+d*x\right]\right]/d+\left(2*(140*a^2*A*b+6*A*b^3+105*a^3*B-21*a*b^2*B)*\operatorname{Sqrt}\left[\cot\left[c+d*x\right]\right]*\operatorname{Sqrt}\left[a+b*\tan\left[c+d*x\right]\right]\right)/(105*a^2*d)+\left(2*(35*a^2*A-3*A*b^2-42*a*b*B)*\cot\left[c+d*x\right]^{(3/2)}*\operatorname{Sqrt}\left[a+b*\tan\left[c+d*x\right]\right]\right)/(105*a*d)-\left(2*(8*A*b+7*a*B)*\cot\left[c+d*x\right]^{(5/2)}*\operatorname{Sqrt}\left[a+b*\tan\left[c+d*x\right]\right]\right)/(35*d)-\left(2*a*A*\cot\left[c+d*x\right]^{(7/2)}*\operatorname{Sqrt}\left[a+b*\tan\left[c+d*x\right]\right]\right)/(7*d)$

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{7d} + \frac{2(8Ab+7aB) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{35d} \\
&= \frac{2(35a^2A-3Ab^2-42abB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{105ad} \\
&= \frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B) \sqrt{a+b \tan(c+dx)}}{105a^2d} \\
&= \frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B) \sqrt{a+b \tan(c+dx)}}{105a^2d} \\
&= \frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B) \sqrt{a+b \tan(c+dx)}}{105a^2d} \\
&= \frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B) \sqrt{a+b \tan(c+dx)}}{105a^2d} \\
&= \frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B) \sqrt{a+b \tan(c+dx)}}{105a^2d} \\
&= \frac{(ia-b)^{3/2}(A+iB) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{105a^2d}
\end{aligned}$$

### Mathematica [A]

time = 3.21, size = 346, normalized size = 0.99

$$\frac{\cot(c+dx) \left( 35a^2A \sqrt{a+b \tan(c+dx)} + 5a^2(8Ab-7aB) \sqrt{a+b \tan(c+dx)} + a \tan(c+dx) \left( 105(-1)^{3/4} \left( (-a+ib)^{3/2}(A-iB) \operatorname{ArcTan}\left(\frac{\sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + (a+ib)^{3/2}(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \right) \tan(c+dx) + 6a^2(8Ab+7aB) \sqrt{a+b \tan(c+dx)} - 2a(35a^2A-3Ab^2-42abB) \tan(c+dx) \sqrt{a+b \tan(c+dx)} - 2(140a^2Ab+6Ab^3+105a^3B-21ab^2B) \tan(c+dx) \sqrt{a+b \tan(c+dx)} \right)}{105a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] -1/105\*(Cot[c + d\*x]^(7/2)\*(35\*a^3\*b\*B\*Sqrt[a + b\*Tan[c + d\*x]] + 5\*a^3\*(6\*a\*A - 7\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] + a\*Tan[c + d\*x]\*(105\*(-1)^(3/4)\*a^2\*((-a + I\*b)^(3/2)\*(A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]]] + (a + I\*b)^(3/2)\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]]])\*Tan[c + d\*x]^(5/2) + 6\*a^2\*(8\*A\*b + 7\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 2\*a\*(35\*a^2\*A

$$\frac{-3A^2b^2 - 42Ab^2B) \tan[c + dx] \sqrt{a + b \tan[c + dx]} - 2(140a^2A^2b + 6A^2b^3 + 105a^3B - 21a^2b^2B) \tan[c + dx]^2 \sqrt{a + b \tan[c + dx]}}{a^3d}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.90, size = 49793, normalized size = 141.86

method	result	size
default	Expression too large to display	49793

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(9/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`



[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorith="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(9/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2), x)

$$3.621 \quad \int \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^{\frac{3}{2}}(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=299

$$\frac{(a+ib)^2(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}(ia+b)^{3/2}(iA+B)\tan(c+dx)}{\sqrt{ia-b}d} + \dots$$

[Out] (I\*a+b)^(3/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+(a+I\*b)^2\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)-2/15\*(6\*A\*b+5\*B\*a)\*cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/5\*a\*A\*cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(1/2)/d+2/15\*(15\*A\*a^2-3\*A\*b^2-20\*B\*a\*b)\*cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d

Rubi [A]

time = 0.86, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2A - 20bbB - 3AP)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15ad} + \frac{(a+ib)^2(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{2(5aB+6Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{15d} + \frac{(b+ia)^{3/2}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(7/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((a + I\*b)^2\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(Sqrt[I\*a - b]\*d) + ((I\*a + b)^(3/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d + (2\*(15\*a^2\*A - 3\*A\*b^2 - 20\*a\*b\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a\*d) - (2\*(6\*A\*b + 5\*a\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*d) - (2\*a\*A\*Cot[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*d)

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f



time = 1.92, size = 286, normalized size = 0.96

$$\frac{\cot^2(c+dx) \left( 30\sqrt{-1} \operatorname{Arctan}\left(\frac{(-a+db)^{3/2}(A-1B)\operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+1B}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - (a+db)^{3/2}(A+1B)\operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+1B}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+b\tan(c+dx)}}\right) \tan^3(c+dx) + 15ab\sqrt{a+b\tan(c+dx)} + 3a(4a-5b)\sqrt{a+b\tan(c+dx)} + 4a(6a+5b)\tan(c+dx)\sqrt{a+b\tan(c+dx)} - 4(15a^2A-3A^2-20abB)\tan^2(c+dx)\sqrt{a+b\tan(c+dx)} \right)}{30ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(7/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] 
$$\begin{aligned} & -1/30*(\operatorname{Cot}[c + d*x]^{(5/2)}*(30*(-1)^{(1/4)}*a*((-a + I*b)^{(3/2)}*(A - I*B)*\operatorname{ArcTan} \\ & [((-1)^{(1/4)}*\operatorname{Sqrt}[-a + I*b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]) \\ & - (a + I*b)^{(3/2)}*(A + I*B)*\operatorname{ArcTan}[((-1)^{(1/4)}*\operatorname{Sqrt}[a + I*b]*\operatorname{Sqrt}[\operatorname{Tan}[c + \\ & d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])]*\operatorname{Tan}[c + d*x]^{(5/2)} + 15*a*b*B*\operatorname{Sqrt}[a + b* \\ & \operatorname{Tan}[c + d*x]] + 3*a*(4*a*A - 5*b*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]] + 4*a*(6*A*b + \\ & 5*a*B)*\operatorname{Tan}[c + d*x]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]] - 4*(15*a^2*A - 3*A*b^2 - 20* \\ & a*b*B)*\operatorname{Tan}[c + d*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])))/(a*d) \end{aligned}$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 29.46, size = 48985, normalized size = 163.83

method	result	size
default	Expression too large to display	48985

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)), x, method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(7/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(7/2)\*(a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(7/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2), x)

$$3.622 \quad \int \cot^2(c+dx)(a+b \tan(c+dx))^3(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=236

$$\frac{(ia-b)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + (ia+b)^{3/2}(A-iB)}{d}$$

[Out] (I\*a-b)^(3/2)\*(A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+(I\*a+b)^(3/2)\*(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d-2/3\*a\*A\*cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(1/2)/d-2/3\*(4\*A\*b+3\*B\*a)\*cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/d

**Rubi [A]**

time = 0.65, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{(-b+ia)^{3/2}(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - \frac{2(3aB+4Ab)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3d} + \frac{(b+ia)^{3/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - \frac{2aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((I\*a - b)^(3/2)\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d + ((I\*a + b)^(3/2)\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/d - (2\*(4\*A\*b + 3\*a\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*d) - (2\*a\*A\*Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*d)

**Rule 95**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)



```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3d} + \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2(4Ab+3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3d} + \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2(4Ab+3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3d} + \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2(4Ab+3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3d} + \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2(4Ab+3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3d} + \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= \frac{(ia-b)^{3/2}(A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d}
\end{aligned}$$

### Mathematica [A]

time = 0.65, size = 244, normalized size = 1.03

$$\frac{\sqrt{\cot(c+dx)} \left( 3\sqrt{-1} \left( (-a+ib)^{3/2}(A+B) \operatorname{ArcTan} \left( \frac{\sqrt{-1}\sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) + i(a+ib)^{3/2}(A+iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1}\sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \right) \sqrt{\tan(c+dx)} - 2(4Ab+3aB) \sqrt{a+b \tan(c+dx)} - 3aB \cot(c+dx) \sqrt{a+b \tan(c+dx)} + (-2aA+3dB) \cot(c+dx) \sqrt{a+b \tan(c+dx)} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] (Sqrt[Cot[c + d\*x]]\*(3\*(-1)^(1/4)\*((-a + I\*b)^(3/2)\*(I\*A + B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]) + I\*(a + I\*b)^(3/2)\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]] - 2\*(4\*A\*b + 3\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]] - 3\*b\*B\*Cot[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]] + (-2\*a\*A + 3\*b\*B)\*Cot[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]])/(3\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.74, size = 24872, normalized size = 105.39

method	result	size
default	Expression too large to display	24872

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(3/2)\*cot(d\*x + c)^(5/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)`

[Out] `int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)`

$$3.623 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{\frac{3}{2}}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=269

$$\frac{(a+ib)^2(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}d} + 2b^{3/2}B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)$$

[Out]  $2*b^{(3/2)}*B*\text{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(I*a+b)^{(3/2)}*(I*A+B)*\text{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(a+I*b)^2*(I*A-B)*\text{arctan}((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/(I*a-b)^{(1/2)}-2*a*A*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d$

**Rubi [A]**

time = 1.32, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4326, 3686, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a+ib)^2(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{(b+ia)^{3/2}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} + \frac{2b^{3/2}B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^{(3/2)}*(a + b*\text{Tan}[c + d*x])^{(3/2)}*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-(((a + I*b)^2*(I*A - B)*\text{ArcTan}[(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]])]/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]/(\text{Sqrt}[I*a - b]*d) + (2*b^{(3/2)}*B*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]])]/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]/d - ((I*a + b)^{(3/2)}*(I*A + B)*\text{ArcTanh}[(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]])]/\text{Sqrt}[a + b*\text{Tan}[c + d*x]])*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]/d - (2*a*A*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[a + b*\text{Tan}[c + d*x]])/d$

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b}))^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] :> \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}*(c + d*x)^n, x], x, (e + f*x)^{(1/q)}, x]]$

$-1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,

A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{3/2}}{\sqrt{\cot(c+dx)}} dx \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2aA \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{d} \\
&= -\frac{2b^{3/2} B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)}}{d} \\
&= -\frac{(a+ib)^2 (iA-B) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)}}{\sqrt{ia}}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 31.30, size = 114092, normalized size = 424.13

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] Result too large to show

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 27.78, size = 42482, normalized size = 157.93

method	result	size
default	Expression too large to display	42482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] result too large to display

**Maxima [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

[Out] Timed out

**Fricas [F(-1)]** Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F(-2)]**  
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

**Giac [F]**  
time = 0.00, size = 0, normalized size = 0.00

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

$$3.624 \quad \int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=264

$$\frac{(ia-b)^{3/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \sqrt{b}(2Ab+3aB) \tan(c+dx)}{d} +$$

[Out]  $-(I*a-b)^{(3/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d}-(I*a+b)^{(3/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d}+(2*A*b+3*B*a)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})*b^{(1/2)}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d}+b*B*(a+b*\tan(d*x+c))^{(1/2)/d}/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 1.31, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4326, 3688, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(-b+ia)^{3/2}(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + \sqrt{b}(3aB+2Ab)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) - (b+ia)^{3/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + \frac{bB\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[\text{Cot}[c+d*x]]*(a+b*\text{Tan}[c+d*x])^{(3/2)}*(A+B*\text{Tan}[c+d*x]),x]$

[Out]  $-\left(\left(\left(I*a-b\right)^{(3/2)}*(A+I*B)*\text{ArcTan}\left[\left(\text{Sqrt}\left[I*a-b\right]*\text{Sqrt}\left[\text{Tan}\left[c+d*x\right]\right]\right)/\text{Sqrt}\left[a+b*\text{Tan}\left[c+d*x\right]\right]\right)*\text{Sqrt}\left[\text{Cot}\left[c+d*x\right]\right]*\text{Sqrt}\left[\text{Tan}\left[c+d*x\right]\right]/d\right) + \left(\text{Sqrt}\left[b\right]*(2*A*b+3*a*B)*\text{ArcTanh}\left[\left(\text{Sqrt}\left[b\right]*\text{Sqrt}\left[\text{Tan}\left[c+d*x\right]\right]\right)/\text{Sqrt}\left[a+b*\text{Tan}\left[c+d*x\right]\right]\right)*\text{Sqrt}\left[\text{Cot}\left[c+d*x\right]\right]*\text{Sqrt}\left[\text{Tan}\left[c+d*x\right]\right]/d - \left(\left(I*a+b\right)^{(3/2)}*(A-I*B)*\text{ArcTanh}\left[\left(\text{Sqrt}\left[I*a+b\right]*\text{Sqrt}\left[\text{Tan}\left[c+d*x\right]\right]\right)/\text{Sqrt}\left[a+b*\text{Tan}\left[c+d*x\right]\right]\right)*\text{Sqrt}\left[\text{Cot}\left[c+d*x\right]\right]*\text{Sqrt}\left[\text{Tan}\left[c+d*x\right]\right]/d + \left(b*B*\text{Sqrt}\left[a+b*\text{Tan}\left[c+d*x\right]\right)/(d*\text{Sqrt}\left[\text{Cot}\left[c+d*x\right]\right])\right)$

**Rule 65**

$\text{Int}[\left((a_.) + (b_.)*(x_)^m\right)*\left((c_.) + (d_.)*(x_)^n\right), x\_Symbol] \rightarrow \text{With}\left[\{p = \text{Denominator}[m]\}, \text{Dist}\left[p/b, \text{Subst}\left[\text{Int}\left[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b})^n), x\right], x, (a + b*x)^{(1/p)}\right], x\right] /; \text{FreeQ}\left[\{a, b, c, d\}, x\right] \&\& \text{NeQ}\left[b*c - a*d, 0\right] \&\& \text{LtQ}\left[-1, m, 0\right] \&\& \text{LeQ}\left[-1, n, 0\right] \&\& \text{LeQ}\left[\text{Denominator}[n], \text{Denominator}[m]\right] \&\& \text{IntLinearQ}\left[a, b, c, d, m, n, x\right]\right]$

**Rule 95**

$\text{Int}[\left(\left((a_.) + (b_.)*(x_)^m\right)*\left((c_.) + (d_.)*(x_)^n\right)\right)/\left((e_.) + (f_.)*(x_)^q\right), x\_Symbol] \rightarrow \text{With}\left[\{q = \text{Denominator}[m]\}, \text{Dist}\left[q, \text{Subst}\left[\text{Int}\left[x^{(q*(m+1)-1)}*(c + d*x^n)/(e + f*x^q), x\right], x, (a + b*x)^{(1/q)}\right], x\right] /; \text{FreeQ}\left[\{a, b, c, d, e, f\}, x\right] \&\& \text{NeQ}\left[e, 0\right] \&\& \text{LeQ}\left[-1, m, 0\right] \&\& \text{LeQ}\left[-1, n, 0\right] \&\& \text{LeQ}\left[\text{Denominator}[n], \text{Denominator}[m]\right] \&\& \text{IntLinearQ}\left[a, b, c, d, m, n, x\right]\right]$

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)  
 ], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]  
 && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3688

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

#### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +

$d^2, 0]$

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{3/2} (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+)}{dx} \\
&= \frac{bB \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \left( \sqrt{\cot(c+dx)} \right) \int \frac{(a+)}{dx} \\
&= \frac{bB \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \right)}{dx} \\
&= \frac{bB \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \right)}{dx} \\
&= \frac{bB \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \right)}{dx} \\
&= \frac{bB \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \frac{\left( \sqrt{\cot(c+dx)} \right)}{dx} \\
&= \frac{bB \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} + \frac{\left( (a+ib)^2 (iA) \right)}{d \sqrt{\cot(c+dx)}} \\
&= \frac{\sqrt{b} (2Ab+3aB) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d} \\
&= \frac{(ia-b)^{3/2} (A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 0.90, size = 263, normalized size = 1.00

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( -\sqrt{-1} (-a+ib)^{3/2} (iA+B) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) - (-1)^{3/4} (a+ib)^{3/2} (A+iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) + bB \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)} + \frac{\sqrt{a} \sqrt{b} (2Ab+3aB) \operatorname{tanh}^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{1+\frac{b \tan(c+dx)}{a}}}{d} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-((-1)^(1/4)*(-a + I*b)^(3/2)*(I*A + B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
```

$$+ d*x]]]) - (-1)^{(3/4)}*(a + I*b)^{(3/2)}*(A + I*B)*ArcTan[((-1)^{(1/4)}*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + b*B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*Sqrt[b]*(2*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]])/d$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 32.96, size = 27748, normalized size = 105.11

method	result	size
default	Expression too large to display	27748

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c)), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}} \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)`

[Out] `Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*sqrt(cot(c + d*x)), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c)), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)`

[Out] `int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)`

$$3.625 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=328

$$\frac{(a+ib)^2(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}d} + \dots$$

[Out] (I\*a+b)^(3/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+(a+I\*b)^2\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)+1/4\*(12\*A\*a\*b+3\*B\*a^2-8\*B\*b^2)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/b^(1/2)+1/2\*b\*B\*(a+b\*tan(d\*x+c))^(1/2)/d/cot(d\*x+c)^(3/2)+1/4\*(4\*A\*b+5\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d/cot(d\*x+c)^(1/2)

**Rubi [A]**

time = 1.72, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(3a^2B+12aAb-8B^2B)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4\sqrt{b}d} + \frac{(a+ib)^2(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{(5aB+4Ab)\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(b+ia)^{3/2}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{bB\sqrt{a+b\tan(c+dx)}}{2d\cot(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]], x]

[Out] ((a + I\*b)^2\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) + ((12\*a\*A\*b + 3\*a^2\*B - 8\*b^2\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(4\*Sqrt[b]\*d) + ((I\*a + b)^(3/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + (b\*B\*Sqrt[a + b\*Tan[c + d\*x]])/(2\*d\*Cot[c + d\*x]^(3/2)) + ((4\*A\*b + 5\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(4\*d\*Sqrt[Cot[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**



```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

#### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

#### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```

b*Tan[e + f*x]]^(m - 1)*(c + d*Tan[e + f*x]]^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{1}{2} \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{bB \sqrt{a + b \tan(c + dx)}}{2d \cot^{3/2}(c + dx)} + \frac{(4Ab + 5aB) \sqrt{a + b \tan(c + dx)}}{4d \sqrt{\cot(c + dx)}} \\
&= \frac{(12aAb + 3a^2B - 8b^2B) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{4\sqrt{b} d} \\
&= \frac{(a + ib)^2 (iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{ia - b} d}
\end{aligned}$$

**Mathematica [A]**

time = 2.12, size = 310, normalized size = 0.95

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( -4\sqrt{-1}(-a + ib)^{3/2}(A - iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1}\sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + 4\sqrt{-1}(a + ib)^{3/2}(A + iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1}\sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + (4Ab + 5aB) \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} + 2bB \tan^3(c + dx) \sqrt{a + b \tan(c + dx)} + \frac{\sqrt{a} (12aAb + 3a^2B - 8b^2B) \operatorname{atanh} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{b} \sqrt{a + b \tan(c + dx)}} \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]], x]

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-4*(-1)^(1/4)*(-a + I*b)^(3/2)*(A -
I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[
c + d*x]]] + 4*(-1)^(1/4)*(a + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt
[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]] + (4*A*b + 5*a*B)*S
qrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*b*B*Tan[c + d*x]^(3/2)*Sqrt[
a + b*Tan[c + d*x]] + (Sqrt[a]*(12*a*A*b + 3*a^2*B - 8*b^2*B)*ArcSinh[(Sqrt
[b]*Sqrt[Tan[c + d*x]]]/Sqrt[a])*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqr
t[a + b*Tan[c + d*x]])))/(4*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.26, size = 31112, normalized size = 94.85

method	result	size
default	Expression too large to display	31112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/sqrt(cot(d*x + c)
), x)
```

**Fricas [F(-1)]** Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^{\frac{3}{2}}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/sqrt(cot(c + d*x)
), x)
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/cot(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/cot(c + d*x)^(1/2), x
)
```

$$3.626 \quad \int \frac{(a+b \tan(c+dx))^{3/2}(A+B \tan(c+dx))}{\cot^2(c+dx)} dx$$

Optimal. Leaf size=383

$$\frac{(ia - b)^{3/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d} (6a^2Ab - 16Ab^3 - a^3B) + \dots$$

[Out] (I\*a-b)^(3/2)\*(A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+1/8\*(6\*A\*a^2\*b-16\*A\*b^3-B\*a^3-24\*B\*a\*b^2)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/b^(3/2)/d+(I\*a+b)^(3/2)\*(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+1/8\*(6\*A\*a\*b-B\*a^2-8\*B\*b^2)\*(a+b\*tan(d\*x+c))^(1/2)/b/d/cot(d\*x+c)^(1/2)+1/12\*(6\*A\*b-B\*a)\*(a+b\*tan(d\*x+c))^(3/2)/b/d/cot(d\*x+c)^(1/2)+1/3\*B\*(a+b\*tan(d\*x+c))^(5/2)/b/d/cot(d\*x+c)^(1/2)

Rubi [A]

time = 1.85, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a^2(-b) + 6a^2b - 6b^2)\sqrt{a + b \tan(c + dx)}}{8d\sqrt{\cot(c + dx)}} + \frac{(a^2(-b) + 6a^2b - 24a^2b^2 - 16a^2b^3)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\text{tanh}^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{8b^2d} + \frac{(-b + ia)^{3/2}(A + iB)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\text{Arctan}\left(\frac{\sqrt{ia - b}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(b + ia)^{3/2}(A - iB)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\text{tanh}^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{8b^2d\sqrt{\cot(c + dx)}} + \frac{B(a + b \tan(c + dx))^{5/2}}{3bd\sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^(3/2)\*(A + B\*Tan[c + d\*x]))/Cot[c + d\*x]^(3/2),x]

[Out] ((I\*a - b)^(3/2)\*(A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + ((6\*a^2\*A\*b - 16\*A\*b^3 - a^3\*B - 24\*a\*b^2\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(8\*b^(3/2)\*d) + ((I\*a + b)^(3/2)\*(A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + ((6\*a\*A\*b - a^2\*B - 8\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(8\*b\*d\*Sqrt[Cot[c + d\*x]]) + ((6\*A\*b - a\*B)\*(a + b\*Tan[c + d\*x])^(3/2))/(12\*b\*d\*Sqrt[Cot[c + d\*x]]) + (B\*(a + b\*Tan[c + d\*x])^(5/2))/(3\*b\*d\*Sqrt[Cot[c + d\*x]])

Rule 65

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3728

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[

```
e + f*x])^(n + 1)/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^{3/2} (A + B \tan(c + dx))}{\cot^{3/2}(c + dx)} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \tan^{3/2}(c + dx) (a + b \tan(c + dx)) dx \\
&= \frac{B(a + b \tan(c + dx))^{5/2}}{3bd \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \tan^{3/2}(c + dx) (a + b \tan(c + dx)) dx}{3bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6Ab - aB)(a + b \tan(c + dx))^{3/2}}{12bd \sqrt{\cot(c + dx)}} + \frac{B(a + b \tan(c + dx))^{5/2}}{3bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6aAb - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{8bd \sqrt{\cot(c + dx)}} + \frac{(6Ab - a^2B - 8b^2B) \sqrt{a + b \tan(c + dx)}}{12bd \sqrt{\cot(c + dx)}} \\
&= \frac{(6a^2Ab - 16Ab^3 - a^3B - 24ab^2B) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{8b^{3/2}d} \\
&= \frac{(ia - b)^{3/2} (A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 3.71, size = 367, normalized size = 0.96

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{240^{1/4} (-a + b)^{3/4} (A + B) \operatorname{ArcTan} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + 24(-1)^{3/4} (a + b)^{3/4} (A + iB) \operatorname{ArcTan} \left( \frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) - 3(-6aAb + a^2B + 8b^2B) \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} + 2(6Ab - a^2B) \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{3/2} + 8B \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^{5/2} - \frac{\sqrt{a} \sqrt{a + b \tan(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{b} \sqrt{a + b \tan(c + dx)}} \left( 1 + \frac{3 \tan(c + dx)}{a} \right)}{8b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(24*(-1)^(1/4)*(-a + I*b)^(3/2)*b*(I*A + B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]] + 24*(-1)^(3/4)*(a + I*b)^(3/2)*b*(A + I*B)*ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]] - 3*(-6*a*A*b + a^2*B + 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*(6*A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2) - (3*Sqrt[a]*(-6*a^2*A*b + 16*A*b^3 + a^3*B + 24*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(24*b*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.39, size = 34370, normalized size = 89.74

method	result	size
default	Expression too large to display	34370

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/cot(d*x + c)^(3/2), x)
```

**Fricas [F(-1)]** Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{\frac{3}{2}}}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(3/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*(3/2)/cot(c + d\*x)\*\*(3/2), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(3/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{3/2}}{\cot(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/cot(c + d\*x)^(3/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(3/2))/cot(c + d\*x)^(3/2), x)

$$3.627 \quad \int \cot^{\frac{13}{2}}(c + dx)(a + b \tan(c + dx))^{5/2}(A + B \tan(c + dx)) dx$$

Optimal. Leaf size=500

$$\frac{(ia - b)^{5/2}(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d} (ia + b)^{5/2}(iA + B)$$

[Out]  $-(I*a-b)^{(5/2)}*(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d}-(I*a+b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d}-2/3465*(1155*A*a^4-1485*A*a^2*b^2-20*A*b^4-2541*B*a^3*b+55*B*a*b^3)*\cot(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))^{(1/2)/a^2/d}+2/1155*(495*A*a^2*b-5*A*b^3+231*B*a^3-275*B*a*b^2)*\cot(d*x+c)^{(5/2)}*(a+b*\tan(d*x+c))^{(1/2)/a/d}+2/693*(99*A*a^2-113*A*b^2-209*B*a*b)*\cot(d*x+c)^{(7/2)}*(a+b*\tan(d*x+c))^{(1/2)/d}-2/99*a*(14*A*b+11*B*a)*\cot(d*x+c)^{(9/2)}*(a+b*\tan(d*x+c))^{(1/2)/d}-2/3465*(8085*A*a^4*b-495*A*a^2*b^3+40*A*b^5+3465*B*a^5-5313*B*a^3*b^2-110*B*a*b^4)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)/a^3/d}-2/11*a*A*\cot(d*x+c)^{(11/2)}*(a+b*\tan(d*x+c))^{(3/2)/d}$

Rubi [A]

time = 1.66, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {4326, 3686, 3726, 3730, 3697, 3696, 95, 209, 212}

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(13/2)}*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $-\left(\frac{(I*a - b)^{(5/2)}*(I*A - B)*\operatorname{ArcTan}\left[\frac{\sqrt{I*a - b}*\sqrt{\operatorname{Tan}[c + d*x]}}{\sqrt{a + b*\operatorname{Tan}[c + d*x]}}\right]}{\sqrt{a + b*\operatorname{Tan}[c + d*x]}}*\sqrt{\operatorname{Cot}[c + d*x]}*\sqrt{\operatorname{Tan}[c + d*x]}\right)/d - \left(\frac{(I*a + b)^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}\left[\frac{\sqrt{I*a + b}*\sqrt{\operatorname{Tan}[c + d*x]}}{\sqrt{a + b*\operatorname{Tan}[c + d*x]}}\right]}{\sqrt{a + b*\operatorname{Tan}[c + d*x]}}*\sqrt{\operatorname{Cot}[c + d*x]}*\sqrt{\operatorname{Tan}[c + d*x]}\right)/d - \frac{2*(8085*a^4*A*b - 495*a^2*A*b^3 + 40*A*b^5 + 3465*a^5*B - 5313*a^3*b^2*B - 110*a*b^4*B)*\sqrt{\operatorname{Cot}[c + d*x]}*\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{(3465*a^3*d)} - \frac{2*(1155*a^4*A - 1485*a^2*A*b^2 - 20*A*b^4 - 2541*a^3*b*B + 55*a*b^3*B)*\operatorname{Cot}[c + d*x]^{(3/2)}*\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{(3465*a^2*d)} + \frac{2*(495*a^2*A*b - 5*A*b^3 + 231*a^3*B - 275*a*b^2*B)*\operatorname{Cot}[c + d*x]^{(5/2)}*\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{(1155*a*d)} + \frac{2*(99*a^2*A - 113*A*b^2 - 209*a*b*B)*\operatorname{Cot}[c + d*x]^{(7/2)}*\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{(693*d)} - \frac{2*a*(14*A*b + 11*a*B)*\operatorname{Cot}[c + d*x]^{(9/2)}*\sqrt{a + b*\operatorname{Tan}[c + d*x]}}{(99*d)} - \frac{2*a*A*\operatorname{Cot}[c + d*x]^{(11/2)}*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}}{(11*d)}$

Rule 95

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
```

st[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3726

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[A\*d\*(b\*d\*m - a\*c\*(n + 1)) + (c\*C - B\*d)\*(b\*c\*m + a\*d\*(n + 1)) - d\*(n + 1)\*((A - C)\*(b\*c - a\*d) + B\*(a\*c + b\*d))\*Tan[e + f\*x] - b\*(d\*(B\*c - A\*d)\*(m + n + 1) - C\*(c^2\*m - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

### Rule 3730

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 4326

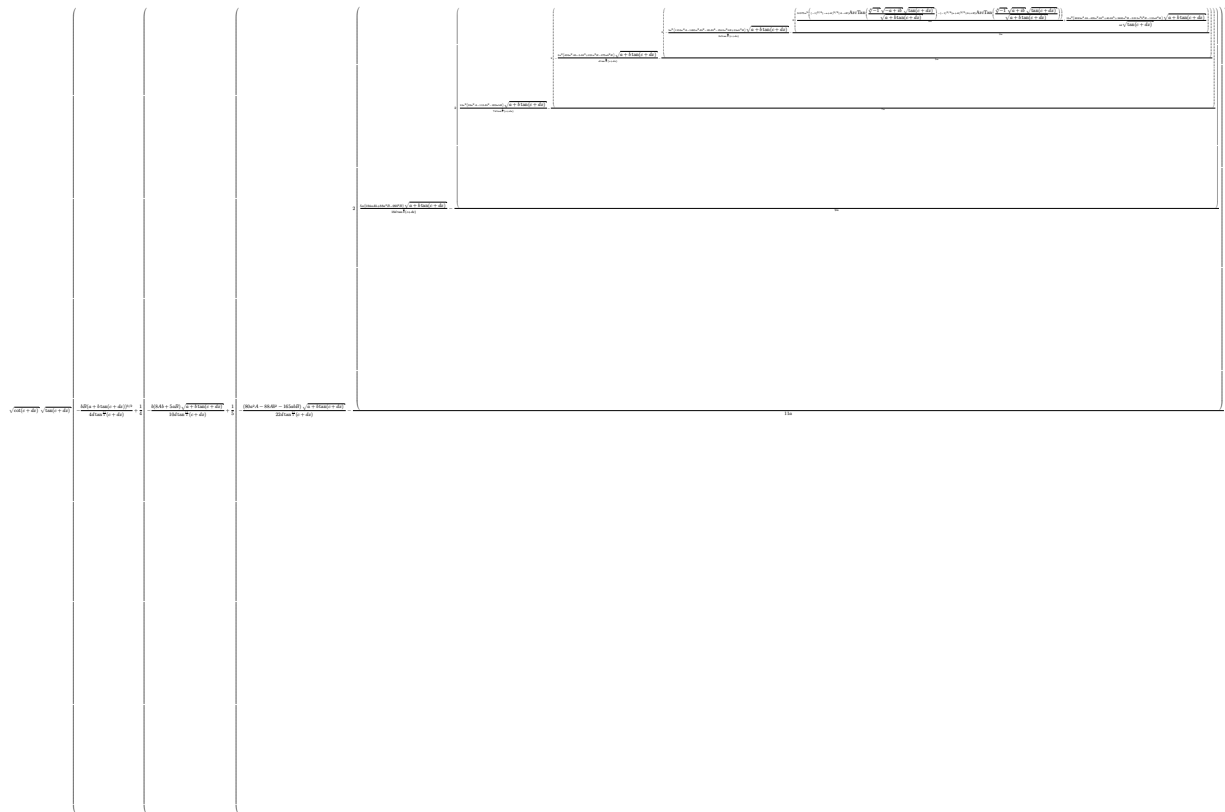
Int[(cot[(a\_.) + (b\_.)\*(x\_)]\*(c\_.))^(m\_.)\*(u\_), x\_Symbol] := Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{13}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{11}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}}{11d} \\
&= -\frac{2a(14Ab+11aB) \cot^{\frac{9}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{99d} \\
&= \frac{2(99a^2A-113Ab^2-209abB) \cot^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{693d} \\
&= \frac{2(495a^2Ab-5Ab^3+231a^3B-275ab^2B) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{1155ad} \\
&= -\frac{2(1155a^4A-1485a^2Ab^2-20Ab^4-2541a^3B+275ab^2B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{1155ad} \\
&= -\frac{2(8085a^4Ab-495a^2Ab^3+40Ab^5+3465a^3B-275ab^2B) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{1155ad} \\
&= -\frac{2(8085a^4Ab-495a^2Ab^3+40Ab^5+3465a^3B-275ab^2B) \sqrt{a+b \tan(c+dx)}}{1155ad} \\
&= -\frac{2(8085a^4Ab-495a^2Ab^3+40Ab^5+3465a^3B-275ab^2B) \sqrt{a+b \tan(c+dx)}}{1155ad} \\
&= -\frac{2(8085a^4Ab-495a^2Ab^3+40Ab^5+3465a^3B-275ab^2B) \sqrt{a+b \tan(c+dx)}}{1155ad} \\
&= -\frac{(ia-b)^{5/2}(iA-B) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{a+b \tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{1155ad}
\end{aligned}$$

Mathematica [A]

time = 6.62, size = 653, normalized size = 1.31



Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(13/2)\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-1/4\*(b\*B\*(a + b\*Tan[c + d\*x])^(3/2))/(d\*Tan[c + d\*x]^(11/2)) + (-1/10\*(b\*(8\*A\*b + 5\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(d\*Tan[c + d\*x]^(11/2)) + (-1/22\*((80\*a^2\*A - 88\*A\*b^2 - 165\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(d\*Tan[c + d\*x]^(11/2)) - (2\*((5\*a\*(184\*a\*A\*b + 88\*a^2\*B - 99\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(18\*d\*Tan[c + d\*x]^(9/2)) - (2\*((10\*a^2\*(99\*a^2\*A - 113\*A\*b^2 - 209\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(7\*d\*Tan[c + d\*x]^(7/2)) - (2\*((-3\*a^2\*(495\*a^2\*A\*b - 5\*A\*b^3 + 231\*a^3\*B - 275\*a\*b^2\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(d\*Tan[c + d\*x]^(5/2)) - (2\*((-5\*a^2\*(1155\*a^4\*A - 1485\*a^2\*A\*b^2 - 20\*A\*b^4 - 2541\*a^3\*b\*B + 55\*a\*b^3\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(2\*d\*Tan[c + d\*x]^(3/2)) - (2\*((51975\*a^5\*((-1)^(3/4))\*(-a + I\*b)^(5/2)\*(A - I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]] - (-1)^(3/4)\*(a + I\*b)^(5/2)\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]/Sqrt[a + b\*Tan[c + d\*x]]]))/(8\*d) + (15\*a^2\*(8085\*a^4\*A\*b - 495\*a^2\*A\*b^3 + 40\*A\*b^5 + 3465\*a^5\*B - 5313\*a^3\*b^2\*B - 110\*a\*b^4\*B)\*Sqrt[a + b\*Tan[c + d\*x]]/(4\*d\*Sqrt[Tan[c + d\*x]]))/(3\*a)))/(5\*a)))/(7\*a)))/(9\*a)))/(11\*a))/5)/4)



**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.44, size = 103896, normalized size = 207.79

method	result	size
default	Expression too large to display	103896

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(13/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(13/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(13/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{13/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(13/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(cot(c + d\*x)^(13/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)



], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]  
&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps

$$\begin{aligned}
\int \cot^{\frac{11}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\cot^{\frac{11}{2}}(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}}{9d} + \frac{2aB \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}}{7d} \\
&= -\frac{2a(4Ab+3aB) \cot^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{21d} + \frac{2b(3aA+2aB) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{5d} \\
&= \frac{2(21a^2A-25Ab^2-45abB) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{105d} + \frac{2(231a^2Ab-5Ab^3+105a^3B-135ab^2B) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} \\
&= -\frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} + \frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} \\
&= -\frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} + \frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} \\
&= -\frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} + \frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} \\
&= -\frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} + \frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB) \cot^{\frac{1}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{315ad} \\
&= \frac{(ia-b)^{5/2}(A+iB) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 6.54, size = 564, normalized size = 1.35



Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]), x]
```

```
[Out] Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-1/3*(b*B*(a + b*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(9/2)) + ((-3*b*(2*A*b + a*B)*Sqrt[a + b*Tan[c + d*x]])/(8*d*Tan[c + d*x]^(9/2)) + (-1/6*((16*a^2*A - 18*A*b^2 - 33*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Tan[c + d*x]^(9/2)) - (2*((6*a*(38*a*A*b + 18*a^2*B - 21*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) - (2*((18*a^2*(21*a^2*A - 25*A*b^2 - 45*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - (2*((-3*a^2*(231*a^2*A*b - 5*A*b^3 + 105*a^3*B - 135*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/2)) - (2*((-2835*a^4*((-1)^(1/4)*(-a + I*b)^(5/2)*(A - I*B)*ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]) + (-1)^(1/4)*(a + I*b)^(5/2)*(A + I*B)*ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])))/(4*d) - (9*a^2*(315*a^4*A - 483*a^2*A*b^2 - 10*A*b^4 - 735*a^3*b*B + 45*a*b^3*B)*Sqrt[a + b*Tan[c + d*x]])/(2*d*Sqrt[Tan[c + d*x]])))/(3*a)))/(5*a))/(7*a)))/(9*a))/4)/3)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 29.57, size = 101204, normalized size = 242.11

method	result	size
default	Expression too large to display	101204

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)), x, method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(11/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(11/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(11/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(11/2)\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(11/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{11/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

$$3.629 \quad \int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=349

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}(ia+b)^{5/2}(iA+B)}{d} + \dots$$

```
[Out] (I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(5/2)*(I*A+B)*arctanh(
(I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan
(d*x+c)^(1/2)/d+2/105*(35*A*a^2-45*A*b^2-77*B*a*b)*cot(d*x+c)^(3/2)*(a+b*ta
n(d*x+c))^(1/2)/d-2/35*a*(10*A*b+7*B*a)*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(
1/2)/d+2/105*(245*A*a^2*b-15*A*b^3+105*B*a^3-161*B*a*b^2)*cot(d*x+c)^(1/2)*
(a+b*tan(d*x+c))^(1/2)/a/d-2/7*a*A*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)/
d
```

**Rubi [A]**

time = 1.09, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {4326, 3686, 3726, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(35A^2a - 77AB - 45A^2b)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + 2(105A^2b + 245A^2b - 161Ab^2 - 15A^2b^3)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)} + (b + ia)^{5/2}(B + iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{tanh}^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + 2A\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}(ia + b\tan(c+dx))^{5/2}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

```
[Out] ((I*a - b)^(5/2)*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + ((I*a + b)^(
5/2)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d + (2*(245*a^2*A*b - 15*A*
b^3 + 105*a^3*B - 161*a*b^2*B)*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
/(105*a*d) + (2*(35*a^2*A - 45*A*b^2 - 77*a*b*B)*Cot[c + d*x]^(3/2)*Sqrt[a
+ b*Tan[c + d*x]])/(105*d) - (2*a*(10*A*b + 7*a*B)*Cot[c + d*x]^(5/2)*Sqrt[
a + b*Tan[c + d*x]])/(35*d) - (2*a*A*Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x]
)^(3/2))/(7*d)
```

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{9}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\cot(c+dx)} dx \\
&= -\frac{2aA \cot^{\frac{7}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}}{7d} + \frac{2aB \cot^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^{3/2}}{5d} \\
&= -\frac{2a(10Ab+7aB) \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{35d} \\
&= \frac{2(35a^2A-45Ab^2-77abB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{105d} \\
&= \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)}{105ad} \\
&= \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)}{105ad} \\
&= \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)}{105ad} \\
&= \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)}{105ad} \\
&= \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)}{105ad} \\
&= \frac{(ia-b)^{5/2}(iA-B) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [A]**

time = 3.41, size = 382, normalized size = 1.09

$$\frac{a^2(b+di)\left(\frac{b^2(a^2+ab)\sqrt{a^2+b^2}\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}} + \frac{2a^2b^2-2ab^2}{\sqrt{a^2+b^2}} + \frac{5ab^2b^2-2b^2}{\sqrt{a^2+b^2}} - \frac{2b^2(b^2+di)\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}} + \frac{21ab^2a+1ab^2+di^2}{\sqrt{a^2+b^2}} - \frac{21ab^2}{\sqrt{a^2+b^2}}\right) + (a+bi)^2(A-B)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + (a+bi)^2(A+B)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + \frac{2a^2b^2-2ab^2}{\sqrt{a^2+b^2}} + \frac{5ab^2b^2-2b^2}{\sqrt{a^2+b^2}} - \frac{2b^2(b^2+di)\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}} + \frac{21ab^2a+1ab^2+di^2}{\sqrt{a^2+b^2}} - \frac{21ab^2}{\sqrt{a^2+b^2}}}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d\*x]^(9/2)\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]), x]

[Out] -1/420\*(Cot[c + d\*x]^(7/2)\*(35\*a\*b\*(4\*A\*b + a\*B)\*Sqrt[a + b\*Tan[c + d\*x]] + 5\*a\*(24\*a^2\*A - 28\*A\*b^2 - 49\*a\*b\*B)\*Sqrt[a + b\*Tan[c + d\*x]] + 6\*a\*(60\*a\*A\*b + 28\*a^2\*B - 35\*b^2\*B)\*Tan[c + d\*x]\*Sqrt[a + b\*Tan[c + d\*x]] + 210\*a\*b\*B\*(a + b\*Tan[c + d\*x])^(3/2) - 4\*Tan[c + d\*x]^2\*(105\*(-1)^(1/4)\*a\*((-a + I\*b)^(5/2)\*(I\*A + B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]) + (a + I\*b)^(5/2)\*((-I)\*A + B)\*ArcTan[(-1)^(1/4)\*S

$$\frac{\sqrt{a + I*b}*\sqrt{\tan[c + d*x]}}{\sqrt{a + b*\tan[c + d*x]}}*\tan[c + d*x]^{(3/2)} + 2*a*(35*a^2*A - 45*A*b^2 - 77*a*b*B)*\sqrt{a + b*\tan[c + d*x]} + 2*(24*5*a^2*A*b - 15*A*b^3 + 105*a^3*B - 161*a*b^2*B)*\tan[c + d*x]*\sqrt{a + b*\tan[c + d*x]}}{(a*d)}$$

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.67, size = 67595, normalized size = 193.68

method	result	size
default	Expression too large to display	67595

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(9/2), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(9/2)\*(a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(9/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{9/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(cot(c + d\*x)^(9/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)

$$3.630 \quad \int \cot^2(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=287

$$\frac{(ia - b)^{5/2}(A + iB)\text{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} (ia + b)^{5/2}(A - iB)}{d} +$$

[Out]  $-(I*a-b)^{(5/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(I*a+b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-2/15*a*(8*A*b+5*B*a)*\cot(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d+2/15*(15*A*a^2-23*A*b^2-35*B*a*b)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/d-2/5*a*A*\cot(d*x+c)^{(5/2)}*(a+b*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 0.85, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$ , Rules used = {4326, 3686, 3726, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(15a^2A - 35abB - 23A^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15d} - \frac{(-b+ia)^{5/2}(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(8aB+5Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{15d} + \frac{(b+ia)^{5/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(7/2)\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left(\left(\left(I*a - b\right)^{(5/2)}*(A + I*B)*\operatorname{ArcTan}\left[\left(\operatorname{Sqrt}\left[I*a - b\right]*\operatorname{Sqrt}\left[\tan\left[c + d*x\right]\right]\right)/\operatorname{Sqrt}\left[a + b*\tan\left[c + d*x\right]\right]\right)*\operatorname{Sqrt}\left[\cot\left[c + d*x\right]\right]*\operatorname{Sqrt}\left[\tan\left[c + d*x\right]\right]/d + \left(\left(I*a + b\right)^{(5/2)}*(A - I*B)*\operatorname{ArcTanh}\left[\left(\operatorname{Sqrt}\left[I*a + b\right]*\operatorname{Sqrt}\left[\tan\left[c + d*x\right]\right]\right)/\operatorname{Sqrt}\left[a + b*\tan\left[c + d*x\right]\right]\right)*\operatorname{Sqrt}\left[\cot\left[c + d*x\right]\right]*\operatorname{Sqrt}\left[\tan\left[c + d*x\right]\right]/d + \left(2*(15*a^2*A - 23*A*b^2 - 35*a*b*B)*\operatorname{Sqrt}\left[\cot\left[c + d*x\right]\right]*\operatorname{Sqrt}\left[a + b*\tan\left[c + d*x\right]\right]/(15*d) - \left(2*a*(8*A*b + 5*a*B)*\cot\left[c + d*x\right]^{(3/2)}*\operatorname{Sqrt}\left[a + b*\tan\left[c + d*x\right]\right)/(15*d) - \left(2*a*A*\cot\left[c + d*x\right]^{(5/2)}*(a + b*\tan\left[c + d*x\right])^{(3/2)}\right)/(5*d)\right)$

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a



, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3726

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*d^2 + c\*(c\*C - B\*d))\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 1)\*(c + d\*Tan[e

```

+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3730

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps



**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.21, size = 66799, normalized size = 232.75

method	result	size
default	Expression too large to display	66799

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(7/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{7/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(cot(c + d\*x)^(7/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)

$$3.631 \quad \int \cot^2(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=300

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2b^{5/2}B \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

[Out]  $-(I*a-b)^{(5/2)}*(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{\wedge}(1/2))*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d+2*b^{(5/2)}*B*\arctanh(b^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{\wedge}(1/2))*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d-(I*a+b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))}^{\wedge}(1/2))*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/d-2*a*(2*A*b+B*a)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))}^{\wedge}(1/2)/d-2/3*a*A*\cot(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))}^{\wedge}(3/2)/d$

Rubi [A]

time = 1.61, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3686, 3726, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(-b+ia)^{5/2}(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(b+2Ab)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} - \frac{(b+ia)^{5/2}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2b^{5/2}B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out]  $-\left(\frac{(I*a-b)^{(5/2)}*(I*A-B)*\text{ArcTan}[(\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a+b*\text{Tan}[c+d*x]]]*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]]}{d} + (2*b^{(5/2)}*B*\text{ArcTanh}[(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a+b*\text{Tan}[c+d*x]]]*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]]}{d} - ((I*a+b)^{(5/2)}*(I*A+B)*\text{ArcTanh}[(\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]])/\text{Sqrt}[a+b*\text{Tan}[c+d*x]]]*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]]}{d} - (2*a*(2*A*b+a*B)*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[a+b*\text{Tan}[c+d*x]]}{d} - (2*a*A*\text{Cot}[c+d*x]^{\wedge}(3/2)*(a+b*\text{Tan}[c+d*x])^{\wedge}(3/2)))/(3*d)\right)$

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-a\*(d/b)+d\*(x^p/b))^n, x], x, (a+b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c-a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 95

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
```

```

f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps





**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
 time = 27.88, size = 47378, normalized size = 157.93

method	result	size
default	Expression too large to display	47378

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{5/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)

$$3.632 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=301

$$\frac{(ia - b)^{5/2}(A + iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + b^{3/2}(2Ab + 5aB) \tan(c + dx)}{d}$$

[Out]  $(I*a-b)^{(5/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+b^{(3/2)}*(2*A*b+5*B*a)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d-(I*a+b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+b*(2*A*a+B*b)*(a+b*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(1/2)}-2*a*A*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(3/2)}/d$

**Rubi [A]**

time = 1.76, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3686, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(-b+ia)^{5/2}(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{ia+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b^{3/2}(2aB+2Ab)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b(2aA+bB)\sqrt{a+b\tan(c+dx)}-\frac{(b+ia)^{5/2}(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+2aA\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cot}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}*(A + B*\operatorname{Tan}[c + d*x]), x]$

[Out]  $((I*a - b)^{(5/2)}*(A + I*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d + (b^{(3/2)}*(2*A*b + 5*a*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d - ((I*a + b)^{(5/2)}*(A - I*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]/d + (b*(2*a*A + b*B)*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])/(d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]) - (2*a*A*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)})/d$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$   $\operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{LtQ}[-1, m, 0] \ \&\& \operatorname{LeQ}[-1, n, 0] \ \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
```

```
e + f*x])^(n + 1)/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^{5/2}}{\cot(c+dx)} dx \\
&= -\frac{2aA \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{3/2}}{d} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b(2aA+bB) \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} - \frac{2aA \sqrt{a+b \tan(c+dx)}}{d \sqrt{\cot(c+dx)}} \\
&= \frac{b^3/2(2Ab+5aB) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d} \\
&= \frac{(ia-b)^{5/2}(A+iB) \tan^{-1}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 39.56, size = 196709, normalized size = 653.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

[Out] Result too large to show

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.63, size = 57756, normalized size = 191.88

method	result	size
default	Expression too large to display	57756

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,algorithm="maxima")`

[Out] Timed out

**Fricas** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)`

[Out] Timed out



**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int(cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2), x)

### 3.633 $\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx)) dx$

**Optimal.** Leaf size=320

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + \sqrt{b}(20aAb+15a^2B)}{d}$$

[Out] (I\*a-b)^(5/2)\*(I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+(I\*a+b)^(5/2)\*(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+1/4\*(20\*A\*a\*b+15\*B\*a^2-8\*B\*b^2)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*b^(1/2)\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d+1/4\*b\*(4\*A\*b+7\*B\*a)\*(a+b\*tan(d\*x+c))^(1/2)/d/cot(d\*x+c)^(1/2)+1/2\*b\*B\*(a+b\*tan(d\*x+c))^(3/2)/d/cot(d\*x+c)^(1/2)

**Rubi [A]**

time = 1.62, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{\sqrt{b}(15a^2B+20aAb-8b^2B)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + (-b+ia)^{5/2}(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + \frac{b(7aB+4Ab)\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{(b+ia)^{5/2}(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + bB(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(5/2)\*(A + B\*Tan[c + d\*x]),x]

[Out] ((I\*a - b)^(5/2)\*(I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + (Sqrt[b]\*(20\*a\*A\*b + 15\*a^2\*B - 8\*b^2\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(4\*d) + ((I\*a + b)^(5/2)\*(I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/d + (b\*(4\*A\*b + 7\*a\*B)\*Sqrt[a + b\*Tan[c + d\*x]])/(4\*d\*Sqrt[Cot[c + d\*x]]) + (b\*B\*(a + b\*Tan[c + d\*x])^(3/2))/(2\*d\*Sqrt[Cot[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^(n), x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
```

```

b*Tan[e + f*x]]^(m - 1)*(c + d*Tan[e + f*x]]^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3736

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rule 6857

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rubi steps



```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(4*(-1)^(1/4)*(-a + I*b)^(5/2)*(I*A
+ B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]) - 4*(-1)^(3/4)*(a + I*b)^(5/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[
a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]) + b*(4*A*b + 7*a*B)*
Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*b*B*Sqrt[Tan[c + d*x]]*(a +
b*Tan[c + d*x])^(3/2) + (Sqrt[a]*Sqrt[b]*(20*a*A*b + 15*a^2*B - 8*b^2*B)*A
rcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/
Sqrt[a + b*Tan[c + d*x]])/(4*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 30.76, size = 32917, normalized size = 102.87

method	result	size
default	Expression too large to display	32917

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c)
), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)`

[Out] `int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)`

$$3.634 \quad \int \frac{(a+b \tan(c+dx))^{5/2}(A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=376

$$\frac{(ia-b)^{5/2}(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}(30a^2Ab-16Ab^3)}{d} +$$

[Out]  $-(I*a-b)^{(5/2)}*(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+(I*a+b)^{(5/2)}*(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+1/8*(30*A*a^2*b-16*A*b^3+5*B*a^3-40*B*a*b^2)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/b^{(1/2)}+1/8*(14*A*a*b+5*B*a^2-8*B*b^2)*(a+b*\tan(d*x+c))^{(1/2)}/d/\cot(d*x+c)^{(1/2)}+1/3*b*B*(a+b*\tan(d*x+c))^{(3/2)}/d/\cot(d*x+c)^{(3/2)}+1/4*(2*A*b+3*B*a)*(a+b*\tan(d*x+c))^{(3/2)}/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 2.21, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(a^2 B + 14 a A b - 8 B^2 b) \sqrt{a + b \tan(c + dx)}}{8 d \sqrt{\cot(c + dx)}} + \frac{(5 a^3 B + 30 a^2 A b - 40 a B^2 b - 16 A b^3) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{8 \sqrt{d}} - \frac{(b + a i)^{5/2} (A + i B) \sqrt{\tan(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{(3 a B + 2 A b) (a + b \tan(c + dx))^{3/2}}{4 d \sqrt{\cot(c + dx)}} + \frac{(b + a i)^{5/2} (A - i B) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{a + b \tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d} + \frac{10 (a + b \tan(c + dx))^{3/2}}{3 d \sqrt{\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x])]/\text{Sqrt}[\text{Cot}[c + d*x]], x]$

[Out]  $-\left(\left(I*a - b\right)^{(5/2)}*(A + I*B)*\text{ArcTan}\left[\left(\text{Sqrt}[I*a - b]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/d + \left(\left(30*a^2*A*b - 16*A*b^3 + 5*a^3*B - 40*a*b^2*B\right)*\text{ArcTanh}\left[\left(\text{Sqrt}[b]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/(8*\text{Sqrt}[b]*d) + \left(\left(I*a + b\right)^{(5/2)}*(A - I*B)*\text{ArcTanh}\left[\left(\text{Sqrt}[I*a + b]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right]*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sqrt}[\text{Tan}[c + d*x]]\right)/d + \left(\left(14*a*A*b + 5*a^2*B - 8*b^2*B\right)*\text{Sqrt}[a + b*\text{Tan}[c + d*x]]\right)/(8*d*\text{Sqrt}[\text{Cot}[c + d*x]]) + (b*B*(a + b*\text{Tan}[c + d*x])^{(3/2)})/(3*d*\text{Cot}[c + d*x]^{(3/2)}) + ((2*A*b + 3*a*B)*(a + b*\text{Tan}[c + d*x])^{(3/2)})/(4*d*\text{Sqrt}[\text{Cot}[c + d*x]])$

**Rule 65**

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x\_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$   $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$



Rule 95

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3688

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3728

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] := Simp[C\*(a + b\*Tan[e + f\*x])^m\*((c + d\*Tan[

```
e + f*x])^(n + 1)/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps



Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(24*(-1)^(1/4)*(-a + I*b)^(5/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(1/4)*(a + I*b)^(5/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]] + 3*(14*a*A*b + 5*a^2*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 6*(2*A*b + 3*a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2) + (3*Sqrt[a]*(30*a^2*A*b - 16*A*b^3 + 5*a^3*B - 40*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(24*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 30.42, size = 36039, normalized size = 95.85

method	result	size
default	Expression too large to display	36039

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/sqrt(cot(d*x + c)), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*(5/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^(5/2)\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^(5/2))/cot(c + d\*x)^(1/2), x)

$$3.635 \quad \int \frac{(a+b \tan(c+dx))^{5/2} (A+B \tan(c+dx))}{\cot^2(c+dx)} dx$$

**Optimal.** Leaf size=457

$$\frac{(ia-b)^{5/2}(iA-B)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(40a^3Ab-320aAb^3-5a^4B+240a^2b^2B+128b^4B)\text{ArcTanh}\left(\frac{b^{1/2}\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(40Aa^3b-320AaAb^3-5Aa^4B+240Aa^2b^2B+128Ab^4B)\text{ArcTanh}\left(\frac{b^{1/2}\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

[Out]  $-(I*a-b)^{(5/2)}*(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+1/64*(40*A*a^3*b-320*A*a*b^3-5*B*a^4-240*B*a^2*b^2+128*B*b^4)*\operatorname{arctanh}(b^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/b^{(3/2)}/d-(I*a+b)^{(5/2)}*(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d+1/64*(40*A*a^2*b-64*A*b^3-5*B*a^3-112*B*a*b^2)*(a+b*\tan(d*x+c))^{(1/2)}/b/d/\cot(d*x+c)^{(1/2)}+1/96*(40*A*a*b-5*B*a^2-48*B*b^2)*(a+b*\tan(d*x+c))^{(3/2)}/b/d/\cot(d*x+c)^{(1/2)}+1/24*(8*A*b-B*a)*(a+b*\tan(d*x+c))^{(5/2)}/b/d/\cot(d*x+c)^{(1/2)}+1/4*B*(a+b*\tan(d*x+c))^{(7/2)}/b/d/\cot(d*x+c)^{(1/2)}$

**Rubi [A]**

time = 2.24, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3688, 3728, 3736, 6857, 65, 223, 212, 95, 211, 214}

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Tan}[c + d*x])^{(5/2)}*(A + B*\text{Tan}[c + d*x])]/\text{Cot}[c + d*x]^{(3/2)}, x]$

[Out]  $-\left(\frac{(I*a-b)^{(5/2)}*(I*A-B)*\text{ArcTan}\left[\frac{\sqrt{I*a-b}*\sqrt{\text{Tan}[c+d*x]}}{\sqrt{a+b*\text{Tan}[c+d*x]}}\right]}{\sqrt{a+b*\text{Tan}[c+d*x]}}*\sqrt{\text{Cot}[c+d*x]}\right)/d + \frac{(40*a^3*A*b-320*a^2*A*b^3-5*a^4*B+240*a^2*b^2*B+128*b^4*B)*\text{ArcTanh}\left[\frac{\sqrt{b}*\sqrt{\text{Tan}[c+d*x]}}{\sqrt{a+b*\text{Tan}[c+d*x]}}\right]}{\sqrt{a+b*\text{Tan}[c+d*x]}}*\sqrt{\text{Cot}[c+d*x]}}/d - \frac{(I*a+b)^{(5/2)}*(I*A+B)*\text{ArcTanh}\left[\frac{\sqrt{I*a+b}*\sqrt{\text{Tan}[c+d*x]}}{\sqrt{a+b*\text{Tan}[c+d*x]}}\right]}{\sqrt{a+b*\text{Tan}[c+d*x]}}*\sqrt{\text{Cot}[c+d*x]}}/d + \frac{(40*a^2*A*b-64*A*b^3-5*a^3*B-112*a*b^2*B)*\sqrt{a+b*\text{Tan}[c+d*x]}}{(64*b*d*\sqrt{\text{Cot}[c+d*x]})} + \frac{(40*a*A*b-5*a^2*B-48*b^2*B)*(a+b*\text{Tan}[c+d*x])^{(3/2)}}{(96*b*d*\sqrt{\text{Cot}[c+d*x]})} + \frac{(8*A*b-a*B)*(a+b*\text{Tan}[c+d*x])^{(5/2)}}{(24*b*d*\sqrt{\text{Cot}[c+d*x]})} + \frac{(B*(a+b*\text{Tan}[c+d*x])^{(7/2)})}{(4*b*d*\sqrt{\text{Cot}[c+d*x]})}$

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_. + (d_.)*(x_.))^{(n_.)}), x\_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) +$

```
d*(x^p/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 95

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3688

```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_))*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 1] &
& (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2), x_Symbol] := Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps





**Mathematica [A]**

time = 3.66, size = 431, normalized size = 0.94

$$\frac{\sqrt{a^2+d^2} \sqrt{a+b \tan(c+dx)} \left( -192(-1)^{1/4} (a+Ib)^{5/2} b (IA+B) \operatorname{ArcTan}\left[\frac{(-1)^{1/4} \sqrt{-a+Ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right] + 192(-1)^{3/4} (a+Ib)^{5/2} b (A+IB) \operatorname{ArcTan}\left[\frac{(-1)^{3/4} \sqrt{-a+Ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right] - 3(-40a^2Ab + 64A^2b^3 + 5a^3B + 112ab^2B) \sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)} - 2(-40a^2Ab + 5a^2B + 48b^2B) \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{3/2} + 8(8Ab - aB) \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{5/2} + 48B \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^{7/2} - (3\sqrt{a} (-40a^3Ab + 320a^2Ab^3 + 5a^4B + 240a^2b^2B - 128b^4B) \operatorname{ArcSinh}\left[\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}}\right] \sqrt{1+(b \tan(c+dx)/a)}}{\sqrt{a+b \tan(c+dx)}}) \right) / (192bd)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-192*(-1)^(1/4)*(-a + I*b)^(5/2)*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]] + 192*(-1)^(3/4)*(a + I*b)^(5/2)*b*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]] - 3*(-40*a^2*A*b + 64*A*b^3 + 5*a^3*B + 112*a*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] - 2*(-40*a*A*b + 5*a^2*B + 48*b^2*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*(8*A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2) + 48*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(7/2) - (3*Sqrt[a]*(-40*a^3*A*b + 320*a*A*b^3 + 5*a^4*B + 240*a^2*b^2*B - 128*b^4*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(192*b*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 31.10, size = 39827, normalized size = 87.15

method	result	size
default	Expression too large to display	39827

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/cot(d*x + c)^(3/2), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorithm="giac")`

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^{5/2}}{\cot(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/cot(c + d*x)^(3/2),x)`

[Out] `int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/cot(c + d*x)^(3/2), x)`

$$3.636 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=296

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + (iA + B) \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a+b)^(1/2)+2/15\*(4\*A\*b-5\*B\*a)\*cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^(1/2)/a^2/d-2/5\*A\*cot(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^(1/2)/a/d+2/15\*(15\*A\*a^2-8\*A\*b^2+10\*B\*a\*b)\*cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^(1/2)/a^3/d

Rubi [A]

time = 0.76, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(4Ab-5aB)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{15a^2d} + \frac{2(15a^2A+10abB-8AB^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15a^2d} + \frac{(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}} - \frac{2A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{5ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(7/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d) + (2\*(15\*a^2\*A - 8\*A\*b^2 + 10\*a\*b\*B)\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a^3\*d) + (2\*(4\*A\*b - 5\*a\*B)\*Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(15\*a^2\*d) - (2\*A\*Cot[c + d\*x]^(5/2)\*Sqrt[a + b\*Tan[c + d\*x]])/(5\*a\*d)

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && (!ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f

```

*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \\
&= -\frac{2A \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{5ad} - \frac{\left( 2 \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} \\
&= \frac{2(4Ab - 5aB) \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{15a^2d} - \frac{2A \cot^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{15a^2d} \\
&= \frac{2(15a^2A - 8Ab^2 + 10abB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^3d} + \frac{2(15a^2A - 8Ab^2 + 10abB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^3d} \\
&= \frac{2(15a^2A - 8Ab^2 + 10abB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^3d} + \frac{2(15a^2A - 8Ab^2 + 10abB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^3d} \\
&= \frac{2(15a^2A - 8Ab^2 + 10abB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^3d} + \frac{2(15a^2A - 8Ab^2 + 10abB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{15a^3d} \\
&= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{ia-b} d}
\end{aligned}$$

**Mathematica [A]**

time = 4.04, size = 244, normalized size = 0.82

$$\frac{\sqrt{\cot(c+dx)} \left( \frac{{}_{15}\sqrt{-1} (A-iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{\sqrt{-a+ib}} + \frac{{}_{15}\sqrt{-1} (A+iB) \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{\sqrt{a+ib}} - \frac{2[-15a^2A+8AB^2-10aB+a(-4Ab+5aB)\cot(c+dx)+3a^2A\cot^2(c+dx)]\sqrt{a+b \tan(c+dx)}}{a^3} \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(7/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x] ], x]

[Out] (Sqrt[Cot[c + d\*x]]\*((-15\*(-1)^(1/4)\*(A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])]/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]])/Sqrt[-a + I\*b] + (15\*(-1)^(1/4)\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])]/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*b] - (2\*(-15\*a^2\*A + 8\*A\*b^2 - 10\*a\*b\*B + a\*(-4\*A\*b + 5\*a\*B)\*Cot[c + d\*x] + 3\*a^2\*A\*Cot[c + d\*x]^2)\*Sqrt[a + b\*Tan[c + d\*x]])/a^3)/(15\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 29.69, size = 29195, normalized size = 98.63

method	result	size
default	Expression too large to display	29195

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(7/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2), x, method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(7/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(7/2)/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
Sympy [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

```
Giac [F(-2)]  
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si
```

```
Mupad [F]  
time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{\cot(c + dx)^{7/2} (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(7/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int((cot(c + d*x)^(7/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2), x)
```



$$3.637 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=243

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} - (A-iB) \tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b}d}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/(I*a-b)^{(1/2)}-(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/(I*a+b)^{(1/2)}-2/3*A*\cot(d*x+c)^{(3/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a/d+2/3*(2*A*b-3*B*a)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a^2/d$

Rubi [A]

time = 0.56, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(2Ab-3aB)\sqrt{\cot(c+dx)}\sqrt{a+b \tan(c+dx)}}{3a^2d} - \frac{(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}} - \frac{2A\cot^3(c+dx)\sqrt{a+b \tan(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]], x]

[Out]  $-(((A+I*B)*\text{ArcTan}[(\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b \tan[c+d*x]])*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]])/(\text{Sqrt}[I*a-b]*d) - ((A-I*B)*\text{ArcTanh}[(\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b \tan[c+d*x]])*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]])/(\text{Sqrt}[I*a+b]*d) + (2*(2*A*b-3*a*B)*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[a+b \tan[c+d*x]])/(3*a^2*d) - (2*A*\text{Cot}[c+d*x]^{(3/2)}*\text{Sqrt}[a+b \tan[c+d*x]])/(3*a*d)$

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m+1)-1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[(((a\_.) + (b\_.)\*(x\_)^(2))^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
+ (f_)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \\
&= -\frac{2A \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3ad} - \frac{\left( 2\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \\
&= \frac{2(2Ab-3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3ad} \\
&= \frac{2(2Ab-3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3ad} \\
&= \frac{2(2Ab-3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3ad} \\
&= \frac{2(2Ab-3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3ad} \\
&= \frac{2(2Ab-3aB) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{3a^2d} - \frac{2A \cot^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}}{3ad} \\
&= -\frac{(A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{\sqrt{ia-b} d}
\end{aligned}$$

### Mathematica [A]

time = 1.79, size = 213, normalized size = 0.88

$$\frac{\sqrt{\cot(c+dx)} \left( \frac{{}_3\sqrt{-1}^{(iA+B)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{\sqrt{-a+ib}} + \frac{{}_3(-1)^{3/4(A+iB)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\tan(c+dx)}}{\sqrt{a+ib}} - \frac{2(-2Ab+3aB+aA \cot(c+dx)) \sqrt{a+b \tan(c+dx)}}{a^2} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] (Sqrt[Cot[c + d\*x]]\*((3\*(-1)^(1/4)\*(I\*A + B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]])/Sqrt[-a + I\*b] + (3\*(-1)^(3/4)\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]])/Sqrt[a + I\*b] - (2\*(-2\*A\*b + 3\*a\*B + a\*A\*Cot[c + d\*x])\*Sqrt[a + b\*Tan[c + d\*x]])/a^2)/(3\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 34.74, size = 14834, normalized size = 61.05

method	result	size
default	Expression too large to display	14834

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(5/2)/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

**Giac** [F(-2)]

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad** [F]

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{\cot(c + dx)^{5/2} (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2), x
)
```

$$3.638 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=199

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + (iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/(I*a-b)^{(1/2)}+(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/d/(I*a+b)^{(1/2)}-2*A*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^{(1/2)}/a/d$

Rubi [A]

time = 0.40, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4326, 3690, 3697, 3696, 95, 209, 212}

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{(B+iA)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\operatorname{tanh}^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}} - \frac{2A\sqrt{\cot(c+dx)}\sqrt{a+b \tan(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In] `Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]`

[Out]  $-\left(\left(I*A - B\right)*\operatorname{ArcTan}\left[\frac{\sqrt{I*a - b}*\sqrt{\tan[c + d*x]}}{\sqrt{a + b*\tan[c + d*x]}}\right]/\sqrt{a + b*\tan[c + d*x]}\right)*\sqrt{\cot[c + d*x]}*\sqrt{\tan[c + d*x]}/\left(\sqrt{I*a - b}*d\right) + \left(\left(I*A + B\right)*\operatorname{ArcTanh}\left[\frac{\sqrt{I*a + b}*\sqrt{\tan[c + d*x]}}{\sqrt{a + b*\tan[c + d*x]}}\right]/\sqrt{a + b*\tan[c + d*x]}\right)*\sqrt{\cot[c + d*x]}*\sqrt{\tan[c + d*x]}/\left(\sqrt{I*a + b}*d\right) - \left(2*A*\sqrt{\cot[c + d*x]}*\sqrt{a + b*\tan[c + d*x]}\right)/\left(a*d\right)$

Rule 95

`Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]`

Rule 209

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx) \sqrt{a+b \tan(c+dx)}} dx \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{ad} - \frac{\left( 2\sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} \right)}{ad} \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{ad} - \frac{1}{2} \left( (iA-B) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} \right) \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{ad} - \frac{\left( (iA-B) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} \right)}{ad} \\
&= -\frac{2A \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{ad} - \frac{\left( (iA-B) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)} \right)}{ad} \\
&= -\frac{(iA-B) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}}{\sqrt{ia-b} d}
\end{aligned}$$

**Mathematica [A]**

time = 1.05, size = 193, normalized size = 0.97

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( -\frac{\sqrt[4]{-1}^{(A-iB)} \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{\sqrt[4]{-1}^{(A+iB)} \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} + \frac{2A \sqrt{a+b \tan(c+dx)}}{a \sqrt{\tan(c+dx)}} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-((( -1)^(1/4)*(A - I*B)*ArcTan[((( -1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + (( -1)^(1/4)*(A + I*B)*ArcTan[((( -1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (2*A*Sqrt[a + b*Tan[c + d*x]])/(a*Sqrt[Tan[c + d*x]])))/d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 34.40, size = 14215, normalized size = 71.43

method	result	size
default	Expression too large to display	14215



---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a), x)`

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)`

[Out] `Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/sqrt(a + b*tan(c + d*x)), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

```
[Out] int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2), x)
```

$$3.639 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx$$

**Optimal.** Leaf size=163

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{ia-b} d} + \frac{(A-iB) \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)+(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a+b)^(1/2)

**Rubi [A]**

time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {4326, 3697, 3696, 95, 209, 212}

$$\frac{(A+iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{-b+ia}} + \frac{(A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/Sqrt[a + b\*Tan[c + d\*x]],x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) + ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d)

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 212**

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

#### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

#### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

#### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{\sqrt{a+b \tan(c+dx)}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)} \sqrt{a+b \tan(c+dx)}} dx \\
&= \frac{1}{2} \left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{1+i \tan(c+dx)}{\sqrt{\tan(c+dx)}} dx \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \text{Subst} \left( \int \frac{1}{(1-ix) \sqrt{x}} dx \right)}{2d} \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \text{Subst} \left( \int \frac{1}{1-(ia+b)x^2} dx \right)}{d} \\
&= \frac{(A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{ia-b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.25, size = 157, normalized size = 0.96

$$\frac{\sqrt[4]{-1} \left( -\frac{(iA+B) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{(-iA+B) \text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]], x]
```

```
[Out] ((-1)^(1/4)*(-(((I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + (((-I)*A + B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 29.87, size = 3484, normalized size = 21.37

method	result	size
default	Expression too large to display	3484

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2), x, method=_RETURNVERBOSE)
```



$c)^{1/2}, -(-b+(a^2+b^2)^{1/2})/(I*a-(a^2+b^2)^{1/2}+b), 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a^2*b-A*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, (-b+(a^2+b^2)^{1/2})/(I*a+(a^2+b^2)^{1/2}-b), 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a^2*b+4*A*EllipticF((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a^2*b+4*A*EllipticF((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*b^3+2*B*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, -(-b+(a^2+b^2)^{1/2})/(I*a-(a^2+b^2)^{1/2}+b), 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a*b*(a^2+b^2)^{1/2}+2*B*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, (-b+(a^2+b^2)^{1/2})/(I*a+(a^2+b^2)^{1/2}-b), 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a*b*(a^2+b^2)^{1/2}-B*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, -(-b+(a^2+b^2)^{1/2})/(I*a-(a^2+b^2)^{1/2}+b), 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a^3-2*B*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^{1/2}*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^{1/2})/sin(d*x+c))^{1/2}, -(-b+(a^2+b^2)^{1/2})/(I*a-(a^2+b^2)^{1/2}+b), 1/2*2^{1/2}*((-b+(a^2+b^2)^{1/2})/(a^2+b^2)^{1/2})^{1/2})*a*b^2-B*EllipticPi((-a...$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/sqrt(b\*tan(d\*x + c) + a), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x))/sqrt(a + b\*tan(c + d\*x)), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2),x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(1/2), x)



$$3.640 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=228

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} - 2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{ia - b} d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{b} d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)+2\*B\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/b^(1/2)-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a+b)^(1/2)

Rubi [A]

time = 0.44, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ ,

Rules used = {4326, 3695, 3697, 3696, 95, 209, 212, 3715, 65, 223}

$$\frac{(-B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d \sqrt{-b + ia}} - \frac{(B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d \sqrt{b + ia}} + \frac{2B \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{\sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]),x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(Sqrt[I\*a - b]\*d) + (2\*B\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(Sqrt[b]\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/(Sqrt[I\*a + b]\*d)

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 95

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3695

Int[(Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])]/Sqrt[(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Int[Simp[a\*A - b\*B + (A\*b + a\*B)\*Tan[e + f\*x], x]/(Sqrt[a + b\*Tan[e + f\*x]]\*Sqrt[c + d\*Tan[e + f\*x]]), x] + Dist[b\*B, Int[(1 + Tan[e + f\*x]^2)/(Sqrt[a + b\*Tan[e + f\*x]]\*Sqrt[c + d\*Tan[e + f\*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{-B + A \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{1}{2} \left( (-iA - B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{1 + \tan(c + dx)}{\sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{\left( (-iA - B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{1}{(1-ix)\sqrt{a+bx}} dx \right)}{2d} \\
&= \frac{\left( (-iA - B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{1}{1-(ia+b)x} dx \right)}{d} \\
&= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} d}
\end{aligned}$$

Mathematica [A]

time = 0.99, size = 225, normalized size = 0.99

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( \sqrt{-1} \left( \frac{(A-iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{(A+iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right) + \frac{2\sqrt{a} B \sinh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a}} \sqrt{1 + \frac{b \tan(c+dx)}{a}} \right)}{\sqrt{b} \sqrt{a + b \tan(c+dx)}} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]),x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-1)^(1/4)\*(-(((A - I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[-a + I\*b]) + ((A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[a + I\*b]) + (2\*Sqrt[a]\*B\*ArcSinh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]]\*Sqrt[1 + (b\*Tan[c + d\*x])/a])/(Sqrt[b]\*Sqrt[a + b\*Tan[c + d\*x]]))/d

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 32.44, size = 6698, normalized size = 29.38

method	result	size
default	Expression too large to display	6698

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)/(sqrt(b\*tan(d\*x + c) + a)\*sqrt(cot(d\*x + c))), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)} \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(1/2)/(a+b\*tan(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))/(sqrt(a + b\*tan(c + d\*x))\*sqrt(cot(c + d\*x))), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError &gt;&gt; An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(1/2)), x)

$$3.641 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx) \sqrt{a+b \tan(c+dx)}} dx$$

Optimal. Leaf size=266

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + (2Ab-aB) \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d}$$

[Out] (2\*A\*b-B\*a)\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/b^(3/2)/d-(A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a+b)^(1/2)+B\*(a+b\*tan(d\*x+c))^(1/2)/b/d/cot(d\*x+c)^(1/2)

Rubi [A]

time = 1.04, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4326, 3688, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{(A+iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + (2Ab-aB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) - (A-iB) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) + \frac{B \sqrt{a+b \tan(c+dx)}}{bd \sqrt{\cot(c+dx)}}}{d \sqrt{-b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]]), x]

[Out] -(((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) + ((2\*A\*b - a\*B)\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(b^(3/2)\*d) - ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d) + (B\*Sqrt[a + b\*Tan[c + d\*x]])/(b\*d\*Sqrt[Cot[c + d\*x]])

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 95

Int((((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)\*(c + d\*x)^(n/q), x], x, (e + f\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e\*f - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)  
 ], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]  
 && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3688

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*B\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(m + n))), x] + Dist[1/(d\*(m + n)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^2\*A\*d\*(m + n) - b\*B\*(b\*c\*(m - 1) + a\*d\*(n + 1)) + d\*(m + n)\*(2\*a\*A\*b + B\*(a^2 - b^2))\*Tan[e + f\*x] - (b\*B\*(b\*c - a\*d)\*(m - 1) - b\*(A\*b + a\*B)\*d\*(m + n))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(IGtQ[n, 1] & & (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

#### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +

$d^2, 0]$

Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{bd \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{-\frac{9}{\sqrt{a + b \tan(c + dx)}}}{b} dx}{b} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{bd \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst}}{b} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{bd \sqrt{\cot(c + dx)}} + \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst}}{b} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{bd \sqrt{\cot(c + dx)}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst}}{b} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{bd \sqrt{\cot(c + dx)}} - \frac{\left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst}}{b} \\
&= \frac{B \sqrt{a + b \tan(c + dx)}}{bd \sqrt{\cot(c + dx)}} - \frac{\left( (iA - B) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst}}{b} \\
&= \frac{(2Ab - aB) \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{b^{3/2} d} \\
&= - \frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} d}
\end{aligned}$$

**Mathematica [A]**

time = 1.68, size = 354, normalized size = 1.33

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( -\sqrt{a} \sqrt{-a + b} \sqrt{a + b} (-2Ab + aB) \sinh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{1 + \frac{b \tan(c + dx)}{a}} + \sqrt{a} \sqrt{-a + b} \sqrt{a + b} (A + B) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{a + b \tan(c + dx)} + \sqrt{-a + b} \left( (-1)^{iA} (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{a + b \tan(c + dx)} + \sqrt{a + b} B \sqrt{\tan(c + dx)} (a + b \tan(c + dx)) \right) \right)}{\sqrt{-a + b} \sqrt{a + b} b^{3/2} d \sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*Sqrt[a + b\*Tan[c + d\*x]]), x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-(Sqrt[a]\*Sqrt[-a + I\*b]\*Sqrt[a + I\*b])\*(-2\*A\*b + a\*B)\*ArcSinh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a]]\*Sqrt[1 + (

$b \cdot \tan[c + d \cdot x]) / a]) + \text{Sqrt}[b] \cdot ((-1)^{(1/4)} \cdot \text{Sqrt}[a + I \cdot b] \cdot b \cdot (I \cdot A + B) \cdot \text{ArcTan}[( (-1)^{(1/4)} \cdot \text{Sqrt}[-a + I \cdot b] \cdot \text{Sqrt}[\text{Tan}[c + d \cdot x]]) / \text{Sqrt}[a + b \cdot \text{Tan}[c + d \cdot x]]] \cdot \text{Sqrt}[a + b \cdot \text{Tan}[c + d \cdot x]] + \text{Sqrt}[-a + I \cdot b] \cdot ((-1)^{(3/4)} \cdot b \cdot (A + I \cdot B) \cdot \text{ArcTan}[( (-1)^{(1/4)} \cdot \text{Sqrt}[a + I \cdot b] \cdot \text{Sqrt}[\text{Tan}[c + d \cdot x]]) / \text{Sqrt}[a + b \cdot \text{Tan}[c + d \cdot x]]] \cdot \text{Sqrt}[a + b \cdot \text{Tan}[c + d \cdot x]] + \text{Sqrt}[a + I \cdot b] \cdot B \cdot \text{Sqrt}[\text{Tan}[c + d \cdot x]] \cdot (a + b \cdot \text{Tan}[c + d \cdot x])])) / (\text{Sqrt}[-a + I \cdot b] \cdot \text{Sqrt}[a + I \cdot b] \cdot b^{(3/2)} \cdot d \cdot \text{Sqrt}[a + b \cdot \text{Tan}[c + d \cdot x]])$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 28.13, size = 21473, normalized size = 80.73

method	result	size
default	Expression too large to display	21473

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2)), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{a + b \tan(c + dx)} \cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**(3/2)), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} \sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)), x)
```

$$3.642 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=316

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} - (iA + B) \tanh^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{3/2}d}$$

```
[Out] -(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/2)/d-2/3*A*cot(d*x+c)^(3/2)/a/d/(a+b*tan(d*x+c))^(1/2)+2/3*b*(5*A*a^2*b+8*A*b^3-3*B*a^3-6*B*a*b^2)/a^3/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)+2/3*(4*A*b-3*B*a)*cot(d*x+c)^(1/2)/a^2/d/(a+b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.88, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(4Ab - 3aB) \sqrt{\cot(c + dx)}}{3a^2 d \sqrt{a + b \tan(c + dx)}} + \frac{2b(-3a^3 B + 5a^2 Ab - 6ab^2 B + 8Ab^3)}{3a^3 d (a^2 + b^2) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{(-B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} - \frac{(B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{3/2}} - \frac{2A \cot^2(c + dx)}{3ad \sqrt{a + b \tan(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

```
[Out] -(((I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a - b)^(3/2)*d) - ((I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/((I*a + b)^(3/2)*d) + (2*b*(5*a^2*A*b + 8*A*b^3 - 3*a^3*B - 6*a*b^2*B))/(3*a^3*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) + (2*(4*A*b - 3*a*B)*Sqrt[Cot[c + d*x]])/(3*a^2*d*Sqrt[a + b*Tan[c + d*x]]) - (2*A*Cot[c + d*x]^(3/2))/(3*a*d*Sqrt[a + b*Tan[c + d*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.))/((e_.) + (f_.)*(x_.)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 3730

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)^2], x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
```

```

f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))} dx \\
&= -\frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad \sqrt{a+b\tan(c+dx)}} - \frac{\left( 2\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))} \\
&= \frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} - \frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad \sqrt{a+b\tan(c+dx)}} + \frac{(4\sqrt{\cot(c+dx)})}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))} \\
&= \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} \\
&= \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}} \\
&= -\frac{(iA-B)\tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 2.74, size = 301, normalized size = 0.95

$$\frac{\left( \frac{\sqrt{\cot(c+dx)} \left( \frac{(a+b(A+B)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right) + (a+b(A+B)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} \right) \sqrt{\tan(c+dx)}}{a^2+b^2} + \frac{8Ab-6aB}{a\sqrt{a+b\tan(c+dx)}} - \frac{2A\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}} + \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)\tan(c+dx)}{a^2(a^2+b^2)\sqrt{a+b\tan(c+dx)}} \right)}{3ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (Sqrt[Cot[c + d\*x]]\*((3\*(-1)^(1/4))\*a\*(((a + I\*b)\*(I\*A + B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[-a + I\*b] + ((I\*a + b)\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d

$$\frac{\sqrt{x}}{\sqrt{a + b \tan[c + dx]}} \sqrt{a + I b} \sqrt{\tan[c + dx]} / (a^2 + b^2) + (8Ab - 6a^2B) / (a \sqrt{a + b \tan[c + dx]}) - (2A \cot[c + dx]) / \sqrt{a + b \tan[c + dx]} + (2b(5a^2Ab + 8A^2b^3 - 3a^3B - 6ab^2B) \tan[c + dx]) / (a^2(a^2 + b^2) \sqrt{a + b \tan[c + dx]}) / (3ad)$$

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 29.59, size = 19809, normalized size = 62.69

method	result	size
default	Expression too large to display	19809

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(3/2), x)
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(cot(d\*x+c)\*\*(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(5/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a  
ssumes constant sign by intervals (correct if the argument is real):Check [  
abs(si

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{5/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int((cot(c + d\*x)^(5/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x  
)

$$3.643 \quad \int \frac{\cot^3(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=256

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)} - (A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a-b)^(3/2)/d-(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a+b)^(3/2)/d-2\*b\*(A\*a^2+2\*A\*b^2-B\*a\*b)/a^2/(a^2+b^2)/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)-2\*A\*cot(d\*x+c)^(1/2)/a/d/(a+b\*tan(d\*x+c))^(1/2)

Rubi [A]

time = 0.64, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2b(a^2A-abB+2Ab^2)}{a^2d(a^2+b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{3/2}} - \frac{(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{3/2}} - \frac{2A\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a - b)^(3/2)\*d) - ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a + b)^(3/2)\*d) - (2\*b\*(a^2\*A + 2\*A\*b^2 - a\*b\*B))/(a^2\*(a^2 + b^2)\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]) - (2\*A\*Sqrt[Cot[c + d\*x]])/(a\*d\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))} dx \\
&= -\frac{2A \sqrt{\cot(c+dx)}}{ad \sqrt{a+b \tan(c+dx)}} - \frac{\left( 2 \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))} dx}{a} \\
&= -\frac{2b(a^2 A + 2Ab^2 - abB)}{a^2 (a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} - \frac{2A \sqrt{\cot(c+dx)}}{ad \sqrt{a+b \tan(c+dx)}} \\
&= -\frac{2b(a^2 A + 2Ab^2 - abB)}{a^2 (a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} - \frac{2A \sqrt{\cot(c+dx)}}{ad \sqrt{a+b \tan(c+dx)}} \\
&= -\frac{2b(a^2 A + 2Ab^2 - abB)}{a^2 (a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} - \frac{2A \sqrt{\cot(c+dx)}}{ad \sqrt{a+b \tan(c+dx)}} \\
&= -\frac{2b(a^2 A + 2Ab^2 - abB)}{a^2 (a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} - \frac{2A \sqrt{\cot(c+dx)}}{ad \sqrt{a+b \tan(c+dx)}} \\
&= \frac{(A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{(ia-b)^{3/2} d}
\end{aligned}$$

**Mathematica [A]**

time = 1.50, size = 255, normalized size = 1.00

$$\frac{\sqrt{\cot(c+dx)} \left( \frac{\sqrt{-1} a \left( \frac{(\alpha+ib)(A-ib) \operatorname{ArcTan}\left(\frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{(\alpha-ib)(A+ib) \operatorname{ArcTan}\left(\frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{a+ib}} \right)}{a^2+b^2} \right)}{\frac{2A}{\sqrt{a+b \tan(c+dx)}} - \frac{2b(a^2A+2Ab^2-abB) \tan(c+dx)}{a(a^2+b^2) \sqrt{a+b \tan(c+dx)}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (Sqrt[Cot[c + d\*x]]\*(((−1)^(1/4)\*a\*(((a + I\*b)\*(A − I\*B)\*ArcTan[(((−1)^(1/4)\*Sqrt[−a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])]/Sqrt[−a + I\*b] − ((a − I\*b)\*(A + I\*B)\*ArcTan[(((−1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])]/Sqrt[a + I\*b])\*Sqrt[Tan[c + d\*x]])/(a^2 + b^2) − (2\*A)/Sqrt[a + b\*Tan[c + d\*x]] − (2\*b\*(a^2\*A + 2\*A\*b^2 − a\*b\*B)\*Tan[c + d\*x])/(a\*(a^2 + b^2)\*Sqrt[a + b\*Tan[c + d\*x]])))/(a\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.14, size = 18997, normalized size = 74.21

method	result	size
default	Expression too large to display	18997

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, method=\_RETU  
RNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(b\*tan(d\*x + c) + a)^(3/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \cot^{\frac{3}{2}}(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2)/(a + b\*tan(c + d\*x))^(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(b\*tan(d\*x + c) + a)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2),x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(3/2), x)

$$3.644 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=215

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{(ia-b)^{3/2} d} + \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d(b+ia)^{3/2}}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a-b)^(3/2)/d+(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a+b)^(3/2)/d+2\*b\*(A\*b-B\*a)/a/(a^2+b^2)/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)

Rubi [A]

time = 0.47, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4326, 3690, 3697, 3696, 95, 209, 212}

$$\frac{2b(Ab - aB)}{ad(a^2 + b^2) \sqrt{\cot(c+dx)} \sqrt{a+b \tan(c+dx)}} + \frac{(-B + iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(-b+ia)^{3/2}} + \frac{(B + iA) \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2),x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/((I\*a - b)^(3/2)\*d) + ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]/((I\*a + b)^(3/2)\*d) + (2\*b\*(A\*b - a\*B))/(a\*(a^2 + b^2)\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3690

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3696

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (A + B \tan(c+dx))}{(a + b \tan(c+dx))^{3/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A + B \tan(c+dx)}{\sqrt{\tan(c+dx)} (a + b \tan(c+dx))^{3/2}} dx \\
&= \frac{2b(Ab - aB)}{a(a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a + b \tan(c+dx)}} + \frac{(2\sqrt{\cot(c+dx)})}{(a + ib)(A - B)} \\
&= \frac{2b(Ab - aB)}{a(a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a + b \tan(c+dx)}} + \frac{((a + ib)(A - B))}{((a + ib)(A - B))} \\
&= \frac{2b(Ab - aB)}{a(a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a + b \tan(c+dx)}} + \frac{((a + ib)(A - B))}{((a + ib)(A - B))} \\
&= \frac{2b(Ab - aB)}{a(a^2 + b^2) d \sqrt{\cot(c+dx)} \sqrt{a + b \tan(c+dx)}} + \frac{((a + ib)(A - B))}{((a + ib)(A - B))} \\
&= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c+dx)}}{\sqrt{a + b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{(a^2 + b^2) d}
\end{aligned}$$

**Mathematica [A]**

time = 0.81, size = 222, normalized size = 1.03

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( \frac{\sqrt{-1}^{-i(a+b)(A-iB)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} + \frac{\sqrt{-1}^{(a-ib)(-iA+B)} \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} + \frac{2b(Ab-aB) \sqrt{\tan(c+dx)}}{a \sqrt{a+b \tan(c+dx)}} \right)}{(a^2 + b^2) d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((( -1)^(1/4)*((-I)*a + b)*(A - I*B)*ArcTan[((( -1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (( -1)^(1/4)*(a - I*b)*((-I)*A + B)*ArcTan[((( -1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (2*b*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a*Sqrt[a + b*Tan[c + d*x]])))/((a^2 + b^2)*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 28.15, size = 9577, normalized size = 44.54

method	result	size
default	Expression too large to display	9577

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^(3/2), x)
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{(a + b \tan(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**(3/2), x)
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2), x)
```

$$3.645 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=210

$$\frac{(A+iB) \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} + (A-iB) \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{(ia-b)^{3/2} d}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/(I*a-b)^{(3/2)/d}+(A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/(I*a+b)^{(3/2)/d}-2*(A*b-B*a)/(a^2+b^2)/d/\cot(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}})$

Rubi [A]

time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {4326, 3689, 3697, 3696, 95, 209, 212}

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{\cot(c+dx)}\sqrt{a+b \tan(c+dx)}} - \frac{(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(-b+ia)^{3/2}} + \frac{(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d(b+ia)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[c+d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*(a+b*\operatorname{Tan}[c+d*x])^{(3/2)}),x]$

[Out]  $-\left(\frac{(A+I*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a-b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]/((I*a-b)^{(3/2)}*d)}{d} + \frac{(A-I*B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a+b]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]])/\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]]*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c+d*x]]/((I*a+b)^{(3/2)}*d)}{d} - \frac{2*(A*b-a*B)}{(a^2+b^2)*d*\operatorname{Sqrt}[\operatorname{Cot}[c+d*x]]*\operatorname{Sqrt}[a+b*\operatorname{Tan}[c+d*x]]}\right)$

Rule 95

$\operatorname{Int}[((a_.)+(b_.)*(x_.))^{(m_.)*((c_.)+(d_.)*(x_.))^{(n_.)}/((e_.)+(f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)/(b*e-a*f-(d*e-c*f)*x^q)}, x], x, (a+b*x)^{(1/q)/(c+d*x)^{(1/q)}}, x]] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[m+n+1, 0] \&\& \operatorname{RationalQ}[n] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{SimplerQ}[a+b*x, c+d*x]$

Rule 209

$\operatorname{Int}[(a_.)+(b_.)*(x_.^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]))*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

Rule 4326

Int[(cot[(a\_) + (b\_)\*(x\_)])\*(c\_)^(m\_)\*(u\_), x\_Symbol] := Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx \\
&= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{(2\sqrt{\cot(c + dx)})}{(a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{((ia + b)(A - B))}{(a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{((ia + b)(A - B))}{(a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{((ia + b)(A - B))}{(a + b \tan(c + dx))^{3/2}} \\
&= -\frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{((ia + b)(A - B))}{(a + b \tan(c + dx))^{3/2}} \\
&= -\frac{(A + iB) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)}}{(ia - b)^{3/2} d}
\end{aligned}$$

**Mathematica [A]**

time = 1.39, size = 259, normalized size = 1.23

$$\frac{\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \left( -\frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{-a + ib}} \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + \frac{\sqrt{-1} \sqrt{a - ib} \sqrt{\tan(c + dx)}}{\sqrt{a + ib}} \operatorname{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) + \frac{2\sqrt{Ab - aB} \tan^3(c + dx)}{\sqrt{a + b \tan(c + dx)}} + 2(-Ab + aB) \sqrt{\tan(c + dx)} \sqrt{a + b \tan(c + dx)} \right)}{a(a^2 + b^2) d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((( (-1)^(1/4)*a*(a + I*b)*(A - I*B)*ArcTan[ ((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]] ]/Sqrt[a + b*Tan[c + d*x]] ])/Sqrt[-a + I*b] + ((-1)^(1/4)*a*(a - I*b)*(A + I*B)*ArcTan[ ((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]] ]/Sqrt[a + b*Tan[c + d*x]] ])/Sqrt[a + I*b] + (2*b*(A*b - a*B)*Tan[c + d*x]^(3/2))/Sqrt[a + b*Tan[c + d*x]] + 2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] )/(a*(a^2 + b^2)*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 30.19, size = 9705, normalized size = 46.21

method	result	size
default	Expression too large to display	9705

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c))), x)
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}} \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*sqrt(cot(c + d*x))), x)
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(1/2)\*(a + b\*tan(c + d\*x))^(3/2)), x)



$$3.646 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=279

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + 2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{3/2}d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/(I*a-b)^{(3/2)/d+2*B*\arctanh(b^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/b^{(3/2)/d}}-(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}}*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)/(I*a+b)^{(3/2)/d+2*a*(A*b-B*a)/b/(a^2+b^2)/d/\cot(d*x+c)^{(1/2)/(a+b*\tan(d*x+c))^{(1/2)}}}$

**Rubi** [A]

time = 1.37, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4326, 3686, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{2a(Ab - aB)}{bd(a^2 + b^2) \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{(-B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} - \frac{(B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{3/2}} + \frac{2B \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{b^{3/2}d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Cot}[c + d*x]^{(3/2)}*(a + b*\operatorname{Tan}[c + d*x])^{(3/2)}), x]$

[Out]  $-(((I*A - B)*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/((I*a - b)^{(3/2)}*d) + (2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/(b^{(3/2)}*d) - ((I*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]])/((I*a + b)^{(3/2)}*d) + (2*a*(A*b - a*B))/(b*(a^2 + b^2)*d*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 95**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}]/((e_.) + (f_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}*(c + d*x)^{(n)}, x], x, (e + f*x)^{(1/q)}, x]]$

$- 1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x,  $(a + b*x)^{(1/q)/(c + d*x)^{(1/q)}$   
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]  
&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

#### Rule 3736

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)^2], x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*(c + d\*ff\*x)^n\*((A + B\*ff\*x + C\*ff^2\*x^2)/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,

$A, B, C, m, n, x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{NeQ}[a^2 + b^2, 0]$  &&  $\text{NeQ}[c^2 + d^2, 0]$

Rule 4326

$\text{Int}[(\cot[a + b*x] + (c + b*x)^m) * (c + b*x)^m * (u), x\_Symbol] \rightarrow \text{Dist}[(c + b*x)^m * (c + b*x)^m * \text{Int}[\text{ActivateTrig}[u] / (c + b*x)^m, x], x] /;$   $\text{FreeQ}\{a, b, c, m, x\}$  &&  $\text{!IntegerQ}[m]$  &&  $\text{KnownTangentIntegrandQ}[u, x]$

Rule 6857

$\text{Int}[u / (a + b*x^n), x\_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u / (a + b*x^n), x]\}, \text{Int}[v, x] /;$   $\text{SumQ}[v]$   $/;$   $\text{FreeQ}\{a, b, x\}$  &&  $\text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{3/2}} dx \\
&= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{(2\sqrt{\cot(c + dx)})}{(a + b \tan(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{(2\sqrt{\cot(c + dx)})}{(a + b \tan(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} + \frac{(2\sqrt{\cot(c + dx)})}{(a + b \tan(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{(\sqrt{\cot(c + dx)})}{(a + b \tan(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{(\sqrt{\cot(c + dx)})}{(a + b \tan(c + dx))^{3/2}} \\
&= \frac{2a(Ab - aB)}{b(a^2 + b^2) d \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{((ia + b)(A + B))}{(a + b \tan(c + dx))^{3/2}} \\
&= \frac{2B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{b^{3/2} d} \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \\
&= - \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{(ia - b)^{3/2} d} \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 39.00, size = 167374, normalized size = 599.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x]
```

[Out] Result too large to show

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 28.83, size = 21787, normalized size = 78.09

method	result	size
default	Expression too large to display	21787

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3/2)), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{3}{2}} \cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)`

[Out] Integral((A + B\*tan(c + d\*x))/((a + b\*tan(c + d\*x))\*\*(3/2)\*cot(c + d\*x)\*\*(3/2)), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [ abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + b \tan(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)), x)

$$3.647 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=399

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}+(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a-b)^(5/2)/d+(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a+b)^(5/2)/d+2/3\*b\*(8\*A\*a^4\*b+30\*A\*a^2\*b^3+16\*A\*b^5-3\*B\*a^5-17\*B\*a^3\*b^2-8\*B\*a\*b^4)/a^4/(a^2+b^2)^2/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)-2/3\*A\*cot(d\*x+c)^(3/2)/a/d/(a+b\*tan(d\*x+c))^(3/2)+2/3\*b\*(7\*A\*a^2\*b+8\*A\*b^3-3\*B\*a^3-4\*B\*a\*b^2)/a^3/(a^2+b^2)/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2)+2\*(2\*A\*b-B\*a)\*cot(d\*x+c)^(1/2)/a^2/d/(a+b\*tan(d\*x+c))^(3/2)

Rubi [A]

time = 1.16, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(2Ab-ab)\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx))^{5/2}} + \frac{2b(-3a^2B+7a^2Ab-4a^2B+8Ab^2)}{3a^2d(a+b^2)\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}} + \frac{2b(-3a^2B+8a^2Ab-17a^2B+30a^2Ab^2-8a^2B+16Ab^3)}{3a^2d(a+b^2)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}} + \frac{(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}} - \frac{2A\cot^3(c+dx)}{3ad(a+b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a - b)^(5/2)\*d) + ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a + b)^(5/2)\*d) + (2\*b\*(7\*a^2\*A\*b + 8\*A\*b^3 - 3\*a^3\*B - 4\*a\*b^2\*B))/(3\*a^3\*(a^2 + b^2)\*d\*Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*(2\*A\*b - a\*B)\*Sqrt[Cot[c + d\*x]])/(a^2\*d\*(a + b\*Tan[c + d\*x])^(3/2)) - (2\*A\*Cot[c + d\*x]^(3/2))/(3\*a\*d\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*b\*(8\*a^4\*A\*b + 30\*a^2\*A\*b^3 + 16\*A\*b^5 - 3\*a^5\*B - 17\*a^3\*b^2\*B - 8\*a\*b^4\*B))/(3\*a^4\*(a^2 + b^2)^2\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)]/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730



```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))} dx \\
&= -\frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad(a+b \tan(c+dx))^{3/2}} - \frac{\left( 2\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\left( 2\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)} \\
&= \frac{2(2Ab-aB) \sqrt{\cot(c+dx)}}{a^2d(a+b \tan(c+dx))^{3/2}} - \frac{2A \cot^{\frac{3}{2}}(c+dx)}{3ad(a+b \tan(c+dx))^{3/2}} + \frac{\left( 4\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)}{\left( 4\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right)} \\
&= \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2(2Ab-aB)}{a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2(2Ab-aB)}{a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2(2Ab-aB)}{a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2(2Ab-aB)}{a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2(2Ab-aB)}{a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2(2Ab-aB)}{a^2d(a+b \tan(c+dx))^{3/2}} \\
&= \frac{(A+iB) \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 2.66, size = 385, normalized size = 0.96

$$\frac{\sqrt{\cot(c+dx)} \left( \frac{6b(4b-3aB)}{3a^2(a+b \tan(c+dx))^{3/2}} - \frac{6A \cot(c+dx)}{(a+b \tan(c+dx))^{3/2}} + \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B) \tan(c+dx)}{a^2(a^2+b^2)(a+b \tan(c+dx))^{3/2}} + \frac{\sqrt{\tan(c+dx)} \left( \frac{\sqrt{-1}\sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{-a+ib}} \operatorname{ArcTan} \left( \frac{\sqrt{-1}\sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{-a+ib}} \right) + \frac{\sqrt{-1}\sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+ib}} \operatorname{ArcTan} \left( \frac{\sqrt{-1}\sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+ib}} \right) \right)}{a^2(a^2+b^2)} \right)}{9ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d\*x]^(5/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

```
[Out] (Sqrt[Cot[c + d*x]]*((6*(6*A*b - 3*a*B))/(a*(a + b*Tan[c + d*x])^(3/2)) - (
6*A*Cot[c + d*x])/(a + b*Tan[c + d*x])^(3/2) + (6*b*(7*a^2*A*b + 8*A*b^3 -
3*a^3*B - 4*a*b^2*B)*Tan[c + d*x])/(a^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^(3
/2)) + (Sqrt[Tan[c + d*x]]*(9*(-1)^(3/4)*a^4*((a + I*b)^2*(A - I*B)*ArcTan
[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/
Sqrt[-a + I*b] + ((a - I*b)^2*(A + I*B)*ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (6*b*(8*a^4*A
*b + 30*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B - 17*a^3*b^2*B - 8*a*b^4*B)*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]))/(a^3*(a^2 + b^2)^2))/(9*a*d)
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 29.98, size = 82067, normalized size = 205.68

method	result	size
default	Expression too large to display	82067

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(5/2
), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{5/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

```
[Out] int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2), x
)
```

$$3.648 \quad \int \frac{\cot^2(c+dx)(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=341

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + (iA + B) \tanh^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{5/2} d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a-b)^(5/2)/d-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a+b)^(5/2)/d-2/3\*b\*(3\*A\*a^4+17\*A\*a^2\*b^2+8\*A\*b^4-8\*B\*a^3\*b-2\*B\*a\*b^3)/a^3/(a^2+b^2)^2/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)-2/3\*b\*(3\*A\*a^2+4\*A\*b^2-B\*a\*b)/a^2/(a^2+b^2)/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2)-2\*A\*cot(d\*x+c)^(1/2)/a/d/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 0.87, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{\frac{2b(3a^2A - abB + 4Ab^2)}{3a^2d(a^2 + b^2)\sqrt{\cot(c + dx)}(a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^4A - 8a^2bB + 17a^2Ab^2 - 2ab^3B + 8Ab^4)}{3a^2d(a^2 + b^2)^2\sqrt{\cot(c + dx)}\sqrt{a + b \tan(c + dx)}} + \frac{(-B + iA)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\operatorname{ArcTan}\left(\frac{\sqrt{-b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} - \frac{(B + iA)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\tanh^{-1}\left(\frac{\sqrt{b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{3/2}} - \frac{2A\sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}}}{(ia - b)^{5/2} d}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d\*x]^(3/2)\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a - b)^(5/2)\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a + b)^(5/2)\*d) - (2\*b\*(3\*a^2\*A + 4\*A\*b^2 - a\*b\*B))/(3\*a^2\*(a^2 + b^2)\*d\*Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)) - (2\*A\*Sqrt[Cot[c + d\*x]])/(a\*d\*(a + b\*Tan[c + d\*x])^(3/2)) - (2\*b\*(3\*a^4\*A + 17\*a^2\*A\*b^2 + 8\*A\*b^4 - 8\*a^3\*b\*B - 2\*a\*b^3\*B))/(3\*a^3\*(a^2 + b^2)^2\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

### Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

### Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
```

```

f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

#### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

#### Rubi steps

$$\begin{aligned}
 \int \frac{\cot^{\frac{3}{2}}(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx \\
 &= -\frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} - \frac{\left( 2\sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))} dx}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{2b(3a^2 A + 4Ab^2 - abB)}{3a^2 (a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{2b(3a^2 A + 4Ab^2 - abB)}{3a^2 (a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{2b(3a^2 A + 4Ab^2 - abB)}{3a^2 (a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{2b(3a^2 A + 4Ab^2 - abB)}{3a^2 (a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{2b(3a^2 A + 4Ab^2 - abB)}{3a^2 (a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{2b(3a^2 A + 4Ab^2 - abB)}{3a^2 (a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2A \sqrt{\cot(c + dx)}}{ad(a + b \tan(c + dx))^{3/2}} \\
 &= -\frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} (a + ib)^2 d}
 \end{aligned}$$

**Mathematica [A]**

time = 2.69, size = 334, normalized size = 0.98

$$\left( \frac{\sqrt{\cot(c + dx)} \left( -\frac{6A}{(a + b \tan(c + dx))^{3/2}} - \frac{2b(3a^2 A + 4Ab^2 - abB) \tan(c + dx)}{a(a^2 + b^2)(a + b \tan(c + dx))^{3/2}} + \frac{\sqrt{\tan(c + dx)} \left( 3\sqrt{-1} a^2 \left( \frac{(-a + ib)^2 (A - iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) - \frac{(-a + ib)^2 (A + iB) \text{ArcTan} \left( \frac{\sqrt{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{-a + ib}} \right) - \frac{2b(3a^4 A + 17a^2 A b^2 + 5a^4 b^2 - 2a^2 b^2 - 2ab^2) \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{a^2 (a^2 + b^2)^2} \right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2), x]
```

```
[Out] (Sqrt[Cot[c + d*x]]*((-6*A)/(a + b*Tan[c + d*x])^(3/2) - (2*b*(3*a^2*A + 4*A*b^2 - a*b*B)*Tan[c + d*x])/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])^(3/2))) + (Sqrt[Tan[c + d*x]]*(3*(-1)^(1/4)*a^3*((a + I*b)^2*(A - I*B)*ArcTan[(-1)^(1/4)*a/(a + I*b)])))/(3*a*d)
```



$$\frac{1}{4} \sqrt{-a + I b} \sqrt{\tan[c + d x]} / \sqrt{a + b \tan[c + d x]} / \sqrt{-a + I b} - \left( (a - I b)^2 (A + I B) \operatorname{ArcTan} \left[ \frac{(-1)^{1/4} \sqrt{a + I b} \sqrt{\tan[c + d x]}}{\sqrt{a + b \tan[c + d x]}} \right] / \sqrt{a + I b} \right) - \frac{2 b (3 a^4 A + 17 a^2 A b^2 + 8 A b^4 - 8 a^3 b B - 2 a b^3 B) \sqrt{\tan[c + d x]} / \sqrt{a + b \tan[c + d x]}}{a^2 (a^2 + b^2)^2} / (3 a d)$$

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.54, size = 54581, normalized size = 160.06

method	result	size
default	Expression too large to display	54581

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,algorithm="maxima")`

[Out] `integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/2), x)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(3/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*cot(d\*x + c)^(3/2)/(b\*tan(d\*x + c) + a)^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c + dx)^{3/2} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int((cot(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2), x)

$$3.649 \quad \int \frac{\sqrt{\cot(c+dx)} (A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=287

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)} - (A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}$$

[Out]  $-(A+I*B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(I*a-b)^{(5/2)}/d - (A-I*B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(I*a+b)^{(5/2)}/d + 2/3*b*(8*A*a^2*b+2*A*b^3-5*B*a^3+B*a*b^2)/a^2/(a^2+b^2)^2/d/\cot(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)} + 2/3*b*(A*b-B*a)/a/(a^2+b^2)/d/\cot(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.71, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3690, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2b(Ab-aB)}{3ad(a^2+b^2)\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}} + \frac{2b(-5a^2B+8a^2Ab+a^2B+2Ab^2)}{3a^2d(a^2+b^2)^2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}} - \frac{(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out]  $-(((A+I*B)*\text{ArcTan}[(\text{Sqrt}[I*a-b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]])/((I*a-b)^{(5/2)}*d) - ((A-I*B)*\text{ArcTanh}[(\text{Sqrt}[I*a+b]*\text{Sqrt}[\text{Tan}[c+d*x]])]/\text{Sqrt}[a+b*\text{Tan}[c+d*x]])*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[\text{Tan}[c+d*x]])/((I*a+b)^{(5/2)}*d) + (2*b*(A*b-a*B))/(3*a*(a^2+b^2)*d*\text{Sqrt}[\text{Cot}[c+d*x]]*(a+b*\text{Tan}[c+d*x])^{(3/2)}) + (2*b*(8*a^2*A*b+2*A*b^3-5*a^3*B+a*b^2*B))/(3*a^2*(a^2+b^2)^2*d*\text{Sqrt}[\text{Cot}[c+d*x]]*\text{Sqrt}[a+b*\text{Tan}[c+d*x]])$

Rule 95

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m+1)-1)/(b\*e-a\*f-(d\*e-c\*f)\*x^q), x], x, (a+b\*x)^(1/q)/(c+d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m+n+1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a+b\*x, c+d\*x]

Rule 209

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3690

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*(n + 1)) + A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) - (A\*b - a\*B)\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f

```

*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}(A+B \tan(c+dx))}{(a+b \tan(c+dx))^{5/2}} dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{A+B \tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b \tan(c+dx))^{3/2}} dx \\
&= \frac{2b(Ab-aB)}{3a(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{(2\sqrt{\cot(c+dx)})}{3a^2(a^2+b^2)} \\
&= \frac{2b(Ab-aB)}{3a(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2}{3a^2(a^2+b^2)} \\
&= \frac{2b(Ab-aB)}{3a(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2}{3a^2(a^2+b^2)} \\
&= \frac{2b(Ab-aB)}{3a(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2}{3a^2(a^2+b^2)} \\
&= \frac{2b(Ab-aB)}{3a(a^2+b^2)d\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^{3/2}} + \frac{2}{3a^2(a^2+b^2)} \\
&= -\frac{(A+iB) \tan^{-1}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)}}{(ia-b)^{5/2}d}
\end{aligned}$$

**Mathematica [A]**

time = 2.39, size = 293, normalized size = 1.02

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( -3\sqrt{-1} \left( \frac{(a+ib)^2(A+B)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{i(a-b)^2(A+B)\text{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} \right) + \frac{2b(a^2+b^2)(A-b)\sqrt{\tan(c+dx)}}{a(a+b\tan(c+dx))^{3/2}} + \frac{2b(8a^2Ab+2A^3-5a^2B+a^2B^2)\sqrt{\tan(c+dx)}}{a^2\sqrt{a+b\tan(c+dx)}} \right)}{3(a^2+b^2)^2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cot[c + d\*x]]\*(A + B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(5/2), x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*(-3\*(-1)^(1/4)\*(((a + I\*b)^2\*(I\*A + B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[-a + I\*b] + (I\*(a - I\*b)^2\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[a + I\*b] + (2\*b\*(a^2 + b^2)\*(A\*b - a\*B)\*Sqrt[Tan[c + d\*x]])/(a\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*b\*(8\*a^2\*A\*b + 2\*A\*b^3 - 5\*a^3\*B + a\*b^2\*B)\*Sqrt[Tan[c + d\*x]])/(a^2\*Sqrt[a + b\*Tan[c + d\*x]])))/(3\*(a^2 + b^2)^2\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 32.16, size = 40999, normalized size = 142.85

method	result	size
default	Expression too large to display	40999

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*sqrt(cot(d\*x + c))/(b\*tan(d\*x + c) + a)^(5/2), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) \sqrt{\cot(c + dx)}}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*sqrt(cot(c + d\*x))/(a + b\*tan(c + d\*x))\*\*(5/2), x)

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^(1/2)\*(A+B\*tan(d\*x+c))/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cot(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2),x)

[Out] int((cot(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x)))/(a + b\*tan(c + d\*x))^(5/2), x)

$$3.650 \quad \int \frac{A+B \tan(c+dx)}{\sqrt{\cot(c+dx)} (a+b \tan(c+dx))^{5/2}} dx$$

Optimal. Leaf size=284

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + (iA + B) \tanh^{-1}\left(\frac{\sqrt{ia + b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{5/2} d}$$

[Out]  $-(I*A-B)*\arctan((I*a-b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(I*a-b)^{(5/2)}/d+(I*A+B)*\operatorname{arctanh}((I*a+b)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)})*\cot(d*x+c)^{(1/2)}*\tan(d*x+c)^{(1/2)}/(I*a+b)^{(5/2)}/d-2/3*(5*A*a^2*b-A*b^3-2*B*a^3+4*B*a*b^2)/a/(a^2+b^2)^2/d/\cot(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(1/2)}-2/3*(A*b-B*a)/(a^2+b^2)/d/\cot(d*x+c)^{(1/2)}/(a+b*\tan(d*x+c))^{(3/2)}$

Rubi [A]

time = 0.74, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3689, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2(Ab - aB)}{3d(a^2 + b^2) \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2(-2a^2B + 5a^2Ab + 4ab^2B - Ab^3)}{3ad(a^2 + b^2)^2 \sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} - \frac{(-B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(-b + ia)^{3/2}} + \frac{(B + iA) \sqrt{\tan(c + dx)} \sqrt{\cot(c + dx)} \tanh^{-1}\left(\frac{\sqrt{b + ia} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{d(b + ia)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[c + d*x])/(\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*(a + b*\operatorname{Tan}[c + d*x])^{(5/2)}), x]$

[Out]  $-\left(\left(\left(I*A - B\right)*\operatorname{ArcTan}\left[\left(\operatorname{Sqrt}\left[I*a - b\right]*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c + d*x\right]\right]\right)/\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right]\right)*\operatorname{Sqrt}\left[\operatorname{Cot}\left[c + d*x\right]\right]*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c + d*x\right]\right]/\left(\left(I*a - b\right)^{(5/2)}*d\right) + \left(\left(I*A + B\right)*\operatorname{ArcTanh}\left[\left(\operatorname{Sqrt}\left[I*a + b\right]*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c + d*x\right]\right]\right)/\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right]\right)*\operatorname{Sqrt}\left[\operatorname{Cot}\left[c + d*x\right]\right]*\operatorname{Sqrt}\left[\operatorname{Tan}\left[c + d*x\right]\right]/\left(\left(I*a + b\right)^{(5/2)}*d\right) - \left(2*(A*b - a*B)\right)/\left(3*(a^2 + b^2)*d*\operatorname{Sqrt}\left[\operatorname{Cot}\left[c + d*x\right]\right]*(a + b*\operatorname{Tan}\left[c + d*x\right])^{(3/2)}\right) - \left(2*(5*a^2*A*b - A*b^3 - 2*a^3*B + 4*a*b^2*B)\right)/\left(3*a*(a^2 + b^2)^2*d*\operatorname{Sqrt}\left[\operatorname{Cot}\left[c + d*x\right]\right]*\operatorname{Sqrt}\left[a + b*\operatorname{Tan}\left[c + d*x\right]\right]\right)$

Rule 95

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*(x_{.})^{\left(n_{.}\right)}\right)/\left(\left(e_{.}\right) + \left(f_{.}\right)*(x_{.})^{\left(p_{.}\right)}\right)\right), x_{\text{Symbol}}] \rightarrow \operatorname{With}\left[\left\{q = \operatorname{Denominator}\left[m_{.}\right]\right\}, \operatorname{Dist}\left[q, \operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(q*(m_{.} + 1) - 1\right)}/\left(b*e - a*f - \left(d*e - c*f\right)*x^q\right), x\right], x, \left(a + b*x\right)^{\left(1/q\right)}/\left(c + d*x\right)^{\left(1/q\right)}\right], x\right] \text{ ; FreeQ}\left[\left\{a, b, c, d, e, f\right\}, x\right] \&\& \operatorname{EqQ}\left[m_{.} + n_{.} + 1, 0\right] \&\& \operatorname{RationalQ}\left[n_{.}\right] \&\& \operatorname{LtQ}\left[-1, m_{.}, 0\right] \&\& \operatorname{SimplerQ}\left[a + b*x, c + d*x\right]$

Rule 209

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*(x_{.})^2\right)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}\left[\left(1/\left(\operatorname{Rt}\left[a, 2\right]*\operatorname{Rt}\left[b, 2\right]\right)\right)*\operatorname{ArcTan}\left[\operatorname{Rt}\left[b, 2\right]*(x/\operatorname{Rt}\left[a, 2\right])\right], x\right] \text{ ; FreeQ}\left[\left\{a, b\right\}, x\right] \&\& \operatorname{PosQ}\left[a/b\right] \&\& \left(\operatorname{GtQ}\left[a, 0\right] \&\& \operatorname{GtQ}\left[b, 0\right]\right)$



, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3689

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(A\*b - a\*B)\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^n/(f\*(m + 1)\*(a^2 + b^2))), x] + Dist[1/(b\*(m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[b\*B\*(b\*c\*(m + 1) + a\*d\*n) + A\*b\*(a\*c\*(m + 1) - b\*d\*n) - b\*(A\*(b\*c - a\*d) - B\*(a\*c + b\*d))\*(m + 1)\*Tan[e + f\*x] - b\*d\*(A\*b - a\*B)\*(m + n + 1)\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || IntegerQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(

```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{5/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx \\
&= -\frac{2(Ab - aB)}{3(a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{(2\sqrt{\cot(c + dx)})}{3a(a^2 + b^2)} \\
&= -\frac{2(Ab - aB)}{3(a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2\sqrt{\cot(c + dx)}}{3a(a^2 + b^2)} \\
&= -\frac{2(Ab - aB)}{3(a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2\sqrt{\cot(c + dx)}}{3a(a^2 + b^2)} \\
&= -\frac{2(Ab - aB)}{3(a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2\sqrt{\cot(c + dx)}}{3a(a^2 + b^2)} \\
&= -\frac{2(Ab - aB)}{3(a^2 + b^2) d \sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} - \frac{2\sqrt{\cot(c + dx)}}{3a(a^2 + b^2)} \\
&= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} (a + ib)^2 d}
\end{aligned}$$

**Mathematica [A]**

time = 2.92, size = 340, normalized size = 1.20

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( \frac{2b(Ab-aB)\tan^2(c+dx)}{(a+b\tan(c+dx))^2} + \frac{6b(2aAb-a^2B+b^2B)\tan^3(c+dx)}{(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{\sqrt{-1} \operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\sqrt{-1} \operatorname{ArcTan}\left(\frac{\sqrt{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}} + \frac{2(-2aAb+a^2B-b^2B)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{a^2+b^2} \right)}{3a(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(5/2)), x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*((2\*b\*(A\*b - a\*B)\*Tan[c + d\*x]^(3/2))/(a + b\*Tan[c + d\*x])^(3/2) + (6\*b\*(2\*a\*A\*b - a^2\*B + b^2\*B)\*Tan[c + d\*x]^(3/2))/((a^2 + b^2)\*Sqrt[a + b\*Tan[c + d\*x]]) + (3\*(-((-1)^(1/4)\*a\*(a + I\*b)^2\*(A - I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[-a + I\*b]) + ((-1)^(1/4)\*a\*(a - I\*b)^2\*(A + I\*B)\*ArcTan[(-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]])/Sqrt[a + I\*b] + 2\*(-2\*a\*A\*b + a^2\*B - b^2\*B)\*Sqrt[Tan[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]]/(a^2 + b^2))/(3\*a\*(a^2 + b^2)\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.22, size = 40915, normalized size = 144.07

method	result	size
default	Expression too large to display	40915

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(5/2), x, method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^(5/2)\*sqrt(cot(d\*x + c))), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{A + B \tan(c + dx)}{(a + b \tan(c + dx))^{\frac{5}{2}} \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(5/2)*sqrt(cot(c + d*x))), x)
```

**Giac [F(-1)]** Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)), x)
```

$$3.651 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=284

$$\frac{(A+iB)\text{ArcTan}\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}+(A-iB)\tanh^{-1}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d}$$

[Out] (A+I\*B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a-b)^(5/2)/d+(A-I\*B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a+b)^(5/2)/d+2/3\*(2\*A\*a^2\*b-4\*A\*b^3+B\*a^3+7\*B\*a\*b^2)/b/(a^2+b^2)^2/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)+2/3\*a\*(A\*b-B\*a)/b/(a^2+b^2)/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(3/2)

**Rubi** [A]

time = 0.73, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$ , Rules used = {4326, 3686, 3730, 3697, 3696, 95, 209, 212}

$$\frac{2a(Ab-aB)}{3bd(a^2+b^2)\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2B+2a^2Ab+7ab^2B-4Ab^3)}{3bd(a^2+b^2)^2\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{(A+iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\text{ArcTan}\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(-b+ia)^{5/2}} + \frac{(A-iB)\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\tanh^{-1}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d(b+ia)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(5/2)),x]

[Out] ((A + I\*B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a - b)^(5/2)\*d) + ((A - I\*B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a + b)^(5/2)\*d) + (2\*a\*(A\*b - a\*B))/(3\*b\*(a^2 + b^2)\*d\*Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*(2\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B + 7\*a\*b^2\*B))/(3\*b\*(a^2 + b^2)^2\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 95**

Int[(((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.))/((e\_.) + (f\_.)\*(x\_.)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

**Rule 209**

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 3686

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)\*(B\*c - A\*d)\*(a + b\*Tan[e + f\*x])^(m - 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(d\*f\*(n + 1)\*(c^2 + d^2))), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a\*A\*d\*(b\*d\*(m - 1) - a\*c\*(n + 1)) + (b\*B\*c - (A\*b + a\*B)\*d)\*(b\*c\*(m - 1) + a\*d\*(n + 1)) - d\*((a\*A - b\*B)\*(b\*c - a\*d) + (A\*b + a\*B)\*(a\*c + b\*d))\*(n + 1)\*Tan[e + f\*x] - b\*(d\*(A\*b\*c + a\*B\*c - a\*A\*d)\*(m + n) - b\*B\*(c^2\*(m - 1) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2\*m, 2\*n])

### Rule 3696

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[A^2/f, Subst[Int[(a + b\*x)^m\*((c + d\*x)^n/(A - B\*x)), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]

### Rule 3697

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[(A + I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(A - I\*B)/2, Int[(a + b\*Tan[e + f\*x])^m\*(c + d\*Tan[e + f\*x])^n\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]

### Rule 3730

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*((c + d\*Tan[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f



time = 2.46, size = 328, normalized size = 1.15

$$\frac{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \left( \frac{-3b \sqrt{\tan(c+dx)}}{(a+b \tan(c+dx))^{3/2}} + \frac{(2a^2b + a^2B + 3b^2B) \sqrt{\tan(c+dx)}}{(a^2 + b^2)(a+b \tan(c+dx))^{3/2}} + \frac{3\sqrt{-1} b \left( \frac{(a+ib)^2 (a+ib) \operatorname{ArcTan}\left(\frac{\sqrt{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{-a+ib}} \right) + \frac{(a-ib)^2 (a+ib) \operatorname{ArcTan}\left(\frac{\sqrt{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{a+ib}} \right) + \frac{2(a^2 ab - a^2 b^2 + a^2 b^2) \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(5/2)),x]

[Out] (Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]]\*((-3\*B\*Sqrt[Tan[c + d\*x]])/(a + b\*Tan[c + d\*x])^(3/2) + ((2\*a\*A\*b + a^2\*B + 3\*b^2\*B)\*Sqrt[Tan[c + d\*x]]/((a^2 + b^2)\*(a + b\*Tan[c + d\*x])^(3/2)) + (3\*(-1)^(1/4)\*b\*((a + I\*b)^2\*(I\*A + B)\*ArcTan[((-1)^(1/4)\*Sqrt[-a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[-a + I\*b] + (I\*(a - I\*b)^2\*(A + I\*B)\*ArcTan[((-1)^(1/4)\*Sqrt[a + I\*b]\*Sqrt[Tan[c + d\*x]]]/Sqrt[a + b\*Tan[c + d\*x]]])/Sqrt[a + I\*b]) + (2\*(2\*a^2\*A\*b - 4\*A\*b^3 + a^3\*B + 7\*a\*b^2\*B)\*Sqrt[Tan[c + d\*x]]/Sqrt[a + b\*Tan[c + d\*x]])/(a^2 + b^2)^2)/(3\*b\*d)

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 30.67, size = 40927, normalized size = 144.11

method	result	size
default	Expression too large to display	40927

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(5/2),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(3/2)), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-2)]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

**Giac [F(-2)]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{3/2} (a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)), x
)
```

$$3.652 \quad \int \frac{A+B \tan(c+dx)}{\cot^2(c+dx)(a+b \tan(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=342

$$\frac{(iA - B) \operatorname{ArcTan}\left(\frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} + 2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}}\right)}{(ia - b)^{5/2}d}$$

[Out] (I\*A-B)\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a-b)^(5/2)/d+2\*B\*arctanh(b^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/b^(5/2)/d-(I\*A+B)\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/(I\*a+b)^(5/2)/d+2\*a\*(2\*A\*b^3-a\*(a^2+3\*b^2)\*B)/b^2/(a^2+b^2)^2/d/cot(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2)+2/3\*a\*(A\*b-B\*a)/b/(a^2+b^2)/d/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2)

**Rubi [A]**

time = 1.78, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {4326, 3686, 3726, 3736, 6857, 65, 223, 212, 95, 211, 214}

$$\frac{2a(Ab - aB)}{3bd(a^2 + b^2)\cot^2(c + dx)(a + b\tan(c + dx))^{5/2}} + \frac{2a(2Ab^2 - aB(a^2 + 3b^2))}{b^2d(a^2 + b^2)^2\sqrt{\cot(c + dx)}\sqrt{a + b\tan(c + dx)}} + \frac{(-B + iA)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\operatorname{ArcTan}\left(\frac{\sqrt{-b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d(-b + ia)^{5/2}} - \frac{(B + iA)\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\tanh^{-1}\left(\frac{\sqrt{b + ia}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{d(b + ia)^{5/2}} + \frac{2B\sqrt{\tan(c + dx)}\sqrt{\cot(c + dx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\tan(c + dx)}}{\sqrt{a + b\tan(c + dx)}}\right)}{b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(5/2)),x]

[Out] ((I\*A - B)\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a - b)^(5/2)\*d) + (2\*B\*ArcTanh[(Sqrt[b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(b^(5/2)\*d) - ((I\*A + B)\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/((I\*a + b)^(5/2)\*d) + (2\*a\*(A\*b - a\*B))/(3\*b\*(a^2 + b^2)\*d\*Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^(3/2)) + (2\*a\*(2\*A\*b^3 - a\*(a^2 + 3\*b^2)\*B))/(b^2\*(a^2 + b^2)^2\*d\*Sqrt[Cot[c + d\*x]]\*Sqrt[a + b\*Tan[c + d\*x]])

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 95**

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

### Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
```

```
f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

### Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f,
A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b \tan(c + dx))^{5/2}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{5}{2}}(c + dx)(A + B \tan(c + dx))}{(a + b \tan(c + dx))^{5/2}} dx \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{(2\sqrt{\cot(c + dx)})}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2a(Ab - aB)}{3b(a^2 + b^2) d \cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} + \frac{2\sqrt{\cot(c + dx)}}{b^2(a^2 + b^2)^{5/2}} \\
&= \frac{2B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{b^{5/2} d} \\
&= \frac{(iA - B) \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} (a + ib)^2 d}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 31.85, size = 250233, normalized size = 731.68

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B\*Tan[c + d\*x])/(Cot[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^(5/2)),x]

[Out] Result too large to show

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 31.63, size = 77867, normalized size = 227.68

method	result	size
default	Expression too large to display	77867

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2),x,method=\_RETURNVERBOSE)

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)/((b\*tan(d\*x + c) + a)^(5/2)\*cot(d\*x + c)^(5/2)), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(5/2)/(a+b\*tan(d\*x+c))\*\*(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(d\*x+c))/cot(d\*x+c)^(5/2)/(a+b\*tan(d\*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \tan(c + dx)}{\cot(c + dx)^{5/2} (a + b \tan(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^(5/2)),x)

[Out] int((A + B\*tan(c + d\*x))/(cot(c + d\*x)^(5/2)\*(a + b\*tan(c + d\*x))^(5/2)), x )

$$3.653 \quad \int \frac{\sqrt{\cot(c+dx)} (aB+bB \tan(c+dx))}{(a+b \tan(c+dx))^{3/2}} dx$$

Optimal. Leaf size=151

$$\frac{B \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{ia-b} d} + \frac{B \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia+b} d}$$

[Out] B\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)+B\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a+b)^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$ , Rules used = {21, 4326, 3656, 926, 95, 211, 214}

$$\frac{B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{-b+ia}} + \frac{B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cot[c + d\*x]]\*(a\*B + b\*B\*Tan[c + d\*x]))/(a + b\*Tan[c + d\*x])^(3/2), x]

[Out] (B\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) + (B\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```



Rule 211

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_)))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 3656

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)} (aB + bB \tan(c+dx))}{(a + b \tan(c+dx))^{3/2}} dx &= B \int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a + b \tan(c+dx)}} dx \\
&= \left( B \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{1}{\sqrt{\tan(c+dx)} \sqrt{a+b}} \\
&= \frac{\left( B \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \text{Subst} \left( \int \frac{1}{\sqrt{x} \sqrt{a+bx}} (1+x) \right)}{d} \\
&= \frac{\left( B \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \text{Subst} \left( \int \left( \frac{i}{2(i-x)\sqrt{x} \sqrt{a+bx}} \right) \right)}{d} \\
&= \frac{\left( iB \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \text{Subst} \left( \int \frac{1}{(i-x)\sqrt{x} \sqrt{a+bx}} \right)}{2d} \\
&= \frac{\left( iB \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \text{Subst} \left( \int \frac{1}{i-(-a+ib)x^2} dx, x \right)}{d} \\
&= \frac{B \tan^{-1} \left( \frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{ia-b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.20, size = 145, normalized size = 0.96

$$\frac{(-1)^{3/4} B \left( -\frac{\text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{-a+ib}} - \frac{\text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a+ib} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}} \right)}{\sqrt{a+ib}} \right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cot[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2), x]
```

```
[Out] ((-1)^(3/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) - ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 31.08, size = 2085, normalized size = 13.81

method	result	size
default	Expression too large to display	2085

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -B/d/(I*a+(a^2+b^2)^(1/2)-b)/(I*a-(a^2+b^2)^(1/2)+b)/a*2^(1/2)*(cos(d*x+c)/
sin(d*x+c))^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))/cos(d*x+c))^(1/2)*(2*I*Ellip
ticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b
^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1
/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b*(a^2+b^2)^(1/2
)-I*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-
b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(
1/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^3-2*I*E
llipticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a
^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-
b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b^2-2*I*Elli
pticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b
),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b*(a^2+b^2)^(1
/2)+I*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)
/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2
)^(1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^3+2*
I*EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b
+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1
/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b^2-Elli
pticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),
1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*(a^2+b^2)^(1/
2)+EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-
b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1
/2)-b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b-Elli
pticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(1/2)+b
),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*(a^2+b^2)^(1
/2)+EllipticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-
b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a-(a^2+b^2)^(
1/2)+b),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2*b+2*
(a^2+b^2)^(1/2)*EllipticF((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(
d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)
^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2+4*(a^2+b^2)^(1/2)*EllipticF((-a*cos(d*
x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*
```

```

x+c))^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2-4
*EllipticF((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(
a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^
2)^(1/2))^(1/2))*a^2*b-4*EllipticF((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+
c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((-b+
(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^3*(a*(-1+cos(d*x+c)))/(-b+(a^2+b
^2)^(1/2))/sin(d*x+c))^(1/2)*((a*cos(d*x+c)+(a^2+b^2)^(1/2)*sin(d*x+c)+b*si
n(d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(-a*cos(d*x+c)-(a^2+b^2)^(1/
2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*sin(d*
x+c)^2/(-1+cos(d*x+c))/(a*cos(d*x+c)+b*sin(d*x+c))

```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, a
lgorithm="maxima")

```

```

[Out] integrate((B*b*tan(d*x + c) + B*a)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^(
3/2), x)

```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, a
lgorithm="fricas")

```

```

[Out] Timed out

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\sqrt{\cot(c + dx)}}{\sqrt{a + b \tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cot(d*x+c)**(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)

```

```

[Out] B*Integral(sqrt(cot(c + d*x))/sqrt(a + b*tan(c + d*x)), x)

```

**Giac [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a*B+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(si
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cot(c+dx)} (Ba + Bb \tan(c+dx))}{(a + b \tan(c+dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

```
[Out] int((cot(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2), x)
```

$$3.654 \quad \int \frac{aB + bB \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} dx$$

Optimal. Leaf size=157

$$\frac{iB \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} - iB \tanh^{-1}\left(\frac{\sqrt{ia+b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{ia-b} d \sqrt{ia+b}}$$

[Out] I\*B\*arctan((I\*a-b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a-b)^(1/2)-I\*B\*arctanh((I\*a+b)^(1/2)\*tan(d\*x+c)^(1/2)/(a+b\*tan(d\*x+c))^(1/2))\*cot(d\*x+c)^(1/2)\*tan(d\*x+c)^(1/2)/d/(I\*a+b)^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$ , Rules used = {21, 4326, 3656, 924, 95, 211, 214}

$$\frac{iB \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{-b+ia}} - \frac{iB \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In] Int[(a\*B + b\*B\*Tan[c + d\*x])/(Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^(3/2)), x]

[Out] (I\*B\*ArcTan[(Sqrt[I\*a - b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a - b]\*d) - (I\*B\*ArcTanh[(Sqrt[I\*a + b]\*Sqrt[Tan[c + d\*x]])/Sqrt[a + b\*Tan[c + d\*x]]]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])/(Sqrt[I\*a + b]\*d)

Rule 21

```
Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 924

Int[((d\_) + (e\_)\*(x\_))^(m\_)/(Sqrt[(f\_) + (g\_)\*(x\_)]\*((a\_) + (c\_)\*(x\_)^2)), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]), (d + e\*x)^(m + 1/2)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m + 1/2, 0]

Rule 3656

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[ff/f, Subst[Int[(a + b\*ff\*x)^m\*((c + d\*ff\*x)^n/(1 + ff^2\*x^2)), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 4326

Int[(cot[(a\_) + (b\_)\*(x\_)]\*(c\_))^(m\_)\*(u\_), x\_Symbol] := Dist[(c\*Cot[a + b\*x])^m\*(c\*Tan[a + b\*x])^m, Int[ActivateTrig[u]/(c\*Tan[a + b\*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\sqrt{\cot(c + dx)} (a + b \tan(c + dx))^{3/2}} dx &= B \int \frac{1}{\sqrt{\cot(c + dx)} \sqrt{a + b \tan(c + dx)}} dx \\
&= \left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{\sqrt{x}}{\sqrt{a + bx} (1+x^2)} dx \right)}{d} \\
&= \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \left( -\frac{1}{2(i-x)\sqrt{x} \sqrt{a - bx}} \right) dx \right)}{d} \\
&= -\frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{1}{(i-x)\sqrt{x} \sqrt{a + bx}} dx \right)}{2d} \\
&= \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{1}{i - (-a+ib)x^2} dx, x, \frac{1}{\sqrt{a + b \tan(c + dx)}} \right)}{d} \\
&= \frac{iB \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} d}
\end{aligned}$$

**Mathematica [A]**

time = 0.13, size = 144, normalized size = 0.92

$$\frac{\sqrt[4]{-1} B \left( -\frac{\text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{-a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{-a + ib}} + \frac{\text{ArcTan} \left( \frac{\sqrt[4]{-1} \sqrt{a + ib} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right)}{\sqrt{a + ib}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)), x]
```

```
[Out] ((-1)^(1/4)*B*(-(ArcTan[(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]]]/Sqrt[a + b*Tan[c + d*x]]/Sqrt[-a + I*b]) + ArcTan[(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]/Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 31.19, size = 1647, normalized size = 10.49



method	result	size
default	Expression too large to display	1647

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,method=_
RETURNVERBOSE)
```

```
[Out] B/d/(-I*a+(a^2+b^2)^(1/2)-b)/(I*a+(a^2+b^2)^(1/2)-b)*(I*EllipticPi((-a*cos
(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin
(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((
-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*(a^2+b^2)^(1/2)-I*EllipticPi(
(-(a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1
/2))/sin(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(
1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*(a^2+b^2)^(1/2)-I*Elli
pticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b)
,1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b+I*EllipticPi
((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(
1/2))/sin(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(
1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a*b-2*EllipticPi((-a*co
s(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/s
in(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*
((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b*(a^2+b^2)^(1/2)-2*EllipticP
i((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(
1/2))/sin(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(
1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b*(a^2+b^2)^(1/2)+Elli
pticPi((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+
b^2)^(1/2))/sin(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b)
,1/2*2^(1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2+2*EllipticPi
((-a*cos(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(
1/2))/sin(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(-I*a+(a^2+b^2)^(1/2)-b),1/2*2^(
1/2)*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2+EllipticPi((-a*co
s(d*x+c)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/si
n(d*x+c)^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((
-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*a^2+2*EllipticPi((-a*cos(d*x+c)
)-(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c
)^(1/2),(-b+(a^2+b^2)^(1/2))/(I*a+(a^2+b^2)^(1/2)-b),1/2*2^(1/2)*((-b+(a^2
+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))*b^2*cos(d*x+c)*sin(d*x+c)*((a*cos(d*x
+c)+b*sin(d*x+c))/cos(d*x+c)^(1/2)*2^(1/2)*(a*(-1+cos(d*x+c)))/(-b+(a^2+b^2
)^(1/2))/sin(d*x+c)^(1/2)*((a*cos(d*x+c)+(a^2+b^2)^(1/2)*sin(d*x+c)+b*sin(
d*x+c)-a)/(a^2+b^2)^(1/2)/sin(d*x+c)^(1/2)*(-(a*cos(d*x+c)-(a^2+b^2)^(1/2)
)*sin(d*x+c)+b*sin(d*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d*x+c)^(1/2)/(a*cos(d
*x+c)+b*sin(d*x+c))/(-1+cos(d*x+c))/(cos(d*x+c)/sin(d*x+c))^(1/2)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c))), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{a + b \tan(c + dx)} \sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(1/(sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x))), x)
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B a + B b \tan(c + d x)}{\sqrt{\cot(c + d x)} (a + b \tan(c + d x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),x)
```

```
[Out] int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)), x)
```

$$3.655 \quad \int \frac{aB + bB \tan(c + dx)}{\cot^2(c + dx)(a + b \tan(c + dx))^{3/2}} dx$$

**Optimal.** Leaf size=215

$$\frac{B \operatorname{ArcTan}\left(\frac{\sqrt{ia-b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}}{\sqrt{ia-b} d} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b} d}$$

[Out]  $-B \arctan\left(\frac{(I*a-b)^{(1/2)} \tan(d*x+c)^{(1/2)} / (a+b \tan(d*x+c))^{(1/2)} \cot(d*x+c)^{(1/2)} \tan(d*x+c)^{(1/2)} / d}{(I*a-b)^{(1/2)} + 2*B \operatorname{arctanh}\left(\frac{b^{(1/2)} \tan(d*x+c)^{(1/2)}}{(a+b \tan(d*x+c))^{(1/2)} \cot(d*x+c)^{(1/2)} \tan(d*x+c)^{(1/2)} / d}\right) - B \operatorname{arctanh}\left(\frac{(I*a+b)^{(1/2)} \tan(d*x+c)^{(1/2)} / (a+b \tan(d*x+c))^{(1/2)} \cot(d*x+c)^{(1/2)} \tan(d*x+c)^{(1/2)} / d}{(I*a+b)^{(1/2)} }\right)}$

**Rubi [A]**

time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$ , Rules used = {21, 4326, 3656, 924, 65, 223, 212, 926, 95, 211, 214}

$$\frac{B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \operatorname{ArcTan}\left(\frac{\sqrt{-b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{-b+ia}} + \frac{2B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{\sqrt{b} d} - \frac{B \sqrt{\tan(c+dx)} \sqrt{\cot(c+dx)} \tanh^{-1}\left(\frac{\sqrt{b+ia} \sqrt{\tan(c+dx)}}{\sqrt{a+b \tan(c+dx)}}\right)}{d \sqrt{b+ia}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a*B + b*B*\operatorname{Tan}[c + d*x]) / (\operatorname{Cot}[c + d*x]^{(3/2)} * (a + b*\operatorname{Tan}[c + d*x])^{(3/2)}), x]$

[Out]  $-(B*\operatorname{ArcTan}[(\operatorname{Sqrt}[I*a - b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] / (\operatorname{Sqrt}[I*a - b]*d) + (2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] / (\operatorname{Sqrt}[b]*d) - (B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[I*a + b]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]]) / \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]])*\operatorname{Sqrt}[\operatorname{Cot}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Tan}[c + d*x]] / (\operatorname{Sqrt}[I*a + b]*d)$

**Rule 21**

$\operatorname{Int}[(u_*)*((a_*) + (b_*)*(v_))^{(m_*)}*((c_*) + (d_*)*(v_))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifyQ[c + d\*x, a + b\*x])

**Rule 65**

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 95

Int[(((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 924

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))/(Sqrt[(f\_.) + (g\_.)\*(x\_)^(n\_)]\*((a\_.) + (c\_.)\*(x\_)^2)), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]), (d + e\*x)^(m + 1/2)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m + 1/2, 0]

### Rule 926

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_)^(n\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)^n, 1/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[m] && !IntegerQ[n]

### Rule 3656

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{aB + bB \tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^{3/2}} dx &= B \int \frac{1}{\cot^{\frac{3}{2}}(c + dx) \sqrt{a + b \tan(c + dx)}} dx \\
&= \left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \frac{\tan^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx \\
&= \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{x^{3/2}}{\sqrt{a + bx} (1+x^2)} dx, x \right)}{d} \\
&= \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \left( \frac{1}{\sqrt{x} \sqrt{a + bx}} - \frac{x}{\sqrt{a + bx}} \right) dx, x \right)}{d} \\
&= \frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{1}{\sqrt{x} \sqrt{a + bx}} dx, x \right)}{d} \\
&= -\frac{\left( B \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \left( \frac{i}{2(i-x)\sqrt{x} \sqrt{a + bx}} - \frac{1}{\sqrt{x} \sqrt{a + bx}} \right) dx, x \right)}{d} \\
&= -\frac{\left( iB \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{1}{(i-x)\sqrt{x} \sqrt{a + bx}} dx, x \right)}{2d} \\
&= \frac{2B \tanh^{-1} \left( \frac{\sqrt{b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{b} d} \\
&= -\frac{B \tan^{-1} \left( \frac{\sqrt{ia - b} \sqrt{\tan(c + dx)}}{\sqrt{a + b \tan(c + dx)}} \right) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)}}{\sqrt{ia - b} d}
\end{aligned}$$







+c)-(a^2+b^2)^(1/2)\*sin(d\*x+c)+b\*sin(d\*x+c)-a)/(-b+(a^2+b^2)^(1/2))/sin(d\*x+c))^(1/2), (-b+(a^2+b^2)^(1/2))/(-b+(a^2+b^2)^(1/2)+a), 1/2\*2^(1/2)\*((-b+(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2))^(1/2))\*a\*b^3-4\*EllipticPi((-a\*cos(d\*x+c)-(a^2+b^2)^(1/2)\*sin(d\*x+c)+b\*sin(d\*x+c)-a)/(-b+(a...

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] Timed out

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sqrt{a + b \tan(c + dx)} \cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2)/(a+b\*tan(d\*x+c))\*\*(3/2), x)

[Out] B\*Integral(1/(sqrt(a + b\*tan(c + d\*x))\*cot(c + d\*x)\*\*(3/2)), x)

**Giac** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*B+b\*B\*tan(d\*x+c))/cot(d\*x+c)^(3/2)/(a+b\*tan(d\*x+c))^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,sageVARx);OUTPUT:Warning, integration of abs or sign a  
 ssumes constant sign by intervals (correct if the argument is real):Check [  
 abs(si

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B a + B b \tan(c + d x)}{\cot(c + d x)^{3/2} (a + b \tan(c + d x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B\*a + B\*b\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)),x)

[Out] int((B\*a + B\*b\*tan(c + d\*x))/(cot(c + d\*x)^(3/2)\*(a + b\*tan(c + d\*x))^(3/2)), x)

$$3.656 \quad \int \cot^m(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

Optimal. Leaf size=195

$$\frac{(A+iB)F_1\left(1-m; -n, 1; 2-m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right) \cot^{-1+m}(c+dx)(a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)}{2d(1-m)}$$

[Out] 1/2\*(A+I\*B)\*AppellF1(1-m,1,-n,2-m,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*cot(d\*x+c)^(-1+m)\*(a+b\*tan(d\*x+c))^n/d/(1-m)/((1+b\*tan(d\*x+c)/a)^n)+1/2\*(A-I\*B)\*AppellF1(1-m,1,-n,2-m,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*cot(d\*x+c)^(-1+m)\*(a+b\*tan(d\*x+c))^n/d/(1-m)/((1+b\*tan(d\*x+c)/a)^n)

Rubi [A]

time = 0.30, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {4326, 3684, 3683, 140, 138}

$$\frac{(A+iB) \cot^{m-1}(c+dx)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(1-m; -n, 1; 2-m; -\frac{b \tan(c+dx)}{a}, -i \tan(c+dx)\right)}{2d(1-m)} + \frac{(A-iB) \cot^{m-1}(c+dx)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(1-m; -n, 1; 2-m; -\frac{b \tan(c+dx)}{a}, i \tan(c+dx)\right)}{2d(1-m)}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A + I\*B)\*AppellF1[1 - m, -n, 1, 2 - m, -((b\*Tan[c + d\*x])/a), (-I)\*Tan[c + d\*x]]\*Cot[c + d\*x]^(-1 + m)\*(a + b\*Tan[c + d\*x])^n)/(2\*d\*(1 - m)\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[1 - m, -n, 1, 2 - m, -((b\*Tan[c + d\*x])/a), I\*Tan[c + d\*x]]\*Cot[c + d\*x]^(-1 + m)\*(a + b\*Tan[c + d\*x])^n)/(2\*d\*(1 - m)\*(1 + (b\*Tan[c + d\*x])/a)^n)

Rule 138

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_ Symbol] :> Simp[c^n\*e^p\*((b\*x)^(m + 1)/(b\*(m + 1)))\*AppellF1[m + 1, -n, -p, m + 2, (-d)\*(x/c), (-f)\*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 140

Int[((b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_)\*((e\_) + (f\_.)\*(x\_))^(p\_), x\_ Symbol] :> Dist[c^IntPart[n]\*((c + d\*x)^FracPart[n]/(1 + d\*(x/c))^FracPart[n]), Int[(b\*x)^m\*(1 + d\*(x/c))^n\*(e + f\*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

#### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

#### Rule 4326

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]
```

#### Rubi steps

$$\begin{aligned}
\int \cot^m(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx &= (\cot^m(c + dx) \tan^m(c + dx)) \int \tan^{-m}(c + dx) \\
&= \frac{1}{2}((A - iB) \cot^m(c + dx) \tan^m(c + dx)) \int (1 - \\
&= \frac{((A - iB) \cot^m(c + dx) \tan^m(c + dx)) \operatorname{Subst}\left(\right)}{2d} \\
&= \frac{\left( (A - iB) \cot^m(c + dx) \tan^m(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) \right)}{2d} \\
&= \frac{(A + iB) F_1\left(1 - m; -n, 1; 2 - m; -\frac{b \tan(c + dx)}{a}\right)}{2d}
\end{aligned}$$

#### Mathematica [F]

time = 4.85, size = 0, normalized size = 0.00

$$\int \cot^m(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Cot[c + d\*x]^m\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple** [F]

time = 0.52, size = 0, normalized size = 0.00

$$\int (\cot^m(dx + c)) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(cot(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^m, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)^m\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*cot(d\*x + c)^m, x)

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*x+c)\*\*m\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^m, x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^m (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)`

[Out] `int(cot(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)`

$$3.657 \quad \int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=169

$$\frac{(A+iB)F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)}{d}$$

[Out]  $-(A+I*B)*\text{AppellF1}(-1/2, 1, -n, 1/2, -I*\tan(d*x+c), -b*\tan(d*x+c)/a)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^n/d/((1+b*\tan(d*x+c)/a)^n)-(A-I*B)*\text{AppellF1}(-1/2, 1, -n, 1/2, I*\tan(d*x+c), -b*\tan(d*x+c)/a)*\cot(d*x+c)^{(1/2)}*(a+b*\tan(d*x+c))^n/d/((1+b*\tan(d*x+c)/a)^n)$

**Rubi [A]**

time = 0.31, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {4326, 3684, 3683, 129, 525, 524}

$$\frac{(A+iB)\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{d} - \frac{(A-iB)\sqrt{\cot(c+dx)}(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[c + d*x]^{(3/2)}*(a + b*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x]), x]$

[Out]  $-\left(\left(\left(A + I*B\right)*\text{AppellF1}\left[-1/2, 1, -n, 1/2, \left(-I\right)*\text{Tan}\left[c + d*x\right], -\left(\left(b*\text{Tan}\left[c + d*x\right]\right)/a\right]\right)*\text{Sqrt}\left[\text{Cot}\left[c + d*x\right]\right]*\left(a + b*\text{Tan}\left[c + d*x\right]\right)^n/\left(d*\left(1 + \left(b*\text{Tan}\left[c + d*x\right]\right)/a\right)^n\right) - \left(\left(A - I*B\right)*\text{AppellF1}\left[-1/2, 1, -n, 1/2, I*\text{Tan}\left[c + d*x\right], -\left(\left(b*\text{Tan}\left[c + d*x\right]\right)/a\right]\right)*\text{Sqrt}\left[\text{Cot}\left[c + d*x\right]\right]*\left(a + b*\text{Tan}\left[c + d*x\right]\right)^n/\left(d*\left(1 + \left(b*\text{Tan}\left[c + d*x\right]\right)/a\right)^n\right)$

**Rule 129**

$\text{Int}\left[\left(\left(e_{.}\right)*\left(x_{.}\right)\right)^{\left(p_{.}\right)}*\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}, x_{\text{Symbol}}\right] :> \text{With}\left[\left\{k = \text{Denominator}\left[p\right]\right\}, \text{Dist}\left[k/e, \text{Subst}\left[\text{Int}\left[x^{\left(k*\left(p + 1\right) - 1\right)}*\left(a + b*\left(x^k/e\right)\right)^m*\left(c + d*\left(x^k/e\right)\right)^n, x\right], x, \left(e*x\right)^{\left(1/k\right)}, x\right] \right]; \text{FreeQ}\left[\{a, b, c, d, e, m, n\}, x\right] \&\& \text{NeQ}\left[b*c - a*d, 0\right] \&\& \text{FractionQ}\left[p\right] \&\& \text{IntegerQ}\left[m\right]$

**Rule 524**

$\text{Int}\left[\left(\left(e_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}\right]^{\left(p_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}\right]^{\left(q_{.}\right)}, x_{\text{Symbol}}\right] :> \text{Simp}\left[a^p*c^q*\left(\left(e*x\right)^{\left(m + 1\right)}/\left(e*\left(m + 1\right)\right)\right)*\text{AppellF1}\left[\left(m + 1\right)/n, -p, -q, 1 + \left(m + 1\right)/n, \left(-b\right)*\left(x^n/a\right), \left(-d\right)*\left(x^n/c\right)\right], x\right]; \text{FreeQ}\left[\{a, b, c, d, e, m, n, p, q\}, x\right] \&\& \text{NeQ}\left[b*c - a*d, 0\right] \&\& \text{NeQ}\left[m, -1\right] \&\& \text{NeQ}\left[m, n - 1\right] \&\& \left(\text{IntegerQ}\left[p\right] \mid\mid \text{GtQ}\left[a, 0\right]\right) \&\& \left(\text{IntegerQ}\left[q\right] \mid\mid \text{GtQ}\left[c, 0\right]\right)$

**Rule 525**

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a)^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

### Rule 3683

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && EqQ[A^2 + B^2, 0]

```

### Rule 3684

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)])*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

### Rubi steps



$$\begin{aligned}
\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \\
&= \frac{1}{2} \left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{2d} \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{d} \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) (a+b \tan(c+dx))^n (A+B \tan(c+dx))}{d} \\
&= - \frac{(A+iB) F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; -i \tan(c+dx)\right)}{d}
\end{aligned}$$

**Mathematica [F]**

time = 5.14, size = 0, normalized size = 0.00

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

[Out] Integrate[Cot[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.53, size = 0, normalized size = 0.00

$$\int \left( \cot^{\frac{3}{2}}(dx+c) \right) (a+b \tan(dx+c))^n (A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

[Out] int(cot(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)), x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x
)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*cot(d*x + c)*tan(d*x + c) + A*cot(d*x + c))*(b*tan(d*x + c) + a
)^n*sqrt(cot(d*x + c)), x)
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2), x
)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

$$3.658 \quad \int \sqrt{\cot(c+dx)} (a+b\tan(c+dx))^n (A+B\tan(c+dx)) dx$$

**Optimal.** Leaf size=167

$$\frac{(A+iB)F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right) (a+b\tan(c+dx))^n \left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\cot(c+dx)}} + \frac{(A-iB)F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right) (a+b\tan(c+dx))^n \left(1+\frac{b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\cot(c+dx)}}$$

[Out] (A+I\*B)\*AppellF1(1/2, 1, -n, 3/2, -I\*tan(d\*x+c), -b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^n/d/cot(d\*x+c)^(1/2)/((1+b\*tan(d\*x+c)/a)^n)+(A-I\*B)\*AppellF1(1/2, 1, -n, 3/2, I\*tan(d\*x+c), -b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^n/d/cot(d\*x+c)^(1/2)/((1+b\*tan(d\*x+c)/a)^n)

**Rubi** [A]

time = 0.29, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {4326, 3684, 3683, 129, 441, 440}

$$\frac{(A+iB)(a+b\tan(c+dx))^n \left(\frac{b\tan(c+dx)}{a}+1\right)^{-n} F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; -i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)}{d\sqrt{\cot(c+dx)}} + \frac{(A-iB)(a+b\tan(c+dx))^n \left(\frac{b\tan(c+dx)}{a}+1\right)^{-n} F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; i\tan(c+dx), -\frac{b\tan(c+dx)}{a}\right)}{d\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A + I\*B)\*AppellF1[1/2, 1, -n, 3/2, (-I)\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*(a + b\*Tan[c + d\*x])^n/(d\*Sqrt[Cot[c + d\*x]]\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[1/2, 1, -n, 3/2, I\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*(a + b\*Tan[c + d\*x])^n/(d\*Sqrt[Cot[c + d\*x]]\*(1 + (b\*Tan[c + d\*x])/a)^n)

Rule 129

Int[((e\_.)\*(x\_))^(p\_)\*((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

Rule 440

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 441

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q},
  x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 3683

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dis
t[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]
], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n]
&& EqQ[A^2 + B^2, 0]
```

### Rule 3684

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &&
!IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

### Rule 4326

```
Int[(cot[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x
]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^n (A+B \tan(c+dx)) dx &= \left( \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \\
&= \frac{1}{2} \left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{2d} \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{d} \\
&= \frac{\left( (A-iB) \sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)} \right) \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)} \sqrt{\tan(c+dx)}} dx}{d} \\
&= \frac{(A+iB) F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; -i \tan(c+dx)\right)}{d \sqrt{\cot(c+dx)}}
\end{aligned}$$

**Mathematica [F]**

time = 4.53, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(c+dx)} (a+b \tan(c+dx))^n (A+B \tan(c+dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Sqrt[Cot[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x]

**Maple [F]**

time = 0.56, size = 0, normalized size = 0.00

$$\int \left( \sqrt{\cot(dx+c)} \right) (a+b \tan(dx+c))^n (A+B \tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(cot(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x
)
```

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \sqrt{\cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*sqrt(cot(c + d*x)), x
)
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x
)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cot(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

$$3.659 \quad \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\cot(c+dx)}} dx$$

**Optimal.** Leaf size=173

$$\frac{(A+iB)F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n}}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{(A-iB)F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n}}{3d \cot^{\frac{3}{2}}(c+dx)}$$

[Out] 1/3\*(A+I\*B)\*AppellF1(3/2,1,-n,5/2,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^n/d/cot(d\*x+c)^(3/2)/((1+b\*tan(d\*x+c)/a)^n)+1/3\*(A-I\*B)\*AppellF1(3/2,1,-n,5/2,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^n/d/cot(d\*x+c)^(3/2)/((1+b\*tan(d\*x+c)/a)^n)

**Rubi** [A]

time = 0.30, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {4326, 3684, 3683, 129, 525, 524}

$$\frac{(A+iB)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{3d \cot^{\frac{3}{2}}(c+dx)} + \frac{(A-iB)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{3d \cot^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Cot[c + d\*x]],x]

[Out] ((A + I\*B)\*AppellF1[3/2, 1, -n, 5/2, (-I)\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*(a + b\*Tan[c + d\*x])^n)/(3\*d\*Cot[c + d\*x]^(3/2)\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[3/2, 1, -n, 5/2, I\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*(a + b\*Tan[c + d\*x])^n)/(3\*d\*Cot[c + d\*x]^(3/2)\*(1 + (b\*Tan[c + d\*x])/a)^n)

**Rule 129**

Int[((e.\_)\*(x.\_))^(p.\_)\*((a.\_) + (b.\_)\*(x.\_))^(m.\_)\*((c.\_) + (d.\_)\*(x.\_))^(n.\_), x\_Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

**Rule 524**

Int[((e.\_)\*(x.\_))^(m.\_)\*((a.\_) + (b.\_)\*(x.\_))^(n.\_))^(p.\_)\*((c.\_) + (d.\_)\*(x.\_))^(n.\_))^(q.\_), x\_Symbol] :> Simp[a^p\*c^q\*((e\*x)^(m + 1)/(e\*(m + 1)))\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 525**

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a)^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

### Rule 3683

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && EqQ[A^2 + B^2, 0]

```

### Rule 3684

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)])*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^n dx \\
&= \frac{1}{2} \left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int (1 + i \tan(c + dx))^n dx \\
&= \frac{\left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{\sqrt{x}}{1-x} dx \right)}{2d} \\
&= \frac{\left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{x^2(a+bx)}{1-x^2} dx \right)}{d} \\
&= \frac{\left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) (a + b \tan(c + dx))^n}{d} \\
&= \frac{(A + iB) F_1 \left( \frac{3}{2}; 1, -n; \frac{5}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a} \right) (a + b \tan(c + dx))^n}{3d \cot^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

**Mathematica [F]**

time = 8.55, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\cot(c + dx)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

```
[Out] Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]], x]
```

**Maple [F]**

time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(dx + c))^n (A + B \tan(dx + c))}{\sqrt{\cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2), x)
```

```
[Out] int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2), x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x
)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx))(a + b \tan(c + dx))^n}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n/sqrt(cot(c + d*x)), x
)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x
)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\sqrt{\cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n)/cot(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n)/cot(c + d\*x)^(1/2), x)

$$3.660 \quad \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=173

$$\frac{(A+iB)F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n}}{5d \cot^{\frac{5}{2}}(c+dx)} + \frac{(A-iB)F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n}}{5d \cot^{\frac{5}{2}}(c+dx)}$$

[Out] 1/5\*(A+I\*B)\*AppellF1(5/2, 1, -n, 7/2, -I\*tan(d\*x+c), -b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^n/d/cot(d\*x+c)^(5/2)/((1+b\*tan(d\*x+c)/a)^n)+1/5\*(A-I\*B)\*AppellF1(5/2, 1, -n, 7/2, I\*tan(d\*x+c), -b\*tan(d\*x+c)/a)\*(a+b\*tan(d\*x+c))^n/d/cot(d\*x+c)^(5/2)/((1+b\*tan(d\*x+c)/a)^n)

**Rubi [A]**

time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {4326, 3684, 3683, 129, 525, 524}

$$\frac{(A+iB)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{5d \cot^{\frac{5}{2}}(c+dx)} + \frac{(A-iB)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{5d \cot^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Cot[c + d\*x]^(3/2), x]

[Out] ((A + I\*B)\*AppellF1[5/2, 1, -n, 7/2, (-I)\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*(a + b\*Tan[c + d\*x])^n)/(5\*d\*Cot[c + d\*x]^(5/2)\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[5/2, 1, -n, 7/2, I\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*(a + b\*Tan[c + d\*x])^n)/(5\*d\*Cot[c + d\*x]^(5/2)\*(1 + (b\*Tan[c + d\*x])/a)^n)

**Rule 129**

Int[((e.\_)\*(x.\_))^(p.\_)\*((a.\_) + (b.\_)\*(x.\_))^(m.\_)\*((c.\_) + (d.\_)\*(x.\_))^(n.\_), x\_Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

**Rule 524**

Int[((e.\_)\*(x.\_))^(m.\_)\*((a.\_) + (b.\_)\*(x.\_))^(n.\_))^(p.\_)\*((c.\_) + (d.\_)\*(x.\_))^(n.\_))^(q.\_), x\_Symbol] := Simp[a^p\*c^q\*((e\*x)^(m + 1)/(e\*(m + 1)))\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 525**

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

### Rule 3683

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && EqQ[A^2 + B^2, 0]

```

### Rule 3684

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]

```

### Rule 4326

```

Int[(cot[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Cot[a + b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u, x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx &= \left( \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int \tan^{\frac{3}{2}}(c + dx) (a + b \tan(c + dx)) dx \\
&= \frac{1}{2} \left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \int (1 + i \tan(c + dx)) dx \\
&= \frac{\left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{x^{3/2}(a+bx)}{1-ix} dx \right)}{2d} \\
&= \frac{\left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) \text{Subst} \left( \int \frac{x^4(a+bx)}{1-ix} dx \right)}{d} \\
&= \frac{\left( (A - iB) \sqrt{\cot(c + dx)} \sqrt{\tan(c + dx)} \right) (a + b \tan(c + dx))}{d} \\
&= \frac{(A + iB) F_1 \left( \frac{5}{2}; 1, -n; \frac{7}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a} \right) (a + b \tan(c + dx))}{5d \cot^{\frac{5}{2}}(c + dx)}
\end{aligned}$$

**Mathematica [F]**

time = 9.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2), x]
```

```
[Out] Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2), x]
```

**Maple [F]**

time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(dx + c))^n (A + B \tan(dx + c))}{\cot(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2), x)
```

```
[Out] int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2), x)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(3/2), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(3/2), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\cot^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)\*\*(3/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n/cot(c + d\*x)\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/cot(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n/cot(d\*x + c)^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\cot(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/cot(c + d*x)^(3/2), x)
```

```
[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/cot(c + d*x)^(3/2), x)
```



$$3.661 \quad \int \tan^{\frac{3}{2}}(c+dx)(a+b \tan(c+dx))^n(A+B \tan(c+dx)) dx$$

**Optimal.** Leaf size=173

$$\frac{(A+iB)F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) \tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n}}{5d}$$

[Out] 1/5\*(A+I\*B)\*AppellF1(5/2,1,-n,7/2,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^n/d/((1+b\*tan(d\*x+c)/a)^n)+1/5\*(A-I\*B)\*AppellF1(5/2,1,-n,7/2,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(5/2)\*(a+b\*tan(d\*x+c))^n/d/((1+b\*tan(d\*x+c)/a)^n)

**Rubi [A]**

time = 0.25, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {3684, 3683, 129, 525, 524}

$$\frac{(A+iB) \tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{5d} + \frac{(A-iB) \tan^{\frac{5}{2}}(c+dx)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A + I\*B)\*AppellF1[5/2, 1, -n, 7/2, (-I)\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^n)/(5\*d\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[5/2, 1, -n, 7/2, I\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*Tan[c + d\*x]^(5/2)\*(a + b\*Tan[c + d\*x])^n)/(5\*d\*(1 + (b\*Tan[c + d\*x])/a)^n)

**Rule 129**

Int[((e\_.)\*(x\_))^(p\_)\*((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

**Rule 524**

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*((e\*x)^(m + 1)/(e\*(m + 1)))\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 525**

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a)^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && EqQ[A^2 + B^2, 0]
```

### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^n(A + B \tan(c + dx)) dx &= \frac{1}{2}(A - iB) \int (1 + i \tan(c + dx)) \tan^{\frac{3}{2}}(c + dx) \\
 &= \frac{(A - iB) \text{Subst}\left(\int \frac{x^{3/2}(a+bx)^n}{1-ix} dx, x, \tan(c + dx)\right)}{2d} \\
 &= \frac{(A - iB) \text{Subst}\left(\int \frac{x^4(a+bx^2)^n}{1-ix^2} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\
 &= \frac{\left((A - iB)(a + b \tan(c + dx))^n \left(1 + \frac{b \tan(c + dx)}{a}\right)\right)}{2} \\
 &= \frac{(A + iB) F_1\left(\frac{5}{2}; 1, -n; \frac{7}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a}\right)}{2}
 \end{aligned}$$

Mathematica [F]

time = 1.97, size = 0, normalized size = 0.00

$$\int \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x  
]

**Maple [F]**

time = 0.49, size = 0, normalized size = 0.00

$$\int \left( \tan^{\frac{3}{2}}(dx + c) \right) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^(3/2), x  
)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c)^2 + A\*tan(d\*x + c))\*(b\*tan(d\*x + c) + a)^n\*sqrt(tan(d\*x + c)), x)

**Sympy [F(-1)]** Timed out  
 time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(3/2)\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Timed out

**Giac [F]**  
 time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*tan(d\*x + c)^(3/2), x)

**Mupad [F]**  
 time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + dx)^{3/2} (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int(tan(c + d\*x)^(3/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)

$$3.662 \quad \int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx$$

**Optimal.** Leaf size=173

$$\frac{(A + iB)F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a}\right) \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^n \left(1 + \frac{b \tan(c + dx)}{a}\right)^{-n}}{3d}$$

[Out] 1/3\*(A+I\*B)\*AppellF1(3/2,1,-n,5/2,-I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n/d/((1+b\*tan(d\*x+c)/a)^n)+1/3\*(A-I\*B)\*AppellF1(3/2,1,-n,5/2,I\*tan(d\*x+c),-b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(3/2)\*(a+b\*tan(d\*x+c))^n/d/((1+b\*tan(d\*x+c)/a)^n)

**Rubi [A]**

time = 0.25, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {3684, 3683, 129, 525, 524}

$$\frac{(A + iB) \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^n \left(\frac{b \tan(c + dx)}{a} + 1\right)^{-n} F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a}\right)}{3d} + \frac{(A - iB) \tan^{\frac{3}{2}}(c + dx)(a + b \tan(c + dx))^n \left(\frac{b \tan(c + dx)}{a} + 1\right)^{-n} F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; i \tan(c + dx), -\frac{b \tan(c + dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] ((A + I\*B)\*AppellF1[3/2, 1, -n, 5/2, (-I)\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n)/(3\*d\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[3/2, 1, -n, 5/2, I\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*Tan[c + d\*x]^(3/2)\*(a + b\*Tan[c + d\*x])^n)/(3\*d\*(1 + (b\*Tan[c + d\*x])/a)^n)

**Rule 129**

Int[((e.\_)\*(x.\_))^(p.\_)\*((a.\_) + (b.\_)\*(x.\_))^(m.\_)\*((c.\_) + (d.\_)\*(x.\_))^(n.\_), x\_Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

**Rule 524**

Int[((e.\_)\*(x.\_))^(m.\_)\*((a.\_) + (b.\_)\*(x.\_))^(n.\_)^(p.\_)\*((c.\_) + (d.\_)\*(x.\_))^(n.\_)^(q.\_), x\_Symbol] :> Simp[a^p\*c^q\*((e\*x)^(m + 1)/(e\*(m + 1)))\*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 525**

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a)^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && EqQ[A^2 + B^2, 0]
```

### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^n (A+B \tan(c+dx)) dx &= \frac{1}{2}(A-iB) \int (1+i \tan(c+dx)) \sqrt{\tan(c+dx)} dx \\ &= \frac{(A-iB) \operatorname{Subst}\left(\int \frac{\sqrt{x} (a+bx)^n}{1-ix} dx, x, \tan(c+dx)\right)}{2d} \\ &= \frac{(A-iB) \operatorname{Subst}\left(\int \frac{x^2 (a+bx^2)^n}{1-ix^2} dx, x, \sqrt{\tan(c+dx)}\right)}{d} \\ &= \frac{\left((A-iB)(a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)\right)}{d} \\ &= \frac{(A+iB) F_1\left(\frac{3}{2}; 1, -n; \frac{5}{2}; -i \tan(c+dx)\right), -b}{d} \end{aligned}$$

**Mathematica [F]**

time = 1.02, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(c + dx)} (a + b \tan(c + dx))^n (A + B \tan(c + dx)) dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]),x]

[Out] Integrate[Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]), x  
]

**Maple** [F]

time = 0.49, size = 0, normalized size = 0.00

$$\int \left( \sqrt{\tan(dx + c)} \right) (a + b \tan(dx + c))^n (A + B \tan(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

[Out] int(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x)

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm  
="maxima")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*sqrt(tan(d\*x + c)), x  
)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm  
="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*sqrt(tan(d\*x + c)), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \tan(c + dx)) (a + b \tan(c + dx))^n \sqrt{\tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)\*\*(1/2)\*(a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c)),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n\*sqrt(tan(c + d\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n\*sqrt(tan(d\*x + c)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan(c + dx)} (A + B \tan(c + dx)) (a + b \tan(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n,x)

[Out] int(tan(c + d\*x)^(1/2)\*(A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n, x)



$$3.663 \quad \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\sqrt{\tan(c+dx)}} dx$$

**Optimal.** Leaf size=167

$$\frac{(A+iB)F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) \sqrt{\tan(c+dx)} (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n}}{d}$$

[Out] (A+I\*B)\*AppellF1(1/2, 1, -n, 3/2, -I\*tan(d\*x+c), -b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n/d/((1+b\*tan(d\*x+c)/a)^n)+(A-I\*B)\*AppellF1(1/2, 1, -n, 3/2, I\*tan(d\*x+c), -b\*tan(d\*x+c)/a)\*tan(d\*x+c)^(1/2)\*(a+b\*tan(d\*x+c))^n/d/((1+b\*tan(d\*x+c)/a)^n)

**Rubi** [A]

time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {3684, 3683, 129, 441, 440}

$$\frac{(A+iB)\sqrt{\tan(c+dx)}(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{d} + \frac{(A-iB)\sqrt{\tan(c+dx)}(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]],x]

[Out] ((A + I\*B)\*AppellF1[1/2, 1, -n, 3/2, (-I)\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n)/(d\*(1 + (b\*Tan[c + d\*x])/a)^n) + ((A - I\*B)\*AppellF1[1/2, 1, -n, 3/2, I\*Tan[c + d\*x], -((b\*Tan[c + d\*x])/a)]\*Sqrt[Tan[c + d\*x]]\*(a + b\*Tan[c + d\*x])^n)/(d\*(1 + (b\*Tan[c + d\*x])/a)^n)

Rule 129

Int[((e\_)\*(x\_))^(p\_)\*((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k\*(p + 1) - 1)\*(a + b\*(x^k/e))^m\*(c + d\*(x^k/e))^n, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[p] && IntegerQ[m]

Rule 440

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)\*(x^n/a), (-d)\*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 441

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[a^IntPart[p]\*((a + b\*x^n)^FracPart[p]/(1 + b\*(x^n/a))^FracPart[p]),

$\text{Int}[(1 + b(x^n/a))^p(c + d x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1] \&\& !(IntegerQ[p] || GtQ[a, 0])$

### Rule 3683

$\text{Int}[(a_.) + (b_.)\text{tan}[e_.) + (f_.)x_)]^{(m_.)}((A_.) + (B_.)\text{tan}[e_.) + (f_.)x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[A^2/f, \text{Subst}[\text{Int}[(a + b*x)^m((c + d*x)^n/(A - B*x)), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& !IntegerQ[m] \&\& !IntegerQ[n] \&\& !IntegersQ[2*m, 2*n] \&\& \text{EqQ}[A^2 + B^2, 0]$

### Rule 3684

$\text{Int}[(a_.) + (b_.)\text{tan}[e_.) + (f_.)x_)]^{(m_.)}((A_.) + (B_.)\text{tan}[e_.) + (f_.)x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(A + I*B)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m(c + d*\text{Tan}[e + f*x])^n(1 - I*\text{Tan}[e + f*x]), x], x] + \text{Dist}[(A - I*B)/2, \text{Int}[(a + b*\text{Tan}[e + f*x])^m(c + d*\text{Tan}[e + f*x])^n(1 + I*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& !IntegerQ[m] \&\& !IntegerQ[n] \&\& !IntegersQ[2*m, 2*n] \&\& \text{NeQ}[A^2 + B^2, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx &= \frac{1}{2} (A - iB) \int \frac{(1 + i \tan(c + dx))(a + b \tan(c + dx))^n}{\sqrt{\tan(c + dx)}} dx + \\ &= \frac{(A - iB) \text{Subst}\left(\int \frac{(a+bx)^n}{(1-ix)\sqrt{x}} dx, x, \tan(c + dx)\right)}{2d} + \frac{(A + iB) \text{Subst}\left(\int \frac{(a+bx)^n}{(1+ix)\sqrt{x}} dx, x, \tan(c + dx)\right)}{2d} \\ &= \frac{(A - iB) \text{Subst}\left(\int \frac{(a+bx^2)^n}{1-ix^2} dx, x, \sqrt{\tan(c + dx)}\right)}{d} + \frac{(A + iB) \text{Subst}\left(\int \frac{(a+bx^2)^n}{1+ix^2} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\ &= \frac{\left((A - iB)(a + b \tan(c + dx))^n \left(1 + \frac{b \tan(c + dx)}{a}\right)^{-n}\right) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\tan(c + dx)}\right)}{d} \\ &= \frac{(A + iB) F_1\left(\frac{1}{2}; 1, -n; \frac{3}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a}\right) \sqrt{\tan(c + dx)}}{d} \end{aligned}$$

### Mathematica [F]

time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Verification is not applicable to the result.

[In] Integrate[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

[Out] Integrate[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Sqrt[Tan[c + d\*x]], x]

**Maple** [F]

time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(dx + c))^n (A + B \tan(dx + c))}{\sqrt{\tan(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2), x)

[Out] int((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2), x)

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2), x, algorithm="maxima")

[Out] Timed out

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n/sqrt(tan(d\*x + c)), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))\*\*n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)\*\*(1/2),x)

[Out] Integral((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))\*\*n/sqrt(tan(c + d\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\sqrt{\tan(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n)/tan(c + d\*x)^(1/2),x)

[Out] int(((A + B\*tan(c + d\*x))\*(a + b\*tan(c + d\*x))^n)/tan(c + d\*x)^(1/2), x)

$$3.664 \quad \int \frac{(a+b \tan(c+dx))^n (A+B \tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=169

$$\frac{(A+iB)F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right) (a+b \tan(c+dx))^n \left(1 + \frac{b \tan(c+dx)}{a}\right)^{-n} (A-iB)}{d \sqrt{\tan(c+dx)}}$$

[Out]  $-(A+I*B)*\text{AppellF1}(-1/2, 1, -n, 1/2, -I*\tan(d*x+c), -b*\tan(d*x+c)/a)*(a+b*\tan(d*x+c))^n/d/\tan(d*x+c)^{(1/2)}/((1+b*\tan(d*x+c)/a)^n)-(A-I*B)*\text{AppellF1}(-1/2, 1, -n, 1/2, I*\tan(d*x+c), -b*\tan(d*x+c)/a)*(a+b*\tan(d*x+c))^n/d/\tan(d*x+c)^{(1/2)}/((1+b*\tan(d*x+c)/a)^n)$

**Rubi** [A]

time = 0.25, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {3684, 3683, 129, 525, 524}

$$\frac{(A+iB)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; -i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{d \sqrt{\tan(c+dx)}} - \frac{(A-iB)(a+b \tan(c+dx))^n \left(\frac{b \tan(c+dx)}{a} + 1\right)^{-n} F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; i \tan(c+dx), -\frac{b \tan(c+dx)}{a}\right)}{d \sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + b*\text{Tan}[c + d*x])^n*(A + B*\text{Tan}[c + d*x])}{\text{Tan}[c + d*x]^{(3/2)}, x]$

[Out]  $-\left(\frac{(A+I*B)*\text{AppellF1}[-1/2, 1, -n, 1/2, (-I)*\text{Tan}[c + d*x], -((b*\text{Tan}[c + d*x])/a)]*(a + b*\text{Tan}[c + d*x])^n}{(d*\text{Sqrt}[\text{Tan}[c + d*x]]*(1 + (b*\text{Tan}[c + d*x])/a))^n}\right) - \left(\frac{(A-I*B)*\text{AppellF1}[-1/2, 1, -n, 1/2, I*\text{Tan}[c + d*x], -((b*\text{Tan}[c + d*x])/a)]*(a + b*\text{Tan}[c + d*x])^n}{(d*\text{Sqrt}[\text{Tan}[c + d*x]]*(1 + (b*\text{Tan}[c + d*x])/a))^n}\right)$

**Rule 129**

$\text{Int}[\frac{(e_*)*(x_*)^{(p_*)}*((a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}))}{\text{Symbol}] :> \text{With}[\{k = \text{Denominator}[p]\}, \text{Dist}[k/e, \text{Subst}[\text{Int}[x^{k*(p+1)-1}*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^{(1/k)}, x]] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{FractionQ}[p] \&\& \text{IntegerQ}[m]$

**Rule 524**

$\text{Int}[\frac{(e_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}}{\text{Symbol}] :> \text{Simp}[a^p*c^q*((e*x)^{(m+1)}/(e*(m+1)))*\text{AppellF1}[(m+1)/n, -p, -q, 1 + (m+1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[m, n - 1] \&\& (\text{IntegerQ}[p] \|\ \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] \|\ \text{GtQ}[c, 0])$

**Rule 525**

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a)^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

### Rule 3683

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && EqQ[A^2 + B^2, 0]
```

### Rule 3684

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Tan[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx &= \frac{1}{2} (A - iB) \int \frac{(1 + i \tan(c + dx))(a + b \tan(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)} dx + \\
 &= \frac{(A - iB) \text{Subst}\left(\int \frac{(a+bx)^n}{(1-ix)x^{3/2}} dx, x, \tan(c + dx)\right)}{2d} + \frac{(A + iB)}{d} \\
 &= \frac{(A - iB) \text{Subst}\left(\int \frac{(a+bx^2)^n}{x^2(1-ix^2)} dx, x, \sqrt{\tan(c + dx)}\right)}{d} + \frac{(A + iB)}{d} \\
 &= \frac{\left((A - iB)(a + b \tan(c + dx))^n \left(1 + \frac{b \tan(c + dx)}{a}\right)^{-n}\right) \text{Subst}}{d} \\
 &= \frac{(A + iB) F_1\left(-\frac{1}{2}; 1, -n; \frac{1}{2}; -i \tan(c + dx), -\frac{b \tan(c + dx)}{a}\right)}{d \sqrt{\tan(c + dx)}}
 \end{aligned}$$

**Mathematica [F]**

time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(c + dx))^n (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification is not applicable to the result.

[In] Integrate[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

[Out] Integrate[((a + b\*Tan[c + d\*x])^n\*(A + B\*Tan[c + d\*x]))/Tan[c + d\*x]^(3/2), x]

**Maple** [F]

time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b \tan(dx + c))^n (A + B \tan(dx + c))}{\tan(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x)

[Out] int((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x)

**Maxima** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x, algorithm="maxima")

[Out] Timed out

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*tan(d\*x+c))^n\*(A+B\*tan(d\*x+c))/tan(d\*x+c)^(3/2), x, algorithm="fricas")

[Out] integral((B\*tan(d\*x + c) + A)\*(b\*tan(d\*x + c) + a)^n/tan(d\*x + c)^(3/2), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)``[Out] Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n/tan(c + d*x)**(3/2), x)`**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="giac")``[Out] Timed out`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(c + dx)) (a + b \tan(c + dx))^n}{\tan(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/tan(c + d*x)^(3/2),x)``[Out] int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/tan(c + d*x)^(3/2), x)`



$$3.665 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^n dx$$

Optimal. Leaf size=63

$$\frac{a(iA + B)(c - ic \tan(e + fx))^n}{fn} - \frac{aB(c - ic \tan(e + fx))^{1+n}}{cf(1 + n)}$$

[Out] a\*(I\*A+B)\*(c-I\*c\*tan(f\*x+e))^n/f/n-a\*B\*(c-I\*c\*tan(f\*x+e))^(1+n)/c/f/(1+n)

Rubi [A]

time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{a(B + iA)(c - ic \tan(e + fx))^n}{fn} - \frac{aB(c - ic \tan(e + fx))^{n+1}}{cf(n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^n,x]

[Out] (a\*(I\*A + B)\*(c - I\*c\*Tan[e + f\*x])^n)/(f\*n) - (a\*B\*(c - I\*c\*Tan[e + f\*x])^(1 + n))/(c\*f\*(1 + n))

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int [ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^n dx = \frac{(ac) \text{Subst}\left(\int (A + Bx)(c - icx)^{-1+n} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int ((A - iB)(c - icx)^{-1+n} dx\right)}{f}$$

$$= \frac{a(iA + B)(c - ic \tan(e + fx))^n}{fn} - \frac{aE}{fn}$$

**Mathematica [A]**

time = 1.77, size = 75, normalized size = 1.19

$$\frac{iae^{n(-\log(c \sec(e+fx)) + \log(c - ic \tan(e+fx)))} (c \sec(e + fx))^n (A - iB + An + Bn \tan(e + fx))}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] (I*a*E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*(c*Sec[e + f*x])^n*(A - I*B + A*n + B*n*Tan[e + f*x]))/(f*n*(1 + n))
```

**Maple [A]**

time = 1.35, size = 80, normalized size = 1.27

method	result	size
norman	$\frac{(iAan+iaA+aB)e^{n \ln(c-ic \tan(fx+e))}}{fn(1+n)} + \frac{iaB \tan(fx+e)e^{n \ln(c-ic \tan(fx+e))}}{f(1+n)}$	80
risch	Expression too large to display	1205

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/n/(1+n)*(I*A*a*n+I*a*A+a*B)*exp(n*ln(c-I*c*tan(f*x+e)))+I*a*B/f/(1+n)*tan(f*x+e)*exp(n*ln(c-I*c*tan(f*x+e)))
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs.  $2(59) = 118$ .

time = 0.58, size = 326, normalized size = 5.17

$$\frac{((A - iB)ae^{n \ln(c - ic \tan(fx + e))} + (A + iB)ae^{n \ln(c - ic \tan(fx + e))}) \cos(2fx + 2e) + ((A + iB)ae^{n \ln(c - ic \tan(fx + e))} - (A - iB)ae^{n \ln(c - ic \tan(fx + e))}) \sin(2fx + 2e)}{(-a^n + (-a^n - in) \cos(2fx + 2e) + (a^n + n) \sin(2fx + 2e) - in) [\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2 \cos(2fx + 2e) + 1]^{1/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorith="maxima")

[Out] (((A - I\*B)\*a\*c^n\*n + (A - I\*B)\*a\*c^n)\*2^n\*cos(-2\*f\*x + n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1) - 2\*e) + ((A + I\*B)\*a\*c^n\*n + (A - I\*B)\*a\*c^n)\*2^n\*cos(n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1)) - ((I\*A + B)\*a\*c^n\*n + (I\*A + B)\*a\*c^n)\*2^n\*sin(-2\*f\*x + n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1) - 2\*e) - ((I\*A - B)\*a\*c^n\*n + (I\*A + B)\*a\*c^n)\*2^n\*sin(n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1))))/((-I\*n^2 + (-I\*n^2 - I\*n)\*cos(2\*f\*x + 2\*e) + (n^2 + n)\*sin(2\*f\*x + 2\*e) - I\*n)\*(cos(2\*f\*x + 2\*e)^2 + sin(2\*f\*x + 2\*e)^2 + 2\*cos(2\*f\*x + 2\*e) + 1)^(1/2\*n)\*f)

**Fricas** [A]

time = 1.64, size = 96, normalized size = 1.52

$$\frac{((iA - B)an + (iA + B)a + ((iA + B)an + (iA + B)a)e^{(2i fx + 2i e)}) \left( \frac{2c}{e^{(2i fx + 2i e)} + 1} \right)^n}{fn^2 + fn + (fn^2 + fn)e^{(2i fx + 2i e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorith="fricas")

[Out] ((I\*A - B)\*a\*n + (I\*A + B)\*a + ((I\*A + B)\*a\*n + (I\*A + B)\*a)\*e^(2\*I\*f\*x + 2\*I\*e)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^n/(f\*n^2 + f\*n + (f\*n^2 + f\*n)\*e^(2\*I\*f\*x + 2\*I\*e))

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(48) = 96.

time = 0.66, size = 394, normalized size = 6.25

$$\begin{cases} x(A + B \tan(e)) (ia \tan(e) + a) (-ic \tan(e) + c)^n & \text{for } f = 0 \\ \frac{2Aa}{2cf \tan(e+fx)+2icf} + \frac{2iBafx \tan(e+fx)}{2cf \tan(e+fx)+2icf} - \frac{2Bafx}{2cf \tan(e+fx)+2icf} - \frac{Ba \log(\tan^2(e+fx)+1) \tan(e+fx)}{2cf \tan(e+fx)+2icf} - \frac{iBa \log(\tan^2(e+fx)+1)}{2cf \tan(e+fx)+2icf} - \frac{2iBa}{2cf \tan(e+fx)+2icf} & \text{for } n = -1 \\ Aax + \frac{iAa \log(\tan^2(e+fx)+1)}{2f} - iBax + \frac{Ba \log(\tan^2(e+fx)+1)}{2f} + \frac{iBa \tan(e+fx)}{f} & \text{for } n = 0 \\ \frac{iAa(-ic \tan(e+fx)+c)^n}{fn^2+fn} + \frac{iAa(-ic \tan(e+fx)+c)^n}{fn^2+fn} + \frac{iBan(-ic \tan(e+fx)+c)^n \tan(e+fx)}{fn^2+fn} + \frac{Ba(-ic \tan(e+fx)+c)^n}{fn^2+fn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x)

[Out] Piecewise((x\*(A + B\*tan(e))\*(I\*a\*tan(e) + a)\*(-I\*c\*tan(e) + c))^n, Eq(f, 0)), (2\*A\*a/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) + 2\*I\*B\*a\*f\*x\*tan(e + f\*x)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 2\*B\*a\*f\*x/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - B\*a\*log(tan(e + f\*x)\*\*2 + 1)\*tan(e + f\*x)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - I\*B\*a\*log(tan(e + f\*x)\*\*2 + 1)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 2\*I\*B\*a/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f), Eq(n, -1)), (A\*a\*x + I\*A\*a\*log(tan(e + f\*x)\*\*2 + 1)/(2\*f) - I\*B\*a\*x + B\*a\*log(tan(e + f\*x)\*\*2 + 1)/(2\*f) + I\*B\*a\*tan(e + f\*x)/f, Eq(n, 0)), (I\*A\*a\*n\*(-I\*c\*tan(e + f\*x) + c))^n/(f\*n\*\*2 + f\*n) + I\*A\*a\*(-I

```
*c*tan(e + f*x) + c)**n/(f*n**2 + f*n) + I*B*a*n*(-I*c*tan(e + f*x) + c)**n
*tan(e + f*x)/(f*n**2 + f*n) + B*a*(-I*c*tan(e + f*x) + c)**n/(f*n**2 + f*n
), True))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algo
rithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) +
c)^n, x)
```

**Mupad [B]**

time = 1.11, size = 128, normalized size = 2.03

$$\frac{(\cos(e + f x) - \sin(e + f x) i) \left( c - \frac{c \sin(e + f x) i}{\cos(e + f x)} \right)^n \left( \frac{a(A - B i + A n + B n i)}{f n (n i + 1)} + \frac{a(A - B i) (\cos(2 e + 2 f x) + \sin(2 e + 2 f x) i) (n + 1)}{f n (n i + 1)} \right)}{2 \cos(e + f x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*i)*(c - c*tan(e + f*x)*i)^n,
x)
```

```
[Out] -((cos(e + f*x) - sin(e + f*x)*i)*(c - (c*sin(e + f*x)*i)/cos(e + f*x))^n
*((a*(A - B*i + A*n + B*n*i))/(f*n*(n*i + 1)) + (a*(A - B*i)*(cos(2*e
+ 2*f*x) + sin(2*e + 2*f*x)*i)*(n + 1))/(f*n*(n*i + 1))))/(2*cos(e + f*x
))
```

$$3.666 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^4 dx$$

Optimal. Leaf size=59

$$\frac{a(iA + B)c^4(1 - i \tan(e + fx))^4}{4f} - \frac{aBc^4(1 - i \tan(e + fx))^5}{5f}$$

[Out]  $1/4*a*(I*A+B)*c^4*(1-I*\tan(f*x+e))^4/f-1/5*a*B*c^4*(1-I*\tan(f*x+e))^5/f$

Rubi [A]

time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{ac^4(B + iA)(1 - i \tan(e + fx))^4}{4f} - \frac{aBc^4(1 - i \tan(e + fx))^5}{5f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^4, x]$

[Out]  $(a*(I*A + B)*c^4*(1 - I*\text{Tan}[e + f*x])^4)/(4*f) - (a*B*c^4*(1 - I*\text{Tan}[e + f*x])^5)/(5*f)$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ictan(e + fx))^4 dx = \frac{(ac)\text{Subst}\left(\int (A + Bx)(c - icx)^3 dx, x\right)}{f}$$

$$= \frac{(ac)\text{Subst}\left(\int ((A - iB)(c - icx)^3 + i(A + B)(c - icx)^2) dx, x\right)}{f}$$

$$= \frac{a(iA + B)c^4(1 - i \tan(e + fx))^4}{4f} - \frac{aBc^4 \tan(e + fx)}{4f}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 226 vs. 2(59) = 118.  
 time = 1.18, size = 226, normalized size = 3.83

$\frac{a^4 \sec(e) \sec^5(c + fx) (5(-5A + 3B) \cos(fx) + 5(-5A + 3B) \cos(2e + fx) - 10A \cos(2e + 3fx) + 10B \cos(2e + 3fx) - 10A \cos(4e + 3fx) + 10B \cos(4e + 3fx) + 25A \sin(fx) + 15B \sin(fx) - 25A \sin(2e + fx) - 15B \sin(2e + fx) + 15A \sin(2e + 3fx) + 5B \sin(2e + 3fx) - 10A \sin(4e + 3fx) - 10B \sin(4e + 3fx) + 5A \sin(4e + 5fx) + 3B \sin(4e + 5fx))}{4f}$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a*c^4*Sec[e]*Sec[e + f*x]^5*(5*((-5*I)*A + 3*B)*Cos[f*x] + 5*((-5*I)*A + 3*B)*Cos[2*e + f*x] - (10*I)*A*Cos[2*e + 3*f*x] + 10*B*Cos[2*e + 3*f*x] - (10*I)*A*Cos[4*e + 3*f*x] + 10*B*Cos[4*e + 3*f*x] + 25*A*Sin[f*x] + (15*I)*B*Sin[f*x] - 25*A*Sin[2*e + f*x] - (15*I)*B*Sin[2*e + f*x] + 15*A*Sin[2*e + 3*f*x] + (5*I)*B*Sin[2*e + 3*f*x] - 10*A*Sin[4*e + 3*f*x] - (10*I)*B*Sin[4*e + 3*f*x] + 5*A*Sin[4*e + 5*f*x] + (3*I)*B*Sin[4*e + 5*f*x]))/(40*f)
```

**Maple [A]**  
 time = 0.10, size = 85, normalized size = 1.44

method	result
risch	$\frac{4a c^4 (5iA e^{2i(fx+e)} + 5B e^{2i(fx+e)} + 5iA - 3B)}{5f(e^{2i(fx+e)} + 1)^5}$
derivativedivides	$\frac{ia c^4 \left( \frac{B(\tan^5(fx+e))}{5} + \frac{(3iB+A)(\tan^4(fx+e))}{4} + \frac{(3iA-3B)(\tan^3(fx+e))}{3} + \frac{(-iB-3A)(\tan^2(fx+e))}{2} - iA \tan(fx+e) \right)}{f}$
default	$\frac{ia c^4 \left( \frac{B(\tan^5(fx+e))}{5} + \frac{(3iB+A)(\tan^4(fx+e))}{4} + \frac{(3iA-3B)(\tan^3(fx+e))}{3} + \frac{(-iB-3A)(\tan^2(fx+e))}{2} - iA \tan(fx+e) \right)}{f}$
norman	$\frac{Aa c^4 \tan(fx+e)}{f} - \frac{(-iAa c^4 + 3Ba c^4)(\tan^4(fx+e))}{4f} + \frac{(-3iAa c^4 + Ba c^4)(\tan^2(fx+e))}{2f} - \frac{(iBa c^4 + Aa c^4)(\tan^3(fx+e))}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

[Out]  $I/f*a*c^4*(1/5*B*\tan(f*x+e)^5+1/4*(A+3*I*B)*\tan(f*x+e)^4+1/3*(-3*B+3*I*A)*\tan(f*x+e)^3+1/2*(-I*B-3*A)*\tan(f*x+e)^2-I*A*\tan(f*x+e))$

**Maxima** [A]

time = 0.50, size = 100, normalized size = 1.69

$$\frac{-4iBac^4 \tan(fx+e)^5 + 5(-iA+3B)ac^4 \tan(fx+e)^4 + 20(A+iB)ac^4 \tan(fx+e)^3 + 10(3iA-B)ac^4 \tan(fx+e)^2 - 20Aac^4 \tan(fx+e)}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")`

[Out]  $-1/20*(-4*I*B*a*c^4*\tan(f*x+e)^5 + 5*(-I*A+3*B)*a*c^4*\tan(f*x+e)^4 + 20*(A+I*B)*a*c^4*\tan(f*x+e)^3 + 10*(3*I*A-B)*a*c^4*\tan(f*x+e)^2 - 20*A*a*c^4*\tan(f*x+e))/f$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs.  $2(51) = 102$ .

time = 1.71, size = 106, normalized size = 1.80

$$\frac{4(5(-iA-B)ac^4e^{(2i fx+2ie)} + (-5iA+3B)ac^4)}{5(fe^{(10i fx+10ie)} + 5fe^{(8i fx+8ie)} + 10fe^{(6i fx+6ie)} + 10fe^{(4i fx+4ie)} + 5fe^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")`

[Out]  $-4/5*(5*(-I*A-B)*a*c^4*e^{(2*I*f*x+2*I*e)} + (-5*I*A+3*B)*a*c^4)/(f*e^{(10*I*f*x+10*I*e)} + 5*f*e^{(8*I*f*x+8*I*e)} + 10*f*e^{(6*I*f*x+6*I*e)} + 10*f*e^{(4*I*f*x+4*I*e)} + 5*f*e^{(2*I*f*x+2*I*e)} + f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs.  $2(46) = 92$ .

time = 0.36, size = 155, normalized size = 2.63

$$\frac{20iAac^4 - 12Bac^4 + (20iAac^4e^{2ie} + 20Bac^4e^{2ie})e^{2ifx}}{5fe^{10ie}e^{10ifx} + 25fe^{8ie}e^{8ifx} + 50fe^{6ie}e^{6ifx} + 50fe^{4ie}e^{4ifx} + 25fe^{2ie}e^{2ifx} + 5f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)`

[Out]  $(20*I*A*a*c**4 - 12*B*a*c**4 + (20*I*A*a*c**4*\exp(2*I*e) + 20*B*a*c**4*\exp(2*I*e))*\exp(2*I*f*x))/(5*f*\exp(10*I*e)*\exp(10*I*f*x) + 25*f*\exp(8*I*e)*\exp(8*I*f*x) + 50*f*\exp(6*I*e)*\exp(6*I*f*x) + 50*f*\exp(4*I*e)*\exp(4*I*f*x) + 25*f*\exp(2*I*e)*\exp(2*I*f*x) + 5*f)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(51) = 102.  
time = 0.93, size = 119, normalized size = 2.02

$$\frac{4(-5iAac^4e^{(2i fx+2ie)} - 5Bac^4e^{(2i fx+2ie)} - 5iAac^4 + 3Bac^4)}{5(fe^{(10i fx+10ie)} + 5fe^{(8i fx+8ie)} + 10fe^{(6i fx+6ie)} + 10fe^{(4i fx+4ie)} + 5fe^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out] -4/5\*(-5\*I\*A\*a\*c^4\*e^(2\*I\*f\*x + 2\*I\*e) - 5\*B\*a\*c^4\*e^(2\*I\*f\*x + 2\*I\*e) - 5\*I\*A\*a\*c^4 + 3\*B\*a\*c^4)/(f\*e^(10\*I\*f\*x + 10\*I\*e) + 5\*f\*e^(8\*I\*f\*x + 8\*I\*e) + 10\*f\*e^(6\*I\*f\*x + 6\*I\*e) + 10\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 5\*f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Mupad [B]**

time = 8.50, size = 100, normalized size = 1.69

$$\frac{\frac{li B a c^4 \tan(e+f x)^5}{5} + \frac{li a (A+B 3i) c^4 \tan(e+f x)^4}{4} + li a (-B + A li) c^4 \tan(e+f x)^3 - \frac{li a (3 A+B li) c^4 \tan(e+f x)^2}{2} + A a c^4 \tan(e+f x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^4, x)

[Out] (A\*a\*c^4\*tan(e + f\*x) + (a\*c^4\*tan(e + f\*x)^4\*(A + B\*3i)\*1i)/4 + (B\*a\*c^4\*tan(e + f\*x)^5\*1i)/5 + a\*c^4\*tan(e + f\*x)^3\*(A\*1i - B)\*1i - (a\*c^4\*tan(e + f\*x)^2\*(3\*A + B\*1i)\*1i)/2)/f



$$3.667 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^3 dx$$

Optimal. Leaf size=59

$$\frac{a(iA + B)c^3(1 - i \tan(e + fx))^3}{3f} - \frac{aBc^3(1 - i \tan(e + fx))^4}{4f}$$

[Out]  $1/3*a*(I*A+B)*c^3*(1-I*\tan(f*x+e))^3/f-1/4*a*B*c^3*(1-I*\tan(f*x+e))^4/f$

Rubi [A]

time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{ac^3(B + iA)(1 - i \tan(e + fx))^3}{3f} - \frac{aBc^3(1 - i \tan(e + fx))^4}{4f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^3, x]$

[Out]  $(a*(I*A + B)*c^3*(1 - I*\text{Tan}[e + f*x])^3)/(3*f) - (a*B*c^3*(1 - I*\text{Tan}[e + f*x])^4)/(4*f)$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^3 dx = \frac{(ac) \text{Subst}\left(\int (A + Bx)(c - icx)^2 dx, x\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left((A - iB)(c - icx)^2 + \frac{i}{f}\right) dx, x\right)}{f}$$

$$= \frac{a(iA + B)c^3(1 - i \tan(e + fx))^3}{3f}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.  
time = 1.30, size = 161, normalized size = 2.73

$$\frac{a^2 \sec(e) \sec^4(e + fx) (3(-2iA + B) \cos(e) + 3(-iA + B) \cos(e + 2fx) - 3iA \cos(3e + 2fx) + 3B \cos(3e + 2fx) - 6A \sin(e) - 3iB \sin(e) + 5A \sin(e + 2fx) + iB \sin(e + 2fx) - 3A \sin(3e + 2fx) - 3iB \sin(3e + 2fx) + 2A \sin(3e + 4fx) + iB \sin(3e + 4fx))}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3, x]

[Out] (a\*c^3\*Sec[e]\*Sec[e + f\*x]^4\*(3\*((-2\*I)\*A + B)\*Cos[e] + 3\*((-I)\*A + B)\*Cos[e + 2\*f\*x] - (3\*I)\*A\*Cos[3\*e + 2\*f\*x] + 3\*B\*Cos[3\*e + 2\*f\*x] - 6\*A\*Sin[e] - (3\*I)\*B\*Sin[e] + 5\*A\*Sin[e + 2\*f\*x] + I\*B\*Sin[e + 2\*f\*x] - 3\*A\*Sin[3\*e + 2\*f\*x] - (3\*I)\*B\*Sin[3\*e + 2\*f\*x] + 2\*A\*Sin[3\*e + 4\*f\*x] + I\*B\*Sin[3\*e + 4\*f\*x]))/(12\*f)

**Maple [A]**

time = 0.10, size = 63, normalized size = 1.07

method	result	size
risch	$\frac{4a c^3 (2iA e^{2i(fx+e)} + 2B e^{2i(fx+e)} + 2iA - B)}{3f (e^{2i(fx+e)} + 1)^4}$	5
derivativedivides	$\frac{a c^3 \left( -\frac{B(\tan^4(fx+e))}{4} - \frac{(2iB+A)(\tan^3(fx+e))}{3} - \frac{(2iA-B)(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	6
default	$\frac{a c^3 \left( -\frac{B(\tan^4(fx+e))}{4} - \frac{(2iB+A)(\tan^3(fx+e))}{3} - \frac{(2iA-B)(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	6
norman	$\frac{Aa c^3 \tan(fx+e)}{f} - \frac{(2iBa c^3 + Aa c^3)(\tan^3(fx+e))}{3f} + \frac{(-2iAa c^3 + Ba c^3)(\tan^2(fx+e))}{2f} - \frac{Ba c^3(\tan^4(fx+e))}{4f}$	9

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x,method=\_RETURNVERBOSE)

[Out]  $1/f*a*c^3*(-1/4*B*\tan(f*x+e)^4-1/3*(A+2*I*B)*\tan(f*x+e)^3-1/2*(2*I*A-B)*\tan(f*x+e)^2+A*\tan(f*x+e))$

**Maxima** [A]

time = 0.52, size = 76, normalized size = 1.29

$$\frac{3Bac^3 \tan(fx + e)^4 + 4(A + 2iB)ac^3 \tan(fx + e)^3 - 6(-2iA + B)ac^3 \tan(fx + e)^2 - 12Aac^3 \tan(fx + e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")`

[Out]  $-1/12*(3*B*a*c^3*\tan(f*x + e)^4 + 4*(A + 2*I*B)*a*c^3*\tan(f*x + e)^3 - 6*(-2*I*A + B)*a*c^3*\tan(f*x + e)^2 - 12*A*a*c^3*\tan(f*x + e))/f$

**Fricas** [A]

time = 1.57, size = 91, normalized size = 1.54

$$\frac{4(2(-iA - B)ac^3e^{2ifx+2ie} + (-2iA + B)ac^3)}{3(fe^{8ifx+8ie} + 4fe^{6ifx+6ie} + 6fe^{4ifx+4ie} + 4fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")`

[Out]  $-4/3*(2*(-I*A - B)*a*c^3*e^{(2*I*f*x + 2*I*e)} + (-2*I*A + B)*a*c^3)/(f*e^{(8*I*f*x + 8*I*e)} + 4*f*e^{(6*I*f*x + 6*I*e)} + 6*f*e^{(4*I*f*x + 4*I*e)} + 4*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs.  $2(46) = 92$ .

time = 0.31, size = 136, normalized size = 2.31

$$\frac{8iAac^3 - 4Bac^3 + (8iAac^3e^{2ie} + 8Bac^3e^{2ie})e^{2ifx}}{3fe^{8ie}e^{8ifx} + 12fe^{6ie}e^{6ifx} + 18fe^{4ie}e^{4ifx} + 12fe^{2ie}e^{2ifx} + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x)`

[Out]  $(8*I*A*a*c^3 - 4*B*a*c^3 + (8*I*A*a*c^3*\exp(2*I*e) + 8*B*a*c^3*\exp(2*I*e))*\exp(2*I*f*x))/(3*f*\exp(8*I*e)*\exp(8*I*f*x) + 12*f*\exp(6*I*e)*\exp(6*I*f*x) + 18*f*\exp(4*I*e)*\exp(4*I*f*x) + 12*f*\exp(2*I*e)*\exp(2*I*f*x) + 3*f)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs.  $2(51) = 102$ .

time = 0.75, size = 105, normalized size = 1.78

$$\frac{4(-2iAac^3e^{2ifx+2ie} - 2Bac^3e^{2ifx+2ie} - 2iAac^3 + Bac^3)}{3(fe^{8ifx+8ie} + 4fe^{6ifx+6ie} + 6fe^{4ifx+4ie} + 4fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$-4/3*(-2*I*A*a*c^3*e^{(2*I*f*x + 2*I*e)} - 2*B*a*c^3*e^{(2*I*f*x + 2*I*e)} - 2*I*A*a*c^3 + B*a*c^3)/(f*e^{(8*I*f*x + 8*I*e)} + 4*f*e^{(6*I*f*x + 6*I*e)} + 6*f*e^{(4*I*f*x + 4*I*e)} + 4*f*e^{(2*I*f*x + 2*I*e)} + f)$$

**Mupad [B]**

time = 8.47, size = 76, normalized size = 1.29

$$-\frac{\frac{B a c^3 \tan(e+f x)^4}{4} + \frac{a(A+B 2i) c^3 \tan(e+f x)^3}{3} + \frac{a(-B+A 2i) c^3 \tan(e+f x)^2}{2} - A a c^3 \tan(e+f x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^3, x)

[Out] 
$$-((a*c^3*\tan(e + f*x)^3*(A + B*2i))/3 - A*a*c^3*\tan(e + f*x) + (B*a*c^3*\tan(e + f*x)^4)/4 + (a*c^3*\tan(e + f*x)^2*(A*2i - B))/2)/f$$

$$3.668 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^2 dx$$

Optimal. Leaf size=66

$$\frac{aAc^2 \tan(e + fx)}{f} - \frac{a(iA - B)c^2 \tan^2(e + fx)}{2f} - \frac{iaBc^2 \tan^3(e + fx)}{3f}$$

[Out] a\*A\*c^2\*tan(f\*x+e)/f-1/2\*a\*(I\*A-B)\*c^2\*tan(f\*x+e)^2/f-1/3\*I\*a\*B\*c^2\*tan(f\*x+e)^3/f

Rubi [A]

time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$-\frac{ac^2(-B + iA) \tan^2(e + fx)}{2f} + \frac{aAc^2 \tan(e + fx)}{f} - \frac{iaBc^2 \tan^3(e + fx)}{3f}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^2,x]

[Out] (a\*A\*c^2\*Tan[e + f\*x])/f - (a\*(I\*A - B)\*c^2\*Tan[e + f\*x]^2)/(2\*f) - ((I/3)\*a\*B\*c^2\*Tan[e + f\*x]^3)/f

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ict \tan(e + fx))^2 dx = \frac{(ac) \text{Subst}(\int (A + Bx)(c - icx) dx, x, \frac{e + fx}{f})}{f}$$

$$= \frac{(ac) \text{Subst}(\int (Ac + (-iA + B)cx - iBx^2) dx, x, \frac{e + fx}{f})}{f}$$

$$= \frac{aAc^2 \tan(e + fx)}{f} - \frac{a(iA - B)c^2 \tan^2(e + fx)}{2f}$$

**Mathematica [A]**

time = 0.90, size = 109, normalized size = 1.65

$$\frac{ac^2 \sec(e) \sec^3(e + fx) (3(-iA + B) \cos(fx) + 3(-iA + B) \cos(2e + fx) + 6A \sin(fx) - 3A \sin(2e + fx) - 3iB \sin(2e + fx) + 3A \sin(2e + 3fx) + iB \sin(2e + 3fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a*c^2*Sec[e]*Sec[e + f*x]^3*(3*((-I)*A + B)*Cos[f*x] + 3*((-I)*A + B)*Cos[2*e + f*x] + 6*A*Sin[f*x] - 3*A*Sin[2*e + f*x] - (3*I)*B*Sin[2*e + f*x] + 3*A*Sin[2*e + 3*f*x] + I*B*Sin[2*e + 3*f*x]))/(12*f)
```

**Maple [A]**

time = 0.09, size = 49, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{ia c^2 \left( \frac{B(\tan^3(fx+e))}{3} + \frac{(iB+A)(\tan^2(fx+e))}{2} + iA \tan(fx+e) \right)}{f}$	49
default	$-\frac{ia c^2 \left( \frac{B(\tan^3(fx+e))}{3} + \frac{(iB+A)(\tan^2(fx+e))}{2} + iA \tan(fx+e) \right)}{f}$	49
risch	$\frac{2a c^2 (3iA e^{2i(fx+e)} + 3B e^{2i(fx+e)} + 3iA - B)}{3f (e^{2i(fx+e)} + 1)^3}$	56
norman	$\frac{aA c^2 \tan(fx+e)}{f} + \frac{(-iAa c^2 + Ba c^2)(\tan^2(fx+e))}{2f} - \frac{iaB c^2 (\tan^3(fx+e))}{3f}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I/f*a*c^2*(1/3*B*tan(f*x+e)^3+1/2*(A+I*B)*tan(f*x+e)^2+I*A*tan(f*x+e))
```

**Maxima [A]**

time = 0.51, size = 58, normalized size = 0.88

$$\frac{-2i Bac^2 \tan (fx + e)^3 - 3(i A - B)ac^2 \tan (fx + e)^2 + 6Aac^2 \tan (fx + e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorith="maxima")

[Out] 1/6\*(-2\*I\*B\*a\*c^2\*tan(f\*x + e)^3 - 3\*(I\*A - B)\*a\*c^2\*tan(f\*x + e)^2 + 6\*A\*a\*c^2\*tan(f\*x + e))/f

**Fricas [A]**

time = 3.61, size = 78, normalized size = 1.18

$$\frac{2(3(-iA - B)ac^2e^{(2i fx + 2ie)} + (-3iA + B)ac^2)}{3(fe^{(6i fx + 6ie)} + 3fe^{(4i fx + 4ie)} + 3fe^{(2i fx + 2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorith="fricas")

[Out] -2/3\*(3\*(-I\*A - B)\*a\*c^2\*e^(2\*I\*f\*x + 2\*I\*e) + (-3\*I\*A + B)\*a\*c^2)/(f\*e^(6\*I\*f\*x + 6\*I\*e) + 3\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 3\*f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(56) = 112.

time = 0.22, size = 117, normalized size = 1.77

$$\frac{6iAac^2 - 2Bac^2 + (6iAac^2e^{2ie} + 6Bac^2e^{2ie})e^{2ifx}}{3fe^{6ie}e^{6ifx} + 9fe^{4ie}e^{4ifx} + 9fe^{2ie}e^{2ifx} + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x)

[Out] (6\*I\*A\*a\*c\*\*2 - 2\*B\*a\*c\*\*2 + (6\*I\*A\*a\*c\*\*2\*exp(2\*I\*e) + 6\*B\*a\*c\*\*2\*exp(2\*I\*e))\*exp(2\*I\*f\*x))/(3\*f\*exp(6\*I\*e)\*exp(6\*I\*f\*x) + 9\*f\*exp(4\*I\*e)\*exp(4\*I\*f\*x) + 9\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + 3\*f)

**Giac [A]**

time = 0.65, size = 92, normalized size = 1.39

$$\frac{2(-3iAac^2e^{(2i fx + 2ie)} - 3Bac^2e^{(2i fx + 2ie)} - 3iAac^2 + Bac^2)}{3(fe^{(6i fx + 6ie)} + 3fe^{(4i fx + 4ie)} + 3fe^{(2i fx + 2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] 
$$-2/3*(-3*I*A*a*c^2*e^{(2*I*f*x + 2*I*e)} - 3*B*a*c^2*e^{(2*I*f*x + 2*I*e)} - 3*I*A*a*c^2 + B*a*c^2)/(f*e^{(6*I*f*x + 6*I*e)} + 3*f*e^{(4*I*f*x + 4*I*e)} + 3*f*e^{(2*I*f*x + 2*I*e)} + f)$$

**Mupad [B]**

time = 8.57, size = 50, normalized size = 0.76

$$\frac{a c^2 \tan(e + f x) (6 A - A \tan(e + f x) 3i + 3 B \tan(e + f x) - B \tan(e + f x)^2 2i)}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^2, x)

[Out] 
$$(a*c^2*\tan(e + f*x)*(6*A - A*\tan(e + f*x)*3i + 3*B*\tan(e + f*x) - B*\tan(e + f*x)^2*2i))/(6*f)$$



$$3.669 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx)) dx$$

Optimal. Leaf size=32

$$\frac{aA \tan(e + fx)}{f} + \frac{aBc \tan^2(e + fx)}{2f}$$

[Out] a\*A\*c\*tan(f\*x+e)/f+1/2\*a\*B\*c\*tan(f\*x+e)^2/f

Rubi [A]

time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$ , Rules used = {3669}

$$\frac{aA \tan(e + fx)}{f} + \frac{aBc \tan^2(e + fx)}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x]),x]

[Out] (a\*A\*c\*Tan[e + f\*x])/f + (a\*B\*c\*Tan[e + f\*x]^2)/(2\*f)

Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx)) dx &= \frac{(ac) \text{Subst}(\int (A + Bx) dx, x, \tan(e + fx))}{f} \\ &= \frac{aA \tan(e + fx)}{f} + \frac{aBc \tan^2(e + fx)}{2f} \end{aligned}$$

Mathematica [A]

time = 0.03, size = 32, normalized size = 1.00

$$\frac{aBc \sec^2(e + fx)}{2f} + \frac{aA \tan(e + fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]),x]
```

```
[Out] (a*B*c*Sec[e + f*x]^2)/(2*f) + (a*A*c*Tan[e + f*x])/f
```

**Maple [A]**

time = 0.05, size = 27, normalized size = 0.84

method	result	size
derivativedivides	$\frac{ac \left( \frac{B(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	27
default	$\frac{ac \left( \frac{B(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	27
norman	$\frac{aA c \tan(fx+e)}{f} + \frac{aB c (\tan^2(fx+e))}{2f}$	31
risch	$\frac{2ac(iA e^{2i(fx+e)} + B e^{2i(fx+e)} + iA)}{f(e^{2i(fx+e)} + 1)^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/f*a*c*(1/2*B*tan(f*x+e)^2+A*tan(f*x+e))
```

**Maxima [A]**

time = 0.50, size = 31, normalized size = 0.97

$$\frac{B a c \tan^2(fx + e) + 2 A a c \tan(fx + e)}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorit
hm="maxima")
```

```
[Out] 1/2*(B*a*c*tan(f*x + e)^2 + 2*A*a*c*tan(f*x + e))/f
```

**Fricas [C]** Result contains complex when optimal does not.

time = 2.82, size = 57, normalized size = 1.78

$$\frac{2((-iA - B)ace^{2ifx+2ie} - iAac)}{fe^{4ifx+4ie} + 2fe^{2ifx+2ie} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)),x, algorithm="fricas")

[Out]  $-2*((-I*A - B)*a*c*e^{(2*I*f*x + 2*I*e)} - I*A*a*c)/(f*e^{(4*I*f*x + 4*I*e)} + 2*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Sympy** [C] Result contains complex when optimal does not.  
time = 0.16, size = 82, normalized size = 2.56

$$\frac{2iAac + (2iAace^{2ie} + 2Bace^{2ie})e^{2ifx}}{fe^{4ie}e^{4ifx} + 2fe^{2ie}e^{2ifx} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)),x)

[Out]  $(2*I*A*a*c + (2*I*A*a*c*\exp(2*I*e) + 2*B*a*c*\exp(2*I*e))*\exp(2*I*f*x))/(f*\exp(4*I*e)*\exp(4*I*f*x) + 2*f*\exp(2*I*e)*\exp(2*I*f*x) + f)$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(32) = 64.  
time = 0.54, size = 113, normalized size = 3.53

$$\frac{Bac \tan(fx)^2 \tan(e)^2 - 2Aac \tan(fx)^2 \tan(e) - 2Aac \tan(fx) \tan(e)^2 + Bac \tan(fx)^2 + Bac \tan(e)^2 + 2Aac \tan(fx) + 2Aac \tan(e) + Bac}{2(f \tan(fx)^2 \tan(e)^2 - 2f \tan(fx) \tan(e) + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)),x, algorithm="giac")

[Out]  $1/2*(B*a*c*\tan(f*x)^2*\tan(e)^2 - 2*A*a*c*\tan(f*x)^2*\tan(e) - 2*A*a*c*\tan(f*x)*\tan(e)^2 + B*a*c*\tan(f*x)^2 + B*a*c*\tan(e)^2 + 2*A*a*c*\tan(f*x) + 2*A*a*c*\tan(e) + B*a*c)/(f*\tan(f*x)^2*\tan(e)^2 - 2*f*\tan(f*x)*\tan(e) + f)$

**Mupad** [B]

time = 8.42, size = 25, normalized size = 0.78

$$\frac{a \tan(e + f x) (2 A + B \tan(e + f x))}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i),x)

[Out]  $(a*c*\tan(e + f*x)*(2*A + B*\tan(e + f*x)))/(2*f)$

### 3.670 $\int (a + ia \tan(e + fx))(A + B \tan(e + fx)) dx$

Optimal. Leaf size=46

$$a(A - iB)x - \frac{a(iA + B) \log(\cos(e + fx))}{f} + \frac{iaB \tan(e + fx)}{f}$$

[Out] a\*(A-I\*B)\*x-a\*(I\*A+B)\*ln(cos(f\*x+e))/f+I\*a\*B\*tan(f\*x+e)/f

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {3606, 3556}

$$-\frac{a(B + iA) \log(\cos(e + fx))}{f} + ax(A - iB) + \frac{iaB \tan(e + fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]),x]

[Out] a\*(A - I\*B)\*x - (a\*(I\*A + B)\*Log[Cos[e + f\*x]])/f + (I\*a\*B\*Tan[e + f\*x])/f

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3606

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(a\*c - b\*d)\*x, x] + (Dist[b\*c + a\*d, Int[Tan[e + f\*x], x], x] + Simp[b\*d\*(Tan[e + f\*x]/f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[b\*c + a\*d, 0]

Rubi steps

$$\begin{aligned} \int (a + ia \tan(e + fx))(A + B \tan(e + fx)) dx &= a(A - iB)x + \frac{iaB \tan(e + fx)}{f} + (a(iA + B)) \int \tan(e + fx) dx \\ &= a(A - iB)x - \frac{a(iA + B) \log(\cos(e + fx))}{f} + \frac{iaB \tan(e + fx)}{f} \end{aligned}$$

Mathematica [A]

time = 0.02, size = 66, normalized size = 1.43

$$aAx - \frac{iaBArcTan(\tan(e + fx))}{f} - \frac{iaA \log(\cos(e + fx))}{f} - \frac{aB \log(\cos(e + fx))}{f} + \frac{iaB \tan(e + fx)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]),x]

[Out] a\*A\*x - (I\*a\*B\*ArcTan[Tan[e + f\*x]])/f - (I\*a\*A\*Log[Cos[e + f\*x]])/f - (a\*B\*Log[Cos[e + f\*x]])/f + (I\*a\*B\*Tan[e + f\*x])/f

**Maple [A]**

time = 0.08, size = 50, normalized size = 1.09

method	result	size
derivativedivides	$\frac{a \left( iB \tan(fx+e) + \frac{(iA+B) \ln(1+\tan^2(fx+e))}{2} + (-iB+A) \arctan(\tan(fx+e)) \right)}{f}$	50
default	$\frac{a \left( iB \tan(fx+e) + \frac{(iA+B) \ln(1+\tan^2(fx+e))}{2} + (-iB+A) \arctan(\tan(fx+e)) \right)}{f}$	50
norman	$(-iaB + aA)x + \frac{iaB \tan(fx+e)}{f} + \frac{(iaA+aB) \ln(1+\tan^2(fx+e))}{2f}$	52
risch	$\frac{2iaBe}{f} - \frac{2aAe}{f} - \frac{2aB}{f(e^{2i(fx+e)}+1)} - \frac{a \ln(e^{2i(fx+e)}+1)B}{f} - \frac{ia \ln(e^{2i(fx+e)}+1)A}{f}$	78

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x,method=\_RETURNVERBOSE)

[Out] 1/f\*a\*(I\*B\*tan(f\*x+e)+1/2\*(I\*A+B)\*ln(1+tan(f\*x+e)^2)+(A-I\*B)\*arctan(tan(f\*x+e)))

**Maxima [A]**

time = 0.56, size = 53, normalized size = 1.15

$$\frac{2(fx+e)(A-iB)a - (-iA-B)a \log(\tan(fx+e)^2+1) + 2iBa \tan(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x, algorithm="maxima")

[Out] 1/2\*(2\*(f\*x + e)\*(A - I\*B)\*a - (-I\*A - B)\*a\*log(tan(f\*x + e)^2 + 1) + 2\*I\*B\*a\*tan(f\*x + e))/f

**Fricas [A]**

time = 3.56, size = 67, normalized size = 1.46

$$\frac{2Ba - ((-iA - B)ae^{(2ifx+2ie)} + (-iA - B)a) \log(e^{(2ifx+2ie)} + 1)}{fe^{(2ifx+2ie)} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x, algorithm="fricas")

[Out]  $-(2*B*a - ((-I*A - B)*a*e^{(2*I*f*x + 2*I*e)} + (-I*A - B)*a)*\log(e^{(2*I*f*x + 2*I*e)} + 1))/(f*e^{(2*I*f*x + 2*I*e)} + f)$

**Sympy [A]**

time = 0.22, size = 53, normalized size = 1.15

$$-\frac{2Ba}{f e^{2ie} e^{2ifx} + f} - \frac{ia(A - iB) \log(e^{2ifx} + e^{-2ie})}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x)

[Out]  $-2*B*a/(f*\exp(2*I*e)*\exp(2*I*f*x) + f) - I*a*(A - I*B)*\log(\exp(2*I*f*x) + \exp(-2*I*e))/f$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs.  $2(42) = 84$ .

time = 0.49, size = 110, normalized size = 2.39

$$\frac{-i A a e^{(2i f x + 2i e)} \log(e^{(2i f x + 2i e)} + 1) - B a e^{(2i f x + 2i e)} \log(e^{(2i f x + 2i e)} + 1) - i A a \log(e^{(2i f x + 2i e)} + 1) - B a \log(e^{(2i f x + 2i e)} + 1) - 2 B a}{f e^{(2i f x + 2i e)} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out]  $(-I*A*a*e^{(2*I*f*x + 2*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) - B*a*e^{(2*I*f*x + 2*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) - I*A*a*\log(e^{(2*I*f*x + 2*I*e)} + 1) - B*a*\log(e^{(2*I*f*x + 2*I*e)} + 1) - 2*B*a)/(f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad [B]**

time = 8.47, size = 38, normalized size = 0.83

$$\frac{\ln(\tan(e + f x) + 1i) (B a + A a 1i)}{f} + \frac{B a \tan(e + f x) 1i}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i),x)

[Out]  $(\log(\tan(e + f*x) + 1i)*(A*a*1i + B*a))/f + (B*a*\tan(e + f*x)*1i)/f$

$$3.671 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{c-ic \tan(e+fx)} dx$$

Optimal. Leaf size=54

$$\frac{iaBx}{c} + \frac{aB \log(\cos(e+fx))}{cf} + \frac{a(A-iB)}{cf(i+\tan(e+fx))}$$

[Out]  $I*a*B*x/c+a*B*\ln(\cos(f*x+e))/c/f+a*(A-I*B)/c/f/(I+\tan(f*x+e))$

Rubi [A]

time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{a(A-iB)}{cf(\tan(e+fx)+i)} + \frac{aB \log(\cos(e+fx))}{cf} + \frac{iaBx}{c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])}, x]$

[Out]  $(I*a*B*x)/c + (a*B*\text{Log}[\text{Cos}[e + f*x]])/(c*f) + (a*(A - I*B))/(c*f*(I + \text{Tan}[e + f*x]))$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{c - ic \tan(e + fx)} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(c-icx)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{-A+iB}{c^2(i+x)^2} - \frac{B}{c^2(i+x)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{iaBx}{c} + \frac{aB \log(\cos(e + fx))}{cf} + \frac{a(A - iB)}{cf(i + \tan(e + fx))}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 123 vs. 2(54) = 108.  
time = 0.84, size = 123, normalized size = 2.28

$$\frac{a(-i \cos(e + fx) + \sin(e + fx)) (\cos(e + fx) (A - iB - 4Bfx + iB \log(\cos^2(e + fx))) + 2B \text{ArcTan}(\tan(2e + fx)) (\cos(e + fx) - i \sin(e + fx)) + (iA + B + 4iBfx + B \log(\cos^2(e + fx))) \sin(e + fx))}{2cf}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x]),x]

[Out] (a\*((-I)\*Cos[e + f\*x] + Sin[e + f\*x])\*(Cos[e + f\*x]\*(A - I\*B - 4\*B\*f\*x + I\*B\*Log[Cos[e + f\*x]^2]) + 2\*B\*ArcTan[Tan[2\*e + f\*x]]\*(Cos[e + f\*x] - I\*Sin[e + f\*x]) + (I\*A + B + (4\*I)\*B\*f\*x + B\*Log[Cos[e + f\*x]^2])\*Sin[e + f\*x]))/(2\*c\*f)

**Maple [A]**

time = 0.18, size = 44, normalized size = 0.81

method	result	size
derivativedivides	$\frac{a\left(-B \ln(i + \tan(fx+e)) - \frac{iB-A}{i + \tan(fx+e)}\right)}{fc}$	44
default	$\frac{a\left(-B \ln(i + \tan(fx+e)) - \frac{iB-A}{i + \tan(fx+e)}\right)}{fc}$	44
risch	$-\frac{e^{2i(fx+e)}Ba}{2cf} - \frac{ie^{2i(fx+e)}aA}{2cf} - \frac{2iaBe}{cf} + \frac{aB \ln(e^{2i(fx+e)}+1)}{cf}$	74
norman	$\frac{\frac{(-iaB+aA) \tan(fx+e) + iaBx}{cf} + \frac{iaBx}{c} + \frac{iaBx(\tan^2(fx+e))}{c} - \frac{iaA+aB}{cf}}{1 + \tan^2(fx+e)} - \frac{aB \ln(1 + \tan^2(fx+e))}{2cf}$	102

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e)),x,method=\_RETURNVERBOSE)

[Out] 1/f\*a/c\*(-B\*ln(I+tan(f\*x+e))-(-A+I\*B)/(I+tan(f\*x+e)))



**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 4.65, size = 45, normalized size = 0.83

$$\frac{(-iA - B)ae^{(2ifx+2ie)} + 2Ba \log(e^{(2ifx+2ie)} + 1)}{2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*A - B)*a*e^(2*I*f*x + 2*I*e) + 2*B*a*log(e^(2*I*f*x + 2*I*e) + 1))
/(c*f)
```

**Sympy [A]**

time = 0.19, size = 88, normalized size = 1.63

$$\frac{Ba \log(e^{2ifx} + e^{-2ie})}{cf} + \begin{cases} \frac{(-iAae^{2ie} - Bae^{2ie})e^{2ifx}}{2cf} & \text{for } cf \neq 0 \\ \frac{x(Aae^{2ie} - iBae^{2ie})}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

```
[Out] B*a*log(exp(2*I*f*x) + exp(-2*I*e))/(c*f) + Piecewise((( -I*A*a*exp(2*I*e) -
B*a*exp(2*I*e))*exp(2*I*f*x)/(2*c*f), Ne(c*f, 0)), (x*(A*a*exp(2*I*e) - I*
B*a*exp(2*I*e))/c, True))
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs.  $2(50) = 100$ .

time = 0.56, size = 130, normalized size = 2.41

$$\frac{Ba \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1)}{c} - \frac{2Ba \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)}{c} + \frac{Ba \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1)}{c} + \frac{3Ba \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 2Aa \tan(\frac{1}{2}fx + \frac{1}{2}e) + 8iBa \tan(\frac{1}{2}fx + \frac{1}{2}e) - 3Ba}{c(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)^2}$$


---


$$f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] (B*a*log(tan(1/2*f*x + 1/2*e) + 1)/c - 2*B*a*log(tan(1/2*f*x + 1/2*e) + I)/c + B*a*log(tan(1/2*f*x + 1/2*e) - 1)/c + (3*B*a*tan(1/2*f*x + 1/2*e)^2 - 2*A*a*tan(1/2*f*x + 1/2*e) + 8*I*B*a*tan(1/2*f*x + 1/2*e) - 3*B*a)/(c*(tan(1/2*f*x + 1/2*e) + I)^2))/f
```

**Mupad [B]**

time = 8.57, size = 51, normalized size = 0.94

$$\frac{\frac{Aa}{c} - \frac{Ba \operatorname{li}}{c}}{f (\tan(e + fx) + 1i)} - \frac{Ba \ln(\tan(e + fx) + 1i)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i), x)
```

```
[Out] ((A*a)/c - (B*a*1i)/c)/(f*(tan(e + f*x) + 1i)) - (B*a*log(tan(e + f*x) + 1i))/(c*f)
```

$$3.672 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx$$

Optimal. Leaf size=46

$$\frac{a(A+B \tan(e+fx))^2}{2(iA+B)c^2 f(1-i \tan(e+fx))^2}$$

[Out] 1/2\*a\*(A+B\*tan(f\*x+e))^2/(I\*A+B)/c^2/f/(1-I\*tan(f\*x+e))^2

Rubi [A]

time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 37}

$$\frac{a(A+B \tan(e+fx))^2}{2c^2 f(B+iA)(1-i \tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^2, x]

[Out] (a\*(A + B\*Tan[e + f\*x])^2)/(2\*(I\*A + B)\*c^2\*f\*(1 - I\*Tan[e + f\*x])^2)

Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp [(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(c-icx)^3} dx, x, \tan(e+fx)\right)}{f} \\ &= \frac{a(A+B \tan(e+fx))^2}{2(iA+B)c^2 f(1-i \tan(e+fx))^2} \end{aligned}$$

**Mathematica [A]**

time = 0.80, size = 62, normalized size = 1.35

$$\frac{a((-3iA + B) \cos(e + fx) - (A + 3iB) \sin(e + fx))(\cos(3(e + fx)) + i \sin(3(e + fx)))}{8c^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a*(((−3*I)*A + B)*Cos[e + f*x] − (A + (3*I)*B)*Sin[e + f*x])*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]))/(8*c^2*f)
```

**Maple [A]**

time = 0.21, size = 46, normalized size = 1.00

method	result	size
derivativedivides	$\frac{a \left( \frac{iB}{i + \tan(fx+e)} - \frac{-iA-B}{2(i + \tan(fx+e))^2} \right)}{f c^2}$	46
default	$\frac{a \left( \frac{iB}{i + \tan(fx+e)} - \frac{-iA-B}{2(i + \tan(fx+e))^2} \right)}{f c^2}$	46
risch	$-\frac{a e^{4i(fx+e)} B}{8c^2 f} - \frac{ia e^{4i(fx+e)} A}{8c^2 f} + \frac{a e^{2i(fx+e)} B}{4c^2 f} - \frac{ia e^{2i(fx+e)} A}{4c^2 f}$	80
norman	$\frac{\frac{aA \tan(fx+e)}{cf} + \frac{iaB(\tan^3(fx+e))}{cf} + \frac{-iaA+ aB}{2cf} + \frac{(iaA+3aB)(\tan^2(fx+e))}{2cf}}{c(1+\tan^2(fx+e))^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a/c^2*(I*B/(I+tan(f*x+e))-1/2*(-I*A-B)/(I+tan(f*x+e))^2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas** [A]

time = 3.19, size = 48, normalized size = 1.04

$$\frac{(-iA - B)ae^{(4ifx+4ie)} - 2(iA - B)ae^{(2ifx+2ie)}}{8c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/8\*((-I\*A - B)\*a\*e^(4\*I\*f\*x + 4\*I\*e) - 2\*(I\*A - B)\*a\*e^(2\*I\*f\*x + 2\*I\*e))/(c^2\*f)

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(36) = 72.

time = 0.20, size = 153, normalized size = 3.33

$$\begin{cases} \frac{(-8iAac^2fe^{2ie}+8Bac^2fe^{2ie})e^{2ifx}+(-4iAac^2fe^{4ie}-4Bac^2fe^{4ie})e^{4ifx}}{32c^4f^2} & \text{for } c^4f^2 \neq 0 \\ \frac{x(Aae^{4ie}+Aae^{2ie}-iBae^{4ie}+iBae^{2ie})}{2c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x)

[Out] Piecewise(((((-8\*I\*A\*a\*c\*\*2\*f\*exp(2\*I\*e) + 8\*B\*a\*c\*\*2\*f\*exp(2\*I\*e))\*exp(2\*I\*f\*x) + (-4\*I\*A\*a\*c\*\*2\*f\*exp(4\*I\*e) - 4\*B\*a\*c\*\*2\*f\*exp(4\*I\*e))\*exp(4\*I\*f\*x)))/(32\*c\*\*4\*f\*\*2), Ne(c\*\*4\*f\*\*2, 0)), (x\*(A\*a\*exp(4\*I\*e) + A\*a\*exp(2\*I\*e) - I\*B\*a\*exp(4\*I\*e) + I\*B\*a\*exp(2\*I\*e))/(2\*c\*\*2), True))

**Giac** [A]

time = 0.66, size = 84, normalized size = 1.83

$$\frac{2\left(Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + iAa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - Ba \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{c^2f\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] -2\*(A\*a\*tan(1/2\*f\*x + 1/2\*e)^3 + I\*A\*a\*tan(1/2\*f\*x + 1/2\*e)^2 - B\*a\*tan(1/2\*f\*x + 1/2\*e)^2 - A\*a\*tan(1/2\*f\*x + 1/2\*e))/(c^2\*f\*(tan(1/2\*f\*x + 1/2\*e) + I)^4)

**Mupad** [B]

time = 8.49, size = 51, normalized size = 1.11

$$\frac{\frac{a(-B+Ai)}{2} + Ba \tan(e + fx) \operatorname{li}}{c^2 f \left( \tan(e + fx)^2 + \tan(e + fx) 2i - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((a*(A*1i - B))/2 + B*a*tan(e + f*x)*1i)/(c^2*f*(tan(e + f*x)*2i + tan(e + f*x)^2 - 1))
```

$$3.673 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^3} dx$$

Optimal. Leaf size=55

$$-\frac{a(A-iB)}{3c^3 f(i+\tan(e+fx))^3} - \frac{aB}{2c^3 f(i+\tan(e+fx))^2}$$

[Out]  $-1/3*a*(A-I*B)/c^3/f/(I+\tan(f*x+e))^3-1/2*a*B/c^3/f/(I+\tan(f*x+e))^2$

Rubi [A]

time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$-\frac{a(A-iB)}{3c^3 f(\tan(e+fx)+i)^3} - \frac{aB}{2c^3 f(\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^3, x]

[Out]  $-1/3*(a*(A - I*B))/(c^3*f*(I + Tan[e + f*x])^3) - (a*B)/(2*c^3*f*(I + Tan[e + f*x])^2)$

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{(c - ict \tan(e + fx))^3} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(c-icx)^4} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{A-iB}{c^4(i+x)^4} + \frac{B}{c^4(i+x)^3} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{a(A - iB)}{3c^3 f (i + \tan(e + fx))^3} - \frac{aB}{2c^3 f (i + \tan(e + fx))^2}$$

**Mathematica [A]**

time = 0.66, size = 72, normalized size = 1.31

$$\frac{a(-3iA + 2(-2iA + B) \cos(2(e + fx)) - 2(A + 2iB) \sin(2(e + fx)))(\cos(4(e + fx)) + i \sin(4(e + fx)))}{24c^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^3,x]
```

```
[Out] (a*((-3*I)*A + 2*((-2*I)*A + B)*Cos[2*(e + f*x)] - 2*(A + (2*I)*B)*Sin[2*(e + f*x)])*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)])/(24*c^3*f)
```

**Maple [A]**

time = 0.21, size = 43, normalized size = 0.78

method	result	size
derivativedivides	$\frac{a \left( -\frac{B}{2(i+\tan(fx+e))^2} - \frac{-iB+A}{3(i+\tan(fx+e))^3} \right)}{f c^3}$	43
default	$\frac{a \left( -\frac{B}{2(i+\tan(fx+e))^2} - \frac{-iB+A}{3(i+\tan(fx+e))^3} \right)}{f c^3}$	43
risch	$-\frac{a e^{6i(fx+e)} B}{24c^3 f} - \frac{ia e^{6i(fx+e)} A}{24c^3 f} - \frac{ia A e^{4i(fx+e)}}{8c^3 f} + \frac{a e^{2i(fx+e)} B}{8c^3 f} - \frac{ia e^{2i(fx+e)} A}{8c^3 f}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a/c^3*(-1/2*B/(I+tan(f*x+e))^2-1/3*(A-I*B)/(I+tan(f*x+e))^3)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 5.66, size = 62, normalized size = 1.13

$$\frac{(-iA - B)ae^{(6ifx+6ie)} - 3iAae^{(4ifx+4ie)} - 3(iA - B)ae^{(2ifx+2ie)}}{24c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")`

[Out]  $\frac{1}{24} * ((-I * A - B) * a * e^{(6 * I * f * x + 6 * I * e)} - 3 * I * A * a * e^{(4 * I * f * x + 4 * I * e)} - 3 * (I * A - B) * a * e^{(2 * I * f * x + 2 * I * e)}) / (c^3 * f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs.  $2(44) = 88$ .

time = 0.26, size = 201, normalized size = 3.65

$$\begin{cases} \frac{-192iAac^6f^2e^{4ie}e^{4ifx} + (-192iAac^6f^2e^{2ie} + 192Bac^6f^2e^{2ie})e^{2ifx} + (-64iAac^6f^2e^{6ie} - 64Bac^6f^2e^{6ie})e^{6ifx}}{1536c^9f^3} & \text{for } c^9f^3 \neq 0 \\ \frac{x(Aae^{6ie} + 2Aae^{4ie} + Aae^{2ie} - iBae^{6ie} + iBae^{2ie})}{4c^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)`

[Out] `Piecewise((( -192*I*A*a*c**6*f**2*exp(4*I*e)*exp(4*I*f*x) + (-192*I*A*a*c**6*f**2*exp(2*I*e) + 192*B*a*c**6*f**2*exp(2*I*e))*exp(2*I*f*x) + (-64*I*A*a*c**6*f**2*exp(6*I*e) - 64*B*a*c**6*f**2*exp(6*I*e))*exp(6*I*f*x))/(1536*c**9*f**3), Ne(c**9*f**3, 0)), (x*(A*a*exp(6*I*e) + 2*A*a*exp(4*I*e) + A*a*exp(2*I*e) - I*B*a*exp(6*I*e) + I*B*a*exp(2*I*e))/(4*c**3), True))`

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs.  $2(47) = 94$ .

time = 0.73, size = 149, normalized size = 2.71

$$\frac{2(3Aa \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 + 6iAa \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 3Ba \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 10Aa \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 2iBa \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 6iAa \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3Ba \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3Aa \tan(\frac{1}{2}fx + \frac{1}{2}e))}{3c^3f(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")`

[Out]  $-2/3*(3*A*a*\tan(1/2*f*x + 1/2*e)^5 + 6*I*A*a*\tan(1/2*f*x + 1/2*e)^4 - 3*B*a*\tan(1/2*f*x + 1/2*e)^4 - 10*A*a*\tan(1/2*f*x + 1/2*e)^3 - 2*I*B*a*\tan(1/2*f*x + 1/2*e)^3 - 6*I*A*a*\tan(1/2*f*x + 1/2*e)^2 + 3*B*a*\tan(1/2*f*x + 1/2*e)^2 + 3*A*a*\tan(1/2*f*x + 1/2*e))/(c^3*f*(\tan(1/2*f*x + 1/2*e) + I)^6)$

Mupad [B]

time = 8.63, size = 63, normalized size = 1.15

$$\frac{\frac{a(2A+B1i)}{6} + \frac{Ba \tan(e+fx)}{2}}{c^3 f (-\tan(e+fx)^3 - \tan(e+fx)^2 3i + 3 \tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)^3,x)`

[Out]  $((a*(2*A + B*1i))/6 + (B*a*\tan(e + f*x))/2)/(c^3*f*(3*\tan(e + f*x) - \tan(e + f*x)^2*3i - \tan(e + f*x)^3 + 1i))$

$$3.674 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^4} dx$$

Optimal. Leaf size=57

$$-\frac{a(iA+B)}{4c^4 f(i+\tan(e+fx))^4} - \frac{iaB}{3c^4 f(i+\tan(e+fx))^3}$$

[Out]  $-1/4*a*(I*A+B)/c^4/f/(I+\tan(f*x+e))^4-1/3*I*a*B/c^4/f/(I+\tan(f*x+e))^3$

Rubi [A]

time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$-\frac{a(B+ia)}{4c^4 f(\tan(e+fx)+i)^4} - \frac{iaB}{3c^4 f(\tan(e+fx)+i)^3}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^4, x]

[Out]  $-1/4*(a*(I*A + B))/(c^4*f*(I + Tan[e + f*x])^4) - ((I/3)*a*B)/(c^4*f*(I + Tan[e + f*x])^3)$

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{(c - ic \tan(e + fx))^4} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(c-icx)^5} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{iA+B}{c^5(i+x)^5} + \frac{iB}{c^5(i+x)^4} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{a(iA + B)}{4c^4 f (i + \tan(e + fx))^4} - \frac{iaB}{3c^4 f (i + \tan(e + fx))^3}$$

**Mathematica [A]**

time = 0.81, size = 97, normalized size = 1.70

$$\frac{a(2(-15iA + B) \cos(e + fx) + 3(-5iA + 3B) \cos(3(e + fx)) - (3A + 5iB)(2 \sin(e + fx) + 3 \sin(3(e + fx))))(\cos(5(e + fx)) + i \sin(5(e + fx)))}{192c^4 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^4,x]
```

```
[Out] (a*(2*((-15*I)*A + B)*Cos[e + f*x] + 3*((-5*I)*A + 3*B)*Cos[3*(e + f*x)] - (3*A + (5*I)*B)*(2*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))*(Cos[5*(e + f*x)] + I*Sin[5*(e + f*x)])/(192*c^4*f)
```

**Maple [A]**

time = 0.23, size = 44, normalized size = 0.77

method	result
derivativedivides	$\frac{a \left( -\frac{iA+B}{4(i+\tan(fx+e))^4} - \frac{iB}{3(i+\tan(fx+e))^3} \right)}{f c^4}$
default	$\frac{a \left( -\frac{iA+B}{4(i+\tan(fx+e))^4} - \frac{iB}{3(i+\tan(fx+e))^3} \right)}{f c^4}$
risch	$-\frac{a e^{8i(fx+e)} B}{64c^4 f} - \frac{ia e^{8i(fx+e)} A}{64c^4 f} - \frac{e^{6i(fx+e)} Ba}{48c^4 f} - \frac{ie^{6i(fx+e)} aA}{16c^4 f} + \frac{e^{4i(fx+e)} Ba}{32c^4 f} - \frac{3ie^{4i(fx+e)} aA}{32c^4 f} + \frac{a e^{2i(fx+e)}}{16c^4 f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a/c^4*(-1/4*(I*A+B)/(I+tan(f*x+e))^4-1/3*I*B/(I+tan(f*x+e))^3)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 13.34, size = 85, normalized size = 1.49

$$\frac{3(iA + B)ae^{(8ifx + 8ie)} + 4(3iA + B)ae^{(6ifx + 6ie)} + 6(3iA - B)ae^{(4ifx + 4ie)} + 12(iA - B)ae^{(2ifx + 2ie)}}{192c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="fricas")

[Out]  $-1/192*(3*(I*A + B)*a*e^{(8*I*f*x + 8*I*e)} + 4*(3*I*A + B)*a*e^{(6*I*f*x + 6*I*e)} + 6*(3*I*A - B)*a*e^{(4*I*f*x + 4*I*e)} + 12*(I*A - B)*a*e^{(2*I*f*x + 2*I*e)})/(c^4*f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs.  $2(46) = 92$ .

time = 0.32, size = 304, normalized size = 5.33

$$\begin{cases} \frac{(-98304iAac^{12}f^3e^{2ie} + 98304Bac^{12}f^3e^{2ie})e^{2ifx} + (-147456iAac^{12}f^3e^{4ie} + 49152Bac^{12}f^3e^{4ie})e^{4ifx} + (-98304iAac^{12}f^3e^{6ie} - 32768Bac^{12}f^3e^{6ie})e^{6ifx} + (-24576iAac^{12}f^3e^{8ie} - 24576Bac^{12}f^3e^{8ie})e^{8ifx}}{1572864c^{16}f^4} & \text{for } c^{16}f^4 \neq 0 \\ \frac{x(Aae^{8ie} + 3Aae^{6ie} + 3Aae^{4ie} + Aae^{2ie} - iBae^{8ie} - iBae^{6ie} + iBae^{4ie} + iBae^{2ie})}{8c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*4,x)

[Out] Piecewise(((((-98304\*I\*A\*a\*c\*\*12\*f\*\*3\*exp(2\*I\*e) + 98304\*B\*a\*c\*\*12\*f\*\*3\*exp(2\*I\*e))\*exp(2\*I\*f\*x) + (-147456\*I\*A\*a\*c\*\*12\*f\*\*3\*exp(4\*I\*e) + 49152\*B\*a\*c\*\*12\*f\*\*3\*exp(4\*I\*e))\*exp(4\*I\*f\*x) + (-98304\*I\*A\*a\*c\*\*12\*f\*\*3\*exp(6\*I\*e) - 32768\*B\*a\*c\*\*12\*f\*\*3\*exp(6\*I\*e))\*exp(6\*I\*f\*x) + (-24576\*I\*A\*a\*c\*\*12\*f\*\*3\*exp(8\*I\*e) - 24576\*B\*a\*c\*\*12\*f\*\*3\*exp(8\*I\*e))\*exp(8\*I\*f\*x))/(1572864\*c\*\*16\*f\*\*4), Ne(c\*\*16\*f\*\*4, 0)), (x\*(A\*a\*exp(8\*I\*e) + 3\*A\*a\*exp(6\*I\*e) + 3\*A\*a\*exp(4\*I\*e) + A\*a\*exp(2\*I\*e) - I\*B\*a\*exp(8\*I\*e) - I\*B\*a\*exp(6\*I\*e) + I\*B\*a\*exp(4\*I\*e) + I\*B\*a\*exp(2\*I\*e))/(8\*c\*\*4), True))

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs.  $2(47) = 94$ .

time = 0.95, size = 213, normalized size = 3.74

$$\frac{2(3Aa \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 + 9iAa \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 3Ba \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 21Aa \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 4Ba \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 24iAa \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 8Ba \tan(\frac{1}{2}fx + \frac{1}{2}e) + 21Aa \tan(\frac{1}{2}fx + \frac{1}{2}e) + 4iBa \tan(\frac{1}{2}fx + \frac{1}{2}e) + 9iAa \tan(\frac{1}{2}fx + \frac{1}{2}e) - 3Ba \tan(\frac{1}{2}fx + \frac{1}{2}e) - 3Aa \tan(\frac{1}{2}fx + \frac{1}{2}e))}{3c^4 f (\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out] 
$$\frac{-2/3*(3*A*a*\tan(1/2*f*x + 1/2*e)^7 + 9*I*A*a*\tan(1/2*f*x + 1/2*e)^6 - 3*B*a*\tan(1/2*f*x + 1/2*e)^6 - 21*A*a*\tan(1/2*f*x + 1/2*e)^5 - 4*I*B*a*\tan(1/2*f*x + 1/2*e)^5 - 24*I*A*a*\tan(1/2*f*x + 1/2*e)^4 + 8*B*a*\tan(1/2*f*x + 1/2*e)^4 + 21*A*a*\tan(1/2*f*x + 1/2*e)^3 + 4*I*B*a*\tan(1/2*f*x + 1/2*e)^3 + 9*I*A*a*\tan(1/2*f*x + 1/2*e)^2 - 3*B*a*\tan(1/2*f*x + 1/2*e)^2 - 3*A*a*\tan(1/2*f*x + 1/2*e))}{(c^4*f*(\tan(1/2*f*x + 1/2*e) + I)^8)}$$

Mupad [B]

time = 8.67, size = 73, normalized size = 1.28

$$\frac{\frac{a(-B+Ai)}{12} + \frac{Ba \tan(e+fx) i}{3}}{c^4 f (\tan(e+fx)^4 + \tan(e+fx)^3 4i - 6 \tan(e+fx)^2 - \tan(e+fx) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i))/(c - c\*tan(e + f\*x)\*1i))^4,x)

[Out] 
$$-\left(\frac{a(A*3i - B)}{12} + \frac{B*a*\tan(e + f*x)*1i}{3}\right) / (c^4*f*(\tan(e + f*x)^3*4i - 6*\tan(e + f*x)^2 - \tan(e + f*x)*4i + \tan(e + f*x)^4 + 1))$$

$$3.675 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^5} dx$$

Optimal. Leaf size=55

$$\frac{a(A-iB)}{5c^5 f(i+\tan(e+fx))^5} + \frac{aB}{4c^5 f(i+\tan(e+fx))^4}$$

[Out] 1/5\*a\*(A-I\*B)/c^5/f/(I+tan(f\*x+e))^5+1/4\*a\*B/c^5/f/(I+tan(f\*x+e))^4

Rubi [A]

time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{a(A-iB)}{5c^5 f(\tan(e+fx)+i)^5} + \frac{aB}{4c^5 f(\tan(e+fx)+i)^4}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^5, x]

[Out] (a\*(A - I\*B))/(5\*c^5\*f\*(I + Tan[e + f\*x])^5) + (a\*B)/(4\*c^5\*f\*(I + Tan[e + f\*x])^4)

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{(c - ict \tan(e + fx))^5} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(c-icx)^6} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{-A+iB}{c^6(i+x)^6} - \frac{B}{c^6(i+x)^5} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{a(A - iB)}{5c^5 f (i + \tan(e + fx))^5} + \frac{aB}{4c^5 f (i + \tan(e + fx))^4}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 124 vs. 2(55) = 110.  
time = 1.28, size = 124, normalized size = 2.25

$$\frac{ia(20A + 5(6A + iB) \cos(2(e + fx)) + 4(3A + 2iB) \cos(4(e + fx)) - 10iA \sin(2(e + fx)) + 15B \sin(2(e + fx)) - 8iA \sin(4(e + fx)) + 12B \sin(4(e + fx)))(\cos(6(e + fx)) + i \sin(6(e + fx)))}{320c^5 f}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^5,x]

[Out] ((-1/320\*I)\*a\*(20\*A + 5\*(6\*A + I\*B)\*Cos[2\*(e + f\*x)] + 4\*(3\*A + (2\*I)\*B)\*Cos[4\*(e + f\*x)] - (10\*I)\*A\*Sin[2\*(e + f\*x)] + 15\*B\*Sin[2\*(e + f\*x)] - (8\*I)\*A\*Sin[4\*(e + f\*x)] + 12\*B\*Sin[4\*(e + f\*x)]\*(Cos[6\*(e + f\*x)] + I\*Sin[6\*(e + f\*x)]))/(c^5\*f)

**Maple [A]**

time = 0.28, size = 45, normalized size = 0.82

method	result
derivativedivides	$\frac{a \left( -\frac{iB-A}{5(i+\tan(fx+e))^5} + \frac{B}{4(i+\tan(fx+e))^4} \right)}{f c^5}$
default	$\frac{a \left( -\frac{iB-A}{5(i+\tan(fx+e))^5} + \frac{B}{4(i+\tan(fx+e))^4} \right)}{f c^5}$
risch	$-\frac{a e^{10i(fx+e)} B}{160c^5 f} - \frac{ia e^{10i(fx+e)} A}{160c^5 f} - \frac{e^{8i(fx+e)} Ba}{64c^5 f} - \frac{ie^{8i(fx+e)} aA}{32c^5 f} - \frac{iaA e^{6i(fx+e)}}{16c^5 f} + \frac{e^{4i(fx+e)} Ba}{32c^5 f} - \frac{ie^{4i(fx+e)}}{16c^5 f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^5,x,method=\_RETURNVERBOSE)

[Out] 1/f\*a/c^5\*(-1/5\*(-A+I\*B)/(I+tan(f\*x+e))^5+1/4\*B/(I+tan(f\*x+e))^4)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs.  $2(47) = 94$ .

time = 3.03, size = 99, normalized size = 1.80

$$\frac{2(iA+B)ae^{(10ifx+10ie)} + 5(2iA+B)ae^{(8ifx+8ie)} + 20iAae^{(6ifx+6ie)} + 10(2iA-B)ae^{(4ifx+4ie)} + 10(iA-B)ae^{(2ifx+2ie)}}{320c^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, algorithm="fricas")`

[Out]  $-1/320*(2*(I*A + B)*a*e^{(10*I*f*x + 10*I*e)} + 5*(2*I*A + B)*a*e^{(8*I*f*x + 8*I*e)} + 20*I*A*a*e^{(6*I*f*x + 6*I*e)} + 10*(2*I*A - B)*a*e^{(4*I*f*x + 4*I*e)} + 10*(I*A - B)*a*e^{(2*I*f*x + 2*I*e)})/(c^5*f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs.  $2(42) = 84$ .

time = 0.43, size = 348, normalized size = 6.33

$$\begin{cases} \frac{-10485760iAac^{20}f^4e^{6ifx} + (-5242880iAac^{20}f^4e^{2ie} + 5242880Bac^{20}f^4e^{2ie})e^{2ifx} + (-10485760iAac^{20}f^4e^{4ie} + 5242880Bac^{20}f^4e^{4ie})e^{4ifx} + (-5242880iAac^{20}f^4e^{6ie} - 2621440Bac^{20}f^4e^{6ie})e^{6ifx} + (-1048576iAac^{20}f^4e^{10ie} - 1048576Bac^{20}f^4e^{10ie})e^{10ifx}}{167772160c^{25}f^5} & \text{for } c^{25}f^5 \neq 0 \\ \frac{z(Aae^{10ie} + 4Aae^{6ie} + 6Aae^{4ie} + 4Aac^{4ie} + Aac^{2ie} - 1Bac^{10ie} - 2iBae^{8ie} + 2iBae^{6ie} + iBac^{2ie})}{16c^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**5,x)`

[Out] `Piecewise((( -10485760*I*A*a*c**20*f**4*exp(6*I*e)*exp(6*I*f*x) + (-5242880*I*A*a*c**20*f**4*exp(2*I*e) + 5242880*B*a*c**20*f**4*exp(2*I*e))*exp(2*I*f*x) + (-10485760*I*A*a*c**20*f**4*exp(4*I*e) + 5242880*B*a*c**20*f**4*exp(4*I*e))*exp(4*I*f*x) + (-5242880*I*A*a*c**20*f**4*exp(8*I*e) - 2621440*B*a*c**20*f**4*exp(8*I*e))*exp(8*I*f*x) + (-1048576*I*A*a*c**20*f**4*exp(10*I*e) - 1048576*B*a*c**20*f**4*exp(10*I*e))*exp(10*I*f*x))/(167772160*c**25*f**5), Ne(c**25*f**5, 0)), (x*(A*a*exp(10*I*e) + 4*A*a*exp(8*I*e) + 6*A*a*exp(6*I*e) + 4*A*a*exp(4*I*e) + A*a*exp(2*I*e) - I*B*a*exp(10*I*e) - 2*I*B*a*exp(8*I*e) + 2*I*B*a*exp(4*I*e) + I*B*a*exp(2*I*e))/(16*c**5), True))`

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs.  $2(47) = 94$ .

time = 1.05, size = 277, normalized size = 5.04

$$\frac{2(5 \operatorname{Arctan}\left(\frac{1}{f+1}\right) + 20 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^2 - 5 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^3 - 60 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^4 - 100 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^5 + 120 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^6 + 24 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^7 + 100 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^8 - 25 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^9 - 60 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^{10} - 100 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^{11} + 20 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^{12} + 5 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^{13} + 5 \operatorname{Arctan}\left(\frac{1}{f+1}\right)^{14})}{5 \sqrt{f^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^5,x, algorithm="giac")

[Out] 
$$\frac{-2/5*(5*A*a*\tan(1/2*f*x + 1/2*e)^9 + 20*I*A*a*\tan(1/2*f*x + 1/2*e)^8 - 5*B*a*\tan(1/2*f*x + 1/2*e)^8 - 60*A*a*\tan(1/2*f*x + 1/2*e)^7 - 10*I*B*a*\tan(1/2*f*x + 1/2*e)^7 - 100*I*A*a*\tan(1/2*f*x + 1/2*e)^6 + 25*B*a*\tan(1/2*f*x + 1/2*e)^6 + 126*A*a*\tan(1/2*f*x + 1/2*e)^5 + 24*I*B*a*\tan(1/2*f*x + 1/2*e)^5 + 100*I*A*a*\tan(1/2*f*x + 1/2*e)^4 - 25*B*a*\tan(1/2*f*x + 1/2*e)^4 - 60*A*a*\tan(1/2*f*x + 1/2*e)^3 - 10*I*B*a*\tan(1/2*f*x + 1/2*e)^3 - 20*I*A*a*\tan(1/2*f*x + 1/2*e)^2 + 5*B*a*\tan(1/2*f*x + 1/2*e)^2 + 5*A*a*\tan(1/2*f*x + 1/2*e))}{c^5*f*(\tan(1/2*f*x + 1/2*e) + I)^{10}}$$

Mupad [B]

time = 8.60, size = 82, normalized size = 1.49

$$\frac{\frac{a(4A+B I)}{20} + \frac{B a \tan(e+f x)}{4}}{c^5 f (\tan(e+f x)^5 + \tan(e+f x)^4 5i - 10 \tan(e+f x)^3 - \tan(e+f x)^2 10i + 5 \tan(e+f x) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i))/(c - c\*tan(e + f\*x)\*1i)^5,x)

[Out] 
$$\left( \frac{a*(4*A + B*1i)}{20} + \frac{B*a*\tan(e + f*x)}{4} \right) / (c^5*f*(5*\tan(e + f*x) - \tan(e + f*x)^2*10i - 10*\tan(e + f*x)^3 + \tan(e + f*x)^4*5i + \tan(e + f*x)^5 + 1i))$$

$$3.676 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ictan(e + fx))^n dx$$

Optimal. Leaf size=109

$$\frac{2a^2(iA + B)(c - ictan(e + fx))^n}{fn} - \frac{a^2(iA + 3B)(c - ictan(e + fx))^{1+n}}{cf(1+n)} + \frac{a^2B(c - ictan(e + fx))^{2+n}}{c^2f(2+n)}$$

[Out]  $2*a^2*(I*A+B)*(c-I*c*tan(f*x+e))^n/f/n - a^2*(I*A+3*B)*(c-I*c*tan(f*x+e))^{(1+n)}/c/f/(1+n) + a^2*B*(c-I*c*tan(f*x+e))^{(2+n)}/c^2/f/(2+n)$

Rubi [A]

time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{2a^2(B + iA)(c - ictan(e + fx))^n}{fn} - \frac{a^2(3B + iA)(c - ictan(e + fx))^{n+1}}{cf(n+1)} + \frac{a^2B(c - ictan(e + fx))^{n+2}}{c^2f(n+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^n, x]$

[Out]  $(2*a^2*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^n)/(f*n) - (a^2*(I*A + 3*B)*(c - I*c*\text{Tan}[e + f*x])^{(1+n)})/(c*f*(1+n)) + (a^2*B*(c - I*c*\text{Tan}[e + f*x])^{(2+n)})/(c^2*f*(2+n))$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)}*((e_. + (f_.)*(x_.))^{(p_.)}), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) || \text{EqQ}[p, 1] || (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] || \text{LeQ}[9*p + 5*(n + 2), 0] || \text{GeQ}[n + p + 1, 0] || (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.))])^{(m_.)}*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.))])^{(n_.)}), x\_Symbol] :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ic \tan(e + fx))^n dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx)(c - icx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (2a(A - iB)(c - icx)) dx\right)}{f}$$

$$= \frac{2a^2(iA + B)(c - ic \tan(e + fx))^n}{fn}$$

**Mathematica [A]**

time = 2.66, size = 146, normalized size = 1.34

$$\frac{a^2 e^{n(-\log(\csc(e+fx)) + \log(c - ic \tan(e+fx)))} \sec^2(e+fx) (c \sec(e+fx))^n ((2+n)(-B(-2+n) + iA(2+n)) + (iA(2+n)^2 + B(4+2n+n^2)) \cos(2(e+fx)) - n(A(2+n) - iB(4+n)) \sin(2(e+fx)))}{2fn(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n,x]
```

```
[Out] (a^2*E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*Sec[e + f*x]^2*(c*Sec[e + f*x])^n*((2 + n)*(-B*(-2 + n)) + I*A*(2 + n)) + (I*A*(2 + n)^2 + B*(4 + 2*n + n^2))*Cos[2*(e + f*x)] - n*(A*(2 + n) - I*B*(4 + n))*Sin[2*(e + f*x)])/(2*f*n*(1 + n)*(2 + n))
```

**Maple [A]**

time = 1.49, size = 167, normalized size = 1.53

method	result
norman	$\frac{(iA a^2 n^2 + 4iA a^2 n + 4iA a^2 + a^2 B n + 4a^2 B) e^{n \ln(c - ic \tan(fx+e))}}{fn(1+n)(2+n)} - \frac{a^2 B (\tan^2(fx+e)) e^{n \ln(c - ic \tan(fx+e))}}{f(2+n)} - \frac{a^2 (-iBn + An - 4iB + 2A)}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/n/(1+n)*(I*A*a^2*n^2+4*I*A*a^2*n+4*I*A*a^2+a^2*B*n+4*a^2*B)/(2+n)*exp(n*ln(c-I*c*tan(f*x+e)))-a^2*B/f/(2+n)*tan(f*x+e)^2*exp(n*ln(c-I*c*tan(f*x+e)))-a^2*(-I*B*n+A*n-4*I*B+2*A)/f/(1+n)/(2+n)*tan(f*x+e)*exp(n*ln(c-I*c*tan(f*x+e)))
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 689 vs.  $2(102) = 204$ .

time = 0.65, size = 689, normalized size = 6.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="maxima")

[Out] 2\*((((A + I\*B)\*a^2\*c^n\*n^2 + 4\*A\*a^2\*c^n\*n + 4\*(A - I\*B)\*a^2\*c^n)\*2^n\*cos(-2\*f\*x + n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1) - 2\*e) + ((A - I\*B)\*a^2\*c^n\*n^2 + 3\*(A - I\*B)\*a^2\*c^n\*n + 2\*(A - I\*B)\*a^2\*c^n)\*2^n\*cos(-4\*f\*x + n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1) - 4\*e) + ((A + I\*B)\*a^2\*c^n\*n + 2\*(A - I\*B)\*a^2\*c^n)\*2^n\*cos(n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1)) - ((I\*A - B)\*a^2\*c^n\*n^2 + 4\*I\*A\*a^2\*c^n\*n + 4\*(I\*A + B)\*a^2\*c^n)\*2^n\*sin(-2\*f\*x + n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1) - 2\*e) - ((I\*A + B)\*a^2\*c^n\*n^2 + 3\*(I\*A + B)\*a^2\*c^n\*n + 2\*(I\*A + B)\*a^2\*c^n)\*2^n\*sin(-4\*f\*x + n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1) - 4\*e) - ((I\*A - B)\*a^2\*c^n\*n + 2\*(I\*A + B)\*a^2\*c^n)\*2^n\*sin(n\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e) + 1)))/((( -I\*n^3 - 3\*I\*n^2 - 2\*I\*n)\*(cos(2\*f\*x + 2\*e)^2 + sin(2\*f\*x + 2\*e)^2 + 2\*cos(2\*f\*x + 2\*e) + 1)^(1/2\*n)\*cos(4\*f\*x + 4\*e) + (n^3 + 3\*n^2 + 2\*n)\*(cos(2\*f\*x + 2\*e)^2 + sin(2\*f\*x + 2\*e)^2 + 2\*cos(2\*f\*x + 2\*e) + 1)^(1/2\*n)\*sin(4\*f\*x + 4\*e) + (-I\*n^3 - 3\*I\*n^2 - 2\*(I\*n^3 + 3\*I\*n^2 + 2\*I\*n)\*cos(2\*f\*x + 2\*e) + 2\*(n^3 + 3\*n^2 + 2\*n)\*sin(2\*f\*x + 2\*e) - 2\*I\*n\*(cos(2\*f\*x + 2\*e)^2 + sin(2\*f\*x + 2\*e)^2 + 2\*cos(2\*f\*x + 2\*e) + 1)^(1/2\*n))\*f)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(102) = 204.

time = 4.97, size = 211, normalized size = 1.94

$$\frac{2((-iA+B)a^2n+2(-iA-B)a^2+((-iA-B)a^2n^2+3(-iA-B)a^2n+2(-iA-B)a^2)e^{4i(fx+4e)}+((-iA+B)a^2n-4iAa^2n+4(-iA-B)a^2)e^{2i(fx+2e)})\left(\frac{2c}{e^{2i(fx+2e)}+1}\right)^n}{fn^3+3fn^2+2fn+(fn^3+3fn^2+2fn)e^{4i(fx+4e)}+2(fn^3+3fn^2+2fn)e^{2i(fx+2e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="fricas")

[Out] -2\*((-I\*A + B)\*a^2\*n + 2\*(-I\*A - B)\*a^2 + ((-I\*A - B)\*a^2\*n^2 + 3\*(-I\*A - B)\*a^2\*n + 2\*(-I\*A - B)\*a^2)\*e^(4\*I\*f\*x + 4\*I\*e) + ((-I\*A + B)\*a^2\*n^2 - 4\*I\*A\*a^2\*n + 4\*(-I\*A - B)\*a^2)\*e^(2\*I\*f\*x + 2\*I\*e)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^n/(f\*n^3 + 3\*f\*n^2 + 2\*f\*n + (f\*n^3 + 3\*f\*n^2 + 2\*f\*n)\*e^(4\*I\*f\*x + 4\*I\*e) + 2\*(f\*n^3 + 3\*f\*n^2 + 2\*f\*n)\*e^(2\*I\*f\*x + 2\*I\*e))

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1482 vs. 2(87) = 174.

time = 1.50, size = 1482, normalized size = 13.60

$$\begin{cases} x(A+B\tan(e))(a\tan(e)+a)^2(-i\tan(e)+c)^2 & \text{for } f=0 \\ -\frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} & \text{for } n=-2 \\ -\frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} & \text{for } n=-1 \\ 2Aa^2 + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - 2Ba^2x + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} & \text{for } n=0 \\ -\frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} - \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} + \frac{2Aa^2f\log(\tan(fx+e))}{2Aa^2f\log(\tan(fx+e))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
[Out] Piecewise((x*(A + B*tan(e))*(I*a*tan(e) + a)**2*(-I*c*tan(e) + c)**n, Eq(f,
0)), (-2*A*a**2*tan(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e
+ f*x) - 2*c**2*f) - 2*I*B*a**2*f*x*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)*
**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 4*B*a**2*f*x*tan(e + f*x)/(2*c**
2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 2*I*B*a**2*f*x/
(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + B*a**2*lo
g(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2
*f*tan(e + f*x) - 2*c**2*f) + 2*I*B*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - B*a**
2*log(tan(e + f*x)**2 + 1)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f
*x) - 2*c**2*f) + 6*I*B*a**2*tan(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c
**2*f*tan(e + f*x) - 2*c**2*f) - 4*B*a**2/(2*c**2*f*tan(e + f*x)**2 + 4*I*c
**2*f*tan(e + f*x) - 2*c**2*f), Eq(n, -2)), (-2*A*a**2*f*x*tan(e + f*x)/(2*
c*f*tan(e + f*x) + 2*I*c*f) - 2*I*A*a**2*f*x/(2*c*f*tan(e + f*x) + 2*I*c*f)
- I*A*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*tan(e + f*x) + 2*I
*c*f) + A*a**2*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e + f*x) + 2*I*c*f) + 4*
A*a**2/(2*c*f*tan(e + f*x) + 2*I*c*f) + 6*I*B*a**2*f*x*tan(e + f*x)/(2*c*f*
tan(e + f*x) + 2*I*c*f) - 6*B*a**2*f*x/(2*c*f*tan(e + f*x) + 2*I*c*f) - 3*B
*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*tan(e + f*x) + 2*I*c*f)
- 3*I*B*a**2*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e + f*x) + 2*I*c*f) - 2*I*
B*a**2*tan(e + f*x)**2/(2*c*f*tan(e + f*x) + 2*I*c*f) - 6*I*B*a**2/(2*c*f*t
an(e + f*x) + 2*I*c*f), Eq(n, -1)), (2*A*a**2*x + I*A*a**2*log(tan(e + f*x)
**2 + 1)/f - A*a**2*tan(e + f*x)/f - 2*I*B*a**2*x + B*a**2*log(tan(e + f*x)
**2 + 1)/f - B*a**2*tan(e + f*x)**2/(2*f) + 2*I*B*a**2*tan(e + f*x)/f, Eq(n
, 0)), (-A*a**2*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(f*n**3 + 3*f*
n**2 + 2*f*n) + I*A*a**2*n**2*(-I*c*tan(e + f*x) + c)**n/(f*n**3 + 3*f*n**2
+ 2*f*n) - 2*A*a**2*n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(f*n**3 + 3*
f*n**2 + 2*f*n) + 4*I*A*a**2*n*(-I*c*tan(e + f*x) + c)**n/(f*n**3 + 3*f*n**
2 + 2*f*n) + 4*I*A*a**2*(-I*c*tan(e + f*x) + c)**n/(f*n**3 + 3*f*n**2 + 2*f
*n) - B*a**2*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**2/(f*n**3 + 3*f*
n**2 + 2*f*n) + I*B*a**2*n**2*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(f*n*
**3 + 3*f*n**2 + 2*f*n) - B*a**2*n*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)**
2/(f*n**3 + 3*f*n**2 + 2*f*n) + 4*I*B*a**2*n*(-I*c*tan(e + f*x) + c)**n*tan
(e + f*x)/(f*n**3 + 3*f*n**2 + 2*f*n) + B*a**2*n*(-I*c*tan(e + f*x) + c)**n
/(f*n**3 + 3*f*n**2 + 2*f*n) + 4*B*a**2*(-I*c*tan(e + f*x) + c)**n/(f*n**3
+ 3*f*n**2 + 2*f*n), True))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^2\*(-I\*c\*tan(f\*x + e) + c)^n, x)

**Mupad [B]**

time = 11.38, size = 193, normalized size = 1.77

$$\frac{e^{-e 2i - f x 2i} \left( c - \frac{c \sin(e + f x) 1i}{\cos(e + f x)} \right)^n \left( \frac{2 a^2 (2 A - B 2i + A n + B n 1i)}{f n (n^2 1i + n 3i + 2i)} + \frac{2 a^2 e^{e 4i + f x 4i} (A - B 1i) (n^2 + 3 n + 2)}{f n (n^2 1i + n 3i + 2i)} + \frac{2 a^2 e^{e 2i + f x 2i} (n + 2) (2 A - B 2i + A n + B n 1i)}{f n (n^2 1i + n 3i + 2i)} \right)}{4 \cos(e + f x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^n,x)

[Out] -(exp(- e\*2i - f\*x\*2i)\*(c - (c\*sin(e + f\*x)\*1i)/cos(e + f\*x))^n\*((2\*a^2\*(2\*A - B\*2i + A\*n + B\*n\*1i))/(f\*n\*(n\*3i + n^2\*1i + 2i)) + (2\*a^2\*exp(e\*4i + f\*x\*4i)\*(A - B\*1i)\*(3\*n + n^2 + 2))/(f\*n\*(n\*3i + n^2\*1i + 2i)) + (2\*a^2\*exp(e\*2i + f\*x\*2i)\*(n + 2)\*(2\*A - B\*2i + A\*n + B\*n\*1i))/(f\*n\*(n\*3i + n^2\*1i + 2i))))/(4\*cos(e + f\*x)^2)

$$3.677 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^5 dx$$

Optimal. Leaf size=99

$$\frac{2a^2(iA + B)c^5(1 - i \tan(e + fx))^5}{5f} - \frac{a^2(iA + 3B)c^5(1 - i \tan(e + fx))^6}{6f} + \frac{a^2Bc^5(1 - i \tan(e + fx))^7}{7f}$$

[Out]  $2/5*a^2*(I*A+B)*c^5*(1-I*\tan(f*x+e))^5/f-1/6*a^2*(I*A+3*B)*c^5*(1-I*\tan(f*x+e))^6/f+1/7*a^2*B*c^5*(1-I*\tan(f*x+e))^7/f$

Rubi [A]

time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^2c^5(3B + iA)(1 - i \tan(e + fx))^6}{6f} + \frac{2a^2c^5(B + iA)(1 - i \tan(e + fx))^5}{5f} + \frac{a^2Bc^5(1 - i \tan(e + fx))^7}{7f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^5, x]$

[Out]  $(2*a^2*(I*A + B)*c^5*(1 - I*\text{Tan}[e + f*x])^5)/(5*f) - (a^2*(I*A + 3*B)*c^5*(1 - I*\text{Tan}[e + f*x])^6)/(6*f) + (a^2*B*c^5*(1 - I*\text{Tan}[e + f*x])^7)/(7*f)$

Rule 78

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(c + d*\text{tan}[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps



$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ictan(e + fx))^5 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx)(c - icx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (2a(A - iB)(c - icx) dx\right)}{f}$$

$$= \frac{2a^2(iA + B)c^5(1 - i \tan(e + fx))^5}{5f}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 254 vs.  $2(99) = 198$ .  
time = 1.90, size = 254, normalized size = 2.57

$a^2c^5 \tan^7(e + fx) (35(-7A + 3B) \cos^2(fx) + 35(-7A + 3B) \cos(2e + 3fx) - 105A \cos(2e + 3fx) + 105B \cos(2e + 3fx) - 105A \cos(4e + 3fx) + 105B \cos(4e + 3fx) + 245A \sin(fx) + 105B \sin(fx) - 245A \sin(2e + fx) - 105B \sin(2e + fx) + 180A \sin(2e + 3fx) + 210B \sin(2e + 3fx) - 105A \sin(4e + 3fx) - 105B \sin(4e + 3fx) + 98A \sin(4e + 5fx) + 42B \sin(4e + 5fx) + 14A \sin(6e + 7fx) + 6B \sin(6e + 7fx)) / (840fx)$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^5,x]

[Out] (a^2\*c^5\*Sec[e]\*Sec[e + f\*x]^7\*(35\*((-7\*I)\*A + 3\*B)\*Cos[f\*x] + 35\*((-7\*I)\*A + 3\*B)\*Cos[2\*e + f\*x] - (105\*I)\*A\*Cos[2\*e + 3\*f\*x] + 105\*B\*Cos[2\*e + 3\*f\*x] - (105\*I)\*A\*Cos[4\*e + 3\*f\*x] + 105\*B\*Cos[4\*e + 3\*f\*x] + 245\*A\*Sin[f\*x] + (105\*I)\*B\*Sin[f\*x] - 245\*A\*Sin[2\*e + f\*x] - (105\*I)\*B\*Sin[2\*e + f\*x] + 189\*A\*Sin[2\*e + 3\*f\*x] + (21\*I)\*B\*Sin[2\*e + 3\*f\*x] - 105\*A\*Sin[4\*e + 3\*f\*x] - (105\*I)\*B\*Sin[4\*e + 3\*f\*x] + 98\*A\*Sin[4\*e + 5\*f\*x] + (42\*I)\*B\*Sin[4\*e + 5\*f\*x] + 14\*A\*Sin[6\*e + 7\*f\*x] + (6\*I)\*B\*Sin[6\*e + 7\*f\*x]))/(840\*f)

**Maple [A]**

time = 0.14, size = 147, normalized size = 1.48

method	result
risch	$\frac{32c^5 a^2 (42iA e^{4i(fx+e)} + 42B e^{4i(fx+e)} + 49iA e^{2i(fx+e)} - 21B e^{2i(fx+e)} + 7iA - 3B)}{105f(e^{2i(fx+e)} + 1)^7}$
derivativedivides	$- \frac{ic^5 a^2 \left( -\frac{B(\tan^7(fx+e))}{7} + \frac{(-3iB-A)(\tan^6(fx+e))}{6} + \frac{(iA+4i(iB-A)+6B)(\tan^5(fx+e))}{5} + \frac{(-2iB+2A)(\tan^4(fx+e))}{4} + \frac{(-6iA+3B)(\tan^3(fx+e))}{3} \right)}{f}$
default	$- \frac{ic^5 a^2 \left( -\frac{B(\tan^7(fx+e))}{7} + \frac{(-3iB-A)(\tan^6(fx+e))}{6} + \frac{(iA+4i(iB-A)+6B)(\tan^5(fx+e))}{5} + \frac{(-2iB+2A)(\tan^4(fx+e))}{4} + \frac{(-6iA+3B)(\tan^3(fx+e))}{3} \right)}{f}$
norman	$\frac{A a^2 c^5 \tan(fx+e)}{f} - \frac{(-iA a^2 c^5 + 3B a^2 c^5)(\tan^6(fx+e))}{6f} - \frac{(2iB a^2 c^5 + 3A a^2 c^5)(\tan^5(fx+e))}{5f} - \frac{(3iB a^2 c^5 + 2A a^2 c^5)(\tan^4(fx+e))}{4f} + \frac{(6iA a^2 c^5 - 3B a^2 c^5)(\tan^3(fx+e))}{3f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5,x,method=\_RE  
TURNVERBOSE)

[Out] -I/f\*c^5\*a^2\*(-1/7\*B\*tan(f\*x+e)^7+1/6\*(-A-3\*I\*B)\*tan(f\*x+e)^6+1/5\*(I\*A+4\*I\*  
(-A+I\*B)+6\*B)\*tan(f\*x+e)^5+1/4\*(2\*A-2\*I\*B)\*tan(f\*x+e)^4+1/3\*(-6\*I\*A-4\*I\*(-A  
+I\*B)-B)\*tan(f\*x+e)^3+1/2\*(3\*A+I\*B)\*tan(f\*x+e)^2+I\*A\*tan(f\*x+e))

**Maxima [A]**

time = 0.51, size = 158, normalized size = 1.60

$$\frac{30iBa^2c^5 \tan(fx+e)^7 + 35(iA-3B)a^2c^5 \tan(fx+e)^6 - 42(3A+2iB)a^2c^5 \tan(fx+e)^5 + 105(-iA-B)a^2c^5 \tan(fx+e)^4 - 70(2A+3iB)a^2c^5 \tan(fx+e)^3 + 105(-3iA+B)a^2c^5 \tan(fx+e)^2 + 210Aa^2c^5 \tan(fx+e)}{210f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5,x, alg  
orithm="maxima")

[Out] 1/210\*(30\*I\*B\*a^2\*c^5\*tan(f\*x + e)^7 + 35\*(I\*A - 3\*B)\*a^2\*c^5\*tan(f\*x + e)^  
6 - 42\*(3\*A + 2\*I\*B)\*a^2\*c^5\*tan(f\*x + e)^5 + 105\*(-I\*A - B)\*a^2\*c^5\*tan(f\*  
x + e)^4 - 70\*(2\*A + 3\*I\*B)\*a^2\*c^5\*tan(f\*x + e)^3 + 105\*(-3\*I\*A + B)\*a^2\*c  
^5\*tan(f\*x + e)^2 + 210\*A\*a^2\*c^5\*tan(f\*x + e))/f

**Fricas [A]**

time = 3.44, size = 161, normalized size = 1.63

$$\frac{32(42(-iA-B)a^2c^5e^{4i fx+4ie}) + 7(-7iA+3B)a^2c^5e^{2i fx+2ie} + (-7iA+3B)a^2c^5)}{105(fe^{14i fx+14ie}) + 7fe^{12i fx+12ie} + 21fe^{10i fx+10ie} + 35fe^{8i fx+8ie} + 35fe^{6i fx+6ie} + 21fe^{4i fx+4ie} + 7fe^{2i fx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5,x, alg  
orithm="fricas")

[Out] -32/105\*(42\*(-I\*A - B)\*a^2\*c^5\*e^(4\*I\*f\*x + 4\*I\*e) + 7\*(-7\*I\*A + 3\*B)\*a^2\*c  
^5\*e^(2\*I\*f\*x + 2\*I\*e) + (-7\*I\*A + 3\*B)\*a^2\*c^5)/(f\*e^(14\*I\*f\*x + 14\*I\*e) +  
7\*f\*e^(12\*I\*f\*x + 12\*I\*e) + 21\*f\*e^(10\*I\*f\*x + 10\*I\*e) + 35\*f\*e^(8\*I\*f\*x +  
8\*I\*e) + 35\*f\*e^(6\*I\*f\*x + 6\*I\*e) + 21\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 7\*f\*e^(2\*I\*  
f\*x + 2\*I\*e) + f)

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than  
twice the leaf count of optimal. 243 vs. 2(80) = 160.

time = 0.77, size = 243, normalized size = 2.45

$$\frac{224iAa^2c^5 - 96Ba^2c^5 + (1568iAa^2c^5e^{2ie} - 672Ba^2c^5e^{2ie})e^{2ifx} + (1344iAa^2c^5e^{4ie} + 1344Ba^2c^5e^{4ie})e^{4ifx}}{105fe^{14ie}e^{14ifx} + 735fe^{12ie}e^{12ifx} + 2205fe^{10ie}e^{10ifx} + 3675fe^{8ie}e^{8ifx} + 3675fe^{6ie}e^{6ifx} + 2205fe^{4ie}e^{4ifx} + 735fe^{2ie}e^{2ifx} + 105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5,x)

[Out] (224\*I\*A\*a\*\*2\*c\*\*5 - 96\*B\*a\*\*2\*c\*\*5 + (1568\*I\*A\*a\*\*2\*c\*\*5\*exp(2\*I\*e) - 672\*  
B\*a\*\*2\*c\*\*5\*exp(2\*I\*e))\*exp(2\*I\*f\*x) + (1344\*I\*A\*a\*\*2\*c\*\*5\*exp(4\*I\*e) + 134

$4*B*a**2*c**5*exp(4*I*e)*exp(4*I*f*x))/(105*f*exp(14*I*e)*exp(14*I*f*x) + 735*f*exp(12*I*e)*exp(12*I*f*x) + 2205*f*exp(10*I*e)*exp(10*I*f*x) + 3675*f*exp(8*I*e)*exp(8*I*f*x) + 3675*f*exp(6*I*e)*exp(6*I*f*x) + 2205*f*exp(4*I*e)*exp(4*I*f*x) + 735*f*exp(2*I*e)*exp(2*I*f*x) + 105*f)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs.  $2(86) = 172$ .  
time = 1.19, size = 191, normalized size = 1.93

$$\frac{32(-42iAa^2c^5e^{(4i fx+4ie)} - 42Ba^2c^5e^{(4i fx+4ie)} - 49iAa^2c^5e^{(2i fx+2ie)} + 21Ba^2c^5e^{(2i fx+2ie)} - 7iAa^2c^5 + 3Ba^2c^5)}{105(fe^{(14i fx+14ie)} + 7fe^{(12i fx+12ie)} + 21fe^{(10i fx+10ie)} + 35fe^{(8i fx+8ie)} + 35fe^{(6i fx+6ie)} + 21fe^{(4i fx+4ie)} + 7fe^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5,x, algorithm="giac")

[Out]  $-32/105*(-42*I*A*a^2*c^5*e^{(4*I*f*x + 4*I*e)} - 42*B*a^2*c^5*e^{(4*I*f*x + 4*I*e)} - 49*I*A*a^2*c^5*e^{(2*I*f*x + 2*I*e)} + 21*B*a^2*c^5*e^{(2*I*f*x + 2*I*e)} - 7*I*A*a^2*c^5 + 3*B*a^2*c^5)/(f*e^{(14*I*f*x + 14*I*e)} + 7*f*e^{(12*I*f*x + 12*I*e)} + 21*f*e^{(10*I*f*x + 10*I*e)} + 35*f*e^{(8*I*f*x + 8*I*e)} + 35*f*e^{(6*I*f*x + 6*I*e)} + 21*f*e^{(4*I*f*x + 4*I*e)} + 7*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad** [B]

time = 8.68, size = 158, normalized size = 1.60

$$\frac{Aa^2c^5\tan(e+fx) + \frac{a^2c^5\tan(e+fx)^3(-3B+A2i)1i}{3} + \frac{a^2c^5\tan(e+fx)^5(-2B+A3i)1i}{5} - \frac{a^2c^5\tan(e+fx)^2(3A+B1i)1i}{2} - \frac{a^2c^5\tan(e+fx)^4(A-B1i)1i}{2} + \frac{a^2c^5\tan(e+fx)^6(A+B3i)1i}{6} + \frac{Ba^2c^5\tan(e+fx)^71i}{7}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^5,x)

[Out]  $((a^2*c^5*\tan(e + f*x)^3*(A*2i - 3*B)*1i)/3 - (a^2*c^5*\tan(e + f*x)^2*(3*A + B*1i)*1i)/2 + (a^2*c^5*\tan(e + f*x)^5*(A*3i - 2*B)*1i)/5 + A*a^2*c^5*\tan(e + f*x) - (a^2*c^5*\tan(e + f*x)^4*(A - B*1i)*1i)/2 + (a^2*c^5*\tan(e + f*x)^6*(A + B*3i)*1i)/6 + (B*a^2*c^5*\tan(e + f*x)^7*1i)/7)/f$

$$3.678 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^4 dx$$

Optimal. Leaf size=99

$$\frac{a^2(iA + B)c^4(1 - i \tan(e + fx))^4}{2f} - \frac{a^2(iA + 3B)c^4(1 - i \tan(e + fx))^5}{5f} + \frac{a^2Bc^4(1 - i \tan(e + fx))^6}{6f}$$

[Out] 1/2\*a^2\*(I\*A+B)\*c^4\*(1-I\*tan(f\*x+e))^4/f-1/5\*a^2\*(I\*A+3\*B)\*c^4\*(1-I\*tan(f\*x+e))^5/f+1/6\*a^2\*B\*c^4\*(1-I\*tan(f\*x+e))^6/f

Rubi [A]

time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^2c^4(3B + iA)(1 - i \tan(e + fx))^5}{5f} + \frac{a^2c^4(B + iA)(1 - i \tan(e + fx))^4}{2f} + \frac{a^2Bc^4(1 - i \tan(e + fx))^6}{6f}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4, x]

[Out] (a^2\*(I\*A + B)\*c^4\*(1 - I\*Tan[e + f\*x])^4)/(2\*f) - (a^2\*(I\*A + 3\*B)\*c^4\*(1 - I\*Tan[e + f\*x])^5)/(5\*f) + (a^2\*B\*c^4\*(1 - I\*Tan[e + f\*x])^6)/(6\*f)

Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ictan(e + fx))^4 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx)(c - ix) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(2a(A - iB)(c - icx)\right) dx\right)}{f}$$

$$= \frac{a^2(iA + B)c^4(1 - i \tan(e + fx))^4}{2f}$$

**Mathematica [A]**

time = 1.44, size = 177, normalized size = 1.79

$$\frac{a^2 c^4 \sec^2(e + fx) (10(-3A + B) \cos(e) + 15(-A + B) \cos(e + 2fx) - 15A \cos(3e + 2fx) + 15B \cos(3e + 2fx) - 30A \sin(e) - 10B \sin(e) + 30A \sin(e + 2fx) - 15A \sin(3e + 2fx) - 15B \sin(3e + 2fx) + 18A \sin(3e + 4fx) + 6B \sin(3e + 4fx) + 3A \sin(5e + 6fx) + iB \sin(5e + 6fx))}{120f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4,x]

[Out] (a^2\*c^4\*Sec[e]\*Sec[e + f\*x]^6\*(10\*((-3\*I)\*A + B)\*Cos[e] + 15\*((-I)\*A + B)\*Cos[e + 2\*f\*x] - (15\*I)\*A\*Cos[3\*e + 2\*f\*x] + 15\*B\*Cos[3\*e + 2\*f\*x] - 30\*A\*Sin[e] - (10\*I)\*B\*Sin[e] + 30\*A\*Sin[e + 2\*f\*x] - 15\*A\*Sin[3\*e + 2\*f\*x] - (15\*I)\*B\*Sin[3\*e + 2\*f\*x] + 18\*A\*Sin[3\*e + 4\*f\*x] + (6\*I)\*B\*Sin[3\*e + 4\*f\*x] + 3\*A\*Sin[5\*e + 6\*f\*x] + I\*B\*Sin[5\*e + 6\*f\*x]))/(120\*f)

**Maple [A]**

time = 0.11, size = 116, normalized size = 1.17

method	result
risch	$\frac{8c^4 a^2 (15iA e^{4i(fx+e)} + 15B e^{4i(fx+e)} + 18iA e^{2i(fx+e)} - 6B e^{2i(fx+e)} + 3iA - B)}{15f(e^{2i(fx+e)} + 1)^6}$
derivativedivides	$c^4 a^2 \left( -\frac{B(\tan^6(fx+e))}{6} + \frac{(-2iB-A)(\tan^5(fx+e))}{5} + \frac{(iA+3i(iB-A)+3B)(\tan^4(fx+e))}{4} - \frac{2iB(\tan^3(fx+e))}{3} + \frac{(-3iA-i(iB-A))}{2} \right) \frac{1}{f}$
default	$c^4 a^2 \left( -\frac{B(\tan^6(fx+e))}{6} + \frac{(-2iB-A)(\tan^5(fx+e))}{5} + \frac{(iA+3i(iB-A)+3B)(\tan^4(fx+e))}{4} - \frac{2iB(\tan^3(fx+e))}{3} + \frac{(-3iA-i(iB-A))}{2} \right) \frac{1}{f}$
norman	$\frac{A a^2 c^4 \tan(fx+e)}{f} - \frac{(2iB a^2 c^4 + A a^2 c^4)(\tan^5(fx+e))}{5f} + \frac{(-2iA a^2 c^4 + B a^2 c^4)(\tan^2(fx+e))}{2f} - \frac{B a^2 c^4 (\tan^6(fx+e))}{6f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4,x,method=\_RE  
TURNVERBOSE)

[Out]  $1/f*c^4*a^2*(-1/6*B*\tan(f*x+e)^6+1/5*(-A-2*I*B)*\tan(f*x+e)^5+1/4*(I*A+3*I*(-A+I*B)+3*B)*\tan(f*x+e)^4-2/3*I*B*\tan(f*x+e)^3+1/2*(-3*I*A-I*(-A+I*B))*\tan(f*x+e)^2+A*\tan(f*x+e))$

**Maxima [A]**

time = 0.53, size = 122, normalized size = 1.23

$$\frac{5Ba^2c^4 \tan(fx+e)^6 + 6(A+2iB)a^2c^4 \tan(fx+e)^5 + 15iAa^2c^4 \tan(fx+e)^4 + 20iBa^2c^4 \tan(fx+e)^3 + 15(2iA-B)a^2c^4 \tan(fx+e)^2 - 30Aa^2c^4 \tan(fx+e)}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")`

[Out]  $-1/30*(5*B*a^2*c^4*\tan(f*x + e)^6 + 6*(A + 2*I*B)*a^2*c^4*\tan(f*x + e)^5 + 15*I*A*a^2*c^4*\tan(f*x + e)^4 + 20*I*B*a^2*c^4*\tan(f*x + e)^3 + 15*(2*I*A - B)*a^2*c^4*\tan(f*x + e)^2 - 30*A*a^2*c^4*\tan(f*x + e))/f$

**Fricas [A]**

time = 3.61, size = 144, normalized size = 1.45

$$\frac{8(15(-iA-B)a^2c^4e^{4ifx+4ie} + 6(-3iA+B)a^2c^4e^{2ifx+2ie} + (-3iA+B)a^2c^4)}{15(fe^{12ifx+12ie} + 6fe^{10ifx+10ie} + 15fe^{8ifx+8ie} + 20fe^{6ifx+6ie} + 15fe^{4ifx+4ie} + 6fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")`

[Out]  $-8/15*(15*(-I*A - B)*a^2*c^4*e^{(4*I*f*x + 4*I*e)} + 6*(-3*I*A + B)*a^2*c^4*e^{(2*I*f*x + 2*I*e)} + (-3*I*A + B)*a^2*c^4)/(f*e^{(12*I*f*x + 12*I*e)} + 6*f*e^{(10*I*f*x + 10*I*e)} + 15*f*e^{(8*I*f*x + 8*I*e)} + 20*f*e^{(6*I*f*x + 6*I*e)} + 15*f*e^{(4*I*f*x + 4*I*e)} + 6*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs.  $2(78) = 156$ .

time = 0.62, size = 224, normalized size = 2.26

$$\frac{24iAa^2c^4 - 8Ba^2c^4 + (144iAa^2c^4e^{2ie} - 48Ba^2c^4e^{2ie})e^{2ifx} + (120iAa^2c^4e^{4ie} + 120Ba^2c^4e^{4ie})e^{4ifx}}{15fe^{12ie}e^{12ifx} + 90fe^{10ie}e^{10ifx} + 225fe^{8ie}e^{8ifx} + 300fe^{6ie}e^{6ifx} + 225fe^{4ie}e^{4ifx} + 90fe^{2ie}e^{2ifx} + 15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)`

[Out]  $(24*I*A*a**2*c**4 - 8*B*a**2*c**4 + (144*I*A*a**2*c**4*\exp(2*I*e) - 48*B*a**2*c**4*\exp(2*I*e))*\exp(2*I*f*x) + (120*I*A*a**2*c**4*\exp(4*I*e) + 120*B*a**2*c**4*\exp(4*I*e))*\exp(4*I*f*x))/(15*f*\exp(12*I*e)*\exp(12*I*f*x) + 90*f*\exp(10*I*e)*\exp(10*I*f*x) + 225*f*\exp(8*I*e)*\exp(8*I*f*x) + 300*f*\exp(6*I*e)*\exp(6*I*f*x) + 225*f*\exp(4*I*e)*\exp(4*I*f*x) + 90*f*\exp(2*I*e)*\exp(2*I*f*x) + 15*f)$

$\exp(6*I*f*x) + 225*f*\exp(4*I*e)*\exp(4*I*f*x) + 90*f*\exp(2*I*e)*\exp(2*I*f*x) + 15*f)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(86) = 172.

time = 1.06, size = 177, normalized size = 1.79

$$\frac{8(-15iAa^2c^4e^{(4i fx+4ie)} - 15Ba^2c^4e^{(4i fx+4ie)} - 18iAa^2c^4e^{(2i fx+2ie)} + 6Ba^2c^4e^{(2i fx+2ie)} - 3iAa^2c^4 + Ba^2c^4)}{15(fe^{(12i fx+12ie)} + 6fe^{(10i fx+10ie)} + 15fe^{(8i fx+8ie)} + 20fe^{(6i fx+6ie)} + 15fe^{(4i fx+4ie)} + 6fe^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out]  $-8/15*(-15*I*A*a^2*c^4*e^{(4*I*f*x + 4*I*e)} - 15*B*a^2*c^4*e^{(4*I*f*x + 4*I*e)} - 18*I*A*a^2*c^4*e^{(2*I*f*x + 2*I*e)} + 6*B*a^2*c^4*e^{(2*I*f*x + 2*I*e)} - 3*I*A*a^2*c^4 + B*a^2*c^4)/(f*e^{(12*I*f*x + 12*I*e)} + 6*f*e^{(10*I*f*x + 10*I*e)} + 15*f*e^{(8*I*f*x + 8*I*e)} + 20*f*e^{(6*I*f*x + 6*I*e)} + 15*f*e^{(4*I*f*x + 4*I*e)} + 6*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad [B]**

time = 8.60, size = 120, normalized size = 1.21

$$\frac{\frac{a^2c^4\tan(e+fx)^2(-B+A2i)}{2} - Aa^2c^4\tan(e+fx) + \frac{a^2c^4\tan(e+fx)^5(A+B2i)}{5} + \frac{Ba^2c^4\tan(e+fx)^6}{6} + \frac{Aa^2c^4\tan(e+fx)^41i}{2} + \frac{Ba^2c^4\tan(e+fx)^32i}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^4,x)

[Out]  $-((a^2*c^4*\tan(e + f*x)^2*(A*2i - B))/2 - A*a^2*c^4*\tan(e + f*x) + (A*a^2*c^4*\tan(e + f*x)^4*1i)/2 + (a^2*c^4*\tan(e + f*x)^5*(A + B*2i))/5 + (B*a^2*c^4*\tan(e + f*x)^3*2i)/3 + (B*a^2*c^4*\tan(e + f*x)^6)/6)/f$

$$3.679 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^3 dx$$

Optimal. Leaf size=99

$$\frac{2a^2(iA + B)c^3(1 - i \tan(e + fx))^3}{3f} - \frac{a^2(iA + 3B)c^3(1 - i \tan(e + fx))^4}{4f} + \frac{a^2Bc^3(1 - i \tan(e + fx))^5}{5f}$$

[Out]  $2/3*a^2*(I*A+B)*c^3*(1-I*\tan(f*x+e))^3/f-1/4*a^2*(I*A+3*B)*c^3*(1-I*\tan(f*x+e))^4/f+1/5*a^2*B*c^3*(1-I*\tan(f*x+e))^5/f$

Rubi [A]

time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^2c^3(3B + iA)(1 - i \tan(e + fx))^4}{4f} + \frac{2a^2c^3(B + iA)(1 - i \tan(e + fx))^3}{3f} + \frac{a^2Bc^3(1 - i \tan(e + fx))^5}{5f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^3, x]$

[Out]  $(2*a^2*(I*A + B)*c^3*(1 - I*\text{Tan}[e + f*x])^3)/(3*f) - (a^2*(I*A + 3*B)*c^3*(1 - I*\text{Tan}[e + f*x])^4)/(4*f) + (a^2*B*c^3*(1 - I*\text{Tan}[e + f*x])^5)/(5*f)$

Rule 78

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(A + B*\text{tan}[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps



$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ictan(e + fx))^3 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx)(c - icx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (2a(A - iB)(c - icx) dx\right)}{f}$$

$$= \frac{2a^2(iA + B)c^3(1 - i \tan(e + fx))^3}{3f}$$

**Mathematica [A]**

time = 1.66, size = 146, normalized size = 1.47

$$\frac{a^2 c^3 \sec(e) \sec^3(e + fx) (15(-iA + B) \cos(fx) + 15(-iA + B) \cos(2e + fx) + 35A \sin(fx) - 5iB \sin(fx) - 15A \sin(2e + fx) - 15iB \sin(2e + fx) + 25A \sin(2e + 3fx) + 5iB \sin(2e + 3fx) + 5A \sin(4e + 5fx) + iB \sin(4e + 5fx))}{120f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3,x]

[Out] (a^2\*c^3\*Sec[e]\*Sec[e + f\*x]^5\*(15\*((-I)\*A + B)\*Cos[f\*x] + 15\*((-I)\*A + B)\*Cos[2\*e + f\*x] + 35\*A\*Sin[f\*x] - (5\*I)\*B\*Sin[f\*x] - 15\*A\*Sin[2\*e + f\*x] - (15\*I)\*B\*Sin[2\*e + f\*x] + 25\*A\*Sin[2\*e + 3\*f\*x] + (5\*I)\*B\*Sin[2\*e + 3\*f\*x] + 5\*A\*Sin[4\*e + 5\*f\*x] + I\*B\*Sin[4\*e + 5\*f\*x]))/(120\*f)

**Maple [A]**

time = 0.13, size = 98, normalized size = 0.99

method	result
risch	$\frac{4c^3 a^2 (20iA e^{4i(fx+e)} + 20B e^{4i(fx+e)} + 25iA e^{2i(fx+e)} - 5B e^{2i(fx+e)} + 5iA - B)}{15f(e^{2i(fx+e)} + 1)^5}$
derivativedivides	$ic^3 a^2 \left( -\frac{B(\tan^5(fx+e))}{5} + \frac{(-iB-A)(\tan^4(fx+e))}{4} + \frac{(iA+2i(iB-A)+B)(\tan^3(fx+e))}{3} + \frac{(-iB-A)(\tan^2(fx+e))}{2} - iA \tan(fx+e) \right) / f$
default	$ic^3 a^2 \left( -\frac{B(\tan^5(fx+e))}{5} + \frac{(-iB-A)(\tan^4(fx+e))}{4} + \frac{(iA+2i(iB-A)+B)(\tan^3(fx+e))}{3} + \frac{(-iB-A)(\tan^2(fx+e))}{2} - iA \tan(fx+e) \right) / f$
norman	$\frac{A a^2 c^3 \tan(fx+e)}{f} + \frac{(-iA a^2 c^3 + B a^2 c^3) (\tan^2(fx+e))}{2f} + \frac{(-iA a^2 c^3 + B a^2 c^3) (\tan^4(fx+e))}{4f} + \frac{(-iB a^2 c^3 + A a^2 c^3) (\tan^6(fx+e))}{6f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x,method=\_RE TURNVERBOSE)

[Out] I/f\*c^3\*a^2\*(-1/5\*B\*tan(f\*x+e)^5+1/4\*(-A-I\*B)\*tan(f\*x+e)^4+1/3\*(I\*A+2\*I\*(-A+I\*B)+B)\*tan(f\*x+e)^3+1/2\*(-A-I\*B)\*tan(f\*x+e)^2-I\*A\*tan(f\*x+e))

**Maxima [A]**

time = 0.51, size = 106, normalized size = 1.07

$$\frac{12iBa^2c^3 \tan(fx+e)^5 - 15(-iA+B)a^2c^3 \tan(fx+e)^4 - 20(A-iB)a^2c^3 \tan(fx+e)^3 - 30(-iA+B)a^2c^3 \tan(fx+e)^2 - 60Aa^2c^3 \tan(fx+e)}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x, algorithm="maxima")

[Out] -1/60\*(12\*I\*B\*a^2\*c^3\*tan(f\*x + e)^5 - 15\*(-I\*A + B)\*a^2\*c^3\*tan(f\*x + e)^4 - 20\*(A - I\*B)\*a^2\*c^3\*tan(f\*x + e)^3 - 30\*(-I\*A + B)\*a^2\*c^3\*tan(f\*x + e)^2 - 60\*A\*a^2\*c^3\*tan(f\*x + e))/f

**Fricas [A]**

time = 4.19, size = 131, normalized size = 1.32

$$\frac{4(20(-iA-B)a^2c^3e^{4ifx+4ie} + 5(-5iA+B)a^2c^3e^{2ifx+2ie} + (-5iA+B)a^2c^3)}{15(fe^{10ifx+10ie} + 5fe^{8ifx+8ie} + 10fe^{6ifx+6ie} + 10fe^{4ifx+4ie} + 5fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x, algorithm="fricas")

[Out] -4/15\*(20\*(-I\*A - B)\*a^2\*c^3\*e^(4\*I\*f\*x + 4\*I\*e) + 5\*(-5\*I\*A + B)\*a^2\*c^3\*e^(2\*I\*f\*x + 2\*I\*e) + (-5\*I\*A + B)\*a^2\*c^3)/(f\*e^(10\*I\*f\*x + 10\*I\*e) + 5\*f\*e^(8\*I\*f\*x + 8\*I\*e) + 10\*f\*e^(6\*I\*f\*x + 6\*I\*e) + 10\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 5\*f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(80) = 160.

time = 0.48, size = 206, normalized size = 2.08

$$\frac{20iAa^2c^3 - 4Ba^2c^3 + (100iAa^2c^3e^{2ie} - 20Ba^2c^3e^{2ie})e^{2ifx} + (80iAa^2c^3e^{4ie} + 80Ba^2c^3e^{4ie})e^{4ifx}}{15fe^{10ie}e^{10ifx} + 75fe^{8ie}e^{8ifx} + 150fe^{6ie}e^{6ifx} + 150fe^{4ie}e^{4ifx} + 75fe^{2ie}e^{2ifx} + 15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x)

[Out] (20\*I\*A\*a\*\*2\*c\*\*3 - 4\*B\*a\*\*2\*c\*\*3 + (100\*I\*A\*a\*\*2\*c\*\*3\*exp(2\*I\*e) - 20\*B\*a\*\*2\*c\*\*3\*exp(2\*I\*e))\*exp(2\*I\*f\*x) + (80\*I\*A\*a\*\*2\*c\*\*3\*exp(4\*I\*e) + 80\*B\*a\*\*2\*c\*\*3\*exp(4\*I\*e))\*exp(4\*I\*f\*x))/(15\*f\*exp(10\*I\*e)\*exp(10\*I\*f\*x) + 75\*f\*exp(8\*I\*e)\*exp(8\*I\*f\*x) + 150\*f\*exp(6\*I\*e)\*exp(6\*I\*f\*x) + 150\*f\*exp(4\*I\*e)\*exp(4\*I\*f\*x) + 75\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + 15\*f)

**Giac [A]**

time = 0.85, size = 164, normalized size = 1.66

$$\frac{4(-20iAa^2c^3e^{4ifx+4ie} - 20Ba^2c^3e^{4ifx+4ie} - 25iAa^2c^3e^{2ifx+2ie} + 5Ba^2c^3e^{2ifx+2ie} - 5iAa^2c^3 + Ba^2c^3)}{15(fe^{10ifx+10ie} + 5fe^{8ifx+8ie} + 10fe^{6ifx+6ie} + 10fe^{4ifx+4ie} + 5fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$\frac{-4/15*(-20*I*A*a^2*c^3*e^{(4*I*f*x + 4*I*e)} - 20*B*a^2*c^3*e^{(4*I*f*x + 4*I*e)} - 25*I*A*a^2*c^3*e^{(2*I*f*x + 2*I*e)} + 5*B*a^2*c^3*e^{(2*I*f*x + 2*I*e)} - 5*I*A*a^2*c^3 + B*a^2*c^3)/(f*e^{(10*I*f*x + 10*I*e)} + 5*f*e^{(8*I*f*x + 8*I*e)} + 10*f*e^{(6*I*f*x + 6*I*e)} + 10*f*e^{(4*I*f*x + 4*I*e)} + 5*f*e^{(2*I*f*x + 2*I*e)} + f)}$$

**Mupad [B]**

time = 9.02, size = 108, normalized size = 1.09

$$\frac{-A a^2 c^3 \tan(e + f x) + \frac{a^2 c^3 \tan(e + f x)^2 (A + B 1i) 1i}{2} + \frac{a^2 c^3 \tan(e + f x)^3 (B + A 1i) 1i}{3} + \frac{a^2 c^3 \tan(e + f x)^4 (A + B 1i) 1i}{4} + \frac{B a^2 c^3 \tan(e + f x)^5 1i}{5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^3,x)

[Out] 
$$-\frac{(a^2*c^3*\tan(e + f*x)^2*(A + B*1i)*1i)}{2} - A*a^2*c^3*\tan(e + f*x) + (a^2*c^3*\tan(e + f*x)^3*(A*1i + B)*1i)/3 + (a^2*c^3*\tan(e + f*x)^4*(A + B*1i)*1i)/4 + (B*a^2*c^3*\tan(e + f*x)^5*1i)/5)/f$$

$$3.680 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^2 dx$$

Optimal. Leaf size=62

$$\frac{a^2 B c^2 \sec^4(e + fx)}{4f} + \frac{a^2 A c^2 \tan(e + fx)}{f} + \frac{a^2 A c^2 \tan^3(e + fx)}{3f}$$

[Out] 1/4\*a^2\*B\*c^2\*sec(f\*x+e)^4/f+a^2\*A\*c^2\*tan(f\*x+e)/f+1/3\*a^2\*A\*c^2\*tan(f\*x+e)^3/f

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 74, 655}

$$\frac{a^2 A c^2 \tan^3(e + fx)}{3f} + \frac{a^2 A c^2 \tan(e + fx)}{f} + \frac{a^2 B c^2 \sec^4(e + fx)}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^2, x]

[Out] (a^2\*B\*c^2\*Sec[e + f\*x]^4)/(4\*f) + (a^2\*A\*c^2\*Tan[e + f\*x])/f + (a^2\*A\*c^2\*Tan[e + f\*x]^3)/(3\*f)

Rule 74

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[(a\*c + b\*d\*x^2)^m\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b\*c + a\*d, 0] && EqQ[n, m] && IntegerQ[m] && (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] || !IntegerQ[p])))

Rule 655

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[e\*((a + c\*x^2)^(p + 1)/(2\*c\*(p + 1))), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^2 dx &= \frac{(ac) \text{Subst}(\int (a + iax)(A + Bx)(c - icx)^2 dx)}{f} \\
&= \frac{(ac) \text{Subst}(\int (A + Bx)(ac + acx^2) dx)}{f} \\
&= \frac{a^2 Bc^2 \sec^4(e + fx)}{4f} + \frac{(aAc) \text{Subst}(\int \tan(x) dx)}{f} \\
&= \frac{a^2 Bc^2 \sec^4(e + fx)}{4f} + \frac{a^2 Ac^2 \tan(e + fx)}{f}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 53, normalized size = 0.85

$$\frac{a^2 Bc^2 \sec^4(e + fx)}{4f} + \frac{a^2 Ac^2 (\tan(e + fx) + \frac{1}{3} \tan^3(e + fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*B*c^2*Sec[e + f*x]^4)/(4*f) + (a^2*A*c^2*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Maple [A]

time = 0.08, size = 53, normalized size = 0.85

method	result	size
derivativedivides	$\frac{c^2 a^2 \left( \frac{B(\tan^4(fx+e))}{4} + \frac{A(\tan^3(fx+e))}{3} + \frac{B(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	53
default	$\frac{c^2 a^2 \left( \frac{B(\tan^4(fx+e))}{4} + \frac{A(\tan^3(fx+e))}{3} + \frac{B(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	53
risch	$\frac{4c^2 a^2 (3iA e^{4i(fx+e)} + 3B e^{4i(fx+e)} + 4iA e^{2i(fx+e)} + iA)}{3f(e^{2i(fx+e)} + 1)^4}$	68
norman	$\frac{a^2 A c^2 \tan(fx+e)}{f} + \frac{B a^2 c^2 (\tan^2(fx+e))}{2f} + \frac{B a^2 c^2 (\tan^4(fx+e))}{4f} + \frac{a^2 A c^2 (\tan^3(fx+e))}{3f}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorithm="giac")

[Out]  $\frac{1}{12} * (3 * B * a^2 * c^2 * \tan(f * x)^4 * \tan(e)^4 - 12 * A * a^2 * c^2 * \tan(f * x)^4 * \tan(e)^3 - 12 * A * a^2 * c^2 * \tan(f * x)^3 * \tan(e)^4 + 6 * B * a^2 * c^2 * \tan(f * x)^4 * \tan(e)^2 + 6 * B * a^2 * c^2 * \tan(f * x)^2 * \tan(e)^4 - 4 * A * a^2 * c^2 * \tan(f * x)^4 * \tan(e) + 24 * A * a^2 * c^2 * \tan(f * x)^3 * \tan(e)^2 + 24 * A * a^2 * c^2 * \tan(f * x)^2 * \tan(e)^3 - 4 * A * a^2 * c^2 * \tan(f * x) * \tan(e)^4 + 3 * B * a^2 * c^2 * \tan(f * x)^4 + 12 * B * a^2 * c^2 * \tan(f * x)^2 * \tan(e)^2 + 3 * B * a^2 * c^2 * \tan(e)^4 + 4 * A * a^2 * c^2 * \tan(f * x)^3 - 24 * A * a^2 * c^2 * \tan(f * x)^2 * \tan(e) - 24 * A * a^2 * c^2 * \tan(f * x) * \tan(e)^2 + 4 * A * a^2 * c^2 * \tan(e)^3 + 6 * B * a^2 * c^2 * \tan(f * x)^2 + 6 * B * a^2 * c^2 * \tan(e)^2 + 12 * A * a^2 * c^2 * \tan(f * x) + 12 * A * a^2 * c^2 * \tan(e) + 3 * B * a^2 * c^2) / (f * \tan(f * x)^4 * \tan(e)^4 - 4 * f * \tan(f * x)^3 * \tan(e)^3 + 6 * f * \tan(f * x)^2 * \tan(e)^2 - 4 * f * \tan(f * x) * \tan(e) + f)$

**Mupad [B]**

time = 8.49, size = 82, normalized size = 1.32

$$\frac{a^2 c^2 \sin(e + f x) (12 A \cos(e + f x)^3 + 6 B \cos(e + f x)^2 \sin(e + f x) + 4 A \cos(e + f x) \sin(e + f x)^2 + 3 B \sin(e + f x)^3)}{12 f \cos(e + f x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^2,x)

[Out]  $(a^2 * c^2 * \sin(e + f * x) * (12 * A * \cos(e + f * x)^3 + 3 * B * \sin(e + f * x)^3 + 4 * A * \cos(e + f * x) * \sin(e + f * x)^2 + 6 * B * \cos(e + f * x)^2 * \sin(e + f * x))) / (12 * f * \cos(e + f * x)^4)$

### 3.681 $\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx))(c - ic \tan(e + fx)) dx$

Optimal. Leaf size=64

$$\frac{a^2 A c \tan(e + fx)}{f} + \frac{a^2 (iA + B) c \tan^2(e + fx)}{2f} + \frac{ia^2 B c \tan^3(e + fx)}{3f}$$

[Out]  $a^2 A c \tan(f*x+e)/f + 1/2 a^2 (I*A+B) c \tan(f*x+e)^2/f + 1/3 I a^2 B c \tan(f*x+e)^3/f$

Rubi [A]

time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{a^2 c (B + iA) \tan^2(e + fx)}{2f} + \frac{a^2 A c \tan(e + fx)}{f} + \frac{ia^2 B c \tan^3(e + fx)}{3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x]),x]$

[Out]  $(a^2*A*c*\text{Tan}[e + f*x])/f + (a^2*(I*A + B)*c*\text{Tan}[e + f*x]^2)/(2*f) + ((I/3)*a^2*B*c*\text{Tan}[e + f*x]^3)/f$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps



$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx)) dx = \frac{(ac) \text{Subst}(\int (a + iax)(A + Bx) dx, ax)}{f}$$

$$= \frac{(ac) \text{Subst}(\int (aA + a(iA + B)x + iaBx^2) dx, ax)}{f}$$

$$= \frac{a^2 A c \tan(e + fx)}{f} + \frac{a^2 (iA + B) c \tan(e + fx)}{2f}$$

**Mathematica [A]**

time = 0.90, size = 109, normalized size = 1.70

$$\frac{a^2 c \sec(e) \sec^3(e + fx) (3(iA + B) \cos(fx) + 3(iA + B) \cos(2e + fx) + 6A \sin(fx) - 3A \sin(2e + fx) + 3iB \sin(2e + fx) + 3A \sin(2e + 3fx) - iB \sin(2e + 3fx))}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x]), x]

[Out] (a^2\*c\*Sec[e]\*Sec[e + f\*x]^3\*(3\*(I\*A + B)\*Cos[f\*x] + 3\*(I\*A + B)\*Cos[2\*e + f\*x] + 6\*A\*Sin[f\*x] - 3\*A\*Sin[2\*e + f\*x] + (3\*I)\*B\*Sin[2\*e + f\*x] + 3\*A\*Sin[2\*e + 3\*f\*x] - I\*B\*Sin[2\*e + 3\*f\*x]))/(12\*f)

**Maple [A]**

time = 0.10, size = 51, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{ia^2c \left( -\frac{B(\tan^3(fx+e))}{3} + \frac{(iB-A)(\tan^2(fx+e))}{2} + iA \tan(fx+e) \right)}{f}$	51
default	$-\frac{ia^2c \left( -\frac{B(\tan^3(fx+e))}{3} + \frac{(iB-A)(\tan^2(fx+e))}{2} + iA \tan(fx+e) \right)}{f}$	51
norman	$\frac{a^2 A c \tan(fx+e)}{f} + \frac{(iA a^2 c + B a^2 c)(\tan^2(fx+e))}{2f} + \frac{ia^2 B c (\tan^3(fx+e))}{3f}$	64
risch	$\frac{2a^2c(6iAe^{4i(fx+e)} + 6Be^{4i(fx+e)} + 9iAe^{2i(fx+e)} + 3Be^{2i(fx+e)} + 3iA + B)}{3f(e^{2i(fx+e)} + 1)^3}$	79

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)), x, method=\_RETURNVERBOSE)

[Out] -I/f\*a^2\*c\*(-1/3\*B\*tan(f\*x+e)^3+1/2\*(-A+I\*B)\*tan(f\*x+e)^2+I\*A\*tan(f\*x+e))

**Maxima [A]**

time = 0.54, size = 56, normalized size = 0.88

$$\frac{-2iBa^2c \tan(fx + e)^3 - 3(iA + B)a^2c \tan(fx + e)^2 - 6Aa^2c \tan(fx + e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(-2*I*B*a^2*c*tan(f*x + e)^3 - 3*(I*A + B)*a^2*c*tan(f*x + e)^2 - 6*A*a^2*c*tan(f*x + e))/f
```

**Fricas [A]**

time = 1.99, size = 103, normalized size = 1.61

$$\frac{2(6(-iA - B)a^2ce^{4ifx+4ie} + 3(-3iA - B)a^2ce^{2ifx+2ie} + (-3iA - B)a^2c)}{3(fe^{6ifx+6ie} + 3fe^{4ifx+4ie} + 3fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(6*(-I*A - B)*a^2*c*e^(4*I*f*x + 4*I*e) + 3*(-3*I*A - B)*a^2*c*e^(2*I*f*x + 2*I*e) + (-3*I*A - B)*a^2*c)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(56) = 112.

time = 0.22, size = 158, normalized size = 2.47

$$\frac{6iAa^2c + 2Ba^2c + (18iAa^2ce^{2ie} + 6Ba^2ce^{2ie})e^{2ifx} + (12iAa^2ce^{4ie} + 12Ba^2ce^{4ie})e^{4ifx}}{3fe^{6ie}e^{6ifx} + 9fe^{4ie}e^{4ifx} + 9fe^{2ie}e^{2ifx} + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

```
[Out] (6*I*A*a**2*c + 2*B*a**2*c + (18*I*A*a**2*c*exp(2*I*e) + 6*B*a**2*c*exp(2*I*e))*exp(2*I*f*x) + (12*I*A*a**2*c*exp(4*I*e) + 12*B*a**2*c*exp(4*I*e))*exp(4*I*f*x))/(3*f*exp(6*I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(59) = 118.

time = 0.59, size = 127, normalized size = 1.98

$$\frac{2(-6iAa^2ce^{4ifx+4ie} - 6Ba^2ce^{4ifx+4ie} - 9iAa^2ce^{2ifx+2ie} - 3Ba^2ce^{2ifx+2ie} - 3iAa^2c - Ba^2c)}{3(fe^{6ifx+6ie} + 3fe^{4ifx+4ie} + 3fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)),x, algorithm="giac")

[Out] 
$$-2/3*(-6*I*A*a^2*c*e^{(4*I*f*x + 4*I*e)} - 6*B*a^2*c*e^{(4*I*f*x + 4*I*e)} - 9*I*A*a^2*c*e^{(2*I*f*x + 2*I*e)} - 3*B*a^2*c*e^{(2*I*f*x + 2*I*e)} - 3*I*A*a^2*c - B*a^2*c)/(f*e^{(6*I*f*x + 6*I*e)} + 3*f*e^{(4*I*f*x + 4*I*e)} + 3*f*e^{(2*I*f*x + 2*I*e)} + f)$$

**Mupad [B]**

time = 8.53, size = 50, normalized size = 0.78

$$\frac{a^2 c \tan(e + f x) (6 A + A \tan(e + f x) 3i + 3 B \tan(e + f x) + B \tan(e + f x)^2 2i)}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i), x)

[Out] 
$$(a^2*c*tan(e + f*x)*(6*A + A*tan(e + f*x)*3i + 3*B*tan(e + f*x) + B*tan(e + f*x)^2*2i))/(6*f)$$

### 3.682 $\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) dx$

Optimal. Leaf size=80

$$2a^2(A - iB)x - \frac{2a^2(iA + B) \log(\cos(e + fx))}{f} - \frac{a^2(A - iB) \tan(e + fx)}{f} + \frac{B(a + ia \tan(e + fx))^2}{2f}$$

[Out] 2\*a^2\*(A-I\*B)\*x-2\*a^2\*(I\*A+B)\*ln(cos(f\*x+e))/f-a^2\*(A-I\*B)\*tan(f\*x+e)/f+1/2\*B\*(a+I\*a\*tan(f\*x+e))^2/f

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3608, 3558, 3556}

$$-\frac{a^2(A - iB) \tan(e + fx)}{f} - \frac{2a^2(B + iA) \log(\cos(e + fx))}{f} + 2a^2x(A - iB) + \frac{B(a + ia \tan(e + fx))^2}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]),x]

[Out] 2\*a^2\*(A - I\*B)\*x - (2\*a^2\*(I\*A + B)\*Log[Cos[e + f\*x]])/f - (a^2\*(A - I\*B)\*Tan[e + f\*x])/f + (B\*(a + I\*a\*Tan[e + f\*x])^2)/(2\*f)

Rule 3556

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3558

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^2, x\_Symbol] := Simp[(a^2 - b^2)\*x, x] + (Dist[2\*a\*b, Int[Tan[c + d\*x], x], x] + Simp[b^2\*(Tan[c + d\*x]/d), x]) /; FreeQ[{a, b, c, d}, x]

Rule 3608

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[d\*((a + b\*Tan[e + f\*x])^m/(f\*m)), x] + Dist[(b\*c + a\*d)/b, Int[(a + b\*Tan[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) dx &= \frac{B(a + ia \tan(e + fx))^2}{2f} - (-A + iB) \int (a + ia \tan(e + fx)) dx \\ &= 2a^2(A - iB)x - \frac{a^2(A - iB) \tan(e + fx)}{f} + \frac{B(a + ia \tan(e + fx))^2}{2f} \\ &= 2a^2(A - iB)x - \frac{2a^2(iA + B) \log(\cos(e + fx))}{f} - \frac{a^2(A - iB)}{f} \end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 263 vs.  $2(80) = 160$ .  
time = 1.16, size = 263, normalized size = 3.29

$\frac{a^2 \sec(e) \cos^2(e + fx) \cos(2fx) + i \sin(2fx) (-iA - iB) \text{ArcTan}(\tan(e + fx)) \cos(e) \cos^2(e + fx) - (iA/f \cos^3(e + 2fx) + 4B/f \cos^3(e + 2fx) + (A + B) \cos(e + 2fx) (4fx - \log(\cos^2(e + fx))) + A \cos^3(e + 2fx) \log(\cos^2(e + fx)) - iB \cos^3(e + 2fx) \log(\cos^2(e + fx)) + 2 \cos(e) (-iB + iA/f + 4B/f + (A - iB) \log(\cos^2(e + fx))) + 2iA \sin(e) + 4B \sin(e) - 2iA \sin(e + 2fx) - 4B \sin(e + 2fx))}{4f \cos^2(e + fx)}$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]),x]

[Out]  $(a^2 \text{Sec}[e] \text{Sec}[e + f*x]^2 (\text{Cos}[2*f*x] + I \text{Sin}[2*f*x]) * (-8*(A - I*B) \text{ArcTan}[\text{Tan}[3*e + f*x]] \text{Cos}[e] \text{Cos}[e + f*x]^2 - I*((4*I)*A*f*x \text{Cos}[3*e + 2*f*x] + 4*B*f*x \text{Cos}[3*e + 2*f*x] + (I*A + B) \text{Cos}[e + 2*f*x] * (4*f*x - I \text{Log}[\text{Cos}[e + f*x]^2]) + A \text{Cos}[3*e + 2*f*x] \text{Log}[\text{Cos}[e + f*x]^2] - I*B \text{Cos}[3*e + 2*f*x] \text{Log}[\text{Cos}[e + f*x]^2] + 2 \text{Cos}[e] * ((-I)*B + (4*I)*A*f*x + 4*B*f*x + (A - I*B) \text{Log}[\text{Cos}[e + f*x]^2]) + (2*I)*A \text{Sin}[e] + 4*B \text{Sin}[e] - (2*I)*A \text{Sin}[e + 2*f*x] - 4*B \text{Sin}[e + 2*f*x])))/(4*f*(\text{Cos}[f*x] + I \text{Sin}[f*x])^2)$

**Maple [A]**

time = 0.11, size = 76, normalized size = 0.95

method	result
derivativedivides	$a^2 \left( -\frac{B(\tan^2(fx+e))}{2} - A \tan(fx+e) + 2iB \tan(fx+e) + \frac{(2iA+2B) \ln\left(\frac{1+\tan^2(fx+e)}{2}\right)}{2} + (-2iB+2A) \arctan(\tan(fx+e)) \right) / f$
default	$a^2 \left( -\frac{B(\tan^2(fx+e))}{2} - A \tan(fx+e) + 2iB \tan(fx+e) + \frac{(2iA+2B) \ln\left(\frac{1+\tan^2(fx+e)}{2}\right)}{2} + (-2iB+2A) \arctan(\tan(fx+e)) \right) / f$
norman	$(-2iB a^2 + 2a^2 A) x - \frac{(-2iB a^2 + a^2 A) \tan(fx+e)}{f} - \frac{a^2 B (\tan^2(fx+e))}{2f} + \frac{(iA a^2 + a^2 B) \ln(1 + \tan^2(fx+e))}{f}$
risch	$\frac{4ia^2 B e}{f} - \frac{4a^2 A e}{f} - \frac{2a^2 (iA e^{2i(fx+e)} + 3B e^{2i(fx+e)} + iA + 2B)}{f(e^{2i(fx+e)} + 1)^2} - \frac{2a^2 \ln(e^{2i(fx+e)} + 1) B}{f} - \frac{2ia^2 \ln(e^{2i(fx+e)} + 1)}{f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e)),x,method=\_RETURNVERBOSE)

[Out]  $\frac{1}{f} a^2 \left( -\frac{1}{2} B \tan(fx+e)^2 - A \tan(fx+e) + 2 I B \tan(fx+e) + \frac{1}{2} (2B + 2IA) \ln(1 + \tan(fx+e)^2) + (2A - 2IB) \arctan(\tan(fx+e)) \right)$

**Maxima** [A]

time = 0.55, size = 75, normalized size = 0.94

$$\frac{Ba^2 \tan(fx+e)^2 - 4(fx+e)(A-iB)a^2 - 2(iA+B)a^2 \log(\tan(fx+e)^2 + 1) + 2(A-2iB)a^2 \tan(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x, algorithm="maxima")`

[Out]  $-\frac{1}{2} (B a^2 \tan(fx+e)^2 - 4(fx+e)(A-iB)a^2 - 2(I A + B)a^2 \log(\tan(fx+e)^2 + 1) + 2(A - 2IB)a^2 \tan(fx+e)) / f$

**Fricas** [A]

time = 1.73, size = 127, normalized size = 1.59

$$\frac{2((iA+3B)a^2 e^{2i fx+2ie} + (iA+2B)a^2 + ((iA+B)a^2 e^{4i fx+4ie} + 2(iA+B)a^2 e^{2i fx+2ie} + (iA+B)a^2) \log(e^{2i fx+2ie} + 1))}{f e^{4i fx+4ie} + 2f e^{2i fx+2ie} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x, algorithm="fricas")`

[Out]  $-2((IA + 3B)a^2 e^{2I fx + 2I e} + (IA + 2B)a^2 + ((IA + B)a^2 e^{4I fx + 4I e} + 2(I A + B)a^2 e^{2I fx + 2I e} + (IA + B)a^2) \log(e^{2I fx + 2I e} + 1)) / (f e^{4I fx + 4I e} + 2f e^{2I fx + 2I e} + f)$

**Sympy** [A]

time = 0.32, size = 122, normalized size = 1.52

$$-\frac{2ia^2(A-iB)\log(e^{2ifx} + e^{-2ie})}{f} + \frac{-2iAa^2 - 4Ba^2 + (-2iAa^2 e^{2ie} - 6Ba^2 e^{2ie}) e^{2ifx}}{f e^{4ie} e^{4ifx} + 2f e^{2ie} e^{2ifx} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e)),x)`

[Out]  $-2IA**2(A - IB) \log(\exp(2Ifx) + \exp(-2Ie)) / f + (-2IA**2 - 4B**2 + (-2IA**2 \exp(2Ie) - 6B**2 \exp(2Ie)) \exp(2Ifx)) / (f \exp(4Ie) \exp(4Ifx) + 2f \exp(2Ie) \exp(2Ifx) + f)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs.  $2(73) = 146$ .

time = 0.58, size = 228, normalized size = 2.85

$$\frac{2(iA^2 e^{4i fx+4ie} \log(e^{2i fx+2ie} + 1) + Ba^2 e^{4i fx+4ie} \log(e^{2i fx+2ie} + 1) + 2iAa^2 e^{2i fx+2ie} \log(e^{2i fx+2ie} + 1) + 2Ba^2 e^{2i fx+2ie} \log(e^{2i fx+2ie} + 1) + iAa^2 e^{2i fx+2ie} + 3Ba^2 e^{2i fx+2ie} + iAa^2 \log(e^{2i fx+2ie} + 1) + Ba^2 \log(e^{2i fx+2ie} + 1) + iAa^2 + 2Ba^2)}{f e^{4i fx+4ie} + 2f e^{2i fx+2ie} + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out]  $-2*(I*A*a^2*e^{(4*I*f*x + 4*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + B*a^2*e^{(4*I*f*x + 4*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 2*I*A*a^2*e^{(2*I*f*x + 2*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 2*B*a^2*e^{(2*I*f*x + 2*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + I*A*a^2*e^{(2*I*f*x + 2*I*e)} + 3*B*a^2*e^{(2*I*f*x + 2*I*e)} + I*A*a^2*\log(e^{(2*I*f*x + 2*I*e)} + 1) + B*a^2*\log(e^{(2*I*f*x + 2*I*e)} + 1) + I*A*a^2 + 2*B*a^2)/(f*e^{(4*I*f*x + 4*I*e)} + 2*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad [B]**

time = 8.54, size = 76, normalized size = 0.95

$$\frac{\ln(\tan(e + f x) + 1) (2 B a^2 + A a^2 2i)}{f} + \frac{\tan(e + f x) (a^2 (B + A 1i) 1i + B a^2 1i)}{f} - \frac{B a^2 \tan(e + f x)^2}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2,x)

[Out]  $(\log(\tan(e + f*x) + 1i)*(A*a^2*2i + 2*B*a^2))/f + (\tan(e + f*x)*(a^2*(A*1i + B)*1i + B*a^2*1i))/f - (B*a^2*\tan(e + f*x)^2)/(2*f)$

$$3.683 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{c-ic \tan(e+fx)} dx$$

Optimal. Leaf size=93

$$-\frac{a^2(A-3iB)x}{c} + \frac{a^2(iA+3B) \log(\cos(e+fx))}{cf} - \frac{ia^2B \tan(e+fx)}{cf} + \frac{2a^2(A-iB)}{cf(i+\tan(e+fx))}$$

[Out]  $-a^2*(A-3*I*B)*x/c+a^2*(I*A+3*B)*\ln(\cos(f*x+e))/c/f-I*a^2*B*\tan(f*x+e)/c/f+2*a^2*(A-I*B)/c/f/(I+\tan(f*x+e))$

Rubi [A]

time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{2a^2(A-iB)}{cf(\tan(e+fx)+i)} + \frac{a^2(3B+iA) \log(\cos(e+fx))}{cf} - \frac{a^2x(A-3iB)}{c} - \frac{ia^2B \tan(e+fx)}{cf}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])}, x]$

[Out]  $-\frac{(a^2*(A - (3*I)*B)*x)/c}{c} + \frac{a^2*(I*A + 3*B)*\text{Log}[\text{Cos}[e + f*x]]}{(c*f)} - \frac{(I*a^2*B*\text{Tan}[e + f*x])}{(c*f)} + \frac{(2*a^2*(A - I*B))}{(c*f*(I + \text{Tan}[e + f*x]))}$

Rule 78

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*((c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps



$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{c - ic \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^2} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( -\frac{iaB}{c^2} - \frac{2a(A-ib)}{c^2(i+x)^2} - \frac{ia(A-3iB)}{c^2(i+x)} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{a^2(A - 3iB)x}{c} + \frac{a^2(iA + 3B) \log(\cos(e + fx))}{cf} - \frac{ia^2}{cf}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 418 vs. 2(93) = 186.  
time = 2.63, size = 418, normalized size = 4.49

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x]), x]

[Out] (a^2\*Sec[e]\*(2\*(A - (3\*I)\*B)\*f\*x\*Cos[e]^3\*Cos[e + f\*x] + A\*f\*x\*Cos[3\*e]\*Cos[e + f\*x] + (2\*I)\*A\*f\*x\*Cos[2\*e]\*Cos[e + f\*x]\*Sin[e] + 6\*B\*f\*x\*Cos[2\*e]\*Cos[e + f\*x]\*Sin[e] - (2\*I)\*Cos[e]^2\*Cos[e + f\*x]\*((5\*A - (9\*I)\*B)\*f\*x + ((-I)\*A - 3\*B)\*Log[Cos[e + f\*x]^2])\*Sin[e] + (2\*I)\*A\*f\*x\*Cos[e + f\*x]\*Sin[e]^3 + 6\*B\*f\*x\*Cos[e + f\*x]\*Sin[e]^3 - 2\*(A - (3\*I)\*B)\*ArcTan[Tan[3\*e + f\*x]]\*Cos[e]\*Cos[e + f\*x]\*(Cos[2\*e] - I\*Sin[2\*e]) - (6\*I)\*B\*f\*x\*Cos[e + f\*x]\*Sin[e]\*Sin[2\*e] + (2\*I)\*B\*Cos[2\*e]\*Sin[f\*x] + 2\*B\*Sin[2\*e]\*Sin[f\*x] + Cos[e]\*Cos[e + f\*x]\*(A\*f\*x + 2\*(I\*A + B)\*Cos[2\*f\*x] - I\*Cos[2\*e]\*(6\*B\*f\*x + (A - (3\*I)\*B)\*Log[Cos[e + f\*x]^2]) - 2\*A\*f\*x\*Sin[e]^2 + (18\*I)\*B\*f\*x\*Sin[e]^2 - 6\*B\*f\*x\*Sin[2\*e] - 2\*A\*Sin[2\*f\*x] + (2\*I)\*B\*Sin[2\*f\*x]))\*((-I)\*Cos[e + f\*x] + Sin[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(2\*c\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [A]**

time = 0.27, size = 62, normalized size = 0.67

method	result
derivativedivides	$\frac{a^2 \left( -iB \tan(fx+e) - \frac{2iB-2A}{i+\tan(fx+e)} + (-iA-3B) \ln(i+\tan(fx+e)) \right)}{fc}$
default	$\frac{a^2 \left( -iB \tan(fx+e) - \frac{2iB-2A}{i+\tan(fx+e)} + (-iA-3B) \ln(i+\tan(fx+e)) \right)}{fc}$
risch	$-\frac{e^{2i(fx+e)} B a^2}{cf} - \frac{ie^{2i(fx+e)} a^2 A}{cf} - \frac{6ia^2 B e}{cf} + \frac{2a^2 A e}{cf} + \frac{2a^2 B}{fc(e^{2i(fx+e)}+1)} + \frac{3a^2 \ln(e^{2i(fx+e)}+1) B}{cf} + \frac{ia^2 \ln(\dots)}{cf}$

norman	$\frac{\frac{(-3iB a^2 + 2a^2 A) \tan(fx+e)}{cf} - \frac{(-3iB a^2 + a^2 A)x}{c} - \frac{2iA a^2 + 2a^2 B}{cf} - \frac{(-3iB a^2 + a^2 A)x(\tan^2(fx+e))}{c} - \frac{iB a^2 (\tan^3(fx+e))}{cf}}{1 + \tan^2(fx+e)} - \frac{iA}{cf}$
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Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^2/c*(-I*B*tan(f*x+e)-(-2*A+2*I*B)/(I+tan(f*x+e))+(-I*A-3*B)*ln(I+tan(f*x+e)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 3.25, size = 116, normalized size = 1.25

$$\frac{(-iA - B)a^2 e^{4i fx + 4ie} + (-iA - B)a^2 e^{2i fx + 2ie} + 2Ba^2 + ((iA + 3B)a^2 e^{2i fx + 2ie} + (iA + 3B)a^2) \log(e^{2i fx + 2ie} + 1)}{c f e^{2i fx + 2ie} + c f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((-I*A - B)*a^2*e^(4*I*f*x + 4*I*e) + (-I*A - B)*a^2*e^(2*I*f*x + 2*I*e) + 2*B*a^2 + ((I*A + 3*B)*a^2*e^(2*I*f*x + 2*I*e) + (I*A + 3*B)*a^2)*log(e^(2*I*f*x + 2*I*e) + 1))/(c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

**Sympy [A]**

time = 0.38, size = 134, normalized size = 1.44

$$\frac{2Ba^2}{c f e^{2ie} e^{2ifx} + c f} + \frac{ia^2(A - 3iB) \log(e^{2ifx} + e^{-2ie})}{c f} + \begin{cases} \frac{(-iAa^2 e^{2ie} - Ba^2 e^{2ie}) e^{2ifx}}{c f} & \text{for } cf \neq 0 \\ \frac{x(2Aa^2 e^{2ie} - 2iBa^2 e^{2ie})}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

[Out]  $2*B*a**2/(c*f*\exp(2*I*e)*\exp(2*I*f*x) + c*f) + I*a**2*(A - 3*I*B)*\log(\exp(2*I*f*x) + \exp(-2*I*e))/(c*f) + \text{Piecewise}((( -I*A*a**2*\exp(2*I*e) - B*a**2*\exp(2*I*e))*\exp(2*I*f*x)/(c*f), \text{Ne}(c*f, 0)), (x*(2*A*a**2*\exp(2*I*e) - 2*I*B*a**2*\exp(2*I*e))/c, \text{True}))$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs.  $2(86) = 172$ .  
time = 0.63, size = 283, normalized size = 3.04

$$\frac{\frac{(Aa^2+3Ba^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)}{c} + 2\frac{(-Aa^2-3Ba^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)}{c} - \frac{(-Aa^2-3Ba^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e)-1)}{c} - \frac{(Aa^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+3Ba^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-2IABa^2\tan(\frac{1}{2}fx+\frac{1}{2}e)-Aa^2-3Ba^2}{(\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+1)c} - \frac{-3IAa^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-9Ba^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+10Aa^2\tan(\frac{1}{2}fx+\frac{1}{2}e)-22IABa^2\tan(\frac{1}{2}fx+\frac{1}{2}e)+3IAa^2+9Ba^2}{c(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="giac")`

[Out]  $((I*A*a^2 + 3*B*a^2)*\log(\tan(1/2*f*x + 1/2*e) + 1)/c + 2*(-I*A*a^2 - 3*B*a^2)*\log(\tan(1/2*f*x + 1/2*e) + I)/c - (-I*A*a^2 - 3*B*a^2)*\log(\tan(1/2*f*x + 1/2*e) - 1)/c - (I*A*a^2*2*\tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*2*\tan(1/2*f*x + 1/2*e)^2 - 2*I*B*a^2*\tan(1/2*f*x + 1/2*e) - I*A*a^2 - 3*B*a^2)/((\tan(1/2*f*x + 1/2*e)^2 - 1)*c) - (-3*I*A*a^2*\tan(1/2*f*x + 1/2*e)^2 - 9*B*a^2*\tan(1/2*f*x + 1/2*e)^2 + 10*A*a^2*\tan(1/2*f*x + 1/2*e) - 22*I*B*a^2*\tan(1/2*f*x + 1/2*e) + 3*I*A*a^2 + 9*B*a^2)/(c*(\tan(1/2*f*x + 1/2*e) + I)^2))/f$

**Mupad** [B]

time = 8.66, size = 105, normalized size = 1.13

$$-\frac{\ln(\tan(e+fx)+1i)\left(\frac{3Ba^2}{c} + \frac{Aa^2 1i}{c}\right)}{f} + \frac{Aa^2 + Ba^2 1i}{c} + \frac{Aa^2 - Ba^2 3i}{c} - \frac{Ba^2 \tan(e+fx) 1i}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1i),x)`

[Out]  $((A*a^2 + B*a^2*1i)/c + (A*a^2 - B*a^2*3i)/c)/(f*(\tan(e + f*x) + 1i)) - (\log(\tan(e + f*x) + 1i)*((A*a^2*1i)/c + (3*B*a^2)/c))/f - (B*a^2*\tan(e + f*x)*1i)/(c*f)$

$$3.684 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=91

$$-\frac{ia^2Bx}{c^2} - \frac{a^2B \log(\cos(e+fx))}{c^2f} + \frac{a^2(iA+B)}{c^2f(i+\tan(e+fx))^2} - \frac{a^2(A-3iB)}{c^2f(i+\tan(e+fx))}$$

[Out]  $-I*a^2*B*x/c^2 - a^2*B*\ln(\cos(f*x+e))/c^2/f + a^2*(I*A+B)/c^2/f/(I+\tan(f*x+e)) - 2*a^2*(A-3*I*B)/c^2/f/(I+\tan(f*x+e))$

**Rubi [A]**

time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^2(A-3iB)}{c^2f(\tan(e+fx)+i)} + \frac{a^2(B+iA)}{c^2f(\tan(e+fx)+i)^2} - \frac{a^2B \log(\cos(e+fx))}{c^2f} - \frac{ia^2Bx}{c^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^2}, x]$

[Out]  $((-I)*a^2*B*x)/c^2 - (a^2*B*\text{Log}[\text{Cos}[e + f*x]])/(c^2*f) + (a^2*(I*A + B))/(c^2*f*(I + \text{Tan}[e + f*x])^2) - (a^2*(A - (3*I)*B))/(c^2*f*(I + \text{Tan}[e + f*x]))$

Rule 78

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)}{(c_.) + (d_.)*(x_.)} \frac{(e_.) + (f_.)*(x_.)}{(e_.) + (f_.)*(x_.)} \frac{(A_.) + (B_.)*(x_.)}{(A_.) + (B_.)*(x_.)}}{(c_.) + (d_.)*(x_.)} \frac{(e_.) + (f_.)*(x_.)}{(e_.) + (f_.)*(x_.)} \frac{(A_.) + (B_.)*(x_.)}{(A_.) + (B_.)*(x_.)}]^n, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[\frac{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]} \frac{(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]}{(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]}}{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]} \frac{(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]}{(A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]}]^m, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^2} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^3} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( -\frac{2ia(A-iB)}{c^3(i+x)^3} + \frac{a(A-3iB)}{c^3(i+x)^2} + \frac{aB}{c^3(i+x)} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{ia^2 Bx}{c^2} - \frac{a^2 B \log(\cos(e + fx))}{c^2 f} + \frac{a^2(iA + B)}{c^2 f(i + \tan(e + fx))}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(91) = 182.  
time = 1.35, size = 184, normalized size = 2.02

$$\frac{a^2(4B - i \cos(2(e + fx))(A - iB + 8Bfx - 2B \log(\cos^2(e + fx))) + A \sin(2(e + fx)) - iB \sin(2(e + fx)) - 8Bfx \sin(2(e + fx)) + 2B \log(\cos^2(e + fx)) \sin(2(e + fx)) + 4B \text{ArcTan}(\tan(3e + fx))(i \cos(2(e + fx)) + \sin(2(e + fx))))}{4c^2 f (\cos(fx) + i \sin(fx))^2} (\cos(2(e + 2fx)) + i \sin(2(e + 2fx)))$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^2,x]

[Out] (a^2\*(4\*B - I\*Cos[2\*(e + f\*x)]\*(A - I\*B + 8\*B\*f\*x - (2\*I)\*B\*Log[Cos[e + f\*x]^2]) + A\*Sin[2\*(e + f\*x)] - I\*B\*Sin[2\*(e + f\*x)] - 8\*B\*f\*x\*Sin[2\*(e + f\*x)] + (2\*I)\*B\*Log[Cos[e + f\*x]^2]\*Sin[2\*(e + f\*x)] + 4\*B\*ArcTan[Tan[3\*e + f\*x]]\*(I\*Cos[2\*(e + f\*x)] + Sin[2\*(e + f\*x)]))\*(Cos[2\*(e + 2\*f\*x)] + I\*Sin[2\*(e + 2\*f\*x)]))/(4\*c^2\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.25, size = 64, normalized size = 0.70

method	result	size
derivativedivides	$\frac{a^2 \left( -\frac{-2iA-2B}{2(i+\tan(fx+e))^2} - \frac{-3iB+A}{i+\tan(fx+e)} + B \ln(i+\tan(fx+e)) \right)}{f c^2}$	64
default	$\frac{a^2 \left( -\frac{-2iA-2B}{2(i+\tan(fx+e))^2} - \frac{-3iB+A}{i+\tan(fx+e)} + B \ln(i+\tan(fx+e)) \right)}{f c^2}$	64
risch	$-\frac{e^{4i(fx+e)} B a^2}{4c^2 f} - \frac{ie^{4i(fx+e)} a^2 A}{4c^2 f} + \frac{a^2 B e^{2i(fx+e)}}{c^2 f} + \frac{2ia^2 B e}{c^2 f} - \frac{a^2 B \ln(e^{2i(fx+e)}+1)}{c^2 f}$	103

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f\*a^2/c^2\*(-1/2\*(-2\*B-2\*I\*A)/(I+tan(f\*x+e))^2-(A-3\*I\*B)/(I+tan(f\*x+e))+B\*  
ln(I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 2.52, size = 65, normalized size = 0.71

$$\frac{(-iA - B)a^2e^{4ifx+4ie} + 4Ba^2e^{(2ifx+2ie)} - 4Ba^2 \log(e^{(2ifx+2ie)} + 1)}{4c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/4\*((-I\*A - B)\*a^2\*e^(4\*I\*f\*x + 4\*I\*e) + 4\*B\*a^2\*e^(2\*I\*f\*x + 2\*I\*e) - 4\*B\*a^2\*log(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c^2\*f)

**Sympy [A]**

time = 0.32, size = 160, normalized size = 1.76

$$-\frac{Ba^2 \log(e^{2ifx} + e^{-2ie})}{c^2f} + \begin{cases} \frac{4Ba^2c^2fe^{2ie}e^{2ifx} + (-iAa^2c^2fe^{4ie} - Ba^2c^2fe^{4ie})e^{4ifx}}{4c^4f^2} & \text{for } c^4f^2 \neq 0 \\ \frac{x(Aa^2e^{4ie} - iBa^2e^{4ie} + 2iBa^2e^{2ie})}{c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*2,x)

[Out] -B\*a\*\*2\*log(exp(2\*I\*f\*x) + exp(-2\*I\*e))/(c\*\*2\*f) + Piecewise(((4\*B\*a\*\*2\*c\*\*2\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + (-I\*A\*a\*\*2\*c\*\*2\*f\*exp(4\*I\*e) - B\*a\*\*2\*c\*\*2\*f\*exp(4\*I\*e))\*exp(4\*I\*f\*x))/(4\*c\*\*4\*f\*\*2), Ne(c\*\*4\*f\*\*2, 0)), (x\*(A\*a\*\*2\*exp(4\*I\*e) - I\*B\*a\*\*2\*exp(4\*I\*e) + 2\*I\*B\*a\*\*2\*exp(2\*I\*e))/c\*\*2, True))

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs.  $2(84) = 168$ .

time = 0.75, size = 201, normalized size = 2.21

$$\frac{6Ba^2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1)}{c^2} - \frac{12Ba^2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)}{c^2} + \frac{6Ba^2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1)}{c^2} + \frac{25Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 + 12Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 112iBa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 198Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 112iBa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 25Ba^2}{c^2(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] 
$$-1/6*(6*B*a^2*\log(\tan(1/2*f*x + 1/2*e) + 1)/c^2 - 12*B*a^2*\log(\tan(1/2*f*x + 1/2*e) + I)/c^2 + 6*B*a^2*\log(\tan(1/2*f*x + 1/2*e) - 1)/c^2 + (25*B*a^2*\tan(1/2*f*x + 1/2*e)^4 + 12*A*a^2*\tan(1/2*f*x + 1/2*e)^3 + 112*I*B*a^2*\tan(1/2*f*x + 1/2*e)^3 - 198*B*a^2*\tan(1/2*f*x + 1/2*e)^2 - 12*A*a^2*\tan(1/2*f*x + 1/2*e) - 112*I*B*a^2*\tan(1/2*f*x + 1/2*e) + 25*B*a^2)/(c^2*(\tan(1/2*f*x + 1/2*e) + I)^4))/f$$

**Mupad [B]**

time = 8.71, size = 104, normalized size = 1.14

$$\frac{a^2 (B 2i + A \tan(e + f x) \operatorname{li} + 3 B \tan(e + f x) + B \ln(\tan(e + f x) + \operatorname{li}) \operatorname{li} + 2 B \ln(\tan(e + f x) + \operatorname{li}) \tan(e + f x) - B \ln(\tan(e + f x) + \operatorname{li}) \tan(e + f x)^2 \operatorname{li}) \operatorname{li}}{c^2 f (-1 + \tan(e + f x) \operatorname{li})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2)/(c - c\*tan(e + f\*x)\*1i)^2,x)

[Out] 
$$-(a^2*(B*2i + A*\tan(e + f*x)*1i + 3*B*\tan(e + f*x) + B*\log(\tan(e + f*x) + 1i)*1i + 2*B*\log(\tan(e + f*x) + 1i)*\tan(e + f*x) - B*\log(\tan(e + f*x) + 1i)*\tan(e + f*x)^2*1i)*1i)/(c^2*f*(\tan(e + f*x)*1i - 1)^2)$$

$$3.685 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^3} dx$$

Optimal. Leaf size=93

$$-\frac{2a^2(A-iB)}{3c^3f(i+\tan(e+fx))^3} - \frac{a^2(iA+3B)}{2c^3f(i+\tan(e+fx))^2} - \frac{ia^2B}{c^3f(i+\tan(e+fx))}$$

[Out]  $-2/3*a^2*(A-I*B)/c^3/f/(I+\tan(f*x+e))^3-1/2*a^2*(I*A+3*B)/c^3/f/(I+\tan(f*x+e))^2-I*a^2*B/c^3/f/(I+\tan(f*x+e))$

Rubi [A]

time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^2(3B+ia)}{2c^3f(\tan(e+fx)+i)^2} - \frac{2a^2(A-iB)}{3c^3f(\tan(e+fx)+i)^3} - \frac{ia^2B}{c^3f(\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^3}, x]$

[Out]  $(-2*a^2*(A - I*B))/(3*c^3*f*(I + \text{Tan}[e + f*x])^3) - (a^2*(I*A + 3*B))/(2*c^3*f*(I + \text{Tan}[e + f*x])^2) - (I*a^2*B)/(c^3*f*(I + \text{Tan}[e + f*x]))$

Rule 78

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]} :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]} :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps



$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^3} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^4} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(A-iB)}{c^4(i+x)^4} + \frac{a(iA+3B)}{c^4(i+x)^3} + \frac{iaB}{c^4(i+x)^2} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{2a^2(A-iB)}{3c^3 f (i + \tan(e + fx))^3} - \frac{a^2(iA+3B)}{2c^3 f (i + \tan(e + fx))^2} - \frac{iaB}{c^3 f (i + \tan(e + fx))}$$

**Mathematica [A]**

time = 1.05, size = 81, normalized size = 0.87

$$\frac{a^2((-5iA + B) \cos(e + fx) - (A + 5iB) \sin(e + fx))(\cos(5e + 7fx) + i \sin(5e + 7fx))}{24c^3 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^3,x]

[Out] (a^2\*(((5\*I)\*A + B)\*Cos[e + f\*x] - (A + (5\*I)\*B)\*Sin[e + f\*x])\*(Cos[5\*e + 7\*f\*x] + I\*Ssin[5\*e + 7\*f\*x]))/(24\*c^3\*f\*(Cos[f\*x] + I\*Ssin[f\*x])^2)

**Maple [A]**

time = 0.30, size = 69, normalized size = 0.74

method	result
derivativedivides	$\frac{a^2 \left( -\frac{-2iB+2A}{3(i+\tan(fx+e))^3} - \frac{iA+3B}{2(i+\tan(fx+e))^2} - \frac{iB}{i+\tan(fx+e)} \right)}{f c^3}$
default	$\frac{a^2 \left( -\frac{-2iB+2A}{3(i+\tan(fx+e))^3} - \frac{iA+3B}{2(i+\tan(fx+e))^2} - \frac{iB}{i+\tan(fx+e)} \right)}{f c^3}$
risch	$-\frac{a^2 e^{6i(fx+e)} B}{12c^3 f} - \frac{ia^2 e^{6i(fx+e)} A}{12c^3 f} + \frac{a^2 e^{4i(fx+e)} B}{8c^3 f} - \frac{ia^2 e^{4i(fx+e)} A}{8c^3 f}$
norman	$\frac{2iA a^2 (\tan^2(fx+e))}{cf} + \frac{a^2 A \tan(fx+e)}{cf} - \frac{iA a^2 + a^2 B}{6cf} - \frac{5(-iB a^2 + a^2 A) (\tan^3(fx+e))}{3cf} - \frac{(iA a^2 + 5a^2 B) (\tan^4(fx+e))}{2cf} - \frac{iB a^2 (\tan^5(fx+e))}{cf}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x,method=\_RE TURNVERBOSE)

[Out] 1/f\*a^2/c^3\*(-1/3\*(2\*A-2\*I\*B)/(I+tan(f\*x+e))^3-1/2\*(I\*A+3\*B)/(I+tan(f\*x+e))^2-I\*B/(I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 3.08, size = 51, normalized size = 0.55

$$\frac{2(iA + B)a^2e^{(6ifx+6ie)} + 3(iA - B)a^2e^{(4ifx+4ie)}}{24c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x, algorithm="fricas")

[Out]  $-1/24*(2*(I*A + B)*a^2*e^{(6*I*f*x + 6*I*e)} + 3*(I*A - B)*a^2*e^{(4*I*f*x + 4*I*e)})/(c^3*f)$

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs.  $2(75) = 150$ .

time = 0.30, size = 167, normalized size = 1.80

$$\begin{cases} \frac{(-12iAa^2c^3fe^{4ie} + 12Ba^2c^3fe^{4ie})e^{4ifx} + (-8iAa^2c^3fe^{6ie} - 8Ba^2c^3fe^{6ie})e^{6ifx}}{96c^6f^2} & \text{for } c^6f^2 \neq 0 \\ \frac{x(Aa^2e^{6ie} + Aa^2e^{4ie} - iBa^2e^{6ie} + iBa^2e^{4ie})}{2c^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x)

[Out] Piecewise(((((-12\*I\*A\*a\*\*2\*c\*\*3\*f\*exp(4\*I\*e) + 12\*B\*a\*\*2\*c\*\*3\*f\*exp(4\*I\*e))\*exp(4\*I\*f\*x) + (-8\*I\*A\*a\*\*2\*c\*\*3\*f\*exp(6\*I\*e) - 8\*B\*a\*\*2\*c\*\*3\*f\*exp(6\*I\*e))\*exp(6\*I\*f\*x))/(96\*c\*\*6\*f\*\*2), Ne(c\*\*6\*f\*\*2, 0)), (x\*(A\*a\*\*2\*exp(6\*I\*e) + A\*a\*\*2\*exp(4\*I\*e) - I\*B\*a\*\*2\*exp(6\*I\*e) + I\*B\*a\*\*2\*exp(4\*I\*e))/(2\*c\*\*3), True))

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 165 vs.  $2(80) = 160$ .

time = 0.93, size = 165, normalized size = 1.77

$$\frac{2(3Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 + 3iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 3Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 8Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 2iBa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 3iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{3c^3f(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$-2/3*(3*A*a^2*\tan(1/2*f*x + 1/2*e)^5 + 3*I*A*a^2*\tan(1/2*f*x + 1/2*e)^4 - 3*B*a^2*\tan(1/2*f*x + 1/2*e)^4 - 8*A*a^2*\tan(1/2*f*x + 1/2*e)^3 + 2*I*B*a^2*\tan(1/2*f*x + 1/2*e)^3 - 3*I*A*a^2*\tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*\tan(1/2*f*x + 1/2*e)^2 + 3*A*a^2*\tan(1/2*f*x + 1/2*e))/(c^3*f*(\tan(1/2*f*x + 1/2*e) + I)^6)$$

**Mupad [B]**

time = 8.69, size = 87, normalized size = 0.94

$$\frac{\frac{a^2(A-B1i)}{6} + \frac{a^2 \tan(e+fx)(-3B+A3i)}{6} + B a^2 \tan(e+fx)^2 1i}{c^3 f (-\tan(e+fx)^3 - \tan(e+fx)^2 3i + 3 \tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2)/(c - c\*tan(e + f\*x)\*1i)^3,x)

[Out] 
$$((a^2*(A - B*1i))/6 + (a^2*\tan(e + f*x)*(A*3i - 3*B))/6 + B*a^2*\tan(e + f*x)^2*1i)/(c^3*f*(3*\tan(e + f*x) - \tan(e + f*x)^2*3i - \tan(e + f*x)^3 + 1i))$$

$$3.686 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^4} dx$$

Optimal. Leaf size=91

$$-\frac{a^2(iA+B)}{2c^4 f(i+\tan(e+fx))^4} + \frac{a^2(A-3iB)}{3c^4 f(i+\tan(e+fx))^3} + \frac{a^2 B}{2c^4 f(i+\tan(e+fx))^2}$$

[Out]  $-1/2*a^2*(I*A+B)/c^4/f/(I+\tan(f*x+e))^4+1/3*a^2*(A-3*I*B)/c^4/f/(I+\tan(f*x+e))^3+1/2*a^2*B/c^4/f/(I+\tan(f*x+e))^2$

Rubi [A]

time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{a^2(A-3iB)}{3c^4 f(\tan(e+fx)+i)^3} - \frac{a^2(B+iA)}{2c^4 f(\tan(e+fx)+i)^4} + \frac{a^2 B}{2c^4 f(\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^4}, x]$

[Out]  $-1/2*(a^2*(I*A + B))/(c^4*f*(I + \text{Tan}[e + f*x])^4) + (a^2*(A - (3*I)*B))/(3*c^4*f*(I + \text{Tan}[e + f*x])^3) + (a^2*B)/(2*c^4*f*(I + \text{Tan}[e + f*x])^2)$

Rule 78

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*(c_. + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^4} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^5} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(iA+B)}{c^5(i+x)^5} - \frac{a(A-3iB)}{c^5(i+x)^4} - \frac{aB}{c^5(i+x)^3} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{a^2(iA + B)}{2c^4 f (i + \tan(e + fx))^4} + \frac{a^2(A - 3iB)}{3c^4 f (i + \tan(e + fx))^3} + \frac{a^2 B}{2c^4 f (i + \tan(e + fx))^2}$$

**Mathematica [A]**

time = 1.13, size = 91, normalized size = 1.00

$$\frac{a^2(-8iA + 3(-3iA + B) \cos(2(e + fx)) - 3(A + 3iB) \sin(2(e + fx)))(\cos(6e + 8fx) + i \sin(6e + 8fx))}{96c^4 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^4,x]

[Out] (a^2\*((-8\*I)\*A + 3\*((-3\*I)\*A + B)\*Cos[2\*(e + f\*x)] - 3\*(A + (3\*I)\*B)\*Sin[2\*(e + f\*x)])\*(Cos[6\*e + 8\*f\*x] + I\*Sin[6\*e + 8\*f\*x])/(96\*c^4\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.25, size = 68, normalized size = 0.75

method	result	size
derivativedivides	$\frac{a^2 \left( -\frac{2iA+2B}{4(i+\tan(fx+e))^4} - \frac{3iB-A}{3(i+\tan(fx+e))^3} + \frac{B}{2(i+\tan(fx+e))^2} \right)}{f c^4}$	68
default	$\frac{a^2 \left( -\frac{2iA+2B}{4(i+\tan(fx+e))^4} - \frac{3iB-A}{3(i+\tan(fx+e))^3} + \frac{B}{2(i+\tan(fx+e))^2} \right)}{f c^4}$	68
risch	$-\frac{a^2 e^{8i(fx+e)} B}{32c^4 f} - \frac{ia^2 e^{8i(fx+e)} A}{32c^4 f} - \frac{ia^2 A e^{6i(fx+e)}}{12c^4 f} + \frac{a^2 e^{4i(fx+e)} B}{16c^4 f} - \frac{ia^2 e^{4i(fx+e)} A}{16c^4 f}$	110

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x,method=\_RE TURNVERBOSE)

[Out] 1/f\*a^2/c^4\*(-1/4\*(2\*B+2\*I\*A)/(I+tan(f\*x+e))^4-1/3\*(-A+3\*I\*B)/(I+tan(f\*x+e))^3+1/2\*B/(I+tan(f\*x+e))^2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, alg  
orithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 5.13, size = 67, normalized size = 0.74

$$\frac{3(iA + B)a^2e^{(8ifx+8ie)} + 8iAa^2e^{(6ifx+6ie)} + 6(iA - B)a^2e^{(4ifx+4ie)}}{96c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, alg  
orithm="fricas")`

[Out]  $-1/96*(3*(I*A + B)*a^2*e^{(8*I*f*x + 8*I*e)} + 8*I*A*a^2*e^{(6*I*f*x + 6*I*e)} + 6*(I*A - B)*a^2*e^{(4*I*f*x + 4*I*e)})/(c^4*f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs.  $2(73) = 146$ .

time = 0.37, size = 218, normalized size = 2.40

$$\begin{cases} \frac{-512iAa^2c^8f^2e^{6ie}e^{6ifx} + (-384iAa^2c^8f^2e^{4ie} + 384Ba^2c^8f^2e^{4ie})e^{4ifx} + (-192iAa^2c^8f^2e^{8ie} - 192Ba^2c^8f^2e^{8ie})e^{8ifx}}{6144c^{12}f^3} & \text{for } c^{12}f^3 \neq 0 \\ \frac{x(Aa^2e^{8ie} + 2Aa^2e^{6ie} + Aa^2e^{4ie} - iBa^2e^{8ie} + iBa^2e^{4ie})}{4c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)`

[Out] `Piecewise((( -512*I*A*a**2*c**8*f**2*exp(6*I*e)*exp(6*I*f*x) + (-384*I*A*a**2*c**8*f**2*exp(4*I*e) + 384*B*a**2*c**8*f**2*exp(4*I*e))*exp(4*I*f*x) + (-192*I*A*a**2*c**8*f**2*exp(8*I*e) - 192*B*a**2*c**8*f**2*exp(8*I*e))*exp(8*I*f*x))/(6144*c**12*f**3), Ne(c**12*f**3, 0)), (x*(A*a**2*exp(8*I*e) + 2*A*a**2*exp(6*I*e) + A*a**2*exp(4*I*e) - I*B*a**2*exp(8*I*e) + I*B*a**2*exp(4*I*e))/(4*c**4), True))`

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs.  $2(78) = 156$ .

time = 1.02, size = 201, normalized size = 2.21

$$\frac{2(3Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 + 6iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 3Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 17Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 + 6Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 17Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 6iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 3Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 3Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{3c^4f(\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out] 
$$\frac{-2/3*(3*A*a^2*\tan(1/2*f*x + 1/2*e)^7 + 6*I*A*a^2*\tan(1/2*f*x + 1/2*e)^6 - 3*B*a^2*\tan(1/2*f*x + 1/2*e)^6 - 17*A*a^2*\tan(1/2*f*x + 1/2*e)^5 - 16*I*A*a^2*\tan(1/2*f*x + 1/2*e)^4 + 6*B*a^2*\tan(1/2*f*x + 1/2*e)^4 + 17*A*a^2*\tan(1/2*f*x + 1/2*e)^3 + 6*I*A*a^2*\tan(1/2*f*x + 1/2*e)^2 - 3*B*a^2*\tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*\tan(1/2*f*x + 1/2*e))/(c^4*f*(\tan(1/2*f*x + 1/2*e) + I)^8)}$$

**Mupad [B]**

time = 8.71, size = 78, normalized size = 0.86

$$\frac{a^2 (3 B \tan(e + f x)^2 + 2 A \tan(e + f x) - A i)}{6 c^4 f (\tan(e + f x)^4 + \tan(e + f x)^3 4i - 6 \tan(e + f x)^2 - \tan(e + f x) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2)/(c - c\*tan(e + f\*x)\*1i)^4,x)

[Out] 
$$(a^2*(2*A*\tan(e + f*x) - A*1i + 3*B*\tan(e + f*x)^2))/(6*c^4*f*(\tan(e + f*x)^3*4i - 6*\tan(e + f*x)^2 - \tan(e + f*x)*4i + \tan(e + f*x)^4 + 1))$$

$$3.687 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ict \tan(e+fx))^5} dx$$

Optimal. Leaf size=95

$$\frac{2a^2(A-iB)}{5c^5 f(i+\tan(e+fx))^5} + \frac{a^2(iA+3B)}{4c^5 f(i+\tan(e+fx))^4} + \frac{ia^2B}{3c^5 f(i+\tan(e+fx))^3}$$

[Out]  $2/5*a^2*(A-I*B)/c^5/f/(I+\tan(f*x+e))^5+1/4*a^2*(I*A+3*B)/c^5/f/(I+\tan(f*x+e))^4+1/3*I*a^2*B/c^5/f/(I+\tan(f*x+e))^3$

Rubi [A]

time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{a^2(3B+ia)}{4c^5 f(\tan(e+fx)+i)^4} + \frac{2a^2(A-iB)}{5c^5 f(\tan(e+fx)+i)^5} + \frac{ia^2B}{3c^5 f(\tan(e+fx)+i)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^5}, x]$

[Out]  $(2*a^2*(A - I*B))/(5*c^5*f*(I + \text{Tan}[e + f*x])^5) + (a^2*(I*A + 3*B))/(4*c^5*f*(I + \text{Tan}[e + f*x])^4) + ((I/3)*a^2*B)/(c^5*f*(I + \text{Tan}[e + f*x])^3)$

Rule 78

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*(c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps



$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^5} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^6} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( -\frac{2a(A-iB)}{c^6(i+x)^6} - \frac{ia(A-3iB)}{c^6(i+x)^5} - \frac{iaB}{c^6(i+x)^4} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{2a^2(A-iB)}{5c^5 f (i + \tan(e + fx))^5} + \frac{a^2(iA + 3B)}{4c^5 f (i + \tan(e + fx))^4} + \frac{a^2 B}{3c^5 f (i + \tan(e + fx))^3}$$

**Mathematica [A]**

time = 1.41, size = 116, normalized size = 1.22

$$\frac{a^2(5(-21iA + B) \cos(e + fx) + 6(-7iA + 3B) \cos(3(e + fx)) - (3A + 7iB)(5 \sin(e + fx) + 6 \sin(3(e + fx))))(\cos(7e + 9fx) + i \sin(7e + 9fx))}{960c^5 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^5, x]
```

```
[Out] (a^2*(5*((-21*I)*A + B)*Cos[e + f*x] + 6*((-7*I)*A + 3*B)*Cos[3*(e + f*x)] - (3*A + (7*I)*B)*(5*Sin[e + f*x] + 6*Sin[3*(e + f*x)]))*(Cos[7*e + 9*f*x] + I*Sin[7*e + 9*f*x]))/(960*c^5*f*(Cos[f*x] + I*Sin[f*x])^2)
```

**Maple [A]**

time = 0.44, size = 69, normalized size = 0.73

method	result
derivativedivides	$\frac{a^2 \left( -\frac{-iA-3B}{4(i+\tan(fx+e))^4} - \frac{2iB-2A}{5(i+\tan(fx+e))^5} + \frac{iB}{3(i+\tan(fx+e))^3} \right)}{f c^5}$
default	$\frac{a^2 \left( -\frac{-iA-3B}{4(i+\tan(fx+e))^4} - \frac{2iB-2A}{5(i+\tan(fx+e))^5} + \frac{iB}{3(i+\tan(fx+e))^3} \right)}{f c^5}$
risch	$-\frac{a^2 e^{10i(fx+e)} B}{80c^5 f} - \frac{ia^2 e^{10i(fx+e)} A}{80c^5 f} - \frac{e^{8i(fx+e)} B a^2}{64c^5 f} - \frac{3ie^{8i(fx+e)} a^2 A}{64c^5 f} + \frac{e^{6i(fx+e)} B a^2}{48c^5 f} - \frac{ie^{6i(fx+e)} a^2 A}{16c^5 f} +$
norman	$\frac{a^2 A \tan(fx+e)}{cf} + \frac{-9iA a^2 + a^2 B}{60cf} - \frac{(-7iB a^2 + 12a^2 A) (\tan^3(fx+e))}{3cf} + \frac{(iA a^2 + 7a^2 B) (\tan^6(fx+e))}{4cf} + \frac{7(-8iB a^2 + 3a^2 A) (\tan^5(fx+e))}{15cf} + \frac{a^2 B}{(1+\tan^2(fx+e))^5 c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5, x, method=_RE TURNVERBOSE)
```

```
[Out] 1/f*a^2/c^5*(-1/4*(-I*A-3*B)/(I+tan(f*x+e))^4-1/5*(-2*A+2*I*B)/(I+tan(f*x+e))^5+1/3*I*B/(I+tan(f*x+e))^3)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 3.94, size = 93, normalized size = 0.98

$$\frac{12(iA+B)a^2e^{(10ifx+10ie)} + 15(3iA+B)a^2e^{(8ifx+8ie)} + 20(3iA-B)a^2e^{(6ifx+6ie)} + 30(iA-B)a^2e^{(4ifx+4ie)}}{960c^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, alg
orithm="fricas")
```

```
[Out] -1/960*(12*(I*A + B)*a^2*e^(10*I*f*x + 10*I*e) + 15*(3*I*A + B)*a^2*e^(8*I*
f*x + 8*I*e) + 20*(3*I*A - B)*a^2*e^(6*I*f*x + 6*I*e) + 30*(I*A - B)*a^2*e^(
4*I*f*x + 4*I*e))/(c^5*f)
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(76) = 152.

time = 0.46, size = 332, normalized size = 3.49

$$\begin{cases} \frac{(-245760iAa^2c^{15}f^3e^{4ie}+245760Ba^2c^{15}f^3e^{4ie})e^{4ifx}+(-491520iAa^2c^{15}f^3e^{6ie}+163840Ba^2c^{15}f^3e^{6ie})e^{6ifx}+(-368640iAa^2c^{15}f^3e^{8ie}-122880Ba^2c^{15}f^3e^{8ie})e^{8ifx}+(-98304iAa^2c^{15}f^3e^{10ie}-98304Ba^2c^{15}f^3e^{10ie})e^{10ifx}}{7864320c^{20}f^4} & \text{for } c^{20}f^4 \neq 0 \\ \frac{x(Aa^2e^{10ie}+3Aa^2e^{8ie}+3Aa^2e^{6ie}+Aa^2e^{4ie}-iBa^2e^{10ie}-iBa^2e^{8ie}+iBa^2e^{6ie}+iBa^2e^{4ie})}{8c^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**5,x)
```

```
[Out] Piecewise(((((-245760*I*A*a**2*c**15*f**3*exp(4*I*e) + 245760*B*a**2*c**15*f
**3*exp(4*I*e))*exp(4*I*f*x) + (-491520*I*A*a**2*c**15*f**3*exp(6*I*e) + 16
3840*B*a**2*c**15*f**3*exp(6*I*e))*exp(6*I*f*x) + (-368640*I*A*a**2*c**15*f
**3*exp(8*I*e) - 122880*B*a**2*c**15*f**3*exp(8*I*e))*exp(8*I*f*x) + (-9830
4*I*A*a**2*c**15*f**3*exp(10*I*e) - 98304*B*a**2*c**15*f**3*exp(10*I*e))*ex
p(10*I*f*x))/(7864320*c**20*f**4), Ne(c**20*f**4, 0)), (x*(A*a**2*exp(10*I*
e) + 3*A*a**2*exp(8*I*e) + 3*A*a**2*exp(6*I*e) + A*a**2*exp(4*I*e) - I*B*a*
**2*exp(10*I*e) - I*B*a**2*exp(8*I*e) + I*B*a**2*exp(6*I*e) + I*B*a**2*exp(4
*I*e))/(8*c**5), True))
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs.  $2(80) = 160$ .  
time = 1.25, size = 309, normalized size = 3.25

$$\frac{2(15A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 - 45A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 15B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 - 150A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 300B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 - 225A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 10B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 + 300A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 + 225B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 - 10B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 150A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 - 150B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 45A^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 + 15B^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6)}{15c^5 f (\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^5,x, algorithm="giac")

[Out]  $-2/15*(15*A*a^2*\tan(1/2*f*x + 1/2*e)^9 + 45*I*A*a^2*\tan(1/2*f*x + 1/2*e)^8 - 15*B*a^2*\tan(1/2*f*x + 1/2*e)^8 - 150*A*a^2*\tan(1/2*f*x + 1/2*e)^7 - 10*I*B*a^2*\tan(1/2*f*x + 1/2*e)^7 - 225*I*A*a^2*\tan(1/2*f*x + 1/2*e)^6 + 55*B*a^2*\tan(1/2*f*x + 1/2*e)^6 + 306*A*a^2*\tan(1/2*f*x + 1/2*e)^5 + 24*I*B*a^2*\tan(1/2*f*x + 1/2*e)^5 + 225*I*A*a^2*\tan(1/2*f*x + 1/2*e)^4 - 55*B*a^2*\tan(1/2*f*x + 1/2*e)^4 - 150*A*a^2*\tan(1/2*f*x + 1/2*e)^3 - 10*I*B*a^2*\tan(1/2*f*x + 1/2*e)^3 - 45*I*A*a^2*\tan(1/2*f*x + 1/2*e)^2 + 15*B*a^2*\tan(1/2*f*x + 1/2*e)^2 + 15*A*a^2*\tan(1/2*f*x + 1/2*e))/(c^5*f*(\tan(1/2*f*x + 1/2*e) + I)^{10})$

**Mupad [B]**

time = 8.77, size = 108, normalized size = 1.14

$$\frac{\frac{a^2(9A+B1i)}{60} + \frac{a^2 \tan(e+fx)(5B+A15i)}{60} + \frac{Ba^2 \tan(e+fx)^2 1i}{3}}{c^5 f (\tan(e+fx)^5 + \tan(e+fx)^4 5i - 10 \tan(e+fx)^3 - \tan(e+fx)^2 10i + 5 \tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2)/(c - c\*tan(e + f\*x)\*1i)^5,x)

[Out]  $((a^2*(9*A + B*1i))/60 + (a^2*\tan(e + f*x)*(A*15i + 5*B))/60 + (B*a^2*\tan(e + f*x)^2*1i)/3)/(c^5*f*(5*\tan(e + f*x) - \tan(e + f*x)^2*10i - 10*\tan(e + f*x)^3 + \tan(e + f*x)^4*5i + \tan(e + f*x)^5 + 1i))$

$$3.688 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ict \tan(e+fx))^6} dx$$

Optimal. Leaf size=91

$$\frac{a^2(iA+B)}{3c^6 f(i+\tan(e+fx))^6} - \frac{a^2(A-3iB)}{5c^6 f(i+\tan(e+fx))^5} - \frac{a^2 B}{4c^6 f(i+\tan(e+fx))^4}$$

[Out] 1/3\*a^2\*(I\*A+B)/c^6/f/(I+tan(f\*x+e))^6-1/5\*a^2\*(A-3\*I\*B)/c^6/f/(I+tan(f\*x+e))^5-1/4\*a^2\*B/c^6/f/(I+tan(f\*x+e))^4

Rubi [A]

time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^2(A-3iB)}{5c^6 f(\tan(e+fx)+i)^5} + \frac{a^2(B+iA)}{3c^6 f(\tan(e+fx)+i)^6} - \frac{a^2 B}{4c^6 f(\tan(e+fx)+i)^4}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^6, x]

[Out] (a^2\*(I\*A + B))/(3\*c^6\*f\*(I + Tan[e + f\*x])^6) - (a^2\*(A - (3\*I)\*B))/(5\*c^6\*f\*(I + Tan[e + f\*x])^5) - (a^2\*B)/(4\*c^6\*f\*(I + Tan[e + f\*x])^4)

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^6} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^7} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( -\frac{2ia(A-iB)}{c^7(i+x)^7} + \frac{a(A-3iB)}{c^7(i+x)^6} + \frac{aB}{c^7(i+x)^5} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{a^2(iA + B)}{3c^6 f (i + \tan(e + fx))^6} - \frac{a^2(A - 3iB)}{5c^6 f (i + \tan(e + fx))^5} - \frac{a^2 B}{4c^6 f (i + \tan(e + fx))^4}$$

**Mathematica [A]**

time = 2.03, size = 143, normalized size = 1.57

$$\frac{ia^2(45A + 8(8A + iB) \cos(2(e + fx)) + 10(2A + iB) \cos(4(e + fx)) - 16iA \sin(2(e + fx)) + 32B \sin(2(e + fx)) - 10iA \sin(4(e + fx)) + 20B \sin(4(e + fx)))(\cos(8e + 10fx) + i \sin(8e + 10fx))}{960c^6 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^6,x]

[Out] ((-1/960\*I)\*a^2\*(45\*A + 8\*(8\*A + I\*B)\*Cos[2\*(e + f\*x)] + 10\*(2\*A + I\*B)\*Cos[4\*(e + f\*x)] - (16\*I)\*A\*Sin[2\*(e + f\*x)] + 32\*B\*Sin[2\*(e + f\*x)] - (10\*I)\*A\*Sin[4\*(e + f\*x)] + 20\*B\*Sin[4\*(e + f\*x)]\*(Cos[8\*e + 10\*f\*x] + I\*Sin[8\*e + 10\*f\*x]))/(c^6\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.30, size = 66, normalized size = 0.73

method	result
derivativedivides	$\frac{a^2 \left( -\frac{-2iA-2B}{6(i+\tan(fx+e))^6} - \frac{B}{4(i+\tan(fx+e))^4} - \frac{-3iB+A}{5(i+\tan(fx+e))^5} \right)}{f c^6}$
default	$\frac{a^2 \left( -\frac{-2iA-2B}{6(i+\tan(fx+e))^6} - \frac{B}{4(i+\tan(fx+e))^4} - \frac{-3iB+A}{5(i+\tan(fx+e))^5} \right)}{f c^6}$
risch	$-\frac{a^2 e^{12i(fx+e)} B}{192c^6 f} - \frac{ia^2 e^{12i(fx+e)} A}{192c^6 f} - \frac{e^{10i(fx+e)} B a^2}{80c^6 f} - \frac{ie^{10i(fx+e)} a^2 A}{40c^6 f} - \frac{3ia^2 A e^{8i(fx+e)}}{64c^6 f} + \frac{e^{6i(fx+e)} B a^2}{48c^6 f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f\*a^2/c^6\*(-1/6\*(-2\*B-2\*I\*A)/(I+tan(f\*x+e))^6-1/4\*B/(I+tan(f\*x+e))^4-1/5\*(A-3\*I\*B)/(I+tan(f\*x+e))^5)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x, alg orithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 2.07, size = 109, normalized size = 1.20

$$\frac{5(iA+B)a^2e^{(12i fx+12ie)} + 12(2iA+B)a^2e^{(10i fx+10ie)} + 45iAa^2e^{(8i fx+8ie)} + 20(2iA-B)a^2e^{(6i fx+6ie)} + 15(iA-B)a^2e^{(4i fx+4ie)}}{960c^6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x, alg orithm="fricas")

[Out] -1/960\*(5\*(I\*A + B)\*a^2\*e^(12\*I\*f\*x + 12\*I\*e) + 12\*(2\*I\*A + B)\*a^2\*e^(10\*I\*f\*x + 10\*I\*e) + 45\*I\*A\*a^2\*e^(8\*I\*f\*x + 8\*I\*e) + 20\*(2\*I\*A - B)\*a^2\*e^(6\*I\*f\*x + 6\*I\*e) + 15\*(I\*A - B)\*a^2\*e^(4\*I\*f\*x + 4\*I\*e))/(c^6\*f)

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(73) = 146.

time = 0.60, size = 379, normalized size = 4.16

$$\left\{ \frac{-141557760Aa^2c^{24}f^4e^{4if} + (-47185920Aa^2c^{24}f^4e^{4ie} + 47185920Ba^2c^{24}f^4e^{4ie})e^{4if} + (-125829120Aa^2c^{24}f^4e^{6ie} + 62914560Ba^2c^{24}f^4e^{6ie})e^{6if} + (-75497472Aa^2c^{24}f^4e^{8ie} - 37748736Ba^2c^{24}f^4e^{8ie})e^{8if} + (-15728640Aa^2c^{24}f^4e^{10ie} - 15728640Ba^2c^{24}f^4e^{10ie})e^{10if} + (-15728640Aa^2c^{24}f^4e^{12ie} - 15728640Ba^2c^{24}f^4e^{12ie})e^{12if}}{3019898880c^{30}f^5} \right\} \text{ for } c^{30}f^5 \neq 0$$

$$\frac{2(Aa^2e^{12ie} + 4Aa^2e^{10ie} + 6Aa^2e^{8ie} + 4Aa^2e^{6ie} + Aa^2e^{4ie} - 12Ba^2e^{12ie} - 21Ba^2e^{10ie} + 21Ba^2e^{8ie} + 12Ba^2e^{6ie})}{16c^6} \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x)

[Out] Piecewise((( -141557760\*I\*A\*a\*\*2\*c\*\*24\*f\*\*4\*exp(8\*I\*e)\*exp(8\*I\*f\*x) + (-47185920\*I\*A\*a\*\*2\*c\*\*24\*f\*\*4\*exp(4\*I\*e) + 47185920\*B\*a\*\*2\*c\*\*24\*f\*\*4\*exp(4\*I\*e) )\*exp(4\*I\*f\*x) + (-125829120\*I\*A\*a\*\*2\*c\*\*24\*f\*\*4\*exp(6\*I\*e) + 62914560\*B\*a\*\*2\*c\*\*24\*f\*\*4\*exp(6\*I\*e))\*exp(6\*I\*f\*x) + (-75497472\*I\*A\*a\*\*2\*c\*\*24\*f\*\*4\*exp(10\*I\*e) - 37748736\*B\*a\*\*2\*c\*\*24\*f\*\*4\*exp(10\*I\*e))\*exp(10\*I\*f\*x) + (-15728640\*I\*A\*a\*\*2\*c\*\*24\*f\*\*4\*exp(12\*I\*e) - 15728640\*B\*a\*\*2\*c\*\*24\*f\*\*4\*exp(12\*I\*e) )\*exp(12\*I\*f\*x))/(3019898880\*c\*\*30\*f\*\*5), Ne(c\*\*30\*f\*\*5, 0)), (x\*(A\*a\*\*2\*exp(12\*I\*e) + 4\*A\*a\*\*2\*exp(10\*I\*e) + 6\*A\*a\*\*2\*exp(8\*I\*e) + 4\*A\*a\*\*2\*exp(6\*I\*e) + A\*a\*\*2\*exp(4\*I\*e) - I\*B\*a\*\*2\*exp(12\*I\*e) - 2\*I\*B\*a\*\*2\*exp(10\*I\*e) + 2\*I\*B\*a\*\*2\*exp(6\*I\*e) + I\*B\*a\*\*2\*exp(4\*I\*e))/(16\*c\*\*6), True))

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(78) = 156.

time = 1.34, size = 381, normalized size = 4.19

2(16A^2a^2e^{12ie} + 4A^2a^2e^{10ie} + 6A^2a^2e^{8ie} + 4A^2a^2e^{6ie} + A^2a^2e^{4ie} - 12Ba^2e^{12ie} - 21Ba^2e^{10ie} + 21Ba^2e^{8ie} + 12Ba^2e^{6ie})/16c^6

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2/15*(15*A*a^2*\tan(1/2*f*x + 1/2*e)^{11} + 60*I*A*a^2*\tan(1/2*f*x + 1/2*e)^{10} \\ & - 15*B*a^2*\tan(1/2*f*x + 1/2*e)^{10} - 235*A*a^2*\tan(1/2*f*x + 1/2*e)^9 - 20*I*B*a^2*\tan(1/2*f*x + 1/2*e)^9 \\ & - 480*I*A*a^2*\tan(1/2*f*x + 1/2*e)^8 + 90*B*a^2*\tan(1/2*f*x + 1/2*e)^8 + 822*A*a^2*\tan(1/2*f*x + 1/2*e)^7 \\ & + 84*I*B*a^2*\tan(1/2*f*x + 1/2*e)^7 + 904*I*A*a^2*\tan(1/2*f*x + 1/2*e)^6 - 158*B*a^2*\tan(1/2*f*x + 1/2*e)^6 \\ & - 822*A*a^2*\tan(1/2*f*x + 1/2*e)^5 - 84*I*B*a^2*\tan(1/2*f*x + 1/2*e)^5 - 480*I*A*a^2*\tan(1/2*f*x + 1/2*e)^4 \\ & + 90*B*a^2*\tan(1/2*f*x + 1/2*e)^4 + 235*A*a^2*\tan(1/2*f*x + 1/2*e)^3 + 20*I*B*a^2*\tan(1/2*f*x + 1/2*e)^3 \\ & + 60*I*A*a^2*\tan(1/2*f*x + 1/2*e)^2 - 15*B*a^2*\tan(1/2*f*x + 1/2*e)^2 - 15*A*a^2*\tan(1/2*f*x + 1/2*e) \\ & )/(c^6*f*(\tan(1/2*f*x + 1/2*e) + I)^{12}) \end{aligned}$$

**Mupad [B]**

time = 8.94, size = 118, normalized size = 1.30

$$\frac{\frac{B a^2 \tan(e+f x)^2}{4} + \frac{a^2 \tan(e+f x) (12 A - B 6 i)}{60} - \frac{a^2 (-B + A 8 i)}{60}}{c^6 f (\tan(e+f x)^6 + \tan(e+f x)^5 6 i - 15 \tan(e+f x)^4 - \tan(e+f x)^3 20 i + 15 \tan(e+f x)^2 + \tan(e+f x) 6 i - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2)/(c - c\*tan(e + f\*x)\*1i)^6,x)

[Out] 
$$\begin{aligned} & -((a^2*\tan(e + f*x)*(12*A - B*6i))/60 - (a^2*(A*8i - B))/60 + (B*a^2*\tan(e + f*x)^2)/4) \\ & / (c^6*f*(\tan(e + f*x)*6i + 15*\tan(e + f*x)^2 - \tan(e + f*x)^3*20i - 15*\tan(e + f*x)^4 + \tan(e + f*x)^5*6i + \tan(e + f*x)^6 - 1)) \end{aligned}$$

$$3.689 \quad \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^n dx$$

**Optimal.** Leaf size=151

$$\frac{4a^3(iA + B)(c - ic \tan(e + fx))^n}{fn} - \frac{4a^3(iA + 2B)(c - ic \tan(e + fx))^{1+n}}{cf(1+n)} + \frac{a^3(iA + 5B)(c - ic \tan(e + fx))^{2+n}}{c^2f(2+n)}$$

[Out]  $4a^3(I*A+B)*(c-I*c*\tan(f*x+e))^n/f/n - 4a^3(I*A+2*B)*(c-I*c*\tan(f*x+e))^{1+n}/c/f/(1+n) + a^3(I*A+5*B)*(c-I*c*\tan(f*x+e))^{2+n}/c^2/f/(2+n) - a^3*B*(c-I*c*\tan(f*x+e))^{3+n}/c^3/f/(3+n)$

**Rubi [A]**

time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ ,

Rules used = {3669, 78}

$$\frac{a^3(5B + iA)(c - ic \tan(e + fx))^{n+2}}{c^2f(n+2)} + \frac{4a^3(B + iA)(c - ic \tan(e + fx))^n}{fn} - \frac{4a^3(2B + iA)(c - ic \tan(e + fx))^{n+1}}{cf(n+1)} - \frac{a^3B(c - ic \tan(e + fx))^{n+3}}{c^3f(n+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^n, x]$

[Out]  $(4*a^3*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^n)/(f*n) - (4*a^3*(I*A + 2*B)*(c - I*c*\text{Tan}[e + f*x])^{1+n})/(c*f*(1+n)) + (a^3*(I*A + 5*B)*(c - I*c*\text{Tan}[e + f*x])^{2+n})/(c^2*f*(2+n)) - (a^3*B*(c - I*c*\text{Tan}[e + f*x])^{3+n})/(c^3*f*(3+n))$

Rule 78

$\text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\tan(e + f*x))^m*(A + B*\tan(e + f*x))*(c + d*\tan(e + f*x))^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]



Rubi steps

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ictan(e + fx))^n dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx)(c - ictan(e + fx))^n dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (4a^2(A - iB)(c - ictan(e + fx))^n dx\right)}{f}$$

$$= \frac{4a^3(iA + B)(c - ictan(e + fx))^n}{fn}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 822 vs.  $2(151) = 302$ .  
time = 8.44, size = 822, normalized size = 5.44

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^n,x]

[Out] (Cos[e + f\*x]^4\*((Sec[e]\*Sec[e + f\*x]^2\*(3\*A\*Cos[e] - (9\*I)\*B\*Cos[e] + A\*n\*Cos[e] - (2\*I)\*B\*n\*Cos[e] + 2\*B\*Sin[e] + B\*n\*Sin[e])\*((-I)\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Cos[3\*e] - E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Sin[3\*e]))/((2 + n)\*(3 + n)) + (Sec[e]\*((12\*I)\*A\*Cos[e] + 12\*B\*Cos[e] + (13\*I)\*A\*n\*Cos[e] + 9\*B\*n\*Cos[e] + (6\*I)\*A\*n^2\*Cos[e] + 6\*B\*n^2\*Cos[e] + I\*A\*n^3\*Cos[e] + B\*n^3\*Cos[e] - 9\*A\*n\*Sin[e] + (13\*I)\*B\*n\*Sin[e] - 6\*A\*n^2\*Sin[e] + (6\*I)\*B\*n^2\*Sin[e] - A\*n^3\*Sin[e] + I\*B\*n^3\*Sin[e])\*((2\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Cos[3\*e])/n - ((2\*I)\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Sin[3\*e])/n))/((1 + n)\*(2 + n)\*(3 + n)) + ((9\*A - (13\*I)\*B + 6\*A\*n - (6\*I)\*B\*n + A\*n^2 - I\*B\*n^2)\*Sec[e]\*Sec[e + f\*x]\*(-2\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Cos[3\*e] + (2\*I)\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Sin[3\*e])\*Sin[f\*x])/((1 + n)\*(2 + n)\*(3 + n)) - (I\*Sec[e]\*Sec[e + f\*x]^3\*(B\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Cos[3\*e] - I\*B\*E^((-I)\*f\*n\*x + n\*(I\*f\*x - Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))\*Sin[3\*e])\*Sin[f\*x])/(3 + n))\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(n - (n\*(-Log[c\*Sec[e + f\*x]] + Log[c - I\*c\*Tan[e + f\*x]]))/Log[c - I\*c\*Tan[e + f\*x]))/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.  
time = 1.32, size = 4339, normalized size = 28.74

method	result	size
risch	Expression too large to display	4339

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_RE
TURNVERBOSE)
```

```
[Out] 4*a^3/(3+n)/f/(exp(2*I*(f*x+e))+1)^3/(1+n)/(2+n)/n*(-n^3*B/((exp(2*I*(f*x+e
))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*
I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(e
xp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*
c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(4*I*f
*x)*exp(4*I*e)+n^3*B/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*
c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*
csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*
(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn
(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*f*x)*exp(6*I*e)+6*n^2*B/((exp(2*I*(f*x+
e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2
*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(
exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I
*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*
f*x)*exp(6*I*e)+11*n*B/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(
I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^
2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*
I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*cs
gn(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I*f*x)*exp(6*I*e)+18*I*A/((exp(2*I*(f*x
+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/
2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/
(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(
I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(2*I
*f*x)*exp(2*I*e)+18*I*A/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn
(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))
^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2
*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*c
sgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(4*I*f*x)*exp(4*I*e)+6*I*A/((exp(2*I*(f*x
+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/
2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/
(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(
I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))*n)*exp(6*I
*f*x)*exp(6*I*e)-2*n^2*B/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csg
n(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1)
)^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(
2*I*(f*x+e))+1))*n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*
```

```

csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(4*I*f*x)*exp(4*I*e)+9*n*B/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c/(exp(2*I*(f*x+e))+1))^n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(4*I*f*x)*exp(4*I*e)-2*n^2*B/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(2*I*f*x)*exp(2*I*e)+2*I*n*A/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^n*(csgn(I*c/(exp(2*I*(f*x+e))+1))-csgn(I/(exp(2*I*(f*x+e))+1)))*(-csgn(I*c/(exp(2*I*(f*x+e))+1))+csgn(I*c)))+6*B/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^n*(csgn(I*c/(exp(2*I*(f*x+e))+1))-csgn(I/(exp(2*I*(f*x+e))+1)))*(-csgn(I*c/(exp(2*I*(f*x+e))+1))+csgn(I*c)))+6*I*A/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^n*(csgn(I*c/(exp(2*I*(f*x+e))+1))-csgn(I/(exp(2*I*(f*x+e))+1)))*(-csgn(I*c/(exp(2*I*(f*x+e))+1))+csgn(I*c)))+I*n^3*A/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(4*I*f*x)*exp(4*I*e)+I*n^3*A/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(6*I*f*x)*exp(6*I*e)+12*I*n*A/((exp(2*I*(f*x+e))+1)^n)*2^n*c^n*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^3*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I*c)*n)*exp(1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))^n)*exp(-1/2*I*Pi*csgn(I*c/(exp(2*I*(f*x+e))+1))*csgn(I*c)*csgn(I/(exp(2*I*(f*x+e))+1))...

```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1099 vs.  $2(141) = 282$ .

time = 0.70, size = 1099, normalized size = 7.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="maxima")

[Out]  $4*(2*((A + I*B)*a^3*c^n*n^2 + 6*A*a^3*c^n*n + 9*(A - I*B)*a^3*c^n)*2^n*\cos(-2*f*x + n*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) - 2*e) + ((A + I*B)*a^3*c^n*n^3 + 2*(4*A + I*B)*a^3*c^n*n^2 + 3*(7*A - 3*I*B)*a^3*c^n*n + 1$

$$\begin{aligned}
& 8*(A - I*B)*a^3*c^n)*2^n*\cos(-4*f*x + n*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x \\
& + 2*e) + 1) - 4*e) + ((A - I*B)*a^3*c^n*n^3 + 6*(A - I*B)*a^3*c^n*n^2 + 11 \\
& *(A - I*B)*a^3*c^n*n + 6*(A - I*B)*a^3*c^n)*2^n*\cos(-6*f*x + n*\arctan2(\sin( \\
& 2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) - 6*e) + 2*((A + I*B)*a^3*c^n*n + 3*(A \\
& - I*B)*a^3*c^n)*2^n*\cos(n*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1)) \\
& - 2*((I*A - B)*a^3*c^n*n^2 + 6*I*A*a^3*c^n*n + 9*(I*A + B)*a^3*c^n)*2^n*\sin \\
& (-2*f*x + n*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) - 2*e) - ((I*A \\
& - B)*a^3*c^n*n^3 + 2*(4*I*A - B)*a^3*c^n*n^2 + 3*(7*I*A + 3*B)*a^3*c^n*n + \\
& 18*(I*A + B)*a^3*c^n)*2^n*\sin(-4*f*x + n*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x \\
& + 2*e) + 1) - 4*e) - ((I*A + B)*a^3*c^n*n^3 + 6*(I*A + B)*a^3*c^n*n^2 + 1 \\
& 1*(I*A + B)*a^3*c^n*n + 6*(I*A + B)*a^3*c^n)*2^n*\sin(-6*f*x + n*\arctan2(\sin \\
& (2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) - 6*e) - 2*((I*A - B)*a^3*c^n*n + 3*(I \\
& *A + B)*a^3*c^n)*2^n*\sin(n*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1)) \\
& )/((( -I*n^4 - 6*I*n^3 - 11*I*n^2 - 6*I*n)*( \cos(2*f*x + 2*e)^2 + \sin(2*f*x + \\
& 2*e)^2 + 2*\cos(2*f*x + 2*e) + 1)^(1/2*n))*\cos(6*f*x + 6*e) - 3*(I*n^4 + 6*I \\
& *n^3 + 11*I*n^2 + 6*I*n)*( \cos(2*f*x + 2*e)^2 + \sin(2*f*x + 2*e)^2 + 2*\cos(2 \\
& *f*x + 2*e) + 1)^(1/2*n))*\cos(4*f*x + 4*e) + (n^4 + 6*n^3 + 11*n^2 + 6*n)*( \c \\
& \cos(2*f*x + 2*e)^2 + \sin(2*f*x + 2*e)^2 + 2*\cos(2*f*x + 2*e) + 1)^(1/2*n)*\si \\
& \sin(6*f*x + 6*e) + 3*(n^4 + 6*n^3 + 11*n^2 + 6*n)*( \cos(2*f*x + 2*e)^2 + \sin(2 \\
& *f*x + 2*e)^2 + 2*\cos(2*f*x + 2*e) + 1)^(1/2*n))*\sin(4*f*x + 4*e) + (-I*n^4 \\
& - 6*I*n^3 - 11*I*n^2 - 3*(I*n^4 + 6*I*n^3 + 11*I*n^2 + 6*I*n)*\cos(2*f*x + 2 \\
& *e) + 3*(n^4 + 6*n^3 + 11*n^2 + 6*n)*\sin(2*f*x + 2*e) - 6*I*n)*( \cos(2*f*x + \\
& 2*e)^2 + \sin(2*f*x + 2*e)^2 + 2*\cos(2*f*x + 2*e) + 1)^(1/2*n))*f)
\end{aligned}$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs.  $2(141) = 282$ .  
time = 2.72, size = 342, normalized size = 2.26

$$\frac{4(2(-A+B)a^{2n}+6(-A-B)a^{2n}+((-A-B)a^{2n}+6(-A-B)a^{2n}+11(-A-B)a^{2n}+6(-A-B)a^{2n})e^{6f*x+6e}+((-A+B)a^{2n}+2(-4A+B)a^{2n}+3(-7A-3B)a^{2n}+18(-A-B)a^{2n})e^{6f*x+6e}+2((-A+B)a^{2n}-6Aa^{2n}+9(-A-B)a^{2n})e^{2n(f*x+e)})}{f^{n^4+6f^{n^3}+11f^{n^2}+6fn+(f^{n^4}+6f^{n^3}+11f^{n^2}+6fn)e^{6f*x+6e}+3(f^{n^4}+6f^{n^3}+11f^{n^2}+6fn)e^{2n(f*x+e)}+3(f^{n^4}+6f^{n^3}+11f^{n^2}+6fn)e^{2n(f*x+e)})} \left(\frac{2e^{2f*x+2e}}{2e^{2f*x+2e}}\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="fricas")

[Out]  $-4*(2*(-I*A + B)*a^3*n + 6*(-I*A - B)*a^3 + ((-I*A - B)*a^3*n^3 + 6*(-I*A - B)*a^3*n^2 + 11*(-I*A - B)*a^3*n + 6*(-I*A - B)*a^3)*e^{(6*I*f*x + 6*I*e)} + ((-I*A + B)*a^3*n^3 + 2*(-4*I*A + B)*a^3*n^2 + 3*(-7*I*A - 3*B)*a^3*n + 18*(-I*A - B)*a^3)*e^{(4*I*f*x + 4*I*e)} + 2*((-I*A + B)*a^3*n^2 - 6*I*A*a^3*n + 9*(-I*A - B)*a^3)*e^{(2*I*f*x + 2*I*e)}*(2*c/(e^{(2*I*f*x + 2*I*e)} + 1))^n/(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n + (f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^{(6*I*f*x + 6*I*e)} + 3*(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^{(4*I*f*x + 4*I*e)} + 3*(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^{(2*I*f*x + 2*I*e)})$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3665 vs.  $2(122) = 244$ .

time = 3.47, size = 3665, normalized size = 24.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
[Out] Piecewise((x*(A + B*tan(e))*(I*a*tan(e) + a)**3*(-I*c*tan(e) + c)**n, Eq(f,
  0)), (6*A*a**3*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan
  (e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 2*A*a**3/(6*c**3*f*ta
  n(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*
  c**3*f) + 6*I*B*a**3*f*x*tan(e + f*x)**3/(6*c**3*f*tan(e + f*x)**3 + 18*I*c
  **3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 18*B*a**3*f*
  x*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 -
  18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 18*I*B*a**3*f*x*tan(e + f*x)/(6*c**
  3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x)
  - 6*I*c**3*f) + 6*B*a**3*f*x/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e
  + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 3*B*a**3*log(tan(e + f*x)
  )**2 + 1)*tan(e + f*x)**3/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f
  *x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 9*I*B*a**3*log(tan(e + f*x)
  )**2 + 1)*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*
  x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 9*B*a**3*log(tan(e + f*x)**2
  + 1)*tan(e + f*x)/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2
  - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 3*I*B*a**3*log(tan(e + f*x)**2 + 1
  )/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e
  + f*x) - 6*I*c**3*f) - 30*I*B*a**3*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)*
  **3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 3
  6*B*a**3*tan(e + f*x)/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)*
  **2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 14*I*B*a**3/(6*c**3*f*tan(e + f
  *x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f)
  ), Eq(n, -3)), (2*A*a**3*f*x*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)**2 + 4*I
  *c**2*f*tan(e + f*x) - 2*c**2*f) + 4*I*A*a**3*f*x*tan(e + f*x)/(2*c**2*f*ta
  n(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - 2*A*a**3*f*x/(2*c**2*
  f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + I*A*a**3*log(tan(
  e + f*x)**2 + 1)*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan
  (e + f*x) - 2*c**2*f) - 2*A*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c
  **2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - I*A*a**3*log(
  tan(e + f*x)**2 + 1)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) -
  2*c**2*f) - 8*A*a**3*tan(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*ta
  n(e + f*x) - 2*c**2*f) - 4*I*A*a**3/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*
  tan(e + f*x) - 2*c**2*f) - 10*I*B*a**3*f*x*tan(e + f*x)**2/(2*c**2*f*tan(e
  + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 20*B*a**3*f*x*tan(e + f*x)
  )/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 10*I*B*
  a**3*f*x/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) +
  5*B*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)**2
```

+ 4\*I\*c\*\*2\*f\*tan(e + f\*x) - 2\*c\*\*2\*f) + 10\*I\*B\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)\*tan(e + f\*x)/(2\*c\*\*2\*f\*tan(e + f\*x)\*\*2 + 4\*I\*c\*\*2\*f\*tan(e + f\*x) - 2\*c\*\*2\*f) - 5\*B\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)/(2\*c\*\*2\*f\*tan(e + f\*x)\*\*2 + 4\*I\*c\*\*2\*f\*tan(e + f\*x) - 2\*c\*\*2\*f) + 2\*I\*B\*a\*\*3\*tan(e + f\*x)\*\*3/(2\*c\*\*2\*f\*tan(e + f\*x)\*\*2 + 4\*I\*c\*\*2\*f\*tan(e + f\*x) - 2\*c\*\*2\*f) + 22\*I\*B\*a\*\*3\*tan(e + f\*x)/(2\*c\*\*2\*f\*tan(e + f\*x)\*\*2 + 4\*I\*c\*\*2\*f\*tan(e + f\*x) - 2\*c\*\*2\*f) - 16\*B\*a\*\*3/(2\*c\*\*2\*f\*tan(e + f\*x)\*\*2 + 4\*I\*c\*\*2\*f\*tan(e + f\*x) - 2\*c\*\*2\*f), Eq(n, -2)), (-8\*A\*a\*\*3\*f\*x\*tan(e + f\*x)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 8\*I\*A\*a\*\*3\*f\*x/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 4\*I\*A\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)\*tan(e + f\*x)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) + 4\*A\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) + 2\*A\*a\*\*3\*tan(e + f\*x)\*\*2/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) + 10\*A\*a\*\*3/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) + 16\*I\*B\*a\*\*3\*f\*x\*tan(e + f\*x)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 16\*B\*a\*\*3\*f\*x/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 8\*B\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)\*tan(e + f\*x)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 8\*I\*B\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) + B\*a\*\*3\*tan(e + f\*x)\*\*3/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 7\*I\*B\*a\*\*3\*tan(e + f\*x)\*\*2/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f) - 16\*I\*B\*a\*\*3/(2\*c\*f\*tan(e + f\*x) + 2\*I\*c\*f), Eq(n, -1)), (4\*A\*a\*\*3\*x + 2\*I\*A\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)/f - I\*A\*a\*\*3\*tan(e + f\*x)\*\*2/(2\*f) - 3\*A\*a\*\*3\*tan(e + f\*x)/f - 4\*I\*B\*a\*\*3\*x + 2\*B\*a\*\*3\*log(tan(e + f\*x)\*\*2 + 1)/f - I\*B\*a\*\*3\*tan(e + f\*x)\*\*3/(3\*f) - 3\*B\*a\*\*3\*tan(e + f\*x)\*\*2/(2\*f) + 4\*I\*B\*a\*\*3\*tan(e + f\*x)/f, Eq(n, 0)), (-I\*A\*a\*\*3\*n\*\*3\*(-I\*c\*tan(e + f\*x) + c)\*\*n\*tan(e + f\*x)\*\*2/(f\*n\*\*4 + 6\*f\*n\*\*3 + 11\*f\*n\*\*2 + 6\*f\*n) - 2\*A\*a\*\*3\*n\*\*3\*(-I\*c\*tan(e + f\*x) + c)\*\*n\*tan(e + f\*x)/(f\*n\*\*4 + 6\*f\*n\*\*3 + 11\*f\*n\*\*2 + 6\*f\*n) + I\*A\*a\*\*3\*n\*\*3\*(-I\*c\*tan(e + f\*x) + c)\*\*n/(f\*n\*\*4 + 6\*f\*n\*\*3 + 11\*f\*n\*\*2 + 6\*f\*n) - 4\*I\*A\*a\*\*3\*n\*\*2\*(-I\*c\*tan(e + f\*x) + c)\*\*n\*tan(e + f\*x)\*\*2/(f\*n\*\*4 + 6\*f\*n\*\*3 + 11\*f\*n\*\*2 + 6\*f\*n) - 12\*A\*a\*\*3\*n\*\*2\*(-I\*c\*tan(e + f\*x) + c)\*\*n\*tan(e + f\*x)/(f\*n\*\*4 + 6\*f\*n\*\*3 + 11\*f\*n\*\*2 + 6\*f\*n) + 8\*I\*A\*a\*\*3\*n\*\*2\*(-I\*c\*tan(e + f\*x) + c)\*\*n/(f\*n\*\*4 + 6\*f\*n\*\*3 + 11\*f\*n\*\*2 + 6\*f\*n)...

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^3\*(-I\*c\*tan(f\*x + e) + c)^n, x)

**Mupad [B]**

time = 13.88, size = 323, normalized size = 2.14

$$\left( c + \frac{c(e^{21+fx} - 1)}{e^{21+fx} + 1} \right)^n \left( \frac{8a^3(3A - B3i + An + Bn1)}{fn(n^3 11 + n^2 6i + n 11i + 6i)} + \frac{4a^3 e^{4i+fx} (n^2 + 5n + 6)(3A - B3i + An + Bn1)}{fn(n^3 11 + n^2 6i + n 11i + 6i)} + \frac{4a^3 e^{6i+fx} (A - B1i)(n^3 + 6n^2 + 11n + 6)}{fn(n^3 11 + n^2 6i + n 11i + 6i)} + \frac{8a^3 e^{21+fx} (n+3)(3A - B3i + An + Bn1)}{fn(n^3 11 + n^2 6i + n 11i + 6i)} \right)$$

$$3e^{21+fx} + 3e^{4i+fx} + e^{6i+fx} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B*\tan(e + f*x))*(a + a*\tan(e + f*x)*1i)^3*(c - c*\tan(e + f*x)*1i)^n, x)$

[Out]  $-\left(\frac{c + (c*(\exp(e*2i + f*x*2i)*1i - 1i)*1i)}{\exp(e*2i + f*x*2i) + 1}\right)^n \left( \frac{8*a^3*(3*A - B*3i + A*n + B*n*1i)}{f*n*(n*11i + n^2*6i + n^3*1i + 6i)} + \frac{4*a^3*\exp(e*4i + f*x*4i)*(5*n + n^2 + 6)*(3*A - B*3i + A*n + B*n*1i)}{f*n*(n*11i + n^2*6i + n^3*1i + 6i)} + \frac{4*a^3*\exp(e*6i + f*x*6i)*(A - B*1i)*(11*n + 6*n^2 + n^3 + 6)}{f*n*(n*11i + n^2*6i + n^3*1i + 6i)} + \frac{8*a^3*\exp(e*2i + f*x*2i)*(n + 3)*(3*A - B*3i + A*n + B*n*1i)}{f*n*(n*11i + n^2*6i + n^3*1i + 6i)} \right) / (3*\exp(e*2i + f*x*2i) + 3*\exp(e*4i + f*x*4i) + \exp(e*6i + f*x*6i) + 1)$

### 3.690 $\int (a+ia \tan(e+fx))^3 (A+B \tan(e+fx))(c-ic \tan(e+fx))^6 dx$

**Optimal.** Leaf size=135

$$\frac{2a^3(iA+B)c^6(1-i \tan(e+fx))^6}{3f} - \frac{4a^3(iA+2B)c^6(1-i \tan(e+fx))^7}{7f} + \frac{a^3(iA+5B)c^6(1-i \tan(e+fx))^8}{8f}$$

[Out]  $2/3*a^3*(I*A+B)*c^6*(1-I*\tan(f*x+e))^6/f-4/7*a^3*(I*A+2*B)*c^6*(1-I*\tan(f*x+e))^7/f+1/8*a^3*(I*A+5*B)*c^6*(1-I*\tan(f*x+e))^8/f-1/9*a^3*B*c^6*(1-I*\tan(f*x+e))^9/f$

**Rubi [A]**

time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ ,

Rules used = {3669, 78}

$$\frac{a^3c^6(5B+iA)(1-i \tan(e+fx))^8}{8f} - \frac{4a^3c^6(2B+iA)(1-i \tan(e+fx))^7}{7f} + \frac{2a^3c^6(B+iA)(1-i \tan(e+fx))^6}{3f} - \frac{a^3Bc^6(1-i \tan(e+fx))^9}{9f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^6, x]$

[Out]  $(2*a^3*(I*A + B)*c^6*(1 - I*\text{Tan}[e + f*x])^6)/(3*f) - (4*a^3*(I*A + 2*B)*c^6*(1 - I*\text{Tan}[e + f*x])^7)/(7*f) + (a^3*(I*A + 5*B)*c^6*(1 - I*\text{Tan}[e + f*x])^8)/(8*f) - (a^3*B*c^6*(1 - I*\text{Tan}[e + f*x])^9)/(9*f)$

**Rule 78**

$\text{Int}[(a + b*x)^n*(c + d*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^n*(c + d*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(A + B*\text{tan}[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps



$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ictan(e + fx))^6 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx) (c - icx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (4a^2(A - iB)(c - icx) dx\right)}{f}$$

$$= \frac{2a^3(iA + B)c^6(1 - i \tan(e + fx))^6}{3f}$$

**Mathematica [A]**

time = 3.07, size = 262, normalized size = 1.94

$e^{iA} \cos(c \tan(e + fx) + 3) (88 - 3A + B) \cos(fx) + 63(-3A + B) \cos(2e + fx) - 84A \cos(2e + 3fx) + 84B \cos(2e + 3fx) - 84A \cos(4e + 3fx) + 84B \cos(4e + 3fx) + 189A \sin(fx) + 63B \sin(fx) - 189A \sin(2e + fx) - 63B \sin(2e + fx) + 189A \sin(2e + 3fx) - 84A \sin(4e + 3fx) - 84B \sin(4e + 3fx) + 189A \sin(4e + 3fx) + 96B \sin(4e + 3fx) + 27A \sin(6e + 7fx) + 96B \sin(6e + 7fx) + 3A \sin(8e + 9fx) + 18 \sin(8e + 9fx)$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^6,x]

[Out] (a^3\*c^6\*Sec[e]\*Sec[e + f\*x]^9\*(63\*((-3\*I)\*A + B)\*Cos[f\*x] + 63\*((-3\*I)\*A + B)\*Cos[2\*e + f\*x] - (84\*I)\*A\*Cos[2\*e + 3\*f\*x] + 84\*B\*Cos[2\*e + 3\*f\*x] - (84\*I)\*A\*Cos[4\*e + 3\*f\*x] + 84\*B\*Cos[4\*e + 3\*f\*x] + 189\*A\*Sin[f\*x] + (63\*I)\*B\*Sin[f\*x] - 189\*A\*Sin[2\*e + f\*x] - (63\*I)\*B\*Sin[2\*e + f\*x] + 168\*A\*Sin[2\*e + 3\*f\*x] - 84\*A\*Sin[4\*e + 3\*f\*x] - (84\*I)\*B\*Sin[4\*e + 3\*f\*x] + 108\*A\*Sin[4\*e + 5\*f\*x] + (36\*I)\*B\*Sin[4\*e + 5\*f\*x] + 27\*A\*Sin[6\*e + 7\*f\*x] + (9\*I)\*B\*Sin[6\*e + 7\*f\*x] + 3\*A\*Sin[8\*e + 9\*f\*x] + I\*B\*Sin[8\*e + 9\*f\*x]))/(1008\*f)

**Maple [A]**

time = 0.14, size = 211, normalized size = 1.56

method	result
risch	$\frac{32c^6 a^3 (84iA e^{6i(fx+e)} + 84B e^{6i(fx+e)} + 108iA e^{4i(fx+e)} - 36B e^{4i(fx+e)} + 27iA e^{2i(fx+e)} - 9B e^{2i(fx+e)} + 3iA - B)}{63f(e^{2i(fx+e)} + 1)^9}$
derivativedivides	$ic^6 a^3 \left( \frac{B(\tan^9(fx+e))}{9} + \frac{(3iB+A)(\tan^8(fx+e))}{8} + \frac{(-11B-2iA+5i(-2iB+A))(\tan^7(fx+e))}{7} + \frac{(-11A+5i(-2iA-B)+10iB)(\tan^6(fx+e))}{6} \right)$
default	$ic^6 a^3 \left( \frac{B(\tan^9(fx+e))}{9} + \frac{(3iB+A)(\tan^8(fx+e))}{8} + \frac{(-11B-2iA+5i(-2iB+A))(\tan^7(fx+e))}{7} + \frac{(-11A+5i(-2iA-B)+10iB)(\tan^6(fx+e))}{6} \right)$
norman	$\frac{A a^3 c^6 \tan(fx+e)}{f} - \frac{(3iB a^3 c^6 + A a^3 c^6) (\tan^3(fx+e))}{3f} - \frac{(5iA a^3 c^6 + B a^3 c^6) (\tan^4(fx+e))}{4f} - \frac{(-iA a^3 c^6 + 3B a^3 c^6) (\tan^5(fx+e))}{8f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^6,x,method=\_RETURNVERBOSE)

[Out]  $I/f*c^6*a^3*(1/9*B*\tan(f*x+e)^9+1/8*(A+3*I*B)*\tan(f*x+e)^8+1/7*(-11*B-2*I*A+5*I*(A-2*I*B))*\tan(f*x+e)^7+1/6*(-11*A+5*I*(-2*I*A-B)+10*I*B)*\tan(f*x+e)^6+1/5*(15*I*A+15*B-10*I*(A-2*I*B))*\tan(f*x+e)^5+1/4*(15*A-10*I*(-2*I*A-B)-9*I*B)*\tan(f*x+e)^4+1/3*(-5*B+I*(A-2*I*B))*\tan(f*x+e)^3+1/2*(-5*A+I*(-2*I*A-B))*\tan(f*x+e)^2-I*A*\tan(f*x+e)$

**Maxima** [A]

time = 0.52, size = 202, normalized size = 1.50

$$\frac{-56iBa^3c^6\tan(fx+e)^9+63(-iA+3B)a^3c^6\tan(fx+e)^8+72(3A+iB)a^3c^6\tan(fx+e)^7+84(iA+5B)a^3c^6\tan(fx+e)^6+504(A+iB)a^3c^6\tan(fx+e)^5+126(5iA+B)a^3c^6\tan(fx+e)^4+168(A+3iB)a^3c^6\tan(fx+e)^3+252(3iA-B)a^3c^6\tan(fx+e)^2-504Aa^3c^6\tan(fx+e)}{504f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x, algorithm="maxima")`

[Out]  $-1/504*(-56*I*B*a^3*c^6*\tan(f*x+e)^9+63*(-I*A+3*B)*a^3*c^6*\tan(f*x+e)^8+72*(3*A+I*B)*a^3*c^6*\tan(f*x+e)^7+84*(I*A+5*B)*a^3*c^6*\tan(f*x+e)^6+504*(A+I*B)*a^3*c^6*\tan(f*x+e)^5+126*(5*I*A+B)*a^3*c^6*\tan(f*x+e)^4+168*(A+3*I*B)*a^3*c^6*\tan(f*x+e)^3+252*(3*I*A-B)*a^3*c^6*\tan(f*x+e)^2-504*A*a^3*c^6*\tan(f*x+e))/f$

**Fricas** [A]

time = 1.32, size = 206, normalized size = 1.53

$$\frac{32(-iA-B)a^3c^6e^{6i fx+6ie}+36(-3iA+B)a^3c^6e^{4i fx+4ie}+9(-3iA+B)a^3c^6e^{2i fx+2ie}+(-3iA+B)a^3c^6}{63(fe^{18i fx+18ie}+9fe^{16i fx+16ie}+36fe^{14i fx+14ie}+84fe^{12i fx+12ie}+126fe^{10i fx+10ie}+126fe^{8i fx+8ie}+84fe^{6i fx+6ie}+36fe^{4i fx+4ie}+9fe^{2i fx+2ie}+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x, algorithm="fricas")`

[Out]  $-32/63*(84*(-I*A-B)*a^3*c^6*e^{(6*I*f*x+6*I*e)}+36*(-3*I*A+B)*a^3*c^6*e^{(4*I*f*x+4*I*e)}+9*(-3*I*A+B)*a^3*c^6*e^{(2*I*f*x+2*I*e)}+(-3*I*A+B)*a^3*c^6)/(f*e^{(18*I*f*x+18*I*e)}+9*f*e^{(16*I*f*x+16*I*e)}+36*f*e^{(14*I*f*x+14*I*e)}+84*f*e^{(12*I*f*x+12*I*e)}+126*f*e^{(10*I*f*x+10*I*e)}+126*f*e^{(8*I*f*x+8*I*e)}+84*f*e^{(6*I*f*x+6*I*e)}+36*f*e^{(4*I*f*x+4*I*e)}+9*f*e^{(2*I*f*x+2*I*e)}+f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs.  $2(110) = 220$ .

time = 1.23, size = 325, normalized size = 2.41

$$\frac{96iAa^3c^6-32Ba^3c^6+(864iAa^3c^6e^{2ie}-288Ba^3c^6e^{2ie})e^{2ifx}+(3456iAa^3c^6e^{4ie}-1152Ba^3c^6e^{4ie})e^{4ifx}+(2688iAa^3c^6e^{6ie}+2688Ba^3c^6e^{6ie})e^{6ifx}}{63fe^{18ie}e^{18ifx}+567fe^{16ie}e^{16ifx}+2268fe^{14ie}e^{14ifx}+5292fe^{12ie}e^{12ifx}+7938fe^{10ie}e^{10ifx}+7938fe^{8ie}e^{8ifx}+5292fe^{6ie}e^{6ifx}+2268fe^{4ie}e^{4ifx}+567fe^{2ie}e^{2ifx}+63f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**6,x)`

[Out]  $(96*I*A*a**3*c**6 - 32*B*a**3*c**6 + (864*I*A*a**3*c**6*\exp(2*I*e) - 288*B*a**3*c**6*\exp(2*I*e))*\exp(2*I*f*x) + (3456*I*A*a**3*c**6*\exp(4*I*e) - 1152*B*a**3*c**6*\exp(4*I*e))*\exp(4*I*f*x) + (2688*I*A*a**3*c**6*\exp(6*I*e) + 2688*B*a**3*c**6*\exp(6*I*e))*\exp(6*I*f*x))/(63*f*\exp(18*I*e)*\exp(18*I*f*x) + 567*f*\exp(16*I*e)*\exp(16*I*f*x) + 2268*f*\exp(14*I*e)*\exp(14*I*f*x) + 5292*f*\exp(12*I*e)*\exp(12*I*f*x) + 7938*f*\exp(10*I*e)*\exp(10*I*f*x) + 7938*f*\exp(8*I*e)*\exp(8*I*f*x) + 5292*f*\exp(6*I*e)*\exp(6*I*f*x) + 2268*f*\exp(4*I*e)*\exp(4*I*f*x) + 567*f*\exp(2*I*e)*\exp(2*I*f*x) + 63*f)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs.  $2(117) = 234$ .

time = 1.44, size = 254, normalized size = 1.88

$$\frac{32(-84i Aa^3 c^6 e^{6i fx+6ie} - 84 Ba^3 c^6 e^{6i fx+6ie} - 108i Aa^3 c^6 e^{4i fx+4ie} + 36 Ba^3 c^6 e^{4i fx+4ie} - 27i Aa^3 c^6 e^{2i fx+2ie} + 9 Ba^3 c^6 e^{2i fx+2ie} - 3i Aa^3 c^6 + Ba^3 c^6)}{63(f e^{(18i fx+18ie)} + 9 f e^{(16i fx+16ie)} + 36 f e^{(14i fx+14ie)} + 84 f e^{(12i fx+12ie)} + 126 f e^{(10i fx+10ie)} + 126 f e^{(8i fx+8ie)} + 84 f e^{(6i fx+6ie)} + 36 f e^{(4i fx+4ie)} + 9 f e^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x, algorithm="giac")`

[Out]  $-32/63*(-84*I*A*a^3*c^6*e^{(6*I*f*x + 6*I*e)} - 84*B*a^3*c^6*e^{(6*I*f*x + 6*I*e)} - 108*I*A*a^3*c^6*e^{(4*I*f*x + 4*I*e)} + 36*B*a^3*c^6*e^{(4*I*f*x + 4*I*e)} - 27*I*A*a^3*c^6*e^{(2*I*f*x + 2*I*e)} + 9*B*a^3*c^6*e^{(2*I*f*x + 2*I*e)} - 3*I*A*a^3*c^6 + B*a^3*c^6)/(f*e^{(18*I*f*x + 18*I*e)} + 9*f*e^{(16*I*f*x + 16*I*e)} + 36*f*e^{(14*I*f*x + 14*I*e)} + 84*f*e^{(12*I*f*x + 12*I*e)} + 126*f*e^{(10*I*f*x + 10*I*e)} + 126*f*e^{(8*I*f*x + 8*I*e)} + 84*f*e^{(6*I*f*x + 6*I*e)} + 36*f*e^{(4*I*f*x + 4*I*e)} + 9*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad** [B]

time = 8.70, size = 208, normalized size = 1.54

$$\frac{Aa^3 c^6 \tan(e + fx) + \frac{a^3 c^6 \tan(e + fx)^2 (-3B + A11)11}{3} + a^3 c^6 \tan(e + fx)^5 (-B + A11)11 - \frac{a^3 c^6 \tan(e + fx)^4 (5A - B11)11}{4} + \frac{a^3 c^6 \tan(e + fx)^7 (-B + A30)11}{7} - \frac{a^3 c^6 \tan(e + fx)^2 (3A + B11)11}{2} - \frac{a^3 c^6 \tan(e + fx)^6 (A - B50)11}{6} + \frac{a^3 c^6 \tan(e + fx)^8 (A + B30)11}{8} + \frac{Ba^3 c^6 \tan(e + fx)^2 11}{9}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^6,x)`

[Out]  $((a^3*c^6*\tan(e + f*x)^3*(A*1i - 3*B)*1i)/3 - (a^3*c^6*\tan(e + f*x)^2*(3*A + B*1i)*1i)/2 + a^3*c^6*\tan(e + f*x)^5*(A*1i - B)*1i - (a^3*c^6*\tan(e + f*x)^4*(5*A - B*1i)*1i)/4 + (a^3*c^6*\tan(e + f*x)^7*(A*3i - B)*1i)/7 + A*a^3*c^6*\tan(e + f*x) - (a^3*c^6*\tan(e + f*x)^6*(A - B*5i)*1i)/6 + (a^3*c^6*\tan(e + f*x)^8*(A + B*3i)*1i)/8 + (B*a^3*c^6*\tan(e + f*x)^9*1i)/9)/f$

### 3.691 $\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx))^5 dx$

**Optimal.** Leaf size=135

$$\frac{4a^3(iA + B)c^5(1 - i \tan(e + fx))^5}{5f} - \frac{2a^3(iA + 2B)c^5(1 - i \tan(e + fx))^6}{3f} + \frac{a^3(iA + 5B)c^5(1 - i \tan(e + fx))^7}{7f}$$

[Out]  $4/5*a^3*(I*A+B)*c^5*(1-I*\tan(f*x+e))^5/f-2/3*a^3*(I*A+2*B)*c^5*(1-I*\tan(f*x+e))^6/f+1/7*a^3*(I*A+5*B)*c^5*(1-I*\tan(f*x+e))^7/f-1/8*a^3*B*c^5*(1-I*\tan(f*x+e))^8/f$

**Rubi [A]**

time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ ,

Rules used = {3669, 78}

$$\frac{a^3c^5(5B + iA)(1 - i \tan(e + fx))^7}{7f} - \frac{2a^3c^5(2B + iA)(1 - i \tan(e + fx))^6}{3f} + \frac{4a^3c^5(B + iA)(1 - i \tan(e + fx))^5}{5f} - \frac{a^3Bc^5(1 - i \tan(e + fx))^8}{8f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^5, x]$

[Out]  $(4*a^3*(I*A + B)*c^5*(1 - I*\text{Tan}[e + f*x])^5)/(5*f) - (2*a^3*(I*A + 2*B)*c^5*(1 - I*\text{Tan}[e + f*x])^6)/(3*f) + (a^3*(I*A + 5*B)*c^5*(1 - I*\text{Tan}[e + f*x])^7)/(7*f) - (a^3*B*c^5*(1 - I*\text{Tan}[e + f*x])^8)/(8*f)$

**Rule 78**

$\text{Int}[(a + b*x)^n*(c + d*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^n*(c + d*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(A + B*\text{tan}[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx))^5 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx)(c - icx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (4a^2(A - iB)(c - icx) dx\right)}{f}$$

$$= \frac{4a^3(iA + B)c^5(1 - i \tan(e + fx))^5}{5f}$$

**Mathematica [A]**

time = 2.03, size = 215, normalized size = 1.59

$$\frac{a^3 c^5 \sec(e + fx) (35(-4A + B) \cos(e) + 70(-A + B) \cos(e + 2fx) - 70A \cos(3e + 2fx) + 70B \cos(3e + 2fx) - 140A \sin(e) - 35B \sin(e) + 154A \sin(e + 2fx) - 14B \sin(3e + 2fx) - 70A \sin(3e + 2fx) - 70B \sin(3e + 2fx) + 112A \sin(3e + 4fx) + 28B \sin(3e + 4fx) + 32A \sin(5e + 6fx) + 8B \sin(5e + 6fx) + 4A \sin(7e + 8fx) + iB \sin(7e + 8fx))}{840f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^5, x]

[Out] (a^3\*c^5\*Sec[e]\*Sec[e + f\*x]^8\*(35\*((-4\*I)\*A + B)\*Cos[e] + 70\*((-I)\*A + B)\*Cos[e + 2\*f\*x] - (70\*I)\*A\*Cos[3\*e + 2\*f\*x] + 70\*B\*Cos[3\*e + 2\*f\*x] - 140\*A\*Sin[e] - (35\*I)\*B\*Sin[e] + 154\*A\*Sin[e + 2\*f\*x] - (14\*I)\*B\*Sin[e + 2\*f\*x] - 70\*A\*Sin[3\*e + 2\*f\*x] - (70\*I)\*B\*Sin[3\*e + 2\*f\*x] + 112\*A\*Sin[3\*e + 4\*f\*x] + (28\*I)\*B\*Sin[3\*e + 4\*f\*x] + 32\*A\*Sin[5\*e + 6\*f\*x] + (8\*I)\*B\*Sin[5\*e + 6\*f\*x] + 4\*A\*Sin[7\*e + 8\*f\*x] + I\*B\*Sin[7\*e + 8\*f\*x]))/(840\*f)

**Maple [A]**

time = 0.15, size = 177, normalized size = 1.31

method	result
risch	$\frac{32c^5 a^3 (84iA e^{6i(fx+e)} + 84B e^{6i(fx+e)} + 112iA e^{4i(fx+e)} - 28B e^{4i(fx+e)} + 32iA e^{2i(fx+e)} - 8B e^{2i(fx+e)} + 4iA - B)}{105f(e^{2i(fx+e)} + 1)^8}$
derivativedivides	$c^5 a^3 \left( -\frac{B(\tan^8(fx+e))}{8} - \frac{(2iB+A)(\tan^7(fx+e))}{7} - \frac{(-7B-2iA+4i(-2iB+A))(\tan^6(fx+e))}{6} - \frac{(-7A+4i(-2iA-B)+8iB)(\tan^5(fx+e))}{5} \right)$
default	$c^5 a^3 \left( -\frac{B(\tan^8(fx+e))}{8} - \frac{(2iB+A)(\tan^7(fx+e))}{7} - \frac{(-7B-2iA+4i(-2iB+A))(\tan^6(fx+e))}{6} - \frac{(-7A+4i(-2iA-B)+8iB)(\tan^5(fx+e))}{5} \right)$
norman	$\frac{A a^3 c^5 \tan(fx+e)}{f} - \frac{(2iA a^3 c^5 + B a^3 c^5)(\tan^6(fx+e))}{6f} - \frac{(2iB a^3 c^5 + A a^3 c^5)(\tan^7(fx+e))}{7f} - \frac{(4iB a^3 c^5 + A a^3 c^5)}{5f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5, x, method=\_RETURNVERBOSE)

[Out]  $1/f*c^5*a^3*(-1/8*B*\tan(f*x+e)^8-1/7*(A+2*I*B)*\tan(f*x+e)^7-1/6*(-7*B-2*I*A+4*I*(A-2*I*B))*\tan(f*x+e)^6-1/5*(-7*A+4*I*(-2*I*A-B)+8*I*B)*\tan(f*x+e)^5-1/4*(8*I*A+7*B-4*I*(A-2*I*B))*\tan(f*x+e)^4-1/3*(7*A-4*I*(-2*I*A-B)-2*I*B)*\tan(f*x+e)^3-1/2*(2*I*A-B)*\tan(f*x+e)^2+A*\tan(f*x+e))$

**Maxima** [A]

time = 0.57, size = 174, normalized size = 1.29

$$\frac{105 B a^5 \tan^8(fx+e) + 120 (A+2iB) a^5 \tan^7(fx+e) - 140 (-2iA-B) a^5 \tan^6(fx+e) + 168 (A+4iB) a^5 \tan^5(fx+e) - 210 (-4iA+B) a^5 \tan^4(fx+e) - 280 (A-2iB) a^5 \tan^3(fx+e) - 420 (-2iA+B) a^5 \tan^2(fx+e) - 840 A a^5 \tan(fx+e)}{840 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, algorithm="maxima")`

[Out]  $-1/840*(105*B*a^3*c^5*\tan(f*x + e)^8 + 120*(A + 2*I*B)*a^3*c^5*\tan(f*x + e)^7 - 140*(-2*I*A - B)*a^3*c^5*\tan(f*x + e)^6 + 168*(A + 4*I*B)*a^3*c^5*\tan(f*x + e)^5 - 210*(-4*I*A + B)*a^3*c^5*\tan(f*x + e)^4 - 280*(A - 2*I*B)*a^3*c^5*\tan(f*x + e)^3 - 420*(-2*I*A + B)*a^3*c^5*\tan(f*x + e)^2 - 840*A*a^3*c^5*\tan(f*x + e))/f$

**Fricas** [A]

time = 2.33, size = 193, normalized size = 1.43

$$\frac{32(84(-iA-B)a^3c^5e^{6ifx+6ie} + 28(-4iA+B)a^3c^5e^{4ifx+4ie} + 8(-4iA+B)a^3c^5e^{2ifx+2ie} + (-4iA+B)a^3c^5)}{105(fe^{16ifx+16ie} + 8fe^{14ifx+14ie} + 28fe^{12ifx+12ie} + 56fe^{10ifx+10ie} + 70fe^{8ifx+8ie} + 56fe^{6ifx+6ie} + 28fe^{4ifx+4ie} + 8fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, algorithm="fricas")`

[Out]  $-32/105*(84*(-I*A - B)*a^3*c^5*e^{(6*I*f*x + 6*I*e)} + 28*(-4*I*A + B)*a^3*c^5*e^{(4*I*f*x + 4*I*e)} + 8*(-4*I*A + B)*a^3*c^5*e^{(2*I*f*x + 2*I*e)} + (-4*I*A + B)*a^3*c^5)/(f*e^{(16*I*f*x + 16*I*e)} + 8*f*e^{(14*I*f*x + 14*I*e)} + 28*f*e^{(12*I*f*x + 12*I*e)} + 56*f*e^{(10*I*f*x + 10*I*e)} + 70*f*e^{(8*I*f*x + 8*I*e)} + 56*f*e^{(6*I*f*x + 6*I*e)} + 28*f*e^{(4*I*f*x + 4*I*e)} + 8*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs.  $2(110) = 220$ .

time = 1.06, size = 306, normalized size = 2.27

$$\frac{128iAa^3c^5 - 32Ba^3c^5 + (1024iAa^3c^5e^{2ie} - 256Ba^3c^5e^{2ie})e^{2ifx} + (3584iAa^3c^5e^{4ie} - 896Ba^3c^5e^{4ie})e^{4ifx} + (2688iAa^3c^5e^{6ie} + 2688Ba^3c^5e^{6ie})e^{6ifx}}{105fe^{16ie}e^{16ifx} + 840fe^{14ie}e^{14ifx} + 2940fe^{12ie}e^{12ifx} + 5880fe^{10ie}e^{10ifx} + 7350fe^{8ie}e^{8ifx} + 5880fe^{6ie}e^{6ifx} + 2940fe^{4ie}e^{4ifx} + 840fe^{2ie}e^{2ifx} + 105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x)`

[Out]  $(128*I*A*a**3*c**5 - 32*B*a**3*c**5 + (1024*I*A*a**3*c**5*exp(2*I*e) - 256*B*a**3*c**5*exp(2*I*e))*exp(2*I*f*x) + (3584*I*A*a**3*c**5*exp(4*I*e) - 896*B*a**3*c**5*exp(4*I*e))*exp(4*I*f*x) + (2688*I*A*a**3*c**5*exp(6*I*e) + 2688*B*a**3*c**5*exp(6*I*e))*exp(6*I*f*x))/(105*f*exp(16*I*e)*exp(16*I*f*x) + 840*f*exp(14*I*e)*exp(14*I*f*x) + 2940*f*exp(12*I*e)*exp(12*I*f*x) + 5880*f*exp(10*I*e)*exp(10*I*f*x) + 7350*f*exp(8*I*e)*exp(8*I*f*x) + 5880*f*exp(6*I*e)*exp(6*I*f*x) + 2940*f*exp(4*I*e)*exp(4*I*f*x) + 840*f*exp(2*I*e)*exp(2*I*f*x) + 105*f)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs.  $2(117) = 234$ .

time = 1.30, size = 241, normalized size = 1.79

$$\frac{32(-84iAa^3c^5e^{(6i fx+6ie)} - 84Ba^3c^5e^{(6i fx+6ie)} - 112iAa^3c^5e^{(4i fx+4ie)} + 28Ba^3c^5e^{(4i fx+4ie)} - 32iAa^3c^5e^{(2i fx+2ie)} + 8Ba^3c^5e^{(2i fx+2ie)} - 4iAa^3c^5 + Ba^3c^5)}{105(fe^{(16i fx+16ie)} + 8fe^{(14i fx+14ie)} + 28fe^{(12i fx+12ie)} + 56fe^{(10i fx+10ie)} + 70fe^{(8i fx+8ie)} + 56fe^{(6i fx+6ie)} + 28fe^{(4i fx+4ie)} + 8fe^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, algorithm="giac")`

[Out]  $-32/105*(-84*I*A*a^3*c^5*e^{(6*I*f*x + 6*I*e)} - 84*B*a^3*c^5*e^{(6*I*f*x + 6*I*e)} - 112*I*A*a^3*c^5*e^{(4*I*f*x + 4*I*e)} + 28*B*a^3*c^5*e^{(4*I*f*x + 4*I*e)} - 32*I*A*a^3*c^5*e^{(2*I*f*x + 2*I*e)} + 8*B*a^3*c^5*e^{(2*I*f*x + 2*I*e)} - 4*I*A*a^3*c^5 + B*a^3*c^5)/(f*e^{(16*I*f*x + 16*I*e)} + 8*f*e^{(14*I*f*x + 14*I*e)} + 28*f*e^{(12*I*f*x + 12*I*e)} + 56*f*e^{(10*I*f*x + 10*I*e)} + 70*f*e^{(8*I*f*x + 8*I*e)} + 56*f*e^{(6*I*f*x + 6*I*e)} + 28*f*e^{(4*I*f*x + 4*I*e)} + 8*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad** [B]

time = 8.65, size = 174, normalized size = 1.29

$$\frac{a^3c^5 \tan(e+fx)^2(-B+A2i) + a^3c^5 \tan(e+fx)^4(-B+A4i) - Aa^3c^5 \tan(e+fx) - \frac{a^3c^5 \tan(e+fx)^3(A-B2i)}{3} + \frac{a^3c^5 \tan(e+fx)^6(B+A2i)}{6} + \frac{a^3c^5 \tan(e+fx)^5(A+B4i)}{5} + \frac{a^3c^5 \tan(e+fx)^7(A+B2i)}{7} + \frac{Ba^3c^5 \tan(e+fx)^8}{8}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^5,x)`

[Out]  $-((a^3*c^5*\tan(e + f*x)^2*(A*2i - B))/2 + (a^3*c^5*\tan(e + f*x)^4*(A*4i - B))/4 - A*a^3*c^5*\tan(e + f*x) - (a^3*c^5*\tan(e + f*x)^3*(A - B*2i))/3 + (a^3*c^5*\tan(e + f*x)^6*(A*2i + B))/6 + (a^3*c^5*\tan(e + f*x)^5*(A + B*4i))/5 + (a^3*c^5*\tan(e + f*x)^7*(A + B*2i))/7 + (B*a^3*c^5*\tan(e + f*x)^8)/8)/f$

### 3.692 $\int (a+ia \tan(e+fx))^3 (A+B \tan(e+fx))(c-ic \tan(e+fx))^4 dx$

**Optimal.** Leaf size=132

$$\frac{a^3(iA+B)c^4(1-i \tan(e+fx))^4}{f} - \frac{4a^3(iA+2B)c^4(1-i \tan(e+fx))^5}{5f} + \frac{a^3(iA+5B)c^4(1-i \tan(e+fx))^6}{6f}$$

[Out]  $a^3*(I*A+B)*c^4*(1-I*\tan(f*x+e))^4/f-4/5*a^3*(I*A+2*B)*c^4*(1-I*\tan(f*x+e))^5/f+1/6*a^3*(I*A+5*B)*c^4*(1-I*\tan(f*x+e))^6/f-1/7*a^3*B*c^4*(1-I*\tan(f*x+e))^7/f$

**Rubi [A]**

time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ ,

Rules used = {3669, 78}

$$\frac{a^3c^4(5B+iA)(1-i \tan(e+fx))^6}{6f} - \frac{4a^3c^4(2B+iA)(1-i \tan(e+fx))^5}{5f} + \frac{a^3c^4(B+iA)(1-i \tan(e+fx))^4}{f} - \frac{a^3Bc^4(1-i \tan(e+fx))^7}{7f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^4, x]$

[Out]  $(a^3*(I*A + B)*c^4*(1 - I*\text{Tan}[e + f*x])^4)/f - (4*a^3*(I*A + 2*B)*c^4*(1 - I*\text{Tan}[e + f*x])^5)/(5*f) + (a^3*(I*A + 5*B)*c^4*(1 - I*\text{Tan}[e + f*x])^6)/(6*f) - (a^3*B*c^4*(1 - I*\text{Tan}[e + f*x])^7)/(7*f)$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^(n_.))*((e_. + (f_.)*(x_.))^(p_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^(m_.))*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*((c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps



$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ictan(e + fx))^4 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx)(c - icx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (4a^2(A - iB)(c - icx) dx\right)}{f}$$

$$= \frac{a^3(iA + B)c^4(1 - i \tan(e + fx))^4}{f}$$

**Mathematica [A]**

time = 1.53, size = 172, normalized size = 1.30

$$\frac{a^3 c^4 \sec^7(e + fx) (70(-iA + B) \cos(fx) + 70(-iA + B) \cos(2e + fx) + 175A \sin(fx) - 35iB \sin(fx) - 70A \sin(2e + fx) - 70iB \sin(2e + fx) + 147A \sin(2e + 3fx) + 21iB \sin(2e + 3fx) + 49A \sin(4e + 5fx) + 7iB \sin(4e + 5fx) + 7A \sin(6e + 7fx) + iB \sin(6e + 7fx))}{840f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4,x]

[Out] (a^3\*c^4\*Sec[e]\*Sec[e + f\*x]^7\*(70\*((-I)\*A + B)\*Cos[f\*x] + 70\*((-I)\*A + B)\*Cos[2\*e + f\*x] + 175\*A\*Sin[f\*x] - (35\*I)\*B\*Sin[f\*x] - 70\*A\*Sin[2\*e + f\*x] - (70\*I)\*B\*Sin[2\*e + f\*x] + 147\*A\*Sin[2\*e + 3\*f\*x] + (21\*I)\*B\*Sin[2\*e + 3\*f\*x] + 49\*A\*Sin[4\*e + 5\*f\*x] + (7\*I)\*B\*Sin[4\*e + 5\*f\*x] + 7\*A\*Sin[6\*e + 7\*f\*x] + I\*B\*Sin[6\*e + 7\*f\*x]))/(840\*f)

**Maple [A]**

time = 0.13, size = 159, normalized size = 1.20

method	result
risch	$\frac{16c^4 a^3 (105iA e^{6i(fx+e)} + 105B e^{6i(fx+e)} + 147iA e^{4i(fx+e)} - 21B e^{4i(fx+e)} + 49iA e^{2i(fx+e)} - 7B e^{2i(fx+e)} + 7iA - B)}{105f(e^{2i(fx+e)} + 1)^7}$
derivativedivides	$- \frac{ic^4 a^3 \left( \frac{B(\tan^7(fx+e))}{7} + \frac{(iB+A)(\tan^6(fx+e))}{6} + \frac{(-4B-2iA+3i(-2iB+A))(\tan^5(fx+e))}{5} + \frac{(-4A+3i(-2iA-B)+5iB)(\tan^4(fx+e))}{4} \right)}{f}$
default	$- \frac{ic^4 a^3 \left( \frac{B(\tan^7(fx+e))}{7} + \frac{(iB+A)(\tan^6(fx+e))}{6} + \frac{(-4B-2iA+3i(-2iB+A))(\tan^5(fx+e))}{5} + \frac{(-4A+3i(-2iA-B)+5iB)(\tan^4(fx+e))}{4} \right)}{f}$
norman	$\frac{A a^3 c^4 \tan(fx+e)}{f} + \frac{(-iA a^3 c^4 + B a^3 c^4)(\tan^2(fx+e))}{2f} + \frac{(-iA a^3 c^4 + B a^3 c^4)(\tan^4(fx+e))}{2f} + \frac{(-iA a^3 c^4 + B a^3 c^4)(\tan^6(fx+e))}{6f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4,x,method=\_RE TURNVERBOSE)

[Out]  $-I/f*c^4*a^3*(1/7*B*\tan(f*x+e)^7+1/6*(A+I*B)*\tan(f*x+e)^6+1/5*(-4*B-2*I*A+3*I*(A-2*I*B))*\tan(f*x+e)^5+1/4*(-4*A+3*I*(-2*I*A-B)+5*I*B)*\tan(f*x+e)^4+1/3*(3*I*A+3*B-I*(A-2*I*B))*\tan(f*x+e)^3+1/2*(3*A-I*(-2*I*A-B))*\tan(f*x+e)^2+I*A*\tan(f*x+e))$

**Maxima** [A]

time = 0.60, size = 158, normalized size = 1.20

$$\frac{30iBa^3c^4 \tan(fx+e)^7 + 35(iA-B)a^3c^4 \tan(fx+e)^6 - 42(A-2iB)a^3c^4 \tan(fx+e)^5 + 105(iA-B)a^3c^4 \tan(fx+e)^4 - 70(2A-iB)a^3c^4 \tan(fx+e)^3 + 105(iA-B)a^3c^4 \tan(fx+e)^2 - 210Aa^3c^4 \tan(fx+e)}{210f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")`

[Out]  $-1/210*(30*I*B*a^3*c^4*\tan(f*x+e)^7 + 35*(I*A - B)*a^3*c^4*\tan(f*x+e)^6 - 42*(A - 2*I*B)*a^3*c^4*\tan(f*x+e)^5 + 105*(I*A - B)*a^3*c^4*\tan(f*x+e)^4 - 70*(2*A - I*B)*a^3*c^4*\tan(f*x+e)^3 + 105*(I*A - B)*a^3*c^4*\tan(f*x+e)^2 - 210*A*a^3*c^4*\tan(f*x+e))/f$

**Fricas** [A]

time = 4.03, size = 180, normalized size = 1.36

$$\frac{16(105(-iA-B)a^3c^4e^{6i fx+6ie} + 21(-7iA+B)a^3c^4e^{4i fx+4ie} + 7(-7iA+B)a^3c^4e^{2i fx+2ie} + (-7iA+B)a^3c^4)}{105(fe^{14i fx+14ie} + 7fe^{12i fx+12ie} + 21fe^{10i fx+10ie} + 35fe^{8i fx+8ie} + 35fe^{6i fx+6ie} + 21fe^{4i fx+4ie} + 7fe^{2i fx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")`

[Out]  $-16/105*(105*(-I*A - B)*a^3*c^4*e^{(6*I*f*x + 6*I*e)} + 21*(-7*I*A + B)*a^3*c^4*e^{(4*I*f*x + 4*I*e)} + 7*(-7*I*A + B)*a^3*c^4*e^{(2*I*f*x + 2*I*e)} + (-7*I*A + B)*a^3*c^4)/(f*e^{(14*I*f*x + 14*I*e)} + 7*f*e^{(12*I*f*x + 12*I*e)} + 21*f*e^{(10*I*f*x + 10*I*e)} + 35*f*e^{(8*I*f*x + 8*I*e)} + 35*f*e^{(6*I*f*x + 6*I*e)} + 21*f*e^{(4*I*f*x + 4*I*e)} + 7*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs.  $2(107) = 214$ .

time = 0.82, size = 287, normalized size = 2.17

$$\frac{112iAa^3c^4 - 16Ba^3c^4 + (784iAa^3c^4e^{2ie} - 112Ba^3c^4e^{2ie})e^{2ifx} + (2352iAa^3c^4e^{4ie} - 336Ba^3c^4e^{4ie})e^{4ifx} + (1680iAa^3c^4e^{6ie} + 1680Ba^3c^4e^{6ie})e^{6ifx}}{105fe^{14ie}e^{14ifx} + 735fe^{12ie}e^{12ifx} + 2205fe^{10ie}e^{10ifx} + 3675fe^{8ie}e^{8ifx} + 3675fe^{6ie}e^{6ifx} + 2205fe^{4ie}e^{4ifx} + 735fe^{2ie}e^{2ifx} + 105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)`

[Out]  $(112*I*A*a**3*c**4 - 16*B*a**3*c**4 + (784*I*A*a**3*c**4*\exp(2*I*e) - 112*B*a**3*c**4*\exp(2*I*e))*\exp(2*I*f*x) + (2352*I*A*a**3*c**4*\exp(4*I*e) - 336*$

$B*a**3*c**4*exp(4*I*e))*exp(4*I*f*x) + (1680*I*A*a**3*c**4*exp(6*I*e) + 1680*B*a**3*c**4*exp(6*I*e))*exp(6*I*f*x)/(105*f*exp(14*I*e)*exp(14*I*f*x) + 735*f*exp(12*I*e)*exp(12*I*f*x) + 2205*f*exp(10*I*e)*exp(10*I*f*x) + 3675*f*exp(8*I*e)*exp(8*I*f*x) + 3675*f*exp(6*I*e)*exp(6*I*f*x) + 2205*f*exp(4*I*e)*exp(4*I*f*x) + 735*f*exp(2*I*e)*exp(2*I*f*x) + 105*f)$

**Giac** [A]

time = 1.10, size = 228, normalized size = 1.73

$$\frac{16(-105iAa^3c^4e^{6i fx+6ie} - 105Ba^3c^4e^{6i fx+6ie} - 147iAa^3c^4e^{4i fx+4ie} + 21Ba^3c^4e^{4i fx+4ie} - 49iAa^3c^4e^{2i fx+2ie} + 7Ba^3c^4e^{2i fx+2ie} - 7iAa^3c^4 + Ba^3c^4)}{105(fe^{14i fx+14ie} + 7fe^{12i fx+12ie} + 21fe^{10i fx+10ie} + 35fe^{8i fx+8ie} + 35fe^{6i fx+6ie} + 21fe^{4i fx+4ie} + 7fe^{2i fx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out]  $-16/105*(-105*I*A*a^3*c^4*e^{(6*I*f*x + 6*I*e)} - 105*B*a^3*c^4*e^{(6*I*f*x + 6*I*e)} - 147*I*A*a^3*c^4*e^{(4*I*f*x + 4*I*e)} + 21*B*a^3*c^4*e^{(4*I*f*x + 4*I*e)} - 49*I*A*a^3*c^4*e^{(2*I*f*x + 2*I*e)} + 7*B*a^3*c^4*e^{(2*I*f*x + 2*I*e)} - 7*I*A*a^3*c^4 + B*a^3*c^4)/(f*e^{(14*I*f*x + 14*I*e)} + 7*f*e^{(12*I*f*x + 12*I*e)} + 21*f*e^{(10*I*f*x + 10*I*e)} + 35*f*e^{(8*I*f*x + 8*I*e)} + 35*f*e^{(6*I*f*x + 6*I*e)} + 21*f*e^{(4*I*f*x + 4*I*e)} + 7*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad** [B]

time = 8.59, size = 156, normalized size = 1.18

$$\frac{\frac{a^3 c^4 \tan(e+f x)^5 (2 B+A 1 i) \operatorname{li}}{5} - A a^3 c^4 \tan(e+f x) + \frac{a^3 c^4 \tan(e+f x)^2 (A+B 1 i) \operatorname{li}}{2} + \frac{a^3 c^4 \tan(e+f x)^3 (B+A 2 i) \operatorname{li}}{3} + \frac{a^3 c^4 \tan(e+f x)^4 (A+B 1 i) \operatorname{li}}{2} + \frac{a^3 c^4 \tan(e+f x)^6 (A+B 1 i) \operatorname{li}}{6} + \frac{B a^3 c^4 \tan(e+f x)^7 \operatorname{li}}{7}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)^4,x)

[Out]  $-((a^3*c^4*\tan(e + f*x)^5*(A*1i + 2*B)*1i)/5 - A*a^3*c^4*\tan(e + f*x) + (a^3*c^4*\tan(e + f*x)^2*(A + B*1i)*1i)/2 + (a^3*c^4*\tan(e + f*x)^3*(A*2i + B)*1i)/3 + (a^3*c^4*\tan(e + f*x)^4*(A + B*1i)*1i)/2 + (a^3*c^4*\tan(e + f*x)^6*(A + B*1i)*1i)/6 + (B*a^3*c^4*\tan(e + f*x)^7*1i)/7)/f$

### 3.693 $\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx))^3 dx$

Optimal. Leaf size=84

$$\frac{a^3 B c^3 \sec^6(e + fx)}{6f} + \frac{a^3 A c^3 \tan(e + fx)}{f} + \frac{2a^3 A c^3 \tan^3(e + fx)}{3f} + \frac{a^3 A c^3 \tan^5(e + fx)}{5f}$$

[Out]  $1/6*a^3*B*c^3*\sec(f*x+e)^6/f+a^3*A*c^3*\tan(f*x+e)/f+2/3*a^3*A*c^3*\tan(f*x+e)^3/f+1/5*a^3*A*c^3*\tan(f*x+e)^5/f$

Rubi [A]

time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$ , Rules used = {3669, 74, 655, 200}

$$\frac{a^3 A c^3 \tan^5(e + fx)}{5f} + \frac{2a^3 A c^3 \tan^3(e + fx)}{3f} + \frac{a^3 A c^3 \tan(e + fx)}{f} + \frac{a^3 B c^3 \sec^6(e + fx)}{6f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^3, x]$

[Out]  $(a^3*B*c^3*\text{Sec}[e + f*x]^6)/(6*f) + (a^3*A*c^3*\text{Tan}[e + f*x])/f + (2*a^3*A*c^3*\text{Tan}[e + f*x]^3)/(3*f) + (a^3*A*c^3*\text{Tan}[e + f*x]^5)/(5*f)$

Rule 74

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x\_Symbol] :> \text{Int}[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[n, m] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{NeQ}[m, -1] \ || \ (\text{EqQ}[e, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ !\text{IntegerQ}[p])))$

Rule 200

$\text{Int}[(a_ + (b_)*(x_))^{(n_)}^{(p_)}], x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 655

$\text{Int}[(d_ + (e_)*(x_))*((a_ + (c_)*(x_)^2)^{(p_)}), x\_Symbol] :> \text{Simp}[e*((a + c*x^2)^{(p+1})/(2*c*(p+1))), x] + \text{Dist}[d, \text{Int}[(a + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ictan(e + fx))^3 dx &= \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx)(c - ictan(e + fx))^3 dx\right)}{f} \\ &= \frac{(ac) \text{Subst}\left(\int (A + Bx) (ac + acx^2)^2 dx\right)}{f} \\ &= \frac{a^3 Bc^3 \sec^6(e + fx)}{6f} + \frac{(aAc) \text{Subst}\left(\int (A + Bx) dx\right)}{f} \\ &= \frac{a^3 Bc^3 \sec^6(e + fx)}{6f} + \frac{(aAc) \text{Subst}\left(\int (A + Bx) dx\right)}{f} \\ &= \frac{a^3 Bc^3 \sec^6(e + fx)}{6f} + \frac{a^3 Ac^3 \tan(e + fx)}{f} \end{aligned}$$

**Mathematica [A]**

time = 0.19, size = 65, normalized size = 0.77

$$\frac{a^3 Bc^3 \sec^6(e + fx)}{6f} + \frac{a^3 Ac^3 (\tan(e + fx) + \frac{2}{3} \tan^3(e + fx) + \frac{1}{5} \tan^5(e + fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^3,x]
```

```
[Out] (a^3*B*c^3*Sec[e + f*x]^6)/(6*f) + (a^3*A*c^3*(Tan[e + f*x] + (2*Tan[e + f*
x]^3)/3 + Tan[e + f*x]^5/5))/f
```

**Maple [A]**

time = 0.09, size = 75, normalized size = 0.89

method	result
derivativedivides	$\frac{c^3 a^3 \left( \frac{B(\tan^6(fx+e))}{6} + \frac{A(\tan^5(fx+e))}{5} + \frac{B(\tan^4(fx+e))}{2} + \frac{2A(\tan^3(fx+e))}{3} + \frac{B(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$

default	$\frac{c^3 a^3 \left( \frac{B(\tan^6(fx+e))}{6} + \frac{A(\tan^5(fx+e))}{5} + \frac{B(\tan^4(fx+e))}{2} + \frac{2A(\tan^3(fx+e))}{3} + \frac{B(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$
risch	$\frac{16c^3 a^3 (10iA e^{6i(fx+e)} + 10B e^{6i(fx+e)} + 15iA e^{4i(fx+e)} + 6iA e^{2i(fx+e)} + iA)}{15f (e^{2i(fx+e)} + 1)^6}$
norman	$\frac{a^3 A c^3 \tan(fx+e)}{f} + \frac{B a^3 c^3 (\tan^2(fx+e))}{2f} + \frac{B a^3 c^3 (\tan^4(fx+e))}{2f} + \frac{B a^3 c^3 (\tan^6(fx+e))}{6f} + \frac{2a^3 A c^3 (\tan^3(fx+e))}{3f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)`

[Out] `1/f*c^3*a^3*(1/6*B*tan(f*x+e)^6+1/5*A*tan(f*x+e)^5+1/2*B*tan(f*x+e)^4+2/3*A*tan(f*x+e)^3+1/2*B*tan(f*x+e)^2+A*tan(f*x+e))`

**Maxima [A]**

time = 0.52, size = 112, normalized size = 1.33

$$\frac{5Ba^3c^3 \tan(fx+e)^6 + 6Aa^3c^3 \tan(fx+e)^5 + 15Ba^3c^3 \tan(fx+e)^4 + 20Aa^3c^3 \tan(fx+e)^3 + 15Ba^3c^3 \tan(fx+e)^2 + 30Aa^3c^3 \tan(fx+e)}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")`

[Out] `1/30*(5*B*a^3*c^3*tan(f*x + e)^6 + 6*A*a^3*c^3*tan(f*x + e)^5 + 15*B*a^3*c^3*tan(f*x + e)^4 + 20*A*a^3*c^3*tan(f*x + e)^3 + 15*B*a^3*c^3*tan(f*x + e)^2 + 30*A*a^3*c^3*tan(f*x + e))/f`

**Fricas [C]** Result contains complex when optimal does not.

time = 13.19, size = 156, normalized size = 1.86

$$\frac{16(10(-iA - B)a^3c^3e^{(6ifx+6ie)} - 15iAa^3c^3e^{(4ifx+4ie)} - 6iAa^3c^3e^{(2ifx+2ie)} - iAa^3c^3)}{15(fe^{(12ifx+12ie)} + 6fe^{(10ifx+10ie)} + 15fe^{(8ifx+8ie)} + 20fe^{(6ifx+6ie)} + 15fe^{(4ifx+4ie)} + 6fe^{(2ifx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")`

[Out] `-16/15*(10*(-I*A - B)*a^3*c^3*e^(6*I*f*x + 6*I*e) - 15*I*A*a^3*c^3*e^(4*I*f*x + 4*I*e) - 6*I*A*a^3*c^3*e^(2*I*f*x + 2*I*e) - I*A*a^3*c^3)/(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)`

**Sympy [C]** Result contains complex when optimal does not.

time = 0.58, size = 224, normalized size = 2.67

$$\frac{240iAa^3c^3e^{4ie}e^{4ifx} + 96iAa^3c^3e^{2ie}e^{2ifx} + 16iAa^3c^3 + (160iAa^3c^3e^{6ie} + 160Ba^3c^3e^{6ie})e^{6ifx}}{15fe^{12ie}e^{12ifx} + 90fe^{10ie}e^{10ifx} + 225fe^{8ie}e^{8ifx} + 300fe^{6ie}e^{6ifx} + 225fe^{4ie}e^{4ifx} + 90fe^{2ie}e^{2ifx} + 15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3,x)`

[Out]  $(240*I*A*a**3*c**3*\exp(4*I*e)*\exp(4*I*f*x) + 96*I*A*a**3*c**3*\exp(2*I*e)*\exp(2*I*f*x) + 16*I*A*a**3*c**3 + (160*I*A*a**3*c**3*\exp(6*I*e) + 160*B*a**3*c**3*\exp(6*I*e))*\exp(6*I*f*x))/(15*f*\exp(12*I*e)*\exp(12*I*f*x) + 90*f*\exp(10*I*e)*\exp(10*I*f*x) + 225*f*\exp(8*I*e)*\exp(8*I*f*x) + 300*f*\exp(6*I*e)*\exp(6*I*f*x) + 225*f*\exp(4*I*e)*\exp(4*I*f*x) + 90*f*\exp(2*I*e)*\exp(2*I*f*x) + 15*f)$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs.  $2(82) = 164$ .

time = 1.10, size = 793, normalized size = 9.44

---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algorithm="giac")`

[Out]  $1/30*(5*B*a^3*c^3*\tan(f*x)^6*\tan(e)^6 - 30*A*a^3*c^3*\tan(f*x)^6*\tan(e)^5 - 30*A*a^3*c^3*\tan(f*x)^5*\tan(e)^6 + 15*B*a^3*c^3*\tan(f*x)^6*\tan(e)^4 + 15*B*a^3*c^3*\tan(f*x)^4*\tan(e)^6 - 20*A*a^3*c^3*\tan(f*x)^6*\tan(e)^3 + 90*A*a^3*c^3*\tan(f*x)^5*\tan(e)^4 + 90*A*a^3*c^3*\tan(f*x)^4*\tan(e)^5 - 20*A*a^3*c^3*\tan(f*x)^3*\tan(e)^6 + 15*B*a^3*c^3*\tan(f*x)^6*\tan(e)^2 + 45*B*a^3*c^3*\tan(f*x)^4*\tan(e)^4 + 15*B*a^3*c^3*\tan(f*x)^2*\tan(e)^6 - 6*A*a^3*c^3*\tan(f*x)^6*\tan(e) + 30*A*a^3*c^3*\tan(f*x)^5*\tan(e)^2 - 180*A*a^3*c^3*\tan(f*x)^4*\tan(e)^3 - 180*A*a^3*c^3*\tan(f*x)^3*\tan(e)^4 + 30*A*a^3*c^3*\tan(f*x)^2*\tan(e)^5 - 6*A*a^3*c^3*\tan(f*x)*\tan(e)^6 + 5*B*a^3*c^3*\tan(f*x)^6 + 45*B*a^3*c^3*\tan(f*x)^4*\tan(e)^2 + 45*B*a^3*c^3*\tan(f*x)^2*\tan(e)^4 + 5*B*a^3*c^3*\tan(e)^6 + 6*A*a^3*c^3*\tan(f*x)^5 - 30*A*a^3*c^3*\tan(f*x)^4*\tan(e) + 180*A*a^3*c^3*\tan(f*x)^3*\tan(e)^2 + 180*A*a^3*c^3*\tan(f*x)^2*\tan(e)^3 - 30*A*a^3*c^3*\tan(f*x)*\tan(e)^4 + 6*A*a^3*c^3*\tan(e)^5 + 15*B*a^3*c^3*\tan(f*x)^4 + 45*B*a^3*c^3*\tan(f*x)^2*\tan(e)^2 + 15*B*a^3*c^3*\tan(e)^4 + 20*A*a^3*c^3*\tan(f*x)^3 - 90*A*a^3*c^3*\tan(f*x)^2*\tan(e) - 90*A*a^3*c^3*\tan(f*x)*\tan(e)^2 + 20*A*a^3*c^3*\tan(e)^3 + 15*B*a^3*c^3*\tan(f*x)^2 + 15*B*a^3*c^3*\tan(e)^2 + 30*A*a^3*c^3*\tan(f*x) + 30*A*a^3*c^3*\tan(e) + 5*B*a^3*c^3)/(f*\tan(f*x)^6*\tan(e)^6 - 6*f*\tan(f*x)^5*\tan(e)^5 + 15*f*\tan(f*x)^4*\tan(e)^4 - 20*f*\tan(f*x)^3*\tan(e)^3 + 15*f*\tan(f*x)^2*\tan(e)^2 - 6*f*\tan(f*x)*\tan(e) + f)$

**Mupad** [B]

time = 8.46, size = 120, normalized size = 1.43

$$\frac{a^3 c^3 \sin(e+f x) (30 A \cos(e+f x)^5 + 15 B \cos(e+f x)^4 \sin(e+f x) + 20 A \cos(e+f x)^3 \sin(e+f x)^2 + 15 B \cos(e+f x)^2 \sin(e+f x)^3 + 6 A \cos(e+f x) \sin(e+f x)^4 + 5 B \sin(e+f x)^5)}{30 f \cos(e+f x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B*\tan(e + f*x))*(a + a*\tan(e + f*x)*i)^3*(c - c*\tan(e + f*x)*i)^3, x)$

[Out]  $(a^3*c^3*\sin(e + f*x)*(30*A*\cos(e + f*x)^5 + 5*B*\sin(e + f*x)^5 + 20*A*\cos(e + f*x)^3*\sin(e + f*x)^2 + 15*B*\cos(e + f*x)^2*\sin(e + f*x)^3 + 6*A*\cos(e + f*x)*\sin(e + f*x)^4 + 15*B*\cos(e + f*x)^4*\sin(e + f*x)))/(30*f*\cos(e + f*x)^6)$



$$3.694 \quad \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^2 dx$$

Optimal. Leaf size=101

$$-\frac{2a^3(iA - B)c^2(1 + i \tan(e + fx))^3}{3f} + \frac{a^3(iA - 3B)c^2(1 + i \tan(e + fx))^4}{4f} + \frac{a^3Bc^2(1 + i \tan(e + fx))^5}{5f}$$

[Out]  $-2/3*a^3*(I*A-B)*c^2*(1+I*\tan(f*x+e))^3/f+1/4*a^3*(I*A-3*B)*c^2*(1+I*\tan(f*x+e))^4/f+1/5*a^3*B*c^2*(1+I*\tan(f*x+e))^5/f$

Rubi [A]

time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{a^3c^2(-3B + iA)(1 + i \tan(e + fx))^4}{4f} - \frac{2a^3c^2(-B + iA)(1 + i \tan(e + fx))^3}{3f} + \frac{a^3Bc^2(1 + i \tan(e + fx))^5}{5f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^2, x]$

[Out]  $(-2*a^3*(I*A - B)*c^2*(1 + I*\text{Tan}[e + f*x])^3)/(3*f) + (a^3*(I*A - 3*B)*c^2*(1 + I*\text{Tan}[e + f*x])^4)/(4*f) + (a^3*B*c^2*(1 + I*\text{Tan}[e + f*x])^5)/(5*f)$

Rule 78

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\tan[(e + f*x)])^m*(A + B*\tan[(e + f*x)])^n, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^2 dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx) (c - ic \tan(e + fx))^2 dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(2(A + iB)c(a + iax)^2\right) dx\right)}{f}$$

$$= -\frac{2a^3(iA - B)c^2(1 + i \tan(e + fx))^3}{3f}$$

**Mathematica [A]**

time = 1.43, size = 146, normalized size = 1.45

$$\frac{a^3 c^2 \sec(e) \sec^5(e + fx) (15(iA + B) \cos(fx) + 15(iA + B) \cos(2e + fx) + 35A \sin(fx) + 5iB \sin(fx) - 15A \sin(2e + fx) + 15iB \sin(2e + fx) + 25A \sin(2e + 3fx) - 5iB \sin(2e + 3fx) + 5A \sin(4e + 5fx) - iB \sin(4e + 5fx))}{120f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*Sec[e]*Sec[e + f*x]^5*(15*(I*A + B)*Cos[f*x] + 15*(I*A + B)*Cos[2*e + f*x] + 35*A*Sin[f*x] + (5*I)*B*Sin[f*x] - 15*A*Sin[2*e + f*x] + (15*I)*B*Sin[2*e + f*x] + 25*A*Sin[2*e + 3*f*x] - (5*I)*B*Sin[2*e + 3*f*x] + 5*A*Sin[4*e + 5*f*x] - I*B*Sin[4*e + 5*f*x]))/(120*f)
```

**Maple [A]**

time = 0.12, size = 94, normalized size = 0.93

method	result
derivativedivides	$\frac{ic^2 a^3 \left( \frac{B(\tan^5(fx+e))}{5} + \frac{(-iB+A)(\tan^4(fx+e))}{4} + \frac{(iA-2i(iB+A)-B)(\tan^3(fx+e))}{3} + \frac{(-iB+A)(\tan^2(fx+e))}{2} - iA \tan(fx+e) \right)}{f}$
default	$\frac{ic^2 a^3 \left( \frac{B(\tan^5(fx+e))}{5} + \frac{(-iB+A)(\tan^4(fx+e))}{4} + \frac{(iA-2i(iB+A)-B)(\tan^3(fx+e))}{3} + \frac{(-iB+A)(\tan^2(fx+e))}{2} - iA \tan(fx+e) \right)}{f}$
risch	$\frac{4c^2 a^3 (30iA e^{6i(fx+e)} + 30B e^{6i(fx+e)} + 50iA e^{4i(fx+e)} + 10B e^{4i(fx+e)} + 25iA e^{2i(fx+e)} + 5B e^{2i(fx+e)} + 5iA + B)}{15f(e^{2i(fx+e)} + 1)^5}$
norman	$\frac{A a^3 c^2 \tan(fx+e)}{f} + \frac{(iA a^3 c^2 + B a^3 c^2) (\tan^2(fx+e))}{2f} + \frac{(iA a^3 c^2 + B a^3 c^2) (\tan^4(fx+e))}{4f} + \frac{(iB a^3 c^2 + A a^3 c^2) (\tan^3(fx+e))}{3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I/f*c^2*a^3*(1/5*B*tan(f*x+e)^5+1/4*(A-I*B)*tan(f*x+e)^4+1/3*(I*A-2*I*(A+I*B)-B)*tan(f*x+e)^3+1/2*(A-I*B)*tan(f*x+e)^2-I*A*tan(f*x+e))
```

**Maxima [A]**

time = 0.56, size = 110, normalized size = 1.09

$$\frac{12iBa^3c^2 \tan(fx+e)^5 - 15(-iA-B)a^3c^2 \tan(fx+e)^4 + 20(A+iB)a^3c^2 \tan(fx+e)^3 - 30(-iA-B)a^3c^2 \tan(fx+e)^2 + 60Aa^3c^2 \tan(fx+e)}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorithm="maxima")

[Out] 1/60\*(12\*I\*B\*a^3\*c^2\*tan(f\*x + e)^5 - 15\*(-I\*A - B)\*a^3\*c^2\*tan(f\*x + e)^4 + 20\*(A + I\*B)\*a^3\*c^2\*tan(f\*x + e)^3 - 30\*(-I\*A - B)\*a^3\*c^2\*tan(f\*x + e)^2 + 60\*A\*a^3\*c^2\*tan(f\*x + e))/f

**Fricas [A]**

time = 8.00, size = 160, normalized size = 1.58

$$\frac{4(30(-iA-B)a^3c^2e^{6ifx+6ie} + 10(-5iA-B)a^3c^2e^{4ifx+4ie} + 5(-5iA-B)a^3c^2e^{2ifx+2ie} + (-5iA-B)a^3c^2)}{15(fe^{10ifx+10ie} + 5fe^{8ifx+8ie} + 10fe^{6ifx+6ie} + 10fe^{4ifx+4ie} + 5fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out] -4/15\*(30\*(-I\*A - B)\*a^3\*c^2\*e^(6\*I\*f\*x + 6\*I\*e) + 10\*(-5\*I\*A - B)\*a^3\*c^2\*e^(4\*I\*f\*x + 4\*I\*e) + 5\*(-5\*I\*A - B)\*a^3\*c^2\*e^(2\*I\*f\*x + 2\*I\*e) + (-5\*I\*A - B)\*a^3\*c^2)/(f\*e^(10\*I\*f\*x + 10\*I\*e) + 5\*f\*e^(8\*I\*f\*x + 8\*I\*e) + 10\*f\*e^(6\*I\*f\*x + 6\*I\*e) + 10\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 5\*f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(80) = 160.

time = 0.48, size = 250, normalized size = 2.48

$$\frac{20iAa^3c^2 + 4Ba^3c^2 + (100iAa^3c^2e^{2ie} + 20Ba^3c^2e^{2ie})e^{2ifx} + (200iAa^3c^2e^{4ie} + 40Ba^3c^2e^{4ie})e^{4ifx} + (120iAa^3c^2e^{6ie} + 120Ba^3c^2e^{6ie})e^{6ifx}}{15fe^{10ie}e^{10ifx} + 75fe^{8ie}e^{8ifx} + 150fe^{6ie}e^{6ifx} + 150fe^{4ie}e^{4ifx} + 75fe^{2ie}e^{2ifx} + 15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x)

[Out] (20\*I\*A\*a\*\*3\*c\*\*2 + 4\*B\*a\*\*3\*c\*\*2 + (100\*I\*A\*a\*\*3\*c\*\*2\*exp(2\*I\*e) + 20\*B\*a\*\*3\*c\*\*2\*exp(2\*I\*e))\*exp(2\*I\*f\*x) + (200\*I\*A\*a\*\*3\*c\*\*2\*exp(4\*I\*e) + 40\*B\*a\*\*3\*c\*\*2\*exp(4\*I\*e))\*exp(4\*I\*f\*x) + (120\*I\*A\*a\*\*3\*c\*\*2\*exp(6\*I\*e) + 120\*B\*a\*\*3\*c\*\*2\*exp(6\*I\*e))\*exp(6\*I\*f\*x))/(15\*f\*exp(10\*I\*e)\*exp(10\*I\*f\*x) + 75\*f\*exp(8\*I\*e)\*exp(8\*I\*f\*x) + 150\*f\*exp(6\*I\*e)\*exp(6\*I\*f\*x) + 150\*f\*exp(4\*I\*e)\*exp(4\*I\*f\*x) + 75\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + 15\*f)

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(88) = 176.

time = 0.81, size = 203, normalized size = 2.01

$$\frac{4(-30iAa^3c^2e^{6ifx+6ie} - 30Ba^3c^2e^{6ifx+6ie} - 50iAa^3c^2e^{4ifx+4ie} - 10Ba^3c^2e^{4ifx+4ie} - 25iAa^3c^2e^{2ifx+2ie} - 5Ba^3c^2e^{2ifx+2ie} - 5iAa^3c^2 - Ba^3c^2)}{15(fe^{10ifx+10ie} + 5fe^{8ifx+8ie} + 10fe^{6ifx+6ie} + 10fe^{4ifx+4ie} + 5fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2,x, algorithm="giac")

[Out]  $-4/15*(-30*I*A*a^3*c^2*e^{(6*I*f*x + 6*I*e)} - 30*B*a^3*c^2*e^{(6*I*f*x + 6*I*e)} - 50*I*A*a^3*c^2*e^{(4*I*f*x + 4*I*e)} - 10*B*a^3*c^2*e^{(4*I*f*x + 4*I*e)} - 25*I*A*a^3*c^2*e^{(2*I*f*x + 2*I*e)} - 5*B*a^3*c^2*e^{(2*I*f*x + 2*I*e)} - 5*I*A*a^3*c^2 - B*a^3*c^2)/(f*e^{(10*I*f*x + 10*I*e)} + 5*f*e^{(8*I*f*x + 8*I*e)} + 10*f*e^{(6*I*f*x + 6*I*e)} + 10*f*e^{(4*I*f*x + 4*I*e)} + 5*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad [B]**

time = 8.42, size = 108, normalized size = 1.07

$$\frac{A a^3 c^2 \tan(e + f x) - \frac{a^3 c^2 \tan(e + f x)^3 (-B + A \operatorname{li}) \operatorname{li}}{3} + \frac{a^3 c^2 \tan(e + f x)^2 (A - B \operatorname{li}) \operatorname{li}}{2} + \frac{a^3 c^2 \tan(e + f x)^4 (A - B \operatorname{li}) \operatorname{li}}{4} + \frac{B a^3 c^2 \tan(e + f x)^5 \operatorname{li}}{5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)^2,x)

[Out]  $(A*a^3*c^2*\tan(e + f*x) - (a^3*c^2*\tan(e + f*x)^3*(A*1i - B)*1i)/3 + (a^3*c^2*\tan(e + f*x)^2*(A - B*1i)*1i)/2 + (a^3*c^2*\tan(e + f*x)^4*(A - B*1i)*1i)/4 + (B*a^3*c^2*\tan(e + f*x)^5*1i)/5)/f$

$$3.695 \quad \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx)) dx$$

Optimal. Leaf size=61

$$-\frac{a^3(iA - B)c(1 + i \tan(e + fx))^3}{3f} - \frac{a^3Bc(1 + i \tan(e + fx))^4}{4f}$$

[Out]  $-1/3*a^3*(I*A-B)*c*(1+I*\tan(f*x+e))^3/f-1/4*a^3*B*c*(1+I*\tan(f*x+e))^4/f$

Rubi [A]

time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$-\frac{a^3c(-B + iA)(1 + i \tan(e + fx))^3}{3f} - \frac{a^3Bc(1 + i \tan(e + fx))^4}{4f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x]),x]$

[Out]  $-1/3*(a^3*(I*A - B)*c*(1 + I*\text{Tan}[e + f*x])^3)/f - (a^3*B*c*(1 + I*\text{Tan}[e + f*x])^4)/(4*f)$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx)) dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx) dx, x\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left((A + iB)(a + iax)^2 - 3(A + iB)ax\right) dx, x\right)}{f}$$

$$= -\frac{a^3(iA - B)c(1 + i \tan(e + fx))^3}{3f}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 161 vs. 2(61) = 122.  
time = 1.37, size = 161, normalized size = 2.64

$$\frac{a^3 c \sec(e) \sec^4(e + fx) (3(2iA + B) \cos(e) + 3(iA + B) \cos(e + 2fx) + 3iA \cos(3e + 2fx) + 3B \cos(3e + 2fx) - 6A \sin(e) + 3iB \sin(e) + 5A \sin(e + 2fx) - iB \sin(e + 2fx) - 3A \sin(3e + 2fx) + 3iB \sin(3e + 2fx) + 2A \sin(3e + 4fx) - iB \sin(3e + 4fx))}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x]), x]

[Out] (a^3\*c\*Sec[e]\*Sec[e + f\*x]^4\*(3\*((2\*I)\*A + B)\*Cos[e] + 3\*(I\*A + B)\*Cos[e + 2\*f\*x] + (3\*I)\*A\*Cos[3\*e + 2\*f\*x] + 3\*B\*Cos[3\*e + 2\*f\*x] - 6\*A\*Sin[e] + (3\*I)\*B\*Sin[e] + 5\*A\*Sin[e + 2\*f\*x] - I\*B\*Sin[e + 2\*f\*x] - 3\*A\*Sin[3\*e + 2\*f\*x] + (3\*I)\*B\*Sin[3\*e + 2\*f\*x] + 2\*A\*Sin[3\*e + 4\*f\*x] - I\*B\*Sin[3\*e + 4\*f\*x])/(12\*f)

**Maple [A]**

time = 0.10, size = 63, normalized size = 1.03

method	result	size
derivativdivides	$\frac{a^3 c \left( -\frac{B(\tan^4(fx+e))}{4} - \frac{(-2iB+A)(\tan^3(fx+e))}{3} - \frac{(-2iA-B)(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	6
default	$\frac{a^3 c \left( -\frac{B(\tan^4(fx+e))}{4} - \frac{(-2iB+A)(\tan^3(fx+e))}{3} - \frac{(-2iA-B)(\tan^2(fx+e))}{2} + A \tan(fx+e) \right)}{f}$	6
norman	$\frac{A a^3 c \tan(fx+e)}{f} - \frac{(-2iB a^3 c + A a^3 c)(\tan^3(fx+e))}{3f} + \frac{(2iA a^3 c + B a^3 c)(\tan^2(fx+e))}{2f} - \frac{B a^3 c(\tan^4(fx+e))}{4f}$	9
risch	$\frac{4a^3 c(6iA e^{6i(fx+e)} + 6B e^{6i(fx+e)} + 12iA e^{4i(fx+e)} + 6B e^{4i(fx+e)} + 8iA e^{2i(fx+e)} + 4B e^{2i(fx+e)} + 2iA + B)}{3f(e^{2i(fx+e)} + 1)^4}$	1

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)), x, method=\_RETURNVERBOSE)

[Out]  $1/f*a^3*c*(-1/4*B*\tan(f*x+e)^4-1/3*(A-2*I*B)*\tan(f*x+e)^3-1/2*(-2*I*A-B)*\tan(f*x+e)^2+A*\tan(f*x+e))$

**Maxima** [A]

time = 0.55, size = 76, normalized size = 1.25

$$\frac{3Ba^3c\tan(fx+e)^4+4(A-2iB)a^3c\tan(fx+e)^3-6(2iA+B)a^3c\tan(fx+e)^2-12Aa^3c\tan(fx+e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="maxima")`

[Out]  $-1/12*(3*B*a^3*c*\tan(f*x+e)^4+4*(A-2*I*B)*a^3*c*\tan(f*x+e)^3-6*(2*I*A+B)*a^3*c*\tan(f*x+e)^2-12*A*a^3*c*\tan(f*x+e))/f$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs.  $2(53) = 106$ .

time = 5.61, size = 139, normalized size = 2.28

$$\frac{4(-iA-B)a^3ce^{6ifx+6ie}+6(-2iA-B)a^3ce^{4ifx+4ie}+4(-2iA-B)a^3ce^{2ifx+2ie}+(-2iA-B)a^3c}{3(fe^{8ifx+8ie}+4fe^{6ifx+6ie}+6fe^{4ifx+4ie}+4fe^{2ifx+2ie}+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algorithm="fricas")`

[Out]  $-4/3*(6*(-I*A-B)*a^3*c*e^{(6*I*f*x+6*I*e)}+6*(-2*I*A-B)*a^3*c*e^{(4*I*f*x+4*I*e)}+4*(-2*I*A-B)*a^3*c*e^{(2*I*f*x+2*I*e)}+(-2*I*A-B)*a^3*c)/(f*e^{(8*I*f*x+8*I*e)}+4*f*e^{(6*I*f*x+6*I*e)}+6*f*e^{(4*I*f*x+4*I*e)}+4*f*e^{(2*I*f*x+2*I*e)}+f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs.  $2(48) = 96$ .

time = 0.31, size = 218, normalized size = 3.57

$$\frac{8iAa^3c+4Ba^3c+(32iAa^3ce^{2ie}+16Ba^3ce^{2ie})e^{2ifx}+(48iAa^3ce^{4ie}+24Ba^3ce^{4ie})e^{4ifx}+(24iAa^3ce^{6ie}+24Ba^3ce^{6ie})e^{6ifx}}{3fe^{8ie}e^{8ifx}+12fe^{6ie}e^{6ifx}+18fe^{4ie}e^{4ifx}+12fe^{2ie}e^{2ifx}+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)`

[Out]  $(8*I*A*a^3*c+4*B*a^3*c+(32*I*A*a^3*c*\exp(2*I*e)+16*B*a^3*c*\exp(2*I*e))*\exp(2*I*f*x)+(48*I*A*a^3*c*\exp(4*I*e)+24*B*a^3*c*\exp(4*I*e))*\exp(4*I*f*x)+(24*I*A*a^3*c*\exp(6*I*e)+24*B*a^3*c*\exp(6*I*e))*\exp(6*I*f*x))/(3*f*\exp(8*I*e)*\exp(8*I*f*x)+12*f*\exp(6*I*e)*\exp(6*I*f*x)+18*f*\exp(4*I*e)*\exp(4*I*f*x)+12*f*\exp(2*I*e)*\exp(2*I*f*x)+3*f)$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(53) = 106.  
time = 0.69, size = 174, normalized size = 2.85

$$\frac{4(-6iAa^3ce^{(6i fx+6ie)} - 6Ba^3ce^{(6i fx+6ie)} - 12iAa^3ce^{(4i fx+4ie)} - 6Ba^3ce^{(4i fx+4ie)} - 8iAa^3ce^{(2i fx+2ie)} - 4Ba^3ce^{(2i fx+2ie)} - 2iAa^3c - Ba^3c)}{3(fe^{(8i fx+8ie)} + 4fe^{(6i fx+6ie)} + 6fe^{(4i fx+4ie)} + 4fe^{(2i fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e)),x, algorithm="giac")

[Out] 
$$-4/3*(-6*I*A*a^3*c*e^{(6*I*f*x + 6*I*e)} - 6*B*a^3*c*e^{(6*I*f*x + 6*I*e)} - 12*I*A*a^3*c*e^{(4*I*f*x + 4*I*e)} - 6*B*a^3*c*e^{(4*I*f*x + 4*I*e)} - 8*I*A*a^3*c*e^{(2*I*f*x + 2*I*e)} - 4*B*a^3*c*e^{(2*I*f*x + 2*I*e)} - 2*I*A*a^3*c - B*a^3*c)/(f*e^{(8*I*f*x + 8*I*e)} + 4*f*e^{(6*I*f*x + 6*I*e)} + 6*f*e^{(4*I*f*x + 4*I*e)} + 4*f*e^{(2*I*f*x + 2*I*e)} + f)$$

**Mupad [B]**

time = 8.48, size = 72, normalized size = 1.18

$$\frac{-\frac{Bca^3 \tan(e+fx)^4}{4} - \frac{c(A-B2i)a^3 \tan(e+fx)^3}{3} + \frac{c(B+A2i)a^3 \tan(e+fx)^2}{2} + Aca^3 \tan(e+fx)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i), x)

[Out] 
$$(A*a^3*c*\tan(e + f*x) + (a^3*c*\tan(e + f*x)^2*(A*2i + B))/2 - (a^3*c*\tan(e + f*x)^3*(A - B*2i))/3 - (B*a^3*c*\tan(e + f*x)^4)/4)/f$$



### 3.696 $\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) dx$

**Optimal.** Leaf size=110

$$4a^3(A - iB)x - \frac{4a^3(iA + B) \log(\cos(e + fx))}{f} - \frac{2a^3(A - iB) \tan(e + fx)}{f} + \frac{a(iA + B)(a + ia \tan(e + fx))^2}{2f}$$

[Out]  $4*a^3*(A - I*B)*x - 4*a^3*(I*A + B)*\ln(\cos(f*x + e))/f - 2*a^3*(A - I*B)*\tan(f*x + e)/f + 1/2*a*(I*A + B)*(a + I*a*\tan(f*x + e))^2/f + 1/3*B*(a + I*a*\tan(f*x + e))^3/f$

**Rubi [A]**

time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {3608, 3559, 3558, 3556}

$$-\frac{2a^3(A - iB) \tan(e + fx)}{f} - \frac{4a^3(B + iA) \log(\cos(e + fx))}{f} + 4a^3x(A - iB) + \frac{a(B + iA)(a + ia \tan(e + fx))^2}{2f} + \frac{B(a + ia \tan(e + fx))^3}{3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x]), x]$

[Out]  $4*a^3*(A - I*B)*x - (4*a^3*(I*A + B)*\text{Log}[\text{Cos}[e + f*x]])/f - (2*a^3*(A - I*B)*\text{Tan}[e + f*x])/f + (a*(I*A + B)*(a + I*a*\text{Tan}[e + f*x])^2)/(2*f) + (B*(a + I*a*\text{Tan}[e + f*x])^3)/(3*f)$

**Rule 3556**

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

**Rule 3558**

$\text{Int}[(a_. + (b_.)*\text{tan}[(c_.) + (d_.)*(x_.)])^2, x\_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[b^2*(\text{Tan}[c + d*x]/d), x]) /; \text{FreeQ}\{a, b, c, d\}, x]$

**Rule 3559**

$\text{Int}[(a_. + (b_.)*\text{tan}[(c_.) + (d_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[b*((a + b*\text{Tan}[c + d*x])^{(n - 1)}/(d*(n - 1))), x] + \text{Dist}[2*a, \text{Int}[(a + b*\text{Tan}[c + d*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[n, 1]$

**Rule 3608**

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[d*((a + b*\text{Tan}[e + f*x])^m/(f*m)), x] + \text{Dist}[(b*c + a*d)/b, \text{Int}[(a + b*\text{Tan}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e,$



norman	$(-4iB a^3 + 4A a^3) x - \frac{(iA a^3 + 3B a^3)(\tan^2(fx+e))}{2f} - \frac{(-4iB a^3 + 3A a^3) \tan(fx+e)}{f} - \frac{iB a^3 (\tan^3(fx+e))}{3f}$
risch	$\frac{8ia^3Be}{f} - \frac{8a^3Ae}{f} - \frac{2a^3(12iAe^{4i(fx+e)} + 24Be^{4i(fx+e)} + 21iAe^{2i(fx+e)} + 33Be^{2i(fx+e)} + 9iA + 13B)}{3f(e^{2i(fx+e)} + 1)^3} - \frac{4a^3 \ln(e^{2i(fx+e)} + 1)}{3f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)`

[Out] `1/f*a^3*(-1/3*I*B*tan(f*x+e)^3-1/2*I*A*tan(f*x+e)^2+4*I*B*tan(f*x+e)-3/2*B*tan(f*x+e)^2-3*A*tan(f*x+e)+1/2*(4*I*A+4*B)*ln(1+tan(f*x+e)^2)+(-4*I*B+4*A)*arctan(tan(f*x+e)))`

**Maxima** [A]

time = 0.50, size = 101, normalized size = 0.92

$$\frac{2iBa^3 \tan(fx+e)^3 + 3(iA+3B)a^3 \tan(fx+e)^2 - 24(fx+e)(A-iB)a^3 + 12(-iA-B)a^3 \log(\tan(fx+e)^2 + 1) + 6(3A-4iB)a^3 \tan(fx+e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x, algorithm="maxima")`

[Out] `-1/6*(2*I*B*a^3*tan(f*x + e)^3 + 3*(I*A + 3*B)*a^3*tan(f*x + e)^2 - 24*(f*x + e)*(A - I*B)*a^3 + 12*(-I*A - B)*a^3*log(tan(f*x + e)^2 + 1) + 6*(3*A - 4*I*B)*a^3*tan(f*x + e))/f`

**Fricas** [A]

time = 6.68, size = 184, normalized size = 1.67

$$\frac{2(12(iA+2B)a^3e^{4i(fx+4ie)} + 3(7iA+11B)a^3e^{2i(fx+2ie)} + (9iA+13B)a^3 + 6((iA+B)a^3e^{6i(fx+6ie)} + 3(iA+B)a^3e^{4i(fx+4ie)} + 3(iA+B)a^3e^{2i(fx+2ie)} + (iA+B)a^3 \log(e^{2i(fx+2ie)} + 1)))}{3(fe^{6i(fx+6ie)} + 3fe^{4i(fx+4ie)} + 3fe^{2i(fx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x, algorithm="fricas")`

[Out] `-2/3*(12*(I*A + 2*B)*a^3*e^(4*I*f*x + 4*I*e) + 3*(7*I*A + 11*B)*a^3*e^(2*I*f*x + 2*I*e) + (9*I*A + 13*B)*a^3 + 6*((I*A + B)*a^3*e^(6*I*f*x + 6*I*e) + 3*(I*A + B)*a^3*e^(4*I*f*x + 4*I*e) + 3*(I*A + B)*a^3*e^(2*I*f*x + 2*I*e) + (I*A + B)*a^3)*log(e^(2*I*f*x + 2*I*e) + 1)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)`

**Sympy** [A]

time = 0.38, size = 184, normalized size = 1.67

$$-\frac{4ia^3(A-iB) \log(e^{2ifx} + e^{-2ie})}{f} + \frac{-18iAa^3 - 26Ba^3 + (-42iAa^3e^{2ie} - 66Ba^3e^{2ie})e^{2ifx} + (-24iAa^3e^{4ie} - 48Ba^3e^{4ie})e^{4ifx}}{3fe^{6ie}e^{6ifx} + 9fe^{4ie}e^{4ifx} + 9fe^{2ie}e^{2ifx} + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x)`

[Out]  $-4*I*a**3*(A - I*B)*\log(\exp(2*I*f*x) + \exp(-2*I*e))/f + (-18*I*A*a**3 - 26*B*a**3 + (-42*I*A*a**3*\exp(2*I*e) - 66*B*a**3*\exp(2*I*e))*\exp(2*I*f*x) + (-24*I*A*a**3*\exp(4*I*e) - 48*B*a**3*\exp(4*I*e))*\exp(4*I*f*x))/(3*f*\exp(6*I*e))*\exp(6*I*f*x) + 9*f*\exp(4*I*e)*\exp(4*I*f*x) + 9*f*\exp(2*I*e)*\exp(2*I*f*x) + 3*f)$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(98) = 196.  
time = 0.65, size = 333, normalized size = 3.03

$\frac{2(18A^2e^{2Ie}\log(e^{2Ifx+1}) + 6Be^{2Ie}\log(e^{2Ifx+1}) + 18Ae^{2Ie}\log(e^{2Ifx+1}) + 18Be^{2Ie}\log(e^{2Ifx+1}) + 18Ae^{2Ie}\log(e^{2Ifx+1}) + 18Be^{2Ie}\log(e^{2Ifx+1}) + 12Ae^{2Ie}\log(e^{2Ifx+1}) + 24Be^{2Ie}\log(e^{2Ifx+1}) + 21Ae^{2Ie}\log(e^{2Ifx+1}) + 33Be^{2Ie}\log(e^{2Ifx+1}) + 6Ae^2\log(e^{2Ifx+1}) + 6Be^2\log(e^{2Ifx+1}) + 9Ae^2 + 13Be^2)}{3(f^{2Ie+3}e^{2Ifx+3} + f^{2Ie+3}e^{2Ifx+3})}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x, algorithm="giac")`

[Out]  $-2/3*(6*I*A*a^3*e^{(6*I*f*x + 6*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 6*B*a^3*e^{(6*I*f*x + 6*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 18*I*A*a^3*e^{(4*I*f*x + 4*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 18*B*a^3*e^{(4*I*f*x + 4*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 18*I*A*a^3*e^{(2*I*f*x + 2*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 18*B*a^3*e^{(2*I*f*x + 2*I*e)}*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 12*I*A*a^3*e^{(4*I*f*x + 4*I*e)} + 24*B*a^3*e^{(4*I*f*x + 4*I*e)} + 21*I*A*a^3*e^{(2*I*f*x + 2*I*e)} + 33*B*a^3*e^{(2*I*f*x + 2*I*e)} + 6*I*A*a^3*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 6*B*a^3*\log(e^{(2*I*f*x + 2*I*e)} + 1) + 9*I*A*a^3 + 13*B*a^3)/(f*e^{(6*I*f*x + 6*I*e)} + 3*f*e^{(4*I*f*x + 4*I*e)} + 3*f*e^{(2*I*f*x + 2*I*e)} + f)$

**Mupad** [B]

time = 8.91, size = 125, normalized size = 1.14

$$-\frac{\tan(e+fx)^2\left(\frac{Ba^3}{2} + \frac{a^3(2B+A1i)}{2}\right)}{f} + \frac{\ln(\tan(e+fx)+1i)(4Ba^3+Aa^34i)}{f} + \frac{\tan(e+fx)(Ba^31i-a^3(2A-B1i)+a^3(2B+A1i)1i)}{f} - \frac{Ba^3\tan(e+fx)^31i}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3,x)`

[Out]  $(\log(\tan(e + f*x) + 1i)*(A*a^3*4i + 4*B*a^3))/f - (\tan(e + f*x)^2*((B*a^3)/2 + (a^3*(A*1i + 2*B))/2))/f + (\tan(e + f*x)*(B*a^3*1i - a^3*(2*A - B*1i) + a^3*(A*1i + 2*B)*1i))/f - (B*a^3*\tan(e + f*x)^3*1i)/(3*f)$

$$3.697 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{c-ic \tan(e+fx)} dx$$

**Optimal.** Leaf size=119

$$-\frac{4a^3(A-2iB)x}{c} + \frac{4a^3(iA+2B) \log(\cos(e+fx))}{cf} + \frac{a^3(A-4iB) \tan(e+fx)}{cf} + \frac{a^3B \tan^2(e+fx)}{2cf} + \frac{4a^3}{cf(i+t)}$$

[Out]  $-4*a^3*(A-2*I*B)*x/c+4*a^3*(I*A+2*B)*\ln(\cos(f*x+e))/c/f+a^3*(A-4*I*B)*\tan(f*x+e)/c/f+1/2*a^3*B*\tan(f*x+e)^2/c/f+4*a^3*(A-I*B)/c/f/(I+\tan(f*x+e))$

**Rubi [A]**

time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ ,

Rules used = {3669, 78}

$$\frac{a^3(A-4iB) \tan(e+fx)}{cf} + \frac{4a^3(A-iB)}{cf(\tan(e+fx)+i)} + \frac{4a^3(2B+iA) \log(\cos(e+fx))}{cf} - \frac{4a^3x(A-2iB)}{c} + \frac{a^3B \tan^2(e+fx)}{2cf}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])}, x]$

[Out]  $(-4*a^3*(A - (2*I)*B)*x)/c + (4*a^3*(I*A + 2*B)*\text{Log}[\text{Cos}[e + f*x]])/(c*f) + (a^3*(A - (4*I)*B)*\text{Tan}[e + f*x])/(c*f) + (a^3*B*\text{Tan}[e + f*x]^2)/(2*c*f) + (4*a^3*(A - I*B))/(c*f*(I + \text{Tan}[e + f*x]))$

Rule 78

$\text{Int}[\frac{(a + b*x)^m * ((c + d*x)^n * ((e + f*x)^p + (e + f*x)^q)]}{(c + d*x)^n}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[\frac{(a + b*\text{tan}[e + f*x])^m * ((c + d*\text{tan}[e + f*x])^n * (A + B*\text{tan}[e + f*x]))}{(c + d*\text{tan}[e + f*x])^n}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1} * (c + d*x)^{n-1} * (A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{c - ic \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^2} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{a^2(A-4iB)}{c^2} + \frac{a^2 Bx}{c^2} - \frac{4a^2(A-iB)}{c^2(i+x)^2} - \frac{4ia^2(A-2iB)}{c^2(i+x)} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{4a^3(A-2iB)x}{c} + \frac{4a^3(iA+2B) \log(\cos(e + fx))}{cf} + \frac{a^3}{cf}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 944 vs. 2(119) = 238.  
time = 7.24, size = 944, normalized size = 7.93

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x]),x]

[Out] ((A - I\*B)\*Cos[2\*f\*x]\*Cos[e + f\*x]^4\*(((2\*I)\*Cos[e])/c - (2\*Sin[e])/c)\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A \*Cos[e + f\*x] + B\*Sin[e + f\*x])) + (Cos[e + f\*x]^2\*((B\*Cos[3\*e])/c - ((I/2)\*B\*Sin[3\*e])/c)\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])) + ((A - (2\*I)\*B)\*Cos[e + f\*x]^4\*(((4\*f\*x)\*Cos[3\*e])/c + ((4\*I)\*f\*x\*Sin[3\*e])/c)\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])) + ((I\*A + 2\*B)\*Cos[e + f\*x]^4\*((2\*Cos[3\*e])\*Log[Cos[e + f\*x]^2])/c - ((2\*I)\*Log[Cos[e + f\*x]^2]\*Sin[3\*e])/c)\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])) + (Cos[e + f\*x]^3\*(Cos[3\*e]/c - (I\*Sin[3\*e])/c)\*(A\*Sin[f\*x] - (4\*I)\*B\*Sin[f\*x])\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(f\*(Cos[e/2] - Sin[e/2])\*(Cos[e/2] + Sin[e/2])\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])) + ((A - I\*B)\*Cos[e + f\*x]^4\*((2\*Cos[e])/c - ((2\*I)\*Sin[e])/c)\*Sin[2\*f\*x]\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])) + (x\*Cos[e + f\*x]^4\*((2\*A\*Cos[e])/c - ((4\*I)\*B\*Cos[e])/c - (2\*A\*Cos[e]^3)/c + ((4\*I)\*B\*Cos[e]^3)/c - ((4\*I)\*A\*Sin[e])/c - (8\*B\*Sin[e])/c + ((8\*I)\*A\*Cos[e]^2\*Sin[e])/c + (16\*B\*Cos[e]^2\*Sin[e])/c + (12\*A\*Cos[e]\*Sin[e]^2)/c - ((24\*I)\*B\*Cos[e]\*Sin[e]^2)/c - ((8\*I)\*A\*Sin[e]^3)/c - (16\*B\*Sin[e]^3)/c - (2\*A\*Sin[e]\*Tan[e])/c + ((4\*I)\*B\*Sin[e]\*Tan[e])/c - (2\*A\*Sin[e]^3\*Tan[e])/c + ((4\*I)\*B\*Sin[e]^3\*Tan[e])/c - I\*(A - (2\*I)\*B)\*((4\*Cos[3\*e])/c - ((4\*I)\*Sin[3\*e])/c)\*Tan[e])\*(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/((Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [A]**

time = 0.21, size = 81, normalized size = 0.68

method	result
derivativedivides	$\frac{a^3 \left( A \tan(fx+e) - 4iB \tan(fx+e) + \frac{B(\tan^2(fx+e))}{2} + (-4iA - 8B) \ln(i + \tan(fx+e)) - \frac{4iB - 4A}{i + \tan(fx+e)} \right)}{fc}$
default	$\frac{a^3 \left( A \tan(fx+e) - 4iB \tan(fx+e) + \frac{B(\tan^2(fx+e))}{2} + (-4iA - 8B) \ln(i + \tan(fx+e)) - \frac{4iB - 4A}{i + \tan(fx+e)} \right)}{fc}$
risch	$-\frac{2e^{2i(fx+e)} B a^3}{cf} - \frac{2ie^{2i(fx+e)} A a^3}{cf} - \frac{16ia^3 B e}{fc} + \frac{8a^3 A e}{fc} + \frac{2a^3 (iA e^{2i(fx+e)} + 5B e^{2i(fx+e)} + iA + 4B)}{cf(e^{2i(fx+e)} + 1)^2} + \frac{8a^3}{c}$
norman	$\frac{\frac{(-4iB a^3 + A a^3)(\tan^3(fx+e))}{cf} + \frac{(-8iB a^3 + 5A a^3) \tan(fx+e)}{cf} - \frac{4(-2iB a^3 + A a^3)x}{c} - \frac{8iA a^3 + 9B a^3}{2cf} - \frac{4(-2iB a^3 + A a^3)x(\tan^2(fx+e))}{c}}{1 + \tan^2(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*a^3/c*(A*tan(f*x+e)-4*I*B*tan(f*x+e)+1/2*B*tan(f*x+e)^2+(-4*I*A-8*B)*ln
(I+tan(f*x+e))-(4*I*B-4*A)/(I+tan(f*x+e)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 5.61, size = 172, normalized size = 1.45

$$\frac{2((iA+B)a^3 e^{6i(fx+4ie)} + 2(iA+B)a^3 e^{4i(fx+4ie)} - 4Ba^3 e^{2i(fx+2ie)} + (-iA-4B)a^3 + 2((-iA-2B)a^3 e^{4i(fx+4ie)} + 2(-iA-2B)a^3 e^{2i(fx+2ie)} + (-iA-2B)a^3 \log(e^{2i(fx+2ie)} + 1))}{cfe^{4i(fx+4ie)} + 2cfe^{2i(fx+2ie)} + cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

```
[Out] -2*((I*A + B)*a^3*e^(6*I*f*x + 6*I*e) + 2*(I*A + B)*a^3*e^(4*I*f*x + 4*I*e)
- 4*B*a^3*e^(2*I*f*x + 2*I*e) + (-I*A - 4*B)*a^3 + 2*((-I*A - 2*B)*a^3*e^(
```

$4*I*f*x + 4*I*e) + 2*(-I*A - 2*B)*a^3*e^{(2*I*f*x + 2*I*e)} + (-I*A - 2*B)*a^3*\log(e^{(2*I*f*x + 2*I*e)} + 1))/(c*f*e^{(4*I*f*x + 4*I*e)} + 2*c*f*e^{(2*I*f*x + 2*I*e)} + c*f)$

**Sympy [A]**

time = 0.50, size = 206, normalized size = 1.73

$$\frac{4ia^3(A - 2iB)\log(e^{2ifx} + e^{-2ie})}{cf} + \frac{2iAa^3 + 8Ba^3 + (2iAa^3e^{2ie} + 10Ba^3e^{2ie})e^{2ifx}}{cfe^{4ie}e^{4ifx} + 2cfe^{2ie}e^{2ifx} + cf} + \begin{cases} \frac{(-2iAa^3e^{2ie} - 2Ba^3e^{2ie})e^{2ifx}}{cf} & \text{for } cf \neq 0 \\ \frac{x(4Aa^3e^{2ie} - 4iBa^3e^{2ie})}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e)),x)

[Out]  $4*I*a**3*(A - 2*I*B)*\log(\exp(2*I*f*x) + \exp(-2*I*e))/(c*f) + (2*I*A*a**3 + 8*B*a**3 + (2*I*A*a**3*\exp(2*I*e) + 10*B*a**3*\exp(2*I*e))*\exp(2*I*f*x))/(c*f*\exp(4*I*e)*\exp(4*I*f*x) + 2*c*f*\exp(2*I*e)*\exp(2*I*f*x) + c*f) + \text{Piecewise}((( -2*I*A*a**3*\exp(2*I*e) - 2*B*a**3*\exp(2*I*e))*\exp(2*I*f*x)/(c*f), \text{Ne}(c*f, 0)), (x*(4*A*a**3*\exp(2*I*e) - 4*I*B*a**3*\exp(2*I*e))/c, \text{True}))$

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs.  $2(111) = 222$ .

time = 0.73, size = 322, normalized size = 2.71

$$\frac{2 \left( \frac{2(-iAa^3 - 2Ba^3)\log(\tan(\frac{1}{2}fx + \frac{1}{2}e))}{c} + \frac{4i(Aa^3 + 2Ba^3)\log(\tan(\frac{1}{2}fx + \frac{1}{2}e))}{c} + \frac{2(-iAa^3 - 2Ba^3)\log(\tan(\frac{1}{2}fx + \frac{1}{2}e))}{c} + \frac{5Aa^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 8Ba^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 2Aa^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 7Ba^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 10Aa^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 14Ba^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 2iAa^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 7Ba^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 5Aa^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 8Ba^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e) + \tan(\frac{1}{2}fx + \frac{1}{2}e) - \tan(\frac{1}{2}fx + \frac{1}{2}e))} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e)),x, algorithm="giac")

[Out]  $-2*(2*(-I*A*a^3 - 2*B*a^3)*\log(\tan(1/2*f*x + 1/2*e) + 1)/c + 4*(I*A*a^3 + 2*B*a^3)*\log(\tan(1/2*f*x + 1/2*e) + I)/c + 2*(-I*A*a^3 - 2*B*a^3)*\log(\tan(1/2*f*x + 1/2*e) - 1)/c + (5*A*a^3*\tan(1/2*f*x + 1/2*e)^5 - 8*I*B*a^3*\tan(1/2*f*x + 1/2*e)^5 + 2*I*A*a^3*\tan(1/2*f*x + 1/2*e)^4 + 7*B*a^3*\tan(1/2*f*x + 1/2*e)^4 - 10*A*a^3*\tan(1/2*f*x + 1/2*e)^3 + 14*I*B*a^3*\tan(1/2*f*x + 1/2*e)^3 - 2*I*A*a^3*\tan(1/2*f*x + 1/2*e)^2 - 7*B*a^3*\tan(1/2*f*x + 1/2*e)^2 + 5*A*a^3*\tan(1/2*f*x + 1/2*e) - 8*I*B*a^3*\tan(1/2*f*x + 1/2*e))/((\tan(1/2*f*x + 1/2*e)^3 + I*\tan(1/2*f*x + 1/2*e)^2 - \tan(1/2*f*x + 1/2*e) - I)^2*c))/f$

**Mupad [B]**

time = 9.00, size = 139, normalized size = 1.17

$$\frac{Ba^3 \tan(e + fx)^2}{2cf} + \frac{4Aa^3 - Ba^3 \operatorname{si} + Ba^3 4i}{f(\tan(e + fx) + i)} - \frac{\tan(e + fx) \left( \frac{Ba^3 2i}{c} + \frac{a^3(2B + A i) i}{c} \right)}{f} - \frac{\ln(\tan(e + fx) + i) \left( \frac{8Ba^3}{c} + \frac{Aa^3 4i}{c} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3)/(c - c\*tan(e + f\*x)\*1i),x)

[Out] ((4\*A\*a^3 - B\*a^3\*8i)/c + (B\*a^3\*4i)/c)/(f\*(tan(e + f\*x) + 1i)) - (log(tan(e + f\*x) + 1i)\*((A\*a^3\*4i)/c + (8\*B\*a^3)/c))/f - (tan(e + f\*x)\*((B\*a^3\*2i)/c + (a^3\*(A\*1i + 2\*B)\*1i)/c))/f + (B\*a^3\*tan(e + f\*x)^2)/(2\*c\*f)

$$3.698 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=123

$$\frac{a^3(A-5iB)x}{c^2} - \frac{a^3(iA+5B) \log(\cos(e+fx))}{c^2 f} + \frac{ia^3 B \tan(e+fx)}{c^2 f} + \frac{2a^3(iA+B)}{c^2 f(i+\tan(e+fx))^2} - \frac{4a^3(A-2iB)}{c^2 f(i+\tan(e+fx))}$$

[Out]  $a^3*(A-5*I*B)*x/c^2 - a^3*(I*A+5*B)*\ln(\cos(f*x+e))/c^2/f + I*a^3*B*\tan(f*x+e)/c^2/f + 2*a^3*(I*A+B)/c^2/f/(I+\tan(f*x+e))^2 - 4*a^3*(A-2*I*B)/c^2/f/(I+\tan(f*x+e))$

**Rubi [A]**

time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{4a^3(A-2iB)}{c^2 f(\tan(e+fx)+i)} + \frac{2a^3(B+iA)}{c^2 f(\tan(e+fx)+i)^2} - \frac{a^3(5B+iA) \log(\cos(e+fx))}{c^2 f} + \frac{a^3 x(A-5iB)}{c^2} + \frac{ia^3 B \tan(e+fx)}{c^2 f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^2, x]$

[Out]  $(a^3*(A - (5*I)*B)*x)/c^2 - (a^3*(I*A + 5*B)*\text{Log}[\text{Cos}[e + f*x]])/(c^2*f) + (I*a^3*B*\text{Tan}[e + f*x])/(c^2*f) + (2*a^3*(I*A + B))/(c^2*f*(I + \text{Tan}[e + f*x])^2) - (4*a^3*(A - (2*I)*B))/(c^2*f*(I + \text{Tan}[e + f*x]))$

**Rule 78**

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^(n_.))*((e_. + (f_.)*(x_.))^(p_.), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^(m_.))*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*((c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^(n_.), x\_Symbol] :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

**Rubi steps**

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^2} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^3} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{ia^2 B}{c^3} - \frac{4ia^2(A-iB)}{c^3(i+x)^3} + \frac{4a^2(A-2iB)}{c^3(i+x)^2} + \frac{a^2(iA+5B)}{c^3(i+x)} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{a^3(A - 5iB)x}{c^2} - \frac{a^3(iA + 5B) \log(\cos(e + fx))}{c^2 f} + \frac{ia^3 B}{c^2}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 285 vs.  $2(123) = 246$ .  
time = 5.34, size = 285, normalized size = 2.32

$\frac{a^3(2(A+3B)\cos(2f)\cos(e+fx)\cos(i-\sin(i))+(A-3B)\cos(4f)\cos(e+fx)(-\cos(i)+\sin(i))+2(A-5iB)f\cos(e+fx)\cos(3i)-i\sin(3i)-i(A-5iB)\cos(e+fx)\log(\cos^2(e+fx))\cos(3i)+2B\cos(i)\cos(3i)+\sin(3i)\sin(f)+(-A-5iB)\cos(e+fx)(-2\cos(i)+2\sin(i)\sin(2f))+(A-3B)\cos(e+fx)\cos(i)+\sin(i)\cos(4f)\cos(e+fx)+i\sin(i)(A+B\tan(e+fx))}{2f(\cos(f)+\sin(f))^2(A\cos(e+fx)+B\sin(e+fx))}$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^2,x]

[Out] (a^3\*(2\*(I\*A + 3\*B)\*Cos[2\*f\*x]\*Cos[e + f\*x]\*(Cos[e] - I\*Sin[e]) + (A - I\*B)\*Cos[4\*f\*x]\*Cos[e + f\*x]\*((-I)\*Cos[e] + Sin[e]) + 2\*(A - (5\*I)\*B)\*f\*x\*Cos[e + f\*x]\*(Cos[3\*e] - I\*Sin[3\*e]) - I\*(A - (5\*I)\*B)\*Cos[e + f\*x]\*Log[Cos[e + f\*x]^2]\*(Cos[3\*e] - I\*Sin[3\*e]) + 2\*B\*Sec[e]\*(I\*Cos[3\*e] + Sin[3\*e])\*Sin[f\*x] + (A - (3\*I)\*B)\*Cos[e + f\*x]\*(-2\*Cos[e] + (2\*I)\*Sin[e])\*Sin[2\*f\*x] + (A - I\*B)\*Cos[e + f\*x]\*(Cos[e] + I\*Sin[e])\*Sin[4\*f\*x]\*(Cos[e + f\*x] + I\*Sin[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(2\*c^2\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [A]**

time = 0.30, size = 83, normalized size = 0.67

method	result
derivativedivides	$\frac{a^3 \left( iB \tan(fx+e) - \frac{-8iB+4A}{i+\tan(fx+e)} + (iA+5B) \ln(i+\tan(fx+e)) - \frac{-4iA-4B}{2(i+\tan(fx+e))^2} \right)}{f c^2}$
default	$\frac{a^3 \left( iB \tan(fx+e) - \frac{-8iB+4A}{i+\tan(fx+e)} + (iA+5B) \ln(i+\tan(fx+e)) - \frac{-4iA-4B}{2(i+\tan(fx+e))^2} \right)}{f c^2}$
risch	$-\frac{e^{4i(fx+e)} B a^3}{2c^2 f} - \frac{ie^{4i(fx+e)} A a^3}{2c^2 f} + \frac{3e^{2i(fx+e)} B a^3}{c^2 f} + \frac{ie^{2i(fx+e)} A a^3}{c^2 f} + \frac{10ia^3 B e}{c^2 f} - \frac{2a^3 A e}{c^2 f} - \frac{2B a^3}{f c^2 (e^{2i(fx+e)})}$
norman	$\frac{\left( -5iB a^3 + A a^3 \right) x}{c} + \frac{2iA a^3 + 6B a^3}{cf} + \frac{\left( -5iB a^3 + A a^3 \right) x (\tan^4(fx+e))}{c} + \frac{iB a^3 (\tan^5(fx+e))}{cf} + \frac{2 \left( -5iB a^3 + A a^3 \right) x (\tan^2(fx+e))}{c} + \frac{2 \left( -5iB a^3 + A a^3 \right) x (\tan^2(fx+e))}{c(1+\tan^2(fx+e))^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f\*a^3/c^2\*(I\*B\*tan(f\*x+e)-(-8\*I\*B+4\*A)/(I+tan(f\*x+e))+(I\*A+5\*B)\*ln(I+tan(  
f\*x+e))-1/2\*(-4\*I\*A-4\*B)/(I+tan(f\*x+e))^2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, alg  
orithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 4.18, size = 144, normalized size = 1.17

$$\frac{(-iA - B)a^3 e^{(6i f x + 6i e)} + (iA + 5B)a^3 e^{(4i f x + 4i e)} - 2(-iA - 3B)a^3 e^{(2i f x + 2i e)} - 4Ba^3 - 2((iA + 5B)a^3 e^{(2i f x + 2i e)} + (iA + 5B)a^3) \log(e^{(2i f x + 2i e)} + 1)}{2(c^2 f e^{(2i f x + 2i e)} + c^2 f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^2,x, alg  
orithm="fricas")

[Out] 1/2\*((-I\*A - B)\*a^3\*e^(6\*I\*f\*x + 6\*I\*e) + (I\*A + 5\*B)\*a^3\*e^(4\*I\*f\*x + 4\*I\*  
e) - 2\*(-I\*A - 3\*B)\*a^3\*e^(2\*I\*f\*x + 2\*I\*e) - 4\*B\*a^3 - 2\*((I\*A + 5\*B)\*a^3\*  
e^(2\*I\*f\*x + 2\*I\*e) + (I\*A + 5\*B)\*a^3)\*log(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c^2\*f  
\*e^(2\*I\*f\*x + 2\*I\*e) + c^2\*f)

**Sympy [A]**

time = 0.54, size = 236, normalized size = 1.92

$$-\frac{2Ba^3}{c^2 f e^{2ie} e^{2ifx} + c^2 f} - \frac{ia^3(A - 5iB) \log(e^{2ifx} + e^{-2ie})}{c^2 f} + \begin{cases} \frac{(2iAa^3 c^2 f e^{2ie} + 6Ba^3 c^2 f e^{2ie}) e^{2ifx} + (-iAa^3 c^2 f e^{4ie} - Ba^3 c^2 f e^{4ie}) e^{4ifx}}{2c^4 f^2} & \text{for } c^4 f^2 \neq 0 \\ \frac{x(2Aa^3 e^{4ie} - 2Aa^3 e^{2ie} - 2iBa^3 e^{4ie} + 6iBa^3 e^{2ie})}{c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*2,x)

[Out] -2\*B\*a\*\*3/(c\*\*2\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + c\*\*2\*f) - I\*a\*\*3\*(A - 5\*I\*B)\*lo  
g(exp(2\*I\*f\*x) + exp(-2\*I\*e))/(c\*\*2\*f) + Piecewise((((2\*I\*A\*a\*\*3\*c\*\*2\*f\*exp  
(2\*I\*e) + 6\*B\*a\*\*3\*c\*\*2\*f\*exp(2\*I\*e))\*exp(2\*I\*f\*x) + (-I\*A\*a\*\*3\*c\*\*2\*f\*exp(  
4\*I\*e) - B\*a\*\*3\*c\*\*2\*f\*exp(4\*I\*e))\*exp(4\*I\*f\*x))/(2\*c\*\*4\*f\*\*2), Ne(c\*\*4\*f\*\*



$$3.699 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=129

$$\frac{ia^3 Bx}{c^3} + \frac{a^3 B \log(\cos(e+fx))}{c^3 f} - \frac{a^3(iA+B)(1+i \tan(e+fx))^3}{6c^3 f(1-i \tan(e+fx))^3} - \frac{2a^3 B}{c^3 f(i+\tan(e+fx))^2} - \frac{4ia^3 B}{c^3 f(i+\tan(e+fx))}$$

[Out]  $I*a^3*B*x/c^3+a^3*B*\ln(\cos(f*x+e))/c^3/f-1/6*a^3*(I*A+B)*(1+I*\tan(f*x+e))^3/c^3/f/(1-I*\tan(f*x+e))^3-2*a^3*B/c^3/f/(I+\tan(f*x+e))^2-4*I*a^3*B/c^3/f/(I+\tan(f*x+e))$

**Rubi [A]**

time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 79, 45}

$$-\frac{a^3(B+ia)(1+i \tan(e+fx))^3}{6c^3 f(1-i \tan(e+fx))^3} - \frac{4ia^3 B}{c^3 f(\tan(e+fx)+i)} - \frac{2a^3 B}{c^3 f(\tan(e+fx)+i)^2} + \frac{a^3 B \log(\cos(e+fx))}{c^3 f} + \frac{ia^3 Bx}{c^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^3, x]$

[Out]  $(I*a^3*B*x)/c^3 + (a^3*B*\text{Log}[\text{Cos}[e + f*x]])/(c^3*f) - (a^3*(I*A + B)*(1 + I*\text{Tan}[e + f*x])^3)/(6*c^3*f*(1 - I*\text{Tan}[e + f*x])^3) - (2*a^3*B)/(c^3*f*(I + \text{Tan}[e + f*x])^2) - ((4*I)*a^3*B)/(c^3*f*(I + \text{Tan}[e + f*x]))$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 79

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x\_Symbol] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ (!\text{LtQ}[n, -1] \ || \ \text{IntegerQ}[p] \ || \ !(\text{IntegerQ}[n] \ || \ !(\text{EqQ}[e, 0] \ || \ !(\text{EqQ}[c, 0] \ || \ \text{LtQ}[p, n])))$

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^2(A+Bx)}{(c-icx)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{a^3(iA + B)(1 + i \tan(e + fx))^3}{6c^3 f(1 - i \tan(e + fx))^3} + \frac{(iaB) \text{Subst}\left(\int \frac{(a+i}{c-i}\right)}{f}$$

$$= -\frac{a^3(iA + B)(1 + i \tan(e + fx))^3}{6c^3 f(1 - i \tan(e + fx))^3} + \frac{(iaB) \text{Subst}\left(\int (-\right)}{f}$$

$$= \frac{ia^3 Bx}{c^3} + \frac{a^3 B \log(\cos(e + fx))}{c^3 f} - \frac{a^3(iA + B)(1 + i \tan(e + fx))^3}{6c^3 f(1 - i \tan(e + fx))^3}$$

**Mathematica [A]**

time = 1.65, size = 167, normalized size = 1.29

$$\frac{a^3(-3B \cos(e + fx) + \cos(3(e + fx))(-iA - B + 6iBfx + 3B \log(\cos^2(e + fx))) + 9iB \sin(e + fx) + A \sin(3(e + fx)) - iB \sin(3(e + fx)) + 6Bfx \sin(3(e + fx)) - 3iB \log(\cos^2(e + fx)) \sin(3(e + fx))) (\cos(3(e + 2fx)) + i \sin(3(e + 2fx)))}{6c^3 f(\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
f*x])^3,x]
```

```
[Out] (a^3*(-3*B*Cos[e + f*x] + Cos[3*(e + f*x)]*((-I)*A - B + (6*I)*B*f*x + 3*B*
Log[Cos[e + f*x]^2]) + (9*I)*B*Sin[e + f*x] + A*Sin[3*(e + f*x)] - I*B*Sin[
3*(e + f*x)] + 6*B*f*x*Sin[3*(e + f*x)] - (3*I)*B*Log[Cos[e + f*x]^2]*Sin[3
*(e + f*x)]*(Cos[3*(e + 2*f*x)] + I*Sin[3*(e + 2*f*x)]))/(6*c^3*f*(Cos[f*x
] + I*Sin[f*x])^3)
```

**Maple [A]**

time = 0.30, size = 88, normalized size = 0.68

method	result
derivativedivides	$\frac{a^3 \left( -\frac{5iB-A}{i+\tan(fx+e)} - \frac{-4iB+4A}{3(i+\tan(fx+e))^3} - \frac{4iA+8B}{2(i+\tan(fx+e))^2} - B \ln(i+\tan(fx+e)) \right)}{f c^3}$

default	$a^3 \left( -\frac{5iB-A}{i+\tan(fx+e)} - \frac{-4iB+4A}{3(i+\tan(fx+e))^3} - \frac{4iA+8B}{2(i+\tan(fx+e))^2} - B \ln(i+\tan(fx+e)) \right)$
risch	$-\frac{e^{6i(fx+e)} B a^3}{6c^3 f} - \frac{ie^{6i(fx+e)} A a^3}{6c^3 f} + \frac{B a^3 e^{4i(fx+e)}}{2c^3 f} - \frac{B a^3 e^{2i(fx+e)}}{c^3 f} - \frac{2iB a^3 e}{c^3 f} + \frac{B a^3 \ln(e^{2i(fx+e)}+1)}{c^3 f}$
norman	$\frac{\frac{(-iB a^3 + A a^3) \tan(fx+e)}{cf} + \frac{(-5iB a^3 + A a^3) (\tan^5(fx+e))}{cf} + \frac{iB a^3 x}{c} + \frac{iB a^3 x (\tan^6(fx+e))}{c} - \frac{iA a^3 + 7B a^3}{3cf} - \frac{2(iB a^3 + 5A a^3) (\tan^2(fx+e))}{3cf}}{c^2(1+\tan^2(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RE  
TURNVERBOSE)`

[Out] `1/f*a^3/c^3*(-(-A+5*I*B)/(I+tan(f*x+e))-1/3*(-4*I*B+4*A)/(I+tan(f*x+e))^3-1  
/2*(4*I*A+8*B)/(I+tan(f*x+e))^2-B*ln(I+tan(f*x+e)))`

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, alg  
orithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 5.15, size = 81, normalized size = 0.63

$$\frac{(-iA - B)a^3 e^{(6i fx + 6ie)} + 3Ba^3 e^{(4i fx + 4ie)} - 6Ba^3 e^{(2i fx + 2ie)} + 6Ba^3 \log(e^{(2i fx + 2ie)} + 1)}{6c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, alg  
orithm="fricas")`

[Out] `1/6*((-I*A - B)*a^3*e^(6*I*f*x + 6*I*e) + 3*B*a^3*e^(4*I*f*x + 4*I*e) - 6*B  
*a^3*e^(2*I*f*x + 2*I*e) + 6*B*a^3*log(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)`

**Sympy** [A]

time = 0.48, size = 212, normalized size = 1.64

$$\frac{Ba^3 \log(e^{2ifx} + e^{-2ie})}{c^3 f} + \begin{cases} \frac{6Ba^3 c^6 f^2 e^{4ie} e^{4ifx} - 12Ba^3 c^6 f^2 e^{2ie} e^{2ifx} + (-2iAa^3 c^6 f^2 e^{6ie} - 2Ba^3 c^6 f^2 e^{6ie}) e^{6ifx}}{12c^9 f^3} & \text{for } c^9 f^3 \neq 0 \\ \frac{x(Aa^3 e^{6ie} - iBa^3 e^{6ie} + 2iBa^3 e^{4ie} - 2iBa^3 e^{2ie})}{c^3} & \text{otherwise} \end{cases}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*3,x)

[Out]  $B*a**3*\log(\exp(2*I*f*x) + \exp(-2*I*e))/(c**3*f) + \text{Piecewise}(((6*B*a**3*c**6*f**2*\exp(4*I*e)*\exp(4*I*f*x) - 12*B*a**3*c**6*f**2*\exp(2*I*e)*\exp(2*I*f*x) + (-2*I*A*a**3*c**6*f**2*\exp(6*I*e) - 2*B*a**3*c**6*f**2*\exp(6*I*e))*\exp(6*I*f*x))/(12*c**9*f**3), \text{Ne}(c**9*f**3, 0)), (x*(A*a**3*\exp(6*I*e) - I*B*a**3*\exp(6*I*e) + 2*I*B*a**3*\exp(4*I*e) - 2*I*B*a**3*\exp(2*I*e))/c**3, \text{True}))$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 255 vs.  $2(118) = 236$ .

time = 1.01, size = 255, normalized size = 1.98

$$\frac{30 B a^3 \log(\tan(\frac{1}{2} f x + \frac{1}{2} e)) - 60 B a^3 \log(\tan(\frac{1}{2} f x + \frac{1}{2} e)) + 30 B a^3 \log(\tan(\frac{1}{2} f x + \frac{1}{2} e)) + 147 B a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^6 - 60 A a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^5 + 942 B a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^4 - 2445 B a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^3 + 200 A a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 - 3620 B a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 + 2445 B a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e) - 60 A a^3 \tan(\frac{1}{2} f x + \frac{1}{2} e) - 147 B a^3}{30 f \tan(\frac{1}{2} f x + \frac{1}{2} e)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x, algorithm="giac")

[Out]  $1/30*(30*B*a^3*\log(\tan(1/2*f*x + 1/2*e) + 1)/c^3 - 60*B*a^3*\log(\tan(1/2*f*x + 1/2*e) + I)/c^3 + 30*B*a^3*\log(\tan(1/2*f*x + 1/2*e) - 1)/c^3 + (147*B*a^3*\tan(1/2*f*x + 1/2*e)^6 - 60*A*a^3*\tan(1/2*f*x + 1/2*e)^5 + 942*I*B*a^3*\tan(1/2*f*x + 1/2*e)^4 - 2445*B*a^3*\tan(1/2*f*x + 1/2*e)^3 + 200*A*a^3*\tan(1/2*f*x + 1/2*e)^2 - 3620*I*B*a^3*\tan(1/2*f*x + 1/2*e)^2 + 2445*B*a^3*\tan(1/2*f*x + 1/2*e) - 60*A*a^3*\tan(1/2*f*x + 1/2*e) + 942*I*B*a^3*\tan(1/2*f*x + 1/2*e) - 147*B*a^3)/(c^3*(\tan(1/2*f*x + 1/2*e) + I)^6))/f$

**Mupad** [B]

time = 8.97, size = 141, normalized size = 1.09

$$\frac{a^3 (15 B \tan(e + f x)^2 - 7 B + B \tan(e + f x) 18 i + A \tan(e + f x)^2 3 i - A 1 i - 3 B \ln(\tan(e + f x) + 1) + B \ln(\tan(e + f x) + 1) \tan(e + f x) 9 i + 9 B \ln(\tan(e + f x) + 1) \tan(e + f x)^2 - B \ln(\tan(e + f x) + 1) \tan(e + f x)^3 3 i)}{3 c^3 f (-1 + \tan(e + f x) 1 i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3)/(c - c\*tan(e + f\*x)\*1i))^3,x)

[Out]  $-(a^3*(B*\tan(e + f*x)*18i - 7*B - A*1i + A*\tan(e + f*x)^2*3i + 15*B*\tan(e + f*x)^2 - 3*B*\log(\tan(e + f*x) + 1i) + B*\log(\tan(e + f*x) + 1i)*\tan(e + f*x)*9i + 9*B*\log(\tan(e + f*x) + 1i)*\tan(e + f*x)^2 - B*\log(\tan(e + f*x) + 1i)*\tan(e + f*x)^3*3i))/(3*c^3*f*(\tan(e + f*x)*1i - 1)^3)$

$$3.700 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ict \tan(e+fx))^4} dx$$

**Optimal.** Leaf size=99

$$-\frac{a^3(iA+B)(1+i \tan(e+fx))^3}{8c^4 f(1-i \tan(e+fx))^4} - \frac{a^3(iA-7B)(1+i \tan(e+fx))^3}{48c^4 f(1-i \tan(e+fx))^3}$$

[Out]  $-1/8*a^3*(I*A+B)*(1+I*\tan(f*x+e))^3/c^4/f/(1-I*\tan(f*x+e))^4-1/48*a^3*(I*A-7*B)*(1+I*\tan(f*x+e))^3/c^4/f/(1-I*\tan(f*x+e))^3$

**Rubi [A]**

time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ ,

Rules used = {3669, 79, 37}

$$-\frac{a^3(-7B+iA)(1+i \tan(e+fx))^3}{48c^4 f(1-i \tan(e+fx))^3} - \frac{a^3(B+iA)(1+i \tan(e+fx))^3}{8c^4 f(1-i \tan(e+fx))^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^4}, x]$

[Out]  $-1/8*(a^3*(I*A + B)*(1 + I*\text{Tan}[e + f*x])^3)/(c^4*f*(1 - I*\text{Tan}[e + f*x])^4) - (a^3*(I*A - 7*B)*(1 + I*\text{Tan}[e + f*x])^3)/(48*c^4*f*(1 - I*\text{Tan}[e + f*x])^3)$

**Rule 37**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{Simp}[\frac{(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)} / ((b*c - a*d)*(m + 1)))}{x}] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

**Rule 79**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] :> \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)} / (f*(p + 1)*(c*f - d*e))), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)) / (f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{LtQ}[p, -1] \&\& (!\text{LtQ}[n, -1] || \text{IntegerQ}[p] || !(\text{IntegerQ}[n] || !(\text{EqQ}[e, 0] || !(\text{EqQ}[c, 0] || \text{LtQ}[p, n])))$

**Rule 3669**

$\text{Int}[\frac{(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Di}$

```
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^4} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^2(A+Bx)}{(c-icx)^5} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{a^3(iA + B)(1 + i \tan(e + fx))^3}{8c^4 f (1 - i \tan(e + fx))^4} + \frac{(a(A + 7iB)) \text{Subst}\left(\int \frac{(a+iax)^2(A+Bx)}{(c-icx)^5} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{a^3(iA + B)(1 + i \tan(e + fx))^3}{8c^4 f (1 - i \tan(e + fx))^4} - \frac{a^3(iA - 7B)(1 + i \tan(e + fx))^3}{48c^4 f (1 - i \tan(e + fx))^4}$$

Mathematica [A]

time = 1.21, size = 81, normalized size = 0.82

$$\frac{a^3((-7iA + B) \cos(e + fx) - (A + 7iB) \sin(e + fx))(\cos(7e + 10fx) + i \sin(7e + 10fx))}{48c^4 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
f*x])^4, x]
```

```
[Out] (a^3*(((7*I)*A + B)*Cos[e + f*x] - (A + (7*I)*B)*Sin[e + f*x])*(Cos[7*e +
10*f*x] + I*Ssin[7*e + 10*f*x]))/(48*c^4*f*(Cos[f*x] + I*Ssin[f*x])^3)
```

Maple [A]

time = 0.34, size = 90, normalized size = 0.91

method	result
risch	$-\frac{a^3 e^{8i(fx+e)} B}{16c^4 f} - \frac{ia^3 e^{8i(fx+e)} A}{16c^4 f} + \frac{a^3 e^{6i(fx+e)} B}{12c^4 f} - \frac{ia^3 e^{6i(fx+e)} A}{12c^4 f}$
derivativedivides	$\frac{a^3 \left( -\frac{-iA-5B}{2(i+\tan(fx+e))^2} - \frac{4iA+4B}{4(i+\tan(fx+e))^4} - \frac{8iB-4A}{3(i+\tan(fx+e))^3} + \frac{iB}{i+\tan(fx+e)} \right)}{f c^4}$
default	$\frac{a^3 \left( -\frac{-iA-5B}{2(i+\tan(fx+e))^2} - \frac{4iA+4B}{4(i+\tan(fx+e))^4} - \frac{8iB-4A}{3(i+\tan(fx+e))^3} + \frac{iB}{i+\tan(fx+e)} \right)}{f c^4}$
norman	$\frac{A a^3 \tan(fx+e)}{cf} + \frac{iB a^3 (\tan^7(fx+e))}{cf} + \frac{-iA a^3 + B a^3}{6cf} + \frac{(iA a^3 + 7B a^3) (\tan^6(fx+e))}{2cf} + \frac{(17iA a^3 + 7B a^3) (\tan^2(fx+e))}{6cf} + \frac{7(-2iA a^3 + B a^3)}{(1+\tan^2(fx+e))^4 c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{f} \frac{a^3}{c^4} \left( -\frac{1}{2} \frac{(-5B - IA)}{(I + \tan(fx + e))^{2 - 1/4}} \frac{(4IA + 4B)}{(I + \tan(fx + e))^{4 - 1/3}} \frac{(8IB - 4A)}{(I + \tan(fx + e))^{3 + IB}} \right)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 3.96, size = 51, normalized size = 0.52

$$\frac{3(iA + B)a^3 e^{(8ifx + 8ie)} + 4(iA - B)a^3 e^{(6ifx + 6ie)}}{48c^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")`

[Out]  $-\frac{1}{48} \frac{(3(IA + B)a^3 e^{(8Ifx + 8Ie)} + 4(IA - B)a^3 e^{(6Ifx + 6Ie)})}{(c^4 f)}$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(80) = 160.

time = 0.43, size = 167, normalized size = 1.69

$$\begin{cases} \frac{(-16iAa^3c^4fe^{6ie} + 16Ba^3c^4fe^{6ie})e^{6ifx} + (-12iAa^3c^4fe^{8ie} - 12Ba^3c^4fe^{8ie})e^{8ifx}}{192c^8f^2} & \text{for } c^8f^2 \neq 0 \\ \frac{x(Aa^3e^{8ie} + Aa^3e^{6ie} - iBa^3e^{8ie} + iBa^3e^{6ie})}{2c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)`

[Out] `Piecewise(((((-16*I*A*a**3*c**4*f*exp(6*I*e) + 16*B*a**3*c**4*f*exp(6*I*e))*exp(6*I*f*x) + (-12*I*A*a**3*c**4*f*exp(8*I*e) - 12*B*a**3*c**4*f*exp(8*I*e))*exp(8*I*f*x))/(192*c**8*f**2), Ne(c**8*f**2, 0)), (x*(A*a**3*exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(8*I*e) + I*B*a**3*exp(6*I*e))/(2*c**4), True))`

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(87) = 174.  
time = 1.15, size = 237, normalized size = 2.39

$$\frac{2(3Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^7 + 3iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^6 - 3Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 17Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 + 4iBa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 10iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 10Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^1 + 17Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^0 - 4iBa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^{-1} + 3iAa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^{-2} - 3Ba^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^{-3} + 3Aa^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^{-4})}{3c^4 f (\tan(\frac{1}{2}fx + \frac{1}{2}e) + i)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out] -2/3\*(3\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^7 + 3\*I\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^6 - 3\*B\*a^3\*tan(1/2\*f\*x + 1/2\*e)^5 - 17\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^4 + 4\*I\*B\*a^3\*tan(1/2\*f\*x + 1/2\*e)^3 - 10\*I\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^2 + 10\*B\*a^3\*tan(1/2\*f\*x + 1/2\*e)^1 + 17\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^0 - 4\*I\*B\*a^3\*tan(1/2\*f\*x + 1/2\*e)^{-1} + 3\*I\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^{-2} - 3\*B\*a^3\*tan(1/2\*f\*x + 1/2\*e)^{-3} + 3\*A\*a^3\*tan(1/2\*f\*x + 1/2\*e)^{-4})/(c^4\*f\*(tan(1/2\*f\*x + 1/2\*e) + I)^8)

**Mupad [B]**

time = 8.93, size = 118, normalized size = 1.19

$$\frac{-\frac{a^3(-B+Ai)}{6} + \frac{a^3 \tan(e+fx)(2A-B4i)}{6} + B a^3 \tan(e+fx)^3 i + \frac{a^3 \tan(e+fx)^2(-3B+Ai)}{6}}{c^4 f (\tan(e+fx)^4 + \tan(e+fx)^3 4i - 6 \tan(e+fx)^2 - \tan(e+fx) 4i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*i)^3)/(c - c\*tan(e + f\*x)\*i)^4,x)

[Out] ((a^3\*tan(e + f\*x)\*(2\*A - B\*4i))/6 - (a^3\*(A\*i - B))/6 + B\*a^3\*tan(e + f\*x)^3\*i + (a^3\*tan(e + f\*x)^2\*(A\*3i - 3\*B))/6)/(c^4\*f\*(tan(e + f\*x)^3\*4i - 6\*tan(e + f\*x)^2 - tan(e + f\*x)\*4i + tan(e + f\*x)^4 + 1))

$$3.701 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ict \tan(e+fx))^5} dx$$

**Optimal.** Leaf size=122

$$\frac{4a^3(A-iB)}{5c^5 f(i+\tan(e+fx))^5} + \frac{a^3(iA+2B)}{c^5 f(i+\tan(e+fx))^4} - \frac{a^3(A-5iB)}{3c^5 f(i+\tan(e+fx))^3} - \frac{a^3 B}{2c^5 f(i+\tan(e+fx))^2}$$

[Out]  $4/5*a^3*(A-I*B)/c^5/f/(I+\tan(f*x+e))^5+a^3*(I*A+2*B)/c^5/f/(I+\tan(f*x+e))^4$   
 $-1/3*a^3*(A-5*I*B)/c^5/f/(I+\tan(f*x+e))^3-1/2*a^3*B/c^5/f/(I+\tan(f*x+e))^2$

**Rubi [A]**

time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^3(A-5iB)}{3c^5 f(\tan(e+fx)+i)^3} + \frac{a^3(2B+iA)}{c^5 f(\tan(e+fx)+i)^4} + \frac{4a^3(A-iB)}{5c^5 f(\tan(e+fx)+i)^5} - \frac{a^3 B}{2c^5 f(\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^5, x]$

[Out]  $(4*a^3*(A - I*B))/(5*c^5*f*(I + \text{Tan}[e + f*x])^5) + (a^3*(I*A + 2*B))/(c^5*f*(I + \text{Tan}[e + f*x])^4) - (a^3*(A - (5*I)*B))/(3*c^5*f*(I + \text{Tan}[e + f*x])^3) - (a^3*B)/(2*c^5*f*(I + \text{Tan}[e + f*x])^2)$

**Rule 78**

$\text{Int}[(a + b*x)^n*(c + d*x)^m*(e + f*x)^p, x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ ((\text{ILtQ}[n, 0] \ \&\& \ \text{ILtQ}[p, 0]) \ || \ \text{EqQ}[p, 1] \ || \ (\text{IGtQ}[p, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ \text{LeQ}[9*p + 5*(n + 2), 0] \ || \ \text{GeQ}[n + p + 1, 0] \ || \ (\text{GeQ}[n + p + 2, 0] \ \&\& \ \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(c + d*\text{tan}[e + f*x])^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

**Rubi steps**

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^5} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^6} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( -\frac{4a^2(A-iB)}{c^6(i+x)^6} - \frac{4ia^2(A-2iB)}{c^6(i+x)^5} + \frac{a^2(A-5iB)}{c^6(i+x)^4} + \frac{a^3}{c^6} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{4a^3(A-iB)}{5c^5 f (i + \tan(e + fx))^5} + \frac{a^3(iA + 2B)}{c^5 f (i + \tan(e + fx))^4} - \frac{a^3}{3c^5}$$

**Mathematica [A]**

time = 1.55, size = 91, normalized size = 0.75

$$\frac{a^3(-15iA + 4(-4iA + B) \cos(2(e + fx)) - 4(A + 4iB) \sin(2(e + fx)))(\cos(8e + 11fx) + i \sin(8e + 11fx))}{240c^5 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^5, x]
```

```
[Out] (a^3*((-15*I)*A + 4*((-4*I)*A + B)*Cos[2*(e + f*x)] - 4*(A + (4*I)*B)*Sin[2*(e + f*x)])*(Cos[8*e + 11*f*x] + I*Sin[8*e + 11*f*x])/(240*c^5*f*(Cos[f*x] + I*Sin[f*x])^3)
```

**Maple [A]**

time = 0.40, size = 87, normalized size = 0.71

method	result
derivativedivides	$\frac{a^3 \left( -\frac{B}{2(i+\tan(fx+e))^2} - \frac{-5iB+A}{3(i+\tan(fx+e))^3} - \frac{4iB-4A}{5(i+\tan(fx+e))^5} - \frac{-4iA-8B}{4(i+\tan(fx+e))^4} \right)}{f c^5}$
default	$\frac{a^3 \left( -\frac{B}{2(i+\tan(fx+e))^2} - \frac{-5iB+A}{3(i+\tan(fx+e))^3} - \frac{4iB-4A}{5(i+\tan(fx+e))^5} - \frac{-4iA-8B}{4(i+\tan(fx+e))^4} \right)}{f c^5}$
risch	$-\frac{a^3 e^{10i(fx+e)} B}{40c^5 f} - \frac{ia^3 e^{10i(fx+e)} A}{40c^5 f} - \frac{ia^3 e^{8i(fx+e)}}{16c^5 f} + \frac{a^3 e^{6i(fx+e)} B}{24c^5 f} - \frac{ia^3 e^{6i(fx+e)} A}{24c^5 f}$
norman	$\frac{A a^3 \tan(fx+e)}{cf} + \frac{-4iA a^3 + B a^3}{30cf} - \frac{(-8iB a^3 + 19A a^3) (\tan^3(fx+e))}{3cf} + \frac{7(-16iB a^3 + 11A a^3) (\tan^5(fx+e))}{15cf} - \frac{(-8iB a^3 + A a^3) (\tan^7(fx+e))}{3cf} + \frac{a^3}{3cf} (1 + \tan(fx+e))$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5, x, method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3/c^5*(-1/2*B/(I+tan(f*x+e))^2-1/3*(A-5*I*B)/(I+tan(f*x+e))^3-1/5*(4*I*B-4*A)/(I+tan(f*x+e))^5-1/4*(-4*I*A-8*B)/(I+tan(f*x+e))^4)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 1.88, size = 67, normalized size = 0.55

$$\frac{6(iA + B)a^3e^{(10ifx+10ie)} + 15iAa^3e^{(8ifx+8ie)} + 10(iA - B)a^3e^{(6ifx+6ie)}}{240c^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, alg
orithm="fricas")
```

```
[Out] -1/240*(6*(I*A + B)*a^3*e^(10*I*f*x + 10*I*e) + 15*I*A*a^3*e^(8*I*f*x + 8*I
*e) + 10*(I*A - B)*a^3*e^(6*I*f*x + 6*I*e))/(c^5*f)
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(100) = 200.

time = 0.56, size = 218, normalized size = 1.79

$$\begin{cases} \frac{-960iAa^3c^{10}f^2e^{8ie}e^{8ifx} + (-640iAa^3c^{10}f^2e^{6ie} + 640Ba^3c^{10}f^2e^{6ie})e^{6ifx} + (-384iAa^3c^{10}f^2e^{10ie} - 384Ba^3c^{10}f^2e^{10ie})e^{10ifx}}{15360c^{15}f^3} & \text{for } c^{15}f^3 \neq 0 \\ \frac{x(Aa^3e^{10ie} + 2Aa^3e^{8ie} + Aa^3e^{6ie} - iBa^3e^{10ie} + iBa^3e^{6ie})}{4c^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**5,x)
```

```
[Out] Piecewise((( -960*I*A*a**3*c**10*f**2*exp(8*I*e)*exp(8*I*f*x) + (-640*I*A*a*
**3*c**10*f**2*exp(6*I*e) + 640*B*a**3*c**10*f**2*exp(6*I*e))*exp(6*I*f*x) +
(-384*I*A*a**3*c**10*f**2*exp(10*I*e) - 384*B*a**3*c**10*f**2*exp(10*I*e))
*exp(10*I*f*x))/(15360*c**15*f**3), Ne(c**15*f**3, 0)), (x*(A*a**3*exp(10*I
*e) + 2*A*a**3*exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(10*I*e) + I*B*
a**3*exp(6*I*e))/(4*c**5), True))
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(106) = 212.

time = 1.41, size = 309, normalized size = 2.53

$\frac{2(15Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2 + 30Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) + 15B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2 - 180Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) + 30B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2 - 170Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) + 65B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2 + 202Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) - 120B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e) + 170Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) - 65B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2 - 140Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) + 50B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2 - 30Aa^3\cos(\frac{1}{2}fx + \frac{1}{2}e) + 15B^2a^3\cos(\frac{1}{2}fx + \frac{1}{2}e)^2)}{1537\cos(\frac{1}{2}fx + \frac{1}{2}e)^3}$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2/15*(15*A*a^3*\tan(1/2*f*x + 1/2*e)^9 + 30*I*A*a^3*\tan(1/2*f*x + 1/2*e)^8 \\ & - 15*B*a^3*\tan(1/2*f*x + 1/2*e)^8 - 140*A*a^3*\tan(1/2*f*x + 1/2*e)^7 + 10*I \\ & *B*a^3*\tan(1/2*f*x + 1/2*e)^7 - 170*I*A*a^3*\tan(1/2*f*x + 1/2*e)^6 + 65*B*a \\ & ^3*\tan(1/2*f*x + 1/2*e)^6 + 282*A*a^3*\tan(1/2*f*x + 1/2*e)^5 - 12*I*B*a^3*t \\ & \tan(1/2*f*x + 1/2*e)^5 + 170*I*A*a^3*\tan(1/2*f*x + 1/2*e)^4 - 65*B*a^3*\tan(1 \\ & /2*f*x + 1/2*e)^4 - 140*A*a^3*\tan(1/2*f*x + 1/2*e)^3 + 10*I*B*a^3*\tan(1/2*f \\ & *x + 1/2*e)^3 - 30*I*A*a^3*\tan(1/2*f*x + 1/2*e)^2 + 15*B*a^3*\tan(1/2*f*x + \\ & 1/2*e)^2 + 15*A*a^3*\tan(1/2*f*x + 1/2*e))/(c^5*f*(\tan(1/2*f*x + 1/2*e) + I) \\ & ^{10}) \end{aligned}$$

**Mupad [B]**

time = 9.01, size = 128, normalized size = 1.05

$$\frac{\frac{a^3(4A+B1i)}{30} + \frac{a^3 \tan(e+fx)(5B+A10i)}{30} - \frac{Ba^3 \tan(e+fx)^3}{2} - \frac{a^3 \tan(e+fx)^2(10A-B5i)}{30}}{c^5 f (\tan(e+fx)^5 + \tan(e+fx)^4 5i - 10 \tan(e+fx)^3 - \tan(e+fx)^2 10i + 5 \tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3)/(c - c\*tan(e + f\*x)\*1i)^5,x)

[Out] 
$$\begin{aligned} & ((a^3*(4*A + B*1i))/30 + (a^3*\tan(e + f*x)*(A*10i + 5*B))/30 - (B*a^3*\tan(e \\ & + f*x)^3)/2 - (a^3*\tan(e + f*x)^2*(10*A - B*5i))/30)/(c^5*f*(5*\tan(e + f*x) \\ & ) - \tan(e + f*x)^2*10i - 10*\tan(e + f*x)^3 + \tan(e + f*x)^4*5i + \tan(e + f* \\ & x)^5 + 1i)) \end{aligned}$$

$$3.702 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^6} dx$$

**Optimal.** Leaf size=127

$$\frac{2a^3(iA+B)}{3c^6 f(i+\tan(e+fx))^6} - \frac{4a^3(A-2iB)}{5c^6 f(i+\tan(e+fx))^5} - \frac{a^3(iA+5B)}{4c^6 f(i+\tan(e+fx))^4} - \frac{ia^3B}{3c^6 f(i+\tan(e+fx))^3}$$

[Out]  $2/3*a^3*(I*A+B)/c^6/f/(I+\tan(f*x+e))^6-4/5*a^3*(A-2*I*B)/c^6/f/(I+\tan(f*x+e))^5-1/4*a^3*(I*A+5*B)/c^6/f/(I+\tan(f*x+e))^4-1/3*I*a^3*B/c^6/f/(I+\tan(f*x+e))^3$

**Rubi [A]**

time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{a^3(5B+ia)}{4c^6 f(\tan(e+fx)+i)^4} - \frac{4a^3(A-2iB)}{5c^6 f(\tan(e+fx)+i)^5} + \frac{2a^3(B+ia)}{3c^6 f(\tan(e+fx)+i)^6} - \frac{ia^3B}{3c^6 f(\tan(e+fx)+i)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^6}, x]$

[Out]  $(2*a^3*(I*A + B))/(3*c^6*f*(I + \text{Tan}[e + f*x])^6) - (4*a^3*(A - (2*I)*B))/(5*c^6*f*(I + \text{Tan}[e + f*x])^5) - (a^3*(I*A + 5*B))/(4*c^6*f*(I + \text{Tan}[e + f*x])^4) - ((I/3)*a^3*B)/(c^6*f*(I + \text{Tan}[e + f*x])^3)$

**Rule 78**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]} :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) || \text{EqQ}[p, 1] || (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] || \text{LeQ}[9*p + 5*(n + 2), 0] || \text{GeQ}[n + p + 1, 0] || (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]} :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^6} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2(A+Bx)}{(c-icx)^7} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( -\frac{4ia^2(A-iB)}{c^7(i+x)^7} + \frac{4a^2(A-2iB)}{c^7(i+x)^6} + \frac{a^2(iA+5B)}{c^7(i+x)^5} + \frac{a^2}{c^7} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{2a^3(iA + B)}{3c^6 f (i + \tan(e + fx))^6} - \frac{4a^3(A - 2iB)}{5c^6 f (i + \tan(e + fx))^5} - \frac{a^3}{4c^6 f (i + \tan(e + fx))^4}$$

**Mathematica [A]**

time = 2.25, size = 112, normalized size = 0.88

$$\frac{a^3(3(-27iA + B) \cos(e + fx) + 10(-3iA + B) \cos(3(e + fx)) - (A + 3iB)(9 \sin(e + fx) + 10 \sin(3(e + fx))))(\cos(9e + 12fx) + i \sin(9e + 12fx))}{960c^6 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^6,x]

[Out] (a^3\*(3\*((-27\*I)\*A + B)\*Cos[e + f\*x] + 10\*((-3\*I)\*A + B)\*Cos[3\*(e + f\*x)] - (A + (3\*I)\*B)\*(9\*Sin[e + f\*x] + 10\*Sin[3\*(e + f\*x)]))\*(Cos[9\*e + 12\*f\*x] + I\*Sin[9\*e + 12\*f\*x]))/(960\*c^6\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.29, size = 90, normalized size = 0.71

method	result
derivativedivides	$\frac{a^3 \left( -\frac{iB}{3(i+\tan(fx+e))^3} - \frac{-4iA-4B}{6(i+\tan(fx+e))^6} - \frac{-8iB+4A}{5(i+\tan(fx+e))^5} - \frac{iA+5B}{4(i+\tan(fx+e))^4} \right)}{f c^6}$
default	$\frac{a^3 \left( -\frac{iB}{3(i+\tan(fx+e))^3} - \frac{-4iA-4B}{6(i+\tan(fx+e))^6} - \frac{-8iB+4A}{5(i+\tan(fx+e))^5} - \frac{iA+5B}{4(i+\tan(fx+e))^4} \right)}{f c^6}$
risch	$-\frac{a^3 e^{12i(fx+e)} B}{96c^6 f} - \frac{ia^3 e^{12i(fx+e)} A}{96c^6 f} - \frac{e^{10i(fx+e)} B a^3}{80c^6 f} - \frac{3ie^{10i(fx+e)} A a^3}{80c^6 f} + \frac{e^{8i(fx+e)} B a^3}{64c^6 f} - \frac{3ie^{8i(fx+e)} A a^3}{64c^6 f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f\*a^3/c^6\*(-1/3\*I\*B/(I+tan(f\*x+e))^3-1/6\*(-4\*I\*A-4\*B)/(I+tan(f\*x+e))^6-1/5\*(-8\*I\*B+4\*A)/(I+tan(f\*x+e))^5-1/4\*(I\*A+5\*B)/(I+tan(f\*x+e))^4)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x, alg orithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 1.01, size = 93, normalized size = 0.73

$$\frac{10(iA+B)a^3e^{(12ifx+12ie)} + 12(3iA+B)a^3e^{(10ifx+10ie)} + 15(3iA-B)a^3e^{(8ifx+8ie)} + 20(iA-B)a^3e^{(6ifx+6ie)}}{960c^6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x, alg orithm="fricas")

[Out]  $-1/960*(10*(I*A + B)*a^3*e^{(12*I*f*x + 12*I*e)} + 12*(3*I*A + B)*a^3*e^{(10*I*f*x + 10*I*e)} + 15*(3*I*A - B)*a^3*e^{(8*I*f*x + 8*I*e)} + 20*(I*A - B)*a^3*e^{(6*I*f*x + 6*I*e)})/(c^6*f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs.  $2(105) = 210$ .

time = 0.63, size = 332, normalized size = 2.61

$$\begin{cases} \frac{(-491520iAa^3c^{18}f^3e^{6ie}+491520Ba^3c^{18}f^3e^{6ie})e^{6ifx}+(-1105920iAa^3c^{18}f^3e^{8ie}+368640Ba^3c^{18}f^3e^{8ie})e^{8ifx}+(-884736iAa^3c^{18}f^3e^{10ie}-294912Ba^3c^{18}f^3e^{10ie})e^{10ifx}+(-245760iAa^3c^{18}f^3e^{12ie}-245760Ba^3c^{18}f^3e^{12ie})e^{12ifx}}{23592960c^{24}f^4} & \text{for } c^{24}f^4 \neq 0 \\ \frac{x(Aa^3e^{12ie}+3Aa^3e^{10ie}+3Aa^3e^{8ie}+Aa^3e^{6ie}-iBa^3e^{12ie}-iBa^3e^{10ie}+iBa^3e^{8ie}+iBa^3e^{6ie})}{8c^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*6,x)

[Out] Piecewise(((((-491520\*I\*A\*a\*\*3\*c\*\*18\*f\*\*3\*exp(6\*I\*e) + 491520\*B\*a\*\*3\*c\*\*18\*f\*\*3\*exp(6\*I\*e))\*exp(6\*I\*f\*x) + (-1105920\*I\*A\*a\*\*3\*c\*\*18\*f\*\*3\*exp(8\*I\*e) + 368640\*B\*a\*\*3\*c\*\*18\*f\*\*3\*exp(8\*I\*e))\*exp(8\*I\*f\*x) + (-884736\*I\*A\*a\*\*3\*c\*\*18\*f\*\*3\*exp(10\*I\*e) - 294912\*B\*a\*\*3\*c\*\*18\*f\*\*3\*exp(10\*I\*e))\*exp(10\*I\*f\*x) + (-245760\*I\*A\*a\*\*3\*c\*\*18\*f\*\*3\*exp(12\*I\*e) - 245760\*B\*a\*\*3\*c\*\*18\*f\*\*3\*exp(12\*I\*e))\*exp(12\*I\*f\*x))/(23592960\*c\*\*24\*f\*\*4), Ne(c\*\*24\*f\*\*4, 0)), (x\*(A\*a\*\*3\*exp(12\*I\*e) + 3\*A\*a\*\*3\*exp(10\*I\*e) + 3\*A\*a\*\*3\*exp(8\*I\*e) + A\*a\*\*3\*exp(6\*I\*e) - I\*B\*a\*\*3\*exp(12\*I\*e) - I\*B\*a\*\*3\*exp(10\*I\*e) + I\*B\*a\*\*3\*exp(8\*I\*e) + I\*B\*a\*\*3\*exp(6\*I\*e))/(8\*c\*\*6), True))

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs.  $2(107) = 214$ .

time = 0.96, size = 345, normalized size = 2.72

$\frac{2}{(15A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{10} - 60A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^8 + 120A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^6 - 120A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^4 + 60A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^2 - 15A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^0) + 20iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^9 - 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^7 + 240iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^5 - 240iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^3 + 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^1 - 20iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-1} + 20iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-3} - 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-5} + 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-7} - 60iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-9} + 15iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-11})}{(15A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{10} - 60A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^8 + 120A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^6 - 120A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^4 + 60A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^2 - 15A^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^0) + 20iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^9 - 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^7 + 240iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^5 - 240iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^3 + 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^1 - 20iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-1} + 20iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-3} - 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-5} + 120iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-7} - 60iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-9} + 15iA^2\cos(\frac{1}{2}f^2x + \frac{1}{2})^{-11})}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^6,x, algorithm="giac")

[Out] 
$$\frac{-2/15*(15*A*a^3*\tan(1/2*f*x + 1/2*e)^{11} + 45*I*A*a^3*\tan(1/2*f*x + 1/2*e)^{10} - 15*B*a^3*\tan(1/2*f*x + 1/2*e)^{10} - 215*A*a^3*\tan(1/2*f*x + 1/2*e)^9 - 390*I*A*a^3*\tan(1/2*f*x + 1/2*e)^8 + 90*B*a^3*\tan(1/2*f*x + 1/2*e)^8 + 738*A*a^3*\tan(1/2*f*x + 1/2*e)^7 + 24*I*B*a^3*\tan(1/2*f*x + 1/2*e)^7 + 746*I*A*a^3*\tan(1/2*f*x + 1/2*e)^6 - 158*B*a^3*\tan(1/2*f*x + 1/2*e)^6 - 738*A*a^3*\tan(1/2*f*x + 1/2*e)^5 - 24*I*B*a^3*\tan(1/2*f*x + 1/2*e)^5 - 390*I*A*a^3*\tan(1/2*f*x + 1/2*e)^4 + 90*B*a^3*\tan(1/2*f*x + 1/2*e)^4 + 215*A*a^3*\tan(1/2*f*x + 1/2*e)^3 + 45*I*A*a^3*\tan(1/2*f*x + 1/2*e)^2 - 15*B*a^3*\tan(1/2*f*x + 1/2*e)^2 - 15*A*a^3*\tan(1/2*f*x + 1/2*e))/(c^6*f*(\tan(1/2*f*x + 1/2*e) + I)^{12})$$

**Mupad [B]**

time = 9.09, size = 140, normalized size = 1.10

$$\frac{-\frac{a^3(-B+Ai)}{60} + \frac{a^3 \tan(e+fx)(18A-B6i)}{60} + \frac{B a^3 \tan(e+fx)^3 i}{3} + \frac{a^3 \tan(e+fx)^2 (15B+Ai)}{60}}{c^6 f (\tan(e+fx)^6 + \tan(e+fx)^5 6i - 15 \tan(e+fx)^4 - \tan(e+fx)^3 20i + 15 \tan(e+fx)^2 + \tan(e+fx) 6i - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3)/(c - c\*tan(e + f\*x)\*1i)^6,x)

[Out] 
$$-\left(\frac{a^3*\tan(e + f*x)*(18*A - B*6i)}{60} - \frac{a^3*(A*7i - B)}{60} + \frac{B*a^3*\tan(e + f*x)^3*1i}{3} + \frac{a^3*\tan(e + f*x)^2*(A*15i + 15*B)}{60}\right)/(c^6*f*(\tan(e + f*x)*6i + 15*\tan(e + f*x)^2 - \tan(e + f*x)^3*20i - 15*\tan(e + f*x)^4 + \tan(e + f*x)^5*6i + \tan(e + f*x)^6 - 1))$$

$$3.703 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^7} dx$$

**Optimal.** Leaf size=125

$$-\frac{4a^3(A-iB)}{7c^7 f(i+\tan(e+fx))^7} - \frac{2a^3(iA+2B)}{3c^7 f(i+\tan(e+fx))^6} + \frac{a^3(A-5iB)}{5c^7 f(i+\tan(e+fx))^5} + \frac{a^3 B}{4c^7 f(i+\tan(e+fx))^4}$$

[Out]  $-4/7*a^3*(A-I*B)/c^7/f/(I+\tan(f*x+e))^7-2/3*a^3*(I*A+2*B)/c^7/f/(I+\tan(f*x+e))^6+1/5*a^3*(A-5*I*B)/c^7/f/(I+\tan(f*x+e))^5+1/4*a^3*B/c^7/f/(I+\tan(f*x+e))^4$

**Rubi [A]**

time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{a^3(A-5iB)}{5c^7 f(\tan(e+fx)+i)^5} - \frac{2a^3(2B+iA)}{3c^7 f(\tan(e+fx)+i)^6} - \frac{4a^3(A-iB)}{7c^7 f(\tan(e+fx)+i)^7} + \frac{a^3 B}{4c^7 f(\tan(e+fx)+i)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^7}, x]$

[Out]  $(-4*a^3*(A - I*B))/(7*c^7*f*(I + \text{Tan}[e + f*x])^7) - (2*a^3*(I*A + 2*B))/(3*c^7*f*(I + \text{Tan}[e + f*x])^6) + (a^3*(A - (5*I)*B))/(5*c^7*f*(I + \text{Tan}[e + f*x])^5) + (a^3*B)/(4*c^7*f*(I + \text{Tan}[e + f*x])^4)$

**Rule 78**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) || \text{EqQ}[p, 1] || (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] || \text{LeQ}[9*p + 5*(n + 2), 0] || \text{GeQ}[n + p + 1, 0] || (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^7} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^8} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(A-iB)}{c^8(i+x)^8} + \frac{4a^2(iA+2B)}{c^8(i+x)^7} - \frac{a^2(A-5iB)}{c^8(i+x)^6} - \frac{a^2}{c^8(i+x)^5} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{4a^3(A-iB)}{7c^7 f (i + \tan(e + fx))^7} - \frac{2a^3(iA+2B)}{3c^7 f (i + \tan(e + fx))^6} + \dots$$

**Mathematica [A]**

time = 2.53, size = 143, normalized size = 1.14

$$\frac{-ia^3(252A + 35(10A + iB) \cos(2(e + fx)) + 20(5A + 2iB) \cos(4(e + fx)) - 70iA \sin(2(e + fx)) + 175B \sin(2(e + fx)) - 40iA \sin(4(e + fx)) + 100B \sin(4(e + fx)))(\cos(10e + 13fx) + i \sin(10e + 13fx))}{6720c^7 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^7, x]

[Out] ((-1/6720\*I)\*a^3\*(252\*A + 35\*(10\*A + I\*B)\*Cos[2\*(e + f\*x)] + 20\*(5\*A + (2\*I)\*B)\*Cos[4\*(e + f\*x)] - (70\*I)\*A\*Sin[2\*(e + f\*x)] + 175\*B\*Sin[2\*(e + f\*x)] - (40\*I)\*A\*Sin[4\*(e + f\*x)] + 100\*B\*Sin[4\*(e + f\*x)]\*(Cos[10\*e + 13\*f\*x] + I\*Sin[10\*e + 13\*f\*x]))/(c^7\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.66, size = 89, normalized size = 0.71

method	result
derivativedivides	$\frac{a^3 \left( -\frac{5iB-A}{5(i+\tan(fx+e))^5} + \frac{B}{4(i+\tan(fx+e))^4} - \frac{4iA+8B}{6(i+\tan(fx+e))^6} - \frac{-4iB+4A}{7(i+\tan(fx+e))^7} \right)}{f c^7}$
default	$\frac{a^3 \left( -\frac{5iB-A}{5(i+\tan(fx+e))^5} + \frac{B}{4(i+\tan(fx+e))^4} - \frac{4iA+8B}{6(i+\tan(fx+e))^6} - \frac{-4iB+4A}{7(i+\tan(fx+e))^7} \right)}{f c^7}$
risch	$-\frac{a^3 e^{14i(fx+e)} B}{224c^7 f} - \frac{ia^3 e^{14i(fx+e)} A}{224c^7 f} - \frac{e^{12i(fx+e)} B a^3}{96c^7 f} - \frac{ie^{12i(fx+e)} A a^3}{48c^7 f} - \frac{3iA a^3 e^{10i(fx+e)}}{80c^7 f} + \frac{e^{8i(fx+e)} B a^3}{64c^7 f}$
norman	$\frac{A a^3 \tan(fx+e)}{cf} + \frac{-44iA a^3 + 5B a^3}{420cf} + \frac{B a^3 (\tan^{10}(fx+e))}{4cf} + \frac{2(-125iB a^3 + 139A a^3) (\tan^5(fx+e))}{15cf} - \frac{6(-85iB a^3 + 36A a^3) (\tan^7(fx+e))}{35cf}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^7, x, method=\_RE TURNVERBOSE)

[Out] 1/f\*a^3/c^7\*(-1/5\*(-A+5\*I\*B)/(I+tan(f\*x+e))^5+1/4\*B/(I+tan(f\*x+e))^4-1/6\*(4\*I\*A+8\*B)/(I+tan(f\*x+e))^6-1/7\*(-4\*I\*B+4\*A)/(I+tan(f\*x+e))^7)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 1.47, size = 109, normalized size = 0.87

$$\frac{30(iA+B)a^3e^{14i fx+14ie} + 70(2iA+B)a^3e^{12i fx+12ie} + 252iAa^3e^{10i fx+10ie} + 105(2iA-B)a^3e^{8i fx+8ie} + 70(iA-B)a^3e^{6i fx+6ie}}{6720c^7f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x, alg
orithm="fricas")
```

```
[Out] -1/6720*(30*(I*A + B)*a^3*e^(14*I*f*x + 14*I*e) + 70*(2*I*A + B)*a^3*e^(12*
I*f*x + 12*I*e) + 252*I*A*a^3*e^(10*I*f*x + 10*I*e) + 105*(2*I*A - B)*a^3*e
^(8*I*f*x + 8*I*e) + 70*(I*A - B)*a^3*e^(6*I*f*x + 6*I*e))/(c^7*f)
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs.  $2(104) = 208$ .

time = 0.87, size = 379, normalized size = 3.03

$$\frac{\left\{ \begin{array}{l} \frac{-396361728Aa^{28}f^{10}e^{10ifx} + (-110100480Aa^{27}f^{9}e^{9ie} + 110100480Ba^{27}f^{9}e^{9ie})e^{6ifx} + (-330301440Aa^{26}f^{8}e^{8ie} + 165150720Ba^{26}f^{8}e^{8ie})e^{4ifx} + (-220200960Aa^{25}f^{7}e^{7ie} - 110100480Ba^{25}f^{7}e^{7ie})e^{2ifx} + (-47185920Aa^{24}f^{6}e^{6ie} - 47185920Ba^{24}f^{6}e^{6ie})e^{0ifx}}{10569646080c^{35}f^5} \text{ for } c^{35}f^5 \neq 0 \\ \frac{2(Aa^{14}e^{14ie} + 4Aa^{13}e^{12ie} + 6Aa^{12}e^{10ie} + 4Aa^{11}e^{8ie} + Aa^{10}e^{6ie} - 1Ba^{13}e^{14ie} - 2Ba^{12}e^{12ie} + 2Ba^{11}e^{10ie} + 1Ba^{10}e^{8ie})}{16c^7} \text{ otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))*7,x)
```

```
[Out] Piecewise((( -396361728*I*A*a**3*c**28*f**4*exp(10*I*e)*exp(10*I*f*x) + (-11
0100480*I*A*a**3*c**28*f**4*exp(6*I*e) + 110100480*B*a**3*c**28*f**4*exp(6*
I*e))*exp(6*I*f*x) + (-330301440*I*A*a**3*c**28*f**4*exp(8*I*e) + 165150720
*B*a**3*c**28*f**4*exp(8*I*e))*exp(8*I*f*x) + (-220200960*I*A*a**3*c**28*f*
**4*exp(12*I*e) - 110100480*B*a**3*c**28*f**4*exp(12*I*e))*exp(12*I*f*x) + (
-47185920*I*A*a**3*c**28*f**4*exp(14*I*e) - 47185920*B*a**3*c**28*f**4*exp(
14*I*e))*exp(14*I*f*x))/(10569646080*c**35*f**5), Ne(c**35*f**5, 0)), (x*(A
*a**3*exp(14*I*e) + 4*A*a**3*exp(12*I*e) + 6*A*a**3*exp(10*I*e) + 4*A*a**3*
exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(14*I*e) - 2*I*B*a**3*exp(12*I
*e) + 2*I*B*a**3*exp(8*I*e) + I*B*a**3*exp(6*I*e))/(16*c**7), True))
```



**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs.  $2(107) = 214$ .  
time = 1.17, size = 453, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^7,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2/105*(105*A*a^3*\tan(1/2*f*x + 1/2*e)^{13} + 420*I*A*a^3*\tan(1/2*f*x + 1/2*e)^{12} - 105*B*a^3*\tan(1/2*f*x + 1/2*e)^{12} - 2170*A*a^3*\tan(1/2*f*x + 1/2*e)^{11} - 70*I*B*a^3*\tan(1/2*f*x + 1/2*e)^{11} - 5180*I*A*a^3*\tan(1/2*f*x + 1/2*e)^{10} + 875*B*a^3*\tan(1/2*f*x + 1/2*e)^{10} + 11431*A*a^3*\tan(1/2*f*x + 1/2*e)^9 + 700*I*B*a^3*\tan(1/2*f*x + 1/2*e)^9 + 15904*I*A*a^3*\tan(1/2*f*x + 1/2*e)^8 - 2380*B*a^3*\tan(1/2*f*x + 1/2*e)^8 - 19436*A*a^3*\tan(1/2*f*x + 1/2*e)^7 - 1340*I*B*a^3*\tan(1/2*f*x + 1/2*e)^7 - 15904*I*A*a^3*\tan(1/2*f*x + 1/2*e)^6 + 2380*B*a^3*\tan(1/2*f*x + 1/2*e)^6 + 11431*A*a^3*\tan(1/2*f*x + 1/2*e)^5 + 700*I*B*a^3*\tan(1/2*f*x + 1/2*e)^5 + 5180*I*A*a^3*\tan(1/2*f*x + 1/2*e)^4 - 875*B*a^3*\tan(1/2*f*x + 1/2*e)^4 - 2170*A*a^3*\tan(1/2*f*x + 1/2*e)^3 - 70*I*B*a^3*\tan(1/2*f*x + 1/2*e)^3 - 420*I*A*a^3*\tan(1/2*f*x + 1/2*e)^2 + 105*B*a^3*\tan(1/2*f*x + 1/2*e)^2 + 105*A*a^3*\tan(1/2*f*x + 1/2*e))/(c^7*f*(\tan(1/2*f*x + 1/2*e) + I)^{14}) \end{aligned}$$

**Mupad [B]**

time = 9.30, size = 151, normalized size = 1.21

$$\frac{\frac{a^3(44A+B5i)}{420} + \frac{a^3 \tan(e+fx)(35B+A112i)}{420} - \frac{Ba^3 \tan(e+fx)^3}{4} - \frac{a^3 \tan(e+fx)^2(84A-B105i)}{420}}{c^7 f (-\tan(e+fx)^7 - \tan(e+fx)^6 7i + 21 \tan(e+fx)^5 + \tan(e+fx)^4 35i - 35 \tan(e+fx)^3 - \tan(e+fx)^2 21i + 7 \tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3)/(c - c\*tan(e + f\*x)\*1i))^7,x)

[Out] 
$$\begin{aligned} & ((a^3*(44*A + B*5i))/420 + (a^3*\tan(e + f*x)*(A*112i + 35*B))/420 - (B*a^3*\tan(e + f*x)^3)/4 - (a^3*\tan(e + f*x)^2*(84*A - B*105i))/420)/(c^7*f*(7*\tan(e + f*x) - \tan(e + f*x)^2*21i - 35*\tan(e + f*x)^3 + \tan(e + f*x)^4*35i + 21*\tan(e + f*x)^5 - \tan(e + f*x)^6*7i - \tan(e + f*x)^7 + 1i)) \end{aligned}$$

$$3.704 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^8} dx$$

**Optimal.** Leaf size=127

$$-\frac{a^3(iA+B)}{2c^8 f(i+\tan(e+fx))^8} + \frac{4a^3(A-2iB)}{7c^8 f(i+\tan(e+fx))^7} + \frac{a^3(iA+5B)}{6c^8 f(i+\tan(e+fx))^6} + \frac{ia^3B}{5c^8 f(i+\tan(e+fx))^5}$$

[Out]  $-1/2*a^3*(I*A+B)/c^8/f/(I+\tan(f*x+e))^8+4/7*a^3*(A-2*I*B)/c^8/f/(I+\tan(f*x+e))^7+1/6*a^3*(I*A+5*B)/c^8/f/(I+\tan(f*x+e))^6+1/5*I*a^3*B/c^8/f/(I+\tan(f*x+e))^5$

**Rubi [A]**

time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{a^3(5B+ia)}{6c^8 f(\tan(e+fx)+i)^6} + \frac{4a^3(A-2iB)}{7c^8 f(\tan(e+fx)+i)^7} - \frac{a^3(B+ia)}{2c^8 f(\tan(e+fx)+i)^8} + \frac{ia^3B}{5c^8 f(\tan(e+fx)+i)^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^8}, x]$

[Out]  $-1/2*(a^3*(I*A + B))/(c^8*f*(I + \text{Tan}[e + f*x])^8) + (4*a^3*(A - (2*I)*B))/(7*c^8*f*(I + \text{Tan}[e + f*x])^7) + (a^3*(I*A + 5*B))/(6*c^8*f*(I + \text{Tan}[e + f*x])^6) + ((I/5)*a^3*B)/(c^8*f*(I + \text{Tan}[e + f*x])^5)$

**Rule 78**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^8} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^9} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(iA+B)}{c^9(i+x)^9} - \frac{4a^2(A-2iB)}{c^9(i+x)^8} - \frac{ia^2(A-5iB)}{c^9(i+x)^7} - \frac{ia^2}{c^9(i+x)^6} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{a^3(iA+B)}{2c^8 f (i + \tan(e + fx))^8} + \frac{4a^3(A-2iB)}{7c^8 f (i + \tan(e + fx))^7} + \frac{ia^2}{6c^8 f (i + \tan(e + fx))^6}$$

**Mathematica [A]**

time = 3.30, size = 182, normalized size = 1.43

$$\frac{ia^2(56(55A+iB)\cos(e+fx)+30(55A+9iB)\cos(3(e+fx))+385A\cos(5(e+fx))+175iB\cos(5(e+fx))-280iA\sin(e+fx)+616B\sin(e+fx)-450iA\sin(3(e+fx))+990B\sin(3(e+fx))-175iA\sin(5(e+fx))+385B\sin(5(e+fx)))(\cos(11e+14fx)+i\sin(11e+14fx))}{53760e^f(\cos(fx)+i\sin(fx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^8,x]
```

```
[Out] ((-1/53760*I)*a^3*(56*(55*A + I*B)*Cos[e + f*x] + 30*(55*A + (9*I)*B)*Cos[3*(e + f*x)] + 385*A*Cos[5*(e + f*x)] + (175*I)*B*Cos[5*(e + f*x)] - (280*I)*A*Sin[e + f*x] + 616*B*Sin[e + f*x] - (450*I)*A*Sin[3*(e + f*x)] + 990*B*Sin[3*(e + f*x)] - (175*I)*A*Sin[5*(e + f*x)] + 385*B*Sin[5*(e + f*x)]*(Cos[11*e + 14*f*x] + I*Sin[11*e + 14*f*x]))/(c^8*f*(Cos[f*x] + I*Sin[f*x])^3)
```

**Maple [A]**

time = 0.35, size = 90, normalized size = 0.71

method	result
derivativedivides	$\frac{a^3 \left( -\frac{-iA-5B}{6(i+\tan(fx+e))^6} - \frac{8iB-4A}{7(i+\tan(fx+e))^7} - \frac{4iA+4B}{8(i+\tan(fx+e))^8} + \frac{iB}{5(i+\tan(fx+e))^5} \right)}{f c^8}$
default	$\frac{a^3 \left( -\frac{-iA-5B}{6(i+\tan(fx+e))^6} - \frac{8iB-4A}{7(i+\tan(fx+e))^7} - \frac{4iA+4B}{8(i+\tan(fx+e))^8} + \frac{iB}{5(i+\tan(fx+e))^5} \right)}{f c^8}$
risch	$-\frac{a^3 e^{16i(fx+e)} B}{512c^8 f} - \frac{ia^3 e^{16i(fx+e)} A}{512c^8 f} - \frac{3e^{14i(fx+e)} B a^3}{448c^8 f} - \frac{5ie^{14i(fx+e)} A a^3}{448c^8 f} - \frac{e^{12i(fx+e)} B a^3}{192c^8 f} - \frac{5ie^{12i(fx+e)} A a^3}{192c^8 f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*a^3/c^8*(-1/6*(-5*B-I*A)/(I+tan(f*x+e))^6-1/7*(8*I*B-4*A)/(I+tan(f*x+e))^7-1/8*(4*I*A+4*B)/(I+tan(f*x+e))^8+1/5*I*B/(I+tan(f*x+e))^5)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 1.12, size = 137, normalized size = 1.08

$$\frac{105(iA+B)a^3e^{16i fx+16i e} + 120(5iA+3B)a^3e^{14i fx+14i e} + 280(5iA+B)a^3e^{12i fx+12i e} + 336(5iA-B)a^3e^{10i fx+10i e} + 210(5iA-3B)a^3e^{8i fx+8i e} + 280(iA-B)a^3e^{6i fx+6i e}}{53760 c^8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x, alg
orithm="fricas")
```

```
[Out] -1/53760*(105*(I*A + B)*a^3*e^(16*I*f*x + 16*I*e) + 120*(5*I*A + 3*B)*a^3*e
^(14*I*f*x + 14*I*e) + 280*(5*I*A + B)*a^3*e^(12*I*f*x + 12*I*e) + 336*(5*I
*A - B)*a^3*e^(10*I*f*x + 10*I*e) + 210*(5*I*A - 3*B)*a^3*e^(8*I*f*x + 8*I
e) + 280*(I*A - B)*a^3*e^(6*I*f*x + 6*I*e))/(c^8*f)
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs.  $2(104) = 208$ .

time = 0.90, size = 496, normalized size = 3.91

$$\frac{(-1803886264320*I*A*a**3*c**40*f**5*\exp(6*I*e) + 1803886264320*B*a**3*c**40*f**5*\exp(6*I*e))*\exp(6*I*f*x) + (-6764573491200*I*A*a**3*c**40*f**5*\exp(8*I*e) + 4058744094720*B*a**3*c**40*f**5*\exp(8*I*e))*\exp(8*I*f*x) + (-10823317585920*I*A*a**3*c**40*f**5*\exp(10*I*e) + 2164663517184*B*a**3*c**40*f**5*\exp(10*I*e))*\exp(10*I*f*x) + (-9019431321600*I*A*a**3*c**40*f**5*\exp(12*I*e) - 1803886264320*B*a**3*c**40*f**5*\exp(12*I*e))*\exp(12*I*f*x) + (-3865470566400*I*A*a**3*c**40*f**5*\exp(14*I*e) - 2319282339840*B*a**3*c**40*f**5*\exp(14*I*e))*\exp(14*I*f*x) + (-6764573491200*I*A*a**3*c**40*f**5*\exp(16*I*e) - 6764573491200*B*a**3*c**40*f**5*\exp(16*I*e))*\exp(16*I*f*x)}{(346346162749440*c**48*f**6), Ne(c**48*f**6, 0)}, (x*(A*a**3*\exp(16*I*e) + 5*A*a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**8,x)
```

```
[Out] Piecewise(((((-1803886264320*I*A*a**3*c**40*f**5*\exp(6*I*e) + 1803886264320*
B*a**3*c**40*f**5*\exp(6*I*e))*\exp(6*I*f*x) + (-6764573491200*I*A*a**3*c**40
*f**5*\exp(8*I*e) + 4058744094720*B*a**3*c**40*f**5*\exp(8*I*e))*\exp(8*I*f*x)
+ (-10823317585920*I*A*a**3*c**40*f**5*\exp(10*I*e) + 2164663517184*B*a**3*
c**40*f**5*\exp(10*I*e))*\exp(10*I*f*x) + (-9019431321600*I*A*a**3*c**40*f**5
*\exp(12*I*e) - 1803886264320*B*a**3*c**40*f**5*\exp(12*I*e))*\exp(12*I*f*x) +
(-3865470566400*I*A*a**3*c**40*f**5*\exp(14*I*e) - 2319282339840*B*a**3*c**
40*f**5*\exp(14*I*e))*\exp(14*I*f*x) + (-6764573491200*I*A*a**3*c**40*f**5*\exp
(16*I*e) - 6764573491200*B*a**3*c**40*f**5*\exp(16*I*e))*\exp(16*I*f*x))/(3463
46162749440*c**48*f**6), Ne(c**48*f**6, 0)), (x*(A*a**3*\exp(16*I*e) + 5*A*a
```

```
**3*exp(14*I*e) + 10*A*a**3*exp(12*I*e) + 10*A*a**3*exp(10*I*e) + 5*A*a**3*
exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(16*I*e) - 3*I*B*a**3*exp(14*I
*e) - 2*I*B*a**3*exp(12*I*e) + 2*I*B*a**3*exp(10*I*e) + 3*I*B*a**3*exp(8*I*
e) + I*B*a**3*exp(6*I*e))/(32*c**8), True))
```

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs.  $2(107) = 214$ .  
time = 1.22, size = 525, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x, alg
orithm="giac")
```

```
[Out] -2/105*(105*A*a^3*tan(1/2*f*x + 1/2*e)^15 + 525*I*A*a^3*tan(1/2*f*x + 1/2*e
)^14 - 105*B*a^3*tan(1/2*f*x + 1/2*e)^14 - 2975*A*a^3*tan(1/2*f*x + 1/2*e)^
13 - 140*I*B*a^3*tan(1/2*f*x + 1/2*e)^13 - 8750*I*A*a^3*tan(1/2*f*x + 1/2*e
)^12 + 1190*B*a^3*tan(1/2*f*x + 1/2*e)^12 + 22365*A*a^3*tan(1/2*f*x + 1/2*e
)^11 + 1596*I*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 39235*I*A*a^3*tan(1/2*f*x + 1
/2*e)^10 - 4711*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 58075*A*a^3*tan(1/2*f*x + 1
/2*e)^9 - 4600*I*B*a^3*tan(1/2*f*x + 1/2*e)^9 - 63300*I*A*a^3*tan(1/2*f*x +
1/2*e)^8 + 7380*B*a^3*tan(1/2*f*x + 1/2*e)^8 + 58075*A*a^3*tan(1/2*f*x + 1
/2*e)^7 + 4600*I*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 39235*I*A*a^3*tan(1/2*f*x +
1/2*e)^6 - 4711*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 22365*A*a^3*tan(1/2*f*x + 1
/2*e)^5 - 1596*I*B*a^3*tan(1/2*f*x + 1/2*e)^5 - 8750*I*A*a^3*tan(1/2*f*x +
1/2*e)^4 + 1190*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 2975*A*a^3*tan(1/2*f*x + 1/2
*e)^3 + 140*I*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 525*I*A*a^3*tan(1/2*f*x + 1/2*
e)^2 - 105*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 105*A*a^3*tan(1/2*f*x + 1/2*e))/(
c^8*f*(tan(1/2*f*x + 1/2*e) + I)^16)
```

**Mupad** [B]

time = 9.47, size = 160, normalized size = 1.26

$$c^8 f (\tan(e + f x)^8 + \tan(e + f x)^7 8i - 28 \tan(e + f x)^6 - \tan(e + f x)^5 56i + 70 \tan(e + f x)^4 + \tan(e + f x)^3 56i - 28 \tan(e + f x)^2 - \tan(e + f x) 8i + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1i
)^8,x)
```

```
[Out] ((a^3*tan(e + f*x)*(50*A - B*16i))/210 - (a^3*(A*20i - 2*B))/210 + (B*a^3*t
an(e + f*x)^3*1i)/5 + (a^3*tan(e + f*x)^2*(A*35i + 49*B))/210)/(c^8*f*(tan(
e + f*x)^3*56i - 28*tan(e + f*x)^2 - tan(e + f*x)*8i + 70*tan(e + f*x)^4 -
tan(e + f*x)^5*56i - 28*tan(e + f*x)^6 + tan(e + f*x)^7*8i + tan(e + f*x)^8
+ 1))
```

$$3.705 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^n}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=115

$$\frac{(iA(1-n) + B(1+n)) {}_2F_1(1, n; 1+n; \frac{1}{2}(1-i \tan(e+fx))) (c-ictan(e+fx))^n}{4afn} + \frac{(iA-B)(c-ictan(e+fx))^n}{2af(1+i \tan(e+fx))}$$

[Out] 1/4\*(I\*A\*(1-n)+B\*(1+n))\*hypergeom([1, n],[1+n],1/2-1/2\*I\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/a/f/n+1/2\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^n/a/f/(1+I\*tan(f\*x+e))

**Rubi [A]**

time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 79, 70}

$$\frac{(B(n+1) + iA(1-n))(c-ictan(e+fx))^n {}_2F_1(1, n; n+1; \frac{1}{2}(1-i \tan(e+fx)))}{4afn} + \frac{(-B+iA)(c-ictan(e+fx))^n}{2af(1+i \tan(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^n)/(a + I\*a\*Tan[e + f\*x]), x]

[Out] ((I\*A\*(1 - n) + B\*(1 + n))\*Hypergeometric2F1[1, n, 1 + n, (1 - I\*Tan[e + f\*x])/2]\*(c - I\*c\*Tan[e + f\*x])^n)/(4\*a\*f\*n) + ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^n)/(2\*a\*f\*(1 + I\*Tan[e + f\*x]))

Rule 70

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 79

Int[((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Di

```
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
  Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
  a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^n}{a + i a \tan(e + fx)} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{-1+n}}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B)(c - i c \tan(e + fx))^n}{2af(1 + i \tan(e + fx))} + \frac{(c(A(1 - n) - iB(1 + \tan(e + fx))))}{4afn}$$

$$= \frac{(iA(1 - n) + B(1 + n)) {}_2F_1(1, n; 1 + n; \frac{1}{2}(1 - i \tan(e + fx)))}{4afn}$$

**Mathematica [A]**

time = 9.53, size = 111, normalized size = 0.97

$$\frac{2^{-1+n} \left(\frac{c}{1+e^{2i(e+fx)}}\right)^n \left((A+iB)(-1+n) + e^{2i(e+fx)}(A(-1+n) + iB(1+n)) {}_2F_1(1, 1-n; 2-n; 1 + e^{2i(e+fx)})\right)}{af(-1+n)(-i + \tan(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
f*x]), x]
```

```
[Out] (2^(-1 + n)*(c/(1 + E^((2*I)*(e + f*x))))^n*((A + I*B)*(-1 + n) + E^((2*I)*
(e + f*x))*(A*(-1 + n) + I*B*(1 + n))*Hypergeometric2F1[1, 1 - n, 2 - n, 1
+ E^((2*I)*(e + f*x))]))/(a*f*(-1 + n)*(-I + Tan[e + f*x]))
```

**Maple [F]**

time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(fx + e))(c - i c \tan(fx + e))^n}{a + i a \tan(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)), x)
```

```
[Out] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)), x)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/2\*((A - I\*B)\*e^(2\*I\*f\*x + 2\*I\*e) + A + I\*B)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^n\*e^(-2\*I\*f\*x - 2\*I\*e)/a, x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A(-ic \tan(e+fx)+c)^n}{\tan(e+fx)-i} dx + \int \frac{B(-ic \tan(e+fx)+c)^n \tan(e+fx)}{\tan(e+fx)-i} dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e)),x)

[Out] -I\*(Integral(A\*(-I\*c\*tan(e + f\*x) + c))^n/(tan(e + f\*x) - I), x) + Integral(B\*(-I\*c\*tan(e + f\*x) + c))^n\*tan(e + f\*x)/(tan(e + f\*x) - I), x)/a

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^n/(I\*a\*tan(f\*x + e) + a), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) \operatorname{li})^n}{a + a \tan(e + f x) \operatorname{li}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1i), x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1i), x)
```

$$3.706 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^4}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=157

$$\frac{4(3A+5iB)c^4x}{a} - \frac{4(3iA-5B)c^4 \log(\cos(e+fx))}{af} - \frac{8(A+iB)c^4}{af(i-\tan(e+fx))} + \frac{(5A+12iB)c^4 \tan(e+fx)}{af} - (i$$

[Out]  $-4*(3*A+5*I*B)*c^4*x/a-4*(3*I*A-5*B)*c^4*\ln(\cos(f*x+e))/a/f-8*(A+I*B)*c^4/a/f/(I-\tan(f*x+e))+5*(A+12*I*B)*c^4*\tan(f*x+e)/a/f-1/2*(I*A-5*B)*c^4*\tan(f*x+e)^2/a/f-1/3*I*B*c^4*\tan(f*x+e)^3/a/f$

**Rubi [A]**

time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{c^4(-5B+iA)\tan^2(e+fx)}{2af} + \frac{c^4(5A+12iB)\tan(e+fx)}{af} - \frac{8c^4(A+iB)}{af(-\tan(e+fx)+i)} - \frac{4c^4(-5B+3iA)\log(\cos(e+fx))}{af} - \frac{4c^4x(3A+5iB)}{a} - \frac{iBc^4\tan^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^4/(a+I*a*\text{Tan}[e+f*x]), x]$

[Out]  $(-4*(3*A+(5*I)*B)*c^4*x)/a - (4*((3*I)*A-5*B)*c^4*\text{Log}[\text{Cos}[e+f*x]])/(a*f) - (8*(A+I*B)*c^4)/(a*f*(I-\text{Tan}[e+f*x])) + ((5*A+(12*I)*B)*c^4*\text{Tan}[e+f*x])/(a*f) - ((I*A-5*B)*c^4*\text{Tan}[e+f*x]^2)/(2*a*f) - ((I/3)*B*c^4*\text{Tan}[e+f*x]^3)/(a*f)$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^4}{a + i a \tan(e + fx)} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^3}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{(5A+12iB)c^3}{a^2} + \frac{(-iA+5B)c^3x}{a^2} - \frac{iBc^3x^2}{a^2} - \frac{8(A+iB)c^3x^3}{a^2}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{4(3A + 5iB)c^4x}{a} - \frac{4(3iA - 5B)c^4 \log(\cos(e + fx))}{af}$$

**Mathematica [A]**

time = 1.60, size = 260, normalized size = 1.66

$\frac{c^4(\cos(fx) + i \sin(fx)) \left( (12i - 3iA + 5iB) \log(\cos(e + fx)) \cos\left(\frac{e}{2}\right) + i \sin\left(\frac{e}{2}\right) \right)^2 - 24(3A + 5iB) \text{ArcTan}(\tan(fx))(\cos(e) + i \sin(e)) + 24(A + iB) \cos(2fx)(\cos(e) + i \sin(e)) + 24(A + iB)(\cos(e) - i \sin(e)) \sin(2fx) + 2(15A + 37iB) \sec(e + fx) \sin(fx) + i \tan(e) + 2B \sec^2(e + fx) \sin(fx) + \cos(e) \sec^2(e + fx) (-1 + \tan(e))(2A + 5iB) + 2B \tan(e) \right) (A + B \tan(e + fx))}{6(A \cos(e + fx) + B \sin(e + fx))(a + i a \tan(e + fx))}$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4)/(a + I\*a\*Tan[e + f\*x]),x]

[Out] (c^4\*(Cos[f\*x] + I\*Sin[f\*x])\*(12\*((-3\*I)\*A + 5\*B)\*Log[Cos[e + f\*x]^2]\*(Cos[e/2] + I\*Sin[e/2])^2 - 24\*(3\*A + (5\*I)\*B)\*ArcTan[Tan[f\*x]]\*(Cos[e] + I\*Sin[e]) + 24\*(A + I\*B)\*Cos[2\*f\*x]\*(I\*Cos[e] + Sin[e]) + 24\*(A + I\*B)\*(Cos[e] - I\*Sin[e])\*Sin[2\*f\*x] + 2\*(15\*A + (37\*I)\*B)\*Sec[e + f\*x]\*Sin[f\*x]\*(1 + I\*Tan[e]) + 2\*B\*Sec[e + f\*x]^3\*Sin[f\*x]\*(-I + Tan[e]) + Cos[e]\*Sec[e + f\*x]^2\*(-I + Tan[e])\*(3\*(A + (5\*I)\*B) + 2\*B\*Tan[e]))\*(A + B\*Tan[e + f\*x]))/(6\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x]))

**Maple [A]**

time = 0.22, size = 106, normalized size = 0.68

method	result
derivativedivides	$c^4 \left( \frac{5B(\tan^2(fx+e))}{2} - \frac{iB(\tan^3(fx+e))}{3} + 5A \tan(fx+e) - \frac{iA(\tan^2(fx+e))}{2} + 12iB \tan(fx+e) - \frac{-8iB-8A}{-i+\tan(fx+e)} + (12iA-20B) \right) / fa$
default	$c^4 \left( \frac{5B(\tan^2(fx+e))}{2} - \frac{iB(\tan^3(fx+e))}{3} + 5A \tan(fx+e) - \frac{iA(\tan^2(fx+e))}{2} + 12iB \tan(fx+e) - \frac{-8iB-8A}{-i+\tan(fx+e)} + (12iA-20B) \right) / fa$
risch	$-\frac{4c^4 e^{-2i(fx+e)} B}{af} + \frac{4ic^4 e^{-2i(fx+e)} A}{af} - \frac{40ic^4 Bx}{a} - \frac{24c^4 Ax}{a} - \frac{40ic^4 Be}{fa} - \frac{24c^4 Ae}{fa} - \frac{2c^4(-12iA e^{4i(fx+e)} + 12iB e^{4i(fx+e)})}{a}$
norman	$\frac{(20ic^4 B + 13A c^4) \tan(fx+e) - 4(5ic^4 B + 3A c^4) x - 17ic^4 A + 21B c^4 + 5(7ic^4 B + 3A c^4) (\tan^3(fx+e)) - 4(5ic^4 B + 3A c^4) x (\tan^2(fx+e))}{af} - \frac{4(5ic^4 B + 3A c^4) x (\tan^2(fx+e))}{a} - \frac{-17ic^4 A + 21B c^4 + 5(7ic^4 B + 3A c^4) (\tan^3(fx+e)) - 4(5ic^4 B + 3A c^4) x (\tan^2(fx+e))}{2af} + \frac{5(7ic^4 B + 3A c^4) (\tan^3(fx+e)) - 4(5ic^4 B + 3A c^4) x (\tan^2(fx+e))}{3af} - \frac{4(5ic^4 B + 3A c^4) x (\tan^2(fx+e))}{a} / (1 + \tan^2(fx+e))$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e)),x,method=\_RETU  
RNVERBOSE)

[Out] 1/f\*c^4/a\*(5/2\*B\*tan(f\*x+e)^2-1/3\*I\*B\*tan(f\*x+e)^3+5\*A\*tan(f\*x+e)-1/2\*I\*A\*t  
an(f\*x+e)^2+12\*I\*B\*tan(f\*x+e)-(-8\*I\*B-8\*A)/(-I+tan(f\*x+e))+(12\*I\*A-20\*B)\*ln  
(-I+tan(f\*x+e)))

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e)),x, algo  
rithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than  
twice the leaf count of optimal. 311 vs. 2(142) = 284.

time = 2.83, size = 311, normalized size = 1.98

$$\frac{2(12(3A+5B)c^2fz^{2n+1}+6(-1A+B)c^2+6(3A+5B)c^2fz+(-3A+5B)c^2e^{2fz+2e})+3(12(3A+5B)c^2fz+5(-3A+5B)c^2e^{2fz+2e})+(12(3A+5B)c^2fz+11(-3A+5B)c^2e^{2fz+2e})+6((3A-5B)c^{2n}z^{2n+1})+3(3A-5B)c^{2n}z^{2n+1})+3(3A-5B)c^{2n}z^{2n+1})\log(e^{2n}z^{2n+1}+1)}{3(af^{2n}z^{2n+1}+3af^{2n}z^{2n+1})+3af^{2n}z^{2n+1}+af^{2n}z^{2n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e)),x, algo  
rithm="fricas")

[Out] -2/3\*(12\*(3\*A + 5\*I\*B)\*c^4\*f\*x\*e^(8\*I\*f\*x + 8\*I\*e) + 6\*(-I\*A + B)\*c^4 + 6\*(  
6\*(3\*A + 5\*I\*B)\*c^4\*f\*x + (-3\*I\*A + 5\*B)\*c^4)\*e^(6\*I\*f\*x + 6\*I\*e) + 3\*(12\*(  
3\*A + 5\*I\*B)\*c^4\*f\*x + 5\*(-3\*I\*A + 5\*B)\*c^4)\*e^(4\*I\*f\*x + 4\*I\*e) + (12\*(3\*A  
+ 5\*I\*B)\*c^4\*f\*x + 11\*(-3\*I\*A + 5\*B)\*c^4)\*e^(2\*I\*f\*x + 2\*I\*e) + 6\*((3\*I\*A  
- 5\*B)\*c^4\*e^(8\*I\*f\*x + 8\*I\*e) + 3\*(3\*I\*A - 5\*B)\*c^4\*e^(6\*I\*f\*x + 6\*I\*e) +  
3\*(3\*I\*A - 5\*B)\*c^4\*e^(4\*I\*f\*x + 4\*I\*e) + (3\*I\*A - 5\*B)\*c^4\*e^(2\*I\*f\*x + 2  
I\*e))\*log(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(a\*f\*e^(8\*I\*f\*x + 8\*I\*e) + 3\*a\*f\*e^(6\*I  
\*f\*x + 6\*I\*e) + 3\*a\*f\*e^(4\*I\*f\*x + 4\*I\*e) + a\*f\*e^(2\*I\*f\*x + 2\*I\*e))

**Sympy** [A]

time = 0.57, size = 328, normalized size = 2.09

$$\frac{30iAc^4 - 74Bc^4 + (54iAc^4e^{2ie} - 114Bc^4e^{2ie})e^{2ifx} + (24iAc^4e^{4ie} - 48Bc^4e^{4ie})e^{4ifx}}{3afe^{2ie}e^{2ifx} + 9afe^{4ie}e^{4ifx} + 9afe^{2ie}e^{2ifx} + 3af} + \begin{cases} \frac{(4iAc^4 - 4Bc^4)e^{-2ie}e^{-2ifx}}{af} & \text{for } af e^{2ie} \neq 0 \\ x \left( \frac{-24Ac^4 - 40Bc^4}{a} + \frac{(-24Ac^4e^{2ie} + 8Ac^4 - 40Bc^4e^{2ie} + 8iBc^4)e^{-2ie}}{a} \right) & \text{otherwise} \end{cases} - \frac{4ie^4 \cdot (3A + 5iB) \log(e^{2ifx} + e^{-2ie})}{af} + \frac{x(-24Ac^4 - 40iBc^4)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e)),x)

```
[Out] (30*I*A*c**4 - 74*B*c**4 + (54*I*A*c**4*exp(2*I*e) - 114*B*c**4*exp(2*I*e))
*exp(2*I*f*x) + (24*I*A*c**4*exp(4*I*e) - 48*B*c**4*exp(4*I*e))*exp(4*I*f*x
))/(3*a*f*exp(6*I*e)*exp(6*I*f*x) + 9*a*f*exp(4*I*e)*exp(4*I*f*x) + 9*a*f*e
xp(2*I*e)*exp(2*I*f*x) + 3*a*f) + Piecewise(((4*I*A*c**4 - 4*B*c**4)*exp(-2
*I*e)*exp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)), (x*(-24*A*c**4 - 40*I*
B*c**4)/a + (-24*A*c**4*exp(2*I*e) + 8*A*c**4 - 40*I*B*c**4*exp(2*I*e) + 8*
I*B*c**4)*exp(-2*I*e)/a), True)) - 4*I*c**4*(3*A + 5*I*B)*log(exp(2*I*f*x)
+ exp(-2*I*e))/(a*f) + x*(-24*A*c**4 - 40*I*B*c**4)/a
```

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs.  $2(142) = 284$ .

time = 0.83, size = 445, normalized size = 2.83

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algo
rithm="giac")
```

```
[Out] 2/3*(6*(-3*I*A*c^4 + 5*B*c^4)*log(tan(1/2*f*x + 1/2*e) + 1)/a - 12*(-3*I*A*
c^4 + 5*B*c^4)*log(tan(1/2*f*x + 1/2*e) - 1)/a - 6*(3*I*A*c^4 - 5*B*c^4)*lo
g(tan(1/2*f*x + 1/2*e) - 1)/a - 6*(9*I*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 15*B*
c^4*tan(1/2*f*x + 1/2*e)^2 + 22*A*c^4*tan(1/2*f*x + 1/2*e) + 34*I*B*c^4*tan
(1/2*f*x + 1/2*e) - 9*I*A*c^4 + 15*B*c^4)/(a*(tan(1/2*f*x + 1/2*e) - 1)^2)
- (-33*I*A*c^4*tan(1/2*f*x + 1/2*e)^6 + 55*B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1
5*A*c^4*tan(1/2*f*x + 1/2*e)^5 + 36*I*B*c^4*tan(1/2*f*x + 1/2*e)^5 + 102*I*
A*c^4*tan(1/2*f*x + 1/2*e)^4 - 180*B*c^4*tan(1/2*f*x + 1/2*e)^4 - 30*A*c^4*
tan(1/2*f*x + 1/2*e)^3 - 76*I*B*c^4*tan(1/2*f*x + 1/2*e)^3 - 102*I*A*c^4*ta
n(1/2*f*x + 1/2*e)^2 + 180*B*c^4*tan(1/2*f*x + 1/2*e)^2 + 15*A*c^4*tan(1/2*
f*x + 1/2*e) + 36*I*B*c^4*tan(1/2*f*x + 1/2*e) + 33*I*A*c^4 - 55*B*c^4)/((t
an(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

**Mupad** [B]

time = 8.80, size = 205, normalized size = 1.31

$$\frac{\ln(\tan(e + f x) - i) \left( -\frac{20 B c^4}{a} + \frac{A c^4 12 i}{a} \right)}{f} - \frac{\tan(e + f x)^2 \left( -\frac{B c^4}{a} + \frac{c^4 (A + B 3 i) 1 i}{2 a} \right)}{f} - \frac{\left( \frac{4 A c^4 + B c^4 12 i}{a} \right) 1 i - \left( \frac{12 A c^4 + B c^4 20 i}{a} \right) 1 i}{f (1 + \tan(e + f x) 1 i)} + \frac{\tan(e + f x) \left( \frac{2 c^4 (A + B 3 i)}{a} + \frac{B c^4 3 i}{a} - \frac{c^4 (-B + A 1 i) 3 i}{a} \right)}{f} - \frac{B c^4 \tan(e + f x)^3 1 i}{3 a f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^4)/(a + a*tan(e + f*x)*1i
),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*((A*c^4*12i)/a - (20*B*c^4)/a))/f - (tan(e + f*x)^2
*((c^4*(A + B*3i)*1i)/(2*a) - (B*c^4)/a))/f - (((4*A*c^4 + B*c^4*12i)*1i)/a
- ((12*A*c^4 + B*c^4*20i)*1i)/a)/(f*(tan(e + f*x)*1i + 1)) + (tan(e + f*x)
*((2*c^4*(A + B*3i))/a + (B*c^4*3i)/a - (c^4*(A*1i - B)*3i)/a))/f - (B*c^4*
tan(e + f*x)^3*1i)/(3*a*f)
```

$$3.707 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^3}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=121

$$\frac{4(A+2iB)c^3x}{a} - \frac{4(iA-2B)c^3 \log(\cos(e+fx))}{af} - \frac{4(A+iB)c^3}{af(i-\tan(e+fx))} + \frac{(A+4iB)c^3 \tan(e+fx)}{af} + \frac{Bc^3 \tan^2(e+fx)}{2af}$$

[Out]  $-4*(A+2*I*B)*c^3*x/a-4*(I*A-2*B)*c^3*\ln(\cos(f*x+e))/a/f-4*(A+I*B)*c^3/a/f/(I-\tan(f*x+e))+ (A+4*I*B)*c^3*\tan(f*x+e)/a/f+1/2*B*c^3*\tan(f*x+e)^2/a/f$

**Rubi [A]**

time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{c^3(A+4iB)\tan(e+fx)}{af} - \frac{4c^3(A+iB)}{af(-\tan(e+fx)+i)} - \frac{4c^3(-2B+iA)\log(\cos(e+fx))}{af} - \frac{4c^3x(A+2iB)}{a} + \frac{Bc^3 \tan^2(e+fx)}{2af}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^3/(a + I*a*\text{Tan}[e + f*x]), x]$

[Out]  $(-4*(A + (2*I)*B)*c^3*x)/a - (4*(I*A - 2*B)*c^3*\text{Log}[\text{Cos}[e + f*x]])/(a*f) - (4*(A + I*B)*c^3)/(a*f*(I - \text{Tan}[e + f*x])) + ((A + (4*I)*B)*c^3*\text{Tan}[e + f*x])/a + (B*c^3*\text{Tan}[e + f*x]^2)/(2*a*f)$

Rule 78

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(c + d*\text{tan}[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^3}{a + i a \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^2}{(a+iax)^2} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{(A+4iB)c^2}{a^2} + \frac{Bc^2x}{a^2} - \frac{4(A+iB)c^2}{a^2(-i+x)^2} + \frac{4i(A+2iB)c^2}{a^2(-i+x)} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{4(A + 2iB)c^3x}{a} - \frac{4(iA - 2B)c^3 \log(\cos(e + fx))}{af} - \frac{4i(A + 2iB)c^3}{af} - \frac{4i(A + 2iB)c^3}{af}$$

**Mathematica [A]**

time = 3.28, size = 212, normalized size = 1.75

$$\frac{c^3(\cos(fx) + i \sin(fx))(-8(A + 2iB) \text{ArcTan}(\tan(fx))(\cos(e) + i \sin(e)) + 4(-iA + 2B) \log(\cos^2(e + fx))(\cos(e) + i \sin(e)) + B \sec^2(e + fx)(\cos(e) + i \sin(e)) + 4(A + iB) \cos(2fx)(i \cos(e) + \sin(e)) + 4(A + iB)(\cos(e) - i \sin(e)) \sin(2fx) + 2(A + 4iB) \sec(e + fx) \sin(fx)(1 + i \tan(e))) (A + B \tan(e + fx))}{2f(A \cos(e + fx) + B \sin(e + fx))(a + ia \tan(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e + f*x]), x]
```

```
[Out] (c^3*(Cos[f*x] + I*Sin[f*x])*(-8*(A + (2*I)*B)*ArcTan[Tan[f*x]]*(Cos[e] + I*Sin[e]) + 4*(-I)*A + 2*B)*Log[Cos[e + f*x]^2*(Cos[e] + I*Sin[e]) + B*Sec[e + f*x]^2*(Cos[e] + I*Sin[e]) + 4*(A + I*B)*Cos[2*f*x]*(I*Cos[e] + Sin[e]) + 4*(A + I*B)*(Cos[e] - I*Sin[e])*Sin[2*f*x] + 2*(A + (4*I)*B)*Sec[e + f*x]*Sin[f*x]*(1 + I*Tan[e])*(A + B*Tan[e + f*x]))/(2*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*(a + I*a*Tan[e + f*x]))
```

**Maple [A]**

time = 0.22, size = 81, normalized size = 0.67

method	result
derivativedivides	$\frac{c^3 \left( A \tan(fx+e) + 4iB \tan(fx+e) + \frac{B(\tan^2(fx+e))}{2} + (4iA-8B) \ln(-i+\tan(fx+e)) - \frac{-4iB-4A}{-i+\tan(fx+e)} \right)}{fa}$
default	$\frac{c^3 \left( A \tan(fx+e) + 4iB \tan(fx+e) + \frac{B(\tan^2(fx+e))}{2} + (4iA-8B) \ln(-i+\tan(fx+e)) - \frac{-4iB-4A}{-i+\tan(fx+e)} \right)}{fa}$
norman	$\frac{\frac{(4iB c^3 + A c^3) (\tan^3(fx+e))}{af} + \frac{(8iB c^3 + 5A c^3) \tan(fx+e)}{af} - \frac{4(2iB c^3 + A c^3) x}{a} - \frac{8ic^3 A + 9B c^3}{2af} - \frac{4(2iB c^3 + A c^3) x (\tan^2(fx+e))}{a}}{1 + \tan^2(fx+e)}$
risch	$-\frac{2c^3 e^{-2i(fx+e)} B}{af} + \frac{2ic^3 e^{-2i(fx+e)} A}{af} - \frac{16ic^3 Bx}{a} - \frac{8c^3 Ax}{a} - \frac{16ic^3 Be}{fa} - \frac{8c^3 Ae}{fa} - \frac{2c^3(-iA e^{2i(fx+e)} + 3B e^{2i(fx+e)})}{af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)), x, method=_RETURNVERBOSE)
```

[Out]  $1/f*c^3/a*(A*\tan(f*x+e)+4*I*B*\tan(f*x+e)+1/2*B*\tan(f*x+e)^2+(4*I*A-8*B)*\ln(-I+\tan(f*x+e))-(-4*I*B-4*A)/(-I+\tan(f*x+e)))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expr: undefined: 0 to a negative exponent.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(111) = 222.

time = 3.24, size = 229, normalized size = 1.89

$$\frac{2(4(A+2iB)c^3fxe^{6i f x+6i e})+(-iA+B)c^3+2(4(A+2iB)c^3fx+(-iA+2B)c^3)e^{4i f x+4i e}+(4(A+2iB)c^3fx+3(-iA+2B)c^3)e^{2i f x+2i e}+2((iA-2B)c^3e^{6i f x+6i e}+2(iA-2B)c^3e^{4i f x+4i e}+(iA-2B)c^3e^{2i f x+2i e})\log(e^{2i f x+2i e}+1))}{af e^{6i f x+6i e}+2af e^{4i f x+4i e}+af e^{2i f x+2i e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algorithm="fricas")`

[Out]  $-2*(4*(A+2*I*B)*c^3*f*x*e^{(6*I*f*x+6*I*e)}+(-I*A+B)*c^3+2*(4*(A+2*I*B)*c^3*f*x+(-I*A+2*B)*c^3)*e^{(4*I*f*x+4*I*e)}+(4*(A+2*I*B)*c^3*f*x+3*(-I*A+2*B)*c^3)*e^{(2*I*f*x+2*I*e)}+2*((I*A-2*B)*c^3*e^{(6*I*f*x+6*I*e)}+2*(I*A-2*B)*c^3*e^{(4*I*f*x+4*I*e)}+(I*A-2*B)*c^3*e^{(2*I*f*x+2*I*e)})*\log(e^{(2*I*f*x+2*I*e)}+1)/(a*f*e^{(6*I*f*x+6*I*e)}+2*a*f*e^{(4*I*f*x+4*I*e)}+a*f*e^{(2*I*f*x+2*I*e)})$

**Sympy** [A]

time = 0.49, size = 265, normalized size = 2.19

$$\frac{2iAc^3-8Bc^3+(2iAc^3e^{2ie}-6Bc^3e^{2ie})e^{2ifx}}{afe^{4ie}e^{4ifx}+2afe^{2ie}e^{2ifx}+af}+\begin{cases} \frac{(2iAc^3-2Bc^3)e^{-2ie}e^{-2ifx}}{af} & \text{for } af e^{2ie} \neq 0 \\ x\left(\frac{-8Ac^3-16iBc^3}{a}+\frac{(-8Ac^3e^{2ie}+4Ac^3-16iBc^3e^{2ie}+4iBc^3)e^{-2ie}}{a}\right) & \text{otherwise} \end{cases}-\frac{4ic^3(A+2iB)\log(e^{2ifx}+e^{-2ie})}{af}+\frac{x(-8Ac^3-16iBc^3)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x)`

[Out]  $(2*I*A*c**3-8*B*c**3+(2*I*A*c**3*\exp(2*I*e)-6*B*c**3*\exp(2*I*e))*\exp(2*I*f*x))/(a*f*\exp(4*I*e)*\exp(4*I*f*x)+2*a*f*\exp(2*I*e)*\exp(2*I*f*x)+a*f)+\text{Piecewise}(((2*I*A*c**3-2*B*c**3)*\exp(-2*I*e)*\exp(-2*I*f*x)/(a*f), \text{Ne}(a*f*\exp(2*I*e), 0)), (x*(-(-8*A*c**3-16*I*B*c**3)/a+(-8*A*c**3*\exp(2*I*e)+4*A*c**3-16*I*B*c**3*\exp(2*I*e)+4*I*B*c**3)*\exp(-2*I*e)/a), \text{True})$



) - 4\*I\*c\*\*3\*(A + 2\*I\*B)\*log(exp(2\*I\*f\*x) + exp(-2\*I\*e))/(a\*f) + x\*(-8\*A\*c\*\*3 - 16\*I\*B\*c\*\*3)/a

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(111) = 222.

time = 0.69, size = 323, normalized size = 2.67

$$\frac{2 \left( \frac{2(-4A^2+2B^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e))}{a} - \frac{4(-4A^2+2B^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e))}{a} - \frac{2(4A^2-2B^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e))}{a} - \frac{5A^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5+8B^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5-2A^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5+7B^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5-10A^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5-14B^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5-20A^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5-7B^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5-5A^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5+8B^2\log(\frac{1}{2}fx+\frac{1}{2}e)^5}{(\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+\tan(\frac{1}{2}fx+\frac{1}{2}e)-\tan(\frac{1}{2}fx+\frac{1}{2}e))^2} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^3/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] 2\*(2\*(-I\*A\*c^3 + 2\*B\*c^3)\*log(tan(1/2\*f\*x + 1/2\*e) + 1)/a - 4\*(-I\*A\*c^3 + 2\*B\*c^3)\*log(tan(1/2\*f\*x + 1/2\*e) - I)/a - 2\*(I\*A\*c^3 - 2\*B\*c^3)\*log(tan(1/2\*f\*x + 1/2\*e) - 1)/a - (5\*A\*c^3\*tan(1/2\*f\*x + 1/2\*e)^5 + 8\*I\*B\*c^3\*tan(1/2\*f\*x + 1/2\*e)^5 - 2\*I\*A\*c^3\*tan(1/2\*f\*x + 1/2\*e)^4 + 7\*B\*c^3\*tan(1/2\*f\*x + 1/2\*e)^4 - 10\*A\*c^3\*tan(1/2\*f\*x + 1/2\*e)^3 - 14\*I\*B\*c^3\*tan(1/2\*f\*x + 1/2\*e)^3 + 2\*I\*A\*c^3\*tan(1/2\*f\*x + 1/2\*e)^2 - 7\*B\*c^3\*tan(1/2\*f\*x + 1/2\*e)^2 + 5\*A\*c^3\*tan(1/2\*f\*x + 1/2\*e) + 8\*I\*B\*c^3\*tan(1/2\*f\*x + 1/2\*e))/((tan(1/2\*f\*x + 1/2\*e)^3 - I\*tan(1/2\*f\*x + 1/2\*e)^2 - tan(1/2\*f\*x + 1/2\*e) + I)^2\*a)/f

**Mupad** [B]

time = 8.70, size = 136, normalized size = 1.12

$$\frac{\ln(\tan(e+fx)-i) \left( -\frac{8Bc^3}{a} + \frac{Ac^3 4i}{a} \right)}{f} + \frac{\tan(e+fx) \left( \frac{c^3(A+B2i)}{a} + \frac{Bc^3 2i}{a} \right)}{f} + \frac{\frac{4Bc^3}{a} + \frac{(4Ac^3+Bc^3 8i) li}{a}}{f(1+\tan(e+fx) li)} + \frac{Bc^3 \tan(e+fx)^2}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^3)/(a + a\*tan(e + f\*x)\*1i),x)

[Out] (log(tan(e + f\*x) - 1i)\*((A\*c^3\*4i)/a - (8\*B\*c^3)/a))/f + (tan(e + f\*x)\*((c^3\*(A + B\*2i))/a + (B\*c^3\*2i)/a))/f + (((4\*A\*c^3 + B\*c^3\*8i)\*1i)/a + (4\*B\*c^3)/a)/(f\*(tan(e + f\*x)\*1i + 1)) + (B\*c^3\*tan(e + f\*x)^2)/(2\*a\*f)

$$3.708 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^2}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=96

$$\frac{(A+3iB)c^2x}{a} - \frac{(iA-3B)c^2 \log(\cos(e+fx))}{af} - \frac{2(A+iB)c^2}{af(i-\tan(e+fx))} + \frac{iBc^2 \tan(e+fx)}{af}$$

[Out]  $-(A+3I*B)*c^2*x/a-(I*A-3*B)*c^2*\ln(\cos(f*x+e))/a/f-2*(A+I*B)*c^2/a/f/(I-\tan(f*x+e))+I*B*c^2*\tan(f*x+e)/a/f$

**Rubi [A]**

time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{2c^2(A+iB)}{af(-\tan(e+fx)+i)} - \frac{c^2(-3B+iA) \log(\cos(e+fx))}{af} - \frac{c^2x(A+3iB)}{a} + \frac{iBc^2 \tan(e+fx)}{af}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^2/(a+I*a*\text{Tan}[e+f*x]), x]$

[Out]  $-(((A+(3*I)*B)*c^2*x)/a) - ((I*A-3*B)*c^2*\text{Log}[\text{Cos}[e+f*x]])/(a*f) - (2*(A+I*B)*c^2)/(a*f*(I-\text{Tan}[e+f*x])) + (I*B*c^2*\text{Tan}[e+f*x])/(a*f)$

**Rule 78**

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^2}{a + i a \tan(e + fx)} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{iBc}{a^2} - \frac{2(A+iB)c}{a^2(-i+x)^2} + \frac{i(A+3iB)c}{a^2(-i+x)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(A + 3iB)c^2 x}{a} - \frac{(iA - 3B)c^2 \log(\cos(e + fx))}{af} - \frac{3c^2 \log(\cos(e + fx))}{af}$$

**Mathematica [A]**

time = 1.75, size = 184, normalized size = 1.92

$$\frac{c^2(\cos(fx) + i \sin(fx))(-2(A + 3iB)\text{ArcTan}(\tan(fx))(\cos(e) + i \sin(e)) + (-iA + 3B)\log(\cos^2(e + fx))(\cos(e) + i \sin(e)) + 2(A + iB)\cos(2fx)(i \cos(e) + \sin(e)) + 2(A + iB)(\cos(e) - i \sin(e))\sin(2fx) - 2B \sec(e + fx)\sin(fx)(-i + \tan(e)))(A + B \tan(e + fx))}{2f(A \cos(e + fx) + B \sin(e + fx))(a + ia \tan(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e + f*x]), x]
```

```
[Out] (c^2*(Cos[f*x] + I*Sin[f*x])*(-2*(A + (3*I)*B)*ArcTan[Tan[f*x]]*(Cos[e] + I*Sin[e]) + ((-I)*A + 3*B)*Log[Cos[e + f*x]^2]*(Cos[e] + I*Sin[e]) + 2*(A + I*B)*Cos[2*f*x]*(I*Cos[e] + Sin[e]) + 2*(A + I*B)*(Cos[e] - I*Sin[e])*Sin[2*f*x] - 2*B*Sec[e + f*x]*Sin[f*x]*(-I + Tan[e]))*(A + B*Tan[e + f*x]))/(2*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*(a + I*a*Tan[e + f*x]))
```

**Maple [A]**

time = 0.23, size = 62, normalized size = 0.65

method	result
derivativedivides	$\frac{c^2 \left( iB \tan(fx+e) - \frac{-2iB-2A}{-i+\tan(fx+e)} + (iA-3B) \ln(-i+\tan(fx+e)) \right)}{fa}$
default	$\frac{c^2 \left( iB \tan(fx+e) - \frac{-2iB-2A}{-i+\tan(fx+e)} + (iA-3B) \ln(-i+\tan(fx+e)) \right)}{fa}$
norman	$\frac{\left( \frac{3ic^2B+2Ac^2}{af} \tan(fx+e) + \frac{ic^2B}{af} (\tan^3(fx+e)) - \frac{(3ic^2B+Ac^2)x}{a} - \frac{-2iAc^2+2Bc^2}{af} - \frac{(3ic^2B+Ac^2)x(\tan^2(fx+e))}{a} \right)}{1+\tan^2(fx+e)} - \frac{(-iA+3B)c^2 \ln(-i+\tan(fx+e))}{af}$
risch	$-\frac{c^2 e^{-2i(fx+e)} B}{af} + \frac{ic^2 e^{-2i(fx+e)} A}{af} - \frac{6ic^2 Bx}{a} - \frac{2c^2 Ax}{a} - \frac{6ic^2 Be}{af} - \frac{2c^2 Ae}{af} - \frac{2c^2 B}{fa(e^{2i(fx+e)}+1)} + \frac{3c^2 \ln(-i+\tan(fx+e))}{af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)), x, method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^2/a*(I*B*tan(f*x+e)-(-2*I*B-2*A)/(-I+tan(f*x+e)))+(I*A-3*B)*ln(-I+tan(f*x+e))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algo
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 4.47, size = 160, normalized size = 1.67

$$\frac{2(A+3iB)c^2fxe^{4ifx+4ie} - (iA-B)c^2 + (2(A+3iB)c^2fx - (iA-3B)c^2)e^{2ifx+2ie} - ((-iA+3B)c^2e^{4ifx+4ie} + (-iA+3B)c^2e^{2ifx+2ie})\log(e^{2ifx+2ie}+1)}{afe^{4ifx+4ie} + afe^{2ifx+2ie}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algo
ithm="fricas")
```

```
[Out] -(2*(A + 3*I*B)*c^2*f*x*e^(4*I*f*x + 4*I*e) - (I*A - B)*c^2 + (2*(A + 3*I*B)
)*c^2*f*x - (I*A - 3*B)*c^2)*e^(2*I*f*x + 2*I*e) - ((-I*A + 3*B)*c^2*e^(4*I
*f*x + 4*I*e) + (-I*A + 3*B)*c^2*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I
e) + 1))/(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))
```

**Sympy [A]**

time = 0.38, size = 194, normalized size = 2.02

$$-\frac{2Bc^2}{afe^{2ie}e^{2ifx} + af} + \begin{cases} \frac{(iAc^2 - Bc^2)e^{-2ie}e^{-2ifx}}{af} & \text{for } afe^{2ie} \neq 0 \\ x\left(-\frac{-2Ac^2 - 6iBc^2}{a} + \frac{(-2Ac^2e^{2ie} + 2Ac^2 - 6iBc^2e^{2ie} + 2iBc^2)e^{-2ie}}{a}\right) & \text{otherwise} \end{cases} - \frac{ic^2(A + 3iB)\log(e^{2ifx} + e^{-2ie})}{af} + \frac{x(-2Ac^2 - 6iBc^2)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -2*B*c**2/(a*f*exp(2*I*e)*exp(2*I*f*x) + a*f) + Piecewise(((I*A*c**2 - B*c*
*2)*exp(-2*I*e)*exp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)), (x*(-(-2*A*c**
2 - 6*I*B*c**2)/a + (-2*A*c**2*exp(2*I*e) + 2*A*c**2 - 6*I*B*c**2*exp(2*I*e
) + 2*I*B*c**2)*exp(-2*I*e)/a), True)) - I*c**2*(A + 3*I*B)*log(exp(2*I*f*x
) + exp(-2*I*e))/(a*f) + x*(-2*A*c**2 - 6*I*B*c**2)/a
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(87) = 174.

time = 0.64, size = 283, normalized size = 2.95

$$\frac{(-1A^2+3B^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1) + 2(1A^2-3B^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e)-1) - (1A^2-3B^2)\log(\tan(\frac{1}{2}fx+\frac{1}{2}e)-1) - 1A^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+3B^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+2iB^2\tan(\frac{1}{2}fx+\frac{1}{2}e)+1A^2-3B^2 - 3iA^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-9B^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^2+10A^2\tan(\frac{1}{2}fx+\frac{1}{2}e)+22iB^2\tan(\frac{1}{2}fx+\frac{1}{2}e)-3iA^2+9B^2}{a(\tan(\frac{1}{2}fx+\frac{1}{2}e)-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] ((-I\*A\*c^2 + 3\*B\*c^2)\*log(tan(1/2\*f\*x + 1/2\*e) + 1)/a + 2\*(I\*A\*c^2 - 3\*B\*c^2)\*log(tan(1/2\*f\*x + 1/2\*e) - I)/a - (I\*A\*c^2 - 3\*B\*c^2)\*log(tan(1/2\*f\*x + 1/2\*e) - 1)/a - (-I\*A\*c^2\*tan(1/2\*f\*x + 1/2\*e)^2 + 3\*B\*c^2\*tan(1/2\*f\*x + 1/2\*e)^2 + 2\*I\*B\*c^2\*tan(1/2\*f\*x + 1/2\*e) + I\*A\*c^2 - 3\*B\*c^2)/((tan(1/2\*f\*x + 1/2\*e)^2 - 1)\*a) - (3\*I\*A\*c^2\*tan(1/2\*f\*x + 1/2\*e)^2 - 9\*B\*c^2\*tan(1/2\*f\*x + 1/2\*e)^2 + 10\*A\*c^2\*tan(1/2\*f\*x + 1/2\*e) + 22\*I\*B\*c^2\*tan(1/2\*f\*x + 1/2\*e) - 3\*I\*A\*c^2 + 9\*B\*c^2)/(a\*(tan(1/2\*f\*x + 1/2\*e) - I)^2)/f

**Mupad [B]**

time = 8.57, size = 110, normalized size = 1.15

$$\frac{\ln(\tan(e + f x) - i) \left( -\frac{3 B c^2}{a} + \frac{A c^2 i}{a} \right)}{f} + \frac{\frac{(A c^2 - B c^2 i) i}{a} + \frac{(A c^2 + B c^2 3 i) i}{a}}{f (1 + \tan(e + f x) i)} + \frac{B c^2 \tan(e + f x) i}{a f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^2)/(a + a\*tan(e + f\*x)\*1i),x)

[Out] (log(tan(e + f\*x) - 1i)\*((A\*c^2\*1i)/a - (3\*B\*c^2)/a))/f + (((A\*c^2 - B\*c^2\*1i)\*1i)/a + ((A\*c^2 + B\*c^2\*3i)\*1i)/a)/(f\*(tan(e + f\*x)\*1i + 1)) + (B\*c^2\*tan(e + f\*x)\*1i)/(a\*f)

$$3.709 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))}{a+ia \tan(e+fx)} dx$$

Optimal. Leaf size=57

$$-\frac{iBcx}{a} + \frac{Bc \log(\cos(e+fx))}{af} - \frac{(A+iB)c}{af(i-\tan(e+fx))}$$

[Out]  $-I*B*c*x/a+B*c*\ln(\cos(f*x+e))/a/f-(A+I*B)*c/a/f/(I-\tan(f*x+e))$

Rubi [A]

time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$-\frac{c(A+iB)}{af(-\tan(e+fx)+i)} + \frac{Bc \log(\cos(e+fx))}{af} - \frac{iBcx}{a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])]/(a + I*a*\text{Tan}[e + f*x]),x]$

[Out]  $((-I)*B*c*x)/a + (B*c*\text{Log}[\text{Cos}[e + f*x]])/(a*f) - ((A + I*B)*c)/(a*f*(I - \text{Tan}[e + f*x]))$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^(m-1)*(c + d*x)^(n-1)*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))}{a + i a \tan(e + fx)} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{-A-iB}{a^2(-i+x)^2} - \frac{B}{a^2(-i+x)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iBcx}{a} + \frac{Bc \log(\cos(e + fx))}{af} - \frac{(A + iB)c}{af(i - \tan(e + fx))}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 124 vs. 2(57) = 114.

time = 0.67, size = 124, normalized size = 2.18

$$\frac{c \cos(e + fx)(A + B \tan(e + fx))(A + iB - iB \log(\cos^2(e + fx)) + (-iA + B + B \log(\cos^2(e + fx))) \tan(e + fx) - 2iB \text{ArcTan}(\tan(fx))(-i + \tan(e + fx)))}{2af(A \cos(e + fx) + B \sin(e + fx))(-i + \tan(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x]))/(a + I\*a\*Tan[e + f\*x]), x]

[Out] (c\*Cos[e + f\*x]\*(A + B\*Tan[e + f\*x])\*(A + I\*B - I\*B\*Log[Cos[e + f\*x]^2] + (-I)\*A + B + B\*Log[Cos[e + f\*x]^2])\*Tan[e + f\*x] - (2\*I)\*B\*ArcTan[Tan[f\*x]]\*(-I + Tan[e + f\*x]))/(2\*a\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(-I + Tan[e + f\*x]))

**Maple [A]**

time = 0.16, size = 44, normalized size = 0.77

method	result	size
derivativedivides	$\frac{c\left(-B \ln(-i + \tan(fx+e)) - \frac{-iB-A}{-i + \tan(fx+e)}\right)}{fa}$	44
default	$\frac{c\left(-B \ln(-i + \tan(fx+e)) - \frac{-iB-A}{-i + \tan(fx+e)}\right)}{fa}$	44
risch	$-\frac{ce^{-2i(fx+e)}B}{2af} + \frac{ice^{-2i(fx+e)}A}{2af} - \frac{2iBcx}{a} - \frac{2iBce}{af} + \frac{Bc \ln(e^{2i(fx+e)}+1)}{af}$	83
norman	$\frac{\frac{(iBc+Ac) \tan(fx+e)}{af} - \frac{-iAc+Bc}{af} - \frac{iBcx}{a} - \frac{icBx(\tan^2(fx+e))}{a}}{1+\tan^2(fx+e)} - \frac{Bc \ln(1+\tan^2(fx+e))}{2af}$	102

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)), x, method=\_RETURNVERBOSE)

[Out] 1/f\*c/a\*(-B\*ln(-I+tan(f\*x+e))-(-A-I\*B)/(-I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 3.35, size = 71, normalized size = 1.25

$$\frac{(-4i Bc f x e^{2i f x + 2i e} + 2 Bc e^{2i f x + 2i e} \log(e^{2i f x + 2i e} + 1) + (i A - B)c) e^{-2i f x - 2i e}}{2 a f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(-4*I*B*c*f*x*e^(2*I*f*x + 2*I*e) + 2*B*c*e^(2*I*f*x + 2*I*e)*log(e^(2*I*f*x + 2*I*e) + 1) + (I*A - B)*c)*e^(-2*I*f*x - 2*I*e)/(a*f)
```

**Sympy [A]**

time = 0.19, size = 112, normalized size = 1.96

$$-\frac{2iBcx}{a} + \frac{Bc \log(e^{2ifx} + e^{-2ie})}{af} + \begin{cases} \frac{(iAc-Bc)e^{-2ie}e^{-2ifx}}{2af} & \text{for } afe^{2ie} \neq 0 \\ x \left( \frac{2iBc}{a} + \frac{(Ac-2iBce^{2ie}+iBc)e^{-2ie}}{a} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -2*I*B*c*x/a + B*c*log(exp(2*I*f*x) + exp(-2*I*e))/(a*f) + Piecewise(((I*A*c - B*c)*exp(-2*I*e)*exp(-2*I*f*x)/(2*a*f), Ne(a*f*exp(2*I*e), 0)), (x*(2*I*B*c/a + (A*c - 2*I*B*c*exp(2*I*e) + I*B*c)*exp(-2*I*e)/a), True))
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs.  $2(50) = 100$ .

time = 0.56, size = 130, normalized size = 2.28

$$\frac{Bc \log(\tan(\frac{1}{2} f x + \frac{1}{2} e) + 1)}{a} - \frac{2 Bc \log(\tan(\frac{1}{2} f x + \frac{1}{2} e) - i)}{a} + \frac{Bc \log(\tan(\frac{1}{2} f x + \frac{1}{2} e) - 1)}{a} + \frac{3 Bc \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 - 2 A c \tan(\frac{1}{2} f x + \frac{1}{2} e) - 8 i B c \tan(\frac{1}{2} f x + \frac{1}{2} e) - 3 B c}{a (\tan(\frac{1}{2} f x + \frac{1}{2} e) - i)^2}$$

$f$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] (B\*c\*log(tan(1/2\*f\*x + 1/2\*e) + 1)/a - 2\*B\*c\*log(tan(1/2\*f\*x + 1/2\*e) - I)/a + B\*c\*log(tan(1/2\*f\*x + 1/2\*e) - 1)/a + (3\*B\*c\*tan(1/2\*f\*x + 1/2\*e)^2 - 2\*A\*c\*tan(1/2\*f\*x + 1/2\*e) - 8\*I\*B\*c\*tan(1/2\*f\*x + 1/2\*e) - 3\*B\*c)/(a\*(tan(1/2\*f\*x + 1/2\*e) - I)^2))/f

**Mupad [B]**

time = 8.46, size = 54, normalized size = 0.95

$$\frac{-\frac{Bc}{a} + \frac{Ac \operatorname{li}}{a}}{f (1 + \tan(e + f x) \operatorname{li})} - \frac{Bc \ln(\tan(e + f x) - i)}{a f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i))/(a + a\*tan(e + f\*x)\*1i), x)

[Out] ((A\*c\*1i)/a - (B\*c)/a)/(f\*(tan(e + f\*x)\*1i + 1)) - (B\*c\*log(tan(e + f\*x) - 1i))/(a\*f)

$$3.710 \quad \int \frac{A+B \tan(e+fx)}{a+ia \tan(e+fx)} dx$$

Optimal. Leaf size=47

$$\frac{(A-iB)x}{2a} + \frac{iA-B}{2f(a+ia \tan(e+fx))}$$

[Out] 1/2\*(A-I\*B)\*x/a+1/2\*(I\*A-B)/f/(a+I\*a\*tan(f\*x+e))

Rubi [A]

time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3607, 8}

$$\frac{-B+iA}{2f(a+ia \tan(e+fx))} + \frac{x(A-iB)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/(a + I\*a\*Tan[e + f\*x]),x]

[Out] ((A - I\*B)\*x)/(2\*a) + (I\*A - B)/(2\*f\*(a + I\*a\*Tan[e + f\*x]))

Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A+B \tan(e+fx)}{a+ia \tan(e+fx)} dx &= \frac{iA-B}{2f(a+ia \tan(e+fx))} + \frac{(A-iB) \int 1 dx}{2a} \\ &= \frac{(A-iB)x}{2a} + \frac{iA-B}{2f(a+ia \tan(e+fx))} \end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 102 vs.  $2(47) = 94$ .

time = 0.26, size = 102, normalized size = 2.17

$$\frac{\cos(e + fx)(A + B \tan(e + fx))(A - 2iAfx + B(i - 2fx) + (B - 2iBfx + A(-i + 2fx)) \tan(e + fx))}{4af(A \cos(e + fx) + B \sin(e + fx))(-i + \tan(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/(a + I\*a\*Tan[e + f\*x]),x]

[Out] (Cos[e + f\*x]\*(A + B\*Tan[e + f\*x])\*(A - (2\*I)\*A\*f\*x + B\*(I - 2\*f\*x) + (B - (2\*I)\*B\*f\*x + A\*(-I + 2\*f\*x))\*Tan[e + f\*x]))/(4\*a\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(-I + Tan[e + f\*x]))

**Maple [A]**

time = 0.16, size = 68, normalized size = 1.45

method	result	size
risch	$-\frac{ixB}{2a} + \frac{xA}{2a} - \frac{e^{-2i(fx+e)}B}{4af} + \frac{ie^{-2i(fx+e)}A}{4af}$	54
derivativedivides	$-\frac{-\frac{A}{2} - \frac{iB}{2}}{-i + \tan(fx+e)} + \left(-\frac{iA}{4} - \frac{B}{4}\right) \ln(-i + \tan(fx+e)) + \frac{i(-iB+A) \ln(i + \tan(fx+e))}{4}$ $fa$	68
default	$-\frac{-\frac{A}{2} - \frac{iB}{2}}{-i + \tan(fx+e)} + \left(-\frac{iA}{4} - \frac{B}{4}\right) \ln(-i + \tan(fx+e)) + \frac{i(-iB+A) \ln(i + \tan(fx+e))}{4}$ $fa$	68
norman	$\frac{(-iB+A)x - \frac{iA+B}{2af} + \frac{(iB+A) \tan(fx+e)}{2af} + \frac{(-iB+A)x (\tan^2(fx+e))}{2a}}{1 + \tan^2(fx+e)}$	81

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x,method=\_RETURNVERBOSE)

[Out] 1/f/a\*(-(-1/2\*A-1/2\*I\*B)/(-I+tan(f\*x+e))+(-1/4\*I\*A-1/4\*B)\*ln(-I+tan(f\*x+e))+1/4\*I\*(A-I\*B)\*ln(I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 4.52, size = 44, normalized size = 0.94

$$\frac{(2(A - iB)fxe^{(2i fx + 2ie)} + iA - B)e^{(-2i fx - 2ie)}}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x, algorithm="fricas")

[Out]  $1/4*(2*(A - I*B)*f*x*e^{(2*I*f*x + 2*I*e)} + I*A - B)*e^{(-2*I*f*x - 2*I*e)}/(a*f)$

**Sympy [A]**

time = 0.12, size = 87, normalized size = 1.85

$$\begin{cases} \frac{(iA-B)e^{-2ie}e^{-2ifx}}{4af} & \text{for } af e^{2ie} \neq 0 \\ x\left(-\frac{A-iB}{2a} + \frac{(Ae^{2ie}+A-iBe^{2ie}+iB)e^{-2ie}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x)

[Out] Piecewise(((I\*A - B)\*exp(-2\*I\*e)\*exp(-2\*I\*f\*x)/(4\*a\*f), Ne(a\*f\*exp(2\*I\*e), 0)), (x\*(-(A - I\*B)/(2\*a) + (A\*exp(2\*I\*e) + A - I\*B\*exp(2\*I\*e) + I\*B)\*exp(-2\*I\*e)/(2\*a)), True)) + x\*(A - I\*B)/(2\*a)

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs.  $2(36) = 72$ .

time = 0.54, size = 90, normalized size = 1.91

$$\frac{\frac{(iA+B)\log(\tan(fx+e)-i)}{a} + \frac{(-iA-B)\log(-i\tan(fx+e)+1)}{a} + \frac{-iA\tan(fx+e)-B\tan(fx+e)-3A-iB}{a(\tan(fx+e)-i)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out]  $-1/4*((I*A + B)*\log(\tan(f*x + e) - I)/a + (-I*A - B)*\log(-I*\tan(f*x + e) + 1)/a + (-I*A*\tan(f*x + e) - B*\tan(f*x + e) - 3*A - I*B)/(a*(\tan(f*x + e) - I)))/f$

**Mupad [B]**

time = 8.59, size = 45, normalized size = 0.96

$$-\frac{x(B + A \operatorname{li}) \operatorname{li}}{2a} + \frac{-\frac{B}{2a} + \frac{A \operatorname{li}}{2a}}{f(1 + \tan(e + fx) \operatorname{li})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/(a + a\*tan(e + f\*x)\*1i),x)

[Out]  $((A*1i)/(2*a) - B/(2*a))/(f*(\tan(e + f*x)*1i + 1)) - (x*(A*1i + B)*1i)/(2*a)$

$$3.711 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))} dx$$

Optimal. Leaf size=45

$$\frac{Ax}{2ac} - \frac{\cos^2(e+fx)(B-A \tan(e+fx))}{2acf}$$

[Out] 1/2\*A\*x/a/c-1/2\*cos(f\*x+e)^2\*(B-A\*tan(f\*x+e))/a/c/f

Rubi [A]

time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$ , Rules used = {3669, 74, 653, 211}

$$\frac{Ax}{2ac} - \frac{\cos^2(e+fx)(B-A \tan(e+fx))}{2acf}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])),x]

[Out] (A\*x)/(2\*a\*c) - (Cos[e + f\*x]^2\*(B - A\*Tan[e + f\*x]))/(2\*a\*c\*f)

Rule 74

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_))\*((e\_) + (f\_)\*(x\_)^(p\_)), x\_Symbol] :> Int[(a\*c + b\*d\*x^2)^m\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b\*c + a\*d, 0] && EqQ[n, m] && IntegerQ[m] && (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] || !IntegerQ[p])))

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 653

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((a\*e - c\*d\*x)/(2\*a\*c\*(p + 1))\*(a + c\*x^2)^(p + 1), x] + Dist[d\*((2\*p + 3)/(2\*a\*(p + 1))), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_))\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x,

$\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))(c - ic \tan(e + fx))} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2(c-icx)^2} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(ac+acx^2)^2} dx, x, \tan(e + fx)\right)}{f} \\ &= -\frac{\cos^2(e + fx)(B - A \tan(e + fx))}{2acf} + \frac{A \text{Subst}\left(\int \frac{1}{ac+acx^2} dx, x, \tan(e + fx)\right)}{2f} \\ &= \frac{Ax}{2ac} - \frac{\cos^2(e + fx)(B - A \tan(e + fx))}{2acf} \end{aligned}$$

**Mathematica [A]**

time = 0.07, size = 43, normalized size = 0.96

$$\frac{-2B \cos^2(e + fx) + A(2(e + fx) + \sin(2(e + fx)))}{4acf}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])),x]

[Out] (-2\*B\*Cos[e + f\*x]^2 + A\*(2\*(e + f\*x) + Sin[2\*(e + f\*x)]))/(4\*a\*c\*f)

**Maple [C]** Result contains complex when optimal does not.

time = 0.17, size = 82, normalized size = 1.82

method	result	size
risch	$\frac{Ax}{2ac} - \frac{B \cos(2fx+2e)}{4acf} + \frac{A \sin(2fx+2e)}{4acf}$	54
norman	$\frac{Ax}{2ac} - \frac{B}{2acf} + \frac{A \tan(fx+e)}{2acf} + \frac{Ax(\tan^2(fx+e))}{2ac}$ $\frac{1+\tan^2(fx+e)}{1+\tan^2(fx+e)}$	73
derivativedivides	$\frac{-\frac{iA \ln(-i+\tan(fx+e))}{4} - \frac{-\frac{A}{4} - \frac{iB}{4}}{-i+\tan(fx+e)} + \frac{iA \ln(i+\tan(fx+e))}{4} - \frac{-\frac{A}{4} + \frac{iB}{4}}{i+\tan(fx+e)}}{fac}$	82
default	$\frac{-\frac{iA \ln(-i+\tan(fx+e))}{4} - \frac{-\frac{A}{4} - \frac{iB}{4}}{-i+\tan(fx+e)} + \frac{iA \ln(i+\tan(fx+e))}{4} - \frac{-\frac{A}{4} + \frac{iB}{4}}{i+\tan(fx+e)}}{fac}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $1/f/a/c*(-1/4*I*A*\ln(-I+\tan(f*x+e))-(-1/4*A-1/4*I*B)/(-I+\tan(f*x+e))+1/4*I*A*\ln(I+\tan(f*x+e))-(-1/4*A+1/4*I*B)/(I+\tan(f*x+e)))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [C] Result contains complex when optimal does not.

time = 3.80, size = 61, normalized size = 1.36

$$\frac{(4Afxe^{2ifx+2ie}) + (-iA - B)e^{4ifx+4ie} + iA - B)e^{-2ifx-2ie}}{8acf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,algorithm="fricas")`

[Out]  $1/8*(4*A*f*x*e^{(2*I*f*x + 2*I*e)} + (-I*A - B)*e^{(4*I*f*x + 4*I*e)} + I*A - B)*e^{(-2*I*f*x - 2*I*e)}/(a*c*f)$

**Sympy** [A]

time = 0.19, size = 165, normalized size = 3.67

$$\frac{Ax}{2ac} + \begin{cases} \frac{((8iAacf-8Bacf)e^{-2ifx} + (-8iAacfe^{4ie}-8Bacfe^{4ie})e^{2ifx})e^{-2ie}}{64a^2c^2f^2} & \text{for } a^2c^2f^2e^{2ie} \neq 0 \\ x\left(-\frac{A}{2ac} + \frac{(Ae^{4ie}+2Ae^{2ie}+A-iBe^{4ie}+iB)e^{-2ie}}{4ac}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)`

[Out]  $A*x/(2*a*c) + \text{Piecewise}(\left(\left(\left(8*I*A*a*c*f - 8*B*a*c*f\right)*\exp(-2*I*f*x) + \left(-8*I*A*a*c*f*\exp(4*I*e) - 8*B*a*c*f*\exp(4*I*e)\right)*\exp(2*I*f*x)\right)*\exp(-2*I*e)/(64*a**2*c**2*f**2), \text{Ne}(a**2*c**2*f**2*\exp(2*I*e), 0)), \left(x*(-A/(2*a*c) + (A*\exp(4*I*e) + 2*A*\exp(2*I*e) + A - I*B*\exp(4*I*e) + I*B)*\exp(-2*I*e)/(4*a*c)\right), \text{True})$

**Giac [A]**

time = 0.55, size = 53, normalized size = 1.18

$$\frac{\frac{(fx+e)A}{ac} + \frac{A \tan(fx+e) - B}{(\tan(fx+e)^2 + 1)ac}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*A/(a*c) + (A*tan(f*x + e) - B)/((tan(f*x + e)^2 + 1)*a*c))/f
```

**Mupad [B]**

time = 8.45, size = 40, normalized size = 0.89

$$\frac{\frac{A \sin(2e+2fx)}{2} - \frac{B \cos(2e+2fx)}{2} + A f x}{2 a c f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)), x)
```

```
[Out] ((A*sin(2*e + 2*f*x))/2 - (B*cos(2*e + 2*f*x))/2 + A*f*x)/(2*a*c*f)
```



$$3.712 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^2} dx$$

Optimal. Leaf size=113

$$\frac{(3A+iB)x}{8ac^2} - \frac{A+iB}{8ac^2 f(i-\tan(e+fx))} + \frac{iA+B}{8ac^2 f(i+\tan(e+fx))^2} + \frac{A}{4ac^2 f(i+\tan(e+fx))}$$

[Out] 1/8\*(3\*A+I\*B)\*x/a/c^2+1/8\*(-A-I\*B)/a/c^2/f/(I-tan(f\*x+e))+1/8\*(I\*A+B)/a/c^2/f/(I+tan(f\*x+e))^2+1/4\*A/a/c^2/f/(I+tan(f\*x+e))

Rubi [A]

time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{A+iB}{8ac^2 f(-\tan(e+fx)+i)} + \frac{B+iA}{8ac^2 f(\tan(e+fx)+i)^2} + \frac{x(3A+iB)}{8ac^2} + \frac{A}{4ac^2 f(\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^2), x]

[Out] ((3\*A + I\*B)\*x)/(8\*a\*c^2) - (A + I\*B)/(8\*a\*c^2\*f\*(I - Tan[e + f\*x])) + (I\*A + B)/(8\*a\*c^2\*f\*(I + Tan[e + f\*x])^2) + A/(4\*a\*c^2\*f\*(I + Tan[e + f\*x]))

Rule 78

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 209

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c +

a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))(c - ic \tan(e + fx))^2} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2(c-icx)^3} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{-A-iB}{8a^2c^3(-i+x)^2} - \frac{i(A-iB)}{4a^2c^3(i+x)^3} - \frac{A}{4a^2c^3(i+x)^2} + \frac{3A}{8a^2c^3}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{A + iB}{8ac^2 f(i - \tan(e + fx))} + \frac{iA + B}{8ac^2 f(i + \tan(e + fx))^2} + \frac{3A}{8ac^2 f}$$

$$= \frac{(3A + iB)x}{8ac^2} - \frac{A + iB}{8ac^2 f(i - \tan(e + fx))} + \frac{iA + B}{8ac^2 f(i + \tan(e + fx))}$$

Mathematica [A]

time = 1.24, size = 166, normalized size = 1.47

$$\frac{(2(A(-3 - 6ifx) + B(i + 2fx)) \cos(e + fx) + (A + 3iB) \cos(3(e + fx)) - (9iA + B + 12Afx + 4iBfx + (6iA - 2B) \cos(2(e + fx))) \sin(e + fx))(\cos(2(e + fx)) + i \sin(2(e + fx)))(A + B \tan(e + fx))}{32a^2 f(A \cos(e + fx) + B \sin(e + fx))(-i + \tan(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^2),x]

[Out] ((2\*(A\*(-3 - (6\*I)\*f\*x) + B\*(I + 2\*f\*x))\*Cos[e + f\*x] + (A + (3\*I)\*B)\*Cos[3\*(e + f\*x)] - ((9\*I)\*A + B + 12\*A\*f\*x + (4\*I)\*B\*f\*x + ((6\*I)\*A - 2\*B)\*Cos[2\*(e + f\*x)])\*Sin[e + f\*x])\*(Cos[2\*(e + f\*x)] + I\*Sin[2\*(e + f\*x)])\*(A + B\*Tan[e + f\*x]))/(32\*a\*c^2\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(-I + Tan[e + f\*x]))

Maple [A]

time = 0.24, size = 106, normalized size = 0.94

method	result	size
derivativedivides	$\frac{A}{4i+4 \tan(fx+e)} + \left(\frac{3iA}{16} - \frac{B}{16}\right) \ln(i+\tan(fx+e)) - \frac{-\frac{iA}{4} - \frac{B}{4}}{2(i+\tan(fx+e))^2} + \left(-\frac{3iA}{16} + \frac{B}{16}\right) \ln(-i+\tan(fx+e)) - \frac{-\frac{A}{8} - \frac{iB}{8}}{-i+\tan(fx+e)}$	10
default	$\frac{A}{4i+4 \tan(fx+e)} + \left(\frac{3iA}{16} - \frac{B}{16}\right) \ln(i+\tan(fx+e)) - \frac{-\frac{iA}{4} - \frac{B}{4}}{2(i+\tan(fx+e))^2} + \left(-\frac{3iA}{16} + \frac{B}{16}\right) \ln(-i+\tan(fx+e)) - \frac{-\frac{A}{8} - \frac{iB}{8}}{-i+\tan(fx+e)}$	10
risch	$\frac{ixB}{8a^2c^2} + \frac{3xA}{8a^2c^2} - \frac{e^{4i(fx+e)}B}{32a^2c^2f} - \frac{ie^{4i(fx+e)}A}{32a^2c^2f} - \frac{\cos(2fx+2e)B}{8a^2c^2f} - \frac{i \cos(2fx+2e)A}{8a^2c^2f} + \frac{A \sin(2fx+2e)}{4a^2c^2f}$	13

norman	$\frac{\frac{(iB+3A)x}{8ac} - \frac{iA+B}{4acf} + \frac{(iB+3A)(\tan^3(fx+e))}{8acf} + \frac{(iB+3A)x(\tan^2(fx+e))}{4ac} + \frac{(iB+3A)x(\tan^4(fx+e))}{8ac} + \frac{(-iB+5A)\tan(fx+e)}{8acf}}{c(1+\tan^2(fx+e))^2}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{f/a/c^2} * \left( \frac{1}{4} * A / (I + \tan(f*x+e)) + \frac{3}{16} * I * A - \frac{1}{16} * B \right) * \ln(I + \tan(f*x+e)) - \frac{1}{2} * \left( -\frac{1}{4} * I * A - \frac{1}{4} * B \right) / (I + \tan(f*x+e))^2 + \left( -\frac{3}{16} * I * A + \frac{1}{16} * B \right) * \ln(-I + \tan(f*x+e)) - \left( -\frac{1}{8} * A - \frac{1}{8} * I * B \right) / (-I + \tan(f*x+e))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 4.97, size = 84, normalized size = 0.74

$$\frac{(4(3A + iB)fxe^{2ifx+2ie} + (-iA - B)e^{6ifx+6ie} - 2(3iA + B)e^{4ifx+4ie} + 2iA - 2B)e^{-2ifx-2ie}}{32ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{32} * (4 * (3 * A + I * B) * f * x * e^{(2 * I * f * x + 2 * I * e)} + (-I * A - B) * e^{(6 * I * f * x + 6 * I * e)} - 2 * (3 * I * A + B) * e^{(4 * I * f * x + 4 * I * e)} + 2 * I * A - 2 * B) * e^{(-2 * I * f * x - 2 * I * e)} / (a * c^2 * f)$

**Sympy** [A]

time = 0.28, size = 284, normalized size = 2.51

$$\begin{cases} \frac{((512iAa^2c^4f^2 - 512Ba^2c^4f^2)e^{-2ifx} + (-1536iAa^2c^4f^2e^{4ie} - 512Ba^2c^4f^2e^{4ie})e^{2ifx} + (-256iAa^2c^4f^2e^{6ie} - 256Ba^2c^4f^2e^{6ie})e^{4ifx})e^{-2ie}}{8192a^3c^6f^3} & \text{for } a^3c^6f^3e^{2ie} \neq 0 \\ x \left( -\frac{3A+iB}{8ac^2} + \frac{(Ae^{6ie} + 3Ae^{4ie} + 3Ae^{2ie} + A - iBe^{6ie} - iBe^{4ie} + iBe^{2ie} + iB)e^{-2ie}}{8ac^2} \right) & \text{otherwise} \end{cases} + \frac{x(3A + iB)}{8ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)`

```
[Out] Piecewise((((512*I*A*a**2*c**4*f**2 - 512*B*a**2*c**4*f**2)*exp(-2*I*f*x) +
(-1536*I*A*a**2*c**4*f**2*exp(4*I*e) - 512*B*a**2*c**4*f**2*exp(4*I*e))*ex
p(2*I*f*x) + (-256*I*A*a**2*c**4*f**2*exp(6*I*e) - 256*B*a**2*c**4*f**2*exp
(6*I*e))*exp(4*I*f*x))*exp(-2*I*e)/(8192*a**3*c**6*f**3), Ne(a**3*c**6*f**3
*exp(2*I*e), 0)), (x*(-(3*A + I*B)/(8*a*c**2) + (A*exp(6*I*e) + 3*A*exp(4*I
*e) + 3*A*exp(2*I*e) + A - I*B*exp(6*I*e) - I*B*exp(4*I*e) + I*B*exp(2*I*e)
+ I*B)*exp(-2*I*e)/(8*a*c**2)), True)) + x*(3*A + I*B)/(8*a*c**2)
```

**Giac [A]**

time = 0.69, size = 169, normalized size = 1.50

$$\frac{\frac{2(3iA-B)\log(\tan(fx+e)+i)}{ac^2} + \frac{2(-3iA+B)\log(\tan(fx+e)-i)}{ac^2} - \frac{2(3A\tan(fx+e)+iB\tan(fx+e)-5iA+3B)}{ac^2(i\tan(fx+e)+1)} + \frac{-9iA\tan(fx+e)^2+3B\tan(fx+e)^2+26A\tan(fx+e)+6iB\tan(fx+e)+21iA+B}{ac^2(\tan(fx+e)+i)^2}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algor
ithm="giac")
```

```
[Out] 1/32*(2*(3*I*A - B)*log(tan(f*x + e) + I)/(a*c^2) + 2*(-3*I*A + B)*log(tan(
f*x + e) - I)/(a*c^2) - 2*(3*A*tan(f*x + e) + I*B*tan(f*x + e) - 5*I*A + 3*
B)/(a*c^2*(I*tan(f*x + e) + 1)) + (-9*I*A*tan(f*x + e)^2 + 3*B*tan(f*x + e)
^2 + 26*A*tan(f*x + e) + 6*I*B*tan(f*x + e) + 21*I*A + B)/(a*c^2*(tan(f*x +
e) + I)^2))/f
```

**Mupad [B]**

time = 9.06, size = 129, normalized size = 1.14

$$\frac{\tan(e + fx) \left(-\frac{B}{8ac^2} + \frac{A3i}{8ac^2}\right) + \tan(e + fx)^2 \left(\frac{3A}{8ac^2} + \frac{B1i}{8ac^2}\right) + \frac{A}{4ac^2} - \frac{B1i}{4ac^2}}{f \left(\tan(e + fx)^3 + \tan(e + fx)^2 1i + \tan(e + fx) + 1i\right)} - \frac{x(-B + A3i) 1i}{8ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^2
),x)
```

```
[Out] (tan(e + f*x)*((A*3i)/(8*a*c^2) - B/(8*a*c^2)) + tan(e + f*x)^2*((3*A)/(8*a
*c^2) + (B*1i)/(8*a*c^2)) + A/(4*a*c^2) - (B*1i)/(4*a*c^2))/(f*(tan(e + f*x
) + tan(e + f*x)^2*1i + tan(e + f*x)^3 + 1i)) - (x*(A*3i - B)*1i)/(8*a*c^2)
```

$$3.713 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=149

$$\frac{(2A+iB)x}{8ac^3} - \frac{A+iB}{16ac^3 f(i-\tan(e+fx))} - \frac{A-iB}{12ac^3 f(i+\tan(e+fx))^3} + \frac{iA}{8ac^3 f(i+\tan(e+fx))^2} + \frac{3A}{16ac^3 f(i+\tan(e+fx))}$$

[Out] 1/8\*(2\*A+I\*B)\*x/a/c^3+1/16\*(-A-I\*B)/a/c^3/f/(I-tan(f\*x+e))+1/12\*(-A+I\*B)/a/c^3/f/(I+tan(f\*x+e))^3+1/8\*I\*A/a/c^3/f/(I+tan(f\*x+e))^2+1/16\*(3\*A+I\*B)/a/c^3/f/(I+tan(f\*x+e))

**Rubi** [A]

time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{A+iB}{16ac^3 f(-\tan(e+fx)+i)} + \frac{3A+iB}{16ac^3 f(\tan(e+fx)+i)} - \frac{A-iB}{12ac^3 f(\tan(e+fx)+i)^3} + \frac{x(2A+iB)}{8ac^3} + \frac{iA}{8ac^3 f(\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3), x]

[Out] ((2\*A + I\*B)\*x)/(8\*a\*c^3) - (A + I\*B)/(16\*a\*c^3\*f\*(I - Tan[e + f\*x])) - (A - I\*B)/(12\*a\*c^3\*f\*(I + Tan[e + f\*x])^3) + ((I/8)\*A)/(a\*c^3\*f\*(I + Tan[e + f\*x])^2) + (3\*A + I\*B)/(16\*a\*c^3\*f\*(I + Tan[e + f\*x]))

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 209**

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 3669**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x,

Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))(c - ic \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2(c-icx)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{-A-iB}{16a^2c^4(-i+x)^2} + \frac{A-iB}{4a^2c^4(i+x)^4} - \frac{iA}{4a^2c^4(i+x)^3} + \frac{-}{16a^2c^4(i+x)^4}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{A + iB}{16ac^3 f(i - \tan(e + fx))} - \frac{A - iB}{12ac^3 f(i + \tan(e + fx))^3} + \frac{(2A + iB)x}{8ac^3} - \frac{A + iB}{16ac^3 f(i - \tan(e + fx))} - \frac{A - iB}{12ac^3 f(i + \tan(e + fx))^3}$$

Mathematica [A]

time = 1.12, size = 203, normalized size = 1.36

(cos(3(e + fx)) + i sin(3(e + fx)))(-18A + 3(A(-2 - 8ifx) + B(i + 4fx)) cos(2(e + fx)) + 2(A + 2iB) cos(4(e + fx)) - 6iA sin(2(e + fx)) - 3B sin(2(e + fx)) - 24Afx sin(2(e + fx)) - 12Bfx sin(2(e + fx)) - 4iA sin(4(e + fx)) + 2B sin(4(e + fx)))(A + B tan(e + fx)) / (96a^2 f(A cos(e + fx) + B sin(e + fx))(-1 + tan(e + fx)))

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3), x]

[Out] ((Cos[3\*(e + f\*x)] + I\*Sin[3\*(e + f\*x)])\*(-18\*A + 3\*(A\*(-2 - (8\*I)\*f\*x) + B\*(I + 4\*f\*x))\*Cos[2\*(e + f\*x)] + 2\*(A + (2\*I)\*B)\*Cos[4\*(e + f\*x)] - (6\*I)\*A\*Sin[2\*(e + f\*x)] - 3\*B\*Sin[2\*(e + f\*x)] - 24\*A\*f\*x\*Sin[2\*(e + f\*x)] - (12\*I)\*B\*f\*x\*Sin[2\*(e + f\*x)] - (4\*I)\*A\*Sin[4\*(e + f\*x)] + 2\*B\*Sin[4\*(e + f\*x)])\*(A + B\*Tan[e + f\*x])/(96\*a\*c^3\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(-I + Tan[e + f\*x]))

Maple [A]

time = 0.31, size = 128, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{iA}{8(i+\tan(fx+e))^2} - \frac{A-iB}{3(i+\tan(fx+e))^3} - \frac{-3A-iB}{i+\tan(fx+e)} + \left(-\frac{B}{16} + \frac{iA}{8}\right) \ln(i+\tan(fx+e)) - \frac{-\frac{A}{16} - \frac{iB}{16}}{-i+\tan(fx+e)} + \left(\frac{B}{16} - \frac{iA}{8}\right) \ln(-i+\tan(fx+e))}{fa c^3}$
default	$\frac{\frac{iA}{8(i+\tan(fx+e))^2} - \frac{A-iB}{3(i+\tan(fx+e))^3} - \frac{-3A-iB}{i+\tan(fx+e)} + \left(-\frac{B}{16} + \frac{iA}{8}\right) \ln(i+\tan(fx+e)) - \frac{-\frac{A}{16} - \frac{iB}{16}}{-i+\tan(fx+e)} + \left(\frac{B}{16} - \frac{iA}{8}\right) \ln(-i+\tan(fx+e))}{fa c^3}$
risch	$\frac{ixB}{8ac^3} + \frac{xA}{4ac^3} - \frac{e^{6i(fx+e)}B}{96ac^3f} - \frac{ie^{6i(fx+e)}A}{96ac^3f} - \frac{e^{4i(fx+e)}B}{32ac^3f} - \frac{ie^{4i(fx+e)}A}{16ac^3f} - \frac{\cos(2fx+2e)B}{32ac^3f} - \frac{5i \cos(2fx+2e)A}{32ac^3f}$

norman	$\frac{\frac{(iB+2A)x}{8ac} - \frac{4iA+B}{12acf} + \frac{B(\tan^2(fx+e))}{4acf} + \frac{(-iB+6A)\tan(fx+e)}{8acf} + \frac{(iB+2A)(\tan^3(fx+e))}{3acf} + \frac{(iB+2A)(\tan^5(fx+e))}{8acf} + \frac{3(iB+2A)x}{8ac}}{(1+\tan^2(fx+e))^3 c^2}$
--------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)`

[Out]  $1/f/a/c^3*(1/8*I*A/(I+\tan(f*x+e))^2-1/3*(1/4*A-1/4*I*B)/(I+\tan(f*x+e))^3-(-3/16*A-1/16*I*B)/(I+\tan(f*x+e))+(-1/16*B+1/8*I*A)*\ln(I+\tan(f*x+e))-(-1/16*A-1/16*I*B)/(-I+\tan(f*x+e))+(1/16*B-1/8*I*A)*\ln(-I+\tan(f*x+e)))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 8.49, size = 97, normalized size = 0.65

$$\frac{(12(2A+iB)fxe^{(2i fx+2i e)} + (-iA-B)e^{(8i fx+8i e)} - 3(2iA+B)e^{(6i fx+6i e)} - 18iAe^{(4i fx+4i e)} + 3iA-3B)e^{(-2i fx-2i e)}}{96ac^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,algorithm="fricas")`

[Out]  $1/96*(12*(2*A + I*B)*f*x*e^{(2*I*f*x + 2*I*e)} + (-I*A - B)*e^{(8*I*f*x + 8*I*e)} - 3*(2*I*A + B)*e^{(6*I*f*x + 6*I*e)} - 18*I*A*e^{(4*I*f*x + 4*I*e)} + 3*I*A - 3*B)*e^{(-2*I*f*x - 2*I*e)}/(a*c^3*f)$

**Sympy** [A]

time = 0.34, size = 328, normalized size = 2.20

$$\begin{cases} \frac{(-294912iAa^3c^3f^3e^{4ie^{2ifx}} + (49152iAa^3c^3f^3 - 49152Ba^3c^3f^3)e^{-2ifx} + (-98304iAa^3c^3f^3e^{6ie} - 49152Ba^3c^3f^3e^{6ie})e^{4ifx} + (-16384iAa^3c^3f^3e^{8ie} - 16384Ba^3c^3f^3e^{8ie})e^{6ifx})e^{-2ie}}{1572864a^4c^{12}f^4} & \text{for } a^4c^{12}f^4e^{2ie} \neq 0 \\ x\left(-\frac{2A+iB}{8ac^3} + \frac{(Ae^{8ie}+4Ae^{6ie}+6Ae^{4ie}+4Ae^{2ie}+A-iBe^{8ie}-2iBe^{6ie}+2iBe^{2ie}+iB)e^{-2ie}}{16ac^3}\right) & \text{otherwise} \end{cases} + \frac{x(2A+iB)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)`

[Out] Piecewise(((−294912\*I\*A\*a\*\*3\*c\*\*9\*f\*\*3\*exp(4\*I\*e)\*exp(2\*I\*f\*x) + (49152\*I\*A\*a\*\*3\*c\*\*9\*f\*\*3 - 49152\*B\*a\*\*3\*c\*\*9\*f\*\*3)\*exp(−2\*I\*f\*x) + (−98304\*I\*A\*a\*\*3\*c\*\*9\*f\*\*3\*exp(6\*I\*e) - 49152\*B\*a\*\*3\*c\*\*9\*f\*\*3\*exp(6\*I\*e))\*exp(4\*I\*f\*x) + (−16384\*I\*A\*a\*\*3\*c\*\*9\*f\*\*3\*exp(8\*I\*e) - 16384\*B\*a\*\*3\*c\*\*9\*f\*\*3\*exp(8\*I\*e))\*exp(6\*I\*f\*x))\*exp(−2\*I\*e)/(1572864\*a\*\*4\*c\*\*12\*f\*\*4), Ne(a\*\*4\*c\*\*12\*f\*\*4\*exp(2\*I\*e), 0)), (x\*(−(2\*A + I\*B)/(8\*a\*c\*\*3) + (A\*exp(8\*I\*e) + 4\*A\*exp(6\*I\*e) + 6\*A\*exp(4\*I\*e) + 4\*A\*exp(2\*I\*e) + A - I\*B\*exp(8\*I\*e) - 2\*I\*B\*exp(6\*I\*e) + 2\*I\*B\*exp(2\*I\*e) + I\*B)\*exp(−2\*I\*e)/(16\*a\*c\*\*3)), True)) + x\*(2\*A + I\*B)/(8\*a\*c\*\*3)

**Giac** [A]

time = 0.79, size = 192, normalized size = 1.29

$$\frac{\frac{6(-2iA+B)\log(\tan(fx+e)+i)}{ac^3} + \frac{6(2iA-B)\log(\tan(fx+e)-i)}{ac^3} + \frac{6(-2iA\tan(fx+e)+B\tan(fx+e)-3A-2iB)}{ac^2(\tan(fx+e)-i)} + \frac{22iA\tan(fx+e)^3-11B\tan(fx+e)^3-84A\tan(fx+e)^2-39iB\tan(fx+e)^2-114iA\tan(fx+e)+45B\tan(fx+e)+60A+9iB}{ac^2(\tan(fx+e)+i)^3}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$-1/96*(6*(-2*I*A + B)*\log(\tan(f*x + e) + I)/(a*c^3) + 6*(2*I*A - B)*\log(\tan(f*x + e) - I)/(a*c^3) + 6*(-2*I*A*\tan(f*x + e) + B*\tan(f*x + e) - 3*A - 2*I*B)/(a*c^3*(\tan(f*x + e) - I)) + (22*I*A*\tan(f*x + e)^3 - 11*B*\tan(f*x + e)^3 - 84*A*\tan(f*x + e)^2 - 39*I*B*\tan(f*x + e)^2 - 114*I*A*\tan(f*x + e) + 45*B*\tan(f*x + e) + 60*A + 9*I*B)/(a*c^3*(\tan(f*x + e) + I)^3))/f$$

**Mupad** [B]

time = 9.11, size = 161, normalized size = 1.08

$$\frac{\frac{B}{12ac^3} + \tan(e+fx)^2\left(-\frac{B}{4ac^3} + \frac{A1i}{2ac^3}\right) + \tan(e+fx)^3\left(\frac{A}{4ac^3} + \frac{B1i}{8ac^3}\right) - \tan(e+fx)\left(\frac{A}{12ac^3} + \frac{B1i}{24ac^3}\right) + \frac{A1i}{3ac^3}}{f(\tan(e+fx)^4 + \tan(e+fx)^3 2i + \tan(e+fx) 2i - 1)} - \frac{x(-B+A2i) 1i}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i))^3), x)

[Out] 
$$(\tan(e + f*x)^2*((A*1i)/(2*a*c^3) - B/(4*a*c^3)) - \tan(e + f*x)*(A/(12*a*c^3) + (B*1i)/(24*a*c^3)) + \tan(e + f*x)^3*(A/(4*a*c^3) + (B*1i)/(8*a*c^3)) + (A*1i)/(3*a*c^3) + B/(12*a*c^3))/(f*(\tan(e + f*x)*2i + \tan(e + f*x)^3*2i + \tan(e + f*x)^4 - 1)) - (x*(A*2i - B)*1i)/(8*a*c^3)$$



$$3.714 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^4} dx$$

**Optimal.** Leaf size=181

$$\frac{(5A+3iB)x}{32ac^4} - \frac{A+iB}{32ac^4 f(i-\tan(e+fx))} - \frac{iA+B}{16ac^4 f(i+\tan(e+fx))^4} - \frac{A}{12ac^4 f(i+\tan(e+fx))^3} + \frac{3}{32ac^4 f(i$$

[Out] 1/32\*(5\*A+3\*I\*B)\*x/a/c^4+1/32\*(-A-I\*B)/a/c^4/f/(I-tan(f\*x+e))+1/16\*(-I\*A-B)/a/c^4/f/(I+tan(f\*x+e))^4-1/12\*A/a/c^4/f/(I+tan(f\*x+e))^3+1/32\*(3\*I\*A-B)/a/c^4/f/(I+tan(f\*x+e))^2+1/16\*(2\*A+I\*B)/a/c^4/f/(I+tan(f\*x+e))

**Rubi [A]**

time = 0.17, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{A+iB}{32ac^4 f(-\tan(e+fx)+i)} + \frac{2A+iB}{16ac^4 f(\tan(e+fx)+i)} + \frac{-B+3iA}{32ac^4 f(\tan(e+fx)+i)^2} - \frac{B+iA}{16ac^4 f(\tan(e+fx)+i)^4} + \frac{x(5A+3iB)}{32ac^4} - \frac{A}{12ac^4 f(\tan(e+fx)+i)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4), x]

[Out] ((5\*A + (3\*I)\*B)\*x)/(32\*a\*c^4) - (A + I\*B)/(32\*a\*c^4\*f\*(I - Tan[e + f\*x])) - (I\*A + B)/(16\*a\*c^4\*f\*(I + Tan[e + f\*x])^4) - A/(12\*a\*c^4\*f\*(I + Tan[e + f\*x])^3) + ((3\*I)\*A - B)/(32\*a\*c^4\*f\*(I + Tan[e + f\*x])^2) + (2\*A + I\*B)/(16\*a\*c^4\*f\*(I + Tan[e + f\*x]))

Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Di

st[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))(c - ic \tan(e + fx))^4} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2(c-icx)^5} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{-A-iB}{32a^2c^5(-i+x)^2} + \frac{iA+B}{4a^2c^5(i+x)^5} + \frac{A}{4a^2c^5(i+x)^4} + \frac{-}{16a^2c^5(i+x)^3}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{A + iB}{32ac^4 f(i - \tan(e + fx))} - \frac{iA + B}{16ac^4 f(i + \tan(e + fx))^4}$$

$$= \frac{(5A + 3iB)x}{32ac^4} - \frac{A + iB}{32ac^4 f(i - \tan(e + fx))} - \frac{iA + B}{16ac^4 f(i + \tan(e + fx))^4}$$

Mathematica [A]

time = 1.14, size = 221, normalized size = 1.22

$$\frac{\sec(e + fx)(\cos(4(e + fx)) + i \sin(4(e + fx))(-12(15A + iB)\cos(e + fx) + 4(-5A + 3iB - 30Afx + 18Bfx)\cos(3(e + fx)) + 9A\cos(5(e + fx)) + 15iB\cos(5(e + fx)) + 60iA\sin(e + fx) - 36B\sin(e + fx) - 20A\sin(3(e + fx)) - 12B\sin(3(e + fx)) - 120Afx\sin(3(e + fx)) - 72Bfx\sin(3(e + fx)) - 15iA\sin(5(e + fx)) + 9B\sin(5(e + fx)))}{768ac^4(-i + \tan(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4), x]

[Out] (Sec[e + f\*x]\*(Cos[4\*(e + f\*x)] + I\*Sin[4\*(e + f\*x)])\*(-12\*(15\*A + I\*B)\*Cos[e + f\*x] + 4\*(-5\*A + (3\*I)\*B - (30\*I)\*A\*f\*x + 18\*B\*f\*x)\*Cos[3\*(e + f\*x)] + 9\*A\*Cos[5\*(e + f\*x)] + (15\*I)\*B\*Cos[5\*(e + f\*x)] + (60\*I)\*A\*Sin[e + f\*x] - 36\*B\*Sin[e + f\*x] - (20\*I)\*A\*Sin[3\*(e + f\*x)] - 12\*B\*Sin[3\*(e + f\*x)] - 120\*A\*f\*x\*Sin[3\*(e + f\*x)] - (72\*I)\*B\*f\*x\*Sin[3\*(e + f\*x)] - (15\*I)\*A\*Sin[5\*(e + f\*x)] + 9\*B\*Sin[5\*(e + f\*x)])/(768\*a\*c^4\*f\*(-I + Tan[e + f\*x]))

Maple [A]

time = 0.34, size = 148, normalized size = 0.82

method	result
derivativedivides	$\left(-\frac{5iA}{64} + \frac{3B}{64}\right) \ln(-i + \tan(fx+e)) - \frac{-\frac{A}{32} - \frac{iB}{32}}{-i + \tan(fx+e)} - \frac{-\frac{3iA}{16} + \frac{B}{16}}{2(i + \tan(fx+e))^2} - \frac{A}{12(i + \tan(fx+e))^3} - \frac{\frac{iA}{4} + \frac{B}{4}}{4(i + \tan(fx+e))^4} - \frac{-\frac{iB}{16} - \frac{A}{8}}{i + \tan(fx+e)} + \frac{f a c^4}{f a c^4}$
default	$\left(-\frac{5iA}{64} + \frac{3B}{64}\right) \ln(-i + \tan(fx+e)) - \frac{-\frac{A}{32} - \frac{iB}{32}}{-i + \tan(fx+e)} - \frac{-\frac{3iA}{16} + \frac{B}{16}}{2(i + \tan(fx+e))^2} - \frac{A}{12(i + \tan(fx+e))^3} - \frac{\frac{iA}{4} + \frac{B}{4}}{4(i + \tan(fx+e))^4} - \frac{-\frac{iB}{16} - \frac{A}{8}}{i + \tan(fx+e)} + \frac{f a c^4}{f a c^4}$

risch	$\frac{3ixB}{32ac^4} + \frac{5xA}{32ac^4} - \frac{e^{8i(fx+e)}B}{256ac^4f} - \frac{ie^{8i(fx+e)}A}{256ac^4f} - \frac{e^{6i(fx+e)}B}{64ac^4f} - \frac{5ie^{6i(fx+e)}A}{192ac^4f} - \frac{e^{4i(fx+e)}B}{64ac^4f} - \frac{5ie^{4i(fx+e)}A}{64ac^4f}$
norman	$\frac{(3iB+5A)x}{32ac} - \frac{iA}{3acf} + \frac{(iA+3B)(\tan^2(fx+e))}{6acf} + \frac{11(3iB+5A)(\tan^5(fx+e))}{96acf} + \frac{(3iB+5A)(\tan^7(fx+e))}{32acf} + \frac{(3iB+5A)x(\tan^2(fx+e))}{8ac} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{f/a/c^4} * ((-5/64*I*A+3/64*B) * \ln(-I+\tan(f*x+e)) - (-1/32*A-1/32*I*B) / (-I+\tan(f*x+e)) - 1/2 * (-3/16*I*A+1/16*B) / (I+\tan(f*x+e))^2 - 1/12*A / (I+\tan(f*x+e))^3 - 1/4 * (1/4*I*A+1/4*B) / (I+\tan(f*x+e))^4 - (-1/16*I*B-1/8*A) / (I+\tan(f*x+e)) + (5/64*I*A-3/64*B) * \ln(I+\tan(f*x+e)))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 6.13, size = 121, normalized size = 0.67

$$\frac{(24(5A+3iB)fxe^{2i fx+2ie} - 3(iA+B)e^{10i fx+10ie} - 4(5iA+3B)e^{8i fx+8ie} - 12(5iA+B)e^{6i fx+6ie} - 24(5iA-B)e^{4i fx+4ie} + 12iA - 12B)e^{-2i fx-2ie}}{768ac^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,algorithm="fricas")`

[Out]  $\frac{1}{768} * (24 * (5 * A + 3 * I * B) * f * x * e^{(2 * I * f * x + 2 * I * e)} - 3 * (I * A + B) * e^{(10 * I * f * x + 10 * I * e)} - 4 * (5 * I * A + 3 * B) * e^{(8 * I * f * x + 8 * I * e)} - 12 * (5 * I * A + B) * e^{(6 * I * f * x + 6 * I * e)} - 24 * (5 * I * A - B) * e^{(4 * I * f * x + 4 * I * e)} + 12 * I * A - 12 * B) * e^{(-2 * I * f * x - 2 * I * e)} / (a * c^4 * f)$

**Sympy** [A]

time = 0.39, size = 439, normalized size = 2.43

$$\left\{ \begin{array}{l} \frac{1}{32ac^4} \left( \frac{100663296iAe^{2i fx} - 100663296iBe^{2i fx} - 100663296iAe^{4i fx} - 100663296iBe^{4i fx} + 201326592iAe^{6i fx} + 201326592iBe^{6i fx} - 503316480iAe^{8i fx} - 503316480iBe^{8i fx} - 100663296iAe^{10i fx} - 100663296iBe^{10i fx} + 100663296iAe^{12i fx} + 100663296iBe^{12i fx} - 25165824iAe^{14i fx} - 25165824iBe^{14i fx} - 25165824iAe^{16i fx} - 25165824iBe^{16i fx}}{643205144i^{20}e^{20i fx}} \right) \text{ for } a^2e^{20i fx} \neq 0 \\ x \left( -\frac{5A+3iB}{32ac^4} + \frac{(Ae^{2i fx} + 5Ae^{4i fx} + 10Ae^{6i fx} + 10Ae^{8i fx} + 5Ae^{10i fx} + A - iB)e^{10i fx} - 3Be^{8i fx} - 2iBe^{6i fx} + 2iBe^{4i fx} + 3Be^{2i fx} + iB}{32ac^4} \right) \text{ otherwise} \end{array} \right. + \frac{x(5A+3iB)}{32ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*4,x)

[Out] Piecewise((((100663296\*I\*A\*a\*\*4\*c\*\*16\*f\*\*4 - 100663296\*B\*a\*\*4\*c\*\*16\*f\*\*4)\*exp(-2\*I\*f\*x) + (-1006632960\*I\*A\*a\*\*4\*c\*\*16\*f\*\*4\*exp(4\*I\*e) + 201326592\*B\*a\*\*4\*c\*\*16\*f\*\*4\*exp(4\*I\*e))\*exp(2\*I\*f\*x) + (-503316480\*I\*A\*a\*\*4\*c\*\*16\*f\*\*4\*exp(6\*I\*e) - 100663296\*B\*a\*\*4\*c\*\*16\*f\*\*4\*exp(6\*I\*e))\*exp(4\*I\*f\*x) + (-167772160\*I\*A\*a\*\*4\*c\*\*16\*f\*\*4\*exp(8\*I\*e) - 100663296\*B\*a\*\*4\*c\*\*16\*f\*\*4\*exp(8\*I\*e))\*exp(6\*I\*f\*x) + (-25165824\*I\*A\*a\*\*4\*c\*\*16\*f\*\*4\*exp(10\*I\*e) - 25165824\*B\*a\*\*4\*c\*\*16\*f\*\*4\*exp(10\*I\*e))\*exp(8\*I\*f\*x))\*exp(-2\*I\*e)/(6442450944\*a\*\*5\*c\*\*20\*f\*\*5), Ne(a\*\*5\*c\*\*20\*f\*\*5\*exp(2\*I\*e), 0)), (x\*(-(5\*A + 3\*I\*B)/(32\*a\*c\*\*4) + (A\*exp(10\*I\*e) + 5\*A\*exp(8\*I\*e) + 10\*A\*exp(6\*I\*e) + 10\*A\*exp(4\*I\*e) + 5\*A\*exp(2\*I\*e) + A - I\*B\*exp(10\*I\*e) - 3\*I\*B\*exp(8\*I\*e) - 2\*I\*B\*exp(6\*I\*e) + 2\*I\*B\*exp(4\*I\*e) + 3\*I\*B\*exp(2\*I\*e) + I\*B)\*exp(-2\*I\*e)/(32\*a\*c\*\*4)), True)) + x\*(5\*A + 3\*I\*B)/(32\*a\*c\*\*4)

**Giac** [A]

time = 0.95, size = 221, normalized size = 1.22

$$\frac{12(5iA-3B)\log(\tan(fx+e)+i) + 12(-5iA+3B)\log(\tan(fx+e)-i) + 12(5A\tan(fx+e)+3iB\tan(fx+e)-7iA+5B) + \frac{-125iA\tan(fx+e)^4+75B\tan(fx+e)^4+596A\tan(fx+e)^3+348iB\tan(fx+e)^3+1110iA\tan(fx+e)^2-618B\tan(fx+e)^2-996A\tan(fx+e)-492iB\tan(fx+e)-405iA+99B}{ac^4(\tan(fx+e)+i)^4}}{768f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out] 1/768\*(12\*(5\*I\*A - 3\*B)\*log(tan(f\*x + e) + I)/(a\*c^4) + 12\*(-5\*I\*A + 3\*B)\*log(tan(f\*x + e) - I)/(a\*c^4) + 12\*(5\*A\*tan(f\*x + e) + 3\*I\*B\*tan(f\*x + e) - 7\*I\*A + 5\*B)/(a\*c^4\*(-I\*tan(f\*x + e) - 1)) + (-125\*I\*A\*tan(f\*x + e)^4 + 75\*B\*tan(f\*x + e)^4 + 596\*A\*tan(f\*x + e)^3 + 348\*I\*B\*tan(f\*x + e)^3 + 1110\*I\*A\*tan(f\*x + e)^2 - 618\*B\*tan(f\*x + e)^2 - 996\*A\*tan(f\*x + e) - 492\*I\*B\*tan(f\*x + e) - 405\*I\*A + 99\*B)/(a\*c^4\*(tan(f\*x + e) + I)^4))/f

**Mupad** [B]

time = 9.26, size = 204, normalized size = 1.13

$$\frac{\tan(e+fx)\left(-\frac{3B}{32ac^4} + \frac{A5i}{32ac^4}\right) + \tan(e+fx)^4\left(\frac{5A}{32ac^4} + \frac{B3i}{32ac^4}\right) + \tan(e+fx)^3\left(-\frac{9B}{32ac^4} + \frac{A15i}{32ac^4}\right) - \tan(e+fx)^2\left(\frac{35A}{96ac^4} + \frac{B7i}{32ac^4}\right) - \frac{A}{3ac^4} - \frac{x(-3B+A5i)1i}{32ac^4}}{f(-\tan(e+fx)^5 - \tan(e+fx)^43i + 2\tan(e+fx)^3 - \tan(e+fx)^22i + 3\tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i))^4),x)

[Out] - (tan(e + f\*x)\*((A\*5i)/(32\*a\*c^4) - (3\*B)/(32\*a\*c^4)) + tan(e + f\*x)^4\*((5\*A)/(32\*a\*c^4) + (B\*3i)/(32\*a\*c^4)) + tan(e + f\*x)^3\*((A\*15i)/(32\*a\*c^4) - (9\*B)/(32\*a\*c^4)) - tan(e + f\*x)^2\*((35\*A)/(96\*a\*c^4) + (B\*7i)/(32\*a\*c^4)) - A/(3\*a\*c^4))/(f\*(3\*tan(e + f\*x) - tan(e + f\*x)^2\*2i + 2\*tan(e + f\*x)^3 - tan(e + f\*x)^4\*3i - tan(e + f\*x)^5 + 1i)) - (x\*(A\*5i - 3\*B)\*1i)/(32\*a\*c^4)

$$3.715 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^n}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=115

$$\frac{(iA(2-n) + B(2+n)) {}_2F_1(2, n; 1+n; \frac{1}{2}(1-i \tan(e+fx))) (c-ic \tan(e+fx))^n}{16a^2fn} + \frac{(iA-B)(c-ic \tan(e+fx))^n}{4a^2f(1+i \tan(e+fx))^2}$$

[Out] 1/16\*(I\*A\*(2-n)+B\*(2+n))\*hypergeom([2, n], [1+n], 1/2-1/2\*I\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/a^2/f/n+1/4\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^n/a^2/f/(1+I\*tan(f\*x+e))^2

**Rubi [A]**

time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 79, 70}

$$\frac{(B(n+2) + iA(2-n))(c-ic \tan(e+fx))^n {}_2F_1(2, n; n+1; \frac{1}{2}(1-i \tan(e+fx)))}{16a^2fn} + \frac{(-B+iA)(c-ic \tan(e+fx))^n}{4a^2f(1+i \tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^n)/(a + I\*a\*Tan[e + f\*x])^2, x]

[Out] ((I\*A\*(2 - n) + B\*(2 + n))\*Hypergeometric2F1[2, n, 1 + n, (1 - I\*Tan[e + f\*x])/2]\*(c - I\*c\*Tan[e + f\*x])^n)/(16\*a^2\*f\*n) + ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^n)/(4\*a^2\*f\*(1 + I\*Tan[e + f\*x])^2)

**Rule 70**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

**Rule 79**

Int[((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

**Rule 3669**

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^n}{(a + ia \tan(e + fx))^2} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{-1+n}}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B)(c - ic \tan(e + fx))^n}{4a^2 f (1 + i \tan(e + fx))^2} + \frac{(c(A(2 - n) - iB(2 + n)))}{16a^2 f n}$$

$$= \frac{(iA(2 - n) + B(2 + n)) {}_2F_1(2, n; 1 + n; \frac{1}{2}(1 - i \tan(e + fx)))}{16a^2 f n}$$

**Mathematica [F]**

time = 67.80, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^n}{(a + ia \tan(e + fx))^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
f*x])^2, x]
```

```
[Out] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
f*x])^2, x]
```

**Maple [F]**

time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(fx + e))(c - ic \tan(fx + e))^n}{(a + ia \tan(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2, x)
```

```
[Out] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2, x)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out] integral(1/4\*((A - I\*B)\*e^(4\*I\*f\*x + 4\*I\*e) + 2\*A\*e^(2\*I\*f\*x + 2\*I\*e) + A + I\*B)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^n\*e^(-4\*I\*f\*x - 4\*I\*e)/a^2, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A(-i c \tan(e+f x)+c)^n}{\tan^2(e+f x)-2 i \tan(e+f x)-1} d x + \int \frac{B(-i c \tan(e+f x)+c)^n \tan(e+f x)}{\tan^2(e+f x)-2 i \tan(e+f x)-1} d x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^2,x)

[Out] -(Integral(A\*(-I\*c\*tan(e + f\*x) + c))^n/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x) + Integral(B\*(-I\*c\*tan(e + f\*x) + c))^n\*tan(e + f\*x)/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x))/a\*\*2

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^n/(I\*a\*tan(f\*x + e) + a)^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) \operatorname{li})^n}{(a + a \tan(e + f x) \operatorname{li})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1i)^2, x)
```



$$3.716 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^5}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=194

$$\frac{8(3A+7iB)c^5x}{a^2} + \frac{8(3iA-7B)c^5 \log(\cos(e+fx))}{a^2f} - \frac{8(iA-B)c^5}{a^2f(i-\tan(e+fx))^2} + \frac{16(2A+3iB)c^5}{a^2f(i-\tan(e+fx))} - \frac{(7A+7iB)c^5}{a^2}$$

[Out]  $8*(3*A+7*I*B)*c^5*x/a^2+8*(3*I*A-7*B)*c^5*\ln(\cos(f*x+e))/a^2/f-8*(I*A-B)*c^5/a^2/f/(I-\tan(f*x+e))^2+16*(2*A+3*I*B)*c^5/a^2/f/(I-\tan(f*x+e))-(7*A+24*I*B)*c^5*\tan(f*x+e)/a^2/f+1/2*(I*A-7*B)*c^5*\tan(f*x+e)^2/a^2/f+1/3*I*B*c^5*\tan(f*x+e)^3/a^2/f$

**Rubi [A]**

time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{c^5(-7B+iA)\tan^2(e+fx)}{2a^2f} - \frac{c^5(7A+24iB)\tan(e+fx)}{a^2f} + \frac{16c^5(2A+3iB)}{a^2f(-\tan(e+fx)+i)} - \frac{8c^5(-B+iA)}{a^2f(-\tan(e+fx)+i)^2} + \frac{8c^5(-7B+3iA)\log(\cos(e+fx))}{a^2f} + \frac{8c^5x(3A+7iB)}{a^2} + \frac{iBc^5\tan^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^5)/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out]  $(8*(3*A+(7*I)*B)*c^5*x)/a^2+(8*((3*I)*A-7*B)*c^5*\text{Log}[\text{Cos}[e+f*x]])/(a^2*f)-(8*(I*A-B)*c^5)/(a^2*f*(I-\text{Tan}[e+f*x])^2)+(16*(2*A+(3*I)*B)*c^5)/(a^2*f*(I-\text{Tan}[e+f*x]))-((7*A+(24*I)*B)*c^5*\text{Tan}[e+f*x])/(a^2*f)+((I*A-7*B)*c^5*\text{Tan}[e+f*x]^2)/(2*a^2*f)+((I/3)*B*c^5*\text{Tan}[e+f*x]^3)/(a^2*f)$

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

## Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^5}{(a + ia \tan(e + fx))^2} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^4}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(-\frac{(7A+24iB)c^4}{a^3} + \frac{i(A+7iB)c^4 x}{a^3} + \frac{iBc^4 x^2}{a^3} + \frac{16i(A+iB)c^4 x^3}{a^3(-i)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{8(3A + 7iB)c^5 x}{a^2} + \frac{8(3iA - 7B)c^5 \log(\cos(e + fx))}{a^2 f} - \frac{f}{a^2}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1357 vs. 2(194) = 388.  
time = 7.53, size = 1357, normalized size = 6.99

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^5)/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] (4\*((-3\*I)\*A + 5\*B)\*c^5\*Cos[2\*f\*x]\*Sec[e + f\*x]\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e + f\*x]\*(3\*A\*c^5\*Cos[e] + (7\*I)\*B\*c^5\*Cos[e] + (3\*I)\*A\*c^5\*Sin[e] - 7\*B\*c^5\*Sin[e])\*(8\*ArcTan[Tan[f\*x]]\*Cos[e] + (8\*I)\*ArcTan[Tan[f\*x]]\*Sin[e]))\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e + f\*x]\*(3\*A\*c^5\*Cos[e] + (7\*I)\*B\*c^5\*Cos[e] + (3\*I)\*A\*c^5\*Sin[e] - 7\*B\*c^5\*Sin[e]))\*((4\*I)\*Cos[e]\*Log[Cos[e + f\*x]^2] - 4\*Log[Cos[e + f\*x]^2]\*Sin[e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e]\*Sec[e + f\*x]^3\*(3\*A\*Cos[e] + (21\*I)\*B\*Cos[e] + 2\*B\*Sin[e]))\*((I/6)\*c^5\*Cos[2\*e] - (c^5\*Sin[2\*e])/6)\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + ((A + I\*B)\*Cos[4\*f\*x]\*Sec[e + f\*x]\*((2\*I)\*c^5\*Cos[2\*e] + 2\*c^5\*Sin[2\*e]))\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + ((3\*A + (7\*I)\*B)\*Sec[e + f\*x]\*(8\*c^5\*f\*x\*Cos[2\*e] + (8\*I)\*c^5\*f\*x\*Sin[2\*e]))\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) - (4\*(3\*A + (5\*I)\*B)\*c^5\*Sec[e + f\*x]\*(Cos[f\*x] + I\*Sin[f\*x])^2\*Sin[2\*f\*x]\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + ((A + I\*B)\*Sec[e + f\*x]\*(2\*c^5\*Cos[2\*e] - (2\*I)\*c^5\*Sin[2\*e]))\*(Cos[f\*x] + I\*Sin[f\*x])^2\*Sin[4\*f\*x]\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e]\*Sec[e +

$$f*x]^4*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^2*(-1/2*(B*c^5*\text{Cos}[2*e - f*x]) + (B*c^5*\text{Cos}[2*e + f*x])/2 - (I/2)*B*c^5*\text{Sin}[2*e - f*x] + (I/2)*B*c^5*\text{Sin}[2*e + f*x])*(A + B*\text{Tan}[e + f*x]))/(3*f*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x])*(a + I*a*\text{Tan}[e + f*x])^2) + (\text{Sec}[e]*\text{Sec}[e + f*x]^2*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^2*((-21*I)/2)*A*c^5*\text{Cos}[2*e - f*x] + (73*B*c^5*\text{Cos}[2*e - f*x])/2 + ((21*I)/2)*A*c^5*\text{Cos}[2*e + f*x] - (73*B*c^5*\text{Cos}[2*e + f*x])/2 + (21*A*c^5*\text{Sin}[2*e - f*x])/2 + ((73*I)/2)*B*c^5*\text{Sin}[2*e - f*x] - (21*A*c^5*\text{Sin}[2*e + f*x])/2 - ((73*I)/2)*B*c^5*\text{Sin}[2*e + f*x])*(A + B*\text{Tan}[e + f*x]))/(3*f*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x])*(a + I*a*\text{Tan}[e + f*x])^2) + (x*\text{Sec}[e + f*x]*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^2*(-24*A*c^5 - (56*I)*B*c^5 - (24*I)*A*c^5*\text{Tan}[e] + 56*B*c^5*\text{Tan}[e] + ((-3*I)*A + 7*B)*(8*c^5*\text{Cos}[2*e] + (8*I)*c^5*\text{Sin}[2*e])* \text{Tan}[e])*(A + B*\text{Tan}[e + f*x]))/((A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x])*(a + I*a*\text{Tan}[e + f*x])^2)$$

**Maple [A]**

time = 0.31, size = 127, normalized size = 0.65

method	result
derivativedivides	$c^5 \left( \frac{iB(\tan^3(fx+e))}{3} + \frac{iA(\tan^2(fx+e))}{2} - 24iB \tan(fx+e) - \frac{7B(\tan^2(fx+e))}{2} - 7A \tan(fx+e) - \frac{48iB+32A}{-i+\tan(fx+e)} - \frac{16iA-16B}{2(-i+\tan(fx+e))} \right) / f a^2$
default	$c^5 \left( \frac{iB(\tan^3(fx+e))}{3} + \frac{iA(\tan^2(fx+e))}{2} - 24iB \tan(fx+e) - \frac{7B(\tan^2(fx+e))}{2} - 7A \tan(fx+e) - \frac{48iB+32A}{-i+\tan(fx+e)} - \frac{16iA-16B}{2(-i+\tan(fx+e))} \right) / f a^2$
risch	$\frac{20c^5 e^{-2i(fx+e)} B}{a^2 f} - \frac{12ic^5 e^{-2i(fx+e)} A}{a^2 f} - \frac{2c^5 e^{-4i(fx+e)} B}{a^2 f} + \frac{2ic^5 e^{-4i(fx+e)} A}{a^2 f} + \frac{112ic^5 Bx}{a^2} + \frac{48c^5 Ax}{a^2} + \frac{112ic^5 B}{a^2} + \frac{48c^5 A}{a^2} + \frac{112ic^5 B}{a^2} + \frac{48c^5 A}{a^2}$
norman	$\frac{-25ic^5 A + 47B c^5}{af} + \frac{8(7ic^5 B + 3A c^5)x}{a} - \frac{(-ic^5 A + 7B c^5)(\tan^6(fx+e))}{2af} - \frac{7(10ic^5 B + 3A c^5)(\tan^5(fx+e))}{3af} + \frac{(-83ic^5 A + 133B c^5)}{2af}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x,method=_RE  
TURNVERBOSE)`

[Out]  $1/f*c^5/a^2*(1/3*I*B*\text{tan}(f*x+e)^3+1/2*I*A*\text{tan}(f*x+e)^2-24*I*B*\text{tan}(f*x+e)-7/2*B*\text{tan}(f*x+e)^2-7*A*\text{tan}(f*x+e)-(48*I*B+32*A)/(-I+\text{tan}(f*x+e))-1/2*(-16*B+16*I*A)/(-I+\text{tan}(f*x+e))^2+(-24*I*A+56*B)*\ln(-I+\text{tan}(f*x+e)))$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x, alg  
orithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
 expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 2.84, size = 343, normalized size = 1.77

$$\frac{2(24(3A+7)B^2f^{10}e^{10f+10e}-3(24A-7)B^2c^{10}e^{10f+10e}-3(-A+B)^2+12(6(3A+7)B)^2f^2-(24A-7)B^2c^{10}e^{10f+10e}+6(12(3A+7)B)^2f^2-5(24A-7)B^2c^{10}e^{10f+10e}+2(12(3A+7)B)^2f^2-11(24A-7)B^2c^{10}e^{10f+10e}-12((-3)A+7)B^2c^{10}e^{10f+10e}+3(-3)A+7)B^2c^{10}e^{10f+10e}+3(-3)A+7)B^2c^{10}e^{10f+10e}+(-3)A+7)B^2c^{10}e^{10f+10e}+1)}{3(a^2f^{10}e^{10f+10e}+3a^2f^{10}e^{10f+10e}+3a^2f^{10}e^{10f+10e}+a^2f^{10}e^{10f+10e})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out]  $\frac{2/3*(24*(3*A + 7*I*B)*c^5*f*x*e^{(10*I*f*x + 10*I*e)} - 3*(3*I*A - 7*B)*c^5*e^{(2*I*f*x + 2*I*e)} - 3*(-I*A + B)*c^5 + 12*(6*(3*A + 7*I*B)*c^5*f*x - (3*I*A - 7*B)*c^5)*e^{(8*I*f*x + 8*I*e)} + 6*(12*(3*A + 7*I*B)*c^5*f*x - 5*(3*I*A - 7*B)*c^5)*e^{(6*I*f*x + 6*I*e)} + 2*(12*(3*A + 7*I*B)*c^5*f*x - 11*(3*I*A - 7*B)*c^5)*e^{(4*I*f*x + 4*I*e)} - 12*((-3*I*A + 7*B)*c^5*e^{(10*I*f*x + 10*I*e)} + 3*(-3*I*A + 7*B)*c^5*e^{(8*I*f*x + 8*I*e)} + 3*(-3*I*A + 7*B)*c^5*e^{(6*I*f*x + 6*I*e)} + (-3*I*A + 7*B)*c^5*e^{(4*I*f*x + 4*I*e)})*\log(e^{(2*I*f*x + 2*I*e)} + 1))/(a^2*f*e^{(10*I*f*x + 10*I*e)} + 3*a^2*f*e^{(8*I*f*x + 8*I*e)} + 3*a^2*f*e^{(6*I*f*x + 6*I*e)} + a^2*f*e^{(4*I*f*x + 4*I*e)})}$

**Sympy** [A]

time = 0.82, size = 445, normalized size = 2.29

$$\frac{-42iAc^5 + 146Bc^5 + (-78iAc^5e^{2e} + 246Bc^5e^{2e})e^{2fx} + (-36iAc^5e^{4e} + 108Bc^5e^{4e})e^{4fx}}{3a^2f^2e^{6e}e^{6fx} + 9a^2f^2e^{4e}e^{4fx} + 9a^2f^2e^{2e}e^{2fx} + 3a^2f} + \begin{cases} \frac{(20Aa^2c^5f^{2e} - 2Ba^2c^5f^{2e})e^{-4fx} + (-12Aa^2c^5f^{4e} + 20Ba^2c^5f^{4e})e^{-2fx}}{a^2f} & \text{for } a^4f^2e^{6e} \neq 0 \\ x\left(-\frac{48A^2c^5 + 112Bc^5}{a^2} + \frac{(48Ac^5e^{4e} - 24A^2c^5 + 8A^2c^5 + 112Bc^5e^{4e} - 40Bc^5e^{2e} + 8Bc^5)e^{-4e}}{a^2}\right) & \text{otherwise} \end{cases} + \frac{8ic^5 \cdot (3A + 7iB) \log(e^{2fx} + e^{-2e})}{a^2f} + \frac{x(48A^2c^5 + 112iBc^5)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*5/(a+I\*a\*tan(f\*x+e))\*\*2,x)

[Out]  $(-42*I*A*c**5 + 146*B*c**5 + (-78*I*A*c**5*\exp(2*I*e) + 246*B*c**5*\exp(2*I*e))*\exp(2*I*f*x) + (-36*I*A*c**5*\exp(4*I*e) + 108*B*c**5*\exp(4*I*e))*\exp(4*I*f*x))/(3*a**2*f*\exp(6*I*e)*\exp(6*I*f*x) + 9*a**2*f*\exp(4*I*e)*\exp(4*I*f*x) + 9*a**2*f*\exp(2*I*e)*\exp(2*I*f*x) + 3*a**2*f) + \text{Piecewise}(\left(\left(\left(2*I*A*a**2*c**5*f*\exp(2*I*e) - 2*B*a**2*c**5*f*\exp(2*I*e)\right)*\exp(-4*I*f*x) + (-12*I*A*a**2*c**5*f*\exp(4*I*e) + 20*B*a**2*c**5*f*\exp(4*I*e))*\exp(-2*I*f*x)\right)*\exp(-6*I*e)/(a**4*f**2), \text{Ne}(a**4*f**2*\exp(6*I*e), 0)), (x*(-(48*A*c**5 + 112*I*B*c**5)/a**2 + (48*A*c**5*\exp(4*I*e) - 24*A*c**5*\exp(2*I*e) + 8*A*c**5 + 112*I*B*c**5*\exp(4*I*e) - 40*I*B*c**5*\exp(2*I*e) + 8*I*B*c**5)*\exp(-4*I*e)/a**2), \text{True})) + 8*I*c**5*(3*A + 7*I*B)*\log(\exp(2*I*f*x) + \exp(-2*I*e))/(a**2*f) + x*(48*A*c**5 + 112*I*B*c**5)/a**2$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than

twice the leaf count of optimal. 517 vs.  $2(172) = 344$ .

time = 1.07, size = 517, normalized size = 2.66

$$\frac{(-42iAc^5 + 146Bc^5 + (-78iAc^5e^{2e} + 246Bc^5e^{2e})e^{2fx} + (-36iAc^5e^{4e} + 108Bc^5e^{4e})e^{4fx})}{3a^2f^2e^{6e}e^{6fx} + 9a^2f^2e^{4e}e^{4fx} + 9a^2f^2e^{2e}e^{2fx} + 3a^2f} + \begin{cases} \frac{(20Aa^2c^5f^{2e} - 2Ba^2c^5f^{2e})e^{-4fx} + (-12Aa^2c^5f^{4e} + 20Ba^2c^5f^{4e})e^{-2fx}}{a^2f} & \text{for } a^4f^2e^{6e} \neq 0 \\ x\left(-\frac{48A^2c^5 + 112Bc^5}{a^2} + \frac{(48Ac^5e^{4e} - 24A^2c^5 + 8A^2c^5 + 112Bc^5e^{4e} - 40Bc^5e^{2e} + 8Bc^5)e^{-4e}}{a^2}\right) & \text{otherwise} \end{cases} + \frac{8ic^5 \cdot (3A + 7iB) \log(e^{2fx} + e^{-2e})}{a^2f} + \frac{x(48A^2c^5 + 112iBc^5)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^5/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] 
$$\frac{2}{3}*(12*(3*I*A*c^5 - 7*B*c^5)*\log(\tan(1/2*f*x + 1/2*e) + 1)/a^2 - 24*(3*I*A*c^5 - 7*B*c^5)*\log(\tan(1/2*f*x + 1/2*e) - I)/a^2 - 12*(-3*I*A*c^5 + 7*B*c^5)*\log(\tan(1/2*f*x + 1/2*e) - 1)/a^2 - (66*I*A*c^5*\tan(1/2*f*x + 1/2*e)^6 - 154*B*c^5*\tan(1/2*f*x + 1/2*e)^6 - 21*A*c^5*\tan(1/2*f*x + 1/2*e)^5 - 72*I*B*c^5*\tan(1/2*f*x + 1/2*e)^5 - 201*I*A*c^5*\tan(1/2*f*x + 1/2*e)^4 + 483*B*c^5*\tan(1/2*f*x + 1/2*e)^4 + 42*A*c^5*\tan(1/2*f*x + 1/2*e)^3 + 148*I*B*c^5*\tan(1/2*f*x + 1/2*e)^3 + 201*I*A*c^5*\tan(1/2*f*x + 1/2*e)^2 - 483*B*c^5*\tan(1/2*f*x + 1/2*e)^2 - 21*A*c^5*\tan(1/2*f*x + 1/2*e) - 72*I*B*c^5*\tan(1/2*f*x + 1/2*e) - 66*I*A*c^5 + 154*B*c^5)/((\tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^2) - 2*(-75*I*A*c^5*\tan(1/2*f*x + 1/2*e)^4 + 175*B*c^5*\tan(1/2*f*x + 1/2*e)^4 - 324*A*c^5*\tan(1/2*f*x + 1/2*e)^3 - 748*I*B*c^5*\tan(1/2*f*x + 1/2*e)^3 + 522*I*A*c^5*\tan(1/2*f*x + 1/2*e)^2 - 1170*B*c^5*\tan(1/2*f*x + 1/2*e)^2 + 324*A*c^5*\tan(1/2*f*x + 1/2*e) + 748*I*B*c^5*\tan(1/2*f*x + 1/2*e) - 75*I*A*c^5 + 175*B*c^5)/(a^2*(\tan(1/2*f*x + 1/2*e) - I)^4)/f$$

**Mupad [B]**

time = 8.81, size = 282, normalized size = 1.45

$$\frac{\ln(\tan(e + fx) - i) \left( -\frac{36Bc^5}{a^2} + \frac{A^2c^{10}}{a^2} \right)}{f} + \frac{\tan(e + fx)^2 \left( -\frac{36Bc^5}{2a^2} + \frac{c^2(A+B^2i)}{2a^2} \right)}{f} - \frac{\tan(e + fx) \left( \frac{3c^2(A+B^2i)}{a^2} + \frac{B^2c^5}{a^2} - \frac{c^2(-3B+A^2i)}{a^2} \right)}{f} + \frac{-\frac{34Bc^5(A^2+6i)}{2a^2} + \frac{18A^2c^2c^5}{2a^2} + \frac{(-56Bc^5+A^2c^5)B}{2a^2} + \tan(e + fx) \left( \frac{16A^2c^2(A^2+6i)}{a^2} - \frac{2(-56Bc^5+A^2c^5)}{a^2} \right)}{f(\tan(e + fx)^2 + 2\tan(e + fx) - i)} + \frac{Bc^5 \tan(e + fx)^3 i}{3a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^5)/(a + a\*tan(e + f\*x)\*1i)^2,x)

[Out] 
$$\frac{(\tan(e + f*x)^2*((c^5*(A + B*4i)*1i)/(2*a^2) - (3*B*c^5)/(2*a^2)))/f - (\log(\tan(e + f*x) - 1i)*((A*c^5*24i)/a^2 - (56*B*c^5)/a^2))/f - (\tan(e + f*x)*((3*c^5*(A + B*4i))/a^2 + (B*c^5*6i)/a^2 - (c^5*(A*2i - 3*B)*2i)/a^2))/f + ((16*A*c^5 + B*c^5*64i)/(2*a^2) - ((A*c^5*8i - 24*B*c^5)*1i)/(2*a^2) + ((A*c^5*24i - 56*B*c^5)*3i)/(2*a^2) + \tan(e + f*x)*(((16*A*c^5 + B*c^5*64i)*1i)/a^2 - (2*(A*c^5*24i - 56*B*c^5))/a^2))/(f*(2*\tan(e + f*x) + \tan(e + f*x)^2*1i - 1i)) + (B*c^5*\tan(e + f*x)^3*1i)/(3*a^2*f)$$

$$3.717 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^4}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=158

$$\frac{6(A+3iB)c^4x}{a^2} + \frac{6(iA-3B)c^4 \log(\cos(e+fx))}{a^2f} - \frac{4(iA-B)c^4}{a^2f(i-\tan(e+fx))^2} + \frac{4(3A+5iB)c^4}{a^2f(i-\tan(e+fx))} - \frac{(A+6iB)}{a^2}$$

[Out] 6\*(A+3\*I\*B)\*c^4\*x/a^2+6\*(I\*A-3\*B)\*c^4\*ln(cos(f\*x+e))/a^2/f-4\*(I\*A-B)\*c^4/a^2/f/(I-tan(f\*x+e))^2+4\*(3\*A+5\*I\*B)\*c^4/a^2/f/(I-tan(f\*x+e))-(A+6\*I\*B)\*c^4\*tan(f\*x+e)/a^2/f-1/2\*B\*c^4\*tan(f\*x+e)^2/a^2/f

**Rubi [A]**

time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{c^4(A+6iB)\tan(e+fx)}{a^2f} + \frac{4c^4(3A+5iB)}{a^2f(-\tan(e+fx)+i)} - \frac{4c^4(-B+iA)}{a^2f(-\tan(e+fx)+i)^2} + \frac{6c^4(-3B+iA)\log(\cos(e+fx))}{a^2f} + \frac{6c^4x(A+3iB)}{a^2} - \frac{Bc^4 \tan^2(e+fx)}{2a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4)/(a + I\*a\*Tan[e + f\*x])^2, x]

[Out] (6\*(A + (3\*I)\*B)\*c^4\*x)/a^2 + (6\*(I\*A - 3\*B)\*c^4\*Log[Cos[e + f\*x]])/(a^2\*f) - (4\*(I\*A - B)\*c^4)/(a^2\*f\*(I - Tan[e + f\*x])^2) + (4\*(3\*A + (5\*I)\*B)\*c^4)/(a^2\*f\*(I - Tan[e + f\*x])) - ((A + (6\*I)\*B)\*c^4\*Tan[e + f\*x])/(a^2\*f) - (B\*c^4\*Tan[e + f\*x]^2)/(2\*a^2\*f)

Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i \tan(e + fx))^4}{(a + ia \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^3}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(ac) \text{Subst}\left(\int \left(-\frac{(A+6iB)c^3}{a^3} - \frac{Bc^3x}{a^3} + \frac{8i(A+iB)c^3}{a^3(-i+x)^3} + \frac{4(3A+5iB)}{a^3(-i+x)}\right) dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{6(A + 3iB)c^4x}{a^2} + \frac{6(iA - 3B)c^4 \log(\cos(e + fx))}{a^2 f} - \frac{6c^4}{a^2 f}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1079 vs. 2(158) = 316.  
time = 7.12, size = 1079, normalized size = 6.83

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4)/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] c^4\*((4\*((-1)\*A + 2\*B)\*Cos[2\*f\*x]\*Sec[e + f\*x]\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e + f\*x]\*(A\*Cos[e] + (3\*I)\*B\*Cos[e] + I\*A\*Sin[e] - 3\*B\*Sin[e]))\*(6\*ArcTan[Tan[f\*x]]\*Cos[e] + (6\*I)\*ArcTan[Tan[f\*x]]\*Sin[e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e + f\*x]\*(A\*Cos[e] + (3\*I)\*B\*Cos[e] + I\*A\*Sin[e] - 3\*B\*Sin[e]))\*((3\*I)\*Cos[e]\*Log[Cos[e + f\*x]^2] - 3\*Log[Cos[e + f\*x]^2]\*Sin[e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + ((A + I\*B)\*Cos[4\*f\*x]\*Sec[e + f\*x]\*(I\*Cos[2\*e] + Sin[2\*e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e + f\*x]^3\*(-1/2\*(B\*Cos[2\*e]) - (I/2)\*B\*Sin[2\*e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + ((A + (3\*I)\*B)\*Sec[e + f\*x]\*(6\*f\*x\*Cos[2\*e] + (6\*I)\*f\*x\*Sin[2\*e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) - (4\*(A + (2\*I)\*B)\*Sec[e + f\*x]\*(Cos[f\*x] + I\*Sin[f\*x])^2\*Sin[2\*f\*x]\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + ((A + I\*B)\*Sec[e + f\*x]\*(Cos[2\*e] - I\*Sin[2\*e])\*(Cos[f\*x] + I\*Sin[f\*x])^2\*Sin[4\*f\*x]\*(A + B\*Tan[e + f\*x]))/(f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2) + (Sec[e + f\*x]^2\*(Cos[f\*x] + I\*Sin[f\*x])^2\*((-1/2\*I)\*A\*Cos[2\*e - f\*x] + 3\*B\*Cos[2\*e - f\*x] + (I/2)\*A\*Cos[2\*e + f\*x] - 3\*B\*Cos[2\*e + f\*x] + (A\*Sin[2\*e - f\*x])/2 + (3\*I)\*B\*Sin[2\*e - f\*x] - (A\*Sin[2\*e + f\*x])/2 - (3\*I)\*B\*Sin[2\*e

$$\frac{(f \cos(e + fx) + B \sin(e + fx)) \cdot (a + I a \tan(e + fx))^2 + (x \sec(e + fx) (\cos(fx) + I \sin(fx))^2 (-6A - (18I)B - (6I)A \tan(e) + 18B \tan(e) + ((-I)A + 3B)(6 \cos(2e) + (6I) \sin(2e)) \tan(e)) \cdot (A + B \tan(e + fx)))}{(f \cos(e + fx) + B \sin(e + fx)) \cdot (a + I a \tan(e + fx))^2}$$

**Maple [A]**

time = 0.27, size = 103, normalized size = 0.65

method	result
derivativedivides	$\frac{c^4 \left( -\frac{B(\tan^2(fx+e))}{2} - 6iB \tan(fx+e) - A \tan(fx+e) + (-6iA+18B) \ln(-i+\tan(fx+e)) - \frac{8iA-8B}{2(-i+\tan(fx+e))^2} - \frac{20iB+12A}{-i+\tan(fx+e)} \right)}{f a^2}$
default	$\frac{c^4 \left( -\frac{B(\tan^2(fx+e))}{2} - 6iB \tan(fx+e) - A \tan(fx+e) + (-6iA+18B) \ln(-i+\tan(fx+e)) - \frac{8iA-8B}{2(-i+\tan(fx+e))^2} - \frac{20iB+12A}{-i+\tan(fx+e)} \right)}{f a^2}$
risch	$\frac{8c^4 e^{-2i(fx+e)} B}{a^2 f} - \frac{4ic^4 e^{-2i(fx+e)} A}{a^2 f} - \frac{c^4 e^{-4i(fx+e)} B}{a^2 f} + \frac{ic^4 e^{-4i(fx+e)} A}{a^2 f} + \frac{36ic^4 Bx}{a^2} + \frac{12c^4 Ax}{a^2} + \frac{36ic^4 Be}{f a^2} +$
norman	$\frac{-8ic^4 A + 17B c^4}{af} + \frac{6(3ic^4 B + A c^4)x}{a} - \frac{(6ic^4 B + A c^4)(\tan^5(fx+e))}{af} + \frac{(-32ic^4 A + 51B c^4)(\tan^2(fx+e))}{2af} + \frac{12(3ic^4 B + A c^4)x(\tan^2(fx+e))}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^2,x,method=\_RETURNVERBOSE)

[Out] 1/f\*c^4/a^2\*(-1/2\*B\*tan(f\*x+e)^2-6\*I\*B\*tan(f\*x+e)-A\*tan(f\*x+e)+(-6\*I\*A+18\*B)\*ln(-I+tan(f\*x+e))-1/2\*(-8\*B+8\*I\*A)/(-I+tan(f\*x+e))^2-(20\*I\*B+12\*A)/(-I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 2.87, size = 260, normalized size = 1.65

$$\frac{12(A+3iB)c^4 f x e^{6i(fx+e)} - 2(iA-3B)c^4 e^{2i(fx+e)} + (iA-B)c^4 + 6(4(A+3iB)c^4 f x - (iA-3B)c^4 e^{6i(fx+e)} + 3(4(A+3iB)c^4 f x - 3(iA-3B)c^4 e^{6i(fx+e)} - 6((-iA+3B)c^4 e^{6i(fx+e)} + 2(-iA+3B)c^4 e^{6i(fx+e)} + (-iA+3B)c^4 e^{6i(fx+e)}) \log(e^{2i(fx+e)} + 1))}{a^2 f e^{6i(fx+e)} + 2a^2 f e^{4i(fx+e)} + a^2 f e^{2i(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out] (12\*(A + 3\*I\*B)\*c^4\*f\*x\*e^(8\*I\*f\*x + 8\*I\*e) - 2\*(I\*A - 3\*B)\*c^4\*e^(2\*I\*f\*x + 2\*I\*e) + (I\*A - B)\*c^4 + 6\*(4\*(A + 3\*I\*B)\*c^4\*f\*x - (I\*A - 3\*B)\*c^4)\*e^(6\*I\*f\*x + 6\*I\*e) + 3\*(4\*(A + 3\*I\*B)\*c^4\*f\*x - 3\*(I\*A - 3\*B)\*c^4)\*e^(4\*I\*f\*x + 4\*I\*e) - 6\*((-I\*A + 3\*B)\*c^4\*e^(8\*I\*f\*x + 8\*I\*e) + 2\*(-I\*A + 3\*B)\*c^4\*e^(6\*I\*f\*x + 6\*I\*e) + (-I\*A + 3\*B)\*c^4\*e^(4\*I\*f\*x + 4\*I\*e))\*log(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(a^2\*f\*e^(8\*I\*f\*x + 8\*I\*e) + 2\*a^2\*f\*e^(6\*I\*f\*x + 6\*I\*e) + a^2\*f\*e^(4\*I\*f\*x + 4\*I\*e))

**Sympy** [A]

time = 0.69, size = 377, normalized size = 2.39

$$\frac{-2iAc^4 + 12Bc^4 + (-2iAc^4e^{2ie} + 10Bc^4e^{2ie})e^{2ifx}}{a^2fe^{4ie}e^{4ifx} + 2a^2fe^{2ie}e^{2ifx} + a^2f} + \begin{cases} \frac{((iAa^2c^4fe^{2ie} - Ba^2c^4fe^{2ie})e^{-4ifx} + (-4iAa^2c^4fe^{4ie} + 8Ba^2c^4fe^{4ie})e^{-2ifx})e^{-6ie}}{a^2f^2} & \text{for } a^4f^2e^{6ie} \neq 0 \\ x\left(-\frac{12Ac^4 + 36iBc^4}{a^2} + \frac{(12Ac^4e^{4ie} - 8Ac^4e^{2ie} + 4Ac^4 + 36iBc^4e^{4ie} - 16iBc^4e^{2ie} + 4iBc^4)e^{-4ie}}{a^2}\right) & \text{otherwise} \end{cases} + \frac{6ic^4(A + 3iB)\log(e^{2ifx} + e^{-2ie})}{a^2f} + \frac{x(12Ac^4 + 36iBc^4)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^2,x)

[Out] (-2\*I\*A\*c\*\*4 + 12\*B\*c\*\*4 + (-2\*I\*A\*c\*\*4\*exp(2\*I\*e) + 10\*B\*c\*\*4\*exp(2\*I\*e))\*exp(2\*I\*f\*x))/(a\*\*2\*f\*exp(4\*I\*e)\*exp(4\*I\*f\*x) + 2\*a\*\*2\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + a\*\*2\*f) + Piecewise((((I\*A\*a\*\*2\*c\*\*4\*f\*exp(2\*I\*e) - B\*a\*\*2\*c\*\*4\*f\*exp(2\*I\*e))\*exp(-4\*I\*f\*x) + (-4\*I\*A\*a\*\*2\*c\*\*4\*f\*exp(4\*I\*e) + 8\*B\*a\*\*2\*c\*\*4\*f\*exp(4\*I\*e))\*exp(-2\*I\*f\*x))\*exp(-6\*I\*e)/(a\*\*4\*f\*\*2), Ne(a\*\*4\*f\*\*2\*exp(6\*I\*e), 0)), (x\*(-(12\*A\*c\*\*4 + 36\*I\*B\*c\*\*4)/a\*\*2 + (12\*A\*c\*\*4\*exp(4\*I\*e) - 8\*A\*c\*\*4\*exp(2\*I\*e) + 4\*A\*c\*\*4 + 36\*I\*B\*c\*\*4\*exp(4\*I\*e) - 16\*I\*B\*c\*\*4\*exp(2\*I\*e) + 4\*I\*B\*c\*\*4)\*exp(-4\*I\*e)/a\*\*2), True)) + 6\*I\*c\*\*4\*(A + 3\*I\*B)\*log(exp(2\*I\*f\*x) + exp(-2\*I\*e))/(a\*\*2\*f) + x\*(12\*A\*c\*\*4 + 36\*I\*B\*c\*\*4)/a\*\*2

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs.  $2(141) = 282$ .

time = 0.95, size = 444, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] (6\*(I\*A\*c^4 - 3\*B\*c^4)\*log(tan(1/2\*f\*x + 1/2\*e) + 1)/a^2 - 12\*(I\*A\*c^4 - 3\*B\*c^4)\*log(tan(1/2\*f\*x + 1/2\*e) - I)/a^2 - 6\*(-I\*A\*c^4 + 3\*B\*c^4)\*log(tan(1/2\*f\*x + 1/2\*e) - 1)/a^2 - (9\*I\*A\*c^4\*tan(1/2\*f\*x + 1/2\*e)^4 - 27\*B\*c^4\*tan(1/2\*f\*x + 1/2\*e)^4 - 2\*A\*c^4\*tan(1/2\*f\*x + 1/2\*e)^3 - 12\*I\*B\*c^4\*tan(1/2\*f\*x + 1/2\*e)^3 - 18\*I\*A\*c^4\*tan(1/2\*f\*x + 1/2\*e)^2 + 56\*B\*c^4\*tan(1/2\*f\*x + 1/2\*e)^2 + 2\*A\*c^4\*tan(1/2\*f\*x + 1/2\*e) + 12\*I\*B\*c^4\*tan(1/2\*f\*x + 1/2\*e) + 9\*I\*A\*c^4 - 27\*B\*c^4)/((tan(1/2\*f\*x + 1/2\*e)^2 - 1)^2\*a^2) - (-25\*I\*A\*c^4\*

$$\frac{\tan(1/2*f*x + 1/2*e)^4 + 75*B*c^4*\tan(1/2*f*x + 1/2*e)^4 - 108*A*c^4*\tan(1/2*f*x + 1/2*e)^3 - 324*I*B*c^4*\tan(1/2*f*x + 1/2*e)^3 + 182*I*A*c^4*\tan(1/2*f*x + 1/2*e)^2 - 514*B*c^4*\tan(1/2*f*x + 1/2*e)^2 + 108*A*c^4*\tan(1/2*f*x + 1/2*e) + 324*I*B*c^4*\tan(1/2*f*x + 1/2*e) - 25*I*A*c^4 + 75*B*c^4}{(a^2*(\tan(1/2*f*x + 1/2*e) - I)^4)}/f$$

**Mupad [B]**

time = 8.77, size = 207, normalized size = 1.31

$$\frac{\ln(\tan(e + f x) - i) \left( -\frac{18 B c^4}{a^2} + \frac{A c^4 6i}{a^2} \right)}{f} - \frac{\tan(e + f x) \left( \frac{c^4 (A + B 3i)}{a^2} + \frac{B c^4 3i}{a^2} \right)}{f} - \frac{\left( -\frac{6 B c^4 + A c^4 2i}{2 a^2} \right) i - \frac{(-18 B c^4 + A c^4 6i) 3i}{2 a^2}}{f (\tan(e + f x)^2 i + 2 \tan(e + f x) - i)} + \frac{\tan(e + f x) \left( \frac{2(-18 B c^4 + A c^4 6i)}{a^2} + \frac{16 B c^4}{a^2} \right) - \frac{B c^4 8i}{a^2}}{2 a^2 f} - \frac{B c^4 \tan(e + f x)^2}{2 a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^4)/(a + a\*tan(e + f\*x)\*1i)^2,x)

[Out] - (log(tan(e + f\*x) - 1i)\*((A\*c^4\*6i)/a^2 - (18\*B\*c^4)/a^2))/f - (tan(e + f\*x)\*((c^4\*(A + B\*3i))/a^2 + (B\*c^4\*3i)/a^2))/f - (((A\*c^4\*2i - 6\*B\*c^4)\*1i)/(2\*a^2) - ((A\*c^4\*6i - 18\*B\*c^4)\*3i)/(2\*a^2) + tan(e + f\*x)\*((2\*(A\*c^4\*6i - 18\*B\*c^4))/a^2 + (16\*B\*c^4)/a^2) - (B\*c^4\*8i)/a^2)/(f\*(2\*tan(e + f\*x) + tan(e + f\*x)^2\*1i - 1i)) - (B\*c^4\*tan(e + f\*x)^2)/(2\*a^2\*f)

$$3.718 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^3}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=128

$$\frac{(A+5iB)c^3x}{a^2} + \frac{(iA-5B)c^3 \log(\cos(e+fx))}{a^2f} - \frac{2(iA-B)c^3}{a^2f(i-\tan(e+fx))^2} + \frac{4(A+2iB)c^3}{a^2f(i-\tan(e+fx))} - \frac{iBc^3 \tan(e+fx)}{a^2f}$$

[Out] (A+5\*I\*B)\*c^3\*x/a^2+(I\*A-5\*B)\*c^3\*ln(cos(f\*x+e))/a^2/f-2\*(I\*A-B)\*c^3/a^2/f/(I-tan(f\*x+e))^2+4\*(A+2\*I\*B)\*c^3/a^2/f/(I-tan(f\*x+e))-I\*B\*c^3\*tan(f\*x+e)/a^2/f

**Rubi [A]**

time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{4c^3(A+2iB)}{a^2f(-\tan(e+fx)+i)} - \frac{2c^3(-B+iA)}{a^2f(-\tan(e+fx)+i)^2} + \frac{c^3(-5B+iA)\log(\cos(e+fx))}{a^2f} + \frac{c^3x(A+5iB)}{a^2} - \frac{iBc^3 \tan(e+fx)}{a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3)/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] ((A + (5\*I)\*B)\*c^3\*x)/a^2 + ((I\*A - 5\*B)\*c^3\*Log[Cos[e + f\*x]])/(a^2\*f) - (2\*(I\*A - B)\*c^3)/(a^2\*f\*(I - Tan[e + f\*x])^2) + (4\*(A + (2\*I)\*B)\*c^3)/(a^2\*f\*(I - Tan[e + f\*x])) - (I\*B\*c^3\*Tan[e + f\*x])/(a^2\*f)

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^3}{(a + ia \tan(e + fx))^2} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^2}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(-\frac{iBc^2}{a^3} + \frac{4i(A+iB)c^2}{a^3(-i+x)^3} + \frac{4(A+2iB)c^2}{a^3(-i+x)^2} + \frac{(-iA+5B)c}{a^3(-i+x)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(A + 5iB)c^3 x}{a^2} + \frac{(iA - 5B)c^3 \log(\cos(e + fx))}{a^2 f} - \frac{2(iA - 5B)c^3}{a^2 f(i - \tan(e + fx))}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 413 vs. 2(128) = 256.  
time = 3.65, size = 413, normalized size = 3.23

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3)/(a + I\*a\*Tan[e + f\*x])^2, x]

[Out] -1/2\*(c^3\*Sec[e]\*Sec[e + f\*x]^2\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(I\*(A + (5\*I)\*B)\*Cos[e]^3\*Log[Cos[e + f\*x]^2] - 2\*(A + (5\*I)\*B)\*Cos[e]^2\*Log[Cos[e + f\*x]^2]\*Sin[e] + 2\*(A + (5\*I)\*B)\*ArcTan[Tan[f\*x]]\*Cos[e]\*(Cos[2\*e] + I\*Sin[2\*e]) + Cos[e]\*(-2\*A\*f\*x - (10\*I)\*B\*f\*x - (2\*I)\*A\*Cos[2\*f\*x] + 6\*B\*Cos[2\*f\*x] - I\*A\*Log[Cos[e + f\*x]^2]\*Sin[e]^2 + 5\*B\*Log[Cos[e + f\*x]^2]\*Sin[e]^2 + (2\*I)\*A\*f\*x\*Sin[2\*e] - 10\*B\*f\*x\*Sin[2\*e] + A\*Cos[4\*f\*x]\*Sin[2\*e] + I\*B\*Cos[4\*f\*x]\*Sin[2\*e] - 2\*A\*Sin[2\*f\*x] - (6\*I)\*B\*Sin[2\*f\*x] - I\*A\*Sin[2\*e]\*Sin[4\*f\*x] + B\*Sin[2\*e]\*Sin[4\*f\*x] + Cos[2\*e]\*(2\*(A + (5\*I)\*B)\*f\*x + I\*(A + I\*B)\*Cos[4\*f\*x] + (A + I\*B)\*Sin[4\*f\*x])) + Sec[e + f\*x]\*(Cos[e] + I\*Sin[e])\*(B\*Cos[e - f\*x] - B\*Cos[e + f\*x] + 2\*Cos[e]\*(I\*(A\*f\*x + B\*(-1 + (5\*I)\*f\*x))\*Sin[f\*x] + ((-I)\*A + 5\*B)\*f\*x\*Sin[2\*e + f\*x])))/(a^2\*f\*(-I + Tan[e + f\*x])^2)

**Maple [A]**

time = 0.24, size = 83, normalized size = 0.65

method	result
derivativedivides	$\frac{c^3 \left( (-iA+5B) \ln(-i+\tan(fx+e)) - \frac{8iB+4A}{-i+\tan(fx+e)} - \frac{4iA-4B}{2(-i+\tan(fx+e))^2} - iB \tan(fx+e) \right)}{f a^2}$
default	$\frac{c^3 \left( (-iA+5B) \ln(-i+\tan(fx+e)) - \frac{8iB+4A}{-i+\tan(fx+e)} - \frac{4iA-4B}{2(-i+\tan(fx+e))^2} - iB \tan(fx+e) \right)}{f a^2}$
risch	$\frac{3c^3 e^{-2i(fx+e)} B}{a^2 f} - \frac{ic^3 e^{-2i(fx+e)} A}{a^2 f} - \frac{c^3 e^{-4i(fx+e)} B}{2a^2 f} + \frac{ic^3 e^{-4i(fx+e)} A}{2a^2 f} + \frac{10ic^3 Bx}{a^2} + \frac{2c^3 Ax}{a^2} + \frac{10ic^3 Be}{a^2 f} + \frac{2c^3 A}{a^2 f}$

norman	$\frac{\frac{(5iBc^3 + Ac^3)x}{a} + \frac{-2ic^3A + 6Bc^3}{af} + \frac{(5iBc^3 + Ac^3)x(\tan^4(fx+e))}{a} + \frac{2(5iBc^3 + Ac^3)x(\tan^2(fx+e))}{a} - \frac{2(5iBc^3 + 2Ac^3)(\tan^3(fx+e))}{af}}{a(1 + \tan^2(fx+e))^2}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out]  $1/f*c^3/a^2*((-I*A+5*B)*\ln(-I+\tan(f*x+e))-(8*I*B+4*A)/(-I+\tan(f*x+e))-1/2*(-4*B+4*I*A)/(-I+\tan(f*x+e))^2-I*B*\tan(f*x+e))$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 3.95, size = 185, normalized size = 1.45

$$\frac{4(A+5iB)c^3fxe^{6i fx+6ie} + (-iA+5B)c^3e^{2i fx+2ie} + (iA-B)c^3 + 2(2(A+5iB)c^3fx - (iA-5B)c^3)e^{4i fx+4ie} - 2((-iA+5B)c^3e^{6i fx+6ie} + (-iA+5B)c^3e^{4i fx+4ie}) \log(e^{2i fx+2ie} + 1)}{2(a^2fe^{6i fx+6ie} + a^2fe^{4i fx+4ie})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")`

[Out]  $1/2*(4*(A+5*I*B)*c^3*f*x*e^{(6*I*f*x+6*I*e)} + (-I*A+5*B)*c^3*e^{(2*I*f*x+2*I*e)} + (I*A-B)*c^3 + 2*(2*(A+5*I*B)*c^3*f*x - (I*A-5*B)*c^3)*e^{(4*I*f*x+4*I*e)} - 2*((-I*A+5*B)*c^3*e^{(6*I*f*x+6*I*e)} + (-I*A+5*B)*c^3*e^{(4*I*f*x+4*I*e)})*\log(e^{(2*I*f*x+2*I*e)} + 1))/(a^2*f*e^{(6*I*f*x+6*I*e)} + a^2*f*e^{(4*I*f*x+4*I*e)})$

**Sympy** [A]

time = 0.55, size = 309, normalized size = 2.41

$$\frac{2Bc^3}{a^2fe^{2ie}e^{2ifx} + a^2f} + \begin{cases} \frac{((iAa^2c^3fe^{2ie} - Ba^2c^3fe^{2ie})e^{-4ifx} + (-2Aa^2c^3fe^{4ie} + 6Ba^2c^3fe^{4ie})e^{-2ifx})e^{-6ie}}{2a^4f^2} & \text{for } a^4f^2e^{6ie} \neq 0 \\ x\left(-\frac{2Ac^3+10iBc^3}{a^2} + \frac{(2Ac^3e^{4ie}-2Ac^3e^{2ie}+2Ac^3+10iBc^3e^{4ie}-6iBc^3e^{2ie}+2iBc^3)e^{-4ie}}{a^2}\right) & \text{otherwise} \end{cases} + \frac{ic^3(A+5iB)\log(e^{2ifx} + e^{-2ie})}{a^2f} + \frac{x(2Ac^3+10iBc^3)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**2,x)`

```
[Out] 2*B*c**3/(a**2*f*exp(2*I*e)*exp(2*I*f*x) + a**2*f) + Piecewise((((I*A*a**2*c**3*f*exp(2*I*e) - B*a**2*c**3*f*exp(2*I*e))*exp(-4*I*f*x) + (-2*I*A*a**2*c**3*f*exp(4*I*e) + 6*B*a**2*c**3*f*exp(4*I*e))*exp(-2*I*f*x))*exp(-6*I*e)/(2*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-(2*A*c**3 + 10*I*B*c**3)/a**2 + (2*A*c**3*exp(4*I*e) - 2*A*c**3*exp(2*I*e) + 2*A*c**3 + 10*I*B*c**3*exp(4*I*e) - 6*I*B*c**3*exp(2*I*e) + 2*I*B*c**3)*exp(-4*I*e)/a**2), True)) + I*c**3*(A + 5*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a**2*f) + x*(2*A*c**3 + 10*I*B*c**3)/a**2
```

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs.  $2(112) = 224$ .

time = 0.81, size = 357, normalized size = 2.79

$$\frac{\frac{6(A^2-5B^2)\operatorname{Im}(\tan\{fz+\frac{1}{2}i\})}{a^2} + \frac{12(-A^2+5B^2)\operatorname{Im}(\tan\{fz+i\})}{a^2} - \frac{6(-A^2+5B^2)\operatorname{Im}(\tan\{fz+i\})}{a^2} - \frac{6(A^2\operatorname{Im}\{fz+\frac{1}{2}i\}^2-5B^2\operatorname{Im}\{fz+\frac{1}{2}i\}-5B^2\operatorname{Im}\{fz+i\}-A^2+5B^2)}{(\operatorname{Im}\{fz+\frac{1}{2}i\})^2} - \frac{25A^2\operatorname{Im}\{fz+i\}^2+125B^2\operatorname{Im}\{fz+i\}^2-100A^2\operatorname{Im}\{fz+i\}-500B^2\operatorname{Im}\{fz+i\}^2-100A^2\operatorname{Im}\{fz+i\}-500B^2\operatorname{Im}\{fz+i\}-25A^2+125B^2}{a^2\operatorname{Im}\{fz+i\}^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(I*A*c^3 - 5*B*c^3)*log(tan(1/2*f*x + 1/2*e) + 1)/a^2 + 12*(-I*A*c^3 + 5*B*c^3)*log(tan(1/2*f*x + 1/2*e) - I)/a^2 - 6*(-I*A*c^3 + 5*B*c^3)*log(tan(1/2*f*x + 1/2*e) - 1)/a^2 - 6*(I*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 5*B*c^3*tan(1/2*f*x + 1/2*e)^2 - 2*I*B*c^3*tan(1/2*f*x + 1/2*e) - I*A*c^3 + 5*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) - (-25*I*A*c^3*tan(1/2*f*x + 1/2*e)^4 + 125*B*c^3*tan(1/2*f*x + 1/2*e)^4 - 100*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 548*I*B*c^3*tan(1/2*f*x + 1/2*e)^3 + 198*I*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 894*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 100*A*c^3*tan(1/2*f*x + 1/2*e) + 548*I*B*c^3*tan(1/2*f*x + 1/2*e) - 25*I*A*c^3 + 125*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) - I)^4))/f
```

**Mupad** [B]

time = 9.01, size = 194, normalized size = 1.52

$$\frac{c^2(6B - A2i + 4A\tan(e + fx) + B\tan(e + fx))7i - A\ln(-1 - \tan(e + fx))3i + 5B\ln(-1 - \tan(e + fx))3i + 2B\tan(e + fx)^2 + B\tan(e + fx)^2 + A\tan(e + fx)^2\ln(-1 - \tan(e + fx))3i - 5B\tan(e + fx)^2\ln(-1 - \tan(e + fx))3i + 2A\tan(e + fx)\ln(-1 - \tan(e + fx))3i + B\tan(e + fx)\ln(-1 - \tan(e + fx))3i)}{a^2f(1 + \tan(e + fx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^3)/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] (c^3*(6*B - A*2i + 4*A*tan(e + f*x) + B*tan(e + f*x)*7i - A*log(-tan(e + f*x)*1i - 1)*1i + 5*B*log(-tan(e + f*x)*1i - 1) + 2*B*tan(e + f*x)^2 + B*tan(e + f*x)^3*1i + A*tan(e + f*x)^2*log(-tan(e + f*x)*1i - 1)*1i - 5*B*tan(e + f*x)^2*log(-tan(e + f*x)*1i - 1) + 2*A*tan(e + f*x)*log(-tan(e + f*x)*1i - 1) + B*tan(e + f*x)*log(-tan(e + f*x)*1i - 1)*10i))/(a^2*f*(tan(e + f*x)*1i + 1)^2)
```



$$\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^2}{(a + ia \tan(e + fx))^2} dx = \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)}{(a+iax)^3} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2i(A+iB)c}{a^3(-i+x)^3} + \frac{(A+3iB)c}{a^3(-i+x)^2} + \frac{Bc}{a^3(-i+x)} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{iBc^2x}{a^2} - \frac{Bc^2 \log(\cos(e + fx))}{a^2 f} - \frac{(iA - B)c^2}{a^2 f (i - \tan(e + fx))^2}$$

**Mathematica [A]**

time = 1.07, size = 140, normalized size = 1.44

$$\frac{c^2 \sec^2(e + fx) (-4B + \cos(2(e + fx)) (-iA + B + 2B \log(\cos^2(e + fx))) - A \sin(2(e + fx)) - iB \sin(2(e + fx)) + 2iB \log(\cos^2(e + fx)) \sin(2(e + fx)) + 4B \text{ArcTan}(\tan(fx)) (-i \cos(2(e + fx)) + \sin(2(e + fx))))}{4a^2 f (-i + \tan(e + fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (c^2*Sec[e + f*x]^2*(-4*B + Cos[2*(e + f*x)]*((-I)*A + B + 2*B*Log[Cos[e + f*x]^2]) - A*Sin[2*(e + f*x)] - I*B*Sin[2*(e + f*x)] + (2*I)*B*Log[Cos[e + f*x]^2]*Sin[2*(e + f*x)] + 4*B*ArcTan[Tan[f*x]]*((-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])))/(4*a^2*f*(-I + Tan[e + f*x])^2)
```

**Maple [A]**

time = 0.29, size = 64, normalized size = 0.66

method	result	size
derivativedivides	$\frac{c^2 \left( B \ln(-i + \tan(fx + e)) - \frac{3iB + A}{-i + \tan(fx + e)} - \frac{2iA - 2B}{2(-i + \tan(fx + e))^2} \right)}{f a^2}$	64
default	$\frac{c^2 \left( B \ln(-i + \tan(fx + e)) - \frac{3iB + A}{-i + \tan(fx + e)} - \frac{2iA - 2B}{2(-i + \tan(fx + e))^2} \right)}{f a^2}$	64
risch	$\frac{B c^2 e^{-2i(fx+e)}}{a^2 f} - \frac{c^2 e^{-4i(fx+e)} B}{4a^2 f} + \frac{ic^2 e^{-4i(fx+e)} A}{4a^2 f} + \frac{2iB c^2 x}{a^2} + \frac{2iB c^2 e}{a^2 f} - \frac{B c^2 \ln(e^{2i(fx+e)} + 1)}{a^2 f}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c^2/a^2*(B*ln(-I+tan(f*x+e))-(A+3*I*B)/(-I+tan(f*x+e))-1/2*(2*I*A-2*B)/(-I+tan(f*x+e))^2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 3.96, size = 93, normalized size = 0.96

$$\frac{(8i Bc^2 f x e^{(4i f x + 4i e)} - 4 Bc^2 e^{(4i f x + 4i e)} \log(e^{(2i f x + 2i e)} + 1) + 4 Bc^2 e^{(2i f x + 2i e)} + (i A - B)c^2) e^{(-4i f x - 4i e)}}{4 a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{4} * (8 * I * B * c^2 * f * x * e^{(4 * I * f * x + 4 * I * e)} - 4 * B * c^2 * e^{(4 * I * f * x + 4 * I * e)} * \log(e^{(2 * I * f * x + 2 * I * e)} + 1) + 4 * B * c^2 * e^{(2 * I * f * x + 2 * I * e)} + (I * A - B) * c^2) * e^{(-4 * I * f * x - 4 * I * e)} / (a^2 * f)$

**Sympy** [A]

time = 0.33, size = 206, normalized size = 2.12

$$\frac{2i Bc^2 x}{a^2} - \frac{Bc^2 \log(e^{2ifx} + e^{-2ie})}{a^2 f} + \begin{cases} \frac{(4Ba^2c^2fe^{4ie}e^{-2ifx} + (iAa^2c^2fe^{2ie} - Ba^2c^2fe^{2ie})e^{-4ifx})e^{-6ie}}{4a^4f^2} & \text{for } a^4f^2e^{6ie} \neq 0 \\ x \left( -\frac{2iBc^2}{a^2} + \frac{(Ac^2 + 2iBc^2e^{4ie} - 2iBc^2e^{2ie} + iBc^2)e^{-4ie}}{a^2} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**2,x)`

[Out]  $2 * I * B * c ** 2 * x / a ** 2 - B * c ** 2 * \log(\exp(2 * I * f * x) + \exp(-2 * I * e)) / (a ** 2 * f) + \text{Piecewise}(((4 * B * a ** 2 * c ** 2 * f * \exp(4 * I * e) * \exp(-2 * I * f * x) + (I * A * a ** 2 * c ** 2 * f * \exp(2 * I * e) - B * a ** 2 * c ** 2 * f * \exp(2 * I * e)) * \exp(-4 * I * f * x)) * \exp(-6 * I * e) / (4 * a ** 4 * f ** 2), \text{Ne}(a ** 4 * f ** 2 * \exp(6 * I * e), 0)), (x * (-2 * I * B * c ** 2 / a ** 2 + (A * c ** 2 + 2 * I * B * c ** 2 * \exp(4 * I * e) - 2 * I * B * c ** 2 * \exp(2 * I * e) + I * B * c ** 2) * \exp(-4 * I * e) / a ** 2), \text{True}))$

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs.  $2(87) = 174$ .

time = 0.69, size = 201, normalized size = 2.07

$$\frac{6 Bc^2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1) - 12 Bc^2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) - i) + 6 Bc^2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1) + 25 Bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 + 12 Ac^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 112i Bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 198 Bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 12 Ac^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 112i Bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 25 Bc^2}{a^2 (\tan(\frac{1}{2}fx + \frac{1}{2}e) - i)^4}$$

6 f

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")`

```
[Out] -1/6*(6*B*c^2*log(tan(1/2*f*x + 1/2*e) + 1)/a^2 - 12*B*c^2*log(tan(1/2*f*x
+ 1/2*e) - 1)/a^2 + 6*B*c^2*log(tan(1/2*f*x + 1/2*e) - 1)/a^2 + (25*B*c^2*t
an(1/2*f*x + 1/2*e)^4 + 12*A*c^2*tan(1/2*f*x + 1/2*e)^3 - 112*I*B*c^2*tan(1
/2*f*x + 1/2*e)^3 - 198*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*c^2*tan(1/2*f*x
+ 1/2*e) + 112*I*B*c^2*tan(1/2*f*x + 1/2*e) + 25*B*c^2)/(a^2*(tan(1/2*f*x
+ 1/2*e) - I)^4))/f
```

**Mupad [B]**

time = 8.51, size = 104, normalized size = 1.07

$$\frac{c^2 (2B + A \tan(e + fx) + B \tan(e + fx) 3i + B \ln(-1 - \tan(e + fx) 1i) - B \tan(e + fx)^2 \ln(-1 - \tan(e + fx) 1i) + B \tan(e + fx) \ln(-1 - \tan(e + fx) 1i) 2i)}{a^2 f (1 + \tan(e + fx) 1i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^2)/(a + a*tan(e + f*x)*1i
)^2,x)
```

```
[Out] (c^2*(2*B + A*tan(e + f*x) + B*tan(e + f*x)*3i + B*log(- tan(e + f*x)*1i -
1) - B*tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1) + B*tan(e + f*x)*log(- tan
(e + f*x)*1i - 1)*2i))/(a^2*f*(tan(e + f*x)*1i + 1)^2)
```

$$3.720 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))}{(a+ia \tan(e+fx))^2} dx$$

Optimal. Leaf size=48

$$-\frac{c(A+B \tan(e+fx))^2}{2a^2(iA-B)f(1+i \tan(e+fx))^2}$$

[Out]  $-1/2*c*(A+B*\tan(f*x+e))^2/a^2/(I*A-B)/f/(1+I*\tan(f*x+e))^2$

**Rubi** [A]

time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 37}

$$-\frac{c(A+B \tan(e+fx))^2}{2a^2f(-B+iA)(1+i \tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x]))/(a + I\*a\*Tan[e + f\*x])^2, x]

[Out]  $-1/2*(c*(A + B*\tan[e + f*x])^2)/(a^2*(I*A - B)*f*(1 + I*\tan[e + f*x])^2$

Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp [(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))}{(a+ia \tan(e+fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3} dx, x, \tan(e+fx)\right)}{f} \\ &= -\frac{c(A+B \tan(e+fx))^2}{2a^2(iA-B)f(1+i \tan(e+fx))^2} \end{aligned}$$

**Mathematica [A]**

time = 0.67, size = 58, normalized size = 1.21

$$\frac{(-3iA - B + (A - 3iB) \tan(e + fx))(c - ic \tan(e + fx))}{8a^2 f (-i + \tan(e + fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((-3*I)*A - B + (A - (3*I)*B)*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(8*a^2*f*(-I + Tan[e + f*x])^2)
```

**Maple [A]**

time = 0.24, size = 46, normalized size = 0.96

method	result	size
derivativedivides	$c \left( -\frac{iA-B}{2(-i+\tan(fx+e))^2} - \frac{iB}{-i+\tan(fx+e)} \right)$	46
default	$c \left( -\frac{iA-B}{2(-i+\tan(fx+e))^2} - \frac{iB}{-i+\tan(fx+e)} \right)$	46
risch	$\frac{ce^{-2i(fx+e)}B}{4a^2f} + \frac{ice^{-2i(fx+e)}A}{4a^2f} - \frac{ce^{-4i(fx+e)}B}{8a^2f} + \frac{ice^{-4i(fx+e)}A}{8a^2f}$	80
norman	$\frac{\frac{Ac \tan(fx+e)}{af} + \frac{iAc+Bc}{2af} + \frac{(-iAc+3Bc)(\tan^2(fx+e))}{2af} - \frac{icB(\tan^3(fx+e))}{af}}{a(1+\tan^2(fx+e))^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c/a^2*(-1/2*(I*A-B)/(-I+tan(f*x+e))^2-I*B/(-I+tan(f*x+e)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas** [A]

time = 2.67, size = 49, normalized size = 1.02

$$\frac{(2(-iA - B)ce^{2ifx+2ie}) - (iA - B)c e^{(-4ifx-4ie)}}{8a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out] -1/8\*(2\*(-I\*A - B)\*c\*e^(2\*I\*f\*x + 2\*I\*e) - (I\*A - B)\*c)\*e^(-4\*I\*f\*x - 4\*I\*e)/(a^2\*f)

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(37) = 74.

time = 0.20, size = 158, normalized size = 3.29

$$\begin{cases} \frac{((4iAa^2cfe^{2ie} - 4Ba^2cfe^{2ie})e^{-4ifx} + (8iAa^2cfe^{4ie} + 8Ba^2cfe^{4ie})e^{-2ifx})e^{-6ie}}{32a^4f^2} & \text{for } a^4f^2e^{6ie} \neq 0 \\ \frac{x(Ace^{2ie} + Ac - iBce^{2ie} + iBc)e^{-4ie}}{2a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x)

[Out] Piecewise((((4\*I\*A\*a\*\*2\*c\*f\*exp(2\*I\*e) - 4\*B\*a\*\*2\*c\*f\*exp(2\*I\*e))\*exp(-4\*I\*f\*x) + (8\*I\*A\*a\*\*2\*c\*f\*exp(4\*I\*e) + 8\*B\*a\*\*2\*c\*f\*exp(4\*I\*e))\*exp(-2\*I\*f\*x))\*exp(-6\*I\*e)/(32\*a\*\*4\*f\*\*2), Ne(a\*\*4\*f\*\*2\*exp(6\*I\*e), 0)), (x\*(A\*c\*exp(2\*I\*e) + A\*c - I\*B\*c\*exp(2\*I\*e) + I\*B\*c)\*exp(-4\*I\*e)/(2\*a\*\*2), True))

**Giac** [A]

time = 0.62, size = 84, normalized size = 1.75

$$\frac{2 \left( A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - i A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - B c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}{a^2 f \left( \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - i \right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] -2\*(A\*c\*tan(1/2\*f\*x + 1/2\*e)^3 - I\*A\*c\*tan(1/2\*f\*x + 1/2\*e)^2 - B\*c\*tan(1/2\*f\*x + 1/2\*e)^2 - A\*c\*tan(1/2\*f\*x + 1/2\*e))/(a^2\*f\*(tan(1/2\*f\*x + 1/2\*e) - I)^4)

**Mupad** [B]

time = 8.54, size = 50, normalized size = 1.04

$$\frac{\frac{c(A-Bli)}{2} + B c \tan(e + fx)}{a^2 f \left( \tan(e + fx)^2 li + 2 \tan(e + fx) - i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i))/(a + a*tan(e + f*x)*1i)^2,x)
```

```
[Out] ((c*(A - B*1i))/2 + B*c*tan(e + f*x))/(a^2*f*(2*tan(e + f*x) + tan(e + f*x)^2*1i - 1i))
```

$$3.721 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2} dx$$

Optimal. Leaf size=80

$$\frac{(A-iB)x}{4a^2} + \frac{iA-B}{4f(a+ia \tan(e+fx))^2} + \frac{iA+B}{4f(a^2+ia^2 \tan(e+fx))}$$

[Out] 1/4\*(A-I\*B)\*x/a^2+1/4\*(I\*A-B)/f/(a+I\*a\*tan(f\*x+e))^2+1/4\*(I\*A+B)/f/(a^2+I\*a^2\*tan(f\*x+e))

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3607, 3560, 8}

$$\frac{B+iA}{4f(a^2+ia^2 \tan(e+fx))} + \frac{x(A-iB)}{4a^2} + \frac{-B+iA}{4f(a+ia \tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] ((A - I\*B)\*x)/(4\*a^2) + (I\*A - B)/(4\*f\*(a + I\*a\*Tan[e + f\*x])^2) + (I\*A + B)/(4\*f\*(a^2 + I\*a^2\*Tan[e + f\*x]))

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[a\*((a + b\*Tan[c + d\*x])^n/(2\*b\*d\*n)), x] + Dist[1/(2\*a), Int[(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2} dx &= \frac{iA - B}{4f(a + ia \tan(e + fx))^2} + \frac{(A - iB) \int \frac{1}{a + ia \tan(e + fx)} dx}{2a} \\ &= \frac{iA - B}{4f(a + ia \tan(e + fx))^2} + \frac{iA + B}{4f(a^2 + ia^2 \tan(e + fx))} + \frac{(A - iB) \int 1 dx}{4a^2} \\ &= \frac{(A - iB)x}{4a^2} + \frac{iA - B}{4f(a + ia \tan(e + fx))^2} + \frac{iA + B}{4f(a^2 + ia^2 \tan(e + fx))} \end{aligned}$$

**Mathematica [A]**

time = 0.36, size = 94, normalized size = 1.18

$$\frac{\sec^2(e + fx)(4iA + (B(-1 - 4ifx) + A(i + 4fx)) \cos(2(e + fx)) + (A + iB + 4iAfx + 4Bfx) \sin(2(e + fx)))}{16a^2 f(-i + \tan(e + fx))^2}$$

Antiderivative was successfully verified.

`[In] Integrate[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]`

```
[Out] -1/16*(Sec[e + f*x]^2*((4*I)*A + (B*(-1 - (4*I)*f*x) + A*(I + 4*f*x))*Cos[2*(e + f*x)] + (A + I*B + (4*I)*A*f*x + 4*B*f*x)*Sin[2*(e + f*x)])/(a^2*f*(-I + Tan[e + f*x])^2)
```

**Maple [A]**

time = 0.20, size = 89, normalized size = 1.11

method	result	size
risch	$-\frac{ixB}{4a^2} + \frac{xA}{4a^2} + \frac{iAe^{-2i(fx+e)}}{4a^2f} - \frac{e^{-4i(fx+e)}B}{16a^2f} + \frac{ie^{-4i(fx+e)}A}{16a^2f}$	73
derivativedivides	$\frac{\left(-\frac{iA}{8} - \frac{B}{8}\right) \ln(-i + \tan(fx+e)) - \frac{\frac{iA}{2} - \frac{B}{2}}{2(-i + \tan(fx+e))^2} - \frac{-\frac{A}{4} + \frac{iB}{4}}{-i + \tan(fx+e)} + \frac{i(-iB+A) \ln(i + \tan(fx+e))}{8}}{fa^2}$	89
default	$\frac{\left(-\frac{iA}{8} - \frac{B}{8}\right) \ln(-i + \tan(fx+e)) - \frac{\frac{iA}{2} - \frac{B}{2}}{2(-i + \tan(fx+e))^2} - \frac{-\frac{A}{4} + \frac{iB}{4}}{-i + \tan(fx+e)} + \frac{i(-iB+A) \ln(i + \tan(fx+e))}{8}}{fa^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)`

```
[Out] 1/f/a^2*((-1/8*I*A-1/8*B)*ln(-I+tan(f*x+e))-1/2*(1/2*I*A-1/2*B)/(-I+tan(f*x+e))^2-(-1/4*A+1/4*I*B)/(-I+tan(f*x+e))+1/8*I*(A-I*B)*ln(I+tan(f*x+e)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 2.03, size = 57, normalized size = 0.71

$$\frac{(4(A - iB)fxe^{(4i fx + 4i e)} + 4i A e^{(2i fx + 2i e)} + i A - B)e^{(-4i fx - 4i e)}}{16 a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")`

[Out]  $1/16*(4*(A - I*B)*f*x*e^{(4*I*f*x + 4*I*e)} + 4*I*A*e^{(2*I*f*x + 2*I*e)} + I*A - B)*e^{(-4*I*f*x - 4*I*e)}/(a^2*f)$

**Sympy** [A]

time = 0.19, size = 162, normalized size = 2.02

$$\begin{cases} \frac{(16iAa^2fe^{4ie}e^{-2ifx} + (4iAa^2fe^{2ie} - 4Ba^2fe^{2ie})e^{-4ifx})e^{-6ie}}{64a^4f^2} & \text{for } a^4f^2e^{6ie} \neq 0 \\ x\left(-\frac{A-iB}{4a^2} + \frac{(Ae^{4ie} + 2Ae^{2ie} + A - iBe^{4ie} + iB)e^{-4ie}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(A - iB)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2,x)`

[Out] `Piecewise(((16*I*A*a**2*f*exp(4*I*e)*exp(-2*I*f*x) + (4*I*A*a**2*f*exp(2*I*e) - 4*B*a**2*f*exp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(64*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-(A - I*B)/(4*a**2) + (A*exp(4*I*e) + 2*A*exp(2*I*e) + A - I*B*exp(4*I*e) + I*B)*exp(-4*I*e)/(4*a**2)), True)) + x*(A - I*B)/(4*a**2)`

**Giac** [A]

time = 0.54, size = 117, normalized size = 1.46

$$\frac{\frac{2(-iA-B)\log(\tan(fx+e)+i)}{a^2} - \frac{2(-iA-B)\log(\tan(fx+e)-i)}{a^2} - \frac{3iA\tan(fx+e)^2 + 3B\tan(fx+e)^2 + 10A\tan(fx+e) - 10iB\tan(fx+e) - 11iA - 3B}{a^2(\tan(fx+e)-i)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")`

[Out]  $-1/16*(2*(-I*A - B)*\log(\tan(f*x + e) + I)/a^2 - 2*(-I*A - B)*\log(\tan(f*x + e) - I)/a^2 - (3*I*A*\tan(f*x + e)^2 + 3*B*\tan(f*x + e)^2 + 10*A*\tan(f*x + e) - 10*I*B*\tan(f*x + e) - 11*I*A - 3*B)/(a^2*(\tan(f*x + e) - I)^2))/f$

**Mupad [B]**

time = 8.54, size = 70, normalized size = 0.88

$$\frac{\frac{A}{2a^2} + \tan(e + fx) \left( \frac{B}{4a^2} + \frac{A1i}{4a^2} \right)}{f (\tan(e + fx)^2 1i + 2 \tan(e + fx) - i)} - \frac{x (B + A 1i) 1i}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/(a + a\*tan(e + f\*x)\*1i)^2,x)

[Out] (A/(2\*a^2) + tan(e + f\*x)\*((A\*1i)/(4\*a^2) + B/(4\*a^2)))/(f\*(2\*tan(e + f\*x) + tan(e + f\*x)^2\*1i - 1i)) - (x\*(A\*1i + B)\*1i)/(4\*a^2)

$$3.722 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))} dx$$

Optimal. Leaf size=117

$$\frac{(3A-iB)x}{8a^2c} - \frac{iA-B}{8a^2cf(i-\tan(e+fx))^2} - \frac{A}{4a^2cf(i-\tan(e+fx))} + \frac{A-iB}{8a^2cf(i+\tan(e+fx))}$$

[Out] 1/8\*(3\*A-I\*B)\*x/a^2/c+1/8\*(-I\*A+B)/a^2/c/f/(I-tan(f\*x+e))^2-1/4\*A/a^2/c/f/(I-tan(f\*x+e))+1/8\*(A-I\*B)/a^2/c/f/(I+tan(f\*x+e))

Rubi [A]

time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$\frac{A-iB}{8a^2cf(\tan(e+fx)+i)} - \frac{-B+iA}{8a^2cf(-\tan(e+fx)+i)^2} + \frac{x(3A-iB)}{8a^2c} - \frac{A}{4a^2cf(-\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])), x]

[Out] ((3\*A - I\*B)\*x)/(8\*a^2\*c) - (I\*A - B)/(8\*a^2\*c\*f\*(I - Tan[e + f\*x])^2) - A/(4\*a^2\*c\*f\*(I - Tan[e + f\*x])) + (A - I\*B)/(8\*a^2\*c\*f\*(I + Tan[e + f\*x]))

Rule 78

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 209

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c +

a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 (c - ic \tan(e + fx))} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3(c-icx)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{i(A+iB)}{4a^3c^2(-i+x)^3} - \frac{A}{4a^3c^2(-i+x)^2} + \frac{-A+iB}{8a^3c^2(i+x)^2} + \frac{A}{8a^3c^2(i+x)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iA - B}{8a^2cf(i - \tan(e + fx))^2} - \frac{A}{4a^2cf(i - \tan(e + fx))} + \frac{A}{8a^2cf(i + \tan(e + fx))} + \frac{A}{8a^2cf}$$

$$= \frac{(3A - iB)x}{8a^2c} - \frac{iA - B}{8a^2cf(i - \tan(e + fx))^2} - \frac{A}{4a^2cf(i - \tan(e + fx))} + \frac{A}{8a^2cf(i + \tan(e + fx))} + \frac{A}{8a^2cf}$$

Mathematica [A]

time = 1.08, size = 129, normalized size = 1.10

$$\frac{-7A + iB + 12iAfx + 4Bfx + 2(A - 3iB) \cos(2(e + fx)) + (3iA + B) \sec(e + fx) \sin(3(e + fx)) + 6iA \tan(e + fx) - 2B \tan(e + fx) - 12Afx \tan(e + fx) + 4iBfx \tan(e + fx)}{32a^2cf(-i + \tan(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])),x]

[Out] -1/32\*(-7\*A + I\*B + (12\*I)\*A\*f\*x + 4\*B\*f\*x + 2\*(A - (3\*I)\*B)\*Cos[2\*(e + f\*x)]) + ((3\*I)\*A + B)\*Sec[e + f\*x]\*Sin[3\*(e + f\*x)] + (6\*I)\*A\*Tan[e + f\*x] - 2\*B\*Tan[e + f\*x] - 12\*A\*f\*x\*Tan[e + f\*x] + (4\*I)\*B\*f\*x\*Tan[e + f\*x]/(a^2\*c\*f\*(-I + Tan[e + f\*x]))

Maple [A]

time = 0.23, size = 106, normalized size = 0.91

method	result
derivativedivides	$\frac{\left(\frac{3iA}{16} + \frac{B}{16}\right) \ln(i + \tan(fx+e)) - \frac{-\frac{A}{8} + \frac{iB}{8}}{i + \tan(fx+e)} - \frac{-\frac{B}{4} + \frac{iA}{4}}{2(-i + \tan(fx+e))^2} + \left(-\frac{3iA}{16} - \frac{B}{16}\right) \ln(-i + \tan(fx+e)) + \frac{A}{-4i + 4 \tan(fx+e)}}{f a^2 c}$
default	$\frac{\left(\frac{3iA}{16} + \frac{B}{16}\right) \ln(i + \tan(fx+e)) - \frac{-\frac{A}{8} + \frac{iB}{8}}{i + \tan(fx+e)} - \frac{-\frac{B}{4} + \frac{iA}{4}}{2(-i + \tan(fx+e))^2} + \left(-\frac{3iA}{16} - \frac{B}{16}\right) \ln(-i + \tan(fx+e)) + \frac{A}{-4i + 4 \tan(fx+e)}}{f a^2 c}$
risch	$-\frac{ixB}{8a^2c} + \frac{3xA}{8a^2c} - \frac{e^{-4i(fx+e)}B}{32a^2cf} + \frac{ie^{-4i(fx+e)}A}{32a^2cf} - \frac{\cos(2fx+2e)B}{8a^2cf} + \frac{i \cos(2fx+2e)A}{8a^2cf} + \frac{A \sin(2fx+2e)}{4a^2cf}$
norman	$\frac{\frac{(-iB+3A)x}{8ac} - \frac{-iA+B}{4acf} + \frac{(-iB+3A)(\tan^3(fx+e))}{8acf} + \frac{(-iB+3A)x(\tan^2(fx+e))}{4ac} + \frac{(-iB+3A)x(\tan^4(fx+e))}{8ac} + \frac{(iB+5A)\tan(fx+e)}{8acf}}{a(1+\tan^2(fx+e))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a^2/c*((3/16*I*A+1/16*B)*ln(I+tan(f*x+e))-(-1/8*A+1/8*I*B)/(I+tan(f*x+e)))-1/2*(-1/4*B+1/4*I*A)/(-I+tan(f*x+e))^2+(-3/16*I*A-1/16*B)*ln(-I+tan(f*x+e))+1/4*A/(-I+tan(f*x+e))
```

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x,algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

**Fricas** [A]

time = 1.80, size = 83, normalized size = 0.71

$$\frac{(4(3A - iB)fxe^{(4i fx + 4i e)} - 2(iA + B)e^{(6i fx + 6i e)} - 2(-3iA + B)e^{(2i fx + 2i e)} + iA - B)e^{(-4i fx - 4i e)}}{32 a^2 c f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x,algorithm="fricas")
```

```
[Out] 1/32*(4*(3*A - I*B)*f*x*e^(4*I*f*x + 4*I*e) - 2*(I*A + B)*e^(6*I*f*x + 6*I*e) - 2*(-3*I*A + B)*e^(2*I*f*x + 2*I*e) + I*A - B)*e^(-4*I*f*x - 4*I*e)/(a^2*c*f)
```

**Sympy** [A]

time = 0.28, size = 296, normalized size = 2.53

$$\left\{ \begin{array}{ll} \frac{((256iAa^4c^2f^2e^{2ie} - 256Ba^4c^2f^2e^{2ie})e^{-4ifx} + (1536iAa^4c^2f^2e^{4ie} - 512Ba^4c^2f^2e^{4ie})e^{-2ifx} + (-512iAa^4c^2f^2e^{8ie} - 512Ba^4c^2f^2e^{8ie})e^{2ifx})e^{-6ie}}{8192a^6c^3f^3} & \text{for } a^6c^3f^3e^{6ie} \neq 0 \\ x\left(-\frac{3A-iB}{8a^2c} + \frac{(Ae^{6ie} + 3Ae^{4ie} + 3Ae^{2ie} + A - iBe^{6ie} - iBe^{4ie} + iBe^{2ie} + iB)e^{-4ie}}{8a^2c}\right) & \text{otherwise} \end{array} \right. + \frac{x(3A - iB)}{8a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x)
```

[Out] Piecewise((((256\*I\*A\*a\*\*4\*c\*\*2\*f\*\*2\*exp(2\*I\*e) - 256\*B\*a\*\*4\*c\*\*2\*f\*\*2\*exp(2\*I\*e))\*exp(-4\*I\*f\*x) + (1536\*I\*A\*a\*\*4\*c\*\*2\*f\*\*2\*exp(4\*I\*e) - 512\*B\*a\*\*4\*c\*\*2\*f\*\*2\*exp(4\*I\*e))\*exp(-2\*I\*f\*x) + (-512\*I\*A\*a\*\*4\*c\*\*2\*f\*\*2\*exp(8\*I\*e) - 512\*B\*a\*\*4\*c\*\*2\*f\*\*2\*exp(8\*I\*e))\*exp(2\*I\*f\*x))\*exp(-6\*I\*e)/(8192\*a\*\*6\*c\*\*3\*f\*\*3), Ne(a\*\*6\*c\*\*3\*f\*\*3\*exp(6\*I\*e), 0)), (x\*(-(3\*A - I\*B)/(8\*a\*\*2\*c) + (A\*exp(6\*I\*e) + 3\*A\*exp(4\*I\*e) + 3\*A\*exp(2\*I\*e) + A - I\*B\*exp(6\*I\*e) - I\*B\*exp(4\*I\*e) + I\*B\*exp(2\*I\*e) + I\*B)\*exp(-4\*I\*e)/(8\*a\*\*2\*c)), True)) + x\*(3\*A - I\*B)/(8\*a\*\*2\*c)

**Giac [A]**

time = 0.66, size = 169, normalized size = 1.44

$$\frac{\frac{2(3iA+B)\log(\tan(fx+e)+i)}{a^2c} + \frac{2(-3iA-B)\log(\tan(fx+e)-i)}{a^2c} - \frac{2(3A\tan(fx+e)-iB\tan(fx+e)+5iA+3B)}{a^2c(-i\tan(fx+e)+1)} + \frac{9iA\tan(fx+e)^2+3B\tan(fx+e)^2+26A\tan(fx+e)-6iB\tan(fx+e)-2iA+B}{a^2c(\tan(fx+e)-i)^2}}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e)),x, algorith="giac")

[Out] 1/32\*(2\*(3\*I\*A + B)\*log(tan(f\*x + e) + I)/(a^2\*c) + 2\*(-3\*I\*A - B)\*log(tan(f\*x + e) - I)/(a^2\*c) - 2\*(3\*A\*tan(f\*x + e) - I\*B\*tan(f\*x + e) + 5\*I\*A + 3\*B)/(a^2\*c\*(-I\*tan(f\*x + e) + 1)) + (9\*I\*A\*tan(f\*x + e)^2 + 3\*B\*tan(f\*x + e)^2 + 26\*A\*tan(f\*x + e) - 6\*I\*B\*tan(f\*x + e) - 21\*I\*A + B)/(a^2\*c\*(tan(f\*x + e) - I)^2))/f

**Mupad [B]**

time = 8.77, size = 129, normalized size = 1.10

$$\frac{\tan(e + fx) \left( \frac{3A}{8a^2c} - \frac{B \operatorname{li}}{8a^2c} \right) + \tan(e + fx)^2 \left( \frac{B}{8a^2c} + \frac{A3i}{8a^2c} \right) - \frac{B}{4a^2c} + \frac{A \operatorname{li}}{4a^2c}}{f \left( \tan(e + fx)^3 \operatorname{li} + \tan(e + fx)^2 + \tan(e + fx) \operatorname{li} + 1 \right)} - \frac{x(B + A3i) \operatorname{li}}{8a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)),x)

[Out] (tan(e + f\*x)\*((3\*A)/(8\*a^2\*c) - (B\*1i)/(8\*a^2\*c)) + tan(e + f\*x)^2\*((A\*3i)/(8\*a^2\*c) + B/(8\*a^2\*c)) + (A\*1i)/(4\*a^2\*c) - B/(4\*a^2\*c))/(f\*(tan(e + f\*x)\*1i + tan(e + f\*x)^2 + tan(e + f\*x)^3\*1i + 1)) - (x\*(A\*3i + B)\*1i)/(8\*a^2\*c)

$$3.723 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^2} dx$$

Optimal. Leaf size=71

$$\frac{3Ax}{8a^2c^2} + \frac{3A \cos(e+fx) \sin(e+fx)}{8a^2c^2f} - \frac{\cos^4(e+fx)(B-A \tan(e+fx))}{4a^2c^2f}$$

[Out] 3/8\*A\*x/a^2/c^2+3/8\*A\*cos(f\*x+e)\*sin(f\*x+e)/a^2/c^2/f-1/4\*cos(f\*x+e)^4\*(B-A\*tan(f\*x+e))/a^2/c^2/f

Rubi [A]

time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$ , Rules used = {3669, 74, 653, 205, 211}

$$-\frac{\cos^4(e+fx)(B-A \tan(e+fx))}{4a^2c^2f} + \frac{3A \sin(e+fx) \cos(e+fx)}{8a^2c^2f} + \frac{3Ax}{8a^2c^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^2), x]

[Out] (3\*A\*x)/(8\*a^2\*c^2) + (3\*A\*Cos[e + f\*x]\*Sin[e + f\*x])/(8\*a^2\*c^2\*f) - (Cos[e + f\*x]^4\*(B - A\*Tan[e + f\*x]))/(4\*a^2\*c^2\*f)

Rule 74

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] :> Int[(a\*c + b\*d\*x^2)^m\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b\*c + a\*d, 0] && EqQ[n, m] && IntegerQ[m] && (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] || !IntegerQ[p])))

Rule 205

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(-x)\*((a + b\*x^n)^(p + 1)/(a\*n\*(p + 1))), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 211

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 653

```
Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e
- c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

### Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 (c - ic \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3(c-icx)^3} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(ac+acx^2)^3} dx, x, \tan(e + fx)\right)}{f} \\ &= -\frac{\cos^4(e + fx)(B - A \tan(e + fx))}{4a^2c^2f} + \frac{(3A) \text{Subst}\left(\int \frac{1}{(ac+acx^2)^3} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{3A \cos(e + fx) \sin(e + fx)}{8a^2c^2f} - \frac{\cos^4(e + fx)(B - A \tan(e + fx))}{4a^2c^2f} \\ &= \frac{3Ax}{8a^2c^2} + \frac{3A \cos(e + fx) \sin(e + fx)}{8a^2c^2f} - \frac{\cos^4(e + fx)(B - A \tan(e + fx))}{4a^2c^2f} \end{aligned}$$

### Mathematica [A]

time = 0.09, size = 53, normalized size = 0.75

$$\frac{-8B \cos^4(e + fx) + A(12(e + fx) + 8 \sin(2(e + fx)) + \sin(4(e + fx)))}{32a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f
*x])^2), x]
```

```
[Out] (-8*B*Cos[e + f*x]^4 + A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f
*x)]))/(32*a^2*c^2*f)
```

### Maple [C] Result contains complex when optimal does not.

time = 0.17, size = 124, normalized size = 1.75



method	result	size
risch	$\frac{3Ax}{8a^2c^2} - \frac{B \cos(4fx+4e)}{32a^2c^2f} + \frac{A \sin(4fx+4e)}{32a^2c^2f} - \frac{B \cos(2fx+2e)}{8a^2c^2f} + \frac{A \sin(2fx+2e)}{4a^2c^2f}$	96
norman	$\frac{\frac{3Ax}{8ac} - \frac{B}{4acf} + \frac{5A \tan(fx+e)}{8acf} + \frac{3A(\tan^3(fx+e))}{8acf} + \frac{3Ax(\tan^2(fx+e))}{4ac} + \frac{3Ax(\tan^4(fx+e))}{8ac}}{(1+\tan^2(fx+e))^2ac}$	11
derivativedivides	$\frac{-\frac{3iA \ln(-i+\tan(fx+e))}{16} - \frac{-\frac{3A}{16} - \frac{iB}{16}}{-i+\tan(fx+e)} - \frac{\frac{iA}{8} - \frac{B}{8}}{2(-i+\tan(fx+e))^2} - \frac{-\frac{iA}{8} - \frac{B}{8}}{2(i+\tan(fx+e))^2} - \frac{-\frac{3A}{16} + \frac{iB}{16}}{i+\tan(fx+e)} + \frac{3iA \ln(i+\tan(fx+e))}{16}}{fa^2c^2}$	12
default	$\frac{-\frac{3iA \ln(-i+\tan(fx+e))}{16} - \frac{-\frac{3A}{16} - \frac{iB}{16}}{-i+\tan(fx+e)} - \frac{\frac{iA}{8} - \frac{B}{8}}{2(-i+\tan(fx+e))^2} - \frac{-\frac{iA}{8} - \frac{B}{8}}{2(i+\tan(fx+e))^2} - \frac{-\frac{3A}{16} + \frac{iB}{16}}{i+\tan(fx+e)} + \frac{3iA \ln(i+\tan(fx+e))}{16}}{fa^2c^2}$	12

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\frac{1}{f/a^2/c^2} * (-3/16 * I * A * \ln(-I + \tan(f * x + e)) - (-3/16 * A - 1/16 * I * B) / (-I + \tan(f * x + e)) - 1/2 * (1/8 * I * A - 1/8 * B) / (-I + \tan(f * x + e))^2 - 1/2 * (-1/8 * I * A - 1/8 * B) / (I + \tan(f * x + e))^2 - (-3/16 * A + 1/16 * I * B) / (I + \tan(f * x + e)) + 3/16 * I * A * \ln(I + \tan(f * x + e)))$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, alg  
orithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [C] Result contains complex when optimal does not.

time = 1.14, size = 95, normalized size = 1.34

$$\frac{(24Afxe^{4ifx+4ie}) + (-iA - B)e^{(8ifx+8ie)} - 4(2iA + B)e^{(6ifx+6ie)} - 4(-2iA + B)e^{(2ifx+2ie)} + iA - B)e^{(-4ifx-4ie)}}{64a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, alg  
orithm="fricas")`

[Out] 
$$\frac{1}{64} * (24 * A * f * x * e^{(4 * I * f * x + 4 * I * e)} + (-I * A - B) * e^{(8 * I * f * x + 8 * I * e)} - 4 * (2 * I * A + B) * e^{(6 * I * f * x + 6 * I * e)} - 4 * (-2 * I * A + B) * e^{(2 * I * f * x + 2 * I * e)} + I * A - B) * e^{(-4 * I * f * x - 4 * I * e)} / (a^2 * c^2 * f)$$

**Sympy [A]**

time = 0.37, size = 360, normalized size = 5.07

$$\frac{3Ax}{8a^2c^2} + \begin{cases} \left( \frac{((16384Aa^6e^6f^3e^{2ie} - 16384Ba^6e^6f^3e^{2ie})e^{-4ifx} + (131072Aa^6e^6f^3e^{4ie} - 65536Ba^6e^6f^3e^{4ie})e^{-2ifx} + (-131072Aa^6e^6f^3e^{8ie} - 65536Ba^6e^6f^3e^{8ie})e^{2ifx} + (-16384Aa^6e^6f^3e^{10ie} - 16384Ba^6e^6f^3e^{10ie})e^{4ifx})e^{-6ie}}{1048576a^8c^8f^4} \right) & \text{for } a^8c^8f^4e^{6ie} \neq 0 \\ x \left( -\frac{3A}{8a^2c^2} + \frac{(Ae^{8ie} + 4Ae^{6ie} + 6Ae^{4ie} + 4Ae^{2ie} + A - iBe^{8ie} - 2iBe^{6ie} + 2iBe^{4ie} + iB)e^{-4ie}}{16a^2c^2} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*2/(c-I\*c\*tan(f\*x+e))\*\*2,x)

**[Out]** 3\*A\*x/(8\*a\*\*2\*c\*\*2) + Piecewise((((16384\*I\*A\*a\*\*6\*c\*\*6\*f\*\*3\*exp(2\*I\*e) - 16384\*B\*a\*\*6\*c\*\*6\*f\*\*3\*exp(2\*I\*e))\*exp(-4\*I\*f\*x) + (131072\*I\*A\*a\*\*6\*c\*\*6\*f\*\*3\*exp(4\*I\*e) - 65536\*B\*a\*\*6\*c\*\*6\*f\*\*3\*exp(4\*I\*e))\*exp(-2\*I\*f\*x) + (-131072\*I\*A\*a\*\*6\*c\*\*6\*f\*\*3\*exp(8\*I\*e) - 65536\*B\*a\*\*6\*c\*\*6\*f\*\*3\*exp(8\*I\*e))\*exp(2\*I\*f\*x) + (-16384\*I\*A\*a\*\*6\*c\*\*6\*f\*\*3\*exp(10\*I\*e) - 16384\*B\*a\*\*6\*c\*\*6\*f\*\*3\*exp(10\*I\*e))\*exp(4\*I\*f\*x))\*exp(-6\*I\*e)/(1048576\*a\*\*8\*c\*\*8\*f\*\*4), Ne(a\*\*8\*c\*\*8\*f\*\*4\*exp(6\*I\*e), 0)), (x\*(-3\*A/(8\*a\*\*2\*c\*\*2) + (A\*exp(8\*I\*e) + 4\*A\*exp(6\*I\*e) + 6\*A\*exp(4\*I\*e) + 4\*A\*exp(2\*I\*e) + A - I\*B\*exp(8\*I\*e) - 2\*I\*B\*exp(6\*I\*e) + 2\*I\*B\*exp(2\*I\*e) + I\*B)\*exp(-4\*I\*e)/(16\*a\*\*2\*c\*\*2)), True))

**Giac [A]**

time = 0.64, size = 67, normalized size = 0.94

$$\frac{\frac{3(fx+e)A}{a^2c^2} + \frac{3A \tan(fx+e)^3 + 5A \tan(fx+e) - 2B}{(\tan(fx+e)^2 + 1)^2 a^2c^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^2,x, algorithm="giac")

**[Out]** 1/8\*(3\*(f\*x + e)\*A/(a^2\*c^2) + (3\*A\*tan(f\*x + e)^3 + 5\*A\*tan(f\*x + e) - 2\*B)/((tan(f\*x + e)^2 + 1)^2\*a^2\*c^2))/f

**Mupad [B]**

time = 8.50, size = 53, normalized size = 0.75

$$\frac{3Ax}{8a^2c^2} + \frac{\cos(e + fx)^4 \left( \frac{3A \tan(e + fx)^3}{8} + \frac{5A \tan(e + fx)}{8} - \frac{B}{4} \right)}{a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int((A + B\*tan(e + f\*x))/(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^2),x)

**[Out]** (3\*A\*x)/(8\*a^2\*c^2) + (cos(e + f\*x)^4\*((5\*A\*tan(e + f\*x))/8 - B/4 + (3\*A\*tan(e + f\*x)^3)/8))/(a^2\*c^2\*f)

$$3.724 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=183

$$\frac{(5A+iB)x}{16a^2c^3} - \frac{iA-B}{32a^2c^3f(i-\tan(e+fx))^2} - \frac{2A+iB}{16a^2c^3f(i-\tan(e+fx))} - \frac{A-iB}{24a^2c^3f(i+\tan(e+fx))^3} + \frac{1}{32a^2c^3f}$$

[Out] 1/16\*(5\*A+I\*B)\*x/a^2/c^3+1/32\*(-I\*A+B)/a^2/c^3/f/(I-tan(f\*x+e))^2+1/16\*(-2\*A-I\*B)/a^2/c^3/f/(I-tan(f\*x+e))+1/24\*(-A+I\*B)/a^2/c^3/f/(I+tan(f\*x+e))^3+1/32\*(3\*I\*A+B)/a^2/c^3/f/(I+tan(f\*x+e))^2+3/16\*A/a^2/c^3/f/(I+tan(f\*x+e))

**Rubi [A]**

time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{2A+iB}{16a^2c^3f(-\tan(e+fx)+i)} - \frac{-B+iA}{32a^2c^3f(-\tan(e+fx)+i)^2} + \frac{B+3iA}{32a^2c^3f(\tan(e+fx)+i)^2} - \frac{A-iB}{24a^2c^3f(\tan(e+fx)+i)^3} + \frac{x(5A+iB)}{16a^2c^3} + \frac{3A}{16a^2c^3f(\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^3), x]

[Out] ((5\*A + I\*B)\*x)/(16\*a^2\*c^3) - (I\*A - B)/(32\*a^2\*c^3\*f\*(I - Tan[e + f\*x])^2) - (2\*A + I\*B)/(16\*a^2\*c^3\*f\*(I - Tan[e + f\*x])) - (A - I\*B)/(24\*a^2\*c^3\*f\*(I + Tan[e + f\*x])^3) + ((3\*I)\*A + B)/(32\*a^2\*c^3\*f\*(I + Tan[e + f\*x])^2) + (3\*A)/(16\*a^2\*c^3\*f\*(I + Tan[e + f\*x]))

Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Di

```
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 (c - ic \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3(c-icx)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{i(A+iB)}{16a^3c^4(-i+x)^3} + \frac{-2A-iB}{16a^3c^4(-i+x)^2} + \frac{A-iB}{8a^3c^4(i+x)^4}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iA - B}{32a^2c^3f(i - \tan(e + fx))^2} - \frac{2A + iB}{16a^2c^3f(i - \tan(e + fx))}$$

$$= \frac{(5A + iB)x}{16a^2c^3} - \frac{iA - B}{32a^2c^3f(i - \tan(e + fx))^2} - \frac{2A}{16a^2c^3f(i - \tan(e + fx))}$$

Mathematica [A]

time = 1.08, size = 217, normalized size = 1.19

$\frac{\sec^2(e + fx) \cos(3(e + fx)) + i \sin(3(e + fx)) (12(B - 2Bfx + A(5i - 10fx)) \cos(e + fx) + 3(-5iA + 9B) \cos(3(e + fx)) - iA \cos(5(e + fx)) + 5B \cos(5(e + fx)) - 60A \sin(e + fx) + 12B \sin(e + fx) + 120Afx \sin(e + fx) - 24Bfx \sin(e + fx) - 45A \sin(3(e + fx)) - 9iB \sin(3(e + fx)) - 5A \sin(5(e + fx)) - iB \sin(5(e + fx)))}{384a^2c^3(-i + \tan(e + fx))^2}$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^3), x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(12*(B - (2*I)*B*f*x + A*(5*I - 10*f*x))*Cos[e + f*x] + 3*((-5*I)*A + 9*B)*Cos[3*(e + f*x)] - I*A*Cos[5*(e + f*x)] + 5*B*Cos[5*(e + f*x)] - 60*A*Sin[e + f*x] + (12*I)*B*Sin[e + f*x] + (120*I)*A*f*x*Sin[e + f*x] - 24*B*f*x*Sin[e + f*x] - 45*A*Sin[3*(e + f*x)] - (9*I)*B*Sin[3*(e + f*x)] - 5*A*Sin[5*(e + f*x)] - I*B*Sin[5*(e + f*x)]))/(384*a^2*c^3*f*(-I + Tan[e + f*x])^2)
```

Maple [A]

time = 0.30, size = 148, normalized size = 0.81

method	result
derivativedivides	$\frac{-\frac{3iA - B}{16} + \frac{3A}{16(i + \tan(fx + e))} - \frac{A - iB}{3(i + \tan(fx + e))^3} + \left(\frac{5iA}{32} - \frac{B}{32}\right) \ln(i + \tan(fx + e)) - \frac{iA - B}{2(-i + \tan(fx + e))^2} + \left(-\frac{5iA}{32} + \frac{B}{32}\right) \ln(i - \tan(fx + e))}{f a^2 c^3}$
default	$\frac{-\frac{3iA - B}{16} + \frac{3A}{16(i + \tan(fx + e))} - \frac{A - iB}{3(i + \tan(fx + e))^3} + \left(\frac{5iA}{32} - \frac{B}{32}\right) \ln(i + \tan(fx + e)) - \frac{iA - B}{2(-i + \tan(fx + e))^2} + \left(-\frac{5iA}{32} + \frac{B}{32}\right) \ln(i - \tan(fx + e))}{f a^2 c^3}$



```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x)
[Out] Piecewise((((50331648*I*A*a**8*c**12*f**4*exp(2*I*e) - 50331648*B*a**8*c**12*f**4*exp(2*I*e))*exp(-4*I*f*x) + (503316480*I*A*a**8*c**12*f**4*exp(4*I*e) - 301989888*B*a**8*c**12*f**4*exp(4*I*e))*exp(-2*I*f*x) + (-1006632960*I*A*a**8*c**12*f**4*exp(8*I*e) - 201326592*B*a**8*c**12*f**4*exp(8*I*e))*exp(2*I*f*x) + (-251658240*I*A*a**8*c**12*f**4*exp(10*I*e) - 150994944*B*a**8*c**12*f**4*exp(10*I*e))*exp(4*I*f*x) + (-33554432*I*A*a**8*c**12*f**4*exp(12*I*e) - 33554432*B*a**8*c**12*f**4*exp(12*I*e))*exp(6*I*f*x))*exp(-6*I*e)/(6442450944*a**10*c**15*f**5), Ne(a**10*c**15*f**5*exp(6*I*e), 0)), (x*(-(5*A + I*B)/(16*a**2*c**3) + (A*exp(10*I*e) + 5*A*exp(8*I*e) + 10*A*exp(6*I*e) + 10*A*exp(4*I*e) + 5*A*exp(2*I*e) + A - I*B*exp(10*I*e) - 3*I*B*exp(8*I*e) - 2*I*B*exp(6*I*e) + 2*I*B*exp(4*I*e) + 3*I*B*exp(2*I*e) + I*B)*exp(-4*I*e)/(32*a**2*c**3)), True)) + x*(5*A + I*B)/(16*a**2*c**3)
```

**Giac** [A]

time = 0.93, size = 219, normalized size = 1.20

$$\frac{\frac{6(-5A+B)\log(\tan(fx+e))+6(5A-B)\log(\tan(fx+e)-1)}{a^2c^3} + \frac{3(15A^2\tan(fx+e)^2-3B\tan(fx+e)^2+38A\tan(fx+e)+10I\tan(fx+e)-25A+9B)}{a^2c^2(i\tan(fx+e)+1)^2} + \frac{55iA\tan(fx+e)^3-11B\tan(fx+e)^3-201A\tan(fx+e)^2-33iB\tan(fx+e)^2-255iA\tan(fx+e)+27B\tan(fx+e)+117A-3iB}{a^2c^3(\tan(fx+e)+1)^2}}{192f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/192*(6*(-5*I*A + B)*log(tan(f*x + e) + I)/(a^2*c^3) + 6*(5*I*A - B)*log(tan(f*x + e) - I)/(a^2*c^3) + 3*(15*I*A*tan(f*x + e)^2 - 3*B*tan(f*x + e)^2 + 38*A*tan(f*x + e) + 10*I*B*tan(f*x + e) - 25*I*A + 9*B)/(a^2*c^3*(I*tan(f*x + e) + 1)^2) + (55*I*A*tan(f*x + e)^3 - 11*B*tan(f*x + e)^3 - 201*A*tan(f*x + e)^2 - 33*I*B*tan(f*x + e)^2 - 255*I*A*tan(f*x + e) + 27*B*tan(f*x + e) + 117*A - 3*I*B)/(a^2*c^3*(tan(f*x + e) + I)^3))/f
```

**Mupad** [B]

time = 0.64, size = 208, normalized size = 1.14

$$\frac{\tan(e+fx)\left(-\frac{5B}{48a^2c^3} + \frac{A25i}{48a^2c^3}\right) + \tan(e+fx)^3\left(-\frac{B}{16a^2c^3} + \frac{A5i}{16a^2c^3}\right) + \tan(e+fx)^4\left(\frac{5A}{16a^2c^3} + \frac{B1i}{16a^2c^3}\right) + \tan(e+fx)^2\left(\frac{25A}{48a^2c^3} + \frac{B5i}{48a^2c^3}\right) + \frac{A}{6a^2c^3} - \frac{B1i}{6a^2c^3} - \frac{x(-B+A5i)1i}{16a^2c^3}}{f(\tan(e+fx)^5 + \tan(e+fx)^41i + 2\tan(e+fx)^3 + \tan(e+fx)^22i + \tan(e+fx) + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^3),x)
```

```
[Out] (tan(e + f*x)*((A*25i)/(48*a^2*c^3) - (5*B)/(48*a^2*c^3)) + tan(e + f*x)^3*((A*5i)/(16*a^2*c^3) - B/(16*a^2*c^3)) + tan(e + f*x)^4*((5*A)/(16*a^2*c^3) + (B*1i)/(16*a^2*c^3)) + tan(e + f*x)^2*((25*A)/(48*a^2*c^3) + (B*5i)/(48*a^2*c^3)) + A/(6*a^2*c^3) - (B*1i)/(6*a^2*c^3))/(f*(tan(e + f*x) + tan(e + f*x)^2*2i + 2*tan(e + f*x)^3 + tan(e + f*x)^4*1i + tan(e + f*x)^5 + 1i)) - (x*(A*5i - B)*1i)/(16*a^2*c^3)
```

$$3.725 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^4} dx$$

Optimal. Leaf size=221

$$\frac{5(3A+iB)x}{64a^2c^4} - \frac{iA-B}{64a^2c^4 f(i-\tan(e+fx))^2} - \frac{5A+3iB}{64a^2c^4 f(i-\tan(e+fx))} - \frac{iA+B}{32a^2c^4 f(i+\tan(e+fx))^4} - \frac{1}{48a^2c^4}$$

[Out] 5/64\*(3\*A+I\*B)\*x/a^2/c^4+1/64\*(-I\*A+B)/a^2/c^4/f/(I-tan(f\*x+e))^2+1/64\*(-5\*A-3\*I\*B)/a^2/c^4/f/(I-tan(f\*x+e))+1/32\*(-I\*A-B)/a^2/c^4/f/(I+tan(f\*x+e))^4+1/48\*(-3\*A+I\*B)/a^2/c^4/f/(I+tan(f\*x+e))^3+3/32\*I\*A/a^2/c^4/f/(I+tan(f\*x+e))^2+1/32\*(5\*A+I\*B)/a^2/c^4/f/(I+tan(f\*x+e))

Rubi [A]

time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{5A+3iB}{64a^2c^4 f(-\tan(e+fx)+i)} + \frac{5A+iB}{32a^2c^4 f(\tan(e+fx)+i)} - \frac{-B+iA}{64a^2c^4 f(-\tan(e+fx)+i)^2} - \frac{3A-iB}{48a^2c^4 f(\tan(e+fx)+i)^3} - \frac{B+iA}{32a^2c^4 f(\tan(e+fx)+i)^4} + \frac{5x(3A+iB)}{64a^2c^4} + \frac{3iA}{32a^2c^4 f(\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^4), x]

[Out] (5\*(3\*A + I\*B)\*x)/(64\*a^2\*c^4) - (I\*A - B)/(64\*a^2\*c^4\*f\*(I - Tan[e + f\*x])^2) - (5\*A + (3\*I)\*B)/(64\*a^2\*c^4\*f\*(I - Tan[e + f\*x])) - (I\*A + B)/(32\*a^2\*c^4\*f\*(I + Tan[e + f\*x])^4) - (3\*A - I\*B)/(48\*a^2\*c^4\*f\*(I + Tan[e + f\*x])^3) + (((3\*I)/32)\*A)/(a^2\*c^4\*f\*(I + Tan[e + f\*x])^2) + (5\*A + I\*B)/(32\*a^2\*c^4\*f\*(I + Tan[e + f\*x]))

Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 (c - ict \tan(e + fx))^4} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3 (c-icx)^5} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{i(A+iB)}{32a^3c^5(-i+x)^3} + \frac{-5A-3iB}{64a^3c^5(-i+x)^2} + \frac{iA+B}{8a^3c^5(i+x)^5} + \dots\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iA - B}{64a^2c^4 f (i - \tan(e + fx))^2} - \frac{5A + 3iB}{64a^2c^4 f (i - \tan(e + fx))} + \dots$$

Mathematica [A]

time = 1.19, size = 232, normalized size = 1.05

$\frac{a^2(c + fx)(-i \cos(e + fx) + \sin(e + fx))(-240A + 30(A(-3 - 12I)fx + B(1 + 4fx)) \cos(2(e + fx)) + 163A + 4iB) \cos(4(e + fx)) + 3A \cos(6(e + fx)) + 9iB \cos(8(e + fx)) - 90A \sin(2(e + fx)) - 30B \sin(2(e + fx)) - 360Afx \sin(2(e + fx)) - 120Bfx \sin(2(e + fx)) - 96A \sin(4(e + fx)) + 32B \sin(4(e + fx)) - 9A \sin(6(e + fx)) + 3B \sin(6(e + fx))}{1536a^2c^4(-i + \tan(e + fx))^2}$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^4), x]
```

```
[Out] (Sec[e + f*x]^2*((-I)*Cos[4*(e + f*x)] + Sin[4*(e + f*x)])*(-240*A + 30*(A*(-3 - (12*I)*f*x) + B*(I + 4*f*x))*Cos[2*(e + f*x)] + 16*(3*A + (4*I)*B)*Cos[4*(e + f*x)] + 3*A*Cos[6*(e + f*x)] + (9*I)*B*Cos[6*(e + f*x)] - (90*I)*A*Sin[2*(e + f*x)] - 30*B*Sin[2*(e + f*x)] - 360*A*f*x*Sin[2*(e + f*x)] - (120*I)*B*f*x*Sin[2*(e + f*x)] - (96*I)*A*Sin[4*(e + f*x)] + 32*B*Sin[4*(e + f*x)] - (9*I)*A*Sin[6*(e + f*x)] + 3*B*Sin[6*(e + f*x)]))/(1536*a^2*c^4*f*(-I + Tan[e + f*x])^2)
```

Maple [A]

time = 0.36, size = 170, normalized size = 0.77

method	result
derivativedivides	$\frac{3iA}{32(i+\tan(fx+e))^2} - \frac{3A-iB}{3(i+\tan(fx+e))^3} + \left(-\frac{5B}{128} + \frac{15iA}{128}\right) \ln(i+\tan(fx+e)) - \frac{\frac{iA}{8} + \frac{B}{8}}{4(i+\tan(fx+e))^4} - \frac{-\frac{5A}{32} - \frac{iB}{32}}{i+\tan(fx+e)} + \left(\frac{5B}{128} - \frac{15iA}{128}\right) \ln\left(\frac{f a^2 c^4}{f a^2 c^4}\right)$



default	$\frac{3iA}{32(i+\tan(fx+e))^2} - \frac{\frac{3A}{16} - \frac{iB}{16}}{3(i+\tan(fx+e))^3} + \left(-\frac{5B}{128} + \frac{15iA}{128}\right) \ln(i+\tan(fx+e)) - \frac{\frac{iA}{8} + \frac{B}{8}}{4(i+\tan(fx+e))^4} - \frac{-\frac{5A}{32} - \frac{iB}{32}}{i+\tan(fx+e)} + \left(\frac{5B}{128} - \frac{15iA}{128}\right) \ln$
norman	$\frac{5(iB+3A)x}{64ac} - \frac{3iA+B}{12acf} + \frac{B(\tan^2(fx+e))}{6acf} + \frac{73(iB+3A)(\tan^3(fx+e))}{192acf} + \frac{55(iB+3A)(\tan^5(fx+e))}{192acf} + \frac{5(iB+3A)(\tan^7(fx+e))}{64acf} + \frac{5(iB+3A)}{a^3(1+\tan^2(fx+e))}$
risch	$\frac{5ixB}{64a^2c^4} + \frac{15xA}{64a^2c^4} - \frac{e^{8i(fx+e)}B}{512a^2c^4f} - \frac{ie^{8i(fx+e)}A}{512a^2c^4f} - \frac{e^{6i(fx+e)}B}{96a^2c^4f} - \frac{ie^{6i(fx+e)}A}{64a^2c^4f} - \frac{3\cos(4fx+4e)B}{128a^2c^4f} - \frac{7i\cos(4fx+4e)A}{128a^2c^4f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f/a^2/c^4*(3/32*I*A/(I+tan(f*x+e))^2-1/3*(3/16*A-1/16*I*B)/(I+tan(f*x+e))^3+(-5/128*B+15/128*I*A)*ln(I+tan(f*x+e))-1/4*(1/8*I*A+1/8*B)/(I+tan(f*x+e))^4-(-5/32*A-1/32*I*B)/(I+tan(f*x+e))+5/128*B-15/128*I*A)*ln(-I+tan(f*x+e))-1/2*(1/32*I*A-1/32*B)/(-I+tan(f*x+e))^2-(-5/64*A-3/64*I*B)/(-I+tan(f*x+e)))
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 0.83, size = 134, normalized size = 0.61

$$\frac{(120(3A+iB)fx^{4i+4ie} - 3(iA+B)e^{12i(fx+12ie)} - 8(3iA+2B)e^{10i(fx+10ie)} - 30(3iA+B)e^{8i(fx+8ie)} - 240iAe^{6i(fx+6ie)} - 24(-3iA+2B)e^{2i(fx+2ie)} + 6iA-6B)e^{(-4i)fx-4ie}}{1536a^2c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, alg
orithm="fricas")
```

```
[Out] 1/1536*(120*(3*A + I*B)*f*x*e^(4*I*f*x + 4*I*e) - 3*(I*A + B)*e^(12*I*f*x + 12*I*e) - 8*(3*I*A + 2*B)*e^(10*I*f*x + 10*I*e) - 30*(3*I*A + B)*e^(8*I*f*x + 8*I*e) - 240*I*A*e^(6*I*f*x + 6*I*e) - 24*(-3*I*A + 2*B)*e^(2*I*f*x + 2*I*e) + 6*I*A - 6*B)*e^(-4*I*f*x - 4*I*e)/(a^2*c^4*f)
```

**Sympy [A]**

time = 0.62, size = 498, normalized size = 2.25

$$\frac{\dots}{1536a^2c^4f} \text{ for } a^2c^4f \neq 0 \text{ otherwise } \frac{x(15A+5iB)}{64a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x)
[Out] Piecewise((( -2061584302080*I*A*a**10*c**20*f**5*exp(8*I*e)*exp(2*I*f*x) + (
51539607552*I*A*a**10*c**20*f**5*exp(2*I*e) - 51539607552*B*a**10*c**20*f**
5*exp(2*I*e))*exp(-4*I*f*x) + (618475290624*I*A*a**10*c**20*f**5*exp(4*I*e)
- 412316860416*B*a**10*c**20*f**5*exp(4*I*e))*exp(-2*I*f*x) + (-7730941132
80*I*A*a**10*c**20*f**5*exp(10*I*e) - 257698037760*B*a**10*c**20*f**5*exp(1
0*I*e))*exp(4*I*f*x) + (-206158430208*I*A*a**10*c**20*f**5*exp(12*I*e) - 13
7438953472*B*a**10*c**20*f**5*exp(12*I*e))*exp(6*I*f*x) + (-25769803776*I*A
*a**10*c**20*f**5*exp(14*I*e) - 25769803776*B*a**10*c**20*f**5*exp(14*I*e))
*exp(8*I*f*x))*exp(-6*I*e)/(13194139533312*a**12*c**24*f**6), Ne(a**12*c**2
4*f**6*exp(6*I*e), 0)), (x*(-(15*A + 5*I*B)/(64*a**2*c**4) + (A*exp(12*I*e)
+ 6*A*exp(10*I*e) + 15*A*exp(8*I*e) + 20*A*exp(6*I*e) + 15*A*exp(4*I*e) +
6*A*exp(2*I*e) + A - I*B*exp(12*I*e) - 4*I*B*exp(10*I*e) - 5*I*B*exp(8*I*e)
+ 5*I*B*exp(4*I*e) + 4*I*B*exp(2*I*e) + I*B)*exp(-4*I*e)/(64*a**2*c**4)),
True)) + x*(15*A + 5*I*B)/(64*a**2*c**4)
```

**Giac [A]**

time = 1.05, size = 241, normalized size = 1.09

$$\frac{60(3A-B)\log(\tan(fx+e))+60(-3A+B)\log(\tan(fx+e)-1)-\frac{e(-45A\tan(fx+e)^5+15B\tan(fx+e)^5-110A\tan(fx+e)-42B\tan(fx+e)+69A-31B)}{a^2c(\tan(fx+e)-1)^2}+\frac{-375A\tan(fx+e)^3+125B\tan(fx+e)^3+1740A\tan(fx+e)^2+548B\tan(fx+e)^2+3114A\tan(fx+e)^2-894B\tan(fx+e)^2-2604A\tan(fx+e)-612B\tan(fx+e)-903A+93B}{a^2c(\tan(fx+e)+1)^2}}{1536f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, alg
orithm="giac")
```

```
[Out] 1/1536*(60*(3*I*A - B)*log(tan(f*x + e) + I)/(a^2*c^4) + 60*(-3*I*A + B)*lo
g(tan(f*x + e) - I)/(a^2*c^4) - 6*(-45*I*A*tan(f*x + e)^2 + 15*B*tan(f*x +
e)^2 - 110*A*tan(f*x + e) - 42*I*B*tan(f*x + e) + 69*I*A - 31*B)/(a^2*c^4*(
tan(f*x + e) - I)^2) + (-375*I*A*tan(f*x + e)^4 + 125*B*tan(f*x + e)^4 + 17
40*A*tan(f*x + e)^3 + 548*I*B*tan(f*x + e)^3 + 3114*I*A*tan(f*x + e)^2 - 89
4*B*tan(f*x + e)^2 - 2604*A*tan(f*x + e) - 612*I*B*tan(f*x + e) - 903*I*A +
93*B)/(a^2*c^4*(tan(f*x + e) + I)^4))/f
```

**Mupad [B]**

time = 10.17, size = 247, normalized size = 1.12

$$\frac{\frac{B}{12a^2c^4} + \tan(e+fx)^4\left(-\frac{5B}{32a^2c^4} + \frac{A15i}{32a^2c^4}\right) + \tan(e+fx)^3\left(\frac{5A}{32a^2c^4} + \frac{B5i}{96a^2c^4}\right) + \tan(e+fx)^2\left(\frac{15A}{64a^2c^4} + \frac{B5i}{64a^2c^4}\right) + \tan(e+fx)\left(-\frac{25B}{96a^2c^4} + \frac{A25i}{32a^2c^4}\right) - \tan(e+fx)\left(\frac{17A}{64a^2c^4} + \frac{B17i}{192a^2c^4}\right) + \frac{A11}{4a^2c^4} + \frac{5x(3A+B1i)}{64a^2c^4}}{f(\tan(e+fx)^6 + \tan(e+fx)^5 2i + \tan(e+fx)^4 + \tan(e+fx)^3 4i - \tan(e+fx)^2 + \tan(e+fx) 2i - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^4),x)
```

```
[Out] (tan(e + f*x)^4*((A*15i)/(32*a^2*c^4) - (5*B)/(32*a^2*c^4)) - tan(e + f*x)*
((17*A)/(64*a^2*c^4) + (B*17i)/(192*a^2*c^4)) + tan(e + f*x)^3*((5*A)/(32*a
```

$$\begin{aligned}
&^2*c^4) + (B*5i)/(96*a^2*c^4) + \tan(e + f*x)^5*((15*A)/(64*a^2*c^4) + (B*5 \\
&i)/(64*a^2*c^4)) + \tan(e + f*x)^2*((A*25i)/(32*a^2*c^4) - (25*B)/(96*a^2*c^ \\
&4) + (A*1i)/(4*a^2*c^4) + B/(12*a^2*c^4))/(f*(\tan(e + f*x)*2i - \tan(e + f* \\
&x)^2 + \tan(e + f*x)^3*4i + \tan(e + f*x)^4 + \tan(e + f*x)^5*2i + \tan(e + f*x \\
&)^6 - 1)) + (5*x*(3*A + B*1i))/(64*a^2*c^4)
\end{aligned}$$

$$3.726 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^5} dx$$

**Optimal.** Leaf size=251

$$\frac{3(7A + 3iB)x}{128a^2c^5} - \frac{iA - B}{128a^2c^5 f(i - \tan(e + fx))^2} - \frac{3A + 2iB}{64a^2c^5 f(i - \tan(e + fx))} + \frac{A - iB}{40a^2c^5 f(i + \tan(e + fx))^5} - \frac{A}{64a^2c^5 f(i + \tan(e + fx))^3}$$

[Out] 3/128\*(7\*A+3\*I\*B)\*x/a^2/c^5+1/128\*(-I\*A+B)/a^2/c^5/f/(I-tan(f\*x+e))^2+1/64\*(-3\*A-2\*I\*B)/a^2/c^5/f/(I-tan(f\*x+e))+1/40\*(A-I\*B)/a^2/c^5/f/(I+tan(f\*x+e))^5+1/64\*(-3\*I\*A-B)/a^2/c^5/f/(I+tan(f\*x+e))^4-1/16\*A/a^2/c^5/f/(I+tan(f\*x+e))^3+1/64\*(5\*I\*A-B)/a^2/c^5/f/(I+tan(f\*x+e))^2+5/128\*(3\*A+I\*B)/a^2/c^5/f/(I+tan(f\*x+e))

**Rubi [A]**

time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$\frac{3A + 2iB}{64a^2c^5 f(-\tan(e + fx) + i)} + \frac{5(3A + iB)}{128a^2c^5 f(\tan(e + fx) + i)} - \frac{-B + iA}{128a^2c^5 f(-\tan(e + fx) + i)^2} + \frac{-B + 5iA}{64a^2c^5 f(\tan(e + fx) + i)^2} - \frac{B + 3iA}{64a^2c^5 f(\tan(e + fx) + i)^4} + \frac{A - iB}{40a^2c^5 f(\tan(e + fx) + i)^5} + \frac{3x(7A + 3iB)}{128a^2c^5} - \frac{A}{16a^2c^5 f(\tan(e + fx) + i)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^5), x]

[Out] (3\*(7\*A + (3\*I)\*B)\*x)/(128\*a^2\*c^5) - (I\*A - B)/(128\*a^2\*c^5\*f\*(I - Tan[e + f\*x])^2) - (3\*A + (2\*I)\*B)/(64\*a^2\*c^5\*f\*(I - Tan[e + f\*x])) + (A - I\*B)/(40\*a^2\*c^5\*f\*(I + Tan[e + f\*x])^5) - ((3\*I)\*A + B)/(64\*a^2\*c^5\*f\*(I + Tan[e + f\*x])^4) - A/(16\*a^2\*c^5\*f\*(I + Tan[e + f\*x])^3) + ((5\*I)\*A - B)/(64\*a^2\*c^5\*f\*(I + Tan[e + f\*x])^2) + (5\*(3\*A + I\*B))/(128\*a^2\*c^5\*f\*(I + Tan[e + f\*x]))

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])



method	result
derivativedivides	$-\frac{\frac{3iA}{16} + \frac{B}{16}}{4(i+\tan(fx+e))^4} - \frac{-\frac{15A}{128} - \frac{5iB}{128}}{i+\tan(fx+e)} - \frac{-\frac{5iA}{32} + \frac{B}{32}}{2(i+\tan(fx+e))^2} + \left(\frac{21iA}{256} - \frac{9B}{256}\right) \ln(i+\tan(fx+e)) - \frac{A}{16(i+\tan(fx+e))^3} - \frac{-\frac{A}{8} + \frac{iB}{8}}{5(i+\tan(fx+e))^5} - \frac{1}{f a^2 c^5}$
default	$-\frac{\frac{3iA}{16} + \frac{B}{16}}{4(i+\tan(fx+e))^4} - \frac{-\frac{15A}{128} - \frac{5iB}{128}}{i+\tan(fx+e)} - \frac{-\frac{5iA}{32} + \frac{B}{32}}{2(i+\tan(fx+e))^2} + \left(\frac{21iA}{256} - \frac{9B}{256}\right) \ln(i+\tan(fx+e)) - \frac{A}{16(i+\tan(fx+e))^3} - \frac{-\frac{A}{8} + \frac{iB}{8}}{5(i+\tan(fx+e))^5} - \frac{1}{f a^2 c^5}$
risch	$\frac{9ixB}{128a^2c^5} + \frac{21xA}{128a^2c^5} - \frac{e^{10i(fx+e)}B}{1280a^2c^5f} - \frac{ie^{10i(fx+e)}A}{1280a^2c^5f} - \frac{5e^{8i(fx+e)}B}{1024a^2c^5f} - \frac{7ie^{8i(fx+e)}A}{1024a^2c^5f} - \frac{3e^{6i(fx+e)}B}{256a^2c^5f} - \frac{7ie^{6i(fx+e)}A}{256a^2c^5f}$
norman	$\frac{3(3iB+7A)x}{128ac} - \frac{11iA+B}{40acf} + \frac{(-9iB+107A)\tan(fx+e)}{128acf} + \frac{(3iB+7A)(\tan^5(fx+e))}{5acf} + \frac{7(3iB+7A)(\tan^7(fx+e))}{64acf} + \frac{3(3iB+7A)(\tan^9(fx+e))}{128acf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f/a^2/c^5*(-1/4*(3/16*I*A+1/16*B)/(I+tan(f*x+e))^4-(-15/128*A-5/128*I*B)/
(I+tan(f*x+e))-1/2*(-5/32*I*A+1/32*B)/(I+tan(f*x+e))^2+(21/256*I*A-9/256*B)
*ln(I+tan(f*x+e))-1/16*A/(I+tan(f*x+e))^3-1/5*(-1/8*A+1/8*I*B)/(I+tan(f*x+
e))^5-1/2*(-1/64*B+1/64*I*A)/(-I+tan(f*x+e))^2-(-1/32*I*B-3/64*A)/(-I+tan(f
x+e))+(-21/256*I*A+9/256*B)*ln(-I+tan(f*x+e)))
```

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas** [A]

time = 2.75, size = 159, normalized size = 0.63

$$\frac{(120(7A+3iB)fxe^{4i(fx+4i)} - 4(iA+B)e^{14i(fx+14i)} - 5(7iA+5B)e^{12i(fx+12i)} - 20(7iA+3B)e^{10i(fx+10i)} - 50(7iA+B)e^{8i(fx+8i)} - 100(7iA-B)e^{6i(fx+6i)} - 20(-7iA+5B)e^{2i(fx+2i)} + 10iA - 10B)e^{-4i(fx-4i)}}{5120a^2c^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x, alg
orithm="fricas")
```

```
[Out] 1/5120*(120*(7*A + 3*I*B)*f*x*e^(4*I*f*x + 4*I*e) - 4*(I*A + B)*e^(14*I*f*x
+ 14*I*e) - 5*(7*I*A + 5*B)*e^(12*I*f*x + 12*I*e) - 20*(7*I*A + 3*B)*e^(10
```

$$*I*f*x + 10*I*e) - 50*(7*I*A + B)*e^(8*I*f*x + 8*I*e) - 100*(7*I*A - B)*e^(6*I*f*x + 6*I*e) - 20*(-7*I*A + 5*B)*e^(2*I*f*x + 2*I*e) + 10*I*A - 10*B)*e^(-4*I*f*x - 4*I*e)/(a^2*c^5*f)$$

**Sympy** [A]

time = 0.71, size = 605, normalized size = 2.41

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^5,x)

[Out] Piecewise((((11258999068426240\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(2\*I\*e) - 11258999068426240\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(2\*I\*e))\*exp(-4\*I\*f\*x) + (157625986957967360\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(4\*I\*e) - 112589990684262400\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(4\*I\*e))\*exp(-2\*I\*f\*x) + (-788129934789836800\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(8\*I\*e) + 112589990684262400\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(8\*I\*e))\*exp(2\*I\*f\*x) + (-394064967394918400\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(10\*I\*e) - 56294995342131200\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(10\*I\*e))\*exp(4\*I\*f\*x) + (-157625986957967360\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(12\*I\*e) - 67553994410557440\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(12\*I\*e))\*exp(6\*I\*f\*x) + (-39406496739491840\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(14\*I\*e) - 28147497671065600\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(14\*I\*e))\*exp(8\*I\*f\*x) + (-4503599627370496\*I\*A\*a\*\*12\*c\*\*30\*f\*\*6\*exp(16\*I\*e) - 4503599627370496\*B\*a\*\*12\*c\*\*30\*f\*\*6\*exp(16\*I\*e))\*exp(10\*I\*f\*x))\*exp(-6\*I\*e)/(5764607523034234880\*a\*\*14\*c\*\*35\*f\*\*7), Ne(a\*\*14\*c\*\*35\*f\*\*7\*exp(6\*I\*e), 0)), (x\*(-(21\*A + 9\*I\*B)/(128\*a\*\*2\*c\*\*5) + (A\*exp(14\*I\*e) + 7\*A\*exp(12\*I\*e) + 21\*A\*exp(10\*I\*e) + 35\*A\*exp(8\*I\*e) + 35\*A\*exp(6\*I\*e) + 21\*A\*exp(4\*I\*e) + 7\*A\*exp(2\*I\*e) + A - I\*B\*exp(14\*I\*e) - 5\*I\*B\*exp(12\*I\*e) - 9\*I\*B\*exp(10\*I\*e) - 5\*I\*B\*exp(8\*I\*e) + 5\*I\*B\*exp(6\*I\*e) + 9\*I\*B\*exp(4\*I\*e) + 5\*I\*B\*exp(2\*I\*e) + I\*B)\*exp(-4\*I\*e)/(128\*a\*\*2\*c\*\*5)), True)) + x\*(21\*A + 9\*I\*B)/(128\*a\*\*2\*c\*\*5)

**Giac** [A]

time = 0.97, size = 269, normalized size = 1.07

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^5,x, algorithm="giac")

[Out] -1/5120\*(60\*(-7\*I\*A + 3\*B)\*log(tan(f\*x + e) + I)/(a^2\*c^5) + 60\*(7\*I\*A - 3\*B)\*log(tan(f\*x + e) - I)/(a^2\*c^5) + 10\*(63\*I\*A\*tan(f\*x + e)^2 - 27\*B\*tan(f\*x + e)^2 + 150\*A\*tan(f\*x + e) + 70\*I\*B\*tan(f\*x + e) - 91\*I\*A + 47\*B)/(a^2\*c^5\*(-I\*tan(f\*x + e) - 1)^2) + (959\*I\*A\*tan(f\*x + e)^5 - 411\*B\*tan(f\*x + e)^5 - 5395\*A\*tan(f\*x + e)^4 - 2255\*I\*B\*tan(f\*x + e)^4 - 12390\*I\*A\*tan(f\*x + e)^3 + 4990\*B\*tan(f\*x + e)^3 + 14710\*A\*tan(f\*x + e)^2 + 5550\*I\*B\*tan(f\*x +

$$e)^2 + 9275*I*A*\tan(f*x + e) - 3015*B*\tan(f*x + e) - 2647*A - 483*I*B)/(a^2 * c^5 * (\tan(f*x + e) + I)^5))/f$$

**Mupad [B]**

time = 10.24, size = 291, normalized size = 1.16

$$\frac{\tan(e + f x) \left( -\frac{3B}{640a^2c^5} + \frac{A7i}{640a^2c^5} \right) + \tan(e + f x)^4 \left( \frac{7A}{32a^2c^5} + \frac{B3i}{32a^2c^5} \right) - \tan(e + f x)^3 \left( -\frac{9B}{32a^2c^5} + \frac{A31i}{32a^2c^5} \right) - \tan(e + f x)^2 \left( \frac{21A}{128a^2c^5} + \frac{B9i}{128a^2c^5} \right) - \tan(e + f x) \left( -\frac{27B}{128a^2c^5} + \frac{A69i}{128a^2c^5} \right) + \tan(e + f x)^2 \left( \frac{469A}{640a^2c^5} + \frac{B201i}{640a^2c^5} \right) + \frac{11A}{40a^2c^5} - \frac{B1i}{40a^2c^5} + \frac{3x(7A + B3i)}{128a^2c^5}}{f \left( -\tan(e + f x)^7 - \tan(e + f x)^6 3i + \tan(e + f x)^5 - \tan(e + f x)^4 5i + 5 \tan(e + f x)^3 - \tan(e + f x)^2 11 + 3 \tan(e + f x) + 11 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^5), x)

[Out] (tan(e + f\*x)\*((A\*7i)/(640\*a^2\*c^5) - (3\*B)/(640\*a^2\*c^5)) + tan(e + f\*x)^4\*((7\*A)/(32\*a^2\*c^5) + (B\*3i)/(32\*a^2\*c^5)) - tan(e + f\*x)^3\*((A\*21i)/(32\*a^2\*c^5) - (9\*B)/(32\*a^2\*c^5)) - tan(e + f\*x)^2\*((21\*A)/(128\*a^2\*c^5) + (B\*9i)/(128\*a^2\*c^5)) - tan(e + f\*x)^5\*((A\*63i)/(128\*a^2\*c^5) - (27\*B)/(128\*a^2\*c^5)) + tan(e + f\*x)^2\*((469\*A)/(640\*a^2\*c^5) + (B\*201i)/(640\*a^2\*c^5)) + (11\*A)/(40\*a^2\*c^5) - (B\*1i)/(40\*a^2\*c^5))/(f\*(3\*tan(e + f\*x) - tan(e + f\*x)^2\*1i + 5\*tan(e + f\*x)^3 - tan(e + f\*x)^4\*5i + tan(e + f\*x)^5 - tan(e + f\*x)^6\*3i - tan(e + f\*x)^7 + 1i)) + (3\*x\*(7\*A + B\*3i))/(128\*a^2\*c^5)



$$3.727 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^n}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=115

$$\frac{(iA(3-n) + B(3+n)) {}_2F_1(3, n; 1+n; \frac{1}{2}(1-i \tan(e+fx))) (c-ic \tan(e+fx))^n}{48a^3fn} + \frac{(iA-B)(c-ic \tan(e+fx))^n}{6a^3f(1+i \tan(e+fx))^3}$$

[Out] 1/48\*(I\*A\*(3-n)+B\*(3+n))\*hypergeom([3, n], [1+n], 1/2-1/2\*I\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/a^3/f/n+1/6\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^n/a^3/f/(1+I\*tan(f\*x+e))^3

**Rubi [A]**

time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 79, 70}

$$\frac{(B(n+3) + iA(3-n))(c-ic \tan(e+fx))^n {}_2F_1(3, n; n+1; \frac{1}{2}(1-i \tan(e+fx)))}{48a^3fn} + \frac{(-B+iA)(c-ic \tan(e+fx))^n}{6a^3f(1+i \tan(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^n)/(a + I\*a\*Tan[e + f\*x])^3, x]

[Out] ((I\*A\*(3 - n) + B\*(3 + n))\*Hypergeometric2F1[3, n, 1 + n, (1 - I\*Tan[e + f\*x])/2]\*(c - I\*c\*Tan[e + f\*x])^n)/(48\*a^3\*f\*n) + ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^n)/(6\*a^3\*f\*(1 + I\*Tan[e + f\*x])^3)

**Rule 70**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(b\*c - a\*d)^n\*((a + b\*x)^(m + 1)/(b^(n + 1)\*(m + 1)))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

**Rule 79**

Int[((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

**Rule 3669**

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^n}{(a + i a \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{-1+n}}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B)(c - i c \tan(e + fx))^n}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{(c(A(3 - n) - iB(3 + n)))}{48a^3 f n}$$

$$= \frac{(iA(3 - n) + B(3 + n)) {}_2F_1(3, n; 1 + n; \frac{1}{2}(1 - i \tan(e + fx)))}{48a^3 f n}$$

**Mathematica** [F]

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
f*x])^3,x]
```

[Out] \$Aborted

**Maple** [F]

time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(fx + e))(c - i c \tan(fx + e))^n}{(a + i a \tan(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

```
[Out] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^3,x, algorithm="fricas")

[Out] integral(1/8\*((A - I\*B)\*e^(6\*I\*f\*x + 6\*I\*e) + (3\*A - I\*B)\*e^(4\*I\*f\*x + 4\*I\*e) + (3\*A + I\*B)\*e^(2\*I\*f\*x + 2\*I\*e) + A + I\*B)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^n\*e^(-6\*I\*f\*x - 6\*I\*e)/a^3, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \frac{A(-ic \tan(e+fx)+c)^n}{\tan^3(e+fx)-3i \tan^2(e+fx)-3 \tan(e+fx)+i} dx + \int \frac{B(-ic \tan(e+fx)+c)^n \tan(e+fx)}{\tan^3(e+fx)-3i \tan^2(e+fx)-3 \tan(e+fx)+i} dx \right) / a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^3,x)

[Out] I\*(Integral(A\*(-I\*c\*tan(e + f\*x) + c))^n/(tan(e + f\*x)\*\*3 - 3\*I\*tan(e + f\*x)\*\*2 - 3\*tan(e + f\*x) + I), x) + Integral(B\*(-I\*c\*tan(e + f\*x) + c))^n\*tan(e + f\*x)/(tan(e + f\*x)\*\*3 - 3\*I\*tan(e + f\*x)\*\*2 - 3\*tan(e + f\*x) + I), x))/a\*\*3

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n/(a+I\*a\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^n/(I\*a\*tan(f\*x + e) + a)^3, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) \operatorname{li})^n}{(a + a \tan(e + f x) \operatorname{li})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*li)^n)/(a + a\*tan(e + f\*x)\*li)^3,x)

[Out] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*li)^n)/(a + a\*tan(e + f\*x)\*li)^3, x)

$$3.728 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^5}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=191

$$\frac{8(A+4iB)c^5x}{a^3} - \frac{8(iA-4B)c^5 \log(\cos(e+fx))}{a^3 f} + \frac{16(A+iB)c^5}{3a^3 f(i-\tan(e+fx))^3} + \frac{8(2iA-3B)c^5}{a^3 f(i-\tan(e+fx))^2} - \frac{8}{a^3 f(i-\tan(e+fx))}$$

[Out]  $-8*(A+4*I*B)*c^5*x/a^3-8*(I*A-4*B)*c^5*\ln(\cos(f*x+e))/a^3/f+16/3*(A+I*B)*c^5/a^3/f/(I-\tan(f*x+e))^3+8*(2*I*A-3*B)*c^5/a^3/f/(I-\tan(f*x+e))^2-8*(3*A+7*I*B)*c^5/a^3/f/(I-\tan(f*x+e))+(A+8*I*B)*c^5*\tan(f*x+e)/a^3/f+1/2*B*c^5*\tan(f*x+e)^2/a^3/f$

**Rubi [A]**

time = 0.16, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{c^5(A+8iB)\tan(e+fx)}{a^3f} - \frac{8c^5(3A+7iB)}{a^3f(-\tan(e+fx)+i)} + \frac{8c^5(-3B+2iA)}{a^3f(-\tan(e+fx)+i)^2} + \frac{16c^5(A+iB)}{3a^3f(-\tan(e+fx)+i)^3} - \frac{8c^5(-4B+iA)\log(\cos(e+fx))}{a^3f} - \frac{8c^5x(A+4iB)}{a^3} + \frac{Bc^5 \tan^2(e+fx)}{2a^3f}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^5)/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out]  $(-8*(A+(4*I)*B)*c^5*x/a^3 - (8*(I*A-4*B)*c^5*\text{Log}[\text{Cos}[e+f*x]])/(a^3*f) + (16*(A+I*B)*c^5)/(3*a^3*f*(I-\text{Tan}[e+f*x])^3) + (8*((2*I)*A-3*B)*c^5)/(a^3*f*(I-\text{Tan}[e+f*x])^2) - (8*(3*A+(7*I)*B)*c^5)/(a^3*f*(I-\text{Tan}[e+f*x])) + ((A+(8*I)*B)*c^5*\text{Tan}[e+f*x])/(a^3*f) + (B*c^5*\text{Tan}[e+f*x]^2)/(2*a^3*f)$

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]



norman	$\frac{(8ic^5B+Ac^5)(\tan^7(fx+e))}{af} + \frac{(32ic^5B+9Ac^5)\tan(fx+e)}{af} + \frac{(80ic^5B+27Ac^5)(\tan^5(fx+e))}{af} - \frac{8(4ic^5B+Ac^5)x}{a} - \frac{-80ic^5A+233}{6af}$
--------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x,method=_RE  
TURNVERBOSE)`

[Out]  $\frac{1}{f}c^5/a^3*(A*\tan(f*x+e)+8*I*B*\tan(f*x+e)+1/2*B*\tan(f*x+e)^2-1/3*(16*A+16*I*B)/(-I+\tan(f*x+e))^3+(8*I*A-32*B)*\ln(-I+\tan(f*x+e))-(-24*A-56*I*B)/(-I+\tan(f*x+e))-1/2*(-32*I*A+48*B)/(-I+\tan(f*x+e))^2)$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x, alg  
orithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

Fricas [A]

time = 3.30, size = 279, normalized size = 1.46

$$\frac{2(24(A+4iB)c^5f^{10}e^{10ix+10ie}+4(-1A+4B)c^5e^{10ix+10ie}+(iA-4B)c^5e^{10ix+10ie}+(-1A+B)c^5+12(4(A+4iB)c^5fx+(-iA+4B)c^5e^{8ix+8ie}+6(4(A+4iB)c^5fx+3(-iA+4B)c^5e^{6ix+6ie}+12((iA-4B)c^5e^{4ix+4ie}+2(iA-4B)c^5e^{2ix+2ie}+(iA-4B)c^5e^{0ix+0ie}))\log(e^{2ix+2ie}+1))}{3(a^3f^{10}e^{10ix+10ie}+2a^2f^{10}e^{10ix+10ie}+af^{10}e^{10ix+10ie})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x, alg  
orithm="fricas")`

[Out]  $-2/3*(24*(A+4*I*B)*c^5*f*x*e^{(10*I*f*x+10*I*e)}+4*(-I*A+4*B)*c^5*e^{(4*I*f*x+4*I*e)}+(I*A-4*B)*c^5*e^{(2*I*f*x+2*I*e)}+(-I*A+B)*c^5+12*(4*(A+4*I*B)*c^5*f*x+(-I*A+4*B)*c^5)*e^{(8*I*f*x+8*I*e)}+6*(4*(A+4*I*B)*c^5*f*x+3*(-I*A+4*B)*c^5)*e^{(6*I*f*x+6*I*e)}+12*((I*A-4*B)*c^5*e^{(10*I*f*x+10*I*e)}+2*(I*A-4*B)*c^5*e^{(8*I*f*x+8*I*e)}+(I*A-4*B)*c^5*e^{(6*I*f*x+6*I*e)})*\log(e^{(2*I*f*x+2*I*e)}+1))/(a^3*f*e^{(10*I*f*x+10*I*e)}+2*a^3*f*e^{(8*I*f*x+8*I*e)}+a^3*f*e^{(6*I*f*x+6*I*e)})$

Sympy [A]

time = 0.94, size = 473, normalized size = 2.48

$$\frac{2iAc^5-16Bc^5+(2iAc^5e^{2ie}-14Bc^5e^{2ie})e^{2ifx}}{a^3f^{10}e^{4ifx}+2a^3f^{10}e^{2ifx}+a^3f} + \begin{cases} \frac{(2iAa^5c^5f^2e^{4ie}-2Ba^5c^5f^2e^{4ie})e^{-6ifx}+(-6iAa^5c^5f^2e^{4ie}+12Ba^5c^5f^2e^{4ie})e^{-4ifx}+(18iAa^5c^5f^2e^{4ie}-54Ba^5c^5f^2e^{4ie})e^{-2ifx}+(-16Aa^5c^5-64Bc^5)}{3a^5f^3} e^{4ifx} & \text{for } a^3f^3e^{12ie} \neq 0 \\ x\left(\frac{-16Aa^5c^5-64Bc^5}{a^3} + \frac{(-16Aa^5c^5+12Aa^5c^5e^{4ie}-8Aa^5c^5e^{2ie}+4Aa^5c^5-64Bc^5e^{4ie}+36Bc^5e^{4ie}-16Bc^5e^{2ie}+4Bc^5)e^{-6ie}}{a^3}\right) & \text{otherwise} \end{cases} - \frac{8ic^5(A+4iB)\log(e^{2ifx}+e^{-2ie})}{a^3f} + \frac{x(-16Aa^5c^5-64Bc^5)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**5/(a+I*a*tan(f*x+e))**3,x)
[Out] (2*I*A*c**5 - 16*B*c**5 + (2*I*A*c**5*exp(2*I*e) - 14*B*c**5*exp(2*I*e))*exp(2*I*f*x))/(a**3*f*exp(4*I*e)*exp(4*I*f*x) + 2*a**3*f*exp(2*I*e)*exp(2*I*f*x) + a**3*f) + Piecewise((((2*I*A*a**6*c**5*f**2*exp(6*I*e) - 2*B*a**6*c**5*f**2*exp(6*I*e))*exp(-6*I*f*x) + (-6*I*A*a**6*c**5*f**2*exp(8*I*e) + 12*B*a**6*c**5*f**2*exp(8*I*e))*exp(-4*I*f*x) + (18*I*A*a**6*c**5*f**2*exp(10*I*e) - 54*B*a**6*c**5*f**2*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(3*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(-(-16*A*c**5 - 64*I*B*c**5)/a**3 + (-16*A*c**5*exp(6*I*e) + 12*A*c**5*exp(4*I*e) - 8*A*c**5*exp(2*I*e) + 4*A*c**5 - 64*I*B*c**5*exp(6*I*e) + 36*I*B*c**5*exp(4*I*e) - 16*I*B*c**5*exp(2*I*e) + 4*I*B*c**5)*exp(-6*I*e)/a**3), True)) - 8*I*c**5*(A + 4*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a**3*f) + x*(-16*A*c**5 - 64*I*B*c**5)/a**3
```

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs.  $2(169) = 338$ .  
time = 1.30, size = 516, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 2/15*(60*(-I*A*c^5 + 4*B*c^5)*log(tan(1/2*f*x + 1/2*e) + 1)/a^3 - 120*(-I*A*c^5 + 4*B*c^5)*log(tan(1/2*f*x + 1/2*e) - I)/a^3 - 60*(I*A*c^5 - 4*B*c^5)*log(tan(1/2*f*x + 1/2*e) - 1)/a^3 + 15*(6*I*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^4 - A*c^5*tan(1/2*f*x + 1/2*e)^3 - 8*I*B*c^5*tan(1/2*f*x + 1/2*e)^3 - 12*I*A*c^5*tan(1/2*f*x + 1/2*e)^2 + 49*B*c^5*tan(1/2*f*x + 1/2*e)^2 + A*c^5*tan(1/2*f*x + 1/2*e) + 8*I*B*c^5*tan(1/2*f*x + 1/2*e) + 6*I*A*c^5 - 24*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^3) - 2*(147*I*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 588*B*c^5*tan(1/2*f*x + 1/2*e)^6 + 942*A*c^5*tan(1/2*f*x + 1/2*e)^5 + 3708*I*B*c^5*tan(1/2*f*x + 1/2*e)^5 - 2445*I*A*c^5*tan(1/2*f*x + 1/2*e)^4 + 9660*B*c^5*tan(1/2*f*x + 1/2*e)^4 - 3460*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 13240*I*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 2445*I*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 9660*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 942*A*c^5*tan(1/2*f*x + 1/2*e) + 3708*I*B*c^5*tan(1/2*f*x + 1/2*e) - 147*I*A*c^5 + 588*B*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) - I)^6))/f
```

**Mupad** [B]

time = 8.98, size = 233, normalized size = 1.22

$$\frac{\ln(\tan(e+f x)-i)\left(-\frac{32 B e^5}{a^3}+\frac{A e^5 8 i}{a^3}\right)}{f}+\frac{\tan(e+f x)\left(\frac{e^5(A+B 4 i)}{a^3}+\frac{B e^5 4 i}{a^3}\right)}{f}+\frac{5(-32 B e^5+A e^5 8 i)}{3 a^3}+\frac{\tan(e+f x)\left(\frac{(-32 B e^5+A e^5 8 i) 4 i}{a^3}+\frac{B e^5 40 i}{a^3}\right)-\tan(e+f x)^2\left(\frac{3(-32 B e^5+A e^5 8 i)}{a^3}+\frac{40 B e^5}{a^3}\right)+\frac{16 B e^5}{a^3}}{f(-\tan(e+f x)^3 1 i-3 \tan(e+f x)^2+\tan(e+f x) 3 i+1)}+\frac{B e^5 \tan(e+f x)^2}{2 a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}(((A + B*\tan(e + f*x))*(c - c*\tan(e + f*x)*1i)^5)/(a + a*\tan(e + f*x)*1i)^3, x)$

[Out]  $(\log(\tan(e + f*x) - 1i)*((A*c^5*8i)/a^3 - (32*B*c^5)/a^3))/f + (\tan(e + f*x))*((c^5*(A + B*4i))/a^3 + (B*c^5*4i)/a^3))/f + ((5*(A*c^5*8i - 32*B*c^5))/(3*a^3) + \tan(e + f*x)*((A*c^5*8i - 32*B*c^5)*4i)/a^3 + (B*c^5*40i)/a^3) - \tan(e + f*x)^2*((3*(A*c^5*8i - 32*B*c^5))/a^3 + (40*B*c^5)/a^3) + (16*B*c^5)/a^3)/(f*(\tan(e + f*x)*3i - 3*\tan(e + f*x)^2 - \tan(e + f*x)^3*1i + 1)) + (B*c^5*\tan(e + f*x)^2)/(2*a^3*f)$

$$3.729 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^4}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=164

$$-\frac{(A+7iB)c^4x}{a^3} - \frac{(iA-7B)c^4 \log(\cos(e+fx))}{a^3 f} + \frac{8(A+iB)c^4}{3a^3 f(i-\tan(e+fx))^3} + \frac{2(3iA-5B)c^4}{a^3 f(i-\tan(e+fx))^2} - \frac{6(A-iB)c^4}{a^3 f(i-\tan(e+fx))}$$

[Out]  $-(A+7*I*B)*c^4*x/a^3 - (I*A-7*B)*c^4*\ln(\cos(f*x+e))/a^3/f + 8/3*(A+I*B)*c^4/a^3/f/(I-\tan(f*x+e))^3 + 2*(3*I*A-5*B)*c^4/a^3/f/(I-\tan(f*x+e))^2 - 6*(A+3*I*B)*c^4/a^3/f/(I-\tan(f*x+e)) + I*B*c^4*\tan(f*x+e)/a^3/f$

**Rubi [A]**

time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$-\frac{6c^4(A+3iB)}{a^3 f(-\tan(e+fx)+i)} + \frac{2c^4(-5B+3iA)}{a^3 f(-\tan(e+fx)+i)^2} + \frac{8c^4(A+iB)}{3a^3 f(-\tan(e+fx)+i)^3} - \frac{c^4(-7B+iA) \log(\cos(e+fx))}{a^3 f} - \frac{c^4 x(A+7iB)}{a^3} + \frac{iBc^4 \tan(e+fx)}{a^3 f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^4/(a+I*a*\text{Tan}[e+f*x])^3, x]$

[Out]  $-(((A+(7*I)*B)*c^4*x)/a^3) - ((I*A-7*B)*c^4*\text{Log}[\text{Cos}[e+f*x]])/(a^3*f) + (8*(A+I*B)*c^4)/(3*a^3*f*(I-\text{Tan}[e+f*x])^3) + (2*((3*I)*A-5*B)*c^4)/(a^3*f*(I-\text{Tan}[e+f*x])^2) - (6*(A+(3*I)*B)*c^4)/(a^3*f*(I-\text{Tan}[e+f*x])) + (I*B*c^4*\text{Tan}[e+f*x])/(a^3*f)$

Rule 78

$\text{Int}[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^{(n_.)}*((e_.) + (f_.)*(x_))^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]^{(m_.)}*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)]^{(n_.)}), x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i \tan(e + fx))^4}{(a + i a \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^3}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{iBc^3}{a^4} + \frac{8(A+iB)c^3}{a^4(-i+x)^4} + \frac{4(-3iA+5B)c^3}{a^4(-i+x)^3} - \frac{6(A+3iB)}{a^4(-i+x)}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(A + 7iB)c^4 x}{a^3} - \frac{(iA - 7B)c^4 \log(\cos(e + fx))}{a^3 f} + \frac{3a^3 j}{3a^3}$$

**Mathematica [A]**

time = 7.07, size = 319, normalized size = 1.95

$c^4 \cos^2(e + fx) \cos(fx) + \cos(fx)^2 (A + 5B) \cos(2fx) \cos(e) - 4 \sin(e) + 3(-iA + 2B) \cos(4fx) \cos(-\sin(e)) + 6(-iA + 7B) \log(\cos(e + fx)) \cos(\frac{3e}{2}) + \cos(\frac{3e}{2})^2 - 6(A + 7B) f \cos(2e) + \cos(2e) + 2(A + iB) \cos(fx) i \cos(2e) + \sin(2e) + 6iB \cos(e) \sin(e + fx) \cos(2e) + \cos(2e) \sin(fx) + 6(A + 5B) \cos(e) + \cos(e) \cos(2f) + (A + 3B) (-3 \cos(e) + 3 \sin(e)) \sin(fx) + 2(A + iB) \cos(2e) - \cos(2e) \cos(fx) (A + B \tan(e + fx))$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^4)/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] (c^4\*Sec[e + f\*x]^2\*(Cos[f\*x] + I\*Sin[f\*x])^3\*((A + (5\*I)\*B)\*Cos[2\*f\*x]\*((6\*I)\*Cos[e] - 6\*Sin[e]) + 3\*(-I)\*A + 3\*B)\*Cos[4\*f\*x]\*(Cos[e] - I\*Sin[e]) + 6\*(-I)\*A + 7\*B)\*Log[Cos[e + f\*x]]\*(Cos[(3\*e)/2] + I\*Sin[(3\*e)/2])^2 - 6\*(A + (7\*I)\*B)\*f\*x\*(Cos[3\*e] + I\*Sin[3\*e]) + 2\*(A + I\*B)\*Cos[6\*f\*x]\*(I\*Cos[3\*e] + Sin[3\*e]) + (6\*I)\*B\*Sec[e]\*Sec[e + f\*x]\*(Cos[3\*e] + I\*Sin[3\*e])\*Sin[f\*x] + 6\*(A + (5\*I)\*B)\*(Cos[e] + I\*Sin[e])\*Sin[2\*f\*x] + (A + (3\*I)\*B)\*(-3\*Cos[e] + (3\*I)\*Sin[e])\*Sin[4\*f\*x] + 2\*(A + I\*B)\*(Cos[3\*e] - I\*Sin[3\*e])\*Sin[6\*f\*x])\*(A + B\*Tan[e + f\*x]))/(6\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^3)

**Maple [A]**

time = 0.32, size = 104, normalized size = 0.63

method	result
derivativedivides	$\frac{c^4 \left( iB \tan(fx+e) - \frac{8iB+8A}{3(-i+\tan(fx+e))^3} - \frac{-18iB-6A}{-i+\tan(fx+e)} + (iA-7B) \ln(-i+\tan(fx+e)) - \frac{-12iA+20B}{2(-i+\tan(fx+e))^2} \right)}{f a^3}$
default	$\frac{c^4 \left( iB \tan(fx+e) - \frac{8iB+8A}{3(-i+\tan(fx+e))^3} - \frac{-18iB-6A}{-i+\tan(fx+e)} + (iA-7B) \ln(-i+\tan(fx+e)) - \frac{-12iA+20B}{2(-i+\tan(fx+e))^2} \right)}{f a^3}$
risch	$-\frac{5c^4 e^{-2i(fx+e)} B}{a^3 f} + \frac{ic^4 e^{-2i(fx+e)} A}{a^3 f} + \frac{3c^4 e^{-4i(fx+e)} B}{2a^3 f} - \frac{ic^4 e^{-4i(fx+e)} A}{2a^3 f} - \frac{c^4 e^{-6i(fx+e)} B}{3a^3 f} + \frac{ic^4 e^{-6i(fx+e)} A}{3a^3 f}$
norman	$\frac{(7ic^4 B + 2A c^4) \tan(fx+e)}{af} + \frac{ic^4 B (\tan^7(fx+e))}{af} - \frac{(7ic^4 B + A c^4) x}{a} - \frac{-8ic^4 A + 32B c^4}{3af} - \frac{3(7ic^4 B + A c^4) x (\tan^2(fx+e))}{a} - \frac{3(7ic^4 B + A c^4) x (\tan^2(fx+e))}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^3,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f\*c^4/a^3\*(I\*B\*tan(f\*x+e)-1/3\*(8\*A+8\*I\*B)/(-I+tan(f\*x+e))^3-(-6\*A-18\*I\*B)  
/(-I+tan(f\*x+e))+I\*A-7\*B)\*ln(-I+tan(f\*x+e))-1/2\*(-12\*I\*A+20\*B)/(-I+tan(f\*x  
+e))^2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^3,x, alg  
orithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 5.03, size = 206, normalized size = 1.26

$$\frac{12(A+7iB)c^d f x e^{8i f x+8ie} + 3(-iA+7B)c^d e^{8i f x+8ie} - (-iA+7B)c^d e^{2i f x+2ie} + 2(-iA+B)c^d + 6(2(A+7iB)c^d f x + (-iA+7B)c^d) e^{6i f x+6ie} + 6((iA-7B)c^d e^{8i f x+8ie} + (iA-7B)c^d e^{6i f x+6ie}) \log(e^{2i f x+2ie} + 1)}{6(a^3 f e^{8i f x+8ie} + a^2 f e^{6i f x+6ie})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^4/(a+I\*a\*tan(f\*x+e))^3,x, alg  
orithm="fricas")

[Out] -1/6\*(12\*(A + 7\*I\*B)\*c^4\*f\*x\*e^(8\*I\*f\*x + 8\*I\*e) + 3\*(-I\*A + 7\*B)\*c^4\*e^(4\*I\*f\*x + 4\*I\*e) - (-I\*A + 7\*B)\*c^4\*e^(2\*I\*f\*x + 2\*I\*e) + 2\*(-I\*A + B)\*c^4 +  
6\*(2\*(A + 7\*I\*B)\*c^4\*f\*x + (-I\*A + 7\*B)\*c^4)\*e^(6\*I\*f\*x + 6\*I\*e) + 6\*((I\*A  
- 7\*B)\*c^4\*e^(8\*I\*f\*x + 8\*I\*e) + (I\*A - 7\*B)\*c^4\*e^(6\*I\*f\*x + 6\*I\*e))\*log(e  
^(2\*I\*f\*x + 2\*I\*e) + 1))/(a^3\*f\*e^(8\*I\*f\*x + 8\*I\*e) + a^3\*f\*e^(6\*I\*f\*x + 6  
I\*e))

**Sympy [A]**

time = 0.75, size = 403, normalized size = 2.46

$$-\frac{2Bc^d}{a^3 f e^{2ie} e^{2ifx} + a^3 f} + \begin{cases} \frac{((2iAa^d c^d f^2 e^{8ie} - 2Ba^d c^d f^2 e^{6ie}) e^{-6ifx} + (-3iAa^d c^d f^2 e^{8ie} + 9Ba^d c^d f^2 e^{6ie}) e^{-4ifx} + (6iAa^d c^d f^2 e^{10ie} - 30Ba^d c^d f^2 e^{10ie}) e^{-2ifx}) e^{-12ie}}{6a^3 f^3} & \text{for } a^d f^3 e^{12ie} \neq 0 \\ x\left(-\frac{2Ac^d - 14iBc^d}{a^3} + \frac{(-2Ac^d e^{8ie} + 2Ac^d e^{6ie} - 2Ac^d e^{2ie} + 2Ac^d - 14iBc^d e^{6ie} + 10iBc^d e^{4ie} - 6iBc^d e^{2ie} + 2iBc^d) e^{-6ie}}{a^3}\right) & \text{otherwise} \end{cases} - \frac{ic^d(A+7iB) \log(e^{2ifx} + e^{-2ie})}{a^3 f} + \frac{x(-2Ac^d - 14iBc^d)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*4/(a+I\*a\*tan(f\*x+e))\*\*3,x)

[Out] -2\*B\*c\*\*4/(a\*\*3\*f\*exp(2\*I\*e)\*exp(2\*I\*f\*x) + a\*\*3\*f) + Piecewise((((2\*I\*A\*a\*  
\*6\*c\*\*4\*f\*\*2\*exp(6\*I\*e) - 2\*B\*a\*\*6\*c\*\*4\*f\*\*2\*exp(6\*I\*e))\*exp(-6\*I\*f\*x) + (-  
3\*I\*A\*a\*\*6\*c\*\*4\*f\*\*2\*exp(8\*I\*e) + 9\*B\*a\*\*6\*c\*\*4\*f\*\*2\*exp(8\*I\*e))\*exp(-4\*I\*f

```
*x) + (6*I*A*a**6*c**4*f**2*exp(10*I*e) - 30*B*a**6*c**4*f**2*exp(10*I*e))*
exp(-2*I*f*x))*exp(-12*I*e)/(6*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (
x*(-(-2*A*c**4 - 14*I*B*c**4)/a**3 + (-2*A*c**4*exp(6*I*e) + 2*A*c**4*exp(4
*I*e) - 2*A*c**4*exp(2*I*e) + 2*A*c**4 - 14*I*B*c**4*exp(6*I*e) + 10*I*B*c
**4*exp(4*I*e) - 6*I*B*c**4*exp(2*I*e) + 2*I*B*c**4)*exp(-6*I*e)/a**3), True
)) - I*c**4*(A + 7*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a**3*f) + x*(-2*A
c**4 - 14*I*B*c**4)/a**3
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs.  $2(143) = 286$ .

time = 1.09, size = 429, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/30*(30*(-I*A*c^4 + 7*B*c^4)*log(tan(1/2*f*x + 1/2*e) + 1)/a^3 + 60*(I*A*c
^4 - 7*B*c^4)*log(tan(1/2*f*x + 1/2*e) - 1)/a^3 - 30*(I*A*c^4 - 7*B*c^4)*lo
g(tan(1/2*f*x + 1/2*e) - 1)/a^3 - 30*(-I*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 7*B
*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*I*B*c^4*tan(1/2*f*x + 1/2*e) + I*A*c^4 - 7*
B*c^4)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - (147*I*A*c^4*tan(1/2*f*x + 1/2*
e)^6 - 1029*B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1002*A*c^4*tan(1/2*f*x + 1/2*e)^
5 + 6534*I*B*c^4*tan(1/2*f*x + 1/2*e)^5 - 2445*I*A*c^4*tan(1/2*f*x + 1/2*e)
^4 + 17115*B*c^4*tan(1/2*f*x + 1/2*e)^4 - 3820*A*c^4*tan(1/2*f*x + 1/2*e)^3
- 23860*I*B*c^4*tan(1/2*f*x + 1/2*e)^3 + 2445*I*A*c^4*tan(1/2*f*x + 1/2*e)
^2 - 17115*B*c^4*tan(1/2*f*x + 1/2*e)^2 + 1002*A*c^4*tan(1/2*f*x + 1/2*e) +
6534*I*B*c^4*tan(1/2*f*x + 1/2*e) - 147*I*A*c^4 + 1029*B*c^4)/(a^3*(tan(1/
2*f*x + 1/2*e) - I)^6))/f
```

**Mupad [B]**

time = 11.01, size = 266, normalized size = 1.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^4)/(a + a*tan(e + f*x)*1i
)^3,x)
```

```
[Out] -(c^4*(25*B*tan(e + f*x) - (B*32i)/3 - A*tan(e + f*x)*6i - (8*A)/3 - A*log(
- tan(e + f*x)*1i - 1) - B*log(- tan(e + f*x)*1i - 1)*7i + 6*A*tan(e + f*x)
^2 + B*tan(e + f*x)^2*15i + 3*B*tan(e + f*x)^3 + B*tan(e + f*x)^4*1i + 3*A*
tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1) + A*tan(e + f*x)^3*log(- tan(e +
f*x)*1i - 1)*1i + B*tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1)*21i - 7*B*tan
(e + f*x)^3*log(- tan(e + f*x)*1i - 1) - A*tan(e + f*x)*log(- tan(e + f*x)*
1i - 1)*3i + 21*B*tan(e + f*x)*log(- tan(e + f*x)*1i - 1))*1i)/(a^3*f*(tan(
e + f*x)*1i + 1)^3)
```

$$3.730 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^3}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=135

$$-\frac{iBc^3x}{a^3} + \frac{Bc^3 \log(\cos(e+fx))}{a^3f} - \frac{2Bc^3}{a^3f(i-\tan(e+fx))^2} - \frac{4iBc^3}{a^3f(i-\tan(e+fx))} + \frac{(iA-B)c^3(1-i \tan(e+fx))}{6a^3f(1+i \tan(e+fx))}$$

[Out]  $-I*B*c^3*x/a^3+B*c^3*\ln(\cos(f*x+e))/a^3/f-2*B*c^3/a^3/f/(I-\tan(f*x+e))^2-4*I*B*c^3/a^3/f/(I-\tan(f*x+e))+1/6*(I*A-B)*c^3*(1-I*\tan(f*x+e))^3/a^3/f/(1+I*\tan(f*x+e))^3$

**Rubi [A]**

time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 79, 45}

$$\frac{c^3(-B+iA)(1-i \tan(e+fx))^3}{6a^3f(1+i \tan(e+fx))^3} - \frac{4iBc^3}{a^3f(-\tan(e+fx)+i)} - \frac{2Bc^3}{a^3f(-\tan(e+fx)+i)^2} + \frac{Bc^3 \log(\cos(e+fx))}{a^3f} - \frac{iBc^3x}{a^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^3)/(a + I\*a\*Tan[e + f\*x])^3, x]

[Out]  $((-I)*B*c^3*x)/a^3 + (B*c^3*Log[Cos[e + f*x]])/(a^3*f) - (2*B*c^3)/(a^3*f*(I - Tan[e + f*x])^2) - ((4*I)*B*c^3)/(a^3*f*(I - Tan[e + f*x])) + ((I*A - B)*c^3*(1 - I*Tan[e + f*x])^3)/(6*a^3*f*(1 + I*Tan[e + f*x])^3)$

**Rule 45**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

**Rule 79**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

**Rule 3669**

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^3}{(a + i a \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^2}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{(iA - B)c^3(1 - i \tan(e + fx))^3}{6a^3 f(1 + i \tan(e + fx))^3} - \frac{(iBc) \text{Subst}\left(\int \frac{(c-icx)}{(a+iax)} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{(iA - B)c^3(1 - i \tan(e + fx))^3}{6a^3 f(1 + i \tan(e + fx))^3} - \frac{(iBc) \text{Subst}\left(\int \left(\frac{4ic}{a^3(-i+ \tan(e + fx))}\right) dx, x, \tan(e + fx)\right)}{f} \\ &= -\frac{iBc^3 x}{a^3} + \frac{Bc^3 \log(\cos(e + fx))}{a^3 f} - \frac{2Bc^3}{a^3 f(i - \tan(e + fx))} \end{aligned}$$

**Mathematica [A]**

time = 1.41, size = 145, normalized size = 1.07

$$\frac{c^3 \sec^3(e + fx)(-3iB \cos(e + fx) - \cos(3(e + fx))(A + iB - 6Bfx - 6iB \log(\cos(e + fx))) + 9B \sin(e + fx) + iA \sin(3(e + fx)) - B \sin(3(e + fx)) + 6iBfx \sin(3(e + fx)) - 6B \log(\cos(e + fx)) \sin(3(e + fx)))}{6a^3 f(-i + \tan(e + fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e +
f*x])^3,x]
```

```
[Out] (c^3*Sec[e + f*x]^3*((-3*I)*B*Cos[e + f*x] - Cos[3*(e + f*x)]*(A + I*B - 6*
B*f*x - (6*I)*B*Log[Cos[e + f*x]]) + 9*B*Sin[e + f*x] + I*A*Sin[3*(e + f*x)]
) - B*Sin[3*(e + f*x)] + (6*I)*B*f*x*Sin[3*(e + f*x)] - 6*B*Log[Cos[e + f*x
]]*Sin[3*(e + f*x)])/(6*a^3*f*(-I + Tan[e + f*x])^3)
```

**Maple [A]**

time = 0.28, size = 88, normalized size = 0.65

method	result
derivativedivides	$\frac{c^3 \left( -\frac{-4iA+8B}{2(-i+\tan(fx+e))^2} - \frac{-5iB-A}{-i+\tan(fx+e)} - B \ln(-i+\tan(fx+e)) - \frac{4iB+4A}{3(-i+\tan(fx+e))^3} \right)}{f a^3}$
default	$\frac{c^3 \left( -\frac{-4iA+8B}{2(-i+\tan(fx+e))^2} - \frac{-5iB-A}{-i+\tan(fx+e)} - B \ln(-i+\tan(fx+e)) - \frac{4iB+4A}{3(-i+\tan(fx+e))^3} \right)}{f a^3}$

risch	$-\frac{B c^3 e^{-2i(fx+e)}}{a^3 f} + \frac{B c^3 e^{-4i(fx+e)}}{2a^3 f} - \frac{c^3 e^{-6i(fx+e)} B}{6a^3 f} + \frac{ic^3 e^{-6i(fx+e)} A}{6a^3 f} - \frac{2iB c^3 x}{a^3} - \frac{2iB c^3 e}{a^3 f} + \frac{B c^3 \ln(e^{2i(fx+e)})}{a^3 f}$
norman	$\frac{\frac{(iB c^3 + A c^3) \tan(fx+e)}{af} + \frac{(5iB c^3 + A c^3) (\tan^5(fx+e))}{af} - \frac{ic^3 A + 7B c^3}{3af} - \frac{2(-iB c^3 + 5A c^3) (\tan^3(fx+e))}{3af} - \frac{3(-ic^3 A + 3B c^3) (\tan^3(fx+e))}{af}}{a^2(1+\tan^2(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x,method=_RE  
TURNVERBOSE)`

[Out]  $\frac{1}{f} c^3 / a^3 * (-1/2 * (-4 * I * A + 8 * B) / (-I + \tan(f * x + e))^2 - (-A - 5 * I * B) / (-I + \tan(f * x + e)) - B * \ln(-I + \tan(f * x + e)) - 1/3 * (4 * I * B + 4 * A) / (-I + \tan(f * x + e))^3)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, alg  
orithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 3.16, size = 109, normalized size = 0.81

$$\frac{(-12i B c^3 f x e^{(6i f x + 6i e)} + 6 B c^3 e^{(6i f x + 6i e)} \log(e^{(2i f x + 2i e)} + 1) - 6 B c^3 e^{(4i f x + 4i e)} + 3 B c^3 e^{(2i f x + 2i e)} + (i A - B) c^3) e^{(-6i f x - 6i e)}}{6 a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, alg  
orithm="fricas")`

[Out]  $\frac{1}{6} * (-12 * I * B * c^3 * f * x * e^{(6 * I * f * x + 6 * I * e)} + 6 * B * c^3 * e^{(6 * I * f * x + 6 * I * e)} * \log(e^{(2 * I * f * x + 2 * I * e)} + 1) - 6 * B * c^3 * e^{(4 * I * f * x + 4 * I * e)} + 3 * B * c^3 * e^{(2 * I * f * x + 2 * I * e)} + (I * A - B) * c^3) * e^{(-6 * I * f * x - 6 * I * e)} / (a^3 * f)$

**Sympy** [A]

time = 0.46, size = 258, normalized size = 1.91

$$-\frac{2iBc^3x}{a^3} + \frac{Bc^3 \log(e^{2ifx} + e^{-2ie})}{a^3 f} + \begin{cases} \frac{(-12Ba^6c^3f^2e^{10ie}e^{-2ifx} + 6Ba^6c^3f^2e^{8ie}e^{-4ifx} + (2iAa^6c^3f^2e^{6ie} - 2Ba^6c^3f^2e^{6ie})e^{-6ifx})e^{-12ie}}{12a^9f^3} & \text{for } a^9f^3e^{12ie} \neq 0 \\ x \left( \frac{2iBc^3}{a^3} + \frac{(Ac^3 - 2iBc^3e^{6ie} + 2iBc^3e^{4ie} - 2iBc^3e^{2ie} + iBc^3)e^{-6ie}}{a^3} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**3,x)
[Out] -2*I*B*c**3*x/a**3 + B*c**3*log(exp(2*I*f*x) + exp(-2*I*e))/(a**3*f) + Piecewise((( -12*B*a**6*c**3*f**2*exp(10*I*e)*exp(-2*I*f*x) + 6*B*a**6*c**3*f**2*exp(8*I*e)*exp(-4*I*f*x) + (2*I*A*a**6*c**3*f**2*exp(6*I*e) - 2*B*a**6*c**3*f**2*exp(6*I*e))*exp(-6*I*f*x))*exp(-12*I*e)/(12*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(2*I*B*c**3/a**3 + (A*c**3 - 2*I*B*c**3*exp(6*I*e) + 2*I*B*c**3*exp(4*I*e) - 2*I*B*c**3*exp(2*I*e) + I*B*c**3)*exp(-6*I*e)/a**3), True))
```

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 255 vs.  $2(118) = 236$ .

time = 0.98, size = 255, normalized size = 1.89

$$\frac{30 B^2 \log(\tan(\frac{1}{2} f x + \frac{1}{2} e)) - 60 B^2 \log(\tan(\frac{1}{2} f x + \frac{1}{2} e)) + 30 B c^3 \log(\tan(\frac{1}{2} f x + \frac{1}{2} e)) - 147 B^2 \tan(\frac{1}{2} f x + \frac{1}{2} e)^6 - 90 A c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^5 - 942 B c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^5 - 2445 B c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^4 + 200 A c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^3 + 3620 B c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^3 + 2445 B c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e)^2 - 60 A c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e) - 942 B c^3 \tan(\frac{1}{2} f x + \frac{1}{2} e) - 147 B c^3}{a^3 f (\tan(\frac{1}{2} f x + \frac{1}{2} e) - 1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(30*B*c^3*log(tan(1/2*f*x + 1/2*e) + 1)/a^3 - 60*B*c^3*log(tan(1/2*f*x + 1/2*e) - I)/a^3 + 30*B*c^3*log(tan(1/2*f*x + 1/2*e) - 1)/a^3 + (147*B*c^3*tan(1/2*f*x + 1/2*e)^6 - 60*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 942*I*B*c^3*tan(1/2*f*x + 1/2*e)^5 - 2445*B*c^3*tan(1/2*f*x + 1/2*e)^4 + 200*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 3620*I*B*c^3*tan(1/2*f*x + 1/2*e)^3 + 2445*B*c^3*tan(1/2*f*x + 1/2*e)^2 - 60*A*c^3*tan(1/2*f*x + 1/2*e) - 942*I*B*c^3*tan(1/2*f*x + 1/2*e) - 147*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) - I)^6)/f
```

**Mupad** [B]

time = 8.96, size = 149, normalized size = 1.10

$$\frac{c^3 (18 B \tan(e + f x) - B^7 i - A - B \log(-\tan(e + f x) * 1i - 1) * 3i + 3 A * \tan(e + f x)^2 + B \tan(e + f x)^2 * 15i + B \tan(e + f x)^2 \ln(-1 - \tan(e + f x) * 1i) * 9i - 3 B \tan(e + f x)^3 \ln(-1 - \tan(e + f x) * 1i) + 9 B \tan(e + f x) \ln(-1 - \tan(e + f x) * 1i)) * 1i}{3 a^3 f (1 + \tan(e + f x) * 1i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^3)/(a + a*tan(e + f*x)*1i))^3,x)
```

```
[Out] -(c^3*(18*B*tan(e + f*x) - B*7i - A - B*log(-tan(e + f*x)*1i - 1)*3i + 3*A*tan(e + f*x)^2 + B*tan(e + f*x)^2*15i + B*tan(e + f*x)^2*log(-tan(e + f*x)*1i - 1)*9i - 3*B*tan(e + f*x)^3*log(-tan(e + f*x)*1i - 1) + 9*B*tan(e + f*x)*log(-tan(e + f*x)*1i - 1))*1i)/(3*a^3*f*(tan(e + f*x)*1i + 1)^3)
```

$$3.731 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^2}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=99

$$\frac{2(A+iB)c^2}{3a^3 f(i-\tan(e+fx))^3} + \frac{(iA-3B)c^2}{2a^3 f(i-\tan(e+fx))^2} - \frac{iBc^2}{a^3 f(i-\tan(e+fx))}$$

[Out]  $2/3*(A+I*B)*c^2/a^3/f/(I-\tan(f*x+e))^3+1/2*(I*A-3*B)*c^2/a^3/f/(I-\tan(f*x+e))^2-I*B*c^2/a^3/f/(I-\tan(f*x+e))$

**Rubi [A]**

time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 78}

$$\frac{c^2(-3B+iA)}{2a^3 f(-\tan(e+fx)+i)^2} + \frac{2c^2(A+iB)}{3a^3 f(-\tan(e+fx)+i)^3} - \frac{iBc^2}{a^3 f(-\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^2}{(a+I*a*\text{Tan}[e+f*x])^3}, x]$

[Out]  $(2*(A+I*B)*c^2)/(3*a^3*f*(I-\text{Tan}[e+f*x])^3) + ((I*A-3*B)*c^2)/(2*a^3*f*(I-\text{Tan}[e+f*x])^2) - (I*B*c^2)/(a^3*f*(I-\text{Tan}[e+f*x]))$

Rule 78

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*(c_. + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) || \text{EqQ}[p, 1] || (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] || \text{LeQ}[9*p + 5*(n + 2), 0] || \text{GeQ}[n + p + 1, 0] || (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^2}{(a + i a \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{2(A+iB)c}{a^4(-i+x)^4} + \frac{(-iA+3B)c}{a^4(-i+x)^3} - \frac{iBc}{a^4(-i+x)^2}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{2(A+iB)c^2}{3a^3 f (i - \tan(e + fx))^3} + \frac{(iA - 3B)c^2}{2a^3 f (i - \tan(e + fx))^2} - \frac{iBc^2}{a^3 f (i - \tan(e + fx))}$$

**Mathematica [A]**

time = 1.18, size = 79, normalized size = 0.80

$$\frac{ic^2 \sec^2(e + fx)(\cos(2(e + fx)) - i \sin(2(e + fx)))(-5iA - B + (A - 5iB) \tan(e + fx))}{24a^3 f (-i + \tan(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^2)/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] ((-1/24\*I)\*c^2\*Sec[e + f\*x]^2\*(Cos[2\*(e + f\*x)] - I\*Sin[2\*(e + f\*x)])\*((-5\*I)\*A - B + (A - (5\*I)\*B)\*Tan[e + f\*x]))/(a^3\*f\*(-I + Tan[e + f\*x])^3)

**Maple [A]**

time = 0.32, size = 69, normalized size = 0.70

method	result
derivativedivides	$\frac{c^2 \left( \frac{iB}{-i + \tan(fx+e)} - \frac{2iB+2A}{3(-i + \tan(fx+e))^3} - \frac{-iA+3B}{2(-i + \tan(fx+e))^2} \right)}{f a^3}$
default	$\frac{c^2 \left( \frac{iB}{-i + \tan(fx+e)} - \frac{2iB+2A}{3(-i + \tan(fx+e))^3} - \frac{-iA+3B}{2(-i + \tan(fx+e))^2} \right)}{f a^3}$
risch	$\frac{c^2 e^{-4i(fx+e)} B}{8a^3 f} + \frac{ic^2 e^{-4i(fx+e)} A}{8a^3 f} - \frac{c^2 e^{-6i(fx+e)} B}{12a^3 f} + \frac{ic^2 e^{-6i(fx+e)} A}{12a^3 f}$
norman	$\frac{-\frac{2iA c^2 (\tan^2(fx+e))}{af} + \frac{A c^2 \tan(fx+e)}{af} + \frac{ic^2 B (\tan^5(fx+e))}{af} - \frac{-iA c^2 + B c^2}{6af} - \frac{5(iB c^2 + A c^2) (\tan^3(fx+e))}{3af} - \frac{(-iA c^2 + 5B c^2)}{2af}}{a^2 (1 + \tan^2(fx+e))^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2/(a+I\*a\*tan(f\*x+e))^3,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f\*c^2/a^3\*(I\*B/(-I+tan(f\*x+e))-1/3\*(2\*I\*B+2\*A)/(-I+tan(f\*x+e))^3-1/2\*(-I\*  
A+3\*B)/(-I+tan(f\*x+e))^2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 4.61, size = 51, normalized size = 0.52

$$\frac{(3(-iA - B)c^2e^{2ifx+2ie} + 2(-iA + B)c^2)e^{(-6ifx-6ie)}}{24a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*(3*(-I*A - B)*c^2*e^(2*I*f*x + 2*I*e) + 2*(-I*A + B)*c^2)*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(78) = 156.

time = 0.30, size = 172, normalized size = 1.74

$$\begin{cases} \frac{((8iAa^3c^2fe^{4ie} - 8Ba^3c^2fe^{4ie})e^{-6ifx} + (12iAa^3c^2fe^{6ie} + 12Ba^3c^2fe^{6ie})e^{-4ifx})e^{-10ie}}{96a^6f^2} & \text{for } a^6f^2e^{10ie} \neq 0 \\ \frac{x(Ac^2e^{2ie} + Ac^2 - iBc^2e^{2ie} + iBc^2)e^{-6ie}}{2a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x)
```

```
[Out] Piecewise((((8*I*A*a**3*c**2*f*exp(4*I*e) - 8*B*a**3*c**2*f*exp(4*I*e))*exp(-6*I*f*x) + (12*I*A*a**3*c**2*f*exp(6*I*e) + 12*B*a**3*c**2*f*exp(6*I*e))*exp(-4*I*f*x))*exp(-10*I*e)/(96*a**6*f**2), Ne(a**6*f**2*exp(10*I*e), 0)), (x*(A*c**2*exp(2*I*e) + A*c**2 - I*B*c**2*exp(2*I*e) + I*B*c**2)*exp(-6*I*e)/(2*a**3), True))
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(80) = 160.

time = 0.87, size = 165, normalized size = 1.67

$$\frac{2(3Ac^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 3iAc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 3Bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 8Ac^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 2iBc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 3iAc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3Bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3Ac^2 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{3a^3f(\tan(\frac{1}{2}fx + \frac{1}{2}e) - i)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^2/(a+I\*a\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$\frac{-2/3*(3*A*c^2*\tan(1/2*f*x + 1/2*e)^5 - 3*I*A*c^2*\tan(1/2*f*x + 1/2*e)^4 - 3*B*c^2*\tan(1/2*f*x + 1/2*e)^4 - 8*A*c^2*\tan(1/2*f*x + 1/2*e)^3 - 2*I*B*c^2*\tan(1/2*f*x + 1/2*e)^3 + 3*I*A*c^2*\tan(1/2*f*x + 1/2*e)^2 + 3*B*c^2*\tan(1/2*f*x + 1/2*e)^2 + 3*A*c^2*\tan(1/2*f*x + 1/2*e))}{(a^3*f*(\tan(1/2*f*x + 1/2*e) - I)^6)}$$

**Mupad [B]**

time = 8.93, size = 87, normalized size = 0.88

$$\frac{\frac{c^2(-B+Ai)}{6} + \frac{c^2 \tan(e+fx)(3A-B3i)}{6} + Bc^2 \tan(e+fx)^2}{a^3 f (-\tan(e+fx)^3 i - 3 \tan(e+fx)^2 + \tan(e+fx) 3i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^2)/(a + a\*tan(e + f\*x)\*1i)^3,x)

[Out] 
$$\frac{(c^2*(A*1i - B))/6 + (c^2*\tan(e + f*x)*(3A - B*3i))/6 + B*c^2*\tan(e + f*x)^2}{(a^3*f*(\tan(e + f*x)*3i - 3*\tan(e + f*x)^2 - \tan(e + f*x)^3*1i + 1))}$$

$$3.732 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))}{(a+ia \tan(e+fx))^3} dx$$

Optimal. Leaf size=59

$$\frac{(A+iB)c}{3a^3 f(i-\tan(e+fx))^3} - \frac{Bc}{2a^3 f(i-\tan(e+fx))^2}$$

[Out] 1/3\*(A+I\*B)\*c/a^3/f/(I-tan(f\*x+e))^3-1/2\*B\*c/a^3/f/(I-tan(f\*x+e))^2

Rubi [A]

time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3669, 45}

$$\frac{c(A+iB)}{3a^3 f(-\tan(e+fx)+i)^3} - \frac{Bc}{2a^3 f(-\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x]))/(a + I\*a\*Tan[e + f\*x])^3, x]

[Out] ((A + I\*B)\*c)/(3\*a^3\*f\*(I - Tan[e + f\*x])^3) - (B\*c)/(2\*a^3\*f\*(I - Tan[e + f\*x])^2)

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))}{(a + i a \tan(e + fx))^3} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{A+iB}{a^4(-i+x)^4} + \frac{B}{a^4(-i+x)^3}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(A + iB)c}{3a^3 f (i - \tan(e + fx))^3} - \frac{Bc}{2a^3 f (i - \tan(e + fx))^2}$$

**Mathematica [A]**

time = 0.60, size = 81, normalized size = 1.37

$$\frac{c \sec^2(e + fx) (3iA + 2(2iA + B) \cos(2(e + fx)) - 2(A - 2iB) \sin(2(e + fx))) (i + \tan(e + fx))}{24a^3 f (-i + \tan(e + fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (c*Sec[e + f*x]^2*((3*I)*A + 2*((2*I)*A + B)*Cos[2*(e + f*x)] - 2*(A - (2*I)*B)*Sin[2*(e + f*x)])*(I + Tan[e + f*x])/(24*a^3*f*(-I + Tan[e + f*x])^3)
```

**Maple [A]**

time = 0.22, size = 43, normalized size = 0.73

method	result	size
derivativedivides	$c \left( -\frac{iB+A}{3(-i+\tan(fx+e))^3} - \frac{B}{2(-i+\tan(fx+e))^2} \right) \frac{1}{f a^3}$	43
default	$c \left( -\frac{iB+A}{3(-i+\tan(fx+e))^3} - \frac{B}{2(-i+\tan(fx+e))^2} \right) \frac{1}{f a^3}$	43
risch	$\frac{c e^{-2i(fx+e)} B}{8a^3 f} + \frac{i c e^{-2i(fx+e)} A}{8a^3 f} + \frac{i A c e^{-4i(fx+e)}}{8a^3 f} - \frac{c e^{-6i(fx+e)} B}{24a^3 f} + \frac{i c e^{-6i(fx+e)} A}{24a^3 f}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*c/a^3*(-1/3*(A+I*B)/(-I+tan(f*x+e))^3-1/2*B/(-I+tan(f*x+e))^2)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 5.19, size = 63, normalized size = 1.07

$$\frac{(3(-iA - B)ce^{(4ifx+4ie)} - 3iAce^{(2ifx+2ie)} - (iA - B)c)e^{(-6ifx-6ie)}}{24a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")`

[Out]  $-1/24*(3*(-I*A - B)*c*e^{(4*I*f*x + 4*I*e)} - 3*I*A*c*e^{(2*I*f*x + 2*I*e)} - (I*A - B)*c)*e^{(-6*I*f*x - 6*I*e)}/(a^3*f)$

**Sympy** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(44) = 88.

time = 0.27, size = 206, normalized size = 3.49

$$\begin{cases} \frac{(192iAa^6cf^2e^{8ie}e^{-4ifx} + (64iAa^6cf^2e^{6ie} - 64Ba^6cf^2e^{6ie})e^{-6ifx} + (192iAa^6cf^2e^{10ie} + 192Ba^6cf^2e^{10ie})e^{-2ifx})e^{-12ie}}{1536a^9f^3} & \text{for } a^9f^3e^{12ie} \neq 0 \\ \frac{x(Ace^{4ie} + 2Ace^{2ie} + Ac - iBce^{4ie} + iBc)e^{-6ie}}{4a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))**3,x)`

[Out] `Piecewise(((192*I*A*a**6*c*f**2*exp(8*I*e)*exp(-4*I*f*x) + (64*I*A*a**6*c*f**2*exp(6*I*e) - 64*B*a**6*c*f**2*exp(6*I*e))*exp(-6*I*f*x) + (192*I*A*a**6*c*f**2*exp(10*I*e) + 192*B*a**6*c*f**2*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(1536*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(A*c*exp(4*I*e) + 2*A*c*exp(2*I*e) + A*c - I*B*c*exp(4*I*e) + I*B*c)*exp(-6*I*e)/(4*a**3), True))`

**Giac** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(47) = 94.

time = 0.77, size = 149, normalized size = 2.53

$$\frac{2(3A\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^5 - 6iA\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^4 - 3B\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^4 - 10A\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^3 + 2iB\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^3 + 6iA\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3B\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3A\operatorname{Actan}(\frac{1}{2}fx + \frac{1}{2}e))}{3a^3f(\tan(\frac{1}{2}fx + \frac{1}{2}e) - i)^6}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$\frac{-2/3*(3*A*c*\tan(1/2*f*x + 1/2*e)^5 - 6*I*A*c*\tan(1/2*f*x + 1/2*e)^4 - 3*B*c*\tan(1/2*f*x + 1/2*e)^4 - 10*A*c*\tan(1/2*f*x + 1/2*e)^3 + 2*I*B*c*\tan(1/2*f*x + 1/2*e)^3 + 6*I*A*c*\tan(1/2*f*x + 1/2*e)^2 + 3*B*c*\tan(1/2*f*x + 1/2*e)^2 + 3*A*c*\tan(1/2*f*x + 1/2*e))/(a^3*f*(\tan(1/2*f*x + 1/2*e) - I)^6}$$

**Mupad [B]**

time = 8.84, size = 62, normalized size = 1.05

$$\frac{\frac{c(B+A2i)}{6} + \frac{Bc\tan(e+fx)1i}{2}}{a^3 f (-\tan(e+fx)^3 1i - 3\tan(e+fx)^2 + \tan(e+fx) 3i + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i))/(a + a\*tan(e + f\*x)\*1i))^3,x)

[Out] 
$$((c*(A*2i + B))/6 + (B*c*\tan(e + f*x)*1i)/2)/(a^3*f*(\tan(e + f*x)*3i - 3*\tan(e + f*x)^2 - \tan(e + f*x)^3*1i + 1))$$

$$3.733 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3} dx$$

Optimal. Leaf size=112

$$\frac{(A-iB)x}{8a^3} + \frac{iA-B}{6f(a+ia \tan(e+fx))^3} + \frac{iA+B}{8af(a+ia \tan(e+fx))^2} + \frac{iA+B}{8f(a^3+ia^3 \tan(e+fx))}$$

[Out] 1/8\*(A-I\*B)\*x/a^3+1/6\*(I\*A-B)/f/(a+I\*a\*tan(f\*x+e))^3+1/8\*(I\*A+B)/a/f/(a+I\*a\*tan(f\*x+e))^2+1/8\*(I\*A+B)/f/(a^3+I\*a^3\*tan(f\*x+e))

Rubi [A]

time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {3607, 3560, 8}

$$\frac{B+iA}{8f(a^3+ia^3 \tan(e+fx))} + \frac{x(A-iB)}{8a^3} + \frac{-B+iA}{6f(a+ia \tan(e+fx))^3} + \frac{B+iA}{8af(a+ia \tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] ((A - I\*B)\*x)/(8\*a^3) + (I\*A - B)/(6\*f\*(a + I\*a\*Tan[e + f\*x])^3) + (I\*A + B)/(8\*a\*f\*(a + I\*a\*Tan[e + f\*x])^2) + (I\*A + B)/(8\*f\*(a^3 + I\*a^3\*Tan[e + f\*x]))

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[a\*((a + b\*Tan[c + d\*x])^n/(2\*b\*d\*n)), x] + Dist[1/(2\*a), Int[(a + b\*Tan[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3607

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3} dx &= \frac{iA - B}{6f(a + ia \tan(e + fx))^3} + \frac{(A - iB) \int \frac{1}{(a + ia \tan(e + fx))^2} dx}{2a} \\
&= \frac{iA - B}{6f(a + ia \tan(e + fx))^3} + \frac{iA + B}{8af(a + ia \tan(e + fx))^2} + \frac{(A - iB) \int \frac{1}{a + ia \tan(e + fx)}}{4a^2} \\
&= \frac{iA - B}{6f(a + ia \tan(e + fx))^3} + \frac{iA + B}{8af(a + ia \tan(e + fx))^2} + \frac{iA + B}{8f(a^3 + ia^3 \tan(e + fx))} \\
&= \frac{(A - iB)x}{8a^3} + \frac{iA - B}{6f(a + ia \tan(e + fx))^3} + \frac{iA + B}{8af(a + ia \tan(e + fx))^2} + \frac{iA + B}{8f(a^3 + ia^3 \tan(e + fx))}
\end{aligned}$$

**Mathematica [A]**

time = 0.47, size = 150, normalized size = 1.34

$$\frac{\sec^3(e + fx)((-27A + 3iB) \cos(e + fx) + 2(-A - iB + 6iAfx + 6Bfx) \cos(3(e + fx)) - 9iA \sin(e + fx) - 9B \sin(e + fx) + 2iA \sin(3(e + fx)) - 2B \sin(3(e + fx)) - 12Afx \sin(3(e + fx)) + 12iBfx \sin(3(e + fx)))}{96a^3 f(-i + \tan(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] (Sec[e + f\*x]^3\*((-27\*A + (3\*I)\*B)\*Cos[e + f\*x] + 2\*(-A - I\*B + (6\*I)\*A\*f\*x + 6\*B\*f\*x)\*Cos[3\*(e + f\*x)] - (9\*I)\*A\*Sin[e + f\*x] - 9\*B\*Sin[e + f\*x] + (2\*I)\*A\*Sin[3\*(e + f\*x)] - 2\*B\*Sin[3\*(e + f\*x)] - 12\*A\*f\*x\*Sin[3\*(e + f\*x)] + (12\*I)\*B\*f\*x\*Sin[3\*(e + f\*x)]))/(96\*a^3\*f\*(-I + Tan[e + f\*x])^3)

**Maple [A]**

time = 0.22, size = 110, normalized size = 0.98

method	result
derivativedivides	$\frac{\left(-\frac{iA}{16} - \frac{B}{16}\right) \ln(-i + \tan(fx + e)) - \frac{-\frac{A}{8} + \frac{iB}{8}}{-i + \tan(fx + e)} - \frac{\frac{iA}{4} + \frac{B}{4}}{2(-i + \tan(fx + e))^2} - \frac{\frac{A}{2} + \frac{iB}{2}}{3(-i + \tan(fx + e))^3} + \frac{i(-iB + A) \ln(i + \tan(fx + e))}{16}}{f a^3}$
default	$\frac{\left(-\frac{iA}{16} - \frac{B}{16}\right) \ln(-i + \tan(fx + e)) - \frac{-\frac{A}{8} + \frac{iB}{8}}{-i + \tan(fx + e)} - \frac{\frac{iA}{4} + \frac{B}{4}}{2(-i + \tan(fx + e))^2} - \frac{\frac{A}{2} + \frac{iB}{2}}{3(-i + \tan(fx + e))^3} + \frac{i(-iB + A) \ln(i + \tan(fx + e))}{16}}{f a^3}$
risch	$-\frac{i x B}{8 a^3} + \frac{x A}{8 a^3} + \frac{e^{-2i(fx+e)} B}{16 a^3 f} + \frac{3i e^{-2i(fx+e)} A}{16 a^3 f} - \frac{e^{-4i(fx+e)} B}{32 a^3 f} + \frac{3i e^{-4i(fx+e)} A}{32 a^3 f} - \frac{e^{-6i(fx+e)} B}{48 a^3 f} + \frac{i e^{-6i(fx+e)} A}{48 a^3 f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3,x,method=\_RETURNVERBOSE)

[Out] 1/f/a^3\*((-1/16\*I\*A-1/16\*B)\*ln(-I+tan(f\*x+e))-(-1/8\*A+1/8\*I\*B)/(-I+tan(f\*x+e))-1/2\*(1/4\*I\*A+1/4\*B)/(-I+tan(f\*x+e))^2-1/3\*(1/2\*A+1/2\*I\*B)/(-I+tan(f\*x+e))^3+1/16\*I\*(A-I\*B)\*ln(I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 6.15, size = 80, normalized size = 0.71

$$\frac{(12(A - iB)fxe^{(6i fx + 6i e)} - 6(-3iA - B)e^{(4i fx + 4i e)} - 3(-3iA + B)e^{(2i fx + 2i e)} + 2iA - 2B)e^{(-6i fx - 6i e)}}{96a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*(A - I*B)*f*x*e^(6*I*f*x + 6*I*e) - 6*(-3*I*A - B)*e^(4*I*f*x + 4*I*e) - 3*(-3*I*A + B)*e^(2*I*f*x + 2*I*e) + 2*I*A - 2*B)*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

**Sympy [A]**

time = 0.27, size = 258, normalized size = 2.30

$$\begin{cases} \frac{((512iAa^6f^2e^{6ie} - 512Ba^6f^2e^{6ie})e^{-6ifx} + (2304iAa^6f^2e^{8ie} - 768Ba^6f^2e^{8ie})e^{-4ifx} + (4608iAa^6f^2e^{10ie} + 1536Ba^6f^2e^{10ie})e^{-2ifx})e^{-12ie}}{24576a^9f^3} & \text{for } a^9f^3e^{12ie} \neq 0 \\ x\left(-\frac{A-iB}{8a^3} + \frac{(Ae^{6ie} + 3Ae^{4ie} + 3Ae^{2ie} + A - iBe^{6ie} - iBe^{4ie} + iBe^{2ie} + iB)e^{-6ie}}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x(A - iB)}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3,x)
```

```
[Out] Piecewise((((512*I*A*a**6*f**2*exp(6*I*e) - 512*B*a**6*f**2*exp(6*I*e))*exp(-6*I*f*x) + (2304*I*A*a**6*f**2*exp(8*I*e) - 768*B*a**6*f**2*exp(8*I*e))*exp(-4*I*f*x) + (4608*I*A*a**6*f**2*exp(10*I*e) + 1536*B*a**6*f**2*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(24576*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(-(A - I*B)/(8*a**3) + (A*exp(6*I*e) + 3*A*exp(4*I*e) + 3*A*exp(2*I*e) + A - I*B*exp(6*I*e) - I*B*exp(4*I*e) + I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(8*a**3)), True)) + x*(A - I*B)/(8*a**3)
```

**Giac [A]**

time = 0.70, size = 140, normalized size = 1.25

$$\frac{6(iA+B)\log(\tan(fx+e)-i)}{a^3} + \frac{6(-iA-B)\log(i\tan(fx+e)-1)}{a^3} + \frac{-11iA\tan(fx+e)^3 - 11B\tan(fx+e)^3 - 45A\tan(fx+e)^2 + 45iB\tan(fx+e)^2 + 69iA\tan(fx+e) + 69B\tan(fx+e) + 51A - 19iB}{a^3(\tan(fx+e)-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3,x, algorithm="giac")

[Out] 
$$-1/96*(6*(I*A + B)*\log(\tan(f*x + e) - I)/a^3 + 6*(-I*A - B)*\log(I*\tan(f*x + e) - 1)/a^3 + (-11*I*A*\tan(f*x + e)^3 - 11*B*\tan(f*x + e)^3 - 45*A*\tan(f*x + e)^2 + 45*I*B*\tan(f*x + e)^2 + 69*I*A*\tan(f*x + e) + 69*B*\tan(f*x + e) + 51*A - 19*I*B)/(a^3*(\tan(f*x + e) - I)^3))/f$$

**Mupad [B]**

time = 9.13, size = 111, normalized size = 0.99

$$-\frac{\tan(e + f x)^2 \left( \frac{B}{8a^3} + \frac{A1i}{8a^3} \right) - \frac{A5i}{12a^3} - \frac{B}{12a^3} + \tan(e + f x) \left( \frac{3A}{8a^3} - \frac{B3i}{8a^3} \right)}{f \left( -\tan(e + f x)^3 1i - 3 \tan(e + f x)^2 + \tan(e + f x) 3i + 1 \right)} - \frac{x (B + A 1i) 1i}{8 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/(a + a\*tan(e + f\*x)\*1i)^3,x)

[Out] 
$$-(\tan(e + f*x)^2*((A*1i)/(8*a^3) + B/(8*a^3)) - (A*5i)/(12*a^3) - B/(12*a^3) + \tan(e + f*x)*((3*A)/(8*a^3) - (B*3i)/(8*a^3)))/(f*(\tan(e + f*x)*3i - 3*\tan(e + f*x)^2 - \tan(e + f*x)^3*1i + 1)) - (x*(A*1i + B)*1i)/(8*a^3)$$

$$3.734 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))} dx$$

**Optimal.** Leaf size=153

$$\frac{(2A-iB)x}{8a^3c} + \frac{A+iB}{12a^3cf(i-\tan(e+fx))^3} - \frac{iA}{8a^3cf(i-\tan(e+fx))^2} - \frac{3A-iB}{16a^3cf(i-\tan(e+fx))} + \frac{A-iB}{16a^3cf(i+\tan(e+fx))}$$

[Out] 1/8\*(2\*A-I\*B)\*x/a^3/c+1/12\*(A+I\*B)/a^3/c/f/(I-tan(f\*x+e))^3-1/8\*I\*A/a^3/c/f/(I-tan(f\*x+e))^2+1/16\*(-3\*A+I\*B)/a^3/c/f/(I-tan(f\*x+e))+1/16\*(A-I\*B)/a^3/c/f/(I+tan(f\*x+e))

**Rubi [A]**

time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{3A-iB}{16a^3cf(-\tan(e+fx)+i)} + \frac{A-iB}{16a^3cf(\tan(e+fx)+i)} + \frac{A+iB}{12a^3cf(-\tan(e+fx)+i)^3} + \frac{x(2A-iB)}{8a^3c} - \frac{iA}{8a^3cf(-\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])), x]

[Out] ((2\*A - I\*B)\*x)/(8\*a^3\*c) + (A + I\*B)/(12\*a^3\*c\*f\*(I - Tan[e + f\*x])^3) - ((I/8)\*A)/(a^3\*c\*f\*(I - Tan[e + f\*x])^2) - (3\*A - I\*B)/(16\*a^3\*c\*f\*(I - Tan[e + f\*x])) + (A - I\*B)/(16\*a^3\*c\*f\*(I + Tan[e + f\*x]))

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

**Rule 3669**

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x,

Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ic \tan(e + fx))} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{A+iB}{4a^4 c^2 (-i+x)^4} + \frac{iA}{4a^4 c^2 (-i+x)^3} + \frac{-3A+iB}{16a^4 c^2 (-i+x)^2} + \frac{A+iB}{12a^3 c f (i - \tan(e + fx))^3} - \frac{iA}{8a^3 c f (i - \tan(e + fx))^2} - \frac{(2A - iB)x}{8a^3 c} + \frac{A+iB}{12a^3 c f (i - \tan(e + fx))^3} - \frac{iA}{8a^3 c f (i - \tan(e + fx))^2}\right) dx, x, \tan(e + fx)\right)}{f}$$

Mathematica [A]

time = 0.98, size = 164, normalized size = 1.07

$$\frac{\sec^2(e + fx)(18iA + 3(B(-1 - 4ifx) + A(2i + 8fx)) \cos(2(e + fx)) + (-2iA - 4B) \cos(4(e + fx)) + 6A \sin(2(e + fx)) + 3iB \sin(2(e + fx)) + 24iAfx \sin(2(e + fx)) + 12Bfx \sin(2(e + fx)) + 4A \sin(4(e + fx)) - 2iB \sin(4(e + fx)))}{96a^3 c f (-i + \tan(e + fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])), x]

[Out] -1/96\*(Sec[e + f\*x]^2\*((18\*I)\*A + 3\*(B\*(-1 - (4\*I)\*f\*x) + A\*(2\*I + 8\*f\*x))\*Cos[2\*(e + f\*x)] + ((-2\*I)\*A - 4\*B)\*Cos[4\*(e + f\*x)] + 6\*A\*Sin[2\*(e + f\*x)] + (3\*I)\*B\*Sin[2\*(e + f\*x)] + (24\*I)\*A\*f\*x\*Sin[2\*(e + f\*x)] + 12\*B\*f\*x\*Sin[2\*(e + f\*x)] + 4\*A\*Sin[4\*(e + f\*x)] - (2\*I)\*B\*Sin[4\*(e + f\*x)]))/(a^3\*c\*f\*(-I + Tan[e + f\*x])^2)

Maple [A]

time = 0.31, size = 128, normalized size = 0.84

method	result
derivativedivides	$\frac{-\frac{iA}{8(-i+\tan(fx+e))^2} - \frac{-\frac{3A+iB}{16} + \frac{iB}{16}}{-i+\tan(fx+e)} - \frac{\frac{A+iB}{4} + \frac{iB}{4}}{3(-i+\tan(fx+e))^3} + \left(-\frac{B}{16} - \frac{iA}{8}\right) \ln(-i+\tan(fx+e)) - \frac{-\frac{A}{16} + \frac{iB}{16}}{i+\tan(fx+e)} + \left(\frac{B}{16} + \frac{iA}{8}\right) \ln(i+\tan(fx+e))}{f a^3 c}$
default	$\frac{-\frac{iA}{8(-i+\tan(fx+e))^2} - \frac{-\frac{3A+iB}{16} + \frac{iB}{16}}{-i+\tan(fx+e)} - \frac{\frac{A+iB}{4} + \frac{iB}{4}}{3(-i+\tan(fx+e))^3} + \left(-\frac{B}{16} - \frac{iA}{8}\right) \ln(-i+\tan(fx+e)) - \frac{-\frac{A}{16} + \frac{iB}{16}}{i+\tan(fx+e)} + \left(\frac{B}{16} + \frac{iA}{8}\right) \ln(i+\tan(fx+e))}{f a^3 c}$
risch	$-\frac{ixB}{8a^3c} + \frac{xA}{4a^3c} - \frac{e^{-4i(fx+e)}B}{32a^3cf} + \frac{ie^{-4i(fx+e)}A}{16a^3cf} - \frac{e^{-6i(fx+e)}B}{96a^3cf} + \frac{ie^{-6i(fx+e)}A}{96a^3cf} - \frac{\cos(2fx+2e)B}{32a^3cf} + \frac{5i \cos(2fx+2e)A}{96a^3cf}$

norman	$\frac{\frac{(-iB+2A)x}{8ac} - \frac{-4iA+B}{12acf} + \frac{B(\tan^2(fx+e))}{4acf} + \frac{(-iB+2A)(\tan^3(fx+e))}{3acf} + \frac{(-iB+2A)(\tan^5(fx+e))}{8acf} + \frac{3(-iB+2A)x(\tan^2(fx+e))}{8ac} + \frac{3(-iB+2A)(\tan^4(fx+e))}{8ac}}{(1+\tan^2(fx+e))^3 a^2}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e)),x,method=\_RETU  
RNVERBOSE)

[Out] 1/f/a^3/c\*(-1/8\*I\*A/(-I+tan(f\*x+e))^2-(-3/16\*A+1/16\*I\*B)/(-I+tan(f\*x+e))-1/  
3\*(1/4\*A+1/4\*I\*B)/(-I+tan(f\*x+e))^3+(-1/16\*B-1/8\*I\*A)\*ln(-I+tan(f\*x+e))-(-1  
/16\*A+1/16\*I\*B)/(I+tan(f\*x+e))+(1/16\*B+1/8\*I\*A)\*ln(I+tan(f\*x+e)))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e)),x, algor  
ithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 6.83, size = 96, normalized size = 0.63

$$\frac{(12(2A-iB)fxe^{(6i fx+6ie)} - 3(iA+B)e^{(8i fx+8ie)} + 18iAe^{(4i fx+4ie)} - 3(-2iA+B)e^{(2i fx+2ie)} + iA-B)e^{(-6i fx-6ie)}}{96a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e)),x, algor  
ithm="fricas")

[Out] 1/96\*(12\*(2\*A - I\*B)\*f\*x\*e^(6\*I\*f\*x + 6\*I\*e) - 3\*(I\*A + B)\*e^(8\*I\*f\*x + 8\*I  
\*e) + 18\*I\*A\*e^(4\*I\*f\*x + 4\*I\*e) - 3\*(-2\*I\*A + B)\*e^(2\*I\*f\*x + 2\*I\*e) + I\*A  
- B)\*e^(-6\*I\*f\*x - 6\*I\*e)/(a^3\*c\*f)

**Sympy [A]**

time = 0.36, size = 340, normalized size = 2.22

$$\left\{ \begin{array}{ll} \frac{(294912iAa^9c^3f^3e^{10ie}-2ifx+(16384iAa^9c^3f^3e^{6ie}-16384Ba^9c^3f^3e^{6ie})e^{-6ifx}+(98304iAa^9c^3f^3e^{8ie}-49152Ba^9c^3f^3e^{8ie})e^{-4ifx}+(-49152iAa^9c^3f^3e^{4ie}-49152Ba^9c^3f^3e^{4ie})e^{2ifx})e^{-12ie}}{1572864a^{12}c^4f^4} & \text{for } a^{12}c^4f^4e^{12ie} \neq 0 \\ x\left(-\frac{2A-iB}{8a^3c} + \frac{(Ae^{8ie}+4Ae^{6ie}+6Ae^{4ie}+4Ae^{2ie}+A-iB)e^{8ie}-2iBe^{6ie}+2iBe^{2ie}+iB}{16a^3c}\right)e^{-6ie} & \text{otherwise} \end{array} \right. + \frac{x(2A-iB)}{8a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e)),x)



```
[Out] Piecewise(((294912*I*A*a**9*c**3*f**3*exp(10*I*e)*exp(-2*I*f*x) + (16384*I*
A*a**9*c**3*f**3*exp(6*I*e) - 16384*B*a**9*c**3*f**3*exp(6*I*e))*exp(-6*I*f
*x) + (98304*I*A*a**9*c**3*f**3*exp(8*I*e) - 49152*B*a**9*c**3*f**3*exp(8*I
*e))*exp(-4*I*f*x) + (-49152*I*A*a**9*c**3*f**3*exp(14*I*e) - 49152*B*a**9*
c**3*f**3*exp(14*I*e))*exp(2*I*f*x))*exp(-12*I*e)/(1572864*a**12*c**4*f**4)
, Ne(a**12*c**4*f**4*exp(12*I*e), 0)), (x*(-(2*A - I*B)/(8*a**3*c) + (A*exp
(8*I*e) + 4*A*exp(6*I*e) + 6*A*exp(4*I*e) + 4*A*exp(2*I*e) + A - I*B*exp(8*
I*e) - 2*I*B*exp(6*I*e) + 2*I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(16*a**3*c)),
True)) + x*(2*A - I*B)/(8*a**3*c)
```

**Giac** [A]

time = 0.78, size = 192, normalized size = 1.25

$$\frac{\frac{6(-2iA-B)\log(\tan(fx+e)+i)}{a^3c} + \frac{6(2iA+B)\log(\tan(fx+e)-i)}{a^3c} + \frac{6(2iA\tan(fx+e)+B\tan(fx+e)-3A+2iB)}{a^3c(\tan(fx+e)+i)} + \frac{-22iA\tan(fx+e)^3-11B\tan(fx+e)^3-84A\tan(fx+e)^2+39iB\tan(fx+e)^2+114iA\tan(fx+e)+45B\tan(fx+e)+60A-9iB}{a^3c(\tan(fx+e)-i)^3}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -1/96*(6*(-2*I*A - B)*log(tan(f*x + e) + I)/(a^3*c) + 6*(2*I*A + B)*log(tan
(f*x + e) - I)/(a^3*c) + 6*(2*I*A*tan(f*x + e) + B*tan(f*x + e) - 3*A + 2*I
*B)/(a^3*c*(tan(f*x + e) + I)) + (-22*I*A*tan(f*x + e)^3 - 11*B*tan(f*x + e
)^3 - 84*A*tan(f*x + e)^2 + 39*I*B*tan(f*x + e)^2 + 114*I*A*tan(f*x + e) +
45*B*tan(f*x + e) + 60*A - 9*I*B)/(a^3*c*(tan(f*x + e) - I)^3))/f
```

**Mupad** [B]

time = 9.06, size = 161, normalized size = 1.05

$$\frac{\frac{A}{3a^3c} + \tan(e+fx)^2\left(\frac{A}{2a^3c} - \frac{B1i}{4a^3c}\right) + \tan(e+fx)^3\left(\frac{B}{8a^3c} + \frac{A1i}{4a^3c}\right) - \tan(e+fx)\left(\frac{B}{24a^3c} + \frac{A1i}{12a^3c}\right) + \frac{B1i}{12a^3c}}{f(\tan(e+fx)^4 1i + 2\tan(e+fx)^3 + 2\tan(e+fx) - i)} - \frac{x(B+A2i) 1i}{8a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
),x)
```

```
[Out] (tan(e + f*x)^2*(A/(2*a^3*c) - (B*1i)/(4*a^3*c)) - tan(e + f*x)*((A*1i)/(12
*a^3*c) + B/(24*a^3*c)) + tan(e + f*x)^3*((A*1i)/(4*a^3*c) + B/(8*a^3*c)) +
A/(3*a^3*c) + (B*1i)/(12*a^3*c))/(f*(2*tan(e + f*x) + 2*tan(e + f*x)^3 + t
an(e + f*x)^4*1i - 1i)) - (x*(A*2i + B)*1i)/(8*a^3*c)
```

$$3.735 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=185

$$\frac{(5A-iB)x}{16a^3c^2} + \frac{A+iB}{24a^3c^2f(i-\tan(e+fx))^3} - \frac{3iA-B}{32a^3c^2f(i-\tan(e+fx))^2} - \frac{3A}{16a^3c^2f(i-\tan(e+fx))} + \frac{3A}{32a^3c^2f(i-\tan(e+fx))} + \frac{3A}{16a^3c^2f(i-\tan(e+fx))} + \frac{3A}{32a^3c^2f(i-\tan(e+fx))}$$

[Out] 1/16\*(5\*A-I\*B)\*x/a^3/c^2+1/24\*(A+I\*B)/a^3/c^2/f/(I-tan(f\*x+e))^3+1/32\*(-3\*I\*A+B)/a^3/c^2/f/(I-tan(f\*x+e))^2-3/16\*A/a^3/c^2/f/(I-tan(f\*x+e))+1/32\*(I\*A+B)/a^3/c^2/f/(I+tan(f\*x+e))^2+1/16\*(2\*A-I\*B)/a^3/c^2/f/(I+tan(f\*x+e))

**Rubi [A]**

time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$\frac{2A-iB}{16a^3c^2f(\tan(e+fx)+i)} - \frac{-B+3iA}{32a^3c^2f(-\tan(e+fx)+i)^2} + \frac{B+iA}{32a^3c^2f(\tan(e+fx)+i)^2} + \frac{A+iB}{24a^3c^2f(-\tan(e+fx)+i)^3} + \frac{x(5A-iB)}{16a^3c^2} - \frac{3A}{16a^3c^2f(-\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^2), x]

[Out] ((5\*A - I\*B)\*x)/(16\*a^3\*c^2) + (A + I\*B)/(24\*a^3\*c^2\*f\*(I - Tan[e + f\*x])^3) - ((3\*I)\*A - B)/(32\*a^3\*c^2\*f\*(I - Tan[e + f\*x])^2) - (3\*A)/(16\*a^3\*c^2\*f\*(I - Tan[e + f\*x])) + (I\*A + B)/(32\*a^3\*c^2\*f\*(I + Tan[e + f\*x])^2) + (2\*A - I\*B)/(16\*a^3\*c^2\*f\*(I + Tan[e + f\*x]))

Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Di

st[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ictan(e + fx))^2} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^3} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{A+iB}{8a^4 c^3 (-i+x)^4} + \frac{i(3A+iB)}{16a^4 c^3 (-i+x)^3} - \frac{3A}{16a^4 c^3 (-i+x)^2}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{A + iB}{24a^3 c^2 f (i - \tan(e + fx))^3} - \frac{3iA - B}{32a^3 c^2 f (i - \tan(e + fx))}$$

$$= \frac{(5A - iB)x}{16a^3 c^2} + \frac{A + iB}{24a^3 c^2 f (i - \tan(e + fx))^3} - \frac{3iA - B}{32a^3 c^2 f (i - \tan(e + fx))}$$

**Mathematica [A]**

time = 1.13, size = 217, normalized size = 1.17

$\frac{\sec^2(e + fx)(\cos(2e + fx) + i \sin(2e + fx))(12(A - 5 + 10fx + B(-1 + 2fx))\cos(e + fx) + 3(5A - 9B)\cos(3e + fx) + A\cos(5e + fx) - 5iB\cos(5e + fx) + 60A\sin(e + fx) - 12B\sin(e + fx) - 120Afx\sin(e + fx) + 24Bfx\sin(e + fx) + 45A\sin(3e + fx) + 9B\sin(3e + fx) + 5A\sin(5e + fx) + B\sin(5e + fx))}{384a^3 c^2 f (-1 + \tan(e + fx))^3}$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^2), x]

[Out] (Sec[e + f\*x]^3\*(Cos[2\*(e + f\*x)] + I\*Sin[2\*(e + f\*x)])\*(12\*(A\*(-5 + (10\*I)\*f\*x) + B\*(-I + 2\*f\*x))\*Cos[e + f\*x] + 3\*(5\*A - (9\*I)\*B)\*Cos[3\*(e + f\*x)] + A\*Cos[5\*(e + f\*x)] - (5\*I)\*B\*Cos[5\*(e + f\*x)] + (60\*I)\*A\*Sin[e + f\*x] - 12\*B\*Sin[e + f\*x] - 120\*A\*f\*x\*Sin[e + f\*x] + (24\*I)\*B\*f\*x\*Sin[e + f\*x] + (45\*I)\*A\*Sin[3\*(e + f\*x)] + 9\*B\*Sin[3\*(e + f\*x)] + (5\*I)\*A\*Sin[5\*(e + f\*x)] + B\*Sin[5\*(e + f\*x)])/(384\*a^3\*c^2\*f\*(-I + Tan[e + f\*x])^3)

**Maple [A]**

time = 0.30, size = 148, normalized size = 0.80

method	result
derivativedivides	$\left(-\frac{5iA}{32} - \frac{B}{32}\right) \ln(-i + \tan(fx + e)) + \frac{3A}{16(-i + \tan(fx + e))} - \frac{\frac{3iA}{16} - \frac{B}{16}}{2(-i + \tan(fx + e))^2} - \frac{\frac{A}{8} + \frac{iB}{8}}{3(-i + \tan(fx + e))^3} - \frac{\frac{iB}{16} - \frac{A}{8}}{i + \tan(fx + e)} + \left(\frac{5iA}{32} + \frac{B}{32}\right) \frac{1}{f a^3 c^2}$
default	$\left(-\frac{5iA}{32} - \frac{B}{32}\right) \ln(-i + \tan(fx + e)) + \frac{3A}{16(-i + \tan(fx + e))} - \frac{\frac{3iA}{16} - \frac{B}{16}}{2(-i + \tan(fx + e))^2} - \frac{\frac{A}{8} + \frac{iB}{8}}{3(-i + \tan(fx + e))^3} - \frac{\frac{iB}{16} - \frac{A}{8}}{i + \tan(fx + e)} + \left(\frac{5iA}{32} + \frac{B}{32}\right) \frac{1}{f a^3 c^2}$

norman	$\frac{\frac{(-iB+5A)x}{16ac} - \frac{-iA+B}{6acf} + \frac{(-iB+5A)(\tan^3(fx+e))}{6acf} + \frac{(-iB+5A)(\tan^5(fx+e))}{16acf} + \frac{3(-iB+5A)x(\tan^2(fx+e))}{(1+\tan^2(fx+e))^3 a^2 c} + \frac{3(-iB+5A)x(\tan^4(fx+e))}{16ac}}$
risch	$-\frac{ixB}{16a^3c^2} + \frac{5xA}{16a^3c^2} - \frac{e^{-6i(fx+e)}B}{192a^3c^2f} + \frac{ie^{-6i(fx+e)}A}{192a^3c^2f} - \frac{\cos(4fx+4e)B}{32a^3c^2f} + \frac{i\cos(4fx+4e)A}{32a^3c^2f} + \frac{i\sin(4fx+4e)B}{64a^3c^2f} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^2,x,method=\_RE  
TURNVERBOSE)

[Out] 1/f/a^3/c^2\*((-5/32\*I\*A-1/32\*B)\*ln(-I+tan(f\*x+e))+3/16\*A/(-I+tan(f\*x+e))-1/  
2\*(3/16\*I\*A-1/16\*B)/(-I+tan(f\*x+e))^2-1/3\*(1/8\*A+1/8\*I\*B)/(-I+tan(f\*x+e))^3  
-(1/16\*I\*B-1/8\*A)/(I+tan(f\*x+e))+(5/32\*I\*A+1/32\*B)\*ln(I+tan(f\*x+e))-1/2\*(-  
1/16\*I\*A-1/16\*B)/(I+tan(f\*x+e))^2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^2,x, alg  
orithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:  
expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 3.01, size = 121, normalized size = 0.65

$$\frac{(24(5A - iB)fxe^{(6i fx + 6i e)} - 3(iA + B)e^{(10i fx + 10i e)} - 6(5iA + 3B)e^{(8i fx + 8i e)} - 12(-5iA + B)e^{(4i fx + 4i e)} - 3(-5iA + 3B)e^{(2i fx + 2i e)} + 2iA - 2B)e^{(-6i fx - 6i e)}}{384 a^3 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^2,x, alg  
orithm="fricas")

[Out] 1/384\*(24\*(5\*A - I\*B)\*f\*x\*e^(6\*I\*f\*x + 6\*I\*e) - 3\*(I\*A + B)\*e^(10\*I\*f\*x +  
10\*I\*e) - 6\*(5\*I\*A + 3\*B)\*e^(8\*I\*f\*x + 8\*I\*e) - 12\*(-5\*I\*A + B)\*e^(4\*I\*f\*x +  
4\*I\*e) - 3\*(-5\*I\*A + 3\*B)\*e^(2\*I\*f\*x + 2\*I\*e) + 2\*I\*A - 2\*B)\*e^(-6\*I\*f\*x -  
6\*I\*e)/(a^3\*c^2\*f)

**Sympy [A]**

time = 0.46, size = 452, normalized size = 2.44

$$\left\{ \begin{array}{l} \frac{(-13554432iA^2B^2f^2e^{6i fx + 6i e} - 33554432B^2f^2e^{6i fx + 6i e} - 1011058240iA^2B^2f^2e^{6i fx + 6i e} - 150994944B^2f^2e^{6i fx + 6i e} - 1100603200iA^2B^2f^2e^{6i fx + 6i e} - 201326592B^2f^2e^{6i fx + 6i e} - 503316480iA^2B^2f^2e^{6i fx + 6i e} - 301989888B^2f^2e^{6i fx + 6i e} - 503316480iA^2B^2f^2e^{6i fx + 6i e} - 301989888B^2f^2e^{6i fx + 6i e})e^{-6i fx - 6i e}}{6442420944i^2c^2f^2} \text{ for } a^{15}e^{10}f^2e^{12i e} \neq 0 \\ x \left( -\frac{5A+iB}{16a^3c^2} + \frac{(A^2B^2+5A^2B^2+10A^2B^2+10A^2B^2+5A^2B^2+4A^2B^2-32B^2e^{6i e}-2iB^2e^{6i e}+2iB^2e^{6i e}+32B^2e^{6i e}+iB^2)e^{-6i e}}{32a^3c^2} \right) \text{ otherwise} + \frac{\pi(5A-iB)}{16a^3c^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**2,x)
[Out] Piecewise((((33554432*I*A*a**12*c**8*f**4*exp(6*I*e) - 33554432*B*a**12*c**8*f**4*exp(6*I*e))*exp(-6*I*f*x) + (251658240*I*A*a**12*c**8*f**4*exp(8*I*e) - 150994944*B*a**12*c**8*f**4*exp(8*I*e))*exp(-4*I*f*x) + (1006632960*I*A*a**12*c**8*f**4*exp(10*I*e) - 201326592*B*a**12*c**8*f**4*exp(10*I*e))*exp(-2*I*f*x) + (-503316480*I*A*a**12*c**8*f**4*exp(14*I*e) - 301989888*B*a**12*c**8*f**4*exp(14*I*e))*exp(2*I*f*x) + (-50331648*I*A*a**12*c**8*f**4*exp(16*I*e) - 50331648*B*a**12*c**8*f**4*exp(16*I*e))*exp(4*I*f*x))/((6442450944*a**15*c**10*f**5), Ne(a**15*c**10*f**5*exp(12*I*e), 0)), (x*(-5*A - I*B)/(16*a**3*c**2) + (A*exp(10*I*e) + 5*A*exp(8*I*e) + 10*A*exp(6*I*e) + 10*A*exp(4*I*e) + 5*A*exp(2*I*e) + A - I*B*exp(10*I*e) - 3*I*B*exp(8*I*e) - 2*I*B*exp(6*I*e) + 2*I*B*exp(4*I*e) + 3*I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(32*a**3*c**2)), True)) + x*(5*A - I*B)/(16*a**3*c**2)
```

**Giac** [A]

time = 0.94, size = 219, normalized size = 1.18

$$\frac{\frac{6(-5iA-B)\log(\tan(fx+e))+6(5iA+B)\log(\tan(fx+e)-1)}{e^{3i}} + \frac{3(-15iA\tan(fx+e)^2-3B\tan(fx+e)+38A\tan(fx+e)-10iB\tan(fx+e)+25iA+9B)}{e^{3i}(-1+\tan(fx+e))^2}}{192f} + \frac{-35iA\tan(fx+e)^3-11B\tan(fx+e)^3-201A\tan(fx+e)^2+33iB\tan(fx+e)^2+255iA\tan(fx+e)+27B\tan(fx+e)+117A+3iB}{e^{3i}(\tan(fx+e)-1)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/192*(6*(-5*I*A - B)*log(tan(f*x + e) + I)/(a^3*c^2) + 6*(5*I*A + B)*log(tan(f*x + e) - I)/(a^3*c^2) + 3*(-15*I*A*tan(f*x + e)^2 - 3*B*tan(f*x + e)^2 + 38*A*tan(f*x + e) - 10*I*B*tan(f*x + e) + 25*I*A + 9*B)/(a^3*c^2*(-I*tan(f*x + e) + 1)^2) + (-55*I*A*tan(f*x + e)^3 - 11*B*tan(f*x + e)^3 - 201*A*tan(f*x + e)^2 + 33*I*B*tan(f*x + e)^2 + 255*I*A*tan(f*x + e) + 27*B*tan(f*x + e) + 117*A + 3*I*B)/(a^3*c^2*(tan(f*x + e) - I)^3))/f
```

**Mupad** [B]

time = 9.47, size = 208, normalized size = 1.12

$$\frac{\tan(e+fx)\left(\frac{25A}{48a^3c^2} - \frac{B5i}{48a^3c^2}\right) + \tan(e+fx)^3\left(\frac{5A}{16a^3c^2} - \frac{B1i}{16a^3c^2}\right) + \tan(e+fx)^4\left(\frac{B}{16a^3c^2} + \frac{A5i}{16a^3c^2}\right) + \tan(e+fx)^2\left(\frac{5B}{48a^3c^2} + \frac{A25i}{48a^3c^2}\right) - \frac{B}{6a^3c^2} + \frac{A1i}{6a^3c^2}}{f(\tan(e+fx)^5 \operatorname{li} + \tan(e+fx)^4 + \tan(e+fx)^3 2i + 2 \tan(e+fx)^2 + \tan(e+fx) \operatorname{li} + 1)} - \frac{x(B+A5i) \operatorname{li}}{16a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^2,x)
```

```
[Out] (tan(e + f*x)*((25*A)/(48*a^3*c^2) - (B*5i)/(48*a^3*c^2)) + tan(e + f*x)^3*((5*A)/(16*a^3*c^2) - (B*1i)/(16*a^3*c^2)) + tan(e + f*x)^4*((A*5i)/(16*a^3*c^2) + B/(16*a^3*c^2)) + B/(16*a^3*c^2)) + tan(e + f*x)^2*((A*25i)/(48*a^3*c^2) + (5*B)/(48*a^3*c^2)) + (A*1i)/(6*a^3*c^2) - B/(6*a^3*c^2))/(f*(tan(e + f*x)*1i + 2*tan(e + f*x)^2 + tan(e + f*x)^3*2i + tan(e + f*x)^4 + tan(e + f*x)^5*1i + 1)) - (x*(A*5i + B)*1i)/(16*a^3*c^2)
```

$$3.736 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=99

$$\frac{5Ax}{16a^3c^3} + \frac{5A \cos(e+fx) \sin(e+fx)}{16a^3c^3f} + \frac{5A \cos^3(e+fx) \sin(e+fx)}{24a^3c^3f} - \frac{\cos^6(e+fx)(B-A \tan(e+fx))}{6a^3c^3f}$$

[Out] 5/16\*A\*x/a^3/c^3+5/16\*A\*cos(f\*x+e)\*sin(f\*x+e)/a^3/c^3/f+5/24\*A\*cos(f\*x+e)^3\*sin(f\*x+e)/a^3/c^3/f-1/6\*cos(f\*x+e)^6\*(B-A\*tan(f\*x+e))/a^3/c^3/f

**Rubi [A]**

time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$ , Rules used = {3669, 74, 653, 205, 211}

$$-\frac{\cos^6(e+fx)(B-A \tan(e+fx))}{6a^3c^3f} + \frac{5A \sin(e+fx) \cos^3(e+fx)}{24a^3c^3f} + \frac{5A \sin(e+fx) \cos(e+fx)}{16a^3c^3f} + \frac{5Ax}{16a^3c^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^3), x]

[Out] (5\*A\*x)/(16\*a^3\*c^3) + (5\*A\*Cos[e + f\*x]\*Sin[e + f\*x])/(16\*a^3\*c^3\*f) + (5\*A\*Cos[e + f\*x]^3\*Sin[e + f\*x])/(24\*a^3\*c^3\*f) - (Cos[e + f\*x]^6\*(B - A\*Tan[e + f\*x]))/(6\*a^3\*c^3\*f)

Rule 74

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[(a\*c + b\*d\*x^2)^m\*(e + f\*x)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b\*c + a\*d, 0] && EqQ[n, m] && IntegerQ[m] && (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] || !IntegerQ[p])))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(-x)\*((a + b\*x^n)^(p + 1)/(a\*n\*(p + 1))), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 653

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((a\*e - c\*d\*x)/(2\*a\*c\*(p + 1))\*(a + c\*x^2)^(p + 1), x] + Dist[d\*((2\*p + 3)/(2\*a\*(p + 1))), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt Q[p, -1] && NeQ[p, -3/2]

Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ic \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^4} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(ac+acx^2)^4} dx, x, \tan(e + fx)\right)}{f} \\ &= -\frac{\cos^6(e + fx)(B - A \tan(e + fx))}{6a^3c^3f} + \frac{(5A) \text{Subst}\left(\int \frac{1}{ac} dx, x, \tan(e + fx)\right)}{6a^3c^3f} \\ &= \frac{5A \cos^3(e + fx) \sin(e + fx)}{24a^3c^3f} - \frac{\cos^6(e + fx)(B - A \tan(e + fx))}{6a^3c^3f} \\ &= \frac{5A \cos(e + fx) \sin(e + fx)}{16a^3c^3f} + \frac{5A \cos^3(e + fx) \sin(e + fx)}{24a^3c^3f} \\ &= \frac{5Ax}{16a^3c^3} + \frac{5A \cos(e + fx) \sin(e + fx)}{16a^3c^3f} + \frac{5A \cos^3(e + fx) \sin(e + fx)}{24a^3c^3f} \end{aligned}$$

Mathematica [A]

time = 0.11, size = 63, normalized size = 0.64

$$\frac{-32B \cos^6(e + fx) + A(60e + 60fx + 45 \sin(2(e + fx)) + 9 \sin(4(e + fx)) + \sin(6(e + fx)))}{192a^3c^3f}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^3), x]

[Out]  $(-32*B*\text{Cos}[e + f*x]^6 + A*(60*e + 60*f*x + 45*\text{Sin}[2*(e + f*x)] + 9*\text{Sin}[4*(e + f*x)] + \text{Sin}[6*(e + f*x)]))/(192*a^3*c^3*f)$

Maple [C] Result contains complex when optimal does not.  
time = 0.24, size = 166, normalized size = 1.68

method	result
risch	$\frac{5Ax}{16a^3c^3} - \frac{B \cos(6fx+6e)}{192f c^3 a^3} + \frac{A \sin(6fx+6e)}{192f c^3 a^3} - \frac{B \cos(4fx+4e)}{32f c^3 a^3} + \frac{3A \sin(4fx+4e)}{64f c^3 a^3} - \frac{5B \cos(2fx+2e)}{64f c^3 a^3} + \frac{15A \sin(2fx+2e)}{64f c^3 a^3}$
norman	$\frac{5Ax}{16ac} - \frac{B}{6acf} + \frac{11A \tan(fx+e)}{16acf} + \frac{5A(\tan^3(fx+e))}{6acf} + \frac{5A(\tan^5(fx+e))}{16acf} + \frac{15Ax(\tan^2(fx+e))}{16ac} + \frac{15Ax(\tan^4(fx+e))}{16ac} + \frac{5Ax(\tan^6(fx+e))}{16ac}$ $(1+\tan^2(fx+e))^3 a^2 c^2$
derivativedivides	$\frac{5iA \ln(i+\tan(fx+e))}{32} - \frac{-\frac{5A}{32} + \frac{iB}{32}}{i+\tan(fx+e)} - \frac{\frac{A}{16} - \frac{iB}{16}}{3(i+\tan(fx+e))^3} - \frac{-\frac{B}{16} - \frac{iA}{8}}{2(i+\tan(fx+e))^2} - \frac{5iA \ln(-i+\tan(fx+e))}{32} - \frac{-\frac{5A}{32} - \frac{iB}{32}}{-i+\tan(fx+e)} - \frac{\frac{A}{16} + \frac{iB}{16}}{3(-i+\tan(fx+e))^3} - \frac{-\frac{B}{16} + \frac{iA}{8}}{2(-i+\tan(fx+e))^2}$ $f a^3 c^3$
default	$\frac{5iA \ln(i+\tan(fx+e))}{32} - \frac{-\frac{5A}{32} + \frac{iB}{32}}{i+\tan(fx+e)} - \frac{\frac{A}{16} - \frac{iB}{16}}{3(i+\tan(fx+e))^3} - \frac{-\frac{B}{16} - \frac{iA}{8}}{2(i+\tan(fx+e))^2} - \frac{5iA \ln(-i+\tan(fx+e))}{32} - \frac{-\frac{5A}{32} - \frac{iB}{32}}{-i+\tan(fx+e)} - \frac{\frac{A}{16} + \frac{iB}{16}}{3(-i+\tan(fx+e))^3} - \frac{-\frac{B}{16} + \frac{iA}{8}}{2(-i+\tan(fx+e))^2}$ $f a^3 c^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x,method=_RETURNVERBOSE)`

[Out]  $1/f/a^3/c^3*(5/32*I*A*\ln(I+\tan(f*x+e))-(-5/32*A+1/32*I*B)/(I+\tan(f*x+e))-1/3*(1/16*A-1/16*I*B)/(I+\tan(f*x+e))^3-1/2*(-1/16*B-1/8*I*A)/(I+\tan(f*x+e))^2-5/32*I*A*\ln(-I+\tan(f*x+e))-(-5/32*A-1/32*I*B)/(-I+\tan(f*x+e))-1/3*(1/16*A+1/16*I*B)/(-I+\tan(f*x+e))^3-1/2*(-1/16*B+1/8*I*A)/(-I+\tan(f*x+e))^2)$

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [C] Result contains complex when optimal does not.

time = 6.85, size = 133, normalized size = 1.34

$$\frac{(120 A f x e^{6i f x+6i e} + (-i A - B) e^{12i f x+12i e} - 3(3i A + 2B) e^{10i f x+10i e} - 15(3i A + B) e^{8i f x+8i e} - 15(-3i A + B) e^{4i f x+4i e} - 3(-3i A + 2B) e^{2i f x+2i e} + i A - B) e^{-6i f x-6i e}}{384 a^3 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, algorithm="fricas")`



```
[Out] 1/384*(120*A*f*x*e^(6*I*f*x + 6*I*e) + (-I*A - B)*e^(12*I*f*x + 12*I*e) - 3
*(3*I*A + 2*B)*e^(10*I*f*x + 10*I*e) - 15*(3*I*A + B)*e^(8*I*f*x + 8*I*e) -
15*(-3*I*A + B)*e^(4*I*f*x + 4*I*e) - 3*(-3*I*A + 2*B)*e^(2*I*f*x + 2*I*e)
+ I*A - B)*e^(-6*I*f*x - 6*I*e)/(a^3*c^3*f)
```

**Sympy** [A]

time = 0.60, size = 508, normalized size = 5.13

$$\frac{5Ax}{\sqrt{a^2 - b^2}} \begin{cases} \frac{1}{\sqrt{a^2 - b^2}} \left( \frac{103079215104 I A a^{15} c^{15} f^{15} \exp(6 I e) - 103079215104 B a^{15} c^{15} f^{15} \exp(6 I e) \right) \exp(-6 I f x) + (927712 935936 I A a^{15} c^{15} f^{15} \exp(8 I e) - 618475290624 B a^{15} c^{15} f^{15} \exp(8 I e)) \exp(-4 I f x) + (4638564679680 I A a^{15} c^{15} f^{15} \exp(10 I e) - 1546188226560 B a^{15} c^{15} f^{15} \exp(10 I e)) \exp(-2 I f x) + (-4638564679 680 I A a^{15} c^{15} f^{15} \exp(14 I e) - 1546188226560 B a^{15} c^{15} f^{15} \exp(14 I e)) \exp(2 I f x) + (-927712935936 I A a^{15} c^{15} f^{15} \exp(16 I e) - 618475290624 B a^{15} c^{15} f^{15} \exp(16 I e)) \exp(4 I f x) + (-103079215104 I A a^{15} c^{15} f^{15} \exp(18 I e) - 103079215104 B a^{15} c^{15} f^{15} \exp(18 I e)) \exp(6 I f x) \exp(-12 I e) / (39582418599936 a^{18} c^{18} f^6), \operatorname{Ne}(a^{18} c^{18} f^6 \exp(12 I e), 0), (x(-5 A / (16 a^3 c^3) + (A \exp(12 I e) + 6 A \exp(10 I e) + 15 A \exp(8 I e) + 20 A \exp(6 I e) + 15 A \exp(4 I e) + 6 A \exp(2 I e) + A - I B \exp(12 I e) - 4 I B \exp(10 I e) - 5 I B \exp(8 I e) + 5 I B \exp(4 I e) + 4 I B \exp(2 I e) + I B) \exp(-6 I e) / (64 a^3 c^3)), \operatorname{True} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x)
```

```
[Out] 5*A*x/(16*a**3*c**3) + Piecewise((((103079215104*I*A*a**15*c**15*f**5*exp(6
*I*e) - 103079215104*B*a**15*c**15*f**5*exp(6*I*e))*exp(-6*I*f*x) + (927712
935936*I*A*a**15*c**15*f**5*exp(8*I*e) - 618475290624*B*a**15*c**15*f**5*ex
p(8*I*e))*exp(-4*I*f*x) + (4638564679680*I*A*a**15*c**15*f**5*exp(10*I*e) -
1546188226560*B*a**15*c**15*f**5*exp(10*I*e))*exp(-2*I*f*x) + (-4638564679
680*I*A*a**15*c**15*f**5*exp(14*I*e) - 1546188226560*B*a**15*c**15*f**5*exp
(14*I*e))*exp(2*I*f*x) + (-927712935936*I*A*a**15*c**15*f**5*exp(16*I*e) -
618475290624*B*a**15*c**15*f**5*exp(16*I*e))*exp(4*I*f*x) + (-103079215104*
I*A*a**15*c**15*f**5*exp(18*I*e) - 103079215104*B*a**15*c**15*f**5*exp(18*I
*e))*exp(6*I*f*x))*exp(-12*I*e)/(39582418599936*a**18*c**18*f**6), Ne(a**18
*c**18*f**6*exp(12*I*e), 0), (x*(-5*A/(16*a**3*c**3) + (A*exp(12*I*e) + 6*
A*exp(10*I*e) + 15*A*exp(8*I*e) + 20*A*exp(6*I*e) + 15*A*exp(4*I*e) + 6*A*exp
(2*I*e) + A - I*B*exp(12*I*e) - 4*I*B*exp(10*I*e) - 5*I*B*exp(8*I*e) + 5*
I*B*exp(4*I*e) + 4*I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(64*a**3*c**3)), True)
)
```

**Giac** [A]

time = 0.90, size = 79, normalized size = 0.80

$$\frac{\frac{15(fx+e)A}{a^3c^3} + \frac{15A \tan(fx+e)^5 + 40A \tan(fx+e)^3 + 33A \tan(fx+e) - 8B}{(\tan(fx+e)^2 + 1)^3 a^3 c^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/48*(15*(f*x + e)*A/(a^3*c^3) + (15*A*tan(f*x + e)^5 + 40*A*tan(f*x + e)^3
+ 33*A*tan(f*x + e) - 8*B)/((tan(f*x + e)^2 + 1)^3*a^3*c^3))/f
```

**Mupad** [B]

time = 8.72, size = 64, normalized size = 0.65

$$\frac{5Ax}{16a^3c^3} + \frac{\cos(e+fx)^6 \left( \frac{5A \tan(e+fx)^5}{16} + \frac{5A \tan(e+fx)^3}{6} + \frac{11A \tan(e+fx)}{16} - \frac{B}{6} \right)}{a^3c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^3),x)
```

```
[Out] (5*A*x)/(16*a^3*c^3) + (cos(e + f*x)^6*((11*A*tan(e + f*x))/16 - B/6 + (5*A*tan(e + f*x)^3)/6 + (5*A*tan(e + f*x)^5)/16))/(a^3*c^3*f)
```

$$3.737 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^4} dx$$

**Optimal.** Leaf size=251

$$\frac{5(7A+iB)x}{128a^3c^4} + \frac{A+iB}{96a^3c^4f(i-\tan(e+fx))^3} - \frac{5iA-3B}{128a^3c^4f(i-\tan(e+fx))^2} - \frac{5(3A+iB)}{128a^3c^4f(i-\tan(e+fx))} - \frac{5A}{64a^3c^4f(i-\tan(e+fx))}$$

[Out] 5/128\*(7\*A+I\*B)\*x/a^3/c^4+1/96\*(A+I\*B)/a^3/c^4/f/(I-tan(f\*x+e))^3+1/128\*(-5\*I\*A+3\*B)/a^3/c^4/f/(I-tan(f\*x+e))^2-5/128\*(3\*A+I\*B)/a^3/c^4/f/(I-tan(f\*x+e))+1/64\*(-I\*A-B)/a^3/c^4/f/(I+tan(f\*x+e))^4+1/48\*(-2\*A+I\*B)/a^3/c^4/f/(I+tan(f\*x+e))^3+1/64\*(5\*I\*A+B)/a^3/c^4/f/(I+tan(f\*x+e))^2+5/32\*A/a^3/c^4/f/(I+tan(f\*x+e))

**Rubi** [A]

time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$\frac{5(3A+iB)}{128a^3c^4f(-\tan(e+fx)+i)} - \frac{-3B+5iA}{128a^3c^4f(-\tan(e+fx)+i)^2} + \frac{B+5iA}{64a^3c^4f(\tan(e+fx)+i)^2} + \frac{A+iB}{96a^3c^4f(-\tan(e+fx)+i)^3} - \frac{2A-iB}{48a^3c^4f(\tan(e+fx)+i)^3} - \frac{B+iA}{64a^3c^4f(\tan(e+fx)+i)^4} + \frac{5x(7A+iB)}{128a^3c^4} + \frac{5A}{32a^3c^4f(\tan(e+fx)+i)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^4), x]

[Out] (5\*(7\*A + I\*B)\*x)/(128\*a^3\*c^4) + (A + I\*B)/(96\*a^3\*c^4\*f\*(I - Tan[e + f\*x])^3) - ((5\*I)\*A - 3\*B)/(128\*a^3\*c^4\*f\*(I - Tan[e + f\*x])^2) - (5\*(3\*A + I\*B))/(128\*a^3\*c^4\*f\*(I - Tan[e + f\*x])) - (I\*A + B)/(64\*a^3\*c^4\*f\*(I + Tan[e + f\*x])^4) - (2\*A - I\*B)/(48\*a^3\*c^4\*f\*(I + Tan[e + f\*x])^3) + ((5\*I)\*A + B)/(64\*a^3\*c^4\*f\*(I + Tan[e + f\*x])^2) + (5\*A)/(32\*a^3\*c^4\*f\*(I + Tan[e + f\*x]))

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

## Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ict \tan(e + fx))^4} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^5} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{A+iB}{32a^4 c^5 (-i+x)^4} + \frac{i(5A+3iB)}{64a^4 c^5 (-i+x)^3} - \frac{5(3A+iB)}{128a^4 c^5 (-i+x)^2}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{A + iB}{96a^3 c^4 f (i - \tan(e + fx))^3} - \frac{5iA - 3B}{128a^3 c^4 f (i - \tan(e + fx))^2}$$

$$= \frac{5(7A + iB)x}{128a^3 c^4} + \frac{A + iB}{96a^3 c^4 f (i - \tan(e + fx))^3} - \frac{5iA - 3B}{128a^3 c^4 f (i - \tan(e + fx))^2}$$

**Mathematica [A]**

time = 1.53, size = 267, normalized size = 1.06

$\frac{a^2 c^2 (c^2 - i a c \tan(e + f x)) - i a c^2 \tan(e + f x) (100 A^2 - 14 f^2) + 8 i^2 f^2 (100 A^2 - 14 f^2) \cos(e + f x) + 18 i^2 A^2 (100 A^2 - 14 f^2) \sin(e + f x) + 14 A^2 \cos^2(e + f x) + 50 B \cos^2(e + f x) + A \sin^2(e + f x) + 7 B \sin^2(e + f x) - 40 A \sin(e + f x) \cos(e + f x) - 60 B \sin(e + f x) \cos(e + f x) - 840 A f \sin(e + f x) - 120 B f \sin(e + f x) - 570 A \sin^3(e + f x) - 54 B \sin^3(e + f x) - 70 A \sin^5(e + f x) + 10 B \sin^5(e + f x) - 5 A \cos^7(e + f x) + B \cos^7(e + f x)}{3072 c^4 (-i + \tan(e + f x))^3}$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f
*x])^4), x]
```

```
[Out] (Sec[e + f*x]^3*(-Cos[4*(e + f*x)] - I*Sin[4*(e + f*x)])*(60*(A*(-7 - (14*I
)*f*x) + B*(I + 2*f*x))*Cos[e + f*x] + 18*(7*A + (9*I)*B)*Cos[3*(e + f*x)]
+ 14*A*Cos[5*(e + f*x)] + (50*I)*B*Cos[5*(e + f*x)] + A*Cos[7*(e + f*x)] +
(7*I)*B*Cos[7*(e + f*x)] - (420*I)*A*Sin[e + f*x] - 60*B*Sin[e + f*x] - 840
*A*f*x*Sin[e + f*x] - (120*I)*B*f*x*Sin[e + f*x] - (378*I)*A*Sin[3*(e + f*x
)] + 54*B*Sin[3*(e + f*x)] - (70*I)*A*Sin[5*(e + f*x)] + 10*B*Sin[5*(e + f
x)] - (7*I)*A*Sin[7*(e + f*x)] + B*Sin[7*(e + f*x)]))/(3072*a^3*c^4*f*(-I +
Tan[e + f*x])^3)
```

**Maple [A]**

time = 0.33, size = 190, normalized size = 0.76

method	result
derivativedivides	$-\frac{\frac{iA}{16} + \frac{B}{16}}{4(i+\tan(fx+e))^4} + \frac{5A}{32(i+\tan(fx+e))} + \left(\frac{35iA}{256} - \frac{5B}{256}\right) \ln(i+\tan(fx+e)) - \frac{-\frac{iB}{16} + \frac{A}{8}}{3(i+\tan(fx+e))^3} - \frac{-\frac{5iA}{32} - \frac{B}{32}}{2(i+\tan(fx+e))^2} - \frac{\frac{A}{32} + \frac{iB}{32}}{3(-i+\tan(fx+e))} - \frac{1}{fa^3c^4}$
default	$-\frac{\frac{iA}{16} + \frac{B}{16}}{4(i+\tan(fx+e))^4} + \frac{5A}{32(i+\tan(fx+e))} + \left(\frac{35iA}{256} - \frac{5B}{256}\right) \ln(i+\tan(fx+e)) - \frac{-\frac{iB}{16} + \frac{A}{8}}{3(i+\tan(fx+e))^3} - \frac{-\frac{5iA}{32} - \frac{B}{32}}{2(i+\tan(fx+e))^2} - \frac{\frac{A}{32} + \frac{iB}{32}}{3(-i+\tan(fx+e))} - \frac{1}{fa^3c^4}$
norman	$\frac{5(iB+7A)x}{128ac} - \frac{iA+B}{8acf} + \frac{(-5iB+93A)\tan(fx+e)}{128acf} + \frac{73(iB+7A)(\tan^3(fx+e))}{384acf} + \frac{55(iB+7A)(\tan^5(fx+e))}{384acf} + \frac{5(iB+7A)(\tan^7(fx+e))}{128acf} - \frac{1}{a^2c^3(1+\tan^2(fx+e))}$
risch	$\frac{5ixB}{128a^3c^4} + \frac{35xA}{128a^3c^4} - \frac{e^{8i(fx+e)}B}{1024a^3c^4f} - \frac{i\sin(4fx+4e)B}{128a^3c^4f} - \frac{\cos(6fx+6e)B}{128a^3c^4f} - \frac{ie^{8i(fx+e)}A}{1024a^3c^4f} - \frac{i\sin(6fx+6e)B}{192a^3c^4f} + \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x,method=_RE  
TURNVERBOSE)`

[Out] 
$$\frac{1}{fa^3c^4} \left( -\frac{1}{4} \left( \frac{1}{16}IA + \frac{1}{16}B \right) (I+\tan(fx+e))^{-4} + \frac{5}{32} \frac{A}{(I+\tan(fx+e))} + \left( \frac{35}{256}IA - \frac{5}{256}B \right) \ln(I+\tan(fx+e)) - \frac{1}{3} \left( -\frac{1}{16}IB + \frac{1}{8}A \right) (I+\tan(fx+e))^{-3} - \frac{1}{2} \left( -\frac{5}{32}IA - \frac{1}{32}B \right) (I+\tan(fx+e))^{-2} - \frac{1}{3} \left( \frac{1}{32}A + \frac{1}{32}IB \right) (-I+\tan(fx+e))^{-3} - \left( -\frac{15}{128}A - \frac{5}{128}IB \right) (-I+\tan(fx+e))^{-1} - \frac{1}{2} \left( \frac{5}{64}IA - \frac{3}{64}B \right) (-I+\tan(fx+e))^{-2} + \left( -\frac{35}{256}IA + \frac{5}{256}B \right) \ln(-I+\tan(fx+e)) \right)$$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 4.46, size = 159, normalized size = 0.63

$$\frac{(120(7A+iB)fxe^{6i(fx+6ie)} - 3(iA+B)e^{14i(fx+14ie)} - 4(7iA+5B)e^{12i(fx+12ie)} - 18(7iA+3B)e^{10i(fx+10ie)} - 60(7iA+B)e^{8i(fx+8ie)} - 36(-7iA+3B)e^{4i(fx+4ie)} - 6(-7iA+5B)e^{2i(fx+2ie)} + 4iA - 4B)e^{-6i(fx-6ie)})}{3072a^3c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, algorithm="fricas")`

[Out] 
$$\frac{1}{3072} \left( 120(7A+I*B)fxxe^{(6I*fx+6I*e)} - 3(I*A+B)e^{(14I*fx+14I*e)} - 4(7I*A+5B)e^{(12I*fx+12I*e)} - 18(7I*A+3B)e^{(10I*fx+10I*e)} - 60(7I*A+B)e^{(8I*fx+8I*e)} - 36(-7I*A+3B)e^{(4I*fx+4I*e)} - 6(-7I*A+5B)e^{(2I*fx+2I*e)} + 4iA - 4B \right) e^{(-6i*fx-6ie)}$$

$$*f*x + 10*I*e) - 60*(7*I*A + B)*e^(8*I*f*x + 8*I*e) - 36*(-7*I*A + 3*B)*e^(4*I*f*x + 4*I*e) - 6*(-7*I*A + 5*B)*e^(2*I*f*x + 2*I*e) + 4*I*A - 4*B)*e^(6*I*f*x - 6*I*e)/(a^3*c^4*f)$$

**Sympy [A]**

time = 0.69, size = 605, normalized size = 2.41

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*3/(c-I\*c\*tan(f\*x+e))\*\*4,x)

[Out] Piecewise((((13510798882111488\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(6\*I\*e) - 13510798882111488\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(6\*I\*e))\*exp(-6\*I\*f\*x) + (141863388262170624\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(8\*I\*e) - 101330991615836160\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(8\*I\*e))\*exp(-4\*I\*f\*x) + (851180329573023744\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(10\*I\*e) - 364791569817010176\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(10\*I\*e))\*exp(-2\*I\*f\*x) + (-1418633882621706240\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(14\*I\*e) - 202661983231672320\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(14\*I\*e))\*exp(2\*I\*f\*x) + (-425590164786511872\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(16\*I\*e) - 182395784908505088\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(16\*I\*e))\*exp(4\*I\*f\*x) + (-94575592174780416\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(18\*I\*e) - 67553994410557440\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(18\*I\*e))\*exp(6\*I\*f\*x) + (-10133099161583616\*I\*A\*a\*\*18\*c\*\*24\*f\*\*6\*exp(20\*I\*e) - 10133099161583616\*B\*a\*\*18\*c\*\*24\*f\*\*6\*exp(20\*I\*e))\*exp(8\*I\*f\*x))\*exp(-12\*I\*e)/(10376293541461622784\*a\*\*21\*c\*\*28\*f\*\*7), Ne(a\*\*21\*c\*\*28\*f\*\*7\*exp(12\*I\*e), 0)), (x\*(-(35\*A + 5\*I\*B)/(128\*a\*\*3\*c\*\*4) + (A\*exp(14\*I\*e) + 7\*A\*exp(12\*I\*e) + 21\*A\*exp(10\*I\*e) + 35\*A\*exp(8\*I\*e) + 35\*A\*exp(6\*I\*e) + 21\*A\*exp(4\*I\*e) + 7\*A\*exp(2\*I\*e) + A - I\*B\*exp(14\*I\*e) - 5\*I\*B\*exp(12\*I\*e) - 9\*I\*B\*exp(10\*I\*e) - 5\*I\*B\*exp(8\*I\*e) + 5\*I\*B\*exp(6\*I\*e) + 9\*I\*B\*exp(4\*I\*e) + 5\*I\*B\*exp(2\*I\*e) + I\*B)\*exp(-6\*I\*e)/(128\*a\*\*3\*c\*\*4)), True)) + x\*(35\*A + 5\*I\*B)/(128\*a\*\*3\*c\*\*4)

**Giac [A]**

time = 0.91, size = 271, normalized size = 1.08

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^4,x, algorithm="giac")

[Out] 1/3072\*(60\*(7\*I\*A - B)\*log(tan(f\*x + e) + I)/(a^3\*c^4) - 60\*(7\*I\*A - B)\*log(-I\*tan(f\*x + e) - 1)/(a^3\*c^4) + 2\*(385\*A\*tan(f\*x + e)^3 + 55\*I\*B\*tan(f\*x + e)^3 - 1335\*I\*A\*tan(f\*x + e)^2 + 225\*B\*tan(f\*x + e)^2 - 1575\*A\*tan(f\*x + e) - 321\*I\*B\*tan(f\*x + e) + 641\*I\*A - 167\*B)/(a^3\*c^4\*(I\*tan(f\*x + e) + 1)^3) + (-875\*I\*A\*tan(f\*x + e)^4 + 125\*B\*tan(f\*x + e)^4 + 3980\*A\*tan(f\*x + e)^3 + 500\*I\*B\*tan(f\*x + e)^3 + 6930\*I\*A\*tan(f\*x + e)^2 - 702\*B\*tan(f\*x + e)^2

- 5548\*A\*tan(f\*x + e) - 340\*I\*B\*tan(f\*x + e) - 1771\*I\*A - 35\*B)/(a^3\*c^4\*(  
tan(f\*x + e) + I)^4))/f

**Mupad [B]**

time = 10.40, size = 286, normalized size = 1.14

$$\frac{\tan(e + fx) \left( -\frac{11B}{128a^3c^4} + \frac{A77i}{128a^3c^4} \right) + \tan(e + fx)^3 \left( -\frac{5B}{48a^3c^4} + \frac{A35i}{48a^3c^4} \right) + \tan(e + fx)^4 \left( \frac{35A}{48a^3c^4} + \frac{B5i}{48a^3c^4} \right) + \tan(e + fx)^5 \left( -\frac{5B}{128a^3c^4} + \frac{A35i}{128a^3c^4} \right) + \tan(e + fx)^6 \left( \frac{35A}{128a^3c^4} + \frac{B5i}{128a^3c^4} \right) + \tan(e + fx)^7 \left( \frac{77A}{128a^3c^4} + \frac{B11i}{128a^3c^4} \right) + \frac{A}{8a^3c^4} - \frac{B11i}{8a^3c^4} + \frac{5x(7A + B11)}{128a^3c^4}}{f \left( \tan(e + fx)^7 + \tan(e + fx)^6 i + 3 \tan(e + fx)^5 + \tan(e + fx)^4 3i + 3 \tan(e + fx)^3 + \tan(e + fx)^2 3i + \tan(e + fx) + i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)^4), x)

[Out] (tan(e + f\*x)\*((A\*77i)/(128\*a^3\*c^4) - (11\*B)/(128\*a^3\*c^4)) + tan(e + f\*x)^3\*((A\*35i)/(48\*a^3\*c^4) - (5\*B)/(48\*a^3\*c^4)) + tan(e + f\*x)^4\*((35\*A)/(48\*a^3\*c^4) + (B\*5i)/(48\*a^3\*c^4)) + tan(e + f\*x)^5\*((A\*35i)/(128\*a^3\*c^4) - (5\*B)/(128\*a^3\*c^4)) + tan(e + f\*x)^6\*((35\*A)/(128\*a^3\*c^4) + (B\*5i)/(128\*a^3\*c^4)) + tan(e + f\*x)^2\*((77\*A)/(128\*a^3\*c^4) + (B\*11i)/(128\*a^3\*c^4)) + A/(8\*a^3\*c^4) - (B\*11i)/(8\*a^3\*c^4))/(f\*(tan(e + f\*x) + tan(e + f\*x)^2\*3i + 3\*tan(e + f\*x)^3 + tan(e + f\*x)^4\*3i + 3\*tan(e + f\*x)^5 + tan(e + f\*x)^6\*1i + tan(e + f\*x)^7 + 1i)) + (5\*x\*(7\*A + B\*11i))/(128\*a^3\*c^4)

$$3.738 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^5} dx$$

**Optimal.** Leaf size=287

$$\frac{7(4A + iB)x}{128a^3c^5} + \frac{A + iB}{192a^3c^5 f(i - \tan(e + fx))^3} - \frac{3iA - 2B}{128a^3c^5 f(i - \tan(e + fx))^2} - \frac{3(7A + 3iB)}{256a^3c^5 f(i - \tan(e + fx))} + \frac{1}{80a^3}$$

[Out]  $7/128*(4*A+I*B)*x/a^3/c^5+1/192*(A+I*B)/a^3/c^5/f/(I-\tan(f*x+e))^3+1/128*(-3*I*A+2*B)/a^3/c^5/f/(I-\tan(f*x+e))^2-3/256*(7*A+3*I*B)/a^3/c^5/f/(I-\tan(f*x+e))+1/80*(A-I*B)/a^3/c^5/f/(I+\tan(f*x+e))^5+1/64*(-2*I*A-B)/a^3/c^5/f/(I+\tan(f*x+e))^4+1/96*(-5*A+I*B)/a^3/c^5/f/(I+\tan(f*x+e))^3+5/64*I*A/a^3/c^5/f/(I+\tan(f*x+e))^2+5/256*(7*A+I*B)/a^3/c^5/f/(I+\tan(f*x+e))$

**Rubi [A]**

time = 0.23, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$\frac{3(7A + 3iB)}{256a^3c^5 f(i - \tan(e + fx))} + \frac{5(7A + iB)}{256a^3c^5 f(\tan(e + fx) + i)} - \frac{-2B + 3iA}{128a^3c^5 f(-\tan(e + fx) + i)^2} + \frac{A + iB}{192a^3c^5 f(-\tan(e + fx) + i)^3} - \frac{5A - iB}{96a^3c^5 f(\tan(e + fx) + i)^3} - \frac{B + 2iA}{64a^3c^5 f(\tan(e + fx) + i)^4} + \frac{A - iB}{80a^3c^5 f(\tan(e + fx) + i)^5} + \frac{7i(4A + iB)}{128a^3c^5} + \frac{5iA}{64a^3c^5 f(\tan(e + fx) + i)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^5), x]

[Out]  $(7*(4*A + I*B)*x)/(128*a^3*c^5) + (A + I*B)/(192*a^3*c^5*f*(I - Tan[e + f*x])^3) - ((3*I)*A - 2*B)/(128*a^3*c^5*f*(I - Tan[e + f*x])^2) - (3*(7*A + (3*I)*B))/(256*a^3*c^5*f*(I - Tan[e + f*x])) + (A - I*B)/(80*a^3*c^5*f*(I + Tan[e + f*x])^5) - ((2*I)*A + B)/(64*a^3*c^5*f*(I + Tan[e + f*x])^4) - (5*A - I*B)/(96*a^3*c^5*f*(I + Tan[e + f*x])^3) + (((5*I)/64)*A)/(a^3*c^5*f*(I + Tan[e + f*x])^2) + (5*(7*A + I*B))/(256*a^3*c^5*f*(I + Tan[e + f*x]))$

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])



Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ictan(e + fx))^5} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^6} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{A+iB}{64a^4 c^6 (-i+x)^4} + \frac{i(3A+2iB)}{64a^4 c^6 (-i+x)^3} - \frac{3(7A+3iB)}{256a^4 c^6 (-i+x)^2}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{A + iB}{192a^3 c^5 f (i - \tan(e + fx))^3} - \frac{3iA - 2B}{128a^3 c^5 f (i - \tan(e + fx))^2}$$

$$= \frac{7(4A + iB)x}{128a^3 c^5} + \frac{A + iB}{192a^3 c^5 f (i - \tan(e + fx))^3} - \frac{3iA - 2B}{128a^3 c^5 f}$$

Mathematica [A]

time = 1.80, size = 280, normalized size = 0.98

Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^5), x]

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^5), x]

[Out] (Sec[e + f\*x]^3\*(Cos[5\*(e + f\*x)] + I\*Sin[5\*(e + f\*x)]\*(2100\*A + 210\*(4\*A\*(1 + (4\*I)\*f\*x) - B\*(I + 4\*f\*x))\*Cos[2\*(e + f\*x)] - 560\*(A + I\*B)\*Cos[4\*(e + f\*x)] - 60\*A\*Cos[6\*(e + f\*x)] - (135\*I)\*B\*Cos[6\*(e + f\*x)] - 4\*A\*Cos[8\*(e + f\*x)] - (16\*I)\*B\*Cos[8\*(e + f\*x)] + (840\*I)\*A\*Sin[2\*(e + f\*x)] + 210\*B\*Sin[2\*(e + f\*x)] + 3360\*A\*f\*x\*Sin[2\*(e + f\*x)] + (840\*I)\*B\*f\*x\*Sin[2\*(e + f\*x)] + (1120\*I)\*A\*Sin[4\*(e + f\*x)] - 280\*B\*Sin[4\*(e + f\*x)] + (180\*I)\*A\*Sin[6\*(e + f\*x)] - 45\*B\*Sin[6\*(e + f\*x)] + (16\*I)\*A\*Sin[8\*(e + f\*x)] - 4\*B\*Sin[8\*(e + f\*x)]))/(15360\*a^3\*c^5\*f\*(-I + Tan[e + f\*x])^3)

Maple [A]

time = 0.44, size = 212, normalized size = 0.74

method	result
derivativedivides	$\frac{5iA}{64(i+\tan(fx+e))^2} - \frac{-\frac{A}{16} + \frac{iB}{16}}{5(i+\tan(fx+e))^5} - \frac{\frac{5A}{32} - \frac{iB}{32}}{3(i+\tan(fx+e))^3} - \frac{-\frac{35A}{256} - \frac{5iB}{256}}{i+\tan(fx+e)} - \frac{\frac{B}{16} + \frac{iA}{8}}{4(i+\tan(fx+e))^4} + \left(-\frac{7B}{256} + \frac{7iA}{64}\right) \frac{\ln(i+\tan(fx+e))}{fa^3c^5} -$
default	$\frac{5iA}{64(i+\tan(fx+e))^2} - \frac{-\frac{A}{16} + \frac{iB}{16}}{5(i+\tan(fx+e))^5} - \frac{\frac{5A}{32} - \frac{iB}{32}}{3(i+\tan(fx+e))^3} - \frac{-\frac{35A}{256} - \frac{5iB}{256}}{i+\tan(fx+e)} - \frac{\frac{B}{16} + \frac{iA}{8}}{4(i+\tan(fx+e))^4} + \left(-\frac{7B}{256} + \frac{7iA}{64}\right) \frac{\ln(i+\tan(fx+e))}{fa^3c^5} -$
norman	$\frac{7(iB+4A)x}{128ac} - \frac{8iA+3B}{40acf} + \frac{B(\tan^2(fx+e))}{8acf} + \frac{79(iB+4A)(\tan^3(fx+e))}{192acf} + \frac{7(iB+4A)(\tan^5(fx+e))}{15acf} + \frac{49(iB+4A)(\tan^7(fx+e))}{192acf} + \frac{7(iB+4A)}{15acf}$
risch	$-\frac{13i \sin(6fx+6e)B}{1536a^3c^5f} + \frac{7xA}{32a^3c^5} - \frac{e^{10i(fx+e)}B}{2560a^3c^5f} - \frac{ie^{8i(fx+e)}A}{256a^3c^5f} - \frac{3e^{8i(fx+e)}B}{1024a^3c^5f} - \frac{i \sin(4fx+4e)B}{128a^3c^5f} - \frac{5 \cos(6fx+6e)B}{512a^3c^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x,method=_RE  
TURNVERBOSE)`

[Out]  $1/f/a^3/c^5*(5/64*I*A/(I+\tan(f*x+e))^2-1/5*(-1/16*A+1/16*I*B)/(I+\tan(f*x+e))^5-1/3*(5/32*A-1/32*I*B)/(I+\tan(f*x+e))^3-(-35/256*A-5/256*I*B)/(I+\tan(f*x+e))-1/4*(1/16*B+1/8*I*A)/(I+\tan(f*x+e))^4+(-7/256*B+7/64*I*A)*\ln(I+\tan(f*x+e))-1/3*(1/64*A+1/64*I*B)/(-I+\tan(f*x+e))^3+(7/256*B-7/64*I*A)*\ln(-I+\tan(f*x+e))-(-21/256*A-9/256*I*B)/(-I+\tan(f*x+e))-1/2*(-1/32*B+3/64*I*A)/(-I+\tan(f*x+e))^2)$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas** [A]

time = 4.10, size = 168, normalized size = 0.59

$$\frac{(840(4A+iB)fx^{6i}e^{6i(fx+e)} - 6(iA+B)e^{16i(fx+10e)} - 15(4iA+3B)e^{14i(fx+14e)} - 140(2iA+B)e^{12i(fx+12e)} - 210(4iA+B)e^{10i(fx+10e)} - 2100iAe^{8i(fx+8e)} - 420(-2iA+B)e^{4i(fx+4e)} - 30(-4iA+3B)e^{2i(fx+2e)} + 10iA - 10B)e^{-6i(fx-6e)}}{15360a^3c^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x, algorithm="fricas")`

[Out]  $1/15360*(840*(4*A + I*B)*f*x*e^{(6*I*f*x + 6*I*e)} - 6*(I*A + B)*e^{(16*I*f*x + 16*I*e)} - 15*(4*I*A + 3*B)*e^{(14*I*f*x + 14*I*e)} - 140*(2*I*A + B)*e^{(12*I*f*x + 12*I*e)} - 210*(4*I*A + B)*e^{(10*I*f*x + 10*I*e)} - 2100*I*A*e^{(8*I*f*x + 8*I*e)} - 420*(-2*I*A + B)*e^{(4*I*f*x + 4*I*e)} - 30*(-4*I*A + 3*B)*e^{(2*I*f*x + 2*I*e)} + 10*I*A - 10*B)$

$$I*f*x + 12*I*e) - 210*(4*I*A + B)*e^{(10*I*f*x + 10*I*e)} - 2100*I*A*e^{(8*I*f*x + 8*I*e)} - 420*(-2*I*A + B)*e^{(4*I*f*x + 4*I*e)} - 30*(-4*I*A + 3*B)*e^{(2*I*f*x + 2*I*e)} + 10*I*A - 10*B)*e^{(-6*I*f*x - 6*I*e)}/(a^3*c^5*f)$$

Sympy [A]

time = 0.91, size = 646, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*3/(c-I\*c\*tan(f\*x+e))\*\*5,x)

[Out] Piecewise(((−7263405479023135948800\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(14\*I\*e)\*exp(2\*I\*f\*x) + (34587645138205409280\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(6\*I\*e) − 34587645138205409280\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(6\*I\*e))\*exp(−6\*I\*f\*x) + (415051741658464911360\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(8\*I\*e) − 311288806243848683520\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(8\*I\*e))\*exp(−4\*I\*f\*x) + (2905362191609254379520\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(10\*I\*e) − 1452681095804627189760\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(10\*I\*e))\*exp(−2\*I\*f\*x) + (−2905362191609254379520\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(16\*I\*e) − 726340547902313594880\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(16\*I\*e))\*exp(4\*I\*f\*x) + (−968454063869751459840\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(18\*I\*e) − 484227031934875729920\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(18\*I\*e))\*exp(6\*I\*f\*x) + (−207525870829232455680\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(20\*I\*e) − 155644403121924341760\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(20\*I\*e))\*exp(8\*I\*f\*x) + (−20752587082923245568\*I\*A\*a\*\*21\*c\*\*35\*f\*\*7\*exp(22\*I\*e) − 20752587082923245568\*B\*a\*\*21\*c\*\*35\*f\*\*7\*exp(22\*I\*e))\*exp(10\*I\*f\*x))\*exp(−12\*I\*e)/(53126622932283508654080\*a\*\*24\*c\*\*40\*f\*\*8), Ne(a\*\*24\*c\*\*40\*f\*\*8\*exp(12\*I\*e), 0)), (x\*(−(28\*A + 7\*I\*B)/(128\*a\*\*3\*c\*\*5) + (A\*exp(16\*I\*e) + 8\*A\*exp(14\*I\*e) + 28\*A\*exp(12\*I\*e) + 56\*A\*exp(10\*I\*e) + 70\*A\*exp(8\*I\*e) + 56\*A\*exp(6\*I\*e) + 28\*A\*exp(4\*I\*e) + 8\*A\*exp(2\*I\*e) + A − I\*B\*exp(16\*I\*e) − 6\*I\*B\*exp(14\*I\*e) − 14\*I\*B\*exp(12\*I\*e) − 14\*I\*B\*exp(10\*I\*e) + 14\*I\*B\*exp(6\*I\*e) + 14\*I\*B\*exp(4\*I\*e) + 6\*I\*B\*exp(2\*I\*e) + I\*B)\*exp(−6\*I\*e)/(256\*a\*\*3\*c\*\*5)), True)) + x\*(28\*A + 7\*I\*B)/(128\*a\*\*3\*c\*\*5)

Giac [A]

time = 1.03, size = 289, normalized size = 1.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^5,x, algorithm="giac")

[Out]  $-1/15360*(420*(-4*I*A + B)*\log(\tan(f*x + e) + I)/(a^3*c^5) + 420*(4*I*A - B)*\log(\tan(f*x + e) - I)/(a^3*c^5) + 10*(-308*I*A*\tan(f*x + e)^3 + 77*B*\tan(f*x + e)^3 - 1050*A*\tan(f*x + e)^2 - 285*I*B*\tan(f*x + e)^2 + 1212*I*A*\tan(f*x + e) - 363*B*\tan(f*x + e) + 478*A + 163*I*B)/(a^3*c^5*(\tan(f*x + e) - I$

$$\begin{aligned} &)^3 + (3836*I*A*\tan(f*x + e)^5 - 959*B*\tan(f*x + e)^5 - 21280*A*\tan(f*x + \\ &e)^4 - 5095*I*B*\tan(f*x + e)^4 - 47960*I*A*\tan(f*x + e)^3 + 10790*B*\tan(f*x \\ &+ e)^3 + 55360*A*\tan(f*x + e)^2 + 11230*I*B*\tan(f*x + e)^2 + 33260*I*A*\tan \\ &(f*x + e) - 5435*B*\tan(f*x + e) - 8608*A - 667*I*B)/(a^3*c^5*(\tan(f*x + e) \\ &+ I)^5))/f \end{aligned}$$

**Mupad [B]**

time = 10.73, size = 319, normalized size = 1.11

$$\frac{\frac{\frac{a^3 B x + \tan(e + f x) (-\frac{7 B x + A^2 I}{4 c^2} + \frac{A^2 I}{4 c^2}) + \tan(e + f x) (-\frac{7 B x + A^2 I}{4 c^2} + \frac{A^2 I}{4 c^2}) + \tan(e + f x) (\frac{7 A^2 x + B^2 I}{4 c^2} + \frac{B^2 I}{4 c^2}) + \tan(e + f x) (\frac{7 A^2 x + B^2 I}{4 c^2} + \frac{B^2 I}{4 c^2}) + \tan(e + f x) (-\frac{7 B x + A^2 I}{4 c^2} + \frac{A^2 I}{4 c^2}) - \tan(e + f x) (\frac{7 B x + A^2 I}{4 c^2} + \frac{A^2 I}{4 c^2}) + \frac{A^2 I}{4 c^2} + \frac{7 x (4 A + B I)}{128 a^3 c^5}}{f (\tan(e + f x)^3 + \tan(e + f x)^2 + 2 \tan(e + f x) + \tan(e + f x)^6 + \tan(e + f x)^6 + \tan(e + f x)^6 - 2 \tan(e + f x)^2 + \tan(e + f x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*I)^3\*(c - c\*tan(e + f\*x)\*I)^5), x)

[Out] (tan(e + f\*x)^4\*((A\*7i)/(6\*a^3\*c^5) - (7\*B)/(24\*a^3\*c^5)) - tan(e + f\*x)\*((61\*A)/(160\*a^3\*c^5) + (B\*61i)/(640\*a^3\*c^5)) + tan(e + f\*x)^6\*((A\*7i)/(16\*a^3\*c^5) - (7\*B)/(64\*a^3\*c^5)) + tan(e + f\*x)^7\*((7\*A)/(32\*a^3\*c^5) + (B\*7i)/(128\*a^3\*c^5)) + tan(e + f\*x)^5\*((35\*A)/(96\*a^3\*c^5) + (B\*35i)/(384\*a^3\*c^5)) + tan(e + f\*x)^2\*((A\*77i)/(80\*a^3\*c^5) - (77\*B)/(320\*a^3\*c^5)) - tan(e + f\*x)^3\*((49\*A)/(480\*a^3\*c^5) + (B\*49i)/(1920\*a^3\*c^5)) + (A\*I)/(5\*a^3\*c^5) + (3\*B)/(40\*a^3\*c^5)/(f\*(tan(e + f\*x)\*2i - 2\*tan(e + f\*x)^2 + tan(e + f\*x)^3\*6i + tan(e + f\*x)^5\*6i + 2\*tan(e + f\*x)^6 + tan(e + f\*x)^7\*2i + tan(e + f\*x)^8 - 1)) + (7\*x\*(4\*A + B\*I))/(128\*a^3\*c^5)

$$3.739 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^6} dx$$

**Optimal.** Leaf size=319

$$\frac{7(3A+iB)x}{128a^3c^6} + \frac{A+iB}{384a^3c^6 f(i-\tan(e+fx))^3} - \frac{7iA-5B}{512a^3c^6 f(i-\tan(e+fx))^2} - \frac{7(2A+iB)}{256a^3c^6 f(i-\tan(e+fx))} + \frac{96a^3c^6}{96a^3c^6}$$

[Out] 7/128\*(3\*A+I\*B)\*x/a^3/c^6+1/384\*(A+I\*B)/a^3/c^6/f/(I-tan(f\*x+e))^3+1/512\*(-7\*I\*A+5\*B)/a^3/c^6/f/(I-tan(f\*x+e))^2-7/256\*(2\*A+I\*B)/a^3/c^6/f/(I-tan(f\*x+e))+1/96\*(I\*A+B)/a^3/c^6/f/(I+tan(f\*x+e))^6+1/80\*(2\*A-I\*B)/a^3/c^6/f/(I+tan(f\*x+e))^5+1/128\*(-5\*I\*A-B)/a^3/c^6/f/(I+tan(f\*x+e))^4-5/96\*A/a^3/c^6/f/(I+tan(f\*x+e))^3+5/512\*(7\*I\*A-B)/a^3/c^6/f/(I+tan(f\*x+e))^2+7/256\*(4\*A+I\*B)/a^3/c^6/f/(I+tan(f\*x+e))

**Rubi** [A]

time = 0.26, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$ , Rules used = {3669, 78, 209}

$$-\frac{7(2A+iB)}{256a^3c^6 f(i-\tan(e+fx))} + \frac{7(4A+iB)}{256a^3c^6 f(\tan(e+fx))} - \frac{-5B+7iA}{512a^3c^6 f(-\tan(e+fx)+i)^2} + \frac{5(-B+7iA)}{512a^3c^6 f(\tan(e+fx)+i)^2} + \frac{A+iB}{384a^3c^6 f(-\tan(e+fx)+i)^3} - \frac{B+5iA}{128a^3c^6 f(\tan(e+fx)+i)^3} + \frac{2A-iB}{80a^3c^6 f(\tan(e+fx)+i)^5} + \frac{B+iA}{96a^3c^6 f(\tan(e+fx)+i)^6} + \frac{7x(3A+iB)}{128a^3c^6} - \frac{5A}{96a^3c^6 f(\tan(e+fx)+i)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^6), x]

[Out] (7\*(3\*A + I\*B)\*x)/(128\*a^3\*c^6) + (A + I\*B)/(384\*a^3\*c^6\*f\*(I - Tan[e + f\*x])^3) - ((7\*I)\*A - 5\*B)/(512\*a^3\*c^6\*f\*(I - Tan[e + f\*x])^2) - (7\*(2\*A + I\*B))/(256\*a^3\*c^6\*f\*(I - Tan[e + f\*x])) + (I\*A + B)/(96\*a^3\*c^6\*f\*(I + Tan[e + f\*x])^6) + (2\*A - I\*B)/(80\*a^3\*c^6\*f\*(I + Tan[e + f\*x])^5) - ((5\*I)\*A + B)/(128\*a^3\*c^6\*f\*(I + Tan[e + f\*x])^4) - (5\*A)/(96\*a^3\*c^6\*f\*(I + Tan[e + f\*x])^3) + (5\*((7\*I)\*A - B))/(512\*a^3\*c^6\*f\*(I + Tan[e + f\*x])^2) + (7\*(4\*A + I\*B))/(256\*a^3\*c^6\*f\*(I + Tan[e + f\*x]))

**Rule 78**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 209**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ic \tan(e + fx))^6} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^7} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{(ac) \text{Subst}\left(\int \left(\frac{A+iB}{128a^4 c^7 (-i+x)^4} + \frac{i(7A+5iB)}{256a^4 c^7 (-i+x)^3} - \frac{7(2A+iB)}{256a^4 c^7 (-i+x)^2}\right) dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{A + iB}{384a^3 c^6 f (i - \tan(e + fx))^3} - \frac{7iA - 5B}{512a^3 c^6 f (i - \tan(e + fx))^2} \\ &= \frac{7(3A + iB)x}{128a^3 c^6} + \frac{A + iB}{384a^3 c^6 f (i - \tan(e + fx))^3} - \frac{7iA - 5B}{512a^3 c^6 f (i - \tan(e + fx))^2} \end{aligned}$$

### Mathematica [A]

time = 2.30, size = 321, normalized size = 1.01

Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^6), x]

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^6), x]

[Out] (Sec[e + f\*x]^3\*(-Cos[6\*(e + f\*x)] - I\*Sin[6\*(e + f\*x)])\*(-210\*(27\*A + I\*B)\*Cos[e + f\*x] + 280\*(-3\*A + I\*B - (18\*I)\*A\*f\*x + 6\*B\*f\*x)\*Cos[3\*(e + f\*x)] + 810\*A\*Cos[5\*(e + f\*x)] + (750\*I)\*B\*Cos[5\*(e + f\*x)] + 81\*A\*Cos[7\*(e + f\*x)] + (147\*I)\*B\*Cos[7\*(e + f\*x)] + 5\*A\*Cos[9\*(e + f\*x)] + (15\*I)\*B\*Cos[9\*(e + f\*x)] + (1890\*I)\*A\*Sin[e + f\*x] - 630\*B\*Sin[e + f\*x] - (840\*I)\*A\*Sin[3\*(e + f\*x)] - 280\*B\*Sin[3\*(e + f\*x)] - 5040\*A\*f\*x\*Sin[3\*(e + f\*x)] - (1680\*I)\*B\*f\*x\*Sin[3\*(e + f\*x)] - (1350\*I)\*A\*Sin[5\*(e + f\*x)] + 450\*B\*Sin[5\*(e + f\*x)] - (189\*I)\*A\*Sin[7\*(e + f\*x)] + 63\*B\*Sin[7\*(e + f\*x)] - (15\*I)\*A\*Sin[9\*(e + f\*x)] + 5\*B\*Sin[9\*(e + f\*x)])/(30720\*a^3\*c^6\*f\*(-I + Tan[e + f\*x])^3)

**Maple [A]**

time = 0.59, size = 232, normalized size = 0.73

method	result
derivativedivides	$\left(-\frac{21iA}{256} + \frac{7B}{256}\right) \ln(-i + \tan(fx+e)) - \frac{\frac{A}{128} + \frac{iB}{128}}{3(-i + \tan(fx+e))^3} - \frac{-\frac{5B}{256} + \frac{7iA}{256}}{2(-i + \tan(fx+e))^2} - \frac{-\frac{7iB}{256} - \frac{7A}{128}}{-i + \tan(fx+e)} - \frac{-\frac{iA}{16} - \frac{B}{16}}{6(i + \tan(fx+e))^6} - \frac{-\frac{35iA}{256}}{2(i + \tan(fx+e))^6} - \frac{-\frac{35iA}{256}}{fa^3c^6}$
default	$\left(-\frac{21iA}{256} + \frac{7B}{256}\right) \ln(-i + \tan(fx+e)) - \frac{\frac{A}{128} + \frac{iB}{128}}{3(-i + \tan(fx+e))^3} - \frac{-\frac{5B}{256} + \frac{7iA}{256}}{2(-i + \tan(fx+e))^2} - \frac{-\frac{7iB}{256} - \frac{7A}{128}}{-i + \tan(fx+e)} - \frac{-\frac{iA}{16} - \frac{B}{16}}{6(i + \tan(fx+e))^6} - \frac{-\frac{35iA}{256}}{2(i + \tan(fx+e))^6} - \frac{-\frac{35iA}{256}}{fa^3c^6}$
norman	$\frac{7(iB+3A)x}{128ac} - \frac{7iA+B}{30acf} + \frac{281(iB+3A)(\tan^5(fx+e))}{320acf} + \frac{231(iB+3A)(\tan^7(fx+e))}{320acf} + \frac{119(iB+3A)(\tan^9(fx+e))}{384acf} + \frac{7(iB+3A)(\tan^{11}(fx+e))}{128acf}$
risch	$-\frac{83i \cos(6fx+6e)A}{3072a^3c^6f} + \frac{21xA}{128a^3c^6} - \frac{e^{12i(fx+e)}B}{6144a^3c^6f} - \frac{45i \cos(2fx+2e)A}{512a^3c^6f} - \frac{7e^{10i(fx+e)}B}{5120a^3c^6f} + \frac{7ixB}{128a^3c^6} - \frac{5e^{8i(fx+e)}B}{1024a^3c^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f/a^3/c^6*((-21/256*I*A+7/256*B)*ln(-I+tan(f*x+e))-1/3*(1/128*A+1/128*I*B
)/(-I+tan(f*x+e))^3-1/2*(-5/256*B+7/256*I*A)/(-I+tan(f*x+e))^2-(-7/256*I*B-
7/128*A)/(-I+tan(f*x+e))-1/6*(-1/16*I*A-1/16*B)/(I+tan(f*x+e))^6-1/2*(-35/2
56*I*A+5/256*B)/(I+tan(f*x+e))^2+(21/256*I*A-7/256*B)*ln(I+tan(f*x+e))-1/4*
(5/32*I*A+1/32*B)/(I+tan(f*x+e))^4-1/5*(1/16*I*B-1/8*A)/(I+tan(f*x+e))^5-(-
7/64*A-7/256*I*B)/(I+tan(f*x+e))-5/96*A/(I+tan(f*x+e))^3)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.
```

**Fricas [A]**

time = 4.26, size = 195, normalized size = 0.61

$$\frac{(1680(3A+iB)fx^{9i}e^{9i(fx+6e)} - 5(iA+B)e^{18i(fx+12e)} - 6(9iA+7B)e^{16i(fx+16e)} - 30(9iA+5B)e^{14i(fx+14e)} - 280(3iA+B)e^{12i(fx+12e)} - 210(9iA+B)e^{10i(fx+10e)} - 420(9iA-B)e^{8i(fx+8e)} - 120(-9iA+5B)e^{6i(fx+6e)} - 15(-9iA+7B)e^{4i(fx+4e)} + 10iA - 10B)e^{-8i(fx-6e)}}{30720a^3c^6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x, alg
orithm="fricas")
```

```
[Out] 1/30720*(1680*(3*A + I*B)*f*x*e^(6*I*f*x + 6*I*e) - 5*(I*A + B)*e^(18*I*f*x
+ 18*I*e) - 6*(9*I*A + 7*B)*e^(16*I*f*x + 16*I*e) - 30*(9*I*A + 5*B)*e^(14
*I*f*x + 14*I*e) - 280*(3*I*A + B)*e^(12*I*f*x + 12*I*e) - 210*(9*I*A + B)*
e^(10*I*f*x + 10*I*e) - 420*(9*I*A - B)*e^(8*I*f*x + 8*I*e) - 120*(-9*I*A +
5*B)*e^(4*I*f*x + 4*I*e) - 15*(-9*I*A + 7*B)*e^(2*I*f*x + 2*I*e) + 10*I*A
- 10*B)*e^(-6*I*f*x - 6*I*e)/(a^3*c^6*f)
```

**Sympy [A]**

time = 0.95, size = 753, normalized size = 2.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x)
```

```
[Out] Piecewise((((6800207735332289107722240*I*A*a**24*c**48*f**8*exp(6*I*e) - 68
00207735332289107722240*B*a**24*c**48*f**8*exp(6*I*e))*exp(-6*I*f*x) + (918
02804426985902954250240*I*A*a**24*c**48*f**8*exp(8*I*e) - 71402181220989035
631083520*B*a**24*c**48*f**8*exp(8*I*e))*exp(-4*I*f*x) + (73442243541588722
3634001920*I*A*a**24*c**48*f**8*exp(10*I*e) - 408012464119937346463334400*B
*a**24*c**48*f**8*exp(10*I*e))*exp(-2*I*f*x) + (-25704785239556052827190067
20*I*A*a**24*c**48*f**8*exp(14*I*e) + 285608724883956142524334080*B*a**24*c
**48*f**8*exp(14*I*e))*exp(2*I*f*x) + (-1285239261977802641359503360*I*A*a*
*24*c**48*f**8*exp(16*I*e) - 142804362441978071262167040*B*a**24*c**48*f**8
*exp(16*I*e))*exp(4*I*f*x) + (-571217449767912285048668160*I*A*a**24*c**48*
f**8*exp(18*I*e) - 190405816589304095016222720*B*a**24*c**48*f**8*exp(18*I*
e))*exp(6*I*f*x) + (-183605608853971805908500480*I*A*a**24*c**48*f**8*exp(2
0*I*e) - 102003116029984336615833600*B*a**24*c**48*f**8*exp(20*I*e))*exp(8*
I*f*x) + (-36721121770794361181700096*I*A*a**24*c**48*f**8*exp(22*I*e) - 28
560872488395614252433408*B*a**24*c**48*f**8*exp(22*I*e))*exp(10*I*f*x) + (-
3400103867666144553861120*I*A*a**24*c**48*f**8*exp(24*I*e) - 34001038676661
44553861120*B*a**24*c**48*f**8*exp(24*I*e))*exp(12*I*f*x))*exp(-12*I*e)/(20
890238162940792138922721280*a**27*c**54*f**9), Ne(a**27*c**54*f**9*exp(12*I
*e), 0)), (x*(-(21*A + 7*I*B)/(128*a**3*c**6) + (A*exp(18*I*e) + 9*A*exp(16
*I*e) + 36*A*exp(14*I*e) + 84*A*exp(12*I*e) + 126*A*exp(10*I*e) + 126*A*exp
(8*I*e) + 84*A*exp(6*I*e) + 36*A*exp(4*I*e) + 9*A*exp(2*I*e) + A - I*B*exp(
18*I*e) - 7*I*B*exp(16*I*e) - 20*I*B*exp(14*I*e) - 28*I*B*exp(12*I*e) - 14*
I*B*exp(10*I*e) + 14*I*B*exp(8*I*e) + 28*I*B*exp(6*I*e) + 20*I*B*exp(4*I*e)
+ 7*I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(512*a**3*c**6)), True)) + x*(21*A +
7*I*B)/(128*a**3*c**6)
```

**Giac [A]**

time = 1.04, size = 319, normalized size = 1.00

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^6,x, algorithm="giac")

[Out]  $\frac{1}{15360} \cdot (420 \cdot (3 \cdot I \cdot A - B) \cdot \log(\tan(f \cdot x + e) + I) / (a^3 \cdot c^6) - 420 \cdot (3 \cdot I \cdot A - B) \cdot \log(I \cdot \tan(f \cdot x + e) + 1) / (a^3 \cdot c^6) - 10 \cdot (231 \cdot A \cdot \tan(f \cdot x + e)^3 + 77 \cdot I \cdot B \cdot \tan(f \cdot x + e)^3 - 777 \cdot I \cdot A \cdot \tan(f \cdot x + e)^2 + 273 \cdot B \cdot \tan(f \cdot x + e)^2 - 882 \cdot A \cdot \tan(f \cdot x + e) - 330 \cdot I \cdot B \cdot \tan(f \cdot x + e) + 340 \cdot I \cdot A - 138 \cdot B) / (a^3 \cdot c^6 \cdot (-I \cdot \tan(f \cdot x + e) - 1)^3) + (-3087 \cdot I \cdot A \cdot \tan(f \cdot x + e)^6 + 1029 \cdot B \cdot \tan(f \cdot x + e)^6 + 20202 \cdot A \cdot \tan(f \cdot x + e)^5 + 6594 \cdot I \cdot B \cdot \tan(f \cdot x + e)^5 + 55755 \cdot I \cdot A \cdot \tan(f \cdot x + e)^4 - 17685 \cdot B \cdot \tan(f \cdot x + e)^4 - 83540 \cdot A \cdot \tan(f \cdot x + e)^3 - 25380 \cdot I \cdot B \cdot \tan(f \cdot x + e)^3 - 72405 \cdot I \cdot A \cdot \tan(f \cdot x + e)^2 + 20415 \cdot B \cdot \tan(f \cdot x + e)^2 + 35106 \cdot A \cdot \tan(f \cdot x + e) + 8442 \cdot I \cdot B \cdot \tan(f \cdot x + e) + 7761 \cdot I \cdot A - 1127 \cdot B) / (a^3 \cdot c^6 \cdot (\tan(f \cdot x + e) + I)^6) / f$

**Mupad [B]**

time = 11.30, size = 352, normalized size = 1.10

$\frac{\tan(e+f \cdot x) \cdot (-\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) - \tan(e+f \cdot x)^2 \cdot (\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) - \tan(e+f \cdot x)^3 \cdot (-\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) + \tan(e+f \cdot x)^4 \cdot (\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) - \tan(e+f \cdot x)^5 \cdot (-\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) + \tan(e+f \cdot x)^6 \cdot (\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) - \tan(e+f \cdot x)^7 \cdot (-\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) + \tan(e+f \cdot x)^8 \cdot (\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) - \tan(e+f \cdot x)^9 \cdot (-\frac{221B}{128a^3c^6} + \frac{cB^2}{128a^3c^6}) + \frac{7x(3A+B)}{128a^3c^6}}{f \cdot (-\tan(e+f \cdot x)^2 - \tan(e+f \cdot x)^3 - \tan(e+f \cdot x)^4 + 6 \tan(e+f \cdot x)^5 - \tan(e+f \cdot x)^6 + 8 \tan(e+f \cdot x)^7 + 3 \tan(e+f \cdot x)^8 + 1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)^6),x)

[Out]  $(\tan(e + f \cdot x) \cdot ((A \cdot 87i) / (640 \cdot a^3 \cdot c^6) - (29 \cdot B) / (640 \cdot a^3 \cdot c^6)) - \tan(e + f \cdot x)^8 \cdot ((21 \cdot A) / (128 \cdot a^3 \cdot c^6) + (B \cdot 7i) / (128 \cdot a^3 \cdot c^6)) - \tan(e + f \cdot x)^7 \cdot ((A \cdot 63i) / (128 \cdot a^3 \cdot c^6) - (21 \cdot B) / (128 \cdot a^3 \cdot c^6)) + \tan(e + f \cdot x)^2 \cdot ((129 \cdot A) / (128 \cdot a^3 \cdot c^6) + (B \cdot 43i) / (128 \cdot a^3 \cdot c^6)) - \tan(e + f \cdot x)^5 \cdot ((A \cdot 147i) / (128 \cdot a^3 \cdot c^6) - (49 \cdot B) / (128 \cdot a^3 \cdot c^6)) + \tan(e + f \cdot x)^6 \cdot ((7 \cdot A) / (128 \cdot a^3 \cdot c^6) + (B \cdot 7i) / (384 \cdot a^3 \cdot c^6)) + \tan(e + f \cdot x)^4 \cdot ((609 \cdot A) / (640 \cdot a^3 \cdot c^6) + (B \cdot 203i) / (640 \cdot a^3 \cdot c^6)) - \tan(e + f \cdot x)^3 \cdot ((A \cdot 413i) / (640 \cdot a^3 \cdot c^6) - (413 \cdot B) / (1920 \cdot a^3 \cdot c^6)) + (7 \cdot A) / (30 \cdot a^3 \cdot c^6) - (B \cdot 1i) / (30 \cdot a^3 \cdot c^6) / (f \cdot (3 \cdot \tan(e + f \cdot x) + 8 \cdot \tan(e + f \cdot x)^3 - \tan(e + f \cdot x)^4 \cdot 6i + 6 \cdot \tan(e + f \cdot x)^5 - \tan(e + f \cdot x)^6 \cdot 8i - \tan(e + f \cdot x)^8 \cdot 3i - \tan(e + f \cdot x)^9 + 1i)) + (7 \cdot x \cdot (3 \cdot A + B \cdot 1i)) / (128 \cdot a^3 \cdot c^6)$

$$3.740 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ictan(e + fx))^{7/2} dx$$

Optimal. Leaf size=62

$$\frac{2a(iA + B)(c - ictan(e + fx))^{7/2}}{7f} - \frac{2aB(c - ictan(e + fx))^{9/2}}{9cf}$$

[Out]  $2/7*a*(I*A+B)*(c-I*c*tan(f*x+e))^{(7/2)}/f-2/9*a*B*(c-I*c*tan(f*x+e))^{(9/2)}/c/f$

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2a(B + iA)(c - ictan(e + fx))^{7/2}}{7f} - \frac{2aB(c - ictan(e + fx))^{9/2}}{9cf}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2), x]

[Out]  $(2*a*(I*A + B)*(c - I*c*Tan[e + f*x])^{(7/2)})/(7*f) - (2*a*B*(c - I*c*Tan[e + f*x])^{(9/2)})/(9*c*f)$

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^{7/2} dx = \frac{(ac) \text{Subst}\left(\int (A + Bx)(c - icx)^{5/2} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left((A - iB)(c - icx)\right)^{5/2} dx\right)}{f}$$

$$= \frac{2a(iA + B)(c - ic \tan(e + fx))^{7/2}}{7f}$$

**Mathematica [A]**

time = 2.11, size = 90, normalized size = 1.45

$$\frac{2ac^3 \sec^3(e + fx)(\cos(fx) - i \sin(fx))(i \cos(3e + 2fx) + \sin(3e + 2fx))(9A - 2iB + 7B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{63f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2), x]
```

```
[Out] (2*a*c^3*Sec[e + f*x]^3*(Cos[f*x] - I*Sin[f*x])*(I*Cos[3*e + 2*f*x] + Sin[3*e + 2*f*x])*(9*A - (2*I)*B + 7*B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(63*f)
```

**Maple [A]**

time = 0.53, size = 55, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2ia \left( \frac{iB(c - ic \tan(fx + e))^{\frac{9}{2}}}{9} + \frac{(-iBc + Ac)(c - ic \tan(fx + e))^{\frac{7}{2}}}{7} \right)}{fc}$	55
default	$\frac{2ia \left( \frac{iB(c - ic \tan(fx + e))^{\frac{9}{2}}}{9} + \frac{(-iBc + Ac)(c - ic \tan(fx + e))^{\frac{7}{2}}}{7} \right)}{fc}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2), x, method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a/c*(1/9*I*B*(c-I*c*tan(f*x+e))^(9/2)+1/7*(-I*B*c+A*c)*(c-I*c*tan(f*x+e))^(7/2))
```

**Maxima [A]**

time = 0.30, size = 50, normalized size = 0.81

$$\frac{2i \left( 7i(-ic \tan(fx + e) + c)^{\frac{9}{2}} Ba + 9(-ic \tan(fx + e) + c)^{\frac{7}{2}} (A - iB)ac \right)}{63cf}$$



[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)\*(-I\*c\*tan(f\*x + e) + c)^(7/2), x)

**Mupad [B]**

time = 14.38, size = 101, normalized size = 1.63

$$\frac{16 a c^3 \sqrt{c + \frac{c (e^{e 2i + f x 2i} 1i - i) 1i}{e^{e 2i + f x 2i} + 1}} (A 9i - 5 B + A e^{e 2i + f x 2i} 9i + 9 B e^{e 2i + f x 2i})}{63 f (e^{e 2i + f x 2i} + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] (16\*a\*c^3\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*(A\*9i - 5\*B + A\*exp(e\*2i + f\*x\*2i)\*9i + 9\*B\*exp(e\*2i + f\*x\*2i)))/(63\*f\*(exp(e\*2i + f\*x\*2i) + 1)^4)

$$3.741 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ictan(e + fx))^{5/2} dx$$

Optimal. Leaf size=62

$$\frac{2a(iA + B)(c - ictan(e + fx))^{5/2}}{5f} - \frac{2aB(c - ictan(e + fx))^{7/2}}{7cf}$$

[Out]  $2/5*a*(I*A+B)*(c-I*c*tan(f*x+e))^{(5/2)}/f-2/7*a*B*(c-I*c*tan(f*x+e))^{(7/2)}/c/f$

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2a(B + iA)(c - ictan(e + fx))^{5/2}}{5f} - \frac{2aB(c - ictan(e + fx))^{7/2}}{7cf}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out]  $(2*a*(I*A + B)*(c - I*c*Tan[e + f*x])^{(5/2)})/(5*f) - (2*a*B*(c - I*c*Tan[e + f*x])^{(7/2)})/(7*c*f)$

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_.)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^{5/2} dx = \frac{(ac) \text{Subst}\left(\int (A + Bx)(c - icx)^{3/2} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left((A - iB)(c - icx)\right)^{3/2} dx\right)}{f}$$

$$= \frac{2a(iA + B)(c - ic \tan(e + fx))^{5/2}}{5f}$$

**Mathematica [A]**

time = 1.43, size = 88, normalized size = 1.42

$$\frac{2ac^2 \sec^2(e + fx)(\cos(fx) - i \sin(fx))(i \cos(2e + fx) + \sin(2e + fx))(7A - 2iB + 5B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{35f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] (2\*a\*c^2\*Sec[e + f\*x]^2\*(Cos[f\*x] - I\*Sin[f\*x])\*(I\*Cos[2\*e + f\*x] + Sin[2\*e + f\*x])\*(7\*A - (2\*I)\*B + 5\*B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(35\*f)

**Maple [A]**

time = 0.46, size = 55, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2ia \left( \frac{iB(c - ic \tan(fx + e))^{\frac{7}{2}}}{7} + \frac{(-iBc + Ae)(c - ic \tan(fx + e))^{\frac{5}{2}}}{5} \right)}{fc}$	55
default	$\frac{2ia \left( \frac{iB(c - ic \tan(fx + e))^{\frac{7}{2}}}{7} + \frac{(-iBc + Ae)(c - ic \tan(fx + e))^{\frac{5}{2}}}{5} \right)}{fc}$	55

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2), x, method=\_RETURNVERBOSE)

[Out] 2\*I/f\*a/c\*(1/7\*I\*B\*(c-I\*c\*tan(f\*x+e))^(7/2)+1/5\*(-I\*B\*c+A\*c)\*(c-I\*c\*tan(f\*x+e))^(5/2))

**Maxima [A]**

time = 0.28, size = 50, normalized size = 0.81

$$\frac{2i \left( 5i(-ic \tan(fx + e) + c)^{\frac{7}{2}} Ba + 7(-ic \tan(fx + e) + c)^{\frac{5}{2}} (A - iB)ac \right)}{35cf}$$





[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)\*(-I\*c\*tan(f\*x + e) + c)^(5/2), x)

**Mupad [B]**

time = 15.53, size = 101, normalized size = 1.63

$$\frac{8ac^2 \sqrt{c + \frac{c(e^{e2i+fx2i} - 1) - i}{e^{e2i+fx2i} + 1}} (A7i - 3B + Ae^{e2i+fx2i}7i + 7Be^{e2i+fx2i})}{35f(e^{e2i+fx2i} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] (8\*a\*c^2\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*(A\*7i - 3\*B + A\*exp(e\*2i + f\*x\*2i)\*7i + 7\*B\*exp(e\*2i + f\*x\*2i)))/(35\*f\*(exp(e\*2i + f\*x\*2i) + 1)^3)

$$3.742 \quad \int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ictan(e + fx))^{3/2} dx$$

Optimal. Leaf size=62

$$\frac{2a(iA + B)(c - ictan(e + fx))^{3/2}}{3f} - \frac{2aB(c - ictan(e + fx))^{5/2}}{5cf}$$

[Out] 2/3\*a\*(I\*A+B)\*(c-I\*c\*tan(f\*x+e))^(3/2)/f-2/5\*a\*B\*(c-I\*c\*tan(f\*x+e))^(5/2)/c/f

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2a(B + iA)(c - ictan(e + fx))^{3/2}}{3f} - \frac{2aB(c - ictan(e + fx))^{5/2}}{5cf}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (2\*a\*(I\*A + B)\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(3\*f) - (2\*a\*B\*(c - I\*c\*Tan[e + f\*x])^(5/2))/(5\*c\*f)

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2} dx = \frac{(ac) \text{Subst}\left(\int (A + Bx) \sqrt{c - icx} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int ((A - iB) \sqrt{c - icx}) dx\right)}{f}$$

$$= \frac{2a(iA + B)(c - ic \tan(e + fx))^{3/2}}{3f}$$

**Mathematica [A]**

time = 1.10, size = 97, normalized size = 1.56

$$\frac{2ac(\cos(e) - i \sin(e))(\cos(fx) - i \sin(fx))(5iA + 2B + 3iB \tan(e + fx))(A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{15f(A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2), x]
```

```
[Out] (2*a*c*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*((5*I)*A + 2*B + (3*I)*B*Tan[e + f*x])*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(15*f*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

**Maple [A]**

time = 0.45, size = 55, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2ia \left( \frac{iB(c - ic \tan(fx + e))^{\frac{5}{2}}}{5} + \frac{(-iBc + Ac)(c - ic \tan(fx + e))^{\frac{3}{2}}}{3} \right)}{fc}$	55
default	$\frac{2ia \left( \frac{iB(c - ic \tan(fx + e))^{\frac{5}{2}}}{5} + \frac{(-iBc + Ac)(c - ic \tan(fx + e))^{\frac{3}{2}}}{3} \right)}{fc}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2), x, method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a/c*(1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)+1/3*(-I*B*c+A*c)*(c-I*c*tan(f*x+e))^(3/2))
```

**Maxima [A]**

time = 0.29, size = 50, normalized size = 0.81

$$\frac{2i \left( 3i(-ic \tan(fx + e) + c)^{\frac{5}{2}} Ba + 5(-ic \tan(fx + e) + c)^{\frac{3}{2}} (A - iB)ac \right)}{15cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="maxima")

[Out]  $\frac{2}{15}I*(3*I*(-I*c*\tan(f*x + e) + c)^{(5/2)}*B*a + 5*(-I*c*\tan(f*x + e) + c)^{(3/2)}*(A - I*B)*a*c)/(c*f)$

**Fricas** [A]

time = 0.98, size = 82, normalized size = 1.32

$$\frac{4\sqrt{2}\left(5(-iA - B)ace^{(2ifx+2ie)} + (-5iA + B)ac\right)\sqrt{\frac{c}{e^{(2ifx+2ie)} + 1}}}{15\left(fe^{(4ifx+4ie)} + 2fe^{(2ifx+2ie)} + f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out]  $\frac{-4}{15}\sqrt{2}*(5*(-I*A - B)*a*c*e^{(2I*f*x + 2I*e)} + (-5*I*A + B)*a*c)*\sqrt{c/(e^{(2I*f*x + 2I*e)} + 1)}/(f*e^{(4I*f*x + 4I*e)} + 2*f*e^{(2I*f*x + 2I*e)} + f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int(-iAc\sqrt{-ictan(e+fx)+c})dx + \int(-iAc\sqrt{-ictan(e+fx)+c}\tan^2(e+fx))dx + \int(-iBc\sqrt{-ictan(e+fx)+c}\tan(e+fx))dx + \int(-iBc\sqrt{-ictan(e+fx)+c}\tan^3(e+fx))dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x)

[Out]  $I*a*(\text{Integral}(-I*A*c*\sqrt{-I*c*\tan(e + f*x) + c}, x) + \text{Integral}(-I*A*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**2, x) + \text{Integral}(-I*B*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x), x) + \text{Integral}(-I*B*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**3, x))$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)\*(-I\*c\*tan(f\*x + e) + c)^(3/2), x)

**Mupad [B]**

time = 11.95, size = 99, normalized size = 1.60

$$\frac{4ac \sqrt{c + \frac{c(e^{e^{2i+fx^{2i}} li - i) li}{e^{e^{2i+fx^{2i}} + 1}}}}{(A5i - B + A e^{e^{2i+fx^{2i}} 5i + 5B e^{e^{2i+fx^{2i}}})}}}{15f(e^{e^{2i+fx^{2i}} + 1})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(3/2),x)`

[Out] `(4*a*c*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*(A*5i - B + A*exp(e*2i + f*x*2i)*5i + 5*B*exp(e*2i + f*x*2i)))/(15*f*(exp(e*2i + f*x*2i) + 1)^2)`

### 3.743 $\int (a+ia \tan(e+fx))(A+B \tan(e+fx)) \sqrt{c-ictan(e+fx)}$

Optimal. Leaf size=60

$$\frac{2a(iA+B)\sqrt{c-ictan(e+fx)}}{f} - \frac{2aB(c-ictan(e+fx))^{3/2}}{3cf}$$

[Out]  $2*a*(I*A+B)*(c-I*c*\tan(f*x+e))^{(1/2)}/f-2/3*a*B*(c-I*c*\tan(f*x+e))^{(3/2)}/c/f$

Rubi [A]

time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2a(B+iA)\sqrt{c-ictan(e+fx)}}{f} - \frac{2aB(c-ictan(e+fx))^{3/2}}{3cf}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])*(A + B*\text{Tan}[e + f*x])*Sqrt[c - I*c*\text{Tan}[e + f*x]], x]$

[Out]  $(2*a*(I*A + B)*Sqrt[c - I*c*\text{Tan}[e + f*x]])/f - (2*a*B*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*c*f)$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 3669

$\text{Int}[(a_. + (b_.)*\tan[(e_. + (f_.)*(x_.))]^{(m_.)*((A_. + (B_.)*\tan[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a + ia \tan(e + fx))(A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{\sqrt{c-icx}} dx, x, \tan(e+fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{A-iB}{\sqrt{c-icx}} + \frac{iB\sqrt{c-icx}}{c} \right) dx, x, \tan(e+fx) \right)}{f}$$

$$= \frac{2a(iA + B) \sqrt{c - ic \tan(e + fx)}}{f}$$

**Mathematica [A]**

time = 0.81, size = 45, normalized size = 0.75

$$\frac{2a(3iA + 2B + iB \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] (2*a*((3*I)*A + 2*B + I*B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*f)
```

**Maple [A]**

time = 0.27, size = 66, normalized size = 1.10

method	result	size
derivativedivides	$\frac{2ia \left( \frac{iB(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} - iBc \sqrt{c - ic \tan(fx + e)} + Ac \sqrt{c - ic \tan(fx + e)} \right)}{fc}$	66
default	$\frac{2ia \left( \frac{iB(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} - iBc \sqrt{c - ic \tan(fx + e)} + Ac \sqrt{c - ic \tan(fx + e)} \right)}{fc}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a/c*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)-I*B*c*(c-I*c*tan(f*x+e))^(1/2)+A*c*(c-I*c*tan(f*x+e))^(1/2))
```

**Maxima [A]**

time = 0.28, size = 50, normalized size = 0.83

$$\frac{2i \left( i(-ic \tan(fx + e) + c)^{\frac{3}{2}} Ba + 3 \sqrt{-ic \tan(fx + e) + c} (A - iB)ac \right)}{3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x, algorithm="maxima")

[Out]  $\frac{2}{3}I*(I*(-I*c*\tan(f*x + e) + c)^{(3/2)}*B*a + 3*\sqrt{-I*c*\tan(f*x + e) + c}*(A - I*B)*a*c)/(c*f)$

**Fricas** [A]

time = 1.18, size = 69, normalized size = 1.15

$$-\frac{2\sqrt{2}\left(3(-iA-B)ae^{(2ifx+2ie)} + (-3iA-B)a\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}}{3\left(fe^{(2ifx+2ie)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x, algorithm="fricas")

[Out]  $\frac{-2/3*\sqrt{2}*(3*(-I*A - B)*a*e^{(2I*f*x + 2I*e)} + (-3*I*A - B)*a)*\sqrt{c/(e^{(2I*f*x + 2I*e)} + 1)}}{(f*e^{(2I*f*x + 2I*e)} + f)}$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int(-iA\sqrt{-i\tan(e+fx)+c})dx + \int A\sqrt{-i\tan(e+fx)+c}\tan(e+fx)dx + \int B\sqrt{-i\tan(e+fx)+c}\tan^2(e+fx)dx + \int(-iB\sqrt{-i\tan(e+fx)+c}\tan(e+fx))dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x)

[Out]  $I*a*(\text{Integral}(-I*A*\sqrt{-I*c*\tan(e + f*x) + c}, x) + \text{Integral}(A*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x), x) + \text{Integral}(B*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**2, x) + \text{Integral}(-I*B*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x), x))$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)\*sqrt(-I\*c\*tan(f\*x + e) + c), x)



**Mupad [B]**

time = 0.69, size = 102, normalized size = 1.70

$$a \sqrt{\frac{c(-2\cos(e+fx)^2 + \sin(2e+2fx)1i)}{2\cos(e+fx)^2}} \frac{(A3i + 2B + A(2\cos(e+fx)^2 - 1)3i + 2B(2\cos(e+fx)^2 - 1) + B\sin(2e+2fx)1i)}{3f\cos(e+fx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^(1/2),x)

[Out] (a\*(-(c\*(sin(2\*e + 2\*f\*x)\*1i - 2\*cos(e + f\*x)^2))/(2\*cos(e + f\*x)^2))^(1/2)\*(A\*3i + 2\*B + A\*(2\*cos(e + f\*x)^2 - 1)\*3i + 2\*B\*(2\*cos(e + f\*x)^2 - 1) + B\*sin(2\*e + 2\*f\*x)\*1i))/(3\*f\*cos(e + f\*x)^2)

$$3.744 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{\sqrt{c-ictan(e+fx)}} dx$$

Optimal. Leaf size=58

$$-\frac{2a(iA+B)}{f\sqrt{c-ictan(e+fx)}} - \frac{2aB\sqrt{c-ictan(e+fx)}}{cf}$$

[Out]  $-2*a*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(1/2)}-2*a*B*(c-I*c*\tan(f*x+e))^{(1/2)}/c/f$

Rubi [A]

time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$-\frac{2a(B+iA)}{f\sqrt{c-ictan(e+fx)}} - \frac{2aB\sqrt{c-ictan(e+fx)}}{cf}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out]  $(-2*a*(I*A + B))/(f*Sqrt[c - I*c*Tan[e + f*x]]) - (2*a*B*Sqrt[c - I*c*Tan[e + f*x]])/(c*f)$

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(m\_.)\*((A\_) + (B\_.)\*tan[(e\_) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{\sqrt{c - ic \tan(e + fx)}} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(c-icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{A-iB}{(c-icx)^{3/2}} + \frac{iB}{c\sqrt{c-icx}} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{2a(iA + B)}{f\sqrt{c - ic \tan(e + fx)}} - \frac{2aB\sqrt{c - ic \tan(e + fx)}}{cf}$$

**Mathematica [A]**

time = 0.83, size = 82, normalized size = 1.41

$$\frac{2a(\cos(fx) - i \sin(fx))(A - 2iB) \cos(e + fx) - B \sin(e + fx)(-i \cos(e + 2fx) + \sin(e + 2fx))\sqrt{c - ic \tan(e + fx)}}{cf}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]],x]

[Out] (2\*a\*(Cos[f\*x] - I\*Sin[f\*x])\*((A - (2\*I)\*B)\*Cos[e + f\*x] - B\*Sin[e + f\*x])\*((-I)\*Cos[e + 2\*f\*x] + Sin[e + 2\*f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(c\*f)

**Maple [A]**

time = 0.22, size = 53, normalized size = 0.91

method	result	size
derivativedivides	$\frac{2ia \left( iB \sqrt{c - ic \tan(fx + e)} - \frac{c(-iB+A)}{\sqrt{c - ic \tan(fx + e)}} \right)}{fc}$	53
default	$\frac{2ia \left( iB \sqrt{c - ic \tan(fx + e)} - \frac{c(-iB+A)}{\sqrt{c - ic \tan(fx + e)}} \right)}{fc}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out] 2\*I/f\*a/c\*(I\*B\*(c-I\*c\*tan(f\*x+e))^(1/2)-c\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.30, size = 50, normalized size = 0.86

$$\frac{2i \left( i \sqrt{-ic \tan(fx + e) + c} Ba - \frac{(A-iB)ac}{\sqrt{-ic \tan(fx + e) + c}} \right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] 2\*I\*(I\*sqrt(-I\*c\*tan(f\*x + e) + c)\*B\*a - (A - I\*B)\*a\*c/sqrt(-I\*c\*tan(f\*x + e) + c))/(c\*f)

**Fricas [A]**

time = 3.11, size = 57, normalized size = 0.98

$$\frac{\sqrt{2} \left( (-iA - B)ae^{(2ifx+2ie)} + (-iA - 3B)a \right) \sqrt{\frac{c}{e^{(2ifx+2ie)} + 1}}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] sqrt(2)\*((-I\*A - B)\*a\*e^(2\*I\*f\*x + 2\*I\*e) + (-I\*A - 3\*B)\*a)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$ia \left( \int \left( -\frac{iA}{\sqrt{-ic \tan(e+fx)+c}} \right) dx + \int \frac{A \tan(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} dx + \int \frac{B \tan^2(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} dx + \int \left( -\frac{iB \tan(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x)

[Out] I\*a\*(Integral(-I\*A/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(A\*tan(e + f\*x)/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(B\*tan(e + f\*x)\*\*2/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-I\*B\*tan(e + f\*x)/sqrt(-I\*c\*tan(e + f\*x) + c), x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)/sqrt(-I\*c\*tan(f\*x + e) + c), x)

Mupad [B]

time = 10.16, size = 164, normalized size = 2.83

$$\frac{a \sqrt{\frac{2c}{e^{e^{2i+fx^{2i}}+1}}} \left( A \operatorname{li} + 3B + A \left( \frac{e^{-e^{2i-fx^{2i}}}}{2} + \frac{e^{e^{2i+fx^{2i}}}}{2} \right) \operatorname{li} - A \left( \frac{e^{-e^{2i-fx^{2i}}}}{2} \operatorname{li} - \frac{e^{e^{2i+fx^{2i}}}}{2} \operatorname{li} \right) + B \left( \frac{e^{-e^{2i-fx^{2i}}}}{2} + \frac{e^{e^{2i+fx^{2i}}}}{2} \right) + B \left( \frac{e^{-e^{2i-fx^{2i}}}}{2} \operatorname{li} - \frac{e^{e^{2i+fx^{2i}}}}{2} \operatorname{li} \right) \operatorname{li} \right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)^(1/2),x)`

[Out] `-(a*((2*c)/(exp(e*2i + f*x*2i) + 1))^(1/2)*(A*1i + 3*B + A*(exp(- e*2i - f*x*2i)/2 + exp(e*2i + f*x*2i)/2)*1i - A*((exp(- e*2i - f*x*2i)*1i)/2 - (exp(e*2i + f*x*2i)*1i)/2) + B*(exp(- e*2i - f*x*2i)/2 + exp(e*2i + f*x*2i)/2) + B*((exp(- e*2i - f*x*2i)*1i)/2 - (exp(e*2i + f*x*2i)*1i)/2)*1i))/(c*f)`

$$3.745 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ictan(e+fx))^{3/2}} dx$$

Optimal. Leaf size=60

$$-\frac{2a(iA+B)}{3f(c-ictan(e+fx))^{3/2}} + \frac{2aB}{cf\sqrt{c-ictan(e+fx)}}$$

[Out]  $2*a*B/c/f/(c-I*c*tan(f*x+e))^{(1/2)}-2/3*a*(I*A+B)/f/(c-I*c*tan(f*x+e))^{(3/2)}$

**Rubi** [A]

time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2aB}{cf\sqrt{c-ictan(e+fx)}} - \frac{2a(B+iA)}{3f(c-ictan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out]  $(-2*a*(I*A + B))/(3*f*(c - I*c*Tan[e + f*x])^{(3/2)}) + (2*a*B)/(c*f*Sqrt[c - I*c*Tan[e + f*x]])$

Rule 45

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{A-iB}{(c-icx)^{5/2}} + \frac{iB}{c(c-icx)^{3/2}} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{2a(iA + B)}{3f(c - ic \tan(e + fx))^{3/2}} + \frac{2aB}{cf \sqrt{c - ic \tan(e + fx)}}$$

**Mathematica [A]**

time = 1.23, size = 98, normalized size = 1.63

$$\frac{2a \cos(e + fx)(\cos(fx) - i \sin(fx))((-iA + 2B) \cos(e + fx) - 3iB \sin(e + fx))(\cos(2e + 3fx) + i \sin(2e + 3fx)) \sqrt{c - ic \tan(e + fx)}}{3c^2 f}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (2\*a\*Cos[e + f\*x]\*(Cos[f\*x] - I\*Sin[f\*x])\*((( -I)\*A + 2\*B)\*Cos[e + f\*x] - (3\*I)\*B\*Sin[e + f\*x])\*(Cos[2\*e + 3\*f\*x] + I\*Sin[2\*e + 3\*f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*c^2\*f)

**Maple [A]**

time = 0.23, size = 53, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2ia \left( -\frac{iB}{\sqrt{c - ic \tan(fx + e)}} - \frac{c(-iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} \right)}{fc}$	53
default	$\frac{2ia \left( -\frac{iB}{\sqrt{c - ic \tan(fx + e)}} - \frac{c(-iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} \right)}{fc}$	53
risch	$-\frac{a(iA e^{2i(fx+e)} + B e^{2i(fx+e)} + iA - 5B) \sqrt{2}}{6c \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2), x, method=\_RETURNVERBOSE)

[Out] 2\*I/f\*a/c\*(-I\*B/(c-I\*c\*tan(f\*x+e))^(1/2)-1/3\*c\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(3/2))

**Maxima [A]**

time = 0.29, size = 47, normalized size = 0.78

$$\frac{2i(3i(-ictan(fx+e)+c)Ba+(A-iB)ac)}{3(-ictan(fx+e)+c)^{\frac{3}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*(3*I*(-I*c*tan(f*x+e)+c)*B*a+(A-I*B)*a*c)/((-I*c*tan(f*x+e)+c)^(3/2)*c*f)
```

**Fricas [A]**

time = 2.37, size = 78, normalized size = 1.30

$$\frac{\sqrt{2}((-iA-B)ae^{(4i fx+4i e)}-2(iA-2B)ae^{(2i fx+2i e)}+(-iA+5B)a)\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}}{6c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*((-I*A-B)*a*e^(4*I*f*x+4*I*e)-2*(I*A-2*B)*a*e^(2*I*f*x+2*I*e)+(-I*A+5*B)*a)*sqrt(c/(e^(2*I*f*x+2*I*e)+1))/(c^2*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \frac{A}{-ic\sqrt{-ictan(e+fx)+c}\tan(e+fx)+c\sqrt{-ictan(e+fx)+c}} dx + \int \frac{A\tan(e+fx)}{-ic\sqrt{-ictan(e+fx)+c}\tan(e+fx)+c\sqrt{-ictan(e+fx)+c}} dx + \int \frac{B\tan^2(e+fx)}{-ic\sqrt{-ictan(e+fx)+c}\tan(e+fx)+c\sqrt{-ictan(e+fx)+c}} dx + \int \frac{iB\tan(e+fx)}{-ic\sqrt{-ictan(e+fx)+c}\tan(e+fx)+c\sqrt{-ictan(e+fx)+c}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

```
[Out] I*a*(Integral(-I*A/(-I*c*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)+c*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(A*tan(e+f*x)/(-I*c*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)+c*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(B*tan(e+f*x)**2/(-I*c*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)+c*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(-I*B*tan(e+f*x)/(-I*c*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)+c*sqrt(-I*c*tan(e+f*x)+c)),x))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

**Mupad [B]**

time = 10.19, size = 170, normalized size = 2.83

$$\frac{a \sqrt{\frac{c(-2\cos(e+fx)^2 + \sin(2e+2fx)1i)}{2\cos(e+fx)^2}} (B(2\cos(2e+2fx)^2 - 1) - 4B(2\cos(e+fx)^2 - 1) - 2A\sin(2e+2fx) - A\sin(4e+4fx) - 5B + A1i + A(2\cos(e+fx)^2 - 1)2i - B\sin(2e+2fx)4i + B\sin(4e+4fx)1i + A(2\cos(2e+2fx)^2 - 1)1i)}{6c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] -(a*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2)*(A*1i - 5*B + A*(2*cos(e + f*x)^2 - 1)*2i - 4*B*(2*cos(e + f*x)^2 - 1) - 2*A*sin(2*e + 2*f*x) - A*sin(4*e + 4*f*x) - B*sin(2*e + 2*f*x)*4i + B*sin(4*e + 4*f*x)*1i + A*(2*cos(2*e + 2*f*x)^2 - 1)*1i + B*(2*cos(2*e + 2*f*x)^2 - 1)))/(6*c^2*f)
```

$$3.746 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ictan(e+fx))^{5/2}} dx$$

Optimal. Leaf size=62

$$-\frac{2a(iA+B)}{5f(c-ictan(e+fx))^{5/2}} + \frac{2aB}{3cf(c-ictan(e+fx))^{3/2}}$$

[Out]  $-2/5*a*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(5/2)}+2/3*a*B/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2aB}{3cf(c-ictan(e+fx))^{3/2}} - \frac{2a(B+iA)}{5f(c-ictan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^{(5/2)}}, x]$

[Out]  $(-2*a*(I*A + B))/(5*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) + (2*a*B)/(3*c*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(c-icx)^{7/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(\frac{A-iB}{(c-icx)^{7/2}} + \frac{iB}{c(c-icx)^{5/2}}\right) dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{2a(iA + B)}{5f(c - ic \tan(e + fx))^{5/2}} + \frac{2aB}{3cf(c - ic \tan(e + fx))^{3/2}}$$

**Mathematica [A]**

time = 2.19, size = 100, normalized size = 1.61

$$\frac{2a \cos^2(e + fx)(\cos(fx) - i \sin(fx))((-3iA + 2B) \cos(e + fx) - 5iB \sin(e + fx))(\cos(3e + 4fx) + i \sin(3e + 4fx)) \sqrt{c - ic \tan(e + fx)}}{15c^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(5/2), x]
```

```
[Out] (2*a*Cos[e + f*x]^2*(Cos[f*x] - I*Sin[f*x])*(((3*I)*A + 2*B)*Cos[e + f*x] - (5*I)*B*Sin[e + f*x])*(Cos[3*e + 4*f*x] + I*Sin[3*e + 4*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(15*c^3*f)
```

**Maple [A]**

time = 0.24, size = 53, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2ia \left( -\frac{c(-iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{iB}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} \right)}{fc}$	53
default	$\frac{2ia \left( -\frac{c(-iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{iB}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} \right)}{fc}$	53
risch	$-\frac{a(3iA e^{4i(fx+e)} + 3B e^{4i(fx+e)} + 6iA e^{2i(fx+e)} - 4B e^{2i(fx+e)} + 3iA - 7B) \sqrt{2}}{60c^2 \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2), x, method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a/c*(-1/5*c*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2)-1/3*I*B/(c-I*c*tan(f*x+e)))^(3/2)
```

**Maxima [A]**

time = 0.28, size = 48, normalized size = 0.77

$$\frac{2i(5i(-ictan(fx+e)+c)Ba+3(A-iB)ac)}{15(-ictan(fx+e)+c)^{\frac{5}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="maxima")

[Out] -2/15\*I\*(5\*I\*(-I\*c\*tan(f\*x+e)+c)\*B\*a+3\*(A-I\*B)\*a\*c)/((-I\*c\*tan(f\*x+e)+c)^(5/2)\*c\*f)

**Fricas [A]**

time = 2.60, size = 96, normalized size = 1.55

$$\frac{\sqrt{2}(3(iA+B)ae^{(6i fx+6i e)} - (-9iA+B)ae^{(4i fx+4i e)} - (-9iA+11B)ae^{(2i fx+2i e)} - (-3iA+7B)a)\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}}{60c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/60\*sqrt(2)\*(3\*(I\*A+B)\*a\*e^(6\*I\*f\*x+6\*I\*e) - (-9\*I\*A+B)\*a\*e^(4\*I\*f\*x+4\*I\*e) - (-9\*I\*A+11\*B)\*a\*e^(2\*I\*f\*x+2\*I\*e) - (-3\*I\*A+7\*B)\*a)\*sqrt(c/(e^(2\*I\*f\*x+2\*I\*e)+1))/(c^3\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \tan(x+e) + B \tan^2(x+e)}{(c - I c \tan(x+e))^{5/2}} dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(5/2),x)

[Out] I\*a\*(Integral(-I\*A/(-c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2 - 2\*I\*c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)),x)+Integral(A\*tan(e+f\*x)/(-c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2 - 2\*I\*c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)),x)+Integral(B\*tan(e+f\*x)\*\*2/(-c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2 - 2\*I\*c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)),x)+Integral(-I\*B\*tan(e+f\*x)/(-c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2 - 2\*I\*c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*2\*sqrt(-I\*c\*tan(e+f\*x)+c)),x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)/(-I\*c\*tan(f\*x + e) + c)^(5/2), x)

**Mupad [B]**

time = 10.95, size = 232, normalized size = 3.74

$$\frac{\sqrt{\frac{c(-2\cos(e+fx)^2 + \sin(2e+2fx))}{2\cos(e+fx)}}}{\sin^2 f} \frac{(7B + 11B(2\cos(e+fx)^2 - 1) + 9A\sin(2e+2fx) + 9A\sin(4e+4fx) + 3A\sin(6e+6fx) + B(2\cos(2e+2fx)^2 - 1) - 3B(2\cos(3e+3fx)^2 - 1) - AB - A(2\cos(e+fx)^2 - 1)B + B\sin(2e+2fx)\sin(4e+4fx) + B\sin(4e+4fx)\sin(6e+6fx) - B\sin(6e+6fx)B - A(2\cos(2e+2fx)^2 - 1)B - A(2\cos(3e+3fx)^2 - 1)B)}{\sin^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i))/(c - c\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] (a\*(-(c\*(sin(2\*e + 2\*f\*x)\*1i - 2\*cos(e + f\*x)^2))/(2\*cos(e + f\*x)^2))^(1/2) \* (7\*B - A\*3i - A\*(2\*cos(e + f\*x)^2 - 1)\*9i + 11\*B\*(2\*cos(e + f\*x)^2 - 1) + 9\*A\*sin(2\*e + 2\*f\*x) + 9\*A\*sin(4\*e + 4\*f\*x) + 3\*A\*sin(6\*e + 6\*f\*x) + B\*sin(2\*e + 2\*f\*x)\*1i + B\*sin(4\*e + 4\*f\*x)\*1i - B\*sin(6\*e + 6\*f\*x)\*3i - A\*(2\*cos(2\*e + 2\*f\*x)^2 - 1)\*9i - A\*(2\*cos(3\*e + 3\*f\*x)^2 - 1)\*3i + B\*(2\*cos(2\*e + 2\*f\*x)^2 - 1) - 3\*B\*(2\*cos(3\*e + 3\*f\*x)^2 - 1)))/(60\*c^3\*f)

$$3.747 \quad \int \frac{(a+ia \tan(e+fx))(A+B \tan(e+fx))}{(c-ictan(e+fx))^{7/2}} dx$$

Optimal. Leaf size=62

$$-\frac{2a(iA+B)}{7f(c-ictan(e+fx))^{7/2}} + \frac{2aB}{5cf(c-ictan(e+fx))^{5/2}}$$

[Out]  $-2/7*a*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(7/2)}+2/5*a*B/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}$

Rubi [A]

time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {3669, 45}

$$\frac{2aB}{5cf(c-ictan(e+fx))^{5/2}} - \frac{2a(B+iA)}{7f(c-ictan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^{(7/2)}}, x]$

[Out]  $(-2*a*(I*A + B))/(7*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) + (2*a*B)/(5*c*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})$

Rule 45

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 3669

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))(A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{7/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(c-icx)^{9/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{A-iB}{(c-icx)^{9/2}} + \frac{iB}{c(c-icx)^{7/2}} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{2a(iA + B)}{7f(c - ic \tan(e + fx))^{7/2}} + \frac{2aB}{5cf(c - ic \tan(e + fx))^{5/2}}$$

**Mathematica [A]**

time = 3.35, size = 100, normalized size = 1.61

$$\frac{2a \cos^3(e + fx)(\cos(fx) - i \sin(fx))((-5iA + 2B) \cos(e + fx) - 7iB \sin(e + fx))(\cos(4e + 5fx) + i \sin(4e + 5fx)) \sqrt{c - ic \tan(e + fx)}}{35c^4 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(7/2), x]
```

```
[Out] (2*a*Cos[e + f*x]^3*(Cos[f*x] - I*Sin[f*x])*((( -5*I)*A + 2*B)*Cos[e + f*x] - (7*I)*B*Sin[e + f*x])*(Cos[4*e + 5*f*x] + I*Sin[4*e + 5*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(35*c^4*f)
```

**Maple [A]**

time = 0.26, size = 53, normalized size = 0.85

method	result	si
derivativedivides	$\frac{2ia \left( -\frac{c(-iB+A)}{7(c-ic \tan(fx+e))^{\frac{7}{2}}} - \frac{iB}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} \right)}{fc}$	53
default	$\frac{2ia \left( -\frac{c(-iB+A)}{7(c-ic \tan(fx+e))^{\frac{7}{2}}} - \frac{iB}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} \right)}{fc}$	53
risch	$-\frac{a(5iA e^{6i(fx+e)} + 5B e^{6i(fx+e)} + 15iA e^{4i(fx+e)} + B e^{4i(fx+e)} + 15iA e^{2i(fx+e)} - 13B e^{2i(fx+e)} + 5iA - 9B) \sqrt{2}}{280c^3 \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2), x, method=_RETURNVERBOSE)
```

```
[Out] 2*I/f*a/c*(-1/7*c*(A-I*B)/(c-I*c*tan(f*x+e))^(7/2)-1/5*I*B/(c-I*c*tan(f*x+e))^(5/2))
```

**Maxima [A]**

time = 0.29, size = 48, normalized size = 0.77

$$\frac{2i(7i(-ictan(fx+e)+c)Ba+5(A-iB)ac)}{35(-ictan(fx+e)+c)^{\frac{7}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="maxima")

[Out] -2/35\*I\*(7\*I\*(-I\*c\*tan(f\*x+e)+c)\*B\*a+5\*(A-I\*B)\*a\*c)/((-I\*c\*tan(f\*x+e)+c)^(7/2)\*c\*f)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(50) = 100.

time = 4.65, size = 118, normalized size = 1.90

$$\frac{\sqrt{2}(5(iA+B)ae^{(8i fx+8ie)}+2(10iA+3B)ae^{(6i fx+6ie)}+6(5iA-2B)ae^{(4i fx+4ie)}+2(10iA-11B)ae^{(2i fx+2ie)}-(-5iA+9B)a)\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}}{280c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="fricas")

[Out] -1/280\*sqrt(2)\*(5\*(I\*A+B)\*a\*e^(8\*I\*f\*x+8\*I\*e)+2\*(10\*I\*A+3\*B)\*a\*e^(6\*I\*f\*x+6\*I\*e)+6\*(5\*I\*A-2\*B)\*a\*e^(4\*I\*f\*x+4\*I\*e)+2\*(10\*I\*A-11\*B)\*a\*e^(2\*I\*f\*x+2\*I\*e)-(-5\*I\*A+9\*B)\*a)\*sqrt(c/(e^(2\*I\*f\*x+2\*I\*e)+1))/(c^4\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(7/2),x)

[Out] I\*a\*(Integral(-I\*A/(I\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*3-3\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2-3\*I\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)),x)+Integral(A\*tan(e+f\*x)/(I\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*3-3\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2-3\*I\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)),x)+Integral(B\*tan(e+f\*x)\*\*2/(I\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*3-3\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)\*\*2-3\*I\*c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)\*tan(e+f\*x)+c\*\*3\*sqrt(-I\*c\*tan(e+f\*x)+c)),x)+Integral



$(-I*B*\tan(e + f*x)/(I*c**3*\sqrt{-I*c*\tan(e + f*x) + c})*\tan(e + f*x)**3 - 3*c**3*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**2 - 3*I*c**3*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x) + c**3*\sqrt{-I*c*\tan(e + f*x) + c})), x)$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)/(-I\*c\*tan(f\*x + e) + c)^(7/2), x)

**Mupad** [B]

time = 12.23, size = 157, normalized size = 2.53

$$-\sqrt{c - \frac{c \sin(e + f x)}{\cos(e + f x)}} \left( \frac{a(5A + B9i)}{280c^4f} \operatorname{li} + \frac{ae^{8i+fx8i}(A - B1i)}{56c^4f} \operatorname{li} + \frac{ae^{4i+fx4i}(5A + B2i)}{140c^4f} \operatorname{li} + \frac{ae^{2i+fx2i}(10A + B11i)}{140c^4f} \operatorname{li} + \frac{ae^{6i+fx6i}(10A - B3i)}{140c^4f} \operatorname{li} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i))/(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out]  $-(c - (c*\sin(e + f*x)*1i)/\cos(e + f*x))^{(1/2)}*((a*(5*A + B*9i)*1i)/(280*c^4*f) + (a*\exp(e*8i + f*x*8i)*(A - B*1i)*1i)/(56*c^4*f) + (a*\exp(e*4i + f*x*4i)*(5*A + B*2i)*3i)/(140*c^4*f) + (a*\exp(e*2i + f*x*2i)*(10*A + B*11i)*1i)/(140*c^4*f) + (a*\exp(e*6i + f*x*6i)*(10*A - B*3i)*1i)/(140*c^4*f))$

$$3.748 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{7/2} dx$$

Optimal. Leaf size=105

$$\frac{4a^2(iA + B)(c - ic \tan(e + fx))^{7/2}}{7f} - \frac{2a^2(iA + 3B)(c - ic \tan(e + fx))^{9/2}}{9cf} + \frac{2a^2B(c - ic \tan(e + fx))^{11/2}}{11c^2f}$$

[Out]  $4/7*a^2*(I*A+B)*(c-I*c*\tan(f*x+e))^{(7/2)}/f-2/9*a^2*(I*A+3*B)*(c-I*c*\tan(f*x+e))^{(9/2)}/c/f+2/11*a^2*B*(c-I*c*\tan(f*x+e))^{(11/2)}/c^2/f$

Rubi [A]

time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$-\frac{2a^2(3B + iA)(c - ic \tan(e + fx))^{9/2}}{9cf} + \frac{4a^2(B + iA)(c - ic \tan(e + fx))^{7/2}}{7f} + \frac{2a^2B(c - ic \tan(e + fx))^{11/2}}{11c^2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $(4*a^2*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(7*f) - (2*a^2*(I*A + 3*B)*(c - I*c*\text{Tan}[e + f*x])^{(9/2)})/(9*c*f) + (2*a^2*B*(c - I*c*\text{Tan}[e + f*x])^{(11/2)})/(11*c^2*f)$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{tan}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{7/2} dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx)(c - icx)^{7/2} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(2a(A - iB)(c - icx)\right) dx\right)}{f}$$

$$= \frac{4a^2(iA + B)(c - ic \tan(e + fx))^{7/2}}{7f}$$

**Mathematica [A]**

time = 3.21, size = 119, normalized size = 1.13

$$\frac{a^2 c^3 \sec^5(e + fx) (121iA - 33B + (121iA + 93B) \cos(2(e + fx)) + (-77A + 105iB) \sin(2(e + fx))) (\cos(3e + fx) - i \sin(3e + fx)) \sqrt{c - ic \tan(e + fx)}}{693f(\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2), x]

[Out] (a^2\*c^3\*Sec[e + f\*x]^5\*((121\*I)\*A - 33\*B + ((121\*I)\*A + 93\*B)\*Cos[2\*(e + f\*x)] + (-77\*A + (105\*I)\*B)\*Sin[2\*(e + f\*x)])\*(Cos[3\*e + f\*x] - I\*Sin[3\*e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(693\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.44, size = 83, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{11/2}}{11} + \frac{(-3iBc+Ac)(c-ic \tan(fx+e))^{9/2}}{9} - \frac{2(-iBc+Ac)c(c-ic \tan(fx+e))^{7/2}}{7} \right)}{fc^2}$	83
default	$-\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{11/2}}{11} + \frac{(-3iBc+Ac)(c-ic \tan(fx+e))^{9/2}}{9} - \frac{2(-iBc+Ac)c(c-ic \tan(fx+e))^{7/2}}{7} \right)}{fc^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2), x, method = \_RETURNVERBOSE)

[Out] -2\*I/f\*a^2/c^2\*(1/11\*I\*B\*(c-I\*c\*tan(f\*x+e))^(11/2)+1/9\*(-3\*I\*B\*c+A\*c)\*(c-I\*c\*tan(f\*x+e))^(9/2)-2/7\*(-I\*B\*c+A\*c)\*c\*(c-I\*c\*tan(f\*x+e))^(7/2))

**Maxima [A]**

time = 0.29, size = 81, normalized size = 0.77

$$\frac{2i \left( 63i(-ic \tan(fx + e) + c)^{11/2} Ba^2 + 77(-ic \tan(fx + e) + c)^{9/2} (A - 3iB)a^2c - 198(-ic \tan(fx + e) + c)^{7/2} (A - iB)a^2c^2 \right)}{693c^2f}$$



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e)
+ c)^(7/2), x)
```

**Mupad [B]**

time = 13.89, size = 132, normalized size = 1.26

$$\frac{32 a^2 c^3 \sqrt{c + \frac{c (e^{2i+fx2i} 1i - i) 1i}{e^{2i+fx2i} + 1}} (A 22i - 6 B + A e^{e^{2i+fx2i} 12i} + A e^{e^{4i+fx4i} 99i} - 33 B e^{e^{2i+fx2i}} + 99 B e^{e^{4i+fx4i}})}{693 f (e^{e^{2i+fx2i} + 1})^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)^(7/2),x)
```

```
[Out] (32*a^2*c^3*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*(A*22i - 6*B + A*exp(e*2i + f*x*2i)*12i + A*exp(e*4i + f*x*4i)*99i - 33*B*exp(e*2i + f*x*2i) + 99*B*exp(e*4i + f*x*4i))/(693*f*(exp(e*2i + f*x*2i) + 1)^5)
```

$$3.749 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{5/2} dx$$

Optimal. Leaf size=105

$$\frac{4a^2(iA + B)(c - ic \tan(e + fx))^{5/2}}{5f} - \frac{2a^2(iA + 3B)(c - ic \tan(e + fx))^{7/2}}{7cf} + \frac{2a^2B(c - ic \tan(e + fx))^{9/2}}{9c^2f}$$

[Out]  $4/5*a^2*(I*A+B)*(c-I*c*\tan(f*x+e))^{(5/2)}/f-2/7*a^2*(I*A+3*B)*(c-I*c*\tan(f*x+e))^{(7/2)}/c/f+2/9*a^2*B*(c-I*c*\tan(f*x+e))^{(9/2)}/c^2/f$

Rubi [A]

time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$-\frac{2a^2(3B + iA)(c - ic \tan(e + fx))^{7/2}}{7cf} + \frac{4a^2(B + iA)(c - ic \tan(e + fx))^{5/2}}{5f} + \frac{2a^2B(c - ic \tan(e + fx))^{9/2}}{9c^2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $(4*a^2*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(5*f) - (2*a^2*(I*A + 3*B)*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(7*c*f) + (2*a^2*B*(c - I*c*\text{Tan}[e + f*x])^{(9/2)})/(9*c^2*f)$

Rule 78

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\tan[(e + f*x)])^m*(A + B*\tan[(e + f*x)])*(c + d*\tan[(e + f*x)])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{5/2} dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx)(c - icx)^{5/2} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(2a(A - iB)(c - icx)\right) dx\right)}{f}$$

$$= \frac{4a^2(iA + B)(c - ic \tan(e + fx))^{5/2}}{5f}$$

**Mathematica [A]**

time = 2.06, size = 112, normalized size = 1.07

$$\frac{a^2 c^2 \sec^4(e + fx) (i \cos(2e) + \sin(2e)) (81A + 9iB + (81A - 61iB) \cos(2(e + fx)) + 5(9iA + 13B) \sin(2(e + fx))) \sqrt{c - ic \tan(e + fx)}}{315f(\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2), x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]^4*(I*Cos[2*e] + Sin[2*e])*(81*A + (9*I)*B + (81*A - (61*I)*B)*Cos[2*(e + f*x)] + 5*((9*I)*A + 13*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(315*f*(Cos[f*x] + I*Sin[f*x])^2)
```

**Maple [A]**

time = 0.45, size = 83, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{9/2}}{9} + \frac{(-3iBc+Ac)(c-ic \tan(fx+e))^{7/2}}{7} - \frac{2(-iBc+Ac)c(c-ic \tan(fx+e))^{5/2}}{5} \right)}{f c^2}$	83
default	$-\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{9/2}}{9} + \frac{(-3iBc+Ac)(c-ic \tan(fx+e))^{7/2}}{7} - \frac{2(-iBc+Ac)c(c-ic \tan(fx+e))^{5/2}}{5} \right)}{f c^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2), x, method = _RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c^2*(1/9*I*B*(c-I*c*tan(f*x+e))^(9/2)+1/7*(-3*I*B*c+A*c)*(c-I*c*tan(f*x+e))^(7/2)-2/5*(-I*B*c+A*c)*c*(c-I*c*tan(f*x+e))^(5/2))
```

**Maxima [A]**

time = 0.30, size = 81, normalized size = 0.77

$$\frac{2i \left( 35i(-ic \tan(fx + e) + c)^{5/2} B a^2 + 45(-ic \tan(fx + e) + c)^{7/2} (A - 3iB) a^2 c - 126(-ic \tan(fx + e) + c)^{5/2} (A - iB) a^2 c^2 \right)}{315 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x,  
algorithm="maxima")

[Out] 
$$\frac{-2/315*I*(35*I*(-I*c*\tan(f*x + e) + c)^{(9/2)}*B*a^2 + 45*(-I*c*\tan(f*x + e) + c)^{(7/2)}*(A - 3*I*B)*a^2*c - 126*(-I*c*\tan(f*x + e) + c)^{(5/2)}*(A - I*B)*a^2*c^2)/(c^2*f)}$$

**Fricas** [A]

time = 5.44, size = 140, normalized size = 1.33

$$\frac{16\sqrt{2}(63(-iA - B)a^2c^2e^{(4i fx + 4i e)} + 9(-9iA + B)a^2c^2e^{(2i fx + 2i e)} + 2(-9iA + B)a^2c^2)\sqrt{\frac{c}{e^{(2i fx + 2i e)} + 1}}}{315(fe^{(8i fx + 8i e)} + 4fe^{(6i fx + 6i e)} + 6fe^{(4i fx + 4i e)} + 4fe^{(2i fx + 2i e)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x,  
algorithm="fricas")

[Out] 
$$\frac{-16/315*\sqrt{2}*(63*(-I*A - B)*a^2*c^2*e^{(4*I*f*x + 4*I*e)} + 9*(-9*I*A + B)*a^2*c^2*e^{(2*I*f*x + 2*I*e)} + 2*(-9*I*A + B)*a^2*c^2)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}}{(f*e^{(8*I*f*x + 8*I*e)} + 4*f*e^{(6*I*f*x + 6*I*e)} + 6*f*e^{(4*I*f*x + 4*I*e)} + 4*f*e^{(2*I*f*x + 2*I*e)} + f)}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int (-Ae^{\sqrt{-ic}\tan(e+fx)+c}) dx + \int (-2Ae^{\sqrt{-ic}\tan(e+fx)+c}\tan^2(e+fx)) dx + \int (-Ae^{\sqrt{-ic}\tan(e+fx)+c}\tan^3(e+fx)) dx + \int (-Be^{\sqrt{-ic}\tan(e+fx)+c}\tan^4(e+fx)) dx + \int (-2Be^{\sqrt{-ic}\tan(e+fx)+c}\tan^5(e+fx)) dx + \int (-Be^{\sqrt{-ic}\tan(e+fx)+c}\tan^6(e+fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(5/2),  
x)

[Out] 
$$-a^{**2}*(\text{Integral}(-A*c^{**2}*\sqrt{-I*c*\tan(e + f*x) + c}), x) + \text{Integral}(-2*A*c^{**2}*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)^{**2}, x) + \text{Integral}(-A*c^{**2}*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)^{**4}, x) + \text{Integral}(-B*c^{**2}*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x), x) + \text{Integral}(-2*B*c^{**2}*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)^{**3}, x) + \text{Integral}(-B*c^{**2}*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)^{**5}, x))$$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^2\*(-I\*c\*tan(f\*x + e)  
+ c)^(5/2), x)

**Mupad [B]**

time = 15.34, size = 132, normalized size = 1.26

$$\frac{16a^2c^2\sqrt{c+\frac{c(e^{2i+fx2i}1i-i)1i}{e^{2i+fx2i}+1}}(A18i-2B+Ae^{e^{2i+fx2i}}81i+Ae^{e^{4i+fx4i}}63i-9Be^{e^{2i+fx2i}}+63Be^{e^{4i+fx4i}})}{315f(e^{e^{2i+fx2i}}+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] (16\*a^2\*c^2\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*(A\*18i - 2\*B + A\*exp(e\*2i + f\*x\*2i)\*81i + A\*exp(e\*4i + f\*x\*4i)\*63i - 9\*B\*exp(e\*2i + f\*x\*2i) + 63\*B\*exp(e\*4i + f\*x\*4i))/(315\*f\*(exp(e\*2i + f\*x\*2i) + 1)^4)

$$3.750 \quad \int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{3/2} dx$$

**Optimal.** Leaf size=105

$$\frac{4a^2(iA + B)(c - ic \tan(e + fx))^{3/2}}{3f} - \frac{2a^2(iA + 3B)(c - ic \tan(e + fx))^{5/2}}{5cf} + \frac{2a^2B(c - ic \tan(e + fx))^{7/2}}{7c^2f}$$

[Out]  $4/3*a^2*(I*A+B)*(c-I*c*\tan(f*x+e))^{(3/2)}/f-2/5*a^2*(I*A+3*B)*(c-I*c*\tan(f*x+e))^{(5/2)}/c/f+2/7*a^2*B*(c-I*c*\tan(f*x+e))^{(7/2)}/c^2/f$

**Rubi [A]**

time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$-\frac{2a^2(3B + iA)(c - ic \tan(e + fx))^{5/2}}{5cf} + \frac{4a^2(B + iA)(c - ic \tan(e + fx))^{3/2}}{3f} + \frac{2a^2B(c - ic \tan(e + fx))^{7/2}}{7c^2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $(4*a^2*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*f) - (2*a^2*(I*A + 3*B)*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(5*c*f) + (2*a^2*B*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(7*c^2*f)$

**Rule 78**

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m*(A + B*\text{tan}[e + f*x])*(c + d*\text{tan}[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

**Rubi steps**

$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) (c - ictan(e + fx))^{3/2} dx = \frac{(ac) \text{Subst}\left(\int (a + iax)(A + Bx) \sqrt{c - ictan(e + fx)} dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(2a(A - iB) \sqrt{c - ictan(e + fx)}\right) dx\right)}{f}$$

$$= \frac{4a^2(iA + B)(c - ictan(e + fx))^3}{3f}$$

**Mathematica [A]**

time = 1.81, size = 116, normalized size = 1.10

$$\frac{a^2 c \sec^3(e + fx) (i \cos(e - fx) + \sin(e - fx)) (49A - 7iB + (49A - 37iB) \cos(2(e + fx)) + 3(7iA + 11B) \sin(2(e + fx))) \sqrt{c - ictan(e + fx)}}{105f(\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (a^2\*c\*Sec[e + f\*x]^3\*(I\*Cos[e - f\*x] + Sin[e - f\*x])\*(49\*A - (7\*I)\*B + (49\*A - (37\*I)\*B)\*Cos[2\*(e + f\*x)] + 3\*((7\*I)\*A + 11\*B)\*Sin[2\*(e + f\*x)])\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(105\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.43, size = 83, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2ia^2 \left( \frac{iB(c-ictan(fx+e))^{7/2}}{7} + \frac{(-3iBc+Ac)(c-ictan(fx+e))^{5/2}}{5} - \frac{2(-iBc+Ac)c(c-ictan(fx+e))^{3/2}}{3} \right)}{f c^2}$	83
default	$-\frac{2ia^2 \left( \frac{iB(c-ictan(fx+e))^{7/2}}{7} + \frac{(-3iBc+Ac)(c-ictan(fx+e))^{5/2}}{5} - \frac{2(-iBc+Ac)c(c-ictan(fx+e))^{3/2}}{3} \right)}{f c^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2), x, method = \_RETURNVERBOSE)

[Out] -2\*I/f\*a^2/c^2\*(1/7\*I\*B\*(c-I\*c\*tan(f\*x+e))^(7/2)+1/5\*(-3\*I\*B\*c+A\*c)\*(c-I\*c\*tan(f\*x+e))^(5/2)-2/3\*(-I\*B\*c+A\*c)\*c\*(c-I\*c\*tan(f\*x+e))^(3/2))

**Maxima [A]**

time = 0.30, size = 81, normalized size = 0.77

$$\frac{2i \left( 15i(-ictan(fx+e)+c)^{7/2}Ba^2 + 21(-ictan(fx+e)+c)^{5/2}(A-3iB)a^2c - 70(-ictan(fx+e)+c)^{3/2}(A-iB)a^2c^2 \right)}{105c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="maxima")
```

```
[Out] -2/105*I*(15*I*(-I*c*tan(f*x + e) + c)^(7/2)*B*a^2 + 21*(-I*c*tan(f*x + e)
+ c)^(5/2)*(A - 3*I*B)*a^2*c - 70*(-I*c*tan(f*x + e) + c)^(3/2)*(A - I*B)*a
^2*c^2)/(c^2*f)
```

**Fricas** [A]

time = 4.14, size = 125, normalized size = 1.19

$$\frac{8\sqrt{2}(35(-iA-B)a^2ce^{4ifx+4ie} + 7(-7iA-B)a^2ce^{2ifx+2ie} + 2(-7iA-B)a^2c)\sqrt{\frac{c}{e^{2ifx+2ie}+1}}}{105(fe^{6ifx+6ie} + 3fe^{4ifx+4ie} + 3fe^{2ifx+2ie} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] -8/105*sqrt(2)*(35*(-I*A - B)*a^2*c*e^(4*I*f*x + 4*I*e) + 7*(-7*I*A - B)*a^
2*c*e^(2*I*f*x + 2*I*e) + 2*(-7*I*A - B)*a^2*c)*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x +
2*I*e) + f)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \int (-Ae^{\sqrt{-c}\tan(e+fx)+c}) dx + \int (-Ae^{\sqrt{-c}\tan(e+fx)+c}\tan^2(e+fx)) dx + \int (-Be^{\sqrt{-c}\tan(e+fx)+c}\tan(e+fx)) dx + \int (-Ae^{\sqrt{-c}\tan(e+fx)+c}\tan^2(e+fx)) dx + \int (-Ae^{\sqrt{-c}\tan(e+fx)+c}\tan(e+fx)) dx + \int (-Be^{\sqrt{-c}\tan(e+fx)+c}\tan^2(e+fx)) dx + \int (-Ae^{\sqrt{-c}\tan(e+fx)+c}\tan(e+fx)) dx + \int (-Be^{\sqrt{-c}\tan(e+fx)+c}\tan^2(e+fx)) dx + \int (-Ae^{\sqrt{-c}\tan(e+fx)+c}\tan(e+fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2),
x)
```

```
[Out] -a**2*(Integral(-A*c*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-A*c*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-B*c*sqrt(-I*c*tan(e +
f*x) + c)*tan(e + f*x), x) + Integral(-B*c*sqrt(-I*c*tan(e + f*x) + c)*tan
(e + f*x)**3, x) + Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)
, x) + Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + In
tegral(-I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-I
*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x))
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^2\*(-I\*c\*tan(f\*x + e)  
+ c)^(3/2), x)

**Mupad [B]**

time = 13.67, size = 130, normalized size = 1.24

$$\frac{8a^2c \sqrt{c + \frac{c(e^{2i+fx2i}1i - i)1i}{e^{2i+fx2i} + 1}} (A14i + 2B + Ae^{e^{2i+fx2i}}49i + Ae^{e^{4i+fx4i}}35i + 7Be^{e^{2i+fx2i}} + 35Be^{e^{4i+fx4i}})}{105f(e^{e^{2i+fx2i}} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^(3/2),x)

[Out] (8\*a^2\*c\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*(A\*14i + 2\*B + A\*exp(e\*2i + f\*x\*2i)\*49i + A\*exp(e\*4i + f\*x\*4i)\*35i + 7\*B\*exp(e\*2i + f\*x\*2i) + 35\*B\*exp(e\*4i + f\*x\*4i))/(105\*f\*(exp(e\*2i + f\*x\*2i) + 1)^3)

### 3.751 $\int (a+ia \tan(e+fx))^2 (A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}$

Optimal. Leaf size=103

$$\frac{4a^2(iA+B)\sqrt{c-ic \tan(e+fx)}}{f} - \frac{2a^2(iA+3B)(c-ic \tan(e+fx))^{3/2}}{3cf} + \frac{2a^2B(c-ic \tan(e+fx))^{5/2}}{5c^2f}$$

[Out]  $4a^2(I*A+B)*(c-I*c*\tan(f*x+e))^{(1/2)}/f-2/3*a^2*(I*A+3*B)*(c-I*c*\tan(f*x+e))^{(3/2)}/c/f+2/5*a^2*B*(c-I*c*\tan(f*x+e))^{(5/2)}/c^2/f$

Rubi [A]

time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ ,

Rules used = {3669, 78}

$$-\frac{2a^2(3B+iA)(c-ic \tan(e+fx))^{3/2}}{3cf} + \frac{4a^2(B+iA)\sqrt{c-ic \tan(e+fx)}}{f} + \frac{2a^2B(c-ic \tan(e+fx))^{5/2}}{5c^2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])*Sqrt[c - I*c*\text{Tan}[e + f*x]], x]$

[Out]  $(4*a^2*(I*A + B)*Sqrt[c - I*c*\text{Tan}[e + f*x]])/f - (2*a^2*(I*A + 3*B)*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*c*f) + (2*a^2*B*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(5*c^2*f)$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol)] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a_. + (b_.)*\tan[(e_. + (f_.)*(x_.))]^{(m_.)*((A_. + (B_.)*\tan[(e_. + (f_.)*(x_.))]^{(n_.)}, x\_Symbol)] :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^2 (A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{\sqrt{c-icx}} dx, x, \tan(e+fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(A-iB)}{\sqrt{c-icx}} - \frac{a(A-3iB)}{\sqrt{c-icx}} \right) dx, x, \tan(e+fx) \right)}{f}$$

$$= \frac{4a^2(iA + B) \sqrt{c - ic \tan(e + fx)}}{f}$$

**Mathematica [A]**

time = 1.31, size = 83, normalized size = 0.81

$$\frac{a^2 \sec^2(e + fx) (5(5iA + 3B) + (25iA + 21B) \cos(2(e + fx)) + (-5A + 9iB) \sin(2(e + fx))) \sqrt{c - ic \tan(e + fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] (a^2*Sec[e + f*x]^2*(5*((5*I)*A + 3*B) + ((25*I)*A + 21*B)*Cos[2*(e + f*x)] + (-5*A + (9*I)*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(15*f)
```

**Maple [A]**

time = 0.44, size = 83, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{\frac{5}{2}}}{5} + \frac{(-3iBc+Ac)(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} - 2(-iBc+Ac)c \sqrt{c - ic \tan(fx + e)} \right)}{f c^2}$	83
default	$-\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{\frac{5}{2}}}{5} + \frac{(-3iBc+Ac)(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} - 2(-iBc+Ac)c \sqrt{c - ic \tan(fx + e)} \right)}{f c^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)), x, method = _RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c^2*(1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)+1/3*(-3*I*B*c+A*c)*(c-I*c*tan(f*x+e))^(3/2)-2*(-I*B*c+A*c)*c*(c-I*c*tan(f*x+e))^(1/2))
```

**Maxima [A]**

time = 0.30, size = 81, normalized size = 0.79

$$\frac{2i \left( 3i(-ic \tan(fx + e) + c)^{\frac{5}{2}} B a^2 + 5(-ic \tan(fx + e) + c)^{\frac{3}{2}} (A - 3iB) a^2 c - 30 \sqrt{-ic \tan(fx + e) + c} (A - iB) a^2 c^2 \right)}{15 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e)),x,  
algorithm="maxima")

[Out] -2/15\*I\*(3\*I\*(-I\*c\*tan(f\*x + e) + c)^(5/2)\*B\*a^2 + 5\*(-I\*c\*tan(f\*x + e) + c)^(3/2)\*(A - 3\*I\*B)\*a^2\*c - 30\*sqrt(-I\*c\*tan(f\*x + e) + c)\*(A - I\*B)\*a^2\*c^2)/(c^2\*f)

**Fricas** [A]

time = 7.07, size = 109, normalized size = 1.06

$$\frac{4\sqrt{2}\left(15(-iA-B)a^2e^{(4ifx+4ie)} + 5(-5iA-3B)a^2e^{(2ifx+2ie)} + 2(-5iA-3B)a^2\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}}{15(fe^{(4ifx+4ie)} + 2fe^{(2ifx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e)),x,  
algorithm="fricas")

[Out] -4/15\*sqrt(2)\*(15\*(-I\*A - B)\*a^2\*e^(4\*I\*f\*x + 4\*I\*e) + 5\*(-5\*I\*A - 3\*B)\*a^2\*e^(2\*I\*f\*x + 2\*I\*e) + 2\*(-5\*I\*A - 3\*B)\*a^2)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(f\*e^(4\*I\*f\*x + 4\*I\*e) + 2\*f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int(-A\sqrt{-ic\tan(e+fz)+c})dz + \int A\sqrt{-ic\tan(e+fz)+c}\tan^2(e+fz)dz + \int(-B\sqrt{-ic\tan(e+fz)+c}\tan(e+fz))dz + \int B\sqrt{-ic\tan(e+fz)+c}\tan^3(e+fz)dz + \int(-2iA\sqrt{-ic\tan(e+fz)+c}\tan(e+fz))dz + \int(-2iB\sqrt{-ic\tan(e+fz)+c}\tan^2(e+fz))dz\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(a+I\*a\*tan(f\*x+e))\*\*2\*(A+B\*tan(f\*x+e)),  
x)

[Out] -a\*\*2\*(Integral(-A\*sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(A\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2, x) + Integral(-B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x), x) + Integral(B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*3, x) + Integral(-2\*I\*A\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x), x) + Integral(-2\*I\*B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2, x))

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e)),x,  
algorithm="giac")



[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^2\*sqrt(-I\*c\*tan(f\*x + e) + c), x)

**Mupad [B]**

time = 11.99, size = 241, normalized size = 2.34

$$2a^2 \sqrt{\frac{c \cos(2e + 2fx) + 1 - \sin(2e + 2fx) 1i}{\cos(2e + 2fx) + 1}} \frac{(A250i + 174B + A \cos(2e + 2fx) 375i + A \cos(4e + 4fx) 150i + A \cos(6e + 6fx) 25i + 267B \cos(2e + 2fx) + 114B \cos(4e + 4fx) + 21B \cos(6e + 6fx) - 25A \sin(2e + 2fx) - 20A \sin(4e + 4fx) - 5A \sin(6e + 6fx) + B \sin(2e + 2fx) 45i + B \sin(4e + 4fx) 36i + B \sin(6e + 6fx) 9i)}{15f(15 \cos(2e + 2fx) + 6 \cos(4e + 4fx) + \cos(6e + 6fx) + 10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2), x)

[Out] (2\*a^2\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*250i + 174\*B + A\*cos(2\*e + 2\*f\*x)\*375i + A\*cos(4\*e + 4\*f\*x)\*150i + A\*cos(6\*e + 6\*f\*x)\*25i + 267\*B\*cos(2\*e + 2\*f\*x) + 114\*B\*cos(4\*e + 4\*f\*x) + 21\*B\*cos(6\*e + 6\*f\*x) - 25\*A\*sin(2\*e + 2\*f\*x) - 20\*A\*sin(4\*e + 4\*f\*x) - 5\*A\*sin(6\*e + 6\*f\*x) + B\*sin(2\*e + 2\*f\*x)\*45i + B\*sin(4\*e + 4\*f\*x)\*36i + B\*sin(6\*e + 6\*f\*x)\*9i))/(15\*f\*(15\*cos(2\*e + 2\*f\*x) + 6\*cos(4\*e + 4\*f\*x) + cos(6\*e + 6\*f\*x) + 10))

$$3.752 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{\sqrt{c-ictan(e+fx)}} dx$$

**Optimal.** Leaf size=101

$$-\frac{4a^2(iA+B)}{f\sqrt{c-ictan(e+fx)}} - \frac{2a^2(iA+3B)\sqrt{c-ictan(e+fx)}}{cf} + \frac{2a^2B(c-ictan(e+fx))^{3/2}}{3c^2f}$$

[Out]  $-4*a^2*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(1/2)}-2*a^2*(I*A+3*B)*(c-I*c*\tan(f*x+e))^{(1/2)}/c/f+2/3*a^2*B*(c-I*c*\tan(f*x+e))^{(3/2)}/c^2/f$

**Rubi [A]**

time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$-\frac{2a^2(3B+iA)\sqrt{c-ictan(e+fx)}}{cf} - \frac{4a^2(B+iA)}{f\sqrt{c-ictan(e+fx)}} + \frac{2a^2B(c-ictan(e+fx))^{3/2}}{3c^2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]}, x]$

[Out]  $(-4*a^2*(I*A + B))/(f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) - (2*a^2*(I*A + 3*B)*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(c*f) + (2*a^2*B*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*c^2*f)$

Rule 78

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]} :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[\frac{(a_. + (b_.)*\tan[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\tan[(e_. + (f_.)*(x_.)])*((c_. + (d_.)*\tan[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]} :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{\sqrt{c - ic \tan(e + fx)}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(A-iB)}{(c-icx)^{3/2}} - \frac{a(A-3iB)}{c\sqrt{c-icx}} - \frac{iaB\sqrt{c-icx}}{c^2} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{4a^2(iA + B)}{f\sqrt{c - ic \tan(e + fx)}} - \frac{2a^2(iA + 3B)\sqrt{c - ic \tan(e + fx)}}{cf}$$

**Mathematica [A]**

time = 1.44, size = 138, normalized size = 1.37

$$\frac{a^2(9A - 15iB + (9A - 13iB) \cos(2(e + fx)) + (-3iA - 7B) \sin(2(e + fx)))(-i \cos(e + 3fx) + \sin(e + 3fx))(A + B \tan(e + fx))\sqrt{c - ic \tan(e + fx)}}{3cf(\cos(fx) + i \sin(fx))^2(A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] (a^2*(9*A - (15*I)*B + (9*A - (13*I)*B)*Cos[2*(e + f*x)] + ((-3*I)*A - 7*B)*Sin[2*(e + f*x)])*((-I)*Cos[e + 3*f*x] + Sin[e + 3*f*x])*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]/(3*c*f*(Cos[f*x] + I*Sin[f*x])^2*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

**Maple [A]**

time = 0.26, size = 93, normalized size = 0.92

method	result
derivativedivides	$\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{3/2}}{3} - 3iBc \sqrt{c - ic \tan(fx + e)} + Ac \sqrt{c - ic \tan(fx + e)} + \frac{2c^2(-i)}{\sqrt{c - ic \tan(fx + e)}} \right)}{fc^2}$
default	$\frac{2ia^2 \left( \frac{iB(c-ic \tan(fx+e))^{3/2}}{3} - 3iBc \sqrt{c - ic \tan(fx + e)} + Ac \sqrt{c - ic \tan(fx + e)} + \frac{2c^2(-i)}{\sqrt{c - ic \tan(fx + e)}} \right)}{fc^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2), x, method = _RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c^2*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)-3*I*B*c*(c-I*c*tan(f*x+e))^(1/2)+A*c*(c-I*c*tan(f*x+e))^(1/2)+2*c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2))
```

**Maxima [A]**

time = 0.29, size = 84, normalized size = 0.83

$$\frac{2i \left( \frac{6(A-iB)a^2c}{\sqrt{-i c \tan(fx+e)+c}} + \frac{i(-i c \tan(fx+e)+c)^{\frac{3}{2}} B a^2 + 3 \sqrt{-i c \tan(fx+e)+c} (A-3iB)a^2 c}{c} \right)}{3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x,  
algorithm="maxima")

[Out] -2/3\*I\*(6\*(A - I\*B)\*a^2\*c/sqrt(-I\*c\*tan(f\*x + e) + c) + (I\*(-I\*c\*tan(f\*x + e) + c)^(3/2)\*B\*a^2 + 3\*sqrt(-I\*c\*tan(f\*x + e) + c)\*(A - 3\*I\*B)\*a^2\*c)/c)/(c\*f)

**Fricas [A]**

time = 4.13, size = 97, normalized size = 0.96

$$\frac{2\sqrt{2} (3(iA+B)a^2e^{(4i fx+4i e)} + 3(3iA+5B)a^2e^{(2i fx+2i e)} + 2(3iA+5B)a^2) \sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}}{3(cfe^{(2i fx+2i e)}+cf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x,  
algorithm="fricas")

[Out] -2/3\*sqrt(2)\*(3\*(I\*A + B)\*a^2\*e^(4\*I\*f\*x + 4\*I\*e) + 3\*(3\*I\*A + 5\*B)\*a^2\*e^(2\*I\*f\*x + 2\*I\*e) + 2\*(3\*I\*A + 5\*B)\*a^2)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c\*f\*e^(2\*I\*f\*x + 2\*I\*e) + c\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \left( \int \left( -\frac{A}{\sqrt{-i c \tan(e+fx)+c}} \right) dx + \int \frac{A \tan^2(e+fx)}{\sqrt{-i c \tan(e+fx)+c}} dx + \int \left( -\frac{B \tan(e+fx)}{\sqrt{-i c \tan(e+fx)+c}} \right) dx + \int \frac{B \tan^3(e+fx)}{\sqrt{-i c \tan(e+fx)+c}} dx + \int \left( -\frac{2iA \tan(e+fx)}{\sqrt{-i c \tan(e+fx)+c}} \right) dx + \int \left( -\frac{2iB \tan^2(e+fx)}{\sqrt{-i c \tan(e+fx)+c}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),  
x)

[Out] -a\*\*2\*(Integral(-A/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(A\*tan(e + f\*x)\*\*2/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-B\*tan(e + f\*x)/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(B\*tan(e + f\*x)\*\*3/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-2\*I\*A\*tan(e + f\*x)/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-2\*I\*B\*tan(e + f\*x)\*\*2/sqrt(-I\*c\*tan(e + f\*x) + c), x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^2/sqrt(-I*c*tan(f*x +
e) + c), x)
```

**Mupad [B]**

time = 10.77, size = 176, normalized size = 1.74

$$\frac{2\sqrt{2}a^2\sqrt{\frac{c}{\cos(2e+2fx)+1+\sin(2e+2fx)}{1i}}(A6i+10B+A\cos(2e+2fx)9i+A\cos(4e+4fx)3i+15B\cos(2e+2fx)+3B\cos(4e+4fx)-9A\sin(2e+2fx)-3A\sin(4e+4fx)+B\sin(2e+2fx)15i+B\sin(4e+4fx)3i)}{3cf(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1i
)^(1/2),x)
```

```
[Out] -(2*2^(1/2)*a^2*(c/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2)*(A*6
i + 10*B + A*cos(2*e + 2*f*x)*9i + A*cos(4*e + 4*f*x)*3i + 15*B*cos(2*e + 2
*f*x) + 3*B*cos(4*e + 4*f*x) - 9*A*sin(2*e + 2*f*x) - 3*A*sin(4*e + 4*f*x)
+ B*sin(2*e + 2*f*x)*15i + B*sin(4*e + 4*f*x)*3i))/(3*c*f*(cos(2*e + 2*f*x)
+ sin(2*e + 2*f*x)*1i + 1))
```

$$3.753 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ictan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=101

$$-\frac{4a^2(iA+B)}{3f(c-ictan(e+fx))^{3/2}} + \frac{2a^2(iA+3B)}{cf\sqrt{c-ictan(e+fx)}} + \frac{2a^2B\sqrt{c-ictan(e+fx)}}{c^2f}$$

[Out]  $2a^2(I*A+3*B)/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}+2a^2*B*(c-I*c*\tan(f*x+e))^{(1/2)}/c^2/f-4/3*a^2*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$\frac{2a^2(3B+iA)}{cf\sqrt{c-ictan(e+fx)}} - \frac{4a^2(B+iA)}{3f(c-ictan(e+fx))^{3/2}} + \frac{2a^2B\sqrt{c-ictan(e+fx)}}{c^2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^{(3/2)}}, x]$

[Out]  $\frac{(-4*a^2*(I*A + B))/(3*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) + (2*a^2*(I*A + 3*B))/(c*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) + (2*a^2*B*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(c^2*f)}$

**Rule 78**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol]}{> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) || \text{EqQ}[p, 1] || (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] || \text{LeQ}[9*p + 5*(n + 2), 0] || \text{GeQ}[n + p + 1, 0] || (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[\frac{(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol]}{> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(A-ib)}{(c-icx)^{5/2}} - \frac{a(A-3ib)}{c(c-icx)^{3/2}} - \frac{iaB}{c^2 \sqrt{c-icx}} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{4a^2(iA + B)}{3f(c - ic \tan(e + fx))^{3/2}} + \frac{2a^2(iA + 3B)}{cf \sqrt{c - ic \tan(e + fx)}}$$

**Mathematica [A]**

time = 2.08, size = 112, normalized size = 1.11

$$\frac{a^2(iA + 7B + (iA + 13B) \cos(2(e + fx)) + 3(A - 5iB) \sin(2(e + fx)))(\cos(2(e + 2fx)) + i \sin(2(e + 2fx))) \sqrt{c - ic \tan(e + fx)}}{3c^2 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (a^2\*(I\*A + 7\*B + (I\*A + 13\*B)\*Cos[2\*(e + f\*x)] + 3\*(A - (5\*I)\*B)\*Sin[2\*(e + f\*x)]\*(Cos[2\*(e + 2\*f\*x)] + I\*Sin[2\*(e + 2\*f\*x)])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*c^2\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.22, size = 80, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2ia^2 \left( iB \sqrt{c - ic \tan(fx + e)} + \frac{2e^{2(-iB+A)}}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{c(-3iB+A)}{\sqrt{c - ic \tan(fx + e)}} \right)}{f c^2}$	80
default	$-\frac{2ia^2 \left( iB \sqrt{c - ic \tan(fx + e)} + \frac{2e^{2(-iB+A)}}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{c(-3iB+A)}{\sqrt{c - ic \tan(fx + e)}} \right)}{f c^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2), x, method = \_RETURNVERBOSE)

[Out] -2\*I/f\*a^2/c^2\*(I\*B\*(c-I\*c\*tan(f\*x+e))^(1/2)+2/3\*c^2\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(3/2)-c\*(A-3\*I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.29, size = 82, normalized size = 0.81

$$\frac{2i \left( \frac{3i \sqrt{-i c \tan(fx + e) + c} B a^2}{c} - \frac{3(-i c \tan(fx + e) + c)(A - 3i B)a^2 - 2(A - i B)a^2 c}{(-i c \tan(fx + e) + c)^{\frac{3}{2}}} \right)}{3 c f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="maxima")
```

```
[Out] -2/3*I*(3*I*sqrt(-I*c*tan(f*x + e) + c)*B*a^2/c - (3*(-I*c*tan(f*x + e) + c)
)*(A - 3*I*B)*a^2 - 2*(A - I*B)*a^2*c)/(-I*c*tan(f*x + e) + c)^(3/2))/(c*f)
```

**Fricas [A]**

time = 4.42, size = 84, normalized size = 0.83

$$\frac{\sqrt{2} \left( (-i A - B)a^2 e^{(4i f x + 4i e)} + (i A + 7 B)a^2 e^{(2i f x + 2i e)} - 2(-i A - 7 B)a^2 \right) \sqrt{\frac{c}{e^{(2i f x + 2i e)} + 1}}}{3 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((-I*A - B)*a^2*e^(4*I*f*x + 4*I*e) + (I*A + 7*B)*a^2*e^(2*I*f*
x + 2*I*e) - 2*(-I*A - 7*B)*a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \left( \frac{A}{-i c \tan(fx + e) + c} + \frac{B \tan(fx + e)}{-i c \tan(fx + e) + c} \right) \frac{1}{(-i c \tan(fx + e) + c)^{3/2}} dx + \int \left( \frac{A}{-i c \tan(fx + e) + c} + \frac{B \tan(fx + e)}{-i c \tan(fx + e) + c} \right) \frac{1}{(-i c \tan(fx + e) + c)^{3/2}} dx + \int \left( \frac{A}{-i c \tan(fx + e) + c} + \frac{B \tan(fx + e)}{-i c \tan(fx + e) + c} \right) \frac{1}{(-i c \tan(fx + e) + c)^{3/2}} dx + \int \left( \frac{A}{-i c \tan(fx + e) + c} + \frac{B \tan(fx + e)}{-i c \tan(fx + e) + c} \right) \frac{1}{(-i c \tan(fx + e) + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),
x)
```

```
[Out] -a**2*(Integral(-A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-
I*c*tan(e + f*x) + c)), x) + Integral(A*tan(e + f*x)**2/(-I*c*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integra
l(-B*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-
I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)**3/(-I*c*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral
(-2*I*A*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqr
t(-I*c*tan(e + f*x) + c)), x) + Integral(-2*I*B*tan(e + f*x)**2/(-I*c*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x))
```



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^2/(-I*c*tan(f*x + e)
+ c)^(3/2), x)
```

**Mupad [B]**

time = 10.15, size = 158, normalized size = 1.56

$$a^2 \sqrt{\frac{c \cos(2e + 2fx) + 1 - \sin(2e + 2fx) i}{\cos(2e + 2fx) + 1}} \frac{(A 2i + 14 B + A \cos(2e + 2fx) i - A \cos(4e + 4fx) i + 7 B \cos(2e + 2fx) - B \cos(4e + 4fx) - A \sin(2e + 2fx) + A \sin(4e + 4fx) + B \sin(2e + 2fx) i - B \sin(4e + 4fx) i)}{3 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1i)
)^(3/2),x)
```

```
[Out] (a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*2i + 14*B + A*cos(2*e + 2*f*x)*1i - A*cos(4*e + 4*f*x)*1i + 7*
B*cos(2*e + 2*f*x) - B*cos(4*e + 4*f*x) - A*sin(2*e + 2*f*x) + A*sin(4*e +
4*f*x) + B*sin(2*e + 2*f*x)*7i - B*sin(4*e + 4*f*x)*1i))/(3*c^2*f)
```

$$3.754 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ictan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=103

$$-\frac{4a^2(iA+B)}{5f(c-ictan(e+fx))^{5/2}} + \frac{2a^2(iA+3B)}{3cf(c-ictan(e+fx))^{3/2}} - \frac{2a^2B}{c^2f\sqrt{c-ictan(e+fx)}}$$

[Out]  $-2*a^2*B/c^2/f/(c-I*c*\tan(f*x+e))^{(1/2)}-4/5*a^2*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(5/2)}+2/3*a^2*(I*A+3*B)/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ ,

Rules used = {3669, 78}

$$\frac{2a^2(3B+iA)}{3cf(c-ictan(e+fx))^{3/2}} - \frac{4a^2(B+iA)}{5f(c-ictan(e+fx))^{5/2}} - \frac{2a^2B}{c^2f\sqrt{c-ictan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^{5/2}}, x]$

[Out]  $(-4*a^2*(I*A + B))/(5*f*(c - I*c*\text{Tan}[e + f*x])^{5/2}) + (2*a^2*(I*A + 3*B))/(3*c*f*(c - I*c*\text{Tan}[e + f*x])^{3/2}) - (2*a^2*B)/(c^2*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

**Rule 78**

$\text{Int}[\frac{(a + b*x)^n*(c + d*x)^p}{(e + f*x)^q}, x]$   $\rightarrow$   $\text{Int}[\text{ExpandIntegrand}[(a + b*x)^n*(c + d*x)^p*(e + f*x)^{-q}, x], x]$   $;$   $\text{FreeQ}\{a, b, c, d, e, f, n\}, x$   $\&\&$   $\text{NeQ}[b*c - a*d, 0]$   $\&\&$   $(\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \parallel \text{EqQ}[p, 1] \parallel (\text{IGtQ}[p, 0] \&\& (\text{!IntegerQ}[n] \parallel \text{LeQ}[9*p + 5*(n + 2), 0]) \parallel \text{GeQ}[n + p + 1, 0] \parallel (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f]))$

**Rule 3669**

$\text{Int}[\frac{(a + b*\tan[e + f*x])^m*(A + B*\tan[e + f*x])^n}{(c + d*\tan[e + f*x])^p}, x]$   $\rightarrow$   $\text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x)^p, x], x, \text{Tan}[e + f*x], x]$   $;$   $\text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x$   $\&\&$   $\text{EqQ}[b*c + a*d, 0]$   $\&\&$   $\text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(A-iB)}{(c-icx)^{7/2}} - \frac{a(A-3iB)}{c(c-icx)^{5/2}} - \frac{iaB}{c^2(c-icx)^{3/2}} \right) dx, x \right)}{f}$$

$$= -\frac{4a^2(iA + B)}{5f(c - ic \tan(e + fx))^{5/2}} + \frac{2a^2(iA + 3B)}{3cf(c - ic \tan(e + fx))^3}$$

**Mathematica [A]**

time = 3.46, size = 118, normalized size = 1.15

$$\frac{a^2 \cos(e + fx)(-iA + 9B + (-iA - 21B) \cos(2(e + fx)) + 5(A + 3iB) \sin(2(e + fx)))(\cos(3e + 5fx) + i \sin(3e + 5fx)) \sqrt{c - ic \tan(e + fx)}}{15c^3 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^2\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] (a^2\*Cos[e + f\*x]\*((-I)\*A + 9\*B + ((-I)\*A - 21\*B)\*Cos[2\*(e + f\*x)] + 5\*(A + (3\*I)\*B)\*Sin[2\*(e + f\*x)])\*(Cos[3\*e + 5\*f\*x] + I\*Sin[3\*e + 5\*f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(15\*c^3\*f\*(Cos[f\*x] + I\*Sin[f\*x])^2)

**Maple [A]**

time = 0.25, size = 80, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2ia^2 \left( -\frac{c(-3iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} + \frac{2c^2(-iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{iB}{\sqrt{c-ic \tan(fx+e)}} \right)}{fc^2}$	80
default	$-\frac{2ia^2 \left( -\frac{c(-3iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} + \frac{2c^2(-iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{iB}{\sqrt{c-ic \tan(fx+e)}} \right)}{fc^2}$	80
risch	$-\frac{a^2(3iA e^{4i(fx+e)} + 3B e^{4i(fx+e)} + iA e^{2i(fx+e)} - 9B e^{2i(fx+e)} - 2iA + 18B) \sqrt{2}}{30c^2 \sqrt{\frac{c}{e^{2i(fx+e)} + 1}} f}$	90

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^2\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2), x, method = \_RETURNVERBOSE)

[Out] -2\*I/f\*a^2/c^2\*(-1/3\*c\*(A-3\*I\*B)/(c-I\*c\*tan(f\*x+e))^(3/2)+2/5\*c^2\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(5/2)-I\*B/(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.29, size = 79, normalized size = 0.77

$$\frac{2i(15i(-ictan(fx+e)+c)^2Ba^2+5(-ictan(fx+e)+c)(A-3iB)a^2c-6(A-iB)a^2c^2)}{15(-ictan(fx+e)+c)^{\frac{5}{2}}c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/15*I*(15*I*(-I*c*tan(f*x+e)+c)^2*B*a^2+5*(-I*c*tan(f*x+e)+c)*(A-3*I*B)*a^2*c-6*(A-I*B)*a^2*c^2)/((-I*c*tan(f*x+e)+c)^(5/2)*c^2*f)
```

**Fricas [A]**

time = 3.61, size = 106, normalized size = 1.03

$$\frac{\sqrt{2}(3(iA+B)a^2e^{6ifx+6ie}+2(2iA-3B)a^2e^{4ifx+4ie}-(iA-9B)a^2e^{2ifx+2ie}+2(-iA+9B)a^2)\sqrt{\frac{c}{e^{2ifx+2ie}+1}}}{30c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="fricas")
```

```
[Out] -1/30*sqrt(2)*(3*(I*A+B)*a^2*e^(6*I*f*x+6*I*e)+2*(2*I*A-3*B)*a^2*e^(4*I*f*x+4*I*e)-(I*A-9*B)*a^2*e^(2*I*f*x+2*I*e)+2*(-I*A+9*B)*a^2)*sqrt(c/(e^(2*I*f*x+2*I*e)+1))/(c^3*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

```
-----
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

```
[Out] -a**2*(Integral(-A/(-c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)**2-2*I*c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)+c**2*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(A*tan(e+f*x)**2/(-c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)**2-2*I*c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)+c**2*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(-B*tan(e+f*x)/(-c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)**2-2*I*c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)+c**2*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(B*tan(e+f*x)**3/(-c**2*sqrt(-I*c*tan(e+f*x)+c))*tan(e+f*x)**2-2*I*c**2*sqrt(-I*c*tan(e+f*x)+c))
```

```
-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x)
+ Integral(-2*I*A*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*
c*tan(e + f*x) + c)), x) + Integral(-2*I*B*tan(e + f*x)**2/(-c**2*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*t
an(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^2/(-I*c*tan(f*x + e)
+ c)^(5/2), x)
```

**Mupad [B]**

time = 11.36, size = 208, normalized size = 2.02

$$a^2 \sqrt{\frac{c(\cos(2e+2fx)+1) - \sin(2e+2fx)i}{\cos(2e+2fx)+1}} \frac{(18B - A2i - A\cos(2e+2fx)i + A\cos(4e+4fx)4i + A\cos(6e+6fx)3i + 9B\cos(2e+2fx) - 6B\cos(4e+4fx) + 3B\cos(6e+6fx) + A\sin(2e+2fx) - 4A\sin(4e+4fx) - 3A\sin(6e+6fx) + B\sin(2e+2fx)9i - B\sin(4e+4fx)6i + B\sin(6e+6fx)3i)}{30c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1i
)^^(5/2),x)
```

```
[Out] -(a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^^(1/2)*(18*B - A*2i - A*cos(2*e + 2*f*x)*1i + A*cos(4*e + 4*f*x)*4i + A
*cos(6*e + 6*f*x)*3i + 9*B*cos(2*e + 2*f*x) - 6*B*cos(4*e + 4*f*x) + 3*B*co
s(6*e + 6*f*x) + A*sin(2*e + 2*f*x) - 4*A*sin(4*e + 4*f*x) - 3*A*sin(6*e +
6*f*x) + B*sin(2*e + 2*f*x)*9i - B*sin(4*e + 4*f*x)*6i + B*sin(6*e + 6*f*x)
*3i))/(30*c^3*f)
```

$$3.755 \quad \int \frac{(a+ia \tan(e+fx))^2(A+B \tan(e+fx))}{(c-ict \tan(e+fx))^{7/2}} dx$$

Optimal. Leaf size=105

$$-\frac{4a^2(iA+B)}{7f(c-ict \tan(e+fx))^{7/2}} + \frac{2a^2(iA+3B)}{5cf(c-ict \tan(e+fx))^{5/2}} - \frac{2a^2B}{3c^2f(c-ict \tan(e+fx))^{3/2}}$$

[Out]  $-4/7*a^2*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(7/2)}+2/5*a^2*(I*A+3*B)/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}-2/3*a^2*B/c^2/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$\frac{2a^2(3B+iA)}{5cf(c-ict \tan(e+fx))^{5/2}} - \frac{4a^2(B+iA)}{7f(c-ict \tan(e+fx))^{7/2}} - \frac{2a^2B}{3c^2f(c-ict \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^2*(A + B*\text{Tan}[e + f*x])/(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $(-4*a^2*(I*A + B))/(7*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) + (2*a^2*(I*A + 3*B))/(5*c*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (2*a^2*B)/(3*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})$

Rule 78

$\text{Int}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^2 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{7/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)(A+Bx)}{(c-icx)^{9/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{2a(A-iB)}{(c-icx)^{9/2}} - \frac{a(A-3iB)}{c(c-icx)^{7/2}} - \frac{iaB}{c^2(c-icx)^{5/2}} \right) dx, x \right)}{f}$$

$$= -\frac{4a^2(iA + B)}{7f(c - ic \tan(e + fx))^{7/2}} + \frac{2a^2(iA + 3B)}{5cf(c - ic \tan(e + fx))^{5/2}}$$

**Mathematica [A]**

time = 5.30, size = 122, normalized size = 1.16

$$\frac{a^2 \cos^2(e + fx)(-9iA + 33B + (-9iA - 37B) \cos(2(e + fx)) + 7(3A + iB) \sin(2(e + fx)))(\cos(4e + 6fx) + i \sin(4e + 6fx)) \sqrt{c - ic \tan(e + fx)}}{105c^4 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(7/2), x]
```

```
[Out] (a^2*Cos[e + f*x]^2*((-9*I)*A + 33*B + ((-9*I)*A - 37*B)*Cos[2*(e + f*x)] + 7*(3*A + I*B)*Sin[2*(e + f*x)]*(Cos[4*e + 6*f*x] + I*SIN[4*e + 6*f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(105*c^4*f*(Cos[f*x] + I*SIN[f*x])^2)
```

**Maple [A]**

time = 0.25, size = 80, normalized size = 0.76

method	result
derivativedivides	$-\frac{2ia^2 \left( -\frac{iB}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{c(-3iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} + \frac{2c^2(-iB+A)}{7(c-ic \tan(fx+e))^{\frac{7}{2}}} \right)}{f c^2}$
default	$-\frac{2ia^2 \left( -\frac{iB}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{c(-3iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} + \frac{2c^2(-iB+A)}{7(c-ic \tan(fx+e))^{\frac{7}{2}}} \right)}{f c^2}$
risch	$-\frac{a^2(15iA e^{6i(fx+e)} + 15B e^{6i(fx+e)} + 24iA e^{4i(fx+e)} - 18B e^{4i(fx+e)} + 3iA e^{2i(fx+e)} - 11B e^{2i(fx+e)} - 6iA + 22B) \sqrt{2}}{420c^3 \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2), x, method = _RETURNVERBOSE)
```

```
[Out] -2*I/f*a^2/c^2*(-1/3*I*B/(c-I*c*tan(f*x+e))^(3/2)-1/5*c*(A-3*I*B)/(c-I*c*tan(f*x+e))^(5/2)+2/7*c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(7/2))
```

**Maxima [A]**

time = 0.29, size = 79, normalized size = 0.75

$$\frac{2i(35i(-ictan(fx+e)+c)^2Ba^2+21(-ictan(fx+e)+c)(A-3iB)a^2c-30(A-iB)a^2c^2)}{105(-ictan(fx+e)+c)^{\frac{7}{2}}c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="maxima")
```

```
[Out] 2/105*I*(35*I*(-I*c*tan(f*x+e)+c)^2*B*a^2+21*(-I*c*tan(f*x+e)+c)*
(A-3*I*B)*a^2*c-30*(A-I*B)*a^2*c^2)/((-I*c*tan(f*x+e)+c)^(7/2)*c^
2*f)
```

**Fricas [A]**

time = 3.48, size = 128, normalized size = 1.22

$$\frac{\sqrt{2}(15(iA+B)a^2e^{8i fx+8ie}+3(13iA-B)a^2e^{6i fx+6ie}-(-27iA+29B)a^2e^{4i fx+4ie}-(3iA-11B)a^2e^{2i fx+2ie}+2(-3iA+11B)a^2)\sqrt{\frac{c}{e^{2i fx+2ie}+1}}}{420c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="fricas")
```

```
[Out] -1/420*sqrt(2)*(15*(I*A+B)*a^2*e^(8*I*f*x+8*I*e)+3*(13*I*A-B)*a^2*
e^(6*I*f*x+6*I*e)-(-27*I*A+29*B)*a^2*e^(4*I*f*x+4*I*e)-(3*I*A-11
*B)*a^2*e^(2*I*f*x+2*I*e)+2*(-3*I*A+11*B)*a^2)*sqrt(c/(e^(2*I*f*x+
2*I*e)+1))/(c^4*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(7/2),
x)
```

```
[Out] -a**2*(Integral(-A/(I*c**3*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)**3-3*
c**3*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)**2-3*I*c**3*sqrt(-I*c*tan(e
+f*x)+c)*tan(e+f*x)+c**3*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(A*tan(e+f*x)**2/(I*c**3*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)**3-3
*c**3*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)**2-3*I*c**3*sqrt(-I*c*tan(e
+f*x)+c)*tan(e+f*x)+c**3*sqrt(-I*c*tan(e+f*x)+c)),x)+Integral(-B*tan(e+f*x)/(I*c**3*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)**3-3*
c**3*sqrt(-I*c*tan(e+f*x)+c)*tan(e+f*x)**2-3*I*c**3*sqrt(-I*c*tan(e
```



```

+ f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Integra
l(B*tan(e + f*x)**3/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 3
*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*tan(
e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Integr
al(-2*I*A*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3
- 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Int
egral(-2*I*B*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x)
)

```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="giac")

```

```

[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^2/(-I*c*tan(f*x + e)
+ c)^(7/2), x)

```

**Mupad [B]**

time = 11.81, size = 167, normalized size = 1.59

$$-\sqrt{c - \frac{c \sin(e + f x) \operatorname{li}^2}{\cos(e + f x)}} \left( -\frac{a^2 (3A + B 11i) \operatorname{li}}{210 c^4 f} - \frac{a^2 e^{2i + f x 2i} (3A + B 11i) \operatorname{li}}{420 c^4 f} + \frac{a^2 e^{6i + f x 6i} (13A + B 1i) \operatorname{li}}{140 c^4 f} + \frac{a^2 e^{4i + f x 4i} (27A + B 29i) \operatorname{li}}{420 c^4 f} + \frac{a^2 e^{8i + f x 8i} (A - B 1i) \operatorname{li}}{28 c^4 f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1i
)^(7/2),x)

```

```

[Out] -(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i))*(13*
A + B*1i)*1i)/(140*c^4*f) - (a^2*exp(e*2i + f*x*2i)*(3*A + B*11i)*1i)/(420*
c^4*f) - (a^2*(3*A + B*11i)*1i)/(210*c^4*f) + (a^2*exp(e*4i + f*x*4i)*(27*A
+ B*29i)*1i)/(420*c^4*f) + (a^2*exp(e*8i + f*x*8i)*(A - B*1i)*1i)/(28*c^4*
f))

```

$$3.756 \quad \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ictan(e + fx))^{7/2} dx$$

**Optimal.** Leaf size=144

$$\frac{8a^3(iA + B)(c - ictan(e + fx))^{7/2}}{7f} - \frac{8a^3(iA + 2B)(c - ictan(e + fx))^{9/2}}{9cf} + \frac{2a^3(iA + 5B)(c - ictan(e + fx))^{11/2}}{11c^2f} - \frac{2a^3B(c - ictan(e + fx))^{13/2}}{13c^3f}$$

[Out]  $8/7*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(7/2)/f-8/9*a^3*(I*A+2*B)*(c-I*c*tan(f*x+e))^(9/2)/c/f+2/11*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(11/2)/c^2/f-2/13*a^3*B*(c-I*c*tan(f*x+e))^(13/2)/c^3/f$

**Rubi [A]**

time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$\frac{2a^3(5B + iA)(c - ictan(e + fx))^{11/2}}{11c^2f} - \frac{8a^3(2B + iA)(c - ictan(e + fx))^{9/2}}{9cf} + \frac{8a^3(B + iA)(c - ictan(e + fx))^{7/2}}{7f} - \frac{2a^3B(c - ictan(e + fx))^{13/2}}{13c^3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $(8*a^3*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(7*f) - (8*a^3*(I*A + 2*B)*(c - I*c*\text{Tan}[e + f*x])^{(9/2)})/(9*c*f) + (2*a^3*(I*A + 5*B)*(c - I*c*\text{Tan}[e + f*x])^{(11/2)})/(11*c^2*f) - (2*a^3*B*(c - I*c*\text{Tan}[e + f*x])^{(13/2)})/(13*c^3*f)$

**Rule 78**

$\text{Int}[(a + b*x)^n * (c + d*x)^m * (e + f*x)^p, x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ ((\text{ILtQ}[n, 0] \ \&\& \ \text{ILtQ}[p, 0]) \ || \ \text{EqQ}[p, 1] \ || \ (\text{IGtQ}[p, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ \text{LeQ}[9*p + 5*(n + 2), 0] \ || \ \text{GeQ}[n + p + 1, 0] \ || \ (\text{GeQ}[n + p + 2, 0] \ \&\& \ \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m * (c + d*\text{tan}[e + f*x])^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{7/2} dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int (4a^2(A - iB)(c - ic \tan(e + fx))^{7/2} dx\right)}{7f}$$

$$= \frac{8a^3(iA + B)(c - ic \tan(e + fx))^{7/2}}{7f}$$

Mathematica [A]

time = 4.04, size = 127, normalized size = 0.88

$$\frac{2a^3 c^3 \sec^5(e + fx) (\cos(3e) - i \sin(3e)) \sqrt{c - ic \tan(e + fx)} (-572iA + 737B + 7(169A - 86iB) \tan(e + fx) + \cos(2(e + fx))(-1391iA - 1279B + 7(169A - 185iB) \tan(e + fx)))}{9009f(\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2), x]
```

```
[Out] (-2*a^3*c^3*Sec[e + f*x]^5*(Cos[3*e] - I*Sin[3*e])*Sqrt[c - I*c*Tan[e + f*x]])*((-572*I)*A + 737*B + 7*(169*A - (86*I)*B)*Tan[e + f*x] + Cos[2*(e + f*x)])*((-1391*I)*A - 1279*B + 7*(169*A - (185*I)*B)*Tan[e + f*x]))/(9009*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [A]

time = 0.50, size = 121, normalized size = 0.84

method	result
derivativedivides	$2ia^3 \left( \frac{iB(c - ic \tan(fx + e))^{13}}{13} + \frac{(-5iBc + Ac)(c - ic \tan(fx + e))^{11}}{11} + \frac{(-4(-iBc + Ac)c + 4iBc^2)(c - ic \tan(fx + e))^9}{9} + \frac{4(-iBc + Ac)c^7}{7} \right) / fc^3$
default	$2ia^3 \left( \frac{iB(c - ic \tan(fx + e))^{13}}{13} + \frac{(-5iBc + Ac)(c - ic \tan(fx + e))^{11}}{11} + \frac{(-4(-iBc + Ac)c + 4iBc^2)(c - ic \tan(fx + e))^9}{9} + \frac{4(-iBc + Ac)c^7}{7} \right) / fc^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2), x, method =_RETURNVERBOSE)
```

```
[Out] 2*I/f*a^3/c^3*(1/13*I*B*(c-I*c*tan(f*x+e))^(13/2)+1/11*(-5*I*B*c+A*c)*(c-I*c*tan(f*x+e))^(11/2)+1/9*(-4*(-I*B*c+A*c)*c+4*I*B*c^2)*(c-I*c*tan(f*x+e))^(9/2)+4/7*(-I*B*c+A*c)*c^2*(c-I*c*tan(f*x+e))^(7/2))
```

**Maxima [A]**

time = 0.29, size = 108, normalized size = 0.75

$$\frac{2i \left( 693i (-i c \tan (fx + e) + c)^{\frac{13}{2}} B a^3 + 819 (-i c \tan (fx + e) + c)^{\frac{11}{2}} (A - 5i B) a^3 c - 4004 (-i c \tan (fx + e) + c)^{\frac{9}{2}} (A - 2i B) a^3 c^2 + 5148 (-i c \tan (fx + e) + c)^{\frac{7}{2}} (A - i B) a^3 c^3 \right)}{9009 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="maxima")
```

```
[Out] 2/9009*I*(693*I*(-I*c*tan(f*x + e) + c)^(13/2)*B*a^3 + 819*(-I*c*tan(f*x + e) + c)^(11/2)*(A - 5*I*B)*a^3*c - 4004*(-I*c*tan(f*x + e) + c)^(9/2)*(A - 2*I*B)*a^3*c^2 + 5148*(-I*c*tan(f*x + e) + c)^(7/2)*(A - I*B)*a^3*c^3)/(c^3*f)
```

**Fricas [A]**

time = 4.95, size = 189, normalized size = 1.31

$$\frac{64 \sqrt{2} (1287 (-i A - B) a^3 c^3 e^{(6i fx + 6i e)} + 143 (-13i A + B) a^3 c^3 e^{(4i fx + 4i e)} + 52 (-13i A + B) a^3 c^3 e^{(2i fx + 2i e)} + 8 (-13i A + B) a^3 c^3) \sqrt{\frac{c}{e^{(2i fx + 2i e)} + 1}}}{9009 (f e^{(12i fx + 12i e)} + 6 f e^{(10i fx + 10i e)} + 15 f e^{(8i fx + 8i e)} + 20 f e^{(6i fx + 6i e)} + 15 f e^{(4i fx + 4i e)} + 6 f e^{(2i fx + 2i e)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="fricas")
```

```
[Out] -64/9009*sqrt(2)*(1287*(-I*A - B)*a^3*c^3*e^(6*I*f*x + 6*I*e) + 143*(-13*I*A + B)*a^3*c^3*e^(4*I*f*x + 4*I*e) + 52*(-13*I*A + B)*a^3*c^3*e^(2*I*f*x + 2*I*e) + 8*(-13*I*A + B)*a^3*c^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^3 \left( \int \sqrt{-i c \tan (e + f x) + c} dx + \int 3i A a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i A a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i A a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i B a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i B a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i B a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i B a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i B a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx + \int 3i B a^2 \sqrt{-i c \tan (e + f x) + c} \tan (e + f x) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2),
x)
```

```
[Out] -I*a**3*(Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(3*I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(3*I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x) + Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**6, x) + Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(3*I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(3*I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x) + Integral(3*I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**6, x))
```

) + c)\*tan(e + f\*x)\*\*3, x) + Integral(3\*I\*B\*c\*\*3\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*5, x) + Integral(I\*B\*c\*\*3\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*7, x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2), x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^3\*(-I\*c\*tan(f\*x + e) + c)^(7/2), x)

**Mupad [B]**

time = 15.07, size = 349, normalized size = 2.42

$$-\frac{\left(\frac{a^3 c^3 (A-B) 64i}{9f} + \frac{a^3 c^3 (A-B) 64i}{9f}\right) \sqrt{c + \frac{c(e^{2i+fx2i} - 1) 1i}{e^{2i+fx2i} + 1}}}{(e^{2i+fx2i} + 1)^3} - \frac{\left(\frac{a^3 c^3 (A-B) 64i}{13f} - \frac{a^3 c^3 (A+B) 64i}{13f}\right) \sqrt{c + \frac{c(e^{2i+fx2i} - 1) 1i}{e^{2i+fx2i} + 1}}}{(e^{2i+fx2i} + 1)^3} + \frac{\left(\frac{256B a^3 c^3}{11f} + \frac{a^3 c^3 (A-B) 64i}{11f}\right) \sqrt{c + \frac{c(e^{2i+fx2i} - 1) 1i}{e^{2i+fx2i} + 1}}}{(e^{2i+fx2i} + 1)^3} + \frac{a^3 c^3 (A-B) 1i \sqrt{c + \frac{c(e^{2i+fx2i} - 1) 1i}{e^{2i+fx2i} + 1}}}{7f (e^{2i+fx2i} + 1)^3} 64i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)^(7/2), x)

[Out] (((a^3\*c^3\*(A - B\*1i)\*64i)/(11\*f) + (256\*B\*a^3\*c^3)/(11\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^5 - (((a^3\*c^3\*(A - B\*1i)\*64i)/(13\*f) - (a^3\*c^3\*(A + B\*1i)\*64i)/(13\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^6 - (((a^3\*c^3\*(A - B\*1i)\*64i)/(9\*f) + (a^3\*c^3\*(A - B\*3i)\*64i)/(9\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^4 + (a^3\*c^3\*(A - B\*1i)\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*64i)/(7\*f\*(exp(e\*2i + f\*x\*2i) + 1)^3)

$$3.757 \quad \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ictan(e + fx))^{5/2} dx$$

**Optimal.** Leaf size=144

$$\frac{8a^3(iA + B)(c - ictan(e + fx))^{5/2}}{5f} - \frac{8a^3(iA + 2B)(c - ictan(e + fx))^{7/2}}{7cf} + \frac{2a^3(iA + 5B)(c - ictan(e + fx))^{9/2}}{9c^2f}$$

[Out]  $8/5*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(5/2)/f-8/7*a^3*(I*A+2*B)*(c-I*c*tan(f*x+e))^(7/2)/c/f+2/9*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(9/2)/c^2/f-2/11*a^3*B*(c-I*c*tan(f*x+e))^(11/2)/c^3/f$

**Rubi [A]**

time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ ,

Rules used = {3669, 78}

$$\frac{2a^3(5B + iA)(c - ictan(e + fx))^{9/2}}{9c^2f} - \frac{8a^3(2B + iA)(c - ictan(e + fx))^{7/2}}{7cf} + \frac{8a^3(B + iA)(c - ictan(e + fx))^{5/2}}{5f} - \frac{2a^3B(c - ictan(e + fx))^{11/2}}{11c^3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{5/2}, x]$

[Out]  $(8*a^3*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^{5/2})/(5*f) - (8*a^3*(I*A + 2*B)*(c - I*c*\text{Tan}[e + f*x])^{7/2})/(7*c*f) + (2*a^3*(I*A + 5*B)*(c - I*c*\text{Tan}[e + f*x])^{9/2})/(9*c^2*f) - (2*a^3*B*(c - I*c*\text{Tan}[e + f*x])^{11/2})/(11*c^3*f)$

**Rule 78**

$\text{Int}[(a + b*x)^n * (c + d*x)^m * (e + f*x)^p, x]$  /;  $\text{FreeQ}\{a, b, c, d, e, f, n\}, x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $(\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \mid \mid \text{EqQ}[p, 1] \mid \mid (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \mid \mid \text{LeQ}[9*p + 5*(n + 2), 0] \mid \mid \text{GeQ}[n + p + 1, 0] \mid \mid (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m * (A + B*\text{tan}[e + f*x])^n, x]$  /;  $\text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x$  &&  $\text{EqQ}[b*c + a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$

**Rubi steps**

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx))^{5/2} dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx) dx\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(4a^2(A - iB)(c - ic \tan(e + fx))\right) dx\right)}{f}$$

$$= \frac{8a^3(iA + B)(c - ic \tan(e + fx))^5}{5f}$$

**Mathematica [A]**

time = 3.76, size = 139, normalized size = 0.97

$$\frac{2a^3 c^2 \sec^4(e + fx) (\cos(2e - fx) - i \sin(2e - fx)) \sqrt{c - ic \tan(e + fx)} (9(-44iA + 31B) + 5(121A - 74iB) \tan(e + fx) + \cos(2(e + fx))(-781iA - 701B + (605A - 685iB) \tan(e + fx)))}{3465 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] (-2\*a^3\*c^2\*Sec[e + f\*x]^4\*(Cos[2\*e - f\*x] - I\*Sin[2\*e - f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]\*(9\*((-44\*I)\*A + 31\*B) + 5\*(121\*A - (74\*I)\*B)\*Tan[e + f\*x] + Cos[2\*(e + f\*x)]\*((-781\*I)\*A - 701\*B + (605\*A - (685\*I)\*B)\*Tan[e + f\*x]))/(3465\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.47, size = 121, normalized size = 0.84

method	result
derivativedivides	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{11}}{11} + \frac{(-5iBc+Ac)(c-ic \tan(fx+e))^{9}}{9} + \frac{(-4(-iBc+Ac)c+4iBc^2)(c-ic \tan(fx+e))^{7}}{7} + \frac{4(-iBc+Ac)c^2}{f c^3} \right)}{f c^3}$
default	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{11}}{11} + \frac{(-5iBc+Ac)(c-ic \tan(fx+e))^{9}}{9} + \frac{(-4(-iBc+Ac)c+4iBc^2)(c-ic \tan(fx+e))^{7}}{7} + \frac{4(-iBc+Ac)c^2}{f c^3} \right)}{f c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2), x, method = \_RETURNVERBOSE)

[Out] 2\*I/f\*a^3/c^3\*(1/11\*I\*B\*(c-I\*c\*tan(f\*x+e))^(11/2)+1/9\*(-5\*I\*B\*c+A\*c)\*(c-I\*c\*tan(f\*x+e))^(9/2)+1/7\*(-4\*(-I\*B\*c+A\*c)\*c+4\*I\*B\*c^2)\*(c-I\*c\*tan(f\*x+e))^(7/2)+4/5\*(-I\*B\*c+A\*c)\*c^2\*(c-I\*c\*tan(f\*x+e))^(5/2))

**Maxima [A]**

time = 0.29, size = 108, normalized size = 0.75

$$\frac{2i(315i(-i c \tan(fx + e) + c)^{\frac{11}{2}} B a^3 + 385(-i c \tan(fx + e) + c)^{\frac{9}{2}}(A - 5iB)a^3 c - 1980(-i c \tan(fx + e) + c)^{\frac{7}{2}}(A - 2iB)a^3 c^2 + 2772(-i c \tan(fx + e) + c)^{\frac{5}{2}}(A - iB)a^3 c^3)}{3465 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/3465*I*(315*I*(-I*c*tan(f*x + e) + c)^(11/2)*B*a^3 + 385*(-I*c*tan(f*x +
e) + c)^(9/2)*(A - 5*I*B)*a^3*c - 1980*(-I*c*tan(f*x + e) + c)^(7/2)*(A - 2
*I*B)*a^3*c^2 + 2772*(-I*c*tan(f*x + e) + c)^(5/2)*(A - I*B)*a^3*c^3)/(c^3*
f)
```

**Fricas [A]**

time = 4.23, size = 182, normalized size = 1.26

$$\frac{32\sqrt{2}(693(-iA - B)a^3c^2e^{(6ifx+6ie)} + 99(-11iA - B)a^3c^2e^{(4ifx+4ie)} + 44(-11iA - B)a^3c^2e^{(2ifx+2ie)} + 8(-11iA - B)a^3c^2)\sqrt{\frac{c}{e^{(2ifx+2ie)} + 1}}}{3465(fe^{(10ifx+10ie)} + 5fe^{(8ifx+8ie)} + 10fe^{(6ifx+6ie)} + 10fe^{(4ifx+4ie)} + 5fe^{(2ifx+2ie)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="fricas")
```

```
[Out] -32/3465*sqrt(2)*(693*(-I*A - B)*a^3*c^2*e^(6*I*f*x + 6*I*e) + 99*(-11*I*A
- B)*a^3*c^2*e^(4*I*f*x + 4*I*e) + 44*(-11*I*A - B)*a^3*c^2*e^(2*I*f*x + 2*
I*e) + 8*(-11*I*A - B)*a^3*c^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(10*
I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f
*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

```
-(f*a**3*(Integral(I*A*c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-A*c**
2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*A*c**2*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-A*c**2*sqrt(-I*c*tan(
e + f*x) + c)*tan(e + f*x)**5, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x)**2, x) + Integral(-2*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**4, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**5/2),
x)
```

```
[Out] -I*a**3*(Integral(I*A*c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-A*c**
2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*A*c**2*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-A*c**2*sqrt(-I*c*tan(
e + f*x) + c)*tan(e + f*x)**5, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x)**2, x) + Integral(-2*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**4, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
```



```
f*x)**6, x) + Integral(2*I*A*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)*
**2, x) + Integral(I*A*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x)
+ Integral(I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral
(2*I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(I*B*
c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**5, x))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e)
+ c)^(5/2), x)
```

**Mupad [B]**

time = 14.48, size = 349, normalized size = 2.42

$$-\frac{\left(\frac{a^3 c^2 (A-B) 32i}{7f} + \frac{a^3 c^2 (A-B) 32i}{7f}\right) \sqrt{c + \frac{c (e^{2i+f x 2i} - 1) 11}{e^{2i+f x 2i} + 1}}}{(e^{2i+f x 2i} + 1)^3} - \frac{\left(\frac{a^3 c^2 (A-B) 32i}{11f} - \frac{a^3 c^2 (A+B) 32i}{11f}\right) \sqrt{c + \frac{c (e^{2i+f x 2i} - 1) 11}{e^{2i+f x 2i} + 1}}}{(e^{2i+f x 2i} + 1)^3} + \frac{\left(\frac{128 B a^3 c^2}{9f} + \frac{a^3 c^2 (A-B) 32i}{9f}\right) \sqrt{c + \frac{c (e^{2i+f x 2i} - 1) 11}{e^{2i+f x 2i} + 1}}}{(e^{2i+f x 2i} + 1)^3} + \frac{a^3 c^2 (A-B) 11}{5f (e^{2i+f x 2i} + 1)^3} \sqrt{c + \frac{c (e^{2i+f x 2i} - 1) 11}{e^{2i+f x 2i} + 1}} 32i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^
(5/2),x)
```

```
[Out] (((a^3*c^2*(A - B*1i)*32i)/(9*f) + (128*B*a^3*c^2)/(9*f))*(c + (c*(exp(e*2i
+ f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i
) + 1)^4 - (((a^3*c^2*(A - B*1i)*32i)/(11*f) - (a^3*c^2*(A + B*1i)*32i)/(11
*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/
2))/(exp(e*2i + f*x*2i) + 1)^5 - (((a^3*c^2*(A - B*1i)*32i)/(7*f) + (a^3*c^
2*(A - B*3i)*32i)/(7*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i
+ f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i) + 1)^3 + (a^3*c^2*(A - B*1i)*(c
+ (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)
/(5*f*(exp(e*2i + f*x*2i) + 1)^2)
```

$$3.758 \quad \int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) (c - ictan(e + fx))^{3/2} dx$$

**Optimal.** Leaf size=144

$$\frac{8a^3(iA + B)(c - ictan(e + fx))^{3/2}}{3f} - \frac{8a^3(iA + 2B)(c - ictan(e + fx))^{5/2}}{5cf} + \frac{2a^3(iA + 5B)(c - ictan(e + fx))^{7/2}}{7c^2f}$$

[Out]  $8/3*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(3/2)/f-8/5*a^3*(I*A+2*B)*(c-I*c*tan(f*x+e))^(5/2)/c/f+2/7*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(7/2)/c^2/f-2/9*a^3*B*(c-I*c*tan(f*x+e))^(9/2)/c^3/f$

**Rubi [A]**

time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ ,

Rules used = {3669, 78}

$$\frac{2a^3(5B + iA)(c - ictan(e + fx))^{7/2}}{7c^2f} - \frac{8a^3(2B + iA)(c - ictan(e + fx))^{5/2}}{5cf} + \frac{8a^3(B + iA)(c - ictan(e + fx))^{3/2}}{3f} - \frac{2a^3B(c - ictan(e + fx))^{9/2}}{9c^3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{3/2}, x]$

[Out]  $(8*a^3*(I*A + B)*(c - I*c*\text{Tan}[e + f*x])^{3/2})/(3*f) - (8*a^3*(I*A + 2*B)*(c - I*c*\text{Tan}[e + f*x])^{5/2})/(5*c*f) + (2*a^3*(I*A + 5*B)*(c - I*c*\text{Tan}[e + f*x])^{7/2})/(7*c^2*f) - (2*a^3*B*(c - I*c*\text{Tan}[e + f*x])^{9/2})/(9*c^3*f)$

**Rule 78**

$\text{Int}[(a + b*x)^n * (c + d*x)^m * (e + f*x)^p, x]$  /;  $\text{FreeQ}\{a, b, c, d, e, f, n\}, x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $(\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \mid \mid \text{EqQ}[p, 1] \mid \mid (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \mid \mid \text{LeQ}[9*p + 5*(n + 2), 0] \mid \mid \text{GeQ}[n + p + 1, 0] \mid \mid (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

**Rule 3669**

$\text{Int}[(a + b*\text{tan}[e + f*x])^m * (A + B*\text{tan}[e + f*x])^n, x]$  /;  $\text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x$  &&  $\text{EqQ}[b*c + a*d, 0]$  &&  $\text{EqQ}[a^2 + b^2, 0]$

**Rubi steps**

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2} dx = \frac{(ac) \text{Subst}\left(\int (a + iax)^2 (A + Bx) dx, \frac{c - ic \tan(e + fx)}{f}\right)}{f}$$

$$= \frac{(ac) \text{Subst}\left(\int \left(4a^2(A - iB)\sqrt{c - ic \tan(e + fx)}\right) dx, \frac{c - ic \tan(e + fx)}{f}\right)}{3f}$$

$$= \frac{8a^3(iA + B)(c - ic \tan(e + fx))^3}{3f}$$

**Mathematica [A]**

time = 3.09, size = 130, normalized size = 0.90

$$\frac{2a^3 c \sec^3(e + fx) (\cos(e - 2fx) - i \sin(e - 2fx)) \sqrt{c - ic \tan(e + fx)} (7(-12iA + B) + (81A - 62iB) \tan(e + fx) + \cos(2(e + fx))(-129iA - 113B + (81A - 97iB) \tan(e + fx)))}{315f(\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (-2\*a^3\*c\*Sec[e + f\*x]^3\*(Cos[e - 2\*f\*x] - I\*Sin[e - 2\*f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]\*(7\*((-12\*I)\*A + B) + (81\*A - (62\*I)\*B)\*Tan[e + f\*x] + Cos[2\*(e + f\*x)]\*((-129\*I)\*A - 113\*B + (81\*A - (97\*I)\*B)\*Tan[e + f\*x]))/(315\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.46, size = 121, normalized size = 0.84

method	result
derivativedivides	$2ia^3 \left( \frac{iB(c - ic \tan(fx + e))^{\frac{9}{2}}}{9} + \frac{(-5iBc + Ac)(c - ic \tan(fx + e))^{\frac{7}{2}}}{7} + \frac{(-4(-iBc + Ac)c + 4iBc^2)(c - ic \tan(fx + e))^{\frac{5}{2}}}{5} + \frac{4(-iBc + Ac)c^2}{f c^3} \right)$
default	$2ia^3 \left( \frac{iB(c - ic \tan(fx + e))^{\frac{9}{2}}}{9} + \frac{(-5iBc + Ac)(c - ic \tan(fx + e))^{\frac{7}{2}}}{7} + \frac{(-4(-iBc + Ac)c + 4iBc^2)(c - ic \tan(fx + e))^{\frac{5}{2}}}{5} + \frac{4(-iBc + Ac)c^2}{f c^3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2), x, method = \_RETURNVERBOSE)

[Out] 2\*I/f\*a^3/c^3\*(1/9\*I\*B\*(c-I\*c\*tan(f\*x+e))^(9/2)+1/7\*(-5\*I\*B\*c+A\*c)\*(c-I\*c\*tan(f\*x+e))^(7/2)+1/5\*(-4\*(-I\*B\*c+A\*c)\*c+4\*I\*B\*c^2)\*(c-I\*c\*tan(f\*x+e))^(5/2)+4/3\*(-I\*B\*c+A\*c)\*c^2\*(c-I\*c\*tan(f\*x+e))^(3/2))



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^3\*(-I\*c\*tan(f\*x + e)  
+ c)^(3/2), x)

**Mupad [B]**

time = 14.08, size = 335, normalized size = 2.33

$$-\frac{\left(\frac{a^3c(A-B)16i}{9f} + \frac{a^3c(A-B)16i}{9f}\right)\sqrt{c + \frac{c(e^{2i+fx}11-1)11}{e^{2i+fx}+1}}}{(e^{2i+fx}+1)^2} - \frac{\left(\frac{a^3c(A-B)16i}{9f} - \frac{a^3c(A+B)16i}{9f}\right)\sqrt{c + \frac{c(e^{2i+fx}11-1)11}{e^{2i+fx}+1}}}{(e^{2i+fx}+1)^4} + \frac{\left(\frac{64B a^3}{9f} + \frac{a^3c(A-B)16i}{9f}\right)\sqrt{c + \frac{c(e^{2i+fx}11-1)11}{e^{2i+fx}+1}}}{(e^{2i+fx}+1)^3} + \frac{a^3c(A-B)16i}{3f}\sqrt{c + \frac{c(e^{2i+fx}11-1)11}{e^{2i+fx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)^(3/2),x)

[Out] (((a^3\*c\*(A - B\*1i)\*16i)/(7\*f) + (64\*B\*a^3\*c)/(7\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^3 - (((a^3\*c\*(A - B\*1i)\*16i)/(9\*f) - (a^3\*c\*(A + B\*1i)\*16i)/(9\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^4 - (((a^3\*c\*(A - B\*1i)\*16i)/(5\*f) + (a^3\*c\*(A - B\*3i)\*16i)/(5\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^2 + (a^3\*c\*(A - B\*1i)\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*16i)/(3\*f\*(exp(e\*2i + f\*x\*2i) + 1))

### 3.759 $\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}$

**Optimal.** Leaf size=142

$$\frac{8a^3(iA + B)\sqrt{c - ic \tan(e + fx)}}{f} - \frac{8a^3(iA + 2B)(c - ic \tan(e + fx))^{3/2}}{3cf} + \frac{2a^3(iA + 5B)(c - ic \tan(e + fx))^{5/2}}{5c^2f}$$

[Out]  $8a^3(I*A+B)*(c-I*c*\tan(f*x+e))^{(1/2)}/f-8/3a^3(I*A+2*B)*(c-I*c*\tan(f*x+e))^{(3/2)}/c/f+2/5a^3(I*A+5*B)*(c-I*c*\tan(f*x+e))^{(5/2)}/c^2/f-2/7a^3*B*(c-I*c*\tan(f*x+e))^{(7/2)}/c^3/f$

**Rubi [A]**

time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$\frac{2a^3(5B + iA)(c - ic \tan(e + fx))^{5/2}}{5c^2f} - \frac{8a^3(2B + iA)(c - ic \tan(e + fx))^{3/2}}{3cf} + \frac{8a^3(B + iA)\sqrt{c - ic \tan(e + fx)}}{f} - \frac{2a^3B(c - ic \tan(e + fx))^{7/2}}{7c^3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])*Sqrt[c - I*c*\text{Tan}[e + f*x]], x]$

[Out]  $(8*a^3*(I*A + B)*Sqrt[c - I*c*\text{Tan}[e + f*x]])/f - (8*a^3*(I*A + 2*B)*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*c*f) + (2*a^3*(I*A + 5*B)*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(5*c^2*f) - (2*a^3*B*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(7*c^3*f)$

**Rule 78**

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol)] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])*(c_. + (d_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol)] :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \tan(e + fx))^3 (A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{\sqrt{c-icx}} dx, x, t \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(A-iB)}{\sqrt{c-icx}} - \frac{4a^2(A-iB)}{\sqrt{c-icx}} \right) dx, x, t \right)}{f}$$

$$= \frac{8a^3(iA + B) \sqrt{c - ic \tan(e + fx)}}{f}$$

**Mathematica [A]**

time = 2.10, size = 124, normalized size = 0.87

$$\frac{a^3 \sec^2(e + fx) (\cos(3fx) + i \sin(3fx)) \sqrt{c - ic \tan(e + fx)} (280iA + 170B + (-98A + 100iB) \tan(e + fx) + \cos(2(e + fx)) (322iA + 290B + (-98A + 130iB) \tan(e + fx)))}{105f(\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out] (a^3\*Sec[e + f\*x]^2\*(Cos[3\*f\*x] + I\*Sin[3\*f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]\*((280\*I)\*A + 170\*B + (-98\*A + (100\*I)\*B)\*Tan[e + f\*x] + Cos[2\*(e + f\*x)]\*((322\*I)\*A + 290\*B + (-98\*A + (130\*I)\*B)\*Tan[e + f\*x]))/(105\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.48, size = 121, normalized size = 0.85

method	result
derivativedivides	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{7/2}}{7} + \frac{(-5iBc+Ac)(c-ic \tan(fx+e))^{5/2}}{5} + \frac{(-4(-iBc+Ac)c+4iBc^2)(c-ic \tan(fx+e))^{3/2}}{3} + 4(-iBc+Ac)c^2 \right)}{f c^3}$
default	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{7/2}}{7} + \frac{(-5iBc+Ac)(c-ic \tan(fx+e))^{5/2}}{5} + \frac{(-4(-iBc+Ac)c+4iBc^2)(c-ic \tan(fx+e))^{3/2}}{3} + 4(-iBc+Ac)c^2 \right)}{f c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e)), x, method =\_RETURNVERBOSE)

[Out] 2\*I/f\*a^3/c^3\*(1/7\*I\*B\*(c-I\*c\*tan(f\*x+e))^(7/2)+1/5\*(-5\*I\*B\*c+A\*c)\*(c-I\*c\*tan(f\*x+e))^(5/2)+1/3\*(-4\*(-I\*B\*c+A\*c)\*c+4\*I\*B\*c^2)\*(c-I\*c\*tan(f\*x+e))^(3/2)+4\*(-I\*B\*c+A\*c)\*c^2\*(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.31, size = 108, normalized size = 0.76

$$\frac{2i(15i(-i c \tan(fx + e) + c)^{\frac{5}{2}} B a^3 + 21(-i c \tan(fx + e) + c)^{\frac{5}{2}}(A - 5iB)a^3 c - 140(-i c \tan(fx + e) + c)^{\frac{3}{2}}(A - 2iB)a^3 c^2 + 420\sqrt{-i c \tan(fx + e) + c}(A - iB)a^3 c^3)}{105 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e)),x,  
algorithm="maxima")

[Out] 2/105\*I\*(15\*I\*(-I\*c\*tan(f\*x + e) + c)^(7/2)\*B\*a^3 + 21\*(-I\*c\*tan(f\*x + e) + c)^(5/2)\*(A - 5\*I\*B)\*a^3\*c - 140\*(-I\*c\*tan(f\*x + e) + c)^(3/2)\*(A - 2\*I\*B)\*a^3\*c^2 + 420\*sqrt(-I\*c\*tan(f\*x + e) + c)\*(A - I\*B)\*a^3\*c^3)/(c^3\*f)

**Fricas [A]**

time = 3.97, size = 144, normalized size = 1.01

$$\frac{8\sqrt{2}(105(-iA - B)a^3 e^{(6i fx + 6i e)} + 35(-7iA - 5B)a^3 e^{(4i fx + 4i e)} + 28(-7iA - 5B)a^3 e^{(2i fx + 2i e)} + 8(-7iA - 5B)a^3) \sqrt{\frac{c}{e^{(2i fx + 2i e)} + 1}}}{105(f e^{(6i fx + 6i e)} + 3 f e^{(4i fx + 4i e)} + 3 f e^{(2i fx + 2i e)} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e)),x,  
algorithm="fricas")

[Out] -8/105\*sqrt(2)\*(105\*(-I\*A - B)\*a^3\*e^(6\*I\*f\*x + 6\*I\*e) + 35\*(-7\*I\*A - 5\*B)\*a^3\*e^(4\*I\*f\*x + 4\*I\*e) + 28\*(-7\*I\*A - 5\*B)\*a^3\*e^(2\*I\*f\*x + 2\*I\*e) + 8\*(-7\*I\*A - 5\*B)\*a^3)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(f\*e^(6\*I\*f\*x + 6\*I\*e) + 3\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 3\*f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^3 \left( \int \frac{1}{\sqrt{-c \tan(e + fx) + c}} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^2(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^3(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^4(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^5(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^6(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^7(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^8(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^9(e + fx)} dx + \int \frac{1}{\sqrt{-c \tan(e + fx) + c} \tan^{10}(e + fx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(a+I\*a\*tan(f\*x+e))\*\*3\*(A+B\*tan(f\*x+e)),  
x)

[Out] -I\*a\*\*3\*(Integral(I\*A\*sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-3\*A\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x), x) + Integral(A\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*3, x) + Integral(-3\*B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2, x) + Integral(B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*4, x) + Integral(-3\*I\*A\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2, x) + Integral(I\*B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x), x) + Integral(-3\*I\*B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*3, x))



**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^3*sqrt(-I*c*tan(f*x +
e) + c), x)
```

**Mupad [B]**

time = 13.58, size = 313, normalized size = 2.20

$$-\frac{\sqrt{c + \frac{c(e^{2i+fx2i}-1)\operatorname{li}}{e^{2i+fx2i}+1}} \left( \frac{a^3(A-B)\operatorname{si}}{3f} + \frac{a^3(A-B)\operatorname{si}}{3f} \right)}{e^{2i+fx2i}+1} - \frac{\sqrt{c + \frac{c(e^{2i+fx2i}-1)\operatorname{li}}{e^{2i+fx2i}+1}} \left( \frac{a^3(A-B)\operatorname{si}}{7f} - \frac{a^3(A+B)\operatorname{si}}{7f} \right)}{(e^{2i+fx2i}+1)^3} + \frac{\sqrt{c + \frac{c(e^{2i+fx2i}-1)\operatorname{li}}{e^{2i+fx2i}+1}} \left( \frac{32B\operatorname{si}}{5f} + \frac{a^3(A-B)\operatorname{si}}{5f} \right)}{(e^{2i+fx2i}+1)^2} + \frac{a^3(A-B)\operatorname{si}}{f} \sqrt{c + \frac{c(e^{2i+fx2i}-1)\operatorname{li}}{e^{2i+fx2i}+1}} \operatorname{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)^(
1/2), x)
```

```
[Out] ((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*
(a^3*(A - B*1i)*8i)/(5*f) + (32*B*a^3)/(5*f))/(exp(e*2i + f*x*2i) + 1)^2 -
((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*
((a^3*(A - B*1i)*8i)/(7*f) - (a^3*(A + B*1i)*8i)/(7*f)))/(exp(e*2i + f*x*2i
) + 1)^3 - ((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) +
1))^(1/2)*((a^3*(A - B*1i)*8i)/(3*f) + (a^3*(A - B*3i)*8i)/(3*f)))/(exp(e*2
i + f*x*2i) + 1) + (a^3*(A - B*1i)*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)
)/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)/f
```

$$3.760 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{\sqrt{c-ictan(e+fx)}} dx$$

Optimal. Leaf size=140

$$-\frac{8a^3(iA+B)}{f\sqrt{c-ictan(e+fx)}} - \frac{8a^3(iA+2B)\sqrt{c-ictan(e+fx)}}{cf} + \frac{2a^3(iA+5B)(c-ictan(e+fx))^{3/2}}{3c^2f} - \frac{2a^3B}{c^2f}$$

[Out]  $-8a^3(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(1/2)}-8a^3(I*A+2*B)*(c-I*c*\tan(f*x+e))^{(1/2)}/c/f+2/3*a^3(I*A+5*B)*(c-I*c*\tan(f*x+e))^{(3/2)}/c^2/f-2/5*a^3*B*(c-I*c*\tan(f*x+e))^{(5/2)}/c^3/f$

Rubi [A]

time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ ,

Rules used = {3669, 78}

$$\frac{2a^3(5B+iA)(c-ictan(e+fx))^{3/2}}{3c^2f} - \frac{8a^3(2B+iA)\sqrt{c-ictan(e+fx)}}{cf} - \frac{8a^3(B+iA)}{f\sqrt{c-ictan(e+fx)}} - \frac{2a^3B(c-ictan(e+fx))^{5/2}}{5c^3f}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out]  $(-8a^3(I*A+B))/(f*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]) - (8a^3(I*A+2*B)*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])/(c*f) + (2a^3(I*A+5*B)*(c-I*c*\text{Tan}[e+f*x])^{(3/2)})/(3*c^2*f) - (2a^3*B*(c-I*c*\text{Tan}[e+f*x])^{(5/2)})/(5*c^3*f)$

Rule 78

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m-1)\*(c + d\*x)^(n-1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{\sqrt{c - ict \tan(e + fx)}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(A-iB)}{(c-icx)^{3/2}} - \frac{4a^2(A-2iB)}{c\sqrt{c-icx}} + \frac{a^2(A-5iB)\sqrt{c-icx}}{c^2} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{8a^3(iA + B)}{f\sqrt{c - ict \tan(e + fx)}} - \frac{8a^3(iA + 2B)\sqrt{c - ict \tan(e + fx)}}{cf}$$

**Mathematica [A]**

time = 2.47, size = 152, normalized size = 1.09

$$\frac{2a^3(\cos(e + 4fx) + i \sin(e + 4fx))(A + B \tan(e + fx))\sqrt{c - ict \tan(e + fx)}(60iA + 87B + (25A - 38iB) \tan(e + fx) + \cos(2(e + fx))(55iA + 71B + (25A - 41iB) \tan(e + fx)))}{15cf(\cos(fx) + i \sin(fx))^3(A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out] (-2\*a^3\*(Cos[e + 4\*f\*x] + I\*Sin[e + 4\*f\*x])\*(A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]\*((60\*I)\*A + 87\*B + (25\*A - (38\*I)\*B)\*Tan[e + f\*x] + Cos[2\*(e + f\*x)]\*((55\*I)\*A + 71\*B + (25\*A - (41\*I)\*B)\*Tan[e + f\*x]))/(15\*c\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [A]**

time = 0.25, size = 135, normalized size = 0.96

method	result
derivativedivides	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{5/2}}{5} - \frac{5iBc(c-ic \tan(fx+e))^{3/2}}{3} + \frac{Ac(c-ic \tan(fx+e))^{3/2}}{3} + 8iBc^2 \sqrt{c - ict \tan(fx + e)} - 4Ac^2 \right)}{fc^3}$
default	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{5/2}}{5} - \frac{5iBc(c-ic \tan(fx+e))^{3/2}}{3} + \frac{Ac(c-ic \tan(fx+e))^{3/2}}{3} + 8iBc^2 \sqrt{c - ict \tan(fx + e)} - 4Ac^2 \right)}{fc^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2), x, method = \_RETURNVERBOSE)

[Out] 2\*I/f\*a^3/c^3\*(1/5\*I\*B\*(c-I\*c\*tan(f\*x+e))^(5/2)-5/3\*I\*B\*c\*(c-I\*c\*tan(f\*x+e))^(3/2)+1/3\*A\*c\*(c-I\*c\*tan(f\*x+e))^(3/2)+8\*I\*B\*c^2\*(c-I\*c\*tan(f\*x+e))^(1/2)-4\*A\*c^2\*(c-I\*c\*tan(f\*x+e))^(1/2)-4\*c^3\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.30, size = 112, normalized size = 0.80

$$\frac{2i \left( \frac{60(A-iB)a^3c}{\sqrt{-ic \tan(fx+e)+c}} - \frac{3i(-ic \tan(fx+e)+c)^{\frac{5}{2}}Ba^3+5(-ic \tan(fx+e)+c)^{\frac{3}{2}}(A-5iB)a^3c-60\sqrt{-ic \tan(fx+e)+c}(A-2iB)a^3c^2}{c^2} \right)}{15cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2), x, algorithm="maxima")

[Out] -2/15\*I\*(60\*(A - I\*B)\*a^3\*c/sqrt(-I\*c\*tan(f\*x + e) + c) - (3\*I\*(-I\*c\*tan(f\*x + e) + c)^(5/2)\*B\*a^3 + 5\*(-I\*c\*tan(f\*x + e) + c)^(3/2)\*(A - 5\*I\*B)\*a^3\*c - 60\*sqrt(-I\*c\*tan(f\*x + e) + c)\*(A - 2\*I\*B)\*a^3\*c^2)/c^2)/(c\*f)

**Fricas [A]**

time = 2.99, size = 133, normalized size = 0.95

$$\frac{4\sqrt{2}(15(iA+B)a^3e^{6i fx+6ie})+15(5iA+7B)a^3e^{4i fx+4ie})+20(5iA+7B)a^3e^{2i fx+2ie})+8(5iA+7B)a^3\sqrt{\frac{c}{e^{2i fx+2ie}+1}}}{15(cfe^{4i fx+4ie})+2cfe^{2i fx+2ie})+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2), x, algorithm="fricas")

[Out] -4/15\*sqrt(2)\*(15\*(I\*A + B)\*a^3\*e^(6\*I\*f\*x + 6\*I\*e) + 15\*(5\*I\*A + 7\*B)\*a^3\*e^(4\*I\*f\*x + 4\*I\*e) + 20\*(5\*I\*A + 7\*B)\*a^3\*e^(2\*I\*f\*x + 2\*I\*e) + 8\*(5\*I\*A + 7\*B)\*a^3)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c\*f\*e^(4\*I\*f\*x + 4\*I\*e) + 2\*c\*f\*e^(2\*I\*f\*x + 2\*I\*e) + c\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-ia^2 \left( \int \frac{IA}{\sqrt{-ic \tan(e+fx)+c}} dx + \int \left( -\frac{3A \tan(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} \right) dx + \int \frac{A \tan^3(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} dx + \int \left( -\frac{3B \tan^2(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} \right) dx + \int \frac{B \tan^4(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} dx + \int \left( -\frac{3iA \tan^2(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} \right) dx + \int \frac{iB \tan(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} dx + \int \left( -\frac{3iB \tan^3(e+fx)}{\sqrt{-ic \tan(e+fx)+c}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(1/2), x)

[Out] -I\*a\*\*3\*(Integral(I\*A/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-3\*A\*tan(e + f\*x)/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(A\*tan(e + f\*x)\*\*3/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-3\*B\*tan(e + f\*x)\*\*2/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(B\*tan(e + f\*x)\*\*4/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-3\*I\*A\*tan(e + f\*x)\*\*2/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(I\*B\*tan(e + f\*x)/sqrt(-I\*c\*tan(e + f\*x) + c), x) + Integral(-3\*I\*B\*tan(e + f\*x)\*\*3/sqrt(-I\*c\*tan(e + f\*x) + c), x))

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^3/sqrt(-I\*c\*tan(f\*x +  
e) + c), x)

**Mupad [B]**

time = 12.43, size = 351, normalized size = 2.51

$$-\sqrt{c + \frac{c(e^{2fx+2e}-1)\operatorname{li}}{e^{2fx+2e}+1}} \left( \frac{a^3(A-B)\operatorname{li}}{cf} + \frac{a^3e^{2fx+2e}(A-B)\operatorname{li}}{cf} \right) - \left( \frac{a^3(A-B)\operatorname{li}}{cf} + \frac{a^3(A-B)\operatorname{li}}{cf} \right) \sqrt{c + \frac{c(e^{2fx+2e}-1)\operatorname{li}}{e^{2fx+2e}+1}} - \frac{\left( \frac{a^3(A-B)\operatorname{li}}{cf} - \frac{a^3(A-B)\operatorname{li}}{cf} \right) \sqrt{c + \frac{c(e^{2fx+2e}-1)\operatorname{li}}{e^{2fx+2e}+1}}}{(e^{2fx+2e}+1)^2} + \frac{\left( \frac{16B}{cf} + \frac{a^3(A-B)\operatorname{li}}{cf} \right) \sqrt{c + \frac{c(e^{2fx+2e}-1)\operatorname{li}}{e^{2fx+2e}+1}}}{e^{2fx+2e}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^3)/(c - c\*tan(e + f\*x)\*1i)  
)^(1/2),x)

[Out] (((a^3\*(A - B\*1i)\*4i)/(3\*c\*f) + (16\*B\*a^3)/(3\*c\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1) - ((a^3\*(A - B\*1i)\*4i)/(c\*f) + (a^3\*(A - B\*3i)\*4i)/(c\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2) - (((a^3\*(A - B\*1i)\*4i)/(5\*c\*f) - (a^3\*(A + B\*1i)\*4i)/(5\*c\*f))\*(c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2))/(exp(e\*2i + f\*x\*2i) + 1)^2 - (c + (c\*(exp(e\*2i + f\*x\*2i)\*1i - 1i)\*1i)/(exp(e\*2i + f\*x\*2i) + 1))^(1/2)\*((a^3\*(A - B\*1i)\*4i)/(c\*f) + (a^3\*exp(e\*2i + f\*x\*2i)\*(A - B\*1i)\*4i)/(c\*f))

$$3.761 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ictan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=140

$$-\frac{8a^3(iA+B)}{3f(c-ictan(e+fx))^{3/2}} + \frac{8a^3(iA+2B)}{cf\sqrt{c-ictan(e+fx)}} + \frac{2a^3(iA+5B)\sqrt{c-ictan(e+fx)}}{c^2f} - \frac{2a^3B(c-ictan(e+fx))^{3/2}}{3c^3f}$$

[Out]  $8a^3(I*A+2*B)/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}+2a^3(I*A+5*B)*(c-I*c*\tan(f*x+e))^{(1/2)}/c^2/f-8/3*a^3(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(3/2)}-2/3*a^3*B*(c-I*c*\tan(f*x+e))^{(3/2)}/c^3/f$

**Rubi [A]**

time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$\frac{2a^3(5B+iA)\sqrt{c-ictan(e+fx)}}{c^2f} + \frac{8a^3(2B+iA)}{cf\sqrt{c-ictan(e+fx)}} - \frac{8a^3(B+iA)}{3f(c-ictan(e+fx))^{3/2}} - \frac{2a^3B(c-ictan(e+fx))^{3/2}}{3c^3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $(-8*a^3*(I*A + B))/(3*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) + (8*a^3*(I*A + 2*B))/(c*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) + (2*a^3*(I*A + 5*B)*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(c^2*f) - (2*a^3*B*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*c^3*f)$

Rule 78

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 3669

$\text{Int}[(a + b*\tan[(e + f*x)])^m*(A + B*\tan[(e + f*x)])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(A-iB)}{(c-icx)^{5/2}} - \frac{4a^2(A-2iB)}{c(c-icx)^{3/2}} + \frac{a^2(A-5iB)}{c^2 \sqrt{c-icx}} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{8a^3(iA + B)}{3f(c - ic \tan(e + fx))^{3/2}} + \frac{8a^3(iA + 2B)}{cf \sqrt{c - ic \tan(e + fx)}}$$

**Mathematica [A]**

time = 3.15, size = 168, normalized size = 1.20

$$\frac{a^3(15(iA + 3B) \cos(e + fx) + (7iA + 23B) \cos(3(e + fx)) + 2(9A - 26iB + (9A - 25iB) \cos(2(e + fx))) \sin(e + fx) (\cos(2e + 5fx) + i \sin(2e + 5fx)) (A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{3c^2 f (\cos(fx) + i \sin(fx))^3 (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (a^3\*(15\*(I\*A + 3\*B)\*Cos[e + f\*x] + ((7\*I)\*A + 23\*B)\*Cos[3\*(e + f\*x)] + 2\*(9\*A - (26\*I)\*B + (9\*A - (25\*I)\*B)\*Cos[2\*(e + f\*x)])\*Sin[e + f\*x]\*(Cos[2\*e + 5\*f\*x] + I\*Sin[2\*e + 5\*f\*x])\*(A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*c^2\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [A]**

time = 0.26, size = 118, normalized size = 0.84

method	result
derivativedivides	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{3/2}}{3} - 5iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - \frac{4c^3(-iB+A)}{3(c-ic \tan(fx+e))^{3/2}} \right)}{f c^3}$
default	$\frac{2ia^3 \left( \frac{iB(c-ic \tan(fx+e))^{3/2}}{3} - 5iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - \frac{4c^3(-iB+A)}{3(c-ic \tan(fx+e))^{3/2}} \right)}{f c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2), x, method = \_RETURNVERBOSE)

[Out] 2\*I/f\*a^3/c^3\*(1/3\*I\*B\*(c-I\*c\*tan(f\*x+e))^(3/2)-5\*I\*B\*c\*(c-I\*c\*tan(f\*x+e))^(1/2)+A\*c\*(c-I\*c\*tan(f\*x+e))^(1/2)-4/3\*c^3\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(3/2)+4\*c^2\*(A-2\*I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.30, size = 109, normalized size = 0.78

$$2i \frac{\left( \frac{4(3(-ic \tan(fx+e)+c)(A-2iB)a^3-(A-iB)a^3c)}{(-ic \tan(fx+e)+c)^{\frac{3}{2}}} + \frac{i(-ic \tan(fx+e)+c)^{\frac{3}{2}}Ba^3+3\sqrt{-ic \tan(fx+e)+c}(A-5iB)a^3c}{c^2} \right)}{3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="maxima")
```

```
[Out] 2/3*I*(4*(3*(-I*c*tan(f*x + e) + c)*(A - 2*I*B)*a^3 - (A - I*B)*a^3*c)/(-I*c*tan(f*x + e) + c)^(3/2) + (I*(-I*c*tan(f*x + e) + c)^(3/2)*B*a^3 + 3*sqrt(-I*c*tan(f*x + e) + c)*(A - 5*I*B)*a^3*c)/c^2)/(c*f)
```

**Fricas [A]**

time = 1.64, size = 122, normalized size = 0.87

$$\frac{2\sqrt{2}((iA+B)a^3e^{6ifx+6ie}) + 3(-iA-3B)a^3e^{4ifx+4ie} + 12(-iA-3B)a^3e^{2ifx+2ie} + 8(-iA-3B)a^3)\sqrt{\frac{c}{e^{2ifx+2ie}+1}}}{3(c^2fe^{2ifx+2ie}+c^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] -2/3*sqrt(2)*((I*A + B)*a^3*e^(6*I*f*x + 6*I*e) + 3*(-I*A - 3*B)*a^3*e^(4*I*f*x + 4*I*e) + 12*(-I*A - 3*B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(-I*A - 3*B)*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

```
-(/((.....)))/((.....))-(/((.....)))/((.....))-(/((.....)))/((.....))-(/((.....)))/((.....))-(/((.....)))/((.....))-(/((.....)))/((.....))
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),
x)
```

```
[Out] -I*a**3*(Integral(I*A/(-I*c*sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*A*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(A*tan(e + f*x)**3/(-I*c*sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*B*tan(e + f*x)**2/(-I*c*sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)**4/(-I*c*sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*A*tan(e + f*x)**2/(-I*c
```



```
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)),
x) + Integral(I*B*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f
*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*B*tan(e + f*x)**3/
(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) +
c)), x))
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^3/(-I*c*tan(f*x + e)
+ c)^(3/2), x)
```

**Mupad [B]**

time = 10.71, size = 221, normalized size = 1.58

$$a^3 \sqrt{\frac{c \cos(2e + 2fx) + 1 - \sin(2e + 2fx) i}{\cos(2e + 2fx) + 1}} \frac{(A 20i + 60B + A \cos(2e + 2fx) 23i + A \cos(4e + 4fx) 2i - A \cos(6e + 6fx) i) + 69B \cos(2e + 2fx) + 8B \cos(4e + 4fx) - B \cos(6e + 6fx) - 7A \sin(2e + 2fx) - 2A \sin(4e + 4fx) + A \sin(6e + 6fx) + B \sin(2e + 2fx) 21i + B \sin(4e + 4fx) 8i - B \sin(6e + 6fx) i)}{3c^2 f (\cos(2e + 2fx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1i
)^3/2),x)
```

```
[Out] (a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*20i + 60*B + A*cos(2*e + 2*f*x)*23i + A*cos(4*e + 4*f*x)*2i -
A*cos(6*e + 6*f*x)*1i + 69*B*cos(2*e + 2*f*x) + 8*B*cos(4*e + 4*f*x) - B*co
s(6*e + 6*f*x) - 7*A*sin(2*e + 2*f*x) - 2*A*sin(4*e + 4*f*x) + A*sin(6*e +
6*f*x) + B*sin(2*e + 2*f*x)*21i + B*sin(4*e + 4*f*x)*8i - B*sin(6*e + 6*f*x
)*1i))/(3*c^2*f*(cos(2*e + 2*f*x) + 1))
```

$$3.762 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=140

$$-\frac{8a^3(iA+B)}{5f(c-ic \tan(e+fx))^{5/2}} + \frac{8a^3(iA+2B)}{3cf(c-ic \tan(e+fx))^{3/2}} - \frac{2a^3(iA+5B)}{c^2f\sqrt{c-ic \tan(e+fx)}} - \frac{2a^3B\sqrt{c-ic \tan(e+fx)}}{c^3f}$$

[Out]  $-2a^3(I*A+5*B)/c^2/f/(c-I*c*\tan(f*x+e))^{(1/2)}-2a^3*B*(c-I*c*\tan(f*x+e))^{(1/2)}/c^3/f-8/5*a^3*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(5/2)}+8/3*a^3*(I*A+2*B)/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ , Rules used = {3669, 78}

$$-\frac{2a^3(5B+iA)}{c^2f\sqrt{c-ic \tan(e+fx)}} + \frac{8a^3(2B+iA)}{3cf(c-ic \tan(e+fx))^{3/2}} - \frac{8a^3(B+iA)}{5f(c-ic \tan(e+fx))^{5/2}} - \frac{2a^3B\sqrt{c-ic \tan(e+fx)}}{c^3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $(-8*a^3*(I*A + B))/(5*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) + (8*a^3*(I*A + 2*B))/(3*c*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - (2*a^3*(I*A + 5*B))/(c^2*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) - (2*a^3*B*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(c^3*f)$

Rule 78

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^{(n_.)*((e_. + (f_.)*(x_.))^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 3669

$\text{Int}[(a_. + (b_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(m_.)*((A_. + (B_.)*\text{tan}[(e_. + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(n-1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(A-iB)}{(c-icx)^{7/2}} - \frac{4a^2(A-2iB)}{c(c-icx)^{5/2}} + \frac{a^2(A-5iB)}{c^2(c-icx)^{3/2}} + \frac{a^2(A-5iB)}{c^3} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{8a^3(iA + B)}{5f(c - ic \tan(e + fx))^{5/2}} + \frac{8a^3(iA + 2B)}{3cf(c - ic \tan(e + fx))^3}$$

**Mathematica [A]**

time = 4.49, size = 135, normalized size = 0.96

$$\frac{a^3(3(A - 11iB) \cos(e + fx) + (11A - 91iB) \cos(3(e + fx)) - 10i(A - 14iB + (A - 17iB) \cos(2(e + fx))) \sin(e + fx))(-i \cos(3(e + 2fx)) + \sin(3(e + 2fx))) \sqrt{c - ic \tan(e + fx)}}{15c^3 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] (a^3\*(3\*(A - (11\*I)\*B)\*Cos[e + f\*x] + (11\*A - (91\*I)\*B)\*Cos[3\*(e + f\*x)] - (10\*I)\*(A - (14\*I)\*B + (A - (17\*I)\*B)\*Cos[2\*(e + f\*x)])\*Sin[e + f\*x]\*((-I)\*Cos[3\*(e + 2\*f\*x)] + Sin[3\*(e + 2\*f\*x)]\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(15\*c^3\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.24, size = 105, normalized size = 0.75

method	result
derivativedivides	$\frac{2ia^3 \left( iB \sqrt{c - ic \tan(fx + e)} + \frac{4c^2(-2iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{4c^3(-iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{c(-5iB+A)}{\sqrt{c - ic \tan(fx + e)}} \right)}{f c^3}$
default	$\frac{2ia^3 \left( iB \sqrt{c - ic \tan(fx + e)} + \frac{4c^2(-2iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{4c^3(-iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{c(-5iB+A)}{\sqrt{c - ic \tan(fx + e)}} \right)}{f c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2), x, method = \_RETURNVERBOSE)

[Out] 2\*I/f\*a^3/c^3\*(I\*B\*(c-I\*c\*tan(f\*x+e))^(1/2)+4/3\*c^2\*(A-2\*I\*B)/(c-I\*c\*tan(f\*x+e))^(3/2)-4/5\*c^3\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(5/2)-c\*(A-5\*I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2))

**Maxima [A]**

time = 0.30, size = 111, normalized size = 0.79

$$\frac{2i \left( -\frac{15i \sqrt{-i c \tan(fx+e) + c} B a^3}{c^2} + \frac{15(-i c \tan(fx+e)+c)(A-5iB)a^3 - 20(-i c \tan(fx+e)+c)(A-2iB)a^3 c + 12(A-iB)a^3 c^2}{(-i c \tan(fx+e)+c)^{\frac{5}{2}} c} \right)}{15 c f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2), x,
algorithm="maxima")
```

```
[Out] -2/15*I*(-15*I*sqrt(-I*c*tan(f*x + e) + c)*B*a^3/c^2 + (15*(-I*c*tan(f*x +
e) + c)^2*(A - 5*I*B)*a^3 - 20*(-I*c*tan(f*x + e) + c)*(A - 2*I*B)*a^3*c +
12*(A - I*B)*a^3*c^2)/((-I*c*tan(f*x + e) + c)^(5/2)*c))/(c*f)
```

**Fricas [A]**

time = 1.64, size = 106, normalized size = 0.76

$$\frac{\sqrt{2} (3(iA + B)a^3 e^{6i f x + 6i e} - (iA + 11B)a^3 e^{4i f x + 4i e} + 4(iA + 11B)a^3 e^{2i f x + 2i e} + 8(iA + 11B)a^3) \sqrt{\frac{c}{e^{2i f x + 2i e} + 1}}}{15 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2), x,
algorithm="fricas")
```

```
[Out] -1/15*sqrt(2)*(3*(I*A + B)*a^3*e^(6*I*f*x + 6*I*e) - (I*A + 11*B)*a^3*e^(4*
I*f*x + 4*I*e) + 4*(I*A + 11*B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(I*A + 11*B)*a^
3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),
x)
```

```
[Out] -I*a**3*(Integral(I*A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 -
2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e +
f*x) + c)), x) + Integral(-3*A*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) +
c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) +
c**2*sqrt(-I*c*tan(e + f*x) + c))), x) + Integral(A*tan(e + f*x)**3/(-c**2*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c))), x) + Integral(-3*B*
tan(e + f*x)**2/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c
```

```

*2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) +
c)), x) + Integral(B*tan(e + f*x)**4/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*ta
n(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sq
rt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*A*tan(e + f*x)**2/(-c**2*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(I*B*ta
n(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)),
x) + Integral(-3*I*B*tan(e + f*x)**3/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*ta
n(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sq
rt(-I*c*tan(e + f*x) + c)), x)

```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="giac")

```

```

[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^3/(-I*c*tan(f*x + e)
+ c)^(5/2), x)

```

**Mupad [B]**

time = 10.28, size = 208, normalized size = 1.49

$$a^3 \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}} (A^8i + 88B + A\cos(2e+2fx)4i - A\cos(4e+4fx)i + A\cos(6e+6fx)3i + 44B\cos(2e+2fx) - 11B\cos(4e+4fx) + 3B\cos(6e+6fx) - 4A\sin(2e+2fx) + A\sin(4e+4fx) - 3A\sin(6e+6fx) + B\sin(2e+2fx)4i - B\sin(4e+4fx)i + B\sin(6e+6fx)3i)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1i
)^5/2),x)

```

```

[Out] -(a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*8i + 88*B + A*cos(2*e + 2*f*x)*4i - A*cos(4*e + 4*f*x)*1i + A
*cos(6*e + 6*f*x)*3i + 44*B*cos(2*e + 2*f*x) - 11*B*cos(4*e + 4*f*x) + 3*B*
cos(6*e + 6*f*x) - 4*A*sin(2*e + 2*f*x) + A*sin(4*e + 4*f*x) - 3*A*sin(6*e
+ 6*f*x) + B*sin(2*e + 2*f*x)*44i - B*sin(4*e + 4*f*x)*11i + B*sin(6*e + 6*
f*x)*3i))/(15*c^3*f)

```

$$3.763 \quad \int \frac{(a+ia \tan(e+fx))^3(A+B \tan(e+fx))}{(c-ictan(e+fx))^{7/2}} dx$$

**Optimal.** Leaf size=142

$$-\frac{8a^3(iA+B)}{7f(c-ictan(e+fx))^{7/2}} + \frac{8a^3(iA+2B)}{5cf(c-ictan(e+fx))^{5/2}} - \frac{2a^3(iA+5B)}{3c^2f(c-ictan(e+fx))^{3/2}} + \frac{2a^3B}{c^3f\sqrt{c-ictan(e+fx)}}$$

[Out]  $2a^3B/c^3/f/(c-I*c*\tan(f*x+e))^{(1/2)}-8/7*a^3*(I*A+B)/f/(c-I*c*\tan(f*x+e))^{(7/2)}+8/5*a^3*(I*A+2*B)/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}-2/3*a^3*(I*A+5*B)/c^2/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$ ,

Rules used = {3669, 78}

$$-\frac{2a^3(5B+iA)}{3c^2f(c-ictan(e+fx))^{3/2}} + \frac{8a^3(2B+iA)}{5cf(c-ictan(e+fx))^{5/2}} - \frac{8a^3(B+iA)}{7f(c-ictan(e+fx))^{7/2}} + \frac{2a^3B}{c^3f\sqrt{c-ictan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^3*(A + B*\text{Tan}[e + f*x])/(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $(-8*a^3*(I*A + B))/(7*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) + (8*a^3*(I*A + 2*B))/(5*c*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (2*a^3*(I*A + 5*B))/(3*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) + (2*a^3*B)/(c^3*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

**Rule 78**

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 3669**

$\text{Int}[(a + b*\tan[e + f*x])^m*(A + B*\tan[e + f*x])^n, x] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{m-1}*(c + d*x)^{n-1}*(A + B*x), x], x, \text{Tan}[e + f*x], x] /;$  FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

**Rubi steps**

$$\int \frac{(a + ia \tan(e + fx))^3 (A + B \tan(e + fx))}{(c - ic \tan(e + fx))^{7/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^2 (A+Bx)}{(c-icx)^{9/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(ac) \text{Subst} \left( \int \left( \frac{4a^2(A-iB)}{(c-icx)^{9/2}} - \frac{4a^2(A-2iB)}{c(c-icx)^{7/2}} + \frac{a^2(A-5iB)}{c^2(c-icx)^{5/2}} + \frac{c^3}{c^3(c-icx)^{3/2}} \right) dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{8a^3(iA + B)}{7f(c - ic \tan(e + fx))^{7/2}} + \frac{8a^3(iA + 2B)}{5cf(c - ic \tan(e + fx))^{5/2}}$$

**Mathematica [A]**

time = 6.20, size = 141, normalized size = 0.99

$$\frac{a^3 \cos(e + fx)(i(A + 13iB) \cos(e + fx) + (-23iA + 89B) \cos(3(e + fx)) + 14(A - 2iB + (A - 17iB) \cos(2(e + fx))) \sin(e + fx))(\cos(4e + 7fx) + i \sin(4e + 7fx)) \sqrt{c - ic \tan(e + fx)}}{105c^4 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^3\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(7/2), x]

[Out] (a^3\*Cos[e + f\*x]\*(I\*(A + (13\*I)\*B)\*Cos[e + f\*x] + ((-23\*I)\*A + 89\*B)\*Cos[3\*(e + f\*x)] + 14\*(A - (2\*I)\*B + (A - (17\*I)\*B)\*Cos[2\*(e + f\*x)])\*Sin[e + f\*x])\*(Cos[4\*e + 7\*f\*x] + I\*Sin[4\*e + 7\*f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(105\*c^4\*f\*(Cos[f\*x] + I\*Sin[f\*x])^3)

**Maple [A]**

time = 0.26, size = 105, normalized size = 0.74

method	result
derivativedivides	$\frac{2ia^3 \left( -\frac{4c^3(-iB+A)}{7(c-ic \tan(fx+e))^{\frac{7}{2}}} + \frac{4c^2(-2iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{c(-5iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{iB}{\sqrt{c-ic \tan(fx+e)}} \right)}{fc^3}$
default	$\frac{2ia^3 \left( -\frac{4c^3(-iB+A)}{7(c-ic \tan(fx+e))^{\frac{7}{2}}} + \frac{4c^2(-2iB+A)}{5(c-ic \tan(fx+e))^{\frac{5}{2}}} - \frac{c(-5iB+A)}{3(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{iB}{\sqrt{c-ic \tan(fx+e)}} \right)}{fc^3}$
risch	$-\frac{a^3(15iA e^{6i(fx+e)} + 15B e^{6i(fx+e)} + 3iA e^{4i(fx+e)} - 39B e^{4i(fx+e)} - 4iA e^{2i(fx+e)} + 52B e^{2i(fx+e)} + 8iA - 104B) \sqrt{2}}{210c^3 \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^3\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2), x, method =\_RETURNVERBOSE)

[Out]  $2*I/f*a^3/c^3*(-4/7*c^3*(A-I*B)/(c-I*c*\tan(f*x+e))^{7/2}+4/5*c^2*(A-2*I*B)/(c-I*c*\tan(f*x+e))^{5/2}-1/3*c*(A-5*I*B)/(c-I*c*\tan(f*x+e))^{3/2}-I*B/(c-I*c*\tan(f*x+e))^{1/2})$

**Maxima [A]**

time = 0.29, size = 106, normalized size = 0.75

$$\frac{2i(105i(-ictan(fx+e)+c)^3Ba^3+35(-ictan(fx+e)+c)^2(A-5iB)a^3c-84(-ictan(fx+e)+c)(A-2iB)a^3c^2+60(A-iB)a^3c^3)}{105(-ictan(fx+e)+c)^{\frac{7}{2}}c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x, algorithm="maxima")`

[Out]  $-2/105*I*(105*I*(-I*c*\tan(f*x+e)+c)^3*B*a^3+35*(-I*c*\tan(f*x+e)+c)^2*(A-5*I*B)*a^3*c-84*(-I*c*\tan(f*x+e)+c)*(A-2*I*B)*a^3*c^2+60*(A-I*B)*a^3*c^3)/((-I*c*\tan(f*x+e)+c)^{7/2}*c^3*f)$

**Fricas [A]**

time = 1.46, size = 128, normalized size = 0.90

$$\frac{\sqrt{2}(15(iA+B)a^3e^{8i fx+8ie}+6(3iA-4B)a^3e^{6i fx+6ie}-(iA-13B)a^3e^{4i fx+4ie}+4(iA-13B)a^3e^{2i fx+2ie}+8(iA-13B)a^3)\sqrt{\frac{c}{e^{2i fx+2ie}+1}}}{210c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x, algorithm="fricas")`

[Out]  $-1/210*\sqrt{2}*(15*(I*A+B)*a^3*e^{(8*I*f*x+8*I*e)}+6*(3*I*A-4*B)*a^3*e^{(6*I*f*x+6*I*e)}-(I*A-13*B)*a^3*e^{(4*I*f*x+4*I*e)}+4*(I*A-13*B)*a^3*e^{(2*I*f*x+2*I*e)}+8*(I*A-13*B)*a^3)*\sqrt{c/(e^{(2*I*f*x+2*I*e)}+1)}(c^4*f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(7/2),x)`

[Out]  $-I*a**3*(\text{Integral}(I*A/(I*c**3*\sqrt{-I*c*\tan(e+f*x)+c})*\tan(e+f*x)**3-3*c**3*\sqrt{-I*c*\tan(e+f*x)+c})*\tan(e+f*x)**2-3*I*c**3*\sqrt{-I*c*\tan(e+f*x)+c})*\tan(e+f*x)+c**3*\sqrt{-I*c*\tan(e+f*x)+c}),x)+\text{Integral}(-3*A*\tan(e+f*x)/(I*c**3*\sqrt{-I*c*\tan(e+f*x)+c})*\tan(e+f*x)**3-3*c**3*\sqrt{-I*c*\tan(e+f*x)+c})*\tan(e+f*x)**2-3*I*c**3*\sqrt{-I*c*\tan(e+f*x)+c}))$



```

an(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Int
egral(A*tan(e + f*x)**3/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3
- 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + In
tegral(-3*B*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x
)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x)
+ Integral(B*tan(e + f*x)**4/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x)
+ Integral(-3*I*A*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(
e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c
)), x) + Integral(I*B*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(
e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c
)), x) + Integral(-3*I*B*tan(e + f*x)**3/(I*c**3*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*
I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*
x) + c)), x))

```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="giac")

```

```

[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^3/(-I*c*tan(f*x + e)
+ c)^(7/2), x)

```

**Mupad [B]**

time = 10.73, size = 161, normalized size = 1.13

$$-\sqrt{c - \frac{c \sin(e + f x)}{\cos(e + f x)} \operatorname{li}} \left( \frac{a^3 (A + B 13i) 4i}{105 c^4 f} + \frac{a^3 e^{6i + f x 6i} (3 A + B 4i) 1i}{35 c^4 f} + \frac{a^3 e^{2i + f x 2i} (A + B 13i) 2i}{105 c^4 f} + \frac{a^3 e^{8i + f x 8i} (A - B 1i) 1i}{14 c^4 f} - \frac{a^3 e^{4i + f x 4i} (A + B 13i) 1i}{210 c^4 f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1i
)^(7/2),x)

```

```

[Out] -(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^3*(A + B*13i)*4i)/(105*c^
4*f) + (a^3*exp(e*6i + f*x*6i)*(3*A + B*4i)*1i)/(35*c^4*f) + (a^3*exp(e*2i
+ f*x*2i)*(A + B*13i)*2i)/(105*c^4*f) + (a^3*exp(e*8i + f*x*8i)*(A - B*1i)*
1i)/(14*c^4*f) - (a^3*exp(e*4i + f*x*4i)*(A + B*13i)*1i)/(210*c^4*f))

```

$$3.764 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{7/2}}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=220

$$\frac{2\sqrt{2} (5iA - 9B)c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{af} + \frac{2(5iA - 9B)c^3 \sqrt{c-ictan(e+fx)}}{af} + \frac{(5iA - 9B)}{af}$$

[Out]  $-2*(5*I*A-9*B)*c^{(7/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*2^{(1/2)}/a/f+2*(5*I*A-9*B)*c^3*(c-I*c*\tan(f*x+e))^{(1/2)}/a/f+1/3*(5*I*A-9*B)*c^2*(c-I*c*\tan(f*x+e))^{(3/2)}/a/f+1/10*(5*I*A-9*B)*c*(c-I*c*\tan(f*x+e))^{(5/2)}/a/f+1/2*(I*A-B)*(c-I*c*\tan(f*x+e))^{(7/2)}/a/f/(1+I*\tan(f*x+e))$

**Rubi [A]**

time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 52, 65, 214}

$$\frac{2\sqrt{2}c^{7/2}(-9B+5iA)\tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{af} + \frac{2c^3(-9B+5iA)\sqrt{c-ictan(e+fx)}}{af} + \frac{c^2(-9B+5iA)(c-ictan(e+fx))^{3/2}}{3af} + \frac{c(-9B+5iA)(c-ictan(e+fx))^{5/2}}{10af} + \frac{(-B+iA)(c-ictan(e+fx))^{7/2}}{2af(1+i\tan(e+fx))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[e+f*x])*(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)}]/(a+I*a*\operatorname{Tan}[e+f*x]),x]$

[Out]  $(-2*\operatorname{Sqrt}[2]*((5*I)*A-9*B)*c^{(7/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]))/(a*f)+(2*((5*I)*A-9*B)*c^3*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]]/(a*f)+(((5*I)*A-9*B)*c^2*(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)})/(3*a*f)+(((5*I)*A-9*B)*c*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)})/(10*a*f)+((I*A-B)*(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)})/(2*a*f*(1+I*\operatorname{Tan}[e+f*x]))$

**Rule 52**

$\operatorname{Int}[(a_.)+(b_.)*(x_.))^{(m_.)}*((c_.)+(d_.)*(x_.))^{(n_.)},x\_Symbol] \rightarrow \operatorname{Simp}[(a+b*x)^{(m+1)}*((c+d*x)^n/(b*(m+n+1))),x]+ \operatorname{Dist}[n*(b*c-a*d)/(b*(m+n+1)),\operatorname{Int}[(a+b*x)^m*(c+d*x)^{(n-1)},x],x] /; \operatorname{FreeQ}\{a,b,c,d\},x \ \&\& \operatorname{NeQ}[b*c-a*d,0] \ \&\& \operatorname{GtQ}[n,0] \ \&\& \operatorname{NeQ}[m+n+1,0] \ \&\& !( \operatorname{IGtQ}[m,0] \ \&\& ( !\operatorname{IntegerQ}[n] || (\operatorname{GtQ}[m,0] \ \&\& \operatorname{LtQ}[m-n,0]) ) ) \ \&\& !\operatorname{ILtQ}[m+n+2,0] \ \&\& \operatorname{IntLinearQ}[a,b,c,d,m,n,x]$

**Rule 65**

$\operatorname{Int}[(a_.)+(b_.)*(x_.))^{(m_.)}*((c_.)+(d_.)*(x_.))^{(n_.)},x\_Symbol] \rightarrow \operatorname{With}\{p=\operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b,\operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)},x],x,(a+b*x)^{(1/p)}],x] /; \operatorname{FreeQ}\{a,b,c,d\},x \ \&\& \operatorname{NeQ}[b*c-a*d,0] \ \&\& \operatorname{LtQ}[-1,m,0] \ \&\& \operatorname{LeQ}[-1,n,0] \ \&\& \operatorname{LeQ}[\operatorname{Denominator}[n],\operatorname{Den}$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{7/2}}{a + ia \tan(e + fx)} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^2} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{7/2}}{2af(1 + i \tan(e + fx))} - \frac{((5A + 9iB)c) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^2} dx, x, \tan(e + fx) \right)}{2af(1 + i \tan(e + fx))} \\
&= \frac{(5iA - 9B)c(c - ic \tan(e + fx))^{5/2}}{10af} + \frac{(iA - B)(c - ic \tan(e + fx))^{7/2}}{2af(1 + i \tan(e + fx))} \\
&= \frac{(5iA - 9B)c^2(c - ic \tan(e + fx))^{3/2}}{3af} + \frac{(5iA - 9B)c(c - ic \tan(e + fx))^{5/2}}{10af} \\
&= \frac{2(5iA - 9B)c^3 \sqrt{c - ic \tan(e + fx)}}{af} + \frac{(5iA - 9B)c^2(c - ic \tan(e + fx))^{5/2}}{10af} \\
&= \frac{2(5iA - 9B)c^3 \sqrt{c - ic \tan(e + fx)}}{af} + \frac{(5iA - 9B)c^2(c - ic \tan(e + fx))^{5/2}}{10af} \\
&= -\frac{2\sqrt{2} (5iA - 9B)c^{7/2} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{af}
\end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(a + I\*a\*Tan[e + f\*x]),x]

[Out] \$Aborted

**Maple [A]**

time = 0.39, size = 192, normalized size = 0.87

method	result
--------	--------

derivativedivides	$2ic \left( \frac{iB(c-ic \tan(fx+e))^{\frac{5}{2}}}{5} + iBc(c-ic \tan(fx+e))^{\frac{3}{2}} + \frac{Ac(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 8iBc^2 \sqrt{c-ic \tan(fx+e)} + 4Ac^2 \right)$
default	$2ic \left( \frac{iB(c-ic \tan(fx+e))^{\frac{5}{2}}}{5} + iBc(c-ic \tan(fx+e))^{\frac{3}{2}} + \frac{Ac(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 8iBc^2 \sqrt{c-ic \tan(fx+e)} + 4Ac^2 \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $2*I/f/a*c*(1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)+I*B*c*(c-I*c*tan(f*x+e))^(3/2)+1/3*A*c*(c-I*c*tan(f*x+e))^(3/2)+8*I*B*c^2*(c-I*c*tan(f*x+e))^(1/2)+4*A*c^2*(c-I*c*tan(f*x+e))^(1/2)-4*c^3*((-1/4*A-1/4*I*B)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/2*(9/2*I*B+5/2*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))$

**Maxima** [A]

time = 0.51, size = 197, normalized size = 0.90

$$i \left( \frac{15 \sqrt{2} (5A+9iB)c^{\frac{3}{2}} \log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right) - 60 \sqrt{-ic \tan(fx+e)+c} (A+iB)c^{\frac{3}{2}} + 2 \left( 3i(-ic \tan(fx+e)+c)^{\frac{5}{2}} Bc^2 + 5(-ic \tan(fx+e)+c)^{\frac{3}{2}} (A+3iB)c^2 + 60 \sqrt{-ic \tan(fx+e)+c} (A+2iB)c^{\frac{3}{2}} \right)}{15cf} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")`

[Out]  $1/15*I*(15*\sqrt{2}*(5*A + 9*I*B)*c^(9/2)*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c}))/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c}))/a - 60*\sqrt{-I*c*\tan(f*x + e) + c}*(A + I*B)*c^5/((-I*c*\tan(f*x + e) + c)*a - 2*a*c) + 2*(3*I*(-I*c*\tan(f*x + e) + c)^(5/2)*B*c^2 + 5*(-I*c*\tan(f*x + e) + c)^(3/2)*(A + 3*I*B)*c^3 + 60*\sqrt{-I*c*\tan(f*x + e) + c}*(A + 2*I*B)*c^4)/a/(c*f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 485 vs.  $2(179) = 358$ .

time = 2.78, size = 485, normalized size = 2.20

$$15 \sqrt{2} \frac{(5A+9iB)c^{\frac{3}{2}} \log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right) - 60 \sqrt{-ic \tan(fx+e)+c} (A+iB)c^{\frac{3}{2}} + 2 \left( 3i(-ic \tan(fx+e)+c)^{\frac{5}{2}} Bc^2 + 5(-ic \tan(fx+e)+c)^{\frac{3}{2}} (A+3iB)c^2 + 60 \sqrt{-ic \tan(fx+e)+c} (A+2iB)c^{\frac{3}{2}} \right)}{15cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] -1/15*(15*sqrt(2)*sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^7/(a^2*f^2))*(a*f*e^
(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))*lo
g(-8*((5*I*A - 9*B)*c^4 + sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^7/(a^2*f^2))
*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f
*x - I*e)/(a*f)) - 15*sqrt(2)*sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^7/(a^2*f
^2))*(a*f*e^(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x
+ 2*I*e))*log(-8*((5*I*A - 9*B)*c^4 - sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^
7/(a^2*f^2))*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) +
1)))*e^(-I*f*x - I*e)/(a*f)) + 2*sqrt(2)*(15*(-5*I*A + 9*B)*c^3*e^(6*I*f*x
+ 6*I*e) + 35*(-5*I*A + 9*B)*c^3*e^(4*I*f*x + 4*I*e) + 23*(-5*I*A + 9*B)*c^
3*e^(2*I*f*x + 2*I*e) + 15*(-I*A + B)*c^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1
)))/(a*f*e^(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x +
2*I*e))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2 \sqrt{-c \tan(e + f x) + c} dx + \int \frac{a^2 \sqrt{-c \tan(e + f x) + c} \cos(e + f x) dx}{\cos(e + f x)} + \int \frac{a^2 \sqrt{-c \tan(e + f x) + c} \sin(e + f x) dx}{\sin(e + f x)} + \int \frac{a^2 \sqrt{-c \tan(e + f x) + c} \cos(e + f x) dx}{\cos(e + f x)} + \int \frac{a^2 \sqrt{-c \tan(e + f x) + c} \sin(e + f x) dx}{\sin(e + f x)} + \int \frac{a^2 \sqrt{-c \tan(e + f x) + c} \cos(e + f x) dx}{\cos(e + f x)} + \int \frac{a^2 \sqrt{-c \tan(e + f x) + c} \sin(e + f x) dx}{\sin(e + f x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*(Integral(A*c**3*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + In
tegral(-3*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)
- I), x) + Integral(B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e
+ f*x) - I), x) + Integral(-3*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**3/(tan(e + f*x) - I), x) + Integral(-3*I*A*c**3*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x)/(tan(e + f*x) - I), x) + Integral(I*A*c**3*sqrt(-I*c*tan(
e + f*x) + c)*tan(e + f*x)**3/(tan(e + f*x) - I), x) + Integral(-3*I*B*c**3
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x) - I), x) + Integ
ral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4/(tan(e + f*x) - I
, x))/a
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x, a
lgorithm="giac")
```

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(7/2)/(I\*a\*tan(f\*x + e) + a), x)

**Mupad [B]**

time = 1.48, size = 298, normalized size = 1.35

$$\frac{4Bc^2\sqrt{c-\tan(e+fx)}\sqrt{a}}{af(c-\tan(e+fx))^{3/2}} + \frac{Ac^2\sqrt{c-\tan(e+fx)}\sqrt{a}}{af} + \frac{A^2(c-\tan(e+fx))^{3/2}\sqrt{a}}{3af} - \frac{16Bc^2\sqrt{c-\tan(e+fx)}\sqrt{a}}{af} - \frac{2Bc^2(c-\tan(e+fx))^{3/2}}{af} - \frac{2Bc(c-\tan(e+fx))^{5/2}}{5af} + \frac{\sqrt{2}A(-c)^{7/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\sqrt{a}}{a\sqrt{-c}}\right)\sqrt{a}}{af} - \frac{\sqrt{2}Bc^{7/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\sqrt{a}}{a\sqrt{-c}}\right)\sqrt{a}}{af} + \frac{A^2\sqrt{c-\tan(e+fx)}\sqrt{a}}{af(c+\tan(e+fx))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(7/2))/(a + a\*tan(e + f\*x)\*1i), x)

[Out] (4\*B\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a\*f\*(c - c\*tan(e + f\*x)\*1i) - 2\*a\*c\*f) + (A\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*8i)/(a\*f) + (A\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(3/2)\*2i)/(3\*a\*f) - (16\*B\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a\*f) - (2\*B\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/(a\*f) - (2\*B\*c\*(c - c\*tan(e + f\*x)\*1i)^(5/2))/(5\*a\*f) + (2^(1/2)\*A\*(-c)^(7/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*10i)/(a\*f) - (2^(1/2)\*B\*c^(7/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*1i)/(2\*c^(1/2)))\*18i)/(a\*f) + (A\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*4i)/(a\*f\*(c + c\*tan(e + f\*x)\*1i))

$$3.765 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^{5/2}}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=180

$$-\frac{\sqrt{2} (3iA - 7B)c^{5/2} \tanh^{-1} \left( \frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{af} + \frac{(3iA - 7B)c^2 \sqrt{c - ict \tan(e + fx)}}{af} + \frac{(3iA - 7B)c(c - ict \tan(e + fx))^{5/2}}{af(1 + i \tan(e + fx))}$$

[Out]  $-(3IA-7B)*c^{(5/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*2^{(1/2)}/a/f+(3IA-7B)*c^2*(c-I*c*\tan(f*x+e))^{(1/2)}/a/f+1/6*(3IA-7B)*c*(c-I*c*\tan(f*x+e))^{(3/2)}/a/f+1/2*(IA-B)*(c-I*c*\tan(f*x+e))^{(5/2)}/a/f/(1+I*\tan(f*x+e))$

**Rubi [A]**

time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 52, 65, 214}

$$-\frac{\sqrt{2} c^{5/2} (-7B + 3iA) \tanh^{-1} \left( \frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{af} + \frac{c^2 (-7B + 3iA) \sqrt{c - ict \tan(e + fx)}}{af} + \frac{c (-7B + 3iA) (c - ict \tan(e + fx))^{3/2}}{6af} + \frac{(-B + iA) (c - ict \tan(e + fx))^{5/2}}{2af(1 + i \tan(e + fx))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)}]/(a + I*a*\operatorname{Tan}[e + f*x]), x]$

[Out]  $-\left(\frac{\operatorname{Sqrt}[2]*((3I)*A - 7*B)*c^{(5/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c])}{(a*f)} + \frac{((3I)*A - 7*B)*c^2*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]]}{(a*f)} + \frac{((3I)*A - 7*B)*c*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)}}{(6*a*f)} + \frac{(I*A - B)*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)}}{(2*a*f*(1 + I*\operatorname{Tan}[e + f*x]))}\right)$

Rule 52

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))], x] + \operatorname{Dist}[n*(b*c - a*d)/(b*(m + n + 1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m + n + 1, 0] \&\& !( \operatorname{IGtQ}[m, 0] \&\& ( !\operatorname{IntegerQ}[n] || (\operatorname{GtQ}[m, 0] \&\& \operatorname{LtQ}[m - n, 0])) ) \&\& !\operatorname{ILtQ}[m + n + 2, 0] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 65

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$



Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^{5/2}}{a + i a \tan(e + fx)} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - i c \tan(e + fx))^{5/2}}{2af(1 + i \tan(e + fx))} - \frac{((3A + 7iB)c) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{2af(1 + i \tan(e + fx))} \\
&= \frac{(3iA - 7B)c(c - i c \tan(e + fx))^{3/2}}{6af} + \frac{(iA - B)(c - i c \tan(e + fx))^{5/2}}{2af(1 + i \tan(e + fx))} \\
&= \frac{(3iA - 7B)c^2 \sqrt{c - i c \tan(e + fx)}}{af} + \frac{(3iA - 7B)c(c - i c \tan(e + fx))^{5/2}}{2af(1 + i \tan(e + fx))} \\
&= \frac{(3iA - 7B)c^2 \sqrt{c - i c \tan(e + fx)}}{af} + \frac{(3iA - 7B)c(c - i c \tan(e + fx))^{5/2}}{2af(1 + i \tan(e + fx))} \\
&= -\frac{\sqrt{2} (3iA - 7B)c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c - i c \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{af}
\end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f*x]),x]
```

[Out] \$Aborted

**Maple [A]**

time = 0.39, size = 150, normalized size = 0.83

method	result
derivativedivides	$2ic \left( \frac{iB(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 3iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - 4c^2 \left( \frac{(-\frac{A}{8} - \frac{iB}{8}) \sqrt{c}}{\frac{c}{2} +} \right) \right) / fa$
default	$2ic \left( \frac{iB(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 3iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - 4c^2 \left( \frac{(-\frac{A}{8} - \frac{iB}{8}) \sqrt{c}}{\frac{c}{2} +} \right) \right) / fa$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a*c*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)+3*I*B*c*(c-I*c*tan(f*x+e))^(1/2)+A*c*(c-I*c*tan(f*x+e))^(1/2)-4*c^2*((-1/8*A-1/8*I*B)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/4*(7/2*I*B+3/2*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

**Maxima [A]**

time = 0.50, size = 173, normalized size = 0.96

$$i \left( \frac{3 \sqrt{2} (3A+7iB)c^{\frac{7}{2}} \log \left( \frac{\sqrt{2} \sqrt{c} - \sqrt{-ic \tan(fx+e) + c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx+e) + c}} \right) - 12 \sqrt{-ic \tan(fx+e) + c} (A+iB)c^4 + \frac{4 (i(-ic \tan(fx+e)+c)^{\frac{3}{2}} Bc^2 + 3 \sqrt{-ic \tan(fx+e) + c} (A+3iB)c^3)}{a}}{6cf} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

[Out]  $\frac{1}{6} I (3 \sqrt{2} (3A + 7I B) c^{7/2} \log(-\sqrt{2} \sqrt{c} - \sqrt{-I c \tan(fx + e) + c}) / (\sqrt{2} \sqrt{c} + \sqrt{-I c \tan(fx + e) + c})) / a - 12 \sqrt{2} \sqrt{-I c \tan(fx + e) + c} (A + I B) c^4 / ((-I c \tan(fx + e) + c) a - 2 a^2 c) + 4 (I (-I c \tan(fx + e) + c)^{3/2} B c^2 + 3 \sqrt{-I c \tan(fx + e) + c}) (A + 3 I B) c^3 / a) / (c f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs.  $2(145) = 290$ .  
time = 3.03, size = 421, normalized size = 2.34

$$\frac{3 \sqrt{2} (3 \sqrt{2} (3A + 7I B) c^{7/2} \log(-\sqrt{2} \sqrt{c} - \sqrt{-I c \tan(fx + e) + c}) / (\sqrt{2} \sqrt{c} + \sqrt{-I c \tan(fx + e) + c})) / a - 12 \sqrt{2} \sqrt{-I c \tan(fx + e) + c} (A + I B) c^4 / ((-I c \tan(fx + e) + c) a - 2 a^2 c) + 4 (I (-I c \tan(fx + e) + c)^{3/2} B c^2 + 3 \sqrt{-I c \tan(fx + e) + c}) (A + 3 I B) c^3 / a) / (c f)}{3 \sqrt{2} (3 \sqrt{2} (3A + 7I B) c^{7/2} \log(-\sqrt{2} \sqrt{c} - \sqrt{-I c \tan(fx + e) + c}) / (\sqrt{2} \sqrt{c} + \sqrt{-I c \tan(fx + e) + c})) / a - 12 \sqrt{2} \sqrt{-I c \tan(fx + e) + c} (A + I B) c^4 / ((-I c \tan(fx + e) + c) a - 2 a^2 c) + 4 (I (-I c \tan(fx + e) + c)^{3/2} B c^2 + 3 \sqrt{-I c \tan(fx + e) + c}) (A + 3 I B) c^3 / a) / (c f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")`

[Out]  $-1/6 (3 \sqrt{2} (a f e^{4 I f x + 4 I e} + a f e^{2 I f x + 2 I e}) \sqrt{-(9 A^2 + 42 I A B - 49 B^2) c^5 / (a^2 f^2)} \log(-4 ((3 I A - 7 B) c^3 + (a f e^{2 I f x + 2 I e} + a f) \sqrt{-(9 A^2 + 42 I A B - 49 B^2) c^5 / (a^2 f^2)}) \sqrt{c / (e^{2 I f x + 2 I e} + 1)}) e^{-I f x - I e} / (a f) - 3 \sqrt{2} (a f e^{4 I f x + 4 I e} + a f e^{2 I f x + 2 I e}) \sqrt{-(9 A^2 + 42 I A B - 49 B^2) c^5 / (a^2 f^2)} \log(-4 ((3 I A - 7 B) c^3 - (a f e^{2 I f x + 2 I e} + a f) \sqrt{-(9 A^2 + 42 I A B - 49 B^2) c^5 / (a^2 f^2)}) \sqrt{c / (e^{2 I f x + 2 I e} + 1)}) e^{-I f x - I e} / (a f) + 2 \sqrt{2} (3 (-3 I A + 7 B) c^2 e^{4 I f x + 4 I e} + 4 (-3 I A + 7 B) c^2 e^{2 I f x + 2 I e} + 3 (-I A + B) c^2) \sqrt{c / (e^{2 I f x + 2 I e} + 1)}) / (a f e^{4 I f x + 4 I e} + a f e^{2 I f x + 2 I e}))$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A^2 \sqrt{-i c \tan(e + f x) + c}}{\tan(e + f x) - i} dx + \int \left( -\frac{A^2 \sqrt{-i c \tan(e + f x) + c} \tan^2(e + f x)}{\tan(e + f x) - i} \right) dx + \int \frac{B^2 \sqrt{-i c \tan(e + f x) + c} \tan^2(e + f x)}{\tan(e + f x) - i} dx + \int \left( -\frac{B^2 \sqrt{-i c \tan(e + f x) + c} \tan^3(e + f x)}{\tan(e + f x) - i} \right) dx + \int \left( -\frac{2 i A^2 \sqrt{-i c \tan(e + f x) + c} \tan^2(e + f x)}{\tan(e + f x) - i} \right) dx + \int \left( -\frac{2 B^2 \sqrt{-i c \tan(e + f x) + c} \tan^3(e + f x)}{\tan(e + f x) - i} \right) dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e)),x)`

[Out]  $-I (\text{Integral}(A c^{**2} \sqrt{-I c \tan(e + f x) + c} / (\tan(e + f x) - I), x) + \text{Integral}(-A c^{**2} \sqrt{-I c \tan(e + f x) + c} \tan^2(e + f x) / (\tan(e + f x) - I), x) + \text{Integral}(B c^{**2} \sqrt{-I c \tan(e + f x) + c} \tan(e + f x) / (\tan(e + f x) - I), x) + \text{Integral}(-B c^{**2} \sqrt{-I c \tan(e + f x) + c} \tan^3(e + f x) / (\tan(e + f x) - I), x) + \text{Integral}(-2 I A c^{**2} \sqrt{-I c \tan(e + f x) + c} \tan^2(e + f x) / (\tan(e + f x) - I), x) + \text{Integral}(-2 I B c^{**2} \sqrt{-I c \tan(e + f x) + c} \tan^3(e + f x) / (\tan(e + f x) - I), x)) / a$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)/(I\*a\*tan(f\*x + e) + a), x)

**Mupad [B]**

time = 1.26, size = 245, normalized size = 1.36

$$\frac{2Bc^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)}}{af(c-\tan(e+fx))^{3/2}-2acf} + \frac{Ac^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)}}{af} - \frac{6Bc^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)}}{af} - \frac{2Bc(c-\tan(e+fx))^{3/2}}{3af} - \frac{\sqrt{2}A(-c)^{5/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)}}{2\sqrt{-c}}\right)}{af} - \frac{\sqrt{2}Bc^{5/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)}}{2\sqrt{c}}\right)}{af} + \frac{Ac^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)}}{af(c+\tan(e+fx))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(5/2))/(a + a\*tan(e + f\*x)\*1i),x)

[Out] (2\*B\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a\*f\*(c - c\*tan(e + f\*x)\*1i) - 2\*a\*c\*f) + (A\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*2i)/(a\*f) - (6\*B\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a\*f) - (2\*B\*c\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/(3\*a\*f) - (2^(1/2)\*A\*(-c)^(5/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*3i)/(a\*f) - (2^(1/2)\*B\*c^(5/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*1i)/(2\*c^(1/2)))\*7i)/(a\*f) + (A\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*2i)/(a\*f\*(c + c\*tan(e + f\*x)\*1i))

$$3.766 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{3/2}}{a+ia \tan(e+fx)} dx$$

**Optimal.** Leaf size=144

$$-\frac{(iA-5B)c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}af} + \frac{(iA-5B)c\sqrt{c-ictan(e+fx)}}{2af} + \frac{(iA-B)(c-ictan(e+fx))^{3/2}}{2af(1+i \tan(e+fx))}$$

[Out]  $-1/2*(I*A-5*B)*c^{(3/2)}*\arctanh(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})/a/f*2^{(1/2)}+1/2*(I*A-5*B)*c*(c-I*c*\tan(f*x+e))^{(1/2)}/a/f+1/2*(I*A-B)*(c-I*c*\tan(f*x+e))^{(3/2)}/a/f/(1+I*\tan(f*x+e))$

**Rubi** [A]

time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 52, 65, 214}

$$-\frac{c^{3/2}(-5B+iA) \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}af} + \frac{c(-5B+iA)\sqrt{c-ictan(e+fx)}}{2af} + \frac{(-B+iA)(c-ictan(e+fx))^{3/2}}{2af(1+i \tan(e+fx))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^{(3/2)}/(a+I*a*\text{Tan}[e+f*x]),x]$

[Out]  $-(((I*A-5*B)*c^{(3/2)}*\text{ArcTanh}[\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[c])])/(\text{Sqrt}[2]*a*f)) + ((I*A-5*B)*c*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])/(2*a*f) + ((I*A-B)*(c-I*c*\text{Tan}[e+f*x])^{(3/2)})/(2*a*f*(1+I*\text{Tan}[e+f*x]))$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m+n+1))), \text{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^{3/2}}{a + i a \tan(e + fx)} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B)(c - i c \tan(e + fx))^{3/2}}{2af(1 + i \tan(e + fx))} - \frac{((A + 5iB)c) \text{Subst}\left(\int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^2} dx, x, \tan(e + fx)\right)}{2af}$$

$$= \frac{(iA - 5B)c\sqrt{c - ic \tan(e + fx)}}{2af} + \frac{(iA - B)(c - i c \tan(e + fx))^{3/2}}{2af(1 + i \tan(e + fx))}$$

$$= \frac{(iA - 5B)c\sqrt{c - ic \tan(e + fx)}}{2af} + \frac{(iA - B)(c - i c \tan(e + fx))^{3/2}}{2af(1 + i \tan(e + fx))}$$

$$= -\frac{(iA - 5B)c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}af} + \frac{(iA - B)(c - i c \tan(e + fx))^{3/2}}{2af(1 + i \tan(e + fx))}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(a + I\*a\*Tan[e + f\*x]),x]

[Out] \$Aborted

Maple [A]

time = 0.37, size = 111, normalized size = 0.77

method	result
derivativedivides	$2ic \left( iB \sqrt{c - ic \tan(fx + e)} + c \left( \frac{\left(\frac{A}{4} + \frac{iB}{4}\right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} - \frac{\left(\frac{A}{2} + \frac{5iB}{2}\right) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(fx + e)}}{2\sqrt{c}}\right)}{fa} \right) \right)$
default	$2ic \left( iB \sqrt{c - ic \tan(fx + e)} + c \left( \frac{\left(\frac{A}{4} + \frac{iB}{4}\right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} - \frac{\left(\frac{A}{2} + \frac{5iB}{2}\right) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(fx + e)}}{2\sqrt{c}}\right)}{fa} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e)),x,method=\_RETURNVERBOSE)

[Out] 2\*I/f/a\*c\*(I\*B\*(c-I\*c\*tan(f\*x+e))^(1/2)+c\*((1/4\*A+1/4\*I\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/(1/2\*c+1/2\*I\*c\*tan(f\*x+e))-1/2\*(1/2\*A+5/2\*I\*B)\*2^(1/2)/c^(1/2)\*arc tanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))))

Maxima [A]

time = 0.51, size = 143, normalized size = 0.99

$$i \left( \frac{\sqrt{2}^{(A+5iB)c^{\frac{5}{2}}} \log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right)}{a} - \frac{4\sqrt{-ic \tan(fx+e)+c}^{(A+iB)c^3}}{(-ic \tan(fx+e)+c)a-2ac} + \frac{8i\sqrt{-ic \tan(fx+e)+c}^{Bc^2}}{a} \right)$$

4cf

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e)),x, algorithm="maxima")

[Out]  $\frac{1}{4}I*(\sqrt{2}*(A + 5*I*B)*c^{5/2}*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c}))/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c}))/a - 4*\sqrt{-I*c*\tan(f*x + e) + c}*(A + I*B)*c^3/((-I*c*\tan(f*x + e) + c)*a - 2*a*c) + 8*I*\sqrt{-I*c*\tan(f*x + e) + c}*B*c^2/a)/(c*f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs.  $2(115) = 230$ .  
time = 3.13, size = 341, normalized size = 2.37

$$\frac{\left(\sqrt{2} \operatorname{erf} \sqrt{\frac{(A^2 + 10AB - 25B^2)c^2}{a^2 f^2}} \operatorname{erf}^{2i+2k+1} \log \left( \frac{(-1 + 5Bc^2 - 10B^2 c^{2i+2k+1} - a^2) \sqrt{\frac{(A^2 + 10AB - 25B^2)c^2}{a^2 f^2}} \sqrt{\frac{c}{2i^2 + 2i + 1}}}{-a} \right) - \sqrt{2} \operatorname{erf} \sqrt{\frac{(A^2 + 10AB - 25B^2)c^2}{a^2 f^2}} \operatorname{erf}^{2i+2k+1} \log \left( \frac{(-1 + 5Bc^2 - 10B^2 c^{2i+2k+1} - a^2) \sqrt{\frac{(A^2 + 10AB - 25B^2)c^2}{a^2 f^2}} \sqrt{\frac{c}{2i^2 + 2i + 1}}}{-a} \right) + 2\sqrt{2}((-1 + 5B)c^{2i+2k+1} + (-1 + B)c) \sqrt{\frac{c}{2i^2 + 2i + 1}} \right) e^{2i f x - 2k}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x, a lgorithm="fricas")`

[Out]  $-1/4*(\sqrt{2}*a*f*\sqrt{-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2)})*e^{(2*I*f*x + 2*I*e)}*\log(-2*((I*A - 5*B)*c^2 + (a*f*e^{(2*I*f*x + 2*I*e)} + a*f)*\sqrt{-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2)})*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})) * e^{(-I*f*x - I*e)/(a*f)} - \sqrt{2}*a*f*\sqrt{-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2)}*e^{(2*I*f*x + 2*I*e)}*\log(-2*((I*A - 5*B)*c^2 - (a*f*e^{(2*I*f*x + 2*I*e)} + a*f)*\sqrt{-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2)})*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})) * e^{(-I*f*x - I*e)/(a*f)} + 2*\sqrt{2}*((-I*A + 5*B)*c*e^{(2*I*f*x + 2*I*e)} + (-I*A + B)*c)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})*e^{(-2*I*f*x - 2*I*e)/(a*f)}$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{Ac \sqrt{-ic \tan(e + fx) + c}}{\tan(e + fx) - i} dx + \int \frac{Bc \sqrt{-ic \tan(e + fx) + c} \tan(e + fx)}{\tan(e + fx) - i} dx + \int \left( -\frac{iAc \sqrt{-ic \tan(e + fx) + c} \tan(e + fx)}{\tan(e + fx) - i} \right) dx + \int \left( -\frac{iBc \sqrt{-ic \tan(e + fx) + c} \tan^2(e + fx)}{\tan(e + fx) - i} \right) dx \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e)),x)`

[Out]  $-I*(\operatorname{Integral}(A*c*\sqrt{-I*c*\tan(e + f*x) + c}/(\tan(e + f*x) - I), x) + \operatorname{Integral}(B*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)/(\tan(e + f*x) - I), x) + \operatorname{Integral}(-I*A*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)/(\tan(e + f*x) - I), x) + \operatorname{Integral}(-I*B*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**2/(\tan(e + f*x) - I), x))/a$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(3/2)/(I\*a\*tan(f\*x + e) + a), x)

**Mupad [B]**

time = 9.77, size = 189, normalized size = 1.31

$$\frac{Bc^2 \sqrt{c - c \tan(e + fx)} \operatorname{Li}}{af(c - c \tan(e + fx) \operatorname{Li}) - 2acf} - \frac{2Bc \sqrt{c - c \tan(e + fx)} \operatorname{Li}}{af} + \frac{\sqrt{2} A(-c)^{3/2} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + fx)} \operatorname{Li}}{2\sqrt{-c}}\right) \operatorname{Li}}{2af} + \frac{5\sqrt{2} Bc^{3/2} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + fx)} \operatorname{Li}}{2\sqrt{c}}\right)}{2af} + \frac{Ac^2 \sqrt{c - c \tan(e + fx)} \operatorname{Li} \operatorname{Li}}{af(c + c \tan(e + fx) \operatorname{Li})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/(a + a\*tan(e + f\*x)\*1i),x)

[Out] (B\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a\*f\*(c - c\*tan(e + f\*x)\*1i) - 2\*a\*c\*f) - (2\*B\*c\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a\*f) + (2^(1/2)\*A\*(-c)^(3/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*1i)/(2\*a\*f) + (5\*2^(1/2)\*B\*c^(3/2)\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2)))/(2\*a\*f) + (A\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*1i)/(a\*f\*(c + c\*tan(e + f\*x)\*1i))

$$3.767 \quad \int \frac{(A+B \tan(e+fx)) \sqrt{c - i c \tan(e+fx)}}{a+ia \tan(e+fx)} dx$$

Optimal. Leaf size=109

$$\frac{(iA + 3B)\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c - i c \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}af} + \frac{(iA - B)\sqrt{c - i c \tan(e+fx)}}{2af(1 + i \tan(e+fx))}$$

[Out] 1/4\*(I\*A+3\*B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))\*c^(1/2)/a/f\*2^(1/2)+1/2\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a/f/(1+I\*tan(f\*x+e))

Rubi [A]

time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$ , Rules used = {3669, 79, 65, 214}

$$\frac{(-B + iA)\sqrt{c - i c \tan(e+fx)}}{2af(1 + i \tan(e+fx))} + \frac{\sqrt{c} (3B + iA) \tanh^{-1}\left(\frac{\sqrt{c - i c \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}af}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a + I\*a\*Tan[e + f\*x]),x]

[Out] ((I\*A + 3\*B)\*Sqrt[c]\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(2\*Sqrt[2]\*a\*f) + ((I\*A - B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(2\*a\*f\*(1 + I\*Tan[e + f\*x]))

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))

))

Rule 214

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{a + ia \tan(e + fx)} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^2 \sqrt{c-icx}} dx, x, \tan(e + fx) \right)}{f}$$

$$= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{2af(1 + i \tan(e + fx))} + \frac{((A - 3iB)c) \text{Subst} \left( \int \frac{1}{2} \right)}{2af(1 + i \tan(e + fx))}$$

$$= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{2af(1 + i \tan(e + fx))} + \frac{(iA + 3B) \text{Subst} \left( \int \frac{1}{2} \right)}{2af(1 + i \tan(e + fx))}$$

$$= \frac{(iA + 3B) \sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{2\sqrt{2} af} + \frac{(iA)}{2af}$$

Mathematica [A]

time = 0.84, size = 168, normalized size = 1.54

$$\frac{(\cos(fx) + i \sin(fx))(A + B \tan(e + fx)) \left( \sqrt{2} (iA + 3B) \sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right) (\cos(e) + i \sin(e)) + 2(A + iB) \cos(e + fx) (i \cos(fx) + \sin(fx)) \sqrt{c - ic \tan(e + fx)} \right)}{4f(A \cos(e + fx) + B \sin(e + fx))(a + ia \tan(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e + f*x]), x]
```

```
[Out] ((Cos[f*x] + I*Sin[f*x])*(A + B*Tan[e + f*x])*(Sqrt[2]*(I*A + 3*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c]])*(Cos[e] + I*Sin[e]) +
```

$$2*(A + I*B)*\text{Cos}[e + f*x]*(I*\text{Cos}[f*x] + \text{Sin}[f*x])* \text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])) / (4*f*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x])*(a + I*a*\text{Tan}[e + f*x]))$$

**Maple [A]**

time = 0.44, size = 90, normalized size = 0.83

method	result	size
derivativedivides	$2ic \left( \frac{\left(\frac{A}{8} + \frac{iB}{8}\right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} + \frac{\left(-\frac{3iB}{2} + \frac{A}{2}\right) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(fx + e)} \sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}} \right)$	90
default	$2ic \left( \frac{\left(\frac{A}{8} + \frac{iB}{8}\right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} + \frac{\left(-\frac{3iB}{2} + \frac{A}{2}\right) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(fx + e)} \sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}} \right)$	90

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $2*I/f/a*c*((1/8*A+1/8*I*B)*(c-I*c*\text{tan}(f*x+e))^{1/2}/(1/2*c+1/2*I*c*\text{tan}(f*x+e))+1/4*(-3/2*I*B+1/2*A)*2^{1/2}/c^{1/2}*\operatorname{arctanh}(1/2*(c-I*c*\text{tan}(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))$

**Maxima [A]**

time = 0.51, size = 120, normalized size = 1.10

$$i \left( \frac{\sqrt{2} (A-3iB)c^{\frac{3}{2}} \log\left(-\frac{\sqrt{2} \sqrt{c} - \sqrt{-ic \tan(fx + e) + c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx + e) + c}}\right)}{a} + \frac{4 \sqrt{-ic \tan(fx + e) + c} (A+iB)c^2}{(-ic \tan(fx + e) + c)a - 2ac} \right) / 8cf$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="maxima")`

[Out]  $-1/8*I*(\text{sqrt}(2)*(A - 3*I*B)*c^{3/2}*\log(-(\text{sqrt}(2)*\text{sqrt}(c) - \text{sqrt}(-I*c*\text{tan}(f*x + e) + c)))/(\text{sqrt}(2)*\text{sqrt}(c) + \text{sqrt}(-I*c*\text{tan}(f*x + e) + c)))/a + 4*\text{sqrt}(-I*c*\text{tan}(f*x + e) + c)*(A + I*B)*c^2/((-I*c*\text{tan}(f*x + e) + c)*a - 2*a*c)/(c*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs.  $2(85) = 170$ .

time = 2.98, size = 337, normalized size = 3.09

$$\left( \sqrt{\frac{1}{2}} \sqrt{\frac{(A^2 - 6AB - 9B^2)c}{a^2 f}} e^{2I(fx+e)} \log \left( \frac{\left( \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{\frac{c}{2(fx+e)+1}} \sqrt{\frac{(A^2 - 6AB - 9B^2)c}{a^2 f}} \right)^{2I(fx+e)}}{\left( \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{\frac{c}{2(fx+e)+1}} \sqrt{\frac{(A^2 - 6AB - 9B^2)c}{a^2 f}} \right)^{2I(fx+e)}} \right) - \sqrt{\frac{1}{2}} \sqrt{\frac{(A^2 - 6AB - 9B^2)c}{a^2 f}} e^{2I(fx+e)} \log \left( \frac{\left( \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{\frac{c}{2(fx+e)+1}} \sqrt{\frac{(A^2 - 6AB - 9B^2)c}{a^2 f}} \right)^{2I(fx+e)}}{\left( \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{\frac{c}{2(fx+e)+1}} \sqrt{\frac{(A^2 - 6AB - 9B^2)c}{a^2 f}} \right)^{2I(fx+e)}} \right) + \sqrt{2} \left( (A - B) e^{2I(fx+e)} + (A + B) \sqrt{\frac{c}{2(fx+e)+1}} \right) e^{-2I(fx+e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x, algorithm="fricas")

[Out] 1/4\*(sqrt(1/2)\*a\*f\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c/(a^2\*f^2))\*e^(2\*I\*f\*x + 2\*I\*e)\*log((sqrt(2)\*sqrt(1/2)\*(a\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c/(a^2\*f^2)) + (I\*A + 3\*B)\*c)\*e^(-I\*f\*x - I\*e)/(a\*f)) - sqrt(1/2)\*a\*f\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c/(a^2\*f^2))\*e^(2\*I\*f\*x + 2\*I\*e)\*log(-(sqrt(2)\*sqrt(1/2)\*(a\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c/(a^2\*f^2)) - (I\*A + 3\*B)\*c)\*e^(-I\*f\*x - I\*e)/(a\*f)) + sqrt(2)\*((I\*A - B)\*e^(2\*I\*f\*x + 2\*I\*e) + I\*A - B)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*e^(-2\*I\*f\*x - 2\*I\*e)/(a\*f)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \frac{A \sqrt{-ictan(e+fx)+c}}{\tan(e+fx)-i} dx + \int \frac{B \sqrt{-ictan(e+fx)+c} \tan(e+fx)}{\tan(e+fx)-i} dx \right)$$

$a$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x)

[Out] -I\*(Integral(A\*sqrt(-I\*c\*tan(e + f\*x) + c)/(tan(e + f\*x) - I), x) + Integral(B\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)/(tan(e + f\*x) - I), x))/a

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e)),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(-I\*c\*tan(f\*x + e) + c)/(I\*a\*tan(f\*x + e) + a), x)

**Mupad** [B]

time = 9.56, size = 159, normalized size = 1.46

$$-\frac{Bc \sqrt{c - c \tan(e + f x)} \operatorname{li}}{2(acf + acf \tan(e + f x) \operatorname{li})} + \frac{Ac \sqrt{c - c \tan(e + f x)} \operatorname{li}}{2af(c + c \tan(e + f x) \operatorname{li})} + \frac{\sqrt{2} A \sqrt{-c} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + f x)} \operatorname{li}}{2\sqrt{-c}}\right) \operatorname{li}}{4af} + \frac{3\sqrt{2} B \sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + f x)} \operatorname{li}}{2\sqrt{c}}\right)}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B \cdot \tan(e + f \cdot x)) \cdot (c - c \cdot \tan(e + f \cdot x) \cdot 1i)^{(1/2)}) / (a + a \cdot \tan(e + f \cdot x) \cdot 1i), x)$

[Out]  $(A \cdot c \cdot (c - c \cdot \tan(e + f \cdot x) \cdot 1i)^{(1/2)} \cdot 1i) / (2 \cdot a \cdot f \cdot (c + c \cdot \tan(e + f \cdot x) \cdot 1i)) - (B \cdot c \cdot (c - c \cdot \tan(e + f \cdot x) \cdot 1i)^{(1/2)}) / (2 \cdot (a \cdot c \cdot f + a \cdot c \cdot f \cdot \tan(e + f \cdot x) \cdot 1i)) + (2^{(1/2)} \cdot A \cdot (-c)^{(1/2)} \cdot \text{atan}((2^{(1/2)} \cdot (c - c \cdot \tan(e + f \cdot x) \cdot 1i)^{(1/2)}) / (2 \cdot (-c)^{(1/2)}))) \cdot 1i) / (4 \cdot a \cdot f) + (3 \cdot 2^{(1/2)} \cdot B \cdot c^{(1/2)} \cdot \text{atanh}((2^{(1/2)} \cdot (c - c \cdot \tan(e + f \cdot x) \cdot 1i)^{(1/2)}) / (2 \cdot c^{(1/2)}))) / (4 \cdot a \cdot f)$

$$3.768 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx)) \sqrt{c-ictan(e+fx)}} dx$$

Optimal. Leaf size=141

$$\frac{(3iA+B) \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}a\sqrt{c}f} - \frac{3iA+B}{4af\sqrt{c-ictan(e+fx)}} + \frac{iA-B}{2af(1+i\tan(e+fx))\sqrt{c-ictan(e+fx)}}$$

[Out] 1/8\*(3\*I\*A+B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a/f\*2^(1/2)/c^(1/2)+1/4\*(-3\*I\*A-B)/a/f/(c-I\*c\*tan(f\*x+e))^(1/2)+1/2\*(I\*A-B)/a/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(1+I\*tan(f\*x+e))

Rubi [A]

time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 53, 65, 214}

$$\frac{-B+iA}{2af(1+i\tan(e+fx))\sqrt{c-ictan(e+fx)}} - \frac{B+3iA}{4af\sqrt{c-ictan(e+fx)}} + \frac{(B+3iA) \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}a\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]), x]

[Out] (((3\*I)\*A + B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(4\*Sqrt[2]\*a\*Sqrt[c]\*f) - ((3\*I)\*A + B)/(4\*a\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + (I\*A - B)/(2\*a\*f\*(1 + I\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])

Rule 53

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2(c-icx)^{3/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{iA - B}{2af(1 + i \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}} + \frac{((3A - iB) \sqrt{c - ic \tan(e + fx)})}{2af(1 + i \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}$$

$$= -\frac{3iA + B}{4af \sqrt{c - ic \tan(e + fx)}} + \frac{iA - B}{2af(1 + i \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}$$

$$= -\frac{3iA + B}{4af \sqrt{c - ic \tan(e + fx)}} + \frac{iA - B}{2af(1 + i \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}$$

$$= \frac{(3iA + B) \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{4\sqrt{2} a \sqrt{c} f} - \frac{3B}{4af \sqrt{c - ic \tan(e + fx)}}$$

Mathematica [A]



time = 1.33, size = 160, normalized size = 1.13

$$\frac{e^{-2i(e+fx)} \sqrt{\frac{c}{1+e^{2i(e+fx)}}} \left( -i(1+e^{2i(e+fx)}) (A(-1+2e^{2i(e+fx)}) - iB(1+2e^{2i(e+fx)})) + (3iA+B)e^{2i(e+fx)} \sqrt{1+e^{2i(e+fx)}} \tanh^{-1} \left( \sqrt{1+e^{2i(e+fx)}} \right) \right)}{4\sqrt{2}acf}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]), x]

[Out] (Sqrt[c/(1 + E^((2\*I)\*(e + f\*x)))]\*((-I)\*(1 + E^((2\*I)\*(e + f\*x)))\*(A\*(-1 + 2\*E^((2\*I)\*(e + f\*x))) - I\*B\*(1 + 2\*E^((2\*I)\*(e + f\*x)))) + ((3\*I)\*A + B)\*E^((2\*I)\*(e + f\*x))\*Sqrt[1 + E^((2\*I)\*(e + f\*x))]\*ArcTanh[Sqrt[1 + E^((2\*I)\*(e + f\*x))]])/(4\*Sqrt[2]\*a\*c\*E^((2\*I)\*(e + f\*x))\*f)

Maple [A]

time = 0.33, size = 121, normalized size = 0.86

method	result
derivativedivides	$2ic \left( -\frac{-iB+A}{4c \sqrt{c - ic \tan(fx + e)}} + \frac{\left(\frac{A+iB}{4}\right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} + \frac{\left(\frac{3A}{2} - \frac{iB}{2}\right) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(fx + e)}}{2\sqrt{c}}\right)}{4c} \right)$
default	$2ic \left( -\frac{-iB+A}{4c \sqrt{c - ic \tan(fx + e)}} + \frac{\left(\frac{A+iB}{4}\right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} + \frac{\left(\frac{3A}{2} - \frac{iB}{2}\right) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(fx + e)}}{2\sqrt{c}}\right)}{4c} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e)), x, method=\_RETURNVERBOSE)

[Out] 2\*I/f/a\*c\*(-1/4/c\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2)+1/4/c\*((1/4\*A+1/4\*I\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/(1/2\*c+1/2\*I\*c\*tan(f\*x+e))+1/2\*(3/2\*A-1/2\*I\*B)\*2^(1/2)/c^(1/2)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))))

Maxima [A]

time = 0.51, size = 148, normalized size = 1.05

$$i \left( \frac{\sqrt{2}^{(3A-iB)} \sqrt{c} \log \left( \frac{\sqrt{2} \sqrt{c} - \sqrt{-i c \tan (f x+e)+c}}{\sqrt{2} \sqrt{c} + \sqrt{-i c \tan (f x+e)+c}} \right)}{a} + \frac{4((-i c \tan (f x+e)+e)(3 A-i B) c-4(A-i B) c^2)}{(-i c \tan (f x+e)+c)^{\frac{3}{2}} a-2 \sqrt{-i c \tan (f x+e)+c} a c} \right) / 16 c f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] -1/16*I*(sqrt(2)*(3*A - I*B)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(
f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a + 4*((-I*
c*tan(f*x + e) + c)*(3*A - I*B)*c - 4*(A - I*B)*c^2)/((-I*c*tan(f*x + e) +
c)^(3/2)*a - 2*sqrt(-I*c*tan(f*x + e) + c)*a*c)/(c*f)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(112) = 224.  
time = 4.29, size = 367, normalized size = 2.60

$$\left( \sqrt{\frac{2}{a^2}} \int \sqrt{\frac{9A^2 - 6AB - B^2}{a^2 c f^2}} e^{2i(fx+e)} \log \left( \frac{\sqrt{2} \sqrt{\frac{c}{2i(fx+e)+1}} \sqrt{\frac{9A^2 - 6AB - B^2}{a^2 c f^2}}}{\sqrt{2} \sqrt{\frac{c}{2i(fx+e)+1}} \sqrt{\frac{9A^2 - 6AB - B^2}{a^2 c f^2}}} \right) - \sqrt{\frac{2}{a^2}} \int \sqrt{\frac{9A^2 - 6AB - B^2}{a^2 c f^2}} e^{2i(fx+e)} \log \left( \frac{\sqrt{2} \sqrt{\frac{c}{2i(fx+e)+1}} \sqrt{\frac{9A^2 - 6AB - B^2}{a^2 c f^2}}}{\sqrt{2} \sqrt{\frac{c}{2i(fx+e)+1}} \sqrt{\frac{9A^2 - 6AB - B^2}{a^2 c f^2}}} \right) - \sqrt{2} (2(A+B)e^{2i(fx+e)} - (A-B)e^{2i(fx+e)} - (A+B) \sqrt{\frac{c}{2i(fx+e)+1}}) e^{i(fx+e)} \right) / c f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] 1/8*(sqrt(1/2)*a*c*f*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^2*c*f^2))*e^(2*I*f*x
+ 2*I*e)*log(1/2*(sqrt(2)*sqrt(1/2)*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^2*c*f^2)) + 3*I
*A + B)*e^(-I*f*x - I*e)/(a*f)) - sqrt(1/2)*a*c*f*sqrt(-(9*A^2 - 6*I*A*B -
B^2)/(a^2*c*f^2))*e^(2*I*f*x + 2*I*e)*log(-1/2*(sqrt(2)*sqrt(1/2)*(a*f*e^(2
*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2 - 6*I
*A*B - B^2)/(a^2*c*f^2)) - 3*I*A - B)*e^(-I*f*x - I*e)/(a*f)) - sqrt(2)*(2*
(I*A + B)*e^(4*I*f*x + 4*I*e) - (-I*A - 3*B)*e^(2*I*f*x + 2*I*e) - I*A + B)
*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*c*f)
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \frac{A}{\sqrt{-i c \tan (e+f x)+c} \tan (e+f x)-i \sqrt{-i c \tan (e+f x)+c}} d x + \int \frac{B \tan (e+f x)}{\sqrt{-i c \tan (e+f x)+c} \tan (e+f x)-i \sqrt{-i c \tan (e+f x)+c}} d x \right) / a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x)
```

[Out]  $-I * (\text{Integral}(A / (\sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) - I * \sqrt{-I * c * \tan(e + f * x) + c}), x) + \text{Integral}(B * \tan(e + f * x) / (\sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) - I * \sqrt{-I * c * \tan(e + f * x) + c}), x) / a$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x, algorithm="giac")`

[Out] `integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f*x + e) + c)), x)`

**Mupad [B]**

time = 9.85, size = 212, normalized size = 1.50

$$\frac{Bc - \frac{B(c - \tan(e + fx))}{4}}{af(c - \tan(e + fx))^{3/2} - 2acf\sqrt{c - \tan(e + fx)}} + \frac{\frac{A(c - \tan(e + fx))^{3i}}{4af} - \frac{Aci}{af}}{2c\sqrt{c - \tan(e + fx)} - (c - \tan(e + fx))^{3/2}} - \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c - \tan(e + fx)}}{2\sqrt{-c}}\right)^{3i}}{8a\sqrt{-c}f} + \frac{\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c - \tan(e + fx)}}{2\sqrt{c}}\right)}{8a\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(1/2)),x)`

[Out]  $(B * c - (B * (c - c * \tan(e + f * x) * 1i)) / 4) / (a * f * (c - c * \tan(e + f * x) * 1i)^{(3/2)} - 2 * a * c * f * (c - c * \tan(e + f * x) * 1i)^{(1/2)}) + ((A * (c - c * \tan(e + f * x) * 1i) * 3i) / (4 * a * f) - (A * c * 1i) / (a * f)) / (2 * c * (c - c * \tan(e + f * x) * 1i)^{(1/2)} - (c - c * \tan(e + f * x) * 1i)^{(3/2)}) - (2^{(1/2)} * A * \operatorname{atan}((2^{(1/2)} * (c - c * \tan(e + f * x) * 1i)^{(1/2)}) / (2 * (-c)^{(1/2)})) * 3i) / (8 * a * (-c)^{(1/2)} * f) + (2^{(1/2)} * B * \operatorname{atanh}((2^{(1/2)} * (c - c * \tan(e + f * x) * 1i)^{(1/2)}) / (2 * c^{(1/2)}))) / (8 * a * c^{(1/2)} * f)$

$$3.769 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ict \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=184

$$\frac{(5iA - B) \tanh^{-1} \left( \frac{\sqrt{c - ict \tan(e+fx)}}{\sqrt{2} \sqrt{c}} \right)}{8\sqrt{2} ac^{3/2} f} - \frac{5iA - B}{12af(c - ict \tan(e+fx))^{3/2}} + \frac{iA - B}{2af(1 + i \tan(e+fx))(c - ict \tan(e+fx))^{3/2}}$$

[Out] 1/16\*(5\*I\*A-B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a/c^(3/2)/f\*2^(1/2)+1/8\*(-5\*I\*A+B)/a/c/f/(c-I\*c\*tan(f\*x+e))^(1/2)+1/12\*(-5\*I\*A+B)/a/f/(c-I\*c\*tan(f\*x+e))^(3/2)+1/2\*(I\*A-B)/a/f/(1+I\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 53, 65, 214}

$$\frac{(-B + 5iA) \tanh^{-1} \left( \frac{\sqrt{c - ict \tan(e+fx)}}{\sqrt{2} \sqrt{c}} \right)}{8\sqrt{2} ac^{3/2} f} + \frac{-B + iA}{2af(1 + i \tan(e+fx))(c - ict \tan(e+fx))^{3/2}} - \frac{-B + 5iA}{8acf \sqrt{c - ict \tan(e+fx)}} - \frac{-B + 5iA}{12af(c - ict \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2)), x]

[Out] (((5\*I)\*A - B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(8\*Sqrt[2]\*a\*c^(3/2)\*f) - ((5\*I)\*A - B)/(12\*a\*f\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + (I\*A - B)/(2\*a\*f\*(1 + I\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2)) - ((5\*I)\*A - B)/(8\*a\*c\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 53**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))(c - ic \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^2(c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{iA - B}{2af(1 + i \tan(e + fx))(c - ic \tan(e + fx))^{3/2}} + \frac{((5A + B) \operatorname{arctanh}\left(\frac{c - ic \tan(e + fx)}{\sqrt{2} \sqrt{c}}\right))}{2af(1 + i \tan(e + fx))^{3/2}} \\
&= -\frac{5iA - B}{12af(c - ic \tan(e + fx))^{3/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{3/2}} \\
&= -\frac{5iA - B}{12af(c - ic \tan(e + fx))^{3/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{3/2}} \\
&= -\frac{5iA - B}{12af(c - ic \tan(e + fx))^{3/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{3/2}} \\
&= \frac{(5iA - B) \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{8\sqrt{2} ac^{3/2} f} - \frac{12af(c - ic \tan(e + fx))^{3/2}}{12af(c - ic \tan(e + fx))^{3/2}}
\end{aligned}$$

**Mathematica [A]**

time = 2.16, size = 239, normalized size = 1.30

$$\frac{e^{-i(e+2fx)} \sqrt{\frac{c}{1+e^{2i(e+fx)}}} \left( (1+e^{2i(e+fx)}) (B(3+2e^{2i(e+fx)}+2e^{4i(e+fx)}) + iA(-3+14e^{2i(e+fx)}+2e^{4i(e+fx)})) + 3(-5iA+B)e^{2i(e+fx)} \sqrt{1+e^{2i(e+fx)}} \operatorname{tanh}^{-1}\left(\frac{\sqrt{1+e^{2i(e+fx)}}}{\sqrt{2}\sqrt{c}}\right) \right) (\cos(fx) + i \sin(fx))(A + B \tan(e + fx))}{24\sqrt{2} c^2 f (A \cos(e + fx) + B \sin(e + fx))(a + ia \tan(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)), x]
```

```
[Out] -1/24*(Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*((1 + E^((2*I)*(e + f*x)))*(B*(3 + 2*E^((2*I)*(e + f*x)) + 2*E^((4*I)*(e + f*x))) + I*A*(-3 + 14*E^((2*I)*(e + f*x)) + 2*E^((4*I)*(e + f*x)))) + 3*((-5*I)*A + B)*E^((2*I)*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])*(Cos[f*x] + I*Sin[f*x])*(A + B*Tan[e + f*x]))/(Sqrt[2]*c^2*E^(I*(e + 2*f*x))*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*(a + I*a*Tan[e + f*x]))
```

**Maple [A]**

time = 0.31, size = 141, normalized size = 0.77

method	result
--------	--------

derivativedivides	$2ic \left( \frac{\left( \frac{A}{8} + \frac{iB}{8} \right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} + \frac{\left( \frac{5A}{2} + \frac{iB}{2} \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{c - ic \tan(fx + e)} \sqrt{2}}{2\sqrt{c}} \right)}{4\sqrt{c}} \right)}{4c^2}$
default	$2ic \left( \frac{\left( \frac{A}{8} + \frac{iB}{8} \right) \sqrt{c - ic \tan(fx + e)}}{\frac{c}{2} + \frac{ic \tan(fx + e)}{2}} + \frac{\left( \frac{5A}{2} + \frac{iB}{2} \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{c - ic \tan(fx + e)} \sqrt{2}}{2\sqrt{c}} \right)}{4\sqrt{c}} \right)}{4c^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $2*I/f/a*c*(1/4/c^2*((1/8*A+1/8*I*B)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/4*(5/2*A+1/2*I*B)*2^(1/2)/c^(1/2)*\operatorname{arctanh}(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))-1/4*A/c^2/(c-I*c*tan(f*x+e))^(1/2)-1/12/c*(A-I*B)/(c-I*c*tan(f*x+e))^(3/2))$

**Maxima** [A]

time = 0.51, size = 173, normalized size = 0.94

$$i \left( \frac{3\sqrt{2}^{(5A+iB)\log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}}{a\sqrt{c}} + \frac{4\left(3(-ic\tan(fx+e)+c)^2(5A+iB)-4(-ic\tan(fx+e)+c)(5A+iB)c-8(A-iB)c^2\right)}{(-ic\tan(fx+e)+c)^{\frac{5}{2}}a-2(-ic\tan(fx+e)+c)^{\frac{3}{2}}ac} \right)$$

96cf

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")`

[Out]  $-1/96*I*(3*\sqrt{2}*(5*A + I*B)*\log(-(\sqrt{2}*\sqrt{c}) - \sqrt{-I*c*\tan(f*x + e) + c})/(\sqrt{2}*\sqrt{c}) + \sqrt{-I*c*\tan(f*x + e) + c}))/a*\sqrt{c}) + 4*(3*(-I*c*\tan(f*x + e) + c)^2*(5*A + I*B) - 4*(-I*c*\tan(f*x + e) + c)*(5*A + I*B)*c - 8*(A - I*B)*c^2)/((-I*c*\tan(f*x + e) + c)^(5/2)*a - 2*(-I*c*\tan(f*x + e) + c)^(3/2)*a*c))/(c*f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 401 vs.  $2(144) = 288$ .

time = 4.66, size = 401, normalized size = 2.18

$$\frac{\left(3\sqrt{\frac{2}{3}}\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}e^{2I*fx+2I*e}\log\left(\frac{\sqrt{2}\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}\sqrt{\frac{c}{25A^2+10AB-B^2}}\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}}{\sqrt{25A^2+10AB-B^2}}\right)-3\sqrt{\frac{2}{3}}\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}e^{2I*fx+2I*e}\log\left(\frac{\sqrt{2}\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}\sqrt{\frac{c}{25A^2+10AB-B^2}}\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}}{\sqrt{25A^2+10AB-B^2}}\right)-\sqrt{2}(2(A+By)^{2I*fx+2I*e}+4(A+By)^{2I*fx+2I*e}-(-11A-5B)^{2I*fx+2I*e}-3A+3B)\sqrt{\frac{25A^2+10AB-B^2}{a^2f^2}}\right)e^{2I*fx+2I*e}}{a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/48\*(3\*sqrt(1/2)\*a\*c^2\*f\*sqrt(-(25\*A^2 + 10\*I\*A\*B - B^2)/(a^2\*c^3\*f^2))\*e^(2\*I\*f\*x + 2\*I\*e)\*log(1/4\*(sqrt(2)\*sqrt(1/2)\*(a\*c\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a\*c\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(25\*A^2 + 10\*I\*A\*B - B^2)/(a^2\*c^3\*f^2)) + 5\*I\*A - B)\*e^(-I\*f\*x - I\*e)/(a\*c\*f)) - 3\*sqrt(1/2)\*a\*c^2\*f\*sqrt(-(25\*A^2 + 10\*I\*A\*B - B^2)/(a^2\*c^3\*f^2))\*e^(2\*I\*f\*x + 2\*I\*e)\*log(-1/4\*(sqrt(2)\*sqrt(1/2)\*(a\*c\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a\*c\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(25\*A^2 + 10\*I\*A\*B - B^2)/(a^2\*c^3\*f^2)) - 5\*I\*A + B)\*e^(-I\*f\*x - I\*e)/(a\*c\*f)) - sqrt(2)\*(2\*(I\*A + B)\*e^(6\*I\*f\*x + 6\*I\*e) + 4\*(4\*I\*A + B)\*e^(4\*I\*f\*x + 4\*I\*e) - (-11\*I\*A - 5\*B)\*e^(2\*I\*f\*x + 2\*I\*e) - 3\*I\*A + 3\*B)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*e^(-2\*I\*f\*x - 2\*I\*e)/(a\*c^2\*f)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\left(\int \frac{A}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-ic\sqrt{-ic\tan(e+fx)+c}}dx + \int \frac{B\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-ic\sqrt{-ic\tan(e+fx)+c}}dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x)

[Out] -I\*(Integral(A/(-I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2 - I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)), x) + Integral(B\*tan(e + f\*x)/(-I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2 - I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)), x))/a

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)\*(-I\*c\*tan(f\*x + e) + c)^(3/2)), x)

Mupad [B]

time = 10.33, size = 261, normalized size = 1.42

$$\frac{\frac{Bc}{3} - \frac{B(c-\tan(e+fx))}{3c} + \frac{B(c-\tan(e+fx))}{3c}}{af(c-\tan(e+fx))^{5/2} - 2acf(c-\tan(e+fx))^{3/2}} - \frac{\frac{A(c-\tan(e+fx))}{af} + \frac{Ae}{3af} - \frac{A(c-\tan(e+fx))}{3af}}{2c(c-\tan(e+fx))^{3/2} - (c-\tan(e+fx))^{5/2}} + \frac{\sqrt{2}A\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}}{2\sqrt{-c}}\right)}{16a(-c)^{3/2}f} - \frac{\sqrt{2}B\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}}{2\sqrt{c}}\right)}{16a^2f}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B*\tan(e + f*x))/((a + a*\tan(e + f*x)*1i)*(c - c*\tan(e + f*x)*1i)^{(3/2})), x)$

[Out]  $((B*c)/3 - (B*(c - c*\tan(e + f*x)*1i))/6 + (B*(c - c*\tan(e + f*x)*1i)^2)/(8*c))/(a*f*(c - c*\tan(e + f*x)*1i)^{(5/2} - 2*a*c*f*(c - c*\tan(e + f*x)*1i)^{(3/2})) - ((A*(c - c*\tan(e + f*x)*1i)*5i)/(6*a*f) + (A*c*1i)/(3*a*f) - (A*(c - c*\tan(e + f*x)*1i)^2*5i)/(8*a*c*f))/(2*c*(c - c*\tan(e + f*x)*1i)^{(3/2} - (c - c*\tan(e + f*x)*1i)^{(5/2})) + (2^{(1/2)}*A*\text{atan}((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(2*(-c)^{(1/2)}))*5i)/(16*a*(-c)^{(3/2)}*f) - (2^{(1/2)}*B*\text{atanh}((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(2*c^{(1/2)})))/(16*a*c^{(3/2)}*f)$

$$3.770 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=223

$$\frac{(7iA - 3B) \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{16\sqrt{2} ac^{5/2} f} - \frac{7iA - 3B}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))(c - ic \tan(e + fx))^{5/2}}$$

[Out] 1/32\*(7\*I\*A-3\*B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a/c^(5/2)/f\*2^(1/2)+1/16\*(-7\*I\*A+3\*B)/a/c^2/f/(c-I\*c\*tan(f\*x+e))^(1/2)+1/20\*(-7\*I\*A+3\*B)/a/f/(c-I\*c\*tan(f\*x+e))^(5/2)+1/2\*(I\*A-B)/a/f/(1+I\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2)+1/24\*(-7\*I\*A+3\*B)/a/c/f/(c-I\*c\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.20, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 53, 65, 214}

$$\frac{(-3B + 7iA) \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{16\sqrt{2} ac^{5/2} f} - \frac{-3B + 7iA}{16ac^2 f \sqrt{c - ic \tan(e + fx)}} - \frac{-3B + 7iA}{24acf(c - ic \tan(e + fx))^{3/2}} - \frac{-3B + 7iA}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{-B + iA}{2af(1 + i \tan(e + fx))(c - ic \tan(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2)), x]

[Out] (((7\*I)\*A - 3\*B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(16\*Sqrt[2]\*a\*c^(5/2)\*f) - ((7\*I)\*A - 3\*B)/(20\*a\*f\*(c - I\*c\*Tan[e + f\*x])^(5/2)) + (I\*A - B)/(2\*a\*f\*(1 + I\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2)) - ((7\*I)\*A - 3\*B)/(24\*a\*c\*f\*(c - I\*c\*Tan[e + f\*x])^(3/2)) - ((7\*I)\*A - 3\*B)/(16\*a\*c^2\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 53**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))(c - ic \tan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^2(c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{iA - B}{2af(1 + i \tan(e + fx))(c - ic \tan(e + fx))^{5/2}} + \frac{((7A + 3B) \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right))}{20af(c - ic \tan(e + fx))^{5/2}} \\
&= -\frac{7iA - 3B}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{5/2}} \\
&= -\frac{7iA - 3B}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{5/2}} \\
&= -\frac{7iA - 3B}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{5/2}} \\
&= -\frac{7iA - 3B}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{5/2}} \\
&= -\frac{7iA - 3B}{20af(c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{2af(1 + i \tan(e + fx))^{5/2}} \\
&= \frac{(7iA - 3B) \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{16\sqrt{2} ac^{5/2} f} - \frac{iA - B}{20af(c - ic \tan(e + fx))^{5/2}}
\end{aligned}$$

**Mathematica [A]**

time = 2.50, size = 213, normalized size = 0.96

$$\frac{e^{-2i(e+fx)} \sqrt{\frac{c}{1+e^{2i(e+fx)}}} \left( (1+e^{2i(e+fx)}) (3B(5-8e^{2i(e+fx)}+4e^{4i(e+fx)}+2e^{6i(e+fx)})+iA(-15+116e^{2i(e+fx)}+32e^{4i(e+fx)}+6e^{6i(e+fx)})) + 15(-7iA+3B)e^{2i(e+fx)} \sqrt{1+e^{2i(e+fx)}} \tanh^{-1} \left( \sqrt{1+e^{2i(e+fx)}} \right) \right)}{240\sqrt{2} ac^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)), x]
```

```
[Out] -1/240*(Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*((1 + E^((2*I)*(e + f*x)))*(3*B*(5 - 8*E^((2*I)*(e + f*x)) + 4*E^((4*I)*(e + f*x)) + 2*E^((6*I)*(e + f*x))) + I*A*(-15 + 116*E^((2*I)*(e + f*x)) + 32*E^((4*I)*(e + f*x)) + 6*E^((6*I)*(e + f*x)))) + 15*((-7*I)*A + 3*B)*E^((2*I)*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])]/(Sqrt[2]*a*c^3*E^((2*I)*(e + f*x))*f)
```

**Maple [A]**

time = 0.33, size = 168, normalized size = 0.75

method	result
derivativedivides	$2ic \left( -\frac{A}{12c^2(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{iB+3A}{16c^3 \sqrt{c-ic \tan(fx+e)}} - \frac{-iB+A}{20c(c-ic \tan(fx+e))^{\frac{5}{2}}} + \frac{(\frac{A}{4} + \frac{iB}{4}) \sqrt{c-ic \tan(fx+e)}}{\frac{c}{2} + \frac{ic \tan(fx+e)}{2}} \right) / fa$
default	$2ic \left( -\frac{A}{12c^2(c-ic \tan(fx+e))^{\frac{3}{2}}} - \frac{iB+3A}{16c^3 \sqrt{c-ic \tan(fx+e)}} - \frac{-iB+A}{20c(c-ic \tan(fx+e))^{\frac{5}{2}}} + \frac{(\frac{A}{4} + \frac{iB}{4}) \sqrt{c-ic \tan(fx+e)}}{\frac{c}{2} + \frac{ic \tan(fx+e)}{2}} \right) / fa$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $2*I/f/a*c*(-1/12*A/c^2/(c-I*c*tan(f*x+e))^(3/2)-1/16/c^3*(3*A+I*B)/(c-I*c*tan(f*x+e))^(1/2)-1/20/c*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2)+1/16/c^3*((1/4*A+1/4*I*B)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/2*(7/2*A+3/2*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))$

**Maxima [A]**

time = 0.52, size = 202, normalized size = 0.91

$$i \left( \frac{4 \left( (15(-i \tan(fx+e)+c)^3(7A+3iB)-20(-i \tan(fx+e)+c)^2(7A+3iB)c-8(-i \tan(fx+e)+c)(7A+3iB)c^2-48(A-iB)c^3) \right)}{(-i \tan(fx+e)+c)^2 ac-2(-i \tan(fx+e)+c)^{\frac{3}{2}} ac^2} + \frac{15 \sqrt{2} (7A+3iB) \log \left( \frac{-\sqrt{2} \sqrt{c}-\sqrt{-i \tan(fx+e)+c}}{\sqrt{2} \sqrt{c}+\sqrt{-i \tan(fx+e)+c}} \right)}{ac^{\frac{3}{2}}} \right) / 960cf$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")`

[Out]  $-1/960*I*(4*(15*(-I*c*tan(f*x+e)+c)^3*(7*A+3*I*B)-20*(-I*c*tan(f*x+e)+c)^2*(7*A+3*I*B)*c-8*(-I*c*tan(f*x+e)+c)*(7*A+3*I*B)*c^2-48*(A-I*B)*c^3)/((-I*c*tan(f*x+e)+c)^(7/2)*a*c-2*(-I*c*tan(f*x+e)+c)^(5/2)*a*c^2)+15*sqrt(2)*(7*A+3*I*B)*log(-sqrt(2)*sqrt(c)-sqrt(-I*c*tan(f*x+e)+c))/(sqrt(2)*sqrt(c)+sqrt(-I*c*tan(f*x+e)+c))/(a*c^(3/2))/c*f$



[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)), x)

**Mupad [B]**

time = 10.83, size = 308, normalized size = 1.38

$$\frac{\frac{Bc}{3} - \frac{B(c - c \tan(e + fx))}{15} - \frac{B(c - c \tan(e + fx))^2}{16c} + \frac{3B(c - c \tan(e + fx))^3}{16c^2}}{af(c - c \tan(e + fx))^{7/2} - 2acf(c - c \tan(e + fx))^{5/2}} - \frac{\frac{A(c - c \tan(e + fx))^{7i}}{30f} + \frac{A(c - c \tan(e + fx))^{5i}}{5c} + \frac{A(c - c \tan(e + fx))^{3i}}{12acf} - \frac{A(c - c \tan(e + fx))^{i}}{16ac^2}}{2c(c - c \tan(e + fx))^{5/2} - (c - c \tan(e + fx))^{7/2}} - \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + fx)}}{2\sqrt{-c}}\right) \pi}{32a(-c)^{5/2}f} - \frac{3\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + fx)}}{2\sqrt{c}}\right)}{32ac^{5/2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)\*(c - c\*tan(e + f\*x)\*1i)^(5/2)), x)

[Out] ((B\*c)/5 - (B\*(c - c\*tan(e + f\*x)\*1i))/10 - (B\*(c - c\*tan(e + f\*x)\*1i)^2)/(4\*c) + (3\*B\*(c - c\*tan(e + f\*x)\*1i)^3)/(16\*c^2))/(a\*f\*(c - c\*tan(e + f\*x)\*1i)^(7/2) - 2\*a\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^(5/2)) - ((A\*(c - c\*tan(e + f\*x)\*1i)\*7i)/(30\*a\*f) + (A\*c\*1i)/(5\*a\*f) + (A\*(c - c\*tan(e + f\*x)\*1i)^2\*7i)/(12\*a\*c\*f) - (A\*(c - c\*tan(e + f\*x)\*1i)^3\*7i)/(16\*a\*c^2\*f))/(2\*c\*(c - c\*tan(e + f\*x)\*1i)^(5/2) - (c - c\*tan(e + f\*x)\*1i)^(7/2)) - (2^(1/2)\*A\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*7i)/(32\*a\*(-c)^(5/2)\*f) - (3\*2^(1/2)\*B\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2)))/(32\*a\*c^(5/2)\*f)

$$3.771 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{9/2}}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=275

$$\frac{7(5iA - 13B)c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c - ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}a^2f} - \frac{7(5iA - 13B)c^4 \sqrt{c - ictan(e+fx)}}{2a^2f} - \frac{7(5iA - 13B)c^5}{2a^2f}$$

[Out]  $7/2*(5*I*A-13*B)*c^{(9/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})/a^2/f*2^{(1/2)}-7/2*(5*I*A-13*B)*c^4*(c-I*c*\tan(f*x+e))^{(1/2)}/a^2/f-7/12*(5*I*A-13*B)*c^3*(c-I*c*\tan(f*x+e))^{(3/2)}/a^2/f-7/40*(5*I*A-13*B)*c^2*(c-I*c*\tan(f*x+e))^{(5/2)}/a^2/f-1/8*(5*I*A-13*B)*c*(c-I*c*\tan(f*x+e))^{(7/2)}/a^2/f/(1+I*\tan(f*x+e))+1/4*(I*A-B)*(c-I*c*\tan(f*x+e))^{(9/2)}/a^2/f/(1+I*\tan(f*x+e))^2$

**Rubi [A]**

time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 43, 52, 65, 214}

$$\frac{7c^{9/2}(-13B+5iA)\tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}a^2f} - \frac{7c^4(-13B+5iA)\sqrt{c-ictan(e+fx)}}{2a^2f} - \frac{7c^3(-13B+5iA)(c-ictan(e+fx))^{3/2}}{12a^2f} - \frac{7c^2(-13B+5iA)(c-ictan(e+fx))^{5/2}}{40a^2f} - \frac{c(-13B+5iA)(c-ictan(e+fx))^{7/2}}{8a^2f(1+i\tan(e+fx))} + \frac{(-B+iA)(c-ictan(e+fx))^{9/2}}{4a^2f(1+i\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(9/2)}]/(a + I*a*\operatorname{Tan}[e + f*x])^2, x]$

[Out]  $(7*((5*I)*A - 13*B)*c^{(9/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]))/(\operatorname{Sqrt}[2]*a^2*f) - (7*((5*I)*A - 13*B)*c^4*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(2*a^2*f) - (7*((5*I)*A - 13*B)*c^3*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)})/(12*a^2*f) - (7*((5*I)*A - 13*B)*c^2*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(40*a^2*f) - ((5*I)*A - 13*B)*c*(c - I*c*\operatorname{Tan}[e + f*x])^{(7/2)}/(8*a^2*f*(1 + I*\operatorname{Tan}[e + f*x])) + ((I*A - B)*(c - I*c*\operatorname{Tan}[e + f*x])^{(9/2)})/(4*a^2*f*(1 + I*\operatorname{Tan}[e + f*x])^2)$

**Rule 43**

$\operatorname{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{ILtQ}[m, -1] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{GtQ}[n, 0]$

**Rule 52**

$\operatorname{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \operatorname{Dist}[n*((b*c - a*d)/($



```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
negerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 214

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{9/2}}{(a + ia \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{7/2}}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{9/2}}{4a^2 f(1 + i \tan(e + fx))^2} - \frac{((5A + 13iB)c) \text{Subst}}{4a^2 f(1 + i \tan(e + fx))^2} \\
&= -\frac{(5iA - 13B)c(c - ic \tan(e + fx))^{7/2}}{8a^2 f(1 + i \tan(e + fx))} + \frac{(iA - B)(c - ic \tan(e + fx))^{9/2}}{4a^2 f(1 + i \tan(e + fx))^2} \\
&= -\frac{7(5iA - 13B)c^2(c - ic \tan(e + fx))^{5/2}}{40a^2 f} - \frac{(5iA - 13B)c(c - ic \tan(e + fx))^{9/2}}{8a^2 f(1 + i \tan(e + fx))^2} \\
&= -\frac{7(5iA - 13B)c^3(c - ic \tan(e + fx))^{3/2}}{12a^2 f} - \frac{7(5iA - 13B)c^2(c - ic \tan(e + fx))^{5/2}}{40a^2 f} \\
&= -\frac{7(5iA - 13B)c^4 \sqrt{c - ic \tan(e + fx)}}{2a^2 f} - \frac{7(5iA - 13B)c^3(c - ic \tan(e + fx))^{3/2}}{12a^2 f} \\
&= -\frac{7(5iA - 13B)c^4 \sqrt{c - ic \tan(e + fx)}}{2a^2 f} - \frac{7(5iA - 13B)c^2(c - ic \tan(e + fx))^{5/2}}{40a^2 f} \\
&= -\frac{7(5iA - 13B)c^4 \sqrt{c - ic \tan(e + fx)}}{2a^2 f} - \frac{7(5iA - 13B)c^3(c - ic \tan(e + fx))^{3/2}}{12a^2 f} \\
&= \frac{7(5iA - 13B)c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{\sqrt{2} a^2 f} - \frac{7(5iA - 13B)c^3(c - ic \tan(e + fx))^{3/2}}{12a^2 f}
\end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(9/2))/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] \$Aborted

**Maple [A]**

time = 0.36, size = 220, normalized size = 0.80

method	result
--------	--------

derivativedivides	$2ic^2 \left( \frac{iB(c-ic \tan(fx+e))^{\frac{5}{2}}}{5} + \frac{5iBc(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + \frac{Ac(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 18iBc^2 \sqrt{c-ic \tan(fx+e)} + 6 \right)$
default	$2ic^2 \left( \frac{iB(c-ic \tan(fx+e))^{\frac{5}{2}}}{5} + \frac{5iBc(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + \frac{Ac(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 18iBc^2 \sqrt{c-ic \tan(fx+e)} + 6 \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x,method =_RETURNVERBOSE)`

[Out]  $-2*I/f/a^2*c^2*(1/5*I*B*(c-I*c*\tan(f*x+e))^{5/2}+5/3*I*B*c*(c-I*c*\tan(f*x+e))^{3/2}+1/3*A*c*(c-I*c*\tan(f*x+e))^{3/2}+18*I*B*c^2*(c-I*c*\tan(f*x+e))^{1/2}+6*A*c^2*(c-I*c*\tan(f*x+e))^{1/2}-8*c^3*(4*((21/64*I*B+13/64*A)*(c-I*c*\tan(f*x+e))^{3/2}+(-19/32*I*B*c-11/32*A*c)*(c-I*c*\tan(f*x+e))^{1/2}))/((c+I*c*\tan(f*x+e))^2+7/8*(13/4*I*B+5/4*A)*2^{1/2}/c^{1/2}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))$

**Maxima [A]**

time = 0.50, size = 253, normalized size = 0.92

$$i \left( \frac{105 \sqrt{2} (5A+13iB) e^{11} \log\left(\frac{-\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}\right)}{60cf} - \frac{60((-i \tan(fx+e)+c)^{\frac{3}{2}}(13A+21iB)c^6-2\sqrt{-ic \tan(fx+e)+c}(11A+19iB)c^7)}{(-i \tan(fx+e)+c)^{\frac{3}{2}}c^2-4(-i \tan(fx+e)+c)^2c+4c^2} + \frac{8(3(-i \tan(fx+e)+c)^{\frac{3}{2}}B^2+5(-i \tan(fx+e)+c)^{\frac{3}{2}}(A+5iB)c^4+90\sqrt{-ic \tan(fx+e)+c}(A+3iB)c^5)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x,algorithm="maxima")`

[Out]  $-1/60*I*(105*\sqrt{2}*(5*A + 13*I*B)*c^{11/2}*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c}))/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c}))/a^2 - 60*((-I*c*\tan(f*x + e) + c)^{3/2}*(13*A + 21*I*B)*c^6 - 2*\sqrt{-I*c*\tan(f*x + e) + c}*(11*A + 19*I*B)*c^7)/((-I*c*\tan(f*x + e) + c)^2*a^2 - 4*(-I*c*\tan(f*x + e) + c)*a^2*c + 4*a^2*c^2) + 8*(3*I*(-I*c*\tan(f*x + e) + c)^{5/2})*B*c^3 + 5*(-I*c*\tan(f*x + e) + c)^{3/2}*(A + 5*I*B)*c^4 + 90*\sqrt{-I*c*\tan(f*x + e) + c}*(A + 3*I*B)*c^5/a^2)/(c*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 533 vs.  $2(224) = 448$ .

time = 3.77, size = 533, normalized size = 1.94

$$i \left( \frac{105 \sqrt{2} (5A+13iB) e^{11} \log\left(\frac{-\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}\right)}{60cf} - \frac{60((-i \tan(fx+e)+c)^{\frac{3}{2}}(13A+21iB)c^6-2\sqrt{-ic \tan(fx+e)+c}(11A+19iB)c^7)}{(-i \tan(fx+e)+c)^{\frac{3}{2}}c^2-4(-i \tan(fx+e)+c)^2c+4c^2} + \frac{8(3(-i \tan(fx+e)+c)^{\frac{3}{2}}B^2+5(-i \tan(fx+e)+c)^{\frac{3}{2}}(A+5iB)c^4+90\sqrt{-ic \tan(fx+e)+c}(A+3iB)c^5)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x,
algorithm="fricas")
```

```
[Out] -1/60*(105*sqrt(2)*sqrt(-(25*A^2 + 130*I*A*B - 169*B^2)*c^9/(a^4*f^2))*(a^2
*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4
*I*e))*log(-14*((-5*I*A + 13*B)*c^5 + sqrt(-(25*A^2 + 130*I*A*B - 169*B^2)*
c^9/(a^4*f^2))*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I
*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f)) - 105*sqrt(2)*sqrt(-(25*A^2 + 130*I*A*
B - 169*B^2)*c^9/(a^4*f^2))*(a^2*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6*I*f*x
+ 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))*log(-14*((-5*I*A + 13*B)*c^5 - sqrt(
-(25*A^2 + 130*I*A*B - 169*B^2)*c^9/(a^4*f^2))*(a^2*f*e^(2*I*f*x + 2*I*e) +
a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f)) + 2*sq
rt(2)*(105*(5*I*A - 13*B)*c^4*e^(8*I*f*x + 8*I*e) + 245*(5*I*A - 13*B)*c^4*
e^(6*I*f*x + 6*I*e) + 161*(5*I*A - 13*B)*c^4*e^(4*I*f*x + 4*I*e) + 15*(5*I*
A - 13*B)*c^4*e^(2*I*f*x + 2*I*e) + 30*(-I*A + B)*c^4)*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1)))/(a^2*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6*I*f*x + 6*I*e) + a
^2*f*e^(4*I*f*x + 4*I*e))
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,
x)
```

```
[Out] Timed out
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(9/2)/(I*a*tan(f*x +
e) + a)^2, x)
```

**Mupad** [B]

time = 9.56, size = 402, normalized size = 1.46

$\frac{30B^2\sqrt{c-\tan(fx+e)}-21B^2(c-\tan(fx+e))^{3/2}}{4a^2f\sqrt{c-\tan(fx+e)}}-\frac{A^2\sqrt{c-\tan(fx+e)}}{af}+\frac{A^2c\sqrt{c-\tan(fx+e)}}{af^2}-\frac{A^2c^2\sqrt{c-\tan(fx+e)}}{af^3}+\frac{A^2c^3\sqrt{c-\tan(fx+e)}}{af^4}-\frac{A^2c^4\sqrt{c-\tan(fx+e)}}{af^5}+\frac{A^2c^5\sqrt{c-\tan(fx+e)}}{af^6}-\frac{A^2c^6\sqrt{c-\tan(fx+e)}}{af^7}+\frac{A^2c^7\sqrt{c-\tan(fx+e)}}{af^8}-\frac{A^2c^8\sqrt{c-\tan(fx+e)}}{af^9}+\frac{A^2c^9\sqrt{c-\tan(fx+e)}}{af^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B*\tan(e + f*x))*(c - c*\tan(e + f*x)*1i)^{(9/2)})/(a + a*\tan(e + f*x)*1i)^2, x)$

[Out]  $(38*B*c^6*(c - c*\tan(e + f*x)*1i)^{(1/2)} - 21*B*c^5*(c - c*\tan(e + f*x)*1i)^{(3/2)})/(4*a^2*c^2*f + a^2*f*(c - c*\tan(e + f*x)*1i)^2 - 4*a^2*c*f*(c - c*\tan(e + f*x)*1i)) - ((A*c^6*(c - c*\tan(e + f*x)*1i)^{(1/2)*22i}/(a^2*f) - (A*c^5*(c - c*\tan(e + f*x)*1i)^{(3/2)*13i}/(a^2*f)))/((c - c*\tan(e + f*x)*1i)^2 - 4*c*(c - c*\tan(e + f*x)*1i) + 4*c^2) - (A*c^4*(c - c*\tan(e + f*x)*1i)^{(1/2)*12i}/(a^2*f) - (A*c^3*(c - c*\tan(e + f*x)*1i)^{(3/2)*2i}/(3*a^2*f) + (36*B*c^4*(c - c*\tan(e + f*x)*1i)^{(1/2)}/(a^2*f) + (10*B*c^3*(c - c*\tan(e + f*x)*1i)^{(3/2)})/(3*a^2*f) + (2*B*c^2*(c - c*\tan(e + f*x)*1i)^{(5/2)})/(5*a^2*f) + (2^{(1/2)}*A*(-c)^{(9/2)}*atan((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)}))/(2*(-c)^{(1/2)}))*35i)/(2*a^2*f) + (2^{(1/2)}*B*c^{(9/2)}*atan((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)}*1i)/(2*c^{(1/2)}))*91i)/(2*a^2*f)$

$$3.772 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=238

$$\frac{5(3iA - 11B)c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c - ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2} a^2 f} - \frac{5(3iA - 11B)c^3 \sqrt{c - ictan(e+fx)}}{4a^2 f} - \frac{5(3iA - 11B)c^5}{4a^2 f}$$

[Out]  $5/4*(3*I*A-11*B)*c^{(7/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})/a^2/f*2^{(1/2)}-5/4*(3*I*A-11*B)*c^3*(c-I*c*\tan(f*x+e))^{(1/2)}/a^2/f-5/24*(3*I*A-11*B)*c^2*(c-I*c*\tan(f*x+e))^{(3/2)}/a^2/f-1/8*(3*I*A-11*B)*c*(c-I*c*\tan(f*x+e))^{(5/2)}/a^2/f/(1+I*\tan(f*x+e))+1/4*(I*A-B)*(c-I*c*\tan(f*x+e))^{(7/2)}/a^2/f/(1+I*\tan(f*x+e))^2$

**Rubi [A]**

time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 43, 52, 65, 214}

$$\frac{5c^{7/2}(-11B+3iA)\tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}a^2f} - \frac{5c^3(-11B+3iA)\sqrt{c-ictan(e+fx)}}{4a^2f} - \frac{5c^5(-11B+3iA)(c-ictan(e+fx))^{3/2}}{24a^2f} - \frac{c(-11B+3iA)(c-ictan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))} + \frac{(-B+iA)(c-ictan(e+fx))^{7/2}}{4a^2f(1+i\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[e+f*x])*(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)}]/(a+I*a*\operatorname{Tan}[e+f*x])^2, x]$

[Out]  $(5*((3*I)*A - 11*B)*c^{(7/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]))/(2*\operatorname{Sqrt}[2]*a^2*f) - (5*((3*I)*A - 11*B)*c^3*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(4*a^2*f) - (5*((3*I)*A - 11*B)*c^2*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)})/(24*a^2*f) - (((3*I)*A - 11*B)*c*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(8*a^2*f*(1 + I*\operatorname{Tan}[e + f*x])) + ((I*A - B)*(c - I*c*\operatorname{Tan}[e + f*x])^{(7/2)})/(4*a^2*f*(1 + I*\operatorname{Tan}[e + f*x])^2)$

**Rule 43**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x]$   
 $\&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{ILtQ}[m, -1] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{GtQ}[n, 0]$

**Rule 52**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \operatorname{Dist}[n*(b*c - a*d)/(b*(m+n+1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x]$   
 $\&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m+n+1, 0] \&\& \operatorname{IntegerQ}[n]$

$[m, 0] \&\& ( !IntegerQ[n] \parallel (GtQ[m, 0] \&\& LtQ[m - n, 0])) \&\& !ILtQ[m + n + 2, 0] \&\& IntLinearQ[a, b, c, d, m, n, x]$

### Rule 65

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] \rightarrow With[\{p = Denominator[m]\}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[\{a, b, c, d\}, x] \&\& NeQ[b*c - a*d, 0] \&\& LtQ[-1, m, 0] \&\& LeQ[-1, n, 0] \&\& LeQ[Denominator[n], Denominator[m]] \&\& IntLinearQ[a, b, c, d, m, n, x]$

### Rule 79

$Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x\_Symbol] \rightarrow Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[\{a, b, c, d, e, f, n\}, x] \&\& LtQ[p, -1] \&\& ( !LtQ[n, -1] \parallel IntegerQ[p] \parallel !(IntegerQ[n] \parallel !(EqQ[e, 0] \parallel !(EqQ[c, 0] \parallel LtQ[p, n])))$

### Rule 214

$Int[((a_) + (b_)*(x_)^2)^(-1), x\_Symbol] \rightarrow Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[\{a, b\}, x] \&\& NegQ[a/b]$

### Rule 3669

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] \rightarrow Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& EqQ[b*c + a*d, 0] \&\& EqQ[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{7/2}}{(a + ia \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{7/2}}{4a^2 f (1 + i \tan(e + fx))^2} - \frac{((3A + 11iB)c) \text{Subst}}{4a^2 f (1 + i \tan(e + fx))^2} \\
&= -\frac{(3iA - 11B)c(c - ic \tan(e + fx))^{5/2}}{8a^2 f (1 + i \tan(e + fx))} + \frac{(iA - B)(c - ic \tan(e + fx))^{7/2}}{4a^2 f (1 + i \tan(e + fx))^2} \\
&= -\frac{5(3iA - 11B)c^2(c - ic \tan(e + fx))^{3/2}}{24a^2 f} - \frac{(3iA - 11B)c(c - ic \tan(e + fx))^{5/2}}{8a^2 f (1 + i \tan(e + fx))} \\
&= -\frac{5(3iA - 11B)c^3 \sqrt{c - ic \tan(e + fx)}}{4a^2 f} - \frac{5(3iA - 11B)c^2(c - ic \tan(e + fx))^{3/2}}{8a^2 f (1 + i \tan(e + fx))} \\
&= -\frac{5(3iA - 11B)c^3 \sqrt{c - ic \tan(e + fx)}}{4a^2 f} - \frac{5(3iA - 11B)c^2(c - ic \tan(e + fx))^{3/2}}{8a^2 f (1 + i \tan(e + fx))} \\
&= \frac{5(3iA - 11B)c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{2\sqrt{2} a^2 f} - \frac{5(3iA - 11B)c^2(c - ic \tan(e + fx))^{3/2}}{8a^2 f (1 + i \tan(e + fx))}
\end{aligned}$$

**Mathematica** [F]

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] \$Aborted

**Maple** [A]

time = 0.36, size = 178, normalized size = 0.75

method	result
--------	--------



derivativedivides	$2ic^2 \left( \frac{iB(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 5iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - 2c^2 \left( \frac{4(\frac{17iB}{32} + \frac{9}{3}}{32} \right) \right)$
default	$2ic^2 \left( \frac{iB(c-ic \tan(fx+e))^{\frac{3}{2}}}{3} + 5iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - 2c^2 \left( \frac{4(\frac{17iB}{32} + \frac{9}{3}}{32} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x,method =_RETURNVERBOSE)`

[Out] 
$$-2*I/f/a^2*c^2*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)+5*I*B*c*(c-I*c*tan(f*x+e))^(1/2)+A*c*(c-I*c*tan(f*x+e))^(1/2)-2*c^2*(4*((17/32*I*B+9/32*A))*(c-I*c*tan(f*x+e))^(3/2)+(-15/16*I*B*c-7/16*A*c)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+5/4*(11/4*I*B+3/4*A)*2^(1/2)/c^(1/2)*\operatorname{arctanh}(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))$$

**Maxima** [A]

time = 0.52, size = 229, normalized size = 0.96

$$\frac{i \left( \frac{15 \sqrt{2} (3A+11iB)c^{\frac{5}{2}} \log \left( \frac{-\sqrt{2} \sqrt{c} - \sqrt{-ic \tan(fx+e) + c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx+e) + c}} \right) - 12 \left( (-ic \tan(fx+e) + c)^{\frac{3}{2}} (9A+17iB)c^{\frac{5}{2}} - 2 \sqrt{-ic \tan(fx+e) + c} (7A+15iB)c^{\frac{5}{2}} \right)}{(-ic \tan(fx+e) + c)^{\frac{3}{2}} a^2 - 4(-ic \tan(fx+e) + c) a^2 c + 4a^2 c^2} + \frac{16 \left( i(-ic \tan(fx+e) + c)^{\frac{3}{2}} Bc^2 + 3 \sqrt{-ic \tan(fx+e) + c} (A+5iB)c^{\frac{5}{2}} \right)}{a^2} \right)}{24cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")`

[Out] 
$$-1/24*I*(15*\sqrt{2}*(3*A + 11*I*B)*c^(9/2)*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c})/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c}))/a^2 - 12*((-I*c*\tan(f*x + e) + c)^(3/2)*(9*A + 17*I*B)*c^5 - 2*\sqrt{-I*c*\tan(f*x + e) + c}*(7*A + 15*I*B)*c^6)/((-I*c*\tan(f*x + e) + c)^2*a^2 - 4*(-I*c*\tan(f*x + e) + c)*a^2*c + 4*a^2*c^2) + 16*(I*(-I*c*\tan(f*x + e) + c)^(3/2)*B*c^3 + 3*\sqrt{-I*c*\tan(f*x + e) + c}*(A + 5*I*B)*c^4)/a^2/(c*f)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs.  $2(192) = 384$ .

time = 8.18, size = 474, normalized size = 1.99

$$\frac{15 \sqrt{2} \left( (3A+11iB)c^{\frac{5}{2}} \log \left( \frac{-\sqrt{2} \sqrt{c} - \sqrt{-ic \tan(fx+e) + c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx+e) + c}} \right) - 12 \left( (-ic \tan(fx+e) + c)^{\frac{3}{2}} (9A+17iB)c^{\frac{5}{2}} - 2 \sqrt{-ic \tan(fx+e) + c} (7A+15iB)c^{\frac{5}{2}} \right) \right)}{24cf} + \frac{16 \left( i(-ic \tan(fx+e) + c)^{\frac{3}{2}} Bc^2 + 3 \sqrt{-ic \tan(fx+e) + c} (A+5iB)c^{\frac{5}{2}} \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2)/(a+I\*a\*tan(f\*x+e))^2,x,  
algorithm="fricas")

[Out] 
$$-1/12*(15*\sqrt{1/2}*(a^2*f*e^{(6*I*f*x + 6*I*e)} + a^2*f*e^{(4*I*f*x + 4*I*e)})$$

$$*\sqrt{-(9*A^2 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2)}*\log(-5*((-3*I*A + 11*B)*$$

$$c^4 + \sqrt{2}*\sqrt{1/2}*(a^2*f*e^{(2*I*f*x + 2*I*e)} + a^2*f)*\sqrt{-(9*A^2 +$$

$$66*I*A*B - 121*B^2)*c^7/(a^4*f^2)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}))e^{(-I$$

$$*f*x - I*e)/(a^2*f)} - 15*\sqrt{1/2}*(a^2*f*e^{(6*I*f*x + 6*I*e)} + a^2*f*e^{(4$$

$$*I*f*x + 4*I*e))*\sqrt{-(9*A^2 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2)}*\log(-5*($$

$$(-3*I*A + 11*B)*c^4 - \sqrt{2}*\sqrt{1/2}*(a^2*f*e^{(2*I*f*x + 2*I*e)} + a^2*f)$$

$$*\sqrt{-(9*A^2 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2)}*\sqrt{c/(e^{(2*I*f*x + 2*I$$

$$*e)} + 1)}))e^{(-I*f*x - I*e)/(a^2*f)} + \sqrt{2}*(15*(3*I*A - 11*B)*c^3*e^{(6*$$

$$I*f*x + 6*I*e)} + 20*(3*I*A - 11*B)*c^3*e^{(4*I*f*x + 4*I*e)} + 3*(3*I*A - 11*$$

$$B)*c^3*e^{(2*I*f*x + 2*I*e)} + 6*(-I*A + B)*c^3)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)}$$

$$+ 1)}))/(a^2*f*e^{(6*I*f*x + 6*I*e)} + a^2*f*e^{(4*I*f*x + 4*I*e)})$$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2)/(a+I\*a\*tan(f\*x+e))^2,  
x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2)/(a+I\*a\*tan(f\*x+e))^2,x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(7/2)/(I\*a\*tan(f\*x +  
e) + a)^2, x)

**Mupad [B]**

time = 9.39, size = 349, normalized size = 1.47

$$\frac{15B^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)+1}}{4a^2e^2f+a^2f(c-\tan(e+fx))^2-4a^2cf(c-\tan(e+fx))} - \frac{15B^2\sqrt{c-\tan(e+fx)+1}}{(c-\tan(e+fx))^2-4c(c-\tan(e+fx))+4c^2} - \frac{A^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)+1}}{a^2f} - \frac{A^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)+1}}{a^2f} - \frac{10B^2\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)+1}}{3a^2f} - \frac{\sqrt{2}A(-e)^{7/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)+1}}{\sqrt{c-\tan(e+fx)+1}}\right)}{4a^2f} - \frac{\sqrt{2}B^2e^{7/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\sqrt{c-\tan(e+fx)+1}}{\sqrt{c-\tan(e+fx)+1}}\right)}{4a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(7/2))/(a + a\*tan(e + f\*x)\*1i)^2,x)

[Out] (15\*B\*c^5\*(c - c\*tan(e + f\*x)\*1i)^(1/2) - (17\*B\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/2)/(4\*a^2\*c^2\*f + a^2\*f\*(c - c\*tan(e + f\*x)\*1i)^2 - 4\*a^2\*c\*f\*(c - c\*tan(e + f\*x)\*1i)) - ((A\*c^5\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*7i)/(a^2\*f) - (A\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(3/2)\*9i)/(2\*a^2\*f))/((c - c\*tan(e + f\*x)\*1i)^2 - 4\*c\*(c - c\*tan(e + f\*x)\*1i) + 4\*c^2) - (A\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*2i)/(a^2\*f) + (10\*B\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a^2\*f) + (2\*B\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/(3\*a^2\*f) - (2^(1/2)\*A\*(-c)^(7/2)\*atan(2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*15i)/(4\*a^2\*f) + (2^(1/2)\*B\*c^(7/2)\*atan(2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*1i)/(2\*c^(1/2)))\*55i)/(4\*a^2\*f)

$$3.773 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=199

$$\frac{3(iA - 9B)c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{4\sqrt{2} a^2 f} - \frac{3(iA - 9B)c^2 \sqrt{c - ict \tan(e + fx)}}{8a^2 f} - \frac{(iA - 9B)c(c - ict \tan(e + fx))^{5/2}}{8a^2 f(1 + i \tan(e + fx))}$$

[Out]  $3/8*(I*A-9*B)*c^{(5/2)*\arctanh(1/2*(c-I*c*\tan(f*x+e))^{(1/2)*2^{(1/2)}/c^{(1/2)})}/a^2/f*2^{(1/2)}-3/8*(I*A-9*B)*c^2*(c-I*c*\tan(f*x+e))^{(1/2)}/a^2/f-1/8*(I*A-9*B)*c*(c-I*c*\tan(f*x+e))^{(3/2)}/a^2/f/(1+I*\tan(f*x+e))+1/4*(I*A-B)*(c-I*c*\tan(f*x+e))^{(5/2)}/a^2/f/(1+I*\tan(f*x+e))^2$

**Rubi [A]**

time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 43, 52, 65, 214}

$$\frac{3c^{5/2}(-9B + iA) \tanh^{-1}\left(\frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{4\sqrt{2} a^2 f} - \frac{3c^2(-9B + iA) \sqrt{c - ict \tan(e + fx)}}{8a^2 f} - \frac{c(-9B + iA)(c - ict \tan(e + fx))^{3/2}}{8a^2 f(1 + i \tan(e + fx))} + \frac{(-B + iA)(c - ict \tan(e + fx))^{5/2}}{4a^2 f(1 + i \tan(e + fx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}]/(a + I*a*\text{Tan}[e + f*x])^2, x]$

[Out]  $(3*(I*A - 9*B)*c^{(5/2)*\text{ArcTanh}[\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]]/(\text{Sqrt}[2]*\text{Sqrt}[c])})/(4*\text{Sqrt}[2]*a^2*f) - (3*(I*A - 9*B)*c^2*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(8*a^2*f) - ((I*A - 9*B)*c*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(8*a^2*f*(1 + I*\text{Tan}[e + f*x])) + ((I*A - B)*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(4*a^2*f*(1 + I*\text{Tan}[e + f*x])^2)$

**Rule 43**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \text{Dist}[d*(n/(b*(m + 1))), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, -1] \&\& !\text{IntegerQ}[n] \&\& \text{GtQ}[n, 0]$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m + n + 1))), \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[m + n + 1, 0] \&\& !( \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{GtQ}[m, 0] \&\& \text{LtQ}[m - n, 0])) ) \&\& !\text{ILtQ}[m + n$

+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[  
 {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) +  
 d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ  
 [b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den  
 ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p  
 \_.), x\_Symbol] := Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1)/  
 (f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c  
 \*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x]  
 , x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I  
 ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))  
 ))

### Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x  
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) +  
 (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Di  
 st[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x,  
 Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c +  
 a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{5/2}}{(a + ia \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{f} \\
 &= \frac{(iA - B)(c - ic \tan(e + fx))^{5/2}}{4a^2 f(1 + i \tan(e + fx))^2} - \frac{((A + 9iB)c) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^3} dx, x, \tan(e + fx)\right)}{4a^2 f(1 + i \tan(e + fx))^2} \\
 &= -\frac{(iA - 9B)c(c - ic \tan(e + fx))^{3/2}}{8a^2 f(1 + i \tan(e + fx))} + \frac{(iA - B)(c - ic \tan(e + fx))^{5/2}}{4a^2 f(1 + i \tan(e + fx))^2} \\
 &= -\frac{3(iA - 9B)c^2 \sqrt{c - ic \tan(e + fx)}}{8a^2 f} - \frac{(iA - 9B)c(c - ic \tan(e + fx))^{3/2}}{8a^2 f(1 + i \tan(e + fx))} \\
 &= -\frac{3(iA - 9B)c^2 \sqrt{c - ic \tan(e + fx)}}{8a^2 f} - \frac{(iA - 9B)c(c - ic \tan(e + fx))^{3/2}}{8a^2 f(1 + i \tan(e + fx))} \\
 &= \frac{3(iA - 9B)c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{4\sqrt{2} a^2 f} - \frac{3(iA - 9B)c^2 \sqrt{c - ic \tan(e + fx)}}{8a^2 f}
 \end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2))/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] \$Aborted

**Maple [A]**

time = 0.35, size = 140, normalized size = 0.70

method	result
derivativedivides	$  \frac{2ic^2 \left( iB \sqrt{c - ic \tan(fx + e)} - c \left( \frac{4 \left( \frac{13iB}{32} + \frac{5A}{32} \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 4 \left( -\frac{11}{16} iBc - \frac{3}{16} Ac \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^2} \right)}{f a^2}  $

default	$2ic^2 \left( iB \sqrt{c - ic \tan(fx + e)} - c \left( \frac{4 \left( \frac{13iB}{32} + \frac{5A}{32} \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 4 \left( -\frac{11}{16} iBc - \frac{3}{16} Ac \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^2} \right) \right) / fa^2$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,method =_RETURNVERBOSE)`

[Out]  $-2*I/f/a^2*c^2*(I*B*(c-I*c*\tan(f*x+e))^{(1/2)}-c*(4*((13/32*I*B+5/32*A)*(c-I*c*\tan(f*x+e))^{(3/2)}+(-11/16*I*B*c-3/16*A*c)*(c-I*c*\tan(f*x+e))^{(1/2)}))/(c+I*c*\tan(f*x+e))^2+3/4*(9/4*I*B+1/4*A)*2^{(1/2)}/c^{(1/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})$

**Maxima [A]**

time = 0.50, size = 200, normalized size = 1.01

$$i \left( \frac{3\sqrt{2}^{(A+9iB)c^{\frac{3}{2}} \log\left(\frac{-\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right)} + \frac{32i\sqrt{-ic \tan(fx+e)+c} Bc^3}{a^2} - \frac{4\left((-ic \tan(fx+e)+c\right)^{\frac{3}{2}}(5A+13iB)c^4-2\sqrt{-ic \tan(fx+e)+c}(3A+11iB)c^5)}{(-ic \tan(fx+e)+c)^2 a^2-4(-ic \tan(fx+e)+c)a^2 c+4a^2 c^2}}{16cf} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,algorithm="maxima")`

[Out]  $-1/16*I*(3*\sqrt{2}*(A+9*I*B)*c^{(7/2)}*\log(-(\sqrt{2}*\sqrt{c}-\sqrt{-I*c*\tan(f*x+e)+c})/(\sqrt{2}*\sqrt{c}+\sqrt{-I*c*\tan(f*x+e)+c}))/a^2+32*I*\sqrt{-I*c*\tan(f*x+e)+c}*B*c^3/a^2-4*((-I*c*\tan(f*x+e)+c)^{(3/2)}*(5*A+13*I*B)*c^4-2*\sqrt{-I*c*\tan(f*x+e)+c}*(3*A+11*I*B)*c^5)/((-I*c*\tan(f*x+e)+c)^2*a^2-4*(-I*c*\tan(f*x+e)+c)*a^2*c+4*a^2*c^2)/(c*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs.  $2(160) = 320$ .

time = 3.74, size = 393, normalized size = 1.97

$$i \sqrt{\frac{1}{2}} \sqrt{\frac{(A^2+18iAB-81B^2)c^2}{a^2 f}} \sqrt{\frac{(c+ic \tan(fx+e))^{\frac{3}{2}} \log\left(\frac{(c+ic \tan(fx+e))^{\frac{3}{2}} \sqrt{\frac{(A^2+18iAB-81B^2)c^2}{20i^2 B^2 c^2+1}}}{(c+ic \tan(fx+e))^{\frac{3}{2}} \sqrt{\frac{(A^2+18iAB-81B^2)c^2}{20i^2 B^2 c^2+1}}}\right)}{1}} - i \sqrt{\frac{1}{2}} \sqrt{\frac{(A^2+18iAB-81B^2)c^2}{a^2 f}} \sqrt{\frac{(c+ic \tan(fx+e))^{\frac{3}{2}} \log\left(\frac{(c+ic \tan(fx+e))^{\frac{3}{2}} \sqrt{\frac{(A^2+18iAB-81B^2)c^2}{20i^2 B^2 c^2+1}}}{(c+ic \tan(fx+e))^{\frac{3}{2}} \sqrt{\frac{(A^2+18iAB-81B^2)c^2}{20i^2 B^2 c^2+1}}}\right)}{1}} + \sqrt{2} (3A-9B) c^2 \sqrt{c+ic \tan(fx+e)} - (-1A+9B) c^2 \sqrt{c-ic \tan(fx+e)} + 2(-1A+9B) c \sqrt{\frac{c}{20i^2 B^2 c^2+1}} \sqrt{c+ic \tan(fx+e)} - 2(-1A+9B) c \sqrt{\frac{c}{20i^2 B^2 c^2+1}} \sqrt{c-ic \tan(fx+e)}) / (c^2 f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,algorithm="fricas")`

[Out]  $-1/8*(3*\sqrt{1/2})*a^2*f*\sqrt{-(A^2+18*I*A*B-81*B^2)*c^5/(a^4*f^2)}*e^{(4*I*f*x+4*I*e)}*\log(-3/2*((-I*A+9*B)*c^3+\sqrt{2}*\sqrt{1/2}*(a^2*f*e^{(2*I*f*x+2*I*e)}))$

$I*f*x + 2*I*e) + a^2*f)*\sqrt{-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1))} * e^{(-I*f*x - I*e)/(a^2*f)} - 3*\sqrt{1/2}*a^2*f*\sqrt{-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2)} * e^{(4*I*f*x + 4*I*e)} * \log(-3/2*((-I*A + 9*B)*c^3 - \sqrt{2}*\sqrt{1/2}*(a^2*f*e^{(2*I*f*x + 2*I*e)} + a^2*f)*\sqrt{-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2)})*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1))} * e^{(-I*f*x - I*e)/(a^2*f)} + \sqrt{2}*(3*(I*A - 9*B)*c^2*e^{(4*I*f*x + 4*I*e)} - (-I*A + 9*B)*c^2*e^{(2*I*f*x + 2*I*e)} + 2*(-I*A + B)*c^2)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1))} * e^{(-4*I*f*x - 4*I*e)/(a^2*f)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A^2 \sqrt{-i c \tan(e + f x) + c}}{\tan^2(e + f x) - 2 i \tan(e + f x) - 1} dx + \int \frac{A^2 \sqrt{-i c \tan(e + f x) + c} \cos(f x)}{\tan^2(e + f x) - 2 i \tan(e + f x) - 1} dx + \int \frac{B^2 \sqrt{-i c \tan(e + f x) + c} \sin(f x)}{\tan^2(e + f x) - 2 i \tan(e + f x) - 1} dx + \int \frac{B^2 \sqrt{-i c \tan(e + f x) + c} \cos(f x)}{\tan^2(e + f x) - 2 i \tan(e + f x) - 1} dx + \int \frac{-3 A^2 \sqrt{-i c \tan(e + f x) + c} \sin(f x)}{\tan^2(e + f x) - 2 i \tan(e + f x) - 1} dx + \int \frac{-3 B^2 \sqrt{-i c \tan(e + f x) + c} \cos(f x)}{\tan^2(e + f x) - 2 i \tan(e + f x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^2, x)

[Out] -(Integral(A\*c\*\*2\*sqrt(-I\*c\*tan(e + f\*x) + c)/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x) + Integral(-A\*c\*\*2\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x) + Integral(B\*c\*\*2\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x) + Integral(-B\*c\*\*2\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*3/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x) + Integral(-2\*I\*A\*c\*\*2\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x) + Integral(-2\*I\*B\*c\*\*2\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2/(tan(e + f\*x)\*\*2 - 2\*I\*tan(e + f\*x) - 1), x))/a\*\*2

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^2, x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)/(I\*a\*tan(f\*x + e) + a)^2, x)

**Mupad [B]**

time = 9.36, size = 294, normalized size = 1.48

$$\frac{11 B^2 \sqrt{c - c \tan(e + f x)} \operatorname{li} \left( \frac{11 B^2 (c - c \tan(e + f x))^{3/2}}{4 a^2 f^2 (c - c \tan(e + f x)) \operatorname{li}^2 - 4 a^2 c f (c - c \tan(e + f x)) \operatorname{li} - \frac{A^2 \sqrt{c - c \tan(e + f x)} \operatorname{li} \operatorname{li} - \frac{A^2 (c - c \tan(e + f x))^{3/2} \operatorname{li}}{4 a^2 f} \right)}{4 a^2 c^2 f + a^2 f (c - c \tan(e + f x)) \operatorname{li}^2 - 4 a^2 c f (c - c \tan(e + f x)) \operatorname{li} - \frac{A^2 \sqrt{c - c \tan(e + f x)} \operatorname{li} \operatorname{li} - \frac{A^2 (c - c \tan(e + f x))^{3/2} \operatorname{li}}{4 a^2 f} \right)} + \frac{2 B^2 \sqrt{c - c \tan(e + f x)} \operatorname{li}}{a^2 f} + \frac{\sqrt{2} A (-c)^{5/2} \operatorname{atan} \left( \frac{\sqrt{2} \sqrt{c - c \tan(e + f x)} \operatorname{li}}{2 \sqrt{-c}} \right)}{8 a^2 f} - \frac{27 \sqrt{2} B^2 \operatorname{atanh} \left( \frac{\sqrt{2} \sqrt{c - c \tan(e + f x)} \operatorname{li}}{2 \sqrt{c}} \right)}{8 a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}(((A + B*\tan(e + f*x))*(c - c*\tan(e + f*x)*1i)^{(5/2)})/(a + a*\tan(e + f*x)*1i)^2, x)$

[Out]  $((11*B*c^4*(c - c*\tan(e + f*x)*1i)^{(1/2)})/2 - (13*B*c^3*(c - c*\tan(e + f*x)*1i)^{(3/2)})/4)/(4*a^2*c^2*f + a^2*f*(c - c*\tan(e + f*x)*1i)^2 - 4*a^2*c*f*(c - c*\tan(e + f*x)*1i)) - ((A*c^4*(c - c*\tan(e + f*x)*1i)^{(1/2)*3i}/(2*a^2*f) - (A*c^3*(c - c*\tan(e + f*x)*1i)^{(3/2)*5i}/(4*a^2*f)))/((c - c*\tan(e + f*x)*1i)^2 - 4*c*(c - c*\tan(e + f*x)*1i) + 4*c^2) + (2*B*c^2*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(a^2*f) + (2^{(1/2)}*A*(-c)^{(5/2)}*\text{atan}((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(2*(-c)^{(1/2)})))*3i)/(8*a^2*f) - (27*2^{(1/2)}*B*c^{(5/2)}*\text{atan}(\text{h}((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(2*c^{(1/2)}))))/(8*a^2*f)$

$$3.774 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=160

$$-\frac{(iA+7B)c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}a^2f} + \frac{(iA+7B)c\sqrt{c-ictan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(iA-B)(c-ictan(e+fx))^{3/2}}{4a^2f(1+i\tan(e+fx))^2}$$

[Out]  $-1/16*(I*A+7*B)*c^{(3/2)*\arctanh(1/2*(c-I*c*\tan(f*x+e))^{(1/2)*2^{(1/2)}/c^{(1/2)}})/a^2/f*2^{(1/2)}+1/8*(I*A+7*B)*c*(c-I*c*\tan(f*x+e))^{(1/2)}/a^2/f/(1+I*\tan(f*x+e))+1/4*(I*A-B)*(c-I*c*\tan(f*x+e))^{(3/2)}/a^2/f/(1+I*\tan(f*x+e))^2$

**Rubi [A]**

time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 43, 65, 214}

$$-\frac{c^{3/2}(7B+iA) \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}a^2f} + \frac{c(7B+iA)\sqrt{c-ictan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(-B+iA)(c-ictan(e+fx))^{3/2}}{4a^2f(1+i\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^{(3/2)}]/(a+I*a*\text{Tan}[e+f*x])^2, x]$

[Out]  $-1/8*((I*A+7*B)*c^{(3/2)*\text{ArcTanh}[\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]]/(\text{Sqrt}[2]*\text{Sqrt}[c])]/(\text{Sqrt}[2]*a^2*f) + ((I*A+7*B)*c*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]/(8*a^2*f*(1+I*\text{Tan}[e+f*x])) + ((I*A-B)*(c-I*c*\text{Tan}[e+f*x])^{(3/2)})/(4*a^2*f*(1+I*\text{Tan}[e+f*x])^2)$

**Rule 43**

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{Simp}[(a + b*x)^{(m+1)}*(c + d*x)^n/(b*(m+1)), x] - \text{Dist}[d*(n/(b*(m+1))), \text{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, -1] \&\& !\text{IntegerQ}[n] \&\& \text{GtQ}[n, 0]$

**Rule 65**

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 79**

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))

```

### Rule 214

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

### Rule 3669

```

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i \tan(e + fx))^{3/2}}{(a + ia \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^3} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - i \tan(e + fx))^{3/2}}{4a^2 f (1 + i \tan(e + fx))^2} + \frac{((A - 7iB)c) \text{Subst} \left( \int \frac{1}{(a+iax)^3} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA + 7B)c \sqrt{c - i \tan(e + fx)}}{8a^2 f (1 + i \tan(e + fx))} + \frac{(iA - B)(c - i \tan(e + fx))^{3/2}}{4a^2 f (1 + i \tan(e + fx))^2} \\
&= \frac{(iA + 7B)c \sqrt{c - i \tan(e + fx)}}{8a^2 f (1 + i \tan(e + fx))} + \frac{(iA - B)(c - i \tan(e + fx))^{3/2}}{4a^2 f (1 + i \tan(e + fx))^2} \\
&= -\frac{(iA + 7B)c^{3/2} \tanh^{-1} \left( \frac{\sqrt{c - i \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{8\sqrt{2} a^2 f} + \frac{(iA - B)(c - i \tan(e + fx))^{3/2}}{4a^2 f (1 + i \tan(e + fx))^2}
\end{aligned}$$

### Mathematica [A]

time = 1.40, size = 205, normalized size = 1.28

$$\frac{\sec(e + fx)(\cos(fx) + i \sin(fx))^2(A + B \tan(e + fx)) \left( \sqrt{2} (A - 7iB)c^{3/2} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right) (-i \cos(2e) + \sin(2e)) + 2c \cos(e + fx)(\cos(2fx) - i \sin(2fx))(3iA + 5B) \cos(e + fx) + (A + 9iB) \sin(e + fx) \sqrt{c - ic \tan(e + fx)} \right)}{16f(A \cos(e + fx) + B \sin(e + fx))(a + i a \tan(e + fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]*(Cos[f*x] + I*Sin[f*x])^2*(A + B*Tan[e + f*x])*(Sqrt[2]*(A - (7*I)*B)*c^(3/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*((-I)*Cos[2*e] + Sin[2*e]) + 2*c*Cos[e + f*x]*(Cos[2*f*x] - I*Sin[2*f*x])*(((3*I)*A + 5*B)*Cos[e + f*x] + (A + (9*I)*B)*Sin[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]))/(16*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*(a + I*a*Tan[e + f*x])^2)
```

**Maple [A]**

time = 0.39, size = 118, normalized size = 0.74

method	result
derivativedivides	$2ic^2 \left( -\frac{4 \left( \left( \frac{9iB}{64} + \frac{A}{64} \right) (c - ic \tan(fx + e)) \right)^{3/2} + \left( -\frac{7}{32} iBc + \frac{1}{32} Ac \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^2} + \frac{\left( -\frac{7iB}{4} + \frac{A}{4} \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{c - ic \tan(fx + e)}}{\sqrt{2} \sqrt{c}} \right)}{fa^2} \right)$
default	$2ic^2 \left( -\frac{4 \left( \left( \frac{9iB}{64} + \frac{A}{64} \right) (c - ic \tan(fx + e)) \right)^{3/2} + \left( -\frac{7}{32} iBc + \frac{1}{32} Ac \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^2} + \frac{\left( -\frac{7iB}{4} + \frac{A}{4} \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{c - ic \tan(fx + e)}}{\sqrt{2} \sqrt{c}} \right)}{fa^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,method =_RETURNVERBOSE)
```

```
[Out] -2*I/f/a^2*c^2*(-4*((9/64*I*B+1/64*A)*(c-I*c*tan(f*x+e))^(3/2)+(-7/32*I*B*c+1/32*A*c)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+1/8*(-7/4*I*B+1/4*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

**Maxima [A]**

time = 0.52, size = 172, normalized size = 1.08

$$i \left( \frac{\sqrt{2}^{(A-7iB)c^{5/2}} \log \left( \frac{\sqrt{2} \sqrt{c} - \sqrt{-ic \tan(fx + e) + c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx + e) + c}} \right)}{a^2} + \frac{4 \left( (-ic \tan(fx + e) + c) \right)^{3/2} (A + 9iB)c^3 + 2 \sqrt{-ic \tan(fx + e) + c} (A - 7iB)c^4}{(-ic \tan(fx + e) + c)^2 a^2 - 4 (-ic \tan(fx + e) + c) a^2 c + 4 a^2 c^2} \right) / 32cf$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e))^2,x,  
algorithm="maxima")

[Out]  $\frac{1}{32}I(\sqrt{2})(A - 7IB)c^{5/2}\log(-(\sqrt{2}\sqrt{c} - \sqrt{-Ic\tan(fx + e) + c})/(\sqrt{2}\sqrt{c} + \sqrt{-Ic\tan(fx + e) + c}))/a^2 + 4((-Ic\tan(fx + e) + c)^{3/2}(A + 9IB)c^3 + 2\sqrt{-Ic\tan(fx + e) + c}(A - 7IB)c^4)/((-Ic\tan(fx + e) + c)^2a^2 - 4(-Ic\tan(fx + e) + c)a^2c + 4a^2c^2)/(c*f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs.  $2(128) = 256$ .

time = 5.05, size = 384, normalized size = 2.40

$$\left( \sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \log\left( \frac{(-1 + 2I\sqrt{2})\sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}}}{2\sqrt{2} + 1} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \right) - \sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \log\left( \frac{(-1 + 2I\sqrt{2})\sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \sqrt{\frac{c}{2}} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}}}{2\sqrt{2} + 1} \sqrt{\frac{LP - 14AB - 49B^2}{a^2f}} \right) + \sqrt{2}(IA + 7B)c^{3/2} + (IA + 5B)c^{3/2} - 2(-IA + B)c \sqrt{\frac{c}{2\sqrt{2} + 1}} \right) / a^2 f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e))^2,x,  
algorithm="fricas")

[Out]  $\frac{1}{16}(\sqrt{1/2})a^2f\sqrt{-(A^2 - 14IA*B - 49B^2)c^3/(a^4f^2)}e^{(4I*f*x + 4I*e)}\log(1/4((-IA - 7B)c^2 + \sqrt{2}\sqrt{1/2}(a^2f*e^{(2I*f*x + 2I*e)} + a^2f)\sqrt{c/(e^{(2I*f*x + 2I*e)} + 1)})\sqrt{-(A^2 - 14IA*B - 49B^2)c^3/(a^4f^2)})e^{(-I*f*x - I*e)/(a^2f)} - \sqrt{1/2}a^2f\sqrt{-(A^2 - 14IA*B - 49B^2)c^3/(a^4f^2)}e^{(4I*f*x + 4I*e)}\log(1/4((-IA - 7B)c^2 - \sqrt{2}\sqrt{1/2}(a^2f*e^{(2I*f*x + 2I*e)} + a^2f)\sqrt{c/(e^{(2I*f*x + 2I*e)} + 1)})\sqrt{-(A^2 - 14IA*B - 49B^2)c^3/(a^4f^2)})e^{(-I*f*x - I*e)/(a^2f)} + \sqrt{2}((IA + 7B)c^2e^{(4I*f*x + 4I*e)} + (3IA + 5B)c^2e^{(2I*f*x + 2I*e)} - 2(-IA + B)c)\sqrt{c/(e^{(2I*f*x + 2I*e)} + 1)})e^{(-4I*f*x - 4I*e)/(a^2f)}$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{Ac\sqrt{-ic\tan(e+fx)+c}}{\tan^2(e+fx)-2i\tan(e+fx)-1} dx + \int \frac{Bc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1} dx + \int \left( \frac{-iAc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1} \right) dx + \int \left( \frac{-iBc\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1} \right) dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e))^2,  
x)

[Out]  $-(\text{Integral}(A*c*\sqrt{-I*c*\tan(e + f*x) + c}/(\tan(e + f*x)**2 - 2*I*\tan(e + f*x) - 1), x) + \text{Integral}(B*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)/(\tan(e + f*x)**2 - 2*I*\tan(e + f*x) - 1), x) + \text{Integral}(-I*A*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)/(\tan(e + f*x)**2 - 2*I*\tan(e + f*x) - 1), x) + \text{Integral}(-I*B*c*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**2/(\tan(e + f*x)**2 - 2*I*\tan(e + f*x) - 1), x))/a**2$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x +
e) + a)^2, x)
```

**Mupad [B]**

time = 9.39, size = 267, normalized size = 1.67

$$\frac{\frac{7Bc^2\sqrt{c-\tan(e+fx)}\operatorname{li}}{4a^2c^2f+a^2f(c-\tan(e+fx))} - \frac{9Bc^2(c-\tan(e+fx))^{3/2}}{8a^2cf} + \frac{Ac^2\sqrt{c-\tan(e+fx)}\operatorname{li}}{4a^2f} + \frac{Ac^2(c-\tan(e+fx))^{3/2}\operatorname{li}}{8a^2f}}{(c-\tan(e+fx))^2-4c(c-\tan(e+fx))+4c^2} + \frac{\sqrt{2}A(-c)^{3/2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\operatorname{li}}{2\sqrt{-c}}\right)\operatorname{li}}{16a^2f} - \frac{7\sqrt{2}Bc^{3/2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\operatorname{li}}{2\sqrt{c}}\right)}{16a^2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*x)
)*1i)^2,x)
```

```
[Out] ((7*B*c^3*(c - c*tan(e + f*x)*1i)^(1/2))/4 - (9*B*c^2*(c - c*tan(e + f*x)*1
i)^(3/2))/8)/(4*a^2*c^2*f + a^2*f*(c - c*tan(e + f*x)*1i)^2 - 4*a^2*c*f*(c
- c*tan(e + f*x)*1i)) + ((A*c^3*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(4*a^2*f)
+ (A*c^2*(c - c*tan(e + f*x)*1i)^(3/2)*1i)/(8*a^2*f))/((c - c*tan(e + f*x)
*1i)^2 - 4*c*(c - c*tan(e + f*x)*1i) + 4*c^2) + (2^(1/2)*A*(-c)^(3/2)*atan(
(2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*1i)/(16*a^2*f) - (7
*2^(1/2)*B*c^(3/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2)
)))/(16*a^2*f)
```

$$3.775 \quad \int \frac{(A+B \tan(e+fx)) \sqrt{c-ictan(e+fx)}}{(a+ia \tan(e+fx))^2} dx$$

**Optimal.** Leaf size=159

$$\frac{(3iA+5B)\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}a^2f} + \frac{(iA-B)\sqrt{c-ictan(e+fx)}}{4a^2f(1+i \tan(e+fx))^2} + \frac{(3iA+5B)\sqrt{c-ictan(e+fx)}}{16a^2f(1+i \tan(e+fx))}$$

[Out] 1/32\*(3\*I\*A+5\*B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))\*c^(1/2)/a^2/f\*2^(1/2)+1/4\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^2/f/(1+I\*tan(f\*x+e))^2+1/16\*(3\*I\*A+5\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^2/f/(1+I\*tan(f\*x+e))

**Rubi [A]**

time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 44, 65, 214}

$$\frac{(-B+iA)\sqrt{c-ictan(e+fx)}}{4a^2f(1+i \tan(e+fx))^2} + \frac{(5B+3iA)\sqrt{c-ictan(e+fx)}}{16a^2f(1+i \tan(e+fx))} + \frac{\sqrt{c}(5B+3iA) \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] (((3\*I)\*A + 5\*B)\*Sqrt[c]\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(16\*Sqrt[2]\*a^2\*f) + ((I\*A - B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(4\*a^2\*f\*(1 + I\*Tan[e + f\*x])^2) + (((3\*I)\*A + 5\*B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(16\*a^2\*f\*(1 + I\*Tan[e + f\*x]))

**Rule 44**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx)) \sqrt{c - i \tan(e + fx)}}{(a + i a \tan(e + fx))^2} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^3 \sqrt{c-icx}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B) \sqrt{c - i \tan(e + fx)}}{4a^2 f (1 + i \tan(e + fx))^2} + \frac{((3A - 5iB)c) \text{Subst} \left( \dots \right)}{16a^2 f (1 + i \tan(e + fx))^2} \\
&= \frac{(iA - B) \sqrt{c - i \tan(e + fx)}}{4a^2 f (1 + i \tan(e + fx))^2} + \frac{(3iA + 5B) \sqrt{c - i \tan(e + fx)}}{16a^2 f (1 + i \tan(e + fx))^2} \\
&= \frac{(iA - B) \sqrt{c - i \tan(e + fx)}}{4a^2 f (1 + i \tan(e + fx))^2} + \frac{(3iA + 5B) \sqrt{c - i \tan(e + fx)}}{16a^2 f (1 + i \tan(e + fx))^2} \\
&= \frac{(3iA + 5B) \sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c - i \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{16\sqrt{2} a^2 f} + \frac{(iA - B) \sqrt{c - i \tan(e + fx)}}{4a^2 f}
\end{aligned}$$

Mathematica [A]



time = 1.10, size = 206, normalized size = 1.30

$$\frac{\sec(e+fx)(\cos(fx)+i\sin(fx))^2(A+B\tan(e+fx))\left(\sqrt{2}(3iA+5B)\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(2e)+i\sin(2e))+2\cos(e+fx)(\cos(2fx)-i\sin(2fx))((7iA+B)\cos(e+fx)+(-3A+5iB)\sin(e+fx))\sqrt{c-ic\tan(e+fx)}\right)}{32f(A\cos(e+fx)+B\sin(e+fx))(a+ia\tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] (Sec[e + f\*x]\*(Cos[f\*x] + I\*Sin[f\*x])^2\*(A + B\*Tan[e + f\*x])\*(Sqrt[2]\*((3\*I)\*A + 5\*B)\*Sqrt[c]\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])]\*(Cos[2\*e] + I\*Sin[2\*e]) + 2\*Cos[e + f\*x]\*(Cos[2\*f\*x] - I\*Sin[2\*f\*x])\*(((7\*I)\*A + B)\*Cos[e + f\*x] + (-3\*A + (5\*I)\*B)\*Sin[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]))/(32\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^2)

**Maple [A]**

time = 0.40, size = 120, normalized size = 0.75

method	result
derivativedivides	$2ic^2 \left( \frac{4 \left( -\frac{(-5iB+3A)(c-ic\tan(fx+e))^{3/2}}{128c} + \left(\frac{5A}{64} - \frac{3iB}{64}\right) \sqrt{c-ic\tan(fx+e)} \right)}{(c+ic\tan(fx+e))^2} \right) \frac{(-5iB+3A)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}}{\sqrt{2}\sqrt{c}}\right)}{fa^2}$
default	$2ic^2 \left( \frac{4 \left( -\frac{(-5iB+3A)(c-ic\tan(fx+e))^{3/2}}{128c} + \left(\frac{5A}{64} - \frac{3iB}{64}\right) \sqrt{c-ic\tan(fx+e)} \right)}{(c+ic\tan(fx+e))^2} \right) \frac{(-5iB+3A)\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}}{\sqrt{2}\sqrt{c}}\right)}{fa^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x,method =\_RETURNVERBOSE)

[Out] -2\*I/f/a^2\*c^2\*(-4\*(-1/128/c\*(3\*A-5\*I\*B)\*(c-I\*c\*tan(f\*x+e))^(3/2)+(5/64\*A-3/64\*I\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2))/(c+I\*c\*tan(f\*x+e))^2-1/64/c^(3/2)\*(3\*A-5\*I\*B)\*2^(1/2)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2)))

**Maxima [A]**

time = 0.54, size = 178, normalized size = 1.12

$$\frac{i \left( \frac{\sqrt{2} (3A-5iB)c^{3/2} \log\left(\frac{-\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2} + \frac{4 \left( (-ic\tan(fx+e)+c)^{3/2} (3A-5iB)c^2 - 2\sqrt{-ic\tan(fx+e)+c} (5A-3iB)c^3 \right)}{(-ic\tan(fx+e)+c)^2 a^2 - 4(-ic\tan(fx+e)+c)a^2 c + 4a^2 c^2} \right)}{64cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x,  
algorithm="maxima")

[Out] 
$$-1/64*I*(\sqrt{2})*(3*A - 5*I*B)*c^{3/2}*\log(-(\sqrt{2})*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c})/(\sqrt{2})*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c})/a^2 + 4*((-I*c*\tan(f*x + e) + c)^{3/2}*(3*A - 5*I*B)*c^2 - 2*\sqrt{-I*c*\tan(f*x + e) + c}*(5*A - 3*I*B)*c^3)/((-I*c*\tan(f*x + e) + c)^2*a^2 - 4*(-I*c*\tan(f*x + e) + c)*a^2*c + 4*a^2*c^2)/(c*f)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs.  $2(127) = 254$ .

time = 4.72, size = 374, normalized size = 2.35

$$\left( \int_0^1 \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}}}{20c^2 + 1}} \log \left( \frac{\sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}}}{20c^2 + 1}} \right) - \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}}}{20c^2 + 1}} \log \left( \frac{\sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}}}{20c^2 + 1}} \right) + \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}}}{20c^2 + 1}} \right) e^{-4I*f*x} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}} \sqrt{\frac{c - \sqrt{2} \sqrt{\frac{19A^2 - 30AB - 25B^2}{a^2}}}{20c^2 + 1}}}{20c^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x,  
algorithm="fricas")

[Out] 
$$1/32*(\sqrt{1/2})*a^2*f*\sqrt{-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2)}*e^{(4*I*f*x + 4*I*e)}*\log(1/8*(\sqrt{2})*\sqrt{1/2}*(a^2*f*e^{(2*I*f*x + 2*I*e)} + a^2*f*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2)} + (3*I*A + 5*B)*c)*e^{(-I*f*x - I*e)/(a^2*f)} - \sqrt{1/2})*a^2*f*\sqrt{-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2)}*e^{(4*I*f*x + 4*I*e)}*\log(-1/8*(\sqrt{2})*\sqrt{1/2}*(a^2*f*e^{(2*I*f*x + 2*I*e)} + a^2*f*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2)} - (3*I*A + 5*B)*c)*e^{(-I*f*x - I*e)/(a^2*f)} + \sqrt{2})*((5*I*A + 3*B)*e^{(4*I*f*x + 4*I*e)} + (7*I*A + B)*e^{(2*I*f*x + 2*I*e)} + 2*I*A - 2*B)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})*e^{(-4*I*f*x - 4*I*e)/(a^2*f)}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sqrt{-ic \tan(e + fx) + c}}{\tan^2(e + fx) - 2i \tan(e + fx) - 1} dx + \int \frac{B \sqrt{-ic \tan(e + fx) + c} \tan(e + fx)}{\tan^2(e + fx) - 2i \tan(e + fx) - 1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*2,  
x)

[Out] 
$$-(\text{Integral}(A*\sqrt{-I*c*\tan(e + f*x) + c}/(\tan(e + f*x)**2 - 2*I*\tan(e + f*x) - 1), x) + \text{Integral}(B*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)/(\tan(e + f*x)**2 - 2*I*\tan(e + f*x) - 1), x))/a**2$$

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e)
) + a)^2, x)
```

**Mupad [B]**

time = 9.18, size = 264, normalized size = 1.66

$$-\frac{3Bc(c-\tan(e+fx))^{3/2}}{4a^2c^2f+a^2f(c-\tan(e+fx))^2}-\frac{3Bc\sqrt{c-\tan(e+fx)}\operatorname{li}}{8}+\frac{A^2\sqrt{c-\tan(e+fx)}\operatorname{li}}{8a^2f}-\frac{Ac(c-\tan(e+fx))^{3/2}\operatorname{li}}{16a^2f}+\frac{\sqrt{2}A\sqrt{-c}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\operatorname{li}}{2\sqrt{-c}}\right)}{32a^2f}+\frac{5\sqrt{2}B\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}\operatorname{li}}{2\sqrt{c}}\right)}{32a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*x)
)*1i)^2,x)
```

```
[Out] ((A*c^2*(c - c*tan(e + f*x)*1i)^(1/2)*5i)/(8*a^2*f) - (A*c*(c - c*tan(e + f
*x)*1i)^(3/2)*3i)/(16*a^2*f))/((c - c*tan(e + f*x)*1i)^2 - 4*c*(c - c*tan(e
+ f*x)*1i) + 4*c^2) - ((5*B*c*(c - c*tan(e + f*x)*1i)^(3/2))/16 - (3*B*c^2
*(c - c*tan(e + f*x)*1i)^(1/2))/8)/(4*a^2*c^2*f + a^2*f*(c - c*tan(e + f*x)
*1i)^2 - 4*a^2*c*f*(c - c*tan(e + f*x)*1i)) + (2^(1/2)*A*(-c)^(1/2)*atan((2
^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*3i)/(32*a^2*f) + (5*2
^(1/2)*B*c^(1/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))
))/(32*a^2*f)
```

$$3.776 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2 \sqrt{c-ic \tan(e+fx)}} dx$$

Optimal. Leaf size=195

$$\frac{3(5iA+3B) \tanh^{-1}\left(\frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}a^2\sqrt{c}f} - \frac{3(5iA+3B)}{32a^2f\sqrt{c-ic \tan(e+fx)}} + \frac{iA-B}{4a^2f(1+i \tan(e+fx))^2\sqrt{c-ic \tan(e+fx)}}$$

[Out] 3/64\*(5\*I\*A+3\*B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a^2/f\*2^(1/2)/c^(1/2)-3/32\*(5\*I\*A+3\*B)/a^2/f/(c-I\*c\*tan(f\*x+e))^(1/2)+1/4\*(I\*A-B)/a^2/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(1+I\*tan(f\*x+e))^2+1/16\*(5\*I\*A+3\*B)/a^2/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(1+I\*tan(f\*x+e))

Rubi [A]

time = 0.17, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 44, 53, 65, 214}

$$\frac{-B+iA}{4a^2f(1+i \tan(e+fx))^2\sqrt{c-ic \tan(e+fx)}} - \frac{3(3B+5iA)}{32a^2f\sqrt{c-ic \tan(e+fx)}} + \frac{3B+5iA}{16a^2f(1+i \tan(e+fx))\sqrt{c-ic \tan(e+fx)}} + \frac{3(3B+5iA) \tanh^{-1}\left(\frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}a^2\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*Sqrt[c - I\*c\*Tan[e + f\*x]]), x]

[Out] (3\*((5\*I)\*A + 3\*B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(32\*Sqrt[2]\*a^2\*Sqrt[c]\*f) - (3\*((5\*I)\*A + 3\*B))/(32\*a^2\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + (I\*A - B)/(4\*a^2\*f\*(1 + I\*Tan[e + f\*x])^2\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + ((5\*I)\*A + 3\*B)/(16\*a^2\*f\*(1 + I\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])

Rule 44

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 53

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ

`[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 79

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))`

### Rule 214

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

### Rule 3669

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]`

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 \sqrt{c - ict \tan(e + fx)}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3(c-icx)^{3/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))^2 \sqrt{c - ict \tan(e + fx)}} + \frac{((5A - B) \tan^{-1}\left(\frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right) - \frac{1}{2} \cos(2(e + fx)))}{16a^2 f \sqrt{c - ict \tan(e + fx)}} \\
&= \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))^2 \sqrt{c - ict \tan(e + fx)}} + \frac{((5A - B) \tan^{-1}\left(\frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right) - \frac{1}{2} \cos(2(e + fx)))}{16a^2 f \sqrt{c - ict \tan(e + fx)}} \\
&= -\frac{3(5iA + 3B)}{32a^2 f \sqrt{c - ict \tan(e + fx)}} + \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))} \\
&= -\frac{3(5iA + 3B)}{32a^2 f \sqrt{c - ict \tan(e + fx)}} + \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))} \\
&= \frac{3(5iA + 3B) \tanh^{-1}\left(\frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{32\sqrt{2} a^2 \sqrt{c} f} - \frac{iA - B}{32a^2 f \sqrt{c - ict \tan(e + fx)}}
\end{aligned}$$

**Mathematica [A]**

time = 2.14, size = 160, normalized size = 0.82

$$\frac{(i \cos(e + fx) + \sin(e + fx)) \left(3(5A - 3iB) e^{i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \tanh^{-1}\left(\frac{\sqrt{1 + e^{2i(e+fx)}}}{\sqrt{2} \sqrt{c}}\right) - 2 \cos(e + fx) (-9A - iB + 2(3A - 5iB) \cos(2(e + fx)) + 2(5iA + 3B) \sin(2(e + fx)))\right) \sqrt{c - ict \tan(e + fx)}}{64a^2 c f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]), x]
```

```
[Out] ((I*Cos[e + f*x] + Sin[e + f*x])*(3*(5*A - (3*I)*B)*E^(I*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]] - 2*Cos[e + f*x]*(-9*A - I*B + 2*(3*A - (5*I)*B)*Cos[2*(e + f*x)] + 2*((5*I)*A + 3*B)*Sin[2*(e + f*x)]))*Sqrt[c - I*c*Tan[e + f*x]]/(64*a^2*c*f)
```

**Maple [A]**

time = 0.35, size = 151, normalized size = 0.77

method	result
--------	--------

derivativedivides	$2ic^2 \left( \frac{iB-A}{sc^2 \sqrt{c-ic \tan(fx+e)}} - \frac{4 \left( \frac{iB}{32} - \frac{7A}{32} \right) (c-ic \tan(fx+e))^{\frac{3}{2}} + 4 \left( \frac{9}{16} Ac + \frac{1}{16} iBc \right) \sqrt{c-ic \tan(fx+e)}}{(c+ic \tan(fx+e))^2} \right)$
default	$2ic^2 \left( \frac{iB-A}{sc^2 \sqrt{c-ic \tan(fx+e)}} - \frac{4 \left( \frac{iB}{32} - \frac{7A}{32} \right) (c-ic \tan(fx+e))^{\frac{3}{2}} + 4 \left( \frac{9}{16} Ac + \frac{1}{16} iBc \right) \sqrt{c-ic \tan(fx+e)}}{(c+ic \tan(fx+e))^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,method =_RETURNVERBOSE)`

[Out]  $-2*I/f/a^2*c^2*(-1/8/c^2*(-A+I*B)/(c-I*c*tan(f*x+e))^{(1/2)}-1/8/c^2*(4*((1/3)2*I*B-7/32*A)*(c-I*c*tan(f*x+e))^{(3/2)}+(9/16*A*c+1/16*I*B*c)*(c-I*c*tan(f*x+e))^{(1/2)})/(c+I*c*tan(f*x+e))^{2+3/4*(-3/4*I*B+5/4*A)*2^{(1/2)}/c^{(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})))$

**Maxima [A]**

time = 0.54, size = 202, normalized size = 1.04

$$i \left( \frac{3\sqrt{2}(5A-3iB)\sqrt{c} \log\left(\frac{-\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right)}{a^2} + \frac{4(3(-ic \tan(fx+e)+c)^2(5A-3iB)c-10(-ic \tan(fx+e)+c)(5A-3iB)c^2+32(A-iB)c^3)}{(-ic \tan(fx+e)+c)^{\frac{3}{2}}a^2-4(-ic \tan(fx+e)+c)^{\frac{3}{2}}a^2c+4\sqrt{-ic \tan(fx+e)+c}a^2c^2} \right)$$

128cf

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,algorithm="maxima")`

[Out]  $-1/128*I*(3*\sqrt{2}*(5*A - 3*I*B)*\sqrt{c}*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*tan(f*x + e) + c}))/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*tan(f*x + e) + c}))/a^2 + 4*(3*(-I*c*tan(f*x + e) + c)^2*(5*A - 3*I*B)*c - 10*(-I*c*tan(f*x + e) + c)*(5*A - 3*I*B)*c^2 + 32*(A - I*B)*c^3)/((-I*c*tan(f*x + e) + c)^{(5/2)}*a^2 - 4*(-I*c*tan(f*x + e) + c)^{(3/2)}*a^2*c + 4*\sqrt{-I*c*tan(f*x + e) + c}*a^2*c^2))/c*f$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 403 vs.  $2(156) = 312$ .

time = 3.92, size = 403, normalized size = 2.07

$$\left( 3\sqrt{\frac{1}{2}} e^{\frac{1}{2} f x} \sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}} e^{2 I e} \operatorname{arctan}\left(\frac{\sqrt{\frac{1}{2}} e^{\frac{1}{2} f x} \sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}}}{\sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}}}\right) - 3\sqrt{\frac{1}{2}} e^{\frac{1}{2} f x} \sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}} \operatorname{arctan}\left(\frac{\sqrt{\frac{1}{2}} e^{\frac{1}{2} f x} \sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}}}{\sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}}}\right) - \sqrt{2} (8(A+B)e^{6 I e} - (A-9B)e^{4 I e} - (11A-3B)e^{2 I e} - 3A+2B)\sqrt{\frac{25 A^2 - 30 A B - 9 B^2}{4 c^2}} \right) e^{-4 I f x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,
algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(1/2)*a^2*c*f*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/(a^4*c*f^2))*e^(
4*I*f*x + 4*I*e)*log(3/16*(sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) +
a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/
(a^4*c*f^2)) + 5*I*A + 3*B)*e^(-I*f*x - I*e)/(a^2*f)) - 3*sqrt(1/2)*a^2*c*f
*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/(a^4*c*f^2))*e^(4*I*f*x + 4*I*e)*log(-3/
16*(sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/(a^4*c*f^2)) - 5*I*A - 3
*B)*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(2)*(8*(I*A + B)*e^(6*I*f*x + 6*I*e) -
(I*A - 9*B)*e^(4*I*f*x + 4*I*e) - (11*I*A - 3*B)*e^(2*I*f*x + 2*I*e) - 2*I*
A + 2*B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-4*I*f*x - 4*I*e)/(a^2*c*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{-i c \tan(e+f x)+c} \tan^2(e+f x)-2 i \sqrt{-i c \tan(e+f x)+c} \tan(e+f x)-\sqrt{-i c \tan(e+f x)+c}} dx + \int \frac{B \tan(e+f x)}{\sqrt{-i c \tan(e+f x)+c} \tan^2(e+f x)-2 i \sqrt{-i c \tan(e+f x)+c} \tan(e+f x)-\sqrt{-i c \tan(e+f x)+c}} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2,
x)
```

```
[Out] -(Integral(A/(sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x)**2 - 2*I*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x) - sqrt(-I*c*tan(e + f*x) + c)), x) + Integral
(B*tan(e + f*x)/(sqrt(-I*c*tan(e + f*x) + c))*tan(e + f*x)**2 - 2*I*sqrt(-I*
c*tan(e + f*x) + c)*tan(e + f*x) - sqrt(-I*c*tan(e + f*x) + c)), x))/a**2
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^2*sqrt(-I*c*tan(f*x
+ e) + c)), x)
```



**Mupad [B]**

time = 9.44, size = 305, normalized size = 1.56

$$\frac{\frac{A(c-\tan(e+fx))^{1/2}B + A^2c^{1/2} - 4c(c-\tan(e+fx))^{1/2}B}{-4(c-\tan(e+fx))^{3/2} + (c-\tan(e+fx))^{5/2} + 4c^2\sqrt{c-\tan(e+fx)}} - \frac{Bc^2 + \frac{9B(c-\tan(e+fx))^{1/2}}{2} - 15B(c-\tan(e+fx))^{3/2}}{a^2f(c-\tan(e+fx))^{3/2} - 4a^2cf(c-\tan(e+fx))^{5/2} + 4a^2c^2f\sqrt{c-\tan(e+fx)}}}{64a^2\sqrt{-c}f} + \frac{\sqrt{2}A\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}}{2\sqrt{-c}}\right)15i}{64a^2\sqrt{-c}f} + \frac{9\sqrt{2}B\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}}{2\sqrt{c}}\right)}{64a^2\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2)),x)

[Out] (9\*2^(1/2)\*B\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2)))/(64\*a^2\*c^(1/2)\*f) - (B\*c^2 + (9\*B\*(c - c\*tan(e + f\*x)\*1i)^2)/32 - (15\*B\*c\*(c - c\*tan(e + f\*x)\*1i))/16)/(a^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(5/2) - 4\*a^2\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^(3/2) + 4\*a^2\*c^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(1/2)) - (2^(1/2)\*A\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*15i)/(64\*a^2\*(-c)^(1/2)\*f) - ((A\*(c - c\*tan(e + f\*x)\*1i)^2\*15i)/(32\*a^2\*f) + (A\*c^2\*1i)/(a^2\*f) - (A\*c\*(c - c\*tan(e + f\*x)\*1i)\*25i)/(16\*a^2\*f))/((c - c\*tan(e + f\*x)\*1i)^(5/2) - 4\*c\*(c - c\*tan(e + f\*x)\*1i)^(3/2) + 4\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(1/2))

$$3.777 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=226

$$\frac{5(7iA+B) \tanh^{-1}\left(\frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2} a^2 c^{3/2} f} - \frac{5(7iA+B)}{96a^2 f (c-ic \tan(e+fx))^{3/2}} + \frac{iA-B}{4a^2 f (1+i \tan(e+fx))^2 (c-ic \tan(e+fx))^{3/2}}$$

[Out] 5/128\*(7\*I\*A+B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a^2/c^(3/2)/f\*2^(1/2)-5/64\*(7\*I\*A+B)/a^2/c/f/(c-I\*c\*tan(f\*x+e))^(1/2)-5/96\*(7\*I\*A+B)/a^2/f/(c-I\*c\*tan(f\*x+e))^(3/2)+1/4\*(I\*A-B)/a^2/f/(1+I\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^(3/2)+1/16\*(7\*I\*A+B)/a^2/f/(1+I\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 44, 53, 65, 214}

$$\frac{5(B+7iA) \tanh^{-1}\left(\frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2} a^2 c^{3/2} f} + \frac{-B+iA}{4a^2 f (1+i \tan(e+fx))^2 (c-ic \tan(e+fx))^{3/2}} - \frac{5(B+7iA)}{64a^2 c f \sqrt{c-ic \tan(e+fx)}} - \frac{5(B+7iA)}{96a^2 f (c-ic \tan(e+fx))^{3/2}} + \frac{B+7iA}{16a^2 f (1+i \tan(e+fx))(c-ic \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(3/2)), x]

[Out] (5\*((7\*I)\*A + B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(64\*Sqrt[2]\*a^2\*c^(3/2)\*f) - (5\*((7\*I)\*A + B))/(96\*a^2\*f\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + (I\*A - B)/(4\*a^2\*f\*(1 + I\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + ((7\*I)\*A + B)/(16\*a^2\*f\*(1 + I\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2)) - (5\*((7\*I)\*A + B))/(64\*a^2\*c\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 44**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

**Rule 53**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !LtQ

`[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 79

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))`

### Rule 214

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

### Rule 3669

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]`

### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^3 (c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{f} \\
 &= \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} + \frac{((7A + B) \operatorname{arctanh}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right))}{16a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} \\
 &= -\frac{5(7iA + B)}{96a^2 f (c - ic \tan(e + fx))^{3/2}} + \frac{i}{4a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} \\
 &= -\frac{5(7iA + B)}{96a^2 f (c - ic \tan(e + fx))^{3/2}} + \frac{i}{4a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} \\
 &= -\frac{5(7iA + B)}{96a^2 f (c - ic \tan(e + fx))^{3/2}} + \frac{i}{4a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} \\
 &= \frac{5(7iA + B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{64\sqrt{2} a^2 c^{3/2} f} - \frac{i}{96a^2 f (c - ic \tan(e + fx))^{3/2}}
 \end{aligned}$$

**Mathematica [A]**

time = 2.35, size = 204, normalized size = 0.90

$$\frac{e^{-4i(e+fx)} \left( -i(1 + e^{2i(e+fx)}) (-iB(6 + 15e^{2i(e+fx)} + 32e^{4i(e+fx)} + 8e^{6i(e+fx)}) + A(-6 - 39e^{2i(e+fx)} + 80e^{4i(e+fx)} + 8e^{6i(e+fx)})) + 15(7iA + B)e^{4i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \operatorname{tanh}^{-1}\left(\frac{\sqrt{1 + e^{2i(e+fx)}}}{\sqrt{2}}\right) \right) \sqrt{c - ic \tan(e + fx)}}{384a^2 c^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)), x]
```

```
[Out] (((-I)*(1 + E^((2*I)*(e + f*x)))*((-I)*B*(6 + 15*E^((2*I)*(e + f*x)) + 32*E^((4*I)*(e + f*x)) + 8*E^((6*I)*(e + f*x))) + A*(-6 - 39*E^((2*I)*(e + f*x)) + 80*E^((4*I)*(e + f*x)) + 8*E^((6*I)*(e + f*x)))) + 15*((7*I)*A + B)*E^((4*I)*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[c - I*c*Tan[e + f*x]])/(384*a^2*c^2*E^((4*I)*(e + f*x))*f)
```

**Maple [A]**

time = 0.35, size = 178, normalized size = 0.79

method	result
--------	--------

derivativedivides	$2ic^2 \left( \frac{4 \left( -\frac{3iB}{32} - \frac{11A}{32} \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 4 \left( \frac{13}{16} Ac + \frac{5}{16} iBc \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^2} + \frac{5 \left( \frac{7A}{4} - \frac{iB}{4} \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{c - ic \tan(fx + e)}}{\sqrt{c + ic \tan(fx + e)}} \right)}{16c^3} \right) \frac{f a^2}{f a^2}$
default	$2ic^2 \left( \frac{4 \left( -\frac{3iB}{32} - \frac{11A}{32} \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 4 \left( \frac{13}{16} Ac + \frac{5}{16} iBc \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^2} + \frac{5 \left( \frac{7A}{4} - \frac{iB}{4} \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{c - ic \tan(fx + e)}}{\sqrt{c + ic \tan(fx + e)}} \right)}{16c^3} \right) \frac{f a^2}{f a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x,method =_RETURNVERBOSE)`

[Out]  $-2*I/f/a^2*c^2*(-1/16/c^3*(4*((-3/32*I*B-11/32*A)*(c-I*c*tan(f*x+e))^(3/2)+(13/16*A*c+5/16*I*B*c)*(c-I*c*tan(f*x+e))^(1/2)))/(c+I*c*tan(f*x+e))^2+5/4*(7/4*A-1/4*I*B)*2^(1/2)/c^(1/2)*\operatorname{arctanh}(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))-1/16/c^3*(-3*A+I*B)/(c-I*c*tan(f*x+e))^(1/2)-1/24*(-A+I*B)/c^2/(c-I*c*tan(f*x+e))^(3/2))$

**Maxima [A]**

time = 0.52, size = 225, normalized size = 1.00

$$i \left( \frac{4 \left( 15(-ic \tan(fx+e)+c)^3(7A-iB) - 50(-ic \tan(fx+e)+c)^2(7A-iB)c + 32(-ic \tan(fx+e)+c)(7A-iB)c^2 + 64(A-iB)c^3 \right)}{(-ic \tan(fx+e)+c)^{\frac{7}{2}} a^2 - 4(-ic \tan(fx+e)+c)^{\frac{5}{2}} a^2 c + 4(-ic \tan(fx+e)+c)^{\frac{3}{2}} a^2 c^2} + \frac{15 \sqrt{2} (7A-iB) \log \left( \frac{\sqrt{2} \sqrt{c} - \sqrt{-ic \tan(fx+e)+c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx+e)+c}} \right)}{a^2 \sqrt{c}} \right) \frac{f a^2}{768 c f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out]  $-1/768*I*(4*(15*(-I*c*tan(f*x+e)+c)^3*(7*A-I*B)-50*(-I*c*tan(f*x+e)+c)^2*(7*A-I*B)*c+32*(-I*c*tan(f*x+e)+c)*(7*A-I*B)*c^2+64*(A-I*B)*c^3)/((-I*c*tan(f*x+e)+c)^(7/2)*a^2-4*(-I*c*tan(f*x+e)+c)^(5/2)*a^2*c+4*(-I*c*tan(f*x+e)+c)^(3/2)*a^2*c^2)+15*\operatorname{sqrt}(2)*(7*A-I*B)*\log(-(\operatorname{sqrt}(2)*\operatorname{sqrt}(c)-\operatorname{sqrt}(-I*c*tan(f*x+e)+c))/(\operatorname{sqrt}(2)*\operatorname{sqrt}(c))+\operatorname{sqrt}(-I*c*tan(f*x+e)+c)))/(a^2*\operatorname{sqrt}(c)))/(c*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs.  $2(188) = 376$ .



[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^2\*(-I\*c\*tan(f\*x + e) + c)^(3/2)), x)

**Mupad [B]**

time = 9.72, size = 353, normalized size = 1.56

$$\frac{\frac{Bc^2}{a^2} - \frac{2B(c-\tan(e+fx))}{a} + \frac{5B(c-\tan(e+fx))^2}{6a^2} + \frac{B(c-\tan(e+fx))}{a}}{a^2 f (c - \tan(e + fx))^{3/2} - 4a^2 c f (c - \tan(e + fx))^{1/2} + 4a^2 c^2 f (c - \tan(e + fx))^{1/2}} - \frac{\frac{A(c-\tan(e+fx))^2}{2a^2 f} + \frac{A^2 c}{4a^2} + \frac{A(c-\tan(e+fx))^2}{4a^2 f} + \frac{A(c-\tan(e+fx))}{4a^2 f}}{-4c(c - \tan(e + fx))^{1/2} + (c - \tan(e + fx))^{1/2} + 4c^2(c - \tan(e + fx))^{1/2}} + \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{2\sqrt{-c}}\right)}{128 a^2 (-c)^{3/2} f} + \frac{35i}{128 a^2 c^{3/2} f} + \frac{5\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{2\sqrt{c}}\right)}{128 a^2 c^{3/2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^(3/2)), x)

[Out] (2^(1/2)\*A\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*35i)/(128\*a^2\*(-c)^(3/2)\*f) - ((A\*c^2\*1i)/(3\*a^2\*f) - (A\*(c - c\*tan(e + f\*x)\*1i)^2\*175i)/(96\*a^2\*f) + (A\*(c - c\*tan(e + f\*x)\*1i)^3\*35i)/(64\*a^2\*c\*f) + (A\*c\*(c - c\*tan(e + f\*x)\*1i)\*7i)/(6\*a^2\*f))/((c - c\*tan(e + f\*x)\*1i)^(7/2) - 4\*c\*(c - c\*tan(e + f\*x)\*1i)^(5/2) + 4\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(3/2)) - ((B\*c^2)/3 - (25\*B\*(c - c\*tan(e + f\*x)\*1i)^2)/96 + (5\*B\*(c - c\*tan(e + f\*x)\*1i)^3)/(64\*c) + (B\*c\*(c - c\*tan(e + f\*x)\*1i))/6)/(a^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(7/2) - 4\*a^2\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^(5/2) + 4\*a^2\*c^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(3/2)) + (5\*2^(1/2)\*B\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2))))/(128\*a^2\*c^(3/2)\*f)

$$3.778 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^2(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=273

$$\frac{7(9iA - B) \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{128\sqrt{2} a^2 c^{5/2} f} - \frac{7(9iA - B)}{160a^2 f (c - ic \tan(e + fx))^{5/2}} + \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}}$$

[Out] 7/256\*(9\*I\*A-B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a^2/c^(5/2)/f\*2^(1/2)-7/128\*(9\*I\*A-B)/a^2/c^2/f/(c-I\*c\*tan(f\*x+e))^(1/2)-7/160\*(9\*I\*A-B)/a^2/f/(c-I\*c\*tan(f\*x+e))^(5/2)+1/4\*(I\*A-B)/a^2/f/(1+I\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^(5/2)+1/16\*(9\*I\*A-B)/a^2/f/(1+I\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2)-7/192\*(9\*I\*A-B)/a^2/c/f/(c-I\*c\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.22, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 44, 53, 65, 214}

$$\frac{7(-B+9iA) \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{128\sqrt{2} a^2 c^{5/2} f} - \frac{7(-B+9iA)}{128a^2 f \sqrt{c - ic \tan(e + fx)}} + \frac{-B+iA}{4a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}} - \frac{7(-B+9iA)}{192a^2 f (c - ic \tan(e + fx))^{5/2}} - \frac{7(-B+9iA)}{160a^2 f (c - ic \tan(e + fx))^{5/2}} + \frac{-B+9iA}{16a^2 f (1 + i \tan(e + fx))^2 (c - ic \tan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(5/2)), x]

[Out] (7\*((9\*I)\*A - B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])]/(128\*Sqrt[2]\*a^2\*c^(5/2)\*f) - (7\*((9\*I)\*A - B))/(160\*a^2\*f\*(c - I\*c\*Tan[e + f\*x])^(5/2)) + (I\*A - B)/(4\*a^2\*f\*(1 + I\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(5/2)) + ((9\*I)\*A - B)/(16\*a^2\*f\*(1 + I\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2)) - (7\*((9\*I)\*A - B))/(192\*a^2\*c\*f\*(c - I\*c\*Tan[e + f\*x])^(3/2)) - (7\*((9\*I)\*A - B))/(128\*a^2\*c^2\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 44**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

**Rule 53**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x]



```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
  {p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
  d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
  [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
  ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
)
```

#### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^3 (c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} + \frac{((9A^2 - B^2) \sqrt{c - ictan(e + fx)})}{16a^2 f (c - ictan(e + fx))^{5/2}} \\
&= \frac{iA - B}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} + \frac{7(9iA - B)}{160a^2 f (c - ictan(e + fx))^{5/2}} + \frac{7(9iA - B)}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} \\
&= -\frac{7(9iA - B)}{160a^2 f (c - ictan(e + fx))^{5/2}} + \frac{7(9iA - B)}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} \\
&= -\frac{7(9iA - B)}{160a^2 f (c - ictan(e + fx))^{5/2}} + \frac{7(9iA - B)}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} \\
&= -\frac{7(9iA - B)}{160a^2 f (c - ictan(e + fx))^{5/2}} + \frac{7(9iA - B)}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} \\
&= -\frac{7(9iA - B)}{160a^2 f (c - ictan(e + fx))^{5/2}} + \frac{7(9iA - B)}{4a^2 f (1 + i \tan(e + fx))^2 (c - ictan(e + fx))^{5/2}} \\
&= \frac{7(9iA - B) \tanh^{-1} \left( \frac{\sqrt{c - ictan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{128 \sqrt{2} a^2 c^{5/2} f} - \frac{7(9iA - B)}{160a^2 f (c - ictan(e + fx))^{5/2}}
\end{aligned}$$

**Mathematica [A]**

time = 3.33, size = 209, normalized size = 0.77

$(\cos(e + fx) + i \sin(e + fx)) \left( 105i(9A + iB)e^{-i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \tanh^{-1} \left( \frac{\sqrt{1 + e^{2i(e+fx)}}}{\sqrt{2}} \right) + 2 \cos(e + fx) (-864iA - 64B + (87iA - 223B) \cos(2(e + fx)) + 6i(A + 9iB) \cos(4(e + fx)) + 423A \sin(2(e + fx)) + 47iB \sin(2(e + fx)) + 54A \sin(4(e + fx)) + 6iB \sin(4(e + fx))) \sqrt{c - ictan(e + fx)} \right) / (3840a^2 c^3 f)$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(5/2)), x]

[Out] ((Cos[e + f\*x] + I\*Sin[e + f\*x])\*(((105\*I)\*(9\*A + I\*B)\*Sqrt[1 + E^((2\*I)\*(e + f\*x))])\*ArcTanh[Sqrt[1 + E^((2\*I)\*(e + f\*x))]])/E^(I\*(e + f\*x)) + 2\*Cos[e + f\*x]\*((-864\*I)\*A - 64\*B + ((87\*I)\*A - 223\*B)\*Cos[2\*(e + f\*x)] + (6\*I)\*(A + (9\*I)\*B)\*Cos[4\*(e + f\*x)] + 423\*A\*Sin[2\*(e + f\*x)] + (47\*I)\*B\*Sin[2\*(e + f\*x)] + 54\*A\*Sin[4\*(e + f\*x)] + (6\*I)\*B\*Sin[4\*(e + f\*x)]))\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3840\*a^2\*c^3\*f)

**Maple [A]**

time = 0.34, size = 198, normalized size = 0.73

method	result
derivativedivides	$2ic^2 \left( \frac{3A}{16c^4 \sqrt{c - ic \tan(fx + e)}} - \frac{iB - 3A}{48c^3 (c - ic \tan(fx + e))^{\frac{3}{2}}} - \frac{iB - A}{40c^2 (c - ic \tan(fx + e))^{\frac{5}{2}}} - \frac{4 \left( -\frac{7iB}{64} - \frac{15A}{64} \right) (c - ic \tan(fx + e))}{\dots} \right)$
default	$2ic^2 \left( \frac{3A}{16c^4 \sqrt{c - ic \tan(fx + e)}} - \frac{iB - 3A}{48c^3 (c - ic \tan(fx + e))^{\frac{3}{2}}} - \frac{iB - A}{40c^2 (c - ic \tan(fx + e))^{\frac{5}{2}}} - \frac{4 \left( -\frac{7iB}{64} - \frac{15A}{64} \right) (c - ic \tan(fx + e))}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x,method =_RETURNVERBOSE)`

[Out]  $-2*I/f/a^2*c^2*(3/16*A/c^4/(c-I*c*tan(f*x+e))^{(1/2)}-1/48*(-3*A+I*B)/c^3/(c-I*c*tan(f*x+e))^{(3/2)}-1/40*(-A+I*B)/c^2/(c-I*c*tan(f*x+e))^{(5/2)}-1/16/c^4*(4*((-7/64*I*B-15/64*A)*(c-I*c*tan(f*x+e))^{(3/2)}+(9/32*I*B*c+17/32*A*c)*(c-I*c*tan(f*x+e))^{(1/2)})/(c+I*c*tan(f*x+e))^{(1/2)}+7/8*(1/4*I*B+9/4*A)*2^{(1/2)}/c^{(1/2)})*arctanh(1/2*(c-I*c*tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))$

**Maxima [A]**

time = 0.51, size = 254, normalized size = 0.93

$$i \left( \frac{4 \left( 105 (-i \tan(fx+e)+c)^4 (9A+iB) - 350 (-i \tan(fx+e)+c)^3 (9A+iB)c + 224 (-i \tan(fx+e)+c)^2 (9A+iB)c^2 + 64 (-i \tan(fx+e)+c) (9A+iB)c^3 + 384 (A-iB)c^4 \right)}{(-i \tan(fx+e)+c)^2 a^2 c - 4 (-i \tan(fx+e)+c)^2 a^2 c^2 + 4 (-i \tan(fx+e)+c)^2 a^2 c^3} + \frac{105 \sqrt{2} (9A+iB) \log \left( \frac{\sqrt{2} \sqrt{c} - \sqrt{-i \tan(fx+e)+c}}{\sqrt{2} \sqrt{c} + \sqrt{-i \tan(fx+e)+c}} \right)}{a^2 c^2} \right)$$

7680 cf

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")`

[Out]  $-1/7680*I*(4*(105*(-I*c*tan(f*x + e) + c)^4*(9*A + I*B) - 350*(-I*c*tan(f*x + e) + c)^3*(9*A + I*B)*c + 224*(-I*c*tan(f*x + e) + c)^2*(9*A + I*B)*c^2 + 64*(-I*c*tan(f*x + e) + c)*(9*A + I*B)*c^3 + 384*(A - I*B)*c^4)/((-I*c*tan(f*x + e) + c)^{(9/2)}*a^2*c - 4*(-I*c*tan(f*x + e) + c)^{(7/2)}*a^2*c^2 + 4*(-I*c*tan(f*x + e) + c)^{(5/2)}*a^2*c^3) + 105*sqrt(2)*(9*A + I*B)*log(-sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c))/(a^2*c^{(3/2)})/(c*f)$



[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^(5/2),x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^2\*(-I\*c\*tan(f\*x + e) + c)^(5/2)), x)

Mupad [B]

time = 10.14, size = 399, normalized size = 1.46

$$\frac{-\frac{Bc^2}{5} + \frac{7B(c-\tan(e+fx))c^2}{30} - \frac{35B(c-\tan(e+fx))c^2}{192c} + \frac{7B(c-\tan(e+fx))c^2}{192c} + \frac{B(c-\tan(e+fx))c^2}{30}}{a^2 f (c - \tan(e + fx) i)^{5/2} - 4a^2 c f (c - \tan(e + fx) i)^{3/2} + 4a^2 c^2 f (c - \tan(e + fx) i)^{1/2}} - \frac{\frac{A(c-\tan(e+fx))c^{1/2} 21i}{20a^2 f} + \frac{4c^2 21i}{5a^2 f} - \frac{A(c-\tan(e+fx))c^{1/2} 109i}{64a^2 c f} + \frac{A(c-\tan(e+fx))c^{1/2} 49i}{128c^2 f} + \frac{A(c-\tan(e+fx))c^{1/2} 3i}{192c^2 f}}{-4c(c - \tan(e + fx) i)^{3/2} + (c - \tan(e + fx) i)^{1/2} + 4c^2(c - \tan(e + fx) i)^{1/2}} - \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx) i}}{2\sqrt{-c}}\right) 63i}{256a^2(-c)^{5/2} f} - \frac{7\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx) i}}{2\sqrt{-c}}\right)}{256a^2 c^{5/2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^2\*(c - c\*tan(e + f\*x)\*1i)^(5/2)),x)

[Out] ((7\*B\*(c - c\*tan(e + f\*x)\*1i)^2)/60 - (B\*c^2)/5 - (35\*B\*(c - c\*tan(e + f\*x)\*1i)^3)/(192\*c) + (7\*B\*(c - c\*tan(e + f\*x)\*1i)^4)/(128\*c^2) + (B\*c\*(c - c\*tan(e + f\*x)\*1i))/30)/(a^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(9/2) - 4\*a^2\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^(7/2) + 4\*a^2\*c^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(5/2)) - ((A\*(c - c\*tan(e + f\*x)\*1i)^2\*21i)/(20\*a^2\*f) + (A\*c^2\*1i)/(5\*a^2\*f) - (A\*(c - c\*tan(e + f\*x)\*1i)^3\*105i)/(64\*a^2\*c\*f) + (A\*(c - c\*tan(e + f\*x)\*1i)^4\*63i)/(128\*a^2\*c^2\*f) + (A\*c\*(c - c\*tan(e + f\*x)\*1i)\*3i)/(10\*a^2\*f))/((c - c\*tan(e + f\*x)\*1i)^(9/2) - 4\*c\*(c - c\*tan(e + f\*x)\*1i)^(7/2) + 4\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(5/2)) - (2^(1/2)\*A\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*63i)/(256\*a^2\*(-c)^(5/2)\*f) - (7\*2^(1/2)\*B\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2))))/(256\*a^2\*c^(5/2)\*f)

$$3.779 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^{9/2}}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=291

$$\frac{35(iA - 5B)c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c - ict \tan(e+fx)}}{\sqrt{2} \sqrt{c}}\right)}{4\sqrt{2} a^3 f} + \frac{35(iA - 5B)c^4 \sqrt{c - ict \tan(e+fx)}}{8a^3 f} + \frac{35(iA - 5B)c^3}{8a^3 f}$$

[Out]  $-35/8*(I*A-5*B)*c^{(9/2)*\arctanh(1/2*(c-I*c*\tan(f*x+e))^{(1/2)*2^{(1/2)}/c^{(1/2)}})/a^3/f*2^{(1/2)}+35/8*(I*A-5*B)*c^4*(c-I*c*\tan(f*x+e))^{(1/2)}/a^3/f+35/48*(I*A-5*B)*c^3*(c-I*c*\tan(f*x+e))^{(3/2)}/a^3/f+7/16*(I*A-5*B)*c^2*(c-I*c*\tan(f*x+e))^{(5/2)}/a^3/f/(1+I*\tan(f*x+e))-1/8*(I*A-5*B)*c*(c-I*c*\tan(f*x+e))^{(7/2)}/a^3/f/(1+I*\tan(f*x+e))^2+1/6*(I*A-B)*(c-I*c*\tan(f*x+e))^{(9/2)}/a^3/f/(1+I*\tan(f*x+e))^3$

**Rubi [A]**

time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 43, 52, 65, 214}

$$\frac{35c^{9/2}(-5B+iA)\tanh^{-1}\left(\frac{\sqrt{c-ict \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}a^3f} + \frac{35c^4(-5B+iA)\sqrt{c-ict \tan(e+fx)}}{8a^3f} + \frac{35c^3(-5B+iA)(c-ict \tan(e+fx))^{3/2}}{48a^3f} + \frac{7c^2(-5B+iA)(c-ict \tan(e+fx))^{5/2}}{16a^3f(1+i \tan(e+fx))} - \frac{c(-5B+iA)(c-ict \tan(e+fx))^{7/2}}{8a^3f(1+i \tan(e+fx))^2} + \frac{(-B+iA)(c-ict \tan(e+fx))^{9/2}}{6a^3f(1+i \tan(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(9/2))/(a + I\*a\*Tan[e + f\*x])^3, x]

[Out]  $(-35*(I*A - 5*B)*c^{(9/2)*\text{ArcTanh}[\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]]/(\text{Sqrt}[2]*\text{Sqrt}[c])})/(4*\text{Sqrt}[2]*a^3*f) + (35*(I*A - 5*B)*c^4*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(8*a^3*f) + (35*(I*A - 5*B)*c^3*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(48*a^3*f) + (7*(I*A - 5*B)*c^2*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(16*a^3*f*(1 + I*\text{Tan}[e + f*x])) - ((I*A - 5*B)*c*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(8*a^3*f*(1 + I*\text{Tan}[e + f*x])^2) + ((I*A - B)*(c - I*c*\text{Tan}[e + f*x])^{(9/2)})/(6*a^3*f*(1 + I*\text{Tan}[e + f*x])^3)$

**Rule 43**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^n/(b\*(m + 1))), x] - Dist[d\*(n/(b\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]

**Rule 52**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^n/(b\*(m + n + 1))), x] + Dist[n\*((b\*c - a\*d)/(

$b*(m + n + 1))$ , Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

### Rule 214

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{9/2}}{(a + ia \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{7/2}}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{9/2}}{6a^3 f(1 + i \tan(e + fx))^3} - \frac{((A + 5iB)c) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{7/2}}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{6a^3 f(1 + i \tan(e + fx))^3} \\
&= -\frac{(iA - 5B)c(c - ic \tan(e + fx))^{7/2}}{8a^3 f(1 + i \tan(e + fx))^2} + \frac{(iA - B)(c - ic \tan(e + fx))^{9/2}}{6a^3 f(1 + i \tan(e + fx))^3} \\
&= \frac{7(iA - 5B)c^2(c - ic \tan(e + fx))^{5/2}}{16a^3 f(1 + i \tan(e + fx))} - \frac{(iA - 5B)c(c - ic \tan(e + fx))^{9/2}}{8a^3 f(1 + i \tan(e + fx))^3} \\
&= \frac{35(iA - 5B)c^3(c - ic \tan(e + fx))^{3/2}}{48a^3 f} + \frac{7(iA - 5B)c^2(c - ic \tan(e + fx))^{5/2}}{16a^3 f(1 + i \tan(e + fx))} \\
&= \frac{35(iA - 5B)c^4 \sqrt{c - ic \tan(e + fx)}}{8a^3 f} + \frac{35(iA - 5B)c^3(c - ic \tan(e + fx))^{3/2}}{16a^3 f(1 + i \tan(e + fx))} \\
&= \frac{35(iA - 5B)c^4 \sqrt{c - ic \tan(e + fx)}}{8a^3 f} + \frac{35(iA - 5B)c^3(c - ic \tan(e + fx))^{3/2}}{16a^3 f(1 + i \tan(e + fx))} \\
&= -\frac{35(iA - 5B)c^{9/2} \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{4\sqrt{2} a^3 f} + \frac{35(iA - 5B)c^4 \sqrt{c - ic \tan(e + fx)}}{8a^3 f}
\end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(9/2))/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] \$Aborted

**Maple [A]**

time = 0.37, size = 207, normalized size = 0.71

method	result
--------	--------



derivativedivides	$2ic^3 \left( \frac{iB(c-ic \tan(fx+e))^{3/2}}{3} + 7iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - 8c^2 \left( \frac{8(-\frac{81iB}{512} - \frac{29}{512})}{3} \right) \right)$
default	$2ic^3 \left( \frac{iB(c-ic \tan(fx+e))^{3/2}}{3} + 7iBc \sqrt{c-ic \tan(fx+e)} + Ac \sqrt{c-ic \tan(fx+e)} - 8c^2 \left( \frac{8(-\frac{81iB}{512} - \frac{29}{512})}{3} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x,method =_RETURNVERBOSE)`

[Out]  $2*I/f/a^3*c^3*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)+7*I*B*c*(c-I*c*tan(f*x+e))^(1/2)+A*c*(c-I*c*tan(f*x+e))^(1/2)-8*c^2*(8*((-81/512*I*B-29/512*A)*(c-I*c*tan(f*x+e))^(5/2)+(53/96*I*B*c+17/96*A*c)*(c-I*c*tan(f*x+e))^(3/2)+(-63/128*I*B*c^2-19/128*A*c^2)*(c-I*c*tan(f*x+e))^(1/2)))/(c+I*c*tan(f*x+e))^3+35/16*(1/8*A+5/8*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))$

**Maxima [A]**

time = 0.53, size = 276, normalized size = 0.95

$$i \left( \frac{105\sqrt{2}(A+5iB)c^{\frac{11}{2}} \log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right)}{a^4} - \frac{4(3(-i \tan(fx+e)+c)^2(29A+81iB)c^6-16(-i \tan(fx+e)+c)^2(17A+53iB)c^7+12\sqrt{-ic \tan(fx+e)+c}(19A+63iB)c^8)}{(-i \tan(fx+e)+c)^3a^3-6(-i \tan(fx+e)+c)^2a^3c+12(-i \tan(fx+e)+c)a^3c^2-8a^3c^3} + \frac{32(i(-i \tan(fx+e)+c)^2Bc^3+3\sqrt{-ic \tan(fx+e)+c}(A+7iB)c^2)}{a^2} \right)$$

48cf

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")`

[Out]  $1/48*I*(105*\sqrt{2}*(A + 5*I*B)*c^(11/2)*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c}))/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c}))/a^3 - 4*(3*(-I*c*\tan(f*x + e) + c)^(5/2)*(29*A + 81*I*B)*c^6 - 16*(-I*c*\tan(f*x + e) + c)^(3/2)*(17*A + 53*I*B)*c^7 + 12*\sqrt{-I*c*\tan(f*x + e) + c}*(19*A + 63*I*B)*c^8)/((-I*c*\tan(f*x + e) + c)^3*a^3 - 6*(-I*c*\tan(f*x + e) + c)^2*a^3*c + 12*(-I*c*\tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3) + 32*(I*(-I*c*\tan(f*x + e) + c)^(3/2)*B*c^4 + 3*\sqrt{-I*c*\tan(f*x + e) + c}*(A + 7*I*B)*c^5)/a^3)/(c*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs.  $2(237) = 474$ .

time = 3.78, size = 488, normalized size = 1.68

$$105\sqrt{2}(A+5iB)c^{\frac{11}{2}} \log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right) - \frac{4(3(-i \tan(fx+e)+c)^2(29A+81iB)c^6-16(-i \tan(fx+e)+c)^2(17A+53iB)c^7+12\sqrt{-ic \tan(fx+e)+c}(19A+63iB)c^8)}{(-i \tan(fx+e)+c)^3a^3-6(-i \tan(fx+e)+c)^2a^3c+12(-i \tan(fx+e)+c)a^3c^2-8a^3c^3} + \frac{32(i(-i \tan(fx+e)+c)^2Bc^3+3\sqrt{-ic \tan(fx+e)+c}(A+7iB)c^2)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x,
algorithm="fricas")
```

```
[Out] -1/24*(105*sqrt(1/2)*(a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6*I*f*x + 6*I*e)
)*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c^9/(a^6*f^2))*log(-35/2*((I*A - 5*B)*c^5
+ sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(-(A^2 + 10*I*
A*B - 25*B^2)*c^9/(a^6*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x -
I*e)/(a^3*f)) - 105*sqrt(1/2)*(a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6*I*f*
x + 6*I*e))*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c^9/(a^6*f^2))*log(-35/2*((I*A
- 5*B)*c^5 - sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(-(A
^2 + 10*I*A*B - 25*B^2)*c^9/(a^6*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e
^(-I*f*x - I*e)/(a^3*f)) + sqrt(2)*(105*(-I*A + 5*B)*c^4*e^(8*I*f*x + 8*I*e
) + 140*(-I*A + 5*B)*c^4*e^(6*I*f*x + 6*I*e) + 21*(-I*A + 5*B)*c^4*e^(4*I*f
*x + 4*I*e) + 6*(I*A - 5*B)*c^4*e^(2*I*f*x + 2*I*e) + 8*(-I*A + B)*c^4)*sqr
t(c/(e^(2*I*f*x + 2*I*e) + 1)))/(a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6*I*f
*x + 6*I*e))
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,
x)
```

```
[Out] Timed out
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(9/2)/(I*a*tan(f*x +
e) + a)^3, x)
```

**Mupad** [B]

time = 9.59, size = 441, normalized size = 1.52

$\frac{A^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2})}{6(c - \tan(e + fx))^3 - 12 B^2 (c - \tan(e + fx))^2 - 6 A^2 (c - \tan(e + fx)) + 8 A^4} - 8 A^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 12 A^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 12 A^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 12 A^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2})}{a^2} - \frac{14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2}) - 14 B^2 \sqrt{c - \tan(e + fx)} \operatorname{Ei}(\frac{A^2 \sqrt{c - \tan(e + fx)}}{a^2})}{3 a^2} - \frac{\sqrt{2} A(-1)^{\frac{1}{2}} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right)}{8 a^2} - \frac{\sqrt{2} B(-1)^{\frac{1}{2}} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right) \operatorname{Ei}\left(\frac{\sqrt{2} \sqrt{c - \tan(e + fx)}}{a}\right)}{176 a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B*\tan(e + f*x))*(c - c*\tan(e + f*x)*1i)^{(9/2)})/(a + a*\tan(e + f*x)*1i)^3, x)$

[Out]  $((A*c^7*(c - c*\tan(e + f*x)*1i)^{(1/2)}*19i)/(a^3*f) - (A*c^6*(c - c*\tan(e + f*x)*1i)^{(3/2)}*68i)/(3*a^3*f) + (A*c^5*(c - c*\tan(e + f*x)*1i)^{(5/2)}*29i)/(4*a^3*f))/(6*c*(c - c*\tan(e + f*x)*1i)^2 - 12*c^2*(c - c*\tan(e + f*x)*1i) - (c - c*\tan(e + f*x)*1i)^3 + 8*c^3) - (63*B*c^7*(c - c*\tan(e + f*x)*1i)^{(1/2)} - (212*B*c^6*(c - c*\tan(e + f*x)*1i)^{(3/2)})/3 + (81*B*c^5*(c - c*\tan(e + f*x)*1i)^{(5/2)})/4)/(8*a^3*c^3*f - a^3*f*(c - c*\tan(e + f*x)*1i)^3 + 6*a^3*c*f*(c - c*\tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*\tan(e + f*x)*1i)) + (A*c^4*(c - c*\tan(e + f*x)*1i)^{(1/2)}*2i)/(a^3*f) - (14*B*c^4*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(a^3*f) - (2*B*c^3*(c - c*\tan(e + f*x)*1i)^{(3/2)})/(3*a^3*f) - (2^{(1/2)}*A*(-c)^{(9/2)}*atan((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)}))/(2*(-c)^{(1/2)}))*35i)/(8*a^3*f) - (2^{(1/2)}*B*c^{(9/2)}*atan((2^{(1/2)}*(c - c*\tan(e + f*x)*1i)^{(1/2)}*1i)/(2*c^{(1/2)}))*175i)/(8*a^3*f)$

$$3.780 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=252

$$-\frac{5(iA-13B)c^{7/2} \tanh^{-1}\left(\frac{\sqrt{c-ict \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}a^3f} + \frac{5(iA-13B)c^3\sqrt{c-ict \tan(e+fx)}}{16a^3f} + \frac{5(iA-13B)c^2}{48a^3f(1+}$$

[Out]  $-5/16*(I*A-13*B)*c^{(7/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})/a^3/f*2^{(1/2)}+5/16*(I*A-13*B)*c^3*(c-I*c*\tan(f*x+e))^{(1/2)}/a^3/f+5/48*(I*A-13*B)*c^2*(c-I*c*\tan(f*x+e))^{(3/2)}/a^3/f/(1+I*\tan(f*x+e))-1/24*(I*A-13*B)*c*(c-I*c*\tan(f*x+e))^{(5/2)}/a^3/f/(1+I*\tan(f*x+e))^2+1/6*(I*A-B)*(c-I*c*\tan(f*x+e))^{(7/2)}/a^3/f/(1+I*\tan(f*x+e))^3$

**Rubi [A]**

time = 0.18, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 43, 52, 65, 214}

$$-\frac{5c^{7/2}(-13B+iA) \tanh^{-1}\left(\frac{\sqrt{c-ict \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}a^3f} + \frac{5c^3(-13B+iA)\sqrt{c-ict \tan(e+fx)}}{16a^3f} + \frac{5c^2(-13B+iA)(c-ict \tan(e+fx))^{3/2}}{48a^3f(1+i \tan(e+fx))} - \frac{c(-13B+iA)(c-ict \tan(e+fx))^{5/2}}{24a^3f(1+i \tan(e+fx))^2} + \frac{(-B+iA)(c-ict \tan(e+fx))^{7/2}}{6a^3f(1+i \tan(e+fx))^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[e+f*x])*(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)}]/(a+I*a*\operatorname{Tan}[e+f*x])^3, x]$

[Out]  $(-5*(I*A-13*B)*c^{(7/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]))/(8*\operatorname{Sqrt}[2]*a^3*f) + (5*(I*A-13*B)*c^3*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])/(16*a^3*f) + (5*(I*A-13*B)*c^2*(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)})/(48*a^3*f*(1+I*\operatorname{Tan}[e+f*x])) - ((I*A-13*B)*c*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)})/(24*a^3*f*(1+I*\operatorname{Tan}[e+f*x])^2) + ((I*A-B)*(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)})/(6*a^3*f*(1+I*\operatorname{Tan}[e+f*x])^3)$

**Rule 43**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x]$   
 $\&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{ILtQ}[m, -1] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{GtQ}[n, 0]$

**Rule 52**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \operatorname{Dist}[n*((b*c - a*d)/(b*(m+n+1))), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x]$   
 $\&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m+n+1, 0] \&\& \operatorname{IntegerQ}[n]$

$[m, 0] \&\& ( !IntegerQ[n] \parallel (GtQ[m, 0] \&\& LtQ[m - n, 0])) \&\& !ILtQ[m + n + 2, 0] \&\& IntLinearQ[a, b, c, d, m, n, x]$

### Rule 65

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] \rightarrow With[\{p = Denominator[m]\}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[\{a, b, c, d\}, x] \&\& NeQ[b*c - a*d, 0] \&\& LtQ[-1, m, 0] \&\& LeQ[-1, n, 0] \&\& LeQ[Denominator[n], Denominator[m]] \&\& IntLinearQ[a, b, c, d, m, n, x]$

### Rule 79

$Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x\_Symbol] \rightarrow Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[\{a, b, c, d, e, f, n\}, x] \&\& LtQ[p, -1] \&\& ( !LtQ[n, -1] \parallel IntegerQ[p] \parallel !(IntegerQ[n] \parallel !(EqQ[e, 0] \parallel !(EqQ[c, 0] \parallel LtQ[p, n]))))$

### Rule 214

$Int[((a_) + (b_)*(x_)^2)^(-1), x\_Symbol] \rightarrow Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[\{a, b\}, x] \&\& NegQ[a/b]$

### Rule 3669

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] \rightarrow Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& EqQ[b*c + a*d, 0] \&\& EqQ[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{7/2}}{(a + ia \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^4} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{7/2}}{6a^3 f (1 + i \tan(e + fx))^3} - \frac{((A + 13iB)c) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^4} dx, x, \tan(e + fx) \right)}{6a^3 f (1 + i \tan(e + fx))^3} \\
&= -\frac{(iA - 13B)c(c - ic \tan(e + fx))^{5/2}}{24a^3 f (1 + i \tan(e + fx))^2} + \frac{(iA - B)(c - ic \tan(e + fx))^{7/2}}{6a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{5(iA - 13B)c^2(c - ic \tan(e + fx))^{3/2}}{48a^3 f (1 + i \tan(e + fx))} - \frac{(iA - 13B)c(c - ic \tan(e + fx))^{5/2}}{24a^3 f (1 + i \tan(e + fx))^2} \\
&= \frac{5(iA - 13B)c^3 \sqrt{c - ic \tan(e + fx)}}{16a^3 f} + \frac{5(iA - 13B)c^2(c - ic \tan(e + fx))^{3/2}}{48a^3 f (1 + i \tan(e + fx))} \\
&= \frac{5(iA - 13B)c^3 \sqrt{c - ic \tan(e + fx)}}{16a^3 f} + \frac{5(iA - 13B)c^2(c - ic \tan(e + fx))^{3/2}}{48a^3 f (1 + i \tan(e + fx))} \\
&= -\frac{5(iA - 13B)c^{7/2} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{8\sqrt{2} a^3 f} + \frac{5(iA - 13B)c^2(c - ic \tan(e + fx))^{3/2}}{48a^3 f (1 + i \tan(e + fx))}
\end{aligned}$$

**Mathematica [F]**

time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] \$Aborted

**Maple [A]**

time = 0.35, size = 168, normalized size = 0.67

method	result
--------	--------

derivativedivides	$2ic^3 \left( iB \sqrt{c - ic \tan(fx + e)} + c \left( \frac{8 \left( \frac{47iB}{128} + \frac{11A}{128} \right) (c - ic \tan(fx + e))^{\frac{5}{2}} + 8 \left( -\frac{29}{24} iBc - \frac{5}{24} Ac \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 8 \left( \frac{33}{32} \right) (c + ic \tan(fx + e))^3}{(c + ic \tan(fx + e))^3} \right) \right)$
default	$2ic^3 \left( iB \sqrt{c - ic \tan(fx + e)} + c \left( \frac{8 \left( \frac{47iB}{128} + \frac{11A}{128} \right) (c - ic \tan(fx + e))^{\frac{5}{2}} + 8 \left( -\frac{29}{24} iBc - \frac{5}{24} Ac \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 8 \left( \frac{33}{32} \right) (c + ic \tan(fx + e))^3}{(c + ic \tan(fx + e))^3} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x,method =_RETURNVERBOSE)`

[Out]  $2*I/f/a^3*c^3*(I*B*(c-I*c*tan(f*x+e))^{(1/2)}+c*(8*((47/128*I*B+11/128*A)*(c-I*c*tan(f*x+e))^{(5/2)}+(-29/24*I*B*c-5/24*A*c)*(c-I*c*tan(f*x+e))^{(3/2)}+(33/32*I*B*c^2+5/32*A*c^2)*(c-I*c*tan(f*x+e))^{(1/2)}))/(c+I*c*tan(f*x+e))^3-5/4*(13/8*I*B+1/8*A)*2^{(1/2)}/c^{(1/2)}*arctanh(1/2*(c-I*c*tan(f*x+e))^{(1/2)}*2^{(1/2)})/c^{(1/2))}$

**Maxima [A]**

time = 0.51, size = 249, normalized size = 0.99

$$i \left( \frac{15 \sqrt{2} (A+13iB)c^2 \log \left( \frac{-\sqrt{2} \sqrt{c} \sqrt{-ic \tan(fx+e)+c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx+e)+c}} \right) + 192i \sqrt{-ic \tan(fx+e)+c} Bc^4 - \frac{4 \left( 3(-ic \tan(fx+e)+c)^{\frac{5}{2}} (11A+47iB)c^5 - 16(-ic \tan(fx+e)+c)^{\frac{3}{2}} (5A+29iB)c^4 + 12 \sqrt{-ic \tan(fx+e)+c} (5A+33iB)c^3 \right)}{(-ic \tan(fx+e)+c)^3 a^3 - 6(-ic \tan(fx+e)+c)^2 a^2 c + 12(-ic \tan(fx+e)+c) a c^2 - 8a^2 c^3}}{96cf} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x,algorithm="maxima")`

[Out]  $1/96*I*(15*\sqrt{2}*(A + 13*I*B)*c^{(9/2)}*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*tan(f*x + e) + c}))/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*tan(f*x + e) + c}))/a^3 + 192*I*\sqrt{-I*c*tan(f*x + e) + c}*B*c^4/a^3 - 4*(3*(-I*c*tan(f*x + e) + c)^{(5/2)}*(11*A + 47*I*B)*c^5 - 16*(-I*c*tan(f*x + e) + c)^{(3/2)}*(5*A + 29*I*B)*c^6 + 12*\sqrt{-I*c*tan(f*x + e) + c}*(5*A + 33*I*B)*c^7)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*a^3*c + 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3)/(c*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 415 vs.  $2(205) = 410$ .

time = 4.53, size = 415, normalized size = 1.65

$$\frac{15 \sqrt{2} \sqrt{c} \sqrt{-ic \tan(fx+e)+c} \log \left( \frac{-\sqrt{2} \sqrt{c} \sqrt{-ic \tan(fx+e)+c}}{\sqrt{2} \sqrt{c} + \sqrt{-ic \tan(fx+e)+c}} \right) + 192i \sqrt{-ic \tan(fx+e)+c} Bc^4 - \frac{4 \left( 3(-ic \tan(fx+e)+c)^{\frac{5}{2}} (11A+47iB)c^5 - 16(-ic \tan(fx+e)+c)^{\frac{3}{2}} (5A+29iB)c^4 + 12 \sqrt{-ic \tan(fx+e)+c} (5A+33iB)c^3 \right)}{(-ic \tan(fx+e)+c)^3 a^3 - 6(-ic \tan(fx+e)+c)^2 a^2 c + 12(-ic \tan(fx+e)+c) a c^2 - 8a^2 c^3}}{96cf}$$





[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(7/2))/(a + a\*tan(e + f\*x)\*1i)^3,x)

[Out] ((A\*c^6\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*5i)/(2\*a^3\*f) - (A\*c^5\*(c - c\*tan(e + f\*x)\*1i)^(3/2)\*10i)/(3\*a^3\*f) + (A\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(5/2)\*11i)/(8\*a^3\*f))/(6\*c\*(c - c\*tan(e + f\*x)\*1i)^2 - 12\*c^2\*(c - c\*tan(e + f\*x)\*1i) - (c - c\*tan(e + f\*x)\*1i)^3 + 8\*c^3) - ((33\*B\*c^6\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/2 - (58\*B\*c^5\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/3 + (47\*B\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(5/2))/8)/(8\*a^3\*c^3\*f - a^3\*f\*(c - c\*tan(e + f\*x)\*1i)^3 + 6\*a^3\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^2 - 12\*a^3\*c^2\*f\*(c - c\*tan(e + f\*x)\*1i)) - (2\*B\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a^3\*f) + (2^(1/2)\*A\*(-c)^(7/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*5i)/(16\*a^3\*f) + (65\*2^(1/2)\*B\*c^(7/2)\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2))))/(16\*a^3\*f)

$$3.781 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=213

$$\frac{(iA + 11B)c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c - ict \tan(e+fx)}}{\sqrt{2} \sqrt{c}}\right)}{16\sqrt{2} a^3 f} - \frac{(iA + 11B)c^2 \sqrt{c - ict \tan(e+fx)}}{16a^3 f(1 + i \tan(e+fx))} + \frac{(iA + 11B)c(c - ict \tan(e+fx))^{5/2}}{24a^3 f(1 + i \tan(e+fx))^3}$$

[Out] 1/32\*(I\*A+11\*B)\*c^(5/2)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2)))/a^3/f\*2^(1/2)-1/16\*(I\*A+11\*B)\*c^2\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^3/f/(1+I\*tan(f\*x+e))+1/24\*(I\*A+11\*B)\*c\*(c-I\*c\*tan(f\*x+e))^(3/2)/a^3/f/(1+I\*tan(f\*x+e))^2+1/6\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(5/2)/a^3/f/(1+I\*tan(f\*x+e))^3

**Rubi [A]**

time = 0.17, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 43, 65, 214}

$$\frac{c^{5/2}(11B + iA) \tanh^{-1}\left(\frac{\sqrt{c - ict \tan(e+fx)}}{\sqrt{2} \sqrt{c}}\right)}{16\sqrt{2} a^3 f} - \frac{c^2(11B + iA) \sqrt{c - ict \tan(e+fx)}}{16a^3 f(1 + i \tan(e+fx))} + \frac{c(11B + iA)(c - ict \tan(e+fx))^{3/2}}{24a^3 f(1 + i \tan(e+fx))^2} + \frac{(-B + iA)(c - ict \tan(e+fx))^{5/2}}{6a^3 f(1 + i \tan(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2))/(a + I\*a\*Tan[e + f\*x])^3, x]

[Out] ((I\*A + 11\*B)\*c^(5/2)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(16\*Sqrt[2]\*a^3\*f) - ((I\*A + 11\*B)\*c^2\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(16\*a^3\*f\*(1 + I\*Tan[e + f\*x])) + ((I\*A + 11\*B)\*c\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(24\*a^3\*f\*(1 + I\*Tan[e + f\*x])^2) + ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^(5/2))/(6\*a^3\*f\*(1 + I\*Tan[e + f\*x])^3)

**Rule 43**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^n/(b\*(m + 1))), x] - Dist[d\*(n/(b\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i \tan(e + fx))^{5/2}}{(a + ia \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - i \tan(e + fx))^{5/2}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{((A - 11iB)c) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^4} dx, x, \tan(e + fx)\right)}{6a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(iA + 11B)c(c - i \tan(e + fx))^{3/2}}{24a^3 f (1 + i \tan(e + fx))^2} + \frac{(iA - B)(c - i \tan(e + fx))^{5/2}}{6a^3 f (1 + i \tan(e + fx))^3} \\
&= -\frac{(iA + 11B)c^2 \sqrt{c - i \tan(e + fx)}}{16a^3 f (1 + i \tan(e + fx))} + \frac{(iA + 11B)c(c - i \tan(e + fx))^{3/2}}{24a^3 f (1 + i \tan(e + fx))^2} \\
&= -\frac{(iA + 11B)c^2 \sqrt{c - i \tan(e + fx)}}{16a^3 f (1 + i \tan(e + fx))} + \frac{(iA + 11B)c(c - i \tan(e + fx))^{3/2}}{24a^3 f (1 + i \tan(e + fx))^2} \\
&= \frac{(iA + 11B)c^{5/2} \tanh^{-1}\left(\frac{\sqrt{c - i \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{16\sqrt{2} a^3 f} - \frac{(iA + 11B)c(c - i \tan(e + fx))^{3/2}}{24a^3 f (1 + i \tan(e + fx))^2}
\end{aligned}$$

**Mathematica [A]**

time = 2.43, size = 227, normalized size = 1.07

$$\frac{\sec^2(e + fx)(\cos(fx) + i \sin(fx))^2(A + B \tan(e + fx)) \left( \sqrt{2} (iA + 11B)c^{5/2} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right) (\cos(3e) + i \sin(3e)) + \frac{3}{2} c^2 \cos(e + fx)(\cos(3fx) - i \sin(3fx))(2iA + 22B + (5iA - 41B) \cos(2(e + fx)) + (11A - 25iB) \sin(2(e + fx))) \sqrt{c - ic \tan(e + fx)} \right)}{32f(A \cos(e + fx) + B \sin(e + fx))(a + ia \tan(e + fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^3*(A + B*Tan[e + f*x])*(Sqrt[2]*(I*A + 11*B)*c^(5/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[3*e] + I*Sin[3*e]) + (2*c^2*Cos[e + f*x]*(Cos[3*f*x] - I*Sin[3*f*x])*((2*I)*A + 22*B + ((5*I)*A - 41*B)*Cos[2*(e + f*x)] + (11*A - (25*I)*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/3)/(32*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*(a + I*a*Tan[e + f*x])^3)
```

**Maple [A]**

time = 0.40, size = 147, normalized size = 0.69

method	result
derivativedivides	$2ic^3 \left( \frac{8 \left( \frac{21iB}{256} + \frac{A}{256} \right) (c - ic \tan(fx + e))^{\frac{5}{2}} + 8 \left( -\frac{11}{48} iBc + \frac{1}{48} Ac \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 8 \left( \frac{11}{64} iBc^2 - \frac{1}{64} Ac^2 \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^3} \right) f a^3$
default	$2ic^3 \left( \frac{8 \left( \frac{21iB}{256} + \frac{A}{256} \right) (c - ic \tan(fx + e))^{\frac{5}{2}} + 8 \left( -\frac{11}{48} iBc + \frac{1}{48} Ac \right) (c - ic \tan(fx + e))^{\frac{3}{2}} + 8 \left( \frac{11}{64} iBc^2 - \frac{1}{64} Ac^2 \right) \sqrt{c - ic \tan(fx + e)}}{(c + ic \tan(fx + e))^3} \right) f a^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x,method =_RETURNVERBOSE)
```

```
[Out] 2*I/f/a^3*c^3*(8*((21/256*I*B+1/256*A)*(c-I*c*tan(f*x+e))^(5/2)+(-11/48*I*B*c+1/48*A*c)*(c-I*c*tan(f*x+e))^(3/2)+(11/64*I*B*c^2-1/64*A*c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+1/8*(-11/8*I*B+1/8*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

**Maxima [A]**

time = 0.51, size = 220, normalized size = 1.03

$$i \left( \frac{3\sqrt{2} (A-11iB)c^{\frac{5}{2}} \log \left( -\frac{\sqrt{2} \sqrt{c - \sqrt{-i c \tan(fx + e) + c}}}{\sqrt{2} \sqrt{c} + \sqrt{-i c \tan(fx + e) + c}} \right)}{a^3} + \frac{4 \left( 3(-i c \tan(fx + e) + c)^{\frac{5}{2}} (A + 21iB)c^4 + 16(-i c \tan(fx + e) + c)^{\frac{3}{2}} (A - 11iB)c^5 - 12 \sqrt{-i c \tan(fx + e) + c} (A - 11iB)c^6 \right)}{(-i c \tan(fx + e) + c)^3 a^3 - 6(-i c \tan(fx + e) + c)^2 a^2 c + 12(-i c \tan(fx + e) + c) a^2 c^2 - 8 a^2 c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="maxima")

[Out] 
$$-1/192*I*(3*\sqrt{2}*(A - 11*I*B)*c^{7/2}*\log(-(\sqrt{2}*\sqrt{c} - \sqrt{-I*c*\tan(f*x + e) + c})/(\sqrt{2}*\sqrt{c} + \sqrt{-I*c*\tan(f*x + e) + c}))/a^3 + 4*(3*(-I*c*\tan(f*x + e) + c)^{5/2}*(A + 21*I*B)*c^4 + 16*(-I*c*\tan(f*x + e) + c)^{3/2}*(A - 11*I*B)*c^5 - 12*\sqrt{-I*c*\tan(f*x + e) + c}*(A - 11*I*B)*c^6)/((-I*c*\tan(f*x + e) + c)^3*a^3 - 6*(-I*c*\tan(f*x + e) + c)^2*a^3*c + 12*(-I*c*\tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3)/(c*f)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 416 vs.  $2(173) = 346$ .  
time = 3.58, size = 416, normalized size = 1.95

$$\left( 3\sqrt{\frac{a^2}{d}} \sqrt{\frac{(A^2 - 20AB - 12B^2)c^2}{d^2}} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{A^2 - 20AB - 12B^2}}{2\sqrt{20A^2 + 1}}\right) \sqrt{\frac{(A^2 - 20AB - 12B^2)c^2}{d^2}} \right) - 3\sqrt{\frac{a^2}{d}} \sqrt{\frac{(A^2 - 20AB - 12B^2)c^2}{d^2}} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{A^2 - 20AB - 12B^2}}{2\sqrt{20A^2 + 1}}\right) \sqrt{\frac{(A^2 - 20AB - 12B^2)c^2}{d^2}} \right) - \sqrt{(11A + 11B)A^{2m+1} - (11A - 11B)B^{2m+1} + 2(-3A - 7B)A^{2m}B + 8(-A + B)B^{2m}} \sqrt{\frac{c^2}{20A^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="fricas")

[Out] 
$$1/96*(3*\sqrt{1/2}*a^3*f*\sqrt{-(A^2 - 22*I*A*B - 121*B^2)*c^5/(a^6*f^2)})*e^{(6*I*f*x + 6*I*e)}*\log(1/8*((I*A + 11*B)*c^3 + \sqrt{2}*\sqrt{1/2}*(a^3*f*e^{(2*I*f*x + 2*I*e)} + a^3*f)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{-(A^2 - 22*I*A*B - 121*B^2)*c^5/(a^6*f^2)})))*e^{(-I*f*x - I*e)/(a^3*f)} - 3*\sqrt{1/2}*a^3*f*\sqrt{-(A^2 - 22*I*A*B - 121*B^2)*c^5/(a^6*f^2)}*e^{(6*I*f*x + 6*I*e)}*\log(1/8*((I*A + 11*B)*c^3 - \sqrt{2}*\sqrt{1/2}*(a^3*f*e^{(2*I*f*x + 2*I*e)} + a^3*f)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{-(A^2 - 22*I*A*B - 121*B^2)*c^5/(a^6*f^2)})))*e^{(-I*f*x - I*e)/(a^3*f)} - \sqrt{2}*(3*(I*A + 11*B)*c^2*e^{(6*I*f*x + 6*I*e)} - (-I*A - 11*B)*c^2*e^{(4*I*f*x + 4*I*e)} + 2*(-5*I*A - 7*B)*c^2*e^{(2*I*f*x + 2*I*e)} + 8*(-I*A + B)*c^2)*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})*e^{(-6*I*f*x - 6*I*e)/(a^3*f)}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\left( \int \frac{A^2 \sqrt{-i \tan(e + f x) + c}}{\tan^2(e + f x) - 3 \tan^2(e + f x) - 3 \tan(e + f x) + 1} dx + \int \frac{A^2 \sqrt{-i \tan(e + f x) + c} \tan^2(e + f x)}{\tan^2(e + f x) - 3 \tan^2(e + f x) - 3 \tan(e + f x) + 1} dx + \int \frac{B^2 \sqrt{-i \tan(e + f x) + c} \tan(e + f x)}{\tan^2(e + f x) - 3 \tan^2(e + f x) - 3 \tan(e + f x) + 1} dx + \int \frac{B^2 \sqrt{-i \tan(e + f x) + c} \tan^2(e + f x)}{\tan^2(e + f x) - 3 \tan^2(e + f x) - 3 \tan(e + f x) + 1} dx + \int \frac{2 A^2 \sqrt{-i \tan(e + f x) + c} \tan(e + f x)}{\tan^2(e + f x) - 3 \tan^2(e + f x) - 3 \tan(e + f x) + 1} dx + \int \frac{2 B^2 \sqrt{-i \tan(e + f x) + c} \tan^2(e + f x)}{\tan^2(e + f x) - 3 \tan^2(e + f x) - 3 \tan(e + f x) + 1} dx \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^3,x)

[Out] 
$$I*(\text{Integral}(A*c**2*\sqrt{-I*c*\tan(e + f*x) + c}/(\tan(e + f*x)**3 - 3*I*\tan(e + f*x)**2 - 3*\tan(e + f*x) + I), x) + \text{Integral}(-A*c**2*\sqrt{-I*c*\tan(e + f*x) + c}*\tan(e + f*x)**2/(\tan(e + f*x)**3 - 3*I*\tan(e + f*x)**2 - 3*\tan(e + f*x) + c)*\tan(e + f*x)**2/(\tan(e + f*x)**3 - 3*I*\tan(e + f*x)**2 - 3*\tan(e + f*x) + c)), x)$$

```
f*x) + I), x) + Integral(B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(
tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(
-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3/(tan(e + f*x)**3 - 3*I*
tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-2*I*A*c**2*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*t
an(e + f*x) + I), x) + Integral(-2*I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan
(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I),
x))/a**3
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x +
e) + a)^3, x)
```

**Mupad [B]**

time = 9.42, size = 360, normalized size = 1.69

$$\frac{-\frac{A^2 \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{4 a^2 f} + \frac{4 A^2 c \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{16 a^2 f} + \frac{4 A^2 c \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{16 a^2 f}}{6(c - c \tan(e + f x)) \operatorname{li} \frac{1}{2} - 12 c^2 (c - c \tan(e + f x)) \operatorname{li} \frac{1}{2} - (c - c \tan(e + f x)) \operatorname{li} \frac{1}{2} + 8 c^2} - \frac{11 B A^2 \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{8 a^2 f} - \frac{11 B A^2 c \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{8 a^2 f} + \frac{21 B A^2 c \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{32 a^2 f}}{8 a^2 c^2 f - a^2 f (c - c \tan(e + f x)) \operatorname{li} \frac{1}{2} + 6 a^2 c f (c - c \tan(e + f x)) \operatorname{li} \frac{1}{2} - 12 a^2 c^2 f (c - c \tan(e + f x)) \operatorname{li} \frac{1}{2}} + \frac{\sqrt{2} A (-c)^{5/2} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{z \sqrt{-c}}\right) \operatorname{li} \frac{1}{2}}{32 a^2 f} + \frac{11 \sqrt{2} B c^{5/2} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{c - c \tan(e + f x)} \operatorname{li} \frac{1}{2}}{z \sqrt{-c}}\right) \operatorname{li} \frac{1}{2}}{32 a^2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*x)
)*1i)^3,x)
```

```
[Out] ((A*c^4*(c - c*tan(e + f*x)*1i)^(3/2)*1i)/(3*a^3*f) - (A*c^5*(c - c*tan(e +
f*x)*1i)^(1/2)*1i)/(4*a^3*f) + (A*c^3*(c - c*tan(e + f*x)*1i)^(5/2)*1i)/(1
6*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i) -
(c - c*tan(e + f*x)*1i)^3 + 8*c^3) - ((11*B*c^5*(c - c*tan(e + f*x)*1i)^(1
/2))/4 - (11*B*c^4*(c - c*tan(e + f*x)*1i)^(3/2))/3 + (21*B*c^3*(c - c*tan(
e + f*x)*1i)^(5/2))/16)/(8*a^3*c^3*f - a^3*f*(c - c*tan(e + f*x)*1i)^3 + 6*
a^3*c*f*(c - c*tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*tan(e + f*x)*1i)) +
(2^(1/2)*A*(-c)^(5/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)
^(1/2)))*1i)/(32*a^3*f) + (11*2^(1/2)*B*c^(5/2)*atanh((2^(1/2)*(c - c*tan(e
+ f*x)*1i)^(1/2))/(2*c^(1/2))))/(32*a^3*f)
```

$$3.782 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^3} dx$$

**Optimal.** Leaf size=211

$$\frac{(iA + 3B)c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c - ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}a^3f} + \frac{(iA + 3B)c\sqrt{c - ictan(e+fx)}}{8a^3f(1 + i \tan(e+fx))^2} - \frac{(iA + 3B)c\sqrt{c - ictan(e+fx)}}{32a^3f(1 + i \tan(e+fx))^3}$$

[Out]  $-1/64*(I*A+3*B)*c^{(3/2)}*\operatorname{arctanh}(1/2*(c-I*c*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})/a^3/f*2^{(1/2)}+1/8*(I*A+3*B)*c*(c-I*c*\tan(f*x+e))^{(1/2)}/a^3/f/(1+I*\tan(f*x+e))^{(1/2)}-1/32*(I*A+3*B)*c*(c-I*c*\tan(f*x+e))^{(1/2)}/a^3/f/(1+I*\tan(f*x+e))+1/6*(I*A-B)*(c-I*c*\tan(f*x+e))^{(3/2)}/a^3/f/(1+I*\tan(f*x+e))^3$

**Rubi** [A]

time = 0.17, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 43, 44, 65, 214}

$$\frac{c^{3/2}(3B + iA) \tanh^{-1}\left(\frac{\sqrt{c - ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}a^3f} - \frac{c(3B + iA)\sqrt{c - ictan(e+fx)}}{32a^3f(1 + i \tan(e+fx))} + \frac{c(3B + iA)\sqrt{c - ictan(e+fx)}}{8a^3f(1 + i \tan(e+fx))^2} + \frac{(-B + iA)(c - ictan(e+fx))^{3/2}}{6a^3f(1 + i \tan(e+fx))^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)}]/(a + I*a*\operatorname{Tan}[e + f*x])^3, x]$

[Out]  $-1/32*((I*A + 3*B)*c^{(3/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]))/(\operatorname{Sqrt}[2]*a^3*f) + ((I*A + 3*B)*c*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(8*a^3*f*(1 + I*\operatorname{Tan}[e + f*x])^2) - ((I*A + 3*B)*c*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(32*a^3*f*(1 + I*\operatorname{Tan}[e + f*x])) + ((I*A - B)*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)})/(6*a^3*f*(1 + I*\operatorname{Tan}[e + f*x])^3)$

**Rule 43**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \operatorname{Dist}[d*(n/(b*(m + 1))), \operatorname{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{ILtQ}[m, -1] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{GtQ}[n, 0]$

**Rule 44**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)})/((b*c - a*d)*(m + 1)), x] - \operatorname{Dist}[d*((m + n + 2)/((b*c - a*d)*(m + 1))), \operatorname{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{ILtQ}[m, -1] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{LtQ}[n, 0]$

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(- (b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^{3/2}}{(a + i a \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^4} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - i c \tan(e + fx))^{3/2}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{((A - 3iB)c) \text{Subst}}{6a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(iA + 3B)c \sqrt{c - i c \tan(e + fx)}}{8a^3 f (1 + i \tan(e + fx))^2} + \frac{(iA - B)(c - i c \tan(e + fx))^{3/2}}{6a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(iA + 3B)c \sqrt{c - i c \tan(e + fx)}}{8a^3 f (1 + i \tan(e + fx))^2} - \frac{(iA + 3B)c \sqrt{c - i c \tan(e + fx)}}{32a^3 f (1 + i \tan(e + fx))^2} \\
&= \frac{(iA + 3B)c \sqrt{c - i c \tan(e + fx)}}{8a^3 f (1 + i \tan(e + fx))^2} - \frac{(iA + 3B)c \sqrt{c - i c \tan(e + fx)}}{32a^3 f (1 + i \tan(e + fx))^2} \\
&= -\frac{(iA + 3B)c^{3/2} \tanh^{-1} \left( \frac{\sqrt{c - i c \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{32\sqrt{2} a^3 f} + \frac{(iA + 3B)c \sqrt{c - i c \tan(e + fx)}}{32a^3 f (1 + i \tan(e + fx))^2}
\end{aligned}$$

**Mathematica [A]**

time = 2.07, size = 224, normalized size = 1.06

$$\frac{\sec^2(e + fx)(\cos(fx) + i \sin(fx))^3(A + B \tan(e + fx)) \left( \sqrt{2} (A - 3iB)c^{3/2} \tanh^{-1} \left( \frac{\sqrt{c - i c \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right) - i \cos(3e) + \sin(3e) \right) + \frac{2}{3} c \cos(e + fx)(\cos(3fx) - i \sin(3fx))(2(7iA + 5B) + (11iA + B) \cos(2(e + fx)) + (5A + 17iB) \sin(2(e + fx))) \sqrt{c - i c \tan(e + fx)}}{64f(A \cos(e + fx) + B \sin(e + fx))(a + i a \tan(e + fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[f*x] + I*Sin[f*x])^3*(A + B*Tan[e + f*x])*(Sqrt[2]*(A - (3*I)*B)*c^(3/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c]])*((-I)*Cos[3*e] + Sin[3*e]) + (2*c*Cos[e + f*x]*(Cos[3*f*x] - I*Sin[3*f*x])*(2*((7*I)*A + 5*B) + ((11*I)*A + B)*Cos[2*(e + f*x)] + (5*A + (17*I)*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/3))/(64*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*(a + I*a*Tan[e + f*x])^3)
```

**Maple [A]**

time = 0.40, size = 139, normalized size = 0.66

method	result
--------	--------



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="fricas")

[Out] 1/192\*(3\*sqrt(1/2)\*a^3\*f\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c^3/(a^6\*f^2))\*e^(6\*I\*f\*x + 6\*I\*e)\*log(1/16\*((-I\*A - 3\*B)\*c^2 + sqrt(2)\*sqrt(1/2)\*(a^3\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a^3\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c^3/(a^6\*f^2)))\*e^(-I\*f\*x - I\*e)/(a^3\*f)) - 3\*sqrt(1/2)\*a^3\*f\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c^3/(a^6\*f^2))\*e^(6\*I\*f\*x + 6\*I\*e)\*log(1/16\*((-I\*A - 3\*B)\*c^2 - sqrt(2)\*sqrt(1/2)\*(a^3\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a^3\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(A^2 - 6\*I\*A\*B - 9\*B^2)\*c^3/(a^6\*f^2)))\*e^(-I\*f\*x - I\*e)/(a^3\*f)) - sqrt(2)\*(3\*(-I\*A - 3\*B)\*c\*e^(6\*I\*f\*x + 6\*I\*e) - (17\*I\*A + 19\*B)\*c\*e^(4\*I\*f\*x + 4\*I\*e) + 2\*(-11\*I\*A - B)\*c\*e^(2\*I\*f\*x + 2\*I\*e) + 8\*(-I\*A + B)\*c)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*e^(-6\*I\*f\*x - 6\*I\*e)/(a^3\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \frac{Ac\sqrt{-ic\tan(e+fx)+c}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i} dx + \int \frac{Bc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i} dx + \int \left( \frac{-iAc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i} \right) dx + \int \left( \frac{-iBc\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i} \right) dx \right)$$

a<sup>3</sup>

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e))^3,x)

[Out] I\*(Integral(A\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)/(tan(e + f\*x)\*\*3 - 3\*I\*tan(e + f\*x)\*\*2 - 3\*tan(e + f\*x) + I), x) + Integral(B\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)/(tan(e + f\*x)\*\*3 - 3\*I\*tan(e + f\*x)\*\*2 - 3\*tan(e + f\*x) + I), x) + Integral(-I\*A\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)/(tan(e + f\*x)\*\*3 - 3\*I\*tan(e + f\*x)\*\*2 - 3\*tan(e + f\*x) + I), x) + Integral(-I\*B\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*2/(tan(e + f\*x)\*\*3 - 3\*I\*tan(e + f\*x)\*\*2 - 3\*tan(e + f\*x) + I), x))/a\*\*3

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2)/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(3/2)/(I\*a\*tan(f\*x + e) + a)^3, x)

**Mupad [B]**

time = 9.41, size = 360, normalized size = 1.71

$$\frac{\frac{A^2 \sqrt{c - \tan(e + f x)} \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) + A^2 c \operatorname{atanh} \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) - A^2 c \operatorname{atanh} \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right)}{6c(c - \tan(e + f x))^3 - 12c^2(c - \tan(e + f x)) - (c - \tan(e + f x))^3 + 8c^3} - \frac{3B^2 \sqrt{c - \tan(e + f x)} \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) + B^2 c \operatorname{atanh} \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) + 3B^2 c \operatorname{atanh} \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right)}{8a^2 c^2 f - a^2 f(c - \tan(e + f x))^3 + 6a^2 c f(c - \tan(e + f x))^3 - 12a^2 c^2 f(c - \tan(e + f x))} + \frac{\sqrt{2} A(-c)^{3/2} \operatorname{atan} \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) + 3\sqrt{2} B c^{3/2} \operatorname{atanh} \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right) \operatorname{li}_2 \left( \frac{\sqrt{2} \sqrt{c - \tan(e + f x)}}{1 + \sqrt{-c}} \right)}{64a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/(a + a\*tan(e + f\*x)\*1i)^3,x)

[Out] ((A\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*1i)/(8\*a^3\*f) + (A\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(3/2)\*1i)/(6\*a^3\*f) - (A\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(5/2)\*1i)/(3\*2\*a^3\*f))/(6\*c\*(c - c\*tan(e + f\*x)\*1i)^2 - 12\*c^2\*(c - c\*tan(e + f\*x)\*1i) - (c - c\*tan(e + f\*x)\*1i)^3 + 8\*c^3) - ((B\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(3/2))/6 - (3\*B\*c^4\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/8 + (3\*B\*c^2\*(c - c\*tan(e + f\*x)\*1i)^(5/2))/32)/(8\*a^3\*c^3\*f - a^3\*f\*(c - c\*tan(e + f\*x)\*1i)^3 + 6\*a^3\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^2 - 12\*a^3\*c^2\*f\*(c - c\*tan(e + f\*x)\*1i)) + (2^(1/2)\*A\*(-c)^(3/2)\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)^(1/2)))\*1i)/(64\*a^3\*f) - (3\*2^(1/2)\*B\*c^(3/2)\*atanh((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2)))/(64\*a^3\*f))

$$3.783 \quad \int \frac{(A+B \tan(e+fx)) \sqrt{c-ictan(e+fx)}}{(a+ia \tan(e+fx))^3} dx$$

Optimal. Leaf size=209

$$\frac{(5iA+7B)\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}a^3f} + \frac{(iA-B)\sqrt{c-ictan(e+fx)}}{6a^3f(1+i \tan(e+fx))^3} + \frac{(5iA+7B)\sqrt{c-ictan(e+fx)}}{48a^3f(1+i \tan(e+fx))^2}$$

[Out] 1/128\*(5\*I\*A+7\*B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))\*c^(1/2)/a^3/f\*2^(1/2)+1/6\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^3/f/(1+I\*tan(f\*x+e))^3+1/48\*(5\*I\*A+7\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^3/f/(1+I\*tan(f\*x+e))^2+1/64\*(5\*I\*A+7\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^3/f/(1+I\*tan(f\*x+e))

Rubi [A]

time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$ , Rules used = {3669, 79, 44, 65, 214}

$$\frac{(-B+iA)\sqrt{c-ictan(e+fx)}}{6a^3f(1+i \tan(e+fx))^3} + \frac{(7B+5iA)\sqrt{c-ictan(e+fx)}}{64a^3f(1+i \tan(e+fx))} + \frac{(7B+5iA)\sqrt{c-ictan(e+fx)}}{48a^3f(1+i \tan(e+fx))^2} + \frac{\sqrt{c}(7B+5iA) \tanh^{-1}\left(\frac{\sqrt{c-ictan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}a^3f}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] (((5\*I)\*A + 7\*B)\*Sqrt[c]\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(64\*Sqrt[2]\*a^3\*f) + ((I\*A - B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(6\*a^3\*f\*(1 + I\*Tan[e + f\*x])^3) + (((5\*I)\*A + 7\*B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(48\*a^3\*f\*(1 + I\*Tan[e + f\*x])^2) + (((5\*I)\*A + 7\*B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(64\*a^3\*f\*(1 + I\*Tan[e + f\*x]))

Rule 44

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{(a + ia \tan(e + fx))^3} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^4 \sqrt{c-icx}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{((5A - 7iB)c) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^4 \sqrt{c-icx}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{(5iA + 7B) \sqrt{c - ic \tan(e + fx)}}{48a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{(5iA + 7B) \sqrt{c - ic \tan(e + fx)}}{48a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{(5iA + 7B) \sqrt{c - ic \tan(e + fx)}}{48a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{6a^3 f (1 + i \tan(e + fx))^3} + \frac{(5iA + 7B) \sqrt{c - ic \tan(e + fx)}}{48a^3 f (1 + i \tan(e + fx))^3} \\
&= \frac{(5iA + 7B) \sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{64\sqrt{2} a^3 f} + \frac{(iA - B) \sqrt{c - ic \tan(e + fx)}}{6a^3 f (1 + i \tan(e + fx))^3}
\end{aligned}$$

### Mathematica [A]

time = 1.47, size = 225, normalized size = 1.08

$$\frac{\sec^2(e + fx)(\cos(fx) + i \sin(fx))^3(A + B \tan(e + fx)) \left( \sqrt{2} (5iA + 7B) \sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right) (\cos(3e) + i \sin(3e)) + \frac{2}{3} \cos(e + fx)(i \cos(3fx) + \sin(3fx))(26A + 2iB + (41A - 19iB) \cos(2(e + fx)) + 5(5iA + 7B) \sin(2(e + fx))) \sqrt{c - ic \tan(e + fx)} \right)}{128f(A \cos(e + fx) + B \sin(e + fx))(a + ia \tan(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a + I\*a\*Tan[e + f\*x])^3,x]

[Out] (Sec[e + f\*x]^2\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A + B\*Tan[e + f\*x])\*(Sqrt[2]\*((5\*I)\*A + 7\*B)\*Sqrt[c]\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])]\*(Cos[3\*e] + I\*Sin[3\*e]) + (2\*Cos[e + f\*x]\*(I\*Cos[3\*f\*x] + Sin[3\*f\*x])\*(26\*A + (2\*I)\*B + (41\*A - (19\*I)\*B)\*Cos[2\*(e + f\*x)] + 5\*((5\*I)\*A + 7\*B)\*Sin[2\*(e + f\*x)])\*Sqrt[c - I\*c\*Tan[e + f\*x]]/3))/(128\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(a + I\*a\*Tan[e + f\*x])^3)

### Maple [A]

time = 0.40, size = 147, normalized size = 0.70

method	result
--------	--------

derivativedivides	$2ic^3 \left( \frac{\frac{(-7iB+5A)(c-ic \tan(fx+e))^{\frac{5}{2}}}{128c^2} - \frac{(-7iB+5A)(c-ic \tan(fx+e))^{\frac{3}{2}}}{24c} + 8\left(\frac{11A}{256} - \frac{9iB}{256}\right) \sqrt{c-ic \tan(fx+e)}}{(c+ic \tan(fx+e))^3} + \frac{(-7iB+5A)}{f a^3} \right)$
default	$2ic^3 \left( \frac{\frac{(-7iB+5A)(c-ic \tan(fx+e))^{\frac{5}{2}}}{128c^2} - \frac{(-7iB+5A)(c-ic \tan(fx+e))^{\frac{3}{2}}}{24c} + 8\left(\frac{11A}{256} - \frac{9iB}{256}\right) \sqrt{c-ic \tan(fx+e)}}{(c+ic \tan(fx+e))^3} + \frac{(-7iB+5A)}{f a^3} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,method
=_RETURNVERBOSE)
```

```
[Out] 2*I/f/a^3*c^3*(8*(1/1024/c^2*(5*A-7*I*B)*(c-I*c*tan(f*x+e))^(5/2)-1/192/c*(
5*A-7*I*B)*(c-I*c*tan(f*x+e))^(3/2)+(11/256*A-9/256*I*B)*(c-I*c*tan(f*x+e)
^(1/2))/(c+I*c*tan(f*x+e))^3+1/256/c^(5/2)*(5*A-7*I*B)*2^(1/2)*arctanh(1/2*
(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

**Maxima [A]**

time = 0.54, size = 228, normalized size = 1.09

$$i \left( \frac{3\sqrt{2}(5A-7iB)c^{\frac{3}{2}} \log\left(\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right)}{a^3} + \frac{4\left(3(-ic \tan(fx+e)+c)^{\frac{3}{2}}(5A-7iB)c^2-16(-ic \tan(fx+e)+c)^{\frac{3}{2}}(5A-7iB)c^3+12\sqrt{-ic \tan(fx+e)+c}(11A-9iB)c^4\right)}{(-ic \tan(fx+e)+c)^3 a^3-6(-ic \tan(fx+e)+c)^2 a^3 c+12(-ic \tan(fx+e)+c) a^3 c^2-8 a^3 c^3} \right)$$

768 cf

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,
algorithm="maxima")
```

```
[Out] -1/768*I*(3*sqrt(2)*(5*A - 7*I*B)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c
*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 +
4*(3*(-I*c*tan(f*x + e) + c)^(5/2)*(5*A - 7*I*B)*c^2 - 16*(-I*c*tan(f*x + e)
+ c)^(3/2)*(5*A - 7*I*B)*c^3 + 12*sqrt(-I*c*tan(f*x + e) + c)*(11*A - 9*I
*B)*c^4)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*a^3*c
+ 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3)/(c*f)
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than the leaf count of optimal. 399 vs. 2(169) = 338.

time = 1.51, size = 399, normalized size = 1.91

$$\left( \frac{3\sqrt{2}i \sqrt{\frac{125A^2-70AB-49B^2}{256}} \log\left(\frac{\sqrt{2}\sqrt{\frac{125A^2-70AB-49B^2}{256}}-\sqrt{\frac{125A^2-70AB-49B^2}{256}}}{\sqrt{2}\sqrt{\frac{125A^2-70AB-49B^2}{256}}+\sqrt{\frac{125A^2-70AB-49B^2}{256}}}\right)}{a^3} + \frac{4\left(3(-ic \tan(fx+e)+c)^{\frac{3}{2}}(5A-7iB)c^2-16(-ic \tan(fx+e)+c)^{\frac{3}{2}}(5A-7iB)c^3+12\sqrt{-ic \tan(fx+e)+c}(11A-9iB)c^4\right)}{(-ic \tan(fx+e)+c)^3 a^3-6(-ic \tan(fx+e)+c)^2 a^3 c+12(-ic \tan(fx+e)+c) a^3 c^2-8 a^3 c^3} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="fricas")

[Out]  $\frac{1}{384} \cdot (3 \sqrt{1/2}) \cdot a^3 \cdot f \cdot \sqrt{-(25A^2 - 70IA*B - 49B^2)c / (a^6 f^2)} \cdot e^{(6I*f*x + 6I*e)} \cdot \log\left(\frac{1}{32} \cdot (\sqrt{2}) \cdot \sqrt{1/2} \cdot (a^3 \cdot f \cdot e^{(2I*f*x + 2I*e)} + a^3 \cdot f) \cdot \sqrt{c / (e^{(2I*f*x + 2I*e)} + 1)} \cdot \sqrt{-(25A^2 - 70IA*B - 49B^2)c / (a^6 f^2)} + (5I*A + 7*B) \cdot c\right) \cdot e^{(-I*f*x - I*e) / (a^3 \cdot f)} - 3 \sqrt{1/2} \cdot a^3 \cdot f \cdot \sqrt{-(25A^2 - 70IA*B - 49B^2)c / (a^6 f^2)} \cdot e^{(6I*f*x + 6I*e)} \cdot \log\left(-\frac{1}{32} \cdot (\sqrt{2}) \cdot \sqrt{1/2} \cdot (a^3 \cdot f \cdot e^{(2I*f*x + 2I*e)} + a^3 \cdot f) \cdot \sqrt{c / (e^{(2I*f*x + 2I*e)} + 1)} \cdot \sqrt{-(25A^2 - 70IA*B - 49B^2)c / (a^6 f^2)} - (5I*A + 7*B) \cdot c\right) \cdot e^{(-I*f*x - I*e) / (a^3 \cdot f)} - \sqrt{2} \cdot (3 \cdot (-11I*A - 9B) \cdot e^{(6I*f*x + 6I*e)} - (59I*A + 25B) \cdot e^{(4I*f*x + 4I*e)} + 2 \cdot (-17I*A + 5B) \cdot e^{(2I*f*x + 2I*e)} - 8I*A + 8B) \cdot \sqrt{c / (e^{(2I*f*x + 2I*e)} + 1)}) \cdot e^{(-6I*f*x - 6I*e) / (a^3 \cdot f)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{i \left( \int \frac{A \sqrt{-ic \tan(e+fx) + c}}{\tan^3(e+fx) - 3i \tan^2(e+fx) - 3 \tan(e+fx) + i} dx + \int \frac{B \sqrt{-ic \tan(e+fx) + c} \tan(e+fx)}{\tan^3(e+fx) - 3i \tan^2(e+fx) - 3 \tan(e+fx) + i} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3,  
x)

[Out]  $I \cdot (\text{Integral}(A \cdot \sqrt{-I \cdot c \cdot \tan(e + f \cdot x) + c} / (\tan(e + f \cdot x)^3 - 3 \cdot I \cdot \tan(e + f \cdot x)^2 - 3 \cdot \tan(e + f \cdot x) + I), x) + \text{Integral}(B \cdot \sqrt{-I \cdot c \cdot \tan(e + f \cdot x) + c} \cdot \tan(e + f \cdot x) / (\tan(e + f \cdot x)^3 - 3 \cdot I \cdot \tan(e + f \cdot x)^2 - 3 \cdot \tan(e + f \cdot x) + I), x) / a^3$

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(-I\*c\*tan(f\*x + e) + c)/(I\*a\*tan(f\*x + e) + a)^3, x)

**Mupad [B]**

time = 9.36, size = 355, normalized size = 1.70

$$\frac{\frac{7B(c-\tan(e+fx))^{5/2}}{48} + \frac{9B^2\sqrt{c-\tan(e+fx)}}{16} - \frac{7B^2(c-\tan(e+fx))^{3/2}}{12} + \frac{A^2\sqrt{c-\tan(e+fx)}}{12a^2} - \frac{A^2(c-\tan(e+fx))^{3/2}}{12a^2} + \frac{A(c-\tan(e+fx))^{5/2}}{8a^2} + \frac{\sqrt{2}A\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}}{2\sqrt{-c}}\right)}{128a^2f} + \frac{7\sqrt{2}B\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-\tan(e+fx)}}{2\sqrt{c}}\right)}{128a^2f}}{8a^3c^2f - a^2f(c-\tan(e+fx))^{3/2} + 6a^2c^2f(c-\tan(e+fx))^{5/2} - 12a^2c^2f(c-\tan(e+fx))^{3/2} + 6c(c-\tan(e+fx))^{5/2} - 12c^2(c-\tan(e+fx))^{3/2} - (c-\tan(e+fx))^{5/2} + 8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B*\tan(e + f*x))*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(a + a*\tan(e + f*x)*1i)^3, x)$

[Out]  $((7*B*c*(c - c*\tan(e + f*x)*1i)^{(5/2)})/64 + (9*B*c^3*(c - c*\tan(e + f*x)*1i)^{(1/2)})/16 - (7*B*c^2*(c - c*\tan(e + f*x)*1i)^{(3/2)})/12)/(8*a^3*c^3*f - a^3*f*(c - c*\tan(e + f*x)*1i)^3 + 6*a^3*c*f*(c - c*\tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*\tan(e + f*x)*1i)) + ((A*c^3*(c - c*\tan(e + f*x)*1i)^{(1/2)*11i)/(16*a^3*f) - (A*c^2*(c - c*\tan(e + f*x)*1i)^{(3/2)*5i)/(12*a^3*f) + (A*c*(c - c*\tan(e + f*x)*1i)^{(5/2)*5i)/(64*a^3*f))/(6*c*(c - c*\tan(e + f*x)*1i)^2 - 12*c^2*(c - c*\tan(e + f*x)*1i) - (c - c*\tan(e + f*x)*1i)^3 + 8*c^3) + (2^{(1/2)*A*(-c)^{(1/2)*\text{atan}((2^{(1/2)*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(2*(-c)^{(1/2))})}*5i)/(128*a^3*f) + (7*2^{(1/2)*B*c^{(1/2)*\text{atanh}((2^{(1/2)*(c - c*\tan(e + f*x)*1i)^{(1/2)})/(2*c^{(1/2))})})})/(128*a^3*f)$

$$3.784 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3 \sqrt{c-ic \tan(e+fx)}} dx$$

**Optimal.** Leaf size=245

$$\frac{5(7iA + 5B) \tanh^{-1} \left( \frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2} \sqrt{c}} \right)}{128\sqrt{2} a^3 \sqrt{c} f} - \frac{5(7iA + 5B)}{128a^3 f \sqrt{c-ic \tan(e+fx)}} + \frac{iA - B}{6a^3 f (1+i \tan(e+fx))^3 \sqrt{c-ic \tan(e+fx)}}$$

[Out] 5/256\*(7\*I\*A+5\*B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a^3/f\*2^(1/2)/c^(1/2)-5/128\*(7\*I\*A+5\*B)/a^3/f/(c-I\*c\*tan(f\*x+e))^(1/2)+1/6\*(I\*A-B)/a^3/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(1+I\*tan(f\*x+e))^3+1/48\*(7\*I\*A+5\*B)/a^3/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(1+I\*tan(f\*x+e))^2+5/192\*(7\*I\*A+5\*B)/a^3/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(1+I\*tan(f\*x+e))

**Rubi** [A]

time = 0.19, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 44, 53, 65, 214}

$$\frac{-B+iA}{6a^3 f (1+i \tan(e+fx))^3 \sqrt{c-ic \tan(e+fx)}} - \frac{5(5B+7iA)}{128a^3 f \sqrt{c-ic \tan(e+fx)}} + \frac{5(5B+7iA)}{192a^3 f (1+i \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}} + \frac{5B+7iA}{48a^3 f (1+i \tan(e+fx))^2 \sqrt{c-ic \tan(e+fx)}} + \frac{5(5B+7iA) \tanh^{-1} \left( \frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2} \sqrt{c}} \right)}{128\sqrt{2} a^3 \sqrt{c} f}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*Sqrt[c - I\*c\*Tan[e + f\*x]]),x]

[Out] (5\*((7\*I)\*A + 5\*B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(128\*Sqrt[2]\*a^3\*Sqrt[c]\*f) - (5\*((7\*I)\*A + 5\*B))/(128\*a^3\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + (I\*A - B)/(6\*a^3\*f\*(1 + I\*Tan[e + f\*x])^3\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + ((7\*I)\*A + 5\*B)/(48\*a^3\*f\*(1 + I\*Tan[e + f\*x])^2\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + (5\*((7\*I)\*A + 5\*B))/(192\*a^3\*f\*(1 + I\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 44**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

**Rule 53**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x]

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 \sqrt{c - ic \tan(e + fx)}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^{3/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 \sqrt{c - ic \tan(e + fx)}} + \frac{((7A - B) \tan^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right) - \frac{5(7iA + 5B)}{128\sqrt{2} a^3 \sqrt{c} f})}{48a^3 f} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 \sqrt{c - ic \tan(e + fx)}} + \frac{5(7iA + 5B) \tan^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{48a^3 f} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 \sqrt{c - ic \tan(e + fx)}} + \frac{5(7iA + 5B)}{128a^3 f \sqrt{c - ic \tan(e + fx)}} + \frac{i}{6a^3 f (1 + i \tan(e + fx))} \\
&= -\frac{5(7iA + 5B)}{128a^3 f \sqrt{c - ic \tan(e + fx)}} + \frac{i}{6a^3 f (1 + i \tan(e + fx))} \\
&= -\frac{5(7iA + 5B) \tan^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{128\sqrt{2} a^3 \sqrt{c} f} - \frac{i}{128a^3 f}
\end{aligned}$$

**Mathematica [A]**

time = 2.47, size = 181, normalized size = 0.74

$$\frac{(\cos(2(e + fx)) - i \sin(2(e + fx))) (15(7iA + 5B) e^{2i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \tanh^{-1}\left(\frac{\sqrt{1 + e^{2i(e+fx)}}}{\sqrt{1 + e^{2i(e+fx)}}}\right) + 2 \cos(e + fx) ((125iA + 7B) \cos(e + fx) + (-40iA - 56B) \cos(3(e + fx)) + (7A - 5iB)(-7 \sin(e + fx) + 8 \sin(3(e + fx)))))}{768a^3 f} \sqrt{c - ic \tan(e + fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]]), x]
```

```
[Out] ((Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*(15*((7*I)*A + 5*B)*E^((2*I)*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]]) + 2*Cos[e + f*x]*(((125*I)*A + 7*B)*Cos[e + f*x] + ((-40*I)*A - 56*B)*Cos[3*(e + f*x)] + (7*A - (5*I)*B)*(-7*Sin[e + f*x] + 8*Sin[3*(e + f*x)])))*Sqrt[c - I*c*Tan[e + f*x]]/(768*a^3*c*f)
```

**Maple [A]**

time = 0.40, size = 178, normalized size = 0.73

method	result
derivativedivides	$2ic^3 \left( \frac{8 \left( -\frac{9iB}{128} + \frac{19A}{128} \right) (c - ic \tan(fx+e))^{\frac{5}{2}} + 8 \left( \frac{7}{24} iBc - \frac{17}{24} Ac \right) (c - ic \tan(fx+e))^{\frac{3}{2}} + 8 \left( -\frac{7}{32} iBc^2 + \frac{29}{32} Ac^2 \right) \sqrt{c - ic \tan(fx+e)}}{(c + ic \tan(fx+e))^3} \right) \frac{1}{16c^3}$
default	$2ic^3 \left( \frac{8 \left( -\frac{9iB}{128} + \frac{19A}{128} \right) (c - ic \tan(fx+e))^{\frac{5}{2}} + 8 \left( \frac{7}{24} iBc - \frac{17}{24} Ac \right) (c - ic \tan(fx+e))^{\frac{3}{2}} + 8 \left( -\frac{7}{32} iBc^2 + \frac{29}{32} Ac^2 \right) \sqrt{c - ic \tan(fx+e)}}{(c + ic \tan(fx+e))^3} \right) \frac{1}{16c^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)`

[Out]  $2*I/f/a^3*c^3*(1/16/c^3*(8*((-9/128*I*B+19/128*A)*(c-I*c*tan(f*x+e))^(5/2)+(7/24*I*B*c-17/24*A*c)*(c-I*c*tan(f*x+e))^(3/2)+(-7/32*I*B*c^2+29/32*A*c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+5/4*(-5/8*I*B+7/8*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))-1/16/c^3*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2))$

**Maxima [A]**

time = 0.53, size = 250, normalized size = 1.02

$$i \left( \frac{4 \left( 15(-ic \tan(fx+e)+c)^3(7A-5iB)c-80(-ic \tan(fx+e)+c)^2(7A-5iB)c^2+132(-ic \tan(fx+e)+c)(7A-5iB)c^3-384(A-iB)c^4 \right)}{(-ic \tan(fx+e)+c)^{\frac{5}{2}}a^3-6(-ic \tan(fx+e)+c)^{\frac{3}{2}}a^3c+12(-ic \tan(fx+e)+c)^{\frac{1}{2}}a^3c^2-8\sqrt{-ic \tan(fx+e)+c}a^3c^3} + \frac{15\sqrt{2}(7A-5iB)\sqrt{c} \log\left(\frac{-\sqrt{2}\sqrt{c}-\sqrt{-ic \tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic \tan(fx+e)+c}}\right)}{a^3} \right)$$

1536 cf

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,algorithm="maxima")`

[Out]  $-1/1536*I*(4*(15*(-I*c*tan(f*x+e)+c)^3*(7*A-5*I*B)*c-80*(-I*c*tan(f*x+e)+c)^2*(7*A-5*I*B)*c^2+132*(-I*c*tan(f*x+e)+c)*(7*A-5*I*B)*c^3-384*(A-I*B)*c^4)/((-I*c*tan(f*x+e)+c)^(7/2)*a^3-6*(-I*c*tan(f*x+e)+c)^(5/2)*a^3*c+12*(-I*c*tan(f*x+e)+c)^(3/2)*a^3*c^2-8*sqr(-I*c*tan(f*x+e)+c)*a^3*c^3)+15*sqrt(2)*(7*A-5*I*B)*sqrt(c)*log((-sqrt(2)*sqrt(c)-sqrt(-I*c*tan(f*x+e)+c))/(sqrt(2)*sqrt(c)+sqrt(-I*c*tan(f*x+e)+c)))/a^3/(c*f)$



[In] integrate((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^3,x,  
algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^3\*sqrt(-I\*c\*tan(f\*x  
+ e) + c)), x)

Mupad [B]

time = 9.67, size = 394, normalized size = 1.61

$$\frac{B^2 \cdot \sqrt{B^2 - c^2} \operatorname{atanh}\left(\frac{B \sqrt{c - \tan(e + f x)}}{B^2 - c^2}\right) + \sqrt{B^2 - c^2} \operatorname{atanh}\left(\frac{B \sqrt{c - \tan(e + f x)}}{B^2 - c^2}\right)}{B^2 f (c - \tan(e + f x))^{3/2} - 6 B^2 c f (c - \tan(e + f x))^{5/2} - 8 a^2 B^2 f \sqrt{c - \tan(e + f x)} + 12 a^2 B^2 f (c - \tan(e + f x))^{3/2} + 6 c (c - \tan(e + f x))^{5/2} - (c - \tan(e + f x))^{7/2} + 8 a^2 \sqrt{c - \tan(e + f x)} - 12 a^2 (c - \tan(e + f x))^{3/2}} + \frac{4 c \tan(e + f x) \sqrt{a^2 f^2 - 4 B^2} - 4 B^2 \sqrt{a^2 f^2 - 4 B^2} + 4 c^2 \tan(e + f x) \sqrt{a^2 f^2 - 4 B^2}}{4 a^2 f \sqrt{a^2 f^2 - 4 B^2}} + \frac{A^2 c \operatorname{atanh}\left(\frac{A \sqrt{c - \tan(e + f x)}}{A^2 - c^2}\right)}{A^2 f \sqrt{a^2 f^2 - 4 B^2}} - \frac{\sqrt{c - \tan(e + f x)} \operatorname{atanh}\left(\frac{\sqrt{c - \tan(e + f x)}}{\sqrt{c - \tan(e + f x)}}\right)}{256 a^2 \sqrt{c - \tan(e + f x)}} + \frac{25 \sqrt{c - \tan(e + f x)} \operatorname{atanh}\left(\frac{\sqrt{c - \tan(e + f x)}}{\sqrt{c - \tan(e + f x)}}\right)}{256 a^2 \sqrt{c - \tan(e + f x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^3\*(c - c\*tan(e + f\*x)\*1i)  
^(1/2)),x)

[Out] (B\*c^3 - (25\*B\*(c - c\*tan(e + f\*x)\*1i)^3)/128 + (25\*B\*c\*(c - c\*tan(e + f\*x)  
\*1i)^2)/24 - (55\*B\*c^2\*(c - c\*tan(e + f\*x)\*1i))/32)/(a^3\*f\*(c - c\*tan(e + f  
\*x)\*1i)^(7/2) - 6\*a^3\*c\*f\*(c - c\*tan(e + f\*x)\*1i)^(5/2) - 8\*a^3\*c^3\*f\*(c -  
c\*tan(e + f\*x)\*1i)^(1/2) + 12\*a^3\*c^2\*f\*(c - c\*tan(e + f\*x)\*1i)^(3/2)) + ((  
A\*(c - c\*tan(e + f\*x)\*1i)^3\*35i)/(128\*a^3\*f) - (A\*c^3\*1i)/(a^3\*f) - (A\*c\*(c  
- c\*tan(e + f\*x)\*1i)^2\*35i)/(24\*a^3\*f) + (A\*c^2\*(c - c\*tan(e + f\*x)\*1i)\*77  
i)/(32\*a^3\*f))/(6\*c\*(c - c\*tan(e + f\*x)\*1i)^(5/2) - (c - c\*tan(e + f\*x)\*1i)  
^(7/2) + 8\*c^3\*(c - c\*tan(e + f\*x)\*1i)^(1/2) - 12\*c^2\*(c - c\*tan(e + f\*x)\*1  
i)^(3/2)) - (2^(1/2)\*A\*atan((2^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(2\*(-c)  
^(1/2)))\*35i)/(256\*a^3\*(-c)^(1/2)\*f) + (25\*2^(1/2)\*B\*atanh((2^(1/2)\*(c - c  
tan(e + f\*x)\*1i)^(1/2))/(2\*c^(1/2))))/(256\*a^3\*c^(1/2)\*f)



$$3.785 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ictan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=274

$$\frac{35(3iA + B) \tanh^{-1}\left(\frac{\sqrt{c - ictan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{256\sqrt{2} a^3 c^{3/2} f} - \frac{35(3iA + B)}{384a^3 f(c - ictan(e + fx))^{3/2}} + \frac{iA - B}{6a^3 f(1 + i \tan(e + fx))^3(c - ictan(e + fx))^{3/2}}$$

[Out] 35/512\*(3\*I\*A+B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a^3/c^(3/2)/f\*2^(1/2)-35/256\*(3\*I\*A+B)/a^3/c/f/(c-I\*c\*tan(f\*x+e))^(1/2)-35/384\*(3\*I\*A+B)/a^3/f/(c-I\*c\*tan(f\*x+e))^(3/2)+1/6\*(I\*A-B)/a^3/f/(1+I\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^(3/2)+1/16\*(3\*I\*A+B)/a^3/f/(1+I\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^(3/2)+7/64\*(3\*I\*A+B)/a^3/f/(1+I\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.21, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 44, 53, 65, 214}

$$\frac{35(B + 3iA) \tanh^{-1}\left(\frac{\sqrt{c - ictan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{256\sqrt{2} a^3 c^{3/2} f} + \frac{-B + iA}{6a^3 f(1 + i \tan(e + fx))^3(c - ictan(e + fx))^{3/2}} - \frac{35(B + 3iA)}{256a^3 f \sqrt{c - ictan(e + fx)}} - \frac{35(B + 3iA)}{384a^3 f(c - ictan(e + fx))^{3/2}} + \frac{7(B + 3iA)}{64a^3 f(1 + i \tan(e + fx))(c - ictan(e + fx))^{3/2}} + \frac{B + 3iA}{16a^3 f(1 + i \tan(e + fx))^2(c - ictan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^(3/2)),x]

[Out] (35\*((3\*I)\*A + B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])]/(256\*Sqrt[2]\*a^3\*c^(3/2)\*f) - (35\*((3\*I)\*A + B))/(384\*a^3\*f\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + (I\*A - B)/(6\*a^3\*f\*(1 + I\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + ((3\*I)\*A + B)/(16\*a^3\*f\*(1 + I\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + (7\*((3\*I)\*A + B))/(64\*a^3\*f\*(1 + I\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2)) - (35\*((3\*I)\*A + B))/(256\*a^3\*c\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 44**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

**Rule 53**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^4 (c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} + \frac{((3}}{16a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} + \frac{16a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}{16a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} + \frac{16a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}{16a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} \\
&= -\frac{35(3iA + B)}{384a^3 f (c - ict \tan(e + fx))^{3/2}} + \frac{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} \\
&= -\frac{35(3iA + B)}{384a^3 f (c - ict \tan(e + fx))^{3/2}} + \frac{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} \\
&= -\frac{35(3iA + B)}{384a^3 f (c - ict \tan(e + fx))^{3/2}} + \frac{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}{6a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}} \\
&= \frac{35(3iA + B) \tanh^{-1} \left( \frac{\sqrt{c - ict \tan(e + fx)}}{\sqrt{2} \sqrt{c}} \right)}{256 \sqrt{2} a^3 c^{3/2} f} - \frac{384a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}{384a^3 f (1 + i \tan(e + fx))^3 (c - ict \tan(e + fx))^{3/2}}
\end{aligned}$$

**Mathematica [A]**

time = 3.31, size = 206, normalized size = 0.75

$$\frac{(i \cos(e + fx) + \sin(e + fx)) (105(3A - iB)e^{i(e+fx)} \sqrt{1 + e^{2i(e+fx)}} \tanh^{-1}(\sqrt{1 + e^{2i(e+fx)}}) - 2 \cos(e + fx)(-165A - 9iB + 2(79A - 69iB) \cos(2(e + fx)) + 8(A - 3iB) \cos(4(e + fx)) + 258iA \sin(2(e + fx)) + 86B \sin(2(e + fx)) + 24iA \sin(4(e + fx)) + 8B \sin(4(e + fx)))) \sqrt{c - ict \tan(e + fx)}}{1536a^3 c^2 f}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^(3/2)), x]

[Out] ((I\*Cos[e + f\*x] + Sin[e + f\*x])\*(105\*(3\*A - I\*B)\*E^(I\*(e + f\*x))\*Sqrt[1 + E^((2\*I)\*(e + f\*x))]\*ArcTanh[Sqrt[1 + E^((2\*I)\*(e + f\*x))]] - 2\*Cos[e + f\*x]\*(-165\*A - (9\*I)\*B + 2\*(79\*A - (69\*I)\*B)\*Cos[2\*(e + f\*x)] + 8\*(A - (3\*I)\*B)\*Cos[4\*(e + f\*x)] + (258\*I)\*A\*Sin[2\*(e + f\*x)] + 86\*B\*Sin[2\*(e + f\*x)] + (24\*I)\*A\*Sin[4\*(e + f\*x)] + 8\*B\*Sin[4\*(e + f\*x)]))\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(1536\*a^3\*c^2\*f)

**Maple [A]**

time = 0.35, size = 205, normalized size = 0.75

method	result
derivativedivides	$2ic^3 \left( -\frac{-iB+2A}{16c^4 \sqrt{c-ic \tan(fx+e)}} - \frac{-iB+A}{48c^3 (c-ic \tan(fx+e))^{\frac{3}{2}}} + \frac{8 \left( -\frac{3iB}{256} + \frac{41A}{256} \right) (c-ic \tan(fx+e))^{\frac{5}{2}} + 8 \left( \frac{1}{48} iBc - \frac{35}{48} Ac \right) (c-ic \tan(fx+e))^{\frac{3}{2}}}{48c^3 (c-ic \tan(fx+e))^{\frac{3}{2}}} \right)$
default	$2ic^3 \left( -\frac{-iB+2A}{16c^4 \sqrt{c-ic \tan(fx+e)}} - \frac{-iB+A}{48c^3 (c-ic \tan(fx+e))^{\frac{3}{2}}} + \frac{8 \left( -\frac{3iB}{256} + \frac{41A}{256} \right) (c-ic \tan(fx+e))^{\frac{5}{2}} + 8 \left( \frac{1}{48} iBc - \frac{35}{48} Ac \right) (c-ic \tan(fx+e))^{\frac{3}{2}}}{48c^3 (c-ic \tan(fx+e))^{\frac{3}{2}}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $2*I/f/a^3*c^3*(-1/16/c^4*(2*A-I*B)/(c-I*c*tan(f*x+e))^{(1/2)}-1/48/c^3*(A-I*B)/(c-I*c*tan(f*x+e))^{(3/2)}+1/16/c^4*(8*((-3/256*I*B+41/256*A)*(c-I*c*tan(f*x+e))^{(5/2)}+(1/48*I*B*c-35/48*A*c)*(c-I*c*tan(f*x+e))^{(3/2)}+(3/64*I*B*c^2+5/64*A*c^2)*(c-I*c*tan(f*x+e))^{(1/2)}))/(c+I*c*tan(f*x+e))^3+35/8*(3/8*A-1/8*I*B)*2^{(1/2)}/c^{(1/2)}*\operatorname{arctanh}(1/2*(c-I*c*tan(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))$

**Maxima [A]**

time = 0.54, size = 273, normalized size = 1.00

$$i \left( \frac{4 \left( 105 (-i \tan(fx+e)+c)^4 (3A-iB) - 560 (-i \tan(fx+e)+c)^3 (3A-iB)c + 924 (-i \tan(fx+e)+c)^2 (3A-iB)c^2 - 384 (-i \tan(fx+e)+c) (3A-iB)c^3 - 256 (A-iB)c^4 \right)}{(-i \tan(fx+e)+c)^{\frac{9}{2}} a^3 - 6 (-i \tan(fx+e)+c)^{\frac{7}{2}} a^2 c + 12 (-i \tan(fx+e)+c)^{\frac{5}{2}} a^2 c^2 - 8 (-i \tan(fx+e)+c)^{\frac{3}{2}} a^2 c^3} + \frac{105 \sqrt{2} (3A-iB) \log \left( \frac{\sqrt{2} \sqrt{c} - \sqrt{-i \tan(fx+e)+c}}{\sqrt{2} \sqrt{c} + \sqrt{-i \tan(fx+e)+c}} \right)}{a^2 \sqrt{c}} \right)$$

3072 cf

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out]  $-1/3072*I*(4*(105*(-I*c*tan(f*x+e)+c)^4*(3*A-I*B)-560*(-I*c*tan(f*x+e)+c)^3*(3*A-I*B)*c+924*(-I*c*tan(f*x+e)+c)^2*(3*A-I*B)*c^2-384*(-I*c*tan(f*x+e)+c)*(3*A-I*B)*c^3-256*(A-I*B)*c^4)/((-I*c*tan(f*x+e)+c)^{(9/2)}*a^3-6*(-I*c*tan(f*x+e)+c)^{(7/2)}*a^3*c+12*(-I*c*tan(f*x+e)+c)^{(5/2)}*a^3*c^2-8*(-I*c*tan(f*x+e)+c)^{(3/2)}*a^3*c^3)+105*sqrt(2)*(3*A-I*B)*log(-sqrt(2)*sqrt(c)-sqrt(-I*c*tan(f*x+e)))$

+ c))/(sqrt(2)\*sqrt(c) + sqrt(-I\*c\*tan(f\*x + e) + c))/(a^3\*sqrt(c)))/(c\*f  
)

**Fricas** [A]

time = 2.36, size = 453, normalized size = 1.65

$$\frac{\int \sqrt{\frac{2c}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} e^{2I*fx + 2I*e} \log\left(\frac{35\sqrt{2}}{128} \sqrt{\frac{2c}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}}\right) - \sqrt{2} \sqrt{\frac{2c}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}}}{\sqrt{2} \sqrt{\frac{2c}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}} \sqrt{\frac{2c - \sqrt{2c}}{a^3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^(3/2),x,  
algorithm="fricas")

[Out] 1/1536\*(105\*sqrt(1/2)\*a^3\*c^2\*f\*sqrt(-(9\*A^2 - 6\*I\*A\*B - B^2)/(a^6\*c^3\*f^2))  
)e^(6\*I\*f\*x + 6\*I\*e)\*log(35/128\*(sqrt(2)\*sqrt(1/2)\*(a^3\*c\*f\*e^(2\*I\*f\*x + 2  
\*I\*e) + a^3\*c\*f)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(9\*A^2 - 6\*I\*A\*B -  
B^2)/(a^6\*c^3\*f^2)) + 3\*I\*A + B)\*e^(-I\*f\*x - I\*e)/(a^3\*c\*f)) - 105\*sqrt(1/  
2)\*a^3\*c^2\*f\*sqrt(-(9\*A^2 - 6\*I\*A\*B - B^2)/(a^6\*c^3\*f^2))\*e^(6\*I\*f\*x + 6\*I\*  
e)\*log(-35/128\*(sqrt(2)\*sqrt(1/2)\*(a^3\*c\*f\*e^(2\*I\*f\*x + 2\*I\*e) + a^3\*c\*f)\*s  
qrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(-(9\*A^2 - 6\*I\*A\*B - B^2)/(a^6\*c^3\*f^2  
)) - 3\*I\*A - B)\*e^(-I\*f\*x - I\*e)/(a^3\*c\*f)) - sqrt(2)\*(16\*(I\*A + B)\*e^(10\*I  
\*f\*x + 10\*I\*e) + 32\*(7\*I\*A + 4\*B)\*e^(8\*I\*f\*x + 8\*I\*e) - (-43\*I\*A - 121\*B)\*e  
^(6\*I\*f\*x + 6\*I\*e) + 5\*(-43\*I\*A + 7\*B)\*e^(4\*I\*f\*x + 4\*I\*e) + 2\*(-29\*I\*A + 1  
7\*B)\*e^(2\*I\*f\*x + 2\*I\*e) - 8\*I\*A + 8\*B)\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*  
e^(-6\*I\*f\*x - 6\*I\*e)/(a^3\*c^2\*f)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A}{-c\sqrt{-i\tan(e+fx)+c} + c\sqrt{\tan(e+fx)-2c\sqrt{-i\tan(e+fx)+c}} + c\sqrt{\tan(e+fx)-2c\sqrt{-i\tan(e+fx)+c}} + c\sqrt{-i\tan(e+fx)+c}} dx + \int \frac{B\tan(e+fx)}{-c\sqrt{-i\tan(e+fx)+c} + c\sqrt{\tan(e+fx)-2c\sqrt{-i\tan(e+fx)+c}} + c\sqrt{\tan(e+fx)-2c\sqrt{-i\tan(e+fx)+c}} + c\sqrt{-i\tan(e+fx)+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^3/2,  
x)

[Out] I\*(Integral(A/(-I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*4 - 2\*c\*sqrt(  
-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*3 - 2\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*ta  
n(e + f\*x) + I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)), x) + Integral(B\*tan(e + f\*x)  
/(-I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x)\*\*4 - 2\*c\*sqrt(-I\*c\*tan(e +  
f\*x) + c)\*tan(e + f\*x)\*\*3 - 2\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)\*tan(e + f\*x) +  
I\*c\*sqrt(-I\*c\*tan(e + f\*x) + c)), x))/a\*\*3

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e)
+ c)^(3/2)), x)
```

**Mupad [B]**

time = 9.98, size = 443, normalized size = 1.62

$$\frac{\frac{A(c - \tan(e + fx))^{3/2} + \frac{4A^2B}{3\sqrt{c}} - \frac{A(c - \tan(e + fx))^{3/2} \tan(e + fx)}{3\sqrt{c}} - \frac{A(c - \tan(e + fx))^{3/2} \tan^2(e + fx)}{3\sqrt{c}} + \frac{A^2(c - \tan(e + fx))^{3/2}}{3\sqrt{c}}}{6(c - \tan(e + fx))^{3/2} - (c - \tan(e + fx))^{5/2} + 8e^2(c - \tan(e + fx))^{3/2} - 12e^2(c - \tan(e + fx))^{5/2}} + \frac{\frac{B^2}{3} + \frac{2B(c - \tan(e + fx))^{3/2}}{3} - \frac{7B(c - \tan(e + fx))^{3/2}}{3} + \frac{B^2(c - \tan(e + fx))^{3/2}}{3} - \frac{10B(c - \tan(e + fx))^{3/2}}{3}}{a^3 f (c - \tan(e + fx))^{3/2} - 6a^2 e f (c - \tan(e + fx))^{5/2} - 8a^2 e^2 f (c - \tan(e + fx))^{3/2} + 12a^2 e^2 f (c - \tan(e + fx))^{5/2}} + \frac{\sqrt{c} A \operatorname{atan}\left(\frac{\sqrt{c - \tan(e + fx)}}{\sqrt{c}}\right)}{512a^3(-c)^{3/2} f} + \frac{105}{512a^3 c^{3/2} f} + \frac{30\sqrt{c} B \operatorname{atanh}\left(\frac{\sqrt{c - \tan(e + fx)}}{\sqrt{c}}\right)}{512a^3 c^{3/2} f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^(3/2)),x)
```

```
[Out] ((B*c^3)/3 + (35*B*(c - c*tan(e + f*x)*1i)^3)/48 - (77*B*c*(c - c*tan(e + f
*x)*1i)^2)/64 + (B*c^2*(c - c*tan(e + f*x)*1i))/2 - (35*B*(c - c*tan(e + f*
x)*1i)^4)/(256*c))/(a^3*f*(c - c*tan(e + f*x)*1i)^(9/2) - 6*a^3*c*f*(c - c*
tan(e + f*x)*1i)^(7/2) - 8*a^3*c^3*f*(c - c*tan(e + f*x)*1i)^(3/2) + 12*a^3
*c^2*f*(c - c*tan(e + f*x)*1i)^(5/2)) - ((A*(c - c*tan(e + f*x)*1i)^3*35i)/
(16*a^3*f) + (A*c^3*1i)/(3*a^3*f) - (A*(c - c*tan(e + f*x)*1i)^4*105i)/(256
*a^3*c*f) - (A*c*(c - c*tan(e + f*x)*1i)^2*231i)/(64*a^3*f) + (A*c^2*(c - c
*tan(e + f*x)*1i)*3i)/(2*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^(7/2) - (c -
c*tan(e + f*x)*1i)^(9/2) + 8*c^3*(c - c*tan(e + f*x)*1i)^(3/2) - 12*c^2*(c
- c*tan(e + f*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e + f*x)*
1i)^(1/2))/(2*(-c)^(1/2)))*105i)/(512*a^3*(-c)^(3/2)*f) + (35*2^(1/2)*B*ata
nh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(512*a^3*c^(3/2)*f
)
```

$$3.786 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^3(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=311

$$\frac{21(11iA + B) \tanh^{-1}\left(\frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{512\sqrt{2} a^3 c^{5/2} f} - \frac{21(11iA + B)}{640a^3 f(c-ic \tan(e+fx))^{5/2}} + \frac{iA - B}{6a^3 f(1+i \tan(e+fx))^3}$$

[Out] 21/1024\*(11\*I\*A+B)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))/a^3/c^(5/2)/f\*2^(1/2)-21/512\*(11\*I\*A+B)/a^3/c^2/f/(c-I\*c\*tan(f\*x+e))^(1/2)-21/640\*(11\*I\*A+B)/a^3/f/(c-I\*c\*tan(f\*x+e))^(5/2)+1/6\*(I\*A-B)/a^3/f/(1+I\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^(5/2)+1/48\*(11\*I\*A+B)/a^3/f/(1+I\*tan(f\*x+e))^2/(c-I\*c\*tan(f\*x+e))^(5/2)+3/64\*(11\*I\*A+B)/a^3/f/(1+I\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2)-7/256\*(11\*I\*A+B)/a^3/c/f/(c-I\*c\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.24, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$ , Rules used = {3669, 79, 44, 53, 65, 214}

$$\frac{21(B+11iA) \tanh^{-1}\left(\frac{\sqrt{c-ic \tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{512\sqrt{2} a^3 c^{5/2} f} - \frac{21(B+11iA)}{512a^3 f \sqrt{c-ic \tan(e+fx)}} + \frac{-B+iA}{6a^3 f(1+i \tan(e+fx))^3(c-ic \tan(e+fx))^{5/2}} - \frac{7(B+11iA)}{256a^3 c f(c-ic \tan(e+fx))^{3/2}} - \frac{21(B+11iA)}{640a^3 f(c-ic \tan(e+fx))^{5/2}} + \frac{3(B+11iA)}{64a^3 f(1+i \tan(e+fx))(c-ic \tan(e+fx))^{3/2}} + \frac{B+11iA}{48a^3 f(1+i \tan(e+fx))^2(c-ic \tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^(5/2)), x]

[Out] (21\*((11\*I)\*A + B)\*ArcTanh[Sqrt[c - I\*c\*Tan[e + f\*x]]/(Sqrt[2]\*Sqrt[c])])/(512\*Sqrt[2]\*a^3\*c^(5/2)\*f) - (21\*((11\*I)\*A + B))/(640\*a^3\*f\*(c - I\*c\*Tan[e + f\*x])^(5/2)) + (I\*A - B)/(6\*a^3\*f\*(1 + I\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^(5/2)) + ((11\*I)\*A + B)/(48\*a^3\*f\*(1 + I\*Tan[e + f\*x])^2\*(c - I\*c\*Tan[e + f\*x])^(5/2)) + (3\*((11\*I)\*A + B))/(64\*a^3\*f\*(1 + I\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2)) - (7\*((11\*I)\*A + B))/(256\*a^3\*c\*f\*(c - I\*c\*Tan[e + f\*x])^(3/2)) - (21\*((11\*I)\*A + B))/(512\*a^3\*c^2\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Rule 44**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*((m + n + 2)/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && LtQ[n, 0]

**Rule 53**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^3 (c - ic \tan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^4 (c-icx)^{7/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 (c - ic \tan(e + fx))^{5/2}} + \frac{1}{48} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 (c - ic \tan(e + fx))^{5/2}} + \frac{1}{48} \\
&= \frac{iA - B}{6a^3 f (1 + i \tan(e + fx))^3 (c - ic \tan(e + fx))^{5/2}} + \frac{1}{48} \\
&= -\frac{21(11iA + B)}{640a^3 f (c - ic \tan(e + fx))^{5/2}} + \frac{1}{6a^3 f (1 + i \tan(e + fx))} \\
&= -\frac{21(11iA + B)}{640a^3 f (c - ic \tan(e + fx))^{5/2}} + \frac{1}{6a^3 f (1 + i \tan(e + fx))} \\
&= -\frac{21(11iA + B)}{640a^3 f (c - ic \tan(e + fx))^{5/2}} + \frac{1}{6a^3 f (1 + i \tan(e + fx))} \\
&= -\frac{21(11iA + B)}{640a^3 f (c - ic \tan(e + fx))^{5/2}} + \frac{1}{6a^3 f (1 + i \tan(e + fx))} \\
&= \frac{21(11iA + B) \tanh^{-1}\left(\frac{\sqrt{c - ic \tan(e + fx)}}{\sqrt{2} \sqrt{c}}\right)}{512\sqrt{2} a^3 c^{5/2} f} - \frac{1}{640}
\end{aligned}$$

**Mathematica [A]**

time = 4.06, size = 256, normalized size = 0.82

$$\frac{e^{-40i(e+fx)} \left( -i(1 + e^{20i(e+fx)}) (-iB(40 + 190e^{20i(e+fx)} + 315e^{40i(e+fx)} + 688e^{60i(e+fx)} + 256e^{80i(e+fx)} + 48e^{100i(e+fx)}) + A(-40 - 310e^{20i(e+fx)} - 1335e^{40i(e+fx)} + 2768e^{60i(e+fx)} + 416e^{80i(e+fx)} + 48e^{100i(e+fx)}) \right) + 315(11iA + B)e^{60i(e+fx)} \sqrt{1 + e^{20i(e+fx)}} \tanh^{-1}\left(\frac{\sqrt{1 + e^{20i(e+fx)}}}{\sqrt{c - ic \tan(e + fx)}}\right)}{15360a^3 c^5 f}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^3\*(c - I\*c\*Tan[e + f\*x])^(5/2)), x]

[Out] (((-I)\*(1 + E^((2\*I)\*(e + f\*x))))\*((-I)\*B\*(40 + 190\*E^((2\*I)\*(e + f\*x)) + 315\*E^((4\*I)\*(e + f\*x)) + 688\*E^((6\*I)\*(e + f\*x)) + 256\*E^((8\*I)\*(e + f\*x)) + 48\*E^((10\*I)\*(e + f\*x)))) + A\*(-40 - 310\*E^((2\*I)\*(e + f\*x)) - 1335\*E^((4\*I)\*(e + f\*x)) + 2768\*E^((6\*I)\*(e + f\*x)) + 416\*E^((8\*I)\*(e + f\*x)) + 48\*E^((

10\*I)\*(e + f\*x)))) + 315\*((11\*I)\*A + B)\*E^((6\*I)\*(e + f\*x))\*Sqrt[1 + E^((2\*I)\*(e + f\*x))]\*ArcTanh[Sqrt[1 + E^((2\*I)\*(e + f\*x))]]]\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(15360\*a^3\*c^3\*E^((6\*I)\*(e + f\*x))\*f)

Maple [A]

time = 0.37, size = 232, normalized size = 0.75

method	result
derivativedivides	$2ic^3 \left( \frac{-iB+5A}{32e^5 \sqrt{c - ic \tan (fx + e)}} - \frac{-iB+2A}{48c^4 (c - ic \tan (fx + e))^{\frac{3}{2}}} - \frac{-iB+A}{80c^3 (c - ic \tan (fx + e))^{\frac{5}{2}}} + \frac{8 \left( \frac{11iB}{256} + \frac{71A}{256} \right) (c - ic \tan (fx + e))}{\dots} \right)$
default	$2ic^3 \left( \frac{-iB+5A}{32e^5 \sqrt{c - ic \tan (fx + e)}} - \frac{-iB+2A}{48c^4 (c - ic \tan (fx + e))^{\frac{3}{2}}} - \frac{-iB+A}{80c^3 (c - ic \tan (fx + e))^{\frac{5}{2}}} + \frac{8 \left( \frac{11iB}{256} + \frac{71A}{256} \right) (c - ic \tan (fx + e))}{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^(5/2),x,method =\_RETURNVERBOSE)

[Out] 2\*I/f/a^3\*c^3\*(-1/32/c^5\*(5\*A-I\*B)/(c-I\*c\*tan(f\*x+e))^(1/2)-1/48/c^4\*(2\*A-I\*B)/(c-I\*c\*tan(f\*x+e))^(3/2)-1/80/c^3\*(A-I\*B)/(c-I\*c\*tan(f\*x+e))^(5/2)+1/32/c^5\*(8\*((11/256\*I\*B+71/256\*A)\*(c-I\*c\*tan(f\*x+e))^(5/2)+(-59/48\*A\*c-11/48\*I\*B\*c)\*(c-I\*c\*tan(f\*x+e))^(3/2)+(21/64\*I\*B\*c^2+89/64\*A\*c^2)\*(c-I\*c\*tan(f\*x+e))^(1/2))/(c+I\*c\*tan(f\*x+e))^3+21/8\*(-1/8\*I\*B+11/8\*A)\*2^(1/2)/c^(1/2)\*arctanh(1/2\*(c-I\*c\*tan(f\*x+e))^(1/2)\*2^(1/2)/c^(1/2))))

Maxima [A]

time = 0.53, size = 302, normalized size = 0.97

$$i \left( \frac{4 \left( 315(-ic \tan (fx + e) + c)^5(11A - iB) - 1680(-ic \tan (fx + e) + c)^4(11A - iB)c + 2772(-ic \tan (fx + e) + c)^3(11A - iB)c^2 - 1152(-ic \tan (fx + e) + c)^2(11A - iB)c^3 - 256(-ic \tan (fx + e) + c)(11A - iB)c^4 - 1536(A - iB)c^5 \right)}{(-ic \tan (fx + e) + c)^5 a^6 c - 6(-ic \tan (fx + e) + c)^4 a^5 c^2 + 12(-ic \tan (fx + e) + c)^3 a^4 c^3 - 8(-ic \tan (fx + e) + c)^2 a^3 c^4} + \frac{315 \sqrt{2} (11A - iB) \log \left( \frac{-\sqrt{2} \sqrt{c - ic \tan (fx + e)} + c}{\sqrt{2} \sqrt{c - ic \tan (fx + e)} + c} \right)}{a^6 c^3} \right)$$

30720 cf

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="maxima")

[Out] -1/30720\*I\*(4\*(315\*(-I\*c\*tan(f\*x + e) + c)^5\*(11\*A - I\*B) - 1680\*(-I\*c\*tan(f\*x + e) + c)^4\*(11\*A - I\*B)\*c + 2772\*(-I\*c\*tan(f\*x + e) + c)^3\*(11\*A - I\*B

$$) * c^2 - 1152 * (-I * c * \tan(f * x + e) + c)^2 * (11 * A - I * B) * c^3 - 256 * (-I * c * \tan(f * x + e) + c) * (11 * A - I * B) * c^4 - 1536 * (A - I * B) * c^5 / ((-I * c * \tan(f * x + e) + c)^{(11/2)} * a^3 * c - 6 * (-I * c * \tan(f * x + e) + c)^{(9/2)} * a^3 * c^2 + 12 * (-I * c * \tan(f * x + e) + c)^{(7/2)} * a^3 * c^3 - 8 * (-I * c * \tan(f * x + e) + c)^{(5/2)} * a^3 * c^4 + 315 * \sqrt{2} * (11 * A - I * B) * \log(-(\sqrt{2} * \sqrt{c} - \sqrt{-I * c * \tan(f * x + e) + c})) / (\sqrt{2} * \sqrt{c} + \sqrt{-I * c * \tan(f * x + e) + c})) / (a^3 * c^{(3/2)})) / (c * f)$$

**Fricas** [A]

time = 1.58, size = 480, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^3/(c-I\*c\*tan(f\*x+e))^(5/2), x, algorithm="fricas")

[Out]  $\frac{1}{15360} * (315 * \sqrt{2} * a^3 * c^3 * f * \sqrt{-(121 * A^2 - 22 * I * A * B - B^2)} / (a^6 * c^5 * f^2)) * e^{(6 * I * f * x + 6 * I * e)} * \log(21 / 256 * (\sqrt{2} * \sqrt{1/2}) * (a^3 * c^2 * f * e^{(2 * I * f * x + 2 * I * e)} + a^3 * c^2 * f) * \sqrt{c / (e^{(2 * I * f * x + 2 * I * e)} + 1)}) * \sqrt{-(121 * A^2 - 22 * I * A * B - B^2)} / (a^6 * c^5 * f^2)) + 11 * I * A + B) * e^{(-I * f * x - I * e)} / (a^3 * c^2 * f) - 315 * \sqrt{2} * a^3 * c^3 * f * \sqrt{-(121 * A^2 - 22 * I * A * B - B^2)} / (a^6 * c^5 * f^2)) * e^{(6 * I * f * x + 6 * I * e)} * \log(-21 / 256 * (\sqrt{2} * \sqrt{1/2}) * (a^3 * c^2 * f * e^{(2 * I * f * x + 2 * I * e)} + a^3 * c^2 * f) * \sqrt{c / (e^{(2 * I * f * x + 2 * I * e)} + 1)}) * \sqrt{-(121 * A^2 - 22 * I * A * B - B^2)} / (a^6 * c^5 * f^2)) - 11 * I * A - B) * e^{(-I * f * x - I * e)} / (a^3 * c^2 * f) - \sqrt{2} * (48 * (I * A + B) * e^{(12 * I * f * x + 12 * I * e)} + 16 * (29 * I * A + 19 * B) * e^{(10 * I * f * x + 10 * I * e)} + 16 * (199 * I * A + 59 * B) * e^{(8 * I * f * x + 8 * I * e)} - (-1433 * I * A - 1003 * B) * e^{(6 * I * f * x + 6 * I * e)} + 5 * (-329 * I * A + 101 * B) * e^{(4 * I * f * x + 4 * I * e)} + 10 * (-35 * I * A + 23 * B) * e^{(2 * I * f * x + 2 * I * e)} - 40 * I * A + 40 * B) * \sqrt{c / (e^{(2 * I * f * x + 2 * I * e)} + 1)}) * e^{(-6 * I * f * x - 6 * I * e)} / (a^3 * c^3 * f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*3/(c-I\*c\*tan(f\*x+e))\*\*(5/2), x)

[Out]  $I * (\text{Integral}(A / (-c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) ** 5 + I * c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) ** 4 - 2 * c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) ** 3 + 2 * I * c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) ** 2 - c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) + I * c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}), x) + \text{Integral}(B * \tan(e + f * x) / (-c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) ** 5 + I * c ** 2 * \sqrt{-I * c * \tan(e + f * x) + c}) * \tan(e + f * x) ** 4 - 2 *$



$$3.787 \quad \int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))(c - ic \tan(e + fx))^{7/2} dx$$

**Optimal.** Leaf size=272

$$\frac{5\sqrt{a} (4iA - 3B)c^{7/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{4f} - \frac{5(4iA - 3B)c^3 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{8f}$$

[Out]  $-5/4*(4*I*A-3*B)*c^{(7/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})*a^{(1/2)}/f-5/8*(4*I*A-3*B)*c^3*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/f-5/24*(4*I*A-3*B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f-1/12*(4*I*A-3*B)*c*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f+1/4*B*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(7/2)}/f$

**Rubi** [A]

time = 0.21, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 52, 65, 223, 209}

$$\frac{5\sqrt{a}c^{7/2}(-3B+4iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} - \frac{5c^2(-3B+4iA)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} - \frac{5c^2(-3B+4iA)\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{24f} - \frac{c(-3B+4iA)\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}{12f} + \frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{7/2}}{4f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $(-5*\text{Sqrt}[a]*((4*I)*A - 3*B)*c^{(7/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])]/(4*f) - (5*((4*I)*A - 3*B)*c^3*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]/(8*f) - (5*((4*I)*A - 3*B)*c^2*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(24*f) - (((4*I)*A - 3*B)*c*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(12*f) + (B*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(4*f)$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m + n + 1))), \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))(c - ic \tan(e + fx))^{7/2} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{\sqrt{a+iax}} dx, \right)}{f} \\
&= \frac{B \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{7/2}}{4f} \\
&= -\frac{(4iA - 3B)c \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}}{12f} \\
&= -\frac{5(4iA - 3B)c^2 \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{24f} \\
&= -\frac{5(4iA - 3B)c^3 \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{1/2}}{8f} \\
&= -\frac{5(4iA - 3B)c^3 \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{1/2}}{8f} \\
&= -\frac{5\sqrt{a} (4iA - 3B)c^{7/2} \tan^{-1} \left( \frac{\sqrt{a + ia \tan(e + fx)}}{\sqrt{a}} \right)}{4f}
\end{aligned}$$

**Mathematica [A]**

time = 3.38, size = 257, normalized size = 0.94

$$\frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx)) \left( \frac{5(-4iA + 3B)c^3 e^{-i(e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} \text{ArcTan}[e^{i(e+fx)}]}{\sqrt{1 + e^{2i(e+fx)}}} + \frac{1}{24} c^3 \sec^3(e + fx) [64(-4iA + 3B) \cos(e + fx) + 96(-iA + B) \cos(3(e + fx)) - 6(12A + 13iB + (12A + 17iB) \cos(2(e + fx))) \sin(e + fx)] \sqrt{c - ic \tan(e + fx)} \right)}{4f \sec^3(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + I\*a\*Tan[e + f\*x]]\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2), x]

[Out] (Sqrt[a + I\*a\*Tan[e + f\*x]]\*(A + B\*Tan[e + f\*x]))\*((5\*((-4\*I)\*A + 3\*B)\*c^4\*Sqrt[E^(I\*(e + f\*x))/(1 + E^((2\*I)\*(e + f\*x)))]\*ArcTan[E^(I\*(e + f\*x))])/(E^

$$(I*(e + f*x))*\text{Sqrt}[c/(1 + E^{((2*I)*(e + f*x))})] + (c^3*\text{Sec}[e + f*x]^{(7/2)}*(64*((-4*I)*A + 3*B)*\text{Cos}[e + f*x] + 96*((-I)*A + B)*\text{Cos}[3*(e + f*x)] - 6*(12*A + (13*I)*B + (12*A + (17*I)*B)*\text{Cos}[2*(e + f*x)])*\text{Sin}[e + f*x]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/24)/(4*f*\text{Sec}[e + f*x]^{(3/2)}*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$$

**Maple [A]**

time = 0.52, size = 349, normalized size = 1.28

method	result
derivativedivides	$\frac{\sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} e^3 \left( 6iB \sqrt{ac(1 + \tan^2(fx + e))} \sqrt{ac} \right)}{\dots}$
default	$\sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} e^3 \left( 6iB \sqrt{ac(1 + \tan^2(fx + e))} \sqrt{ac} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^3*(6*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+8*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+45*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c-45*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-24*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-88*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+60*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c-36*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+72*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1419 vs.  $2(216) = 432$ .

time = 1.53, size = 1419, normalized size = 5.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,algorithm="maxima")
```

```
[Out] -96*(60*(4*A + 3*I*B)*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 220*(4*A + 3*I*B)*c^3*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 292*(4*A + 3*I*B)*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
```



$$\begin{aligned}
& 2*e))) + 12*(44*A + 49*I*B)*c^3*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 60*(4*I*A - 3*B)*c^3*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 220*(4*I*A - 3*B)*c^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 292*(4*I*A - 3*B)*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 12*(44*I*A - 49*B)*c^3*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 30*((4*A + 3*I*B)*c^3*\cos(8*f*x + 8*e) + 4*(4*A + 3*I*B)*c^3*\cos(6*f*x + 6*e) + 6*(4*A + 3*I*B)*c^3*\cos(4*f*x + 4*e) + 4*(4*A + 3*I*B)*c^3*\cos(2*f*x + 2*e) + (4*I*A - 3*B)*c^3*\sin(8*f*x + 8*e) + 4*(4*I*A - 3*B)*c^3*\sin(6*f*x + 6*e) + 6*(4*I*A - 3*B)*c^3*\sin(4*f*x + 4*e) + 4*(4*I*A - 3*B)*c^3*\sin(2*f*x + 2*e) + (4*A + 3*I*B)*c^3)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 30*((4*A + 3*I*B)*c^3*\cos(8*f*x + 8*e) + 4*(4*A + 3*I*B)*c^3*\cos(6*f*x + 6*e) + 6*(4*A + 3*I*B)*c^3*\cos(4*f*x + 4*e) + 4*(4*A + 3*I*B)*c^3*\cos(2*f*x + 2*e) + (4*I*A - 3*B)*c^3*\sin(8*f*x + 8*e) + 4*(4*I*A - 3*B)*c^3*\sin(6*f*x + 6*e) + 6*(4*I*A - 3*B)*c^3*\sin(4*f*x + 4*e) + 4*(4*I*A - 3*B)*c^3*\sin(2*f*x + 2*e) + (4*A + 3*I*B)*c^3)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((4*I*A - 3*B)*c^3*\cos(8*f*x + 8*e) + 4*(4*I*A - 3*B)*c^3*\cos(6*f*x + 6*e) + 6*(4*I*A - 3*B)*c^3*\cos(4*f*x + 4*e) + 4*(4*I*A - 3*B)*c^3*\cos(2*f*x + 2*e) - (4*A + 3*I*B)*c^3*\sin(8*f*x + 8*e) - 4*(4*A + 3*I*B)*c^3*\sin(6*f*x + 6*e) - 6*(4*A + 3*I*B)*c^3*\sin(4*f*x + 4*e) - 4*(4*A + 3*I*B)*c^3*\sin(2*f*x + 2*e) + (4*I*A - 3*B)*c^3)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((-4*I*A + 3*B)*c^3*\cos(8*f*x + 8*e) + 4*(-4*I*A + 3*B)*c^3*\cos(6*f*x + 6*e) + 6*(-4*I*A + 3*B)*c^3*\cos(4*f*x + 4*e) + 4*(-4*I*A + 3*B)*c^3*\cos(2*f*x + 2*e) + (4*A + 3*I*B)*c^3*\sin(8*f*x + 8*e) + 4*(4*A + 3*I*B)*c^3*\sin(6*f*x + 6*e) + 6*(4*A + 3*I*B)*c^3*\sin(4*f*x + 4*e) + 4*(4*A + 3*I*B)*c^3*\sin(2*f*x + 2*e) + (-4*I*A + 3*B)*c^3)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1))*\sqrt{a}*\sqrt{c}/(f*(-4608*I*\cos(8*f*x + 8*e) - 18432*I*\cos(6*f*x + 6*e) - 27648*I*\cos(4*f*x + 4*e) - 18432*I*\cos(2*f*x + 2*e) + 4608*\sin(8*f*x + 8*e) + 18432*\sin(6*f*x + 6*e) + 27648*\sin(4*f*x + 4*e) + 18432*\sin(2*f*x + 2*e) - 4608*I))
\end{aligned}$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs.  $2(216) = 432$ .

time = 2.94, size = 631, normalized size = 2.32

$$\frac{\sqrt{a} \sqrt{c} \left( \frac{1}{2} \arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right) \right)^2 + \sin\left(\frac{1}{2} \arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)^2 + 2 \sin\left(\frac{1}{2} \arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right) + 1}{f \left( -4608 I \cos(8fx+8e) - 18432 I \cos(6fx+6e) - 27648 I \cos(4fx+4e) - 18432 I \cos(2fx+2e) + 4608 \sin(8fx+8e) + 18432 \sin(6fx+6e) + 27648 \sin(4fx+4e) + 18432 \sin(2fx+2e) - 4608 I \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="fricas")

```
[Out] 1/48*(15*sqrt((16*A^2 + 24*I*A*B - 9*B^2)*a*c^7/f^2)*(f*e^(6*I*f*x + 6*I*e)
+ 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((4*I*A
- 3*B)*c^3*e^(3*I*f*x + 3*I*e) + (4*I*A - 3*B)*c^3*e^(I*f*x + I*e)))*sqrt(a/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((16*A^2
+ 24*I*A*B - 9*B^2)*a*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((4*I*A - 3*B)
*c^3*e^(2*I*f*x + 2*I*e) + (4*I*A - 3*B)*c^3) - 15*sqrt((16*A^2 + 24*I*A*B
- 9*B^2)*a*c^7/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f
*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((4*I*A - 3*B)*c^3*e^(3*I*f*x + 3*I*e) +
(4*I*A - 3*B)*c^3*e^(I*f*x + I*e)))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(
c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((16*A^2 + 24*I*A*B - 9*B^2)*a*c^7/f^2)*
(f*e^(2*I*f*x + 2*I*e) - f))/((4*I*A - 3*B)*c^3*e^(2*I*f*x + 2*I*e) + (4*I*
A - 3*B)*c^3) - 4*(15*(4*I*A - 3*B)*c^3*e^(7*I*f*x + 7*I*e) + 55*(4*I*A -
3*B)*c^3*e^(5*I*f*x + 5*I*e) + 73*(4*I*A - 3*B)*c^3*e^(3*I*f*x + 3*I*e) + 3
*(44*I*A - 49*B)*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4
*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) \sqrt{a + a \tan(e + f x)} \operatorname{li}(c - c \tan(e + f x))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*  
1i)^(7/2),x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*  
1i)^(7/2), x)
```

$$3.788 \quad \int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))(c - ic \tan(e + fx))^{5/2} dx$$

**Optimal.** Leaf size=217

$$\frac{\sqrt{a} (3iA - 2B)c^{5/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{f} - \frac{(3iA - 2B)c^2 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f}$$

[Out]  $-(3IA-2B)*c^{(5/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)})/(c-I*c*\tan(f*x+e))^{(1/2)}*a^{(1/2)}/f-1/2*(3IA-2B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/f-1/6*(3IA-2B)*c*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/3*B*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f$

**Rubi [A]**

time = 0.19, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 52, 65, 223, 209}

$$\frac{\sqrt{a} c^{5/2} (-2B + 3iA) \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{f} - \frac{c^{(-2B + 3iA)} \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f} - \frac{c^{(-2B + 3iA)} \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{6f} + \frac{B \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $-\left(\frac{\text{Sqrt}[a]*((3I)*A - 2*B)*c^{(5/2)}*\text{ArcTan}[\frac{\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]}{\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]}]}{f} - \frac{((3I)*A - 2*B)*c^2*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]}{(2*f)} - \frac{(((3I)*A - 2*B)*c*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})}{(6*f)} + \frac{(B*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})}{(3*f)}\right)$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m + n + 1))), \text{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) +$

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))(c - ic \tan(e + fx))^{5/2} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{3/2}}{\sqrt{a+iax}} dx, x \right)}{f} \\
&= \frac{B \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}}{3f} \\
&= -\frac{(3iA - 2B)c \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}}{6f} \\
&= -\frac{(3iA - 2B)c^2 \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}}{2f} \\
&= -\frac{(3iA - 2B)c^2 \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}}{2f} \\
&= -\frac{(3iA - 2B)c^2 \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}}{2f} \\
&= -\frac{\sqrt{a} (3iA - 2B)c^{5/2} \tan^{-1} \left( \frac{\sqrt{c}}{\sqrt{a}} \right)}{f}
\end{aligned}$$

**Mathematica [A]**

time = 2.40, size = 226, normalized size = 1.04

$$\frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx)) \left( \frac{(-3iA + 2B)c^3 e^{-i(e+fx)} \sqrt{\frac{e^{(e+fx)}}{1 + e^{2i(e+fx)}}} \text{ArcTan}[e^{i(e+fx)}]}{\frac{c}{\sqrt{1 + e^{2i(e+fx)}}}} + \frac{1}{12} c^2 \sec^3(e + fx) (-12iA + 8B + 12(-iA + B) \cos(2(e + fx)) - 3(A + 2iB) \sin(2(e + fx))) \sqrt{c - ic \tan(e + fx)} \right)}{f \sec^3(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + I\*a\*Tan[e + f\*x]]\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] (Sqrt[a + I\*a\*Tan[e + f\*x]]\*(A + B\*Tan[e + f\*x])\*((((-3\*I)\*A + 2\*B)\*c^3\*Sqrt[E^(I\*(e + f\*x))/(1 + E^((2\*I)\*(e + f\*x)))]\*ArcTan[E^(I\*(e + f\*x))])/(E^(I\*(e + f\*x))\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x))])) + (c^2\*Sec[e + f\*x])^(5/2)\*((-12\*I)\*A + 8\*B + 12\*((-I)\*A + B)\*Cos[2\*(e + f\*x)] - 3\*(A + (2\*I)\*B)\*Sin[2\*(

$e + f*x]])*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/12)))/(f*\text{Sec}[e + f*x]^(3/2)*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$

**Maple [A]**

time = 0.39, size = 285, normalized size = 1.31

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} c^2 \left(-6iB \ln\left(\frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}}\right) \sqrt{ac} \frac{1}{\sqrt{ac}}\right)}{1}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} c^2 \left(-6iB \ln\left(\frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}}\right) \sqrt{ac} \frac{1}{\sqrt{ac}}\right)}{1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/6/f*(a*(1+I*\text{tan}(f*x+e)))^(1/2)*(-c*(I*\text{tan}(f*x+e)-1))^(1/2)*c^2*(-6*I*B*\ln((a*c*\text{tan}(f*x+e)+(a*c)^(1/2)*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+6*I*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)+2*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)^2+12*I*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)-9*A*\ln((a*c*\text{tan}(f*x+e)+(a*c)^(1/2)*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+3*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)-10*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)/(a*c)^(1/2)}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1147 vs.  $2(172) = 344$ .

time = 0.91, size = 1147, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")`

[Out] 
$$-6*(12*(3*A + 2*I*B)*c^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 32*(3*A + 2*I*B)*c^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 12*(5*A + 6*I*B)*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 12*(3*I*A - 2*B)*c^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 32*(3*I*A - 2*B)*c^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 12*(5*I*A - 6*B)*c^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 6*((3*A + 2*I*B)*c^2*\cos(6*f*x + 6*e) + 3*(3*A + 2*I*B)*c^2*\cos(4$$

```

*f*x + 4*e) + 3*(3*A + 2*I*B)*c^2*cos(2*f*x + 2*e) + (3*I*A - 2*B)*c^2*sin(
6*f*x + 6*e) + 3*(3*I*A - 2*B)*c^2*sin(4*f*x + 4*e) + 3*(3*I*A - 2*B)*c^2*si
in(2*f*x + 2*e) + (3*A + 2*I*B)*c^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 1) + 6*((3*A + 2*I*B)*c^2*cos(6*f*x + 6*e) + 3*(3*A + 2*I*B)*c^2*cos(4*
f*x + 4*e) + 3*(3*A + 2*I*B)*c^2*cos(2*f*x + 2*e) + (3*I*A - 2*B)*c^2*sin(6
*f*x + 6*e) + 3*(3*I*A - 2*B)*c^2*sin(4*f*x + 4*e) + 3*(3*I*A - 2*B)*c^2*si
n(2*f*x + 2*e) + (3*A + 2*I*B)*c^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 1) + 3*((3*I*A - 2*B)*c^2*cos(6*f*x + 6*e) + 3*(3*I*A - 2*B)*c^2*cos(4*
f*x + 4*e) + 3*(3*I*A - 2*B)*c^2*cos(2*f*x + 2*e) - (3*A + 2*I*B)*c^2*sin(6
*f*x + 6*e) - 3*(3*A + 2*I*B)*c^2*sin(4*f*x + 4*e) - 3*(3*A + 2*I*B)*c^2*si
n(2*f*x + 2*e) + (3*I*A - 2*B)*c^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 3*((-3*I*
A + 2*B)*c^2*cos(6*f*x + 6*e) + 3*(-3*I*A + 2*B)*c^2*cos(4*f*x + 4*e) + 3*(
-3*I*A + 2*B)*c^2*cos(2*f*x + 2*e) + (3*A + 2*I*B)*c^2*sin(6*f*x + 6*e) + 3
*(3*A + 2*I*B)*c^2*sin(4*f*x + 4*e) + 3*(3*A + 2*I*B)*c^2*sin(2*f*x + 2*e)
+ (-3*I*A + 2*B)*c^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1))*sqrt(a)*sqrt(c)/(f*(-72*
I*cos(6*f*x + 6*e) - 216*I*cos(4*f*x + 4*e) - 216*I*cos(2*f*x + 2*e) + 72*s
in(6*f*x + 6*e) + 216*sin(4*f*x + 4*e) + 216*sin(2*f*x + 2*e) - 72*I))

```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs.  $2(172) = 344$ .  
time = 3.34, size = 570, normalized size = 2.63

$$\frac{\sqrt{\frac{9A^2 + 12AB - 4B^2}{f^2}} \sqrt{\frac{a}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{c}{e^{2Ifx + 2Ie} + 1}} \left( \frac{1}{2} \log\left( \frac{e^{2Ifx + 2Ie} + f}{e^{2Ifx + 2Ie} - f} \right) \right) - \sqrt{\frac{9A^2 + 12AB - 4B^2}{f^2}} \sqrt{\frac{a}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{c}{e^{2Ifx + 2Ie} + 1}} \left( \frac{1}{2} \log\left( \frac{e^{2Ifx + 2Ie} + f}{e^{2Ifx + 2Ie} - f} \right) \right) + 4(3A - 2B)e^{5Ifx + 5Ie} + 3(3A - 2B)e^{4Ifx + 4Ie} + 3(3A - 2B)e^{3Ifx + 3Ie} + 3(3A - 2B)e^{2Ifx + 2Ie} + 3(3A - 2B)e^{Ifx + Ie} + 3(3A - 2B)e^0 + 3(3A - 2B)e^{-Ifx - Ie} + 3(3A - 2B)e^{-2Ifx - 2Ie} + 3(3A - 2B)e^{-3Ifx - 3Ie} + 3(3A - 2B)e^{-4Ifx - 4Ie} + 3(3A - 2B)e^{-5Ifx - 5Ie}}{2(3A - 2B)e^{5Ifx + 5Ie} + 3(3A - 2B)e^{4Ifx + 4Ie} + 3(3A - 2B)e^{3Ifx + 3Ie} + 3(3A - 2B)e^{2Ifx + 2Ie} + 3(3A - 2B)e^{Ifx + Ie} + 3(3A - 2B)e^0 + 3(3A - 2B)e^{-Ifx - Ie} + 3(3A - 2B)e^{-2Ifx - 2Ie} + 3(3A - 2B)e^{-3Ifx - 3Ie} + 3(3A - 2B)e^{-4Ifx - 4Ie} + 3(3A - 2B)e^{-5Ifx - 5Ie}}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2
),x, algorithm="fricas")

```

```

[Out] 1/12*(3*sqrt((9*A^2 + 12*I*A*B - 4*B^2)*a*c^5/f^2)*(f*e^(4*I*f*x + 4*I*e) +
2*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((3*I*A - 2*B)*c^2*e^(3*I*f*x + 3*I
*e) + (3*I*A - 2*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((9*A^2 + 12*I*A*B - 4*B^2)*a*c^5/f
^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-3*I*A + 2*B)*c^2*e^(2*I*f*x + 2*I*e) +
(-3*I*A + 2*B)*c^2) - 3*sqrt((9*A^2 + 12*I*A*B - 4*B^2)*a*c^5/f^2)*(f*e^(4
*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((3*I*A - 2*B)*c^2
*e^(3*I*f*x + 3*I*e) + (3*I*A - 2*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((9*A^2 + 12*I*A*B
- 4*B^2)*a*c^5/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-3*I*A + 2*B)*c^2*e^(2*
I*f*x + 2*I*e) + (-3*I*A + 2*B)*c^2) - 4*(3*(3*I*A - 2*B)*c^2*e^(5*I*f*x +

```



$$\frac{5Ie + 8(3IA - 2B)c^2e^{(3Ifx + 3Ie)} + 3(5IA - 6B)c^2e^{(Ifx + Ie)} \sqrt{a/(e^{(2Ifx + 2Ie)} + 1)} \sqrt{c/(e^{(2Ifx + 2Ie)} + 1)}}{(fe^{(4Ifx + 4Ie)} + 2fe^{(2Ifx + 2Ie)} + f)}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(e + fx) - i)} (-ic(\tan(e + fx) + i))^{\frac{5}{2}} (A + B \tan(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(5/2),x)

[Out] Integral(sqrt(I\*a\*(tan(e + f\*x) - I))\*(-I\*c\*(tan(e + f\*x) + I))\*\*(5/2)\*(A + B\*tan(e + f\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + fx)) \sqrt{a + a \tan(e + fx) li} (c - c \tan(e + fx) li)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*li)^(1/2)\*(c - c\*tan(e + f\*x)\*li)^(5/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*li)^(1/2)\*(c - c\*tan(e + f\*x)\*li)^(5/2), x)

$$3.789 \quad \int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{3/2} dx$$

**Optimal.** Leaf size=164

$$\frac{\sqrt{a} (2iA - B)c^{3/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{f} - \frac{(2iA - B)c \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f}$$

[Out]  $-(2IA-B)c^{3/2} \arctan(c^{1/2}(a+Ia*\tan(f*x+e))^{1/2}/a^{1/2}/(c-Ic*\tan(f*x+e))^{1/2})a^{1/2}/f-1/2*(2IA-B)c*(a+Ia*\tan(f*x+e))^{1/2}*(c-Ic*\tan(f*x+e))^{1/2}/f+1/2*B*(a+Ia*\tan(f*x+e))^{1/2}*(c-Ic*\tan(f*x+e))^{3/2}/f$

**Rubi [A]**

time = 0.17, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 52, 65, 223, 209}

$$\frac{\sqrt{a} c^{3/2} (-B + 2iA) \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{f} - \frac{c(-B + 2iA) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f} + \frac{B \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{3/2}, x]$

[Out]  $-((\text{Sqrt}[a]*((2I)*A - B)*c^{3/2}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])])/f) - (((2I)*A - B)*c*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(2*f) + (B*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{3/2})/(2*f)$

**Rule 52**

$\text{Int}[(a + b*x)^m * ((c + d*x)^n)^m, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * ((c + d*x)^n / (b*(m+n+1))), x] + \text{Dist}[n * ((b*c - a*d) / (b*(m+n+1))), \text{Int}[(a + b*x)^m * (c + d*x)^{n-1}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILTQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

$\text{Int}[(a + b*x)^m * ((c + d*x)^n)^m, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1} * (c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{1/p}], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)\sqrt{c-icx}}{\sqrt{a+iax}} dx, \right)}{f} \\
&= \frac{B \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{2f} \\
&= -\frac{(2iA - B)c \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{2f} \\
&= -\frac{(2iA - B)c \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{2f} \\
&= -\frac{(2iA - B)c \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{3/2}}{2f} \\
&= -\frac{\sqrt{a} (2iA - B)c^{3/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}} \right)}{f}
\end{aligned}$$

**Mathematica [A]**

time = 2.35, size = 159, normalized size = 0.97

$$\frac{c^2 e^{-ie} \left( -i \cos\left(\frac{e}{2}\right) + \sin\left(\frac{e}{2}\right) \right) \left( \cos\left(\frac{e}{2} + fx\right) - i \sin\left(\frac{e}{2} + fx\right) \right) \left( (4A + 2iB) \text{ArcTan}\left(e^{i(e+fx)}\right) + \sec(e + fx)(2A + 2iB + B \sec(e) \sec(e + fx) \sin(fx) + B \tan(e)) \right) \sqrt{a + ia \tan(e + fx)}}{2\sqrt{2} \sqrt{\frac{c}{1 + e^{2i(e+fx)}}} f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2), x]
```

```
[Out] (c^2*((-I)*Cos[e/2] + Sin[e/2])*(Cos[e/2 + f*x] - I*Sin[e/2 + f*x])*((4*A + (2*I)*B)*ArcTan[E^(I*(e + f*x))] + Sec[e + f*x]*(2*A + (2*I)*B + B*Sec[e]*Sec[e + f*x]*Sin[f*x] + B*Tan[e]))*Sqrt[a + I*a*Tan[e + f*x]])/(2*Sqrt[2]*E^(I*e)*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f)
```

**Maple [A]**

time = 0.40, size = 223, normalized size = 1.36

method	result
--------	--------

derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} c \left( iB \sqrt{ac(1+\tan^2(fx+e))} \sqrt{a} \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} c \left( iB \sqrt{ac(1+\tan^2(fx+e))} \sqrt{a} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/2/f*(a*(1+I*\tan(f*x+e)))^{1/2}*(-c*(I*\tan(f*x+e)-1))^{1/2}*c*(I*B*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}*\tan(f*x+e)-I*B*\ln((a*c*\tan(f*x+e)+(a*c)^{1/2}*(a*c*(1+\tan(f*x+e)^2))^{1/2}))/((a*c)^{1/2})*a*c+2*I*A*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}-2*A*\ln((a*c*\tan(f*x+e)+(a*c)^{1/2}*(a*c*(1+\tan(f*x+e)^2))^{1/2}))/((a*c)^{1/2})*a*c-2*B*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}))/((a*c*(1+\tan(f*x+e)^2))^{1/2}))/((a*c)^{1/2})$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 819 vs.  $2(125) = 250$ .  
time = 0.70, size = 819, normalized size = 4.99

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out] 
$$-4*(4*(2*A + I*B)*c*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(2*A + 3*I*B)*c*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(2*I*A - B)*c*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(2*I*A - 3*B)*c*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*((2*A + I*B)*c*\cos(4*f*x + 4*e) + 2*(2*A + I*B)*c*\cos(2*f*x + 2*e) + (2*I*A - B)*c*\sin(4*f*x + 4*e) + 2*(2*I*A - B)*c*\sin(2*f*x + 2*e) + (2*A + I*B)*c)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 2*((2*A + I*B)*c*\cos(4*f*x + 4*e) + 2*(2*A + I*B)*c*\cos(2*f*x + 2*e) + (2*I*A - B)*c*\sin(4*f*x + 4*e) + 2*(2*I*A - B)*c*\sin(2*f*x + 2*e) + (2*A + I*B)*c)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + ((2*I*A - B)*c*\cos(4*f*x + 4*e) + 2*(2*I*A - B)*c*\cos(2*f*x + 2*e) - (2*A + I*B)*c*\sin(4*f*x + 4*e) - 2*(2*A + I*B)*c*\sin(2*f*x + 2*e) + (2*I*A - B)*c)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\sin(1/2*$$

$\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1) + ((-2IA + B)*c*\cos(4fx + 4e) + 2*(-2IA + B)*c*\cos(2fx + 2e) + (2A + IB)*c*\sin(4fx + 4e) + 2*(2A + IB)*c*\sin(2fx + 2e) + (-2IA + B)*c)*\log(\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + \sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 - 2*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1))*\sqrt{a}*\sqrt{c}/(f*(-16I*\cos(4fx + 4e) - 32I*\cos(2fx + 2e) + 16*\sin(4fx + 4e) + 32*\sin(2fx + 2e) - 16I))$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(125) = 250.

time = 3.58, size = 478, normalized size = 2.91

$$\frac{\sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \log\left(\frac{\sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}}}{\sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}}}\right) - \sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \log\left(\frac{\sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}}}{\sqrt{\frac{4A^2 + 4IAB - B^2}{f}} \sqrt{\frac{4A^2 + 4IAB - B^2}{f}}}\right)}{4(f\sqrt{a}\sqrt{c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/4\*(sqrt((4A^2 + 4IAB - B^2)\*a\*c^3/f^2)\*(f\*e^(2I\*f\*x + 2I\*e) + f)\*log(-4\*(2\*((2IA - B)\*c\*e^(3I\*f\*x + 3I\*e) + (2IA - B)\*c\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2I\*f\*x + 2I\*e) + 1))\*sqrt(c/(e^(2I\*f\*x + 2I\*e) + 1)) + sqrt((4A^2 + 4IAB - B^2)\*a\*c^3/f^2)\*(f\*e^(2I\*f\*x + 2I\*e) - f))/((-2IA + B)\*c\*e^(2I\*f\*x + 2I\*e) + (-2IA + B)\*c) - sqrt((4A^2 + 4IAB - B^2)\*a\*c^3/f^2)\*(f\*e^(2I\*f\*x + 2I\*e) + f)\*log(-4\*(2\*((2IA - B)\*c\*e^(3I\*f\*x + 3I\*e) + (2IA - B)\*c\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2I\*f\*x + 2I\*e) + 1))\*sqrt(c/(e^(2I\*f\*x + 2I\*e) + 1)) - sqrt((4A^2 + 4IAB - B^2)\*a\*c^3/f^2)\*(f\*e^(2I\*f\*x + 2I\*e) - f))/((-2IA + B)\*c\*e^(2I\*f\*x + 2I\*e) + (-2IA + B)\*c) - 4\*((2IA - B)\*c\*e^(3I\*f\*x + 3I\*e) + (2IA - 3B)\*c\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2I\*f\*x + 2I\*e) + 1))\*sqrt(c/(e^(2I\*f\*x + 2I\*e) + 1)))/(f\*e^(2I\*f\*x + 2I\*e) + f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(e + fx) - i)} (-ic(\tan(e + fx) + i))^{\frac{3}{2}} (A + B \tan(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Integral(sqrt(I\*a\*(tan(e + f\*x) - I))\*(-I\*c\*(tan(e + f\*x) + I))\*\*(3/2)\*(A + B\*tan(e + f\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.01
```

$$\int (A + B \tan(e + f x)) \sqrt{a + a \tan(e + f x) \operatorname{li}(c - c \tan(e + f x) \operatorname{li})}^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(3/2), x)
```

### 3.790 $\int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx)) \sqrt{c - ictan(e + fx)}$

Optimal. Leaf size=104

$$-\frac{2i\sqrt{a} A\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{f} + \frac{B\sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{f}$$

[Out]  $-2*I*A*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})*a^{(1/2)}*c^{(1/2)}/f+B*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/f$

Rubi [A]

time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3669, 81, 65, 223, 209}

$$\frac{B\sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{f} - \frac{2i\sqrt{a} A\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]],x]`

[Out]  $((-2*I)*\operatorname{Sqrt}[a]*A*\operatorname{Sqrt}[c]*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])])/f + (B*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/f$

Rule 65

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 81

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]`

Rule 209



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx)) \sqrt{c - ict \tan(e + fx)} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{\sqrt{a+iax} \sqrt{c-icx}} dx \right)}{f} \\ &= \frac{B \sqrt{a + ia \tan(e + fx)} \sqrt{c - ict \tan(e + fx)}}{f} \\ &= \frac{B \sqrt{a + ia \tan(e + fx)} \sqrt{c - ict \tan(e + fx)}}{f} \\ &= \frac{B \sqrt{a + ia \tan(e + fx)} \sqrt{c - ict \tan(e + fx)}}{f} \\ &= -\frac{2i\sqrt{a} A\sqrt{c} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ict \tan(e+fx)}} \right)}{f} \end{aligned}$$

### Mathematica [A]

time = 1.33, size = 102, normalized size = 0.98

$$\frac{\sqrt{2} e^{-i(e+fx)} \sqrt{\frac{c}{1+e^{2i(e+fx)}}} (B e^{i(e+fx)} - iA(1+e^{2i(e+fx)}) \text{ArcTan}(e^{i(e+fx)})) \sqrt{a+ia \tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] (Sqrt[2]*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*(B*E^(I*(e + f*x)) - I*A*(1 + E^((2*I)*(e + f*x))))*ArcTan[E^(I*(e + f*x))])*Sqrt[a + I*a*Tan[e + f*x]]/(E^(I*(e + f*x))*f)
```

**Maple [A]**

time = 0.39, size = 121, normalized size = 1.16

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} \left( A \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1+\tan^2(fx+e))} \right)}{f \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac}} \right)}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} \left( A \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1+\tan^2(fx+e))} \right)}{f \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac}} \right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)), x, method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*(A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs.  $2(82) = 164$ .

time = 0.63, size = 481, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)), x, algorithm="maxima")
```

```
[Out] -(2*(A*cos(2*f*x + 2*e) + I*A*sin(2*f*x + 2*e) + A)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 2*(A*cos(2*f*x + 2*e) + I*A*sin(2*f*x + 2*e) + A)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 4*I*B*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (-I*A*cos(2*f*x + 2*e) + A*sin(2*f*x + 2*e) - I*A)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1
```

$$\frac{1}{2} \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e)) \wedge 2 + 2 \sin(\frac{1}{2} \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1 - (IA \cos(2fx + 2e) - A \sin(2fx + 2e) + IA) \log(\cos(\frac{1}{2} \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) \wedge 2 + \sin(\frac{1}{2} \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) \wedge 2 - 2 \sin(\frac{1}{2} \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1 - 4B \sin(\frac{1}{2} \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) \sqrt{a} \sqrt{c} / (f(-2I \cos(2fx + 2e) + 2 \sin(2fx + 2e) - 2I))$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 308 vs.  $2(82) = 164$ .

time = 5.10, size = 308, normalized size = 2.96

$$\frac{4B \sqrt{\frac{a}{e^{2i(fx+2e)}+1}} \sqrt{\frac{c}{e^{2i(fx+2e)}+1}} e^{i(fx+e)} + \sqrt{\frac{A^2ac}{f^2}} f \log \left( \frac{2(Ae^{2i(fx+2e)} + Ae^{i(fx+e)}) \sqrt{\frac{a}{e^{2i(fx+2e)}+1}} \sqrt{\frac{c}{e^{2i(fx+2e)}+1}} - \sqrt{\frac{A^2ac}{f^2}} (Ae^{2i(fx+2e)} - f)}{Ae^{2i(fx+2e)} + A} \right) - \sqrt{\frac{A^2ac}{f^2}} f \log \left( \frac{2(Ae^{2i(fx+2e)} + Ae^{i(fx+e)}) \sqrt{\frac{a}{e^{2i(fx+2e)}+1}} \sqrt{\frac{c}{e^{2i(fx+2e)}+1}} - \sqrt{\frac{A^2ac}{f^2}} (-Ae^{2i(fx+2e)} + f)}{Ae^{2i(fx+2e)} + A} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e)),x, algorithm="fricas")

[Out]  $\frac{1}{2} (4B \sqrt{a/(e^{2Ifx+2Ie}+1)} \sqrt{c/(e^{2Ifx+2Ie}+1)}) e^{Ifx+Ie} + \sqrt{A^2ac/f^2} f \log(4*(Ae^{3Ifx+3Ie}+Ae^{Ifx+Ie}) \sqrt{a/(e^{2Ifx+2Ie}+1)} \sqrt{c/(e^{2Ifx+2Ie}+1)}) - \sqrt{A^2ac/f^2} (Ifx e^{2Ifx+2Ie} - Ifx) / (Ae^{2Ifx+2Ie}+A) - \sqrt{A^2ac/f^2} f \log(4*(Ae^{3Ifx+3Ie}+Ae^{Ifx+Ie}) \sqrt{a/(e^{2Ifx+2Ie}+1)} \sqrt{c/(e^{2Ifx+2Ie}+1)}) - \sqrt{A^2ac/f^2} (-Ifx e^{2Ifx+2Ie} + Ifx) / (Ae^{2Ifx+2Ie}+A) / f$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia(\tan(e+fx)-i)} \sqrt{-ic(\tan(e+fx)+i)} (A+B \tan(e+fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(1/2)\*(c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e)),x)

[Out] Integral(sqrt(I\*a\*(tan(e+f\*x)-I))\*sqrt(-I\*c\*(tan(e+f\*x)+I))\*(A+B\*tan(e+f\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(I\*a\*tan(f\*x + e) + a)\*sqrt(-I\*c\*tan(f\*x + e) + c), x)

Mupad [B]

time = 11.09, size = 133, normalized size = 1.28

$$\frac{A \sqrt{a} \sqrt{c} \operatorname{atan}\left(\frac{\sqrt{c}(\sqrt{a+a \tan(e+f x) 1 i}-\sqrt{a})}{\sqrt{a}(\sqrt{c-c \tan(e+f x) 1 i}-\sqrt{c})}\right)}{f} + \frac{\sqrt{2} B \sqrt{\frac{c}{2 \cos(e+f x)^2+\sin(2 e+2 f x) 1 i}} \sqrt{\frac{a(2 \cos(e+f x)^2+\sin(2 e+2 f x) 1 i)}{2 \cos(e+f x)^2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2),x)

[Out] (2^(1/2)\*B\*(c/(sin(2\*e + 2\*f\*x)\*1i + 2\*cos(e + f\*x)^2))^(1/2)\*((a\*(sin(2\*e + 2\*f\*x)\*1i + 2\*cos(e + f\*x)^2))/(2\*cos(e + f\*x)^2))^(1/2))/f - (A\*a^(1/2)\*c^(1/2)\*atan((c^(1/2)\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(a^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))))\*4i)/f

$$3.791 \quad \int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{\sqrt{c - ictan(e + fx)}} dx$$

Optimal. Leaf size=109

$$\frac{2\sqrt{a} B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{\sqrt{c} f} - \frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}}$$

[Out] 2\*B\*arctan(c^(1/2)\*(a+I\*a\*tan(f\*x+e))^(1/2)/a^(1/2)/(c-I\*c\*tan(f\*x+e))^(1/2))\*a^(1/2)/f/c^(1/2)-(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^(1/2)/f/(c-I\*c\*tan(f\*x+e))^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3669, 79, 65, 223, 209}

$$\frac{2\sqrt{a} B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{\sqrt{c} f} - \frac{(B + iA) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + I\*a\*Tan[e + f\*x]]\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out] (2\*Sqrt[a]\*B\*ArcTan[(Sqrt[c]\*Sqrt[a + I\*a\*Tan[e + f\*x]])/(Sqrt[a]\*Sqrt[c - I\*c\*Tan[e + f\*x]])])/(Sqrt[c]\*f) - ((I\*A + B)\*Sqrt[a + I\*a\*Tan[e + f\*x]])/(f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I

```
IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{\sqrt{c - ictan(e + fx)}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{\sqrt{a + ia x} (c-ix)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
 &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} + \frac{(iaB) \text{Subst} \left( \int \frac{1}{\sqrt{a + ia x}} dx, x, \tan(e + fx) \right)}{f} \\
 &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} + \frac{(2B) \text{Subst} \left( \int \frac{1}{\sqrt{2c - 2ia x}} dx, x, \tan(e + fx) \right)}{f} \\
 &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} + \frac{(2B) \text{Subst} \left( \int \frac{1}{1 + \frac{cx^2}{a}} dx, x, \tan(e + fx) \right)}{f} \\
 &= \frac{2\sqrt{a} B \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right)}{\sqrt{c} f} - \frac{(iA + B)}{f \sqrt{c}}
 \end{aligned}$$

**Mathematica [A]**

time = 1.57, size = 127, normalized size = 1.17

$$\frac{(\cos(\frac{1}{2}(e+fx)) - i \sin(\frac{1}{2}(e+fx))) (-i \cos(\frac{1}{2}(e+fx)) + \sin(\frac{1}{2}(e+fx))) (A - iB + 2B \text{ArcTan}(e^{i(e+fx)})) (i \cos(e+fx) + \sin(e+fx)) \sqrt{a + ia \tan(e+fx)}}{f \sqrt{c - i \tan(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + I\*a\*Tan[e + f\*x]]\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]],x]

[Out] ((Cos[(e + f\*x)/2] - I\*Sin[(e + f\*x)/2])\*((-I)\*Cos[(e + f\*x)/2] + Sin[(e + f\*x)/2])\*(A - I\*B + 2\*B\*ArcTan[E^(I\*(e + f\*x))])\*(I\*Cos[e + f\*x] + Sin[e + f\*x])\*Sqrt[a + I\*a\*Tan[e + f\*x]]/(f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(88) = 176.

time = 0.46, size = 321, normalized size = 2.94

method	result
derivativedivides	$-\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} \left( -2iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \frac{\sqrt{ac}(1 - \dots)}{\sqrt{ac}} \right) \right)}{\dots}$
default	$-\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} \left( -2iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \frac{\sqrt{ac}(1 - \dots)}{\sqrt{ac}} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out] -I/f\*(a\*(1+I\*tan(f\*x+e)))^(1/2)\*(-c\*(I\*tan(f\*x+e)-1))^(1/2)/c\*(-2\*I\*B\*ln((a\*c\*tan(f\*x+e)+(a\*c)^(1/2)\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2))/(a\*c)^(1/2))\*a\*c\*tan(f\*x+e)-B\*ln((a\*c\*tan(f\*x+e)+(a\*c)^(1/2)\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2))/(a\*c)^(1/2))\*a\*c\*tan(f\*x+e)^2+I\*A\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2)\*(a\*c)^(1/2)\*tan(f\*x+e)+I\*B\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2)\*(a\*c)^(1/2)+B\*ln((a\*c\*tan(f\*x+e)+(a\*c)^(1/2)\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2))/(a\*c)^(1/2))\*a\*c+B\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2)\*(a\*c)^(1/2)\*tan(f\*x+e)-A\*(a\*c\*(1+tan(f\*x+e)^2))^(1/2)\*(a\*c)^(1/2))/(a\*c\*(1+tan(f\*x+e)^2))^(1/2)/(I+tan(f\*x+e))^2/(a\*c)^(1/2)

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(87) = 174.  
time = 4.14, size = 348, normalized size = 3.19

$$\frac{cf\sqrt{\frac{B^2a}{c^2f^2}} \log\left(\frac{e^{\frac{1}{2}(Bc^{2n}f^{2n+1} + Bc^{2n}f^{2n+1})} \sqrt{\frac{a}{e^{2n}f^{2n} + 1}} \sqrt{\frac{c}{e^{2n}f^{2n} + 1}} + (cf)^{2n}f^{2n-1} \sqrt{\frac{B^2a}{c^2f^2}}}{Bc^{2n}f^{2n+1} + B}\right) - cf\sqrt{\frac{B^2a}{c^2f^2}} \log\left(\frac{e^{\frac{1}{2}(Bc^{2n}f^{2n+1} + Bc^{2n}f^{2n+1})} \sqrt{\frac{a}{e^{2n}f^{2n} + 1}} \sqrt{\frac{c}{e^{2n}f^{2n} + 1}} - (cf)^{2n}f^{2n-1} \sqrt{\frac{B^2a}{c^2f^2}}}{Bc^{2n}f^{2n+1} + B}\right)}{2cf} + 2((iA + B)e^{2n}f^{2n} + (iA + B)e^{2n}f^{2n}) \sqrt{\frac{a}{e^{2n}f^{2n} + 1}} \sqrt{\frac{c}{e^{2n}f^{2n} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] 
$$-1/2*(cf*\sqrt{-B^2*a/(cf^2)})*\log(4*(2*(B*e^{(3*I*f*x + 3*I*e)} + B*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + (cf*e^{(2*I*f*x + 2*I*e)} - cf)*\sqrt{-B^2*a/(cf^2)})/(B*e^{(2*I*f*x + 2*I*e)} + B) - cf*\sqrt{-B^2*a/(cf^2)}*\log(4*(2*(B*e^{(3*I*f*x + 3*I*e)} + B*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - (cf*e^{(2*I*f*x + 2*I*e)} - cf)*\sqrt{-B^2*a/(cf^2)})/(B*e^{(2*I*f*x + 2*I*e)} + B) + 2*((I*A + B)*e^{(3*I*f*x + 3*I*e)} + (I*A + B)*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})/(cf)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia(\tan(e + fx) - i)}(A + B \tan(e + fx))}{\sqrt{-ic(\tan(e + fx) + i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(I\*a\*(tan(e + f\*x) - I))\*(A + B\*tan(e + f\*x))/sqrt(-I\*c\*(tan(e + f\*x) + I)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(I\*a\*tan(f\*x + e) + a)/sqrt(-I\*c\*tan(f\*x + e) + c), x)

**Mupad [B]**

time = 12.41, size = 266, normalized size = 2.44

$$\frac{4B\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{c}(\sqrt{a+a\tan(e+fx)}\operatorname{li}-\sqrt{a})}{\sqrt{a}(\sqrt{c-c\tan(e+fx)}\operatorname{li}-\sqrt{c})}\right)}{\sqrt{c}f} + \frac{A\sqrt{a+a\tan(e+fx)}\operatorname{li}-\sqrt{c-c\tan(e+fx)}\operatorname{li}}{cf(\tan(e+fx)+\operatorname{li})} - \frac{4Ba(\sqrt{a+a\tan(e+fx)}\operatorname{li}-\sqrt{a})}{cf(\sqrt{c-c\tan(e+fx)}\operatorname{li}-\sqrt{c})\left(\frac{a}{\sqrt{c-c\tan(e+fx)}\operatorname{li}-\sqrt{c}} + \frac{2\sqrt{a}(\sqrt{a+a\tan(e+fx)}\operatorname{li}-\sqrt{a})}{\sqrt{c}(\sqrt{c-c\tan(e+fx)}\operatorname{li}-\sqrt{c})}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(1/2))/(c - c\*tan(e + f\*x)\*1i)^(1/2),x)

[Out] (4\*B\*a^(1/2)\*atan((c^(1/2)\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(a^(1/2)\*((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))))/(c^(1/2)\*f) + (A\*(a + a\*tan(e + f\*x)\*1i)^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(c\*f\*(tan(e + f\*x) + 1i)) - (4\*B\*a\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(c\*f\*((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))\*(a/c - ((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2))^2/((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))^2 + (2\*a^(1/2)\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(c^(1/2)\*((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2)))))

$$3.792 \quad \int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{3/2}} dx$$

Optimal. Leaf size=102

$$-\frac{(iA + B)\sqrt{a + ia \tan(e + fx)}}{3f(c - ictan(e + fx))^{3/2}} - \frac{(iA - 2B)\sqrt{a + ia \tan(e + fx)}}{3cf\sqrt{c - ictan(e + fx)}}$$

[Out]  $-1/3*(I*A-2*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}-1/3*(I*A+B)*(a+I*a*\tan(f*x+e))^{(1/2)}/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {3669, 79, 37}

$$-\frac{(-2B + iA)\sqrt{a + ia \tan(e + fx)}}{3cf\sqrt{c - ictan(e + fx)}} - \frac{(B + iA)\sqrt{a + ia \tan(e + fx)}}{3f(c - ictan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(A + B*\text{Tan}[e + f*x]))/(c - I*c*\text{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $-1/3*((I*A + B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - ((I*A - 2*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(3*c*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)})/((b*c - a*d)*(m + 1)), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 79

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)})/(f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ (!\text{LtQ}[n, -1] \ || \ \text{IntegerQ}[p] \ || \ !(\text{IntegerQ}[n] \ || \ !(\text{EqQ}[e, 0] \ || \ !(\text{EqQ}[c, 0] \ || \ \text{LtQ}[p, n])))$

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{\sqrt{a + ia x} (c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{3f(c - ictan(e + fx))^{3/2}} + \frac{(a(A + 2iB)) \text{Subst} \left( \int \frac{1}{\sqrt{a + ia x} (c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{3cf \sqrt{c - ictan(e + fx)}}$$

$$= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{3f(c - ictan(e + fx))^{3/2}} - \frac{(iA - 2B) \sqrt{a + ia \tan(e + fx)}}{3cf \sqrt{c - ictan(e + fx)}}$$

**Mathematica [A]**

time = 1.65, size = 101, normalized size = 0.99

$$\frac{\cos(e + fx)((-2iA + B) \cos(e + fx) - (A + 2iB) \sin(e + fx))(\cos(2(e + fx)) + i \sin(2(e + fx))) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{3c^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e
+ f*x])^(3/2), x]
```

```
[Out] (Cos[e + f*x]*((-2*I)*A + B)*Cos[e + f*x] - (A + (2*I)*B)*Sin[e + f*x])*(C
os[2*(e + f*x)] + I*Ssin[2*(e + f*x)])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I
*c*Tan[e + f*x]]/(3*c^2*f)
```

**Maple [A]**

time = 0.42, size = 100, normalized size = 0.98

method	result
risch	$-\frac{\sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (iA e^{2i(fx+e)} + B e^{2i(fx+e)} + 3iA - 3B)}{6c \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$\frac{\sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} (2iB(\tan^2(fx + e)) + 3iA \tan(fx + e) + A(\tan^2(fx + e) + 1))}{3f c^2 (i + \tan(fx + e))^3}$

default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} (2iB(\tan^2(fx+e))+3iA\tan(fx+e)+A(\tan^2(fx+e)-1))}{3fc^2(i+\tan(fx+e))^3}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{3}f*(a*(1+I*\tan(f*x+e)))^{1/2}*(-c*(I*\tan(f*x+e)-1))^{1/2}/c^2*(2*I*B*\tan(f*x+e)^2+3*I*A*\tan(f*x+e)+A*\tan(f*x+e)^2-I*B-3*B*\tan(f*x+e)-2*A)/(I+\tan(f*x+e))^3$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [A]

time = 7.75, size = 101, normalized size = 0.99

$$\frac{((-iA - B)e^{(5ifx+5ie)} - 2(2iA - B)e^{(3ifx+3ie)} - 3(iA - B)e^{(ifx+ie)}) \sqrt{\frac{a}{e^{(2ifx+2ie)} + 1}} \sqrt{\frac{c}{e^{(2ifx+2ie)} + 1}}}{6c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,algorithm="fricas")`

[Out]  $\frac{1}{6}*((-I*A - B)*e^{(5*I*f*x + 5*I*e)} - 2*(2*I*A - B)*e^{(3*I*f*x + 3*I*e)} - 3*(I*A - B)*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}/(c^2*f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia(\tan(e+fx)-i)}(A+B\tan(e+fx))}{(-ic(\tan(e+fx)+i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Integral(sqrt(I\*a\*(tan(e + f\*x) - I))\*(A + B\*tan(e + f\*x))/(-I\*c\*(tan(e + f\*x) + I))\*\*(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(I\*a\*tan(f\*x + e) + a)/(-I\*c\*tan(f\*x + e) + c)^(3/2), x)

**Mupad [B]**

time = 1.24, size = 145, normalized size = 1.42

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}(A3i-3B+A\cos(2e+2fx)1i+B\cos(2e+2fx)-A\sin(2e+2fx)+B\sin(2e+2fx)1i)}{6cf\sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(1/2))/(c - c\*tan(e + f\*x)\*1i)^(3/2),x)

[Out] -(((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*3i - 3\*B + A\*cos(2\*e + 2\*f\*x)\*1i + B\*cos(2\*e + 2\*f\*x) - A\*sin(2\*e + 2\*f\*x) + B\*sin(2\*e + 2\*f\*x)\*1i))/(6\*c\*f\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2))

$$3.793 \quad \int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{5/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{(iA + B)\sqrt{a + ia \tan(e + fx)}}{5f(c - ictan(e + fx))^{5/2}} - \frac{(2iA - 3B)\sqrt{a + ia \tan(e + fx)}}{15cf(c - ictan(e + fx))^{3/2}} - \frac{(2iA - 3B)\sqrt{a + ia \tan(e + fx)}}{15c^2f\sqrt{c - ictan(e + fx)}}$$

[Out]  $-1/15*(2*I*A-3*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(1/2)}-1/5*(I*A+B)*(a+I*a*\tan(f*x+e))^{(1/2)}/f/(c-I*c*\tan(f*x+e))^{(5/2)}-1/15*(2*I*A-3*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{(-3B + 2iA)\sqrt{a + ia \tan(e + fx)}}{15c^2f\sqrt{c - ictan(e + fx)}} - \frac{(-3B + 2iA)\sqrt{a + ia \tan(e + fx)}}{15cf(c - ictan(e + fx))^{3/2}} - \frac{(B + iA)\sqrt{a + ia \tan(e + fx)}}{5f(c - ictan(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(A + B*\text{Tan}[e + f*x]))/(c - I*c*\text{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $-1/5*((I*A + B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (((2*I)*A - 3*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(15*c*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - (((2*I)*A - 3*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(15*c^2*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

**Rule 37**

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * (c + d*x)^{n+1} / ((b*c - a*d)*(m+1)), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

**Rule 47**

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * (c + d*x)^{n+1} / ((b*c - a*d)*(m+1)), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2] / ((b*c - a*d)*(m+1))), \text{Int}[(a + b*x)^{\text{Simplify}[m+1]} * (c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && I LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

## Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+iax}(c-icx)^{7/2}} dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(iA+B)\sqrt{a+ia \tan(e+fx)}}{5f(c-ictan(e+fx))^{5/2}} + \frac{(a(2A+3iB)) \text{Subst}\left(\int \frac{1}{\sqrt{a+iax}(c-icx)^{7/2}} dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(iA+B)\sqrt{a+ia \tan(e+fx)}}{5f(c-ictan(e+fx))^{5/2}} - \frac{(2iA-3B)\sqrt{a+ia \tan(e+fx)}}{15cf(c-ictan(e+fx))^{5/2}}$$

$$= -\frac{(iA+B)\sqrt{a+ia \tan(e+fx)}}{5f(c-ictan(e+fx))^{5/2}} - \frac{(2iA-3B)\sqrt{a+ia \tan(e+fx)}}{15cf(c-ictan(e+fx))^{5/2}}$$

## Mathematica [A]

time = 2.74, size = 114, normalized size = 0.74

$$\frac{\cos(e+fx)(-5iA+(-9iA+6B)\cos(2(e+fx))-3(2A+3iB)\sin(2(e+fx)))(\cos(3(e+fx))+i\sin(3(e+fx)))\sqrt{a+ia \tan(e+fx)}\sqrt{c-ictan(e+fx)}}{30c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e
+ f*x])^(5/2), x]
```

```
[Out] (Cos[e + f*x]*((-5*I)*A + ((-9*I)*A + 6*B)*Cos[2*(e + f*x)] - 3*(2*A + (3*I)*B)*Sin[2*(e + f*x)])*(Cos[3*(e + f*x)] + I*Ssin[3*(e + f*x)])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]]/(30*c^3*f)
```

**Maple [A]**

time = 0.42, size = 125, normalized size = 0.81

method	result
risch	$-\frac{\sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (3iA e^{4i(fx+e)}+3B e^{4i(fx+e)}+10iA e^{2i(fx+e)}+15iA-15B)}{60c^2 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$-\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} (2iA(\tan^3(fx+e))-12iB(\tan^2(fx+e))-3B \tan(fx+e))}{15f c^3(i+\tan(fx+e))^4}$
default	$-\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} (2iA(\tan^3(fx+e))-12iB(\tan^2(fx+e))-3B \tan(fx+e))}{15f c^3(i+\tan(fx+e))^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/c^3*(2*I*A*tan(f*x+e)^3-12*I*B*tan(f*x+e)^2-3*B*tan(f*x+e)^3-13*I*A*tan(f*x+e)-8*A*tan(f*x+e)^2+3*I*B+12*B*tan(f*x+e)+7*A)/(I+tan(f*x+e))^4
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [A]**

time = 9.58, size = 119, normalized size = 0.77

$$-\frac{(3(iA+B)e^{7i fx+7i e}) - (-13iA-3B)e^{5i fx+5i e} + 5(5iA-3B)e^{3i fx+3i e} + 15(iA-B)e^{i fx+i e}) \sqrt{\frac{a}{e^{2i fx+2i e}+1}} \sqrt{\frac{c}{e^{2i fx+2i e}+1}}}{60c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="fricas")
```



[Out]  $-1/60*(3*(I*A + B)*e^{(7*I*f*x + 7*I*e)} - (-13*I*A - 3*B)*e^{(5*I*f*x + 5*I*e)} + 5*(5*I*A - 3*B)*e^{(3*I*f*x + 3*I*e)} + 15*(I*A - B)*e^{(I*f*x + I*e)})*\sqrt{t(a/(e^{(2*I*f*x + 2*I*e)} + 1))*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}}/(c^3*f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia(\tan(e + fx) - i)}(A + B \tan(e + fx))}{(-ic(\tan(e + fx) + i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),x)`

[Out] `Integral(sqrt(I*a*(tan(e + f*x) - I))*(A + B*tan(e + f*x))/(-I*c*(tan(e + f*x) + I))**(5/2), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac")`

[Out] `integrate((B*tan(f*x + e) + A)*sqrt(I*a*tan(f*x + e) + a)/(-I*c*tan(f*x + e) + c)^(5/2), x)`

**Mupad [B]**

time = 9.86, size = 171, normalized size = 1.10

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}(A15i-15B+A\cos(2e+2fx)10i+A\cos(4e+4fx)3i+3B\cos(4e+4fx)-10A\sin(2e+2fx)-3A\sin(4e+4fx)+B\sin(4e+4fx)3i)}{60c^2f\sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2))/(c - c*tan(e + f*x)*1i)^(5/2),x)`

[Out]  $-(((a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{\frac{1}{2}}*(A*15i - 15*B + A*\cos(2*e + 2*f*x)*10i + A*\cos(4*e + 4*f*x)*3i + 3*B*\cos(4*e + 4*f*x) - 10*A*\sin(2*e + 2*f*x) - 3*A*\sin(4*e + 4*f*x) + B*\sin(4*e + 4*f*x)*3i))/(60*c^2*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{\frac{1}{2}})$

$$3.794 \quad \int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{7/2}} dx$$

Optimal. Leaf size=208

$$\frac{(iA + B)\sqrt{a + ia \tan(e + fx)}}{7f(c - ictan(e + fx))^{7/2}} - \frac{(3iA - 4B)\sqrt{a + ia \tan(e + fx)}}{35cf(c - ictan(e + fx))^{5/2}} - \frac{2(3iA - 4B)\sqrt{a + ia \tan(e + fx)}}{105c^2f(c - ictan(e + fx))^{3/2}}$$

[Out]  $-2/105*(3*I*A-4*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/c^3/f/(c-I*c*\tan(f*x+e))^{(1/2)}-1/7*(I*A+B)*(a+I*a*\tan(f*x+e))^{(1/2)}/f/(c-I*c*\tan(f*x+e))^{(7/2)}-1/35*(3*I*A-4*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}-2/105*(3*I*A-4*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.18, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$-\frac{2(-4B+3iA)\sqrt{a+ia\tan(e+fx)}}{105c^3f\sqrt{c-ictan(e+fx)}} - \frac{2(-4B+3iA)\sqrt{a+ia\tan(e+fx)}}{105c^2f(c-ictan(e+fx))^{3/2}} - \frac{(-4B+3iA)\sqrt{a+ia\tan(e+fx)}}{35cf(c-ictan(e+fx))^{5/2}} - \frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{7f(c-ictan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(A + B*\text{Tan}[e + f*x]))/(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $-1/7*((I*A + B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) - (((3*I)*A - 4*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(35*c*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (2*((3*I)*A - 4*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(105*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - (2*((3*I)*A - 4*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(105*c^3*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 47

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}], x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ (\text{SumSimpler}$

Q[m, 1] || !SumSimplerQ[n, 1])

### Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + ia \tan(e + fx)} (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^{7/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+iax} (c-icx)^{9/2}} dx, x, \tan(e + fx)\right)}{f} \\ &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{7f(c - ict \tan(e + fx))^{7/2}} + \frac{(a(3A + 4iB)) \text{Subst}\left(\int \frac{1}{\sqrt{a+iax} (c-icx)^{9/2}} dx, x, \tan(e + fx)\right)}{f} \\ &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{7f(c - ict \tan(e + fx))^{7/2}} - \frac{(3iA - 4B) \sqrt{a + ia \tan(e + fx)}}{35cf(c - ict \tan(e + fx))^{7/2}} \\ &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{7f(c - ict \tan(e + fx))^{7/2}} - \frac{(3iA - 4B) \sqrt{a + ia \tan(e + fx)}}{35cf(c - ict \tan(e + fx))^{7/2}} \\ &= -\frac{(iA + B) \sqrt{a + ia \tan(e + fx)}}{7f(c - ict \tan(e + fx))^{7/2}} - \frac{(3iA - 4B) \sqrt{a + ia \tan(e + fx)}}{35cf(c - ict \tan(e + fx))^{7/2}} \end{aligned}$$

### Mathematica [A]

time = 4.32, size = 136, normalized size = 0.65

$$\frac{\cos(e + fx)(7(-12iA + B) \cos(e + fx) + 15(-4iA + 3B) \cos(3(e + fx)) - (3A + 4iB)(7 \sin(e + fx) + 15 \sin(3(e + fx))))(\cos(4(e + fx)) + i \sin(4(e + fx))) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ict \tan(e + fx)}}{420c^4 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(7/2), x]
```

```
[Out] (Cos[e + f*x]*(7*((-12*I)*A + B)*Cos[e + f*x] + 15*((-4*I)*A + 3*B)*Cos[3*(e + f*x)] - (3*A + (4*I)*B)*(7*Sin[e + f*x] + 15*Sin[3*(e + f*x)]))*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]]/(420*c^4*f)
```

**Maple [A]**

time = 0.40, size = 147, normalized size = 0.71

method	result
risch	$-\frac{\sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (15iA e^{6i(fx+e)}+15B e^{6i(fx+e)}+63iA e^{4i(fx+e)}+21B e^{4i(fx+e)}+105iA e^{2i(fx+e)}-35B e^{2i(fx+e)}+15iA e^{0i(fx+e)}+15B e^{0i(fx+e)})}{840c^3 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} (8iB(\tan^4(fx+e))+30iA(\tan^3(fx+e))+6A(\tan^2(fx+e))+3A)}{105f c^4(i+\tan(fx+e))}$
default	$\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} (8iB(\tan^4(fx+e))+30iA(\tan^3(fx+e))+6A(\tan^2(fx+e))+3A)}{105f c^4(i+\tan(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2), x, method=_RETURNVERBOSE)
```

```
[Out] 1/105/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/c^4*(8*I*B*tan(f*x+e)^4+30*I*A*tan(f*x+e)^3+6*A*tan(f*x+e)^2-84*I*B*tan(f*x+e)^2-40*B*tan(f*x+e)^3-75*I*A*tan(f*x+e)-63*A*tan(f*x+e)^2+13*I*B+65*B*tan(f*x+e)+36*A)/(I+tan(f*x+e))^5
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2), x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [A]**

time = 6.38, size = 138, normalized size = 0.66

$$\frac{(15(iA+B)e^{9i(fx+9ie)}+6(13iA+6B)e^{7i(fx+7ie)}+14(12iA-B)e^{5i(fx+5ie)}+70(3iA-2B)e^{3i(fx+3ie)}+105(iA-B)e^{i(fx+ie)})\sqrt{\frac{a}{e^{2i(fx+2ie)}+1}}\sqrt{\frac{c}{e^{2i(fx+2ie)}+1}}}{840c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="fricas")

[Out] -1/840\*(15\*(I\*A + B)\*e^(9\*I\*f\*x + 9\*I\*e) + 6\*(13\*I\*A + 6\*B)\*e^(7\*I\*f\*x + 7\*I\*e) + 14\*(12\*I\*A - B)\*e^(5\*I\*f\*x + 5\*I\*e) + 70\*(3\*I\*A - 2\*B)\*e^(3\*I\*f\*x + 3\*I\*e) + 105\*(I\*A - B)\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c^4\*f)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia(\tan(e+fx) - i)}(A + B \tan(e+fx))}{(-ic(\tan(e+fx) + i))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x)

[Out] Integral(sqrt(I\*a\*(tan(e + f\*x) - I))\*(A + B\*tan(e + f\*x))/(-I\*c\*(tan(e + f\*x) + I))^(7/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(I\*a\*tan(f\*x + e) + a)/(-I\*c\*tan(f\*x + e) + c)^(7/2), x)

**Mupad** [B]

time = 10.80, size = 246, normalized size = 1.18

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}(A105i-105B+A\cos(2e+2fx)105i+A\cos(4e+4fx)63i+A\cos(6e+6fx)15i-35B\cos(2e+2fx)+21B\cos(4e+4fx)+15B\cos(6e+6fx)-105A\sin(2e+2fx)-63A\sin(4e+4fx)-15A\sin(6e+6fx)-B\sin(2e+2fx)35i+B\sin(4e+4fx)21i+B\sin(6e+6fx)15i)}{840e^f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(1/2))/(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] -(((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*105i - 105\*B + A\*cos(2\*e + 2\*f\*x)\*105i + A\*cos(4\*e + 4\*f\*x)\*63i +

$$\frac{A \cos(6e + 6fx) \cdot 15i - 35B \cos(2e + 2fx) + 21B \cos(4e + 4fx) + 15B \cos(6e + 6fx) - 105A \sin(2e + 2fx) - 63A \sin(4e + 4fx) - 15A \sin(6e + 6fx) - B \sin(2e + 2fx) \cdot 35i + B \sin(4e + 4fx) \cdot 21i + B \sin(6e + 6fx) \cdot 15i}{(840c^3f((c(\cos(2e + 2fx) - \sin(2e + 2fx)) \cdot 1i + 1)) / (\cos(2e + 2fx) + 1))^{1/2}}$$

$$3.795 \quad \int (a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) (c - ictan(e + fx))^{7/2} dx$$

**Optimal.** Leaf size=279

$$\frac{a^{3/2}(5iA - 2B)c^{7/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{4f} + \frac{a(5A + 2iB)c^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{8f}$$

[Out]  $-1/4*a^{(3/2)}*(5*I*A-2*B)*c^{(7/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f+1/8*a*(5*A+2*I*B)*c^3*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f-1/12*(5*I*A-2*B)*c^2*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f-1/20*(5*I*A-2*B)*c*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f+1/5*B*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(7/2)}/f$

**Rubi [A]**

time = 0.23, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 81, 51, 38, 65, 223, 209}

$$\frac{a^{3/2}c^{7/2}(-2B + 5iA)\operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{4f} + \frac{a^2(5A + 2iB)\tan(e + fx)\sqrt{a + ia \tan(e + fx)}\sqrt{c - ictan(e + fx)}}{8f} - \frac{c^2(-2B + 5iA)(a + ia \tan(e + fx))^{3/2}(c - ictan(e + fx))^{3/2}}{12f} - \frac{c(-2B + 5iA)(a + ia \tan(e + fx))^{5/2}(c - ictan(e + fx))^{3/2}}{20f} + \frac{B(a + ia \tan(e + fx))^{3/2}(c - ictan(e + fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $-1/4*(a^{(3/2)}*((5*I)*A - 2*B)*c^{(7/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])])/f + (a*(5*A + (2*I)*B)*c^3*\operatorname{Tan}[e + f*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(8*f) - ((5*I)*A - 2*B)*c^2*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)}/(12*f) - (((5*I)*A - 2*B)*c*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(20*f) + (B*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(7/2)})/(5*f)$

**Rule 38**

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x] := \operatorname{Simp}[x*(a + b*x)^m*(c + d*x)^n/(2*m + 1), x] + \operatorname{Dist}[2*a*c*(m/(2*m + 1)), \operatorname{Int}[(a + b*x)^{m-1}*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 51**

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x] := \operatorname{Simp}[(a + b*x)^{m+1}*(c + d*x)^n/(b*(m + n + 1)), x] + \operatorname{Dist}[2*c*(n/(m + n + 1))$

), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]

### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps



$$\begin{aligned}
\int (a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{7/2} dx &= \frac{(ac) \text{Subst}\left(\int \sqrt{a + iax} (A + B \tan(e + fx)) dx\right)}{5f} \\
&= \frac{B(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))}{5f} \\
&= -\frac{(5iA - 2B)c(a + ia \tan(e + fx))^{3/2}}{20} \\
&= -\frac{(5iA - 2B)c^2(a + ia \tan(e + fx))^{3/2}}{12} \\
&= \frac{a(5A + 2iB)c^3 \tan(e + fx) \sqrt{a - ic \tan(e + fx)}}{4f} \\
&= \frac{a(5A + 2iB)c^3 \tan(e + fx) \sqrt{a - ic \tan(e + fx)}}{4f} \\
&= \frac{a^{3/2}(5iA - 2B)c^{7/2} \tan^{-1}\left(\frac{\sqrt{a - ic \tan(e + fx)}}{\sqrt{a + ic \tan(e + fx)}}\right)}{4f}
\end{aligned}$$

**Mathematica [A]**

time = 5.48, size = 257, normalized size = 0.92

$$\frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) \left( \frac{(-5iA + 2B)c^2 e^{-2i(e + fx)} \sqrt{\frac{e^{i(e + fx)}}{1 + e^{2i(e + fx)}}} \text{ArcTan}\left(\frac{e^{i(e + fx)}}{1 + e^{2i(e + fx)}}\right)}{c} - \frac{1}{20} c^3 \sec^2(e + fx) (320(A + iB) \cos(2(e + fx)) + 30(iA + 6B) \sin(2(e + fx)) + (5A + 2iB)(64 + 15i \sin(4(e + fx))))(i + \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{4f \sec^2(e + fx) (A \cos(e + fx) + B \sin(e + fx))} \right)}{4f \sec^2(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

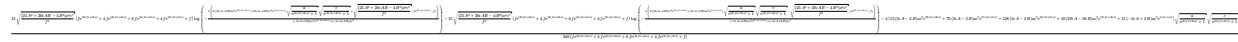
[In] Integrate[(a + I\*a\*Tan[e + f\*x])^(3/2)\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2), x]

[Out] ((a + I\*a\*Tan[e + f\*x])^(3/2)\*(A + B\*Tan[e + f\*x])\*((( -5\*I)\*A + 2\*B)\*c^4\*Sqrt[E^(I\*(e + f\*x))/(1 + E^((2\*I)\*(e + f\*x))])\*ArcTan[E^(I\*(e + f\*x))])/(E^((2\*I)\*(e + f\*x))\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x))])) - (c^3\*Sec[e + f\*x])^(7/2)\*(320\*(A + I\*B)\*Cos[2\*(e + f\*x)] + 30\*(I\*A + 6\*B)\*Sin[2\*(e + f\*x)] + (5\*



$$\begin{aligned}
& 2*f*x + 2*e)) + 40*(29*A + 50*I*B)*a*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) - 60*(5*A + 2*I*B)*a*c^3*\cos(1/2*\arctan2(\sin(2*f*x + 2* \\
& e), \cos(2*f*x + 2*e))) + 60*(5*I*A - 2*B)*a*c^3*\sin(9/2*\arctan2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e))) + 280*(5*I*A - 2*B)*a*c^3*\sin(7/2*\arctan2(\sin(2*f \\
& *x + 2*e), \cos(2*f*x + 2*e))) + 512*(5*I*A - 2*B)*a*c^3*\sin(5/2*\arctan2(\sin \\
& (2*f*x + 2*e), \cos(2*f*x + 2*e))) + 40*(29*I*A - 50*B)*a*c^3*\sin(3/2*\arctan \\
& 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 60*(-5*I*A + 2*B)*a*c^3*\sin(1/2*\ar \\
& ctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 30*((5*A + 2*I*B)*a*c^3*\cos(10 \\
& *f*x + 10*e) + 5*(5*A + 2*I*B)*a*c^3*\cos(8*f*x + 8*e) + 10*(5*A + 2*I*B)*a* \\
& c^3*\cos(6*f*x + 6*e) + 10*(5*A + 2*I*B)*a*c^3*\cos(4*f*x + 4*e) + 5*(5*A + 2 \\
& *I*B)*a*c^3*\cos(2*f*x + 2*e) + (5*I*A - 2*B)*a*c^3*\sin(10*f*x + 10*e) + 5*( \\
& 5*I*A - 2*B)*a*c^3*\sin(8*f*x + 8*e) + 10*(5*I*A - 2*B)*a*c^3*\sin(6*f*x + 6* \\
& e) + 10*(5*I*A - 2*B)*a*c^3*\sin(4*f*x + 4*e) + 5*(5*I*A - 2*B)*a*c^3*\sin(2* \\
& f*x + 2*e) + (5*A + 2*I*B)*a*c^3)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + \\
& 1) + 30*((5*A + 2*I*B)*a*c^3*\cos(10*f*x + 10*e) + 5*(5*A + 2*I*B)*a*c^3*\co \\
& s(8*f*x + 8*e) + 10*(5*A + 2*I*B)*a*c^3*\cos(6*f*x + 6*e) + 10*(5*A + 2*I*B) \\
& *a*c^3*\cos(4*f*x + 4*e) + 5*(5*A + 2*I*B)*a*c^3*\cos(2*f*x + 2*e) + (5*I*A - \\
& 2*B)*a*c^3*\sin(10*f*x + 10*e) + 5*(5*I*A - 2*B)*a*c^3*\sin(8*f*x + 8*e) + 1 \\
& 0*(5*I*A - 2*B)*a*c^3*\sin(6*f*x + 6*e) + 10*(5*I*A - 2*B)*a*c^3*\sin(4*f*x + \\
& 4*e) + 5*(5*I*A - 2*B)*a*c^3*\sin(2*f*x + 2*e) + (5*A + 2*I*B)*a*c^3)*\arcta \\
& n2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(s \\
& in(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) + 15*((5*I*A - 2*B)*a*c^3*\cos(10*f \\
& *x + 10*e) + 5*(5*I*A - 2*B)*a*c^3*\cos(8*f*x + 8*e) + 10*(5*I*A - 2*B)*a*c^ \\
& 3*\cos(6*f*x + 6*e) + 10*(5*I*A - 2*B)*a*c^3*\cos(4*f*x + 4*e) + 5*(5*I*A - 2 \\
& *B)*a*c^3*\cos(2*f*x + 2*e) - (5*A + 2*I*B)*a*c^3*\sin(10*f*x + 10*e) - 5*(5* \\
& A + 2*I*B)*a*c^3*\sin(8*f*x + 8*e) - 10*(5*A + 2*I*B)*a*c^3*\sin(6*f*x + 6*e) \\
& - 10*(5*A + 2*I*B)*a*c^3*\sin(4*f*x + 4*e) - 5*(5*A + 2*I*B)*a*c^3*\sin(2*f* \\
& x + 2*e) + (5*I*A - 2*B)*a*c^3)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2 \\
& *f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \\
& 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) + 15*((-5*I*A + \\
& 2*B)*a*c^3*\cos(10*f*x + 10*e) + 5*(-5*I*A + 2*B)*a*c^3*\cos(8*f*x + 8*e) + \\
& 10*(-5*I*A + 2*B)*a*c^3*\cos(6*f*x + 6*e) + 10*(-5*I*A + 2*B)*a*c^3*\cos(4*f* \\
& x + 4*e) + 5*(-5*I*A + 2*B)*a*c^3*\cos(2*f*x + 2*e) + (5*A + 2*I*B)*a*c^3*\si \\
& n(10*f*x + 10*e) + 5*(5*A + 2*I*B)*a*c^3*\sin(8*f*x + 8*e) + 10*(5*A + 2*I*B) \\
& )*a*c^3*\sin(6*f*x + 6*e) + 10*(5*A + 2*I*B)*a*c^3*\sin(4*f*x + 4*e) + 5*(5*A \\
& + 2*I*B)*a*c^3*\sin(2*f*x + 2*e) + (-5*I*A + 2*B)*a*c^3)*\log(\cos(1/2*\arctan \\
& 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e) \\
& , \cos(2*f*x + 2*e)))^2 - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2* \\
& e))) + 1))*\sqrt{a}*\sqrt{c}/(f*(-115200*I*\cos(10*f*x + 10*e) - 576000*I*\cos( \\
& 8*f*x + 8*e) - 1152000*I*\cos(6*f*x + 6*e) - 1152000*I*\cos(4*f*x + 4*e) - 57 \\
& 6000*I*\cos(2*f*x + 2*e) + 115200*\sin(10*f*x + 10*e) + 576000*\sin(8*f*x + 8* \\
& e) + 1152000*\sin(6*f*x + 6*e) + 1152000*\sin(4*f*x + 4*e) + 576000*\sin(2*f*x \\
& + 2*e) - 115200*I)
\end{aligned}$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs.  $2(224) = 448$ .  
time = 3.96, size = 713, normalized size = 2.56



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((5*I*A - 2*B)*a*c^3*e^(3*I*f*x + 3*I*e) + (5*I*A - 2*B)*a*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-5*I*A + 2*B)*a*c^3*e^(2*I*f*x + 2*I*e) + (-5*I*A + 2*B)*a*c^3) - 15*sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((5*I*A - 2*B)*a*c^3*e^(3*I*f*x + 3*I*e) + (5*I*A - 2*B)*a*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-5*I*A + 2*B)*a*c^3*e^(2*I*f*x + 2*I*e) + (-5*I*A + 2*B)*a*c^3) - 4*(15*(5*I*A - 2*B)*a*c^3*e^(9*I*f*x + 9*I*e) + 70*(5*I*A - 2*B)*a*c^3*e^(7*I*f*x + 7*I*e) + 128*(5*I*A - 2*B)*a*c^3*e^(5*I*f*x + 5*I*e) + 10*(29*I*A - 50*B)*a*c^3*e^(3*I*f*x + 3*I*e) + 15*(-5*I*A + 2*B)*a*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{3/2} (c - c \tan(e + f x) i)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(7/2), x)

$$3.796 \quad \int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$$

**Optimal.** Leaf size=226

$$\frac{a^{3/2}(4iA - B)c^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{4f} + \frac{a(4A + iB)c^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{8f}$$

[Out]  $-1/4*a^{(3/2)}*(4*I*A-B)*c^{(5/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)})/(c-I*c*\tan(f*x+e))^{(1/2)}/f+1/8*a*(4*A+I*B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f-1/12*(4*I*A-B)*c*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/4*B*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f$

**Rubi [A]**

time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 81, 51, 38, 65, 223, 209}

$$\frac{a^{3/2}c^{5/2}(-B + 4iA)\operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{4f} + \frac{ac^2(4A + iB)\tan(e + fx)\sqrt{a + ia \tan(e + fx)}\sqrt{c - ic \tan(e + fx)}}{8f} - \frac{c(-B + 4iA)(a + ia \tan(e + fx))^{3/2}(c - ic \tan(e + fx))^{3/2}}{12f} + \frac{B(a + ia \tan(e + fx))^{3/2}(c - ic \tan(e + fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $-1/4*(a^{(3/2)}*((4*I)*A - B)*c^{(5/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])])/f + (a*(4*A + I*B)*c^2*\operatorname{Tan}[e + f*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(8*f) - (((4*I)*A - B)*c*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)})/(12*f) + (B*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(4*f)$

**Rule 38**

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(m_)}), x\_Symbol] \rightarrow \operatorname{Simp}[x*(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + \operatorname{Dist}[2*a*c*(m/(2*m + 1)), \operatorname{Int}[(a + b*x)^{(m-1)}*(c + d*x)^{(m-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 51**

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \operatorname{Dist}[2*c*(n/(m + n + 1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{5/2} dx &= \frac{(ac) \text{Subst}\left(\int \sqrt{a + iax} (A + Bx) dx\right)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))}{4f} \\
&= -\frac{(4iA - B)c(a + ia \tan(e + fx))}{12f} \\
&= \frac{a(4A + iB)c^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{4f} \\
&= \frac{a(4A + iB)c^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{4f} \\
&= \frac{a(4A + iB)c^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{4f} \\
&= -\frac{a^{3/2} (4iA - B) c^{5/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a}}\right)}{4f}
\end{aligned}$$

### Mathematica [A]

time = 4.34, size = 241, normalized size = 1.07

$$\frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) \left( \frac{(-4iA + B)c^2 e^{-2i(e + fx)} \sqrt{\frac{e^{i(e + fx)}}{1 + e^{2i(e + fx)}}} \text{ArcTan}(e^{i(e + fx)})}{\frac{c}{\sqrt{1 + e^{2i(e + fx)}}}} + \frac{1}{12} c^2 \sec^3(e + fx) (1 - i \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} (16(-iA + B) + 3(4A - 3iB + (4A + iB) \cos(2(e + fx))) \tan(e + fx)) \right)}{4f \sec^3(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^(3/2)\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] ((a + I\*a\*Tan[e + f\*x])^(3/2)\*(A + B\*Tan[e + f\*x])\*((((-4\*I)\*A + B)\*c^3\*Sqrt[E^(I\*(e + f\*x))/(1 + E^((2\*I)\*(e + f\*x)))]\*ArcTan[E^(I\*(e + f\*x))])/(E^((2\*I)\*(e + f\*x))\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x))])) + (c^2\*Sec[e + f\*x]^(3/2)\*(1 - I\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]\*(16\*((-I)\*A + B) + 3\*(4\*A - (3\*I)\*B + (4\*A + I\*B)\*Cos[2\*(e + f\*x)])\*Tan[e + f\*x]))/12))/(4\*f\*Sec[e + f\*x]^(5/2)\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))



**Maple [A]**

time = 0.44, size = 350, normalized size = 1.55

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} c^{2a} \left( {}_{6iB} \sqrt{ac(1+\tan^2(fx+e))} \right)}{\dots}$
default	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} c^{2a} \left( {}_{6iB} \sqrt{ac(1+\tan^2(fx+e))} \right) \dots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^2*a*(6*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+8*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-3*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+3*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-8*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+8*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-12*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-12*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-8*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1451 vs. 2(179) = 358.

time = 1.54, size = 1451, normalized size = 6.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -96*(12*(4*A + I*B)*a*c^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 44*(4*A + I*B)*a*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(20*A + 53*I*B)*a*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(4*A + I*B)*a*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(4*I*A - B)*a*c^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 44*(4*I*A - B)*a*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(20*I*A - 53*B)*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(-4*I*A + B)*a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*((4*A + I*B)*a*c^2*cos(8*f*x + 8*e) + 4*(4*A + I*B)*a
```

```

*c^2*cos(6*f*x + 6*e) + 6*(4*A + I*B)*a*c^2*cos(4*f*x + 4*e) + 4*(4*A + I*B
)*a*c^2*cos(2*f*x + 2*e) + (4*I*A - B)*a*c^2*sin(8*f*x + 8*e) + 4*(4*I*A -
B)*a*c^2*sin(6*f*x + 6*e) + 6*(4*I*A - B)*a*c^2*sin(4*f*x + 4*e) + 4*(4*I*A
- B)*a*c^2*sin(2*f*x + 2*e) + (4*A + I*B)*a*c^2)*arctan2(cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))) + 1) + 6*((4*A + I*B)*a*c^2*cos(8*f*x + 8*e) + 4*(4*A + I*B)
*a*c^2*cos(6*f*x + 6*e) + 6*(4*A + I*B)*a*c^2*cos(4*f*x + 4*e) + 4*(4*A + I
*B)*a*c^2*cos(2*f*x + 2*e) + (4*I*A - B)*a*c^2*sin(8*f*x + 8*e) + 4*(4*I*A
- B)*a*c^2*sin(6*f*x + 6*e) + 6*(4*I*A - B)*a*c^2*sin(4*f*x + 4*e) + 4*(4*I
*A - B)*a*c^2*sin(2*f*x + 2*e) + (4*A + I*B)*a*c^2)*arctan2(cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))) + 1) + 3*((4*I*A - B)*a*c^2*cos(8*f*x + 8*e) + 4*(4*I*A -
B)*a*c^2*cos(6*f*x + 6*e) + 6*(4*I*A - B)*a*c^2*cos(4*f*x + 4*e) + 4*(4*I*
A - B)*a*c^2*cos(2*f*x + 2*e) - (4*A + I*B)*a*c^2*sin(8*f*x + 8*e) - 4*(4*A
+ I*B)*a*c^2*sin(6*f*x + 6*e) - 6*(4*A + I*B)*a*c^2*sin(4*f*x + 4*e) - 4*(
4*A + I*B)*a*c^2*sin(2*f*x + 2*e) + (4*I*A - B)*a*c^2)*log(cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 1) + 3*((-4*I*A + B)*a*c^2*cos(8*f*x + 8*e) + 4*(-4*I*A + B)*a*c^2*cos
(6*f*x + 6*e) + 6*(-4*I*A + B)*a*c^2*cos(4*f*x + 4*e) + 4*(-4*I*A + B)*a*c^
2*cos(2*f*x + 2*e) + (4*A + I*B)*a*c^2*sin(8*f*x + 8*e) + 4*(4*A + I*B)*a*c
^2*sin(6*f*x + 6*e) + 6*(4*A + I*B)*a*c^2*sin(4*f*x + 4*e) + 4*(4*A + I*B)*
a*c^2*sin(2*f*x + 2*e) + (-4*I*A + B)*a*c^2)*log(cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1))*s
qrt(a)*sqrt(c)/(f*(-4608*I*cos(8*f*x + 8*e) - 18432*I*cos(6*f*x + 6*e) - 27
648*I*cos(4*f*x + 4*e) - 18432*I*cos(2*f*x + 2*e) + 4608*sin(8*f*x + 8*e) +
18432*sin(6*f*x + 6*e) + 27648*sin(4*f*x + 4*e) + 18432*sin(2*f*x + 2*e) -
4608*I))

```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs.  $2(179) = 358$ .  
time = 4.33, size = 641, normalized size = 2.84

$$\sqrt{\frac{(4A^2 + 8IAB - B^2)a^3c^5/f^2}{(f^2e^{2Ifx+2Ie} + 1)}} \left( \frac{(4A + IB)a^2c^2 \arctan\left(\frac{\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)}{\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)}\right)}{\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)}\right) + 1 \right) + 6 \left( (4A + IB)a^2c^2 \cos(8fx+8e) + 4(4A + IB)a^2c^2 \cos(6fx+6e) + 6(4A + IB)a^2c^2 \cos(4fx+4e) + 4(4A + IB)a^2c^2 \cos(2fx+2e) + (4IA - B)a^2c^2 \sin(8fx+8e) + 4(4IA - B)a^2c^2 \sin(6fx+6e) + 6(4IA - B)a^2c^2 \sin(4fx+4e) + 4(4IA - B)a^2c^2 \sin(2fx+2e) + (4A + IB)a^2c^2 \arctan\left(\frac{\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)}{\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)}\right) + 1 \right) + 3 \left( (-4IA + B)a^2c^2 \cos(8fx+8e) + 4(-4IA + B)a^2c^2 \cos(6fx+6e) + 6(-4IA + B)a^2c^2 \cos(4fx+4e) + 4(-4IA + B)a^2c^2 \cos(2fx+2e) + (4A + IB)a^2c^2 \sin(8fx+8e) + 4(4A + IB)a^2c^2 \sin(6fx+6e) + 6(4A + IB)a^2c^2 \sin(4fx+4e) + 4(4A + IB)a^2c^2 \sin(2fx+2e) + (-4IA + B)a^2c^2 \log\left(\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)\right)^2 + \sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right)^2 - 2\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}\right)\right) + 1 \right) \right) \sqrt{a} \sqrt{c} / (f(-4608I \cos(8fx+8e) - 18432I \cos(6fx+6e) - 27648I \cos(4fx+4e) - 18432I \cos(2fx+2e) + 4608 \sin(8fx+8e) + 18432 \sin(6fx+6e) + 27648 \sin(4fx+4e) + 18432 \sin(2fx+2e) - 4608I))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{48} \cdot (3 \sqrt{(16A^2 + 8IAB - B^2)a^3c^5/f^2}) \cdot (f^2e^{(6Ifx+6Ie)} + 3f^2e^{(4Ifx+4Ie)} + 3f^2e^{(2Ifx+2Ie)} + f) \cdot \log(-4 \cdot (2 \cdot ((4IA - B)a^2c^2e^{(3Ifx+3Ie)} + (4IA - B)a^2c^2e^{(Ifx+Ie)})) \sqrt{a/(e^{(2Ifx+2Ie)} + 1)}) \sqrt{c/(e^{(2Ifx+2Ie)} + 1)} + \sqrt{(16A^2 + 8IAB - B^2)a^3c^5/f^2}) \cdot (f^2e^{(2Ifx+2Ie)} - f) / (((-4IA + B)a^2c^2$

$$c^2 e^{(2I f x + 2I e)} + (-4I A + B) a c^2) - 3 \sqrt{(16A^2 + 8I A B - B^2) a^3 c^5 / f^2} (f e^{(6I f x + 6I e)} + 3f e^{(4I f x + 4I e)} + 3f e^{(2I f x + 2I e)} + f) \log(-4(2((4I A - B) a c^2 e^{(3I f x + 3I e)} + (4I A - B) a c^2 e^{(I f x + I e)}) \sqrt{a / (e^{(2I f x + 2I e)} + 1)}) \sqrt{c / (e^{(2I f x + 2I e)} + 1)} - \sqrt{(16A^2 + 8I A B - B^2) a^3 c^5 / f^2} (f e^{(2I f x + 2I e)} - f)) / ((-4I A + B) a c^2 e^{(2I f x + 2I e)} + (-4I A + B) a c^2) - 4(3(4I A - B) a c^2 e^{(7I f x + 7I e)} + 11(4I A - B) a c^2 e^{(5I f x + 5I e)} - (-20I A + 53B) a c^2 e^{(3I f x + 3I e)} + 3(-4I A + B) a c^2 e^{(I f x + I e)}) \sqrt{a / (e^{(2I f x + 2I e)} + 1)}) \sqrt{c / (e^{(2I f x + 2I e)} + 1)}) / (f e^{(6I f x + 6I e)} + 3f e^{(4I f x + 4I e)} + 3f e^{(2I f x + 2I e)} + f)$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{3/2} (c - c \tan(e + f x) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(5/2), x)

$$3.797 \quad \int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx))(c-ictan(e+fx))^{3/2} dx$$

**Optimal.** Leaf size=157

$$\frac{ia^{3/2}Ac^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{f} + \frac{aA\text{ctan}(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{2f}$$

[Out]  $-I*a^{(3/2)}*A*c^{(3/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f+1/2*a*A*c*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+1/3*B*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f$

**Rubi [A]**

time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 38, 65, 223, 209}

$$\frac{ia^{3/2}Ac^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{f} + \frac{aA\text{ctan}(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{2f} + \frac{B(a+ia\tan(e+fx))^{3/2}(c-ictan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $((-I)*a^{(3/2)}*A*c^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])])/f + (a*A*c*\text{Tan}[e + f*x]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(2*f) + (B*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(3*f)$

**Rule 38**

$\text{Int}[(a + b*x)^m*(c + d*x)^m, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*x)^m*(c + d*x)^m/(2*m + 1), x] + \text{Dist}[2*a*c*(m/(2*m + 1)), \text{Int}[(a + b*x)^{m-1}*(c + d*x)^{m-1}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 65**

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) (c - ictan(e + fx))^{3/2} dx &= \frac{(ac) \text{Subst} \left( \int \sqrt{a + iax} (A + Bx) dx \right)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{3/2} (c - ictan(e + fx))^{3/2}}{3f} \\
&= \frac{aA \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{2f} \\
&= \frac{aA \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{2f} \\
&= \frac{aA \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{2f} \\
&= - \frac{ia^{3/2} A c^{3/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right)}{f}
\end{aligned}$$

**Mathematica [A]**

time = 3.13, size = 109, normalized size = 0.69

$$\frac{iac^2(4B - 12iAArcTan(e^{i(e+fx)}) \cos^3(e + fx) + 3A \sin(2(e + fx))) (-i + \tan(e + fx))(i + \tan(e + fx))^2 \sqrt{a + ia \tan(e + fx)}}{12f \sqrt{c - ictan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2), x]
```

```
[Out] ((-1/12*I)*a*c^2*(4*B - (12*I)*A*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x]^3 + 3*A*Sin[2*(e + f*x)])*(-I + Tan[e + f*x])*(I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]])/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

**Maple [A]**

time = 0.41, size = 186, normalized size = 1.18

method	result
--------	--------

derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} ac \left( {}_{2B} \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)}{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} ac \left( {}_{2B} \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)}$
default	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} ac \left( {}_{2B} \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{6} \frac{f (a(1+i \tan(fx+e)))^{1/2} (-c(I \tan(fx+e)-1))^{1/2} a^2 c^2 B (a^2 c^2 (1+\tan^2(fx+e))^{1/2} (a^2 c^2)^{1/2} \tan^2(fx+e) + 3A \ln((a^2 c^2 \tan(fx+e) + (a^2 c^2)^{1/2} (a^2 c^2 (1+\tan^2(fx+e))^{1/2}))) / (a^2 c^2)^{1/2} a^2 c^2 + 3A (a^2 c^2 (1+\tan^2(fx+e))^{1/2} (a^2 c^2)^{1/2} \tan(fx+e) + 2B (a^2 c^2 (1+\tan^2(fx+e))^{1/2} (a^2 c^2)^{1/2})) / (a^2 c^2 (1+\tan^2(fx+e))^{1/2} (a^2 c^2)^{1/2})}{(a^2 c^2 (1+\tan^2(fx+e))^{1/2} (a^2 c^2)^{1/2})}$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs.  $2(126) = 252$ .  
time = 0.67, size = 915, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -(12Aa^2c \cos(5/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))) + 32IBa^2c \cos(3/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))) - 12Aa^2c \cos(1/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))) + 12IAa^2c \sin(5/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))) - 32Ba^2c \sin(3/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))) - 12IAa^2c \sin(1/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))) + 6(Aa^2c \cos(6fx+6e) + 3Aa^2c \cos(4fx+4e) + 3Aa^2c \cos(2fx+2e) + IAa^2c \sin(6fx+6e) + 3IAa^2c \sin(4fx+4e) + 3IAa^2c \sin(2fx+2e) + Aa^2c) \arctan^2(\cos(1/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))), \sin(1/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e)))) + 1) + 6(Aa^2c \cos(6fx+6e) + 3Aa^2c \cos(4fx+4e) + 3Aa^2c \cos(2fx+2e) + IAa^2c \sin(6fx+6e) + 3IAa^2c \sin(4fx+4e) + 3IAa^2c \sin(2fx+2e) + Aa^2c) \arctan^2(\cos(1/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e))), -\sin(1/2 \arctan^2(\sin(2fx+2e), \cos(2fx+2e)))) + 1) + 3(IAa^2c \cos(6fx+6e) + 3IAa^2c \cos(4fx+4e) + 3IAa^2c \cos(2fx+2e) - Aa^2c \sin(6fx+6e) - 3Aa^2c \sin(4fx+4e) - 3Aa^2c \sin(2fx+2e) + IAa^2c) \log(\cos(1/2 \arctan^2(\sin(2fx+2e), c \end{aligned}$$

$\cos(2fx + 2e))^{1/2} + \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^{1/2} + 2 \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1 + 3(-I A a c \cos(6fx + 6e) - 3 I A a c \cos(4fx + 4e) - 3 I A a c \cos(2fx + 2e) + A a c \sin(6fx + 6e) + 3 A a c \sin(4fx + 4e) + 3 A a c \sin(2fx + 2e) - I A a c) \log(\cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^{1/2} + \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^{1/2} - 2 \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1) \sqrt{a} \sqrt{c} / (f(-12 I \cos(6fx + 6e) - 36 I \cos(4fx + 4e) - 36 I \cos(2fx + 2e) + 12 \sin(6fx + 6e) + 36 \sin(4fx + 4e) + 36 \sin(2fx + 2e) - 12 I))$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs.  $2(126) = 252$ .  
time = 4.41, size = 456, normalized size = 2.90

$$\frac{3 \sqrt{\frac{A^2 a^3 c^3}{f^2}} \sqrt{f^2 \cos^2(e + fx) + 2 f^2 \sin^2(e + fx)} \log\left(\frac{\left(\frac{1 + \sqrt{1 + \frac{A^2 a^3 c^3}{f^2}}}{2} \sqrt{\frac{a}{2 f^2 \cos^2(e + fx) + 1}} \sqrt{\frac{c}{2 f^2 \sin^2(e + fx) + 1}} \sqrt{\frac{A^2 a^3 c^3}{f^2}}\right)^{1/2} - \frac{1 - \sqrt{1 + \frac{A^2 a^3 c^3}{f^2}}}{2} \sqrt{\frac{a}{2 f^2 \cos^2(e + fx) + 1}} \sqrt{\frac{c}{2 f^2 \sin^2(e + fx) + 1}} \sqrt{\frac{A^2 a^3 c^3}{f^2}}}{\sqrt{1 + \frac{A^2 a^3 c^3}{f^2}}}\right) - 3 \sqrt{\frac{A^2 a^3 c^3}{f^2}} \sqrt{f^2 \cos^2(e + fx) + 2 f^2 \sin^2(e + fx)} \log\left(\frac{\left(\frac{1 + \sqrt{1 + \frac{A^2 a^3 c^3}{f^2}}}{2} \sqrt{\frac{a}{2 f^2 \cos^2(e + fx) + 1}} \sqrt{\frac{c}{2 f^2 \sin^2(e + fx) + 1}} \sqrt{\frac{A^2 a^3 c^3}{f^2}}\right)^{1/2} - \frac{1 - \sqrt{1 + \frac{A^2 a^3 c^3}{f^2}}}{2} \sqrt{\frac{a}{2 f^2 \cos^2(e + fx) + 1}} \sqrt{\frac{c}{2 f^2 \sin^2(e + fx) + 1}} \sqrt{\frac{A^2 a^3 c^3}{f^2}}}{\sqrt{1 + \frac{A^2 a^3 c^3}{f^2}}}\right)}{12 \sqrt{f^2 \cos^2(e + fx) + 2 f^2 \sin^2(e + fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} (3 \sqrt{A^2 a^3 c^3 / f^2}) (f e^{4 I f x + 4 I e} + 2 f e^{2 I f x + 2 I e} + f) \log(4 (2 (A a c e^{3 I f x + 3 I e} + A a c e^{I f x + I e})) \sqrt{a / (e^{2 I f x + 2 I e} + 1)} \sqrt{c / (e^{2 I f x + 2 I e} + 1)} - \sqrt{A^2 a^3 c^3 / f^2} (I f e^{2 I f x + 2 I e} - I f)) / (A a c e^{2 I f x + 2 I e} + A a c) - 3 \sqrt{A^2 a^3 c^3 / f^2} (f e^{4 I f x + 4 I e} + 2 f e^{2 I f x + 2 I e} + f) \log(4 (2 (A a c e^{3 I f x + 3 I e} + A a c e^{I f x + I e})) \sqrt{a / (e^{2 I f x + 2 I e} + 1)} \sqrt{c / (e^{2 I f x + 2 I e} + 1)} - \sqrt{A^2 a^3 c^3 / f^2} (-I f e^{2 I f x + 2 I e} + I f)) / (A a c e^{2 I f x + 2 I e} + A a c) - 4 (3 I A a c e^{5 I f x + 5 I e} - 8 B a c e^{3 I f x + 3 I e} - 3 I A a c e^{I f x + I e}) \sqrt{a / (e^{2 I f x + 2 I e} + 1)} \sqrt{c / (e^{2 I f x + 2 I e} + 1))} / (f e^{4 I f x + 4 I e} + 2 f e^{2 I f x + 2 I e} + f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e + fx) - i))^{\frac{3}{2}} (-ic(\tan(e + fx) + i))^{\frac{3}{2}} (A + B \tan(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))\*\*(3/2)\*(-I\*c\*(tan(e + f\*x) + I))\*\*(3/2)\*(A + B\*tan(e + f\*x)), x)



**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{3/2} (c - c \tan(e + f x) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(3/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(3/2), x)

### 3.798 $\int (a+ia \tan(e+fx))^{3/2} (A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}$

Optimal. Leaf size=160

$$\frac{a^{3/2}(2iA+B)\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{f} + \frac{a(2iA+B)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{2f}$$

[Out]  $-a^{(3/2)}*(2*I*A+B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})*c^{(1/2)}/f+1/2*a*(2*I*A+B)*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/f+1/2*B*(c-I*c*\tan(f*x+e))^{(1/2)}*(a+I*a*\tan(f*x+e))^{(3/2)}/f$

Rubi [A]

time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 52, 65, 223, 209}

$$\frac{a^{3/2}\sqrt{c}(B+2iA)\operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{f} + \frac{a(B+2iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{2f} + \frac{B(a+ia \tan(e+fx))^{3/2}\sqrt{c-ic \tan(e+fx)}}{2f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(A + B*\operatorname{Tan}[e + f*x])*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]], x]$

[Out]  $-\left(\frac{a^{(3/2)}*((2*I)*A + B)*Sqrt[c]*\operatorname{ArcTan}[(Sqrt[c]*Sqrt[a + I*a*\operatorname{Tan}[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])]}{f} + \frac{a*((2*I)*A + B)*Sqrt[a + I*a*\operatorname{Tan}[e + f*x])*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]]}{(2*f)} + \frac{B*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]]}{(2*f)}\right)$

Rule 52

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \operatorname{Dist}[n*(b*c - a*d)/(b*(m+n+1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m+n+1, 0] \&\& !( \operatorname{IGtQ}[m, 0] \&\& ( !\operatorname{IntegerQ}[n] || (\operatorname{GtQ}[m, 0] \&\& \operatorname{LtQ}[m-n, 0]) ) ) \&\& !\operatorname{ILtQ}[m+n+2, 0] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} dx &= \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + ia x} (A+Bx)}{\sqrt{c - icx}} dx, \right. \\
&= \frac{B(a + ia \tan(e + fx))^{3/2} \sqrt{c - ic \tan(e + fx)}}{2f} \\
&= \frac{a(2iA + B) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f} \\
&= \frac{a(2iA + B) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f} \\
&= \frac{a(2iA + B) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2f} \\
&= \frac{a^{3/2} (2iA + B) \sqrt{c} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}} \right)}{f}
\end{aligned}$$

**Mathematica [A]**

time = 2.99, size = 220, normalized size = 1.38

$$\frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx)) \left( -\frac{i(2A - iB) c e^{-2i(e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} \text{ArcTan}(e^{i(e+fx)})}{\sqrt{\frac{c}{1 + e^{2i(e+fx)}}}} + \frac{\cos(e) \sqrt{\sec(e + fx)} (i + \tan(e)) (2A - 2iB + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{2 \cos(fx) + 2i \sin(fx)} \right)}{f \sec^{\frac{3}{2}}(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] ((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*((( -I)*(2*A - I*B)*c*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))])/(E^((2*I)*(e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x))])) + (Cos[e]*Sqrt[Sec[e + f*x]]*(I + Tan[e])*(2*A - (2*I)*B + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(2*Cos[f*x] + (2*I)*Sin[f*x]))/(f*Sec[e + f*x]^(5/2)*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

**Maple [A]**

time = 0.42, size = 223, normalized size = 1.39

method	result
derivativedivides	$\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} a \left( iB \sqrt{ac(1 + \tan^2(fx + e))} \sqrt{ac} \right)$
default	$\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} a \left( iB \sqrt{ac(1 + \tan^2(fx + e))} \sqrt{ac} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*a*(I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c+2*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+2*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c+2*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 819 vs.  $2(126) = 252$ .  
time = 0.67, size = 819, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 4*(4*(2*A - 3*I*B)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(2*A - I*B)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(2*I*A + 3*B)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(2*I*A + B)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 2*((2*A - I*B)*a*cos(4*f*x + 4*e) + 2*(2*A - I*B)*a*cos(2*f*x + 2*e) - (-2*I*A - B)*a*sin(4*f*x + 4*e) - 2*(-2*I*A - B)*a*sin(2*f*x + 2*e) + (2*A - I*B)*a*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) - 2*((2*A - I*B)*a*cos(4*f*x + 4*e) + 2*(2*A - I*B)*a*cos(2*f*x + 2*e) - (-2*I*A - B)*a*sin(4*f*x + 4*e) - 2*(-2*I*A - B)*a*sin(2*f*x + 2*e) + (2*A - I*B)*a*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + ((-2*I*A - B)*a*cos(4*f*x + 4*e) + 2*(-2*I*A - B)*a*cos(2*f*x + 2*e) + (2*A - I*B)*a*sin(4*f*x + 4*e) + 2*(2*A - I*B)*a*sin(2*f*x + 2*e) + (-2*I*A - B)*a*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
```

+ 2\*e)))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))^2 + 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 1) + ((2\*I\*A + B)\*a\*cos(4\*f\*x + 4\*e) + 2\*(2\*I\*A + B)\*a\*cos(2\*f\*x + 2\*e) - (2\*A - I\*B)\*a\*sin(4\*f\*x + 4\*e) - 2\*(2\*A - I\*B)\*a\*sin(2\*f\*x + 2\*e) + (2\*I\*A + B)\*a\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))^2 - 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 1))\*sqrt(a)\*sqrt(c)/(f\*(-16\*I\*cos(4\*f\*x + 4\*e) - 32\*I\*cos(2\*f\*x + 2\*e) + 16\*sin(4\*f\*x + 4\*e) + 32\*sin(2\*f\*x + 2\*e) - 16\*I))

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(126) = 252.

time = 3.71, size = 478, normalized size = 2.99

$$\frac{\sqrt{\frac{4A^2 - 4IAB - B^2}{f}} \log\left(\frac{\sqrt{\frac{4A^2 - 4IAB - B^2}{f}} \sqrt{\frac{4A^2 - 4IAB - B^2}{f}} \sqrt{\frac{4A^2 - 4IAB - B^2}{f}}}{\sqrt{\frac{4A^2 - 4IAB - B^2}{f}}}\right) - \sqrt{\frac{4A^2 - 4IAB - B^2}{f}} \log\left(\frac{\sqrt{\frac{4A^2 - 4IAB - B^2}{f}} \sqrt{\frac{4A^2 - 4IAB - B^2}{f}} \sqrt{\frac{4A^2 - 4IAB - B^2}{f}}}{\sqrt{\frac{4A^2 - 4IAB - B^2}{f}}}\right) + 4((-2A - 3B)\sqrt{\frac{4A^2 - 4IAB - B^2}{f}} + (-2A - B)\sqrt{\frac{4A^2 - 4IAB - B^2}{f}})}{4(f^{2I+1} + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e)),x, algorithm="fricas")

[Out] -1/4\*(sqrt((4\*A^2 - 4\*I\*A\*B - B^2)\*a^3\*c/f^2)\*(f\*e^(2\*I\*f\*x + 2\*I\*e) + f)\*log(-4\*(2\*((-2\*I\*A - B)\*a\*e^(3\*I\*f\*x + 3\*I\*e) + (-2\*I\*A - B)\*a\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)) + sqrt((4\*A^2 - 4\*I\*A\*B - B^2)\*a^3\*c/f^2)\*(f\*e^(2\*I\*f\*x + 2\*I\*e) - f))/((2\*I\*A + B)\*a\*e^(2\*I\*f\*x + 2\*I\*e) + (2\*I\*A + B)\*a) - sqrt((4\*A^2 - 4\*I\*A\*B - B^2)\*a^3\*c/f^2)\*(f\*e^(2\*I\*f\*x + 2\*I\*e) + f)\*log(-4\*(2\*((-2\*I\*A - B)\*a\*e^(3\*I\*f\*x + 3\*I\*e) + (-2\*I\*A - B)\*a\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)) - sqrt((4\*A^2 - 4\*I\*A\*B - B^2)\*a^3\*c/f^2)\*(f\*e^(2\*I\*f\*x + 2\*I\*e) - f))/((2\*I\*A + B)\*a\*e^(2\*I\*f\*x + 2\*I\*e) + (2\*I\*A + B)\*a) + 4\*((-2\*I\*A - 3\*B)\*a\*e^(3\*I\*f\*x + 3\*I\*e) + (-2\*I\*A - B)\*a\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)))/(f\*e^(2\*I\*f\*x + 2\*I\*e) + f)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e + fx) - i))^{3/2} \sqrt{-ic(\tan(e + fx) + i)} (A + B \tan(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(a+I\*a\*tan(f\*x+e))\*\*(3/2)\*(A+B\*tan(f\*x+e)),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))\*\*(3/2)\*sqrt(-I\*c\*(tan(e + f\*x) + I))\*(A + B\*tan(e + f\*x)), x)

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs.  $2(126) = 252$ .  
time = 1.70, size = 566, normalized size = 3.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{8}(-3I B a^{3/2} \sqrt{c}) e^{(8I f x + 8I e)} \log(e^{(I f x + I e)} + I) - 12I B a^{3/2} \sqrt{c} e^{(6I f x + 6I e)} \log(e^{(I f x + I e)} + I) - 18I B a^{3/2} \sqrt{c} e^{(4I f x + 4I e)} \log(e^{(I f x + I e)} + I) - 12I B a^{3/2} \sqrt{c} e^{(2I f x + 2I e)} \log(e^{(I f x + I e)} + I) + 3I B a^{3/2} \sqrt{c} e^{(8I f x + 8I e)} \log(e^{(I f x + I e)} - I) + 12I B a^{3/2} \sqrt{c} e^{(6I f x + 6I e)} \log(e^{(I f x + I e)} - I) + 18I B a^{3/2} \sqrt{c} e^{(4I f x + 4I e)} \log(e^{(I f x + I e)} - I) + 12I B a^{3/2} \sqrt{c} e^{(2I f x + 2I e)} \log(e^{(I f x + I e)} - I) + 10B a^{3/2} \sqrt{c} e^{(7I f x + 7I e)} + 26B a^{3/2} \sqrt{c} e^{(5I f x + 5I e)} + 22B a^{3/2} \sqrt{c} e^{(3I f x + 3I e)} + 6B a^{3/2} \sqrt{c} e^{(I f x + I e)} - 3I B a^{3/2} \sqrt{c} \log(e^{(I f x + I e)} + I) + 3I B a^{3/2} \sqrt{c} \log(e^{(I f x + I e)} - I) / (f e^{(8I f x + 8I e)} + 4f e^{(6I f x + 6I e)} + 6f e^{(4I f x + 4I e)} + 4f e^{(2I f x + 2I e)} + f) - 1/4(I(8A a^{3/2} \sqrt{c}) - I B a^{3/2} \sqrt{c}) \arctan(e^{(I f x + I e)}) - I(8A a^{3/2} \sqrt{c}) e^{(3I f x + 3I e)} - 7I B a^{3/2} \sqrt{c} e^{(3I f x + 3I e)} + 8A a^{3/2} \sqrt{c} e^{(I f x + I e)} - I B a^{3/2} \sqrt{c} e^{(I f x + I e)} / (e^{(2I f x + 2I e)} + 1)^2 / f$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{3/2} \sqrt{c - c \tan(e + f x) \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*li)^(3/2)\*(c - c\*tan(e + f\*x)\*li)^(1/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*li)^(3/2)\*(c - c\*tan(e + f\*x)\*li)^(1/2), x)

$$3.799 \quad \int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{\sqrt{c-ictan(e+fx)}} dx$$

**Optimal.** Leaf size=169

$$\frac{2a^{3/2}(iA+2B)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{3/2}}{f\sqrt{c-ictan(e+fx)}} - \frac{a(iA+2B)\sqrt{a+ia \tan(e+fx)}}{f\sqrt{c-ictan(e+fx)}}$$

[Out] 2\*a^(3/2)\*(I\*A+2\*B)\*arctan(c^(1/2)\*(a+I\*a\*tan(f\*x+e))^(1/2)/a^(1/2)/(c-I\*c\*tan(f\*x+e))^(1/2))/f/c^(1/2)-a\*(I\*A+2\*B)\*(a+I\*a\*tan(f\*x+e))^(1/2)\*(c-I\*c\*tan(f\*x+e))^(1/2)/c/f-(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^(3/2)/f/(c-I\*c\*tan(f\*x+e))^(1/2)

**Rubi [A]**

time = 0.18, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 52, 65, 223, 209}

$$\frac{2a^{3/2}(2B+iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{\sqrt{c}f} - \frac{a(2B+iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ictan(e+fx)}}{cf} - \frac{(B+iA)(a+ia \tan(e+fx))^{3/2}}{f\sqrt{c-ictan(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])^(3/2)\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out] (2\*a^(3/2)\*(I\*A + 2\*B)\*ArcTan[(Sqrt[c]\*Sqrt[a + I\*a\*Tan[e + f\*x]])/(Sqrt[a]\*Sqrt[c - I\*c\*Tan[e + f\*x]])])/(Sqrt[c]\*f) - ((I\*A + B)\*(a + I\*a\*Tan[e + f\*x])^(3/2))/(f\*Sqrt[c - I\*c\*Tan[e + f\*x]]) - (a\*(I\*A + 2\*B)\*Sqrt[a + I\*a\*Tan[e + f\*x]]\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(c\*f)

**Rule 52**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^n/(b\*(m + n + 1))), x] + Dist[n\*(b\*c - a\*d)/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ



`[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 79

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))`

### Rule 209

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

### Rule 223

`Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

### Rule 3669

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]`

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx))}{\sqrt{c - ictan(e + fx)}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + ia x} (A + Bx)}{(c - icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{(a(A - 2iB)) \text{Subst} \left( \int \frac{\sqrt{a + ia x}}{(c - icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{a(iA + 2B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{a(iA + 2B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{a(iA + 2B) \sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= \frac{2a^{3/2} (iA + 2B) \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right)}{\sqrt{c} f}
\end{aligned}$$

**Mathematica [A]**

time = 3.02, size = 190, normalized size = 1.12

$$\frac{2ae^{-2i(e+fx)} \sqrt{\frac{c}{1+e^{2i(e+fx)}}} \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} (e^{i(e+fx)} (A(1+e^{2i(e+fx)}) - iB(2+e^{2i(e+fx)})) - (A-2iB)(1+e^{2i(e+fx)}) \text{ArcTan}(e^{i(e+fx)})) (-i + \tan(e+fx)) \sqrt{a+ia \tan(e+fx)}}{cf \sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] (2*a*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*(E^(I*(e + f*x))*(A*(1 + E^((2*I)*(e + f*x))) - I*B*(2 + E^((2*I)*(e + f*x)))) - (A - (2*I)*B)*(1 + E^((2*I)*(e + f*x)))*ArcTan[E^(I*(e + f*x))])*(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]/(c*E^((2*I)*(e + f*x))*f*Sec[e + f*x]^(3/2))
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs.  $2(140) = 280$ .

time = 0.42, size = 497, normalized size = 2.94



\*e), cos(2\*f\*x + 2\*e))) + ((I\*A + 2\*B)\*a\*cos(2\*f\*x + 2\*e) - (A - 2\*I\*B)\*a\*sin(2\*f\*x + 2\*e) + (I\*A + 2\*B)\*a\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 1) + ((-I\*A - 2\*B)\*a\*cos(2\*f\*x + 2\*e) + (A - 2\*I\*B)\*a\*sin(2\*f\*x + 2\*e) + (-I\*A - 2\*B)\*a\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 - 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 1) - 4\*((I\*A + B)\*a\*cos(2\*f\*x + 2\*e) - (A - I\*B)\*a\*sin(2\*f\*x + 2\*e) + (I\*A + 2\*B)\*a\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*sqrt(a)\*sqrt(c)/((-2\*I\*c\*cos(2\*f\*x + 2\*e) + 2\*c\*sin(2\*f\*x + 2\*e) - 2\*I\*c)\*f)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs.  $2(137) = 274$ .

time = 4.43, size = 454, normalized size = 2.69

$$\frac{\sqrt{\frac{(A^2 - 4AB - 4B^2)a^3}{c^2 f^2}} \operatorname{Im} \left( \frac{\left( \frac{(A + B) \tan(e + fx) - I}{\sqrt{2B^2 \tan^2(e + fx) + 1}} \sqrt{\frac{(A - 4AB - 4B^2)a^3}{c^2 f^2}} \right) \sqrt{\frac{(A - 4AB - 4B^2)a^3}{c^2 f^2}}}{\sqrt{2B^2 \tan^2(e + fx) + 1}} \sqrt{\frac{(A - 4AB - 4B^2)a^3}{c^2 f^2}} \right)}{\sqrt{\frac{(A^2 - 4AB - 4B^2)a^3}{c^2 f^2}} \operatorname{Im} \left( \frac{\left( \frac{(A + B) \tan(e + fx) - I}{\sqrt{2B^2 \tan^2(e + fx) + 1}} \sqrt{\frac{(A - 4AB - 4B^2)a^3}{c^2 f^2}} \right) \sqrt{\frac{(A - 4AB - 4B^2)a^3}{c^2 f^2}}}{\sqrt{2B^2 \tan^2(e + fx) + 1}} \sqrt{\frac{(A - 4AB - 4B^2)a^3}{c^2 f^2}} \right)} - 4((A + B) \tan(e + fx) + (A - 2B) \tan^2(e + fx)) \sqrt{\frac{a}{2B^2 \tan^2(e + fx) + 1}} \sqrt{\frac{c}{2B^2 \tan^2(e + fx) + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/2\*(c\*sqrt((A^2 - 4\*I\*A\*B - 4\*B^2)\*a^3/(c\*f^2))\*f\*log(-4\*(2\*((-I\*A - 2\*B)\*a\*e^(3\*I\*f\*x + 3\*I\*e) + (-I\*A - 2\*B)\*a\*e^(I\*f\*x + I\*e)))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)) + (c\*f\*e^(2\*I\*f\*x + 2\*I\*e) - c\*f)\*sqrt((A^2 - 4\*I\*A\*B - 4\*B^2)\*a^3/(c\*f^2)))/((I\*A + 2\*B)\*a\*e^(2\*I\*f\*x + 2\*I\*e) + (I\*A + 2\*B)\*a) - c\*sqrt((A^2 - 4\*I\*A\*B - 4\*B^2)\*a^3/(c\*f^2))\*f\*log(-4\*(2\*((-I\*A - 2\*B)\*a\*e^(3\*I\*f\*x + 3\*I\*e) + (-I\*A - 2\*B)\*a\*e^(I\*f\*x + I\*e)))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)) - (c\*f\*e^(2\*I\*f\*x + 2\*I\*e) - c\*f)\*sqrt((A^2 - 4\*I\*A\*B - 4\*B^2)\*a^3/(c\*f^2)))/((I\*A + 2\*B)\*a\*e^(2\*I\*f\*x + 2\*I\*e) + (I\*A + 2\*B)\*a) - 4\*((I\*A + B)\*a\*e^(3\*I\*f\*x + 3\*I\*e) + (I\*A + 2\*B)\*a\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)))/(c\*f)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(i a (\tan(e + f x) - i))^{\frac{3}{2}} (A + B \tan(e + f x))}{\sqrt{-i c (\tan(e + f x) + i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(3/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(1/2),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))\*\*(3/2)\*(A + B\*tan(e + f\*x))/sqrt(-I\*c\*(tan(e + f\*x) + I)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)/sqrt(-I*c*tan(f*x + e) + c), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{3/2}}{\sqrt{c - c \tan(e + f x) \operatorname{li}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(3/2))/(c - c*tan(e + f*x)*li)^(1/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(3/2))/(c - c*tan(e + f*x)*li)^(1/2), x)
```

$$3.800 \quad \int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{2a^{3/2}B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ic \tan(e+fx)}}\right)}{c^{3/2}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{3/2}}{3f(c-ic \tan(e+fx))^{3/2}} + \frac{2aB \sqrt{a+ia \tan(e+fx)}}{cf \sqrt{c-ic \tan(e+fx)}}$$

[Out]  $-2*a^{(3/2)}*B*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/c^{(3/2)}/f+2*a*B*(a+I*a*\tan(f*x+e))^{(1/2)}/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}-1/3*(I*A+B)*(a+I*a*\tan(f*x+e))^{(3/2)}/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 49, 65, 223, 209}

$$\frac{2a^{3/2}B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ic \tan(e+fx)}}\right)}{c^{3/2}f} - \frac{(B+ia)(a+ia \tan(e+fx))^{3/2}}{3f(c-ic \tan(e+fx))^{3/2}} + \frac{2aB \sqrt{a+ia \tan(e+fx)}}{cf \sqrt{c-ic \tan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)}*(A+B*\operatorname{Tan}[e+f*x])]/(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)},x]$

[Out]  $(-2*a^{(3/2)}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])])/c^{(3/2)}*f - ((I*A+B)*(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)})/(3*f*(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)}) + (2*a*B*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])/(c*f*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])$

**Rule 49**

$\operatorname{Int}[(a_.)+(b_.)*(x_)^{(m_)}*((c_.)+(d_.)*(x_)^{(n_)}, x\_Symbol] :> \operatorname{Simp}[(a+b*x)^{(m+1)}*((c+d*x)^n/(b*(m+1))), x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a+b*x)^{(m+1)}*(c+d*x)^{(n-1)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(IntegerQ[n] \&\& !IntegerQ[m]) \&\& !(IntegerQ[m+n+2, 0] \&\& (FractionQ[m] || GeQ[2*n+m+1, 0])) \& \& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 65**

$\operatorname{Int}[(a_.)+(b_.)*(x_)^{(m_)}*((c_.)+(d_.)*(x_)^{(n_)}, x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)}, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + ia x} (A + Bx)}{(c - icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{(iaB) \text{Subst} \left( \int \frac{\sqrt{a + ia x}}{(c - icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2aB \sqrt{a + ia \tan(e + fx)}}{cf \sqrt{c - ict \tan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2aB \sqrt{a + ia \tan(e + fx)}}{cf \sqrt{c - ict \tan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2aB \sqrt{a + ia \tan(e + fx)}}{cf \sqrt{c - ict \tan(e + fx)}} \\
&= -\frac{2a^{3/2} B \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ict \tan(e + fx)}} \right)}{c^{3/2} f} - \frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{3f(c - ict \tan(e + fx))^{3/2}}
\end{aligned}$$

**Mathematica [A]**

time = 2.84, size = 123, normalized size = 0.79

$$\frac{ae^{-i(e+fx)}(iAe^{3i(e+fx)} + Be^{i(e+fx)}(-6 + e^{2i(e+fx)}) + 6BArcTan(e^{i(e+fx)}))\sqrt{a + ia \tan(e + fx)}}{3\sqrt{2}c\sqrt{\frac{c}{1 + e^{2i(e+fx)}}}f}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^(3/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] -1/3\*(a\*(I\*A\*E^((3\*I)\*(e + f\*x)) + B\*E^(I\*(e + f\*x))\*(-6 + E^((2\*I)\*(e + f\*x)))) + 6\*B\*ArcTan[E^(I\*(e + f\*x))]\*Sqrt[a + I\*a\*Tan[e + f\*x]]/(Sqrt[2]\*c\*E^(I\*(e + f\*x))\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x)))]\*f)

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(126) = 252.

time = 0.43, size = 406, normalized size = 2.62



method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a \left( 3iB \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \frac{\sqrt{ac(1+i\tan(fx+e))}}{\sqrt{ac}} \right) \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a \left( 3iB \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \frac{\sqrt{ac(1+i\tan(fx+e))}}{\sqrt{ac}} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/3/f*(a*(1+I*\tan(f*x+e)))^{1/2}*(-c*(I*\tan(f*x+e)-1))^{1/2}*a/c^2*(3*I*B*\ln((a*c*\tan(f*x+e)+(a*c)^{1/2}*(a*c*(1+\tan(f*x+e)^2))^{1/2})/(a*c)^{1/2}))*a*c*\tan(f*x+e)^3-9*I*B*\ln((a*c*\tan(f*x+e)+(a*c)^{1/2}*(a*c*(1+\tan(f*x+e)^2))^{1/2})/(a*c)^{1/2}))*a*c*\tan(f*x+e)-7*I*B*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}*\tan(f*x+e)^2-9*B*\ln((a*c*\tan(f*x+e)+(a*c)^{1/2}*(a*c*(1+\tan(f*x+e)^2))^{1/2})/(a*c)^{1/2}))*a*c*\tan(f*x+e)^2+A*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}*\tan(f*x+e)^2+5*I*B*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}+3*B*\ln((a*c*\tan(f*x+e)+(a*c)^{1/2}*(a*c*(1+\tan(f*x+e)^2))^{1/2})/(a*c)^{1/2}))*a*c+12*B*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2}*\tan(f*x+e)+A*(a*c*(1+\tan(f*x+e)^2))^{1/2}*(a*c)^{1/2})/(a*c*(1+\tan(f*x+e)^2))^{1/2}/(I+\tan(f*x+e))^3/(a*c)^{1/2}$$

**Maxima** [A]

time = 0.60, size = 184, normalized size = 1.19

$$\frac{(6Ba \arctan(\cos(fx+e), \sin(fx+e)+1) + 6Ba \arctan(\cos(fx+e), -\sin(fx+e)+1) - 2(-A-B)a \cos(3fx+3e) - 12Ba \cos(fx+e) + 3iB a \log(\cos(fx+e)^2 + \sin(fx+e)^2 + 2\sin(fx+e)+1) - 3iB a \log(\cos(fx+e)^2 + \sin(fx+e)^2 - 2\sin(fx+e)+1) - 2(A-iB)a \sin(3fx+3e) - 12iBa \sin(fx+e)) \sqrt{a}}{6c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] 
$$-1/6*(6*B*a*\arctan2(\cos(f*x+e), \sin(f*x+e)+1) + 6*B*a*\arctan2(\cos(f*x+e), -\sin(f*x+e)+1) - 2*(-I*A-B)*a*\cos(3*f*x+3*e) - 12*B*a*\cos(f*x+e) + 3*I*B*a*\log(\cos(f*x+e)^2 + \sin(f*x+e)^2 + 2*\sin(f*x+e)+1) - 3*I*B*a*\log(\cos(f*x+e)^2 + \sin(f*x+e)^2 - 2*\sin(f*x+e)+1) - 2*(A-I*B)*a*\sin(3*f*x+3*e) - 12*I*B*a*\sin(f*x+e))*\sqrt{a}/(c^{3/2}*f)$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 397 vs.  $2(127) = 254$ .

time = 4.07, size = 397, normalized size = 2.56

$$\frac{3c^2 f \sqrt{\frac{B a^2}{c^2 f}} \log \left( \frac{\left( \frac{2 \left[ \frac{B a^2}{c^2 f} \sqrt{\frac{a}{2 B^2 f^2 \cos^2(fx+e)+1}} \sqrt{\frac{c}{2 B^2 f^2 \cos^2(fx+e)+1}} + \frac{c}{2 B^2 f^2 \cos^2(fx+e)+1} \right) \sqrt{\frac{B a^2}{c^2 f}} \right)}{\frac{B a^2}{c^2 f}} \right) - 3c^2 f \sqrt{\frac{B a^2}{c^2 f}} \log \left( \frac{\left( \frac{2 \left[ \frac{B a^2}{c^2 f} \sqrt{\frac{a}{2 B^2 f^2 \cos^2(fx+e)+1}} \sqrt{\frac{c}{2 B^2 f^2 \cos^2(fx+e)+1}} - \frac{c}{2 B^2 f^2 \cos^2(fx+e)+1} \right) \sqrt{\frac{B a^2}{c^2 f}} \right)}{\frac{B a^2}{c^2 f}} \right) - 2 \left( (I+A+B)a e^{2i(fx+e)} + (iA-5B)a e^{2i(fx+e)} - 6B a e^{i(fx+e)} \right) \sqrt{\frac{a}{2 B^2 f^2 \cos^2(fx+e)+1}} \sqrt{\frac{c}{2 B^2 f^2 \cos^2(fx+e)+1}}}{6c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*f*sqrt(-B^2*a^3/(c^3*f^2))*log(4*(2*(B*a*e^(3*I*f*x + 3*I*e) + B*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (c^2*f*e^(2*I*f*x + 2*I*e) - c^2*f)*sqrt(-B^2*a^3/(c^3*f^2)))/(B*a*e^(2*I*f*x + 2*I*e) + B*a)) - 3*c^2*f*sqrt(-B^2*a^3/(c^3*f^2))*log(4*(2*(B*a*e^(3*I*f*x + 3*I*e) + B*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^2*f*e^(2*I*f*x + 2*I*e) - c^2*f)*sqrt(-B^2*a^3/(c^3*f^2)))/(B*a*e^(2*I*f*x + 2*I*e) + B*a)) - 2*((I*A + B)*a*e^(5*I*f*x + 5*I*e) + (I*A - 5*B)*a*e^(3*I*f*x + 3*I*e) - 6*B*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e + fx) - i))^{\frac{3}{2}}(A + B \tan(e + fx))}{(-ic(\tan(e + fx) + i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e + f*x) + I))**(3/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)/(-I*c*tan(f*x + e) + c)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{3/2}}{(c - c \tan(e + f x) \operatorname{li})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*x)*1i)^(3/2), x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*x)*1i)^(3/2), x)
```

$$3.801 \quad \int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$$

Optimal. Leaf size=102

$$-\frac{(iA+B)(a+ia \tan(e+fx))^{3/2}}{5f(c-ic \tan(e+fx))^{5/2}} - \frac{(iA-4B)(a+ia \tan(e+fx))^{3/2}}{15cf(c-ic \tan(e+fx))^{3/2}}$$

[Out]  $-1/5*(I*A+B)*(a+I*a*\tan(f*x+e))^{(3/2)}/f/(c-I*c*\tan(f*x+e))^{(5/2)}-1/15*(I*A-4*B)*(a+I*a*\tan(f*x+e))^{(3/2)}/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ ,

Rules used = {3669, 79, 37}

$$-\frac{(-4B+iA)(a+ia \tan(e+fx))^{3/2}}{15cf(c-ic \tan(e+fx))^{3/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{3/2}}{5f(c-ic \tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $-1/5*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - ((I*A - 4*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(15*c*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \text{ :> Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)})/((b*c - a*d)*(m + 1)), x] \text{ /; FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 79

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \text{ :> Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)})/(f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, n\}, x\} \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ (\ !\text{LtQ}[n, -1] \ || \ \text{IntegerQ}[p] \ || \ \!(\text{IntegerQ}[n] \ || \ \!(\text{EqQ}[e, 0] \ || \ \!(\text{EqQ}[c, 0] \ || \ \text{LtQ}[p, n]))))$

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + iax} (A+Bx)}{(c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{5f(c - ict \tan(e + fx))^{5/2}} + \frac{(a(A + 4iB)) \text{Subst} \left( \int \frac{\sqrt{a + iax}}{(c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{15cf(c - ict \tan(e + fx))^{5/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{5f(c - ict \tan(e + fx))^{5/2}} - \frac{(iA - 4B)(a + ia \tan(e + fx))^{3/2}}{15cf(c - ict \tan(e + fx))^{5/2}}$$

**Mathematica [A]**

time = 3.54, size = 117, normalized size = 1.15

$$\frac{a \cos(e + fx)(\cos(fx) - i \sin(fx))((-4iA + B) \cos(e + fx) - (A + 4iB) \sin(e + fx))(\cos(4e + 5fx) + i \sin(4e + 5fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ict \tan(e + fx)}}{15c^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[
e + f*x])^(5/2), x]
```

```
[Out] (a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*((-4*I)*A + B)*Cos[e + f*x] - (A +
(4*I)*B)*Sin[e + f*x]*(Cos[4*e + 5*f*x] + I*Sin[4*e + 5*f*x])*Sqrt[a + I*
a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]]/(15*c^3*f)
```

**Maple [A]**

time = 0.40, size = 90, normalized size = 0.88

method	result
derivativedivides	$\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} a(1 + \tan^2(fx + e))(-4A + iA \tan(fx + e) - iB - 4B \tan(fx + e))}{15f c^3 (i + \tan(fx + e))^4}$
default	$\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} a(1 + \tan^2(fx + e))(-4A + iA \tan(fx + e) - iB - 4B \tan(fx + e))}{15f c^3 (i + \tan(fx + e))^4}$

risch	$-\frac{a \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (3iA e^{4i(fx+e)} + 3B e^{4i(fx+e)} + 5iA e^{2i(fx+e)} - 5B e^{2i(fx+e)})}{30c^2 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
-------	---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^3*(1+tan(f*x+e)^2)*(-4*A+I*A*tan(f*x+e)-I*B-4*B*tan(f*x+e))/(I+tan(f*x+e))^4
```

**Maxima [A]**

time = 0.67, size = 162, normalized size = 1.59

$$\frac{30(3(A-iB)a\cos(7fx+7e)+2(4A+iB)a\cos(5fx+5e)+5(A+iB)a\cos(3fx+3e)-3(-iA-B)a\sin(7fx+7e)-2(-4iA+B)a\sin(5fx+5e)-5(-iA+B)a\sin(3fx+3e))\sqrt{a}\sqrt{c}}{-900(i^2c^2\cos(2fx+2e)-c^2\sin(2fx+2e)+i^2c^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")
```

```
[Out] -30*(3*(A - I*B)*a*cos(7*f*x + 7*e) + 2*(4*A + I*B)*a*cos(5*f*x + 5*e) + 5*(A + I*B)*a*cos(3*f*x + 3*e) - 3*(-I*A - B)*a*sin(7*f*x + 7*e) - 2*(-4*I*A + B)*a*sin(5*f*x + 5*e) - 5*(-I*A + B)*a*sin(3*f*x + 3*e))*sqrt(a)*sqrt(c)/((-900*I*c^3*cos(2*f*x + 2*e) + 900*c^3*sin(2*f*x + 2*e) - 900*I*c^3)*f)
```

**Fricas [A]**

time = 3.87, size = 103, normalized size = 1.01

$$\frac{(3(iA+B)ae^{(7ifx+7ie)}+2(4iA-B)ae^{(5ifx+5ie)}+5(iA-B)ae^{(3ifx+3ie)})\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}}{30c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="fricas")
```

```
[Out] -1/30*(3*(I*A + B)*a*e^(7*I*f*x + 7*I*e) + 2*(4*I*A - B)*a*e^(5*I*f*x + 5*I*e) + 5*(I*A - B)*a*e^(3*I*f*x + 3*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}(A+B\tan(e+fx))}{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(3/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e + f*x) + I))**(5/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)/(-I*c*tan(f*x + e) + c)^(5/2), x)
```

**Mupad [B]**

time = 10.26, size = 190, normalized size = 1.86

$$a \sqrt{\frac{a (\cos(2e + 2fx) + 1 + \sin(2e + 2fx) 1i)}{\cos(2e + 2fx) + 1}} \frac{(A \cos(2e + 2fx) 5i + A \cos(4e + 4fx) 3i - 5B \cos(2e + 2fx) + 3B \cos(4e + 4fx) - 5A \sin(2e + 2fx) - 3A \sin(4e + 4fx) - B \sin(2e + 2fx) 5i + B \sin(4e + 4fx) 3i)}{30c^2 f \sqrt{\frac{c (\cos(2e + 2fx) + 1 - \sin(2e + 2fx) 1i)}{\cos(2e + 2fx) + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] -(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*cos(2*e + 2*f*x)*5i + A*cos(4*e + 4*f*x)*3i - 5*B*cos(2*e + 2*f*x) + 3*B*cos(4*e + 4*f*x) - 5*A*sin(2*e + 2*f*x) - 3*A*sin(4*e + 4*f*x) - B*sin(2*e + 2*f*x)*5i + B*sin(4*e + 4*f*x)*3i))/(30*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

$$3.802 \quad \int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{(iA+B)(a+ia \tan(e+fx))^{3/2}}{7f(c-ic \tan(e+fx))^{7/2}} - \frac{(2iA-5B)(a+ia \tan(e+fx))^{3/2}}{35cf(c-ic \tan(e+fx))^{5/2}} - \frac{(2iA-5B)(a+ia \tan(e+fx))^{3/2}}{105c^2f(c-ic \tan(e+fx))^{3/2}}$$

[Out]  $-1/7*(I*A+B)*(a+I*a*\tan(f*x+e))^{(3/2)}/f/(c-I*c*\tan(f*x+e))^{(7/2)}-1/35*(2*I*A-5*B)*(a+I*a*\tan(f*x+e))^{(3/2)}/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}-1/105*(2*I*A-5*B)*(a+I*a*\tan(f*x+e))^{(3/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$-\frac{(-5B+2iA)(a+ia \tan(e+fx))^{3/2}}{105c^2f(c-ic \tan(e+fx))^{3/2}} - \frac{(-5B+2iA)(a+ia \tan(e+fx))^{3/2}}{35cf(c-ic \tan(e+fx))^{5/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{3/2}}{7f(c-ic \tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $-1/7*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) - (((2*I)*A - 5*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(35*c*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (((2*I)*A - 5*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(105*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})$

**Rule 37**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^{(n+1)})/((b*c - a*d)*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

**Rule 47**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^{(n+1)})/((b*c - a*d)*(m+1)), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m+1))), \text{Int}[(a + b*x)^{\text{Simplify}[m+1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[\text{Simplify}[m + n + 2], 0] \&\& \text{NeQ}[m, -1] \&\& !(\text{LtQ}[m, -1] \&\& \text{LtQ}[n, -1] \&\& (\text{EqQ}[a, 0] || (\text{NeQ}[c, 0] \&\& \text{LtQ}[m - n, 0] \&\& \text{IntegerQ}[n]))) \&\& (\text{SumSimplerQ}[m, 1] || !\text{SumSimplerQ}[n, 1])$

**Rule 79**



```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{7/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + ia x} (A + Bx)}{(c - icx)^{9/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{7f(c - ictan(e + fx))^{7/2}} + \frac{(a(2A + 5iB))S}{35cf(c - ictan(e + fx))^{7/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{7f(c - ictan(e + fx))^{7/2}} - \frac{(2iA - 5B)(a + ia \tan(e + fx))^{3/2}}{35cf(c - ictan(e + fx))^{7/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{7f(c - ictan(e + fx))^{7/2}} - \frac{(2iA - 5B)(a + ia \tan(e + fx))^{3/2}}{35cf(c - ictan(e + fx))^{7/2}}$$

### Mathematica [A]

time = 5.58, size = 131, normalized size = 0.85

$$\frac{a \cos(e + fx)(\cos(fx) - i \sin(fx))(-21iA + 5(-5iA + 2B) \cos(2(e + fx)) - 5(2A + 5iB) \sin(2(e + fx)))(\cos(5e + 6fx) + i \sin(5e + 6fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{210c^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[
e + f*x])^(7/2), x]
```

```
[Out] (a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*((-21*I)*A + 5*((-5*I)*A + 2*B)*Cos
[2*(e + f*x)] - 5*(2*A + (5*I)*B)*Sin[2*(e + f*x)])*(Cos[5*e + 6*f*x] + I*S
```

`in[5*e + 6*f*x])*Sqrt[a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]/(210*c^4*f)`

**Maple [A]**

time = 0.43, size = 113, normalized size = 0.73

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a(1+\tan^2(fx+e))(5B-25iB\tan(fx+e)-5B\tan(fx+e))}{105f c^4(i+\tan(fx+e))^5}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a(1+\tan^2(fx+e))(5B-25iB\tan(fx+e)-5B\tan(fx+e))}{105f c^4(i+\tan(fx+e))^5}$
risch	$\frac{a\sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}(15iA e^{6i(fx+e)}+15B e^{6i(fx+e)}+42iA e^{4i(fx+e)}+35iA e^{2i(fx+e)}-35B e^{2i(fx+e)})}{420c^3\sqrt{\frac{c}{e^{2i(fx+e)}+1}}f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)`

[Out] `1/105*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^4*(1+tan(f*x+e)^2)*(5*B-25*I*B*tan(f*x+e)-5*B*tan(f*x+e)^2-23*I*A-10*A*tan(f*x+e)+2*I*A*tan(f*x+e)^2)/(I+tan(f*x+e))^5`

**Maxima [A]**

time = 0.61, size = 198, normalized size = 1.28

(15(-A - B)a cos(1/2 arctan(sin(2fx + 2e), cos(2fx + 2e))) - 42Aa cos(3/2 arctan(sin(2fx + 2e), cos(2fx + 2e))) + 35(-A + B)a cos(5/2 arctan(sin(2fx + 2e), cos(2fx + 2e))) + 15(A - I\*B)a sin(7/2 arctan(sin(2fx + 2e), cos(2fx + 2e))) + 42Aa sin(5/2 arctan(sin(2fx + 2e), cos(2fx + 2e))) + 35(A + I\*B)a sin(3/2 arctan(sin(2fx + 2e), cos(2fx + 2e))))sqrt(a)/(c^(7/2)\*f

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x, algorithm="maxima")`

[Out] `1/420*(15*(-I*A - B)*a*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 42*I*A*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 35*(-I*A + B)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(A - I*B)*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 42*A*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 35*(A + I*B)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(7/2)*f)`

**Fricas [A]**

time = 5.39, size = 123, normalized size = 0.79

$$\frac{(15(iA + B)ae^{9i fx + 9ie} + 3(19iA + 5B)ae^{7i fx + 7ie} + 7(11iA - 5B)ae^{5i fx + 5ie} + 35(iA - B)ae^{3i fx + 3ie})\sqrt{\frac{a}{e^{2i fx + 2ie} + 1}}\sqrt{\frac{c}{e^{2i fx + 2ie} + 1}}}{420c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="fricas")

[Out] 
$$-1/420*(15*(I*A + B)*a*e^{(9*I*f*x + 9*I*e)} + 3*(19*I*A + 5*B)*a*e^{(7*I*f*x + 7*I*e)} + 7*(11*I*A - 5*B)*a*e^{(5*I*f*x + 5*I*e)} + 35*(I*A - B)*a*e^{(3*I*f*x + 3*I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} / (c^4*f)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e + fx) - i))^{\frac{3}{2}}(A + B \tan(e + fx))}{(-ic(\tan(e + fx) + i))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(3/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(7/2),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))\*\*(3/2)\*(A + B\*tan(e + f\*x))/(-I\*c\*(tan(e + f\*x) + I))\*\*(7/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(3/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(3/2)/(-I\*c\*tan(f\*x + e) + c)^(7/2), x)

**Mupad [B]**

time = 10.55, size = 215, normalized size = 1.39

$$a \sqrt{\frac{\cos(2e+2fx)+1+\sin(2e+2fx)1i}{\cos(2e+2fx)+1}} (A \cos(2e+2fx)35i + A \cos(4e+4fx)42i + A \cos(6e+6fx)15i - 35B \cos(2e+2fx) + 15B \cos(6e+6fx) - 35A \sin(2e+2fx) - 42A \sin(4e+4fx) - 15A \sin(6e+6fx) - B \sin(2e+2fx)35i + B \sin(6e+6fx)15i) \sqrt{\frac{c \cos(2e+2fx)+1-\sin(2e+2fx)1i}{\cos(2e+2fx)+1}} / (420c^3f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2))/(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] 
$$-(a*((a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1)))^{(1/2)}*(A*\cos(2*e + 2*f*x)*35i + A*\cos(4*e + 4*f*x)*42i + A*\cos(6*e + 6*f*x)*15i - 35*B*\cos(2*e + 2*f*x) + 15*B*\cos(6*e + 6*f*x) - 35*A*\sin(2*e + 2*f*x) - 42*A*\sin(4*e + 4*f*x) - 15*A*\sin(6*e + 6*f*x) - B*\sin(2*e + 2*f*x)*35i + B*\sin(6*e + 6*f*x)*15i)/((420*c^3*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1)))^{(1/2)})$$

$$3.803 \quad \int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{9/2}} dx$$

**Optimal.** Leaf size=208

$$\frac{(iA+B)(a+ia \tan(e+fx))^{3/2}}{9f(c-ic \tan(e+fx))^{9/2}} - \frac{(iA-2B)(a+ia \tan(e+fx))^{3/2}}{21cf(c-ic \tan(e+fx))^{7/2}} - \frac{2(iA-2B)(a+ia \tan(e+fx))^{3/2}}{105c^2f(c-ic \tan(e+fx))^{5/2}}$$

[Out]  $-1/9*(I*A+B)*(a+I*a*\tan(f*x+e))^{(3/2)}/f/(c-I*c*\tan(f*x+e))^{(9/2)}-1/21*(I*A-2*B)*(a+I*a*\tan(f*x+e))^{(3/2)}/c/f/(c-I*c*\tan(f*x+e))^{(7/2)}-2/105*(I*A-2*B)*(a+I*a*\tan(f*x+e))^{(3/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(5/2)}-2/315*(I*A-2*B)*(a+I*a*\tan(f*x+e))^{(3/2)}/c^3/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ ,

Rules used = {3669, 79, 47, 37}

$$\frac{2(-2B+iA)(a+ia \tan(e+fx))^{3/2}}{315c^3f(c-ic \tan(e+fx))^{3/2}} - \frac{2(-2B+iA)(a+ia \tan(e+fx))^{3/2}}{105c^2f(c-ic \tan(e+fx))^{5/2}} - \frac{(-2B+iA)(a+ia \tan(e+fx))^{3/2}}{21cf(c-ic \tan(e+fx))^{7/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{3/2}}{9f(c-ic \tan(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(9/2)}, x]$

[Out]  $-1/9*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(9/2)}) - ((I*A - 2*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(21*c*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) - (2*(I*A - 2*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(105*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (2*(I*A - 2*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)})/(315*c^3*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})$

**Rule 37**

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)})/((b*c - a*d)*(m + 1)), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

**Rule 47**

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)})/((b*c - a*d)*(m + 1)), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n])) \ \&\& \ (\text{SumSimplerQ}[m, 1] \ || \ !\text{SumSimplerQ}[n, 1])$

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

## Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{9/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + iax} (A+Bx)}{(c-icx)^{11/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{9f(c - ictan(e + fx))^{9/2}} + \frac{(a(A + 2iB)) \text{Subst} \left( \int \frac{\sqrt{a + iax}}{(c-icx)^{11/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{9f(c - ictan(e + fx))^{9/2}} - \frac{(iA - 2B)(a + ia \tan(e + fx))^{3/2}}{21cf(c - ictan(e + fx))^{9/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{9f(c - ictan(e + fx))^{9/2}} - \frac{(iA - 2B)(a + ia \tan(e + fx))^{3/2}}{21cf(c - ictan(e + fx))^{9/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{9f(c - ictan(e + fx))^{9/2}} - \frac{(iA - 2B)(a + ia \tan(e + fx))^{3/2}}{21cf(c - ictan(e + fx))^{9/2}}$$

**Mathematica [A]**

time = 3.57, size = 148, normalized size = 0.71

$$\frac{a \cos(e + fx)(\cos(fx) - i \sin(fx))(9(-18iA + B) \cos(e + fx) + 35(-2iA + B) \cos(3(e + fx)) - (A + 2iB)(27 \sin(e + fx) + 35 \sin(3(e + fx))))(\cos(6e + 7fx) + i \sin(6e + 7fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{1260c^2 f}$$

Antiderivative was successfully verified.





```
[Out] -(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*cos(2*e + 2*f*x)*105i + A*cos(4*e + 4*f*x)*189i + A*cos(6*e + 6*f*x)*135i + A*cos(8*e + 8*f*x)*35i - 105*B*cos(2*e + 2*f*x) - 63*B*cos(4*e + 4*f*x) + 45*B*cos(6*e + 6*f*x) + 35*B*cos(8*e + 8*f*x) - 105*A*sin(2*e + 2*f*x) - 189*A*sin(4*e + 4*f*x) - 135*A*sin(6*e + 6*f*x) - 35*A*sin(8*e + 8*f*x) - B*sin(2*e + 2*f*x)*105i - B*sin(4*e + 4*f*x)*63i + B*sin(6*e + 6*f*x)*45i + B*sin(8*e + 8*f*x)*35i))/(2520*c^4*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```



$$3.804 \quad \int \frac{(a+ia \tan(e+fx))^{3/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{11/2}} dx$$

**Optimal.** Leaf size=261

$$\frac{(iA+B)(a+ia \tan(e+fx))^{3/2}}{11f(c-ic \tan(e+fx))^{11/2}} - \frac{(4iA-7B)(a+ia \tan(e+fx))^{3/2}}{99cf(c-ic \tan(e+fx))^{9/2}} - \frac{(4iA-7B)(a+ia \tan(e+fx))^{3/2}}{231c^2f(c-ic \tan(e+fx))^{7/2}}$$

```
[Out] -1/11*(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(11/2)-1/99*(4*I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(9/2)-1/231*(4*I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*tan(f*x+e))^(7/2)-2/1155*(4*I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c^3/f/(c-I*c*tan(f*x+e))^(5/2)-2/3465*(4*I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c^4/f/(c-I*c*tan(f*x+e))^(3/2)
```

**Rubi** [A]

time = 0.22, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$-\frac{2(-7B+4iA)(a+ia \tan(e+fx))^{3/2}}{3465c^4f(c-ic \tan(e+fx))^{3/2}} - \frac{2(-7B+4iA)(a+ia \tan(e+fx))^{3/2}}{1155c^3f(c-ic \tan(e+fx))^{5/2}} - \frac{(-7B+4iA)(a+ia \tan(e+fx))^{3/2}}{231c^2f(c-ic \tan(e+fx))^{7/2}} - \frac{(-7B+4iA)(a+ia \tan(e+fx))^{3/2}}{99cf(c-ic \tan(e+fx))^{9/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{3/2}}{11f(c-ic \tan(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(11/2), x]
```

```
[Out] -1/11*((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(f*(c - I*c*Tan[e + f*x])^(11/2)) - (((4*I)*A - 7*B)*(a + I*a*Tan[e + f*x])^(3/2))/(99*c*f*(c - I*c*Tan[e + f*x])^(9/2)) - (((4*I)*A - 7*B)*(a + I*a*Tan[e + f*x])^(3/2))/(231*c^2*f*(c - I*c*Tan[e + f*x])^(7/2)) - (2*((4*I)*A - 7*B)*(a + I*a*Tan[e + f*x])^(3/2))/(1155*c^3*f*(c - I*c*Tan[e + f*x])^(5/2)) - (2*((4*I)*A - 7*B)*(a + I*a*Tan[e + f*x])^(3/2))/(3465*c^4*f*(c - I*c*Tan[e + f*x])^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
```

(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

### Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

### Rule 3669

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(e + fx))^{3/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{11/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{\sqrt{a + ia x} (A + Bx)}{(c - icx)^{13/2}} dx, x, \tan(e + fx) \right)}{f} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{11f(c - ictan(e + fx))^{11/2}} + \frac{(a(4A + 7iB)) \text{Subst} \left( \int \frac{\sqrt{a + ia x}}{(c - icx)^{13/2}} dx, x, \tan(e + fx) \right)}{99cf(c - ictan(e + fx))^{11/2}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(4iA - 7B)(a + ia \tan(e + fx))^{3/2}}{99cf(c - ictan(e + fx))^{11/2}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(4iA - 7B)(a + ia \tan(e + fx))^{3/2}}{99cf(c - ictan(e + fx))^{11/2}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(4iA - 7B)(a + ia \tan(e + fx))^{3/2}}{99cf(c - ictan(e + fx))^{11/2}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{3/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(4iA - 7B)(a + ia \tan(e + fx))^{3/2}}{99cf(c - ictan(e + fx))^{11/2}}
 \end{aligned}$$

**Mathematica [A]**

time = 6.14, size = 179, normalized size = 0.69

$$\frac{i a \cos(e+f x) \cos(f x) - i \sin(f x) (1485 A + 308(7 A + i B) \cos(2(e+f x)) + 105(7 A + 4 i B) \cos(4(e+f x)) - 616 i A \sin(2(e+f x)) + 1078 B \sin(2(e+f x)) - 420 i A \sin(4(e+f x)) + 735 B \sin(4(e+f x))) (\cos(7 e + 8 f x) + i \sin(7 e + 8 f x)) \sqrt{a + i a \tan(e+f x)} \sqrt{c - i c \tan(e+f x)}}{27720 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(11/2), x]
```

```
[Out] ((-1/27720*I)*a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(1485*A + 308*(7*A + I*B)*Cos[2*(e + f*x)] + 105*(7*A + (4*I)*B)*Cos[4*(e + f*x)] - (616*I)*A*Sin[2*(e + f*x)] + 1078*B*Sin[2*(e + f*x)] - (420*I)*A*Sin[4*(e + f*x)] + 735*B*Sin[4*(e + f*x)])*(Cos[7*e + 8*f*x] + I*Sin[7*e + 8*f*x])*Sqrt[a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]/(c^6*f)
```

**Maple [A]**

time = 0.41, size = 158, normalized size = 0.61

method	result
derivativedivides	$-\frac{\sqrt{a(1+i \tan (f x+e))} \sqrt{-c(i \tan (f x+e)-1)}}{a(1+\tan ^2(f x+e))(14 i B(\tan ^4(f x+e))+56 i A)}$
default	$-\frac{\sqrt{a(1+i \tan (f x+e))} \sqrt{-c(i \tan (f x+e)-1)}}{a(1+\tan ^2(f x+e))(14 i B(\tan ^4(f x+e))+56 i A)}$
risch	$-\frac{a \sqrt{\frac{a e^{2 i(f x+e)}}{e^{2 i(f x+e)}+1}}}{55440 c^5 \sqrt{\frac{c}{e^{2 i(f x+e)}+1}}} f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2), x, method=_RETURNVERBOSE)
```

```
[Out] -1/3465/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^6*(1+tan(f*x+e)^2)*(14*I*B*tan(f*x+e)^4+56*I*A*tan(f*x+e)^3+8*A*tan(f*x+e)^4-315*I*B*tan(f*x+e)^2-98*B*tan(f*x+e)^3-364*I*A*tan(f*x+e)-180*A*tan(f*x+e)^2+91*I*B+637*B*tan(f*x+e)+547*A)/(I+tan(f*x+e))^7
```

**Maxima [A]**

time = 0.62, size = 332, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2), x, algorithm="maxima")
```

[Out]  $\frac{1}{55440} \cdot (315 \cdot (-I \cdot A - B) \cdot a \cdot \cos(\frac{11}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 770 \cdot (-2 \cdot I \cdot A - B) \cdot a \cdot \cos(\frac{9}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) - 2970 \cdot I \cdot A \cdot a \cdot \cos(\frac{7}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 1386 \cdot (-2 \cdot I \cdot A + B) \cdot a \cdot \cos(\frac{5}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 1155 \cdot (-I \cdot A + B) \cdot a \cdot \cos(\frac{3}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 315 \cdot (A - I \cdot B) \cdot a \cdot \sin(\frac{11}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 770 \cdot (2 \cdot A - I \cdot B) \cdot a \cdot \sin(\frac{9}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 2970 \cdot A \cdot a \cdot \sin(\frac{7}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 1386 \cdot (2 \cdot A + I \cdot B) \cdot a \cdot \sin(\frac{5}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e))) + 1155 \cdot (A + I \cdot B) \cdot a \cdot \sin(\frac{3}{2} \cdot \arctan_2(\sin(2 \cdot f \cdot x + 2 \cdot e), \cos(2 \cdot f \cdot x + 2 \cdot e)))) \cdot \sqrt{a} / (c^{\frac{11}{2}} \cdot f)$

**Fricas** [A]

time = 2.22, size = 163, normalized size = 0.62

$$\frac{(315(iA + B)ae^{(13i f x + 13i e)} + 35(53iA + 31B)ae^{(11i f x + 11i e)} + 110(41iA + 7B)ae^{(9i f x + 9i e)} + 198(29iA - 7B)ae^{(7i f x + 7i e)} + 231(17iA - 11B)ae^{(5i f x + 5i e)} + 1155(iA - B)ae^{(3i f x + 3i e)}) \sqrt{\frac{a}{e^{(2i f x + 2i e)} + 1}} \sqrt{\frac{c}{e^{(2i f x + 2i e)} + 1}}}{55440 c^6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x, algorithm="fricas")`

[Out]  $-1/55440 \cdot (315 \cdot (I \cdot A + B) \cdot a \cdot e^{(13 \cdot I \cdot f \cdot x + 13 \cdot I \cdot e)} + 35 \cdot (53 \cdot I \cdot A + 31 \cdot B) \cdot a \cdot e^{(11 \cdot I \cdot f \cdot x + 11 \cdot I \cdot e)} + 110 \cdot (41 \cdot I \cdot A + 7 \cdot B) \cdot a \cdot e^{(9 \cdot I \cdot f \cdot x + 9 \cdot I \cdot e)} + 198 \cdot (29 \cdot I \cdot A - 7 \cdot B) \cdot a \cdot e^{(7 \cdot I \cdot f \cdot x + 7 \cdot I \cdot e)} + 231 \cdot (17 \cdot I \cdot A - 11 \cdot B) \cdot a \cdot e^{(5 \cdot I \cdot f \cdot x + 5 \cdot I \cdot e)} + 1155 \cdot (I \cdot A - B) \cdot a \cdot e^{(3 \cdot I \cdot f \cdot x + 3 \cdot I \cdot e)}) \cdot \sqrt{a / (e^{(2 \cdot I \cdot f \cdot x + 2 \cdot I \cdot e)} + 1)} \cdot \sqrt{c / (e^{(2 \cdot I \cdot f \cdot x + 2 \cdot I \cdot e)} + 1)} / (c^6 \cdot f)$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(11/2),x)`

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x, algorithm="giac")`

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(3/2)/(-I\*c\*tan(f\*x + e) + c)^(11/2), x)

**Mupad [B]**

time = 14.51, size = 315, normalized size = 1.21

$$\frac{\sqrt{\frac{a \cos(2e + 2fx) + \sin(2e + 2fx) + 1}{\cos(2e + 2fx) + 1}} (A \cos(2e + 2fx) 1155i + A \cos(4e + 4fx) 2772i + A \cos(6e + 6fx) 2970i + A \cos(8e + 8fx) 1540i + A \cos(10e + 10fx) 315i - 1155B \cos(2e + 2fx) - 1386B \cos(4e + 4fx) + 770B \cos(8e + 8fx) + 315B \cos(10e + 10fx) - 1155A \sin(2e + 2fx) - 2772A \sin(4e + 4fx) - 2970A \sin(6e + 6fx) - 1540A \sin(8e + 8fx) - 315A \sin(10e + 10fx) - B \sin(2e + 2fx) 1155i - B \sin(4e + 4fx) 1386i - B \sin(8e + 8fx) 770i - B \sin(10e + 10fx) 315i)}{55440c^5 f \sqrt{\frac{a \cos(2e + 2fx) + \sin(2e + 2fx) + 1}{\cos(2e + 2fx) + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(3/2))/(c - c\*tan(e + f\*x)\*1i)^(11/2), x)

[Out] 
$$-(a*((a*(\cos(2e + 2fx) + \sin(2e + 2fx)*1i + 1))/(\cos(2e + 2fx) + 1)))^{1/2} * (A \cos(2e + 2fx) 1155i + A \cos(4e + 4fx) 2772i + A \cos(6e + 6fx) 2970i + A \cos(8e + 8fx) 1540i + A \cos(10e + 10fx) 315i - 1155B \cos(2e + 2fx) - 1386B \cos(4e + 4fx) + 770B \cos(8e + 8fx) + 315B \cos(10e + 10fx) - 1155A \sin(2e + 2fx) - 2772A \sin(4e + 4fx) - 2970A \sin(6e + 6fx) - 1540A \sin(8e + 8fx) - 315A \sin(10e + 10fx) - B \sin(2e + 2fx) 1155i - B \sin(4e + 4fx) 1386i + B \sin(8e + 8fx) 770i + B \sin(10e + 10fx) 315i) / (55440c^5 f ((c*(\cos(2e + 2fx) - \sin(2e + 2fx)*1i + 1))/(\cos(2e + 2fx) + 1)))^{1/2}$$



), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]

### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{7/2} dx &= \frac{(ac) \text{Subst}\left(\int (a + iax)^{3/2} (A + Bx) dx\right)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{7/2}}{6f} \\
&= -\frac{(6iA - B)c(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{7/2}}{30f} \\
&= \frac{a(6A + iB)c^2 \tan(e + fx)(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{7/2}}{30f} \\
&= \frac{a^2(6A + iB)c^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{7/2}}{30f} \\
&= \frac{a^2(6A + iB)c^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{7/2}}{30f} \\
&= \frac{a^2(6A + iB)c^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{7/2}}{30f} \\
&= -\frac{a^{5/2}(6iA - B)c^{7/2} \tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{a + ia \tan(e + fx)}}\right)}{8f}
\end{aligned}$$

**Mathematica [A]**

time = 9.21, size = 568, normalized size = 1.97

```

(* Mathematica output showing the antiderivative result *)

```

Warning: Unable to verify antiderivative.

```

[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2), x]

```

```

[Out] (((-6*I)*A + B)*c^4*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(8*E^(I*(3*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(7/2)*(Cos[f*x] + I*Sin[f*x])^(5/2)*(A*Cos[e + f*x] + B*Sin[e + f*x])) + (Cos[e + f*x]^3*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(Sec[e]*Sec[e + f*x]^4*((-6*I)*A*Cos[e] + 6*B*Cos[e] - (5*I)*B*Sin[e])*((c^3*Cos[2*e])/30 - (I/30)*c^3*Sin[2*e]) - I*B*c^3*Sec[e]*Sec[e + f*x]^5*(Cos[2*

```



$$e]/6 - (I/6)*\text{Sin}[2*e])*\text{Sin}[f*x] + \text{Sec}[e]*\text{Sec}[e + f*x]^3*(\text{Cos}[2*e]/24 - (I/24)*\text{Sin}[2*e])*(6*A*c^3*\text{Sin}[f*x] + I*B*c^3*\text{Sin}[f*x]) + \text{Sec}[e]*\text{Sec}[e + f*x]*(\text{Cos}[2*e]/16 - (I/16)*\text{Sin}[2*e])*(6*A*c^3*\text{Sin}[f*x] + I*B*c^3*\text{Sin}[f*x]) + (6*A + I*B)*\text{Sec}[e + f*x]^2*((c^3*\text{Cos}[2*e])/24 - (I/24)*c^3*\text{Sin}[2*e])*\text{Tan}[e] + (6*A + I*B)*((c^3*\text{Cos}[2*e])/16 - (I/16)*c^3*\text{Sin}[2*e])*\text{Tan}[e]*(a + I*a*\text{Tan}[e + f*x])^(5/2)*(A + B*\text{Tan}[e + f*x]))/(f*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^2*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(236) = 472.

time = 0.44, size = 478, normalized size = 1.66

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 c^3 \left( 40iB \sqrt{ac(1+\tan^2(fx+e))} \right)}{\dots}$
default	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 c^3 \left( 40iB \sqrt{ac(1+\tan^2(fx+e))} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/240/f*(a*(1+I*\text{tan}(f*x+e)))^(1/2)*(-c*(I*\text{tan}(f*x+e)-1))^(1/2)*a^2*c^3*(40 \\ & *I*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)^5+48*I*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)* \\ & (a*c)^(1/2)*\text{tan}(f*x+e)^4+70*I*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)^3-48*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)* \\ & (a*c)^(1/2)*\text{tan}(f*x+e)^2+96*I*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)^2 \\ & -60*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)^3-15*I*B*\ln((a*c*\text{tan}(f*x+e)+(a*c)^(1/2)*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)) \\ & /((a*c)^(1/2))*a*c+15*I*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)-96*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)* \\ & (a*c)^(1/2)*\text{tan}(f*x+e)^2+48*I*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)-90*A*\ln((a*c*\text{tan}(f*x+e)+(a*c)^(1/2)*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)) \\ & /((a*c)^(1/2))*a*c-150*A*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)*(a*c)^(1/2)*\text{tan}(f*x+e)-48*B*(a*c*(1+\text{tan}(f*x+e)^2))^(1/2)* \\ & (a*c)^(1/2))/((a*c)^(1/2))/(a*c*(1+\text{tan}(f*x+e)^2))^(1/2) \end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2137 vs. 2(232) = 464.

time = 5.90, size = 2137, normalized size = 7.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="maxima")

[Out]  $-3840*(60*(6*A + I*B)*a^2*c^3*\cos(11/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 340*(6*A + I*B)*a^2*c^3*\cos(9/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 792*(6*A + I*B)*a^2*c^3*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 24*(58*A + 223*I*B)*a^2*c^3*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 340*(6*A + I*B)*a^2*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 60*(6*A + I*B)*a^2*c^3*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 60*(6*I*A - B)*a^2*c^3*\sin(11/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 340*(6*I*A - B)*a^2*c^3*\sin(9/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 792*(6*I*A - B)*a^2*c^3*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 24*(58*I*A - 223*B)*a^2*c^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 340*(-6*I*A + B)*a^2*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 60*(-6*I*A + B)*a^2*c^3*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 30*((6*A + I*B)*a^2*c^3*\cos(12*f*x + 12*e) + 6*(6*A + I*B)*a^2*c^3*\cos(10*f*x + 10*e) + 15*(6*A + I*B)*a^2*c^3*\cos(8*f*x + 8*e) + 20*(6*A + I*B)*a^2*c^3*\cos(6*f*x + 6*e) + 15*(6*A + I*B)*a^2*c^3*\cos(4*f*x + 4*e) + 6*(6*A + I*B)*a^2*c^3*\cos(2*f*x + 2*e) + (6*I*A - B)*a^2*c^3*\sin(12*f*x + 12*e) + 6*(6*I*A - B)*a^2*c^3*\sin(10*f*x + 10*e) + 15*(6*I*A - B)*a^2*c^3*\sin(8*f*x + 8*e) + 20*(6*I*A - B)*a^2*c^3*\sin(6*f*x + 6*e) + 15*(6*I*A - B)*a^2*c^3*\sin(4*f*x + 4*e) + 6*(6*I*A - B)*a^2*c^3*\sin(2*f*x + 2*e) + (6*A + I*B)*a^2*c^3*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 30*((6*A + I*B)*a^2*c^3*\cos(12*f*x + 12*e) + 6*(6*A + I*B)*a^2*c^3*\cos(10*f*x + 10*e) + 15*(6*A + I*B)*a^2*c^3*\cos(8*f*x + 8*e) + 20*(6*A + I*B)*a^2*c^3*\cos(6*f*x + 6*e) + 15*(6*A + I*B)*a^2*c^3*\cos(4*f*x + 4*e) + 6*(6*A + I*B)*a^2*c^3*\cos(2*f*x + 2*e) + (6*I*A - B)*a^2*c^3*\sin(12*f*x + 12*e) + 6*(6*I*A - B)*a^2*c^3*\sin(10*f*x + 10*e) + 15*(6*I*A - B)*a^2*c^3*\sin(8*f*x + 8*e) + 20*(6*I*A - B)*a^2*c^3*\sin(6*f*x + 6*e) + 15*(6*I*A - B)*a^2*c^3*\sin(4*f*x + 4*e) + 6*(6*I*A - B)*a^2*c^3*\sin(2*f*x + 2*e) + (6*A + I*B)*a^2*c^3*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((6*I*A - B)*a^2*c^3*\cos(12*f*x + 12*e) + 6*(6*I*A - B)*a^2*c^3*\cos(10*f*x + 10*e) + 15*(6*I*A - B)*a^2*c^3*\cos(8*f*x + 8*e) + 20*(6*I*A - B)*a^2*c^3*\cos(6*f*x + 6*e) + 15*(6*I*A - B)*a^2*c^3*\cos(4*f*x + 4*e) + 6*(6*I*A - B)*a^2*c^3*\cos(2*f*x + 2*e) - (6*A + I*B)*a^2*c^3*\sin(12*f*x + 12*e) - 6*(6*A + I*B)*a^2*c^3*\sin(10*f*x + 10*e) - 15*(6*A + I*B)*a^2*c^3*\sin(8*f*x + 8*e) - 20*(6*A + I*B)*a^2*c^3*\sin(6*f*x + 6*e) - 15*(6*A + I*B)*a^2*c^3*\sin(4*f*x + 4*e) - 6*(6*A + I*B)*a^2*c^3*\sin(2*f*x + 2*e) + (6*I*A - B)*a^2*c^3*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((-6*I*A + B)*a^2*c^3*\cos(12*f*x + 12*e) + 6*(-6*I*A + B)*a^2*c^3*\cos(10*f*x + 10*e) + 15*(-6*I*A + B)*a^2*c^3*\cos(8*f*x + 8*e) + 20*(-6*I*A + B)*a^2*c^3*\cos(6*f*x + 6*e) + 15*(-$

```

6*I*A + B)*a^2*c^3*cos(4*f*x + 4*e) + 6*(-6*I*A + B)*a^2*c^3*cos(2*f*x + 2*
e) + (6*A + I*B)*a^2*c^3*sin(12*f*x + 12*e) + 6*(6*A + I*B)*a^2*c^3*sin(10*
f*x + 10*e) + 15*(6*A + I*B)*a^2*c^3*sin(8*f*x + 8*e) + 20*(6*A + I*B)*a^2*
c^3*sin(6*f*x + 6*e) + 15*(6*A + I*B)*a^2*c^3*sin(4*f*x + 4*e) + 6*(6*A + I
*B)*a^2*c^3*sin(2*f*x + 2*e) + (-6*I*A + B)*a^2*c^3*log(cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 1))*sqrt(a)*sqrt(c)/(f*(-1843200*I*cos(12*f*x + 12*e) - 11059200*I*cos(1
0*f*x + 10*e) - 27648000*I*cos(8*f*x + 8*e) - 36864000*I*cos(6*f*x + 6*e) -
27648000*I*cos(4*f*x + 4*e) - 11059200*I*cos(2*f*x + 2*e) + 1843200*sin(12
*f*x + 12*e) + 11059200*sin(10*f*x + 10*e) + 27648000*sin(8*f*x + 8*e) + 36
864000*sin(6*f*x + 6*e) + 27648000*sin(4*f*x + 4*e) + 11059200*sin(2*f*x +
2*e) - 1843200*I))

```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 791 vs.  $2(232) = 464$ .  
time = 2.72, size = 791, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2
),x, algorithm="fricas")

```

```

[Out] 1/480*(15*sqrt((36*A^2 + 12*I*A*B - B^2)*a^5*c^7/f^2)*(f*e^(10*I*f*x + 10*I
*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x
+ 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((6*I*A - B)*a^2*c^3*e^(3
*I*f*x + 3*I*e) + (6*I*A - B)*a^2*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((36*A^2 + 12*I*A*B -
B^2)*a^5*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-6*I*A + B)*a^2*c^3*e^(2*
I*f*x + 2*I*e) + (-6*I*A + B)*a^2*c^3)) - 15*sqrt((36*A^2 + 12*I*A*B - B^2)
*a^5*c^7/f^2)*(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(
6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)*
log(-4*(2*((6*I*A - B)*a^2*c^3*e^(3*I*f*x + 3*I*e) + (6*I*A - B)*a^2*c^3*e^
(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e
) + 1)) - sqrt((36*A^2 + 12*I*A*B - B^2)*a^5*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e
) - f))/((-6*I*A + B)*a^2*c^3*e^(2*I*f*x + 2*I*e) + (-6*I*A + B)*a^2*c^3))
- 4*(15*(6*I*A - B)*a^2*c^3*e^(11*I*f*x + 11*I*e) + 85*(6*I*A - B)*a^2*c^3*
e^(9*I*f*x + 9*I*e) + 198*(6*I*A - B)*a^2*c^3*e^(7*I*f*x + 7*I*e) + 6*(58*I
*A - 223*B)*a^2*c^3*e^(5*I*f*x + 5*I*e) + 85*(-6*I*A + B)*a^2*c^3*e^(3*I*f*
x + 3*I*e) + 15*(-6*I*A + B)*a^2*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(10*I*f*x + 10*I*e) +
5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*
e) + 5*f*e^(2*I*f*x + 2*I*e) + f)

```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(5/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(7/2),x)

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{5/2} (c - c \tan(e + f x) i)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(7/2), x)

$$3.806 \quad \int (a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) (c - ictan(e + fx))^{5/2} dx$$

**Optimal.** Leaf size=213

$$\frac{3ia^{5/2}Ac^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{4f} + \frac{3a^2Ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{8f}$$

[Out]  $-3/4*I*a^{(5/2)}*A*c^{(5/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f+3/8*a^2*A*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+1/4*a*A*c*\tan(f*x+e)*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/5*B*(a+I*a*\tan(f*x+e))^{(5/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f$

**Rubi [A]**

time = 0.18, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 38, 65, 223, 209}

$$\frac{3ia^{5/2}Ac^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{4f} + \frac{3a^2Ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{8f} + \frac{aAc\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ictan(e+fx))^{3/2}}{4f} + \frac{B(a+ia\tan(e+fx))^{5/2}(c-ictan(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $(((-3*I)/4)*a^{(5/2)}*A*c^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])])/f + (3*a^2*A*c^2*\text{Tan}[e + f*x]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(8*f) + (a*A*c*\text{Tan}[e + f*x]*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(4*f) + (B*(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(5*f)$

**Rule 38**

$\text{Int}[(a + b*x)^m * (c + d*x)^m, x\_Symbol] := \text{Simp}[x*(a + b*x)^m * (c + d*x)^m / (2*m + 1), x] + \text{Dist}[2*a*c*m / (2*m + 1), \text{Int}[(a + b*x)^{m-1} * (c + d*x)^{m-1}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 65**

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1} * (c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{5/2} dx &= \frac{(ac) \text{Subst}(\int (a + iax)^{3/2} (A + B \tan(e + fx)) dx)}{5f} \\
&= \frac{B(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}}{5f} \\
&= \frac{aAc \tan(e + fx) (a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}}{4f} \\
&= \frac{3a^2 Ac^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{8} \\
&= \frac{3a^2 Ac^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{8} \\
&= \frac{3a^2 Ac^2 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{8} \\
&= -\frac{3ia^{5/2} Ac^{5/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}} \right)}{4f}
\end{aligned}$$

**Mathematica [A]**

time = 4.92, size = 119, normalized size = 0.56

$$\frac{a^2 c^3 \sec^4(e + fx) (240A \text{ArcTan}(e^{i(e+fx)}) \cos^5(e + fx) + i(64B + 70A \sin(2(e + fx)) + 15A \sin(4(e + fx)))) (i + \tan(e + fx)) \sqrt{a + ia \tan(e + fx)}}{320f \sqrt{c - ic \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/320*(a^2*c^3*Sec[e + f*x]^4*(240*A*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x]^5 + I*(64*B + 70*A*Sin[2*(e + f*x)] + 15*A*Sin[4*(e + f*x)]))*(I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

**Maple [A]**

time = 0.43, size = 252, normalized size = 1.18

method	result
--------	--------

derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 c^2 \left( 8B \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)}{\dots}$
default	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 c^2 \left( 8B \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c^2*(8*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+10*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+16*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+15*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c+25*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+8*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1535 vs.  $2(173) = 346$ .  
time = 1.19, size = 1535, normalized size = 7.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")
```

```
[Out] -(60*A*a^2*c^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 280*A*a^2*c^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 512*I*B*a^2*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 280*A*a^2*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*A*a^2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 60*I*A*a^2*c^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 280*I*A*a^2*c^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 512*B*a^2*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 280*I*A*a^2*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*I*A*a^2*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(A*a^2*c^2*cos(10*f*x + 10*e) + 5*A*a^2*c^2*cos(8*f*x + 8*e) + 10*A*a^2*c^2*cos(6*f*x + 6*e) + 10*A*a^2*c^2*cos(4*f*x + 4*e) + 5*A*a^2*c^2*cos(2*f*x + 2*e) + I*A*a^2*c^2*sin(10*f*x + 10*e) + 5*I*A*a^2*c^2*sin(8*f*x + 8*e) + 10*I*A*a^2*c^2*sin(6*f*x + 6*e) + 10*I*A*a^2*c^2*sin(4*f*x + 4*e) + 5*I*A*a^2*c^2*sin(2*f*x + 2*e) + A*a^2*c^2)*arctan2(cos(1
```





```
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^5*c^5/f^2)*(-I*f
*e^(2*I*f*x + 2*I*e) + I*f))/(A*a^2*c^2*e^(2*I*f*x + 2*I*e) + A*a^2*c^2)) +
  4*(-15*I*A*a^2*c^2*e^(9*I*f*x + 9*I*e) - 70*I*A*a^2*c^2*e^(7*I*f*x + 7*I*e
) + 128*B*a^2*c^2*e^(5*I*f*x + 5*I*e) + 70*I*A*a^2*c^2*e^(3*I*f*x + 3*I*e)
+ 15*I*A*a^2*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e
) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{5/2} (c - c \tan(e + f x) \operatorname{li})^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(5/2)*(c - c*tan(e + f*x)*
li)^(5/2),x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(5/2)*(c - c*tan(e + f*x)*
li)^(5/2), x)
```

$$3.807 \quad \int (a+ia \tan(e+fx))^{5/2} (A+B \tan(e+fx))(c-ictan(e+fx))^{3/2} dx$$

**Optimal.** Leaf size=222

$$\frac{a^{5/2}(4iA+B)c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{4f} + \frac{a^2(4A-iB)c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}}{8f}$$

[Out]  $-1/4*a^{(5/2)}*(4*I*A+B)*c^{(3/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)})/(c-I*c*\tan(f*x+e))^{(1/2)}/f+1/8*a^2*(4*A-I*B)*c*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+1/12*a*(4*I*A+B)*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/4*B*(a+I*a*\tan(f*x+e))^{(5/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f$

**Rubi** [A]

time = 0.20, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 81, 51, 38, 65, 223, 209}

$$\frac{a^{5/2}c^{3/2}(B+4iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{4f} + \frac{a^2c(4A-iB)\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{8f} + \frac{a(B+4iA)(a+ia\tan(e+fx))^{3/2}(c-ictan(e+fx))^{3/2}}{12f} + \frac{B(a+ia\tan(e+fx))^{5/2}(c-ictan(e+fx))^{3/2}}{4f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $-1/4*(a^{(5/2)}*((4*I)*A + B)*c^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])]/f + (a^2*(4*A - I*B)*c*\text{Tan}[e + f*x]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(8*f) + (a*((4*I)*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(12*f) + (B*(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(4*f)$

**Rule 38**

$\text{Int}[(a_) + (b_)*(x_)^{(m_)}*((c_) + (d_)*(x_)^{(m_)}, x\_Symbol] := \text{Simp}[x*(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + \text{Dist}[2*a*c*(m/(2*m + 1)), \text{Int}[(a + b*x)^{m-1}*(c + d*x)^{m-1}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{IGtQ}[m + 1/2, 0]$

**Rule 51**

$\text{Int}[(a_) + (b_)*(x_)^{(m_)}*((c_) + (d_)*(x_)^{(n_)}, x\_Symbol] := \text{Simp}[(a + b*x)^{m+1}*((c + d*x)^n/(b*(m + n + 1))), x] + \text{Dist}[2*c*(n/(m + n + 1)), \text{Int}[(a + b*x)^m*(c + d*x)^{n-1}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{IGtQ}[m + 1/2, 0] \ \&\& \ \text{IGtQ}[n + 1/2, 0] \ \&\& \ \text{LtQ}[m, n]$

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{3/2} dx &= \frac{(ac) \text{Subst}\left(\int (a + iax)^{3/2} (A + B \tan(e + fx)) dx\right)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))}{4f} \\
&= \frac{a(4iA + B)(a + ia \tan(e + fx))}{12f} \\
&= \frac{a^2(4A - iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{12f} \\
&= \frac{a^2(4A - iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{12f} \\
&= \frac{a^2(4A - iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{12f} \\
&= -\frac{a^{5/2}(4iA + B)c^{3/2} \tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{a + ia \tan(e + fx)}}\right)}{4f}
\end{aligned}$$

**Mathematica [A]**

time = 5.73, size = 145, normalized size = 0.65

$$\frac{a^2 c^2 \sec^2(e + fx) (1 - i \tan(e + fx)) \sqrt{a + ia \tan(e + fx)} (16(iA + B) - 12i(4A - iB) \text{ArcTan}(e^{i(e+fx)}) \cos^3(e + fx) + 3(4A + 3iB + (4A - iB) \cos(2(e + fx))) \tan(e + fx))}{48f \sqrt{c - ic \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^(5/2)\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] (a^2\*c^2\*Sec[e + f\*x]^2\*(1 - I\*Tan[e + f\*x])\*Sqrt[a + I\*a\*Tan[e + f\*x]]\*(16\*(I\*A + B) - (12\*I)\*(4\*A - I\*B)\*ArcTan[E^(I\*(e + f\*x))]\*Cos[e + f\*x]^3 + 3\*(4\*A + (3\*I)\*B + (4\*A - I\*B)\*Cos[2\*(e + f\*x)])\*Tan[e + f\*x])/(48\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Maple [A]**

time = 0.42, size = 350, normalized size = 1.58

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^{2c} \left( {}_{6iB} \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)}{\dots}$
default	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^{2c} \left( {}_{6iB} \sqrt{ac(1+\tan^2(fx+e))} \sqrt{ac} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c*(6*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+8*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-3*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+3*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+8*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+8*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+12*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+12*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+8*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1451 vs.  $2(177) = 354$ .

time = 1.43, size = 1451, normalized size = 6.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -96*(12*(4*A - I*B)*a^2*c*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(20*A - 53*I*B)*a^2*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 44*(4*A - I*B)*a^2*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(4*A - I*B)*a^2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(-4*I*A - B)*a^2*c*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(20*I*A + 53*B)*a^2*c*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 44*(4*I*A + B)*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(4*I*A + B)*a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*((4*A - I*B)*a^2*c*cos(8*f*x + 8*e) + 4*(4*A - I*B)*a^2*c*cos(6*f*x + 6*e) + 6*(4*A - I*B)*a^2*c*cos(4*f*x + 4*e) + 4*(4*A - I*B)*a^2*c*cos(2*f*x + 2*e) - (-4*I*A - B)*a^2*c*sin(8*f*x + 8*e) - 4*(-4*I*A
```

$$\begin{aligned}
& - B)a^2c\sin(6fx + 6e) - 6(-4IA - B)a^2c\sin(4fx + 4e) - 4(-4 \\
& IA - B)a^2c\sin(2fx + 2e) + (4A - IB)a^2c\arctan2(\cos(1/2\arctan2(\sin(2fx + 2e), \\
& \cos(2fx + 2e))), \sin(1/2\arctan2(\sin(2fx + 2e), \\
& \cos(2fx + 2e))) + 1) + 6((4A - IB)a^2c\cos(8fx + 8e) + 4(4A - \\
& IB)a^2c\cos(6fx + 6e) + 6(4A - IB)a^2c\cos(4fx + 4e) + 4(4A \\
& - IB)a^2c\cos(2fx + 2e) - (-4IA - B)a^2c\sin(8fx + 8e) - 4(- \\
& 4IA - B)a^2c\sin(6fx + 6e) - 6(-4IA - B)a^2c\sin(4fx + 4e) - \\
& 4(-4IA - B)a^2c\sin(2fx + 2e) + (4A - IB)a^2c\arctan2(\cos(1/2 \\
& \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))), -\sin(1/2\arctan2(\sin(2fx + \\
& 2e), \cos(2fx + 2e))) + 1) - 3((-4IA - B)a^2c\cos(8fx + 8e) + 4 \\
& (-4IA - B)a^2c\cos(6fx + 6e) + 6(-4IA - B)a^2c\cos(4fx + 4e) \\
& ) + 4(-4IA - B)a^2c\cos(2fx + 2e) + (4A - IB)a^2c\sin(8fx + 8 \\
& e) + 4(4A - IB)a^2c\sin(6fx + 6e) + 6(4A - IB)a^2c\sin(4fx + \\
& 4e) + 4(4A - IB)a^2c\sin(2fx + 2e) + (-4IA - B)a^2c\log(\cos \\
& (1/2\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + \sin(1/2\arctan2(\sin(2 \\
& fx + 2e), \cos(2fx + 2e)))^2 + 2\sin(1/2\arctan2(\sin(2fx + 2e), \cos \\
& (2fx + 2e))) + 1) - 3((4IA + B)a^2c\cos(8fx + 8e) + 4(4IA + B) \\
& )a^2c\cos(6fx + 6e) + 6(4IA + B)a^2c\cos(4fx + 4e) + 4(4IA + B) \\
& )a^2c\cos(2fx + 2e) - (4A - IB)a^2c\sin(8fx + 8e) - 4(4A - \\
& IB)a^2c\sin(6fx + 6e) - 6(4A - IB)a^2c\sin(4fx + 4e) - 4(4A \\
& - IB)a^2c\sin(2fx + 2e) + (4IA + B)a^2c\log(\cos(1/2\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + \sin(1/2\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 - 2\sin(1/2\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1))\sqrt{a}\sqrt{c}/(f(-4608I\cos(8fx + 8e) - 18432I\cos(6fx + 6e) - 27648I\cos(4fx + 4e) - 18432I\cos(2fx + 2e) + 4608\sin(8fx + 8e) + 18432\sin(6fx + 6e) + 27648\sin(4fx + 4e) + 18432\sin(2fx + 2e) - 4608I))
\end{aligned}$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs.  $2(177) = 354$ .  
time = 2.41, size = 641, normalized size = 2.89

$$\frac{\sqrt{\frac{16A^2 - 8IA^*B - B^2}{f^2}} \arctan2\left(\frac{\sin\left(\frac{1}{2}\arctan2\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)}{\cos\left(\frac{1}{2}\arctan2\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)}\right)}{\sqrt{\frac{16A^2 - 8IA^*B - B^2}{f^2}}} \sqrt{\frac{c}{(e^{2I*fx + 2I*e} + 1)}} \sqrt{\frac{c}{(e^{2I*fx + 2I*e} + 1)}} + \sqrt{\frac{16A^2 - 8IA^*B - B^2}{f^2}} \sqrt{\frac{c}{(e^{2I*fx + 2I*e} + 1)}} \sqrt{\frac{c}{(e^{2I*fx + 2I*e} + 1)}} + \sqrt{\frac{16A^2 - 8IA^*B - B^2}{f^2}} \sqrt{\frac{c}{(e^{2I*fx + 2I*e} + 1)}} \sqrt{\frac{c}{(e^{2I*fx + 2I*e} + 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -1/48*(3\sqrt{(16A^2 - 8IA^*B - B^2)}a^5c^3/f^2)*(f*e^{(6I*fx + 6I*e)} \\
& + 3f*e^{(4I*fx + 4I*e)} + 3f*e^{(2I*fx + 2I*e)} + f)*\log(-4*(2*((-4IA \\
& - B)a^2c*e^{(3I*fx + 3I*e)} + (-4IA - B)a^2c*e^{(I*fx + I*e)})\sqrt{c} \\
& /((e^{(2I*fx + 2I*e)} + 1))\sqrt{c}/((e^{(2I*fx + 2I*e)} + 1)) + \sqrt{(16A \\
& ^2 - 8IA^*B - B^2)}a^5c^3/f^2)*(f*e^{(2I*fx + 2I*e)} - f))/((4IA + B) \\
& a^2c*e^{(2I*fx + 2I*e)} + (4IA + B)a^2c) - 3\sqrt{(16A^2 - 8IA^*B \\
& - B^2)}a^5c^3/f^2)*(f*e^{(6I*fx + 6I*e)} + 3f*e^{(4I*fx + 4I*e)} + 3f*
\end{aligned}$$

$$e^{(2I*fx + 2I*e) + f} * \log(-4*(2*((-4I*A - B)*a^2*c*e^{(3I*fx + 3I*e)} + (-4I*A - B)*a^2*c*e^{(I*fx + I*e)})) * \sqrt{a/(e^{(2I*fx + 2I*e) + 1})} * \sqrt{c/(e^{(2I*fx + 2I*e) + 1})} - \sqrt{(16A^2 - 8I*AB - B^2)*a^5*c^3/f^2} * (f*e^{(2I*fx + 2I*e) - f}) / ((4I*A + B)*a^2*c*e^{(2I*fx + 2I*e)} + (4I*A + B)*a^2*c)) + 4*(3*(4I*A + B)*a^2*c*e^{(7I*fx + 7I*e)} - (20I*A + 53*B)*a^2*c*e^{(5I*fx + 5I*e)} + 11*(-4I*A - B)*a^2*c*e^{(3I*fx + 3I*e)} + 3*(-4I*A - B)*a^2*c*e^{(I*fx + I*e)}) * \sqrt{a/(e^{(2I*fx + 2I*e) + 1})} * \sqrt{c/(e^{(2I*fx + 2I*e) + 1})}) / (f*e^{(6I*fx + 6I*e)} + 3*f*e^{(4I*fx + 4I*e)} + 3*f*e^{(2I*fx + 2I*e)} + f)$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(5/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{5/2} (c - c \tan(e + f x) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(3/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(3/2), x)



### 3.808 $\int (a+ia \tan(e+fx))^{5/2} (A+B \tan(e+fx)) \sqrt{c-icta}$

Optimal. Leaf size=217

$$-\frac{a^{5/2}(3iA+2B)\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-icta \tan(e+fx)}}\right)}{f} + \frac{a^2(3iA+2B)\sqrt{a+ia \tan(e+fx)}\sqrt{c-icta}}{2f}$$

[Out]  $-a^{5/2}*(3*I*A+2*B)*\arctan(c^{1/2}*(a+I*a*\tan(f*x+e))^{1/2}/a^{1/2}/(c-I*c*\tan(f*x+e))^{1/2})*c^{1/2}/f+1/2*a^2*(3*I*A+2*B)*(a+I*a*\tan(f*x+e))^{1/2}*(c-I*c*\tan(f*x+e))^{1/2}/f+1/6*a*(3*I*A+2*B)*(c-I*c*\tan(f*x+e))^{1/2}*(a+I*a*\tan(f*x+e))^{3/2}/f+1/3*B*(c-I*c*\tan(f*x+e))^{1/2}*(a+I*a*\tan(f*x+e))^{5/2}/f$

Rubi [A]

time = 0.19, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 52, 65, 223, 209}

$$-\frac{a^{5/2}\sqrt{c}(2B+3iA)\operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-icta \tan(e+fx)}}\right)}{f} + \frac{a^2(2B+3iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-icta \tan(e+fx)}}{2f} + \frac{a(2B+3iA)(a+ia \tan(e+fx))^{3/2}\sqrt{c-icta \tan(e+fx)}}{6f} + \frac{B(a+ia \tan(e+fx))^{5/2}\sqrt{c-icta \tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\operatorname{Tan}[e+f*x])^{5/2}*(A+B*\operatorname{Tan}[e+f*x])*Sqrt[c-I*c*\operatorname{Tan}[e+f*x]],x]$

[Out]  $-((a^{5/2}*((3*I)*A+2*B)*Sqrt[c]*\operatorname{ArcTan}[(Sqrt[c]*Sqrt[a+I*a*\operatorname{Tan}[e+f*x]])/(Sqrt[a]*Sqrt[c-I*c*\operatorname{Tan}[e+f*x]])])/f) + (a^2*((3*I)*A+2*B)*Sqrt[a+I*a*\operatorname{Tan}[e+f*x]]*Sqrt[c-I*c*\operatorname{Tan}[e+f*x]])/(2*f) + (a*((3*I)*A+2*B)*(a+I*a*\operatorname{Tan}[e+f*x])^{3/2}*Sqrt[c-I*c*\operatorname{Tan}[e+f*x]])/(6*f) + (B*(a+I*a*\operatorname{Tan}[e+f*x])^{5/2}*Sqrt[c-I*c*\operatorname{Tan}[e+f*x]])/(3*f)$

Rule 52

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))], x] + \operatorname{Dist}[n*((b*c - a*d)/(b*(m+n+1))), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{3/2}(A+Bx)}{\sqrt{c-icx}} dx, \frac{a+ia \tan(e+fx)}{f} \right)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{5/2} \sqrt{c - ic \tan(e + fx)}}{3f} \\
&= \frac{a(3iA + 2B)(a + ia \tan(e + fx))}{6f} \\
&= \frac{a^2(3iA + 2B) \sqrt{a + ia \tan(e + fx)}}{2f} \\
&= \frac{a^2(3iA + 2B) \sqrt{a + ia \tan(e + fx)}}{2f} \\
&= \frac{a^2(3iA + 2B) \sqrt{a + ia \tan(e + fx)}}{2f} \\
&= \frac{a^5/2(3iA + 2B) \sqrt{c} \tan^{-1} \left( \frac{\sqrt{c}}{\sqrt{a + ia \tan(e + fx)}} \right)}{f}
\end{aligned}$$

**Mathematica [A]**

time = 3.95, size = 253, normalized size = 1.17

$$\frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) \left( -\frac{i(3A - 2iB) e^{-3i(e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} \text{ArcTan}(e^{i(e+fx)})}{\sqrt{1 + e^{2i(e+fx)}}} + \frac{\sec^{5/2}(e+fx)(i \cos(2e) + \sin(2e))(12A - 8iB + 12(A - iB) \cos(2(e+fx)) + (3iA + 6B) \sin(2(e+fx))) \sqrt{c - ic \tan(e + fx)}}{12(\cos(fx) + i \sin(fx))^2} \right)}{f \sec^{5/2}(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]], x]
```

```
[Out] ((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*((( -I)*(3*A - (2*I)*B)*c*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))])/(E^((3*I)*(e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x))])) + (Sec[e + f*x])^(5/2)*(I*Cos[2*e] + Sin[2*e])*(12*A - (8*I)*B + 12*(A - I*B)*Cos[2*(e + f*x)] +
```

```
((3*I)*A + 6*B)*Sin[2*(e + f*x)]*Sqrt[c - I*c*Tan[e + f*x]]/(12*(Cos[f*x] + I*Sin[f*x])^2))/(f*Sec[e + f*x]^(7/2)*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

**Maple [A]**

time = 0.41, size = 285, normalized size = 1.31

method	result
derivativedivides	$\frac{\sqrt{-c(i \tan (fx + e) - 1)} \sqrt{a(1 + i \tan (fx + e))} a^2 \left( -6iB \ln \left( \frac{ac \tan (fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + i \tan (fx + e))} \right)}{\sqrt{ac}} \right)}{\sqrt{-c(i \tan (fx + e) - 1)} \sqrt{a(1 + i \tan (fx + e))} a^2 \left( -6iB \ln \left( \frac{ac \tan (fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + i \tan (fx + e))} \right)}{\sqrt{ac}} \right)}$
default	$\sqrt{-c(i \tan (fx + e) - 1)} \sqrt{a(1 + i \tan (fx + e))} a^2 \left( -6iB \ln \left( \frac{ac \tan (fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + i \tan (fx + e))} \right)}{\sqrt{ac}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*a^2*(-6*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)))^(1/2))/(a*c)^(1/2))*a*c+6*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-2*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+12*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+9*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)))^(1/2))/(a*c)^(1/2))*a*c-3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+10*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1149 vs.  $2(173) = 346$ .

time = 0.87, size = 1149, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] 6*(12*(5*A - 6*I*B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(3*A - 2*I*B)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(3*A - 2*I*B)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(5*I*A + 6*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*(3*I*A + 2*B)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*(3*I*A + 2*B)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
```

$$\begin{aligned}
& 2*e))) - 6*((3*A - 2*I*B)*a^2*\cos(6*f*x + 6*e) + 3*(3*A - 2*I*B)*a^2*\cos(4* \\
& f*x + 4*e) + 3*(3*A - 2*I*B)*a^2*\cos(2*f*x + 2*e) - (-3*I*A - 2*B)*a^2*\sin( \\
& 6*f*x + 6*e) - 3*(-3*I*A - 2*B)*a^2*\sin(4*f*x + 4*e) - 3*(-3*I*A - 2*B)*a^2 \\
& *\sin(2*f*x + 2*e) + (3*A - 2*I*B)*a^2)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e \\
& ))) + 1) - 6*((3*A - 2*I*B)*a^2*\cos(6*f*x + 6*e) + 3*(3*A - 2*I*B)*a^2*\cos( \\
& 4*f*x + 4*e) + 3*(3*A - 2*I*B)*a^2*\cos(2*f*x + 2*e) - (-3*I*A - 2*B)*a^2*\si \\
& n(6*f*x + 6*e) - 3*(-3*I*A - 2*B)*a^2*\sin(4*f*x + 4*e) - 3*(-3*I*A - 2*B)*a \\
& ^2*\sin(2*f*x + 2*e) + (3*A - 2*I*B)*a^2)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x \\
& + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + \\
& 2*e))) + 1) + 3*((-3*I*A - 2*B)*a^2*\cos(6*f*x + 6*e) + 3*(-3*I*A - 2*B)*a^2 \\
& *\cos(4*f*x + 4*e) + 3*(-3*I*A - 2*B)*a^2*\cos(2*f*x + 2*e) + (3*A - 2*I*B)*a \\
& ^2*\sin(6*f*x + 6*e) + 3*(3*A - 2*I*B)*a^2*\sin(4*f*x + 4*e) + 3*(3*A - 2*I*B \\
& )*a^2*\sin(2*f*x + 2*e) + (-3*I*A - 2*B)*a^2)*\log(\cos(1/2*\arctan2(\sin(2*f*x \\
& + 2*e), \cos(2*f*x + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x \\
& + 2*e)))^2 + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) + \\
& 3*((3*I*A + 2*B)*a^2*\cos(6*f*x + 6*e) + 3*(3*I*A + 2*B)*a^2*\cos(4*f*x + 4*e \\
& ) + 3*(3*I*A + 2*B)*a^2*\cos(2*f*x + 2*e) - (3*A - 2*I*B)*a^2*\sin(6*f*x + 6* \\
& e) - 3*(3*A - 2*I*B)*a^2*\sin(4*f*x + 4*e) - 3*(3*A - 2*I*B)*a^2*\sin(2*f*x + \\
& 2*e) + (3*I*A + 2*B)*a^2)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x \\
& + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 2*\sin \\
& (1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1)*\sqrt{a}*\sqrt{c}/(f* \\
& (-72*I*\cos(6*f*x + 6*e) - 216*I*\cos(4*f*x + 4*e) - 216*I*\cos(2*f*x + 2*e) + \\
& 72*\sin(6*f*x + 6*e) + 216*\sin(4*f*x + 4*e) + 216*\sin(2*f*x + 2*e) - 72*I))
\end{aligned}$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal.  $570$  vs.  $2(173) = 346$ .  
time = 2.33, size = 570, normalized size = 2.63

$$\frac{\sqrt{\frac{3A-2IB-4B^2}{f^2}} \sqrt{\frac{c}{e^{2Ifx+2Ie}+1}} \log\left(\frac{-4(2((-3IA-2B)a^2e^{3Ifx+3Ie} + (-3IA-2B)a^2e^{Ifx+Ie}))\sqrt{a/(e^{2Ifx+2Ie}+1)}}{e^{2Ifx+2Ie}+1}\right) + \sqrt{\frac{3A-2IB-4B^2}{f^2}} \sqrt{\frac{c}{e^{2Ifx+2Ie}+1}} \sqrt{\frac{a}{e^{2Ifx+2Ie}+1}} + \sqrt{\frac{3A-2IB-4B^2}{f^2}} \sqrt{\frac{c}{e^{2Ifx+2Ie}+1}} \sqrt{\frac{a}{e^{2Ifx+2Ie}+1}}}{\sqrt{\frac{3A-2IB-4B^2}{f^2}} \sqrt{\frac{c}{e^{2Ifx+2Ie}+1}} \sqrt{\frac{a}{e^{2Ifx+2Ie}+1}} + \sqrt{\frac{3A-2IB-4B^2}{f^2}} \sqrt{\frac{c}{e^{2Ifx+2Ie}+1}} \sqrt{\frac{a}{e^{2Ifx+2Ie}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** `integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)),x, algorithm="fricas")`

**[Out]** 
$$\begin{aligned}
& -1/12*(3*\sqrt{(9*A^2 - 12*I*A*B - 4*B^2)}*a^5*c/f^2)*(f*e^{(4*I*f*x + 4*I*e)} \\
& + 2*f*e^{(2*I*f*x + 2*I*e)} + f)*\log(-4*(2*((-3*I*A - 2*B)*a^2*e^{(3*I*f*x + 3 \\
& *I*e)} + (-3*I*A - 2*B)*a^2*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1 \\
& )}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + \sqrt{(9*A^2 - 12*I*A*B - 4*B^2)}*a^5*c/f^2 \\
& *(f*e^{(2*I*f*x + 2*I*e)} - f))/((3*I*A + 2*B)*a^2*e^{(2*I*f*x + 2*I*e)} \\
& + (3*I*A + 2*B)*a^2) - 3*\sqrt{(9*A^2 - 12*I*A*B - 4*B^2)}*a^5*c/f^2*(f*e^{(4 \\
& *I*f*x + 4*I*e)} + 2*f*e^{(2*I*f*x + 2*I*e)} + f)*\log(-4*(2*((-3*I*A - 2*B)*a \\
& ^2*e^{(3*I*f*x + 3*I*e)} + (-3*I*A - 2*B)*a^2*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I \\
& *f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - \sqrt{(9*A^2 - 12*I* \\
& A*B - 4*B^2)}*a^5*c/f^2*(f*e^{(2*I*f*x + 2*I*e)} - f))/((3*I*A + 2*B)*a^2*e^{(
\end{aligned}$$

$$2*I*f*x + 2*I*e) + (3*I*A + 2*B)*a^2)) + 4*(3*(-5*I*A - 6*B)*a^2*e^(5*I*f*x + 5*I*e) + 8*(-3*I*A - 2*B)*a^2*e^(3*I*f*x + 3*I*e) + 3*(-3*I*A - 2*B)*a^2 * e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e + fx) - i))^{\frac{5}{2}} \sqrt{-ic(\tan(e + fx) + i)} (A + B \tan(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(a+I\*a\*tan(f\*x+e))\*\*(5/2)\*(A+B\*tan(f\*x+e)),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))\*\*(5/2)\*sqrt(-I\*c\*(tan(e + f\*x) + I))\*(A + B\*tan(e + f\*x)), x)

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + fx)) (a + a \tan(e + fx) 1i)^{5/2} \sqrt{c - c \tan(e + fx) 1i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2), x)

$$3.809 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{\sqrt{c-ictan(e+fx)}} dx$$

**Optimal.** Leaf size=227

$$\frac{3a^{5/2}(2iA+3B)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{f\sqrt{c-ictan(e+fx)}} - \frac{3a^2(2iA+3B)}{f\sqrt{c-ictan(e+fx)}}$$

[Out]  $3a^{5/2}(2iA+3B)\text{arctan}(c^{1/2}(a+Ia*\tan(f*x+e))^{1/2}/a^{1/2}/(c-I*c*\tan(f*x+e))^{1/2})/f/c^{1/2}-3/2*a^2*(2iA+3B)*(a+Ia*\tan(f*x+e))^{1/2}*(c-I*c*\tan(f*x+e))^{1/2}/c/f-1/2*a*(2iA+3B)*(c-I*c*\tan(f*x+e))^{1/2}*(a+Ia*\tan(f*x+e))^{3/2}/c/f-(iA+B)*(a+Ia*\tan(f*x+e))^{5/2}/f/(c-I*c*\tan(f*x+e))^{1/2}$

**Rubi [A]**

time = 0.20, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 52, 65, 223, 209}

$$\frac{3a^{5/2}(3B+2iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{\sqrt{c}f} - \frac{3a^2(3B+2iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ictan(e+fx)}}{2cf} - \frac{a(3B+2iA)(a+ia \tan(e+fx))^{3/2}\sqrt{c-ictan(e+fx)}}{2cf} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{f\sqrt{c-ictan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+Ia*\text{Tan}[e+fx])^{5/2}*(A+B*\text{Tan}[e+fx])/ \text{Sqrt}[c-I*c*\text{Tan}[e+fx]], x]$

[Out]  $(3a^{5/2}*((2i)*A+3B)*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a+Ia*\text{Tan}[e+fx]])/(\text{Sqrt}[a]*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])])/( \text{Sqrt}[c]*f) - ((iA+B)*(a+Ia*\text{Tan}[e+fx])^{5/2})/(f*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]]) - (3a^2*((2i)*A+3B)*\text{Sqrt}[a+Ia*\text{Tan}[e+fx]*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])/(2*c*f) - (a*((2i)*A+3B)*(a+Ia*\text{Tan}[e+fx])^{3/2}*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])/(2*c*f)$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m+n+1))), \text{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) +$

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{\sqrt{c - ictan(e + fx)}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{3/2} (A+Bx)}{(c-icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{(a(2A - 3iB))S}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{a(2iA + 3B)(a - ia \tan(e + fx))^{5/2}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{3a^2(2iA + 3B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{3a^2(2iA + 3B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{3a^2(2iA + 3B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= \frac{3a^{5/2}(2iA + 3B) \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right)}{\sqrt{c} f}
\end{aligned}$$

**Mathematica [A]**

time = 4.72, size = 239, normalized size = 1.05

$$\frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) \left( \frac{3(2iA + 3B)e^{-3i(e + fx)} \sqrt{\frac{e^{i(e + fx)}}{1 + e^{2i(e + fx)}}} \text{ArcTan}(e^{i(e + fx)})}{\frac{c}{\sqrt{1 + e^{2i(e + fx)}}}} - \frac{\sqrt{\sec(e + fx)} (5(2A - 3iB) + (10A - 13iB) \cos(2(e + fx)) + (-2iA - 5B) \sin(2(e + fx)))}{4} \sqrt{c - ictan(e + fx)}}{f \sec^{\frac{7}{2}}(e + fx) (A \cos(e + fx) + B \sin(e + fx))} \right)}{f \sec^{\frac{7}{2}}(e + fx) (A \cos(e + fx) + B \sin(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e + f*x]],x]
```

```
[Out] ((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*((3*((2*I)*A + 3*B)*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))])/(E^((3*I)*(e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x))])) - (Sqrt[Sec[e + f*x]]*(5*(2*A - (3*I)*B) + (10*A - (13*I)*B)*Cos[2*(e + f*x)] + ((-2*I)*A - 5*B)*Sin
```

$[2*(e + f*x)]*(I + \tan[e + f*x])*Sqrt[c - I*c*\tan[e + f*x]]/(4*c))/((f*Se$   
 $c[e + f*x]^{(7/2)}*(A*\cos[e + f*x] + B*\sin[e + f*x]))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(187) = 374.

time = 0.42, size = 565, normalized size = 2.49

method	result
derivativedivides	$i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac(1+tan(fx+e))}}{\sqrt{ac}}\right)\right)$
default	$i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac(1+tan(fx+e))}}{\sqrt{ac}}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{2}I/f*(a*(1+I*\tan(f*x+e)))^{(1/2)}*(-c*(I*\tan(f*x+e)-1))^{(1/2)}*a^2/c*(6*I*A*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c*\tan(f*x+e)^2+18*I*B*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c*\tan(f*x+e)+9*B*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c*\tan(f*x+e)^2+4*I*B*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^2-B*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^3-6*I*A*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c-12*A*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c*\tan(f*x+e)-12*I*A*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)-2*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^2-9*B*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c-14*I*B*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}-19*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)+10*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)})/(a*c*(1+\tan(f*x+e)^2))^{(1/2)}/(I+\tan(f*x+e))^{(1/2)}/(a*c)^{(1/2)}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1049 vs. 2(183) = 366.

time = 0.73, size = 1049, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,algorithm="maxima")`



```
*I*e) + c*f)*log(4*(2*((-2*I*A - 3*B)*a^2*e^(3*I*f*x + 3*I*e) + (-2*I*A - 3*B)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((4*A^2 - 12*I*A*B - 9*B^2)*a^5/(c*f^2))*(c*f*e^(2*I*f*x + 2*I*e) - c*f))/((-2*I*A - 3*B)*a^2*e^(2*I*f*x + 2*I*e) + (-2*I*A - 3*B)*a^2)) - 4*(4*(I*A + B)*a^2*e^(5*I*f*x + 5*I*e) + 5*(2*I*A + 3*B)*a^2*e^(3*I*f*x + 3*I*e) + 3*(2*I*A + 3*B)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e + fx) - i))^{\frac{5}{2}}(A + B \tan(e + fx))}{\sqrt{-ic(\tan(e + fx) + i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(e + f*x) - I))**(5/2)*(A + B*tan(e + f*x))/sqrt(-I*c*(tan(e + f*x) + I)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)/sqrt(-I*c*tan(f*x + e) + c), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + fx)) (a + a \tan(e + fx) li)^{5/2}}{\sqrt{c - c \tan(e + fx) li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(5/2))/(c - c*tan(e + f*x)*li)^(1/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(5/2))/(c - c*tan(e + f*x)*li)^(1/2), x)
```

$$3.810 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=226

$$\frac{2a^{5/2}(iA+4B)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{c^{3/2}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{3f(c-ic \tan(e+fx))^{3/2}} + \frac{2a(iA+4B)(a+ia \tan(e+fx))^{5/2}}{3cf\sqrt{c-ic \tan(e+fx)}}$$

[Out]  $-2*a^{(5/2)}*(I*A+4*B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/c^{(3/2)}/f+a^{(5/2)}*(I*A+4*B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)})/c^{(3/2)}/f+2/3*a*(I*A+4*B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(3/2)}/c/f/(c-I*c*\tan(f*x+e))^{(1/2)})-1/3*(I*A+B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(5/2)}/f/(c-I*c*\tan(f*x+e))^{(3/2)})$

**Rubi [A]**

time = 0.20, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ ,

Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{2a^{5/2}(4B+iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2(4B+iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{c^2f} + \frac{2a(4B+iA)(a+ia \tan(e+fx))^{3/2}}{3cf\sqrt{c-ic \tan(e+fx)}} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{3f(c-ic \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+I*a*\text{Tan}[e+f*x])^{(5/2)}*(A+B*\text{Tan}[e+f*x])]/(c-I*c*\text{Tan}[e+f*x])^{(3/2)},x]$

[Out]  $(-2*a^{(5/2)}*(I*A+4*B)*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a+I*a*\text{Tan}[e+f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])])/c^{(3/2)}*f - ((I*A+B)*(a+I*a*\text{Tan}[e+f*x])^{(5/2)})/(3*f*(c-I*c*\text{Tan}[e+f*x])^{(3/2)}) + (2*a*(I*A+4*B)*(a+I*a*\text{Tan}[e+f*x])^{(3/2)})/(3*c*f*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]) + (a^{(5/2)}*(I*A+4*B)*\text{Sqrt}[a+I*a*\text{Tan}[e+f*x]]*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])/c^{(3/2)}*f$

**Rule 49**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \text{Dist}[d*(n/(b*(m+1))), \text{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1] \&\& !(IntegerQ[n] \&\& !IntegerQ[m]) \&\& !(IntegerQ[m+n+2, 0] \&\& (FractionQ[m] || GeQ[2*n+m+1, 0])) \& \& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \text{Dist}[n*((b*c - a*d)/($

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || (EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n])))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{3/2} (A+Bx)}{(c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{3f(c - ict \tan(e + fx))^{3/2}} - \frac{(a(A - 4iB)) \text{Su}}{3cf \sqrt{c - i}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2a(iA + 4B)(a - i)}{3cf \sqrt{c - i}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2a(iA + 4B)(a - i)}{3cf \sqrt{c - i}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2a(iA + 4B)(a - i)}{3cf \sqrt{c - i}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{3f(c - ict \tan(e + fx))^{3/2}} + \frac{2a(iA + 4B)(a - i)}{3cf \sqrt{c - i}} \\
&= -\frac{2a^{5/2}(iA + 4B) \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ict \tan(e + fx)}} \right)}{c^{3/2} f}
\end{aligned}$$

**Mathematica [A]**

time = 4.91, size = 227, normalized size = 1.00

$$\frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx)) \left( -\frac{6i(A-4iB)a^{-3i(e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \text{ArcTan}(e^{i(e+fx)})}{\sqrt{1+e^{2i(e+fx)}}} + \sqrt{\sec(e+fx)} (2iA+8B+(2iA+11B)\cos(2(e+fx))+(4A-13iB)\sin(2(e+fx))) \sqrt{c-ict \tan(e+fx)} \right)}{3c^2 f \sec^{\frac{3}{2}}(e+fx)(A \cos(e+fx) + B \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^(5/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(3/2), x]

[Out] ((a + I\*a\*Tan[e + f\*x])^(5/2)\*(A + B\*Tan[e + f\*x])\*((( -6\*I)\*(A - (4\*I)\*B)\*c \*Sqrt[E^(I\*(e + f\*x))/(1 + E^((2\*I)\*(e + f\*x))]]\*ArcTan[E^(I\*(e + f\*x))])/(E^((3\*I)\*(e + f\*x))\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x))])) + Sqrt[Sec[e + f\*x]] \*((2\*I)\*A + 8\*B + ((2\*I)\*A + 11\*B)\*Cos[2\*(e + f\*x)] + (4\*A - (13\*I)\*B)\*Sin[

$2*(e + f*x)])*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(3*c^2*f*\text{Sec}[e + f*x]^(7/2)*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs.  $2(186) = 372$ .

time = 0.42, size = 667, normalized size = 2.95

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 \left( -12iB \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \frac{\sqrt{ac}(1+i\tan(fx+e))}{\sqrt{ac}} \right) \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 \left( -12iB \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \frac{\sqrt{ac}(1+i\tan(fx+e))}{\sqrt{ac}} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{3} f (a(1+i \tan(fx+e)))^{1/2} (-c(i \tan(fx+e)-1))^{1/2} a^2/c^2 (-12i B \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac \tan(fx+e)^3 + 9i A \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac \tan(fx+e)^2 + 3A \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac \tan(fx+e)^3 + 36i B \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac \tan(fx+e)^2 + 29i B (ac(1+\tan(fx+e)^2))^{1/2} (ac)^{1/2} \tan(fx+e)^2 + 36B \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac \tan(fx+e)^2 + 3B (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2} \tan(fx+e)^3 - 3i A \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac - 12i A (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2} \tan(fx+e) - 9A \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac \tan(fx+e) - 8A (ac(1+\tan(fx+e)^2))^{1/2} (ac)^{1/2} \tan(fx+e)^2 - 19i B (ac(1+\tan(fx+e)^2))^{1/2} (ac)^{1/2} - 12B \ln((ac \tan(fx+e) + (ac)^{1/2} (ac(1+\tan(fx+e)^2))^{1/2})/(ac)^{1/2})) + ac - 45B (ac(1+\tan(fx+e)^2))^{1/2} (ac)^{1/2} \tan(fx+e) + 4A (ac(1+\tan(fx+e)^2))^{1/2} (ac)^{1/2} / (ac(1+\tan(fx+e)^2))^{1/2} / (ac)^{1/2} / (I + \tan(fx+e))^3$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 841 vs.  $2(184) = 368$ .

time = 0.65, size = 841, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] 
$$-3*(6*((A - 4*I*B)*a^2*\cos(2*f*x + 2*e) - (-I*A - 4*B)*a^2*\sin(2*f*x + 2*e) + (A - 4*I*B)*a^2)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 6*((A - 4*I*B)*a^2*\cos(2*f*x + 2*e) - (-I*A - 4*B)*a^2*\sin(2*f*x + 2*e) + (A - 4*I*B)*a^2)*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 4*((A - I*B)*a^2*\cos(2*f*x + 2*e) - (-I*A - B)*a^2*\sin(2*f*x + 2*e) + (A - I*B)*a^2)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*((A - 3*I*B)*a^2*\cos(2*f*x + 2*e) + (I*A + 3*B)*a^2*\sin(2*f*x + 2*e) + (A - 4*I*B)*a^2)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 3*((-I*A - 4*B)*a^2*\cos(2*f*x + 2*e) + (A - 4*I*B)*a^2*\sin(2*f*x + 2*e) + (-I*A - 4*B)*a^2)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) - 3*((I*A + 4*B)*a^2*\cos(2*f*x + 2*e) - (A - 4*I*B)*a^2*\sin(2*f*x + 2*e) + (I*A + 4*B)*a^2)*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) - 4*((-I*A - B)*a^2*\cos(2*f*x + 2*e) + (A - I*B)*a^2*\sin(2*f*x + 2*e) + (-I*A - B)*a^2)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*((I*A + 3*B)*a^2*\cos(2*f*x + 2*e) - (A - 3*I*B)*a^2*\sin(2*f*x + 2*e) + (I*A + 4*B)*a^2)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sqrt{a}*\sqrt{c}/((-18*I*c^2*\cos(2*f*x + 2*e) + 18*c^2*\sin(2*f*x + 2*e) - 18*I*c^2)*f)$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs.  $2(184) = 368$ .

time = 4.76, size = 510, normalized size = 2.26

$$\frac{\sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \operatorname{Im} \left( \frac{\sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}}}{\sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}}} \right) - \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \operatorname{Re} \left( \frac{\sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}}}{\sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}}} \right) + 4(A + B)\sqrt{a^5} + 2(-A - 4B)\sqrt{a^5} + 3(-A - 4B)\sqrt{a^5} + 2(-A - 4B)\sqrt{a^5}}{\sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}} \sqrt{\frac{A^2 - 8AB - 16B^2}{c^3 f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] 
$$-1/6*(3*c^2*\sqrt{(A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2)}*f*\log(-4*(2*((-I*A - 4*B)*a^2*e^{(3*I*f*x + 3*I*e)} + (-I*A - 4*B)*a^2*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + (c^2*f*e^{(2*I*f*x + 2*I*e)} - c^2*f)*\sqrt{(A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2)}))/((I*A + 4*B)*a^2*e^{(2*I*f*x + 2*I*e)} + (I*A + 4*B)*a^2) - 3*c^2*\sqrt{(A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2)}*f*\log(-4*(2*((-I*A - 4*B)*a^2*e^{(3*I*f*x + 3*I*e)} + (-I*A - 4*B)*a^2*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - (c^2*f*e^{(2*I*f*x + 2*I*e)} - c^2*f)*\sqrt{(A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2)}))/((I*A + 4*B)*a^2*e^{(2*I*f*x + 2*I*e)} + (I*A + 4*B)*a^2) + 4*((I*A + B)*a^2*e^{(5*I*f*x + 5*I*e)} + 2*(-I*A -$$

$4*B)*a^2*e^{(3*I*f*x + 3*I*e)} + 3*(-I*A - 4*B)*a^2*e^{(I*f*x + I*e)})*\text{sqrt}(a/(\text{e}^{(2*I*f*x + 2*I*e)} + 1))*\text{sqrt}(c/(\text{e}^{(2*I*f*x + 2*I*e)} + 1)))/(c^2*f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e + fx) - i))^{\frac{5}{2}}(A + B \tan(e + fx))}{(-ic(\tan(e + fx) + i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))\*\*(5/2)\*(A + B\*tan(e + f\*x))/(-I\*c\*(tan(e + f\*x) + I))\*\*(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(5/2)/(-I\*c\*tan(f\*x + e) + c)^(3/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + fx)) (a + a \tan(e + fx) 1i)^{5/2}}{(c - c \tan(e + fx) 1i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(3/2),x)

[Out] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(3/2), x)

$$3.811 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=203

$$\frac{2a^{5/2}B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ic \tan(e+fx)}}\right)}{c^{5/2}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{5f(c-ic \tan(e+fx))^{5/2}} + \frac{2aB(a+ia \tan(e+fx))^3}{3cf(c-ic \tan(e+fx))^3}$$

[Out]  $2*a^{5/2}*B*\arctan(c^{1/2}*(a+I*a*\tan(f*x+e))^{1/2}/a^{1/2}/(c-I*c*\tan(f*x+e))^{1/2})/c^{5/2}/f-2*a^2*B*(a+I*a*\tan(f*x+e))^{1/2}/c^2/f/(c-I*c*\tan(f*x+e))^{1/2}-1/5*(I*A+B)*(a+I*a*\tan(f*x+e))^{5/2}/f/(c-I*c*\tan(f*x+e))^{5/2}+2/3*a*B*(a+I*a*\tan(f*x+e))^{3/2}/c/f/(c-I*c*\tan(f*x+e))^{3/2}$

**Rubi [A]**

time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 49, 65, 223, 209}

$$\frac{2a^{5/2}B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ic \tan(e+fx)}}\right)}{c^{5/2}f} - \frac{2a^2B \sqrt{a+ia \tan(e+fx)}}{c^2f \sqrt{c-ic \tan(e+fx)}} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{5f(c-ic \tan(e+fx))^{5/2}} + \frac{2aB(a+ia \tan(e+fx))^{3/2}}{3cf(c-ic \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\tan(e+fx))^{5/2}*(A+B*\tan(e+fx))]/(c-I*c*\tan(e+fx))^{5/2}, x]$

[Out]  $(2*a^{5/2}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a+I*a*\tan(e+fx)])]/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-I*c*\tan(e+fx)])))/(c^{5/2}*f) - ((I*A+B)*(a+I*a*\tan(e+fx))^{5/2})/(5*f*(c-I*c*\tan(e+fx))^{5/2}) + (2*a*B*(a+I*a*\tan(e+fx))^{3/2})/(3*c*f*(c-I*c*\tan(e+fx))^{3/2}) - (2*a^2*B*\operatorname{Sqrt}[a+I*a*\tan(e+fx)])/(c^2*f*\operatorname{Sqrt}[c-I*c*\tan(e+fx)])$

**Rule 49**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{IntegerQ}[n] \&\& !\operatorname{IntegerQ}[m]) \&\& !(I\operatorname{LeQ}[m+n+2, 0] \&\& (\operatorname{FractionQ}[m] \mid\mid \operatorname{GeQ}[2*n+m+1, 0])) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}$

`[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 79

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))`

### Rule 209

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

### Rule 223

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

### Rule 3669

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]`

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{3/2} (A+Bx)}{(c-icx)^{7/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}} + \frac{(iaB) \text{Subst} \left( \int \frac{1}{(c-icx)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}} + \frac{2aB(a + ia \tan(e + fx))^{5/2}}{3cf(c - ictan(e + fx))^{5/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}} + \frac{2aB(a + ia \tan(e + fx))^{5/2}}{3cf(c - ictan(e + fx))^{5/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}} + \frac{2aB(a + ia \tan(e + fx))^{5/2}}{3cf(c - ictan(e + fx))^{5/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}} + \frac{2aB(a + ia \tan(e + fx))^{5/2}}{3cf(c - ictan(e + fx))^{5/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}} + \frac{2aB(a + ia \tan(e + fx))^{5/2}}{3cf(c - ictan(e + fx))^{5/2}} \\
&= \frac{2a^{5/2} B \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right)}{c^{5/2} f} - \frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{5f(c - ictan(e + fx))^{5/2}}
\end{aligned}$$

### Mathematica [A]

time = 6.52, size = 203, normalized size = 1.00

$$\frac{a^2 \cos^2(e + fx) (\cos(\frac{1}{3}(e - 2fx)) - i \sin(\frac{1}{3}(e - 2fx))) (\cos(\frac{1}{3}(e - 2fx)) + i \sin(\frac{1}{3}(e - 2fx))) (-10B + (3IA + 33B) \cos(2(e + fx)) - 3A \sin(2(e + fx)) - 27iB \sin(2(e + fx)) - 30B \text{ArcTan}(e^{i(e+fx)})) (\cos(3(e + fx)) - i \sin(3(e + fx))) (-i + \tan(e + fx))^2 \sqrt{a + ia \tan(e + fx)}}{15c^2 f \sqrt{c - ictan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^(5/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(5/2), x]

[Out] (a^2\*Cos[e + f\*x]^2\*(Cos[(e - 2\*f\*x)/2] - I\*Sin[(e - 2\*f\*x)/2])\*(Cos[(e - 2\*f\*x)/2] + I\*Sin[(e - 2\*f\*x)/2])\*(-10\*B + ((3\*I)\*A + 33\*B)\*Cos[2\*(e + f\*x)] - 3\*A\*Sin[2\*(e + f\*x)] - (27\*I)\*B\*Sin[2\*(e + f\*x)] - 30\*B\*ArcTan[E^(I\*(e + f\*x))]\*(Cos[3\*(e + f\*x)] - I\*Sin[3\*(e + f\*x)]))\*(-I + Tan[e + f\*x])^2\*Sqrt[a + I\*a\*Tan[e + f\*x]]/(15\*c^2\*f\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(166) = 332.

time = 0.41, size = 555, normalized size = 2.73

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 \left( -15iB \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{\frac{ac(1+i\tan(fx+e))}{ac}} \right) \right)}{1}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^2 \left( -15iB \ln \left( \frac{ac \tan(fx+e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{\frac{ac(1+i\tan(fx+e))}{ac}} \right) \right)}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^3*(-15*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4+90*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+43*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+60*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+3*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-15*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-77*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-60*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-97*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+3*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+23*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*x+e))^4/(a*c)^(1/2)
```

**Maxima [A]**

time = 0.61, size = 230, normalized size = 1.13

$\frac{30 B^2 \arctan(\cos(fx+e), \sin(fx+e)+1) + 30 B^2 \arctan(\cos(fx+e), -\sin(fx+e)+1) - 6(IA+B)a^2 \cos(5fx+5e) + 20 B^2 \cos(3fx+3e) - 60 B^2 \cos(fx+e) + 15 I B a^2 \log(\cos(fx+e)^2 + \sin(fx+e)^2 + 2 \sin(fx+e)+1) - 15 I B a^2 \log(\cos(fx+e)^2 + \sin(fx+e)^2 - 2 \sin(fx+e)+1) + 6(A-IB)a^2 \sin(5fx+5e) + 20 B^2 \sin(3fx+3e) - 60 B^2 \sin(fx+e) \sqrt{c}}{30 f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(30*B*a^2*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 30*B*a^2*arctan2(cos(f*x + e), -sin(f*x + e) + 1) - 6*(I*A + B)*a^2*cos(5*f*x + 5*e) + 20*B*a^2*cos(3*f*x + 3*e) - 60*B*a^2*cos(f*x + e) + 15*I*B*a^2*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 15*I*B*a^2*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + 6*(A - I*B)*a^2*sin(5*f*x + 5*e) + 20*I*B*a^2*sin(3*f*x + 3*e) - 60*I*B*a^2*sin(f*x + e))*sqrt(a)/(c^(5/2)*f)
```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 436 vs.  $2(167) = 334$ .  
time = 3.08, size = 436, normalized size = 2.15

$$\frac{15ef\sqrt{\frac{Bc^3}{2f^2}} \log\left(\frac{\sqrt{\frac{a}{2B^2f^2+1}} \sqrt{\frac{c}{2B^2f^2+1}} \sqrt{\frac{Bc^3}{2f^2}}}{\sqrt{\frac{a}{2B^2f^2+1}} \sqrt{\frac{c}{2B^2f^2+1}} \sqrt{\frac{Bc^3}{2f^2}}}\right) - 15ef\sqrt{\frac{Bc^3}{2f^2}} \log\left(\frac{\sqrt{\frac{a}{2B^2f^2+1}} \sqrt{\frac{c}{2B^2f^2+1}} \sqrt{\frac{Bc^3}{2f^2}}}{\sqrt{\frac{a}{2B^2f^2+1}} \sqrt{\frac{c}{2B^2f^2+1}} \sqrt{\frac{Bc^3}{2f^2}}}\right) + 2(3(A+B)c^{2f+2} + (3A-7B)c^{2f+1} + 20Bc^{2f} + 30Bc^{2f+1}) \sqrt{\frac{a}{2B^2f^2+1}} \sqrt{\frac{c}{2B^2f^2+1}}}{30ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out]  $-1/30*(15*c^3*f*\sqrt{-B^2*a^5/(c^5*f^2)}*\log(4*(2*(B*a^2*e^{(3*I*f*x + 3*I*e)} + B*a^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + (c^3*f*e^{(2*I*f*x + 2*I*e)} - c^3*f)*\sqrt{-B^2*a^5/(c^5*f^2)}))/(B*a^2*e^{(2*I*f*x + 2*I*e)} + B*a^2) - 15*c^3*f*\sqrt{-B^2*a^5/(c^5*f^2)}*\log(4*(2*(B*a^2*e^{(3*I*f*x + 3*I*e)} + B*a^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - (c^3*f*e^{(2*I*f*x + 2*I*e)} - c^3*f)*\sqrt{-B^2*a^5/(c^5*f^2)}))/(B*a^2*e^{(2*I*f*x + 2*I*e)} + B*a^2) + 2*(3*(I*A + B)*a^2*e^{(7*I*f*x + 7*I*e)} + (3*I*A - 7*B)*a^2*e^{(5*I*f*x + 5*I*e)} + 20*B*a^2*e^{(3*I*f*x + 3*I*e)} + 30*B*a^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)))/(c^3*f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(e + fx) - i))^{\frac{5}{2}} (A + B \tan(e + fx))}{(-ic(\tan(e + fx) + i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))^(5/2)\*(A + B\*tan(e + f\*x))/(-I\*c\*(tan(e + f\*x) + I))^(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(5/2)/(-I\*c\*tan(f\*x + e) + c)^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{5/2}}{(c - c \tan(e + f x) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(5/2), x)



$$3.812 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$$

Optimal. Leaf size=102

$$\frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{7f(c-ic \tan(e+fx))^{7/2}} - \frac{(iA-6B)(a+ia \tan(e+fx))^{5/2}}{35cf(c-ic \tan(e+fx))^{5/2}}$$

[Out]  $-1/7*(I*A+B)*(a+I*a*\tan(f*x+e))^{(5/2)}/f/(c-I*c*\tan(f*x+e))^{(7/2)}-1/35*(I*A-6*B)*(a+I*a*\tan(f*x+e))^{(5/2)}/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}$

Rubi [A]

time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ ,

Rules used = {3669, 79, 37}

$$\frac{(-6B+iA)(a+ia \tan(e+fx))^{5/2}}{35cf(c-ic \tan(e+fx))^{5/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{7f(c-ic \tan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $-1/7*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(5/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) - (((I*A - 6*B)*(a + I*a*\text{Tan}[e + f*x])^{(5/2)})/(35*c*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}))$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

Rule 79

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)}/(f*(p + 1)*(c*f - d*e))), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{LtQ}[p, -1] \&\& (!\text{LtQ}[n, -1] \|\ \text{IntegerQ}[p] \|\ !(\text{IntegerQ}[n] \|\ !(\text{EqQ}[e, 0] \|\ !(\text{EqQ}[c, 0] \|\ \text{LtQ}[p, n])))$

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{7/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{3/2}(A+Bx)}{(c-icx)^{9/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{7f(c - ictan(e + fx))^{7/2}} + \frac{(a(A + 6iB)) \text{Subst}\left(\int \frac{1}{(c-icx)^{9/2}} dx, x, \tan(e + fx)\right)}{35cf(c - ictan(e + fx))^{7/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{7f(c - ictan(e + fx))^{7/2}} - \frac{(iA - 6B)(a + ia \tan(e + fx))^{5/2}}{35cf(c - ictan(e + fx))^{7/2}}$$

Mathematica [A]

time = 6.41, size = 121, normalized size = 1.19

$$\frac{a^2 \cos(e + fx)((-6iA + B) \cos(e + fx) - (A + 6iB) \sin(e + fx))(\cos(6e + 8fx) + i \sin(6e + 8fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{35c^4 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(7/2), x]
```

```
[Out] (a^2 * Cos[e + f*x] * (((-6*I)*A + B) * Cos[e + f*x] - (A + (6*I)*B) * Sin[e + f*x]) * (Cos[6*e + 8*f*x] + I * Sin[6*e + 8*f*x]) * Sqrt[a + I*a*Tan[e + f*x]] * Sqrt[c - I*c*Tan[e + f*x]]) / (35*c^4*f*(Cos[f*x] + I * Sin[f*x])^2)
```

Maple [A]

time = 0.45, size = 115, normalized size = 1.13

method	result
risch	$-\frac{a^2 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)} + 1}} (5iA e^{6i(fx+e)} + 5B e^{6i(fx+e)} + 7iA e^{4i(fx+e)} - 7B e^{4i(fx+e)})}{70c^3 \sqrt{\frac{c}{e^{2i(fx+e)} + 1}} f}$
derivativedivides	$-\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} a^2 (1 + \tan^2(fx + e)) (iA(\tan^2(fx + e)) + 5iB \tan(fx + e))}{35f c^4 (i + \tan(fx + e))^5}$
default	$-\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} a^2 (1 + \tan^2(fx + e)) (iA(\tan^2(fx + e)) + 5iB \tan(fx + e))}{35f c^4 (i + \tan(fx + e))^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/35*I/f*(a*(1+I*\tan(f*x+e)))^{1/2}*(-c*(I*\tan(f*x+e)-1))^{1/2}*a^2/c^4*(1+\tan(f*x+e)^2)*(I*A*\tan(f*x+e)^2+5*I*B*\tan(f*x+e)-6*B*\tan(f*x+e)^2+6*I*A-5*A*\tan(f*x+e)-B)/(I+\tan(f*x+e))^5$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs.  $2(82) = 164$ .  
time = 0.62, size = 174, normalized size = 1.71

$$\frac{-70(5(A-iB)a^2\cos(9fx+9e)+2(6A+iB)a^2\cos(7fx+7e)+7(A+iB)a^2\cos(5fx+5e)-5(-iA-B)a^2\sin(9fx+9e)-2(-6iA+B)a^2\sin(7fx+7e)-7(-iA+B)a^2\sin(5fx+5e))\sqrt{a}\sqrt{c}}{-4900(i^2c^2\cos(2fx+2e)-c^2\sin(2fx+2e)+i^2c^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,algorithm="maxima")`

[Out] 
$$-70*(5*(A-I*B)*a^2*\cos(9*f*x+9*e)+2*(6*A+I*B)*a^2*\cos(7*f*x+7*e)+7*(A+I*B)*a^2*\cos(5*f*x+5*e)-5*(-I*A-B)*a^2*\sin(9*f*x+9*e)-2*(-6*I*A+B)*a^2*\sin(7*f*x+7*e)-7*(-I*A+B)*a^2*\sin(5*f*x+5*e))*\sqrt{a}*\sqrt{c}/((-4900*I*c^4*\cos(2*f*x+2*e)+4900*c^4*\sin(2*f*x+2*e)-4900*I*c^4)*f)$$

**Fricas** [A]

time = 5.73, size = 109, normalized size = 1.07

$$\frac{(5(iA+B)a^2e^{(9i fx+9ie)}+2(6iA-B)a^2e^{(7i fx+7ie)}+7(iA-B)a^2e^{(5i fx+5ie)})\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}}{70c^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,algorithm="fricas")`

[Out] 
$$-1/70*(5*(I*A+B)*a^2*e^{(9*I*f*x+9*I*e)}+2*(6*I*A-B)*a^2*e^{(7*I*f*x+7*I*e)}+7*(I*A-B)*a^2*e^{(5*I*f*x+5*I*e)})*\sqrt{a/(e^{(2*I*f*x+2*I*e)}+1)}*\sqrt{c/(e^{(2*I*f*x+2*I*e)}+1)}/(c^4*f)$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(7/2),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(5/2)/(-I\*c\*tan(f\*x + e) + c)^(7/2), x)

**Mupad [B]**

time = 10.28, size = 192, normalized size = 1.88

$$\frac{a^2 \sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)i)}{\cos(2e+2fx)+1}} (A \cos(4e+4fx) 7i + A \cos(6e+6fx) 5i - 7B \cos(4e+4fx) + 5B \cos(6e+6fx) - 7A \sin(4e+4fx) - 5A \sin(6e+6fx) - B \sin(4e+4fx) 7i + B \sin(6e+6fx) 5i)}{70c^3 f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] -(a^2\*((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*cos(4\*e + 4\*f\*x)\*7i + A\*cos(6\*e + 6\*f\*x)\*5i - 7\*B\*cos(4\*e + 4\*f\*x) + 5\*B\*cos(6\*e + 6\*f\*x) - 7\*A\*sin(4\*e + 4\*f\*x) - 5\*A\*sin(6\*e + 6\*f\*x) - B\*sin(4\*e + 4\*f\*x)\*7i + B\*sin(6\*e + 6\*f\*x)\*5i))/(70\*c^3\*f\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2))

$$3.813 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ictan(e+fx))^{9/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{9f(c-ictan(e+fx))^{9/2}} - \frac{(2iA-7B)(a+ia \tan(e+fx))^{5/2}}{63cf(c-ictan(e+fx))^{7/2}} - \frac{(2iA-7B)(a+ia \tan(e+fx))^{5/2}}{315c^2f(c-ictan(e+fx))^{5/2}}$$

[Out]  $-1/9*(I*A+B)*(a+I*a*\tan(f*x+e))^{(5/2)}/f/(c-I*c*\tan(f*x+e))^{(9/2)}-1/63*(2*I*A-7*B)*(a+I*a*\tan(f*x+e))^{(5/2)}/c/f/(c-I*c*\tan(f*x+e))^{(7/2)}-1/315*(2*I*A-7*B)*(a+I*a*\tan(f*x+e))^{(5/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(5/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$-\frac{(-7B+2iA)(a+ia \tan(e+fx))^{5/2}}{315c^2f(c-ictan(e+fx))^{5/2}} - \frac{(-7B+2iA)(a+ia \tan(e+fx))^{5/2}}{63cf(c-ictan(e+fx))^{7/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{9f(c-ictan(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(9/2)}, x]$

[Out]  $-1/9*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(5/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(9/2)}) - (((2*I)*A - 7*B)*(a + I*a*\text{Tan}[e + f*x])^{(5/2)})/(63*c*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}) - (((2*I)*A - 7*B)*(a + I*a*\text{Tan}[e + f*x])^{(5/2)})/(315*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})$

**Rule 37**

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*(c + d*x)^{n+1}/((b*c - a*d)*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

**Rule 47**

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*(c + d*x)^{n+1}/((b*c - a*d)*(m+1)), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m+1))), \text{Int}[(a + b*x)^{\text{Simplify}[m+1]}*(c + d*x)^n, x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ (\text{SumSimplerQ}[m, 1] \ || \ !\text{SumSimplerQ}[n, 1])$

**Rule 79**

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_.)*((A_) + (B_.)*tan[(e_) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{9/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{3/2}(A+Bx)}{(c-icx)^{11/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{9f(c - ictan(e + fx))^{9/2}} + \frac{(a(2A + 7iB)) \text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{11/2}} dx, x, \tan(e + fx)\right)}{63cf(c - ictan(e + fx))^{9/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{9f(c - ictan(e + fx))^{9/2}} - \frac{(2iA - 7B)(a + ia \tan(e + fx))^{5/2}}{63cf(c - ictan(e + fx))^{9/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{9f(c - ictan(e + fx))^{9/2}} - \frac{(2iA - 7B)(a + ia \tan(e + fx))^{5/2}}{63cf(c - ictan(e + fx))^{9/2}}$$

### Mathematica [A]

time = 3.63, size = 135, normalized size = 0.87

$$\frac{a^2 \cos(e + fx) (-45iA + 7(-7iA + 2B) \cos(2(e + fx)) - 7(2A + 7iB) \sin(2(e + fx))) (\cos(7e + 9fx) + i \sin(7e + 9fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{630c^5 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[
e + f*x])^(9/2), x]
```

```
[Out] (a^2*Cos[e + f*x]*((-45*I)*A + 7*((-7*I)*A + 2*B)*Cos[2*(e + f*x)] - 7*(2*A
+ (7*I)*B)*Sin[2*(e + f*x)]*(Cos[7*e + 9*f*x] + I*Sin[7*e + 9*f*x])*Sqrt[
a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(630*c^5*f*(Cos[f*x] + I*
Sin[f*x])^2)
```

**Maple [A]**

time = 0.42, size = 138, normalized size = 0.89

method	result
risch	$-\frac{a^2 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (35iA e^{8i(fx+e)} + 35B e^{8i(fx+e)} + 90iA e^{6i(fx+e)} + 63iA e^{4i(fx+e)} - 63B e^{4i(fx+e)})}{1260c^4 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$-\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2 (1+\tan^2(fx+e)) (-47A-33iA \tan(fx+e))}{315f c^5 (i+\tan(fx+e))}$
default	$-\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2 (1+\tan^2(fx+e)) (-47A-33iA \tan(fx+e))}{315f c^5 (i+\tan(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^5*(1+tan(f*x+e)^2)*(-47*A-33*I*A*tan(f*x+e)-12*A*tan(f*x+e)^2-7*I*B-42*I*B*tan(f*x+e)^2-42*B*tan(f*x+e)+2*I*A*tan(f*x+e)^3-7*B*tan(f*x+e)^3)/(I+tan(f*x+e))^6
```

**Maxima [A]**

time = 0.64, size = 210, normalized size = 1.35

$$\frac{(35(-A-B)a^2 \cos(\frac{9}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) - 90Aa^2 \cos(\frac{7}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 63(-A+B)a^2 \cos(\frac{5}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 35(A-I*B)a^2 \sin(\frac{9}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 90Aa^2 \sin(\frac{7}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 63(A+I*B)a^2 \sin(\frac{5}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) \sqrt{a}}{1260c^5 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] 1/1260*(35*(-I*A - B)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 90*I*A*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 63*(-I*A + B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 35*(A - I*B)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 90*A*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 63*(A + I*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(9/2)*f)
```

**Fricas [A]**

time = 3.96, size = 131, normalized size = 0.85

$$\frac{(35(iA+B)a^2 e^{(11i fx+11ie)} + 5(25iA+7B)a^2 e^{(9i fx+9ie)} + 9(17iA-7B)a^2 e^{(7i fx+7ie)} + 63(iA-B)a^2 e^{(5i fx+5ie)}) \sqrt{\frac{a}{e^{(2i fx+2ie)}+1}} \sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}}{1260 c^5 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(9/2),x, algorithm="fricas")

[Out] 
$$-1/1260*(35*(I*A + B)*a^2*e^{(11*I*f*x + 11*I*e)} + 5*(25*I*A + 7*B)*a^2*e^{(9*I*f*x + 9*I*e)} + 9*(17*I*A - 7*B)*a^2*e^{(7*I*f*x + 7*I*e)} + 63*(I*A - B)*a^2*e^{(5*I*f*x + 5*I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} / (c^5*f)$$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(9/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(5/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(9/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(5/2)/(-I\*c\*tan(f\*x + e) + c)^(9/2), x)

**Mupad [B]**

time = 11.00, size = 217, normalized size = 1.40

$$\frac{a^2 \sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}} (A \cos(4e+4fx)63i + A \cos(6e+6fx)90i + A \cos(8e+8fx)35i - 63B \cos(4e+4fx) + 35B \cos(8e+8fx) - 63A \sin(4e+4fx) - 90A \sin(6e+6fx) - 35A \sin(8e+8fx) - B \sin(4e+4fx)63i + B \sin(8e+8fx)35i)}{1260c^4f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(9/2),x)

[Out] 
$$-(a^2*((a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{(1/2)}*(A*\cos(4*e + 4*f*x)*63i + A*\cos(6*e + 6*f*x)*90i + A*\cos(8*e + 8*f*x)*35i - 63*B*\cos(4*e + 4*f*x) + 35*B*\cos(8*e + 8*f*x) - 63*A*\sin(4*e + 4*f*x) - 90*A*\sin(6*e + 6*f*x) - 35*A*\sin(8*e + 8*f*x) - B*\sin(4*e + 4*f*x)*63i + B*\sin(8*e + 8*f*x)*35i))/((1260*c^4*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{(1/2)}))$$



$$3.814 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{11/2}} dx$$

**Optimal.** Leaf size=208

$$\frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{11f(c-ictan(e+fx))^{11/2}} - \frac{(3iA-8B)(a+ia \tan(e+fx))^{5/2}}{99cf(c-ictan(e+fx))^{9/2}} - \frac{2(3iA-8B)(a+ia \tan(e+fx))^{5/2}}{693c^2f(c-ictan(e+fx))^{7/2}}$$

[Out] -1/11\*(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/f/(c-I\*c\*tan(f\*x+e))^(11/2)-1/99\*(3\*I\*A-8\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c/f/(c-I\*c\*tan(f\*x+e))^(9/2)-2/693\*(3\*I\*A-8\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c^2/f/(c-I\*c\*tan(f\*x+e))^(7/2)-2/3465\*(3\*I\*A-8\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c^3/f/(c-I\*c\*tan(f\*x+e))^(5/2)

**Rubi [A]**

time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ ,

Rules used = {3669, 79, 47, 37}

$$\frac{2(-8B+3iA)(a+ia \tan(e+fx))^{5/2}}{3465c^3f(c-ictan(e+fx))^{5/2}} - \frac{2(-8B+3iA)(a+ia \tan(e+fx))^{5/2}}{693c^2f(c-ictan(e+fx))^{7/2}} - \frac{(-8B+3iA)(a+ia \tan(e+fx))^{5/2}}{99cf(c-ictan(e+fx))^{9/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{11f(c-ictan(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])^(5/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(11/2), x]

[Out] -1/11\*((I\*A + B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(f\*(c - I\*c\*Tan[e + f\*x])^(11/2)) - (((3\*I)\*A - 8\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(99\*c\*f\*(c - I\*c\*Tan[e + f\*x])^(9/2)) - (2\*((3\*I)\*A - 8\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(693\*c^2\*f\*(c - I\*c\*Tan[e + f\*x])^(7/2)) - (2\*((3\*I)\*A - 8\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(3465\*c^3\*f\*(c - I\*c\*Tan[e + f\*x])^(5/2))

**Rule 37**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

**Rule 47**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(Simplify[m + 1]\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[m + n + 2] && !IntegerQ[m] && !IntegerQ[n] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

## Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{11/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{3/2}(A+Bx)}{(c-icx)^{13/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{11f(c - ictan(e + fx))^{11/2}} + \frac{(a(3A + 8iB)) \text{Subst}\left(\int \frac{(a+iax)^{3/2}}{(c-icx)^{13/2}} dx, x, \tan(e + fx)\right)}{99cf(c - ictan(e + fx))^{11/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(3iA - 8B)(a + ia \tan(e + fx))^{5/2}}{99cf(c - ictan(e + fx))^{11/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(3iA - 8B)(a + ia \tan(e + fx))^{5/2}}{99cf(c - ictan(e + fx))^{11/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(3iA - 8B)(a + ia \tan(e + fx))^{5/2}}{99cf(c - ictan(e + fx))^{11/2}}$$

**Mathematica [A]**

time = 6.56, size = 156, normalized size = 0.75

$$\frac{a^2 \cos(e + fx) (55(-24iA + B) \cos(e + fx) + 63(-8iA + 3B) \cos(3(e + fx)) - (3A + 8iB)(55 \sin(e + fx) + 63 \sin(3(e + fx)))) (\cos(8e + 10fx) + i \sin(8e + 10fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{13860e^6 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[
e + f*x])^(11/2), x]
```

```
[Out] (a^2*cos[e + f*x]*(55*((-24*I)*A + B)*cos[e + f*x] + 63*((-8*I)*A + 3*B)*cos[3*(e + f*x)] - (3*A + (8*I)*B)*(55*sin[e + f*x] + 63*sin[3*(e + f*x)]))*(cos[8*e + 10*f*x] + I*sin[8*e + 10*f*x])*sqrt[a + I*a*tan[e + f*x]]*sqrt[c - I*c*tan[e + f*x]]/(13860*c^6*f*(cos[f*x] + I*sin[f*x])^2)
```

**Maple [A]**

time = 0.40, size = 161, normalized size = 0.77

method	result
risch	$-\frac{a^2 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (315iA e^{10i(fx+e)}+315B e^{10i(fx+e)}+1155iA e^{8i(fx+e)}+385B e^{8i(fx+e)}+1485iA e^{6i(fx+e)}-495B e^{4i(fx+e)}+1485iA e^{2i(fx+e)}+315B e^{2i(fx+e)}+315iA e^{0i(fx+e)}+315B e^{0i(fx+e)})}{27720c^5 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$-\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(6iA(\tan^4(fx+e))-112iB \tan^3(fx+e)+36iB \tan^2(fx+e)+61iB)}{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(6iA(\tan^4(fx+e))-112iB \tan^3(fx+e)+36iB \tan^2(fx+e)+61iB)}$
default	$-\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(6iA(\tan^4(fx+e))-112iB \tan^3(fx+e)+36iB \tan^2(fx+e)+61iB)}{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(6iA(\tan^4(fx+e))-112iB \tan^3(fx+e)+36iB \tan^2(fx+e)+61iB)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3465*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^6*(1+tan(f*x+e)^2)*(6*I*A*tan(f*x+e)^4-112*I*B*tan(f*x+e)^3-16*B*tan(f*x+e)^4-135*I*A*tan(f*x+e)^2-42*A*tan(f*x+e)^3-427*I*B*tan(f*x+e)+360*B*tan(f*x+e)^2-456*I*A+273*A*tan(f*x+e)+61*B)/(I+tan(f*x+e))^7
```

**Maxima [A]**

time = 0.63, size = 292, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] 1/27720*(315*(-I*A - B)*a^2*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 385*(-3*I*A - B)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 495*(-3*I*A + B)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 693*(-I*A + B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 315*(A - I*B)*a^2*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 385*(3*A - I*B)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 495*(3*A + I*B)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 693*(A + I*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(11/2)*f)
```

**Fricas [A]**

time = 4.78, size = 153, normalized size = 0.74

$$\frac{(315(iA + B)a^2e^{(13iAx+13ie)} + 70(2iA + 10B)a^2e^{(11iAx+11ie)} + 110(24iA - B)a^2e^{(9iAx+9ie)} + 198(11iA - 6B)a^2e^{(7iAx+7ie)} + 693(iA - B)a^2e^{(5iAx+5ie)})\sqrt{\frac{a}{e^{(2iAx+2ie)} + 1}}\sqrt{\frac{c}{e^{(2iAx+2ie)} + 1}}}{27720c^6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] -1/27720*(315*(I*A + B)*a^2*e^(13*I*f*x + 13*I*e) + 70*(21*I*A + 10*B)*a^2*e^(11*I*f*x + 11*I*e) + 110*(24*I*A - B)*a^2*e^(9*I*f*x + 9*I*e) + 198*(11*I*A - 6*B)*a^2*e^(7*I*f*x + 7*I*e) + 693*(I*A - B)*a^2*e^(5*I*f*x + 5*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^6*f)
```

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)/(-I*c*tan(f*x + e) + c)^(11/2), x)
```

**Mupad [B]**

time = 12.85, size = 292, normalized size = 1.40

$$\frac{a^{\frac{5}{2}} \sqrt{\frac{\cos(2f x + 2e) + 1}{\cos(2f x + 2e) - 1}} (A \cos(4fx + 4e) \cos(8fx + 8e) + A \cos(8fx + 8e) + 1405A \cos(12fx + 12e) + A \cos(16fx + 16e) + 1152A \cos(20fx + 20e) - 692B \cos(4fx + 4e) - 692B \cos(8fx + 8e) + 392B \cos(12fx + 12e) + 312B \cos(16fx + 16e) - 692A \sin(4fx + 4e) - 1405A \sin(8fx + 8e) - 1152A \sin(12fx + 12e) - 312A \sin(16fx + 16e) - B \sin(4fx + 4e) \cos(8fx + 8e) + B \sin(8fx + 8e) \cos(12fx + 12e) + B \sin(12fx + 12e) \cos(16fx + 16e) + B \sin(16fx + 16e) \cos(20fx + 20e) - 27720c^6 \sqrt{\frac{\cos(2f x + 2e) + 1}{\cos(2f x + 2e) - 1}}}{27720c^6 \sqrt{\frac{\cos(2f x + 2e) + 1}{\cos(2f x + 2e) - 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*x)*1i)^(11/2),x)
```

```
[Out] -(a^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*cos(4*e + 4*f*x)*693i + A*cos(6*e + 6*f*x)*1485i + A*cos(8*e + 8*f*x)*1155i + A*cos(10*e + 10*f*x)*315i - 693*B*cos(4*e + 4*f*x) - 495*B*cos(6*e + 6*f*x) + 385*B*cos(8*e + 8*f*x) + 315*B*cos(10*e + 10*f*x) - 693*A*sin(4*e + 4*f*x) - 1485*A*sin(6*e + 6*f*x) - 1155*A*sin(8*e + 8*f*x) - 315*A*sin(10*e + 10*f*x) - B*sin(4*e + 4*f*x)*693i - B*sin(6*e + 6*f*x)*495i + B*sin(8*e + 8*f*x)*385i + B*sin(10*e + 10*f*x)*315i))/(27720*c^5*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

$$3.815 \quad \int \frac{(a+ia \tan(e+fx))^{5/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{13/2}} dx$$

**Optimal.** Leaf size=261

$$\frac{(iA+B)(a+ia \tan(e+fx))^{5/2}}{13f(c-ic \tan(e+fx))^{13/2}} - \frac{(4iA-9B)(a+ia \tan(e+fx))^{5/2}}{143cf(c-ic \tan(e+fx))^{11/2}} - \frac{(4iA-9B)(a+ia \tan(e+fx))^{5/2}}{429c^2f(c-ic \tan(e+fx))^{9/2}}$$

[Out] -1/13\*(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/f/(c-I\*c\*tan(f\*x+e))^(13/2)-1/143\*(4\*I\*A-9\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c/f/(c-I\*c\*tan(f\*x+e))^(11/2)-1/429\*(4\*I\*A-9\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c^2/f/(c-I\*c\*tan(f\*x+e))^(9/2)-2/3003\*(4\*I\*A-9\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c^3/f/(c-I\*c\*tan(f\*x+e))^(7/2)-2/15015\*(4\*I\*A-9\*B)\*(a+I\*a\*tan(f\*x+e))^(5/2)/c^4/f/(c-I\*c\*tan(f\*x+e))^(5/2)

**Rubi [A]**

time = 0.22, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{2(-9B+4iA)(a+ia \tan(e+fx))^{5/2}}{15015c^4f(c-ic \tan(e+fx))^{5/2}} - \frac{2(-9B+4iA)(a+ia \tan(e+fx))^{5/2}}{3003c^3f(c-ic \tan(e+fx))^{7/2}} - \frac{(-9B+4iA)(a+ia \tan(e+fx))^{5/2}}{429c^2f(c-ic \tan(e+fx))^{9/2}} - \frac{(-9B+4iA)(a+ia \tan(e+fx))^{5/2}}{143cf(c-ic \tan(e+fx))^{11/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{5/2}}{13f(c-ic \tan(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])^(5/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(13/2), x]

[Out] -1/13\*((I\*A + B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(f\*(c - I\*c\*Tan[e + f\*x])^(13/2)) - (((4\*I)\*A - 9\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(143\*c\*f\*(c - I\*c\*Tan[e + f\*x])^(11/2)) - (((4\*I)\*A - 9\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(429\*c^2\*f\*(c - I\*c\*Tan[e + f\*x])^(9/2)) - (2\*((4\*I)\*A - 9\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(3003\*c^3\*f\*(c - I\*c\*Tan[e + f\*x])^(7/2)) - (2\*((4\*I)\*A - 9\*B)\*(a + I\*a\*Tan[e + f\*x])^(5/2))/(15015\*c^4\*f\*(c - I\*c\*Tan[e + f\*x])^(5/2))

**Rule 37**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

**Rule 47**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^Simplify[m + 1]\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&

(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

### Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*(c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{5/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{13/2}} dx = \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{3/2} (A+Bx)}{(c-icx)^{15/2}} dx, x, \tan(e + fx) \right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{13f(c - ictan(e + fx))^{13/2}} + \frac{(a(4A + 9iB))S}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(4iA - 9B)(a + ia \tan(e + fx))^{5/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(4iA - 9B)(a + ia \tan(e + fx))^{5/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(4iA - 9B)(a + ia \tan(e + fx))^{5/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{5/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(4iA - 9B)(a + ia \tan(e + fx))^{5/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

### Mathematica [A]

time = 11.32, size = 183, normalized size = 0.70

$$\frac{ia^2 \cos(e + fx)(5005A + 780(9A + iB) \cos(2(e + fx)) + 231(9A + 4iB) \cos(4(e + fx)) - 1560iA \sin(2(e + fx)) + 3510B \sin(2(e + fx)) - 924iA \sin(4(e + fx)) + 2079B \sin(4(e + fx)))(\cos(9e + 11fx) + i \sin(9e + 11fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{120120c^2 f (\cos(fx) + i \sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(13/2), x]
```

```
[Out] ((-1/120120*I)*a^2*cos[e + f*x]*(5005*A + 780*(9*A + I*B)*cos[2*(e + f*x)] + 231*(9*A + (4*I)*B)*cos[4*(e + f*x)] - (1560*I)*A*sin[2*(e + f*x)] + 3510*B*sin[2*(e + f*x)] - (924*I)*A*sin[4*(e + f*x)] + 2079*B*sin[4*(e + f*x)])*(cos[9*e + 11*f*x] + I*sin[9*e + 11*f*x])*sqrt[a + I*a*Tan[e + f*x])*sqrt[c - I*c*Tan[e + f*x]]/(c^7*f*(cos[f*x] + I*sin[f*x])^2)
```

**Maple [A]**

time = 0.47, size = 183, normalized size = 0.70

method	result
risch	$-\frac{a^2 \sqrt{\frac{ae^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (1155iAe^{12i(fx+e)}+1155Be^{12i(fx+e)}+5460iAe^{10i(fx+e)}+2730Be^{10i(fx+e)}+10010iAe^{8i(fx+e)}+240240c^6 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(18iB(\tan^5(fx+e))+64iA(\tan^3(fx+e)+1704iB \tan(fx+e)+1224B \tan^2(fx+e)-1763iA+911A \tan(fx+e)+213B)/(1+\tan(fx+e))^8)}$
derivativedivides	$\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(18iB(\tan^5(fx+e))+64iA(\tan^3(fx+e)+1704iB \tan(fx+e)+1224B \tan^2(fx+e)-1763iA+911A \tan(fx+e)+213B)/(1+\tan(fx+e))^8)}{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(18iB(\tan^5(fx+e))+64iA(\tan^3(fx+e)+1704iB \tan(fx+e)+1224B \tan^2(fx+e)-1763iA+911A \tan(fx+e)+213B)/(1+\tan(fx+e))^8)}$
default	$\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(18iB(\tan^5(fx+e))+64iA(\tan^3(fx+e)+1704iB \tan(fx+e)+1224B \tan^2(fx+e)-1763iA+911A \tan(fx+e)+213B)/(1+\tan(fx+e))^8)}{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^2(1+\tan^2(fx+e))(18iB(\tan^5(fx+e))+64iA(\tan^3(fx+e)+1704iB \tan(fx+e)+1224B \tan^2(fx+e)-1763iA+911A \tan(fx+e)+213B)/(1+\tan(fx+e))^8)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2), x, method=_RETURNVERBOSE)
```

```
[Out] 1/15015/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^7*(1+tan(f*x+e)^2)*(18*I*B*tan(f*x+e)^5+64*I*A*tan(f*x+e)^4+8*A*tan(f*x+e)^5-531*I*B*tan(f*x+e)^3-144*B*tan(f*x+e)^4-544*I*A*tan(f*x+e)^2-236*A*tan(f*x+e)^3-1704*I*B*tan(f*x+e)+1224*B*tan(f*x+e)^2-1763*I*A+911*A*tan(f*x+e)+213*B)/(I+tan(f*x+e))^8
```

**Maxima [A]**

time = 0.64, size = 352, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2), x, algorithm="maxima")
```

```
[Out] 1/240240*(1155*(-I*A - B)*a^2*cos(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2730*(-2*I*A - B)*a^2*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10010*I*A*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
```



```
*e))) + 4290*(-2*I*A + B)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 3003*(-I*A + B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1155*(A - I*B)*a^2*sin(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 2730*(2*A - I*B)*a^2*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 10010*A*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 4290*(2*A + I*B)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 3003*(A + I*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))*sqrt(a)/(c^(13/2)*f)
```

**Fricas** [A]

time = 3.10, size = 175, normalized size = 0.67

$$\frac{(1155(iA + B)a^2e^{15ifx + 15ie} + 105(63iA + 37B)a^2e^{13ifx + 13ie} + 910(17iA + 3B)a^2e^{11ifx + 11ie} + 1430(13iA - 3B)a^2e^{9ifx + 9ie} + 429(27iA - 17B)a^2e^{7ifx + 7ie} + 3003(iA - B)a^2e^{5ifx + 5ie})\sqrt{\frac{a}{e^{2ifx + 2ie} + 1}}\sqrt{\frac{c}{e^{2ifx + 2ie} + 1}}}{240240c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] -1/240240*(1155*(I*A + B)*a^2*e^(15*I*f*x + 15*I*e) + 105*(63*I*A + 37*B)*a
^2*e^(13*I*f*x + 13*I*e) + 910*(17*I*A + 3*B)*a^2*e^(11*I*f*x + 11*I*e) + 1
430*(13*I*A - 3*B)*a^2*e^(9*I*f*x + 9*I*e) + 429*(27*I*A - 17*B)*a^2*e^(7*I
*f*x + 7*I*e) + 3003*(I*A - B)*a^2*e^(5*I*f*x + 5*I*e))*sqrt(a/(e^(2*I*f*x
+ 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^7*f)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1
3/2),x)
```

```
[Out] Timed out
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)/(-I*c*tan(f*x +
e) + c)^(13/2), x)
```

Mupad [B]

time = 13.73, size = 191, normalized size = 0.73

$$\frac{\sqrt{a + \frac{a \sin(e + f x) \operatorname{li}}{\cos(e + f x)} \left( \frac{a^2 e^{e 6i + f x 6i} (2A + B 1i) \operatorname{li}}{56 c^6 f} + \frac{a^2 e^{e 10i + f x 10i} (2A - B 1i) \operatorname{li}}{88 c^6 f} + \frac{A a^2 e^{e 8i + f x 8i} \operatorname{li}}{24 c^6 f} + \frac{a^2 e^{e 4i + f x 4i} (A + B 1i) \operatorname{li}}{80 c^6 f} + \frac{a^2 e^{e 12i + f x 12i} (A - B 1i) \operatorname{li}}{208 c^6 f} \right)}}{\sqrt{c - \frac{c \sin(e + f x) \operatorname{li}}{\cos(e + f x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(5/2))/(c - c\*tan(e + f\*x)\*1i)^(13/2),x)

[Out] -((a + (a\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)\*((a^2\*exp(e\*6i + f\*x\*6i))\*(2\*A + B\*1i)\*1i)/(56\*c^6\*f) + (a^2\*exp(e\*10i + f\*x\*10i)\*(2\*A - B\*1i)\*1i)/(88\*c^6\*f) + (A\*a^2\*exp(e\*8i + f\*x\*8i)\*1i)/(24\*c^6\*f) + (a^2\*exp(e\*4i + f\*x\*4i)\*(A + B\*1i)\*1i)/(80\*c^6\*f) + (a^2\*exp(e\*12i + f\*x\*12i)\*(A - B\*1i)\*1i)/(208\*c^6\*f)))/(c - (c\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)

$$3.816 \quad \int (a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{9/2} dx$$

**Optimal.** Leaf size=350

$$\frac{5a^{7/2}(8iA - B)c^{9/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{64f} + \frac{5a^3(8A + iB)c^4 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{128f}$$

[Out]  $-5/64*a^{(7/2)}*(8*I*A-B)*c^{(9/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f+5/128*a^3*(8*A+I*B)*c^4*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+5/192*a^2*(8*A+I*B)*c^3*\tan(f*x+e)*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/48*a*(8*A+I*B)*c^2*\tan(f*x+e)*(a+I*a*\tan(f*x+e))^{(5/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f-1/56*(8*I*A-B)*c*(a+I*a*\tan(f*x+e))^{(7/2)}*(c-I*c*\tan(f*x+e))^{(7/2)}/f+1/8*B*(a+I*a*\tan(f*x+e))^{(7/2)}*(c-I*c*\tan(f*x+e))^{(9/2)}/f$

**Rubi** [A]

time = 0.24, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 81, 51, 38, 65, 223, 209}

$$\frac{5a^{7/2}(-B+8iA)\operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{64f} + \frac{5a^3(8A+iB)\tan(e+fx)\sqrt{a+ia\tan(e+fx)}}{128f} + \frac{5a^2(8A+iB)\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{192f} - \frac{a(8A+iB)c^2\tan(e+fx)(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{48f} - \frac{(8I)A-B}{56f}c(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{7/2} + \frac{B}{8f}(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{9/2}$$

Antiderivative was successfully verified.

[In]  $\int (a + I*a*\tan[e + f*x])^{(7/2)}*(A + B*\tan[e + f*x])*(c - I*c*\tan[e + f*x])^{(9/2)}, x]$

[Out]  $(-5*a^{(7/2)}*((8*I)*A - B)*c^{(9/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + I*a*\tan[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c - I*c*\tan[e + f*x]])]/(64*f) + (5*a^3*(8*A + I*B)*c^4*\tan[e + f*x]*\operatorname{Sqrt}[a + I*a*\tan[e + f*x]]*\operatorname{Sqrt}[c - I*c*\tan[e + f*x]])/(128*f) + (5*a^2*(8*A + I*B)*c^3*\tan[e + f*x]*(a + I*a*\tan[e + f*x])^{(3/2)}*(c - I*c*\tan[e + f*x])^{(3/2)})/(192*f) + (a*(8*A + I*B)*c^2*\tan[e + f*x]*(a + I*a*\tan[e + f*x])^{(5/2)}*(c - I*c*\tan[e + f*x])^{(5/2)})/(48*f) - (((8*I)*A - B)*c*(a + I*a*\tan[e + f*x])^{(7/2)}*(c - I*c*\tan[e + f*x])^{(7/2)})/(56*f) + (B*(a + I*a*\tan[e + f*x])^{(7/2)}*(c - I*c*\tan[e + f*x])^{(9/2)})/(8*f)$

**Rule 38**

$\operatorname{Int}[(a + b*x)^m * (c + d*x)^m, x\_Symbol] := \operatorname{Simp}[x*(a + b*x)^m * (c + d*x)^m / (2*m + 1), x] + \operatorname{Dist}[2*a*c*m / (2*m + 1), \operatorname{Int}[(a + b*x)^{m-1} * (c + d*x)^{m-1}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 51**

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a
+ b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{9/2} dx &= \frac{(ac) \text{Subst}(\int (a + iax)^{5/2} (A + B \tan(e + fx)) dx)}{8f} \\
&= \frac{B(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))^{9/2}}{8f} \\
&= -\frac{(8iA - B)c(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))^{9/2}}{56f} \\
&= \frac{a(8A + iB)c^2 \tan(e + fx)(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))^{9/2}}{56f} \\
&= \frac{5a^2(8A + iB)c^3 \tan(e + fx)(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))^{9/2}}{56f} \\
&= \frac{5a^3(8A + iB)c^4 \tan(e + fx) \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{9/2}}{56f} \\
&= \frac{5a^3(8A + iB)c^4 \tan(e + fx) \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{9/2}}{56f} \\
&= \frac{5a^3(8A + iB)c^4 \tan(e + fx) \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{9/2}}{56f} \\
&= -\frac{5a^{7/2}(8iA - B)c^{9/2} \tan^{-1}\left(\frac{\sqrt{a + ia \tan(e + fx)}}{\sqrt{a - ia \tan(e + fx)}}\right)}{64f}
\end{aligned}$$

**Mathematica [A]**

time = 10.98, size = 666, normalized size = 1.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2), x]
```

```
[Out] (5*((-8*I)*A + B)*c^5*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])/(64*E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(9/2)*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e + f*x])
```

)) + (Cos[e + f\*x]^4\*Sqrt[Sec[e + f\*x]\*(c\*Cos[e + f\*x] - I\*c\*Sin[e + f\*x])] \* (Sec[e]\*Sec[e + f\*x]^6\*((-8\*I)\*A\*Cos[e] + 8\*B\*Cos[e] - (7\*I)\*B\*Sin[e])\*((c^4\*Cos[3\*e])/56 - (I/56)\*c^4\*Sin[3\*e]) - I\*B\*c^4\*Sec[e]\*Sec[e + f\*x]^7\*(Cos[3\*e]/8 - (I/8)\*Sin[3\*e])\*Sin[f\*x] + Sec[e]\*Sec[e + f\*x]^5\*(Cos[3\*e]/48 - (I/48)\*Sin[3\*e])\*(8\*A\*c^4\*Sin[f\*x] + I\*B\*c^4\*Sin[f\*x]) + Sec[e]\*Sec[e + f\*x]^3\*((5\*Cos[3\*e])/192 - ((5\*I)/192)\*Sin[3\*e])\*(8\*A\*c^4\*Sin[f\*x] + I\*B\*c^4\*Sin[f\*x]) + Sec[e]\*Sec[e + f\*x]\*((5\*Cos[3\*e])/128 - ((5\*I)/128)\*Sin[3\*e])\*(8\*A\*c^4\*Sin[f\*x] + I\*B\*c^4\*Sin[f\*x]) + (8\*A + I\*B)\*Sec[e + f\*x]^4\*((c^4\*Cos[3\*e])/48 - (I/48)\*c^4\*Sin[3\*e])\*Tan[e] + (8\*A + I\*B)\*Sec[e + f\*x]^2\*((5\*c^4\*Cos[3\*e])/192 - ((5\*I)/192)\*c^4\*Sin[3\*e])\*Tan[e] + (8\*A + I\*B)\*((5\*c^4\*Cos[3\*e])/128 - ((5\*I)/128)\*c^4\*Sin[3\*e])\*Tan[e])\*(a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x])/(f\*(Cos[f\*x] + I\*Sin[f\*x])^3\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]))

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 603 vs.  $2(289) = 578$ .  
time = 0.43, size = 604, normalized size = 1.73

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^4 \left( 952iB\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^4 \left( 952iB\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(9/2), x, method=\_RETURNVERBOSE)

[Out] 
$$\begin{aligned} & -1/2688/f*(a*(1+I*\tan(f*x+e)))^{(1/2)}*(-c*(I*\tan(f*x+e)-1))^{(1/2)}*a^3*c^4*(9 \\ & 52*I*B*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^5+826*I*B*(a*c)^{(1/2)} \\ & *(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^3+1152*I*A*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)} \\ & *\tan(f*x+e)^4-384*B*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^6+384*I*A*(a*c)^{(1/2)} \\ & *(a*c*(1+\tan(f*x+e)^2))^{(1/2)}-448*A*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^5+336*I*B*(a*c)^{(1/2)} \\ & *(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^7-1152*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)} \\ & *(a*c)^{(1/2)}*\tan(f*x+e)^4+105*I*B*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)} \\ & *\tan(f*x+e)-1456*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^3+3 \\ & 84*I*A*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^6-105*I*B*\ln((a*c*\tan(f*x+e) \\ & +(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c-1152*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)} \\ & *(a*c)^{(1/2)}*\tan(f*x+e)^2+1152*I*A*(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*\tan(f*x+e)^2 \\ & -840*A*\ln((a*c*\tan(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c-1848*A*(a*c*(1+\tan(f \end{aligned}$$

$(x+e)^2)^{1/2} * (a*c)^{1/2} * \tan(f*x+e) - 384*B*(a*c*(1+\tan(f*x+e)^2))^{1/2} * (a*c)^{1/2} / (a*c)^{1/2} / (a*c*(1+\tan(f*x+e)^2))^{1/2}$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2735 vs.  $2(285) = 570$ .

time = 23.31, size = 2735, normalized size = 7.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(9/2),x, algorithm="maxima")

[Out]  $-86016*(420*(8*A + I*B)*a^3*c^4*\cos(15/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 3220*(8*A + I*B)*a^3*c^4*\cos(13/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 10724*(8*A + I*B)*a^3*c^4*\cos(11/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 20212*(8*A + I*B)*a^3*c^4*\cos(9/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(8728*A + 44099*I*B)*a^3*c^4*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 10724*(8*A + I*B)*a^3*c^4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 3220*(8*A + I*B)*a^3*c^4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 420*(8*A + I*B)*a^3*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 420*(8*I*A - B)*a^3*c^4*\sin(15/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 3220*(8*I*A - B)*a^3*c^4*\sin(13/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 10724*(8*I*A - B)*a^3*c^4*\sin(11/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 20212*(8*I*A - B)*a^3*c^4*\sin(9/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(8728*I*A - 44099*B)*a^3*c^4*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 10724*(-8*I*A + B)*a^3*c^4*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 3220*(-8*I*A + B)*a^3*c^4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 420*(-8*I*A + B)*a^3*c^4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 210*((8*A + I*B)*a^3*c^4*\cos(16*f*x + 16*e) + 8*(8*A + I*B)*a^3*c^4*\cos(14*f*x + 14*e) + 28*(8*A + I*B)*a^3*c^4*\cos(12*f*x + 12*e) + 56*(8*A + I*B)*a^3*c^4*\cos(10*f*x + 10*e) + 70*(8*A + I*B)*a^3*c^4*\cos(8*f*x + 8*e) + 56*(8*A + I*B)*a^3*c^4*\cos(6*f*x + 6*e) + 28*(8*A + I*B)*a^3*c^4*\cos(4*f*x + 4*e) + 8*(8*A + I*B)*a^3*c^4*\cos(2*f*x + 2*e) + (8*I*A - B)*a^3*c^4*\sin(16*f*x + 16*e) + 8*(8*I*A - B)*a^3*c^4*\sin(14*f*x + 14*e) + 28*(8*I*A - B)*a^3*c^4*\sin(12*f*x + 12*e) + 56*(8*I*A - B)*a^3*c^4*\sin(10*f*x + 10*e) + 70*(8*I*A - B)*a^3*c^4*\sin(8*f*x + 8*e) + 56*(8*I*A - B)*a^3*c^4*\sin(6*f*x + 6*e) + 28*(8*I*A - B)*a^3*c^4*\sin(4*f*x + 4*e) + 8*(8*I*A - B)*a^3*c^4*\sin(2*f*x + 2*e) + (8*A + I*B)*a^3*c^4*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) + 210*((8*A + I*B)*a^3*c^4*\cos(16*f*x + 16*e) + 8*(8*A + I*B)*a^3*c^4*\cos(14*f*x + 14*e) + 28*(8*A + I*B)*a^3*c^4*\cos(12*f*x + 12*e) + 56*(8*A + I*B)*a^3*c^4*\cos(10*f*x + 10*e) + 70*(8*A + I*B)*a^3*c^4*\cos(8*f*x + 8*e) + 56*(8*A + I*B)*a^3*c^4*\cos(6*f*x + 6*e) + 28$

```

*(8*A + I*B)*a^3*c^4*cos(4*f*x + 4*e) + 8*(8*A + I*B)*a^3*c^4*cos(2*f*x + 2
*e) + (8*I*A - B)*a^3*c^4*sin(16*f*x + 16*e) + 8*(8*I*A - B)*a^3*c^4*sin(14
*f*x + 14*e) + 28*(8*I*A - B)*a^3*c^4*sin(12*f*x + 12*e) + 56*(8*I*A - B)*a
^3*c^4*sin(10*f*x + 10*e) + 70*(8*I*A - B)*a^3*c^4*sin(8*f*x + 8*e) + 56*(8
*I*A - B)*a^3*c^4*sin(6*f*x + 6*e) + 28*(8*I*A - B)*a^3*c^4*sin(4*f*x + 4*e
) + 8*(8*I*A - B)*a^3*c^4*sin(2*f*x + 2*e) + (8*A + I*B)*a^3*c^4)*arctan2(c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 105*((8*I*A - B)*a^3*c^4*cos(16*f*x
+ 16*e) + 8*(8*I*A - B)*a^3*c^4*cos(14*f*x + 14*e) + 28*(8*I*A - B)*a^3*c^4
*cos(12*f*x + 12*e) + 56*(8*I*A - B)*a^3*c^4*cos(10*f*x + 10*e) + 70*(8*I*A
- B)*a^3*c^4*cos(8*f*x + 8*e) + 56*(8*I*A - B)*a^3*c^4*cos(6*f*x + 6*e) +
28*(8*I*A - B)*a^3*c^4*cos(4*f*x + 4*e) + 8*(8*I*A - B)*a^3*c^4*cos(2*f*x +
2*e) - (8*A + I*B)*a^3*c^4*sin(16*f*x + 16*e) - 8*(8*A + I*B)*a^3*c^4*sin(
14*f*x + 14*e) - 28*(8*A + I*B)*a^3*c^4*sin(12*f*x + 12*e) - 56*(8*A + I*B)
*a^3*c^4*sin(10*f*x + 10*e) - 70*(8*A + I*B)*a^3*c^4*sin(8*f*x + 8*e) - 56*
(8*A + I*B)*a^3*c^4*sin(6*f*x + 6*e) - 28*(8*A + I*B)*a^3*c^4*sin(4*f*x + 4
*e) - 8*(8*A + I*B)*a^3*c^4*sin(2*f*x + 2*e) + (8*I*A - B)*a^3*c^4)*log(cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))) + 1) + 105*((-8*I*A + B)*a^3*c^4*cos(16*f*x + 16*e) + 8*(-8
*I*A + B)*a^3*c^4*cos(14*f*x + 14*e) + 28*(-8*I*A + B)*a^3*c^4*cos(12*f*x +
12*e) + 56*(-8*I*A + B)*a^3*c^4*cos(10*f*x + 10*e) + 70*(-8*I*A + B)*a^3*c
^4*cos(8*f*x + 8*e) + 56*(-8*I*A + B)*a^3*c^4*cos(6*f*x + 6*e) + 28*(-8*I*A
+ B)*a^3*c^4*cos(4*f*x + 4*e) + 8*(-8*I*A + B)*a^3*c^4*cos(2*f*x + 2*e) +
(8*A + I*B)*a^3*c^4*sin(16*f*x + 16*e) + 8*(8*A + I*B)*a^3*c^4*sin(14*f*x +
14*e) + 28*(8*A + I*B)*a^3*c^4*sin(12*f*x + 12*e) + 56*(8*A + I*B)*a^3*c^4
*sin(10*f*x + 10*e) + 70*(8*A + I*B)*a^3*c^4*sin(8*f*x + 8*e) + 56*(8*A + I
*B)*a^3*c^4*sin(6*f*x + 6*e) + 28*(8*A + I*B)*a^3*c^4*sin(4*f*x + 4*e) + 8*
(8*A + I*B)*a^3*c^4*sin(2*f*x + 2*e) + (-8*I*A + B)*a^3*c^4)*log(cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))) + 1)))*sqrt(a)*sqrt(c)/(f*(-462422016*I*cos(16*f*x + 16*e) - 369937
6128*I*cos(14*f*x + 14*e) - 12947816448*I*cos(12*f*x + 12*e) - 25895632896*
I*cos(10*f*x + 10*e) - 32369541120*I*cos(8*f*x + 8*e) - 25895632896*I*cos(6
*f*x + 6*e) - 12947816448*I*cos(4*f*x + 4*e) - ...

```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 925 vs.  $2(285) = 570$ .

time = 5.65, size = 925, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2
),x, algorithm="fricas")

```



```
[Out] 1/5376*(105*sqrt((64*A^2 + 16*I*A*B - B^2)*a^7*c^9/f^2)*(f*e^(14*I*f*x + 14
*I*e) + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(8*
I*f*x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7*f*
e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((8*I*A - B)*a^3*c^4*e^(3*I*f*x + 3*I*e)
+ (8*I*A - B)*a^3*c^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((64*A^2 + 16*I*A*B - B^2)*a^7*c^9/f^
2)*(f*e^(2*I*f*x + 2*I*e) - f))/((8*I*A - B)*a^3*c^4*e^(2*I*f*x + 2*I*e) +
(8*I*A - B)*a^3*c^4) - 105*sqrt((64*A^2 + 16*I*A*B - B^2)*a^7*c^9/f^2)*(f*
e^(14*I*f*x + 14*I*e) + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I
*e) + 35*f*e^(8*I*f*x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x
+ 4*I*e) + 7*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((8*I*A - B)*a^3*c^4*e^(3
*I*f*x + 3*I*e) + (8*I*A - B)*a^3*c^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))) - sqrt((64*A^2 + 16*I*A*B -
B^2)*a^7*c^9/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((8*I*A - B)*a^3*c^4*e^(2*I
*f*x + 2*I*e) + (8*I*A - B)*a^3*c^4) - 4*(105*(8*I*A - B)*a^3*c^4*e^(15*I*
f*x + 15*I*e) + 805*(8*I*A - B)*a^3*c^4*e^(13*I*f*x + 13*I*e) + 2681*(8*I*A
- B)*a^3*c^4*e^(11*I*f*x + 11*I*e) + 5053*(8*I*A - B)*a^3*c^4*e^(9*I*f*x +
9*I*e) - (-8728*I*A + 44099*B)*a^3*c^4*e^(7*I*f*x + 7*I*e) + 2681*(-8*I*A
+ B)*a^3*c^4*e^(5*I*f*x + 5*I*e) + 805*(-8*I*A + B)*a^3*c^4*e^(3*I*f*x + 3*
I*e) + 105*(-8*I*A + B)*a^3*c^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(14*I*f*x + 14*I*e) + 7*f*e
^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(8*I*f*x + 8*I*e
) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7*f*e^(2*I*f*x +
2*I*e) + f)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9
/2),x)
```

[Out] Timed out

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2
),x, algorithm="giac")
```

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) 1i)^{7/2} (c - c \tan(e + f x) 1i)^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*
1i)^(9/2),x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*
1i)^(9/2), x)
```

$$3.817 \quad \int (a+ia \tan(e+fx))^{7/2} (A+B \tan(e+fx))(c-ictan(e+fx))^{7/2} dx$$

**Optimal.** Leaf size=267

$$\frac{5ia^{7/2}Ac^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{8f} + \frac{5a^3Ac^3\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{16f}$$

[Out]  $-5/8*I*a^{(7/2)}*A*c^{(7/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f+5/16*a^3*A*c^3*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+5/24*a^2*A*c^2*\tan(f*x+e)*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/6*a*A*c*\tan(f*x+e)*(a+I*a*\tan(f*x+e))^{(5/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f+1/7*B*(a+I*a*\tan(f*x+e))^{(7/2)}*(c-I*c*\tan(f*x+e))^{(7/2)}/f$

**Rubi [A]**

time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 38, 65, 223, 209}

$$\frac{5ia^{7/2}Ac^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{8f} + \frac{5a^3Ac^3\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ictan(e+fx)}}{16f} + \frac{5a^2Ac^2\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ictan(e+fx))^{3/2}}{24f} + \frac{aAc\tan(e+fx)(a+ia\tan(e+fx))^{5/2}(c-ictan(e+fx))^{5/2}}{6f} + \frac{B(a+ia\tan(e+fx))^{7/2}(c-ictan(e+fx))^{7/2}}{7f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(7/2)}, x]$

[Out]  $(((-5*I)/8)*a^{(7/2)}*A*c^{(7/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])])/f + (5*a^3*A*c^3*\text{Tan}[e + f*x]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(16*f) + (5*a^2*A*c^2*\text{Tan}[e + f*x]*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(24*f) + (a*A*c*\text{Tan}[e + f*x]*(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(6*f) + (B*(a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})/(7*f)$

**Rule 38**

$\text{Int}[(a + b*x)^m*(c + d*x)^m, x\_Symbol] := \text{Simp}[x*(a + b*x)^m*(c + d*x)^m/(2*m + 1), x] + \text{Dist}[2*a*c*(m/(2*m + 1)), \text{Int}[(a + b*x)^{m-1}*(c + d*x)^{m-1}, x], x] /; \text{FreeQ}[a, b, c, d, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{IGtQ}[m + 1/2, 0]$

**Rule 65**

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] := \text{With}[p = \text{Denominator}[m], \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1}*(c - a*(d/b) +$

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)/(d*f*(n + p +
2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{7/2} dx &= \frac{(ac) \text{Subst}(\int (a + iax)^{5/2} (A + B \tan(e + fx)) dx)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))}{7f} \\
&= \frac{aAc \tan(e + fx) (a + ia \tan(e + fx))^{7/2}}{6f} \\
&= \frac{5a^2 Ac^2 \tan(e + fx) (a + ia \tan(e + fx))^{7/2}}{2} \\
&= \frac{5a^3 Ac^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{16} \\
&= \frac{5a^3 Ac^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{16} \\
&= \frac{5a^3 Ac^3 \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{16} \\
&= -\frac{5ia^{7/2} Ac^{7/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{8f}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 535 vs. 2(267) = 534.  
time = 9.67, size = 535, normalized size = 2.00

Integrate[(a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2), x]

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2), x]

[Out] (((-5\*I)/8)\*A\*c^4\*Sqrt[E^(I\*f\*x)]\*Sqrt[E^(I\*(e + f\*x))]/(1 + E^((2\*I)\*(e + f\*x))))\*ArcTan[E^(I\*(e + f\*x))]\*(a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x])/E^(I\*(4\*e + f\*x))\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x)))]\*f\*Sec[e + f\*x]^(9/2)\*(Cos[f\*x] + I\*Sin[f\*x])^(7/2)\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x]) + (Cos[e + f\*x]^4\*Sqrt[Sec[e + f\*x]\*(c\*Cos[e + f\*x] - I\*c\*Sin[e + f\*x])]\*(Sec[e + f\*x]^6\*((B\*c^3\*Cos[3\*e])/7 - (I/7)\*B\*c^3\*Sin[3\*e]) + A\*c^3\*Sec[e]\*Sec[e

$$+ f*x]^5*(\text{Cos}[3*e]/6 - (I/6)*\text{Sin}[3*e])*\text{Sin}[f*x] + A*c^3*\text{Sec}[e]*\text{Sec}[e + f*x]^3*((5*\text{Cos}[3*e])/24 - ((5*I)/24)*\text{Sin}[3*e])*\text{Sin}[f*x] + A*c^3*\text{Sec}[e]*\text{Sec}[e + f*x]*((5*\text{Cos}[3*e])/16 - ((5*I)/16)*\text{Sin}[3*e])*\text{Sin}[f*x] + \text{Sec}[e + f*x]^4*((A*c^3*\text{Cos}[3*e])/6 - (I/6)*A*c^3*\text{Sin}[3*e])*\text{Tan}[e] + \text{Sec}[e + f*x]^2*((5*A*c^3*\text{Cos}[3*e])/24 - ((5*I)/24)*A*c^3*\text{Sin}[3*e])*\text{Tan}[e] + ((5*A*c^3*\text{Cos}[3*e])/16 - ((5*I)/16)*A*c^3*\text{Sin}[3*e])*\text{Tan}[e]*(a + I*a*\text{Tan}[e + f*x])^(7/2)*(A + B*\text{Tan}[e + f*x]))/(f*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^3*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$$

**Maple [A]**

time = 0.45, size = 314, normalized size = 1.18

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^3 \left( 48B\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)}{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^3 \left( 48B\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^3 \left( 48B\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)}{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^3 \left( 48B\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{336} f (a(1+I\tan(fx+e)))^{1/2} (-c(I\tan(fx+e)-1))^{1/2} a^3 c^3 (48B + B(a*c)^{1/2} (a*c(1+\tan(fx+e)^2))^{1/2} \tan(fx+e)^6 + 56A(a*c)^{1/2} (a*c(1+\tan(fx+e)^2))^{1/2} \tan(fx+e)^5 + 144B(a*c(1+\tan(fx+e)^2))^{1/2} (a*c)^{1/2} \tan(fx+e)^4 + 182A(a*c(1+\tan(fx+e)^2))^{1/2} (a*c)^{1/2} \tan(fx+e)^3 + 144B(a*c(1+\tan(fx+e)^2))^{1/2} (a*c)^{1/2} \tan(fx+e)^2 + 105A \ln((a*c \tan(fx+e) + (a*c)^{1/2} (a*c(1+\tan(fx+e)^2))^{1/2}) / (a*c)^{1/2}) * a*c + 231A(a*c(1+\tan(fx+e)^2))^{1/2} (a*c)^{1/2} \tan(fx+e) + 48B(a*c(1+\tan(fx+e)^2))^{1/2} (a*c)^{1/2}) / (a*c)^{1/2} / (a*c(1+\tan(fx+e)^2))^{1/2}$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2019 vs. 2(220) = 440.

time = 3.30, size = 2019, normalized size = 7.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,algorithm="maxima")`

[Out]  $-(420Aa^3c^3\cos(13/2\arctan2(\sin(2fx+2e), \cos(2fx+2e))) + 2800Aa^3c^3\cos(11/2\arctan2(\sin(2fx+2e), \cos(2fx+2e)))) + 7924A*$

$$\begin{aligned}
& a^3 c^3 \cos(9/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 12288 I B a^3 c^3 \cos(7/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 7924 A a^3 c^3 \cos(5/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 2800 A a^3 c^3 \cos(3/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 420 A a^3 c^3 \cos(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 420 I A a^3 c^3 \sin(13/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 2800 I A a^3 c^3 \sin(11/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 7924 I A a^3 c^3 \sin(9/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 12288 B a^3 c^3 \sin(7/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 7924 I A a^3 c^3 \sin(5/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 2800 I A a^3 c^3 \sin(3/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) - 420 I A a^3 c^3 \sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 210 (A a^3 c^3 \cos(14fx + 14e) + 7 A a^3 c^3 \cos(12fx + 12e) + 21 A a^3 c^3 \cos(10fx + 10e) + 35 A a^3 c^3 \cos(8fx + 8e) + 35 A a^3 c^3 \cos(6fx + 6e) + 21 A a^3 c^3 \cos(4fx + 4e) + 7 A a^3 c^3 \cos(2fx + 2e) + I A a^3 c^3 \sin(14fx + 14e) + 7 I A a^3 c^3 \sin(12fx + 12e) + 21 I A a^3 c^3 \sin(10fx + 10e) + 35 I A a^3 c^3 \sin(8fx + 8e) + 35 I A a^3 c^3 \sin(6fx + 6e) + 21 I A a^3 c^3 \sin(4fx + 4e) + 7 I A a^3 c^3 \sin(2fx + 2e) + A a^3 c^3) \arctan 2(\cos(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))), \sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1) + 210 (A a^3 c^3 \cos(14fx + 14e) + 7 A a^3 c^3 \cos(12fx + 12e) + 21 A a^3 c^3 \cos(10fx + 10e) + 35 A a^3 c^3 \cos(8fx + 8e) + 35 A a^3 c^3 \cos(6fx + 6e) + 21 A a^3 c^3 \cos(4fx + 4e) + 7 A a^3 c^3 \cos(2fx + 2e) + I A a^3 c^3 \sin(14fx + 14e) + 7 I A a^3 c^3 \sin(12fx + 12e) + 21 I A a^3 c^3 \sin(10fx + 10e) + 35 I A a^3 c^3 \sin(8fx + 8e) + 35 I A a^3 c^3 \sin(6fx + 6e) + 21 I A a^3 c^3 \sin(4fx + 4e) + 7 I A a^3 c^3 \sin(2fx + 2e) + A a^3 c^3) \arctan 2(\cos(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))), -\sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1) + 105 (I A a^3 c^3 \cos(14fx + 14e) + 7 I A a^3 c^3 \cos(12fx + 12e) + 21 I A a^3 c^3 \cos(10fx + 10e) + 35 I A a^3 c^3 \cos(8fx + 8e) + 35 I A a^3 c^3 \cos(6fx + 6e) + 21 I A a^3 c^3 \cos(4fx + 4e) + 7 I A a^3 c^3 \cos(2fx + 2e) - A a^3 c^3 \sin(14fx + 14e) - 7 A a^3 c^3 \sin(12fx + 12e) - 21 A a^3 c^3 \sin(10fx + 10e) - 35 A a^3 c^3 \sin(8fx + 8e) - 35 A a^3 c^3 \sin(6fx + 6e) - 21 A a^3 c^3 \sin(4fx + 4e) - 7 A a^3 c^3 \sin(2fx + 2e) + I A a^3 c^3) \log(\cos(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))))^2 + \sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))))^2 + 2 \sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1) + 105 (-I A a^3 c^3 \cos(14fx + 14e) - 7 I A a^3 c^3 \cos(12fx + 12e) - 21 I A a^3 c^3 \cos(10fx + 10e) - 35 I A a^3 c^3 \cos(8fx + 8e) - 35 I A a^3 c^3 \cos(6fx + 6e) - 21 I A a^3 c^3 \cos(4fx + 4e) - 7 I A a^3 c^3 \cos(2fx + 2e) + A a^3 c^3 \sin(14fx + 14e) + 7 A a^3 c^3 \sin(12fx + 12e) + 21 A a^3 c^3 \sin(10fx + 10e) + 35 A a^3 c^3 \sin(8fx + 8e) + 35 A a^3 c^3 \sin(6fx + 6e) + 21 A a^3 c^3 \sin(4fx + 4e) + 7 A a^3 c^3 \sin(2fx + 2e) - I A a^3 c^3) \log(\cos(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))))^2 + \sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))))^2 - 2 \sin(1/2 \arctan 2(\sin(2fx + 2e), \cos(2fx + 2e))) + 1)
\end{aligned}$$

```
)*sqrt(a)*sqrt(c)/(f*(-672*I*cos(14*f*x + 14*e) - 4704*I*cos(12*f*x + 12*e)
- 14112*I*cos(10*f*x + 10*e) - 23520*I*cos(8*f*x + 8*e) - 23520*I*cos(6*f*
x + 6*e) - 14112*I*cos(4*f*x + 4*e) - 4704*I*cos(2*f*x + 2*e) + 672*sin(14*
f*x + 14*e) + 4704*sin(12*f*x + 12*e) + 14112*sin(10*f*x + 10*e) + 23520*si
n(8*f*x + 8*e) + 23520*sin(6*f*x + 6*e) + 14112*sin(4*f*x + 4*e) + 4704*sin
(2*f*x + 2*e) - 672*I))
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 732 vs.  $2(220) = 440$ .  
time = 2.64, size = 732, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/672*(105*sqrt(A^2*a^7*c^7/f^2)*(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(A*a^3*c^3*e^(3*I*f*x + 3*I*e) + A*a^3*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^7*c^7/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(A*a^3*c^3*e^(2*I*f*x + 2*I*e) + A*a^3*c^3) - 105*sqrt(A^2*a^7*c^7/f^2)*(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(A*a^3*c^3*e^(3*I*f*x + 3*I*e) + A*a^3*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^7*c^7/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(A*a^3*c^3*e^(2*I*f*x + 2*I*e) + A*a^3*c^3) + 4*(-105*I*A*a^3*c^3*e^(13*I*f*x + 13*I*e) - 700*I*A*a^3*c^3*e^(11*I*f*x + 11*I*e) - 1981*I*A*a^3*c^3*e^(9*I*f*x + 9*I*e) + 3072*B*a^3*c^3*e^(7*I*f*x + 7*I*e) + 1981*I*A*a^3*c^3*e^(5*I*f*x + 5*I*e) + 700*I*A*a^3*c^3*e^(3*I*f*x + 3*I*e) + 105*I*A*a^3*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)
```

**Sympy** [F(-1)] Timed out  
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2),x)
```



[Out] Timed out

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{7/2} (c - c \tan(e + f x) i)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2)\*(c - c\*tan(e + f\*x)\*1i)^(7/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2)\*(c - c\*tan(e + f\*x)\*1i)^(7/2), x)

$$3.818 \quad \int (a+ia \tan(e+fx))^{7/2} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{5/2} dx$$

**Optimal.** Leaf size=284

$$\frac{a^{7/2}(6iA+B)c^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{8f} + \frac{a^3(6A-iB)c^2 \tan(e+fx) \sqrt{a+ia \tan(e+fx)}}{16f}$$

[Out]  $-1/8*a^{(7/2)}*(6*I*A+B)*c^{(5/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)})/(c-I*c*\tan(f*x+e))^{(1/2)}/f+1/16*a^3*(6*A-I*B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+1/24*a^2*(6*A-I*B)*c*\tan(f*x+e)*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/30*a*(6*I*A+B)*(a+I*a*\tan(f*x+e))^{(5/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f+1/6*B*(a+I*a*\tan(f*x+e))^{(7/2)}*(c-I*c*\tan(f*x+e))^{(5/2)}/f$

**Rubi [A]**

time = 0.22, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 81, 51, 38, 65, 223, 209}

$$\frac{a^{7/2}c^{5/2}(B+6iA)\operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{8f} + \frac{a^3(6A-iB)\tan(e+fx)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{16f} + \frac{a^2c(6A-iB)\tan(e+fx)(a+ia \tan(e+fx))^{3/2}(c-ic \tan(e+fx))^{3/2}}{24f} + \frac{a(B+6iA)(a+ia \tan(e+fx))^{5/2}(c-ic \tan(e+fx))^{3/2}}{30f} + \frac{B(a+ia \tan(e+fx))^{7/2}(c-ic \tan(e+fx))^{5/2}}{6f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[e + f*x])^{(7/2)}*(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)}, x]$

[Out]  $-1/8*(a^{(7/2)}*((6*I)*A + B)*c^{(5/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])])/f + (a^3*(6*A - I*B)*c^2*\operatorname{Tan}[e + f*x]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(16*f) + (a^2*(6*A - I*B)*c*\operatorname{Tan}[e + f*x]*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)})/(24*f) + (a*((6*I)*A + B)*(a + I*a*\operatorname{Tan}[e + f*x])^{(5/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(30*f) + (B*(a + I*a*\operatorname{Tan}[e + f*x])^{(7/2)}*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(6*f)$

**Rule 38**

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[x*(a + b*x)^m*((c + d*x)^{m/(2*m + 1)}), x] + \operatorname{Dist}[2*a*c*(m/(2*m + 1)), \operatorname{Int}[(a + b*x)^{(m - 1)}*(c + d*x)^{(m - 1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 51**

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + n + 1))), x] + \operatorname{Dist}[2*c*(n/(m + n + 1))$

), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]

### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps



$$\begin{aligned} &^2*\cos[3*e])/30 - (I/30)*c^2*\sin[3*e]) + I*B*c^2*\sec[e]*\sec[e + f*x]^5*(\cos \\ &[3*e]/6 - (I/6)*\sin[3*e])*\sin[f*x] + \sec[e]*\sec[e + f*x]^3*(\cos[3*e]/24 - ( \\ &I/24)*\sin[3*e])*(6*A*c^2*\sin[f*x] - I*B*c^2*\sin[f*x]) + \sec[e]*\sec[e + f*x] \\ &*(\cos[3*e]/16 - (I/16)*\sin[3*e])*(6*A*c^2*\sin[f*x] - I*B*c^2*\sin[f*x]) + (6 \\ &*A - I*B)*\sec[e + f*x]^2*((c^2*\cos[3*e])/24 - (I/24)*c^2*\sin[3*e])* \tan[e] + \\ &(6*A - I*B)*((c^2*\cos[3*e])/16 - (I/16)*c^2*\sin[3*e])* \tan[e]*(a + I*a*\tan \\ &[e + f*x])^{(7/2)}*(A + B*\tan[e + f*x]))/(f*(\cos[f*x] + I*\sin[f*x])^3*(A*\cos[ \\ &e + f*x] + B*\sin[e + f*x])) \end{aligned}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs.  $2(232) = 464$ .

time = 0.44, size = 478, normalized size = 1.68

method	result
derivativedivides	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^2 \left( 40iB\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)$
default	$\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c^2 \left( 40iB\sqrt{ac} \sqrt{ac(1+\tan^2(fx+e))} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} &1/240/f*(a*(1+I*\tan(f*x+e)))^{(1/2)}*(-c*(I*\tan(f*x+e)-1))^{(1/2)}*a^3*c^2*(40* \\ &I*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^5+48*I*A*(a*c*(1+\tan \\ &(\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^4+70*I*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)} \\ &*(a*c)^{(1/2)}*\tan(f*x+e)^3+48*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)} \\ &*\tan(f*x+e)^4+96*I*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^2+ \\ &60*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^3-15*I*B*\ln((a*c*\tan \\ &(\tan(f*x+e)+a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)})/(a*c)^{(1/2)})*a*c+15*I*B \\ &*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)+96*B*(a*c*(1+\tan(f*x+e) \\ &)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e)^2+48*I*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a \\ &)*c)^{(1/2)}+90*A*\ln((a*c*\tan(f*x+e)+a*c)^{(1/2)}*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}) \\ &/(a*c)^{(1/2)})*a*c+150*A*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\tan(f*x+e) \\ &+48*B*(a*c*(1+\tan(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)})/(a*c)^{(1/2)}/(a*c*(1+\tan(f*x \\ &+e)^2))^{(1/2)} \end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2137 vs.  $2(230) = 460$ .

time = 5.84, size = 2137, normalized size = 7.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -3840*(60*(6*A - I*B)*a^3*c^2*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 340*(6*A - I*B)*a^3*c^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 24*(58*A - 223*I*B)*a^3*c^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 792*(6*A - I*B)*a^3*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 340*(6*A - I*B)*a^3*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(6*A - I*B)*a^3*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(-6*I*A - B)*a^3*c^2*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 340*(-6*I*A - B)*a^3*c^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 24*(58*I*A + 223*B)*a^3*c^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 792*(6*I*A + B)*a^3*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 340*(6*I*A + B)*a^3*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(6*I*A + B)*a^3*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*((6*A - I*B)*a^3*c^2*cos(12*f*x + 12*e) + 6*(6*A - I*B)*a^3*c^2*cos(10*f*x + 10*e) + 15*(6*A - I*B)*a^3*c^2*cos(8*f*x + 8*e) + 20*(6*A - I*B)*a^3*c^2*cos(6*f*x + 6*e) + 15*(6*A - I*B)*a^3*c^2*cos(4*f*x + 4*e) + 6*(6*A - I*B)*a^3*c^2*cos(2*f*x + 2*e) - (-6*I*A - B)*a^3*c^2*sin(12*f*x + 12*e) - 6*(-6*I*A - B)*a^3*c^2*sin(10*f*x + 10*e) - 15*(-6*I*A - B)*a^3*c^2*sin(8*f*x + 8*e) - 20*(-6*I*A - B)*a^3*c^2*sin(6*f*x + 6*e) - 15*(-6*I*A - B)*a^3*c^2*sin(4*f*x + 4*e) - 6*(-6*I*A - B)*a^3*c^2*sin(2*f*x + 2*e) + (6*A - I*B)*a^3*c^2*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 30*((6*A - I*B)*a^3*c^2*cos(12*f*x + 12*e) + 6*(6*A - I*B)*a^3*c^2*cos(10*f*x + 10*e) + 15*(6*A - I*B)*a^3*c^2*cos(8*f*x + 8*e) + 20*(6*A - I*B)*a^3*c^2*cos(6*f*x + 6*e) + 15*(6*A - I*B)*a^3*c^2*cos(4*f*x + 4*e) + 6*(6*A - I*B)*a^3*c^2*cos(2*f*x + 2*e) - (-6*I*A - B)*a^3*c^2*sin(12*f*x + 12*e) - 6*(-6*I*A - B)*a^3*c^2*sin(10*f*x + 10*e) - 15*(-6*I*A - B)*a^3*c^2*sin(8*f*x + 8*e) - 20*(-6*I*A - B)*a^3*c^2*sin(6*f*x + 6*e) - 15*(-6*I*A - B)*a^3*c^2*sin(4*f*x + 4*e) - 6*(-6*I*A - B)*a^3*c^2*sin(2*f*x + 2*e) + (6*A - I*B)*a^3*c^2*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) - 15*((-6*I*A - B)*a^3*c^2*cos(12*f*x + 12*e) + 6*(-6*I*A - B)*a^3*c^2*cos(10*f*x + 10*e) + 15*(-6*I*A - B)*a^3*c^2*cos(8*f*x + 8*e) + 20*(-6*I*A - B)*a^3*c^2*cos(6*f*x + 6*e) + 15*(-6*I*A - B)*a^3*c^2*cos(4*f*x + 4*e) + 6*(-6*I*A - B)*a^3*c^2*cos(2*f*x + 2*e) + (6*A - I*B)*a^3*c^2*sin(12*f*x + 12*e) + 6*(6*A - I*B)*a^3*c^2*sin(10*f*x + 10*e) + 15*(6*A - I*B)*a^3*c^2*sin(8*f*x + 8*e) + 20*(6*A - I*B)*a^3*c^2*sin(6*f*x + 6*e) + 15*(6*A - I*B)*a^3*c^2*sin(4*f*x + 4*e) + 6*(6*A - I*B)*a^3*c^2*sin(2*f*x + 2*e) + (-6*I*A - B)*a^3*c^2*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 15*((6*I*A +
```

$$\begin{aligned}
& B)a^3c^2\cos(12fx + 12e) + 6(6IA + B)a^3c^2\cos(10fx + 10e) + \\
& 15(6IA + B)a^3c^2\cos(8fx + 8e) + 20(6IA + B)a^3c^2\cos(6fx \\
& + 6e) + 15(6IA + B)a^3c^2\cos(4fx + 4e) + 6(6IA + B)a^3c^2c \\
& \text{os}(2fx + 2e) - (6A - IB)a^3c^2\sin(12fx + 12e) - 6(6A - IB)a^ \\
& 3c^2\sin(10fx + 10e) - 15(6A - IB)a^3c^2\sin(8fx + 8e) - 20(6A \\
& - IB)a^3c^2\sin(6fx + 6e) - 15(6A - IB)a^3c^2\sin(4fx + 4e) \\
& - 6(6A - IB)a^3c^2\sin(2fx + 2e) + (6IA + B)a^3c^2\log(\cos(1/ \\
& 2\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + \sin(1/2\arctan2(\sin(2fx \\
& + 2e), \cos(2fx + 2e)))^2 - 2\sin(1/2\arctan2(\sin(2fx + 2e), \cos(2 \\
& fx + 2e))) + 1)\sqrt{a}\sqrt{c}/(f(-1843200I\cos(12fx + 12e) - 1105 \\
& 9200I\cos(10fx + 10e) - 27648000I\cos(8fx + 8e) - 36864000I\cos(6 \\
& fx + 6e) - 27648000I\cos(4fx + 4e) - 11059200I\cos(2fx + 2e) + 18 \\
& 43200\sin(12fx + 12e) + 11059200\sin(10fx + 10e) + 27648000\sin(8fx \\
& + 8e) + 36864000\sin(6fx + 6e) + 27648000\sin(4fx + 4e) + 11059200 \\
& \sin(2fx + 2e) - 1843200I)
\end{aligned}$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 791 vs.  $2(230) = 460$ .  
time = 2.27, size = 791, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned}
& -1/480*(15\sqrt{(36A^2 - 12IA*B - B^2)}a^7c^5/f^2)*(f*e^{(10I*fx + 10I \\
& I*e) + 5f*e^{(8I*fx + 8I*e) + 10f*e^{(6I*fx + 6I*e) + 10f*e^{(4I*fx \\
& + 4I*e) + 5f*e^{(2I*fx + 2I*e) + f}}\log(-4*(2*((-6IA - B)a^3c^2e^{ \\
& (3I*fx + 3I*e) + (-6IA - B)a^3c^2e^{(I*fx + I*e)})\sqrt{a/(e^{(2I*fx \\
& x + 2I*e) + 1)}\sqrt{c/(e^{(2I*fx + 2I*e) + 1)}} + \sqrt{(36A^2 - 12IA*A \\
& B - B^2)}a^7c^5/f^2)*(f*e^{(2I*fx + 2I*e) - f})/((6IA + B)a^3c^2e^{( \\
& 2I*fx + 2I*e) + (6IA + B)a^3c^2)) - 15\sqrt{(36A^2 - 12IA*A*B - B^2} \\
& )a^7c^5/f^2)*(f*e^{(10I*fx + 10I*e) + 5f*e^{(8I*fx + 8I*e) + 10f*e^{ \\
& (6I*fx + 6I*e) + 10f*e^{(4I*fx + 4I*e) + 5f*e^{(2I*fx + 2I*e) + f}} \\
& * \log(-4*(2*((-6IA - B)a^3c^2e^{(3I*fx + 3I*e) + (-6IA - B)a^3c^2 \\
& *e^{(I*fx + I*e)})\sqrt{a/(e^{(2I*fx + 2I*e) + 1)}}\sqrt{c/(e^{(2I*fx + 2I \\
& I*e) + 1)}} - \sqrt{(36A^2 - 12IA*A*B - B^2)}a^7c^5/f^2)*(f*e^{(2I*fx + 2I \\
& I*e) - f})/((6IA + B)a^3c^2e^{(2I*fx + 2I*e) + (6IA + B)a^3c^2)) \\
& + 4*(15(6IA + B)a^3c^2e^{(11I*fx + 11I*e) + 85(6IA + B)a^3c^2 \\
& *e^{(9I*fx + 9I*e) + 6(-58IA - 223B)a^3c^2e^{(7I*fx + 7I*e) + 19 \\
& 8(-6IA - B)a^3c^2e^{(5I*fx + 5I*e) + 85(-6IA - B)a^3c^2e^{(3I \\
& *fx + 3I*e) + 15(-6IA - B)a^3c^2e^{(I*fx + I*e)})\sqrt{a/(e^{(2I*fx \\
& + 2I*e) + 1)}}\sqrt{c/(e^{(2I*fx + 2I*e) + 1}})/(f*e^{(10I*fx + 10I*e) \\
& + 5f*e^{(8I*fx + 8I*e) + 10f*e^{(6I*fx + 6I*e) + 10f*e^{(4I*fx + 4 \\
& *I*e) + 5f*e^{(2I*fx + 2I*e) + f}}
\end{aligned}$$

**Sympy [F(-1)]** Timed out  
 time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(5/2),x)

[Out] Timed out

**Giac [F(-1)]** Timed out  
 time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**  
 time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{7/2} (c - c \tan(e + f x) i)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2)\*(c - c\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] int((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2)\*(c - c\*tan(e + f\*x)\*1i)^(5/2), x)



$$3.819 \quad \int (a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx)) (c - ictan(e + fx))^{3/2} dx$$

**Optimal.** Leaf size=279

$$\frac{a^{7/2}(5iA + 2B)c^{3/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{4f} + \frac{a^3(5A - 2iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{8f}$$

[Out]  $-1/4*a^{(7/2)}*(5*I*A+2*B)*c^{(3/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f+1/8*a^3*(5*A-2*I*B)*c*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}*\tan(f*x+e)/f+1/12*a^2*(5*I*A+2*B)*(a+I*a*\tan(f*x+e))^{(3/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/20*a*(5*I*A+2*B)*(a+I*a*\tan(f*x+e))^{(5/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f+1/5*B*(a+I*a*\tan(f*x+e))^{(7/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/f$

**Rubi [A]**

time = 0.23, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 81, 51, 38, 65, 223, 209}

$$\frac{a^{7/2}c^{3/2}(2B + 5iA)\text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{4f} + \frac{a^3(5A - 2iB)\tan(e + fx) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{8f} + \frac{a^2(2B + 5iA)(a + ia \tan(e + fx))^{3/2}(c - ictan(e + fx))^{3/2}}{12f} + \frac{a(2B + 5iA)(a + ia \tan(e + fx))^{5/2}(c - ictan(e + fx))^{3/2}}{20f} + \frac{B(a + ia \tan(e + fx))^{7/2}(c - ictan(e + fx))^{3/2}}{5f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $-1/4*(a^{(7/2)}*((5*I)*A + 2*B)*c^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])])/f + (a^3*(5*A - (2*I)*B)*c*\text{Tan}[e + f*x]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(8*f) + (a^2*((5*I)*A + 2*B)*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(12*f) + (a*((5*I)*A + 2*B)*(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(20*f) + (B*(a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(5*f)$

**Rule 38**

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x] := \text{Simp}[x*(a + b*x)^m*(c + d*x)^n/(2*m + 1), x] + \text{Dist}[2*a*c*(m/(2*m + 1)), \text{Int}[(a + b*x)^{m-1}*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

**Rule 51**

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x] := \text{Simp}[(a + b*x)^{m+1}*(c + d*x)^n/(b*(m + n + 1)), x] + \text{Dist}[2*c*(n/(m + n + 1))$

), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]

### Rule 65

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 81

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 209

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[b, 2]))\*ArcTan[Rt[b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 223

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 3669

Int[((a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int (a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{3/2} dx &= \frac{(ac) \text{Subst}\left(\int (a + iax)^{5/2} (A + B \tan(e + fx)) dx\right)}{f} \\
&= \frac{B(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))}{5f} \\
&= \frac{a(5iA + 2B)(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))}{20f} \\
&= \frac{a^2(5iA + 2B)(a + ia \tan(e + fx))^{7/2} (c - ic \tan(e + fx))}{12f} \\
&= \frac{a^3(5A - 2iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{12f} \\
&= \frac{a^3(5A - 2iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{12f} \\
&= \frac{a^3(5A - 2iB)c \tan(e + fx) \sqrt{a + ia \tan(e + fx)}}{12f} \\
&= \frac{a^{7/2}(5iA + 2B)c^{3/2} \tan^{-1}\left(\frac{\sqrt{a + ia \tan(e + fx)}}{\sqrt{a - ia \tan(e + fx)}}\right)}{4f}
\end{aligned}$$

**Mathematica [A]**

time = 8.72, size = 507, normalized size = 1.82

```

(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2), x

```

Warning: Unable to verify antiderivative.

```

[In] Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2), x]

```

```

[Out] ((-1/4*I)*(5*A - (2*I)*B)*c^2*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))]/(1 + E^((2*I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])/(E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*Sec[e + f*x]^(9/2)*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e + f*x])) + (Cos[e + f*x]^4*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]

```

$$*x)]*(\text{Sec}[e]*\text{Sec}[e + f*x]^2*((8*I)*A*\text{Cos}[e] + 8*B*\text{Cos}[e] - 3*A*\text{Sin}[e] + (6*I)*B*\text{Sin}[e])*((c*\text{Cos}[3*e])/12 - (I/12)*c*\text{Sin}[3*e]) + \text{Sec}[e + f*x]^4*(-1/5*(B*c*\text{Cos}[3*e]) + (I/5)*B*c*\text{Sin}[3*e]) + \text{Sec}[e]*\text{Sec}[e + f*x]*(\text{Cos}[3*e]/8 - (I/8)*\text{Sin}[3*e])*(5*A*c*\text{Sin}[f*x] - (2*I)*B*c*\text{Sin}[f*x]) + \text{Sec}[e]*\text{Sec}[e + f*x]^3*(\text{Cos}[3*e]/4 - (I/4)*\text{Sin}[3*e])*(-(A*c*\text{Sin}[f*x]) + (2*I)*B*c*\text{Sin}[f*x]) + (5*A - (2*I)*B)*((c*\text{Cos}[3*e])/8 - (I/8)*c*\text{Sin}[3*e])*\text{Tan}[e]*(a + I*a*\text{Tan}[e + f*x])^(7/2)*(A + B*\text{Tan}[e + f*x]))/(f*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^3*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$$

**Maple [A]**

time = 0.40, size = 412, normalized size = 1.48

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c \left(60iB \sqrt{ac(1+\tan^2(fx+e))} \sqrt{a}\right)}{\dots}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} a^3 c \left(60iB \sqrt{ac(1+\tan^2(fx+e))} \sqrt{a}\right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*c*(60*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-24*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+80*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-30*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-30*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2)*a*c+30*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+32*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+80*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+75*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2)*a*c+45*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+56*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1747 vs.  $2(224) = 448$ .

time = 2.93, size = 1747, normalized size = 6.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```

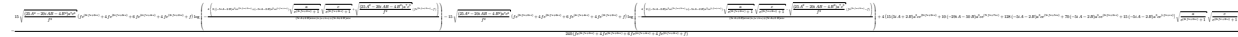
[Out] -480*(60*(5*A - 2*I*B)*a^3*c*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 40*(29*A - 50*I*B)*a^3*c*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 512*(5*A - 2*I*B)*a^3*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 280*(5*A - 2*I*B)*a^3*c*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 60*(5*A - 2*I*B)*a^3*c*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 60*(-5*I*A - 2*B)*a^3*c*sin(9/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 40*(29*I*A + 50*B)*a^3*c*sin(7/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 512*(5*I*A + 2*B)*a^3*c*sin(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 280*(5*I*A + 2*B)*a^3*c*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(5*I*A + 2*B)*a^3*c*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*((5*A - 2*I*B)*a^3*c*cos(10
*f*x + 10*e) + 5*(5*A - 2*I*B)*a^3*c*cos(8*f*x + 8*e) + 10*(5*A - 2*I*B)*a^
3*c*cos(6*f*x + 6*e) + 10*(5*A - 2*I*B)*a^3*c*cos(4*f*x + 4*e) + 5*(5*A - 2
*I*B)*a^3*c*cos(2*f*x + 2*e) - (-5*I*A - 2*B)*a^3*c*sin(10*f*x + 10*e) - 5*
(-5*I*A - 2*B)*a^3*c*sin(8*f*x + 8*e) - 10*(-5*I*A - 2*B)*a^3*c*sin(6*f*x +
6*e) - 10*(-5*I*A - 2*B)*a^3*c*sin(4*f*x + 4*e) - 5*(-5*I*A - 2*B)*a^3*c*s
in(2*f*x + 2*e) + (5*A - 2*I*B)*a^3*c)*arctan2(cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 30*((5*A - 2*I*B)*a^3*c*cos(10*f*x + 10*e) + 5*(5*A - 2*I*B)*a^3
*c*cos(8*f*x + 8*e) + 10*(5*A - 2*I*B)*a^3*c*cos(6*f*x + 6*e) + 10*(5*A - 2
*I*B)*a^3*c*cos(4*f*x + 4*e) + 5*(5*A - 2*I*B)*a^3*c*cos(2*f*x + 2*e) - (-5
*I*A - 2*B)*a^3*c*sin(10*f*x + 10*e) - 5*(-5*I*A - 2*B)*a^3*c*sin(8*f*x + 8
*e) - 10*(-5*I*A - 2*B)*a^3*c*sin(6*f*x + 6*e) - 10*(-5*I*A - 2*B)*a^3*c*si
n(4*f*x + 4*e) - 5*(-5*I*A - 2*B)*a^3*c*sin(2*f*x + 2*e) + (5*A - 2*I*B)*a^
3*c)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 15*((-5*I*A - 2*B)*a^3
*c*cos(10*f*x + 10*e) + 5*(-5*I*A - 2*B)*a^3*c*cos(8*f*x + 8*e) + 10*(-5*I*
A - 2*B)*a^3*c*cos(6*f*x + 6*e) + 10*(-5*I*A - 2*B)*a^3*c*cos(4*f*x + 4*e)
+ 5*(-5*I*A - 2*B)*a^3*c*cos(2*f*x + 2*e) + (5*A - 2*I*B)*a^3*c*sin(10*f*x
+ 10*e) + 5*(5*A - 2*I*B)*a^3*c*sin(8*f*x + 8*e) + 10*(5*A - 2*I*B)*a^3*c*s
in(6*f*x + 6*e) + 10*(5*A - 2*I*B)*a^3*c*sin(4*f*x + 4*e) + 5*(5*A - 2*I*B)
*a^3*c*sin(2*f*x + 2*e) + (-5*I*A - 2*B)*a^3*c)*log(cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)
- 15*((5*I*A + 2*B)*a^3*c*cos(10*f*x + 10*e) + 5*(5*I*A + 2*B)*a^3*c*cos(8
*f*x + 8*e) + 10*(5*I*A + 2*B)*a^3*c*cos(6*f*x + 6*e) + 10*(5*I*A + 2*B)*a^
3*c*cos(4*f*x + 4*e) + 5*(5*I*A + 2*B)*a^3*c*cos(2*f*x + 2*e) - (5*A - 2*I*
B)*a^3*c*sin(10*f*x + 10*e) - 5*(5*A - 2*I*B)*a^3*c*sin(8*f*x + 8*e) - 10*(
5*A - 2*I*B)*a^3*c*sin(6*f*x + 6*e) - 10*(5*A - 2*I*B)*a^3*c*sin(4*f*x + 4*
e) - 5*(5*A - 2*I*B)*a^3*c*sin(2*f*x + 2*e) + (5*I*A + 2*B)*a^3*c)*log(cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 1))*sqrt(a)*sqrt(c)/(f*(-115200*I*cos(10*f*x + 10*e) - 576
000*I*cos(8*f*x + 8*e) - 1152000*I*cos(6*f*x + 6*e) - 1152000*I*cos(4*f*x +
4*e) - 576000*I*cos(2*f*x + 2*e) + 115200*sin(10*f*x + 10*e) + 576000*sin(

```

$8*f*x + 8*e) + 1152000*\sin(6*f*x + 6*e) + 1152000*\sin(4*f*x + 4*e) + 576000$   
 $*\sin(2*f*x + 2*e) - 115200*I)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs.  $2(224) = 448$ .

time = 4.76, size = 713, normalized size = 2.56



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out]  $-1/240*(15*\sqrt{(25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2}*(f*e^{(8*I*f*x + 8*I*e) + 4*f*e^{(6*I*f*x + 6*I*e) + 6*f*e^{(4*I*f*x + 4*I*e) + 4*f*e^{(2*I*f*x + 2*I*e) + f}}*\log(-4*(2*((-5*I*A - 2*B)*a^3*c*e^{(3*I*f*x + 3*I*e) + (-5*I*A - 2*B)*a^3*c*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e) + 1}})*\sqrt{c/(e^{(2*I*f*x + 2*I*e) + 1}) + \sqrt{(25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2}*(f*e^{(2*I*f*x + 2*I*e) - f})/((5*I*A + 2*B)*a^3*c*e^{(2*I*f*x + 2*I*e) + (5*I*A + 2*B)*a^3*c})} - 15*\sqrt{(25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2}*(f*e^{(8*I*f*x + 8*I*e) + 4*f*e^{(6*I*f*x + 6*I*e) + 6*f*e^{(4*I*f*x + 4*I*e) + 4*f*e^{(2*I*f*x + 2*I*e) + f}}*\log(-4*(2*((-5*I*A - 2*B)*a^3*c*e^{(3*I*f*x + 3*I*e) + (-5*I*A - 2*B)*a^3*c*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e) + 1}})*\sqrt{c/(e^{(2*I*f*x + 2*I*e) + 1})} - \sqrt{(25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2}*(f*e^{(2*I*f*x + 2*I*e) - f})/((5*I*A + 2*B)*a^3*c*e^{(2*I*f*x + 2*I*e) + (5*I*A + 2*B)*a^3*c})} + 4*(15*(5*I*A + 2*B)*a^3*c*e^{(9*I*f*x + 9*I*e) + 10*(-29*I*A - 50*B)*a^3*c*e^{(7*I*f*x + 7*I*e) + 128*(-5*I*A - 2*B)*a^3*c*e^{(5*I*f*x + 5*I*e) + 70*(-5*I*A - 2*B)*a^3*c*e^{(3*I*f*x + 3*I*e) + 15*(-5*I*A - 2*B)*a^3*c*e^{(I*f*x + I*e)}}*\sqrt{a/(e^{(2*I*f*x + 2*I*e) + 1}})*\sqrt{c/(e^{(2*I*f*x + 2*I*e) + 1})})/(f*e^{(8*I*f*x + 8*I*e) + 4*f*e^{(6*I*f*x + 6*I*e) + 6*f*e^{(4*I*f*x + 4*I*e) + 4*f*e^{(2*I*f*x + 2*I*e) + f}}$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(7/2)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{7/2} (c - c \tan(e + f x) i)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*1i)^(3/2), x)
```

### 3.820 $\int (a+ia \tan(e+fx))^{7/2} (A+B \tan(e+fx)) \sqrt{c-ic \tan(e+fx)}$

Optimal. Leaf size=272

$$\frac{5a^{7/2}(4iA+3B)\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{4f} + \frac{5a^3(4iA+3B)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{8f}$$

[Out]  $-5/4*a^{(7/2)}*(4*I*A+3*B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})*c^{(1/2)}/f+5/8*a^3*(4*I*A+3*B)*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/f+5/24*a^2*(4*I*A+3*B)*(c-I*c*\tan(f*x+e))^{(1/2)}*(a+I*a*\tan(f*x+e))^{(3/2)}/f+1/12*a*(4*I*A+3*B)*(c-I*c*\tan(f*x+e))^{(1/2)}*(a+I*a*\tan(f*x+e))^{(5/2)}/f+1/4*B*(c-I*c*\tan(f*x+e))^{(1/2)}*(a+I*a*\tan(f*x+e))^{(7/2)}/f$

Rubi [A]

time = 0.21, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 81, 52, 65, 223, 209}

$$\frac{5a^{7/2}\sqrt{c}(3B+4iA)\operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{4f} + \frac{5a^3(3B+4iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{8f} + \frac{5a^2(3B+4iA)(a+ia \tan(e+fx))^{3/2}\sqrt{c-ic \tan(e+fx)}}{24f} + \frac{a(3B+4iA)(a+ia \tan(e+fx))^{5/2}\sqrt{c-ic \tan(e+fx)}}{12f} + \frac{B(a+ia \tan(e+fx))^{7/2}\sqrt{c-ic \tan(e+fx)}}{4f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Tan}[e + f*x])^{(7/2)}*(A + B*\operatorname{Tan}[e + f*x])*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]], x]$

[Out]  $(-5*a^{(7/2)}*((4*I)*A + 3*B)*Sqrt[c]*\operatorname{ArcTan}[(Sqrt[c]*Sqrt[a + I*a*\operatorname{Tan}[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])]/(4*f) + (5*a^3*((4*I)*A + 3*B)*Sqrt[a + I*a*\operatorname{Tan}[e + f*x]]*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])/(8*f) + (5*a^2*((4*I)*A + 3*B)*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])/(24*f) + (a*((4*I)*A + 3*B)*(a + I*a*\operatorname{Tan}[e + f*x])^{(5/2)}*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])/(12*f) + (B*(a + I*a*\operatorname{Tan}[e + f*x])^{(7/2)}*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])/(4*f)$

Rule 52

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))], x] + \operatorname{Dist}[n*((b*c - a*d)/(b*(m+n+1))], \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) +$



```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned} & ec[e + f*x]^{(9/2)} * (\cos[f*x] + I * \sin[f*x])^{(7/2)} * (A * \cos[e + f*x] + B * \sin[e + f*x]) \\ & + (\cos[e + f*x]^4 * (\sec[e] * \sec[e + f*x]^2 * (4 * A * \cos[e] - (12 * I) * B * \cos[e] \\ & + 3 * B * \sin[e]) * ((-1/12 * I) * \cos[3 * e] - \sin[3 * e] / 12) + \sec[e] * ((32 * I) * A * \cos[e] \\ & + 32 * B * \cos[e] - 12 * A * \sin[e] + (17 * I) * B * \sin[e]) * (\cos[3 * e] / 8 - (I / 8) * \sin[3 * e]) \\ & - I * B * \sec[e] * \sec[e + f*x]^3 * (\cos[3 * e] / 4 - (I / 4) * \sin[3 * e]) * \sin[f*x] + \\ & \sec[e] * \sec[e + f*x] * (\cos[3 * e] / 8 - (I / 8) * \sin[3 * e]) * (-12 * A * \sin[f*x] + (17 * I) * \\ & B * \sin[f*x])) * \sqrt{\sec[e + f*x] * (c * \cos[e + f*x] - I * c * \sin[e + f*x])} * (a + I * \\ & a * \tan[e + f*x])^{(7/2)} * (A + B * \tan[e + f*x]) / (f * (\cos[f*x] + I * \sin[f*x])^3 * (A * \cos[e + f*x] + B * \sin[e + f*x])) \end{aligned}$$

**Maple [A]**

time = 0.47, size = 349, normalized size = 1.28

method	result
derivativedivides	$\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} a^3 \left(6iB \sqrt{ac(1 + \tan^2(fx + e))} \sqrt{\dots}}{\dots}$
default	$\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} a^3 \left(6iB \sqrt{ac(1 + \tan^2(fx + e))} \sqrt{\dots}}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -1/24/f * (-c * (I * \tan(f*x+e) - 1))^{(1/2)} * (a * (1 + I * \tan(f*x+e)))^{(1/2)} * a^3 * (6 * I * B * \\ & a * c * (1 + \tan(f*x+e)^2))^{(1/2)} * (a * c)^{(1/2)} * \tan(f*x+e)^3 + 8 * I * A * (a * c * (1 + \tan(f*x+ \\ & e)^2))^{(1/2)} * (a * c)^{(1/2)} * \tan(f*x+e)^2 + 45 * I * B * \ln((a * c * \tan(f*x+e) + (a * c)^{(1/2)} \\ & * (a * c * (1 + \tan(f*x+e)^2))^{(1/2)}) / (a * c)^{(1/2)}) * a * c - 45 * I * B * (a * c * (1 + \tan(f*x+e)^2 \\ & ))^{(1/2)} * (a * c)^{(1/2)} * \tan(f*x+e) + 24 * B * (a * c * (1 + \tan(f*x+e)^2))^{(1/2)} * (a * c)^{(1/2)} \\ & * \tan(f*x+e)^2 - 88 * I * A * (a * c * (1 + \tan(f*x+e)^2))^{(1/2)} * (a * c)^{(1/2)} - 60 * A * \ln((a * \\ & c * \tan(f*x+e) + (a * c)^{(1/2)} * (a * c * (1 + \tan(f*x+e)^2))^{(1/2)}) / (a * c)^{(1/2)}) * a * c + 36 * \\ & A * (a * c * (1 + \tan(f*x+e)^2))^{(1/2)} * (a * c)^{(1/2)} * \tan(f*x+e) - 72 * B * (a * c * (1 + \tan(f*x+ \\ & e)^2))^{(1/2)} * (a * c)^{(1/2)} / (a * c * (1 + \tan(f*x+e)^2))^{(1/2)} / (a * c)^{(1/2)} \end{aligned}$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1421 vs.  $2(216) = 432$ .

time = 1.48, size = 1421, normalized size = 5.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e)),x,algorithm="maxima")`



[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e)),x, algorithm="fricas")

[Out] 
$$-1/48*(15*\sqrt{(16*A^2 - 24*I*A*B - 9*B^2)*a^7*c/f^2}*(f*e^{(6*I*f*x + 6*I*e)} + 3*f*e^{(4*I*f*x + 4*I*e)} + 3*f*e^{(2*I*f*x + 2*I*e)} + f)*\log(4*(2*((-4*I*A - 3*B)*a^3*e^{(3*I*f*x + 3*I*e)} + (-4*I*A - 3*B)*a^3*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + \sqrt{(16*A^2 - 24*I*A*B - 9*B^2)*a^7*c/f^2}*(f*e^{(2*I*f*x + 2*I*e)} - f)))/((-4*I*A - 3*B)*a^3*e^{(2*I*f*x + 2*I*e)} + (-4*I*A - 3*B)*a^3) - 15*\sqrt{(16*A^2 - 24*I*A*B - 9*B^2)*a^7*c/f^2}*(f*e^{(6*I*f*x + 6*I*e)} + 3*f*e^{(4*I*f*x + 4*I*e)} + 3*f*e^{(2*I*f*x + 2*I*e)} + f)*\log(4*(2*((-4*I*A - 3*B)*a^3*e^{(3*I*f*x + 3*I*e)} + (-4*I*A - 3*B)*a^3*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - \sqrt{(16*A^2 - 24*I*A*B - 9*B^2)*a^7*c/f^2}*(f*e^{(2*I*f*x + 2*I*e)} - f)))/((-4*I*A - 3*B)*a^3*e^{(2*I*f*x + 2*I*e)} + (-4*I*A - 3*B)*a^3) + 4*(3*(-44*I*A - 49*B)*a^3*e^{(7*I*f*x + 7*I*e)} + 7*3*(-4*I*A - 3*B)*a^3*e^{(5*I*f*x + 5*I*e)} + 55*(-4*I*A - 3*B)*a^3*e^{(3*I*f*x + 3*I*e)} + 15*(-4*I*A - 3*B)*a^3*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)))/(f*e^{(6*I*f*x + 6*I*e)} + 3*f*e^{(4*I*f*x + 4*I*e)} + 3*f*e^{(2*I*f*x + 2*I*e)} + f)$$

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e)),x, algorithm="giac")

[Out] Timed out

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{7/2} \sqrt{c - c \tan(e + f x) \operatorname{li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*  
1i)^(1/2), x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)*  
1i)^(1/2), x)
```

$$3.821 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{\sqrt{c-ictan(e+fx)}} dx$$

**Optimal.** Leaf size=283

$$\frac{5a^{7/2}(3iA+4B)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{f\sqrt{c-ictan(e+fx)}} - \frac{5a^3(3iA+4B)}{f\sqrt{c-ictan(e+fx)}}$$

[Out]  $5*a^{(7/2)}*(3*I*A+4*B)*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f/c^{(1/2)}-5/2*a^3*(3*I*A+4*B)*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/c/f-5/6*a^2*(3*I*A+4*B)*(c-I*c*\tan(f*x+e))^{(1/2)}*(a+I*a*\tan(f*x+e))^{(3/2)}/c/f-1/3*a*(3*I*A+4*B)*(c-I*c*\tan(f*x+e))^{(1/2)}*(a+I*a*\tan(f*x+e))^{(5/2)}/c/f-(I*A+B)*(a+I*a*\tan(f*x+e))^{(7/2)}/f/(c-I*c*\tan(f*x+e))^{(1/2)}$

**Rubi** [A]

time = 0.22, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 52, 65, 223, 209}

$$\frac{5a^{7/2}(4B+3iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ictan(e+fx)}}\right)}{\sqrt{c}f} - \frac{5a^2(4B+3iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ictan(e+fx)}}{2cf} - \frac{5a^2(4B+3iA)(a+ia \tan(e+fx))^{3/2}\sqrt{c-ictan(e+fx)}}{6cf} - \frac{a(4B+3iA)(a+ia \tan(e+fx))^{3/2}\sqrt{c-ictan(e+fx)}}{3cf} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{f\sqrt{c-ictan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+I*a*\text{Tan}[e+fx])^{(7/2)}*(A+B*\text{Tan}[e+fx])/(\text{Sqrt}[c-I*c*\text{Tan}[e+fx]]),x]$

[Out]  $(5*a^{(7/2)}*((3*I)*A+4*B)*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a+I*a*\text{Tan}[e+fx]])/(\text{Sqrt}[a]*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])])/(f*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]]) - ((I*A+B)*(a+I*a*\text{Tan}[e+fx])^{(7/2)})/(f*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]]) - (5*a^3*((3*I)*A+4*B)*\text{Sqrt}[a+I*a*\text{Tan}[e+fx]]*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])/(2*c*f) - (5*a^2*((3*I)*A+4*B)*(a+I*a*\text{Tan}[e+fx])^{(3/2)}*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])/(6*c*f) - (a*((3*I)*A+4*B)*(a+I*a*\text{Tan}[e+fx])^{(5/2)}*\text{Sqrt}[c-I*c*\text{Tan}[e+fx]])/(3*c*f)$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))), x] + \text{Dist}[n*((b*c - a*d)/(b*(m+n+1))], \text{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx))}{\sqrt{c - ictan(e + fx)}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{5/2} (A+Bx)}{(c-icx)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{(a(3A - 4iB))S}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{a(3iA + 4B)(a - ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{5a^2(3iA + 4B)(a - ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{5a^3(3iA + 4B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{5a^3(3iA + 4B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{5a^3(3iA + 4B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}} \\
&= \frac{5a^{7/2}(3iA + 4B) \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right)}{\sqrt{c} f}
\end{aligned}$$

**Mathematica [A]**

time = 8.91, size = 481, normalized size = 1.70

$$\frac{5a^4 + 4B\sqrt{c} \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{\sqrt{c} f} \text{ArcTan} \left[ \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}} \right] - \frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{f \sqrt{c - ictan(e + fx)}} - \frac{5a^3(3iA + 4B)\sqrt{a + ia \tan(e + fx)}}{f \sqrt{c - ictan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x]))/Sqrt[c - I\*c\*Tan[e + f\*x]], x]

[Out] (5\*((3\*I)\*A + 4\*B)\*Sqrt[E^(I\*f\*x)]\*Sqrt[E^(I\*(e + f\*x))]/(1 + E^((2\*I)\*(e + f\*x))))\*ArcTan[E^(I\*(e + f\*x))]\*(a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e +

$$\begin{aligned} & f*x)))/(E^{(I*(4*e + f*x))*\text{Sqrt}[c/(1 + E^{((2*I)*(e + f*x))})*f*\text{Sec}[e + f*x]} \\ & ^{(9/2)*(Cos[f*x] + I*\text{Sin}[f*x])^{(7/2)*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x])} + ( \\ & Cos[e + f*x]^4*((A - I*B)*\text{Cos}[2*f*x]*((-4*I)*\text{Cos}[e])/c - (4*\text{Sin}[e])/c) + S \\ & ec[e]*(16*A*\text{Cos}[e] - (24*I)*B*\text{Cos}[e] + I*A*\text{Sin}[e] + 4*B*\text{Sin}[e])*((-1/2*I)* \\ & Cos[3*e])/c - \text{Sin}[3*e]/(2*c)) + \text{Sec}[e + f*x]^2*((B*\text{Cos}[3*e])/(3*c) - ((I/3) \\ & *B*\text{Sin}[3*e])/c) + \text{Sec}[e]*\text{Sec}[e + f*x]*(\text{Cos}[3*e]/(2*c) - ((I/2)*\text{Sin}[3*e])/c) \\ & *(A*\text{Sin}[f*x] - (4*I)*B*\text{Sin}[f*x]) + (A - I*B)*((4*\text{Cos}[e])/c - ((4*I)*\text{Sin}[e] \\ & /c)*\text{Sin}[2*f*x])* \text{Sqrt}[\text{Sec}[e + f*x]*(c*\text{Cos}[e + f*x] - I*c*\text{Sin}[e + f*x])]*(a + \\ & I*a*\text{Tan}[e + f*x])^{(7/2)*(A + B*\text{Tan}[e + f*x])}/(f*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^3 \\ & *(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x])) \end{aligned}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs.  $2(234) = 468$ .  
time = 0.45, size = 627, normalized size = 2.22

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a^3\left(-60iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac}(1)}{\sqrt{ac}}\right)}{\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a^3\left(-60iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac}(1)}{\sqrt{ac}}\right)}\right)}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a^3\left(-60iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac}(1)}{\sqrt{ac}}\right)}{\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}a^3\left(-60iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac}(1)}{\sqrt{ac}}\right)}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c*(-60*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+8*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3-2*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+90*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+18*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+45*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+60*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+128*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+120*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+24*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-72*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-45*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-93*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-94*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*x+e))^2/(a*c)^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1403 vs.  $2(229) = 458$ .  
time = 0.96, size = 1403, normalized size = 4.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -6*(12*(9*A - 20*I*B)*a^3*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 32*(6*A - 11*I*B)*a^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 12*(9*I*A + 20*B)*a^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 32*(6*I*A + 11*B)*a^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 30*((3*A - 4*I*B)*a^3*\cos(6*f*x + 6*e) + 3*(3*A - 4*I*B)*a^3*\cos(4*f*x + 4*e) + 3*(3*A - 4*I*B)*a^3*\cos(2*f*x + 2*e) - (-3*I*A - 4*B)*a^3*\sin(6*f*x + 6*e) - 3*(-3*I*A - 4*B)*a^3*\sin(4*f*x + 4*e) - 3*(-3*I*A - 4*B)*a^3*\sin(2*f*x + 2*e) + (3*A - 4*I*B)*a^3*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) - 30*((3*A - 4*I*B)*a^3*\cos(6*f*x + 6*e) + 3*(3*A - 4*I*B)*a^3*\cos(4*f*x + 4*e) + 3*(3*A - 4*I*B)*a^3*\cos(2*f*x + 2*e) - (-3*I*A - 4*B)*a^3*\sin(6*f*x + 6*e) - 3*(-3*I*A - 4*B)*a^3*\sin(4*f*x + 4*e) - 3*(-3*I*A - 4*B)*a^3*\sin(2*f*x + 2*e) + (3*A - 4*I*B)*a^3*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 12*(8*(A - I*B)*a^3*\cos(6*f*x + 6*e) + 24*(A - I*B)*a^3*\cos(4*f*x + 4*e) + 24*(A - I*B)*a^3*\cos(2*f*x + 2*e) + 8*(I*A + B)*a^3*\sin(6*f*x + 6*e) + 24*(I*A + B)*a^3*\sin(4*f*x + 4*e) + 24*(I*A + B)*a^3*\sin(2*f*x + 2*e) + 5*(3*A - 4*I*B)*a^3*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 15*((-3*I*A - 4*B)*a^3*\cos(6*f*x + 6*e) + 3*(-3*I*A - 4*B)*a^3*\cos(4*f*x + 4*e) + 3*(-3*I*A - 4*B)*a^3*\cos(2*f*x + 2*e) + (3*A - 4*I*B)*a^3*\sin(6*f*x + 6*e) + 3*(3*A - 4*I*B)*a^3*\sin(4*f*x + 4*e) + 3*(3*A - 4*I*B)*a^3*\sin(2*f*x + 2*e) + (-3*I*A - 4*B)*a^3*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((3*I*A + 4*B)*a^3*\cos(6*f*x + 6*e) + 3*(3*I*A + 4*B)*a^3*\cos(4*f*x + 4*e) + 3*(3*I*A + 4*B)*a^3*\cos(2*f*x + 2*e) - (3*A - 4*I*B)*a^3*\sin(6*f*x + 6*e) - 3*(3*A - 4*I*B)*a^3*\sin(4*f*x + 4*e) - 3*(3*A - 4*I*B)*a^3*\sin(2*f*x + 2*e) + (3*I*A + 4*B)*a^3*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 12*(8*(I*A + B)*a^3*\cos(6*f*x + 6*e) + 24*(I*A + B)*a^3*\cos(4*f*x + 4*e) + 24*(I*A + B)*a^3*\cos(2*f*x + 2*e) - 8*(A - I*B)*a^3*\sin(6*f*x + 6*e) - 24*(A - I*B)*a^3*\sin(4*f*x + 4*e) - 24*(A - I*B)*a^3*\sin(2*f*x + 2*e) + 5*(3*I*A + 4*B)*a^3*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-72*I*c*\cos(6*f*x + 6*e) - 216*I*c*\cos(4*f*x + 4*e) - 216*I*c*\cos(2*f*x + 2*e) + \end{aligned}$$

$72*c*\sin(6*f*x + 6*e) + 216*c*\sin(4*f*x + 4*e) + 216*c*\sin(2*f*x + 2*e) - 72*I*c*f$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 614 vs.  $2(229) = 458$ .

time = 3.44, size = 614, normalized size = 2.17

$$\frac{\sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} \sqrt{\frac{a}{e^{2Ifx} + 2Ie} + 1}}{\sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} \sqrt{\frac{a}{e^{2Ifx} + 2Ie} + 1}}} + \frac{\sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} \sqrt{\frac{a}{e^{2Ifx} + 2Ie} + 1}}{\sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} \sqrt{\frac{a}{e^{2Ifx} + 2Ie} + 1}}} + \frac{\sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} \sqrt{\frac{a}{e^{2Ifx} + 2Ie} + 1}}{\sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} \sqrt{\frac{a}{e^{2Ifx} + 2Ie} + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} * (15 * \sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} * a^{7/2} / (c * f^2)) * (c * f * e^{4Ifx} + 4I * e) + 2 * c * f * e^{2Ifx} + 2I * e + c * f) * \log(4 * (2 * ((-3IA - 4B) * a^3 * e^{3Ifx} + 3I * e) + (-3IA - 4B) * a^3 * e^{Ifx + I * e})) * \sqrt{a / (e^{2Ifx} + 2I * e) + 1}) * \sqrt{c / (e^{2Ifx} + 2I * e) + 1}) + \sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} * a^{7/2} / (c * f^2) * (c * f * e^{2Ifx} + 2I * e) - c * f) / ((-3IA - 4B) * a^3 * e^{2Ifx} + 2I * e) + (-3IA - 4B) * a^3) - 15 * \sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} * a^{7/2} / (c * f^2) * (c * f * e^{4Ifx} + 4I * e) + 2 * c * f * e^{2Ifx} + 2I * e + c * f) * \log(4 * (2 * ((-3IA - 4B) * a^3 * e^{3Ifx} + 3I * e) + (-3IA - 4B) * a^3 * e^{Ifx + I * e})) * \sqrt{a / (e^{2Ifx} + 2I * e) + 1}) * \sqrt{c / (e^{2Ifx} + 2I * e) + 1}) - \sqrt{\frac{9A^2 - 24IAB - 16B^2}{c^2 f^2}} * a^{7/2} / (c * f^2) * (c * f * e^{2Ifx} + 2I * e) - c * f) / ((-3IA - 4B) * a^3 * e^{2Ifx} + 2I * e) + (-3IA - 4B) * a^3) - 4 * (24 * (IA + B) * a^3 * e^{7Ifx} + 7I * e) + 33 * (3IA + 4B) * a^3 * e^{5Ifx} + 5I * e) + 40 * (3IA + 4B) * a^3 * e^{3Ifx} + 3I * e) + 15 * (3IA + 4B) * a^3 * e^{Ifx + I * e}) * \sqrt{a / (e^{2Ifx} + 2I * e) + 1}) * \sqrt{c / (e^{2Ifx} + 2I * e) + 1}) / (c * f * e^{4Ifx} + 4I * e) + 2 * c * f * e^{2Ifx} + 2I * e + c * f)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(1/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)/sqrt(-I*c*tan(f*x + e) + c), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{7/2}}{\sqrt{c - c \tan(e + f x) i}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*x)*1i)^(1/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*x)*1i)^(1/2), x)
```

$$3.822 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=285

$$\frac{5a^{7/2}(2iA + 5B)\text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{c^{3/2}f} - \frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ic \tan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(c - ic \tan(e + fx))^{5/2}}{3cf \sqrt{c - ic \tan(e + fx)}}$$

[Out]  $-5a^{7/2}(2iA+5B)\text{arctan}(c^{1/2}(a+Ia*\tan(f*x+e))^{1/2}/a^{1/2}/(c-Ic*\tan(f*x+e))^{1/2})/c^{3/2}/f+5/2*a^3*(2iA+5B)*(a+Ia*\tan(f*x+e))^{1/2}/(c-Ic*\tan(f*x+e))^{1/2}/c^2/f+5/6*a^2*(2iA+5B)*(c-Ic*\tan(f*x+e))^{1/2}/c^2/(a+Ia*\tan(f*x+e))^{3/2}/c^2/f+2/3*a*(2iA+5B)*(a+Ia*\tan(f*x+e))^{5/2}/c/f/(c-Ic*\tan(f*x+e))^{1/2}-1/3*(I*A+B)*(a+Ia*\tan(f*x+e))^{7/2}/f/(c-Ic*\tan(f*x+e))^{3/2}$

**Rubi [A]**

time = 0.23, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{5a^{7/2}(5B+2iA)\text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{c^{3/2}f} + \frac{5a^3(5B+2iA)\sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{2c^2f} + \frac{5a^2(5B+2iA)(a + ia \tan(e + fx))^{3/2} \sqrt{c - ic \tan(e + fx)}}{6c^2f} + \frac{2a(5B+2iA)(a + ia \tan(e + fx))^{5/2}}{3cf \sqrt{c - ic \tan(e + fx)}} - \frac{(B+iA)(a + ia \tan(e + fx))^{7/2}}{3f(c - ic \tan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + I*a*\text{Tan}[e + f*x])^{7/2}*(A + B*\text{Tan}[e + f*x])}{(c - I*c*\text{Tan}[e + f*x])^{3/2}}, x]$

[Out]  $(-5a^{7/2}*((2*I)*A + 5*B)*\text{ArcTan}[\frac{\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]}{\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]}])/c^{3/2}*f - ((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{7/2})/(3*f*(c - I*c*\text{Tan}[e + f*x])^{3/2}) + (2*a*((2*I)*A + 5*B)*(a + I*a*\text{Tan}[e + f*x])^{5/2})/(3*c*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) + (5*a^3*((2*I)*A + 5*B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(2*c^2*f) + (5*a^2*((2*I)*A + 5*B)*(a + I*a*\text{Tan}[e + f*x])^{3/2}*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(6*c^2*f)$

**Rule 49**

$\text{Int}[\frac{(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}}{(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))}, x] - \text{Dist}[d*(n/(b*(m + 1))), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2\*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

**Rule 52**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
negerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \tan(e + fx))^{7/2}(A + B \tan(e + fx))}{(c - ictan(e + fx))^{3/2}} dx &= \frac{(ac)\text{Subst}\left(\int \frac{(a+iax)^{5/2}(A+Bx)}{(c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{f} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} - \frac{(a(2A - 5iB))\text{Subst}\left(\int \frac{(a+iax)^{5/2}}{(c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{3f(c - ictan(e + fx))^{3/2}} + \frac{2a(2iA + 5B)(a + ia \tan(e + fx))^{5/2}}{3cf\sqrt{c - ictan(e + fx)}} \\
 &= -\frac{5a^{7/2}(2iA + 5B) \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{c^{3/2}f}
 \end{aligned}$$

**Mathematica [A]**

time = 9.89, size = 517, normalized size = 1.81

5024 - 5429... ArcTan[...]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(3/2), x]
```

```
[Out] ((-5*I)*(2*A - (5*I)*B)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))]/(1 + E^((2*I)*(e + f*x))))*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c*I*E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e
```



$$+ f*x]^{(9/2)}*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^{(7/2)}*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]) + (\text{Cos}[e + f*x]^4*((5*I)*A + 11*B)*\text{Cos}[2*f*x]*((2*\text{Cos}[e])/3*c^2) - ((2*I)/3)*\text{Sin}[e])/c^2 + (A - I*B)*\text{Cos}[4*f*x]*((( -2*I)/3)*\text{Cos}[e])/c^2 + (2*\text{Sin}[e])/3*c^2) + \text{Sec}[e]*((10*I)*A*\text{Cos}[e] + 26*B*\text{Cos}[e] + I*B*\text{Sin}[e])* (\text{Cos}[3*e]/(2*c^2) - ((I/2)*\text{Sin}[3*e])/c^2) + I*B*\text{Sec}[e]*\text{Sec}[e + f*x]*(\text{Cos}[3*e]/(2*c^2) - ((I/2)*\text{Sin}[3*e])/c^2)*\text{Sin}[f*x] + (5*A - (11*I)*B)*((-2*\text{Cos}[e])/3*c^2) + (((2*I)/3)*\text{Sin}[e])/c^2)*\text{Sin}[2*f*x] + (A - I*B)*((2*\text{Cos}[e])/3*c^2) + (((2*I)/3)*\text{Sin}[e])/c^2)*\text{Sin}[4*f*x]]*\text{Sqrt}[\text{Sec}[e + f*x]*(c*\text{Cos}[e + f*x] - I*c*\text{Sin}[e + f*x])]*(a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(A + B*\text{Tan}[e + f*x])]/(f*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^3*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]))$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs.  $2(234) = 468$ .

time = 0.42, size = 731, normalized size = 2.56 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $1/6/f*(a*(1+I*\text{tan}(f*x+e)))^{(1/2)}*(-c*(I*\text{tan}(f*x+e)-1))^{(1/2)}*a^3/c^2*(-114*I*A*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\text{tan}(f*x+e)-118*I*B*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}-30*I*A*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c+225*I*B*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c*\text{tan}(f*x+e)+30*A*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c*\text{tan}(f*x+e)^2+225*B*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c*\text{tan}(f*x+e)^2+21*B*(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*\text{tan}(f*x+e)^3+6*I*A*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\text{tan}(f*x+e)^3+3*I*B*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\text{tan}(f*x+e)^4-90*A*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c*\text{tan}(f*x+e)-74*A*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\text{tan}(f*x+e)^2-75*I*B*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c*\text{tan}(f*x+e)^3-75*B*\ln((a*c*\text{tan}(f*x+e)+(a*c)^{(1/2)}*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}))/(a*c)^{(1/2)}*a*c-279*B*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}*\text{tan}(f*x+e)+46*A*(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}*(a*c)^{(1/2)}/(a*c*(1+\text{tan}(f*x+e)^2))^{(1/2)}/(a*c)^{(1/2)}/(I+\text{tan}(f*x+e))^3$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1240 vs.  $2(231) = 462$ .

time = 0.74, size = 1240, normalized size = 4.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(30*((2*A - 5*I*B)*a^3*cos(4*f*x + 4*e) + 2*(2*A - 5*I*B)*a^3*cos(2*f*x + 2*e) - (-2*I*A - 5*B)*a^3*sin(4*f*x + 4*e) - 2*(-2*I*A - 5*B)*a^3*sin(2*f*x + 2*e) + (2*A - 5*I*B)*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 30*((2*A - 5*I*B)*a^3*cos(4*f*x + 4*e) + 2*(2*A - 5*I*B)*a^3*cos(2*f*x + 2*e) - (-2*I*A - 5*B)*a^3*sin(4*f*x + 4*e) - 2*(-2*I*A - 5*B)*a^3*sin(2*f*x + 2*e) + (2*A - 5*I*B)*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 4*(4*(A - I*B)*a^3*cos(4*f*x + 4*e) + 8*(A - I*B)*a^3*cos(2*f*x + 2*e) - 4*(-I*A - B)*a^3*sin(4*f*x + 4*e) - 8*(-I*A - B)*a^3*sin(2*f*x + 2*e) - (2*A - 29*I*B)*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 12*(8*(A - 2*I*B)*a^3*cos(4*f*x + 4*e) + 16*(A - 2*I*B)*a^3*cos(2*f*x + 2*e) + 8*(I*A + 2*B)*a^3*sin(4*f*x + 4*e) + 16*(I*A + 2*B)*a^3*sin(2*f*x + 2*e) + 5*(2*A - 5*I*B)*a^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 15*((-2*I*A - 5*B)*a^3*cos(4*f*x + 4*e) + 2*(-2*I*A - 5*B)*a^3*cos(2*f*x + 2*e) + (2*A - 5*I*B)*a^3*sin(4*f*x + 4*e) + 2*(2*A - 5*I*B)*a^3*sin(2*f*x + 2*e) + (-2*I*A - 5*B)*a^3)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) - 15*((2*I*A + 5*B)*a^3*cos(4*f*x + 4*e) + 2*(2*I*A + 5*B)*a^3*cos(2*f*x + 2*e) - (2*A - 5*I*B)*a^3*sin(4*f*x + 4*e) - 2*(2*A - 5*I*B)*a^3*sin(2*f*x + 2*e) + (2*I*A + 5*B)*a^3)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) - 4*(4*(-I*A - B)*a^3*cos(4*f*x + 4*e) + 8*(-I*A - B)*a^3*cos(2*f*x + 2*e) + 4*(A - I*B)*a^3*sin(4*f*x + 4*e) + 8*(A - I*B)*a^3*sin(2*f*x + 2*e) + (2*I*A + 29*B)*a^3)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 12*(8*(I*A + 2*B)*a^3*cos(4*f*x + 4*e) + 16*(I*A + 2*B)*a^3*cos(2*f*x + 2*e) - 8*(A - 2*I*B)*a^3*sin(4*f*x + 4*e) - 16*(A - 2*I*B)*a^3*sin(2*f*x + 2*e) + 5*(2*I*A + 5*B)*a^3)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-24*I*c^2*cos(4*f*x + 4*e) - 48*I*c^2*cos(2*f*x + 2*e) + 24*c^2*sin(4*f*x + 4*e) + 48*c^2*sin(2*f*x + 2*e) - 24*I*c^2)*f)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(231) = 462.

time = 2.06, size = 592, normalized size = 2.08

$$\frac{\sqrt{a} \sqrt{c} \left( \frac{(A - I B) a^3 \cos(4 f x + 4 e) + 2 (2 A - 5 I B) a^3 \cos(2 f x + 2 e) - (-2 I A - 5 B) a^3 \sin(4 f x + 4 e) - 2 (-2 I A - 5 B) a^3 \sin(2 f x + 2 e) + (2 A - 5 I B) a^3}{\cos(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e)))} + 1 \right) + 30 \left( \frac{(A - I B) a^3 \cos(4 f x + 4 e) + 2 (2 A - 5 I B) a^3 \cos(2 f x + 2 e) - (-2 I A - 5 B) a^3 \sin(4 f x + 4 e) - 2 (-2 I A - 5 B) a^3 \sin(2 f x + 2 e) + (2 A - 5 I B) a^3}{\cos(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e)))} - \sin(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))) \right) + 4 \left( 4 (A - I B) a^3 \cos(4 f x + 4 e) + 8 (A - I B) a^3 \cos(2 f x + 2 e) - 4 (-I A - B) a^3 \sin(4 f x + 4 e) - 8 (-I A - B) a^3 \sin(2 f x + 2 e) - (2 A - 29 I B) a^3 \cos(3/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))) \right) - 12 \left( 8 (A - 2 I B) a^3 \cos(4 f x + 4 e) + 16 (A - 2 I B) a^3 \cos(2 f x + 2 e) + 8 (I A + 2 B) a^3 \sin(4 f x + 4 e) + 16 (I A + 2 B) a^3 \sin(2 f x + 2 e) + 5 (2 A - 5 I B) a^3 \cos(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))) \right) - 15 \left( (-2 I A - 5 B) a^3 \cos(4 f x + 4 e) + 2 (-2 I A - 5 B) a^3 \cos(2 f x + 2 e) + (2 A - 5 I B) a^3 \sin(4 f x + 4 e) + 2 (2 A - 5 I B) a^3 \sin(2 f x + 2 e) + (-2 I A - 5 B) a^3 \right) \log(\cos(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))))^2 + \sin(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))))^2 + 2 \sin(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e)))) + 1) - 15 \left( (2 I A + 5 B) a^3 \cos(4 f x + 4 e) + 2 (2 I A + 5 B) a^3 \cos(2 f x + 2 e) - (2 A - 5 I B) a^3 \sin(4 f x + 4 e) - 2 (2 A - 5 I B) a^3 \sin(2 f x + 2 e) + (2 I A + 5 B) a^3 \right) \log(\cos(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))))^2 + \sin(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e))))^2 - 2 \sin(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e)))) + 1) - 4 \left( 4 (-I A - B) a^3 \cos(4 f x + 4 e) + 8 (-I A - B) a^3 \cos(2 f x + 2 e) + 4 (A - I B) a^3 \sin(4 f x + 4 e) + 8 (A - I B) a^3 \sin(2 f x + 2 e) + (2 I A + 29 B) a^3 \right) \sin(3/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e)))) - 12 \left( 8 (I A + 2 B) a^3 \cos(4 f x + 4 e) + 16 (I A + 2 B) a^3 \cos(2 f x + 2 e) - 8 (A - 2 I B) a^3 \sin(4 f x + 4 e) - 16 (A - 2 I B) a^3 \sin(2 f x + 2 e) + 5 (2 I A + 5 B) a^3 \right) \sin(1/2 \arctan(2 \sin(2 f x + 2 e) / \cos(2 f x + 2 e)))) \sqrt{a} \sqrt{c} / ((-24 I c^2 \cos(4 f x + 4 e) - 48 I c^2 \cos(2 f x + 2 e) + 24 c^2 \sin(4 f x + 4 e) + 48 c^2 \sin(2 f x + 2 e) - 24 I c^2) f)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(15*(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)*sqrt((4*A^2 - 20*I*A*B - 25*B
^2)*a^7/(c^3*f^2))*log(4*(2*((-2*I*A - 5*B)*a^3*e^(3*I*f*x + 3*I*e) + (-2*I
*A - 5*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1)) + (c^2*f*e^(2*I*f*x + 2*I*e) - c^2*f)*sqrt((4*A^2 -
20*I*A*B - 25*B^2)*a^7/(c^3*f^2)))/((-2*I*A - 5*B)*a^3*e^(2*I*f*x + 2*I*e)
+ (-2*I*A - 5*B)*a^3)) - 15*(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)*sqrt((4*A^
2 - 20*I*A*B - 25*B^2)*a^7/(c^3*f^2))*log(4*(2*((-2*I*A - 5*B)*a^3*e^(3*I*f
*x + 3*I*e) + (-2*I*A - 5*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^2*f*e^(2*I*f*x + 2*I*e) - c
^2*f)*sqrt((4*A^2 - 20*I*A*B - 25*B^2)*a^7/(c^3*f^2)))/((-2*I*A - 5*B)*a^3*
e^(2*I*f*x + 2*I*e) + (-2*I*A - 5*B)*a^3)) + 4*(4*(I*A + B)*a^3*e^(7*I*f*x
+ 7*I*e) + 8*(-2*I*A - 5*B)*a^3*e^(5*I*f*x + 5*I*e) + 25*(-2*I*A - 5*B)*a^3
*e^(3*I*f*x + 3*I*e) + 15*(-2*I*A - 5*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*
I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2*f*e^(2*I*f*x +
2*I*e) + c^2*f)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)/(-I*c*tan(f*x +
e) + c)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{7/2}}{(c - c \tan(e + f x) \operatorname{li})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*x)  
)*1i)^(3/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*x)  
)*1i)^(3/2), x)
```

$$3.823 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=283

$$\frac{2a^{7/2}(iA+6B)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{c^{5/2}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{5f(c-ic \tan(e+fx))^{5/2}} + \frac{2a(iA+6B)(a+ia \tan(e+fx))^{5/2}}{15cf(c-ic \tan(e+fx))^{3/2}}$$

[Out]  $2a^{7/2}(iA+6B)\text{arctan}(c^{1/2}(a+I*a*\tan(f*x+e))^{1/2}/a^{1/2}/(c-I*c*\tan(f*x+e))^{1/2})/c^{5/2}/f - a^3(I*A+6*B)*(a+I*a*\tan(f*x+e))^{3/2}/c^2/f/(c-I*c*\tan(f*x+e))^{1/2} - 1/5*(I*A+B)*(a+I*a*\tan(f*x+e))^{7/2}/f/(c-I*c*\tan(f*x+e))^{5/2} + 2/15*a*(I*A+6*B)*(a+I*a*\tan(f*x+e))^{5/2}/c/f/(c-I*c*\tan(f*x+e))^{3/2}$

**Rubi** [A]

time = 0.23, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{2a^{7/2}(6B+iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia \tan(e+fx)}}{\sqrt{a}\sqrt{c-ic \tan(e+fx)}}\right)}{c^{5/2}f} - \frac{a^3(6B+iA)\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}}{c^2f} - \frac{2a^2(6B+iA)(a+ia \tan(e+fx))^{3/2}}{3c^2f\sqrt{c-ic \tan(e+fx)}} + \frac{2a(6B+iA)(a+ia \tan(e+fx))^{5/2}}{15cf(c-ic \tan(e+fx))^{3/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{5f(c-ic \tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a+I*a*\text{Tan}[e+f*x])^{7/2}(A+B*\text{Tan}[e+f*x])}{(c-I*c*\text{Tan}[e+f*x])^{5/2}}, x]$

[Out]  $(2*a^{7/2}(I*A+6*B)*\text{ArcTan}[\frac{\text{Sqrt}[c]*\text{Sqrt}[a+I*a*\text{Tan}[e+f*x]]}{\text{Sqrt}[a]*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]}])/c^{5/2}/f - ((I*A+B)*(a+I*a*\text{Tan}[e+f*x])^{7/2})/(5*f*(c-I*c*\text{Tan}[e+f*x])^{5/2}) + (2*a*(I*A+6*B)*(a+I*a*\text{Tan}[e+f*x])^{5/2})/(15*c*f*(c-I*c*\text{Tan}[e+f*x])^{3/2}) - (2*a^2*(I*A+6*B)*(a+I*a*\text{Tan}[e+f*x])^{3/2})/(3*c^2*f*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]]) - (a^3*(I*A+6*B)*\text{Sqrt}[a+I*a*\text{Tan}[e+f*x]]*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])/c^3*f$

**Rule 49**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \text{Dist}[d*(n/(b*(m+1))), \text{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1] \&\& !(\text{IntegerQ}[n] \&\& !\text{IntegerQ}[m]) \&\& !(I\text{LeQ}[m+n+2, 0] \&\& (\text{FractionQ}[m] || \text{GeQ}[2*n+m+1, 0])) \& \& \text{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 52**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned} & \left( \frac{9}{2} \right) * (\cos[f*x] + I * \sin[f*x])^{7/2} * (A * \cos[e + f*x] + B * \sin[e + f*x]) + ( \\ & \cos[e + f*x]^4 * ((A - (6*I)*B) * \cos[2*f*x] * (((-2*I)/3) * \cos[e]) / c^3 - (2 * \sin[ \\ & e]) / (3 * c^3)) + (I * A + 6 * B) * \cos[4*f*x] * ((2 * \cos[e]) / (15 * c^3) + (((2*I)/15) * \sin[ \\ & e]) / c^3) + (A - (6*I)*B) * (((-I) * \cos[3*e]) / c^3 - \sin[3*e] / c^3) + (A - I * B) \\ & * \cos[6*f*x] * (((-1/5 * I) * \cos[3*e]) / c^3 + \sin[3*e] / (5 * c^3)) + (A - (6*I)*B) * (( \\ & 2 * \cos[e]) / (3 * c^3) - (((2*I)/3) * \sin[e]) / c^3) * \sin[2*f*x] + (A - (6*I)*B) * ((-2 \\ & * \cos[e]) / (15 * c^3) - (((2*I)/15) * \sin[e]) / c^3) * \sin[4*f*x] + (A - I * B) * (\cos[3 * \\ & e] / (5 * c^3) + ((I/5) * \sin[3*e]) / c^3) * \sin[6*f*x] * \sqrt{\sec[e + f*x] * (c * \cos[e + \\ & f*x] - I * c * \sin[e + f*x])} * (a + I * a * \tan[e + f*x])^{7/2} * (A + B * \tan[e + f*x] \\ & )) / (f * (\cos[f*x] + I * \sin[f*x])^3 * (A * \cos[e + f*x] + B * \sin[e + f*x])) \end{aligned}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 832 vs.  $2(234) = 468$ .

time = 0.43, size = 833, normalized size = 2.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^3*(246
*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-90*I*B*ln((a*c*t
an(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*
x+e)^4+15*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a
*c)^(1/2))*a*c*tan(f*x+e)^4+26*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)
+60*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(
1/2))*a*c*tan(f*x+e)^3+360*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*
x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+15*B*(a*c*(1+tan(f*x+e)^2))^(
1/2)*(a*c)^(1/2)*tan(f*x+e)^4-474*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1
/2)*tan(f*x+e)+540*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-90*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*
(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-46*A*(a*c*(1+ta
n(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-90*I*B*ln((a*c*tan(f*x+e)+(a*c)
^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-94*I*A*(a*c*(1+tan(f*
x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-360*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)
)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-564*B*(a*c*(1+t
an(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-60*I*A*ln((a*c*tan(f*x+e)+(a*c)
)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+15*A*ln((
a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+7
4*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+141*B*(a*c*(1+tan(f
*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*x+e))^4/
(a*c)^(1/2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 990 vs.  $2(231) = 462$ .

time = 0.66, size = 990, normalized size = 3.50



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="maxima")

[Out] 15\*(30\*((A - 6\*I\*B)\*a^3\*cos(2\*f\*x + 2\*e) - (-I\*A - 6\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (A - 6\*I\*B)\*a^3)\*arctan2(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))), sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) + 30\*((A - 6\*I\*B)\*a^3\*cos(2\*f\*x + 2\*e) - (-I\*A - 6\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (A - 6\*I\*B)\*a^3)\*arctan2(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))), -sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) - 12\*((A - I\*B)\*a^3\*cos(2\*f\*x + 2\*e) + (I\*A + B)\*a^3\*sin(2\*f\*x + 2\*e) + (A - I\*B)\*a^3)\*cos(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 20\*((A - 3\*I\*B)\*a^3\*cos(2\*f\*x + 2\*e) - (-I\*A - 3\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (A - 3\*I\*B)\*a^3)\*cos(3/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - 60\*((A - 5\*I\*B)\*a^3\*cos(2\*f\*x + 2\*e) + (I\*A + 5\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (A - 6\*I\*B)\*a^3)\*cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - 15\*((-I\*A - 6\*B)\*a^3\*cos(2\*f\*x + 2\*e) + (A - 6\*I\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (-I\*A - 6\*B)\*a^3)\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) - 15\*((I\*A + 6\*B)\*a^3\*cos(2\*f\*x + 2\*e) - (A - 6\*I\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (I\*A + 6\*B)\*a^3)\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 - 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) - 12\*((I\*A + B)\*a^3\*cos(2\*f\*x + 2\*e) - (A - I\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (I\*A + B)\*a^3)\*sin(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - 20\*((-I\*A - 3\*B)\*a^3\*cos(2\*f\*x + 2\*e) + (A - 3\*I\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (-I\*A - 3\*B)\*a^3)\*sin(3/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - 60\*((I\*A + 5\*B)\*a^3\*cos(2\*f\*x + 2\*e) - (A - 5\*I\*B)\*a^3\*sin(2\*f\*x + 2\*e) + (I\*A + 6\*B)\*a^3)\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*sqrt(a)\*sqrt(c)/((-450\*I\*c^3\*cos(2\*f\*x + 2\*e) + 450\*c^3\*sin(2\*f\*x + 2\*e) - 450\*I\*c^3)\*f)

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 533 vs.  $2(231) = 462$ .

time = 1.62, size = 533, normalized size = 1.88

$$\frac{\sqrt{a} \sqrt{c} \sqrt{A^2 - 12IA^*B - 36B^2} \left( \frac{\sqrt{A^2 - 12IA^*B - 36B^2}}{2\sqrt{a}} \sqrt{\frac{A - 12IA^*B - 36B^2}{4a}} - \frac{\sqrt{A^2 - 12IA^*B - 36B^2}}{2\sqrt{a}} \sqrt{\frac{A - 12IA^*B - 36B^2}{4a}} \right) - 43(IA + 5B)a^{3/2}e^{2Ie} - 43(-IA - 6B)a^{3/2}e^{Ie} + 43(IA + 6B)a^{3/2}e^{Ie} + 43(IA + 6B)a^{3/2}e^{Ie}}{20\sqrt{a} \sqrt{c} \sqrt{A^2 - 12IA^*B - 36B^2} \sqrt{a} \sqrt{c} \sqrt{A^2 - 12IA^*B - 36B^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] 1/30\*(15\*c^3\*sqrt((A^2 - 12\*I\*A\*B - 36\*B^2)\*a^7/(c^5\*f^2))\*f\*log(-4\*(2\*((-I\*A - 6\*B)\*a^3\*e^(3\*I\*f\*x + 3\*I\*e) + (-I\*A - 6\*B)\*a^3\*e^(I\*f\*x + I\*e)))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)) + (c^3\*f\*e^(2\*I\*f\*x + 2\*I\*e) - c^3\*f)\*sqrt((A^2 - 12\*I\*A\*B - 36\*B^2)\*a^7/(c^5\*f^2)))/((

```
I*A + 6*B)*a^3*e^(2*I*f*x + 2*I*e) + (I*A + 6*B)*a^3)) - 15*c^3*sqrt((A^2 -
12*I*A*B - 36*B^2)*a^7/(c^5*f^2))*f*log(-4*(2*((-I*A - 6*B)*a^3*e^(3*I*f*x
+ 3*I*e) + (-I*A - 6*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^3*f*e^(2*I*f*x + 2*I*e) - c^3*f
)*sqrt((A^2 - 12*I*A*B - 36*B^2)*a^7/(c^5*f^2)))/((I*A + 6*B)*a^3*e^(2*I*f*
x + 2*I*e) + (I*A + 6*B)*a^3)) - 4*(3*(I*A + B)*a^3*e^(7*I*f*x + 7*I*e) + 2
*(-I*A - 6*B)*a^3*e^(5*I*f*x + 5*I*e) + 10*(I*A + 6*B)*a^3*e^(3*I*f*x + 3*I
*e) + 15*(I*A + 6*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^3*f)
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)/(-I*c*tan(f*x +
e) + c)^(5/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{7/2}}{(c - c \tan(e + f x) \operatorname{li})^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(7/2))/(c - c*tan(e + f*x
)*li)^(5/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*li)^(7/2))/(c - c*tan(e + f*x
)*li)^(5/2), x)
```

$$3.824 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{7/2}} dx$$

Optimal. Leaf size=251

$$\frac{2a^{7/2}B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ic \tan(e+fx)}}\right)}{c^{7/2}f} - \frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{7f(c-ic \tan(e+fx))^{7/2}} + \frac{2aB(a+ia \tan(e+fx))}{5cf(c-ic \tan(e+fx))}$$

[Out]  $-2*a^{(7/2)}*B*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/c^{(7/2)}/f+2*a^3*B*(a+I*a*\tan(f*x+e))^{(1/2)}/c^3/f/(c-I*c*\tan(f*x+e))^{(1/2)}-1/7*(I*A+B)*(a+I*a*\tan(f*x+e))^{(7/2)}/f/(c-I*c*\tan(f*x+e))^{(7/2)}+2/5*a*B*(a+I*a*\tan(f*x+e))^{(5/2)}/c/f/(c-I*c*\tan(f*x+e))^{(5/2)}-2/3*a^2*B*(a+I*a*\tan(f*x+e))^{(3/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.21, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 49, 65, 223, 209}

$$-\frac{2a^{7/2}B \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ic \tan(e+fx)}}\right)}{c^{7/2}f} + \frac{2a^3B \sqrt{a+ia \tan(e+fx)}}{c^3f \sqrt{c-ic \tan(e+fx)}} - \frac{2a^2B(a+ia \tan(e+fx))^{3/2}}{3c^2f(c-ic \tan(e+fx))^{3/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{7f(c-ic \tan(e+fx))^{7/2}} + \frac{2aB(a+ia \tan(e+fx))^{5/2}}{5cf(c-ic \tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+I*a*\operatorname{Tan}[e+f*x])^{(7/2)}*(A+B*\operatorname{Tan}[e+f*x])]/(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)},x]$

[Out]  $(-2*a^{(7/2)}*B*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])])/c^{(7/2)}*f - ((I*A+B)*(a+I*a*\operatorname{Tan}[e+f*x])^{(7/2)})/(7*f*(c-I*c*\operatorname{Tan}[e+f*x])^{(7/2)}) + (2*a*B*(a+I*a*\operatorname{Tan}[e+f*x])^{(5/2)})/(5*c*f*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)}) - (2*a^2*B*(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)})/(3*c^2*f*(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)}) + (2*a^3*B*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])/(c^3*f*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])$

Rule 49

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+1))), x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x$  &&  $\operatorname{NeQ}[b*c - a*d, 0]$  &&  $\operatorname{GtQ}[n, 0]$  &&  $\operatorname{LtQ}[m, -1]$  &&  $!(\operatorname{IntegerQ}[n] \&\& \operatorname{IntegerQ}[m])$  &&  $!(\operatorname{ILeQ}[m+n+2, 0])$  &&  $(\operatorname{FractionQ}[m] \mid\mid \operatorname{GeQ}[2*n+m+1, 0])$  &&  $\operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) +$

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps





[Out]  $-1/210*(210*B*a^3*\arctan2(\cos(f*x + e), \sin(f*x + e) + 1) + 210*B*a^3*\arctan2(\cos(f*x + e), -\sin(f*x + e) + 1) - 30*(-I*A - B)*a^3*\cos(7*f*x + 7*e) - 84*B*a^3*\cos(5*f*x + 5*e) + 140*B*a^3*\cos(3*f*x + 3*e) - 420*B*a^3*\cos(f*x + e) + 105*I*B*a^3*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 + 2*\sin(f*x + e) + 1) - 105*I*B*a^3*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 - 2*\sin(f*x + e) + 1) - 30*(A - I*B)*a^3*\sin(7*f*x + 7*e) - 84*I*B*a^3*\sin(5*f*x + 5*e) + 140*I*B*a^3*\sin(3*f*x + 3*e) - 420*I*B*a^3*\sin(f*x + e))*\sqrt{a}/(c^{7/2}*f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs.  $2(207) = 414$ .

time = 1.78, size = 453, normalized size = 1.80

$$\frac{105 \sqrt{\frac{B a^3}{c^7 f}} \operatorname{arctan} \left( \frac{\sqrt{\frac{a}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{c}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{B a^3}{c^7 f}}}{\sqrt{\frac{a}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{c}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{B a^3}{c^7 f}}} \right) - 105 \sqrt{\frac{B a^3}{c^7 f}} \operatorname{arctan} \left( \frac{\sqrt{\frac{a}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{c}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{B a^3}{c^7 f}}}{\sqrt{\frac{a}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{c}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{B a^3}{c^7 f}}} \right) - 2(15(A + B)a^3e^{9I f x + 9I e} + 3(5I A - 9B)a^3e^{7I f x + 7I e} + 28B a^3e^{5I f x + 5I e} - 140B a^3e^{3I f x + 3I e} - 210B a^3e^{I f x + I e}) \sqrt{\frac{a}{2 B^2 c^2 f^2 + 1}} \sqrt{\frac{c}{2 B^2 c^2 f^2 + 1}}}{210 c^7 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x, algorithm="fricas")`

[Out]  $1/210*(105*c^4*f*\sqrt{-B^2*a^7/(c^7*f^2)}*\log(4*(2*(B*a^3*e^{(3*I*f*x + 3*I*e)} + B*a^3*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + (c^4*f*e^{(2*I*f*x + 2*I*e)} - c^4*f)*\sqrt{-B^2*a^7/(c^7*f^2)})/(B*a^3*e^{(2*I*f*x + 2*I*e)} + B*a^3)) - 105*c^4*f*\sqrt{-B^2*a^7/(c^7*f^2)}*\log(4*(2*(B*a^3*e^{(3*I*f*x + 3*I*e)} + B*a^3*e^{(I*f*x + I*e)}))*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - (c^4*f*e^{(2*I*f*x + 2*I*e)} - c^4*f)*\sqrt{-B^2*a^7/(c^7*f^2)})/(B*a^3*e^{(2*I*f*x + 2*I*e)} + B*a^3)) - 2*(15*(I*A + B)*a^3*e^{(9*I*f*x + 9*I*e)} + 3*(5*I*A - 9*B)*a^3*e^{(7*I*f*x + 7*I*e)} + 28*B*a^3*e^{(5*I*f*x + 5*I*e)} - 140*B*a^3*e^{(3*I*f*x + 3*I*e)} - 210*B*a^3*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})/(c^4*f)$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(7/2),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)/(-I*c*tan(f*x + e) + c)^(7/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) i)^{7/2}}{(c - c \tan(e + f x) i)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*x)*1i)^(7/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*x)*1i)^(7/2), x)
```



$$3.825 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{9/2}} dx$$

Optimal. Leaf size=102

$$\frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{9f(c-ic \tan(e+fx))^{9/2}} - \frac{(iA-8B)(a+ia \tan(e+fx))^{7/2}}{63cf(c-ic \tan(e+fx))^{7/2}}$$

[Out]  $-1/9*(I*A+B)*(a+I*a*\tan(f*x+e))^{(7/2)}/f/(c-I*c*\tan(f*x+e))^{(9/2)}-1/63*(I*A-8*B)*(a+I*a*\tan(f*x+e))^{(7/2)}/c/f/(c-I*c*\tan(f*x+e))^{(7/2)}$

Rubi [A]

time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ ,

Rules used = {3669, 79, 37}

$$\frac{(-8B+iA)(a+ia \tan(e+fx))^{7/2}}{63cf(c-ic \tan(e+fx))^{7/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{9f(c-ic \tan(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(A + B*\text{Tan}[e + f*x])]/(c - I*c*\text{Tan}[e + f*x])^{(9/2)}, x]$

[Out]  $-1/9*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(7/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(9/2)}) - ((I*A - 8*B)*(a + I*a*\text{Tan}[e + f*x])^{(7/2)})/(63*c*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 79

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*((e + f*x)^{(p + 1)}/(f*(p + 1)*(c*f - d*e))), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{9/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{5/2}(A+Bx)}{(c-icx)^{11/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{9f(c - ictan(e + fx))^{9/2}} + \frac{(a(A + 8iB)) \text{Subst}\left(\int \frac{1}{(c-icx)^{11/2}} dx, x, \tan(e + fx)\right)}{63cf(c - ictan(e + fx))^{9/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{9f(c - ictan(e + fx))^{9/2}} - \frac{(iA - 8B)(a + ia \tan(e + fx))^{7/2}}{63cf(c - ictan(e + fx))^{9/2}}$$

### Mathematica [A]

time = 4.51, size = 121, normalized size = 1.19

$$\frac{a^3 \cos(e + fx)((-8iA + B) \cos(e + fx) - (A + 8iB) \sin(e + fx))(\cos(8e + 11fx) + i \sin(8e + 11fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{63c^5 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[
e + f*x])^(9/2), x]
```

```
[Out] (a^3 * Cos[e + f*x] * (((-8*I)*A + B) * Cos[e + f*x] - (A + (8*I)*B) * Sin[e + f*x]
) * (Cos[8*e + 11*f*x] + I * Sin[8*e + 11*f*x]) * Sqrt[a + I*a*Tan[e + f*x]] * Sqrt
[c - I*c*Tan[e + f*x]]) / (63*c^5*f*(Cos[f*x] + I*Sin[f*x])^3)
```

### Maple [A]

time = 0.42, size = 134, normalized size = 1.31

method	result
risch	$-\frac{a^3 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (7iA e^{8i(fx+e)} + 7B e^{8i(fx+e)} + 9iA e^{6i(fx+e)} - 9B e^{6i(fx+e)})}{126c^4 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$-\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(8iB(\tan^3(fx+e))+6iA(\tan^2(fx+e)))}{63f c^5(i+\tan(fx+e))^6}$
default	$-\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(8iB(\tan^3(fx+e))+6iA(\tan^2(fx+e)))}{63f c^5(i+\tan(fx+e))^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/63/f*(a*(1+I*\tan(f*x+e)))^{(1/2)}*(-c*(I*\tan(f*x+e)-1))^{(1/2)}*a^3/c^5*(1+\tan(f*x+e)^2)*(8*I*B*\tan(f*x+e)^3+6*I*A*\tan(f*x+e)^2+A*\tan(f*x+e)^3-6*I*B*\tan(f*x+e)+15*B*\tan(f*x+e)^2-8*I*A+15*A*\tan(f*x+e)+B)/(I+\tan(f*x+e))^6$$

**Maxima** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs.  $2(82) = 164$ .

time = 0.63, size = 174, normalized size = 1.71

$$\frac{-126(7(A-iB)a^3\cos(11fx+11e)+2(8A+iB)a^3\cos(9fx+9e)+9(A+iB)a^3\cos(7fx+7e)-7(-iA-B)a^3\sin(11fx+11e)-2(-8iA+B)a^3\sin(9fx+9e)-9(-iA+B)a^3\sin(7fx+7e))\sqrt{a}\sqrt{c}}{-15876(i^5\cos(2fx+2e)-c^5\sin(2fx+2e)+i^5)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,algorithm="maxima")`

[Out] 
$$-126*(7*(A-I*B)*a^3*\cos(11*f*x+11*e)+2*(8*A+I*B)*a^3*\cos(9*f*x+9*e)+9*(A+I*B)*a^3*\cos(7*f*x+7*e)-7*(-I*A-B)*a^3*\sin(11*f*x+11*e)-2*(-8*I*A+B)*a^3*\sin(9*f*x+9*e)-9*(-I*A+B)*a^3*\sin(7*f*x+7*e))*\sqrt{a}*\sqrt{c}/((-15876*I*c^5*\cos(2*f*x+2*e)+15876*c^5*\sin(2*f*x+2*e)-15876*I*c^5)*f)$$

**Fricas** [A]

time = 2.39, size = 109, normalized size = 1.07

$$\frac{(7(iA+B)a^3e^{(11ifx+11ie)}+2(8iA-B)a^3e^{(9ifx+9ie)}+9(iA-B)a^3e^{(7ifx+7ie)})\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}}{126c^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,algorithm="fricas")`

[Out] 
$$-1/126*(7*(I*A+B)*a^3*e^{(11*I*f*x+11*I*e)}+2*(8*I*A-B)*a^3*e^{(9*I*f*x+9*I*e)}+9*(I*A-B)*a^3*e^{(7*I*f*x+7*I*e)})*\sqrt{a/(e^{(2*I*f*x+2*I*e)}+1)}*\sqrt{c/(e^{(2*I*f*x+2*I*e)}+1)}/(c^5*f)$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(9/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(9/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(7/2)/(-I\*c\*tan(f\*x + e) + c)^(9/2), x)

**Mupad [B]**

time = 11.48, size = 192, normalized size = 1.88

$$\frac{a^3 \sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)i)}{\cos(2e+2fx)+1}} (A \cos(6e+6fx) 9i + A \cos(8e+8fx) 7i - 9B \cos(6e+6fx) + 7B \cos(8e+8fx) - 9A \sin(6e+6fx) - 7A \sin(8e+8fx) - B \sin(6e+6fx) 9i + B \sin(8e+8fx) 7i)}{126 e^4 f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2))/(c - c\*tan(e + f\*x)\*1i)^(9/2),x)

[Out] -(a^3\*((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*cos(6\*e + 6\*f\*x)\*9i + A\*cos(8\*e + 8\*f\*x)\*7i - 9\*B\*cos(6\*e + 6\*f\*x) + 7\*B\*cos(8\*e + 8\*f\*x) - 9\*A\*sin(6\*e + 6\*f\*x) - 7\*A\*sin(8\*e + 8\*f\*x) - B\*sin(6\*e + 6\*f\*x)\*9i + B\*sin(8\*e + 8\*f\*x)\*7i))/(126\*c^4\*f\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2))

$$3.826 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ictan(e+fx))^{11/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{11f(c-ictan(e+fx))^{11/2}} - \frac{(2iA-9B)(a+ia \tan(e+fx))^{7/2}}{99cf(c-ictan(e+fx))^{9/2}} - \frac{(2iA-9B)(a+ia \tan(e+fx))^{7/2}}{693c^2f(c-ictan(e+fx))^{7/2}}$$

[Out]  $-1/11*(I*A+B)*(a+I*a*\tan(f*x+e))^{(7/2)}/f/(c-I*c*\tan(f*x+e))^{(11/2)}-1/99*(2*I*A-9*B)*(a+I*a*\tan(f*x+e))^{(7/2)}/c/f/(c-I*c*\tan(f*x+e))^{(9/2)}-1/693*(2*I*A-9*B)*(a+I*a*\tan(f*x+e))^{(7/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(7/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$-\frac{(-9B+2iA)(a+ia \tan(e+fx))^{7/2}}{693c^2f(c-ictan(e+fx))^{7/2}} - \frac{(-9B+2iA)(a+ia \tan(e+fx))^{7/2}}{99cf(c-ictan(e+fx))^{9/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{11f(c-ictan(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{((a + I*a*\text{Tan}[e + f*x])^{(7/2)}*(A + B*\text{Tan}[e + f*x]))}{(c - I*c*\text{Tan}[e + f*x])^{(11/2)}}, x]$

[Out]  $-1/11*((I*A + B)*(a + I*a*\text{Tan}[e + f*x])^{(7/2)})/(f*(c - I*c*\text{Tan}[e + f*x])^{(11/2)}) - (((2*I)*A - 9*B)*(a + I*a*\text{Tan}[e + f*x])^{(7/2)})/(99*c*f*(c - I*c*\text{Tan}[e + f*x])^{(9/2)}) - (((2*I)*A - 9*B)*(a + I*a*\text{Tan}[e + f*x])^{(7/2)})/(693*c^2*f*(c - I*c*\text{Tan}[e + f*x])^{(7/2)})$

**Rule 37**

$\text{Int}[\frac{((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}}{(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}*((b*c - a*d)*(m + 1)))}, x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

**Rule 47**

$\text{Int}[\frac{((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}}{(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}*((b*c - a*d)*(m + 1)))}, x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m + n + 2] \&\& \text{IntegerQ}[n] \&\& (\text{EqQ}[a, 0] || (\text{NeQ}[c, 0] \&\& \text{LtQ}[m - n, 0] \&\& \text{IntegerQ}[n])) \&\& (\text{SumSimplerQ}[m, 1] || !\text{SumSimplerQ}[n, 1])$

**Rule 79**

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_.)*((A_) + (B_.)*tan[(e_) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{11/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{5/2}(A+Bx)}{(c-icx)^{13/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{11f(c - ictan(e + fx))^{11/2}} + \frac{(a(2A + 9iB)) \text{Subst}\left(\int \frac{(a+iax)^{3/2}(A+Bx)}{(c-icx)^{11/2}} dx, x, \tan(e + fx)\right)}{99cf(c - ictan(e + fx))^{11/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(2iA - 9B)(a + ia \tan(e + fx))^{5/2}}{99cf(c - ictan(e + fx))^{11/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{11f(c - ictan(e + fx))^{11/2}} - \frac{(2iA - 9B)(a + ia \tan(e + fx))^{5/2}}{99cf(c - ictan(e + fx))^{11/2}}$$

### Mathematica [A]

time = 7.62, size = 135, normalized size = 0.87

$$\frac{a^3 \cos(e + fx)(-77iA + 9(-9iA + 2B) \cos(2(e + fx)) - 9(2A + 9iB) \sin(2(e + fx)))(\cos(9e + 12fx) + i \sin(9e + 12fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{1386c^6 f (\cos(fx) + i \sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[
e + f*x])^(11/2), x]
```

```
[Out] (a^3*Cos[e + f*x]*((-77*I)*A + 9*((-9*I)*A + 2*B)*Cos[2*(e + f*x)] - 9*(2*A
+ (9*I)*B)*Sin[2*(e + f*x)]*(Cos[9*e + 12*f*x] + I*Ssin[9*e + 12*f*x])*Sqr
t[a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]/(1386*c^6*f*(Cos[f*x] +
I*Ssin[f*x])^3)
```

**Maple [A]**

time = 0.42, size = 161, normalized size = 1.04

method	result
risch	$\frac{a^3 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (63iA e^{10i(fx+e)} + 63B e^{10i(fx+e)} + 154iA e^{8i(fx+e)} + 99iA e^{6i(fx+e)} - 99B e^{6i(fx+e)})}{2772c^5 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(2iA(\tan^4(fx+e))-63iB(t$
default	$\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(2iA(\tan^4(fx+e))-63iB(t$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x,m
method=_RETURNVERBOSE)
```

```
[Out] 1/693*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^6*(1
+tan(f*x+e)^2)*(2*I*A*tan(f*x+e)^4-63*I*B*tan(f*x+e)^3-9*B*tan(f*x+e)^4-45*
I*A*tan(f*x+e)^2-14*A*tan(f*x+e)^3+63*I*B*tan(f*x+e)-144*B*tan(f*x+e)^2+79*
I*A-140*A*tan(f*x+e)-9*B)/(I+tan(f*x+e))^7
```

**Maxima [A]**

time = 0.67, size = 210, normalized size = 1.35

$$\frac{(63(-A-B)a^3 \cos(\frac{11}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) - 154iAa^3 \cos(\frac{9}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 99(-A+B)a^3 \cos(\frac{7}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 63(A-Ia^3 \sin(\frac{11}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 154Aa^3 \sin(\frac{9}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)})) + 99(A+Ia^3 \sin(\frac{7}{2} \arctan(\frac{\sin(2fx+2e)}{\cos(2fx+2e)}))) \sqrt{a}}{2772c^5 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] 1/2772*(63*(-I*A - B)*a^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 154*I*A*a^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 9
9*(-I*A + B)*a^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 63*
(A - I*B)*a^3*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 154*A
*a^3*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 99*(A + I*B)*a^
3*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(11/2)*f
)
```

**Fricas [A]**

time = 3.14, size = 131, normalized size = 0.85

$$\frac{(63(iA+B)a^3 e^{(13i fx+13ie)} + 7(31iA+9B)a^3 e^{(11i fx+11ie)} + 11(23iA-9B)a^3 e^{(9i fx+9ie)} + 99(iA-B)a^3 e^{(7i fx+7ie)}) \sqrt{\frac{a}{e^{(2i fx+2ie)}+1}} \sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}}{2772 c^5 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(11/2),x, algorithm="fricas")

[Out] 
$$-1/2772*(63*(I*A + B)*a^3*e^{(13*I*f*x + 13*I*e)} + 7*(31*I*A + 9*B)*a^3*e^{(11*I*f*x + 11*I*e)} + 11*(23*I*A - 9*B)*a^3*e^{(9*I*f*x + 9*I*e)} + 99*(I*A - B)*a^3*e^{(7*I*f*x + 7*I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} / (c^6*f)$$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(11/2),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(11/2),x, algorithm="giac")

[Out] 
$$\int (B*\tan(f*x + e) + A)*(I*a*\tan(f*x + e) + a)^{7/2} / (-I*c*\tan(f*x + e) + c)^{11/2}, x$$

**Mupad [B]**

time = 11.75, size = 217, normalized size = 1.40

$$\frac{a^3 \sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}} (A \cos(6e+6fx)99i + A \cos(8e+8fx)154i + A \cos(10e+10fx)63i - 99B \cos(6e+6fx) + 63B \cos(10e+10fx) - 99A \sin(6e+6fx) - 154A \sin(8e+8fx) - 63A \sin(10e+10fx) - B \sin(6e+6fx)99i + B \sin(10e+10fx)63i)}{2772e^f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2))/(c - c\*tan(e + f\*x)\*1i)^(11/2),x)

[Out] 
$$-(a^3*((a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{1/2}*(A*\cos(6*e + 6*f*x)*99i + A*\cos(8*e + 8*f*x)*154i + A*\cos(10*e + 10*f*x)*63i - 99*B*\cos(6*e + 6*f*x) + 63*B*\cos(10*e + 10*f*x) - 99*A*\sin(6*e + 6*f*x) - 154*A*\sin(8*e + 8*f*x) - 63*A*\sin(10*e + 10*f*x) - B*\sin(6*e + 6*f*x)*99i + B*\sin(10*e + 10*f*x)*63i))/((2772*c^5*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{1/2}))$$



$$3.827 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{13/2}} dx$$

**Optimal.** Leaf size=208

$$\frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{13f(c-ic \tan(e+fx))^{13/2}} - \frac{(3iA-10B)(a+ia \tan(e+fx))^{7/2}}{143cf(c-ic \tan(e+fx))^{11/2}} - \frac{2(3iA-10B)(a+ia \tan(e+fx))^{7/2}}{1287c^2f(c-ic \tan(e+fx))^{9/2}}$$

[Out]  $-1/13*(I*A+B)*(a+I*a*\tan(f*x+e))^{(7/2)}/f/(c-I*c*\tan(f*x+e))^{(13/2)}-1/143*(3*I*A-10*B)*(a+I*a*\tan(f*x+e))^{(7/2)}/c/f/(c-I*c*\tan(f*x+e))^{(11/2)}-2/1287*(3*I*A-10*B)*(a+I*a*\tan(f*x+e))^{(7/2)}/c^2/f/(c-I*c*\tan(f*x+e))^{(9/2)}-2/9009*(3*I*A-10*B)*(a+I*a*\tan(f*x+e))^{(7/2)}/c^3/f/(c-I*c*\tan(f*x+e))^{(7/2)}$

**Rubi [A]**

time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ ,

Rules used = {3669, 79, 47, 37}

$$\frac{2(-10B+3iA)(a+ia \tan(e+fx))^{7/2}}{9009c^3f(c-ic \tan(e+fx))^{7/2}} - \frac{2(-10B+3iA)(a+ia \tan(e+fx))^{7/2}}{1287c^2f(c-ic \tan(e+fx))^{9/2}} - \frac{(-10B+3iA)(a+ia \tan(e+fx))^{7/2}}{143cf(c-ic \tan(e+fx))^{11/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{13f(c-ic \tan(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+I*a*\text{Tan}[e+f*x])^{(7/2)}*(A+B*\text{Tan}[e+f*x])]/(c-I*c*\text{Tan}[e+f*x])^{(13/2)},x]$

[Out]  $-1/13*((I*A+B)*(a+I*a*\text{Tan}[e+f*x])^{(7/2)})/(f*(c-I*c*\text{Tan}[e+f*x])^{(13/2)}) - (((3*I)*A-10*B)*(a+I*a*\text{Tan}[e+f*x])^{(7/2)})/(143*c*f*(c-I*c*\text{Tan}[e+f*x])^{(11/2)}) - (2*((3*I)*A-10*B)*(a+I*a*\text{Tan}[e+f*x])^{(7/2)})/(1287*c^2*f*(c-I*c*\text{Tan}[e+f*x])^{(9/2)}) - (2*((3*I)*A-10*B)*(a+I*a*\text{Tan}[e+f*x])^{(7/2)})/(9009*c^3*f*(c-I*c*\text{Tan}[e+f*x])^{(7/2)})$

**Rule 37**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^{(n+1)}/((b*c - a*d)*(m+1))), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

**Rule 47**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^{(n+1)}/((b*c - a*d)*(m+1))), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m+1))), \text{Int}[(a + b*x)^{\text{Simplify}[m+1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& (\text{EqQ}[a, 0] || (\text{NeQ}[c, 0] \&\& \text{LtQ}[m - n, 0] \&\& \text{IntegerQ}[n])) \&\& (\text{SumSimplerQ}[m, 1] || !\text{SumSimplerQ}[n, 1])$

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^(n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

## Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx))}{(c - ictan(e + fx))^{13/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{5/2}(A+Bx)}{(c-icx)^{15/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{13f(c - ictan(e + fx))^{13/2}} + \frac{(a(3A + 10iB)) \text{St}}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(3iA - 10B)(a + ia \tan(e + fx))^{7/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(3iA - 10B)(a + ia \tan(e + fx))^{7/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{13f(c - ictan(e + fx))^{13/2}} - \frac{(3iA - 10B)(a + ia \tan(e + fx))^{7/2}}{143cf(c - ictan(e + fx))^{13/2}}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 495 vs. 2(208) = 416.

time = 14.52, size = 495, normalized size = 2.38

Integrate[(a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x])/(c - I\*c\*Tan[e + f\*x])^(13/2), x]

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(13/2), x]

```
[Out] (Cos[e + f*x]^4*((-I)*A + B)*Cos[6*f*x]*(Cos[3*e]/(112*c^7) + ((I/112)*Sin[3*e])/c^7) + ((-15*I)*A + 8*B)*Cos[8*f*x]*(Cos[5*e]/(504*c^7) + ((I/504)*Sin[5*e])/c^7) + ((-30*I)*A + B)*Cos[10*f*x]*(Cos[7*e]/(792*c^7) + ((I/792)*Sin[7*e])/c^7) + (25*A - (12*I)*B)*Cos[12*f*x]*((-1/1144*I)*Cos[9*e])/c^7 + Sin[9*e]/(1144*c^7) + (A - I*B)*Cos[14*f*x]*((-1/208*I)*Cos[11*e])/c^7 + Sin[11*e]/(208*c^7) + (A + I*B)*(Cos[3*e]/(112*c^7) + ((I/112)*Sin[3*e])/c^7)*Sin[6*f*x] + (15*A + (8*I)*B)*(Cos[5*e]/(504*c^7) + ((I/504)*Sin[5*e])/c^7)*Sin[8*f*x] + (30*A + I*B)*(Cos[7*e]/(792*c^7) + ((I/792)*Sin[7*e])/c^7)*Sin[10*f*x] + (25*A - (12*I)*B)*(Cos[9*e]/(1144*c^7) + ((I/1144)*Sin[9*e])/c^7)*Sin[12*f*x] + (A - I*B)*(Cos[11*e]/(208*c^7) + ((I/208)*Sin[11*e])/c^7)*Sin[14*f*x])*Sqrt[Sec[e + f*x]*(c*cos[e + f*x] - I*c*sin[e + f*x])*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])]/(f*(Cos[f*x] + I*Sin[f*x]))^3*(A*cos[e + f*x] + B*sin[e + f*x]))
```

**Maple [A]**

time = 0.46, size = 184, normalized size = 0.88

method	result
risch	$\frac{a^3 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (693iA e^{12i(fx+e)}+693B e^{12i(fx+e)}+2457iA e^{10i(fx+e)}+819B e^{10i(fx+e)}+3003iA e^{8i(fx+e)}-172072c^6 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}{72072c^6 \sqrt{\frac{c}{e^{2i(fx+e)}+1}} f}$
derivativedivides	$\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(6iA(\tan^5(fx+e))-160iB)}{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(6iA(\tan^5(fx+e))-160iB)}$
default	$\frac{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(6iA(\tan^5(fx+e))-160iB)}{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(6iA(\tan^5(fx+e))-160iB)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9009*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^7*(1+tan(f*x+e)^2)*(6*I*A*tan(f*x+e)^5-160*I*B*tan(f*x+e)^4-20*B*tan(f*x+e)^5-177*I*A*tan(f*x+e)^3-48*A*tan(f*x+e)^4-1643*I*B*tan(f*x+e)^2+590*B*tan(f*x+e)^3-1569*I*A*tan(f*x+e)+408*A*tan(f*x+e)^2-97*I*B-776*B*tan(f*x+e)-930*A)/(I+tan(f*x+e))^8
```

**Maxima [A]**

time = 0.67, size = 292, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x, algorithm="maxima")
```

```
[Out] 1/72072*(693*(-I*A - B)*a^3*cos(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 819*(-3*I*A - B)*a^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1001*(-3*I*A + B)*a^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1287*(-I*A + B)*a^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 693*(A - I*B)*a^3*sin(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 819*(3*A - I*B)*a^3*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1001*(3*A + I*B)*a^3*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1287*(A + I*B)*a^3*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(13/2)*f)
```

**Fricas** [A]

time = 3.79, size = 153, normalized size = 0.74

$$\frac{(693(iA + B)a^3e^{(15i f x + 15i e)} + 126(25iA + 12B)a^3e^{(13i f x + 13i e)} + 182(30iA - B)a^3e^{(11i f x + 11i e)} + 286(15iA - 8B)a^3e^{(9i f x + 9i e)} + 1287(iA - B)a^3e^{(7i f x + 7i e)})\sqrt{\frac{a}{e^{(2i f x + 2i e)} + 1}}\sqrt{\frac{c}{e^{(2i f x + 2i e)} + 1}}}{72072 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x, algorithm="fricas")
```

```
[Out] -1/72072*(693*(I*A + B)*a^3*e^(15*I*f*x + 15*I*e) + 126*(25*I*A + 12*B)*a^3*e^(13*I*f*x + 13*I*e) + 182*(30*I*A - B)*a^3*e^(11*I*f*x + 11*I*e) + 286*(15*I*A - 8*B)*a^3*e^(9*I*f*x + 9*I*e) + 1287*(I*A - B)*a^3*e^(7*I*f*x + 7*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^7*f)
```

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x, algorithm="giac")
```

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(7/2)/(-I\*c\*tan(f\*x + e) + c)^(13/2), x)

Mupad [B]

time = 13.49, size = 167, normalized size = 0.80

$$\frac{\sqrt{a + \frac{a \sin(e + f x) \operatorname{li}}{\cos(e + f x)}} \left( \frac{a^3 e^{e 8i + f x 8i} (3A + B 1i) \operatorname{li}}{72 c^6 f} + \frac{a^3 e^{e 10i + f x 10i} (3A - B 1i) \operatorname{li}}{88 c^6 f} + \frac{a^3 e^{e 6i + f x 6i} (A + B 1i) \operatorname{li}}{56 c^6 f} + \frac{a^3 e^{e 12i + f x 12i} (A - B 1i) \operatorname{li}}{104 c^6 f} \right)}{\sqrt{c - \frac{c \sin(e + f x) \operatorname{li}}{\cos(e + f x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2))/(c - c\*tan(e + f\*x)\*1i)^(13/2), x)

[Out] -((a + (a\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)\*((a^3\*exp(e\*8i + f\*x\*8i))\*(3\*A + B\*1i)\*1i)/(72\*c^6\*f) + (a^3\*exp(e\*10i + f\*x\*10i))\*(3\*A - B\*1i)\*1i)/(88\*c^6\*f) + (a^3\*exp(e\*6i + f\*x\*6i)\*(A + B\*1i)\*1i)/(56\*c^6\*f) + (a^3\*exp(e\*12i + f\*x\*12i)\*(A - B\*1i)\*1i)/(104\*c^6\*f))/(c - (c\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)

$$3.828 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ic \tan(e+fx))^{15/2}} dx$$

**Optimal.** Leaf size=261

$$\frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{15f(c-ic \tan(e+fx))^{15/2}} - \frac{(4iA-11B)(a+ia \tan(e+fx))^{7/2}}{195cf(c-ic \tan(e+fx))^{13/2}} - \frac{(4iA-11B)(a+ia \tan(e+fx))^{7/2}}{715c^2f(c-ic \tan(e+fx))^{11/2}}$$

[Out] -1/15\*(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^(7/2)/f/(c-I\*c\*tan(f\*x+e))^(15/2)-1/195\*(4\*I\*A-11\*B)\*(a+I\*a\*tan(f\*x+e))^(7/2)/c/f/(c-I\*c\*tan(f\*x+e))^(13/2)-1/715\*(4\*I\*A-11\*B)\*(a+I\*a\*tan(f\*x+e))^(7/2)/c^2/f/(c-I\*c\*tan(f\*x+e))^(11/2)-2/6435\*(4\*I\*A-11\*B)\*(a+I\*a\*tan(f\*x+e))^(7/2)/c^3/f/(c-I\*c\*tan(f\*x+e))^(9/2)-2/45045\*(4\*I\*A-11\*B)\*(a+I\*a\*tan(f\*x+e))^(7/2)/c^4/f/(c-I\*c\*tan(f\*x+e))^(7/2)

**Rubi [A]**

time = 0.21, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{2(-11B+4iA)(a+ia \tan(e+fx))^{7/2}}{45045c^4f(c-ic \tan(e+fx))^{7/2}} - \frac{2(-11B+4iA)(a+ia \tan(e+fx))^{7/2}}{6435c^3f(c-ic \tan(e+fx))^{9/2}} - \frac{(-11B+4iA)(a+ia \tan(e+fx))^{7/2}}{715c^2f(c-ic \tan(e+fx))^{11/2}} - \frac{(-11B+4iA)(a+ia \tan(e+fx))^{7/2}}{195cf(c-ic \tan(e+fx))^{13/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{15f(c-ic \tan(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(15/2), x]

[Out] -1/15\*((I\*A + B)\*(a + I\*a\*Tan[e + f\*x])^(7/2))/(f\*(c - I\*c\*Tan[e + f\*x])^(15/2)) - (((4\*I)\*A - 11\*B)\*(a + I\*a\*Tan[e + f\*x])^(7/2))/(195\*c\*f\*(c - I\*c\*Tan[e + f\*x])^(13/2)) - (((4\*I)\*A - 11\*B)\*(a + I\*a\*Tan[e + f\*x])^(7/2))/(715\*c^2\*f\*(c - I\*c\*Tan[e + f\*x])^(11/2)) - (2\*((4\*I)\*A - 11\*B)\*(a + I\*a\*Tan[e + f\*x])^(7/2))/(6435\*c^3\*f\*(c - I\*c\*Tan[e + f\*x])^(9/2)) - (2\*((4\*I)\*A - 11\*B)\*(a + I\*a\*Tan[e + f\*x])^(7/2))/(45045\*c^4\*f\*(c - I\*c\*Tan[e + f\*x])^(7/2))

**Rule 37**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

**Rule 47**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^Simplify[m + 1]\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && I

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\int \frac{(a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^{15/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(a+iax)^{5/2}(A+Bx)}{(c-icx)^{17/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{15f(c - ict \tan(e + fx))^{15/2}} + \frac{(a(4A + 11iB))}{195cf(c - ict \tan(e + fx))^{15/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{15f(c - ict \tan(e + fx))^{15/2}} - \frac{(4iA - 11B)(a - ia \tan(e + fx))^{7/2}}{195cf(c - ict \tan(e + fx))^{15/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{15f(c - ict \tan(e + fx))^{15/2}} - \frac{(4iA - 11B)(a - ia \tan(e + fx))^{7/2}}{195cf(c - ict \tan(e + fx))^{15/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{15f(c - ict \tan(e + fx))^{15/2}} - \frac{(4iA - 11B)(a - ia \tan(e + fx))^{7/2}}{195cf(c - ict \tan(e + fx))^{15/2}}$$

$$= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{15f(c - ict \tan(e + fx))^{15/2}} - \frac{(4iA - 11B)(a - ia \tan(e + fx))^{7/2}}{195cf(c - ict \tan(e + fx))^{15/2}}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 577 vs.  $2(261) = 522$ .





)<sup>4</sup>-7260\*I\*B\*tan(f\*x+e)<sup>2</sup>+2145\*B\*tan(f\*x+e)<sup>3</sup>-6858\*I\*A\*tan(f\*x+e)+1455\*A\*tan(f\*x+e)<sup>2</sup>-407\*I\*B-3663\*B\*tan(f\*x+e)-4243\*A)/(I+tan(f\*x+e))<sup>9</sup>

**Maxima** [A]

time = 0.70, size = 352, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))<sup>(7/2)</sup>\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))<sup>(15/2)</sup>),x, algorithm="maxima")

[Out] 1/720720\*(3003\*(-I\*A - B)\*a<sup>3</sup>\*cos(15/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 6930\*(-2\*I\*A - B)\*a<sup>3</sup>\*cos(13/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - 24570\*I\*A\*a<sup>3</sup>\*cos(11/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 10010\*(-2\*I\*A + B)\*a<sup>3</sup>\*cos(9/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 6435\*(-I\*A + B)\*a<sup>3</sup>\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 3003\*(A - I\*B)\*a<sup>3</sup>\*sin(15/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 6930\*(2\*A - I\*B)\*a<sup>3</sup>\*sin(13/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 24570\*A\*a<sup>3</sup>\*sin(11/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 10010\*(2\*A + I\*B)\*a<sup>3</sup>\*sin(9/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 6435\*(A + I\*B)\*a<sup>3</sup>\*sin(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*sqrt(a)/(c<sup>(15/2)</sup>\*f)

**Fricas** [A]

time = 5.76, size = 175, normalized size = 0.67

$$\frac{(3003(iA + B)a^3e^{17i f x + 17i e} + 231(73iA + 43B)a^3e^{15i f x + 15i e} + 630(61iA + 11B)a^3e^{13i f x + 13i e} + 910(49iA - 11B)a^3e^{11i f x + 11i e} + 715(37iA - 23B)a^3e^{9i f x + 9i e} + 6435(iA - B)a^3e^{7i f x + 7i e})\sqrt{\frac{a}{e^{2i f x + 2i e} + 1}}\sqrt{\frac{c}{e^{2i f x + 2i e} + 1}}}{720720 c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))<sup>(7/2)</sup>\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))<sup>(15/2)</sup>),x, algorithm="fricas")

[Out] -1/720720\*(3003\*(I\*A + B)\*a<sup>3</sup>\*e<sup>(17\*I\*f\*x + 17\*I\*e)</sup> + 231\*(73\*I\*A + 43\*B)\*a<sup>3</sup>\*e<sup>(15\*I\*f\*x + 15\*I\*e)</sup> + 630\*(61\*I\*A + 11\*B)\*a<sup>3</sup>\*e<sup>(13\*I\*f\*x + 13\*I\*e)</sup> + 910\*(49\*I\*A - 11\*B)\*a<sup>3</sup>\*e<sup>(11\*I\*f\*x + 11\*I\*e)</sup> + 715\*(37\*I\*A - 23\*B)\*a<sup>3</sup>\*e<sup>(9\*I\*f\*x + 9\*I\*e)</sup> + 6435\*(I\*A - B)\*a<sup>3</sup>\*e<sup>(7\*I\*f\*x + 7\*I\*e)</sup>)\*sqrt(a/(e<sup>(2\*I\*f\*x + 2\*I\*e)</sup> + 1))\*sqrt(c/(e<sup>(2\*I\*f\*x + 2\*I\*e)</sup> + 1))/(c<sup>8</sup>\*f)

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(15/2),x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(15/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(7/2)/(-I\*c\*tan(f\*x + e) + c)^(15/2), x)

**Mupad [B]**

time = 13.48, size = 191, normalized size = 0.73

$$\frac{\sqrt{a + \frac{a \sin(e + f x) \operatorname{li}}{\cos(e + f x)}} \left( \frac{a^3 e^{e 8i + f x 8i} (2A + B \operatorname{li}) \operatorname{li}}{72 c^7 f} + \frac{a^3 e^{e 12i + f x 12i} (2A - B \operatorname{li}) \operatorname{li}}{104 c^7 f} + \frac{A a^3 e^{e 10i + f x 10i} 3i}{88 c^7 f} + \frac{a^3 e^{e 6i + f x 6i} (A + B \operatorname{li}) \operatorname{li}}{112 c^7 f} + \frac{a^3 e^{e 14i + f x 14i} (A - B \operatorname{li}) \operatorname{li}}{240 c^7 f} \right)}{\sqrt{c - \frac{c \sin(e + f x) \operatorname{li}}{\cos(e + f x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(a + a\*tan(e + f\*x)\*1i)^(7/2))/(c - c\*tan(e + f\*x)\*1i)^(15/2),x)

[Out] -((a + (a\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)\*((a^3\*exp(e\*8i + f\*x\*8i))\*(2\*A + B\*1i)\*1i)/(72\*c^7\*f) + (a^3\*exp(e\*12i + f\*x\*12i))\*(2\*A - B\*1i)\*1i)/(104\*c^7\*f) + (A\*a^3\*exp(e\*10i + f\*x\*10i)\*3i)/(88\*c^7\*f) + (a^3\*exp(e\*6i + f\*x\*6i)\*(A + B\*1i)\*1i)/(112\*c^7\*f) + (a^3\*exp(e\*14i + f\*x\*14i)\*(A - B\*1i)\*1i)/(240\*c^7\*f)))/(c - (c\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)

$$3.829 \quad \int \frac{(a+ia \tan(e+fx))^{7/2}(A+B \tan(e+fx))}{(c-ictan(e+fx))^{17/2}} dx$$

**Optimal.** Leaf size=314

$$\frac{(iA+B)(a+ia \tan(e+fx))^{7/2}}{17f(c-ictan(e+fx))^{17/2}} - \frac{(5iA-12B)(a+ia \tan(e+fx))^{7/2}}{255cf(c-ictan(e+fx))^{15/2}} - \frac{4(5iA-12B)(a+ia \tan(e+fx))^{7/2}}{3315c^2f(c-ictan(e+fx))^{13/2}}$$

```
[Out] -1/17*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(17/2)-1/255*(5
*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(15/2)-4/3315*(5
*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^2/f/(c-I*c*tan(f*x+e))^(13/2)-4/12155
*(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^3/f/(c-I*c*tan(f*x+e))^(11/2)-8/10
9395*(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^4/f/(c-I*c*tan(f*x+e))^(9/2)-8
/765765*(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^5/f/(c-I*c*tan(f*x+e))^(7/2
)
```

**Rubi [A]**

time = 0.24, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{8(-12B+5iA)(a+ia \tan(e+fx))^{7/2}}{765765c^2f(c-ictan(e+fx))^{17/2}} - \frac{8(-12B+5iA)(a+ia \tan(e+fx))^{7/2}}{109395c^2f(c-ictan(e+fx))^{15/2}} - \frac{4(-12B+5iA)(a+ia \tan(e+fx))^{7/2}}{12155c^2f(c-ictan(e+fx))^{13/2}} - \frac{4(-12B+5iA)(a+ia \tan(e+fx))^{7/2}}{3315c^2f(c-ictan(e+fx))^{11/2}} - \frac{(-12B+5iA)(a+ia \tan(e+fx))^{7/2}}{255cf(c-ictan(e+fx))^{9/2}} - \frac{(B+iA)(a+ia \tan(e+fx))^{7/2}}{17f(c-ictan(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
x])^(17/2), x]
```

```
[Out] -1/17*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(f*(c - I*c*Tan[e + f*x])^(1
7/2)) - (((5*I)*A - 12*B)*(a + I*a*Tan[e + f*x])^(7/2))/(255*c*f*(c - I*c*T
an[e + f*x])^(15/2)) - (4*((5*I)*A - 12*B)*(a + I*a*Tan[e + f*x])^(7/2))/(3
315*c^2*f*(c - I*c*Tan[e + f*x])^(13/2)) - (4*((5*I)*A - 12*B)*(a + I*a*Tan
[e + f*x])^(7/2))/(12155*c^3*f*(c - I*c*Tan[e + f*x])^(11/2)) - (8*((5*I)*A
- 12*B)*(a + I*a*Tan[e + f*x])^(7/2))/(109395*c^4*f*(c - I*c*Tan[e + f*x])
^(9/2)) - (8*((5*I)*A - 12*B)*(a + I*a*Tan[e + f*x])^(7/2))/(765765*c^5*f*(
c - I*c*Tan[e + f*x])^(7/2))
```

**Rule 37**

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

**Rule 47**

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
```

```

simplify[m + n + 2]/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
+ d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])

```

### Rule 79

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])
))

```

### Rule 3669

```

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \tan(e + fx))^{7/2} (A + B \tan(e + fx))}{(c - ict \tan(e + fx))^{17/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(a+iax)^{5/2} (A+Bx)}{(c-icx)^{19/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{17f(c - ict \tan(e + fx))^{17/2}} + \frac{(a(5A + 12iB))}{255cf(c - ict \tan(e + fx))^{17/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{17f(c - ict \tan(e + fx))^{17/2}} - \frac{(5iA - 12B)(a - ic \tan(e + fx))^{7/2}}{255cf(c - ict \tan(e + fx))^{17/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{17f(c - ict \tan(e + fx))^{17/2}} - \frac{(5iA - 12B)(a - ic \tan(e + fx))^{7/2}}{255cf(c - ict \tan(e + fx))^{17/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{17f(c - ict \tan(e + fx))^{17/2}} - \frac{(5iA - 12B)(a - ic \tan(e + fx))^{7/2}}{255cf(c - ict \tan(e + fx))^{17/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{17f(c - ict \tan(e + fx))^{17/2}} - \frac{(5iA - 12B)(a - ic \tan(e + fx))^{7/2}}{255cf(c - ict \tan(e + fx))^{17/2}} \\
&= -\frac{(iA + B)(a + ia \tan(e + fx))^{7/2}}{17f(c - ict \tan(e + fx))^{17/2}} - \frac{(5iA - 12B)(a - ic \tan(e + fx))^{7/2}}{255cf(c - ict \tan(e + fx))^{17/2}}
\end{aligned}$$

**Mathematica [B]** Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 655 vs. 2(314) = 628.  
time = 17.07, size = 655, normalized size = 2.09

Antiderivative was successfully verified.

[In] Integrate[((a + I\*a\*Tan[e + f\*x])^(7/2)\*(A + B\*Tan[e + f\*x]))/(c - I\*c\*Tan[e + f\*x])^(17/2),x]

[Out] (Cos[e + f\*x]^4\*(((I)\*A + B)\*Cos[6\*f\*x]\*(Cos[3\*e]/(448\*c^9) + ((I/448)\*Sin[3\*e])/c^9) + ((-22\*I)\*A + 15\*B)\*Cos[8\*f\*x]\*(Cos[5\*e]/(2016\*c^9) + ((I/2016)\*Sin[5\*e])/c^9) + ((-145\*I)\*A + 51\*B)\*Cos[10\*f\*x]\*(Cos[7\*e]/(6336\*c^9) + ((I/6336)\*Sin[7\*e])/c^9) + ((-60\*I)\*A + B)\*Cos[12\*f\*x]\*(Cos[9\*e]/(2288\*c^9) + ((I/2288)\*Sin[9\*e])/c^9) + (215\*A - (69\*I)\*B)\*Cos[14\*f\*x]\*(((I/12480)\*Cos[11\*e])/c^9 + Sin[11\*e]/(12480\*c^9)) + (50\*A - (33\*I)\*B)\*Cos[16\*f\*x]\*(((I/8160)\*Cos[13\*e])/c^9 + Sin[13\*e]/(8160\*c^9)) + (A - I\*B)\*Cos[18\*f\*x]\*(((I/1088)\*Cos[15\*e])/c^9 + Sin[15\*e]/(1088\*c^9)) + (A + I\*B)\*(Cos[3\*e]/(448\*c^9) + ((I/448)\*Sin[3\*e])/c^9)\*Sin[6\*f\*x] + (22\*A + (15\*I)\*B)\*(Cos[5\*e]/(2016\*c^9) + ((I/2016)\*Sin[5\*e])/c^9)\*Sin[8\*f\*x] + (145\*A + (51\*I)\*B)\*(Cos[7\*e]/(6336\*c^9) + ((I/6336)\*Sin[7\*e])/c^9)\*Sin[10\*f\*x] + (60\*A + I\*B)\*(Cos

$$\begin{aligned} & [9*e]/(2288*c^9) + ((I/2288)*\text{Sin}[9*e])/c^9*\text{Sin}[12*f*x] + (215*A - (69*I)*B) \\ & *( \text{Cos}[11*e]/(12480*c^9) + ((I/12480)*\text{Sin}[11*e])/c^9*\text{Sin}[14*f*x] + (50*A - \\ & (33*I)*B)*( \text{Cos}[13*e]/(8160*c^9) + ((I/8160)*\text{Sin}[13*e])/c^9*\text{Sin}[16*f*x] + \\ & (A - I*B)*( \text{Cos}[15*e]/(1088*c^9) + ((I/1088)*\text{Sin}[15*e])/c^9*\text{Sin}[18*f*x] ) * \text{Sqrt} \\ & [\text{Sec}[e + f*x]*(c*\text{Cos}[e + f*x] - I*c*\text{Sin}[e + f*x])] * (a + I*a*\text{Tan}[e + f*x]) \\ & ^{(7/2)} * (A + B*\text{Tan}[e + f*x]) / (f*(\text{Cos}[f*x] + I*\text{Sin}[f*x])^3 * (A*\text{Cos}[e + f*x] + \\ & B*\text{Sin}[e + f*x])) \end{aligned}$$

**Maple [A]**

time = 0.46, size = 230, normalized size = 0.73

method	result
risch	$-\frac{a^3 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)}+1}} (45045iA e^{16i(fx+e)}+45045B e^{16i(fx+e)}+255255iA e^{14i(fx+e)}+153153B e^{14i(fx+e)}+589050iA e^{12i(fx+e)}+153153B e^{10i(fx+e)}+45045iA e^{8i(fx+e)}+45045B e^{8i(fx+e)}+153153iA e^{6i(fx+e)}+153153B e^{6i(fx+e)}+45045iA e^{4i(fx+e)}+45045B e^{4i(fx+e)}+153153iA e^{2i(fx+e)}+153153B e^{2i(fx+e)}+45045iA+45045B)}{i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(11175iA(\tan^3(fx+e))+12960iB \tan^2(fx+e)+103165iA \tan(fx+e)-400A \tan(fx+e)^2+109881iB \tan(fx+e)^2+4464B \tan(fx+e)^5+5871iB+5400A \tan(fx+e)^4+40iA \tan(fx+e)^7-26820B \tan(fx+e)^3-1860iA \tan(fx+e)^5-18030A \tan(fx+e)^2-960iB \tan(fx+e)^6+58710B \tan(fx+e)+66260A)/(I+\tan(fx+e))^{10}}$
derivativedivides	$i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(11175iA(\tan^3(fx+e))+12960iB \tan^2(fx+e)+103165iA \tan(fx+e)-400A \tan(fx+e)^2+109881iB \tan(fx+e)^2+4464B \tan(fx+e)^5+5871iB+5400A \tan(fx+e)^4+40iA \tan(fx+e)^7-26820B \tan(fx+e)^3-1860iA \tan(fx+e)^5-18030A \tan(fx+e)^2-960iB \tan(fx+e)^6+58710B \tan(fx+e)+66260A)/(I+\tan(fx+e))^{10}$
default	$i \sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} a^3(1+\tan^2(fx+e))(11175iA(\tan^3(fx+e))+12960iB \tan^2(fx+e)+103165iA \tan(fx+e)-400A \tan(fx+e)^2+109881iB \tan(fx+e)^2+4464B \tan(fx+e)^5+5871iB+5400A \tan(fx+e)^4+40iA \tan(fx+e)^7-26820B \tan(fx+e)^3-1860iA \tan(fx+e)^5-18030A \tan(fx+e)^2-960iB \tan(fx+e)^6+58710B \tan(fx+e)+66260A)/(I+\tan(fx+e))^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{765765} \frac{I}{f} \frac{a^3 (1 + I \tan(fx+e))^{1/2} (-c(I \tan(fx+e) - 1))^{1/2} a^3 c^9 (1 + \tan^2(fx+e))^2 (11175 I A \tan^3(fx+e) + 12960 I B \tan^2(fx+e) + 103165 I A \tan(fx+e) - 400 A \tan^2(fx+e) + 109881 I B \tan^2(fx+e) + 4464 B \tan^5(fx+e) + 5871 I B + 5400 A \tan^4(fx+e) + 40 I A \tan^7(fx+e) - 26820 B \tan^3(fx+e) - 1860 I A \tan^5(fx+e) - 18030 A \tan^2(fx+e) - 960 I B \tan^6(fx+e) + 58710 B \tan(fx+e) + 66260 A)}{(I + \tan(fx+e))^{10}}$$

**Maxima [A]**

time = 0.74, size = 436, normalized size = 1.39

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17/2),x, algorithm="maxima")`

[Out] 
$$\frac{1}{24504480} (45045 * (-I * A - B) * a^3 * \cos(17/2 * \arctan^2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) + 51051 * (-5 * I * A - 3 * B) * a^3 * \cos(15/2 * \arctan^2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) + 117810 * (-5 * I * A - B) * a^3 * \cos(13/2 * \arctan^2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) + 139230 * (-5 * I * A + B) * a^3 * \cos(11/2 * \arctan^2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))$$

+ 2\*e), cos(2\*f\*x + 2\*e))) + 85085\*(-5\*I\*A + 3\*B)\*a^3\*cos(9/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 109395\*(-I\*A + B)\*a^3\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 45045\*(A - I\*B)\*a^3\*sin(17/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 51051\*(5\*A - 3\*I\*B)\*a^3\*sin(15/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 117810\*(5\*A - I\*B)\*a^3\*sin(13/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 139230\*(5\*A + I\*B)\*a^3\*sin(11/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 85085\*(5\*A + 3\*I\*B)\*a^3\*sin(9/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + 109395\*(A + I\*B)\*a^3\*sin(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*sqrt(a)/(c^(17/2)\*f)

**Fricas** [A]

time = 3.42, size = 197, normalized size = 0.63

$$\frac{(45045(iA+B)a^{15}e^{15fx+15e}) + 6006(50iA+33B)a^{15}e^{15fx+15e}) + 3927(215iA+69B)a^{15}e^{15fx+15e}) + 21420(60iA-B)a^{15}e^{15fx+15e}) + 7735(145iA-51B)a^{15}e^{15fx+15e}) + 24310(22iA-15B)a^{15}e^{15fx+15e}) + 109395(iA-B)a^{15}e^{15fx+15e}) \sqrt{\frac{a}{e^{2fx+2e}+1}} \sqrt{\frac{c}{e^{2fx+2e}+1}}}{24504480c^9f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(17/2),x, algorithm="fricas")

[Out] -1/24504480\*(45045\*(I\*A + B)\*a^3\*e^(19\*I\*f\*x + 19\*I\*e) + 6006\*(50\*I\*A + 33\*B)\*a^3\*e^(17\*I\*f\*x + 17\*I\*e) + 3927\*(215\*I\*A + 69\*B)\*a^3\*e^(15\*I\*f\*x + 15\*I\*e) + 21420\*(60\*I\*A - B)\*a^3\*e^(13\*I\*f\*x + 13\*I\*e) + 7735\*(145\*I\*A - 51\*B)\*a^3\*e^(11\*I\*f\*x + 11\*I\*e) + 24310\*(22\*I\*A - 15\*B)\*a^3\*e^(9\*I\*f\*x + 9\*I\*e) + 109395\*(I\*A - B)\*a^3\*e^(7\*I\*f\*x + 7\*I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))/(c^9\*f)

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(17/2),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(7/2)\*(A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(17/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(7/2)/(-I\*c\*tan(f\*x + e) + c)^(17/2), x)

Mupad [B]

time = 14.48, size = 229, normalized size = 0.73

$$\frac{\sqrt{a + \frac{a \sin(e + f x) \operatorname{li}}{\cos(e + f x)}} \left( \frac{a^3 e^{e 8 i + f x 8 i} (5 A + B 3 i) \operatorname{li}}{288 c^8 f} + \frac{a^3 e^{e 10 i + f x 10 i} (5 A + B 1 i) \operatorname{li}}{176 c^8 f} + \frac{a^3 e^{e 12 i + f x 12 i} (5 A - B 1 i) \operatorname{li}}{208 c^8 f} + \frac{a^3 e^{e 14 i + f x 14 i} (5 A - B 3 i) \operatorname{li}}{480 c^8 f} + \frac{a^3 e^{e 6 i + f x 6 i} (A + B 1 i) \operatorname{li}}{224 c^8 f} + \frac{a^3 e^{e 16 i + f x 16 i} (A - B 1 i) \operatorname{li}}{544 c^8 f} \right)}{\sqrt{c - \frac{c \sin(e + f x) \operatorname{li}}{\cos(e + f x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*a + a\*tan(e + f\*x)\*1i)^(7/2))/(c - c\*tan(e + f\*x)\*1i)^(17/2),x)

[Out] -((a + (a\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2))\*((a^3\*exp(e\*8i + f\*x\*8i))\*(5\*A + B\*3i)\*1i)/(288\*c^8\*f) + (a^3\*exp(e\*10i + f\*x\*10i)\*(5\*A + B\*1i)\*1i)/(176\*c^8\*f) + (a^3\*exp(e\*12i + f\*x\*12i)\*(5\*A - B\*1i)\*1i)/(208\*c^8\*f) + (a^3\*exp(e\*14i + f\*x\*14i)\*(5\*A - B\*3i)\*1i)/(480\*c^8\*f) + (a^3\*exp(e\*6i + f\*x\*6i)\*(A + B\*1i)\*1i)/(224\*c^8\*f) + (a^3\*exp(e\*16i + f\*x\*16i)\*(A - B\*1i)\*1i)/(544\*c^8\*f)))/(c - (c\*sin(e + f\*x)\*1i)/cos(e + f\*x))^(1/2)



$$3.830 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{5/2}}{\sqrt{a+ia \tan(e+fx)}} dx$$

**Optimal.** Leaf size=228

$$\frac{3(2iA - 3B)c^{5/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{\sqrt{a} f} + \frac{3(2iA - 3B)c^2 \sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}{2af}$$

[Out]  $3*(2*I*A-3*B)*c^{(5/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/f/a^{(1/2)}+3/2*(2*I*A-3*B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/a/f+1/2*(2*I*A-3*B)*c*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/a/f+(I*A-B)*(c-I*c*\tan(f*x+e))^{(5/2)}/f/(a+I*a*\tan(f*x+e))^{(1/2)}$

**Rubi** [A]

time = 0.20, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 52, 65, 223, 209}

$$\frac{3c^{5/2}(-3B+2iA)\text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{\sqrt{a} f} + \frac{3c^2(-3B+2iA)\sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}{2af} + \frac{c(-3B+2iA)\sqrt{a+ia \tan(e+fx)}(c-ictan(e+fx))^{3/2}}{2af} + \frac{(-B+iA)(c-ictan(e+fx))^{5/2}}{f\sqrt{a+ia \tan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}/\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]], x]$

[Out]  $(3*((2*I)*A - 3*B)*c^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])]/(\text{Sqrt}[a]*f) + (3*((2*I)*A - 3*B)*c^2*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(2*a*f) + (((2*I)*A - 3*B)*c*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(3/2)})/(2*a*f) + ((I*A - B)*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})/(f*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])$

**Rule 52**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*((c + d*x)^n/(b*(m+n+1))], x] + \text{Dist}[n*((b*c - a*d)/(b*(m+n+1))], \text{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) +$

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i \tan(e + fx))^{5/2}}{\sqrt{a + i \tan(e + fx)}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^{3/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - i \tan(e + fx))^{5/2}}{f \sqrt{a + i \tan(e + fx)}} - \frac{((2A + 3iB)c) \text{Subst}\left(\int \frac{1}{\sqrt{a + i \tan(e + fx)}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(2iA - 3B)c \sqrt{a + i \tan(e + fx)} (c - i \tan(e + fx))^{3/2}}{2af} \\
&= \frac{3(2iA - 3B)c^2 \sqrt{a + i \tan(e + fx)} \sqrt{c - i \tan(e + fx)}}{2af} \\
&= \frac{3(2iA - 3B)c^2 \sqrt{a + i \tan(e + fx)} \sqrt{c - i \tan(e + fx)}}{2af} \\
&= \frac{3(2iA - 3B)c^2 \sqrt{a + i \tan(e + fx)} \sqrt{c - i \tan(e + fx)}}{2af} \\
&= \frac{3(2iA - 3B)c^{5/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + i \tan(e + fx)}}{\sqrt{a} \sqrt{c - i \tan(e + fx)}}\right)}{\sqrt{a} f}
\end{aligned}$$

**Mathematica [A]**

time = 2.94, size = 185, normalized size = 0.81

$$\frac{c^3 \sec(e + fx) (\cos(fx) + i \sin(fx)) (6(-2A + 3B) \text{ArcTan}(\cos(e + fx) + i \sin(e + fx)) (\cos(fx) - i \sin(fx)) + \frac{1}{2} \sec^2(e + fx) (5(-2A + 3B) + (-10iA + 13B) \cos(2(e + fx)) + (2A + 5iB) \sin(2(e + fx))) (\cos(e + 2fx) - i \sin(e + 2fx)))}{2f \sqrt{a + i \tan(e + fx)} \sqrt{c - i \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2))/Sqrt[a + I\*a\*Tan[e + f\*x]],x]

[Out] -1/2\*(c^3\*Sec[e + f\*x]\*(Cos[f\*x] + I\*Sin[f\*x])\*(6\*((-2\*I)\*A + 3\*B)\*ArcTan[Cos[e + f\*x] + I\*Sin[e + f\*x]]\*(Cos[f\*x] - I\*Sin[f\*x]) + (Sec[e + f\*x]^2\*(5\*((-2\*I)\*A + 3\*B) + ((-10\*I)\*A + 13\*B)\*Cos[2\*(e + f\*x)] + (2\*A + (5\*I)\*B)\*Sin[2\*(e + f\*x)]\*(Cos[e + 2\*f\*x] - I\*Sin[e + 2\*f\*x]))/2))/(f\*Sqrt[a + I\*a\*Tan[e + f\*x]]\*Sqrt[c - I\*c\*Tan[e + f\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 565 vs.  $2(188) = 376$ .  
time = 0.47, size = 566, normalized size = 2.48

method	result
derivativedivides	$i\sqrt{-c(i\tan(fx+e)-1)}\sqrt{a(1+i\tan(fx+e))}c^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac(1+\tan(fx+e))}}{\sqrt{ac}}\right)\right)$
default	$i\sqrt{-c(i\tan(fx+e)-1)}\sqrt{a(1+i\tan(fx+e))}c^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}}{\sqrt{ac}}\frac{\sqrt{ac(1+\tan(fx+e))}}{\sqrt{ac}}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{2}I/f(-c(I\tan(fx+e)-1))^{1/2}(a(1+I\tan(fx+e)))^{1/2}c^2/a(6IA*\ln((ac*\tan(fx+e)+(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}))/((ac)^{1/2})*ac*\tan(fx+e)^2+18IB*\ln((ac*\tan(fx+e)+(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}))/((ac)^{1/2})*ac*\tan(fx+e)+4IA*(ac*(1+\tan(fx+e)^2))^{1/2}*(ac)^{1/2}*\tan(fx+e)^2-9B*\ln((ac*\tan(fx+e)+(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}))/((ac)^{1/2})*ac*\tan(fx+e)^2+B*(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}*\tan(fx+e)^3-6IA*\ln((ac*\tan(fx+e)+(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}))/((ac)^{1/2})*ac-12IA*(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}*\tan(fx+e)+12A*\ln((ac*\tan(fx+e)+(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}))/((ac)^{1/2})*ac*\tan(fx+e)+2A*(ac*(1+\tan(fx+e)^2))^{1/2}*(ac)^{1/2}*\tan(fx+e)^2-14IB*(ac*(1+\tan(fx+e)^2))^{1/2}*(ac)^{1/2}+9B*\ln((ac*\tan(fx+e)+(ac)^{1/2}*(ac*(1+\tan(fx+e)^2))^{1/2}))/((ac)^{1/2})*ac+19B*(ac*(1+\tan(fx+e)^2))^{1/2}*(ac)^{1/2}*\tan(fx+e)-10A*(ac*(1+\tan(fx+e)^2))^{1/2}*(ac)^{1/2}))/((ac*(1+\tan(fx+e)^2))^{1/2}/(I-\tan(fx+e))^2/(ac)^{1/2})$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1403 vs.  $2(184) = 368$ .  
time = 1.01, size = 1403, normalized size = 6.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x,algorithm="maxima")`

[Out] 
$$4*(12*(2*A + 3*I*B)*c^2*\cos(4*f*x + 4*e) + 20*(2*A + 3*I*B)*c^2*\cos(2*f*x + 2*e) + 12*(2*I*A - 3*B)*c^2*\sin(4*f*x + 4*e) + 20*(2*I*A - 3*B)*c^2*\sin(2*f*x + 2*e))$$

$$\begin{aligned}
& f*x + 2*e) + 16*(A + I*B)*c^2 + 6*((2*A + 3*I*B)*c^2*\cos(5/2*\arctan2(\sin(2* \\
& f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*A + 3*I*B)*c^2*\cos(3/2*\arctan2(\sin(2* \\
& f*x + 2*e), \cos(2*f*x + 2*e))) + (2*A + 3*I*B)*c^2*\cos(1/2*\arctan2(\sin(2*f* \\
& x + 2*e), \cos(2*f*x + 2*e))) + (2*I*A - 3*B)*c^2*\sin(5/2*\arctan2(\sin(2*f*x \\
& + 2*e), \cos(2*f*x + 2*e))) + 2*(2*I*A - 3*B)*c^2*\sin(3/2*\arctan2(\sin(2*f*x \\
& + 2*e), \cos(2*f*x + 2*e))) + (2*I*A - 3*B)*c^2*\sin(1/2*\arctan2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e))))*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f \\
& *x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 6* \\
& ((2*A + 3*I*B)*c^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2 \\
& *(2*A + 3*I*B)*c^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + ( \\
& 2*A + 3*I*B)*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2* \\
& I*A - 3*B)*c^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2* \\
& I*A - 3*B)*c^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*I* \\
& A - 3*B)*c^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\arctan2( \\
& \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin( \\
& 2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) + 3*((2*I*A - 3*B)*c^2*\cos(5/2*\arctan \\
& 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*I*A - 3*B)*c^2*\cos(3/2*\arctan \\
& 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*I*A - 3*B)*c^2*\cos(1/2*\arctan2( \\
& \sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - (2*A + 3*I*B)*c^2*\sin(5/2*\arctan2(si \\
& n(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(2*A + 3*I*B)*c^2*\sin(3/2*\arctan2(si \\
& n(2*f*x + 2*e), \cos(2*f*x + 2*e))) - (2*A + 3*I*B)*c^2*\sin(1/2*\arctan2(\sin( \\
& 2*f*x + 2*e), \cos(2*f*x + 2*e))))*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos \\
& (2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 \\
& + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1) + 3*((-2*I*A \\
& + 3*B)*c^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(-2*I*A \\
& + 3*B)*c^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (-2*I*A \\
& + 3*B)*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*A + 3* \\
& I*B)*c^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*A + 3* \\
& I*B)*c^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*A + 3*I* \\
& B)*c^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\log(\cos(1/2*ar \\
& ctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e)))^2 - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x \\
& + 2*e))) + 1))*\sqrt{a}*\sqrt{c}/((-16*I*a*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) - 32*I*a*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2 \\
& *e))) - 16*I*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*a* \\
& \sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 32*a*\sin(3/2*\arctan2 \\
& (\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*a*\sin(1/2*\arctan2(\sin(2*f*x + 2* \\
& e), \cos(2*f*x + 2*e))))*f)
\end{aligned}$$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs.  $2(184) = 368$ .

time = 4.42, size = 570, normalized size = 2.50

$$\frac{\sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}}}{\sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}}} + \frac{3(1 - 3A + 3B)\sqrt{a^2 + b^2} + 5(-3A + 3B)\sqrt{a^2 + b^2} + 4(-1A + 3B)\sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}}}{\sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}} \sqrt{\frac{16a^2 - 16ab - 8b^2}{a^2 + b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] 
$$-1/4*(3*\sqrt{(4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f^2)}*(a*f*e^{(3*I*f*x + 3*I*e)} + a*f*e^{(I*f*x + I*e)})*\log(4*(2*((2*I*A - 3*B)*c^2*e^{(3*I*f*x + 3*I*e)} + (2*I*A - 3*B)*c^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + \sqrt{(4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f^2)}*(a*f*e^{(2*I*f*x + 2*I*e)} - a*f))/((2*I*A - 3*B)*c^2*e^{(2*I*f*x + 2*I*e)} + (2*I*A - 3*B)*c^2)) - 3*\sqrt{(4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f^2)}*(a*f*e^{(3*I*f*x + 3*I*e)} + a*f*e^{(I*f*x + I*e)})*\log(4*(2*((2*I*A - 3*B)*c^2*e^{(3*I*f*x + 3*I*e)} + (2*I*A - 3*B)*c^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - \sqrt{(4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f^2)}*(a*f*e^{(2*I*f*x + 2*I*e)} - a*f))/((2*I*A - 3*B)*c^2*e^{(2*I*f*x + 2*I*e)} + (2*I*A - 3*B)*c^2)) + 4*(3*(-2*I*A + 3*B)*c^2*e^{(4*I*f*x + 4*I*e)} + 5*(-2*I*A + 3*B)*c^2*e^{(2*I*f*x + 2*I*e)} + 4*(-I*A + B)*c^2)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)))/(a*f*e^{(3*I*f*x + 3*I*e)} + a*f*e^{(I*f*x + I*e)})$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e + fx) + i))^{5/2} (A + B \tan(e + fx))}{\sqrt{ia(\tan(e + fx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(1/2),x)

[Out] Integral((-I\*c\*(tan(e + f\*x) + I))^(5/2)\*(A + B\*tan(e + f\*x))/sqrt(I\*a\*(tan(e + f\*x) - I)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)/sqrt(I\*a\*tan(f\*x + e) + a), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) \operatorname{li})^{5/2}}{\sqrt{a + a \tan(e + f x) \operatorname{li}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*x)  
)*1i)^(1/2), x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*x)  
)*1i)^(1/2), x)
```

$$3.831 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{3/2}}{\sqrt{a+ia \tan(e+fx)}} dx$$

**Optimal.** Leaf size=169

$$\frac{2(iA-2B)c^{3/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{\sqrt{a} f} + \frac{(iA-2B)c \sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}{af}$$

[Out] 2\*(I\*A-2\*B)\*c^(3/2)\*arctan(c^(1/2)\*(a+I\*a\*tan(f\*x+e))^(1/2)/a^(1/2)/(c-I\*c\*tan(f\*x+e))^(1/2))/f/a^(1/2)+(I\*A-2\*B)\*c\*(a+I\*a\*tan(f\*x+e))^(1/2)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a/f+(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(3/2)/f/(a+I\*a\*tan(f\*x+e))^(1/2)

**Rubi [A]**

time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 52, 65, 223, 209}

$$\frac{2c^{3/2}(-2B+iA) \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{\sqrt{a} f} + \frac{c(-2B+iA) \sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}{af} + \frac{(-B+iA)(c-ictan(e+fx))^{3/2}}{f \sqrt{a+ia \tan(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2))/Sqrt[a + I\*a\*Tan[e + f\*x]],x]

[Out] (2\*(I\*A - 2\*B)\*c^(3/2)\*ArcTan[(Sqrt[c]\*Sqrt[a + I\*a\*Tan[e + f\*x]])/(Sqrt[a]\*Sqrt[c - I\*c\*Tan[e + f\*x]])]/(Sqrt[a]\*f) + ((I\*A - 2\*B)\*c\*Sqrt[a + I\*a\*Tan[e + f\*x]]\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a\*f) + ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(f\*Sqrt[a + I\*a\*Tan[e + f\*x]])

**Rule 52**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^n/(b\*(m + n + 1))), x] + Dist[n\*(b\*c - a\*d)/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 65**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - a\*(d/b) + d\*(x^p/b))^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ



$[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 79

$\text{Int}[(a_.) + (b_.)(x_.)]((c_.) + (d_.)(x_.))^{(n_.)}((e_.) + (f_.)(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)}/(f*(p + 1)*(c*f - d*e)), x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{LtQ}[p, -1] \&\& (!\text{LtQ}[n, -1] || \text{IntegerQ}[p] || !(\text{IntegerQ}[n] || !(\text{EqQ}[e, 0] || !(\text{EqQ}[c, 0] || \text{LtQ}[p, n])))$

### Rule 209

$\text{Int}[(a_.) + (b_.)(x_.)^2]^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[b, 2]))* \text{ArcTan}[\text{Rt}[b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{GtQ}[b, 0])$

### Rule 223

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)(x_.)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

### Rule 3669

$\text{Int}[(a_.) + (b_.)\tan[(e_.) + (f_.)(x_.))]^{(m_.)}((A_.) + (B_.)\tan[(e_.) + (f_.)(x_.)])*((c_.) + (d_.)\tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m - 1)}*(c + d*x)^{(n - 1)}*(A + B*x), x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2}}{\sqrt{a + ia \tan(e + fx)}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{f \sqrt{a + ia \tan(e + fx)}} - \frac{((A + 2iB)c) \text{Subst} \left( \int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^{3/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - 2B)c \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{af} \\
&= \frac{(iA - 2B)c \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{af} \\
&= \frac{(iA - 2B)c \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{af} \\
&= \frac{2(iA - 2B)c^{3/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}} \right)}{\sqrt{a} f} + \dots
\end{aligned}$$

**Mathematica [A]**

time = 2.57, size = 161, normalized size = 0.95

$$\frac{c^2(\cos(fx) + i \sin(fx))(i \cos(fx) + \sin(fx))(A + B \tan(e + fx))(2(A + 2iB) \text{ArcTan}(\cos(e + fx) + i \sin(e + fx)) + \cos(e + fx)(i + \tan(e + fx))(-2iA + 3B + iB \tan(e + fx)))}{f(A \cos(e + fx) + B \sin(e + fx)) \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/Sqrt[a + I*a*
Tan[e + f*x]], x]
```

```
[Out] (c^2*(Cos[f*x] + I*Sin[f*x])*(I*Cos[f*x] + Sin[f*x])*(A + B*Tan[e + f*x])*(
2*(A + (2*I)*B)*ArcTan[Cos[e + f*x] + I*Sin[e + f*x]] + Cos[e + f*x]*(I + T
an[e + f*x])*((-2*I)*A + 3*B + I*B*Tan[e + f*x]))/(f*(A*Cos[e + f*x] + B*S
in[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs.  $2(140) = 280$ .

time = 0.46, size = 499, normalized size = 2.95



```

*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 2*I*B)*c*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + (I*A - 2*B)*c*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + (I*A - 2*B)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) * arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
, -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - ((-I*A + 2*B
)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (-I*A + 2*B)*c*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 2*I*B)*c*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 2*I*B)*c*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) * log(cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - ((I*
A - 2*B)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (I*A - 2*
B)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (A + 2*I*B)*c*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (A + 2*I*B)*c*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) * log(cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1))
*sqrt(a)*sqrt(c)/((-2*I*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) - 2*I*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*a*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*a*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*f)

```

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs.  $2(137) = 274$ .  
time = 5.03, size = 474, normalized size = 2.80

$$\left( \frac{\sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2}} \operatorname{arctan}\left( \frac{\sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2}}}{\sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2} + 1}} \right) - \sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2}} \operatorname{arctan}\left( \frac{\sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2}}}{\sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2} - 1}} \right)}{2f} + 4((-I + 2B)c^{3/2} + (-I + B)c) \sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2}} \sqrt{\frac{A^2 + 4AB - 4B^2}{4f^2} + 1} \right) e^{I f x + I e}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2
),x, algorithm="fricas")

```

```

[Out] -1/2*(a*sqrt((A^2 + 4*I*A*B - 4*B^2)*c^3/(a*f^2))*f*e^(I*f*x + I*e)*log(-4*
(2*((I*A - 2*B)*c*e^(3*I*f*x + 3*I*e) + (I*A - 2*B)*c*e^(I*f*x + I*e))*sqrt
(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a*f*e^(2
*I*f*x + 2*I*e) - a*f)*sqrt((A^2 + 4*I*A*B - 4*B^2)*c^3/(a*f^2)))/((-I*A +
2*B)*c*e^(2*I*f*x + 2*I*e) + (-I*A + 2*B)*c) - a*sqrt((A^2 + 4*I*A*B - 4*B
^2)*c^3/(a*f^2))*f*e^(I*f*x + I*e)*log(-4*(2*((I*A - 2*B)*c*e^(3*I*f*x + 3*
I*e) + (I*A - 2*B)*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (a*f*e^(2*I*f*x + 2*I*e) - a*f)*sqrt((A^2
+ 4*I*A*B - 4*B^2)*c^3/(a*f^2)))/((-I*A + 2*B)*c*e^(2*I*f*x + 2*I*e) + (-I*
A + 2*B)*c) + 4*((-I*A + 2*B)*c*e^(2*I*f*x + 2*I*e) + (-I*A + B)*c)*sqrt(a
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x -
I*e)/(a*f)

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e + fx) + i))^{\frac{3}{2}} (A + B \tan(e + fx))}{\sqrt{ia(\tan(e + fx) - i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))**(3/2)*(A + B*tan(e + f*x))/sqrt(I*a*(tan(e + f*x) - I)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(3/2)/sqrt(I*a*tan(f*x + e) + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + fx)) (c - c \tan(e + fx) li)^{3/2}}{\sqrt{a + a \tan(e + fx) li}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*li)^(3/2))/(a + a*tan(e + f*x)*li)^(1/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*li)^(3/2))/(a + a*tan(e + f*x)*li)^(1/2), x)
```

$$3.832 \quad \int \frac{(A+B \tan(e+fx)) \sqrt{c - i c \tan(e + f x)}}{\sqrt{a + i a \tan(e + f x)}} dx$$

Optimal. Leaf size=110

$$-\frac{2B\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + i a \tan(e + f x)}}{\sqrt{a} \sqrt{c - i c \tan(e + f x)}}\right)}{\sqrt{a} f} + \frac{(iA - B)\sqrt{c - i c \tan(e + f x)}}{f \sqrt{a + i a \tan(e + f x)}}$$

[Out]  $-2*B*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})*c^{(1/2)}/f/a^{(1/2)}+(I*A-B)*(c-I*c*\tan(f*x+e))^{(1/2)}/f/(a+I*a*\tan(f*x+e))^{(1/2)}$

Rubi [A]

time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3669, 79, 65, 223, 209}

$$\frac{(-B + iA)\sqrt{c - i c \tan(e + f x)}}{f \sqrt{a + i a \tan(e + f x)}} - \frac{2B\sqrt{c} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + i a \tan(e + f x)}}{\sqrt{a} \sqrt{c - i c \tan(e + f x)}}\right)}{\sqrt{a} f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[e + f*x])*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]]/Sqrt[a + I*a*\operatorname{Tan}[e + f*x]], x]$

[Out]  $(-2*B*Sqrt[c]*\operatorname{ArcTan}[(Sqrt[c]*Sqrt[a + I*a*\operatorname{Tan}[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])]/(Sqrt[a]*f) + ((I*A - B)*Sqrt[c - I*c*\operatorname{Tan}[e + f*x]])/(f*Sqrt[a + I*a*\operatorname{Tan}[e + f*x]])$

Rule 65

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^{p/b})^n), x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 79

$\operatorname{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-b*e - a*f)*(c + d*x)^{(n+1)}*((e + f*x)^{(p+1)}/(f*(p+1)*(c*f - d*e))), x] - \operatorname{Dist}[(a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1))]/(f*(p+1)*(c*f - d*e)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^{(p+1)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \operatorname{LtQ}[p, -1] \&\& (!\operatorname{LtQ}[n, -1] || I$

```

IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))

```

### Rule 209

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

```

### Rule 223

```

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

```

### Rule 3669

```

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx)) \sqrt{c - i c \tan(e + fx)}}{\sqrt{a + i a \tan(e + fx)}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{A+Bx}{(a+iax)^{3/2} \sqrt{c-icx}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{f \sqrt{a + i a \tan(e + fx)}} - \frac{(iBc) \text{Subst} \left( \int \frac{1}{\sqrt{a + i a \tan(e + fx)}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{f \sqrt{a + i a \tan(e + fx)}} - \frac{(2Bc) \text{Subst} \left( \int \frac{1}{\sqrt{2c}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{f \sqrt{a + i a \tan(e + fx)}} - \frac{(2Bc) \text{Subst} \left( \int \frac{1}{1 + \frac{cx^2}{a}} dx, x, \tan(e + fx) \right)}{f} \\
&= -\frac{2B\sqrt{c} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + i a \tan(e + fx)}}{\sqrt{a} \sqrt{c - i c \tan(e + fx)}} \right)}{\sqrt{a} f} + \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{f \sqrt{a + i a \tan(e + fx)}}
\end{aligned}$$

**Mathematica [A]**

time = 1.60, size = 152, normalized size = 1.38

$$\frac{c \sec(e + fx) ((A + iB) (\cos(\frac{1}{2}(e + fx)) - i \sin(\frac{1}{2}(e + fx))) + 2iB \operatorname{ArcTan}(\cos(e + fx) + i \sin(e + fx) (\cos(\frac{1}{2}(e + fx)) + i \sin(\frac{1}{2}(e + fx)))) (i \cos(\frac{1}{2}(e + fx)) + \sin(\frac{1}{2}(e + fx)))}{f \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/Sqrt[a + I*a*Tan[e + f*x]],x]
```

```
[Out] (c*Sec[e + f*x]*((A + I*B)*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/2]) + (2*I)*B*ArcTan[Cos[e + f*x] + I*Sin[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2]))*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(89) = 178.

time = 0.42, size = 323, normalized size = 2.94

method	result
derivativedivides	$-\frac{i \sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} \left( -2iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + \dots)} \right) \right)}{\dots}$
default	$-\frac{i \sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} \left( -2iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + \dots)} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a*(-2*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)+B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c-B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I-tan(f*x+e))^2/(a*c)^(1/2)
```

**Maxima [A]**

time = 0.56, size = 150, normalized size = 1.36

$$\frac{(2B \arctan(\cos(fx + e), \sin(fx + e) + 1) + 2B \arctan(\cos(fx + e), -\sin(fx + e) + 1) - 2(iA - B) \cos(fx + e) + iB \log(\cos(fx + e)^2 + \sin(fx + e)^2 + 2 \sin(fx + e) + 1) - iB \log(\cos(fx + e)^2 + \sin(fx + e)^2 - 2 \sin(fx + e) + 1) - 2(A + iB) \sin(fx + e)) \sqrt{c}}{2 \sqrt{a} f}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2),x, algorithm="maxima")

[Out]  $-1/2*(2*B*\arctan2(\cos(f*x + e), \sin(f*x + e) + 1) + 2*B*\arctan2(\cos(f*x + e), -\sin(f*x + e) + 1) - 2*(I*A - B)*\cos(f*x + e) + I*B*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 + 2*\sin(f*x + e) + 1) - I*B*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 - 2*\sin(f*x + e) + 1) - 2*(A + I*B)*\sin(f*x + e))*\sqrt{c}/(\sqrt{a}*f)$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs.  $2(88) = 176$ .

time = 13.11, size = 366, normalized size = 3.33

$$\left( a f \sqrt{\frac{Bc}{af^2}} e^{i(fx+e)} \log \left( \frac{e^{i(2(B\cos(fx+e)+B\cos(fx+e))\sqrt{\frac{a}{e^{2B(fx+e)}+1}}\sqrt{\frac{c}{e^{2B(fx+e)}+1}}+e^{i(2(B\cos(fx+e)-B\cos(fx+e))\sqrt{\frac{a}{e^{2B(fx+e)}+1}}\sqrt{\frac{c}{e^{2B(fx+e)}+1}})}\sqrt{\frac{Bc}{af^2}})}{2a}} \right) - a f \sqrt{\frac{Bc}{af^2}} e^{i(fx+e)} \log \left( \frac{e^{i(2(B\cos(fx+e)+B\cos(fx+e))\sqrt{\frac{a}{e^{2B(fx+e)}+1}}\sqrt{\frac{c}{e^{2B(fx+e)}+1}}-e^{i(2(B\cos(fx+e)-B\cos(fx+e))\sqrt{\frac{a}{e^{2B(fx+e)}+1}}\sqrt{\frac{c}{e^{2B(fx+e)}+1}})}\sqrt{\frac{Bc}{af^2}})}{2a}} \right) - 2((-iA+B)e^{2B(fx+e)}-iA+B)\sqrt{\frac{a}{e^{2B(fx+e)}+1}}\sqrt{\frac{c}{e^{2B(fx+e)}+1}} \right) e^{i(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2),x, algorithm="fricas")

[Out]  $1/2*(a*f*\sqrt{-B^2*c/(a*f^2)}*e^{(I*f*x + I*e)}*\log(4*(2*(B*e^{(3*I*f*x + 3*I*e)} + B*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + (a*f*e^{(2*I*f*x + 2*I*e)} - a*f)*\sqrt{-B^2*c/(a*f^2)}))/(B*e^{(2*I*f*x + 2*I*e)} + B) - a*f*\sqrt{-B^2*c/(a*f^2)}*e^{(I*f*x + I*e)}*\log(4*(2*(B*e^{(3*I*f*x + 3*I*e)} + B*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - (a*f*e^{(2*I*f*x + 2*I*e)} - a*f)*\sqrt{-B^2*c/(a*f^2)}))/(B*e^{(2*I*f*x + 2*I*e)} + B) - 2*((-i*A + B)*e^{(2*I*f*x + 2*I*e)} - I*A + B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)})*e^{(-I*f*x - I*e)}/(a*f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ic(\tan(e+fx)+i)}(A+B\tan(e+fx))}{\sqrt{ia(\tan(e+fx)-i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2),x)

[Out] Integral(sqrt(-I\*c\*(tan(e + f\*x) + I))\*(A + B\*tan(e + f\*x))/sqrt(I\*a\*(tan(e + f\*x) - I)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(-I\*c\*tan(f\*x + e) + c)/sqrt(I\*a\*tan(f\*x + e) + a), x)

**Mupad [B]**

time = 12.16, size = 250, normalized size = 2.27

$$\frac{A \sqrt{c - c \tan(e + f x)} \operatorname{Li} \operatorname{Li}}{f \sqrt{a + a \tan(e + f x)} \operatorname{Li}} - \frac{4 B \sqrt{c} \operatorname{atan}\left(\frac{\sqrt{c} (\sqrt{a + a \tan(e + f x)} \operatorname{Li} - \sqrt{a})}{\sqrt{a} (\sqrt{c - c \tan(e + f x)} \operatorname{Li} - \sqrt{c})}\right)}{\sqrt{a} f} - \frac{4 B (\sqrt{a + a \tan(e + f x)} \operatorname{Li} - \sqrt{a})}{f (\sqrt{c - c \tan(e + f x)} \operatorname{Li} - \sqrt{c}) \left(-\frac{a}{c} + \frac{(\sqrt{a + a \tan(e + f x)} \operatorname{Li} - \sqrt{a})^2}{(\sqrt{c - c \tan(e + f x)} \operatorname{Li} - \sqrt{c})^2} + \frac{2 \sqrt{a} (\sqrt{a + a \tan(e + f x)} \operatorname{Li} - \sqrt{a})}{\sqrt{c} (\sqrt{c - c \tan(e + f x)} \operatorname{Li} - \sqrt{c})}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a + a\*tan(e + f\*x)\*1i)^(1/2),x)

[Out] (A\*(c - c\*tan(e + f\*x)\*1i)^(1/2)\*1i)/(f\*(a + a\*tan(e + f\*x)\*1i)^(1/2)) - (4\*B\*c^(1/2)\*atan((c^(1/2)\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(a^(1/2))\*((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))))/(a^(1/2)\*f) - (4\*B\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(f\*((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2))^2/((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2))^2 - a/c + (2\*a^(1/2)\*((a + a\*tan(e + f\*x)\*1i)^(1/2) - a^(1/2)))/(c^(1/2)\*((c - c\*tan(e + f\*x)\*1i)^(1/2) - c^(1/2)))))

$$3.833 \quad \int \frac{A+B \tan(e+fx)}{\sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}} dx$$

Optimal. Leaf size=92

$$-\frac{iA+B}{f\sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}} + \frac{iA\sqrt{c-ictan(e+fx)}}{cf\sqrt{a+ia \tan(e+fx)}}$$

[Out]  $(-I*A-B)/f/(a+I*a*\tan(f*x+e))^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)}+I*A*(c-I*c*\tan(f*x+e))^{(1/2)}/c/f/(a+I*a*\tan(f*x+e))^{(1/2)}$

Rubi [A]

time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ ,

Rules used = {3669, 79, 37}

$$\frac{iA\sqrt{c-ictan(e+fx)}}{cf\sqrt{a+ia \tan(e+fx)}} - \frac{B+iA}{f\sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])/(\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]), x]$

[Out]  $-((I*A + B)/(f*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])) + (I*A*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])/(c*f*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/(b*c - a*d)*(m + 1))}, x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

Rule 79

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)*((e + f*x)^{(p + 1)/(f*(p + 1)*(c*f - d*e))}, x] - \text{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{LtQ}[p, -1] \&\& (!\text{LtQ}[n, -1] || \text{IntegerQ}[p] || !(\text{IntegerQ}[n] || !(\text{EqQ}[e, 0] || !(\text{EqQ}[c, 0] || \text{LtQ}[p, n])))$

Rule 3669

$\text{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Di}$

st[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{\sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{3/2}(c-icx)^{3/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iA + B}{f \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}} + \frac{(aA)}{cf \sqrt{a + ia \tan(e + fx)}}$$

$$= -\frac{iA + B}{f \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}} + \frac{iA \sqrt{a + ia \tan(e + fx)}}{cf \sqrt{a + ia \tan(e + fx)}}$$

**Mathematica [A]**

time = 1.10, size = 77, normalized size = 0.84

$$\frac{(\cos(e + fx) + i \sin(e + fx))(B \cos(e + fx) - A \sin(e + fx)) \sqrt{c - ictan(e + fx)}}{cf \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Tan[e + f\*x])/(Sqrt[a + I\*a\*Tan[e + f\*x]]\*Sqrt[c - I\*c\*Tan[e + f\*x]]), x]

[Out] -(((Cos[e + f\*x] + I\*Sin[e + f\*x])\*(B\*Cos[e + f\*x] - A\*Sin[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(c\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]]))

**Maple [A]**

time = 0.45, size = 99, normalized size = 1.08

method	result
risch	$-\frac{iA e^{2i(fx+e)} + B e^{2i(fx+e)} - iA + B}{2 \sqrt{\frac{a e^{2i(fx+e)}}{e^{2i(fx+e)} + 1}} (e^{2i(fx+e)} + 1) \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$
derivativedivides	$\frac{\sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} (A(\tan^3(fx + e)) - B(\tan^2(fx + e)) + A \tan(fx + e))}{fac(i + \tan(fx + e))^2 (i - \tan(fx + e))^2}$
default	$\frac{\sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} (A(\tan^3(fx + e)) - B(\tan^2(fx + e)) + A \tan(fx + e))}{fac(i + \tan(fx + e))^2 (i - \tan(fx + e))^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $1/f*(a*(1+I*\tan(f*x+e)))^{1/2}*(-c*(I*\tan(f*x+e)-1))^{1/2}/a/c*(A*\tan(f*x+e))^3-B*\tan(f*x+e)^2+A*\tan(f*x+e)-B/(I+\tan(f*x+e))^2/(I-\tan(f*x+e))^2$

**Maxima [A]**

time = 0.60, size = 132, normalized size = 1.43

$$\frac{2((A-iB)\cos(4fx+4e)-2iB\cos(2fx+2e)-(-iA-B)\sin(4fx+4e)+2B\sin(2fx+2e)-A-iB)\sqrt{a}\sqrt{c}}{-4(iac\cos(3fx+3e)+iaccos(fx+e)-ac\sin(3fx+3e)-ac\sin(fx+e))f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x,algorithm="maxima")`

[Out]  $-2*((A-I*B)*\cos(4*f*x+4*e)-2*I*B*\cos(2*f*x+2*e)-(-I*A-B)*\sin(4*f*x+4*e)+2*B*\sin(2*f*x+2*e)-A-I*B)*\sqrt{a}*\sqrt{c}/((-4*I*a*c*\cos(3*f*x+3*e)-4*I*a*c*\cos(f*x+e)+4*a*c*\sin(3*f*x+3*e)+4*a*c*\sin(f*x+e))*f)$

**Fricas [A]**

time = 6.15, size = 121, normalized size = 1.32

$$\frac{((-iA-B)e^{(4i fx+4i e)}+2Be^{(3i fx+3i e)}-2Be^{(2i fx+2i e)}+2Be^{(i fx+i e)}+iA-B)\sqrt{\frac{a}{e^{(2i fx+2i e)}+1}}\sqrt{\frac{c}{e^{(2i fx+2i e)}+1}}e^{(-i fx-i e)}}{2acf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x,algorithm="fricas")`

[Out]  $1/2*((-I*A-B)*e^{(4*I*f*x+4*I*e)}+2*B*e^{(3*I*f*x+3*I*e)}-2*B*e^{(2*I*f*x+2*I*e)}+2*B*e^{(I*f*x+I*e)}+I*A-B)*\sqrt{a/(e^{(2*I*f*x+2*I*e)}+1)}*\sqrt{c/(e^{(2*I*f*x+2*I*e)}+1)}*e^{(-I*f*x-I*e)/(a*c*f)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan(e+fx)}{\sqrt{ia(\tan(e+fx)-i)}\sqrt{-ic(\tan(e+fx)+i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x)`

[Out] `Integral((A+B*tan(e+f*x))/(sqrt(I*a*(tan(e+f*x)-I))*sqrt(-I*c*(tan(e+f*x)+I))), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)/(sqrt(I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f*x + e) + c)), x)
```

**Mupad [B]**

time = 0.80, size = 143, normalized size = 1.55

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}(A1i+B-A\cos(2e+2fx)1i+B\cos(2e+2fx)-A\sin(2e+2fx)-B\sin(2e+2fx)1i)}{2af\sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2)),x)
```

```
[Out] -(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*1i + B - A*cos(2*e + 2*f*x)*1i + B*cos(2*e + 2*f*x) - A*sin(2*e + 2*f*x) - B*sin(2*e + 2*f*x)*1i))/(2*a*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

$$3.834 \quad \int \frac{A+B \tan(e+fx)}{\sqrt{a+ia \tan(e+fx)} (c-ictan(e+fx))^{3/2}} dx$$

Optimal. Leaf size=157

$$\frac{iA-B}{f\sqrt{a+ia \tan(e+fx)} (c-ictan(e+fx))^{3/2}} - \frac{(2iA-B)\sqrt{a+ia \tan(e+fx)}}{3af(c-ictan(e+fx))^{3/2}} - \frac{(2iA-B)\sqrt{a+ia \tan(e+fx)}}{3acf\sqrt{c-ictan(e+fx)}}$$

[Out]  $-1/3*(2*I*A-B)*(a+I*a*\tan(f*x+e))^{(1/2)}/a/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}+(I*A-B)/f/(a+I*a*\tan(f*x+e))^{(1/2)}/(c-I*c*\tan(f*x+e))^{(3/2)}-1/3*(2*I*A-B)*(a+I*a*\tan(f*x+e))^{(1/2)}/a/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{-B+iA}{f\sqrt{a+ia \tan(e+fx)} (c-ictan(e+fx))^{3/2}} - \frac{(-B+2iA)\sqrt{a+ia \tan(e+fx)}}{3acf\sqrt{c-ictan(e+fx)}} - \frac{(-B+2iA)\sqrt{a+ia \tan(e+fx)}}{3af(c-ictan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])/(Sqrt[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}), x]$

[Out]  $(I*A - B)/(f*Sqrt[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - (((2*I)*A - B)*Sqrt[a + I*a*\text{Tan}[e + f*x]])/(3*a*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - (((2*I)*A - B)*Sqrt[a + I*a*\text{Tan}[e + f*x]])/(3*a*c*f*Sqrt[c - I*c*\text{Tan}[e + f*x]])$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{NeQ}[m, -1]$

Rule 47

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}), x] - \text{Dist}[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{I} \text{LtQ}[\text{Simplify}[m + n + 2], 0] \&\& \text{NeQ}[m, -1] \&\& !(\text{LtQ}[m, -1] \&\& \text{LtQ}[n, -1] \&\& (\text{EqQ}[a, 0] || (\text{NeQ}[c, 0] \&\& \text{LtQ}[m - n, 0] \&\& \text{IntegerQ}[n]))) \&\& (\text{SumSimplerQ}[m, 1] || !\text{SumSimplerQ}[n, 1])$

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

## Rule 3669

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{A + B \tan(e + fx)}{\sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{3/2}(c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{iA - B}{f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{3/2}} + \frac{((2A - Bc))}{3c}$$

$$= \frac{iA - B}{f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{3/2}} - \frac{(2iA - 3c)}{3c}$$

$$= \frac{iA - B}{f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{3/2}} - \frac{(2iA - 3c)}{3c}$$

## Mathematica [A]

time = 1.85, size = 103, normalized size = 0.66

$$\frac{i(\cos(2(e + fx)) + i \sin(2(e + fx)))(-3A + (A + 2iB) \cos(2(e + fx)) + (-2iA + B) \sin(2(e + fx))) \sqrt{c - ictan(e + fx)}}{6c^2 f \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e +
f*x])^(3/2)), x]
```



```
[Out] ((I/6)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(-3*A + (A + (2*I)*B)*Cos[2*(e + f*x)] + ((-2*I)*A + B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(c^2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple [A]**

time = 0.42, size = 151, normalized size = 0.96

method	result
risch	$-\frac{iAe^{4i(fx+e)} + Be^{4i(fx+e)} + 6iAe^{2i(fx+e)} - 3iA + 3B}{12c \sqrt{\frac{ae^{2i(fx+e)}}{e^{2i(fx+e)} + 1}} (e^{2i(fx+e)} + 1) \sqrt{\frac{c}{e^{2i(fx+e)} + 1}}} f$
derivativedivides	$-\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} (2iA(\tan^4(fx + e)) - iB(\tan^3(fx + e)) - B(\tan^2(fx + e)))}{3fa c^2 (i - \tan(fx + e))^2 (i + \tan(fx + e))}$
default	$-\frac{i \sqrt{a(1 + i \tan(fx + e))} \sqrt{-c(i \tan(fx + e) - 1)} (2iA(\tan^4(fx + e)) - iB(\tan^3(fx + e)) - B(\tan^2(fx + e)))}{3fa c^2 (i - \tan(fx + e))^2 (i + \tan(fx + e))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a/c^2*(2*I*A*tan(f*x+e)^4-I*B*tan(f*x+e)^3-B*tan(f*x+e)^4+3*I*A*tan(f*x+e)^2-2*A*tan(f*x+e)^3-I*B*tan(f*x+e)+I*A-2*A*tan(f*x+e)+B)/(I-tan(f*x+e))^2/(I+tan(f*x+e))^3
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [A]**

time = 4.68, size = 155, normalized size = 0.99

$$\frac{((-iA - B)e^{6ifx+6ie} + (-7iA - B)e^{4ifx+4ie} - 4(-iA - B)e^{3ifx+3ie} - 3(iA + B)e^{2ifx+2ie} - 4(-iA - B)e^{ifx+ie} + 3iA - 3B) \sqrt{\frac{a}{e^{2ifx+2ie} + 1}} \sqrt{\frac{c}{e^{2ifx+2ie} + 1}} e^{-ifx-ie}}{12ac^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

[Out]  $1/12*((-I*A - B)*e^{(6*I*f*x + 6*I*e)} + (-7*I*A - B)*e^{(4*I*f*x + 4*I*e)} - 4*(-I*A - B)*e^{(3*I*f*x + 3*I*e)} - 3*(I*A + B)*e^{(2*I*f*x + 2*I*e)} - 4*(-I*A - B)*e^{(I*f*x + I*e)} + 3*I*A - 3*B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-I*f*x - I*e)}/(a*c^{2*f})$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{\sqrt{ia(\tan(e + fx) - i)} (-ic(\tan(e + fx) + i))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(3/2),x)`

[Out] `Integral((A + B*tan(e + f*x))/(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(3/2)), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="giac")`

[Out] `integrate((B*tan(f*x + e) + A)/(sqrt(I*a*tan(f*x + e) + a))*(-I*c*tan(f*x + e) + c)^(3/2)), x)`

**Mupad [B]**

time = 0.73, size = 146, normalized size = 0.93

$$\frac{\sqrt{\frac{a(\cos(2e + 2fx) + 1 + \sin(2e + 2fx)1i)}{\cos(2e + 2fx) + 1}} (2A \sin(2e + 2fx) + A \cos(2e + 2fx)1i - 2B \cos(2e + 2fx) - A3i + B \sin(2e + 2fx)1i)}{6acf \sqrt{\frac{c(\cos(2e + 2fx) + 1 - \sin(2e + 2fx)1i)}{\cos(2e + 2fx) + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(3/2)),x)`

[Out] `((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*cos(2*e + 2*f*x)*1i - A*3i - 2*B*cos(2*e + 2*f*x) + 2*A*sin(2*e + 2*f*x) + B*sin(2*e + 2*f*x)*1i))/(6*a*c*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))`

$$3.835 \quad \int \frac{A+B \tan(e+fx)}{\sqrt{a+ia \tan(e+fx)} (c-ictan(e+fx))^{5/2}} dx$$

Optimal. Leaf size=213

$$\frac{iA-B}{f\sqrt{a+ia \tan(e+fx)} (c-ictan(e+fx))^{5/2}} - \frac{(3iA-2B)\sqrt{a+ia \tan(e+fx)}}{5af(c-ictan(e+fx))^{5/2}} - \frac{2(3iA-2B)\sqrt{a+ia \tan(e+fx)}}{15acf(c-ictan(e+fx))^{5/2}}$$

[Out]  $-2/15*(3*I*A-2*B)*(a+I*a*\tan(f*x+e))^{(1/2)}/a/c^2/f/(c-I*c*\tan(f*x+e))^{(1/2)}$   
 $+ (I*A-B)/f/(a+I*a*\tan(f*x+e))^{(1/2)}/(c-I*c*\tan(f*x+e))^{(5/2)} - 1/5*(3*I*A-2*B)$   
 $*(a+I*a*\tan(f*x+e))^{(1/2)}/a/f/(c-I*c*\tan(f*x+e))^{(5/2)} - 2/15*(3*I*A-2*B)*(a$   
 $+I*a*\tan(f*x+e))^{(1/2)}/a/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.19, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{2(-2B+3iA)\sqrt{a+ia \tan(e+fx)}}{15ac^2f\sqrt{c-ictan(e+fx)}} - \frac{2(-2B+3iA)\sqrt{a+ia \tan(e+fx)}}{15acf(c-ictan(e+fx))^{3/2}} - \frac{(-2B+3iA)\sqrt{a+ia \tan(e+fx)}}{5af(c-ictan(e+fx))^{5/2}} + \frac{-B+iA}{f\sqrt{a+ia \tan(e+fx)}(c-ictan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])/(Sqrt[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}), x]$

[Out]  $(I*A - B)/(f*Sqrt[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (((3*I)*A - 2*B)*Sqrt[a + I*a*\text{Tan}[e + f*x]])/(5*a*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) - (2*((3*I)*A - 2*B)*Sqrt[a + I*a*\text{Tan}[e + f*x]])/(15*a*c*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) - (2*((3*I)*A - 2*B)*Sqrt[a + I*a*\text{Tan}[e + f*x]])/(15*a*c^2*f*Sqrt[c - I*c*\text{Tan}[e + f*x]])$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 47

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)*((c + d*x)^{(n + 1)/((b*c - a*d)*(m + 1))}], x] - \text{Dist}[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[m + n + 2] && !IntegerQ[m] && !IntegerQ[n] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimpler

Q[m, 1] || !SumSimplerQ[n, 1])

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\int \frac{A + B \tan(e + fx)}{\sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{3/2}(c-icx)^{7/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{iA - B}{f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}} + \frac{((3A - B) \cos(3(e + fx)) + 3 \sin(3(e + fx))) \sqrt{c - ictan(e + fx)}}{5 f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}}$$

$$= \frac{iA - B}{f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}} - \frac{(3iA - B) \cos(3(e + fx)) + 3 \sin(3(e + fx))}{5 f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}}$$

$$= \frac{iA - B}{f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}} - \frac{(3iA - B) \cos(3(e + fx)) + 3 \sin(3(e + fx))}{5 f \sqrt{a + ia \tan(e + fx)} (c - ictan(e + fx))^{5/2}}$$

### Mathematica [A]

time = 3.01, size = 128, normalized size = 0.60

$$\frac{(\cos(3(e + fx)) + i \sin(3(e + fx)))(5(-6iA + B) \cos(e + fx) + (6iA - 9B) \cos(3(e + fx)) + (3A + 2iB)(-5 \sin(e + fx) + 3 \sin(3(e + fx)))) \sqrt{c - ictan(e + fx)}}{60c^3 f \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(5*((-6*I)*A + B)*Cos[e + f*x] + (6*I)*A - 9*B)*Cos[3*(e + f*x)] + (3*A + (2*I)*B)*(-5*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(60*c^3*f*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple [A]**

time = 0.40, size = 184, normalized size = 0.86

method	result
risch	$\frac{-3iAe^{6i(fx+e)}+3Be^{6i(fx+e)}+15iAe^{4i(fx+e)}+5Be^{4i(fx+e)}+45iAe^{2i(fx+e)}-15Be^{2i(fx+e)}-15iA+15B}{120c^2 \sqrt{\frac{ae^{2i(fx+e)}}{e^{2i(fx+e)}+1}} \sqrt{\frac{c}{e^{2i(fx+e)}+1}} (e^{2i(fx+e)}+1)f}$
derivativedivides	$\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} (4iB(\tan^5(fx+e))+12iA(\tan^4(fx+e))+6A(\tan^3(fx+e)))}{15f}$
default	$\frac{\sqrt{a(1+i \tan(fx+e))} \sqrt{-c(i \tan(fx+e)-1)} (4iB(\tan^5(fx+e))+12iA(\tan^4(fx+e))+6A(\tan^3(fx+e)))}{15f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a/c^3*(4*I*B*tan(f*x+e)^5+12*I*A*tan(f*x+e)^4+6*A*tan(f*x+e)^5+2*I*B*tan(f*x+e)^3-8*B*tan(f*x+e)^4+18*I*A*tan(f*x+e)^2+3*A*tan(f*x+e)^3-2*I*B*tan(f*x+e)-7*B*tan(f*x+e)^2+6*I*A-3*A*tan(f*x+e)+B)/(I+tan(f*x+e))^4/(I-tan(f*x+e))^2
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x,algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [A]**

time = 3.41, size = 170, normalized size = 0.80

$$\frac{(3(iA+B)e^{8i(fx+3e)}+2(9iA+4B)e^{6i(fx+6e)}+10(6iA-B)e^{4i(fx+4e)}+8(-6iA-B)e^{3i(fx+3e)}+30iAe^{2i(fx+2e)}+8(-6iA-B)e^{i(fx+e)}-15iA+15B)\sqrt{\frac{a}{e^{2i(fx+2e)}+1}}\sqrt{\frac{c}{e^{2i(fx+2e)}+1}}e^{(-ifx-i)c}}{120ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] 
$$-1/120*(3*(I*A + B)*e^{(8*I*f*x + 8*I*e)} + 2*(9*I*A + 4*B)*e^{(6*I*f*x + 6*I*e)} + 10*(6*I*A - B)*e^{(4*I*f*x + 4*I*e)} + 8*(-6*I*A - B)*e^{(3*I*f*x + 3*I*e)} + 30*I*A*e^{(2*I*f*x + 2*I*e)} + 8*(-6*I*A - B)*e^{(I*f*x + I*e)} - 15*I*A + 15*B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-I*f*x - I*e)/(a*c^3*f)}$$

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{\sqrt{ia (\tan(e + fx) - i)} (-ic (\tan(e + fx) + i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x)

[Out] Integral((A + B\*tan(e + f\*x))/(sqrt(I\*a\*(tan(e + f\*x) - I))\*(-I\*c\*(tan(e + f\*x) + I))^(5/2)), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(1/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/(sqrt(I\*a\*tan(f\*x + e) + a)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)), x)

Mupad [B]

time = 9.89, size = 186, normalized size = 0.87

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}} (A45i - 15B + A \cos(4e+4fx) 3i + 20B \cos(2e+2fx) + 3B \cos(4e+4fx) - 30A \sin(2e+2fx) - 3A \sin(4e+4fx) - B \sin(2e+2fx) 10i + B \sin(4e+4fx) 3i)}{120 a^2 f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^(1/2)\*(c - c\*tan(e + f\*x)\*1i)^(5/2)),x)

[Out] 
$$-(((a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{(1/2)}*(A*45i - 15*B + A*\cos(4*e + 4*f*x)*3i + 20*B*\cos(2*e + 2*f*x) + 3*B*\cos(4*e + 4*f*x) - 30*A*\sin(2*e + 2*f*x) - 3*A*\sin(4*e + 4*f*x) - B*\sin(2*e + 2*f*x)*10i + B*\sin(4*e + 4*f*x)*3i))/((120*a*c^2*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{(1/2)})$$

$$3.836 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=287

$$\frac{5(2iA - 5B)c^{7/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{a^{3/2} f} - \frac{5(2iA - 5B)c^3 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{2a^2 f}$$

[Out]  $-5*(2*I*A-5*B)*c^{(7/2)}*\arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)})/(c-I*c*\tan(f*x+e))^{(1/2)}/a^{(3/2)}/f-5/2*(2*I*A-5*B)*c^3*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(1/2)}/a^2/f-5/6*(2*I*A-5*B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)}*(c-I*c*\tan(f*x+e))^{(3/2)}/a^2/f-2/3*(2*I*A-5*B)*c*(c-I*c*\tan(f*x+e))^{(5/2)}/a/f/(a+I*a*\tan(f*x+e))^{(1/2)}+1/3*(I*A-B)*(c-I*c*\tan(f*x+e))^{(7/2)}/f/(a+I*a*\tan(f*x+e))^{(3/2)}$

**Rubi** [A]

time = 0.23, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{5c^{7/2}(-5B+2iA)\operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ictan(e + fx)}}\right)}{a^{3/2} f} - \frac{5c^3(-5B+2iA)\sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}}{2a^2 f} - \frac{5c^2(-5B+2iA)\sqrt{a + ia \tan(e + fx)}(c - ictan(e + fx))^{3/2}}{6a^2 f} - \frac{2c(-5B+2iA)(c - ictan(e + fx))^{5/2}}{3af\sqrt{a + ia \tan(e + fx)}} + \frac{(-B+iA)(c - ictan(e + fx))^{7/2}}{3f(a + ia \tan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A + B*\operatorname{Tan}[e + f*x])*(c - I*c*\operatorname{Tan}[e + f*x])^{(7/2)}/(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)}, x]$

[Out]  $(-5*((2*I)*A - 5*B)*c^{(7/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])]/(a^{(3/2)}*f) - (5*((2*I)*A - 5*B)*c^3*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]*\operatorname{Sqrt}[c - I*c*\operatorname{Tan}[e + f*x]])/(2*a^2*f) - (5*((2*I)*A - 5*B)*c^2*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]*(c - I*c*\operatorname{Tan}[e + f*x])^{(3/2)})/(6*a^2*f) - (2*((2*I)*A - 5*B)*c*(c - I*c*\operatorname{Tan}[e + f*x])^{(5/2)})/(3*a*f*\operatorname{Sqrt}[a + I*a*\operatorname{Tan}[e + f*x]]) + ((I*A - B)*(c - I*c*\operatorname{Tan}[e + f*x])^{(7/2)})/(3*f*(a + I*a*\operatorname{Tan}[e + f*x])^{(3/2)})$

**Rule 49**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^n/(b*(m + 1))), x] - \operatorname{Dist}[d*(n/(b*(m + 1))), \operatorname{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n - 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{IntegerQ}[n] \&\& !\operatorname{IntegerQ}[m]) \&\& !(I\operatorname{LeQ}[m + n + 2, 0] \&\& (\operatorname{FractionQ}[m] || \operatorname{GeQ}[2*n + m + 1, 0])) \& \& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 52**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
)
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^{7/2}}{(a + i a \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - i c \tan(e + fx))^{7/2}}{3f(a + i a \tan(e + fx))^{3/2}} - \frac{((2A + 5iB)c) \text{Subst} \left( \int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^{5/2}} dx, x, \tan(e + fx) \right)}{3f(a + i a \tan(e + fx))^{3/2}} \\
&= -\frac{2(2iA - 5B)c(c - i c \tan(e + fx))^{5/2}}{3af \sqrt{a + i a \tan(e + fx)}} + \frac{(iA - B)(c - i c \tan(e + fx))^{7/2}}{3f(a + i a \tan(e + fx))^{3/2}} \\
&= -\frac{5(2iA - 5B)c^2 \sqrt{a + i a \tan(e + fx)} (c - i c \tan(e + fx))^{5/2}}{6a^2 f} \\
&= -\frac{5(2iA - 5B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^2 f} \\
&= -\frac{5(2iA - 5B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^2 f} \\
&= -\frac{5(2iA - 5B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^2 f} \\
&= -\frac{5(2iA - 5B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^2 f} \\
&= -\frac{5(2iA - 5B)c^{7/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + i a \tan(e + fx)}}{\sqrt{a} \sqrt{c - i c \tan(e + fx)}} \right)}{a^{3/2} f}
\end{aligned}$$

**Mathematica [A]**

time = 5.22, size = 255, normalized size = 0.89

$$\frac{\sqrt{\sec(e + fx)} (A + B \tan(e + fx)) \left( \frac{\frac{5(-2iA + 5B)c^4 e^{i(e + fx)}}{1 + e^{2i(e + fx)}} \text{ArcTan}(e^{i(e + fx)})}{\frac{c}{\sqrt{1 + e^{2i(e + fx)}}}} + \frac{1}{12} c^3 \sec^2(e + fx) (33(-2iA + 5B) \cos(e + fx) + (-26iA + 71B) \cos(3(e + fx)) + 2(34A + 82iB) + (34A + 79iB) \cos(2(e + fx))) \sin(e + fx) \sqrt{c - i c \tan(e + fx)}}{f(A \cos(e + fx) + B \sin(e + fx))(a + i a \tan(e + fx))^{3/2}} \right)}{f(A \cos(e + fx) + B \sin(e + fx))(a + i a \tan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(a + I\*a\*Tan[e + f\*x])^(3/2), x]

[Out] (Sqrt[Sec[e + f\*x]]\*(A + B\*Tan[e + f\*x])\*((5\*((-2\*I)\*A + 5\*B)\*c^4\*E^(I\*(e + f\*x)))\*Sqrt[E^(I\*(e + f\*x))/(1 + E^((2\*I)\*(e + f\*x)))]\*ArcTan[E^(I\*(e + f\*x))])/(a + I\*a\*Tan[e + f\*x])^(3/2)

$$\left. \right) \left. \right) / \text{Sqrt}[c / (1 + E^{((2*I)*(e + f*x))}] + (c^3 \text{Sec}[e + f*x]^{(3/2)} * (33 * ((-2*I) * A + 5*B) * \text{Cos}[e + f*x] + ((-26*I)*A + 71*B) * \text{Cos}[3*(e + f*x)] + 2*(34*A + (82*I)*B + (34*A + (79*I)*B) * \text{Cos}[2*(e + f*x)]) * \text{Sin}[e + f*x]) * \text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) / 12) / (f*(A*\text{Cos}[e + f*x] + B*\text{Sin}[e + f*x]) * (a + I*a*\text{Tan}[e + f*x]))^{(3/2)}$$

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 732 vs.  $2(236) = 472$ .

time = 0.43, size = 733, normalized size = 2.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{6} f^{-1} (-c(I \tan(fx+e) - 1))^{1/2} (a(1 + I \tan(fx+e)))^{1/2} c^3 a^{-2} (-114 I A (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} \tan(fx+e) - 118 I B (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} - 30 I A \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c + 225 I B \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c \tan(fx+e) - 30 A \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c \tan(fx+e)^3 + 185 I B (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} \tan(fx+e)^2 + 90 I A \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c \tan(fx+e)^2 - 225 B \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c \tan(fx+e)^2 - 21 B (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} \tan(fx+e)^3 + 6 I A (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} \tan(fx+e)^3 + 3 I B (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} \tan(fx+e)^4 + 90 A \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c \tan(fx+e) + 74 A (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} \tan(fx+e)^2 - 75 I B \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c \tan(fx+e)^3 + 75 B \ln((a^2 c^2 \tan(fx+e) + (a^2 c)^{1/2} (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2}) / (a^2 c)^{1/2}) * a^2 c + 279 B (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2} \tan(fx+e) - 46 A (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} (a^2 c)^{1/2}) / (a^2 c^2 (1 + \tan(fx+e))^2)^{1/2} / (a^2 c)^{1/2} / (I - \tan(fx+e))^3$$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1459 vs.  $2(229) = 458$ .

time = 1.21, size = 1459, normalized size = 5.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out] 
$$-12*(60*(2*A + 5*I*B)*c^3*\cos(6*f*x + 6*e) + 100*(2*A + 5*I*B)*c^3*\cos(4*f*x + 4*e) + 32*(2*A + 5*I*B)*c^3*\cos(2*f*x + 2*e) + 60*(2*I*A - 5*B)*c^3*\sin$$

$$\begin{aligned}
& (6*f*x + 6*e) + 100*(2*I*A - 5*B)*c^3*\sin(4*f*x + 4*e) + 32*(2*I*A - 5*B)*c^3*\sin(2*f*x + 2*e) - 16*(A + I*B)*c^3 + 30*((2*A + 5*I*B)*c^3*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*A + 5*I*B)*c^3*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*A + 5*I*B)*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*I*A - 5*B)*c^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 30*((2*A + 5*I*B)*c^3*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*A + 5*I*B)*c^3*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*A + 5*I*B)*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + (2*I*A - 5*B)*c^3*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 2*(2*I*A - 5*B)*c^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + (2*I*A - 5*B)*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\arctan2(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), -\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((2*I*A - 5*B)*c^3*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*I*A - 5*B)*c^3*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - (2*A + 5*I*B)*c^3*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(2*A + 5*I*B)*c^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - (2*A + 5*I*B)*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 15*((-2*I*A + 5*B)*c^3*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(-2*I*A + 5*B)*c^3*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + (-2*I*A + 5*B)*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + (2*A + 5*I*B)*c^3*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(2*A + 5*I*B)*c^3*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + (2*A + 5*I*B)*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\log(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))^2 - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(2*f*x + 2*e))) + 1)*\sqrt{a}*\sqrt{c}/((-144*I*a^2*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 288*I*a^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 144*I*a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 144*a^2*\sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 288*a^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 144*a^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*f)
\end{aligned}$$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 612 vs.  $2(229) = 458$ .

time = 3.36, size = 612, normalized size = 2.13

$$\frac{\left( \frac{144 a^2 \cos\left(\frac{7}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right) - 288 a^2 \cos\left(\frac{5}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right) - 144 a^2 \cos\left(\frac{3}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right) + 144 a^2 \sin\left(\frac{7}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right) + 288 a^2 \sin\left(\frac{5}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right) + 144 a^2 \sin\left(\frac{3}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right) \right) \sqrt{a} \sqrt{c}}{\left( \cos\left(\frac{1}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right)^2 + \sin\left(\frac{1}{2} \arctan\left(\frac{\sin(2 f x + 2 e)}{\cos(2 f x + 2 e)}\right)\right)^2 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(15*(a^2*f*e^(5*I*f*x + 5*I*e) + a^2*f*e^(3*I*f*x + 3*I*e))*sqrt((4*A^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f^2))*log(4*(2*((2*I*A - 5*B)*c^3*e^(3*I*f*x + 3*I*e) + (2*I*A - 5*B)*c^3*e^(I*f*x + I*e)))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt((4*A^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f^2)))/((2*I*A - 5*B)*c^3*e^(2*I*f*x + 2*I*e) + (2*I*A - 5*B)*c^3) - 15*(a^2*f*e^(5*I*f*x + 5*I*e) + a^2*f*e^(3*I*f*x + 3*I*e))*sqrt((4*A^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f^2))*log(4*(2*((2*I*A - 5*B)*c^3*e^(3*I*f*x + 3*I*e) + (2*I*A - 5*B)*c^3*e^(I*f*x + I*e)))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt((4*A^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f^2)))/((2*I*A - 5*B)*c^3*e^(2*I*f*x + 2*I*e) + (2*I*A - 5*B)*c^3) - 4*(15*(2*I*A - 5*B)*c^3*e^(6*I*f*x + 6*I*e) + 25*(2*I*A - 5*B)*c^3*e^(4*I*f*x + 4*I*e) + 8*(2*I*A - 5*B)*c^3*e^(2*I*f*x + 2*I*e) + 4*(-I*A + B)*c^3)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(a^2*f*e^(5*I*f*x + 5*I*e) + a^2*f*e^(3*I*f*x + 3*I*e))
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(7/2)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) \operatorname{li})^{7/2}}{(a + a \tan(e + f x) \operatorname{li})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

$$3.837 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=229

$$\frac{2(iA-4B)c^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{a^{3/2} f} - \frac{(iA-4B)c^2 \sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}{a^2 f}$$

[Out]  $-2*(I*A-4*B)*c^{(5/2)*\arctan(c^{(1/2)*(a+I*a*\tan(f*x+e))^{(1/2)/a^{(1/2)/(c-I*c*\tan(f*x+e))^{(1/2)}}/a^{(3/2)/f-(I*A-4*B)*c^2*(a+I*a*\tan(f*x+e))^{(1/2)*(c-I*c*\tan(f*x+e))^{(1/2)/a^2/f-2/3*(I*A-4*B)*c*(c-I*c*\tan(f*x+e))^{(3/2)/a/f/(a+I*a*\tan(f*x+e))^{(1/2)+1/3*(I*A-B)*(c-I*c*\tan(f*x+e))^{(5/2)/f/(a+I*a*\tan(f*x+e))^{(3/2)}}}$

**Rubi [A]**

time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{2c^{5/2}(-4B+iA)\operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{a^{3/2} f} - \frac{c^2(-4B+iA)\sqrt{a+ia \tan(e+fx)} \sqrt{c-ictan(e+fx)}}{a^2 f} - \frac{2c(-4B+iA)(c-ictan(e+fx))^{3/2}}{3af \sqrt{a+ia \tan(e+fx)}} + \frac{(-B+iA)(c-ictan(e+fx))^{5/2}}{3f(a+ia \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[e+f*x])*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)}]/(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)}, x]$

[Out]  $(-2*(I*A-4*B)*c^{(5/2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])] ])/(a^{(3/2)*f} - ((I*A-4*B)*c^2*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]]*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])/(a^2*f) - (2*(I*A-4*B)*c*(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)})/(3*a*f*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]]) + ((I*A-B)*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)})/(3*f*(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)})$

Rule 49

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{Simp}[(a + b*x)^{(m+1)*((c + d*x)^n/(b*(m+1)))}, x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)*((c + d*x)^{(n-1))}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(IntegerQ[n] \&\& !IntegerQ[m]) \&\& !(IntegerQ[m+n+2, 0] \&\& (FractionQ[m] || GeQ[2*n+m+1, 0])) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 52

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \operatorname{Simp}[(a + b*x)^{(m+1)*((c + d*x)^n/(b*(m+n+1)))}, x] + \operatorname{Dist}[n*((b*c - a*d)/(b*(m+n+1))), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /; \operatorname{FreeQ}\{a, b,$

$c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[m + n + 1, 0] \&\& \text{!(IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \mid (\text{GtQ}[m, 0] \&\& \text{LtQ}[m - n, 0]))) \&\& \text{!ILtQ}[m + n + 2, 0] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 65

$\text{Int}[(a_.) + (b_.)(x_)^{(m_)} * ((c_.) + (d_.)(x_)^{(n_)}), x\_Symbol] \text{ :> With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1) - 1)} * (c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] \text{ /; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 79

$\text{Int}[(a_.) + (b_.)(x_) * ((c_.) + (d_.)(x_)^{(n_.)} * ((e_.) + (f_.)(x_)^{(p_.)}), x\_Symbol] \text{ :> Simp}[(-b*e - a*f) * (c + d*x)^{(n+1)} * (e + f*x)^{(p+1)} / (f*(p+1) * (c*f - d*e)), x] - \text{Dist}[(a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1))] / (f*(p+1) * (c*f - d*e)), \text{Int}[(c + d*x)^n * (e + f*x)^{(p+1)}, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{LtQ}[p, -1] \&\& (\text{!LtQ}[n, -1] \mid \text{IntegerQ}[p] \mid \text{!(IntegerQ}[n] \mid \text{!(EqQ}[e, 0] \mid \text{!(EqQ}[c, 0] \mid \text{LtQ}[p, n]))}))$

### Rule 209

$\text{Int}[(a_.) + (b_.)(x_)^2]^{-1}, x\_Symbol] \text{ :> Simp}[(1/(\text{Rt}[a, 2] * \text{Rt}[b, 2])) * \text{ArcTan}[\text{Rt}[b, 2] * (x/\text{Rt}[a, 2])], x] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \mid \text{GtQ}[b, 0])$

### Rule 223

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)(x_)^2], x\_Symbol] \text{ :> Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

### Rule 3669

$\text{Int}[(a_.) + (b_.) * \tan[(e_.) + (f_.)(x_)])^{(m_.)} * ((A_.) + (B_.) * \tan[(e_.) + (f_.)(x_)]) * ((c_.) + (d_.) * \tan[(e_.) + (f_.)(x_)])^{(n_.)}, x\_Symbol] \text{ :> Dist}[a*(c/f), \text{Subst}[\text{Int}[(a + b*x)^{(m-1)} * (c + d*x)^{(n-1)} * (A + B*x), x], x, \text{Tan}[e + f*x]], x] \text{ /; FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{5/2}}{(a + ia \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^{5/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{5/2}}{3f(a + ia \tan(e + fx))^{3/2}} - \frac{((A + 4iB)c) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{3/2}}{(a+iax)^{5/2}} dx, x, \tan(e + fx)\right)}{3f(a + ia \tan(e + fx))^{3/2}} \\
&= -\frac{2(iA - 4B)c(c - ic \tan(e + fx))^{3/2}}{3af \sqrt{a + ia \tan(e + fx)}} + \frac{(iA - B)(c - ic \tan(e + fx))^{5/2}}{3f(a + ia \tan(e + fx))^{3/2}} \\
&= -\frac{(iA - 4B)c^2 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{a^2 f} \\
&= -\frac{(iA - 4B)c^2 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{a^2 f} \\
&= -\frac{(iA - 4B)c^2 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{a^2 f} \\
&= -\frac{2(iA - 4B)c^{5/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{a^{3/2} f}
\end{aligned}$$

**Mathematica [A]**

time = 3.74, size = 174, normalized size = 0.76

$$-\frac{4\sqrt{2} \left(\frac{c}{1+e^{2i(e+fx)}}\right)^{5/2} (A(-1+2e^{2i(e+fx)}+3e^{4i(e+fx)})+iB(-1+8e^{2i(e+fx)}+12e^{4i(e+fx)})+3(A+4iB)e^{3i(e+fx)}(1+e^{2i(e+fx)}) \text{ArcTan}(e^{i(e+fx)}))}{3af(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(5/2))/(a + I\*a\*Tan[e + f\*x])^(3/2), x]

[Out] (-4\*Sqrt[2]\*(c/(1 + E^((2\*I)\*(e + f\*x))))^(5/2)\*(A\*(-1 + 2\*E^((2\*I)\*(e + f\*x))) + 3\*E^((4\*I)\*(e + f\*x))) + I\*B\*(-1 + 8\*E^((2\*I)\*(e + f\*x))) + 12\*E^((4\*I)\*(e + f\*x))) + 3\*(A + (4\*I)\*B)\*E^((3\*I)\*(e + f\*x))\*(1 + E^((2\*I)\*(e + f\*x)))\*ArcTan[E^(I\*(e + f\*x))])/(3\*a\*f\*(-I + Tan[e + f\*x])\*Sqrt[a + I\*a\*Tan[e + f\*x]])



**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 668 vs.  $2(189) = 378$ .  
time = 0.41, size = 669, normalized size = 2.92

method	result
derivativedivides	$\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} c^2 \left( -12iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{\frac{ac(1 - \dots)}{\dots}} \right) \right)}{\dots}$
default	$\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} c^2 \left( -12iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{\frac{ac(1 - \dots)}{\dots}} \right) \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{1}{3} f \frac{(-c(I \tan(fx + e) - 1))^{1/2} (a(1 + I \tan(fx + e)))^{1/2} c^2 a^{-2} (-12 I B \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac \tan(fx + e)^3 + 9 I A \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac \tan(fx + e)^2 - 3 A \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac \tan(fx + e)^3 + 36 I B \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac \tan(fx + e) + 29 I B (ac(1 + \tan(fx + e)^2))^{1/2} (ac)^{1/2} \tan(fx + e)^2 - 36 B \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac \tan(fx + e)^2 - 3 B (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2} \tan(fx + e)^3 - 3 I A \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac - 12 I A (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2} \tan(fx + e) + 9 A \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac \tan(fx + e) + 8 A (ac(1 + \tan(fx + e)^2))^{1/2} (ac)^{1/2} \tan(fx + e)^2 - 19 I B (ac(1 + \tan(fx + e)^2))^{1/2} (ac)^{1/2} + 12 B \ln((ac \tan(fx + e) + (ac)^{1/2} (ac(1 + \tan(fx + e)^2))^{1/2})) / (ac)^{1/2}) * ac + 45 B (ac(1 + \tan(fx + e)^2))^{1/2} (ac)^{1/2} \tan(fx + e) - 4 A (ac(1 + \tan(fx + e)^2))^{1/2} (ac)^{1/2}) / (ac(1 + \tan(fx + e)^2))^{1/2} / (ac)^{1/2} / (I - \tan(fx + e))^3$$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 530 vs.  $2(183) = 366$ .  
time = 4.49, size = 530, normalized size = 2.31

$$\frac{\left( \frac{\sqrt{A^2 + 8AB - 16B^2}}{2A} \sqrt{\frac{A + B \tan(fx + e)}{2A + 2B \tan(fx + e)}} \sqrt{\frac{A - B \tan(fx + e)}{2A - 2B \tan(fx + e)}} \right) - 3A \sqrt{\frac{A^2 + 8AB - 16B^2}}{2A} \sqrt{\frac{A + B \tan(fx + e)}{2A + 2B \tan(fx + e)}} \sqrt{\frac{A - B \tan(fx + e)}{2A - 2B \tan(fx + e)}}}{4(3A - 4B) \sqrt{A^2 + 8AB - 16B^2} \sqrt{\frac{A + B \tan(fx + e)}{2A + 2B \tan(fx + e)}} \sqrt{\frac{A - B \tan(fx + e)}{2A - 2B \tan(fx + e)}}} + \frac{3A \sqrt{\frac{A^2 + 8AB - 16B^2}}{2A} \sqrt{\frac{A + B \tan(fx + e)}{2A + 2B \tan(fx + e)}} \sqrt{\frac{A - B \tan(fx + e)}{2A - 2B \tan(fx + e)}}}{4(3A - 4B) \sqrt{A^2 + 8AB - 16B^2} \sqrt{\frac{A + B \tan(fx + e)}{2A + 2B \tan(fx + e)}} \sqrt{\frac{A - B \tan(fx + e)}{2A - 2B \tan(fx + e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{6} (3a^2 \sqrt{(A^2 + 8IAB - 16B^2)c^5/(a^3 f^2)}) f e^{(3Ifx + 3Ie)} \log(-4(2((IA - 4B)c^2 e^{(3Ifx + 3Ie)} + (IA - 4B)c^2 e^{(Ifx + Ie)}) \sqrt{a/(e^{(2Ifx + 2Ie)} + 1)}) \sqrt{c/(e^{(2Ifx + 2Ie)} + 1)} + (a^2 f e^{(2Ifx + 2Ie)} - a^2 f) \sqrt{(A^2 + 8IAB - 16B^2)c^5/(a^3 f^2)}) / ((-IA + 4B)c^2 e^{(2Ifx + 2Ie)} + (-IA + 4B)c^2) - 3a^2 \sqrt{(A^2 + 8IAB - 16B^2)c^5/(a^3 f^2)}) f e^{(3Ifx + 3Ie)} \log(-4(2((IA - 4B)c^2 e^{(3Ifx + 3Ie)} + (IA - 4B)c^2 e^{(Ifx + Ie)}) \sqrt{a/(e^{(2Ifx + 2Ie)} + 1)}) \sqrt{c/(e^{(2Ifx + 2Ie)} + 1)} - (a^2 f e^{(2Ifx + 2Ie)} - a^2 f) \sqrt{(A^2 + 8IAB - 16B^2)c^5/(a^3 f^2)}) / ((-IA + 4B)c^2 e^{(2Ifx + 2Ie)} + (-IA + 4B)c^2) - 4(3(IA - 4B)c^2 e^{(4Ifx + 4Ie)} + 2(IA - 4B)c^2 e^{(2Ifx + 2Ie)} + (-IA + B)c^2) \sqrt{a/(e^{(2Ifx + 2Ie)} + 1)}) \sqrt{c/(e^{(2Ifx + 2Ie)} + 1)}) e^{(-3Ifx - 3Ie)}/(a^2 f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e + fx) + i))^{5/2} (A + B \tan(e + fx))}{(ia(\tan(e + fx) - i))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(3/2),x)

[Out] Integral((-I\*c\*(tan(e + f\*x) + I))^(5/2)\*(A + B\*tan(e + f\*x))/(I\*a\*(tan(e + f\*x) - I))^(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(5/2)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) i)^{5/2}}{(a + a \tan(e + f x) i)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

$$3.838 \quad \int \frac{(A+B \tan(e+fx))(c-ict \tan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=157

$$\frac{2Bc^{3/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ict \tan(e+fx)}}\right)}{a^{3/2} f} + \frac{2Bc \sqrt{c-ict \tan(e+fx)}}{af \sqrt{a+ia \tan(e+fx)}} + \frac{(iA-B)(c-ict \tan(e+fx))^{3/2}}{3f(a+ia \tan(e+fx))^{3/2}}$$

[Out]  $2*B*c^{(3/2)}*arctan(c^{(1/2)}*(a+I*a*\tan(f*x+e))^{(1/2)}/a^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)})/a^{(3/2)}/f+2*B*c*(c-I*c*\tan(f*x+e))^{(1/2)}/a/f/(a+I*a*\tan(f*x+e))^{(1/2)}+1/3*(I*A-B)*(c-I*c*\tan(f*x+e))^{(3/2)}/f/(a+I*a*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 49, 65, 223, 209}

$$\frac{2Bc^{3/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ict \tan(e+fx)}}\right)}{a^{3/2} f} + \frac{(-B+ia)(c-ict \tan(e+fx))^{3/2}}{3f(a+ia \tan(e+fx))^{3/2}} + \frac{2Bc \sqrt{c-ict \tan(e+fx)}}{af \sqrt{a+ia \tan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Tan}[e+f*x])*(c-I*c*\text{Tan}[e+f*x])^{(3/2)}/(a+I*a*\text{Tan}[e+f*x])^{(3/2)},x]$

[Out]  $(2*B*c^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[a+I*a*\text{Tan}[e+f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])])/a^{(3/2)}*f+(2*B*c*\text{Sqrt}[c-I*c*\text{Tan}[e+f*x]])/(a*f*\text{Sqrt}[a+I*a*\text{Tan}[e+f*x]])+((I*A-B)*(c-I*c*\text{Tan}[e+f*x])^{(3/2)})/(3*f*(a+I*a*\text{Tan}[e+f*x])^{(3/2)})$

**Rule 49**

$\text{Int}[(a_.)+(b_.)*(x_)^{(m_)}*((c_.)+(d_.)*(x_)^{(n_)}),x\_Symbol] :> \text{Simp}[(a+b*x)^{(m+1)}*((c+d*x)^n/(b*(m+1))),x]-\text{Dist}[d*(n/(b*(m+1))),\text{Int}[(a+b*x)^{(m+1)}*(c+d*x)^{(n-1)},x],x] /; \text{FreeQ}[\{a,b,c,d\},x] \&\& \text{NeQ}[b*c-a*d,0] \&\& \text{GtQ}[n,0] \&\& \text{LtQ}[m,-1] \&\& !(IntegerQ[n] \&\& !IntegerQ[m]) \&\& !(ILeQ[m+n+2,0] \&\& (FractionQ[m] || GeQ[2*n+m+1,0])) \&\& \text{IntLinearQ}[a,b,c,d,m,n,x]$

**Rule 65**

$\text{Int}[(a_.)+(b_.)*(x_)^{(m_)}*((c_.)+(d_.)*(x_)^{(n_)}),x\_Symbol] :> \text{With}[\{p=\text{Denominator}[m]\},\text{Dist}[p/b,\text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c-a*(d/b)+d*(x^p/b))^{(n)},x],x,(a+b*x)^{(1/p)}],x]] /; \text{FreeQ}[\{a,b,c,d\},x] \&\& \text{NeQ}[b*c-a*d,0] \&\& \text{LtQ}[-1,m,0] \&\& \text{LeQ}[-1,n,0] \&\& \text{LeQ}[\text{Denominator}[n],\text{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2}}{(a + ia \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst} \left( \int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{3f(a + ia \tan(e + fx))^{3/2}} - \frac{(iBc) \text{Subst} \left( \int \frac{\sqrt{c-icx}}{(a+iax)^{5/2}} dx, x, \tan(e + fx) \right)}{f} \\
&= \frac{2Bc \sqrt{c - ic \tan(e + fx)}}{af \sqrt{a + ia \tan(e + fx)}} + \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{3f(a + ia \tan(e + fx))^{3/2}} \\
&= \frac{2Bc \sqrt{c - ic \tan(e + fx)}}{af \sqrt{a + ia \tan(e + fx)}} + \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{3f(a + ia \tan(e + fx))^{3/2}} \\
&= \frac{2Bc \sqrt{c - ic \tan(e + fx)}}{af \sqrt{a + ia \tan(e + fx)}} + \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{3f(a + ia \tan(e + fx))^{3/2}} \\
&= \frac{2Bc^{3/2} \tan^{-1} \left( \frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}} \right)}{a^{3/2} f} + \frac{2Bc \sqrt{c - ic \tan(e + fx)}}{af \sqrt{a + ia \tan(e + fx)}}
\end{aligned}$$

**Mathematica [A]**

time = 2.65, size = 114, normalized size = 0.73

$$\frac{\sqrt{2} c e^{-2i(e+fx)} \sqrt{\frac{c}{1 + e^{2i(e+fx)}}} (iA + B(-1 + 6e^{2i(e+fx)}) + 6B e^{3i(e+fx)} \text{ArcTan}(e^{i(e+fx)}))}{3af \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(a + I\*a\*Tan[e + f\*x])^(3/2), x]

[Out] (Sqrt[2]\*c\*Sqrt[c/(1 + E^((2\*I)\*(e + f\*x)))]\*(I\*A + B\*(-1 + 6\*E^((2\*I)\*(e + f\*x)))) + 6\*B\*E^((3\*I)\*(e + f\*x))\*ArcTan[E^(I\*(e + f\*x))])/(3\*a\*E^((2\*I)\*(e + f\*x))\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 407 vs.  $2(128) = 256$ .

time = 0.42, size = 408, normalized size = 2.60

method	result
derivativedivides	$\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} c \left( -3iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{\frac{ac(1 + \dots)}{\sqrt{ac}}} \right) \right)$
default	$\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} c \left( -3iB \ln \left( \frac{ac \tan(fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{\frac{ac(1 + \dots)}{\sqrt{ac}}} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*c*(-3*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+9*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+7*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-5*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+3*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+12*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I-tan(f*x+e))^3/(a*c)^(1/2)
```

**Maxima** [A]

time = 0.59, size = 182, normalized size = 1.16

$$\frac{(6B \arctan(\cos(fx + e) \sin(fx + e) + 1) + 6B \arctan(\cos(fx + e) - \sin(fx + e) + 1) - 2(-iA + B) \cos(3fx + 3e) + 12B \cos(fx + e) + 3iB \log(\cos(fx + e)^2 + \sin(fx + e)^2 + 2 \sin(fx + e) + 1) - 3iB \log(\cos(fx + e)^2 + \sin(fx + e)^2 - 2 \sin(fx + e) + 1) + 2(A + iB) \sin(3fx + 3e) - 12iB \sin(fx + e)) \sqrt{c}}{6a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*B*c*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 6*B*c*arctan2(cos(f*x + e), -sin(f*x + e) + 1) - 2*(-I*A + B)*c*cos(3*f*x + 3*e) + 12*B*c*cos(f*x + e) + 3*I*B*c*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 3*I*B*c*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + 2*(A + I*B)*c*sin(3*f*x + 3*e) - 12*I*B*c*sin(f*x + e))*sqrt(c)/(a^(3/2)*f)
```

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(125) = 250.

time = 4.58, size = 419, normalized size = 2.67

$$\frac{\left( 3a^2 f \sqrt{\frac{Bc^2}{a^2 f}} e^{iB \tan(fx + e)} \log \left( \frac{\left( \sqrt{\frac{a}{2B \cos(fx + e) + 1}} \sqrt{\frac{c}{2B \cos(fx + e) + 1}} \sqrt{\frac{Bc^2}{a^2 f}} \right)^{1/2} e^{iB \tan(fx + e)} \sqrt{\frac{Bc^2}{a^2 f}} \right) - 3a^2 f \sqrt{\frac{Bc^2}{a^2 f}} e^{iB \tan(fx + e)} \log \left( \frac{\left( \sqrt{\frac{a}{2B \cos(fx + e) - 1}} \sqrt{\frac{c}{2B \cos(fx + e) - 1}} \sqrt{\frac{Bc^2}{a^2 f}} \right)^{1/2} e^{iB \tan(fx + e)} \sqrt{\frac{Bc^2}{a^2 f}} \right) \right)}{6a^2 f} - 2(6B \cos(fx + e) - (-iA - 5B) \cos^2(fx + e) - (-iA + B) \cos(fx + e) \sqrt{\frac{a}{2B \cos(fx + e) + 1}} \sqrt{\frac{c}{2B \cos(fx + e) + 1}}) e^{iB \tan(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*f*sqrt(-B^2*c^3/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(4*(2*(B*c*e^(3*I*f*x + 3*I*e) + B*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt(-B^2*c^3/(a^3*f^2)))/(B*c*e^(2*I*f*x + 2*I*e) + B*c) - 3*a^2*f*sqrt(-B^2*c^3/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(4*(2*(B*c*e^(3*I*f*x + 3*I*e) + B*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt(-B^2*c^3/(a^3*f^2)))/(B*c*e^(2*I*f*x + 2*I*e) + B*c) - 2*(6*B*c*e^(4*I*f*x + 4*I*e) - (-I*A - 5*B)*c*e^(2*I*f*x + 2*I*e) - (-I*A + B)*c)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e + fx) + i))^{\frac{3}{2}}(A + B \tan(e + fx))}{(ia(\tan(e + fx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x)
```

```
[Out] Integral((-I*c*(tan(e + f*x) + I))^(3/2)*(A + B*tan(e + f*x))/(I*a*(tan(e + f*x) - I))^(3/2), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) i)^{3/2}}{(a + a \tan(e + f x) i)^{3/2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*x)*1i)^(3/2), x)
```

$$3.839 \quad \int \frac{(A+B \tan(e+fx)) \sqrt{c - i c \tan(e + fx)}}{(a+ia \tan(e+fx))^{3/2}} dx$$

Optimal. Leaf size=104

$$\frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{3f(a + ia \tan(e + fx))^{3/2}} + \frac{(iA + 2B) \sqrt{c - i c \tan(e + fx)}}{3af \sqrt{a + ia \tan(e + fx)}}$$

[Out] 1/3\*(I\*A+2\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a/f/(a+I\*a\*tan(f\*x+e))^(1/2)+1/3\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/f/(a+I\*a\*tan(f\*x+e))^(3/2)

Rubi [A]

time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {3669, 79, 37}

$$\frac{(-B + iA) \sqrt{c - i c \tan(e + fx)}}{3f(a + ia \tan(e + fx))^{3/2}} + \frac{(2B + iA) \sqrt{c - i c \tan(e + fx)}}{3af \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a + I\*a\*Tan[e + f\*x])^(3/2), x]

[Out] ((I\*A - B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*f\*(a + I\*a\*Tan[e + f\*x])^(3/2)) + ((I\*A + 2\*B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*a\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]])

Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx)) \sqrt{c - i c \tan(e + fx)}}{(a + i a \tan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{5/2} \sqrt{c-icx}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{3f(a + i a \tan(e + fx))^{3/2}} + \frac{((A - 2iB)c) \text{Subst}\left(\int \frac{1}{(a + i a \tan(e + fx))^{5/2}} dx, x, \tan(e + fx)\right)}{3f \sqrt{a + i a \tan(e + fx)}}$$

$$= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{3f(a + i a \tan(e + fx))^{3/2}} + \frac{(iA + 2B) \sqrt{c - i c \tan(e + fx)}}{3af \sqrt{a + i a \tan(e + fx)}}$$

**Mathematica [A]**

time = 1.19, size = 81, normalized size = 0.78

$$\frac{(2A - iB + (iA + 2B) \tan(e + fx)) \sqrt{c - i c \tan(e + fx)}}{3af(-i + \tan(e + fx)) \sqrt{a + i a \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
+ f*x])^(3/2),x]
```

```
[Out] ((2*A - I*B + (I*A + 2*B)*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*a*f*
(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple [A]**

time = 0.39, size = 103, normalized size = 0.99

method	result
derivativedivides	$\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} (2iB(\tan^2(fx + e)) + 3iA \tan(fx + e) - A(\tan^2(fx + e)))}{3fa^2(i - \tan(fx + e))^3}$
default	$\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} (2iB(\tan^2(fx + e)) + 3iA \tan(fx + e) - A(\tan^2(fx + e)))}{3fa^2(i - \tan(fx + e))^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{3}f*(-c*(I*\tan(f*x+e)-1))^{1/2}*(a*(1+I*\tan(f*x+e)))^{1/2}/a^2*(2*I*B*\tan(f*x+e)^2+3*I*A*\tan(f*x+e)-A*\tan(f*x+e)^2-I*B+3*B*\tan(f*x+e)+2*A)/(I-\tan(f*x+e))^3$

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [A]

time = 1.98, size = 97, normalized size = 0.93

$$\frac{(3(-iA - B)e^{4i fx + 4i e} + 2(-2iA - B)e^{2i fx + 2i e} - iA + B)\sqrt{\frac{a}{e^{2i fx + 2i e} + 1}}\sqrt{\frac{c}{e^{2i fx + 2i e} + 1}}e^{(-3i fx - 3i e)}}{6a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x,algorithm="fricas")`

[Out]  $-1/6*(3*(-I*A - B)*e^{(4*I*f*x + 4*I*e)} + 2*(-2*I*A - B)*e^{(2*I*f*x + 2*I*e)} - I*A + B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-3*I*f*x - 3*I*e)}/(a^2*f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ic(\tan(e + fx) + i)}(A + B \tan(e + fx))}{(ia(\tan(e + fx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(3/2),x)`

[Out] `Integral(sqrt(-I*c*(tan(e + f*x) + I))*(A + B*tan(e + f*x))/(I*a*(tan(e + f*x) - I))**(3/2), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*sqrt(-I*c*tan(f*x + e) + c)/(I*a*tan(f*x + e) + a)^(3/2), x)
```

**Mupad [B]**

time = 10.13, size = 195, normalized size = 1.88

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)i)}{\cos(2e+2fx)+1}} \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)i)}{\cos(2e+2fx)+1}} (A3i+3B+A\cos(2e+2fx)4i+A\cos(4e+4fx)i+2B\cos(2e+2fx)-B\cos(4e+4fx)+4A\sin(2e+2fx)+A\sin(4e+4fx)-B\sin(2e+2fx)2i+B\sin(4e+4fx)i)}{12a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*x)*1i)^(3/2),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*3i + 3*B + A*cos(2*e + 2*f*x)*4i + A*cos(4*e + 4*f*x)*1i + 2*B*cos(2*e + 2*f*x) - B*cos(4*e + 4*f*x) + 4*A*sin(2*e + 2*f*x) + A*sin(4*e + 4*f*x) - B*sin(2*e + 2*f*x)*2i + B*sin(4*e + 4*f*x)*1i))/(12*a^2*f)
```

$$3.840 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{3/2} \sqrt{c-ictan(e+fx)}} dx$$

Optimal. Leaf size=152

$$-\frac{ia+B}{f(a+ia \tan(e+fx))^{3/2} \sqrt{c-ictan(e+fx)}} + \frac{(2iA+B) \sqrt{c-ictan(e+fx)}}{3cf(a+ia \tan(e+fx))^{3/2}} + \frac{(2iA+B) \sqrt{c-ictan(e+fx)}}{3acf \sqrt{a+ia \tan(e+fx)}}$$

[Out] 1/3\*(2\*I\*A+B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a/c/f/(a+I\*a\*tan(f\*x+e))^(1/2)+(-I\*A-B)/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^(3/2)+1/3\*(2\*I\*A+B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/c/f/(a+I\*a\*tan(f\*x+e))^(3/2)

Rubi [A]

time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$-\frac{B+ia}{f(a+ia \tan(e+fx))^{3/2} \sqrt{c-ictan(e+fx)}} + \frac{(B+2iA) \sqrt{c-ictan(e+fx)}}{3acf \sqrt{a+ia \tan(e+fx)}} + \frac{(B+2iA) \sqrt{c-ictan(e+fx)}}{3cf(a+ia \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^(3/2)\*Sqrt[c - I\*c\*Tan[e + f\*x]]), x]

[Out] -((I\*A + B)/(f\*(a + I\*a\*Tan[e + f\*x])^(3/2)\*Sqrt[c - I\*c\*Tan[e + f\*x]])) + (((2\*I)\*A + B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*c\*f\*(a + I\*a\*Tan[e + f\*x])^(3/2)) + (((2\*I)\*A + B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(3\*a\*c\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]])

Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 47

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^Simplify[m + 1]\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && I LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

## Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^{3/2} \sqrt{c - ictan(e + fx)}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{5/2}(c-icx)^{3/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iA + B}{f(a + ia \tan(e + fx))^{3/2} \sqrt{c - ictan(e + fx)}} + \dots$$

$$= -\frac{iA + B}{f(a + ia \tan(e + fx))^{3/2} \sqrt{c - ictan(e + fx)}} + \dots$$

$$= -\frac{iA + B}{f(a + ia \tan(e + fx))^{3/2} \sqrt{c - ictan(e + fx)}} + \dots$$

## Mathematica [A]

time = 1.39, size = 85, normalized size = 0.56

$$\frac{i(-3A + (A - 2iB) \cos(2(e + fx)) + (2iA + B) \sin(2(e + fx))) \sqrt{c - ictan(e + fx)}}{6acf \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*T
an[e + f*x]]),x]
```

[Out]  $((-1/6*I)*(-3*A + (A - (2*I)*B)*\text{Cos}[2*(e + f*x)] + ((2*I)*A + B)*\text{Sin}[2*(e + f*x)])*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]/(a*c*f*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])$

**Maple [A]**

time = 0.43, size = 152, normalized size = 1.00

method	result
derivativedivides	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\sqrt{a(1+i\tan(fx+e))}}{3fa^2c(i-\tan(fx+e))^3(i+\tan(fx+e))} (2iA(\tan^4(fx+e))-iB(\tan^3(fx+e))+B(\tan^4(fx+e)))$
default	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\sqrt{a(1+i\tan(fx+e))}}{3fa^2c(i-\tan(fx+e))^3(i+\tan(fx+e))} (2iA(\tan^4(fx+e))-iB(\tan^3(fx+e))+B(\tan^4(fx+e)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{3}I/f*(-c*(I*\tan(f*x+e)-1))^{1/2}*(a*(1+I*\tan(f*x+e)))^{1/2}/a^2/c*(2*I*A*\tan(f*x+e)^4-I*B*\tan(f*x+e)^3+B*\tan(f*x+e)^4+3*I*A*\tan(f*x+e)^2+2*A*\tan(f*x+e)^3-I*B*\tan(f*x+e)+I*A+2*A*\tan(f*x+e)-B)/(I-\tan(f*x+e))^3/(I+\tan(f*x+e))^2$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas [A]**

time = 0.92, size = 153, normalized size = 1.01

$$\frac{(3(iA+B)e^{6i fx+6ie}+4(iA-B)e^{5i fx+5ie}+3(-iA+B)e^{4i fx+4ie}+4(iA-B)e^{3i fx+3ie}-(7iA-B)e^{2i fx+2ie}-iA+B)\sqrt{\frac{a}{e^{2i fx+2ie}+1}}\sqrt{\frac{c}{e^{2i fx+2ie}+1}}e^{-3i fx-3ie}}{12a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,algorithm="fricas")`

[Out]  $-1/12*(3*(I*A + B)*e^{(6*I*f*x + 6*I*e)} + 4*(I*A - B)*e^{(5*I*f*x + 5*I*e)} + 3*(-I*A + B)*e^{(4*I*f*x + 4*I*e)} + 4*(I*A - B)*e^{(3*I*f*x + 3*I*e)} - (7*I*A$



$- B) * e^{(2 * I * f * x + 2 * I * e)} - I * A + B) * \sqrt{a / (e^{(2 * I * f * x + 2 * I * e)} + 1)} * \sqrt{c / (e^{(2 * I * f * x + 2 * I * e)} + 1)} * e^{(-3 * I * f * x - 3 * I * e)} / (a^2 * c * f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{(ia(\tan(e + fx) - i))^{\frac{3}{2}} \sqrt{-ic(\tan(e + fx) + i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))\*\*(1/2)/(a+I\*a\*tan(f\*x+e))\*\*(3/2),x)

[Out] Integral((A + B\*tan(e + f\*x))/((I\*a\*(tan(e + f\*x) - I))\*\*(3/2)\*sqrt(-I\*c\*(tan(e + f\*x) + I))), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^(3/2)\*sqrt(-I\*c\*tan(f\*x + e) + c)), x)

**Mupad [B]**

time = 9.77, size = 170, normalized size = 1.12

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}} (6A \sin(2e+2fx) - 3B + A \cos(2e+2fx) 6i + A \cos(4e+4fx) 1i - B \cos(4e+4fx) - A 3i + A \sin(4e+4fx) + B \sin(4e+4fx) 1i)}{12a^2 f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2)),x)

[Out] (((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*cos(2\*e + 2\*f\*x)\*6i - 3\*B - A\*3i + A\*cos(4\*e + 4\*f\*x)\*1i - B\*cos(4\*e + 4\*f\*x) + 6\*A\*sin(2\*e + 2\*f\*x) + A\*sin(4\*e + 4\*f\*x) + B\*sin(4\*e + 4\*f\*x)\*1i))/(12\*a^2\*f\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2))

$$3.841 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{3/2}(c-ictan(e+fx))^{3/2}} dx$$

Optimal. Leaf size=152

$$\frac{-iA - B}{3f(a + ia \tan(e + fx))^{3/2}(c - ictan(e + fx))^{3/2}} + \frac{iA}{3cf(a + ia \tan(e + fx))^{3/2}\sqrt{c - ictan(e + fx)}} + \frac{1}{3acf\sqrt{c - ictan(e + fx)}}$$

[Out] 2/3\*A\*tan(f\*x+e)/a/c/f/(a+I\*a\*tan(f\*x+e))^(1/2)/(c-I\*c\*tan(f\*x+e))^(1/2)+1/3\*I\*A/c/f/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^(3/2)+1/3\*(-I\*A-B)/f/(a+I\*a\*tan(f\*x+e))^(3/2)/(c-I\*c\*tan(f\*x+e))^(3/2)

Rubi [A]

time = 0.16, antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 39}

$$\frac{B + iA}{3f(a + ia \tan(e + fx))^{3/2}(c - ictan(e + fx))^{3/2}} + \frac{2A \tan(e + fx)}{3acf \sqrt{a + ia \tan(e + fx)} \sqrt{c - ictan(e + fx)}} + \frac{iA}{3cf(a + ia \tan(e + fx))^{3/2}\sqrt{c - ictan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^(3/2)\*(c - I\*c\*Tan[e + f\*x])^(3/2)), x]

[Out] -1/3\*(I\*A + B)/(f\*(a + I\*a\*Tan[e + f\*x])^(3/2)\*(c - I\*c\*Tan[e + f\*x])^(3/2)) + ((I/3)\*A)/(c\*f\*(a + I\*a\*Tan[e + f\*x])^(3/2)\*Sqrt[c - I\*c\*Tan[e + f\*x]]) + (2\*A\*Tan[e + f\*x])/(3\*a\*c\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]]\*Sqrt[c - I\*c\*Tan[e + f\*x]])

Rule 39

Int[1/(((a\_) + (b\_.)\*(x\_))^(3/2)\*((c\_) + (d\_.)\*(x\_))^(3/2)), x\_Symbol] :> Simp[x/(a\*c\*Sqrt[a + b\*x]\*Sqrt[c + d\*x]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0]

Rule 47

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^Simplify[m + 1]\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[m + n + 2] && !LtQ[m, -1] && !LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 79

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))

```

### Rule 3669

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{3/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{5/2}(c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{3/2}} \\
&= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{3/2}} \\
&= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{3/2}}
\end{aligned}$$

### Mathematica [A]

time = 2.23, size = 120, normalized size = 0.79

$$\frac{(-i \cos(2(e + fx)) + \sin(2(e + fx)))(-2B - 2B \cos(2(e + fx)) + A \sec(e + fx) \sin(3(e + fx)) + 9A \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{12ac^2 f (-i + \tan(e + fx)) \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e
+ f*x])^(3/2)),x]

```

```

[Out] (((-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)])*(-2*B - 2*B*Cos[2*(e + f*x)] +
A*Sec[e + f*x]*Sin[3*(e + f*x)] + 9*A*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*
x]])/(12*a*c^2*f*(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])

```

**Maple [A]**

time = 0.39, size = 113, normalized size = 0.74

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}}{3fa^2c^2(i+\tan(fx+e))^3(i-\tan(fx+e))^3} (2A(\tan^5(fx+e))+5A(\tan^3(fx+e))-B(\tan^2(fx+e)))$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}}{3fa^2c^2(i+\tan(fx+e))^3(i-\tan(fx+e))^3} (2A(\tan^5(fx+e))+5A(\tan^3(fx+e))-B(\tan^2(fx+e)))$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c^2*(2*A*tan(f*x+e)^5+5*A*tan(f*x+e)^3-B*tan(f*x+e)^2+3*A*tan(f*x+e)-B)/(I+tan(f*x+e))^3/(I-tan(f*x+e))^3
```

**Maxima [A]**

time = 0.60, size = 212, normalized size = 1.39

$$\frac{(3(3A - B)\cos(2fx + 2e) - 3(3A + I)B\sin(2fx + 2e) - 2B)\cos(\frac{1}{2}\arctan(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)})) + 3(-3A - B)\cos(\frac{1}{2}\arctan(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)})) + (3(3A + I)B)\cos(2fx + 2e) + 3(3A - B)\sin(2fx + 2e) + 2A\sin(\frac{1}{2}\arctan(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)})) + 3(3A - I)B\sin(\frac{1}{2}\arctan(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}))}{24a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*((3*(3*I*A - B)*cos(2*f*x + 2*e) - 3*(3*A + I*B)*sin(2*f*x + 2*e) - 2*B)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(-3*I*A - B)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*(3*A + I*B)*cos(2*f*x + 2*e) + 3*(3*I*A - B)*sin(2*f*x + 2*e) + 2*A)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(3*A - I*B)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))/(a^(3/2)*c^(3/2)*f)
```

**Fricas [A]**

time = 1.61, size = 159, normalized size = 1.05

$$\frac{((-iA - B)e^{(8i fx + 8e)} - 2(5iA + 2B)e^{(6i fx + 6e)} + 8Be^{(5i fx + 5e)} - 6Be^{(4i fx + 4e)} + 8Be^{(3i fx + 3e)} - 2(-5iA + 2B)e^{(2i fx + 2e)} + iA - B)\sqrt{\frac{a}{e^{(2i fx + 2e)} + 1}}\sqrt{\frac{c}{e^{(2i fx + 2e)} + 1}}e^{(-3i fx - 3e)}}{24a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((-I*A - B)*e^(8*I*f*x + 8*I*e) - 2*(5*I*A + 2*B)*e^(6*I*f*x + 6*I*e) + 8*B*e^(5*I*f*x + 5*I*e) - 6*B*e^(4*I*f*x + 4*I*e) + 8*B*e^(3*I*f*x + 3*I*e)
```

$e) - 2*(-5*I*A + 2*B)*e^{(2*I*f*x + 2*I*e)} + I*A - B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-3*I*f*x - 3*I*e)/(a^2*c^2*f)}$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{(ia(\tan(e + fx) - i))^{\frac{3}{2}}(-ic(\tan(e + fx) + i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*(3/2)/(c-I\*c\*tan(f\*x+e))\*\*(3/2),x)

[Out] Integral((A + B\*tan(e + f\*x))/((I\*a\*(tan(e + f\*x) - I))\*\*(3/2)\*(-I\*c\*(tan(e + f\*x) + I))\*\*(3/2)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2)/(c-I\*c\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^(3/2)\*(-I\*c\*tan(f\*x + e) + c)^(3/2)), x)

**Mupad [B]**

time = 9.94, size = 198, normalized size = 1.30

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}(10A\sin(2e+2fx)-3B+A\cos(2e+2fx)8i+A\cos(4e+4fx)1i-4B\cos(2e+2fx)-B\cos(4e+4fx)-A9i+A\sin(4e+4fx)+B\sin(2e+2fx)2i+B\sin(4e+4fx)1i)}{24a^2cf\sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)1i)}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^(3/2)\*(c - c\*tan(e + f\*x)\*1i)^(3/2)),x)

[Out] (((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*cos(2\*e + 2\*f\*x)\*8i - 3\*B - A\*9i + A\*cos(4\*e + 4\*f\*x)\*1i - 4\*B\*cos(2\*e + 2\*f\*x) - B\*cos(4\*e + 4\*f\*x) + 10\*A\*sin(2\*e + 2\*f\*x) + A\*sin(4\*e + 4\*f\*x) + B\*sin(2\*e + 2\*f\*x)\*2i + B\*sin(4\*e + 4\*f\*x)\*1i))/(24\*a^2\*c\*f\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2))

$$3.842 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{3/2}(c-ic \tan(e+fx))^{5/2}} dx$$

Optimal. Leaf size=269

$$\frac{iA - B}{3f(a + ia \tan(e + fx))^{3/2}(c - ic \tan(e + fx))^{5/2}} + \frac{4iA - B}{3af \sqrt{a + ia \tan(e + fx)} (c - ic \tan(e + fx))^{5/2}} - \frac{(4iA - B)}{5a^2 f}$$

[Out]  $-2/15*(4*I*A-B)*(a+I*a*\tan(f*x+e))^{(1/2)}/a^2/c^2/f/(c-I*c*\tan(f*x+e))^{(1/2)}$   
 $+1/3*(4*I*A-B)/a/f/(a+I*a*\tan(f*x+e))^{(1/2)}/(c-I*c*\tan(f*x+e))^{(5/2)}-1/5*(4$   
 $*I*A-B)*(a+I*a*\tan(f*x+e))^{(1/2)}/a^2/f/(c-I*c*\tan(f*x+e))^{(5/2)}+1/3*(I*A-B)$   
 $/f/(a+I*a*\tan(f*x+e))^{(3/2)}/(c-I*c*\tan(f*x+e))^{(5/2)}-2/15*(4*I*A-B)*(a+I*a*$   
 $\tan(f*x+e))^{(1/2)}/a^2/c/f/(c-I*c*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.22, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{2(-B+4iA)\sqrt{a+ia \tan(e+fx)}}{15a^2 f \sqrt{c-ic \tan(e+fx)}} - \frac{2(-B+4iA)\sqrt{a+ia \tan(e+fx)}}{15a^2 c f (c-ic \tan(e+fx))^{3/2}} - \frac{(-B+4iA)\sqrt{a+ia \tan(e+fx)}}{5a^2 f (c-ic \tan(e+fx))^{3/2}} + \frac{-B+iA}{3f(a+ia \tan(e+fx))^{3/2}(c-ic \tan(e+fx))^{5/2}} + \frac{-B+4iA}{3af \sqrt{a+ia \tan(e+fx)} (c-ic \tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])/((a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(5/2)})], x]$

[Out]  $(I*A - B)/(3*f*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) +$   
 $((4*I)*A - B)/(3*a*f*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) -$   
 $((4*I)*A - B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]/(5*a^2*f*(c - I*c*\text{Tan}[e + f*x])^{(5/2)}) -$   
 $(2*((4*I)*A - B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(15*a^2*c*f*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) -$   
 $(2*((4*I)*A - B)*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])/(15*a^2*c^2*f*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

Rule 37

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \&\& \ \text{NeQ}[m, -1]$

Rule 47

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{I} \ \text{LtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1]) \ \&\&$

(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

### Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(-(b\*e - a\*f))\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

### Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{5/2}(c-icx)^{7/2}} dx, x, \tan(e + fx)\right)}{f} \\ &= \frac{iA - B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{5/2}} + \\ &= \frac{iA - B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{5/2}} + \\ &= \frac{iA - B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{5/2}} + \\ &= \frac{iA - B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{5/2}} + \\ &= \frac{iA - B}{3f(a + ia \tan(e + fx))^{3/2} (c - ic \tan(e + fx))^{5/2}} + \end{aligned}$$

**Mathematica [A]**

time = 4.04, size = 170, normalized size = 0.63

$$\frac{\sec(e+fx)(\cos(3(e+fx)) + i \sin(3(e+fx)))(-45A + 20(A+iB)\cos(2(e+fx)) + (A+4iB)\cos(4(e+fx)) - 40iA\sin(2(e+fx)) + 10B\sin(2(e+fx)) - 4iA\sin(4(e+fx)) + B\sin(4(e+fx)))\sqrt{c-i\tan(e+fx)}}{120ac^3f(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2)), x]
```

```
[Out] (Sec[e + f*x]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(-45*A + 20*(A + I*B)*Cos[2*(e + f*x)] + (A + (4*I)*B)*Cos[4*(e + f*x)] - (40*I)*A*Sin[2*(e + f*x)] + 10*B*Sin[2*(e + f*x)] - (4*I)*A*Sin[4*(e + f*x)] + B*Sin[4*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(120*a*c^3*f*(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple [A]**

time = 0.40, size = 199, normalized size = 0.74

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}}{(8iA(\tan^6(fx+e))-2iB(\tan^5(fx+e))-2B(\tan^6$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}}{(8iA(\tan^6(fx+e))-2iB(\tan^5(fx+e))-2B(\tan^6$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2), x, method=_RETURNVERBOSE)
```

```
[Out] 1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c^3*(8*I*A*tan(f*x+e)^6-2*I*B*tan(f*x+e)^5-2*B*tan(f*x+e)^6+20*I*A*tan(f*x+e)^4-8*A*tan(f*x+e)^5-5*I*B*tan(f*x+e)^3-5*B*tan(f*x+e)^4+15*I*A*tan(f*x+e)^2-20*A*tan(f*x+e)^3-3*I*B*tan(f*x+e)+3*I*A-12*A*tan(f*x+e)+3*B)/(I-tan(f*x+e))^3/(I+tan(f*x+e))^4
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2), x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```



**Fricas [A]**

time = 1.24, size = 191, normalized size = 0.71

$$\frac{(3(iA+B)e^{10iA+10iB} - (-23iA-13B)e^{8iA+8iB} + 10(11iA+B)e^{6iA+6iB} + 48(-iA-B)e^{4iA+4iB} + 30(iA+B)e^{2iA+2iB} + 48(-iA-B)e^{2iA+2iB} + 5(-13iA+7B)e^{2iA+2iB} - 5iA+5B)\sqrt{\frac{a}{e^{2iA+2iB}+1}}\sqrt{\frac{c}{e^{2iA+2iB}+1}}e^{(-3iA-3iB)}}{240a^2e^f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/240\*(3\*(I\*A + B)\*e^(10\*I\*f\*x + 10\*I\*e) - (-23\*I\*A - 13\*B)\*e^(8\*I\*f\*x + 8\*I\*e) + 10\*(11\*I\*A + B)\*e^(6\*I\*f\*x + 6\*I\*e) + 48\*(-I\*A - B)\*e^(5\*I\*f\*x + 5\*I\*e) + 30\*(I\*A + B)\*e^(4\*I\*f\*x + 4\*I\*e) + 48\*(-I\*A - B)\*e^(3\*I\*f\*x + 3\*I\*e) + 5\*(-13\*I\*A + 7\*B)\*e^(2\*I\*f\*x + 2\*I\*e) - 5\*I\*A + 5\*B)\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*e^(-3\*I\*f\*x - 3\*I\*e)/(a^2\*c^3\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{(ia(\tan(e + fx) - i))^{\frac{3}{2}}(-ic(\tan(e + fx) + i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x)

[Out] Integral((A + B\*tan(e + f\*x))/((I\*a\*(tan(e + f\*x) - I))^(3/2)\*(-I\*c\*(tan(e + f\*x) + I))^(5/2)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^(3/2)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)), x)

**Mupad [B]**

time = 9.99, size = 196, normalized size = 0.73

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)\operatorname{li})}{\cos(2e+2fx)+1}}(40A\sin(2e+2fx)+A\cos(2e+2fx)20i+A\cos(4e+4fx)\operatorname{li}-20B\cos(2e+2fx)-4B\cos(4e+4fx)-A45i+4A\sin(4e+4fx)+B\sin(2e+2fx)10i+B\sin(4e+4fx)\operatorname{li})}{120a^2e^f\sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx)\operatorname{li})}{\cos(2e+2fx)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(5/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(
1/2)*(A*cos(2*e + 2*f*x)*20i - A*45i + A*cos(4*e + 4*f*x)*1i - 20*B*cos(2*
e + 2*f*x) - 4*B*cos(4*e + 4*f*x) + 40*A*sin(2*e + 2*f*x) + 4*A*sin(4*e + 4
*f*x) + B*sin(2*e + 2*f*x)*10i + B*sin(4*e + 4*f*x)*1i))/(120*a^2*c^2*f*((c
*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2
))
```

$$3.843 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{9/2}}{(a+ia \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=343

$$\frac{7(2iA - 7B)c^{9/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e+fx)}}{\sqrt{a} \sqrt{c - ic \tan(e+fx)}}\right)}{a^{5/2} f} + \frac{7(2iA - 7B)c^4 \sqrt{a + ia \tan(e+fx)} \sqrt{c - ic \tan(e+fx)}}{2a^3 f}$$

```
[Out] 7*(2*I*A-7*B)*c^(9/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x+e))^(1/2))/a^(5/2)/f+7/2*(2*I*A-7*B)*c^4*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/a^3/f+7/6*(2*I*A-7*B)*c^3*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(3/2)/a^3/f+14/15*(2*I*A-7*B)*c^2*(c-I*c*tan(f*x+e))^(5/2)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)-2/15*(2*I*A-7*B)*c*(c-I*c*tan(f*x+e))^(7/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)+1/5*(I*A-B)*(c-I*c*tan(f*x+e))^(9/2)/f/(a+I*a*tan(f*x+e))^(5/2)
```

**Rubi [A]**

time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{7c^{9/2}(-7B+2iA)\operatorname{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{5/2}f} + \frac{7c^4(-7B+2iA)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2a^3f} + \frac{7c^3(-7B+2iA)\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{6a^2f} + \frac{14c^2(-7B+2iA)(c-ic\tan(e+fx))^{5/2}}{15a^2f\sqrt{a+ia\tan(e+fx)}} - \frac{2c(-7B+2iA)(c-ic\tan(e+fx))^{7/2}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan[e + f*x])^(5/2), x]
```

```
[Out] (7*((2*I)*A - 7*B)*c^(9/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])]/(a^(5/2)*f) + (7*((2*I)*A - 7*B)*c^4*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(2*a^3*f) + (7*((2*I)*A - 7*B)*c^3*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2))/(6*a^3*f) + (14*((2*I)*A - 7*B)*c^2*(c - I*c*Tan[e + f*x])^(5/2))/(15*a^2*f*Sqrt[a + I*a*Tan[e + f*x]]) - (2*((2*I)*A - 7*B)*c*(c - I*c*Tan[e + f*x])^(7/2))/(15*a*f*(a + I*a*Tan[e + f*x])^(3/2)) + ((I*A - B)*(c - I*c*Tan[e + f*x])^(9/2))/(5*f*(a + I*a*Tan[e + f*x])^(5/2))
```

**Rule 49**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) & IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^{9/2}}{(a + i a \tan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{7/2}}{(a+iax)^{7/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - i c \tan(e + fx))^{9/2}}{5f(a + i a \tan(e + fx))^{5/2}} - \frac{((2A + 7iB)c) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{7/2}}{(a+iax)^{7/2}} dx, x, \tan(e + fx)\right)}{5f(a + i a \tan(e + fx))^{5/2}} \\
&= -\frac{2(2iA - 7B)c(c - i c \tan(e + fx))^{7/2}}{15af(a + i a \tan(e + fx))^{3/2}} + \frac{(iA - B)(c - i c \tan(e + fx))^{9/2}}{5f(a + i a \tan(e + fx))^{5/2}} \\
&= \frac{14(2iA - 7B)c^2(c - i c \tan(e + fx))^{5/2}}{15a^2 f \sqrt{a + i a \tan(e + fx)}} - \frac{2(2iA - 7B)c(c - i c \tan(e + fx))^{7/2}}{15af(a + i a \tan(e + fx))^{3/2}} \\
&= \frac{7(2iA - 7B)c^3 \sqrt{a + i a \tan(e + fx)} (c - i c \tan(e + fx))^{5/2}}{6a^3 f} \\
&= \frac{7(2iA - 7B)c^4 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^3 f} \\
&= \frac{7(2iA - 7B)c^4 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^3 f} \\
&= \frac{7(2iA - 7B)c^4 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^3 f} \\
&= \frac{7(2iA - 7B)c^4 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{2a^3 f} \\
&= \frac{7(2iA - 7B)c^{9/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + i a \tan(e + fx)}}{\sqrt{a} \sqrt{c - i c \tan(e + fx)}}\right)}{a^{5/2} f}
\end{aligned}$$

**Mathematica [A]**

time = 8.92, size = 247, normalized size = 0.72

$$\frac{\sqrt{2} c^4 e^{-4i(e+fx)} \sqrt{\frac{c}{1+e^{2i(e+fx)}}} \left( -2iA(6 - 8e^{2i(e+fx)} + 56e^{4i(e+fx)} + 175e^{6i(e+fx)} + 105e^{8i(e+fx)}) + B(12 - 56e^{2i(e+fx)} + 392e^{4i(e+fx)} + 1225e^{6i(e+fx)} + 735e^{8i(e+fx)} + 105(-2iA + 7B)c^{2i(e+fx)}(1 + e^{2i(e+fx)})^2 \text{ArcTan}(e^{i(e+fx)})) \right)}{15a^2 (1 + e^{2i(e+fx)})^2 f \sqrt{a + i a \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(9/2))/(a + I\*a\*Tan[e + f\*x])^(5/2), x]

```
[Out] -1/15*(Sqrt[2]*c^4*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*((-2*I)*A*(6 - 8*E^((2*I)*(e + f*x)) + 56*E^((4*I)*(e + f*x)) + 175*E^((6*I)*(e + f*x)) + 105*E^((8*I)*(e + f*x))) + B*(12 - 56*E^((2*I)*(e + f*x)) + 392*E^((4*I)*(e + f*x)) + 1225*E^((6*I)*(e + f*x)) + 735*E^((8*I)*(e + f*x))) + 105*((-2*I)*A + 7*B)*E^((5*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*ArcTan[E^(I*(e + f*x))])/(a^2*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*f*Sqrt[a + I*a*Tan[e + f*x]]])
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 898 vs.  $2(283) = 566$ .

time = 0.43, size = 899, normalized size = 2.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^4/a^3*(-735*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^4+2014*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+334*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+840*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-210*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^4+15*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^5+4410*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-2940*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-150*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4-3881*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-735*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c+1260*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+584*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+30*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4-1316*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+2940*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)+4576*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2-840*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c*tan(f*x+e)-210*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)))/(a*c)^(1/2))*a*c-1096*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-1154*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)/(I-tan(f*x+e))^4
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(9/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 634 vs.  $2(275) = 550$ .  
time = 2.33, size = 634, normalized size = 1.85

$$\frac{\left( \frac{c^2 - 1}{c^2 + 1} \right)^{9/2} \sqrt{\frac{a^2 - 1}{a^2 + 1}} \log\left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(I f x + I e)}}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right) \sqrt{\frac{a^2 - 1}{a^2 + 1}} \sqrt{\frac{c^2 - 1}{c^2 + 1}} \sqrt{\frac{4a^2 + 28I A B - 49B^2}{a^5 f^2}}}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} - 105(a^3 f e^{(7I f x + 7I e)} + a^3 f e^{(5I f x + 5I e)}) \sqrt{\frac{4a^2 + 28I A B - 49B^2}{a^5 f^2}} \log\left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(3I f x + 3I e)} + (2a^2 - 7c^2)c^4 e^{(I f x + I e)}}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right) \sqrt{\frac{a^2 - 1}{a^2 + 1}} \sqrt{\frac{c^2 - 1}{c^2 + 1}} \sqrt{\frac{4a^2 + 28I A B - 49B^2}{a^5 f^2}} \left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + (2a^2 - 7c^2)c^4}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right) - 105(a^3 f e^{(7I f x + 7I e)} + a^3 f e^{(5I f x + 5I e)}) \sqrt{\frac{4a^2 + 28I A B - 49B^2}{a^5 f^2}} \log\left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(3I f x + 3I e)} + (2a^2 - 7c^2)c^4 e^{(I f x + I e)}}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right) \sqrt{\frac{a^2 - 1}{a^2 + 1}} \sqrt{\frac{c^2 - 1}{c^2 + 1}} \sqrt{\frac{4a^2 + 28I A B - 49B^2}{a^5 f^2}} \left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + (2a^2 - 7c^2)c^4}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right) + 4(105(-2I A + 7B)c^4 e^{(8I f x + 8I e)} + 175(-2I A + 7B)c^4 e^{(6I f x + 6I e)} + 56(-2I A + 7B)c^4 e^{(4I f x + 4I e)} + 8(2I A - 7B)c^4 e^{(2I f x + 2I e)} + 12(-I A + B)c^4) \sqrt{\frac{a^2 - 1}{a^2 + 1}} \sqrt{\frac{c^2 - 1}{c^2 + 1}} \sqrt{\frac{4a^2 + 28I A B - 49B^2}{a^5 f^2}} \left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + (2a^2 - 7c^2)c^4}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right) \left( \frac{4c^2(a^2 - 1) + (2a^2 - 7c^2)c^4 e^{(7I f x + 7I e)} + a^3 f e^{(5I f x + 5I e)}}{(2a^2 - 7c^2)c^4 e^{(2I f x + 2I e)} + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(9/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/60*(105*(a^3*f*e^{(7*I*f*x + 7*I*e)} + a^3*f*e^{(5*I*f*x + 5*I*e)})*\sqrt{(4*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^5*f^2)}*\log(4*(2*((2*I*A - 7*B)*c^4*e^{(3*I*f*x + 3*I*e)} + (2*I*A - 7*B)*c^4*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} + (a^3*f*e^{(2*I*f*x + 2*I*e)} - a^3*f)*\sqrt{(4*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^5*f^2)}))/((2*I*A - 7*B)*c^4*e^{(2*I*f*x + 2*I*e)} + (2*I*A - 7*B)*c^4) - 105*(a^3*f*e^{(7*I*f*x + 7*I*e)} + a^3*f*e^{(5*I*f*x + 5*I*e)})*\sqrt{(4*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^5*f^2)} \\ & * \log(4*(2*((2*I*A - 7*B)*c^4*e^{(3*I*f*x + 3*I*e)} + (2*I*A - 7*B)*c^4*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)} - (a^3*f*e^{(2*I*f*x + 2*I*e)} - a^3*f)*\sqrt{(4*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^5*f^2)}))/((2*I*A - 7*B)*c^4*e^{(2*I*f*x + 2*I*e)} + (2*I*A - 7*B)*c^4) \\ & ) + 4*(105*(-2*I*A + 7*B)*c^4*e^{(8*I*f*x + 8*I*e)} + 175*(-2*I*A + 7*B)*c^4*e^{(6*I*f*x + 6*I*e)} + 56*(-2*I*A + 7*B)*c^4*e^{(4*I*f*x + 4*I*e)} + 8*(2*I*A - 7*B)*c^4*e^{(2*I*f*x + 2*I*e)} + 12*(-I*A + B)*c^4)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1))}/(a^3*f*e^{(7*I*f*x + 7*I*e)} + a^3*f*e^{(5*I*f*x + 5*I*e)}) \end{aligned}$$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(9/2)/(a+I\*a\*tan(f\*x+e))\*\*(5/2),x)

[Out] Timed out

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(9/2)/(I*a*tan(f*x + e) + a)^(5/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) i)^{9/2}}{(a + a \tan(e + f x) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(9/2))/(a + a*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(9/2))/(a + a*tan(e + f*x)*1i)^(5/2), x)
```



$$3.844 \quad \int \frac{(A+B \tan(e+fx))(c-ic \tan(e+fx))^{7/2}}{(a+ia \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=284

$$\frac{2(iA - 6B)c^{7/2} \text{ArcTan}\left(\frac{\sqrt{c} \sqrt{a + ia \tan(e + fx)}}{\sqrt{a} \sqrt{c - ic \tan(e + fx)}}\right)}{a^{5/2} f} + \frac{(iA - 6B)c^3 \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}}{a^3 f}$$

[Out] 2\*(I\*A-6\*B)\*c^(7/2)\*arctan(c^(1/2)\*(a+I\*a\*tan(f\*x+e))^(1/2)/a^(1/2)/(c-I\*c\*tan(f\*x+e))^(1/2))/a^(5/2)/f+(I\*A-6\*B)\*c^3\*(a+I\*a\*tan(f\*x+e))^(1/2)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^3/f+2/3\*(I\*A-6\*B)\*c^2\*(c-I\*c\*tan(f\*x+e))^(3/2)/a^2/f/(a+I\*a\*tan(f\*x+e))^(1/2)-2/15\*(I\*A-6\*B)\*c\*(c-I\*c\*tan(f\*x+e))^(5/2)/a/f/(a+I\*a\*tan(f\*x+e))^(3/2)+1/5\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(7/2)/f/(a+I\*a\*tan(f\*x+e))^(5/2)

**Rubi** [A]

time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3669, 79, 49, 52, 65, 223, 209}

$$\frac{2c^{7/2}(-6B+iA)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{5/2}f} + \frac{c^3(-6B+iA)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a^3f} + \frac{2c^2(-6B+iA)(c-ic\tan(e+fx))^{3/2}}{3a^2f\sqrt{a+ia\tan(e+fx)}} - \frac{2c(-6B+iA)(c-ic\tan(e+fx))^{5/2}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(a + I\*a\*Tan[e + f\*x])^(5/2), x]

[Out] (2\*(I\*A - 6\*B)\*c^(7/2)\*ArcTan[(Sqrt[c]\*Sqrt[a + I\*a\*Tan[e + f\*x]])/(Sqrt[a]\*Sqrt[c - I\*c\*Tan[e + f\*x]])]/(a^(5/2)\*f) + ((I\*A - 6\*B)\*c^3\*Sqrt[a + I\*a\*Tan[e + f\*x]]\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(a^3\*f) + (2\*(I\*A - 6\*B)\*c^2\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(3\*a^2\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]]) - (2\*(I\*A - 6\*B)\*c\*(c - I\*c\*Tan[e + f\*x])^(5/2))/(15\*a\*f\*(a + I\*a\*Tan[e + f\*x])^(3/2)) + ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(5\*f\*(a + I\*a\*Tan[e + f\*x])^(5/2))

**Rule 49**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^n/(b\*(m + 1))), x] - Dist[d\*(n/(b\*(m + 1))), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2\*n + m + 1, 0])) & IntLinearQ[a, b, c, d, m, n, x]

**Rule 52**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
)
```

### Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 3669

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \tan(e + fx))(c - i c \tan(e + fx))^{7/2}}{(a + i a \tan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)(c-icx)^{5/2}}{(a+iax)^{7/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= \frac{(iA - B)(c - i c \tan(e + fx))^{7/2}}{5f(a + i a \tan(e + fx))^{5/2}} - \frac{((A + 6iB)c) \text{Subst}}{f} \\
&= -\frac{2(iA - 6B)c(c - i c \tan(e + fx))^{5/2}}{15af(a + i a \tan(e + fx))^{3/2}} + \frac{(iA - B)(c - i c \tan(e + fx))^{7/2}}{5f(a + i a \tan(e + fx))^{5/2}} \\
&= \frac{2(iA - 6B)c^2(c - i c \tan(e + fx))^{3/2}}{3a^2 f \sqrt{a + i a \tan(e + fx)}} - \frac{2(iA - 6B)c(c - i c \tan(e + fx))^{5/2}}{15af(a + i a \tan(e + fx))^{3/2}} \\
&= \frac{(iA - 6B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{a^3 f} \\
&= \frac{(iA - 6B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{a^3 f} \\
&= \frac{(iA - 6B)c^3 \sqrt{a + i a \tan(e + fx)} \sqrt{c - i c \tan(e + fx)}}{a^3 f} \\
&= \frac{2(iA - 6B)c^{7/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{a + i a \tan(e + fx)}}{\sqrt{a} \sqrt{c - i c \tan(e + fx)}}\right)}{a^{5/2} f} +
\end{aligned}$$

**Mathematica [A]**

time = 6.26, size = 205, normalized size = 0.72

$$\frac{2\sqrt{2} c^2 e^{-4i(e+fx)} \left(\frac{c}{1+e^{2i(e+fx)}}\right)^{3/2} (iA(3 - 2e^{2i(e+fx)} + 10e^{4i(e+fx)} + 15e^{6i(e+fx)}) - 3B(1 - 4e^{2i(e+fx)} + 20e^{4i(e+fx)} + 30e^{6i(e+fx)}) + 15i(A + 6iB)e^{5i(e+fx)}(1 + e^{2i(e+fx)}) \text{ArcTan}(e^{i(e+fx)}))}{15a^2 f \sqrt{a + i a \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(7/2))/(a + I\*a\*Tan[e + f\*x])^(5/2), x]

[Out] (2\*sqrt[2]\*c^2\*(c/(1 + E^((2\*I)\*(e + f\*x))))^(3/2)\*(I\*A\*(3 - 2\*E^((2\*I)\*(e + f\*x)) + 10\*E^((4\*I)\*(e + f\*x)) + 15\*E^((6\*I)\*(e + f\*x))) - 3\*B\*(1 - 4\*E^((2\*I)\*(e + f\*x)) + 20\*E^((4\*I)\*(e + f\*x)) + 30\*E^((6\*I)\*(e + f\*x))) + (15\*I

)\*(A + (6\*I)\*B)\*E^((5\*I)\*(e + f\*x))\*(1 + E^((2\*I)\*(e + f\*x)))\*ArcTan[E^(I\*(e + f\*x)))]/(15\*a^2\*E^((4\*I)\*(e + f\*x))\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]])

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 834 vs.  $2(235) = 470$ .

time = 0.42, size = 835, normalized size = 2.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x,method=\_RETURNVERBOSE)

[Out]  $\frac{1}{15} \frac{1}{f} (-c(I \tan(fx+e) - 1))^{1/2} (a(1 + I \tan(fx+e)))^{1/2} c^3/a^3 (-90 I B \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e)^4 + 246 I B (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e)^3 - 15 A \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e)^4 + 26 I A (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} + 60 I A \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e)^3 - 360 B \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e)^3 - 15 B (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e)^4 - 474 I B (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e) + 540 I B \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e)^2 + 90 A \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e)^2 + 46 A (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e)^3 - 90 I B \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c - 94 I A (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e)^2 + 360 B \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e) + 564 B (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e)^2 - 60 I A \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c \tan(fx+e) - 15 A \ln((a c \tan(fx+e) + (a c)^{1/2} (a c (1 + \tan(fx+e)^2))^{1/2}) / (a c)^{1/2})) a c - 74 A (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} \tan(fx+e) - 141 B (a c (1 + \tan(fx+e)^2))^{1/2} (a c)^{1/2} / (a c (1 + \tan(fx+e)^2))^{1/2} / (a c)^{1/2} / (I \tan(fx+e))^4$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1091 vs.  $2(228) = 456$ .

time = 0.79, size = 1091, normalized size = 3.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x,algorithm="maxima")

[Out]  $15(60(A + 6I*B)*c^3 \cos(6f*x + 6e) + 40(A + 6I*B)*c^3 \cos(4f*x + 4e) - 8(A + 6I*B)*c^3 \cos(2f*x + 2e) + 60(I*A - 6*B)*c^3 \sin(6f*x + 6e)$

e) + 40\*(I\*A - 6\*B)\*c^3\*sin(4\*f\*x + 4\*e) + 8\*(-I\*A + 6\*B)\*c^3\*sin(2\*f\*x + 2\*e) + 12\*(A + I\*B)\*c^3 + 30\*((A + 6\*I\*B)\*c^3\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + (A + 6\*I\*B)\*c^3\*cos(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + (I\*A - 6\*B)\*c^3\*sin(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + (I\*A - 6\*B)\*c^3\*sin(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*arctan2(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))), sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) + 30\*((A + 6\*I\*B)\*c^3\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + (A + 6\*I\*B)\*c^3\*cos(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + (I\*A - 6\*B)\*c^3\*sin(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + (I\*A - 6\*B)\*c^3\*sin(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*arctan2(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))), -sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) + 15\*((I\*A - 6\*B)\*c^3\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + (I\*A - 6\*B)\*c^3\*cos(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) - (A + 6\*I\*B)\*c^3\*sin(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - (A + 6\*I\*B)\*c^3\*sin(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1) + 15\*((-I\*A + 6\*B)\*c^3\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) + (-I\*A + 6\*B)\*c^3\*cos(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*log(cos(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 + sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))^2 - 2\*sin(1/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 1)\*sqrt(a)\*sqrt(c)/((-450\*I\*a^3\*cos(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))) - 450\*I\*a^3\*cos(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 450\*a^3\*sin(7/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e)))) + 450\*a^3\*sin(5/2\*arctan2(sin(2\*f\*x + 2\*e), cos(2\*f\*x + 2\*e))))\*f)

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 553 vs.  $2(228) = 456$ .  
time = 2.78, size = 553, normalized size = 1.95

$$\left( \frac{15a \sqrt{\frac{A^2 + 12AB - 36B^2}{4f^2}} \sqrt{\frac{c}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{a}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{A^2 + 12AB - 36B^2}{4f^2}}}{\sqrt{a} \sqrt{c}} \right) - 15a \sqrt{\frac{A^2 + 12AB - 36B^2}{4f^2}} \sqrt{\frac{c}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{a}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{A^2 + 12AB - 36B^2}{4f^2}} + 4(15c(A + 6IB)\sqrt{a} + 30c(A + 6IB)\sqrt{a} + 30c(A + 6IB)\sqrt{a} + 30c(A + 6IB)\sqrt{a}) \sqrt{\frac{c}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{a}{e^{2Ifx + 2Ie} + 1}} \sqrt{\frac{A^2 + 12AB - 36B^2}{4f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(7/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/30\*(15\*a^3\*sqrt((A^2 + 12\*I\*A\*B - 36\*B^2)\*c^7/(a^5\*f^2))\*f\*e^(5\*I\*f\*x + 5\*I\*e)\*log(-4\*(2\*((I\*A - 6\*B)\*c^3\*e^(3\*I\*f\*x + 3\*I\*e) + (I\*A - 6\*B)\*c^3\*e^(I\*f\*x + I\*e))\*sqrt(a/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)) + (a^3\*f\*e^(2\*I\*f\*x + 2\*I\*e) - a^3\*f)\*sqrt((A^2 + 12\*I\*A\*B - 36\*B^2)\*c^7/(a^5\*f^2)))/((-I\*A + 6\*B)\*c^3\*e^(2\*I\*f\*x + 2\*I\*e) + (-I\*A + 6\*B)\*c^3)

```

- 15*a^3*sqrt((A^2 + 12*I*A*B - 36*B^2)*c^7/(a^5*f^2))*f*e^(5*I*f*x + 5*I*
e)*log(-4*(2*((I*A - 6*B)*c^3*e^(3*I*f*x + 3*I*e) + (I*A - 6*B)*c^3*e^(I*f*
x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1
)) - (a^3*f*e^(2*I*f*x + 2*I*e) - a^3*f)*sqrt((A^2 + 12*I*A*B - 36*B^2)*c^7
/(a^5*f^2)))/((-I*A + 6*B)*c^3*e^(2*I*f*x + 2*I*e) + (-I*A + 6*B)*c^3)) + 4
*(15*(-I*A + 6*B)*c^3*e^(6*I*f*x + 6*I*e) + 10*(-I*A + 6*B)*c^3*e^(4*I*f*x
+ 4*I*e) + 2*(I*A - 6*B)*c^3*e^(2*I*f*x + 2*I*e) + 3*(-I*A + B)*c^3)*sqrt(a
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-5*I*f*x
- 5*I*e)/(a^3*f)

```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(7/2)/(I*a*tan(f*x +
e) + a)^(5/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) i)^{7/2}}{(a + a \tan(e + f x) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*x
)*1i)^(5/2),x)
```

```
[Out] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*x
)*1i)^(5/2), x)
```

$$3.845 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{5/2}}{(a+ia \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=205

$$-\frac{2Bc^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{a^{5/2} f} - \frac{2Bc^2 \sqrt{c-ictan(e+fx)}}{a^2 f \sqrt{a+ia \tan(e+fx)}} + \frac{2Bc(c-ictan(e+fx))^{3/2}}{3af(a+ia \tan(e+fx))^{3/2}} +$$

[Out]  $-2*B*c^{(5/2)*\arctan(c^{(1/2)*(a+I*a*\tan(f*x+e))^{(1/2)/a^{(1/2)/(c-I*c*\tan(f*x+e))^{(1/2)})/a^{(5/2)/f-2*B*c^2*(c-I*c*\tan(f*x+e))^{(1/2)/a^2/f/(a+I*a*\tan(f*x+e))^{(1/2)+2/3*B*c*(c-I*c*\tan(f*x+e))^{(3/2)/a/f/(a+I*a*\tan(f*x+e))^{(3/2)+1/5*(I*A-B)*(c-I*c*\tan(f*x+e))^{(5/2)/f/(a+I*a*\tan(f*x+e))^{(5/2)}$

**Rubi [A]**

time = 0.19, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3669, 79, 49, 65, 223, 209}

$$-\frac{2Bc^{5/2} \operatorname{ArcTan}\left(\frac{\sqrt{c} \sqrt{a+ia \tan(e+fx)}}{\sqrt{a} \sqrt{c-ictan(e+fx)}}\right)}{a^{5/2} f} - \frac{2Bc^2 \sqrt{c-ictan(e+fx)}}{a^2 f \sqrt{a+ia \tan(e+fx)}} + \frac{(-B+IA)(c-ictan(e+fx))^{5/2}}{5f(a+ia \tan(e+fx))^{5/2}} + \frac{2Bc(c-ictan(e+fx))^{3/2}}{3af(a+ia \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[e+f*x])*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)}/(a+I*a*\operatorname{Tan}[e+f*x])^{(5/2)},x]$

[Out]  $(-2*B*c^{(5/2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])]}/(a^{(5/2)*f} - (2*B*c^2*\operatorname{Sqrt}[c-I*c*\operatorname{Tan}[e+f*x]])/(a^{(5/2)*f*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]]) + (2*B*c*(c-I*c*\operatorname{Tan}[e+f*x])^{(3/2)})/(3*a*f*(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)}) + ((I*A-B)*(c-I*c*\operatorname{Tan}[e+f*x])^{(5/2)})/(5*f*(a+I*a*\operatorname{Tan}[e+f*x])^{(5/2)})$

**Rule 49**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{Simp}[(a + b*x)^{(m+1)*((c+d*x)^n/(b*(m+1)))}, x] - \operatorname{Dist}[d*(n/(b*(m+1))), \operatorname{Int}[(a + b*x)^{(m+1)*(c+d*x)^{(n-1)}}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{IntegerQ}[n] \&\& !\operatorname{IntegerQ}[m]) \&\& !(I\operatorname{LeQ}[m+n+2, 0] \&\& (\operatorname{FractionQ}[m] \mid\mid \operatorname{GeQ}[2*n+m+1, 0])) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

**Rule 65**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-a*(d/b)+d*(x^p/b)^n}, x], x, (a + b*x)^{(1/p)}], x]] /;$   $\operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}$

`[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 79

`Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))`

### Rule 209

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

### Rule 223

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

### Rule 3669

`Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]`

### Rubi steps





**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 556 vs.  $2(168) = 336$ .  
time = 0.42, size = 557, normalized size = 2.72

method	result
derivativedivides	$\frac{\sqrt{-c(i \tan (fx + e) - 1)} \sqrt{a(1 + i \tan (fx + e))} c^2 \left( -15iB \ln \left( \frac{ac \tan (fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + i \tan (fx + e))} \right) \right)}{\sqrt{-c(i \tan (fx + e) - 1)} \sqrt{a(1 + i \tan (fx + e))} c^2 \left( -15iB \ln \left( \frac{ac \tan (fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + i \tan (fx + e))} \right) \right)}$
default	$\sqrt{-c(i \tan (fx + e) - 1)} \sqrt{a(1 + i \tan (fx + e))} c^2 \left( -15iB \ln \left( \frac{ac \tan (fx + e) + \sqrt{ac}}{\sqrt{ac}} \sqrt{ac(1 + i \tan (fx + e))} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^3*(-15*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4+90*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+43*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-60*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+3*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-15*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-77*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+60*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+97*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+3*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-23*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I-tan(f*x+e))^4/(a*c)^(1/2)
```

**Maxima [A]**

time = 0.59, size = 232, normalized size = 1.13

$\frac{30 B^2 \arctan(\cos(fx + e), \sin(fx + e) + 1) + 30 B^2 \arctan(\cos(fx + e), -\sin(fx + e) + 1) - 6(I A - B) \cos(5fx + 5e) - 20 B^2 \cos(3fx + 3e) + 60 B^2 \cos(fx + e) + 15 I B^2 \log(\cos(fx + e)^2 + \sin(fx + e)^2 + 2 \sin(fx + e) + 1) - 15 I B^2 \log(\cos(fx + e)^2 + \sin(fx + e)^2 - 2 \sin(fx + e) + 1) - 6(A + B) \sin(5fx + 5e) + 20 B^2 \sin(3fx + 3e) - 60 B^2 \sin(fx + e)}{30 a^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x,algorithm="maxima")
```

```
[Out] -1/30*(30*B*c^2*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 30*B*c^2*arctan2(cos(f*x + e), -sin(f*x + e) + 1) - 6*(I*A - B)*c^2*cos(5*f*x + 5*e) - 20*B*c^2*cos(3*f*x + 3*e) + 60*B*c^2*cos(f*x + e) + 15*I*B*c^2*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 15*I*B*c^2*log(cos(f*x + e)^2 +
```

$\sin(f*x + e)^2 - 2*\sin(f*x + e) + 1) - 6*(A + I*B)*c^2*\sin(5*f*x + 5*e) + 2$   
 $0*I*B*c^2*\sin(3*f*x + 3*e) - 60*I*B*c^2*\sin(f*x + e))*\sqrt{c}/(a^{(5/2)*f})$

**Fricas** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs.  $2(165) = 330$ .

time = 3.54, size = 456, normalized size = 2.22

$$\left( \frac{11a^2f\sqrt{\frac{Bc^2}{a^2f}}e^{2I*f*x}\log\left(\frac{1+\sqrt{\frac{a}{2B^2c^2+1}}\sqrt{\frac{c}{2B^2c^2+1}}e^{2I*f*x}\sqrt{\frac{Bc^2}{a^2f}}}{1-\sqrt{\frac{a}{2B^2c^2+1}}\sqrt{\frac{c}{2B^2c^2+1}}e^{2I*f*x}\sqrt{\frac{Bc^2}{a^2f}}}\right)-11a^2f\sqrt{\frac{Bc^2}{a^2f}}e^{2I*f*x}\log\left(\frac{1+\sqrt{\frac{a}{2B^2c^2+1}}\sqrt{\frac{c}{2B^2c^2+1}}e^{2I*f*x}\sqrt{\frac{Bc^2}{a^2f}}}{1-\sqrt{\frac{a}{2B^2c^2+1}}\sqrt{\frac{c}{2B^2c^2+1}}e^{2I*f*x}\sqrt{\frac{Bc^2}{a^2f}}}\right)-2(30Bc^2e^{2I*f*x}+20Bc^2e^{4I*f*x}+(-3I*A-7B)c^2e^{2I*f*x}+3(-I*A+B)c^2)\sqrt{\frac{a}{2B^2c^2+1}}\sqrt{\frac{c}{2B^2c^2+1}}e^{2I*f*x}}{30a^2f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{30}*(15*a^3*f*\sqrt{-B^2*c^5/(a^5*f^2)})*e^{(5*I*f*x + 5*I*e)}*\log(4*(2*(B*c^2*e^{(3*I*f*x + 3*I*e)} + B*c^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}) + (a^3*f*e^{(2*I*f*x + 2*I*e)} - a^3*f)*\sqrt{-B^2*c^5/(a^5*f^2)})/(B*c^2*e^{(2*I*f*x + 2*I*e)} + B*c^2)) - 15*a^3*f*\sqrt{-B^2*c^5/(a^5*f^2)})*e^{(5*I*f*x + 5*I*e)}*\log(4*(2*(B*c^2*e^{(3*I*f*x + 3*I*e)} + B*c^2*e^{(I*f*x + I*e)})*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}) - (a^3*f*e^{(2*I*f*x + 2*I*e)} - a^3*f)*\sqrt{-B^2*c^5/(a^5*f^2)})/(B*c^2*e^{(2*I*f*x + 2*I*e)} + B*c^2)) - 2*(30*B*c^2*e^{(6*I*f*x + 6*I*e)} + 20*B*c^2*e^{(4*I*f*x + 4*I*e)} + (-3*I*A - 7*B)*c^2*e^{(2*I*f*x + 2*I*e)} + 3*(-I*A + B)*c^2)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)))*e^{(-5*I*f*x - 5*I*e)}/(a^3*f)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e + fx) + i))^{\frac{5}{2}}(A + B \tan(e + fx))}{(ia(\tan(e + fx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(5/2)/(a+I\*a\*tan(f\*x+e))\*\*(5/2),x)

[Out] Integral((-I\*c\*(tan(e + f\*x) + I))\*\*(5/2)\*(A + B\*tan(e + f\*x))/(I\*a\*(tan(e + f\*x) - I))\*\*(5/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(5/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)/(I\*a\*tan(f\*x + e) + a)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \tan(e + f x)) (c - c \tan(e + f x) i)^{5/2}}{(a + a \tan(e + f x) i)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(5/2))/(a + a\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(5/2))/(a + a\*tan(e + f\*x)\*1i)^(5/2), x)

$$3.846 \quad \int \frac{(A+B \tan(e+fx))(c-ictan(e+fx))^{3/2}}{(a+ia \tan(e+fx))^{5/2}} dx$$

Optimal. Leaf size=104

$$\frac{(iA - B)(c - ictan(e + fx))^{3/2}}{5f(a + ia \tan(e + fx))^{5/2}} + \frac{(iA + 4B)(c - ictan(e + fx))^{3/2}}{15af(a + ia \tan(e + fx))^{3/2}}$$

[Out] 1/5\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(3/2)/f/(a+I\*a\*tan(f\*x+e))^(5/2)+1/15\*(I\*A+4\*B)\*(c-I\*c\*tan(f\*x+e))^(3/2)/a/f/(a+I\*a\*tan(f\*x+e))^(3/2)

Rubi [A]

time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ ,

Rules used = {3669, 79, 37}

$$\frac{(4B + iA)(c - ictan(e + fx))^{3/2}}{15af(a + ia \tan(e + fx))^{3/2}} + \frac{(-B + iA)(c - ictan(e + fx))^{3/2}}{5f(a + ia \tan(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(a + I\*a\*Tan[e + f\*x])^(5/2), x]

[Out] ((I\*A - B)\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(5\*f\*(a + I\*a\*Tan[e + f\*x])^(5/2)) + ((I\*A + 4\*B)\*(c - I\*c\*Tan[e + f\*x])^(3/2))/(15\*a\*f\*(a + I\*a\*Tan[e + f\*x])^(3/2))

Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 79

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(-b\*e - a\*f)\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(f\*(p + 1)\*(c\*f - d\*e))), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2}}{(a + ia \tan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^{7/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{5f(a + ia \tan(e + fx))^{5/2}} + \frac{((A - 4iB)c) \text{Subst}\left(\int \frac{(A+Bx)\sqrt{c-icx}}{(a+iax)^{7/2}} dx, x, \tan(e + fx)\right)}{15af(a + ia \tan(e + fx))^{5/2}}$$

$$= \frac{(iA - B)(c - ic \tan(e + fx))^{3/2}}{5f(a + ia \tan(e + fx))^{5/2}} + \frac{(iA + 4B)(c - ic \tan(e + fx))^{3/2}}{15af(a + ia \tan(e + fx))^{5/2}}$$

Mathematica [A]

time = 2.27, size = 92, normalized size = 0.88

$$\frac{c(1 - i \tan(e + fx))(-4iA - B + (A - 4iB) \tan(e + fx)) \sqrt{c - ic \tan(e + fx)}}{15a^2 f (-i + \tan(e + fx))^2 \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[
e + f*x])^(5/2), x]
```

```
[Out] (c*(1 - I*Tan[e + f*x])*((-4*I)*A - B + (A - (4*I)*B)*Tan[e + f*x])*Sqrt[c
- I*c*Tan[e + f*x]]/(15*a^2*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f
*x]])
```

Maple [A]

time = 0.55, size = 92, normalized size = 0.88

method	result
derivativedivides	$\frac{i \sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} c(1 + \tan^2(fx + e))(iA \tan(fx + e) - iB + 4B \tan(fx + e))}{15f a^3 (i - \tan(fx + e))^4}$
default	$\frac{i \sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))} c(1 + \tan^2(fx + e))(iA \tan(fx + e) - iB + 4B \tan(fx + e))}{15f a^3 (i - \tan(fx + e))^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{15} I / f (-c(I \tan(fx+e)-1))^{1/2} (a(1+I \tan(fx+e)))^{1/2} / a^3 c (1+\tan(fx+e)^2) (I A \tan(fx+e) - I B + 4 B \tan(fx+e) + 4 A) / (I - \tan(fx+e))^4$

**Maxima [A]**

time = 0.62, size = 161, normalized size = 1.55

$$\frac{30(5(A-iB)c \cos(4fx+4e) + 2(4A-iB)c \cos(2fx+2e) - 5(-iA-B)c \sin(4fx+4e) - 2(-4iA-B)c \sin(2fx+2e) + 3(A+iB)c) \sqrt{a} \sqrt{c}}{-900(i a^3 \cos(7fx+7e) + i a^3 \cos(5fx+5e) - a^3 \sin(7fx+7e) - a^3 \sin(5fx+5e)) f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="maxima")`

[Out]  $30*(5*(A - I*B)*c*\cos(4*f*x + 4*e) + 2*(4*A - I*B)*c*\cos(2*f*x + 2*e) - 5*(-I*A - B)*c*\sin(4*f*x + 4*e) - 2*(-4*I*A - B)*c*\sin(2*f*x + 2*e) + 3*(A + I*B)*c)*\sqrt{a}*\sqrt{c}/((-900*I*a^3*\cos(7*f*x + 7*e) - 900*I*a^3*\cos(5*f*x + 5*e) + 900*a^3*\sin(7*f*x + 7*e) + 900*a^3*\sin(5*f*x + 5*e))*f)$

**Fricas [A]**

time = 4.64, size = 103, normalized size = 0.99

$$\frac{(5(-iA-B)ce^{4ifx+4ie} + 2(-4iA-B)ce^{2ifx+2ie} + 3(-iA+B)c)\sqrt{\frac{a}{e^{2ifx+2ie}+1}}\sqrt{\frac{c}{e^{2ifx+2ie}+1}}e^{-5ifx-5ie}}{30a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="fricas")`

[Out]  $-1/30*(5*(-I*A - B)*c*e^{(4*I*f*x + 4*I*e)} + 2*(-4*I*A - B)*c*e^{(2*I*f*x + 2*I*e)} + 3*(-I*A + B)*c)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-5*I*f*x - 5*I*e)}/(a^3*f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ic(\tan(e+fx)+i))^{\frac{3}{2}}(A+B\tan(e+fx))}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(5/2),x)`

[Out] `Integral((-I*c*(tan(e + f*x) + I))**(3/2)*(A + B*tan(e + f*x))/(I*a*(tan(e + f*x) - I))**(5/2), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^(3/2)/(I*a*tan(f*x + e) + a)^(5/2), x)
```

**Mupad [B]**

time = 11.07, size = 240, normalized size = 2.31

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx))}{\cos(2e+2fx)+1}} \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx))}{\cos(2e+2fx)+1}} (A \cos(2e+2fx) 5i + A \cos(4e+4fx) 8i + A \cos(6e+6fx) 3i + 5B \cos(2e+2fx) + 2B \cos(4e+4fx) - 3B \cos(6e+6fx) + 5A \sin(2e+2fx) + 8A \sin(4e+4fx) + 3A \sin(6e+6fx) - B \sin(2e+2fx) 5i - B \sin(4e+4fx) 2i + B \sin(6e+6fx) 3i)}{60a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*x)*1i)^(5/2),x)
```

```
[Out] (c*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2)*(A*cos(2*e + 2*f*x)*5i + A*cos(4*e + 4*f*x)*8i + A*cos(6*e + 6*f*x)*3i + 5*B*cos(2*e + 2*f*x) + 2*B*cos(4*e + 4*f*x) - 3*B*cos(6*e + 6*f*x) + 5*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x) + 3*A*sin(6*e + 6*f*x) - B*sin(2*e + 2*f*x)*5i - B*sin(4*e + 4*f*x)*2i + B*sin(6*e + 6*f*x)*3i))/(60*a^3*f)
```



$$3.847 \quad \int \frac{(A+B \tan(e+fx)) \sqrt{c - i c \tan(e+fx)}}{(a+ia \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=157

$$\frac{(iA - B) \sqrt{c - i c \tan(e+fx)}}{5f(a + ia \tan(e+fx))^{5/2}} + \frac{(2iA + 3B) \sqrt{c - i c \tan(e+fx)}}{15af(a + ia \tan(e+fx))^{3/2}} + \frac{(2iA + 3B) \sqrt{c - i c \tan(e+fx)}}{15a^2 f \sqrt{a + ia \tan(e+fx)}}$$

[Out] 1/15\*(2\*I\*A+3\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a^2/f/(a+I\*a\*tan(f\*x+e))^(1/2)+1/5\*(I\*A-B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/f/(a+I\*a\*tan(f\*x+e))^(5/2)+1/15\*(2\*I\*A+3\*B)\*(c-I\*c\*tan(f\*x+e))^(1/2)/a/f/(a+I\*a\*tan(f\*x+e))^(3/2)

**Rubi [A]**

time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{(3B + 2iA) \sqrt{c - i c \tan(e+fx)}}{15a^2 f \sqrt{a + ia \tan(e+fx)}} + \frac{(-B + iA) \sqrt{c - i c \tan(e+fx)}}{5f(a + ia \tan(e+fx))^{5/2}} + \frac{(3B + 2iA) \sqrt{c - i c \tan(e+fx)}}{15af(a + ia \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(a + I\*a\*Tan[e + f\*x]))^(5/2), x]

[Out] (((I\*A - B)\*Sqrt[c - I\*c\*Tan[e + f\*x]])/(5\*f\*(a + I\*a\*Tan[e + f\*x])^(5/2)) + (((2\*I)\*A + 3\*B)\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(15\*a\*f\*(a + I\*a\*Tan[e + f\*x])^(3/2))) + (((2\*I)\*A + 3\*B)\*Sqrt[c - I\*c\*Tan[e + f\*x]]/(15\*a^2\*f\*Sqrt[a + I\*a\*Tan[e + f\*x]]))

**Rule 37**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

**Rule 47**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(Simplify[m + 1])\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[m + n + 2] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

## Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
))
```

## Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_.)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

## Rubi steps

$$\int \frac{(A + B \tan(e + fx)) \sqrt{c - i c \tan(e + fx)}}{(a + i a \tan(e + fx))^{5/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{7/2} \sqrt{c-icx}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{5f(a + i a \tan(e + fx))^{5/2}} + \frac{((2A - 3iB)c) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{7/2} \sqrt{c-icx}} dx, x, \tan(e + fx)\right)}{f}$$

$$= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{5f(a + i a \tan(e + fx))^{5/2}} + \frac{(2iA + 3B) \sqrt{c - i c \tan(e + fx)}}{15af(a + i a \tan(e + fx))^{5/2}}$$

$$= \frac{(iA - B) \sqrt{c - i c \tan(e + fx)}}{5f(a + i a \tan(e + fx))^{5/2}} + \frac{(2iA + 3B) \sqrt{c - i c \tan(e + fx)}}{15af(a + i a \tan(e + fx))^{5/2}}$$

**Mathematica [A]**

time = 1.40, size = 106, normalized size = 0.68

$$\frac{\sec^2(e + fx)(-5iA + (-9iA - 6B) \cos(2(e + fx)) + (6A - 9iB) \sin(2(e + fx))) \sqrt{c - i c \tan(e + fx)}}{30a^2 f (-i + \tan(e + fx))^2 \sqrt{a + i a \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
+ f*x])^(5/2), x]
```

[Out]  $(\text{Sec}[e + f*x]^2*((-5*I)*A + ((-9*I)*A - 6*B)*\text{Cos}[2*(e + f*x)] + (6*A - (9*I)*B)*\text{Sin}[2*(e + f*x)])*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]/(30*a^2*f*(-I + \text{Tan}[e + f*x])^2*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]])$

**Maple [A]**

time = 0.42, size = 127, normalized size = 0.81

method	result
derivativedivides	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\sqrt{a(1+i\tan(fx+e))}}{15fa^3(i-\tan(fx+e))^4} (2iA(\tan^3(fx+e))-12iB(\tan^2(fx+e))+3B)$
default	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\sqrt{a(1+i\tan(fx+e))}}{15fa^3(i-\tan(fx+e))^4} (2iA(\tan^3(fx+e))-12iB(\tan^2(fx+e))+3B)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $-1/15*I/f*(-c*(I*\tan(f*x+e)-1))^{1/2}*(a*(1+I*\tan(f*x+e)))^{1/2}/a^3*(2*I*A*\tan(f*x+e)^3-12*I*B*\tan(f*x+e)^2+3*B*\tan(f*x+e)^3-13*I*A*\tan(f*x+e)+8*A*\tan(f*x+e)^2+3*I*B-12*B*\tan(f*x+e)-7*A)/(I-\tan(f*x+e))^4$

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2),x,algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**Fricas [A]**

time = 3.07, size = 118, normalized size = 0.75

$$\frac{(15(-iA-B)e^{(6i fx+6ie)}+5(-5iA-3B)e^{(4i fx+4ie)}-(13iA-3B)e^{(2i fx+2ie)}-3iA+3B)\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i fx+2ie)}+1}}e^{(-5i fx-5ie)}}{60a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2),x,algorithm="fricas")`

[Out]  $-1/60*(15*(-I*A - B)*e^{(6*I*f*x + 6*I*e)} + 5*(-5*I*A - 3*B)*e^{(4*I*f*x + 4*I*e)} - (13*I*A - 3*B)*e^{(2*I*f*x + 2*I*e)} - 3*I*A + 3*B)*\text{sqrt}(a/(e^{(2*I*f*x$

+ 2\*I\*e) + 1))\*sqrt(c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))\*e^(-5\*I\*f\*x - 5\*I\*e)/(a^3\*f)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ic(\tan(e + fx) + i)} (A + B \tan(e + fx))}{(ia(\tan(e + fx) - i))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))\*\*(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))\*\*(5/2),x)

[Out] Integral(sqrt(-I\*c\*(tan(e + f\*x) + I))\*(A + B\*tan(e + f\*x))/(I\*a\*(tan(e + f\*x) - I))\*\*(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-I\*c\*tan(f\*x+e))^(1/2)\*(A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*sqrt(-I\*c\*tan(f\*x + e) + c)/(I\*a\*tan(f\*x + e) + a)^(5/2), x)

**Mupad [B]**

time = 10.96, size = 246, normalized size = 1.57

$$\frac{\sqrt{\frac{a(\cos(2e + 2fx) + 1 + \sin(2e + 2fx)i)}{\cos(2e + 2fx) + 1}} \sqrt{\frac{c(\cos(2e + 2fx) - 1 - \sin(2e + 2fx)i)}{\cos(2e + 2fx) + 1}} (A15i + 15B - A \cos(2e + 2fx)25i + A \cos(4e + 4fx)13i + A \cos(6e + 6fx)3i + 15B \cos(2e + 2fx) - 3B \cos(4e + 4fx) - 3B \cos(6e + 6fx) + 25A \sin(2e + 2fx) + 13A \sin(4e + 4fx) + 3A \sin(6e + 6fx) - B \sin(2e + 2fx)15i + B \sin(4e + 4fx)3i + B \sin(6e + 6fx)3i)}{120a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c - c\*tan(e + f\*x)\*1i)^(1/2))/(a + a\*tan(e + f\*x)\*1i)^(5/2),x)

[Out] (((a\*(cos(2\*e + 2\*f\*x) + sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*((c\*(cos(2\*e + 2\*f\*x) - sin(2\*e + 2\*f\*x)\*1i + 1))/(cos(2\*e + 2\*f\*x) + 1))^(1/2)\*(A\*15i + 15\*B + A\*cos(2\*e + 2\*f\*x)\*25i + A\*cos(4\*e + 4\*f\*x)\*13i + A\*cos(6\*e + 6\*f\*x)\*3i + 15\*B\*cos(2\*e + 2\*f\*x) - 3\*B\*cos(4\*e + 4\*f\*x) - 3\*B\*cos(6\*e + 6\*f\*x) + 25\*A\*sin(2\*e + 2\*f\*x) + 13\*A\*sin(4\*e + 4\*f\*x) + 3\*A\*sin(6\*e + 6\*f\*x) - B\*sin(2\*e + 2\*f\*x)\*15i + B\*sin(4\*e + 4\*f\*x)\*3i + B\*sin(6\*e + 6\*f\*x)\*3i))/(120\*a^3\*f)

$$3.848 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{5/2} \sqrt{c-ictan(e+fx)}} dx$$

Optimal. Leaf size=212

$$-\frac{iA+B}{f(a+ia \tan(e+fx))^{5/2} \sqrt{c-ictan(e+fx)}} + \frac{(3iA+2B) \sqrt{c-ictan(e+fx)}}{5cf(a+ia \tan(e+fx))^{5/2}} + \frac{2(3iA+2B) \sqrt{c-ictan(e+fx)}}{15acf(a+ia \tan(e+fx))^{5/2}}$$

[Out]  $2/15*(3*I*A+2*B)*(c-I*c*\tan(f*x+e))^{(1/2)}/a^2/c/f/(a+I*a*\tan(f*x+e))^{(1/2)}+(-I*A-B)/f/(c-I*c*\tan(f*x+e))^{(1/2)}/(a+I*a*\tan(f*x+e))^{(5/2)}+1/5*(3*I*A+2*B)*(c-I*c*\tan(f*x+e))^{(1/2)}/c/f/(a+I*a*\tan(f*x+e))^{(5/2)}+2/15*(3*I*A+2*B)*(c-I*c*\tan(f*x+e))^{(1/2)}/a/c/f/(a+I*a*\tan(f*x+e))^{(3/2)}$

Rubi [A]

time = 0.19, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ , Rules used = {3669, 79, 47, 37}

$$\frac{2(2B+3iA)\sqrt{c-ictan(e+fx)}}{15a^2cf\sqrt{a+ia \tan(e+fx)}} - \frac{B+iA}{f(a+ia \tan(e+fx))^{5/2}\sqrt{c-ictan(e+fx)}} + \frac{2(2B+3iA)\sqrt{c-ictan(e+fx)}}{15acf(a+ia \tan(e+fx))^{3/2}} + \frac{(2B+3iA)\sqrt{c-ictan(e+fx)}}{5cf(a+ia \tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Tan[e + f\*x])/((a + I\*a\*Tan[e + f\*x])^(5/2)\*Sqrt[c - I\*c\*Tan[e + f\*x]]), x]

[Out]  $-((I*A+B)/(f*(a+I*a*\tan(e+f*x))^{5/2}*\sqrt{c-I*c*\tan(e+f*x)})) + (((3*I)*A+2*B)*\sqrt{c-I*c*\tan(e+f*x)})/(5*c*f*(a+I*a*\tan(e+f*x))^{5/2}) + (2*((3*I)*A+2*B)*\sqrt{c-I*c*\tan(e+f*x)})/(15*a*c*f*(a+I*a*\tan(e+f*x))^{3/2}) + (2*((3*I)*A+2*B)*\sqrt{c-I*c*\tan(e+f*x)})/(15*a^2*c*f*\sqrt{a+I*a*\tan(e+f*x)})$

Rule 37

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 47

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*((c + d\*x)^(n + 1)/((b\*c - a\*d)\*(m + 1))), x] - Dist[d\*(Simplify[m + n + 2]/((b\*c - a\*d)\*(m + 1))), Int[(a + b\*x)^(m + 1)\*((c + d\*x)^n, x), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && !LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler



Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]]),x]
```

```
[Out] -1/60*(Sec[e + f*x]*((-30*A + (5*I)*B)*Cos[e + f*x] + (6*A - (9*I)*B)*Cos[3*(e + f*x)] - I*(3*A - (2*I)*B)*(5*Sin[e + f*x] - 3*Sin[3*(e + f*x)]))*Sqrt[c - I*c*Tan[e + f*x]]/(a^2*c*f*(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple [A]**

time = 0.42, size = 186, normalized size = 0.88

method	result
derivativedivides	$-\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))}}{(4iB(\tan^5(fx+e))+12iA(\tan^4(fx+e))-6A)}$
default	$-\frac{\sqrt{-c(i \tan(fx + e) - 1)} \sqrt{a(1 + i \tan(fx + e))}}{(4iB(\tan^5(fx+e))+12iA(\tan^4(fx+e))-6A)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3/c*(4*I*B*tan(f*x+e)^5+12*I*A*tan(f*x+e)^4-6*A*tan(f*x+e)^5+2*I*B*tan(f*x+e)^3+8*B*tan(f*x+e)^4+18*I*A*tan(f*x+e)^2-3*A*tan(f*x+e)^3-2*I*B*tan(f*x+e)+7*B*tan(f*x+e)^2+6*I*A+3*A*tan(f*x+e)-B)/(I-tan(f*x+e))^4/(I+tan(f*x+e))^2
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas [A]**

time = 3.77, size = 170, normalized size = 0.80

$$\frac{(15(iA + B)e^{8ifx+8ie} + 8(6iA - B)e^{7ifx+7ie} - 30iAe^{6ifx+6ie} + 8(6iA - B)e^{5ifx+5ie} + 10(-6iA - B)e^{4ifx+4ie} + 2(-9iA + 4B)e^{2ifx+2ie} - 3iA + 3B)\sqrt{\frac{a}{e^{2ifx+2ie} + 1}}\sqrt{\frac{c}{e^{2ifx+2ie} + 1}}e^{(-5ifx-5ie)}}{120a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out] 
$$-1/120*(15*(I*A + B)*e^{(8*I*f*x + 8*I*e)} + 8*(6*I*A - B)*e^{(7*I*f*x + 7*I*e)} - 30*I*A*e^{(6*I*f*x + 6*I*e)} + 8*(6*I*A - B)*e^{(5*I*f*x + 5*I*e)} + 10*(-6*I*A - B)*e^{(4*I*f*x + 4*I*e)} + 2*(-9*I*A + 4*B)*e^{(2*I*f*x + 2*I*e)} - 3*I*A + 3*B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-5*I*f*x - 5*I*e)}/(a^3*c*f)$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{(ia(\tan(e + fx) - i))^{5/2} \sqrt{-ic(\tan(e + fx) + i)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x)

[Out] Integral((A + B\*tan(e + f\*x))/((I\*a\*(tan(e + f\*x) - I))^(5/2)\*sqrt(-I\*c\*(tan(e + f\*x) + I))), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(c-I\*c\*tan(f\*x+e))^(1/2)/(a+I\*a\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^(5/2)\*sqrt(-I\*c\*tan(f\*x + e) + c)), x)

**Mupad [B]**

time = 10.72, size = 246, normalized size = 1.16

$$\frac{\sqrt{\frac{a(\cos(2e + 2fx) + 1 + \sin(2e + 2fx)i)}{\cos(2e + 2fx) + 1}} (15B \cos(2e + 2fx) - 15B + A \cos(2e + 2fx) 45i + A \cos(4e + 4fx) 15i + A \cos(6e + 6fx) 3i - A 15i - 5B \cos(4e + 4fx) - 3B \cos(6e + 6fx) + 45A \sin(2e + 2fx) + 15A \sin(4e + 4fx) + 3A \sin(6e + 6fx) - B \sin(2e + 2fx) 15i + B \sin(4e + 4fx) 5i + B \sin(6e + 6fx) 3i)}{120a^3 f \sqrt{\frac{c(\cos(2e + 2fx) + 1 - \sin(2e + 2fx)i)}{\cos(2e + 2fx) + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*tan(e + f\*x))/((a + a\*tan(e + f\*x)\*1i)^(5/2)\*(c - c\*tan(e + f\*x)\*1i)^(1/2)),x)

[Out] 
$$\left( \frac{(a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))}{(\cos(2*e + 2*f*x) + 1)} \right)^{(1/2)} * (A*\cos(2*e + 2*f*x)*45i - 15*B - A*15i + A*\cos(4*e + 4*f*x)*15i + A*\cos(6*e + 6*f*x)*3i + 15*B*\cos(2*e + 2*f*x) - 5*B*\cos(4*e + 4*f*x) - 3*B*\cos(6*e + 6*f*x) + 45*A*\sin(2*e + 2*f*x) + 15*A*\sin(4*e + 4*f*x) + 3*A*\sin(6*e + 6*f*x) - B*\sin(2*e + 2*f*x)*15i + B*\sin(4*e + 4*f*x)*5i + B*\sin(6*e + 6*f*x)*3i) / (120*a^3*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1)) / (\cos(2*e + 2*f*x) + 1))^{(1/2)})$$



$$3.849 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{5/2}(c-ic \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=218

$$\frac{-iA - B}{3f(a + ia \tan(e + fx))^{5/2}(c - ic \tan(e + fx))^{3/2}} + \frac{4iA + B}{15cf(a + ia \tan(e + fx))^{5/2}\sqrt{c - ic \tan(e + fx)}} + \frac{15ac}{15ac}$$

[Out]  $2/15*(4*A-I*B)*\tan(f*x+e)/a^2/c/f/(a+I*a*\tan(f*x+e))^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)}+1/15*(4*I*A+B)/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}/(a+I*a*\tan(f*x+e))^{(5/2)}+1/15*(4*I*A+B)/a/c/f/(c-I*c*\tan(f*x+e))^{(1/2)}/(a+I*a*\tan(f*x+e))^{(3/2)}+1/3*(-I*A-B)/f/(a+I*a*\tan(f*x+e))^{(5/2)}/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.20, antiderivative size = 216, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$ ,

Rules used = {3669, 79, 47, 39}

$$\frac{2(4A - iB) \tan(e + fx)}{15a^2cf \sqrt{a + ia \tan(e + fx)} \sqrt{c - ic \tan(e + fx)}} - \frac{B + iA}{3f(a + ia \tan(e + fx))^{5/2}(c - ic \tan(e + fx))^{3/2}} + \frac{B + 4iA}{15acf(a + ia \tan(e + fx))^{5/2}\sqrt{c - ic \tan(e + fx)}} + \frac{B + 4iA}{15cf(a + ia \tan(e + fx))^{5/2}\sqrt{c - ic \tan(e + fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Tan}[e + f*x])/((a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}), x]$

[Out]  $-1/3*(I*A + B)/(f*(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*(c - I*c*\text{Tan}[e + f*x])^{(3/2)}) + ((4*I)*A + B)/(15*c*f*(a + I*a*\text{Tan}[e + f*x])^{(5/2)}*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) + ((4*I)*A + B)/(15*a*c*f*(a + I*a*\text{Tan}[e + f*x])^{(3/2)}*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]]) + (2*(4*A - I*B)*\text{Tan}[e + f*x])/(15*a^2*c*f*\text{Sqrt}[a + I*a*\text{Tan}[e + f*x]]*\text{Sqrt}[c - I*c*\text{Tan}[e + f*x]])$

**Rule 39**

$\text{Int}[1/(((a_) + (b_)*(x_))^{(3/2)}*((c_) + (d_)*(x_))^{(3/2)}), x\_Symbol] := \text{Simp}[x/(a*c*\text{Sqrt}[a + b*x]*\text{Sqrt}[c + d*x]), x] /;$   $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[b*c + a*d, 0]$

**Rule 47**

$\text{Int}(((a_) + (b_)*(x_))^{(m_)}*((c_) + (d_)*(x_))^{(n_)}), x\_Symbol] := \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)})/((b*c - a*d)*(m + 1)), x] - \text{Dist}[d*(\text{Simplify}[m + n + 2]/((b*c - a*d)*(m + 1))), \text{Int}[(a + b*x)^{\text{Simplify}[m + 1]}*(c + d*x)^n, x], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[\text{Simplify}[m + n + 2], 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !(\text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ (\text{SumSimplerQ}[m, 1] \ || \ !\text{SumSimplerQ}[n, 1])$

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^(n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3669

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_.)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^{5/2} (c - ictan(e + fx))^{3/2}} dx = \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{7/2}(c-icx)^{5/2}} dx, x, \tan(e + fx)\right)}{f}$$

$$= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{5/2} (c - ictan(e + fx))^{3/2}} +$$

$$= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{5/2} (c - ictan(e + fx))^{3/2}} +$$

$$= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{5/2} (c - ictan(e + fx))^{3/2}} +$$

$$= -\frac{iA + B}{3f(a + ia \tan(e + fx))^{5/2} (c - ictan(e + fx))^{3/2}} +$$

Mathematica [A]

time = 3.38, size = 133, normalized size = 0.61

$$\frac{i(-45A + 20(A - iB) \cos(2(e + fx)) + (A - 4iB) \cos(4(e + fx)) + 40iA \sin(2(e + fx)) + 10B \sin(2(e + fx)) + 4iA \sin(4(e + fx)) + B \sin(4(e + fx))) \sqrt{c - ictan(e + fx)}}{120a^2c^2f \sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e
+ f*x])^(3/2)), x]
```

```
[Out] ((-1/120*I)*(-45*A + 20*(A - I*B)*Cos[2*(e + f*x)] + (A - (4*I)*B)*Cos[4*(e + f*x)] + (40*I)*A*Sin[2*(e + f*x)] + 10*B*Sin[2*(e + f*x)] + (4*I)*A*Sin[4*(e + f*x)] + B*Sin[4*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(a^2*c^2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple** [A]

time = 0.40, size = 199, normalized size = 0.91

method	result
derivativedivides	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}(8iA(\tan^6(fx+e))-2iB(\tan^5(fx+e))+2B)}{...}$
default	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\sqrt{-c(i\tan(fx+e)-1)}(8iA(\tan^6(fx+e))-2iB(\tan^5(fx+e))+2B)}{...}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c^2*(8*I*A*tan(f*x+e)^6-2*I*B*tan(f*x+e)^5+2*B*tan(f*x+e)^6+20*I*A*tan(f*x+e)^4+8*A*tan(f*x+e)^5-5*I*B*tan(f*x+e)^3+5*B*tan(f*x+e)^4+15*I*A*tan(f*x+e)^2+20*A*tan(f*x+e)^3-3*I*B*tan(f*x+e)+3*I*A+12*A*tan(f*x+e)-3*B)/(I-tan(f*x+e))^4/(I+tan(f*x+e))^3
```

**Maxima** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

**Fricas** [A]

time = 2.15, size = 191, normalized size = 0.88

$$\frac{(5(iA+B)e^{10i(fx+e)}+5(13iA+7B)e^{8i(fx+e)}+48(iA-B)e^{7i(fx+e)}+30(-iA+B)e^{6i(fx+e)}+48(iA-B)e^{5i(fx+e)}+10(-11iA+B)e^{4i(fx+e)}-(23iA-13B)e^{2i(fx+e)}-3iA+3B)\sqrt{\frac{a}{e^{2i(fx+e)}+1}}\sqrt{\frac{c}{e^{2i(fx+e)}+1}}e^{(-5i(fx-5i)e)}}{240a^3c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(5*(I*A + B)*e^(10*I*f*x + 10*I*e) + 5*(13*I*A + 7*B)*e^(8*I*f*x + 8
*I*e) + 48*(I*A - B)*e^(7*I*f*x + 7*I*e) + 30*(-I*A + B)*e^(6*I*f*x + 6*I*e
) + 48*(I*A - B)*e^(5*I*f*x + 5*I*e) + 10*(-11*I*A + B)*e^(4*I*f*x + 4*I*e)
- (23*I*A - 13*B)*e^(2*I*f*x + 2*I*e) - 3*I*A + 3*B)*sqrt(a/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-5*I*f*x - 5*I*e)/(a^3*c^
2*f)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + fx)}{(ia(\tan(e + fx) - i))^{5/2} (-ic(\tan(e + fx) + i))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(3
/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(5/2)*(-I*c*(tan(e
+ f*x) + I))**(3/2)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x
+ e) + c)^(3/2)), x)
```

**Mupad [B]**

time = 10.64, size = 249, normalized size = 1.14

$$\frac{\sqrt{\frac{a(\cos(2e + 2fx) + 1 + \sin(2e + 2fx))}{\cos(2e + 2fx) + 1}} (95A \sin(2e + 2fx) - 30B + A \cos(2e + 2fx) 85i + A \cos(4e + 4fx) 20i + A \cos(6e + 6fx) 3i - 5B \cos(2e + 2fx) - 10B \cos(4e + 4fx) - 3B \cos(6e + 6fx) - A 60i + 20A \sin(4e + 4fx) + 3A \sin(6e + 6fx) - B \sin(2e + 2fx) 5i + B \sin(4e + 4fx) 10i + B \sin(6e + 6fx) 3i)}{240a^3cf \sqrt{\frac{c(\cos(2e + 2fx) + 1 - \sin(2e + 2fx))}{\cos(2e + 2fx) + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(3/2)),x)
```

```
[Out] (((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(
1/2)*(A*cos(2*e + 2*f*x)*85i - 30*B - A*60i + A*cos(4*e + 4*f*x)*20i + A*c
os(6*e + 6*f*x)*3i - 5*B*cos(2*e + 2*f*x) - 10*B*cos(4*e + 4*f*x) - 3*B*cos
(6*e + 6*f*x) + 95*A*sin(2*e + 2*f*x) + 20*A*sin(4*e + 4*f*x) + 3*A*sin(6*e
+ 6*f*x) - B*sin(2*e + 2*f*x)*5i + B*sin(4*e + 4*f*x)*10i + B*sin(6*e + 6*
f*x)*3i))/(240*a^3*c*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(c
os(2*e + 2*f*x) + 1))^(1/2))
```

$$3.850 \quad \int \frac{A+B \tan(e+fx)}{(a+ia \tan(e+fx))^{5/2}(c-ic \tan(e+fx))^{5/2}} dx$$

**Optimal.** Leaf size=206

$$\frac{-iA - B}{5f(a + ia \tan(e + fx))^{5/2}(c - ic \tan(e + fx))^{5/2}} + \frac{iA}{5cf(a + ia \tan(e + fx))^{5/2}(c - ic \tan(e + fx))^{3/2}} + \frac{1}{15ac}$$

[Out]  $8/15*A*\tan(f*x+e)/a^2/c^2/f/(a+I*a*\tan(f*x+e))^{(1/2)}/(c-I*c*\tan(f*x+e))^{(1/2)}+1/5*(-I*A-B)/f/(a+I*a*\tan(f*x+e))^{(5/2)}/(c-I*c*\tan(f*x+e))^{(5/2)}+1/5*I*A/c/f/(a+I*a*\tan(f*x+e))^{(5/2)}/(c-I*c*\tan(f*x+e))^{(3/2)}+4/15*A*\tan(f*x+e)/a/c/f/(a+I*a*\tan(f*x+e))^{(3/2)}/(c-I*c*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.18, antiderivative size = 204, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 45,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3669, 79, 47, 40, 39}

$$\frac{8A \tan(e+fx)}{15a^2c^2f\sqrt{a+ia \tan(e+fx)}\sqrt{c-ic \tan(e+fx)}} - \frac{B+iA}{5f(a+ia \tan(e+fx))^{5/2}(c-ic \tan(e+fx))^{5/2}} + \frac{4A \tan(e+fx)}{15acf(a+ia \tan(e+fx))^{3/2}(c-ic \tan(e+fx))^{3/2}} + \frac{iA}{5cf(a+ia \tan(e+fx))^{5/2}(c-ic \tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2)), x]`

[Out]  $-1/5*(I*A + B)/(f*(a + I*a*\tan[e + f*x])^{(5/2)}*(c - I*c*\tan[e + f*x])^{(5/2)}) + ((I/5)*A)/(c*f*(a + I*a*\tan[e + f*x])^{(5/2)}*(c - I*c*\tan[e + f*x])^{(3/2)}) + (4*A*\tan[e + f*x])/(15*a*c*f*(a + I*a*\tan[e + f*x])^{(3/2)}*(c - I*c*\tan[e + f*x])^{(3/2)}) + (8*A*\tan[e + f*x])/(15*a^2*c^2*f*\sqrt{a + I*a*\tan[e + f*x]})*\sqrt{c - I*c*\tan[e + f*x]}$

**Rule 39**

`Int[1/(((a_) + (b_)*(x_))^(3/2)*((c_) + (d_)*(x_))^(3/2)), x_Symbol] := Simp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]`

**Rule 40**

`Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m + 3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]`

**Rule 47**

`Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S`

```

simplify[m + n + 2]/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
+ d*x)^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])

```

### Rule 79

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])
))

```

### Rule 3669

```

Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_.)*((A_) + (B_.)*tan[(e_) +
(f_.)*(x_)])*((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \tan(e + fx)}{(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}} dx &= \frac{(ac) \text{Subst}\left(\int \frac{A+Bx}{(a+iax)^{7/2}(c-icx)^{7/2}} dx, x, \tan(e + fx)\right)}{f} \\
&= -\frac{iA + B}{5f(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}} + \\
&= -\frac{iA + B}{5f(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}} + \\
&= -\frac{iA + B}{5f(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}} + \\
&= -\frac{iA + B}{5f(a + ia \tan(e + fx))^{5/2} (c - ic \tan(e + fx))^{5/2}} +
\end{aligned}$$

### Mathematica [A]

time = 4.58, size = 151, normalized size = 0.73

$$\frac{\sec^2(e + fx)(\cos(3(e + fx)) + i \sin(3(e + fx)))(30B \cos(e + fx) + 15B \cos(3(e + fx)) + 3B \cos(5(e + fx)) - 150A \sin(e + fx) - 25A \sin(3(e + fx)) - 3A \sin(5(e + fx)))\sqrt{c - ic \tan(e + fx)}}{240a^2c^3f(-i + \tan(e + fx))^2\sqrt{a + ia \tan(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^2*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*(30*B*Cos[e + f*x] + 15*B*Cos[3*(e + f*x)] + 3*B*Cos[5*(e + f*x)] - 150*A*Sin[e + f*x] - 25*A*Sin[3*(e + f*x)] - 3*A*Sin[5*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]/(240*a^2*c^3*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]])
```

**Maple [A]**

time = 0.42, size = 124, normalized size = 0.60

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} (8A(\tan^7(fx+e))+28A(\tan^5(fx+e))+35A(\tan^3(fx+e))-3A)}{15fa^3c^3(i-\tan(fx+e))^4(i+\tan(fx+e))^4}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))} \sqrt{-c(i\tan(fx+e)-1)} (8A(\tan^7(fx+e))+28A(\tan^5(fx+e))+35A(\tan^3(fx+e))-3A)}{15fa^3c^3(i-\tan(fx+e))^4(i+\tan(fx+e))^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c^3*(8*A*tan(f*x+e)^7+28*A*tan(f*x+e)^5+35*A*tan(f*x+e)^3-3*B*tan(f*x+e)^2+15*A*tan(f*x+e)-3*B)/(I-tan(f*x+e))^4/(I+tan(f*x+e))^4
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs.  $2(170) = 340$ .

time = 0.62, size = 354, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/480*((30*(5*I*A - B)*cos(4*f*x + 4*e) + 5*(5*I*A - 3*B)*cos(2*f*x + 2*e) - 30*(5*A + I*B)*sin(4*f*x + 4*e) - 5*(5*A + 3*I*B)*sin(2*f*x + 2*e) - 6*B)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*(-5*I*A - 3*B)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(-5*I*A - B)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (30*(5*A + I*B)*cos(4*f*x + 4*e) + 5*(5*A + 3*I*B)*cos(2*f*x + 2*e) + 30*(5*I*A - B)*sin(4*f*x + 4*e) + 5*(5*I*A - 3*B)*sin(2*f*x + 2*e) + 6*A)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*(5*A - 3*I*B)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
```

$s(2*f*x + 2*e))) + 30*(5*A - I*B)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))/ (a^{(5/2)}*c^{(5/2)}*f)$

**Fricas [A]**

time = 3.59, size = 196, normalized size = 0.95

$$\frac{(3(A+B)e^{12i f x + 12i e} + 2(14i A + 9B)e^{10i f x + 10i e} + 5(35i A + 9B)e^{8i f x + 8i e} - 96B e^{7i f x + 7i e} + 60B e^{6i f x + 6i e} - 96B e^{5i f x + 5i e} + 5(-35i A + 9B)e^{4i f x + 4i e} + 2(-14i A + 9B)e^{2i f x + 2i e} - 3i A + 3B)\sqrt{\frac{a}{c^{2i f x + 2i e} + 1}}\sqrt{\frac{c}{e^{2i f x + 2i e} + 1}}e^{(-5i f x - 5i e)}}{480 a^3 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(5/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="fricas")

[Out]  $-1/480*(3*(I*A + B)*e^{(12*I*f*x + 12*I*e)} + 2*(14*I*A + 9*B)*e^{(10*I*f*x + 10*I*e)} + 5*(35*I*A + 9*B)*e^{(8*I*f*x + 8*I*e)} - 96*B*e^{(7*I*f*x + 7*I*e)} + 60*B*e^{(6*I*f*x + 6*I*e)} - 96*B*e^{(5*I*f*x + 5*I*e)} + 5*(-35*I*A + 9*B)*e^{(4*I*f*x + 4*I*e)} + 2*(-14*I*A + 9*B)*e^{(2*I*f*x + 2*I*e)} - 3*I*A + 3*B)*\sqrt{a/(e^{(2*I*f*x + 2*I*e)} + 1)}*\sqrt{c/(e^{(2*I*f*x + 2*I*e)} + 1)}*e^{(-5*I*f*x - 5*I*e)}/(a^3*c^3*f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan(e + f x)}{(i a (\tan(e + f x) - i))^{5/2} (-i c (\tan(e + f x) + i))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(5/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x)

[Out] Integral((A + B\*tan(e + f\*x))/((I\*a\*(tan(e + f\*x) - I))^(5/2)\*(-I\*c\*(tan(e + f\*x) + I))^(5/2)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(5/2)/(c-I\*c\*tan(f\*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)/((I\*a\*tan(f\*x + e) + a)^(5/2)\*(-I\*c\*tan(f\*x + e) + c)^(5/2)), x)

**Mupad [B]**

time = 10.94, size = 249, normalized size = 1.21

$$\frac{\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx))^{11}}{\cos(2e+2fx)+1}}(175A\sin(2e+2fx)-30B+A\cos(2e+2fx)125+A\cos(4e+4fx)221+A\cos(6e+6fx)38-45B\cos(2e+2fx)-18B\cos(4e+4fx)-3B\cos(6e+6fx)-A150+28A\sin(4e+4fx)+3A\sin(6e+6fx)+B\sin(2e+2fx)15+B\sin(4e+4fx)12+B\sin(6e+6fx)3)}}{480 a^3 c^3 f \sqrt{\frac{c(\cos(2e+2fx)+1-\sin(2e+2fx))^{11}}{\cos(2e+2fx)+1}}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B*\tan(e + f*x))/((a + a*\tan(e + f*x)*1i)^{(5/2)}*(c - c*\tan(e + f*x)*1i)^{(5/2)}),x)$

[Out]  $((a*(\cos(2*e + 2*f*x) + \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{(1/2)}*(A*\cos(2*e + 2*f*x)*125i - 30*B - A*150i + A*\cos(4*e + 4*f*x)*22i + A*\cos(6*e + 6*f*x)*3i - 45*B*\cos(2*e + 2*f*x) - 18*B*\cos(4*e + 4*f*x) - 3*B*\cos(6*e + 6*f*x) + 175*A*\sin(2*e + 2*f*x) + 28*A*\sin(4*e + 4*f*x) + 3*A*\sin(6*e + 6*f*x) + B*\sin(2*e + 2*f*x)*15i + B*\sin(4*e + 4*f*x)*12i + B*\sin(6*e + 6*f*x)*3i)/(480*a^3*c^2*f*((c*(\cos(2*e + 2*f*x) - \sin(2*e + 2*f*x)*1i + 1))/(\cos(2*e + 2*f*x) + 1))^{(1/2)})$

### 3.851 $\int (a+ia \tan(e+fx))^m (A+B \tan(e+fx))(c-ic \tan(e+fx))^n dx$

**Optimal.** Leaf size=150

$$\frac{(iA+B)(a+ia \tan(e+fx))^m (c-ic \tan(e+fx))^n}{2fn} - \frac{2^{-1+n}(B(m-n)+iA(m+n)) {}_2F_1(m, -n; 1+m; \frac{1}{2}(a+ia \tan(e+fx)))}{f m}$$

[Out] 1/2\*(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^m\*(c-I\*c\*tan(f\*x+e))^n/f/n-2^(-1+n)\*(B\*(-n+m)+I\*A\*(n+m))\*hypergeom([m, -n], [1+m], 1/2+1/2\*I\*tan(f\*x+e))\*(a+I\*a\*tan(f\*x+e))^m\*(c-I\*c\*tan(f\*x+e))^n/f/m/n/((1-I\*tan(f\*x+e))^n)

**Rubi [A]**

time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$ , Rules used = {3669, 80, 72, 71}

$$\frac{(B+iA)(a+ia \tan(e+fx))^m (c-ic \tan(e+fx))^n}{2fn} - \frac{2^{-1+n}(B(m-n)+iA(m+n))(1-i \tan(e+fx))^{-n}(a+ia \tan(e+fx))^m (c-ic \tan(e+fx))^n {}_2F_1(m, -n; m+1; \frac{1}{2}(i \tan(e+fx)+1))}{f m}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])^m\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^n, x]

[Out] ((I\*A + B)\*(a + I\*a\*Tan[e + f\*x])^m\*(c - I\*c\*Tan[e + f\*x])^n)/(2\*f\*n) - (2^(-1 + n)\*(B\*(m - n) + I\*A\*(m + n))\*Hypergeometric2F1[m, -n, 1 + m, (1 + I\*Tan[e + f\*x])/2]\*(a + I\*a\*Tan[e + f\*x])^m\*(c - I\*c\*Tan[e + f\*x])^n)/(f\*m\*n\*(1 - I\*Tan[e + f\*x])^n)

Rule 71

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)/(b\*(m + 1)\*(b\*(b\*c - a\*d))^n))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b\*c - a\*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b\*c - a\*d), 0]))

Rule 72

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Dist[(c + d\*x)^FracPart[n]/((b/(b\*c - a\*d))^IntPart[n]\*(b\*((c + d\*x)/(b\*c - a\*d)))^FracPart[n]), Int[(a + b\*x)^m\*Simp[b\*(c/(b\*c - a\*d)) + b\*d\*(x/(b\*c - a\*d)), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

### Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int (a + ia \tan(e + fx))^m (A + B \tan(e + fx)) (c - ic \tan(e + fx))^n dx &= \frac{(ac) \text{Subst}(\int (a + iax)^{-1+m} (A + B \tan(e + fx))^n dx)}{2fn} \\ &= \frac{(iA + B)(a + ia \tan(e + fx))^m (c - ic \tan(e + fx))^n}{2fn} \\ &= \frac{(iA + B)(a + ia \tan(e + fx))^m (c - ic \tan(e + fx))^n}{2fn} \\ &= \frac{(iA + B)(a + ia \tan(e + fx))^m (c - ic \tan(e + fx))^n}{2fn} \end{aligned}$$

### Mathematica [A]

time = 16.70, size = 216, normalized size = 1.44

$$\frac{2^{-1+m+n} (e^{fx})^m \left(\frac{c}{1+e^{2i(fx)}}\right)^n \left(\frac{c+e^{2i(fx)}}{1+e^{2i(fx)}}\right)^m (1+e^{2i(fx)})^{m+n} ((-iA+B)(1+m) {}_2F_1(m, 1+m+n; 1+m; -e^{2i(fx)}) - i(A-iB)e^{2i(fx)} {}_2F_1(1+m, 1+m+n; 2+m; -e^{2i(fx)})) \sec^{-m}(e+fx) (\cos(fx) + i \sin(fx))^{-m} (a+ia \tan(e+fx))^m}{fm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^m*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^n,x]
```

```
[Out] (2^(-1 + m + n)*(E^(I*f*x))^m*(c/(1 + E^((2*I)*(e + f*x))))^n*(E^(I*(e + f*
x))/(1 + E^((2*I)*(e + f*x))))^m*(1 + E^((2*I)*(e + f*x)))^(m + n)*((-I)*A
+ B)*(1 + m)*Hypergeometric2F1[m, 1 + m + n, 1 + m, -E^((2*I)*(e + f*x))]
- I*(A - I*B)*E^((2*I)*(e + f*x))*m*Hypergeometric2F1[1 + m, 1 + m + n, 2 +
m, -E^((2*I)*(e + f*x))]*(a + I*a*Tan[e + f*x])^m)/(f*m*(1 + m)*Sec[e + f
*x]^m*(Cos[f*x] + I*Sin[f*x])^m)
```

**Maple [F]**

time = 0.98, size = 0, normalized size = 0.00

$$\int (a + ia \tan(fx + e))^m (A + B \tan(fx + e)) (c - ic \tan(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^m\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x)

[Out] int((a+I\*a\*tan(f\*x+e))^m\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^m\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^m\*(-I\*c\*tan(f\*x + e) + c)^n, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^m\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x, algorithm="fricas")

[Out] integral(((A - I\*B)\*e^(2\*I\*f\*x + 2\*I\*e) + A + I\*B)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^n\*e^(2\*I\*f\*m\*x + 2\*I\*m\*e + m\*log(a/c) + m\*log(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)))/(e^(2\*I\*f\*x + 2\*I\*e) + 1), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(e + fx) - i))^m (-ic(\tan(e + fx) + i))^n (A + B \tan(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^m\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^n,x)

[Out] Integral((I\*a\*(tan(e + f\*x) - I))^m\*(-I\*c\*(tan(e + f\*x) + I))^n\*(A + B\*tan(e + f\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^m*(-I*c*tan(f*x + e) + c)^n, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \tan(e + f x)) (a + a \tan(e + f x) i)^m (c - c \tan(e + f x) i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^n,x)
```

```
[Out] int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)^n, x)
```

$$3.852 \quad \int (a+ia \tan(e+fx))^{1+m} (A+B \tan(e+fx))(c-ic \tan(e+fx))^{-1-m} dx$$

Optimal. Leaf size=147

$$\frac{(iA+B)(a+ia \tan(e+fx))^{1+m}(c-ic \tan(e+fx))^{-1-m}}{2f(1+m)} + \frac{2^m a B {}_2F_1(-m, -m; 1-m; \frac{1}{2}(1-i \tan(e+fx)))}{2f(m+1)}$$

[Out] -1/2\*(I\*A+B)\*(a+I\*a\*tan(f\*x+e))^(1+m)\*(c-I\*c\*tan(f\*x+e))^(-1-m)/f/(1+m)+2^m\*a\*B\*hypergeom([-m, -m], [1-m], 1/2-1/2\*I\*tan(f\*x+e))\*(a+I\*a\*tan(f\*x+e))^m/c/f/m/((1+I\*tan(f\*x+e))^m)/((c-I\*c\*tan(f\*x+e))^m)

Rubi [A]

time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 47,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.085$ , Rules used = {3669, 80, 72, 71}

$$\frac{aB2^m(1+i \tan(e+fx))^{-m}(a+ia \tan(e+fx))^m(c-ic \tan(e+fx))^{-m} {}_2F_1(-m, -m; 1-m; \frac{1}{2}(1-i \tan(e+fx)))}{cfm} - \frac{(B+iA)(a+ia \tan(e+fx))^{m+1}(c-ic \tan(e+fx))^{-m-1}}{2f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Tan[e + f\*x])^(1 + m)\*(A + B\*Tan[e + f\*x])\*(c - I\*c\*Tan[e + f\*x])^(-1 - m), x]

[Out] -1/2\*((I\*A + B)\*(a + I\*a\*Tan[e + f\*x])^(1 + m)\*(c - I\*c\*Tan[e + f\*x])^(-1 - m))/(f\*(1 + m)) + (2^m\*a\*B\*Hypergeometric2F1[-m, -m, 1 - m, (1 - I\*Tan[e + f\*x])/2]\*(a + I\*a\*Tan[e + f\*x])^m)/(c\*f\*m\*(1 + I\*Tan[e + f\*x])^m\*(c - I\*c\*Tan[e + f\*x])^m)

Rule 71

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)/(b\*(m + 1)\*(b\*(b\*c - a\*d))^n))\*Hypergeometric2F1[-n, m + 1, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b\*c - a\*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b\*c - a\*d), 0]))

Rule 72

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Dist[(c + d\*x)^FracPart[n]/((b/(b\*c - a\*d))^IntPart[n]\*(b\*((c + d\*x)/(b\*c - a\*d)))^FracPart[n]), Int[(a + b\*x)^m\*Simp[b\*(c/(b\*c - a\*d)) + b\*d\*(x/(b\*c - a\*d)), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

### Rule 3669

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Di
st[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x,
Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int (a + ia \tan(e + fx))^{1+m} (A + B \tan(e + fx)) (c - ic \tan(e + fx))^{-1-m} dx &= \frac{(ac) \text{Subst}(\int (a + iax)^m (A + B \tan(e + fx)) dx)}{2f} \\ &= -\frac{(iA + B)(a + ia \tan(e + fx))^{1+m}}{2f} \\ &= -\frac{(iA + B)(a + ia \tan(e + fx))^{1+m}}{2f} \\ &= -\frac{(iA + B)(a + ia \tan(e + fx))^{1+m}}{2f} \end{aligned}$$

### Mathematica [A]

time = 59.47, size = 177, normalized size = 1.20

$$\frac{ae^{i(e+2fx)} (e^{ifx})^m \left(\frac{c}{1+e^{2i(e+fx)}}\right)^{-m} \left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^m (A - iB + 2iB {}_2F_1(1, 1+m; 2+m; -e^{2i(e+fx)})) \sec^{-1-m}(e + fx) (\cos(fx) + i \sin(fx))^{-1-m} (-i + \tan(e + fx))(a + ia \tan(e + fx))^m}{2cf(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^(1 + m)*(A + B*Tan[e + f*x])*(c - I*c*Tan[
e + f*x])^(-1 - m), x]
```

```
[Out] (a*E^(I*(e + 2*f*x))*(E^(I*f*x))^m*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
))))^m*(A - I*B + (2*I)*B*Hypergeometric2F1[1, 1 + m, 2 + m, -E^((2*I)*(e +
f*x))])*Sec[e + f*x]^(-1 - m)*(Cos[f*x] + I*Sin[f*x])^(-1 - m)*(-I + Tan[e
+ f*x])*(a + I*a*Tan[e + f*x])^m/(2*c*(c/(1 + E^((2*I)*(e + f*x))))^m*f*(1
+ m))
```

**Maple [F]**

time = 1.57, size = 0, normalized size = 0.00

$$\int (a + ia \tan (fx + e))^{1+m} (A + B \tan (fx + e)) (c - ic \tan (fx + e))^{-1-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*tan(f\*x+e))^(1+m)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(-1-m),x)

[Out] int((a+I\*a\*tan(f\*x+e))^(1+m)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(-1-m),x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1+m)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(-1-m),x, algorithm="maxima")

[Out] 
$$-(2*(-I*B*a^{(m+1)}*m - I*B*a^{(m+1)})*\cos(2*f*m*x + 2*m*e) + ((A - I*B)*a^{(m+1)}*m^2 - (A - I*B)*a^{(m+1)}*m)*\cos(2*(f*m + 2*f)*x + 2*m*e + 4*e) + ((A + I*B)*a^{(m+1)}*m^2 - (A - I*B)*a^{(m+1)}*m - 2*I*B*a^{(m+1)})*\cos(2*(f*m + f)*x + 2*m*e + 2*e) - 4*(B*a^{(m+1)}*c^{(m+1)}*f*m^3 - B*a^{(m+1)}*c^{(m+1)}*f*m + (B*a^{(m+1)}*c^{(m+1)}*f*m^3 - B*a^{(m+1)}*c^{(m+1)}*f*m)*\cos(2*f*x + 2*e) - (-I*B*a^{(m+1)}*c^{(m+1)}*f*m^3 + I*B*a^{(m+1)}*c^{(m+1)}*f*m)*\sin(2*f*x + 2*e))*integrate(((\cos(4*f*x + 4*e) + 2*\cos(2*f*x + 2*e) + 1)*\cos(2*f*m*x + 2*m*e) + (\sin(4*f*x + 4*e) + 2*\sin(2*f*x + 2*e))*\sin(2*f*m*x + 2*m*e))/((c^{(m+1)}*m - c^{(m+1)})*\cos(4*f*x + 4*e)^2 + 4*(c^{(m+1)}*m - c^{(m+1)})*\cos(2*f*x + 2*e)^2 + (c^{(m+1)}*m - c^{(m+1)})*\sin(4*f*x + 4*e)^2 + 4*(c^{(m+1)}*m - c^{(m+1)})*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 4*(c^{(m+1)}*m - c^{(m+1)})*\sin(2*f*x + 2*e)^2 + c^{(m+1)}*m + 2*(c^{(m+1)}*m + 2*(c^{(m+1)}*m - c^{(m+1)})*\cos(2*f*x + 2*e) - c^{(m+1)})*\cos(4*f*x + 4*e) + 4*(c^{(m+1)}*m - c^{(m+1)})*\cos(2*f*x + 2*e) - c^{(m+1)}), x) + 4*(-I*B*a^{(m+1)}*c^{(m+1)}*f*m^3 + I*B*a^{(m+1)}*c^{(m+1)}*f*m + (-I*B*a^{(m+1)}*c^{(m+1)}*f*m^3 + I*B*a^{(m+1)}*c^{(m+1)}*f*m)*\cos(2*f*x + 2*e) + (B*a^{(m+1)}*c^{(m+1)}*f*m^3 - B*a^{(m+1)}*c^{(m+1)}*f*m)*\sin(2*f*x + 2*e))*integrate(-((\sin(4*f*x + 4*e) + 2*\sin(2*f*x + 2*e))*\cos(2*f*m*x + 2*m*e) - (\cos(4*f*x + 4*e) + 2*\cos(2*f*x + 2*e) + 1)*\sin(2*f*m*x + 2*m*e))/((c^{(m+1)}*m - c^{(m+1)})*\cos(4*f*x + 4*e)^2 + 4*(c^{(m+1)}*m - c^{(m+1)})*\cos(2*f*x + 2*e)^2 + (c^{(m+1)}*m - c^{(m+1)})*\sin(4*f*x + 4*e)^2 + 4*(c^{(m+1)}*m - c^{(m+1)})*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 4*(c^{(m+1)}*m - c^{(m+1)})*\sin(2*f*x + 2*e)^2 + c^{(m+1)}*m + 2*(c^{(m+1)}*m + 2*(c^{(m+1)}*m - c^{(m+1)})*\cos(2*f*x + 2*e) - c^{(m+1)})*\cos(4*f*x + 4*e) + 4*(c^{(m+1)}*m - c^{(m+1)})*\cos(2*f*x + 2*e) - c^{(m+1)}), x) + 2*(B*a^{(m+1)}*m + B*a^{(m+1)})*\sin(2*f*m$$



$$*x + 2*m*e) - ((-I*A - B)*a^{(m+1)*m^2} + (I*A + B)*a^{(m+1)*m})*\sin(2*(f*m + 2*f)*x + 2*m*e + 4*e) - ((-I*A + B)*a^{(m+1)*m^2} + (I*A + B)*a^{(m+1)*m} - 2*B*a^{(m+1)})*\sin(2*(f*m + f)*x + 2*m*e + 2*e))/(-2*I*c^{(m+1)*f*m^3} + 2*I*c^{(m+1)*f*m} - 2*(I*c^{(m+1)*f*m^3} - I*c^{(m+1)*f*m})*\cos(2*f*x + 2*e) + 2*(c^{(m+1)*f*m^3} - c^{(m+1)*f*m})*\sin(2*f*x + 2*e))$$

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1+m)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(-1-m),x, algorithm="fricas")

[Out] integral(((A - I\*B)\*e^(2\*I\*f\*x + 2\*I\*e) + A + I\*B)\*(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1))^(-m - 1)\*e^(-2\*(-I\*f\*m - I\*f)\*x - 2\*(-I\*m - I)\*e + (m + 1)\*log(a/c) + (m + 1)\*log(2\*c/(e^(2\*I\*f\*x + 2\*I\*e) + 1)))/(e^(2\*I\*f\*x + 2\*I\*e) + 1), x)

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))\*\*(1+m)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))\*\*(-1-m),x)

[Out] Exception raised: TypeError >> Invalid NaN comparison

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*tan(f\*x+e))^(1+m)\*(A+B\*tan(f\*x+e))\*(c-I\*c\*tan(f\*x+e))^(-1-m),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(I\*a\*tan(f\*x + e) + a)^(m + 1)\*(-I\*c\*tan(f\*x + e) + c)^(-m - 1), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \tan(e + f x)) (a + a \tan(e + f x) \operatorname{li})^{m+1}}{(c - c \tan(e + f x) \operatorname{li})^{m+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(m + 1))/(c - c*tan(e + f*x)*1i)^(m + 1), x)
```

```
[Out] int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(m + 1))/(c - c*tan(e + f*x)*1i)^(m + 1), x)
```

$$3.853 \quad \int \frac{(c - ic \tan(e + fx))^n (-i(2+n) + (-2+n) \tan(e + fx))}{(-i + \tan(e + fx))^2} dx$$

Optimal. Leaf size=33

$$\frac{(c - ic \tan(e + fx))^n}{f(i - \tan(e + fx))^2}$$

[Out] (c-I\*c\*tan(f\*x+e))^n/f/(I-tan(f\*x+e))^2

Rubi [A]

time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {3669, 75}

$$\frac{(c - ic \tan(e + fx))^n}{f(-\tan(e + fx) + i)^2}$$

Antiderivative was successfully verified.

[In] Int[((c - I\*c\*Tan[e + f\*x])^n\*((-I)\*(2 + n) + (-2 + n)\*Tan[e + f\*x]))/(-I + Tan[e + f\*x])^2,x]

[Out] (c - I\*c\*Tan[e + f\*x])^n/(f\*(I - Tan[e + f\*x])^2)

Rule 75

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[b\*(c + d\*x)^(n + 1)\*((e + f\*x)^(p + 1)/(d\*f\*(n + p + 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

Rule 3669

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[a\*(c/f), Subst[Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(A + B\*x), x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(c - ic \tan(e + fx))^n (-i(2+n) + (-2+n) \tan(e + fx))}{(-i + \tan(e + fx))^2} dx = -\frac{(ic) \text{Subst}\left(\int \frac{(c-icx)^{-1+n}(-i(2+n)+(-2+n)x}{(-i+x)^3}\right)}{f} = \frac{(c - ic \tan(e + fx))^n}{f(i - \tan(e + fx))^2}$$

**Mathematica [A]**

time = 1.23, size = 56, normalized size = 1.70

$$\frac{e^{n(-\log(c \sec(e+fx)) + \log(c - i c \tan(e+fx)))} (c \sec(e+fx))^n}{f(-i + \tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c - I*c*Tan[e + f*x])^n*((-I)*(2 + n) + (-2 + n)*Tan[e + f*x]))
/(-I + Tan[e + f*x])^2,x]
```

```
[Out] (E^(n*(-Log[c*Sec[e + f*x]] + Log[c - I*c*Tan[e + f*x]]))*(c*Sec[e + f*x])^
n)/(f*(-I + Tan[e + f*x])^2)
```

**Maple [F]**

time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(c - i c \tan(fx + e))^n (-i(2 + n) + (-2 + n) \tan(fx + e))}{(-i + \tan(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))^2,x)
```

```
[Out] int((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))^2,x)
```

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**Fricas [A]**

time = 6.66, size = 58, normalized size = 1.76

$$-\frac{\left(\frac{2c}{e^{(2i fx + 2i e)} + 1}\right)^n (e^{(4i fx + 4i e)} + 2e^{(2i fx + 2i e)} + 1)e^{(-4i fx - 4i e)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))
^2,x, algorithm="fricas")
```

[Out]  $-1/4*(2*c/(e^{(2*I*f*x + 2*I*e)} + 1))^n*(e^{(4*I*f*x + 4*I*e)} + 2*e^{(2*I*f*x + 2*I*e)} + 1)*e^{(-4*I*f*x - 4*I*e)}/f$

**Sympy [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs.  $2(24) = 48$ .

time = 0.69, size = 66, normalized size = 2.00

$$\begin{cases} \frac{(-ic \tan(e+fx)+c)^n}{f \tan^2(e+fx)-2if \tan(e+fx)-f} & \text{for } f \neq 0 \\ \frac{x((n-2) \tan(e)-i(n+2))(-ic \tan(e)+c)^n}{(\tan(e)-i)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))**n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))**2,x)`

[Out] `Piecewise(((c-I*c*tan(e+f*x)+c)**n/(f*tan(e+f*x)**2-2*I*f*tan(e+f*x)-f), Ne(f, 0)), (x*((n-2)*tan(e)-I*(n+2))*(-I*c*tan(e)+c)**n/(tan(e)-I)**2, True))`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))^2,x, algorithm="giac")`

[Out] `integrate(((n-2)*tan(f*x+e)-I*n-2*I)*(-I*c*tan(f*x+e)+c)^n/(tan(f*x+e)-I)^2,x)`

**Mupad [B]**

time = 9.38, size = 90, normalized size = 2.73

$$\frac{\left(-\frac{c(-2\cos(e+fx)^2+\sin(2e+2fx)1i)}{2\cos(e+fx)^2}\right)^n (-4\cos(e+fx)^2-2\cos(2e+2fx)^2+\sin(2e+2fx)2i+\sin(4e+4fx)1i+2)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-((c-c*tan(e+f*x)*1i)^n*(n*1i-tan(e+f*x)*(n-2)+2i))/(tan(e+f*x)-1i)^2,x)`

[Out] `((-(c*(sin(2*e+2*f*x)*1i-2*cos(e+f*x)^2))/(2*cos(e+f*x)^2))^n*(sin(2*e+2*f*x)*2i+sin(4*e+4*f*x)*1i-2*cos(2*e+2*f*x)^2-4*cos(e+f*x)^2+2))/(4*f)`

$$3.854 \quad \int \frac{(A+B \tan(e+fx))(c+d \tan(e+fx))}{(a+ia \tan(e+fx))^2} dx$$

Optimal. Leaf size=104

$$\frac{(A-iB)(c-id)x}{4a^2} + \frac{B(c+3id)+A(ic+d)}{4a^2 f(1+i \tan(e+fx))} + \frac{(iA-B)(c+id)}{4f(a+ia \tan(e+fx))^2}$$

[Out] 1/4\*(A-I\*B)\*(c-I\*d)\*x/a^2+1/4\*(B\*(c+3\*I\*d)+A\*(I\*c+d))/a^2/f/(1+I\*tan(f\*x+e))+1/4\*(I\*A-B)\*(c+I\*d)/f/(a+I\*a\*tan(f\*x+e))^2

Rubi [A]

time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {3671, 3607, 8}

$$\frac{A(d+ic)+B(c+3id)}{4a^2 f(1+i \tan(e+fx))} + \frac{x(A-iB)(c-id)}{4a^2} + \frac{(-B+iA)(c+id)}{4f(a+ia \tan(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*Tan[e + f\*x])\*(c + d\*Tan[e + f\*x]))/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] ((A - I\*B)\*(c - I\*d)\*x)/(4\*a^2) + (B\*(c + (3\*I)\*d) + A\*(I\*c + d))/(4\*a^2\*f\*(1 + I\*Tan[e + f\*x])) + ((I\*A - B)\*(c + I\*d))/(4\*f\*(a + I\*a\*Tan[e + f\*x])^2)

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3607

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-b\*c - a\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a\*f\*m)), x] + Dist[(b\*c + a\*d)/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3671

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-A\*b - a\*B)\*((a\*c + b\*d)\*((a + b\*Tan[e + f\*x])^m/(2\*a^2\*f\*m)), x] + Dist[1/(2\*a\*b), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[A\*b\*c + a\*B\*c + a\*A\*d + b\*B\*d + 2\*a\*B\*d\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]

## Rubi steps

$$\int \frac{(A + B \tan(e + fx))(c + d \tan(e + fx))}{(a + ia \tan(e + fx))^2} dx = \frac{(iA - B)(c + id)}{4f(a + ia \tan(e + fx))^2} - \frac{i \int \frac{a(B(c+id)+A(ic+d))+2aBd \tan(e+fx)}{a+ia \tan(e+fx)} dx}{2a^2}$$

$$= \frac{B(c + 3id) + A(ic + d)}{4a^2 f(1 + i \tan(e + fx))} + \frac{(iA - B)(c + id)}{4f(a + ia \tan(e + fx))^2} + \frac{(A - iB)(c - id)x}{4a^2} + \frac{B(c + 3id) + A(ic + d)}{4a^2 f(1 + i \tan(e + fx))} + \frac{(iA - B)(c + id)}{4f(a + ia \tan(e + fx))^2}$$

## Mathematica [A]

time = 1.14, size = 201, normalized size = 1.93

$$\frac{(4i(Ac + Bd) + (A(d(-1 - 4ifx) + c(i + 4fx)) - B(c + 4icfx + d(i + 4fx))) \cos(2(e + fx)) + (B(ic - d + 4cfx - 4idf) + A(e + id + 4icfx + 4df)) \sin(2(e + fx)))(A + B \tan(e + fx))(c + d \tan(e + fx))}{16a^2 f(A \cos(e + fx) + B \sin(e + fx))(c \cos(e + fx) + d \sin(e + fx))(-i + \tan(e + fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*Tan[e + f\*x])\*(c + d\*Tan[e + f\*x]))/(a + I\*a\*Tan[e + f\*x])^2,x]

[Out] -1/16\*(((4\*I)\*(A\*c + B\*d) + (A\*(d\*(-1 - (4\*I)\*f\*x) + c\*(I + 4\*f\*x)) - B\*(c + (4\*I)\*c\*f\*x + d\*(I + 4\*f\*x)))\*Cos[2\*(e + f\*x)] + (B\*(I\*c - d + 4\*c\*f\*x - (4\*I)\*d\*f\*x) + A\*(c + I\*d + (4\*I)\*c\*f\*x + 4\*d\*f\*x))\*Sin[2\*(e + f\*x)]\*(A + B\*Tan[e + f\*x])\*(c + d\*Tan[e + f\*x]))/(a^2\*f\*(A\*Cos[e + f\*x] + B\*Sin[e + f\*x])\*(c\*Cos[e + f\*x] + d\*Sin[e + f\*x])\*(-I + Tan[e + f\*x])^2)

## Maple [A]

time = 0.24, size = 134, normalized size = 1.29

method	result
derivativedivides	$\frac{(-\frac{1}{8}iAc + \frac{1}{8}iBd - \frac{1}{8}Ad - \frac{1}{8}Bc) \ln(-i + \tan(fx+e)) - \frac{\frac{1}{4}iAd + \frac{1}{4}iBc - \frac{1}{4}Ac - \frac{3}{4}Bd}{-i + \tan(fx+e)} - \frac{-\frac{1}{2}Ad - \frac{1}{2}Bc + \frac{1}{2}iAc - \frac{1}{2}iBd}{2(-i + \tan(fx+e))^2} - \frac{i(iAd + iBc - Ac - Bd)}{f a^2}}{f a^2}$
default	$\frac{(-\frac{1}{8}iAc + \frac{1}{8}iBd - \frac{1}{8}Ad - \frac{1}{8}Bc) \ln(-i + \tan(fx+e)) - \frac{\frac{1}{4}iAd + \frac{1}{4}iBc - \frac{1}{4}Ac - \frac{3}{4}Bd}{-i + \tan(fx+e)} - \frac{-\frac{1}{2}Ad - \frac{1}{2}Bc + \frac{1}{2}iAc - \frac{1}{2}iBd}{2(-i + \tan(fx+e))^2} - \frac{i(iAd + iBc - Ac - Bd)}{f a^2}}{f a^2}$
risch	$-\frac{ixAd}{4a^2} - \frac{ixBc}{4a^2} + \frac{xAc}{4a^2} - \frac{x Bd}{4a^2} + \frac{ie^{-2i(fx+e)}Ac}{4fa^2} + \frac{ie^{-2i(fx+e)}Bd}{4fa^2} - \frac{e^{-4i(fx+e)}Ad}{16fa^2} - \frac{e^{-4i(fx+e)}Bc}{16fa^2} + \frac{ie^{-4i(fx+e)}(A+B \tan(fx+e))(c+d \tan(fx+e))}{16fa^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x,method=\_RETURNVERBOSE)

[Out] 1/f/a^2\*((-1/8\*I\*A\*c+1/8\*I\*B\*d-1/8\*A\*d-1/8\*B\*c)\*ln(-I+tan(f\*x+e))-(1/4\*I\*A\*d+1/4\*I\*B\*c-1/4\*A\*c-3/4\*B\*d)/(-I+tan(f\*x+e))-1/2\*(-1/2\*A\*d-1/2\*B\*c+1/2\*I\*A\*

$c - 1/2 * I * B * d) / (-I + \tan(f * x + e)) ^ 2 - 1/8 * I * (-A * c + B * d + I * A * d + I * B * c) * \ln(I + \tan(f * x + e))$   
 ))

**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**Fricas [A]**

time = 7.25, size = 87, normalized size = 0.84

$$\frac{(4((A - iB)c - (iA + B)d)fxe^{(4i fx + 4ie)} + (iA - B)c - (A + iB)d - 4(-iAc - iBd)e^{(2i fx + 2ie)})e^{(-4i fx - 4ie)}}{16a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="fricas")

[Out]  $1/16 * (4 * ((A - I * B) * c - (I * A + B) * d) * f * x * e^{(4 * I * f * x + 4 * I * e)} + (I * A - B) * c - (A + I * B) * d - 4 * (-I * A * c - I * B * d) * e^{(2 * I * f * x + 2 * I * e)}) * e^{(-4 * I * f * x - 4 * I * e)} / (a^{2 * f})$

**Sympy [A]**

time = 0.28, size = 296, normalized size = 2.85

$$\begin{cases} \left( \frac{(16iAa^2cfe^{4ie} + 16iBa^2dfe^{4ie})e^{-2ifx} + (4iAa^2cfe^{2ie} - 4Aa^2dfe^{2ie} - 4Ba^2cfe^{2ie} - 4iBa^2dfe^{2ie})e^{-4ifx}}{64a^4f^2} \right) e^{-6ie} & \text{for } a^4 f^2 e^{6ie} \neq 0 \\ x \left( -\frac{Ac - iAd - iBc - Bd}{4a^2} + \frac{(Ace^{4ie} + 2Ace^{2ie} + Ac - iAde^{4ie} + iAd - iBce^{4ie} + iBc - Bde^{4ie} + 2Bde^{2ie} - Bd)e^{-4ie}}{4a^2} \right) & \text{otherwise} \end{cases} + \frac{x(Ac - iAd - iBc - Bd)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x)

[Out] Piecewise((((16\*I\*A\*a\*\*2\*c\*f\*exp(4\*I\*e) + 16\*I\*B\*a\*\*2\*d\*f\*exp(4\*I\*e))\*exp(-2\*I\*f\*x) + (4\*I\*A\*a\*\*2\*c\*f\*exp(2\*I\*e) - 4\*A\*a\*\*2\*d\*f\*exp(2\*I\*e) - 4\*B\*a\*\*2\*c\*f\*exp(2\*I\*e) - 4\*I\*B\*a\*\*2\*d\*f\*exp(2\*I\*e))\*exp(-4\*I\*f\*x))\*exp(-6\*I\*e)/(64\*a\*\*4\*f\*\*2), Ne(a\*\*4\*f\*\*2\*exp(6\*I\*e), 0)), (x\*(-(A\*c - I\*A\*d - I\*B\*c - B\*d)/(4\*a\*\*2) + (A\*c\*exp(4\*I\*e) + 2\*A\*c\*exp(2\*I\*e) + A\*c - I\*A\*d\*exp(4\*I\*e) + I\*A\*d - I\*B\*c\*exp(4\*I\*e) + I\*B\*c - B\*d\*exp(4\*I\*e) + 2\*B\*d\*exp(2\*I\*e) - B\*d)\*exp(-4\*I\*e)/(4\*a\*\*2)), True)) + x\*(A\*c - I\*A\*d - I\*B\*c - B\*d)/(4\*a\*\*2)



**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(82) = 164.  
time = 0.58, size = 200, normalized size = 1.92

$$\frac{2(-iAc - Bc - Ad + iBd)\log(-i\tan(fx+e)+1) + 2(iAc + Bc + Ad - iBd)\log(-i\tan(fx+e)-1) + \frac{-3iAc\tan(fx+e)^2 - 3Bc\tan(fx+e)^2 - 3Ad\tan(fx+e)^2 + 3iBd\tan(fx+e)^2 - 10Ac\tan(fx+e) + 10iBc\tan(fx+e) + 10iAd\tan(fx+e) - 6Bd\tan(fx+e) + 11iAc + 3Bc + 3Ad + 5iBd}{a^2(\tan(fx+e)-i)^2}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/16*(2*(-I*A*c - B*c - A*d + I*B*d)*\log(-I*\tan(f*x + e) + 1)/a^2 + 2*(I*A*c + B*c + A*d - I*B*d)*\log(-I*\tan(f*x + e) - 1)/a^2 + (-3*I*A*c*\tan(f*x + e)^2 - 3*B*c*\tan(f*x + e)^2 - 3*A*d*\tan(f*x + e)^2 + 3*I*B*d*\tan(f*x + e)^2 \\ & - 10*A*c*\tan(f*x + e) + 10*I*B*c*\tan(f*x + e) + 10*I*A*d*\tan(f*x + e) - 6*B*d*\tan(f*x + e) + 11*I*A*c + 3*B*c + 3*A*d + 5*I*B*d)/(a^2*(\tan(f*x + e) - I)^2)/f \end{aligned}$$

**Mupad [B]**

time = 9.50, size = 159, normalized size = 1.53

$$\frac{\frac{Bdfx - Acfx + Adfxli + Bcfxli}{4a^2f} + \frac{(Ac + 3Bd - Adli - Bcli)\tan(e + fx)^3 + (2Ad + 2Bc + Bd4i)\tan(e + fx)^2 + (3Ac + Bd + Adli + Bcli)\tan(e + fx) + Ac2i + Bd2i}{f(4a^2\tan(e + fx)^4 + 8a^2\tan(e + fx)^2 + 4a^2)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B\*tan(e + f\*x))\*(c + d\*tan(e + f\*x)))/(a + a\*tan(e + f\*x)\*1i)^2,x)

[Out] 
$$\begin{aligned} & (A*c*2i + B*d*2i + \tan(e + f*x)*(3*A*c + A*d*1i + B*c*1i + B*d) + \tan(e + f*x)^2*(2*A*d + 2*B*c + B*d*4i) + \tan(e + f*x)^3*(A*c - A*d*1i - B*c*1i + 3*B*d))/(f*(4*a^2 + 8*a^2*\tan(e + f*x)^2 + 4*a^2*\tan(e + f*x)^4)) - (A*d*f*x*1i - A*c*f*x + B*c*f*x*1i + B*d*f*x)/(4*a^2*f) \end{aligned}$$

$$3.855 \quad \int \frac{(A+B \tan(e+fx))(c+d \tan(e+fx))}{(a+ia \tan(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=147

$$-\frac{(iA+B)(c-id) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{(iA-B)(c+id)}{3f(a+ia \tan(e+fx))^{3/2}} + \frac{B(c+3id)+A(ic+d)}{2af\sqrt{a+ia \tan(e+fx)}}$$

[Out]  $-1/4*(I*A+B)*(c-I*d)*\operatorname{arctanh}(1/2*(a+I*a*\tan(f*x+e))^{(1/2)*2^{(1/2)}/a^{(1/2)}}/a^{(3/2)}/f*2^{(1/2)}+1/2*(B*(c+3*I*d)+A*(I*c+d))/a/f/(a+I*a*\tan(f*x+e))^{(1/2)}+1/3*(I*A-B)*(c+I*d)/f/(a+I*a*\tan(f*x+e))^{(3/2)}$

**Rubi [A]**

time = 0.22, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3671, 3607, 3561, 212}

$$-\frac{(B+iA)(c-id) \tanh^{-1}\left(\frac{\sqrt{a+ia \tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{(-B+iA)(c+id)}{3f(a+ia \tan(e+fx))^{3/2}} + \frac{A(d+ic)+B(c+3id)}{2af\sqrt{a+ia \tan(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(A+B*\operatorname{Tan}[e+f*x])*(c+d*\operatorname{Tan}[e+f*x])/(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)},x]$

[Out]  $-1/2*((I*A+B)*(c-I*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])])/(\operatorname{Sqrt}[2]*a^{(3/2)}*f)+((I*A-B)*(c+I*d))/(3*f*(a+I*a*\operatorname{Tan}[e+f*x])^{(3/2)})+(B*(c+(3*I)*d)+A*(I*c+d))/(2*a*f*\operatorname{Sqrt}[a+I*a*\operatorname{Tan}[e+f*x]])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3561

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\tan[(c_+ + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/d), \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, \operatorname{Sqrt}[a + b*\operatorname{Tan}[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

Rule 3607

$\operatorname{Int}[(a_+ + (b_+)*\tan[(e_+ + (f_+)*(x_+)])^{(m_+)}*((c_+ + (d_+)*\tan[(e_+ + (f_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Simp}[(-b*c - a*d)*((a + b*\operatorname{Tan}[e + f*x])^m/(2*a$

```
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[m, 0]
```

### Rule 3671

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]) , x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Dist[1/(
2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*d
+ 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(A + B \tan(e + fx))(c + d \tan(e + fx))}{(a + ia \tan(e + fx))^{3/2}} dx &= \frac{(iA - B)(c + id)}{3f(a + ia \tan(e + fx))^{3/2}} - \frac{i \int \frac{a(B(c+id)+A(ic+d))+2aBd \tan(e+fx)}{\sqrt{a + ia \tan(e + fx)}}}{2a^2} \\ &= \frac{(iA - B)(c + id)}{3f(a + ia \tan(e + fx))^{3/2}} + \frac{B(c + 3id) + A(ic + d)}{2af \sqrt{a + ia \tan(e + fx)}} + \\ &= \frac{(iA - B)(c + id)}{3f(a + ia \tan(e + fx))^{3/2}} + \frac{B(c + 3id) + A(ic + d)}{2af \sqrt{a + ia \tan(e + fx)}} - \\ &= -\frac{(iA + B)(c - id) \tanh^{-1} \left( \frac{\sqrt{a + ia \tan(e + fx)}}{\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} a^{3/2} f} + \frac{3}{3} \end{aligned}$$

### Mathematica [A]

time = 3.25, size = 206, normalized size = 1.40

$$\frac{(-i(A - iB)(c - id)e^{(e+fx)} \sqrt{1 + e^{2(e+fx)}} \sinh^{-1}(e^{(e+fx)}) + \frac{2}{3} \cos(e + fx)((B(c + 7id) + A(5ic + d)) \cos(e + fx) - 3(Ac - iBc - iAd + 3Bd) \sin(e + fx))) (A + B \tan(e + fx))(c + d \tan(e + fx))}{4f(A \cos(e + fx) + B \sin(e + fx))(c \cos(e + fx) + d \sin(e + fx))(a + ia \tan(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Tan[e + f*x])*(c + d*Tan[e + f*x]))/(a + I*a*Tan[e + f*x]
)^(3/2), x]
```

```
[Out] (((-I)*(A - I*B)*(c - I*d)*E^(I*(e + f*x))*Sqrt[1 + E^((2*I)*(e + f*x))]*Ar
cSinh[E^(I*(e + f*x))] + (2*Cos[e + f*x]*((B*(c + (7*I)*d) + A*((5*I)*c + d
))*Cos[e + f*x] - 3*(A*c - I*B*c - I*A*d + 3*B*d)*Sin[e + f*x]))/3)*(A + B*
Tan[e + f*x])*(c + d*Tan[e + f*x]))/(4*f*(A*Cos[e + f*x] + B*Sin[e + f*x])*
(c*Cos[e + f*x] + d*Sin[e + f*x])*(a + I*a*Tan[e + f*x])^(3/2))
```

**Maple [A]**

time = 0.26, size = 131, normalized size = 0.89

method	result
derivativedivides	$2i \frac{\left( \frac{1}{4}iAd + \frac{1}{4}iBc - \frac{1}{4}Ac + \frac{1}{4}Bd \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{a + ia \tan(fx + e)} \sqrt{2}}{2\sqrt{a}} \right)}{2\sqrt{a}} - \frac{-\frac{1}{4}iAd - \frac{1}{4}iBc + \frac{1}{4}Ac + \frac{3}{4}Bd}{\sqrt{a + ia \tan(fx + e)}}$
default	$2i \frac{\left( \frac{1}{4}iAd + \frac{1}{4}iBc - \frac{1}{4}Ac + \frac{1}{4}Bd \right) \sqrt{2} \operatorname{arctanh} \left( \frac{\sqrt{a + ia \tan(fx + e)} \sqrt{2}}{2\sqrt{a}} \right)}{2\sqrt{a}} - \frac{-\frac{1}{4}iAd - \frac{1}{4}iBc + \frac{1}{4}Ac + \frac{3}{4}Bd}{\sqrt{a + ia \tan(fx + e)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x,method=_RE  
TURNVERBOSE)`

[Out]  $-2*I/f/a*(-1/2*(1/4*I*A*d+1/4*I*B*c-1/4*A*c+1/4*B*d)*2^{(1/2)}/a^{(1/2)}*\operatorname{arctan}$   
 $h(1/2*(a+I*a*\tan(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})-(-1/4*I*A*d-1/4*I*B*c+1/4*A$   
 $*c+3/4*B*d)/(a+I*a*\tan(f*x+e))^{(1/2)}-1/6*a*(-B*d+I*A*d+I*B*c+A*c)/(a+I*a*\tan$   
 $(f*x+e))^{(3/2)}$

**Maxima [A]**

time = 0.52, size = 146, normalized size = 0.99

$$i \left( \frac{3\sqrt{2}^{(A-iB)(c-id)} \log \left( \frac{-\sqrt{2}\sqrt{a}-\sqrt{ia \tan(fx+e)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia \tan(fx+e)+a}} \right)}{\sqrt{a}} + \frac{4(2(A+iB)ac+2(iA-B)ad+3((A-iB)c+(-iA+3B)d)(ia \tan(fx+e)+a))}{(ia \tan(fx+e)+a)^{\frac{3}{2}}} \right)$$

24af

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x, alg  
orithm="maxima")`

[Out]  $1/24*I*(3*\sqrt{2}*(A - I*B)*(c - I*d)*\log(-(\sqrt{2}*\sqrt{a} - \sqrt{I*a*\tan}$   
 $(f*x + e) + a))/(\sqrt{2}*\sqrt{a} + \sqrt{I*a*\tan(f*x + e) + a}))/\sqrt{a} + 4*$   
 $(2*(A + I*B)*a*c + 2*(I*A - B)*a*d + 3*((A - I*B)*c + (-I*A + 3*B)*d)*(I*a*$   
 $\tan(f*x + e) + a))/(I*a*\tan(f*x + e) + a)^{(3/2)}/(a*f)$

**Fricas [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 633 vs.  $2(111) = 222$ .

time = 6.66, size = 633, normalized size = 4.31



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} \cdot (3 \sqrt{\frac{1}{2}} \cdot a^2 \cdot f \cdot \sqrt{-((A^2 - 2IA^*B - B^2) \cdot c^2 + 2 \cdot (-IA^2 - 2A^*B + IB^2) \cdot c \cdot d - (A^2 - 2IA^*B - B^2) \cdot d^2) / (a^3 \cdot f^2)}) \cdot e^{(3I \cdot f \cdot x + 3I \cdot e)} \cdot \log(4 \cdot (\sqrt{2} \cdot \sqrt{\frac{1}{2}} \cdot (I \cdot a^2 \cdot f \cdot e^{(2I \cdot f \cdot x + 2I \cdot e)} + I \cdot a^2 \cdot f) \cdot \sqrt{a / (e^{(2I \cdot f \cdot x + 2I \cdot e)} + 1)}) \cdot \sqrt{-((A^2 - 2IA^*B - B^2) \cdot c^2 + 2 \cdot (-IA^2 - 2A^*B + IB^2) \cdot c \cdot d - (A^2 - 2IA^*B - B^2) \cdot d^2) / (a^3 \cdot f^2)}) + ((A - I \cdot B) \cdot a \cdot c + (-I \cdot A - B) \cdot a \cdot d) \cdot e^{(I \cdot f \cdot x + I \cdot e)} \cdot e^{(-I \cdot f \cdot x - I \cdot e)} / ((A - I \cdot B) \cdot c - (I \cdot A + B) \cdot d)) - 3 \sqrt{\frac{1}{2}} \cdot a^2 \cdot f \cdot \sqrt{-((A^2 - 2IA^*B - B^2) \cdot c^2 + 2 \cdot (-IA^2 - 2A^*B + IB^2) \cdot c \cdot d - (A^2 - 2IA^*B - B^2) \cdot d^2) / (a^3 \cdot f^2)}) \cdot e^{(3I \cdot f \cdot x + 3I \cdot e)} \cdot \log(4 \cdot (\sqrt{2} \cdot \sqrt{\frac{1}{2}} \cdot (-I \cdot a^2 \cdot f \cdot e^{(2I \cdot f \cdot x + 2I \cdot e)} - I \cdot a^2 \cdot f) \cdot \sqrt{a / (e^{(2I \cdot f \cdot x + 2I \cdot e)} + 1)}) \cdot \sqrt{-((A^2 - 2IA^*B - B^2) \cdot c^2 + 2 \cdot (-IA^2 - 2A^*B + IB^2) \cdot c \cdot d - (A^2 - 2IA^*B - B^2) \cdot d^2) / (a^3 \cdot f^2)}) + ((A - I \cdot B) \cdot a \cdot c + (-I \cdot A - B) \cdot a \cdot d) \cdot e^{(I \cdot f \cdot x + I \cdot e)} \cdot e^{(-I \cdot f \cdot x - I \cdot e)} / ((A - I \cdot B) \cdot c - (I \cdot A + B) \cdot d)) + \sqrt{2} \cdot ((I \cdot A - B) \cdot c - (A + I \cdot B) \cdot d - 2 \cdot ((-2I \cdot A - B) \cdot c - (A + 4I \cdot B) \cdot d)) \cdot e^{(4I \cdot f \cdot x + 4I \cdot e)} + ((5I \cdot A + B) \cdot c + (A + 7I \cdot B) \cdot d) \cdot e^{(2I \cdot f \cdot x + 2I \cdot e)} \cdot \sqrt{a / (e^{(2I \cdot f \cdot x + 2I \cdot e)} + 1)}) \cdot e^{(-3I \cdot f \cdot x - 3I \cdot e)} / (a^2 \cdot f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \tan(e + fx))(c + d \tan(e + fx))}{(ia(\tan(e + fx) - i))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2),x)

[Out] Integral((A + B\*tan(e + f\*x))\*(c + d\*tan(e + f\*x))/(I\*a\*(tan(e + f\*x) - I))^(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*tan(f\*x+e))\*(c+d\*tan(f\*x+e))/(a+I\*a\*tan(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B\*tan(f\*x + e) + A)\*(d\*tan(f\*x + e) + c)/(I\*a\*tan(f\*x + e) + a)^(3/2), x)

**Mupad [B]**

time = 10.27, size = 245, normalized size = 1.67

$$\frac{\frac{(Ac+Ad1) \operatorname{li} + (Ac-Ad1) \operatorname{li} + \tan(e+fx) \operatorname{li}}{3f} - \frac{Bc+Bd1}{3f} - \frac{(Bc+Bd3) \operatorname{li} + \tan(e+fx) \operatorname{li}}{2af}}{(a + a \tan(e + fx) \operatorname{li})^{3/2}} + \frac{\sqrt{2} B \operatorname{atanh}\left(\frac{\sqrt{2} B (d+c1) \sqrt{a + a \tan(e + fx) \operatorname{li}}}{2 \sqrt{-a} (Bc-Bd1)}\right) (d + c1)}{4(-a)^{3/2} f} + \frac{\sqrt{2} A \operatorname{atan}\left(\frac{\sqrt{2} A (d+c1) \sqrt{a + a \tan(e + fx) \operatorname{li}}}{2 \sqrt{a} (Ac-Ad1)}\right) (d + c1) \operatorname{li}}{4a^{3/2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((A + B*\tan(e + f*x))*(c + d*\tan(e + f*x)))/(a + a*\tan(e + f*x)*1i)^{(3/2}), x)$

[Out]  $((A*c + A*d*1i)*1i)/(3*f) + ((A*c - A*d*1i)*(a + a*\tan(e + f*x)*1i)*1i)/(2*a*f)/(a + a*\tan(e + f*x)*1i)^{(3/2)} - ((B*c + B*d*1i)/(3*f) - ((B*c + B*d*3i)*(a + a*\tan(e + f*x)*1i))/(2*a*f))/(a + a*\tan(e + f*x)*1i)^{(3/2)} + (2^{(1/2)}*B*\text{atanh}((2^{(1/2)}*B*(c*1i + d)*(a + a*\tan(e + f*x)*1i)^{(1/2)}))/(2*(-a)^{(1/2)}*(B*c - B*d*1i)))*(c*1i + d)/(4*(-a)^{(3/2)}*f) + (2^{(1/2)}*A*\text{atan}((2^{(1/2)}*A*(c*1i + d)*(a + a*\tan(e + f*x)*1i)^{(1/2)}))/(2*a^{(1/2)}*(A*c - A*d*1i)))*(c*1i + d)*1i)/(4*a^{(3/2)}*f)$

# Chapter 4

## Appendix

### Local contents

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## 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica\_syntax.zip

**Maple and Mupad format** Maple\_syntax.zip

**Sympy format** SYMPY\_syntax.zip

**Sage math format** SAGE\_syntax.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```



```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```



```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

## 4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
  fi;

  leaf_count_optimal := leafcount(optimal);
  ExpnType_result := ExpnType(result);
  ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex

```

```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string),"$ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```

```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```



```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

#### 4.2.4 SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```